10

ESSENTIAL MATHEMATICS CORE

FOR THE VICTORIAN CURRICULUM

David Greenwood
Sara Woolley
Jenny Goodman
Jennifer Vaughan
Stuart Palmer

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Contents

Essential Mathematics for the Victorian Curriculum CORE Year 10 ISBN 978-1-108-87859-3

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About the Authors

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 25+ years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and is currently a Head of Mathematics. She has written more than 15 mathematics titles and specialises in lesson design and differentiation.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

Jenny Goodman has taught in schools for over 25 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke Prize for Mathematics. She has written for *CambridgeMATHS NSW* and was involved in the *Spectrum* and *Spectrum Gold* series.

Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and The University of Western Sydney.

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Introduction

Essential Mathematics CORE for the Victorian Curriculum is the successor to the prior *GOLD* series. The new name better reflects the nature of the series: a set of books that focuses on covering the basics of the curriculum in an accessible, straightforward manner. It has been tailored to the Victorian Curriculum and is best suited for students aiming to undertake General/Further Mathematics, a VET course or Foundation Mathematics in Years 11 and 12.

Compared to previous editions, the *CORE* series features some substantial new features in the print and digital versions of the textbook, as well as in the Online Teaching Suite. The main ones are listed below.

Learning intentions and chapter checklist

At the beginning of every lesson is a set of learning intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a chapter checklist of "I can..." statements; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics looks like. These checklists can also be downloaded and printed off so that students can physically check them off as they accomplish their goals.

Now you try

Every worked example now contains additional questions, without solutions, called 'Now you try'. We anticipate many uses of these questions, first and foremost to give students immediate practice at what they've just seen demonstrated in a worked example, rather than expecting students to simply absorb the example by reading through it. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution or answer before.

Workspaces and self-assessment

In the Interactive Textbook, students can complete almost any question from the textbook inside the platform via workspaces. Questions can be answered with full worked solutions using three input tools: 'handwriting' using a stylus, inputting text via a keyboard and in-built symbol palette, or uploading an image of work completed elsewhere. Then students can critically engage with their own work using the self-assessment tools, which allow them to rate their confidence with their work and also red-flag to the teacher any questions they have not understood. All work is saved, and teachers will be able to see both students' working-out and how they've assessed their own work via the Online Teaching Suite.

Note that the workspaces and self-assessment feature is intended to be used as much or as little as the teacher wishes, including not at all (the feature can be turned off). However, the ease with which useful data can be collected will make this feature a powerful teaching and learning tool when used creatively and strategically.

Algorithmic Thinking

Previously included as an appendix chapter, Algorithmic Thinking now becomes the last chapter of each book in the series. Instead of exercises and worked examples, this chapter contains a range of activities that show how algorithms and programming can be used as powerful tools for solving mathematical problems across all three Victorian Curriculum content strands (Number and Algebra, Measurement and Geometry, Statistics and Probability). The activities utilise a range of readily-available technologies, can be completed at any time during the year, and assume no prior knowledge of algorithms or coding.

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Guide to the working programs

Essential Mathematics CORE for the Victorian Curriculum contains working programs that are subtly embedded in the exercises. The suggested working programs provide two pathways through the book to allow differentiation for Building and Progressing students.

Each exercise is structured in subsections that match the Victorian Curriculum proficiency strands (with Problem-solving and Reasoning combined into one section to reduce exercise length), as well as 'Gold star' (★). The questions* suggested for each pathway are listed in two columns at the top of each subsection.

- The left column (lightest shade) shows the questions in the Building working program.
- The right column (darkest shade) shows the questions in the Progressing working program.

Gradients within exercises and proficiency strands

The working programs make use of two gradients that have been carefully integrated into the exercises. A gradient runs through the overall structure of each exercise – where there's an increasing level of sophistication required as a student progresses through the proficiency strands and then on to the 'Gold Star' question(s) – but also within each proficiency strand; the first few questions in Fluency are easier than the last few, for example, and the first few Problem-solving and reasoning questions are easier than the last few.

	Building	Progressing
Understanding	1–3	3
Fluency	4–6	4–6(½)
Problem-solving and reasoning	7–9	8–11
★	—	12

The right mix of questions

Questions in the working programs have been selected to give the most appropriate mix of types of questions for each learning pathway. Students going through the Building pathway are given extra practice at the Understanding and basic Fluency questions and only the easiest Problem-solving and reasoning questions. The Progressing pathway, while not challenging, spends a little less time on basic Understanding questions and a little more on Fluency and Problem-solving and reasoning questions.) The Progressing pathway also includes the 'Gold star' question(s).

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works best for them. If required, the Warm-up quiz at the start of each chapter can be used as a diagnostic to. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before doing the Building questions, and may require further assistance.
- A student who gets between 40% and 75% should do the Building questions.
- A student who gets 75% and higher should do the Progressing questions.

For schools that have classes grouped according to ability, teachers may wish to set either the Building or Progressing pathways as the default pathway for an entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- – : complete none of the questions in this section.

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Guide to this resource

PRINT TEXTBOOK FEATURES

1 **Victorian Curriculum:** content strands, sub-strands and content descriptions are listed at the beginning of the chapter (see the teaching program for more detailed curriculum documents)

2 **In this chapter:** an overview of the chapter contents

3 **Chapter introduction:** sets context for students about how the topic connects with the real world and the history of mathematics

4 **Warm-up quiz:** a quiz for students on the prior knowledge and essential skills required before beginning each chapter

5 **Sections labelled to aid planning:** All non-core sections are labelled as 'Consolidating' (indicating a revision section) or with a gold star (indicating a topic that could be considered challenging) to help teachers decide on the most suitable way of approaching the course for their class or for individual students.

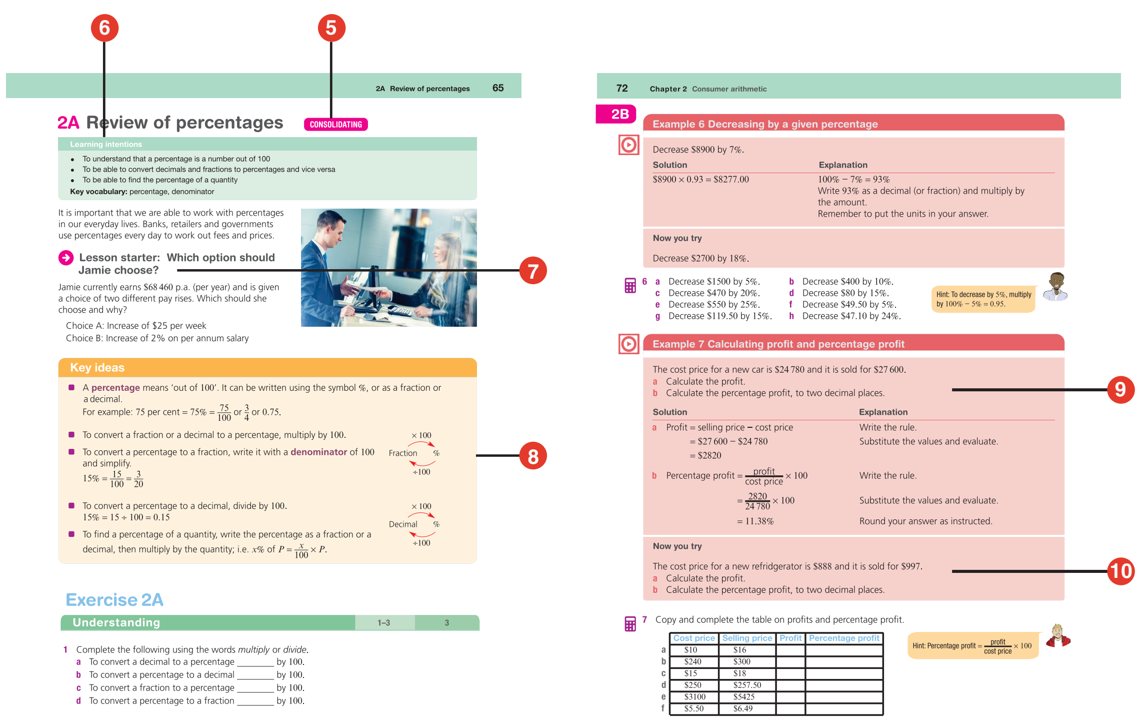
6 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

7 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

8 **Key ideas:** summarises the knowledge and skills for the section

9 **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example. Worked examples are placed within the exercise so they can be referenced quickly, with each example followed by the questions that directly relate to it.

10 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give students immediate practice



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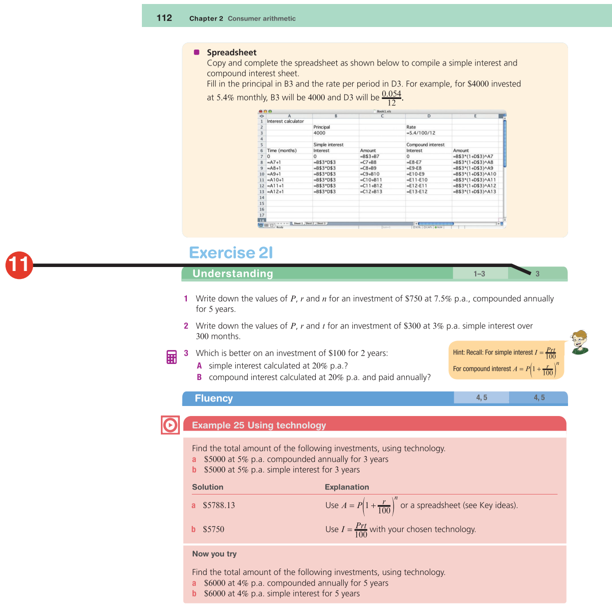
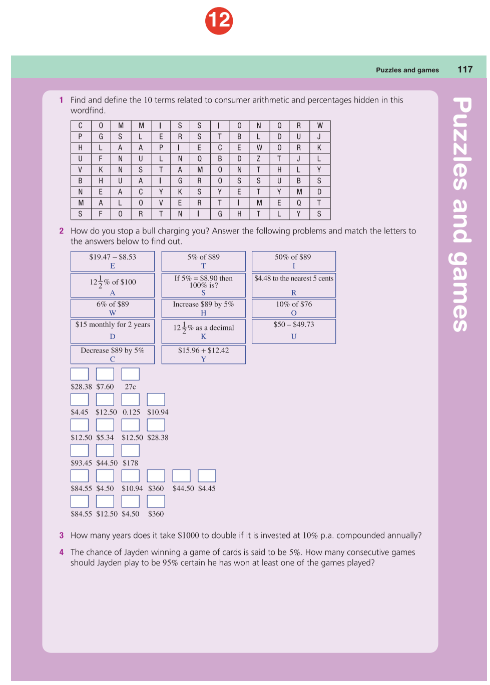
11 **Working programs:** differentiated question sets for two ability levels in exercises

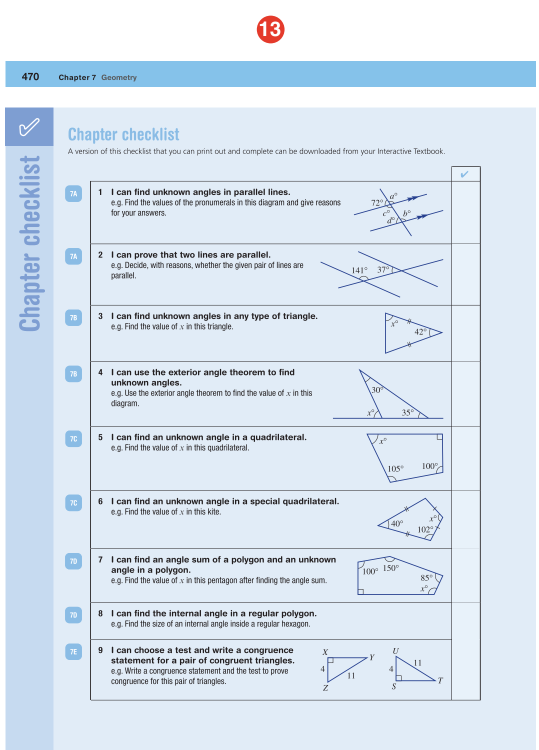
12 **Puzzles and games:** in each chapter provide problem-solving practice in the context of puzzles and games connected with the topic

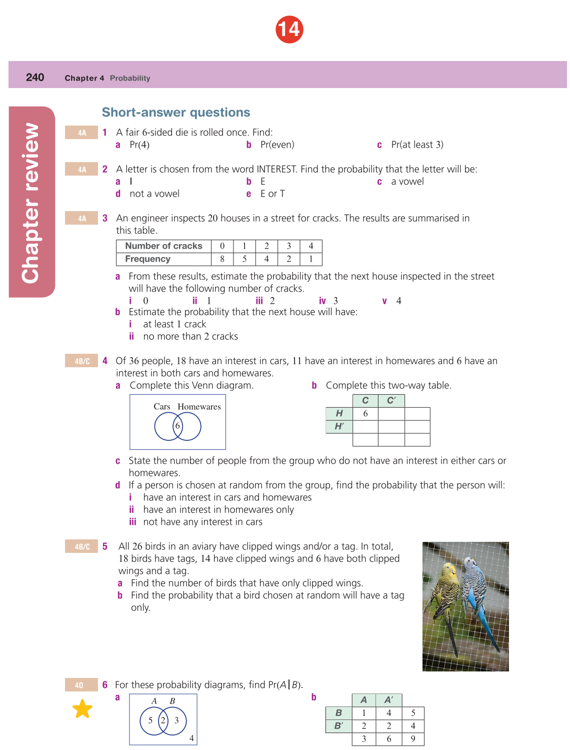




13 NEW **Chapter checklist:** a checklist of the learning intentions for the chapter, with example questions

14 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are 'Gold Star' (extension) are clearly signposted





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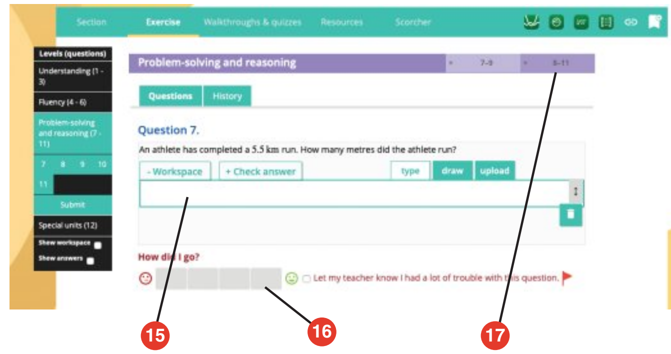
INTERACTIVE TEXTBOOK FEATURES



15 NEW **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work

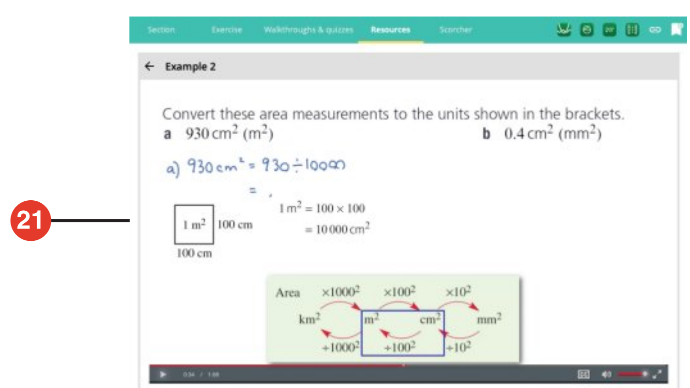
16 NEW **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page x for more information

17 **Interactive question tabs** can be clicked on so that only questions included in that working program are shown on the screen

18 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

19 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

20 **Scorcher:** the popular competitive game



21 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom

22 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

23 **Auto-marked maths** literacy activities test students on their ability to understand and use the key mathematical language used in the chapter

24 **Auto-marked prior knowledge pre-test** (the 'Warm-up quiz' of the print book) for testing the knowledge that students will need before starting the chapter

25 NEW **Auto-marked diagnostic pre-test** for setting a baseline of knowledge of chapter content

26 **Auto-marked progress quizzes and chapter review multiple-choice questions** in the chapter reviews can now be completed online

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DOWNLOADABLE PDF TEXTBOOK

27 In addition to the Interactive Textbook, a **PDF version of the textbook** has been retained for times when users cannot go online. PDF search and commenting tools are enabled.

ONLINE TEACHING SUITE

28 **Learning Management System** with class and student analytics, including reports and communication tools

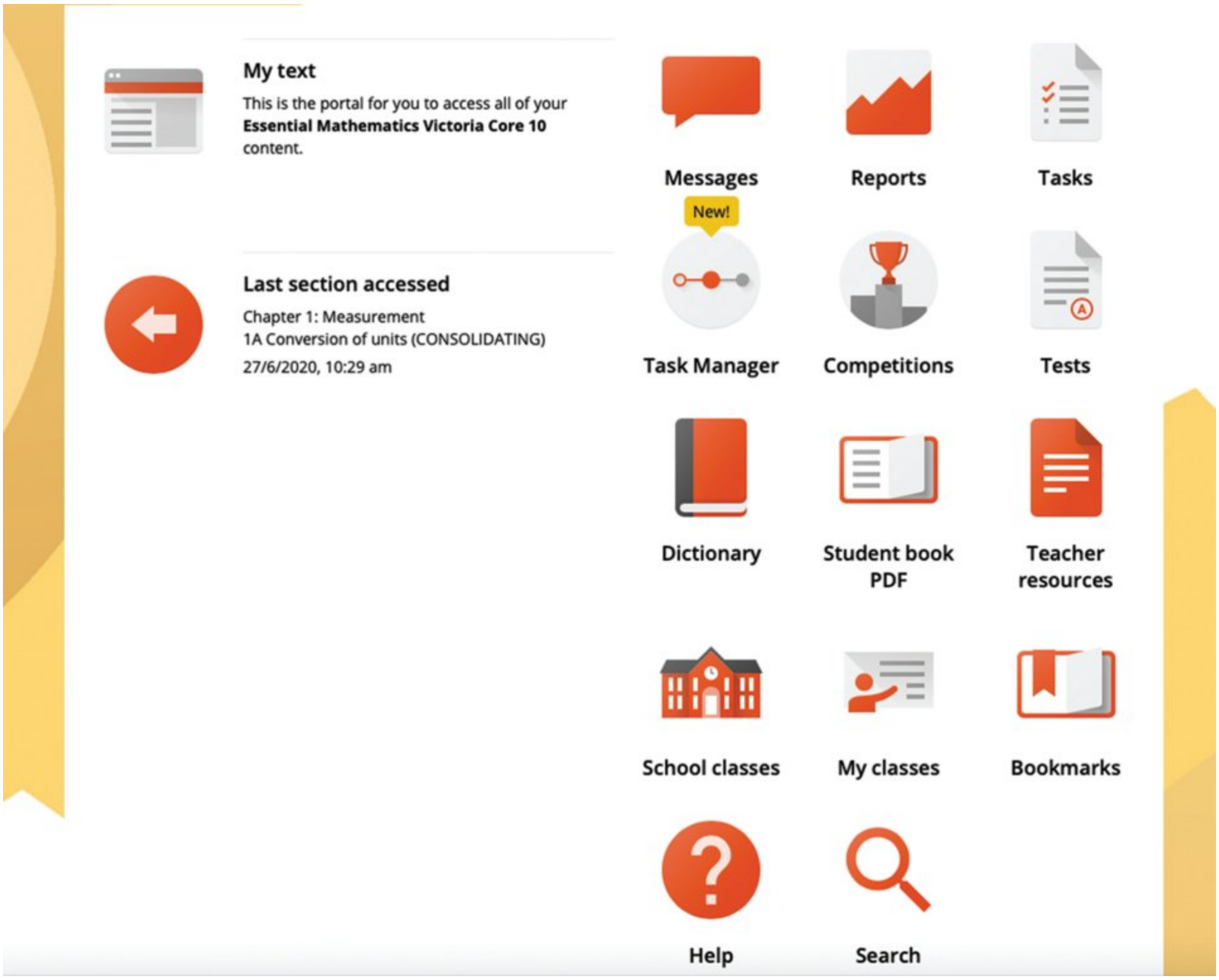
29 NEW **Teacher view of students' work and self-assessment** allows the teacher to see their class's workout, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher

30 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

31 NEW **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes

32 **Worksheets, skillsheets, maths literacy worksheets**, and **two differentiated chapter tests in every chapter**, provided in editable Word documents

33 NEW **More printable resources:** all Pre-tests and Progress quizzes and Applications and problem-solving tasks are provided in printable worksheet versions



Chapter 1 Measurement

Essential mathematics: why measurement skills are important

Our lives rely on people who can calculate accurate measurements. Qualified experts design and construct our houses, vehicles, water supply systems, and crucially, they grow our food. Some examples are below.

- Everyday users of area calculations include carpet layers, floor tilers, curtain makers, furniture upholsterers, house painters, landscapers, renovators, carpenters, builders and surveyors.
- Mechanical engineers and sheet metal workers calculate surface areas when designing and constructing commercial kitchens and air-conditioning ducts.
- Architects and builders calculate sector areas when designing and constructing steps for a spiral staircase.
- Plumbers calculate water pressures using a pipe's circular cross-sectional area and its volume.
- Workers in the boat-building trades calculate perimeters, surface areas and volumes when constructing and repairing small craft, cruise ships and naval boats.

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In this chapter

Victorian Curriculum

MEASUREMENT

Using units of measurement

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (VCMMG343)

NUMBER AND ALGEBRA

Patterns and algebra

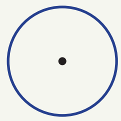
Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

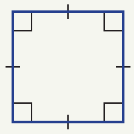
Online resources

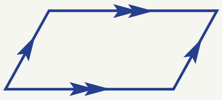
A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

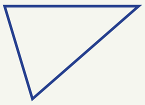
Warm-up quiz

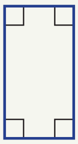
1 Name these shapes. Choose from the words *trapezium*, *triangle*, *circle*, *rectangle*, *square*, *semicircle*, *parallelogram* and *rhombus*.

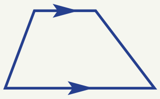
a 

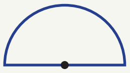
b 

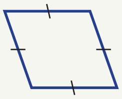
c 

d 

e 

f 

g 

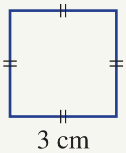
h 

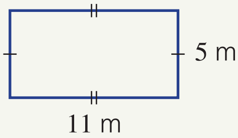
2 Write the missing number.

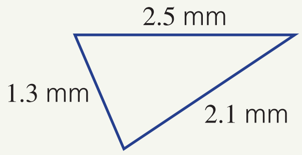
a 1 km = ☐ m **b** 1 m = ☐ cm **c** 1 cm = ☐ mm

d 1 L = ☐ mL **e** 0.5 km = ☐ m **f** 2.5 cm = ☐ mm

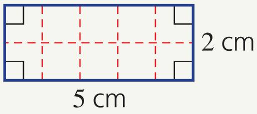
3 Find the perimeter of these shapes.

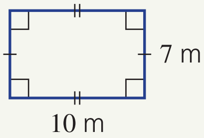
a 

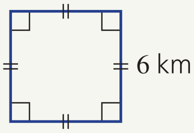
b 

c 

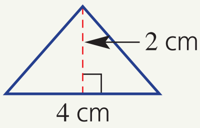
4 Find the area of these shapes.

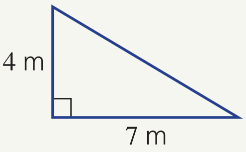
a 

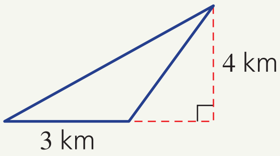
b 

c 

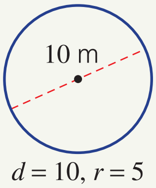
5 Find the area of these triangles using $A = \frac{1}{2}bh$.

a 

b 

c 

6 Use $C = \pi d$ and $A = \pi r^2$ to find the circumference and area of this circle. Round your answer to two decimal places.



$d = 10,\ r = 5$

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1A Conversion of units

CONSOLIDATING

Learning intentions

- To review the metric units of measurement
- To be able to convert between metric units for length, area and volume

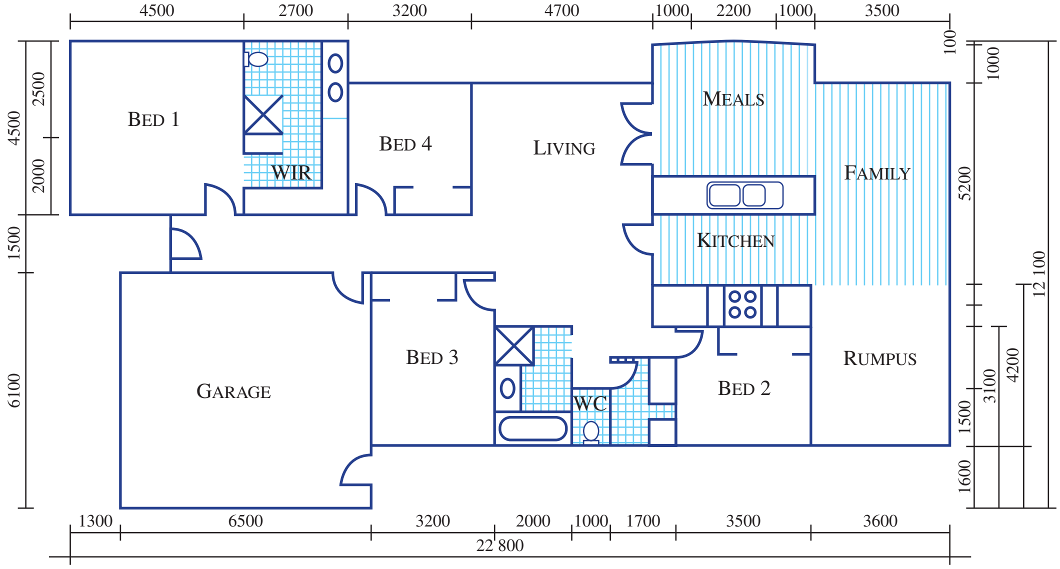
Key vocabulary: unit, length, area, volume

To work with length, area or volume measurements, it is important to be able to convert between different units. For example, timber is widely used in buildings for frames, roof trusses and windows, therefore it is important to order the correct amount so that the cost of the house is minimised. Although plans give measurements in millimetres and centimetres, timber is ordered in metres (often referred to as lineal metres), so we have to convert all our measurements to metres.

Building a house also involves many area and volume calculations and unit conversions.

Lesson starter: House plans

All homes start from a plan, which is usually designed by an architect and shows most of the basic features and measurements that are needed to build the house. Measurements are given in millimetres.



- How many bedrooms are there?
- What are the dimensions of the master bedroom (i.e. BED 1)?
- What are the dimensions of the master bedroom, in metres?
- Will the rumpus room fit a pool table that measures $2.5\,\text{m} \times 1.2\,\text{m}$, and still have room to play?
- How many cars do you think will fit in the garage?

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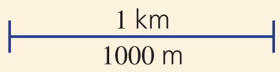
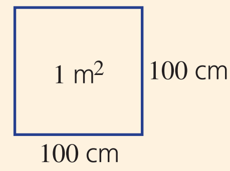
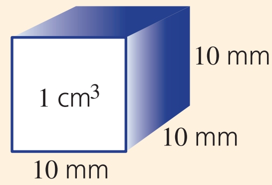
Key ideas

- To convert units, draw an appropriate diagram and use it to find the conversion factor.
  For example:





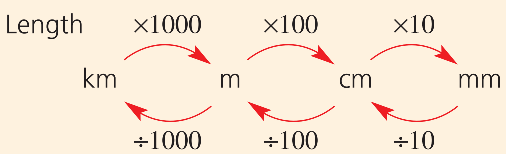


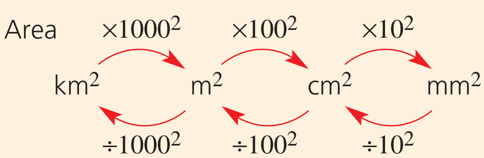
$1\text{ km} = 1000\text{ m}$

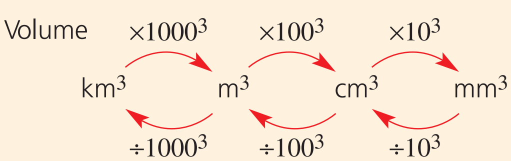
$1\text{ m}^2 = 100 \times 100$
$= 10\,000\text{ cm}^2$

$1\text{ cm}^3 = 10 \times 10 \times 10$
$= 1000\text{ mm}^3$

- Conversions:







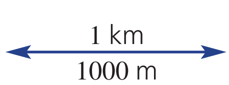
- To multiply by 10, 100, 1000 etc. move the decimal point one place to the right for each zero;
  e.g. $3.425 \times 100 = 342.5$
- To divide by 10, 100, 1000 etc. move the decimal point one place to the left for each zero;
  e.g. $4.10 \div 1000 = 0.0041$
- $10^2 = 10 \times 10 = 100$
  $100^2 = 100 \times 100 = 10\,000$
  $1000^2 = 1000 \times 1000 = 1\,000\,000$
  $10^3 = 10 \times 10 \times 10 = 1000$
  $100^3 = 100 \times 100 \times 100 = 1\,000\,000$
  $1000^3 = 1000 \times 1000 \times 1000$
  $= 1\,000\,000\,000$

Exercise 1A

Understanding

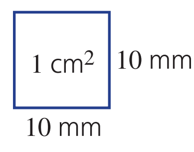
1–3 | 3

1 Write the missing numbers in these sentences involving length.



a There are ☐ m in 1 km.
b There are ☐ mm in 1 cm.
c There are ☐ cm in 1 m.

2 Write the missing numbers in these sentences involving area units.



a There are ☐ mm^2 in 1 cm^2.
b There are ☐ cm^2 in 1 m^2.
c There are ☐ m^2 in 1 km^2.

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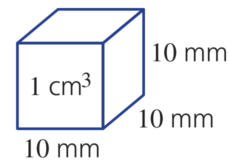
3 Write the missing numbers in these sentences involving volume units.

a There are ☐ mm^3 in 1 cm^3.

b There are ☐ m^3 in 1 km^3.

c There are ☐ cm^3 in 1 m^3.



Fluency

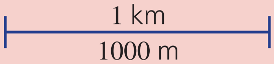
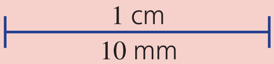
4–6 | 4–6(½)

Example 1 Converting length measurements

Convert these length measurements to the units shown in the brackets.

a 8.2 km (m)

b 45 mm (cm)

Solution	Explanation
a $8.2\text{ km} = 8.2 \times 1000$ $= 8200\text{ m}$	 $1\text{ km} = 1000\text{ m}$ Multiply when converting to a smaller unit.
b $45\text{ mm} = 45 \div 10$ $= 4.5\text{ cm}$	 $1\text{ cm} = 10\text{ mm}$ Divide when converting to a larger unit.

Now you try

Convert these length measurements to the units shown in the brackets.

a 4.6 m (cm)

b 3200 m (km)

4 Convert the following measurements of length to the units given in the brackets.

a 4.32 cm (mm)

b 327 m (km)

c 834 cm (m)

d 0.096 m (mm)

e 297.5 m (km)

f 0.0127 m (cm)

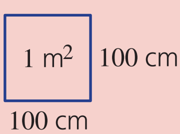
Hint: When converting to a smaller unit, multiply. Otherwise, divide.

Example 2 Converting area measurements

Convert these area measurements to the units shown in the brackets.

a 930 cm^2 (m^2)

b 0.4 cm^2 (mm^2)

Solution	Explanation
a $930\text{ cm}^2 = 930 \div 10\,000$ $= 0.093\text{ m}^2$	 $1\text{ m}^2 = 100 \times 100$ $= 10\,000\text{ cm}^2$ When dividing by 10 000, move the decimal point 4 places to the left.

Continued on next page

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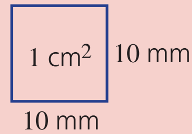
1A

b $0.4\text{ cm}^2 = 0.4 \times 100$

$= 40\text{ mm}^2$



$1\text{ cm}^2 = 10 \times 10$

$= 100\text{ mm}^2$

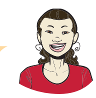
Now you try

Convert these area measurements to the units shown in the brackets.

a 320 mm^2 (cm^2) **b** 0.00024 km^2 (m^2)

5 Convert the following area measurements to the units given in the brackets.

a 3000 cm^2 (mm^2) **b** 0.5 m^2 (cm^2)
c 5 km^2 (m^2) **d** $2\,980\,000\text{ mm}^2$ (cm^2)
e 537 cm^2 (mm^2) **f** 0.023 m^2 (cm^2)



Hint:
$1\text{ cm}^2 = 100\text{ mm}^2$
$1\text{ m}^2 = 10\,000\text{ cm}^2$
$1\text{ km}^2 = 1\,000\,000\text{ m}^2$

Example 3 Converting volume measurements

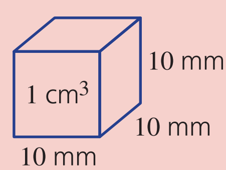
Convert these volume measurements to the units shown in the brackets.

a 3.72 cm^3 (mm^3) **b** 4300 cm^3 (m^3)

Solution | **Explanation**

a $3.72\text{ cm}^3 = 3.72 \times 1000$

$= 3720\text{ mm}^3$

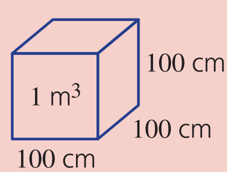


$1\text{ cm}^3 = 10 \times 10 \times 10$

$= 1000\text{ mm}^3$

b $4300\text{ cm}^3 = 4300 \div 1\,000\,000$

$= 0.0043\text{ m}^3$



$1\text{ m}^3 = 100 \times 100 \times 100$

$= 1\,000\,000\text{ cm}^3$

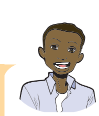
Now you try

Convert these volume measurements to the units shown in the brackets.

a 0.21 m^3 (cm^3) **b** $94\,000\text{ mm}^3$ (cm^3)

6 Convert these volume measurements to the units given in the brackets.

a 2 cm^3 (mm^3) **b** 0.2 m^3 (cm^3)
c 5700 mm^3 (cm^3) **d** 0.015 km^3 (m^3)
e $28\,300\,000\text{ m}^3$ (km^3) **f** $762\,000\text{ cm}^3$ (m^3)



Hint:
$1\text{ cm}^3 = 1000\text{ mm}^3$
$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$
$1\text{ km}^3 = 1\,000\,000\,000\text{ m}^3$

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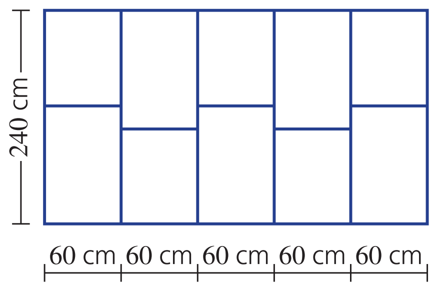
Problem-solving and reasoning | 7–9 | 8–11

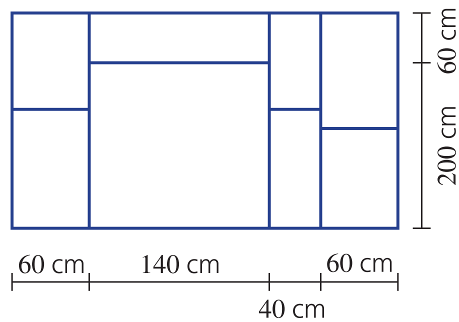
7 An athlete has completed a 5.5 km run. How many metres did the athlete run?

8 Determine the metres of timber needed to construct the following frames.

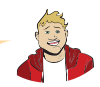
a



b



9 Find the total sum of the measurements given, expressing your answer in the units given in the brackets.



Hint: Convert to the units in brackets. Add up to find the sum.

a 10 cm, 18 mm (mm)
b 1.2 m, 19 cm, 83 mm (cm)
c 453 km, 258 m (km)
d 400 mm^2, 11.5 cm^2 (cm^2)
e 0.3 m^2, 251 cm^2 (cm^2)
f 0.000 03 km^2, 9 m^2, 37 000 000 cm^2 (m^2)
g 482 000 mm^3, 2.5 cm^3 (mm^3)
h 0.000 51 km^3, 27 300 m^3 (m^3)

10 A snail is moving at a rate of 43 mm every minute. How many centimetres will the snail move in 5 minutes?

11 Why do you think that builders measure many of their lengths using only millimetres, even their long lengths?

Special units | — | 12

12 Many units of measurement apart from those relating to mm, cm, m and km are used in our society. Some of these are described here.

Length	Inches	1 inch ≈ 2.54 cm = 25.4 mm
	Feet	1 foot = 12 inches ≈ 30.48 cm
	Miles	1 mile ≈ 1.609 km = 1609 m
Area	Squares	1 square = 100 square feet
	Hectares (ha)	1 hectare = 10 000 m^2
Volume	Millilitres (mL)	1 millilitre = 1 cm^3
	Litres (L)	1 litre = 1000 cm^3

Convert these special measurements to the units given in the brackets. Use the conversion information given above to help.

a 5.5 miles (km)
b 54 inches (feet)
c 10.5 inches (cm)
d 2000 m (miles)
e 5.7 ha (m^2)
f 247 cm^3 (L)
g 8.2 L (mL)
h 5.5 m^3 (mL)
i 10 squares (sq. feet)
j 2 m^3 (L)
k 1 km^2 (ha)
l 152 000 mL (m^3)

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1B Perimeter CONSOLIDATING

Learning intentions

- To be able to calculate the perimeter of a shape
- To be able to find an unknown length given the perimeter

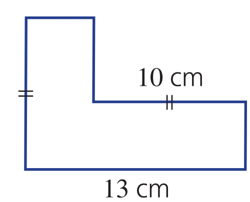
Key vocabulary: perimeter

Perimeter is a measure of length around the outside of a shape. We calculate perimeter when ordering ceiling cornices for a room or materials for fencing a paddock or when designing a house.

Lesson starter: L-shaped perimeters

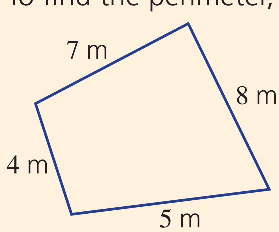
The L-shaped figure on the right includes only right (90°) angles. Only two measurements are given.

- Can you figure out any other side lengths?
- Is it possible to find its perimeter? Why?



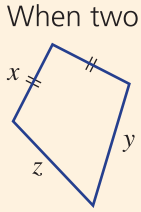
Key ideas

- **Perimeter** is the distance around the outside of a two-dimensional shape.
  - To find the perimeter, we add all the lengths of the sides in the same units.



$$P = 4 + 5 + 7 + 8 = 24\text{ m}$$

  - When two sides of a shape are the same length they are labelled with the same markings.



$$P = 2x + y + z$$

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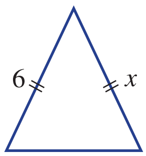
Exercise 1B

Understanding	1, 2	2

1 Write the missing word: The distance around the outside of a shape is called the ________.

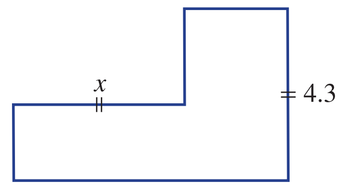
2 Write down the value of x for these shapes.

a



b

c

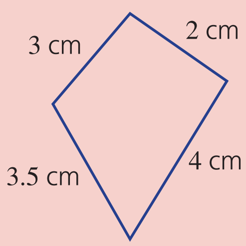


Fluency	3, 4	3–4(½)

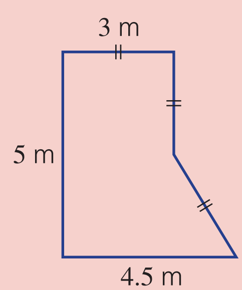
Example 4 Finding perimeters of basic shapes

Find the perimeter of these shapes.

a



b

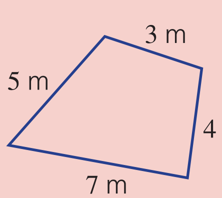


Solution	Explanation
a Perimeter $= 3 + 2 + 4 + 3.5$ $= 12.5\text{ cm}$	Add all the lengths of the sides together.
b Perimeter $= 5 + 4.5 + 3 \times 3$ $= 18.5\text{ m}$	Three lengths have the same markings and therefore are the same length.

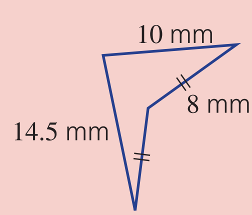
Now you try

Find the perimeter of these shapes.

a



b



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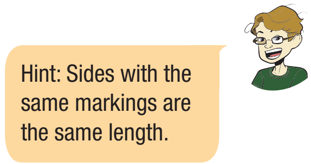
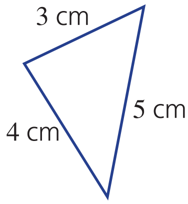
1B

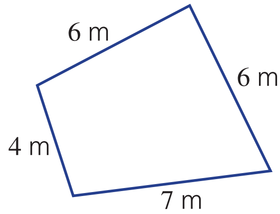
3 Find the perimeter of these shapes.



a



b



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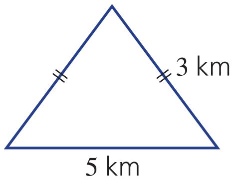

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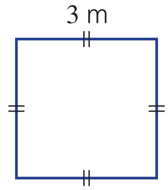

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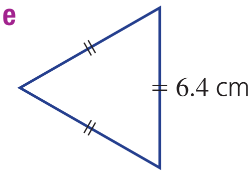

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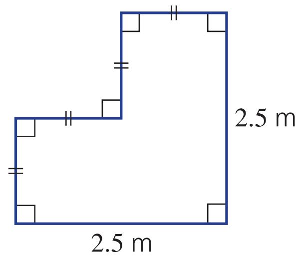
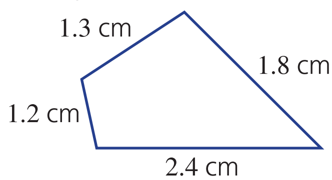


4 Find the perimeter of these shapes.

a



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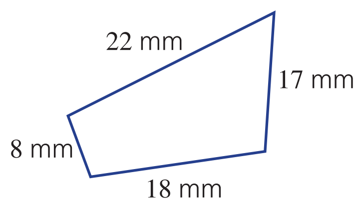

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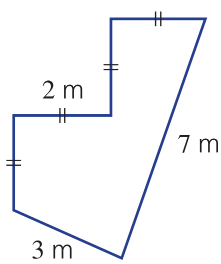

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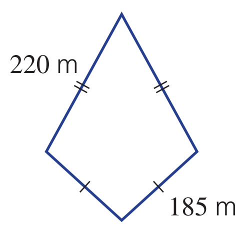

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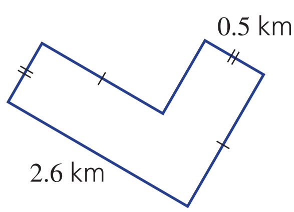

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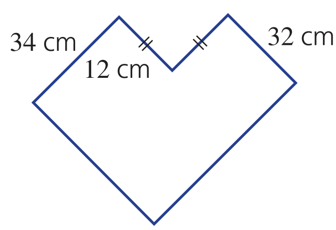

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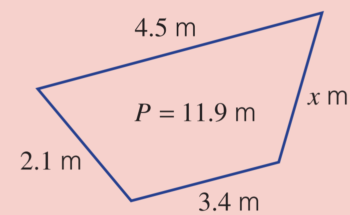
Problem-solving and reasoning | 5–7 | 6–10

Example 5 Finding a missing side length

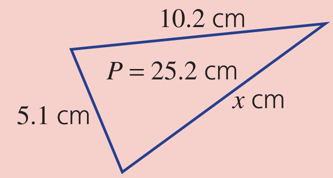
Find the value of x for this shape with the given perimeter.



Solution	Explanation
$4.5 + 2.1 + 3.4 + x = 11.9$	All the sides add to 11.9 in length.
$10 + x = 11.9$	Simplify.
$x = 1.9$	Subtract 10 from both sides to find the value of x.

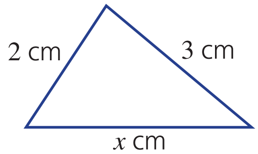
Now you try

Find the value of x for this shape with the given perimeter.



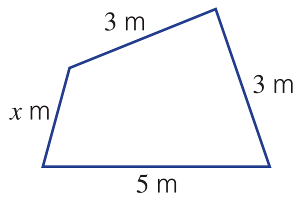
5 Find the value of x for these shapes with the given perimeters.

a



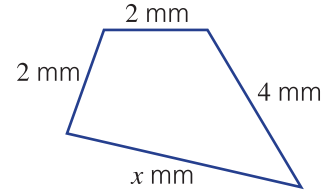
Perimeter = 9 cm

b



Perimeter = 13 m

c

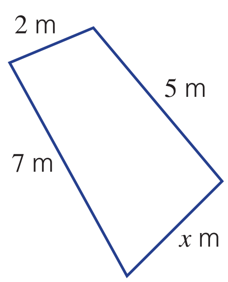


Perimeter = 14 mm

Hint: Add up all the sides and then determine the value of x to suit the given perimeters.

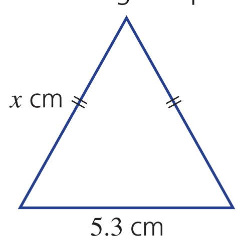
6 Find the value of x for these shapes with the given perimeters.

a



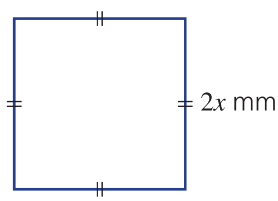
Perimeter = 17 m

b



Perimeter = 22.9 cm

c



Perimeter = 0.8 mm

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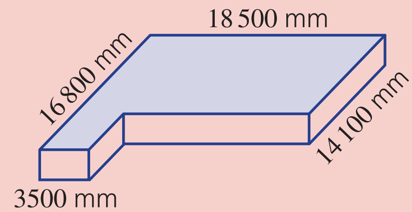
1B

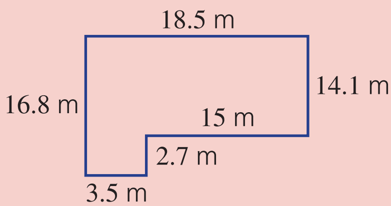
Example 6 Working with perimeter with three dimensions

A concrete slab has the measurements shown.

a Draw a new diagram, showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.



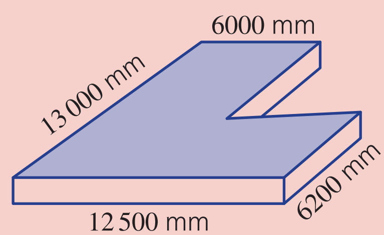
Solution	Explanation
a 	Convert your measurements and place them all on the diagram. $1\text{ m} = 100 \times 10 = 1000\text{ mm}$ Add or subtract to find the missing measurements.
b $\text{Perimeter} = 18.5 + 16.8 + 3.5 + 2.7 + 15 + 14.1$ $= 70.6\text{ m}$	Add all the measurements.
The lineal metres of timber needed is 70.6 m.	Write your answer in words.

Now you try

A concrete slab has the measurements shown.

a Draw a new diagram showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.

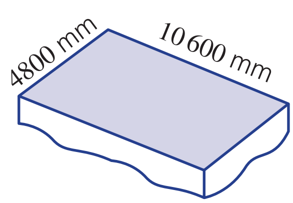


7 Six concrete slabs are shown below.

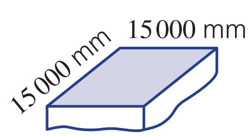
i Draw a new diagram for each with the measurements in metres.

ii Determine the lineal metres of timber needed for each to surround it.

a



b



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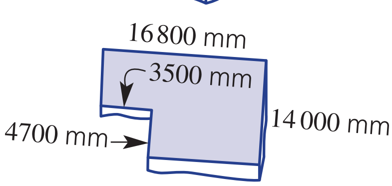

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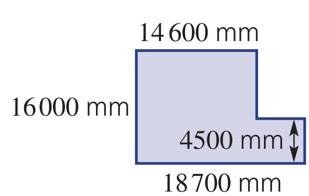

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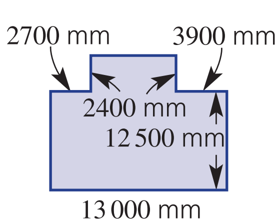

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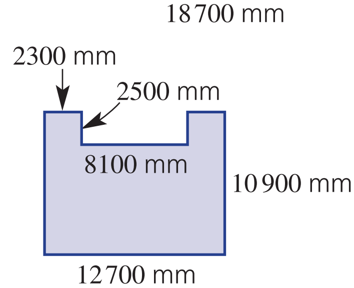

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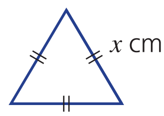
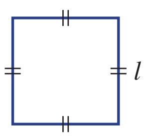
8 A rectangular paddock has perimeter 100 m. Find the width of the paddock if its length is 30 m.

9 The equilateral triangle shown has perimeter 45 cm. Find its side length.



10 Write formulas for the perimeter of these shapes, using the pronumerals given.

a



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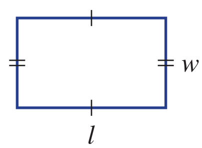

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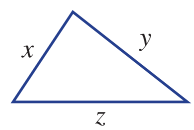

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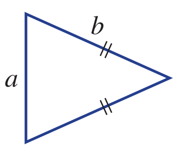
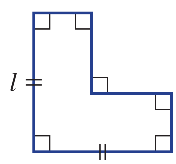


e



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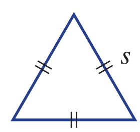
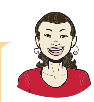


Hint: A formula for perimeter could be $P = l + 2w$ or $P = a + b + c$.



How many different tables? | — | 11, 12

11 A large dining table is advertised with a perimeter of 12 m. The length and width are a whole number of metres (e.g. 1 m, 2 m, …). How many different-sized tables are possible?

12 How many rectangles (using whole number lengths) have perimeters between 16 m and 20 m, inclusive?

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1C Circumference CONSOLIDATING

Learning intentions

- To know the formula for the circumference of a circle
- To be able to find the circumference of a circle
- To be able to find the circumference of circle portions and simple composite shapes

Key vocabulary: circumference, pi, radius, diameter, circle

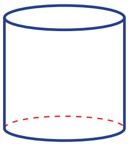
To find the distance around the outside of a circle – the circumference – we use the special number called pi (π). Pi provides a direct link between the diameter of a circle and the circumference of that circle.

The wheel is one of the most useful components in many forms of machinery and its shape, of course, is a circle. One revolution of a vehicle's wheel moves the vehicle a distance equal to the wheel's circumference.

Lesson starter: When circumference = height

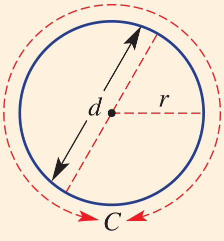
Here is an example of a cylinder.

- Try drawing your own cylinder so that its height is equal to the circumference of the circular top.
- How would you check that you have drawn a cylinder with the correct dimensions? Discuss.



Key ideas

- The **radius (r)** is the distance from the centre of a **circle** to a point on the circle.
- The **diameter (d)** is the distance across a circle through its centre.
  - Radius $= \frac{1}{2}$ diameter or diameter $= 2 \times$ radius
- **Circumference (C)** is the distance around a circle.
  - $C = 2\pi \times$ radius
    $= 2\pi r$
    or $C = \pi \times$ diameter
    $= \pi d$
  - **π (pi)** is a special number and can be found on your calculator.
    It can be approximated by $\pi \approx 3.142$.



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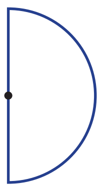
Exercise 1C

Understanding 1–3 3

1 **a** The distance from the centre of a circle to its outside edge is called the _________.
b The distance across a circle, through its centre is called the _________.
c The distance around a circle is called the _________.

2 Write the formula for the circumference of a circle using:
a d for diameter **b** r for radius

3 What fraction of a circle is shown here?

a


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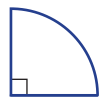

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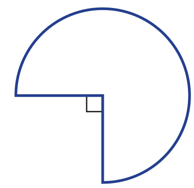
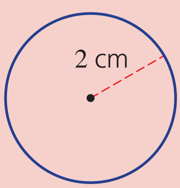


Fluency 4, 5 4–5(½)

Example 7 Finding the circumference of a circle

Find the circumference of these circles to two decimal places.

a


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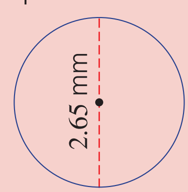
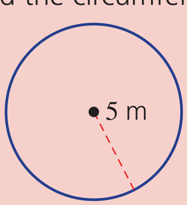


Solution	Explanation
a $C = 2\pi r$	Write the formula involving the radius, r.
$= 2\pi(2)$	Substitute $r = 2$.
$= 12.57\text{ cm}$ (to 2 d.p.)	Round your answer to two decimal places.
b $C = \pi d$	Write the formula involving diameter.
$= \pi(2.65)$	Substitute $d = 2.65$.
$= 8.33\text{ mm}$ (to 2 d.p.)	Round your answer to two decimal places.

Now you try

Find the circumference of these circles to two decimal places.

a


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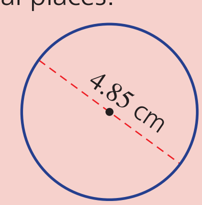

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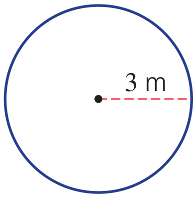
1C

4 Find the circumference of these circles, to two decimal places.

a



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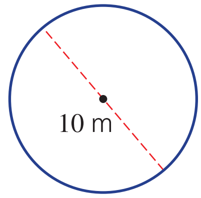

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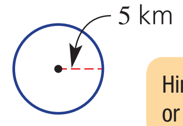
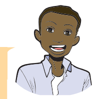
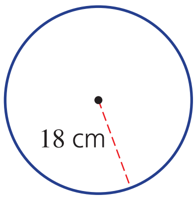




Hint: Use $C = 2\pi r$ or $C = \pi d$.

d



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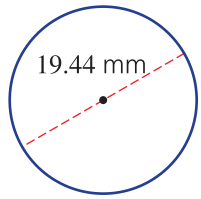

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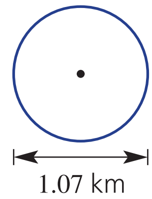
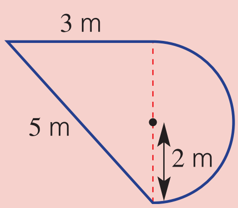


Example 8 Finding perimeters of composite shapes

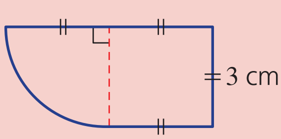
Find the perimeter of this composite shape, to two decimal places.



Solution	**Explanation**
$P = 3 + 5 + \frac{1}{2} \times 2\pi(2)$	Add all the sides, including half a circle.
$= 8 + 2\pi$	Simplify.
$= 14.28$ m (to 2 d.p.)	Round your answer as instructed.

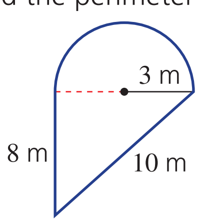
Now you try

Find the perimeter of this composite shape, to two decimal places.



5 Find the perimeter of these composite shapes, correct to two decimal places.

a



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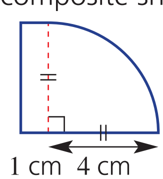

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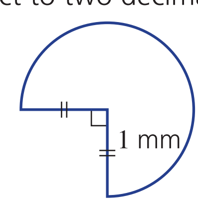
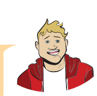


Hint: Don't forget to add the straight sides to the fraction $\left(\frac{1}{4}, \frac{1}{2} \text{ or } \frac{3}{4}\right)$ of the circumference.



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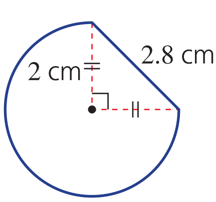

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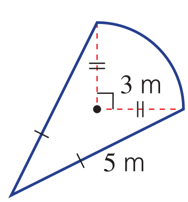

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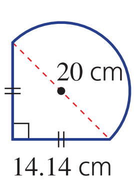

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Problem-solving and reasoning | 6–8 | 7–10

6 David wishes to build a circular fish pond. The diameter of the pond is to be 3 m.

a How many lineal metres of bricks are needed to surround it? Round your answer to two decimal places.

b What is the cost if the bricks are $45 per metre? (Use your answer from part **a**.)

7 The wheels of a bike have a diameter of 1 m.

a How many metres will the bike travel (to two decimal places) after:

i one full turn of the wheels?

ii 15 full turns of the wheels?

b How many kilometres will the bike travel after 1000 full turns of the wheels? (Give your answer correct to two decimal places.)

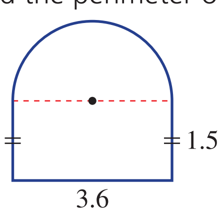
Hint: For one revolution, use $C = \pi d$.

8 What is the minimum number of times a wheel of diameter 1 m needs to spin to cover a distance of 1 km? You will need to find the circumference of the wheel first. Give your answer as a whole number.

9 Find the perimeter of these composite shapes, correct to two decimal places.

Hint: Make sure you know the radius or diameter of the circle you are dealing with.

a



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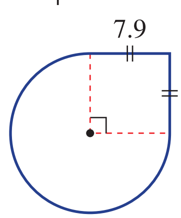

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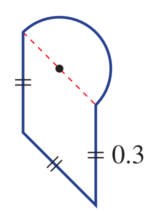

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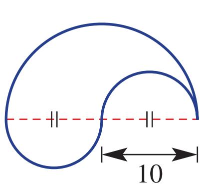

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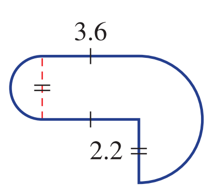

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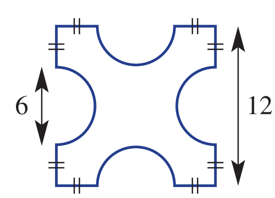

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1C

10 **a** Rearrange the formula for the circumference of a circle, $C = 2\pi r$, to write r in terms of C.

b Find, to two decimal places, the radius of a circle with the given circumference.

i 35 cm

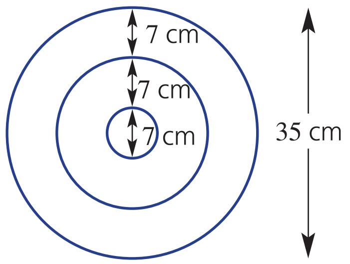
ii 1.85 m

iii 0.27 km

Hint: To make r the subject, divide both sides by 2π.

Target practice — 11

11 A target is made up of three rings, as shown.



a Find the radius of the smallest ring.

b Find, to two decimal places, the circumference of the:

i smallest ring

ii middle ring

iii outside ring

c If the circumference of a different ring is 80 cm, what would be its radius, correct to two decimal places?

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1D Area

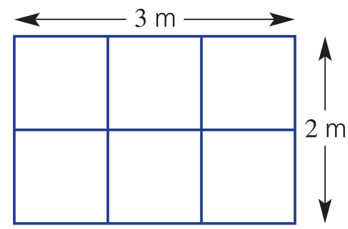
Learning intentions

- To know the formulas for the areas of simple shapes
- To be able to find the area of simple shapes

Key vocabulary: area, square, rectangle, triangle, rhombus, parallelogram, trapezium, perpendicular

In this simple diagram, a rectangle, with side lengths 2 m and 3 m, has an area of 6 square metres or 6 m^2. This is calculated by counting the number of squares (each measuring a square metre) that make up the rectangle.

We use formulas to help us quickly count the number of square units contained within a shape. For this rectangle, for example, the formula $A = lw$ simply tells us to multiply the length by the width to find the area.

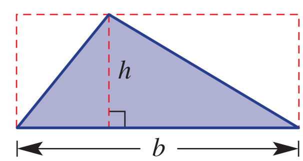


Area = 6 m^2

Lesson starter: How does $A = \frac{1}{2}bh$ work for a triangle?

Look at this triangle, including its rectangular red dashed lines.

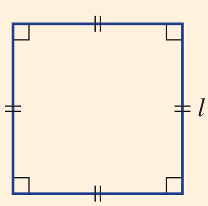
- How does the shape of the triangle relate to the shape of the outside rectangle?
- How can you use the formula for a rectangle to help find the area of the triangle (or parts of the triangle)?
- Why is the rule for the area of a triangle given by $A = \frac{1}{2}bh$?



Key ideas

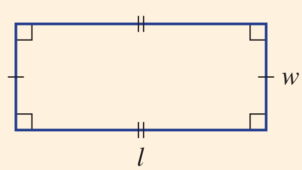
- The **area** of a two-dimensional shape is the number of square units contained within its boundaries.
- Some of the common area formulas are as follows.

Square



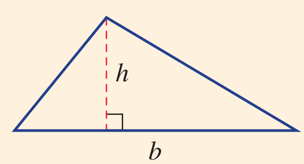
Area = l^2

Rectangle



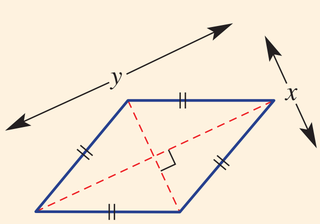
Area = lw

Triangle



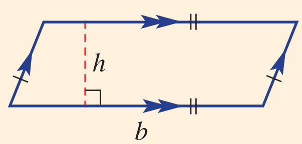
Area = $\frac{1}{2}bh$

Rhombus



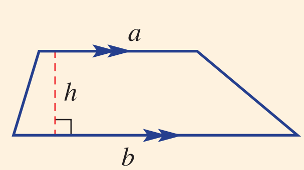
Area = $\frac{1}{2}xy$

Parallelogram



Area = bh

Trapezium



Area = $\frac{1}{2}(a + b)h$

- The 'height' in a triangle, parallelogram or trapezium should be **perpendicular** (at 90°) to the base.

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Exercise 1D

Understanding	1, 2	2

1 Match each shape (**a**–**f**) with its area formula (**A**–**F**).

a square — **A** $A = \frac{1}{2}bh$

b rectangle — **B** $A = lw$

c rhombus — **C** $A = bh$

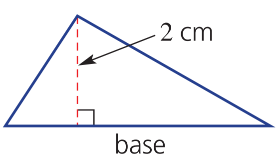
d parallelogram — **D** $A = \frac{1}{2}(a+b)h$

e trapezium — **E** $A = l^2$

f triangle — **F** $A = \frac{1}{2}xy$

2 These shapes show the base and a height length. Write down the given height of each shape.

a



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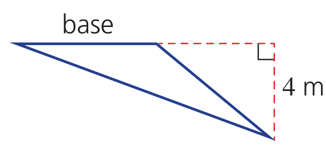
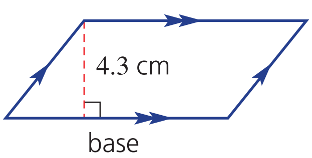
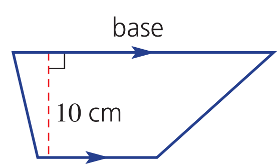


c



d



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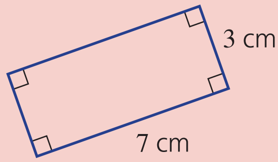
Fluency

3, 4 | 3–4(½)

Example 9 Using area formulas

Find the area of these basic shapes.

a


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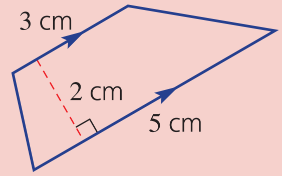

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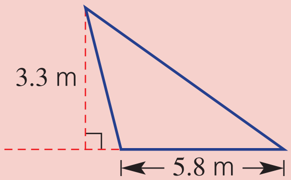
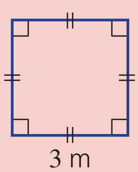


Solution	Explanation
a Area $= lw$	Write the formula for the area of a rectangle.
$= 7 \times 3$	Substitute the lengths $l = 7$ and $w = 3$.
$= 21\text{ cm}^2$	Simplify and include the units.
b Area $= \frac{1}{2}(a+b)h$	Write the formula for the area of a trapezium.
$= \frac{1}{2}(3+5) \times 2$	Substitute the lengths $a = 3$, $b = 5$ and $h = 2$.
$= 8\text{ cm}^2$	Simplify and include the units.
c Area $= \frac{1}{2}bh$	Write the formula for the area of a triangle.
$= \frac{1}{2} \times 5.8 \times 3.3$	Substitute the lengths $b = 5.8$ and $h = 3.3$.
$= 9.57\text{ m}^2$	Simplify and include the units.

Now you try

Find the area of these basic shapes.

a


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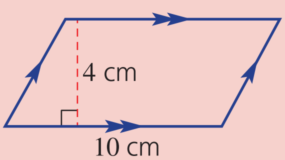

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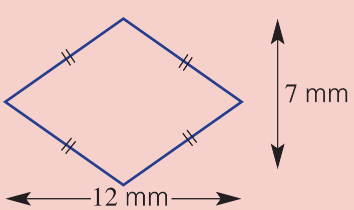
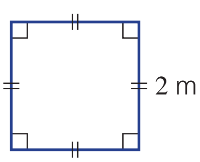


3 Find the area of these basic shapes.

a


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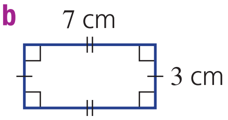

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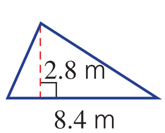

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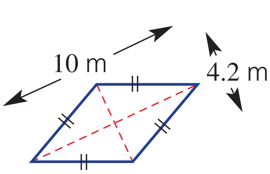
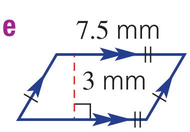


e


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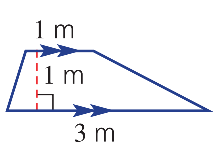

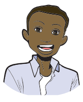

Hint: First, choose the correct formula and substitute for each pronumeral (letter).

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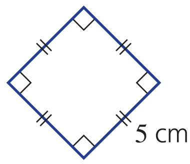
1D

4 Find the area of these basic shapes.

a



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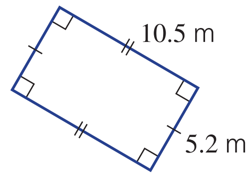

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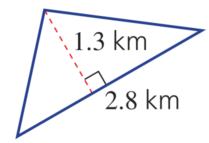

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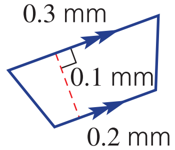

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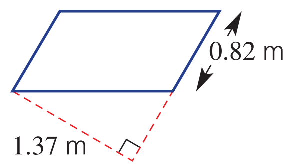

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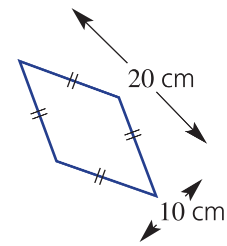


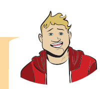
Problem-solving and reasoning | 5–8 | 7–11

5 A rectangular table top is 1.2 m long and 80 cm wide. Find the area of the table top using:

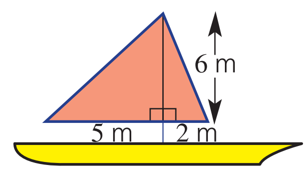
a square metres (m^2)

b square centimetres (cm^2)

Hint: First convert to the units that you want to work with.



6 Two triangular sails have side lengths as shown. Find the total area of the two sails.

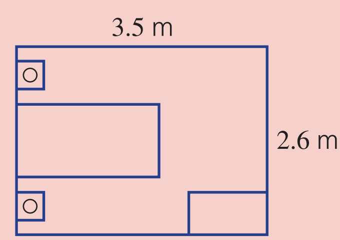


Example 10 Applying area formulas

Christine decides to use carpet to cover the floor of her rectangular bedroom, shown at right. Determine:

a the area of floor to be covered

b the total cost if the carpet costs \$32 per square metre



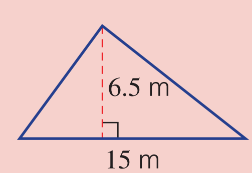
Solution	Explanation
a Area of floor $= l \times w$ $= 3.5 \times 2.6$ $= 9.1\ m^2$	The room is a rectangle, so use $A = l \times w$ to calculate the total floor space.
b Cost of carpet $= 9.1 \times 32$ $= \$291.20$	Every square metre of carpet costs \$32.

Now you try

Richo decides to lay lawn on his triangular backyard, shown at right. Determine:

a the area of lawn to be laid

b the total cost if lawn costs \$11 per square metre



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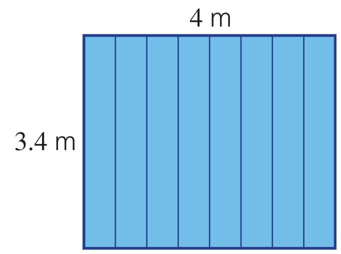
7 Jack's shed is to have a flat rectangular roof, which he decides to cover with metal sheets.

a Determine the total area of the roof.

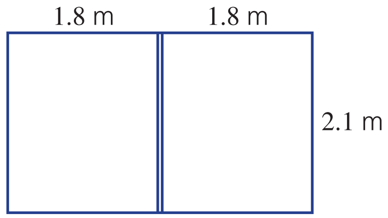
b If the metal roofing costs \$11 a square metre, how much will it cost in total?



8 A sliding door has two glass panels. Each of these is 2.1 m high and 1.8 m wide.

a How many square metres of glass are needed?

b What is the total cost of the glass if the price is \$65 per square metre?

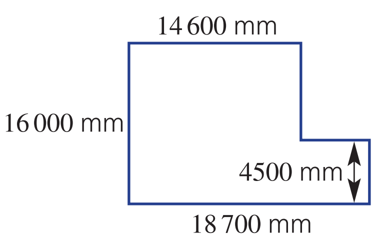


9 A rectangular window has a whole number measurement for its length and width and its area is 24 m^2. Write down the possible lengths and widths for the window.

10 Determine the area of the houses shown (if all angles are right angles), in square metres (correct to two decimal places).

Hint: Note that there are 1000 mm in 1 m.

a



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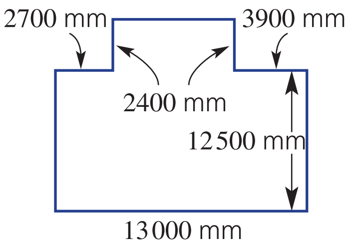

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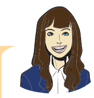
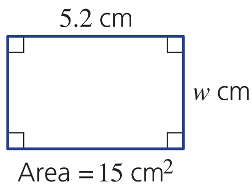
1D

11 Find the value of the pronumeral in these shapes, rounding your answer to two decimal places each time.



Hint: First, write the appropriate formula and substitute for the area and length pronumerals. Then solve for the unknown.

a


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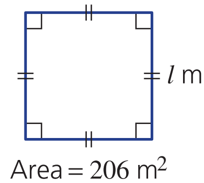

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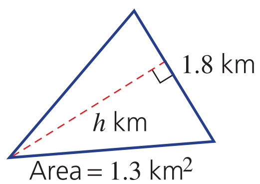

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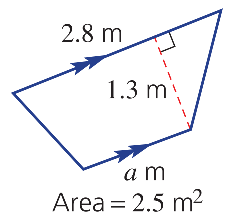

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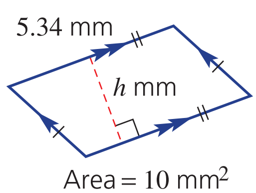

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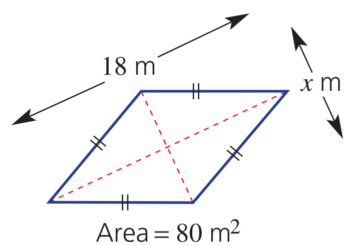
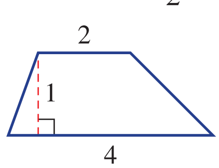


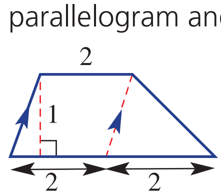
Four ways to find the area of a trapezium — 12

12 Find the area of this trapezium using each of the suggested methods.

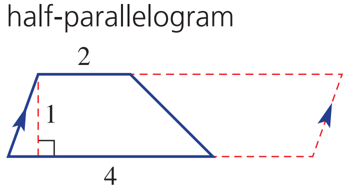
a formula $A = \frac{1}{2}(a + b)h$



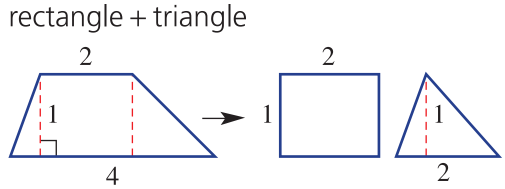
b parallelogram and triangle



c half-parallelogram



d rectangle + triangle



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1E Area of circles and sectors

Learning intentions

- To know the formula for the area of a circle
- To be able to calculate what fraction of a circle is represented by a sector
- To be able to find the area of circles and sectors

Key vocabulary: sector, circle, radius, diameter, pi

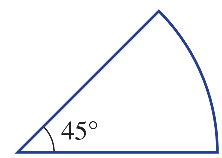
Like its circumference, a circle's area is linked to the special number pi (π). The area is the product of pi and the square of the radius, so $A = \pi r^2$.

Knowing the formula for the area of a circle helps us build circular objects, plan water sprinkler systems and estimate the damage caused by an oil slick from a ship in calm seas.

Lesson starter: What fraction is that?

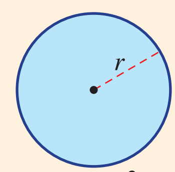
When finding areas of sectors, we first need to decide what fraction of a circle we are dealing with. This sector, for example, has a radius of 4 cm and a 45° angle.

- What fraction of a full circle is shown in this sector?
- How can you use this fraction to help find the area of this sector?
- How would you set out your working to find its area?



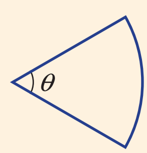
Key ideas

- The formula for finding the area (A) of a circle of radius r is given by the equation: $A = \pi r^2$.
- When the diameter (d) of the circle is given, determine the radius before calculating the area of the circle: $r = d \div 2$.



$A = \pi r^2$

- A **sector** is a portion of a circle including two radii.
- The angle of a sector of a circle determines the fraction of the circle. A full circle is 360°.
  - This sector is $\frac{\theta}{360}$ of a circle.



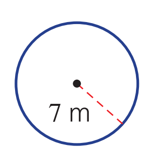
  - The area of a sector is given by $A = \frac{\theta}{360} \times \pi r^2$

Exercise 1E

Understanding	1–3	3

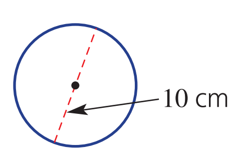
1 Which is the correct working step for the area of this circle?

A $A = \pi(7)$ **B** $A = 2\pi(7)$ **C** $A = \pi(14)^2$
D $A = (\pi 7)^2$ **E** $A = \pi(7)^2$



2 Which is the correct working step for the area of this circle?

A $A = \pi(10)^2$ **B** $A = (\pi 10)^2$
C $A = \pi(5)^2$ **D** $A = 2\pi(5)$
E $A = 5\pi$



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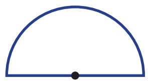
1E

3 What fraction of a circle is shown by these sectors? Simplify your fraction.

a


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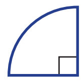

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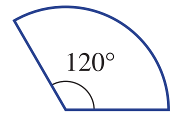

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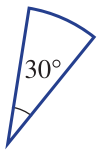

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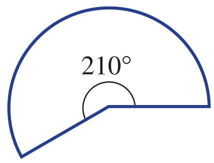

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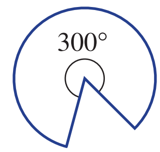
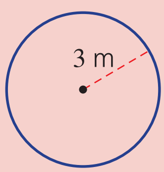


Fluency

	4, 5	4–5(½)

Example 11 Finding areas of circles

Find the area of these circles, correct to two decimal places.

a


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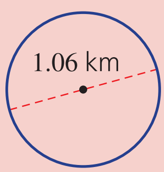
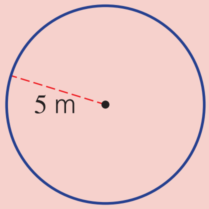


Solution	Explanation
a $A = \pi r^2$	Write the formula.
$= \pi(3)^2$	Substitute $r = 3$.
$= \pi \times 9$	Evaluate $3^2 = 9$ and then multiply by π.
$= 28.27\ \text{m}^2$ (to 2 d.p.)	
b Radius $r = 1.06 \div 2 = 0.53$ km	Find the radius, given the diameter of 1.06.
$A = \pi r^2$	Write the formula.
$= \pi(0.53)^2$	Substitute $r = 0.53$.
$= 0.88\ \text{km}^2$ (to 2 d.p.)	Write your answer to two decimal places with units.

Now you try

Find the area of these circles, correct to two decimal places.

a


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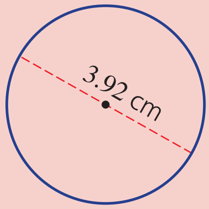

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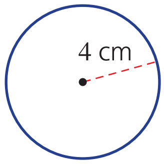
4 Find the area of these circles, correct to two decimal places.

a



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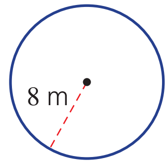

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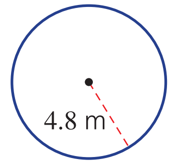

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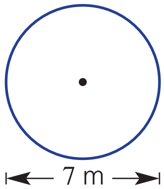

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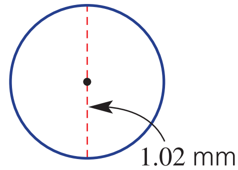

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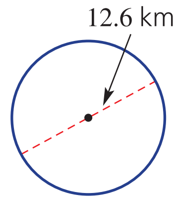
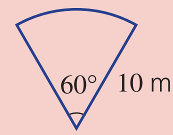


Example 12 Finding areas of sectors

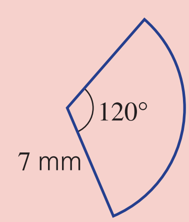
Find the area of this sector, correct to two decimal places.



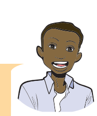
Solution	Explanation
Fraction of circle $= \frac{60}{360} = \frac{1}{6}$	The sector uses 60° out of the 360° in a whole circle.
Area $= \frac{1}{6} \times \pi r^2$	Write the formula, including the fraction.
$= \frac{1}{6} \times \pi(10)^2$	Substitute $r = 10$.
$= 52.36\ \text{m}^2$ (to 2 d.p.)	Write your answer to two decimal places.

Now you try

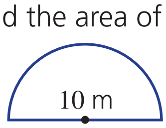
Find the area of this sector, correct to two decimal places.



5 Find the area of these sectors, correct to two decimal places.



a



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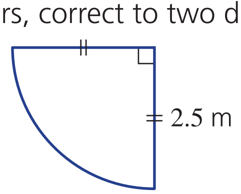

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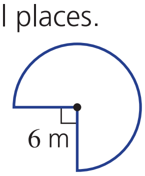

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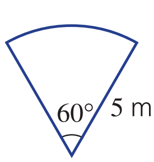

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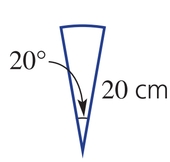

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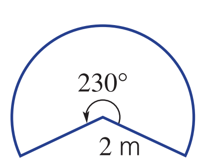

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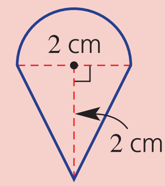
1E

Problem-solving and reasoning 6–8 7, 8, 9(½)

6 A pizza with 40 cm diameter is divided into eight equal parts. Find the area of each portion, correct to one decimal place.

Example 13 Finding areas of composite shapes

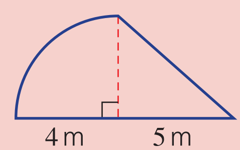
Find the area of this composite shape, correct to two decimal places.



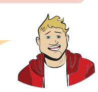
Solution	Explanation
$A = \frac{1}{2}\pi r^2 + \frac{1}{2}bh$	The shape is made up of a semicircle and a triangle. Write the formulas for both shapes.
$= \frac{1}{2}\pi(1)^2 + \frac{1}{2}(2)(2)$	Substitute $r = 1$, $b = 2$ and $h = 2$.
$= 1.5707... + 2$	
$= 3.57\text{ cm}^2$ (to 2 d.p.)	Write your answer to two decimal places with units.

Now you try

Find the area of this composite shape, correct to two decimal places.

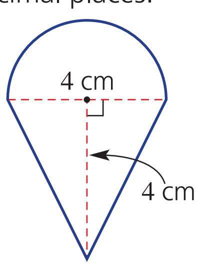


Hint: For Question 7, find the area of each shape that makes up the larger shape, then add them. For example, triangle + semicircle.



7 Find the area of these composite shapes, correct to two decimal places.

a



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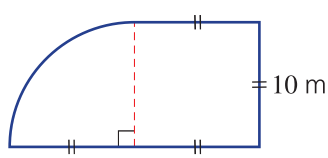

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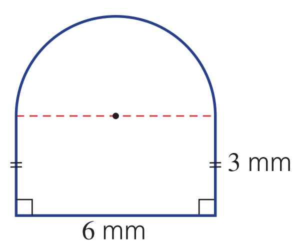
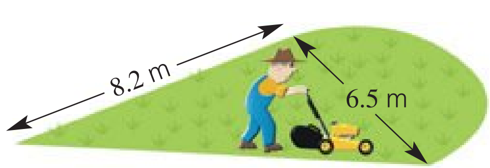


8 The lawn area in a backyard is made up of a semicircular region with diameter 6.5 m and a right-angled triangular region of length 8.2 m, as shown. Find the total area of lawn in the backyard, correct to two decimal places.



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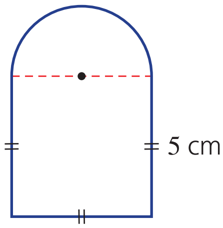
9 Find the area of these composite shapes, correct to one decimal place.

a



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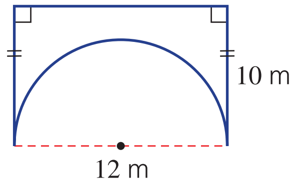

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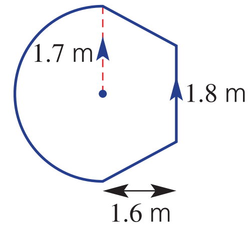
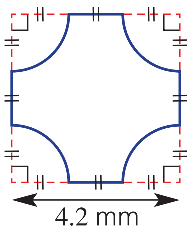


Hint: Use addition or subtraction, depending on the shape given.

d



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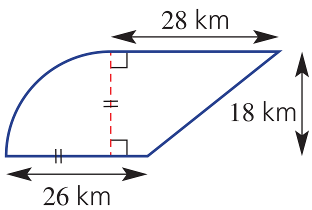

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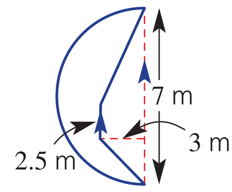
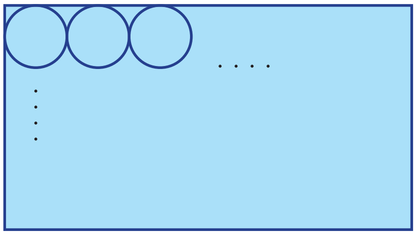


Circular pastries | — | 10

10 A rectangular piece of pastry is used to create small circular pastry discs for the base of Christmas tarts. The rectangular piece of pastry is 30 cm long and 24 cm wide, and each circular piece has a diameter of 6 cm.



- **a** How many circular pieces of pastry can be removed from the rectangle?
- **b** Find the total area removed from the original rectangle, correct to two decimal places.
- **c** Find the total area of pastry remaining, correct to two decimal places.
- **d** If the remaining pastry was collected and re-rolled to the same thickness, how many circular pieces could be cut? (Assume that the pastry can be re-rolled and cut many times.)

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1F Surface area of prisms

Learning intentions

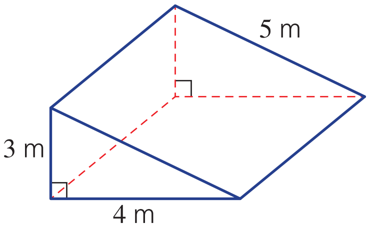
- To know that the surface area of a solid can be represented using a net
- To be able to calculate the total surface area of a prism

Key vocabulary: total surface area, prism, net, cross-section

The total surface area of a three-dimensional object can be found by finding the sum of the areas of each of the shapes that make up the surface of the object.

Lesson starter: Which net?

The solid below is a triangular prism with a right-angled triangle as its cross-section.

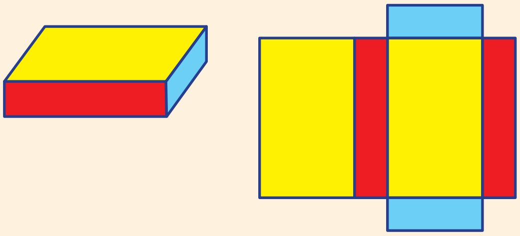


- How many different types of shapes make up its outside surface?
- What is a possible net for the solid? Is there more than one?
- How would you find the total surface area?

Key ideas

- A **prism** is a solid with a constant **cross-section** shape.
- To calculate the **total surface area (TSA)** of a solid or prism:
  - Draw a **net** (i.e. a two-dimensional drawing that includes all the surfaces).
  - Determine the area of each shape inside the net.
  - Add the areas of each shape together.

Solid **Net**



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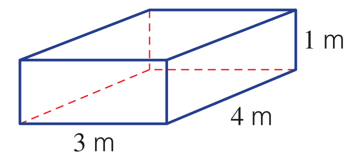
Exercise 1F

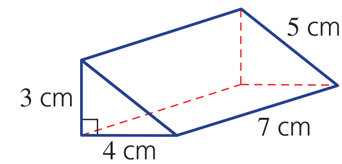
Understanding 1–3 3

1 A two-dimensional drawing of all the faces of a solid is called a __________.

2 For a rectangular prism, answer the following.
  a How many faces does the prism have?
  b How many *different* rectangles form the surface of the prism?



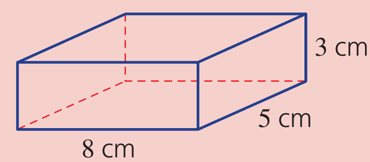
3 For this triangular prism, answer the following.
  a What is the area of the largest surface rectangle?
  b What is the area of the smallest surface rectangle?
  c What is the combined area of the two triangles?
  d What is the total surface area?



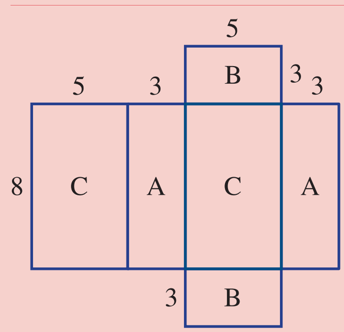
Fluency 4, 5 4–6

Example 14 Finding the TSA of a rectangular prism

Find the total surface area (TSA) of this rectangular prism by first drawing its net.



Solution



TSA = 2 × area of A + 2 × area of B
  + 2 × area of C
$= 2 \times (8 \times 3) + 2 \times (5 \times 3) + 2 \times (8 \times 5)$
$= 158\text{ cm}^2$

Explanation

Draw the net of the solid, labelling the lengths and shapes of equal areas.

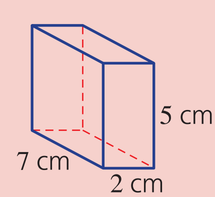
Describe each area.

Substitute the correct lengths.

Simplify and include units.

Now you try

Find the total surface area (TSA) of this rectangular prism by first drawing its net.



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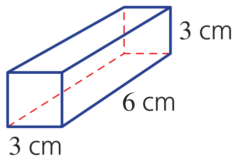
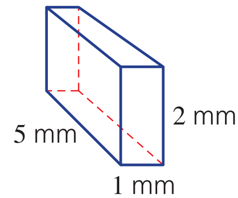
1F

4 Find the total surface area (TSA) of these rectangular prisms by first drawing their nets.

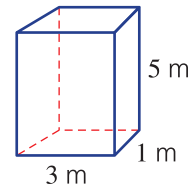
a



b

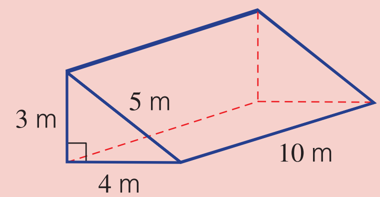


c

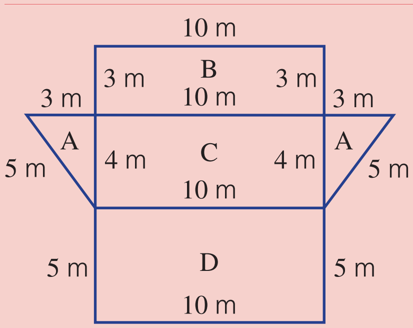


Example 15 Finding the TSA of a triangular prism

Find the TSA of the triangular prism shown.



Solution



Total surface area

$= 2 \times \text{area A} + \text{area B} + \text{area C} + \text{area D}$

$= 2 \times \left(\frac{1}{2} \times 3 \times 4\right) + (3 \times 10) + (4 \times 10)$

$+ (5 \times 10)$

$= 12 + 30 + 40 + 50$

$= 132\text{ m}^2$

Explanation

Draw a net of the object with all the measurements and label the sections to be calculated.

There are two triangles with the same area and three different rectangles.

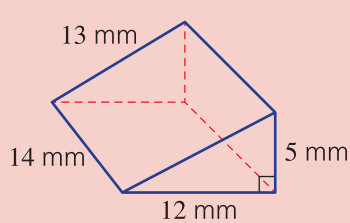
Substitute the correct lengths.

Calculate the area of each shape.

Add the areas together.

Now you try

Find the TSA of the triangular prism shown.



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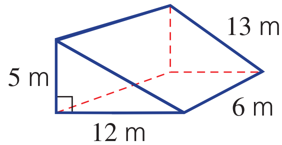
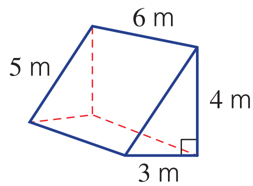
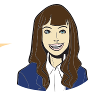
5 Find the total surface area of the following prisms.

a



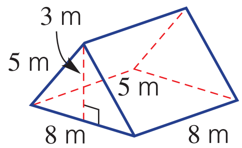
b





Hint: There are three rectangles and two identical triangles.

c



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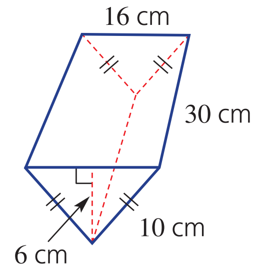
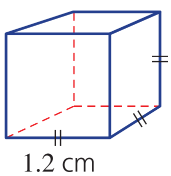


6 Find the TSA of these objects by first drawing a net.

a



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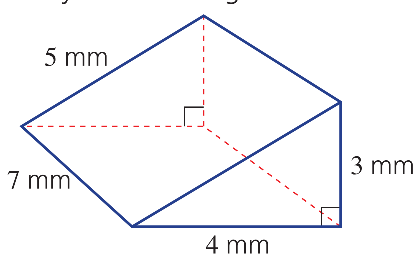

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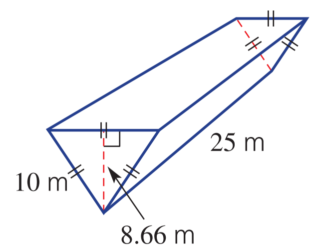
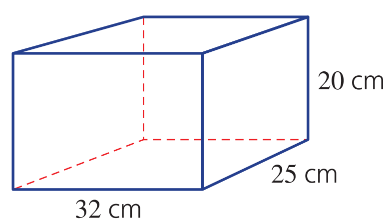


Problem-solving and reasoning 7–9 9–12

7 A cube with side lengths of 8 cm is to be painted all over with bright red paint. What is the total surface area that is to be painted?

8 What is the minimum amount of paper required to wrap a box with dimensions 25 cm wide, 32 cm long and 20 cm high?



9 An open-topped box is to be covered inside and out with a special material. If the box is 40 cm long, 20 cm wide and 8 cm high, find the minimum amount of material required to cover the box.

Hint: Count both inside and outside but do not include the top.

10 David wants to paint his bedroom. The ceiling and walls are to be the same colour. If the room measures 3.3 m × 4 m and the ceiling is 2.6 m high, find the amount of paint needed if:

a each litre covers 10 square metres

b each litre covers 5 square metres

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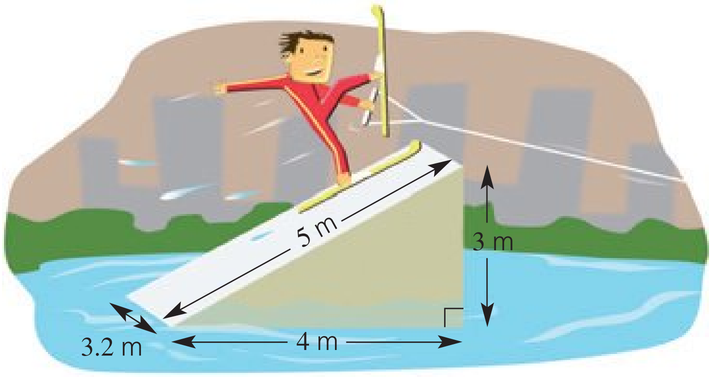
1F

11 A ski ramp in the shape of a triangular prism needs to be painted before the Moomba Classic waterskiing competition in Melbourne is held. The base and sides of the ramp require a fully waterproof paint, which covers 2.5 square metres per litre. The top needs special smooth paint, which covers only 0.7 square metres per litre.

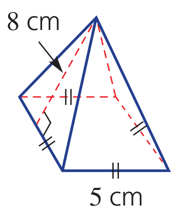


a Determine the amount of each type of paint required. Round your answers to two decimal places where necessary.

b If the waterproof paint is \$7 per litre and the special smooth paint is \$20 per litre, calculate the total cost of painting the ramp, to the nearest cent. (Use the exact answers from part **a** to help.)

12 Find the total surface area (TSA) of these right square-based pyramids.

a



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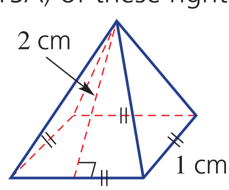

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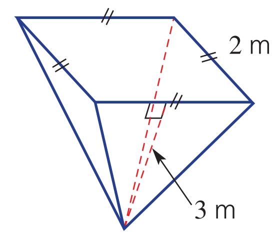
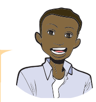
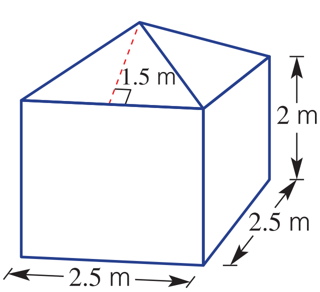


Hint: There is one square and four identical triangles.



Will I have enough paint? — 13

13 I have 6 litres of paint and on the tin it says that the coverage is $5.5\,m^2$ per litre. I wish to paint the four outside walls of a shed and the roof, which has four identical triangular sections. Will I have enough paint to complete the job?



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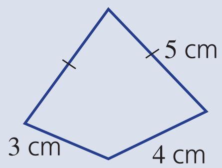
Progress quiz

1A **1** Convert the given measurements to the units shown in brackets.

a 9 km (m) **b** 4.5 m^2 (cm^2) **c** 145 cm^3 (mm^3)

1B **2** Find the perimeter of these shapes.

a



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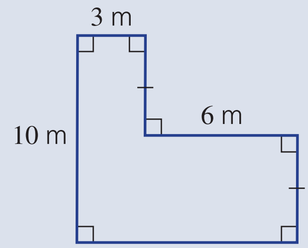
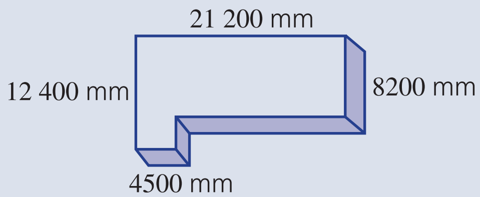


1B **3** A concrete slab is shown below.

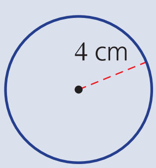
a Draw a new diagram, showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.



1C/E **4** Find the circumference (C) and area (A) of these circles, correct to two decimal places.

a



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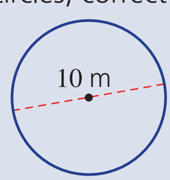
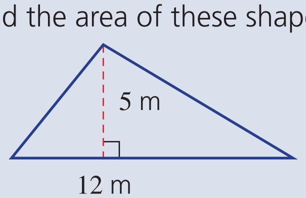


1D/E **5** Find the area of these shapes. Round your answer to one decimal place in part **d**.

a



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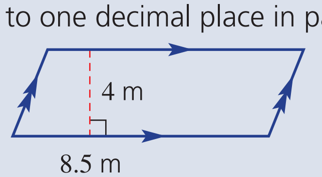

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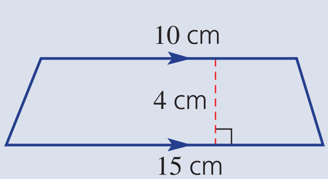

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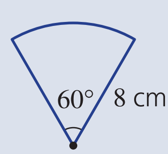

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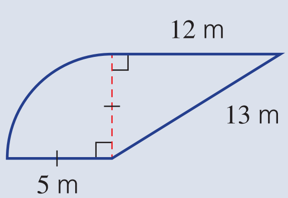
1D

6 A rectangular kitchen floor is to be replaced with wooden floorboards. If the floorboards cost \$46 per square metre, determine the cost to cover the kitchen floor if its dimensions are 4.4 m by 3 m.

1C/E

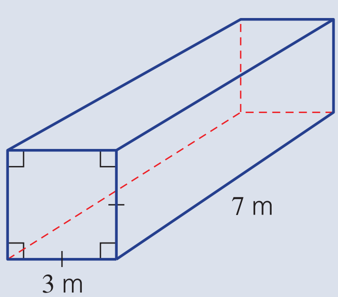
7 Find the area (A) and perimeter (P) of the composite shape shown. Round each answer to one decimal place.



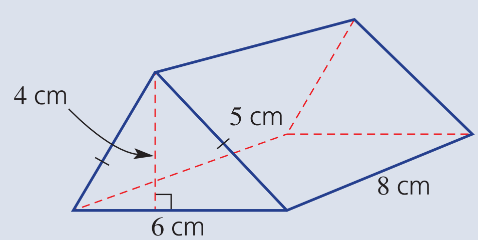
1F

8 Find the TSA of the following prisms. (Drawing a net may help you.)

a



b



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1G Surface area of a cylinder ★

Learning intentions

- To understand how the net of a cylinder can be drawn to show the total surface area
- To know the formula for the total surface area of a cylinder
- To be able to calculate the total surface area of a cylinder

Key vocabulary: cylinder, area, prism, circumference, net, cross-section

Like a prism, a cylinder has a uniform cross-section with identical circles as its two ends. The curved surface of a cylinder can be rolled out to form a rectangle that has a length equal to the circumference of the circle.

A can is a good example of a cylinder. We need to know the area of the ends and the curved surface area in order to cut sections from a sheet of aluminium to manufacture the can.

Lesson starter: Why $2\pi rh$?

We can see from the net of a cylinder (see the diagram below in Key ideas) that the total area of the two circular ends is $2 \times \pi r^2$ or $2\pi r^2$. For the curved part, though, consider the following.

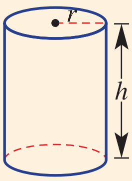
- Why can it be drawn as a rectangle? Can you explain this using a piece of paper?
- Why are the dimensions of this rectangle h and $2\pi r$?
- Where does the formula TSA $= 2\pi r^2 + 2\pi rh$ come from?

Key ideas

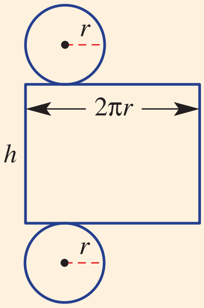
- A **cylinder** is a solid with a circular **cross-section**.
  - The net contains two equal circles and a rectangle. The rectangle has one side length equal to the circumference of the circle.
  - TSA = 2 circles + 1 rectangle

    $= 2\pi r^2 + 2\pi rh$
  - Another way of writing $2\pi r^2 + 2\pi rh$ is $2\pi r(r + h)$.

Diagram



Net



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1G

Exercise 1G

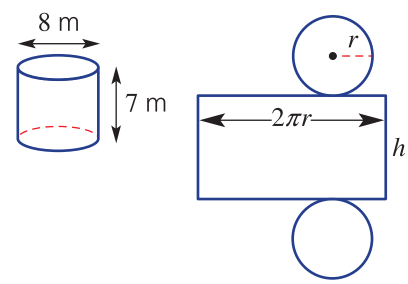
Understanding — 1, 2 — 2

1 Write the missing word/expression.

a The shape of the cross-section of a cylinder is a ___________.

b The TSA of a cylinder is TSA = $2\pi r^2 +$ ___________.

2 A cylinder and its net are shown here.



a What is the value of:

i r? ii h?

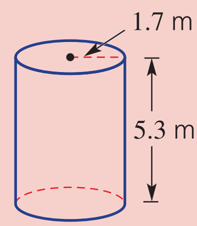
b Find the value of $2\pi r$, correct to two decimal places.

c Use TSA = $2\pi r^2 + 2\pi rh$ to find the total surface area, correct to two decimal places.

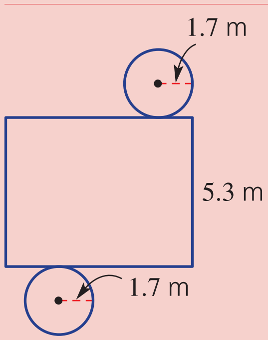
Fluency — 3, 4 — 3–5

Example 16 Finding the surface area of a cylinder

By first drawing a net, find the total surface area of this cylinder, correct to two decimal places.



Solution



TSA = 2 circles + 1 rectangle

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(1.7)^2 + 2\pi(1.7)(5.3)$

$= 74.77\text{ m}^2$ (to 2 d.p.)

Explanation

Draw the net and label the appropriate lengths.

Write what you need to calculate.

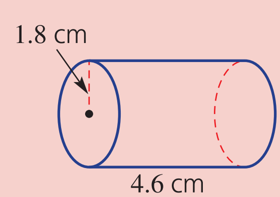
Write the formula.

Substitute the correct values: $r = 1.7$ and $h = 5.3$.

Round your answer to two decimal places.

Now you try

By first drawing a net, find the total surface area of this cylinder, correct to two decimal places.



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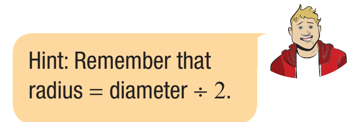
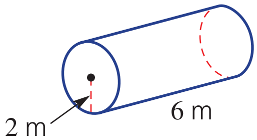
3 By first drawing a net, find the total surface area of these cylinders, to two decimal places.



a



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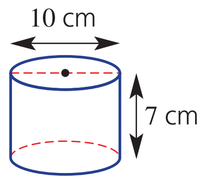

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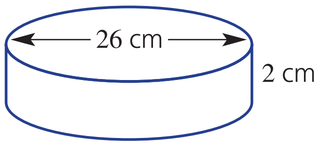

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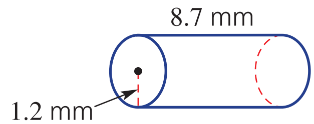
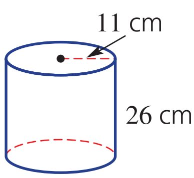


4 Use the formula TSA $= 2\pi r^2 + 2\pi rh$ to find the total surface area of these cylinders, to one decimal place.

a



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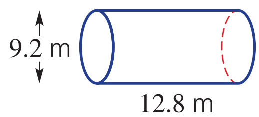
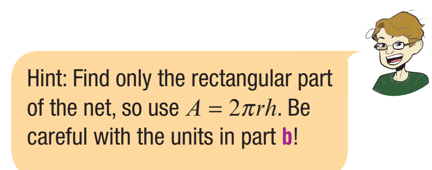
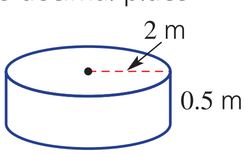


5 Find the area of only the curved surface of these cylinders, to one decimal place.



a



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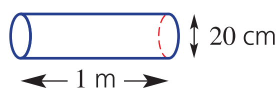

Problem-solving and reasoning

6, 7 | 6–9

6 Find the outside surface area of a pipe of radius 85 cm and length 4.5 m, to one decimal place. Give your answer in m^2.

7 The base and sides of a circular cake tin are to be lined on the inside with baking paper. The tin has a base diameter of 20 cm and is 5 cm high. What is the minimum amount of baking paper required, to one decimal place?

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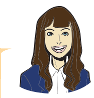
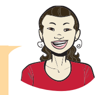
1G

8 The inside and outside of an open-topped cylindrical concrete tank is to be coated with a special waterproofing paint. The tank has diameter 4 m and height 2 m. Find the total area to be coated with the paint. Round your answer to one decimal place.



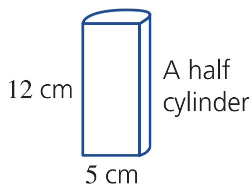
Hint: Include the base but not the top.

9 Find the TSA of these cylindrical portions, to one decimal place.



Hint: Carefully consider the fraction of a circle made up by the ends, and the fraction of a full cylinder made up by the curved part.

a



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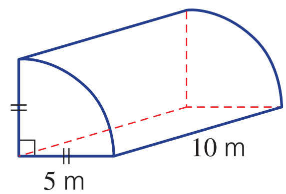

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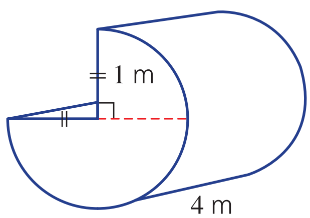

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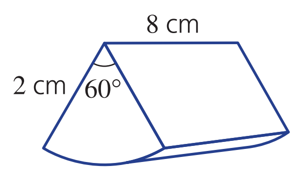

The steamroller — 10

10 A steamroller has a large, heavy cylindrical barrel that is 4 m wide and has a diameter of 2 m.

a Find the area of the curved surface of the barrel, to two decimal places.

b After 10 complete turns of the barrel, how much ground would be covered, to two decimal places?

c Find the circumference of one end of the barrel, to two decimal places.

d How many times would the barrel turn after 1 km of distance, to two decimal places?

e What area of ground would be covered if the steamroller travels 1 km?

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1H Volume of solids

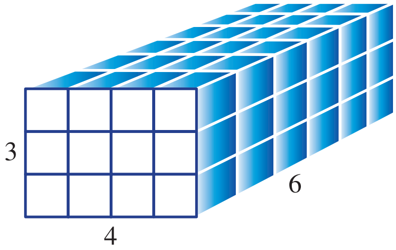
Learning intentions

- To understand how the volume of solids relates to its constant cross-section and height
- To know the common units for capacity
- To know the formula for the volume of a solid with a uniform cross-section
- To be able to calculate the volume of a solid with a uniform cross-section

Key vocabulary: solid, volume, cross-section, uniform, prism, cylinder, perpendicular, capacity

The volume of a solid is the amount of space it occupies within its outside surface. It is measured in cubic units.

For solids with a uniform cross-section, the area of the cross-section multiplied by the perpendicular height gives the volume. Consider the rectangular prism below.



Number of cubic units (base) $= 4 \times 6 = 24$

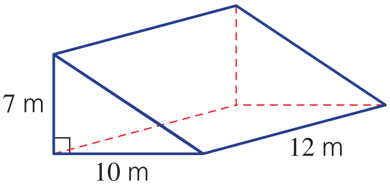
Area (base) $= 4 \times 6 = 24$ units^2

Volume = area (base) $\times 3 = 24 \times 3 = 72$ units^3

Lesson starter: Volume of a triangular prism

This prism has a triangular cross-section.

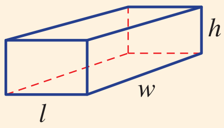
- What is the area of the cross-section?
- What is the 'height' of the prism?
- How can $V = A \times h$ be applied to this prism, where A is the area of the cross-section?



Key ideas

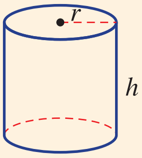
- **Volume** is the amount of three-dimensional space within an object.
- The volume of a solid with a uniform cross-section is given by $V = A \times h$, where:
  - A is the area of the cross-section.
  - h is the perpendicular (at 90°) height.

Rectangular prism



$V = lwh$

Cylinder



$V = \pi r^2 h$

- **Capacity** is the volume of a given object measured in litres or millilitres.
- Units for capacity include:
  - $1\text{ L} = 1000\text{ mL}$
  - $1\text{ cm}^3 = 1\text{ mL}$

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Exercise 1H

Understanding 1–3 3

1 Match the solid (**a**–**c**) with the volume formula (**A**–**C**).

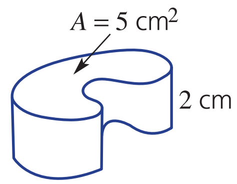
a cylinder
b rectangular prism
c triangular prism

A $V = lwh$
B $V = \frac{1}{2}bh \times \text{length}$
C $V = \pi r^2 h$

2 Write the missing number.

a There are __________ mL in 1 L.
b There are __________ cm^3 in 1 L.

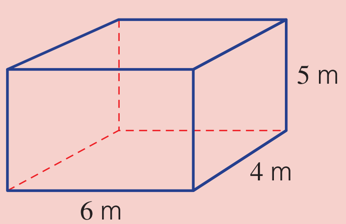
3 The area of the cross-section of this solid is given. Find the solid's volume, using $V = A \times h$.



Fluency 4–6 4–7

Example 17 Finding the volume of a rectangular prism

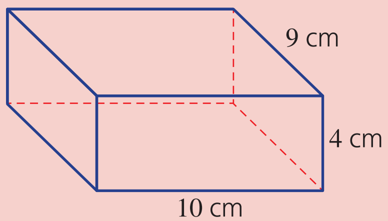
Find the volume of this rectangular prism.



Solution	Explanation
$V = A \times h$	Write the general formula.
$= 6 \times 4 \times 5$	$A = 6 \times 4$ and $h = 5$.
$= 120\text{ m}^3$	Simplify and include units.

Now you try

Find the volume of this rectangular prism.



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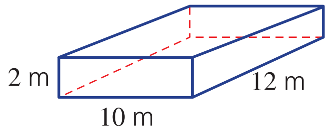
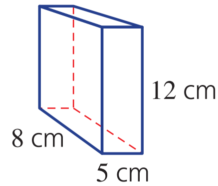
4 Find the volume of these rectangular prisms.

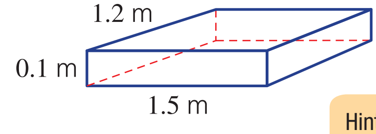
a



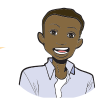
b



c

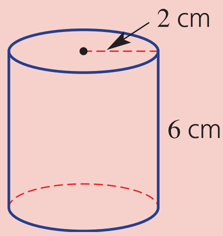


Hint: Use $V = lwh$.



Example 18 Finding the volume of a cylinder

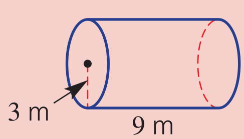
Find the volume of this cylinder, correct to two decimal places.



Solution	Explanation
$V = A \times h$	Write the general formula.
$= \pi r^2 \times h$	The cross-section is a circle.
$= \pi(2)^2 \times 6$	Substitute $r = 2$ and $h = 6$.
$= 75.40\text{ cm}^3$ (to 2 d.p.)	Simplify and write your answer as required, with units.

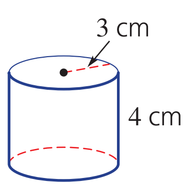
Now you try

Find the volume of this cylinder, correct to two decimal places.



5 Find the volume of these cylinders, correct to two decimal places.

a



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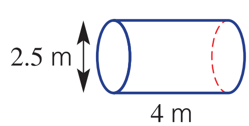

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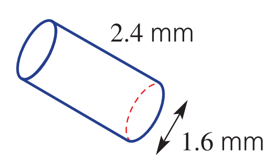
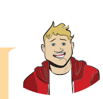


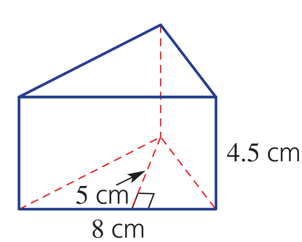
Hint: For a cylinder: $V = \pi r^2 \times h$



6 A triangle with base 8 cm and height 5 cm forms the base of a prism, as shown. If the prism stands 4.5 cm high, find:

a the area of the triangular base

b the volume of the prism



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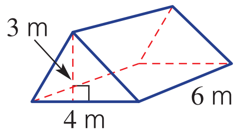
1H

7 Find the volume of these triangular prisms.

a


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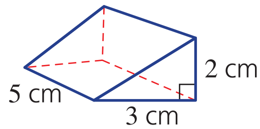
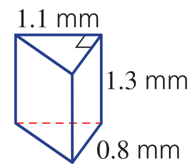


c


Hint: Use $V = A \times h$, where A is the area of a triangle.

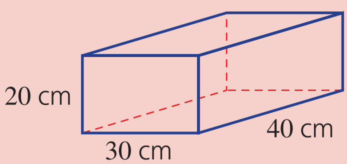
Problem-solving and reasoning

8–10 | 9–12

8 A cylindrical drum stands on one end with a diameter of 25 cm and water is filled to a height of 12 cm. Find the volume of water in the drum, in cm^3, correct to two decimal places.

Example 19 Working with capacity

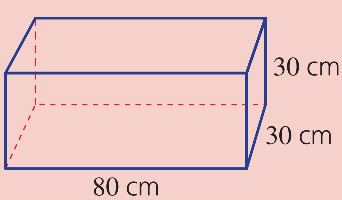
Find the number of litres of water that this container can hold.



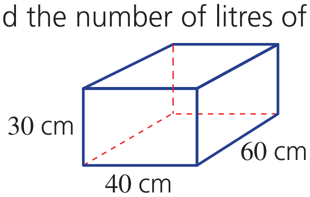
Solution	Explanation
$V = 30 \times 40 \times 20$	First work out the volume in cm^3.
$= 24\,000\ cm^3$	Then divide by 1000 to convert to litres, since $1\ cm^3 = 1$ mL
$= 24$ L	and there are 1000 mL in 1 litre.

Now you try

Find the number of litres of water that this container can hold.



9 Find the number of litres of water that these containers can hold.

a


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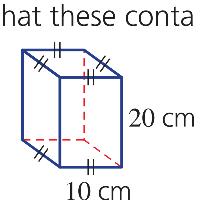

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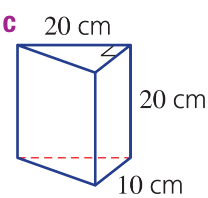
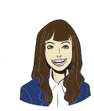


Hint: Use $1\ L = 1000\ cm^3$.



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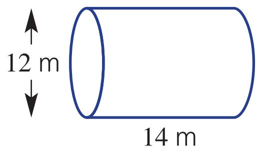
10 Find the volume of these solids, rounding your answers to two decimal places where necessary.

Hint: Find the area of the cross-section first.

a



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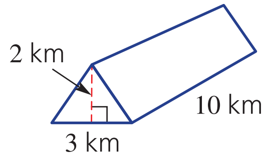
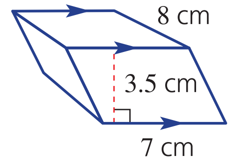
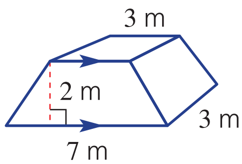


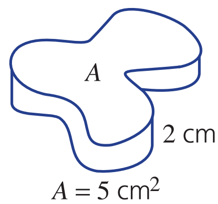
c



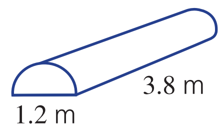
d



e



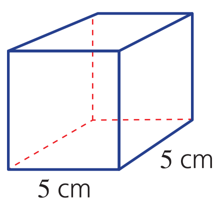
f



11 $100\,\text{cm}^3$ of water is to be poured into this container.

a Find the area of the base of the container.

b Find the depth of water in the container.



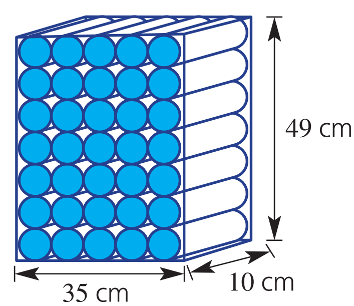
12 In a scientific experiment, solid cylinders of ice are removed from a solid block carved out of a glacier. The ice cylinders have diameter 7 cm and length 10 cm. The dimensions of the solid block are shown in the diagram.

a Find the volume of ice in the original ice block.

b Find the volume of ice in one ice cylinder, to two decimal places.

c Find the number of ice cylinders that can be removed from the ice block, using the configuration shown.

d Find the volume of ice remaining after the ice cylinders are removed from the block, to two decimal places.

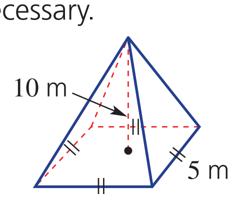


Volume of pyramids and cones — 13

13 The volume of a pyramid or cone is exactly one-third the volume of the prism with the same base area and height; i.e. $V = \frac{1}{3} \times A \times h$.

Find the volume of these pyramids and cones. Round your answer to one decimal place where necessary.

a



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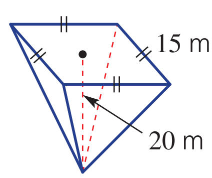

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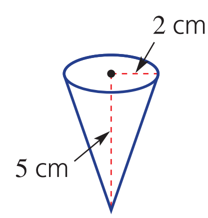

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1I Accuracy of measuring instruments ★

Learning intentions

- To understand that accuracy depends on how measurements are recorded
- To know the limits of accuracy for a given recorded measurement
- To be able to calculate the limits of accuracy for given measurements

Key vocabulary: accuracy, precision, rounding

Humans and machines measure many different things, such as the time taken to swim a race, a length of timber needed for a building and the volume of cement needed to lay a concrete path around a swimming pool. The degree or level of accuracy required usually depends on what is being measured and what the information is being used for.

All measurements are approximate. Errors can come from the equipment being used or the person using the measuring device.

Accuracy is the measure of how true to the 'real' the measure is, whereas **precision** is the ability to obtain the same result over and over again.

Lesson starter: Rounding a decimal

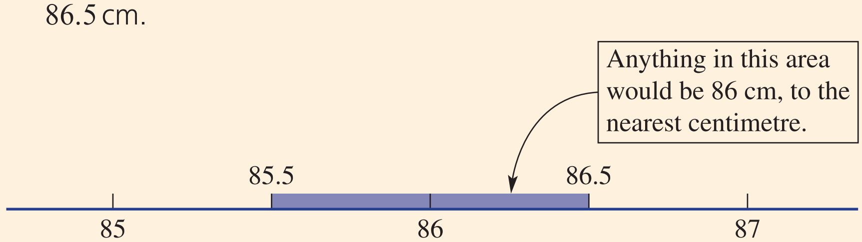
1 A piece of timber is measured to be 86 cm, correct to the nearest centimetre. What is the smallest decimal that it could be rounded from and what is the largest decimal that, when rounded to the nearest whole, is recorded as 86 cm?

2 If a measurement is recorded as 6.0 cm, correct to the nearest millimetre, then:
- a What units were used when measuring?
- b What is the smallest decimal that could be rounded to this value?
- c What is the largest decimal that would have resulted in 6.0 cm?

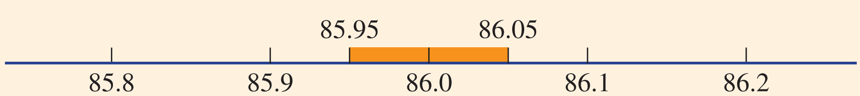
3 Consider a square with sides given as 7.8941 km each. What is the perimeter of the square if the side length is:
- a left unchanged with four decimal places?
- b rounded to one decimal place?
- c truncated (i.e. chopped off) at one decimal place?

Key ideas

- The limits of **accuracy** tell you what the upper and lower boundaries are for the true measurement.
  - Usually, it is $\pm 0.5 \times$ the smallest unit of measurement.
  - Note that values are stated to 1 decimal place beyond that of the given measurement. For example, when measuring to the nearest centimetre, 86 cm has limits from 85.5 cm up to 86.5 cm.



- When measuring to the nearest millimetre, the limits of accuracy for 86.0 cm are 85.95 cm to 86.05 cm.



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Exercise 1I

Understanding | 1–3 | 3

1 Write down a decimal that, when rounded from two decimal places, gives 3.4.

2 Write down a measurement of 3467 mm, correct to the nearest:
a centimetre **b** metre

3 What is the smallest decimal that, when rounded to one decimal place, could result in an answer of 6.7?

Fluency | 4, 5 | 4, 5

Example 20 Stating the smallest unit of measurement

For each of the following, write down the smallest unit of measurement.
a 89.8 cm **b** 56.85 m

Solution	Explanation
a 0.1 cm or 1 mm	The measurement is given to one decimal place. That means the smallest unit of measurement is tenths.
b 0.01 m or 1 cm	This measurement is given to two decimal places. Therefore, the smallest unit of measurement is hundredths or 0.01.

Now you try

For each of the following, write down the smallest unit of measurement.
a 24.3 m **b** 4.75 km

4 For each of the following, state the smallest unit of measurement.
a 45 cm **b** 6.8 mm **c** 12 m
d 15.6 kg **e** 56.8 g **f** 10 m
g 673 h **h** 9.84 m **i** 12.34 km

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1I

Example 21 Finding limits of accuracy

Give the limits of accuracy for these measurements.

a 72 cm **b** 86.6 mm

Solution	Explanation
a $72 \pm 0.5 \times 1$ cm $= 72 - 0.5$ cm to $72 + 0.5$ cm $= 71.5$ cm to 72.5 cm	Smallest unit of measurement is one whole cm. Error $= 0.5 \times 1$ cm This error is subtracted and added to the given measurement to find the limits of accuracy.
b $86.6 \pm 0.5 \times 0.1$ mm $= 86.6 \pm 0.05$ mm $= 86.6 - 0.05$ mm to $86.6 + 0.05$ mm $= 86.55$ mm to 86.65 mm	Smallest unit of measurement is 0.1 mm. Error $= 0.5 \times 0.1$ mm This error is subtracted and added to the given measurement to find the limits of accuracy.

Now you try

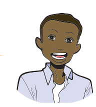
Give the limits of accuracy for these measurements.

a 36 cm **b** 15.1 m

5 Give the limits of accuracy for each of these measurements.

a 5 m **b** 8 cm **c** 78 mm
d 5 ns **e** 2 km **f** 34.2 cm
g 3.9 kg **h** 19.4 kg **i** 457.9 t
j 18.65 m **k** 7.88 km **l** 5.05 s

Hint: Give measurement $\pm 0.5 \times$ smallest unit of measurement.



Problem-solving and reasoning

6–8 | 6, 8–11

6 Write the following as a measurement, given that the lower and upper limits of the measurements are as follows.

a 29.5 m to 30.5 m **b** 144.5 g to 145.5 g **c** 4.55 km to 4.65 km

7 Martha writes down the length of her fabric as 150 cm. As Martha does not give her level of accuracy, give the limits of accuracy of her fabric if it was measured correct to the nearest:

a centimetre **b** 10 centimetres **c** millimetre

8 A length of copper pipe is given as 25 cm, correct to the nearest centimetre.

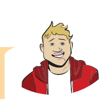
a What are the limits of accuracy for this measurement?

b Ten pieces of pipe, each with a given length of 25 cm, are joined.

i What is the minimum length that it could be?

ii What is the maximum length that it could be?

Hint: Use upper and lower limits of accuracy for max and min lengths.



9 The sides of a square are recorded as 9.2 cm, correct to one decimal place.

a What is the minimum length that each side of this square could be?

b What is the maximum length that each side of this square could be?

c Find the upper and lower boundaries for this square's perimeter.

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10 Johan measures the mass of an object to be 6 kg. Amy says the same object is 5.8 kg and Thomas gives his answer as 5.85 kg.

- **a** Explain how all three people could have different answers for the same measurement.
- **b** Write down the level of accuracy being used by each person.
- **c** Are all their answers correct? Discuss.

11 Write down a sentence explaining the need to accurately measure items in our everyday lives and the accuracy that is needed for each of your examples. Give three examples of items that need to be measured correct to the nearest:

a kilometre **b** millimetre **c** millilitre **d** litre

Practical measurement — 12

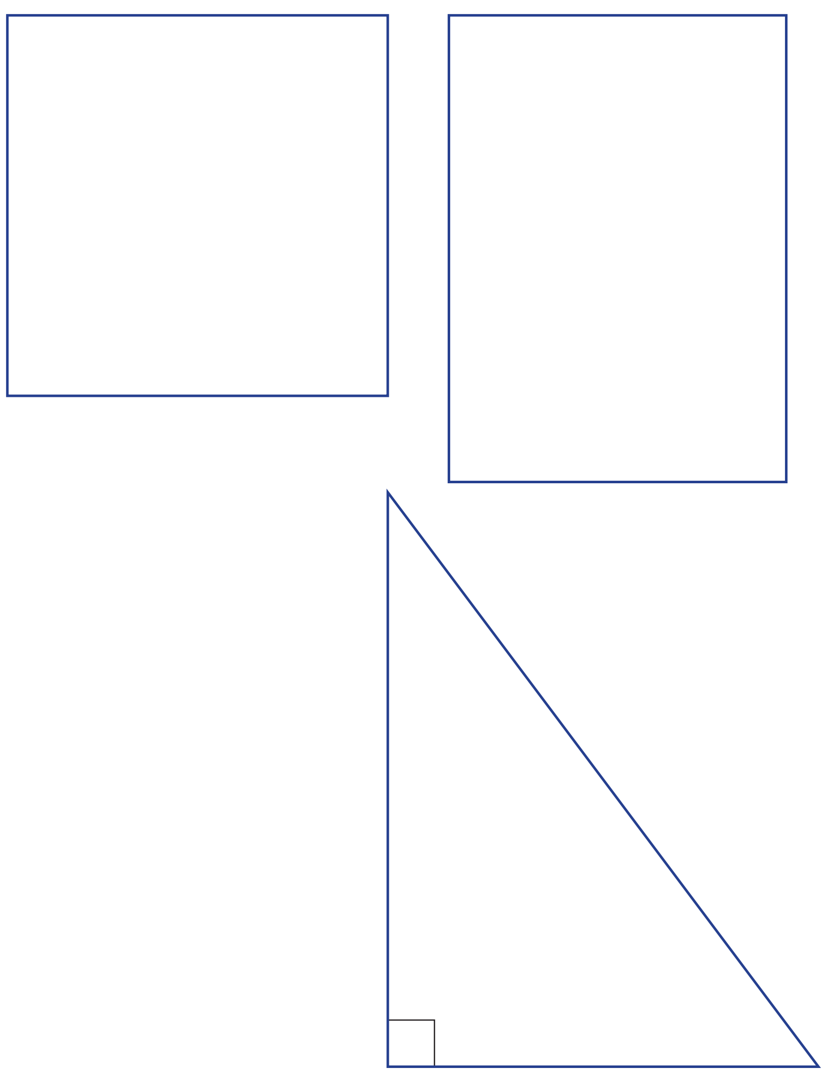
12 **a** Measure each of the shapes below, correct to the nearest:

- **i** cm
- **ii** mm

b Use your measurements to find the perimeter and area of each shape.

c After collating your classmates' measurements, find the average perimeter for each shape.

d By how much did the lowest and highest perimeters vary? How can this difference be explained?



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Maths@Work: Bricklayer

A bricklayer has a physically challenging job that requires stamina and strength and also good communication skills, as they often work as part of a team.

Bricklayers must have a solid understanding of how the construction process works and the ability to read plans and blueprints.

Mathematical skills are essential in this trade. Bricklayers must understand ratios for mixing mortar and cement. Good measurement skills are also important, as bricklayers must be able to work out the number of bricks required for a job, convert between different units and take accurate measurements at the work site, using the most appropriate tool. An understanding of geometry and trigonometry is also required.

Complete these questions that a bricklayer may face in their day-to-day job.

1 A standard house brick has dimensions $l \times w \times h = 230$ mm $\times$ 76 mm $\times$ 110 mm and the standard thickness of mortar when laying bricks is 10 mm. The bricks are laid so that $l \times h$ is the outer face.

> Hint: $A = l \times w$
> $V = l \times w \times h$

- **a** What is the length and height of each brick in centimetres and metres?
- **b** Determine the area in cm^2 and m^2 of the outer face of one brick.
- **c** Determine the volume in cubic centimetres of each brick.
- **d** Calculate the length, in metres (to two decimal places), when laying the following number of bricks in a line with mortar between each join.
  - **i** 10 bricks
  - **ii** 100 bricks

> Hint: For 10 bricks, there would be 9 mortar joins.

- **e** Calculate the height, in millimetres, of a wall of 25 rows of standard house bricks. Remember to consider the thickness of the mortar.
- **f** Estimate how many standard house bricks are needed to build a wall 4 m by 1.5 m, by dividing the area of the wall by the area of a brick's face.

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2 Ready-mix mortar comes in 20 kg bags that cost \$7.95 per bag. One bag of mortar is used to lay 20 standard house bricks.
Using standard house bricks (see dimension details in Question **1**), a brick wall is to be built that has a finished length of 8630 mm, a height of 2750 mm and is one brick deep.

- **a** Calculate the exact number of standard house bricks needed to build this wall. Remember to consider the thickness of the mortar.
- **b** How many Ready-mix mortar bags must be purchased for this wall?
- **c** If each house brick costs 60 cents, find the total cost, to the nearest dollar, of the materials needed for this wall.

3 A type of large brick is chosen for an outside retaining wall. These bricks are sold only in whole packs and each pack covers 12.5 square metres when laid. How many whole packs of these bricks must be bought to build a wall with dimensions:

- **a** 6 m by 1.5 m?
- **b** 9 m by 2 m?

Hint: You can't buy half a pack, so round up to the next whole number.

4 Fastwall house bricks are larger and lighter than standard bricks and can be used for single-storey constructions. They are sold in pallets of 1000 for \$1258.21, including delivery and GST. Each Fastwall brick has dimensions $l \times w \times h = 305 \text{ mm} \times 90 \text{ mm} \times 162 \text{ mm}$ and the standard thickness of mortar is 10 mm.

- **a** If a pallet has 125 bricks per layer, how many layers does each pallet have?
- **b** If the wooden base of the pallet has a height of 30 cm, what is the total height of the pallet, in cm, when loaded with bricks?
- **c** Find the exact number of bricks needed to build a wall that is 20.15 m long and 6.87 m high. Remember to consider the thickness of the mortar.
- **d** Determine the cost of the bricks, to the nearest dollar, required to be bought for building the wall in part **c**.

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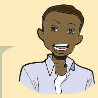
Using technology



Hint: For E5 the formula would be $= G5 \times A5 + (G5 - 1) \times 10$
For G6 the formula would be $= (E6 + 10)/(230 + 10)$

5 Copy the following table into a spreadsheet. Then enter formulas into the shaded cells and, hence, determine the missing values.

	A	B	C	D	E	F	G	H	I
1	Brick Wall Calculations								
2	Note: All dimensions are in mm								
3	Single brick dimensions			Mortar	Brick wall dimensions		Number of bricks used		
4	Length	Width	Height	Width	Length	Height	Per layer (row)	Number of layers	Total
5	230	76	110	10			24	18	
6	230	76	110	10	21350	6230			

Hint: Copy the total number of bricks used from the first table.

6 Copy the following table into your spreadsheet underneath the table from Question **5**. Enter the formulas into the shaded cells and, hence, determine the missing values. Assume there are 1000 bricks per pallet and that one bag of mortar is used per 20 bricks laid.

	A	B	C	D	E	F	G	H	I	J
	Cost calculations									
	Bricks		Cost of pallets				Cost of mortar			
	Number of bricks used	Decimal number of pallets	number of whole pallets	cost per pallet (including GST)	total cost of pallets	Decimal number of mortar bags	number of whole bags	cost per bag (including GST)	total cost of mortar	Total cost of mortar and pallets
12				$1,358.24				$7.95		
13				$1,541.56				$8.45		

In cell B13 enter = ROUNDUP (B12, 0), which will round the number from cell B12 up to the nearest whole number; e.g. 1.3 will be rounded up to 2.

7 Use your spreadsheet tables to find the total cost of materials for the following brick walls made from Fastwall bricks. (See Question **4** for Fastwall brick dimensions.) The spreadsheet formulas will not need to be changed.

a A wall of 44 bricks per layer (row) and 30 layers (rows) if pallets cost $1258.36, including GST, and mortar is $7.55 per bag.

b A wall 20.15 m long by 7.79 m high if pallets cost $1364.32, including GST, and mortar is $9.25 per bag.

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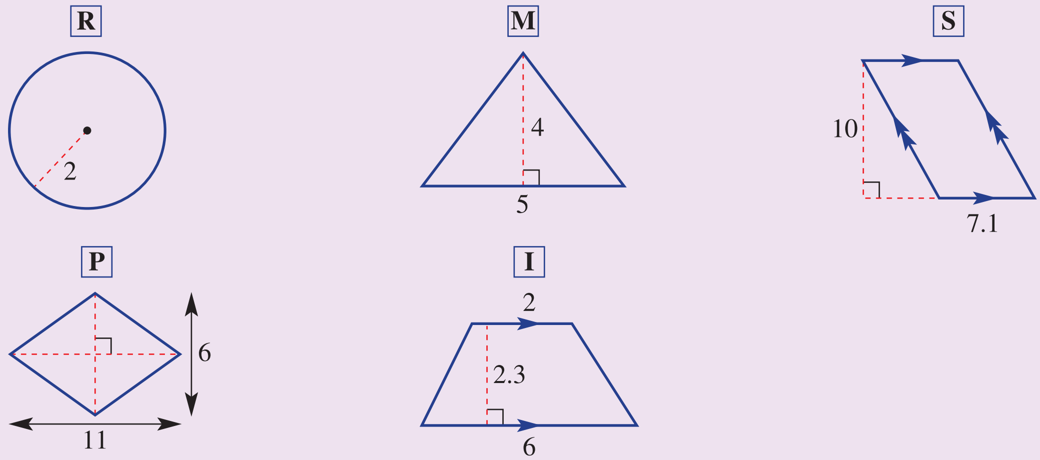
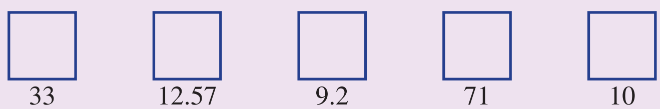
Puzzles and games

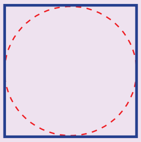
1 'I am the same shape all the way through. What am I?' Find the area of each shape. Match the letters to the answers below to solve the riddle.



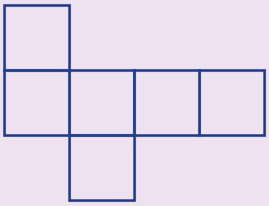


2 One litre of water is poured into a container in the shape of a rectangular prism. The dimensions of the prism are 8 cm by 12 cm by 11 cm. Will the water overflow?

3 A circular piece of pastry is removed from a square sheet with side length 30 cm. What percentage of pastry remains?

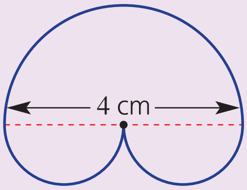


4 How many different nets are there for a cube? Do not count reflections or rotations of the same net. Here is one example.



5 Give the radius of a circle whose value for the circumference is equal to the value for the area.

6 Find the area of this special shape.



7 A cube's surface area is 54 cm^2. What is its volume?

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Chapter summary

Measurement

Conversion of units

Length

km → m → cm → mm: $\times 1000$, $\times 100$, $\times 10$

mm → cm → m → km: $\div 10$, $\div 100$, $\div 1000$

Area

km^2 → m^2 → cm^2 → mm^2: $\times 1000^2$, $\times 100^2$, $\times 10^2$

mm^2 → cm^2 → m^2 → km^2: $\div 10^2$, $\div 100^2$, $\div 1000^2$

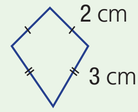
Volume

km^3 → m^3 → cm^3 → mm^3: $\times 1000^3$, $\times 100^3$, $\times 10^3$

mm^3 → cm^3 → m^3 → km^3: $\div 10^3$, $\div 100^3$, $\div 1000^3$

Perimeter

The distance around the outside of a shape.

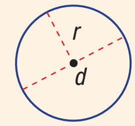


$P = 2 \times 2 + 2 \times 3$
$= 10$ cm

Circumference

The distance around the outside of a circle.

$C = 2\pi r$ or $C = \pi d$



Area of basic shapes

Square: $A = l^2$

Rectangle: $A = lw$

Triangle: $A = \frac{1}{2}bh$

Rhombus: $A = \frac{1}{2}xy$

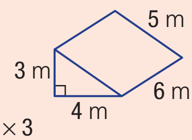
Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$

Total surface area

Draw a net and sum the surface areas.

Triangular prism

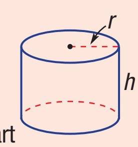


$TSA = 2 \times \frac{1}{2} \times 4 \times 3$
$+ 6 \times 4 + 6 \times 3 + 6 \times 5$
$= 84\ m^2$

Cylinder ★

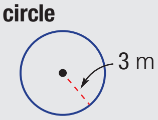
$TSA = 2\pi r^2 + 2\pi rh$

2 ends, curved part



Area of a circle

$A = \pi r^2$
$= \pi \times 3^2$
$= 28.27\ m^2$ (to 2 d.p.)



Volume

Rectangular prism $V = lwh$

Cylinder $V = \pi r^2 h$

Capacity: 1 L = 1000 mL
$1\ cm^3 = 1$ mL

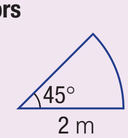
Accuracy ★

Accuracy depends on any error associated with the measuring instruments and how they are used.

Limits of accuracy are usually $\pm 0.5 \times$ the smallest unit.

Area of sectors

$A = \frac{45}{360} \times \pi r^2$
$= \frac{1}{8} \times \pi \times 2^2$
$= 1.57\ m^2$ (to 2 d.p.)



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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

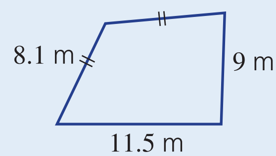
✔

1A **1 I can convert between metric units of length.**
e.g. Convert these measurements to the units shown in the brackets.
a 2.3 m (cm) **b** 270 000 cm (km)

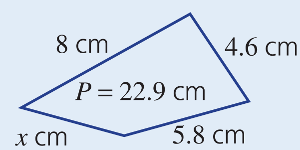
1A **2 I can convert between metric units of area.**
e.g. Convert these measurements to the units shown in the brackets.
a 32 000 m^2 (km^2) **b** 7.12 cm^2 (mm^2)

1A **3 I can convert between metric units of volume.**
e.g. Convert these measurements to the units shown in the brackets.
a 3.7 cm^3 (mm^3) **b** 5 900 000 cm^3 (m^3)

1B **4 I can find the perimeter of basic shapes.**
e.g. Find the perimeter of this shape.

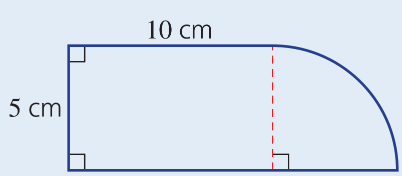


1B **5 I can find a missing side length given the perimeter.**
e.g. Find the value of x for this shape with the given perimeter.

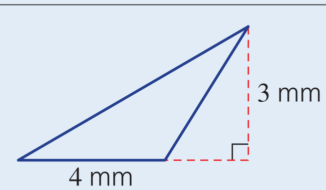


1C **6 I can find the circumference of a circle.**
e.g. Find the circumference of a circle with a diameter of 5 m, correct to two decimal places.

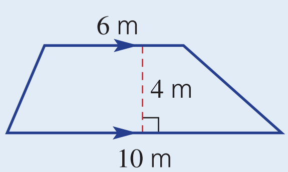
1C **7 I can find the perimeter of simple composite shapes.**
e.g. Find the perimeter of this composite shape, correct to two decimal places.



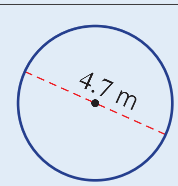
1D **8 I can find the area of squares, rectangles and triangles.**
e.g. Find the area of this triangle.



1D **9 I can find the area of rhombuses, parallelograms and trapeziums.**
e.g. Find the area of this trapezium.



1E **10 I can find the area of a circle.**
e.g. Find the area of this circle, correct to two decimal places.



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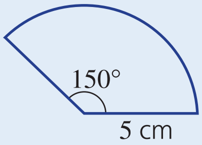
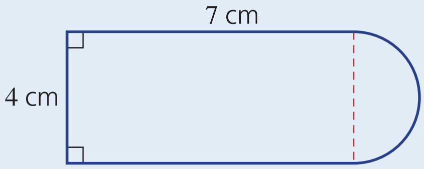
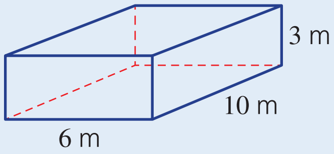
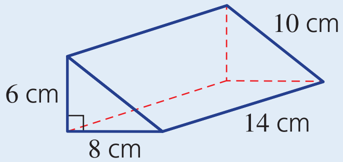
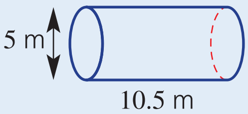
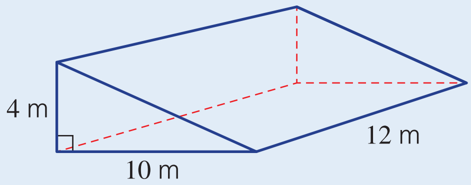
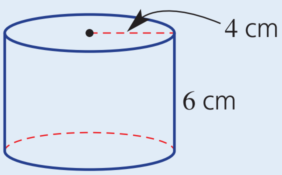
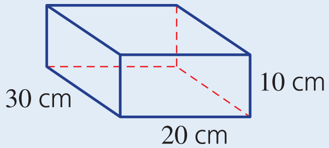
Section		✔
1E	**11 I can find the area of a sector.** e.g. Find the area of this sector, correct to two decimal places. 	
1E	**12 I can find the area of simple composite shapes involving sectors.** e.g. Find the area of this composite shape, correct to two decimal places. 	
1F	**13 I can find the total surface area of a rectangular prism using a net.** e.g. Find the total surface area of this rectangular prism. 	
1F	**14 I can find the total surface area of a triangular prism using a net.** e.g. Find the total surface area of this triangular prism. 	
1G	**15 I can find the total surface area of a cylinder.** e.g. Find the total surface area of this cylinder, correct to two decimal places. 	
1H	**16 I can find the volume of a prism.** e.g. Find the volume of this triangular prism. 	
1H	**17 I can find the volume of a cylinder.** e.g. Find the volume of this cylinder, correct to two decimal places. 	
1H	**18 I can find the volume of a prism, giving an answer in L or mL.** e.g. Find the volume of this triangular prism in litres. 	
1I	**19 I can state the smallest unit for a given measurement.** e.g. Write down the smallest unit of measurement for 27.3 cm.	
1I	**20 I can find the limits of accuracy for a given measurement.** e.g. Give the limits of accuracy for the measurement 65.3 m.	

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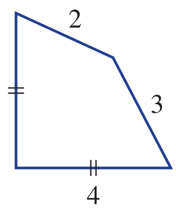
Short-answer questions

1A **1** Convert these measurements to the units shown in the brackets.

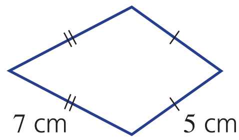
a 5.3 km (m) **b** $27\,000\text{ cm}^2$ (m^2) **c** 0.04 cm^3 (mm^3)

1B **2** Find the perimeter of these shapes.

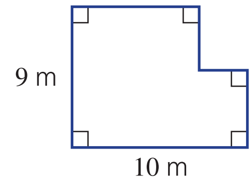
a



b

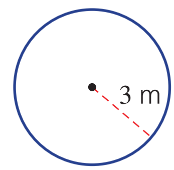


c



1C/E **3** For the circle shown, find, to two decimal places:

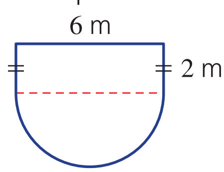
a the circumference **b** the area



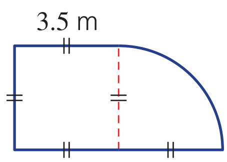
1E **4** For these composite shapes, find, to two decimal places:

i the perimeter **ii** the area

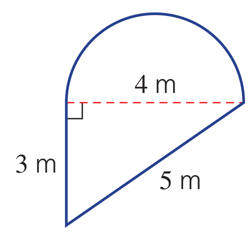
a



b

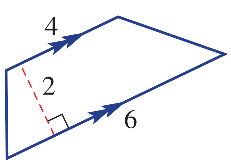


c

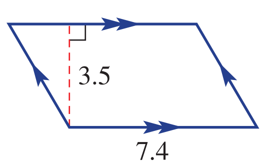


1D **5** Find the area of these shapes.

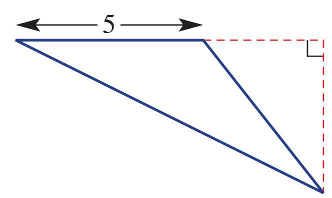
a



b

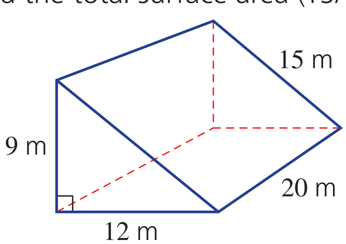


c

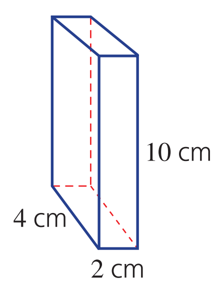


1F **6** Find the total surface area (TSA) of these prisms.

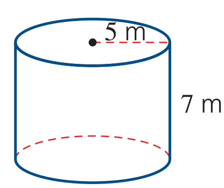
a



b



1G **7** Determine the total surface area of this cylinder, to two decimal places.



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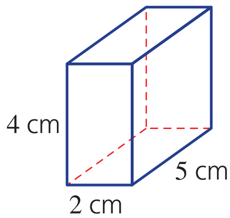
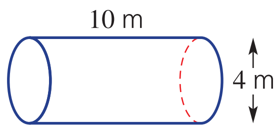
Chapter review

1H **8** Find the volume of these solids, to two decimal places where necessary.

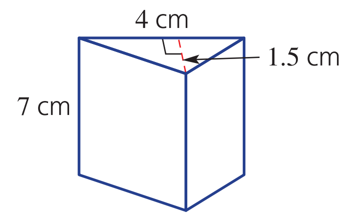
a



b



c



1I **9** Give the limits of accuracy for these measurements.

a 6 cm **b** 4.2 kg **c** 16.21 m

Multiple-choice questions

1A **1** The number of centimetres in a kilometre is:

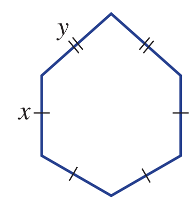
A 10 **B** 100 **C** 1000 **D** 10 000 **E** 100 000

1B **2** The perimeter of a square with side lengths 2 cm is:

A 4 cm **B** 8 cm **C** 4 cm^2 **D** 8 cm^2 **E** 16 cm

1B **3** The perimeter of the shape shown is given by the formula:

A $x - y$ **B** $2x + y$ **C** $4x + 2y$
D $x - 2y$ **E** $4x + y$



1C **4** A correct expression for determining the circumference of a circle with diameter 6 cm is:

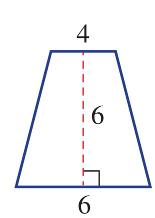
A $\pi \times 6$ **B** $\pi \times 3$ **C** $2 \times \pi \times 6$ **D** 2×6 **E** $\pi \times 6^2$

1D **5** The area of a rectangle with side lengths 3 cm and 4 cm is:

A 12 cm^2 **B** 12 cm **C** 7 cm^2 **D** 14 cm **E** 14 cm^2

1D **6** The correct expression for calculating the area of this trapezium is:

A $(6 - 4) \times 6$ **B** $\frac{1}{2}(6 + 4) \times 6$ **C** $\frac{1}{2} \times 6 \times 4$ **D** $6 \times 6 - 4$
E $6 \times 6 + 6 \times 4$



1E **7** A sector's centre angle measures 90°. This is equivalent to:

A $\frac{1}{5}$ of a circle **B** $\frac{1}{2}$ of a circle **C** $\frac{3}{4}$ of a circle
D $\frac{2}{3}$ of a circle **E** $\frac{1}{4}$ of a circle

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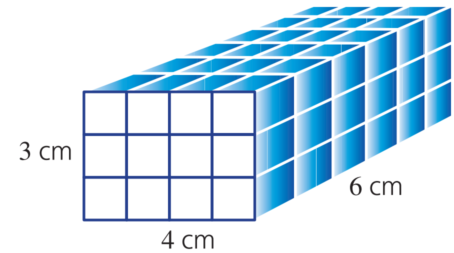
1H **8** The volume of the shape shown is:

A 13 cm^3 **B** 27 cm^3
C 72 cm^2 **D** 72 cm^3
E 27 cm^2

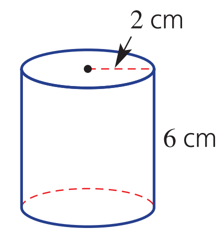


1H **9** The volume of a cube of side length 3 cm is:

A 9 cm^3 **B** 27 cm^3 **C** 54 cm^2 **D** 54 cm^3 **E** 27 cm^2

1G **10** The curved surface area for this cylinder is closest to:

A 87.96 cm^2 **B** 12.57 cm^2
C 75.40 cm^2 **D** 75.39 cm^2
E 113.10 cm^2



Extended-response question

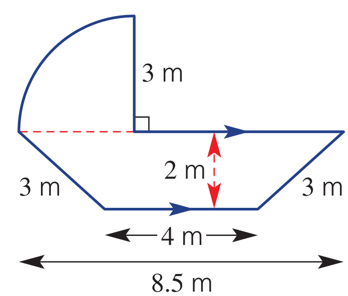
1 A new playground is being built with the shape and dimensions as shown below.

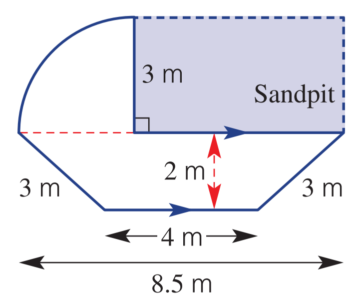
a The playground will be surrounded by wooden planks.
  i Determine the perimeter of the playground correct to two decimal places.
  ii If the wood to be used costs \$16.50/m, what will be the cost of surrounding the play area to the nearest dollar?

b The playground area is to be covered with a layer of woodchips. Find the area of the playground correct to one decimal place.

c If a bag of woodchips from the hardware store covers 7.5 m^2, how many bags would be required to cover the playground area?

d A rectangular sandpit is to be included as shown. If sand is to be spread flat and filled to a height of 40 cm, determine the volume of sand required in m^3.





2 A cylindrical tank has diameter 8 m and height 2 m.

a Find the surface area of the curved part of the tank, to two decimal places.
b Find the TSA, including the top and the base, to two decimal places.
c Find the total volume of the tank, to two decimal places.
d Find the total volume of the tank in litres, to two decimal places.
Note: There are 1000 litres in 1 m^3.

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Chapter 2

Consumer arithmetic

Essential mathematics: why skills with percentages and consumer arithmetic are important

Money management skills are essential for achieving personal financial independence and security, and for business success. The exchange of money is the basis of our economy, and percentages are used universally to describe changes of value.

- Essential skills for personal money management include making a personal budget, calculating the interest and repayments on loans, comparing discounted prices, and checking pay and tax amounts.
- Service businesses, such as web designers, hairdressers, mobile car mechanics and cake makers, all increase costs by a profit percentage and 10% GST.
- Successful food businesses (e.g. pizza shop, cafe) regularly calculate food costs as a percentage of sales revenue.
- Training for employment in finance, e.g. a bank teller, includes percentage calculations for loans, investments, insurance, pay scales, commission and tax.
- Investment advisors assist clients to make a budget; analyse investment growth forecasts and loan costs using simple and compound interest rates; and to compare insurance rates and cover.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Money and financial mathematics

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (VCMNA328)

Patterns and algebra

Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Find the following totals.

a $\$15.92 + \$27.50 + \$56.20$
b $\$134 + \$457 + \$1021$
c $\$457 \times 6$
d $\$56.34 \times 1\frac{1}{2}$
e $87 560 ÷ 52 (to the nearest cent)

2 Express the following fractions with denominators of 100.

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{1}{5}$ **d** $\frac{17}{25}$ **e** $\frac{9}{20}$

3 Write each of the following fractions as decimals.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{20}{50}$ **e** $\frac{1}{3}$

4 Round the following decimals to two decimal places.

a 16.7893 **b** 7.347 **c** 45.3444 **d** 6.8389 **e** 102.8999

5 Give the values of the pronumerals in the following table.

Hint: Net Income = gross income − deductions

Gross income ($)	Deductions ($)	Net income ($)
4976	456.72	a
72 156	21 646.80	b
92 411	c	62 839
156 794	d	101 916
e	18 472.10	79 431.36

6 Calculate the following annual incomes for each of these people.

a Tom: $1256 per week
b Sally: $15 600 per month
c Anthony: $1911 per fortnight
d Crystal: $17.90 per hour, for 40 hours per week, for 50 weeks per year

7 Without a calculator, find:

a 10% of $400
b 5% of $5000
c 2% of $100
d 25% of $844
e 20% of $12.80
f 75% of $1000

8 Find the simple interest on the following amounts.

a $400 at 5% p.a. for 1 year
b $5000 at 6% p.a. for 1 year
c $800 at 4% p.a. for 2 years

Hint: Simple Interest = $\frac{Prt}{100}$

9 Complete the following table, giving the values of the pronumerals.

Cost price	Deduction	Sale price
$34	$16	a
$460	$137	b
$500	c	$236
d	$45	$67
e	$12.65	$45.27

10 The following amounts include the 10% GST. By dividing each one by 1.1, find the original costs before the GST was added to each.

a $55 **b** $61.60 **c** $605

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2A Review of percentages CONSOLIDATING

Learning intentions

- To understand that a percentage is a number out of 100
- To be able to convert decimals and fractions to percentages and vice versa
- To be able to find the percentage of a quantity

Key vocabulary: percentage, denominator

It is important that we are able to work with percentages in our everyday lives. Banks, retailers and governments use percentages every day to work out fees and prices.

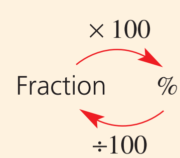
Lesson starter: Which option should Jamie choose?

Jamie currently earns $68 460 p.a. (per year) and is given a choice of two different pay rises. Which should she choose and why?

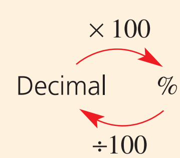
Choice A: Increase of $25 per week
Choice B: Increase of 2% on per annum salary

Key ideas

- A **percentage** means 'out of 100'. It can be written using the symbol %, or as a fraction or a decimal.
  For example: 75 per cent $= 75\% = \frac{75}{100}$ or $\frac{3}{4}$ or 0.75.
- To convert a fraction or a decimal to a percentage, multiply by 100.
- To convert a percentage to a fraction, write it with a **denominator** of 100 and simplify.
  $15\% = \frac{15}{100} = \frac{3}{20}$



- To convert a percentage to a decimal, divide by 100.
  $15\% = 15 \div 100 = 0.15$
- To find a percentage of a quantity, write the percentage as a fraction or a decimal, then multiply by the quantity; i.e. $x\%$ of $P = \frac{x}{100} \times P$.



Exercise 2A

Understanding	1–3	3

1 Complete the following using the words *multiply* or *divide*.
  a To convert a decimal to a percentage ________ by 100.
  b To convert a percentage to a decimal ________ by 100.
  c To convert a fraction to a percentage ________ by 100.
  d To convert a percentage to a fraction ________ by 100.

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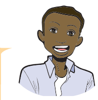
2A

2 Complete the following to express as a fraction in part **a** and a decimal in part **b**.

a **i** $7\% = \frac{7}{\square}$ **ii** $23\% = \frac{\square}{\square}$

b **i** $18\% = \square$ **ii** $5\% = \square$

3 Complete the following



Hint: Cancel any fractions before multiplying.

a 10% of $50 = \frac{\square}{100} \times 50$

$= \square$

b 25% of $412 = \frac{\square}{100} \times \square$

$= \square$

c 2% of $60 = \frac{\square}{100} \times 60$

$= \square$

Fluency

4–7(½) 4–8(½)

Example 1 Converting to a percentage

Write each of the following as a percentage.

a $\frac{19}{20}$ **b** $\frac{3}{8}$ **c** 0.07

Solution	Explanation
a $\frac{19}{20} = \frac{95}{100}$	Write using a denominator of 100.
$= 95\%$	Alternatively, multiply the fraction by 100.
	$\frac{19}{\cancel{20}_1} \times \frac{\cancel{100}^5}{1} = 19 \times 5 = 95$
b $\frac{3}{\cancel{8}_2} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{2}$	Multiply the fraction by 100.
$= 37.5$	Cancel common factors, then simplify.
So $\frac{3}{8} = 37.5\%$	
c $0.07 \times 100 = 7$	Multiply the decimal by 100.
So $0.07 = 7\%$	Move the decimal point two places to the right.

Now you try

Write each of the following as a percentage.

a $\frac{6}{25}$ **b** $\frac{7}{16}$ **c** 0.15

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4 Convert each fraction to a percentage.

a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$ **d** $\frac{1}{10}$

e $\frac{1}{100}$ **f** $\frac{7}{25}$ **g** $\frac{15}{50}$ **h** $\frac{3}{4}$

i $\frac{5}{8}$ **j** $\frac{19}{25}$ **k** $\frac{99}{100}$ **l** $\frac{47}{50}$

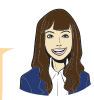
Hint: First write using a denominator of 100 or, alternatively, multiply by 100.

5 Write these decimals as percentages.

a 0.17 **b** 0.73 **c** 0.48 **d** 0.09

e 0.06 **f** 0.13 **g** 1.13 **h** 1.01

i 0.8 **j** 0.9 **k** 0.99 **l** 0.175



Hint: To multiply by 100, move the decimal point two places to the right.

Example 2 Writing percentages as simplified fractions

Write each of the following percentages as a simplified fraction.

a 37% **b** 58% **c** $6\frac{1}{2}\%$

Solution	Explanation
a $37\% = \frac{37}{100}$	Write the percentage with a denominator of 100.
b $58\% = \frac{58}{100}$	Write the percentage with a denominator of 100.
$= \frac{29}{50}$	Simplify $\frac{58}{100}$ by cancelling, using the HCF of 58 and 100, which is 2. $\frac{\cancel{58}^{29}}{\cancel{100}_{50}} = \frac{29}{50}$
c $6\frac{1}{2}\% = \frac{6\frac{1}{2}}{100}$	Write the percentage with a denominator of 100.
$= \frac{13}{200}$	Double the numerator $\left(6\frac{1}{2}\right)$ and the denominator (100) so that the numerator is a whole number.

Now you try

Write each of the following percentages as a simplified fraction.

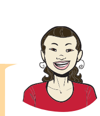
a 23% **b** 64% **c** $10\frac{1}{2}\%$

6 Write each percentage as a simplified fraction.

a 71% **b** 80% **c** 25% **d** 55%

e 40% **f** 88% **g** 15% **h** $16\frac{1}{2}\%$

i $17\frac{1}{2}\%$ **j** $2\frac{1}{4}\%$ **k** $5\frac{1}{4}\%$ **l** $52\frac{1}{2}\%$



Hint: Write with a denominator of 100, then simplify if possible.

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2A

Example 3 Writing a percentage as a decimal

Convert these percentages to decimals.

a 93% b 7% c 30%

Solution	Explanation
a $93\% = 93 \div 100$ $= 0.93$	Divide the percentage by 100. This is the same as moving the decimal point two places to the left.
b $7\% = 7 \div 100$ $= 0.07$	Divide the percentage by 100.
c $30\% = 30 \div 100$ $= 0.3$	Divide the percentage by 100. Write 0.30 as 0.3.

Now you try

Convert these percentages to decimals.

a 28% b 3% c 60%

7 Convert these percentages to decimals.

a 61% b 83% c 75% d 45% e 9% f 90%
g 50% h 16.5% i 7.3% j 200% k 430% l 0.5%

Example 4 Finding a percentage of a quantity

Find 42% of \$1800.

Solution	Explanation
42% of $\$1800 = 0.42 \times 1800$ $= \$756$	Remember that 'of' means to multiply. Write 42% as a decimal or a fraction: $42\% = \frac{42}{100} = 0.42$ Then multiply by the amount. If using a calculator, enter 0.42×1800. Without a calculator: $\frac{42}{\cancel{100}_1} \times 18\cancel{00} = 42 \times 18 = 756$

Now you try

Find 36% of \$2300.

8 Use a calculator to find the following.

a 10% of \$250 b 50% of \$300 c 75% of \$80
d 12% of \$750 e 9% of \$240 f 43% of 800 grams
g 90% of \$56 h 110% of \$98 i $17\frac{1}{2}\%$ of 2000 m

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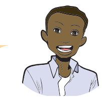
Problem-solving and reasoning | 9–11 | 11–14

9 A 300 g pie contains 15 g of saturated fat.

a What fraction of the pie is saturated fat?

b What percentage of the pie is saturated fat?

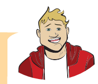


Hint: 15 g out of 300 g.

10 About 80% of the mass of a human body is water. If Hugo is 85 kg, how many kilograms of water are in his body?

11 Rema spends 12% of the 6.6 hour school day in maths. How many minutes are spent in the maths classroom?

12 In a cricket match, Brett spent 35 minutes bowling.
His team's total fielding time was $3\frac{1}{2}$ hours.
What percentage of the fielding time, correct to two decimal places, did Brett spend bowling?



Hint: First convert hours to minutes, and then write a fraction comparing times.

13 Malcolm lost 8 kg, and now weighs 64 kg. What percentage of his original weight did he lose?

14 47.9% of a local council's budget is spent on garbage collection. If a rate payer pays $107.50 per quarter in total rate charges, how much do they contribute in a year to garbage collection?

Australia's statistics | — | 15

15 Below is the preliminary data on Australia's population growth, as gathered by the Australian Bureau of Statistics for June 2015.

	Population at end June quarter 2015 ('000)	Change over previous year ('000)	Change over previous year (%, one decimal place)
New South Wales	7618.2	104.3	
Victoria	5938.1	99.4	
Queensland	4779.4	58.4	
South Australia	1698.6	13.1	
Western Australia	2591.6	33.2	
Tasmania	516.6	1.9	
Northern Territory	244.6	0.9	
Australian Capital Territory	390.8	5.4	
Australia	23 777.9	316.6	

a Calculate the percentage change for each state and territory shown using the previous year's population, and complete the table.

b What percentage of Australia's overall population, correct to one decimal place, is living in:

i NSW? **ii** Vic? **iii** WA?

Hint: % Change = $\frac{\text{change}}{\text{population}} \times \frac{100}{1}$

c Use a spreadsheet to draw a pie chart (i.e. sector graph) showing the populations of the eight states and territories in the table. What percentage of the total is represented by each state/territory? Round your answer to the nearest per cent.

d In your pie chart for part **c**, what is the angle size of the sector representing Victoria?

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2B Applications of percentages

Learning intentions

- To understand what a percentage increase or decrease of a quantity represents
- To be able to increase and decrease an amount by a given percentage
- To be able to use percentage increase and decrease to calculate a selling price or a discounted price
- To be able to determine the profit made on an item and calculate this as a percentage profit

Key vocabulary: discount, profit, selling price, cost price

There are many applications of percentages. Prices are often increased by a percentage to create a profit or decreased by a percentage when on sale.

When goods are purchased by a store, the cost to the owner is called the cost price. The price of the goods sold to the customer is called the selling price. This price will vary according to whether the store is having a sale or decides to make a certain percentage profit.

Lesson starter: Discounts

Discuss as a class:

- Which is better: 20% off or a $\$20$ discount?
- If a discount of 20% or $\$20$ resulted in the same price, what was the original price?
- Why are percentages used to show discounts, rather than a dollar amount?

Key ideas

- To increase by a given percentage, multiply by the sum of 100% and the given percentage. For example: To increase by 12%, multiply by 112% or 1.12.
- To decrease by a given percentage, multiply by 100% minus the given percentage. For example: To decrease by 20%, multiply by 80% or 0.8.
- Profits and discounts:
  - The normal price of the goods recommended by the manufacturer is called the retail price.
  - When there is a sale and the goods are priced less than the retail price, they are said to be **discounted**.
  - **Profit** is the amount of money made by selling an item or service for more than its cost.
  - Profit = selling price − cost price, where **selling price** is the amount the item is sold for and **cost price** is the original cost to the seller.
  - Percentage profit $= \frac{\text{profit}}{\text{cost price}} \times 100$
  - Percentage discount $= \frac{\text{discount}}{\text{cost price}} \times 100$

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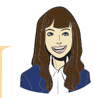
Exercise 2B

Understanding 1–4 4



Hint: $100\% + 10\% = 1.1$
$100\% - 5\% = 0.95$

1 By what number do you multiply to increase an amount by:
a 10%? **b** 20%? **c** 2%? **d** 18%?

2 By what number do you multiply to decrease an amount by:
a 5%? **b** 30%? **c** 15%? **d** 50%?

3 Use the words *selling price* or *cost price* to complete the following.
a A profit is made when the ________________ is more than the ________________.
b A discount in a store reduces the ________________.
c Profit = ________________ − ________________.

4 Decide how much profit or loss is made in each of the following situations.
a cost price = $15 selling price = $20
b cost price = $17.50 selling price = $20
c cost price = $250 selling price = $234

Fluency 5–7(½), 8–10 5–7(½), 8, 9, 11, 12

Example 5 Increasing by a given percentage

Increase $370 by 8%.

Solution	Explanation
$\$370 \times 1.08 = \$399.60$	$100\% + 8\% = 108\%$ Write 108% as a decimal (or fraction) and multiply by the amount. Show two decimal places to represent the cents.

Now you try

Increase $650 by 12%.

5 **a** Increase $90 by 5%.
b Increase $400 by 10%.
c Increase $55 by 20%.
d Increase $490 by 8%.
e Increase $50 by 12%.
f Increase $7000 by 3%.
g Increase $49.50 by 14%.
h Increase $1.50 by 140%.

Hint: To increase by 5%, multiply by $100\% + 5\% = 1.05$.

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2B

Example 6 Decreasing by a given percentage

Decrease \$8900 by 7%.

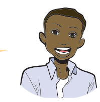
Solution	Explanation
$\$8900 \times 0.93 = \$8277.00$	$100\% - 7\% = 93\%$ Write 93% as a decimal (or fraction) and multiply by the amount. Remember to put the units in your answer.

Now you try

Decrease \$2700 by 18%.

6 **a** Decrease \$1500 by 5%. **b** Decrease \$400 by 10%.
c Decrease \$470 by 20%. **d** Decrease \$80 by 15%.
e Decrease \$550 by 25%. **f** Decrease \$49.50 by 5%.
g Decrease \$119.50 by 15%. **h** Decrease \$47.10 by 24%.

Hint: To decrease by 5%, multiply by $100\% - 5\% = 0.95$.



Example 7 Calculating profit and percentage profit

The cost price for a new car is \$24 780 and it is sold for \$27 600.
a Calculate the profit.
b Calculate the percentage profit, to two decimal places.

Solution	Explanation
a Profit = selling price − cost price	Write the rule.
$= \$27\,600 - \$24\,780$	Substitute the values and evaluate.
$= \$2820$	
b Percentage profit $= \dfrac{\text{profit}}{\text{cost price}} \times 100$	Write the rule.
$= \dfrac{2820}{24\,780} \times 100$	Substitute the values and evaluate.
$= 11.38\%$	Round your answer as instructed.

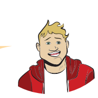
Now you try

The cost price for a new refrigerator is \$888 and it is sold for \$997.
a Calculate the profit.
b Calculate the percentage profit, to two decimal places.

7 Copy and complete the table on profits and percentage profit.

	Cost price	Selling price	Profit	Percentage profit
a	\$10	\$16		
b	\$240	\$300		
c	\$15	\$18		
d	\$250	\$257.50		
e	\$3100	\$5425		
f	\$5.50	\$6.49		

Hint: Percentage profit $= \dfrac{\text{profit}}{\text{cost price}} \times 100$



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Example 8 Finding the selling price

A retailer buys some calico material for \$43.60 a roll. He wishes to make a 35% profit.

a What will be the selling price per roll?
b If he sells 13 rolls, what profit will he make?

Solution	Explanation
a Selling price = 135% of \$43.60	For a 35% profit the unit price is 135%.
= 1.35 × \$43.60	Write 135% as a decimal (1.35) and evaluate.
= \$58.86 per roll	
b Profit per roll = \$58.86 − \$43.60	Selling price − cost price
= \$15.26	
Total profit = \$15.26 × 13	There are 13 rolls at \$15.26 profit per roll.
= \$198.38	

Now you try

A retailer buys swimsuits for \$32 per suit. She wishes to make a 30% profit.

a What will be the selling price of each swimsuit?
b If she sells 20 swimsuits, what profit will she make?

8 A retailer buys some christmas snow globes for \$41.80 each. She wishes to make a 25% profit.
a What will be the selling price per snow globe?
b If she sells a box of 25 snow globes, what profit will she make?

9 A second-hand car dealer bought a trade-in car for \$1200 and wishes to resell it for a 28% profit. What is the resale price?

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2B

Example 9 Finding the discounted price

A shirt worth $\$25$ is discounted by 15%.

a What is the selling price?

b How much is the saving?

Solution	Explanation
a Selling price $= 85\%$ of $\$25$	15% discount means there must be 85% left $(100\% - 15\%)$.
$= 0.85 \times \$25$	Convert 85% to 0.85 and multiply by the amount.
$= \$21.25$	
b Saving $= 15\%$ of $\$25$	You save 15% of the original price.
$= 0.15 \times \$25$	Convert 15% to 0.15 and multiply by the original price.
$= \$3.75$	
or saving $= \$25 - \$21.25$	Saving = original price − discounted price
$= \$3.75$	

Now you try

A suitcase worth $\$220$ is discounted by 35%.

a What is the selling price?

b How much is the saving?

10 Samantha buys a wetsuit from the sports store where she works. Its original price was $\$79.95$. Employees receive a 15% discount.

a What is the selling price?

b How much will Samantha save?

11 A travel agent offers a 12.5% discount on airfares if you travel during May or June. The normal fare to London (return trip) is $\$2446$.

a What is the selling price?

b How much is the saving?

12 A store sells second-hand goods at 40% off the recommended retail price. A lawn mower is valued at $\$369$.

a What is the selling price?

b How much would you save?

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Problem-solving and reasoning | 13–16 | 15–18

13 Ski jackets are delivered to a shop in packs of 50 for \$3500. The shop owner wishes to make a 35% profit.

a What will be the total profit made on a pack?

b What is the profit on each jacket?

14 A pair of sports shoes is discounted by 47%. The recommended price was \$179.

a What is the amount of the discount?

b What will be the discounted price?

15 Jeans are priced at a May sale for \$89. If this is a saving of 15% off the selling price, what do the jeans normally sell for?

Hint: 85% of amount = \$89. Find 1% then × 100 to find 100%.

16 Discounted tyres are reduced in price by 35%. They now sell for \$69 each. Determine:

a the normal price of one tyre

b the saving if you buy one tyre

17 The local shop purchases a carton of containers for \$54. Each container is sold for \$4. If the carton had 30 containers, determine:

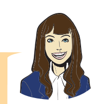
a the profit per container

b the percentage profit per container, to two decimal places

c the overall profit per carton

d the overall percentage profit, to two decimal places

18 A retailer buys a book for \$50 and wants to sell it for a 26% profit. The 10% GST must then be added onto the cost of the book.



Hint: $\% \text{ Increase} = \frac{\text{increase}}{\text{cost price}} \times 100$

a Calculate the profit on the book.

b How much GST is added to the cost of the book?

c What is the advertised price of the book, including the GST?

d Find the overall percentage increase of the final selling price compared to the \$50 cost price.

Building a gazebo | — | 19

19 Christopher designs a gazebo for a new house. He buys the timber from a retailer, who sources it at wholesale price and then marks it up before selling to Christopher at retail price. The table below shows the wholesale prices as well as the mark-up for each type of timber.

Quantity	Description	Cost/unit	Mark-up
6	Treated pine posts	\$23	20%
11	300 × 50 oregon beams	\$75	10%
5	Sheet lattice work	\$86	15%
2	300 × 25 oregon fascias	\$46	12%
8	Laserlite sheets	\$32	10%

a Determine Christopher's overall cost for the material, including the mark-up.

b Determine the profit made by the retailer.

c Determine the retailer's overall percentage profit, to two decimal places.

d If the retailer pays 27% of his profits in tax, how much tax does he pay on this sale?

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2C Income

Learning intentions

- To understand a range of different ways in which employees can be paid
- To know how net income is calculated from gross income and deductions
- To be able to calculate wages for overtime and shift work
- To be able to calculate commission

Key vocabulary: wages, commission, salary, fees, gross income, overtime, deductions, net income, time and a half, double time, deductions

You may have earned money for baby-sitting or delivering newspapers or have a part-time job. As you move into the workforce it is important that you understand how you are paid.

Lesson starter: Who earns what?

As a class, discuss the different types of jobs held by different members of each person's family, and discuss how they are paid.

- What are the different ways that people can be paid?
- What does it mean when you work fewer than full-time hours?
- What does it mean when you work longer than full-time hours?

What other types of income can people in the class think of?

Key ideas

Methods of payment

- Hourly **wages**: You are paid a certain amount per hour worked.
- **Commission**: You are paid a percentage of the total amount of sales.
- **Salary**: You are paid a set amount per year, regardless of how many hours you work.
- **Fees**: You are paid according to the charges you set; e.g. doctors, lawyers, contractors.
- Some terms you should be familiar with include:
  - **Gross income**: the total amount of money you earn before taxes and other deductions
  - **Deductions**: money taken from your income before you are paid; e.g. taxation, union fees, superannuation
  - **Net income**: the amount of money you actually receive after the deductions are taken from your gross income

    Net income = gross income − deductions

Payments by hourly rate

- If you are paid by the hour you will be paid an amount per hour for your normal working time. If you work **overtime** (hours beyond the normal working hours), the rates may be different.

  Usually, normal working time is 38 hours per week.

  Normal: $1.0 \times$ normal rate

  Time and a half: $1.5 \times$ normal rate

  Double time: $2.0 \times$ normal rate

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- If you work shift work the hourly rates may differ from shift to shift.

  For example:

6 a.m.–2 p.m.	$24.00/hour	(regular rate)
2 p.m.–10 p.m.	$27.30/hour	(afternoon shift rate)
10 p.m.–6 a.m.	$36.80/hour	(night shift rate)

Exercise 2C

Understanding 1–3 3

1 Match the job description on the left with the method of payment on the right.

a Jennie is paid $85 600 per year — **A** hourly wage
b Danielle earns 3% of all the sales she makes — **B** fee
c Jett earns $18.90 per hour worked — **C** commission
d Stuart charges $450 for a consultation — **D** salary

2 Callum earns $1090 a week and has annual deductions of $19 838. What is Callum's net income for the year? Assume 52 weeks in a year.

Hint: Net = total − deductions

3 If Tao earns $15.20 per hour, calculate his:

a time-and-a-half rate **b** double-time rate

Hint: Time-and-a-half rate $= 1.5 \times$ hourly rate

Fluency 4–8 5–9

Example 10 Finding gross and net income (including overtime)

Pauline is paid $13.20 per hour at the local stockyard to muck out the stalls. Her normal hours of work are 38 hours per week. She receives time and a half for the next 4 hours worked and double time after that.

a What will be her gross income if she works 50 hours?

b If she pays $220 per week in taxation and $4.75 in union fees, what will be her weekly net income?

Solution	Explanation
a Gross income $= 38 \times \$13.20$	Normal 38 hours
$+4 \times 1.5 \times \$13.20$	Overtime rate for next 4 hours: time and a half $= 1.5 \times$ normal
$+8 \times 2 \times \$13.20$	
$= \$792$	Overtime rate for next 8 hours: double time $= 2 \times$ normal
b Net income $= \$792 - (\$220 + \$4.75)$	Net income = gross income − deductions
$= \$567.25$	

Now you try

Toby is paid $17.50 per hour at his supermarket job. His normal hours of work are 38 hours per week. He receives time and a half for the next 6 hours worked and double time after that.

a What will be his gross income if he works 48 hours in a week?

b If he pays $240 per week in taxation and $6.50 in union fees, what will be his weekly net income?

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2C

4 Jack is paid $14.70 per hour. His normal hours of work are 38 hours per week. He receives time and a half for the next 2 hours worked and double time after that.

a What will be his gross income if he works 43 hours?

b If he has $207.20 of deductions, what will be his weekly net income?

5 Copy and complete this table.

	Hourly rate	Normal hours worked	Time and a half hours	Double time hours	Gross income	Deductions	Net income
a	$15	38	0	0		$155	
b	$24	38	2	0		$220	
c	$13.15	38	4	1		$300	
d	$70	40	2	3		$510	
e	$17.55	35	4	6		$184	

Example 11 Calculating shift work

Michael is a shift worker and is paid $31.80 per hour for the morning shift, $37.02 per hour for the afternoon shift and $50.34 per hour for the night shift. Each shift is 8 hours. In a given fortnight he works four morning, two afternoon and three night shifts. Calculate his gross income.

Solution	Explanation
Gross income $= 4 \times 31.80 \times 8$	4 morning shifts at $31.80 per hour for 8 hours
$+ 2 \times 37.02 \times 8$	2 afternoon shifts at $37.02 per hour for 8 hours
$+ 3 \times 50.34 \times 8$	3 night shifts at $50.34 per hour for 8 hours
$= \$2818.08$	Gross income because tax has not been paid.

Now you try

Kate is a shift worker and is paid $26.20 per hour for the morning shift, $32.40 per hour for the afternoon shift and $54.25 per hour for the night shift. Each shift is 8 hours. In a given fortnight she works five morning, three afternoon and two night shifts. Calculate her gross income.

6 Greg works shifts at a processing plant. In a given rostered fortnight he works:

- 3 day shifts ($31.80 per hour)
- 4 afternoon shifts ($37.02 per hour)
- 4 night shifts ($50.34 per hour).

a If each shift is 8 hours long, determine Greg's gross income for the fortnight.

b If the answer to part **a** is Greg's average fortnightly income, what will be his gross income for a year (i.e. 52 weeks)?

Hint: A fortnight = 2 weeks

Many hospital workers work shift work.

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Example 12 Calculating income involving commission

Jeff sells memberships to a gym and receives $225 per week plus 5.5% commission on his sales. Calculate his gross income after a 5-day week.

Day	1	2	3	4	5
Sales ($)	680	450	925	1200	1375

Solution	**Explanation**
Total sales = $4630	Determine the total sales: 680 + 450 + 925 + 1200 + 1375.
Commission = 5.5% of $4630 = 0.055 × $4630 = $254.65	Determine the commission on the total sales at 5.5% by multiplying 0.055 by the total sales.
Gross income = $225 + $254.65 = $479.65	Gross income is $225 plus commission.

Now you try

Jin sells vacuum cleaners and receives $250 per week plus 4.3% commission on her sales. Calculate her gross income after a 5-day week.

Day	1	2	3	4	5
Sales ($)	1230	690	1422	1590	2648

7 A car salesman earns $5000 a month plus 3.5% commission on all sales. In the month of January his sales total was $56 000. Calculate:

a his commission for January
b his gross income for January

8 A real estate agent receives 2.75% commission on the sale of a house valued at $1 250 000. Find the commission earned.

9 Sarah earns an annual salary of $77 000 plus 2% commission on all sales. Find:

a her weekly base salary before sales
b her commission for a week when her sales totalled $7500
c her gross weekly income for the week in part **b**
d her annual gross income if over the year her sales totalled $571 250

Problem-solving and reasoning — 10, 11 — 10–12

10 If Simone receives $10 000 on the sale of a property worth $800 000, calculate her rate of commission.

Hint: What percentage of $800 000 is $10 000?

11 Jonah earns a commission on his sales of fashion items. For goods to the value of $2000 he receives 6% and for sales over $2000 he receives 9% on the amount in excess of $2000. In a given week he sold $4730 worth of goods. Find the commission earned.

12 William earns 1.75% commission on all sales at the electrical goods store where he works. If William earns $35 in commission on the sale of one television, how much did the TV sell for?

Hint: 1.75% is $35. Find 1%, then 100%.

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2C

Elmo's payslip — 13

13 Refer to the payslip below to answer the following questions.

Kuger Incorporated			
Employee ID: 75403A			Page: 1
Name: Elmo Rodriguez			Pay Period: 21/05/2016
Pay Method: EFT		Tax Status: Gen Exempt	
Bank account name: E. Rodriguez			
Bank: Mathsville Credit Union			
BSB: 102-196 Account No: 00754031			
Payment Details this pay:			
Amount	Days	Payment Description	Rate/Frequency
2 777.15	14.00	Normal time	$72 454/annum
Before tax deductions:			
This pay		Description	
170		Salary sacrifice: car pre-tax deduction	
Miscellaneous deductions:			
This pay		Description	
52.90		Health fund	
23.10		Union fees	
76.00			
Reconciliation details:			
This pay	YTD	Description	
2 607.15	62 571.60	Taxable gross pay	
616.00	14 784.00	less income tax	
76.00	1 824.00	less miscellaneous deductions	
1 915.15	45 963.60		

a Which company does Elmo work for?

b What is the name of Elmo's bank and what is his account number?

c How much gross pay does Elmo earn in 1 year?

d How often does Elmo get paid?

e How much, per year, does Elmo salary sacrifice?

f How much is Elmo's health fund contribution each week?

g Calculate 1 year's union fees.

h Using the information on this payslip, calculate Elmo's annual tax and also his annual net income.

i If Elmo works Monday to Friday from 9 a.m. to 5 p.m. each day for an entire year, calculate his effective hourly rate of pay. Use Elmo's fortnightly payment as a starting point.

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2D Income taxation ★

Learning intentions

- To understand how the key components of the Australian taxation system work
- To be able to calculate a person's taxable income
- To be able to calculate a person's tax payable using Australian tax brackets

Key vocabulary: taxation, employer, employee, tax return, taxable income, tax bracket, levy, deductions, p.a. (per annum)

It has been said that there are only two sure things in life: death and taxes! The Australian Taxation Office (ATO) collects taxes on behalf of the government to pay for education, hospitals, roads, railways, airports and services, such as the police and fire brigades.

In Australia, the financial year runs from July 1 to June 30 the following year. People engaged in paid employment are normally paid weekly or fortnightly. Most of them pay some income tax every time they are paid for their work. This is known as the Pay-As-You-Go system (PAYG).

At the end of the financial year (June 30), people who earned an income complete an income tax return to determine if they have paid the correct amount of income tax during the year.
If they paid too much, they will receive a refund. If they did not pay enough, they will be required to pay more.

The Australian tax system is very complex and the laws change frequently. This section covers the main aspects only.

Lesson starter: The ATO website

The Australian Taxation Office website has some income tax calculators. Use one to find out how much income tax you would need to pay if your taxable income is:

$5200 per annum (i.e. $100 per week)

$10 400 per annum (i.e. $200 per week)

$15 600 per annum (i.e. $300 per week)

$20 800 per annum (i.e. $400 per week)

$26 000 per annum (i.e. $500 per week)

Does a person earning $1000 per week pay twice as much tax as a person earning $500 per week?

Does a person earning $2000 per week pay twice as much tax as a person earning $1000 per week?

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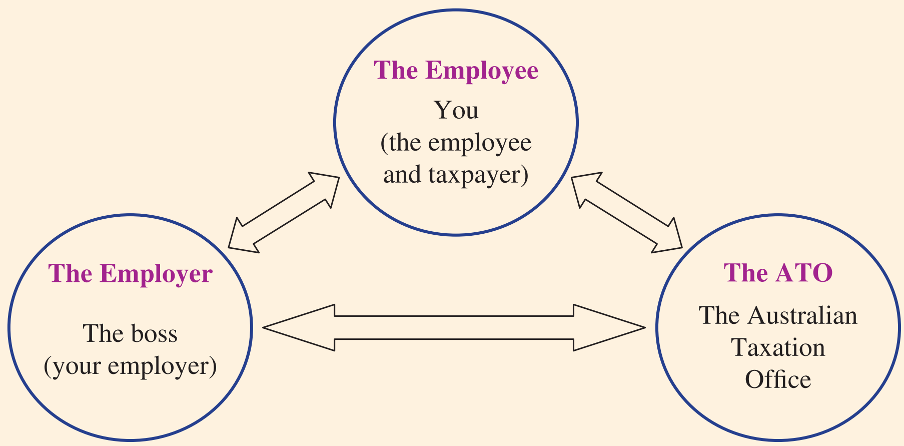
Key ideas



- The PAYG tax system works in the following way.
  - The employee works for and gets paid by the employer every week, fortnight or month.
  - The employer calculates the tax that the employee should pay for the amount earned by the employee.
  - The employer sends that tax to the ATO every time the employee gets paid.
  - The ATO passes the income tax to the federal government.
  - On June 30, the employer gives the employee a payment summary to confirm the amount of tax that has been paid to the ATO on behalf of the employee.
  - Between July 1 and October 31, the employee completes a **tax return** and sends it to the ATO. Some people pay a registered tax agent to do this return for them.
  - On this tax return, the employee lists the following.
    - *All* forms of income, including interest from investments.
    - Legitimate deductions shown on receipts and invoices, such as work-related expenses and donations.
  - **Taxable income** is calculated using the formula:
    Taxable income = gross income − deductions
  - There are tables and calculators on the ATO website, such as the following.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

    This table can be used to calculate the amount of tax you *should have* paid (i.e. the tax payable), as opposed to the tax you *did* pay during the year (i.e. the tax withheld). Each row in the table is called a **tax bracket**.
  - You may also need to pay the Medicare **levy**. This is a scheme in which all Australian taxpayers share in the cost of running the medical system. For many people this is currently 2% of their taxable income.
  - It is possible that you may have paid too much tax during the year and will receive a tax refund.
  - It is also possible that you may have paid too little tax and will receive a letter from the ATO asking for the tax liability to be paid.

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Exercise 2D

Understanding 1–3 2, 3

Note: The questions in this exercise relate to the tax table given in Key ideas, unless stated otherwise.

1 Complete this statement: Taxable income = ____________ income minus ____________.

2 Based on the table in the key ideas, determine if the following statements are true or false?
  a A taxable income of $10 400 requires no tax to be paid.
  b The highest income earners in Australia pay 45 cents tax for every dollar they earn.

3 In the 2019/2020 financial year, Ann's taxable income was $80 000, which puts her at the very top of the middle tax bracket in the tax table. Ben's taxable income was $80 001, which puts him in a higher tax bracket. Ignoring the Medicare levy, how much extra tax did Ben pay compared to Ann?

Fluency 4, 5 4–6

Example 13 Calculating income tax payable

During the 2019/2020 financial year, Richard earned $1050 per week ($54 600 per annum) from his employer and other sources, such as interest on investments. He has receipts for $375 for work-related expenses and donations.

a Calculate Richard's taxable income.

b Use this tax table to calculate Richard's tax payable amount.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

c Richard must also pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy amounts.

e Express the total tax in part **d** as a percentage of Richard's taxable income, to one decimal place.

f During the financial year, Richard's employer sent a total of $7797 in tax to the ATO. Has Richard paid too much tax or not enough? Calculate his refund or liability.

Solution	Explanation
a Gross income = $54 600 Deductions = $375 Taxable income = $54 225	Taxable income = gross income – deductions
b Tax payable: $\$3572 + 0.325 \times (\$54\,225 - \$37\,000)$ $= \$9170.13$	Richard is in the middle tax bracket in the table, in which it says: $3572 plus 32.5c for each $1 over $37 000 Note: 32.5 cents is $0.325.
c $\frac{2}{100} \times 54\,225 = \$1084.50$	Medicare levy is 2% of the taxable income.

Continued on next page

ISBN 978-1-108-87859-3

2D

d $\$9170.13 + \$1084.50 = \$10\,254.63$

This is the total amount of tax that Richard should have paid.

e $\frac{10\,254.63}{54\,225} \times 100 = 18.9\%$ (to 1 d.p.)

This implies that Richard paid approximately 18.9% tax on every dollar. This is sometimes read as '18.9 cents in the dollar'.

f Richard paid \$7797 in tax during the year. He should have paid \$10 254.63. Richard has not paid enough tax. He must pay another \$2457.63 in tax.

This is known as a shortfall or a liability. He will receive a letter from the ATO requesting payment of the difference.
$\$10\,254.63 - \$7797 = \$2457.63$

Now you try

During the 2019/2020 financial year, Francesca earned \$82 300 per annum from her employer and other sources, such as interest on investments. She has receipts for \$530 for work-related expenses and donations.

a Calculate Francesca's taxable income.
b Use the tax table from the Key ideas to calculate Francesca's tax payable amount.
c Francesca must also pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy amounts.
e Express the total tax in part **d** as a percentage of Francesca's taxable income, to one decimal place.
f During the financial year, Francesca's employer sent a total of \$17 445 in tax to the ATO. Has Francesca paid too much tax or not enough? Calculate her refund or liability.

4 During the 2019/2020 financial year, Liam earned \$94 220 per annum from his employer and other sources, such as interest on investments. He has receipts for \$615 for work-related expenses and donations.
a Calculate Liam's taxable income.
b Use the tax table from the Key ideas to calculate Liam's tax payable amount.
c Liam must also pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy amounts.
e Express the total tax in part **d** as a percentage of Liam's taxable income, to one decimal place.
f During the financial year, Liam's employer sent a total of \$25 249 in tax to the ATO. Has Liam paid too much tax or not enough? Calculate his refund or liability.

5 Use the tax table in the key ideas to calculate the income tax payable on these taxable incomes.
a \$30 000 **b** \$60 000 **c** \$150 000 **d** \$200 000

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6 Lee has come to the end of her first financial year employed as a website developer. On June 30 she made the following notes about the financial year.

Gross income from employer	\$58 725
Gross income from casual job	\$7500
Interest on investments	\$75
Donations	\$250
Work-related expenses	\$425
Tax paid during the financial year	\$13 070

Hint: Taxable income = all incomes − deductions

- **a** Calculate Lee's taxable income.
- **b** Use the tax table shown in Example 13 to calculate Lee's tax payable amount.
- **c** Lee must also pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?
- **d** Add the tax payable and the Medicare levy.
- **e** Express the total tax in part **d** as a percentage of Lee's taxable income, to one decimal place.
- **f** Has Lee paid too much tax or not enough? Calculate her refund or liability.

Problem-solving and reasoning

7, 8, 10, 11 | 7, 9, 11–13

7 Alec's Medicare levy is \$1750. This is 2% of his taxable income. What is Alec's taxable income?

8 Tara is saving for an overseas trip. Her taxable income is usually about \$20 000. She estimates that she will need \$5000 for the trip, so she is going to do some extra work to raise the money. How much extra will Tara need to earn in order to save the extra \$5000 after tax?

9 When Saled used the tax table to calculate his income tax payable, it turned out to be \$23 097. What is his taxable income?

Hint: Use the tax table given in Example 13 to determine in which tax bracket Saled falls.

10 Explain the difference between a tax refund and a tax liability.

11 Gordana looked at the last row of the tax table and said, 'It is so unfair that people in that tax bracket must pay 45 cents in every dollar in tax.' Explain why Gordana is incorrect.

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2D

12 The most recent significant change to Australian income tax rates was first applied in the 2012/2013 financial year. Consider the tax tables for the two consecutive financial years 2011/2012 and 2012/2013. Note that the amounts listed first in each table are often called the tax-free threshold (i.e. the amount that a person can earn before they must pay tax).

2011/2012	
Taxable income	**Tax on this income**
0 – $6000	Nil
$6001 – $37 000	15c for each $1 over $6000
$37 001 – $80 000	$4650 plus 30c for each $1 over $37 000
$80 001 – $180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000
2012/2013	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

a There are some significant changes between the financial years 2011/2012 and 2012/2013. Describe three of them.

b The following people had the same taxable income during both financial years. Find the difference in their tax payable amounts and state whether they were advantaged or disadvantaged by the changes, or not affected at all?

i Ali: Taxable income = $5000
ii Xi: Taxable income = $15 000
iii Charlotte: Taxable income = $30 000
iv Diego: Taxable income = $50 000

13 Below is the tax table for people who are not residents of Australia but are working in Australia.

Taxable income	Tax on this income
$1 – $90 000	32.5c for each $1
$90 001 – $180 000	$29 250 plus 37c for each $1 over $90 000
$180 001 and over	$62 550 plus 45c for each $1 over $180 000

Compare this table to the one in the example for Australian residents.
What difference would it make to the amount of tax paid by these people if they were non-residents rather than residents?

a Ali: Taxable income = $5000
b Xi: Taxable income = $15 000
c Charlotte: Taxable income = $30 000
d Diego: Taxable income = $100 000

What are legitimate tax deductions? — 14

14 **a** Choose an occupation or career in which you are interested. Imagine that you are working in that job. During the year you will need to keep receipts for items you have bought that are legitimate work-related expenses. Do some research on the internet and write down some of the things that you will be able to claim as work-related expenses in your chosen occupation.

b **i** Imagine your taxable income is $80 000. What is your tax payable amount?
ii You just found a receipt for a $100 donation to a registered charity. This decreases your taxable income by $100. By how much does it decrease your tax payable amount?

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2E Budgeting

Learning intentions

- To know the types of expenses that are included in a budget
- To understand how a budget is affected by fixed and variable expenses
- To be able to calculate savings and other expenses based on the information in a budget
- To be able to calculate the best buy (cheapest deal) from a range of options

Key vocabulary: budget, fixed expenses, variable expenses

Once people have been paid their income for the week, fortnight or month, they must plan how to spend it. Most families work on a budget, allocating money for fixed expenses such as the mortgage or rent and the varying (i.e. changing) expenses of petrol, food and clothing.

Lesson starter: Expenses for the month

Write down everything that you think your family would spend money on for the week and the month, and estimate how much those things might cost for the entire year. Where do you think savings could be made? What would be some additional annual expenses?

Key ideas

- A **budget** is an estimate of income and expenses for a period of time.
- Managing money for an individual is similar to operating a small business. Expenses can be divided into two areas:
  - **Fixed expenses** (these do not change during a time period): payment of loans, mortgages, regular bills etc.
  - **Variable expenses** (these costs change over a time period): clothing, entertainment, food etc. (these are estimates)
- When your budget is completed you should always check that your figures are reasonable estimates.
- By looking at the budget you should be able to see how much money is remaining; this can be used as savings or to buy non-essential items.

Exercise 2E

Understanding 1–3 3

1 Classify each expense listed below as most likely a fixed expense or a variable expense.
   a monthly rent
   b monthly phone bill payment plan
   c take away food
   d stationery supplies for work

2 Binh has an income of \$956 a week. His expenses, both fixed and variable, total \$831.72 of his income. How much money can Binh save each week?

3 Roslyn has the following monthly expenses. Mortgage = \$1458, mobile phone = \$49, internet = \$60, council rates = \$350, water = \$55, electricity = \$190. What is the total of Roslyn's monthly expenses?

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2E

Fluency

4–7 | 4, 5, 7, 8

Example 14 Budgeting using percentages

Fiona has a net annual income of $36 000 after deductions. She allocates her budget on a percentage basis.

	Mortgage	Car loan	Food	Education	Sundries	Savings
Expenses (%)	20	15	25	20	10	10

a Determine the amount of fixed expenses, including the mortgage, car loan and education.
b How much should Fiona save?
c Is the amount allocated for food reasonable?

Solution

a Fixed expenses $= 55\%$ of $\$36\,000$
$= 0.55 \times \$36\,000$
$= \$19\,800$

b Savings $= 10\%$ of $\$36\,000$
$= 0.1 \times \$36\,000$
$= \$3600$

c Food $= 25\%$ of $\$36\,000$
$= 0.25 \times \$36\,000$
$= \$9000$ per year, or
$\$173$ per week

This seems reasonable.

Explanation

a The mortgage, car loan and education are 55% in total.
Change 55% to a decimal and multiply by the net income.

b Savings are 10% of the budget.
Change 10% to a decimal and multiply by the net income.

c Food is 25% of the budget.
Change 25% to a decimal and calculate.
Divide the yearly expenditure by 52 to make a decision on the reasonableness of your answer.

Now you try

Kyle has a net annual income of $64 200 after deductions. He allocates his budget on a percentage basis.

	Rent	Food	Entertainment	Bills	Transport	Sundries	Savings
Expenses (%)	30	10	10	15	5	10	20

a Determine the amount of fixed expenses including the rent and bills.
b How much should Kyle save?
c Is the amount allocated for transport reasonable?

4 Paul has an annual income of $75 000 after deductions. He allocates his budget on a percentage basis.

	Mortgage	Car loan	Personal loan	Clothing	Food	Other
Expenses (%)	20	10	25	5	10	30

a Determine the amount of fixed expenses, including the mortgage and loans.
b How much should Paul have left over after paying for his mortgage, car loan and personal loan?
c Is the amount allocated for food reasonable?

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5 Lachlan has an income of \$2120 per month. If he budgets 5% for clothes, how much will he actually have to spend on clothes each month?

Example 15 Budgeting using fixed values

Running a certain type of car involves yearly, monthly and weekly expenditure. Consider the following vehicle's costs.

- lease — \$210 per month
- registration — \$475 per year
- insurance — \$145 per quarter
- servicing — \$1800 per year
- petrol — \$37 per week

a Determine the overall cost to run this car for a year.

b What percentage of a \$70 000 salary would this be, correct to one decimal place?

Solution	Explanation
a Overall cost = 210×12	Leasing cost: 12 months in a year
$+ \quad 475$	Registration cost
$+ \quad 145 \times 4$	Insurance cost: 4 quarters in a year
$+ \quad 1800$	Servicing cost
$+ \quad 37 \times 52$	Petrol cost: 52 weeks in a year
$= \quad \$7299$	The overall cost is found by adding the individual totals.
The overall cost to run the car is \$7299.	
b $\%$ of salary $= \dfrac{7299}{70\,000} \times 100$	Percentage $= \dfrac{\text{car cost}}{\text{total salary}} \times 100$
$= 10.4\%$ (to 1 d.p.)	Round as required.

Now you try

Running a boat involves yearly, monthly and weekly expenditure. Consider the following boat's costs.

- registration — \$342 per year
- insurance — \$120 per quarter
- servicing — \$360 per year
- fuel — \$300 per month
- storing boat — \$2400 per year

a Determine the overall cost to run this boat for a year.

b What percentage of a \$82 000 salary would this be, correct to one decimal place?

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2E

6 Eliana is a student and has the following expenses in her budget.

- rent \$270 per week
- electricity \$550 per quarter
- phone and internet \$109 per month
- car \$90 per week
- food \$170 per week
- insurance \$2000 a year

Hint: Use 52 weeks in a year, 12 months in a year and 4 quarters in a year.

a Determine Eliana's costs for a year.
b What percentage of Eliana's net annual salary of \$45 000 would this be, correct to one decimal place?

7 The costs of sending a student to Modkin Private College are as follows.

- fees per term (4 terms) \$1270
- subject levies per year \$489
- building fund per week \$35
- uniforms and books per year \$367

a Determine the overall cost per year.
b If the school bills twice a year, covering all the items above, what would be the amount of each payment?
c How much should be saved per week to make the biannual payments?

8 A small business owner has the following expenses to budget for.

- rent \$1400 a month
- phone line \$59 a month
- wages \$1200 a week
- electricity \$430 a quarter
- water \$120 a quarter
- insurance \$50 a month

a What is the annual budget for the small business?
b How much does the business owner need to make each week just to break even?
c If the business earns \$5000 a week, what percentage of this needs to be spent on wages?

Problem-solving and reasoning | 9, 10, 12–14 | 10–12, 14, 15

9 Francine's petrol budget is \$47 from her weekly income of \$350.
a What percentage of her budget is this? Give your answer to two decimal places.
b If petrol costs \$1.59 per litre, how many litres of petrol, correct to two decimal places, is Francine budgeting for in a week?

10 Grant works a 34-hour week at \$15.50 per hour. His net income is 65% of his gross income.
a Determine his net weekly income.
b If Grant spends 12% of his net income on entertainment, determine the amount he actually spends per year on entertainment.
c Grant saves \$40 per week. What percentage of his net income is this (to two decimal places)?

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11 Dario earns \$432 per fortnight at a take-away pizza shop. He budgets 20% for food, 10% for recreation, 13% for transport, 20% for savings, 25% for taxation and 12% for clothing.

a Determine the actual amount budgeted for each category every fortnight.

Dario's wage increases by 30%.

b Determine how much he would now save each week.

c What percentage increase is the answer to part **c** on the original amount saved?

d Determine the extra amount of money Dario saves per year after his wage increase.

e If transport is a fixed expense, its percentage of Dario's budget will change. Determine the new percentage.

Example 16 Calculating best buys

Soft drink is sold in three convenient packs at the local store.

- carton of 36 (375 mL) cans at \$22.50
- a six-pack of (375 mL) cans at \$5.00
- 2-litre bottles at \$2.80

Determine the cheapest way to buy the soft drink.

Solution	Explanation
Buying by the carton:	
Cost = \$22.50 ÷ (36 × 375)	Total mL = 36 × 375
= \$0.0017 per mL	Divide to work out the cost per mL.
Buying by the six-pack:	
Cost = \$5 ÷ (6 × 375)	Total mL = 6 × 375
= \$0.0022 per mL	
Buying by the bottle:	
Cost = \$2.80 ÷ 2000	Total mL = 2 × 1000, since 1 L = 1000 mL.
= \$0.0014 per mL	
∴ The cheapest way to buy the soft drink is to buy the 2-litre bottle.	Compare the three costs per mL.

Now you try

A brand of toilet rolls are sold in three pack types at the supermarket.

- a pack of 18 rolls for \$8.82
- a pack of 6 rolls for \$3.30
- a pack of 4 double length rolls for \$3.68

Determine the cheapest way to buy the toilet rolls.

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2E

12 Tea bags can be purchased from the supermarket in three forms.
- 25 tea bags at \$2.36
- 50 bags at \$4.80
- 200 bags at \$15.00

What is the cheapest way to buy tea bags?

13 A weekly train concession ticket costs \$16. A day ticket costs \$3.60. If you are going to school only 4 days next week, is it cheaper to buy one ticket per day or a weekly ticket?

14 A holiday caravan park offers its cabins at the following rates.
- \$87 per night (Sunday–Thursday)
- \$187 for a weekend (Friday and Saturday)
- \$500 per week

a Determine the nightly rate in each case.
b Which price is the best value?

15 Tomato sauce is priced at:
- 200 mL bottle \$2.35
- 500 mL bottle \$5.24

a Find the cost per mL of the tomato sauce in each case.
b Which is the cheapest way to buy tomato sauce?
c What would be the cost of 200 mL at the 500 mL rate?
d How much would be saved by buying the 200 mL bottle at this rate?
e Suggest why the 200 mL bottle is not sold at this price.

Minimum cost of tennis balls — 16

16 Safeserve has a sale on tennis balls for one month.
When you buy:
- 1 container, it costs \$5
- 6 containers, it costs \$28
- 12 containers, it costs \$40
- 24 containers, it costs \$60

You need 90 containers for your club to have enough for a season.

a Determine the minimum cost if you buy exactly 90 containers.
b Determine the overall minimum cost, and the number of extra containers you will have in this situation.

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2F Simple interest

Learning intentions

- To understand how simple interest is calculated
- To be able to calculate interest using the simple interest formula
- To be able to determine the rate of interest based on the interest earned
- To be able to calculate the amount owing on a loan and calculate repayments

Key vocabulary: principal, rate of interest, simple interest, annual, invest, borrow

Borrowed or invested money usually has an associated interest rate. The consumer needs to establish the type of interest they are paying and the effects it has on the amount borrowed or invested over time. Some loans or investments deliver the full amount of interest using only the initial loan or investment amount in the interest calculations. These types are said to use simple interest.

Lesson starter: How long to invest?

Marcus and Brittney each have $\$200$ in their bank accounts. Marcus earns $\$10$ a year in interest. Brittney earns 10% p.a. simple interest.

For how long must each of them invest their money for it to double in value?

Key ideas

- **Simple interest** is a type of interest that is calculated on the amount **invested** or **borrowed**.
- The terms needed to understand simple interest are:
  - **Principal (P)**: the amount of money borrowed or invested
  - **Rate of interest (r)**: the **annual** (yearly) percentage rate of interest (e.g. 3% p.a.)
  - Time (t): the number of years for which the principal is borrowed or invested
  - Interest (I): the amount of interest accrued over a given time.
- The formula for calculating simple interest is:

  $$I = \text{principal} \times \text{rate} \times \text{time}$$

  $$I = \frac{Prt}{100} \text{ (Since the rate is a percentage)}$$

- Total repaid = amount borrowed + interest

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Exercise 2F

Understanding — 1, 2 | 2

1 In the formula $I = \frac{Prt}{100}$:

a I is the ________

b P is the ________

c r is the ________

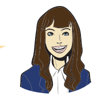
d t is the ________.

2 Calculate interest earned (I) if:

a $P = 1000,\ r = 4,\ t = 5$

b $P = 2500,\ r = 3,\ t = 4$

Hint: Use $I = \frac{Prt}{100}$



Fluency — 3–6 | 4–7

Example 17 Using the simple interest formula

Use the simple interest formula, $I = \frac{Prt}{100}$, to find:

a the interest (I) when \$600 is invested at 8% p.a. for 18 months

b the annual interest rate (r) when \$5000 earns \$150 interest in 2 years

Solution	Explanation
a $P = 600$ $r = 8$ $t = 18 \text{ months} = \frac{18}{12} = 1.5 \text{ years}$	Write out the information that you know and the formula.
$I = \frac{Prt}{100}$ $= \frac{600 \times 8 \times 1.5}{100}$ $= 72$ The interest is \$72 in 18 months.	Substitute into the formula using years for t.
b $P = 5000$ $I = 150$ $t = 2 \text{ years}$ $I = \frac{Prt}{100}$ $150 = \frac{5000 \times r \times 2}{100}$ $150 = 100 \times r$ $r = 1.5$	Write the formula and the information known. Substitute the values into the formula and solve the equation to find r. $\frac{50\cancel{00} \times r \times 2}{1\cancel{00}} = 100r$
The simple interest rate is 1.5% per year.	Write the rate as a percentage.

Now you try

Use the simple interest formula, $I = \frac{Prt}{100}$, to find:

a the interest (I) when \$450 is invested at 5% p.a. for 30 months

b the annual interest rate (r) when \$3500 earns \$210 interest in 3 years

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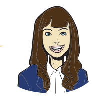
3 Use the simple interest formula, $I = \frac{Prt}{100}$, to find:

a the interest (I) when \$500 is invested at 6% p.a. for 24 months

b the annual interest rate (r) when \$3000 earns \$270 interest in 3 years

4 Copy and complete this table of values for I, P, r and t.

Hint: Use $I = \frac{Prt}{100}$



	P	Rate	Time	I
a	\$700	5% p.a.	4 years	
b	\$2000	7% p.a.	3 years	
c	\$3500	3% p.a.	22 months	
d	\$750	$2\frac{1}{2}$% p.a.	30 months	
e	\$22 500		3 years	\$2025
f	\$1770		5 years	\$354

Example 18 Calculating repayments with simple interest

\$3000 is borrowed at 12% p.a. simple interest for 2 years.

a What is the total amount owed over the 2 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Solution	Explanation
a $P = \$3000,\ r = 12,\ t = 2$	List the information you know.
$I = \frac{Prt}{100}$	Write the formula.
$= \frac{3000 \times 12 \times 2}{100}$	Substitute the values and evaluate.
$= \$720$	
Total amount $= \$3000 + \$720$ $= \$3720$	Total amount is the original amount *plus* the interest.
b Amount of each payment $= \$3720 \div 24$	2 years = 24 months
$= \$155$ per month	There are 24 payments to be made. Divide the total by 24.

Now you try

\$5400 is borrowed at 9% p.a. simple interest for 4 years.

a What is the total amount owed over the 4 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

5 \$5000 is borrowed at 11% p.a. simple interest for 3 years.

a What is the total amount owed over the 3 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Hint: Calculate the interest first.

6 Under hire purchase, John bought a second-hand car for \$11 500. He paid no deposit and decided to pay the loan off in 7 years. If the simple interest is 6.45%, determine:

a the total interest paid

b the total amount of the repayment

c the payments per month

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2F

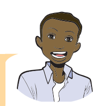
7 \$10 000 is borrowed to buy a second-hand BMW. The interest is calculated at a simple interest rate of 19% p.a. over 4 years.
   a What is the total interest on the loan?
   b How much is to be repaid?
   c What is the monthly repayment on this loan?

Problem-solving and reasoning | 8–10 | 10–13

8 How much interest will Giorgio receive if he invests \$7000 in stocks at 3.6% p.a. simple interest for 4 years?

9 Rebecca invests \$4000 for 3 years at 5.7% p.a. simple interest paid yearly.
   a How much interest will she receive in the first year?
   b What is the total amount of interest Rebecca will receive over the 3 years?
   c How much money will Rebecca have after the 3-year investment?

10 An investment of \$15 000 receives an interest payment over 3 years of \$7200. What was the rate of simple interest per annum?



Hint: Substitute into the formula $I = \frac{Prt}{100}$ and solve the resulting equation.

11 Jonathon wishes to invest \$3000 at 8% per annum. How long will he need to invest for his total investment to double?

12 Ivan wishes to invest some money for 5 years at 4.5% p.a. paid yearly. If he wishes to receive \$3000 in interest payments per year, how much should he invest? Round your answer to the nearest dollar.

13 Gretta's interest payment on her loan totalled \$1875. If the interest rate was 5% p.a. and the loan had a life of 5 years, what amount did she borrow?

Which way is best? | — | 14

14 A shed manufacturer offers finance with a rate of 3.5% p.a. paid at the end of 5 years with a deposit of 10%, or a rate of 6.4% repaid over 3 years with a deposit of 20%.
Melania and Donald decide to purchase a fully erected four-square shed for \$12 500.
   a How much deposit will they need to pay in each case?
   b What is the total interest they will incur in each case?
   c If they decided to pay per month, what would be their monthly repayment?
   d Discuss the benefits of the different types of purchasing methods.

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Progress quiz

2A

1 Express:

a 32% as a decimal

b 8% as a simplified fraction

c $\frac{11}{25}$ as a percentage

d $\frac{5}{16}$ as a percentage

e 0.252 as a percentage

f $15\frac{1}{2}\%$ as a fraction

2A

2 Gina puts 36% of her \$6000 monthly salary in a savings account. How much does she have left over?

2B

3 Complete the following.

a Increase \$230 by 24%

b Increase 180 mL by 8%

c Decrease 156 cm by 15%

2B

4 A \$299 coffee machine is discounted by 35%. What is the discounted price?

2B

5 An illegal scalper buys a concert ticket for \$150 and sells it for \$210. What is the percentage profit?

2C

6 Find the gross income for a particular week in the following work situations.

a Pippa is a door-to-door sales representative for an air conditioning company. She earns \$300 per week plus 8% commission on her sales. In a particular week she makes \$8200 worth of sales.

b Ari is paid \$15.70 per hour in his job as a shop assistant. The first 36 hours he works in a week are paid at the normal hourly rate, the next 4 hours at time and a half and then double time after that. Ari works 42 hours in a particular week.

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2D

7 During the 2019/2020 financial year, Cameron earned $76 300 per annum. He had receipts for $425 for donations and work-related expenses.

a Calculate Cameron's taxable income.

b Use this tax table to calculate Cameron's tax payable amount, to the nearest cent.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

c Cameron also must pay the Medicare levy of 2% of his taxable income. How much is the levy, to the nearest cent?

d During the financial year, Cameron's employer sent a total of $15 255 in tax to the ATO on his behalf. By adding together your answers from parts **b** and **c**, calculate the amount Cameron must pay or will be refunded on his tax return.

2E

8 Charli has the following expenses in her household budget.

- rent $320 per week
- phone and internet $119 per month
- electricity $72 per quarter
- car registration $700 per year
- other car costs $120 per month
- food $110 per week
- clothing $260 per month
- medical and other insurance $160 per month

a Determine the overall cost for running the household for the year. (Use 52 weeks in a year.)

b What percentage of an $82 000 annual salary does your answer to part **a** represent? Round your answer to one decimal place.

2F

9 Use the simple interest formula $I = \frac{Prt}{100}$ to find:

a the amount owed when $4000 is borrowed at 6% p.a. for 3 years

b the investment period, in years, if an investment of $2500 at 4% p.a. earns $450 in interest

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2G Compound interest

Learning intentions

- To understand how compound interest is calculated
- To be able to apply the compound interest formula to calculate the total amount
- To be able to use the compound interest formula with different time periods such as months

Key vocabulary: compound interest, principal, rate of interest

For simple interest, the interest is always calculated on the principal amount. Sometimes, however, interest is calculated on the actual amount present in an account at each time period that interest is calculated. This means that the interest is added to the amount, then the next lot of interest is calculated again using this new amount. This process is called compound interest.

Compound interest can be calculated using updated applications of the simple interest formula or by using the compound interest formula.

Lesson starter: Investing using updated simple interest

Consider investing $\$400$ at 12% per annum. What is the balance at the end of 4 years if interest is added to the amount at the end of each year?

Copy and complete the table to find out.

Time	Amount (A)	Interest (I)	New amount
1st year	$\$400$	$\$48$	$\$448$
2nd year	$\$448$	$\$53.76$	$\$501.76$
3rd year	$\$501.76$		
4th year			

As you can see, the amount from which interest is calculated is continually changing.

Key ideas

- **Compound interest** is a type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal but also on the interest accumulated during the loan/investment period.
- Compound interest can be found by using updated applications of the simple interest formula. For example, $\$100$ compounded at 10% p.a. for 2 years.

  Year 1: $100 + 10\%$ of $100 = \$110$

  Year 2: $110 + 10\%$ of $110 = \$121$, so compound interest $= \$21$.
- The total amount in an account using compound interest for a given number of time periods is given by:

  $$A = P\left(1 + \frac{r}{100}\right)^n,$$ where:

  - Principal (P) = the amount of money borrowed or invested
  - Rate of interest (r) = the percentage applied to the principal per period of investment
  - Periods (n) = the number of time periods the principal is invested
  - Amount (A) = the total amount of your investment
- Interest = amount (A) − principal (P)

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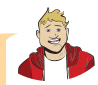
Exercise 2G

Understanding 1–3 3

1 Consider \$500 invested at 10% p.a. compounded annually.

- **a** How much interest is earned in the first year?
- **b** What is the balance of the account once the first year's interest is added?
- **c** How much interest is earned in the second year?
- **d** What is the balance of the account at the end of the second year?



Hint: For the second year, you need to use \$500 plus the interest from the first year.

2 \$1200 is invested at 4% p.a. compounded annually for 3 years. Complete the following.

- **a** The value of the principal P is ____________
- **b** 4% is the ____________, r.
- **c** The number of time periods the money is invested is ______.

3 Fill in the missing numbers.

- **a** \$700 invested at 8% p.a. compounded annually for 2 years.

  $A = \square\,(1.08)^{\square}$
- **b** \$1000 invested at 15% p.a. compounded annually for 6 years.

  $A = 1000\,(\square)^6$
- **c** \$850 invested at 6% p.a. compounded annually for 4 years.

  $A = 850\,(\square)^{\square}$

Hint: For compound interest, $A = P\left(1 + \frac{r}{100}\right)^n$.

Fluency 4, 5–6(½), 7, 8(½) 4–6(½), 8(½)

Example 19 Using the compound interest formula

Determine the amount after 5 years when \$4000 is compounded annually at 8%.

Solution	Explanation
$P = 4000,\ n = 5,\ r = 8$	List the values for the terms you know.
$A = P\left(1 + \frac{r}{100}\right)^n$	Write the formula.
$= 4000\left(1 + \frac{8}{100}\right)^5$	Substitute the values.
$= 4000(1.08)^5$	Simplify and evaluate.
$= \$5877.31$	Write your answer to two decimal places, (nearest cent).

Now you try

Determine the amount after 4 years when \$3000 is compounded annually at 6%.

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4 Determine the amount after 5 years when:

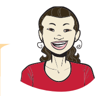
a \$4000 is compounded annually at 5%

b \$8000 is compounded annually at 8.35%

c \$6500 is compounded annually at 16%

d \$6500 is compounded annually at 8%

Hint: $A = P\left(1 + \frac{r}{100}\right)^n$



5 Determine the amount when \$100 000 is compounded annually at 6% for:

a 1 year **b** 2 years **c** 3 years

d 5 years **e** 10 years **f** 15 years

Example 20 Converting rates and time periods

Calculate the number of periods and the rates of interest offered per period for each of the following.

a 6% p.a. over 4 years paid monthly

b 18% p.a. over 3 years paid quarterly

Solution	Explanation
a $n = 4 \times 12 = 48$, $r = 6 \div 12 = 0.5$	4 years is the same as 48 months, as 12 months = 1 year. 6% p.a. = 6% in one year. Divide by 12 to find the monthly rate.
b $n = 3 \times 4 = 12$, $r = 18 \div 4 = 4.5$	There are four quarters in 1 year.

Now you try

Calculate the number of periods and the rates of interest offered per period for each of the following.

a 3% p.a. over 2 years paid monthly

b 7% p.a. over 4 years paid bi-annually (twice yearly)

6 Calculate the number of periods (n) and the rates of interest (r) offered per period for the following. (Round the interest rate to three decimal places where necessary.)

a 6% p.a. over 3 years paid biannually

b 12% p.a. over 5 years paid monthly

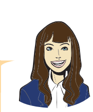
c 4.5% p.a. over 2 years paid fortnightly

d 10.5% p.a. over 3.5 years paid quarterly

e 15% p.a. over 8 years paid quarterly

f 9.6% p.a. over 10 years paid monthly

Hint: 'Bi-annually' means 'twice a year'. 26 fortnights = 1 year



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2G

Example 21 Finding compounded amounts using months

Tony's investment of $\$4000$ is compounded at 8.4% p.a. over 5 years. Determine the amount he will have after 5 years if the interest is paid monthly.

Solution	Explanation
$P = 4000$	List the values of the terms you know.
$n = 5 \times 12$ $= 60$	Convert the time in years to the number of periods (in this question, months); i.e. 60 months = 5 years.
$r = 8.4 \div 12$ $= 0.7$	Convert the rate per year to the rate per period (i.e. months) by dividing by 12.
$A = P\left(1 + \frac{r}{100}\right)^n$	Write the formula.
$= 4000(1 + 0.007)^{60}$	Substitute the values.
$= 4000(1.007)^{60}$ $= \$6078.95$	Simplify and evaluate.

Now you try

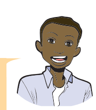
Sally's investment of $\$6000$ is compounded at 4.8% p.a. over 4 years. Determine the amount she will have after 4 years if the interest is paid monthly.

7 An investment of $\$8000$ is compounded at 12% p.a. over 3 years. Determine the amount the investor will have after 3 years if the interest is compounded monthly.

8 Calculate the value of the following investments if interest is compounded monthly.

- **a** $\$2000$ at 6% p.a. for 2 years
- **b** $\$34\,000$ at 24% p.a. for 4 years
- **c** $\$350$ at 18% p.a. for 8 years
- **d** $\$670$ at 6.6% p.a. for $2\frac{1}{2}$ years
- **e** $\$250$ at 7.2% p.a. for 12 years

Hint: Convert years to months and the annual rate to the monthly rate.



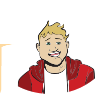
Problem-solving and reasoning

9, 10 | 10–12

9 Shafiq invests $\$5000$ compounded monthly at 18% p.a. Determine the value of the investment after:

- **a** 1 month
- **b** 3 months
- **c** 5 months

Hint: 18% p.a. $= 1.5\%$ each month



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10 **a** Calculate the amount of compound interest paid on \$8000 at the end of 3 years for each rate below.

   i 12% compounded annually
   ii 12% compounded biannually (twice a year)
   iii 12% compounded monthly
   iv 12% compounded weekly
   v 12% compounded daily

b What is the difference in the interest paid between annual and daily compounding in this case?

Hint: Use: 1 year = 12 months
1 year = 52 weeks
1 year = 365 days

11 The following are expressions relating to compound interest calculations. Determine the principal (P), number of periods (n), rate of interest per period (r), annual rate of interest (R) and the overall time (t).

a $300(1.07)^{12}$, biannually
b $5000(1.025)^{24}$, monthly
c $1000(1.00036)^{65}$, fortnightly
d $3500(1.000053)^{30}$, daily
e $10\,000\,(1.078)^{10}$, annually

Hint: For 12 time periods with interest paid twice a year, this is 6 years

12 Ellen needs to decide whether to invest her \$13 500 for 6 years at 4.2% p.a. compounded monthly or 5.3% compounded biannually. Decide which investment would be the best for Ellen.

Double your money | — | 13

13 You have \$100 000 to invest and wish to double that amount. Use trial and error in the following.

a Determine, to the nearest whole number of years, the length of time it will take to do this using the compound interest formula at rates of:

i 12% p.a.	**ii** 6% p.a.	**iii** 8% p.a.
iv 16% p.a.	**v** 10% p.a.	**vi** 20% p.a.

b If the amount of investment is \$200 000 and you wish to double it, determine the time it will take using the same interest rates as above.

c Are the lengths of time to double your investment the same in part **a** and part **b**?

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2H Investments and loans ★

Learning intentions

- To understand that a loan can be repaid in instalments that include interest
- To be able to calculate the total payment for a purchase or loan involving repayments
- To be able to calculate bank interest using the minimum monthly balance

Key vocabulary: investment, loan, repayment, interest, deposit, debit

When you borrow money, interest is charged, and when you invest money, interest is earned. When you invest money, the institution in which you invest (e.g. bank or credit union) pays you interest. However, when you borrow money, the institution from which you borrow charges you interest, so that you must pay back the money you initially borrowed, plus the interest.

Lesson starter: Credit card statements

Refer to Allan's credit card statement below.

a How many days were there between the closing balance and the due date?

b What is the minimum payment due?

c If Allan pays only the minimum, on what balance is the interest charged?

d How much interest is charged if Allan pays \$475.23 on 25/5?

Statement Issue Date: 2/5/20		
Date of purchase	**Details**	**Amount**
3/4/20	Opening balance	314.79
5/4/20	Dean's Jeans	59.95
16/4/20	Tyre Warehouse	138.50
22/4/20	Payment made—thank you	−100.00
27/4/20	Cottonworth's Grocery Store	58.64
30/4/20	Interest charges	3.35
2/5/20	Closing balance	475.23
Percentage rate	**Due date**	**Min. payment**
18.95%	25/5/20	23.75

Key ideas

- Interest rates are associated with many loan and savings accounts.
- Bank accounts:
  - accrue interest each month on the minimum monthly balance
  - may incur account-keeping fees each month
- **Investments** are amounts put into a bank account or similar with the aim of earning interest on the money.
- **Loans** (money borrowed) have interest charged to them on the amount left owing (i.e. the balance).
- **Repayments** are amounts paid to the bank, usually each month, to repay a loan plus interest within an agreed time period.

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Exercise 2H

Understanding — 1–3 | 3

1 State if the following are examples of investments, loans or repayments.
- **a** Kara pays \$160 per month to pay off her holiday loan.
- **b** Sam deposits a \$2000 prize in an account with 3% p.a. interest.
- **c** Georgia borrows \$6500 from the bank to finance setting up her small business.

2 Donna can afford to repay \$220 a month. How much does she repay over:
- **a** 1 year?
- **b** 18 months?
- **c** 5 years?

3 Sarafina buys a new bed on a 'buy now, pay later' offer. No interest is charged if she pays for the bed in 2 years. Sarafina's bed costs \$2490 and she pays it back over a period of 20 months in 20 equal instalments. How much is each instalment?

Fluency — 4–8 | 4, 6–9

Example 22 Repaying a loan

Wendy takes out a personal loan of \$7000 to fund her trip to South Africa. Repayments are made monthly for 3 years at \$275 a month. Find:
- **a** the total cost of Wendy's trip
- **b** the interest charged on the loan

Solution	Explanation
a Total cost $= \$275 \times 36$ $= \$9900$	3 years $= 3 \times 12 = 36$ months Cost = 36 lots of \$275
b Interest $= \$9900 - \$7000$ $= \$2900$	Interest = total paid − amount borrowed

Now you try

Jacob takes out a personal loan of \$13 000 to buy a car. He makes repayments monthly for 2 years at \$680 a month. Find:
- **a** the total cost of the car
- **b** the interest charged on the loan

4 Jason has a personal loan of \$10 000. He is repaying the loan over 5 years. The monthly repayment is \$310.
- **a** Calculate the total amount Jason repays over the 5 year loan.
- **b** How much interest is he charged?

Hint: How many monthly repayments in 5 years?

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2H

5 Robert borrows \$5500 to buy a second-hand motorbike. He repays the loan in 36 equal monthly instalments of \$155.
- **a** Calculate the total cost of the loan.
- **b** How much interest does Robert pay?

6 Alma borrows \$250 000 to buy a house. The repayments are \$1736 a month for 30 years.
- **a** How many repayments does Alma make?
- **b** What is the total amount Alma pays for the house?
- **c** How much interest is paid over the 30 years?

Example 23 Paying off a purchase

Harry buys a new \$2100 computer on the following terms.
- 20% deposit
- monthly repayments of \$90 for 2 years

Find:
- **a** the deposit paid
- **b** the total paid for the computer
- **c** the interest charged

Solution	Explanation
a Deposit $= 0.2 \times 2100$ $= \$420$	Find 20% of 2100.
b Repayments $= \$90 \times 24$ $= \$2160$	2 years = 24 months. Repay 24 lots of \$90.
Total paid $= \$2160 + \$420$ $= \$2580$	Repay = deposit + repayments
c Interest $= \$2580 - \$2100$ $= \$480$	Interest = total paid − original price

Now you try

Sophie pays \$3180 for a holiday apartment rental on the following terms.
- 30% deposit
- monthly repayments of \$195 for 1 year

Find:
- **a** the deposit paid
- **b** the total paid for the apartment
- **c** the interest charged

7 George buys a car for \$12 750 on the following terms: 20% deposit and monthly repayments of \$295 for 3 years.
- **a** Calculate the deposit.
- **b** Find the total of all the repayments.
- **c** Find the cost of buying the car on these terms.
- **d** Find the interest George pays on these terms.

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Example 24 Calculating interest

An account has a minimum monthly balance of $200 and interest is credited monthly on this amount at 1.5%.

a Determine the amount of interest to be credited at the end of the month.

b If the bank charges a fixed administration fee of $5 per month and other fees totalling $1.07, what will be the net amount credited or debited to the account at the end of the month?

Solution	Explanation
a Interest = 1.5% of $200	Interest is 1.5% per month.
$= 0.015 \times \$200$	Change 1.5% to a decimal and calculate.
$= \$3$	
b Net amount $= 3 - (5 + 1.07)$	Subtract the deductions from the interest.
$= -3.07$	
$3.07 will be debited from the account.	A negative amount is called a debit.

Now you try

An account has a minimum monthly balance of $180 and interest is credited monthly on this amount at 2.2%.

a Determine the amount of interest to be credited at the end of the month.

b If the bank charges a fixed administration fee of $4.50 per month and other fees totalling $1.18, what will be the net amount credited or debited to the account at the end of the month?

8 A bank account has a minimum monthly balance of $300 and interest is credited monthly at 1.5%.

a Determine the amount of interest to be credited each month.

b If the bank charges a fixed administration fee of $3 per month and fees of $0.24, what will be the net amount credited to the account at the end of the month?

9 An account has no administration fee. The monthly balances for May–October are in the table below. If the interest payable on the minimum monthly balance is 1%, how much interest will be added:

a for each separate month? **b** over the 6-month period?

May	June	July	August	September	October
$240	$300	$12	$500	$208	$73

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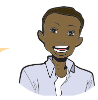
2H

Problem-solving and reasoning | 10, 12, 13 | 11, 13, 14

10 Supersound offers the following two deals on a sound system worth $7500.

- Deal A: no deposit, interest free and nothing to pay for 18 months
- Deal B: 15% off for cash



Hint: 15% off is 85% of the original amount.

a Nick chooses deal A. Find:
  - **i** the deposit he must pay
  - **ii** the interest charged
  - **iii** the total cost if Nick pays the system off within 18 months

b Phil chooses deal B. What does Phil pay for the same sound system?

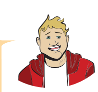
c How much does Phil save by paying cash?

11 Camden Finance Company charges 35% flat interest on all loans.

a Mei borrows $15 000 from Camden Finance over 6 years.
  - **i** Calculate the interest on the loan.
  - **ii** What is the total repaid (i.e. loan + interest)?
  - **iii** What is the value of each monthly repayment?

b Lancelle borrows $24 000 from the same company over 10 years.
  - **i** Calculate the interest on her loan.
  - **ii** What is the total repaid?
  - **iii** What is the value of each monthly instalment?

12 A list of transactions that Emma made over a 1-month period is shown. The bank calculates interest *daily* at 0.01% and adds the total to the account balance at the end of this period. It has an administrative fee of $7 per month and other fees over this time total $0.35.



Hint: In part **b**, interest is calculated on the end-of-the-day balance.

a Copy and complete the balance column of the table.

Date	Deposit	Withdrawal	Balance
1 May			$3010
3 May	$490		
5 May		$2300	
17 May	$490		
18 May		$150	
20 May		$50	
25 May		$218	
31 May	$490		

b Determine the amount of interest added over this month.

c Determine the final balance after all calculations have been made.

d Suggest what the regular deposits might be for.

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13 The table below shows the interest and monthly repayments on loans when the simple interest rate is 8.5% p.a.

	18-month term		24-month term		36-month term	
Loan amount	Interest ($)	Monthly payments ($)	Interest ($)	Monthly payments ($)	Interest ($)	Monthly payments ($)
1000	127.50	62.64	170.00	48.75	255.00	34.86
1100	140.25	68.90	187.00	53.63	280.50	38.35
1200	153.00	75.17	204.00	58.50	306.00	41.83
1300	165.75	81.43	221.00	63.38	331.50	45.32
1400	178.50	87.69	238.00	68.25	357.00	48.81
1500	191.25	93.96	255.00	73.13	382.50	52.29
1600	204.00	100.22	272.00	78.00	408.00	55.78
1700	216.75	106.49	289.00	82.88	433.50	59.26
1800	229.50	112.75	306.00	87.75	459.00	62.75
1900	242.25	119.01	323.00	92.63	484.50	66.24
2000	255.00	125.28	340.00	97.50	510.00	69.72

a Use the table to find the monthly repayments for a loan of:
- **i** $1500 over 2 years
- **ii** $2000 over 3 years
- **iii** $1200 over 18 months

b Damien and Lisa can afford monthly repayments of $60. What is the most they can borrow and on what terms?

14 Part of a credit card statement is shown here.

Understanding your account

CLOSING BALANCE
$403.80

→ CLOSING BALANCE
This is the amount you owe at the end of the statement period

MINIMUM PAYMENT DUE
$10.00

→ MINIMUM PAYMENT DUE
This is the minimum payment that must be made towards this account

PAYABLE TO MINIMISE FURTHER INTEREST CHARGES
$403.80

→ PAYABLE TO MINIMISE FURTHER INTEREST CHARGES
This amount you must pay to minimise interest charges for the next statement period

a What is the closing balance?

b What is due on the card if only the minimum payment is made on the due date?

c This card charges 21.9% p.a. interest calculated daily on the unpaid balance. To find the daily interest amount, the company multiplies this balance by 0.0006. What does it cost in interest per day if only the minimum payment is made?

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2H

Reducing the balance of loans (a spreadsheet approach) — 15

15 When you take out a loan from a lending institution you will be asked to make regular payments (usually monthly) for a certain period of time to repay the loan completely. The larger the repayment, the shorter the term of the loan.

Loans work mostly on a reducing balance and you can find out how much balance is owing at the end of each month from a statement, which is issued on a regular basis.

Let's look at an example of how the balance is reducing.

If you borrow $15 000 at 17% p.a. and make repayments of $260 per month, at the end of the first month your statement would be calculated as shown.

$$\begin{aligned}\text{Interest due} &= \frac{15\,000 \times 0.17}{12}\\ &= \$212.50\\ \text{Repayment} &= \$260\\ \text{Amount owing} &= \$15\,000 + \$212.50 - \$260\\ &= \$14\,952.50\end{aligned}$$

This process would be repeated for the next month:

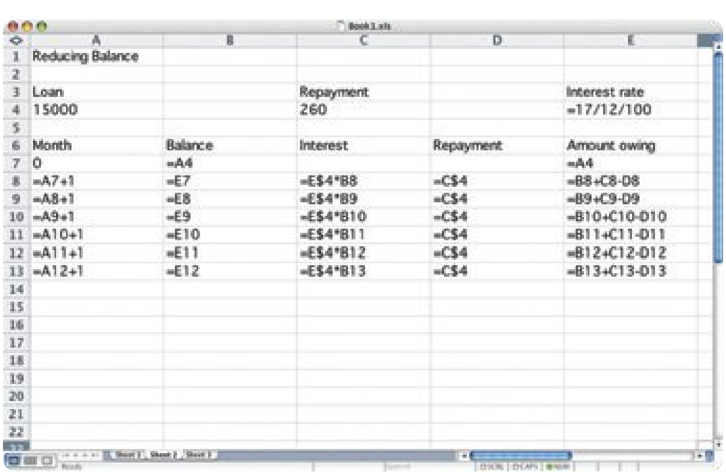
$$\begin{aligned}\text{Interest due} &= \frac{14\,952.50 \times 0.17}{12}\\ &= \$211.83\\ \text{Repayment} &= \$260\\ \text{Amount owing} &= \$14\,952.50 + \$211.83 - \$260\\ &= \$14\,904.33\end{aligned}$$

As you can see, the amount owing is decreasing and so is the interest owed each month. Meanwhile, more of your repayment is actually reducing the balance of the loan.

A statement might look like this:

Balance	Interest	Repayment	Amount owing
15 000	212.50	260	14 952.50
14 952.50	211.83	260	14 904.33
14 904.33	211.14	260	14 855.47
14 855.47	210.45	260	14 805.92
14 805.92	209.75	260	14 755.67

Check to see that all the calculations are correct on the statement above.
As this process is repetitive, the calculations are best done by means of a spreadsheet. To create a spreadsheet for the process, copy the following, extending your sheet to cover 5 years.



	A	B	C	D	E
1	Reducing Balance				
2					
3	Loan		Repayment		Interest rate
4	15000		260		=17/12/100
5					
6	Month	Balance	Interest	Repayment	Amount owing
7	0	=A4			=A4
8	=A7+1	=E7	=E$4*B8	=C$4	=B8+C8-D8
9	=A8+1	=E8	=E$4*B9	=C$4	=B9+C9-D9
10	=A9+1	=E9	=E$4*B10	=C$4	=B10+C10-D10
11	=A10+1	=E10	=E$4*B11	=C$4	=B11+C11-D11
12	=A11+1	=E11	=E$4*B12	=C$4	=B12+C12-D12
13	=A12+1	=E12	=E$4*B13	=C$4	=B13+C13-D13

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2I Comparing interest using technology ★

Learning intentions

- To understand how technology can be used to efficiently compare interest calculations
- To be able to use technology to calculate interest and final amounts and compare interest plans

Key vocabulary: simple interest, compound interest

Both compound interest and simple interest calculations involve formulas. Technology including scientific and CAS calculators, spreadsheets or even computer programs can be used to make simple and compound interest calculations.

These allow for quick, repeated calculations where values can be adjusted and the interest from different accounts compared.

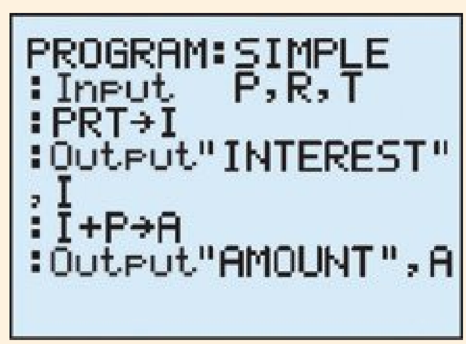
Lesson starter: Who earns the most?

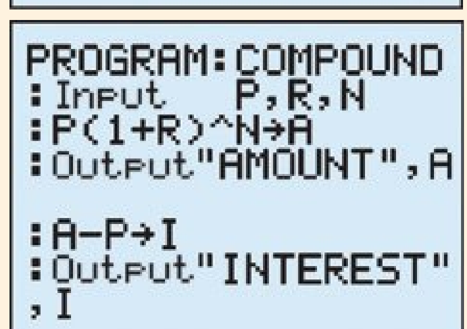
- Ceanna invests $\$500$ at 8% p.a. compounded monthly over 3 years.
- Huxley invests $\$500$ at 10% p.a. compounded annually over 3 years.
- Loreli invests $\$500$ at 15% p.a. simple interest over 3 years.
  - How much does each person have at the end of the 3 years?
  - Who earned the most?

Key ideas

You can calculate the total amount of your investment for either form of interest using technology.

- Using formulas in calculators
  - Simple interest $I = \frac{Prt}{100}$
  - Compound interest $A = P\left(1 + \frac{r}{100}\right)^n$
- **Simple code**
  To create programs for the two types of interest, enter the data shown at right.
  This will allow you to calculate both types of interest for a given time period. If you invest $\$100\,000$ at 8% p.a. paid monthly for 2 years, you will be asked for P, $R = \frac{r}{100}$, t or n and the calculator will do the work for you.
  Note: Some modifications may be needed for the CAS or other calculators or other technology.





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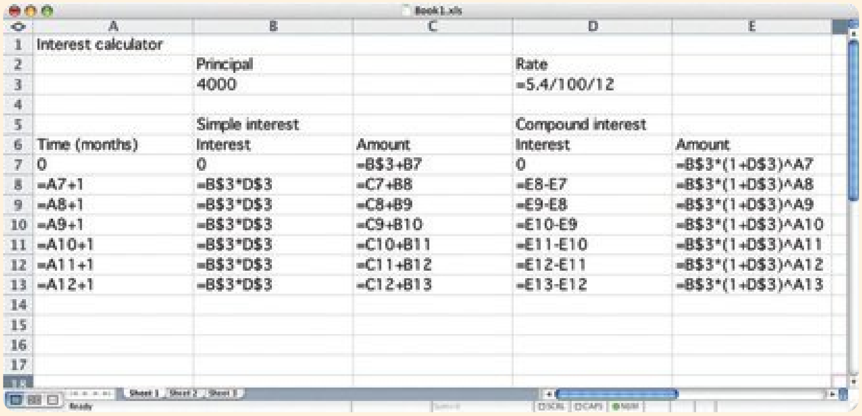
- **Spreadsheet**
  Copy and complete the spreadsheet as shown below to compile a simple interest and compound interest sheet.
  Fill in the principal in B3 and the rate per period in D3. For example, for \$4000 invested at 5.4% monthly, B3 will be 4000 and D3 will be $\frac{0.054}{12}$.



	A	B	C	D	E
1	Interest calculator				
2		Principal		Rate	
3		4000		=5.4/100/12	
4					
5		Simple interest		Compound interest	
6	Time (months)	Interest	Amount	Interest	Amount
7	0	0	=B$3+B7	0	=B$3*(1+D$3)^A7
8	=A7+1	=B$3*D$3	=C7+B8	=E8-E7	=B$3*(1+D$3)^A8
9	=A8+1	=B$3*D$3	=C8+B9	=E9-E8	=B$3*(1+D$3)^A9
10	=A9+1	=B$3*D$3	=C9+B10	=E10-E9	=B$3*(1+D$3)^A10
11	=A10+1	=B$3*D$3	=C10+B11	=E11-E10	=B$3*(1+D$3)^A11
12	=A11+1	=B$3*D$3	=C11+B12	=E12-E11	=B$3*(1+D$3)^A12
13	=A12+1	=B$3*D$3	=C12+B13	=E13-E12	=B$3*(1+D$3)^A13
14					
15					
16					
17					

Exercise 2I

Understanding — 1–3 — 3

1 Write down the values of P, r and n for an investment of \$750 at 7.5% p.a., compounded annually for 5 years.

2 Write down the values of P, r and t for an investment of \$300 at 3% p.a. simple interest over 300 months.

3 Which is better on an investment of \$100 for 2 years:
  A simple interest calculated at 20% p.a.?
  B compound interest calculated at 20% p.a. and paid annually?

Hint: Recall: For simple interest $I = \frac{Prt}{100}$
For compound interest $A = P\left(1 + \frac{r}{100}\right)^n$

Fluency — 4, 5 — 4, 5

Example 25 Using technology

Find the total amount of the following investments, using technology.
a \$5000 at 5% p.a. compounded annually for 3 years
b \$5000 at 5% p.a. simple interest for 3 years

Solution	Explanation
a \$5788.13	Use $A = P\left(1 + \frac{r}{100}\right)^n$ or a spreadsheet (see Key ideas).
b \$5750	Use $I = \frac{Prt}{100}$ with your chosen technology.

Now you try

Find the total amount of the following investments, using technology.
a \$6000 at 4% p.a. compounded annually for 5 years
b \$6000 at 4% p.a. simple interest for 5 years

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4 **a** Find the total amount of the following investments, using technology.
- **i** \$6000 at 6% p.a. compounded annually for 3 years
- **ii** \$6000 at 3% p.a. compounded annually for 5 years
- **iii** \$6000 at 3.4% p.a. compounded annually for 4 years
- **iv** \$6000 at 10% p.a. compounded annually for 2 years
- **v** \$6000 at 5.7% p.a. compounded annually for 5 years

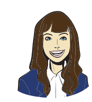
b Which of the above yields the most interest?

5 **a** Find the total amount of the following investments, using technology where possible.
- **i** \$6000 at 6% p.a. simple interest for 3 years
- **ii** \$6000 at 3% p.a. simple interest for 6 years
- **iii** \$6000 at 3.4% p.a. simple interest for 7 years
- **iv** \$6000 at 10% p.a. simple interest for 2 years
- **v** \$6000 at 5.7% p.a. simple interest for 5 years

b Which of the above yields the most interest?

Problem-solving and reasoning | 6, 7 | 6–8

6 **a** Determine the total simple and compound interest accumulated on the following.
- **i** \$4000 at 6% p.a. payable annually for:
  I 1 year **II** 2 years **III** 5 years **IV** 10 years
- **ii** \$4000 at 6% p.a. payable biannually for:
  I 1 year **II** 2 years **III** 5 years **IV** 10 years
- **iii** \$4000 at 6% p.a. payable monthly for:
  I 1 year **II** 2 years **III** 5 years **IV** 10 years



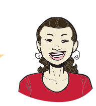
Hint: 6% p.a. paid biannually is 3% per 6 months.
6% p.a. paid monthly is $\frac{6}{12} = 0.5\%$ per month.

b Would you prefer the same rate of compound interest or simple interest if you were investing money and paying off the loan in instalments?

c Would you prefer the same rate of compound interest or simple interest if you were borrowing money?

7 **a** Copy and complete the following table if simple interest is applied.

Principal	Rate	Overall time	Interest	Amount
\$7000		5 years		\$8750
\$7000		5 years		\$10 500
	10%	3 years	\$990	
	10%	3 years	\$2400	
\$9000	8%	2 years		
\$18 000	8%	2 years		



Hint: $I = \frac{Prt}{100}$
$A = P + I$

b Explain the effect on the interest when we double the:
i rate **ii** period **iii** overall time

8 Copy and complete the following table if compound interest is applied. You may need to use a calculator and trial and error to find some of the missing values.

Principal	Rate	Period	Overall time	Interest	Amount
\$7000		Annually	5 years		\$8750
\$7000		Annually	5 years		\$10 500
\$9000	8%	Fortnightly	2 years		
\$18 000	8%	Fortnightly	2 years		

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2I

Changing the parameters — 9, 10

9 If you invest $5000, determine the interest rate per annum (to two decimal places) if the total amount is approximately $7500 after 5 years and if interest is:

a compounded annually
b compounded quarterly
c compounded weekly

Comment on the effect of changing the period for each payment on the rate needed to achieve the same total amount in a given time.

10 a Determine, to one decimal place, the equivalent simple interest rate for the following investments over 3 years.

i $8000 at 4% compounded annually
ii $8000 at 8% compounded annually

b If you double or triple the compound interest rate, how is the simple interest rate affected?

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Maths@Work: Finance manager

A bookkeeper and an accounts manager are both occupations that deal with numbers and budgets. They require employees to have good communication and mathematical skills. Employees also need a commitment to detail and to be honest, as they deal with other people's money.

Excellent number skills are essential in these fields. Bookkeepers need to work with spreadsheets, percentages, tax systems and business plans.

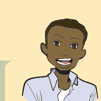
Complete these questions that a finance manager may face in their day-to-day job.

1 Consider the information supplied in a section of a business budget for a 3-month period. Round all answers to two decimal places.

- **a** Calculate the total income for the month of July.
- **b** Calculate the total income for the month of August.
- **c** Calculate the total income for the month of September.
- **d** Which month had the highest income and by how much?
- **e** What contributed to this increase in income?
- **f** What percentage of the total income for the 3 months shown came from a fixed fee?
- **g** What was the monthly fixed fee before the 25% reduction occurred?

Income	July	August	September
Fixed fee (with 25% reduction)	\$52 813	\$52 813	\$52 813
Variable fee (with 25% reduction)	\$53 906	\$53 906	\$53 906
Associate members fees (with 25% reduction)	\$1563	\$1563	\$1563
Shared costs billed EAL	\$0	\$0	\$0
CBA interest earned	\$25	\$25	\$25
Operating costs	\$167	\$167	\$167
Term deposit	\$0	\$2500	\$0
Miscellaneous income	\$0	\$0	\$0
Total Income	**a**	**b**	**c**

Hint: $\% \text{ of total} = \frac{\text{amount}}{\text{total}} \times \frac{100}{1}$



2 The office expenses for the same company for the same 3-month period are given below.

- **a** Calculate the percentage of the total office expenses for July spent in rent.
- **b** What is the cost of electricity shown in the table, and in what month is it shown?
- **c** Why does the electricity not appear in the other two months?
- **d** What is the projected cost of electricity for the year?

Office expenses	July	August	September
Rent	\$7200	\$7200	\$7200
Property outgoings (costs)	\$220	\$220	\$220
Cleaning	\$250	\$250	\$250
Maintenance	\$50	\$50	\$50
Capital expenditure	\$100	\$100	\$100
Electricity	\$0	\$1200	\$0
Total office expenses	**\$7820**	**\$9020**	**\$7820**

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3 The employment expenses for the three months of October, November and December are shown.

Employment expenses	October	November	December
Superannuation	$4800	$4800	$4800
Employees' salaries	$50 424	$50 424	$50 424
Payroll tax	$1200	$1200	$4500
Consultant	$0	$0	$0
Workers compensation	$0	$0	$0
Performance review	$0	$0	$75 000
Training	$335	$335	$335
Parking	$400	$400	$400
Total employment expenses	**$57 159**	**$57 159**	**a**

a Calculate the total employment expenses for the month of December.

b What is the whole number percentage increase of November's total employment expenses from November to December? What was the cause of this increase?

c The company has 11 full-time employees. What is an employee's average:

- **i** salary per month?
- **ii** annual salary?

d The company has total expenses for the month of November of $92 117. What percentage of the total expenses for November comes from the employment expenses?

Using technology

4 A trucking business has invested in a new prime mover for hauling cattle by road train. It has a bank loan of $230 000 at 9% per annum charged monthly. The business requires an Excel spreadsheet to show the progress of the debt repayment.

a Develop the following table in an Excel spreadsheet by entering formulas into the yellow shaded cells to calculate their values. Use the notes below to help you.

Hint: After entering your formulas, check specific results with a calculator.

	B	C	D	E	F	G
1	Debt repayment table					
2	Payment date	Starting balance	Scheduled payment	Interest due	Principal paid	Ending balance
3	1-Jan	$230,000.00	$5,780.00			
4	1-Feb		$5,780.00			
5	1-Mar		$5,780.00			

Notes:

The interest due per month is $\frac{1}{12}$ of 9% of the starting balance for that month.

The principal (i.e. debt) paid will be the scheduled payment minus the interest due.
The ending balances will equal the starting balance minus the principal paid.
The next month's starting balance equals the previous month's ending balance.

b Extend the table for 12 payments and answer the following questions.

- **i** What is the amount of debt remaining on July 1?
- **ii** What is the interest paid in October?
- **iii** Use an Excel formula to find the difference between the principal paid in December and the principal paid in January.
- **iv** Enter 'sum' formulas to determine the total interest paid in the year and the total principal paid off in the year.

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1 Find and define the 10 terms related to consumer arithmetic and percentages hidden in this wordfind.

C	O	M	M	I	S	S	I	O	N	Q	R	W
P	G	S	L	E	R	S	T	B	L	D	U	J
H	L	A	A	P	I	E	C	E	W	O	R	K
U	F	N	U	L	N	Q	B	D	Z	T	J	L
V	K	N	S	T	A	M	O	N	T	H	L	Y
B	H	U	A	I	G	R	O	S	S	U	B	S
N	E	A	C	Y	K	S	Y	E	T	Y	M	D
M	A	L	O	V	E	R	T	I	M	E	Q	T
S	F	O	R	T	N	I	G	H	T	L	Y	S

2 How do you stop a bull charging you? Answer the following problems and match the letters to the answers below to find out.

\$19.47 – \$8.53 E	5% of \$89 T	50% of \$89 I
$12\frac{1}{2}$% of \$100 A	If 5% = \$8.90 then 100% is? S	\$4.48 to the nearest 5 cents R
6% of \$89 W	Increase \$89 by 5% H	10% of \$76 O
\$15 monthly for 2 years D	$12\frac{1}{2}$% as a decimal K	\$50 – \$49.73 U
Decrease \$89 by 5% C	\$15.96 + \$12.42 Y	

\$28.38 \$7.60 27c

\$4.45 \$12.50 0.125 \$10.94

\$12.50 \$5.34 \$12.50 \$28.38

\$93.45 \$44.50 \$178

\$84.55 \$4.50 \$10.94 \$360 \$44.50 \$4.45

\$84.55 \$12.50 \$4.50 \$360

3 How many years does it take \$1000 to double if it is invested at 10% p.a. compounded annually?

4 The chance of Jayden winning a game of cards is said to be 5%. How many consecutive games should Jayden play to be 95% certain he has won at least one of the games played?

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Chapter summary

Consumer arithmetic

Conversions

Fraction → Percentage: $\times 100$

Percentage → Fraction: $\div 100$

Decimal → Percentage: $\times 100$

Percentage → Decimal: $\div 100$

Percentages

'Out of 100'

79 per cent $= 79\% = \frac{79}{100}$

Percentage work

15% *of* 459

$= \frac{15}{100}$ or 0.15×459

Increase 20 by 8%

$= 20 \times 108\%$

$= 20 \times 1.08$

Decrease 20 by 8%

$= 20 \times 92\%$

$= 20 \times 0.92$

Profit

Profit = selling price − cost price

$\%\ \text{Profit} = \frac{\text{profit}}{\text{cost price}} \times 100$

$\%\ \text{Discount} = \frac{\text{discount}}{\text{cost price}} \times 100$

Income

Hourly rate = \$/h

Time and a half $= 1\frac{1}{2} \times$ hrly rate

Double time $= 2 \times$ hrly rate

Commission is % of total sales

Gross income = total of all money earned

Net income = gross income − deductions

Simple (flat rate) interest

$$I = \frac{Prt}{100}$$

I = simple interest

P = principal (\$ invested)

r = rate per year, as a percentage

t = number of years

Compound interest

$$A = P\left(1 + \frac{r}{100}\right)^n$$

A = final balance

P = principal (\$ invested)

r = rate per time period, as a percentage

n = number of time periods money is invested for

Spreadsheets can be used to manage money or compare loans.

Budgets

Deciding how income is spent on *fixed* and *variable* expenses.

Income taxation ⭐

Money given from wages (income tax) to the government (Refer to the income tax table on page 82.)

Loans ⭐

Balance owing = amount left to repay

Repayment = money given each month to repay the loan amount and the interest

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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
2A	**1 I can convert to a percentage.** e.g. Write each of the following as a percentage. **a** $\frac{7}{40}$ **b** 0.24	
2A	**2 I can write percentages as simplified fractions and decimals.** e.g. Write each of the following percentages as both a simplified fraction and a decimal. **a** 53% **b** 4% **c** 10.5%	
2A	**3 I can find the percentage of a quantity.** e.g. Find 64% of $\$1400$.	
2B	**4 I can increase and decrease by a given percentage.** e.g. For the amount of $\$800$: **a** increase $\$800$ by 6% **b** decrease $\$800$ by 15%	
2B	**5 I can calculate percentage profit.** e.g. Jimmy buys a second-hand desk for $\$145$ and restores it to a good condition. If he sells if for $\$210$, calculate his profit and the percentage profit, correct to one decimal place.	
2B	**6 I can find the selling price.** e.g. Jo buys t-shirts for $\$24$ each and wishes to make a 28% profit on the purchase. What should be her selling price and what will be the profit on the sale of 20 t-shirts?	
2B	**7 I can calculate a discount.** e.g. A $\$849$ television is discounted by 18%. What is the selling price of the television?	
2C	**8 I can find gross and net income involving overtime.** e.g. Anika earns $\$21.40$ per hour and has normal working hours of 38 hours per week. She earns time and a half for the next 4 hours worked and double time after that. She pays $\$190$ per week in tax and other deductions. Calculate her gross and net income for a week in which she works 45 hours.	
2C	**9 I can calculate income involving commission.** e.g. Tia earns $\$300$ per week plus a commission of 6% on her sales of solar panels. If she sells $\$8200$ worth of solar panels in a week, what is her gross income for the week?	
2D	**10 I can calculate income tax payable.** e.g. Noah earns $\$78\,406$ per year, including interest on investments. He has receipts for donations and work related expenses of $\$445$. **a** Calculate Noah's taxable income. **b** Use the tax table in the Key ideas on page 82 to calculate Noah's tax payable amount, to the nearest cent. **c** If Noah also has to pay $\$1559$ for the Medicare levy, calculate his tax refund if his employer sent $\$19\,200$ to the ATO.	
2E	**11 I can budget using percentages.** e.g. Ash has a net annual income of $\$54\,800$ after deductions. She allocates her budget on a percentage basis. (see table below) **a** Determine the amount of fixed expenses (rent and the loan). **b** Determine how much she budgets to save each month.	

	Rent	Holiday loan	Clothing	Food	Other	Savings
Expenses (%)	25	10	5	10	35	15

Chapter checklist

ISBN 978-1-108-87859-3

Chapter checklist

Section	Checklist item	✔
2E	**12 I can budget from fixed values.** e.g. Running a certain type of motorbike involves the following costs: registration $520 per year insurance $120 per quarter servicing $310 per year petrol $64 per month Determine the overall cost to run the bike for a year and what percentage of an $80 000 salary this would be, correct to one decimal place.	
2E	**13 I can calculate a best buy.** e.g. Packets of chips can be bought in the following ways at the store: • 20 packs (20 grams each) for $5.50 • 6 packs (20 grams each) for $3.35 • 2 share bags (60 grams each) for $4 Determine the cheapest way to buy the chips.	
2F	**14 I can use the simple interest formula to find interest.** e.g. Use the simple interest formula to calculate the interest when $800 is invested at 5% p.a. for 3 years.	
2F	**15 I can calculate repayments using simple interest.** e.g. If a simple interest loan of $4000 is borrowed for 2 years at a simple interest rate of 4% p.a., what is the total amount owed over the 2 years and if repayments are made monthly, how much would each payment need to be?	
2F	**16 I can use the simple interest formula to find the rate of interest.** e.g. Use the simple interest formula to calculate the rate of interest when $2800 earns $294 interest in 3 years.	
2G	**17 I can use the compound interest formula.** e.g. Determine the amount after 6 years when $8000 is compounded annually at 3%.	
2G	**18 I can use compound interest with different time periods.** e.g. An investment of $5500 is compounded at 6% p.a. over 4 years. Determine the amount he will have after 4 years if interest is paid monthly.	
2H	**19 I can work with repayments to calculate a purchase cost.** e.g. Vanessa pays for a $8600 travel package with a travel agent with a 30% deposit and monthly repayments of $300 for 2 years. Calculate: **a** the deposit paid, **b** the total amount paid for the travel package and hence the interest paid.	
2H	**20 I can calculate interest earned on an account.** e.g. An account has a minimum monthly balance of $140 and interest is credited monthly on this amount at 1.8%. Determine the amount of interest to be credited at the end of the month and the total amount credited or debited if the bank charges $5 per month in account keeping fees.	
2I	**21 I can use technology to calculate interest and final amounts.** e.g. Use technology to find the total amount on the following investments. **a** $7000 at 4% p.a. compounded annually for 5 years **b** $7000 at 4% p.a. simple interest for 5 years	

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Short-answer questions

2A
1 Find 16% of \$9000.

2B
2 a Increase \$968 by 12%.
b Decrease \$4900 by 7%.

2B
3 The cost price of an item is \$7.60. If this is increased by 50%, determine:
a the retail price
b the profit made

2B
4 An airfare of \$7000 is discounted 40% if you fly off-peak. What would be the discounted price?

2B
5 A sofa is discounted to \$375. If this is a 35% discount, find the recommended retail price.

2C
6 Josephine budgets 20% of her income for entertainment. If her yearly income is \$37 000, how much could be spent on entertainment in:
a a year?
b a month?
c a week (taking 52 weeks in a year)?

2C
7 Mariah works a 34-hour week at \$25.43 per hour. Her net income is 62% of her wage.
a Work out her weekly net income.
b If 15% is spent on clothing, determine the amount she can spend each week.
c If she saves \$100, what percentage (to two decimal places) of her gross weekly income is this?

2E
8 Frank has the following expenses to run his car:
- hire purchase payment — \$350 per month
- registration — \$885 per year
- insurance — \$315 per quarter
- servicing — \$1700 per year
- petrol — \$90 per week

a Find the total cost of running his vehicle for 1 year.
b What percentage (to the nearest percentage) of the overall cost to run the car is the cost of the petrol?

2E
9 Ronan works 36 hours in a week at \$39.20 per hour. He pays \$310 in tax and \$20.50 in superannuation in the week. Determine:
a his gross wage in a week
b his net pay in a week

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Chapter review

2D

10 Lil receives an annual taxable income of $90 000.

a Using the tax table shown, calculate the amount of tax she pays over the year.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

b If Lil pays the 2% Medicare levy on her taxable income, find this amount.

2C

11 Zane receives 4.5% commission on sales of $790. Determine the amount of his commission.

2F

12 Find the interest paid on a $5000 loan under the following conditions.

a 8% p.a. simple interest over 4 years

b 7% p.a. simple interest over 3 years and 4 months

2G

13 Find the interest paid on a $5000 loan under the following conditions.

a 4% p.a. compounded annually over 3 years

b 9.75% p.a. compounded annually over 2 years

c 6% p.a. compounded monthly over 2 years

2H

14 A vehicle worth $7000 is purchased on a finance package. The purchaser pays 15% deposit and $250 per month over 4 years.

a How much deposit is paid?

b What are the total repayments?

c How much interest is paid over the term of the loan?

Multiple-choice questions

2A

1 28% of $89 is closest to:

A $28.00 **B** $64.08 **C** $113.92 **D** $2492 **E** $24.92

2A

2 As a percentage, $\frac{21}{60}$ is:

A 21% **B** 3.5% **C** 60% **D** 35% **E** 12.6%

2E

3 If a budget allows 30% for car expenses, how much is allocated from a weekly wage of $560?

A $201 **B** $145 **C** $100 **D** $168 **E** $109

2C

4 The gross income for 30 hours at $5.26 per hour is:

A $35.26 **B** $389.24 **C** $157.80 **D** $249.20 **E** $24.92

2C

5 If Simon receives $2874 on the sale of a property worth $195 800, his rate of commission, to one decimal place, is:

A 21% **B** 1.5% **C** 60% **D** 15% **E** 12.6%

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2C **6** In a given rostered fortnight, Bilal works the following number of 8-hour shifts:

- three day shifts ($10.60 per hour)
- three afternoon shifts ($12.34 per hour)
- five night shifts ($16.78 per hour).

His total income for the fortnight is:

A $152.72 **B** $1457.34 **C** $1000 **D** $168.84 **E** $1221.76

2B **7** A computer tablet is discounted by 26%. What is the price if it was originally $329?

A $85.54 **B** $243.46 **C** $156.05 **D** $77.78 **E** $206.90

2H **8** A $5000 loan is repaid by monthly instalments of $200 for 5 years. The amount of interest charged is:

A $300 **B** $7000 **C** $12 000 **D** $2400 **E** $6000

2F **9** The simple interest earned on $600 invested at 5% p.a. for 4 years is:

A $570 **B** $630 **C** $120 **D** $720 **E** $30

2G **10** The compound interest earned on $4600 invested at 12% p.a. for 2 years is:

A $1104 **B** $5704 **C** $4600 **D** $5770.24 **E** $1170.24

Extended-response questions

1 $5000 is invested at 4% p.a. compounding annually for 3 years.

a What is the value of the investment after the 3 years?
b How much interest is earned in the 3 years?
c Using $r = \frac{100I}{Pt}$, what simple interest rate results in the same amount?
d How much interest is earned on the investment if it is compounded monthly at 4% p.a. for the 3 years?

2 Your bank account has an opening July monthly balance of $217.63. You have the following transactions over the month.

Date	Withdrawals	Date	Deposits
7 July	$64.00	July 9th	$140
11 July	$117.34	July 20th	$20
20 July	$12.93	July 30th	$140

a Design a statement of your records if $0.51 is taken out as a fee on 15 July.
b Find the minimum balance.
c If interest is credited monthly on the minimum balance at 0.05%, determine the interest for July, rounded to the nearest cent.

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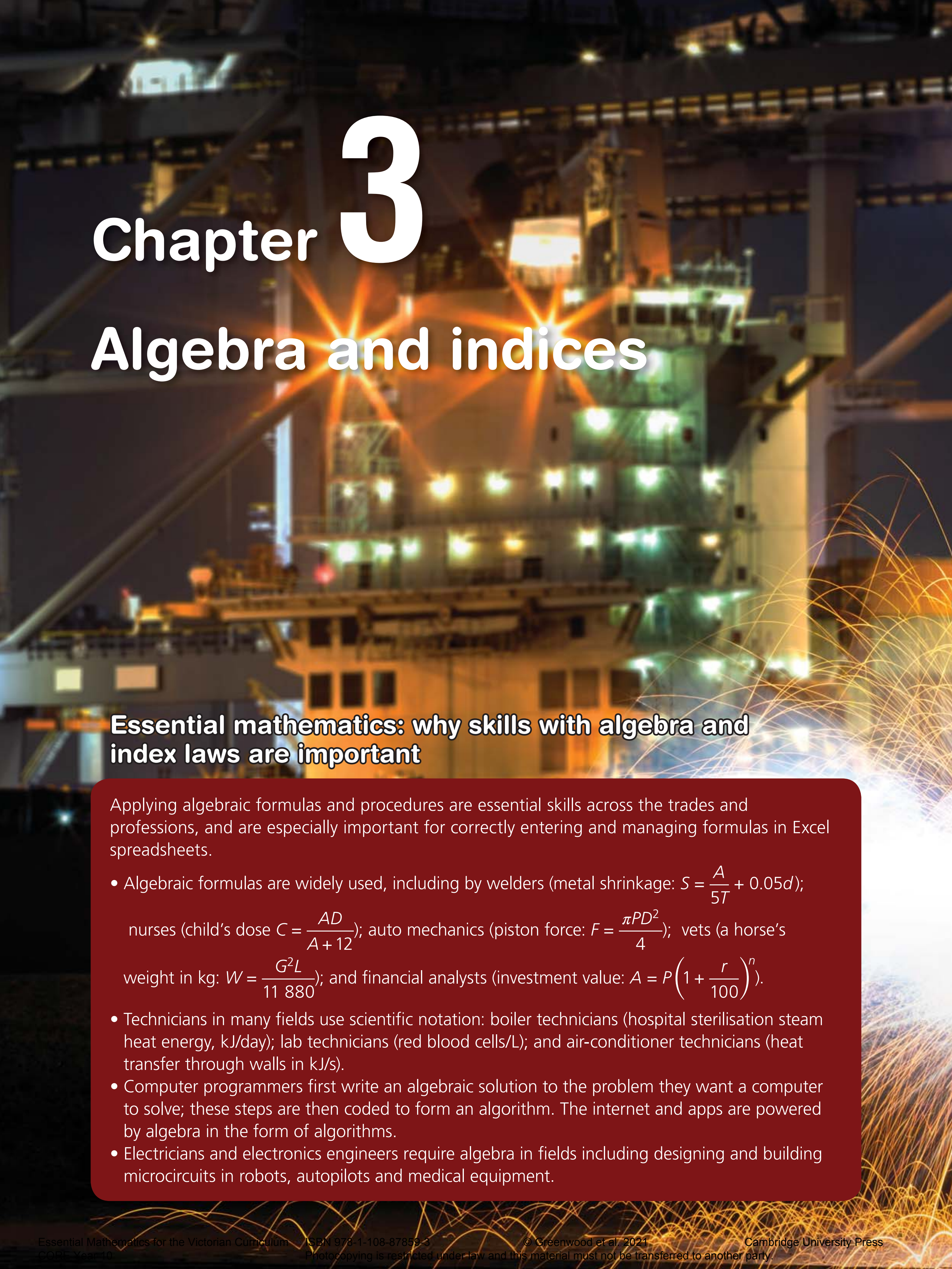

Chapter 3

Algebra and indices

Essential mathematics: why skills with algebra and index laws are important

Applying algebraic formulas and procedures are essential skills across the trades and professions, and are especially important for correctly entering and managing formulas in Excel spreadsheets.

- Algebraic formulas are widely used, including by welders (metal shrinkage: $S = \frac{A}{5T} + 0.05d$); nurses (child's dose $C = \frac{AD}{A+12}$); auto mechanics (piston force: $F = \frac{\pi PD^2}{4}$); vets (a horse's weight in kg: $W = \frac{G^2L}{11\,880}$); and financial analysts (investment value: $A = P\left(1 + \frac{r}{100}\right)^n$).
- Technicians in many fields use scientific notation: boiler technicians (hospital sterilisation steam heat energy, kJ/day); lab technicians (red blood cells/L); and air-conditioner technicians (heat transfer through walls in kJ/s).
- Computer programmers first write an algebraic solution to the problem they want a computer to solve; these steps are then coded to form an algorithm. The internet and apps are powered by algebra in the form of algorithms.
- Electricians and electronics engineers require algebra in fields including designing and building microcircuits in robots, autopilots and medical equipment.

In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor (VCMNA329)

Simplify algebraic products and quotients using index laws (VCMNA330)

Apply the four operations to simple algebraic fractions with numerical denominators (VCMNA331)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Write algebraic expressions for the following.

a 3 lots of x
b one more than a
c 5 less than $2m$
d 4 times the sum of x and y

2 Find the value of the following when $x = 4$ and $y = 7$.

a $5x$
b $2y + 3$
c $xy - 5$
d $2(x + y)$

3 Decide whether the following pairs of terms are like terms.

a $6x$ and 8
b $3a$ and $7a$
c $4xy$ and $2yx$
d $3x^2$ and $10x$

4 Simplify:

a $3m + 5m$
b $8ab - 3ab$
c $4x + 3y + 2x + 5y$
d $2 \times 4 \times x$
e $5 \times a \times 3 \times b$
f $6y \div 2$

5 Expand:

a $2(x + 5)$
b $3(y - 2)$
c $4(2x - 3)$
d $x(3x + 1)$

6 Find the HCF (highest common factor) of these pairs of terms.

a 8, 12
b 18, 30
c $7a$, $14a$
d $2xy$ and $8xz$
e $5x$ and $8x^2$
f $7x^2y$ and $21xy^2$

7 Simplify:

a $\frac{3}{8} + \frac{2}{5}$
b $\frac{6}{7} - \frac{1}{3}$
c $\frac{5}{9} \times \frac{6}{25}$
d $\frac{2}{3} \div \frac{4}{9}$

8 Write each of the following in index form (e.g. $5 \times 5 \times 5 = 5^3$).

a $7 \times 7 \times 7 \times 7$
b $m \times m \times m$
c $x \times x \times y \times y \times y$
d $3a \times 3a \times 3a \times 3a \times 3a$

9 Evaluate:

a 7^2
b 3^3
c 2^4
d 4^3

10 Write the following as 3 raised to a single power.

a $3^4 \times 3^5$
b $3^7 \div 3^5$
c $(3^2)^5$
d 1
e $\frac{1}{3^2}$

11 Complete the following.

a $3.8 \times 10 =$ __________
b $2.31 \times 1000 =$ __________
c $17.2 \div 100 =$ __________
d $0.18 \div 100 =$ __________
e $3827 \div$ __________ $= 3.827$
f $6.49 \times$ __________ $= 64\,900$

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3A Algebraic expressions

CONSOLIDATING

Learning intentions

- To know the names of the parts of an algebraic expression
- To be able to form algebraic expressions from simple word problems
- To be able to evaluate expressions by substituting given values

Key vocabulary: expression, pronumeral, variable, term, constant term, coefficient, substitute, evaluate

Algebra involves the use of pronumerals (also called variables), which are letters that represent numbers. Numbers and pronumerals connected by multiplication or division form *terms*, and *expressions* are one or more terms connected by addition or subtraction.

If a ticket to an art gallery costs $\$12$, then the cost for y visitors is the expression $12 \times y = 12y$. By substituting values for y we can find the costs for different numbers of visitors. For example, if there are five visitors, then $y = 5$ and $12y = 12 \times 5 = 60$. So total cost $= \$60$.

Lesson starter: Expressions at the gallery

Ben, Alea and Victoria are visiting the art gallery. The three of them combined have $\$c$ between them. Drinks cost $\$d$ and Ben has bought x postcards in the gift shop.

Write expressions for the following.

- The cost of two drinks
- The amount of money each person has if the money is shared equally
- The number of postcards Alea and Victoria bought if Alea bought three more than Ben and Victoria bought five less than twice the number Ben bought

Key ideas

- A **pronumeral** (or **variable**) is a letter used to represent an unknown number.
- Algebraic **expressions** are made up of one or more terms connected by addition or subtraction; e.g. $3a + 7b$, $\frac{x}{2} + 3y$, $3x - 4$.
  - A **term** is a group of numbers and pronumerals connected by multiplication and division; e.g. $2x$, $\frac{y}{4}$, $5x^2$.
  - A **constant term** is a number with no attached pronumerals; e.g. 7, −3.
  - The **coefficient** is the number multiplied by the pronumerals in the term;
    e.g. 3 is the coefficient of y in $2x + 3y$.
    −4 is the coefficient of x in $5 - 4x$.
    1 is the coefficient of x^2 in $2x + x^2$.

    This expression has 3 terms: $3x$, $2y$ and 4.

    $$3x - 2y + 4$$

    3 is the coefficient of x; −2 is the coefficient of y; 4 is the constant term

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- Operations in algebraic expressions:
  - The operations for addition and subtraction are written with '+' and '−'.
  - Multiplication is written without the sign; e.g. $3 \times y = 3y$.
  - Division is written as a fraction; e.g. $y \div 4 = \frac{y}{4}$ or $\frac{1}{4}y$.
- To find the value of an expression (or to **evaluate**), **substitute** a value for each pronumeral. The order of operations (BODMAS) is followed. For example, if $x = 2$ and $y = 3$:

$$\begin{aligned} 4xy - y^2 &= 4 \times 2 \times 3 - 3^2 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

Exercise 3A

Understanding | 1–3 | 1

1 Fill in the missing word(s) in the sentences, using the words *expression, term, constant term* or *coefficient*.

a An algebraic ____________ is made up of one or more terms connected by addition and subtraction.

b A term without a pronumeral part is a ____________.

c A number multiplied by the pronumerals in a term is a ____________.

d Numbers and pronumerals connected by multiplication and division form a ____________.

2 Express in simplified mathematical form

a x plus 3 **b** $5 \times y$ **c** $a \div 5$ **d** $2 \times x \times y$

3 Substitute the value 3 for the pronumeral x in the following and evaluate.

a $x + 4$ **b** $5x$ **c** $8 - x$ **d** x^2 **e** $\frac{18}{x}$

Fluency | 4, 5, 6(½) | 4, 5–6(½)

Example 1 Naming parts of an expression

Consider the expression $\frac{xy}{2} - 4x + 3y^2 - 2$. Determine:

a the number of terms **b** the constant term

c the coefficient of:

i y^2 **ii** x

Solution	Explanation
a 4	There are four terms with different combinations of pronumerals and numbers, separated by + or −.
b −2	The term with no pronumerals is −2. The negative is included.
c i 3	The number multiplied by y^2 in $3y^2$ is 3.
ii −4	The number multiplied by x in $-4x$ is −4. The negative sign belongs to the term that follows.

Continued on next page

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Now you try

Consider the expression $4y - \frac{x}{3} - 2x^2 + 1$. Determine:

a the number of terms
b the constant term
c the coefficient of:
  i y
  ii x^2

4 For these algebraic expressions, determine:
  i the number of terms
  ii the constant term
  iii the coefficient of y

a $4xy + 5y + 8$
b $2xy + \frac{1}{2}y^2 - 3y + 2$
c $2x^2 - 4 + y$

Hint: The coefficient is the number multiplied by the pronumerals in each term. The constant term has no pronumerals.

Example 2 Writing algebraic expressions

Write algebraic expressions for the following.

a three more than x
b 4 less than 5 times y
c the sum of c and d is divided by 3
d the product of a and the square of b

Solution	Explanation
a $x + 3$	More than means add (+).
b $5y - 4$	Times means multiply ($5 \times y = 5y$) and less than means subtract (−).
c $\frac{c+d}{3}$	Sum c and d first (+), then divide by 3 (÷). Division is written as a fraction.
d ab^2	'Product' means 'multiply'. The square of b is b^2 (i.e. $b \times b$). $a \times b^2 = ab^2$

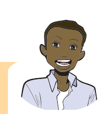
Now you try

Write algebraic expressions for the following.

a five more than y
b 7 less than 3 times x
c the sum of a and b is divided by 5
d the product of x and the square of y

5 Write an expression for the following.

a two more than x
b four less than y
c the sum of ab and y
d three less than 2 lots of x
e the product of x and 5
f twice m
g three times the value of r
h half of x
i three-quarters of m
j the quotient of x and y
k the sum of a and b is divided by 4
l the product of the square of x and y



Hint: Quotient means ÷. Product means ×
$\frac{1}{3}y = \frac{y}{3}$.

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3A

Example 3 Substituting values

Find the value of these expressions when $x = 2$, $y = 3$ and $z = -5$.

a $xy + 3y$ b $y^2 - \frac{8}{x}$ c $2x - yz$

Solution	Explanation
a $xy + 3y = 2 \times 3 + 3 \times 3$ $= 6 + 9$ $= 15$	Substitute for each pronumeral: $x = 2$ and $y = 3$. Recall that $xy = x \times y$ and $3y = 3 \times y$. Simplify, following order of operations, by multiplying first.
b $y^2 - \frac{8}{x} = 3^2 - \frac{8}{2}$ $= 9 - 4$ $= 5$	Substitute $y = 3$ and $x = 2$. $3^2 = 3 \times 3$ and $\frac{8}{2} = 8 \div 2$. Do subtraction last.
c $2x - yz = 2 \times 2 - 3 \times (-5)$ $= 4 - (-15)$ $= 4 + 15$ $= 19$	Substitute for each pronumeral. $3 \times (-5) = -15$ To subtract a negative number, add its opposite.

Now you try

Find the value of these expressions when $x = 6$, $y = -2$ and $z = 4$.

a $xz + 2x$ b $x^2 + \frac{z}{2}$ c $3z - xy$

6 Find the value of these expressions when $a = 4$, $b = -2$ and $c = 3$.

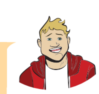
a ac b $2a - 5$ c $3a - c$

d $a^2 - 2c$ e $ac + b$ f $3b + a$

g $ab + c^2$ h $\frac{a}{2} - b$ i $\frac{ac}{b}$

j $2a - b$ k $a + bc$ l $\frac{6bc}{a}$

Hint:
$12 + (-2) = 12 - 2$
$2 - (-2) = 2 + 2$



Problem-solving and reasoning 7–9 8–11

7 Write an expression for the following.

a The cost of 5 pencils at x cents each
b The cost of y apples at 35 cents each
c One person's share when \$500 is divided among n people
d The cost of a pizza (\$11) equally shared between m people
e Parvinda's age in x years' time if he is 11 years old now

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8 A taxi in Sydney has a pick-up charge (i.e. flagfall) of \$3.40 and charges \$2 per km.

a Write an expression for the taxi fare for a trip of d kilometres.

b Use your expression in part **a** to find the cost of a trip that is:

i 10 km

ii 22 km

Hint: The taxi fare has initial cost + cost per km × number of km.

9 **a** Ye thinks of a number, which we will call x.
Now write an expression for each of the following stages.

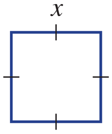
i He doubles the number.

ii He decreases the result by 3.

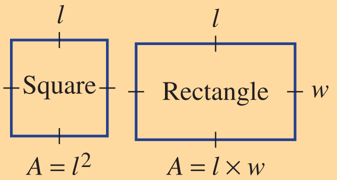
iii He multiplies the result by 3.

b If $x = 5$, use your answer to part **a iii** to find the final number.

10 A square with side length x is changed to a rectangle by increasing the length by 1 and decreasing the width by 1.



Hint: Perimeter is the sum of the side lengths.



a Write an expression for the new length and width of the rectangle.

b Is there any change in the perimeter of the shape?

c **i** Write an expression for the area of the rectangle.

ii Use trial and error to determine whether the area of the rectangle is more or less than the original square. By how much?

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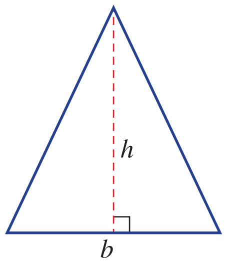
3A

11 The area of a triangle is given by $A = \frac{1}{2}bh$.



a If $b = 6$ and $h = 7$, what is the area?

b If the area is 9, what are the possible whole number values for b if h is also a whole number?

Area and perimeter

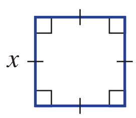
Area and perimeter	—	12

12 For the shapes shown, write an expression for:

i the perimeter

ii the area

a



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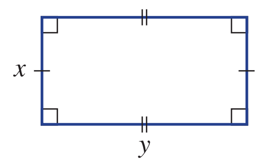

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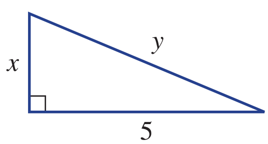

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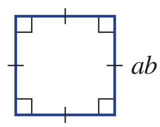

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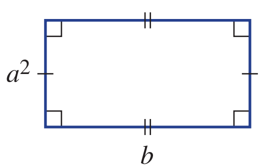

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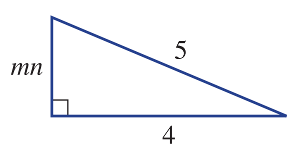

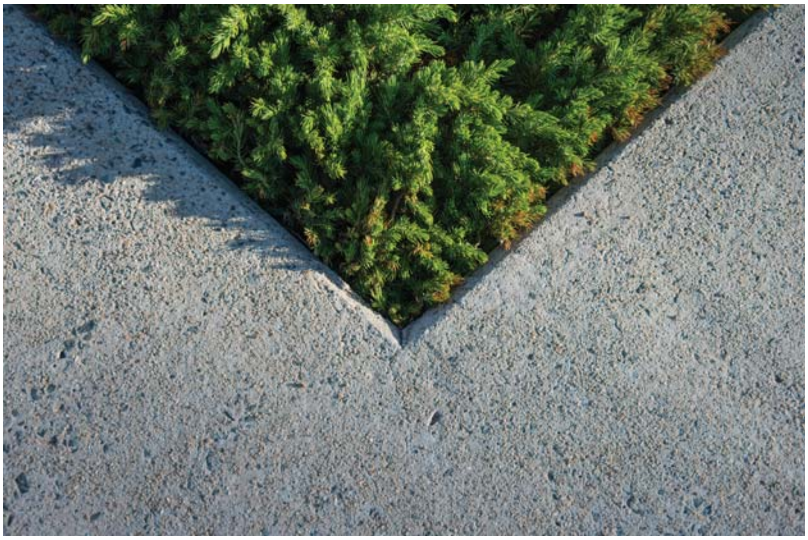

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3B Simplifying algebraic expressions

Learning intentions

- To be able to identify like terms
- To know that only like terms can be combined under addition and subtraction
- To be able to simplify algebraic expressions using the four operations: +, −, × and ÷

Key vocabulary: like terms, pronumeral

Many areas of finance and industry involve complex algebraic expressions. Often these expressions can be made simpler by applying the rules of addition, subtraction, multiplication and division.

Just as we would write $3 + 3 + 3 + 3$ as 4×3, we write $x + x + x + x$ as $4 \times x$ or $4x$. Similarly, $3x + 2x = 5x$ and $3x - 2x = 1x$ ($1x$ is written as x).

We also know that $2 \times 3 = 3 \times 2$ and $(2 \times 3) \times 4 = 2 \times 3 \times 4 = 3 \times 4 \times 2$ etc., so $2 \times x \times 4 = 2 \times 4 \times x = 8x$. By writing a division as a fraction we can also cancel common factors. For example, $9x \div 3 = \frac{9x}{3} = 3x$.

Lesson starter: Equivalent expressions

Split these expressions into two groups that are equivalent by simplifying them first.

$3x + 6x$	$17x - 5x$	$x + 7x + x$	$4x + 3 + 5x - 3$
$2 \times 6x$	$\frac{24xy}{2y}$	$3x \times 3$	$3x - 2y + 9x + 2y$
$8x + 6x - 2x$	$18x \div 2$	$\frac{9x^2}{x}$	$6x - (-6x)$

Key ideas

- **Like terms** have the exact same pronumeral factors, including powers; e.g. $3x$ and $7x$, and $4x^2y$ and $-3x^2y$.
  - Since $x \times y = y \times x$, $3xy$ and $2yx$ are like terms.
- Addition and subtraction apply to like terms only.
  For example, $5x + 7x = 12x$
  $7ab - 6ab = 1ab = ab$
  $3x + 2y$ cannot be simplified
- Multiplication and division apply to all terms.
  - In multiplication, deal with numerals and pronumerals separately:
    $2 \times 8a = 2 \times 8 \times a = 16a$
    $6x \times 3y = 6 \times 3 \times x \times y = 18xy$
- When dividing, write as a fraction and cancel common factors:

$$\frac{\cancel{8}^4 x}{\cancel{2}^1} = 4x$$

$$6x^2 \div (3x) = \frac{6x^2}{3x} = \frac{\cancel{6}^2 \times \cancel{x}^1 \times x}{\cancel{3}^1 \times \cancel{x}^1} = 2x$$

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Exercise 3B

Understanding | 1–4 | 4

1 Are the following sets of terms like terms? Answer yes (Y) or no (N).

a $3x,\ 2x,\ -5x$

b $2ax,\ 3xa,\ -ax$

c $2ax^2,\ 2ax,\ 62a^2x$

d $\frac{3}{4}x^2,\ 2x^2,\ \frac{x^2}{3}$

2 Simplify the following.

a $8g + 2g$

b $3f + 2f$

c $12e - 4e$

d $3h - 3h$

e $5x + x$

f $14st + 3st$

Hint: Add or subtract the numerals in like terms.

3 Simplify the following.

a $3 \times 2x$

b $4 \times 3a$

c $2 \times 5m$

d $-3 \times 6y$

4 Simplify these fractions by cancelling.

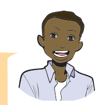
a $\frac{4}{8}$

b $\frac{12}{3}$

c $\frac{14}{21}$

d $\frac{35}{15}$

Hint: Choose the highest common factor to cancel.



Fluency | 5–8(½) | 5–8(½)

Example 4 Identifying like terms

Write down the like terms in the following lists.

a $3x,\ 6a,\ 2ax,\ 3a,\ 5xa$

b $-2ax,\ 3x^2a,\ 3a,\ -5x^2a,\ 3x$

Solution	Explanation
a $6a$ and $3a$	Both terms contain a.
$5xa$ and $2ax$	Both terms contain ax; $x \times a = a \times x$.
b $3x^2a$ and $-5x^2a$	Both terms contain x^2a.

Now you try

Write down the like terms in the following lists.

a $4a,\ 3b,\ 5ab,\ 2a,\ 2ba$

b $-x^2y,\ 3x^2,\ 2xy,\ 4x,\ 4x^2y$

5 Write down the like terms in the following lists.

a $3ac,\ 2a,\ 5x,\ -2ac$

b $4pq,\ 3qp,\ 2p^2,\ -4p^2q$

c $7x^2y,\ -3xy^2,\ 2xy^2,\ 4yx^2$

d $2r^2,\ 3rx,\ -r^2,\ 4r^2x$

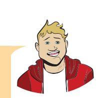
e $-2ab,\ 5bx,\ 4ba,\ 7xa$

f $3p^2q,\ -4pq^2,\ \frac{1}{2}pq,\ 4qp^2$

g $\frac{1}{3}lm,\ 2l^2m,\ \frac{lm}{4},\ 2lm^2$

h $x^2y,\ yx^2,\ -xy,\ yx$

Hint: Like terms have the same pronumeral factors. $x \times y = y \times x$, so $3xy$ and $5yx$ are like terms.



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Example 5 Collecting like terms

Simplify the following.

a $4a + 5a + 3$ b $3x + 2y + 5x - 3y$ c $5xy + 2xy^2 - 2xy + xy^2$

Solution	Explanation
a $4a + 5a + 3 = 9a + 3$	Collect like terms ($4a$ and $5a$) and add coefficients.
b $3x + 2y + 5x - 3y = 3x + 5x + 2y - 3y$ $= 8x - y$	Collect like terms in x $(3 + 5 = 8)$ and y $(2 - 3 = -1)$. Note: $-1y$ is written as $-y$.
c $5xy + 2xy^2 - 2xy + xy^2$ $= 5xy - 2xy + 2xy^2 + xy^2$ $= 3xy + 3xy^2$	Collect like terms. In xy, the negative belongs to $2xy$. In xy^2, recall that xy^2 is $1xy^2$.

Now you try

Simplify the following.

a $7x + 3x + 2$ b $2a + 4b + 3a - 2b$ c $4mn + 3m^2n - mn + 2m^2n$

6 Simplify the following by collecting like terms.

a $4t + 3t + 10$ b $5g - g + 1$ c $3x - 5 + 4x$
d $4m + 2 - 3m$ e $2x + 3y + x$ f $3x + 4y - x + 2y$
g $8a + 4b - 3a - 6b$ h $2m - 3n - 5m + n$ i $3de + 3de^2 + 2de + 4de^2$
j $6kl - 4k^2l - 6k^2l - 3kl$ k $3x^2y + 2xy^2 - xy^2 + 4x^2y$ l $4fg - 5g^2f + 4fg^2 - fg$

Example 6 Multiplying algebraic terms

Simplify the following.

a $2a \times 7d$ b $-3m \times 8mn$

Solution	Explanation
a $2a \times 7d = 2 \times 7 \times a \times d$ $= 14ad$	Multiply coefficients and collect the pronumerals: $2 \times a \times 7 \times d = 2 \times 7 \times a \times d$. Multiplication can be done in any order.
b $-3m \times 8mn = -3 \times 8 \times m \times m \times n$ $= -24m^2n$	Multiply coefficients $(-3 \times 8 = -24)$ and pronumerals. Recall: $m \times m$ can be written as m^2.

Now you try

Simplify the following.

a $4x \times 5w$ b $-2a \times 6ac$

7 Simplify the following.

a $3r \times 2s$ b $2h \times 3u$ c $4w \times 4h$
d $2r^2 \times 3s$ e $-2e \times 4s$ f $5h \times (-2v)$
g $-3c \times (-4m^2)$ h $-7f \times (-5l)$ i $2x \times 4xy$
j $3ab \times 8a$ k $xy \times 3y$ l $-2a \times 8ab$
m $-3m^2n \times 4n$ n $-5xy^2 \times (-4x)$ o $5ab \times 4ab$

Hint: Multiply the numerals and collect the pronumerals.
$a \times b = ab$

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3B

Example 7 Dividing algebraic terms

Simplify the following.

a $\frac{18x}{6}$

b $12a^2b \div (8ab)$

Solution

a $\frac{\cancel{18}^{3}x}{\cancel{6}^{1}} = 3x$

Explanation: Cancel highest common factor of numerals; i.e. 6.

b $12a^2b \div (8ab) = \frac{12a^2b}{8ab}$

Explanation: Write division as a fraction.

$= \frac{{}^{3}\cancel{12} \times a \times \cancel{a}_1 \times \cancel{b}_1}{{}_{2}\cancel{8} \times \cancel{a}_1 \times \cancel{b}_1}$

Explanation: Cancel the highest common factor of 12 and 8 and cancel an a and b.

$= \frac{3a}{2}$

Now you try

Simplify the following.

a $\frac{20x}{4}$

b $9s^2t \div (15st)$

8 Simplify by cancelling common factors.

a $\frac{6a}{2}$

b $\frac{7x}{14}$

c $3a \div 9$

d $2ab \div 8$

e $\frac{4ab}{2a}$

f $\frac{15xy}{5y}$

g $4xy \div (8x)$

h $28ab \div (35b)$

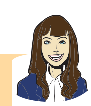
i $\frac{8x^2}{20x}$

j $\frac{12xy^2}{18y}$

k $30a^2b \div (10a)$

l $12mn^2 \div (36mn)$

Hint: Write each division as a fraction first where necessary.



Problem-solving and reasoning

9, 10 | 9–12

9 A rectangle's length is three times its width, x. Write a simplified expression for:

a the rectangle's perimeter

b the rectangle's area

Hint: Draw a rectangle and label the width x and the length $3 \times x = 3x$.

10 Fill in the missing term to make the following true.

a $8x + 4 - \square = 3x + 4$

b $3x + 2y - \square + 4y = 3x - 2y$

c $3b \times \square = 12ab$

d $4xy \times (\square) = -24x^2y$

e $12xy \div (\square) = 6y$

f $\square \div (15ab) = \frac{2a}{3}$

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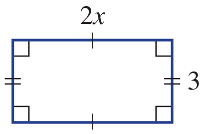
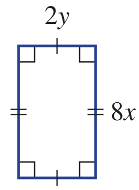
11 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes.

a



b



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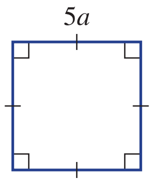
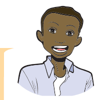
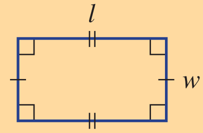
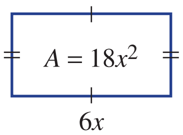




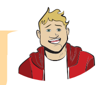
Hint: Perimeter is the sum of all the sides.
Area = $l \times w$



12 A rectangular garden bed has length given by $6x$ and area $18x^2$.
What is the width of the garden bed?



Hint: The opposite of × is ÷.



Order of operations | — | 13

13 Simplify the following expressions, using order of operations.

a $4 \times 3x \div 2$

b $2 + 4a \times 2 + 5a \div a$

c $5a \times 2b \div a - 6b$

d $8x^2 \div (4x) + 3 \times 3x$

e $2x \times (4x + 5x) \div 6$

f $5xy - 4x^2y \div (2x) + 3x \times 4y$

g $(5x - x) \times (16xy \div (8y))$

h $9x^2y \div (3y) + 4x \times (-8x)$

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3C Expanding algebraic expressions

Learning intentions

- To understand the distributive law for expanding brackets
- To be able to expand expressions involving brackets

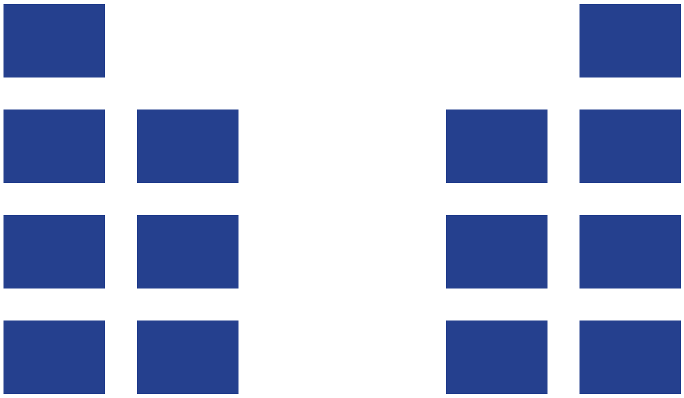
Key vocabulary: distributive law, expand

When an expression is multiplied by a term, each term in the expression must be multiplied by the term. Brackets are used to show this. For example, to double $4+3$ we write $2\times(4+3)$, and each term within the brackets (both 4 and 3) must be doubled. The expanded version of this expression is $2\times4+2\times3$.

Similarly, to double the expression $x+1$, we write $2(x+1)=2\times x+2\times 1$. This expansion of brackets uses the distributive law.

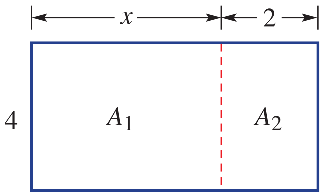
In this diagram, 7 blue blocks are doubled in groups of 4 and 3.

$2\times(4+3)=2\times4+2\times3$



Lesson starter: Rectangle brackets

Consider the diagram shown.



- Write an expression for the rectangle area A_1.
- Write an expression for the rectangle area A_2.
- Add your results for A_1 and A_2 to give the area of the rectangle.
- Write an expression for the total length of the rectangle.
- Using the total length, write an expression for the area of the rectangle.
- Combine your results to complete this statement: $4(x+2)=\square+\square$.

Key ideas

- The **distributive law** is used to **expand** and remove brackets:
  - The terms inside the brackets are multiplied by the term outside the brackets.

$$a(b+c)=ab+ac \qquad a(b-c)=ab-ac$$

For example, $2(x+4)=2\times x+2\times 4$
$\qquad\qquad\qquad\quad =2x+8$

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Exercise 3C

Understanding | 1, 2 | 2

1 The distributive law says that each term inside the ____________ is multiplied by the term ____________ the brackets.

2 Complete the following.

a $3(x+4) = 3 \times \square + 3 \times \square$
$= 3x + \square$

b $2(x-5) = 2 \times \square + \square \times (-5)$
$= \square - 10$

c $2(4x+3) = 2 \times \square + \square \times 3$
$= \square + 6$

d $x(x-3) = x \times \square + \square \times \square$
$= \square - \square$

Fluency | 3–5(½) | 3–5(½)

Example 8 Expanding expressions with brackets

Expand the following.

a $2(x+5)$ **b** $3(2x-3)$ **c** $3y(2x+4y)$

Solution	Explanation
a $2(x+5) = 2 \times x + 2 \times 5$ $= 2x + 10$	Multiply each term inside the brackets by 2.
b $3(2x-3) = 3 \times 2x + 3 \times (-3)$ $= 6x - 9$	Multiply $2x$ and -3 by 3. $3 \times 2x = 3 \times 2 \times x = 6x$.
c $3y(2x+4y) = 3y \times 2x + 3y \times 4y$ $= 6xy + 12y^2$	Multiply $2x$ and $4y$ by $3y$. $3y \times 2x = 3 \times 2 \times x \times y$ and $3y \times 4y = 3 \times 4 \times y \times y$. Recall: $y \times y$ is written as y^2.

Now you try

Expand the following.

a $3(x+4)$ **b** $5(3x-2)$ **c** $4a(2a+5b)$

3 Expand the following.

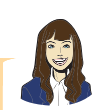
a $2(x+4)$ **b** $3(x+7)$ **c** $4(y-3)$

d $5(y-2)$ **e** $2(3x+2)$ **f** $4(2x+5)$

g $3(3a-4)$ **h** $7(2y-5)$ **i** $5(2a+b)$

j $3(4a-3b)$ **k** $2x(x+5)$ **l** $3x(x-4)$

m $2a(3a+2b)$ **n** $2y(3x-4y)$ **o** $3b(2a-5b)$



Hint: Use the distributive law:

$a(b+c) = a \times b + a \times c$
$= ab + ac$

$a(b-c) = a \times b + a \times (-c)$
$= ab - ac$

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3C

Example 9 Expanding expressions with a negative out the front

Expand the following.

a $-3(x-4)$

b $-2x(3x-2y)$

Solution

a $-3(x-4) = -3 \times x + (-3) \times (-4)$
$= -3x + 12$

Explanation

Multiply each term inside the brackets by -3.
$-3 \times (-4) = +12$
If there is a negative sign outside the bracket, the sign of each term inside the brackets is changed when expanded.

Solution

b $-2x(3x-2y) = -2x \times 3x + (-2x) \times (-2y)$
$= -6x^2 + 4xy$

Explanation

$-2x \times 3x = -2 \times 3 \times x \times x$ and $-2x \times (-2y)$
$= -2 \times (-2) \times x \times y$

Now you try

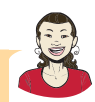
Expand the following.

a $-4(x-5)$

b $-3y(2x-4y)$

4 Expand the following.

a $-2(x+3)$
b $-5(m+2)$
c $-3(w+4)$
d $-4(x-3)$
e $-2(m-7)$
f $-7(w-5)$
g $-(x+y)$
h $-(x-y)$
i $-2x(3x+4)$
j $-3x(2x+5)$
k $-4x(2x-2)$
l $-3y(2y-9)$
m $-2x(3x-5y)$
n $-3x(3x+2y)$
o $-6y(2x+3y)$



Hint: A negative out the front will change the sign of each term in the brackets when expanded.
$-2(x-3) = -2x+6$

Example 10 Simplifying expressions by removing brackets

Expand and simplify the following.

a $8+3(2x-3)$

b $3(2x+2)-4(x+4)$

Solution

a $8+3(2x-3) = 8+6x-9$
$= 6x-1$

Explanation

Expand the brackets first: $3 \times 2x + 3 \times (-3) = 6x - 9$.

Collect like terms: $8 - 9 = -1$.

Solution

b $3(2x+2)-4(x+4) = 6x+6-4x-16$
$= 2x-10$

Explanation

Expand the brackets first. Note that
$-4(x+4) = -4 \times x + (-4) \times 4 = -4x - 16$.

Collect like terms: $6x - 4x = 2x$ and $6 - 16 = -10$.

Now you try

Expand and simplify the following.

a $5+2(4a-3)$

b $5(y+3)-2(2y+5)$

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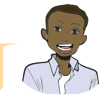
5 Expand and simplify the following.



Hint: Expand first, then collect like terms.

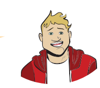
a $2+5(x+3)$ **b** $3+7(x+2)$
c $5+2(x-3)$ **d** $7-2(x+3)$
e $21-5(x+4)$ **f** $4+3(2x-1)$
g $3+2(3x+4)$ **h** $8-2(2x-3)$ **i** $12-3(2x-5)$
j $3(x+2)+4(x+3)$ **k** $2(p+2)+5(p-3)$ **l** $4(x-3)+2(3x+4)$
m $3(2s+3)-2(s+2)$ **n** $4(3f+2)-2(6f+2)$ **o** $3(2x-5)-2(2x-4)$

Problem-solving and reasoning | 6, 7 | 6–9

6 Fill in the missing term/number to make each statement true.

a $\square(x+4)=2x+8$ **b** $\square(2x-3)=8x-12$ **c** $\square(2x+3)=6x^2+9x$
d $4(\square+5)=12x+20$ **e** $4y(\square-\square)=4y^2-4y$ **f** $-2x(\square+\square)=-4x^2-6xy$

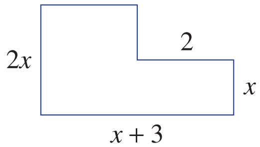
7 Four rectangular rooms in a house have floor side lengths listed below. Find an expression for the area of each floor in expanded form.

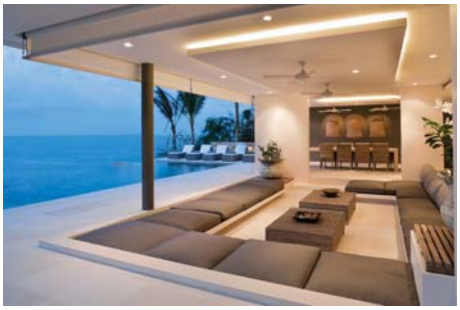


Hint: Area of a rectangle = length × width

a 2 and $x-5$ **b** x and $x+3$
c $2x$ and $x+4$ **d** $3x$ and $2x-1$

8 The deck on a house is constructed in the shape shown. Find the area of the deck in expanded form. (All lines meet at 90°.)





Hint: To find 10% of an amount, multiply by $\frac{10}{100}=0.1$.

9 Virat earns $\$x$ but does not have to pay tax on the first $\$18\,200$.

a Write an expression for the amount of money Virat is taxed on.
b Virat is taxed 10% of his earnings in part **a**. Write an expanded expression for how much tax he pays.

Expanding binomial products | — | 10

10 A rectangle has dimensions $(x+2)$ by $(x+3)$, as shown. The area can be found by summing the individual areas:

	x	3
x	x^2	$3x$
2	$2x$	6

$$(x+2)(x+3)=x^2+3x+2x+6$$
$$=x^2+5x+6$$

This can also be done using the distributive law:

$$(x+2)(x+3)=x(x+3)+2(x+3)$$
$$=x^2+3x+2x+6$$
$$=x^2+5x+6$$

Expand and simplify these binomial products using this method.

a $(x+4)(x+3)$ **b** $(x+3)(x+1)$ **c** $(x+2)(x+5)$
d $(x+2)(x-4)$ **e** $(x+5)(x-2)$ **f** $(x+4)(2x+3)$
g $(2x+3)(x-2)$ **h** $(x-3)(x+4)$ **i** $(4x-2)(x+5)$

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3D Factorising algebraic expressions

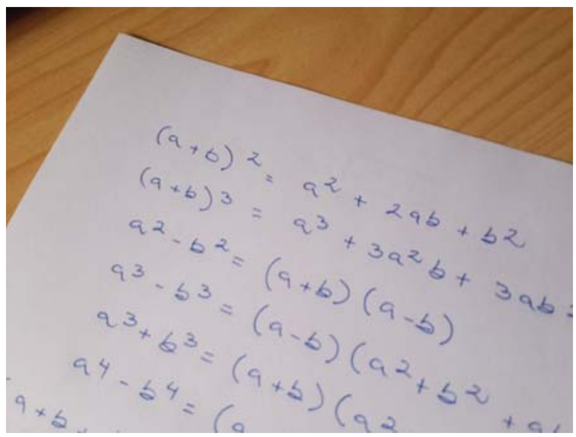
Learning intentions

- To be able to identify the highest common factor of terms
- To know the form of a factorised expression
- To understand that factorising and expanding are reverse processes
- To be able to factorise algebraic expressions involving a common factor

Key vocabulary: highest common factor, factorise, term

Factorising is an important step in solving many types of equations and in simplifying algebraic expressions.

Just as 15 can be expanded and written as 3×5, we can factorise to write an algebraic expression as the product of its factors. Factorising is therefore the opposite of expanding.



Lesson starter: Products of factors

- Expand the product $6(2x + 4)$.
- Write as many products as you can (using whole numbers) that give the same result as $6(2x + 4)$ when expanded.
- Which of your products has the highest number in front of the brackets? What is this number?
- How does this number relate to the two terms in the expanded form?
- Write a product of factors that expand to $18x + 24$, using the highest common factor.

Key ideas

- **Factorising** involves writing an expression as a product.
- Factorisation is the opposite process of expansion.
- To factorise an expression, take out the **highest common factor (HCF)** of each of the terms. The highest common factor is the largest number, pronumeral or product of these that divides into each term.
  - Divide each term by the HCF and leave the expression in the brackets.
  - A factorised expression can be checked by expanding to get the original expression.
  - If the HCF has been removed, the terms in the brackets should have no common factors; e.g. $2(x + 3)$ is fully factorised, but $2(4x + 6)$ is not because 2 can still be divided into both 4 and 6 within the brackets.

    For example: $3x + 12 = 3(x + 4)$ HCF: 3

    $2x^2 + 8x = 2x(x + 4)$ HCF: $2x$

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Exercise 3D

Understanding

	1–3	3

1 Write down the highest common factor (HCF) of these pair of numbers.
- a 10 and 16
- b 9 and 27
- c 14 and 35
- d 36 and 48

2 State true (T) or false (F) if the first expression is the factorised form of the second expression. Confirm by expanding.
- a $3(x+2)$, $3x+6$
- b $-2(x-4)$, $-2x-8$

3 Consider the expression $4x^2+8x$.
- a Which of the following factorised forms uses the HCF?
  A $2(2x^2+4x)$ B $4(x^2+8x)$ C $4x(x+2)$ D $2x(2x+4)$
- b What can be said about the terms inside the brackets once the HCF is removed, which is not the case for the other forms?

Fluency

	4–7(½)	4–7(½)

Example 11 Finding the HCF

Determine the HCF of the following.
a $8a$ and 20 b $3x$ and $6x$ c $10a^2$ and $15ab$

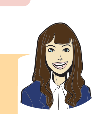
Solution	Explanation
a HCF of $8a$ and 20 is 4.	Compare numerals and pronumerals separately. The highest common factor (HCF) of 8 and 20 is 4. a is not a common factor.
b HCF of $3x$ and $6x$ is $3x$.	HCF of 3 and 6 is 3. x is also a common factor.
c HCF of $10a^2$ and $15ab$ is $5a$.	HCF of 10 and 15 is 5. HCF of a^2 and ab is a.

Now you try

Determine the HCF of the following.
a $10x$ and 25 b $7x$ and $14x$ c $9yz$ and $15y^2$

4 Determine the HCF of the following.
- a $6x$ and 12
- b 10 and $15y$
- c $8a$ and $12b$
- d $9x$ and $18y$
- e $5a$ and $20a$
- f $10m$ and $22m$
- g $14x$ and $21x$
- h $8a$ and $40ab$
- i $3a^2$ and $9ab$
- j $4x^2$ and $10x$
- k $16y$ and $24xy$
- l $15x^2y$ and $25xy$

Hint: Find the HCF of the numeral and variable factors.



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3D

Example 12 Factorising simple expressions

Factorise the following.

a $4x + 20$

b $6a - 15b$

Solution	Explanation
a $4x + 20 = 4(x + 5)$	HCF of $4x$ and 20 is 4. Place 4 in front of the brackets and divide each term by 4. Expand to check: $4(x + 5) = 4x + 20$.
b $6a - 15b = 3(2a - 5b)$	HCF of $6a$ and $15b$ is 3. Place 3 in front of the brackets and divide each term by 3.

Now you try

Factorise the following.

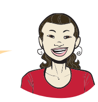
a $3x + 15$

b $12m - 18n$

5 Factorise the following.

a $3x + 9$ b $4x - 8$ c $10y - 20$
d $6a + 30$ e $5x + 5y$ f $12a + 4b$
g $18m - 27n$ h $36x - 48y$ i $8x + 44y$
j $24a - 18b$ k $121m + 55n$ l $14k - 63l$

Hint: Check your answer by expanding.
$3(x + 3) = 3x + 9$



Example 13 Factorising expressions with pronumeral common factors

Factorise the following.

a $8y + 12xy$

b $4x^2 - 10x$

Solution	Explanation
a $8y + 12xy = 4y(2 + 3x)$	HCF of 8 and 12 is 4, HCF of y and xy is y. Place $4y$ in front of the brackets and divide each term by $4y$. Check that $4y(2 + 3x) = 8y + 12xy$.
b $4x^2 - 10x = 2x(2x - 5)$	HCF of $4x^2$ and $10x$ is $2x$. Place $2x$ in front of the brackets and divide each term by $2x$. Recall: $x^2 = x \times x$.

Now you try

Factorise the following.

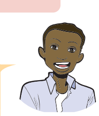
a $9a + 24ab$

b $15x^2 - 35x$

6 Factorise the following.

a $14x + 21xy$ b $6ab - 15b$ c $32y - 40xy$
d $5x^2 - 5x$ e $x^2 + 7x$ f $2a^2 + 8a$
g $12a^2 + 42ab$ h $9y^2 - 63y$ i $6x^2 + 14x$
j $9x^2 - 6x$ k $16y^2 + 40y$ l $10m - 40m^2$

Hint: Place the HCF in front of the brackets and divide each term by the HCF: $14x + 21xy = 7x(\_\_\_ + \_\_\_)$



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Example 14 Factorising expressions by removing a common negative

Factorise $-10x^2 - 18x$.

Solution	Explanation
$-10x^2 - 18x = -2x(5x + 9)$	The HCF of $-10x^2$ and $-18x$ is $-2x$, including the common negative. Place $-2x$ in front of the brackets and divide each term by $-2x$. Dividing by a negative changes the sign of each term.

Now you try

Factorise $-8y^2 - 36y$.

7 Factorise the following, including the common negative.

a $-2x - 6$ **b** $-4a - 8$ **c** $-3x - 6y$
d $-7a - 14ab$ **e** $-x - 10xy$ **f** $-3b - 12ab$
g $-x^2 - 7x$ **h** $-4x^2 - 12x$ **i** $-2y^2 - 10y$
j $-8x^2 - 14x$ **k** $-12x^2 - 8x$ **l** $-15a^2 - 5a$

Hint: Dividing by a negative changes the sign of the term.

Problem-solving and reasoning

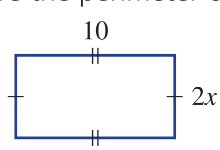
	8, 9	8(½), 9–11

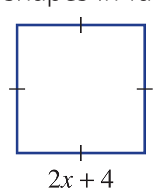
8 Factorise these mixed expressions.

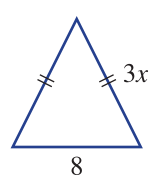
a $7a^2b + ab$ **b** $4a^2b + 20a^2$ **c** $xy - xy^2$
d $x^2y + 4x^2y^2$ **e** $6mn + 18mn^2$ **f** $5x^2y + 10xy^2$
g $-y^2 - 8yz$ **h** $-3a^2b - 6ab$ **i** $-ab^2 - a^2b$

Hint: Be sure to find the highest common factor first.

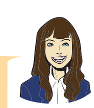
9 Give the perimeter of these shapes in factorised form.

a


b


c


Hint: Find the perimeter first, then factorise.



10 A square sandpit has perimeter $(4x + 12)$ metres. What is the side length of the square?

11 Common factors from expressions involving more than two terms can be removed in a similar way. Factorise these by taking out the HCF.

a $2x + 4y + 6z$ **b** $3x^2 + 12x + 6$ **c** $4x^2 + 8xy + 12$
d $6x^2 + 3xy - 9x$ **e** $10xy - 5xz + 5x$ **f** $4y^2 - 18y + 14xy$

Hint: $4a + 6b + 10c = 2(2a + 3b + 5c)$

Taking out a binomial factor

	—	12

12 A common factor may be a binomial term, such as $(x + 1)$.
For example, $3(x + 1) + x(x + 1)$ has HCF $= (x + 1)$, so $3(x + 1) + x(x + 1) = (x + 1)(3 + x)$, where $(3 + x)$ is what remains when $3(x + 1)$ and $x(x + 1)$ are divided by $(x + 1)$.
Use the method above to factorise the following.

a $4(x + 2) + x(x + 2)$ **b** $x(x + 3) + 2(x + 3)$ **c** $x(x + 4) - 7(x + 4)$
d $x(2x + 1) - 3(2x + 1)$ **e** $2x(y - 3) + 4(y - 3)$ **f** $2x(x - 1) - 3(x - 1)$

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3E Multiplying and dividing algebraic fractions ★

Learning intentions

- To know that expressions must be factorised before common factors can be cancelled
- To be able to simplify algebraic fractions by cancelling common factors
- To be able to multiply and divide algebraic fractions

Key vocabulary: algebraic fraction, common factor, factorise, numerator, denominator, reciprocal

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors to simplify the calculation and dividing by multiplying by the reciprocal of a fraction.

The process of cancelling requires cancelling of factors, for example:

$$\frac{8}{12} = \frac{2 \times \cancel{4}^{1}}{3 \times \cancel{4}_{1}} = \frac{2}{3}$$

For algebraic fractions, you need to factorise the expressions to identify and cancel common factors.

Lesson starter: Expressions as products of their factors

Factorise these expressions to write them as a product of their factors. Fill in the blanks and simplify.

$$\frac{2x+4}{2} = \frac{\square(\square)}{2} = \square \qquad \frac{6x+9}{3} = \frac{\square(\square)}{3} = \square$$

$$\frac{x^2+2x}{x} = \frac{\square(\square)}{x} = \square \qquad \frac{4x+4}{4} = \frac{\square(\square)}{4} = \square$$

Describe the errors made in these factorisations.

$$\frac{{}^{1}\cancel{3}x+2}{\cancel{3}^{1}} = x+2 \qquad \frac{x^2+\cancel{3x}^{1}}{\cancel{3x}^{1}} = x^2+1 \qquad \frac{6\cancel{x}^{1}+6}{{}^{1}\cancel{x}+1} = \frac{12}{1} = 12$$

Key ideas

- An **algebraic fraction** is a fraction containing pronumerals as well as numbers.
- Simplify algebraic fractions by cancelling common factors in factorised form.

  For example, $\frac{4x+6}{2} = \frac{\cancel{2}_{1}(2x+3)}{\cancel{2}_{1}}$

  $= 2x+3$

- To multiply algebraic fractions:
  - Factorise expressions if possible.
  - Cancel common factors.
  - Multiply numerators and denominators together.

  $$\frac{\cancel{(x+1)}^{1}}{\cancel{10}_{2}} \times \frac{{}^{1}\cancel{5}x}{4\cancel{(x+1)}_{1}} = \frac{x}{8}$$

- To divide algebraic fractions:
  - Multiply by the **reciprocal** of the fraction following the division sign (e.g. the reciprocal of 6 is $\frac{1}{6}$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$).
  - Follow the rules for multiplication.

  $$\frac{2}{(x-2)} \div \frac{8}{3(x-2)}$$
  $$= \frac{\cancel{2}_{1}}{\cancel{(x-2)}_{1}} \times \frac{3\cancel{(x-2)}_{1}}{\cancel{8}_{4}}$$
  $$= \frac{3}{4}$$

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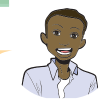
Exercise 3E

Understanding

1–3 | 2, 3

1 Write these fractions in simplest form by cancelling common factors.

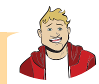
a $\frac{14}{21}$ b $\frac{9}{12}$ c $\frac{8x}{20}$ d $\frac{4x}{10}$



Hint: Be sure to cancel the *highest* common factor.

2 Write the reciprocal of these fractions.

a $\frac{3}{2}$ b $\frac{5x}{3}$ c 7 d $\frac{x+3}{4}$



Hint: The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

3 Follow the rules for multiplication and division to simplify these numeric fractions. Cancel common factors before multiplying.

a $\frac{15}{21}\times\frac{14}{25}$ b $\frac{4}{27}\div\frac{16}{9}$

Fluency

4–7(½) | 4–7(½)

Example 15 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{8xy}{12x}$ b $\frac{3(x+2)}{6(x+2)}$

Solution	Explanation
a $\frac{8xy}{12x}=\frac{{}^{2}\cancel{8}\times\cancel{x}^{1}\times y}{{}^{3}\cancel{12}\times\cancel{x}^{1}}$ $=\frac{2y}{3}$	Cancel the highest common factor of 8 and 12 (i.e. 4) and cancel the x.
b $\frac{3(x+2)}{6(x+2)}=\frac{{}^{1}\cancel{3}\times\cancel{(x+2)}^{1}}{{}^{2}\cancel{6}\times\cancel{(x+2)}^{1}}$ $=\frac{1}{2}$	Cancel the highest common factors: 3 and $(x+2)$.

Now you try

Simplify by cancelling common factors.

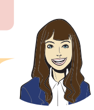
a $\frac{18ab}{8b}$ b $\frac{5(x-1)}{15(x-1)}$

4 Simplify by cancelling common factors.

a $\frac{6xy}{12x}$ b $\frac{12ab}{30b}$ c $\frac{8x^2}{40x}$ d $\frac{25x^2}{5x}$

e $\frac{3(x+1)}{3}$ f $\frac{7(x-5)}{7}$ g $\frac{4(x+1)}{8}$ h $\frac{5(x-2)}{x-2}$

i $\frac{4(x-3)}{x-3}$ j $\frac{6(x+2)}{12(x+2)}$ k $\frac{9(x+3)}{3(x+3)}$ l $\frac{15(x-4)}{10(x-4)}$



Hint: Cancel the HCF of the numerals and pronumerals.

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Example 16 Simplifying by factorising

Simplify these fractions by factorising first.

a $\dfrac{9x-12}{3}$ **b** $\dfrac{4x+8}{x+2}$

Solution	Explanation
a $\dfrac{9x-12}{3} = \dfrac{{}^{1}\cancel{3}(3x-4)}{\cancel{3}_{1}}$ $= 3x-4$	Factorise the expression in the numerator, which has HCF = 3. Then cancel the common factor of 3.
b $\dfrac{4x+8}{x+2} = \dfrac{4\cancel{(x+2)}_{1}}{\cancel{x+2}_{1}}$ $= 4$	4 is the HCF in the numerator. After factorising, $(x+2)$ can be seen as a common factor and can be cancelled.

Now you try

Simplify these fractions by factorising first.

a $\dfrac{16x-8}{8}$ **b** $\dfrac{3x-6}{x-2}$

5 Simplify these fractions by factorising first.

a $\dfrac{4x+8}{4}$ **b** $\dfrac{6a-30}{6}$ **c** $\dfrac{8y-12}{4}$

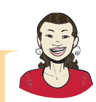
d $\dfrac{14b-21}{7}$ **e** $\dfrac{3x+9}{x+3}$ **f** $\dfrac{4x-20}{x-5}$

g $\dfrac{6x+9}{2x+3}$ **h** $\dfrac{12x-4}{3x-1}$ **i** $\dfrac{x^2+2x}{x}$

j $\dfrac{x^2-5x}{x}$ **k** $\dfrac{2x^2+6x}{2x}$ **l** $\dfrac{x^2+4x}{x+4}$

m $\dfrac{x^2-7x}{x-7}$ **n** $\dfrac{2x^2-4x}{x-2}$ **o** $\dfrac{3x^2+6x}{x+2}$

Hint: Cancel after you have factorised the numerator.



Example 17 Multiplying algebraic fractions

Simplify these products.

a $\dfrac{12}{5x} \times \dfrac{10x}{9}$ **b** $\dfrac{3(x-1)}{10} \times \dfrac{15}{x-1}$

Solution	Explanation
a $\dfrac{\cancel{12}_{4}}{\cancel{5x}_{1}} \times \dfrac{\cancel{10x}^{2}}{\cancel{9}_{3}}$ $= \dfrac{8}{3}\left(= 2\dfrac{2}{3}\right)$	Cancel common factors between numerators and denominators: $5x$ and 3. Then multiply the numerators and the denominators.
b $\dfrac{3\cancel{(x-1)}^{1}}{\cancel{10}^{2}} \times \dfrac{\cancel{15}_{3}}{\cancel{x-1}_{1}}$ $= \dfrac{9}{2}\left(= 4\dfrac{1}{2}\right)$	Cancel the common factors, which are $(x-1)$ and 5. Multiply numerators and denominators.

Continued on next page

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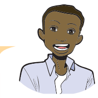
Now you try

Simplify these products.

a $\frac{20}{3x} \times \frac{6x}{25}$

b $\frac{4(x+2)}{9} \times \frac{12}{x+2}$



Hint: Cancel any common factors between numerators and denominators before multiplying.

6 Simplify these products.

a $\frac{3}{x} \times \frac{2x}{9}$

b $\frac{4x}{5} \times \frac{15}{8x}$

c $\frac{9a}{14} \times \frac{7}{6a}$

d $\frac{2x^2}{5} \times \frac{25}{6x}$

e $\frac{4y^2}{7} \times \frac{21}{8y}$

f $\frac{x+1}{6} \times \frac{5}{x+1}$

g $\frac{x+3}{9} \times \frac{4}{x+3}$

h $\frac{4(y-7)}{2} \times \frac{5}{y-7}$

i $\frac{10}{a+6} \times \frac{3(a+6)}{4}$

j $\frac{4(x-2)}{7} \times \frac{14}{5(x-2)}$

k $\frac{3(x+2)}{2x} \times \frac{8}{9(x+2)}$

l $\frac{4(2x+1)}{3x} \times \frac{9x}{2x+1}$

Example 18 Dividing algebraic fractions

Simplify the following.

a $\frac{3x^2}{8} \div \frac{9x}{4}$

b $\frac{2(x-2)}{3} \div \frac{x-2}{6}$

Solution

a $\frac{3x^2}{8} \div \frac{9x}{4} = \frac{\cancel{3}x^{\cancel{2}1}}{\cancel{8}_2} \times \frac{\cancel{4}_1}{_3\cancel{9}\cancel{x}^1}$

$= \frac{x}{6}$

Explanation

Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{9x}{4}$ is $\frac{4}{9x}$.

Cancel common factors: $3x$ and 4.

Note: $\frac{3x^2}{9x} = \frac{\cancel{3}^1 \times x \times \cancel{x}^1}{_3\cancel{9} \times \cancel{x}^1}$

Multiply the numerators and the denominators.

Solution

b $\frac{2(x-2)}{3} \div \frac{x-2}{6} = \frac{2\cancel{(x-2)}^1}{\cancel{3}^1} \times \frac{\cancel{6}^2}{\cancel{x-2}^1}$

$= 4$

Explanation

The reciprocal of $\frac{x-2}{6}$ is $\frac{6}{x-2}$.

Cancel the common factors $(x-2)$ and 3, and multiply. Recall: $\frac{4}{1} = 4$.

Now you try

Simplify the following.

a $\frac{7x^2}{10} \div \frac{14x}{5}$

b $\frac{3(x+1)}{4} \div \frac{x+1}{12}$

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3E

7 Simplify the following.

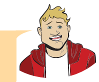
a $\frac{x}{5} \div \frac{x}{15}$ b $\frac{3x}{10} \div \frac{x}{20}$

c $\frac{4a^2}{9} \div \frac{a}{18}$ d $\frac{3x^2}{10} \div \frac{6x}{5}$

e $\frac{4a}{9} \div \frac{5a^2}{6}$ f $\frac{2x}{7} \div \frac{x^2}{14}$

g $\frac{x+4}{2} \div \frac{x+4}{6}$ h $\frac{5(x-2)}{8} \div \frac{x-2}{4}$ i $\frac{3(x+4)}{10} \div \frac{6(x+4)}{15}$

j $\frac{2}{5(2x-1)} \div \frac{10}{2x-1}$ k $\frac{2(x-3)}{x-4} \div \frac{x-3}{5(x-4)}$ l $\frac{3(x+1)}{14(x-1)} \div \frac{6(x+1)}{35(x-1)}$



Hint: To divide, multiply by the reciprocal of the fraction following the division sign. $\frac{x}{5} \div \frac{x}{15} = \frac{x}{5} \times \frac{15}{x}$

Problem-solving and reasoning

8, 9(½) | 8–10(½)

8 Find the error in the simplifying of these fractions and correct it.

a $\frac{3x+\cancel{6}^{2}}{\cancel{3}_{1}} = 3x+2$ b $\frac{x^2+2\cancel{x}^{1}}{\cancel{x}_{1}} = x^2+2$

c $\frac{4x}{5} \div \frac{10x}{3} = \frac{4x}{\cancel{5}_{1}} \times \frac{^{2}\cancel{10}x}{3} = \frac{8x^2}{3}$ d $\frac{\cancel{x}_{1}+4}{\cancel{15}_{5}} \times \frac{\cancel{3}_{1}}{\cancel{x}_{1}} = \frac{4}{5}$

Hint: Remember that common factors can be easily identified when expressions are in factorised form.

9 Simplify these algebraic fractions by factorising expressions first.

a $\frac{7a+14a^2}{21a}$ b $\frac{4x+8}{5x+10}$ c $\frac{x^2+3x}{4x+12}$

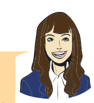
d $\frac{2m+4}{15} \times \frac{3}{m+2}$ e $\frac{5-x}{12} \times \frac{14}{15-3x}$ f $\frac{x^2+2x}{4} \times \frac{8}{3x+6}$

g $\frac{2x-1}{10} \div \frac{4x-2}{25}$ h $\frac{2x+4}{6x} \div \frac{3x+6}{x^2}$ i $\frac{2x^2-4x}{3x-6} \div \frac{6x}{x+5}$

10 By removing a negative factor, further simplifying is sometimes possible.

For example, $\frac{-2x-4}{x+2} = \frac{-2\cancel{(x+2)}_{1}}{\cancel{x+2}_{1}} = -2$.

Use this idea to simplify the following.



Hint: Taking out a negative factor changes the sign of each term inside the brackets.

a $\frac{-3x-9}{x+3}$ b $\frac{-4x-10}{2x+5}$ c $\frac{-x^2-4x}{x+4}$

d $\frac{-3x^2-6x}{-9x}$ e $\frac{-2x+12}{-2}$ f $\frac{-10x+15}{-5}$

Cancelling of powers

— | 11

11 Just as $\frac{x^{\cancel{2}1}}{\cancel{x}^{1}} = x$, $\frac{(x+1)^{\cancel{2}1}}{\cancel{x+1}_{1}} = x+1$. Use this idea to simplify these algebraic fractions. Some will need factorising first.

a $\frac{(x+1)^2}{8} \times \frac{4}{x+1}$ b $\frac{(x+1)^2}{7x} \times \frac{14x}{3(x+1)}$ c $\frac{9}{x-2} \div \frac{18}{(x-2)^2}$

d $\frac{(x+2)^2}{10} \times \frac{5}{4x+8}$ e $\frac{(x-3)^2}{9x} \times \frac{3x}{4x-12}$ f $\frac{15}{8x+4} \div \frac{6}{(2x+1)^2}$

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3F Adding and subtracting algebraic fractions

Learning intentions

- To know that the steps for adding and subtracting algebraic fractions are the same as for numerical fractions
- To be able to find the lowest common denominator of fractions
- To be able to add and subtract algebraic fractions

Key vocabulary: lowest common denominator, equivalent fraction, algebraic fraction, numerator, denominator

As with multiplying and dividing, the steps for adding and subtracting numerical fractions can be applied to algebraic fractions. A lowest common denominator is required before the fractions can be combined.

Lesson starter: Steps for adding fractions

- Write out the list of steps you would give to someone to show them how to add $\frac{3}{5}$ and $\frac{2}{7}$.
- Follow your steps to add the fractions $\frac{3x}{5}$ and $\frac{2x}{7}$.
- What is different when these steps are applied to $\frac{x+2}{5}$ and $\frac{x}{7}$?

Key ideas

- To add or subtract **algebraic fractions**:
  - Determine the **lowest common denominator (LCD)** – the smallest common multiple of the denominators.
    For example, the LCD of 3 and 5 is 15 and the LCD of 4 and 12 is 12.
  - Write each fraction as an equivalent fraction by multiplying the denominator(s) to equal the LCD. When denominators are multiplied, numerators should also be multiplied.
    For example, $\frac{x}{3}+\frac{2x}{5}$ (LCD of 3 and 5 = 15.)
    $$=\frac{x_{(\times 5)}}{3_{(\times 5)}}+\frac{2x_{(\times 3)}}{5_{(\times 3)}}$$
    $$=\frac{5x}{15}+\frac{6x}{15}$$
    and $\frac{2x}{4}-\frac{x}{12}$ (LCD of 4 and 12 = 12.)
    $$=\frac{2x_{(\times 3)}}{4_{(\times 3)}}-\frac{x}{12}$$
    $$=\frac{6x}{12}-\frac{x}{12}$$
  - Add or subtract the numerators.
    For example, $\frac{5x}{15}+\frac{6x}{15}=\frac{11x}{15}$ and $\frac{6x}{12}-\frac{x}{12}=\frac{5x}{12}$
- To express $\frac{x+1}{3}$ with a denominator of 12, both the numerator and denominator must be multiplied by 4 with brackets required to multiply the numerator:
  $$\frac{(x+1)_{(\times 4)}}{3_{(\times 4)}}=\frac{4x+4}{12}$$

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Exercise 3F

Understanding | 1–3 | 3

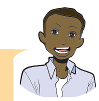
1 Write down the lowest common denominator (LCD) for these pairs of fractions.

a $\frac{2x}{5}, \frac{x}{4}$ b $\frac{x}{3}, \frac{x}{12}$ c $\frac{3x}{10}, \frac{2x}{15}$

Hint: The LCD is not always the two denominators multiplied together; e.g. $3 \times 6 = 18$ but the LCD of 3 and 6 is 6.

2 Complete these equivalent fractions by giving the missing term.

a $\frac{x}{4} = \frac{\square}{12}$ b $\frac{2x}{5} = \frac{\square}{15}$ c $\frac{x-1}{4} = \frac{\square(x-1)}{20}$



Hint: For equivalent fractions, whatever the denominator is multiplied by, the numerator must be multiplied by the same amount.

3 Complete the following by filling in the boxes.

a $\frac{x}{4} + \frac{x}{5} = \frac{\square}{20} + \frac{\square}{20}$
$= \frac{\square}{20}$

b $\frac{2x}{5} - \frac{x}{10} = \frac{\square}{10} - \frac{\square}{10}$
$= \frac{\square}{10}$

Fluency | 4–6(½) | 4–6(½)

Example 19 Adding and subtracting algebraic fractions

Simplify the following.

a $\frac{x}{2} + \frac{x}{3}$ b $\frac{4x}{5} - \frac{x}{2}$ c $\frac{x}{2} - \frac{5}{6}$

Solution	Explanation
a $\frac{x_{(\times 3)}}{2_{(\times 3)}} + \frac{x_{(\times 2)}}{3_{(\times 2)}} = \frac{3x}{6} + \frac{2x}{6}$ $= \frac{5x}{6}$	The LCD of 2 and 3 is 6. Express each fraction with a denominator of 6 and add numerators.
b $\frac{4x_{(\times 2)}}{5_{(\times 2)}} - \frac{x_{(\times 5)}}{2_{(\times 5)}} = \frac{8x}{10} - \frac{5x}{10}$ $= \frac{3x}{10}$	The LCD of 5 and 2 is 10. Express each fraction with a denominator of 10 and subtract $5x$ from $8x$.
c $\frac{x_{(\times 3)}}{2_{(\times 3)}} - \frac{5}{6} = \frac{3x}{6} - \frac{5}{6}$ $= \frac{3x-5}{6}$	The LCD of 2 and 6 is 6. Multiply the numerator and denominator of $\frac{x}{2}$ by 3 to express with a denominator of 6. Write as a single fraction; $3x - 5$ cannot be simplified.

Now you try

Simplify the following.

a $\frac{x}{4} + \frac{x}{5}$ b $\frac{3x}{2} - \frac{x}{7}$ c $\frac{x}{3} + \frac{7}{9}$

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4 Simplify the following.

a $\frac{x}{3}+\frac{x}{4}$ **b** $\frac{x}{5}+\frac{x}{2}$ **c** $\frac{x}{3}-\frac{x}{9}$

d $\frac{x}{5}-\frac{x}{7}$ **e** $\frac{2x}{3}+\frac{x}{5}$ **f** $\frac{3x}{4}+\frac{5x}{12}$

g $\frac{5x}{6}-\frac{4x}{9}$ **h** $\frac{7x}{10}-\frac{3x}{8}$ **i** $\frac{x}{7}-\frac{x}{2}$

j $\frac{x}{10}-\frac{2x}{5}$ **k** $\frac{5x}{6}-\frac{13x}{15}$ **l** $\frac{3x}{10}-\frac{3x}{2}$

Hint: Express each fraction with a common denominator using the LCD, then add or subtract numerators.

5 Simplify the following.

a $\frac{x}{2}+\frac{3}{4}$ **b** $\frac{x}{5}+\frac{2}{3}$ **c** $\frac{2x}{15}+\frac{7}{20}$

d $\frac{x}{4}-\frac{2}{5}$ **e** $\frac{2x}{3}-\frac{5}{9}$ **f** $\frac{5}{6}-\frac{x}{4}$

Example 20 Adding and subtracting with binomial numerators

Simplify the following algebraic expressions.

a $\frac{x+2}{4}-\frac{x}{6}$ **b** $\frac{x+3}{3}+\frac{x-4}{7}$

Solution	Explanation
a $\frac{(x+2)^{(\times 3)}}{4_{(\times 3)}}-\frac{x^{(\times 2)}}{6_{(\times 2)}}=\frac{3(x+2)}{12}-\frac{2x}{12}$ $=\frac{3x+6-2x}{12}$ $=\frac{x+6}{12}$	The LCD of 4 and 6 is 12. Express each fraction with a denominator of 12. When multiplying $(x+2)$ by 3, brackets are required. Expand the brackets and collect the terms: $3x+6-2x=3x-2x+6$
b $\frac{(x+3)^{(\times 7)}}{3_{(\times 7)}}+\frac{(x-4)^{(\times 3)}}{7_{(\times 3)}}=\frac{7(x+3)}{21}+\frac{3(x-4)}{21}$ $=\frac{7x+21+3x-12}{21}$ $=\frac{10x+9}{21}$	The LCD of 3 and 7 is 21. Express each fraction with a denominator of 21. Expand each pair of brackets first and sum by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\frac{x-3}{10}-\frac{x}{15}$ **b** $\frac{x+2}{4}+\frac{x-3}{5}$

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3F

6 Simplify these algebraic expressions.

a $\frac{x+2}{3}+\frac{x}{2}$

b $\frac{x+4}{5}+\frac{2x}{3}$

c $\frac{x-2}{4}+\frac{3x}{8}$

d $\frac{x+4}{3}-\frac{x}{6}$

e $\frac{x+2}{2}-\frac{2x}{5}$

f $\frac{6x+7}{12}-\frac{3x}{8}$

g $\frac{x+3}{5}+\frac{x+2}{4}$

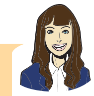
h $\frac{2x+3}{7}+\frac{x+1}{2}$

i $\frac{x+8}{6}+\frac{x-3}{4}$

j $\frac{2x+5}{3}+\frac{x-2}{4}$

k $\frac{x-3}{5}+\frac{x+4}{10}$

l $\frac{2x+1}{8}+\frac{x-2}{3}$



Hint: LCD of 2 and 3 is 6:

$\frac{x+2}{3}+\frac{x}{2}=\frac{\square(x+2)}{6}+\frac{\square x}{6}$

Problem-solving and reasoning

	7, 8	7–9(½)

7 Find the error in each of the following and then correct it.

a $\frac{2x}{3}+\frac{3x}{4}=\frac{5x}{12}$

b $\frac{3x}{5}-\frac{x}{2}=\frac{2x}{3}$

c $\frac{x+2}{5}+\frac{x+4}{3}=\frac{3x+2+5x+4}{15}$
$=\frac{8x+6}{15}$

d $\frac{x+4}{2}+\frac{x-3}{6}=\frac{3x+12+x+3}{6}$
$=\frac{4x+15}{6}$

8 Recall that the expansion of $-5(x-2)$ is $-5x+10$,
so $6(x+1)-5(x-2)=6x+6-5x+10=x+16$.
Use this method to simplify these subtractions.

a $\frac{x+1}{5}-\frac{x-2}{6}$

b $\frac{x+2}{3}-\frac{x-4}{5}$

c $\frac{x-3}{4}-\frac{x+2}{5}$

d $\frac{x+8}{2}-\frac{x+7}{4}$

9 The LCD of the fractions $\frac{4}{x}+\frac{2}{3}$ is $3\times x=3x$.

Use this to find the LCD and simplify these fractions.

a $\frac{4}{x}+\frac{2}{3}$

b $\frac{3}{4}+\frac{2}{x}$

c $\frac{2}{5}+\frac{3}{x}$

d $\frac{3}{7}-\frac{2}{x}$

e $\frac{1}{5}-\frac{4}{x}$

f $\frac{3}{x}-\frac{5}{8}$

Hint:

$\frac{4}{x}+\frac{2}{3}=\frac{\square}{3x}+\frac{\square}{3x}$

$=\frac{\square}{3x}$

Pronumerals in the denominator

	—	10

10 As seen in Question **9**, pronumerals may form part of the LCD.

The fractions $\frac{5}{2x}$ and $\frac{3}{4}$ would have a LCD of $4x$, whereas the fractions $\frac{3}{x}$ and $\frac{5}{x^2}$ would have a LCD of x^2.

By first finding the LCD, simplify these algebraic fractions.

a $\frac{3}{4}+\frac{5}{2x}$

b $\frac{1}{6}+\frac{5}{2x}$

c $\frac{3}{10}-\frac{1}{4x}$

d $\frac{3}{x}+\frac{5}{x^2}$

e $\frac{4}{x}+\frac{1}{x^2}$

f $\frac{3}{x^2}-\frac{5}{x}$

g $\frac{3}{2x}+\frac{2}{x^2}$

h $\frac{4}{x}+\frac{7}{3x^2}$

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Progress quiz

3A **1** For the expression $2x + \frac{y}{2} - 3x^2 + 5$, determine:

- **a** the number of terms
- **b** the constant term
- **c** the coefficient of:
  - **i** x^2
  - **ii** y

3A **2** Find the value of the following expressions if $a = 2$, $b = -5$ and $c = 8$.

a $ab + 2c$ **b** $b^2 - ac$ **c** $\frac{c}{a} - 2b$

3B **3** Simplify the following by collecting like terms.

- **a** $4x - 3 + 2x$
- **b** $7x - 3y - 2x + 8y$
- **c** $3x^2y + 5xy^2 - x^2y + 2xy^2$

3B **4** Simplify the following.

a $3r \times 4rs$ **b** $\frac{2x}{6}$ **c** $15mn^2 \div (6mn)$

3C **5** Expand the following.

- **a** $3(2x + 3)$
- **b** $4x(5x - 2)$
- **c** $-6(2x - 3)$

3C **6** Expand and simplify the following.

- **a** $4 + 2(4x - 5)$
- **b** $4(2x + 3) - 5(x - 4)$

3D **7** Factorise the following by first identifying the highest common factor. (Include any common negatives.)

a $6m + 12$ **b** $15a - 20ab$
c $4xy + x$ **d** $6x^2 - 10x$
e $-8x - 20$ **f** $-3y^2 - 6y$

3E **8** Simplify by cancelling common factors. You will need to factorise first in parts **b** and **c**.

a $\frac{8(x-1)}{4(x-1)}$ **b** $\frac{15x - 35}{5}$ **c** $\frac{2x^2 + 6x}{x + 3}$

3E ★ **9** Simplify the following algebraic fractions.

a $\frac{8x}{7} \times \frac{21}{16x}$ **b** $\frac{9x}{4(x-1)} \div \frac{12}{x-1}$

3F ★ **10** Simplify the following algebraic fractions.

a $\frac{3x}{5} + \frac{x}{4}$ **b** $\frac{2x}{3} - \frac{4}{9}$

3F ★ **11** Simplify $\frac{x+1}{4} + \frac{x-2}{6}$.

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3G Index notation and index laws 1 and 2

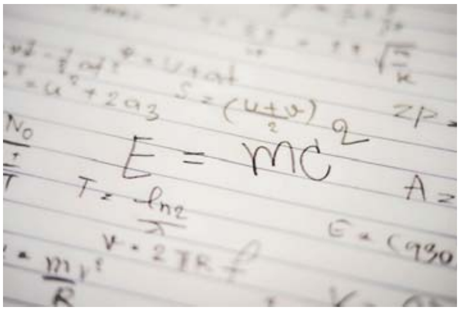
Learning intentions

- To understand index notation and know that it is a shorthand way of writing multiplication of the same base
- To know the index laws for multiplication and division involving a common base
- To be able to apply the index laws for multiplication and division
- To be able to use a calculator to evaluate a power

Key vocabulary: index form, exponent, power, base, index

When 9 is written as 3^2, we are using index form. The 2 in 3^2 is called the index, exponent or power, and the 3 is called the base.

When the same factor is multiplied repeatedly, index form provides an efficient way to represent this. For example, $5 \times 5 \times 5 \times 5$ is written as 5^4 instead. For terms with the same base, calculations can be carried out in index form using the index laws.



Lesson starter: Simplifying in index form

- Express the following calculations in expanded form and then simplify the result in index form.
  a $5^3 \times 5^4$ **b** $6^2 \times 6^3$
- Describe how you could write these in index form without first expanding them.
- Express the following in expanded form and then cancel common factors to write the result as a single term in index form.
  a $\frac{4^5}{4^2}$ **b** $\frac{7^6}{7^3}$
- Describe how you could write these in index form without first expanding them.

Key ideas

- For a number in **index form**; e.g. a^m, we say 'this is a to the power of m', where a is the **base** and m is the **index**, **exponent** or **power**.
  - The exponent, index or power tells us how many times to multiply the base number by itself. For example:

index form		expanded form		basic numeral
5^3	=	$5 \times 5 \times 5$	=	125

  - Note that $a = a^1$
  - The power button (^) on a calculator allows you to evaluate 5^4 using 5(^)4.
- The first two index laws deal with multiplication and division of numbers expressed in index form.
  - Index law 1: $a^m \times a^n = a^{m+n}$; e.g. $5^3 \times 5^4 = 5^{3+4} = 5^7$.
  - Index law 2: $a^m \div a^n = a^{m-n}$
    or $\frac{a^m}{a^n} = a^{m-n}$; e.g. $7^5 \div 7^2 = 7^{5-2} = 7^3$.

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Exercise 3G

Understanding 1–4 3, 4

1 Fill in the missing words, using *index, power, multiply, expanded* and *base*.

a In 3^5, 3 is the ____________ and 5 is the ____________.

b 4^6 is read as 4 to the ____________ of 6.

c 7^4 is the ____________ form of $7 \times 7 \times 7 \times 7$.

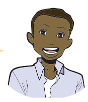
d The power tells you how many times to ____________ the base number by itself.

e $6 \times 6 \times 6$ is the ____________ form of 6^3.

2 Write the following in expanded form.

a 8^3 **b** 7^5

c x^6 **d** $(ab)^4$



Hint: 4 times

$5^{④} = 5 \times 5 \times 5 \times 5$

index form expanded form

3 Complete the following to write each as a single term in index form.

a $7^3 \times 7^4 = 7 \times 7 \times 7 \times \square$

$= 7^{\square}$

b $\dfrac{5^6}{5^2} = \dfrac{5 \times 5 \times \square}{5 \times 5}$

$= 5^{\square}$

4 Choose from the words *add* or *subtract* to fill in the missing words.

a Index law 1 says that when two terms with the same base are multiplied, _______ the powers.

b Index law 2 says that when two terms with the same base are divided, _______ the powers.

Fluency 5–7(½) 5–7(½)

Example 21 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5$ **b** $4 \times x \times x \times x \times 4 \times x$ **c** $a \times b \times b \times a \times b \times b$

Solution	Explanation
a $5 \times 5 \times 5 = 5^3$	The factor 5 is repeated 3 times.
b $4 \times x \times x \times x \times 4 \times x = 4 \times 4 \times x \times x \times x$	Group the factors of 4 and the factors of x together.
$= 4^2 x^3$	The factor x is repeated 3 times; 4 is repeated twice.
c $a \times b \times b \times a \times b \times b$	Group the like pronumerals.
$= a \times a \times b \times b \times b \times b$ $= a^2 b^4$	The factor a is repeated twice and the factor b is repeated 4 times.

Now you try

Write each of the following in index form.

a $3 \times 3 \times 3 \times 3$ **b** $7 \times y \times 7 \times y \times y$ **c** $m \times m \times n \times m \times n$

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3G

5 Write the following in index form.

a $9 \times 9 \times 9 \times 9$

b $3 \times 3 \times 3 \times 3 \times 3 \times 3$

c $15 \times 15 \times 15$

d $5 \times x \times x \times x \times 5$

e $4 \times a \times 4 \times a \times 4 \times a \times a$

f $b \times 7 \times b \times b \times b$

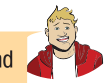
g $x \times y \times x \times x \times y$

h $a \times b \times a \times b \times b \times b$

i $3 \times x \times 3 \times y \times x \times 3 \times y \times y$

j $4 \times x \times z \times 4 \times z \times x$

Hint: Group different bases together and write each base in index form.



Example 22 Using index law 1: $a^m \times a^n = a^{m+n}$

Simplify the following, using the first index law.

a $x^7 \times x^4$ b $a^2b^2 \times ab^3$ c $3x^2y^3 \times 4x^3y^4$

Solution	Explanation
a $x^7 \times x^4 = x^{7+4}$ $= x^{11}$	Use law 1, $a^m \times a^n = a^{m+n}$, to add the indices.
b $a^2b^2 \times ab^3 = a^{2+1}b^{2+3}$ $= a^3b^5$	Add the indices of base a and base b. Recall that $a = a^1$.
c $3x^2y^3 \times 4x^3y^4 = (3 \times 4)x^{2+3}y^{3+4}$ $= 12x^5y^7$	Multiply the coefficients and add indices of the common bases x and y.

Now you try

Simplify the following, using the first index law.

a $x^5 \times x^3$ b $x^2y \times x^3y^4$ c $5ab^2 \times 2a^3b^4$

6 Simplify the following, using the first index law.

a $x^3 \times x^4$ b $p^5 \times p^2$

c $t^7 \times t^2$ d $d^4 \times d$

e $g \times g^3$ f $f^2 \times f$

g $2p^2 \times p^3$ h $3c^4 \times c^4$

i $2s^4 \times 3s^7$ j $a^2b^3 \times a^3b^5$

k $d^7f^3 \times d^2f^2$ l $v^3z^5 \times v^2z^3$

m $3a^2b \times 5ab^5$ n $2x^2y \times 3xy^2$

o $3e^7r^2 \times 6e^2r$ p $-4p^3c^2 \times 2pc$

q $-2r^2s^3 \times 5r^5s^5$ r $-3d^4f^2 \times (-2f^2d^2)$

Hint: Index law 1: $a^m \times a^n = a^{m+n}$
Group common bases and add indices when multiplying.

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Example 23 Using index law 2: $a^m \div a^n = a^{m-n}$

Simplify the following, using the second index law.

a $p^5 \div p^3$ **b** $12m^8 \div (6m^3)$ **c** $\dfrac{4x^2y^4}{8xy^2}$

Solution

a $p^5 \div p^3 = p^{5-3}$
$= p^2$

Explanation

Use law 2, $a^m \div a^n = a^{m-n}$, to subtract the indices.

b $12m^8 \div (6m^3) = \dfrac{{}^{2}\cancel{12}m^8}{{}^{1}\cancel{6}m^3}$
$= 2m^{8-3}$
$= 2m^5$

Write in fraction form.

Cancel the highest common factor of 12 and 6.

Use law 2 to subtract indices.

c $\dfrac{4x^2y^4}{8xy^2} = \dfrac{{}^{1}\cancel{4} \times x^2 \times y^4}{{}^{2}\cancel{8} \times x \times y^2}$
$= \dfrac{x^{2-1}y^{4-2}}{2}$
$= \dfrac{xy^2}{2}\left(\text{or } \dfrac{1}{2}xy^2\right)$

Cancel the common factors of the numerals and subtract the indices of base x and base y.

Now you try

Simplify the following, using the second index law.

a $b^7 \div b^2$ **b** $20a^6 \div (8a^2)$ **c** $\dfrac{6a^3b^4}{9ab^3}$

7 Simplify the following, using the second index law.

a $a^4 \div a^2$ **b** $d^7 \div d^6$ **c** $r^3 \div r$

d $\dfrac{c^{10}}{c^6}$ **e** $\dfrac{l^4}{l^3}$ **f** $\dfrac{b^5}{b^2}$

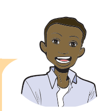
g $\dfrac{4d^4}{d^2}$ **h** $\dfrac{f^2}{2f^2}$ **i** $\dfrac{9n^4}{3n}$

j $6p^4 \div (3p^2)$ **k** $24m^7 \div (16m^3)$ **l** $10d^3 \div (30d)$

m $\dfrac{8t^4r^3}{2tr^2}$ **n** $\dfrac{5h^6d^4}{3d^3h^2}$ **o** $\dfrac{2p^2q^3}{p^2q}$

p $\dfrac{4x^2y^3}{8xy}$ **q** $\dfrac{3r^5s^2}{9r^3s}$ **r** $6c^4d^6 \div (15c^3d)$

s $2a^4y^2 \div (4ay)$ **t** $13m^4n^6 \div (26m^4n^5)$ **u** $18x^4y^3 \div (-3x^2y)$



Hint: Index law 2:
$a^m \div a^n = a^{m-n}$
or
$\dfrac{a^m}{a^n} = a^{m-n}$
When dividing, subtract indices of common bases.

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3G

Problem-solving and reasoning | 8 | 8(½), 9

Example 24 Combining laws 1 and 2

Simplify $\frac{2a^3b \times 3a^2b^3}{12a^4b^2}$ using index laws 1 and 2.

Solution	Explanation
$\frac{2a^3b \times 3a^2b^3}{12a^4b^2} = \frac{(2 \times 3)a^{3+2}b^{1+3}}{12a^4b^2}$	Simplify numerator first by multiplying coefficients and using law 1 to add indices of a and b.
$= \frac{\cancel{6}^{1} a^5b^4}{{}_{2}\cancel{12}a^4b^2}$	Cancel common factor of numerals and use law 2 to subtract indices of common bases.
$= \frac{a^{5-4}b^{4-2}}{2}$	
$= \frac{ab^2}{2}$	

Now you try

Simplify $\frac{4ab^2 \times 3a^5b^4}{18a^3b^5}$ using index laws 1 and 2.

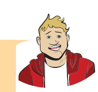
8 Simplify the following, using index laws 1 and 2.

a $\frac{x^2y^3 \times x^2y^4}{x^3y^5}$ **b** $\frac{m^2w \times m^3w^2}{m^4w^3}$

c $\frac{r^4s^7 \times r^4s^7}{r^6s^{10}}$ **d** $\frac{16a^8b \times 4ab^7}{32a^7b^6}$

e $\frac{9x^2y^3 \times 6x^7y^7}{12xy^6}$ **f** $\frac{4e^2w^2 \times 12e^2w^3}{12e^4w}$

Hint: Simplify the numerator first using index law 1, then apply index law 2.



9 When Stuart uses a calculator to raise -2 to the power 4 he gets -16, when in fact the answer is actually 16. What has he done wrong?

Index laws and calculations | — | 10, 11

10 Consider the following use of negative numbers.

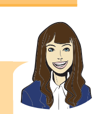
a Evaluate:
 i $(-3)^2$ **ii** -3^2

b What is the difference between your two answers in part **a**?

c Evaluate:
 i $(-2)^3$ **ii** -2^3

d What do you notice about your answers in part **c**? Explain.

Hint:
$(-3)^2 = -3 \times (-3)$
$(-2)^3 = -2 \times (-2) \times (-2)$
Consider order of operations for -3^2 and -2^3.



11 Use index law 2 to evaluate these expressions without the use of a calculator.

a $\frac{13^3}{13^2}$ **b** $\frac{18^7}{18^6}$ **c** $\frac{9^8}{9^6}$ **d** $\frac{3^{10}}{3^7}$ **e** $\frac{4^8}{4^5}$ **f** $\frac{2^{12}}{2^8}$

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3H Index laws 3–5 and the zero power

Learning intentions

- To know how to apply indices to terms in brackets
- To know the rule for the zero power
- To be able to simplify expressions involving indices and brackets
- To be able to simplify using the zero power

Key vocabulary: index/indices, base

Using index laws 1 and 2, we can work out four other index laws to simplify expressions, especially those using brackets.

For example, $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= 4^{2+2+2} = 4^6$ (Add indices using law 1.)

Therefore, $(4^2)^3 = 4^{2 \times 3} = 4^6$.

We also have a result for the zero power. Consider $5^3 \div 5^3$, which clearly equals 1.

Using index law 2, we can see that $5^3 \div 5^3 = 5^{3-3} = 5^0$

Therefore, $5^0 = 1$, leading to the zero power rule: $a^0 = 1$, $(a \neq 0)$.

Lesson starter: Indices with brackets

Brackets are used to show that the power outside the brackets applies to each factor inside the brackets.

Consider $(2x)^3 = 2x \times 2x \times 2x$.

- Write this in index form without using brackets.
- Can you suggest the index form of $(3y)^4$ without brackets?

Consider $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$.

- Write the numerator and denominator of this expanded form in index form.
- Can you suggest the index form of $\left(\frac{x}{4}\right)^5$ without brackets?

Write a rule for removing the brackets of the following.

- $(ab)^m$
- $\left(\frac{a}{b}\right)^m$

Key ideas

- Index law 3: $(a^m)^n = a^{m \times n}$
  Remove brackets and multiply indices. For example, $(x^3)^4 = x^{3 \times 4} = x^{12}$.
- Index law 4: $(a \times b)^m = a^m \times b^m$
  Apply the index to each factor in the brackets. For example, $(3x)^4 = 3^4x^4$.
- Index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  Apply the index to the numerator and denominator. For example, $\left(\frac{y}{3}\right)^5 = \frac{y^5}{3^5}$.
- The zero power: a^0
  Any number (except zero) to the power of zero is 1. For example, $5^0 = 1$, $y^0 = 1$, $4y^0 = 4 \times 1 = 4$.

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Exercise 3H

Understanding

	1, 2	2

1 Complete the following index laws.

a Any number (except 0) to the power of zero is equal to ______.

b Index law 3 states $(a^m)^n =$ ______.

c Index law 4: $(a \times b)^m =$ ______ $\times$ ______.

d Index law 5: $\left(\frac{a}{b}\right)^m = \frac{\square}{\square}$.

2 Copy and complete the following.

a $(2a)^3 = 2a \times \square \times \square$
$= 2 \times \square \times \square \times a \times \square \times \square$
$= 2^{\square} a^{\square}$

b $\left(\frac{4}{7}\right)^4 = \frac{4}{7} \times \square \times \square \times \square$
$= \frac{4 \times \square \times \square \times \square}{7 \times \square \times \square \times \square}$
$= \frac{4^{\square}}{7^{\square}}$

Fluency

	3–6(½)	3–6(½)

Example 25 Using the zero power: $a^0 = 1$

Evaluate the following by using the zero power.

a $4^0 + 2^0$ b $3a^0$ c $(-3)^0 + 6x^0$

Solution	Explanation
a $4^0 + 2^0 = 1 + 1$ $= 2$	Zero power: $a^0 = 1$, any number to the power zero (except zero) is 1.
b $3a^0 = 3 \times a^0$ $= 3 \times 1$ $= 3$	The zero power applies only to a, so $a^0 = 1$.
c $(-3)^0 + 6x^0 = 1 + 6 \times 1$ $= 7$	Any number to the power of zero is 1. $(-3)^0 = 1, 6x^0 = 6 \times x^0 = 6 \times 1$

Now you try

Evaluate the following by using the zero power.

a $5^0 - 3^0$ b $4y^0$ c $(-2)^0 + 3b^0$

3 Evaluate the following, using the zero power.

a 4^0 b 5^0 c x^0 d a^0

e $3e^0$ f $4y^0$ g $3^0 + 6^0$ h $10 - 10x^0$

i $3d^0 - 2$ j $(-4)^0 + 2x^0$ k $\frac{2}{m^0}$ l $5a^0 + 4b^0$

Hint: Any number (except zero) to the power of 0 is 1.
$4a^0 = 4 \times a^0 = 4 \times 1$

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Example 26 Using index law 3: $(a^m)^n = a^{m \times n}$

Simplify the following by using the third index law.

a $(x^5)^7$ b $3(f^4)^3$

Solution	Explanation
a $(x^5)^7 = x^{5 \times 7}$ $= x^{35}$	Apply index law 3: $(a^m)^n = a^{m \times n}$ to multiply indices.
b $3(f^4)^3 = 3f^{4 \times 3}$ $= 3f^{12}$	Apply index law 3 to the value inside the bracket only.

Now you try

Simplify the following by using the third index law.

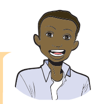
a $(y^3)^6$ b $5(m^2)^4$

4 Simplify the following by using the third index law.

a $(b^3)^4$ b $(f^5)^4$ c $(k^3)^7$

d $3(x^2)^3$ e $5(c^9)^2$ f $4(s^6)^3$

Hint:
Index law 3:
$(a^m)^n = a^{m \times n}$



Example 27 Using index laws 3, 4 and 5: $(a^m)^n = a^{m \times n}$, $(a \times b)^m = a^m \times b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplify the following by using the third, fourth and fifth index laws.

a $(2s)^4$ b $(x^2y^3)^5$ c $\left(\frac{x}{4}\right)^3$

Solution	Explanation
a $(2s)^4 = 2^4 \times s^4$ $= 16s^4$	Apply index law 4: $(a \times b)^m = a^m \times b^m$. Evaluate $2^4 = 2 \times 2 \times 2 \times 2$.
b $(x^2y^3)^5 = (x^2)^5 \times (y^3)^5$ $= x^{10}y^{15}$	Using index law 4, apply the index 5 to each factor in the brackets. Using index law 3, multiply indices: $(x^2)^5 = x^{2 \times 5}$, $(y^3)^5 = y^{3 \times 5}$
c $\left(\frac{x}{4}\right)^3 = \frac{x^3}{4^3}$ $= \frac{x^3}{64}$	Apply index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. Evaluate $4^3 = 4 \times 4 \times 4$.

Now you try

Simplify the following by using the third, fourth and fifth index laws.

a $(3m)^3$ b $(a^3b^2)^4$ c $\left(\frac{y}{3}\right)^4$

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3H

5 Simplify the following by using the third, fourth and fifth index laws.

a $(3x)^2$ **b** $(4m)^3$ **c** $(5y)^3$

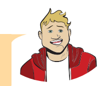
d $(2x^3)^4$ **e** $(x^2y)^5$ **f** $(3a^3)^3$

g $(x^4y^2)^6$ **h** $(a^2b)^3$ **i** $(m^3n^3)^4$

j $\left(\frac{x}{5}\right)^2$ **k** $\left(\frac{y}{3}\right)^4$ **l** $\left(\frac{m}{2}\right)^4$

m $\left(\frac{x^2}{y}\right)^3$ **n** $\left(\frac{x^3}{y^2}\right)^4$ **o** $\left(\frac{x}{y^5}\right)^3$

Hint:
Index law 4 says $(3 \times x)^2 = 3^2 \times x^2$
Index law 5 says $\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2}$



Example 28 Combining index laws

Simplify the following by using index laws.

a $\frac{3x^2y \times 2x^3y^2}{10xy^3}$ **b** $\left(\frac{2x^2}{y}\right)^4$ **c** $(2x^2)^3 + (3x)^0$

Solution | **Explanation**

a $\frac{3x^2y \times 2x^3y^2}{10xy^3}$

Simplify the numerator by multiplying coefficients and adding indices, using index law 1.

$= \frac{6x^5y^3}{10xy^3}$

Cancel the common factor of 6 and 10 and apply index law 2 to subtract indices of common bases.

$= \frac{3x^4y^0}{5}$

The zero power says $y^0 = 1$.

$= \frac{3x^4}{5}$

b $\left(\frac{2x^2}{y}\right)^4 = \frac{(2x^2)^4}{y^4}$

Apply index law 5 to apply the index to the numerator and denominator.

$= \frac{2^4 \times (x^2)^4}{y^4}$

Apply laws 3 and 4 to multiply indices.

$= \frac{16x^8}{y^4}$

c $(2x^2)^3 + (3x)^0 = 2^3 \times x^6 + 3^0 \times x^0$
$= 8x^6 + 1 \times 1$
$= 8x^6 + 1$

Using index law 4, apply the power to each factor inside the brackets:
$(x^2)^3 = x^{2\times 3} = x^6$
Any number to the power of zero is 1.

Now you try

Simplify the following by using index laws.

a $\frac{4a^2b^3 \times 3a^2b}{6a^3b^4}$ **b** $\left(\frac{3x}{y^2}\right)^3$ **c** $(5m^4)^2 + (5m)^0$

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6 Simplify the following by using index laws.

a $\dfrac{m^7w \times m^3w^2}{m^4w^3}$ **b** $\dfrac{x^3y^2 \times x^2y^7}{10x^5y^4}$

c $\dfrac{b^3c^5 \times 4b^5c^3}{3b^4c^8}$ **d** $\dfrac{9c^4s^2 \times 3c^3s^5}{2c^3s^7}$

Hint: First simplify the numerator, then consider the denominator.

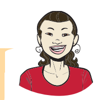
e $\dfrac{(5r^6)^2}{3r^8}$ **f** $\dfrac{(2p^4)^3}{3p^7}$

g $\left(\dfrac{2s^2}{t^3}\right)^4$ **h** $\left(\dfrac{r^2}{5s^3}\right)^4$

Problem-solving and reasoning 7–8(½) 7–8(½)

7 Evaluate the following without the use of a calculator.

a $\dfrac{(5^2)^2}{5^4}$ **b** $\dfrac{36^2}{6^4}$ **c** $\dfrac{27^2}{3^4}$ **d** $\dfrac{16^2}{4^3}$



Hint: $36^2 = (6^2)^2$

8 Simplify the following.

a $2p^2q^4 \times pq^3$ **b** $4(a^2b)^3 \times (3ab)^3$

c $(4r^2y)^2 \times r^2y^4 \times 3(ry^2)^3$ **d** $2(m^3n)^4 \div m^3$

e $\dfrac{(7s^2y)^2 \times 3sy^2}{7(sy)^2}$ **f** $\dfrac{3(d^4c^3)^3 \times 4dc}{(2c^2d)^3}$

g $\dfrac{4r^2t \times 3(r^2t)^3}{6r^2t^4}$ **h** $\dfrac{(2xy)^2 \times 2(x^2y)^3}{8xy \times x^7y^3}$

All laws together — 9

9 Simplify the following, expressing your answer with positive indices.

a $(a^3b^2)^3 \times a^2b^4$ **b** $2x^2y \times (xy^4)^3$

c $2(p^2)^4 \times (3p^2q)^2$ **d** $\dfrac{2a^3b^2}{a^3} \times \dfrac{2a^2b^5}{b^4}$

e $\dfrac{(3rs^2)^4}{r^3s^4} \times \dfrac{(2r^2s)^2}{s^7}$ **f** $\dfrac{4(x^2y^4)^2}{x^2y^3} \times \dfrac{xy^4}{2s^2y}$

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3I Negative indices

Learning intentions

- To know how negative indices can be equivalently expressed using positive indices
- To be able to express negative indices in terms of positive indices
- To be able to use the index laws with negative indices

Key vocabulary: index/indices, base

We have seen how positive indices are used as a shorthand way of writing repeated multiplication of the same base but what do negative powers represent; e.g. 3^{-2} and x^{-1}. Negative powers are used in many areas of science.

Consider $\frac{3^3}{3^5}$:

Expanding and simplifying gives

$$\frac{3^3}{3^5} = \frac{\cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1}{3 \times 3 \times \cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1}$$

$$= \frac{1}{3 \times 3}$$

$$= \frac{1}{3^2}$$

Using index law 2, however, we get

$$\frac{3^3}{3^5} = 3^{3-5}$$

$$= 3^{-2}$$

$$\therefore \quad 3^{-2} = \frac{1}{3^2}.$$

Scientists use negative powers when describing the mass or size of very small objects.

Lesson starter: Continuing the pattern

Complete this table to consider the value of powers of 3 including negative powers.

Index form	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	27	9				$\frac{1}{9} = \frac{1}{3^2}$	

$\div 3$ $\div 3$ $\div 3$

Complete a similar table for powers of 5.

- What do you notice about the fractions in the second row compared to the numbers with negative indices in the top row in each table?
- Can you write this connection as a rule?
- What would be a way of writing 3^{-5} and 5^{-4} with a positive index?

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Key ideas

- Negative indices can be expressed as positive indices using the following rules:
  $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$
- The negative index only applies to the pronumeral or term it is a power of.
  For example, $2a^{-4}b^5 = 2 \times \frac{1}{a^4} \times b^5 = \frac{2b^5}{a^4}$
- All the index laws can be applied to negative indices.

Exercise 3I

Understanding | 1, 2 | 2

1 Complete the following, to express with positive indices.

a $a^{-m} = \square$ **b** $\frac{1}{a^{-m}} = \square$

2 Complete the following by filling in the boxes to express with positive indices.

a $x^{-3} = \frac{1}{x^{\square}}$ **b** $5 \times m^{-2} = 5 \times \frac{1}{\square}$ $= \frac{5}{\square}$ **c** $\frac{1}{a^{-4}} = a^{\square}$

Fluency | 3–5(½) | 3–5(½)

Example 29 Expressing negative indices in positive index form

Express the following with positive indices.

a x^{-2} **b** $4y^{-2}$ **c** $2a^{-3}b^2$

Solution	Explanation
a $x^{-2} = \frac{1}{x^2}$	Use $a^{-m} = \frac{1}{a^m}$.
b $4y^{-2} = 4 \times \frac{1}{y^2}$	The negative index applies only to y; i.e. $y^{-2} = \frac{1}{y^2}$.
$= \frac{4}{y^2}$	$4 \times \frac{1}{y^2} = \frac{4}{1} \times \frac{1}{y^2} = \frac{4}{y^2}$
c $2a^{-3}b^2 = 2 \times \frac{1}{a^3} \times b^2$	$a^{-3} = \frac{1}{a^3}$, $\frac{2}{1} \times \frac{1}{a^3} \times \frac{b^2}{1} = \frac{2b^2}{a^3}$
$= \frac{2b^2}{a^3}$	Multiply the numerators and the denominators.

Now you try

Express the following with positive indices.

a b^{-4} **b** $3a^{-3}$ **c** $4x^{-2}y^4$

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3I

3 Express the following with positive indices.

a y^{-3} **b** x^{-4} **c** x^{-2} **d** a^{-5}
e $3x^{-2}$ **f** $5b^{-3}$ **g** $4x^{-1}$ **h** $2m^{-9}$
i $2x^2y^{-3}$ **j** $3xy^{-4}$ **k** $3a^{-2}b^4$ **l** $5m^{-3}n^2$

Hint: Use $a^{-m} = \frac{1}{a^m}$.

Example 30 Using $\frac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices only.

a $\frac{1}{x^{-3}}$ **b** $\frac{4}{x^{-5}}$ **c** $\frac{5}{a^2b^{-4}}$

Solution	**Explanation**
a $\frac{1}{x^{-3}} = x^3$	Use $\frac{1}{a^{-m}} = a^m$.
b $\frac{4}{x^{-5}} = 4 \times \frac{1}{x^{-5}}$ $= 4 \times x^5$ $= 4x^5$	The 4 remains unchanged. Note: $\frac{1}{x^{-5}} = x^5$.
c $\frac{5}{a^2b^{-4}} = \frac{5}{a^2} \times \frac{1}{b^{-4}}$ $= \frac{5}{a^2} \times b^4$ $= \frac{5b^4}{a^2}$	The negative index applies to b only; i.e. $\frac{1}{b^{-4}} = b^4$. $\frac{5}{a^2} \times b^4 = \frac{5}{a^2} \times \frac{b^4}{1} = \frac{5b^4}{a^2}$

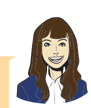
Now you try

Rewrite the following with positive indices only.

a $\frac{1}{y^{-2}}$ **b** $\frac{3}{a^{-4}}$ **c** $\frac{7}{x^{-2}y^3}$

4 Rewrite the following with positive indices only.

a $\frac{1}{b^{-4}}$ **b** $\frac{1}{x^{-7}}$ **c** $\frac{1}{y^{-1}}$
d $\frac{5}{m^{-3}}$ **e** $\frac{2}{y^{-2}}$ **f** $\frac{3}{x^{-4}}$
g $\frac{5a^2}{b^{-3}}$ **h** $\frac{4}{x^2y^{-5}}$ **i** $\frac{10}{a^{-2}b^4}$



Hint: Use $\frac{1}{a^{-m}} = a^m$.

5 Rewrite the following with positive indices only.

a $\frac{4x^{-2}}{y^3}$ **b** $\frac{b^{-3}}{5a^2}$ **c** $\frac{2a^3}{b^{-2}}$
d $\frac{a^4}{3b^{-5}}$ **e** $\frac{y^{-2}}{x^{-3}}$ **f** $\frac{xy^{-3}}{x^{-2}y}$

Hint: For part **e**, $\frac{y^{-2}}{x^{-3}} = y^{-2} \times \frac{1}{x^{-3}} = \ldots$

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Problem-solving and reasoning

6–7(½), 8	6–7(½), 8, 9(½)

Example 31 Combining index laws with negative indices

Simplify the following, using index laws. Express answers with positive indices.

a $\dfrac{x^4y^3 \times x^{-2}y^5}{x^5y^4}$ **b** $\dfrac{4(x^2y^{-1})^{-2}}{y^5}$

Solution

a $\dfrac{x^4y^3 \times x^{-2}y^5}{x^5y^4} = \dfrac{x^{4+(-2)}y^{3+5}}{x^5y^4}$

$= \dfrac{x^2y^8}{x^5y^4}$

$= x^{2-5}y^{8-4}$

$= x^{-3}y^4$

$= \dfrac{y^4}{x^3}$

Explanation

Use law 1 to add indices of x and y in numerator:
For x: $4 + (-2) = 4 - 2 = 2$
For y: $3 + 5 = 8$

Express with positive indices; i.e. $x^{-3} = \dfrac{1}{x^3}$.

$x^{-3}y^4 = \dfrac{1}{x^3} \times \dfrac{y^4}{1}$

Solution

b $\dfrac{4(x^2y^{-1})^{-2}}{y^5} = \dfrac{4x^{-4}y^2}{y^5}$

$= 4x^{-4}y^{-3}$

$= \dfrac{4}{x^4y^3}$

Explanation

Remove the brackets by applying index laws 3 and 4 to distribute the power to each pronumeral: $x^{2\times(-2)}$ and $y^{-1\times(-2)}$.

Apply index law 2 to subtract the powers with a base of y.

Express with positive indices: $4x^{-4}y^{-3} = 4 \times \dfrac{1}{x^4} \times \dfrac{1}{y^3}$

Now you try

Simplify the following, using index laws. Express answers with positive indices.

a $\dfrac{x^2y^{-1} \times x^2y^4}{x^6y^2}$ **b** $\dfrac{9(x^{-3}y^2)^{-2}}{x^7}$

6 Simplify the following, expressing answers using positive indices.

a $\dfrac{a^6b^2 \times a^{-2}b^3}{a^7b}$ **b** $\dfrac{x^5y^3 \times x^2y^{-1}}{x^3y^5}$

c $\dfrac{x^4y^7 \times x^{-2}y^{-5}}{x^4y^6}$ **d** $\dfrac{a^5b^{-2} \times a^{-3}b^4}{a^6b}$

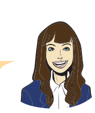
Hint: Index laws 1 and 2 apply to negative indices also.
$x^5 \times x^{-2} = x^{5+(-2)} = x^3$
$\dfrac{x^4}{x^6} = x^{4-6} = x^{-2} = \dfrac{1}{x^2}$

7 Simplify, using index laws, and express with positive indices.

a $(x^{-4})^2$ **b** $(x^3)^{-2}$ **c** $(x^{-2})^0$

d $(2y^{-2})^3$ **e** $(ay^{-3})^2$ **f** $(4x^{-3})^{-2}$

g $\dfrac{3(x^{-4}y^3)^{-2}}{4x^7}$ **h** $(a^{-3}b^2)^{-2} \times (a^{-1}b^{-2})^3$ **i** $\dfrac{(2m^{-3}n)^2}{4m^2n^{-3}}$



Hint: Remove brackets using index laws, then use $a^{-m} = \dfrac{1}{a^m}$ to express with a positive index.

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3I

8 The mass of a small insect is 3^{-6} kg. How many grams is this, correct to two decimal places?

9 Evaluate the following without the use of a calculator.

Hint: Express each one with a positive index first.

a 2^{-2} **b** 5^{-3} **c** $\dfrac{4}{3^{-2}}$ **d** $\dfrac{5}{2^{-3}}$

e -3×2^{-2} **f** $6^4 \times 6^{-6}$ **g** $\dfrac{2^3}{2^{-3}}$ **h** $8 \times (2^2)^{-2}$

The power of −1 | — | 10

10 Consider the number $\left(\dfrac{3}{4}\right)^{-1}$. Using a positive index this becomes $\dfrac{1}{\left(\frac{3}{4}\right)} = 1 \div \dfrac{3}{4} = 1 \times \dfrac{4}{3} = \dfrac{4}{3}$.

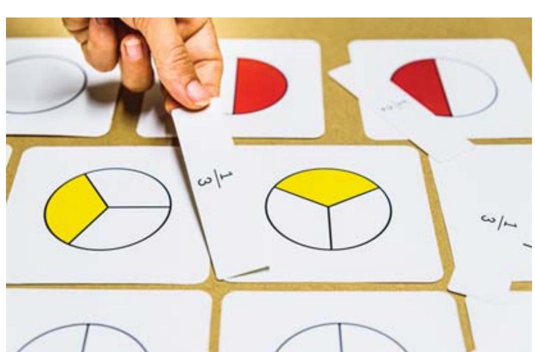
a Complete similar working to simplify the following.

i $\left(\dfrac{5}{3}\right)^{-1}$ **ii** $\left(\dfrac{1}{4}\right)^{-1}$ **iii** $\left(\dfrac{x}{2}\right)^{-1}$ **iv** $\left(\dfrac{a}{b}\right)^{-1}$

b What conclusion can you come to regarding the simplification of fractions raised to the power of −1?

c Simplify these fractions.

i $\left(\dfrac{3}{2}\right)^{-2}$ **ii** $\left(\dfrac{3}{5}\right)^{-2}$ **iii** $\left(\dfrac{1}{3}\right)^{-3}$ **iv** $\left(\dfrac{2}{3}\right)^{-4}$



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3J Scientific notation

Learning intentions

- To know that scientific notation is a way of representing very large and very small numbers
- To know the form of numbers written in scientific notation
- To be able to express numbers using scientific notation and as a basic numeral
- To be able to use and interpret scientific notation on a calculator
- To know how significant figures are counted
- To be able to round to a number of significant figures

Key vocabulary: scientific notation, significant figures

Scientific notation is useful when working with very large or very small numbers. Combined with the use of significant figures, numbers can be written down with an appropriate degree of accuracy and without the need to write all the zeroes that define the position of the decimal point. The approximate distance between Earth and the Sun is 150 million kilometres or 1.5×10^8 km, when written in scientific notation. Negative indices can be used for very small numbers, such as 0.0000382 g $= 3.82 \times 10^{-5}$ g.

Lesson starter: Amazing facts large and small

Think of an object, place or living thing that is associated with a very large or small number.

- Give three examples of very large numbers.
- Give three examples of very small numbers.
- Can you remember how to write these numbers using scientific notation? List the rules you remember.

Key ideas

- **Scientific notation** is a way to express very large and very small numbers.
- A number written using scientific notation is of the form $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer.
- To write numbers using scientific notation, place the decimal point after the first non-zero digit and then multiply by the power of 10 that corresponds to how many places the decimal point is moved.
  - Large numbers will use positive powers of 10.
    For example, $24\,800\,000 = 2.48 \times 10^7$
    $9\,020\,000\,000 = 9.02 \times 10^9$
  - Small numbers will use negative powers of 10.
    For example, $0.00307 = 3.07 \times 10^{-3}$
    $0.000001\,2 = 1.2 \times 10^{-6}$
- **Significant figures** are counted from left to right, starting at the first non-zero digit. Rounding occurs by considering the digit following the last significant digit; 5 or more round up, less than 5 round down. For example:
  - $47\,086\,120$ is written $47\,086\,000$ using five significant figures.
  - 2.03684 is written 2.037 using four significant figures.
  - 0.00143 is written 0.0014 using two significant figures.
  - 0.0014021 is written 0.00140 using three significant figures.
    Zeroes at the end of a number are counted for decimals (see 0.00140 above) but not whole numbers (see $47\,086\,000$ above).

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- When using scientific notation, the first significant figure sits to the left of the decimal point. For example:
  - 20 190 000 is written 2.02×10^7 using three significant figures.
- The EE or Exp keys on calculators can be used to enter numbers that use scientific notation: 2.3E−4 means 2.3×10^{-4}.

Exercise 3J

Understanding — 1–3 | 1, 3

1 State yes (Y) or no (N) as to whether the following numbers are written in scientific notation form.
a 1.27×10^3 **b** 15.2×10^{-2} **c** 0.8×10^2 **d** -4.1×10^{-3}

2 State whether these numbers would have positive or negative indices when written in scientific notation.
a 7800 **b** 0.0024 **c** 27 000 **d** 0.0009

3 Write the number 4.8721, using the following numbers of significant figures.
a three **b** four **c** two

Hint: Start counting significant digits from the 4.

Fluency — 4–5(½) | 4–5(½)

Example 32 Converting from scientific notation to a basic numeral

Write these numbers as a basic numeral.
a 5.016×10^5 **b** 3.2×10^{-7}

Solution	Explanation
a $5.016 \times 10^5 = 501\,600$	Move the decimal point 5 places to the right, inserting zeroes after the last digit. 5.01600
b $3.2 \times 10^{-7} = 0.00000032$	Move the decimal point 7 places to the left due to the −7, and insert zeroes where necessary.

Now you try

Write these numbers as a basic numeral.
a 3.27×10^4 **b** 1.2×10^{-3}

4 Write these numbers as a basic numeral.
a 3.12×10^3 **b** 5.4293×10^4 **c** 7.105×10^5
d 8.213×10^6 **e** 5.95×10^4 **f** 8.002×10^5
g 1.012×10^4 **h** 9.99×10^6 **i** 2.105×10^8
j 4.5×10^{-3} **k** 2.72×10^{-2} **l** 3.085×10^{-4}
m 7.83×10^{-3} **n** 9.2×10^{-5} **o** 2.65×10^{-1}
p 1.002×10^{-4} **q** 6.235×10^{-6} **r** 9.8×10^{-1}

Hint: For a positive index, move the decimal point right (number gets bigger). For a negative index, move the decimal point left (number gets smaller).

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Example 33 Writing numbers using scientific notation

Write these numbers in scientific notation.

a 5 700 000 **b** 0.0000006

Solution	Explanation
a $5\,700\,000 = 5.7 \times 10^6$	Place the decimal point after the first non-zero digit (5) and then multiply by 10^6, as the decimal point has been moved 6 places to the left.
b $0.0000006 = 6 \times 10^{-7}$	6 is the first non-zero digit. Multiply by 10^{-7} since the decimal point has been moved 7 places to the right.

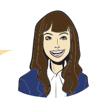
Now you try

Write these numbers in scientific notation.

a 320 000 **b** 0.0002

5 Write these numbers in scientific notation.

a 43 000 **b** 712 000 **c** 901 200
d 10 010 **e** 23 900 **f** 703 000 000
g 0.00078 **h** 0.00101 **i** 0.00003
j 0.03004 **k** 0.112 **l** 0.00192



Hint: For scientific notation, place the decimal point after the first non-zero digit and multiply by the power of 10.

Example 34 Converting to scientific notation using significant figures

Write these numbers in scientific notation using three significant figures.

a 5 218 300 **b** 0.0042031

Solution	Explanation
a $5\,218\,300 = 5.22 \times 10^6$	Put the decimal point after 5 and multiply by 10^6: 5.218300 The digit following the third digit (8) is at least 5, so round the 1 up to 2.
b $0.0042031 = 4.20 \times 10^{-3}$	Put the decimal point after 4 and multiply by 10^{-3}: 0.0042031 Round down in this case, since the digit following the third digit (3) is less than 5, but retain the zero to show the value of the third significant figure.

Now you try

Write these numbers in scientific notation using three significant figures.

a 53 721 **b** 0.0003625

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3J

Problem-solving and reasoning

6–8 | 6–10

6 Write these numbers in scientific notation using three significant figures.

a 6241 **b** 572 644

c 30 248 **d** 423 578

e 10 089 **f** 34 971 863

g 72 477 **h** 356 088

i 110 438 523 **j** 0.002423

k 0.018754 **l** 0.000125

m 0.0078663 **n** 0.0007082

o 0.11396 **p** 0.000006403

q 0.00007892 **r** 0.000129983

Hint: For three significant figures, count from the first non-zero digit. Look at the digit after the third digit to determine whether you should round up or down.

7 Write the following numerical facts using scientific notation.

a The area of Australia is about 7 700 000 km^2.

b The number of stones used to build the Pyramid of Khufu is about 2 500 000.

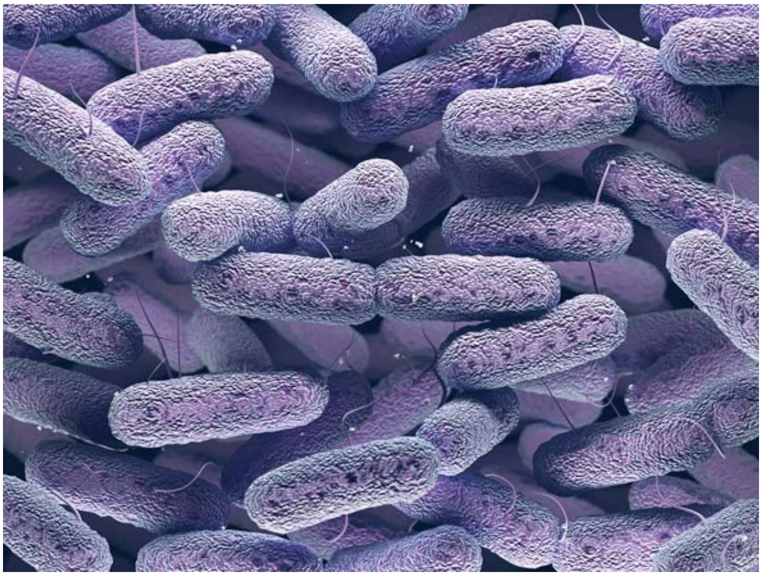
c The greatest distance of Pluto from the Sun is about 7 400 000 000 km.

d A human hair is about 0.01 cm wide.

e The mass of a neutron is about 0.000000000000000000000000001675 kg.

f The mass of a bacteria cell is about 0.00000000000095 g.

Hint: Large numbers have a positive index. Small numbers have a negative index.



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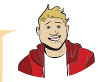
8 Explain why 38×10^7 is not written using scientific notation and then convert it to scientific notation.

9 Use a calculator to evaluate the following, giving the answers in scientific notation using three significant figures.

a $(2.31)^{-7}$
b $(5.04)^{-4}$
c $(2.83 \times 10^2)^{-3}$
d $5.1 \div (8 \times 10^2)$
e $9.3 \times 10^{-2} \times 8.6 \times 10^8$
f $(3.27 \times 10^4) \div (9 \times 10^{-5})$
g $\sqrt{3.23 \times 10^{-6}}$
h $\pi(3.3 \times 10^7)^2$

Hint: Locate the $\times 10^x$ or EE or Exp button on your calculator.



10 The speed of light is approximately 3×10^5 km/s and the average distance between Pluto and the Sun is about 5.9×10^9 km. How long does it take for light from the Sun to reach Pluto? Answer correct to the nearest minute. (Divide by 60 to convert seconds to minutes.)

Hint: Time = $\frac{\text{distance}}{\text{speed}}$

$E = mc^2$ — 11

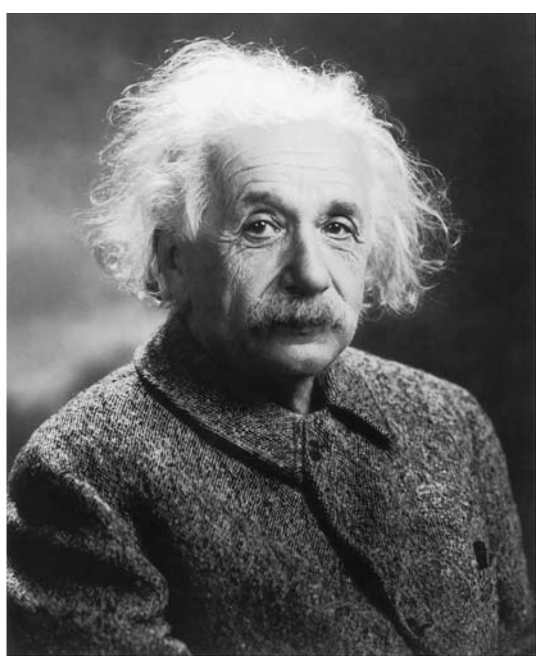
11 $E = mc^2$ is a formula derived by Albert Einstein (1879–1955). The formula relates the energy (E joules) of an object to its mass (m kg), where c is the speed of light (approximately 3×10^8 m/s).

Use $E = mc^2$ to answer these questions using scientific notation.

a Find the energy, in joules, contained inside an object with these given masses.
i 10 kg
ii 26 000 kg
iii 0.03 kg
iv 0.00001 kg

b Find the mass, in kilograms, of an object that contains the given amounts of energy. Give your answer using 3 significant figures.
i 1×10^{25} J
ii 3.8×10^{16} J
iii 8.72×10^4 J
iv 1.7×10^{-2} J

c The mass of Earth is about 6×10^{24} kg. How much energy does this convert to?



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3K Exponential growth and decay ★

Learning intentions

- To understand the concept of exponential growth and decay
- To know the rule that models exponential growth and decay
- To be able to form a rule for exponential growth or decay
- To be able to apply an exponential rule including compound interest

Key vocabulary: exponential growth, exponential decay, compound interest, principal

Exponential change occurs when a quantity is continually affected by a constant multiplying factor. The change in quantity is not the same amount each time.
If you have a continual percentage increase, it is called exponential growth. If you have a continual percentage decrease, it is called exponential decay.
Some examples include:

- compound interest at a rate of 5% per year, where the interest is calculated as 5% of the investment value each year, including the previous year's interest
- a radioactive element has a 'half-life' of 5 years, which means the element decays at a rate of 50% every 5 years.

Lesson starter: A compound rule

Imagine that you have an investment valued at $100 000 and you hope that it will return 10% p.a. (per annum).
The 10% return is to be added to the investment balance each year.

- Discuss how to calculate the investment balance in the first year.
- Discuss how to calculate the investment balance in the second year.
- Complete this table.

Year	0	1	2	3
Balance ($)	100 000	$100\,000 \times 1.1 =$ ______	$100\,000 \times 1.1 \times$ ____ $=$ ______	______ $=$ ______

- Recall how indices can be used to calculate the balance after the second year.
- Discuss how indices can be used to calculate the balance after the 10th year.
- What might be the rule connecting the investment balance ($$A$) and the time, n years?

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Key ideas

- **Exponential growth** and **decay** is a repeated increase or decrease of a quantity by a constant percentage over time. It can be modelled by the rule $A = A_0\left(1 \pm \frac{r}{100}\right)^n$.
  - A is the amount.
  - A_0 is the initial amount (the subscript zero represents time zero).
  - r is the percentage rate of increase or decrease.
  - n is time; i.e. how many times the percentage increase/decrease is applied.
- For a growth rate of $r\%$ p.a., use $1 + \frac{r}{100}$.
  - For example, for a population increasing at 2% per year, $P = P_0(1.02)^n$.
- For a decay rate of $r\%$ p.a., use $1 - \frac{r}{100}$.
  - For a population decreasing at 3% per year, $P = P_0(0.97)^n$.
- Compound interest involves *adding* any interest earned to the balance at the end of each year or other period. The rule for the investment amount ($\$A$) is given by: $A = P\left(1 + \frac{r}{100}\right)^n$.
  - P is the initial amount or principal.
  - r is the interest rate expressed as a percentage.
  - n is the time.

Exercise 3K

Understanding | 1–3 | 3

1 An investment of $1000 is increasing at 5% per year.

a Find the value of the investment at the end of the first year.

b Copy and complete the rule for the value of the investment ($\$V$) after n years.

$$V = 1000(1 + \_\_\_\_\_\_\_\_)^n \quad = 1000 \times \_\_\_\_\_\_^n$$

c Use your rule to calculate the value of the investment after 4 years, correct to two decimal places.

2 The mass of a 5 kg limestone rock exposed to the weather is decreasing at a rate of 2% per annum.

Hint: For decrease in part **b**, use $1 - \frac{r}{100}$.

a Find the mass of the rock at the end of the first year.

b Copy and complete the rule for the mass of the rock (M kg) after n years.
$M = 5(1 - \_\_\_\_\_\_)^n = 5 \times \_\_\_\_\_\_^n$

c Use your rule to calculate the mass of the rock after 5 years, correct to two decimal places.

3 Decide whether the following represent exponential *growth* or exponential *decay*.

a $A = 1000 \times 1.3^n$

b $A = 200 \times 1.78^n$

c $A = 350 \times 0.9^n$

d $P = 50\,000 \times 0.85^n$

e $P = P_0\left(1 + \frac{3}{100}\right)^n$

f $T = T_0\left(1 - \frac{7}{100}\right)^n$

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3K

Fluency 4–6 4(½),5–7

Example 35 Writing exponential rules

Form exponential rules for the following situations.

a Paloma invests her $100 000 in savings at a rate of 14% per annum.
b A city's initial population of 50 000 is decreasing by 12% per year.

Solution	Explanation
a Let A = the amount of money at any time n = the number of years the money is invested $r = 14$ $A_0 = 100\,000$ (initial amount)	Define your variables. The basic formula is $A = A_0\left(1 \pm \frac{r}{100}\right)^n$.
$A = 100\,000\left(1 + \frac{14}{100}\right)^n$ $\therefore A = 100\,000(1.14)^n$	Substitute $r = 14$ and $A_0 = 100\,000$ and use '+' since we have growth. $\frac{14}{100} = 0.14$.
b Let P = the population at any time n = the number of years the population decreases $P_0 = 50\,000$ (starting population) $r = 12$	Define your variables. The basic formula is $P = P_0\left(1 \pm \frac{r}{100}\right)^n$.
$P = 50\,000\left(1 - \frac{12}{100}\right)^n$ $\therefore P = 50\,000(0.88)^n$	Substitute $r = 12$ and $P_0 = 50\,000$ and use '−' since we have decay. $\frac{12}{100} = 0.12$ and $1 - 0.12 = 0.88$.

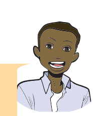
Now you try

Form exponential rules for the following situations.

a A town population of 3000 is increasing by 2% per year.
b A car purchased for $36 000 is losing value at 6% per year.

4 Define variables and form exponential rules for the following situations.

a $200 000 is invested at 17% per annum.
b A house initially valued at $530 000 is losing value at 5% per annum.
c The value of a car, bought for $14 200, is decreasing at 3% per annum.
d A population, initially 172 500, is increasing at 15% per year.
e A tank with 1200 litres of water is leaking at a rate of 10% of the water in the tank every hour.
f A cell of area 0.01 cm^2 doubles its size every minute.
g An oil spill, initially covering an area of 2 square metres, is increasing at 5% per minute.
h A substance of mass 30 g is decaying at a rate of 8% per hour.



Hint: The exponential rule is of the form $A = A_0\left(1 \pm \frac{r}{100}\right)^n$

- A is the amount
- A_0 is the initial amount
- r is the percentage increase/decrease
- n is the time

Use + for growth and − for decay.

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Example 36 Applying exponential rules

House prices are rising at 9% per year and Zoe's flat is currently valued at $600 000.

a Determine a rule for the value of Zoe's flat ($\$V$) in n years' time.

b What will be the value of her flat:

  i next year? **ii** in 3 years' time?

c Use trial and error to find when Zoe's flat will be valued at $900 000, to one decimal place.

Solution | **Explanation**

a Let V = value of Zoe's flat at any time

V_0 = starting value $600 000

n = number of years from now

$r = 9$

$V = V_0(1.09)^n$

$\therefore\ V = 600\,000(1.09)^n$

Define your variables.

$V = V_0\left(1 \pm \frac{r}{100}\right)^n$

Use '+' since we have growth.

b **i** When $n = 1$, $V = 600\,000(1.09)^1$

$= 654\,000$

Zoe's flat would be valued at $654 000 next year.

Substitute $n = 1$ for next year.

ii When $n = 3$, $V = 600\,000(1.09)^3$

$= 777\,017.40$

In 3 years' time Zoe's flat will be valued at about $777 017.

For 3 years, substitute $n = 3$.

c

n	4	5	4.6	4.8	4.7
V	846 949	923 174	891 894	907 399	899 613

Zoe's flat will be valued at $900 000 in about 4.7 years' time.

Try a value of n in the rule. If V is too low, increase your n value; if V is too high, decrease your n value. Continue this process until you get close to 900 000.

Now you try

An investment is increasing at 4% per year and is currently valued at $20 000.

a Determine a rule for the value of the investment ($\$V$) in n years' time.

b What will be the value of the investment

  i next year? **ii** in 5 years' time?

c Use trial and error to find when the investment will be valued at $25 000, to one decimal place.

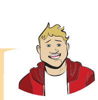
5 The value of a house purchased for $500 000 is expected to grow by 10% per year. Let $\$A$ be the value of the house after n years.

a Copy and complete the rule connecting A and n. $A = 500\,000 \times \_\_\_\_\_^n$

b Use your rule to find the expected value of the house after the following number of years. Round your answers to the nearest cent.

  i 3 years **ii** 10 years **iii** 20 years

Hint: An increase of 10% is $1 + \frac{10}{100}$.



c Use trial and error to estimate when the house will be worth $1 million. Round your answer to one decimal place.

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3K

6 A share portfolio initially worth \$300 000 is reduced by 15% p.a. over a number of years. Let \$$A$ be the share portfolio value after n years.

Hint: A decrease of 15% is $1 - \frac{15}{100}$.

a Copy and complete the rule connecting A and n.

$A = \_\_\_\_\_\_\_\_ \times 0.85^n$

b Use your rule to find the value of the shares after the following number of years. Round your answers to the nearest cent.

i 2 years **ii** 7 years **iii** 12 years

c Use trial and error to estimate when the share portfolio will be valued at \$180 000. Round your answer to one decimal place.

7 A water tank containing 15 000 L has a small hole that reduces the amount of water by 6% per hour.

a Determine a rule for the volume of water (V litres) left after n hours.

b Calculate (to the nearest litre) the amount of water left in the tank after:

i 3 hours **ii** 7 hours

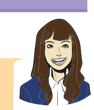
c How much water is left after 2 days? Round your answer to two decimal places.

d Using trial and error, determine when the tank holds less than 500 L of water, to one decimal place.

Problem-solving and reasoning

Problem-solving and reasoning	8, 9	8, 10

8 A certain type of bacteria grows according to the equation $N = 3000(2.6)^n$, where N is the number of cells present after n hours.



Hint: 'At the start' is $n = 0$ and $a^0 = 1$.

a How many bacteria are there at the start?

b Determine the number of cells (round to the whole number) present after:

i 0 hours **ii** 2 hours **iii** 4.6 hours

c If 5 000 000 bacteria are needed to make a drop of serum, determine by trial and error how long you will have to wait to make a drop (to the nearest minute).

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9 A car tyre has 10 mm of tread when new. It is considered unroadworthy when there is only 3 mm left. The rubber wears at 12.5% every 10 000 km.

a Write an equation relating the depth of tread (D) for every 10 000 km travelled.

b If a tyre lasts 80 000 km ($n = 8$) before becoming unroadworthy, it is considered to be a 'good' tyre. Is this a good tyre?

c Using trial and error, determine when the tyre becomes unroadworthy ($D = 3$), to the nearest 10 000 km.

Hint: Use $D = D_0\left(1 - \frac{12.5}{100}\right)^n$ where n is $\frac{\text{number of km}}{10\,000}$ and D_0 is initial tread. In part **b**, is D greater than 3 when $n = 8$?

10 A cup of coffee has an initial temperature of 90°C.

a If the temperature reduces by 8% every minute, determine a rule for the temperature of the coffee (T) after n minutes.

b What is the temperature of the coffee (to one decimal place) after:

i 2 minutes? **ii** 90 seconds?

c Using trial and error, when is the coffee suitable to drink if it is best consumed at a temperature of 68.8°C? Give your answer to the nearest second.

Hint: The rule is of the form: $T = T_0\left(1 - \frac{r}{100}\right)^n$

Time periods

	—	11, 12

11 Interest on investments can be calculated using different time periods. Consider \$1000 invested at 10% p.a. over 5 years.

- If interest is compounded annually, then $r = 10$ and $n = 5$, so $A = 1000(1.1)^5$.
- If interest is compounded monthly, then $r = \frac{10}{12}$ and $n = 5 \times 12 = 60$, so $A = 1000\left(1 + \frac{10}{1200}\right)^{60}$.

a If interest is calculated annually, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

b If interest is calculated monthly, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

12 You are given \$2000 and you invest it in an account that offers 7% p.a. compound interest. What will the investment be worth, to the nearest cent, after 5 years if interest is compounded:

a annually? **b** monthly? **c** weekly? (Assume 52 weeks in the year.)

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Maths@Work: Electrical trades

Electricians must be able to work in teams and also independently. They need to be good at calculating with decimals, as well as using scientific notation. Understanding and working with electrical charges is one example where this is important.

When evaluating academic readiness for apprenticeship training in the construction trades, which include electricians, plumbers and air conditioning mechanics, scientific notation is seen as important and appears in different areas of their courses.

Complete these questions, which an apprentice electrician may face during their training.

1 The electrical charge (Q) of an object is determined by the number of electrons it has in excess to the number of protons it has.
The unit for measuring electrical charge is the coulomb (C). One coulomb (1C) is approximately 6.24 quintillion electrons (*e*).

$$1\text{ C} = 6\,240\,000\,000\,000\,000\,000\text{ e}$$

a Convert the following electrical charges, in coulombs, to the number of electrons for each. Use scientific notation using three significant figures.

i 1 C **ii** 2 C **iii** 3 C
iv 250 C **v** $\frac{1}{2}$ C **vi** 12 C

b The charge on one electron in coulombs is $(1 \div 6\,240\,000\,000\,000\,000\,000)$ C. Write down the value of the charge of one electron in scientific notation, using two significant figures.

c Amperes (or amps) are a measure of how much electrical charge in coulombs per second is being transmitted. We call this flow of charge electrical current. This means that an electrical current of 1 A (ampere) has 1 coulomb of charge per second, which is exactly $6.24150975 \times 10^{18}$ electrons per second flowing through a point in the wire at any given time.
Calculate the exact number of electrons per second (e/s) flowing through a wire if the current is:

i 2 A **ii** 10 A **iii** 20 A
iv $\frac{1}{2}$ A **v** 5 A

2 When working with metal it is important to know how it behaves under increases in temperature. For each degree Celsius increase in temperature of hard steel, it has a linear expansion by a factor of 0.0000132.
Write the following scientific notation answers, using four significant figures.

a Express the value 0.0000132 in scientific notation.

b If a section of hard steel measuring 12 mm thick is subject to a 2°C increase in temperature, what is its increase in length, in mm?

Hint: Use $12 \times (1 + 0.0000132)^2$, then substract 12 to find the increase.

c Give one example to illustrate the importance of this information when working with steel.

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Using technology

Hint: The formula for cell D3 $:= (1.60217646 \times 10^{-19}) \times$ C3. Excel uses capital E to represent a power of 10.

3 Using an Excel spreadsheet, set up a conversion table between electrical charges measured in coulombs and in electrons, as shown below.

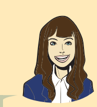
D3 fx =(1.60217646*(10^-19))*C3

	A	B	C	D
1	**Conversion table between charge (Q) in coulombs (C) and electrons (e)**			
2	Charge Q (in coulombs, C)	Charge Q (in number of electrons, e)	Charge Q (in number of electrons, e)	Charge Q (in coulombs, C)
3	1	6.24150975E+18	1	1.60217646E-19
4	10		10	
5	100		100	
6	1000		1000	
7	10000		10000	
8	100000		100000	
9	1000000		1000000	

Use your spreadsheet to find the answers to the following questions and write them in scientific notation to nine significant figures.

i State the charge of 100 C as the number of electrons.
ii State the charge in coulombs of 1 million electrons.
iii What is the increase in the number of electrons between charges of 1000 C and 1 000 000 C?

4 Using an Excel spreadsheet, set up a conversion table between electrical current in amperes (A), time in seconds and charge in units of C and e, as shown below.
Note: Charge in coulombs = amperes × time in seconds



Hint: When referring to cell A22, type $A$22.

Formulas using amperes will need \$ signs since $A22$ is a fixed cell.

	A	B	C	D
20	**Conversion table between amperes, time and electrical charge measured in coulombs and numbers of electrons**			
21	Electrical current (A)	Time (s)	Charge Q (in coulombs, C)	Charge Q (in number of electrons, e)
22	0.5	1		
23		30		
24		60		
25		90		
26		120		
27		150		
28		180		

Use your spreadsheet to find the answers to the following questions and write them in scientific notation, to nine significant figures where possible.

a If a current of 0.50 A flows through a circuit for 90 seconds, how much charge will have passed into the circuit:
  i in coulombs?
  ii in number of electrons?

b If a current of 1.5 A flows through a circuit for 150 seconds, how much charge will have passed into the circuit:
  i in coulombs?
  ii in number of electrons?

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Puzzles and games

1 In this magic square, each row and column adds to a sum that is an algebraic expression. Complete the square to find the sum.

$\frac{4x^2}{2x}$	$-y$	$x+3y$
$x-2y$		$2y$

2 Write $3^{n-1} \times 3^{n-1} \times 3^{n-1}$ as a single power of 3.

3 You are offered a choice of two prizes:
- One million dollars right now, or
- You can receive 1 cent on the first day of a 30-day month, double your money every day for 30 days and receive the total amount on the 30th day.

Which prize offers the most money?

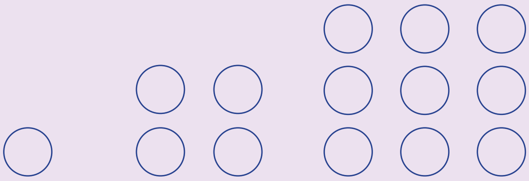
4 Simplify $\frac{25^6 \times 5^4}{125^5}$ without the use of a calculator.

5 Write $(((2^1)^2)^3)^4$ as a single power of 2.

6 How many zeroes are there in 100^{100} in expanded form?

7 Simplify $\frac{x}{2} + \frac{3x}{5} - \frac{4x}{3} + \frac{x+1}{6}$.

8 Write a rule for the number of counters in the nth term of the pattern below. Use this to find the number of counters in the 15th term.



Term 1 Term 2 Term 3

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Chapter summary

Algebra

Expanding
$a(b+c) = ab + ac$
$a(b-c) = ab - ac$
e.g. $2(4x+3) = 8x + 6$
$-3x(2x-y) = -6x^2 + 3xy$

Simplifying expressions
Add/subtract like terms only. Like terms have the same pronumeral factors.
e.g. $3x$ and $7x$, $2xy$ and $4yx$,
not $2x$ and x^2 or $3y$ and $4xy$.
For example,
$3x + 2y - x + 7y = 3x - x + 2y + 7y$
$= 2x + 9y$

Multiply/Divide
$3a \times 2b = 3 \times 2 \times a \times b$
$= 6ab$
$\frac{{}^{1}\cancel{7}x\cancel{y}^{1}}{{}^{2}\cancel{14}\cancel{y}_{1}} = \frac{x}{2}$

Algebraic expressions
$3xy + 7x - 4y + 3$
4 terms
3 is the constant term.
7 is the coefficient of x.
-4 is the coefficient of y.

Adding/subtracting algebraic fractions ★
Find the lowest common denominator and combine.
e.g. $\frac{x}{2} + \frac{2x}{3}$ LCD is 6.
$= \frac{3x}{6} + \frac{4x}{6}$
$= \frac{7x}{6}$

Factorising
This is the opposite of expanding

Factorised form		Expanded form
$2(x+4)$	=	$2x + 8$

Look for highest common factor of terms.
e.g. $3ab + 6a$ HCF is $3a$.
$= 3a(b+2)$
$-2x^2 - 6x$ HCF is $-2x$.
$= -2x(x+3)$

Multiplying/dividing algebraic fractions ★
Multiply: Cancel common factors in factorised form and then multiply.
e.g. $\frac{3\cancel{(x+4)}^{1}}{\cancel{2}_{1}} \times \frac{{}^{4}\cancel{8}x}{\cancel{x+4}_{1}}$
$= 12x$
Divide: Multiply by the reciprocal of the fraction following the ÷ sign.
Reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Indices

Index form a^m
a is the base.
m is the index/power/exponent.

Index form		Expanded form
5^4	=	$5 \times 5 \times 5 \times 5$

e.g. $3 \times a \times b \times a \times a \times b = 3a^3b^2$

Scientific notation
Used for very large and small numbers in the form $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$.
$3\,870\,000 = 3.87 \times 10^6$
$0.000\,21 = 2.1 \times 10^{-4}$

Exponential growth and decay ★
For a repeated percentage increase or decrease, the rule can be
$A = A_0\left(1 \pm \frac{r}{100}\right)^n$, where
A_0 is the initial amount
r is % rate
n is the amount of time.
Use '+' for growth and '–' for decay.

Significant Figures
These are counted from left to right from the first non-zero digit
e.g. 213 270 is 2.13×10^5 in scientific notation to 3 significant figures.
0.000429 1 is 4.3×10^{-4} in scientific notation to 2 significant figures

Negative indices
$a^{-m} = \frac{1}{a^m}$
e.g. $x^{-3} = \frac{1}{x^3}$
$\frac{1}{a^{-m}} = a^m$
e.g. $\frac{4}{2^{-3}} = 4 \times 2^3 = 32$

Index laws
Law 1: $a^m \times a^n = a^{m+n}$
Law 2: $a^m \div a^n = a^{m-n}$
Law 3: $(a^m)^n = a^{m \times n}$
Law 4: $(ab)^m = a^m \times b^m$
Law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero power: $a^0 = 1$ when $a \neq 0$.

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
3A	**1 I can identify the parts of an algebraic expression.** e.g. For the expression $4a + 3b - 7$, state the: **a** number of terms **b** constant term **c** coefficient of b	
3A	**2 I can form an algebraic expression.** e.g. Write an algebraic expression for: **a** 3 more than 2 lots of a **b** the product of x and y, divided by 2	
3A	**3 I can evaluate an algebraic expression using substitution.** e.g. If $x = 2$, $y = 5$ and $z = -3$, evaluate: **a** $xy + 2z$ **b** $y^2 - xz$	
3B	**4 I can identify like terms.** e.g. Write down the like terms in the following list: $ax, 7b, 6x, -3b, -2xa$	
3B	**5 I can collect like terms.** e.g. Simplify: **a** $5b + 4b - 2$ **b** $4xy + 3x - 5xy + 3x$	
3B	**6 I can multiply and divide algebraic terms.** e.g. Simplify: **a** $3a \times 5ab$ **b** $9xy \div (18x)$	
3C	**7 I can expand expressions with brackets.** e.g. Expand the following. **a** $4(3x - 2)$ **b** $-2y(5x - 7y)$	
3C	**8 I can simplify expressions by removing brackets.** e.g. Expand and simplify $3(2x + 5) - 2(x + 2)$.	
3D	**9 I can determine the HCF.** e.g. Determine the HCF of the following. **a** $6x$ and $24x$ **b** $8ab$ and $20b^2$	
3D	**10 I can factorise expressions with common factors.** e.g. Factorise the following. **a** $8a + 12$ **b** $6x^2 - 10xy$ **c** $-4ab - 18a$ (including common negative)	
3E	**11 I can simplify algebraic fractions.** e.g. Simplify this fraction by factorising first: $\frac{4x - 12}{x - 3}$.	
3E	**12 I can multiply algebraic fractions.** e.g. Simplify this product: $\frac{3(x - 2)}{4x} \times \frac{10x}{x - 2}$.	

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Chapter checklist

			✔
3E	13	**I can divide algebraic fractions.** e.g. Simplify $\frac{5x^2}{9} \div \frac{10x}{3}$.	
3F	14	**I can add and subtract simple algebraic fractions.** e.g. Simplify $\frac{3x}{4} - \frac{x}{16}$.	
3F	15	**I can add and subtract with binomial numerators.** e.g. Simplify $\frac{x+2}{4} - \frac{x}{10}$.	
3G	16	**I can write in index form.** e.g. Write the following in index form. **a** $7 \times 7 \times 7 \times 7$ **b** $x \times y \times x \times x \times y$	
3G	17	**I can use index law 1.** e.g. Simplify the following using the first index law. **a** $y^3 \times y^5$ **b** $3a^3b \times 5a^2b^3$	
3G	18	**I can use index law 2.** e.g. Simplify the following using the second index law. **a** $\frac{4m^5}{2m^3}$ **b** $5x^5y^4 \div (15xy^2)$	
3G	19	**I can combine index laws 1 and 2.** e.g. Simplify $\frac{2x^5y^3 \times 3x^2y^2}{9x^3y^4}$.	
3H	20	**I can use the zero power.** e.g. Evaluate using the zero power: $5^0 + a^0$.	
3H	21	**I can use index law 3.** e.g. Simplify using the third index law: $4(x^2)^5$.	
3H	22	**I can use index laws 4 and 5.** e.g. Simplify, using index laws. **a** $(2m)^3$ **b** $\left(\frac{x^2}{3}\right)^4$	
3H	23	**I can combine index laws.** e.g. Simplify: **a** $\frac{(m^2n)^5}{4m^3n^5}$ **b** $(3a)^0 - (4a^3)^2$	
3I	24	**I can express negative indices in positive index form.** e.g. Rewrite the following with positive indices only. **a** $3y^{-2}$ **b** $\frac{4}{x^2y^{-3}}$	

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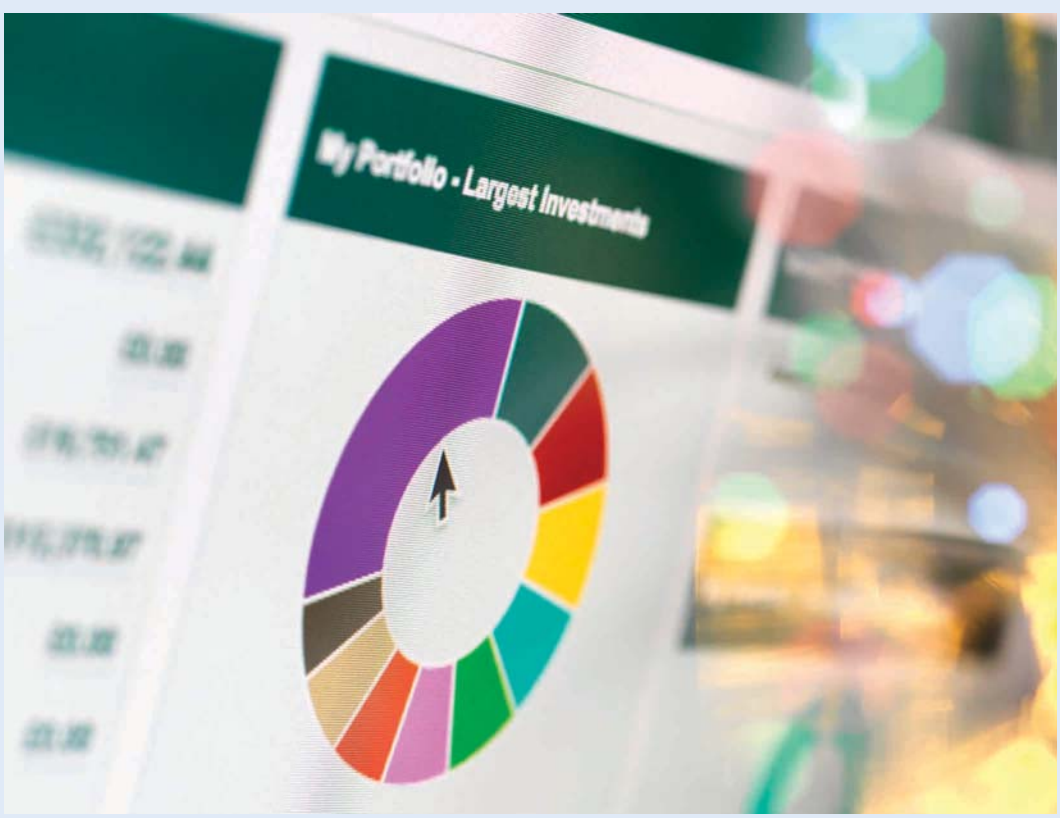
Chapter checklist

		✔
3I	**25 I can use index laws with negative indices.** e.g. Simplify, expressing with positive indices: $\dfrac{a^4b^3 \times a^{-2}(b^2)^{-3}}{a^6}$.	
3J	**26 I can convert from scientific notation to a basic numeral.** e.g. Write these numbers as a basic numeral. **a** 3.02×10^4 **b** 7.21×10^{-5}	
3J	**27 I can write numbers using scientific notation.** e.g. Write these numbers in scientific notation. **a** $64\,000$ **b** 0.000035	
3J	**28 I can write in scientific notation rounding to significant figures.** e.g. Write these numbers in scientific notation using three significant figures. **a** $472\,815$ **b** 0.0053821	
3K	**29 I can form exponential rules.** e.g. Write a rule for this statement: A substance of mass 450 g is decaying at a rate of 14% per day.	
3K	**30 I can apply exponential rules.** e.g. A share portfolio valued at $\$80\,000$ is expected to grow by 6% per year. Determine a rule for the value of the portfolio ($\$V$) in n years' time then use this to find the value in 4 years' time. Use trial and error to find when the portfolio will be valued at $\$115\,000$, correct to one decimal place.	



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Chapter review

Short-answer questions

3A **1** Consider the expression $3xy - 3b + 4x^2 + 5$.
- **a** How many terms are in the expression?
- **b** What is the constant term?
- **c** State the coefficient of:
  - **i** x^2 **ii** b

3A **2** Write an algebraic expression for the following.
- **a** 3 more than y
- **b** 5 less than the product of x and y
- **c** the sum of a and b is divided by 4

3A **3** Evaluate the following if $x = 3$, $y = 5$ and $z = -2$.
- **a** $3x + y$ **b** xyz **c** $y^2 - 5z$

3B **4** Simplify the following expressions.
- **a** $4x - 5 + 3x$ **b** $4a - 5b + 9a + 3b$ **c** $3xy + xy^2 - 2xy - 4y^2x$
- **d** $3m \times 4n$ **e** $-2xy \times 7x$ **f** $\frac{8ab}{12a}$

3C **5** Expand the following and collect like terms where necessary.
- **a** $5(2x + 4)$ **b** $-2(3x - 4y)$ **c** $3x(2x + 5y)$
- **d** $3 + 4(a + 3)$ **e** $3(y + 3) + 2(y + 2)$ **f** $5(2t + 3) - 2(t + 2)$

3D **6** Factorise the following expressions.
- **a** $16x - 40$ **b** $10x^2y + 35xy^2$
- **c** $4x^2 - 10x$ **d** $-2xy - 18x$ (include the common negative)

3F **7** Simplify the following algebraic fractions involving addition and subtraction.
- **a** $\frac{2x}{3} + \frac{4x}{15}$ **b** $\frac{3}{7} - \frac{a}{2}$ **c** $\frac{x+4}{4} + \frac{x-3}{5}$

3E **8** Simplify these algebraic fractions by first cancelling common factors in factorised form.
- **a** $\frac{5x}{12} \times \frac{9}{10x}$ **b** $\frac{x+2}{4} \times \frac{16x}{x+2}$ **c** $\frac{12x-4}{4}$ **d** $\frac{x-3}{4} \div \frac{3(x-3)}{8}$

3G **9** Simplify the following, using index laws 1 and 2.
- **a** $3x^5 \times 4x^2$ **b** $4xy^6 \times 2x^3y^2$ **c** $\frac{b^7}{b^3}$ **d** $\frac{4a^3b^5}{6ab^2}$

3H **10** Simplify the following, using the third, fourth and fifth index laws.
- **a** $(b^2)^4$ **b** $(2m^2)^3$ **c** $\left(\frac{x}{7}\right)^2$ **d** $\left(\frac{4y^2}{z^4}\right)^3$

3H **11** Simplify the following, using the zero power.
- **a** 7^0 **b** $4x^0$ **c** $5a^0 + (2y)^0$ **d** $(x^2 + 4y)^0$

3I **12** Express the following, using positive indices.
- **a** $4x^{-3}$ **b** $3r^4s^{-2}$ **c** $\frac{2x^{-3}y^4}{3}$ **d** $\frac{4}{m^{-5}}$

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Chapter review

3I **13** Simplify the following, using index laws. Express all answers with positive indices.

a $\dfrac{3x^2y^4 \times 5xy^7}{12x^3y^5}$ **b** $\dfrac{(5x^2y)^2 \times 4xy^2}{8(xy)^2}$

c $\dfrac{2x^3y^2 \times 5xy^2}{x^7y^4}$ **d** $\dfrac{4x^2y^5}{8(x^2)^{-3}y^8}$

3J **14** Write these numbers as a basic numeral.

a 4.25×10^3 **b** 3.7×10^7 **c** 2.1×10^{-2} **d** 7.25×10^{-5}

3J **15** Convert these numbers to scientific notation, using three significant figures.

a 123 574 **b** 39 452 178 **c** 0.0000090241 **d** 0.00045986

3K **16** Form an exponential equation for the following.

a The population of a colony of kangaroos, which starts at 20 and is increasing at a rate of 10%.

b The amount of petrol in a petrol tank fuelling a generator if it starts with 100 000 litres and uses 15% of its fuel every hour.

Multiple-choice questions

3A **1** The coefficient of x in $3xy - 4x + 7$ is:

A 4 **B** 7 **C** -4 **D** 3 **E** -1

3B **2** The simplified form of $7ab + 2b - 5ab + b$ is:

A $2ab + 2b^2$ **B** $2ab + 3b$ **C** $5ab$ **D** $2ab + b$ **E** $12ab + 3b$

3C **3** The expanded form of $2x(3x - 5)$ is:

A $6x^2 - 5$ **B** $6x - 10$ **C** $6x^2 - 10x$ **D** $5x^2 - 10x$ **E** $-4x$

3D **4** The fully factorised form of $8xy - 24y$ is:

A $4y(2x - 6y)$ **B** $8(xy - 3y)$ **C** $8y(x - 24)$ **D** $8y(x - 3)$ **E** $8x(y - 24)$

3E **5** The simplified form of $\dfrac{2(x+1)}{5x} \times \dfrac{15}{x+1}$ is:

A $\dfrac{6}{x+1}$ **B** $\dfrac{6}{x}$ **C** $\dfrac{3(x+1)}{x}$ **D** $6x$ **E** $\dfrac{3x}{x+1}$

3F **6** The sum of the algebraic fractions $\dfrac{3x}{8} + \dfrac{x}{12}$ is:

A $\dfrac{x}{5}$ **B** $\dfrac{x}{6}$ **C** $\dfrac{x}{24}$ **D** $\dfrac{11x}{24}$ **E** $\dfrac{9x}{24}$

3G **7** $3x^3y \times 2x^5y^3$ is equal to:

A $5x^{15}y^3$ **B** $6x^{15}y^3$ **C** $6x^8y^4$ **D** $5x^8y^4$ **E** $6x^8y^3$

3H **8** $(2x^4)^3$ can be written as:

A $2x^{12}$ **B** $2x^7$ **C** $6x^{12}$ **D** $8x^{12}$ **E** $8x^7$

3H **9** $5x^0 - (2x)^0$ is equal to:

A 4 **B** 0 **C** 3 **D** 2 **E** -1

3I **10** $12a^4 \div (4a^7)$ simplifies to:

A $3a^3$ **B** $8a^3$ **C** $3a^{11}$ **D** $\dfrac{8}{a^3}$ **E** $\dfrac{3}{a^3}$

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3J **11** 417 000 converted to scientific notation is:

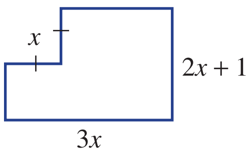
A 4.17×10^{-5} **B** 417×10^3 **C** 4.17×10^5
D 0.417×10^6 **E** 41.7×10^{-2}

3K **12** A rule for the amount of money, A, in an account after n years, if \$1200 is invested at 4% per year, is:

A $A = 1200(4)^n$ **B** $A = 1200(1.4)^n$ **C** $A = 1200(0.96)^n$
D $A = 1200(1.04)^n$ **E** $A = 1200(0.04)^n$

Extended-response questions

1 A room in a house has the shape and dimensions, in metres, shown. All lines meet at 90°.



a Find the perimeter of the room, in factorised form.

b If $x = 3$, what is the room's perimeter?

The floor of the room is to be recarpeted.

c Give the area of the floor in terms of x and in expanded form.

d If the carpet costs \$20 per square metre and $x = 3$, what is the cost of laying the carpet?

2 During the growing season, a certain type of water lily spreads by 9% per week. The water lily covers an area of 2 m^2 at the start of the growing season.

a Write a rule for the area, $A\text{ m}^2$, covered by the water lily after n weeks.

b Calculate the area covered, correct to four decimal places, after:

i 2 weeks
ii 5 weeks

c Use trial and error to determine, to one decimal place, when there will be a coverage of 50 m^2.

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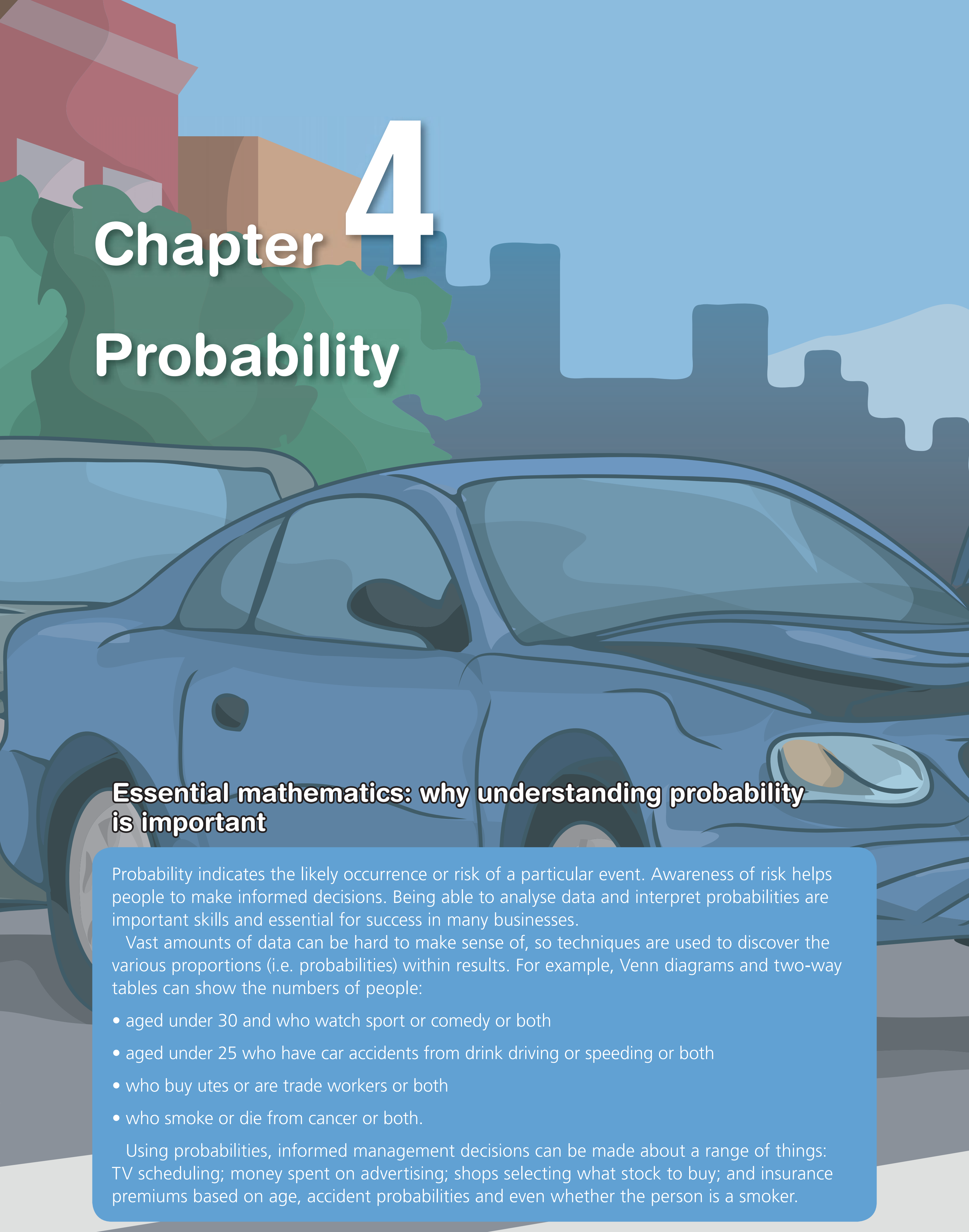

Chapter 4 Probability

Essential mathematics: why understanding probability is important

Probability indicates the likely occurrence or risk of a particular event. Awareness of risk helps people to make informed decisions. Being able to analyse data and interpret probabilities are important skills and essential for success in many businesses.

Vast amounts of data can be hard to make sense of, so techniques are used to discover the various proportions (i.e. probabilities) within results. For example, Venn diagrams and two-way tables can show the numbers of people:

- aged under 30 and who watch sport or comedy or both
- aged under 25 who have car accidents from drink driving or speeding or both
- who buy utes or are trade workers or both
- who smoke or die from cancer or both.

Using probabilities, informed management decisions can be made about a range of things: TV scheduling; money spent on advertising; shops selecting what stock to buy; and insurance premiums based on age, accident probabilities and even whether the person is a smoker.

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In this chapter

Victorian Curriculum

STATISTICS AND PROBABILITY

Chance

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (VCMSP347)

Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language (VCMSP348)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

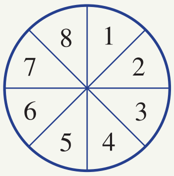
1 A letter is selected from the word PROBABILITY.

a How many letters are there in total?

b Find the chance (i.e. probability) of selecting:

i the letter R **ii** the letter B **iii** a vowel
iv not a vowel **v** a T or an I **vi** neither a B nor a P

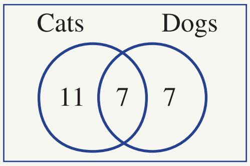
2 A spinning wheel has eight equal sectors numbered 1 to 8. On one spin of the wheel, find the following probabilities.



a Pr(5) **b** Pr(even)
c Pr(not even) **d** Pr(multiple of 3)
e Pr(factor of 12) **f** Pr(odd or a factor of 12)
g Pr(both odd and a factor of 12)

3 Arrange from lowest to highest: $\frac{1}{2}$, 0.4, 1 in 5, 39%, $\frac{3}{4}$, 1, 0, $\frac{9}{10}$, 0.62, 71%.

4 This Venn diagram shows the number of people in a group of 25 who own cats and/or dogs.



a State the number of people who own:

i a dog **ii** a cat or a dog (including both)
iii only a cat

b If a person is selected at random from this group, find the probability that they will own:

i a cat **ii** a cat and a dog **iii** only a dog

5 Drew shoots from the free-throw line on a basketball court. After 80 shots he counts 35 successful throws.

a Estimate the probability, in simplified form, that his next throw will be successful.

b Estimate the probability, in simplified form, that his next throw will not be successful.

6 Two 4-sided dice are rolled and the sum of the two numbers obtained is noted. Copy and complete this grid to help answer the following.

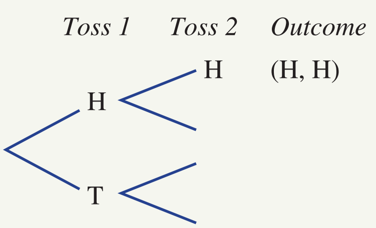
Roll 2 \ Roll 1	1	2	3	4
1				
2				
3				
4				

a What is the total number of outcomes?

b Find the probability that the total sum is:

i 2
ii 4
iii less than 5
iv less than or equal to 5
v at most 6
vi no more than 3

7 Two coins are tossed. Copy and complete this tree diagram to help answer the following.



a State the total number of outcomes.

b Find the probability of obtaining:

i 2 heads
ii no heads
iii 1 tail
iv at least 1 tail
v 1 of each, a head and a tail
vi at most 2 heads

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4A Review of probability CONSOLIDATING

Learning intentions

- To understand the idea of chance and how to describe it numerically
- To know that the level of chance is based on a numerical value
- To be able to find the probability of an event for equally likely outcomes
- To be able to calculate an experimental probability

Key vocabulary: theoretical probability, experimental probability, trial, sample space, outcome, event, chance, long run proportion

Probability is an area of mathematics concerned with the likelihood of particular random events. In some situations, such as rolling a die, we can determine theoretical probabilities because we know the total number of outcomes and the number of favourable outcomes. In other cases we can use statistics and experimental results to describe the chance that an event will occur. The chance that a particular soccer team will win its next match, for example, could be estimated using various results from previous games.

Lesson starter: Name the event

For each number below, describe an event that has that exact or approximate probability. If you think it is exact, give a reason.

$\frac{1}{2}$ $\quad$ 25% $\quad$ 0.2 $\quad$ 0.00001 $\quad$ $\frac{99}{100}$

Key ideas

- Definitions:
  - **Probability** is the likelihood of an event happening.
  - A **trial** is a single undertaking of an experiment, such as a single roll of a die.
  - The **sample space** is the list of outcomes from an experiment, such as $\{1, 2, 3, 4, 5, 6\}$ from rolling a 6-sided die.
  - An **outcome** is a possible result of an experiment, such as a 6 on the roll of a die.
  - An **event** is the list of favourable outcomes, such as a 5 or 6 from the roll of a die.
  - Equally likely outcomes are possible results that have the same chance of occurring.
- In the study of probability, a numerical value based on a scale from 0 to 1 is used to describe levels of **chance** – the likelihood of an event happening.

  0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

  impossible (0), unlikely (0.1), even chance (0.5), likely (0.85), certain (1)
- A probability can be written as a decimal, fraction or percentage; e.g. 0.125, $\frac{1}{8}$ or 12.5%.

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- The **theoretical probability** of an event in which outcomes are equally likely is calculated as follows:

$$\text{Pr(Event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- **Experimental probability** is calculated in the same way as theoretical probability but uses the results of an experiment.

$$\text{Pr(Event)} = \frac{\text{number of favourable outcomes}}{\text{total number of trials}}$$

  - The **long run proportion** is the experimental probability for a sufficiently large number of trials.

Exercise 4A

Understanding | 1–3 | 3

1 Complete the following by filling in the blanks.

a A list of all the possible outcomes from an experiment is called the ________________.

b The numerical values used to describe levels of chance are between ________ and ________.

c An event with a probability of 0.8 would be described as ________ to occur.

d Obtaining a tail from the toss of a coin is called an ________ of the experiment.

2 Order these events (**A–D**) from least likely to most likely.

A The chance that it will rain every day for the next 10 days.

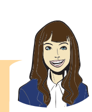
B The chance that a member of class is ill on the next school day.

C The chance that school is cancelled next year.

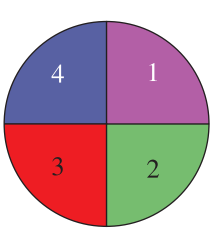
D The chance that the Sun comes up tomorrow.

3 For the following spinners, find the probability that the outcome will be a 4.

Hint: Since each section is equal, $\text{Pr}(4) = \frac{\text{number of 4s}}{\text{total number of sections}}$



a



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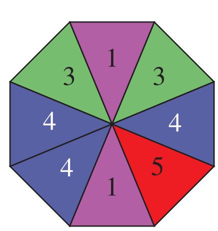

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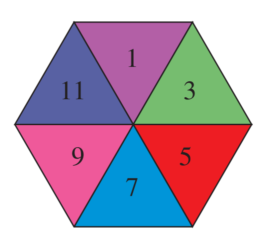

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Fluency

4–6 | 4, 6, 7

Example 1 Calculating simple theoretical probabilities

A letter is chosen randomly from the word TELEVISION.

a How many letters are there in the word TELEVISION?

b Find the probability that the letter is:

i a V
ii an E
iii not an E
iv an E or a V

Solution	Explanation
a 10	The sample space includes 10 letters.
b **i** $\Pr(V) = \frac{1}{10} (= 0.1)$	$\Pr(V) = \frac{\text{number of Vs}}{\text{total number of letters}}$
ii $\Pr(E) = \frac{2}{10}$	There are 2 Es in the word TELEVISION.
$= \frac{1}{5} (= 0.2)$	Simplify the fraction.
iii $\Pr(\text{not an E}) = \frac{8}{10}$	If there are 2 Es in the word TELEVISION with 10 letters, then there must be 8 letters that are not E.
$= \frac{4}{5} (= 0.8)$	
iv $\Pr(\text{an E or a V}) = \frac{3}{10} (= 0.3)$	The number of letters that are either E or V is 3.

Now you try

A letter is chosen randomly from the word DINNER.

a How many letters are there in the word DINNER?

b Find the probability that the letter is:

i a D
ii an N
iii not an N
iv a D or an N

4 A letter is chosen randomly from the word TEACHER.

a How many letters are there in the word TEACHER?

b Find the probability that the letter is:

i an R
ii an E
iii not an E
iv an R or an E

5 A letter is chosen randomly from the word EXPERIMENT. Find the probability that the letter is:

a an E
b a vowel
c not a vowel
d an X or a vowel

Hint: The vowels are A, E, I, O and U.

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4A

Example 2 Calculating simple experimental probabilities

An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

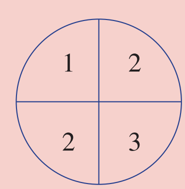
Number of heads	0	1	2	3
Frequency	11	40	36	13

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads
  ii 2 heads
  iii fewer than 2 heads
  iv at least 1 head

Solution	Explanation
a 36	From the table, you can see that 2 heads has a frequency of 36.
b $11 + 40 = 51$	Fewer than 2 means obtaining 0 heads or 1 head.
c **i** $\text{Pr(0 heads)} = \frac{11}{100} = 0.11$	$\text{Pr(0 heads)} = \frac{\text{number of times 0 heads is observed}}{\text{total number of trials}}$
ii $\text{Pr(2 heads)} = \frac{36}{100} = 0.36$	$\text{Pr(2 heads)} = \frac{\text{number of times 2 heads is observed}}{\text{total number of trials}}$
iii Pr(fewer than 2 heads) $= \frac{11 + 40}{100} = \frac{51}{100} = 0.51$	Fewer than 2 heads means to observe 0 or 1 head.
iv Pr(at least 1 head) $= \frac{40 + 36 + 13}{100} = \frac{89}{100} = 0.89$	At least 1 head means that 1, 2 or 3 heads can be observed.

Now you try

An experiment involves spinning the spinner shown 3 times and counting the number of 2s. Here are the results after running the experiment 100 times.



Number of 2s	0	1	2	3
Frequency	15	34	42	9

a How many times did one 2 occur?
b How many times did more than one 2 occur?
c Find the experimental probability of obtaining:
  i no 2s
  ii three 2s
  iii fewer than two 2s
  iv at least one 2

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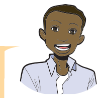


Hint: The total number of outcomes is 100.

6 An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	9	38	43	10

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads
  ii 2 heads
  iii fewer than 2 heads
  iv at least 1 head

7 An experiment involves rolling two dice and counting the number of 6s. Here are the results after running the experiment 100 times.

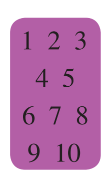
Number of 6s	0	1	2
Frequency	62	35	3

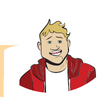
Find the experimental probability of obtaining:
a no 6s
b two 6s
c fewer than two 6s
d at least one 6

Problem-solving and reasoning

8, 9 | 9–11

8 A 10-sided die, numbered 1 to 10, is rolled once. Find these probabilities.
a Pr(8)
b Pr(odd)
c Pr(even)
d Pr(less than 6)
e Pr(prime)
f Pr(3 or 8)
g Pr(8, 9 or 10)





Hint: Prime numbers less than 10 are 2, 3, 5 and 7.

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4A

9 Amelia is a prizewinner in a competition and will be randomly awarded a single prize chosen from a collection of 50 prizes. The type and number of prizes to be handed out are listed below.

Hint: Remember that the total number of prizes is 50.

Prize	Car	Holiday	iPad	Blu-ray player
Number	1	4	15	30

Find the probability that Amelia will be awarded the following.

a a car
b an iPad
c a prize that is not a car

10 Many of the 50 cars inspected at an assembly plant contain faults. The results of the inspection are as follows.

Number of faults	0	1	2	3	4
Number of cars	30	12	4	3	1

Find the experimental probability that a car selected from the assembly plant will have:

a 1 fault
b 4 faults
c fewer than 2 faults
d 1 or more faults
e 3 or 4 faults
f at least 2 faults

11 A bag contains red and yellow counters. A counter is drawn from the bag and then replaced. This happens 100 times and 41 of the counters drawn were red.

Hint: $\frac{41}{100}$ were red.

a How many counters drawn were yellow?
b If there were 10 counters in the bag, how many do you expect were red? Give a reason.
c If there were 20 counters in the bag, how many do you expect were red? Give a reason.

Cards probability — 12

12 A card is chosen from a standard deck of 52 playing cards that includes 4 aces, 4 kings, 4 queens and 4 jacks. Find the following probabilities.

Hint: There are 4 suits in a deck of cards: hearts, diamonds, spades and clubs.

a Pr(heart)
b Pr(king)
c Pr(king of hearts)
d Pr(heart or club)
e Pr(king or jack)
f Pr(heart or king)
g Pr(not a king)
h Pr(neither a heart nor a king)

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4B Venn diagrams

Learning intentions

- To understand how a Venn diagram is used to show the distribution of the sample space among events
- To know the notation and regions of a Venn diagram that represent the union, intersection and complement
- To be able to use a Venn diagram to display the distribution of two sets
- To be able to use a Venn diagram to calculate probabilities of events

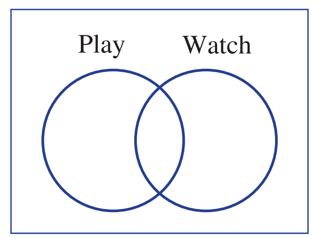
Key vocabulary: Venn diagram, union, intersection, complement, mutually exclusive

Sometimes we need to work with situations where there are overlapping events. A TV station, for example, might be collecting statistics regarding whether or not a person watches cricket and/or tennis or neither over a certain period of time. The estimated probability that a person will watch cricket *or* tennis will therefore depend on how many people responded *yes* to watching both cricket *and* tennis. Venn diagrams are a useful tool when dealing with such events.

Lesson starter: How many like both?

Of 20 students in a class, 12 people like to play tennis and 15 people like to watch tennis. Two people like neither playing nor watching tennis. Some like both playing and watching tennis.

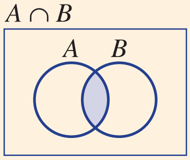
- Is it possible to represent this information in a Venn diagram?
- How many students like to play and watch tennis?
- How many students like to watch tennis only?
- From the group of 20 students, what would be the probability of selecting a person that likes watching tennis only?



Key ideas

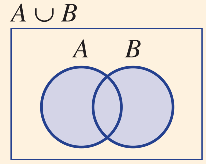
- A **Venn diagram** illustrates how all elements in the sample space are distributed among the events.
  - All elements that belong to both A *and* B make up the **intersection**: $A \cap B$.

    $A \cap B$

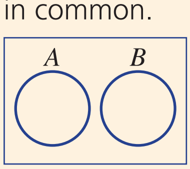
    

  - All elements that belong to either events A *or* B make up the **union**: $A \cup B$.

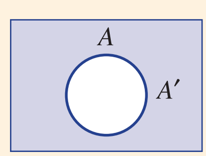
    $A \cup B$

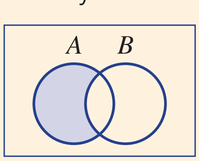
  - Two sets A and B are **mutually exclusive** if they have no elements in common.

  - For an event A, the **complement** of A is A' (or 'not A'). $\Pr(A') = 1 - \Pr(A)$

  - 'A only' is defined as all the elements in A but not in any other set.

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Exercise 4B

Understanding | 1–3 | 3

1 Match the words in the left column with the description in the right column for two sets.

a union — **A** no elements in both sets

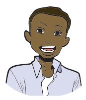
b intersection — **B** elements not in the set

c complement — **C** elements in both sets

d mutually exclusive — **D** elements in either set

2 Decide whether the events A and B are mutually exclusive.

Hint: Mutually exclusive events have nothing in common.



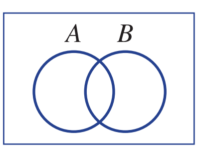
a $A = \{1, 3, 5, 7\}$
$B = \{5, 8, 11, 14\}$

b $A = \{-3, -2, \ldots, 4\}$
$B = \{-11, -10, \ldots, -4\}$

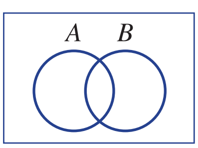
c $A = \{\text{prime numbers}\}$
$B = \{\text{even numbers}\}$

3 Copy these Venn diagrams and shade the region described by each of the following.

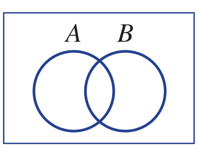
a A



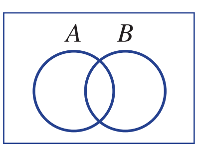
b $A \cap B$ (i.e. A and B)



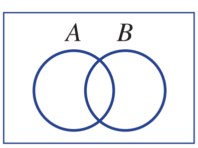
c $A \cup B$ (i.e. A or B)



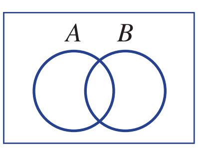
d B only



e A' (not A)



f neither A nor B



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Fluency

4–6 | 5–7

Example 3 Listing sets

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 3, 4, 5, 6\}$ $B = \{1, 3, 7, 8\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets.

i $A \cap B$ (i.e. A and B) **ii** $A \cup B$ (i.e. A or B)

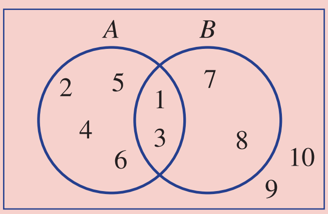
c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why or why not?

Solution | **Explanation**

a



The elements 1 and 3 are common to both sets A and B. The elements 9 and 10 belong to neither set A nor set B.

b **i** $A \cap B = \{1, 3\}$

$A \cap B$ is the intersection of sets A and B.

ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cup B$ contains elements in either A or B.

c **i** $\Pr(A) = \frac{6}{10} = \frac{3}{5}$

There are 6 numbers in A.

ii $\Pr(A \cap B) = \frac{2}{10} = \frac{1}{5}$

$A \cap B$ contains 2 numbers.

iii $\Pr(A \cup B) = \frac{8}{10} = \frac{4}{5}$

$A \cup B$ contains 8 numbers.

d The sets A and B are not mutually exclusive since there are numbers inside $A \cap B$.

The set $A \cap B$ contains at least 1 number.

Now you try

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$A = \{2, 4, 6, 8, 10\}$ $B = \{2, 3, 5, 7\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets.

i $A \cap B$ (i.e. A and B) **ii** $A \cup B$ (i.e. A or B)

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why or why not?

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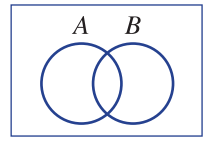
4B

4 Consider the given events A and B, which involve numbers taken from the first 10 positive integers.

$$A = \{1, 2, 4, 5, 7, 8, 10\} \qquad B = \{2, 3, 5, 6, 8, 9\}$$

Hint: $A \cap B$ means A *and* B. $A \cup B$ means A *or* B.



a Represent events A and B in a Venn diagram.

b List the following sets.

i $A \cap B$ **ii** $A \cup B$

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why or why not?

5 The elements of the events A and B described below are numbers taken from the first 10 prime numbers.

$$A = \{2, 5, 7, 11, 13\} \qquad B = \{2, 3, 13, 17, 19, 23, 29\}$$

a Represent events A and B in a Venn diagram.

b List the elements belonging to the following.

i A and B **ii** A or B

c If a number from the first 10 prime numbers is selected, find the probability that these events occur.

i A **ii** B **iii** $A \cap B$ **iv** $A \cup B$

Example 4 Using Venn diagrams

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.

a Illustrate this information in a Venn diagram.

b State the number of students who enjoy:

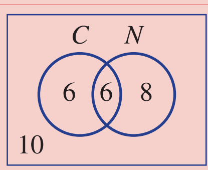
i netball only **ii** neither cricket nor netball

c Find the probability that a person chosen randomly from the class will enjoy:

i netball **ii** netball only **iii** both cricket and netball

Solution | **Explanation**

a



First place the 6 in the intersection (i.e. 6 enjoy cricket and netball), then determine the other values according to the given information.

The total must be 30, with 12 in the cricket circle and 14 in netball.

b **i** 8 — Includes students in N but not in C.

ii 10 — These are the students outside both C and N.

c **i** $\Pr(N) = \frac{14}{30} = \frac{7}{15}$ — 14 of the 30 students enjoy netball.

ii $\Pr(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$ — 8 of the 30 students enjoy netball but not cricket.

iii $\Pr(C \cap N) = \frac{6}{30} = \frac{1}{5}$ — 6 students like both cricket and netball.

Continued on next page

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Now you try

From a survey of 20 families, 7 enjoy camping (C), 10 enjoy beach holidays (B) and 2 enjoy both camping and beach holidays.

a Illustrate this information in a Venn diagram.
b State the number of families who enjoy:
 i camping only
 ii neither camping nor beach holidays
c Find the probability that a randomly chosen family from the survey enjoys:
 i camping
 ii camping only
 iii both camping and beach holidays

6 From a group of 50 adults, 35 enjoy reading fiction (F), 20 enjoy reading non-fiction (N) and 10 enjoy reading both fiction and non-fiction.

Hint: First enter the '10' in the intersection, then balance all the other regions.

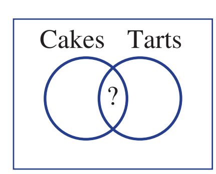
a Illustrate the information in a Venn diagram.
b State the number of people who enjoy:
 i fiction only
 ii neither fiction nor non-fiction
c Find the probability that a person chosen randomly from the group will enjoy reading:
 i non-fiction
 ii non-fiction only
 iii both fiction and non-fiction

7 At a show, 45 children have the choice of riding on the Ferris wheel (F) and/or the Big Dipper (B). 35 of the children wish to ride on the Ferris wheel, 15 children want to ride on the Big Dipper and 10 children want to ride on both.

a Illustrate the information in a Venn diagram.
b State the number of children who want to:
 i ride on the Ferris wheel only
 ii ride on neither the Ferris wheel nor the Big Dipper
c For a child chosen at random from the group, find the probability that they will want to ride on:
 i the Ferris wheel
 ii both the Ferris wheel and the Big Dipper
 iii the Ferris wheel or the Big Dipper
 iv not the Ferris wheel
 v neither the Ferris wheel nor the Big Dipper

Problem-solving and reasoning | 8, 9 | 8–10

8 In a group of 12 chefs, all enjoy baking cakes and/or tarts. In fact, 7 enjoy baking cakes and 8 enjoy baking tarts. Find out how many chefs from the group enjoy baking both cakes and tarts.



9 In a group of 32 car enthusiasts, all collect either vintage cars or modern sports cars. 18 collect vintage cars and 19 collect modern sports cars. How many from the group collect both vintage cars and modern sports cars?

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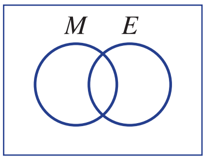
4B

10 Mario and Elisa are choosing a colour to paint the interior walls of their house. They have six colours to choose from: white (w), cream (c), navy (n), sky blue (s), maroon (m) and violet (v).

Mario would be happy with white or cream and Elisa would be happy with cream, navy or sky blue. As they can't decide, a colour is chosen at random for them.

Let M be the event that Mario will be happy with the colour and let E be the event that Elisa will be happy with the colour.

a Represent the events M and E in a Venn diagram.



b Find the probability that the following events occur.

- **i** Mario will be happy with the colour choice; i.e. find Pr(M).
- **ii** Mario will not be happy with the colour choice.
- **iii** Both Mario and Elisa will be happy with the colour choice.
- **iv** Mario or Elisa will be happy with the colour choice.
- **v** Neither Mario nor Elisa will be happy with the colour choice.

Courier companies — 11

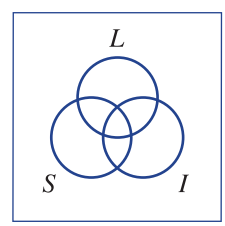
11 Of 15 chosen courier companies, 9 offer a local service (L), 7 offer an interstate service (S) and 6 offer an international service (I). Two companies offer all three services, 3 offer both local and interstate services, 5 offer only local services and 1 offers only an international service.

a Draw a Venn diagram displaying the given information.

b Find the number of chosen courier companies that offer neither local, interstate nor international services.

c If a courier is chosen at random from the 15 initially examined, find the following probabilities.

- **i** Pr(L)
- **ii** Pr(L only)
- **iii** Pr(L or S)
- **iv** Pr(L and S only)



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4C Two-way tables

Learning intentions

- To know that a two-way table is an alternate way of representing the information in a Venn diagram
- To understand how the rows and columns of a two-way table work
- To be able to fill out a two-way table either from a problem or from a Venn diagram
- To be able to use a two-way table to find associated probabilities

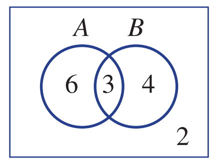
Key vocabulary: two-way table, Venn diagram

Like a Venn diagram, two-way tables are useful tools for the organisation of overlapping events. The totals at the end of each column and row help to find the unknown numbers required to solve various problems.

Lesson starter: Comparing Venn diagrams with two-way tables

Here is a Venn diagram and an incomplete two-way table.

- First, can you complete the two-way table?
- Describe what each box in the two-way table means.
- Was it possible to find all the missing numbers in the two-way table without referring to the Venn diagram?

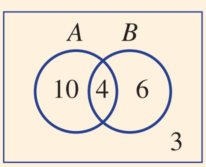


	A	A'	
B		4	
B'			8
	9		15

Key ideas

- **Two-way tables** use rows and columns to describe the number of elements in different regions of overlapping events. Each row and column sums to the total at the end.

Venn diagram



Two-way table

	A	A'	
B	4	6	10
B'	10	3	13
	14	9	23

$A \cap B$ → 4; B only → 6; A only → 10 (in row B'); Neither A nor B → 3; 10 → Total for B; 13 → Total for not B; 23 → Total; 14 → Total for A; 9 → Total for not A

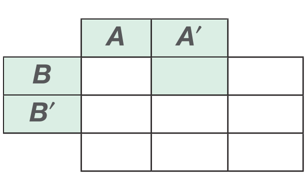
Exercise 4C

Understanding 1, 2 | 2

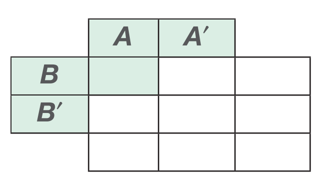
1 Match the shaded two-way tables (**A–D**) with each description (**a–d**).

a $A \cap B$ **b** B only **c** A **d** $A \cup B$

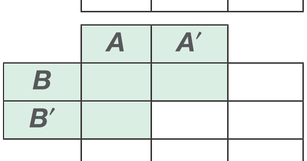
A



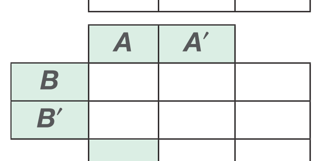
B



C



D



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4C

2 Look at this two-way table.

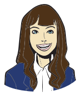
	A	*A′*	
B	4	3	7
B′	6	1	7
	10	4	14

a State the number of elements in these events.

i *A* and *B* **ii** *A* only
iii *B* only **iv** neither *A* nor *B*
v *A* **vi** *B*
vii *A′* **viii** B′

b $A \cup B$ (i.e. *A* or *B*) includes $A \cap B$, *A* only and *B* only.
Find the total number of elements in $A \cup B$.

Hint: *A* only is at the intersection of column *A* and row *B′*.

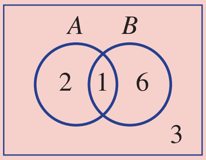


Fluency

3, 4 3–5

Example 5 Using two-way tables

The Venn diagram shows the distribution of elements in two sets, *A* and *B*.



a Transfer the information in the Venn diagram to a two-way table.

b Find the number of elements for these regions.

i *A* and *B* **ii** *B* only **iii** *A* only
iv neither *A* nor *B* **v** *A* **vi** not *B*
vii *A* or *B*

c Find:

i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

Solution

a

	A	*A′*	
B	1	6	7
B′	2	3	5
	3	9	12

Explanation

	A	*A′*	
B	$A \cap B$	*B* only	Total the row
B′	*A* only	Neither *A* nor *B*	Total the row
	Total the column	Total the column	Overall total

b i 1 — In both *A* and *B*

ii 6 — In *B* but not *A*

iii 2 — In *A* but not *B*

iv 3 — In neither *A* nor *B*

v 3 — Total of *A*

vi 5 — Total not in *B*

vii $2 + 1 + 6 = 9$ — In *A* only or *B* only or both (3 regions)

c i $\Pr(A \cap B) = \frac{1}{12}$

ii $\Pr(A') = \frac{9}{12} = \frac{3}{4}$

iii $\Pr(A \text{ only}) = \frac{2}{12} = \frac{1}{6}$

When calculating probabilities, you will need to divide the number of elements in each set by the number of elements in the sample space, which is 12.

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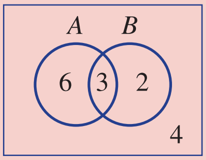
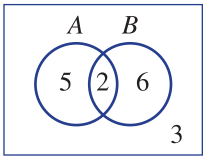
Now you try

The Venn diagram shows the distribution of elements in two sets, A and B.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements for these regions.
  i A and B **ii** B only **iii** A only
  iv neither A nor B **v** A **vi** not B
  vii A or B
c Find:
  i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

3 The Venn diagram shows the distribution of elements in two sets, A and B.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements in these regions.
  i A and B **ii** B only **iii** A only **iv** neither A nor B
  v A **vi** not B **vii** A or B
c Find:
  i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

4 From a total of 10 people, 5 like apples (A), 6 like bananas (B) and 4 like both apples and bananas.
a Draw a Venn diagram for the 10 people.
b Draw a two-way table for the 10 people.
c Find the number of people who like:
  i only bananas **ii** apples
  iii apples and bananas **iv** apples or bananas
d Find:
  i $\Pr(B)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A \text{ only})$ **iv** $\Pr(B')$ **v** $\Pr(A \cup B)$

Hint: Once you have your Venn diagram, you can transfer to the two-way table.

5 Of 12 people interviewed at a train station, 7 like staying in hotels, 8 like staying in apartments and 4 like staying in hotels and apartments.
a Draw a two-way table for the 12 people.
b Find the number of people who like:
  i only hotels
  ii neither hotels nor apartments
c Find the probability that one of the people interviewed likes:
  i hotels or apartments
  ii only apartments

Problem-solving and reasoning 6–8 7–10

6 Complete the following two-way tables.

a

	A	A'	
B		3	6
B'			
		4	11

b

	A	A'	
B	2	7	
B'			3
	4		

Hint: All the rows and columns should add up correctly.

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4C

7 In a class of 24 students, 13 like Mathematics, 9 like English and 3 like both.
  a Find the probability that a randomly selected student from the class likes both Mathematics and English.
  b Find the probability that a randomly selected student from the class likes neither Mathematics nor English.

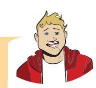
8 Two sets, A and B, are mutually exclusive.
  a Find $\Pr(A \cap B)$.
  b Now complete this two-way table.

	A	A'	
B		6	
B'			12
	10		18

9 Of 32 cars at a show, 18 cars have four-wheel drive, 21 are sports cars and 27 have four-wheel drive or are sports cars.
  a Find the probability that a randomly selected car at the show is both four-wheel drive and a sports car.
  b Find the probability that a randomly selected car at the show is neither four-wheel drive nor a sports car.

10 A card is selected from a deck of 52 playing cards. Find the probability that the card is:
  a a heart or a king
  b a club or a queen
  c a black card or an ace
  d a red card or a jack

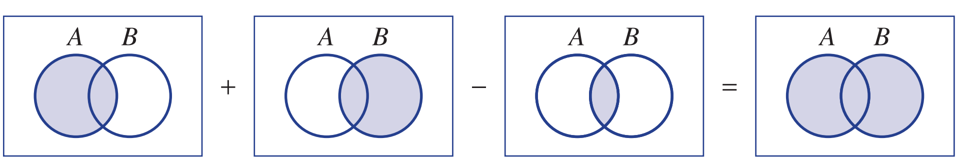
Hint: Make sure you don't count some cards twice; e.g. the king of hearts in part **a**.



The addition rule — 11

11 For some of the problems above you will have noticed the following, which is called the addition rule.

$A \quad + \quad B \quad - \quad A \cap B \quad = \quad A \cup B$



Use the addition rule to find $A \cup B$ in these problems.
  a Of 20 people at a sports day, 12 people like archery (A), 14 like basketball (B) and 8 like both archery and basketball ($A \cap B$). How many from the group like archery or basketball?
  b Of 100 households, 84 have wide-screen TVs, 32 have high-definition TVs and 41 have both. How many of the households have wide-screen or high-definition TVs?

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4D Conditional probability

Learning intentions

- To understand the notion of conditional probability
- To know the notation of conditional probability and how to calculate it
- To be able to calculate simple conditional probabilities from a Venn diagram or two-way table

Key vocabulary: conditional probability

The mathematics associated with the probability that an event occurs, given that another event has already occurred, is called conditional probability.

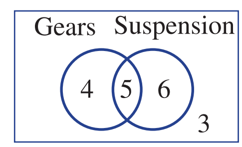
Consider, for example, a group of primary school students who own bicycles. Some of the bicycles have gears, some have suspension and some have both gears and suspension. Consider these two questions.

- What is the probability that a randomly selected bicycle has gears?
- What is the probability that a randomly selected bicycle has gears, given that it has suspension?

The second question is conditional, in that we already know that the bicycle has suspension.

Lesson starter: Gears and suspension

Suppose that, in a group of 18 bicycles, 9 have gears, 11 have suspension and 5 have both gears and suspension. Discuss the solution to the following question by considering the points below.

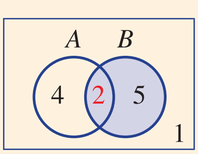


What is the probability that a randomly selected bicycle will have gears, given that it has suspension?

- First look at the information set out in a Venn diagram.
- How many of the bicycles that have suspension also have gears?
- Out of the 11 that have suspension, what is the probability that a bike will have gears?
- What would be the answer to the question in reverse; i.e. what is the probability that a bicycle will have suspension, given that it has gears?

Key ideas

- **Conditional probability** is the probability of an event occurring given that another event has already occurred.
- The probability of event A occurring given that event B has occurred is denoted by $\Pr(A|B)$, which reads 'the probability of A given B'.
- $\Pr(A \text{ given } B) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } B}$ for equally likely outcomes



$\Pr(A|B) = \frac{2}{7}$

	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

$\Pr(A|B) = \frac{2}{7}$

- $\Pr(B \text{ given } A) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } A}$ for equally likely outcomes

  For the diagrams above, $\Pr(B|A) = \frac{2}{6} = \frac{1}{3}$.

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Exercise 4D

Understanding

1–3 | 3

1 Complete the following by filling in the blanks

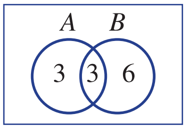
a The probability of A given B is denoted by ________

b $\Pr(A \text{ given } B) = \dfrac{\text{number of elements in } \_\_\_\_\_\_}{\text{number of elements in } \_\_\_\_\_\_}$

2 Consider this Venn diagram.

a What fraction of the elements in A are also in B? (This finds $\Pr(B|A)$.)

b What fraction of the elements in B are also in A? (This finds $\Pr(A|B)$.)



3 Use this two-way table to answer these questions.

a What fraction of the elements in A are also in B? (This finds $\Pr(B|A)$.)

b What fraction of the elements in B are also in A? (This finds $\Pr(A|B)$.)

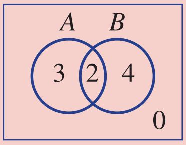
	A	**A′**	
B	7	5	12
B′	3	1	4
	10	6	16

Fluency

4, 5 | 4–6

Example 6 Finding conditional probabilities using a Venn diagram

Consider this Venn diagram, displaying the number of elements belonging to the events A and B.



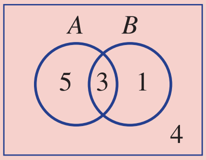
Find the following probabilities.

a $\Pr(A)$ **b** $\Pr(A \cap B)$ **c** $\Pr(A|B)$ **d** $\Pr(B|A)$

Solution	Explanation
a $\Pr(A) = \frac{5}{9}$	There are 5 elements in A and 9 in total.
b $\Pr(A \cap B) = \frac{2}{9}$	There are 2 elements common to A and B.
c $\Pr(A\|B) = \frac{2}{6} = \frac{1}{3}$	2 of the 6 elements in B are in A.
d $\Pr(B\|A) = \frac{2}{5}$	2 of the 5 elements in A are in B.

Now you try

Consider this Venn diagram, displaying the number of elements belonging to the events A and B.



Find the following probabilities.

a $\Pr(A)$ **b** $\Pr(A \cap B)$ **c** $\Pr(A|B)$ **d** $\Pr(B|A)$

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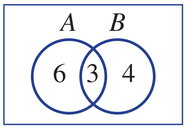
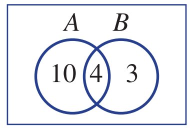
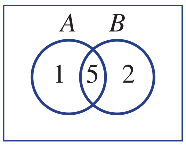
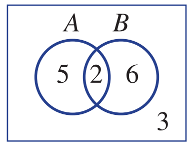
4 The following Venn diagrams display information about the number of elements associated with the events A and B. For each Venn diagram, find:

i $\Pr(A)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A|B)$ **iv** $\Pr(B|A)$

a


b


c


d


Hint:
- $A \cap B$ means both A and B.
- $\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$
- $\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

Example 7 Finding conditional probabilities using a two-way table

From a group of 15 hockey players at a game of hockey, 13 played on the field, 7 sat on the bench and 5 both played and sat on the bench.
A hockey player is chosen at random from the team.
Let A be the event 'the person played on the field' and B be the event 'the person sat on the bench'.

a Represent the information in a two-way table.
b Find the probability that the person only sat on the bench.
c Find the probability that the person sat on the bench, given that they played on the field.
d Find the probability that the person played on the field, given that they sat on the bench.

Solution

a

	A	A'	
B	5	2	7
B'	8	0	8
	13	2	15

Explanation

$A \cap B$ has 5 elements, A has a total of 13 and B a total of 7. There are 15 players in total.

b $\Pr(\text{bench only}) = \dfrac{2}{15}$

Two people sat on the bench and did not play on the field.

c $\Pr(B|A) = \dfrac{5}{13}$

$\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

d $\Pr(A|B) = \dfrac{5}{7}$

$\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

Now you try

From a class of 20 students, 12 own a cat, 10 own a dog and 5 owned both a cat and a dog.
A student is chosen at random from the class.
Let A be the event 'the student owns a cat' and B be the event 'the student owns a dog'.

a Represent the information in a two-way table.
b Find the probability that the student only owns a dog.
c Find the probability that the student owns a dog, given that they own a cat.
d Find the probability that the student owns a cat, given that they own a dog.

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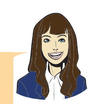
4D

5 Of a group of 20 English cricket fans at a match, 13 purchased a pie, 15 drank beer and 9 purchased a pie *and* drank beer.
Let A be the event 'the fan purchases a pie'.
Let B be the event 'the fan drank beer'.

a Copy and complete this two-way table.
b Find the probability that a fan only purchased a pie (and did not drink beer).
c Find the probability that a fan purchased a pie, given that they drank beer.
d Find the probability that a fan drank beer, given that they purchased a pie.

	A	A'	
B	9		
B'			
			20

6 The following two-way tables show information about the number of elements in the events A and B.
For each two-way table, find:



Hint: First decide on the total that gives the denominator of your fraction.

i $\Pr(A)$
ii $\Pr(A \cap B)$
iii $\Pr(A|B)$
iv $\Pr(B|A)$

a

	A	A'	
B	2	8	10
B'	5	3	8
	7	11	18

b

	A	A'	
B	1	4	5
B'	3	1	4
	4	5	9

c

	A	A'	
B	7	3	10
B'	1	6	7
	8	9	17

d

	A	A'	
B	4	2	6
B'	8	2	10
	12	4	16

Problem-solving and reasoning 7 7–9

7 Of 15 musicians surveyed to find out whether they play the violin or the piano, 5 play the violin, 8 play the piano and 2 play both instruments.

a Represent the information in a Venn diagram.
b How many of the musicians surveyed do not play either the violin or the piano?
c Find the probability that one of the 15 musicians surveyed plays piano, given that they play the violin.
d Find the probability that one of the 15 musicians surveyed plays the violin, given that they play piano.

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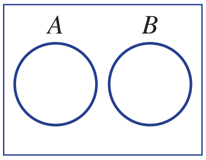
8 A card is drawn from a deck of 52 playing cards.
Find the probability that:
a the card is a king given that it is a heart
b the card is a jack given that it is a red card

Hint: 13 of the cards are hearts. There are 4 kings, including one king of hearts.

9 Two events, A and B, are mutually exclusive. What can be said about the probability of A given B (i.e. $\Pr(A|B)$) or the probability of B given A (i.e. $\Pr(B|A)$)? Give a reason.



Cruise control and airbags — 10

10 On a car production line, 30 cars are due to be completed by the end of the day. Fifteen of the cars have cruise control and 20 have airbags, and 6 have both cruise control and airbags.
a Represent the information provided in a Venn diagram or two-way table.
b Find the probability that a car chosen at random will have:
  i cruise control only **ii** airbags only
c Given that the car chosen has cruise control, find the probability that the car will have airbags.
d Given that the car chosen has airbags, find the probability that the car will have cruise control.

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4E Using tables for two-step experiments

Learning intentions

- To know how to list the sample space of a two-step experiment in a table
- To understand the difference between experiments carried out with replacement and without replacement
- To be able to construct tables for two-step experiments with and without replacement and find associated probabilities

Key vocabulary: with replacement, without replacement, two-step experiments, sample space

Some experiments contain more than one step and are called multi-stage experiments. Examples include rolling a die twice, selecting a number of chocolates from a box or choosing people at random to fill positions on a committee. Tables can be used to list all the outcomes from two-step experiments. The number of outcomes depend on whether or not the experiment is conducted with or without replacement.

Lesson starter: Two prizes, three people

Two special prizes are to be awarded in some way to Bill, May and Li for their efforts in helping at the school fete. This table shows how the prizes might be awarded.

		2nd prize		
		Bill	May	Li
	Bill	(B, B)	(B, M)	(B, L)
1st prize	May	(M, B)		
	Li			

- Complete the table to show how the two prizes can be awarded.
- Does the table show that the same person can be awarded both prizes?
- What is the probability that Bill and Li are both awarded a prize?
- How would the table change if the same person could not be awarded both prizes?
- How do the words 'with replacement' and 'without replacement' relate to the situation above? Discuss.

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Key ideas

- A **two-step experiment** involves two stages of an experiment eg. tossing a coin twice.
- Tables are used to list the sample space for two-step experiments.
- If replacement is allowed, then outcomes from each selection can be repeated, and such experiments are called **with replacement**.
- If selections are made **without replacement**, then outcomes from each selection cannot be repeated.
  For example, two selections are made from the digits $\{1, 2, 3\}$.

With replacement

		1st: 1	2	3
2nd	1	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

9 outcomes

Without replacement

		1st: 1	2	3
2nd	1	×	(2, 1)	(3, 1)
	2	(1, 2)	×	(3, 2)
	3	(1, 3)	(2, 3)	×

6 outcomes

Exercise 4E

Understanding | 1–3 | 1, 3

1 Choose either *with replacement* or *without replacement* to complete the following.

a Two chocolates are selected from a box and eaten. This is an example of ________________.

b Two cards are selected from a pack one after the other and their suit recorded. Each card is returned to the pack after its suit is recorded. This is an example of ________________.

2 Two letters are chosen from the word DOG.

a Complete a table listing the sample space if selections are made:

i with replacement

		1st: D	O	G
2nd	D	(D, D)	(O, D)	
	O			
	G			

ii without replacement

		1st: D	O	G
2nd	D	×	(O, D)	
	O		×	
	G			×

b State the total number of outcomes if selection is made:

i with replacement **ii** without replacement

3 Two digits are selected from the set $\{2, 3, 4\}$ to form a two-digit number. Find the number of two-digit numbers that can be formed if the digits are selected:

a with replacement

	2	3	4
2	22	32	
3			
4			

b without replacement

	2	3	4
2	×	32	
3		×	
4			×

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4E

Fluency 4–6 4–6

Example 8 Constructing a table with replacement

A fair 6-sided die is rolled twice.

a List all the outcomes, using a table.
b State the total number of outcomes.
c Find the probability of obtaining the outcome $(1, 5)$.
d Find:
  i Pr(double) **ii** Pr(sum of at least 10) **iii** Pr(sum not equal to 7)

Solution | **Explanation**

a

		Roll 2					
		1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Roll 1	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Be sure to place the number from roll 1 in the first position for each outcome.

b 36 outcomes

There is a total of $6 \times 6 = 36$ outcomes.

c $\text{Pr}(1, 5) = \frac{1}{36}$

Only one outcome is $(1, 5)$.

d **i** $\text{Pr(double)} = \frac{6}{36}$

$= \frac{1}{6}$

Six outcomes have the same number repeated.

ii $\text{Pr(sum of at least 10)} = \frac{6}{36} = \frac{1}{6}$

Six outcomes have a sum of either 10, 11 or 12.

iii $\text{Pr(sum not equal to 7)} = 1 - \frac{6}{36}$

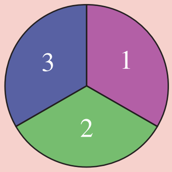
$= \frac{30}{36}$

$= \frac{5}{6}$

This is the complement of having a sum of 7.
Six outcomes have a sum of 7.
$\text{Pr(not } A) = 1 - \text{Pr}(A)$

Now you try

The spinner shown is spun twice.



a List all the outcomes, using a table.
b State the total number of outcomes.
c Find the probability of obtaining the outcome $(3, 2)$.
d Find:
  i Pr(same 2 numbers) **ii** Pr(product is more than 3) **iii** Pr(product is not equal to 9)

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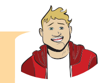
4 A fair 4-sided die is rolled twice.

a List all the outcomes, using a table.

		1st			
		1	2	3	4
2nd	1	(1, 1)	(2, 1)		
	2				
	3				
	4				



Hint: Questions **4** and **5** are making selections 'with replacement' because outcomes can be repeated.

b State the total number of possible outcomes.

c Find the probability of obtaining the outcome $(2, 4)$.

d Find the probability of:

i a double **ii** a sum of at least 5 **iii** a sum not equal to 4

5 Two coins are tossed, each landing with a head (H) or tail (T).

a List all the outcomes, using a table.

b State the total number of possible outcomes.

c Find the probability of obtaining the outcome (H, T).

d Find the probability of obtaining:

i exactly one tail **ii** at least one tail

e If the two coins are tossed 1000 times, how many times would you expect to get two tails?

		1st	
		H	T
2nd	H	(H, H)	(T, H)
	T		

Example 9 Constructing a table without replacement

Two letters are chosen from the word KICK, without replacement.

a Construct a table to list the sample space.

b Find the probability of:

i obtaining the outcome (K, C) **ii** selecting two letters that are K **iii** selecting a K and a C

Solution

a

		1st			
		K	I	C	K
2nd	K	×	(I, K)	(C, K)	(K, K)
	I	(K, I)	×	(C, I)	(K, I)
	C	(K, C)	(I, C)	×	(K, C)
	K	(K, K)	(I, K)	(C, K)	×

Explanation

Selection is without replacement, so the same letter (from the same position) cannot be chosen twice.

b **i** $\Pr(\text{K, C}) = \frac{2}{12}$

$= \frac{1}{6}$

Two of the 12 outcomes are (K, C).

ii $\Pr(\text{K, K}) = \frac{2}{12}$

$= \frac{1}{6}$

Two of the outcomes are K and K, which use different Ks from the word KICK.

iii $\Pr(\text{K and C}) = \frac{4}{12}$

$= \frac{1}{3}$

Four outcomes contain a K and a C.

Continued on next page

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4E

Now you try

Two letters are chosen from the word TREE, without replacement.

a Construct a table to list the sample space.

b Find the probability of:

i obtaining the outcome (T, E) **ii** selecting two letters that are E **iii** selecting an E and a T

6 Two letters are chosen from the word SETS, without replacement.

a Complete this table to list the sample space.

b Find the probability of:

i obtaining the outcome (E, S)

ii selecting one T

iii selecting two letters that are S

iv selecting an S and a T

v selecting an S or a T

		1st			
		S	E	T	S
2nd	S	×	(E, S)	(T, S)	(S, S)
	E		×		
	T			×	
	S				×

Problem-solving and reasoning

7, 8 | 8–10

7 A letter is chosen from the word LEVEL without replacement and then a second letter is chosen from the same word.

a Draw a table displaying the sample space for the pair of letters chosen.

b State the total number of outcomes possible.

c State the number of outcomes that contain exactly one of the following letters.

i V **ii** L **iii** E

d Find the probability that the outcome will contain exactly one of the following letters.

i V **ii** L **iii** E

e Find the probability that the two letters chosen will be the same.

Hint: Remember that this is 'without replacement'.

8 In a quiz, Min guessed that the probability of rolling a sum of 10 or more from 2 six-sided dice is 10%. Complete the following to decide whether or not this guess is correct.

a Copy and complete the table representing all the outcomes for possible totals that can be obtained.

b State the total number of outcomes.

c Find the number of the outcomes that represent a sum of:

i 3 **ii** 7 **iii** less than 7

d Find the probability that the following sums are obtained.

i 7

ii less than 5

iii greater than 2

iv at least 11

e Find the probability that the sum is at least 10, and decide whether or not Min's guess is correct.

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	…			
	2	3	…				
	3	4					
	4	⋮					
	5	⋮					
	6						

9 A coin and a six-sided die are tossed. Heads on the coin is worth 2 points and a tail is worth 4 points. These points are added to the score on the die. Use a table to find the probability that the total score is:

a 4

b 6

c greater than 7

d at most 6

10 Decide whether the following situations would naturally involve selections with replacement or without replacement.

a selecting two people to play in a team

b tossing a coin twice

c rolling two dice

d choosing two chocolates to eat

Random weights — 11

11 In a gym, Justine considers choosing two weights to fit onto a leg weights machine to make the load heavier. She can choose from 2.5 kg, 5 kg, 10 kg or 20 kg, and there are plenty of each weight available. Justine's friend randomly chooses both weights, with equal probability that she will choose each weight, and places them on the machine. Justine then attempts to operate the machine without knowing which weights were chosen.

a Complete a table that displays all possible total weights that could be placed on the machine.

b State the total number of outcomes.

c How many of the outcomes deliver a total weight described by the following?

i equal to 10 kg

ii less than 20 kg

iii at least 20 kg

d Find the probability that Justine will be attempting to lift the following weights?

i 20 kg

ii 30 kg

iii no more than 10 kg

iv less than 10 kg

e If Justine is unable to lift more than 22 kg, what is the probability that she will not be able to operate the leg weights machine?

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Progress quiz

4A

1 A letter is chosen from the word ELEPHANT. Find the probability that the letter is:

a an E **b** a T or an E **c** not an E **d** a vowel

4A

2 An experiment involves rolling two dice and counting the number of even numbers. Here are the results after running the experiment 100 times.

Number of even numbers	0	1	2
Frequency	21	46	33

a How many times did more than 1 even number occur?

b Find the experimental probability of obtaining:

i 0 even numbers

ii fewer than 2 even numbers

iii at least 1 even number

4B/D

3 From a group of 25 students on a school camp, 18 enjoy sailing (S), 15 enjoy bushwalking (B) and 8 enjoy both sailing and bushwalking.

a Illustrate this information in a Venn diagram.

b State the number of students who enjoy:

i sailing only

ii neither sailing nor bushwalking

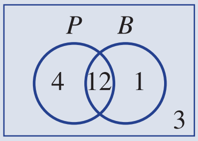
c Find the following probabilities for a student chosen at random from the group.

i $\Pr(B)$ **ii** $\Pr(S \text{ only})$ **iii** $\Pr(B \cap S)$

iv $\Pr(B')$ **v** $\Pr(S|B)$

4C/D

4 The Venn diagram shown shows the distribution of 20 guests staying at a resort in Noosa. Some guests liked to swim at the hotel pool (P), others liked to swim at the beach (B) and others liked both.



a Transfer the information to a two-way table.

b Find the number of guests who like:

i only swimming at the hotel pool

ii swimming at either the beach or the hotel pool

c Find:

i $\Pr(P)$ **ii** $\Pr(B \text{ only})$ **iii** $\Pr(P \cap B)$ **iv** $\Pr(P|B)$

4E

5 A fair six-sided die is rolled and a coin is tossed.

	1	2	3	4	5	6
H	(1, H)	(2, H)…				
T						

a List all the outcomes, using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome (3, H).

d Find the probability of obtaining:

i an even number and a tail

ii at least a 2 and a head

4E

6 Two counters are chosen randomly from a bag containing 2 red, 1 blue and 1 green counter, without replacement.

a Construct a table to list the sample space and state the number of outcomes.

b Find the probability of:

i obtaining the outcome (R, G)

ii selecting 2 red counters

iii selecting 1 blue and 1 red counter

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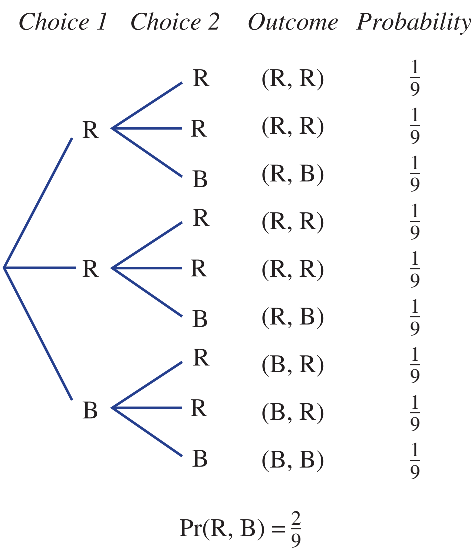
4F Using tree diagrams

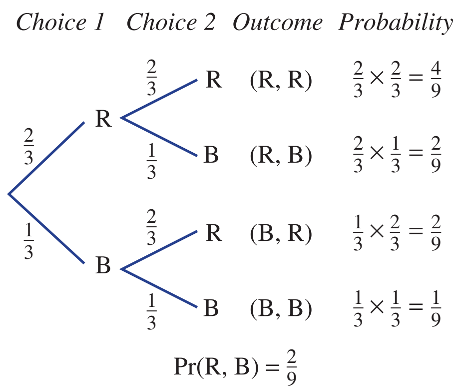
Learning intentions

- To know how to use a tree diagram to list the sample space from experiments with two or more components
- To understand the difference between the probabilities on tree diagrams for experiments with replacement and those without replacement
- To be able to use a tree diagram to find the probability of outcomes in experiments

Key vocabulary: tree diagram, with replacement, without replacement, sample space

Tree diagrams can also be used to help list outcomes for multi-stage experiments. Suppose that a bag contains two red counters and one blue counter and that two counters are selected at random with replacement. One way to display the outcomes is with a tree diagram in which all equally likely outcomes are listed in columns, as shown below left. A more efficient way, however, is to group similar outcomes and write their corresponding probabilities on the branches, as shown below right.



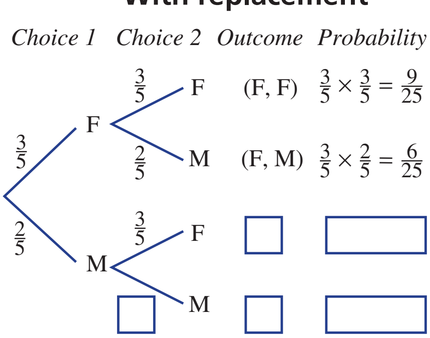


In the tree diagram on the right, the probability of each outcome is obtained by multiplying the branch probabilities. This also applies when selection is made without replacement.

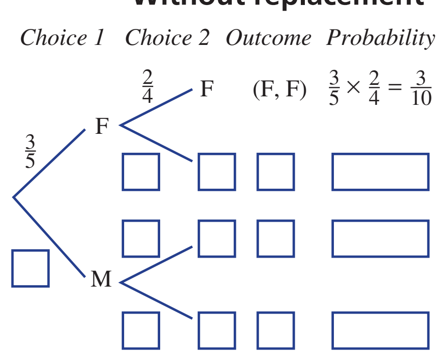
Lesson starter: Trees with and without replacement

Suppose that two selections are made from a group of 2 male and 3 female workers to complete two extra tasks.

With replacement



Without replacement



- Complete these two tree diagrams to show how these selections can be made, both with and without replacement.
- Explain where the branch probabilities come from on each branch of the tree diagrams.
- What is the total of all the probabilities on each tree diagram?

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Key ideas

- **Tree diagrams** can be used to list the sample space for experiments involving two or more components.
  - Branch probabilities are used to describe the chance of each outcome at each step.
  - The probability of each outcome for the experiment is obtained by multiplying the branch probabilities.
  - Branch probabilities will depend on whether selection is made with or without replacement.
    For experiments *with replacement*, probabilities do not change.
    For experiments *without replacement*, probabilities do change.

Exercise 4F

Understanding | 1, 2 | 2

1 A coin is tossed three times and a head or tail is obtained each time as shown in the tree diagram.

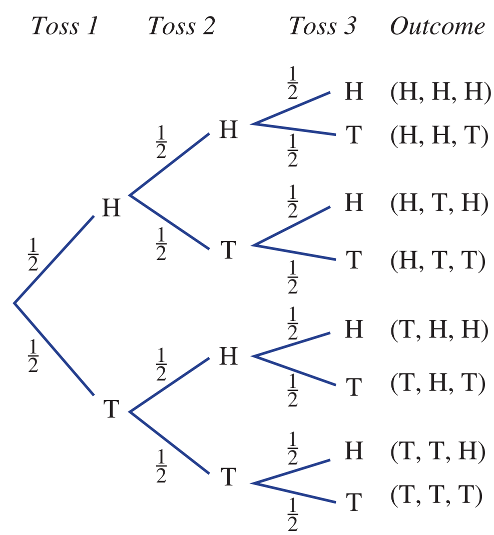
a How many outcomes are there?

b What is the probability of the outcome HHH?

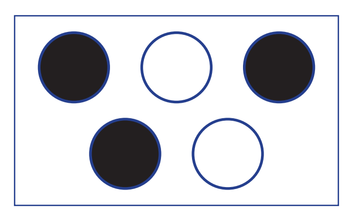
c How many outcomes obtain:

i 2 tails?

ii 2 or 3 heads?



2 A box contains 2 white (W) and 3 black (B) counters.



a A single counter is drawn at random. Find the probability that it is:

i white **ii** black

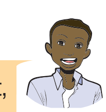
b Two counters are now drawn at random. The first one is replaced before the second one is drawn. Find the probability that the second counter is:

i white **ii** black

c Two counters are drawn and the first counter is not replaced before the second one is drawn. If the first counter is white, find the probability that the second counter is:

i white **ii** black

Hint: After one white counter is taken out, how many of each remain?



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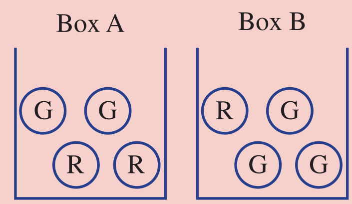
Fluency | 3, 4 | 3–5

Example 10 Constructing a tree diagram for multi-stage experiments

Boxes A and B contain 4 counters each. Box A contains 2 red and 2 green counters and box B contains 1 red and 3 green counters. A box is chosen at random and then a single counter is selected.

a What is the probability of selecting a red counter from box A?
b What is the probability of selecting a red counter from box B?
c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.
d What is the probability of selecting box B and a red counter?
e What is the probability of selecting a red counter?

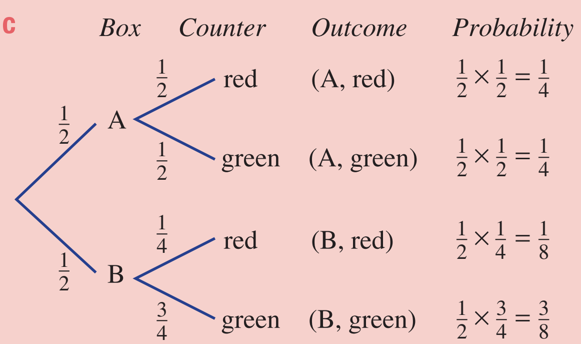


Solution

a Pr(red from box A) $= \frac{2}{4} = \frac{1}{2}$

b Pr(red from box B) $= \frac{1}{4}$

c



d Pr(B, red) $= \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$

e Pr(1 red) $= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{4} + \frac{1}{8}$
$= \frac{3}{8}$

Explanation

a 2 of the 4 counters in box A are red.

b 1 of the 4 counters in box B is red.

c First selection is a box followed by a counter.
Multiply each of the probabilities along the branch pathways to find the probability of each outcome.

d The probability of choosing box B is $\frac{1}{2}$ and a red counter from box B is $\frac{1}{4}$, so multiply the probabilities for these two outcomes together.

e The outcomes (A, red) and (B, red) both contain 1 red counter, so add together the probabilities for these two outcomes.

Continued on next page

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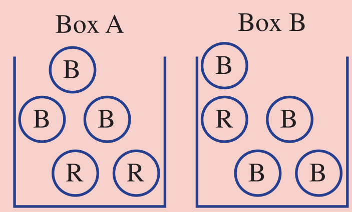
4F

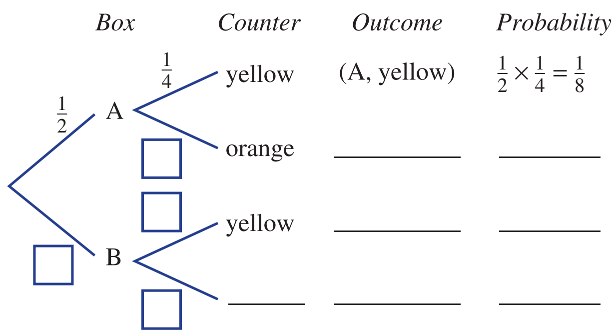
Now you try

Boxes A and B contain 5 counters each. Box A contains 2 red and 3 blue counters and box B contains 1 red and 4 blue counters.

a What is the probability of selecting a red counter from box A?
b What is the probability of selecting a red counter from box B?
c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.
d What is the probability of selecting box B and a red counter?
e What is the probability of selecting a red counter?

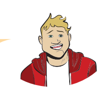


3 Boxes A and B contain 4 counters each. Box A contains 1 yellow and 3 orange counters and box B contains 3 yellow and 1 orange counter. A box is chosen at random and then a single counter is selected.
a If box A is chosen, what is the probability of selecting a yellow counter?
b If box B is chosen, what is the probability of selecting a yellow counter?
c Represent the options available by completing this tree diagram.



d What is the probability of selecting box B and a yellow counter?
e What is the probability of selecting 1 yellow counter?

Hint: For part **e**, add the probabilities for both outcomes that have a yellow counter.

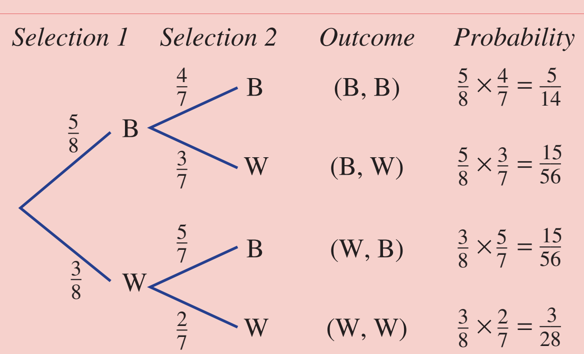


Example 11 Using a tree diagram without replacement

A bag contains 5 blue (B) and 3 white (W) marbles and two marbles are selected without replacement.
a Draw a tree diagram showing all outcomes and probabilities.
b Find the probability of selecting:
 i a blue marble followed by a white marble; i.e. the outcome (B, W)
 ii 2 blue marbles
 iii exactly one blue marble
c If the experiment is repeated with replacement, find the answers to each question in part **b**.

Solution

a



Explanation

After one blue marble is selected there are 7 marbles remaining: 4 blue and 3 white.

After one white marble is selected there are 7 marbles remaining: 5 blue and 2 white.

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b **i** $\Pr(B,\ W) = \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{56}$

Multiply the probabilities on the (B, W) pathway.

ii $\Pr(B,\ B) = \frac{5}{8} \times \frac{4}{7}$

$= \frac{5}{14}$

Only 4 blue marbles remain after the first selection. Multiply the probabilities on the (B, B) pathway.

iii $\Pr(1 \text{ blue}) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$

$= \frac{15}{28}$

The outcomes (B, W) and (W, B) both have 1 blue marble. Multiply probabilities to find individual probabilities, then sum for the final result.

c **i** $\Pr(B,\ W) = \frac{5}{8} \times \frac{3}{8}$

$= \frac{15}{64}$

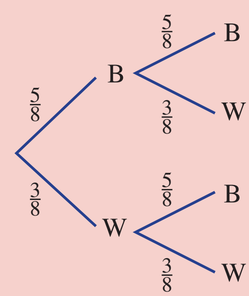
When selecting objects with replacement, remember that the number of marbles in the bag remains the same for each selection.

ii $\Pr(B,\ B) = \frac{5}{8} \times \frac{5}{8}$

$= \frac{25}{64}$

iii $\Pr(1 \text{ blue}) = \frac{5}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8}$

$= \frac{15}{32}$



Now you try

A jar contains 4 toffees (T) and 3 mints (M) and two lollies are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.

b Find the probability of selecting:

i a toffee followed by a mint: i.e. the outcome (T, M)

ii 2 mints

iii exactly one toffee

c If the experiment is repeated with replacement, find the answers to each question in part **b**.

4 A bag contains 4 red (R) and 2 white (W) marbles, and two marbles are selected without replacement.

a Complete this tree diagram, showing all outcomes and probabilities.

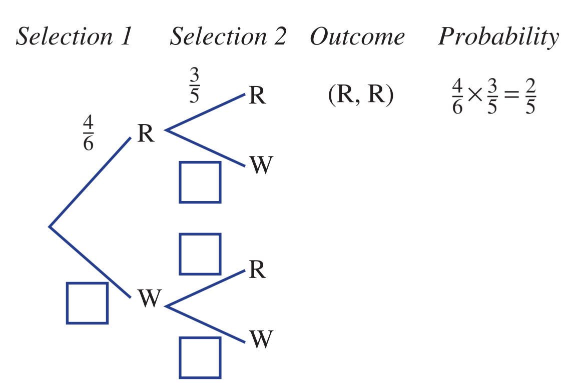
b Find the probability of selecting:

i a red marble and then a white marble (R, W)

ii 2 red marbles

iii exactly 1 red marble

c If the experiment is repeated with replacement, find the answers to each question in part **b**. You may need to redraw the tree diagram.



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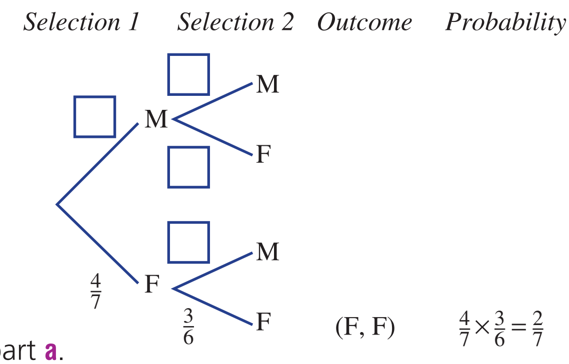
5 Two students are selected from a group of 3 males (M) and 4 females (F), without replacement.

a Complete this tree diagram to help find the probability of selecting:

- **i** 2 males
- **ii** 2 females
- **iii** 1 male and 1 female
- **iv** 2 people either both male or both female

b If the experiment is repeated with replacement, find the answers to each question in part **a**.



Problem-solving and reasoning

6, 7 | 6–8

6 A fair 4-sided die is rolled twice and the pair of numbers is recorded.

a Complete this tree diagram to list the outcomes.

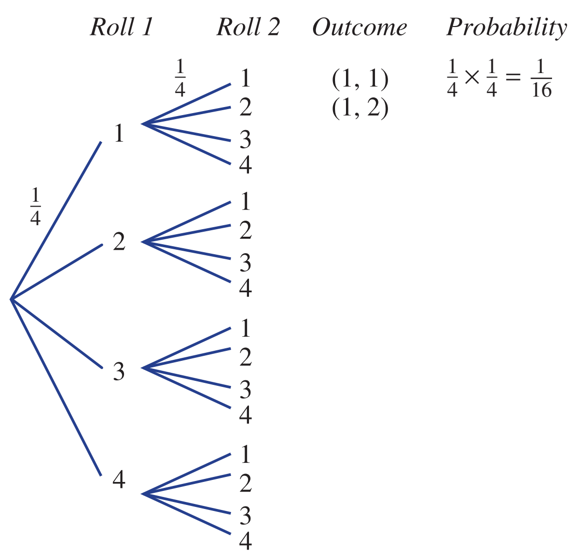
b State the total number of outcomes.

c Find the probability of obtaining:

- **i** a 4 then a 1; i.e. the outcome (4, 1)
- **ii** a double

d Find the probability of obtaining a sum described by the following:

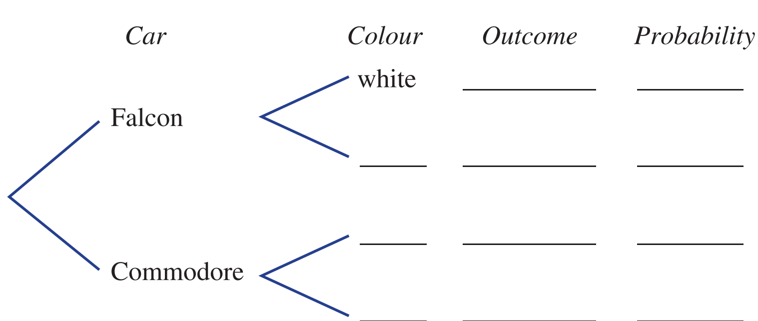
- **i** equal to 2
- **ii** equal to 5
- **iii** less than or equal to 5



7 As part of a salary package, a person can select either a Falcon or a Commodore. There are 3 white Falcons, 1 silver Falcon, 2 white Commodores and 1 red Commodore to choose from.

a Complete a tree diagram showing a random selection of a car make, then a colour.

Hint: Since car make selection is random, Pr(Falcon) $= \frac{1}{2}$.



b Find the probability that the person chooses:

- **i** a white Falcon
- **ii** a red Commodore
- **iii** a white car
- **iv** a car that is not white
- **v** a silver car or a white car
- **vi** a car that is neither a Falcon nor red

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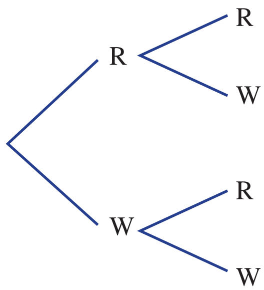
8 Two bottles of wine are randomly selected for tasting from a box containing 2 red and 2 white wines. Use a tree diagram to help answer the following.



a If the first bottle is replaced before the second is selected, find:
- **i** Pr(2 red)
- **ii** Pr(1 red)
- **iii** Pr(not 2 white)
- **iv** Pr(at least 1 white)

b If the first bottle is not replaced before the second is selected, find:
- **i** Pr(2 red)
- **ii** Pr(1 red)
- **iii** Pr(not 2 white)
- **iv** Pr(at least 1 white)

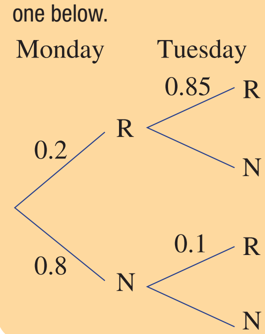
Rainy days — 9

9 Imagine that the probability of rain next Monday is 0.2. The probability of rain on a day after a rainy day is 0.85, whereas the probability of rain on a day after a non-rainy day is 0.1.

a Next Monday and Tuesday, find the probability of having:
- **i** 2 rainy days
- **ii** exactly 1 rainy day
- **iii** at least 1 dry day

b Next Monday, Tuesday and Wednesday, find the probability of having:
- **i** 3 rainy days
- **ii** exactly 1 dry day
- **iii** at most 2 rainy days

Hint: Draw a tree diagram, like the one below.



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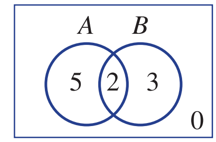
4G Independent events

Learning intentions

- To understand the concept of independent events
- To be able to determine if two events are independent using a Venn diagram or two-way table
- To know how with and without replacement affects independent events

Key vocabulary: independent events, with replacement, without replacement, conditional probability

In previous sections we have looked at problems involving conditional probability. This Venn diagram, for example, gives the following results.

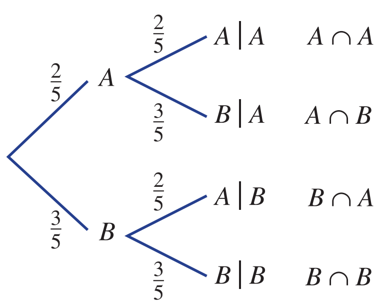


$$\Pr(A) = \frac{7}{10} \text{ and } \Pr(A \mid B) = \frac{2}{5}$$

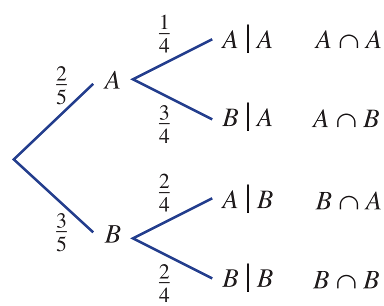
The condition B in $\Pr(A \mid B)$ has changed the probability of A. The events A and B are therefore not independent.

For multi-stage experiments we can consider events either with or without replacement. These tree diagrams, for example, show two selections of marbles from a bag of 2 aqua (A) and 3 blue (B) marbles.

With replacement



Without replacement



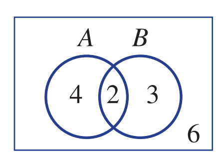
In the first tree diagram $\Pr(A \mid B) = \Pr(A)$, so the events are independent.

In the second tree diagram $\Pr(A \mid B) \neq \Pr(A)$, so the events are not independent.

Lesson starter: Is it the same to be mutually exclusive and independent?

Use the Venn diagram to consider the following questions.

- Are the events mutually exclusive? Why?
- Find $\Pr(A)$ and $\Pr(A \mid B)$. Does this mean that the events A and B are independent?



Key ideas

- Two events are **independent** if the outcome of one event does not change the probability of obtaining the other event.
  - $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
  - $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- For multi-stage experiments where selection is made with replacement, successive events are independent.
- For multi-stage experiments where selection is made without replacement, successive events are not independent.

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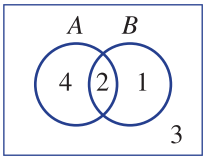
Exercise 4G

Understanding — 1–3 | 2, 3

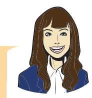
1 State if events A and B are independent for the following probabilities.
   a $\Pr(A) = 0.4$, $\Pr(A|B) = 0.6$
   b $\Pr(A) = 0.7$, $\Pr(A|B) = 0.7$

Hint: $\Pr(A|B) = \Pr(A)$ if A and B are independent

2 This Venn diagram shows the number of elements in events A and B.



   a Find:
      i $\Pr(B)$
      ii $\Pr(B|A)$
   b Is $\Pr(B|A) = \Pr(B)$?
   c Are the events A and B independent?



Hint: Recall: $\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

3 Complete each sentence.
   a For multi-stage experiments, successive events are independent if selections are made ________ replacement.
   b For multi-stage experiments, successive events are not independent if selections are made ________ replacement.

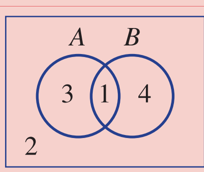
Hint: Choose from 'with' or 'without'.

Fluency — 4 | 4, 5

Example 12 Using Venn diagrams to consider independence

A selection of 10 mobile phone plans includes 4 with free connection and 5 with a free second battery. 1 plan has both free connection and a free second battery.
Let A be the event 'choosing a mobile phone plan with free connection'.
Let B be the event 'choosing a mobile phone plan with a free second battery'.

a Summarise the information about the 10 mobile phone plans in a Venn diagram.
b Find: i $\Pr(A)$ ii $\Pr(A|B)$
c State whether or not the events A and B are independent.

Solution	Explanation
a 	Start with the 1 element that belongs to both A and B and complete according to the given information.
b i $\Pr(A) = \frac{4}{10} = \frac{2}{5}$	4 of the 10 elements belong to A.
ii $\Pr(A\|B) = \frac{1}{5}$	1 of the 5 elements in B belongs to A.
c The events A and B are not independent.	$\Pr(A\|B) \neq \Pr(A)$

Continued on next page

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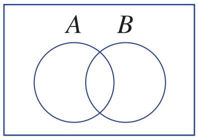
Now you try

A selection of 8 hotel deals includes 3 with a free breakfast and 4 with free parking. 2 deals have both a free breakfast and free parking.
Let A be the event 'choosing a hotel deal with a free breakfast'.
Let B be the event 'choosing a hotel deal with free parking'.

a Summarise the information about the 8 hotel deals in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A|B)$
c State whether or not the events A and B are independent.

4 A selection of 8 offers for computer printers includes 3 with a free printer cartridge and 4 with a free box of paper. 2 have both a free printer cartridge and a free box of paper.
Let A be the event 'choosing a printer with a free printer cartridge'.
Let B be the event 'choosing a printer with a free box of paper'.



a Summarise the given information about the 8 computer printer offers in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A|B)$
c State whether or not the events A and B are independent.

Hint: If $\Pr(A) = \Pr(A|B)$, then the events A and B are independent.

5 A selection of 6 different baby strollers includes 3 with a free rain cover and 4 with a free sunshade. 2 offer both a free rain cover and a free sunshade.
Let A be the event 'choosing a stroller with a free sunshade'.
Let B be the event 'choosing a stroller with a free rain cover'.

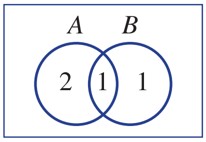
a Summarise the given information about the six baby strollers in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A|B)$
c State whether or not the events A and B are independent.

Problem-solving and reasoning | 6, 7 | 6–9

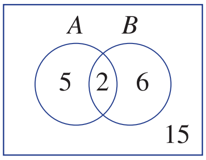
6 Events A and B are given in the Venn diagrams below.
i Find $\Pr(A)$ and $\Pr(A|B)$.
ii Hence, decide whether or not events A and B are independent.

Hint: Remember:
$\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

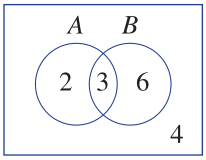
a



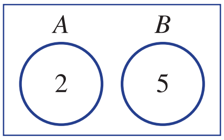
b



c



d



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7 For the events A and B with details provided in the given two-way tables, find $\Pr(A)$ and $\Pr(A \mid B)$. Decide whether or not the events A and B are independent.

a

	A	A'	
B	1	1	2
B'	3	3	6
	4	4	8

b

	A	A'	
B	1	3	4
B'	2	4	6
	3	7	10

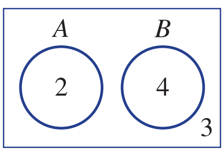
c

	A	A'	
B	3	17	20
B'	12	4	16
	15	21	36

d

	A	A'	
B	1		9
B'			
	5		45

8 Use the diagram below to help decide if this statement is true or false:
If two events, A and B, are mutually exclusive, then they are also independent.



9 A coin is tossed 5 times. Find the probability of obtaining:

a 5 heads
b at least 1 tail
c at least 1 head

Hint: Coin tosses are independent. From two coins, the probability of two heads is $\frac{1}{2} \times \frac{1}{2}$.

Tax and investment advice — 10

10 Of 17 leading accountants, 15 offer advice on tax (T) and 10 offer advice on business growth (G). Eight of the accountants offer advice on both tax and business growth. One of the 17 accountants is chosen at random.

a Use a Venn diagram or two-way table to help find:
i $\Pr(T)$ **ii** $\Pr(T \text{ only})$ **iii** $\Pr(T \mid G)$

b Are the events T and G independent?

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Maths@Work: Business analyst

Businesses employ analysts to look at how their businesses run, and how they can run more efficiently. Analysts use probabilities, relative frequencies and graphs to look at data, undertake calculations and make recommendations.

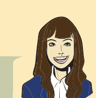
Being able to summarise and interpret data and understand likelihoods are important skills for analysts and business owners to have.

Complete these questions that a business analyst may face in their day-to-day job.

1 An online photo printing company wishes to analyse their customer orders and needs based on the products that are currently purchased from them. They looked at the results for a typical week over two areas of the business: photos and other services, such as canvases, mugs and calendars. The results of one week's orders are shown in the table below.

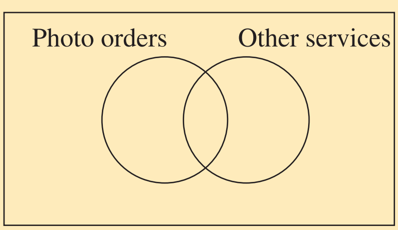
	Other services	No other services	Total orders
Photos ordered	561	1179	1740
No photos ordered	260	–	260
Totals	821	1179	2000

a Percentages are often used in probability. Rewrite the table above using percentages, where 100% equals the total orders for the week.



$\frac{561}{2000} \times \frac{100}{1} = 28.05\%$

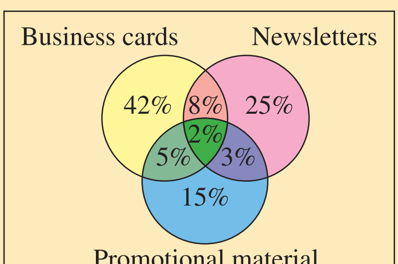
b Show this information in a Venn diagram, using percentages.



c Each customer ordering photos averages 30 prints (size 4×6). The company charges 12 cents a print. How much is the weekly income generated by photos, using the week's orders shown above?

d The company wishes to invest money in some advertising. Should this advertising be for the promotion of their photo printing service or should it be aimed at the other services they supply? Explain your answer.

2 Consider the information of the services offered by a different company wishing to look into how its business operates. They offer three types of products: business cards, newsletter printing and promotional lines, which include calendars, mugs etc. This company does not offer separate photo printing. The breakdown of their business can be seen in the Venn diagram on the right.



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a If 1000 customers were surveyed, how many of them purchased:
   i only one of the three product lines on offer?
   ii exactly two of the product lines?
   iii at least two of the product lines?
   iv all three product lines available?

b What is the single most important product that the company offers? State the statistics for your selection and explain the significance to the company of this information.

c If you were the owners or share holders in this company, ideally how would you like to see the Venn diagram change over time to see growth within the company?

Using technology

3 'Get Set' is an online business selling sportswear. It has been very successful over the past year and its quarterly sales are shown in this table.

	A	B	C	D	E	F	G
1	Sportswear sales from Get Set online business						
2	Sport	QUARTER 1	QUARTER 2	QUARTER 3	QUARTER 4	Totals	Percentages
3	Golf	159	98	104	137		
4	Running	278	312	320	287		
5	Cycling	209	276	248	268		
6	Dance	94	77	107	68		
7	Gym	169	176	195	284		
8	Yoga	29	47	26	32		
9	Totals						
10	Percentages						

Use dollar signs (e.g. $H$12) in a formula that links to a fixed cell.

a Copy the data into an Excel spreadsheet and enter appropriate formulas into the shaded cells to find their values.

b Which type of sportswear is:
   i most likely to be sold?
   ii least likely to be sold?

Select the sports and sales area of the table and click on Insert and then Column.

c Insert a column graph showing the quarterly sales for each sport.

d In the fourth quarter, Get Set offered special prices for one line of sportswear as part of an advertising drive. Which sportswear do you think was on special? Was the advertising successful? Give reasons for your answer.

Hold the Ctrl button while selecting the titles Quarter 1 to Quarter 4 and also their total percentages row.

e Insert a column graph showing the total percentages for each quarter of sales.

f Businesses use graphs to show comparisons between data sets.
What is the increase in total percentage from Quarter 1 to Quarter 4? Comment on whether your graph in part **e** exaggerates this increase and, if so, explain why. Make an adjustment on the graph so that it is not misleading.

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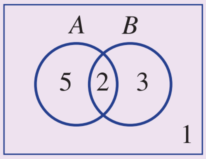
Puzzles and games

1 'I have nothing in common.' Match the answers to the letters in parts **a** and **b** to uncover the code.

___	___	___	___	___	___	___	___	
$\frac{5}{14}$	5	2	5	7	10	10	$\frac{7}{11}$	

___	___	___	___	___	___	___	___	___
$\frac{5}{11}$	$\frac{3}{14}$	$\frac{1}{2}$	10	5	$\frac{10}{11}$	$\frac{1}{7}$	3	$\frac{5}{11}$

a These questions relate to the Venn diagram at right.



- T How many elements in $A \cap B$?
- L How many elements in $A \cup B$?
- V How many elements in B only?
- Y Find Pr(A).
- S Find Pr($A \cup B$).
- E Find Pr(A only).

b These questions relate to the two-way table at right.

	P	P'	
Q	U	4	9
Q'	2		
		A	14

- U What number should be in place of the letter U?
- A What number should be in place of the letter A?
- M Find Pr($P \cap Q$).
- C Find Pr(P').
- X Find Pr(neither P nor Q).
- I Find Pr(P only).

2 What is the chance of rolling a sum of at least 10 from rolling two 6-sided dice?

3 *Game for two people*: You will need a bag or pocket and coloured counters.

- One person places 8 counters of 3 different colours in a bag or pocket. The second person must not look!
- The second person then selects a counter from the bag. The colour is noted, then the counter is returned to the bag. This is repeated 100 times.
- Complete this table.

Colour	Tally	Frequency	Guess
Total:	100	100	

- Using the experimental results, the second person now tries to guess how many counters of each colour are in the bag.

4 Two digits are chosen without replacement from the set $\{1, 2, 3, 4\}$ to form a two-digit number. Find the probability that the two-digit number is:

a 32 **b** even **c** less than 40 **d** at least 22

5 A coin is tossed 4 times. What is the probability that at least 1 tail is obtained?

6 Two leadership positions are to be filled from a group of 2 girls and 3 boys. What is the probability that the positions will be filled by 1 girl and 1 boy?

7 The letters of the word DOOR are jumbled randomly. What is the probability that the final arrangement will spell DOOR?

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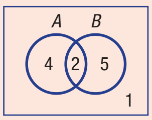
Chapter summary

Probability

Review

- Sample space is the list of all possible outcomes
- $\Pr(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Venn diagram



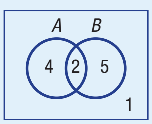
Two-way table

	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

Conditional probability ★

$\Pr(A \mid B)$ is read as the probability of A given B

$\Pr(A \mid B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$



	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

$\Pr(A \mid B) = \frac{2}{7}$

$\Pr(B \mid A) = \frac{2}{6} = \frac{1}{3}$

Independent events ★

- $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
- $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Tables

With replacement

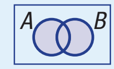
	A	B	C
A	(A, A)	(B, A)	(C, A)
B	(A, B)	(B, B)	(C, B)
C	(A, C)	(B, C)	(C, C)

Without replacement

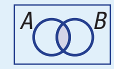
	A	B	C
A	×	(B, A)	(C, A)
B	(A, B)	×	(C, B)
C	(A, C)	(B, C)	×

Notation

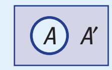
- Union $A \cup B$ (A or B)



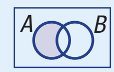
- Intersection $A \cap B$ (A and B)



- Complement of A is A' (not A)



- A only

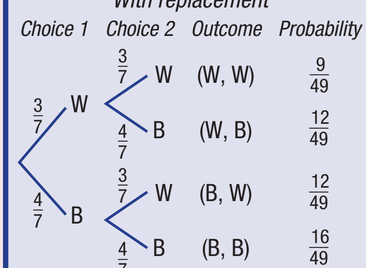


- Mutually exclusive events $\Pr(A \cap B) = 0$

Tree diagrams

3 white
4 black

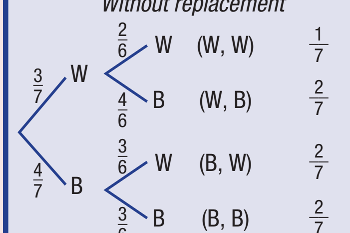
With replacement



$\Pr(\text{W, B}) = \frac{3}{7} \times \frac{4}{7}$

$= \frac{12}{49}$

Without replacement



$\Pr(1 \text{ white}) = \frac{2}{7} + \frac{2}{7}$

$= \frac{4}{7}$

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
4A	**1 I can calculate a theoretical probability.** e.g. A letter is chosen from the word ALLIGATOR. Find the probability that the letter is: **a** an A **b** not a G **c** an A or a T	

4A

2 I can find an experimental probability.

e.g. An experiment involves tossing 3 coins and counting the number of tails. The results from running the experiment 100 times are shown.

Number of tails	0	1	2	3
Frequency	15	39	37	9

Find the experimental probability of obtaining:

a 1 tail
b at least 1 tail

4B

3 I can list sets from a Venn diagram.

e.g. Events A and B involve numbers taken from the first 10 positive integers:
$A = \{1, 4, 5, 7\}$ and $B = \{4, 5, 6, 7, 8\}$.
Represent the two events in a Venn diagram and hence:

a list the set $A \cup B$
b find the probability that a randomly selected number from the first 10 positive integers is in $A \cap B$
c decide if the events are mutually exclusive

4B

4 I can use a Venn diagram.

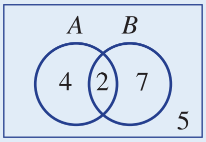
e.g. From a group of 20 people in a swimming squad, 12 train for freestyle (*F*), 6 train for butterfly (*B*) and 3 train for both freestyle and butterfly.
Illustrate this information in a Venn diagram and hence:

a state the number in the squad who train for butterfly only
b find the probability a randomly selected swimmer from the squad trains for neither freestyle nor butterfly

4C

5 I can use a two-way table.

e.g. The Venn diagram shows the distribution of elements from two events A and B.



Transfer the information into a two-way table and hence, find:

a the number of elements in A and B
b the number of elements in neither A nor B
c $\Pr(B')$ and $\Pr(A \text{ only})$

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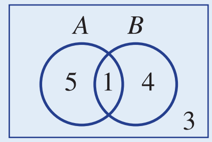
Chapter checklist ✔

4D

6 I can find conditional probability using a Venn diagram.
e.g. The Venn diagram shows the distribution of elements belonging to two events A and B.



Find the following probabilities:
a $\Pr(A \mid B)$
b $\Pr(B \mid A)$

4D

7 I can find a conditional probability in a word problem using a two-way table.
e.g. From a team of 11 cricketers in a cricket match, 9 batted in the match, 6 bowled in the match and 4 both batted and bowled. A cricketer is chosen at random from the team. Let A be the event 'the cricketer batted in the match' and B be the event 'the cricketer bowled in the match'. Represent the information in a two-way table and hence, find the probability that:
a the cricketer batted, given that they bowled
b the cricketer bowled, given that they batted

4E

8 I can construct a table with replacement.
e.g. A fair 5-sided die is rolled twice. List all the outcomes using a table and find:
a Pr(2 even numbers)
b Pr(sum of at least 7)
c Pr(sum not equal to 9)

4E

9 I can construct a table without replacement.
e.g. Two letters are chosen from the word TENT, without replacement. Construct a table to list the sample space and find the probability of:
a obtaining the outcome (T, E)
b selecting a T and an N

4F

10 I can construct a tree diagram.
e.g. Two jars contain 4 jelly beans each. Jar A has 3 black and 1 pink jelly bean and Jar B contains 2 black and 2 pink jelly beans. A jar is chosen at random followed by a single jelly bean. Represent the options available in a tree diagram that shows all outcomes and the associated probabilities. Find the probability of selecting:
a jar A and a black jelly bean
b a black jelly bean

4F

11 I can use a tree diagram without replacement.
e.g. A mixed bag of chocolates contains 2 Mars bars (M) and 4 Snickers (S). Draw a tree diagram to show the outcomes and probabilities of the selection of two chocolates without replacement. Find the probability of selecting:
a 2 Snickers
b exactly 1 Mars bar

4G

12 I can determine if events are independent.
e.g. A selection of 10 soccer club memberships found that 5 included a free scarf and 3 included a free soccer ball. One had both a free scarf and a free soccer ball.
Let A be the event 'soccer membership had a free scarf', and B be the event 'soccer membership had a free soccer ball'. Summarise the information in a Venn diagram and find $\Pr(A \mid B)$ and $\Pr(B \mid A)$ to determine whether or not events A and B are independent.

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Chapter review

Short-answer questions

4A **1** A fair 6-sided die is rolled once. Find:

a Pr(4) **b** Pr(even) **c** Pr(at least 3)

4A **2** A letter is chosen from the word INTEREST. Find the probability that the letter will be:

a I **b** E **c** a vowel
d not a vowel **e** E or T

4A **3** An engineer inspects 20 houses in a street for cracks. The results are summarised in this table.

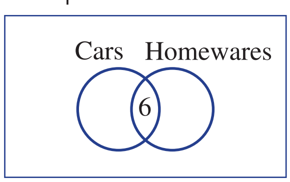
Number of cracks	0	1	2	3	4
Frequency	8	5	4	2	1

a From these results, estimate the probability that the next house inspected in the street will have the following number of cracks.
 i 0 **ii** 1 **iii** 2 **iv** 3 **v** 4

b Estimate the probability that the next house will have:
 i at least 1 crack
 ii no more than 2 cracks

4B/C **4** Of 36 people, 18 have an interest in cars, 11 have an interest in homewares and 6 have an interest in both cars and homewares.

a Complete this Venn diagram.



b Complete this two-way table.

	C	***C′***	
H	6		
H′			

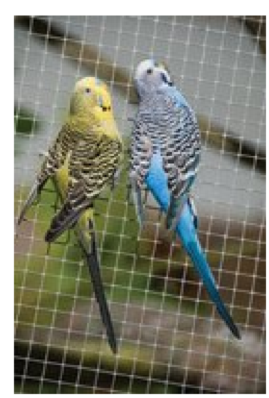
c State the number of people from the group who do not have an interest in either cars or homewares.

d If a person is chosen at random from the group, find the probability that the person will:
 i have an interest in cars and homewares
 ii have an interest in homewares only
 iii not have any interest in cars

4B/C **5** All 26 birds in an aviary have clipped wings and/or a tag. In total, 18 birds have tags, 14 have clipped wings and 6 have both clipped wings and a tag.

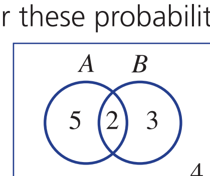
a Find the number of birds that have only clipped wings.

b Find the probability that a bird chosen at random will have a tag only.



4D **6** For these probability diagrams, find Pr($A|B$).

a



b

	A	***A′***	
B	1	4	5
B′	2	2	4
	3	6	9

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4E **7** A letter is chosen at random from the word HAPPY and a second letter is chosen from the word HEY.

a List the sample space by completing this table.

b State the total number of outcomes.

c Find the probability that the two letters chosen will be:

i H then E **ii** the same **iii** not the same

		1st				
		H	A	P	P	Y
2nd	H	(H, H)	(A, H)	(P, H)		
	E					
	Y					

4F **8** A fair 4-sided die is rolled twice and the total is noted.

a Complete this tree diagram to list the sample space.

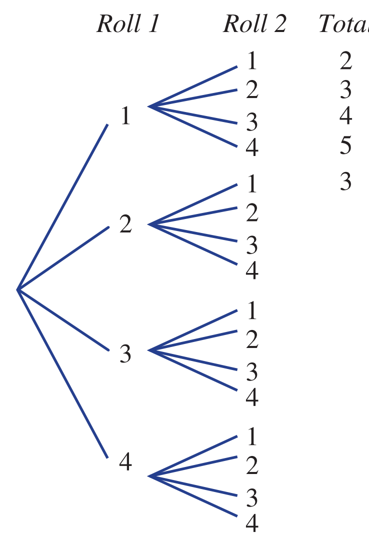
b Find these probabilities.

i Pr(2)

ii Pr(5)

iii Pr(1)

iv Pr(not 1)



4F **9** Two people are selected from a group of 2 females and 3 males, without replacement. Use a tree diagram to find the probability of selecting:

a a female on the first selection

b a male on the second selection given that a female was chosen on the first selection

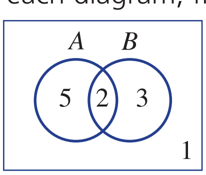
c 2 males

d 1 male

e at least 1 female

4D/G **10** For each diagram, find Pr(A) and Pr($A|B$), then decide if events A and B are independent.

a



b

	A	A'	
B	3		6
B'			
	5		10

Multiple-choice questions

4A **1** A letter is chosen from the word SUCCESS. The probability that the letter is not a C is:

A $\frac{2}{7}$ **B** $\frac{3}{5}$ **C** $\frac{5}{7}$ **D** $\frac{4}{7}$ **E** $\frac{3}{7}$

4A **2** The number of manufacturing errors spotted in a car plant on 20 randomly selected days is given by this table.

Number of errors	0	1	2	3	Total
Frequency	11	6	2	1	20

An estimate of the probability that on the next day no errors will be observed is:

A $\frac{3}{10}$ **B** $\frac{9}{20}$ **C** $\frac{11}{20}$ **D** $\frac{17}{20}$ **E** $\frac{3}{20}$

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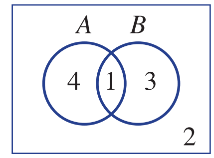
Chapter review

4B **3** For this Venn diagram, $\Pr(A \cup B)$ is equal to:



A $\frac{4}{5}$ **B** $\frac{1}{2}$ **C** $\frac{5}{8}$
D $\frac{1}{4}$ **E** $\frac{1}{10}$

4B **4** 15 people like apples or bananas. Of those 15 people, 10 like apples and 3 like both apples and bananas. How many from the group like only apples?

A 5 **B** 3 **C** 13 **D** 7 **E** 10

4E **5** A letter is chosen from each of the words CAN and TOO. The probability that the pair of letters will not have an O is:

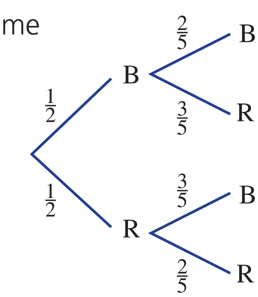
	C	A	N
T	(C, T)	(A, T)	(N, T)
O	(C, O)	(A, O)	(N, O)
O	(C, O)	(A, O)	(N, O)

A $\frac{2}{3}$ **B** $\frac{1}{2}$ **C** $\frac{1}{3}$
D $\frac{1}{9}$ **E** $\frac{5}{9}$

4B **6** The sets A and B are known to be mutually exclusive. Which of the following is therefore true?

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A \cap B) = 0$ **C** $\Pr(A) = 0$
D $\Pr(A \cap B) = 1$ **E** $\Pr(A \cup B) = 0$

4F **7** For this tree diagram, what is the probability of the outcome (B, R)?



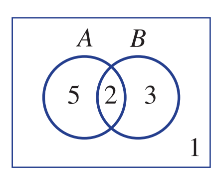
A $\frac{1}{5}$ **B** $\frac{3}{10}$ **C** $\frac{3}{7}$
D $\frac{1}{10}$ **E** $\frac{6}{11}$

4C **8** For this two-way table, $\Pr(A \cap B)$ is:

	A	A'	
B		1	3
B'			4
		4	

A $\frac{2}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{7}$
D $\frac{1}{3}$ **E** $\frac{2}{7}$

4D ★ **9** For this Venn diagram, $\Pr(A \mid B)$ is:



A $\frac{5}{7}$ **B** $\frac{2}{5}$ **C** $\frac{5}{8}$
D $\frac{5}{3}$ **E** $\frac{3}{11}$

4G **10** Two events are independent when:

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A') = 0$ **C** $\Pr(A \cup B) = 0$
D $\Pr(A \mid B) = \Pr(B)$ **E** $\Pr(A) = \Pr(A \mid B)$

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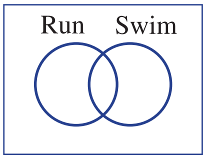
Extended-response questions

1 Of 15 people surveyed to find out if they run or swim for exercise, 6 said they run, 4 said they swim and 3 said they both run and swim.



a How many people surveyed neither run nor swim?

b One of the 15 people is selected at random. Find the probability that they:

i run or swim **ii** only swim

c Represent the information in a two-way table.

★ **d** Find the probability that:

i a person swims, given that they run

ii a person runs, given that they swim

2 A bakery sells three types of bread: raisin (R) at \$2 each, sourdough (S) at \$3 each, and white (W) at \$1.50 each. Judy is in a hurry. She randomly selects 2 loaves and quickly takes them to the counter. (Assume an unlimited loaf supply.)

a Complete this table, showing the possible combination of loaves that Judy could have selected.

		1st		
		R	**S**	**W**
2nd	**R**			
	S			
	W			

b Find the probability that Judy selects:

i 2 raisin loaves **ii** 2 loaves that are the same

iii at least 1 white loaf **iv** not a sourdough loaf

Judy has only \$4 in her purse.

c How many different combinations of bread will Judy be able to afford?

d Find the probability that Judy will not be able to afford her two chosen loaves.

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Chapter 5

Statistics

Essential mathematics: why skills with statistics are important

Vast quantities of data are collected from online records and surveys. To find meaning, statistics such as typical measures and graphs are used to show trends and allow comparisons.

Statistical calculations and interpretations provide essential information for decision making by governments, farmers, research scientists, technicians, and business and finance personnel.

- The Australian Government uses statistics to compare the health of Indigenous and non-Indigenous Australians to improve policy; for example, time-series graphs of average lifespan versus year.
- Farmers use digital sensors to record data which is then statistically analysed. For example, crop and soil moisture levels are used to automate irrigation and improve crop production.
- Stem-and-leaf plots can be used to record fuel economies and tailpipe emissions of carbon dioxide for various vehicle models.
- Medical researchers use parallel box plots that show significantly lower birth weights for babies with mothers who smoke compared to non-smoking mothers. Low birth weights can cause health problems.

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In this chapter

Victorian Curriculum

STATISTICS AND PROBABILITY

Data representation and interpretation

Determine quartiles and interquartile range and investigate the effect of individual data values, including outliers on the interquartile range (VCMSP349)

Construct and interpret box plots and use them to compare data sets (VCMSP350)

Compare shapes of box plots to corresponding histograms and dot plots and discuss the distribution of data (VCMSP351)

Use scatter plots to investigate and comment on relationships between two numerical variables (VCMSP352)

Investigate and describe bivariate numerical data, including where the independent variable is time (VCMSP353)

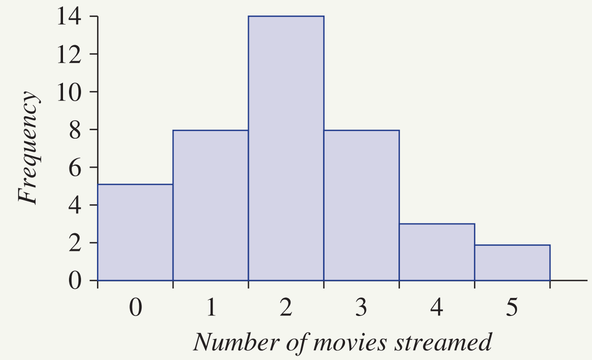
Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (VCMSP354)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 The number of movies streamed in a month online by a number of people is shown in this graph.



a How many people streamed three movies?
b How many people were surveyed?
c How many movies were streamed during the survey?
d How many people streamed fewer than two movies?

2 This table shows the frequency of scores in a test.

Score	Frequency
0–	2
20–	3
40–	6
60–	12
80–100	7

a How many scores were in the 40 to less than 60 range?
b How many scores were:
  i at least 60? **ii** less than 80?
c How many scores were there in total?
d What percentage of scores were in the 20 to less than 40 range?

3 Calculate:

a $\frac{6+10}{2}$ **b** $\frac{8+9}{2}$ **c** $\frac{2+4+5+9}{4}$ **d** $\frac{3+5+8+10+14}{5}$

4 For each of these data sets, find:
  i the mean (i.e. average)
  ii the mode (i.e. most frequent)
  iii the median (i.e. middle value of ordered data)
  iv the range (i.e. difference between highest and lowest)

a 38, 41, 41, 47, 58
b 2, 2, 2, 4, 6, 6, 7, 9, 10, 12

5 This stem-and-leaf plot shows the weight, in grams, of some small calculators.

a How many calculators are represented in the plot?
b What is the mode (i.e. most frequent)?
c What is the minimum calculator weight and maximum weight?
d Find the range (i.e. maximum value – minimum value).

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

13 | 6 means 136 grams

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5A Collecting data

Learning intentions

- To understand what is required to construct a good survey
- To know the difference between a population and a sample
- To be able to categorise types of statistical data

Key vocabulary: survey, statistical data, categorical data, numerical data, nominal data, ordinal data, discrete data, continuous data, population, sample, census

A statistician is a person who is employed to design surveys. They also collect, analyse and interpret data. They assist the government, companies and other organisations to make decisions and plan for the future.

Statisticians:

- **Formulate** and **refine** questions for a survey.
- Choose some **subjects** (i.e. people) to complete the survey.
- **Collect** the data.
- **Organise and display** the data using the most appropriate graphs and tables.
- **Analyse** the data.
- **Interpret the data** and draw conclusions.

There are many reports in the media that begin with the words 'A recent study has found that…'. These are usually the result of a survey or investigation that a researcher has conducted to collect information about an important issue, such as unemployment, crime or obesity.

Lesson starter: Critiquing survey questions

Here is a short survey. It is not very well constructed.
Question 1: How old are you?
Question 2: How much time did you spend sitting in front of the television or a computer yesterday?
Question 3: Some people say that teenagers like you are lazy and spend too much time sitting around when you should be outside exercising. What do you think of that comment?

Have a class discussion about the following.

- What will the answers to Question 1 look like? How could they be displayed?
- What will the answers to Question 2 look like? How could they be displayed?
- Is Question 2 going to give a realistic picture of your normal daily activity?
- How could Question 2 be improved?
- What will the answers to Question 3 look like? How could they be displayed?
- How could Question 3 be improved?

Key ideas

- **Surveys** are used to collect statistical data by asking randomly selected people questions.
  - Survey questions need to be constructed carefully so that the person knows exactly what sort of answer to give. They should use simple language and should not be ambiguous.
  - Survey questions should not be worded so that they deliberately try to provoke a certain kind of response.
  - If the question contains an option to be chosen from a list, the number of options should be an odd number, so that there is a 'neutral' choice. For example, the options could be:

strongly disagree	disagree	unsure	agree	strongly agree

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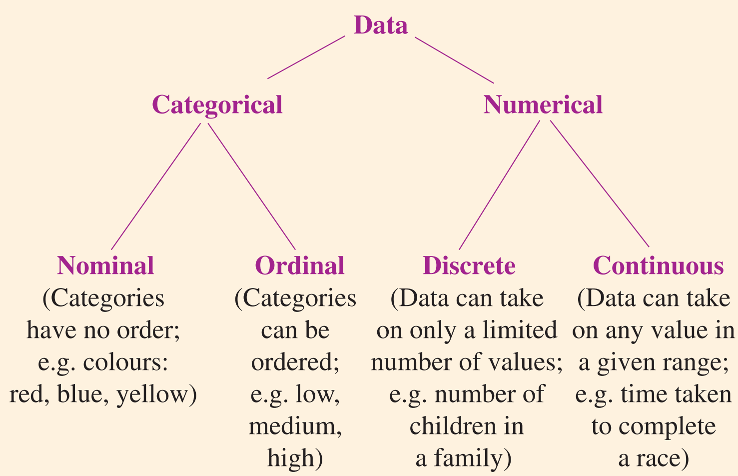
- A **population** is a group of people, animals or objects with something in common. Some examples of populations are:
  - all the people in Australia on Census night (a **census** is a set of statistics collected from the entire population)
  - all the students in your school
  - all the boys in your Maths class
  - all the tigers in the wild in Sumatra
  - all the cars in Sydney
  - all the wheat farms in NSW
- A **sample** is a group that has been chosen from a population. Sometimes information from a sample is used to describe the whole population, so it is important to choose the sample carefully.
- **Statistical data** is information gathered by observation, survey or measurement. It can be categorised as follows.



Exercise 5A

Understanding | 1–3 | 2, 3

1 What are some of the considerations needed when constructing a survey?

2 Match each word (**a**–**f**) with its definition (**A**–**F**).

a Population	**A** A group chosen from a population
b Census	**B** A tool used to collect statistical data
c Sample	**C** All the people or objects in question
d Survey	**D** Statistics collected from an entire population
e Data	**E** The practice of collecting and analysing data
f Statistics	**F** The factual information collected from a survey or other source

3 Match each word (**a**–**f**) with its definition (**A**–**F**).

a Numerical	**A** Categorical data that has no order
b Continuous	**B** Data that are numbers
c Discrete	**C** Numerical data that take on a limited number of values
d Categorical	**D** Data that can be divided into categories
e Ordinal	**E** Numerical data that take any value in a given range
f Nominal	**F** Categorical data that can be ordered

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Fluency | 4–6 | 4, 5, 7

Example 1 Describing types of data

What type of data would the following survey questions generate?

a How many televisions do you have in your home?
b To what type of music do you most like to listen?

Solution	Explanation
a Numerical and discrete	The answer to the question is a number with a limited number of values; in this case, a whole number.
b Categorical and nominal	The answer is a type of music and these categories have no order.

Now you try

What type of data would the following survey questions generate?

a How would you rate your hotel stay (poor, satisfactory, good, excellent)?
b How many times in a week do you exercise?

4 Which one of the following survey questions would generate numerical data?

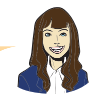
A What is your favourite colour?
B What type of car does your family own?
C How long does it take for you to travel to school?
D What type of dog do you own?

5 Which one of the following survey questions would generate categorical data?

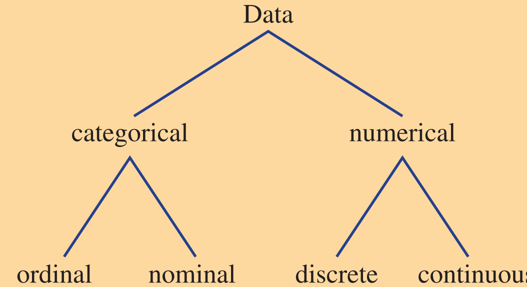
A How many times do you eat at your favourite fast-food place in a typical week?
B How much money do you usually spend buying your favourite fast food?
C How many items did you buy last time you went to your favourite fast-food place?
D Which is your favourite fast food?

6 Year 10 students were asked the following questions in a survey. Describe what type of data each question generates.

a How many people under the age of 18 years are there in your immediate family?
b How many letters are there in your first name?
c Which company is the carrier of your mobile telephone calls?
Optus/Telstra/Vodafone/Boost/Other (Please specify.)
d What is your height?
e How would you describe your level of application in Maths? (Choose from *very high*, *high*, *medium* or *low*.)



Hint: Recall the four types of data:



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5A

7 Every student in Years 7 to 12 votes in the prefect elections. The election process is an example of:
- **A** a population
- **B** continuous data
- **C** a representative sample
- **D** a census

Problem-solving and reasoning | 8, 9 | 9–11

8 A popular Australian 'current affairs' television show recently investigated the issue of spelling. They suspected that people in their twenties are not as good at spelling as people in their fifties, so they decided to conduct a statistical investigation. They chose a sample of 12 people aged 50–59 years and 12 people aged 20–29 years.
Answer the following questions on paper, then discuss in a small group or as a whole class.
- **a** Do you think that the number of people surveyed is enough?
- **b** Do you think it is fair and reasonable to compare the spelling ability of these two groups of people?
- **c** How would you go about comparing the spelling ability of these two groups of people?
- **d** Would you give the two groups the same set of words to spell?
- **e** How could you give the younger people an unfair advantage?
- **f** What sorts of words would you include in a spelling test for the survey?
- **g** How and where would you choose the people to do the spelling test?

9 The principal decides to survey Year 10 students to determine their opinion of Mathematics. In order to increase the chance of choosing a representative sample, the principal should:
- **A** Give a survey form to the first 30 Year 10 students who arrive at school.
- **B** Give a survey form to all the students studying the most advanced maths subject.
- **C** Give a survey form to five students in every Maths class.
- **D** Give a survey form to 20% of the students in every class.

10 Discuss some of the problems with the selection of a survey sample for each given topic.
- **a** A survey at the train station of how Australians get to work.
- **b** An email survey on people's use of computers.
- **c** Phoning people on the electoral roll to determine Australia's favourite sport.

11 Choose a topic in which you are especially interested, such as football, cricket, movies, music, cooking, food, computer games or social media.
Make up a survey about your topic that you could give to the other students in your class. It must have *four* questions.
Question 1 must produce data that are categorical and ordinal.
Question 2 must produce data that are categorical and nominal.
Question 3 must produce data that are numerical and discrete.
Question 4 must produce data that are numerical and continuous.

The Australian Census | — | 12

12 The Australian Census is conducted by the Australian Bureau of Statistics every five years. Research either the 2011 or 2016 Australian Census on the website of the Australian Bureau of Statistics. Find out something interesting from the results of the Census.
Write a short news report or record a 3-minute news report on your computer.

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5B Frequency tables, column graphs and histograms CONSOLIDATING

Learning intentions

- To be able to construct a frequency table from a set of data
- To know the difference between a column graph and histogram when choosing to represent data from a frequency table
- To be able to construct and analyse column graphs and histograms
- To be able to describe data as symmetrical or skewed

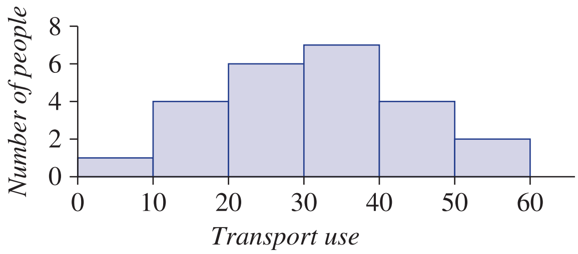
Key vocabulary: frequency table, column graph, histogram, symmetrical, skewed, data

As a simple list, data can be difficult to interpret. Sorting the data into a frequency table allows us to make sense of it and draw conclusions from it.

Statistical graphs are an essential part of the analysis and representation of data. By looking at statistical graphs, we can draw conclusions about the numbers or categories in the data set.

Lesson starter: Public transport analysis

A survey was carried out to find out how many times people in a particular group had used public transport in the past month. The results are shown in this histogram.



Discuss what the histogram tells you about this group of people and their use of public transport.

You may wish to include these points:

- How many people were surveyed?
- Is the data symmetrical or skewed?
- Is it possible to work out all the individual data values from this graph?
- Do you think these people were selected from a group in your own community? Give reasons.

Key ideas

- A **frequency table** displays data by showing the number of values within a set of categories or class intervals. It may include a tally column to help count the data.

Favourite ice-cream flavour	Tally	Frequency
Vanilla	||	2
Chocolate	𝍸	5
Mint choc chip	||||	4
Strawberry	|||	3
Cookies and cream	𝍸 |	6

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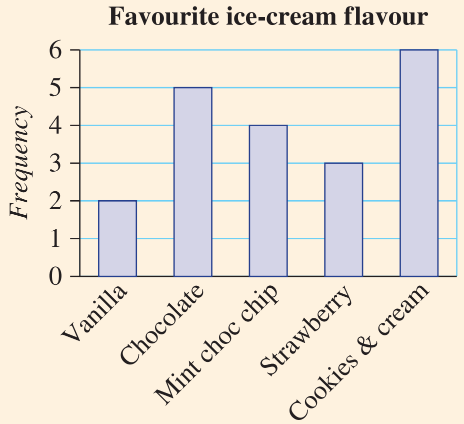
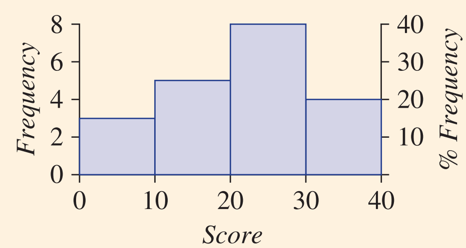
- A **column graph** can be used for a single set of categorical or discrete data to show the frequency.



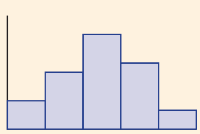
- **Histograms** can be used for grouped discrete or continuous numerical data. The frequency of particular class intervals is recorded.
  - The interval 10– (in the table below) includes all numbers from 10 (including 10) to less than 20.
  - The percentage frequency is calculated as $\%\text{Frequency} = \frac{\text{frequency}}{\text{total}} \times 100$.

Class interval	Frequency	Percentage frequency
0–	3	$\frac{3}{20} \times 100 = 15\%$
10–	5	$\frac{5}{20} \times 100 = 25\%$
20–	8	40%
30–40	4	20%
Total	20	100%

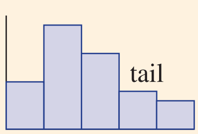


- Data can be **symmetrical** (same shape either side of the middle) or **skewed** (data weighted to the left or the right).

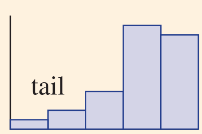
Symmetrical



Positively skewed



Negatively skewed



Exercise 5B

Understanding 1–3 3

1 Decide whether a histogram or column graph would be best used to display the following types of data
  a heights of students in a class
  b favourite colour
  c rating of hotel service (low, medium, high)
  d hourly wage at a number of restaurants

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2 Complete these frequency tables.

Hint: In the tally, 卌 is 5.

a

Car colour	Tally	Frequency
Red	|||	
White	卌	
Green	||	
Silver	||	
Total		

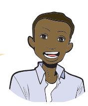
b

Class interval	Frequency	Percentage frequency
80–	8	$\frac{8}{50} \times 100 = 16\%$
85–	23	
90–	13	
95–100		
Total	**50**	

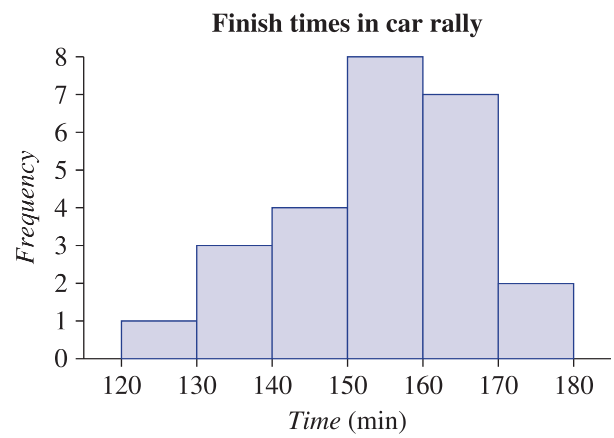
3 This frequency histogram shows how many competitors in a car rally race finished within a given time interval.

a How many competitors finished in a time between 150 and 160 minutes?

b How many cars were there in the race?



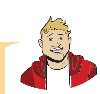
Hint: There was one competitor in the 120–130 interval, three competitors in the 130–140 interval etc.



c Determine the following.

i How many competitors finished in fewer than 150 minutes?

ii What percentage of competitors finished in fewer than 150 minutes?



Hint: Percentage $= \frac{\text{number} < 150}{\text{total}} \times 100$

Fluency 4–6 4, 5, 7

Example 2 Constructing a frequency table and column graph

Twenty people checking out at a hotel were surveyed on the level of service provided by the hotel staff. The results were:

Poor	First class	Poor	Average	Good
Good	Average	Good	First class	First class
Good	Good	First class	Good	Average
Average	Good	Poor	First class	Good

a Construct a frequency table to record the data, with headings Category, Tally and Frequency.

b Construct a column graph for the data.

Continued on next page

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5B

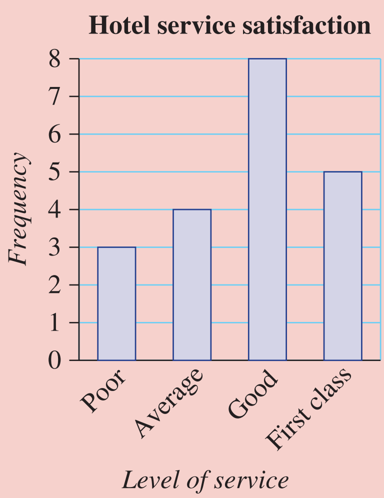
Solution | **Explanation**

a

<table>
<tr><th>Category</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>Poor</td><td>|||</td><td>3</td></tr>
<tr><td>Average</td><td>||||</td><td>4</td></tr>
<tr><td>Good</td><td>𝍸 |||</td><td>8</td></tr>
<tr><td>First class</td><td>𝍸</td><td>5</td></tr>
<tr><td>Total</td><td>20</td><td>20</td></tr>
</table>

Construct a table with the headings Category, Tally, Frequency.
Fill in each category shown in the data. Work through the data in order, recording a tally mark (|) next to the category. It is a good idea to tick the data as you go, to keep track.
On the 5th occurrence of a category, place a diagonal line through the tally marks (𝍸). Then start again on the 6th. Do this every five values, as it makes the tally marks easy to count up.
Once all data is recorded, count the tally marks for the frequency.
Check that the frequency total adds up to the number of people surveyed (20).

b



Draw a set of axes with frequency going up to 8.
For each category, draw a column with height up to its frequency value.
Leave gaps between each column.
Give your graph an appropriate heading.

Now you try

A class of 24 students was surveyed on their favourite genre of movie. The results were:

Action	Comedy	Comedy	Romance	Action	Sci-Fi
Horror	Sci-Fi	Comedy	Action	Comedy	Romance
Comedy	Action	Romance	Horror	Action	Comedy
Action	Romance	Horror	Action	Comedy	Action

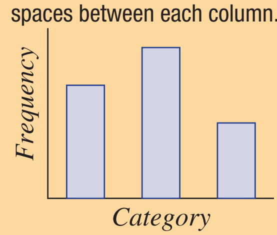
a Construct a frequency table to record the data, with headings Category, Tally and Frequency.
b Construct a column graph for the data.

4 For the data on the next page, which was obtained from surveys:
i Copy and complete this frequency table.

Category	Tally	Frequency
⋮	⋮	⋮

ii Construct a column graph for the data and include a heading.

Hint: In the column graph, leave spaces between each column.



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a The results from 10 subjects on a student's school report showing their level of application are:

Good	Low	Good	Good	Excellent
Very Low	Low	Good	Good	Low

b The favourite sports of a class of students are:

Football	Tennis	Basketball	Tennis	Football
Netball	Football	Tennis	Football	Basketball
Basketball	Tennis	Netball	Football	Football
Football	Basketball	Football	Netball	Tennis

Example 3 Constructing and analysing a histogram

Twenty people were surveyed to find out how many times they use the internet in a week. The raw data are listed.

21, 19, 5, 10, 15, 18, 31, 40, 32, 25
11, 28, 31, 29, 16, 2, 13, 33, 14, 24

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of people used the internet 20 times or more?

Solution / **Explanation**

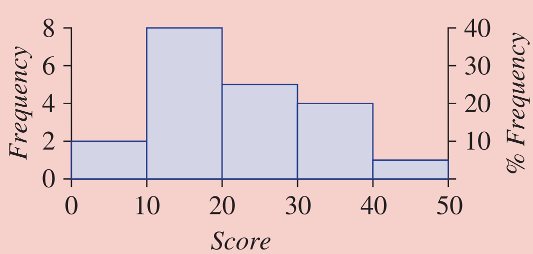
a

Class interval	Tally	Frequency	Percentage frequency
0–	||	2	10%
10–	~~||||~~ |||	8	40%
20–	~~||||~~	5	25%
30–	||||	4	20%
40–49	|	1	5%
Total	20	20	100%

The interval 10– includes all numbers from 10 (including 10) to less than 20, so 10 is in this interval but 20 is not.

Count the tally marks to record the frequency. Add the frequency column to ensure all 20 values have been recorded. Calculate each percentage frequency by dividing the frequency by the total (20) and multiplying by 100%; i.e. $\frac{2}{20} \times 100 = 10$.

b **Number of times the internet is accessed**



Transfer the data from the frequency table to the histogram. Axes scales are evenly spaced and the histogram bar is placed across the boundaries of the class interval. There is no space between the bars.

c The 10– interval is the most frequent.

The frequency (8) is highest for this interval. It is the highest bar on the histogram.

d 50% of those surveyed used the internet 20 or more times.

Sum the percentages for the class intervals from 20– and above.
$25 + 20 + 5 = 50$

Continued on next page

ISBN 978-1-108-87859-3

5B

Now you try

The number of points scored by a basketball team in its 25-game season are shown below:

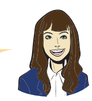
74	82	77	101	91
66	86	87	90	88
79	108	94	89	70
75	81	72	89	97
86	78	78	82	88

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of their games did they score 80 or more?

5 The Maths test results of a class of 25 students were recorded as:

74	65	54	77	85	68	93	59	75
71	82	57	98	73	66	88	76	
92	70	77	65	68	81	79	80	

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d If an A is awarded for a mark of 80 or more, what percentage of the class received an A?



Hint: Construct a frequency table like this:

Class interval	Tally	Frequency	Percentage frequency
50–	|||	3	$\frac{\text{freq.}}{\text{total}} \times 100$
60–			
70–			
80–			
90–100			
Total			

6 The number of wins scored this season is given for 20 hockey teams. Here are the raw data.
4, 8, 5, 12, 15, 9, 9, 7, 3, 7
10, 11, 1, 9, 13, 0, 6, 4, 12, 5
a Organise the data into a frequency table, using class intervals of 5. Start with 0–, then 5– etc. and include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of teams scored 5 or more wins?

7 This frequency table displays the way in which 40 people travel to and from work.

Type of transport	Frequency	Percentage frequency
Car	16	
Train	6	
Tram	8	
Walking	5	
Bicycle	2	
Bus	3	
Total	**40**	

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a Copy and complete the table.

b Use the table to find the:

i frequency of people who travel by train

ii most popular form of transport

iii percentage of people who travel by car

iv percentage of people who walk or cycle to work

v percentage of people who travel by public transport, including trains, buses and trams

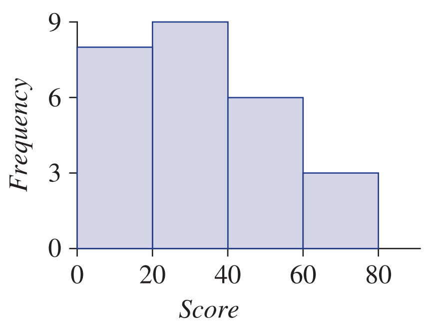
Hint: Percentage frequency:

$$= \frac{\text{frequency}}{\text{total}} \times 100$$

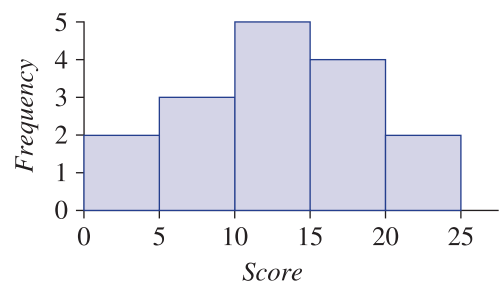
Problem-solving and reasoning — 8, 9 — 8, 10, 11

8 Which of these histograms shows a symmetrical data set and which one shows a skewed data set?

a



b



9 This tally records the number of mice that were weighed and categorised into particular mass intervals for a scientific experiment.

a Construct a table using these column headings: Mass, Frequency and Percentage frequency.

b Find the total number of mice weighed in the experiment.

c State the percentage of mice that were in the 20– gram interval.

d Which was the most common weight interval?

e What percentage of mice were in the most common mass interval?

f What percentage of mice had a mass of 15 grams or more?

Mass (grams)	Tally
10–	|||
15–	卌 |
20–	卌 卌 卌 |
25–	卌 卌 卌 卌 |
30–34	||||

10 A school orchestra contains four musical sections: string, woodwind, brass and percussion. The number of students playing in each section is summarised in this tally.

a Construct and complete a percentage frequency table for the data.

b What is the total number of students in the school orchestra?

c What percentage of students play in the string section?

d What percentage of students do not play in the string section?

e If the number of students in the string section increased by three, what would be the percentage of students who play in the percussion section? (Round your answer to one decimal place.)

Section	Tally
String	卌 卌 卌 卌 |
Woodwind	卌 |||
Brass	卌 ||
Percussion	||||

11 Describe the information that is lost when displaying data using a histogram.

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5B

Interpreting other graphical displays — 12, 13

12 The graph shown compares the life expectancy of males and females in 10 different countries. Use the graph to answer the following questions.

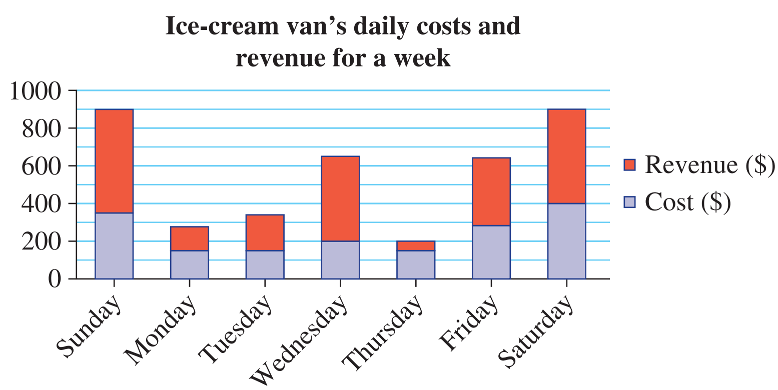
a Which country has the biggest difference in life expectancy for males and females? Approximately how many years is this difference?

b Which country appears to have the smallest difference in life expectancy between males and females?

c From the information in the graph, write a statement comparing the life expectancy of males and females.

d South Africa is clearly below the other countries. Provide some reasons why you think this may be the case.

13 This graph shows the amount spent (Cost) on the purchase and storage of ice-cream each day by an ice-cream vendor, and the amount of money made from the daily sales of ice-cream (Revenue) over the course of a week.



a On which days was the cost highest for the purchase and storage of ice-cream? Why do you think the vendor chose these days to spend the most?

b Wednesday had the greatest revenue for any weekday. What factors may have led to this?

c Daily profit is determined by the difference in revenue and cost. Identify:

  i on which day the largest profit was made and state this profit (in dollars)

  ii on which day the vendor suffered the biggest financial loss

d Describe some problems associated with this type of graph.

Using technology 5B: Using calculators to graph grouped data
This activity is available on the companion website as a printable PDF.

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5C Dot plots and stem-and-leaf plots CONSOLIDATING

Learning intentions

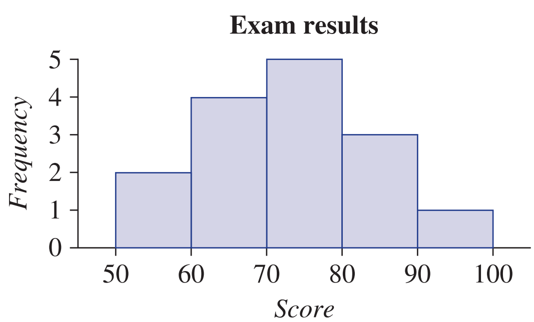
- To be able to construct and interpret a dot plot
- To be able to construct and interpret a stem-and-leaf plot and back-to-back stem-and-leaf-plots
- To know when it is appropriate to use a dot plot or stem-and-leaf plot to represent a set of data
- To be able to interpret the shape of these graphs to describe the distribution of the data as symmetrical or skewed

Key vocabulary: dot plot, stem-and-leaf plot, symmetrical data, skewed data

In addition to column graphs, dot plots and stem-and-leaf plots can be used to display categorical or discrete data. They can also display two related sets for comparison. Like a histogram, they help to show how the data are distributed. A stem-and-leaf plot has the advantage of still displaying all the individual data items.

Lesson starter: Alternative representations

The histogram and stem-and-leaf plot below represent the same set of data. They show the scores achieved by a class in an exam.



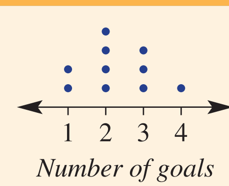
Stem	Leaf
5	1 3
6	5 7 8
7	1 2 4 6
8	3 4 7
9	6

6 | 8 means 68

- Describe the similarities in what the two graphs display.
- What does the stem-and-leaf provide that the histogram does not? What is the advantage of this?
- Which graph do you prefer?
- Discuss any other types of graphs that could be used to present the data.

Key ideas

- A **dot plot** records the frequency of each category or discrete value in a data set.
  - Each occurrence of the value is marked with a dot.



- A **stem-and-leaf plot** displays each value in the data set using a stem number and a leaf number.
  - The data are displayed in two parts: a stem and a leaf.
  - The 'key' tells you how to interpret the stem and leaf parts.
  - The graph is similar to a histogram with class intervals, but the original data values are not lost.

Stem	Leaf
1	0 1 1 5
2	3 7
3	4 4 6
4	2 9

2 | 3 means 23
key

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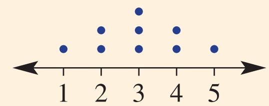
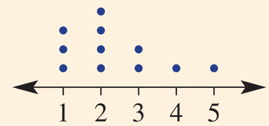
- The stem-and-leaf plot is ordered to allow for further statistical calculations.
- Back-to-back stem and leaf plots, with leaves either side of the stem, can be used to compare two related sets of data.

■ The shape of each of these graphs gives information about the distribution of the data.

- A graph that is even either side of the centre is symmetrical.



- A graph that is bunched to one side of the centre is skewed.

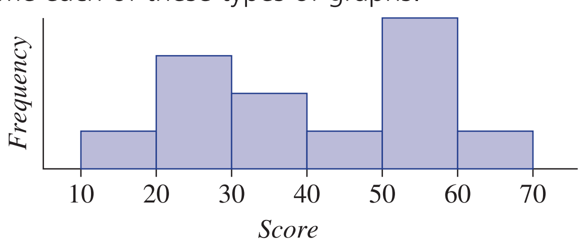


Exercise 5C

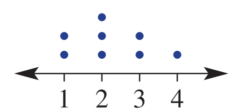
Understanding 1–3 3

1 Name each of these types of graphs.

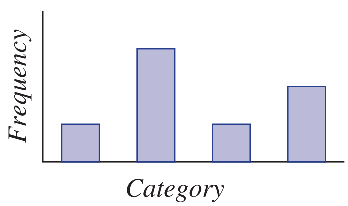
a



b



c



d

Stem	Leaf
0	1 1 3
1	2 4 7
2	0 2 2 5 8
3	1 3

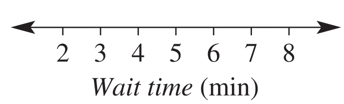
2 | 5 means 25

2 A student records the following wait times, in minutes, for his school bus over 4 school weeks.

5 4 2 8 4 2 7 5 3 3

5 4 2 5 4 5 8 7 2 6

Copy and complete this dot plot of the data by placing a dot for each occurrence of a value.



3 List all the data shown in these stem-and-leaf plots (e.g. 32, 35, …).

a

Stem	Leaf
3	2 5
4	1 3 7
5	4 4 6
6	0 2
7	1 1

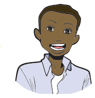
4 | 1 means 41

b

Stem	Leaf
0	2 3 7
1	4 4 8 9
2	3 6 6
3	0 5

2 | 3 means 2.3

Hint: Look at the key '4 | 1 means 41' to see how the stems and leaves go together.



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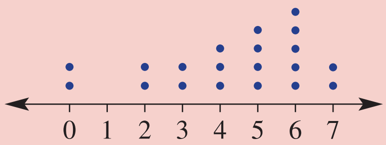
Fluency

4, 5, 6–7(½) | 4, 5, 6–7(½)

Example 4 Interpreting a dot plot

This dot plot shows the number of goals per game scored by a team during the soccer season.

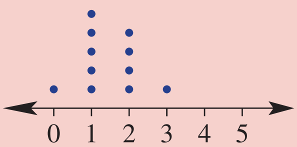


a How many games were played?
b What was the most common number of goals per game?
c How many goals were scored for the season?
d Describe the data in the dot plot.

Solution	Explanation
a There were 20 matches played.	Each dot represents a match. Count the number of dots.
b 6 goals in a game occurred most often.	The most common number of goals has the most dots.
c $2 \times 0 + 2 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 2 \times 7$ $= 0 + 4 + 6 + 12 + 20 + 30 + 14$ $= 86$ goals	Count the number of games (dots) for each number of goals and multiply by the number of goals. Add these together.
d Two games resulted in no goals but the data were generally skewed towards a higher number of goals.	Consider the shape of the graph; it is bunched towards the 6 end of the goal scale.

Now you try

This dot plot shows the number of pets owned by a number of households in a street.



a How many households were surveyed?
b What was the most common number of pets?
c How many pets are there in the street?
d Describe the data in the dot plot.

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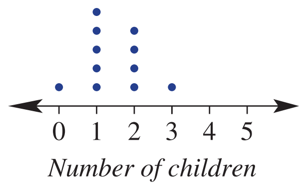
5C

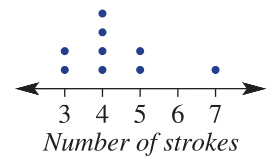
4 A number of families were surveyed to find the number of children in each. The results are shown in this dot plot.



Hint: 4 families had 2 children (4 dots), so that represents 8 children from these families.

- **a** How many families were surveyed?
- **b** What was the most common number of children in a family?
- **c** How many children were there in total?
- **d** Describe the data in the dot plot.

5 This dot plot shows the number of strokes a golfer played, each hole, in his round of golf.



- **a** How many holes did he play?
- **b** How many strokes did he play in the round?
- **c** Describe his round of golf.

Example 5 Constructing a stem-and-leaf plot

Consider this data set.

22 62 53 44 35 47 51 64 72
32 43 57 64 70 33 51 68 59

- **a** Organise the data into an ordered stem-and-leaf plot.
- **b** Describe the distribution of the data as symmetrical or skewed.

Solution

a

Stem	Leaf
2	2
3	2 3 5
4	3 4 7
5	1 1 3 7 9
6	2 4 4 8
7	0 2

5 | 1 means 51

Explanation

For two-digit numbers, select the tens value as the stem and the units as the leaves.

The data ranges from 22 to 72, so the graph will need stems 2 to 7.

Work through the data and record the leaves in the order of the data.

Stem	Leaf
2	2
3	5 2 3
4	4 7 3
5	3 1 7 1 9
6	2 4 4 8
7	2 0

5 | 1 means 51

51 occurs twice, so the leaf 1 is recorded twice in the 5 stem row.

Once data are recorded, redraw and order the leaves from smallest to largest.

Include a key to explain how the stem and leaf go together; i.e. 5 | 1 means 51.

b The distribution of the data is symmetrical.

The shape of the graph is symmetrical (i.e. evenly spread) either side of the centre.

Now you try

Consider this data set.

15 11 18 22 31 24 24 26
14 63 54 40 44 32 28 10

- **a** Organise the data into an ordered stem-and-leaf plot.
- **b** Describe the distribution of the data as symmetrical or skewed.

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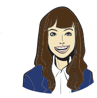
6 Consider the following sets of data.

- **i** Organise the data into an ordered stem-and-leaf plot.
- **ii** Describe the distribution of the data as symmetrical or skewed.



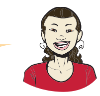
Hint: Remember to include a key such as '4 | 6 means 46'.

a 46 22 37 15 26 38 52 24
31 20 15 37 21 25 26

b 35 16 23 55 38 44 12 48 21 42
53 36 35 25 40 51 27 31 40 36 32

c 153 121 124 117 125 118 135 137 162
145 147 119 127 149 116 133 160 158

d 4.9 3.7 4.5 5.8 3.8 4.3 5.2 7.0 4.7
4.4 5.5 6.5 6.1 3.3 5.4 2.0 6.3 4.8



Hint:

Symmetrical

Stem	Leaf
1	1 2
2	1 2 3
3	1 2 3 4
4	1 2 7
5	3

Skewed

Stem	Leaf
1	2 5 7 8
2	3 4 7 6
3	1 2
4	5

Example 6 Constructing back-to-back stem-and-leaf plots

Two television sales representatives sell the following number of televisions each week over a 15-week period.

Employee 1
23 38 35 21 45 27 43 36
19 35 49 20 39 58 18

Employee 2
28 32 37 20 30 45 48 17
32 37 29 17 49 40 46

a Construct an ordered back-to-back stem-and-leaf plot.
b Describe the distribution of each employee's sales.

Solution

a

Employee 1 Leaf	Stem	Employee 2 Leaf
9 8	1	7 7
7 3 1 0	2	0 8 9
9 8 6 5 5	3	0 2 2 7 7
9 5 3	4	0 5 6 8 9
8	5	

3 | 7 means 37

Explanation

Construct an ordered stem-and-leaf plot with the sales by employee 1 on the left-hand side and the sales by employee 2 on the right-hand side. Include a key.

b Sales by employee 1 are symmetrical, whereas sales by employee 2 are skewed.

Observe the shape of each employee's graph. If appropriate, use the words symmetrical (i.e. spread evenly around the centre) or skewed (i.e. bunched to one side of the centre).

Now you try

Two friends spent the following amounts in dollars on take-away food over a 12-week period.

Friend 1
54 44 30 32 46 62
22 66 41 36 57 48

Friend 2
16 24 30 44 29 19
22 28 18 52 32 41

a Construct an ordered back-to-back stem-and-leaf plot.
b Describe the distribution of each friend's spending.

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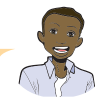
5C

7 Consider the following sets of data.
   i Draw a back-to-back stem-and-leaf plot.
   ii Comment on the distribution of the two data sets.



Hint: For part **a** use a key like ‘3 | 7 means 37’ and for part **b** use a key like ‘15 | 6 means 156’.

a Set 1: 61 38 40 53 48 57 64
39 42 59 46 42 53 43

Set 2: 41 55 64 47 35 63 61
52 60 52 56 47 67 32

b Set 1: 176 164 180 168 185 187 195 166 201
199 171 188 175 192 181 172 187 208

Set 2: 190 174 160 170 186 163 182 171
167 187 171 165 194 182 163 178

Problem-solving and reasoning

8, 9 | 8, 10, 11

8 Two football players, Logan and Max, compare their personal tallies of the number of goals scored for their team over a 12-match season. Their tallies are as follows.

Game	1	2	3	4	5	6	7	8	9	10	11	12
Logan	0	2	2	0	3	1	2	1	2	3	0	1
Max	0	0	4	1	0	5	0	3	1	0	4	0

a Draw a dot plot to display Logan’s goal-scoring achievement.
b Draw a dot plot to display Max’s goal-scoring achievement.
c How would you describe Logan’s scoring habits?
d How would you describe Max’s scoring habits?

9 This stem-and-leaf plot shows the times, in minutes, that Chris has achieved in the past 14 fun runs she competed in.
a What is the difference between her slowest and fastest times?
b Just by looking at the stem-and-leaf plot, what would you estimate to be Chris’ average time?
c If Chris records another time of 24.9 minutes, how would this affect your answer to part **b**?

Stem	Leaf
20	5 7
21	1 2 6
22	2 4 6 8
23	4 5 6
24	3 6

22 | 4 means 22.4

10 The data below show the distances travelled (in km) by students at an inner-city and an outer-suburb school.

Inner city: 3 10 9 14 21 6
1 12 24 1 19 4

Outer suburb: 12 21 18 9 34 19
24 3 23 41 18 4

a Draw a back-to-back stem-and-leaf plot for the data.
b Comment on the distribution of distances for each school.
c Give a practical reason for the distribution of the data.

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11 Determine the possible values of the pronumerals in the following ordered stem-and-leaf plots.

a

Stem	Leaf
1	2 4
2	3 6 9 b
a	1 4
4	7 c 8

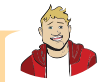
2 | 3 means 2.3

b

Stem	Leaf
20	a 1 4
21	2 2 9
22	0 b 5 7
23	1 4

22 | 7 means 227

Hint: The stems and leaves are ordered from smallest to largest. A leaf can appear more than once.



Splitting stems — 12

12 The back-to-back stem-and-leaf plot below shows the maximum daily temperature for two cities over a 2-week period.

Maximum temperature

City A leaf	Stem	City B leaf
	0	
9 8 8	0^*	
4 3 3 1 1 1	1	
8 8 6 6 5	1^*	7 9
	2	0 2 2 3 4 4
	2^*	5 6 7 7 8
	3	1

1 | 4 means 14
1^* | 5 means 15

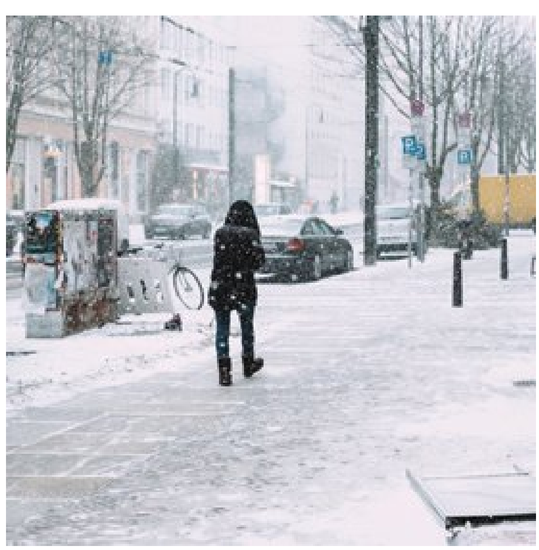
a Describe the difference between the stems 1 and 1^*.

b To which stem would these numbers be allocated?
i 12°C **ii** 5°C

c Why might you use this process of splitting stems, like that used for 1 and 1^*?

d Compare and comment on the differences in temperatures between the two cities.

e What might be a reason for these different temperatures?



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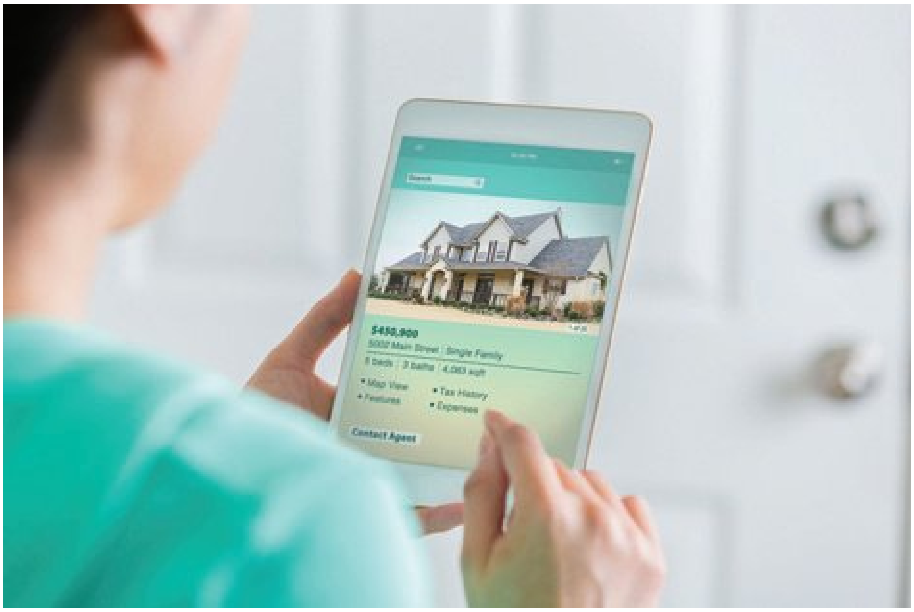
5D Range and measures of centre

Learning intentions

- To know some measures of centre and spread used to summarise a data set.
- To know how the median is found differently for data sets with an odd or even number of values
- To be able to find the mean, median, mode and range of a set of data in list or graphical display form
- To understand the effect of extreme values on the values of the summary statistics

Key vocabulary: mode, bimodal, mean, range, median, stem-and-leaf plot, dot plot

In the previous sections you have seen how to summarise data in the form of a frequency table and to display data using graphs. Key summary statistics also allow us to describe the data using a single numerical value. The mean (i.e. average), for example, may be used to describe a student's performance over a series of tests. The median (i.e. middle value when data are ordered) is often used when describing the house prices in a suburb. These are termed *measures of centre*. Providing information about the spread of the data is the range, which measures the difference between the maximum and minimum values.



Lesson starter: Mean, median or mode?

The following data represent the number of goals scored by Ellie in each game of a 9-game netball season.

24 18 25 16 3 23 27 19 25

It is known that the figures below represent, in some order, the mean, median and mode.

25 20 23

- Without doing any calculations, can you suggest which statistic is which? Explain.
- From the data, what gives an indication that the mean (i.e. average) will be less than the median (i.e. middle value)?
- Describe how you would calculate the mean, median and mode from the data values.

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Key ideas

- The **mean** (or average) is calculated by summing all the data values and dividing by the total number of values.

$$\text{Mean } (\bar{x}) = \frac{\text{sum of all data values}}{\text{number of data values}}$$

  - The mean is affected by extreme values in the data.
- The **mode** is the most commonly occurring value in the data set.
  - A data set can have two modes (called **bimodal**) or no unique mode at all.
- The **median** is the middle value of a data set when the data are arranged in order.
  - When the data set has an even number of values, the median is the average of the two middle values.
    For example,

    2 3 (6) 8 12 — Median = 6

    4 7 8 10 13 17 — $\text{Median} = \frac{8+10}{2} = 9$

  - The median is not significantly affected by extreme values in the data.
- The **range** is a measure of how spread out the data is.
  - Range = maximum value − minimum value

Exercise 5D

Understanding

1–3 | 1, 3

1 Use the words from the list below to fill in the missing word in these sentences.

mean, median, mode, bimodal, range

a The ________ is the most frequently occurring value in a data set.
b Dividing the sum of all the data values by the total number of values gives the ________.
c The middle value of a data set ordered from smallest to largest is the ________ .
d A data set with two most common values is ________ .
e A data set has a maximum value of 7 and a minimum value of 2. The ________ is 5.

2 Circle the middle value(s) of these ordered data sets.
a 2 4 6 7 8 10 11
b 6 9 10 14 17 20

Hint: Recall that an even number of data values will have two middle values.

3 Aaron drinks the following number of cups of coffee each day in a week.

4 5 3 6 4 3 3

a How many cups of coffee does he drink in the week (sum of the data values)?
b How many days are in the week (total number of data values)?
c What is the mean number of cups of coffee Aaron drinks each day (part **a** ÷ part **b**)?

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5D

Fluency

4–5(½) | 4–5(½), 6

Example 7 Finding the mean, mode and range

For the following data sets, find:

i the mean
ii the mode
iii the range

a 2, 4, 5, 8, 8 **b** 3, 15, 12, 9, 12, 15, 6, 8

Solution	Explanation
a i Mean $= \frac{2+4+5+8+8}{5}$ $= \frac{27}{5}$ $= 5.4$	Mean $= \frac{\text{sum of all data values}}{\text{number of data values}}$ Add all the data and divide by the number of values (5).
ii The mode is 8.	The mode is the most common value in the data.
iii Range $= 8 - 2$ $= 6$	Range = maximum value − minimum value
b i Mean $= \frac{3+15+12+9+12+15+6+8}{8}$ $= \frac{80}{8}$ $= 10$	Mean $= \frac{\text{sum of all data values}}{\text{number of data values}}$ Add all the data and divide by the number of values (8).
ii There are two modes: 12 and 15.	The data set is bimodal: 12 and 15 are the most common data values.
iii Range $= 15 - 3$ $= 12$	Range = maximum value − minimum value

Now you try

For the following data sets, find:

i the mean
ii the mode
iii the range

a 8, 10, 2, 6, 12, 3, 10, 1 **b** 22, 25, 18, 22, 3, 18

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4 For each of the following data sets, find:

i the mean **ii** the mode **iii** the range

a 2 4 5 8 8

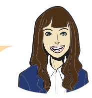
b 5 8 10 15 20 12 10 50

c 55 70 75 50 90 85 50 65 90

d 27 30 28 29 24 12

e 2.0 1.9 2.7 2.9 2.6 1.9 2.7 1.9

f 1.7 1.2 1.4 1.6 2.4 1.3



Hint:
$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$
Mode is the most common value.
Range = maximum − minimum

Example 8 Finding the median

Find the median of each data set.

a 4, 7, 12, 2, 9, 15, 1 **b** 16, 20, 8, 5, 21, 14

Solution	Explanation
a 1 2 4 (7) 9 12 15 Median = 7	The data must first be ordered from smallest to largest. The median is the middle value. For an odd number of data values, there will be one middle value.
b 5 8 14 16 20 21 $\text{Median} = \frac{14+16}{2}$ $= 15$	Order the data from smallest to largest. For an even number of data values, there will be two middle values. The median is the average of these two values (i.e. the value halfway between the two middle numbers).

Now you try

Find the median of each data set.

a 10, 16, 2, 7, 1, 18, 5, 10, 14 **b** 10, 35, 18, 24, 12, 28, 16, 31

5 Find the median of each data set.

a 1 4 7 8 12

b 1 2 2 4 4 7 9

c 11 13 6 10 14 13 11

d 62 77 56 78 64 73 79 75 77

e 2 4 4 5 6 8 8 10 12 22

f 1 2 2 3 7 12 12 18

g 30 36 31 38 27 40

h 2.4 2.0 3.2 2.8 3.5 3.1 3.7 3.9

Hint: First make sure that the data values are in order. For two middle values, find their average.

6 Nine people watch the following number of hours of television on a weekend.

4 4 6 6 6 8 9 9 11

a Find the mean number of hours of television watched.

b Find the median number of hours of television watched.

c Find the range of the television hours watched.

d What is the mode number of hours of television watched?

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5D

Problem-solving and reasoning 7–9 9–11

7 Eight students compare the amount of pocket money they receive. The data are as follows.

$12 $15 $12 $24 $20 $8 $50 $25

a Find the range of pocket money received.
b Find the median amount of pocket money.
c Find the mean amount of pocket money.
d Why is the mean larger than the median?

Example 9 Calculating summary statistics from a stem-and-leaf plot

For the data in this stem-and-leaf plot, find the:

a range
b mode
c mean
d median

Stem	Leaf
2	5 8
3	1 2 2 2 6
4	0 3 3
5	2 6

5 | 2 means 52

Solution

a Minimum value = 25
Maximum value = 56
Range = 56 − 25
= 31

Explanation

In an ordered stem-and-leaf plot, the first data item is the minimum and the last is the maximum. Use the key '5 | 2 means 52' to see how to put the stem and leaf together.
Range = maximum value − minimum value

b Mode = 32

The mode is the most common value. The leaf 2 appears three times with the stem 3.

c $\text{Mean} = \dfrac{25 + 28 + 31 + 32 + 32 + 32 + 36 + 40 + 43 + 43 + 52 + 56}{12}$

$= \dfrac{450}{12}$

$= 37.5$

Form each data value from the graph and add them all together. Then divide by the number of data values in the stem-and-leaf plot.

d $\text{Median} = \dfrac{32 + 36}{2}$

$= 34$

There is an even number of data values: 12. The median will be the average of the middle two values (i.e. the 6th and 7th data values).

Now you try

For the data in this stem-and-leaf plot, find the:

a range
b mode
c mean
d median

Stem	Leaf
2	1 3 7
3	2 8 9 9
4	4 6 8

3 | 2 means 32

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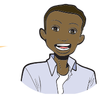
8 For the data in these stem-and-leaf plots, find the:
- **i** range
- **ii** mode
- **iii** mean (rounded to one decimal place)
- **iv** median



Hint: Use the key to see how the stem and leaf go together.

a

Stem	Leaf
0	4 4
1	0 2 5 9
2	1 7 8
3	2

2 | 7 means 27

b

Stem	Leaf
10	1 2 4
11	2 6
12	5

11 | 6 means 116

c

Stem	Leaf
3	0 0 5
4	2 7
5	1 3 3
6	0 2

3 | 2 means 3.2

9 This back-to-back stem-and-leaf plot shows the results of two students, Hugh and Mark, on their end-of-year examination in each subject.

a For each student, find:
- **i** the mean
- **ii** the median
- **iii** the range

b Compare the performance of the two students using your answers to part **a**.

Hugh leaf	Stem	Mark leaf
8 8 5	6	4
7 3	7	4 7
5 4 2 1 1	8	2 4 6 8
	9	2 4 5

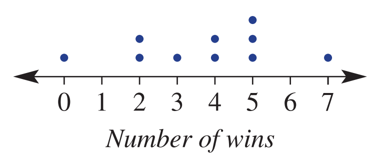
7 | 4 means 74%

10 A real estate agent recorded the following amounts for the sale of five properties:

$120 000 $210 000 $280 000 $370 000 $1 700 000

The mean is $536 000 and the median is $280 000.
Which is a better measure of the centre of the five property prices: the mean or the median? Give a reason.

11 This dot plot shows the number of wins recorded by a school sports team in the past 10 eight-game seasons.



a What was the median number of wins?

b What was the mean number of wins?

c The following season, the team records 3 wins. What effect will this have (i.e. increase/decrease/no change) on the:
- **i** median?
- **ii** mean?

Moving run average — 12

12 A moving average is determined by calculating the average of all data values up to a particular time or place in the data set.
Consider a batter in cricket with the following runs scored from 10 completed innings.

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32								

In the table, 26 is the average after 1 innings and 32 is the average after 2 innings.

a Complete the table by calculating the moving average for innings 3–10. Round your answer to the nearest whole number where required.

b Plot the score and moving averages for the batter on the same set of axes, with the innings number on the horizontal axis. Join the points to form two line graphs.

c Describe the behaviour of the:
- **i** score graph
- **ii** moving average graph

d Describe the main difference in the behaviour of the two graphs. Give reasons.

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5E Quartiles and outliers

Learning intentions

- To know which statistics make up the five-figure summary
- To be able to calculate the quartiles of a data set
- To be able to find the interquartile range and know what it represents
- To know what is meant by an outlier and be able to find any outliers in a data set
- To understand the impact of outliers on various statistics

Key vocabulary: five-figure summary, upper quartile, lower quartile, median, interquartile range, outlier, upper fence, lower fence

In addition to the median of a single set of data, there are two related statistics called the upper and lower quartiles. If data are placed in order, then the lower quartile is central to the lower half of the data. The upper quartile is central to the upper half of the data. These quartiles are used to calculate the interquartile range, which helps to describe the spread of the data, and show whether or not any data points do not fit the rest of the data (outliers).

Lesson starter: House prices

A real estate agent tells you that the median house price for a suburb in 2020 was \$753 000 and the mean was \$948 000.

- Is it possible for the median and the mean to differ by so much?
- Under what circumstances could this occur? Discuss.

Key ideas

- The **five-figure summary** uses the following statistical measures to summarise a set of data:
  - Minimum value (min) — the lowest value
  - **Lower quartile** (Q_1) — the number above 25% of the ordered data
  - Median (Q_2) — the middle value above 50% of the ordered data
  - **Upper quartile** (Q_3) — the number above 75% of the ordered data
  - Maximum value (max) — the highest value

Odd number of data values

1 2 2 3) 5 (6 6 7 9

$Q_1 = \frac{2+2}{2} = 2$ $\quad Q_2 \quad$ $Q_3 = \frac{6+7}{2} = 6.5$

Even number of data values

2 3 3 4 7 | 8 8 9 9 9

$Q_1 = 3 \quad Q_2 = 7.5 \quad Q_3 = 9$

- Another measure of the spread of the data is the **interquartile range (IQR)**.

  IQR = upper quartile − lower quartile

  $= Q_3 - Q_1$

- **Outliers** are data elements outside the vicinity of the rest of the data.
  A data point is an outlier if it is either:
  - less than the **lower fence**, where lower fence = $Q_1 - 1.5 \times$ IQR or
  - greater than the **upper fence**, where upper fence = $Q_3 + 1.5 \times$ IQR
- Outliers significantly affect the range of a data set but have limited to no effect on the IQR.

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Exercise 5E

Understanding

	1–3	3

1 **a** State the five values that need to be calculated for a five-figure summary.
b Explain the difference between the range and the interquartile range.
c What is an *outlier*?

2 The data show, in order, the numbers of cars owned by 10 families surveyed.

$0, 1, 1, 1, 1, 2, 2, 2, 3, 3$

a Find the median (the middle value).
b By splitting the data in half, determine the:
i lower quartile Q_1 (middle of lower half)
ii upper quartile Q_3 (middle of upper half)

Hint: For an even number of data values, average the two middle values for the median.

3 For the data set with $Q_1 = 3$ and $Q_3 = 8$:
a Find $IQR = Q_3 - Q_1$.
b Calculate $Q_1 - 1.5 \times IQR$ (lower fence) and $Q_3 + 1.5 \times IQR$ (upper fence).
c Identify the name that would be given to the value 18 in the data set.

Fluency

	4–6	4–6, 7(½)

Example 10 Finding quartiles and IQR for an even number of data values

Consider this data set:
$2, 2, 4, 5, 6, 8, 10, 13, 16, 20$

a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

Solution

a 2 2 (4) 5 6 | 8 10 (13) 16 20

$Q_2 = \frac{6+8}{2} = 7$

Q_1 (4), Q_3 (13)

$Q_1 = 4$ and $Q_3 = 13$

Explanation

The data are already ordered. Since there is an even number of values, split the data in half to locate the median.
Q_1 is the middle value of the lower half:
2 2 (4) 5 6
Q_3 is the middle value of the upper half:
8 10 (13) 16 20

b $IQR = 13 - 4$
$= 9$

$IQR = Q_3 - Q_1$

Now you try

Consider this data set:
$3, 6, 6, 10, 12, 13, 15, 19, 24, 27, 28, 31, 33, 33$

a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

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5E

4 For these data sets, find the:

  i upper quartile (Q_3) and the lower quartile (Q_1)

  ii IQR

a 3, 4, 6, 8, 8, 10

b 10, 10, 11, 14, 14, 15, 16, 18, 20, 21

c 41, 49, 53, 58, 59, 62, 62, 65

d 1.2, 1.7, 1.9, 2.2, 2.4, 2.5, 2.9, 3.2

Hint: For an even number of data values, split the ordered data in half:

2 4 7 | 8 10 12

↑ Q_1 (4) ↑ Q_3 (10)

$IQR = Q_3 - Q_1$

Example 11 Finding quartiles and IQR for an odd number of data values

Consider this data set:
2.2, 1.6, 3.0, 2.7, 1.8, 3.6, 3.9, 2.8, 3.8

a Find the upper quartile (Q_3) and the lower quartile (Q_1).

b Determine the IQR.

Solution	Explanation
a (1.6 1.8 | 2.2 2.7) 2.8 (3.0 3.6 | 3.8 3.9), with 2.8 = Q_2	First order the data and locate the median (Q_2).
$Q_1 = \frac{1.8 + 2.2}{2} = \frac{4.0}{2} = 2.0$	Split the data in half; i.e. either side of the median.
$Q_3 = \frac{3.6 + 3.8}{2} = \frac{7.4}{2} = 3.7$	Q_1 is the middle value of the lower half; for two middle values, average the two numbers. Q_3 is the middle value of the upper half.
b $IQR = 3.7 - 2.0 = 1.7$	$IQR = Q_3 - Q_1$

Now you try

Consider this data set:
14.4, 15.2, 16.0, 13.7, 18.2, 21.4, 19.7, 19.9, 12.8, 20.6, 21.4

a Find the upper quartile (Q_3) and the lower quartile (Q_1).

b Determine the IQR.

5 For these data sets, find:

  i the upper quartile (Q_3) and the lower quartile (Q_1)

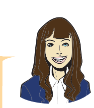
  ii the IQR

a 1, 2, 4, 8, 10, 11, 14

b 10, 7, 14, 2, 5, 8, 3, 9, 2, 12, 1

c 0.9, 1.3, 1.1, 1.2, 1.7, 1.5, 1.9, 1.1, 0.8

d 21, 7, 15, 9, 18, 16, 24, 33, 4, 12, 13, 18, 24



Hint: For an odd number of data values, split ordered data in half, leaving out the middle value.

0 (2) 4) 7 (9 (14) 16

↑ Q_1 (2) ↑ Q_3 (14)

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Example 12 Finding the five-figure summary and outliers

The following data set represents the number of flying geese spotted on each day of a 13-day tour of England.
5, 1, 2, 6, 3, 3, 18, 4, 4, 1, 7, 2, 4

a For the data, find the:
  - **i** minimum and maximum number of geese spotted
  - **ii** median
  - **iii** upper and lower quartiles
  - **iv** IQR

b Find any outliers.
c Can you give a possible reason for why the outlier occurred?
d If the outlier's value is corrected to 8, determine the range and IQR of the data set. Explain the similarities or differences compared to the original data set.

Solution

a i Min = 1, max = 18

ii 1, 1, 2, 2, 3, 3, (4), 4, 4, 5, 6, 7, 18
∴ Median = 4

iii Lower quartile $(Q_1) = \frac{2+2}{2}$
$= 2$
Upper quartile$(Q_3) = \frac{5+6}{2}$
$= 5.5$

iv IQR = 5.5 − 2
= 3.5

Explanation

Look for the largest and smallest numbers and order the data:

1 1 2 | 2 3 3) 4 (4 4 5 | 6 7 18
↑ Q_1 ↑ Q_2 ↑ Q_3

Since Q_2 falls on a data value, it is not included in the lower or higher halves when Q_1 and Q_3 are calculated.

IQR = $Q_3 - Q_1$

Solution

b Lower fence = $Q_1 - 1.5 \times$ IQR
$= 2 - 1.5 \times 3.5$
$= 2 - 5.25$
$= -3.25$

Upper fence = $Q_3 + 1.5 \times$ IQR
$= 5.5 + 1.5 \times 3.5$
$= 5.5 + 5.25$
$= 10.75$
∴ The outlier is 18.

Explanation

A data point is an outlier if it is less than the lower fence = $Q_1 - 1.5 \times$ IQR or greater than the upper fence = $Q_3 + 1.5 \times$ IQR.

There are no numbers less than −3.25 but 18 is greater than 10.75.

c Perhaps a flock of geese was spotted that day.

Continued on next page

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5E

d 1, 1, 2, 2, 3, 4, 4, 4, 5, 6, 7, 8

$Range = 8 - 1$
$= 7$
$IQR = 5.5 - 2$
$= 3.5$

The range is significantly reduced by correcting the outlier. The IQR here is unchanged. Extreme values do not significantly affect the IQR.

18 is replaced in the data set with 8 and the range and IQR calculated.

The outlier affected the range as the maximum value is changed but the IQR is not affected by the value of the maximum.

Now you try

The following data set represents the number of hours a ball girl worked at the Australian Open over the 14-day event.

4, 6, 6, 5, 8, 7, 8, 14, 5, 4, 2, 3, 5, 6

a For the data, find the:
- **i** minimum and maximum number of hours worked
- **ii** median
- **iii** upper and lower quartiles
- **iv** IQR

b Find any outliers.

c Can you give a possible reason for why the outlier occurred?

d If the outlier's value is corrected to 9, determine the range and IQR of the data set. Explain the similarities or differences compared to the original data set.

6 The following numbers of cars were counted on each day for 15 days, travelling on a quiet suburban street.

10, 9, 15, 14, 10, 17, 15, 0, 12, 14, 8, 15, 15, 11, 13

a For the given data, find the:
- **i** minimum and maximum number of cars counted
- **ii** median
- **iii** lower and upper quartiles (Q_1 and Q_3)
- **iv** IQR

b Find any outliers.

c Give a possible reason for the outlier.

d The outlier value is changed to 8. Find the new range and IQR and comment on any similarities or differences compared to the original data set.

Hint:
Outliers are more than $Q_3 + 1.5 \times IQR$
or
less than $Q_1 - 1.5 \times IQR$

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7 Summarise the data sets below by finding:
- **i** the minimum and maximum values
- **ii** the median (Q_2)
- **iii** the lower and upper quartiles (Q_1 and Q_3)
- **iv** the IQR
- **v** any outliers

a 4, 5, 10, 7, 5, 14, 8, 5, 9, 9
b 24, 21, 23, 18, 25, 29, 31, 16, 26, 25, 27
c 10, 13, 2, 11, 10, 8, 24, 12, 13, 15, 12
d 3, 6, 10, 11, 17, 4, 4, 1, 8, 4, 10, 8

Problem-solving and reasoning — 8, 9 — 8, 10–12

8 Twelve different calculators had the following numbers of buttons:
36, 48, 52, 43, 46, 53, 25, 60, 128, 32, 52, 40

a For the given data, find:
- **i** the minimum and maximum number of buttons on the calculators
- **ii** the median
- **iii** the lower and upper quartiles (Q_1 and Q_3)
- **iv** the IQR
- **v** any outliers
- **vi** the mean

b Can you give a possible reason why the outlier has occurred?
c Which is a better measure of the centre of the data: the mean or the median? Explain.
d Which is a better measure of the spread of the data: the range or the IQR? Explain.

9 At an airport, Paul checks the weight of 20 luggage items. If the weight of a piece of luggage is an outlier, then the contents undergo a further check. The weights in kilograms are:

1	4	5	5	6	7	7	7	8	8
10	10	10	13	15	16	17	19	32	33

How many luggage items will undergo a further check?

10 The prices of nine fridges are displayed in a sale catalogue. They are:
\$350 \$1000 \$850 \$900 \$1100 \$1200
\$1100 \$1000 \$1700
How many of the fridge prices could be considered outliers?

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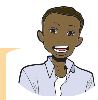
5E

11 For the data in this stem-and-leaf plot, find:
 a the IQR
 b any outliers
 c any outliers if the number 32 was added to the list

Stem	Leaf
0	1
1	6 8
2	0 4 6 8
3	0

2 | 4 means 24

Hint: Split the data in half to find Q_2, then find Q_1 and Q_3.



12 For the data in this stem-and-leaf plot the value 42 was incorrectly recorded. What are the possible values it should have been if the:
 a median is not changed
 b IQR is not changed

Stem	Leaf
1	7 8
2	1 4 6
3	5 5
4	2 9
5	0 3

4 | 2 means 42

Some research — 13

13 Use the internet to search for data about a topic that interests you. Try to choose a single set of data that includes between 15 and 50 values.
 a Organise the data using a:
  i stem-and-leaf plot
  ii frequency table and histogram
 b Find the mean and the median.
 c Find the range and the interquartile range.
 d Write a brief report describing the centre and spread of the data, referring to parts **a** to **c** above.
 e Present your findings to your class or a classmate.

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5F Box plots

Learning intentions

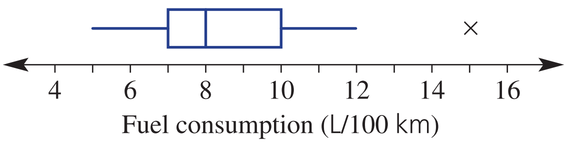
- To know that a box plot can be used to represent the five-figure summary of a data set
- To understand that the sections of a box plot each represent approximately 25% of the data and show the spread of the data
- To be able to construct box plots both with and without outliers
- To know that parallel box plots can be used to compare two or more sets of data in the same context

Key vocabulary: box plot, parallel box plot, outlier, five-figure summary, upper quartile, lower quartile, median, interquartile range, upper fence, lower fence

The five-figure summary (min, Q_1, Q_2, Q_3, max) can be represented in graphical form as a box plot. Box plots are graphs that summarise single data sets. They clearly display the minimum and maximum values, the median, the quartiles and any outliers. Q_1, Q_2 and Q_3 divide the data into quarters. Box plots also give a clear indication of how data are spread, as the IQR (interquartile range) is shown by the width of the central box.

Lesson starter: Fuel consumption

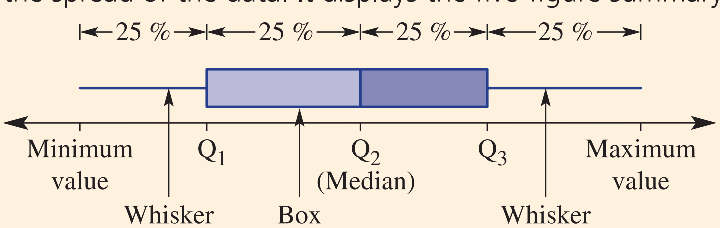
This box plot summarises the average fuel consumption (litres per 100 km) for a group of European-made cars.



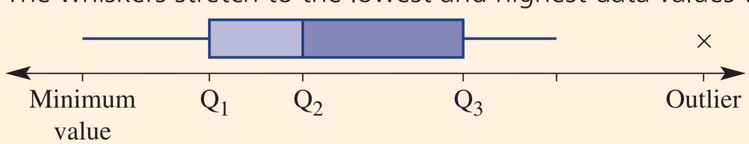
- What does each part of the box plot represent in terms of the five-figure summary?
- What do you think the cross (×) represents?
- Describe how you can use the box plot to find the IQR.
- For the top 25% of cars, what would you expect the fuel consumption to be above?

Key ideas

- A **box plot** (also called a box-and-whisker plot) can be used to summarise a data set and show the spread of the data. It displays the five-figure summary (min, Q_1, Q_2, Q_3, max), as shown.



- An outlier is marked with a cross (×).
  - An outlier is greater than $Q_3 + 1.5 \times \text{IQR}$ (upper fence) or less than $Q_1 - 1.5 \times \text{IQR}$ (lower fence).
  - $\text{IQR} = Q_3 - Q_1$
  - The whiskers stretch to the lowest and highest data values that are not outliers.



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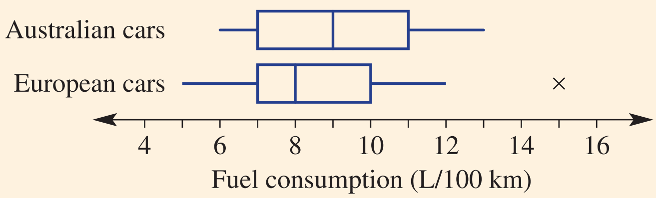
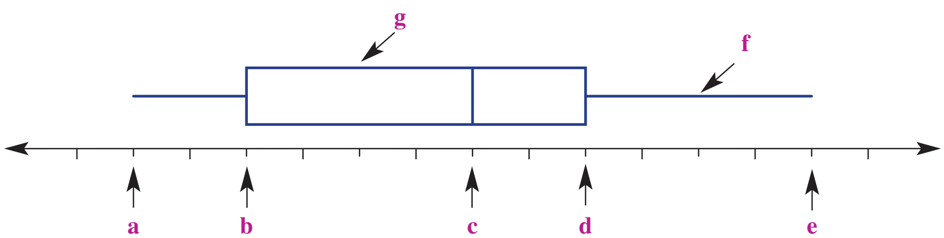
- **Parallel box plots** are box plots drawn on the same scale. They are used to compare data sets within the same context.



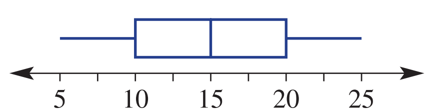
Exercise 5F

Understanding | 1–4 | 4

1 Label the parts **a–g** of the box plot below.



2 For this simple box plot, find the:



a median (Q_2)

b minimum

c maximum

d range

e lower quartile (Q_1)

f upper quartile (Q_3)

g interquartile range (IQR)

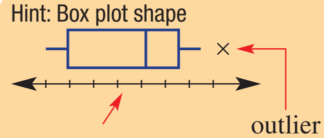
3 Construct a box plot showing these features.

a Min $= 1$, $Q_1 = 3$, $Q_2 = 4$, $Q_3 = 7$, max $= 8$

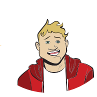
b Outlier $= 5$, minimum above outlier $= 10$,

$Q_1 = 12$, $Q_2 = 14$, $Q_3 = 15$, max $= 17$

Hint: Box plot shape



include an even scale.



4 Select from the list below to fill in the blanks.
minimum, Q_1, Q_2, Q_3, *maximum*.

a The top 25% of data are above _______.

b The middle 50% of data are between _______ and _______.

c The lowest or first 25% of data are between the _______ and _______.

d The highest or last 25% of data are between _______ and the _______.

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Fluency 5–6(½) 5–6(½)

Example 13 Constructing box plots with no outliers

Consider the given data set.

12 26 14 11 15 10 18 17 21 27

a Find the five-figure summary (i.e. the minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the maximum).

b Draw a box plot to summarise the data.

Solution

a 10 11 (12) 14 15 | 17 18 (21) 26 27

min = 10, Q_1 = 12, $Q_2 = \frac{15+17}{2} = 16$, Q_3 = 21, max = 27

Minimum = 10

$Q_1 = 12$, $Q_2 = 16$, $Q_3 = 21$

Maximum = 27

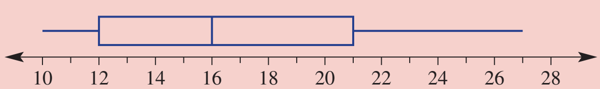
Explanation

Order the data from smallest to largest.

Locate the median (Q_2) first. For 10 data values there are two middle values, 15 and 17. Average these to find Q_2.

Split the data in half at the median. Q_1 is the middle value of the lower half. Q_3 is the middle value of the upper half. The minimum is the smallest value and the maximum the largest value.

b



Draw an even scale, covering the minimum and maximum values.

Mark the minimum (10), Q_1 (12), Q_2 (16), Q_3 (21) and the maximum (27) to draw the box plot.

Now you try

Consider the given data set.

56 32 44 45 60 55 54 38 40 44 54 52

a Find the five-figure summary (i.e. the minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the maximum).

b Draw a box plot to summarise the data.

5 Consider the data sets below.

  i Find the five-figure summary (minimum, Q_1, Q_2, Q_3, maximum).

  ii Draw a box plot to summarise the data.

a 11, 15, 18, 17, 1, 2, 8, 12, 19, 15

b 0, 1, 5, 4, 4, 4, 2, 3, 3, 1, 4, 3

c 124, 118, 119, 117, 120, 120, 121, 118, 122

d 62, 85, 20, 34, 40, 66, 47, 82, 25, 32, 28, 49, 41, 30, 22

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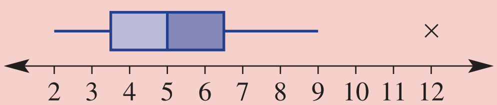
5F

Example 14 Constructing box plots with outliers

Consider the given data set.
5, 9, 4, 3, 5, 6, 6, 5, 7, 12, 2, 3, 5

a Determine the quartiles Q_1, Q_2 and Q_3.
b Determine whether any outliers exist.
c Draw a box plot to summarise the data, marking outliers if they exist.

Solution	Explanation
a 2 3 3 4 5 5) ⑤ (5 6 6 7 9 12 Q_1 (between 3 and 4), Q_2 (the circled 5), Q_3 (between 6 and 7) $Q_1 = \frac{3+4}{2} = 3.5$ $Q_3 = \frac{6+7}{2} = 6.5$	Order the data to help find the quartiles. Locate the median Q_2 (i.e. the middle value), then split the data in half above and below this value. Q_1 is the middle value of the lower half and Q_3 is the middle value of the upper half. Average the two middle values to find the median.
b $\text{IQR} = 6.5 - 3.5 = 3$ $Q_1 - 1.5 \times \text{IQR} = 3.5 - 1.5 \times 3 = -1$ $Q_3 + 1.5 \times \text{IQR} = 6.5 + 1.5 \times 3 = 11$	Determine $\text{IQR} = Q_3 - Q_1$. Check for any outliers; i.e. numbers below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.
$\therefore$ 12 is an outlier.	There are no data below -1 but $12 > 11$.
c 	Draw a line and mark in a uniform scale, reaching from 2 to 12. Sketch the box plot by marking the minimum 2 and the outlier 12, and Q_1, Q_2 and Q_3. The end of the five-point summary is the nearest value below 11; i.e. 9.

Now you try

Consider the given data set.
10, 8, 11, 9, 8, 22, 15, 10, 12

a Determine the quartiles Q_1, Q_2 and Q_3.
b Determine whether any outliers exist.
c Draw a box plot to summarise the data, marking outliers if they exist.

6 Consider the data sets below.
  i Determine the quartiles Q_1, Q_2 and Q_3.
  ii Determine whether any outliers exist.
  iii Draw a box plot to summarise the data, marking outliers if they exist.

a 4, 6, 5, 2, 3, 4, 4, 13, 8, 7, 6
b 1.8, 1.7, 1.8, 1.9, 1.6, 1.8, 2.0, 1.1, 1.4, 1.9, 2.2
c 21, 23, 18, 11, 16, 19, 24, 21, 23, 22, 20, 31, 26, 22
d 37, 48, 52, 51, 51, 42, 48, 47, 39, 41, 65

Hint:
Outliers are more than $Q_3 + 1.5 \times \text{IQR}$
or
less than $Q_1 - 1.5 \times \text{IQR}$.
Mark with a X.
The next value above or below an outlier is used as the new end of the whisker.

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Problem-solving and reasoning | 7, 8 | 7–9

7 A butcher records the weight (in kilograms) of a dozen parcels of sausages sold on one morning.

1.6 1.9 2.0 2.0 2.1 2.2
2.2 2.4 2.5 2.7 3.8 3.9

a Write down the value of:
- **i** the minimum **ii** Q_1
- **iii** Q_2 **iv** Q_3
- **v** the maximum **vi** IQR

b Find any outliers.

c Draw a box plot for the weight of the parcels of sausages.

8 Joel the gardener records the number of days that it takes for 11 special bulbs to germinate. The results are:

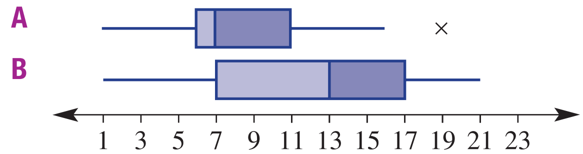
8 14 15 15 16 16 16 17 19 19 24

a Write down the value of:
- **i** the minimum **ii** Q_1 **iii** Q_2
- **iv** Q_3 **v** the maximum **vi** IQR

b Are there any outliers? If so, what are they?

c Draw a box plot for the number of days it takes for the bulbs to germinate.

9 Consider these parallel box plots, A and B.



Hint: Parallel box plots are two box plots that can be compared using the same scale.
Compare the box plots at each point of the five-figure summary.

a What statistical measure do these box plots have in common?

b Which data set (A or B) has a wider range of values?

c Find the IQR for:
- **i** data set A **ii** data set B

d How would you describe the main difference between the two sets of data from which the parallel box plots have been drawn?

Creating parallel box plots | — | 10

10 Fifteen essays are marked for spelling errors by a particular examiner and the following numbers of spelling errors are counted.

3, 2, 4, 6, 8, 4, 6, 7, 6, 1, 7, 12, 7, 3, 8

The same 15 essays are marked for spelling errors by a second examiner and the following numbers of spelling errors are counted.

12, 7, 9, 11, 15, 5, 14, 16, 9, 11, 8, 13, 14, 15, 13

a Draw parallel box plots for the data.

b Do you believe there is a major difference in the way the essays were marked by the two examiners? If yes, describe this difference.

Using technology 5F: Using calculators to draw box plots

This activity can be found in the interactive textbook in the form of a printable PDF.

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Progress quiz

5A

1 Name the type of data that would be generated by the following survey questions.

a What is your favourite sport?

b How many times did you exercise in the past week?

5B

2 Twenty-five people were surveyed as to the number of hours of sleep they had the previous night. The data are listed below.

6	5	8	9	7	10	5	8	8
11	7	9	4	2	9	8	7	
10	8	7	5	7	10	6	9	

a Organise the data into a frequency table using class intervals of 3, starting with 0–, then 3– etc. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c What percentage of people surveyed had fewer than 6 hours sleep?

d Which interval is the most frequent?

5C/D

3 This dot plot shows the number of days a group of students did homework in a week.

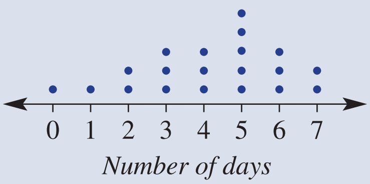
a How many students were surveyed?

b What was the mode number of days of homework?

c What was the median number of days of homework?

d What was the mean number of days of homework?

e What was the range of the number of days of homework?



5C/D

4 Consider the given data set.

7	11	46	42	34	40	24	45	15	22
21	16	49	25	33	30	47	30	3	48

a Organise the data into an ordered stem-and-leaf plot.

b Describe the distribution of the data as symmetrical or skewed.

c Use the stem-and-leaf plot to find:

i the mode **ii** the median

5D

5 For the following data set, find the:

8, 15, 23, 12, 3, 19, 42, 33

a mean **b** median **c** range

5E

6 Consider the given data set.

10 13 8 15 24 18 11 20

a Find the upper quartile (Q_3) and lower quartile (Q_1).

b Determine the IQR.

5F

7 The data below show the number of cars that travel down a particular road between 5:30 p.m. and 6 p.m. each day for a week.

0 82 75 49 102 110 97

a Determine the quartiles Q_1, Q_2 and Q_3.

b Determine whether any outliers exist.

c Draw a box plot to summarise the data.

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5G Time-series data

Learning intentions

- To understand that time-series data is data recorded at regular time intervals
- To be able to plot a time-series graph
- To be able to describe any trends in the data of a time-series graph

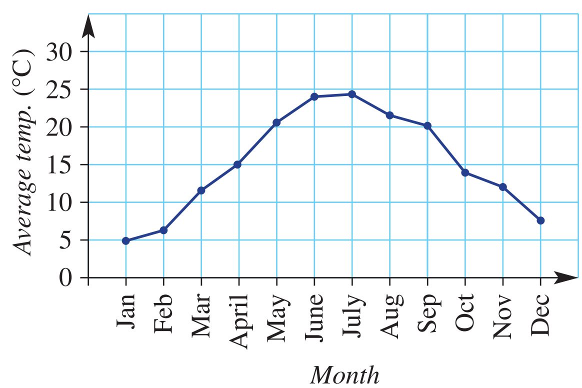
Key vocabulary: time-series data, linear, trend

A time series is a sequence of data values that are recorded at regular time intervals. Examples include temperature recorded on the hour, speed recorded every second, population recorded every year and profit recorded every month. A line graph can be used to represent time-series data. This can help to analyse the data, describe trends and make predictions about the future.

Lesson starter: Changing temperatures

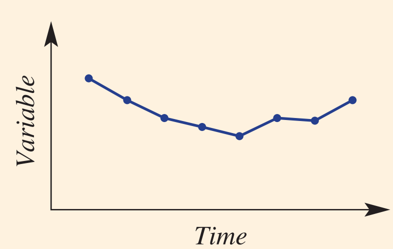
The average monthly maximum temperature for a city is illustrated by this graph.

- Describe the trend in the data at different times of the year.
- Explain why the average maximum temperature for December is close to the average maximum temperature for January.
- Do you think this graph is for an Australian city? Explain.
- If another year of temperatures was included on this graph, what would you expect the shape of the graph to look like?
- Do you think this city is in the Northern Hemisphere or the Southern Hemisphere? Give a reason.

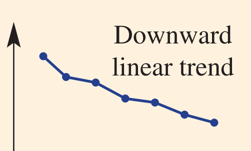


Key ideas

- **Time-series data** are recorded at regular time intervals.
- The graph or plot of a time series uses:
  - time on the horizontal axis
  - line segments connecting points on the graph



- If the time-series plot results in points being on or near a straight line, then we say that the **trend** is **linear**.



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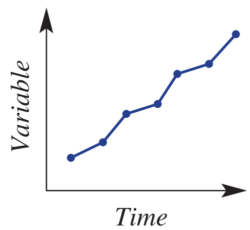
Exercise 5G

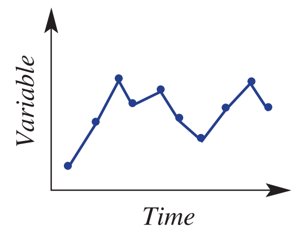
Understanding | 1, 2 | 2

1 Describe the following time-series plots as having a linear (i.e. straight line) trend, non-linear trend (i.e. a smooth curve) or no trend.

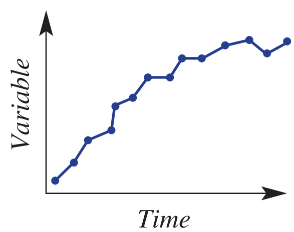
a



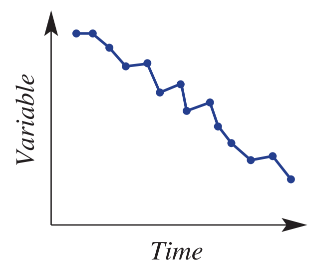
b



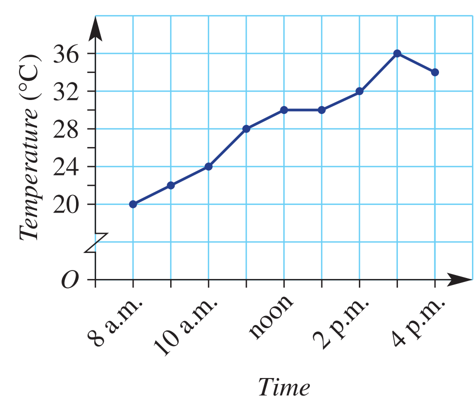
c



d



2 This time-series graph shows the temperature over the course of 8 hours of a day.



- **a** State the temperature at:
  - **i** 8 a.m.
  - **ii** noon
- **b** What was the maximum temperature?
- **c** During what times did the temperature:
  - **i** stay the same?
  - **ii** decrease?
- **d** Describe the general trend in the temperature for the 8 hours.

Fluency | 3, 4 | 3, 4

Example 15 Plotting and interpreting a time-series plot

The approximate population of a small town was recorded from 2000 to 2015.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Population	1300	1250	1250	1150	1000	950	900	950	800	700	800	750	850	950	800	850

a Plot the time-series graph.
b Describe the trend in the data over the 16 years.

Continued on next page

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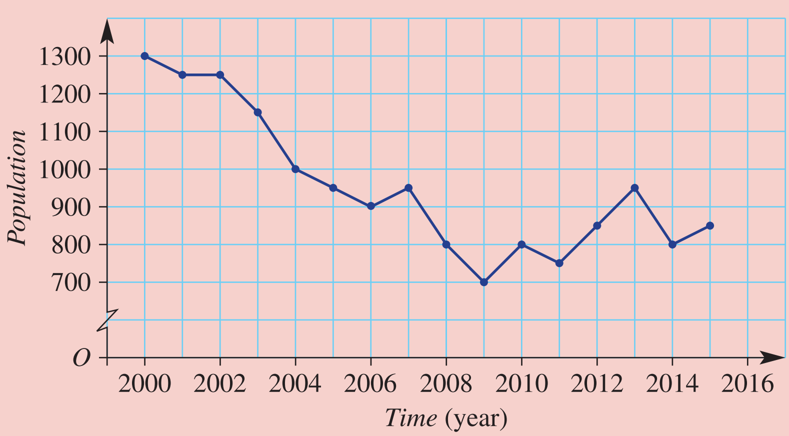
Solution

Explanation

a



Use time on the horizontal axis. Break the y-axis so as to not include 0–700. Label an even scale on each axis. Join points with line segments.

b The population declines steadily for the first 10 years. The population rises and falls in the last 6 years, resulting in a slight upwards trend.

Interpret the overall rise and fall of the lines on the graph.

Now you try

The approximate population of a small town was recorded from 2005 to 2015.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Population	550	500	550	600	700	650	750	750	850	950	900

a Plot the time-series graph. Break the y-axis so it does not include 0–500.

b Describe the general trend in the data over the 11 years.

3 A company's share price over 12 months is recorded in this table.

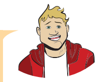
Month	J	F	M	A	M	J	J	A	S	O	N	D
Price ($)	1.30	1.32	1.35	1.34	1.40	1.43	1.40	1.38	1.30	1.25	1.22	1.23

a Plot the time-series graph. Break the y-axis to exclude values from \$0 to \$1.20.

b Describe the way in which the share price has changed over the 12 months.

c What is the difference between the maximum and minimum share price in the 12 months?

Hint: The scale on the vertical axis will need to include from \$1.20 to \$1.43. Choose an appropriate scale. Month will be on the horizontal axis.



4 The pass rate (%) for a particular examination is given in this table.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Pass rate (%)	74	71	73	79	85	84	87	81	84	83

a Plot the time-series graph for the 10 years.

b Describe the way in which the pass rate for the examination has changed in the given time period.

c In what year is the pass rate a maximum?

d By how much has the pass rate improved from 2006 to 2010?

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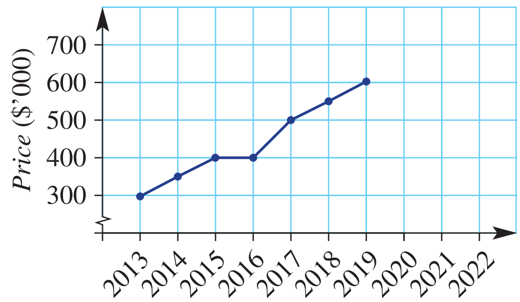
5G

Problem-solving and reasoning 5, 6 5, 7, 8

5 This time-series plot shows the upwards trend of house prices in an Adelaide suburb over 7 years from 2013 to 2019.



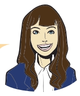
Hint: Recall that a linear trend has the points on or near a straight line.

a Would you say that the general trend in house prices is linear or non-linear?

b Assuming that the trend in house prices continues for this suburb, what would you expect the house price to be in:

i 2020?

ii 2022?



Hint: $'000s means 22 represents $22 000.

6 The following data show the monthly sales of strawberries ($'000s) for a particular year.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ($'000s)	22	14	9	11	12	9	7	9	8	10	18	25

a Plot the time-series graph for the year.

b Describe any trends in the data over the year.

c Give a reason why you think the trends you observed may have occurred.

7 The two top-selling book stores for a company list their sales figures for the first 6 months of the year. Sales amounts are in thousands of dollars.

	July	August	September	October	November	December
City Central ($'000)	12	13	12	10	11	13
Southbank ($'000)	17	19	16	12	13	9

a What is the difference in the sales volume for:

i August?

ii December?

b How many months did the City Central store sell more books than the Southbank store?

c Construct a time-series plot for both stores on the same set of axes.

d Describe the trend of sales for the 6 months for:

i City Central

ii Southbank

Hint: Use different colours for the two line graphs.

e Based on the trend for the sales for the Southbank store, what would you expect the approximate sales volume to be in January?

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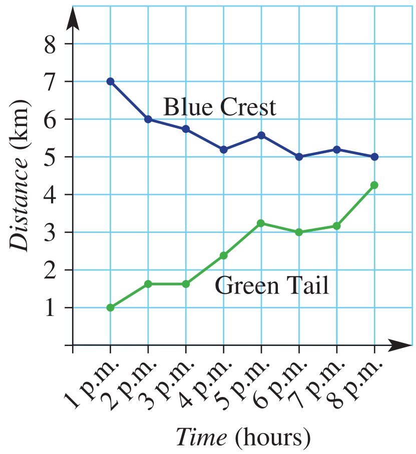
8 Two pigeons (Green Tail and Blue Crest) each have a beacon that communicates with a recording machine. The distance of each pigeon from the machine is recorded every hour for 8 hours.



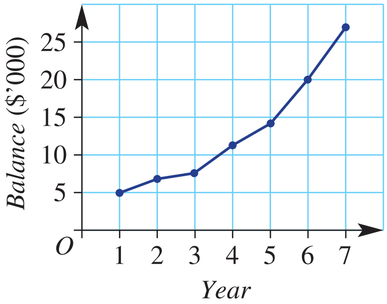
- **a** State the distance from the machine at 3 p.m. of:
  - **i** Blue Crest **ii** Green Tail
- **b** Describe the trend in the distance from the recording machine for:
  - **i** Blue Crest **ii** Green Tail
- **c** Assuming that the given trends continue, predict the time when the pigeons will be the same distance from the recording machine.

Non-linear trends

	—	9, 10

9 The balance of an investment account is shown in this time-series plot.

- **a** Describe the trend in the account balance over the 7 years.
- **b** Give a practical reason for the shape of the curve that models the trend in the graph.



10 A drink at room temperature is placed in a fridge that is at 4°C.

- **a** Sketch a time-series plot that might show the temperature of the drink after it has been placed in the fridge.
- **b** Would the temperature of the drink ever get to 3°C? Why?
- **c** Record the temperature at regular intervals of a drink at room temperature that is placed in a fridge. Plot your results and compare them to your answer in part **a**.

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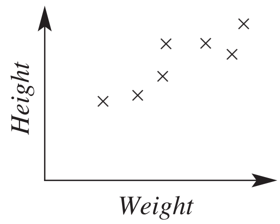
5H Bivariate data and scatter plots

Learning intentions

- To know that bivariate data is data that involves two variables
- To be able to construct a scatterplot for bivariate data
- To be able to describe the correlation of the data in a scatter plot

Key vocabulary: bivariate data, scatter plot, correlation, association, outlier

When we collect information about two variables in a given context we are collecting bivariate data. As there are two variables involved in bivariate data, we use a number plane to graph the data. These graphs are called scatter plots and are used to show a relationship that may exist between the variables. Scatter plots make it very easy to see the strength of the relationship between the two variables.



Lesson starter: A relationship or not?

Consider the two variables in each part below.

- Would you expect there to be some relationship between the two variables in each of these cases?
- If you feel that a relationship exists, would you expect the second-listed variable to increase or to decrease as the first variable increases?

a *Height of person* and *Weight of person*
b *Temperature* and *Life of milk*
c *Length of hair* and *IQ*
d *Depth of topsoil* and *Brand of motorcycle*
e *Years of education* and *Income*
f *Spring rainfall* and *Crop yield*
g *Size of ship* and *Cargo capacity*
h *Fuel economy* and *CD track number*
i *Amount of traffic* and *Travel time*
j *Cost of 2 litres of milk* and *Ability to swim*
k *Background noise* and *Amount of work completed*

Key ideas

- **Bivariate data** is data that involves two variables.
  - The two variables are usually related; for example, height and weight.
- A **scatter plot** is a graph on a number plane in which the axes variables correspond to the two variables from the bivariate data. Points are marked with a cross.
- The words *relationship*, **correlation** and **association** are used to describe the way in which the variables are related.

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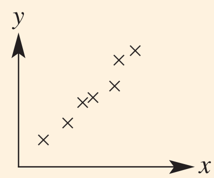
- Types of correlation:
  - The correlation is positive if the y variable generally increases as the x variable increases.
  - The correlation is negative if the y variable generally decreases as the x variable increases.

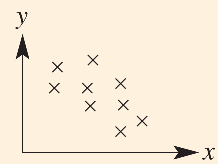
  Examples:

  Strong positive correlation

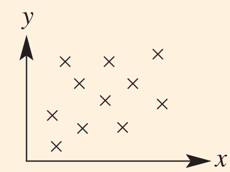
  As x increases, y clearly increases.

  Weak negative correlation

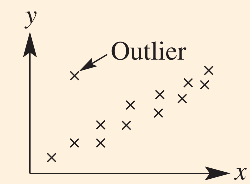
  As x increases, y generally decreases.

  No correlation

  As x increases, there is no particular effect on y.

- An outlier can be clearly identified as a data point that is isolated from the rest of the data.



Exercise 5H

Understanding 1–3 3

1 Decide whether it is likely or unlikely that there will be a strong relationship between these pairs of variables.

a *Height of door* and *Width of door*

b *Weight of car* and *Fuel consumption*

c *Temperature* and *Length of phone calls*

d *Colour of flower* and *Strength of perfume*

e *Amount of rain* and *Size of vegetables in the vegetable garden*

2 Complete the following using the word *increases* or *decreases*.

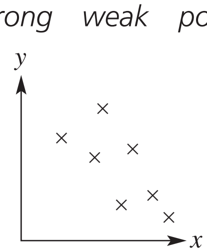
a For a positive correlation the y variable generally ________ as the x variable increases.

b For a negative correlation the y variable generally ________ as the x variable increases.

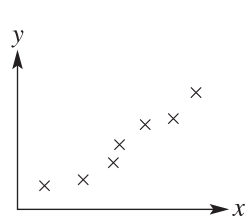
3 For these scatter plots, choose two words from those listed below to best describe the correlation between the two variables.

strong weak positive negative

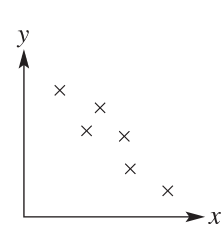
a



b



c



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5H

Fluency 4, 5 4, 6

Example 16 Constructing and interpreting scatter plots

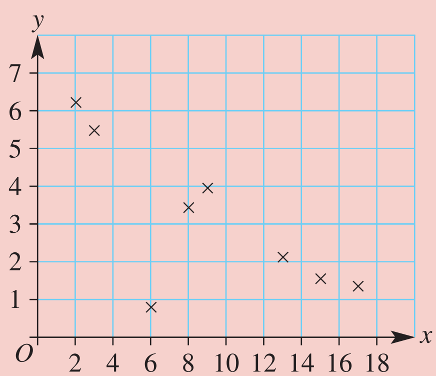
Consider this simple bivariate data set.

x	13	9	2	17	3	6	8	15
y	2.1	4.0	6.2	1.3	5.5	0.9	3.5	1.6

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

Solution

a



Explanation

Draw an appropriate scale on each axis by looking at the data:
- x is up to 17
- y is up to 6.2

The scale must be spread evenly on each axis. Plot each point using a cross symbol on graph paper.

b Negative correlation

Looking at the scatter plot, as x increases y decreases.

c Strong correlation

The downwards trend in the data is clearly defined.

d The outlier is $(6, 0.9)$.

This point defies the trend.

Now you try

Consider this simple bivariate data set.

x	9	6	4	3	8	2	1	10
y	11	8	4	4	3	2	3	12

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

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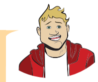

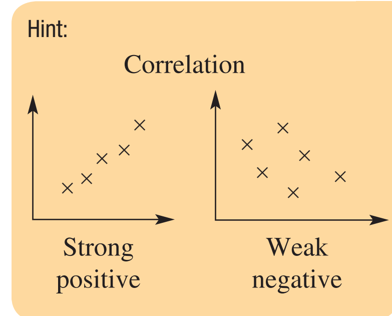

4 Consider this simple bivariate data set.

x	1	2	3	4	5	6	7	8
y	1.0	1.1	1.3	1.3	1.4	1.6	1.8	1.0

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

5 Consider this simple bivariate data set.

x	14	8	7	10	11	15	6	9	10
y	4	2.5	2.5	1.5	1.5	0.5	3	2	2

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

6 By completing scatter plots for each of the following data sets, describe the correlation between x and y as *positive*, *negative* or *none*.

a

x	1.1	1.8	1.2	1.3	1.7	1.9	1.6	1.6	1.4	1.0	1.5
y	22	12	19	15	10	9	14	13	16	23	16

b

x	4	3	1	7	8	10	6	9	5	5
y	115	105	105	135	145	145	125	140	120	130

c

x	28	32	16	19	21	24	27	25	30	18
y	13	25	22	21	16	9	19	25	15	12

Problem-solving and reasoning 7, 8 7, 9, 10

7 A tomato grower experiments with a new organic fertiliser and sets up five separate garden beds: A, B, C, D and E. The grower applies different amounts of fertiliser to each bed and records the diameter of each tomato picked.
The average diameter of a tomato from each garden bed and the corresponding amount of fertiliser are recorded below.

Bed	A	B	C	D	E
Fertiliser (grams per week)	20	25	30	35	40
Average diameter (cm)	6.8	7.4	7.6	6.2	8.5

a Draw a scatter plot for the data with 'Diameter' on the vertical axis and 'Fertiliser' on the horizontal axis. Label the points A, B, C, D and E.
b Which garden bed appears to go against the trend?
c According to the given results, would you be confident saying that the amount of fertiliser fed to tomato plants does affect the size of the tomato produced?

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5H

8 For common motor vehicles, consider the two variables *Engine size* (cylinder volume) and *Fuel economy* (number of kilometres travelled for every litre of petrol).

a Do you expect there to be some relationship between these two variables?

b As the engine size increases, would you expect the fuel economy to increase or decrease?

c The following data were collected for 10 vehicles.

Car	A	B	C	D	E	F	G	H	I	J
Engine size	1.1	1.2	1.2	1.5	1.5	1.8	2.4	3.3	4.2	5.0
Fuel economy	21	18	19	18	17	16	15	20	14	11

i Do the data generally support your answers to parts **a** and **b** above?

ii Which car gives a fuel economy reading that does not support the general trend?

9 On 14 consecutive days a local council measures the volume of sound heard from a freeway at various points in a local suburb. The volume (V) of sound, in decibels, is recorded against the distance (d), in metres, between the freeway and the point in the suburb.

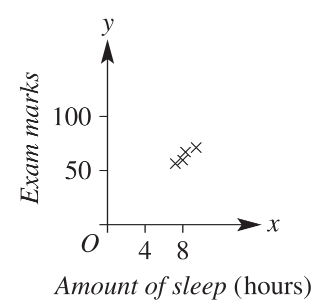
d	200	350	500	150	1000	850	200	450	750	250	300	1500	700	1250
V	4.3	3.7	2.9	4.5	2.1	2.3	4.4	3.3	2.8	4.1	3.6	1.7	3.0	2.2

a Draw a scatter plot of V against d, plotting V on the vertical axis and d on the horizontal axis.

b Describe the correlation between d and V as positive, negative or none.

c Generally, as d increases, does V increase or decrease?

10 A person presents you with this scatter plot and suggests to you that there is a strong correlation between the amount of sleep and exam marks. What do you suggest is the problem with the person's graph and conclusions?



Crime rates and police — 11

11 A government department is interested in convincing the electorate that a large number of police on patrol leads to lower crime rates. Two separate surveys are completed over a 1-week period and the results are listed in the table below.

	Area	A	B	C	D	E	F	G
Survey 1	Number of police	15	21	8	14	19	31	17
	Incidence of crime	28	16	36	24	24	19	21
Survey 2	Number of police	12	18	9	12	14	26	21
	Incidence of crime	26	25	20	24	22	23	19

a Using scatter plots, determine whether or not there is a relationship between the number of police on patrol and the incidence of crime, using the data in:

i survey 1 **ii** survey 2

b Which survey results do you think the government will use to make its point? Why?

Hint: *Number of police* will be on the horizontal axis.

Using technology 5H: Using calculators to draw scatter plots

This activity is available on the companion website as a printable PDF.

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5I Line of best fit by eye

Learning intentions

- To know that a straight line can be fitted by eye to bivariate data with a positive or negative linear correlation
- To know how to fit a line of best fit by eye
- To be able use a line of best fit to find unknown points using both interpolation and extrapolation

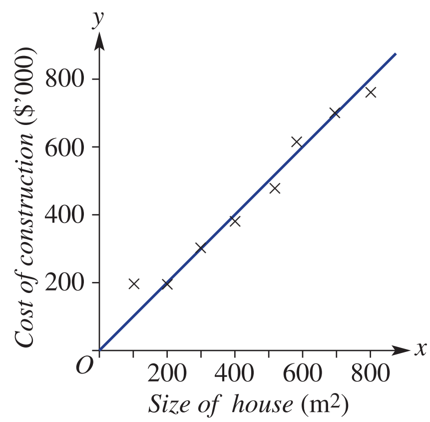
Key vocabulary: bivariate data, line of best fit (trend line), interpolation, extrapolation, linear, correlation

When bivariate data have a strong linear correlation, we can model the data with a straight line. This line is called a trend line or line of best fit. When we fit the line 'by eye', we try to balance the number of data points above the line with the number of points below the line. This trend line can then be used to construct other data points inside and outside the existing data points.

Lesson starter: Size versus cost

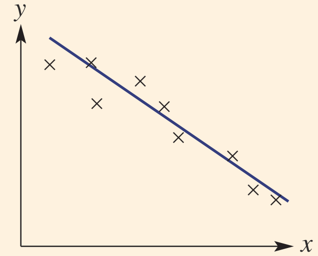
This scatter plot shows the estimated cost of building a house of a given size by a building company. A trend line has been added to the scatter plot.

- Why is it appropriate to fit a trend line to this data?
- Do you think the trend line is a good fit to the points on the scatter plot? Why?
- How can you predict the cost of a house of 1000 m^2 with this building company?



Key ideas

- For bivariate data showing a clearly defined positive or negative correlation, a straight line can be fitted by eye.
- A **line of best fit** (or trend line) is positioned by eye by balancing the number of points above the line with the number of points below the line.
  - The distance of each point from the trend line also needs to be taken into account.
  - Outliers should be ignored.
- The line of best fit can be used for:
  - **interpolation:** finding unknown points within the given data range
  - **extrapolation:** finding unknown points outside the given data range



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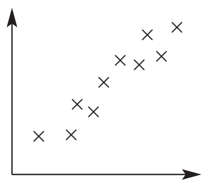
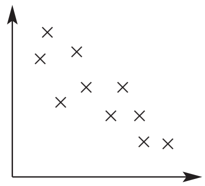
Exercise 5I

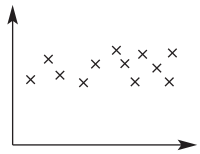
Understanding | 1–3 | 2(½), 3

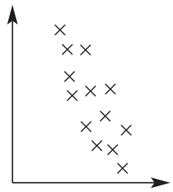
1 **a** When is it suitable to add a line of best fit to a scatterplot?
b Describe the general guideline for placing a line of best fit.

2 Practise fitting a line of best fit on these scatter plots by trying to balance the number of points above the line with the number of points below the line. (Using a pencil might help.)

a


b


c


d


3 For the graph, with the given line of best fit shown, choose from *interpolation* or *extrapolation* to complete the following.

a Estimating the y value when $x = 6$ is an example of ______________________.
b Estimating the y value when $x = 10$ is an example of ______________________.

Fluency | 4–6 | 4, 6, 7

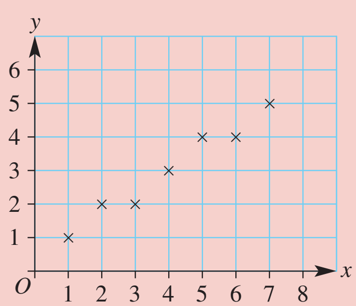
Example 17 Fitting a line of best fit

Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	1	2	2	3	4	4	5

a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
i y when $x = 3.5$ **ii** y when $x = 0$
iii x when $y = 1.5$ **iv** x when $y = 5.5$

Solution | **Explanation**

a


Plot the points on graph paper.

Continued on next page

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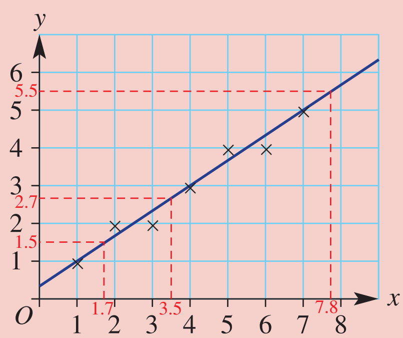
b Positive correlation

As x increases, y increases.

c



Since a relationship exists, draw a line on the plot, keeping as many points above as below the line. (There are no outliers in this case.)

d **i** $y \approx 2.7$
ii $y \approx 0.4$
iii $x \approx 1.7$
iv $x \approx 7.8$

Start at $x = 3.5$. Draw a vertical line to the line of best fit, then draw a horizontal line to the y-axis and read off your solution.
Extend vertical and horizontal lines from the values given and read off your solution.
As they are approximations, we use the ≈ sign and not the = sign.

Now you try

Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6
y	10	7	6	6	5	3

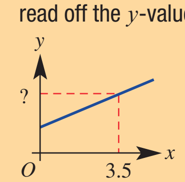
a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
i y when $x = 4.5$ **ii** y when $x = 0$
iii x when $y = 7.5$ **iv** x when $y = 1.5$

4 Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	2	2	3	4	4	5	5

a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
i y when $x = 3.5$ **ii** y when $x = 0$ **iii** x when $y = 2$ **iv** x when $y = 5.5$

Hint: Locate $x = 3.5$ and read off the y-value for part **di**.



5 Consider the variables x and y and the corresponding data below.

x	1	2	4	5	7	8	10	12
y	20	16	17	16	14	13	9	10

a Draw a scatter plot for the data.
b Is there a positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
i y when $x = 7.5$ **ii** y when $x = 0$ **iii** x when $y = 12$ **iv** x when $y = 15$

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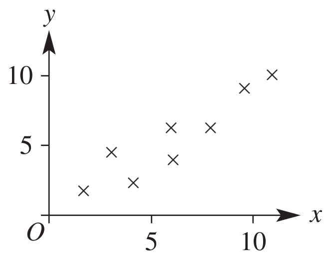
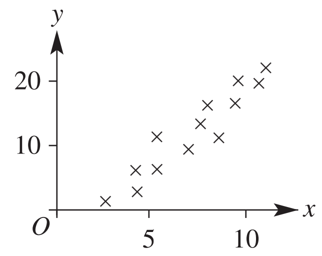
6 For the following scatter plots, pencil in a line of best fit by eye, and then use your line to estimate the value of y when $x = 5$.

a



b



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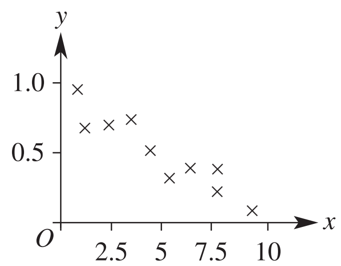
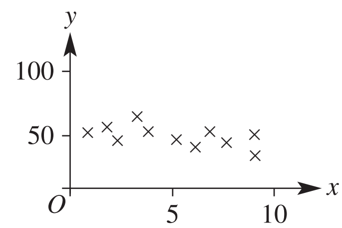
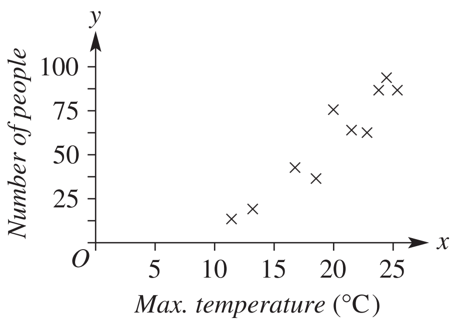


d



7 This chart shows data for the *number of people* entering a suburban park and the corresponding *maximum temperature* for 10 spring days.



a Generally, as the maximum daily temperature increases, does the number of people who enter the park increase or decrease?

b Draw a line of best fit by eye on the given chart.

c Use your line of best fit to estimate:

i the number of people expected to enter the park if the maximum daily temperature is 20°C

ii the maximum daily temperature when the total number of people who visit the park on a particular day is 25

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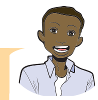
Problem-solving and reasoning | 8, 9 | 8, 10

8 A small bookshop records its profit and number of customers for the past 8 days.

Number of customers	6	12	15	9	8	5	8
Profit ($)	200	450	550	300	350	250	300



Hint: For 17 customers, you will need to extend your line beyond the data. This is called extrapolation.

a Draw a scatter plot for the data, using profit on the vertical axis.
b Fit a line of best fit by eye.
c Use your line of best fit to predict the profit for 17 customers.
d Use your line of best fit to predict the number of customers for a $100 profit.

9 Over eight consecutive years, a city nursery has measured the growth of an outdoor bamboo species for that year. The annual rainfall in the area where the bamboo is growing was also recorded. The data are listed in the table.

Rainfall (mm)	450	620	560	830	680	650	720	540
Growth (cm)	25	45	25	85	50	55	50	20

a Draw a scatter plot for the data, showing growth on the vertical axis.
b Fit a line of best fit by eye.
c Use your line of best fit to estimate the growth expected for the following rainfall readings.
  i 500 mm **ii** 900 mm
d Use your line of best fit to estimate the rainfall for a given year if the growth of the bamboo is:
  i 30 cm **ii** 60 cm

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5I

10 At a suburban sports club, the distance record for the hammer throw has increased over time. The first recorded value was 72.3 m in 1967. The most recent record was 118.2 m in 1996. Further details are in this table.

Year	1967	1968	1969	1976	1978	1983	1987	1996
New record (m)	72.3	73.4	82.7	94.2	99.1	101.2	111.6	118.2

a Draw a scatter plot for the data.

b Fit a line of best fit by eye.

c Use your line of best fit to estimate the distance record for the hammer throw for:
i 2000 **ii** 2015

d Would you say that it is realistic to use your line of best fit to estimate distance records beyond 2015? Why?

Heart rate and age — 11

11 Two independent scientific experiments confirmed a correlation between *Maximum heart rate* and *Age*. The data for the two experiments are in this table.

Experiment 1													
Age (years)	15	18	22	25	30	34	35	40	40	52	60	65	71
Max. heart rate (bpm)	190	200	195	195	180	185	170	165	165	150	125	128	105
Experiment 2													
Age (years)	20	20	21	26	27	32	35	41	43	49	50	58	82
Max. heart rate (bpm)	205	195	180	185	175	160	160	145	150	150	135	140	90

a Sketch separate scatter plots for experiment 1 and experiment 2, with age on the horizontal axis.

b By fitting a line of best fit by eye to your scatter plots, estimate the maximum heart rate for a person aged 55 years, using the results from:
i experiment 1 **ii** experiment 2

c Estimate the age of a person who has a maximum heart rate of 190 bpm, using the results from:
i experiment 1 **ii** experiment 2

d For a person aged 25 years, which experiment estimates a lower maximum heart rate?

e Research the average maximum heart rate of people by age and compare with the results given above.

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Maths@Work: Project manager on a building site

Project managers can be responsible for coordinating a building site. They need to be organised, must be able to read and interpret plans, manage staff, coordinate the supply and costs of labour and equipment, as well as solve problems when unexpected situations arise on the worksite.

Project managers also need to evaluate their staff. Performance reviews are often used to evaluate staff performance. In some companies, bonuses and other incentives are linked to these reviews.

The performance review below is from a worksite, and used by the project manager to review their staff.

Use the performance review data in the table on the next page to answer the following questions that a project manager may face in their day-to-day job.

1 Consider trainee A.
- **a** Calculate their mean score on this review.
- **b** What is their median score?
- **c** Write down the five-figure summary from the data given in this review.

Hint: Don't forget to order the data before finding the median.

2 Consider trainee B.
- **a** Calculate their mean score on this review.
- **b** What is their median score?
- **c** Write down the five-figure summary from the data given in this review.

Hint: The five-figure summary includes min, Q_1, Q_2, Q_3 and max.

3 Display the data for all three trainees in parallel box plots on the same scale.

4 Compare the competency of trainees A, B and C by listing the following for *each* trainee. Then decide who is the most competent trainee, giving reasons.
- **a** mean
- **b** median
- **c** lowest score
- **d** highest score
- **e** interquartile range
- **f** range

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Performance review

Competency - please score 0 to 10	Trainee A	Trainee B	Trainee C
1. Please score the individual's punctuality when arriving to work.	9	8	8
2. Please score the individual's attitude when performing required tasks.	8.5	9	6.5
3. Once the individual has completed a task, do they ask for additional work?	8	8	5
4. How would you score the individual's willingness to learn?	9.5	8	6
5. How would you score the individual's application to competencies or requirements, as taught by the site team?	8	7	6
6. Please score the individual's accuracy when performing required tasks.	8.5	7.5	6.5
7. Please score the individual's skills in email, faxes and letter writing.	7	8	8.5
8. Please score the individual's skills in drafting site instructions.	7.5	7	5
9. Please score the individual's skills in drafting or issuing RFIs.	8	6.5	5
10. Please score the individual's respect earned within the site/department team for their required role.	8	7.5	5.5
11. Please score the individual's respect earned with subcontractors for their required role.	7.5	7	5.5
12. Please score the individual's respect earned with consultants for their required role.	8	7	5
13. How would you score the individual's application/focus to the required tasks you have assigned?	8.5	8	5
14. Do you consider that the individual has integrated well with the site/department team?	9	8	4.5
15. Do you find that the individual has improved their skills while under your supervision?	8	7.5	5.5
16. Would you take the individual to your next project to continue their competency training in other aspects of construction? Yes - score 10 Maybe - score 5 No - score 0	10	5	0
17. How would you score the individual's progression through their required tasks?	9	8.5	3

Using technology

5 The following data gives the competency scores (C) for trainee D.

C 1	C 2	C 3	C 4	C 5	C 6	C 7	C 8	C 9
9	7	7	7	6	7	7	6	6
C 10	C 11	C 12	C 13	C 14	C 15	C 16	C 17	
7	8	6	7	9	8	5	8	

a Using a CAS calculator, enter the data for trainees B and D and draw parallel box plots. Note that an activity ('Using calculators to draw box plots') showing how to enter data and draw box plots for both the TI-Nspire and Class Pad is available in the interactive textbook.

b For trainees B and D, write down the five-figure summaries from the box plots.

c Compare the IQRs for trainee B and D. Which IQR shows more consistent competency? Give reasons for your answer.

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Puzzles and games

1 The mean mass of six boys is 71 kg. The mean mass of five girls is 60 kg. Find the mean mass of all 11 children together.

2 Sean has a current four-topic average of 78% for Mathematics. What score does he need in the fifth topic to have an overall average of 80%?

3 I am a data set made up of five whole number values. My mode is 2 and both my mean and median are 5. What is my biggest possible range?

4 A single data set has 3 added to every value. Describe the change in the:
a mean
b median
c range

5 Find the interquartile range for a set of data if 75% of the data is above 2.6 and 25% of the data is above 3.7.

6 I am a data set with four whole number values.
- I have a range of 8.
- I have a mode of 3.
- I have a median of 6.

What are my four values?

7 A single-ordered data set includes the following data.
$2, 4, 5, 6, 8, 10, x$
What is the largest possible value of x if it is not an outlier?

8 Describe what happens to the mean, median and mode of a data set if each value in the set is:
a increased by 10
b multiplied by 10

9 At the end of 2019, the average rainfall in a town for the ten-year period just ended was 546 mm. A year later, the ten-year average was 562 mm and 654 mm had fallen in 2020.
What was the rainfall in mm in 2010?

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Chapter summary

Statistics

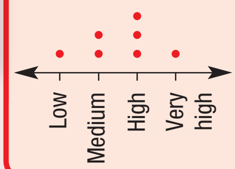
Data

Categorical
- Nominal (red, blue, ...)
- Ordinal (low, medium, ...)

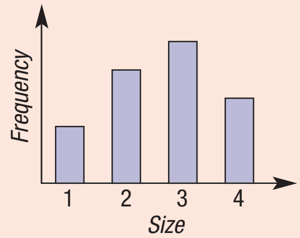
Numerical
- Discrete (1, 2, 3, ...)
- Continuous (0.31, 0.481, ...)

Graphs for a single set of categorical or discrete data

Dot plot



Column graphs



Stem-and-leaf plot

Stem	Leaf
0	1 6
1	2 7 9
2	3 8
3	4

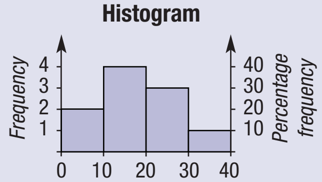
2 | 3 means 23

Frequency tables

Class interval	Frequency	Percentage frequency
0–	2	20%
10–	4	40%
20–	3	30%
30–40	1	10%
Total	10	100%

$$\text{Percentage frequency} = \frac{\text{frequency}}{\text{total}} \times 100$$

Histogram



Measures of centre

- Mean $(\bar{x}) = \dfrac{\text{sum of all values}}{\text{number of scores}}$
- Median (Q_2) = middle value of ordered data

odd number: 1 4 (6) 9 12 — median is 6

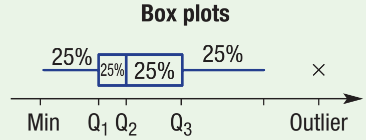
even number: 1 2 4 6 7 11 — $\text{Median} = \dfrac{4+6}{2} = 5$

- Mode = most common value
- A data set with two modes is bimodal.

Measures of spread

- Range = max − min
- Interquartile range (IQR) $= Q_3 - Q_1$

Box plots



Quartiles

Q_1: above 25% of the data (middle value of lower half)

Q_3: above 75% of the data (middle value of upper half)

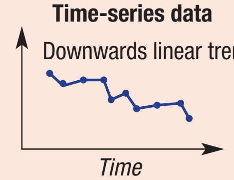
2 3 5 7 8 | 9 11 12 14 15

$Q_1 = 5$ $Q_2 = 8.5$ $Q_3 = 12$

1 2 3) 6 (7 10 13

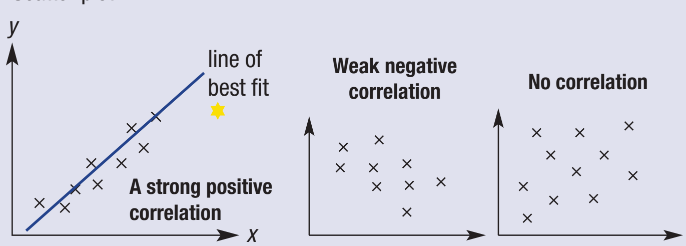
$Q_1 = 2$ Q_2 $Q_3 = 10$

Time-series data



Bivariate data

- Two related variables
- Scatter plot



Outliers

- Single data set
  - less than $Q_1 - 1.5 \times \text{IQR}$ or more than $Q_3 + 1.5 \times \text{IQR}$
- Bivariate
  - not in the vicinity of the rest of the data

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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

5A

1 I can describe the type of data generated by a survey question.
e.g. What type of data is generated by the survey questions, ‘What is your favourite fruit?’ and ‘How many mobile phones are there in your household?’

5B

2 I can construct a frequency table and column graph.
e.g. Twenty students were surveyed on their favourite school subject. The results were:

Maths	PE	Science	Maths	Language
Maths	History	English	PE	History
Science	English	History	Language	Science
Maths	PE	Maths	History	Maths

Construct a frequency table including the tally and frequency and construct a column graph for the data.

5B

3 I can work with a histogram.
e.g. A sample of 25 people leaving a movie are asked their age. The data are listed:

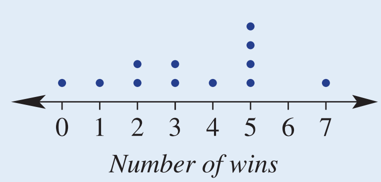
16	19	22	23	42	44	36	41	28
26	28	34	52	55	46	20	22	
34	35	48	46	50	29	26	35	

Organise the data into a frequency table, using class intervals of 10 and include a percentage frequency column. Construct a histogram showing both the frequency and percentage frequency and determine the percentage of people surveyed who were 30 years of age or older.

5C

4 I can interpret a dot plot.
e.g. This dot plot shows the number of wins by a tennis player in each of the grand slams they entered over 3 years.

a How many grand slams did they play in?
b What was the most common number of wins they had and the total number of wins in the 3 years?
c Describe the data in the dot plot.



5C

5 I can construct a stem-and-leaf plot or back-to-back stem-and-leaf plot.
e.g. Two sausage sizzles at two hardware stores sell the following number of sausages each week for a 10-week period.

Store 1

32	44	28	36	52
56	31	45	42	47

Store 2

27	35	32	43	52
34	29	24	38	22

Construct an ordered back-to-back stem-and-leaf plot and describe the distribution of each store’s sausage sales.

5D

6 I can find the mean, mode and range.
e.g. For the data set: $12, 7, 11, 14, 3, 7$, find the:
a mode **b** mean **c** range

5D

7 I can find the median.
e.g. Find the median for the following data sets.
a $18, 14, 25, 28, 7, 11, 15$
b $14, 19, 25, 16, 8, 1, 10, 30$

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Chapter checklist

		✔

5D

8 I can calculate summary statistics from a graphical display.
e.g. For the data in this stem-and-leaf plot, find the range, mode, median and mean.

Stem	Leaf
1	2 5
2	1 3 4 7
3	0 0 5
4	2

2 | 3 means 23

5E

9 I can find the upper and lower quartiles and the interquartile range of a data set.
e.g. For the data sets listed below, find the upper quartile (Q_3) and the lower quartile (Q_1) and hence the IQR.
a 4, 8, 12, 18, 16, 20, 24, 19
b 10.2, 11.4, 12.6, 9.8, 10.0, 15.6, 18

5E

10 I can find the five-figure summary and outliers.
e.g. The following data set represents the price of 10 different refrigerators in a department store.

\$620	\$540	\$642	\$457	\$585
\$877	\$1599	\$918	\$724	\$840

For the data set, find:
a the five-figure summary **b** the IQR
c any outliers and give a possible reason why they are outliers

5F

11 I can construct a box plot with and without outliers.
e.g. For the data sets below, find the five-figure summary and whether any outliers exist. Draw a box plot to summarise the data, including outliers if they exist.
a 15 20 21 16 15 22 18 23 25 20
b 2.9 3.5 2.6 4.1 3.2 2.4 1.8 3.1 4.4 9.2 5.6 3.0 2.8

5G

12 I can plot and interpret a time-series graph.
e.g. The approximate rainfall (in mm) for the 12 months of a year was recorded for Brisbane.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	110	125	115	130	95	70	60	40	30	80	95	105

Plot the time-series graph and describe the trend in the data over the 12 months.

5H

13 I can construct and interpret a scatter plot.
e.g. For the bivariate data set below, draw a scatter plot of the data, identify any outliers and describe the correlation between x and y choosing from the words: *positive*, *negative*, *weak* or *strong*.

x	0.5	1.1	0.9	1.6	1.2	1.8	0.1	2.5	2.0
y	8	7	6	6	7	9	10	1	3

5I

14 I can fit a line of best fit by eye.
e.g. Consider the bivariate data in the table.

x	1	2	4	5	8	10	12	15
y	2	4	4	6	10	9	12	16

Draw a scatter plot for the data and fit a line of best fit by eye. Use the line of best fit to estimate:
a y when $x = 6$
b x when $y = 18$

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Short-answer questions

5B **1** A group of 16 people was surveyed to find the number of hours of television they watch in a week. The raw data are listed:
$6, 5, 11, 13, 24, 8, 1, 12, 7, 6, 14, 10, 9, 16, 8, 3$
- **a** Organise the data into a table with class intervals of 5. Start at 0−, 5− etc. Include a tally, frequency and percentage frequency column.
- **b** Construct a histogram for the data, showing both the frequency and percentage frequency on the graph.
- **c** Would you describe the data as symmetrical or skewed?

5C **2** A basketball team scores the following points per match for a season.
$20, 19, 24, 37, 42, 34, 38, 49, 28, 15, 38, 32, 50, 29$
- **a** Construct an ordered stem-and-leaf plot for the data.
- **b** Describe the distribution of scores.

5D **3** For the following sets of data, determine:
i the mean **ii** the range **iii** the median
- **a** $2, 7, 4, 8, 3, 6, 5$
- **b** $10, 55, 67, 24, 11, 16$
- **c** $1.7, 1.2, 1.4, 1.6, 2.4, 1.3$

5D **4** Thirteen adults compare their ages at a party. They are:
$40, 41, 37, 32, 48, 43, 32, 76, 29, 33, 26, 38, 87$
- **a** Find the mean age of the adults, to one decimal place.
- **b** Find the median age of the adults.
- **c** Why do you think the mean age is larger than the median age?

5E **5** Determine Q_1, Q_2 and Q_3 for these sets of data.
- **a** $4, 5, 8, 10, 10, 11, 12, 14, 15, 17, 21$
- **b** $14, 6, 2, 23, 11, 6, 15, 14, 12, 18, 16, 10$

5E **6** For the data set: $3, 7, 2, 10, 6, 21, 5, 9, 6, 2, 8, 10$.
- **a** Find the range and IQR of the data set
- **b** Find any outlier in the data set
- **c** Remove the outlier from the data set and find the new range and IQR. What do you notice?

5F **7** For each set of data below, complete the following tasks.
- **i** Find the lower quartile (Q_1) and the upper quartile (Q_3).
- **ii** Find the interquartile range ($IQR = Q_3 - Q_1$).
- **iii** Locate any outliers.
- **iv** Draw a box plot.

- **a** $2, 2, 3, 3, 3, 4, 5, 6, 12$
- **b** $11, 12, 15, 15, 17, 18, 20, 21, 24, 27, 28$
- **c** $2.4, 0.7, 2.1, 2.8, 2.3, 2.6, 2.6, 1.9, 3.1, 2.2$

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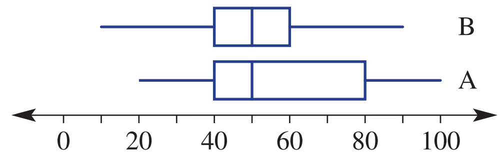
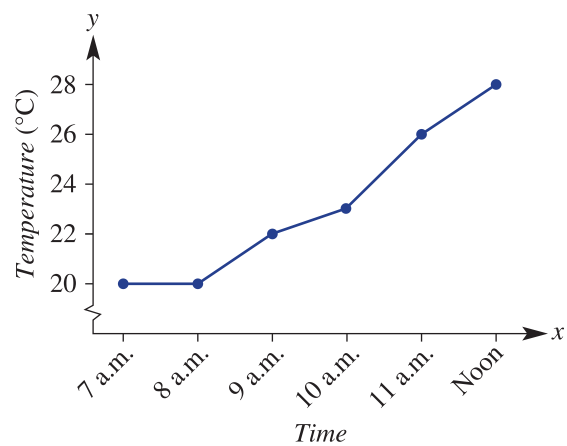
5F **8** Compare these parallel box plots, A and B, and answer the following as true or false.



a The range for A is greater than the range for B.
b The median for A is equal to the median for B.
c The interquartile range is smaller for B.
d 75% of the data for A sits below 80.

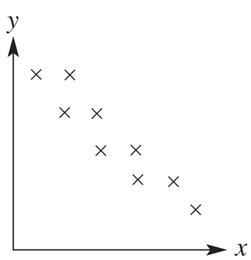
5G **9** This time series plot shows six temperature readings taken over 5 hours.



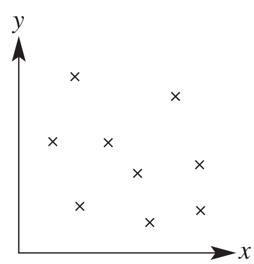
a Would you describe the trend as linear or non-linear?
b During which hour does the largest temperature increase occur?

5H **10** For the scatter plots below, describe the correlation between x and y as positive, negative or none.

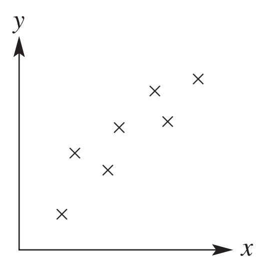
a



b



c

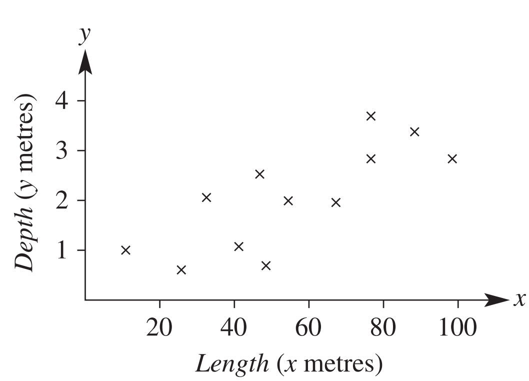


5H **11** Consider the simple bivariate data set.

x	1	4	3	2	1	4	3	2	5	5
y	24	15	16	20	22	11	5	17	6	8

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

5I ★ **12** The given scatter plot shows the maximum *length* (x metres) and *depth* (y metres) of 11 public pools around town.



a Draw a line of best fit by eye.
b Use your line to estimate the maximum depth of a pool that is 50 m in length.

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Multiple-choice questions

5A **1** The number of pets in 10 households are recorded after a survey is conducted. The type of data recorded could be described as:

A categorical and discrete
B categorical and ordinal
C categorical and nominal
D numerical and discrete
E numerical and continuous

Questions **2** and **3** refer to the stem-and-leaf plot below, at right.

Stem	Leaf
2	4 9
3	1 1 7 8
4	2 4 6
5	0 4

4 | 2 means 42

5C **2** The minimum score in the data is:

A 4
B 0
C 24
D 38
E 54

5C **3** The mode is:

A 3
B 31
C 4
D 38
E 30

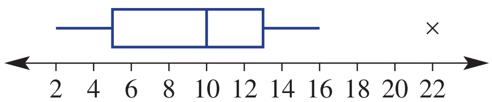
5D **4** The range and mean of $2, 4, 3, 5, 10$ and 6 are:

A range $= 8$, mean $= 5$
B range $= 4$, mean $= 5$
C range $= 8$, mean $= 4$
D range $= 2 - 10$, mean $= 6$
E range $= 8$, mean $= 6$

5D **5** The median of $29, 12, 18, 26, 15$ and 22 is:

A 18 **B** 22 **C** 20 **D** 17 **E** 26

Questions **6–8** refer to the box plot below.



5E/F **6** The interquartile range (IQR) is:

A 8 **B** 5 **C** 3
D 20 **E** 14

5F **7** The outlier is:

A 2 **B** 0 **C** 20 **D** 16 **E** 22

5F **8** The median is:

A 2 **B** 3 **C** 10 **D** 13 **E** 16

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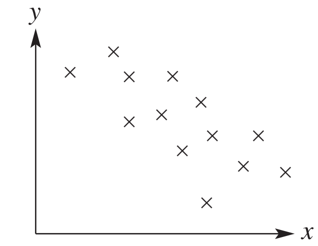
5H

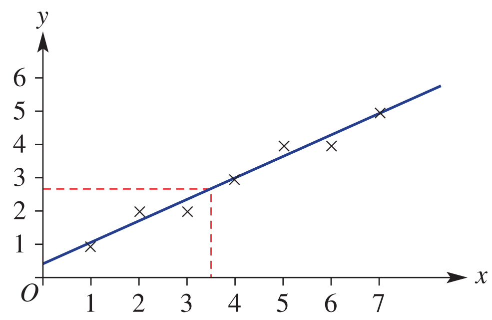
9 The variables x and y in this scatter plot could be described as having:



A no correlation
B strong positive correlation
C strong negative correlation
D weak negative correlation
E weak positive correlation

5I

10 According to this scatter plot, when x is 3.5, y is approximately:



A 4.4 **B** 2.7
C 2.5 **D** 3.5
E 5

Extended-response questions

1 The number of flying foxes taking refuge in a fig tree is recorded over a period of 14 days. The data collected is given here.

Number of flying foxes	73	50	36	82	15	24	73	57	65	86	51	32	21	39

i Find the IQR.
ii Identify any outliers.
iii Draw a box plot for the data.

2 A newsagency records the *number of customers* and *profit* for 14 working days.

Number of customers	18	13	15	24	29	12	18	16	15	11	4	32	26	21
Profit ($)	150	70	100	210	240	90	130	110	120	80	30	240	200	190

a Draw a scatter plot for the data and draw a line of best fit by eye. Place *Number of customers* on the horizontal axis.
b Use your line of best fit to predict the profit for:
 i 10 customers **ii** 20 customers **iii** 30 customers
c Use your line of best fit to predict the number of customers for a:
 i $50 profit **ii** $105 profit **iii** $220 profit

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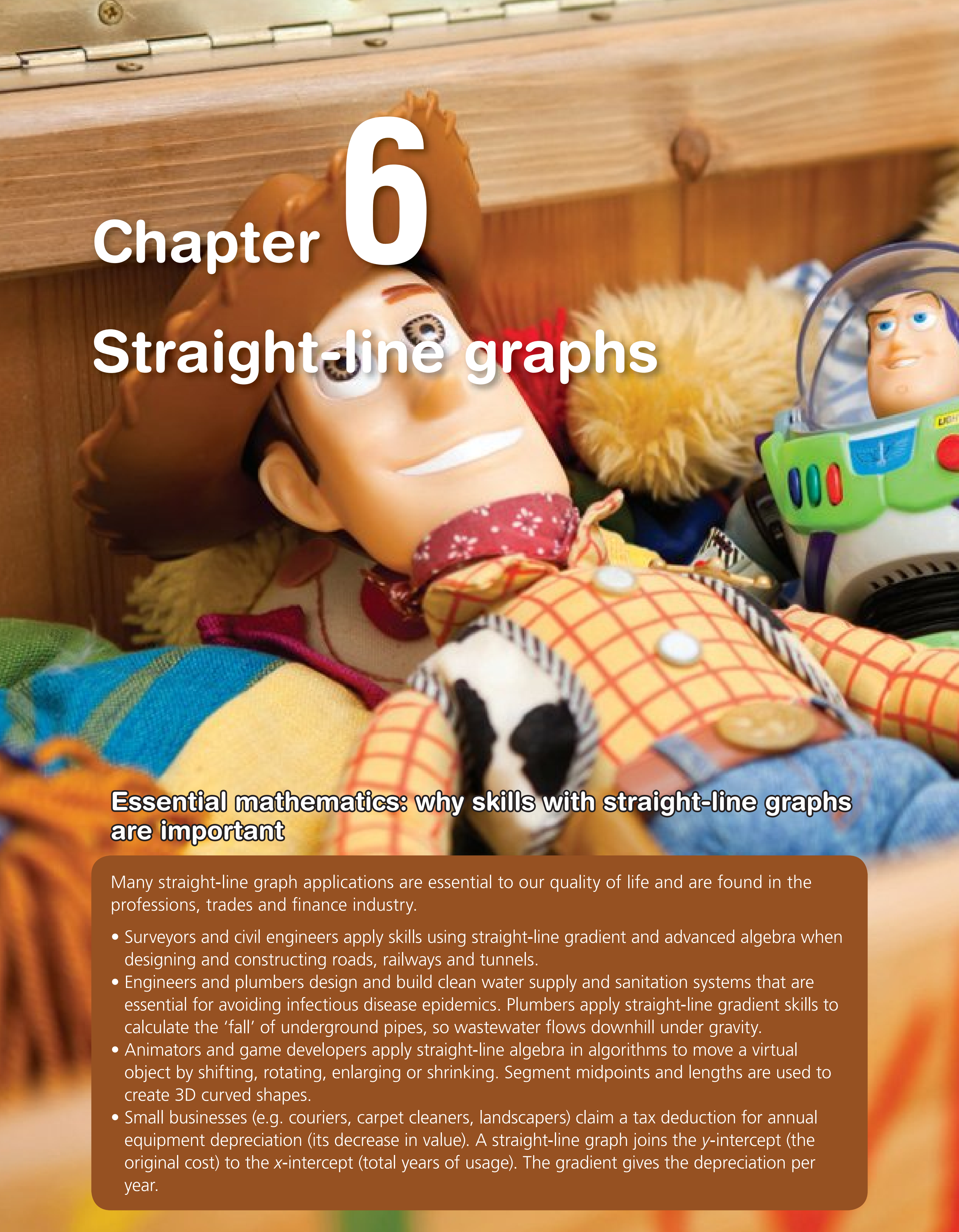


Chapter 6

Straight-line graphs

Essential mathematics: why skills with straight-line graphs are important

Many straight-line graph applications are essential to our quality of life and are found in the professions, trades and finance industry.

- Surveyors and civil engineers apply skills using straight-line gradient and advanced algebra when designing and constructing roads, railways and tunnels.
- Engineers and plumbers design and build clean water supply and sanitation systems that are essential for avoiding infectious disease epidemics. Plumbers apply straight-line gradient skills to calculate the 'fall' of underground pipes, so wastewater flows downhill under gravity.
- Animators and game developers apply straight-line algebra in algorithms to move a virtual object by shifting, rotating, enlarging or shrinking. Segment midpoints and lengths are used to create 3D curved shapes.
- Small businesses (e.g. couriers, carpet cleaners, landscapers) claim a tax deduction for annual equipment depreciation (its decrease in value). A straight-line graph joins the y-intercept (the original cost) to the x-intercept (total years of usage). The gradient gives the depreciation per year.

In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (VCMNA335)

Solve problems involving parallel and perpendicular lines (VCMNA338)

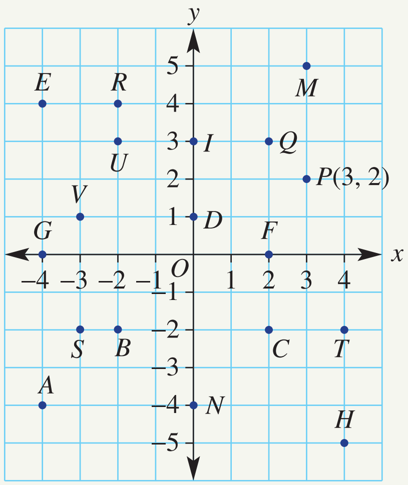
Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 The coordinates of P on this graph are (3, 2). Write down the coordinates of:

a M **b** T **c** A
d V **e** C **f** F



2 Name the point with coordinates:

A $(-4, 0)$ **B** $(0, 1)$ **C** $(-2, -2)$
D $(-3, -2)$ **E** $(0, -4)$ **F** $(2, 3)$

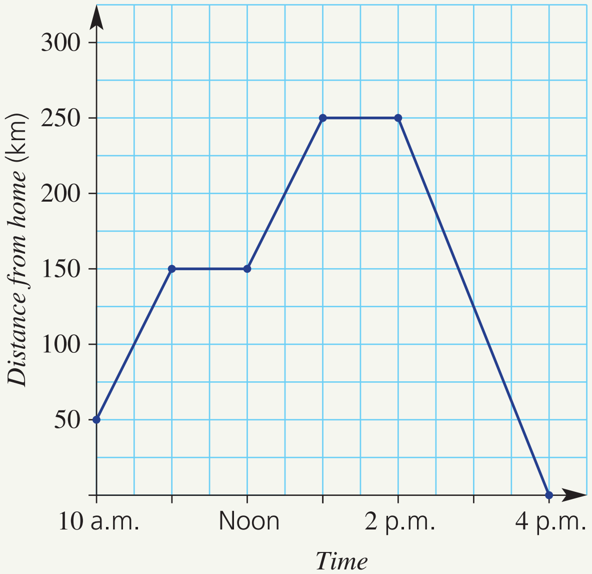
3 Draw up a four-quadrant number plane and plot the following points.
What shape do they form? (Choose from: *hexagon*, *rectangle*, *square*, *pentagon*, *equilateral triangle*, *isosceles triangle*.)

a $(0, 0), (0, 5), (5, 5), (5, 0)$
b $(-3, -1), (-3, 1), (4, 0)$
c $(-2, 3), (-4, 1), (-2, -3), (2, -3), (4, 1), (2, 3)$

4 Find the mean (i.e. average) of the following pairs.

a 10 and 12 **b** 15 and 23 **c** 6 and 14 **d** 3 and 4
e −6 and 6 **f** −3 and 1 **g** 0 and 7 **h** −8 and −10

5 a For how many minutes did the Heart family stop on their trip, according to their journey shown in this travel graph?



b How far had the Heart family travelled by 1 p.m., after starting at 10 a.m.?
c What was their speed in the first hour of travel?

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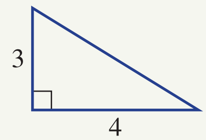
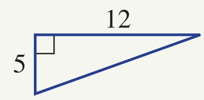
6 Find the length of the hypotenuse in each right-angled triangle. Use $a^2 + b^2 = c^2$. Round your answers to two decimal places in parts **e** and **f**.

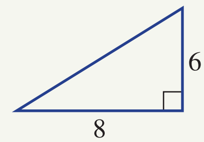
a



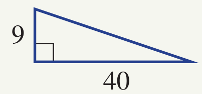
b



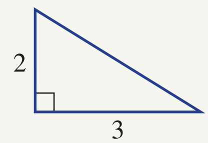
c



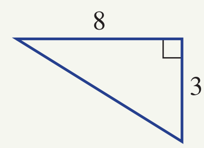
d



e



f



7 Copy and complete the table of values for each rule given.

a $y = x + 3$

x	0	1	2	3
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

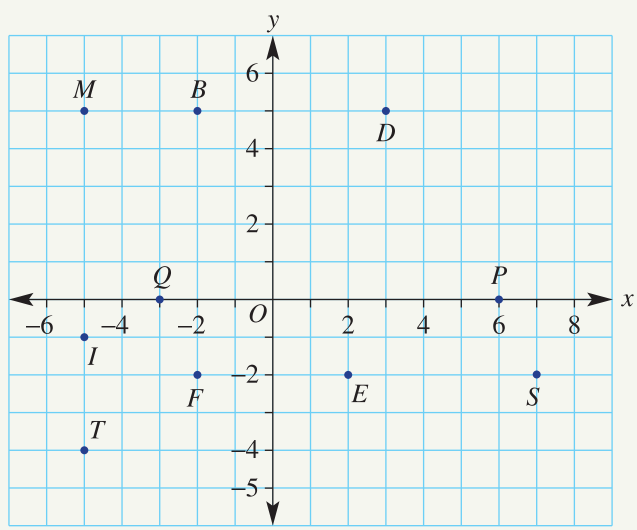
x	0	1	2
y			

d $y = 4 - x$

x	−2	−1	0
y			

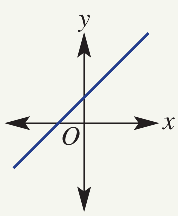
8 Use the graph to find the following distances.

a OP
b QP
c MB
d FS
e BD
f TM

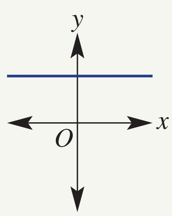


9 Select a word (i.e. *positive*, *negative*, *zero or undefined*) to describe the gradient (slope) of the following lines.

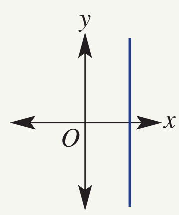
a



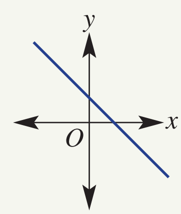
b



c



d



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6A Interpretation of straight-line graphs CONSOLIDATING

Learning intentions

- To understand that two variables with a linear relationship can be represented with a straight-line graph
- To be able to interpret information from a graph using the given variables
- To be able to read off values from a graph both within the graph and using an extended graph

Key vocabulary: variable, linear relationship, interpolation, extrapolation

When two variables are related, we can use mathematical rules to describe the relationship. The simplest kind of relationship forms a straight-line graph and the rule is called a linear equation.

Information can be easily read from within a linear graph – this is called interpolation. A straight line also can be extended to determine information outside of the original data – this is called extrapolation.

For example, if a swimming pool is filled at 1000 L per hour, the relationship between volume and time is linear because the volume is increasing at the constant rate of 1000 L/h.

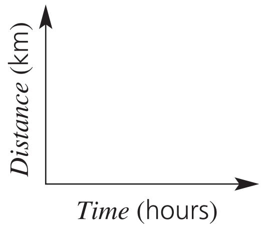
$$\text{Volume} = 1000 \times \text{number of hours}$$

This rule is a linear equation and the graph of volume versus time will be a straight line.

Lesson starter: Graphing a straight line

Jozef is an athlete who trains by running 24 km in 2 hours at a constant rate. Draw a straight-line graph to show this linear relation.

- Draw axes with time (up to 2 hours) on the horizontal axis and distance (up to 24 km) on the vertical axis.



- How far has Jozef run at zero hours? Mark this point on your graph.
- Mark the point on the graph that shows the end of Jozef's run.
- Join these two points with a straight line.
- Mark the point on the graph that shows Jozef's position after half an hour. How far had he run?
- Mark the point on the graph that shows Jozef's position after 18 km. For how long had Jozef been running?
- Name the variables shown on the graph.
- Discuss some advantages of showing information on a graph.

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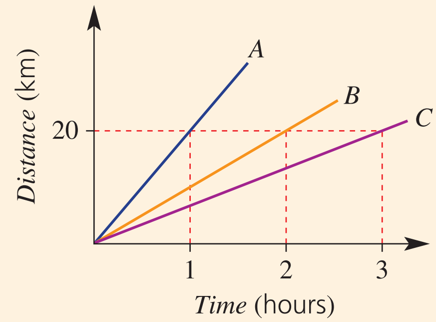
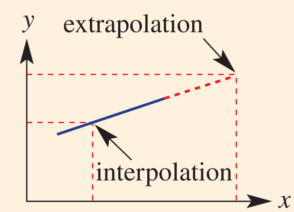
Key ideas

- A **variable** is an unknown that can take on many different values.
- When two variables have a **linear relationship** they can be represented as a straight-line graph.
  Information about one of the variables based on information about the other variable is easily determined by reading from the graph:
  - A: 20 km in 1 hour
  - B: 20 km in 2 hours
  - C: 20 km in 3 hours



- Information can be found from:
  - reading within a graph (**interpolation**) or
  - reading off an extended graph (**extrapolation**)



Exercise 6A

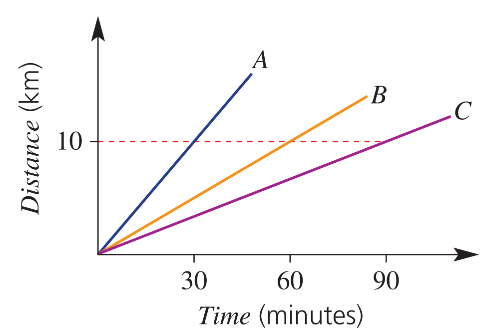
Understanding | 1, 2 | 2

1 For the graph shown, determine how long it takes each cyclist to travel 10 km.

a cyclist A

b cyclist B

c cyclist C



2 Choose the correct word: *interpolation* or *extrapolation*, to complete the following.

a ______________ is reading information from within a graph

b ______________ is reading information from an extended graph

Fluency | 3–5 | 3–5

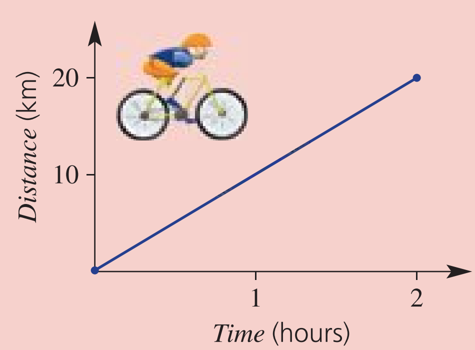
Example 1 Reading information from a graph

This graph shows the journey of a cyclist from one place (A) to another (B).

a How far did the cyclist travel?

b How long did it take the cyclist to complete the journey?

c If the cyclist were to ride from A to B and then halfway back to A, how far would the journey be?



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6A

Solution	Explanation
a 20 km	Draw an imaginary line from the end point (B) to the vertical axis; i.e. 20 km.
b 2 hours	Draw an imaginary line from the end point (B) to the horizontal axis; i.e. 2 hours.
c $20 + 10 = 30$ km	Cyclist would ride 20 km out and 10 km back.

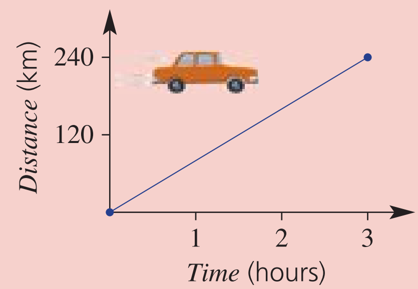
Now you try

This graph shows a car journey from one place (A) to another (B).

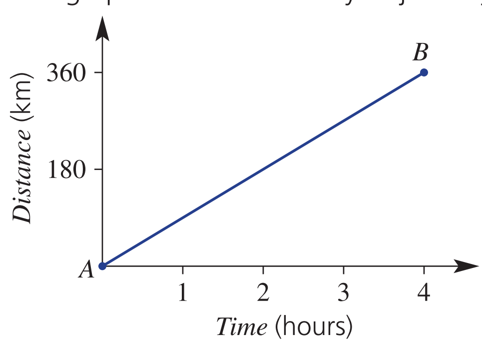
a How far did the car travel?

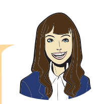
b How long did it take to complete the journey?

c If the car were to be driven from A to B, then halfway back to A, how far would the journey be?



3 This graph shows a motorcycle journey from one place (A) to another (B).





Hint: To find the total time taken to go from A to B, look on the time scale that is aligned with the right end point.

a How far did the motorcycle travel?

b How long did it take to complete the journey?

c If the motorcycle were to be driven from A to B, then halfway back to A, how far would the journey be?

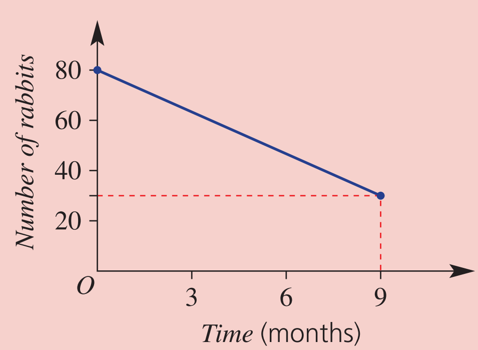
Example 2 Interpreting information from a graph

The number of rabbits in a colony has decreased according to this graph.

a How many rabbits were there in the colony to begin with?

b How many rabbits were there after 9 months?

c How many rabbits disappeared from the colony during the 9-month period?



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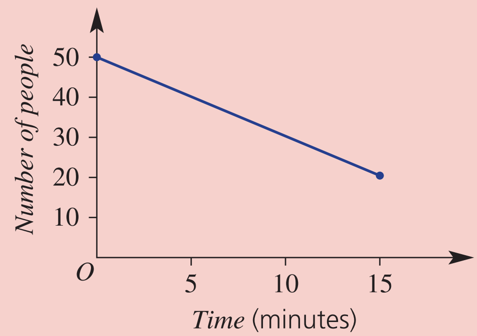
Solution	**Explanation**
a 80 rabbits	At $t = 0$ there were 80 rabbits.
b 30 rabbits	Read the number of rabbits from the graph at $t = 9$.
c $80 - 30 = 50$ rabbits	There were 80 rabbits at the start and 30 after 9 months.

Now you try

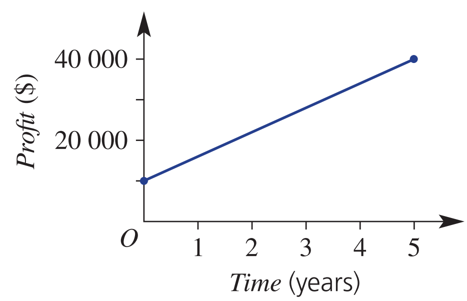
The number of people in a queue has decreased according to this graph.

a How many people were in the queue to begin with?
b How many people were in the queue after 15 minutes?
c How many people left the queue during the 15 minutes shown?



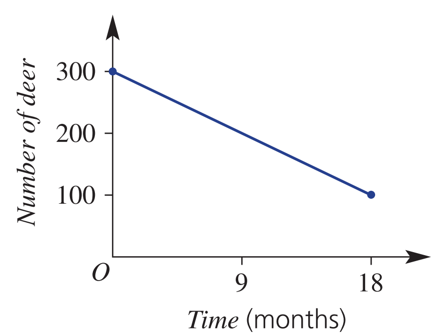
4 This graph shows a company's profit result over a 5-year period.

a What is the company's profit at:
  i the beginning of the 5-year period?
  ii the end of the 5-year period?
b Has the profit increased or decreased over the 5-year period?
c How much has the profit increased over the 5 years?



5 The number of deer in a particular forest has decreased over recent months according to the graph shown.

a How many deer were there to begin with?
b How many deer were there after 18 months?
c How many deer disappeared from the colony during the 18-month period?



Hint: 'To begin with' means time = 0.

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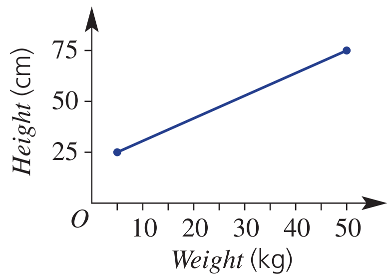
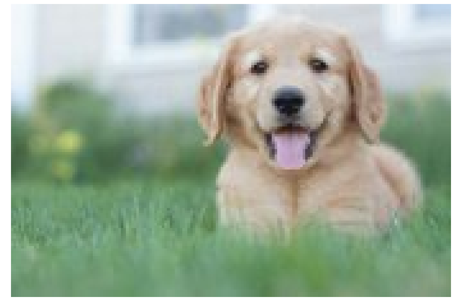
6A

Problem-solving and reasoning 6–9 7, 9, 10

6 A height versus weight graph for a golden retriever dog breed is shown.





a From the smallest to the largest dog, use the graph to find the total increase in:
  i height **ii** weight

b Fill in the missing numbers.
  i The largest weight is ___ times the smallest weight.
  ii The largest height is ___ times the smallest height.

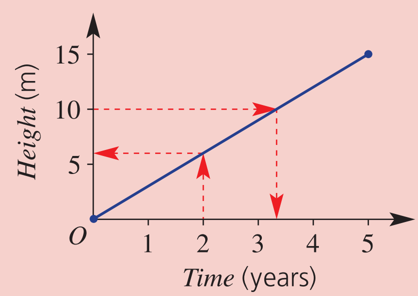
Example 3 Reading within a graph (interpolation)

This graph shows the growth of a tree over 5 years.

a How many metres has the tree grown over the 5 years?

b Use the graph to find how tall the tree is after 2 years.

c Use the graph to find how long it took for the tree to grow to 10 metres.



Solution	Explanation
a 15 metres	The end point of the graph is at 15 metres.
b 6 metres	Draw a dotted line at 2 years and read the height.
c 3.3 years	Draw a dotted line at 10 metres and read the time.

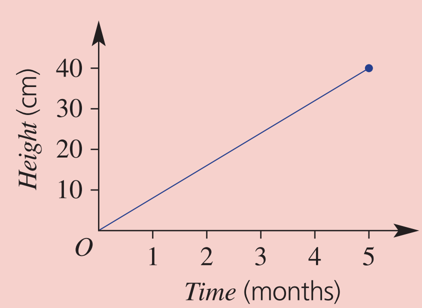
Now you try

This graph shows the growth of a seedling over 5 months.

a How many cm has the seed grown over the 5 months?

b Use the graph to find the height of the seedling after 2 months.

c Use the graph to find how long it took for the seedling to grow to 30 cm.



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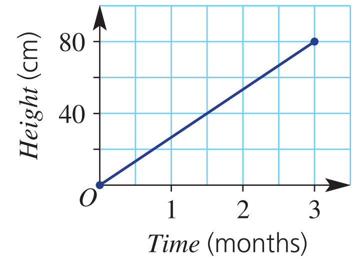
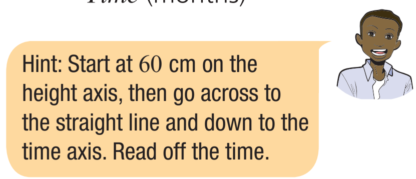
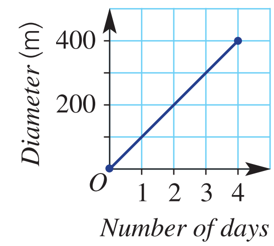
7 This graph shows the height of a tomato plant over 3 months.
  a How many centimetres has the plant grown over 3 months?
  b Use the graph to find how tall the tomato plant is after $1\frac{1}{2}$ months.
  c Use the graph to find how long it took for the plant to grow to 60 centimetres.



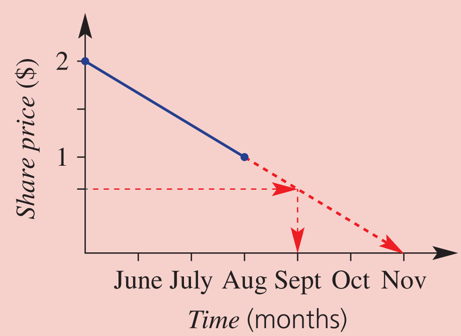


8 The diameter of an oil slick increased every day after an oil tanker hit some rocks. Use the graph to answer the following.
  a How wide is the oil slick after 4 days?
  b How wide is the oil slick after 2.5 days?
  c How many days does it take for the oil slick to reach a diameter of 350 m?



Example 4 Reading off an extended graph (extrapolation)

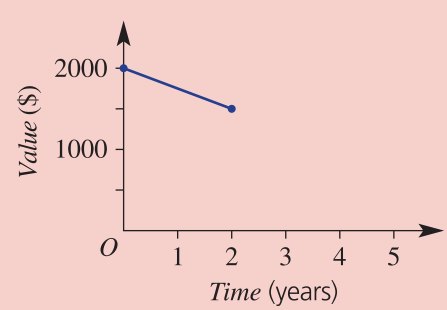
Due to poor performance, the value of a company's share price is falling.
a By the end of August, how much has the share price fallen?
b At the end of November what would you estimate the share price to be?
c Near the end of which month would you estimate the share price to be 70 cents?



Solution	Explanation
a Price has dropped by $1.	By August the price has changed from $2 to $1.
b $0	Use a ruler to extend your graph (shown by the dotted line) and read the share price for November.
c September	Move across from 70 cents to the extended line and read the month.

Now you try

The value of a computer is decreasing over time.
a By the end of 2 years, how much has the value fallen?
b At the end of 4 years, what would the estimated value be?
c Near the end of which year would you estimate the value to be $1200?



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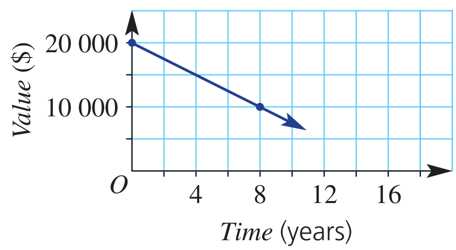
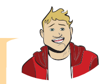
6A

9 The value of a car decreases with time.



Hint: Use your ruler on the line to 'extend' it.

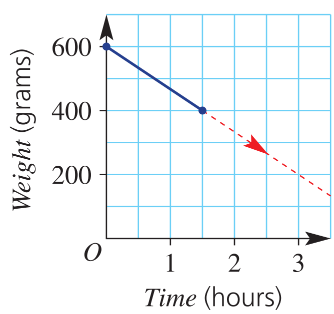


a By the end of 8 years, how much has the car's value fallen?

b At the end of 16 years, what would you estimate the car's value to be?

c Near the end of which year would you estimate the car's value to be $5000?

10 The weight of a wet sponge is reduced after it is left in the sun to dry.



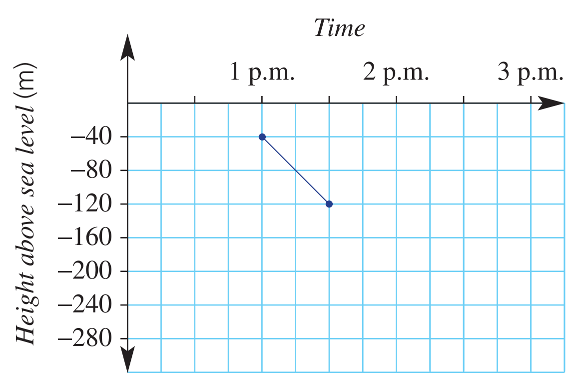
a The weight of the sponge has been reduced by how many grams over the first 1.5 hours?

b What would you estimate the weight of the sponge to be after 3 hours?

c How many hours would it take for the sponge to weigh 300 g?

Submarine depth — 11

11 A submarine goes to depths below sea level, as shown in this graph.



a How long does it take for the submarine to drop from 40 m to 120 m below sea level?

b At what time of day is the submarine at:

i −40 m? **ii** −80 m?

iii −60 m? **iv** −120 m?

c What is the submarine's depth at:

i 1:30 p.m.?

ii 1:15 p.m.?

d Extend the graph to find the submarine's depth at:

i 12:45 p.m. **ii** 1:45 p.m. **iii** 2:30 p.m.

e Use your extended graph to estimate the time when the submarine is at:

i 0 m **ii** −200 m **iii** −320 m

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6B Distance–time graphs

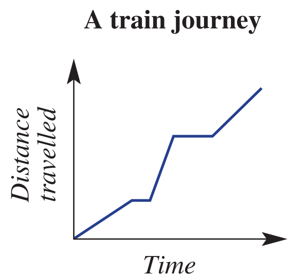
Learning intentions

- To understand that a distance–time graph can be made up of line segments joined together
- To be able to interpret the features of a distance–time graph
- To be able to sketch a distance–time graph with movement at a constant rate

Key vocabulary: line segment, stationary, horizontal axis, vertical axis

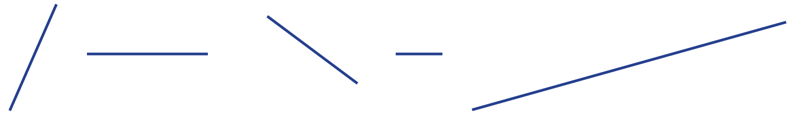
Some of the graphs considered in the previous section were distance–time graphs, which show the distance on the vertical axis and the time on the horizontal axis. Many important features of a journey can be displayed on such graphs. Each section of a journey that is at a constant rate of movement can be graphed with a straight-line segment. Several different line segments can make up a total journey.

For example, a train journey could be graphed with a series of sloping line segments that show travel between stations and flat line segments that show when the train is stopped at a station.



Lesson starter: An imaginary journey

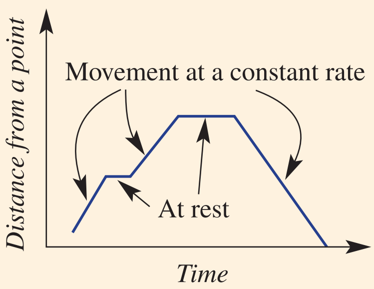
Here are five line segments.



- Use five similar line segments, arranged in any order you choose, and draw a distance–time graph. Each segment must be joined to the one next to it.
- Write a summary of the journey shown by your distance–time graph.
- Swap graphs with a classmate and explain the journey that you think your classmate's graph is showing.

Key ideas

- Graphs of *distance* versus *time* usually consist of **line segments**.
- Each segment shows whether the object is moving or at rest (**stationary**).
- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.



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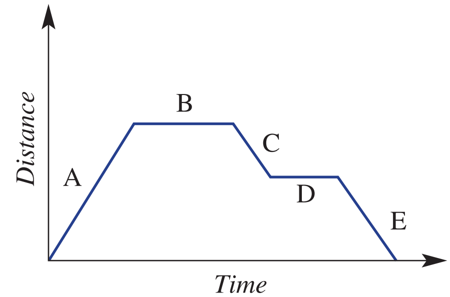
6B

Exercise 6B

Understanding | 1, 2 | 2

1 For the distance versus time graph shown, for each line segment A – E, describe the movement of the object as either moving at a constant rate or at rest.

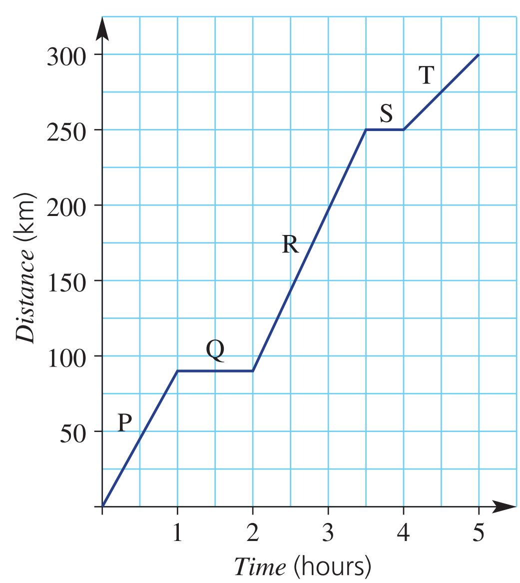


2 The Martino family makes a 300-km car journey, which takes 5 hours. The distance–time graph of this journey is shown. For each description below, choose the line segment of the graph that matches it.

Hint: A flat line segment shows that the car has stopped.

- **a** A half-hour rest break is taken after travelling 250 km.
- **b** In the first hour, the car travels 90 km.
- **c** The car is at rest for 1 hour, 90 km from the start.
- **d** The car takes 1.5 hours to travel from 90 km to 250 km.
- **e** The distance from 250 km to 350 km takes 1 hour.
- **f** The distance travelled stays constant at 250 km for half an hour.

Distance–time graph of car journey

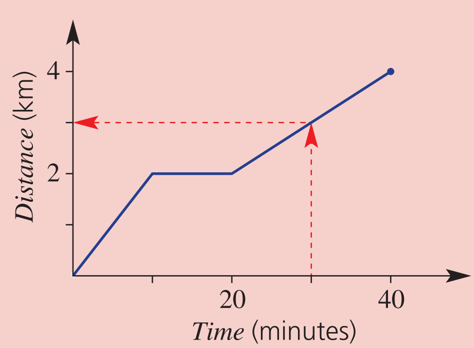


Fluency | 3–5 | 3, 5, 6

Example 5 Interpreting a distance–time graph

This distance–time graph shows a car's journey from home, to school and then to the local shopping centre.

- **a** What is the total distance travelled?
- **b** How long is the car resting while at the school?
- **c** What is the total distance travelled after 30 minutes?



Continued on next page

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Solution	Explanation
a 4 km	Read the distance from the end point of the graph.
b 10 minutes	The rest starts at 10 minutes and finishes at 20 minutes.
c 3 km	Draw a line from 30 minutes and read off the distance.

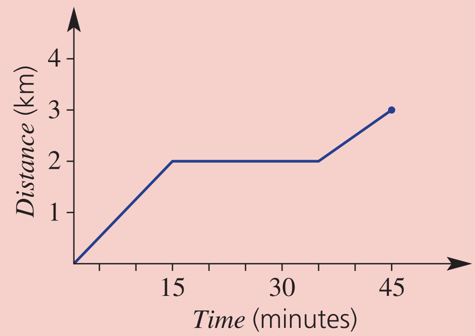
Now you try

The distance–time graph shows a person's walk from their house to the cafe for a rest and then to the post office.

a What is the total distance travelled?

b How long is the person resting at the cafe?

c What is the total distance travelled after 40 minutes?

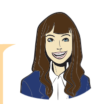


3 A bicycle journey is shown on this distance–time graph.

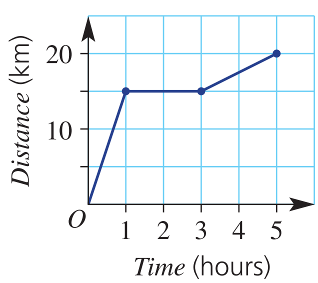
a What is the total distance travelled?

b How long is the cyclist at rest?

c How far has the cyclist travelled after 4 hours?



Hint: From the end of the line, go across to the distance scale. This will show the total distance travelled.



4 The weight of a water container increases while water is poured into it from a tap.

a What is the total weight of the container after:

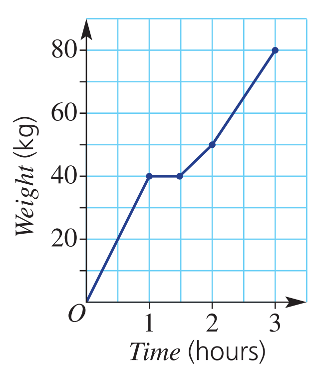
i 1 hour?

ii 2 hours?

iii 3 hours?

b During the 3 hours, how long is the container not actually being filled with water?

c During which hour is the container filling the fastest?



Hint: A flat line segment shows that the weight is not changing, so no water is being poured in at that time.

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6B

5 This graph shows a shopper's short walk in a shopping mall.

a What is the total distance the shopper travelled?

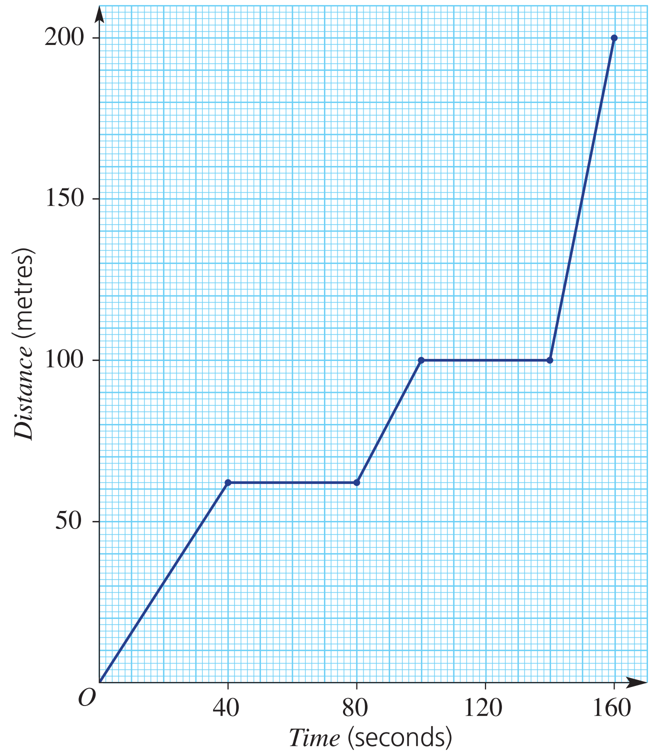
b How long was the shopper not walking?

c What was the total distance the shopper travelled by the following times?

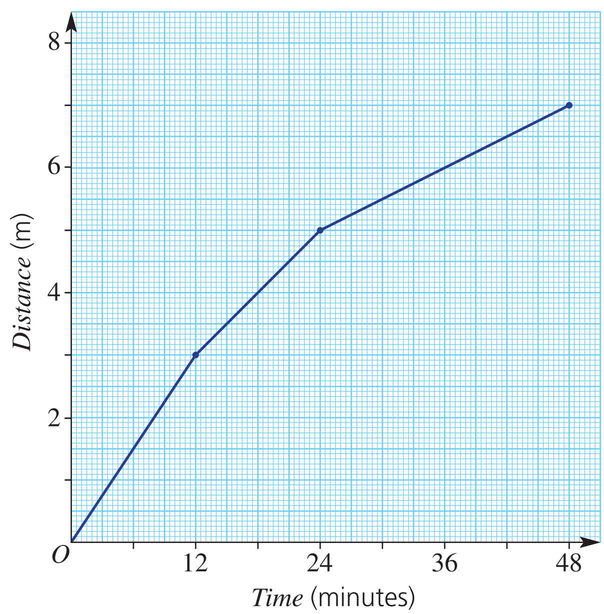
i 20 seconds

ii 80 seconds

iii 2.5 minutes



6 A snail makes its way across a footpath, garden bed and lawn according to this graph.



a How far does the snail travel on:

i the footpath?

ii the garden bed?

iii the lawn?

b On which surface does the snail spend the most time?

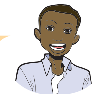
c Use your graph to find how far the snail travelled after:

i 6 minutes

ii 18 minutes

iii 42 minutes

Hint: The line segment that has the largest horizontal change is the surface that the snail spent most time on.



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Problem-solving and reasoning

7–9 | 9–11

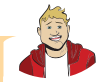
7 This graph shows the distance of a train from the city station over a period of time.

a What is the farthest distance the train travelled from the station?

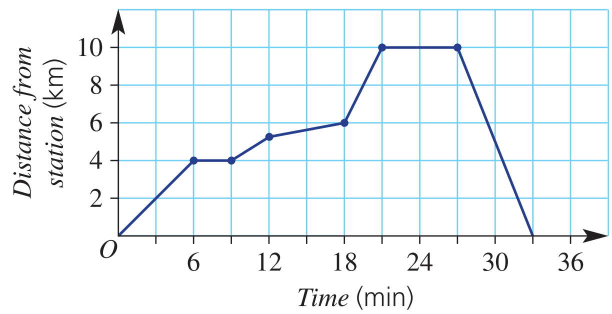
b What is the total distance travelled?

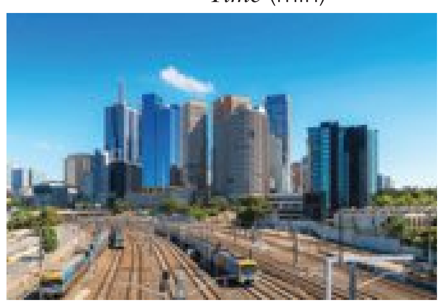
c After how many minutes does the train begin to return to the station?

d What is the total number of minutes the train was at rest?



Hint: Remember to include the return trip in the total distance travelled.



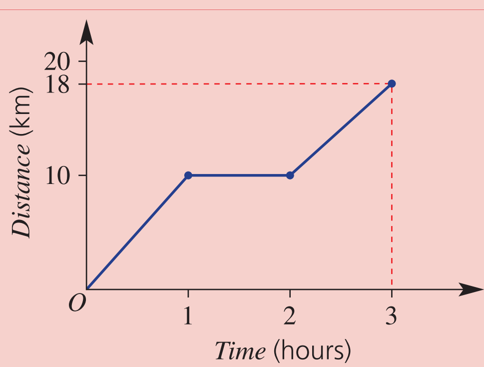


Example 6 Sketching a distance–time graph

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 18 km in 3 hours
- 10 km covered in the first hour
- a 1-hour long rest after the first hour

Solution



Explanation

Draw axes with time on the horizontal (up to 3 hours) and distance on the vertical (up to 18 km).

Start at time zero.

Draw the first hour with 10 km covered.

Draw the rest stop, which lasts for 1 hour.

Draw the remainder of the journey, so that 18 km is completed after 3 hours.

Now you try

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 20 km in 2 hours
- 14 km covered in the first hour
- a half-hour rest stop after the first hour

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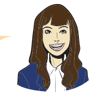
6B

8 Sketch a distance–time graph displaying all of the following information.
- total distance covered is 100 km in 2 hours
- 50 km covered in the first hour
- a half-hour rest stop after the first hour

Hint: Draw axes with time on the horizontal (up to 2 hours) and distance on the vertical (up to 100 km).

9 Sketch a graph to illustrate a journey described by the following.
- total distance covered is 15 metres in 40 seconds
- 10 metres covered in the first 10 seconds
- a 25-second rest after the first 10 seconds

Hint: Always use a ruler to draw line segments.



10 A bus travels 5 km in 6 minutes, stops for 2 minutes, then travels 10 km in 8 minutes, stops for another 2 minutes and then completes the journey by travelling 5 km in 4 minutes.

a What is the total distance travelled?

b What is the total time taken?

c Sketch a distance–time graph for the journey.

Hint: Find the total time taken to determine the scale for the horizontal axis. Find the total distance travelled to determine the scale for the vertical axis.

11 A 1-day, 20 km bush hike included the following features.
- a 3-hour hike to waterfalls (10 km distance)
- a half-hour rest at the falls
- a 2-hour hike to the mountain peak (5 km distance)
- a $1\frac{1}{2}$-hour hike to the camp site

Sketch a distance–time graph for the journey.

Pigeon flight — 12

12 The distance travelled by a pigeon is described by these points.
- a half-hour flight, covering a distance of 18 km
- a 15-minute rest
- a 15-minute flight, covering 12 km
- a half-hour rest
- turning and flying 10 km back towards 'home' over the next half hour
- a rest for a quarter of an hour
- reaching 'home' after another 45-minute flight

a Sketch a graph illustrating the points above using *Distance* on the vertical axis.

b What is the fastest speed (in km/h) that the pigeon flew? $\left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$

c Determine the pigeon's average speed, in km/h. $\left(\text{Average speed} = \frac{\text{total distance}}{\text{total flying time}}\right)$

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6C Plotting straight lines CONSOLIDATING

Learning intentions

- To review the Cartesian plane and how to position and describe points using coordinates
- To understand that a straight-line graph is formed from a linear rule
- To be able to plot a graph of a rule using a table of values
- To be able to use a straight line graph to determine the values of variables

Key vocabulary: Cartesian or number plane, x-coordinate, y-coordinate, point of intersection, origin, linear

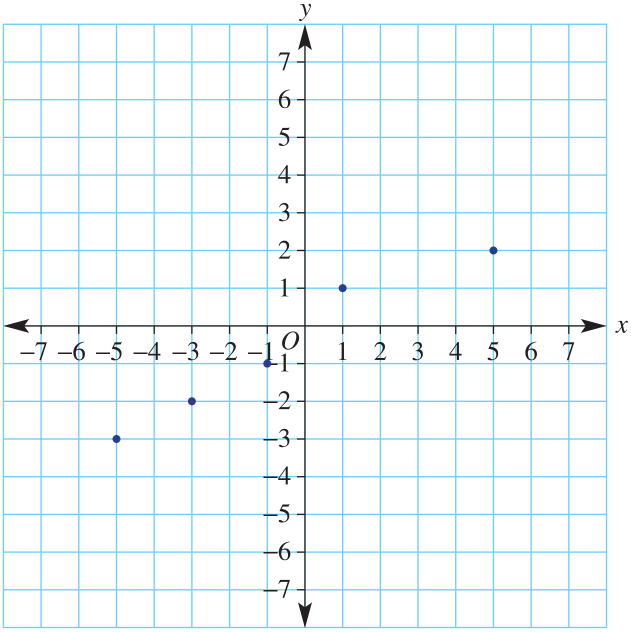
On a number plane, a pair of coordinates gives the exact position of a point. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin (O). It provides a reference point for all other points on the plane.

A rule that relates two variables can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight-line graphs are described as being linear.

Architects apply their knowledge of two-dimensional straight lines and geometric shapes to form interesting three-dimensional surfaces. Computers use line equations to produce visual models.

Lesson starter: What's the error?

- Which point is not in line with the rest of the points on this graph? What should its coordinates be so it is in line? List two other points that would be in line with the points on this graph.



- This table shows coordinates for the rule $y = 4x + 3$. Which y value has been calculated incorrectly in the table? What would be the correct y value?

x	0	1	2	3	4
y	3	7	11	12	19

- Which two points in the list below will not be in the same line as the other points?
  $(-2, 4), (-1, 2), (0, 0), (1, -2), (2, 4), (3, 6)$
  What would be the correct coordinates for these two points, using the given x values?
- Points that follow a linear rule will always be in a straight line. Discuss some ways of checking whether the coordinates of a point have been calculated incorrectly.

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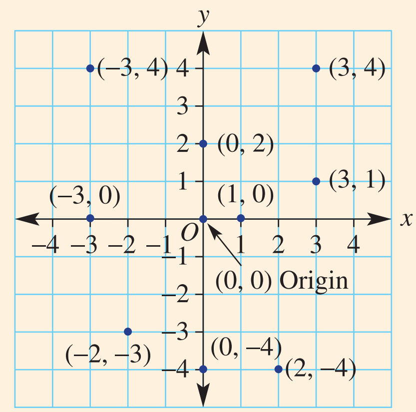
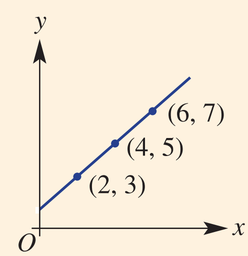
Key ideas



- A **number plane** or **Cartesian plane** includes a vertical ***y*-axis** and a horizontal ***x*-axis** intersecting at right angles.
- A point on a number plane has coordinates (x, y).
  - The ***x*-coordinate** is listed first, followed by the ***y*-coordinate**.
- The point $(0, 0)$ is called the **origin** (O).
- $(x, y) = \left(\begin{array}{c}\text{horizontal}\\ \text{units from}\\ \text{origin}\end{array}, \begin{array}{c}\text{vertical}\\ \text{units from}\\ \text{origin}\end{array}\right)$
- A rule is an equation connecting two or more variables.
- A straight-line graph will result from a rule that is **linear**.
- For two variables, a linear rule is often written with y as the subject. For example, $y = 2x - 3$ or $y = -x + 7$.
- To graph a linear relationship using a rule:
  - Construct a table of values finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

x	2	4	6
y	3	5	7

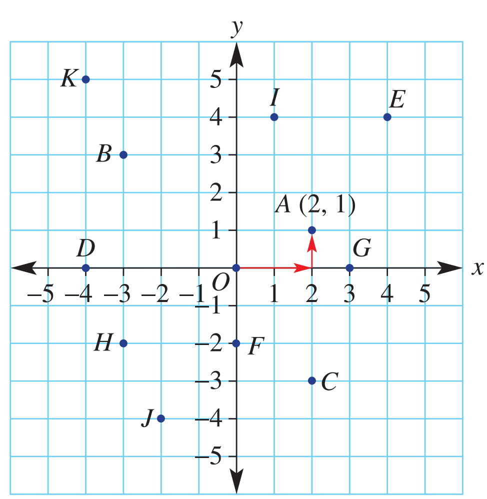


- The **point of intersection** of two lines is the point where the lines cross over each other.

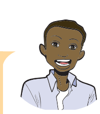
Exercise 6C

Understanding | 1–3 | 3

1 **a** List the coordinates of each point (A–K) plotted on the number plane.
b Which points are on the x-axis?
c Which points are on the y-axis?
d What are the coordinates of the point called the 'origin'?



Hint:
$(x, y) = \left(\begin{array}{c}\text{right}\\ \text{or}\\ \text{left}\end{array}, \begin{array}{c}\text{up}\\ \text{or}\\ \text{down}\end{array}\right)$
The 'origin' is the point where the x-axis and y-axis meet.



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2 Write the coordinates for each point listed in this table.

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

Hint: Coordinates are written as (x, y).

x	−2
y	1

} $(-2, 1)$

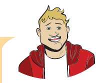
3 Ethan is finding the coordinates of some points that are on the line $y = -2x + 4$. Copy and complete these calculations, stating the coordinates for each point.

a $x = -3, y = -2 \times (-3) + 4 = 6 + 4 = (-3,\ \ )$

b $x = -1, y = -2 \times (-1) + 4 = \quad = (\ ,\ )$

c $x = 0, y = -2 \times 0 + 4 = \quad = (\ ,\ )$

d $x = 2, y = -2 \times 2 + 4 = \quad = (\ ,\ )$



Hint: In the order of operations, first do any multiplication, then do addition or subtraction from left to right. When multiplying, same signs make a positive and different signs make a negative.

Fluency 4, 5(½) 4–5(½), 6

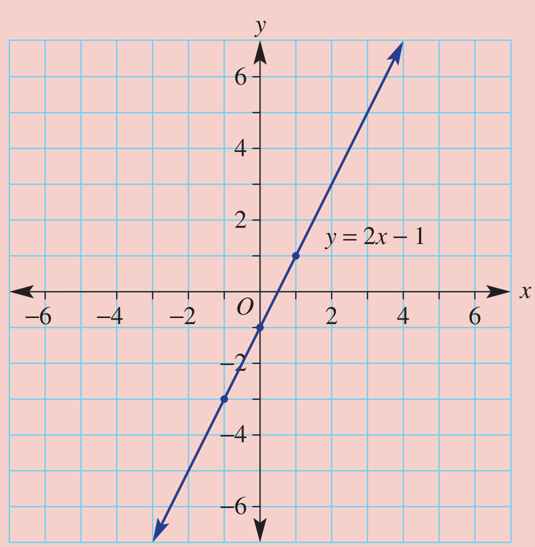
Example 7 Plotting a graph from a rule

Plot the graph of $y = 2x - 1$ by first completing the table of values.

x	−1	0	1
y			

Solution

x	−1	0	1
y	−3	−1	1



Explanation

Substitute each value into the equation:

$x = -1, y = 2 \times (-1) - 1 = -3 \quad (-1, -3)$

$x = 0, y = 2 \times 0 - 1 = -1 \quad (0, -1)$

$x = 1, y = 2 \times 1 - 1 = 1 \quad (1, 1)$

Plot the points and draw the line with a ruler.

When labelling axes, put the numbers on the grid lines, not in the spaces.

Now you try

Plot the graph of $y = 3x + 2$ by first completing the table of values.

x	−1	0	1
y			

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6C

4 Complete the following tables, then plot the graph of each one on a separate number plane.

a $y = 2x$

x	-1	0	1
y			

b $y = x + 4$

x	0	1	2
y			

Hint: When multiplying, same signs make a positive; e.g. $-2 \times (-1) = 2$

c $y = 2x - 3$

x	0	1	2
y			

d $y = -2x$

x	-1	0	1
y			

e $y = x - 4$

x	1	2	3
y			

f $y = 6 - x$

x	0	1	2
y			

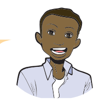
5 Complete the following tables, then plot the graph of each pair on the same axes.

a **i** $y = x + 2$

x	0	2	4
y			

ii $y = -x + 2$

x	0	2	4
y			



Hint: For each part, draw line **i** and line **ii** on the same axes.

b **i** $y = x - 4$

x	0	1	2
y			

ii $y = 4 - x$

x	0	1	2
y			

c **i** $y = 2 + 3x$

x	-3	0	3
y			

ii $y = 3x - 4$

x	-3	0	3
y			

6 By plotting the graphs of each of the following pairs of lines on the same axes, find the coordinates of the point of intersection. Use a table of values, with x from -2 to 2.

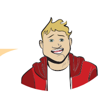
a $y = 2x$ and $y = x$

b $y = x + 3$ and $y = 2x + 2$

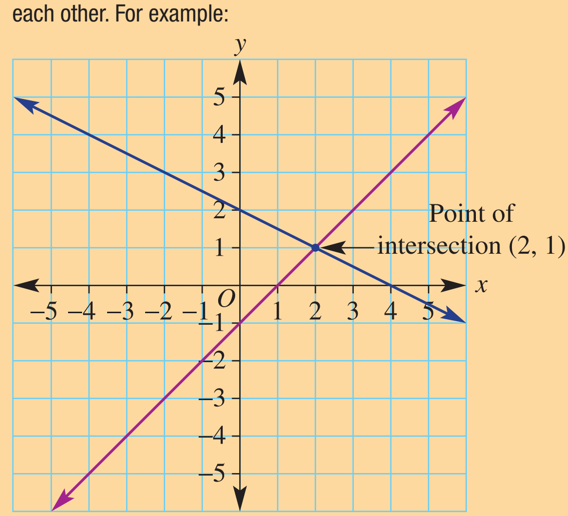
c $y = 2 - x$ and $y = 2x + 5$

d $y = 2 - x$ and $y = x + 2$

e $y = 2x - 3$ and $y = x - 4$



Hint: The point of intersection of two lines is where they cross each other. For example:



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Problem-solving and reasoning

7, 9 | 8, 10

Example 8 Interpreting a graph when given a table of values

Jasmine is organising a school dance. The venue is chosen and the costs are shown in the table.

Number of students (*S*)	0	100	200	300
Total cost in dollars (*C*)	500	1000	1500	2000

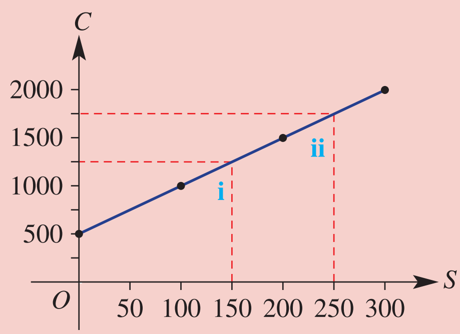
a Plot a graph of the total cost against the number of students.

b Use the graph to determine:

- **i** the total cost for 150 students
- **ii** how many students could attend the dance if Jasmine has $1750 to spend

Solution

a



b **i** The total cost for 150 students is $1250.

ii 250 students could attend the dance if the budget is $1750.

Explanation

Construct a set of axes using S between 0 and 300 and C between 0 and 2000.

Number of students is placed on the horizontal axis.

Plot each point using the information in the table.

Draw a vertical dotted line at $S = 150$ to meet the graph, then draw another dotted line horizontally to the C-axis.

Draw a horizontal dotted line at $C = 1750$ to meet the graph, then draw a dotted line vertically to the S-axis.

Now you try

Hayley is organising a party for her child. The venue is chosen and the costs are shown in the table.

Number of children (*n*)	0	10	20	30
Total cost in dollars (*C*)	200	400	600	800

a Plot a graph of the total cost against the number of children.

b Use the graph to determine:

- **i** the total cost for 25 children
- **ii** how many children could be invited if Hayley has $500 to spend

7 A furniture removalist charges per hour. His rates are shown in the table below.

No. of hours (*n*)	0	1	2	3	4	5
Cost (*C*)	200	240	280	320	360	400

a Plot a graph of cost against hours.

b Use the graph to determine the:

- **i** total cost for 2.5 hours of work
- **ii** number of hours the removalist will work for $380

Hint: Place 'Number of hours' on the horizontal axis.

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6C

8 Olive oil is sold in bulk for $8 per litre.

No. of litres (*L*)	1	2	3	4	5
Cost (*C*)	8	16	24	32	40

a Plot a graph of cost against number of litres.

b Use the graph to determine the:

i total cost for 3.5 litres of oil

ii number of litres of oil you can buy for $20

Example 9 Constructing a table and graph for interpretation

An electrician charges $50 for a service call and $60 an hour for labour.

a Complete the table of values.

No. of hours (*n*)	1	2	3	4	5
Cost (*C*)					

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 4.5 hours of work

ii how long the electrician will work for $260

Solution

a

No. of hours (*n*)	1	2	3	4	5
Cost (*C*)	110	170	230	290	350

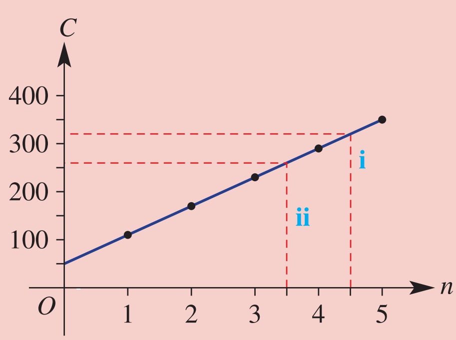
Explanation

Cost for 1 hour = $50 + $60 = $110

Cost for 2 hours = $50 + 2 × $60 = $170

Cost for 3 hours = $50 + 3 × $60 = $230 etc.

b



Plot the points from the table using C on the vertical axis and n on the horizontal axis. Join all the points to form the straight line.

c **i** The cost is $320.

Draw a vertical dotted line at $n = 4.5$ to meet the graph, then draw a line horizontally to the C-axis.

ii The electrician will work for 3.5 hours.

Draw a horizontal dotted line at $C = 260$ to meet the graph, then draw vertically to the n-axis.

Continued on next page

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Now you try

A balloon artist charges \$120 to attend a child's party and \$60 per hour in attendance.

a Complete the table of values.

No. of hours (n)	1	2	3	4
Cost (C)				

b Plot a graph of cost against number of hours.
c Use the graph to determine:
  i the cost for 2.5 hours of hire
  ii how long the balloon artist will work for \$330

9 A car rental firm charges \$200 plus \$1 for each kilometre travelled.
a Complete the table of values below.

No. of km (k)	100	200	300	400	500
Cost (C)					

b Plot a graph of cost against kilometres.
c Use the graph to determine:
  i the cost if you travel 250 km
  ii how many kilometres you can travel on a budget of \$650

10 Matthew delivers pizza for a fast-food outlet. He is paid \$20 a shift plus \$3 per delivery.
a Complete the table of values below.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of Matthew's pay against number of deliveries.
c Use the graph to determine the:
  i amount of pay for 12 deliveries
  ii number of deliveries made if Matthew is paid \$74

Which mechanic? — 11

11 Two mechanics charge different rates for their labour. Yuri charges \$75 for a service call plus \$50 per hour. Sherry charges \$90 for a service call plus \$40 per hour.
a Create a table for each mechanic for up to 5 hours of work.
b Plot a graph for the total charge against the number of hours worked for Yuri and Sherry, on the same axes.
c Use the graph to determine the:
  i cost of hiring Yuri for 3.5 hours
  ii cost of hiring Sherry for 1.5 hours
  iii number of hours of work if Yuri charges \$100
  iv number of hours of work if Sherry charges \$260
  v number of hours of work if the cost from Yuri and Sherry is the same
d Write a sentence describing who is cheaper for different hours of work.

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6D Midpoint and length of a line segment

Learning intentions

- To understand what is meant by the midpoint and length of a line segment
- To be able to determine the midpoint of a line segment from a graph and using coordinates
- To understand how the length of a line segment can be found using Pythagoras' theorem
- To be able to find the length of a line segment from a graph and using coordinates

Key vocabulary: line segment, length, midpoint, coordinates, Pythagoras' theorem, square root, average, hypotenuse

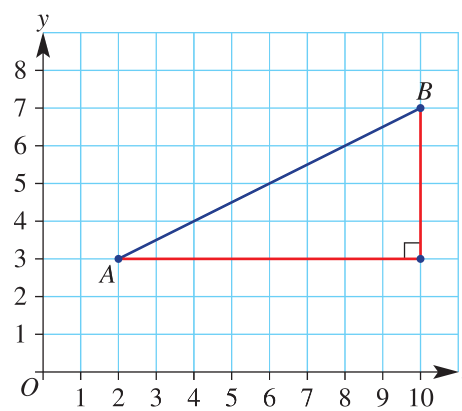
A line segment has a definite length and also has a point in the middle of the segment called the midpoint. Both the midpoint and length can be found by using the coordinates of the end points.

Builders use mathematical calculations to determine the length, midpoint and angle of inclination of wooden beams when constructing the timber frame of a house.

Lesson starter: Finding a method

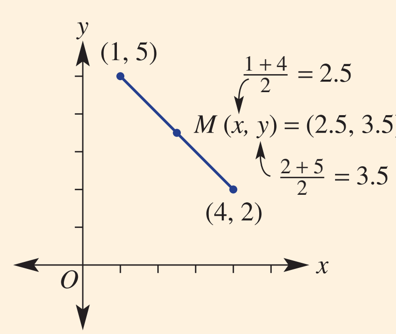
Below right is a graph of the line segment AB. A right-angled triangle has been drawn so that AB is the hypotenuse.

- How many units long are the horizontal and vertical sides of this right-angled triangle?
- Discuss and explain a method for finding the length of the line segment AB.
- What is the x value of the middle point of the horizontal side of the right-angled triangle?
- What is the y value of the middle point of the vertical side of the right-angled triangle?
- What are the coordinates of the point in the middle of the line segment AB?
- Discuss and explain a method for finding the midpoint of a line segment.



Key ideas

- A **line segment** is a part of a line and has two end points.
- The **midpoint** (M) of a line segment is the halfway point between the two end points.
  - We find the mean of the two x values at the end points and the mean of the two y values at the end points.
  - $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
  - When finding the mean, add the values in the numerator before dividing by 2.



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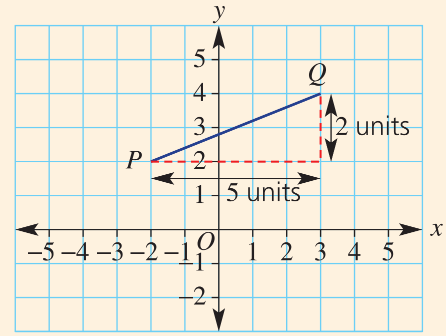
- The length of a line segment is found using **Pythagoras' theorem**.
  To find the length of the line segment PQ, follow these steps.
  - Draw a right-angled triangle with the line segment PQ as the hypotenuse (i.e. longest side).
  - Count the grid squares to find the length of each smaller side.
  - Apply Pythagoras' theorem:

$$PQ^2 = 5^2 + 2^2$$
$$= 25 + 4$$
$$= 29 \text{ (take the square root)}$$
$$PQ = \sqrt{29} \text{ units}$$



  - $\sqrt{29}$ is the length of line segment PQ, in square root form.

Exercise 6D

Understanding | 1–3 | 3

1 Complete the following.

a The ___________ of a line segment is the halfway point between the two end points.

b The ___________ of a line segment is found using Pythagoras' theorem.

2 Complete the working to find the midpoint of the line segment with the following end point coordinates.

a $(1, 4)$ and $(3, 8)$

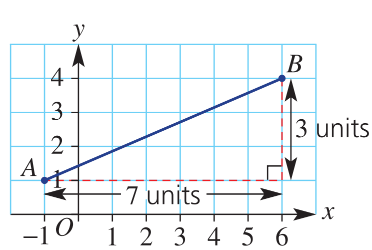
$$M = \left(\frac{1 + \square}{2}, \frac{4 + \square}{2}\right)$$
$$M = (\square, \square)$$

b $(-1, 3)$ and $(5, -3)$

$$M = \left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right)$$
$$M = (\square, 0)$$

3 For the line segment shown, complete the Pythagoras' theorem working below to find the length.

$$AB^2 = \square^2 + \square^2$$
$$= \square + \square$$
$$= 58$$
$$AB = \sqrt{58}$$



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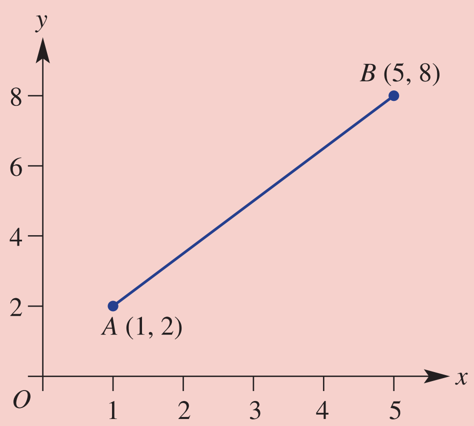
6D

Fluency 4–8(½) 4–9(½)

Example 10 Finding the midpoint of a line segment from a graph

Find the midpoint of the interval between $A(1, 2)$ and $B(5, 8)$.



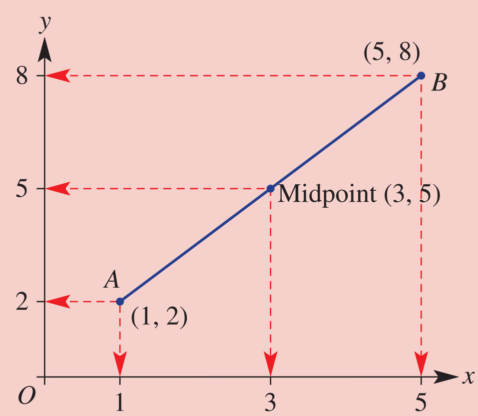
Solution

$$\text{Average of } x \text{ values} = \frac{1+5}{2} = \frac{6}{2} = 3$$

$$\text{Average of } y \text{ values} = \frac{2+8}{2} = \frac{10}{2} = 5$$

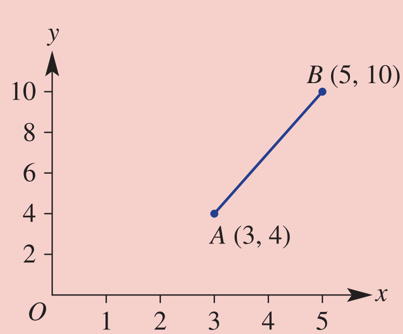
Midpoint is $(3, 5)$.

Explanation



Now you try

Find the midpoint of the interval between $A(3, 4)$ and $B(5, 10)$.



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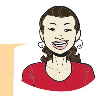
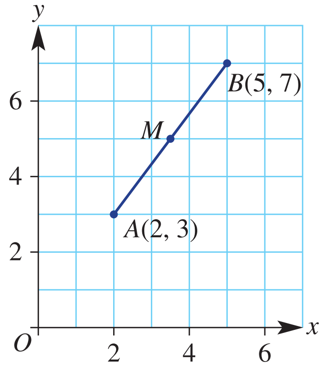
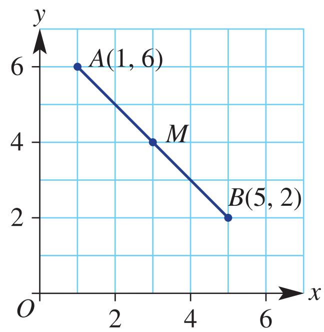
4 Find the midpoint, M, of each of the following intervals.



a



b



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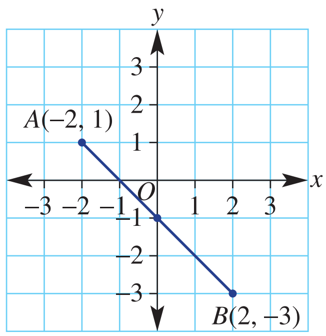

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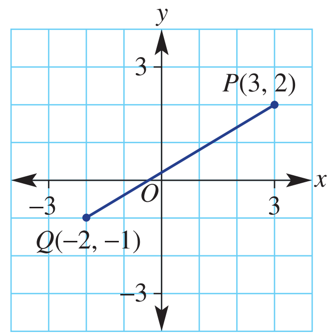

Example 11 Finding the midpoint of a line segment when given the coordinates of the end points

Find the midpoint of the line segment joining $P(-3, 1)$ and $Q(5, -4)$.

Solution	**Explanation**
$x = \frac{-3+5}{2}$	Average the x-coordinates.
$= \frac{2}{2}$	Calculate the numerator before dividing by 2; i.e. $-3+5=2$.
$= 1$	
$y = \frac{1+(-4)}{2}$	Average the y-coordinates.
$= \frac{-3}{2}$	Calculate the numerator before dividing by 2; i.e. $1+(-4)=1-4=-3$.
$= -1.5$	
Midpoint is $(1, -1.5)$.	Write the coordinates of the midpoint.

Now you try

Find the midpoint of the line segment joining $P(3, 1)$ and $Q(6, -5)$.

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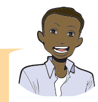
6D

5 Find the midpoint of the line segment joining the following points.

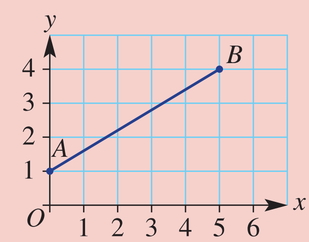
a $(1, 4)$ and $(3, 6)$ **b** $(3, 7)$ and $(5, 9)$
c $(0, 4)$ and $(6, 6)$ **d** $(2, 4)$ and $(3, 5)$
e $(7, 2)$ and $(5, 3)$ **f** $(1, 6)$ and $(4, 2)$
g $(0, 0)$ and $(-2, -4)$ **h** $(-2, -3)$ and $(-4, -5)$
i $(-3, -1)$ and $(-5, -5)$ **j** $(-3, -4)$ and $(5, 6)$
k $(0, -8)$ and $(-6, 0)$ **l** $(3, -4)$ and $(-3, 4)$

Hint: Check that your answer appears to be halfway between the end points.



Example 12 Finding the length of a line segment from a graph

Find the length of the line segment between $A(0, 1)$ and $B(5, 4)$.



Solution

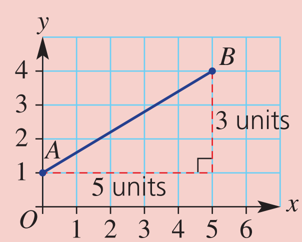
$AB^2 = 5^2 + 3^2$

$AB^2 = 25 + 9$

$AB^2 = 34$

$AB = \sqrt{34}$

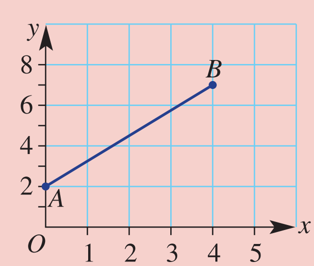
Explanation



Create a right-angled triangle and use Pythagoras' theorem.
For $AB^2 = 34$, take the square root of both sides to find AB.
$\sqrt{34}$ is the exact answer.

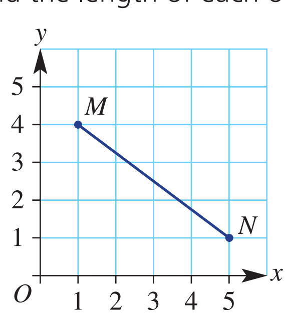
Now you try

Find the length of the line segment between $A(0, 2)$ and $B(4, 7)$.

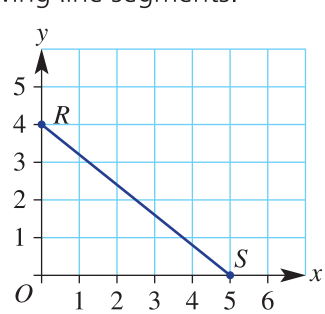


6 Find the length of each of the following line segments.

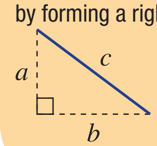
a

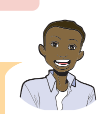


b



Hint: Use Pythagoras' theorem ($c^2 = a^2 + b^2$) by forming a right-angled triangle:





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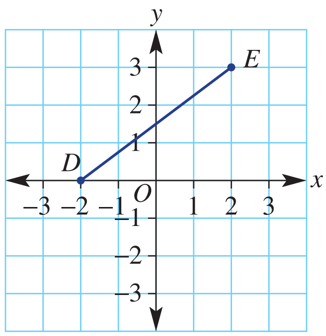
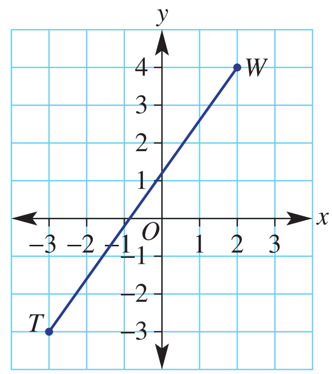
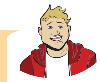
c



d

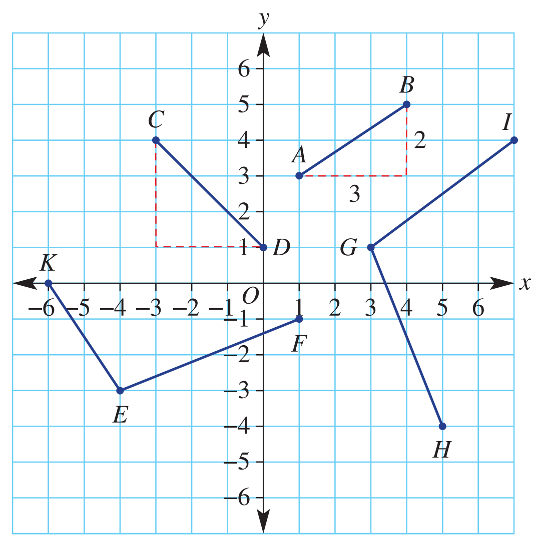




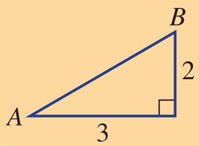
Hint: Write the answer in square root form if it is not a known square root.

7 Find the length of each line segment on the following number plane. Leave your answers in square root form.

a AB **b** CD
c EF **d** GH
e KE **f** GI



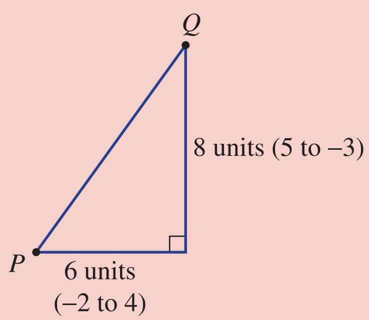
Hint: First sketch a right-angled triangle for each line segment, labelling the known sides. For example:



Example 13 Finding the length of a line segment when given the coordinates of the end points

Find the distance between the points P and Q if P is at $(-2, -3)$ and Q is at $(4, 5)$.

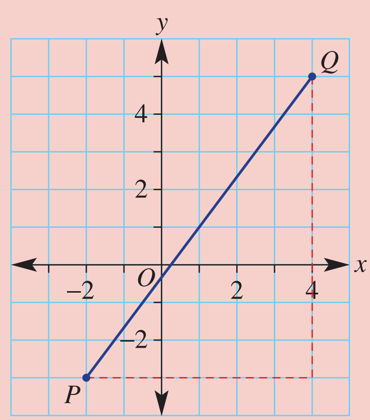
Solution



$PQ^2 = 6^2 + 8^2$
$PQ^2 = 36 + 64$
$PQ^2 = 100$
$PQ = \sqrt{100}$
$PQ = 10$ units

Explanation

Use Pythagoras' theorem to find PQ, the hypotenuse.



If you know the value of the square root, write its value.

Now you try

Find the distance between the points P and Q if P is at $(-1, -4)$ and Q is at $(3, 2)$.

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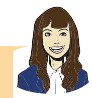
6D

8 Plot each of the following pairs of points and find the distance between them, correct to one decimal place where necessary.

a $(2, 3)$ and $(5, 7)$
b $(0, 1)$ and $(6, 9)$
c $(0, 0)$ and $(-5, 10)$
d $(-4, -1)$ and $(0, -5)$
e $(-3, 0)$ and $(0, 4)$
f $(0, -1)$ and $(2, -4)$

Hint: First rule up your axes with x from -5 to 10 and y from -5 to 10.



9 Find the exact length between these pairs of points.

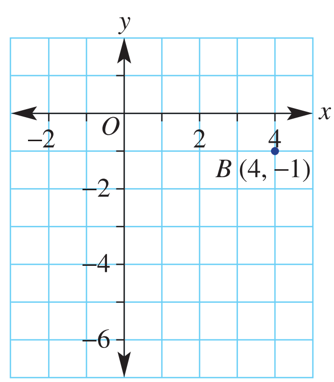
a $(1, 3)$ and $(2, 2)$
b $(4, 1)$ and $(7, 3)$
c $(-3, -1)$ and $(0, 4)$
d $(-2, -3)$ and $(3, 5)$
e $(-1, 0)$ and $(-6, 1)$
f $(1, -3)$ and $(4, -2)$

Hint: Exact length means leave the $\sqrt{\ }$ sign in the answers.

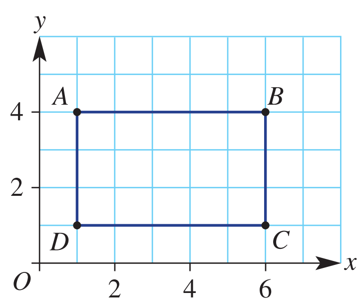
Problem-solving and reasoning

10, 11	10–12

10 Copy the diagram below. Mark the point $B(4, -1)$, as shown, then mark the point $M(1, -3)$. Find the coordinates of A if M is the midpoint of the interval AB.



11 Copy the diagram of rectangle $ABCD$.



a What are the coordinates of each vertex?
b Find the midpoint of the diagonal AC.
c Find the midpoint of the diagonal BD.
d What does this tell us about the diagonals of a rectangle?

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12 **a** Draw up a four-quadrant number plane like the one shown.

b Plot the points $A(-4, 0)$, $B(0, 3)$ and $C(0, -3)$ and form the triangle ABC.

c What is the length of:

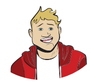
i AB? **ii** AC?

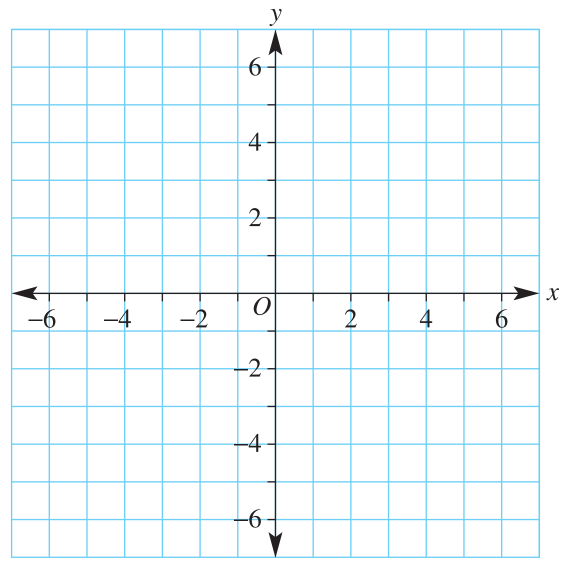
d What type of triangle is ABC?

e Calculate its perimeter and area.

f Write down the coordinates of D such that $ABDC$ is a rhombus.

Hint: A rhombus has all sides of equal length.





Features of a circle — 13

13 The diameter of a circle is shown on this graph.

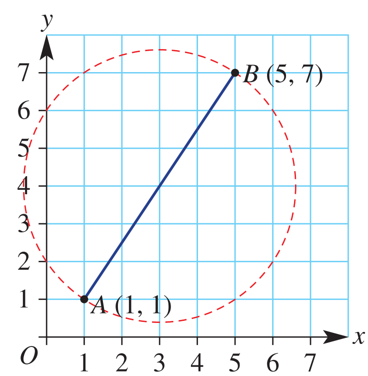
a What are the coordinates of X, the centre of the circle? Mark this point on your graph.

b What is the length of the radius XA?

c Find the distance from X to the point $(5, 1)$. How can we tell that $(5, 1)$ lies on the circle?

d Use $C = 2\pi r$ to find the circumference of the circle shown. Round your answer to one decimal place.

e Calculate the area of this circle using $A = \pi r^2$, correct to one decimal place.



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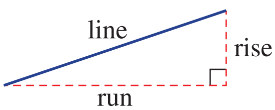
6E Exploring gradient

Learning intentions

- To know that gradient is a numerical measure of the slope of a line
- To understand that the gradient of a straight line is constant
- To be able to identify on a graph if the gradient is positive, negative, zero or undefined
- To be able to use the rise and the run between two points to calculate the gradient

Key vocabulary: gradient, slope, rise, run

The gradient of a line is a measure of its slope. It is a number that shows the steepness of a line. It is calculated by knowing how far a line rises or falls (called the *rise*) within a certain horizontal distance (called the *run*). The gradient is equal to the *rise* divided by the *run*. The letter m is used to represent gradient.

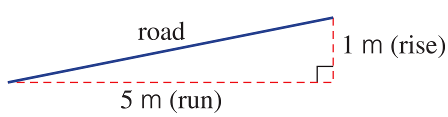


$$\text{Gradient of line} = m = \frac{\text{rise}}{\text{run}}$$

Engineers apply their knowledge of gradients when designing roads, bridges, railway lines and buildings. Some mountain railways have a gradient greater than 1, which is a slope far too steep for a normal train or even a powerful car.

For example, a train takes tourists to the Matterhorn, a mountain in Switzerland. To cope with the very steep slopes it has an extra wheel with teeth, which grips a central notched line.

Lesson starter: What's the gradient?



A road that rises by 1 m for each 5 m of horizontal distance has a gradient of 0.2 or 20%.

Trucks would find this gradient very steep.

The gradient is calculated by finding the rise divided by the run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1}{5} = 0.2 = 20\%$$

1 Find the gradient for each of these roads. Give the answer as a decimal and a percentage.

- **a** Baldwin Street, Dunedin, New Zealand is known as the steepest street in the world. For each 2.86 m of horizontal (run) distance, the road rises by 1 m.
- **b** Gower Street, Toowong, is Brisbane's steepest street. For each 3.2 m of horizontal (run) distance, the road rises by 1 m.

2 The Scenic Railway, Katoomba, NSW has a maximum gradient of 122% as it passes through a gorge in the cliff. What is its vertical distance (rise) for each 1 m of horizontal distance (run)?

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Use computer software (dynamic geometry) to produce a set of axes and grid.

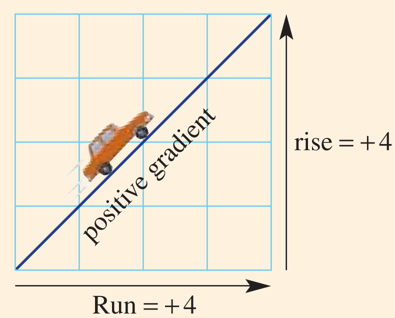
- Construct a line segment with end points on the grid. Show the coordinates of the end points.
- Calculate the rise (i.e. vertical distance between the end points) and the run (i.e. horizontal distance between the end points).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the end points and explore the effect on the gradient.
- Can you drag the end points but retain the same gradient value? Explain why this is possible.
- Can you drag the end points so that the gradient is zero or undefined? Describe how this can be achieved.

Key ideas

- **Gradient (m)** $= \frac{\text{rise}}{\text{run}}$, it describes the steepness of a **slope**.

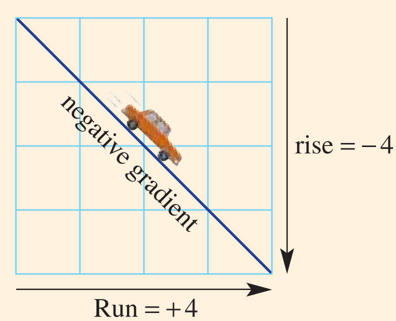
  Always move from left to right when considering the rise and the run.
  - The horizontal **'run'** always goes to the right and is always positive. The vertical **'rise'** can go up (positive) or down (negative).
  - If the line slopes up from left to right, the rise is positive and the gradient is positive.

    e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{+4}{+4} = 1$

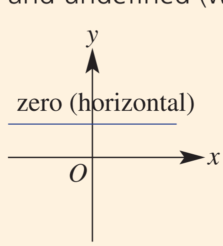


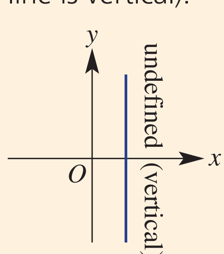
  - If the line slopes down from left to right, the rise is considered to be negative and the gradient is negative.

    e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{+4} = -1$



- The gradient can also be zero (when a line is horizontal) and undefined (when a line is vertical).





- Between two points (x_1, y_1) and (x_2, y_2), the gradient (m) is $m = \frac{y_2 - y_1 \text{ (rise)}}{x_2 - x_1 \text{ (run)}}$.

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6E

Exercise 6E

Understanding — 1, 2 | 1

1 Use the words *positive*, *negative*, *zero* or *undefined* to complete each sentence.

a The gradient of a horizontal line is ______________.

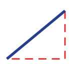
b The gradient of the line joining $(0, 3)$ and $(5, 0)$ is ______________.

c The gradient of the line joining $(-6, 0)$ and $(1, 1)$ is ______________.

d The gradient of a vertical line is ______________.

Hint: Lines going downhill from left to right have a negative gradient.

2 Decide whether each of the following lines would have a positive or negative gradient.

a


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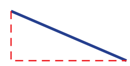
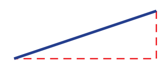
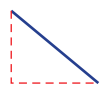
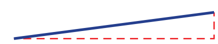


c


d

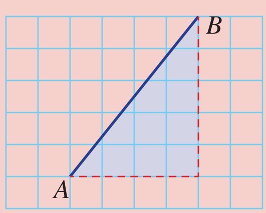

e


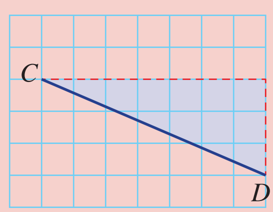
f


Fluency — 3–5, 7 | 3, 4, 5(½), 6, 7

Example 14 Finding the gradient from a grid

Find the gradient of the following line segments, where each grid box equals 1 unit.

a


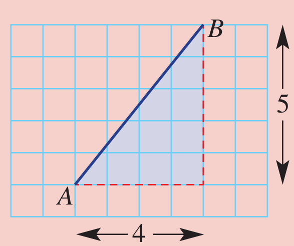
b


Solution

a Gradient of $AB = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{4}$

Explanation

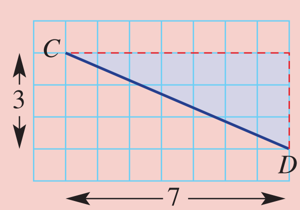


The slope is upwards, therefore the gradient is positive.
The rise is 5 and the run is 4.

b Gradient of $CD = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{7}$

$= -\frac{3}{7}$



The slope is downwards, therefore the gradient is negative.
The fall is 3, so we write rise $= -3$, and the run is 7.

Continued on next page

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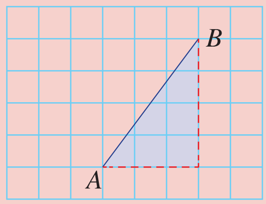
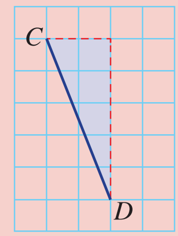
Now you try

Find the gradient of the following line segments, where each grid box equals 1 unit.

a

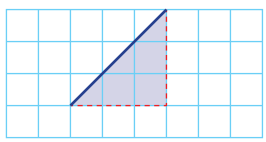


b

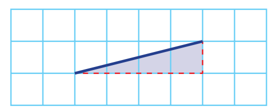


3 Find the gradient of the following line segments.

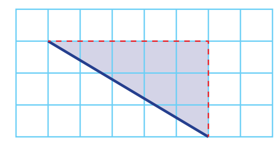
a



b

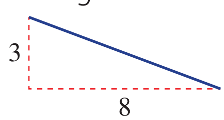


c

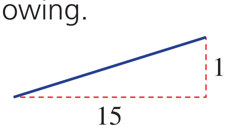


4 Find the gradient of the following.

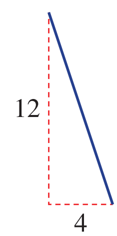
a

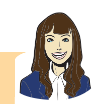


b



c



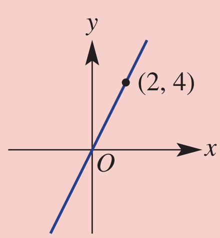


Hint: The gradient is written as a fraction or a whole number.

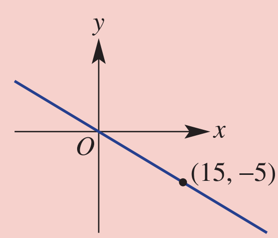
Example 15 Finding the gradient from graphs

Find the gradient of the following lines.

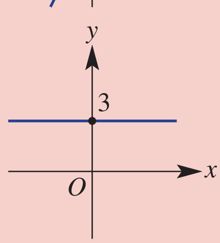
a



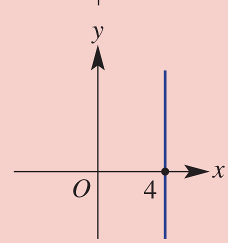
b



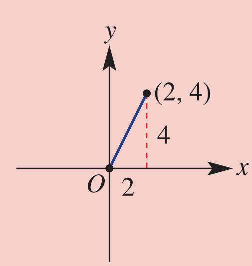
c



d



Solution	Explanation
a Gradient $= \frac{\text{rise}}{\text{run}}$	Write the rule each time.
$= \frac{4}{2}$	The rise is 4 and the run is 2 between the two points $(0, 0)$ and $(2, 4)$.
$= 2$	Simplify by cancelling.



Continued on next page

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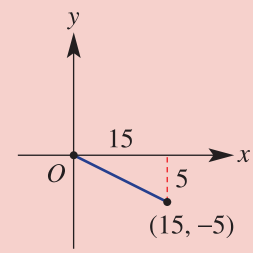
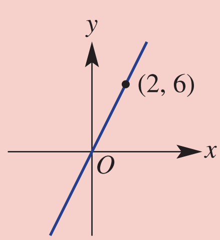
6E

Solution	Explanation
b Gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{-5}{15}$ $= \frac{-1}{3}$ $= -\frac{1}{3}$	Note this time that, when working from left to right, there will be a slope downwards. The fall is 5 (rise $= -5$) and the run is 15. Simplify. 
c Gradient $= 0$	Horizontal lines have a zero gradient.
d Gradient is undefined.	Vertical lines have an undefined gradient.

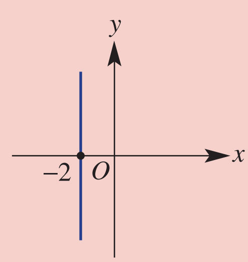
Now you try

Find the gradient of the following lines.

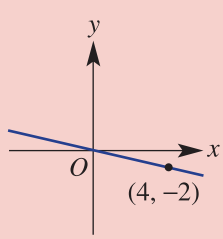
a



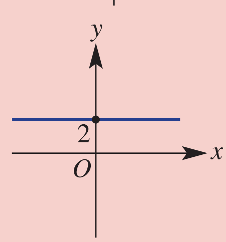
b



c

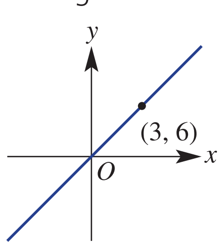


d

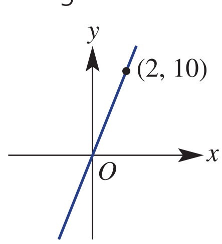


5 Find the gradient of the following lines.

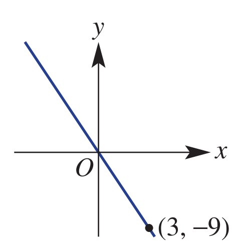
a



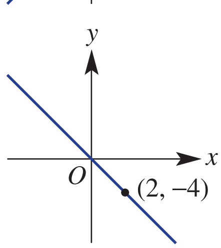
b



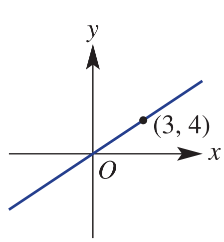
c



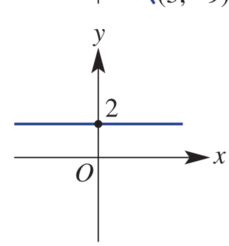
d



e



f



Hint: A horizontal line has zero rise, so its gradient is zero.

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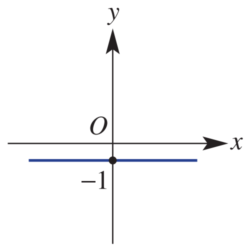
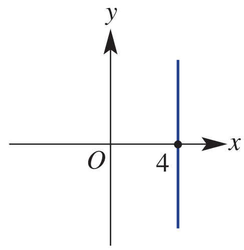
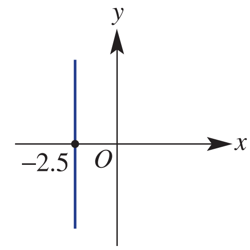
g



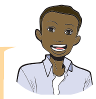
h



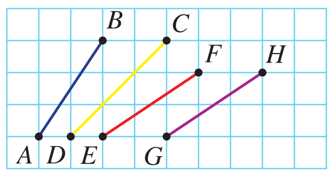
i



Hint: A vertical line has no 'run', so it has undefined gradient.

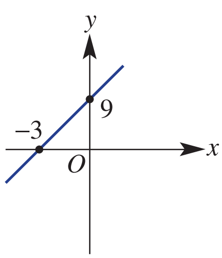


6 Use the grid to find the gradient of these line segments. Then order the segments from least to steepest gradient.



7 Determine the gradient of the following lines.

a



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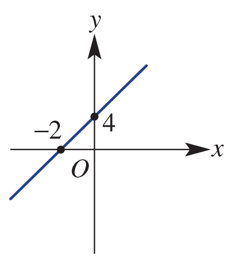

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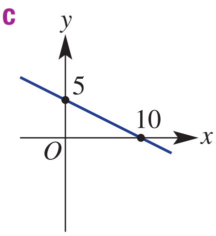
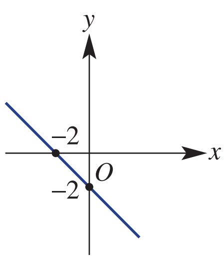
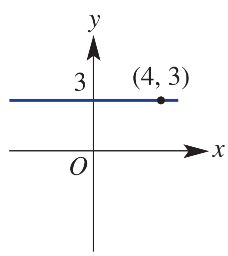


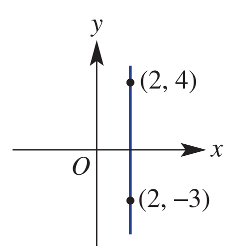
d



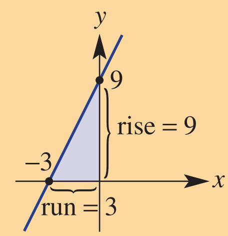
e

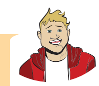


f



Hint:

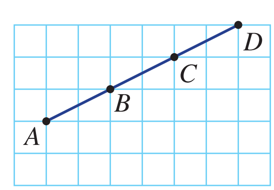




Problem-solving and reasoning | 8, 9 | 9–10(½), 11

8 **a** Copy and complete the table below.

Line segment	Rise	Run	Gradient
AB			
AC			
AD			
BC			
BD			
CD			



b What do you notice about the gradient between points on the same line?

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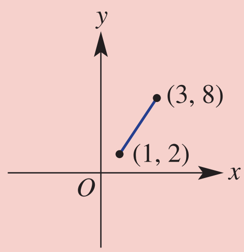
6E

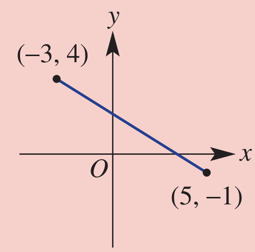
Example 16 Using a formula to calculate gradient

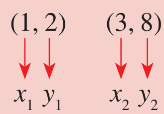
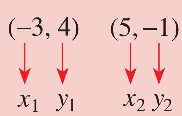
Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b

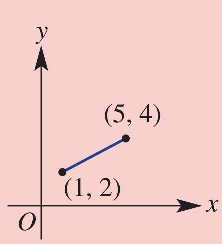


Solution	Explanation
a $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Write the rule.
$= \dfrac{8 - 2}{3 - 1}$	
$= \dfrac{6}{2}$	
$= 3$	It does not matter which point is labelled (x_1, y_1) and which is (x_2, y_2).
b $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Write the rule.
$= \dfrac{-1 - 4}{5 - (-3)}$	
$= \dfrac{-5}{8}$	Remember that $5 - (-3) = 5 + 3 = 8$.
$= -\dfrac{5}{8}$	

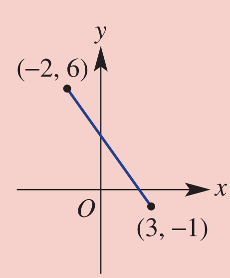
Now you try

Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b



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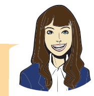
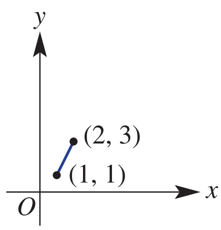
9 Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between these pairs of points.

Hint: First copy the coordinates and label them.
e.g. (1, 1) (2, 3) → x_1 y_1 x_2 y_2

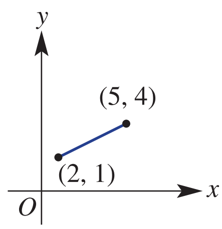


Hint: You can choose either point to be (x_1, y_1).

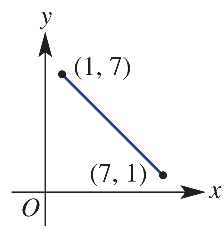
a



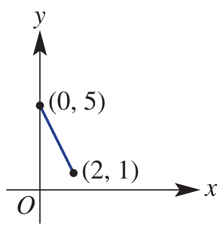
b



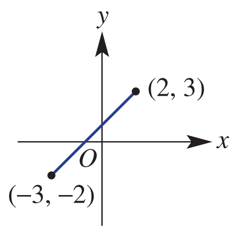
c



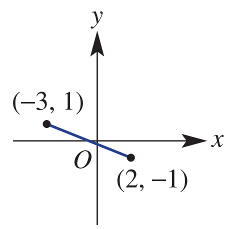
d



e

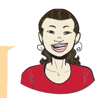


f



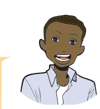
10 Find the gradient between the following pairs of points.

a $(1, 3)$ and $(5, 7)$
b $(-1, -1)$ and $(3, 3)$
c $(-3, 4)$ and $(2, 1)$
d $(-6, -1)$ and $(3, -1)$
e $(1, -4)$ and $(2, 7)$
f $(-4, -2)$ and $(-1, -1)$



Hint: Use $m = \frac{y_2 - y_1}{x_2 - x_1}$.

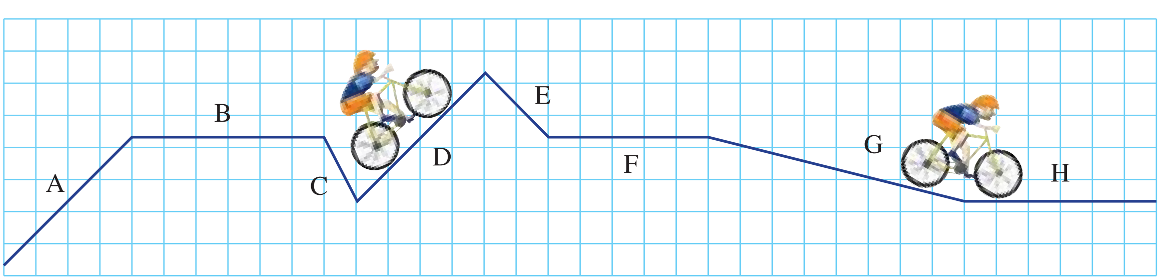
11 The first section of the Cairns Skyrail travels from Caravonica terminal at 5 m above sea level to Red Peak terminal, which is 545 m above sea level. This is across a horizontal distance of approximately 1.57 km. What is the overall gradient of this section of the Skyrail? Round your answer to three decimal places.



Hint: Both distances must be in the same units.

From Bakersville to Rolland | — | 12

12 A transversal map for a bike ride from Bakersville to Rolland is shown.



a Which sections, A, B, C, D, E, F, G or H, indicate travelling a positive gradient?
b Which sections indicate travelling a negative gradient?
c Which will be the hardest section to ride?
d Which sections show a zero gradient?
e Which section is the flattest of the downhill rides?

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6F Rates from graphs

Learning intentions

- To know that a rate compares two quantities
- To understand the connection between rate and the gradient of a straight line
- To be able to calculate speed (using distance ÷ time) and other rates from a graph using the gradient

Key vocabulary: rate, gradient, speed

The speed or rate at which something changes can be analysed by looking at the gradient (steepness) of a graph. Two common rates are kilometres per hour (km/h) and litres per second (L/s).

Graphs of a patient's records provide valuable information for a doctor. For example, from a graph of temperature versus time, the rate of temperature change in °C/min can be calculated. This rate provides important information to help a doctor diagnose an illness.

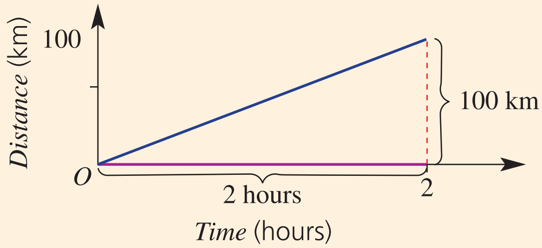
Lesson starter: What's the rate?

Calculate each of these rates.

a \$60 000 for 200 tonnes of wheat.
b Lee travels 840 km in 12 hours.
c A foal grows 18 cm in height in 3 months.
d Petrol costs \$96 for 60 litres.
e Before take-off, a hot-air balloon of volume 6000 m^3 is filled in 60 seconds.

Key ideas

- A **rate** compares two quantities. Many rates show how a quantity changes over *time*.



$$\text{Rate} = \frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50 \text{ km/h}$$

- Rate = change in quantity ÷ change in time
  (L, kg, ...) (seconds, hours, ...)
- The gradient of a line gives the rate.
- A common rate is **speed**.
  - Speed = change in distance ÷ change in time
    (cm, km, ...) (seconds, hours, ...)
- To determine a rate from a linear graph, calculate the gradient and include the units (y unit/x unit); e.g. km/h.

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Exercise 6F

Understanding | 1, 2 | 2

1 Complete the sentences.

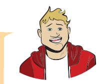
a A rate is found from a linear graph by calculating the ________ of the line.

b A rate compares ________ quantities.

c A rate has two ________.

d A speed of 60 kilometres per hour is written as 60 ________.

e If the rate of filling a bath is 50 litres per minute, this is written as 50 ________.

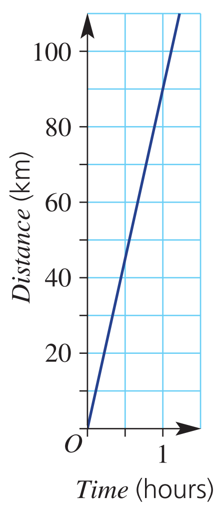


Hint: Choose from: *km/h, gradient, units, L/min, two.*

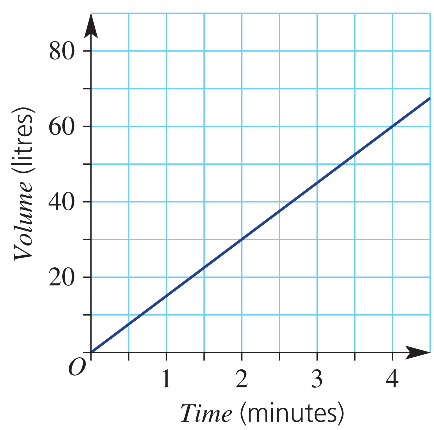
2 Write down the rate by calculating the gradient of each line graph. Include units.

Hint: A rate = gradient with units.
Gradient = $\frac{\text{rise}}{\text{run}}$

a



b



Fluency | 3–5 | 3, 5, 6

Example 17 Calculating speed from a graph

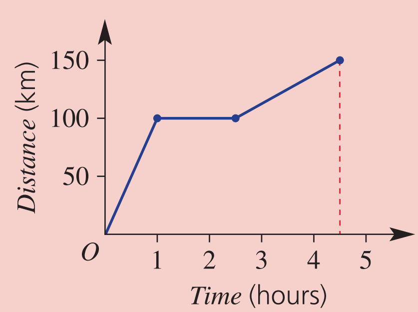
A 4WD vehicle completes a journey, which is described by this graph.

a For the first hour, find:

- **i** the total distance travelled
- **ii** the speed

b How fast was the 4WD travelling during:

- **i** the first hour?
- **ii** the second section?
- **iii** the third section?



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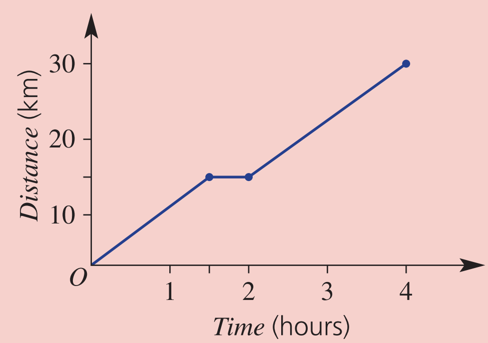
Solution	Explanation
a **i** 100 km	Read the distance at 1 hour.
ii 100 km/1 h = 100 km/h	Speed = distance ÷ time
b **i** 100 km/1 h = 100 km/h	Speed = distance ÷ time
ii 0 km/1.5 h = 0 km/h	The vehicle is at rest.
iii (150 − 100) km/(4.5 − 2.5) h = 50 km/2 h = 25 km/h	Determine the distance travelled and the amount of time, then apply the rate formula. Speed = distance ÷ time 50 km in 2 hours is $\frac{50}{2}$ = 25 km in 1 hour.

Now you try

A marathon entrant completes a training run which is described by this graph.

a For the first hour, find:
  i the total distance travelled
  ii the speed

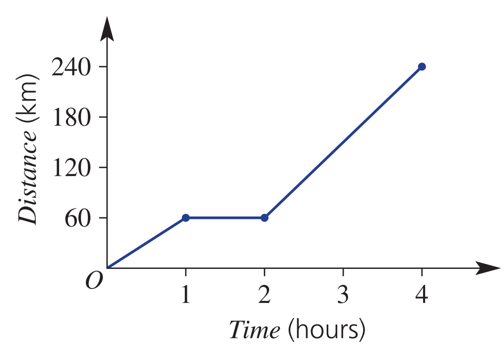
b How fast was the runner travelling during:
  i the first hour?
  ii the second section?
  iii the third section?



3 A car completes a journey, which is described by this graph.

a For the first hour, find:
  i the total distance travelled
  ii the speed

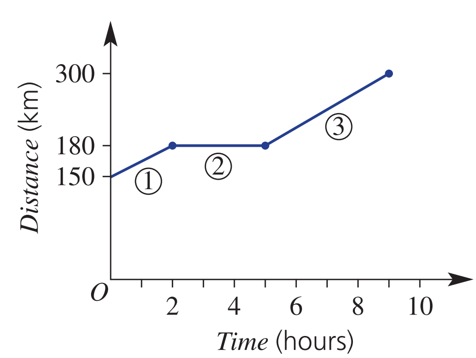
b How fast was the car travelling during:
  i the first hour?
  ii the second section?
  iii the third section?



4 A cyclist training for a professional race includes a rest stop between two travelling sections.

a For the first hour, find:
  i the total distance travelled
  ii the speed

b How fast was the cyclist travelling during:
  i the second section?
  ii the third section?



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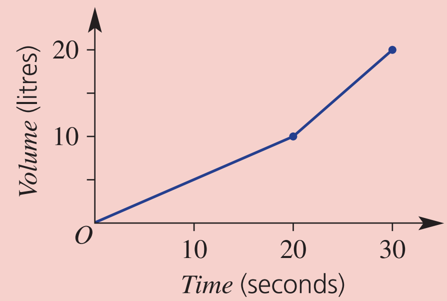
Example 18 Calculating the rate of change of volume in L/s

A container is being filled with water from a hose.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?



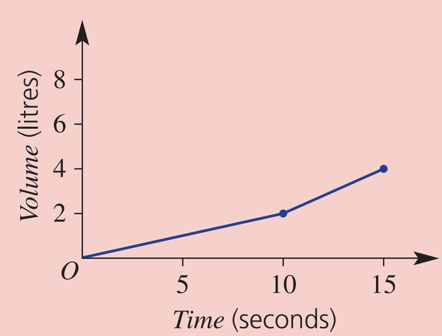
Solution	Explanation
a **i** 5 litres	Read the number of litres after 10 seconds.
ii 10 litres	Read the change in litres from 20 to 30 seconds.
b **i** 5 L/10 s = 0.5 L/s	5 litres is added in the first 10 seconds.
ii 10 L/10 s = 1 L/s	10 litres is added in the final 10 seconds.
iii 5 L/10 s = 0.5 L/s	5 litres is added between 10 and 20 seconds.

Now you try

A bucket is being filled with water from a tap.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 5 seconds?

b How fast (i.e. what rate in L/s) is the bucket being filled:
- **i** during the final 5 seconds?
- **ii** between the 5 and 10 second marks?

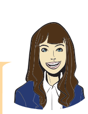


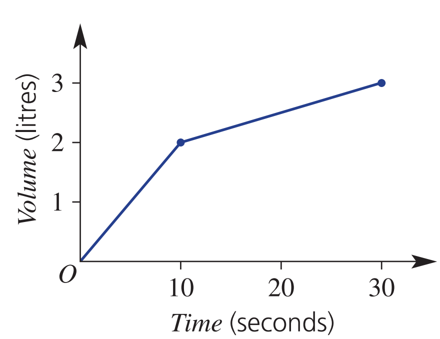
5 A large carton is being filled with milk.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

Hint:
Rate = volume ÷ time





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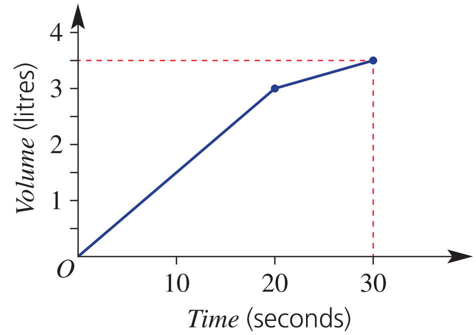
6F

6 A large bottle with a long narrow neck is being filled with water.



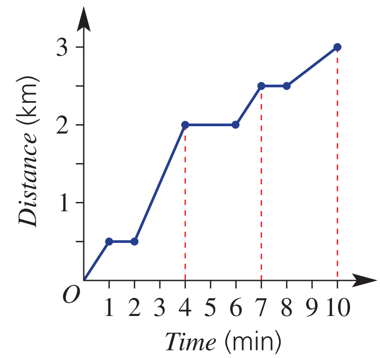
a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the bottle being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

Problem-solving and reasoning

	7, 8	8–10

7 A postal worker stops to deliver mail to each of three houses along a country lane.



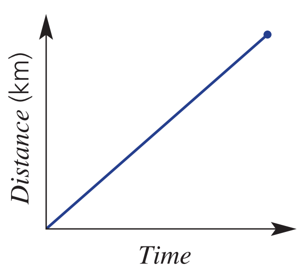
a What is the total length of the country lane?

b What is the total time the postal worker spends standing still?

c Find the speed (use km/min) of the postal worker at the following times.
- **i** before the first house
- **ii** between the first and the second house
- **iii** between the second and the third house
- **iv** after his delivery to the third house

8 Three friends, Anna, Billy and Cianne, travel 5 km from school to the library. Their journeys are displayed in the three graphs below. All three graphs are drawn to the same scale.

i



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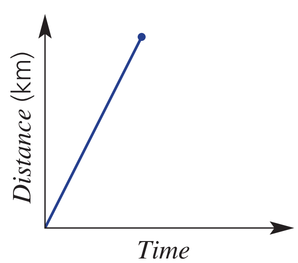
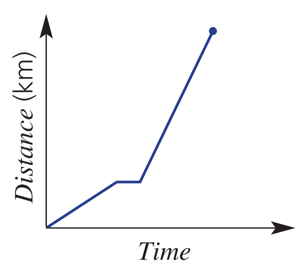


iii



a If Anna walked a short distance before getting picked up by her mum, which graph represents her trip?

b If Cianne arrived at the library last, which graph best represents her journey?

c Which graph represents the fastest journey? Explain your answer.

9 **a** Draw your own graph to show the following journey: 10 km/h for 2 hours, then rest for 1 hour, and then 20 km/h for 2 hours.

b Then use your graph to find the total distance travelled.

Hint:
Mark each segment one at a time. 10 km/h for 2 hours covers a distance of $10 \times 2 = 20$ km.

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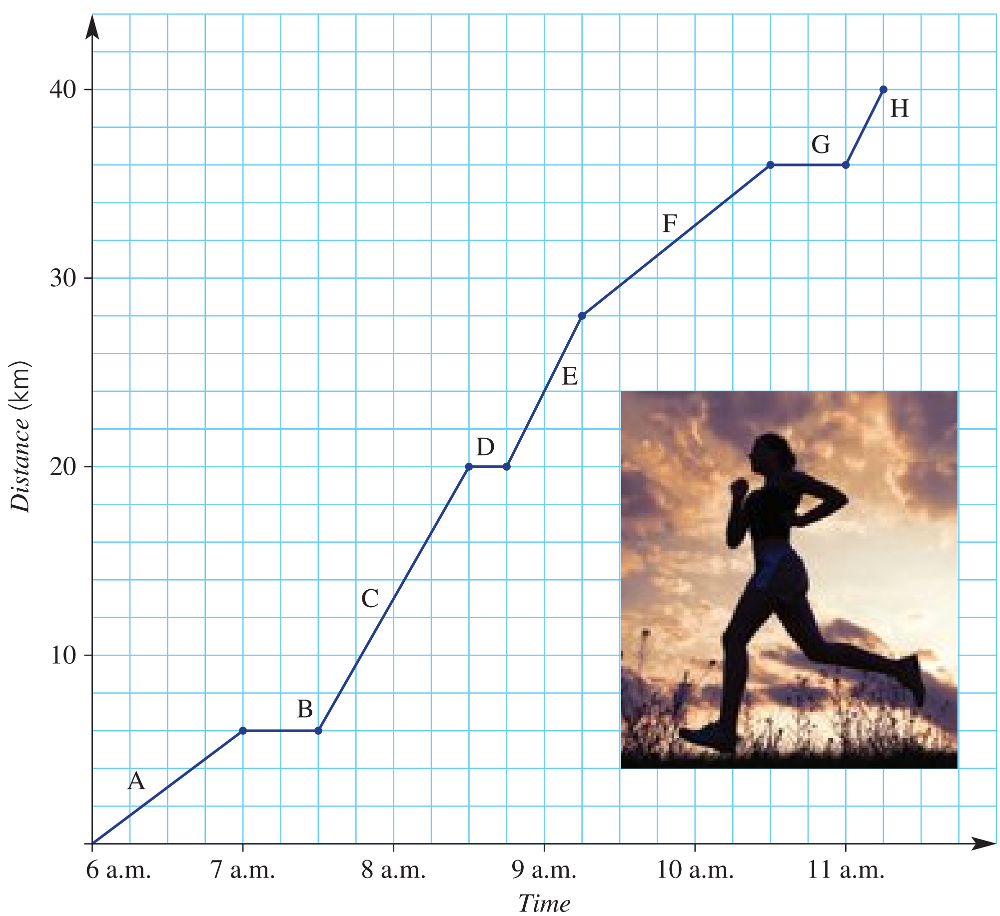
10 A lift starts on the ground floor (height 0 m) and moves to floor 3 at a rate of 3 m/s for 5 seconds. After waiting at floor 3 for 9 seconds, the lift rises 45 m to floor 9 in 9 seconds. The lift rests for 11 seconds before returning to ground level at a rate of 6 m/s. Draw a graph to help find the total time taken to complete the movements described. Use time, in seconds, on the horizontal axis.

Sienna's training — 11

11 Sienna is training for the Sydney Marathon. Her distance–time graph is shown below.

- **a** How many stops does Sienna make?
- **b** How far does she jog between:
  - **i** 6 a.m. and 7 a.m.?
  - **ii** 7:30 a.m. and 8:30 a.m.?
- **c** Which sections of the graph have a zero gradient?
- **d** Which sections of the graph have the steepest gradient?
- **e** At what speed does Sienna run in section:
  i A? **ii** C? **iii** E? **iv** F? **v** H?
- **f** In which sections is Sienna travelling at the same speed? How does the graph show this?
- **g** How long does the training session last?
- **h** What is the total distance travelled by Sienna during the training session?
- **i** What is her average speed for the entire trip, excluding rest periods?

Sienna's training



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Progress quiz

6A

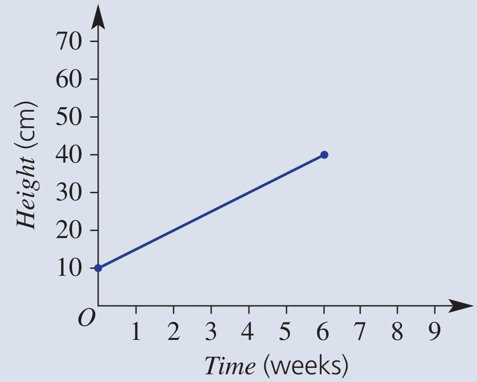
1 The graph at right shows the growth of a plant measured at weekly intervals.

a What is the initial height of the plant?

b How much does the plant grow in 6 weeks?

c Use the graph to find the height of the plant after 4 weeks.

d Extend the graph to estimate the height of the plant after 8 weeks.



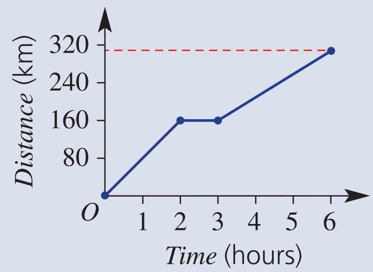
6B/F

2 The graph at right shows the journey of a car.

a What is the distance travelled in the first 2 hours?

b What is the speed in the first 2 hours?

c What is the speed in the final section after the rest break?



6B

3 Sketch a distance–time graph for a runner's training session. Display the following information in your graph:

- covers 6 km in 30 minutes
- 4 km covered in first 15 minutes
- a 5-minute rest after 15 minutes.

6C

4 Plot the graphs of the following by first completing the table of values.

a $y = 3x + 1$

x	-2	-1	0	1	2
y					

b $y = -2x + 3$

x	-2	-1	0	1	2
y					

6C

5 A sailing boat can be hired for \$60 per hour with an initial fee of \$40.

a Complete the table of values.

No. of hours (*n*)	1	2	3	4	5
Cost (\$*C*)					

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 2.5 hours of hire

ii how long the sailing boat was hired if the cost was \$250

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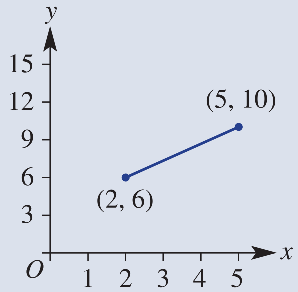
6D **6** Find the midpoint and length of the following.

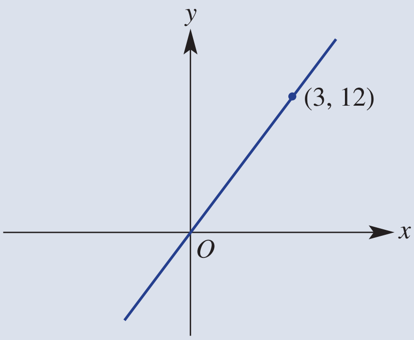
a



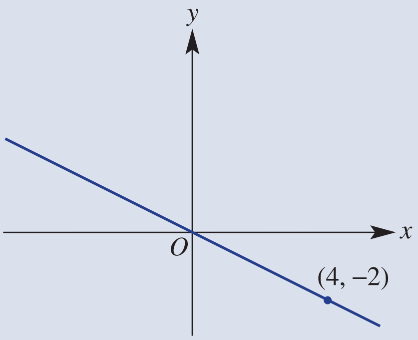
b a line segment joining $(-2, 7)$ and $(4, 12)$

6E **7** Find the gradient of the following lines.

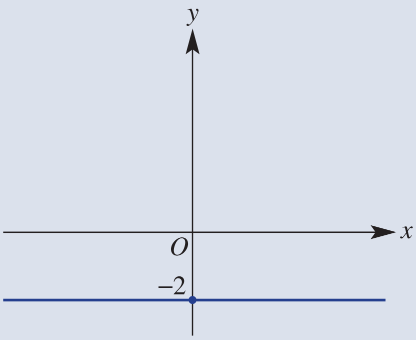
a



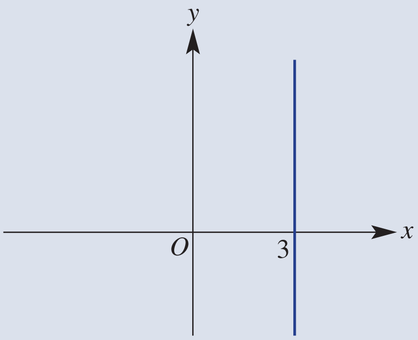
b



c



d



6E **8** Calculate the gradient of the line segment with end points $(-3, 2)$ and $(2, 6)$.

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6G $y = mx + c$ and special lines

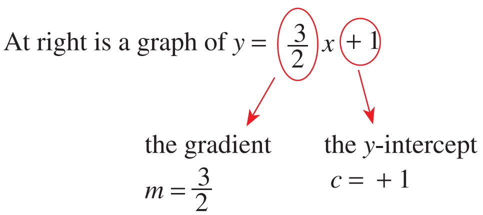
Learning intentions

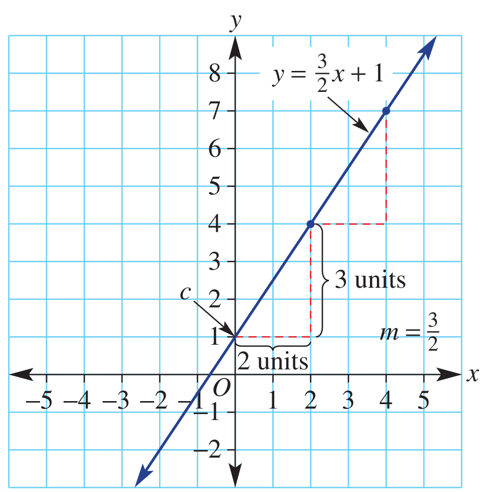
- To know that the y-intercept is the point where a graph crosses the y-axis
- To know how the gradient and y-intercept can be determined from a straight line equation
- To be able to sketch a straight-line graph using the gradient and the y-intercept
- To be able to sketch vertical and horizontal lines and lines passing through the origin from their equation
- To be able to determine if a point is on a line

Key vocabulary: gradient–intercept form, gradient, coefficient, y-intercept, horizontal, vertical

Most straight-line graphs can be described by a linear equation $y = mx + c$.

The gradient, m, is the coefficient of x, and c is the y-intercept. This is why this rule is called the gradient–intercept form.





Mathematicians use rules and graphs to help determine how many items should be manufactured to make the maximum profit. For example, profit would be reduced by making too many of a certain style of mobile phone that soon will be outdated. A knowledge of graphs is important in business.

Lesson starter: Matching lines with equations

Below are some equations of lines and some graphs. Work with a classmate and help each other to match each equation with its correct line graph.

a $y = 2x - 3$

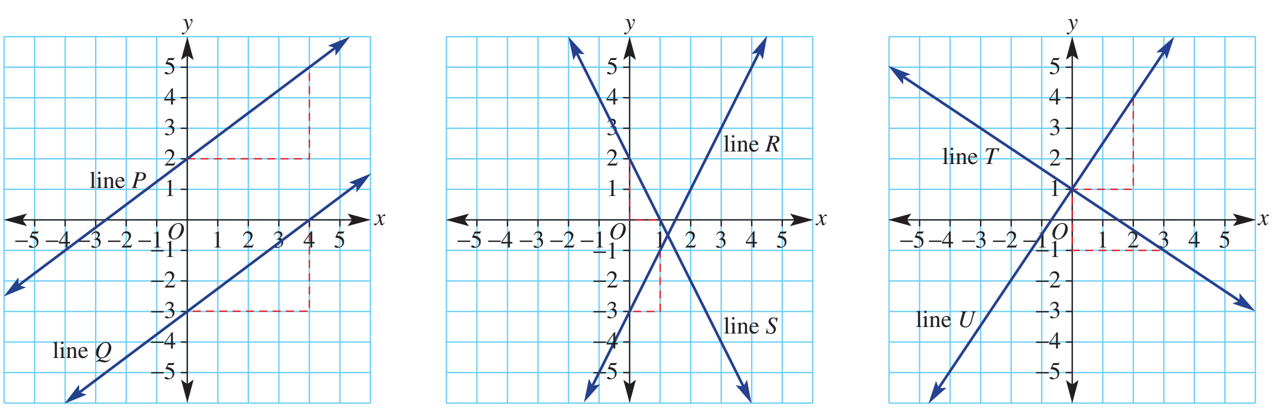
b $y = \frac{3}{4}x + 2$

c $y = -\frac{2}{3}x + 1$

d $y = -2x + 2$

e $y = \frac{3}{4}x - 3$

f $y = \frac{3}{2}x + 1$



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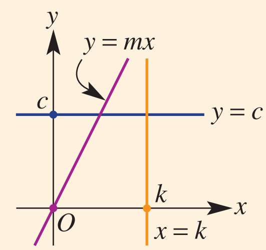
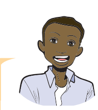
Key ideas

- $y = mx + c$ is called the **gradient–intercept form**, where m and c are constants.
  Examples are $y = 3x + 4$, $y = \frac{1}{2}x - 3$ and $y = 2$.
  - The **gradient** or slope equals m (the **coefficient** of x).
  - The ***y*-intercept** is the y value at the point where the line cuts the y-axis.
    In $y = mx + c$, the y-intercept is c.
- Some special lines include:
  - **horizontal** lines: $y = c$ $(m = 0)$
  - **vertical** lines: $x = k$ (m is undefined)
  - lines passing through the origin $(0, 0)$: $y = mx$ $(c = 0)$



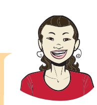
Exercise 6G

Understanding | 1–3 | 2, 3



1 Complete the sentences.
- **a** $y = mx + c$ is called the __________ — __________ form of a straight line.
- **b** The symbol m stands for the __________.
- **c** In the equation, the gradient, m, is the __________ of x.
- **d** The symbol c stands for the __________.
- **e** A horizontal line has a __________ gradient.

Hint:
Choose from: *gradient, intercept, zero, y-intercept, coefficient.*



2 Form rules of the form $y = mx + c$ for the following straight-line graphs:
- **a** gradient $= 3$, y-intercept $= -1$
- **b** gradient $= -\frac{3}{2}$, y-intercept $= 2$
- **c** gradient $= -1$, y-intercept $= 0$

Hint: In $y = mx + c$
m is the gradient
c is the y-intercept.

3 Identify the following special lines as: *horizontal*, *vertical* or *passes though the origin*.

a $y = 2$ **b** $x = -3$ **c** $y = 2x$ **d** $x = 0$ **e** $y = -x$

Fluency | 4, 5, 6(½) | 4–7(½)

Example 19 Reading the gradient and *y*-intercept from an equation

For the following equations, state:

i the gradient **ii** the y-intercept

a $y = 3x + 4$

b $y = -\frac{3}{4}x - 7$

Continued on next page

ISBN 978-1-108-87859-3

6G

Solution	Explanation
a **i** Gradient is 3.	The coefficient of x is 3; i.e. $m = 3$.
ii y-intercept is 4.	The value of c is $+4$.
b **i** Gradient is $-\frac{3}{4}$.	The coefficient of x is $-\frac{3}{4}$; i.e. $m = -\frac{3}{4}$.
ii y-intercept is -7.	The value of c is -7; don't forget to include the – sign.

Now you try

For the following equations, state the:

i gradient **ii** y-intercept

a $y = 2x - 5$

b $y = -\frac{1}{3}x + 2$

4 For each of the following equations, state the:

i gradient **ii** y-intercept

a $y = 2x + 4$ **b** $y = 6x - 7$

c $y = -\frac{2}{3}x + 7$ **d** $y = -7x - 3$

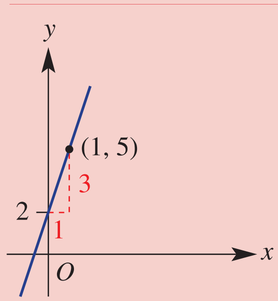
e $y = \frac{3}{5}x - 8$ **f** $y = 9x - 5$

Hint: The gradient is the coefficient of x, which is the number multiplied by x. It does not include the x. Write the y-intercept, including the sign of c.

Example 20 Sketching a line using the y-intercept and gradient

Sketch the graph of $y = 3x + 2$ by considering the y-intercept and the gradient.

Solution



Explanation

Consider $y = mx + c$; the value of c is 2 and therefore the y-intercept is 2.

The value of m is 3 and therefore the gradient is 3 or $\frac{3}{1}$.

Start at the y-intercept 2 and, with the gradient of $\frac{3}{1}$, move 1 right (run) and 3 up (rise) to the point (1, 5).

Join the points in a line.

Now you try

Sketch the graph of $y = 4x - 1$ by considering the y-intercept and the gradient.

5 Sketch the graph of the following by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = 3x - 12$

c $y = x + 4$ **d** $y = -2x + 5$

e $y = -5x - 7$ **f** $y = -x - 4$

Hint: Plot the y-intercept first. For a line with $m = -2$: $m = -2 = \frac{-2}{1} = \frac{\text{down } 2}{\text{right } 1}$. From the y-intercept, go right 1 then down 2 to plot the next point.

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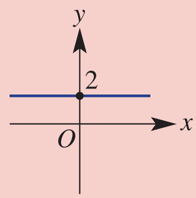
Example 21 Sketching special lines

Sketch the graphs of these equations.

a $y = 2$ **b** $x = -3$ **c** $y = -2x$

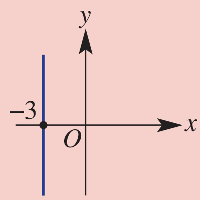
Solution | **Explanation**

a



Sketch a horizontal line passing through $(0, 2)$.

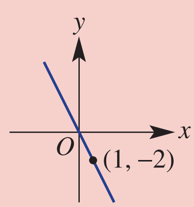
b



Sketch a vertical line passing through $(-3, 0)$.

c When $x = 0$, $y = -2(0) = 0$.

When $x = 1$, $y = -2(1) = -2$.



The line passes through the origin $(0, 0)$.

Use $x = 1$ to find another point.

Sketch the graph passing through $(0, 0)$ and $(1, -2)$.

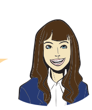
Now you try

Sketch the graphs of these equations.

a $y = -3$ **b** $x = 1$ **c** $y = 4x$

6 Sketch the following lines.

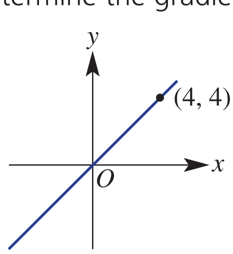
a $y = 4$ **b** $y = -2$ **c** $y = 5$
d $x = 5$ **e** $x = -2$ **f** $x = 9$
g $y = 3x$ **h** $y = 6x$ **i** $y = -2x$



Hint: For the line equation $y = 2$, every point on the line has a y value of 2; e.g. $(-3, 2)$ $(0, 2)$ $(1, 2)$ $(3, 2)$. For the line equation $x = -3$, every point on the line has an x value of -3; e.g. $(-3, 1)$ $(-3, 0)$ $(-3, -4)$.

7 Determine the gradient and y-intercept for the following lines.

a



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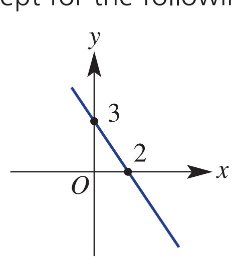

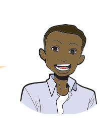

Hint: Use $m = \frac{\text{rise}}{\text{run}}$ for the gradient between two known points.

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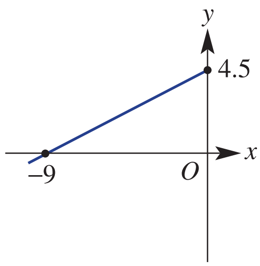
c



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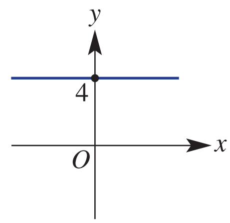

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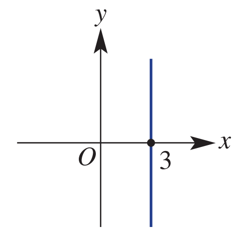


Problem-solving and reasoning

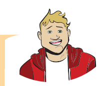
8, 9, 10(½), 12 | 8, 11–13

8 Match each of the following linear equations to one of the sketches shown.

a $y = -\frac{2}{3}x + 2$ **b** $y = -x + 4$ **c** $y = x + 3$

d $y = 2x + 4$ **e** $y = 4$ **f** $y = 7x$

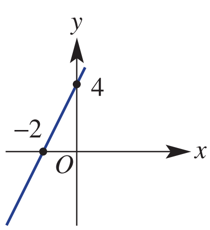
g $y = -3x + 6$ **h** $x = 2$ **i** $y = -3x$



Hint: A linear equation is an equation that gives a straight-line graph.

Hint: For a negative gradient, move the negative sign to the numerator.
$m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{down } 2}{\text{right } 3}$

i



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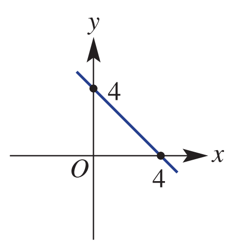

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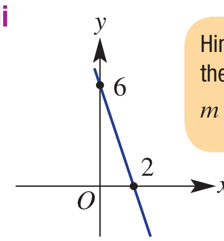

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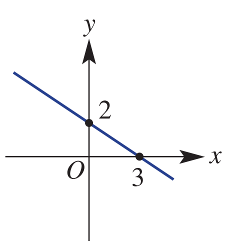

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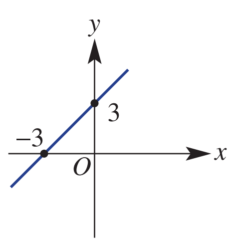

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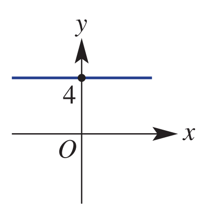

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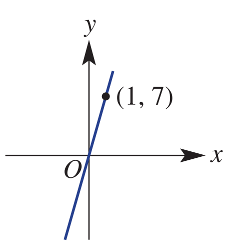

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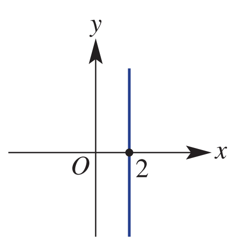

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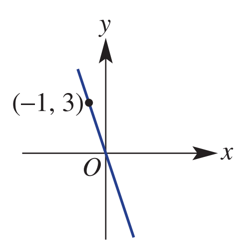

9 **a** Write down three different equations that have a graph with a y-intercept of 5.
b Write down three different equations that have a graph with a y-intercept of −2.

10 **a** Write down three different equations that have a graph with a gradient of 3.
b Write down three different equations that have a graph with a gradient of −1.
c Write down three different equations that have a graph with a gradient of 0.
d Write down three different equations that have a graph with an undefined gradient.

11 **a** Which of the following points lie on the line $y = 2$?
i (2, 3) **ii** (1, 2) **iii** (5, 2) **iv** (−2, −2)
b Which of the following points lie on the line $x = 5$?
i (5, 3) **ii** (3, 5) **iii** (1, 7) **iv** (5, −2)

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Example 22 Identifying points on a line

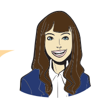
Does the point $(3, -4)$ lie on the line $y = 2x - 7$?

Solution	Explanation
$y = 2x - 7$	Copy the equation and substitute $x = 3$.
$y = 2 \times 3 - 7$	The y value for $x = 3$ is $y = -1$.
$y = -1$	Compare the y values.
$\neq -4$	The point $(3, -1)$ is *on* the line.
No, $(3, -4)$ is not on the line.	So $(3, -4)$ is *not* on the line.

Now you try

Does the point $(-2, 5)$ lie on the line $y = 3x + 2$?

12 **a** Does the point $(3, 2)$ lie on the line $y = x + 2$?
b Does the point $(-2, 0)$ lie on the line $y = x + 2$?
c Does the point $(1, -5)$ lie on the line $y = 3x + 2$?
d Does the point $(2, 2)$ lie on the line $y = x$?
e Does the line $y - 2x = 0$ pass through the origin?



Hint: Substitute the x value into the equation and compare the two y values. When the y values are the same, the point is on the line.

13 Draw each of the following on a number plane and write down the equation of the line.

a

x	0	1	2	3
y	4	5	6	7

b

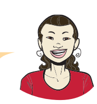
x	0	1	2	3
y	−1	0	1	2

c

x	−2	0	4	6
y	−1	0	2	3

d

x	−2	0	2	4
y	−3	1	5	9



Hint: Use your graph to find the gradient between two points (m) and locate the y-intercept (c). Then use $y = mx + c$.

Graphs using technology — 14, 15

14 Use technology to sketch a graph of these equations.

a $y = x + 2$ **b** $y = -4x - 3$ **c** $y = \frac{1}{2}x - 1$ **d** $y = 1.5x + 3$

e $y = 2x - 5$ **f** $y = 0.5x + 5$ **g** $y = -0.2x - 3$ **h** $y = 0.1x - 1.4$

15 **a** On the same set of axes, plot graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$ and $y = 2x - 3$, using a calculator.
Discuss what you see and describe the connection with the given equations.

b On the same set of axes, plot graphs of $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$, $y = \frac{1}{2}x - 1$ and $y = \frac{3}{4}x - 1$, using a calculator.
Discuss what you see and describe the connection with the given equations.

c The equations of families of graphs can be entered into a calculator using one line only. For example, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$ can be entered as $y = 2x + \{1, 2, 3\}$ using set brackets. Use this notation to draw the graphs of the rules in parts **a** and **b**.

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6H Parallel and perpendicular lines

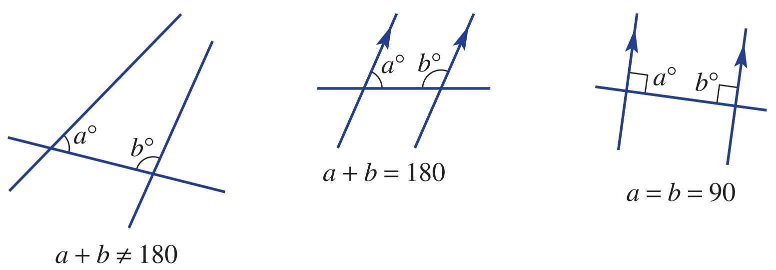
Learning intentions

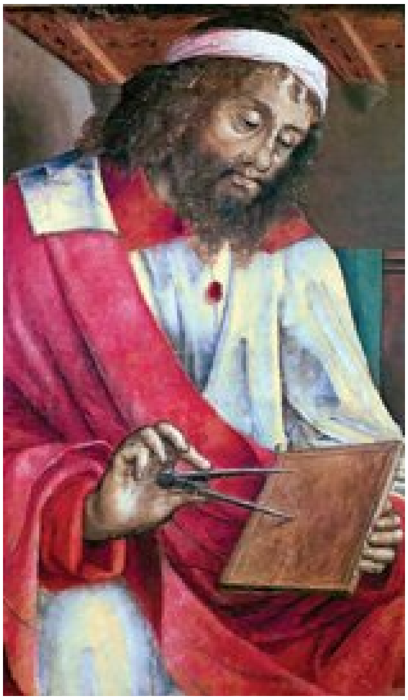
- To know that parallel lines have the same gradient
- To know the relationship between the gradients of perpendicular lines
- To be able to determine if lines are parallel or perpendicular using their gradients
- To be able to find the equation of parallel or perpendicular lines

Key vocabulary: parallel, perpendicular, gradient, reciprocal, y-intercept

Euclid of Alexandria (300 BCE) was a Greek mathematician and is known as the 'Father of geometry'. In his texts, known as *Euclid's Elements*, his work is based on five simple axioms.

His fifth axiom, the Parallel Postulate, says that if cointerior angles do not sum to 180° then the two lines are not parallel. Furthermore, if the two interior angles are equal and also sum to 180°, then the third line must be perpendicular to the other two.



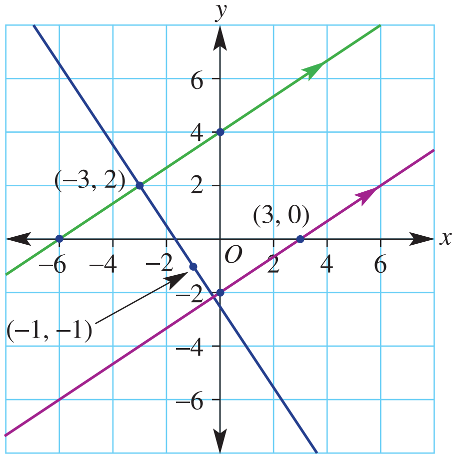


Parallel and perpendicular lines, including their gradient and rule, are the focus of this section.

Lesson starter: Gradient connection

Shown here is a pair of parallel lines and a third line that is perpendicular to the other two lines.

- Find the gradient of each line using the coordinates shown on the graph.
- What is common about the gradients for the two parallel lines?
- Is there any connection between the gradients of the parallel lines and the perpendicular line? Can you write down this connection as a formula?



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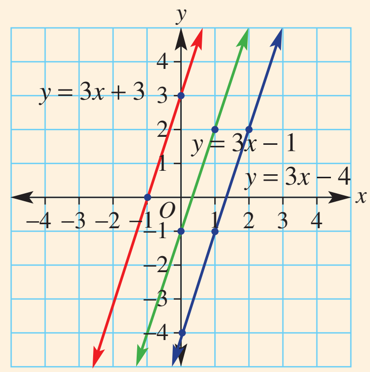
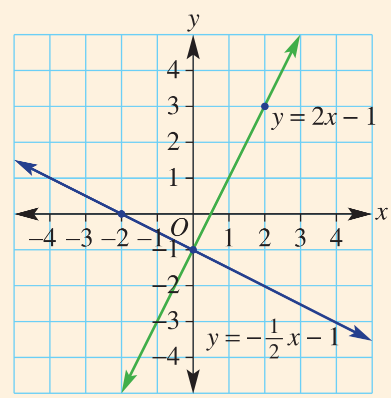
Key ideas

- Two **parallel lines** have the same gradient as they point in the same direction.
For example, $y = 3x - 1$ and $y = 3x + 3$ have the same gradient of 3.



- Two **perpendicular lines** (lines that are at right angles to each other) with gradients m_1 and m_2 satisfy the following rule:
$m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$ (i.e. m_2 is the negative reciprocal of m_1).
In the graph shown, $m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$.



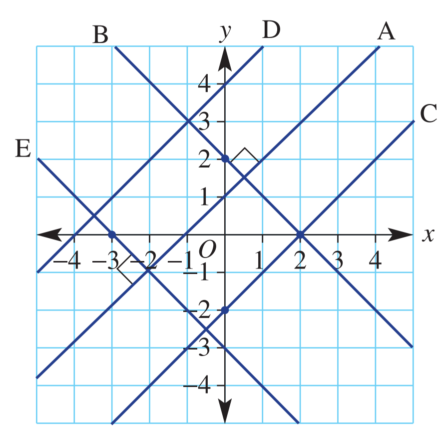
- Equations of parallel or perpendicular lines can be found by:
  - first finding the gradient (m)
  - then substituting a point to find c in $y = mx + c$

Exercise 6H

Understanding

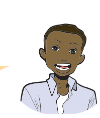
1–3 | 3

1 For this diagram:
- **a** Which lines, B, C, D or E, are parallel to line A?
- **b** Which lines, B, C, D or E, are perpendicular to line A?
- **c** Are lines C and D parallel?
- **d** Are lines B and E parallel?
- **e** Are lines E and C perpendicular?



2 Write down the gradient of a line that is parallel to the graph of these equations.
- **a** $y = 4x - 6$
- **b** $y = -7x - 1$
- **c** $y = -\frac{3}{4}x + 2$
- **d** $y = \frac{8}{7}x - \frac{1}{2}$

3 Use $m_2 = -\frac{1}{m_1}$ to find the gradient of the line that is perpendicular to the graphs of these equations.
- **a** $y = 3x - 1$
- **b** $y = -2x + 6$
- **c** $y = \frac{7}{8}x - \frac{2}{3}$
- **d** $y = -\frac{4}{9}x - \frac{4}{7}$



Hint: In part **a**, $m_1 = 3$ so find m_2, which is the perpendicular gradient. Note:
$-\frac{1}{\left(\frac{7}{8}\right)} = -1 \div \frac{7}{8} = -1 \times \frac{8}{7}$

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6H

Fluency 4–6(½) 4–6(½)

Example 23 Deciding if lines are parallel or perpendicular

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = 4x + 2$ and $y = 4x - 6$

b $y = -3x - 8$ and $y = \frac{1}{3}x + 1$

Solution	**Explanation**
a $y = 4x + 2, m = 4$ (1) $y = 4x - 6, m = 4$ (2)	Note that both equations are in the form $y = mx + c$.
So the lines are parallel.	Both lines have a gradient of 4, so the lines are parallel.
b $y = -3x - 8, m = -3$ (1)	Both equations are in the form $y = mx + c$.
$y = \frac{1}{3}x + 1, m = \frac{1}{3}$ (2)	
$-3 \times \frac{1}{3} = -1$	Test $m_1 \times m_2 = -1$.
So the lines are perpendicular.	

Now you try

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = -3x + 2$ and $y = -3x - 7$

b $y = 4x - 2$ and $y = -\frac{1}{4}x + 1$

4 Decide if the line graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = 2x - 1$ and $y = 2x + 1$

b $y = 3x + 3$ and $y = -\frac{1}{3}x + 1$

c $y = 5x + 2$ and $y = 6x + 2$

d $y = -4x - 1$ and $y = \frac{1}{5}x - 1$

e $y = 3x - 1$ and $y = 3x + 7$

f $y = \frac{1}{2}x - 6$ and $y = \frac{1}{2}x - 4$

g $y = -\frac{2}{3}x + 1$ and $y = \frac{2}{3}x - 3$

h $y = -4x - 2$ and $y = x - 7$

i $y = -\frac{3}{7}x - \frac{1}{2}$ and $y = \frac{7}{3}x + 2$

j $y = -8x + 4$ and $y = \frac{1}{8}x - 2$

Hint: If the gradients are equal, then the lines are parallel. If $m_1 \times m_2 = -1$, then the lines are perpendicular.

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Example 24 Finding the equation of a parallel or perpendicular line when given the y-intercept

Find the equation of the line, given the following description.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 3.
b A line passes through $(0, -1)$ and is perpendicular to another line with gradient -2.

Solution	Explanation
a $m = 3$ and $c = 2$	A parallel line has the same gradient.
So $y = 3x + 2$.	Use $y = mx + c$ with $m = 3$ and y-intercept 2.
b $m = -\left(\frac{1}{-2}\right) = \frac{1}{2}$ and $c = -1$	Being perpendicular, use $m_2 = -\frac{1}{m_1}$.
So $y = \frac{1}{2}x - 1$.	Note also that the y-intercept is -1.

Now you try

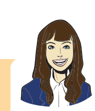
Find the equation of the line, given the following description.

a A line passes through $(0, 4)$ and is parallel to another line with gradient 2.
b A line passes through $(0, 3)$ and is perpendicular to another line with gradient 4.

5 Find the equation of the lines with the following description.

Hint: In $y = mx + c$, m is the gradient and c is the y-intercept.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 4.
b A line passes through $(0, 4)$ and is parallel to another line with gradient 2.
c A line passes through $(0, -3)$ and is parallel to another line with gradient -1.
d A line passes through $(0, 3)$ and is perpendicular to another line with gradient 2.
e A line passes through $(0, -5)$ and is perpendicular to another line with gradient 3.
f A line passes through $(0, -10)$ and is perpendicular to another line with gradient $\frac{1}{2}$.
g A line passes through $(0, 6)$ and is perpendicular to another line with gradient $\frac{1}{6}$.
h A line passes through $(0, -7)$ and is perpendicular to another line with gradient $-\frac{1}{4}$.



Hint: $-\frac{1}{\left(\frac{1}{2}\right)} = -2$

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6H

Example 25 Finding the equation of a parallel or perpendicular line

Find the equation of the line that is:

a parallel to $y = -2x - 7$ and passes through $(1, 9)$

b perpendicular to $y = \frac{1}{4}x - 1$ and passes through $(3, -2)$

Solution	Explanation
a $y = mx + c$ $m = -2$ $y = -2x + c$	Since the line is parallel to $y = -2x - 7$, $m = -2$. Write the equation $y = mx + c$ with $m = -2$.
Substitute $(1, 9)$: $9 = -2(1) + c$ $11 = c$ $\therefore y = -2x + 11$	Substitute the given point $(1, 9)$ where $x = 1$ and $y = 9$ and solve for c.
b $y = mx + c$	
$m = -\dfrac{1}{\frac{1}{4}}$	The gradient is the negative reciprocal of $\frac{1}{4}$.
$= -1 \times \frac{4}{1}$	$-1 \div \frac{1}{4} = -1 \times \frac{4}{1}$.
$= -4$ $y = -4x + c$	
Substitute $(3, -2)$: $-2 = -4(3) + c$ $-2 = -12 + c$ $c = 10$ $\therefore y = -4x + 10$	Substitute $(3, -2)$ and solve for c.

Now you try

Find the equation of the line that is:

a parallel to $y = 3x - 4$ and passes through $(2, 3)$

b perpendicular to $y = -\frac{1}{2}x + 2$ and passes through $(-1, 4)$

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6 Find the equation of the line that is:

a parallel to $y = x + 3$ and passes through $(1, 5)$

b parallel to $y = -x - 5$ and passes through $(1, 7)$

c parallel to $y = -4x - 1$ and passes through $(-1, 3)$

d parallel to $y = \frac{2}{3}x + 1$ and passes through $(3, -4)$

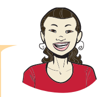
e perpendicular to $y = 2x + 3$ and passes through $(2, 5)$

f perpendicular to $y = -4x + 1$ and passes through $(-4, -3)$

g perpendicular to $y = \frac{2}{3}x - 4$ and passes through $(4, -1)$

h perpendicular to $y = -\frac{2}{7}x - \frac{3}{4}$ and passes through $(-8, 3)$

Hint: First find m. Then write $y = mx + c$. Substitute a point to find c; e.g. for $(1, 5)$ substitute $x = 1$ and $y = 5$.



Problem-solving and reasoning

	7, 8	7, 9, 10

7 The lines on this grid are either parallel or perpendicular to each other.

a What is the gradient of each of these lines?

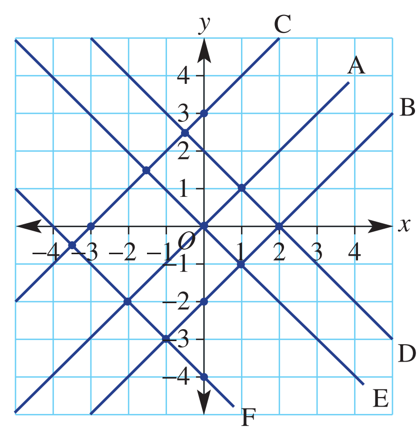
i A **ii** D

b What is the equation for the following lines?

i A **ii** B

iii C **iv** D

v E **vi** F



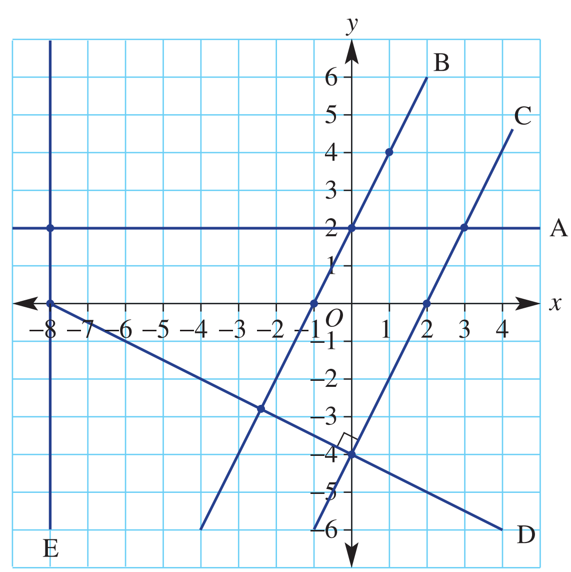
8 In its original position, the line A has equation $y = 2$, as shown.

a Line A is rotated to form line B. What is its new rule?

b Line B is shifted to form line C. What is its new rule?

c Line C is rotated 90° to form line D. What is its new rule?

d Line A is rotated 90° to form line E. What is its new rule?



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6H

9 Recall that the negative reciprocal of, say, $\frac{2}{3}$ is $-\frac{3}{2}$.
Use this to help find the equation of a line that:

a passes through $(0, 7)$ and is perpendicular to $y = \frac{2}{3}x + 3$

b passes through $(0, -2)$ and is perpendicular to $y = \frac{3}{2}x + 1$

c passes through $(0, 2)$ and is perpendicular to $y = -\frac{4}{5}x - 3$

d passes through $(1, -2)$ and is perpendicular to $y = -\frac{2}{3}x - 1$

10 Decide if the graphs of each pair of rules will be parallel, perpendicular or neither.

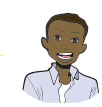
a $2y + x = 2$ and $y = -\frac{1}{2}x - 3$

b $x - y = 4$ and $y = x + \frac{1}{2}$

c $8y + 2x = 3$ and $y = 4x + 1$

d $3x - y = 2$ and $x + 3y = 5$

Hint: First write each rule in the form $y = mx + c$.



Perpendicular and parallel geometry — 11, 12

11 A quadrilateral, $ABCD$, has vertex coordinates $A(2, 7)$, $B(4, 9)$, $C(6, 3)$ and $D(8, 5)$.

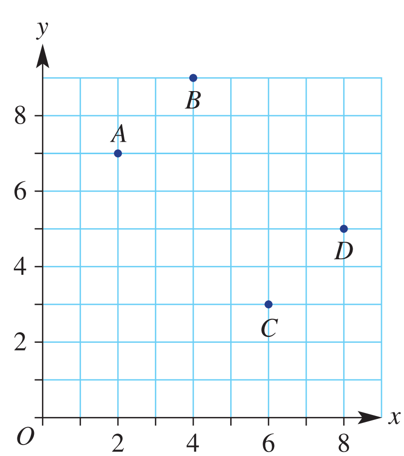
a Find the gradient of these line segments.

i AB **ii** CD

iii BD **iv** AC

b What do you notice about the gradient of the opposite sides?

c What type of quadrilateral is $ABCD$?



12 The vertices of triangle ABC are $A(0, 0)$, $B(3, 4)$ and $C\left(\frac{25}{3}, 0\right)$.

a Find the gradient of these line segments.

i AB **ii** BC **iii** CA

b What type of triangle is $\triangle ABC$?

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6I Sketching with *x*- and *y*-intercepts

Learning intentions

- To know that the x-intercept is the point where a graph crosses the x-axis
- To know how to find the x and y intercepts of a straight-line graph from its equation
- To know that only two points are needed to sketch a straight line
- To be able to sketch a linear graph using its x and y intercepts

Key vocabulary: x-intercept, y-intercept, x-axis, y-axis

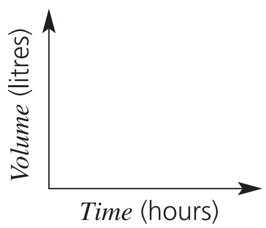
Only two points are required to define a straight line. Two convenient points are the x- and y-intercepts. These are the points where the graph crosses the x- and y-axis, respectively.

The axis intercepts are quite significant in practical situations. For example, imagine that 200 m^3 of dirt needs to be removed from a construction site before foundations for a new building can be laid. The graph of volume remaining versus time taken to remove the dirt has a y-intercept of 200, showing the total volume to be removed, and an x-intercept showing the time taken for the job.

Lesson starter: Leaking water

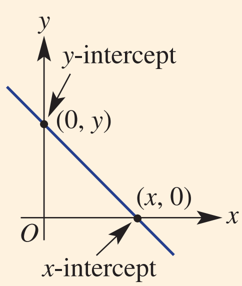
A family that is camping has 20 litres of water in a plastic container. The water begins to slowly leak at a rate of 2 litres per hour.

- Sketch two axes labelled 'Volume' and 'Time'.
- At $t = 0$, what is the volume of water in the container? Mark and label this point on the Volume axis.
- How long will it take the water container to be empty? Mark this point on the Time axis.
- Join these two points with a straight line.
- Write the coordinates of the Volume axis intercept and the Time axis intercept. Follow the order (i.e. Time, Volume).
- Can you suggest a rule for finding the volume of the water in the container after t hours?



Key ideas

- A straight line can be sketched by finding axis intercepts.
- The ***x*-intercept** (where the line cuts the x-axis) is where $y = 0$.
  Find the x-intercept by substituting $y = 0$ into the equation.
- The ***y*-intercept** is where $x = 0$.
  Find the y-intercept by substituting $x = 0$ into the equation.
- Once the axis intercepts are found, plot the points and join to form the straight-line graph.



Exercise 6I

Understanding	1, 2	1

1 Copy and complete these sentences.

a The x-intercept is where _____ $= 0$. **b** The y-intercept is where _____ $= 0$.

2 Plot the following x- and y-intercept coordinates and join in a straight line to form the graph.

a $(0, 3)$ and $(-2, 0)$ **b** $(0, -1)$ and $(2, 0)$ **c** $(0, -4)$ and $(-1, 0)$ **d** $(0, 3)$ and $(5, 0)$

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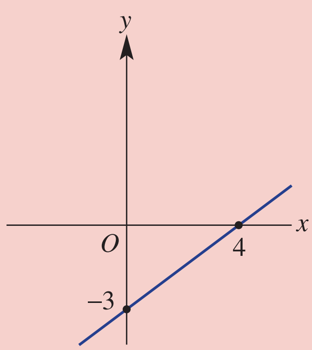
Fluency

3(½), 4, 5 | 3(½), 4, 5(½), 6

Example 26 Sketching lines in the form $ax + by = d$, using x- and y-intercepts

Sketch a graph of $3x - 4y = 12$ by finding the x- and y-intercepts.

Solution		Explanation
$3x - 4y = 12$		
x-intercept ($y = 0$):	$3x - 4(0) = 12$	Find the x-intercept by substituting $y = 0$.
	$3x = 12$	Simplify. Any number multiplied by 0 is 0.
	$x = 4$	Divide both sides by 3 to solve for x.
y-intercept ($x = 0$):	$3(0) - 4y = 12$	Find the y-intercept by substituting $x = 0$.
	$-4y = 12$	Simplify and retain the negative sign.
	$y = -3$	Divide both sides by −4 to solve for y.



Sketch the graph by first marking the x-intercept (4, 0) and the y-intercept (0, −3) and join them with a line.

Now you try

Sketch a graph of $5x + 3y = 15$ by finding the x- and y-intercepts.

3 Sketch graphs of the following equations by finding the x- and y-intercepts.

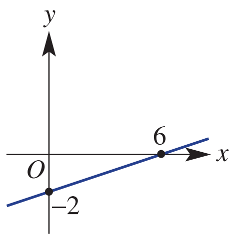
a $3x - 2y = 6$ **b** $2x + 6y = 12$
c $3x - 4y = 12$ **d** $5x - 2y = 20$
e $-2x + 7y = 14$ **f** $-x + 3y = 3$
g $-x - 2y = 8$ **h** $-5x - 9y = 90$

Hint: Two calculations are required: Substitute $y = 0$ and solve for x-intercept. Substitute $x = 0$ and solve for y-intercept.

4 Match each of the following linear equations of the form $ax + by = d$ to one of the graphs shown.

a $x + y = 3$ **b** $2x - y = 4$ **c** $x - 3y = 6$

i



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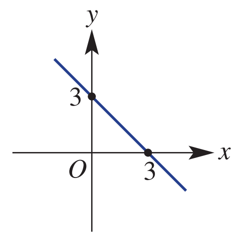

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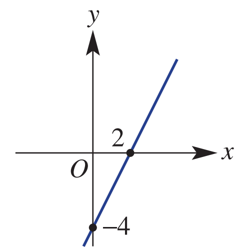

Hint: First find the x- and y-intercepts for the line equation.

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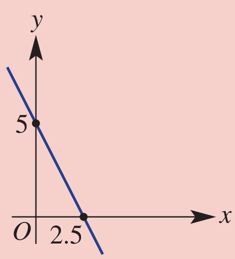
Example 27 Sketching lines in the form $y = mx + c$, using x- and y-intercepts

Sketch the graph of $y = -2x + 5$ by finding the x- and y-intercepts.

Solution		Explanation
$y = -2x + 5$		
x-intercept ($y = 0$):	$0 = -2x + 5$	Substitute $y = 0$.
	$-5 = -2x$	Subtract 5 from both sides.
	$x = 2.5$	Divide both sides by -2.
y-intercept is ($x = 0$):	$y = -2(0) + 5$	Substitute $x = 0$.
	$= 5$	Simplify.



Sketch the graph by first marking the x-intercept $(2.5, 0)$ and the y-intercept $(0, 5)$.

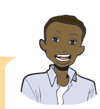
Now you try

Sketch the graph of $y = 3x - 9$ by finding the x- and y-intercepts.

5 Sketch graphs of the following equations by finding the x- and y-intercepts.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = -4x - 3$

d $y = -x + 2$ **e** $y = -\frac{1}{2}x + 1$ **f** $y = \frac{3}{2}x - 3$



Hint: **5e:** $0 = -\frac{1}{2}x + 1$

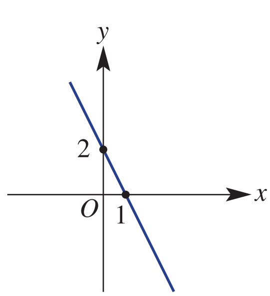
$\frac{1}{2}x = 1$

Now multiply both sides by 2.

6 Match each of the following linear equations of the form $y = mx + c$ to one of the sketches shown.

a $y = x + 1$ **b** $y = 3x - 1$ **c** $y = -2x + 2$

i



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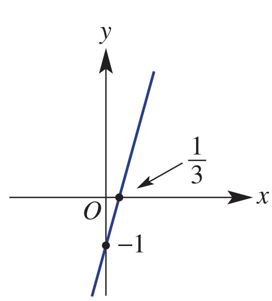

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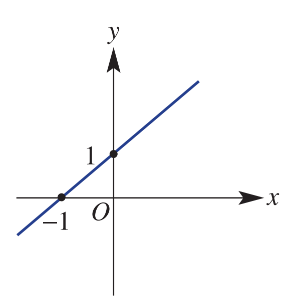

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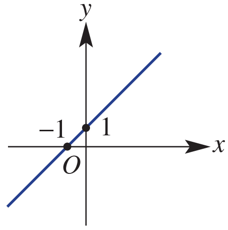
6I

Problem-solving and reasoning 7–9 8–11

7 Match each of the following linear equations to one of the graphs shown.

a $2x + y = 4$
b $x - y = 3$
c $y = x + 1$
d $y = -2x - 3$
e $3x - 5y = 15$
f $y = \frac{2}{5}x - 1$

i


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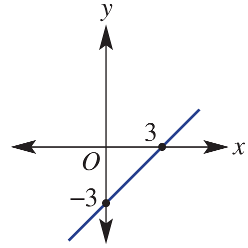

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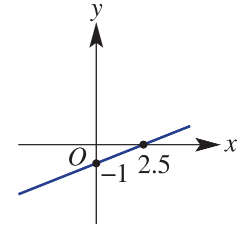

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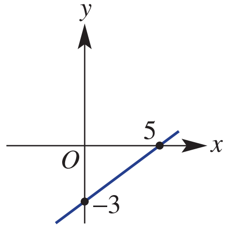

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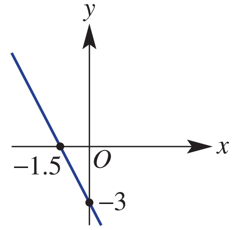

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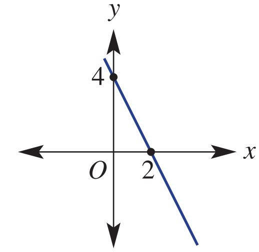
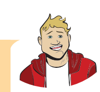


8 By first finding the x- and y-intercepts of the graphs of these equations, find the gradient in each case.

a $2x + y = 4$
b $x - 5y = 10$
c $4x - 2y = 5$
d $-1.5x + 3y = 4$



Hint: A quick sketch of each line and the axis intercepts will help to show the rise and run.

9 For the graphs of each of the following equations, find:

i the x- and y-intercepts
ii the area of the triangle enclosed by the x- and y-axes and the graph of each equation

Remember that the area of a triangle is $A = \frac{1}{2}bh$.

a $2x - y = 4$
b $-3x + 3y = 6$
c $y = -2x - 3$
d $y = \frac{1}{2}x + 2$

10 The height, h, in metres, of a lift above ground after t seconds is given by $h = 90 - 12t$.

a How high is the lift initially (i.e. at $t = 0$)?
b How long does it take for the lift to reach the ground (i.e. at $h = 0$)?

11 If $ax + by = d$, find a set of numbers for a, b and d that give an x-intercept of $(2, 0)$ and y-intercept of $(0, 4)$.

Hint: Use trial and error to start.

Axes intercepts using technology — 12

12 For the following rules, use technology to sketch a graph and find the x- and y-intercepts.

a $y = 2x - 4$
b $y = -2x - 10$
c $y = -x + 1$
d $y + 2x = 4$
e $2y - 3x = 12$
f $3y - 2x = 2$

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6J Linear modelling ★

Learning intentions

- To be able to find the equation of a straight-line graph
- To be able to form a linear model from a word problem relating two variables and sketch its graph
- To be able to use a linear model to solve a problem

Key vocabulary: linear, gradient, y-intercept, modelling

When given at least two points, you can find the equation of a straight line.
If the relationship between two variables is linear, then:

- the graph of the relation is a straight line
- a rule can be written in the form $y = mx + c$.

Lesson starter: Trainee pay

Isabella has a trainee scholarship to complete her apprenticeship as a mechanic. She is paid $\$50$ per week plus $\$8$/h for work at the garage.

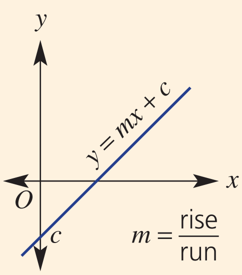
Isabella's weekly wage can be modelled by the rule:

Wage $= 8t + 50$, where t is the number of hours worked in a week.

- Explain why the rule for Isabella's wage is Wage $= 8t + 50$.
- Show how the rule can be used to find Isabella's wage after $10, 20$ and 35 hours.
- Show how the rule can be used to find how long Isabella worked if she earned $\$114$, $\$202$ and $\$370$.

Key ideas

- The equation of a straight line can be determined using:
  - $y = mx + c$
  - gradient $= m = \frac{\text{rise}}{\text{run}}$
  - y-intercept $= c$



- If the y-intercept is not obvious, then it can be found by substituting a point.
- Vertical and horizontal lines:
  - vertical lines have the equation $x = k$, where k is the x-intercept
  - horizontal lines have the equation $y = c$, where c is the y-intercept
- **Modelling** may involve:
  - writing a rule linking two variables
  - sketching a graph
  - using the rule or the graph to help solve related problems

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6J

Exercise 6J

Understanding 1, 2 2

1 Each week Ava gets paid \$30 plus \$15 per hour. Decide which rule shows the relationship between Ava's total weekly pay, P, and the number of hours she works, n.

A $P = 30 + n$ B $P = 15n$ C $P = 30 + 15$ D $P = 30 + 15n$

2 Riley is 100 km from home and is cycling home at 20 km/h. Decide which rule shows the relationship between Riley's distance from home, d km, and the number of hours he has been cycling, t.

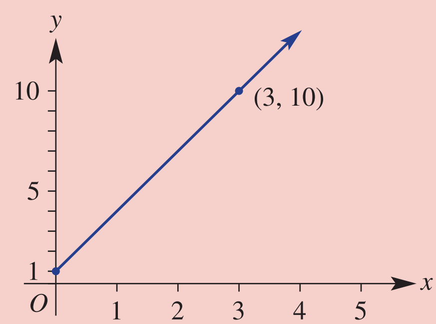
A $P = 100t$ B $P = 100 - 20$ C $P = 100 - 20t$ D $P = 100 + 20t$

Fluency 3–6(½) 3–7(½)

Example 28 Finding the equation of a line from a graph with a known y-intercept

A straight line passes through the points shown.

a Determine its gradient.
b Find the y-intercept.
c Write the equation of the line.



Solution

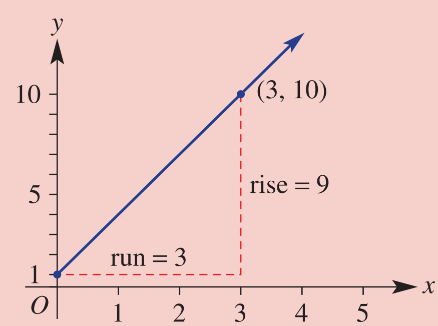
Explanation

a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{9}{3}$

$= 3$

Draw a triangle on the graph and decide whether the gradient is positive or negative.



b The y-intercept is 1, so $c = 1$.

Look at where the graph meets the y-axis.

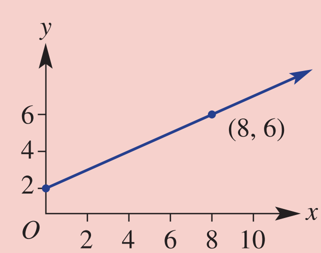
c $y = 3x + 1$

Substitute m and c into $y = mx + c$.

Now you try

A straight line passes through the points shown.

a Determine its gradient.
b Find the y-intercept.
c Write the equation of the line.



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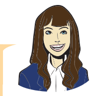
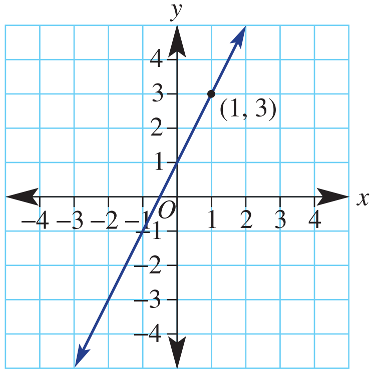
3 The graphs below show straight lines.
- **i** Determine the gradient of each.
- **ii** Find the y-intercept.
- **iii** Write the equation of the line.

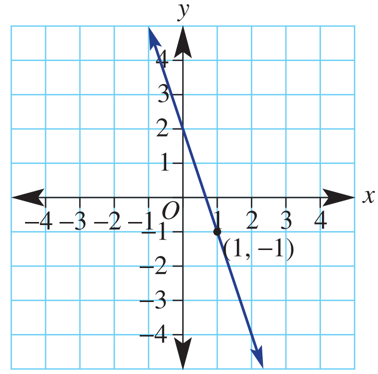


Hint: To find the rise and run, form a right-angled triangle using the y-intercept and the second point.

a



b



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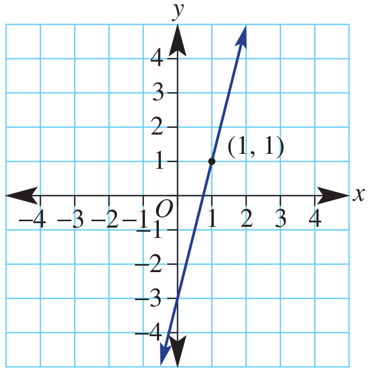

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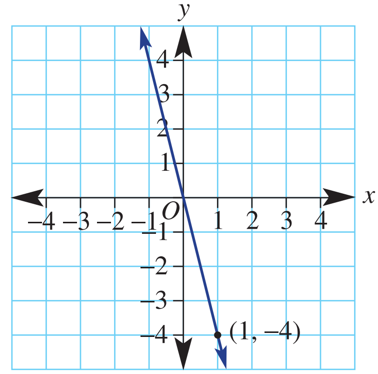

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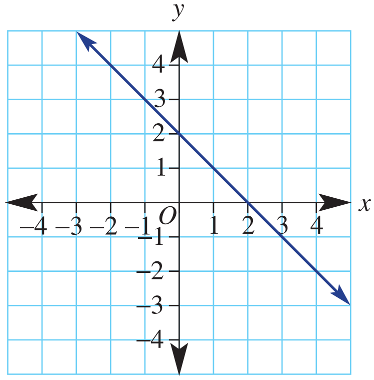

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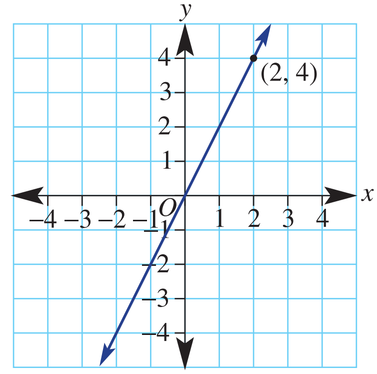
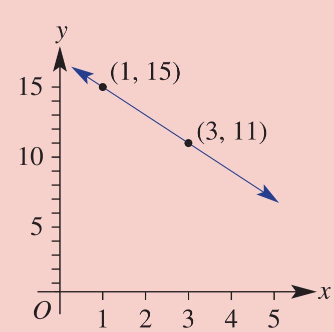


Example 29 Finding the equation of a line when given a graph with two known points

A straight line passes through the points shown.
- **a** Determine the gradient.
- **b** Find the y-intercept.
- **c** Write the equation of the line.



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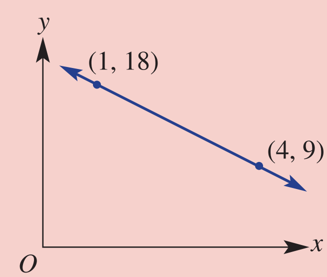
6J

Solution	Explanation
a $m = \frac{\text{rise}}{\text{run}}$	The gradient is negative.
$= \frac{-4}{2}$	Run $= 3 - 1 = 2$ Rise $= 11 - 15 = -4$ A 'fall' of 4 means rise $= -4$.
$= -2$	Simplify.
b $y = -2x + c$	Write $y = mx + c$ using $m = -2$.
$11 = -2(3) + c$ $11 = -6 + c$	Substitute a chosen point into $y = -2x + c$, (use $(3, 11)$ or $(1, 15)$). Here, $x = 3$ and $y = 11$.
$17 = c$, so the y-intercept is 17.	Simplify and solve for c.
c $y = -2x + 17$	Substitute $m = -2$ and $c = 17$ into $y = mx + c$.

Now you try

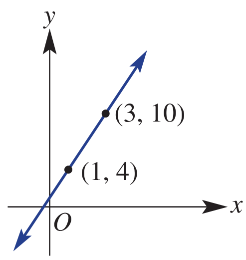
A straight line passes through the points shown.

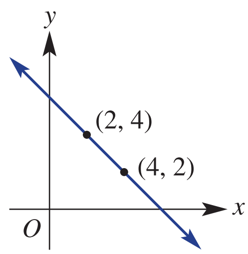
a Determine the gradient.
b Find the y-intercept.
c Write the equation of the line.

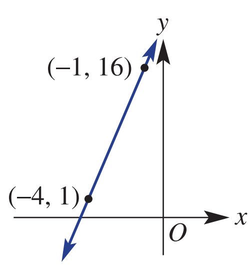


4 Straight lines passing through two points are shown below.
  i Determine the gradient.
  ii Find the y-intercept.
  iii Write the equation of the line.

Hint:
For m, find the rise and run between the two given points.
Choose either point to substitute when finding c. If $m = 4$ and $(3, 5) = (x, y)$:
$y = mx + c.$
$5 = 4 \times 3 + c$
Solve for c.

a


b


c


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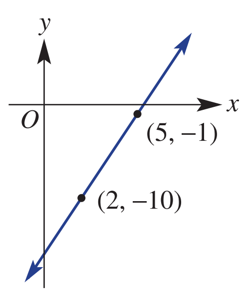

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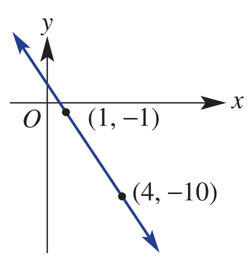

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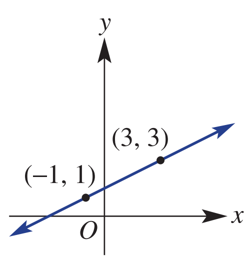

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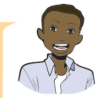
5 Determine the equation of each of the following lines. Remember from Section 6G that vertical and horizontal lines have special equations.

Hint: Vertical lines cut the x-axis and have an equation such as $x = 3$. Horizontal lines cut the y-axis and have an equation such as $y = -4$.



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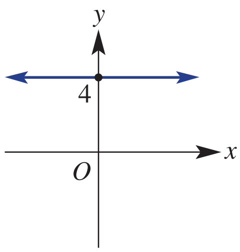

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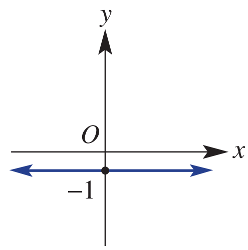

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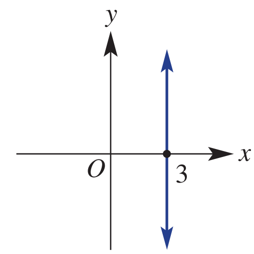

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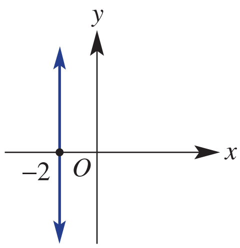

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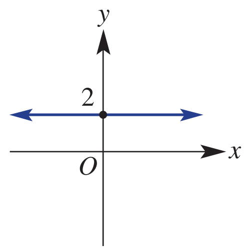

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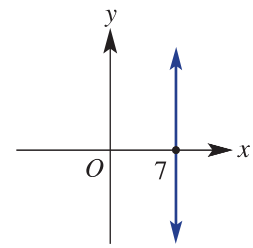
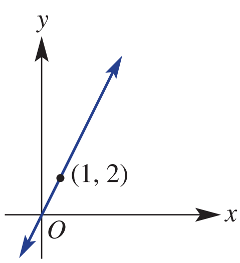


6 Remember that equations of graphs that pass through the origin are of the form $y = mx$ (since $c = 0$). Find the equation of each of these graphs.

a


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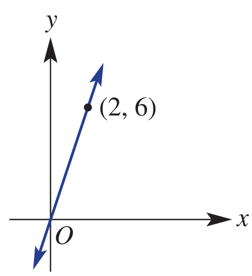

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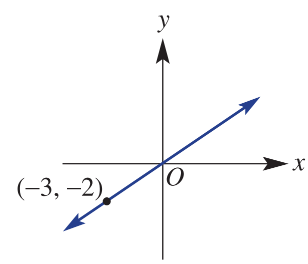

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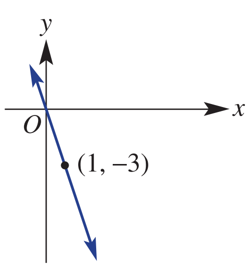

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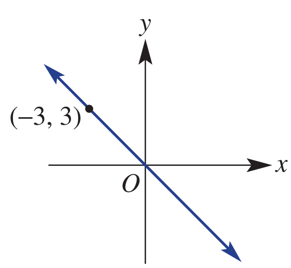

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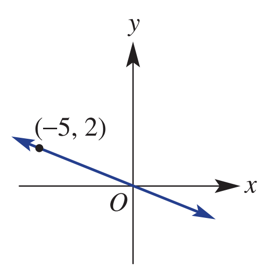
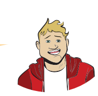


7 For the line joining the following pairs of points, find:

i the gradient **ii** the equation of the line

Hint: Substitute the gradient and a point in $y = mx + c$ to find c in part **ii**.



a $(0, 0)$ and $(1, 7)$ **b** $(0, 0)$ and $(2, -3)$
c $(-1, 1)$ and $(1, 3)$ **d** $(-2, 3)$ and $(2, -3)$
e $(-4, 2)$ and $(7, 2)$ **f** $(3, -3)$ and $(3, 1)$

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6J

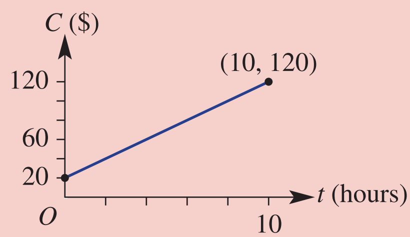
Problem-solving and reasoning

8, 9 | 8, 10, 11

Example 30 Modelling with linear graphs

An employee gets paid \$20 plus \$10 for each hour of work. If he earns \$$C$ for t hours of work, complete the following.

a Write a rule for C in terms of t.
b Sketch a graph, using t between 0 and 10.
c Use your rule to find:
  i the amount earned after working 8 hours
  ii the number of hours worked if \$180 is earned

Solution	Explanation
a $C = 10t + 20$	\$10 is earned for each hour and \$20 is a fixed amount.
b 	20 is the y-intercept and the gradient is $10 = \frac{10}{1}$. For $t = 10$, $C = 10(10) + 20 = 120$.
c i $C = 10(8) + 20$	Substitute $t = 8$ into $C = 10t + 20$.
$= 100$	Simplify.
\$100 is earned.	Write your answer in words.
ii $180 = 10t + 20$	Substitute $C = 180$ into $C = 10t + 20$.
$160 = 10t$	Subtract 20 from both sides.
$t = 16$	Divide both sides by 10.
16 hours of work is completed.	Write your answer in words.

Now you try

A tennis court costs \$100 to hire plus \$20 per hour of use. If \$$C$ is the cost for t hours of hire, complete the following.

a Write a rule for C in terms of t.
b Sketch a graph, using t between 0 and 5.
c Use your rule to find:
  i the cost of hiring for 3 hours
  ii the number of hours of hire if the cost is \$260

8 A seasonal worker gets paid \$10 plus \$2 per kg of tomatoes that they pick. If the worker earns \$$P$ for n kg of tomatoes picked, complete the following.
  a Write a rule for P in terms of n.
  b Sketch a graph of P against n for n between 0 and 10.
  c Use your rule to find:
    i the amount earned after picking 9 kg of tomatoes
    ii the number of kilograms of tomatoes picked if the worker earns \$57

Hint: rate of pay → $y = mx + c$; y ← P, x ← n, c ← fixed amount

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9 An architect charges $100 for the initial consultation plus $60 per hour thereafter. If the architect earns $A for t hours of work, complete the following.

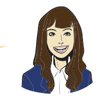
a Write a rule for A in terms of t.

b Sketch a graph of A against t for t between 0 and 15.

c Use your rule to find:

i the amount earned after working for 12 hours

ii the number of hours worked if the architect earns $700



Hint:
Draw a line between the points at $t = 0$ and $t = 15$.

10 A man's weight when holding two empty buckets of water is 80 kg. 1 kg is added for each litre of water poured into the buckets. If the man's total weight is W kg with l litres of water, complete the following.

a Write a rule for W in terms of l.

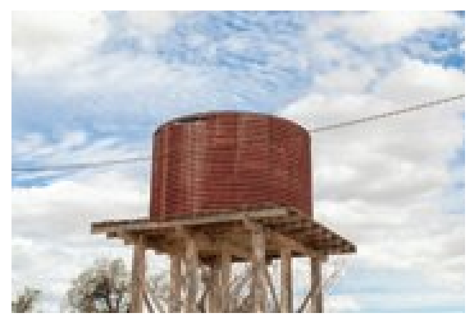
b Sketch a graph of W against l for l between 0 and 20.

c Use your rule to find:

i the man's weight after 7 litres of water are added

ii the number of litres of water added if the man's weight is 109 kg

11 The amount of water (W litres) in a leaking tank after t hours is given by the rule $W = -2t + 1000$.



a State the gradient and y-intercept for the graph of the rule.

b Sketch a graph of W against t for t between 0 and 500.

c State the initial water volume at $t = 0$.

d Find the volume of water after:

i 320 hours **ii** 1 day **iii** 1 week

e Find the time taken, in hours, for the water volume to fall to:

i 300 litres **ii** 185 litres

Production lines — 12

12 An assembly plant needs to order some new parts. Three companies can supply them but at different rates.

- Mandy's Millers charge: set-up fee $0 + $1.40 per part
- Terry's Turners charge: set-up fee $3000 + $0.70 per part
- Lenny's Lathes charge: set-up fee $4000 + $0.50 per part

a Complete a table of values similar to the following for each of the companies.

No. of parts (p)	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Cost (C)										

b Plot a graph of the total cost against the number of parts for each company on the same set of axes. Make your axes quite large as there are three graphs to complete.

c Use the graphs to find the lowest price for:

i 1500 parts **ii** 1000 parts

iii 6500 parts **iv** 9500 parts

d Advise the assembly plant when it is best to use Mandy's, Terry's or Lenny's company.

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6K Direct proportion

Learning intentions

- To understand what it means if two variables are directly proportional
- To know the form of the equation and graph of two variables in direct proportion
- To be able to form and use an equation for two directly proportional variables
- To be able to find the constant of proportionality for two variables in direct proportion

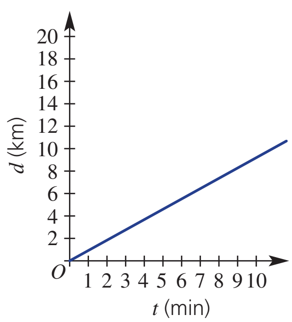
Key vocabulary: directly proportional, constant of proportionality

Two variables are said to be directly proportional when the rate of change of one variable with respect to the other is constant. So if one variable increases, then the other also increases and at the same rate. For example, distance is directly proportional to speed because, in a given time, if the speed is doubled then the distance travelled is doubled also.

Lesson starter: Discovering the features of direct variation

Here are five different travel graphs showing how the distance from home varies with time.

A



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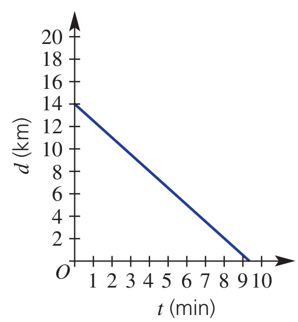

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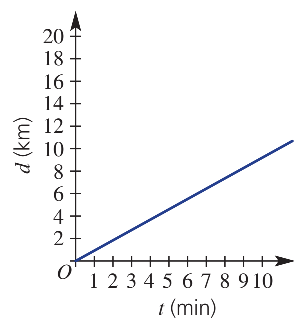

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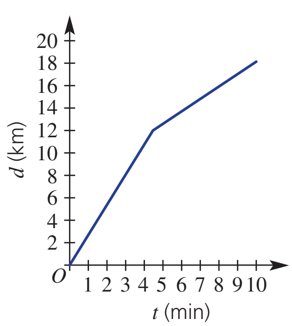

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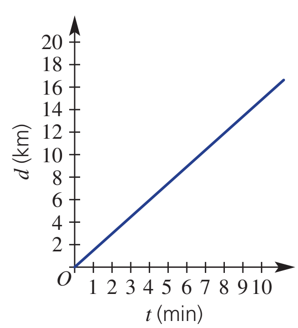

List all the graphs that show the following features.

- The distance from home increases as the time increases.
- The distance from home decreases as the time increases.
- The person travels at a constant speed throughout the trip.
- The person starts from home.
- The person starts from home and also travels at a constant speed throughout the trip.
- The graph starts at $(0, 0)$ and the gradient is constant.
- The distance is directly proportional to the time (i.e. the equation is of the form $y = mx$).

List three features of graphs that show when two variables are directly proportional to each other.

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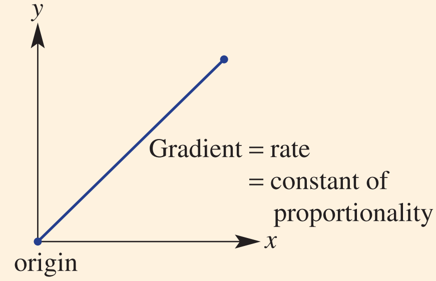
Key ideas

- For two variables that are **directly proportional**:
  - Both variables will increase together or decrease together at the same rate. For example, the cost of buying some sausages is directly proportional to the weight of the sausages. If the weight increases, then the cost increases; when the weight decreases, the cost decreases.

Graph of direct variation



  - The rate of change of one variable with respect to the other is constant.
  - The graph is a straight line passing through the origin $(0, 0)$; i.e. the rule is of the form $y = mx$.
  - The rule is usually written as $y = kx$, where k is the constant of proportionality.
  - The **constant of proportionality**, k, is the gradient with units, it is the same as the rate.

Exercise 6K

Understanding — 1–3 — 2

1 Write in the missing words for these statements: *increases* or *decreases*.

a As the volume of fuel decreases, the distance a car can travel _______.

b As the volume of fuel decreases, the cost of filling the tank _______.

2 a Which of these equations show that y is directly proportional to x?

i $y = 2x + 4$ **ii** $y = 3x$ **iii** $y = x - 2$

iv $y = 8x + 5$ **v** $y = 70x$

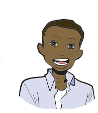
Hint: The direct proportion rule is $y = kx$, where k is the constant of proportionality.

b State the value of k, the constant of proportionality, for each of these direct proportion equations.

i $y = 5x$ **ii** $y = 12x$ **iii** $d = 40t$

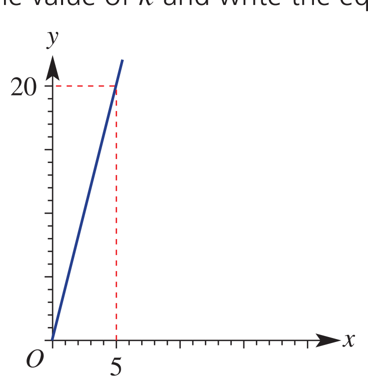
iv $V = 22t$ **v** $C = 7.5w$

3 For each of the following graphs of direct variation, determine the value of k and write the equation in the form $y = kx$.



Hint: k = gradient = $\frac{\text{rise}}{\text{run}}$

a



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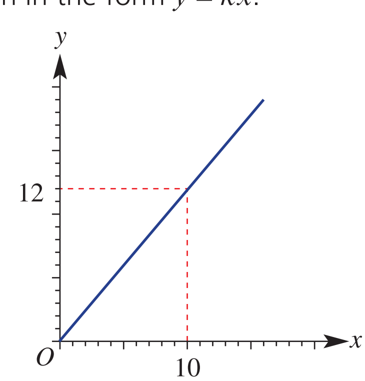

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6K

Fluency

4–6, 8 | 5–7, 8(½)

Example 31 Forming direct proportion equations when given the constant of proportionality, *k*

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres.

a Write the direct proportion equation, given that $k = \$1.45$/L.
b Use this equation to calculate the cost of 63 L.

Solution	Explanation
a $C = 1.45n$	$y = kx$ becomes $C = kn$, where $k = 1.45$.
b $C = 1.45 \times 63$	Substitute $n = 63$ into the equation.
$= \$91.35$	Write the answer in dollars.

Now you try

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres of fuel pumped.

a Write the direct proportion equation, given that $k = \$1.38$/L.
b Use this equation to calculate the cost of 48 L.

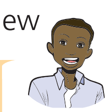
4 For a fixed rate of pay, wages (W) in dollars are directly proportional to the number (n) of hours worked.

a Write the direct proportion equation, given that $k = \$11.50$/h.
b Use this equation to calculate the wages earned for 37.5 hours worked.

5 At a fixed flow rate, the volume (V) in litres of water flowing from a tap is directly proportional to the amount of time (t) the tap has been turned on.

a Write the direct proportion equation, given that $k = 6$ L/min.
b Use this equation to calculate the volume of water, in litres, flowing from a tap for 4 hours.
c Change the constant of proportionality to units of L/day and rewrite the equation with this new value of k.
d Use this equation to calculate the volume of water, in litres, flowing from a tap for 1 week.

Hint: There are $60 \times 24 = 1440$ minutes in a day.



Example 32 Forming direct proportion equations from given information

The amount of wages Sonali earns is in direct proportion to the number of hours she works.

a Find the constant of proportionality, k, given that Sonali earned \$166.50 in 18 hours.
b Write the direct proportion equation relating Sonali's wages (W) in dollars and the number of hours (n) that she worked.
c Calculate the wages earned for 8 hours and 45 minutes of work.
d Calculate the number of hours Sonali must work to earn \$259.

Continued on next page

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Solution	Explanation
a $k = \dfrac{166.50}{18}$ $= \$9.25/\text{h}$	The constant of proportionality, k, is the rate of pay. Include units in the answer.
b $W = 9.25n$	$y = kx$ becomes $W = kn$, where $k = 9.25$.
c $W = 9.25n$ $= 9.25 \times 8.75$ $= \$80.94$	Write the equation. 45 min $= 45 \div 60 = 0.75$ h. Substitute $n = 8.75$. Write \$ in the answer and round to two decimal places.
d $W = 9.25n$ $259 = 9.25n$ $\dfrac{259}{9.25} = n$ $n = 28$	Write the equation. Substitute $W = 259$. Divide both sides by 9.25.
$\therefore$ Sonali must work for 28 h.	Write the answer in words.
Check: $W = 9.25 \times 28 = \$259$	Check that your answer is correct.

Now you try

Daniel's wages earned are in direct proportion to the hours he works at the local service station.

a Find the constant of proportionality, k, given that Daniel earned \$200 in 16 hours.
b Write the direct proportion equation relating Daniel's wages (W) in dollars and the number of hours (n) worked.
c Calculate the wage earned for 6 hours of work.
d Calculate the number of hours Daniel must work to earn a wage of \$237.50.

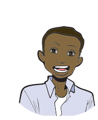
6 The amount that a farmer earns from selling wheat is in direct proportion to the number of tonnes harvested.

a Find the constant of proportionality, k, given that a farmer receives \$8296 for 34 tonnes of wheat.
b Write the direct proportion equation relating selling price (P) in dolllars and number of tonnes (n).
c Calculate the selling price of 136 tonnes of wheat.
d Calculate the number of tonnes of harvested wheat that is sold for \$286 700.

7 When flying at a constant speed, the distance that an aeroplane has travelled is in direct proportion to the time it has been flying.

a Find the constant of proportionality, k, given that the plane flies 1161 km in 1.5 h.
b Write the direct proportion equation relating distance (d) in km and time (t) in hours.
c Calculate the time taken, in hours, for the aeroplane to fly from Sydney to Perth, a distance of around 3300 km. Round your answer to two decimal places.
d Calculate the distance that the plane would fly in 48 minutes.

Hint: Convert 48 mins to hours in part **d**.



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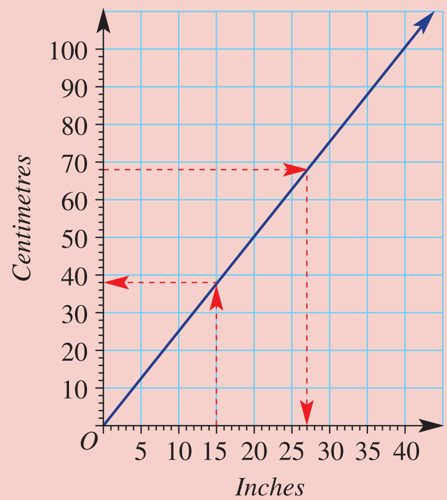
6K

Example 33 Using a unit conversion graph

A length, measured in centimetres, is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 15 inches to cm **b** 68 cm to inches

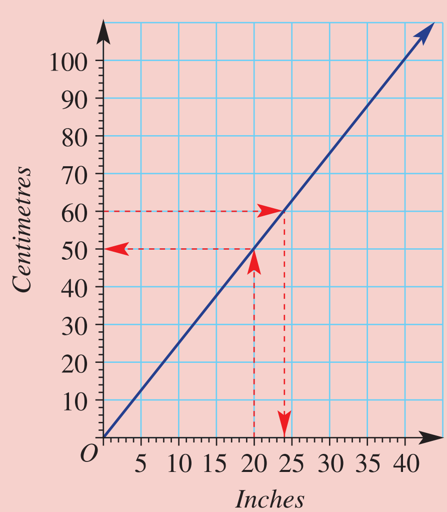


Solution	Explanation
a 38 cm	Start at 15 inches. Now move up to the line and then across to the centimetre scale.
b 27 inches	Start at 68 cm. Now move across to the line and then down to the inches scale. Round your answer to the nearest whole number.

Now you try

A length, measured in centimetres, is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 20 inches to cm **b** 60 cm to inches



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8 Use the graph in Example 33 to make the following unit conversions. Round your answers to the nearest whole number.

- **a** 19 inches to cm
- **b** 25 cm to inches
- **c** 1 foot (12 inches) to cm
- **d** 1 hand (4 inches) to cm
- **e** the height, in inches, of the world's shortest living man, who is 54.6 cm tall
- **f** the height, in cm, of a miniature pony that is 7.5 hands high
- **g** the height, in cm, of the world's shortest living woman, who is about 2 feet and 1 inch tall
- **h** the length, in cm, of a giant Australian earthworm that is 39.5 inches long

Problem-solving and reasoning

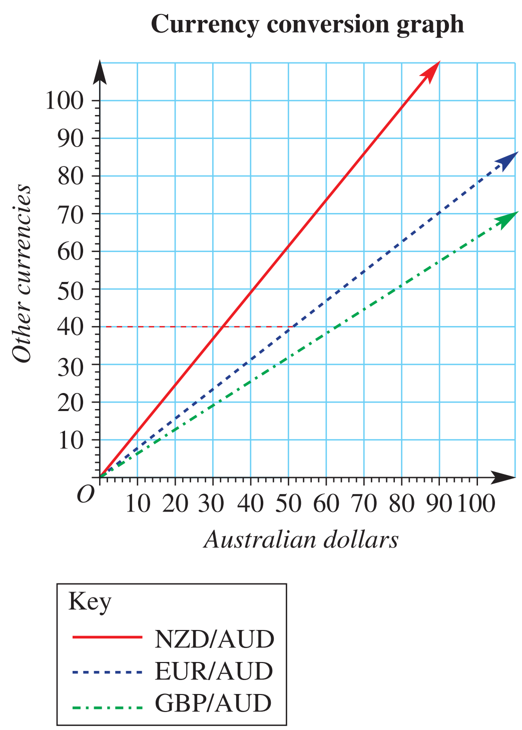
	9–11	10–12, 13(½), 14

9 Use the length unit conversion graph in Example 33 to answer these questions.

> Hint: Gradient = $\frac{\text{rise}}{\text{run}} = \frac{\text{cm}}{\text{inches}}$
> k = rate
> = gradient with units

- **a** Convert 94 cm to inches and use these values to find the gradient of the line, to two decimal places.
- **b** State the conversion rate in cm/inch, to two decimal places.
- **c** State the value of k, the constant of proportionality, to two decimal places.
- **d** Write the direct proportion equation between centimetres (y) and inches (x).
- **e** Use the equation to calculate the number of centimetres in 50 inches.

10 At any given time, an amount in Australian dollars is directly proportional to that amount in a foreign currency. This graph shows the direct variation (at a particular time) between Australian dollars (AUD) and New Zealand dollars (NZD), European euros (EUR) and Great British pounds (GBP).



- **a** Use the graph to make these currency conversions.
  - **i** 80 AUD to NZD
  - **ii** 80 AUD to EUR
  - **iii** 80 AUD to GBP
  - **iv** 50 NZD to AUD
  - **v** 32 EUR to AUD
  - **vi** 26 GBP to AUD
- **b** Answer these questions, using the line that shows the direct variation between the euro (EUR) and the Australian dollar (AUD).
  - **i** Find 40 EUR in AUD and, hence, find the gradient of the line, to one decimal place.
  - **ii** State the conversion rate in EUR/AUD, to one decimal place.
  - **iii** State the value of k, the constant of proportionality, to one decimal place.
  - **iv** Write the direct proportion equation between EUR (y) and AUD (x).
  - **v** Use the equation to calculate the value in euros of 625 Australian dollars.

> Hint: The rate in EUR/AUD = ? euros, per 1 Australian dollar. The rate = the constant of proportionality, k.

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6K

11 The cost (C) of buying fuel is directly proportional to the volume (V) of fuel pumped.

a Copy and complete this table for the cost of diesel, using the rule $C = 1.5V$.

Volume (V) of diesel, in litres	0	10	20	30	40	50
Cost (C), in dollars						

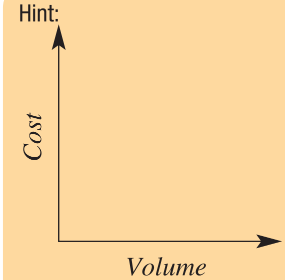
b Plot these points and then use a ruler to join them to form a neat graph of Volume (V) vs Cost (C).

c Find the gradient, m, of the line.

d At what rate is the cost of fuel increasing, in \$/L?

e What is the constant of proportionality, k, between cost and volume?

Hint:



12 For each of the following pair of variables, describe, using sentences, why the variables are in direct proportion to each other or why they are not in direct proportion.

a The number of *hours* worked and *wages* earned at a fixed rate per hour.

b The *cost* of buying tomatoes and the *number of kilograms* at a fixed price per kilogram.

c The *speed* and *time* taken to travel a certain distance.

d The *size* of a movie file and the *time* taken to download it to a computer at a constant rate of kB/s.

e The *cost* of a taxi ride and the *distance* travelled. The cost includes flag fall (i.e. a starting charge) and a fixed rate of \$/km.

13 Convert the following rates to the units given in brackets.

a \$9/h (cents/min) **b** \$24/h (cents/min)

c 10.8 L/h (mL/s) **d** 18 L/h (mL/s)

e 72 km/h (m/s) **f** 18 km/h (m/s)

g \$15/kg (cents/g) **h** \$32/kg (cents/g)

i 400 g/month (kg/year) **j** 220 g/day (kg/week)

Hint:
\$9/h = 900 cents in 1 hour
= 900 cents in 60 mins
= 15 cents in 1 min
= 15 cents/min

14 For a fixed speed, the distance (d) that a car travels is directly proportional to time (t).

a Write the direct proportion equation, given that $k = 90$ km/h.

b Change the constant of proportionality to units of m/s and rewrite the equation with this new value of k.

c Use this equation to calculate the distance, in metres, that a car would travel in 4 seconds.

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Currency conversions — 15

15 At any given time, an amount of money in a foreign currency is in direct proportion to the corresponding amount in Australian dollars.
For example, if 8 Hong Kong dollars is equivalent to 1 Australian dollar, the conversion rate is HK \$8/AUD and the direct proportion equation is HK $= 8 \times$ AUD.

- To change A\$24 to Hong Kong dollars, we must substitute 24 for AUD:
  HK $= 8 \times 24$
  $= \$192$
- To change HK \$24 to Australian dollars, we substitute 24 for HK:
  $24 = 8 \times$ AUD
  $\frac{24}{8} =$ AUD
  AUD $= \$3$

Follow the example above to complete the following questions.

a Singapore dollar (SGD)
- **i** Write the direct proportion equation, given the conversion rate is 1.2 SGD/AUD.
- **ii** Convert AUD 240 to SGD.
- **iii** Convert SGD 240 to AUD.

b Chinese yuan (CNY)
- **i** Write the direct proportion equation, given the conversion rate is 6.47 CNY/AUD.
- **ii** Convert AUD 75 to CNY.
- **iii** Convert CNY 75 to AUD.

c South African rand (ZAR)
- **i** Write the direct proportion equation, given the conversion rate is 9.5 ZAR/AUD.
- **ii** Convert AUD 50 to ZAR.
- **iii** Convert ZAR 50 to AUD.

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6L Inverse proportion ★

Learning intentions

- To understand what it means if two variables are inversely proportional
- To be able to find the constant of proportionality for two inversely proportional variables
- To be able to form and use an equation for inversely proportional variables

Key vocabulary: inversely proportional, constant of proportionality, variables

Two variables are said to be inversely proportional when an increase in one variable causes the other variable to decrease. For example, when a pizza is shared equally, the size of each pizza slice is inversely (or indirectly) proportional to the number of people sharing it. If the number of people sharing increases, then the size of each pizza slice decreases.

Lesson starter: Bushwalking age groups

Imagine you are planning a 12 km bush hike for people of various age groups. You estimate that the average speed for each group is as follows.

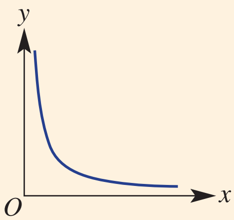
Age (years)	Speed (km/h)
20–35	6
36–50	4
51–65	3
66+	2

- How long will it take for each group to complete the hike?
- Plot points to form a graph of time (t hours) to complete the hike for different age group speeds (s km/h). Use t hours on the vertical axis.
- Do you think that the variables t and s are inversely proportional? Explain why.

Key ideas

- For two **variables** that are **inversely** or **indirectly proportional**:
  - When one variable increases, then the other variable decreases.
  - The graph is a curve showing that as x increases, then y decreases. For example, speed is indirectly or inversely proportional to travelling time. As the speed decreases, the time taken to travel a particular distance increases; as the speed increases, the time taken to travel a particular distance decreases.
  - The rule is usually written as $y = \frac{k}{x}$, where k is the constant of proportionality.

Graph of indirect or inverse variation



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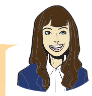
Exercise 6L

Understanding | 1–3 | 2, 3

1 For each part, state whether the variables are directly or indirectly (i.e. inversely) proportional.
- **a** As the number of questions correct increases, the total mark for the test increases.
- **b** As the speed decreases, the time taken to travel a particular distance increases.
- **c** As the number of hours worked increases, the pay for that work increases.
- **d** As the size of a computer file decreases, the time required to transfer it decreases.
- **e** As the rate of typing words per minute increases, the time needed to type an assignment decreases.

2 Which of the following equations shows inverse proportion?
- **a** $y = \frac{2}{x}$
- **b** $y = 3x$
- **c** $T = \frac{2.7}{s}$



Hint: Inverse proportion equation is of the form $y = \frac{k}{x}$.

3 Determine the value of k in $y = \frac{k}{x}$ when:
- **a** $x = 2$ and $y = 3$
- **b** $y = 2$ and $x = 0.5$

Fluency | 4, 5 | 4–6

Example 34 Working with inverse proportion

The length of a rectangle, l metres, with a fixed area of 2 m^2, varies inversely with the width, w metres, such that $l = \frac{2}{w}$.

a Complete this table of values.

w	1	2	3	4
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.

c Find the length of the rectangle if the width is 5 m.

d Find the width of the rectangle if the length is 0.5 m.

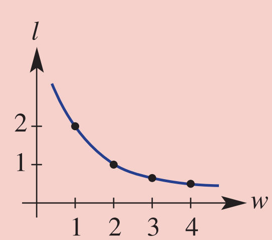
Solution / **Explanation**

a

w	1	2	3	4
l	2	1	$\frac{2}{3}$	$\frac{1}{2}$

Substitute each value of w into the rule $l = \frac{2}{w}$.

b



Plot the points from the table and join the points in a smooth curve.

c

$l = \frac{2}{w}$

$l = \frac{2}{5}$

length = 0.4 metres

Substitute $w = 5$ into the given rule.

Continued on next page

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6L

d $l = \frac{2}{w}$

$0.5 = \frac{2}{w}$

$0.5w = 2$

$w = 4$

width = 4 metres

Substitute $l = 0.5$ into the given rule then solve for w. Note that $2 \div 0.5 = 4$.

Now you try

The length of a rectangle, l metres, with a fixed area of $12\,\text{m}^2$, varies inversely with the width, w metres, such that $l = \frac{12}{w}$.

a Complete this table of values.

w	1	3	6	12
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
c Find the length of the rectangle if the width is 4 m.
d Find the width of the rectangle if the length is 2.5 m.

4 The length of a rectangle, l metres, with a fixed area of $6\,\text{m}^2$, varies inversely with the width, w metres, such that $l = \frac{6}{w}$.

a Complete this table of values.

w	1	2	3	6
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
c Find the length of the rectangle if the width is 8 m.
d Find the width of the rectangle if the length is 1.5 m.

5 The time taken (t days) to paint a house is inversely proportional to the number of painters (n) such that $t = \frac{16}{n}$.

a Find the time taken to paint a house for the following number of painters, n.
- **i** 1
- **ii** 4
- **iii** 8

b Sketch a graph of t vs n with n on the x-axis.
c Find how long it will take for 10 painters to paint a house.
d Find the number of painters required to paint a house in 8 days.

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6 y is inversely proportional to x such that $y = \frac{20}{x}$.

a Find the value of y if:

i $x = 5$

ii $x = 8$

b Find the value of x if:

i $y = 10$

ii $y = 40$

Problem-solving and reasoning 7–9 7–10

Example 35 Finding and using the constant of proportionality

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 2$, $y = 3$.

b Find the value of y if $x = 6$.

Solution	Explanation
a $y = \frac{k}{x}$ $3 = \frac{k}{2}$ $k = 2 \times 3$ $= 6$	Substitute $x = 2$ and $y = 3$ into the rule and solve for k.
b $y = \frac{6}{x}$ $y = \frac{6}{6}$ $y = 1$	Write the rule using $k = 6$. Substitute $x = 6$ and simplify to find the value of y.

Now you try

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 4$, $y = 2$.

b Find the value of y if $x = 1$.

7 y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 5$, $y = 10$.

b Find the value of y if $x = 2$.

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6L

8 For each of the following determine the constant of proportionality, k, if $y = \frac{k}{x}$. Then complete the table of values.

a

x	1	2	3	4
y	12	6		

b

x	0.5	1	2	4
y	8		2	

9 The volume, V cm^3, of gas in a container is inversely proportional to the pressure, P kg/cm^3. When the pressure is 2 kg/cm^3 the volume is 20 cm^3.

a Find the constant of proportionality k if $V = \frac{k}{P}$.

b Find the volume if the pressure is 4 kg/cm^3.

c Find the pressure if the volume is 16 cm^3.

10 The cost to rent a house for a week is \$500.

a Find the cost per person if:

i 2 people rent the house

ii 5 people rent the house

b Find a rule linking the cost per person, \$$C$, with the number of people, n.

c Find the number of people sharing the house if the cost per person is \$62.50.

Electrical current — 11

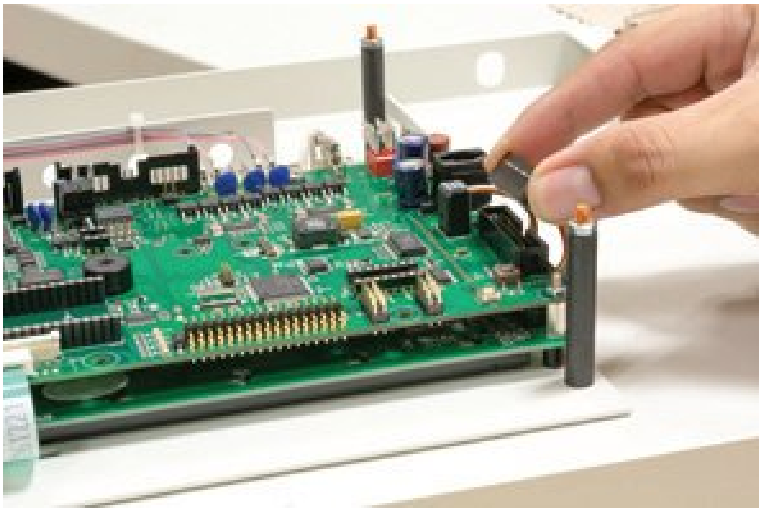
11 The current (I amps) that flows from a device is inversely proportional to the resistance (R ohms). When the current is 4 amps, the resistance is 60 ohms.

a Find a rule linking I and R.

b Find the current if the resistance is 40 ohms.

c Find the resistance if the current is 120 amps.

d What percentage increase in resistance causes a 50% reduction in current?



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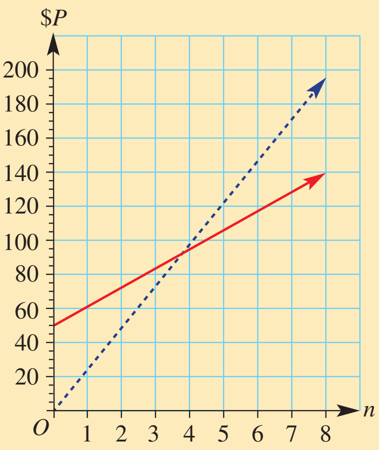
Maths@Work: Accountant or small business owner

Accountants and small business owners need to have an understanding of operating costs and income. It is important to understand break-even points, which are when the costs or expenses equal the income. This is essential knowledge for small business owners, given that many start-up businesses fail in their first 3 years.

Reading and drawing graphs helps accountants and business owners understand break-even points. Many business situations can be displayed using linear functions.

Complete these questions that an accountant or small business owner may face in their day-to-day job.

1 Chloe buys and sells necklaces at a local community weekend market. She sells each necklace for \$25 and her costs, C, are calculated using the formula $C = 50 + 12n$, where n is the number of necklaces Chloe buys and \$50 is the fixed cost of hiring the stand each weekend.



- **a** For the graph shown at right, which line represents Chloe's costs and which line represents Chloe's income?
- **b** State the possible numbers of necklaces to be sold each weekend that will result in Chloe making a loss.
- **c** What is the loss at the break-even point? What is the profit at the break-even point?
- **d** State the minimum number of necklaces to be sold each weekend for Chloe to start making a profit.
- **e** Sketch a copy of the graph and shade the region that represents the area of profit.
- **f** Write down a rule, in terms of n, for calculating Chloe's profit, \$$P$, each weekend.
- **g** How much profit is made by selling:
  - **i** 10 necklaces?
  - **ii** 15 necklaces?
  - **iii** 30 necklaces?
  - **iv** 50 necklaces?

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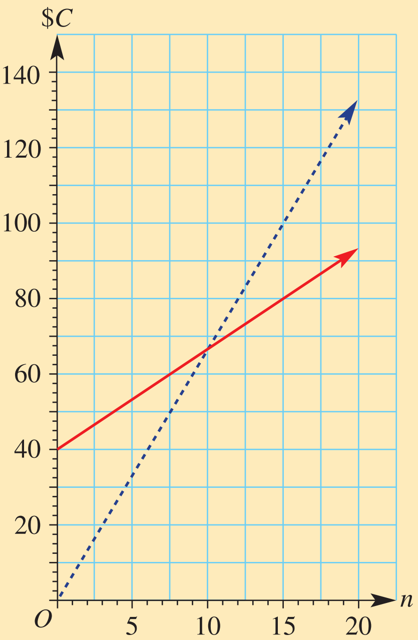
2 Consider this graph for a company creating retro drinking glasses.

a What is the value of the y-intercept for the costs relationship, and what could it represent in this situation?
b What is the wholesale price of each glass?
c What is the gradient of the income line and what does this represent?
d How many glasses must be sold for the company to break even?
e Write an equation for the:
i cost
ii income
iii profit



Using technology

3 An industrial plant produces car parts, which they sell to car manufacturers for \$80 per car part. The costs of production for the plant are \$200 plus \$50 per car part produced each day.

a Write an equation for the costs (\$$C$) of production of n car parts.
b Write an equation for the income (\$$I$) generated.
c Use a graphics calculator or digital graphing tool to graph these two equations.
d From the graphs, what is the break-even point for the plant each day?
e Determine the profit when 32 car parts are sold on any given day.

4 Alex has a start-up company. He estimates his costs, including staff, rent, electricity and insurance, to be \$3000 per month. He buys computer parts at \$4 per part and sells them on at a retail price of \$$a$ per part.
Use a graphics calculator or a digital graphing tool to answer these questions.

a By drawing pairs of graphs, find the number of parts, x, and Alex's costs, at each break-even point when the retail price, \$$a$, per part is:
i \$10 **ii** \$12 **iii** \$14 **iv** \$16
Hint: Enter all the graph equations into the graphics calculator or computer graphing tool and then select two equations at a time to graph.
b Alex buys a batch of 500 parts. Write an equation for Alex's profit when these parts are sold at the retail price \$$a$ per part.
c Calculate the profit made when the parts are sold at each value of \$$a$ given in part **a** above.
d Give one reason why Alex must limit the retail price charged per part.

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1 What is not so devious? Solve the puzzle to find the answer.
Match the letter beside each question to the answers below.

Find where each line cuts the x-axis:

O $y = 3x - 24$

! $y = -\frac{3}{2}x - 9$

N $4x - 2y = -20$

I This line joins $(0, 4)$ to $(5, -1)$.

S $x = -7$

Find the gradient of each line:

E $y = 3x - 4$

L $y = -\frac{5}{2}x + 7$

P This line joins the origin to $(3, -6)$.

G This line joins $(2, 5)$ to $(-4, 11)$.

T $y = 5$

__ -2 __ -2.5 __ 8 __ 0 __ 0 __ 4 __ -5 __ -1 __ -2.5 __ 4 __ -5 __ 3 __ -7 __ -6

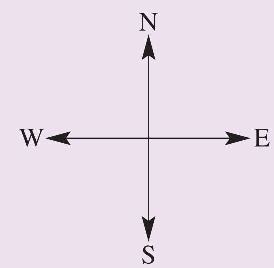
2 Solve the wordfind below.

T	F	Z	T	V	M	V	Z	J	H	E	R
O	W	I	M	J	G	R	J	O	A	L	A
T	M	G	T	E	R	K	R	U	T	B	T
E	C	N	A	T	S	I	D	N	E	A	E
S	T	M	P	R	Z	A	E	S	I	I	F
R	P	H	G	O	X	M	E	O	Z	R	E
D	Y	E	N	Z	G	J	U	R	X	A	F
H	G	T	E	E	J	Q	W	G	C	V	H
I	A	L	S	D	R	U	S	G	U	N	A
L	P	G	R	A	P	H	A	P	R	P	I

- DISTANCE
- GRAPH
- HORIZONTAL
- INCREASE
- RATE
- SEGMENT
- SPEED
- TIME
- VARIABLE

3 Cooper and Sophie are in a cycling orienteering competition.
- From the starting point, Cooper cycles 7 km east, then 3 km south to checkpoint 1. From there, Cooper cycles 5 km east and 8 km north to checkpoint 2.
- Sophie cycles 10 km north from the starting point to checkpoint 3.

Use calculations to show that the distance between where Sophie and Cooper are now is the same as the direct distance that Cooper is now from the starting point.



4 Lucas and Charlotte want to raise money for their school environment club, so they have volunteered to run a strawberry ice-cream stall at their town's annual show.
It costs \$200 to hire the stall and they make \$1.25 profit on each ice-cream sold.

a How many ice-creams must be sold to make zero profit (i.e. not a loss)?

b If they make \$416.25 profit, how many ice-creams were sold?

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Chapter summary

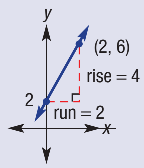
Straight-line graphs

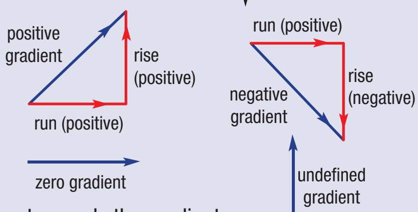
Gradient of a line

Gradient measures the slope of a line

Gradient, $m = \frac{\text{rise}}{\text{run}}$ e.g.

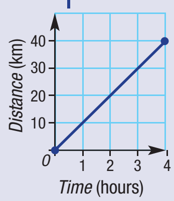
$m = \frac{4}{2} = 2$





A rate equals the gradient with units.

e.g. Speed = $\frac{40}{4} = 10$ km/h



Equation of a line

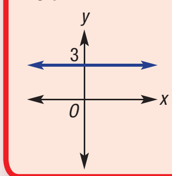
$y = mx + c$

gradient y-intercept

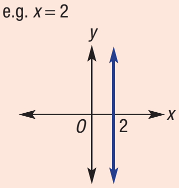
- The rule is a linear equation.
- The graph is made up of points in a straight line.

Special lines

Horizontal lines e.g. $y = 3$



Vertical lines e.g. $x = 2$



Parallel and perpendicular lines

- Parallel-lines have the same gradient; e.g. $y = 3x - 4$ and $y = 3x + 1$
- For perpendicular lines, the product of their gradients is -1, so $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

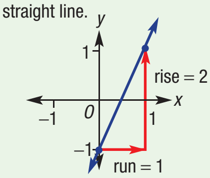
Sketching a line

Plotting straight-line graphs:
- Complete a table of values.
- Plot points and join them to form a straight line.

Using the y-intercept and gradient:
- Plot the y-intercept (c).
- Use the gradient to plot the next point.
- Join points to form a straight line.

e.g. $y = 2x - 1$

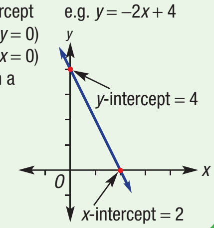
$c = -1$ $m = \frac{2}{1}$



Using the axes intercepts

- Plot each axis intercept
  x-intercept (when $y = 0$)
  y-intercept (when $x = 0$)
- Join points to form a straight line.

e.g. $y = -2x + 4$

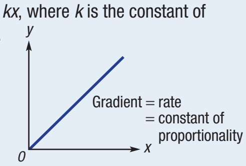


Linear modelling ⭐

- Find a rule in the form $y = mx + c$, using the appropriate pronumerals.
- Sketch a graph.
- Apply the rule to solve problems.
- Answer the problem in words.

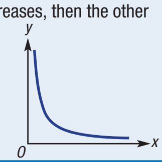
For two variables that are directly proportional ⭐

- Both variables will increase or decrease together at the same rate.
- The rule is $y = kx$, where k is the constant of proportionality.



For two variables that are inversely (or indirectly) proportional ⭐

- When one variable increases, then the other variable decreases.
- The graph is a curve.



Midpoint of a line segment

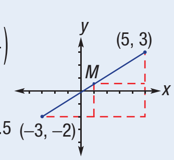
Find the average of the end point coordinates.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$x = \frac{-3 + 5}{2} = \frac{2}{2} = 1$

$y = \frac{-2 + 3}{2} = \frac{1}{2} = 0.5$

$\therefore M = (1, 0.5)$



Length of a line segment

Use Pythagoras' theorem.

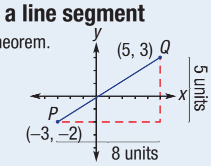
$PQ^2 = 8^2 + 5^2$

$PQ^2 = 64 + 25$

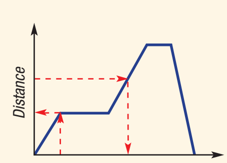
$PQ^2 = 89$

$PQ = \sqrt{89}$

$\sqrt{89}$ is an exact length.



Distance–time graph



- Flat segment means the object is at rest.

Reading from a graph:
- Start on given distance; move across to line and then down to time scale (or in reverse).

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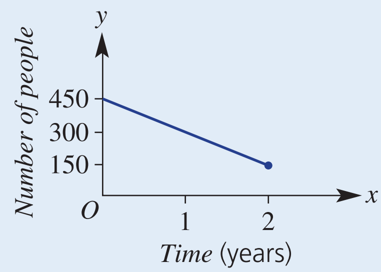
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

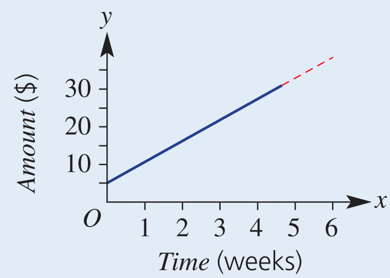
6A

1 I can interpret information from a graph.
e.g. The number of people living on a small island has decreased over recent years according to the graph shown.
- **a** How many people were there to begin with?
- **b** How many people left the island during the 2-year period?



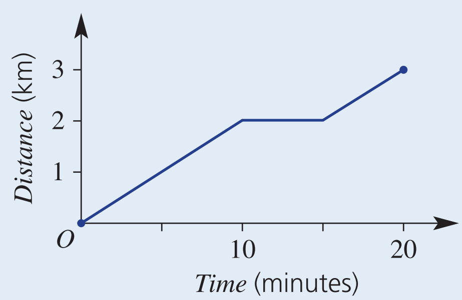
6A

2 I can read off a graph using interpolation and extrapolation.
e.g. This graph shows the increase in Chloe's pocket money savings over 4 weeks.
- **a** How much has she saved over the 4 weeks?
- **b** Use the graph to find out how much she saved after 2 weeks.
- **c** After how long does the graph suggest she will have saved $35?



6B

3 I can interpret a distance–time graph.
e.g. The distance–time graph shows a student's bike ride from school, to the corner store for an ice cream and then to home.
Determine:
- **a** the total distance covered
- **b** how long the student was stopped at the store
- **c** the total distance travelled after 17.5 minutes



6B

4 I can sketch a distance–time graph.
e.g. Sketch a distance–time graph displaying all the following information.
- total distance covered is 12 km in 3 hours
- 6 km covered in the first hour
- a half-hour rest stop after the first hour

6C

5 I can plot a graph from a rule.
e.g. Plot the graph of $y = 3x - 2$ by first completing the table of values.

x	−1	0	1
y			

6C

6 I can construct a table and graph and interpret it.
e.g. A tv technician charges $110 for a service call and $70 per hour for labour. Complete the table of values and plot a graph of cost against number of hours.

No. of hours (n)	0	1	2	3	4
Cost (C)					

Use the graph to determine:
- **a** the cost for 3.5 hours of work
- **b** how long the technician worked on a job that cost $285

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Chapter checklist

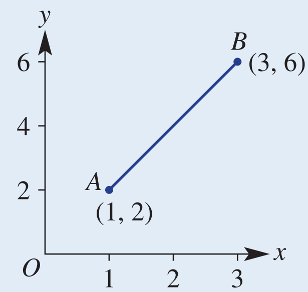
✔

6D

7 I can find the midpoint of a line segment from a graph and from coordinates.

e.g. Find the midpoint of the line segment joining the following points.

a A and B **b** $(-3, 6)$ and $(5, 1)$

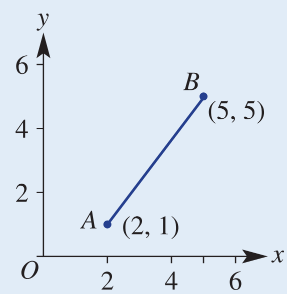


6D

8 I can find the length of a line segment from a graph and from coordinates.

e.g. Find the distance between the following pairs of points.

a A and B **b** $(-2, 3)$ and $(2, 8)$

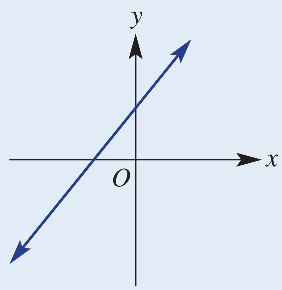


6E

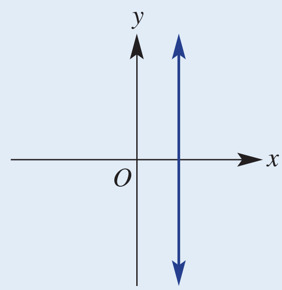
9 I can describe the gradient of a graph.

e.g. Describe the gradients of these lines as positive, negative, zero or undefined.

a



b

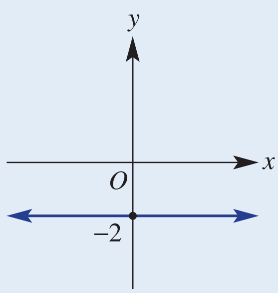


6E

10 I can calculate the gradient from a graph.

e.g. Find the gradient of the following lines and line segments.

a



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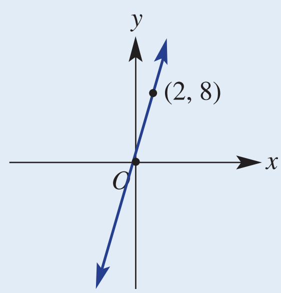
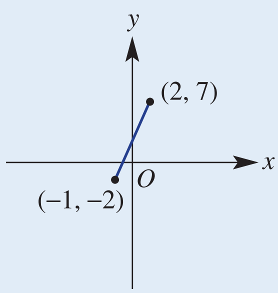


c



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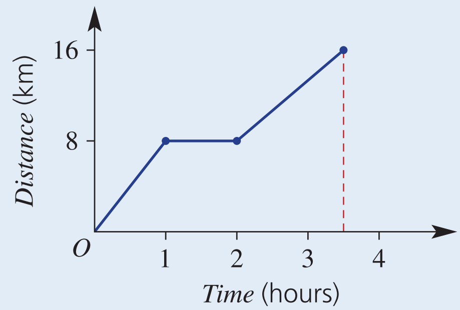
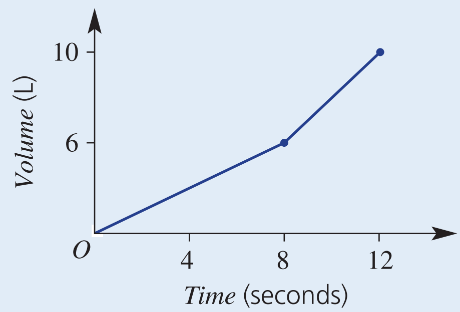
Chapter checklist

✓

		✓
6F	**11 I can calculate speed from a distance–time graph.** e.g. A cyclist completes a journey which is described by this graph. Find how fast the cyclist was travelling during the: **a** first hour **b** second section **c** third section	
6F	**12 I can calculate a rate from a graph.** e.g. A kettle is being filled with water from a tap as shown. How fast (in L/s) is the kettle being filled in the first 4 seconds, final 4 seconds and between the 4- and 8-second mark?	
6G	**13 I can determine the gradient and y-intercept from an equation.** e.g. For $y = \frac{1}{2}x - 4$, state the gradient and the y-intercept.	
6G	**14 I can sketch a line using the y-intercept and gradient.** e.g. Sketch the graph of $y = 2x + 3$ by considering the y-intercept and gradient.	
6G	**15 I can sketch horizontal and vertical lines.** e.g. Sketch the graph of the equations: **a** $x = 4$ **b** $y = -3$.	
6G	**16 I can sketch lines passing through the origin.** e.g. Sketch the graph of $y = -3x$.	
6H	**17 I can determine if lines are parallel or perpendicular.** e.g. State whether the following pair of lines are parallel, perpendicular or neither: $y = -2x - 3$ and $y = \frac{1}{2}x + 1$	
6H	**18 I can find the equation of a line that is parallel or perpendicular to another line given the y-intercept.** e.g. A line passes through $(0, -3)$. Give the equation of the line if it is: **a** parallel to a line with gradient 2 **b** perpendicular to another line with gradient 3	
6H	**19 I can find the equation of a line that is parallel or perpendicular to a line.** e.g. A line passes through $(2, -3)$. Find the equation of the line if it is: **a** parallel to the line with equation $y = -3x + 1$ **b** perpendicular to the line with equation $y = \frac{1}{2}x + 1$	





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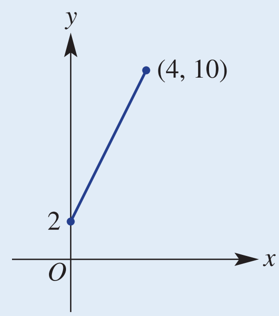
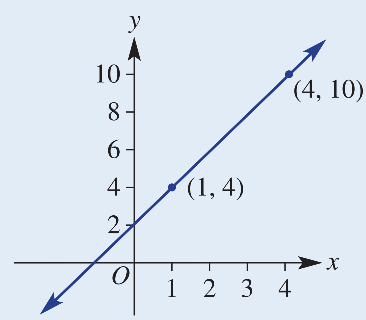
Chapter checklist

		✔
6I	**20 I can sketch lines in the form $ax + by = d$ using x- and y-intercepts.** e.g. Sketch a graph of $2x - 5y = 15$ by finding the x- and y-intercepts.	
6I	**21 I can sketch lines in the form $y = mx + c$ using x- and y-intercepts.** e.g. Sketch a graph of $y = -3x + 12$ by finding the x- and y-intercepts.	
6J	**22 I can find the equation of a line from a graph with a known y-intercept.** e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line. 	
6J	**23 I can find the equation of a line from a graph given two points.** e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line. 	
6J	**24 I can form a linear model and graph for a problem.** e.g. An employee gets paid $\$50$ plus $\$15$ for each hour of work. If she earns $\$C$ for t hours of work, write a rule for C in terms of t and sketch the graph for t between 0 and 8. Use the rule to find: **a** the amount earned after working 6 hours **b** the number of hours worked if $\$200$ is earned	
6K	**25 I can determine the constant of proportionality and find a rule connecting two variables which are directly proportional.** e.g. The volume of water in a bucket is in direct proportion to the number of seconds it has been filled for. The bucket was filled with 6 L of water in 12 seconds. **a** Find the constant of proportionality, k. **b** Use this to write the direct proportion equation relating the volume (V litres) of water in the bucket and the number of seconds (s) it is filled for. **c** Use the rule to find the volume after 8 seconds and the number of seconds to fill the bucket with 20 L of water.	
6L	**26 I can use an equation connecting inversely proportional variables to sketch a graph and find the value of an unknown.** e.g. In a rectangle of area 8 cm^2, the length, l cm, is inversely proportional to the width, w cm, such that $l = \frac{8}{w}$. Sketch a graph of l vs w and find the length if $w = 2.5$.	
6L	**27 I can determine the constant of proportionality and find a rule connecting two variables which are inversely proportional.** e.g. If y is inversely proportional to x and when $x = 3$, $y = 4$, find a rule linking y and x. Then find the value of x if $y = 6$.	

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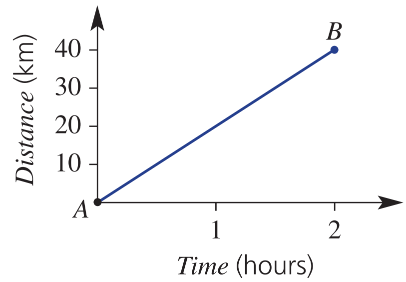
Short-answer questions

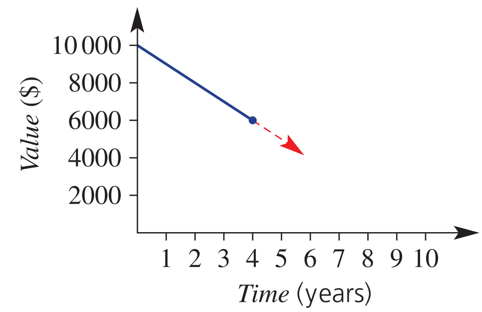
6A/B **1** This graph shows the journey of a cyclist from place A to place B.

- **a** How far does the cyclist travel?
- **b** How long does it take the cyclist to complete the journey?
- **c** If the cyclist were to ride from A to B and then halfway back to A, how far would the journey be?



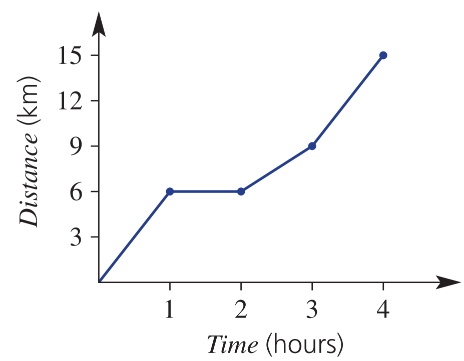
6A **2** The value of a poor investment has decreased according to this graph.

- **a** Find the value of the investment after:
  - **i** 4 years
  - **ii** 2 years
  - **iii** 1 year
- **b** Extend the graph and use it to estimate the value of the investment after:
  - **i** 8 years
  - **ii** 6 years
  - **iii** 5 years
- **c** After how many years will the investment be valued at \$0?



6B **3** The distance travelled by a walker is described by this graph.

- **a** What is the total distance walked?
- **b** For how long does the person actually walk?
- **c** How far has the person walked after:
  - **i** 1 hour?
  - **ii** 2 hours?
  - **iii** 3 hours?
  - **iv** 4 hours?
- **d** How long does it take the walker to walk a distance of 12 km?



6B **4** Sketch a graph to show a journey described by:

- a total distance of 60 m in 15 seconds
- 30 m covered in the first 6 seconds
- a 5 second rest after the first 6 seconds

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6C **5** Francene delivers burgers for a fast-food outlet. She is paid $10 a shift plus $5 per delivery.

a Complete the table of values.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of amount paid against number of deliveries.

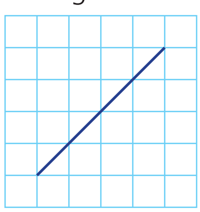
c Use the graph to determine:

i the amount of pay for 12 deliveries

ii the number of deliveries made if Francene is paid $95

6E **6** Find the gradient of the following lines.

a



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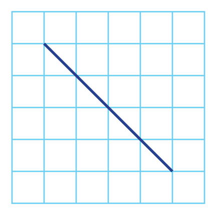
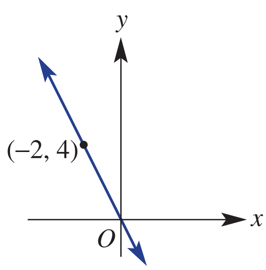
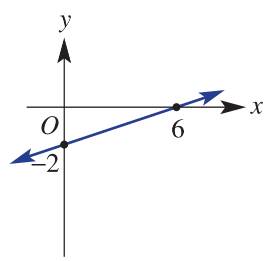


c

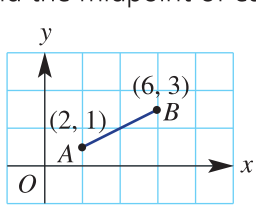


d



6D **7** Find the midpoint of each line segment.

a

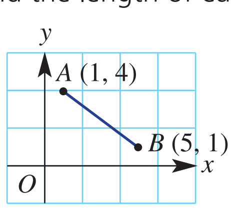


b $P(5, 7)$ to $Q(-1, -2)$

c $G(-3, 8)$ to $H(6, -10)$

6D **8** Find the length of each line segment.

a



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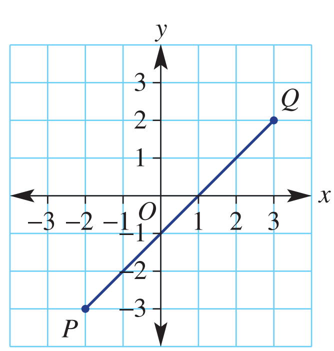

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6G **9** State the gradient and y-intercept of the following lines.

a $y = 3x + 4$ **b** $y = -2x$

6G **10** Sketch the following lines by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = -4x$ **c** $y = 2$ **d** $x = -1$

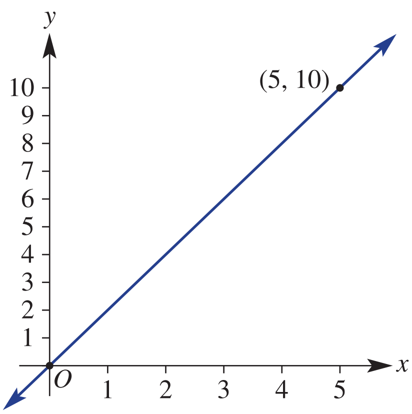
6I **11** Sketch the following lines by considering the x- and y-intercepts.

a $3x + 4y = 12$ **b** $2x - y = 6$ **c** $y = 3x - 9$

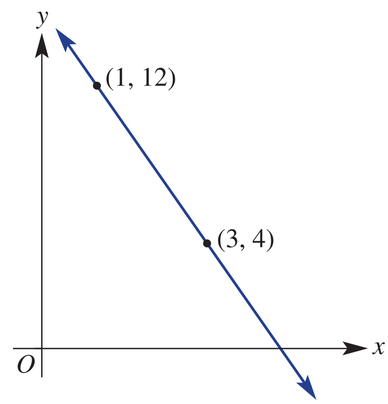
6G **12** For each of the straight lines shown:

i Determine the gradient.
ii Find the y-intercept.
iii Write the equation of the line.

a



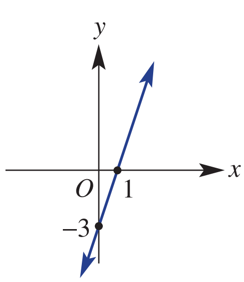
b



6G/I **13** Match each of the linear equations to the lines shown.

a $y = 3x - 3$ **b** $y = 5x$ **c** $5x + 4y = 20$
d $x = 2$ **e** $-2x + 5y = 10$ **f** $y = -4$

i



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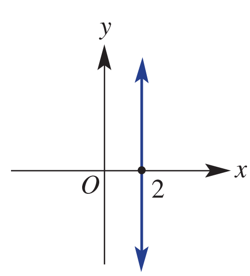

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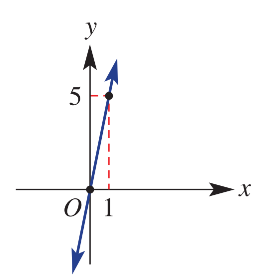

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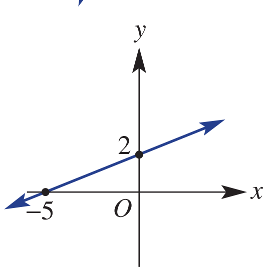

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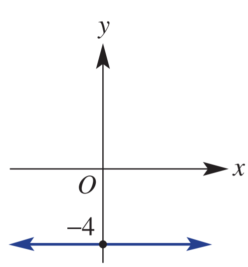

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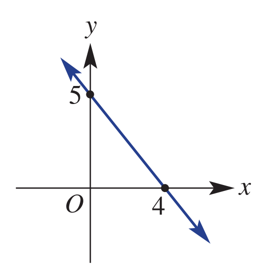

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Chapter review

6J

14 A fruit picker earns $\$50$ plus $\$20$ per bin of fruit picked. If the picker earns $\$E$ for n bins picked, complete the following.

a Write a rule for E in terms of n.

b Sketch a graph for n between 0 and 6.

c Use your rule to find:

i the amount earned after picking four bins of fruit

ii the number of bins of fruit picked if $\$160$ is earned

6H

15 Find the equation of the lines with the given description.

a A line passes through $(0, 3)$ and is parallel to another line with gradient 2.

b A line passes through $(0, -1)$ and is parallel to another line with gradient $\frac{1}{2}$.

c A line passes through $(0, 2)$ and is perpendicular to another line with gradient 1.

d A line passes through $(0, -7)$ and is perpendicular to another line with gradient $\frac{3}{4}$.

e A line passes through $(1, 2)$ and is parallel to another line with gradient -4.

f A line passes through $(-2, 5)$ and is perpendicular to another line with gradient 2.

6K

16 An employee earns wages in direct proportion to the hours he works.

a Find the constant of proportionality, k, given that he earned $\$198$ in 12 hours.

b Write the direct proportion equation relating the wages (W) in dollars and the number of hours (n) worked.

c Use the rule in part **b** to calculate:

i the wages earned for 8 hours of work

ii the number of hours the employee must work to earn a wage of $\$264$

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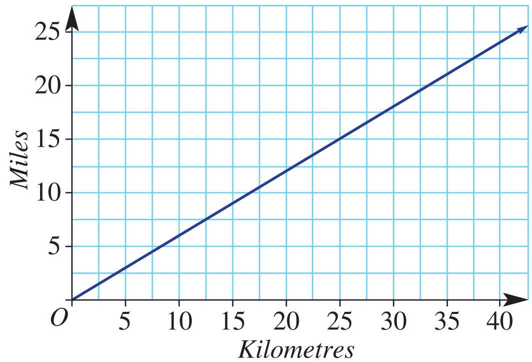
6L

17 This graph shows the direct proportional relationship between miles and kilometres.



a Use the graph to convert 5 miles to kilometres.
b Use the graph to convert 35 kilometres to miles.
c Given that 15 miles is 24.14 km, find the gradient, to three decimal places.
d State the conversion rate in miles/km, to three decimal places.
e Determine the constant of proportionality, k, to three decimal places.
f Write the direct proportion equation between miles (y) and kilometres (x).
g Use this equation to find the number of miles in 100 km.
h Use this equation to find the number of kilometres in 100 miles.

6K

18 State whether these variables are in direct or indirect proportion and give a reason why.
a Cost of buying cricket balls and the number of balls.
b Cost per person of renting a beach house for a week and the number of people sharing it.

6K

19 The length of a rectangle, l metres, with a fixed area, varies inversely with the width, w metres, such that $l = \frac{k}{w}$.
a If when $w = 3$, $l = 5$ find the value of k, the constant of proportionality.
b Complete this table of values.

w	1	3	5	15
l				

c Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
d Find the length of the rectangle if the width is 10 m.
e Find the width of the rectangle if the length is 4 m.

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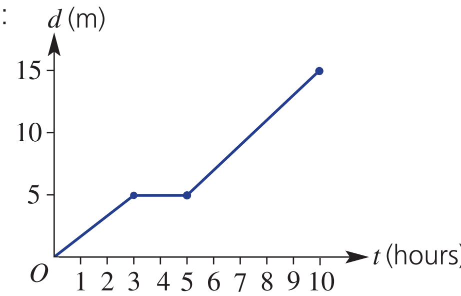
Multiple-choice questions

Questions 1–4 refer to the following graph of the movement of a snail.



6A **1** The total number of hours the snail is at rest is:

A 2 **B** 4 **C** 5
D 6 **E** 10

6B **2** The distance travelled in the first 3 hours is:

A 3 m **B** 3 hours **C** 7 m
C 4 m **D** 5 m

6F **3** The speed of the snail in the last 5 hours is:

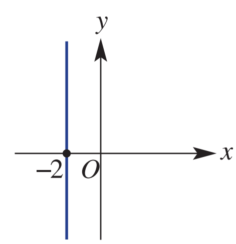
A 5 hours **B** 10 m **C** 10 m/h **D** 2 m/h **E** 5 m/h

6B **4** The total distance travelled by the snail is:

A 15 m **B** 10 m **C** 5 m **D** 12 m **E** 8 m

6G **5** The equation of the line shown at the right is:

A $x = 2$ **B** $x = -2$ **C** $y = 1$
D $y = -2$ **E** $y = -2x$



6G **6** The graph of $C = 10t + 5$ would pass through which of the following points?

A $(1, 10)$ **B** $(1, 20)$ **C** $(2, 20)$ **D** $(4, 50)$ **E** $(5, 55)$

6E **7** The gradient of the line joining $(0, 0)$ and $(2, -6)$ is:

A 2 **B** 3 **C** -3 **D** 6 **E** -6

6E **8** A vertical line has gradient:

A undefined **B** zero **C** positive **D** negative **E** 1

6E **9** A line passes through $(-2, 7)$ and $(1, 2)$. The gradient of the line is:

A -3 **B** $-\frac{5}{3}$ **C** 3 **D** $\frac{5}{3}$ **E** $-\frac{3}{5}$

6I **10** The x- and y-intercepts of the graph of the rule $3x - y = 4.5$ are, respectively:

A $(0, 3.5)$ and $(4.5, 0)$ **B** $(-1.5, 0)$ and $(4.5, 0)$ **C** $(1.5, 0)$ and $(0, 4.5)$
D $(1.5, 0)$ and $(0, -4.5)$ **E** $(0, 3.5)$ and $(-4.5, 0)$

6G **11** Which of the following equations has a gradient of 2 and a y-intercept of -1?

A $2y + x = 2$ **B** $y - 2x = 1$ **C** $y = -2x + 1$ **D** $y = 2x - 1$ **E** $2x + y = 1$

6I **12** A line has x- and y-intercepts of, respectively, 1 and 2. Its equation is:

A $2x - y = 2$ **B** $y = -x + 2$ **C** $y = 2x + 2$ **D** $x + 2y = 1$ **E** $y = -2x + 2$

6H **13** The gradient of the line that is perpendicular to the line with equation $y = -2x - 5$ is:

A 2 **B** -2 **C** $\frac{1}{2}$ **D** $-\frac{1}{2}$ **E** $\frac{1}{5}$

6K **14** Which equation shows that y is directly proportional to x?

A $y = 5x - 6$ **B** $y = \frac{6}{x}$ **C** $y = 2x + 4$ **D** $y = 12x$ **E** $y = 20 - 3x$

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Extended-response questions

1 David and Kaylene travel from Melton to Moorbank army base to watch their daughter's march-out parade. The total distance for the trip is 720 km, and they travel an average of 90 km per hour.

a Complete the table of values below from 0 to 8 hours.

Time in hours (t)	0	2	4	6	8
km from Moorbank	720				

b Plot a graph of the number of kilometres from Moorbank army base against time.

c David and Kaylene start their trip at 6 a.m. If they decide to stop for breakfast at Albury and Albury is 270 km from Melton, what time would they stop for breakfast?

d If the car they are driving needs refilling every 630 km, how long could they drive for before refilling the car?

e What would be the total driving time if they didn't stop at all?

f If the total number of breaks, including food and petrol stops, is 2 hours, when would they arrive at the army base?

2 A young maths whiz in the back seat of a car is counting down the distance to the nearest town, which initially is 520 km away. The car is travelling at an average speed of 80 km per hour.

a Find the distance to the town after:

i 1 hour **ii** 3 hours

b D km is the distance to the town after t hours.

i Write a rule for D in terms of t.

ii Sketch a graph for t between 0 and 6.5.

c Use your rule to find:

i the distance to the town after 4.5 hours

ii the time it takes for the distance to the town to be 340 km

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Measurement

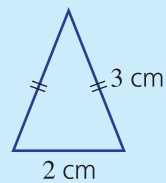
Short-answer questions

1 Convert these measurements to the units shown in the brackets.

a 0.43 m (cm) **b** 32 000 mm^2 (cm^2) **c** 0.03 m^3 (cm^3)

2 Find the perimeter of each of these shapes.

a



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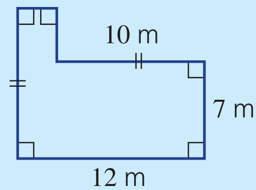

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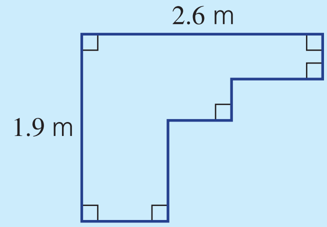
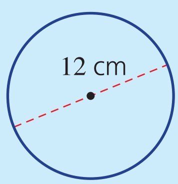


3 For these shapes, find, correct to two decimal places:

i the perimeter **ii** the area

a



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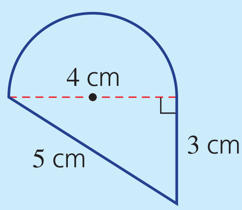

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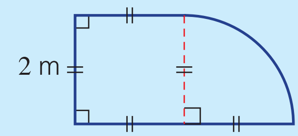
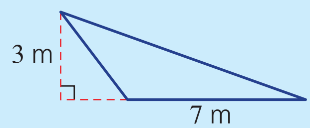


4 Find the area of each of these shapes.

a



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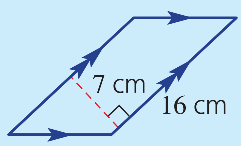

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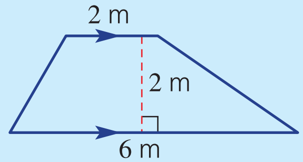
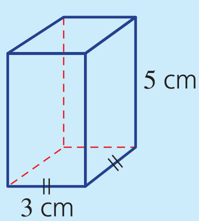


5 For these solids, find (correct to two decimal places where necessary):

i the volume **ii** the total surface area

a



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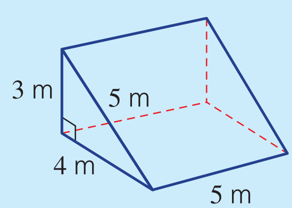

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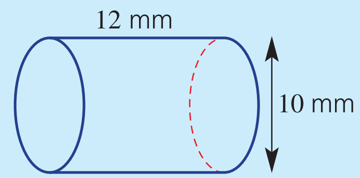


Multiple-choice questions

1 The number of centimetres in 2.8 metres is:

A 0.28 **B** 28 **C** 280 **D** 2.8 **E** 2800

2 A rectangle has length 7 cm and perimeter 22 cm. Its width is:

A 7.5 cm **B** 15 cm **C** 14 cm **D** 8 cm **E** 4 cm

3 The area of a circle with diameter 10 cm is given by:

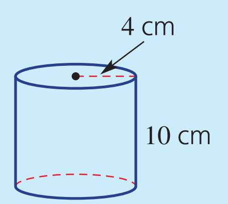
A $\pi(10)^2$ cm^2 **B** $\pi(5)^2$ cm^2 **C** 10π cm^2

D $5 \times \pi$ cm^2 **E** 25 cm^2

4 The surface area of this cylinder is closest to:

A 351.9 cm^2 **B** 301.6 cm^2 **C** 175.9 cm^2

D 276.5 cm^2 **E** 183.4 cm^2



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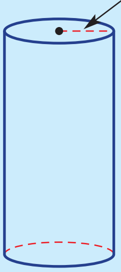
5 The area of the triangular cross-section of a prism is 8 mm^2 and the prism's height is 3 mm. The prism's volume is:

A 48 mm^3 B 12 mm^3 C 24 mm^2
D 24 mm^3 E 12 mm^2

Extended-response question

A concrete cylindrical pole has radius 0.5 m and height 10 m. The outside curved surface only is to be painted. Answer the following, to two decimal places.

a What volume of concrete was used to make the pole?
b What area is to be painted?
c A litre of paint covers 6 m^2. Paint costs $12 per litre and there are 18 poles to be painted. What is the cost of paint required? Round your answer to the nearest $10.



Consumer arithmetic

Short-answer questions

1 Sven earns $28.40 per hour working in a bar. What would he earn for working a 6-hour shift?

2 Tobias earns $55.20 an hour as a mechanic. Calculate his:
a time-and-a-half rate
b double-time rate
c weekly wage for 38 hours at normal rate
d weekly wage for 38 hours at normal rate plus 3 hours at time and a half

3 Fatima earns $60.80 an hour on weekdays and double time on the weekends. Calculate her weekly pay if she works 9 a.m.–3 p.m. Monday to Friday and 9 a.m. till 11:30 a.m. on Saturday.

4 Cara invests 10% of her net annual salary for 1 year into an investment account earning 4% p.a. simple interest for 5 years. Calculate the simple interest earned if her annual net salary is $77 500.

5 A $120 Blu-ray player is discounted by 15%. What is the sale price?

6 Marina has a taxable income of $42 600. Calculate her income tax if she falls into the following tax bracket.

$3572 plus 32.5c for each $1 over $37 000

7 Each fortnight, Raj earns $1430 gross income and pays $34.94 in superannuation, $23.40 in union fees and $493.60 in tax.
a What is Raj's annual gross income?
b How much tax does Raj pay each year?
c What is Raj's net annual income?
d What is Raj's net weekly income?

8 Find the final value of an investment of $7000 at 6% p.a., compounded annually for 4 years.

9 Jason earns $664.20 in a week, where he works 35 hours at his normal hourly rate and 4 hours at time and a half. Calculate Jason's normal hourly rate.

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Multiple-choice questions

1 Nigel earns $1256 a week. His annual income is:
A $24.15 B $32 656 C $65 312 D $15 072 E $12 560

2 Who earns the most in a year?
A Priya: $56 982 p.a.
B Suresh: $1986 per fortnight
C Henry: $1095 per week
D Yasmin: $32.57 per hour, 38-hour weeks for 44 weeks
E Bill: $20 000 p.a.

3 Adrian works 35 hours a week, earning $575.75. His wage for a 38-hour week is:
A $16.45 B $21 878.50 C $625.10 D $530.30 E $575.75

4 Jake earns a retainer of $420 per week plus a 2% commission on all sales. His fortnightly pay when his sales total $56 000 for the fortnight is:
A $2240 B $840 C $1540 D $1960 E $56 420

5 Antonia earns $4700 gross a month. She has annual deductions of $14 100 in tax and $1664 in health insurance. Her net monthly income is:
A $3386.33 B $11 064 C $40 636 D $72 164 E $10 000

Extended-response question

A computer tablet with a recommended retail price of $749 is offered for sale in three different ways:

Method A	Method B	Method C
5% discount for cash	3% fee for a credit card payment	20% deposit and then $18.95 per month for 3 years

a Jai buys a tablet for cash. How much does he pay?
b Thalia buys a tablet using her mother's credit card. How much more does Tahlia pay for her tablet compared to Jai?
c Georgia chooses to pay for her tablet using method C.
i Calculate the deposit Georgia must pay.
ii What is the final cost of purchasing the tablet on these terms?
iii How much interest does Georgia pay on her purchase?
iv What percentage of the recommended retail price is Georgia's interest? Round your answer to two decimal places.

Algebra and indices

Short-answer questions

1 Simplify the following.
a $2xy + 7x + 5xy - 3x$ b $-3a \times 7b$ c $\frac{4a^2b}{8ab}$

2 a Expand and simplify the following.
i $-4(x - 3)$ ii $3x(5x + 2)$ iii $4(2x + 1) + 5(x - 2)$

b Factorise the following.
i $18 - 6b$ ii $3x^2 + 6x$ iii $-8xy - 12y$

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3 Simplify these algebraic fractions.

a $\frac{6x+18}{6}$ b $\frac{3(x-1)}{8x} \div \frac{x-1}{2x}$ c $\frac{x}{2}+\frac{2x}{5}$ d $\frac{x}{4}-\frac{3}{8}$

4 Use index laws to simplify the following. Express with positive indices.

a $2x^2 \times 5x^4$ b $\frac{12x^3y^2}{3xy^5}$ c $(2m^4)^3$ d $3x^0+(4x)^0$

e $\left(\frac{3a}{b^4}\right)^2$ f $3a^{-5}b^2$ g $\frac{4}{t^{-5}}$ h $\frac{4x^5y^3 \times 5x^{-2}y}{10x^7y^2}$

5 a Write the following as basic numerals.

i 4.73×10^5 ii 5.21×10^{-3}

b Convert these to scientific notation, using three significant figures.

i 0.000027561 ii 8 707 332

Multiple-choice questions

1 The expanded and simplified form of $4(2x-3)-4$ is:

A $8x-7$ B $6x-11$ C $8x-16$ D $8x-8$ E $6x-7$

2 The fully factorised form of $4x^2+12x$ is:

A $4(x^2+3x)$ B $4x(x+12)$ C $4(x^2+12x)$ D $4x(x+3)$ E $2x(x+6)$

3 $\frac{5(x-2)}{3} \times \frac{12}{x-2}$ simplifies to:

A 20 B -6 C $\frac{20}{x}$ D $16(x-2)$ E $\frac{(x-2)}{6}$

4 Using index laws, $\frac{3x^2y \times 2x^3y^2}{xy^3}$ simplifies to:

A $\frac{5x^5}{y}$ B $6x^5$ C $\frac{6x^2}{y}$ D $6x^4$ E $\frac{6x^4}{y}$

5 $(a^3)^4b^{-2}$ expressed with positive indices is:

A $\frac{a^7}{b^2}$ B $\frac{a^{12}}{b^2}$ C $\frac{1}{a^7b^2}$ D $\frac{a^{12}}{b^{-2}}$ E a^7b^2

Extended-response question

Julie invests \$3000 at an interest rate of 6% per year, compounded.

a Write a rule for the amount of money, \$$A$, in her account after n years.

b How much will be in her account, correct to two decimal places, in:

i 2 years' time? ii 6 years' time?

c Use trial and error to determine how long it will take her to double her initial investment. Answer to one decimal place.

Probability

Short-answer questions

1 A keen birdwatcher records the number of different species of birds in her backyard over a 20-day period.

Number of species	0	1	2	3	4	5	6
Frequency	0	2	3	8	4	2	1

From these results, estimate the probability that on the next day the birdwatcher will observe the following number of species.

a 3 b 2 or 3 c fewer than 5 d at least 2

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2 Of 25 students, 18 are wearing jackets, 14 are wearing hats and 10 are wearing both jackets and hats.

a Represent this information in a Venn diagram.

b Represent this information in a two-way table.

c How many students are wearing neither a hat nor a jacket?

d If a student is chosen randomly from the group, find the probability that the student will be wearing:

i a hat and not a jacket ii a hat or a jacket

iii a hat and a jacket iv a hat, given that they are wearing a jacket

3 Two 6-sided dice are rolled and the total is recorded.

a Complete the table to find the total number of outcomes.

b Find:

i $\Pr(5)$ ii $\Pr(7)$

iii $\Pr(\text{at least } 7)$ iv $\Pr(\text{at most } 4)$

Die 1 (columns), *Die 2* (rows)

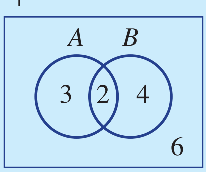
	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3						
4						
5						
6						

4 Two people are chosen without replacement from a group of 2 males and 4 females. Use a tree diagram to help find the probability of selecting:

a 2 males b 1 male and 1 female c at least 1 female

5 For each diagram, find $\Pr(A)$ and $\Pr(A \mid B)$ and then decide if events A and B are independent.

a



b

	A	A'	
B	3	2	5
B'	2	1	3
	5	3	8

Multiple-choice questions

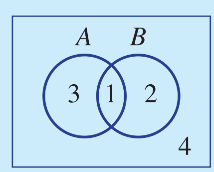
1 A letter is chosen from the word PROBABILITY. What is the probability that it will not be a vowel?

A $\frac{3}{11}$ B $\frac{4}{11}$ C $\frac{7}{11}$ D $\frac{1}{2}$ E $\frac{8}{11}$

2 For this Venn diagram, $\Pr(A \cup B)$ is equal to:

A 1 B $\frac{1}{6}$ C $\frac{1}{10}$

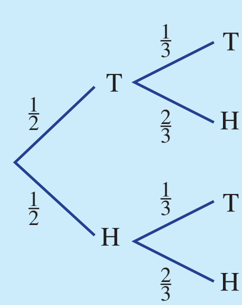
D $\frac{3}{10}$ E $\frac{3}{5}$



3 For this tree diagram, what is the probability of the outcome (T, H)?

A $\frac{1}{3}$ B $\frac{1}{6}$ C $\frac{1}{2}$

D $\frac{1}{4}$ E $\frac{2}{3}$



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4 The number of faults in a computer network over a period of 10 days is recorded in this table.

Number of faults	0	1	2	3
Frequency	1	5	3	1

An estimate for the probability that on the next day there would be at least two errors is:

A $\frac{3}{10}$ B $\frac{1}{5}$ C $\frac{4}{5}$ D $\frac{2}{5}$ E $\frac{1}{10}$

5 Two events are mutually exclusive when:

A $\Pr(A) = 0$ B $\Pr(A \cap B) = 0$ C $\Pr(A \cup B) = 0$
D $\Pr(A|B) = \Pr(A)$ E $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Extended-response question

A hot dog stall produces two types of hot dogs: traditional (T) at $4 each and Aussie (A) at $5 each. Gareth randomly selects two hot dogs.

a Complete this table to show the possible selections.

	T	A
T	(T, T)	
A		

b Find the probability of selecting:
i 2 Aussie hot dogs
ii at least 1 Aussie hot dog

c Gareth has only $8. What is the probability that he will be able to afford two hot dogs?

Statistics

Short-answer questions

1 Twenty people were surveyed to find out how many days in the past completed month they used public transport. The results are as follows:
7, 16, 22, 23, 28, 12, 18, 4, 0, 5, 8, 19, 20, 22, 14, 9, 21, 24, 11, 10

a Organise the data into a frequency table with class intervals of 5 and include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and the percentage frequency on the one graph.
c i State the frequency of people who used public transport on 10 or more days.
ii State the percentage of people who used public transport on fewer than 15 days.
iii State the most common interval of days for which public transport was used. Can you think of a reason for this?

2 The data set shows the number of Apps owned by students in a school class.
12 24 36 17 8 24 9 4 15 32 41 26 15 18 7

a Display this data using a stem-and-leaf plot.
b Describe the distribution of the data as symmetrical or skewed.

3 For the data set 8, 10, 2, 17, 6, 30, 12, 7, 12, 15, 4:

a Order the data.
b Determine:
i the minimum and maximum values
ii the median
iii the lower quartile (Q_1) and the upper quartile (Q_3)
iv IQR ($= Q_3 - Q_1$)
v any outliers

c Draw a box plot of the data.

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4 Farsan's bank balance over 12 months is recorded below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Balance ($)	1500	2100	2300	2500	2200	1500	1200	1600	2000	2200	1700	2000

a Plot the time series for the 12 months.
b Describe the way in which the bank balance has changed over the 12 months.
c Between which consecutive months did the biggest change in the bank balance occur?
d What is the overall change in the bank balance over the year?

Multiple-choice questions

1 The values of a and b in this frequency table are:

A $a = 3,\ b = 28$
B $a = 4,\ b = 28$
C $a = 4,\ b = 19$
D $a = 6,\ b = 20$
E $a = 3,\ b = 30$

Colour	Frequency	Percentage frequency
Blue	4	16
Red	7	b
Green	a	12
White	6	24
Black	5	20
Total	25	

2 The mean, median and mode of the data set 3, 11, 11, 7, 1, 9 are:
A mean = 7, median = 9, mode = 11
B mean = 6, median = 9, mode = 11
C mean = 7, median = 8, mode = 11
D mean = 7, median = 11, mode = 8
E mean = 8, median = 7, mode = 11

3 For the given stem-and-leaf plot, the range and median, respectively, of the data are:

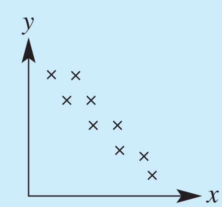
A 20, 12.5
B 7, 12
C 27, 12.5
D 29.3
E 27, 13

Stem	Leaf
0	2 2 6 7
1	0 1 2 3 5 8
2	3 3 5 7 9

1 | 5 means 15

4 The interquartile range (IQR) for the data set 2, 3, 3, 7, 8, 8, 10, 13, 15 is:
A 5
B 8.5
C 7
D 13
E 8

5 The best description of the correlation between the variables for the scatter plot shown is:
A weak negative
B strong positive
C strong negative
D weak positive
E no correlation



Extended-response question

The heights of plants in a group of the same species after a month of watering with a set number of millimetres of water per day are recorded below.

Water (mL)	8	5	10	14	12	15	18
Height (cm)	25	27	34	40	35	38	45

a Draw a scatter plot for the data, using 'Water' for the x-axis.
b Describe the correlation between water and height as positive, negative or none.
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate the:
  i height of a plant watered with 16 mL of water per day
  ii daily amount of water given to a plant of height 50 cm

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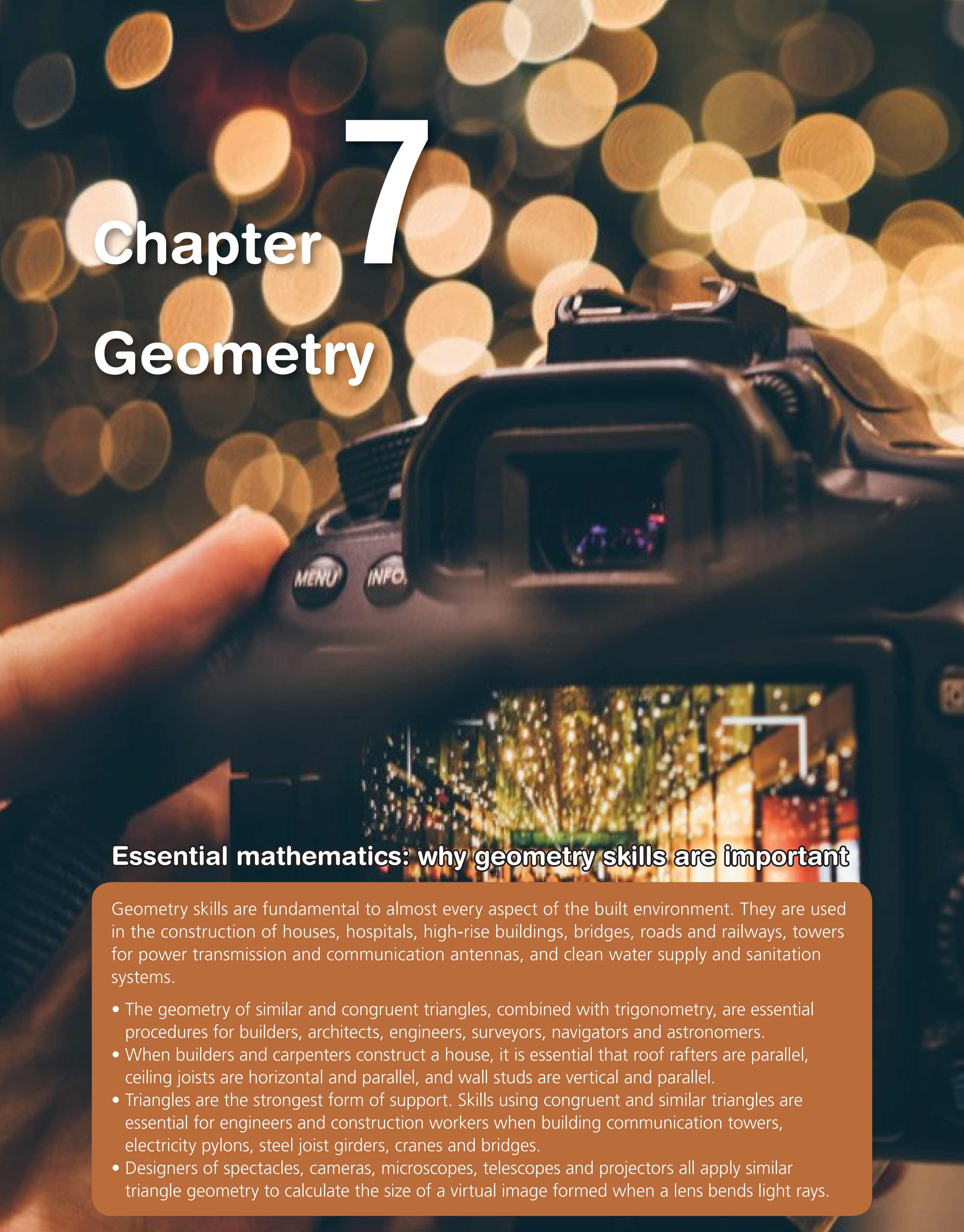

Chapter 7

Geometry

Essential mathematics: why geometry skills are important

Geometry skills are fundamental to almost every aspect of the built environment. They are used in the construction of houses, hospitals, high-rise buildings, bridges, roads and railways, towers for power transmission and communication antennas, and clean water supply and sanitation systems.

- The geometry of similar and congruent triangles, combined with trigonometry, are essential procedures for builders, architects, engineers, surveyors, navigators and astronomers.
- When builders and carpenters construct a house, it is essential that roof rafters are parallel, ceiling joists are horizontal and parallel, and wall studs are vertical and parallel.
- Triangles are the strongest form of support. Skills using congruent and similar triangles are essential for engineers and construction workers when building communication towers, electricity pylons, steel joist girders, cranes and bridges.
- Designers of spectacles, cameras, microscopes, telescopes and projectors all apply similar triangle geometry to calculate the size of a virtual image formed when a lens bends light rays.

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In this chapter

Victorian Curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Formulate proofs involving congruent triangles and angle properties (VCMMG344)

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (VCMMG345)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 Write the missing word. Choose from: *right*, *reflex*, *straight*, *complementary*, *acute*, *revolution*, *obtuse*, *supplementary*.

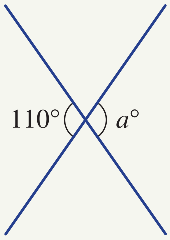
a ________ angles are between $0°$ and $90°$.
b A ________ angle is $90°$.
c An ________ angle is between $90°$ and $180°$.
d A $180°$ angle is called a ________ angle.
e A ________ angle is between $180°$ and $360°$.
f A ________ is $360°$.
g ________ angles sum to $90°$.
h ________ angles sum to $180°$.

2 Match the type of triangle (**A**–**F**) with the given properties (**a**–**f**).

a all sides of different length
b two sides of the same length
c one right angle
d one obtuse angle
e three sides of equal length
f all angles acute

A obtuse angled
B isosceles
C equilateral
D acute angled
E scalene
F right angled

3 Find the values of the pronumerals.

a


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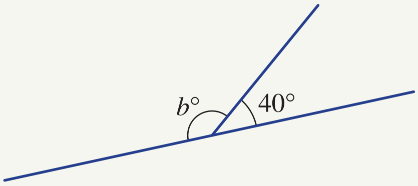

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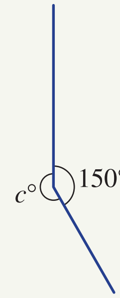

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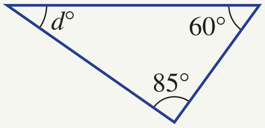
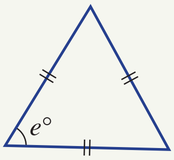


e


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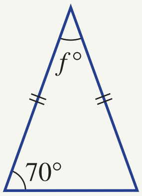
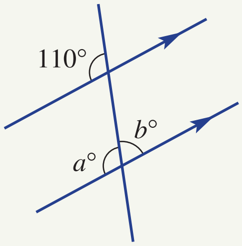


4 Find the value of the pronumerals in these sets of parallel lines.

a


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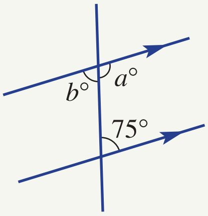
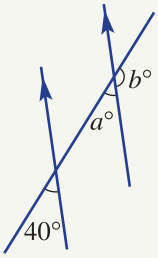


c


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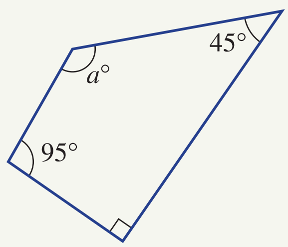
5 How many special quadrilaterals have these properties?

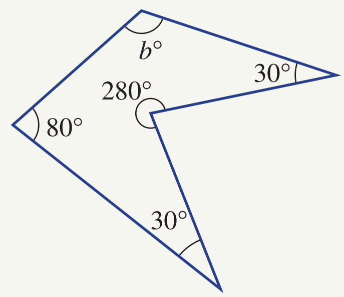
- **a** All sides equal and all angles $90°$
- **b** Two pairs of parallel sides
- **c** Two pairs of parallel sides and all angles $90°$
- **d** Two pairs of parallel sides and all sides equal
- **e** One pair of parallel sides
- **f** Two pairs of equal-length sides and no sides parallel

6 Use the angle sum formula, $S = 180° \times (n - 2)$, to find the angle sum of these polygons and the value of the pronumeral.

a



b



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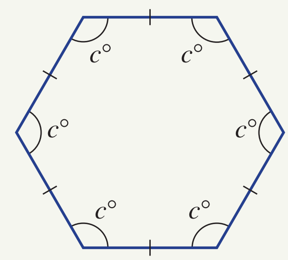
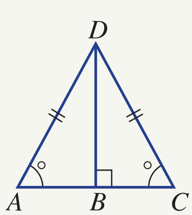


7 Select the option from **A**–**E** that represents:

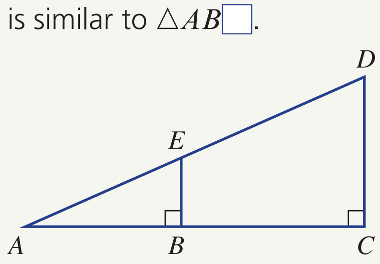
- **a** the tests for congruence of triangles
- **b** the tests for similarity of triangles

- **A** SSS, SA, AA, RHS
- **B** S, SAS, A, RAS
- **C** SSS, SAS, AAA, RHS
- **D** SSS, SAS, AAS, RHS
- **E** SSS, AAA, RS

8 Complete this sentence for the given diagram. $\triangle AB\square$ is congruent to $\triangle\square BD$.



9 Complete this sentence for the given diagram.
$\triangle A\square D$ is similar to $\triangle AB\square$.



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7A Parallel lines CONSOLIDATING

Learning intentions

- To know the special pairs of angles formed when a transversal cuts two other lines
- To know the relationship between pairs of angles formed when a pair of parallel lines are cut by a transversal
- To be able to calculate unknown angles associated with parallel lines

Key vocabulary: parallel, transversal, corresponding, alternate, cointerior, vertically opposite, supplementary

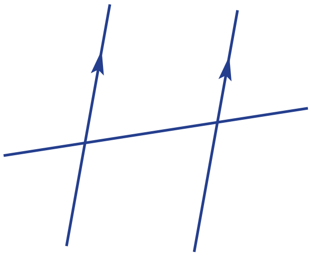
Parallel lines are everywhere – in buildings, in nature and on clothing patterns. Steel or concrete uprights at road intersections are an example.

Parallel lines are always the same distance apart and never meet. In diagrams, arrows are used to show that lines are parallel.

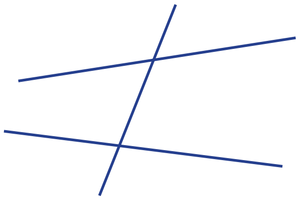
Lesson starter: 2, 4 or 8 different angles

Here are two pairs of lines crossed by a transversal. One pair is parallel and the other is not.

A



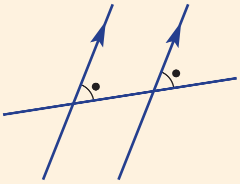
B



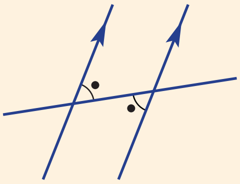
- How many angles of different size are in set A?
- How many angles of different size are in set B?
- If only one angle is known in set A, can you determine all other angles? Give reasons.

Key ideas

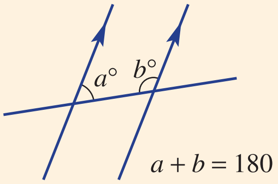
- A **transversal** is a line cutting two or more other lines.
- For parallel lines:
  - **Corresponding** angles are equal.



  - **Alternate** angles are equal.



  - **Cointerior** angles are **supplementary** (sum to 180°).



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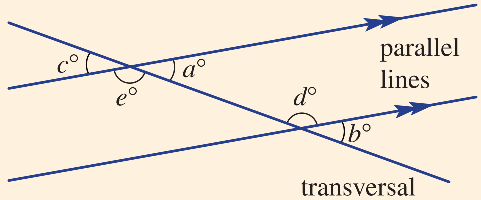
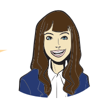


$a = b$	Corresponding angles
$a = c$	**Vertically opposite** angles
$d = e$	Alternate angles
$a + e = 180$	Supplementary angles
$a + d = 180$	Cointerior angles

Exercise 7A

Understanding — 1, 2 — 2



Hint: Choose from: 180°, *equal, supplementary*

1 Write the missing word or number.

a Supplementary angles add to __________.

b Vertically opposite angles are __________.

c When two lines are parallel and are crossed by a transversal, then:

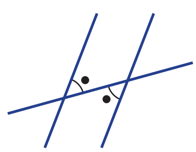
i Corresponding angles are __________.

ii Alternate angles are __________.

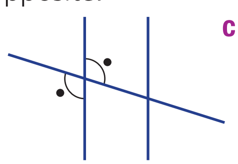
iii Cointerior angles are __________.

2 For the given diagrams, decide whether the given pair of marked angles are corresponding, alternate, cointerior or vertically opposite.

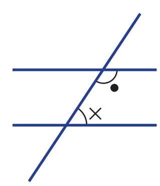
a



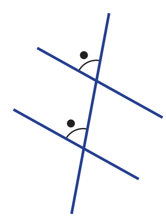
b



c



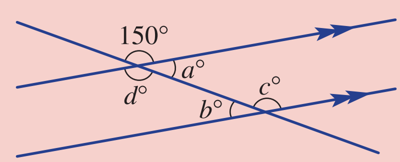
d



Fluency — 3–5 — 3, 4, 5(½)

Example 1 Finding angles in parallel lines

Find the values of the pronumerals in this diagram.
Write down the reason in each case.



Solution	Explanation
$a = 180 - 150 = 30$ The angles marked as $a°$ and 150° are supplementary.	Two angles on a straight line sum to 180°.
$b = 30$ The angles marked as $b°$ and $a°$ are alternate.	Alternate angles are equal in parallel lines.

Continued on next page

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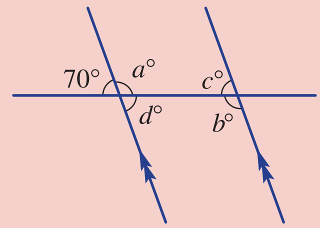
$c = 150$
The angles marked as $c°$ and $150°$ are corresponding.
Corresponding angles are equal in parallel lines.

OR, the angles marked as $c°$ and $a°$ are cointerior.
Cointerior angles are supplementary in parallel lines.

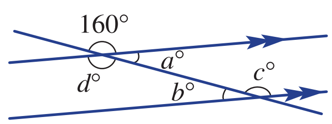
$d = 150$
The angles marked as $d°$ and $150°$ are vertically opposite.
Vertically opposite angles are equal.

Now you try

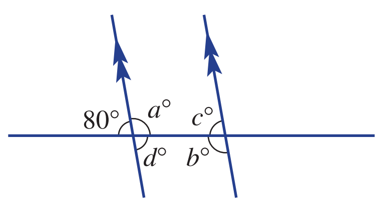
Find the values of the pronumerals in this diagram.
Write down the reason in each case.



3 Find the values of the pronumerals in this diagram. Write down the reason in each case.

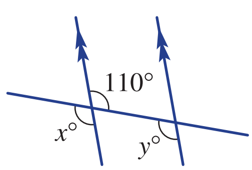


4 Find the values of the pronumerals in this diagram. Write down the reason in each case.

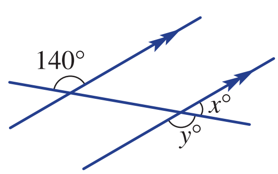


5 Find the value of x and y in these diagrams.

a



b



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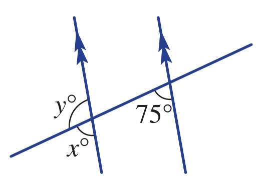
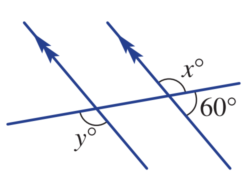
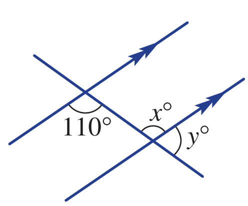


Hint: When lines are parallel:
- Corresponding angles are equal.
- Alternate angles are equal.
- Cointerior angles add to $180°$.

d



e



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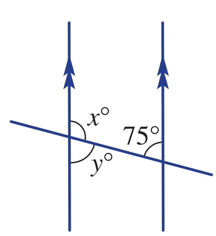

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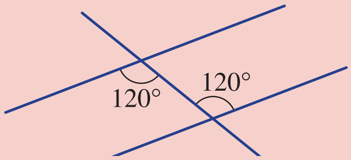
Problem-solving and reasoning

6, 7–8(½), 9 | 6–8(½), 9, 10

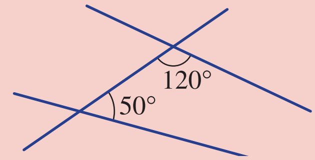
Example 2 Proving that two lines are parallel

Decide, with reasons, whether the given pairs of lines are parallel.

a



b

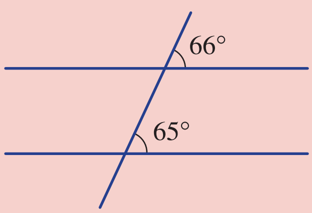


Solution	Explanation
a Yes – alternate angles are equal.	If alternate angles are equal, then lines are parallel.
b No – cointerior angles are not supplementary.	If lines are parallel, then cointerior angles should add to 180°, but $120^\circ + 50^\circ = 170^\circ$.

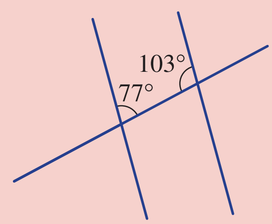
Now you try

Decide, with reasons, whether the given pairs of lines are parallel.

a

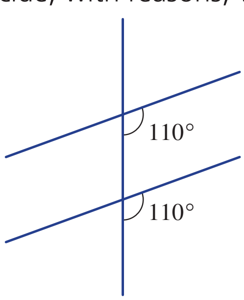


b



6 Decide, with reasons, whether the given pairs of lines are parallel.

a



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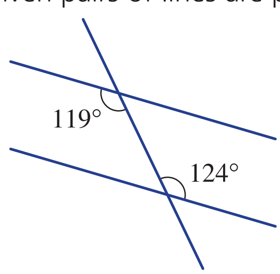

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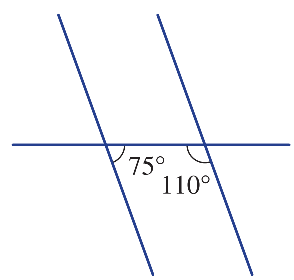

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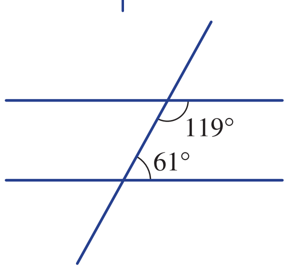

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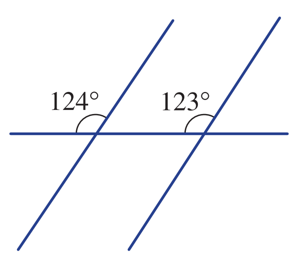

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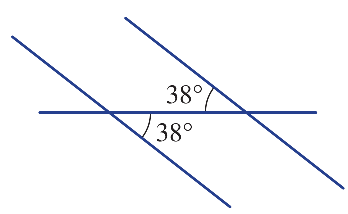
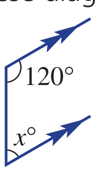


7 These diagrams have a pair of parallel lines. Find the unknown angle, x.

a



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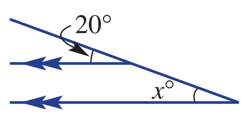

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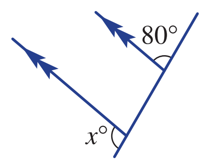

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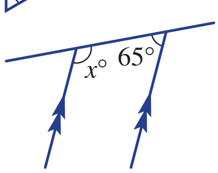

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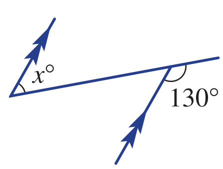

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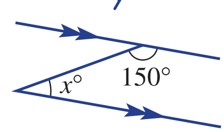

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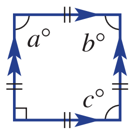
7A

8 These common shapes consist of parallel lines. One or more internal angles are given. Find the values of the pronumerals.

a



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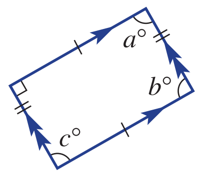

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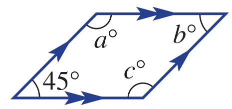

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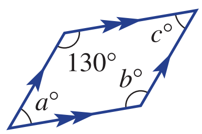

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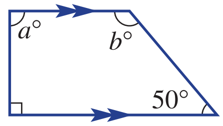

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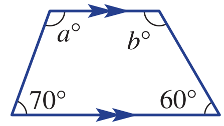
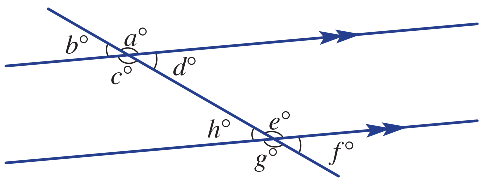


9 For this diagram, list all pairs of angles that are:

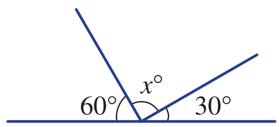
a corresponding
b alternate
c cointerior
d vertically opposite



Hint: One example for part **a** is (a, e).

10 Find the unknown value, x, in each of these cases.

a



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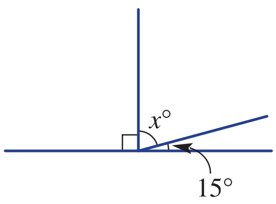
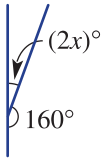
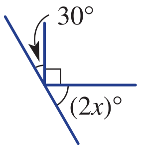


Hint: Angles on a straight line add to 180°.

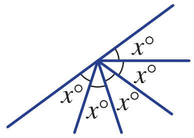
c



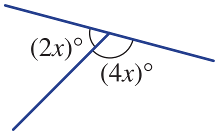
d



e



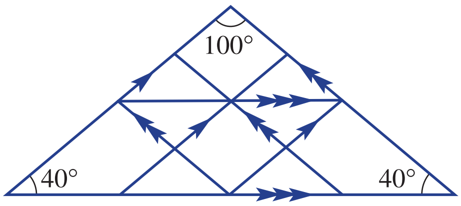
f



The roof truss — 11

11 This diagram is of a roof truss with three groups of parallel supports.
How many of the angles are:

a 100° in size?
b 40° in size?
c 140° in size?



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7B Triangles CONSOLIDATING

Learning intentions

- To know the angle sum of a triangle
- To know the properties of special types of triangles
- To know the exterior angle theorem
- To be able to calculate unknown angles inside a triangle
- To be able to calculate unknown angles using the exterior angle theorem

Key vocabulary: triangle, acute, right, obtuse, scalene, isosceles, equilateral, exterior angle theorem

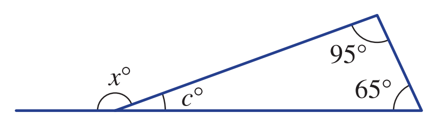
The triangle is at the foundation of geometry, and its properties are used to work with more complex geometry.

One of the best known and most useful properties of triangles is the internal angle sum (180°). You can check this by measuring and adding up the three internal angles of any triangle.

Lesson starter: Explore the exterior angle

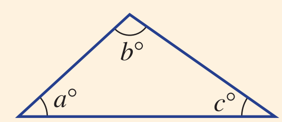
Consider this triangle with exterior angle x°.

- Use the angle sum of a triangle to find the value of c.
- Now find the value of x.
- What do you notice about x° and the two given angles? Is this true for other triangles?
  Give examples and reasons.

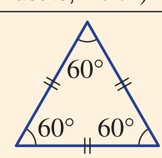
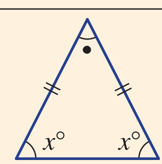
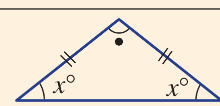
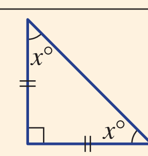
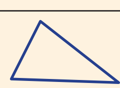
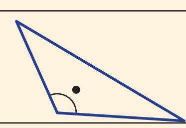
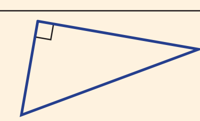


Key ideas

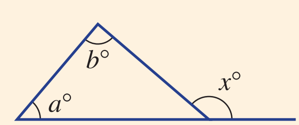
- The sum of all three internal angles of a triangle is 180°.
- Triangles can be classified by their side lengths or their internal angles.



$a + b + c = 180$

		Classified by internal angles		
		Acute-angled triangles (all angles acute, $< 90^\circ$)	Obtuse-angled triangles (one angle obtuse, $> 90^\circ$)	Right-angled triangles (one right angle, 90°)
Classified by side lengths	Equilateral triangles (three equal side lengths)		Not possible	Not possible
	Isosceles triangles (two equal side lengths)			
	Scalene triangles (no equal side lengths)			

- The **exterior angle theorem**:
  The exterior angle is equal to the sum of the two opposite interior angles.



$x = a + b$

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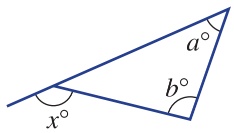
Exercise 7B

Understanding 1–3 3

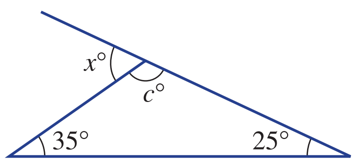
1 State the missing word or angle size.
  a The angle sum of a triangle is ________.
  b An ________ triangle has two equal side lengths.
  c An equilateral triangle's interior angles are all ________.
  d An ________ -angled triangle has one obtuse angle.
  e A ________ triangle has all sides of different length.
  f An acute-angled triangle has all angles less than ________.

2 Choose the correct expression for this exterior angle.



A $a = x + b$ B $b = x + a$
C $x = a + b$ D $a + b = 180$
E $2a + b = 2x$

3 The two given interior angles for this triangle are 25° and 35°.

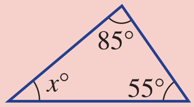


  a Use the angle sum (180°) to find the value of c.
  b Hence, find the value of x.
  c What do you notice about the value of x and the two given interior angles?

Fluency 4–6(½) 4–6(½)

Example 3 Using the angle sum

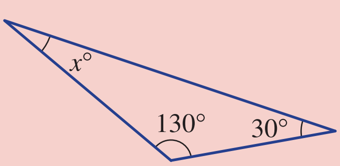
Find the value of the unknown angle, x, in this triangle.



Solution	Explanation
$x + 85 + 55 = 180$	The sum of the three internal angles in a triangle is 180°.
$x + 140 = 180$	Simplify before solving for x.
$x = 40$	Solve for x by subtracting 140 from both sides of the 'equals' sign.
∴ The unknown angle is 40°.	

Now you try

Find the value of the unknown angle, x, in this triangle.



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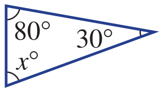
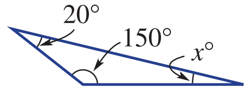
4 Find the value of the unknown angle, x, in these triangles.

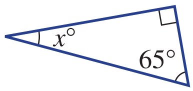
a



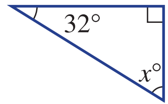
b



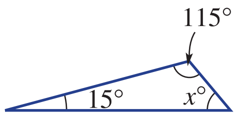
c



d



e



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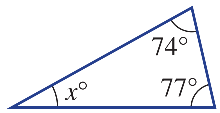
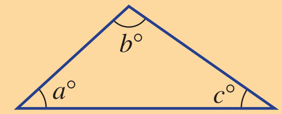


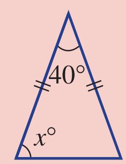
Hint:



$a + b + c = 180$

Example 4 Working with an isosceles triangle

Find the value of x in this isosceles triangle.

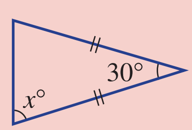


Solution	Explanation
$x + x + 40 = 180$	The triangle is isosceles and therefore the two base angles are equal.
$2x + 40 = 180$	Collect like terms.
$2x = 140$	Subtract 40 from both sides.
$x = 70$	Divide both sides by 2.

∴ The unknown angle is 70°.

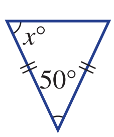
Now you try

Find the value of x in this isosceles triangle.



5 Find the value of the unknown angle, x, in these triangles.

a



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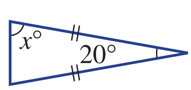

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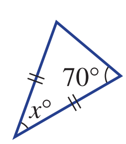

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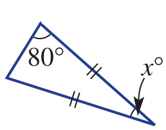

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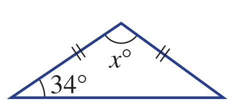

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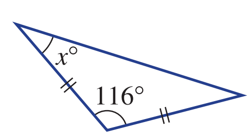

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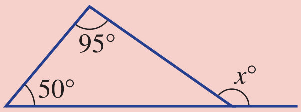
7B

Example 5 Using the exterior angle theorem

Use the exterior angle theorem to find the value of x.

a



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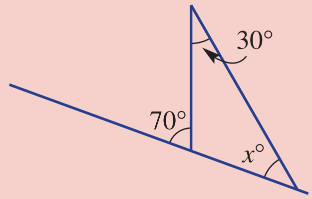
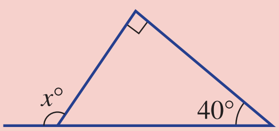


Solution	Explanation
a $x = 95 + 50$ $= 145$	The exterior angle $x°$ is the sum of the two opposite interior angles.
b $x + 30 = 70$ $x = 40$	The two opposite interior angles are $x°$ and $30°$, and $70°$ is the exterior angle.

Now you try

Use the exterior angle theorem to find the value of x.

a



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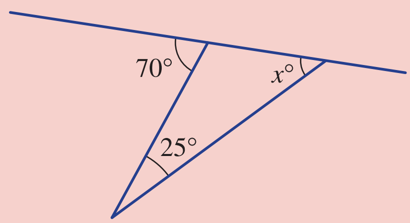
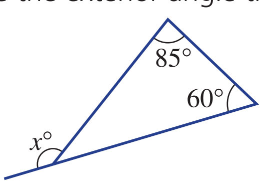


6 Use the exterior angle theorem to find the value of x.

a



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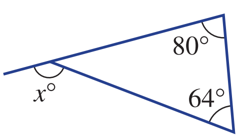
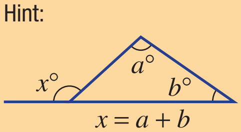
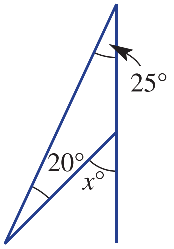


Hint:



c



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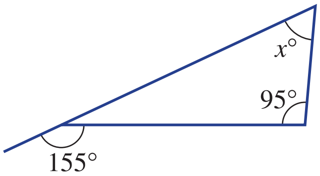

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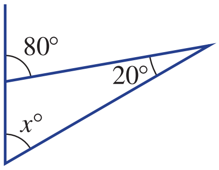

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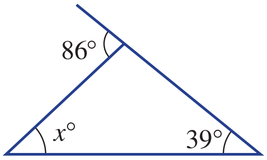

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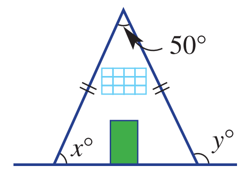
Problem-solving and reasoning — 7, 8 — 7, 9, 10

7 Decide whether the following are possible. If so, make a drawing.

a acute scalene triangle
b acute isosceles triangle
c obtuse equilateral triangle
d acute equilateral triangle
e obtuse isosceles triangle
f obtuse scalene triangle
g right equilateral triangle
h right isosceles triangle
i right scalene triangle

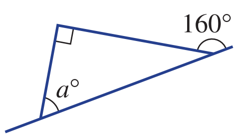
8 An architect draws the cross-section of a new ski lodge, which includes a very steep roof, as shown. The angle at the top is $50°$. Find:

a the acute angle the roof makes with the floor $(x°)$
b the obtuse angle the roof makes with the floor $(y°)$

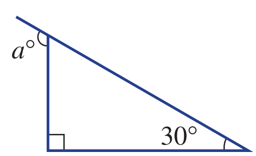


9 Use your knowledge of parallel lines and triangles to find out the value of the pronumerals in these diagrams.

a



b



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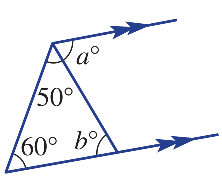

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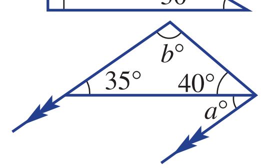

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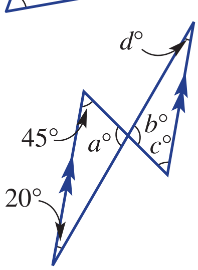

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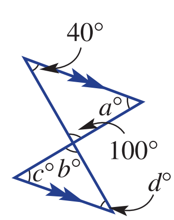
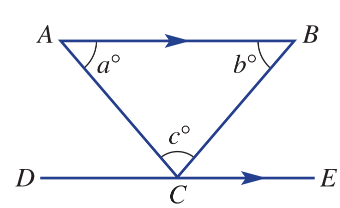


10 For this diagram, AB is parallel to DE.

a What is the size of $\angle ACD$? Use a pronumeral and give a reason.
b What is the size of $\angle BCE$? Use a pronumeral and give a reason.
c Since $\angle DCE = 180°$, what does this tell us about a, b and c?



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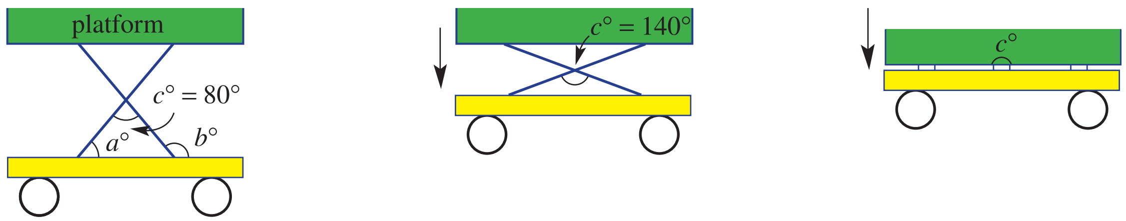
7B

The hydraulic platform — 11

11 A hydraulic platform includes a movable 'X' shape support system, as shown. When the platform is at its highest point, the angle at the centre ($c°$) of the 'X' is $80°$, as shown.



a Find the following when the platform is at its highest position.
- **i** the acute angle the 'X' makes with the platform ($a°$)
- **ii** the obtuse angle the 'X' makes with the platform ($b°$)

b The platform now moves down so that the angle at the centre ($c°$) of the 'X' changes from $80°$ to $140°$. With this platform position, find the values of:
- **i** the acute angle the 'X' makes with the platform ($a°$)
- **ii** the obtuse angle the 'X' makes with the platform ($b°$)

c The platform now moves down to the base so that the angle at the centre ($c°$) of the 'X' is now $180°$. Find:
- **i** the acute angle the 'X' makes with the platform ($a°$)
- **ii** the obtuse angle the 'X' makes with the platform ($b°$)

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7C Quadrilaterals

Learning intentions

- To know the properties of special quadrilaterals
- To know the angle sum of a quadrilateral
- To be able to calculate unknown angles inside a quadrilateral

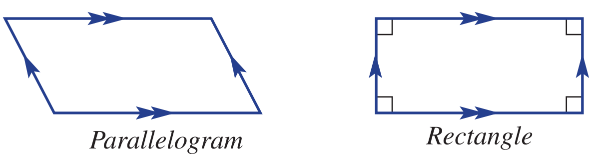
Key vocabulary: quadrilateral, parallelogram, square, rectangle. rhombus, trapezium, kite, diagonal, parallel

Quadrilaterals are shapes that have four straight sides with a special angle sum of 360°. There are six special quadrilaterals, each with their own special set of properties.

When you look around any old or modern building, you will see examples of these shapes.

Lesson starter: Why is a rectangle a parallelogram?

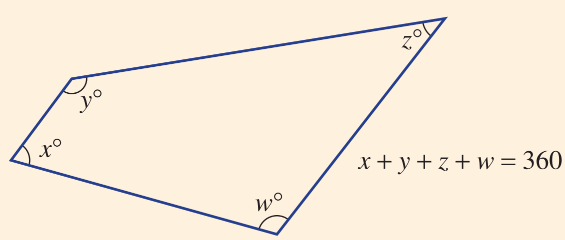
By definition, a parallelogram is a quadrilateral with two pairs of parallel sides.



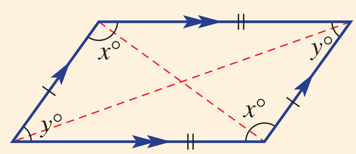
- Using this definition, do you think that a rectangle is also a parallelogram? Why?
- What properties does a rectangle have that a general parallelogram does not?
- What other special shapes are parallelograms? What are their properties?

Key ideas

- The sum of the interior angles of any quadrilateral is 360°.



- **Parallelogram** Two pairs of parallel lines
  Two pairs of equal length sides
  Opposite angles equal
  Diagonals are not equal in length



  - Parallelograms are quadrilaterals with two pairs of parallel sides. These include the square, rectangle and rhombus.

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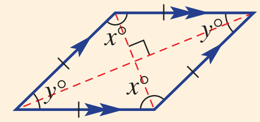
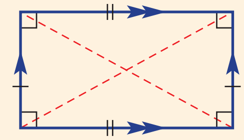
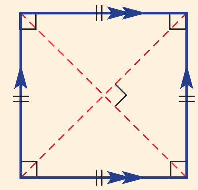
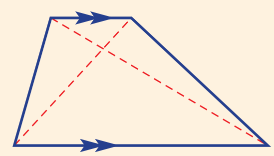
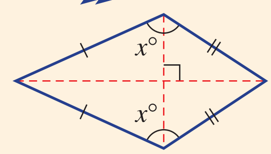
Parallelogram	**Properties**	**Drawing**
Rhombus	Two pairs of parallel lines All sides of equal length Opposite angles equal Diagonals intersect at right angles	
Rectangle	Two pairs of parallel lines Two pairs of equal-length sides All angles 90° Diagonals equal in length	
Square	Two pairs of parallel lines All sides of equal length All angles 90° Diagonals equal and intersect at right angles.	

Other quadrilaterals	**Properties**	**Drawing**
Trapezium	At least one pair of parallel sides	
Kite	Two pairs of equal-length adjacent sides One pair of equal angles Diagonals intersect at right angles	

Exercise 7C

Understanding — 1–3 | 2, 3

1 State the missing word or angle.
- **a** The angle sum of a quadrilateral is ________ .
- **b** The two special quadrilaterals which are not parallelograms are the ________ and the ________.

2 Which three special quadrilaterals are parallelograms?

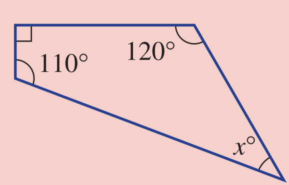
Hint: Refer to the Key ideas for help.

3 List all the quadrilaterals that have the following properties.
- **a** two pairs of parallel sides
- **b** two pairs of equal-length sides
- **c** equal opposite angles
- **d** one pair of parallel sides
- **e** one pair of equal angles
- **f** all angles 90°
- **g** equal-length diagonals
- **h** diagonals intersecting at right angles

Fluency — 4–5(½) | 4–5(½)

Example 6 Using the angle sum of a quadrilateral

Find the unknown angle in this quadrilateral.



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Solution

$x + 110 + 120 + 90 = 360$

$x + 320 = 360$

$x = 40$

$\therefore$ The unknown angle is 40°.

Explanation

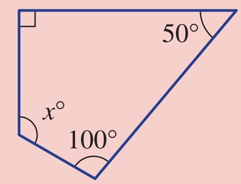
The sum of internal angles is 360°.

Simplify.

Subtract 320 from both sides.

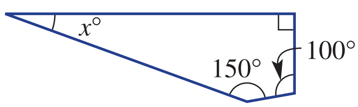
Now you try

Find the unknown angle in this quadrilateral.

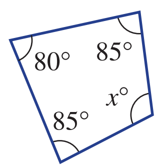


4 Find the unknown angles in these quadrilaterals.

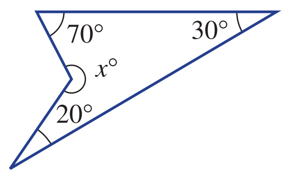
a



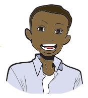
b



c



Hint: The angle sum of a quadrilateral is 360°.



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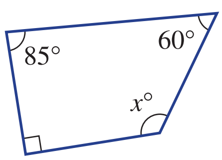
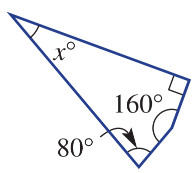


e



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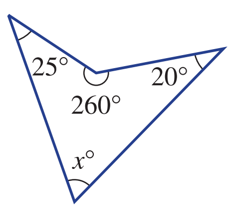
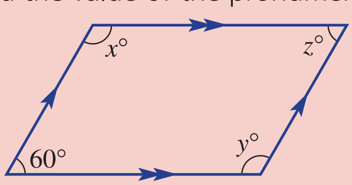


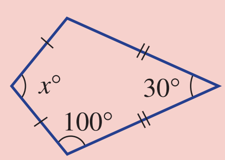
Example 7 Finding angles in special quadrilaterals

Find the value of the pronumerals in these special quadrilaterals.

a



b



Solution

a $x + 60 = 180$

$x = 120$

$\therefore y = 120$

$\therefore z = 60$

Explanation

$x°$ and 60° are cointerior angles and sum to 180°.

Subtract 60 from both sides.

$y°$ is opposite and equal to $x°$.

$z°$ is opposite and equal to 60°.

b $x + 100 + 100 + 30 = 360$

$x + 230 = 360$

$x = 130$

A kite has a pair of equal opposite angles, so there are two 100° angles.
The total sum is still 360°.

Continued on next page

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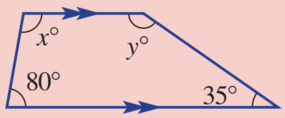
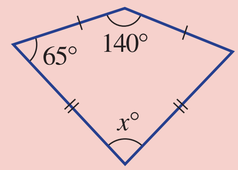
Now you try

Find the value of the pronumerals in these special quadrilaterals.

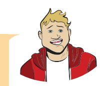
a



b

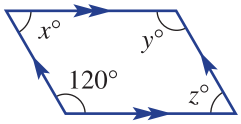


5 Find the value of the pronumerals in these special quadrilaterals.

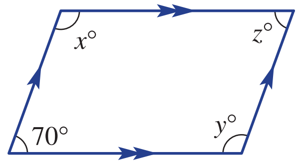


Hint: Refer to the properties of special quadrilaterals for help.

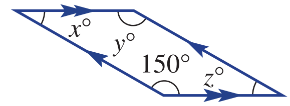
a



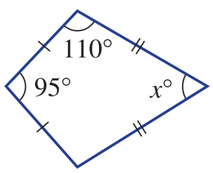
b



c



d



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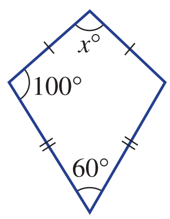

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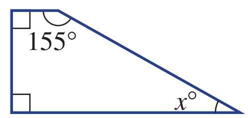
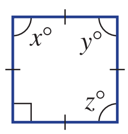
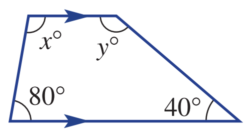


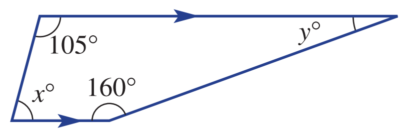
g



h



i

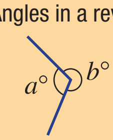


Problem-solving and reasoning

6, 7 | 6–9

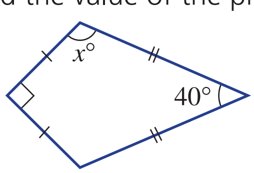
6 Find the value of the pronumerals in these shapes.

Hint:
Angles in a revolution add to 360°.

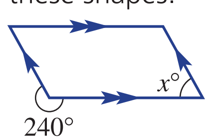


$a + b = 360$

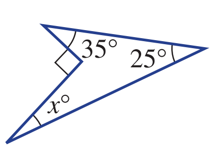
a



b



c



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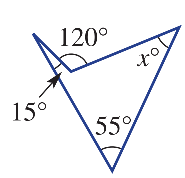

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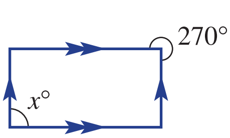

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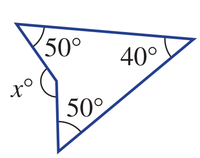

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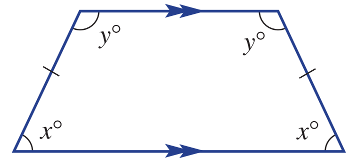
7 This shape is called an isosceles trapezium.



a Why do you think it is called an isosceles trapezium?

b i When $x = 60$, find y.

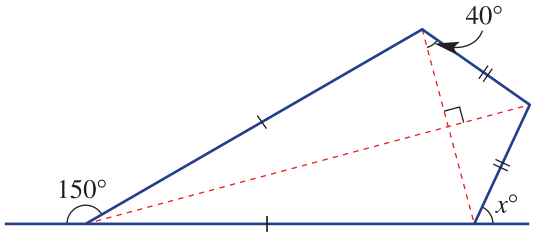
ii When $y = 140$, find x.

c List the properties of an isosceles trapezium.

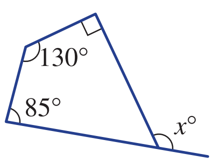
8 A modern hotel is in the shape of a kite.
Some angles are given in the diagram.

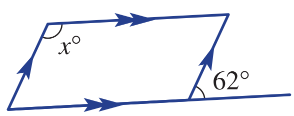
a Draw a copy of only the kite shape, including the diagonals.

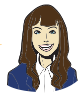
b Find the angle that the right-hand wall makes with the ground ($x°$).



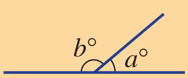
9 These quadrilaterals also include exterior angles. Find the value of x.

a


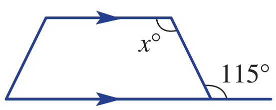
b




Hint:



$a + b = 180$

c


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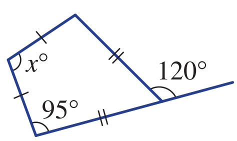
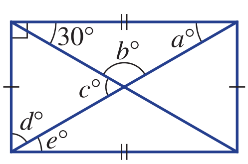


Quadrilaterals and triangles — 10

10 The following shapes combine quadrilaterals with triangles. Find the value of the pronumerals.

a


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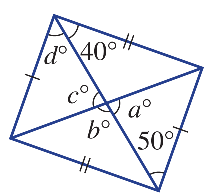

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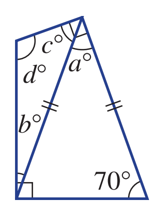
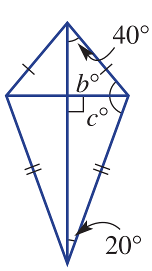


d


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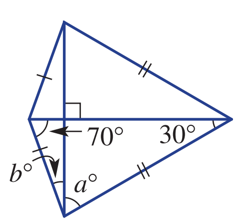

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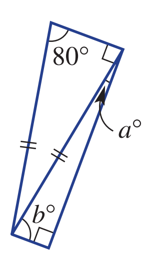

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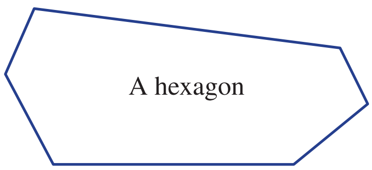
7D Polygons ★

Learning intentions

- To know the rule for the angle sum of a polygon
- To be able to calculate unknown angles inside a polygon
- To be able to calculate the interior angle of regular polygons

Key vocabulary: polygon, regular polygon

A closed shape with all straight sides is called a polygon. Like triangles and quadrilaterals (which are both polygons), they all have a special angle sum.



Lesson starter: Remember the names

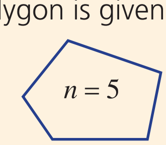
From previous years you should remember some of the names for polygons. See if you can remember them by completing this table.

Number of sides	Name
3	
4	
5	
6	
7	Heptagon

Number of sides	Name
8	
9	
10	
11	
12	

Key ideas

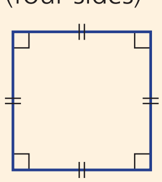
- A **polygon** is a shape with straight sides.
  - They are named by their number of sides.
- The sum of internal angles (S) of a polygon is given by this rule:
  $S = 180^\circ \times (n - 2)$
  where n is the number of sides



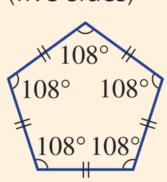
$S = 180^\circ \times (n - 2)$
$S = 180^\circ \times (5 - 2)$
$\quad = 180^\circ \times 3$
$\quad = 540^\circ$

- **A regular polygon** has sides of equal length and equal angles.

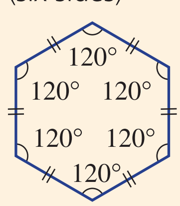
regular quadrilateral (square) (four sides)



regular pentagon (five sides)



regular hexagon (six sides)



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Exercise 7D

Understanding

	1–3	2(½), 3

1 How many sides do these shapes have?

a quadrilateral **b** octagon **c** decagon **d** heptagon
e nonagon **f** hexagon **g** pentagon **h** dodecagon

2 Use the angle sum rule, $S = 180° \times (n - 2)$, to find the angle sum of these polygons.

a pentagon $(n = 5)$ **b** hexagon $(n = 6)$ **c** heptagon $(n = 7)$
d octagon $(n = 8)$ **e** nonagon $(n = 9)$ **f** decagon $(n = 10)$

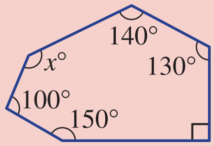
3 What is always true about a polygon that is regular?

Fluency

	4(½), 5	4(½), 6

Example 8 Finding and using the angle sum of a polygon

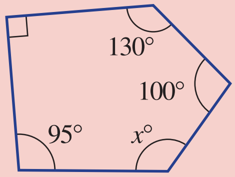
For this polygon, find the angle sum and then the value of x.



Solution	Explanation
$S = 180° \times (n - 2)$ $= 180° \times (6 - 2)$ $= 720°$	Use the angle sum rule first, with $n = 6$ since there are 6 sides. Find the angle sum.
$x + 100 + 150 + 90 + 130 + 140 = 720$ $x + 610 = 720$ $x = 110$	Use the total angle sum to find the value of x. Solve for the value of x.

Now you try

For this polygon, find the angle sum and then the value of x.



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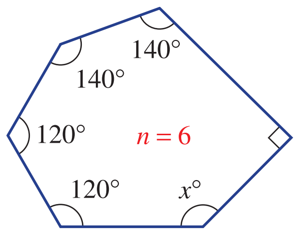
7D

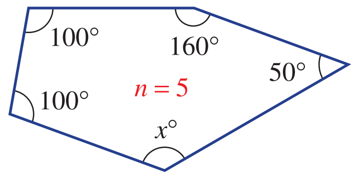
4 For these polygons, find the angle sum and then find the value of x.

Hint: First use $S = 180° \times (n - 2)$

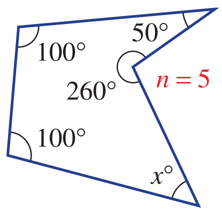
a



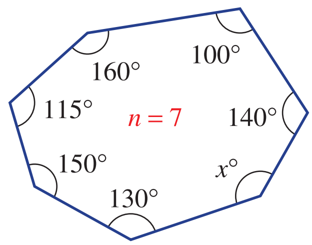
b



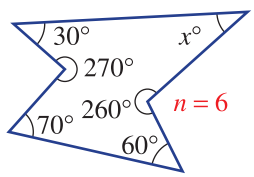
c



d



e



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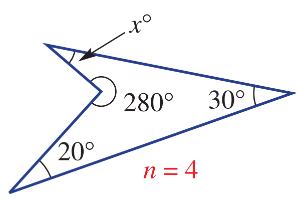
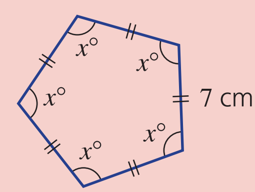


Example 9 Working with regular polygons

Shown here is a regular pentagon with a straight edge side length of 7 cm.

a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $x°$.

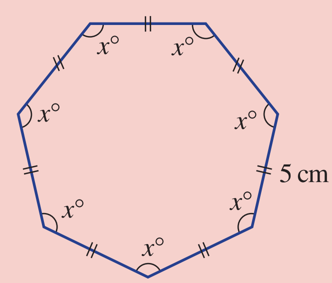


Solution	Explanation
a 35 cm	There are five sides at 7 cm each.
b $S = 180° \times (n - 2)$ $= 180° \times (5 - 2)$ $= 180° \times 3$ $= 540°$	Write the general rule for the sum of internal angles for a polygon. $n = 5$ since there are five sides. Simplify and evaluate.
c $540° \div 5 = 108°$ $x° = 108°$	There are five equally sized angles since it is a regular pentagon.

Now you try

Shown here is a regular heptagon with a straight edge side length of 5 cm.

a Find the perimeter of the heptagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $x°$, correct two decimal places.



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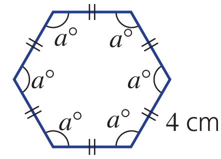
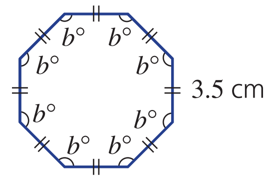
5 Shown here is a regular hexagon with straight-edge side length 4 cm.
a Find the perimeter of the hexagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $a°$.

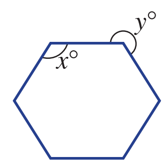


6 Shown here is a regular octagon with straight-edge side length 3.5 cm.
a Find the perimeter of the octagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $b°$.



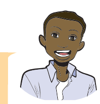
Problem-solving and reasoning | 7, 8, 10 | 9–12

7 The cross-section of a pencil is a regular hexagon.
a Find the interior angle ($x°$).
b Find the outside angle ($y°$).



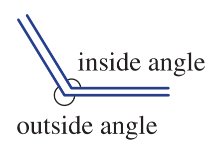
8 Find the total internal angle sum for a polygon with:
a 11 sides
b 20 sides

Hint: Remember: $S = 180° \times (n - 2)$

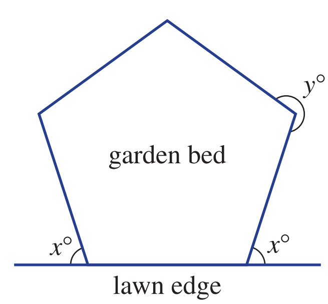


9 Find the size of a single internal angle for a regular polygon with:
a 10 sides
b 25 sides

10 A castle turret is in the shape of a regular hexagon. At each of the six corners, find:
a the inside angle
b the outside angle



11 A garden bed is to be designed in the shape of a regular pentagon and sits adjacent to a lawn edge, as shown.
a Find the angle the lawn edge makes with the garden bed ($x°$).
b Find the outside angle for each corner ($y°$).



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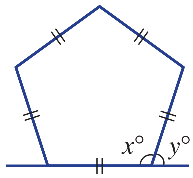
7D

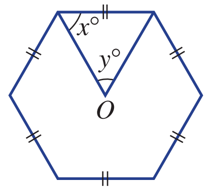
12 For these diagrams, find the values of the unknowns. The shapes are regular.

Hint: In parts **b** and **c**, the point at O is the centre.

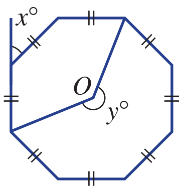
a



b



c

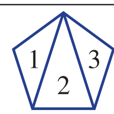


Develop the angle sum rule

— 13

13 a Copy and complete this table.

Hint: For the diagram, choose one vertex and draw lines to all other vertices.

Polygon	Number of sides	Diagram	Number of triangles	Total angle sum (S)	Regular polygon internal angle (A)
Triangle					
Quadrilateral					
Pentagon	5		3	$3 \times 180 = 540$	$540 \div 5 = 108$
Hexagon					
…					
n-gon	n				

b Complete these sentences by writing the rule.

i For a polygon with n sides, the total angle sum, S, is given by $S =$ _________.

ii For a regular polygon with n sides, a single internal angle, A, is given by $A =$ _________.

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7E Congruent triangles

Learning intentions

- To understand the four tests for congruence of triangles
- To be able to recognise a pair of congruent triangles using one of the four tests
- To be able to prove that two triangles are congruent

Key vocabulary: congruent, corresponding, hypotenuse

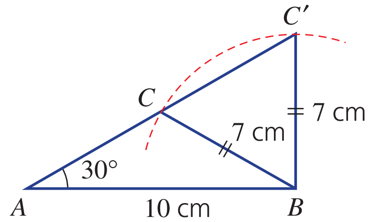
In solving problems or in the building of structures, for example, it is important to know whether or not two expressions or objects are identical. The mathematical word used to describe identical objects is *congruence*.

For congruent triangles there are four important tests that can be used to prove congruence.

Lesson starter: Why are AAA and SSA not tests for congruence?

AAA and SSA are not tests for the congruence of triangles.

- For AAA, can you draw two different triangles using the same three angles? Why does this mean that AAA is not a test for congruence?
- Look at this diagram, showing triangle ABC and triangle ABC'. Both triangles have a $30°$ angle and two sides of length 10 cm and 7 cm. Explain how this diagram shows that SSA is not a test for congruence of triangles.

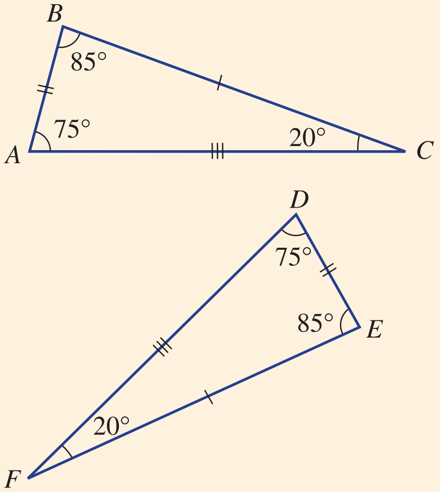


Key ideas

- Two triangles are said to be **congruent** if they are exactly the same 'size' and 'shape'.

  Corresponding sides and angles will be of the same size, as shown in the triangles below.

- If triangle ABC is congruent to triangle DEF, we write $\triangle ABC \equiv \triangle DEF$.
  - This is called a congruence statement.
  - Letters are usually written in matching order.
  - If two triangles are not congruent, we write: $\triangle ABC \not\equiv \triangle DEF$.



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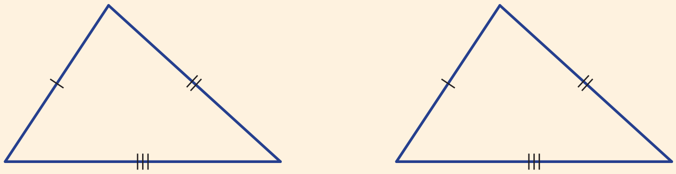
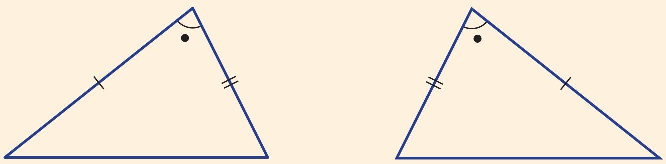
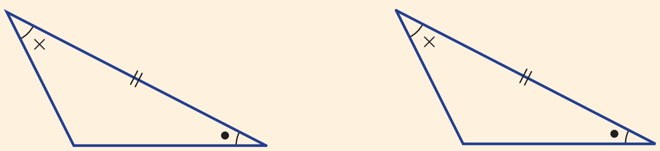
- Two triangles can be tested for congruence by considering the following necessary conditions.
  1. Three pairs of corresponding sides are equal (**SSS**).



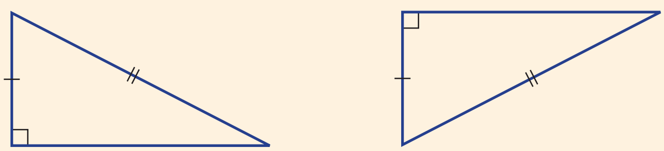
  2. Two pairs of corresponding sides and the angle between them are equal (**SAS**).



  3. Two angles and any pair of corresponding sides are equal (**AAS**).



  4. A right angle, the hypotenuse and one other pair of corresponding sides are equal (**RHS**).



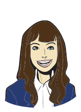
Exercise 7E

Understanding 1–3 3

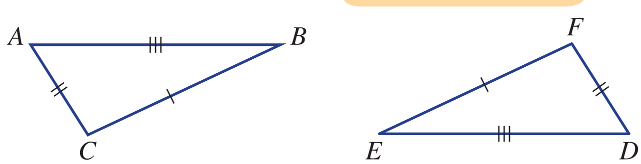
1 True or false?
- **a** SSA is a test for the congruence of triangles.
- **b** AAA is a test for the congruence of triangles.
- **c** Two congruent triangles are the same shape and size.
- **d** If $\triangle ABC \equiv \triangle DEF$, then triangle ABC is congruent to triangle DEF.

2 Write the four tests for congruence, using their abbreviated names.

Hint: SAS is one answer.



3 Here is a pair of congruent triangles.
- **a** Which point on $\triangle DEF$ corresponds to point B on $\triangle ABC$?
- **b** Which side on $\triangle ABC$ corresponds to side DF on $\triangle DEF$?
- **c** Which angle on $\triangle DEF$ corresponds to $\angle BAC$ on $\triangle ABC$?



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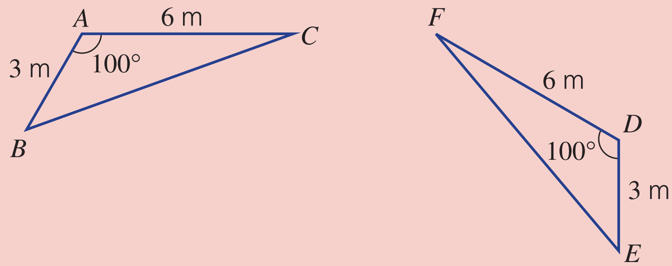
Fluency

4, 5(½) | 4–5(½)

Example 10 Choosing a test for congruence

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



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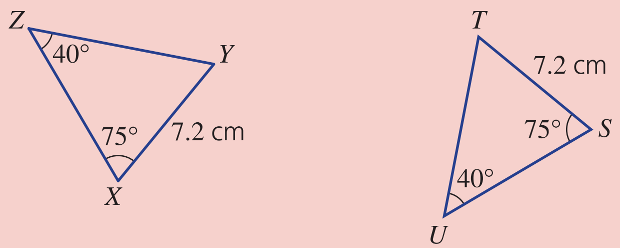
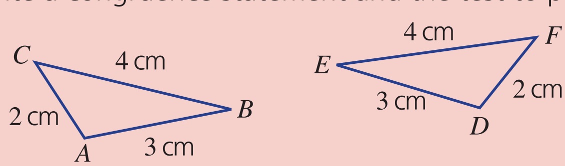


Solution	Explanation
a $\triangle ABC \equiv \triangle DEF$ (SAS)	Write letters in corresponding (matching) order. Two pairs of sides are equal as well as the angle between.
b $\triangle XYZ \equiv \triangle STU$ (AAS)	X matches S, Y matches T, and Z matches U. Two angles and one pair of matching sides are equal.

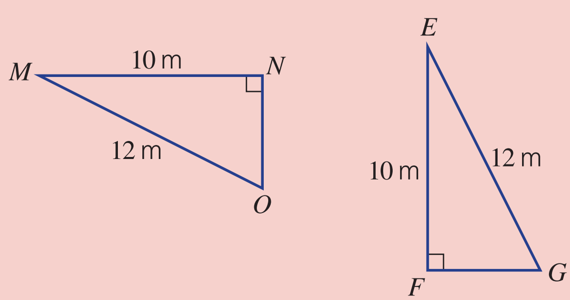
Now you try

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



b



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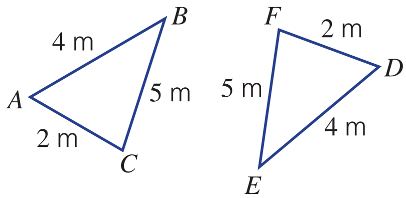
7E

4 Write a congruence statement and the test to prove congruence for these pairs of triangles.

Hint: $\triangle ABC \equiv \triangle DEF$ is a congruence statement.
Choose one of the tests SSS, SAS, AAS or RHS.

a



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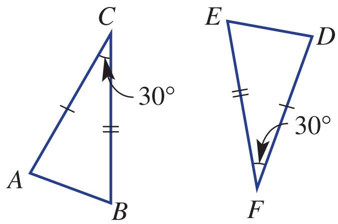

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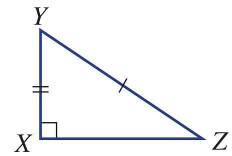

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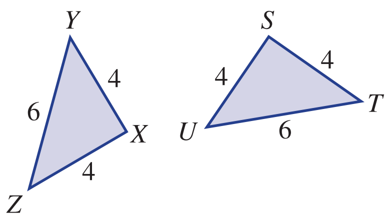

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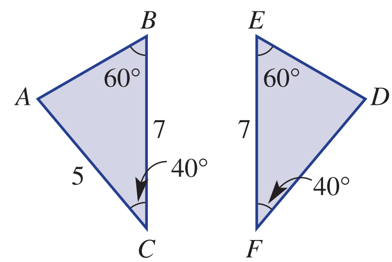

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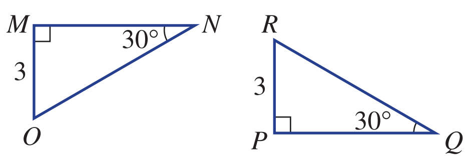
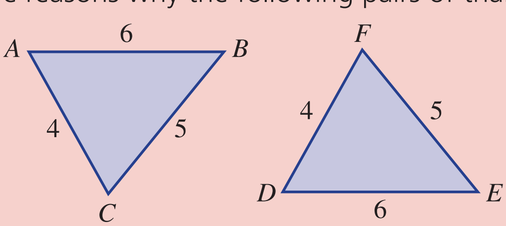


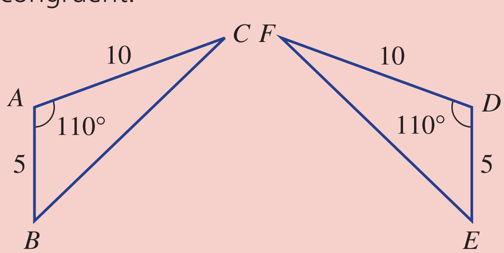
Example 11 Proving that a pair of triangles are congruent

Give reasons why the following pairs of triangles are congruent.

a



b



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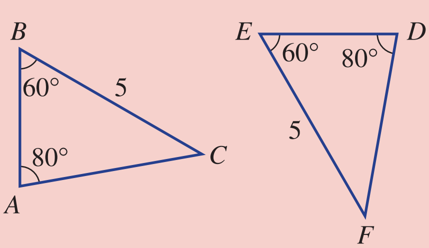

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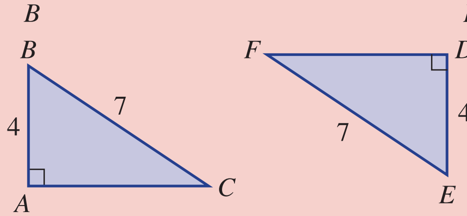

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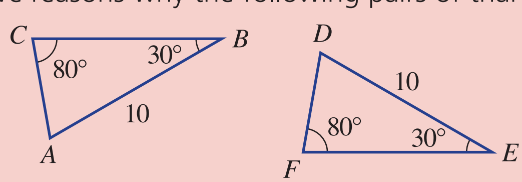
Solution		Explanation
a $AB = DE$	(S)	First choose all the corresponding side lengths.
$AC = DF$	(S)	Corresponding side lengths will have the same length.
$BC = EF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(SSS)	Write the congruence statement and the abbreviated reason.
b $AB = DE$	(S)	Note that two corresponding side lengths are equal and the included angles are equal.
$\angle BAC = \angle EDF$	(A)	
$AC = DF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(SAS)	Write the congruence statement and the abbreviated reason.
c $\angle ABC = \angle DEF$	(A)	Two of the angles are equal and one of the pairs of corresponding sides are equal. Write the congruence statement and the abbreviated reason.
$\angle BAC = \angle EDF$	(A)	
$BC = EF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(AAS)	
d $\angle BAC = \angle EDF = 90°$	(R)	Note that both triangles are right angled, the hypotenuse of each triangle is of the same length and another pair of corresponding sides is of the same length.
$BC = EF$	(H)	
$AB = DE$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(RHS)	Write the congruence statement and the abbreviated reason.

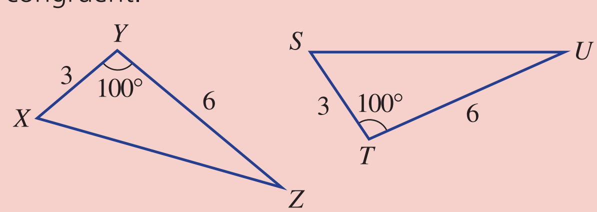
Now you try

Give reasons why the following pairs of triangles are congruent.

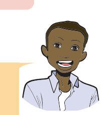
a



b

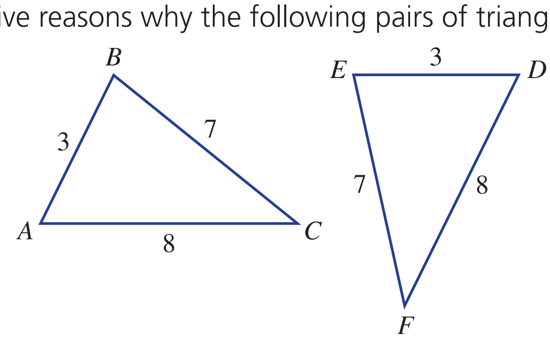


5 Give reasons why the following pairs of triangles are congruent.



Hint: List reasons as in the examples to establish SSS, SAS, AAS or RHS.

a



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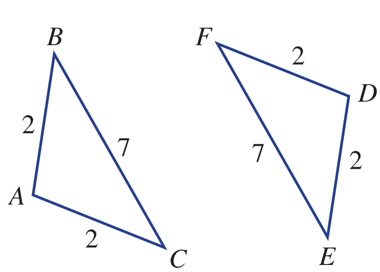

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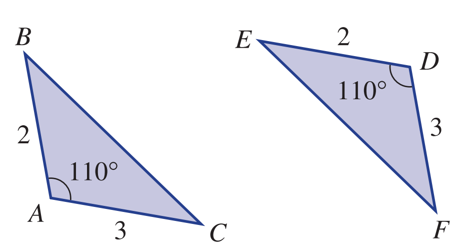

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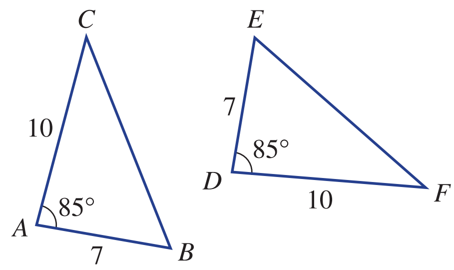
7E

d



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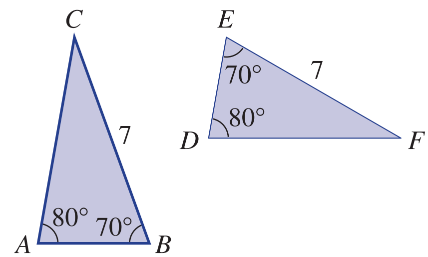

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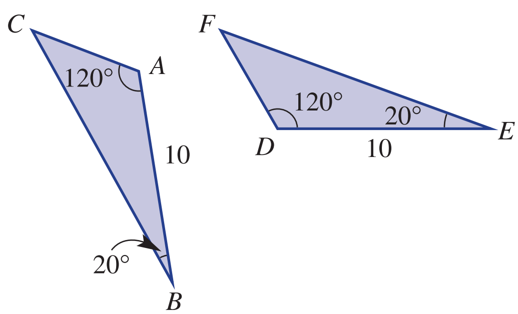

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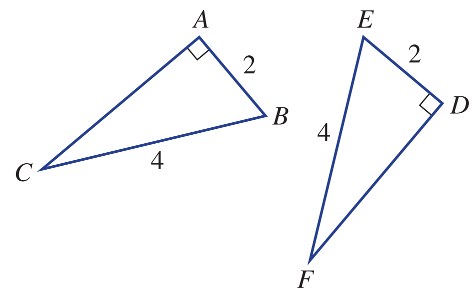
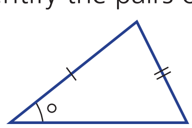


Problem-solving and reasoning

6, 7 | 6, 7, 8(½)

6 Identify the pairs of congruent triangles from those below.

a



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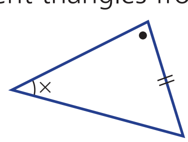

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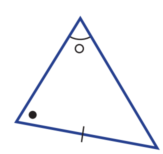
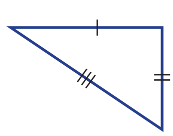


Hint: Sides with the same markings and angles with the same mark are equal.

d



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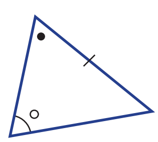

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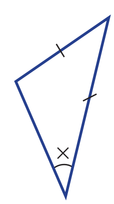

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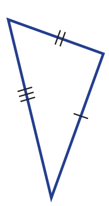

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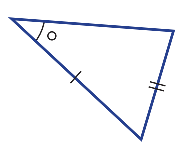
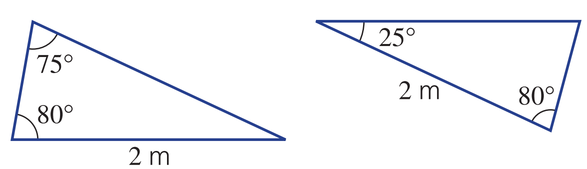


7 Two triangular windows have the given dimensions.

a Find the missing angle in each triangle.

b Are the two triangles congruent? Give a reason.



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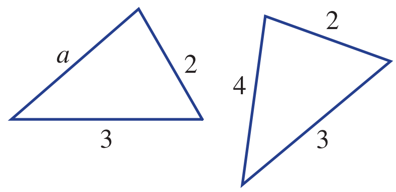
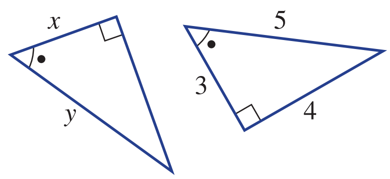
8 For the pairs of congruent triangles, find the values of the pronumerals.

Hint: Given that these triangles are congruent, corresponding sides are equal, as are corresponding angles.

a



b



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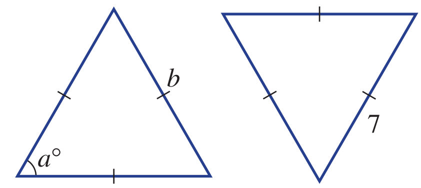

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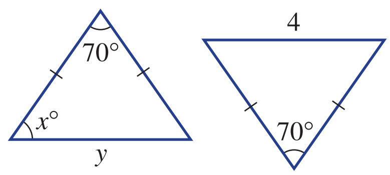

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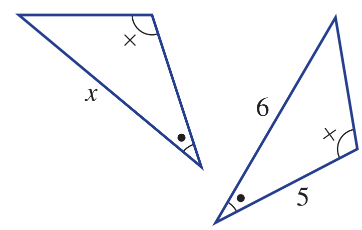

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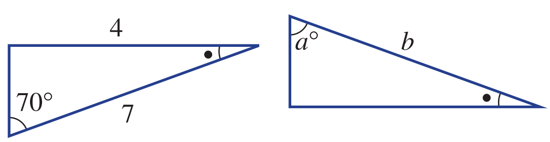

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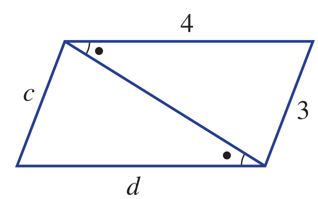

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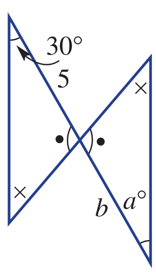
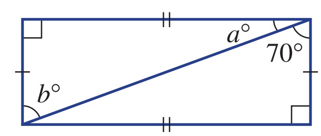


i



Lawn landscaping — 9

9 A new garden design includes two triangular lawn areas, as shown.

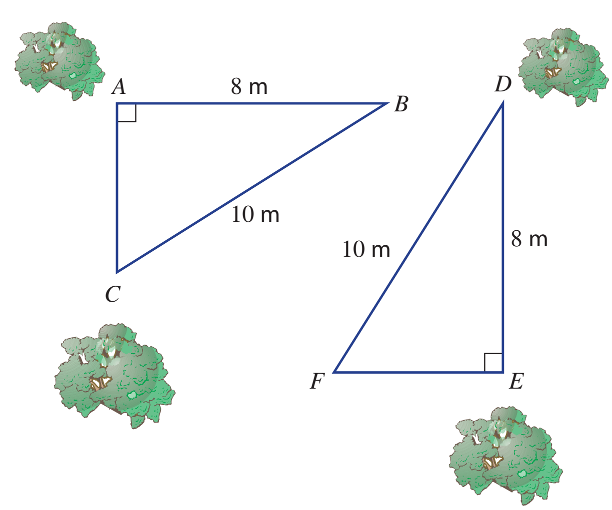
a Give reasons why the two triangular lawn areas are congruent.

b If the length of AC is 6 m, find the length of EF.

c If the angle $ABC = 37°$, find the angles:

i $\angle EDF$

ii $\angle DFE$



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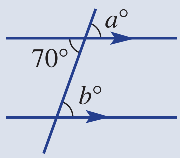
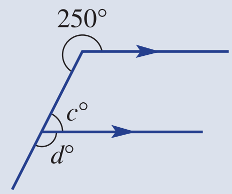
Progress quiz

7A **1** Find the values of the pronumerals in these diagrams with parallel lines, giving reasons.

a

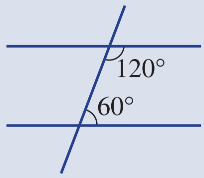


b

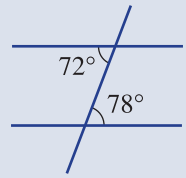


7A **2** State, with reasons, if the following pairs of lines are parallel.

a

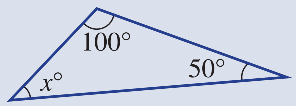


b



7B/C **3** Find the values of the pronumerals in these triangles and quadrilaterals.

a



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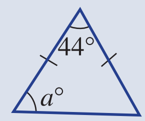

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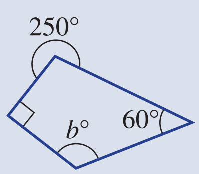
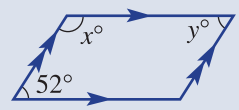
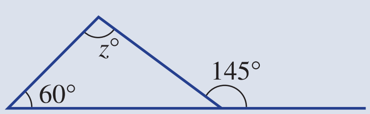


d

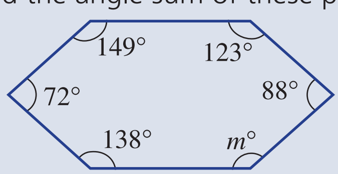


7B **4** Use the exterior angle theorem to find the value of z.

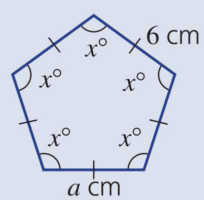


7D **5** Find the angle sum of these polygons and then find the value of each pronumeral.

a

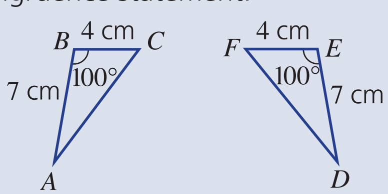


b

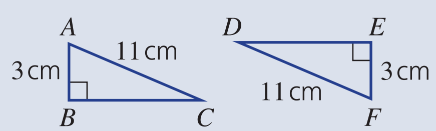


7E **6** Give reasons why the following pairs of triangles are congruent and write a congruence statement.

a



b



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7F Similar triangles

Learning intentions

- To know what it means for triangles to be similar
- To understand the four tests for similarity of triangles
- To be able to recognise a pair of similar triangles using one of the four tests
- To be able to prove that two triangles are similar
- To be able to calculate and use the scale factor to find an unknown length

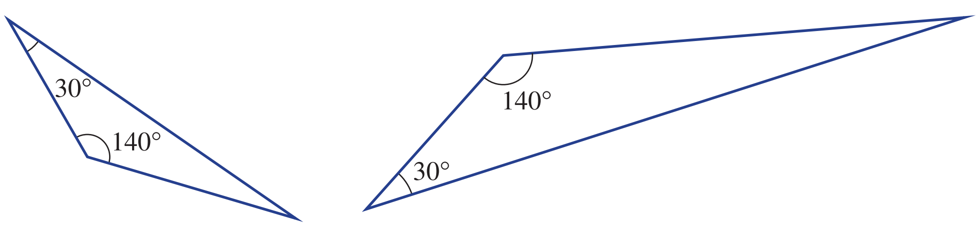
Key vocabulary: similar, scale factor, ratio, corresponding, hypotenuse

When two objects are similar, they are the same shape but of different size. For example, a computer image reproduced on a large screen with the same aspect ratio (e.g. 16:9) will show all aspects of the image in the same way except in size.

The computer image and screen image are said to be similar figures.

Lesson starter: Is AA the same as AAA?

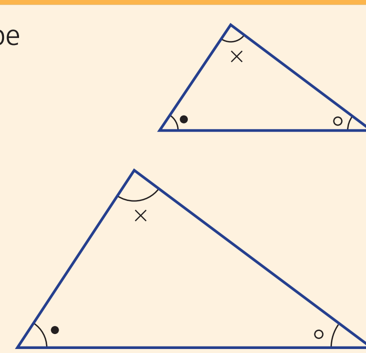
Look at these two triangles.



- What is the missing angle in each triangle?
- Do you think the triangles are similar? Why?
- Is the AA test equivalent to the AAA test?

Key ideas

- Two triangles are said to be **similar** if they are the same shape but different in size. Corresponding angles will be equal and corresponding side lengths will be in the same ratio.
- If $\triangle ABC$ is similar to $\triangle DEF$, then we write $\triangle ABC \,|||\, \triangle DEF$ or $\triangle ABC \sim \triangle DEF$.



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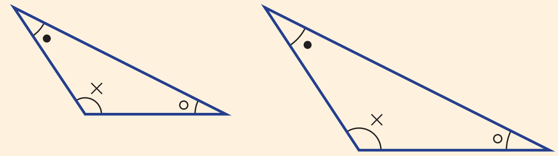
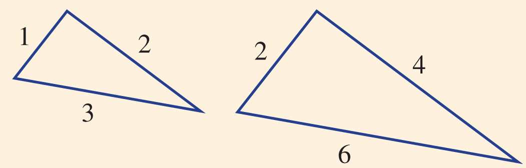
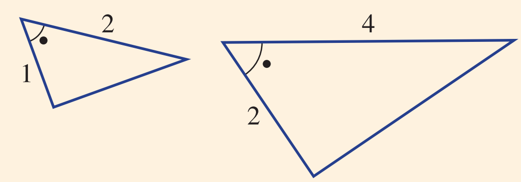
- Two triangles can be tested for similarity by considering the following necessary conditions.
  1. All three pairs of corresponding angles are equal (**AAA**). (Remember that if two pairs of corresponding angles are equal then the third pair of corresponding angles is also equal.)



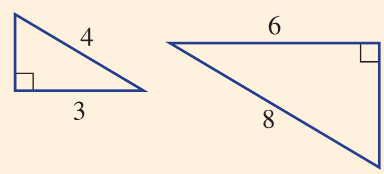
  2. All three pairs of corresponding sides are in the same ratio (**SSS**).
     $\frac{6}{3} = \frac{4}{2} = \frac{2}{1} = 2$



  3. Two pairs of corresponding sides are in the same ratio and the included corresponding angles between these sides are equal (**SAS**).
     $\frac{4}{2} = \frac{2}{1} = 2$



  4. The hypotenuses and a pair of corresponding sides in a right-angled triangle are in the same ratio (**RHS**).
     $\frac{8}{4} = \frac{6}{3} = 2$



- The **scale factor** is calculated using a pair of corresponding sides.
  In the three examples above, the scale factor is 2.

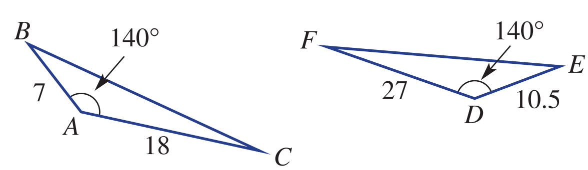
Exercise 7F

Understanding | 1–3 | 3

1 Which of the following is not a test for the similarity of triangles?
SSS, SAS, RHS, SSA, AAA

2 Why is the AA test the same as the AAA test for similar triangles?

3 Consider this pair of triangles.



a Work out $\frac{DE}{AB}$.

b Work out $\frac{DF}{AC}$. What do you notice?

c What is the scale factor?

d Which of SSS, SAS, AAA or RHS would be used to explain their similarity?

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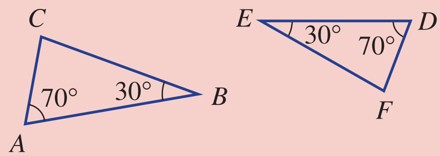
Fluency 4(½), 5 4(½), 5

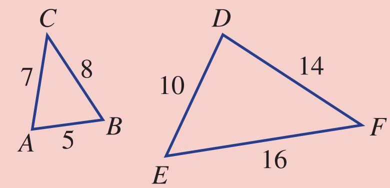
Example 12 Proving similar triangles

Decide whether the pairs of triangles are similar, giving reasons.

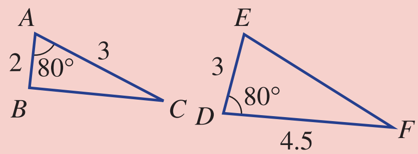
a



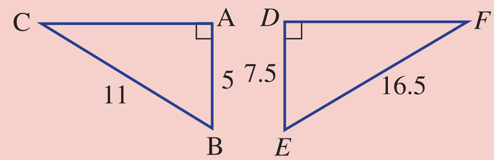
b



c



d



Solution	**Explanation**

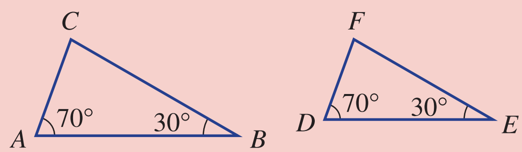
a $\angle BAC = \angle EDF$ (A)

$\angle ABC = \angle DEF$ (A)

$\angle ACB = \angle DFE$ (A)

$\therefore \triangle ABC \mathbin{|||} \triangle DEF$ (AAA)

Two corresponding angles are equal and therefore the third corresponding angle is also equal.



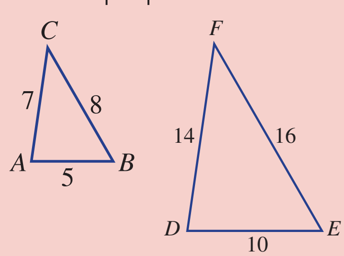
b $\dfrac{DE}{AB} = \dfrac{10}{5} = 2$ (S)

$\dfrac{EF}{BC} = \dfrac{16}{8} = 2$ (S)

$\dfrac{DF}{AC} = \dfrac{14}{7} = 2$ (S)

$\therefore \triangle ABC \mathbin{|||} \triangle DEF$ (SSS)

All three corresponding sides are in the same ratio or proportion.



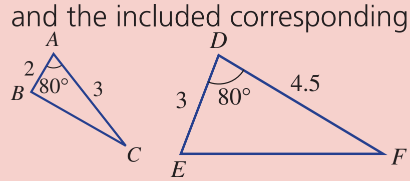
c $\dfrac{DE}{AB} = \dfrac{3}{2} = 1.5$ (S)

$\angle BAC = \angle EDF$ (A)

$\dfrac{DF}{AC} = \dfrac{4.5}{3} = 1.5$ (S)

$\therefore \triangle ABC \mathbin{|||} \triangle DEF$ (SAS)

Two corresponding sides are in the same ratio and the included corresponding angles are equal.



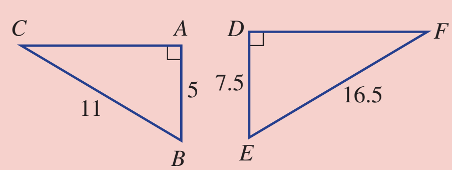
d $\angle BAC = \angle EDF = 90°$ (R)

$\dfrac{EF}{BC} = \dfrac{16.5}{11} = 1.5$ (H)

$\dfrac{DE}{AB} = \dfrac{7.5}{5} = 1.5$ (S)

$\therefore \triangle ABC \mathbin{|||} \triangle DEF$ (RHS)

They are right-angled triangles with the hypotenuses and one other pair of corresponding sides in the same ratio.



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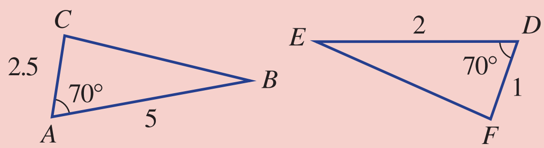
7F

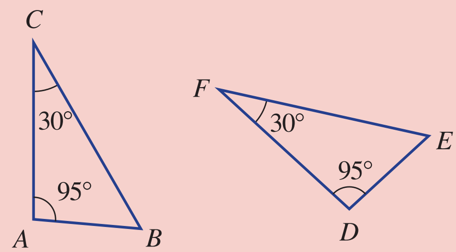
Now you try

Decide whether the pairs of triangles are similar, giving reasons.

a

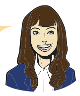


b

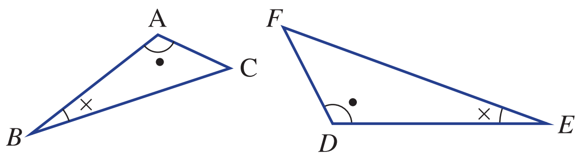


4 Decide whether the pairs of triangles are similar, giving reasons.

Hint: List all the equal angles and corresponding pairs of sides, as in Example 12.



a



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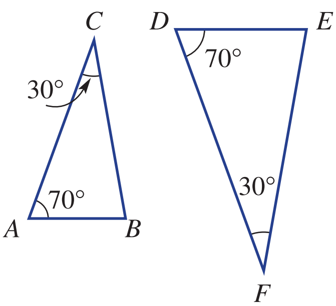

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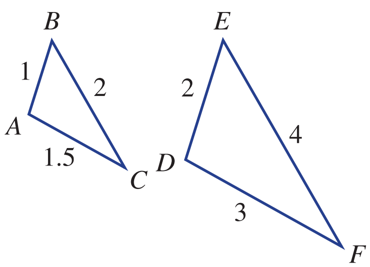

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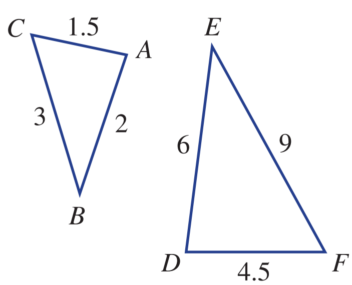

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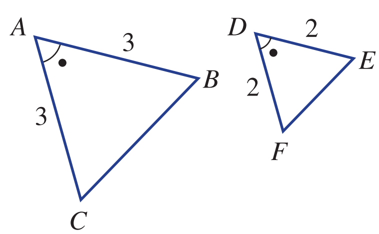

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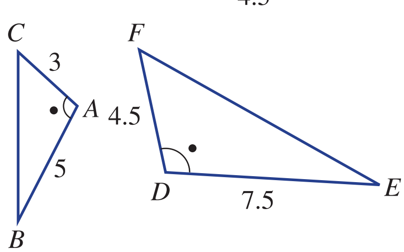

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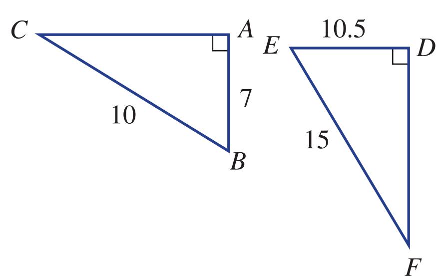

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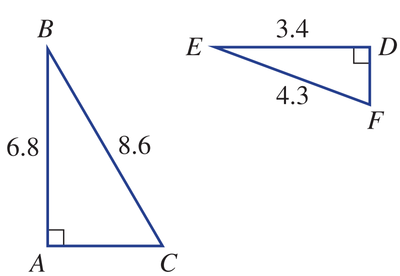

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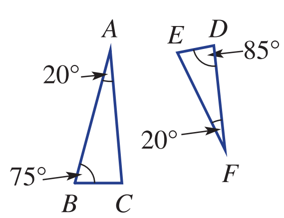

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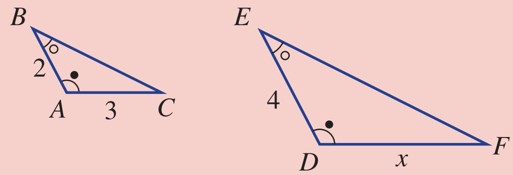
Example 13 Using similar triangles to find an unknown length

If the given pair of triangles are known to be similar, find the value of x.



Solution

Scale factor $= \frac{DE}{AB} = \frac{4}{2} = 2$

$x = 3 \times 2$

$= 6$

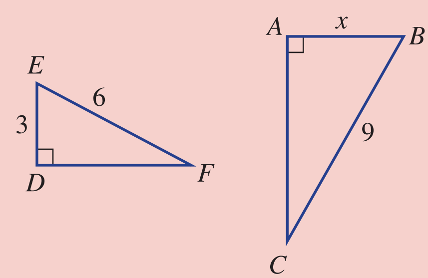
Explanation

First find the scale factor using a pair of corresponding sides. Divide the larger number by the smaller number.

Multiply the corresponding length on the smaller triangle using the scale factor.

Now you try

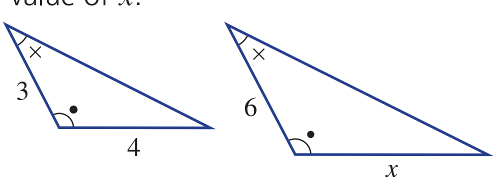
If the given pair of triangles are known to be similar, find the value of x.



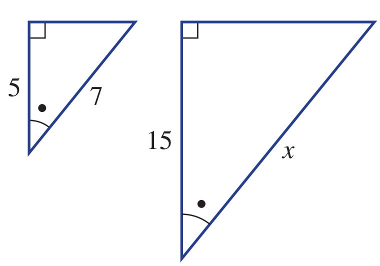
Hint: For parts **c** and **d**, use division to find x.

5 If the given pair of triangles are known to be similar, find the value of x.

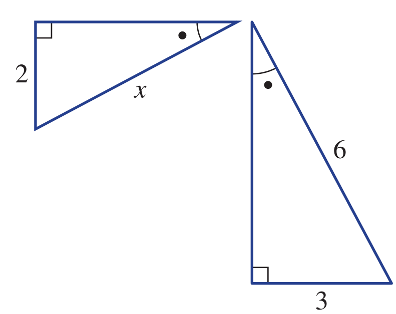
a



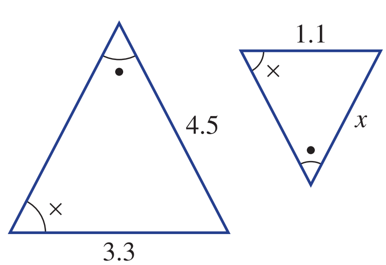
b



c



d



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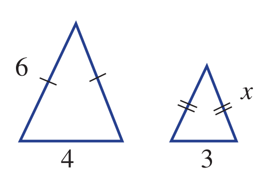

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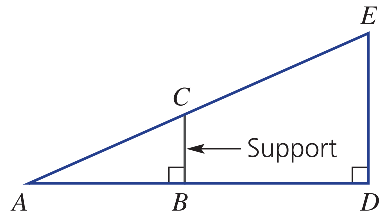
7F

Problem-solving and reasoning | 6 | 6, 7

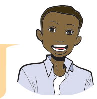
6 A ski ramp has a vertical support, as shown.

a List the two triangles that are similar.

b Why are the two triangles similar?



Hint: List triangles like this: $\triangle STU$.

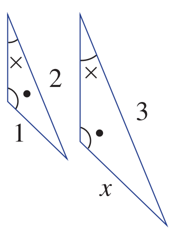


c If $AB = 4$ m and $AD = 10$ m, find the scale factor.

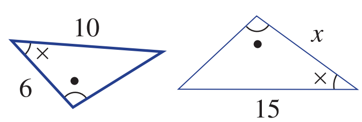
d If $BC = 1.5$ m, find the height of the ramp, DE, using the scale factor from part **c**.

7 State why the pairs of triangles are similar (give the abbreviated reason) and determine the value of x in each case.

a

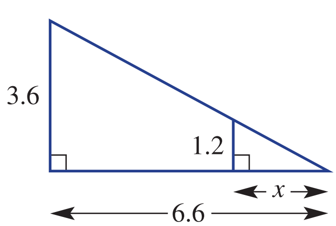


b

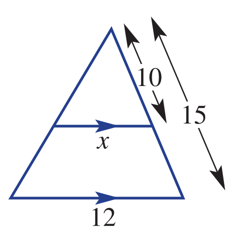


Hint: They all have the same reason.

c



d



Triangles in parallel lines | — | 8

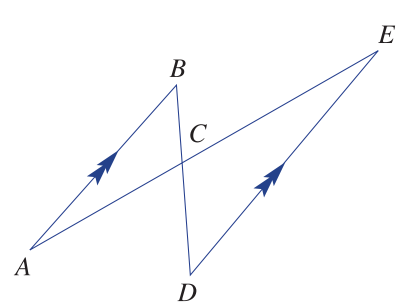
8 In the given diagram, AB is parallel to DE.

a List the three pairs of angles that are equal and give a reason.

b If $AB = 8$ cm and $DE = 12$ cm, find:

i DC if $BC = 4$ cm

ii AC if $CE = 9$ cm



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7G Applying similar triangles

Learning intentions

- To be able to identify a pair of similar triangles in a given context
- To be able to calculate and use the scale factor to find an unknown length in a real situation

Key vocabulary: similar, scale factor

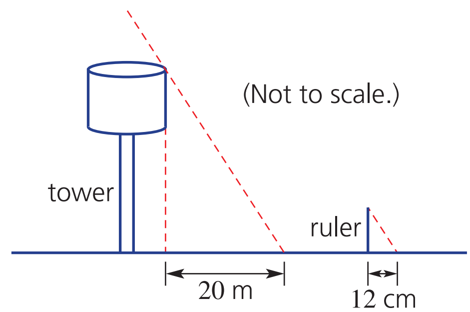
Once it is established that two triangles for a particular situation are similar, the ratio or scale factor between side lengths can be used to find unknown side lengths.

Similar triangles have many applications in the real world. One application is finding an inaccessible distance, like the height of a tall object or the distance across a deep ravine.

Lesson starter: The tower and the ruler

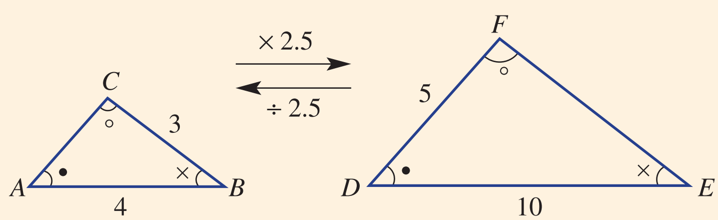
Franklin wants to know how tall a water tower is in his town. At a particular time of day he measures its shadow to be 20 m long. At the same time he stands a 30 cm ruler near the tower, which gives a 12 cm shadow.

- Explain why the two formed triangles are similar.
- What is the scale factor?
- What is the height of the tower?



Key ideas

- For two similar triangles, the ratio of the corresponding side lengths, written as a single number, is called the scale factor.



- Once the scale factor is known, it can be used to find unknown side lengths.

$$\frac{DE}{AB} = \frac{10}{4} = \frac{5}{2} = 2.5$$

$\therefore$ Scale factor is 2.5.

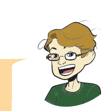
$\therefore\ EF = 3 \times 2.5 = 7.5$

$AC = 5 \div 2.5 = 2$

Exercise 7G

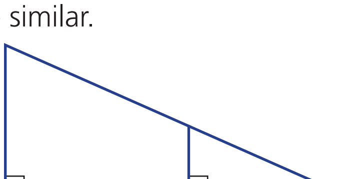
Understanding	1, 2	2

1 Give reasons why the pairs of triangles in each diagram are similar.



Hint: Think about all the corresponding pairs of angles. How many equal pairs are there?

a



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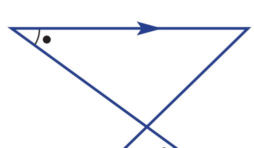

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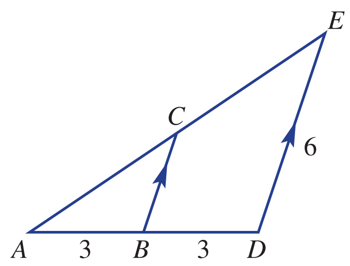
7G

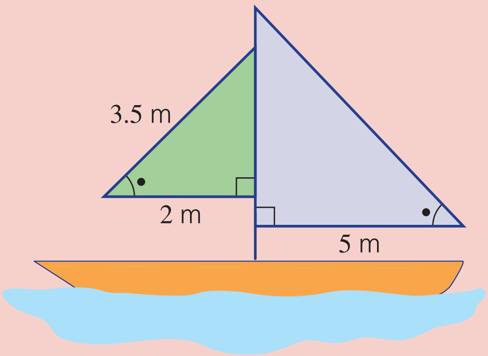
2 For the pair of triangles in the given diagram:
  a Which reason would be chosen to explain their similarity: SSS, SAS, AAA or RHS?
  b What is the scale factor?
  c What is the length of BC?



Fluency	3, 4	3, 5

Example 14 Applying similar triangles

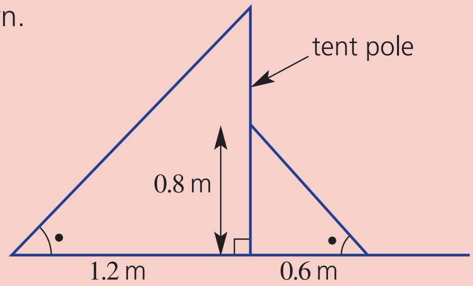
A home-made raft consists of two sails with measurements and angles as shown in this diagram.
a Give reasons why the two sails are similar in shape.
b Find the scale factor for the side lengths of the sails.
c Find the length of the longest side of the large sail.



Solution	Explanation
a AAA (there are three equal pairs of angles)	Two of the three angles are clearly equal, so the third must be equal.
b Scale factor $= \frac{5}{2} = 2.5$	Choose two corresponding sides with known lengths and divide the larger by the smaller.
c Longest side $= 3.5 \times 2.5$ $= 8.75$ m	Multiply the corresponding side on the smaller triangle by the scale factor.

Now you try

A vertical tent pole is held in place with two guy ropes as shown.
a Give reasons why the two triangles formed by the guy ropes are similar.
b Find the scale factor for the guy ropes.
c Find the height of the tent pole.



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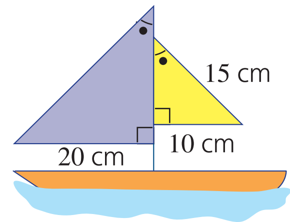
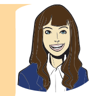
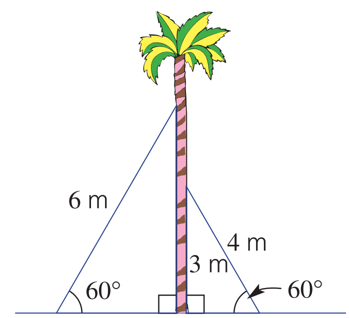
3 A toy yacht consists of two sails with measurements and angles as shown in this diagram.
   a Give reasons why the two sails are similar in shape.
   b Find the scale factor for the side lengths of the sails.
   c Find the length of the longest side of the large sail.



Hint: You can't choose SSS, SAS or RHS because only one pair of corresponding sides is given.

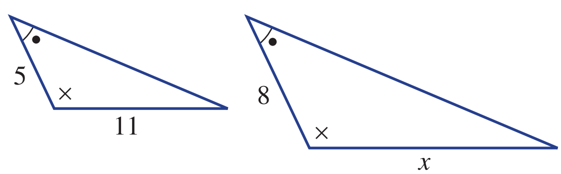


4 A tall palm tree is held in place with two cables of length 6 m and 4 m, as shown.
   a Give reasons why the two triangles created by the cables are similar in shape.
   b Find the scale factor for the side lengths of the cables.
   c Find the height of the point above the ground where the longer cable is attached to the palm tree.

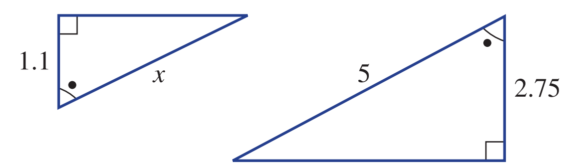


5 These pairs of triangles are known to be similar. By finding the scale factor, find the value of x.

a



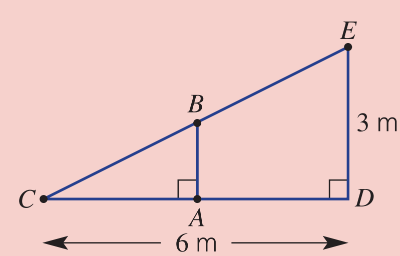
b



Problem-solving and reasoning 6–8, 10 7–10

Example 15 Working with combined triangles

A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 6$ m and that the ramp is 3 m high.
a Using the letters given, name the two triangles that are similar and give your reason.
b Find the length of the stud AB.



Solution	Explanation
a $\triangle ABC$ and $\triangle DEC$ (AAA)	The angle at C is common to both triangles and they both have a right angle.
b $AC = 3$ m	Since A is in the centre of CD, then AC is half of CD.
Scale factor $= \frac{6}{3} = 2$	$CD = 6$ m and $AC = 3$ m.
$\therefore AB = 3 \div 2$ $= 1.5$ m	Divide the larger side length, DE, by the scale factor.

Continued on next page

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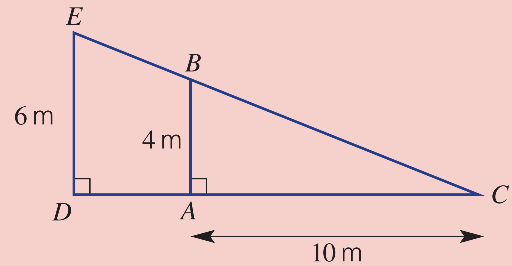
Now you try

A 'lean–to' shelter is made with two vertical poles *AB* and *DE* as shown.

a Using the letters given, name the two triangles that are similar and give your reason.

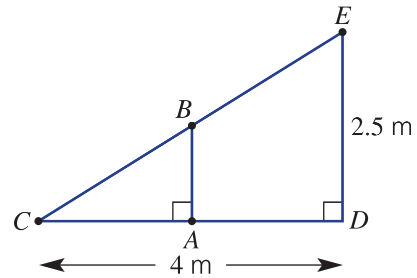
b Find the length *CD*.



6 A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 4$ m and that the ramp is 2.5 m high.

a Using the letters given, name the two triangles that are similar and give your reason.

b Find the length of the stud AB.



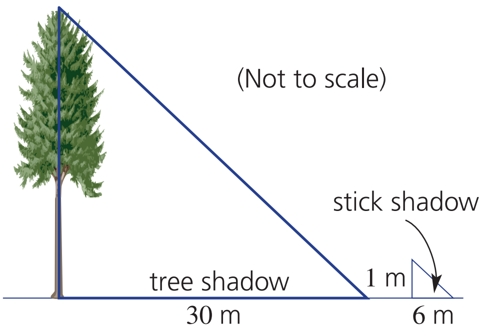
7 A 1 m vertical stick and a tree cast their shadows at a particular time in the day. The shadow lengths are shown in this diagram.

a Give reasons why the two triangles shown are similar in shape.

b Find the scale factor for the side lengths of the triangles.

c Find the height of the tree.

Hint: At the same time of day, the angle that the light makes with the ground will be the same.



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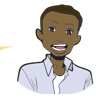
8 From a place on the river (C), a tree (E) is spotted on the opposite bank. The distances between selected trees A, B, C and D are measured as shown.

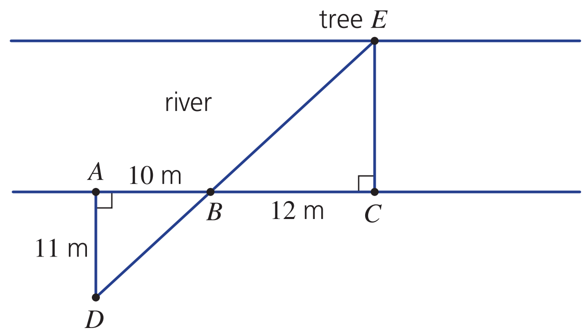


Hint: AB corresponds to CB and AD corresponds to CE.

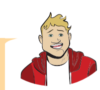
a List two similar triangles and give a reason why they are similar.

b Find the scale factor.

c Find the width of the river.



9 At a particular time of day, Leon casts a shadow 1.3 m long, whereas Jackson, who is 1.75 m tall, casts a shadow 1.2 m long. Find the height of Leon, to two decimal places.



Hint: Draw a diagram to find the scale factor.

10 Try this activity with a classmate but ensure that at least one person knows their height.

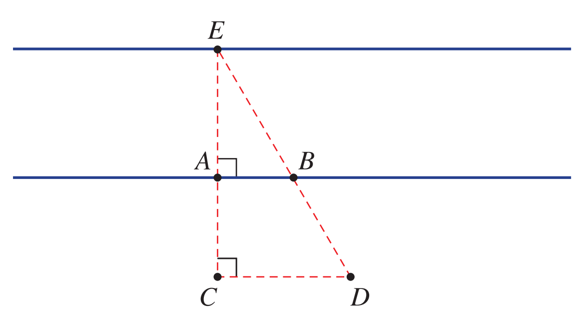
a Go out into the sun and measure the length of each person's shadow.

b Use these measurements plus the known height of one person to find the height of the other person.

Gorge challenge — 11

11 Mandy sets up a series of rocks alongside a straight section of a deep gorge. She places rocks A, B, C and D as shown. Rock E sits naturally on the other side of the gorge. Mandy then measures the following distances.

- $AB = 10$ m
- $AC = 10$ m
- $CD = 15$ m



a Explain why $\triangle ABE \,|||\, \triangle CDE$.

b What is the scale factor?

c Use trial and error to find the distance across the gorge from rocks A to E.

d Can you find the length AE by setting up an equation?

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7H Applications of similarity in measurement

Learning intentions

- To know how the length, area and volume ratios are related in similar objects
- To be able to calculate and use the scale factor to find an unknown area or volume given similar objects

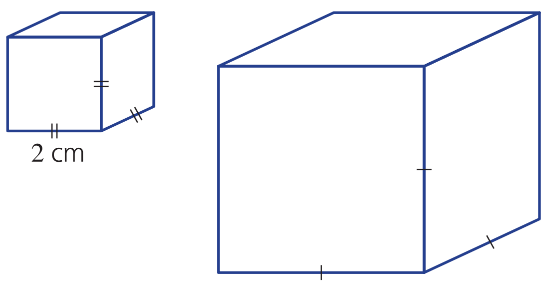
Key vocabulary: length, area, volume, ratio, scale factor

Shapes or objects that are similar have a special length, area and volume ratio relationship.

For example, if the lengths on a model of a building are one-hundredth of the actual structure, then the length ratio is $1:100$. From this, the surface area and volume ratios are $1^2:100^2$ $(1:10\ 000)$ and $1^3:100^3$ $(1:1\ 000\ 000)$, respectively. These ratios can be used to calculate the amount of material that is needed for the construction of the building.

Lesson starter: Cube analysis

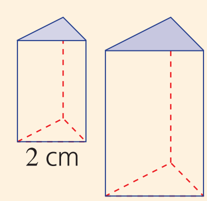
These two cubes have a 2 cm and 6 cm side length.



- What is the side length ratio when comparing the two cubes?
- What are the surface areas of the two cubes?
- What is the surface area ratio? What do you notice?
- What are the volumes of the two cubes?
- What is the volume ratio? What do you notice?

Key ideas

- When two objects are similar and have a length ratio of $a:b$, then:
  - Length ratio $= a:b$ Scale factor $= \frac{b}{a}$
  - Area ratio $= a^2:b^2$ Scale factor $= \frac{b^2}{a^2}$
  - Volume ratio $= a^3:b^3$ Scale factor $= \frac{b^3}{a^3}$
  - one dimension: length ratio $= 2^1:3^1 = 2:3$
  - two dimensions: area ratio $= 2^2:3^2 = 4:9$
  - three dimensions: volume ratio $= 2^3:3^3 = 8:27$



Exercise 7H

Understanding 1, 2 | 2

1 The length ratio for two objects is $2:5$.

a What would be the area ratio?

b What would be the volume ratio?

Hint:

Length ratio	$a:b$
Area ratio	$a^2:b^2$
Volume ratio	$a^3:b^3$

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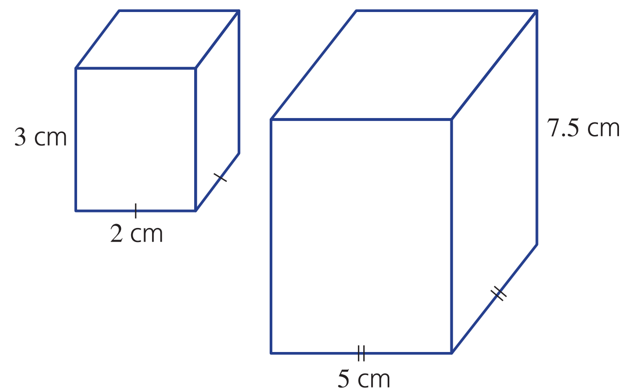
2 These two rectangular prisms are similar.
- **a** What is the side length ratio?
- **b** What is the surface area of each prism?
- **c** What is the surface area ratio? What do you notice?
- **d** What is the volume of each prism?
- **e** What is the volume ratio? What do you notice?

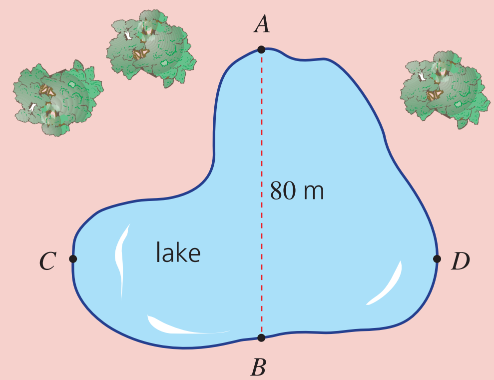


Fluency

	3, 4, 7	4–8

Example 16 Measuring to find actual lengths

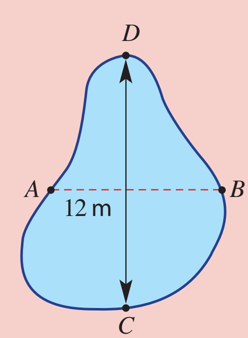
The given diagram is a simple map of a park lake.
- **a** Use a ruler to measure the distance across the lake (AB). (Answer in cm.)
- **b** Find the scale factor between the map and ground distance.
- **c** Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)
- **d** Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



Solution	Explanation
a 4 cm	Check with your ruler.
b $\frac{8000}{4} = 2000$	Using the same units, divide the real distance (80 m = 8000 cm) by the measured distance (4 cm).
c 5 cm	Check with your ruler.
d $5 \times 2000 = 10\,000$ cm = 100 m	Multiply the measured distance by the scale factor and convert to metres by dividing by 100.

Now you try

The given diagram is a simple map of a swimming pool.
- **a** Use a ruler to measure the distance across the pool (AB). (Answer in cm.)
- **b** Find the scale factor between the map and ground distance.
- **c** Use a ruler to measure the map distance across the pool (CD). (Answer in cm.)
- **d** Use your scale factor to find the real length of the pool (CD). (Answer in m.)



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7H

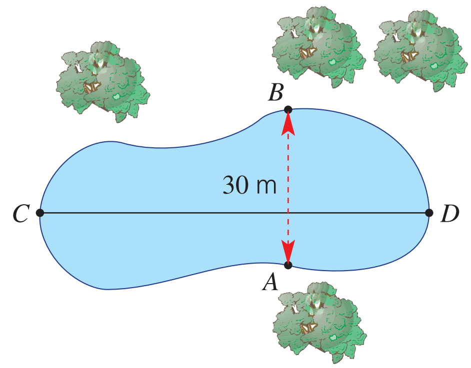
3 The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



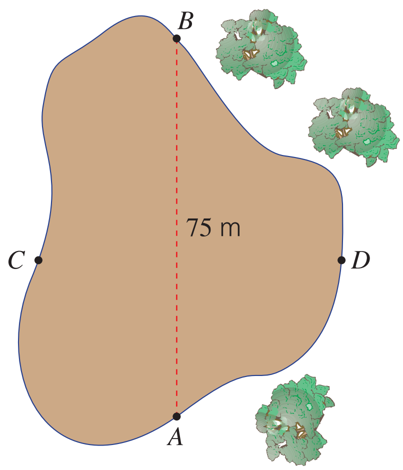
4 The given diagram is a simple map of a children's play area.

a Use a ruler to measure the distance across the children's play area (AB). (Answer in cm.)

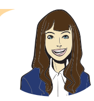
b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the children's play area (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the children's play area (CD). (Answer in m.)

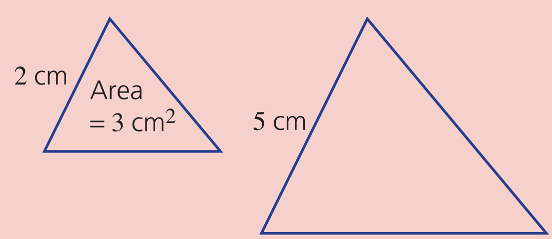


Hint: Use the measured distance AB and the actual distance AB to find the scale factor.



Example 17 Using similarity to find areas

The two given triangles are known to be similar.
Find the area of the larger triangle.



Solution	Explanation
Length ratio $= 2^1 : 5^1 = 2 : 5$	First, write the length ratio.
Area ratio $= 2^2 : 5^2 = 4 : 25$	Square each number in the length ratio to get the area ratio.
Area scale factor $= \frac{25}{4} = 6.25$	Divide the two numbers in the area ratio to get the scale factor.
$\therefore$ Area of larger triangle $= 3 \times 6.25$ $= 18.75\text{ cm}^2$	Multiply the area of the smaller triangle by the scale factor.

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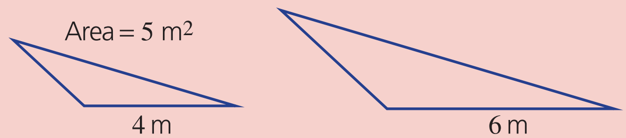
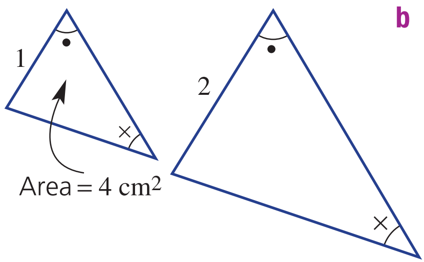
Now you try

The two given triangles are known to be similar.
Find the area of the larger triangle.

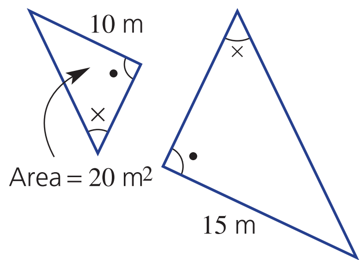


5 The two given triangles are known to be similar. Find the area of the larger triangle.

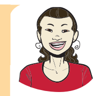
a



b

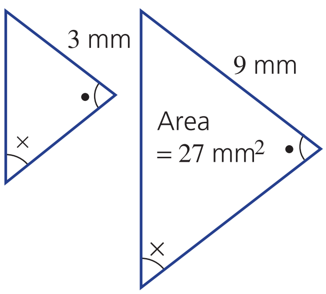


Hint:
Length ratio = $a : b$
Area ratio = $a^2 : b^2$
Area scale factor = $\frac{b^2}{a^2}$

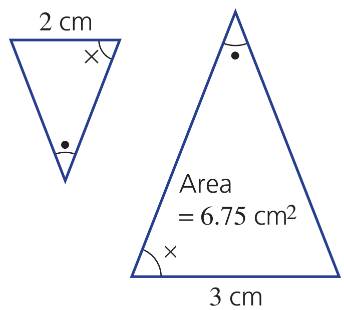


6 The two given triangles are known to be similar. Find the area of the smaller triangle.

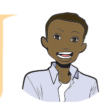
a



b

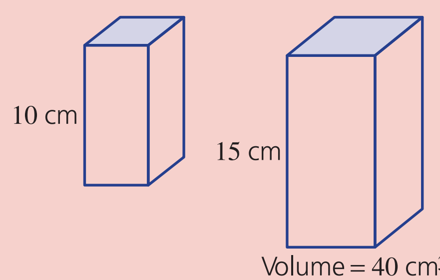


Hint: You will need to divide the larger area by the area scale factor.



Example 18 Using similarity to find volume

The two given prisms are known to be similar.
Find the volume of the smaller prism, correct to two decimal places.



Solution	Explanation
Length ratio = $10^1 : 15^1 = 2 : 3$	First, write the length ratio and simplify.
Volume ratio = $2^3 : 3^3 = 8 : 27$	Cube each number in the length ratio to get the volume ratio.
Volume scale factor = $\frac{27}{8} = 3.375$	Divide the two numbers in the volume ratio to get the scale factor.
$\therefore$ Volume of smaller prism = $40 \div 3.375$ $= 11.85\ \text{cm}^3$ (to 2 d.p.)	Divide the volume of the larger prism by the scale factor and round as required.

Continued on next page

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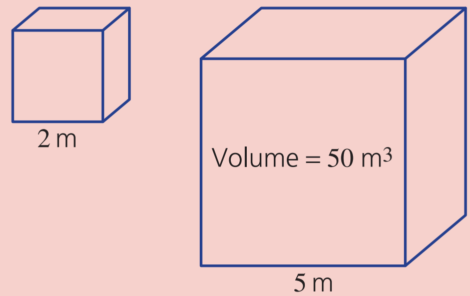
7H

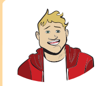
Now you try

The two given prisms are known to be similar.
Find the volume of the smaller prism, correct to two decimal places.

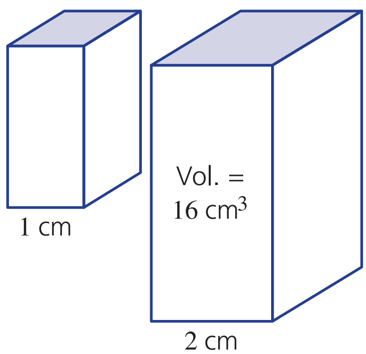


7 The two given prisms are known to be similar. Find the volume of the smaller prism (to two decimal places).

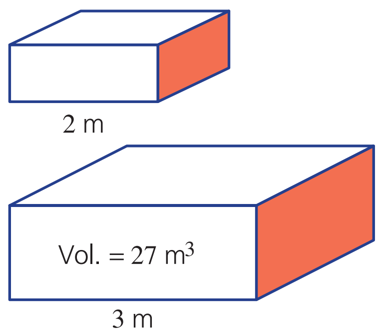
Hint: Volume scale factor = $\frac{b^3}{a^3}$ if the length ratio is $a:b$.



a

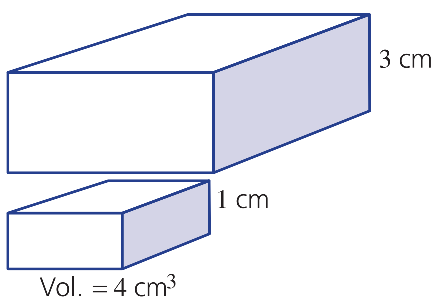


b



8 The two given prisms are known to be similar. Find the volume of the larger prism.

a



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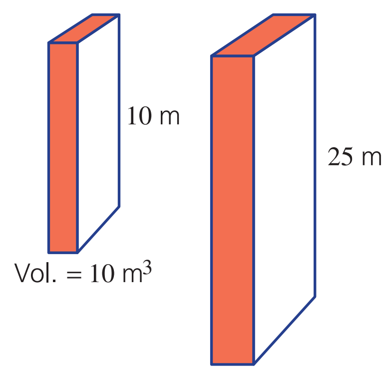


Problem-solving and reasoning

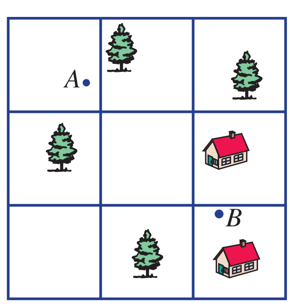
9, 10 | 10–12

9 The given map has a scale factor of 50 000 (i.e. ratio 1 : 50 000).

a How far on the ground, in km, is represented by these map distances?
i 2 cm **ii** 6 cm

b How far on the map, in cm, is represented by these ground distances?
i 5 km **ii** 0.5 km

c What is the actual ground distance between the two points A and B? Use your ruler to measure the distance between A and B first.



Hint:
1 m = 100 cm
1 km = 1000 m

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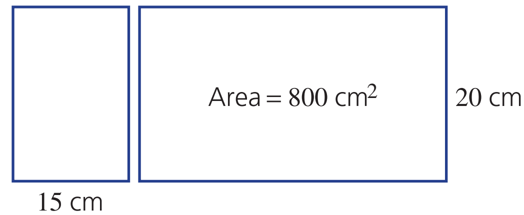
10 Two pieces of paper are similar in shape, as shown.

a What is:

i the length ratio?

ii the area ratio?

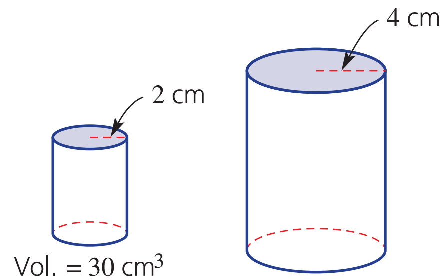
b Find the area of the smaller piece of paper.



11 Two cylinders are similar in shape, as shown.

a Find the volume ratio.

b Find the volume of the larger cylinder.



12 Two rectangular prisms are known to be similar.

a Find the following ratios.

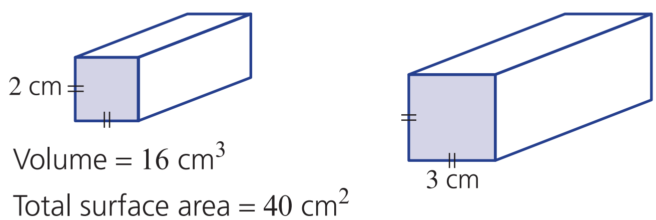
i length

ii area

iii volume

b Find the total surface area of the larger prism.

c Find the volume of the larger prism.



Skyscraper model — 13

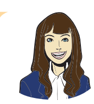
13 A scale model of a skyscraper is 1 m tall and the volume is 2 m^3. The actual height of the skyscraper is 300 m tall.

a Find the volume ratio between the model and actual skyscraper.

b Find the volume of the actual skyscraper.

c If the area of a window on the model is 1 cm^2, find the area of the actual window, in m^2.

Hint:
$1 \text{ m}^2 = 100 \times 100$
$= 10\,000 \text{ cm}^2$

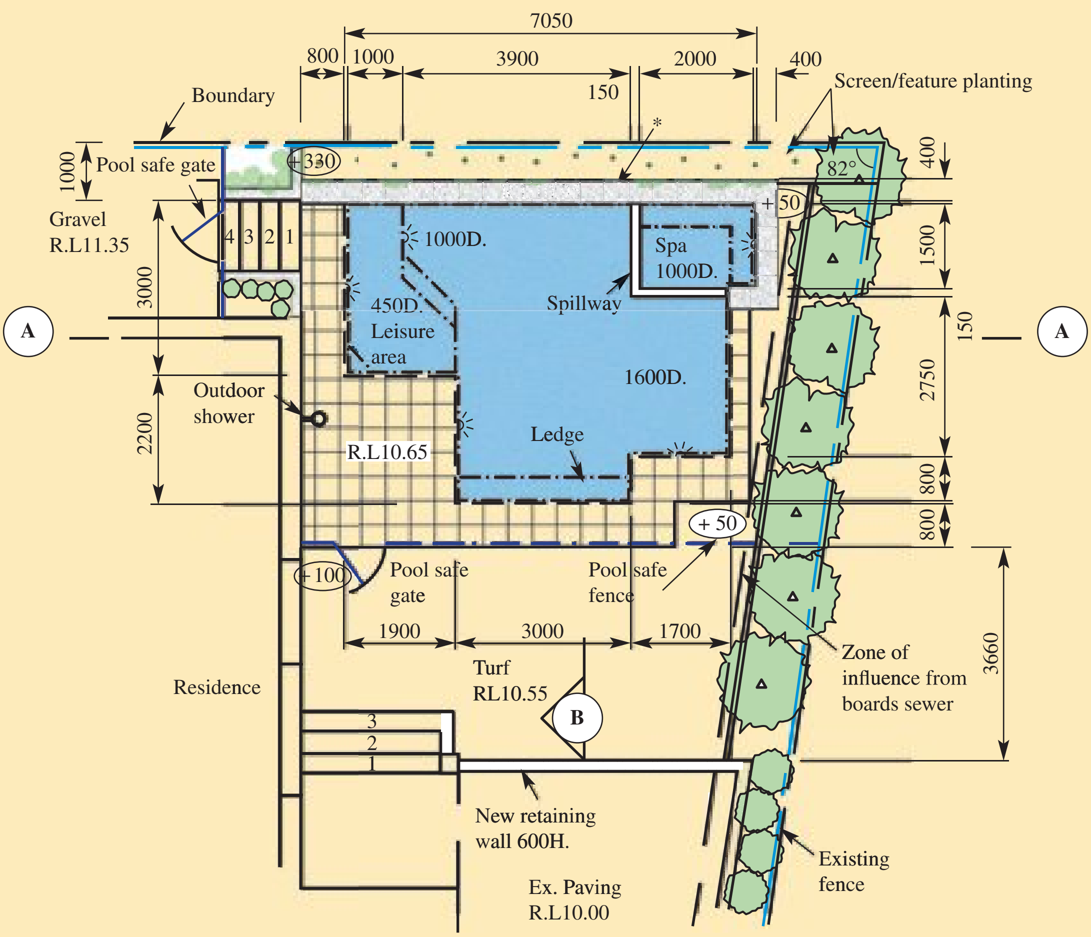


 ISBN 978-1-108-87859-3 Cambridge University Press

Maths@Work: Pool builder

Pool builders and designers need to understand many aspects of measurement and geometry to be able to accurately undertake the task of designing and building pools that meet the specifications of the client. They must be able to read plans, measure the angles, create similar figures and work with parallel lines, especially when they create and work from the pool plans and drawings.

Below are the plans for a pool drawn by a designer. All lengths are given in mm. Answer the questions on the next page relating to the design of this pool.



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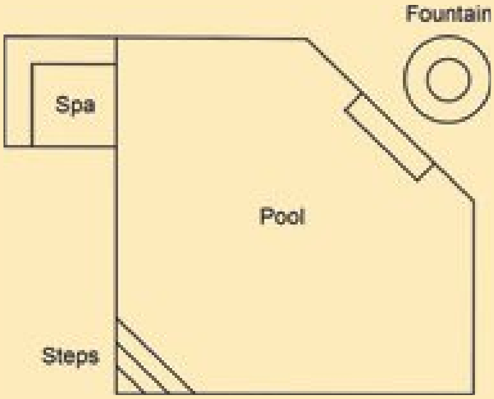
1 Refer to the plans to answer the following questions.
 a What is the total length of the pool when looking across the top of the plans, in metres?
 b How deep is the proposed spa, in metres?
 c What is the height of the new retaining wall, in cm?
 d What is the area taken up by the internal dimensions of the spa? Include the seated area and give your answer in square metres.
 e What is the angle between the back fence and the right side fence?
 f If you walked around the outside of the pool/spa area, how many right angles would you turn? Assume you start and finish at the same point, facing in the same direction.

2 Consider the section of the pool called Leisure area, including the two steps which are a part of this area.
 a By counting the number of outside edges, decide what is the shape of the proposed leisure area.
 b What is the sum of the interior angles of the leisure area, using your knowledge of the angle sum of any polygon?
 c Use a protractor to measure the angles of the leisure area to check that the total is what you expect from part **b**.
 d Using your geometrical equipment, ruler and protractor, create an enlargement of the leisure area. Use the enlargement factor of 4. Your drawing will be mathematically similar to the original. Add all the labelled measurements from the original drawing.

3 Consider this pool tile, which will be used in the spa of this pool.
 a What are two shapes being used in its design?
 b Are both shapes regular?
 c What are the internal angles of the two shapes?
 d This pattern is a tessellation of two shapes. Why does there need to be two different shapes used here? Explain.
 e Design your own tessellation using two regular shapes, ensuring they are different from the ones used here.

Using technology

4 Use a geometry package, like GeoGebra, Desmos, Cabri or Geometer's sketchpad, to come up with your own pool design. You may wish to add length and angle measurements to add detail. Here is a simple example.



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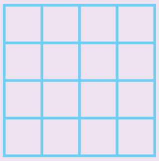
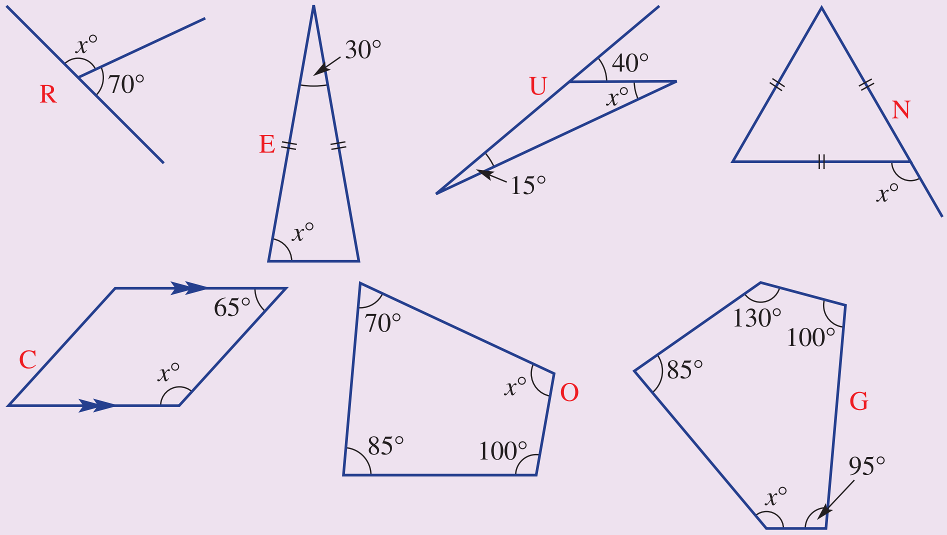
Puzzles and games

1 How many squares can you see in this diagram?

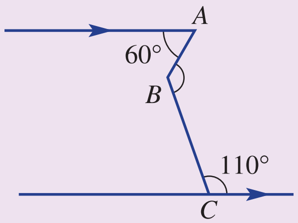


2 'I think of this when I look in the mirror.' Find the value of x in each diagram, then match the letters beside the diagrams to the answers below.



$\overline{115}$ $\overline{105}$ $\overline{120}$ $\overline{130}$ $\overline{110}$ $\overline{25}$ $\overline{75}$ $\overline{120}$ $\overline{115}$ $\overline{75}$

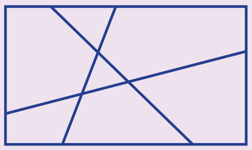
3 What is the size of the obtuse angle $\angle ABC$ in this diagram?



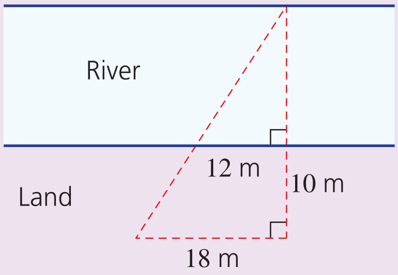
4 This rectangle is subdivided by three straight lines.

a How many regions are formed?

b What is the maximum number of regions formed if four lines are used instead of three?



5 Find the distance across the river.



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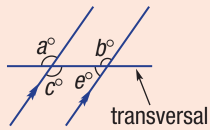
Chapter summary

Geometry

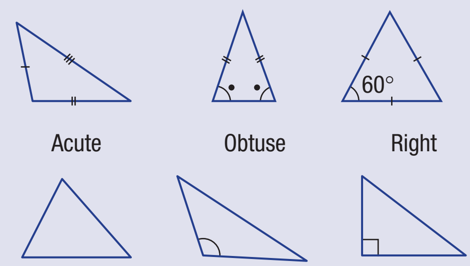
Parallel lines



$a = b$ corresponding angles
$b = c$ alternate angles
$c + e = 180$ cointerior angles
$a = c$ vertically opposite angles

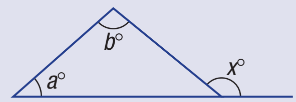
Triangles

Angle sum = 180°



Exterior angles

$x = a + b$



Congruent triangles

Equal in size and shape

Tests: SSS, SAS, AAS, RHS

A congruence statement:
$\triangle ABC \equiv \triangle DEF$

Similar triangles

Equal in shape and sides are in the same ratio

Tests: SSS, SAS, AAA, RHS

A similarity statement:
$\triangle ABC ||| \triangle DEF$

Quadrilaterals

Angle sum = 360°

Special types include:
- parallelogram
- square
- rectangle
- rhombus
- kite
- trapezium

Applications of similarity

When $\triangle ABC ||| \triangle DEF$, then:
- Scale factor $= \frac{DE}{AB}$ or $\frac{DF}{AC}$ or $\frac{EF}{BC}$
- Length ratio $a : b$, Scale factor $= \frac{b}{a}$
- Area ratio $a^2 : b^2$, Scale factor $= \frac{b^2}{a^2}$
- Volume ratio $a^3 : b^3$, Scale factor $= \frac{b^3}{a^3}$

Polygons

Angle sum, $S = 180° \times (n - 2)$

Regular polygons have:
- equal side lengths
- equal angles

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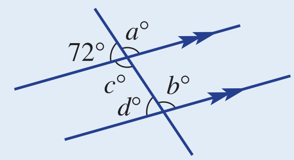
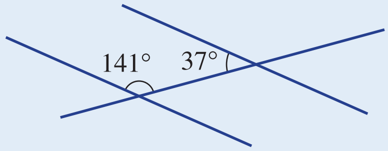
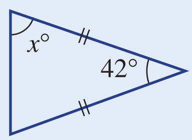
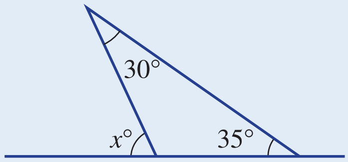
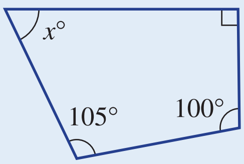
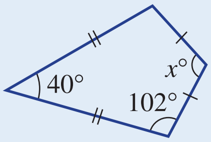
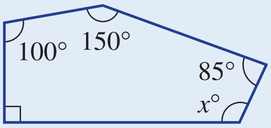
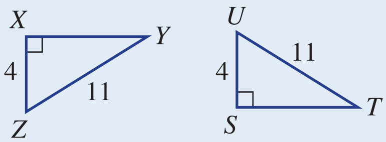
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
7A	**1 I can find unknown angles in parallel lines.** e.g. Find the values of the pronumerals in this diagram and give reasons for your answers.  	
7A	**2 I can prove that two lines are parallel.** e.g. Decide, with reasons, whether the given pair of lines are parallel.  	
7B	**3 I can find unknown angles in any type of triangle.** e.g. Find the value of x in this triangle.  	
7B	**4 I can use the exterior angle theorem to find unknown angles.** e.g. Use the exterior angle theorem to find the value of x in this diagram.  	
7C	**5 I can find an unknown angle in a quadrilateral.** e.g. Find the value of x in this quadrilateral.  	
7C	**6 I can find an unknown angle in a special quadrilateral.** e.g. Find the value of x in this kite.  	
7D	**7 I can find an angle sum of a polygon and an unknown angle in a polygon.** e.g. Find the value of x in this pentagon after finding the angle sum.  	
7D	**8 I can find the internal angle in a regular polygon.** e.g. Find the size of an internal angle inside a regular hexagon.	
7E	**9 I can choose a test and write a congruence statement for a pair of congruent triangles.** e.g. Write a congruence statement and the test to prove congruence for this pair of triangles.  	

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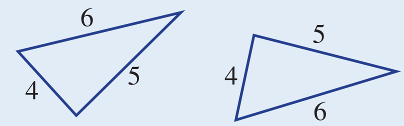
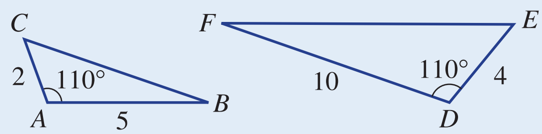
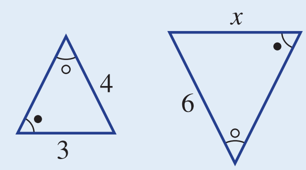
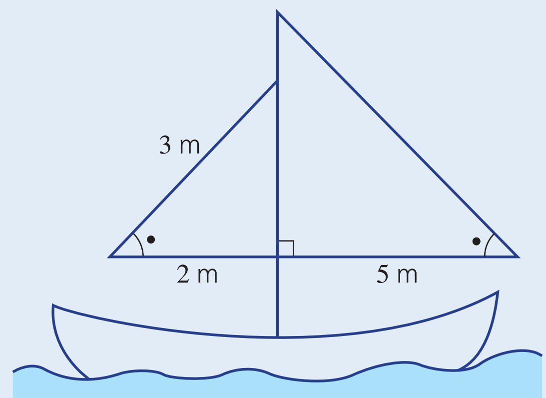
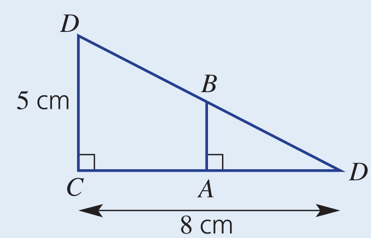
		✔
7E	**10 I can prove that a pair of triangles are congruent.** e.g. Prove that this pair of triangles are congruent, giving full reasons.	
7F	**11 I can prove that a pair of triangles are similar.** e.g. Prove that this pair of triangles are similar, giving full reasons.	
7F	**12 I can find a scale factor and use this to find an unknown length.** e.g. If the given pair of triangles are known to be similar, find the value of x.	
7G	**13 I can use the scale factor for similar triangles to find an unknown length in a real context.** e.g. For the triangles formed from these yacht sails, give a reason why they are similar then use the scale factor to find the length of the hypotenuse on the larger sail.	
7G	**14 I can use the scale factor for similar triangles to find an unknown length inside combined triangles.** e.g. If the point A is at the centre of CD in this diagram, find the length AB.	
7H	**15 I can use measurement and the map scale factor to find a real distance on a map.** e.g. Use measurement and the map scale factor to find the distance across the pool (CD). Scale $1 : 300$	
7H	**16 I can use an area or volume ratio to find an area or volume of a similar object.** e.g. Find the volume of the larger prism if it is known that they are similar.	

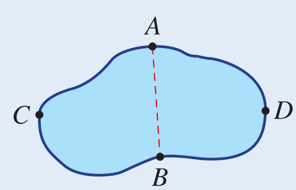


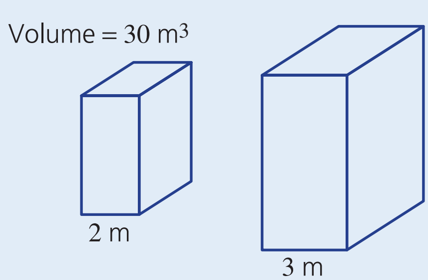












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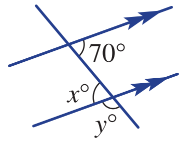
Short-answer questions

7A **1** Find the value of x and y in these diagrams.

a


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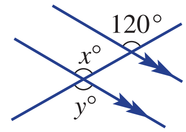

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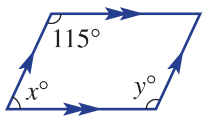

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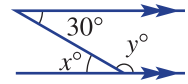

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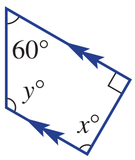

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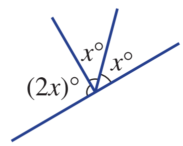
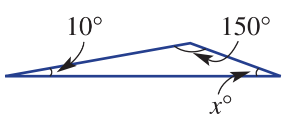


7B **2** Find the value of x in these triangles.

a


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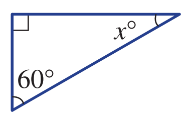

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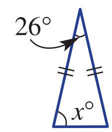

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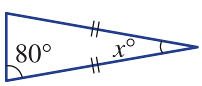

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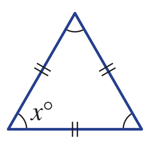

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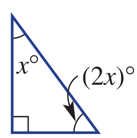

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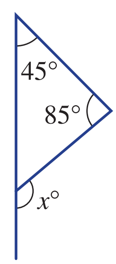

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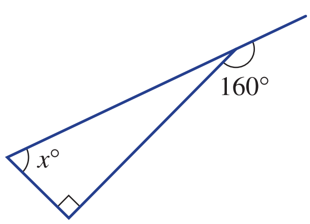

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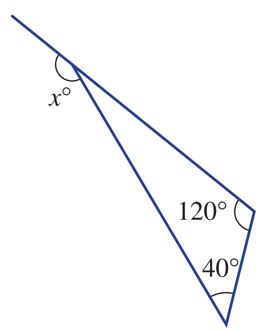

7C **3** List all the quadrilaterals that have:

a two pairs of parallel lines
b opposite angles that are equal
c one pair of equal angles
d diagonals intersecting at right angles

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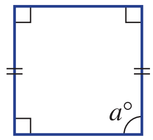
7C **4** Find the values of the pronumerals in these quadrilaterals.

a



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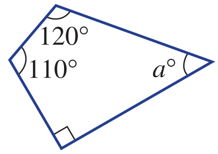

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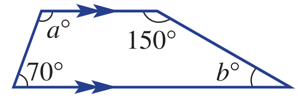

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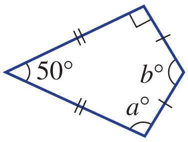

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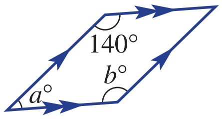

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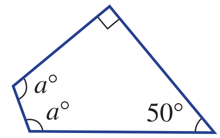
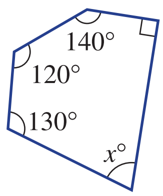


7D **5** Find the value of x by first finding the angle sum. Use $S = 180° \times (n - 2)$.

a



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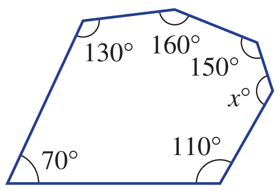

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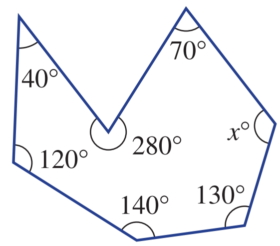
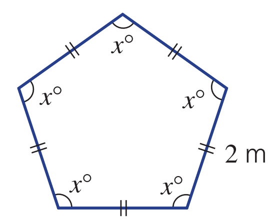


7D **6** Shown here is an example of a regular pentagon ($n = 5$) with side lengths 2 m.

a Find the perimeter of the pentagon.

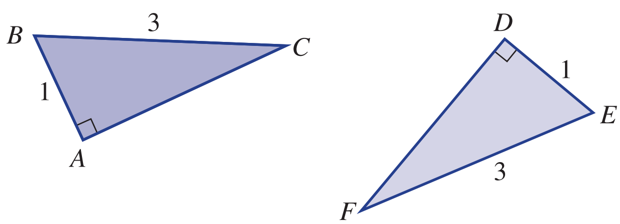
b Find the total internal angle sum (S).

c Find the size of each internal angle (x).



7E **7** Give reasons why the following pairs of triangles are congruent.

a



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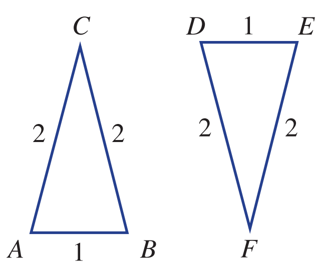

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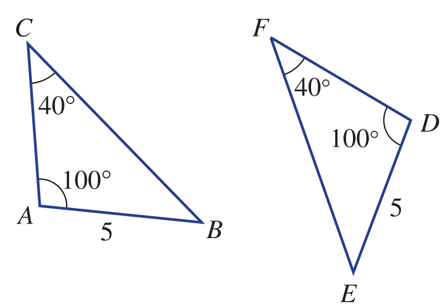

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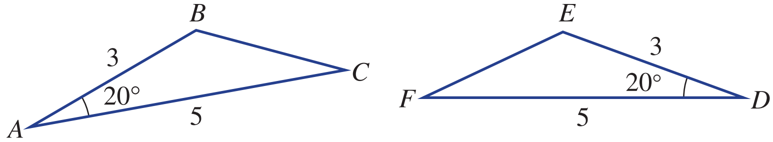

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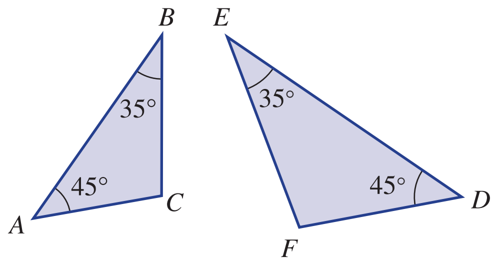
7F **8** Decide whether the given pairs of triangles are similar and give your reasons.

a



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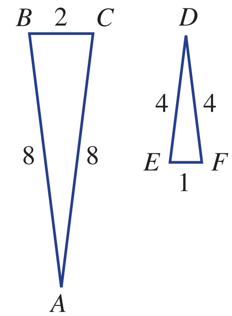

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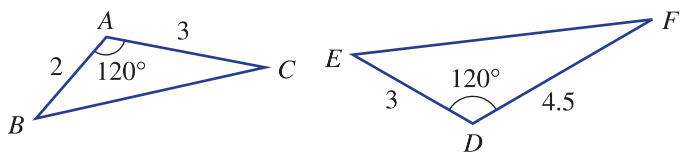

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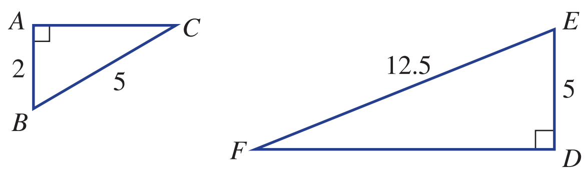
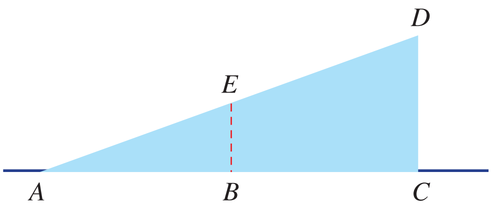


7G **9** A skateboard ramp is supported by two vertical struts, BE (2 m) and CD (5 m).



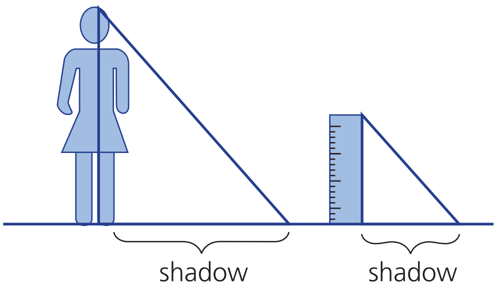
a Name two triangles that are similar, using the letters A, B, C, D and E.

b Give a reason why the triangles are similar.

c Find the scale factor from the smallest to the larger triangle.

d If the length AB is 3 m, find the horizontal length of the ramp AC.

7G **10** The shadow of Clara standing in the sun is 1.5 m long, whereas the shadow of a 30 cm ruler is 24 cm.



a Give a reason why the two created triangles are similar.

b Find the scale factor between the two triangles.

c How tall is Clara?

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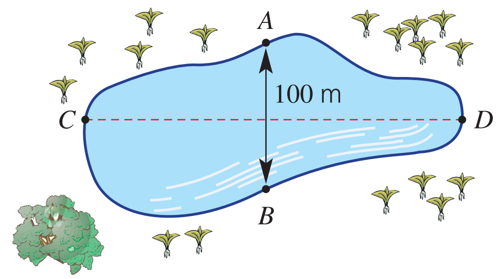
7H

11 The given diagram is a simple map of a swamp in bushland.



a Use a ruler to measure the distance across the swamp (AB). (Answer in cm.)

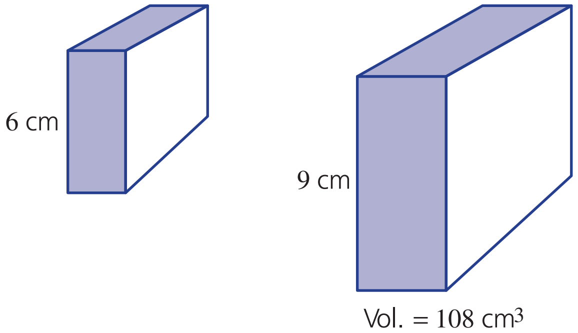
b Find the scale factor between the map and actual ground distance.

c Use a ruler to find the map distance across the swamp (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the swamp (CD). (Answer in m.)

7H

12 The two rectangular prisms shown are known to be similar.



a Find:

  i the length ratio

  ii the area ratio

  iii the volume ratio

b Find the volume of the smaller prism.

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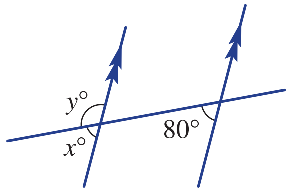
Chapter review

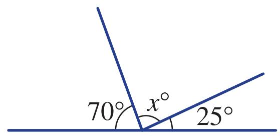
Multiple-choice questions

7A **1** The values of x and y in this diagram are, respectively:



A 100, 100 **B** 80, 100
C 80, 80 **D** 60, 120
E 80, 60

7A **2** The unknown value x in this diagram is:

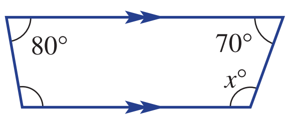


A 85 **B** 105 **C** 75
D 80 **E** 90

7B **3** A triangle has one angle of 60° and another angle of 70°. The third angle is:

A 60° **B** 30° **C** 40°
D 50° **E** 70°

7C **4** The value of x in this quadrilateral is:



A 130 **B** 90 **C** 100
D 120 **E** 110

7D **5** The sum of the internal angles of a hexagon is:

A 180° **B** 900° **C** 360°
D 540° **E** 720°

7E **6** Which abbreviated reason is not relevant for proving congruent triangles?

A AAS **B** RHS **C** SSS
D AAA **E** SAS

7F **7** Two similar triangles have a length ratio of 2 : 3. If one side on the smaller triangle is 5 cm, the length of the corresponding side on the larger triangle is:

A 3 cm **B** 7.5 cm **C** 9 cm
D 8 cm **E** 6 cm

7G **8** A stick of length 2 m and a tree of unknown height stand vertically in the sun. The shadow lengths cast by each are 1.5 m and 30 m, respectively. The height of the tree is:

A 40 m **B** 30 m **C** 15 m **D** 20 m **E** 60 m

7H **9** Two similar triangles have a length ratio of 1 : 3 and the area of the large triangle is 27 cm^2. The area of the smaller triangle is:

A 12 cm^2 **B** 1 cm^2 **C** 3 cm^2 **D** 9 cm^2 **E** 27 cm^2

7H **10** Two similar prisms have a length ratio of 2 : 3. The volume ratio is:

A 4 : 9 **B** 8 : 27 **C** 2 : 27 **D** 2 : 9 **E** 4 : 27

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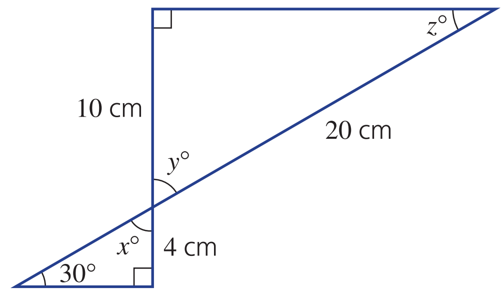
Extended-response questions

1 A company logo contains two triangles, as shown.



a Write down the value of x, y and z.
b Explain why the two triangles are similar.
c Write down the scale factor for length.
d Find the length of the longest side of the smaller triangle.
e Write down the area ratio of the two triangles.
f Write down the area scale factor of the two triangles.

2 A toy model of a car is 8 cm long and the actual car is 5 m long.
a Write down the length ratio of the toy car to the actual car.
b If the toy car is 4.5 cm wide, what is the width of the actual car?
c What is the surface area ratio?
d If the actual car needs 5 litres of paint, what amount of paint would be needed for the toy car?

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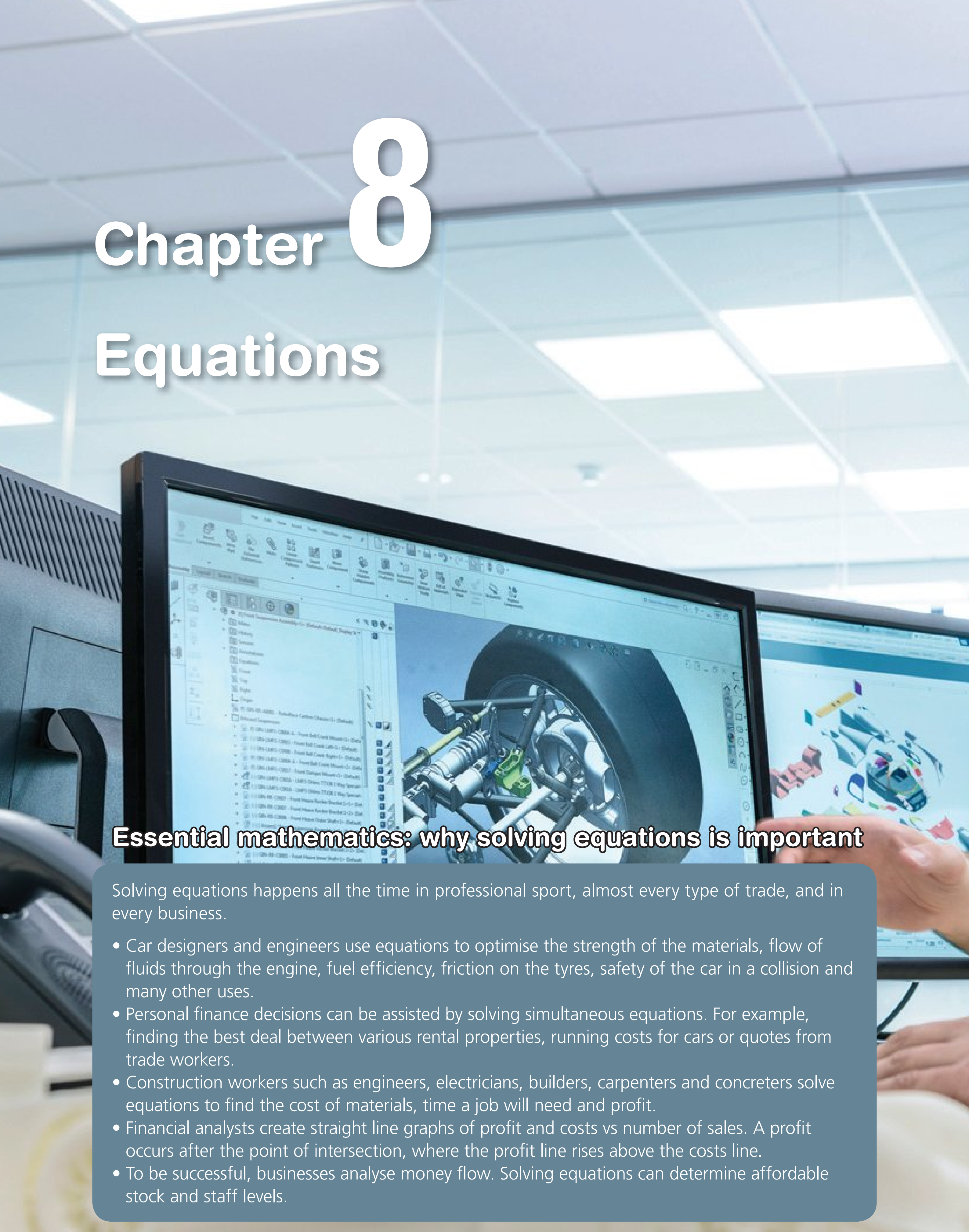

Chapter 8

Equations

Essential mathematics: why solving equations is important

Solving equations happens all the time in professional sport, almost every type of trade, and in every business.

- Car designers and engineers use equations to optimise the strength of the materials, flow of fluids through the engine, fuel efficiency, friction on the tyres, safety of the car in a collision and many other uses.
- Personal finance decisions can be assisted by solving simultaneous equations. For example, finding the best deal between various rental properties, running costs for cars or quotes from trade workers.
- Construction workers such as engineers, electricians, builders, carpenters and concreters solve equations to find the cost of materials, time a job will need and profit.
- Financial analysts create straight line graphs of profit and costs vs number of sales. A profit occurs after the point of intersection, where the profit line rises above the costs line.
- To be successful, businesses analyse money flow. Solving equations can determine affordable stock and staff levels.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (VCMNA335)

Solve linear inequalities and graph their solutions on a number line (VCMNA336)

Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology (VCMNA337)

Solve linear equations involving simple algebraic fractions (VCMNA340)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Warm-up quiz

1 When $a = 6$ and $b = -3$, evaluate the following.

a $a - b$ **b** ab **c** b^2 **d** $3(a + 2b)$

2 When $m = 4$, $n = 7$ and $p = -2$, evaluate the following.

a $4m + p$ **b** $p(4 - n)$ **c** $\frac{8m}{p}$ **d** $2m^2$

3 Simplify the following.

a $a + 2a$ **b** $4m - m$ **c** $6p + 2p$ **d** $7m - 7m$
e $2m - 7m$ **f** $8x + y - x$ **g** $8p + 4p - 3p$ **h** $7m - 4m + 3m$

4 Simplify the following.

a $5x \times 3$ **b** $4p \times 4$ **c** $8x \times 4y$
d $6a \times (-5)$ **e** $a \times b$ **f** $6x \div 6$
g $m \div m$ **h** $6a \div 3$ **i** $\frac{15a}{5a}$

5 Complete the following.

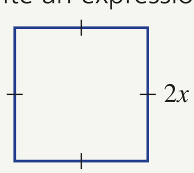
a $x + 5 - \square = x$ **b** $w - 3 + \square = w$ **c** $p - 5 + \square = p$
d $z + 1 - \square = z$ **e** $w \times 4 \div \square = w$ **f** $a \div 2 \times \square = a$
g $m - 3 + \square = m$ **h** $2m \div \square = m$ **i** $\frac{x}{4} \times \square = x$
j $\frac{m}{3} \times \square = m$ **k** $6a \div \square = a$ **l** $10x \div \square = x$

6 Write an expression for each of the following.

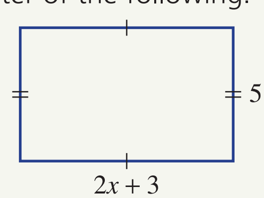
a the sum of x and 3 **b** six more than n
c double w **d** half of x
e six more than double x **f** seven less than x
g three more than x and then doubled **h** one more than triple x

7 Write an expression for the perimeter of the following.

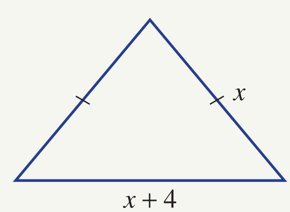
a



b



c



8 State true (T) or false (F) for whether the following are equations.

a $x + 3$ **b** $3x - 6 = 9$ **c** $x^2 - 8$
d $2x$ **e** $3a = 12$ **f** $x^2 = 100$
g $1 = x - 3$ **h** $m - m$ **i** $2p = 0$

9 Solve the following simple equations.

a $m + 3 = 10$ **b** $y - 7 = 12$ **c** $3x = 15$ **d** $\frac{b}{4} = 3$

10 Answer true (T) or false (F) to the following.

a $5 > 3$ **b** $-2 < 7$ **c** $4 \le 2$

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8A Solving linear equations CONSOLIDATING

Learning intentions

- To know what a solution to an equation is
- To be able to solve a simple linear equation
- To be able to verify a solution to an equation

Key vocabulary: equation, linear equation, solve, variable, pronumeral, backtracking, verify, substitute, solution

A cricket batsman will put on socks, then cricket shoes and, finally, pads in that order. When the game is over, these items are removed in reverse order: first the pads, then the shoes and finally the socks. Nobody takes their socks off before their shoes. A similar reversal occurs when solving equations.

We can undo the operations around x by doing the opposite operation in the reverse order to how they have been applied to x. To keep each equation balanced, we always apply the same operation to both sides of an equation.

For example:

Applying operations to $x = 7$

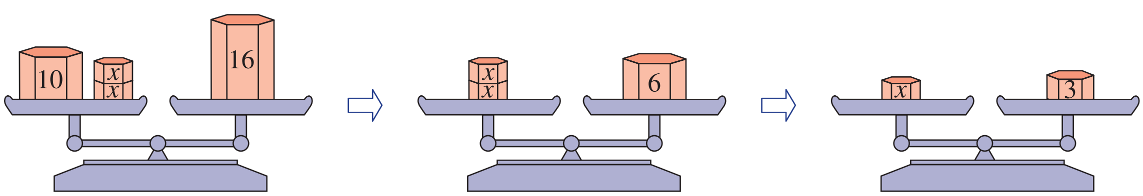
$x = 7$
$\times 2$ $\quad$ $\times 2$
$2x = 14$
$+12$ $\quad$ $+12$
$2x + 12 = 26$

Undoing the operations around x

$2x + 12 = 26$
-12 $\quad$ -12
$2x = 14$
$\div 2$ $\quad$ $\div 2$
$x = 7$

Lesson starter: Keeping it balanced

Three weighing scales are each balanced with various weights on the left and right pans.



Balanced scales 1 $\quad$ Balanced scales 2 $\quad$ Balanced scales 3

- What weight has been removed from each side of scales 1 to get to scales 2?
- What has been done to both the left and right sides of scales 2 to get to scales 3?
- What equations are represented in each of the balanced scales shown above?
- What methods can you recall for solving equations?

Key ideas

- An **equation** is a mathematical statement that includes an equals sign. The equation will be true only for certain value(s) of the pronumeral(s) that make the left-hand side equal to the right-hand side.
  For example: $\frac{5x}{6} = -2$, $3p + 2t = 6$ are equations; $6x - 13$ is not an equation.
- A **linear equation** contains a variable (e.g. x) to the power of 1 and no other powers.
  For example: $3x - 5 = 7$, $4(m - 3) = m + 6$ are linear equations; $x^2 = 49$ is not linear.

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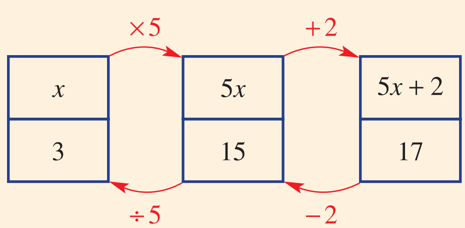
- To **solve** an equation, undo the operations built around x by doing the opposite operation in the reverse order.
  - Always perform the same operation to both sides of an equation so it remains balanced. For example:
    For $5x + 2 = 17$, we observe operations that have been applied to x:
    $$x \xrightarrow{\times 5} 5x \xrightarrow{+2} 5x + 2$$
    So we solve the equation by 'undoing' them in reverse order on both sides of the equation:
    $$5x + 2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x \text{ and } 17 \xrightarrow{-2} 15 \xrightarrow{\div 5} 3$$
    This gives the solution:
    $$\begin{aligned} 5x + 2 &= 17 \quad (-2 \text{ both sides})\\ 5x &= 15 \quad (\div 5 \text{ both sides})\\ x &= 3 \end{aligned}$$
  - Alternatively, a solution need not show the operations applied to each side. These can be done mentally. For example:
    $$\begin{aligned} 5x + 2 &= 17\\ 5x &= 15\\ x &= 3 \end{aligned}$$

- A flow chart can be used to solve equations. First, the equation is built up following the order of operations applied to x and then the solution for x is found by undoing these operations in the reverse order.
  For example, here is a flow chart solution to $5x + 2 = 17$.



  Solution $x = 3$.
  - **Backtracking** is the process of undoing the operations applied to x.

- To **verify** an answer means to check that the solution is correct by substituting the answer to see if it makes the equation true.
  e.g. Verify that $x = 3$ is a solution to $5x + 2 = 17$.
  $$\begin{aligned} \text{LHS} &= 5x + 2 \qquad \text{RHS} = 17\\ &= 5(3) + 2\\ &= 17 \qquad \therefore\ x = 3 \text{ is a solution.} \end{aligned}$$

Exercise 8A

Understanding 1–3 3

1 State the missing word or number.
- **a** An equation is a statement that contains an ______ sign.
- **b** A linear equation contains a variable to the power of ______.

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2 Consider the equation $2x + 3 = 7$.

a Complete this table by evaluating $2x + 3$ for the given values of x.

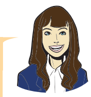
x	0	1	2	3
$2x + 3$				

b By looking at your table of values, which value of x is the solution to $2x + 3 = 7$?

3 Decide whether $x = 2$ is a solution to these equations.

a $x + 3 = 5$ **b** $2x = 7$ **c** $x - 1 = 4$
d $2x - 1 = 10$ **e** $3x + 2 = 8$ **f** $2 - x = 0$

Hint: Substitute $x = 2$ to see whether LHS = RHS.



Fluency

4–9(½) 6–9(½)

Example 1 Solving one-step equations

Solve:

a $x + 7 = 12$ **b** $x - 9 = 3$ **c** $3x = 12$ **d** $\frac{x}{4} = 20$

Solution	Explanation
a $x + 7 = 12$	Write the equation. The opposite of $+7$ is -7.
$x = 12 - 7$	Subtract 7 from both sides.
$x = 5$	Simplify.
Verify: LHS $= 5 + 7$ $= 12$ RHS $= 12$	Check that your answer is correct.
b $x - 9 = 3$	Write the equation. The opposite of -9 is $+9$.
$x = 3 + 9$	Add 9 to both sides.
$x = 12$	Simplify.
Verify: LHS $= 12 - 9$ $= 3$ RHS $= 3$	Check that your answer is correct.
c $3x = 12$	Write the equation. The opposite of $\times 3$ is $\div 3$.
$x = \frac{12}{3}$	Divide both sides by 3.
$x = 4$	Simplify.
Verify: LHS $= 3 \times 4$ $= 12$ RHS $= 12$	Check that your answer is correct.
d $\frac{x}{4} = 20$	Write the equation. The opposite of $\div 4$ is $\times 4$.
$x = 20 \times 4$	Multiply both sides by 4.
$x = 80$	Simplify.
Verify: LHS $= \frac{80}{4}$ $= 20$ RHS $= 20$	Check that your answer is correct.

Now you try

Solve:

a $x + 5 = 21$ **b** $x - 6 = 12$ **c** $4x = 36$ **d** $\frac{x}{3} = -4$

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8A

4 Solve the following.

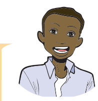
a $t+5=8$ **b** $m+4=10$ **c** $8+x=14$
d $m+8=40$ **e** $a+1=-5$ **f** $16=m+1$
g $x-3=3$ **h** $x-7=2$ **i** $x-8=9$
j $x-3=0$ **k** $x-2=-8$ **l** $x-5=7$

Hint:
$8+x=14$ is the same as $x+8=14$.
$16=m+1$ is the same as $m+1=16$.

5 Solve the following.

a $8p=24$ **b** $5c=30$ **c** $27=3d$
d $15p=15$ **e** $6m=-42$ **f** $-10=20p$
g $\frac{x}{5}=10$ **h** $\frac{m}{3}=7$ **i** $\frac{a}{6}=-2$
j $\frac{z}{7}=0$ **k** $\frac{w}{3}=\frac{1}{2}$ **l** $\frac{m}{2}=\frac{1}{4}$

Hint: $27=3d$ is the same as $3d=27$.



Hint: $3\times\frac{1}{2}=\frac{3}{1}\times\frac{1}{2}=\frac{3}{2}$

6 Solve the following equations.

a $x+9=12$ **b** $x+3=12$ **c** $x+15=4$
d $x-7=3$ **e** $x-2=12$ **f** $x-5=5$
g $3x=9$ **h** $4x=16$ **i** $2x=100$
j $\frac{x}{5}=4$ **k** $\frac{x}{3}=7$ **l** $\frac{x}{7}=1$

Hint: Carry out the 'opposite' operation to solve for x.

Example 2 Solving two-step equations

Solve $4x+5=17$.

Solution	Explanation
$4x+5=17$	Write the equation.
$4x=12$	Subtract 5 from both sides first.
$x=\frac{12}{4}$	Divide both sides by 4.
$x=3$	Simplify.
Verify: LHS $=4(3)+5$ $=17$ RHS $=17$	Check your answer.

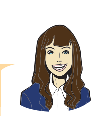
Now you try

Solve $5x-1=19$.

7 Solve the following equations.

a $2x+5=7$ **b** $3x+2=11$
c $4x-3=9$ **d** $6x+13=1$
e $8x+16=8$ **f** $10x+92=2$
g $3x-4=8$ **h** $2x-7=9$
i $5x-4=36$ **j** $2x-6=-10$
k $7x-3=-24$ **l** $6x-3=27$

Hint: First choose to add or subtract a number from both sides and then divide by the coefficient of x.



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Example 3 Solving two-step equations involving simple fractions

Solve $\frac{x}{5} - 3 = 4$.

Solution	Explanation
$\frac{x}{5} - 3 = 4$	Write the equation.
$\frac{x}{5} = 7$	Add 3 to both sides.
$x = 35$	Multiply both sides by 5.
Verify: LHS $= \frac{35}{5} - 3$ $= 4$ RHS $= 4$	Check that your answer is correct.

Now you try

Solve $\frac{x}{7} + 2 = 6$.

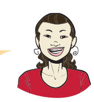
8 Solve the following equations.

a $\frac{x}{3} + 2 = 5$ **b** $\frac{x}{6} + 3 = 3$ **c** $\frac{x}{7} + 4 = 12$

d $\frac{x}{4} - 3 = 2$ **e** $\frac{x}{5} - 4 = 3$ **f** $\frac{x}{10} - 2 = 7$

g $\frac{x}{8} - 2 = -6$ **h** $\frac{x}{4} - 3 = -8$ **i** $\frac{x}{2} - 1 = -10$

Hint: When solving equations, the order of steps is important. For $\frac{x}{3} - 5$, undo the -5 first, then undo the $\div 3$.



Example 4 Solving more two-step equations

Solve $\frac{x+4}{2} = 6$.

Solution	Explanation
$\frac{x+4}{2} = 6$	Write the equation.
$x + 4 = 12$	In $\frac{x+4}{2}$ we first add 4 and then divide by 2. So to undo we first multiply both sides by 2.
$x = 8$	Subtract 4 from both sides.
Verify: LHS $= \frac{8+4}{2}$ $= 6$ RHS $= 6$	Check that your answer is correct.

Now you try

Solve $\frac{x-3}{4} = 1$.

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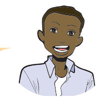
8A

9 Solve the following equations.

a $\frac{m+1}{2} = 3$
b $\frac{a-1}{3} = 2$
c $\frac{x+5}{2} = 3$
d $\frac{x+5}{3} = 2$
e $\frac{n-4}{5} = 1$
f $\frac{m-6}{2} = 8$
g $\frac{w+4}{3} = -1$
h $\frac{m+3}{5} = 2$
i $\frac{w-6}{3} = 7$
j $\frac{a+7}{4} = 2$
k $\frac{a-3}{8} = -5$
l $\frac{m+5}{8} = 0$

Hint: When solving equations, the order of steps is important. For $\frac{x+7}{3}$, undo the $\div 3$ first, then undo the $+7$. Never cancel a number joined by $+$ or $-$ to an x. In $\frac{x+8}{4}$, you cannot cancel the 4 into the 8.



Problem-solving and reasoning 10, 11 11–13

Example 5 Writing equations from word problems

For each of the following statements, write an equation and solve for the pronumeral.

a When 7 is subtracted from x, the result is 12.
b When x is divided by 5 and then 6 is added, the result is 10.
c When 4 is subtracted from x and that answer is divided by 2, the result is 9.

Solution	Explanation
a $x - 7 = 12$ $x = 19$	Subtract 7 from x means to start with x and then subtract 7. 'The result' means '='.
b $\frac{x}{5} + 6 = 10$ $\frac{x}{5} = 4$ $x = 20$	Divide x by 5, then add 6 and make it equal to 10. Solve the equation by subtracting 6 from both sides first.
c $\frac{x-4}{2} = 9$ $x - 4 = 18$ $x = 22$	Subtracting 4 from x gives $x - 4$, and divide that answer by 2. Undo $\div$ 2 by multiplying both sides by 2, then add 4 to both sides.

Now you try

For each of the following statements, write an equation and solve for the pronumeral.

a When 3 is added to x, the result is 9.
b When x is divided by 3 then 7 is subtracted, the result is 0.
c When 6 is subtracted from x and that answer is divided by 3, the result is 10.

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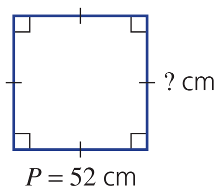
10 For each of the following statements, write an equation and solve for the pronumeral.

- **a** When 4 is added to x, the result is 6.
- **b** When x is added to 12, the result is 8.
- **c** When 5 is subtracted from x, the result is 5.
- **d** When x is divided by 3 and then 2 is added, the result is 8.
- **e** Twice the value of x is added to 3 and the result is 9.
- **f** $(x - 3)$ is divided by 5 and the result is 6.
- **g** 3 times x plus 4 is equal to 16.

Hint: 5 subtracted from x is $x - 5$.

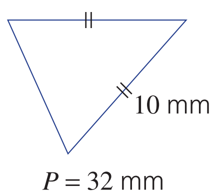
11 Write an equation and solve it for each of these questions.

- **a** The perimeter of a square is 52 cm. Determine the length of the side.



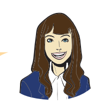
Hint: Draw a diagram and choose a pronumeral to represent the unknown side. Then write an equation and solve it.

- **b** The perimeter of an isosceles triangle is 32 mm. If the equal sides are both 10 mm, determine the length of the other side.



12 Convert the following into equations, then solve them for the unknown number.

- **a** n is multiplied by 2, then 5 is added. The result is 11.
- **b** Four times a certain number is added to 9 and the result is 29. What is the number?
- **c** Half of a number less 2 equals 12. What is the number?
- **d** A number plus 6 has been divided by 4. The result is 12. What is the number?
- **e** 12 is subtracted from a certain number and the result is divided by 5. If the answer is 14, what is the number?



Hint: Choose a pronumeral to represent the unknown number, then write an equation using the pronumeral. $\frac{1}{2}$ of x can be written as $\frac{x}{2}$.

13 Write an equation and solve it for each of these questions.

- **a** The sum of two consecutive whole numbers is 23. What are the numbers?
- **b** If I add 5 to twice a number, the result is 17. What is the number?
- **c** Three less than five times a number is 12. What is the number?
- **d** One person is 19 years older than another person. Their age sum is 69. What are their ages?
- **e** Andrew threw the shot-put 3 m more than twice the distance Barry threw it. If Andrew threw the shot-put 19 m, how far did Barry throw it?

Hint: Consecutive whole numbers are one number apart; e.g. 3, 4, 5, 6 etc. The next consecutive number after x is $x + 1$.

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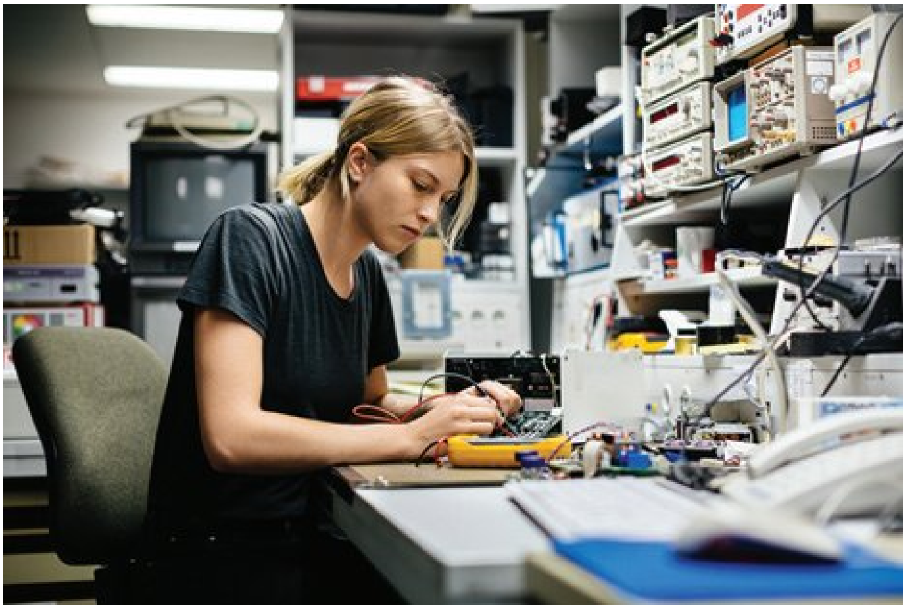
8A

Modelling with equations — 14, 15

14 A service technician charges \$40 up front and \$60 for each hour she works.

- **a** Find a linear equation for the total charge, $\$C$, of any job for h hours worked.
- **b** What will a 4-hour job cost?
- **c** If the technician works on a job for 3 days and averages 6 hours per day, what will be the overall cost?
- **d** If a customer is charged \$400, how long did the job take?



15 A petrol tank holds 71 litres of fuel. It originally contained 5 litres. If a petrol pump fills it at 6 litres per minute, find:

- **a** a linear equation for the amount of fuel (V litres) in the tank at time t minutes
- **b** how long it will take to fill the tank to 23 litres
- **c** how long it will take to fill the tank

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8B Solving more difficult linear equations ★

Learning intentions

- To be able to expand brackets and collect like terms when solving a linear equation
- To be able to collect pronumerals to one side in order to solve a linear equation
- To be able to solve a simple word problem by setting up and solving a linear equation

Key vocabulary: expand, like terms, product, equivalent

More complex linear equations may have variables on both sides of the equation and/or brackets. Examples are $6x = 2x - 8$ or $5(x + 3) = 12x + 4$.

Brackets can be removed by expanding. Equations with variables on both sides can be solved by collecting variables to one side, using addition or subtraction of a term.

More complex linear equations of this type are used when constructing buildings and in science and engineering.

Lesson starter: Steps in the wrong order

The steps to solve $8(x + 2) = 2(3x + 12)$ are listed here in the incorrect order.

$$8(x + 2) = 2(3x + 12)$$
$$x = 4$$
$$2x + 16 = 24$$
$$8x + 16 = 6x + 24$$
$$2x = 8$$

- Arrange them in the correct order, working from the problem to the solution.
- By considering all the steps in the correct order, write what has happened in each step.

Key ideas

- When solving complicated linear equations:
  1. First, **expand** any brackets.
     In this example, multiply the 3 into the first bracket and the −2 into the second bracket.
     $$3(2x - 1) - 2(x - 2) = 22$$
     $$6x - 3 - 2x + 4 = 22$$
  2. Collect any **like terms** on the LHS and any like terms on the RHS.
     Collecting like terms on each side of this example:
     $$5x - 4 - 3x - 9 = x - 5 + 2x + 10$$
     $$2x - 13 = 3x + 5$$
     $5x - 3x = 2x$, $-4 - 9 = -13$, $x + 2x = 3x$ and $-5 + 10 = 5$
  3. If an equation has variables on both sides, collect to one side by adding or subtracting one of the terms.
     For example, when solving the equation $12x + 7 = 5x + 19$, first subtract $5x$ from both sides:
     LHS: $12x + 7 - 5x = 7x + 7$, RHS: $5x + 19 - 5x = 19$:
     $$12x + 7 = 5x + 19 \quad (-5x \text{ both sides})$$
     $$7x + 7 = 19$$
  4. Start to perform the opposite operation to both sides of the equation.
  5. Repeat Step **4** until the equation is solved.
  6. Verify that the answer is correct.

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8B

- To solve a word problem using algebra:
  - Read the problem and find out what the question is asking for.
  - Define a pronumeral and write a statement such as: 'Let x be the number of …'. The pronumeral is often what you have been asked to find in the question.
  - Write an equation using your defined pronumeral.
  - Solve the equation.
  - Answer the question in words.

Exercise 8B

Understanding 1–3 | 2, 3

1 Choose from the words *collect*, *expand* and *one* to complete the following when solving linear equations.

a First _______ any brackets.
b _______ any like terms.
c If variables are on both sides, collect to _______ side.

2 When $-2(x-1)$ is expanded, the result is:

A $-2x-2$ **B** $-2x+1$ **C** $-2x+2$
D $2x+2$ **E** $2x+1$

3 When $2x$ is subtracted from both sides, $5x+1=2x-3$ becomes:

A $3x-1=3$ **B** $7x+1=-3$ **C** $7x+1=3$
D $3x+1=3$ **E** $3x+1=-3$

Fluency 4–9(½) | 4–9(½)

Example 6 Solving equations with brackets

Solve $4(x-1)=16$.

Solution	Explanation
$4(x-1)=16$	
$4x-4=16$	Expand the brackets: $4\times x$ and $4\times(-1)$.
$4x=20$	Add 4 to both sides.
$x=5$	Divide both sides by 4.

Now you try

Solve $3(x+1)=15$.

4 Solve each of the following equations by first expanding the brackets.

a $3(x+2)=9$ **b** $4(x-1)=16$
c $3(x+5)=12$ **d** $4(a-2)=12$
e $5(a+1)=10$ **f** $2(x-10)=10$
g $6(m-3)=6$ **h** $3(d+4)=15$
i $7(a-8)=14$ **j** $10(a+2)=20$
k $5(3+x)=15$ **l** $2(a-3)=0$

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Example 7 Solving equations with two sets of brackets

Solve $3(2x + 4) + 2(3x - 2) = 20$.

Solution	Explanation
$3(2x + 4) + 2(3x - 2) = 20$	Use the distributive law to expand each set of brackets.
$6x + 12 + 6x - 4 = 20$	
$12x + 8 = 20$	Collect like terms on the LHS.
$12x = 12$	Subtract 8 from both sides.
$x = 1$	Divide both sides by 12.

Now you try

Solve $2(3x - 1) - 3(x - 4) = 16$.

5 Solve the following equations.

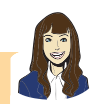
a $3(2x + 3) + 2(x + 4) = 25$
b $2(2x + 3) + 4(3x + 1) = 42$
c $2(2x + 3) + 3(4x - 1) = 51$
d $3(2x - 2) + 5(x + 4) = 36$
e $4(2x - 3) + 2(x - 4) = 10$
f $2(3x - 1) + 3(2x - 3) = 13$
g $2(x - 4) + 3(x - 1) = -21$
h $4(2x - 1) + 2(2x - 3) = -22$

Hint: Expand each pair of brackets and collect like terms before solving.

6 Solve the following equations.

a $3(2x + 4) - 4(x + 2) = 6$
b $2(5x + 4) - 3(2x + 1) = 9$
c $2(3x - 2) - 3(x + 1) = -7$
d $2(x + 1) - 3(x - 2) = 8$
e $8(x - 1) - 2(3x - 2) = 2$
f $5(2x - 3) - 2(3x - 1) = -9$
g $5(2x + 1) - 3(x - 3) = 35$
h $4(2x - 3) - 2(3x - 1) = -14$

Hint:
$-4(x + 2) = -4x - 8$
$-4(x - 2) = -4x + 8$



Example 8 Solving equations with variables on both sides

Solve $7x + 9 = 2x - 11$ for x.

Solution	Explanation
$7x + 9 = 2x - 11$	
$5x + 9 = -11$	Subtract $2x$ from both sides.
$5x = -20$	Subtract 9 from both sides.
$x = -4$	Divide both sides by 5.

Now you try

Solve $10x + 3 = 8x - 1$ for x.

7 Find the value of x in the following.

a $7x = 2x + 10$
b $10x = 9x + 12$
c $8x = 4x - 12$
d $6x = 2x + 80$
e $2x = 12 - x$
f $2x = 8 + x$
g $3x + 4 = x + 12$
h $4x + 9 = x - 3$
i $2x - 9 = x - 10$
j $6x - 10 = 12 + 4x$
k $9x = 10 - x$
l $1 - x = x + 3$

Hint: Remove the term containing x on the RHS. For parts **e**, **k** and **l**, you will need to add x to both sides.

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8B

Example 9 Solving equations with fractions

Solve $\frac{2x+3}{4} = 2$ for x.

Solution	Explanation
$\frac{2x+3}{4} = 2$	
$2x + 3 = 8$	Multiply both sides by 4.
$2x = 5$	Subtract 3 from both sides.
$x = 2.5$	Divide both sides by 2.

Now you try

Solve $\frac{4x-3}{2} = 4$.

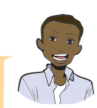
8 Solve the following equations.

a $\frac{x+2}{3} = 5$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{x-1}{3} = 4$

d $\frac{x-5}{3} = 2$ **e** $\frac{2x+1}{7} = 3$ **f** $\frac{2x+2}{3} = 4$

g $\frac{5x-3}{3} = 9$ **h** $\frac{3x-6}{2} = 9$ **i** $\frac{5x-2}{4} = -3$

Hint: First multiply by the denominator.



Example 10 Solving equations with more difficult fractions

Solve $\frac{3x}{2} - 4 = 2$ for x.

Solution	Explanation
$\frac{3x}{2} - 4 = 2$	
$\frac{3x}{2} = 6$	Add 4 to both sides.
$3x = 12$	Multiply both sides by 2.
$x = 4$	Divide both sides by 3.

Now you try

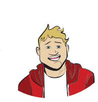
Solve $\frac{5x}{3} + 1 = 6$ for x.

9 Solve the following equations.

a $\frac{x}{3} + 1 = 5$ **b** $\frac{x}{3} + 1 = 7$ **c** $\frac{x}{4} - 5 = 10$

d $\frac{3x}{4} - 2 = 5$ **e** $\frac{2x}{5} - 3 = -1$ **f** $\frac{3x}{2} - 5 = -14$

Hint: First add or subtract a number from both sides.



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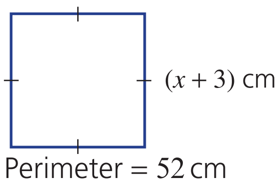
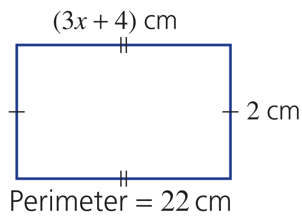
Problem-solving and reasoning | 10–13 | 11–14

10 For each of these questions, write an equation and solve it for x.

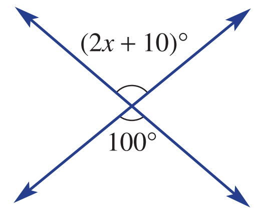
a



b



c



Hint: Vertically opposite angles are equal.

11 Solve the following equations using trial and error (guess, check and refine). Substitute your chosen values of x until you have found a value that makes the equation true.

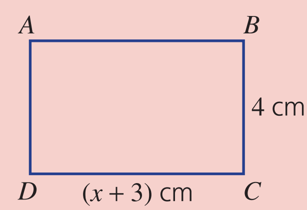
a $\frac{x+22}{3} = 4x$

b $5(3 - x) = 2(x + 7.5)$

c $\frac{2x-1}{4} = 2 - x$

Example 11 Solving a word problem

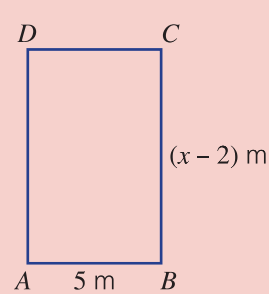
Find the value of x if the area of rectangle $ABCD$ shown is 24 cm^2.



Solution	Explanation
$A = l \times w$	Write an equation for area.
$24 = (x + 3) \times 4$	Substitute: $l = (x + 3)$, $w = 4$, $A = 24$.
$24 = 4x + 12$	Expand the brackets: $(x + 3) \times 4 = 4(x + 3)$.
$12 = 4x$	Subtract 12 from both sides.
$3 = x$	Divide both sides by 4.
$x = 3$	Write the answer.

Now you try

Find the value of x if the area of rectangle $ABCD$ shown is 40 m^2.



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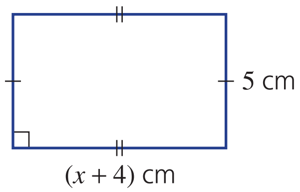
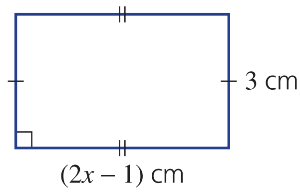
8B

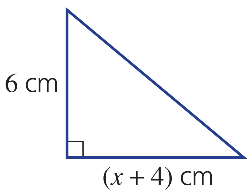
12 **a** Find the value of x if the area is 35 cm^2.



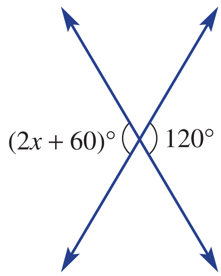
b Find the value of x if the area is 27 cm^2.



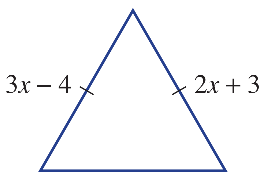
c Find the value of x if the area is 42 cm^2.



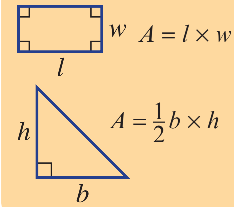
d Vertically opposite angles are equal. Find the value of x.



e Find the value of x.



Hint: Form the area equation first.



13 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 5 and 1 more than a number is 40.
b The product of 5 and 6 less than a number is -15.
c When 6 less than 3 lots of a number is doubled, the result is 18.
d When 8 more than 2 lots of a number is tripled, the result is 36.
e 10 more than 4 lots of a number is equivalent to 6 lots of the number.
f 5 more than 4 times a number is equivalent to 1 less than 5 times the number.
g 6 more than a doubled number is equivalent to 5 less than 3 lots of the number.

Hint:
- 'Product' means 'to multiply'
- The product of 5 and 1 more than a number means $5(x + 1)$.
- '6 less than 3 lots of a number is doubled' will require brackets.
- 'Tripled' means three times a number.
- 'Equivalent' means 'equal to'.

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14 Valentina and Harrison are planning to hire a car for their wedding day. 'Vehicles For You' have the following deal: \$850 hiring fee plus a charge of \$156 per hour.

a Write an equation for the cost (\$$C$) of hiring a car for h hours.

b If Valentina and Harrison have budgeted for the car to cost a maximum of \$2000, find the maximum number of full hours they can hire the car.

c If the car picks up the bride at 1:15 p.m., at what time must the event finish if the cost is to remain within budget?

More than one fraction — 15

15 Consider:

$$\frac{4x-2}{3}=\frac{3x-1}{2}$$

$$\frac{{}^{2}\cancel{6}(4x-2)}{\cancel{3}_{1}}=\frac{{}^{3}\cancel{6}(3x-1)}{\cancel{2}_{1}}$$ (Multiply both sides by 6 (LCM of 2 and 3) to get rid of the fractions.)

$2(4x-2)=3(3x-1)$ (Simplify.)

$8x-4=9x-3$ (Expand both sides.)

$-4=x-3$ (Subtract $8x$ from both sides.)

$-1=x$ (Add 3 to both sides.)

$\therefore x=-1$

Solve the following equations using the method shown above.

a $\frac{x+2}{3}=\frac{x+1}{2}$ **b** $\frac{x+1}{2}=\frac{x}{3}$ **c** $\frac{3x+4}{4}=\frac{x+6}{3}$

d $\frac{5x+2}{3}=\frac{3x+4}{2}$ **e** $\frac{2x+1}{7}=\frac{3x-5}{4}$ **f** $\frac{5x-1}{3}=\frac{x-4}{4}$

Using technology 8B: Solving linear equations
This activity is available on the companion website as a printable PDF.

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8C Using formulas

Learning intentions

- To understand that a relationship between variables can be described using formulas
- To be able to substitute into a formula and evaluate
- To be able to solve an equation after substitution into a formula

Key vocabulary: subject, formula, variable, substitute, evaluate

A formula (or rule) is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns.

You will already be familiar with many formulas. For example, $C = 2\pi r$ is the formula for finding the circumference, C, of a circle when given its radius, r.

$F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

$s = \frac{d}{t}$ is the formula for finding the speed, s, when given the distance, d, and time, t.

C, F and s are said to be the subjects of the formulas given above.

Lesson starter: Jumbled solution

Problem: The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.

Xavier was asked to find a, given that $A = 126$, $b = 10$ and $h = 14$, and to write the explanation beside each step of the solution.

Xavier's solution and explanation are below. His solution is correct but he has jumbled up the steps in the explanation. Copy Xavier's solution and write the correct instruction(s) beside each step.

Solution	Jumbled explanation
$A = \frac{h}{2}(a + b)$	Subtract 70 from both sides.
$126 = \frac{14}{2}(a + 10)$	Divide both sides by 7.
$126 = 7(a + 10)$	Substitute the given values.
$126 = 7a + 70$	Copy the formula.
$56 = 7a$	Simplify $\frac{14}{2}$.
$a = 8$	Expand the brackets.

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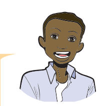
Key ideas

- A **formula** is an equation that relates two or more variables.
- The **subject** of a formula is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be rearranged to make another variable the subject. $C = 2\pi r$ can be rearranged to give $r = \frac{C}{2\pi}$.
- Note that $\sqrt{a^2} = a$ when $a \geq 0$ and $\sqrt{a^2 + b^2} \neq a + b$.

Exercise 8C

Understanding	1, 2	2

1 State the letter that is the subject in these formulas.

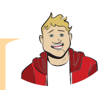


Hint: The subject of a formula is the letter on its own, on the left-hand side.

a $I = \frac{Prt}{100}$ **b** $F = ma$ **c** $V = \frac{4}{3}\pi r^3$

d $A = \pi r^2$ **e** $c = \sqrt{a^2 + b^2}$ **f** $P = 2x + 2y$

2 Substitute the given values into each of the following formulas to find the value of each subject. Round the answer to one decimal place where appropriate.



Hint: Copy each formula, substitute the given values and then calculate the answer.

a $m = \frac{F}{a}$, when $F = 180$ and $a = 3$

b $A = lw$, when $l = 6$ and $w = 8$

c $A = \frac{1}{2}(a + b)h$, when $a = 6$, $b = 12$ and $h = 4$

d $v^2 = u^2 + 2as$, when $u = 6$, $a = 12$ and $s = 7$ **e** $m = \sqrt{\frac{x}{y}}$, when $x = 56$ and $y = 4$

Fluency	3–8(½)	4–5(½), 7–9(½)

Example 12 Substituting values and solving equations

If $v = u + at$, find t when $v = 16$, $u = 4$ and $a = 3$.

Solution	Explanation
$v = u + at$	Substitute each value into the formula. $v = 16, u = 4, a = 3$
$16 = 4 + 3t$	An equation now exists. Solve this equation for t.
$12 = 3t$	Subtract 4 from both sides.
$4 = t$	Divide both sides by 3.
$t = 4$	Answer with the pronumeral on the left-hand side.

Now you try

If $A = \frac{1}{2}xy$, find y when $A = 12$ and $x = 4$.

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3 If $v = u + at$, find t when:
a $v = 16$, $u = 8$ and $a = 2$
b $v = 20$, $u = 8$ and $a = 3$
c $v = 100$, $u = 10$ and $a = 9$
d $v = 84$, $u = 4$ and $a = 10$

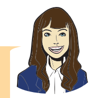
Hint: First copy the formula. Then substitute the given values. Then solve the equation.

4 If $P = 2(l + 2b)$, find b when:
a $P = 60$ and $l = 10$
b $P = 48$ and $l = 6$
c $P = 96$ and $l = 14$
d $P = 12.4$ and $l = 3.6$

5 If $V = lwh$, find h when:
a $V = 100$, $l = 5$ and $w = 4$
b $V = 144$, $l = 3$ and $w = 4$
c $V = 108$, $l = 3$ and $w = 12$
d $V = 280$, $l = 8$ and $w = 5$

6 If $A = \frac{1}{2}bh$, find b when:
a $A = 90$ and $h = 12$
b $A = 72$ and $h = 9$
c $A = 108$ and $h = 18$
d $A = 96$ and $h = 6$

Hint: For $90 = \frac{1}{2} \times b \times 12$,
$\frac{1}{2} \times b \times 12 = \frac{1}{2} \times 12 \times b$
$= 6b$
So, $90 = 6b$
Solve for b.

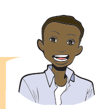


7 If $A = \frac{h}{2}(a + b)$, find h when:
a $A = 20$, $a = 4$ and $b = 1$
b $A = 48$, $a = 5$ and $b = 7$
c $A = 108$, $a = 9$ and $b = 9$
d $A = 196$, $a = 9$ and $b = 5$

Hint: When solving the equation first undo the division by 2 by multiplying both sides by 2.

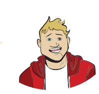
8 $E = mc^2$. Find m when:
a $E = 100$ and $c = 5$
b $E = 4000$ and $c = 10$
c $E = 72$ and $c = 1$
d $E = 144$ and $c = 6$

Hint: Square the c value before solving the equation.



9 If $V = \pi r^2 h$, find h (to one decimal place) when:
a $V = 160$ and $r = 3$
b $V = 400$ and $r = 5$
c $V = 1460$ and $r = 9$
d $V = 314$ and $r = 2.5$

Hint: For $160 = 9\pi h$, divide both sides by 9π to find h:
$h = \frac{160}{9\pi}$
Then evaluate on a calculator.



Problem-solving and reasoning

10–12 | 10, 12–14

10 The formula $F = \frac{9C}{5} + 32$ is used to convert temperature from degrees Celsius (°C) (which is used in Australia) to degrees Fahrenheit (°F) (which is used in the USA).
a When it is 30°C in Sydney, what is the temperature in Fahrenheit?
b How many degrees Celsius is 30° Fahrenheit? Answer to one decimal place.
c Water boils at 100°C. What is this temperature in degrees Fahrenheit?
d What is 0°F in degrees Celsius? Answer to one decimal place.

Hint: When finding C, you will have an equation to solve.

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11 The cost, in dollars, of a taxi is $C = 3 + 1.45d$, where d is the distance travelled, in kilometres.

a What is the cost of a 20 km trip?

b How many kilometres can be travelled for \$90?

12 $I = \frac{Prt}{100}$ calculates interest on an investment. Find:

a P when $I = 60$, $r = 8$ and $t = 1$

b t when $I = 125$, $r = 5$ and $P = 800$

c r when $I = 337.50$, $P = 1500$ and $t = 3$

13 The number of tablets a nurse must give a patient is found using the formula:

$$\text{Tablets} = \frac{\text{strength required}}{\text{tablet strength}}$$

a 750 milligrams of a drug must be administered to a patient. How many 500 milligram tablets should the nurse give the patient?

b If the nurse administers 2.5 of these tablets to another patient, how much of the drug did the patient take?

14 A drip is a way of pumping a liquid drug into a patient's blood. The flow rate of the pump, in millilitres per hour, is calculated using the formula: $\text{Rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$.

a A patient needs 300 mL of the drug administered over 4 hours. Calculate the rate, in mL/h, which needs to be delivered by the pump.

b A patient was administered 100 mL of the drug at a rate of 300 mL/h. How long was the pump running?

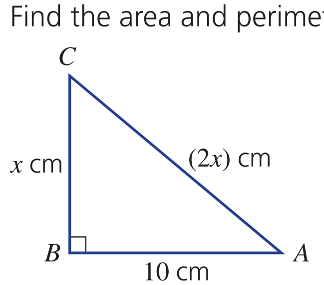
Calculation challenges — 15–17

15 A tax agent charges \$680 for an 8-hour day. The agent uses the formula $F = \frac{680x}{8}$ to calculate a fee to a client, in dollars.

a What does the x represent?

b If the fee charged to a client is \$637.50, how many hours, to one decimal place, did the agent spend working on the client's behalf?

16 Find the area and perimeter of triangle ABC, shown. Round to two decimal places.



Hint: Use Pythagoras' theorem to find x.

17 Iqra is 10 years older than Urek. In 3 years' time, she will be twice as old as Urek. How old are they now?

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8D Linear inequalities

Learning intentions

- To know the four inequality symbols and what they mean
- To be able to illustrate an inequality using a number line
- To know when to reverse an inequality sign

Key vocabulary: inequality, inequality sign, linear inequality

There are many situations in which a solution to the problem is best described using one of the symbols <, ≤, > or ≥. For example, a medical company will publish the lowest and highest amounts for a safe dose of a particular medicine; e.g. 20 mg/day ≤ dose ≤ 55 mg/day, meaning that the dose should be between 20 and 55 mg/day.

An inequality is a mathematical statement that uses an 'is less than' (<), an 'is less than or equal to' (≤), an 'is greater than' (>) or an 'is greater than or equal to' (≥) symbol. Inequalities may result in an infinite number of solutions. These can be illustrated using a number line.

You can solve inequalities in a similar way to solving equations.

Lesson starter: What does it mean for *x*?

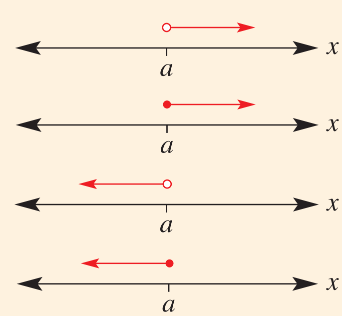
The following inequalities provide some information about the number x.

a $x < 6$ **b** $x \ge 4$ **c** $-5 \ge x$ **d** $-2 < x$

- Can you describe the possible values for x that satisfy each inequality?
- Test some values to check.
- How would you write the solution for x? Illustrate each on a number line.

Key ideas

- The four **inequality signs** are <, ≤, > and ≥.
  - $x > a$ means x is greater than a.
  - $x \ge a$ means x is greater than or equal to a.
  - $x < a$ means x is less than a.
  - $x \le a$ means x is less than or equal to a.



- On the number line, a closed circle (•) indicates that the number is included. An open circle (◦) indicates that the number is not included.
- Solving **linear inequalities** follows the same rules as solving linear equations, except:
  - We reverse an inequality sign if we multiply or divide by a negative number. For example, $-5 < -3$ and $5 > 3$, and if $-2x < 4$ then $x > -2$.
  - We reverse the inequality sign if the sides are switched. For example, if $2 \ge x$, then $x \le 2$.

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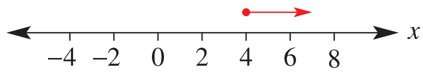
Exercise 8D

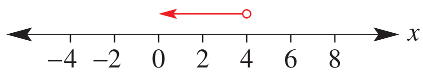
Understanding | 1, 2(½) | 2(½)

1 Match each inequality given with the correct number line.

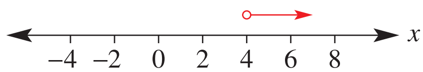
a $x > 4$

A 

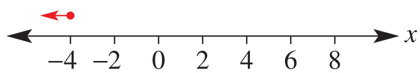
b $x < 4$

B 

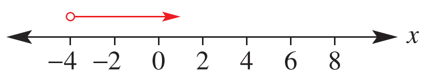
c $x \geq 4$

C 

d $x > -4$

D 

e $x \leq -4$

E 

Hint: Look back at the Key ideas. The direction of the arrowhead is the same as the direction of the inequality sign.

2 Match each inequality with the correct description.

a $x < 2$ **A** x is greater than 2

b $x \geq 2$ **B** x is less than or equal to 2

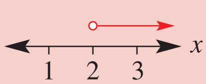
c $x \leq 2$ **C** x is less than 2

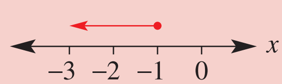
d $x > 2$ **D** x is greater than or equal to 2

Fluency | 3–5(½), 6, 7 | 4–5(½), 7, 8

Example 13 Writing inequalities from number lines

Write each number line as an inequality.

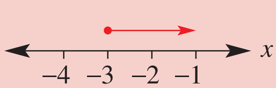
a 

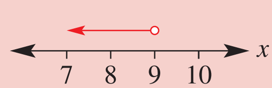
b 

Solution	Explanation
a $x > 2$	An open circle means 2 is not included.
b $x \leq -1$	A closed circle means -1 is included.

Now you try

Write each number line as an inequality.

a 

b 

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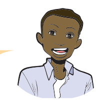
8D

3 Write each graph as an inequality.



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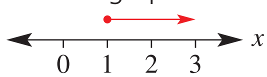

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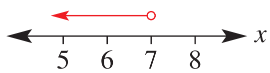

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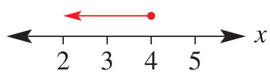

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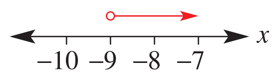

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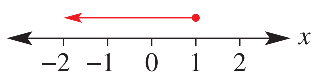

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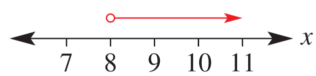

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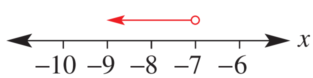

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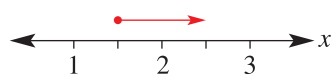

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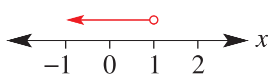

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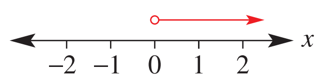
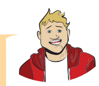
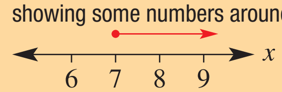


4 Show each of the following on separate number lines.

a $x \geq 7$	**b** $x > 1$	**c** $x < 1$
d $x \leq 1$	**e** $x \geq -1$	**f** $a \geq 0$
g $p \geq -2$	**h** $a > -15$	**i** $h < 5$

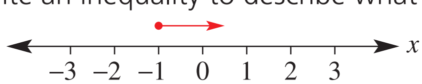


Hint: For $x \geq 7$, draw a number line showing some numbers around 7.



Use a closed circle (•) for ≥ and ≤.
Use an open circle (∘) for > and <.

5 Write an inequality to describe what is shown on each of the following number lines.

a


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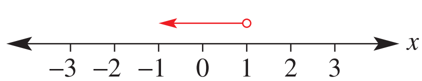

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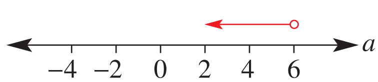

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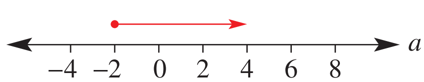

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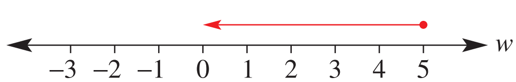

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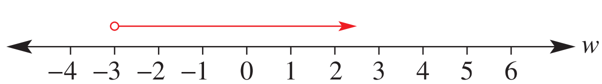

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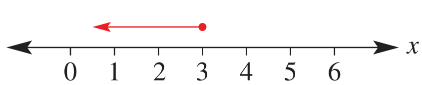

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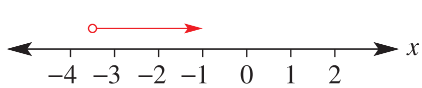

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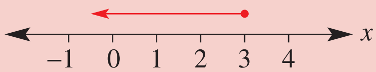
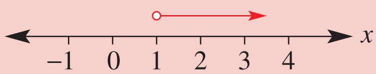
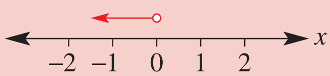
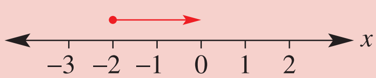
Example 14 Writing and graphing inequalities

Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 3
b x is greater than 1
c x is less than 0
d x is greater than or equal to -2

Solution	Explanation
a $x \leq 3$ 	Less than or equal to, ≤, closed circle
b $x > 1$ 	Greater than, >, open circle
c $x < 0$ 	Less than, <, open circle
d $x \geq -2$ 	Greater than or equal to, ≥, closed circle

Now you try

Write each of the following as an inequality and then show each solution on a number line.

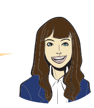
a x is greater than -4
b x is less than or equal to 6

6 Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 6
b x is greater than 4
c x is less than 2
d x is greater than or equal to 5

7 Write each of the following as an inequality, using the pronumeral n.

Hint: 'At least 50' means '50 or more'.



a The number of people who visit the Sydney Opera House each year is more than 100 000.

b The number of lollies in a bag should be at least 50.

c A factory worker must pack more than three boxes a minute.

d More than 100 penguins take part in the nightly parade on Phillip Island.

e The weight of a suitcase is 30 kg or less.

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8D

8 Write each of the following statements as an inequality and determine which of the numbers below make each inequality true.

$-6, -2, -\frac{1}{2}, 0, 2, 5, 7, 10, 15, 24$

Hint: Write the inequality, then list the given numbers that make it true.

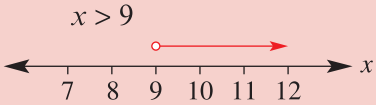
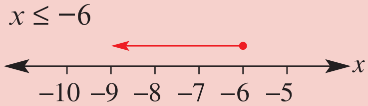
a x is less than zero
b x is greater than 10
c x is greater than or equal to 10
d x is less than or equal to zero
e x is greater than or equal to -1
f x is less than 10

Problem-solving and reasoning 9–12(½) 10–13(½)

Example 15 Solving and graphing inequalities

Solve the following and show your solution on a number line.

a $2x - 1 > 17$
b $\frac{x}{3} \le -2$

Solution	Explanation
a $2x - 1 > 17$	Add 1 to both sides.
$2x > 18$	Divide both sides by 2.
$x > 9$	
	> uses an open circle.
b $\frac{x}{3} \le -2$	Multiply both sides by 3.
$x \le -6$	
	≤ uses a closed circle.

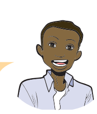
Now you try

Solve the following and show your solution on a number line.

a $3x + 2 \le 11$
b $\frac{x-1}{2} > -3$

9 Solve each of the following inequalities and show your solution on a number line.

a $2x > 10$
b $x + 2 < 7$
c $3x > 15$
d $\frac{x}{2} \ge 8$
e $x - 3 > 4$
f $x - 3 < 4$
g $p + 8 \le 0$
h $3a > 0$
i $x - 7 < 0$
j $2x \le 14$
k $5m > -15$
l $d - 3 > 2.4$
m $\frac{x}{7} \le 0.1$
n $\frac{1}{2}x \le 6$
o $5 + x > 9$



Hint: Keep the inequality sign the same when:
- adding or subtracting a number from both sides
- multiplying or dividing both sides by a positive number.

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10 Solve the following.

a $2 + 4a \le 10$ **b** $5 + 2y > 11$ **c** $3p - 1 > 14$

d $3x - 2 \ge 10$ **e** $3x - 2 < 1$ **f** $5 + 2w \ge 8$

g $5x + 5 < 10$ **h** $5x - 5 \ge 0$ **i** $10p - 2 < 8$

11 Give the solution set for each of the following.

a $\frac{x+2}{4} \le 1$ **b** $\frac{a-3}{2} \le -1$

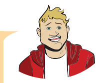
c $\frac{x}{4} - 1 \ge 6$ **d** $\frac{x}{3} + 7 > 2$

e $5 + \frac{x}{2} < 7$ **f** $\frac{x+2}{4} < 8$

g $\frac{2x-7}{3} > 4$ **h** $\frac{2x+1}{5} < 0$

i $\frac{3x}{2} + 1 \ge -3$ **j** $5x - 4 > 2 - x$

k $4(2x + 1) \ge 16$ **l** $3x + 7 < x - 2$



Hint: For $\frac{x+2}{4} \le 1$, first multiply both sides by 4.
For $\frac{x}{4} - 1 \ge 6$, first add 1 to both sides.

12 For each of the following, write an inequality and solve it to find the possible values of x.

a When a number, x, is multiplied by 3, the result is less than 9.

b When a number, x, is multiplied by 3 and the result divided by 4, it creates an answer less than 6.

c When a number, x, is doubled and then 15 is added, the result is greater than 20.

d Thuong is x years old and Gary is 4 years older. The sum of their ages is less than 24.

e Kaitlyn has x rides on the Ferris wheel at \$4 a ride and spends \$7 on food. The total amount she spends is less than or equal to \$27.

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8D

Example 16 Solving inequalities when the pronumeral has a negative coefficient

Solve $4 - x \geq 6$.

Solution	Explanation
$4 - x \geq 6$	Subtract 4 from both sides.
$-x \geq 2$	Divide both sides by -1.
$x \leq -2$	When we divide both sides by a *negative* number, the inequality sign is reversed.
Alternative solution:	
$4 - x \geq 6$	Add the x to both sides so that it is positive.
$4 \geq 6 + x$	Subtract 6 from both sides.
$-2 \geq x$	Switch the sides to have the x on the left-hand side.
$x \leq -2$	Reverse the inequality sign. Note that the inequality sign still 'points' to the x.

Now you try

Solve $5 - 2x < 17$.

13 Choose an *appropriate strategy* to solve the following.

a $5 - x < 6$
b $7 - x \geq 10$
c $-p \leq 7$
d $9 - a < -10$
e $-w \geq 6$
f $-3 - 2p < 9$
g $5 - 2x < 7$
h $-2 - 7a \geq 4$

Hint: Remember to reverse the inequality sign when multiplying or dividing by a negative number.
e.g.
$\times(-1)$ $-x < 7$ → $x > -7$ $\times(-1)$
$\div(-2)$ $-2x \geq 20$ → $x \leq -10$ $\div(-2)$

Investigating inequalities — 14

14 **a** Let us start with the numbers 4 and 6 and the true relationship $4 < 6$. Copy and complete the following table.

4 and 6	4	6	4 < 6	True or false? True
Add 3	$4 + 3$	$6 + 3$	$7 < 9$	True
Subtract 3	$4 - 3$		$1 < 3$	True
Multiply by 2				
Divide by 2				
Multiply by -2				False $(-8 > -12)$
Divide by -2	$4 \div (-2)$	$6 \div (-2)$		

b Copy and complete the following.
When solving an inequality, you can add or _______ a number from both sides and the inequality remains true. You can multiply or _______ by a _______ number and the _______ also remains true. However, if you _______ or _______ by a negative _______ the inequality sign must be reversed for the inequality to remain _______.

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8E Solving simultaneous equations graphically

Learning intentions

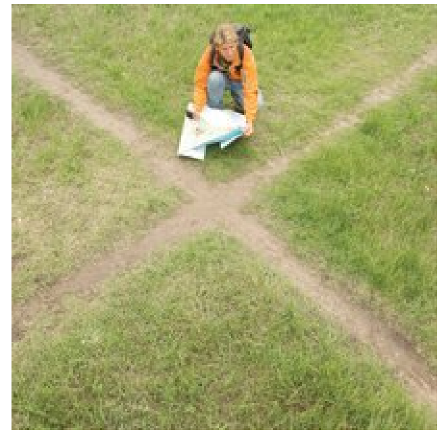
- To understand that an intersection point represents the solution to simultaneous linear equations
- To be able to locate an intersection point graphically
- To be able to interpret an intersection point as the solution to a real problem involving simultaneous equations

Key vocabulary: intersection point, coordinates, parallel, gradient, simultaneous

When we approach an intersection while driving, we near the shared position of two or more roads.

Like two roads, two straight lines in the same plane will always intersect unless they are parallel.

If we try to find the point of intersection, we are said to be solving the equations simultaneously.



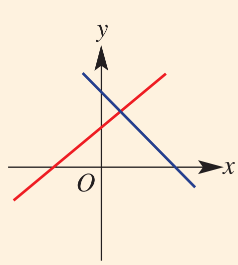
Lesson starter: Which job has better pay?

You start working as a delivery person for the Hasty Tasty Pizza Company. You're paid $\$25$ per shift and $\$4$ per pizza delivery. A second pizza company, More-2-Munch Pizzas, offers you a job at $\$15$ per shift and $\$5$ per pizza delivery.

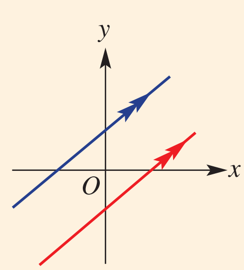
- How much does each company pay for delivery of 7 pizzas in one shift? How much does each company pay for delivery of 12 pizzas in one shift?
- For each pizza company, draw up a table to show the money you could earn for delivery of up to 15 pizzas delivered in one shift.
- On the same sheet, draw a graph of the information in your tables for each pizza company. Draw the graph for each pizza company on the same set of axes.
- What does the point of intersection show us?
- Write a sentence describing which job pays better for different numbers of pizzas delivered.
- Write down one advantage of using a graph to compare these two wages.

Key ideas

- At a point of **intersection**, two lines will have the same coordinates.
- The point of intersection represents the solution of two linear simultaneous equations.
- To find the point of intersection, sketch each straight line and read off the coordinates of where the lines meet.
- When two lines are **parallel**, they have the same gradient and there is no point of intersection.



1 point of intersection



0 points of intersection

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8E

Exercise 8E

Understanding

1–3 | 2, 3

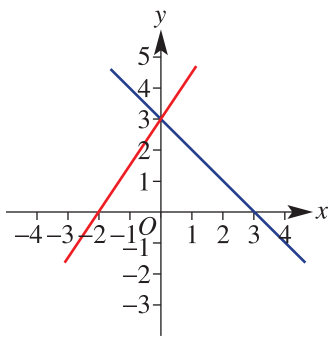
1 State the missing number.

a If two lines are parallel, then there are _______ points of intersection.

b If two lines are not parallel, then there is _______ point of intersection.

2 State the point of intersection (x, y) for the following lines, if there is one.

a



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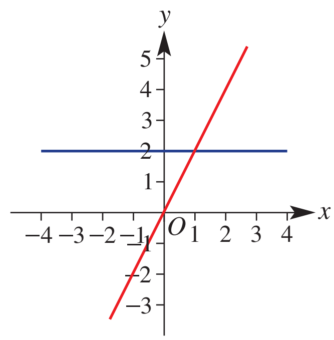

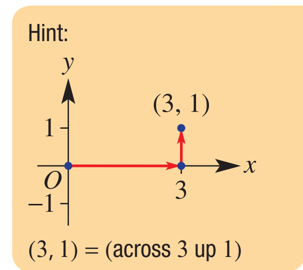

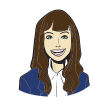

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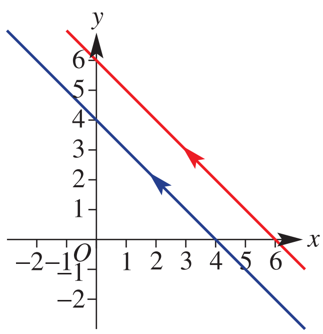

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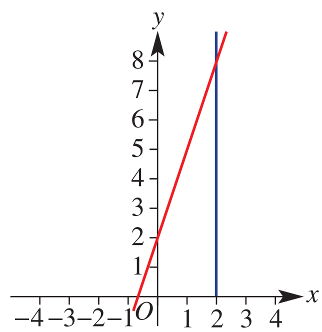

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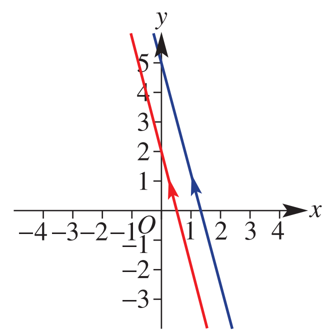

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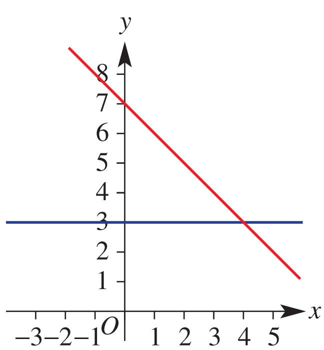

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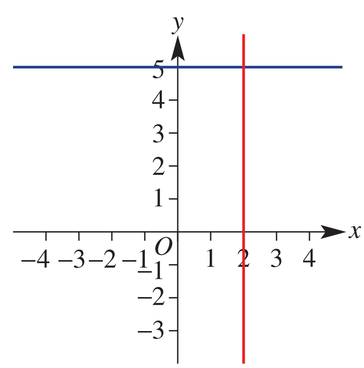

3 Use the method of trial and error (guess, check and refine) to find the point of intersection for these pairs of linear equations. Remember, your chosen point must satisfy both equations, i.e. substitute your x and y values into the equations to see if both are true.

a $y = 2x - 1$ and $y = 5 - x$

b $y = x - 3$ and $2x + y = -6$

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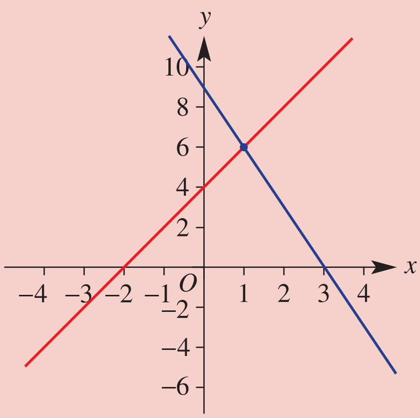
Fluency

4(½), 5, 6 | 4(½), 5–7

Example 17 Finding the point of intersection by graphing

Find the point of intersection (x, y) of $y = 2x + 4$ and $3x + y = 9$ by sketching accurate graphs on the same axes.

Solution	Explanation
$y = 2x + 4$	First, find the x- and y-intercepts of each graph.
y-intercept at $x = 0$: $y = 2(0) + 4 = 4$	Substitute $x = 0$.
x-intercept at $y = 0$: $0 = 2x + 4$	Substitute $y = 0$.
$2x = -4$	Subtract 4 from both sides.
$x = -2$	Divide both sides by 2.
$3x + y = 9$	
y-intercept at $x = 0$: $3(0) + y = 9$	Substitute $x = 0$.
$y = 9$	Simplify.
x-intercept at $y = 0$: $3x + (0) = 9$	Substitute $y = 0$.
$3x = 9$	Simplify.
$x = 3$	Divide both sides by 3.



Sketch the graphs using the x- and y-intercepts.
For $y = 2x + 4$, the x-intercept $= -2$ and the y-intercept $= 4$.
For $3x + y = 9$, the x-intercept $= 3$ and the y-intercept $= 9$.

The point of intersection is $(1, 6)$. — Read off the intersection point, listing x followed by y.

Now you try

Find the point of intersection (x, y) of $y = x - 2$ and $2x - y = 3$ by sketching accurate graphs on the same axes.

4 Find the point of intersection (x, y) of each pair of equations by plotting an accurate graph.
- a $y = x + 1$ and $3x + 2y = 12$
- b $y = 3x + 2$ and $2x + y = 12$
- c $y = 2x + 9$ and $3x + 2y = 18$
- d $y = x + 11$ and $4x + 3y = 12$

Hint:
y-intercept: $x = 0$
x-intercept: $y = 0$

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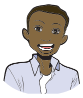
5 Find the point of intersection of each pair of equations by plotting an accurate graph.
 a $y = 3$ and $x = 2$
 b $y = -2$ and $x = 3$

Hint: $y = 3$ cuts the y-axis at 3 and is horizontal.
$x = 2$ cuts the x-axis at 2 and is vertical.



6 Find the point of intersection of each pair of equations by plotting an accurate graph.
 a $y = 3x$ and $y = 2x + 3$
 b $y = -3x$ and $y = 2x - 5$

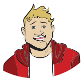
7 Find the point of intersection of each pair of equations by plotting an accurate graph.
 a $y = 2x - 6$ and $y = 3x - 7$
 b $y = -2x + 3$ and $y = 3x - 2$

Problem-solving and reasoning

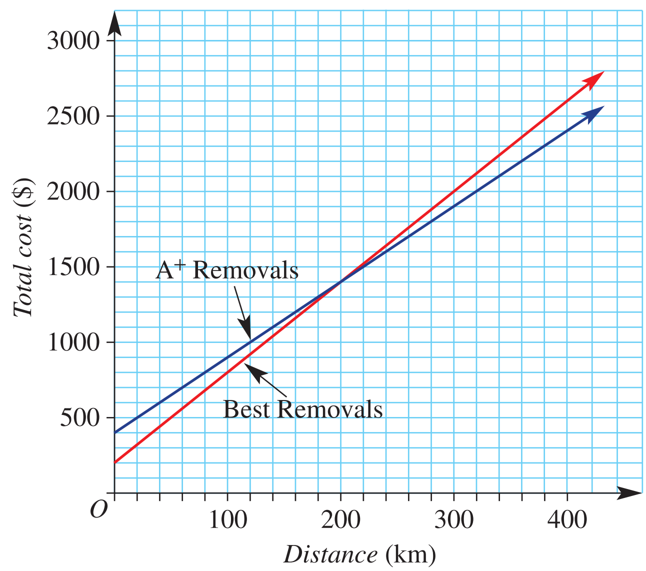
8, 9 | 9–11

8 This graph represents the cost of hiring two different removalist companies to move a person's belongings for various distances.
 a Determine the number of kilometres for which the total cost of the removalists is the same.

Hint: The cost is the same at the point of intersection.



 b What is the price when the total cost is equal?
 c If a person wanted to move 100 km, which company would be cheaper and by how much?
 d If a person wanted to move 400 km, which company would be cheaper and by how much?



9 The pay structures for baking companies A and B are given by the following.
Company A: $20 per hour
Company B: $45 plus $15 per hour
 a Complete two tables, showing the pay by each company for up to 12 hours.
 b Draw a graph of the pay by each company (on the vertical axis) versus time, in hours (on the horizontal axis). Draw the graphs for both companies on the same set of axes.
 c State the number of hours worked for which the earnings are the same for the two companies.
 d State the amount earned when the earnings are the same for the two companies.

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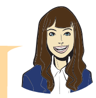
10 **a** Graph these three lines on the same coordinate axes by plotting the axis intercepts for each: $y = 3$, $y = x + 1$, $y = 1 - x$.
b Write the coordinates of the points of intersection.
c Find the length of each line segment formed between the intersection points.
d What type of triangle is formed by these line segments?

Hint: Use Pythagoras' theorem to find the length of a line segment.

11 The value of two cars is depreciating (i.e. decreasing) at a constant rate according to the information in this table.

Car	Initial value	Annual depreciation
Luxury sports coupe	\$70 000	\$5000
Family sedan	\$50 000	\$3000



Hint: Annual depreciation means how much the car's value goes down by each year.

a Complete two tables showing the value of each car every second year from zero to 12 years.
b Draw a graph of the value of each car (on the vertical axis) versus time, in years (on the horizontal axis). Draw both graphs on the same set of axes.
c From the graph, determine the time taken for the cars to have the same value.
d State the value of the cars when they have the same value.

Multiple intersections | — | 12, 13

Use a calculator to complete these questions.

12 On the same axes, plot the graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$.
a Are there any points of intersection?
b Suggest a reason for your answer to part **a**.
c Plot the graph of $y = 3x + 6$.
d Determine the points of intersection of the graphs already drawn and $y = 3x + 6$.

13 On the same axes, plot $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$ and $y = 4x - 1$.
a Are there any points of intersection?
b Suggest a reason for your answer to part **a**.
c Plot the graph of $y = 2x + 1$.
d Determine the points of intersection of the graphs already drawn and $y = 2x + 1$.

Using technology 8E: Finding intersections
This activity is available on the companion website as a printable PDF.

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Progress quiz

8A **1** Solve the following one-step equations.

a $x + 12 = 18$ **b** $y - 9 = 8$ **c** $3m = 21$ **d** $\frac{x}{5} = -2$

8A/B **2** Solve the following equations.

a $2x - 3 = 15$ **b** $1 - 2y = 9$ **c** $\frac{x}{3} + 4 = 7$

d $\frac{x-2}{4} = 1$ **e** $\frac{2y+2}{5} = 4$ **f** $2 - \frac{x}{5} = 3$

8B **3** Solve these equations with brackets.

a $3(x - 1) = 6$ **b** $5(2a + 3) = -45$

c $3(y + 4) + 2(2y - 3) = 27$ **d** $4(2x - 1) - 3(x - 2) = 22$

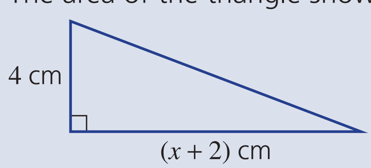
8A/B **4** Write equations for the following and solve for the pronumeral.

a When x is doubled and then 3 is added, the result is 25.

b When 3 is subtracted from x and the answer is divided by 2, the result is 6.

c When 2 is subtracted from x and the answer is multiplied by 3, the result is 12.

d The area of the triangle shown is 16 cm^2.



8B **5** Solve these equations with variables on both sides.

a $8x = 2x - 12$ **b** $9x + 4 = 4x + 14$ **c** $3x + 4 = 20 - x$

8C **6** Find the value of the unknown in these formulas by substituting the given values.

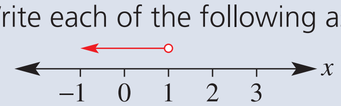
a For $A = \frac{1}{2}bh$, find b when $A = 28$ and $h = 14$.

b For $s = ut + \frac{1}{2}at^2$, find u when $a = 2$, $t = 3$ and $s = 24$.

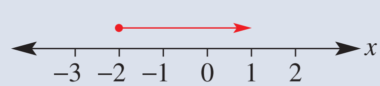
c For $S = 2\pi rh$, find h (to one decimal place) when $S = 30$ and $r = 2$.

8D **7** Write each of the following as inequalities.

a



b



8D **8** Solve the following inequalities and show the solution on a number line.

a $3x + 1 < 10$ **b** $\frac{x+1}{3} \ge 2$ **c** $3 - 2x < 13$

8E **9** Find the point of intersection of $x + y = 4$ and $y = 2x + 1$ by sketching accurate graphs on the same axes.

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8F Solving simultaneous equations using substitution ★

Learning intentions

- To understand that a solution to a pair of simultaneous equations can be found algebraically
- To be able to use the method of substitution to find a solution to a pair of simultaneous equations
- To be able to apply the method of substitution to solve simultaneous equations in a real context

Key vocabulary: substitute, subject, define

A pair of simultaneous equations is formed when there are two unknown quantities (i.e. variables) and two pieces of information relating these quantities. The solution to these simultaneous equations gives the variable values that make both equations true.

In the previous section, the solution was found from the point of intersection of two line graphs. In this section, you will learn how to find the solution using the algebraic method of *substitution*.

An example of two variables is the cost of a wedding reception and the number of invited guests. Two simultaneous equations could be made from two different catering companies. The solution will be the number of guests that make the costs equal for the two companies. Using equations helps to accurately compare the two deals.

Lesson starter: Equations and solutions

Albert is 11 years older than Jenny and the sum of their ages is 69. What are the ages of Albert and Jenny? Here are the steps to solve this problem but they are in the wrong order. Decide on the correct order.

A $x + (x + 11) = 69$
$2x + 11 = 69$
$2x = 58$
$x = 29$

B Let x = Jenny's age
Let y = Albert's age

C Jenny is 29 years old.
Albert is 40 years old.

D $x + y = 69$
$y = x + 11$

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Key ideas

- The algebraic method of **substitution** is generally used when at least one of the linear equations has x or y as the subject;

  e.g. $y = 3x + 4$ and $3x + y = 2$ or $y = -2x + 6$ and $y = -x - 1$ or $x = 2$ and $2x - y = 5$

- The method involves:
  - substituting one equation into the other
  - solving for the remaining variable
  - substituting to find the value of the second variable

- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two unknowns using pronumerals
  - Write down two equations using your pronumerals
  - Solve the equations using the method of substitution
  - Answer the original question in words

Exercise 8F

Understanding 1, 2 | 2

1 Write the missing words to complete each statement. Choose from *intersection*, *substituted*, *simultaneous* and *substitution*.

a __________ equations involve two equations and two variables.

b When two equations have been graphed, the x and y values that make both equations true are the coordinates of the point of __________.

c If x (or y) is replaced with a number, then we have __________ that number for x.

d If x (or y) is replaced with an algebraic expression, then we have __________ that expression for x (or y).

e When we algebraically substitute one equation into another, this is called solving simultaneous equations by the method of __________.

Hint: In part a, replace y in $2x + y = 6$ with $x + 1$.

2 Choose the correct option.

a When $y = x - 1$ is substituted into $2x + y = 6$, the result is:

A $2x + (x - 1) = 6$
B $2x - 1 = 6$
C $x - 1 = 6$
D $2x - x + 1 = 6$
E $3x = 6$

b When $y = 2x + 3$ is substituted into $x - 3y = 1$, the result is:

A $x + 3(2x + 3) = 1$
B $3(2x + 3) = 1$
C $x - 3(2x + 3) = 1$
D $x - (2x + 3) = 1$
E $2x - 3 = 1$

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Fluency

3–4(½), 6 | 3–4(½), 5–7

Example 18 Using the substitution method to solve simultaneous equations

Determine the point of intersection of $y = 5x$ and $y = 2x + 6$.

Solution	Explanation
$y = 5x$ [1] $y = 2x + 6$ [2]	Label the two equations.
Substitute [1] into [2]:	Explain how you are substituting the equations.
$5x = 2x + 6$	Replace y in the second equation with $5x$.
$3x = 6$	Subtract $2x$ from both sides.
$x = 2$	Divide both sides by 3.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$y = 5(2)$	Replace x with the number 2.
$y = 10$	Simplify.
The point of intersection is $(2, 10)$.	Write the solution.
Check: $10 = 2(2) + 6$	Substitute your solution into the other equation to check.

Now you try

Determine the point of intersection of $y = 7x$ and $y = 2x + 5$.

3 Determine the point of intersection for the following pairs of lines.

a $y = 5x$
$y = 3x + 4$

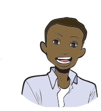
b $y = 3x$
$y = 2x - 5$

c $y = 2x$
$y = 4x + 8$

d $y = 4x$
$y = -3x + 7$

e $y = x$
$y = -5x + 12$

f $y = 6x$
$y = -2x + 16$



Hint:
$y = 5x$
$y = 3x + 4$
$5x = 3x + 4$

Example 19 Solving simultaneous equations with the substitution method

Solve the simultaneous equations $y = x + 3$ and $2x + 3y = 19$ using the substitution method; i.e. find the point of intersection.

Solution	Explanation
$y = x + 3$ [1] $2x + 3y = 19$ [2]	Label the two equations.

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ISBN 978-1-108-87859-3

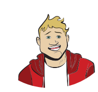
Substitute [1] into [2]:	Explain how you are substituting the equations.
$2x + 3(x + 3) = 19$	Replace y in the second equation with $(x + 3)$.
$2x + 3x + 9 = 19$	Expand the brackets.
$5x + 9 = 19$	Simplify.
$5x = 10$	Subtract 9 from both sides.
$x = 2$	Divide both sides by 5.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation (2).
$y = 2 + 3$	Replace x with the number 2.
$y = 5$	Simplify.
The point of intersection is (2, 5).	Write the solution.
Check: $2(2) + 3(5) = 19$	Substitute your solution into the other equation to check.

Now you try

Solve the simultaneous equations $y = x - 1$ and $3x + 2y = -12$ using the substitution method; i.e. find the point of intersection.

4 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = x + 3$ and $2x + 3y = 19$
b $y = x + 2$ and $3x + y = 6$
c $y = x - 1$ and $3x + 2y = 8$
d $y = x - 1$ and $3x + 5y = 27$
e $y = x + 2$ and $2x + 3y = -19$
f $y = x + 5$ and $5x - y = -1$
g $y = x - 3$ and $5x - 2y = 18$
h $y = x - 4$ and $3x - y = 2$



Hint: In part **a**, replace y in the second equation with $(x + 3)$. It is important to use brackets.
$y = x + 3$
$2x + 3y = 19$
Remember that
$3(x + 3) = 3x + 9$

5 Solve the following pairs of simultaneous equations; i.e. find the point of intersection.

a $y = 2$
$y = 2x + 4$

b $y = -1$
$y = 2x - 7$

c $y = 4$
$2x + 3y = 20$

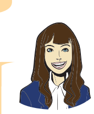
Hint: Replace y in the second equation with 2.
$y = 2$
$y = 2x + 4$

6 Determine the point of intersection for the following.

a $x = 2$
$3x + 2y = 14$

b $x = -3$
$y = -2x - 4$

c $x = 7$
$4x - 3y = 31$



Hint: Replace x in the second equation with 2.
$x = 2$
$3x + 2y = 14$
Remember that $3x$ means $3 \times x$.

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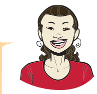
7 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 2x + 3$
$11x - 5y = -14$

b $y = 3x - 2$
$7x - 2y = 8$

c $y = 3x - 5$
$3x + 5y = 11$

d $y = 4x + 1$
$2x - 3y = -23$



Hint: Be careful with signs when expanding brackets.
$-5 \times (+3) = -15$
$11x - 5(2x + 3)$
$= 11x - 10x - 15$
When multiplying numbers with different signs, the answer is negative.

Problem-solving and reasoning

8, 9	8, 10, 11

Example 20 Solving word problems with simultaneous equations (substitution)

Jade is 5 years older than Marian. If their combined age is 33, find their ages.

Solution	**Explanation**
Let j be Jade's age and m be Marian's age.	Define two pronumerals using words.
$j = m + 5$ [1]	The first piece of information is that Jade is 5 years older than Marian.
$j + m = 33$ [2]	The second piece of information is that their combined age is 33.
$(m + 5) + m = 33$	Substitute $m + 5$ for j in the second equation.
$2m + 5 = 33$	Collect any like terms, so $m + m = 2m$.
$2m = 28$	Subtract 5 from both sides.
$m = 14$	Divide both sides by 2.
$j = m + 5$ [1] $j = 14 + 5$ $j = 19$	Use the first equation, $j = m + 5$, to find j.
Jade is 19 years old and Marian is 14 years old.	Answer the original question in words.

Now you try

A rectangle's length is 3 cm more than its width. If its perimeter is 32 cm, determine its dimensions.

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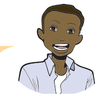
8 Kye is 5 years older than Viviana. If their combined age is 81, determine their ages.

Hint: First define a pronumeral for Kye's age and another pronumeral for Viviana's age. Then write two equations before solving.



9 The length of a rectangle is three times the width. If the perimeter of the rectangle is 48 cm, determine its dimensions.

Hint: Draw a diagram to help form the perimeter equation.

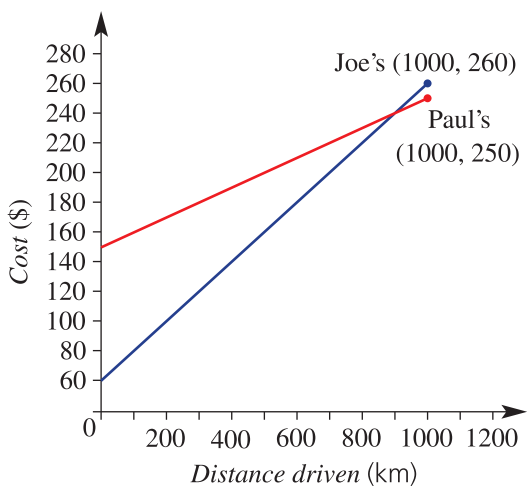
10 A vanilla thick shake is $2 more than a fruity twirl. If three vanilla thick shakes and five fruity twirls cost $30, determine their individual prices.

Hint: If a fruity twirl costs $\$x$, then 5 will cost $\$5x$.

11 Carlos is 3 more than twice Ella's age. If the sum of their ages is 54 years, determine their ages.

Rentals — 12

12 The given graph represents the rental cost of a new car from two car rental firms called Paul's Motor Mart and Joe's Car Rental.



a Determine:
- **i** the initial rental cost from each company
- **ii** the cost per kilometre when renting from each company
- **iii** the linear equations for the total cost from each company
- **iv** the number of kilometres at which the total cost is the same for both rental firms, using the method of substitution.

b Describe when you would use Joe's or Paul's rental firm.

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8G Solving simultaneous equations using elimination ★

Learning intentions

- To be able to use the method of elimination to find a solution to a pair of simultaneous equations
- To be able to apply the method of elimination to solve simultaneous equations in a real context

Key vocabulary: elimination, simultaneous, multiple

A second method for solving simultaneous equations, called elimination, can sometimes be more efficient, depending on how the equations are structured in the first place.

When setting up equations for real situations, we should define the unknown quantities using pronumerals. When solving simultaneous linear equations there should be only two unknown quantities for two equations formed from the given information.

For example, two related variables are the cost of owning a car and the number of kilometres driven. For two different cars, two equations could be made relating these variables. The simultaneous solution gives the number of kilometres that makes the total running costs of each car equal. Solving simultaneous equations provides information for an accurate comparison of costs between two vehicles.

Lesson starter: Eliminating a variable

One step in the elimination method involves adding or subtracting two equations in order to eliminate one of the variables. When adding, we write [1] + [2]; when subtracting, we write [1] − [2].

- A student has either added or subtracted pairs of equations, but has many incorrect answers.
- Determine which answers are incorrect and write the correct answer for these. (Note: Do not solve the equations for x or y.)

A $5x + 3y = 34$ [1]
$7x - 3y = 26$ [2]
[1] + [2] gives:
$12x + 0 = 60$

B $3x + 2y = 18$ [1]
$2x - 2y = 2$ [2]
[1] + [2] gives:
$5x - 4y = 20$

C $3x - 3y = 9$ [1]
$2x - 3y = 4$ [2]
[1] − [2] gives:
$5x + 0 = 5$

D $2x - 2y = 8$ [1]
$4x - 2y = 24$ [2]
[1] − [2] gives:
$2x - 4y = 16$

E $4x + 3y = 16$ [1]
$-4x + 2y = 3$ [2]
[1] + [2] gives:
$0 + y = 19$

F $3x + 2y = 25$ [1]
$2x + 2y = 18$ [2]
[1] − [2] gives:
$x + 0 = 43$

G $5x + 3y = 31$ [1]
$5x - 3y = 19$ [2]
[1] + [2] gives:
$0 + 0 = 12$

H $x + 3y = 15$ [1]
$x + 2y = 12$ [2]
[1] − [2] gives:
$2x + y = 3$

I $7x - y = 5$ [1]
$3x - y = -2$ [2]
[1] − [2] gives:
$4x = 3$

Key ideas

- **Elimination** is generally used to solve simultaneous equations when both equations are in the form $ax + by = d$.
e.g. $2x - y = 6$ and $3x + y = 10$ or $-5x + y = -2$ and $6x + 3y = 5$

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8G

- Adding or subtracting multiples of these two equations allows one of the variables to be eliminated.
  - Add $x - y = 10$ and $3x + y = 34$ to eliminate y.
  - Subtract $5x + 2y = 7$ and $5x + y = 6$ to eliminate the x.
  - Form a matching pair by multiplying by a chosen factor.
    For example, $2x - y = 3 \quad \Rightarrow \times 2 \Rightarrow \quad 4x - 2y = 6$
    $5x + 2y = 12 \quad \Rightarrow \quad 5x + 2y = 12$
- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two variables using letters.
  - Write down two equations using your variables.
  - Solve the equations using the method of elimination.
  - Answer the original question in words.

Exercise 8G

Understanding — 1–3 | 2, 3

1 What operation (i.e. + or −) will make these equations true?
a $2x$ _____ $2x = 0$ **b** $-3y$ _____ $3y = 0$ **c** $4x$ _____ $(-4x) = 0$

2 Multiply both sides of the equation $3x - 2y = -1$ by the following numbers. Write the new equations.
a 2 **b** 3 **c** 4

3 Choose the correct option.
a When $2x + y = 3$ is added to $5x - y = 11$ the result is:
A $7x = 11$ **B** $7x = 14$ **C** $3x = 14$ **D** $3x = 1$ **E** $7x = 8$
b When $x + y = 5$ is subtracted from $3x + y = 7$ the result is:
A $2x = 7$ **B** $4x = 2$ **C** $4x = 12$ **D** $x = 2$ **E** $2x = 2$

Fluency — 4, 5, 6–9(½) | 6–11(½)

Example 21 Eliminating a variable by addition of equations then solving

Add equation [1] to equation [2] and then solve for x and y.

$x + 2y = 10 \quad [1]$
$x - 2y = 2 \quad [2]$

Solution	Explanation
$x + 2y = 10$ [1] $x - 2y = 2$ [2]	Copy equations with the labels [1] and [2].
[1] + [2] gives:	Write the instruction to add: [1] + [2].
$2x + 0 = 12$	Add the x column: $x + x = 2x$.
$2x = 12$	Add the y column: $2y + (-2y) = 0$.
$x = 6$	Add the RHS: $10 + 2 = 12$ and then divide both sides by 2.

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Substitute $x = 6$ into [1]:	In equation [1], replace x with 6. Equation [2] could also have been used.
$6 + 2y = 10$	
$2y = 4$	Subtract 6 from both sides.
$y = 2$	Divide both sides by 2.
Solution is $(6, 2)$.	Write the solution as an ordered pair.
Check: [2] $6 - 2 \times 2 = 2$, true	Check that the solution satisfies equation [2].

Now you try

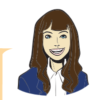
Add equation [1] to equation [2] then solve for x and y.

$3x + y = 11$ [1]

$x - y = 5$ [2]

4 Copy each pair of equations, add equation [1] to [2], then solve for x and y.

- **a** $x + y = 7$ [1]
  $x - y = 5$ [2]
  [1] + [2]
- **b** $x + 2y = 11$ [1]
  $x - 2y = -5$ [2]
  [1] + [2]
- **c** $3x + 2y = 20$ [1]
  $-3x + y = 1$ [2]
  [1] + [2]



Hint:
Adding equations:
[1] + [2]
$x + y = 7$ [1]
$x - y = 5$ [2]
$2x + 0 = 12$
Remember that
$+y + (-y) = +y - y = 0$

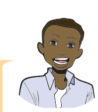
5 Copy each pair of equations, subtract equation [2] from equation [1] and then solve for x and y, showing all steps.

- **a** $2x + y = 16$ [1]
  $x + y = 9$ [2]
  [1] − [2]
- **b** $3x + 5y = 49$ [1]
  $3x + 2y = 25$ [2]
  [1] − [2]
- **c** $5x - 4y = 16$ [1]
  $2x - 4y = 4$ [2]
  [1] − [2]

Hint:
Subtracting equations:
[1] − [2]
$5x - 2y = 16$ [1]
$2x - 2y = 4$ [2]
$3x + 0 = 12$
Remember that
$-2y - (-2y)$
$= -2y + 2y = 0$

6 Determine the point of intersection of the following lines, using the elimination method.

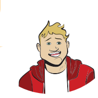
- **a** $x + y = 7$ and $5x - y = 5$
- **b** $x + y = 5$ and $3x - y = 3$
- **c** $x - y = 2$ and $2x + y = 10$
- **d** $x - y = 0$ and $4x + y = 10$



Hint: Label the two equations, one under the other, and decide whether to eliminate x or y; i.e. eliminate whichever variable has the same number in each equation.
Remember that $+y + (-y) = 0$.
The point of intersection is the same as the simultaneous solution of the equations.

7 Solve the following pairs of simultaneous equations, using the elimination method. You will need to subtract the equations to eliminate one of the variables.

- **a** $3x + 4y = 7$
  $2x + 4y = 6$
- **b** $4x + 3y = 11$
  $x + 3y = 5$
- **c** $2x + 3y = 1$
  $2x + 5y = -1$



Hint: Always label the equations and write the instruction; e.g. [1] − [2] or [2] − [1].

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8G

Example 22 Forming a matching pair

Determine the point of intersection of the lines $x + y = 6$ and $3x + 2y = 14$, using the elimination method.

Solution	Explanation
$x + y = 6$ [1] $3x + 2y = 14$ [2]	Label the two equations and decide where to form a matching pair.
$[1] \times 2$: $2x + 2y = 12$ [3] $[2]$: $3x + 2y = 14$ [2]	
$[2] - [3]$: $x = 2$	Subtract the two equations because $2y - 2y = 0$, $3x - 2x = x$ and $14 - 12 = 2$
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$2 + y = 6$	Replace x with the number 2.
$y = 4$	Subtract 2 from both sides.
Point of intersection is $(2, 4)$.	Write the solution as an ordered pair.
Check: $3(2) + 2(4) = 14$	Check that the solution satisfies the other equation.

Now you try

Determine the point of intersection of the lines $x + y = 4$ and $5x + 2y = 11$, using the elimination method.

8 Solve these simultaneous equations by first forming a matching pair.

a $x - 3y = 1$
$2x + y = 9$

b $4x + 2y = 10$
$x + 3y = 10$

c $3x + 4y = 19$
$x - 3y = 2$

Hint: Multiply one equation by a number to form a matching pair.

Example 23 Forming a matching pair by multiplying both equations

Solve the simultaneous equations $3x + 2y = 6$ and $5x + 3y = 11$, using the elimination method.

Solution	Explanation
$3x + 2y = 6$ [1] $5x + 3y = 11$ [2]	Label the two equations and decide whether to eliminate x or y.
$[1] \times 3$: $9x + 6y = 18$ [3] $[2] \times 2$: $10x + 6y = 22$ [4]	Multiplying the first equation by 3 and the second by 2 results in a matching pair $6y$ in each equation.
$[4] - [3]$: $x = 4$	Subtract the equations since $6y - 6y = 0$.

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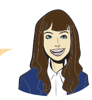
Substitute $x = 4$ into [1]:	Alternatively, substitute into equation [2].
$3(4) + 2y = 6$	Replace x with the number 4.
$2y = -6$	Subtract 12 from both sides, since $3 \times 4 = 12$.
$y = -3$	Divide both sides by 2.
Solution is $(4, -3)$.	Write the solution as an ordered pair.
Check: $5(4) + 3(-3) = 11$	Check the solution with the other equation.

Now you try

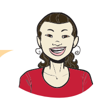
Solve the simultaneous equations $5x + 3y = 1$ and $2x + 5y = 8$, using the elimination method.

9 Solve the following pairs of simultaneous equations, using the elimination method.
- **a** $3x + 2y = 6$ and $5x + 3y = 11$
- **b** $3x + 2y = 5$ and $2x + 3y = 5$
- **c** $2x + y = 4$ and $5x + 2y = 10$
- **d** $2x + 5y = 7$ and $x + 3y = 4$



Hint: When multiplying an equation by a number, multiply every term on the LHS and RHS by that number. Always write the instruction for multiplying; e.g. $[2] \times 4$.

10 Solve the following pairs of simultaneous equations, using the elimination method.
- **a** $3x + 5y = 8$
  $x - 2y = -1$
- **b** $2x + y = 10$
  $3x - 2y = 8$
- **c** $4x - 3y = 0$
  $3x + 4y = 25$



Hint: Choose to eliminate x or y. The coefficients need to be the same size (with + or −); e.g. $-4x$ and $4x$ or $-5y$ and $5y$. Choose to add or subtract the equations to eliminate one variable.

11 Solve the following pairs of simultaneous equations.
- **a** $5x + 3y = 18$ and $3y - x = 0$
- **b** $3x - y = 13$ and $x + y = -9$
- **c** $2x + 7y = -25$ and $5x + 7y = -31$
- **d** $2x + 6y = 6$ and $3x - 2y = -2$
- **e** $4x - 5y = -14$ and $7x + y = -5$
- **f** $7x - 3y = 41$ and $3x - y = 17$

Problem-solving and reasoning 12–15 | 12, 13, 16–18

Example 24 Solving word problems with simultaneous equations (elimination)

Kathy is older than Blake. The sum of their ages is 17 years and the difference is 5 years. Find Kathy and Blake's ages.

Solution	Explanation
Let k be Kathy's age and b be Blake's age.	Define two variables.
$k + b = 17$ [1]	The first piece of information is 'the sum of their ages is 17'.
$k - b = 5$ [2]	The second is 'the difference is 5 and Kathy is older than Blake'.

Continued on next page

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[1] + [2]: $2k = 22$ $k = 11$	Add the two equations to eliminate b or, alternatively, subtract to eliminate k.
Substitute $k = 11$ into [1]: $11 + b = 17$	Alternatively, substitute into [2].
$b = 6$	Subtract 11 from both sides.
Kathy is 11 years old and Blake is 6.	Answer the original question in words.

Now you try

The sum of two numbers is 97 and their difference is 13. Find the two numbers.

12 Ayden is older than Tamara. The sum of their ages is 56 years and the difference is 16 years. Use simultaneous equations to find Ayden and Tamara's ages.

Example 25 Problem solving with simultaneous equations

Reese purchases three daffodils and five petunias from the local nursery and the cost is \$25.
Giuliana buys four daffodils and three petunias and the cost is \$26.
Determine the cost of each type of flower.

Solution	Explanation
Let \$$d$ be the cost of a daffodil and \$$p$ be the cost of a petunia.	Define your variables.
$3d + 5p = 25$ [1]	Three daffodils and five petunias from the local nursery cost \$25.
$4d + 3p = 26$ [2]	Four daffodils and three petunias cost \$26.
[1] × 4: $12d + 20p = 100$ [3] [2] × 3: $12d + 9p = 78$ [4]	Multiply [1] by 4 and [2] by 3 to obtain a matching pair ($12d$ and $12d$).
[3] − [4]: $11p = 22$	Subtract the equations to eliminate d.
$p = 2$	Divide both sides by 2.
Substitute $p = 2$ into [1]:	Alternatively, substitute into [2].
$3d + 5(2) = 25$	Replace p with the number 2.
$3d + 10 = 25$	Simplify.
$3d = 15$	Subtract 10 from both sides.
$d = 5$	Divide both sides by 3.
Check: $4(5) + 3(2) = 26$	Check your solutions by substituting into the second equation.
Daffodils cost \$5 and petunias cost \$2 each.	Answer the question in words.

Now you try

Jess purchases 4 buckets of chips and 3 drinks for \$23.50, while Nigel purchases 3 buckets of chips and 4 drinks for \$22. Find the price of a bucket of chips and a drink.

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13 A market stall sells two fruit packs:
Pack 1: 10 apples and 5 mangoes ($12.50)
Pack 2: 15 apples and 4 mangoes ($13.50)

a Define two pronumerals and set up a pair of linear equations to eventually find the cost of each fruit.

b Solve the two simultaneous equations to determine the individual prices of each piece of fruit.

c Determine the cost of one apple and five mangoes.

14 Tickets to a basketball game cost $3 for children and $7 for adults. If 5000 people attended the game and the total takings at the door was $25 000, determine the number of children and adults who attended the game.

Hint: What you are being asked to find is often how you define your variables.

15 A Maths test contains multiple-choice questions worth 2 marks each and short-answer questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.

Hint: Total marks is 50. Number of questions is 22.

a Define two pronumerals to represent the number of each question type.

b Set up two linear equations.

c Solve the two equations simultaneously to determine the number of multiple-choice questions.

16 Let x and y be two numbers that satisfy the following statements. Set up two linear equations according to the information and solve them simultaneously to determine the numbers in each case.

a Their sum is 16 but their difference is 2.

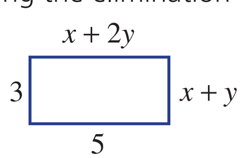
b Their sum is 30 but their difference is 10.

c Twice the larger number plus the smaller is 12 and their sum is 7.

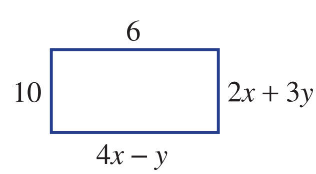
17 Find the value of x and y in the following rectangles. You will need to write two equations and solve using the elimination method.

Hint: Opposite sides of rectangles are equal.

a



b



18 Gordon is currently 31 years older than his daughter. In 30 years' time he will be twice his daughter's age. Using g for Gordon's current age and d for Gordon's daughter's current age, complete the following.

a Write down expressions for:

i Gordon's age in 30 years' time **ii** Gordon's daughter's age in 30 years' time

b Write down two linear equations, using the information at the start.

c Solve the equations to find the current ages of Gordon and his daughter.

Using technology — 19

19 Use technology to solve these simultaneous equations.

a $3x + 2y = 6$ and $5x + 3y = 11$

b $3x + 2y = 5$ and $2x + 3y = 5$

c $4x - 3y = 0$ and $3x + 4y = 25$

d $2x + 3y = 10$ and $3x - 4y = -2$

e $-2y - 4x = 0$ and $3y + 2x = -2$

f $-7x + 3y = 22$ and $3x - 6y = -11$

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Maths@Work: Nurse

Nursing is a career that is both challenging and rewarding. It requires a person to be caring and empathetic. Good communication skills, as well as an understanding of mathematics and science, are also important.

Nurses need to be competent in many mathematical areas, including fractions, ratios, converting units of measurement and equations. They must be able to calculate medical dosages, substitute into equations and also know how to program the correct flow rate for intravenous (IV) drips.

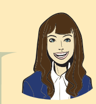
Complete these questions that are typical of a nurse's job administering medication.

1 Use these formulas to answer each of the following questions.

$$\text{Volume required} = \frac{\text{strength needed}}{\text{strength in stock}} \times \text{volume of stock solution}$$

$$\text{Number of tablets} = \frac{\text{strength required}}{\text{strength per tablet in stock}}$$

Hint: For 100 mg in 2 mL, need 75 mg in ? mL.



- **a** What volume, in mL, of Pethidine should be given if the patient is prescribed 75 mg and the existing stock contains 100 mg in 2 mL?
- **b** Calculate the volume, in mL, of insulin that is required for a patient who has been prescribed 60 units of the drug, if the stock is 100 units/1 mL.
- **c** Pethidine 50 mg has been ordered to alleviate a patient's pain. The stock strength is 75 mg/1.5 mL. How much Pethidine should be given?
- **d** How many tablets does a nurse need to give for a prescription of 500 mg of amoxicillin per day, if the stock available in the ward is 250 mg per capsule?
- **e** How many tablets are needed for a dosage of 125 mg, if the stock available is labelled 25 mg per tablet?

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2 Paediatrics is a branch of medicine dealing with young children. Different formulas are used to calculate the doses suitable for children. Apply the rules given below to complete the following calculations and state each answer to the nearest mg.

Clarke's body weight rule:

$$\text{Child's dose} = \frac{\text{weight of child (kg)}}{\text{average adult weight (70 kg)}} \times \text{adult dose}$$

Clarke's body surface area rule:

$$\text{Child's dose} = \frac{\text{surface area of child (m}^2\text{)}}{\text{average adult surface area (1.7 m}^2\text{)}} \times \text{adult dose}$$

Fried's rule (used for infants under 1 year old):

$$\text{Child's dose} = \frac{\text{age in months}}{150} \times \text{adult dose}$$

Young's rule (used for children aged 2 to 12 years):

$$\text{Child's dose} = \frac{\text{age in years}}{\text{age} + 12} \times \text{adult dose}$$

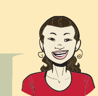
a Use Young's rule to calculate the Amoxil dose needed for a 10-year-old boy, if the adult dose of the drug Amoxil is 250 mg.

b Use Clarke's body weight rule to find a child's dose for the drug Ampicillin, given the child's weight is 15 kg and an adult's dose is 500 mg.

c Use Fried's rule to calculate the dose required for an 8-month-old baby girl for the drug amoxicillan, given that an adult's dose is 500 mg.

d Use Clarke's body surface area formula to find the dose of penicillin, in mg, required for a child whose surface area is 8000 cm^2, given that the adult dose is 1 gram.

Hint: Recall 1 m^2 = 10 000 cm^2



3 Drugs that are given with an intravenous (IV) drip use a different set of equations to calculate the time needed or the drop rate per minute.

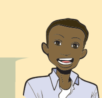
$$\text{Time (in minutes)} = \frac{\text{volume (mL)}}{\text{flow rate (drops/min)}} \times \text{drip factor}$$

$$\text{Flow rate (drops/minute)} = \frac{\text{volume (mL)}}{\text{time (mins)}} \times \text{drip factor}$$

Use the equations above to answer these questions about IV drug dosage. The drip factor is in drops/mL. State all answers rounded to one decimal place.

a Find the flow rate, in drops per minute, when a $\frac{1}{2}$ litre bag of saline solution is run over 4 hours with the IV machine set at a drip factor of 20 drops per mL.

Hint: Recall 1 L = 1000 mL



b An IV drip of saline solution started at 4:15 p.m. Tuesday. The machine has 700 mL to run and is set at 40 drops/min with a drip factor of 18 drops per mL. At what time will the IV be finished?

c How long will it take 180 mL of zero negative blood to flow through an IV at 36 drops/minute when the blood supply machine is set at a drip factor of 15 drops/mL?

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Using technology

4 Set up this Excel worksheet to calculate the ending times of intravenous drips for various patients. You will need to copy the given data and enter formulas into the shaded cells.

	A	B	C	D	E	F	G
1	**Intravenous drip administration**						
2	**IV bag**	**IV machine settings**		**Times**			
3	Total volume in mL	Flow rate in drops per minute	Drip factor drops per mL	Starting time	Time in minutes for IV	Time in hours and minutes for IV	Ending time
4	700	40	18	4:15 PM			
5	500	36	16	10:00 AM			
6	800	30	24	1:25 PM			
7	1500	40	20	6:00 PM			
8	2000	45	22	2:00 AM			

Hint:
- Format starting and ending times as *Number Category: Custom, Type: h:mm AM/PM*.
- To calculate time in hours and minutes, divide the time in minutes by the number of minutes in 24 hours, and format cells as *Number Category: Custom and Type: h:mm*.

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Puzzles and games

1 The answers to these equations will form a magic square, where each row, column and diagonal will add to the same number. Draw a 4 by 4 square for your answers and check that they do make a magic square.

$x - 3 = 6$	$x + 15 = 10$	$\frac{x}{2} = -2$	$5x = 30$
$3x + 7 = 1$	$\frac{x}{4} - 8 = -7$	$\frac{x + 7}{2} = 5$	$3(x + 4) = x + 14$
$\frac{x}{2} - 5 = -4$	$4x - 9 = -9$	$x + 7 = 4x + 10$	$2(3x - 12) - 5 = 1$
$\frac{9 - 3x}{3} = 6$	$-2(3 - x) = x + 1$	$x - 16 = -x$	$5x + 30 - 3x = -3x$

2 Write an equation and solve it to help you find each unknown number in these puzzles.

a Three-quarters of a number plus 16 is equal to 64.

b A number is increased by 6, then that answer is doubled and the result is four more than triple the number.

c The average of a number and its triple is equal to 58.6.

d In 4 years' time, Ahmed's age will be double the age he was 7 years ago. How old is Ahmed now?

3 By applying at least two operations to x, write three different equations so that each equation has the solution $x = -2$. Verify that $x = -2$ makes each equation true.
For example, $3 \times (-2) + 10 = 4$, so one possible equation would be $3x + 10 = 4$.

4 Write two sets of simultaneous equations so that each pair has the solution $(3, -2)$.

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Puzzles and games

5 Which Australian city has its centre on the intersection of the Warrego Highway and the New England Highway?

To decode this puzzle, solve the inequalities and simultaneous equations below, and match them to a number line or graph. Place the corresponding letters above the matching numbers to find the answer.

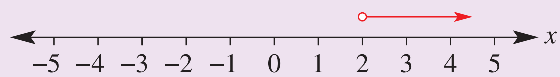
___ ___ ___ ___ ___ ___ ___ ___ ___
6 4 4 3 4 4 5 1 2

Solve these inequalities and match the solution to a number line (**1–3**).

W $2 - 3x > 8$

A $3x + 10 \geq 1$

B $x + 5 > 7$

1


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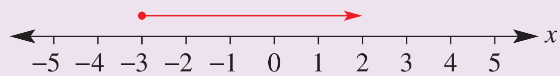

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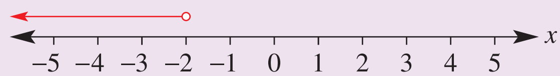
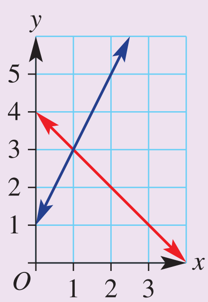


Solve these simultaneous equations and match the solution to a graph (**4–6**).

M $3x - y = 7$
$2x + y = 3$

O $y = 2x + 1$
$x + y = 4$

T $2x - y = -1$
$x - 2y = 4$

4


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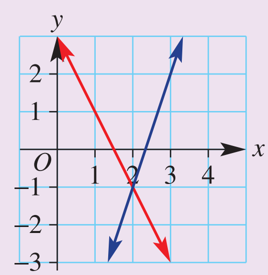

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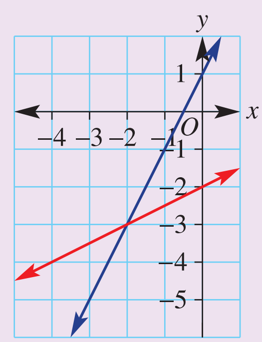

6 Jules and Enzo are participating in a long-distance bike race. Jules rides at 18 km/h and has a 2 hour head start. Enzo travels at 26 km/h.
  a How long does it take for Enzo to catch up to Jules? (Use: Distance = speed × time.)
  b How far did they both ride before Enzo caught up to Jules?

7 Emily travelled a distance of 138 km by jogging for 2 hours and cycling for 5 hours. She could have travelled the same distance by jogging for 4 hours and cycling for 4 hours. Find the speed at which she was jogging and the speed at which she was cycling.

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Chapter summary

Equations

Solving linear equations that have brackets ⭐
- Expand all brackets.
- Collect like terms on each side of the equation.
- Collect terms with a pronumeral to one side (usually the LHS).
- Solve for unknown.

e.g.

$12(x + 1) - 2(3x - 3) = 4(x + 10)$
$12x + 12 - 6x + 6 = 4x + 40$
$6x + 18 = 4x + 40$
$2x + 18 = 40$
$2x = 22$
$x = 11$

Solving linear equations
Solving involves finding the value that makes an equation true.

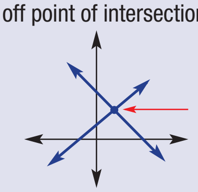
e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)

Equations with fractions ⭐
e.g.
$\frac{3x}{4} - 2 = 7$
$\frac{3x}{4} = 9$ (first + 2 to both sides)
$3x = 36$ (× 4 both sides)
$x = 12$ (÷ 3 both sides)

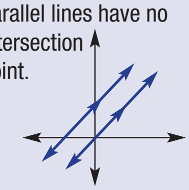
e.g.
$\frac{2x - 5}{3} = 7$
$2x - 5 = 21$ (first ×3 to both sides)
$2x = 26$ (+5 to both sides)
$x = 13$ (÷2 to both sides)

Solving word problems
1 Define variable(s).
2 Set up equation(s).
3 Solve equation(s).
4 Check each answer and write in words.

Graphical solutions of simultaneous equations
Graph each line and read off point of intersection.

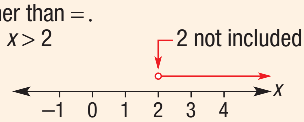


Parallel lines have no intersection point.

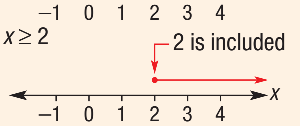


Formulas
Some common formulas
e.g. $A = \pi r^2$, $C = 2\pi r$
An unknown value can be found by substituting values for the other variables.
A formula can be rearranged to make a different variable the subject; i.e. the variable is out the front on its own.
e.g. $E = mc^2$, find m when $E = 320$ and $c = 4$.
$320 = m \times 4^2$ (substitute values)
$320 = 16m$
$20 = m$ (divide both sides by 16)
$m = 20$ (Write the answer with m on the left.)

Inequalities
These are represented using $>$, $<$, $\geq$, $\leq$ rather than $=$.
e.g. $x > 2$



e.g. $x \geq 2$



Solving inequalities uses the same steps as solving equations, except when multiplying or dividing by a negative number. In this case, the inequality sign must be reversed.
e.g. $4 - 2x > 10$ (–4)
$-2x > 6$ (÷ –2)
$x < -3$ (reverse sign)

Simultaneous equations ⭐
Use substitution or elimination to find the solution that satisfies two equations.

Substitution
e.g. $2x + y = 12$ [1]
$y = x + 3$ [2]

In [1] replace y with [2]:
$2x + (x + 3) = 12$
$3x + 3 = 12$
$3x = 9$
$x = 3$
Sub. $x = 3$ to find y.
In [2] $y = 3 + 3 = 6$
Solution is (3, 6).

Elimination
Ensure both equations have a matching pair.
Add two equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ [1]
$2x + 3y = 5$ [2]
[1] × 2: $2x + 4y = 4$ [3]
[3] – [2]: $y = -1$
In [1]: $x + 2(-1) = 2$
$x - 2 = 2$
$x = 4$
Solution is (4, –1).

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

8A 1 **I can solve simple linear equations.**
e.g. Solve these linear equations.
a $2x - 1 = 7$ **b** $\frac{x+4}{2} = 12$

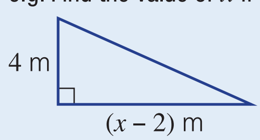
8A 2 **I can set up and solve worded problems using linear equations.**
e.g. 5 less than twice a number is equal to 31. Find the number.

8B 3 **I can solve equations with brackets.**
e.g. Solve $3(x - 2) = 21$.

8B 4 **I can solve equations with variables on both sides.**
e.g. Solve $5x - 2 = 3x + 6$.

8B 5 **I can solve equations with fractions.**
e.g. Solve $\frac{2x - 1}{4} = -1$.

8B 6 **I can set up and solve an equation with brackets from a real situation.**
e.g. Find the value of x if the area of this triangle is 32 m^2.



8C 7 **I can substitute into a formula and solve for a variable.**
e.g. If $A = \frac{h}{2}(a + b)$ and $A = 20$, $h = 4$ and $b = 3$ find the value of a.

8D 8 **I can illustrate an inequality on a number line.**
e.g. Represent $x > -2$ on a number line.

8D 9 **I can solve a simple inequality.**
e.g. Solve $3x - 2 > 7$.

8D 10 **I can solve an inequality when the pronumeral has a negative coefficient.**
e.g. Solve $6 - 3x \geq 18$.

8E 11 **I can find an intersection point of two linear relations which represents the solution to a pair of simultaneous equations.**
e.g. Find the point of intersection (x, y) of these equations by plotting a graph.
$y = 4x - 9$ and $2x + y = 3$

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		✔
8F	**12 I can use the method of substitution to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $y = x - 2$ and $2x + 3y = -1$ using the substitution method; i.e. find the point of intersection.	
8F	**13 I can use the method of substitution to find a solution to a real problem involving simultaneous equations.** e.g. Jonah is 9 years older then Penny and their combined ages is 47. Find their ages.	
8G	**14 I can use the method of elimination to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $x + y = 1$ and $4x + 3y = 5$ using the elimination method.	
8G	**15 I can use the method of elimination to find a solution to a real problem involving simultaneous equations.** e.g. Jill buys 5 pens and 2 pencils from her favourite store for $\$13$, while Michael buys 4 pens and 3 pencils from the same store for $\$12.50$. Find the cost of a pen and a pencil from this store.	

Chapter checklist

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Chapter review

Short-answer questions

8A **1** Solve the following.

a $4a = 32$ **b** $\frac{m}{5} = -6$ **c** $x + 9 = 1$

d $x + x = 16$ **e** $9m = 0$ **f** $w - 6 = 9$

g $8m = -1.6$ **h** $\frac{w}{4} = 1$ **i** $r - 3 = 3$

8A **2** Find the solution to the following.

a $2m + 7 = 11$ **b** $3w - 6 = 18$ **c** $\frac{m}{2} + 1 = 6$

d $\frac{5w}{4} - 3 = 7$ **e** $\frac{m-6}{2} = 4$ **f** $\frac{3m+2}{6} = 1$

g $6a - 9 = 0$ **h** $4 - x = 3$ **i** $9 = x + 6$

8B **3** Solve the following by first expanding the brackets.

a $3(m + 1) = 12$ **b** $4(a - 3) = 16$ **c** $5(2 + x) = 30$

d $4(2x + 1) = 16$ **e** $2(3m - 3) = 9$ **f** $2(1 + 4x) = 9$

g $2(2x + 3) + 3(5x - 1) = 41$ **h** $3(2x + 4) - 4(x - 7) = 56$

8B **4** Find the value of p in the following.

a $7p = 5p + 8$ **b** $2p = 12 - p$ **c** $6p + 9 = 5p$

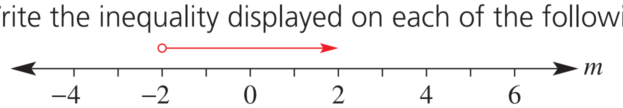
d $2p + 10 = p + 8$ **e** $3p + 1 = p - 9$ **f** $4p - 8 = p - 2$

8A **5** Write an equation for the following and then solve it.

a Six times a number equals 420. What is the number?
b Eight more than a number equals 5. What is the number?
c A number divided by 9 gives 12. What is the number?
d Seven more than a number gives 3. What is the number?
e The sum of a number and 2.3 equals 7. What is the number?

8C **6** **a** For $A = \frac{1}{2}hb$, find b when $A = 24$ and $h = 6$.
b For $V = lwh$, find w when $V = 84$, $l = 6$ and $h = 4$.
c For $A = \frac{x+y}{2}$, find x when $A = 3.2$ and $y = 4$.
d For $E = mc^2$, find m when $E = 40$ and $c = 2$.
e For $F = \frac{9}{5}C + 32$, find C when $F = 95$.

8D **7** Write the inequality displayed on each of the following number lines.

a


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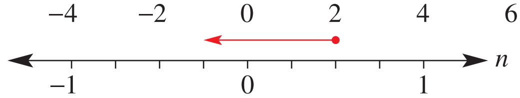

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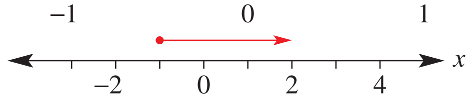

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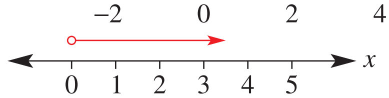

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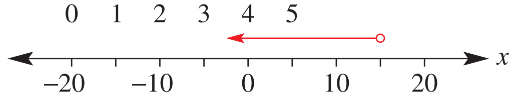

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8D **8** Solve the following.

a $x + 8 \geq -10$ **b** $2m < 7$ **c** $2x + 6 > 10$

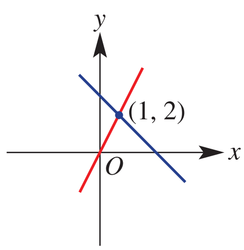
d $x - 3 < 0$ **e** $\frac{x}{4} + 1 \leq 3$ **f** $m - 6 \geq 4$

8D **9** Solve the following.

a $-6x \leq 12$ **b** $8 - x \leq 10$ **c** $-x > 0$

8E **10** Determine the point of intersection of the following lines.

a



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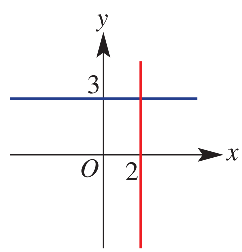

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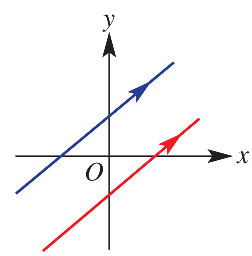

8E/F **11** Find the point of intersection (x, y) of the following by plotting an accurate graph.

a $y = 2x + 4$
$3x + y = 9$

b $y = 2$
$x = 3$

c $y = 3x$
$y = -3x$

8F **12** Solve the simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 5x - 13$
$2x + 3y = 12$

b $y = -1$
$y = 2x - 11$

8G **13** Determine the point of intersection of the following lines, using the elimination method.

a $2x + 7y = -25$
$5x + 7y = -31$

b $3x + 2y = 8$
$x - 2y = 0$

8G **14** The sum of two numbers is 15 and their difference is 7. Use simultaneous equations to find the two numbers.

8G **15** A money box contains 20 cent and 50 cent coins. The amount in the money box is \$50 and there are 160 coins.

a Define two variables and set up a pair of linear equations.

b Solve the two simultaneous equations to determine the number of 20 cent and 50 cent coins.

8F **16** There are twice as many adults as children at a local grand final football match. It costs \$10 for adults and \$2 for children to attend the match. If the football club collected \$1100 at the entrance gates, how many children went to see the match?

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Chapter review

Multiple-choice questions

8A **1** The solution to $x + 7 = 9$ is:
A $x = 16$ **B** $x = -2$ **C** $x = 2$ **D** $x = 1$ **E** $x = -16$

8B **2** To solve the equation $3(2x + 4) - 4(x + 2) = 6$, you would first:
A divide both sides by 12
B expand the brackets
C subtract 6 from both sides
D multiply both sides by 6
E add $4(x + 2)$ to both sides

8A **3** A number is increased by 6 and then doubled. The result is 36. This translates to:
A $6x + 2 = 36$ **B** $2x + 6 = 36$ **C** $2(x + 6) = 36$
D $2(x - 6) = 36$ **E** $x + 12 = 36$

8B **4** If $4a - 6 = 2a$, then a equals:
A -1 **B** 1 **C** 6 **D** 3 **E** -3

8D **5** $x \le 4$ is a solution to:
A $x + 1 < 3$ **B** $3x - 1 \le 11$ **C** $\frac{x}{2} - 1 \ge 0$
D $x - 1 \ge 1$ **E** $-x \le -4$

8D **6** Which number line shows $x + 4 < 6$?

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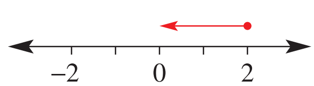

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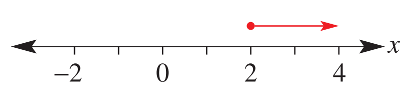

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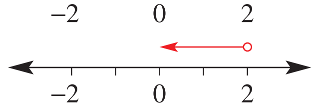

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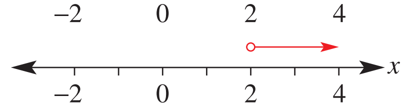

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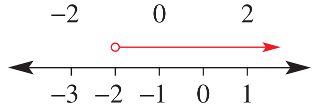

8B **7** The solution to $\frac{5x}{9} - 4 = 1$ is:
A $x = 6$ **B** $x = -9$ **C** $x = -5$ **D** $x = 9$ **E** $x = 5$

8E **8** If two lines are not parallel, the number of intersection points they will have is:
A 0 **B** 1 **C** 2 **D** 3 **E** 4

8E **9** The intersection point for the graphs of $y = 2$ and $x = 3$ is:
A $(-1, 2)$ **B** $(2, 2)$ **C** $(3, 2)$ **D** $(3, 3)$ **E** $(2, 3)$

8B **10** The solution to $3(x - 1) = 12$ is:
A $x = -1$ **B** $x = 2$ **C** $x = 0$ **D** $x = 5$ **E** $x = 4$

8F **11** $y = 3x$ and $x + y = 4$ has the solution:
A $(1, 3)$ **B** $(3, 1)$ **C** $(2, 6)$ **D** $(2, 2)$ **E** $(-1, 5)$

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8F

12 Substituting $y = x - 1$ into $x + 2y = 3$ gives:

A $x - 2x - 2 = 3$ **B** $x + 2y - 2 = 3$ **C** $x - x - 1 = 3$

D $x + 2x - 1 = 3$ **E** $x + 2(x - 1) = 3$

8G

13 Adding $x + y = 3$ to $x - y = 4$ gives:

A $2x - 2y = 7$ **B** $2x = 7$ **C** $x = 7$ **D** $y = 7$ **E** $2y = 7$

Extended-response questions

1 All the lines meet at 90° in this shape.

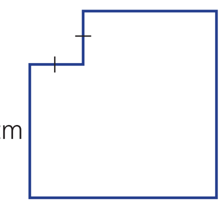
a Determine the equation of its perimeter, P.

b **i** If the perimeter is 128 cm, determine the value of x.

ii Find the actual side lengths.

c Repeat part **b** for perimeters of:

i 152 cm **ii** 224 cm



2 Two computer consultants have an up-front fee plus an hourly rate. Rhys charges \$50 plus \$70 per hour, whereas Agnes charges \$100 plus \$60 per hour.

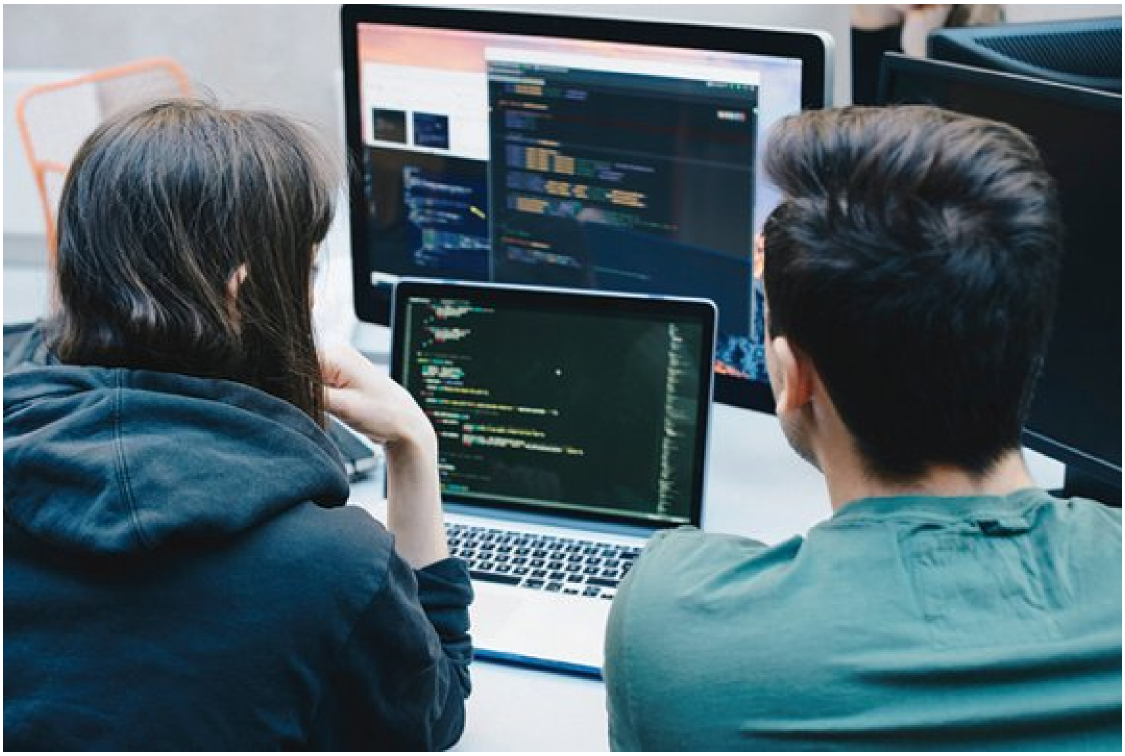
a Using $\$C$ for the cost and t hours for the time, write a rule for the cost of hiring:

i Rhys **ii** Agnes

b If Agnes charges \$280, solve an equation to find how long she was hired for.

c By drawing a graph of C versus t for both Rhys and Agnes on the same set of axes, find the coordinates of the intersection point.

d Use the algebraic method of substitution to solve the simultaneous equations and confirm your answer to part **c**.



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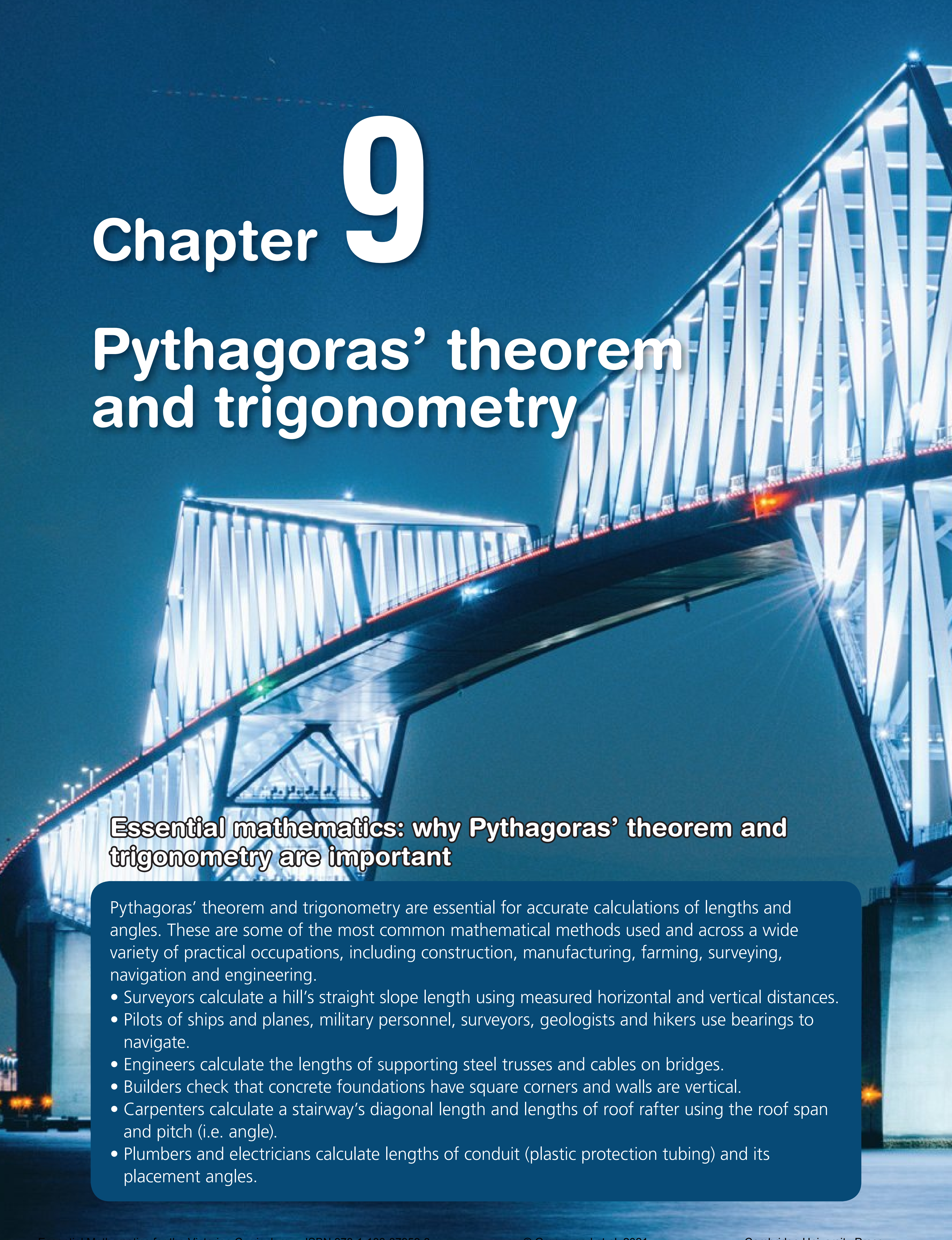

Chapter 9

Pythagoras’ theorem and trigonometry

Essential mathematics: why Pythagoras’ theorem and trigonometry are important

Pythagoras’ theorem and trigonometry are essential for accurate calculations of lengths and angles. These are some of the most common mathematical methods used and across a wide variety of practical occupations, including construction, manufacturing, farming, surveying, navigation and engineering.

- Surveyors calculate a hill’s straight slope length using measured horizontal and vertical distances.
- Pilots of ships and planes, military personnel, surveyors, geologists and hikers use bearings to navigate.
- Engineers calculate the lengths of supporting steel trusses and cables on bridges.
- Builders check that concrete foundations have square corners and walls are vertical.
- Carpenters calculate a stairway’s diagonal length and lengths of roof rafter using the roof span and pitch (i.e. angle).
- Plumbers and electricians calculate lengths of conduit (plastic protection tubing) and its placement angles.

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In this chapter

Victorian Curriculum

MEASUREMENT AND GEOMETRY

Pythagoras and trigonometry

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (VCMMG346)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Essential Mathematics for the Victorian Curriculum CORE Year 10 ISBN 978-1-108-87859-3

Warm-up quiz

1 Round the following decimals, correct to two decimal places.

a 15.84312 **b** 164.8731 **c** 0.86602 **d** 0.57735
e 0.173648 **f** 0.7071 **g** 12.99038 **h** 14.301

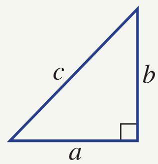
2 Find the value of each of the following.

a 5^2 **b** 6.8^2 **c** 19^2 **d** $9^2 + 12^2$ **e** $3.1^2 + 5.8^2$ **f** $41^2 - 40^2$

3 Write the following as a decimal, correct to one decimal place.

a $\sqrt{8}$ **b** $\sqrt{7}$ **c** $\sqrt{15}$ **d** $\sqrt{10}$
e $\sqrt{12.9}$ **f** $\sqrt{8.915}$ **g** $\sqrt{3.8}$ **h** $\sqrt{200}$

4 Write down the letter or letters matching the hypotenuse (i.e. the side opposite the right angle) on the following triangles.

a


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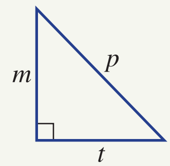

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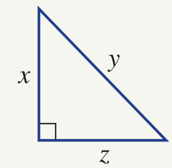

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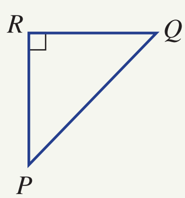

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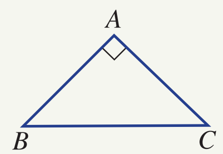

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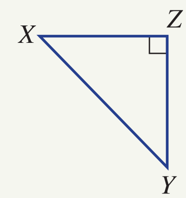


5 Solve for x.

a $3x = 9$ **b** $4x = 16$ **c** $\frac{x}{5} = 7$ **d** $\frac{2x}{3} = 6$

6 Solve for m.

a $7m = 25.55$ **b** $9m = 10.8$ **c** $\frac{m}{1.3} = 4$ **d** $\frac{m}{5.4} = 1.06$

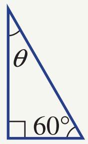
7 Solve each of the following equations, correct to one decimal place.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 17$ **c** $\frac{32}{x} = 15$ **d** $\frac{3.8}{x} = 9.2$ **e** $\frac{15}{x} = 6.2$ **f** $\frac{29.3}{x} = 3.2$

8 If x is a positive integer, solve:

a $x^2 = 16$ **b** $x^2 = 400$ **c** $x^2 = 5^2 + 12^2$ **d** $x^2 + 3^2 = 5^2$

9 Find the size of the angle θ in the following diagrams.

a


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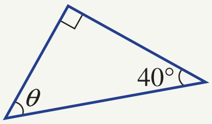

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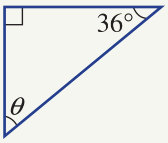

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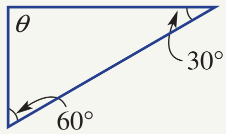

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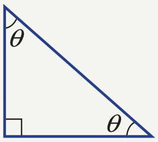

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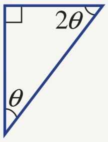

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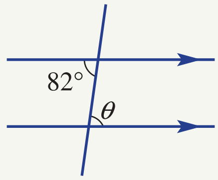

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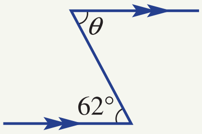

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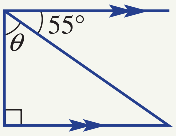

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9A Reviewing Pythagoras' theorem

CONSOLIDATING

Learning intentions

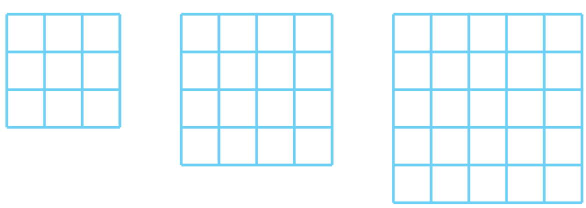
- To know that Pythagoras' theorem connects the three side lengths of a right-angled triangle
- To be able to find the length of the hypotenuse of a right-angled triangle given the other two sides.
- To be able to apply Pythagoras' theorem in finding the length of the hypotenuse in a simple application

Key vocabulary: Pythagoras' theorem, right angle, hypotenuse, square

The ancient Egyptians knew of the relationship between the numbers 3, 4 and 5 and how they could be used to form a right-angled triangle.

Greek philosopher and mathematician Pythagoras expanded on this idea and the theorem we use today is named after him.

Lesson starter: Three, four and five

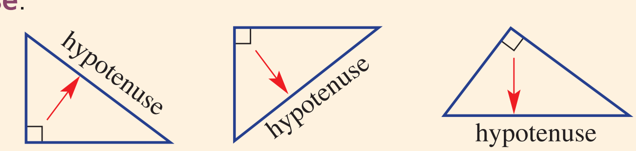


- On square grid paper, construct three squares as shown above.
- Cut them out and place the middle-sized square on top of the largest square. Then cut the smallest square into 9 smaller squares and also place them onto the largest square to finish covering it.
- What does this show about the numbers 3, 4 and 5?

Key ideas

- A right-angled triangle has its longest side opposite the right angle. This side is called the **hypotenuse**.

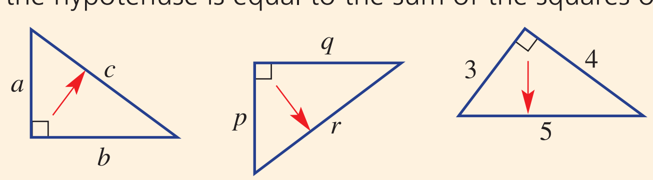
  For example:



- **Pythagoras' theorem** states:

  The square of the hypotenuse is equal to the sum of the squares on the other two sides.

  For example:



$c^2 = a^2 + b^2$ $\quad r^2 = p^2 + q^2$ $\quad 5^2 = 3^2 + 4^2$

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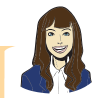
9A

Exercise 9A

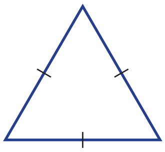
Understanding | 1–4 | 3, 4

1 Which of the following triangles have a side known as the hypotenuse?



Hint: Only right-angled triangles have a hypotenuse.

a



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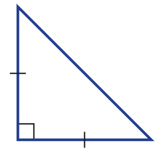

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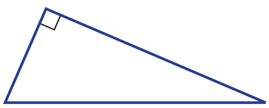

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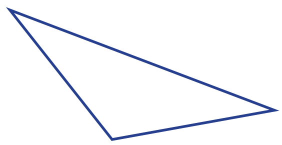
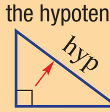
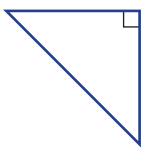


2 Copy these triangles into your workbook and label the hypotenuse.

Hint: Draw an arrow across from the right angle to find the hypotenuse (hyp).



a



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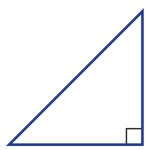
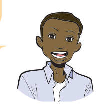
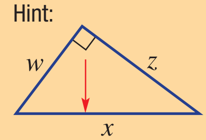


3 Write the relationship between the sides of these triangles.

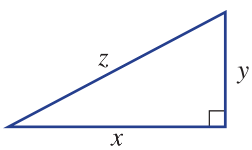


Hint:



$x^2 = w^2 + z^2$

a



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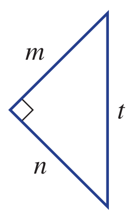
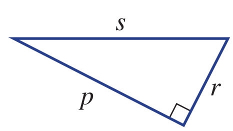


c



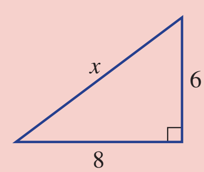
4 Find the value of $a^2 + b^2$ when:

a $a = 3$ and $b = 4$ **b** $a = 3$ and $b = 5$ **c** $a = 3$ and $b = 6$

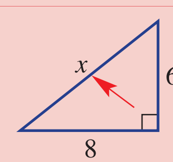
Fluency | 5–6(½) | 5–7(½)

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse (x) of the triangle shown.



Solution	Explanation
$x^2 = 6^2 + 8^2$ $= 36 + 64$ $= 100$	Write the relationship for the given triangle using Pythagoras' theorem.
$x = \sqrt{100}$ $= 10$	Take the square root to find x.



Continued on next page

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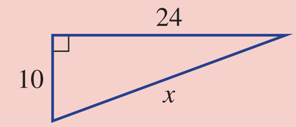
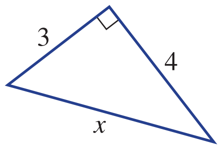
Now you try

Find the length of the hypotenuse (x) of the triangle shown.

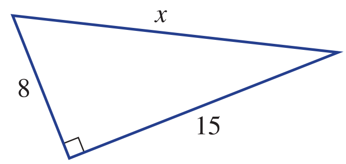


5 Find the length of the hypotenuse in these right-angled triangles.

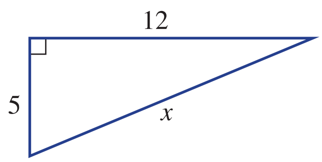
a



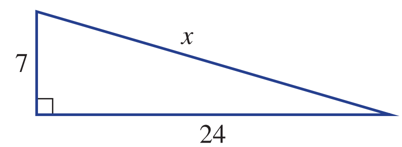
b



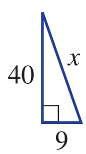
c



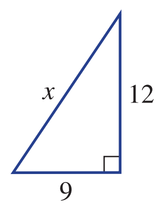
d



e

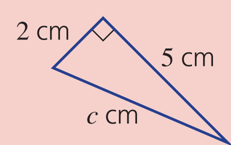


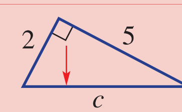
f



Example 2 Finding the length of the hypotenuse as a decimal

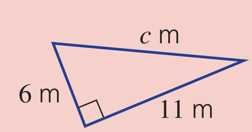
Find the length of the hypotenuse in this triangle, correct to one decimal place.



Solution	Explanation
$c^2 = 5^2 + 2^2$	Write the relationship for this triangle, where c is the length of the hypotenuse. 
$= 25 + 4$	
$= 29$	
$c = \sqrt{29}$	Simplify.
$c = 5.38516\ldots$	Take the square root to find c.
$c = 5.4$ (to 1 d.p.)	Round 5.3 ⑧ 516… to one decimal place by rounding up.

Now you try

Find the length of the hypotenuse in this triangle, correct to one decimal place.



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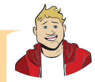
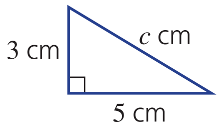
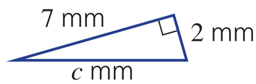
9A

6 Find the length of the hypotenuse in these triangles, correct to one decimal place.



Hint: If $c^2 = 34$, then $c = \sqrt{34}$. Use a calculator to find the decimal.

a


b


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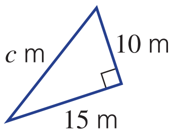
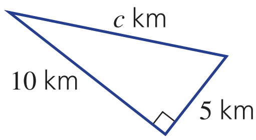


d


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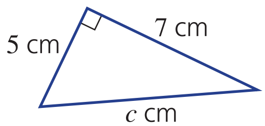

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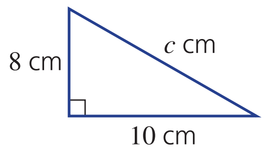
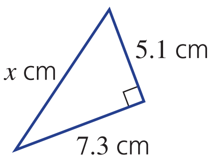


7 Find the value of the hypotenuse in these triangles, correct to two decimal places.

a


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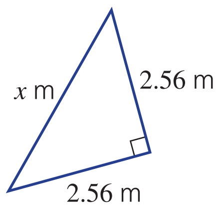

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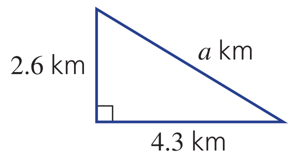

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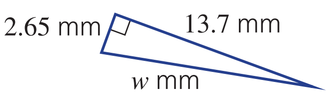

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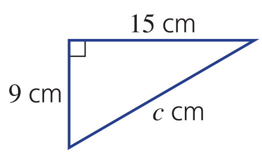

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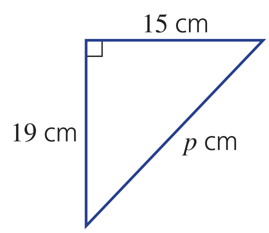
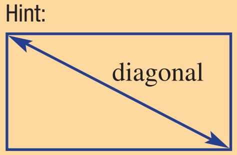


Problem-solving and reasoning

8–11 | 11–14

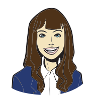
8 A LCD plasma TV is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to one decimal place.

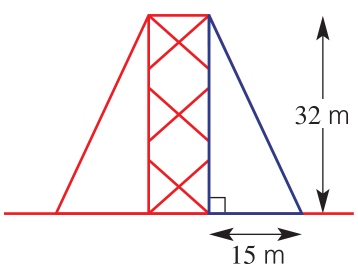
Hint:



9 A 32 m tower is supported by cables from the top to a position on the ground 15 m from the base of the tower. Determine the length of each cable needed to support the tower, correct to one decimal place.

Hint: Set up and solve using Pythagoras' theorem.





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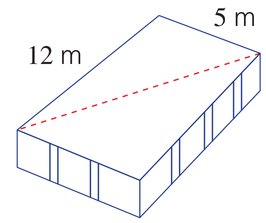
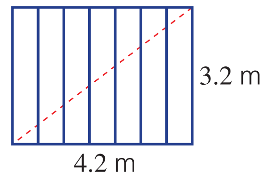
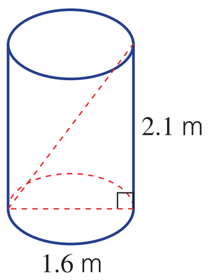
10 Boris the builder uses Pythagoras' theorem to check the corners of his concrete slab. What will be the length of the diagonal when the angle is 90°?



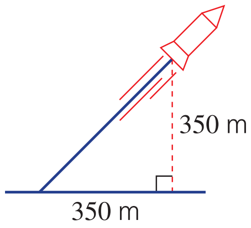
11 Find the length of the diagonal steel brace needed to support a gate of length 4.2 m and width 3.2 m, correct to two decimal places.



12 Find the length of the longest rod that will fit in a cylindrical container of height 2.1 m and diameter 1.6 m, correct to two decimal places.



13 A rocket blasts off and after a few seconds it is 350 m above the ground. At this time it has covered a horizontal distance of 350 m. How far has the rocket travelled, correct to two decimal places?



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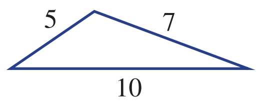
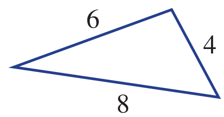
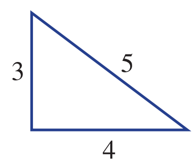
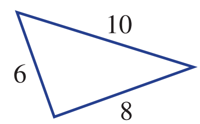
9A

14 Determine whether these triangles contain a right angle.

a


b


c


d


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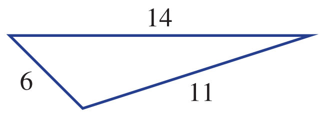

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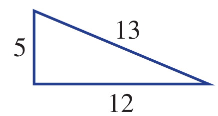
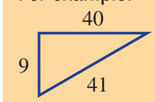


Hint: If Pythagoras' theorem works, then the triangle has a right angle.
For example:



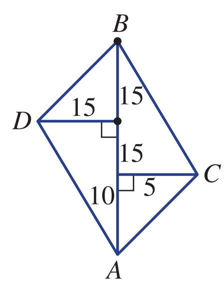
$41^2 = 1681$
$40^2 + 9^2 = 1681$
$\therefore 41^2 = 40^2 + 9^2$ and the triangle has a right angle, opposite the 41.

An offset survey — 15, 16

15 An offset survey measures distances perpendicular to the baseline offset. A notebook entry is made showing these distances, and then perimeters and areas are calculated.

	B		
	40		
D 15	25		
	10	5	C
	0		
	A		

Notebook entry



Field diagram
(Not to scale)

a Using the diagrams above, find these lengths, correct to one decimal place.
i AC **ii** BC **iii** DB **iv** AD

b Find the perimeter of the field $ACBD$, correct to the nearest metre.

c Find the area of the field.

16 At right is a notebook entry. Draw the field diagram and find the perimeter of the field, to one decimal place.

	B		
	60		
	40	10	E
D 25	30		
C 15	10		
	0		
	A		

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9B Finding the length of a shorter side

Learning intentions

- To be able to find the length of a shorter side of a right-angled triangle given the other two sides
- To be able to find the length of a shorter side of a right-angled triangle in a simple application

Key vocabulary: hypotenuse

Using Pythagoras' theorem, we can determine the length of the shorter sides of a right-angled triangle. The angled support beams on a rollercoaster ride, for example, create right-angled triangles with the ground. The vertical and horizontal distances are the shorter sides of the triangle.

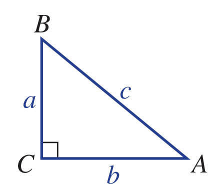
Lesson starter: Choosing the correct numbers

For the triangle ABC, Pythagoras' theorem is written $c^2 = a^2 + b^2$.

Choose the 3 numbers from each group that work for $c^2 = a^2 + b^2$.

Group 1: 6, 7, 8, 9, 10 Group 2: 15, 16, 20, 25

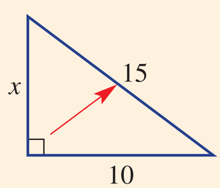
Group 3: 9, 10, 12, 15 Group 4: 9, 20, 21, 40, 41



Key ideas

- We can use Pythagoras' theorem to determine the length of one of the shorter sides if we know the length of the hypotenuse and the other side.

For example:



$15^2 = x^2 + 10^2$ becomes $x^2 = 15^2 - 10^2$.

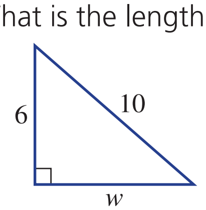
So $x^2 = 125$ and $x = \sqrt{125}$.

Exercise 9B

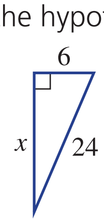
Understanding 1–3 3

1 What is the length of the hypotenuse in each of these triangles?

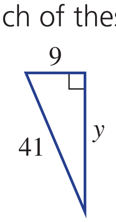
a



b



c



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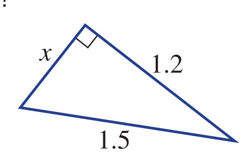
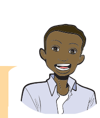


2 Copy and complete:

a When $10^2 = 6^2 + w^2$, then $w^2 = 10^2 - \square$.

b When $13^2 = 5^2 + x^2$, then $x^2 = 13^2 - \square$.

c When $30^2 = p^2 + 18^2$, then $p^2 = \square - 18^2$.



Hint: Follow a step as if you were solving an equation.

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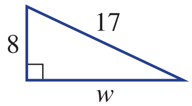
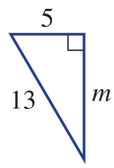
9B

3 Substitute the numbers and pronumerals into Pythagoras' theorem $c^2 = a^2 + b^2$, for each of these triangles. Do not solve for the unknown.

a


b


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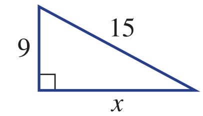
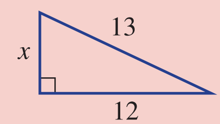


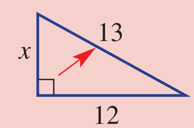
Fluency

4–5(½) | 4–6(½)

Example 3 Calculating a shorter side

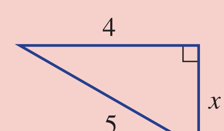
Determine the value of x in the triangle shown, using Pythagoras' theorem.



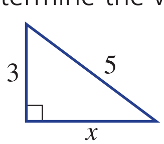
Solution	Explanation
$13^2 = x^2 + 12^2$	Write the relationship for this triangle using Pythagoras' theorem, with 13 as the hypotenuse. 
$x^2 = 13^2 - 12^2$ $= 169 - 144$	Rewrite the rule with the x^2 on the left-hand side.
$= 25$	Simplify.
$x = \sqrt{25}$ $\therefore x = 5$	Find the square root to find x.

Now you try

Determine the value of x in the triangle shown, using Pythagoras' theorem.



4 Determine the value of x in these triangles, using Pythagoras' theorem.

a


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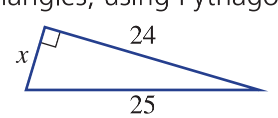
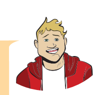


Hint: In $c^2 = a^2 + b^2$, c is always the hypotenuse.



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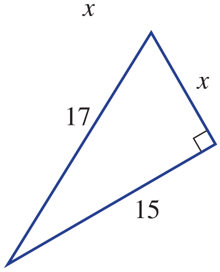

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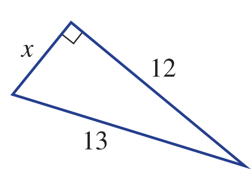

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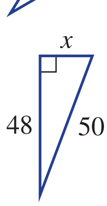

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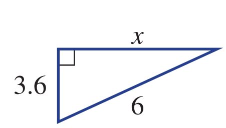

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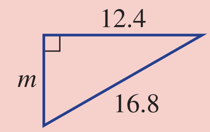
Example 4 Finding a shorter side length as a decimal value

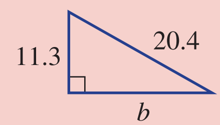
Determine the value of m in the triangle, correct to one decimal place.



Solution	Explanation
$16.8^2 = m^2 + 12.4^2$	Write the relationship for this triangle.
$m^2 = 16.8^2 - 12.4^2$	Make m^2 the subject.
$= 128.48$	Simplify, using your calculator.
$m = \sqrt{128.48}$	Take the square root of both sides to find m.
$= 11.3349\ldots$	
$m = 11.3$ (to 1 d.p.)	Round your answer to one decimal place.

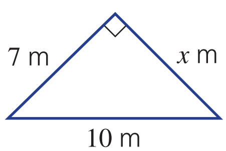
Now you try

Determine the value of b in the triangle, correct to one decimal place.



5 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to one decimal place.

a



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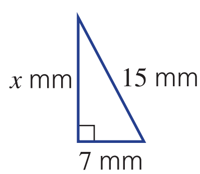
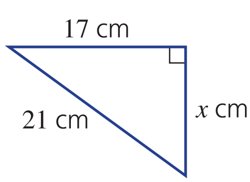


c



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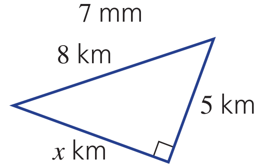

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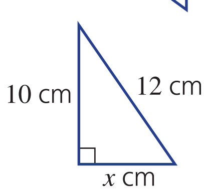
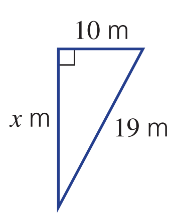
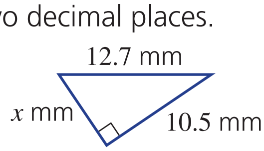


f



6 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to two decimal places.

a



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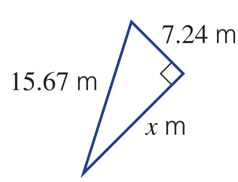

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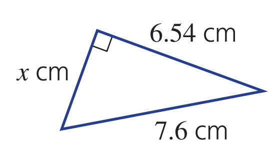

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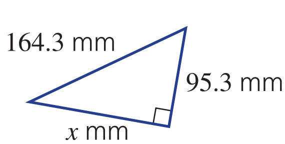

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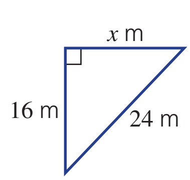

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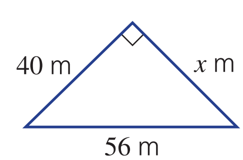

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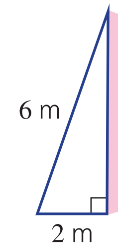
9B

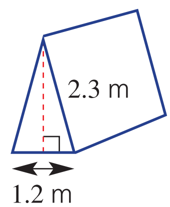
Problem-solving and reasoning

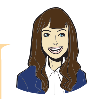
7–9	9–11

7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to two decimal places.

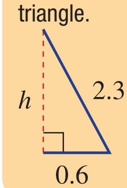


8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to one decimal place.

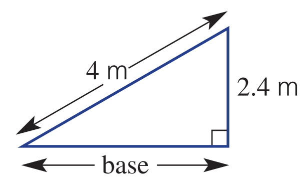




Hint: Identify the right-angled triangle.



9 A city council wants to build a skateboard ramp measuring 4 m long and 2.4 m high. How long should the base of the ramp be, correct to one decimal place?



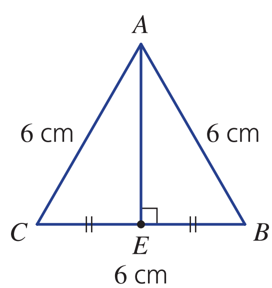
10 Triangle ABC is equilateral. AE is an axis of symmetry.

a Find the length of:

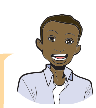
i EB

ii AE, to one decimal place

b Find the area of triangle ABC, to one decimal place.

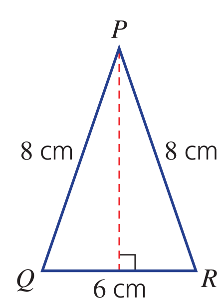


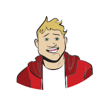
Hint: An equilateral triangle has 3 equal sides.



Hint: Remember: $A = \frac{1}{2}bh$ is the area of a triangle.

11 What is the height of this isosceles triangle, to one decimal place?





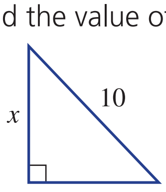
Hint: Pythagoras' theorem applies only to right-angled triangles.

More than one pronumeral

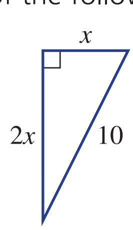
—	12

12 Find the value of x in each of the following. Answer to one decimal place.

a



b



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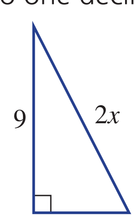

Hint: Remember to square the entire side. The square of $2x$ is $(2x)^2$ or $4x^2$.

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9C Applications of Pythagoras' theorem

Learning intentions

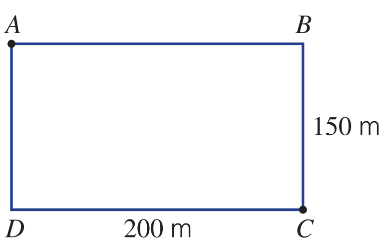
- To be able to identify right-angled triangles in simple applications
- To be able to apply Pythagoras' theorem in a real situation to find an unknown length

Key vocabulary: identify, pronumeral

Pythagoras' theorem has many applications, some of which you may have noticed already in this chapter. Some areas where Pythagoras' theorem is useful include drafting, building and navigation.

Lesson starter: Finding the shortest path

A rectangular field is 200 m by 150 m. Marco wants to walk from the corner of the field marked A to the corner of the field marked C. How many metres are saved by walking along the diagonal AC rather than walking along AB and then BC?



Key ideas

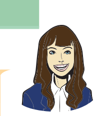
- When applying Pythagoras' theorem follow these steps.
  - Identify and draw the right-angled triangle or triangles you need in order to solve the problem.
  - Label the triangle and place a pronumeral (letter) on the side length that is unknown.
  - Use Pythagoras' theorem to find the value of the pronumeral.
  - Answer the question. (Written questions should have written answers.)

Exercise 9C

Understanding	1–3	3

1 Draw a diagram for each of the following questions. You don't need to answer the question.

Hint: Each one involves a right-angled triangle.



a A 2.4 m ladder is placed 1 m from the foot of a building. How far up the building will the ladder reach?

b The diagonal of a rectangle with length 18 cm is 24 cm. How wide is the rectangle?

c Sebastian walks 5 km north, then 3 km west. How far is he from his starting point?

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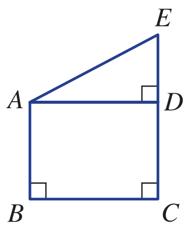
9C

2 Name the right-angled triangles in each of the following diagrams; e.g. $\triangle ABC$.

a



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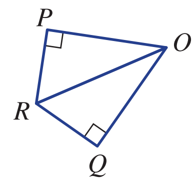

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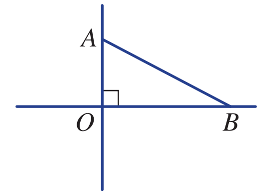
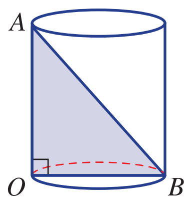


3 Name the hypotenuse in each of the shaded right-angled triangles found within these three-dimensional shapes; e.g. FG.

a



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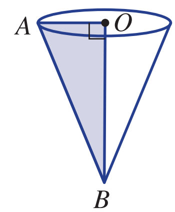

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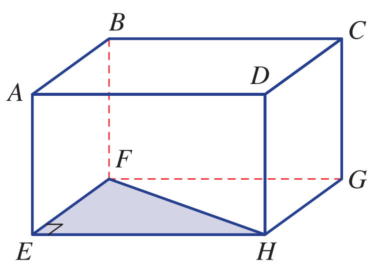

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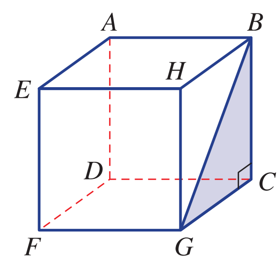
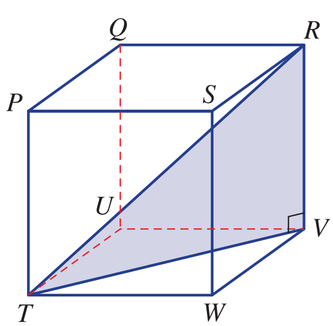


e



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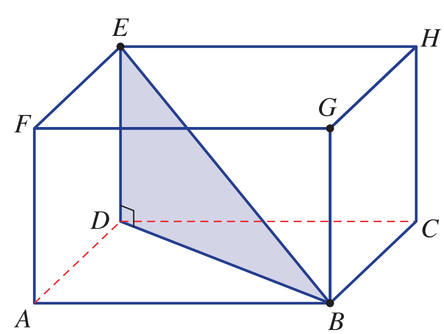
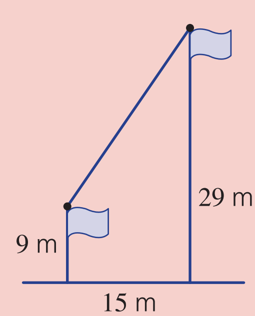


Fluency	4–7	4, 5, 7, 8

Example 5 Applying Pythagoras' theorem

Two flag poles are 15 m apart and a rope links the tops of both poles. Find the length of the rope if one flag pole is 9 m tall and the other is 29 m tall.



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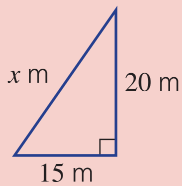
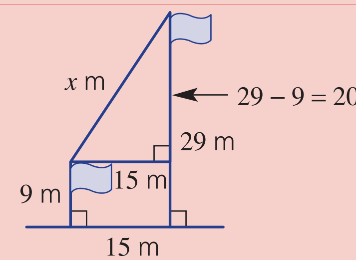
Solution	Explanation
Let x metres be the length of rope.	Locate and draw the right-angled triangle, showing all measurements. Introduce a pronumeral for the missing side.





$$x^2 = 15^2 + 20^2$$
$$= 225 + 400$$
$$= 625$$

Write the relationship, using Pythagoras' theorem.

Simplify.

$$x = \sqrt{625}$$
$$= 25$$

Take the square root to find x.

The rope is 25 m long.

Answer the question.

Now you try

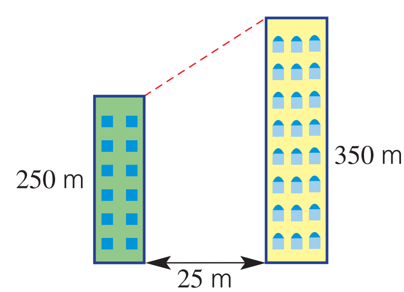
Two vertical poles are 2 m and 5 m high and are 4 m apart. Find the distance between the top of the two poles.

4 Two skyscrapers are 25 m apart and a cable runs from the top of one building to the top of the other. One building is 350 m tall and the other is 250 m.

a Determine the difference in the heights of the buildings.

b Draw an appropriate right-angled triangle you could use to find the length of the cable.

c Find the length of the cable, correct to two decimal places.

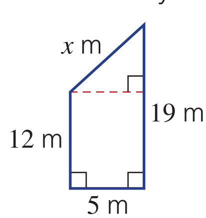


(Not to scale)

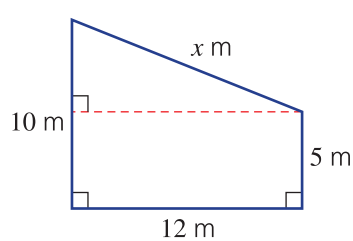
5 Find the value of x in each of the following, correct to one decimal place where necessary.

Hint: Label the two known lengths of each triangle first.

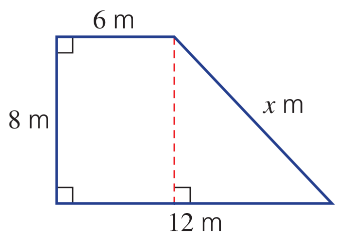
a



b



c



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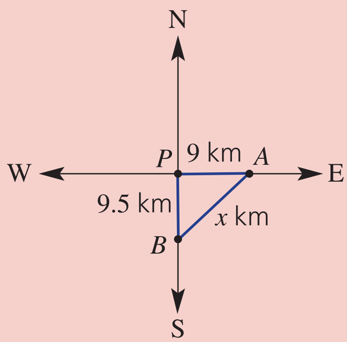
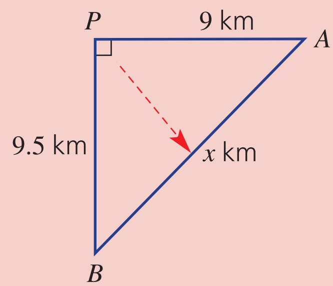
9C

Example 6 Using direction with Pythagoras' theorem

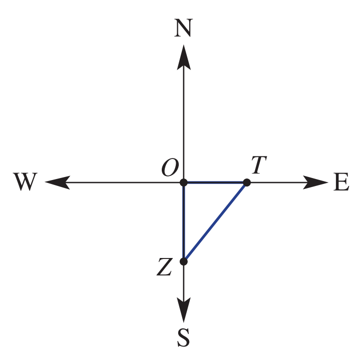
Two hikers leave their camp (P) at the same time. One walks due east for 9 km; the other walks due south for 9.5 km. How far apart are the two hikers at this point? (Answer to one decimal place.)

Solution	Explanation
	Draw a diagram. Consider $\triangle PAB$. 
$\therefore x^2 = 9^2 + 9.5^2$	Write Pythagoras' theorem and evaluate.
$x^2 = 171.25$	Square root to find x.
$x = \sqrt{171.25}$	
$= 13.086$	
$= 13.1$ (to 1 d.p.)	Round to one decimal place.
$\therefore$ The hikers are 13.1 km apart.	Answer the question in words.

Now you try

From a floating oil rig, one ship travels due north for 20 km and another travels due west for 30 km. How far apart are the two ships at this point? (Answer to one decimal place.)

6 Tranh (T) walks 4.5 km east while Zara (Z) walks 5.2 km south. How far from Tranh is Zara? Answer to one decimal place.



7 Find the distance between Sui and Kevin if:

a Sui walks 6 km north from camp O and Kevin walks 8 km west from camp O.

b Sui walks 40 km east from point A and Kevin walks 9 km south from point A.

c Kevin walks 15 km north-west from O and Sui walks 8 km south-west also from O.

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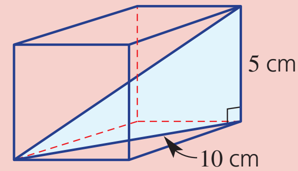
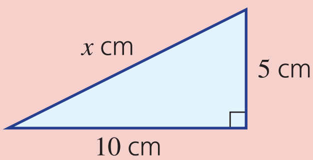
Example 7 Using Pythagoras' theorem in 3D

Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



Solution



$x^2 = 5^2 + 10^2$

$= 25 + 100$

$x = \sqrt{125}$

$= 11.18$ cm (to 2 d.p.)

∴ The distance between the opposite corners is 11.18 cm.

Explanation

Draw the triangle you need and mark the lengths.

Write the relationship for this triangle.

Simplify.

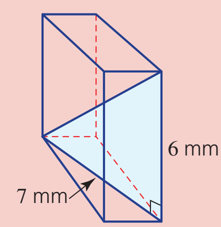
Take the square root to find x.

Round your answer to two decimal places.

Write the answer.

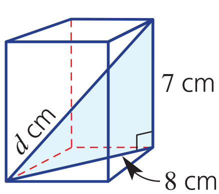
Now you try

Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.

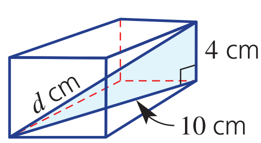


8 Find the distance of d from one corner to the opposite corner in the following rectangular prisms, correct to one decimal place.

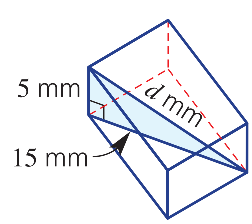
a



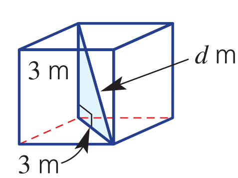
b



c



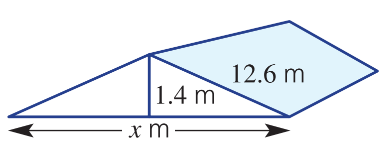
d



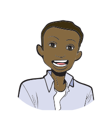
Problem-solving and reasoning

9–11 | 11–14

9 The height of a roof is 1.4 m. If the length of a gable (the diagonal) is to be 12.6 m, determine the length of the horizontal beam needed to support the roof, correct to two decimal places.



Hint: Find the base length of the right-angled triangle first.



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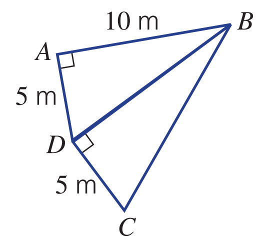
9C

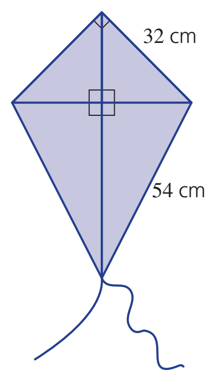
10 For the diagram shown, find the lengths of:

a BD, correct to two decimal places

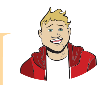
b BC, correct to one decimal place



11 A kite is constructed with six pieces of wooden dowel and covered in fabric. The four pieces around the edge have two 32 cm rods and two 54 cm rods. If the top of the kite is right angled, find the length of the horizontal and vertical rods, correct to two decimal places.

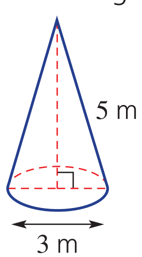


Hint: Find the length of the horizontal rod first. What type of triangle is the top of the kite? Find the length of the vertical rod using two calculations.

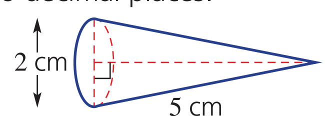


12 Find the height of the following cones, correct to two decimal places.

a

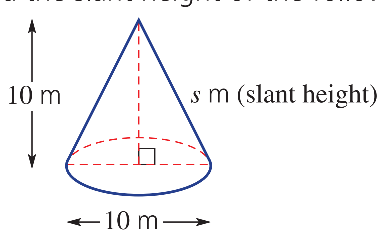


b

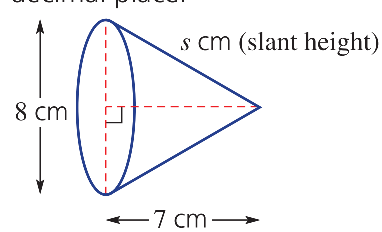


13 Find the slant height of the following, correct to one decimal place.

a

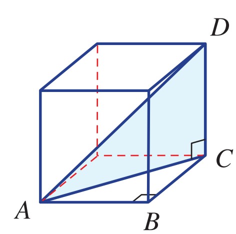


b



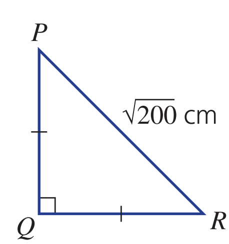
14 This cube has 1 cm sides. Find, correct to two decimal places, the lengths of:

a AC **b** AD



How much do you know? — 15

15 Write down everything you know about $\triangle PQR$, including the things that you can calculate.



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9D Trigonometric ratios CONSOLIDATING

Learning intentions

- To know the three trigonometric ratios for a right-angled triangle
- To be able to write down the ratio for sine, cosine and tangent for a triangle with given side lengths

Key vocabulary: trigonometry, sine, cosine, tangent, hypotenuse, opposite, adjacent, angle of reference

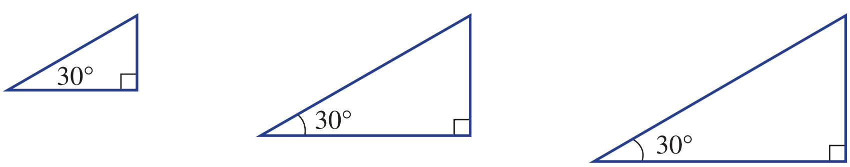
Trigonometry deals with the relationship between the sides and angles of triangles.

In this section we look at the relationship between right-angled triangles and the three trigonometric ratios: sine (sin), cosine (cos) and tangent (tan).

Using your calculator and knowing how to label the sides of right-angled triangles, you can use trigonometry to find missing sides and angles.

Lesson starter: Thirty degrees

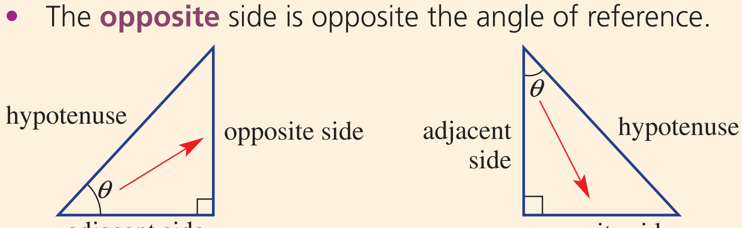
- Draw three different right-angled triangles that each have a 30° angle.



- Measure each side of each triangle and add these measurements to your diagrams.
- The hypotenuse, as we know, is opposite the right angle. The side opposite the 30° is called the opposite side. For each of your three triangles, write down the ratio of the opposite side divided by the hypotenuse. What do you notice?
- Type '$\sin 30^\circ$' into your calculator. What do you notice?

Key ideas

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The **angle of reference** is the angle in a right-angled triangle that is used to determine the opposite side and the adjacent side.
  - The **hypotenuse** is always opposite the right angle.
  - The **adjacent** side is next to the angle of reference.
  - The **opposite** side is opposite the angle of reference.



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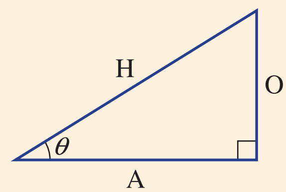
9D

- For a right-angled triangle with a given angle θ (theta), the three trigonometric ratios of **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine of angle θ: $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ: $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ: $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$
- When working with right-angled triangles, label each side of the triangle O (opposite), A (adjacent) and H (hypotenuse).



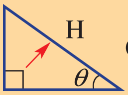
- The three trigonometric ratios are:
  $\sin\theta = \dfrac{O}{H}$ $\cos\theta = \dfrac{A}{H}$ $\tan\theta = \dfrac{O}{A}$
  We can remember this as **SOH CAH TOA**.

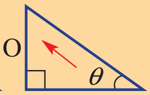
Exercise 9D

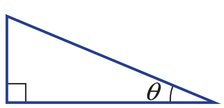
Understanding 1–4 4

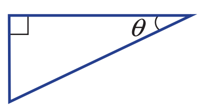
1 By referring to the angles marked, copy each triangle and label the sides opposite, adjacent and hypotenuse, using O, A and H.

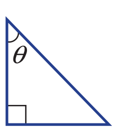
Hint: Arrows help you find the hypotenuse and the opposite side:

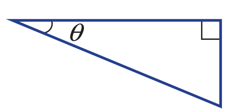


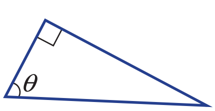


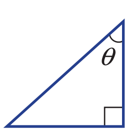
a


b


c


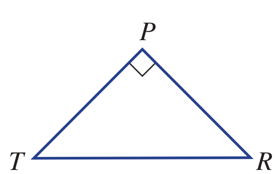
d


e


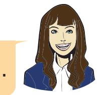
f


2 Referring to triangle PTR, name the:

a side opposite the angle at T
b side adjacent to the angle at T
c side opposite the angle at R
d side adjacent to the angle at R
e hypotenuse
f angle opposite the side PR



Hint: 'Adjacent' means 'next to'.



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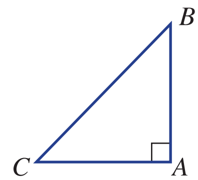
3 Referring to triangle ABC, name the:

a hypotenuse

b side opposite the angle at B

c side opposite the angle at C

d side adjacent to the angle at B

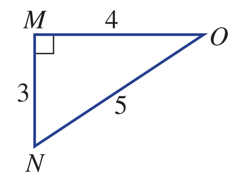


4 In triangle MNO, write the ratio (i.e. fraction) of:

a $\dfrac{\text{the side oposite angle } O}{\text{hypotenuse}}$

b $\dfrac{\text{the side opposite angle } N}{\text{hypotenuse}}$

c $\dfrac{\text{the side adjacent angle } O}{\text{hypotenuse}}$



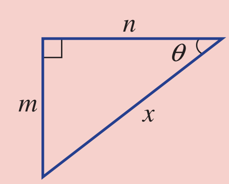
Fluency

5–6(½) | 5–6(½)

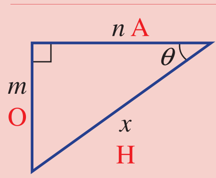
Example 8 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ b $\cos\theta$ c $\tan\theta$



Solution



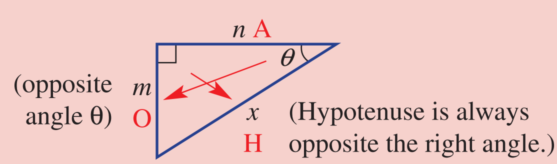
a $\sin\theta = \dfrac{m}{x}$

b $\cos\theta = \dfrac{n}{x}$

c $\tan\theta = \dfrac{m}{n}$

Explanation

Use arrows to label the sides correctly.



SOH CAH TOA

$\sin\theta = \dfrac{\text{O}}{\text{H}} = \dfrac{m}{x}$

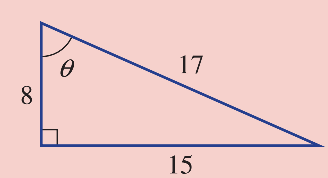
$\cos\theta = \dfrac{\text{A}}{\text{H}} = \dfrac{n}{x}$

$\tan\theta = \dfrac{\text{O}}{\text{A}} = \dfrac{m}{n}$

Now you try

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ b $\cos\theta$ c $\tan\theta$



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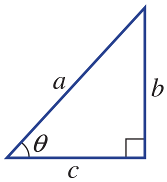
9D

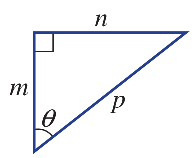
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

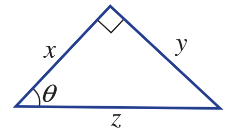
a

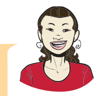


b



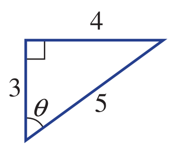
c



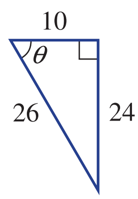


Hint: Use SOH CAH TOA after labelling the sides as O, A and H.

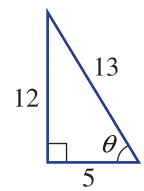
d



e

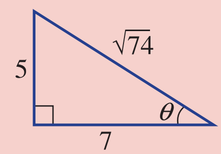


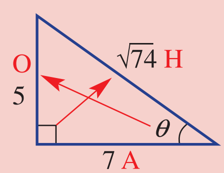
f



Example 9 Writing a trigonometric ratio

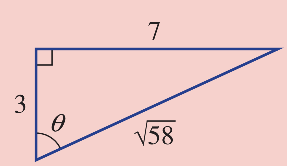
Write down the ratio of $\cos\theta$ for this triangle.



Solution	Explanation
$\cos\theta = \frac{\text{A}}{\text{H}}$	Label the sides of the triangle.
$\cos\theta = \frac{7}{\sqrt{74}}$	
	SOH CAH TOA tells us $\cos\theta$ is $\frac{\text{adjacent}}{\text{hypotenuse}}$.
	Substitute the values for the adjacent (A) and hypotenuse (H).

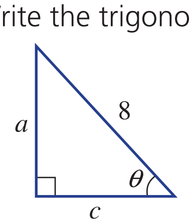
Now you try

Write down the ratio of $\sin\theta$ for this triangle.



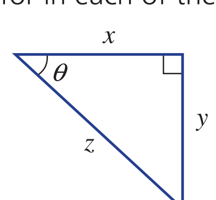
6 Write the trigonometric ratio asked for in each of the following.

a



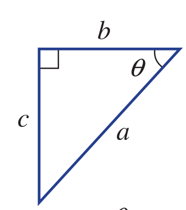
$\tan\theta =$

b



$\sin\theta =$

c



$\cos\theta =$

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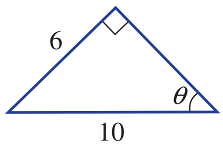
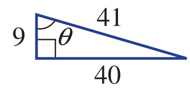
d



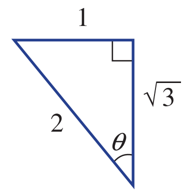
$\sin\theta =$

e



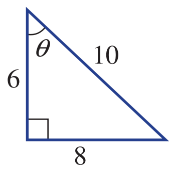
$\sin\theta =$

f



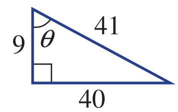
$\tan\theta =$

g



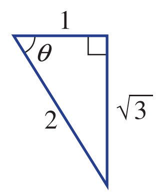
$\tan\theta =$

h



$\cos\theta =$

i



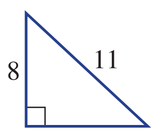
$\tan\theta =$

Problem-solving and reasoning

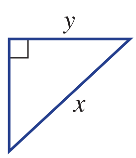
7–9 | 8-11

7 Copy each of these triangles and mark the angle θ that will enable you to write a ratio for $\sin\theta$.

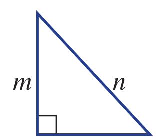
a



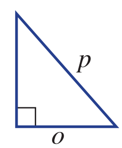
b



c



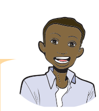
d

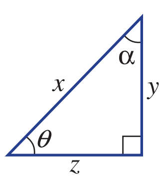


8 For the triangle shown, write a ratio for:

a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$
d $\cos\alpha$ **e** $\tan\theta$ **f** $\tan\alpha$

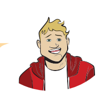
Hint: θ and α are letters of the Greek alphabet that are used to mark angles.





9 For each of the triangles below, decide which trigonometric ratio (sin, cos or tan) you would use for the given information.

Hint: First decide which two sides are involved.



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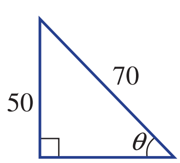

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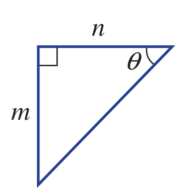

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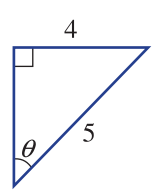

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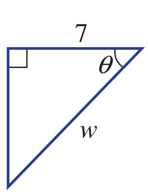

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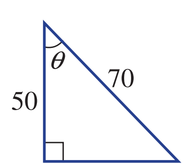

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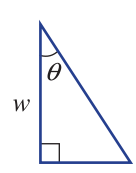

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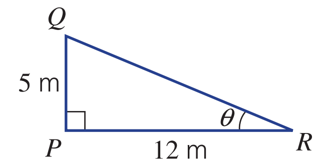
9D

10 Consider the triangle PQR.

a Use Pythagoras' theorem to find the length of QR.

b Write down the ratio of $\sin\theta$.



11 For a given right-angled triangle, $\sin\theta = \frac{1}{2}$.

a Draw a right-angled triangle and show this information.

b What is the length of the third side? Use Pythagoras' theorem.

c Find the value of:

i $\cos\theta$ **ii** $\tan\theta$

Relationship between sine and cosine — 12

12 Use your calculator to complete the table, answering to three decimal places where necessary.

a For what angle is $\sin\theta = \cos\theta$?

b Copy and complete:

i $\sin 5° = \cos$ ____°

ii $\sin 10° = \cos$ ____°

iii $\sin 60° = \cos$ ____°

iv $\sin 90° = \cos$ ____°

c Write down a relationship, in words, between sin and cos.

d Why do you think it's called cosine?

Angle (θ)	$\sin\theta$	$\cos\theta$
0°		
5°		
10°		
15°		
20°		
25°		
30°		
35°		
40°		
45°		
50°		
55°		
60°		
65°		
70°		
75°		
80°		
85°		
90°		

Hint: For most calculators, you enter the values in the same order as they are written. That is, $\sin 30° \rightarrow$ [sin] $30 = 0.5$.

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9E Finding side lengths

Learning intentions

- To be able to identify which trigonometric ratio can be used to set up an equation
- To be able to find a missing length on a right-angled triangle given an angle and another side

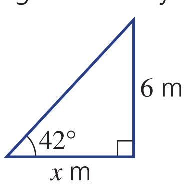
Key vocabulary: sine, cosine, tangent, hypotenuse, opposite, adjacent

In any right-angled triangle, given one of the acute angles and a side length, you can find the length of the other two sides. This can help builders find special lengths in right-angled triangles if they know an angle and the length of another side.

Lesson starter: Is it sin, cos or tan?

Of the six triangles below, only two provide enough information to directly use the sine ratio. Which two triangles are they?

A



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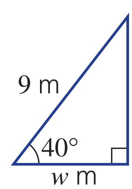
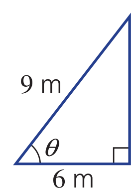


C



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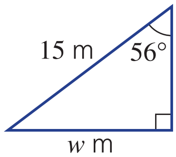

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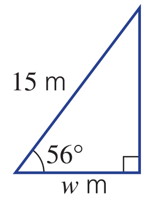

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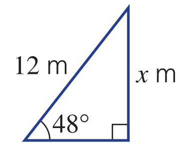
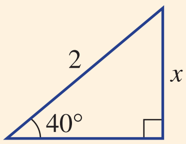


Key ideas

- To find a missing side when given a right-angled triangle with one acute angle and one of the sides:
  - Label the triangle using O (opposite), A (adjacent) and H (hypotenuse).
  - Use SOH CAH TOA to decide on the correct trigonometric ratio.
  - Write down the equation, using your chosen ratio.
  - Solve the equation, using your calculator, to find the unknown.

Write → $\frac{x}{2} = \sin 40°$

Solve → $x = 2 \times \sin 40°$



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9E

Exercise 9E

Understanding 1–2(½), 3 2(½), 3

Hint: Locate the sin, cos and tan buttons on your calculator.

1 Use a calculator to find the value of each of the following, correct to four decimal places.

a $\sin 10^\circ$ **b** $\cos 10^\circ$ **c** $\tan 10^\circ$
d $\tan 30^\circ$ **e** $\cos 40^\circ$ **f** $\sin 70^\circ$
g $\cos 80^\circ$ **h** $\tan 40^\circ$ **i** $\sin 80^\circ$

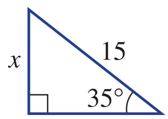
Hint: On your calculator, enter $12 \tan 10^\circ$ as $12 \times \tan 10$.

2 Evaluate each of the following, correct to two decimal places.

a $12 \tan 10^\circ$ **b** $12 \sin 25^\circ$ **c** $18 \tan 60^\circ$
d $56 \sin 56^\circ$ **e** $8 \tan 45^\circ$ **f** $20 \sin 70^\circ$

Hint: Remember to label the triangle and think SOH CAH TOA. Consider which two sides are involved.

3 Decide which of the three trigonometric ratios ($\sin$, $\cos$ or $\tan$) is suitable for these triangles. Do not solve.

a


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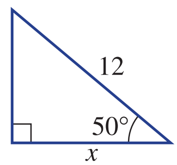

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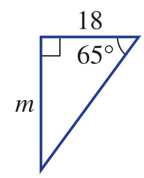

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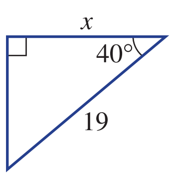

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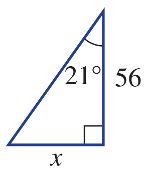

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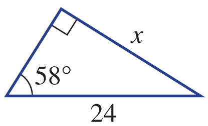

Fluency 4–8(½) 4–9(½)

Example 10 Solving a trigonometric equation

Find the value of x, correct to two decimal places, for $\cos 30^\circ = \frac{x}{12}$.

Solution	Explanation
$\cos 30^\circ = \frac{x}{12}$	Multiply both sides by 12 to get x on its own. $12 \times \cos 30^\circ = \frac{x}{\cancel{12}_1} \times \cancel{12}_1$
$x = 12 \times \cos 30^\circ$	
$= 10.39230\ldots$	Use your calculator.
$= 10.39$ (to 2 d.p.)	Round as indicated.

Now you try

Find the value of x, correct to two decimal places, for $\sin 40^\circ = \frac{x}{6}$.

4 Find the value of x in these equations, correct to two decimal places.

a $\sin 20^\circ = \frac{x}{4}$ **b** $\cos 43^\circ = \frac{x}{7}$ **c** $\tan 85^\circ = \frac{x}{8}$
d $\tan 30^\circ = \frac{x}{24}$ **e** $\sin 50^\circ = \frac{x}{12}$ **f** $\cos 40^\circ = \frac{x}{12}$

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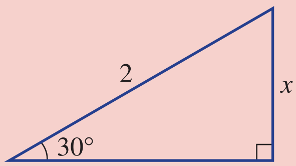
Example 11 Finding a missing side, using SOH CAH TOA

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

a



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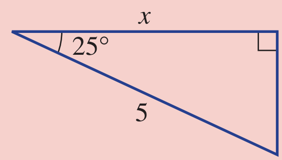

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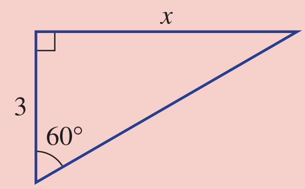


Solution | **Explanation**

a

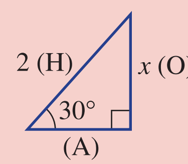
$$\sin\theta = \frac{\text{O}}{\text{H}}$$

$$\sin 30° = \frac{x}{2}$$

$$2 \times \sin 30° = x$$

$$\therefore\ x = 1$$

Label the triangle and decide on your trigonometric ratio SOH CAH TOA.



Write the ratio.

Substitute values and solve the equation, using your calculator.

b

$$\cos\theta = \frac{\text{A}}{\text{H}}$$

$$\cos 25° = \frac{x}{5}$$

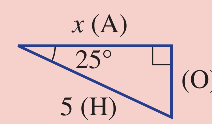
$$5 \times \cos 25° = x$$

$$x = 4.5315\ldots$$

$$\therefore\ x = 4.53 \text{ (to 2 d.p.)}$$

Label the triangle.

SOH CAH TOA



Write the ratio.

Substitute values and solve the equation, using your calculator.

Round to two decimal places.

c

$$\tan\theta = \frac{\text{O}}{\text{A}}$$

$$\tan 60° = \frac{x}{3}$$

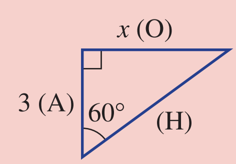
$$3 \times \tan 60° = x$$

$$x = 5.1961\ldots$$

$$\therefore\ x = 5.20 \text{ (to 2 d.p.)}$$

Label the triangle.

SOH CAH TOA



Write the ratio.

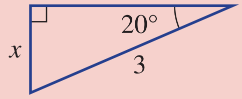
Substitute values and solve the equation, using your calculator.

Round to two decimal places.

Now you try

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

a



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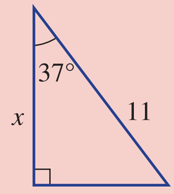

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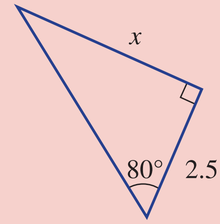

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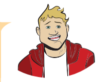
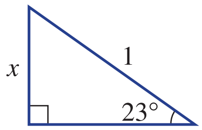
9E

5 Triangles with one unknown side are given below.
  i Copy each one and label the three sides opposite (O), adjacent (A) and hypotenuse (H).
  ii Decide on a trigonometric ratio.
  iii Find the value of each pronumeral, correct to two decimal places.



Hint: Use SOH CAH TOA to help you decide which ratio to use. If O and H are involved, use sin etc.

a


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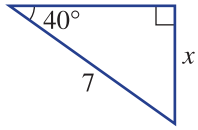

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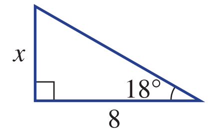

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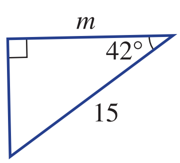

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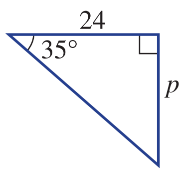

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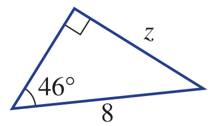

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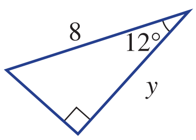

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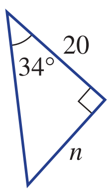

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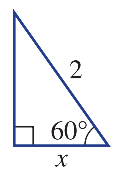
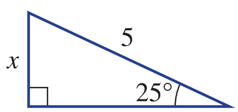


6 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

Hint: What ratio did you use for each of these?

a


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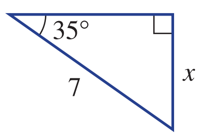

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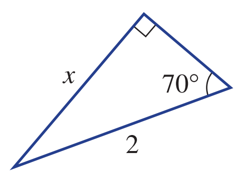
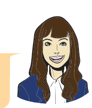
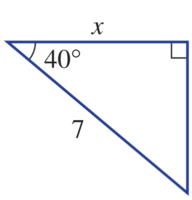


7 Find the value of the unknown length (x) in these triangles. Round to two decimal places.



Hint: These three all use cos.

a


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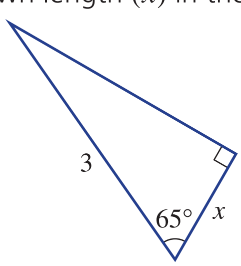

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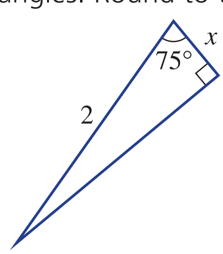
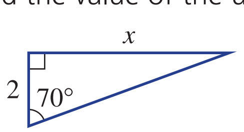


8 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

Hint: These all use tan.

a


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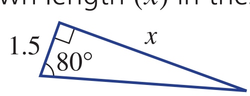

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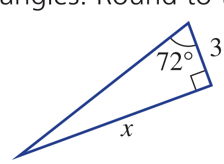

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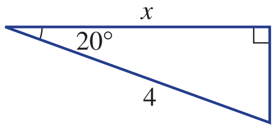
9 Decide whether to use sin, cos or tan, then find the value of x in these triangles. Round to two decimal places.

a


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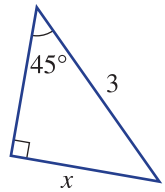

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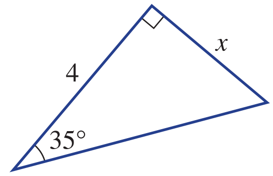

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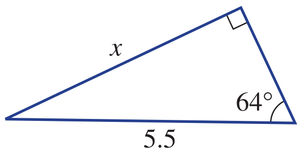

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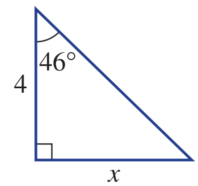

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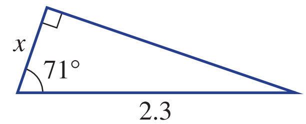
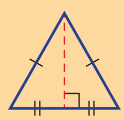


Problem-solving and reasoning

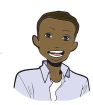
	10, 11	10, 12

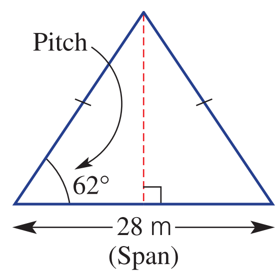
10 **a** Find the height of this isosceles triangle, which is similar to a roof truss, to two decimal places.

b If the span doubles to 56 m, what is the height of the roof, to two decimal places?

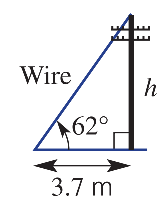


Hint: In an isosceles triangle, the perpendicular cuts the base in half.

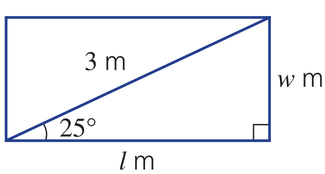




11 The stay wire of a power pole joins the top to the ground. It makes an angle of 62° with the ground. It is fixed to the ground 3.7 m from the bottom of the pole. How high is the pole, correct to two decimal places?



12 Find the length and width of these rectangles, to two decimal places.

a


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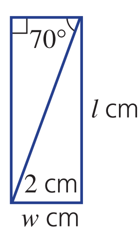

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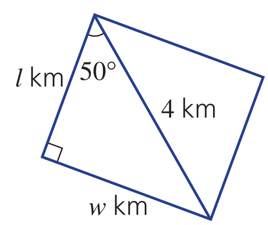
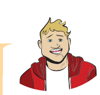


Hint: Use the hypotenuse in each calculation.



Accuracy and errors

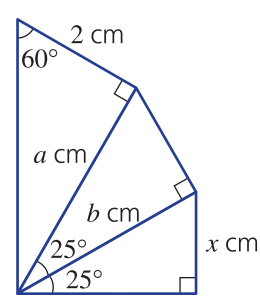
	—	13

13 Our aim is to find the value of x, correct to two decimal places, by first finding the value of a and b.

a Find the value of a, then b and then x, using one decimal place for a and b.

b Repeat this process, finding a and b, correct to three decimal places each, before finding x.

c Does it make any difference to your final answer for x if you round off the values of a and b during calculations?



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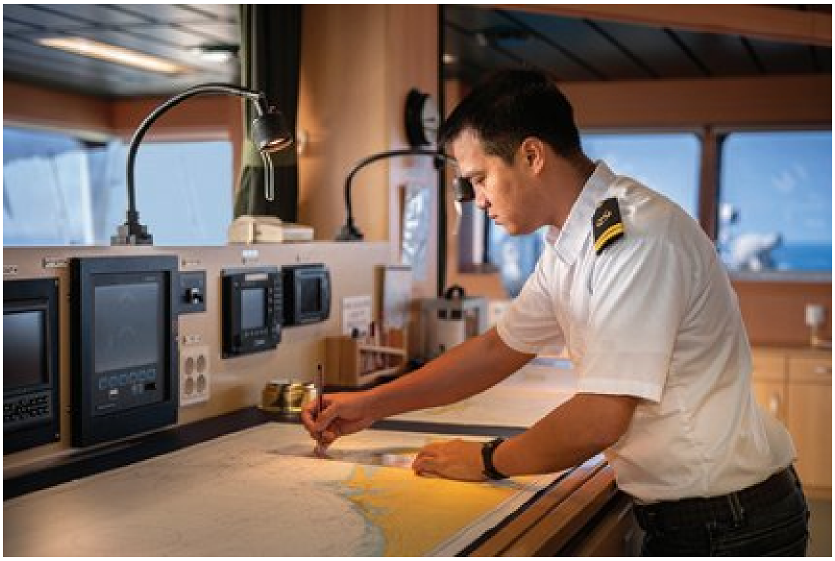
9F Solving for the denominator

Learning intentions

- To be able to identify which trigonometric ratio can be used to set up an equation
- To be able to find a missing length on a right-angled triangle if the variable sits in the denominator of the equation

Key vocabulary: denominator, hypotenuse, opposite, adjacent

So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator and these problems can be solved with an extra step in your mathematical working.



Lesson starter: Solving equations with x in the denominator

Consider the equations $\frac{x}{3} = 4$ and $\frac{3}{x} = 4$.

- Do the equations have the same solution?
- What steps are used to solve the equations?
- Now solve $\frac{4}{x} = \sin 30°$ and $\frac{2}{x} = \cos 40°$.

Key ideas

- When the unknown value is in the **denominator**, you need to do two algebraic steps to find the unknown. For example,

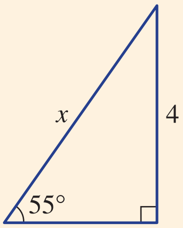
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55° = \frac{4}{x}$$

$$x \times \sin 55° = 4$$

$$x = \frac{4}{\sin 55°}$$

$$= 4.88 \text{ (to two decimal places)}$$



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Exercise 9F

Understanding | 1–3 | 2, 3

Hint: For part **a**, enter 10 ÷ tan 30 into your calculator.

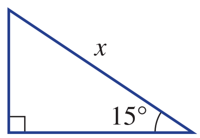
1 Find the value, correct to two decimal places, of:

a $\dfrac{10}{\tan 30^\circ}$ **b** $\dfrac{12}{\sin 60^\circ}$ **c** $\dfrac{15}{\tan 8^\circ}$

d $\dfrac{12.4}{\tan 32^\circ}$ **e** $\dfrac{15.2}{\sin 38^\circ}$ **f** $\dfrac{9}{\cos 47^\circ}$

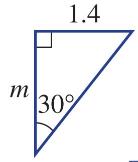
2 For each of these triangles, complete the required trigonometric ratio.

a



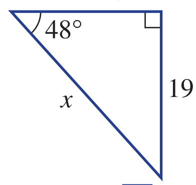
$\cos 15^\circ = \dfrac{\square}{\square}$

b



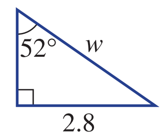
$\tan 30^\circ = \dfrac{\square}{\square}$

c



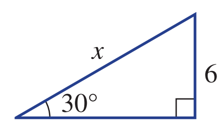
$\sin 48^\circ = \dfrac{\square}{\square}$

d



$\sin 52^\circ = \dfrac{\square}{\square}$

3 Which one of the following is the correct working to solve for x in this triangle?



A
$$\cos 30^\circ = \frac{6}{x}$$
$$x\cos 30^\circ = 6$$
$$x = \frac{6}{\cos 30^\circ}$$
$$= 6.93$$

B
$$\sin 30^\circ = \frac{6}{x}$$
$$x = 6 \times \sin 30^\circ$$
$$= 3$$

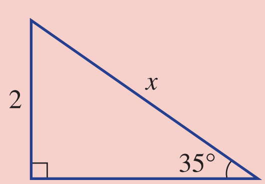
C
$$\sin 30^\circ = \frac{6}{x}$$
$$x\sin 30^\circ = 6$$
$$x = \frac{6}{\sin 30^\circ}$$
$$= 12$$

Fluency | 4–6 | 4–6, 7(½)

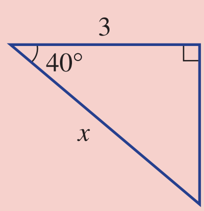
Example 12 Finding the value of the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

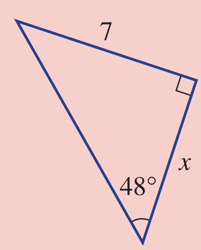
a



b



c



Solution	Explanation
a $\sin 35^\circ = \dfrac{2}{x}$	Use $\sin\theta = \dfrac{\text{O}}{\text{H}}$, as we can use the opposite (2) and hypotenuse (x).
$x \times \sin 35^\circ = 2$	Multiply both sides by x.
$x = \dfrac{2}{\sin 35^\circ}$	Divide both sides by $\sin 35^\circ$ to get x on its own. Recall that $\sin 35^\circ$ is just a number.
$\therefore\ x = 3.49$ (to 2 d.p.)	Evaluate and round your answer.

Continued on next page

 ISBN 978-1-108-87859-3

b $\cos 40° = \dfrac{3}{x}$ — Use $\cos\theta = \dfrac{\text{A}}{\text{H}}$, as we can use the adjacent (3) and hypotenuse (x).

$x \times \cos 40° = 3$ — Multiply both sides by x.

$x = \dfrac{3}{\cos 40°}$ — Divide both sides by $\cos\ 40°$ to get x on its own.

$\therefore\ x = 3.92$ (to 2 d.p.) — Evaluate and round your answer.

c $\tan 48° = \dfrac{7}{x}$ — Use $\tan\theta = \dfrac{\text{O}}{\text{A}}$, as we can use the adjacent (x) and opposite (7).

$x \times \tan 48° = 7$ — Multiply both sides by x.

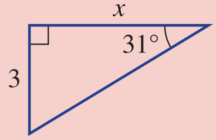
$x = \dfrac{7}{\tan 48°}$ — Divide both sides by $\tan 48°$ to get x on its own.

$\therefore\ x = 6.30$ (to 2 d.p.) — Evaluate and round your answer.

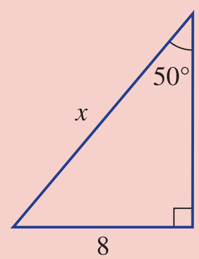
Now you try

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

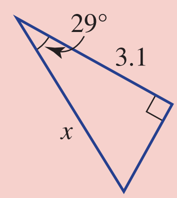
a



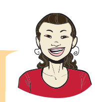
b



c

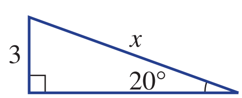


4 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.



Hint: In $\sin 20° = \dfrac{3}{x}$, multiply both sides by x first.

a



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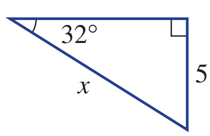

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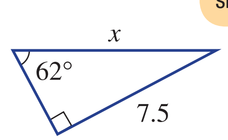
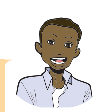
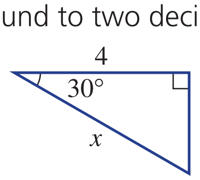


5 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.



Hint: $\cos\theta = \dfrac{\text{A}}{\text{H}}$

a



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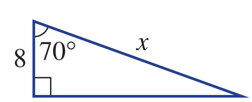

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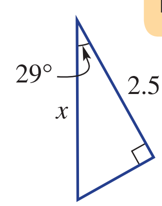

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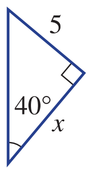
6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

a


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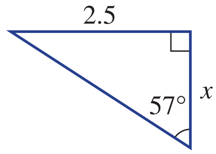

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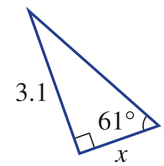
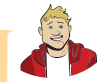
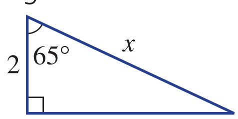


Hint: $\tan\theta = \frac{O}{A}$



7 By first deciding whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$, find the value of x in these triangles. Round to two decimal places.

Hint: SOH CAH TOA

a


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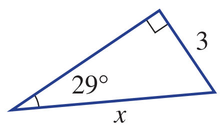

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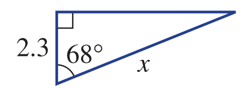

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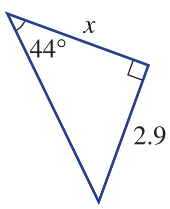

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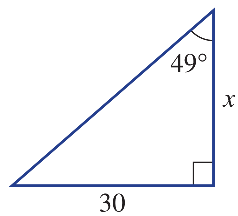

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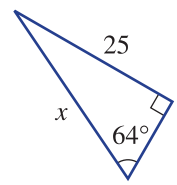
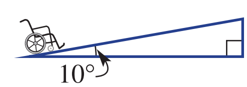


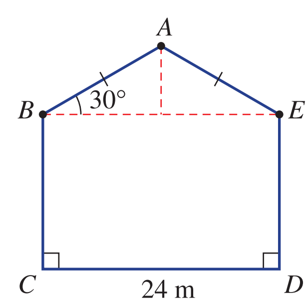
Problem-solving and reasoning

8–10 | 9–12

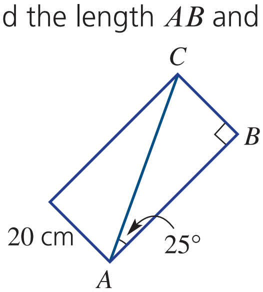
8 The recommended angle of a wheelchair ramp to the horizontal is approximately 10°. How long is the ramp if the horizontal distance is 2.5 metres? Round your answer to two decimal places.



9 The roof of this barn has a pitch of 30°, as shown. Find the length of roof section AB, to one decimal place.



10 Find the length AB and BC in these shapes. Round your answers to two decimal places.

a


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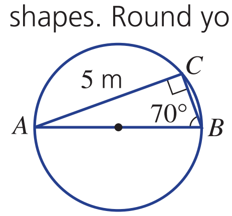

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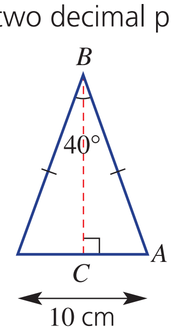

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Cambridge University Press

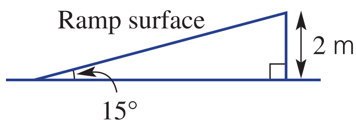
9F

11 The ramp shown has an incline angle of 15° and a height of 2 m. Find, correct to three decimal places:

a the base length of the ramp

b the length of the ramp surface

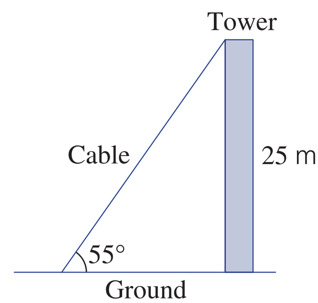
Hint: The 'incline' is the angle to the horizontal.



12 For this communications tower, find, correct to one decimal place:

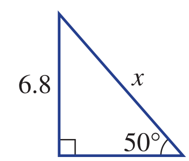
a the length of the cable

b the distance from the base of the tower to the point where the cable is attached to the ground



Inverting the fraction — 13

Shown below is another way of solving trigonometric equations with x in the denominator.



Find x, to two decimal places.

$\sin 50^\circ = \dfrac{6.8}{x}$ First, write the correct ratio.

$\dfrac{1}{\sin 50^\circ} = \dfrac{x}{6.8}$ Invert both fractions so x is in the numerator.

$x = \dfrac{1}{\sin 50^\circ} \times 6.8$ $\dfrac{1}{\sin 50^\circ} \times 6.8 = \dfrac{x}{6.8} \times 6.8$

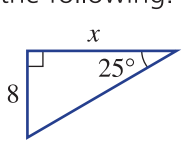
$x = \dfrac{6.8}{\sin 50^\circ}$ Multiply both sides by 6.8 to get x on its own.

$x = 8.876769\ldots$ Use your calculator.

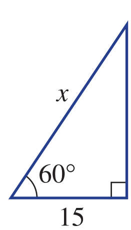
$x = 8.88$ (to 2 d.p.) Round your answer as required.

13 Use the method shown above to find the value of x, to two decimal places where necessary, in each of the following.

a



b



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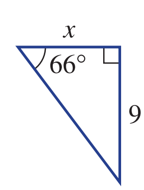
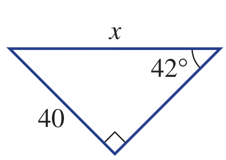
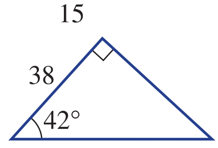


d



e



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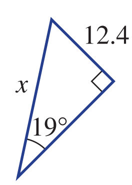

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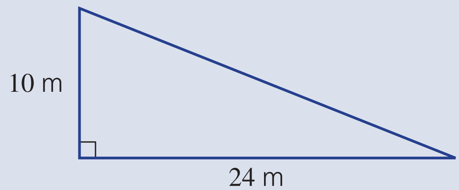
Progress quiz

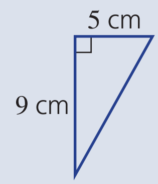
9A

1 Find the length of the hypotenuse in the following right-angled triangles. Round to one decimal place in part **b**.

a



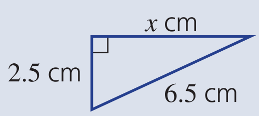
b



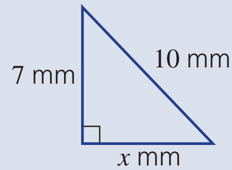
9B

2 Find the value of the pronumeral in the following triangles. Round to one decimal place in part **b**.

a

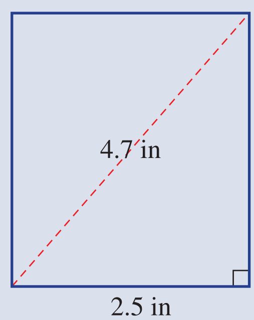


b



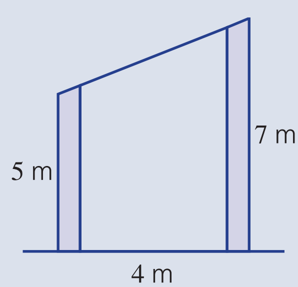
9B

3 A phone has a screen size of 4.7 inches (i.e. this is the distance from corner to corner). If the screen has a width of 2.5 inches, what is the screen height, correct to one decimal place?



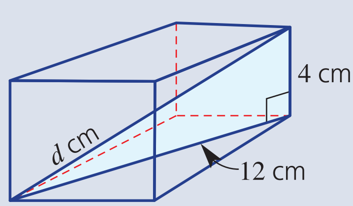
9C

4 A straight wire connects the tops of two telegraph poles. The poles are 7 m and 5 m high and are located 4 m apart. What is the length of the wire, correct to two decimal places?



9C

5 Find the distance (d cm) from one corner to the opposite corner of this rectangular prism, correct to one decimal place.



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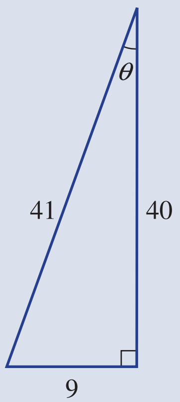
9D

6 For the triangle shown, write down the ratio for:

a $\cos\theta$

b $\sin\theta$

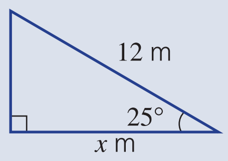
c $\tan\theta$



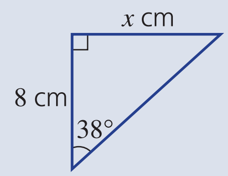
9E

7 Find the value of x in these triangles. Round your answers to one decimal place.

a



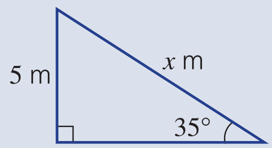
b



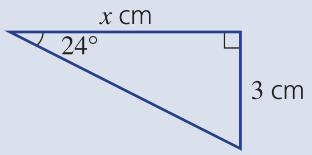
9F

8 Find the value of x in these triangles. Round your answers to one decimal place.

a



b

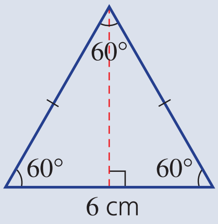


9B/F

9 Use each of the following methods to find the area of the equilateral triangle shown. Give your answer to one decimal place.
(Hint: First you will need to find the height of the triangle.)

a Pythagoras' theorem

b Trigonometry



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9G Finding angles

Learning intentions

- To know that angles can be found using inverse trigonometric functions
- To be able to find an angle in a right-angled triangle given any two side lengths

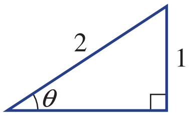
Key vocabulary: inverse trigonometric function or ratio

Given two side lengths of a right-angled triangle, you can find either of the acute angles. There are many different situations where you might be given two side lengths of a right-angled triangle and be asked to find the associated angles. Angles associated with flight for example, can be calculated using trigonometry.

Lesson starter: Knowing the angle

Imagine a triangle that produces $\sin\theta = 0.5$.

- Use your calculator and trial and error to find a value of θ for which $\sin\theta = 0.5$.
- Repeat for $\tan\theta = 1$ and $\cos\theta = \frac{\sqrt{3}}{2}$.
- Do you know of a quicker method, rather than using trial and error?



Key ideas

- To find an angle, you use **inverse trigonometric ratios** on your calculator.

  If $\sin\theta = x$, then $\theta = \sin^{-1}(x)$; $\sin^{-1}$ is inverse sin.
  If $\cos\theta = y$, then $\theta = \cos^{-1}(y)$; $\cos^{-1}$ is inverse cos.
  If $\tan\theta = z$, then $\theta = \tan^{-1}(z)$; $\tan^{-1}$ is inverse tan.

- Look for the three calculator buttons/functions:

  $\boxed{\sin^{-1}}$ $\boxed{\cos^{-1}}$ $\boxed{\tan^{-1}}$

Exercise 9G

Understanding	1–3	3

1 Use your calculator to find $\sin 30°$ and $\sin^{-1}(0.5)$. What do you notice?

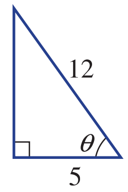
2 Use your calculator to evaluate the following, correct to the nearest whole degree.

a $\sin^{-1}(0.71)$ **b** $\cos^{-1}(0.866)$ **c** $\tan^{-1}(1.6)$

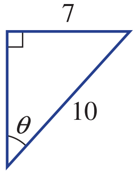
Hint: Many calculators use $\boxed{\text{shift}}$ to access $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$.

3 Write down the trigonometric ratio for these triangles, e.g. $\sin\theta = \frac{2}{3}$.

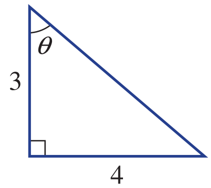
a



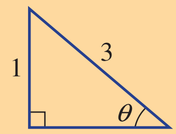
b



c



Hint: Look for: SOH CAH TOA



$\sin\theta = \frac{1}{3}$

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9G

Fluency

4–5(½) 4–6(½)

Example 13 Finding an angle

Find the angle θ, correct to the nearest degree where necessary, in each of the following.

a $\sin\theta = \frac{2}{3}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1.7$

Solution	Explanation
a $\sin\theta = \frac{2}{3}$ $\theta = \sin^{-1}\left(\frac{2}{3}\right)$ $\theta = 42°$ (to nearest degree)	Look for the $\sin^{-1}$ button on your calculator. Round as required.
b $\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta = 60°$	Look for the $\cos^{-1}$ button on your calculator.
c $\tan\theta = 1.7$ $\theta = \tan^{-1}(1.7)$ $\theta = 60°$ (to nearest degree)	Look for the $\tan^{-1}$ button on your calculator. Round to the nearest degree.

Now you try

Find the angle θ, correct to the nearest degree where necessary, in each of the following.

a $\cos\theta = 0.7$ **b** $\tan\theta = 2.5$ **c** $\sin\theta = \frac{4}{5}$

4 Find the angle θ, to the nearest degree, for the following.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{3}{5}$ **c** $\sin\theta = \frac{7}{8}$

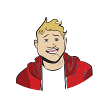
d $\tan\theta = 1$ **e** $\tan\theta = \frac{7}{8}$ **f** $\sin\theta = \frac{8}{10}$

g $\cos\theta = \frac{2}{3}$ **h** $\sin\theta = \frac{1}{10}$ **i** $\cos\theta = \frac{4}{5}$

j $\tan\theta = 6$ **k** $\cos\theta = \frac{3}{10}$ **l** $\tan\theta = \sqrt{3}$

m $\sin\theta = \frac{4}{6}$ **n** $\cos\theta = \frac{4}{6}$ **o** $\tan\theta = \frac{4}{6}$

Hint:
Remember: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on the calculator.



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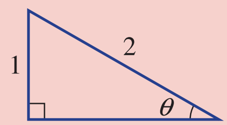
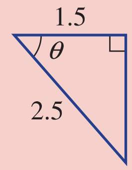
Example 14 Using SOH CAH TOA to find angles

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

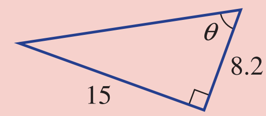
a



b



c

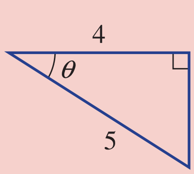


Solution	Explanation
a $\sin\theta = \frac{O}{H}$	Use $\sin\theta$ since we know the opposite and the hypotenuse.
$\sin\theta = \frac{1}{2}$	Substitute O = 1 and H = 2.
$\theta = \sin^{-1}\left(\frac{1}{2}\right)$	Use your calculator to find $\sin^{-1}\left(\frac{1}{2}\right)$.
$= 30°$	
b $\cos\theta = \frac{A}{H}$	Use $\cos\theta$ since we know the adjacent and the hypotenuse.
$\cos\theta = \frac{1.5}{2.5}$	Substitute A = 1.5 and H = 2.5.
$\theta = \cos^{-1}\left(\frac{1.5}{2.5}\right)$	Use your calculator to find $\cos^{-1}\left(\frac{1.5}{2.5}\right)$.
$= 53.13°$ (to 2 d.p.)	Round to two decimal places.
c $\tan\theta = \frac{O}{A}$	Use $\tan\theta$ since we know the opposite and the adjacent.
$\tan\theta = \frac{15}{8.2}$	Substitute O = 15 and A = 8.2.
$\theta = \tan^{-1}\left(\frac{15}{8.2}\right)$	Use your calculator to find $\tan^{-1}\left(\frac{15}{8.2}\right)$.
$= 61.34°$ (to 2 d.p.)	Round to two decimal places.

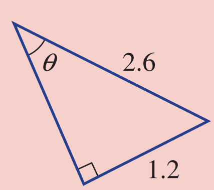
Now you try

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

a



b



c

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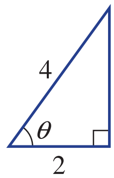
9G

5 Use one of sin, cos or tan to find θ in these triangles, rounding to two decimal places where necessary.

a


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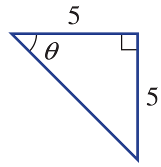

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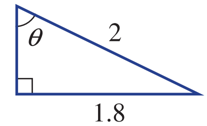

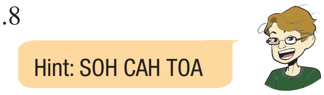

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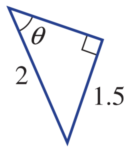

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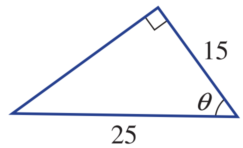

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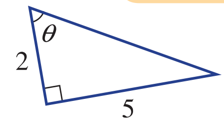
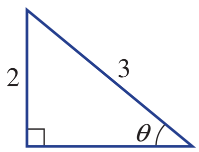


6 Find the angle θ, correct to the nearest degree, in these triangles. You will need to decide whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$.

a


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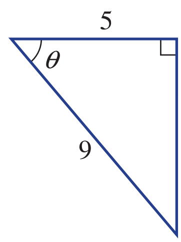

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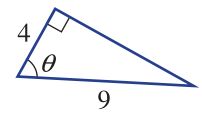

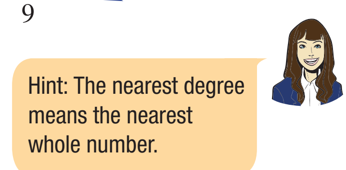

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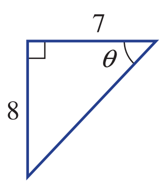

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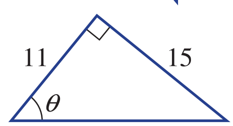

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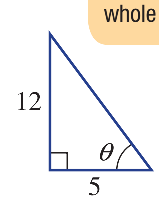

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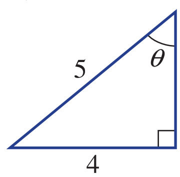

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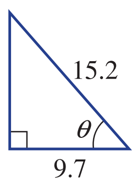

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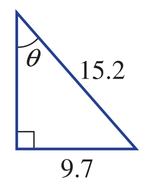

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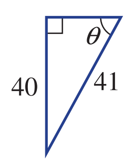

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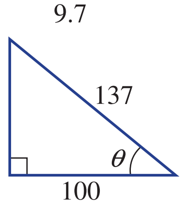

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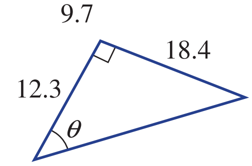
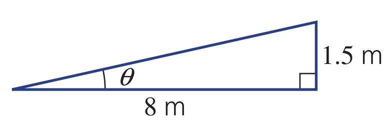
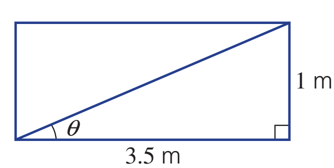


Problem-solving and reasoning | 7, 8 | 8–10

7 A ramp is 8 m long horizontally and 1.5 m high. Find the angle the ramp makes with the ground, correct to two decimal places.



8 A rectangular piece of wood 1 m wide and 3.5 m long is to be cut across the diagonal. Find the angle the cut makes with the long side (correct to two decimal places).



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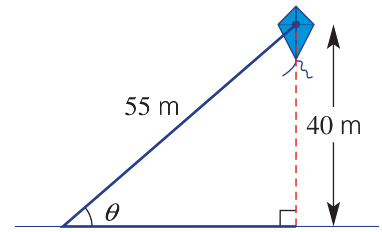
9 At what angle to the ground is a kite (shown) with height 40 m and string length 55 m? Round to two decimal places.



10 Find the two acute angles in a right-angled triangle with the given side lengths, correct to one decimal place.

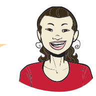
a hypotenuse 5 cm, other side 3 cm

b hypotenuse 7 m, other side 4 m

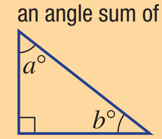
c hypotenuse 0.5 mm, other side 0.3 mm

d the two shorter side lengths are 3 cm and 6 cm

e the two shorter side lengths are 10 m and 4 m



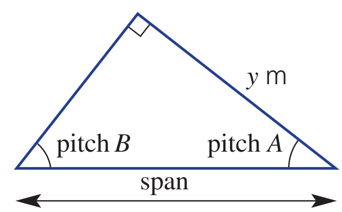
Hint:
- Draw a picture.
- Use SOH CAH TOA.
- Find one acute angle using trigonometry.
- Remember: all triangles have an angle sum of 180°.



$a + b + 90 = 180$

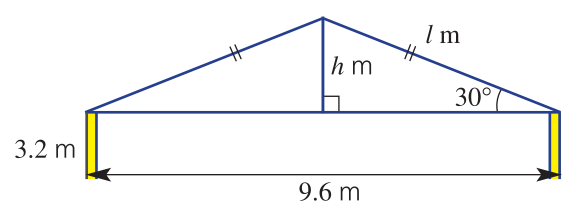
Building and construction — 11–13

11 A roof is pitched so that the angle at its peak is 90°. If each roof truss spans 10.5 m and distance y is 7.2 m, find the pitch angles A and B, to the nearest whole number.

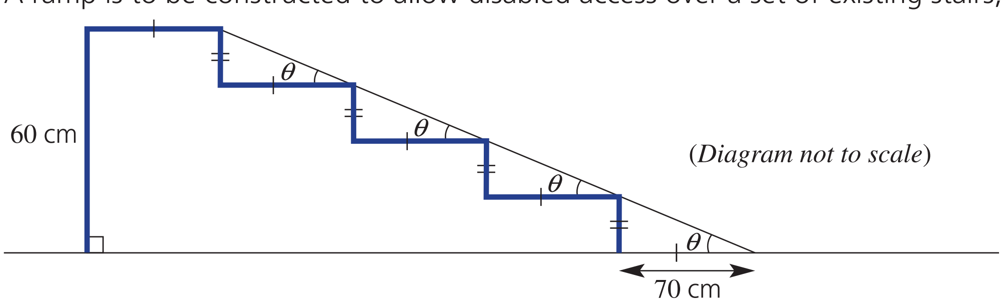


12 **a** Find the length of the slats (l metres) needed along each hypotenuse for this roof cross-section, correct to two decimal places.

b Find the height of the highest point of the roof above ground level, correct to two decimal places.



13 A ramp is to be constructed to allow disabled access over a set of existing stairs, as shown.



a What angle does the ramp make with the ground, to the nearest degree?

b Government regulations state that the ramp cannot be more than 13° to the horizontal. Does this ramp meet these requirements?

c How long is the ramp? Round your answer to one decimal place.

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9H Angles of elevation and depression

Learning intentions

- To know how angles of elevation and depression can be identified in a real context
- To be able to use trigonometry to solve problems involving angles of elevation and depression

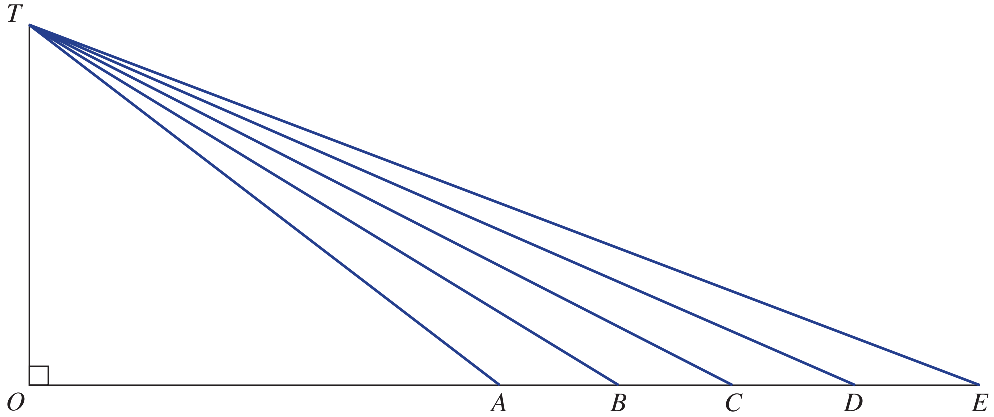
Key vocabulary: elevation, depression, horizontal

Many applications of trigonometry involve angles of elevation and angles of depression. These angles are measured up or down from a horizontal level. Whenever you view something above or below the horizontal, you form an angle of elevation or depression.

Lesson starter: How close should you sit?

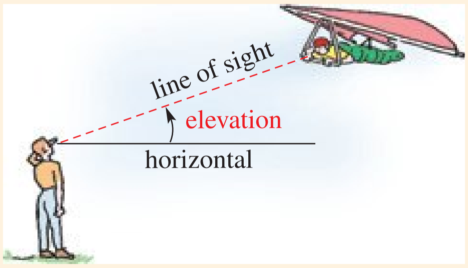
The diagram below shows an outdoor movie screen (OT). The point T is the top of the screen. The points $A-E$ are the five rows of seats in the theatre, from which a person's line of sight is taken. The line OE is the horizontal line of sight.

- Use your protractor to measure the angle of elevation from each point along the horizontal to the top of the movie screen.
- Where should you sit if you wish to have an angle of elevation between 25° and 20° and not be in the first or last row of the theatre?



Key ideas

- Looking up at an object forms an **angle of elevation** above the horizontal.



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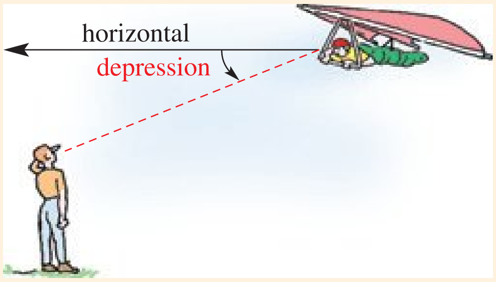
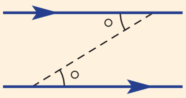
- Looking down at an object forms an **angle of depression** below the horizontal.



- The angle of elevation will equal the angle of depression in the same context.



Exercise 9H

Understanding | 1–3 | 2, 3

1 Give the missing word.

a Looking down at an object forms an angle of ________________.

b Looking up at an object forms an angle of ________________.

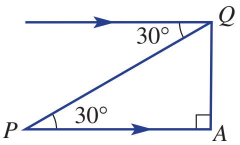
c Angles of elevation and depression are measured up or down from the ________________.

2 This diagram shows angles of elevation and depression.

a What is the angle of elevation of Q from P?

b What is the angle of depression of P from Q?

c What is the size of $\angle PQA$?

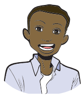


3 For each description, draw a triangle diagram that matches the information given.

a The angle of elevation to the top of a tower from a point 50 m from its base is 55°.

b The angle of depression from the top of a 200 m cliff to a boat out at sea is 22°.

c The angle of elevation of the top of a castle wall from a point on the ground 30 m from the castle wall is 33°.



Hint: Always measure angles of elevation and depression from the horizontal.

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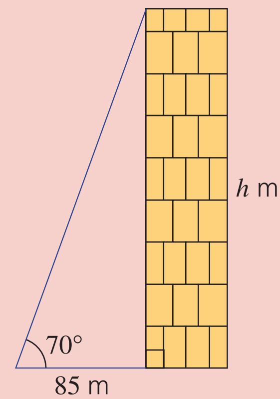
9H

Fluency 4–6(½) 4–6(½)

Example 15 Using an angle of elevation

To find the height of a tall building, Johal stands 85 m away from its base and measures the angle of elevation at the top of the building as 70°. Find the height of the building, correct to the nearest metre.



Solution

$\tan\theta = \frac{\text{O}}{\text{A}}$

$\tan 70° = \frac{h}{85}$

$h = 85 \times \tan 70°$

$= 233.53558\ldots$

$= 234$ m (to the nearest metre)

∴ The building is 234 m tall.

Explanation

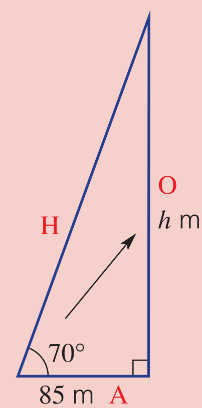
Label the triangle with O, A and H.

Use tan since the opposite and adjacent are given.

Find h by solving the equation.

$85 \times \tan 70° = \frac{h}{85} \times 85$

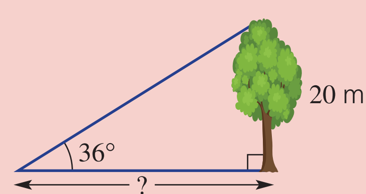
Round to the nearest metre.



Now you try

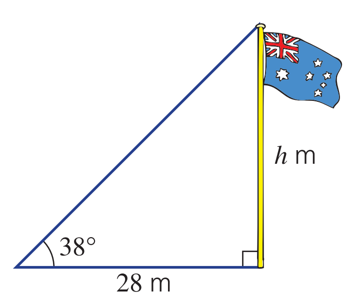
A 20 m high tree casts a shadow of unknown length.

Find the length of the shadow if the angle of elevation of the top of the tree from the end of the shadow is 36°. Round to the nearest metre.



4 Answer the following questions about angles of elevation.

a The angle of elevation to the top of a flagpole from a point 28 m from its base is 38°. How tall is the flagpole, correct to two decimal places?



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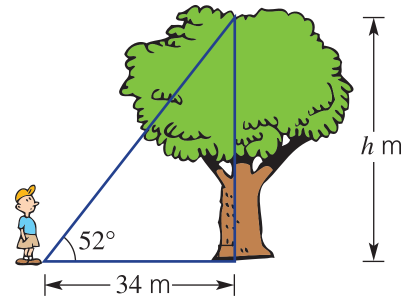
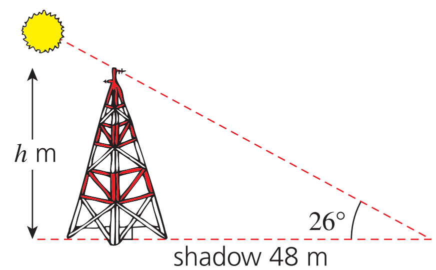
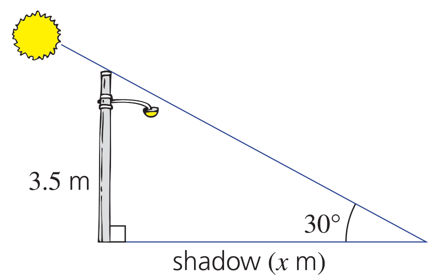
b Alvin is 34 m away from a tree and the angle of elevation to the top of the tree from the ground is 52°. What is the height of the tree, correct to one decimal place?



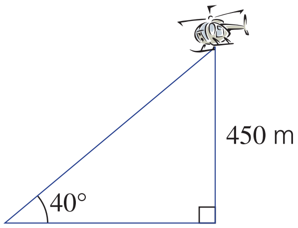
c The Sun's rays shining over a tower make an angle of elevation of 26° and cast a 48 m shadow on the ground. How tall, to two decimal places, is the tower?



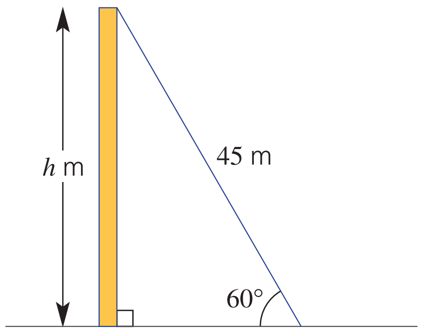
d The Sun makes an angle of elevation of 30° with a lamp post that is 3.5 m tall. How long is the shadow on the ground, correct to two decimal places?



e The altitude of a hovering helicopter is 450 m, and the angle of elevation from the helipad to the helicopter is 40°. Find the horizontal distance from the helicopter to the helipad, correct to two decimal places.



f A cable of length 45 m is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, correct to two decimal places.



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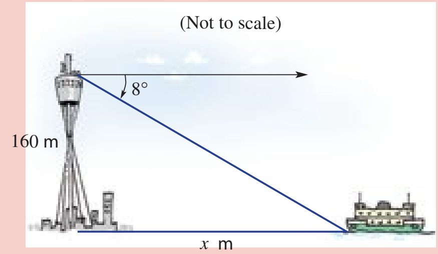
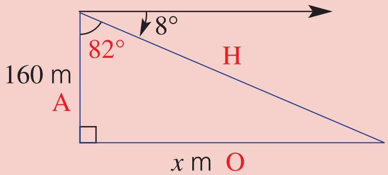
Example 16 Using an angle of depression

From the observation room of Centrepoint Tower in Sydney, which has a height of 160 m, the angle of depression of a boat moored at Circular Quay is observed to be 8°. How far from the base of the tower is the boat, correct to the nearest metre?



Solution



$$\tan\theta = \frac{O}{A}$$

$$\tan 82° = \frac{x}{160}$$

$$x = 160 \times \tan 82°$$
$$= 1138.459\ldots$$
$$= 1138 \text{ (to the nearest metre)}$$

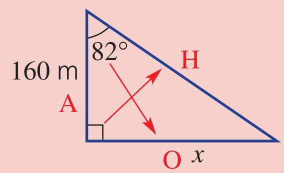
∴ The boat is about 1138 m from the base of the tower.

Explanation

Draw the triangle and find the angle inside the triangle:
$90° - 8° = 82°$

Use this angle to label the triangle.

Use tan since we have the opposite and adjacent.



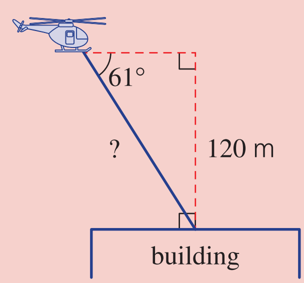
Find x by solving the equation.

Round to the nearest metre.

Now you try

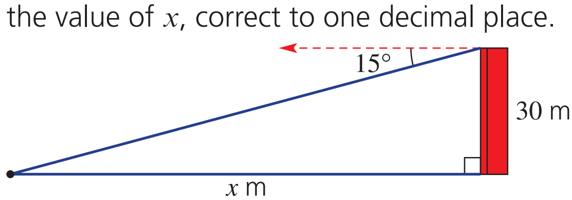
The angle of depression from a hovering helicopter to a building on the ground is 61°.

If the vertical height of the helicopter above the building is 120 m, find the direct distance from the helicopter to the building. Round to one decimal place.

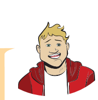


5 Answer these problems relating to angles of depression.

a The angle of depression from the top of a tower 30 m tall to a point x m from its base is 15°. Find the value of x, correct to one decimal place.



Hint: Use 15° to label an angle inside the triangle.



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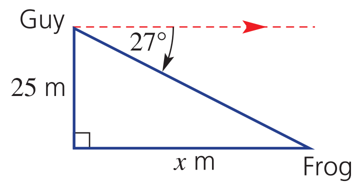
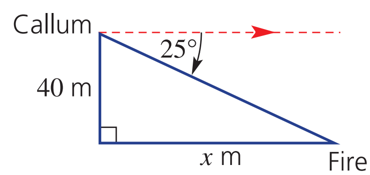
b From a bridge 25 m above a stream, Guy spots two frogs on a lilypad. He estimates the angle of depression to the frogs to be $27°$. How far from the bridge are the frogs, to the nearest metre?

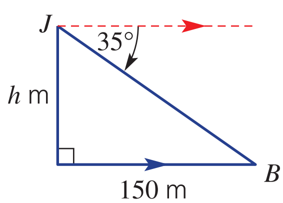


Hint: The angle of depression is the angle below the horizontal, looking down at an object.

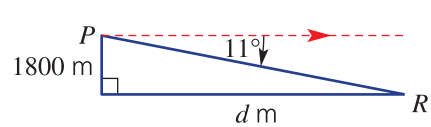
c From a lookout tower, Callum spots a bushfire at an angle of depression of $25°$. If the lookout tower is 40 m tall, how far away (to the nearest metre) is the bushfire from the base of the tower?



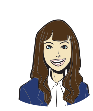
d From the top of a vertical cliff, Jung spots a boat 150 m out to sea. The angle of depression from Jung to the boat is $35°$. How many metres (to the nearest whole number) above sea level is Jung?



e A plane is flying 1800 m above the ground. At the time the pilots spot the runway, the angle of depression to the edge of the runway is $11°$. How far does the plane have to fly to be above the edge of the runway at its current altitude? Answer to the nearest whole number.



Hint: 'Altitude' means height.



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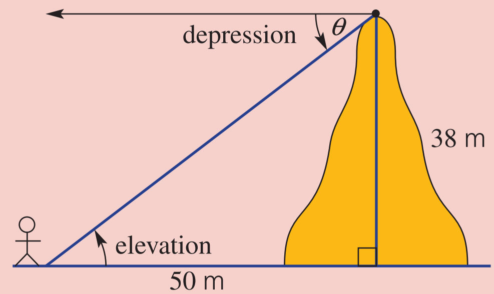
9H

Example 17 Finding angles of elevation and depression

a Find the angle of depression from the top of the hill to a point on the ground 50 m from the middle of the hill. Answer to the nearest degree.

b What is the angle of elevation from the point on the ground to the top of the 38 m hill? Answer to the nearest degree.



Solution

a $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{38}{50}$

$\theta = \tan^{-1}\left(\frac{38}{50}\right)$

$\theta = 37.2348\ldots$

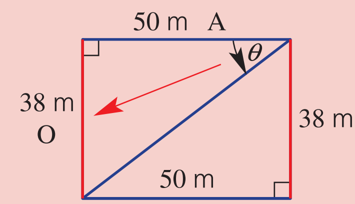
$\theta = 37°$ (to the nearest degree)

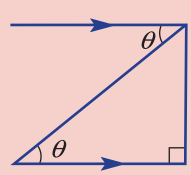
Angle of depression is 37°.

b Angle of elevation is 37°.

Explanation

Aim to find θ. Redraw the diagram as a rectangle so that θ is inside the triangle. Label the triangle, opposite and adjacent. Use tan.

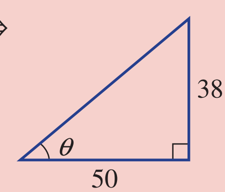




Alternate angles are equal when lines are parallel.

angle of elevation = angle of depression

Alternatively ⇒



$\Rightarrow \tan\theta = \frac{38}{50}$

Now you try

From a 20 m high lookout platform, you can see a fire that is 150 m away horizontally. Find the angle of depression from the lookout platform to the fire, correct to the nearest degree.

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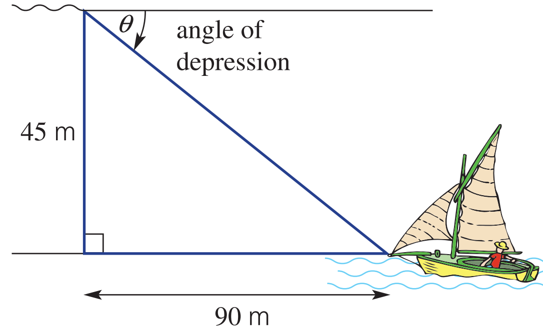
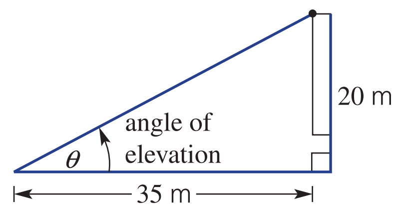
6 Answer these questions about finding angles of elevation and depression. Round all answers to one decimal place.

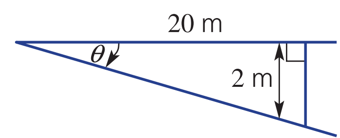
a From the top of a vertical cliff, Jacquie spots a boat 90 m out to sea. If the top of the cliff is 45 m above sea level, find the angle of depression from the top of the cliff to the boat.



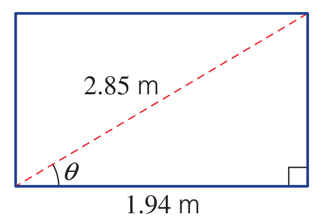
b Find the angle of elevation from a person sitting 35 m from a movie screen to the top of the screen at 20 m above the ground.



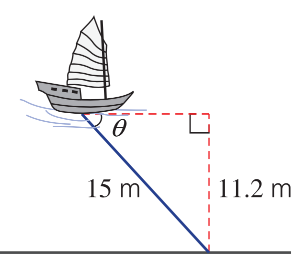
c A person sits 20 m away from a stage that is 2 m below the horizontal viewing level. Find the angle of depression of the person's viewing level to the stage.



d A diagonal cut 2.85 m long is to be made on a piece of plaster board attached to a wall, as shown. The base of the plaster board measures 1.94 m. Find the angle of elevation of the diagonal cut from the base.

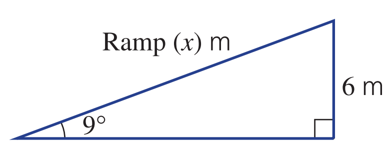


f A 15 m chain with an anchor attached, as shown, is holding a boat in a position against a current. If the water depth is 11.2 m, find the angle of depression from the boat to where the anchor is fixed to the seabed.

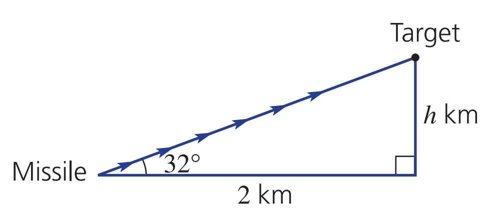


Problem-solving and reasoning | 7, 9 | 8–10

7 A ramp for wheelchairs is to be constructed into a footbridge 6 m high. The angle of elevation is to be 9°. What will be the length of the ramp, correct to two decimal places?



8 A missile is launched at an angle of elevation of 32°. If the target is 2 km away on the horizontal, how far above ground level is the target, correct to two decimal places?



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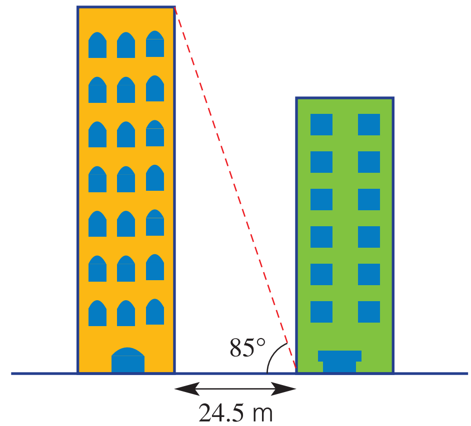
9H

9 The distance between two buildings is 24.5 m, as shown. Find the height of the taller building, correct to two decimal places, if the angle of elevation from the base of the shorter building to the top of the taller building is 85°.



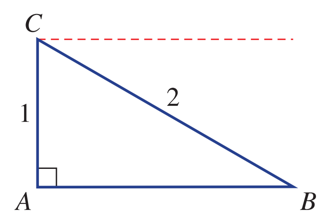
10 Right-angled triangle ABC is shown:

a Find the angle of elevation from B to C.

b State the angle of depression from C to B.

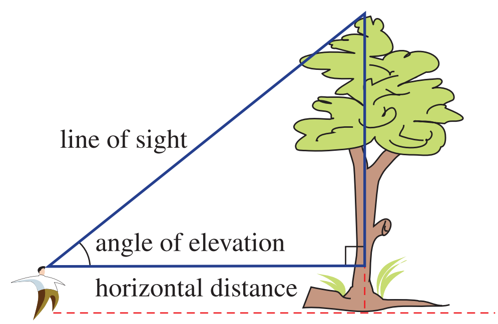
c Describe the relationship that exists between these two angles.

d Find the length AB, correct to one decimal place.



Practical trigonometry: measuring heights — 11

11 It is not always possible or practical to measure the height of an object directly. Here you will find the height of an object that is difficult to measure. Select a building or other structure (e.g. a statue or flagpole) for height calculation. You must be able to measure right up to the base of the structure.



a Choose a position from which you can see the top of your structure and measure the angle of elevation, θ, from your eye level. (Use an inclinometer, if your teacher has one, or simply estimate the angle using a protractor.)

b Measure the distance along the ground (d) from your location to the base of the structure.

c Calculate the height of the structure. *Remember to make an adjustment for the height of your eye level from the ground*.

d Move to another position and repeat the measurements. Calculate the height using your new measurements.

e Was there much difference between the calculated heights? Suggest reasons for any differences.

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9I Direction and bearings

Learning intentions

- To know the 8-point compass rose and how true bearings are measured
- To be able to draw a diagram including a right-angled triangle for a problem involving bearings
- To be able to find an unknown distance in a problem involving bearings

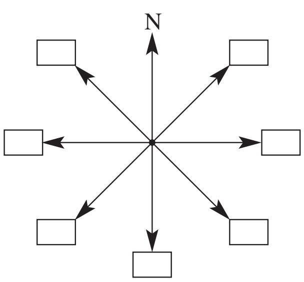
Key vocabulary: true bearing, clockwise

True bearings are used to communicate a direction and are important in navigation. Ships, planes, bushwalkers and the military all use bearings when communicating direction.

Lesson starter: Compass bearings

Work together as a class to label the 8-point compass rose using letters/words in **1** and angles in **2**.

1



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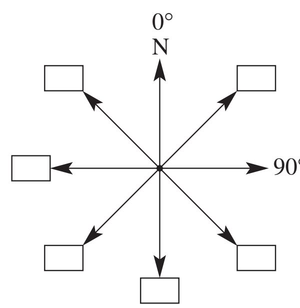
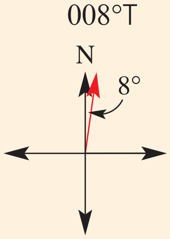


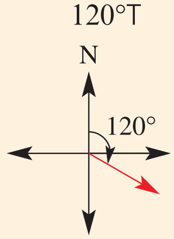
Key ideas

- A **true bearing** (°T) is an angle measured clockwise from north.
  - It is written using three digits.
    For example,

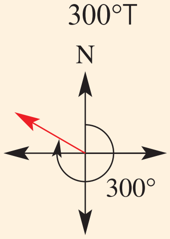
008°T

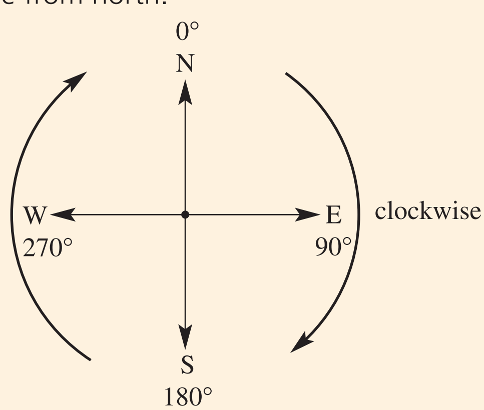


120°T



300°T





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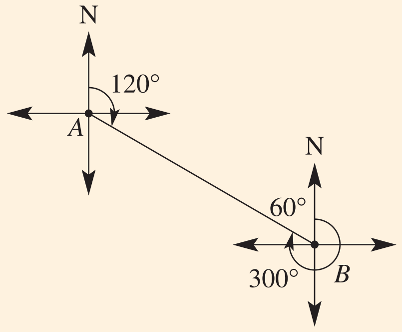
9I

- The word *from* indicates the direction from which a bearing is being taken. For example,



The bearing of B from A is 120°T.

The bearing of A from B is 300°T.

- When solving problems relating to bearings, always draw a diagram using N, S, E and W each time a bearing is used.

Exercise 9I

Understanding | 1–4 | 3, 4

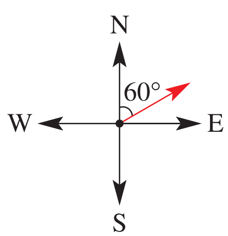
1 What is the opposite direction to:

a north (N)?
b east (E)?
c south (S)?
d north-east (NE)?

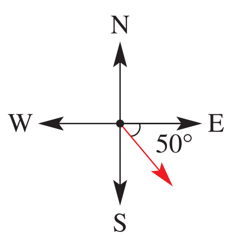
2 Match each diagram below with the correct true bearing from the list below.

i 300° **ii** 260° **iii** 225°
iv 140° **v** 060° **vi** 350°

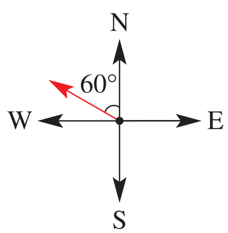
a



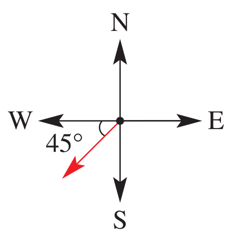
b



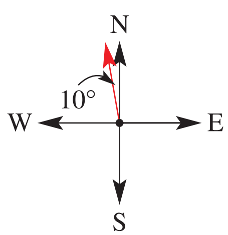
c



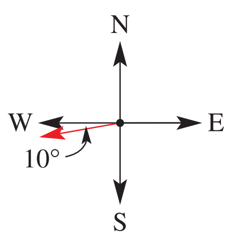
d



e



f



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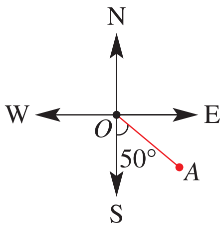
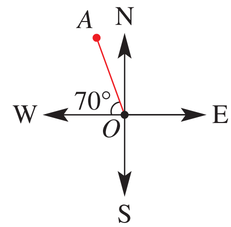
3 Write down the true bearings of A from O, as shown in these diagrams.

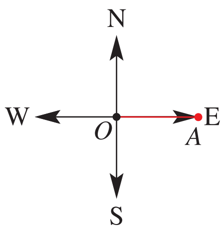
a



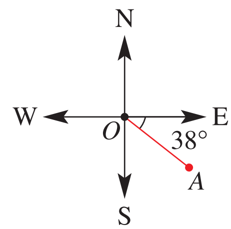
b



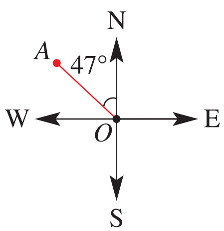
c



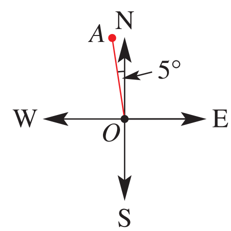
d

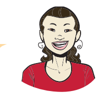


e

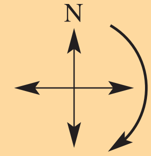


f





Hint: Remember to use 3 digits and to go clockwise from north.



4 Fill in the missing terms and values for the diagram shown.

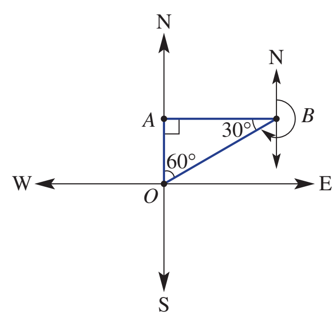
a A is due ______ of O.

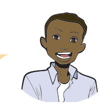
b B is due ______ of A.

c A is due ______ of B.

d The true bearing of B from O is ______.

e The true bearing of O from B is ______.





Hint: For part **e**, start from north and move clockwise to the line BO.

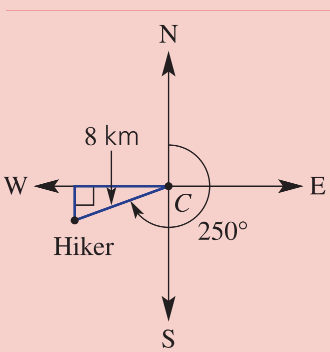
Fluency

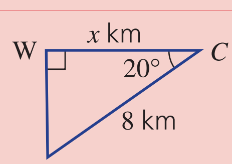
5–7 | 5, 6, 8

Example 18 Drawing a diagram

A hiker leaves camp (C) and walks on a bearing of 250°T for 8 km. How far west of camp (x km) is the hiker? Show all this information on a right-angled triangle. You do not need to solve for x.

Solution





Explanation

Draw the compass points first.

Start your diagram with the camp at the centre.

Mark in 250° clockwise from north, 8 km.

Draw a line from the hiker to the west line at right angles.

Redraw the triangle, showing any angles and lengths known (270° − 250° = 20°). Place a pronumeral on the required side.

Continued on next page

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Now you try

A plane leaves the airport and heads on a bearing of 165° T for 100 km. How far south of the airport (x km) is the plane?

Show all this information on a right-angled triangle. You do not need to solve for x.

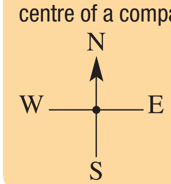
5 Draw a right-angled triangle for each of the situations outlined below.

a Zahra runs on a true bearing of 300° from her home for 6 km. How far north of home is she when she stops?

b Bailey walks 12.5 km from camp, C, on a bearing of 135°T. How far south is he now from camp?

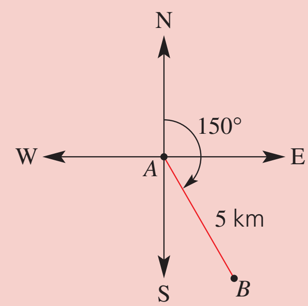
c Samir walks due south 10 km, then turns and walks due east 12 km. What is his bearing from O, his starting point?

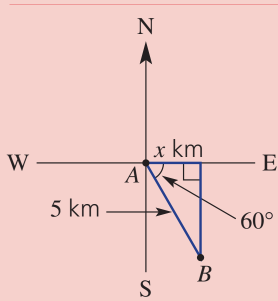
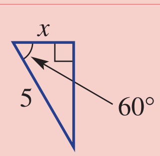
Hint: Start by making the starting point at the centre of a compass.



Example 19 Finding distances with bearings

A bushwalker walks 5 km on a true bearing of 150° from point A to point B. Find how far east point B is from point A.



Solution	Explanation
 	Copy the diagram and draw a line from B up to the east line. Use the pronumeral x along the east line. Find the angle within the triangle: $150° - 90° = 60°$ Redraw the triangle.
$\cos\theta = \frac{\text{A}}{\text{H}}$	Since the adjacent (A) and hypotenuse (H) are given, use cos.
$\cos 60° = \frac{x}{5}$	
$x = 5 \times \cos 60°$	
$x = 2.5$	Solve the equation to find x.
∴ Point B is 2.5 km east of point A.	Answer the question.

Now you try

A ship travels 200 km on a true bearing of 290° from point A to point B. Find how far west the point B is from point A. Round to one decimal place.

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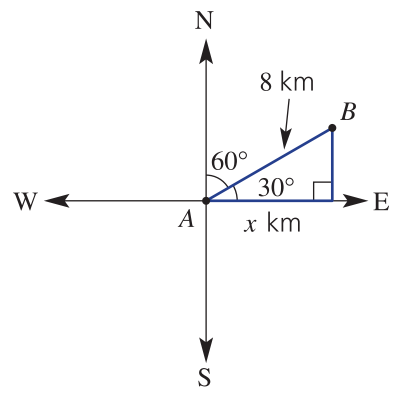
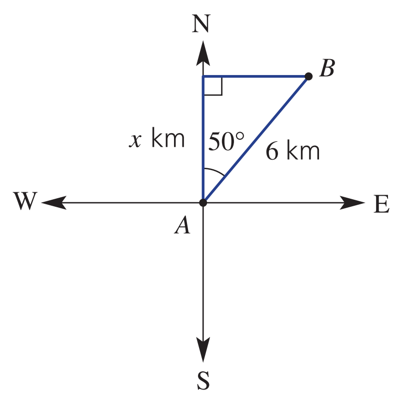
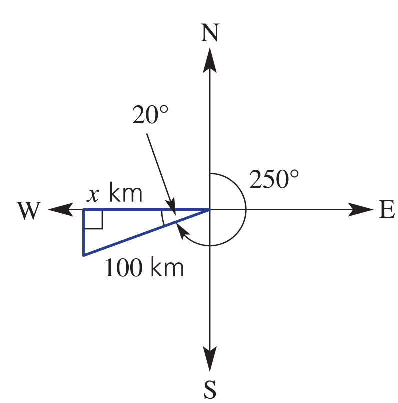
6 Rihanna walks 8 km from point A to point B on a true bearing of 060°. How far east, correct to one decimal place, is point B from point A?



7 A bushwalker walks 6 km on a true bearing of 050° from point A to point B. Find how far north point B is from point A, correct to two decimal places.



8 A speed boat travels 100 km on a true bearing of 250°. Find how far west of its starting point the speed boat is, correct to two decimal places.



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9I

Problem-solving and reasoning

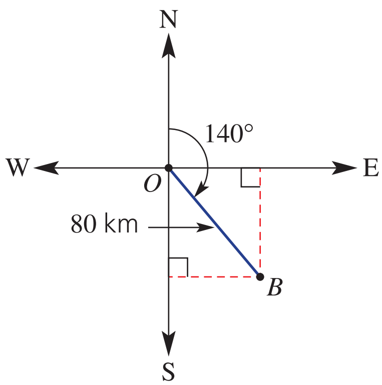
9, 10	9, 11

9 A fishing boat starts from point O and sails 80 km on a true bearing of $140°$ to point B, as shown below.

a How far east of point O is point B? (Answer to two decimal places.)

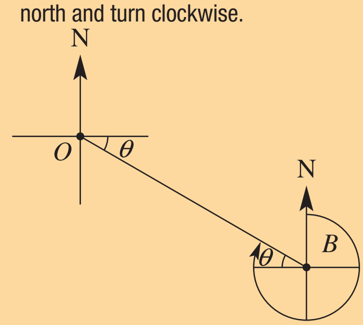
b How far south of point O is point B? (Answer to two decimal places.)

c What is the bearing of point O from point B?



Hint:
Remember what 'from' means!

Hint:
Remember: to find a bearing, face north and turn clockwise.

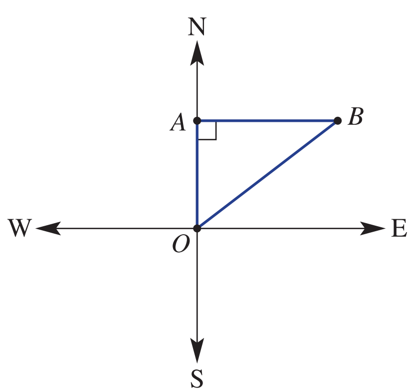


Bearings are given as a 3-digit angle.

10 A plane flies from point O 50 km due north to point A, and then turns and flies 60 km east to point B.

a Copy the diagram below and mark in the lengths 50 km and 60 km.

Hint: Pythagoras' theorem: $c^2 = a^2 + b^2$



b Use Pythagoras' theorem to find the distance of B from O, correct to two decimal places.

c Use trigonometry to find the size of angle AOB. Round to the nearest degree.

d What is the bearing of B from O?

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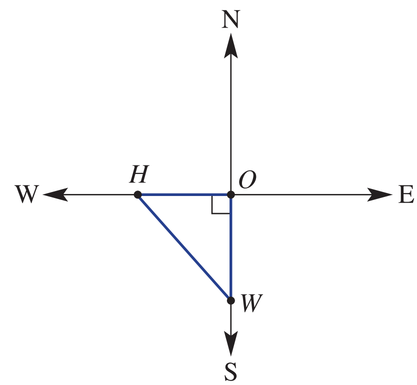
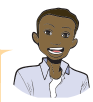
11 William and Harry both leave camp, O, at the same time. William walks south from O for 10 km. Harry walks west from O for 8 km.

- **a** Copy and complete the diagram for this question.
- **b** How far is Harry from William (to one decimal place)?
- **c** Find the size of angle OWH, correct to the nearest degree.
- **d** What is the bearing of Harry from William?





Hint: You can use Pythagoras' theorem here.

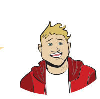
Drawing your own diagrams — 12–14

12 Huang walks on a true bearing of $210°$ for 6.5 km. How far west of his starting point is he?

13 A plane flies on a true bearing of $320°$ from an airport, A, for 150 km. At this time how far north of the airport is the plane? Answer to the nearest kilometre.

14 Point A is 10 km due east of point O, and point B is 15 km due south of point A.

- **a** How far is it, correct to two decimal places, from point B to point O?
- **b** What is the bearing, to the nearest degree, of point B from point O?



Hint: Remember: The word 'from' indicates where the bearing is being taken.

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Maths@Work: Surveyor

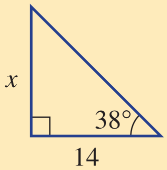
Surveyors collect and measure data about land and the environment, including measurements for subdivisions of new housing estates, new roads and open-pit mines. They work with plans, files, charts and reports.

To become a surveyor you can either do a university course, requiring Year 12 mathematics, or apply to TAFE and complete the training there. All courses require a student to have studied mathematics.

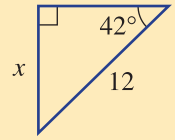
Mathematical skills needed in becoming a land surveyor include geometry and trigonometry. Complete these questions that a surveyor may face in their day-to-day job.

1 Refresh your trigonometry skills by finding the value of each pronumeral in the following triangles. All measurements are in metres, and give answers correct to two decimal places.

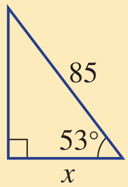
a



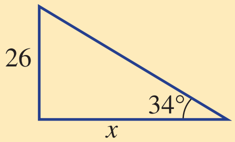
b



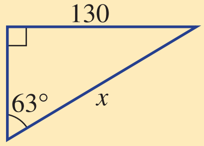
c



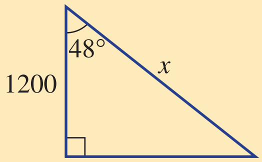
d



e

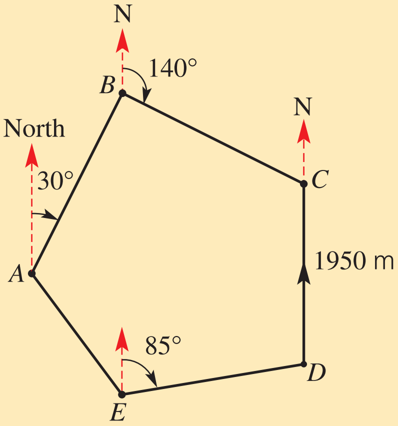


f



In surveying, there are two types of traverse surveys: the closed traverse survey and the open traverse survey.

2 The example below is of a closed traverse survey that could be around a large contained development, such as a school or airport. The diagram is not to scale.



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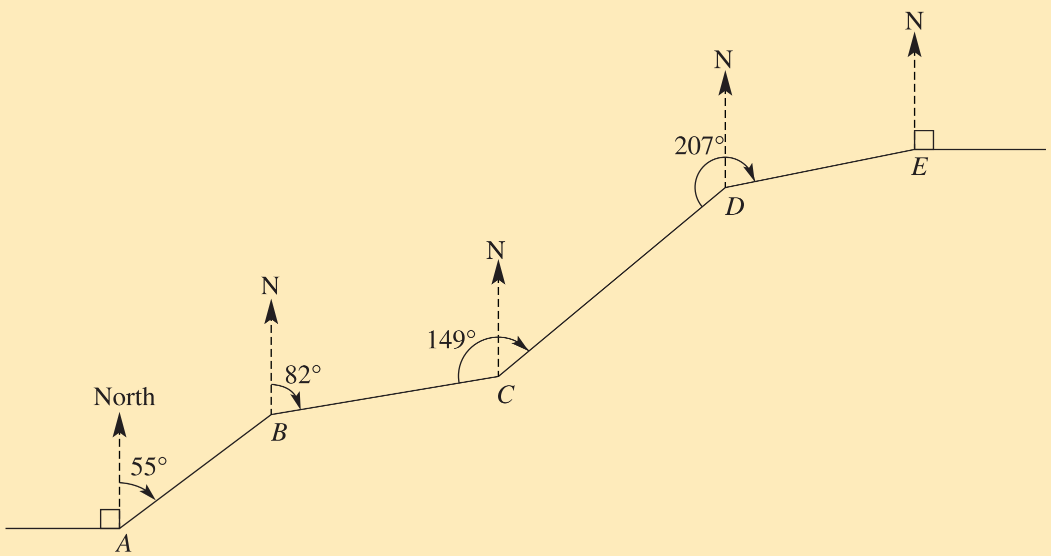
State answers to the following questions in metres, to two decimal places unless told otherwise.

Hint: For part **a**, start by forming a right-angled triangle with AB as the hypotenuse.

a Given that marker B is 650 m east of A, calculate the distance AB.
b Given that marker C is 900 m south of B, calculate the distance BC.
c Given that marker E is 160 m south of D, calculate the distance DE.
d Marker E is 600 m east of A and 900 m south of A. Calculate the distance AE.
e Calculate the perimeter of the figure $ABCDE$, correct to the nearest metre.
f Calculate the true bearing of B from C.
g Calculate the true bearing of E from D.

3 Below is an example of an open traverse survey that could be the simplified model of a section of road. The diagram is not to scale.



a Complete this table of measurements for the open traverse survey above.
Hints: Draw a large, ruled and labelled diagram with a north–south line at each marker. Recall the rules for angles between parallel lines.

Marker	Distance to the next marker (metres)	True bearing of the next marker from the current marker	Angle between line segments at marker (clockwise)	Distance East to the next marker (in m to 2 d.p.)	Distance North to the next marker (in m to 2 d.p.)
A	98.00	055°			
B	125.36	082°			
C	157.80		149°		
D	104.42		207°		
E	–	–		–	–

b How far east is marker E from A? Give your answer in metres to two decimal places.
c How far north of marker A is E? Give your answer in metres to two decimal places.
d By considering a right-angle triangle with hypotenuse AE, what is the true bearing of E from A? Give your answer to the nearest degree.

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Using technology

4 Use FxDraw or CAD software or another digital drawing program to create a labelled diagram of a closed traverse survey for a town park. Your diagram of the park must include:
- a true north line on each marker
- true north bearings of each next marker from each marker
- the length, in metres, of each boundary segment
- internal angles between line segments (these need to be calculated)
- the perimeter of the park

Your diagram doesn't need to be to scale but it should be a close representation of the survey data.

Closed Traverse survey data for Penny Park			
Marker	True north bearing of next marker from current marker	Distance from current marker to next marker	Internal angle between line segments at marker
A	30°	394 m	
B	99°	658 m	
C	180°	500 m	
D	266°	590 m	
E	324°	360 m	

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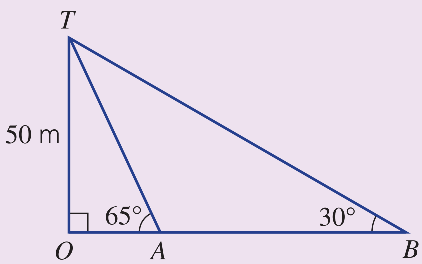
Puzzles and games

1 What is the opposite direction to:
a East? **b** NE? **c** SE? **d** 018°? **e** 300°?

2 Use two different right-angled triangles to find the distance from A to B in this diagram, correct to two decimal places.

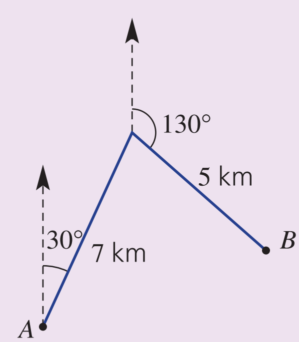


3 Make up your own saying using the letters in SOH CAH TOA as the first letter of each word.
S___ O___ H___ C___ A___ H___ T___ O___ A___

4 In the wordfind below there are 17 terms that are used in this chapter. See if you can locate all 17 terms and write a definition or draw a diagram for each of them.

S	I	D	R	T	Y	I	P	Y	T	S	O	H	T	H
I	D	E	P	R	E	S	S	I	O	N	D	Y	O	Y
N	T	E	I	N	T	E	W	P	Y	A	D	J	H	P
E	I	O	E	E	D	S	V	B	Y	T	P	U	W	O
Q	U	O	T	R	S	I	A	D	J	A	C	E	N	T
A	D	A	N	G	E	D	E	P	R	A	N	G	L	E
S	E	N	A	R	T	E	E	G	R	I	L	K	O	N
D	P	T	R	I	A	N	G	L	E	H	E	I	P	U
C	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
B	A	C	G	B	L	I	N	O	Y	A	A	W	O	E
H	D	O	H	T	E	A	N	G	L	N	T	H	S	H
U	J	S	T	R	G	A	O	K	Y	G	I	I	I	Y
J	E	I	W	F	L	N	R	M	K	E	O	D	T	P
K	N	N	T	J	T	R	N	I	O	N	P	Z	E	O
E	L	E	V	A	T	I	O	N	N	T	P	A	M	B
E	T	B	A	S	P	Y	T	H	A	G	O	R	A	S

5 Find the bearing from A to B in this diagram.



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Chapter summary

Pythagoras' theorem and trigonometry

Trigonometric ratios

SOH → $\sin\theta = \frac{O}{H}$

CAH → $\cos\theta = \frac{A}{H}$

TOA → $\tan\theta = \frac{O}{A}$

Hypotenuse

The *hypotenuse* is the longest side in a right-angled triangle. It is opposite the right angle.

hypotenuse

Opposite and Adjacent

In a right-angled triangle the sides marked 'opposite' and 'adjacent' are named according to the angle used.

O, A, θ — A, θ, O

Finding angles

(Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$)

4, 3, θ

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$= 37°$

(to the nearest degree)

Finding lengths with trigonometry

x, 10, 30°

$\sin 30° = \frac{x}{10}$

$x = 10 \sin 30°$

$x = 5$

12, x, 30°

$\sin 30° = \frac{12}{x}$

$x \times \sin 30° = 12$

$x = \frac{12}{\sin 30°}$

$x = 24$

Pythagoras' theorem

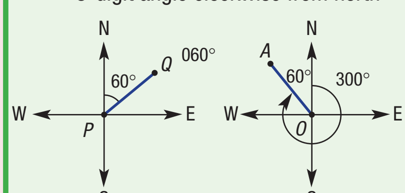
For the triangle shown, *Pythagoras' theorem* is

a, b, c

$c^2 = a^2 + b^2$

True bearings

3-digit angle clockwise from north



Bearing of *Q* from *P* is 060°.

Bearing of *A* from *O* is 300°.

Finding the hypotenuse

3, 1, c

$c^2 = 1^2 + 3^2$

$c^2 = 10$

$c = \sqrt{10}$

$= 3.2$ (to 1 d.p.)

Finding a shorter side

x, 3, 5

$5^2 = x^2 + 3^2$

$x^2 = 5^2 - 3^2$

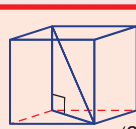
$x^2 = 16$

$x = 4$

Applications

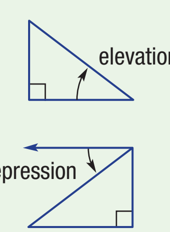
Start by identifying right-angled diagrams

c, b, a



(Cube)

Elevation and depression



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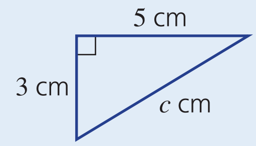
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

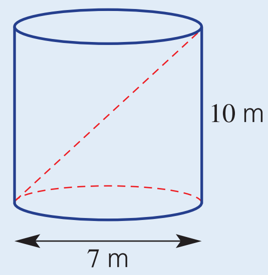
9A

1 I can find the length of the hypotenuse for a right-angled triangle given the length of the other two sides.
e.g. Find the length of the hypotenuse of this triangle, correct to one decimal place.



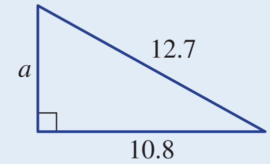
9A

2 I can apply Pythagoras' theorem and find the length of the hypotenuse in a real situation.
e.g. Find the length of the longest rod that will fit into this cylinder, correct to two decimal places.



9B

3 I can use Pythagoras' theorem to find the length of a shorter side in a right-angled triangle.
e.g. Find the value of a in this triangle, correct to one decimal place.

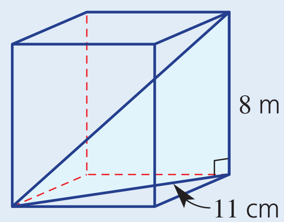


9C

4 I can apply Pythagoras' theorem to solve a range of real-life problems.
e.g. Two bushwalkers leave their camp at the same time. One walks due south for 5 km and the other walks due west for 4 km. How far apart are the bushwalkers at this point. Round to two decimal places.

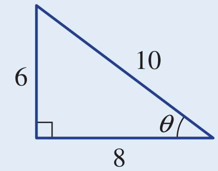
9C

5 I can apply Pythagoras' theorem to find a length in a three-dimensional solid.
e.g. Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



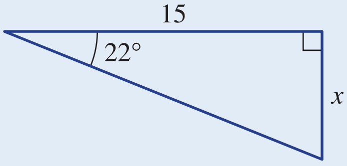
9D

6 I can write a ratio for sine, cosine and tangent of an angle in a triangle with given side lengths.
e.g. Write a ratio for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for this triangle.



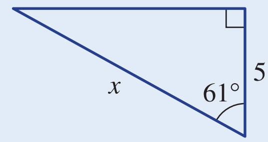
9E

7 I can find a missing length on a right-angled triangle given an angle and another side.
e.g. Find the value of x in this triangle, correct to two decimal places.



9F

8 I can find a missing length in a right-angled triangle if the pronumeral sits in the denominator of the fraction.
e.g. Find the value of x in this triangle, correct to two decimal places.



Chapter checklist

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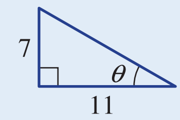
Chapter checklist

			✔
9G	9	**I can find an angle in a right-angled triangle given any two side lengths.** e.g. Find the angle θ in this triangle, correct to one decimal place.  	
9H	10	**I can solve problems in trigonometry using angles of elevation and depression.** e.g. The angle of elevation to the top of a vertical pole from a point on the ground 50 m away horizontally is $27°$. Find the height of the pole, correct to the nearest metre.	
9H	11	**I can find angles of elevation or depression in a real situation if given two other lengths from a right-angled triangle.** e.g. Find the angle of elevation of a 40 m high tower from a point which is 30 m horizontally from the base of the tower. Round to one decimal place.	
9I	12	**I can interpret a bearings problem and draw a suitable right-angled triangle.** e.g. A yacht travels 10 km on a true bearing of $210°$. How far east of the starting point is the yacht? Show all this information on a diagram using a right-angled triangle. You do not need to solve the problem.	
9I	13	**I can find an unknown distance in a problem involving bearings.** e.g. A hiker leaves base camp and walks 8 km on a true bearing of $235°$. How far west of base camp is the hiker at this point? Round to the nearest degree.	

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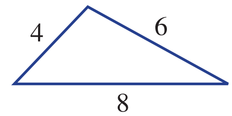
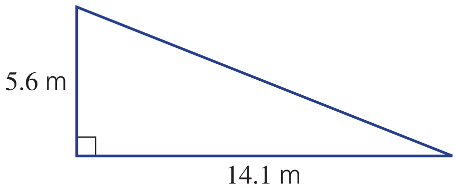
Short-answer questions

9A **1** Determine whether the triangle shown contains a right angle.

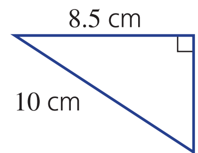


9A/B **2** Find the missing length, correct to two decimal places, in these triangles.

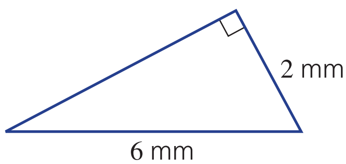
a



b



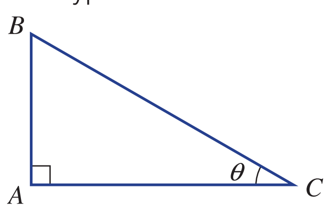
c



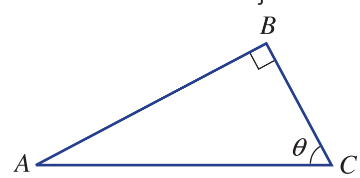
9D **3** Which side (AB, AC or BC) of these triangles is:

i the hypotenuse? **ii** opposite to θ? **iii** adjacent to θ?

a



b

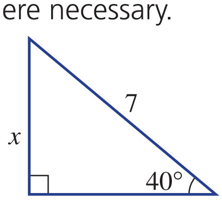


9D **4** Use a calculator to find the value of the following, rounding to two decimal places.

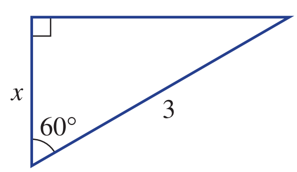
a $\sin 35°$ **b** $\cos 17°$ **c** $\tan 83°$

9E **5** Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

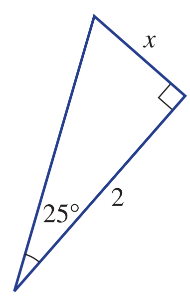
a



b

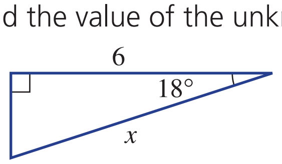


c

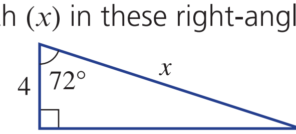


9F **6** Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

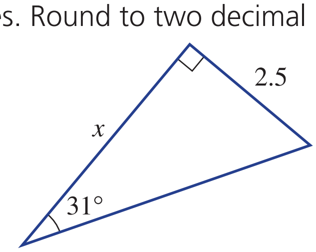
a



b



c



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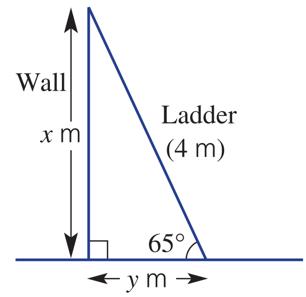
Chapter review

9E

7 A 4 m ladder leans, as shown, against a wall at an angle of $65°$ to the horizontal.

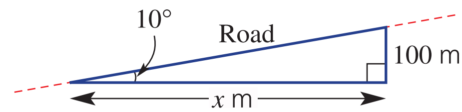
a Find how high up the wall the ladder reaches (x m), correct to two decimal places.

b Find how far the bottom of the ladder is from the wall (y m), correct to two decimal places.



9F

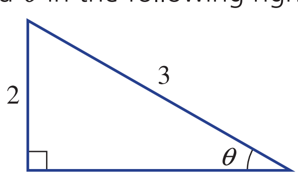
8 A section of road has a slope of $10°$ and gains 100 m in height. Find the horizontal length of the road (x m), correct to two decimal places.



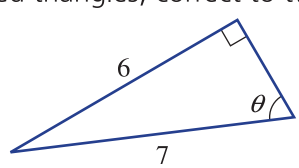
9G

9 Find θ in the following right-angled triangles, correct to two decimal places.

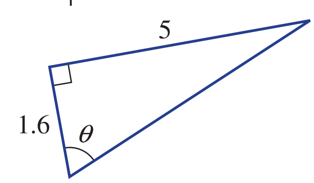
a



b



c

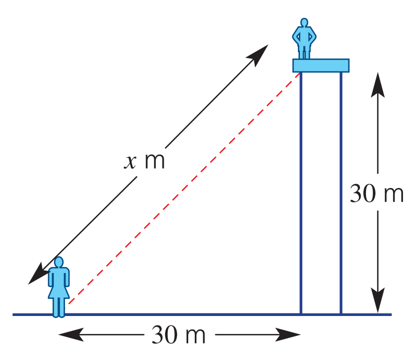


9H

10 Raymond and Sofia view each other from two different places, as shown. Raymond is on a viewing platform, whereas Sofia is 30 m from the base of the platform, on the ground. The platform is 30 m above the ground.

a Find the angle of elevation from Sofia's feet to Raymond's feet.

b Using your answer to part **a**, find the distance (x) between Sofia and Raymond, correct to one decimal place.



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9I

11 A hiker walks on a true bearing of 220° from point A for 3 km to point B.

a Find how far south the hiker has walked, correct to one decimal place.

b Find how far west the hiker has walked, correct to one decimal place.

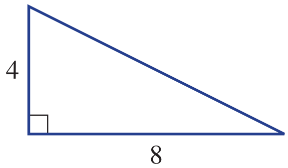
c What is the bearing of point A from B?

Multiple-choice questions

9A

1 The length of the hypotenuse in the triangle shown is closest to:

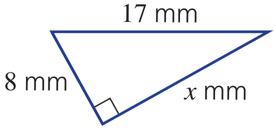
A 10 **B** 9 **C** 4
D 100 **E** 64



9B

2 The length of the side marked x in the triangle shown is:

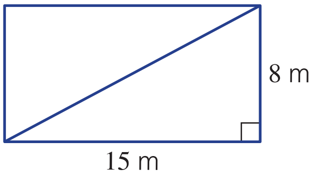
A 23 **B** 17 **C** 12
D 19 **E** 15



9C

3 For the shape shown to be a rectangle, the length of the diagonal must be:

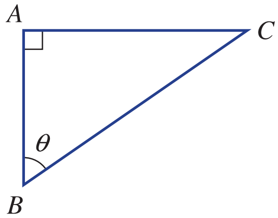
A 15 m **B** 8 m **C** 17 m
D 23 m **E** 32 m



9D

4 Which side (AB, AC or BC) is adjacent to θ in this triangle?

A AC **B** AB **C** BC
D hypotenuse **E** opposite



9D

5 The value of $\cos 21^\circ$ is closest to:

A -0.55 **B** 0.9 **C** 0.9336
D 0.93 **E** 0.934

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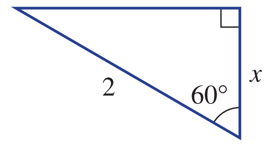
Chapter review

9E

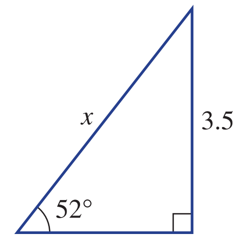
6 The value of x in this triangle is:



A $2 \div \cos 60°$ **B** $2 \div \sin 60°$
C $2 \times \cos 60°$ **D** $2 \times \sin 60°$
E $2 \times \tan 60°$

9F

7 The value of x in this triangle is closest to:



A 2.76 **B** 4.48 **C** 5.68
D 4.44 **E** 2.73

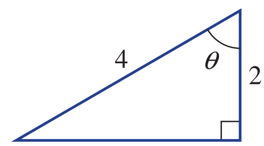
9F

8 A metal brace sits at 55° to the horizontal and reaches 4.2 m up a wall. The distance between the base of the wall and the base of the brace is closest to:

A 6.00 m **B** 2.41 m **C** 7.32 m
D 5.13 m **E** 2.94 m

9G

9 The angle θ in this triangle is:



A 60° **B** 30° **C** 26.57°
D 20° **E** none of the above

9H

10 The angle of depression from the roof of a building to a trampoline is 75°. If the roof is 12 m above the level of the trampoline, then the distance of the trampoline from the building is closest to:

A 12.42 m **B** 11.59 m **C** 3.22 m
D 44.78 m **E** 3.11 m

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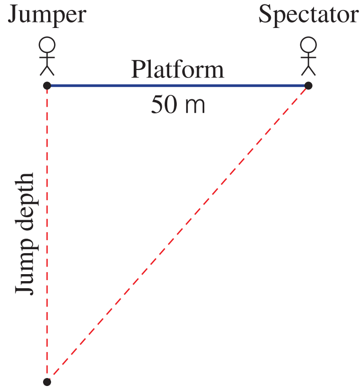
Extended-response questions

1 A spectator is viewing bungee jumping from a point 50 m to the side but level with the jumping platform.



a The first bungee jumper has a maximum fall of 70 m. Find the angle of depression from the spectator to the bungee jumper at the maximum depth, correct to two decimal places.

b The second bungee jumper's maximum angle of depression from the spectator is 69°. Find the jumper's maximum depth, correct to two decimal places.

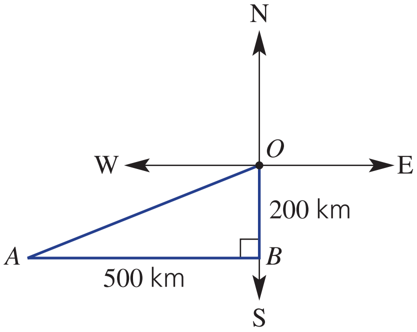
c The third jumper wants to do the 'Head Dunk' into the river below. This occurs when the spectator's angle of depression to the river is 75°. Find, correct to the nearest metre, the height of the platform above the river.

2 A military plane flies 200 km from point O to point B, then west 500 km to point A.

a How far is A from O, to the nearest kilometre?

b What is angle BOA, correct to the nearest degree?

c What is the bearing of A from O?



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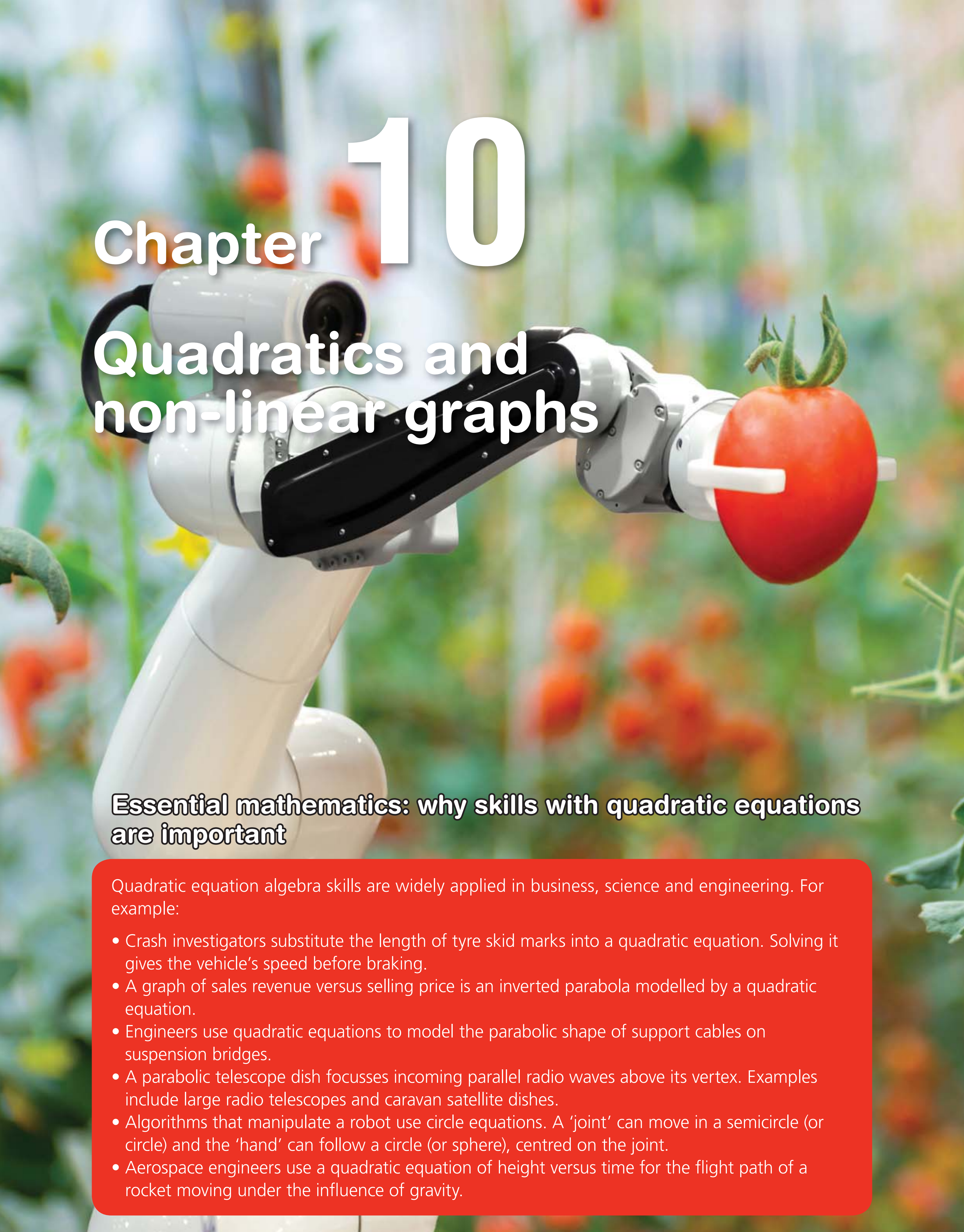

Chapter 10

Quadratics and non-linear graphs

Essential mathematics: why skills with quadratic equations are important

Quadratic equation algebra skills are widely applied in business, science and engineering. For example:

- Crash investigators substitute the length of tyre skid marks into a quadratic equation. Solving it gives the vehicle's speed before braking.
- A graph of sales revenue versus selling price is an inverted parabola modelled by a quadratic equation.
- Engineers use quadratic equations to model the parabolic shape of support cables on suspension bridges.
- A parabolic telescope dish focusses incoming parallel radio waves above its vertex. Examples include large radio telescopes and caravan satellite dishes.
- Algorithms that manipulate a robot use circle equations. A 'joint' can move in a semicircle (or circle) and the 'hand' can follow a circle (or sphere), centred on the joint.
- Aerospace engineers use a quadratic equation of height versus time for the flight path of a rocket moving under the influence of gravity.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Expand binomial products and factorise monic quadratic expressions using a variety of strategies (VCMNA332)

Linear and non-linear relationships

Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate (VCMNA339)

Solve simple quadratic equations using a range of strategies (VCMNA341)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Consider the expression $5 + 2ab - b$.
- **a** How many terms are there?
- **b** What is the coefficient of the second term?
- **c** What is the value of the constant term?

2 Simplify each of the following by collecting like terms.
- **a** $7x + 2y - 3x$
- **b** $3xy + 4x - xy - 5x$
- **c** $4ab - 2ba$

3 Simplify:
- **a** $\frac{4a}{2}$
- **b** $\frac{-24mn}{12n}$
- **c** $6a \times 3a$
- **d** $-2x \times 3xy$
- **e** $x \times (-3) \div (9x)$
- **f** $4x^2 \div (2x)$

4 Expand and simplify by collecting like terms where possible.
- **a** $4(m + n)$
- **b** $-3(2x - 4)$
- **c** $2x(3x + 1)$
- **d** $4a(1 - 2a)$
- **e** $5 + 3(x - 4)$
- **f** $5 - 2(x + 3) + 2$

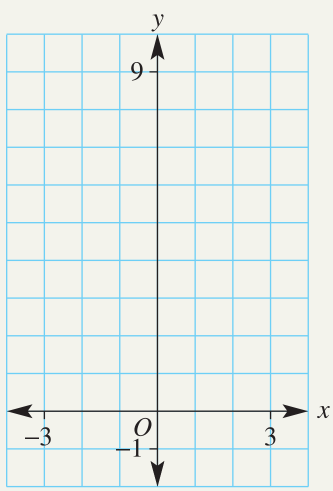
5 Factorise each of the following by taking out a common factor.
- **a** $7x + 7$
- **b** $-9x - 27x^2$
- **c** $a^2 + ab$

6 Solve:
- **a** $2x + 1 = 0$
- **b** $2(x - 3) = 0$
- **c** $\frac{3x + 1}{4} = 4$

7 **a** Complete this table for $y = x^2$.

x	-3	-2	-1	0	1	2	3
y	9						

b Plot a graph of $y = x^2$ and state if its shape is:
- **A** linear (i.e. a straight line)
- **B** non-linear (i.e. not a straight line)



8 Answer true (T) or false (F) to the following.
- **a** $x = 3$ is a solution of $x^2 = 9$.
- **b** $x = -2$ is a solution of $x^2 = -4$.
- **c** $x = 0$ is a solution of $x(x + 4) = 0$.
- **d** $x = -5$ is a solution of $x^2 = 25$.
- **e** $x = 2$ is a solution of $x(x + 2) = 0$.

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10A Expanding binomial products ★

Learning intentions

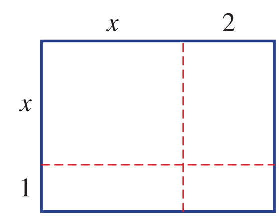
- To know the distributive law in order to expand binomial products
- To be able to expand and simplify expressions including brackets
- To be able to expand perfect squares and be able to form a difference of perfect squares

Key vocabulary: expand, binomial, product, like terms, distributive law, perfect square, difference of perfect squares

Expressions that include numerals and variables (or pronumerals) are central to the topic of algebra. Sound skills in algebra are essential for solving most mathematical problems and this includes the ability to expand expressions involving brackets, such as binomial products, perfect squares and the difference of perfect squares. Exploring how projectiles fly subject to Earth's gravity, for example, can be modelled with expressions with and without brackets.

Lesson starter: Why does $(x + 1)(x + 2) = x^2 + 3x + 2$?

Look at this rectangle with side lengths $x + 1$ and $x + 2$.



- What are the areas of the four regions?
- Add up the areas to find an expression for the total area.
- Why does this explain that $(x + 1)(x + 2) = x^2 + 3x + 2$?

Key ideas

- **Like terms** have the same pronumeral part.
  - They can be collected (added and subtracted) to form a single term. For example: $7x - 11x = -4x$ and $4a^2b - 7ba^2 = -3a^2b$.
- The **distributive law** is used to expand brackets.

$a(b + c) = ab + ac \qquad (a + b)(c + d) = ac + ad + bc + bd$

and $a(b - c) = ab - ac$

	a
b	ab
c	ac

	a	b
c	ac	bc
d	ad	bd

$(a + b)(c + d)$ is called a **binomial** product because each expression in the brackets has two terms.

- **Perfect squares**

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2$$
$$= a^2 + 2ab + b^2$$
$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ba + b^2$$
$$= a^2 - 2ab + b^2$$

- **Difference of perfect squares (DOPS)**

$$(a + b)(a - b) = a^2 - \cancel{ab} + \cancel{ba} - b^2$$
$$= a^2 - b^2$$

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10A

Exercise 10A

Understanding | 1–3 | 3

1 Decide whether each of the following is a perfect square (PS) or a difference of perfect squares (DOPS).

a $(x+1)^2$ **b** x^2-16 **c** $4x^2-25$ **d** $(2x-3)^2$

2 Copy and complete.

a $a(b+c) =$ ____________________

b $a(b-c) =$ ____________________

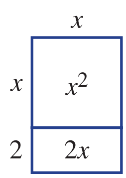
c $(a+b)(c+d) =$ ____________________

d $(a+b)(a-b) =$ ____________________

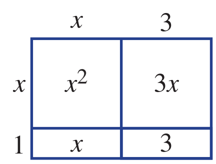
3 Use each diagram to help expand the expressions.

Hint: Simply add up all the areas inside the rectangular diagram.

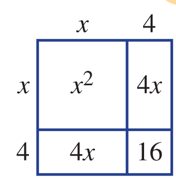
a $x(x+2)$



b $(x+3)(x+1)$



c $(x+4)^2$



Fluency | 4–7(½) | 4–7(½)

Example 1 Expanding simple expressions

Expand and simplify.

a $-3(x-5)$ **b** $-2x(1-x)$

Solution	Explanation
a $-3(x-5) = -3 \times x - (-3) \times 5$ $= -3x + 15$	Use the distributive law $a(b-c) = ab - ac$. A negative times a negative is a positive.
b $-2x(1-x) = -2x \times 1 - (-2x) \times x$ $= -2x + 2x^2$	Recall: $x \times x = x^2$.

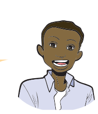
Now you try

Expand and simplify.

a $-2(x+3)$ **b** $-5x(3x-1)$

4 Expand and simplify where possible.

Hint: $a(b+c) = ab + ac$



a $2(x+5)$ **b** $3(x-4)$ **c** $-5(x+3)$

d $-4(x-2)$ **e** $3(2x-1)$ **f** $4(3x+1)$

g $-2(5x-3)$ **h** $-5(4x+3)$ **i** $x(2x+5)$

j $x(3x-1)$ **k** $2x(1-x)$ **l** $3x(2-x)$

m $-2x(3x+2)$ **n** $-3x(6x-2)$ **o** $-5x(2-2x)$

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Example 2 Expanding binomial products

Expand the following.

a $(x+5)(x+4)$

b $(2x-1)(3x+5)$

Solution	Explanation
a $(x+5)(x+4) = x^2 + 4x + 5x + 20$ $= x^2 + 9x + 20$	For binomial products use $(a+b)(c+d) = ac + ad + bc + bd$. Simplify by collecting like terms: $4x + 5x = 9x$
b $(2x-1)(3x+5) = 6x^2 + 10x - 3x - 5$ $= 6x^2 + 7x - 5$	Expand using the distributive law and simplify. Note that $2x \times 3x = 2 \times 3 \times x \times x = 6x^2$ and $-1 \times 3x = -3x$, $-1 \times 5 = -5$.

Now you try

Expand the following.

a $(x-1)(x+7)$

b $(3x-2)(4x-1)$

5 Expand the following.

a $(x+2)(x+8)$ b $(x+3)(x+4)$

c $(x+7)(x+5)$ d $(x+8)(x-3)$

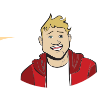
e $(x+6)(x-5)$ f $(x-2)(x+3)$

g $(x-7)(x+3)$ h $(x-4)(x-6)$ i $(x-8)(x-5)$

j $(2x+1)(3x+5)$ k $(4x+5)(3x+2)$ l $(5x+3)(2x+7)$

m $(3x+2)(3x-5)$ n $(5x+3)(4x-2)$ o $(2x+5)(3x-5)$

Hint: $(a+b)(c+d) = ac + ad + bc + bd$



Example 3 Expanding perfect squares

Expand these perfect squares.

a $(x+2)^2$

b $(x-4)^2$

Solution	Explanation
a $(x+2)^2 = (x+2)(x+2)$ $= x^2 + 2x + 2x + 4$ $= x^2 + 4x + 4$	First write in expanded form, then use the distributive law.
or $(x+2)^2 = x^2 + 2(x)(2) + 2^2$ $= x^2 + 4x + 4$	$(a+b)^2 = a^2 + 2ab + b^2$ with $a = x$ and $b = 2$.
b $(x-4)^2 = (x-4)(x-4)$ $= x^2 - 4x - 4x + 16$ $= x^2 - 8x + 16$	Rewrite and expand, using the distributive law. $-4 \times (-4) = 16$
or $(x-4)^2 = x^2 - 2(x)(4) + 4^2$ $= x^2 - 8x + 16$	Alternatively for perfect squares $(a-b)^2 = a^2 - 2ab + b^2$. Here $a = x$ and $b = 4$.

Now you try

Expand these perfect squares.

a $(x+3)^2$

b $(2x-1)^2$

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10A

6 Expand these perfect squares.

a $(x+5)^2$ **b** $(x+7)^2$ **c** $(x+6)^2$
d $(x-3)^2$ **e** $(x-8)^2$ **f** $(x-10)^2$
g $(2x+5)^2$ **h** $(5x+6)^2$ **i** $(7x-1)^2$

Hint: Recall:
$(x+5)^2 = (x+5)(x+5)$
$= \ldots$

Example 4 Expanding to form a difference of perfect squares

Expand to form a difference of perfect squares.

a $(x-3)(x+3)$ **b** $(2x+1)(2x-1)$

Solution	Explanation
a $(x-3)(x+3) = x^2 + 3x - 3x - 9$ $= x^2 - 9$	$x \times x = x^2$, $x \times 3 = 3x$, $-3 \times x = -3x$, $-3 \times 3 = -9$ Note that the two middle terms cancel.
or $(x-3)(x+3) = x^2 - 3^2$ $= x^2 - 9$	$(a-b)(a+b) = a^2 - b^2$
b $(2x+1)(2x-1) = 4x^2 - 2x + 2x - 1$ $= 4x^2 - 1$	Expand, recalling that $2x \times 2x = 4x^2$. Cancel the $-2x$ and $+2x$ terms.
or $(2x+1)(2x-1) = (2x)^2 - (1)^2$ $= 4x^2 - 1$	Alternatively for difference of perfect squares $(a-b)(a+b) = a^2 - b^2$. Here $a = 2x$ and $b = 1$ and $(2x)^2 = 2x \times 2x = 4x^2$.

Now you try

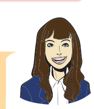
Expand to form a difference of perfect squares.

a $(x+5)(x-5)$ **b** $(3x-2)(3x+2)$

7 Expand to form a difference of perfect squares.

a $(x+4)(x-4)$ **b** $(x+9)(x-9)$
c $(x+8)(x-8)$ **d** $(3x+4)(3x-4)$
e $(2x-3)(2x+3)$ **f** $(8x-7)(8x+7)$
g $(4x-5)(4x+5)$ **h** $(2x-9)(2x+9)$
i $(5x-7)(5x+7)$ **j** $(7x+11)(7x-11)$

Hint: The two middle terms will cancel to give
$(a+b)(a-b) = a^2 - b^2$



Problem-solving and reasoning

8-9(½) | 8-9(½), 10, 11

8 Write the missing number.

a $(x+2)(x-3) = x^2 - x - \square$
b $(x-4)(x-3) = x^2 - \square x + 12$
c $(x-4)(x+4) = x^2 - \square$
d $(2x-1)(2x+1) = \square x^2 - 1$
e $(x+2)^2 = x^2 + \square x + 4$
f $(3x-1)^2 = 9x^2 - \square x + 1$

Hint: Expand if you need to.

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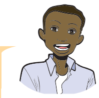
9 Write the missing number.

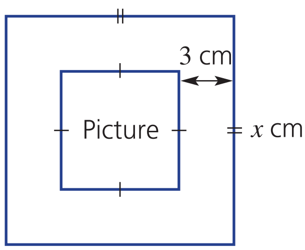
a $(x+\square)(x+2)=x^2+5x+6$
b $(x+\square)(x+5)=x^2+8x+15$
c $(x+7)(x-\square)=x^2+4x-21$
d $(x+4)(x-\square)=x^2-4x-32$
e $(x-6)(x-\square)=x^2-7x+6$
f $(x-\square)(x-8)=x^2-10x+16$



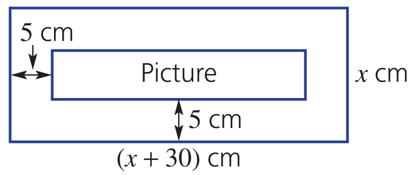
Hint: Notice how the two numerals in the brackets multiply to give the constant term.

10 Find an expanded expression for the area of the pictures centred in these square and rectangular frames.

a



b



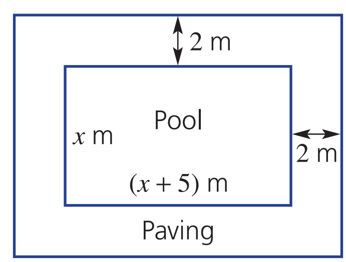
Hint: For part a, the side length of the picture will be $(x-6)$ cm.

11 Each problem below has an incorrect answer. Find the error and give the correct answer.

a $-x(x-7)=-x^2-7x$
b $3a-7(4-a)=-4a-28$
c $(x-9)(x+9)=x^2-18x-81$
d $(2x+3)^2=4x^2+9$

Swimming pool algebra — 12

12 A pool company builds rectangular pools that are 5 m longer than they are wide. The company then paves around the outside of the pool using a width of 2 m.



a Find an expanded expression for the:
   i pool area
   ii total area (including the pool and paving)
   iii paved area

b Find the area of the following when $x=4$.
   i the pool
   ii the paved area

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10B Factorising a difference of perfect squares ★

Learning intentions

- To be able to identify a highest common factor and factorise simple expressions
- To be able to factorise a difference of perfect squares

Key vocabulary: factorise, highest common factor, difference of perfect squares

A common and key step in the simplification and solution of equations involves factorisation. Factorisation is the process of writing a number or expression as a product of its factors. For example: $6 = 2 \times 3$, $2x + 6 = 2(x + 3)$, $x^2 - x = x(x - 1)$ and $x^2 - 9 = (x + 3)(x - 3)$.

In this section we look at expressions in which each term has a common factor and expressions that are a difference of perfect squares.

Lesson starter: It's just a DOPS expansion in reverse

Complete each column to see the connection when expanding or factorising a DOPS.

Expand

$(x + 2)(x - 2) = x^2 - 4$

$(x - 3)(x + 3) = x^2 - 9$

$(2x + 3)(2x - 3) = 4x^2 - 9$

$(7x - 6)(7x + 6) =$ _____

Factorise

$x^2 - 4 = (x$______$)(x$______$)$

$x^2 - 9 = (x$______$)($______$)$

$4x^2 - 9 = (2x$______$)($______$)$

_____ $= ($______$)($______$)$

Key ideas

- **Factorise** expressions with common factors by 'taking out' the highest common factor. For example: $-5x - 20 = -5(x + 4)$ and $4x^2 - 8x = 4x(x - 2)$.
- Factorise a difference of perfect squares (DOPS) using

$$a^2 - b^2 = (a + b)(a - b)$$

For example:

$$x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$$

and

$$9x^2 - 25 = (3x)^2 - 5^2 = (3x + 5)(3x - 5)$$

Exercise 10B

Understanding	1, 2	2

1 Complete these statements.

a $2(x + 3) = 2x + 6$, so $2x + 6 = 2($______$)$

b $-4(x - 1) = -4x + 4$, so $-4x + 4 = -4($______$)$

c $(x + 2)(x - 2) = x^2 - 4$, so $x^2 - 4 = ($______$)($______$)$

d $(3x + 2)(3x - 2) = 9x^2 - 4$, so $9x^2 - 4 = ($______$)($______$)$

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2 Determine the highest common factor of these pairs of terms.

a $7x$ and 14 **b** $12x$ and 30 **c** $-8y$ and 40
d $-5y$ and -25 **e** $4a^2$ and $2a$ **f** $12a^2$ and $9a$
g $-5a^2$ and $-50a$ **h** $-3x^2y$ and $-6xy$ **i** $-2ab$ and $-6a^2b$

Hint: Include a common negative.

Fluency 3–4(½) 3–5(½)

Example 5 Taking out common factors

Factorise by taking out the highest common factor.

a $-3x - 12$ **b** $20a^2 + 30a$

Solution	Explanation
a $-3x - 12 = -3(x + 4)$	-3 is common to both $-3x$ and -12.
b $20a^2 + 30a = 10a(2a + 3)$	The HCF of $20a^2$ and $30a$ is $10a$. Place $10a$ out the front and divide each term by $10a$.

Now you try

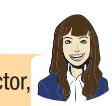
Factorise by taking out the highest common factor.

a $-4x - 36$ **b** $14b^2 - 21b$

3 Factorise by taking out the highest common factor.

a $3x - 18$ **b** $4x + 20$ **c** $7a + 7b$
d $9a - 15$ **e** $-5x - 30$ **f** $-4y - 2$
g $-12a - 3$ **h** $-2ab - bc$ **i** $4x^2 + x$
j $5x^2 - 2x$ **k** $6b^2 - 18b$ **l** $14a^2 - 21a$
m $10a - 5a^2$ **n** $12x - 30x^2$ **o** $-2x - x^2$

Hint: Find the highest common factor, then take it out.



Example 6 Factorising a difference of perfect squares

Factorise the following differences of perfect squares.

a $x^2 - 16$ **b** $9a^2 - 4b^2$

Solution	Explanation
a $x^2 - 16 = (x)^2 - (4)^2$ $= (x - 4)(x + 4)$	Use $a^2 - b^2 = (a - b)(a + b)$ with $a = x$ and $b = 4$.
b $9a^2 - 4b^2 = (3a)^2 - (2b)^2$ $= (3a - 2b)(3a + 2b)$	$9a^2 = (3a)^2$ since $3a \times 3a = 9a^2$ and $4b^2 = (2b)^2$.

Now you try

Factorise the following differences of perfect squares.

a $x^2 - 100$ **b** $16a^2 - 81b^2$

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10B

4 Factorise the following differences of perfect squares.

Hint: $a^2 - b^2 = (a + b)(a - b)$

a $x^2 - 9$ b $x^2 - 25$ c $y^2 - 49$
d $y^2 - 1$ e $a^2 - 16$ f $b^2 - 36$
g $y^2 - 144$ h $z^2 - 400$ i $4x^2 - 9$
j $36a^2 - 25$ k $1 - 81y^2$ l $100 - 9x^2$
m $25x^2 - 4y^2$ n $64x^2 - 25y^2$ o $9a^2 - 49b^2$

5 Factorise these differences of perfect squares.

Hint: $4 - x^2 = 2^2 - x^2$ now use $a = 2$ and $b = x$.

a $4 - x^2$ b $9 - y^2$
c $36 - a^2$ d $100 - 9x^2$
e $b^2 - a^2$ f $400 - 25a^2$
g $4a^2 - 9b^2$ h $16y^2 - 121x^2$

Problem-solving and reasoning 6(½), 7 6(½), 7, 8(½)

Example 7 Factorising by first taking out a common factor

Factorise $12y^2 - 1200$ by first taking out a common factor.

Solution	Explanation
$12y^2 - 1200 = 12(y^2 - 100)$	First take out the common factor of 12.
$= 12(y - 10)(y + 10)$	$100 = (10)^2$, use $a^2 - b^2 = (a - b)(a + b)$.

Now you try

Factorise $4a^2 - 36$ by first taking out a common factor.

6 Factorise the following by first taking out a common factor.

Hint: As a first step, take out a common factor and then factorise the DOPS.

a $2x^2 - 32$ b $5x^2 - 45$ c $6y^2 - 24$
d $3y^2 - 48$ e $3x^2 - 75y^2$ f $3a^2 - 300b^2$
g $12x^2 - 27y^2$ h $63a^2 - 112b^2$ i $108x^2 - 147y^2$

7 The height (in metres) of a falling object above ground level is given by $100 - t^2$, where t is in seconds.

a Find the height of the object:
  i initially (i.e. at $t = 0$)
  ii after 2 seconds
  iii after 8 seconds
b Factorise the expression $100 - t^2$.
c Use your factorised expression from part b to find the height of the object:
  i initially (i.e. at $t = 0$)
  ii after 2 seconds
  iii after 8 seconds
d How long does it take for the object to hit the ground?

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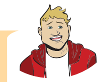
8 We can work out problems such as $19^2 - 17^2$ without a calculator, like this:



Hint: Factorise first, using $a^2 - b^2 = (a + b)(a - b)$, then evaluate.

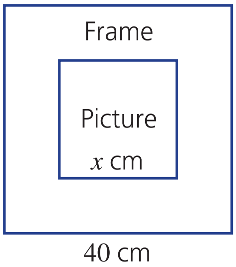
$$
\begin{aligned}
19^2 - 17^2 &= (19 + 17)(19 - 17)\\
&= 36 \times 2\\
&= 72
\end{aligned}
$$

Use this idea to evaluate the following by first factorising, without the use of a calculator.

a $16^2 - 14^2$ **b** $18^2 - 17^2$ **c** $13^2 - 10^2$ **d** $15^2 - 11^2$
e $17^2 - 15^2$ **f** $11^2 - 9^2$ **g** $27^2 - 24^2$ **h** $52^2 - 38^2$

Flexible framing — 9

9 A special square picture frame can hold a square picture of any size up to 40 cm.



a Using a picture side length of x cm, write expressions for the area of:
  i the picture
  ii the frame (not including the picture)

b Factorise your expression for the frame area.

c Find the frame area when:
  i $x = 20$ **ii** $x = 10$

d Using trial and error, what value of x is required if the frame area is to be 700 cm^2?

Using technology 10B: Expanding and factorising
This activity is available on the companion website as a printable PDF.

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10C Factorising trinomials of the form $x^2 + bx + c$

Learning intentions

- To know the form of a monic quadratic trinomial
- To be able to factorise a monic quadratic trinomial

Key vocabulary: monic quadratic trinomial, factor, constant, coefficient, integer

A quadratic trinomial of the form $x^2 + bx + c$ is called a monic quadratic because the coefficient of x^2 is 1. ('Monic' comes from the word 'mono', which means 'one'.)

Now consider: $(x+m)(x+n) = x^2 + xn + mx + mn$

$$= x^2 + (m+n)x + mn$$

We can see from this expansion that mn gives the constant term (c) and $m+n$ is the coefficient of x. This tells us that to factorise a monic quadratic, we should look for factors of the constant term (c) that add to give the coefficient of the middle term (b).

Lesson starter: So many choices of factors?

We know that to factorise $x^2 - 5x - 24$ we must choose a pair of numbers that multiply to give −24. Look at the following equations and discuss which of them are true.

$x^2 - 5x - 24 = (x+8)(x+3)$ $\quad$ $x^2 - 5x - 24 = (x+6)(x-4)$

$x^2 - 5x - 24 = (x-12)(x-2)$ $\quad$ $x^2 - 5x - 24 = (x-6)(x+4)$

$x^2 - 5x - 24 = (x-12)(x+2)$ $\quad$ $x^2 - 5x - 24 = (x-8)(x+3)$

$x^2 - 5x - 24 = (x-8)(x-3)$ $\quad$ $x^2 - 5x - 24 = (x-24)(x+1)$

Key ideas

- **Monic quadratics** have a coefficient of x^2 equal to 1.
- Monic quadratics of the form $x^2 + bx + c$ can be factorised by finding the two numbers that multiply to give the constant term (c) and add to give the coefficient of x (b).

$x^2 + 5x + 6 = (x+3)(x+2)$ — 5: $2 + 3$; 6: 2×3

$x^2 - x - 6 = (x-3)(x+2)$ — -1: $-3 + 2$; -6: -3×2

$x^2 - 5x + 6 = (x-3)(x-2)$ — -5: $-2 + (-3)$; 6: $-2 \times (-3)$

$x^2 + x - 6 = (x+3)(x-2)$ — 1: $3 + (-2)$; -6: $3 \times (-2)$

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Exercise 10C

Understanding 1–3 3

1 Give the missing words or letters.
  a A _________ quadratic has the coefficient of x^2 equal to 1.
  b To factorise a monic quadratic, we look for factors of the _________ term (c) that add to the _________ of x (b).

2 Find two integers that multiply to give the first number and add to give the second number.

a $18, 11$	**b** $20, 12$	**c** $-15, 2$
d $-12, 1$	**e** $-24, -5$	**f** $-30, -7$
g $10, -7$	**h** $36, -15$	**i** $-8, 7$

Hint: The integers include $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

3 **a** **i** Which two numbers multiply to give 15 and add to give 8?
  ii Complete $x^2 + 8x + 15 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$
b **i** Which two numbers multiply to give -10 and add to give 3?
  ii Complete $x^2 + 3x - 10 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$
c **i** Which two numbers multiply to give 8 and add to give -6?
  ii Complete $x^2 - 6x + 8 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$

Fluency 4–5(½) 4–5(½)

Example 8 Factorising trinomials of the form $x^2 + bx + c$

Factorise:
a $x^2 + 7x + 12$ **b** $x^2 + x - 6$ **c** $x^2 - 5x + 6$

Solution	Explanation
a $x^2 + 7x + 12 = (x + 4)(x + 3)$	$3 \times 4 = 12$ and $3 + 4 = 7$. Check: $(x + 4)(x + 3) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$.
b $x^2 + x - 6 = (x - 2)(x + 3)$	Since the numbers must multiply to -6, one must be positive and one negative. $-2 \times 3 = -6$ and $-2 + 3 = 1$. Check: $(x - 2)(x + 3) = x^2 + 3x - 2x - 6 = x^2 + x - 6$.
c $x^2 - 5x + 6 = (x - 3)(x - 2)$	$-3 \times (-2) = 6$ and $-3 + (-2) = -5$. Check: $(x - 3)(x - 2) = x^2 - 2x - 3x + 6 = x^2 - 5x + 6$.

Now you try

Factorise:
a $x^2 + 13x + 36$ **b** $x^2 + 7x - 8$ **c** $x^2 - 8x - 20$

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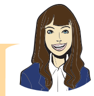
10C

4 Factorise these quadratic trinomials.

a $x^2 + 7x + 6$ **b** $x^2 + 5x + 6$ **c** $x^2 + 6x + 9$
d $x^2 + 7x + 10$ **e** $x^2 + 7x + 12$ **f** $x^2 + 11x + 18$
g $x^2 + 5x - 6$ **h** $x^2 + x - 6$ **i** $x^2 + 2x - 8$
j $x^2 + 3x - 4$ **k** $x^2 + 7x - 30$ **l** $x^2 + 9x - 22$
m $x^2 - 7x + 10$ **n** $x^2 - 6x + 8$ **o** $x^2 - 7x + 12$
p $x^2 - 2x + 1$ **q** $x^2 - 9x + 18$ **r** $x^2 - 11x + 18$
s $x^2 - 4x - 12$ **t** $x^2 - x - 20$ **u** $x^2 - 5x - 14$
v $x^2 - x - 12$ **w** $x^2 + 4x - 32$ **x** $x^2 - 3x - 10$

Hint: For $x^2 + bx + c$, look for factors of c that add to give b.



Example 9 Factorising perfect squares

Factorise $x^2 - 8x + 16$ to form a perfect square.

Solution	Explanation
$x^2 - 8x + 16 = (x - 4)(x - 4)$ $= (x - 4)^2$	$-4 \times (-4) = 16$ and $-4 + (-4) = -8$. $(x - 4)(x - 4) = (x - 4)^2$ is a perfect square.

Now you try

Factorise $x^2 - 10x + 25$ to form a perfect square.

5 Factorise these perfect squares.

a $x^2 - 4x + 4$ **b** $x^2 + 6x + 9$
c $x^2 + 12x + 36$ **d** $x^2 - 14x + 49$
e $x^2 - 18x + 81$ **f** $x^2 - 20x + 100$
g $x^2 + 8x + 16$ **h** $x^2 + 20x + 100$

Hint: Factorise perfect squares just like any trinomial but finish by writing them in the form $(x + a)^2$.

Problem-solving and reasoning

6, 7(½) | 6–7(½), 8

Example 10 Factorising by first taking out a common factor

Factorise $2x^2 - 10x - 28$ by first taking out a common factor.

Solution	Explanation
$2x^2 - 10x - 28 = 2(x^2 - 5x - 14)$ $= 2(x - 7)(x + 2)$	First take out the common factor of 2. $-7 \times 2 = -14$ and $-7 + 2 = -5$.

Now you try

Factorise $3x^2 - 18x + 15$ by first taking out a common factor.

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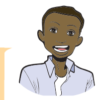
6 Factorise by first taking out the common factor.

a $2x^2 + 14x + 20$ **b** $3x^2 + 21x + 36$
c $2x^2 + 22x + 36$ **d** $5x^2 - 5x - 10$
e $4x^2 - 16x - 20$ **f** $3x^2 - 9x - 30$
g $-2x^2 - 14x - 24$ **h** $-3x^2 + 9x - 6$
i $-2x^2 + 10x + 28$ **j** $-4x^2 + 4x + 8$
k $-5x^2 - 20x - 15$ **l** $-7x^2 + 49x - 42$

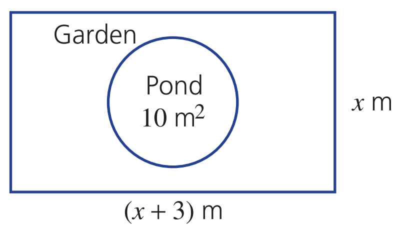
Hint: Factor out the coefficient of x^2.



7 Factorise these as perfect squares after first taking out the common factor.

a $2x^2 + 44x + 242$ **b** $3x^2 - 24x + 48$
c $5x^2 - 50x + 125$ **d** $-3x^2 + 36x - 108$
e $-2x^2 + 28x - 98$ **f** $-4x^2 - 72x - 324$

8 A rectangular garden has length 3 m more than its width, x m. There is a pond of area 10 m^2 in the centre.



a Find an expression for:
i the entire area (Expand your answer.)
ii the garden area, excluding the pond

b Factorise your answer from part **a ii**.

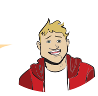
c What is the area of the garden, excluding the pond, when:
i $x = 5$? **ii** $x = 7$?

Algebraic fractions — 9(½)

9 Some algebraic fractions can be simplified using factorisation.
Here is an example:

$$\frac{x^2 - x - 12}{x - 4} = \frac{(x-4)(x+3)}{x-4} = x + 3$$

Hint: First factorise the numerator or denominator, then cancel.



Use this idea to simplify these fractions.

a $\frac{x^2 - 3x - 54}{x - 9}$ **b** $\frac{x^2 + x - 12}{x + 4}$ **c** $\frac{x^2 - 6x + 9}{x - 3}$

d $\frac{x + 2}{x^2 + 9x + 14}$ **e** $\frac{x - 3}{x^2 - 8x + 15}$ **f** $\frac{x + 1}{x^2 - 5x - 6}$

g $\frac{x^2 - 4x + 4}{x - 2}$ **h** $\frac{x^2 + 2x + 1}{x + 1}$ **i** $\frac{x^2 - 16x + 64}{x - 8}$

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10D Solving equations of the form $ax^2 = c$

Learning intentions

- To know that a quadratic equation can have 0, 1 or 2 solutions
- To be able to solve a quadratic equation of the form $ax^2 = c$

Key vocabulary: quadratic equation, solve, exact, surd, square root

A quadratic equation can be expressed in the general form $ax^2 + bx + c = 0$, where a, b and c are real numbers with $a \neq 0$. In a quadratic equation the highest power is 2. The simplest quadratic equation can be expressed in the form $x^2 = c$ and these will be considered in this section.

We will see that since $5^2 = 5 \times 5 = 25$ and $(-5)^2 = -5 \times (-5) = 25$, there are two possible solutions to the equation $x^2 = 25$; i.e. $x = 5$ or $x = -5$. However, note that $x^2 = -25$ has no solutions.

This simplest form of a quadratic equation arises in many circumstances, including in Pythagoras' theorem and in area and volume formulas. Formulas involving motion, including the velocity of a constantly accelerating object and the time taken for an object to fall due to gravity, also give rise to quadratic equations.

Lesson starter: How many possible solutions?

- List all the pairs of numbers that multiply to 16.
- List all the pairs of numbers that multiply to −16.
- What do you notice about the signs of your pairs of numbers that multiply to 16? What about the signs of the numbers that multiply to −16?
- How many of your listed pairs satisfy $x \times x = 16$?
- Are there any pairs that satisfy $x \times x = -16$?
- From what you have found above, how many values of x would make the following true?

  a $x^2 = 9$ **b** $x^2 = -9$

Key ideas

- A **quadratic equation** is an equation in which the highest power is 2.
  For example: $x^2 = 9$, $2x^2 - 6 = 0$ and $x^2 - 2x = 8$.
  - A quadratic equation may have 0, 1 or 2 possible solutions.
- The square of a number is always positive or 0; i.e. $x^2 \geq 0$ for all values of x.
  For example: $(2)^2 = 2 \times 2 = 4$ and $(-3)^2 = -3 \times (-3) = 9$.
- The inverse operation of squaring is the square root, $\sqrt{\ }$.
  - $\sqrt{4} = 2$ since $2 \times 2 = 4$.
  - $\sqrt{7} \times \sqrt{7} = 7$
- When $x^2 = c$, then $x = \pm\sqrt{c}$ provided that $c \geq 0$.
  - $\pm$ represents two possible solutions: $+\sqrt{c}$ and $-\sqrt{c}$.
  - $c = 0$ gives just one solution since $\sqrt{0} = 0$.
- When $x^2 = c$ and $c < 0$, there are no real solutions.
- Square roots that do not reduce to whole numbers are called **surds** and can be left in this exact form; i.e. $\sqrt{2}$, $\sqrt{10}$, … etc.
  For example: $\sqrt{25}$ is not a surd as 25 is a square number and $\sqrt{25} = 5$.

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- To solve equations of the form $ax^2 = c$, make x^2 the subject and then take the square root of both sides to solve for x.
- In practical problems such as those involving measurement, it may make sense to reject the negative solution obtained.

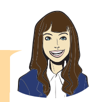
Exercise 10D

Understanding | 1–4 | 4

1 List the first 12 square numbers.

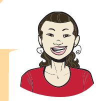
Hint: Start at $1^2 = 1,\ 2^2 = \dots$

2 Evaluate the following.
a 4^2
b 9^2
c 7^2
d $(-3)^2$
e $(-8)^2$
f $(-1)^2$



Hint: $4^2 = 4 \times 4 = \dots$
$(-3)^2 = -3 \times (-3) = \dots$

3 Evaluate the following.
a $\sqrt{36}$
b $-\sqrt{49}$
c $-\sqrt{100}$
d $\sqrt{0}$
e $\sqrt{400}$
f $\sqrt{1600}$



Hint: $\sqrt{4} = 2$ since $2^2 = 4$.

4 Complete the following. The equation $x^2 = c$ has:
a two solutions when ________
b one solution when ________
c no solutions when ________

Fluency | 5–7(½) | 5–7(½)

Example 11 Solving equations of the form $ax^2 = c$

Find all possible solutions to the following equations.
a $x^2 = 16$
b $3x^2 = 12$
c $x^2 = -25$

Solution	Explanation
a $x^2 = 16$ $\therefore x = \pm\sqrt{16}$ $x = \pm 4$	Take the square root of both sides to solve for x. If $x^2 = c$, where $c > 0$, then $x = \pm\sqrt{c}$. Since 16 is a square number, $\sqrt{16}$ is a whole number. $\pm$ represents two solutions: $+4$ since $(+4)^2 = 16$ and -4 since $(-4)^2 = 16$.
b $3x^2 = 12$ $x^2 = 4$ $\therefore x = \pm\sqrt{4}$ $x = \pm 2$	First make x^2 the subject by dividing both sides by 3. Take the square root of both sides to solve for x. This gives two possible values for x: $+2$ and -2.
c $x^2 = -25$ There are no solutions.	Since $x^2 \ge 0$ for all values of x, there are no values of x that will make $x^2 = -25$.

Continued on next page

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10D

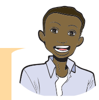
Now you try

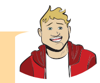
Find all possible solutions to the following equations.

a $x^2 = 64$ b $2x^2 = 18$ c $x^2 = -36$

5 Find the possible solutions for x in the following equations.

a $x^2 = 25$ b $x^2 = 81$ c $x^2 = 36$
d $x^2 = 49$ e $x^2 = -16$ f $x^2 = -100$
g $x^2 = 400$ h $x^2 = 144$ i $2x^2 = 50$
j $3x^2 = 48$ k $-5x^2 = -5$ l $-5x^2 = 20$
m $\frac{x^2}{2} = 32$ n $\frac{x^2}{3} = 27$ o $\frac{1}{2}x^2 = 18$


Hint: When taking the square root, don't forget ±.


Hint: $\frac{1}{2}x^2$ is the same as $\frac{x^2}{2}$.

Example 12 Solving equations with answers that are surds

Find all possible solutions to the following equations.

a $x^2 = 10$ (Give your answer in exact surd form.)
b $3x^2 = 9$ (Give your answer to one decimal place.)

Solution	Explanation
a $x^2 = 10$	Take the square root of both sides.
$\therefore x = \pm\sqrt{10}$	Since 10 is not a square number, leave your answer in exact surd form $\pm\sqrt{10}$, as required.
b $3x^2 = 9$	Divide both sides by 3.
$x^2 = 3$	Take the square root of both sides.
$\therefore x = \pm\sqrt{3}$	Use a calculator to evaluate $\pm\sqrt{3} = 1.73205\ldots$ and round your answer to one decimal place, as required.
$x = \pm 1.7$ (to 1 d.p.)	

Now you try

Find all possible solutions to the following equations.

a $x^2 = 29$ (Give your answer in exact surd form.)
b $5x^2 = 55$ (Give your answer to one decimal place.)

6 a Solve the following equations, where possible. Leave your answer in exact surd form.

i $x^2 = 14$ ii $x^2 = 22$ iii $x^2 = 17$
iv $x^2 = -7$ v $3x^2 = 15$ vi $4x^2 = 24$
vii $\frac{x^2}{3} = 7$ viii $\frac{x^2}{2} = 3$ ix $\frac{1}{5}x^2 = 3$


Hint: $\sqrt{14}$ is exact surd form.

b Solve these equations, giving your answers to one decimal place.

i $x^2 = 12$
ii $x^2 = 35$
iii $3x^2 = 90$
iv $\frac{x^2}{9} = 7$

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Example 13 Solving equations of the form $ax^2 + b = 0$

Find the exact solution(s), where possible, to the following equations.

a $x^2 - 9 = 0$ **b** $x^2 + 12 = 0$

c $\frac{x^2}{2} - 3 = 4,\ x > 0$ **d** $7 - x^2 = 6,\ x < 0$

Solution	Explanation
a $x^2 - 9 = 0$	Make x^2 the subject by adding 9 to both sides.
$x^2 = 9$	Solve for x by taking the square root of both sides.
$\therefore x = \pm\sqrt{9}$	
$x = \pm 3$	
b $x^2 + 12 = 0$	Make x^2 the subject by subtracting 12 from both sides.
$x^2 = -12$	Since x^2 is positive for all values of x, $x^2 = c$, where c is negative has no real solution.
There is no real solution.	
c $\frac{x^2}{2} - 3 = 4$	Solve for x^2 by adding 3 to both sides and then multiplying both sides by 2.
$\frac{x^2}{2} = 7$	
$x^2 = 14$	Take the square root of both sides to solve for x. $\sqrt{14}$ is the exact form of the answer.
$\therefore x = \pm\sqrt{14}$	
$x = \sqrt{14}$ since $x > 0$.	Note the restriction that x must be a positive value (i.e. > 0), so reject the solution $x = -\sqrt{14}$.
d $7 - x^2 = 6$	Subtract 7 from both sides and then divide both sides by -1. Alternatively, add x^2 to both sides to give $7 = x^2 + 6$ and then solve.
$-x^2 = -1$	
$x^2 = 1$	
$\therefore x = \pm\sqrt{1}$	
$x = \pm 1$	$\sqrt{1} = 1$ since $1^2 = 1$.
$x = -1$ since $x < 0$.	Reject the solution $x = 1$ because $x < 0$.

Now you try

Find the exact solution(s), where possible, to the following equations.

a $x^2 - 25 = 0$ **b** $x^2 + 7 = 0$

c $\frac{x^2}{3} - 1 = 4,\ x > 0$ **d** $3 - x^2 = -6,\ x < 0$

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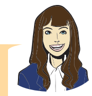
10D

7 Find the exact solution(s), where possible, of these equations.

a $x^2 - 4 = 0$ **b** $x^2 - 81 = 0$ **c** $x^2 + 16 = 0$
d $x^2 + 10 = 0$ **e** $x^2 + 6 = 15$ **f** $x^2 - 5 = 10$
g $2x^2 - 6 = 0,\ x > 0$ **h** $3x^2 + 10 = 13$ **i** $\dfrac{x^2}{2} + 4 = 5,\ x > 0$
j $10 - x^2 = 3,\ x < 0$ **k** $5 - x^2 = 7$ **l** $4 - x^2 = -1,\ x > 0$

Hint: First get your equation in the form $x^2 = c$.



Problem-solving and reasoning

	8–11	8(½), 11, 13–16

8 Solve these equations by first collecting like terms.

a $3x^2 = 2x^2 + 16$ **b** $4x^2 = 2x^2 + 18$
c $4x^2 - 11 = x^2 + 16$ **d** $x^2 = 50 - x^2$
e $2x^2 + 5 = 17 - x^2$ **f** $4x^2 + 8 = 2x^2 + 5$

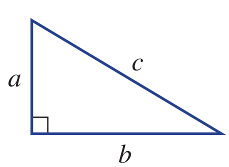
Hint: Rewrite the equation in the form $x^2 = c$ by collecting x^2 terms on one side and the number on the other.

$-2x^2$ $3x^2 = 2x^2 + 16$ $-2x^2$
$x^2 = 16$

9 The area of a square is 36 m^2. What is its perimeter?

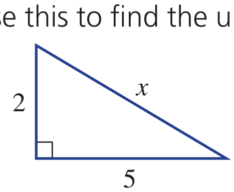
10 The distance, d metres, that a bird dives in t seconds is given by $d = 5t^2$. How long does the bird take to dive 80 m?

11 Recall Pythagoras' theorem: $c^2 = a^2 + b^2$.

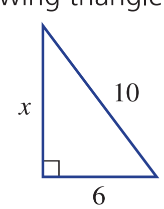


Use this to find the unknown side length in the following triangles. Give your answers in exact form.

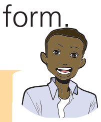
a



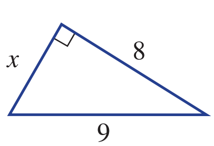
b



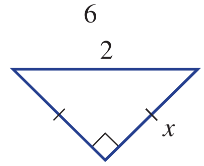
Hint: Set up the Pythagorean equation first.



c



d



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12 The volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. If the volume of a cylindrical water tank is 285 m^3 and its height is 15 m, determine its radius, to one decimal place.

Hint: $\pi r^2(15)$ can be written as $15\pi r^2$.

13 **a** Solve the equation $\pi r^2 = 24$, to find two possible values for r, rounded to one decimal place.

b Given that $A = \pi r^2$ is the formula for the area of a circle of radius r, what restriction does this place on your answer to part **a** if the area of the circle is 24 units^2?

14 Determine for which values of b the equation $x^2 - b = 0$ has:

a two solutions **b** one solution **c** no solutions

Hint: For Questions **14** & **15**, first solve for x^2.

15 Determine for which values of b the equation $x^2 + b = 0$ has:

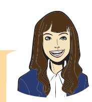
a two solutions **b** one solution **c** no solutions

16 **a** Solve the equation $x^2 - 25 = 0$.

b Hence, determine the range of values of x for which:

i $x^2 - 25 < 0$ **ii** $x^2 - 25 > 0$

Hint: In part **b**, make use of your answer to part **a**.



Square powers and brackets	—	17(½)

17 Consider the following example.

Solve $(x+1)^2 = 16$

$x + 1 = \pm\sqrt{16}$ (Take the square root of both sides.)

$x + 1 = \pm 4$

$x + 1 = 4$ or $x + 1 = -4$ (Write each equation separately.)

$x = 3$ or $x = -5$ (Subtract 1 from both sides.)

Use this method to solve these equations.

a $(x+1)^2 = 9$ **b** $(x+3)^2 = 49$ **c** $(x-2)^2 = 4$ **d** $(x-5)^2 = 16$

e $(2x+1)^2 = 25$ **f** $(2x-1)^2 = 1$ **g** $(3x-2)^2 = 4$ **h** $(4x-3)^2 = 81$

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10E Solving equations using the null factor law ★

Learning intentions

- To understand how the null factor law gives solutions to an equation
- To be able to factorise and solve a quadratic equation using the null factor law

Key vocabulary: null factor law, product, standard form, factor

In previous chapters you would have solved linear equations such as $3x = 9$ and $2x - 1 = 5$, and you may have used 'back tracking' or inverse operations to solve them.

For quadratic equations such as $x^2 - x = 0$ or $x^2 - x - 20 = 0$, we need a new method, because there are different powers of x involved and 'back tracking' isn't useful.

The result of multiplying a number by zero is zero. Therefore, if an expression equals zero then at least one of its factors must be zero. This is called the null factor law and it provides us with an important method that can be utilised to solve a range of mathematical problems involving quadratic equations.

Such equations relate to the engineering of bridges, for example, which use parabolic arches.

Lesson starter: How does the null factor law work?

Start this investigation by completing this table.

x	-5	-4	-3	-2	-1	0	1	2
$(x-1)(x+4)$	6							

- Which values of x make $(x - 1)(x + 4) = 0$? Why?
- Could you work out what values of x make $(x - 1)(x + 4) = 0$ without doing a table? Explain.
- What values of x make $(x - 2)(x + 3) = 0$ or $(x + 5)(x - 7) = 0$?

Key ideas

- The **null factor law** states that if the product of two numbers is zero, then either or both of the two numbers is zero.
  - If $a \times b = 0$, then $a = 0$ and/or $b = 0$.
- To solve a quadratic equation, write it in standard form (i.e. $ax^2 + bx + c = 0$) and factorise. Then use the null factor law.

  For example: $x^2 - 2x - 8 = 0$
  $(x - 4)(x + 2) = 0$
  $x - 4 = 0$ or $x + 2 = 0$
  $x = 4$ or $x = -2$
  - If the coefficients of all the terms have a common factor, then first divide by that common factor.

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Exercise 10E

Understanding | 1–3 | 3

1 Complete the following.
  a If $a \times b = 0$, the null factor law states that _______ or _______.
  b If the coefficients of all terms in an equation have a _______ _______ then divide by that _______ _______.

2 a Complete this table for the given values of x.

x	-3	-2	-1	0	1	2
$(x+2)(x-1)$						

  b What values of x make $(x+2)(x-1)=0$?
  c What values of x would make $(x+3)(x-2)=0$?

3 Copy and complete:
  a $x(x-2)=0$
  $x = 0$ or _____ $= 0$
  $x = 0$ or $x =$ _____

  b $(x-1)(x+4)=0$
  $x-1=0$ or _____ $= 0$
  $x =$ _____ or $x =$ _____

  c $(x+6)(2x-7)=0$
  _____ $= 0$ or _____ $= 0$
  $x =$ ____ or $2x =$ _____
  $x=-6$ or $x =$ ___

Fluency | 4–7(½) | 4–7(½)

Example 14 Using the null factor law

Use the null factor law to solve these equations.
a $x(x-1)=0$
b $(x-1)(2x+5)=0$

Solution	Explanation
a $x(x-1)=0$	
$x=0$ or $x-1=0$	Set each factor equal to zero.
$x=0$ or $x=1$	For $x-1=0$, add 1 to both sides to finish.
b $(x-1)(2x+5)=0$	Set each factor equal to zero and then solve each linear equation.
$x-1=0$ or $2x+5=0$	
$x=1$ or $2x=-5$	
$x=1$ or $x=-\frac{5}{2}$	

Now you try

Use the null factor law to solve these equations.
a $x(x+7)=0$
b $(x+6)(3x-2)=0$

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10E

4 Use the null factor law to solve these equations.

a $x(x+1)=0$
b $x(x-5)=0$
c $2x(x-4)=0$
d $(x-3)(x+2)=0$
e $(x+5)(x-4)=0$
f $(x+1)(x-1)=0$
g $(2x-4)(x+1)=0$
h $(3x-2)(x-7)=0$
i $3x(4x+5)=0$
j $(2x-1)(3x+7)=0$
k $(4x-5)(5x+2)=0$
l $(8x+3)(4x+3)=0$

Hint: Null factor law: if $a \times b = 0$, then either $a = 0$ or $b = 0$.

Example 15 Solving quadratic equations with a common factor

Solve $x^2 - 2x = 0$.

Solution	Explanation
$x^2 - 2x = 0$	Factorise by taking out the common factor x. Apply the null factor law: if $a \times b = 0$, then $a = 0$ or $b = 0$.
$x(x-2)=0$	Solve for x.
$\therefore\ x = 0$ or $x - 2 = 0$	
$\therefore\ x = 0$ or $x = 2$	

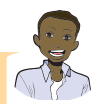
Now you try

Solve $5x^2 + 30x = 0$.

5 Solve the following quadratic equations.

a $x^2 - 4x = 0$
b $x^2 - 3x = 0$
c $x^2 + 2x = 0$
d $3x^2 - 12x = 0$
e $2x^2 - 10x = 0$
f $4x^2 + 8x = 0$

Hint: First take out the common factor, then use the null factor law.



Example 16 Solving with DOPS

Solve $x^2 - 16 = 0$ by factorising the DOPS.

Solution	Explanation
$x^2 - 16 = 0$	Note that $x^2 - 16$ is a difference of perfect squares.
$(x+4)(x-4)=0$	$x^2 - 16 = x^2 - 4^2 = (x+4)(x-4)$
$x + 4 = 0$ or $x - 4 = 0$	Solve each linear factor equal to zero to finish.
$x = -4$ or $x = 4$	

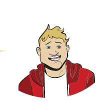
Now you try

Solve $x^2 - 81 = 0$ by factorising the DOPS.

6 Solve the following by factorising the DOPS.

a $x^2 - 25 = 0$
b $x^2 - 36 = 0$
c $x^2 - 100 = 0$
d $4x^2 - 9 = 0$
e $9x^2 - 16 = 0$
f $49x^2 - 81 = 0$

Hint: Recall $4x^2 = (2x)^2$



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Example 17 Solving quadratic equations

Solve the following quadratic equations.

a $x^2 - 5x + 6 = 0$ **b** $x^2 + 2x + 1 = 0$

Solution	Explanation
a $x^2 - 5x + 6 = 0$ $(x - 3)(x - 2) = 0$	Factorise by finding two numbers that multiply to 6 and add to −5: $-3 \times (-2) = 6$ and $-3 + (-2) = -5$.
$\therefore x - 3 = 0$ or $x - 2 = 0$ $\therefore x = 3$ or $x = 2$	Apply the null factor law and solve for x.
b $x^2 + 2x + 1 = 0$ $(x + 1)(x + 1) = 0$	$1 \times 1 = 1$ and $1 + 1 = 2$.
$(x + 1)^2 = 0$	$(x + 1)(x + 1) = (x + 1)^2$ is a perfect square.
$\therefore x + 1 = 0$ $\therefore x = -1$	This gives one solution for x.

Now you try

Solve the following quadratic equations.

a $x^2 + 11x + 30 = 0$ **b** $x^2 - 8x + 16 = 0$

7 Solve the following quadratic equations.

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 5x + 6 = 0$
c $x^2 - 6x + 8 = 0$ **d** $x^2 - 7x + 10 = 0$
e $x^2 + 4x - 12 = 0$ **f** $x^2 + 2x - 15 = 0$
g $x^2 - x - 20 = 0$ **h** $x^2 - 5x - 24 = 0$
i $x^2 - 12x + 32 = 0$ **j** $x^2 + 4x + 4 = 0$
k $x^2 + 10x + 25 = 0$ **l** $x^2 - 8x + 16 = 0$
m $x^2 - 14x + 49 = 0$ **n** $x^2 - 24x + 144 = 0$

Hint: Parts **j** to **n** are perfect squares, so you will find only one solution.

Problem-solving and reasoning 8(½), 9 8(½), 9, 10

8 How many different solutions for x will these equations have?

a $(x - 2)(x - 1) = 0$ **b** $(x + 7)(x + 3) = 0$ **c** $(x + 1)(x + 1) = 0$
d $(x - 3)(x - 3) = 0$ **e** $(x + \sqrt{2})(x - \sqrt{2}) = 0$ **f** $(x + 8)(x - \sqrt{5}) = 0$
g $(x + 2)^2 = 0$ **h** $(x + 3)^2 = 0$ **i** $3(2x + 1)^2 = 0$

9 The height of a paper plane above floor level, in metres, is given by $-\frac{1}{5}t(t - 10)$, where t is in seconds.

a Find the height of the plane after:
  i 2 seconds
  ii 6 seconds

b Solve $-\frac{1}{5}t(t - 10) = 0$ for t.

c How long does it take for the plane to hit the ground after it is launched?

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10E

10 Solve by first taking out a common factor.

a $2x^2 + 16x + 24 = 0$

b $2x^2 - 20x - 22 = 0$

c $3x^2 - 18x + 27 = 0$

d $5x^2 - 20x + 20 = 0$

Hint: First take out the common factor, then factorise before using the null factor law.

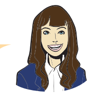
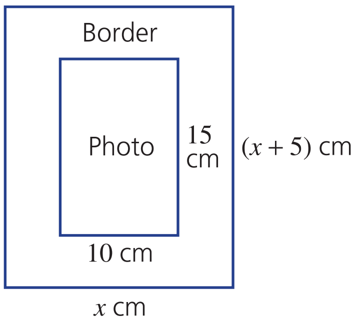


Photo albums — 11

11 A printer produces rectangular photo albums. Each page has a length 5 cm more than the width, and includes a spot in the middle for a standard 10 cm by 15 cm photo.



a Find the area of the photo.

b Find an expression for:

- **i** the total area of a page
- **ii** the border area of a page (i.e. excluding the photo)

c Factorise your expression for the border area.

d For what value of x is the border area equal to zero?

e For what value of x is the border area equal to 350 cm^2?

Using technology 10E: Solving equations using the null factor law.
This activity is available on the companion website as a printable PDF.

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10F Applications of quadratics

Learning intentions

- To be able to set up a quadratic equation from a simple worded problem
- To be able to solve a problem in a real context using a quadratic equation and the null factor law

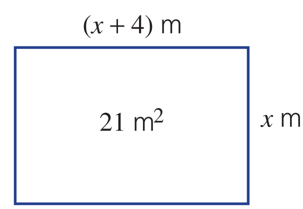
Key vocabulary: define, pronumeral, solve

Defining variables, setting up equations, solving equations and interpreting solutions are all important elements of applying quadratics in problem solving. For example, the area of a rectangular paddock that can be fenced off using a limited length of fencing can be found by setting up a quadratic equation, solving it and then interpreting the solutions.

Lesson starter: Rectangular quadratics

The length of a rectangular room is 4 m longer than its width, which is x metres. Its area is 21 m^2.

- Write an expression for the area using the variable x.
- Use the 21 m^2 area fact to set up an equation equal to zero.
- Factorise and solve the equation to find x and then find the dimensions of the rectangle.



Key ideas

- When applying quadratic equations:
  - Define a pronumeral; i.e. 'Let x be …'.
  - Write an equation.
  - Solve the equation.
  - Choose the solution(s) that solves the equation and answers the question.

Exercise 10F

Understanding	1–3	3

1 Write expressions for each of the following.

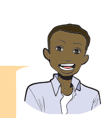
a The length of a rectangle if it is 4 more than its width, x.
b The length of a rectangle if it is 10 more than its width, x.
c The length of a rectangle if it is 7 less than its width, x.
d The height of a triangle if it is 2 less than its base, x.
e The height of a triangle if it is 6 more than its base, x.

Hint: An example of an expression is $x + 3$.

2 Rearrange these equations so that there is a zero on the right-hand side. Do not try to solve the equation.

a $x^2 + 2x = 3$
b $x^2 - 3x = 5$
c $x^2 + 7x = 4$

Hint: Subtract from both sides to give a zero on the right-hand side.



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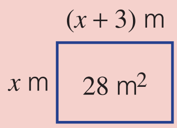
10F

3 The given steps for solving a mathematical problem are in the wrong order. Give the correct order.
- **A** Solve the equation
- **B** Define a pronumeral
- **C** Choose the solution to answer the question
- **D** Write an equation

Fluency | 4–6 | 4–7

Example 18 Finding dimensions

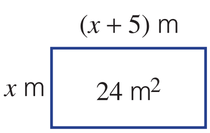
The area of a rectangle is fixed at 28 m^2 and its length is 3 m more than its width. Find the dimensions of the rectangle.

Solution	**Explanation**
Let x m be the width of the rectangle. Length $= (x + 3)$ m	
$x(x + 3) = 28$	Write an equation for area using the given information.
$x^2 + 3x - 28 = 0$	Then write with a zero on the right and solve for x.
$(x + 7)(x - 4) = 0$ $x = -7$ or $x = 4$	Disregard $x = -7$ because x must be greater than zero.
Choose $x = 4$.	
Rectangle has width 4 m and length 7 m.	Answer the question in full.

Now you try

The area of a rectangle is fixed at 72 m^2 and its length is 6 m more than its width. Find the dimensions of the rectangle.

4 A rectangle has an area of 24 m^2. Its length is 5 m longer than its width.



- **a** Copy this sentence: 'Let x m be the width of the rectangle.'
- **b** Write an expression for the rectangle's length.
- **c** Write an equation using the rectangle's area.
- **d** Write your equation from part **c** with a zero on the right-hand side, and solve for x.
- **e** Find the dimensions of the rectangle.

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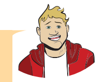
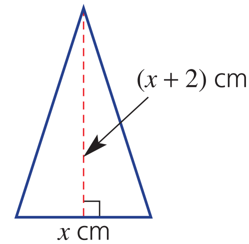
5 Repeat all the steps in Question 4 to find the dimensions of a rectangle with the following properties.
  - a Its area is 60 m^2 and its length is 4 m more than its width.
  - b Its area is 63 m^2 and its length is 2 m less than its width.
  - c Its area is 154 mm^2 and its length is 3 mm less than its width.

Hint: Carefully set out each step, as in Example 18 and Question 4.



6 A triangle's area is 4 cm^2 and its height is 2 cm more than its base.
  - a Write an expression for the area of the triangle, using $A = \frac{1}{2}bh$.
  - b Write an equation using the 4 cm^2 area fact.
  - c Multiply both sides by 2 and write your equation with a zero on the right side.
  - d Solve your equation to find the base and height dimensions of the triangle.

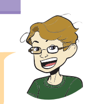


7 Find the height and base lengths of a triangle that has an area of 7 m^2 and height 5 m less than its base.

Problem-solving and reasoning — 8, 9 — 9–11

8 The product of two consecutive whole numbers (x and $x + 1$) is 132, so $x(x + 1) = 132$.
  - a Expand the equation and make a zero on the right side.
  - b Solve the equation to find the two values of x.
  - c List the two pairs of consecutive numbers that multiply to 132.

Hint: 'Product' means 'multiply'. 'Consecutive' means 'next to'; e.g. 4 and 5 are consecutive whole numbers.

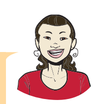


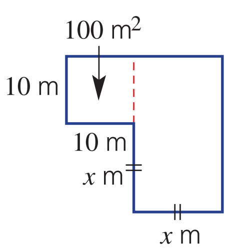
9 The product of two consecutive numbers is 72. Use a quadratic equation to find the two sets of numbers.

Hint: Let x be the smaller number.

10 A 100 m^2 hay shed is to be expanded to give 475 m^2 of floor space in total, as shown in the diagram. Find the value of x.

Hint: Find an expression for the total area using x, then set equal to 475.





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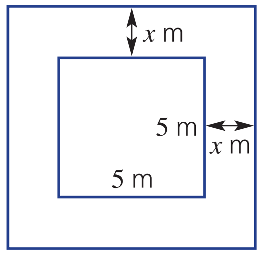
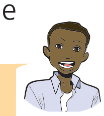
10F

11 A square hut of side length 5 m is to be surrounded with a verandah of width x metres. Find the width of the verandah if its area is to be 24 m^2.





Hint: What is the side length of the verandah?

Projectile maths — 12, 13

12 A ball is thrust vertically upwards from a machine on the ground. The height (h metres) after t seconds is given by $h = t(4 - t)$.

a Find the height after 1.5 seconds.

b Find when the ball is at a height of 3 m.

c Why are there two solutions to part **b**?

d Find when the ball is at ground level.

e Find when the ball is at a height of 4 m.

f Why is there only one solution for part **e**?

g Is there a time when the ball is at a height of 5 m? Explain.

13 The height, h (in metres), of a rocket is given by $h = -x^2 + 100x$, where x metres is the horizontal distance from where the rocket was launched.

a Find the values of x when $h = 0$.

b Interpret your answer from part **a**.

c Find how far the rocket has travelled horizontally when the height is 196 m.

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Progress quiz

10A ★

1 Expand the following.

a $(x + 2)(x + 6)$

b $(2x + 3)(3x - 5)$

c $(3x - 1)(2x - 5)$

10A ★

2 Expand the following.

a $(x + 6)^2$

b $(2x - 3)^2$

c $(x - 5)(x + 5)$

d $(3x + 4)(3x - 4)$

10B ★

3 Factorise these differences of perfect squares.

a $x^2 - 4$

b $y^2 - 81$

c $9x^2 - 25$

d $16x^2 - 49y^2$

10C ★

4 Factorise these trinomials.

a $x^2 + 9x + 14$

b $x^2 - 2x - 15$

c $x^2 - 5x + 4$

d $x^2 + 10x + 25$

10B/C ★

5 Factorise the following by first taking out the common factor.

a $2x^2 - 72$

b $12y^2 - 27$

c $2x^2 + 10x - 28$

10D ★

6 Find all possible solutions to the following equations. Give exact answers.

a $x^2 = 25$

b $2x^2 = 72$

c $x^2 = 14$

d $x^2 + 9 = 0$

e $x^2 - 2 = 9$

f $6 - x^2 = 2, x < 0$

10E ★

7 Use the null factor law to solve these equations.

a $x(x + 2) = 0$

b $(x - 3)(x + 3) = 0$

c $(2x - 1)(x + 4) = 0$

10E ★

8 Factorise these quadratic equations and then apply the null factor law to solve.

a $x^2 - 4x = 0$

b $x^2 - 36 = 0$

c $x^2 + 3x - 40 = 0$

d $6x^2 - 12x = 0$

e $4x^2 - 1 = 0$

f $x^2 - 8x + 16 = 0$

10F ★

9 The area of a rectangle is fixed at 21 m^2. If its length is 4 m more than its width, find the dimensions of the rectangle.

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10G Exploring parabolas

Learning intentions

- To understand that a parabola is a graph of a quadratic relation
- To be able to identify key features of a parabola
- To be able to sketch transformations of $y = x^2$ using simple dilations, reflections and translations

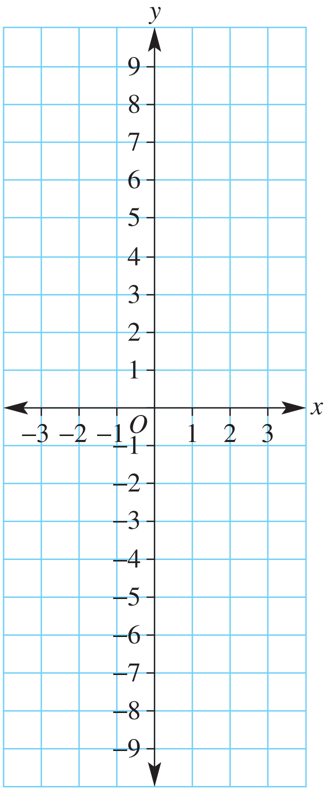
Key vocabulary: parabola, non-linear, plot, turning point, intercept, vertex, minimum, maximum, transformations, dilation, reflection, translation, axis of symmetry

One of the simplest and most important non-linear graphs is the parabola. When a ball is thrown or water streams up and out from a garden hose or fountain, the path followed has a parabolic shape. The parabola is the graph of a quadratic relation with the basic rule $y = x^2$. Quadratic rules, such as $y = (x - 1)^2$ and $y = 2x^2 - x - 3$, also give graphs that are parabolas and are transformations of the graph of $y = x^2$.

Lesson starter: To what effect?

To see how different quadratic rules compare to the graph of $y = x^2$, complete this table and plot the graph of each equation on the same set of axes.

x	-3	-2	-1	0	1	2	3
$y_1 = x^2$	9	4					
$y_2 = -x^2$	-9						
$y_3 = (x - 2)^2$	25	16	9				
$y_4 = x^2 - 3$	6						



- For all the graphs, find such features as the:
  - turning point
  - axis of symmetry
  - y-intercept
  - x-intercepts
- Discuss how each of the graphs of y_2, y_3 and y_4 compare to the graph of $y = x^2$. Compare the rule with the position of the graph.

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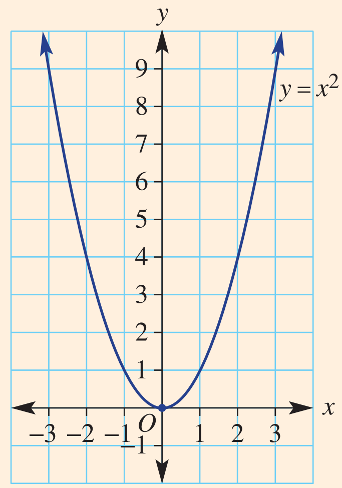
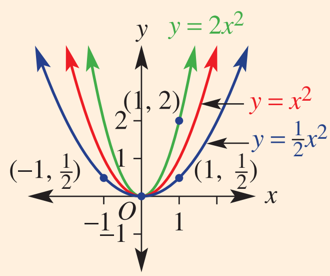
Key ideas

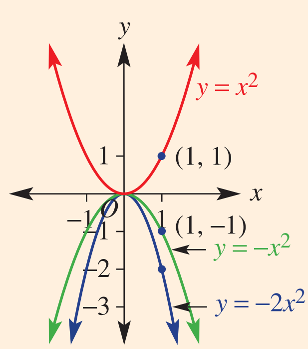
- A **parabola** is the graph of a quadratic relation. The basic parabola has the rule $y = x^2$.
  - The **vertex** (or **turning point**) is $(0, 0)$.
  - It is a minimum turning point.
  - **Axis of symmetry** is $x = 0$.
  - y-intercept is 0.
  - x-intercept is 0.



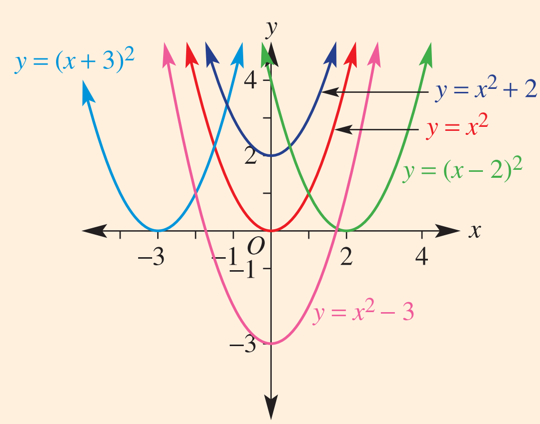
- Simple **transformations** of the graph of $y = x^2$ include:
  - **dilation**



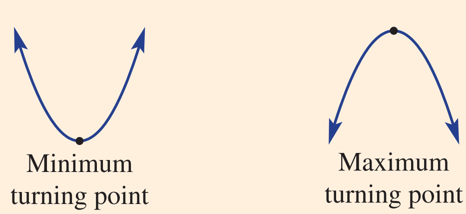
  - **reflection**



  - **translation**



- Turning points can be a maximum or a minimum.



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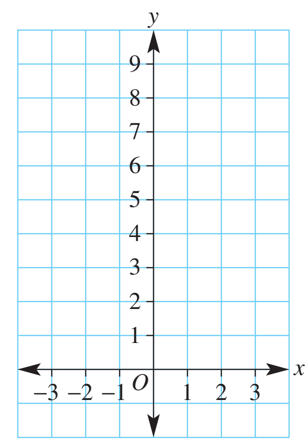
10G

Exercise 10G

Understanding | 1, 2 | 2

1 Complete this table and grid to plot the graph of $y = x^2$.

x	-3	-2	-1	0	1	2	3
y	9						



2 Write the missing features for this graph in the sentences below.

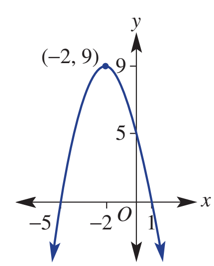
a The parabola has a _________ (maximum or minimum) turning point.

b The coordinates of the turning point are ______ .

c The y-intercept is ______.

d The x-intercepts are ______ and _______.

e The axis of symmetry is ______.



Fluency | 3–6 | 3–6

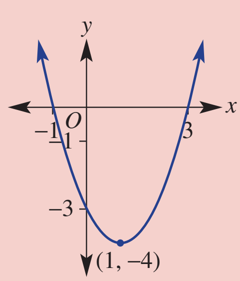
Example 19 Identifying key features of parabolas

Determine the following key features of each of the given graphs:

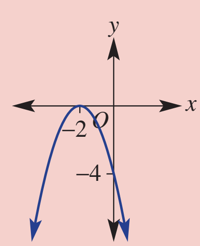
i turning point and whether it is a maximum or minimum

ii axis of symmetry **iii** x-intercepts **iv** y-intercept

a



b



Solution

a **i** Turning point is a minimum at $(1, -4)$.

ii Axis of symmetry is $x = 1$.

iii x-intercepts at -1 and 3.

iv y-intercept at -3.

Explanation

Lowest point of graph is at $(1, -4)$.

Line of symmetry is through the x-coordinate of the turning point.

x-intercepts lie on the x-axis $(y = 0)$ and the y-intercept on the y-axis $(x = 0)$.

Continued on next page

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b **i** Turning point is a maximum at $(-2, 0)$.
Graph has a highest point at $(-2, 0)$.

ii Axis of symmetry is $x = -2$.
Line of symmetry is through the x-coordinate of the turning point.

iii x-intercept at -2.
Turning point is also the one x-intercept.

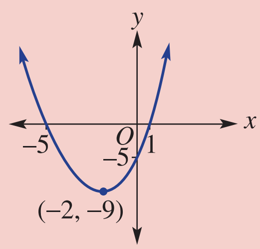
iv y-intercept at -4.

Now you try

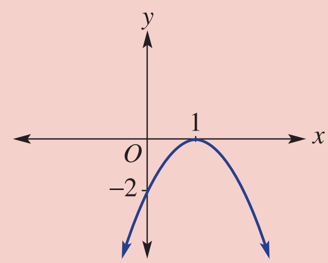
Determine the following key features of each of the given graphs:

i turning point and whether it is a maximum or minimum
ii axis of symmetry **iii** x-intercepts **iv** y-intercept

a



b

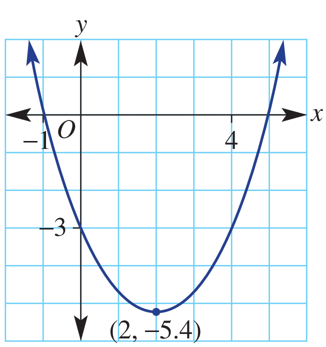


3 Determine these key features of the following graphs.

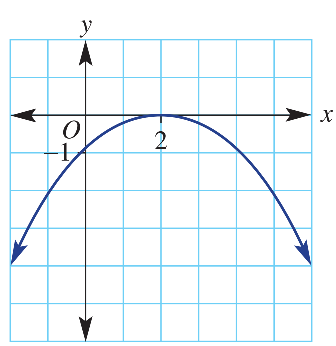
i turning point and whether it is a maximum or minimum
ii axis of symmetry
iii x-intercepts
iv y-intercept

Hint: An axis of symmetry is described by a rule such as $x = 2$ or $x = -3$.

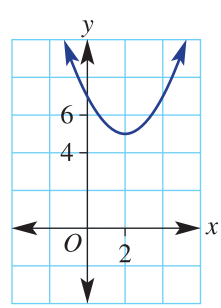
a



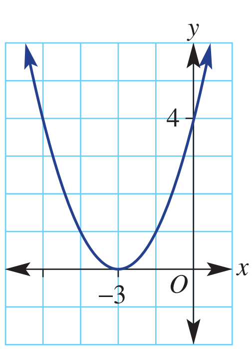
b



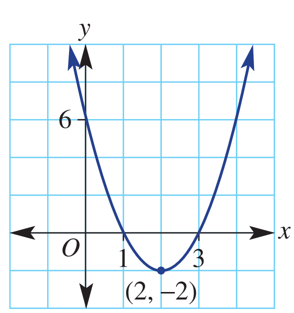
c



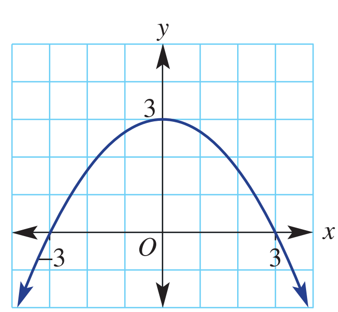
d



e



f



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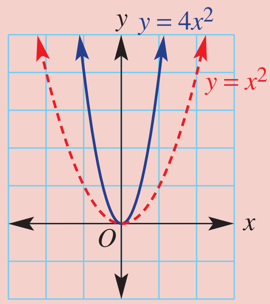
10G

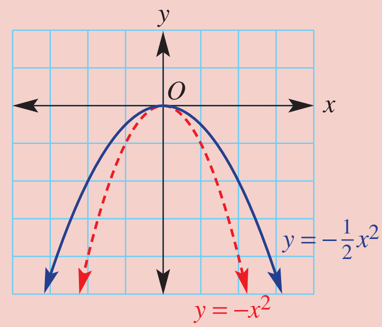
Example 20 Dilating and reflecting parabolas

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$					
b	$y = -\frac{1}{2}x^2$					

Solution

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$	Minimum	No	$(0, 0)$	4	Narrower
b	$y = -\frac{1}{2}x^2$	Maximum	Yes	$(0, 0)$	$-\frac{1}{2}$	Wider

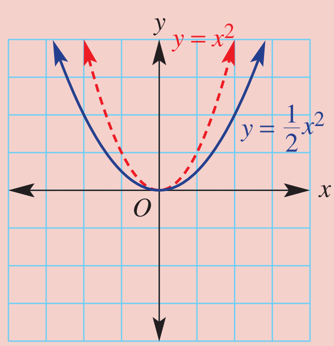
Explanation

Read features from graphs and consider the effect of each change in equation on the graph.

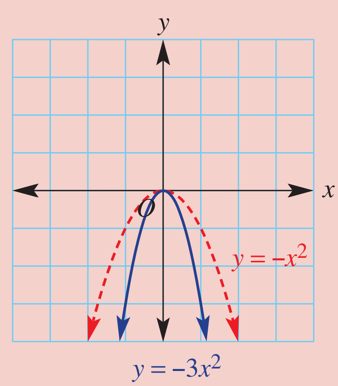
Now you try

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = \frac{1}{2}x^2$					
b	$y = -3x^2$					

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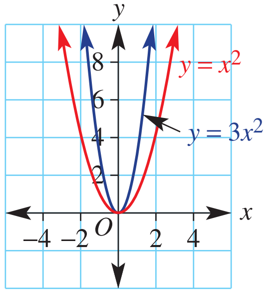
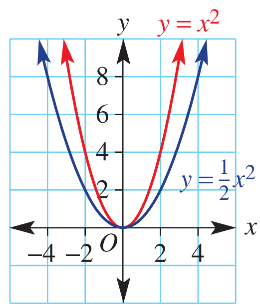
4 Copy and complete the table below for the graphs that follow.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = \frac{1}{2}x^2$					
c	$y = 2x^2$					
d	$y = -4x^2$					
e	$y = -\frac{1}{3}x^2$					
f	$y = -2x^2$					

a



b



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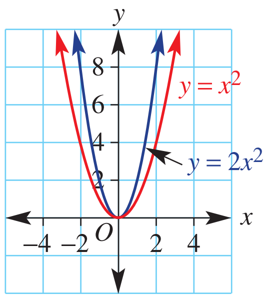
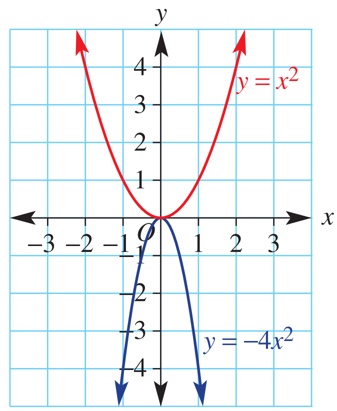
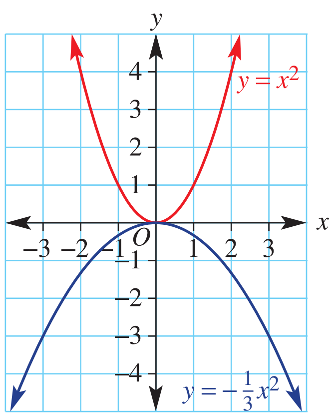


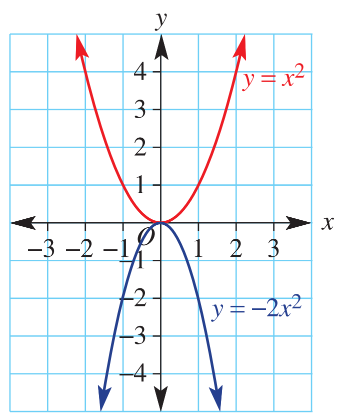
d



e



f



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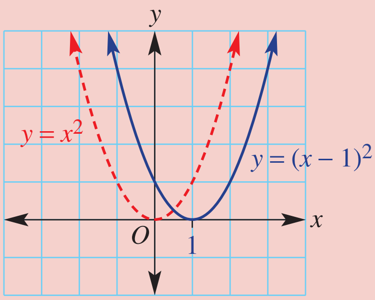
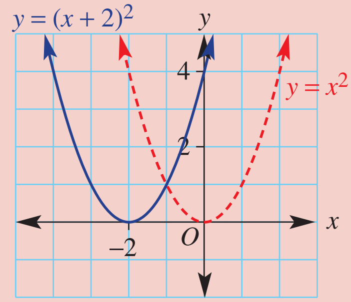
Example 21 Translating horizontally

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x - 1)^2$					
b	$y = (x + 2)^2$					

Solution

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x - 1)^2$	Minimum	No	$(1, 0)$	0	Same
b	$y = (x + 2)^2$	Minimum	No	$(-2, 0)$	9	Same

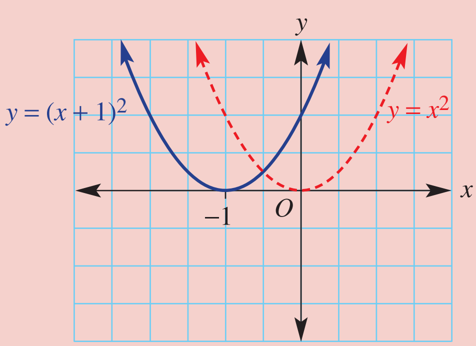
Explanation

The effect is to shift right or left; right for part **a** and left for part **b**.

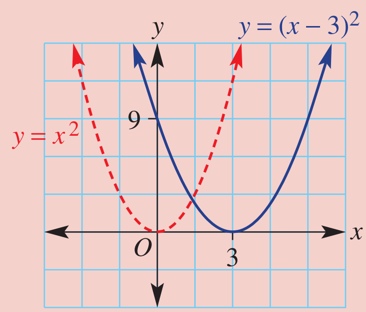
Now you try

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x + 1)^2$					
b	$y = (x - 3)^2$					

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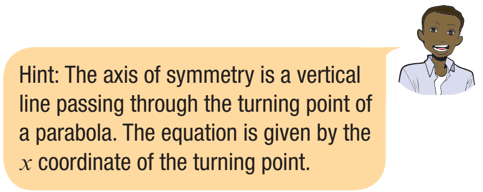
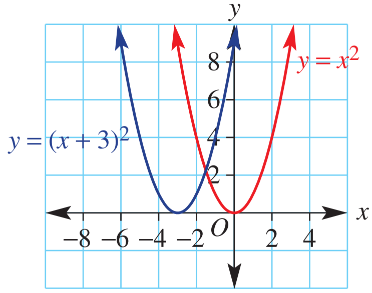
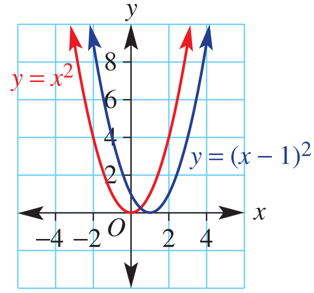
5 Copy and complete the table below for the graphs that follow.

	Formula	Turning point	Axis of symmetry	y-intercept ($x = 0$)	x-intercept
a	$y = (x+3)^2$				
b	$y = (x-1)^2$				
c	$y = (x-2)^2$				
d	$y = (x+4)^2$				



a


b


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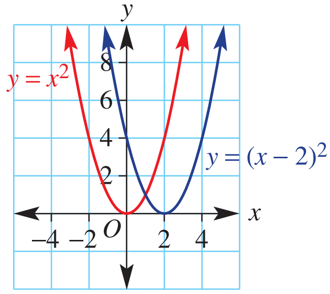

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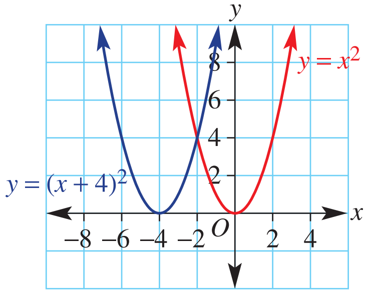
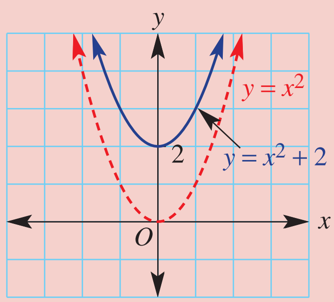
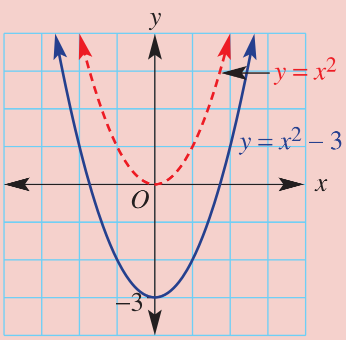


Example 22 Translating vertically

Copy and complete the table for the following graphs.

a


b


	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$					
b	$y = x^2 - 3$					

Continued on next page

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10G

Solution

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$	Minimum	No	$(0, 2)$	3	Same
b	$y = x^2 - 3$	Minimum	No	$(0, -3)$	-2	Same

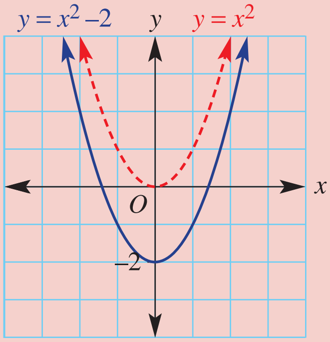
Explanation

The effect is to shift up or down; up for $y = x^2 + 2$ and down for $y = x^2 - 3$.

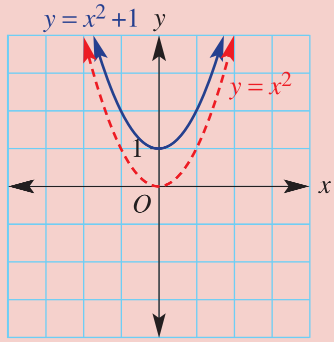
Now you try

Copy and complete the table for the following graphs.

a



b

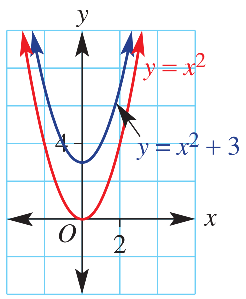


	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 - 2$					
b	$y = x^2 + 1$					

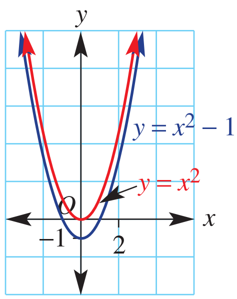
6 Copy and complete the table for the graphs that follow.

	Formula	Turning point	y-intercept $(x = 0)$	y-value when $x = 1$
a	$y = x^2 + 3$			
b	$y = x^2 - 1$			
c	$y = x^2 + 2$			
d	$y = x^2 - 4$			

a



b



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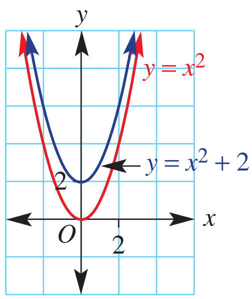

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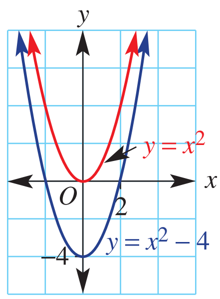

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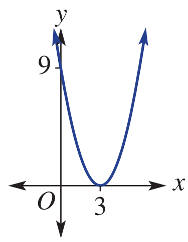
Problem-solving and reasoning | 7 | 7, 8

7 Match each of the following equations to one of the graphs below.

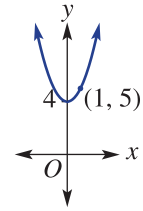
a $y = 2x^2$ **b** $y = x^2 - 6$ **c** $y = (x + 2)^2$

d $y = -5x^2$ **e** $y = (x - 3)^2$ **f** $y = x^2 + 4$

i



ii



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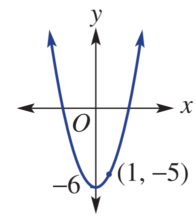

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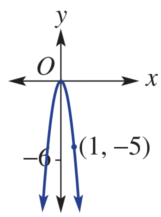
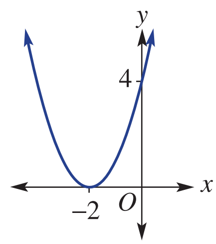
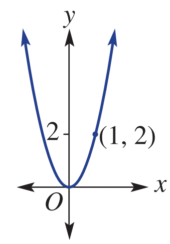


v



vi



8 Write a rule for a parabola with each feature.

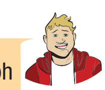
a Same shape as $y = x^2$, minimum turning point $(0, 2)$

b Same shape as $y = x^2$, maximum turning point $(0, 0)$

c Same shape as $y = x^2$, minimum turning point $(-1, 0)$

d Same shape as $y = x^2$, minimum turning point $(5, 0)$

Hint: What turns $y = x^2$ into a graph with a maximum turning point?



Parabolas with technology | — | 9–11

9 **a** Using technology, plot the following pairs of graphs on the same set of axes for $-5 \le x \le 5$ and compare their tables of values.

i $y = x^2$ and $y = 4x^2$ **ii** $y = x^2$ and $y = \frac{1}{3}x^2$ **iii** $y = x^2$ and $y = 6x^2$

iv $y = x^2$ and $y = \frac{1}{4}x^2$ **v** $y = x^2$ and $y = 7x^2$ **vi** $y = x^2$ and $y = \frac{2}{5}x^2$

b Suggest how the constant a in $y = ax^2$ transforms the graph of $y = x^2$.

10 **a** Using technology, plot the following sets of graphs on the same set of axes for $-5 \le x \le 5$ and compare the turning point of each.

i $y = x^2, y = (x + 1)^2, y = (x + 2)^2, y = (x + 3)^2$

ii $y = x^2, y = (x - 1)^2, y = (x - 2)^2, y = (x - 3)^2$

b Explain how the constant h in $y = (x + h)^2$ transforms the graph of $y = x^2$.

11 **a** Using technology, plot the following sets of graphs on the same set of axes for $-5 \le x \le 5$ and compare the turning point of each.

i $y = x^2, y = x^2 + 1, y = x^2 + 2, y = x^2 + 3$

ii $y = x^2, y = x^2 - 1, y = x^2 - 3, y = x^2 - 5$

b Explain how the constant k in $y = x^2 + k$ transforms the graph of $y = x^2$.

Using technology 10G: Sketching parabolas

This activity is available on the companion website as a printable PDF.

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10H Graphs of circles and exponentials ★

Learning intentions

- To know the general rule for a circle with centre (0, 0) and an exponential relation
- To be able to identify key features of a circle and an exponential graph
- To be able to sketch simple circles and exponentials given the rule

Key vocabulary: circle, exponential, radius, asymptote

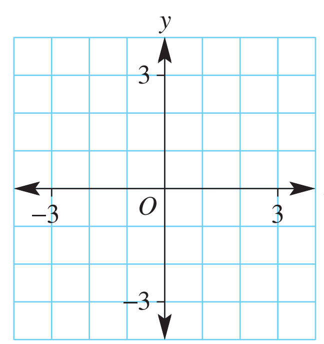
We know the circle as a common shape in geometry. We can also describe a circle using a rule and as a graph on the Cartesian plane.

We can also use graphs to illustrate exponential relationships. For example, the population of the world or the balance of an investment account can be described using exponential rules that include indices. The rule $A = 100\,000(1.05)^t$ describes the account balance of $\$100\,000$ invested at 5% p.a. compound interest for t years.

Lesson starter: Plotting non-linear curves

A graph has the rule $x^2 + y^2 = 9$.

- When $x = 0$ what are the two values of y?
- When $x = 1$ what are the two values of y?
- When $x = 4$ are there any values of y? Discuss.



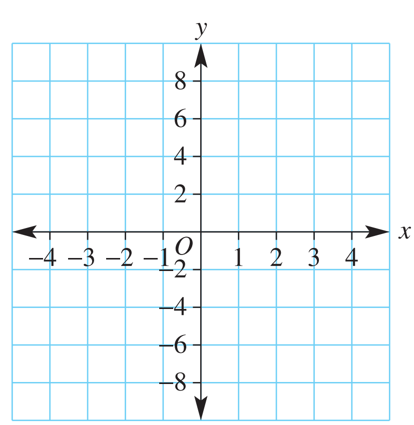
Complete this table of values.

x	-3	-2	-1	0	1	2	3
y		$\pm\sqrt{5}$					

- Now plot all your points on a number plane and join them to form a smooth curve.
- What shape have you drawn and what are its features?
- How does the radius of your circle relate to the equation?

Complete this table and graph the rule $y = 2^x$ before discussing the points below.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$			1		4	



- Discuss the shape of the graph.
- Where does the graph cut the y-axis?
- Does the graph have an x-intercept? Why not?

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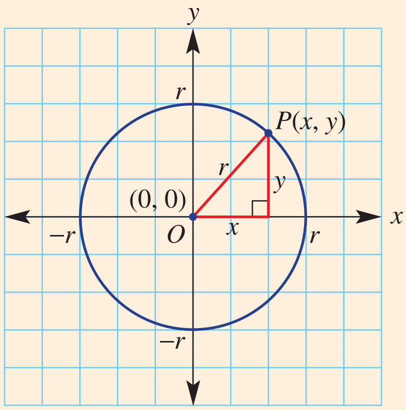
Key ideas

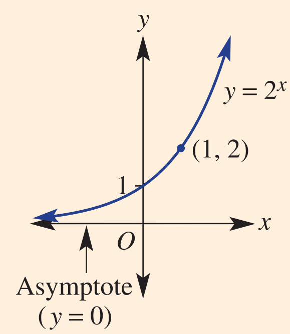
- The Cartesian equation of a circle with centre $(0, 0)$ and radius r is given by $x^2 + y^2 = r^2$.



Using Pythagoras' theorem: $a^2 + b^2 = c^2$ gives $x^2 + y^2 = r^2$.

- A simple **exponential** rule is of the form $y = a^x$, where $a > 0$ and $a \neq 1$.
  - y-intercept is 1.
  - $y = 0$ is the equation of the asymptote.

- An **asymptote** is a line that a curve approaches but never touches. The curve gets closer and closer to the line so that the distance between the curve and the line approaches zero, but the curve never meets the line so the distance is never zero.

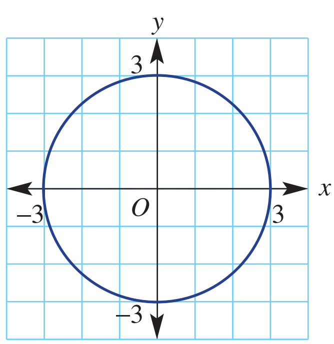


Exercise 10H

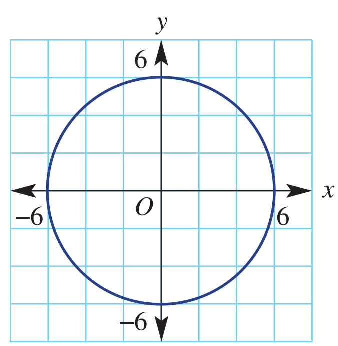
Understanding 1–4 4

1 Write the coordinates of the centre and give the radius of these circles.

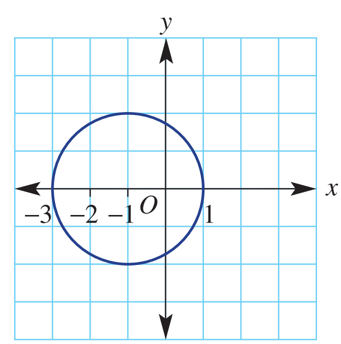
a



b



c



2 Which of the following is an exponential relation?

A $y = 2x$ **B** $y = 2^x$ **C** $y = x^2$ **D** $y^2 + x^2 = 2$

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10H

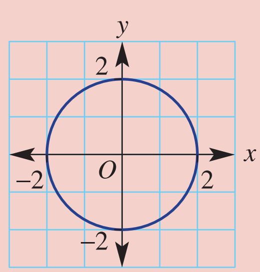
3 Which of the following is a circle equation?
A $y = 3^x$ B $x + y = 3$ C $x^2 + y^2 = 9$ D $y = 9x$

4 A circle has equation $x^2 + y^2 = r^2$. Complete these sentences.
a The centre of the circle is _______.
b The radius of the circle is _______.

Fluency 5–7 5, 7, 8

Example 23 Sketching a circle

Complete the following for the equation $x^2 + y^2 = 4$.
a State the coordinates of the centre.
b State the radius.
c Find the values of y when $x = 1$, correct to one decimal place.
d Find the values of x when $y = 0$.
e Sketch a graph showing intercepts.

Solution	Explanation
a $(0, 0)$	$(0, 0)$ is the centre for all circles $x^2 + y^2 = r^2$.
b $r = 2$	$x^2 + y^2 = r^2$ so $r^2 = 4$.
c $x^2 + y^2 = 4$	Substitute $x = 1$ and solve for y.
$1^2 + y^2 = 4$	$y^2 = 3$, so $y = \pm\sqrt{3}$.
$y^2 = 3$	$\sqrt{3} \approx 1.7$
$y = \pm 1.7$ (to 1 d.p.)	
d $x^2 + 0^2 = 4$	Substitute $y = 0$.
$x^2 = 4$	Solve for x.
$x = \pm 2$	Both $(-2)^2$ and $2^2 = 4$.
e 	Draw a circle with centre $(0, 0)$ and radius 2. Label intercepts.

Now you try

Complete the following for the equation $x^2 + y^2 = 36$.
a State the coordinates of the centre.
b State the radius.
c Find the values of y when $x = 1$, correct to one decimal place.
d Find the values of x when $y = 0$.
e Sketch a graph showing intercepts.

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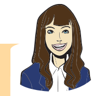
5 A circle has equation $x^2 + y^2 = 9$. Complete the following.

a State the coordinates of the centre.
b State the radius.
c Find the values of y when $x = 2$, correct to one decimal place.
d Find the values of x when $y = 0$.
e Sketch a graph showing intercepts.

Hint: If $x^2 + y^2 = r^2$, then r is the radius.



6 Complete the following for the equation $x^2 + y^2 = 25$.

a State the coordinates of the centre.
b State the radius.
c Find the values of y when $x = 4$.
d Find the values of x when $y = 0$.
e Sketch a graph showing intercepts.

Hint: If $y^2 = 5$, then $y = \pm\sqrt{5}$.

Example 24 Plotting an exponential graph

Consider the rule $y = 2^x$.

a Complete this table.
b Plot points to form its graph.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$				

Solution

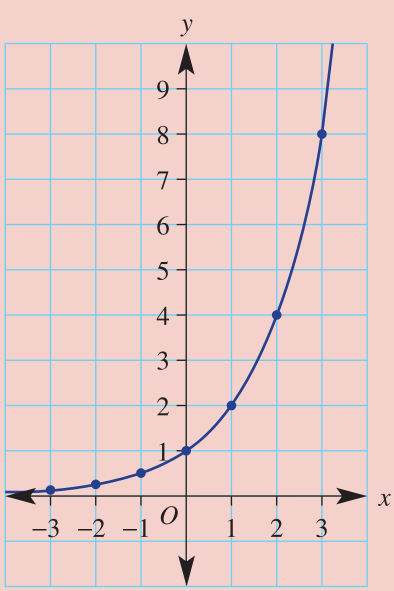
a

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Explanation

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8$.

b



Plot each point and join to form a smooth curve.

Now you try

Consider the rule $y = 5^x$.

a Complete this table.
b Plot points to form its graph.

x	-2	-1	0	1	2
y	$\frac{1}{25}$	$\frac{1}{5}$			

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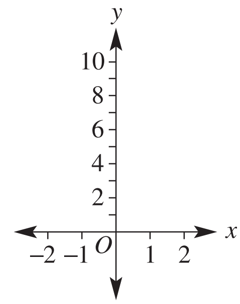
10H

7 Consider the exponential rule $y = 3^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$			

b Plot the points in the table to form the graph of $y = 3^x$.

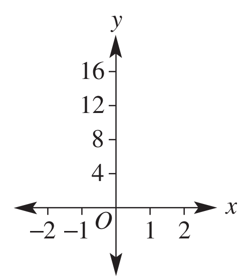


8 Consider the exponential rule $y = 4^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$			

b Plot the points in the table to form the graph of $y = 4^x$.



Problem-solving and reasoning 9–11 10–13

9 **a** Sketch the graph of the following on the same set of axes, labelling the y-intercept and the point where $x = 1$.

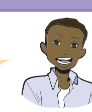
i $y = 2^x$

ii $y = 4^x$

iii $y = 5^x$

b What do you notice about the y-intercept on each graph?

c What does increasing the base number do to each graph?



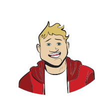
Hint: Use this table to help.

x	-1	0	1	2
$y = 2^x$	$\frac{1}{2}$	1		
$y = 4^x$	$\frac{1}{4}$			
$y = 5^x$	$\frac{1}{5}$			

10 Give the radius of the circles with these equations.

a $x^2 + y^2 = 36$ **b** $x^2 + y^2 = 81$ **c** $x^2 + y^2 = 144$

d $x^2 + y^2 = 5$ **e** $x^2 + y^2 = 14$ **f** $x^2 + y^2 = 20$



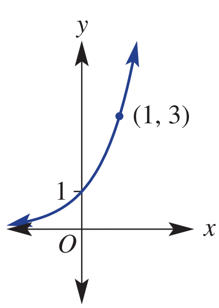
Hint:
Remember: $x^2 + y^2 = r^2$

11 Write the equation of a circle with centre $(0, 0)$ and radius 7.

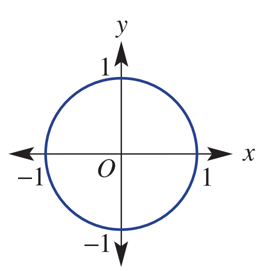
12 Match equations **a**–**c** with graphs **A**–**C**.

a $y = -x - 2$ **b** $y = 3^x$ **c** $x^2 + y^2 = 1$

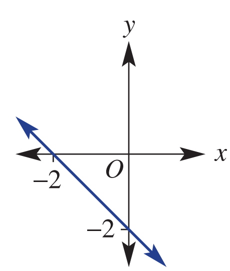
A



B



C



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13 A study shows that the population of a town is modelled by the rule $P = 2^t$, where t is in years and P is in thousands of people.

a State the number of people in the town at the start of the study (i.e. $t = 0$).

b State the number of people in the town after:

i 1 year **ii** 3 years

c When is the town's population expected to reach:

i 4000 people? **ii** 16 000 people?

Hint: If $P = 3$, there are 3000 people.

Graphs of hyperbolas — 14, 15

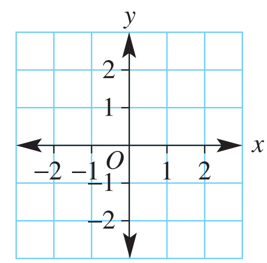
Another type of graph is called a hyperbola and it comes from the rule $y = \frac{1}{x}$.

14 A hyperbola has the rule $y = \frac{1}{x}$.

a Complete this table.

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y						

b Plot the points to form the graph of $y = \frac{1}{x}$.

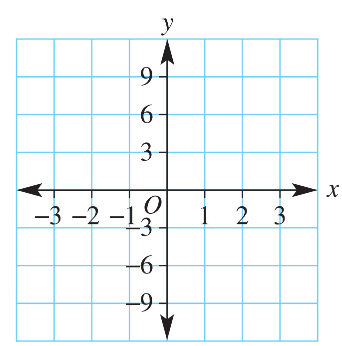


15 A hyperbola has the rule $y = \frac{3}{x}$.

a Complete this table.

x	-3	-1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3
y						

b Plot the points to form the graph of $y = \frac{3}{x}$.



Using technology 10H: Graphing circles and other graphs
This activity is available on the companion website as a printable PDF.

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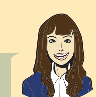
Maths@Work: Driving instructor

Driving instructors have a challenging job, teaching often young and inexperienced drivers how to be safe on the roads. Instructors must have a valid driver's licence and a Certificate IV in Transport and logistics for Road transport–Car Driving instruction.

Workplace skills needed by a driving instructor include good communication skills, patience, the ability to teach and instruct, as well as a good knowledge of motor vehicle operation and safety. Good driving instructors understand the mathematics involved in speed, fuel consumption, angles for parking, braking and stopping distances. They also need to calculate hourly rates of pay and charge clients correctly.

Understanding braking distances and stopping distances is important when training to be an instructor. Answer the following questions relating to the rules used to calculate these distances.

1 Show your working for the following calculations.
- **a** By what fraction do you multiply when converting 1 km/h to m/s (metres per second)?
- **b** By what fraction do you multiply when converting 1 m/s to km/h?



Hint: 1 km = 1000 m
1 h = 3600 s

2 Convert the following speeds to m/s. Give your answers to two decimal places.
a 40 km/h **b** 50 km/h **c** 60 km/h **d** 70 km/h **e** 80 km/h **f** 100 km/h

3 A driver's reaction time to danger before applying a vehicle's brakes is approximately 0.7 seconds at best. The distance travelled while reacting to a situation can be found by rearranging the formula for speed.
Find the formula for the reaction distance (R), in metres, travelled in 0.7 seconds at a speed of s metres per second.

Hint: Distance = speed × time

4 Use your answer to Question **3** to find the distance covered in the reaction time at each of the following speeds. Give your answers in metres, to two decimal places.
a 40 km/h **b** 50 km/h **c** 60 km/h **d** 70 km/h **e** 80 km/h **f** 100 km/h

5 The formula for the braking distance, B, in metres, of a car in dry conditions is $B = \frac{s^2}{20}$, where s is the speed in m/s. Calculate the braking distance for each of the following speeds. Give your answers in metres to two decimal places.
a 40 km/h **b** 50 km/h **c** 60 km/h **d** 70 km/h **e** 80 km/h **f** 100 km/h

6
- **a** Write an equation for the total stopping distance (D) by adding the reaction time distance (R) to the braking distance (B).
- **b** What type of equation is formed for total stopping distance?
- **c** How would you explain to a learner driver how the total stopping distance increases as speed increases?

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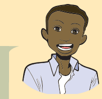
Using technology



Hint: Recall: reaction distance $= 0.7 \times$ speed

7 **a** Set up an Excel worksheet to calculate stopping distances for the various speeds, as shown in this snapshot below. You will need to enter appropriate formulas in the shaded cells.

	A	B	C	D	E
1	Stopping distances at various speeds				
2	Speed, in km/h	Speed, in m/s	Reaction distance, in m	Braking distance, in m	Total stopping distance, in m
3	0	0	0	0	0
4	10				
5	20				
6	30				
7	40				
8	50				
9	60				
10	70				
11	80				
12	90				
13	100				
14	110				

b At what speed is the reaction distance almost equal to the braking distance? Give the total stopping distance for this speed.

c Given that the average car length is 4.8 m, at what speed is the total braking distance about:

i 4 car lengths?

ii 10 car lengths?

d Extend your Excel table to help answer the following questions.

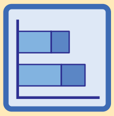
i When your speed increases by 10%, what is the $\%$ increase in braking distance? Try some different starting speeds to check your answer.

ii What is the $\%$ increase in braking distance when you increase your speed by:

I 20%?

II 30%?

e Insert a stacked bar graph in your worksheet, showing reaction and braking distances for various speeds. Follow these steps:

- Insert a new column A titled 'Speeds' and enter each speed with units; e.g. 10 km/h, 20 km/h, 30 km/h etc.
- Hold Ctrl and select this new column of speeds, the columns of distances of reaction and braking, including each of their column headings.
- Click on Insert/2D stacked bar: 
- Format the horizontal axis with a minor unit of 5 and show minor gridlines.

f Driving instructors can explain to learner drivers the mathematical reasons for speed limits. From your graph, state the highest speed that would allow a driver to stop before a collision if an object (e.g. a kangaroo or another vehicle) suddenly appeared in front of the car at these distances:

i 20 m **ii** 35 m **iii** 50 m **iv** 60 m

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Puzzles and games

1 I am a beautiful curve! Solve the equations and then match the letters to the answers to find out what I am.

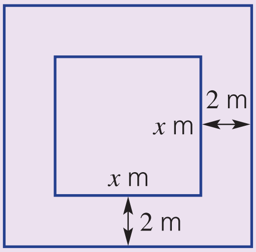
O	B	A	L	P	R
2^3	Radius of $x^2 + y^2 = 16$	Solution to $(x - 3)^2 = 0$	$x^2 - x$	$-2(x - 1)$	Solution to $x^2 - 2x - 8 = 0$

___	___	___	___	___	___	___	___
$-2x + 2$	3	$4, -2$	3	4	8	$x(x - 1)$	3

2 A square pool of side length x metres is surrounded by a 2 m wide tiled edge.

a Find an expression for the total area.

b For what value of x is the total area equal to 100 m^2?



3 A book's length is 6 cm longer than its width and its total cover area is 280 cm^2. What are its dimensions?

4 The product of two consecutive even numbers is 168. Find the two numbers.

5 A father's age is the square of his son's age (x). In 20 years' time the father will be 3 times as old as his son. What are the ages of the father and son?

6 A rectangular painting is to have a total area (including the frame) of 1200 cm^2. The painting is 30 cm long and 20 cm wide. Find the width of the frame.

7 Simplify these expressions.

a $4x - 3(2 - x)$

b $(x - 1)^2 - (x + 1)^2$

c $\dfrac{x^2 - x - 6}{x + 2}$

8 A cyclist in a charity ride rides 300 km at a constant average speed. If the average speed had been 5 km/h faster, the ride would have taken 2 hours less. What was the average speed of the cyclist?

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Chapter summary

Quadratic equations ⭐

Factorising $x^2 + bx + c$

Two numbers + to b.
Two numbers × to c.
e.g. $x^2 - 7x - 18 = (x - 9)(x + 2)$
$-9 \times 2 = -18$
$-9 + 2 = -7$

Expanding brackets

$a(b + c) = ab + ac$
$(a + b)(c + d) = ac + ad + bc + bd$
$(a + b)(a - b) = a^2 - b^2$ (DOPS)
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

Solving quadratic equations

- **Solving $ax^2 = c$**
  e.g. $2x^2 = 8$
  $x^2 = 4$
  $x = \pm 2$
- **Null factor law:**
  If $ab = 0$, then $a = 0$ or $b = 0$.
  Write each quadratic in standard form $ax^2 + bx + c = 0$, factorise and then apply the null factor law to solve.
  e.g. **1** $x^2 - 4x = 0$
  $x(x - 4) = 0$
  $x = 0$ or $x - 4 = 0$
  $x = 0$ or $x = 4$
  2 $x^2 - 3x + 10 = 0$
  $(x - 5)(x + 2) = 0$
  $x - 5 = 0$ or $x + 2 = 0$
  $x = 5$ or $x = -2$

Factorising and DOPS

Always take out common factors first.
Difference of perfect squares
$a^2 - b^2 = (a - b)(a + b)$
e.g. **1** $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$
2 $4x^2 - 9 = (2x)^2 - (3)^2$
$= (2x - 3)(2x + 3)$

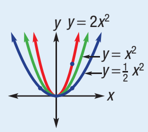
Applications

1. Define the variable.
2. Set up the equation.
3. Solve by factorising and using the null factor law.
4. Determine the suitable answer(s).

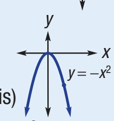
Parabolas ⭐

Transformations of a parabola

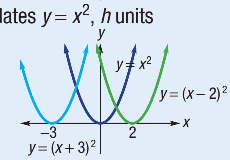
$y = ax^2$ dilates graph compared to $y = x^2$
$0 < a < 1$ graph is wider
$a > 1$ graph is narrower



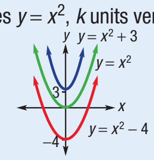
$a < 0$ graph is inverted
(i.e reflected in x-axis)



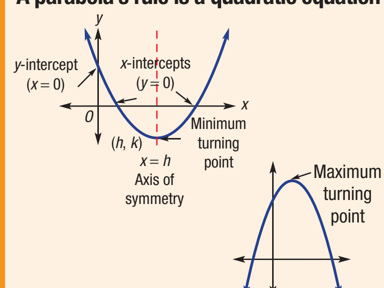
$y = (x - h)^2$ translates $y = x^2$, h units horizontally
$h > 0$ move right
$h < 0$ move left



$y = x^2 + k$ translates $y = x^2$, k units vertically
$k > 0$ move up
$k < 0$ move down



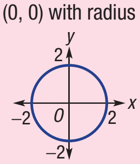
A parabola's rule is a quadratic equation



Other graphs ⭐

Circles

$x^2 + y^2 = r^2$ is a circle centred at (0, 0) with radius r.

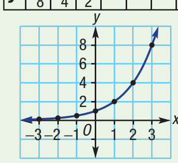


$x^2 + y^2 = 4$
$r^2 = 4$
$\therefore r = 2$

Exponential graphs

$y = 2^x$

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
10A	**1 I can expand binomial products including perfect squares.** e.g. Expand the following. **a** $(2x - 1)(x + 5)$ **b** $(2x + 3)^2$	
10A	**2 I can expand binomial products to form a difference of perfect squares.** e.g. Expand $(3x - 1)(3x + 1)$.	
10B	**3 I can factorise an expression by taking out a common factor.** e.g. Factorise $-3a^2 - 9a$.	
10B	**4 I can factorise a difference of perfect squares.** e.g. Factorise $x^2 - 25$.	
10C	**5 I can factorise a monic quadratic.** e.g. Factorise $x^2 - x - 12$.	
10C	**6 I can factorise a perfect square.** e.g. Factorise $a^2 - 6a + 9$.	
10D	**7 I can solve quadratic equations of the form $ax^2 = c$.** e.g. Solve $2x^2 = 72$.	
10D	**8 I can solve quadratic equations of the form $ax^2 + b = c$.** e.g. Solve $2x^2 - 2 = 10$.	
10E	**9 I can factorise and use the null factor law to solve a simple quadratic equation.** e.g. Solve: **a** $2x^2 - 4x = 0$ **b** $x^2 - 25 = 0$	
10E	**10 I can use the null factor law to solve a quadratic equation of the form $x^2 + bx + c = 0$.** e.g. Solve $x^2 - x - 6 = 0$.	
10F	**11 I can set up and solve a quadratic equation to find the solution to a real problem.** e.g. The area of a rectangle is fixed at 36 m^2 and its length is 5 m more than its width. Find the dimensions of the rectangle.	

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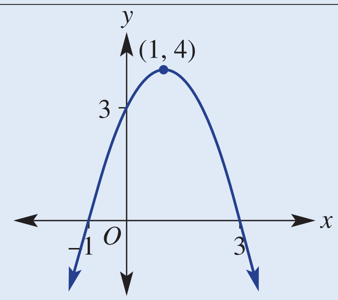
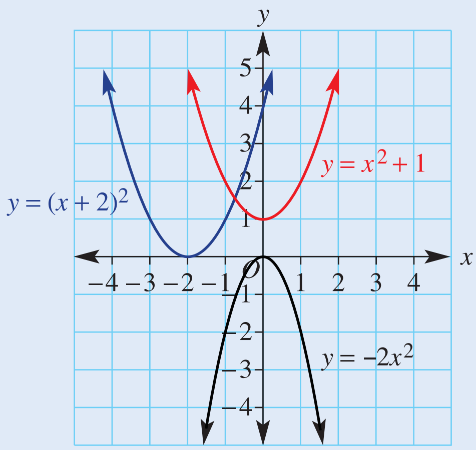
		✔
10G	**12 I can identify key features of a parabola.** e.g. List the key features of this parabola including the type and coordinates of the turning point, the axis of symmetry and the x- and y-intercepts.	
10G	**13 I can identify key features of a parabola after $y = x^2$ is reflected, dilated or translated.** e.g. List the key features of the following parabolas, including the type and coordinates of the turning point, axis of symmetry and x- and y-intercepts. **a** $y = -2x^2$ **b** $y = (x+2)^2$ **c** $y = x^2 + 1$	
10H	**14 I can identify key features and am able to sketch a simple circle.** e.g. Sketch the graph of $x^2 + y^2 = 9$.	
10H	**15 I can identify key features and am able to sketch a simple exponential relation.** e.g. Sketch the graph of $y = 3^x$.	





Chapter checklist

Chapter review

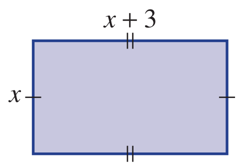
Short-answer questions

10A **1** Expand the following and simplify where possible.

a $-2(x+1)$ **b** $x(x+3)$ **c** $(x+2)(x-1)$
d $(x+5)(3x-4)$ **e** $(x+4)(x-4)$ **f** $(5x-2)(5x+2)$
g $(x+2)^2$ **h** $(x-6)^2$ **i** $(3x-2)(4x-5)$

10A **2** Write, in expanded form, an expression for the shaded areas. All angles are 90°.

a



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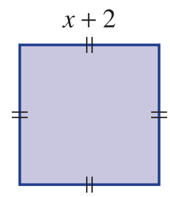

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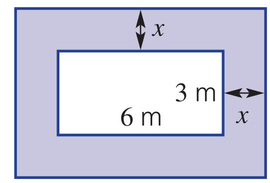

10B **3** Factorise by removing a common factor.

a $3x-9$ **b** $-4x-16$ **c** x^2+2x
d $ab-b$ **e** $7x-14x^2$ **f** $-a^2b-6ab$

10B **4** Factorise the following by using difference of perfect squares. Remember to look for a common factor first.

a x^2-49 **b** $9x^2-16$ **c** $4x^2-1$
d $3x^2-75$ **e** $2x^2-18$ **f** $4x^2-81$

10C **5** Factorise these quadratic trinomials. Some are perfect squares.

a x^2+5x+6 **b** x^2-x-6 **c** $x^2-8x+12$
d $x^2+10x-24$ **e** $x^2+5x-50$ **f** $x^2-12x+32$
g x^2-6x+9 **h** $x^2+20x+100$ **i** $x^2+40x+400$

10D **6** Solve these quadratics, giving answers with exact values.

a $x^2=11$ **b** $2x^2=6$ **c** $3x^2-1=14$

10E **7** Solve using the null factor law.

a $(x+1)(x-2)=0$ **b** $(x-3)(x+7)=0$ **c** $(2x-1)(x+4)=0$
d $x(x-3)=0$ **e** $-4x(x+6)=0$ **f** $7x(2x-5)=0$

10E **8** Solve these quadratic equations by factorising and applying the null factor law.

a $x^2+4x=0$ **b** $3x^2-9x=0$ **c** $x^2-25=0$
d $9x^2-16=0$ **e** $x^2+8x+15=0$ **f** $x^2-10x+21=0$
g $x^2-8x+16=0$ **h** $x^2+10x+25=0$ **i** $x^2+5x-36=0$

10F **9** A large rectangular sandpit is 2 m longer than it is wide. If it occupies an area of 48 m^2, determine the dimensions of the sandpit by solving a suitable equation.

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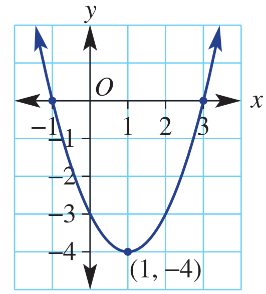
10G **10** State the following features of the quadratic graph shown:

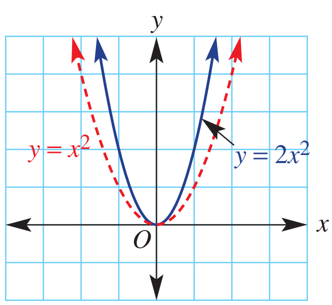
a turning point and whether it is a maximum or a minimum
b axis of symmetry
c x-intercepts
d y-intercept



10G **11** Copy and complete the table for the following graphs.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 2x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = (x - 2)^2$					
d	$y = x^2 - 2$					

a



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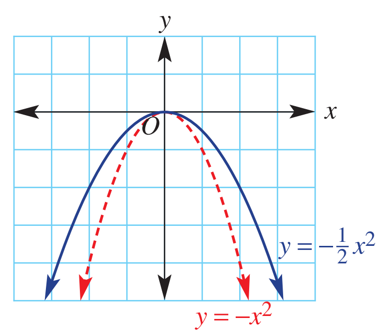

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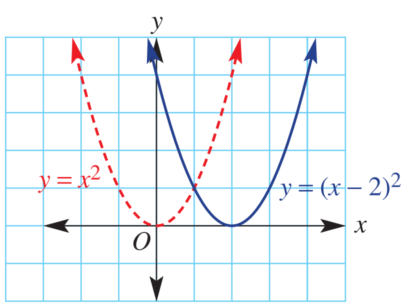

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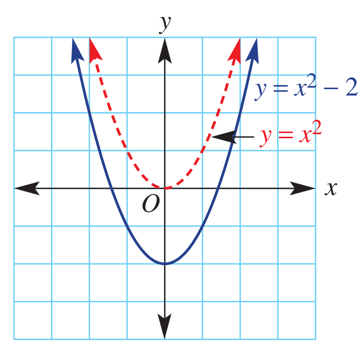

10H **12** Sketch these circles. Label the centre and axes intercepts.

a $x^2 + y^2 = 25$ **b** $x^2 + y^2 = 4$

10H **13** Sketch the following graphs, labelling the y-intercept and the point where $x = 1$.

a $y = 2^x$ **b** $y = 4^x$

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Multiple-choice questions

10A **1** $-2x(1-x)$ expands to:
A $-2+2x^2$ **B** $-2x-2x^2$ **C** $2x+2x^2$ **D** $-3x^2$ **E** $-2x+2x^2$

10A **2** $(x+5)^2$ is the same as:
A x^2+25 **B** x^2+5x **C** $x^2+5x+25$ **D** $x^2+10x+25$ **E** x^2+50

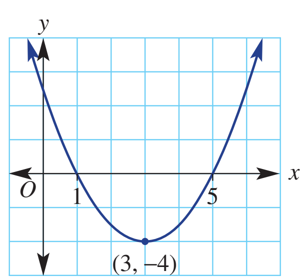
10A **3** $(2x-1)(x+4)$ is equal to:
A $2x^2+11x-2$ **B** $2x^2+7x-4$ **C** $4x^2+14x-8$
D $4x^2+9x-2$ **E** $2x^2+5x+4$

10B **4** In factorised form, $4x^2-25$ is:
A $4(x-5)(x+5)$ **B** $(2x-5)^2$ **C** $(2x-5)(2x+5)$
D $(4x+5)(x-5)$ **E** $2(2x+1)(x-25)$

10E **5** The solutions to $2x^2-8x=0$ are:
A $x=0, x=-4$ **B** $x=2$ **C** $x=0, x=4$
D $x=4$ **E** $x=0, x=2$

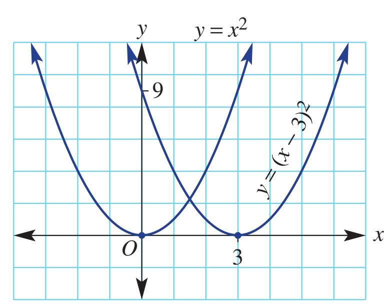
10E **6** The solutions to $(2x-1)(x+1)=0$ are:
A $x=0, x=1$ **B** $x=-1, x=\frac{1}{2}$ **C** $x=-1, x=2$
D $x=-1, x=-2$ **E** $x=1, x=\frac{1}{2}$

10G **7** The equation of the axis of symmetry of the graph shown is:



A $y=-4$ **B** $x=3$ **C** $x=-4$ **D** $y=3$ **E** $y=3x$

10G **8** Compared to the graph of $y=x^2$, the graph of $y=(x-3)^2$ is:
A 3 units down
B 3 units left
C in the same place
D 3 units right
E 3 units up



10H **9** The equation of a circle centred at the origin with radius 4 units is:
A $y=4x^2$ **B** $x^2+y^2=4$ **C** $x^2+y^2=8$
D $y=4x$ **E** $x^2+y^2=16$

10H **10** The graph of $y=3^x$ has y-intercept with coordinates:
A $(0, 3)$ **B** $(3, 0)$ **C** $(0, 1)$
D $(1, 3)$ **E** $\left(0, \frac{1}{3}\right)$

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Extended-response questions

1 A square spa is to be built in the middle of a 10 m by 10 m paved area. The builder does not yet know the size of the spa, so on the plan the spa size is variable. Its side length is x metres.

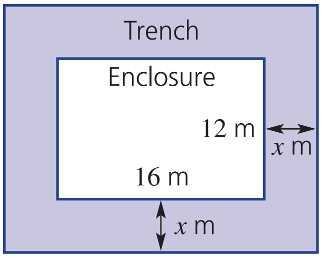
a Write expressions for the area of:
  i the spa ii paving

b Factorise your expression from part a ii.

c What will be the area of the paving if:
  i $x = 2$? ii $x = 4$?

d What value of x makes the paving area equal to 75 m^2?

2 A zoo enclosure for a rare tiger is rectangular in shape and has a trench of width x m all the way around it to ensure the tiger doesn't get far if it tries to escape. The dimensions are as shown.



a Write an expression in terms of x for:
  i the length of the enclosure
  ii the width of the enclosure

b Use your answers from part a to find the area of the enclosure (including the trench), in expanded form.

c Hence, find an expression for the area of the trench alone.

d Zoo restrictions state that the trench must have an area of at least 128 m^2. Find the minimum width of the trench.

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Straight-line graphs

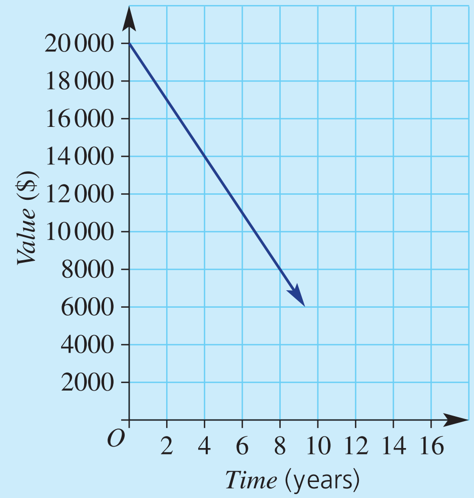
Short-answer questions

1 This graph shows how the value of a car decreases with time.

a By the end of 4 years, how much has its value fallen?

b After how many years is the car worth $8000?

c Estimate the car's worth after 13 years.



2 This distance–time graph shows the journey of a cyclist from home to a location and back again.

a How many km has the cyclist travelled after:

i 1 hour?

ii 1.5 hours?

iii 3 hours?

b Calculate the cyclist's speed over the first 2 hours.

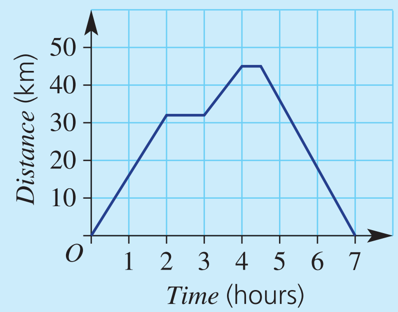
c What is the total time in rest breaks?

d What is the cyclist's greatest distance from home?

e How long does the return trip take?

f Calculate the cyclist's speed for the return journey.

g What is the total distance cycled?



3 **a** Copy and complete this table for the rule $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						

b List the coordinates for each point in the table.

c Draw x- and y-axes, each labelled between -5 and 5. Plot the points and join them with a ruler.

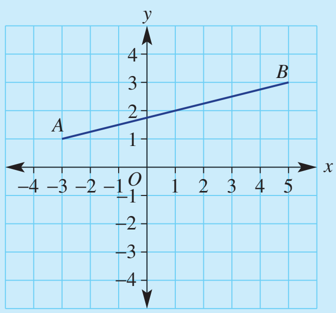
4 For each of the graphs that follow, find the:

i midpoint of the line segment

ii exact length of the line segment (i.e. give the answer in square root form)

iii gradient of the line segment

a



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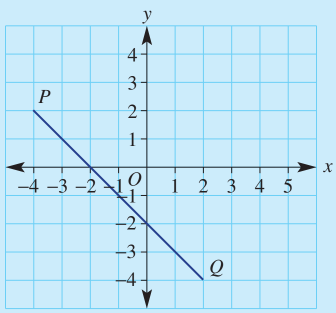

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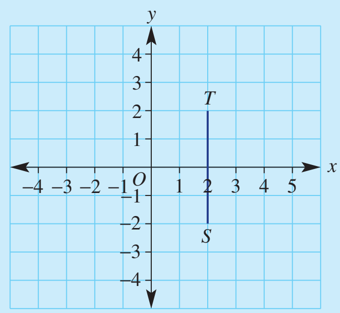

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5 Plot and join each pair of points and find the gradient, m, of the line segment joining these points.

a $A(3, 2)$ and $B(5, 6)$ **b** $K(1, -3)$ and $L(-2, 6)$

6 Plot and join each pair of points and find the length of each line segment. Write the answer in square root form if it is not a whole number.

a $P(3, 4)$ and $Q(-1, 9)$ **b** $R(-4, 2)$ and $M(1, 10)$

7 Without plotting these points, find the coordinates of the midpoint of the line segment joining each pair of points.

a $T(3, 4)$ and $K(5, 12)$ **b** $F(4, 5)$ and $G(-2, -9)$

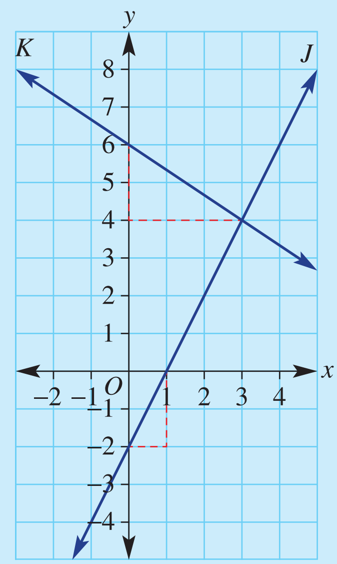
8 **a** Consider the lines, J and K, graphed here.

i Find the y-intercept.

ii Determine the gradient.

iii Write the equation of the line.

b State the coordinates of the point of intersection of lines J and K.



9 Sketch each of these lines by considering the y-intercept and gradient.

a $y = -3x + 4$ **b** $y = \frac{2}{3}x + 1$

c $y = -2$ **d** $x = -3$

10 Find the x- and y-intercepts for each of these lines and sketch them. Label all axes intercepts.

a $y = 2x - 6$ **b** $2x - 3y = 12$

Multiple-choice questions

1 If a straight line has a gradient of -3 and a y-intercept of 5, its equation is:

A $y = 5$ **B** $y = 3x + 5$ **C** $y = 5x - 3$
D $y = -3x + 5$ **E** $m = -3$

2 The gradient of the line joining $(0, 6)$ and $(2, -4)$ is:

A -2 **B** 6 **C** 5
D $\frac{1}{5}$ **E** -5

3 The midpoint of the line segment between $(-3, 8)$ and $(7, 2)$ has coordinates:

A $(4, 10)$ **B** $(2, 5)$ **C** $(2.5, 4.5)$
D $(0.5, 9)$ **E** $(-5, 3)$

4 The equation and gradient of the vertical line through the point $(1, 3)$ are:

A $x = 1$; gradient undefined **B** $x = 1$; gradient zero
C $y = 3$; gradient positive **D** $y = 3$; gradient negative
E $y = 3$; gradient undefined

5 A landscape company charges \$80 delivery plus \$73 per cubic metre of soil. If \$$C$ is the cost of n cubic metres of soil, then:

A $C = 80n + 73$ **B** $C = 73n - 80$ **C** $n = 73C + 80$
D $C = 73n + 80$ **E** $C = 80n - 73$

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Extended-response question

You have a $\$100$ gift voucher for downloading movies from the internet. Each movie costs $\$2.40$. After downloading n movies you have a balance of $\$B$ on your voucher.

a Write a rule for the balance, B, on your voucher in terms of n.

b Use your rule to find:

  i the balance on the voucher after 10 movies are downloaded

  ii the number of movies bought that will leave a balance of $\$28$

c Copy and complete this table.

Number of movies (n)	0	5	10	15	20
Balance (B)					

d Sketch a graph of B versus n, using the values in the table above.

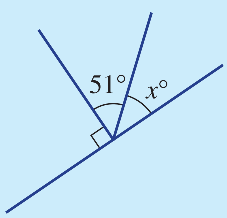
e What is the maximum number of movies you could buy and how much would be left on your voucher?

Geometry

Short-answer questions

1 Find the value of x in these diagrams.

a



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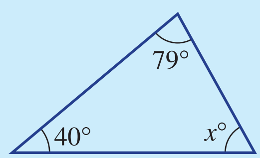

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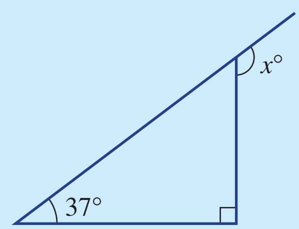

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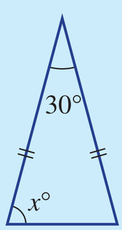

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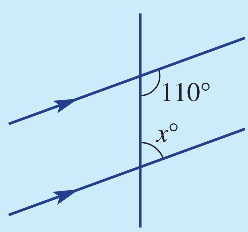

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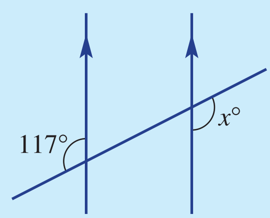

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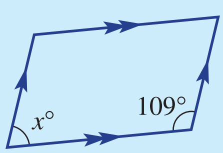

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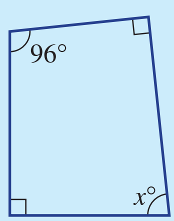

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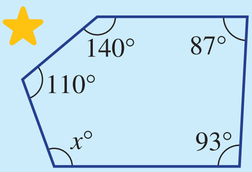

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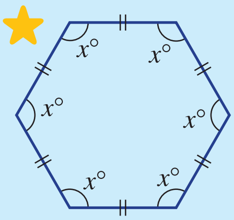

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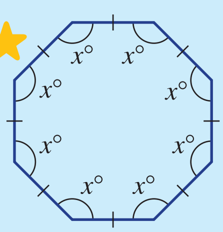

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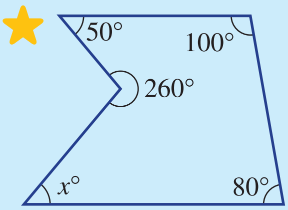

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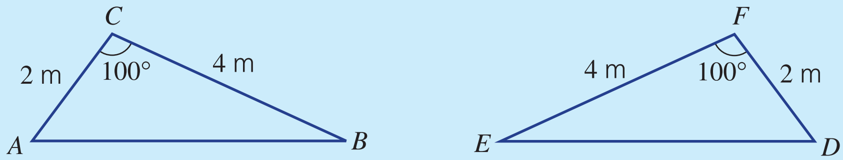
2 Write a congruence statement and the test to prove congruence in these pairs of triangles.

a



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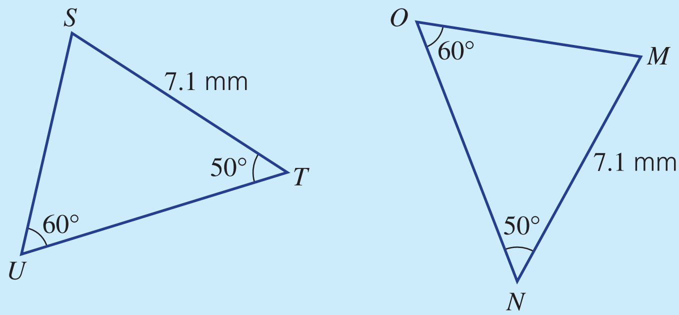
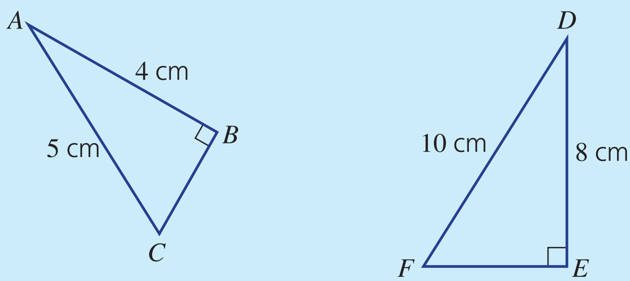


3 Decide whether the pairs of triangles are similar, giving reasons.

a



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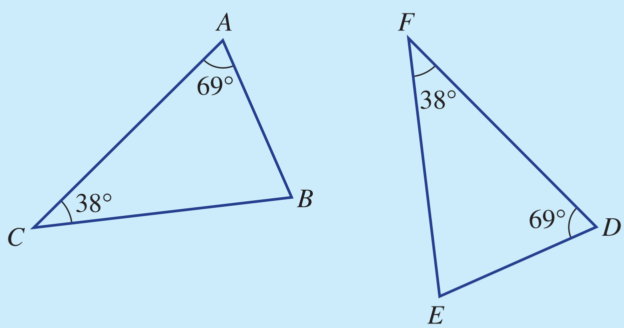

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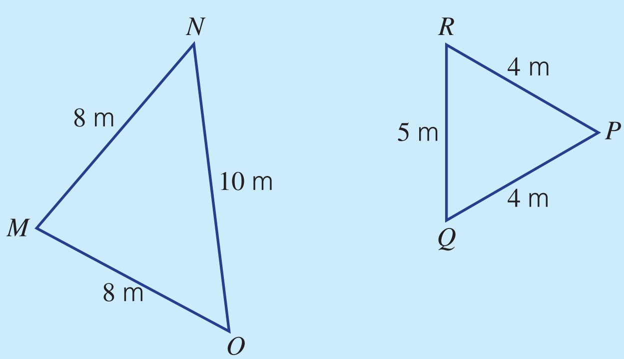
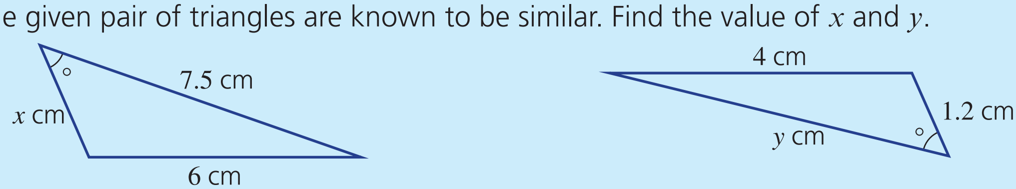


4 The given pair of triangles are known to be similar. Find the value of x and y.



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Multiple-choice questions

1 If two lines are parallel, then cointerior angles will:

A be equal **B** sum to 90° **C** sum to 180° **D** sum to 360° **E** sum to 270°

2 The sum of the internal angles of a hexagon is:

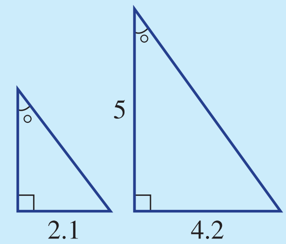
A 360° **B** 540° **C** 1080° **D** 900° **E** 720°

3 Which of the following is not a test for congruent triangles?

A SSS **B** SAS **C** AAA **D** AAS **E** RHS

4 The scale factor for these similar triangles is:

A 2 **B** 4 **C** 5
D 0.1 **E** 0.4

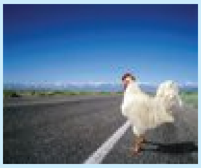


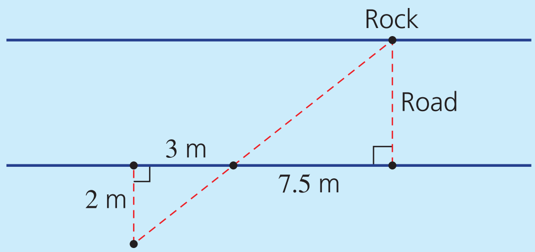
5 The length ratio for two similar solid objects is 2 : 3. The volume ratio is:

A 16 : 81 **B** 2 : 3 **C** 4 : 9
D 8 : 27 **E** 1 : 5

Extended-response question

A chicken wants to know the distance across the road without having to cross it. The chicken places 4 pebbles in various positions on its own side of the road, as shown. There is a rock on the other side of the road aligned with one of the pebbles.





a What reason would be given to explain why the two triangles are similar?

b Find the scale factor.

c What is the distance across the road?

Equations

Short-answer questions

1 Solve the following one-step equations.

a $x + 12 = 3$ **b** $x - 5 = 21$ **c** $\frac{m}{4} = 8$

d $5a = 20$ **e** $-2k = 12$ **f** $-x = 8$

2 Solve the following two-step equations.

a $2p + 3 = 7$ **b** $3a - 10 = 2$ **c** $\frac{x}{2} + 3 = 9$ **d** $\frac{x - 8}{4} = 3$

3 Solve the following equations.

a $2(x - 4) = 8$ **b** $3(k - 2) + 4k = 15$

c $3m - 13 = m + 5$ **d** $\frac{3x + 1}{2} = 8$

e $\frac{3a - 2}{7} = -2$ **f** $4x + 7 + 3x - 12 = 5x + 3$

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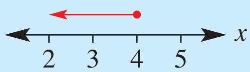
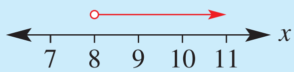
4 For each of the following statements, write an equation and then solve it for the pronumeral.
  a When 5 is subtracted from x, the result is 8
  b When 8 is added to the product of 4 and x, the result is 20.
  c When 6 less than 3 lots of x is doubled, the result is 18.

5 Find the value of the unknown in each of the following formulas.
  a For $A = \frac{1}{2}bh$, find b when $A = 120$ and $h = 24$.
  b For $I = \frac{Prt}{100}$, find P when $I = 80$, $r = 5$ and $t = 4$.

6 Write the inequality displayed on each of the following number lines.
  a 

  b 

7 Solve each of the following inequalities and graph the solution on a number line.
  a $\frac{x}{3} \le 2$ b $3x - 2 > 4$ c $-3x \ge 12$

8 Find the point of intersection (x, y) of each pair of equations below by plotting an accurate graph. First draw x- and y-axes, each labelled from -5 to 5.
  a $x = 3$, $y = 2$ b $y = 2x - 4$ and $3x + 2y = 6$

9 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.
  a $y = 2x$, $x + y = 3$
  b $x + y = 12$, $y = x + 6$
  c $y = 3 - x$, $3x + 2y = 5$

10 Solve the following pairs of simultaneous equations using the elimination method; i.e. find the point of intersection.
  a $x + 2y = 3$, $-x + 3y = 2$
  b $3x + y = 10$, $x + y = 6$
  c $2x - 3y = 3$, $3x - 2y = 7$

11 Oliver is older than Ruby. The sum of their ages is 45 years and the difference of their ages is 7 years.
  a Define pronumerals to represent each person's age.
  b Set up a pair of simultaneous equations based on the information given.
  c Solve the simultaneous equations to find Oliver and Ruby's ages.

Multiple-choice questions

1 Which of the following is *not* an equation?
  A $x - 3 = 5$ B $2x + 4 = 5x$ C $y + 7x - 4$ D $y = 3x - 5$ E $y = 8$

2 A number is decreased by 8 and then doubled. The result is equal to 24. This can be written as:
  A $2x - 8 = 24$ B $x - 8 \times 2 = 24$ C $x - 8 = 2 \times 24$
  D $2(x - 8) = 24$ E $\frac{x-8}{2} = 24$

3 The solution to $\frac{x-9}{3} = 6$ is:
  A $x = 27$ B $x = 45$ C $x = 9$ D $x = 11$ E $x = 3$

4 The solution to $3(x - 1) = 5x + 7$ is:
  A $x = -4$ B $x = -5$ C $x = 5$ D $x = 3$ E $x = 1$

5 The solution to the inequality $1 - 2x > 9$ is:
  A $x > 5$ B $x < 4$ C $x < -5$ D $x > -4$ E $x < -4$

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Extended-response question

Ishan and Mia normally have an electricity bill of \$200 per month. Now that they have installed solar panels (which cost \$6000, including installation), the solar energy is providing for all their power usage with some left over. Their excess power is sold to the town's electricity supplier. On average, they receive a cheque for \$50 per month for the sale of this solar-generated power.

a Copy and complete this table, which compares the total cost of normal town electricity to solar power.

Number of months, *n*	0	6	12	18	24	30	36	42	48
Total cost of electricity (at \$200 per month), *E*	0								
Total cost of solar power (reducing by \$50 per month), *S*	\$6000								

b Draw a graph of the information given in the table.

c On the graph, show the point at which the total cost of solar power is the same as the total cost of electricity for that period of time. State the value of n and the cost.

d Write an equation for E (i.e. total cost of electricity) in terms of n and another equation for S (i.e. total cost of solar power) in terms of n.

e Solve the equations in part **d** for n, when $E = S$ (i.e. the total costs of each power supply are equal).

f After 4 years, how much money has been saved by using solar power?

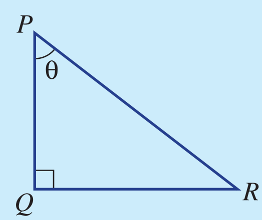
g Suppose that, in a wet climate, the cheque for excess power sold to the town supply is reduced to \$25 per month. Using equations, find how many months it takes for the total costs to be equal. (Round to the nearest month.)

Pythagoras' theorem and trigonometry

Short-answer questions

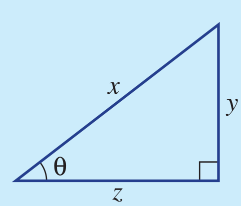
1 Refer to the angle marked θ and name the:

a hypotenuse
b side opposite θ
c side adjacent to θ



2 Use the triangle shown to help you write a fraction for:

a $\sin \theta$ **b** $\cos \theta$ **c** $\tan \theta$



3 Use a calculator to find the value of each of the following, correct to two decimal places.

a $\sin 40°$ **b** $\cos 51°$ **c** $\tan 18°$
d $12 \tan 32°$ **e** $40 \tan 38°$ **f** $5.6 \sin 55°$
g $\dfrac{15}{\sin 24°}$ **h** $\dfrac{28}{\cos 30°}$ **i** $\dfrac{12.5}{\tan 52°}$

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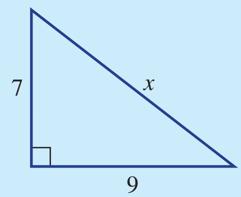
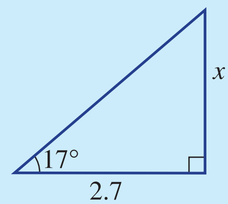
4 Find the value of each pronumeral, correct to two decimal places.

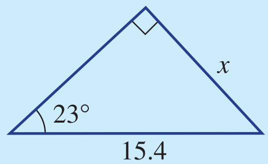
a



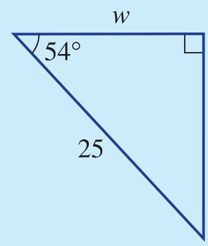
b



c

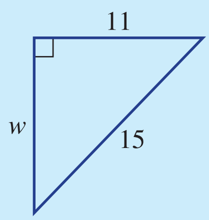


d

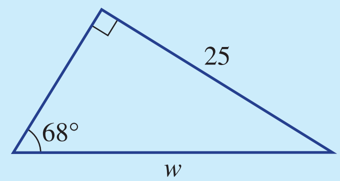


5 Find the value of each pronumeral, correct to one decimal place.

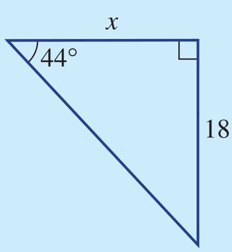
a



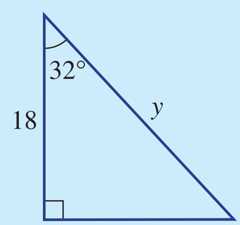
b



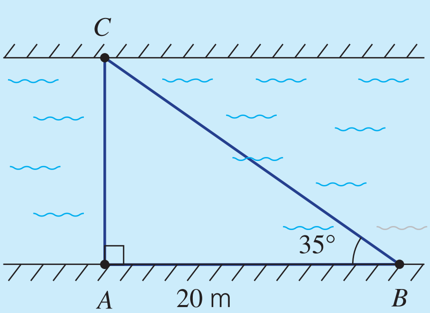
c



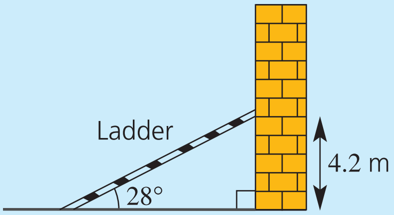
d



6 Kara wants to measure the width of a river. She places two markers, A and B, 20 m apart along one side. C is a point directly opposite marker A. Kara measures angle ABC as 35°. How wide is the river, to the nearest metre?

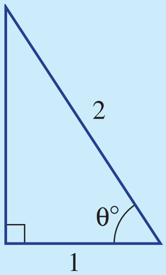


7 A ladder is inclined at an angle of 28° to the ground. If the ladder reaches 4.2 m up the wall, what is the length of the ladder, correct to two decimal places?

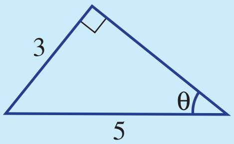


8 Find the angle θ in the following triangles, correct to the nearest degree.

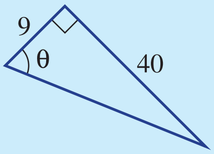
a



b



c



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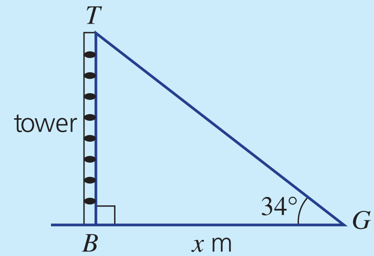
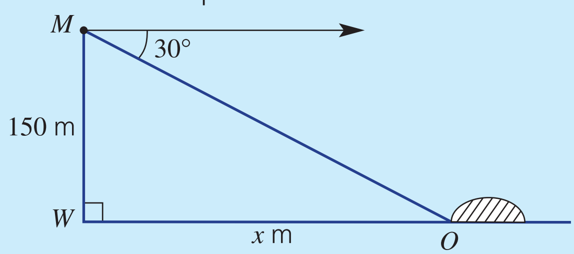
9 Elliott measures the angle of elevation to the top of a 120 m tower to be 34°. How many metres is Elliott from the base of the tower? Round your answer to one decimal place.



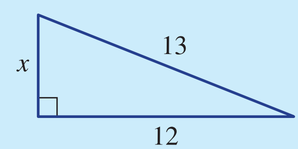
10 Xi-Rui is sitting on top of a bridge 150 m above the water level of the river. He notices an object floating on the river some distance away. If the angle of depression to the object is thought to be 30°, how many metres (x) from the bridge is the object? Round your answer to one decimal place.



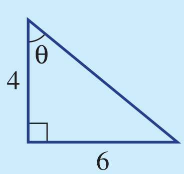
Multiple-choice questions

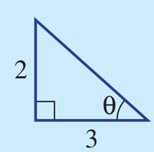
1 The value of x in the triangle shown is:

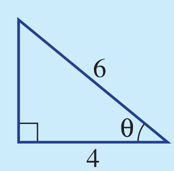
A 1 **B** 11 **C** 4
D 10 **E** 5

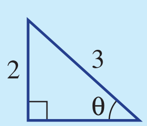


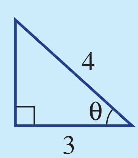
2 In which of the following triangles does $\cos\theta = \frac{2}{3}$?

A


B


C


D


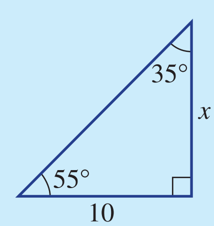
E


3 Choose the correct trigonometric statement for the diagram shown.

A $\tan 55° = \frac{x}{10}$ **B** $\tan 35° = \frac{x}{10}$

C $\sin 55° = \frac{x}{10}$ **D** $\sin 35° = \frac{x}{10}$

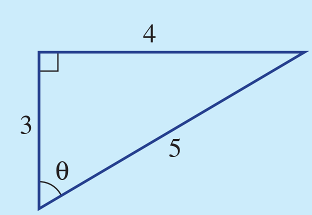
E $\cos 35° = \frac{x}{10}$



4 For the triangle shown, $\sin\theta$ is equal to:

A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{5}{3}$

D $\frac{3}{4}$ **E** $\frac{4}{3}$



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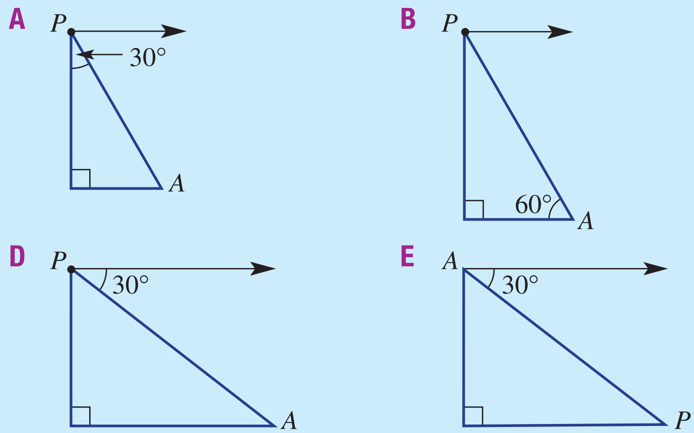
5 In which diagram is the angle of depression of A from P equal to $30°$?



Extended-response question

A plane flies from the airport on a bearing of 136° for 450 km.

a Draw a diagram showing the plane's flight.
b How far east of the airport is the plane? Round to one decimal place.
c How far south of the airport is the plane? Round to one decimal place.
d What is the true bearing of the airport from the plane, correct to the nearest degree?

Quadratics and non-linear graphs

Short-answer questions

1 Expand the following expressions.

a $-2(x-1)$ b $(x+2)(x-3)$ c $(2x-7)(x+3)$
d $(x+2)(x-2)$ e $(x-3)^2$ f $(2x+1)^2$

2 Factorise these expressions.

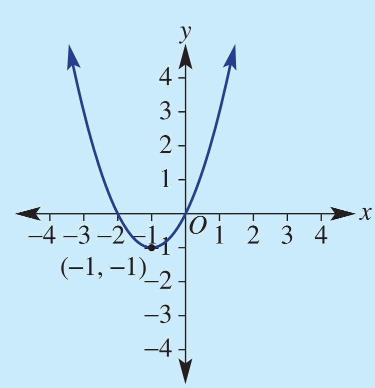
a $3x-12$ b $-2x-x^2$ c x^2-25
d $9x^2-100$ e $x^2+7x+12$ f x^2-x-6
g x^2+2x-8 h $x^2-8x+16$ i x^2+6x+9

3 Solve these equations.

a $x(x-3)=0$ b $x^2+2x=0$ c $x^2-4=0$
d $4x^2-9=0$ e $(x-3)(2x-1)=0$ f $x^2-x-20=0$
g $x^2+10x+21=0$ h $x^2+8x+16=0$ i $x^2-14x+49=0$

4 State the following features of the quadratic graph shown:

a turning point and whether it is a minimum or a maximum
b equation of the axis of symmetry
c x-intercepts
d y-intercept



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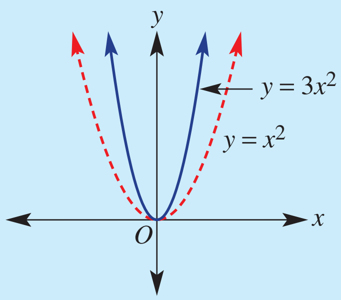
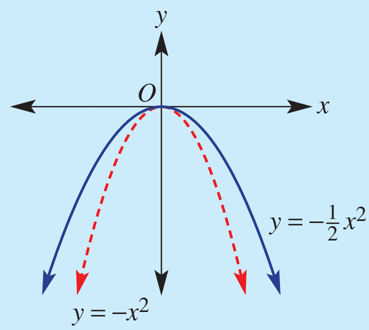
5 Copy and complete the table for these parabolas.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = (x + 2)^2$					
d	$y = x^2 - 3$					

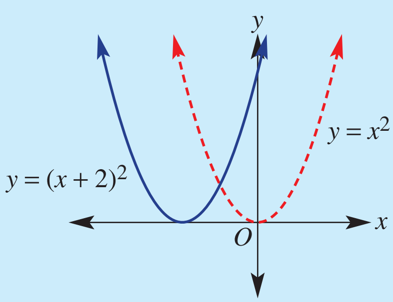
a



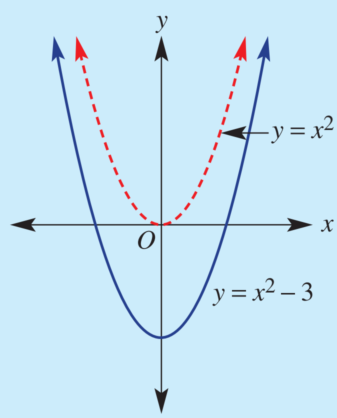
b



c



d



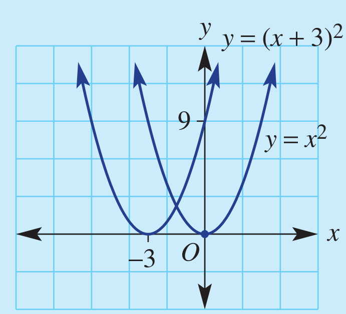
Multiple-choice questions

1 $-x(x - 1) + 2x^2$ simplifies to:

A $x^2 - 1$ **B** $x^2 + x$ **C** $3x^2 + 1$ **D** $3x^2 - x$ **E** $3x^2 + 3$

2 Compared to the graph of $y = x^2$, the graph of $y = (x + 3)^2$ is:

A 3 units down **B** 3 units up
C 3 units left **D** in the same place
E 3 units right



3 The solutions to $(2x - 1)(x + 3) = 0$ are:

A $x = -\frac{1}{2}, x = -3$ **B** $x = 2, x = 3$

C $x = 2, x = -3$ **D** $x = \frac{1}{2}, x = 3$

E $x = \frac{1}{2}, x = -3$

4 The radius of the circle with equation $x^2 + y^2 = 25$ is:

A 5 **B** 25 **C** −5 **D** 0 **E** 1

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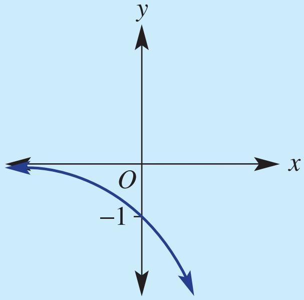
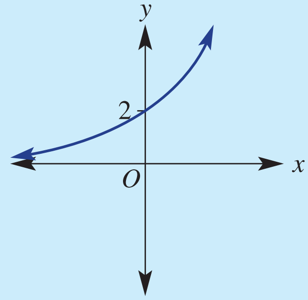
5 The graph of $y = 2^x$ could be:

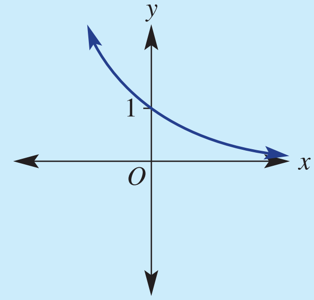
A



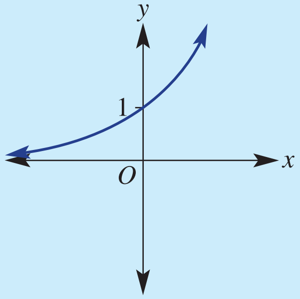
B



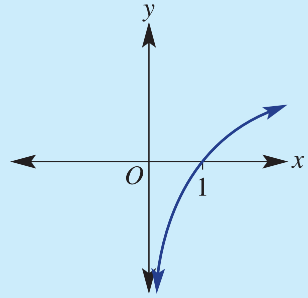
C



D

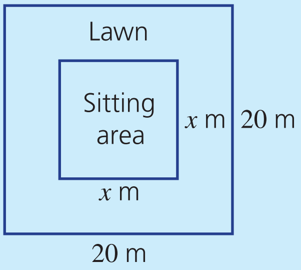


E



Extended-response question

In a garden, a square sitting area paved with stone is to be placed in the centre of a square area of lawn that is 20 m by 20 m.



a Write expressions for the area of the:
- **i** sitting area
- **ii** lawn

b Factorise your expression from part **a ii**.

c What will be the lawn area if:
- **i** $x = 4$?
- **ii** $x = 9$?

d What value of x makes the lawn area equal to:
- **i** 175 m^2?
- **ii** 75% of the total area?

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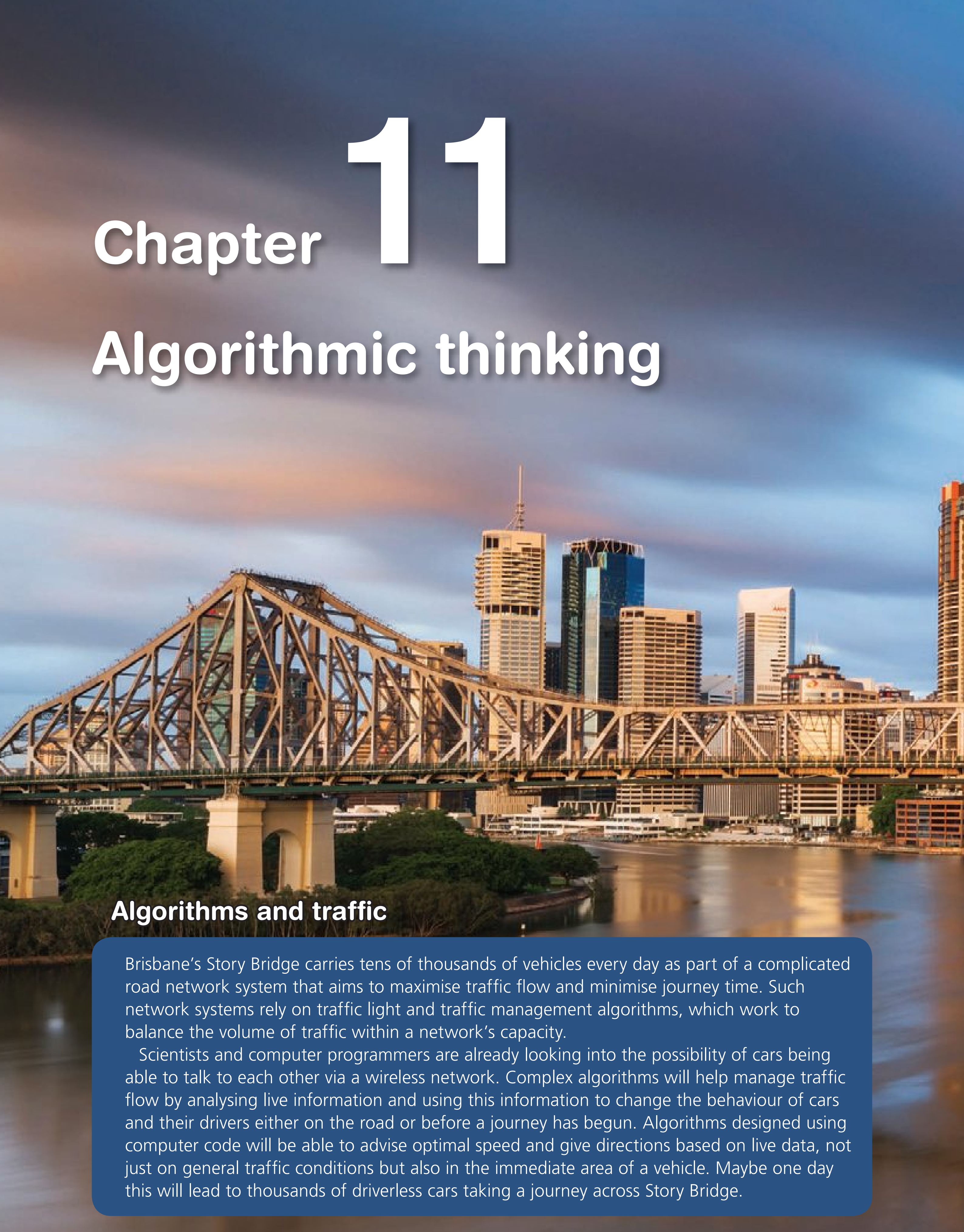

Chapter 11

Algorithmic thinking

Algorithms and traffic

Brisbane's Story Bridge carries tens of thousands of vehicles every day as part of a complicated road network system that aims to maximise traffic flow and minimise journey time. Such network systems rely on traffic light and traffic management algorithms, which work to balance the volume of traffic within a network's capacity.

Scientists and computer programmers are already looking into the possibility of cars being able to talk to each other via a wireless network. Complex algorithms will help manage traffic flow by analysing live information and using this information to change the behaviour of cars and their drivers either on the road or before a journey has begun. Algorithms designed using computer code will be able to advise optimal speed and give directions based on live data, not just on general traffic conditions but also in the immediate area of a vehicle. Maybe one day this will lead to thousands of driverless cars taking a journey across Story Bridge.

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In this chapter

Victorian Curriculum

CHAPTER 11: ALGORITHMIC THINKING

Implement algorithms using data structures in a general purpose programming language.

Solve equations using systematic guess and check and refine with digital technology.

Devise and use algorithms and simulations to solve mathematical problems.

Solve simultaneous equations using systematic guess and check and refine with digital technology.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

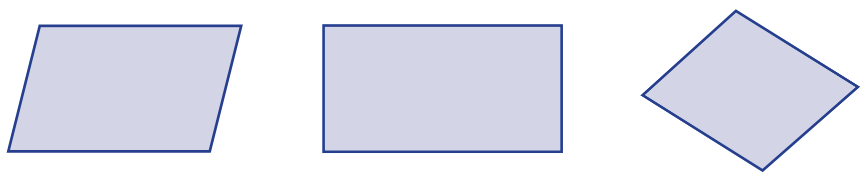
Introduction

An **algorithm** is a sequence of steps that when followed, lead to the solution of a problem. It has a defined set of inputs and delivers an output. Each step in the algorithm leads to another step or completes the algorithm.

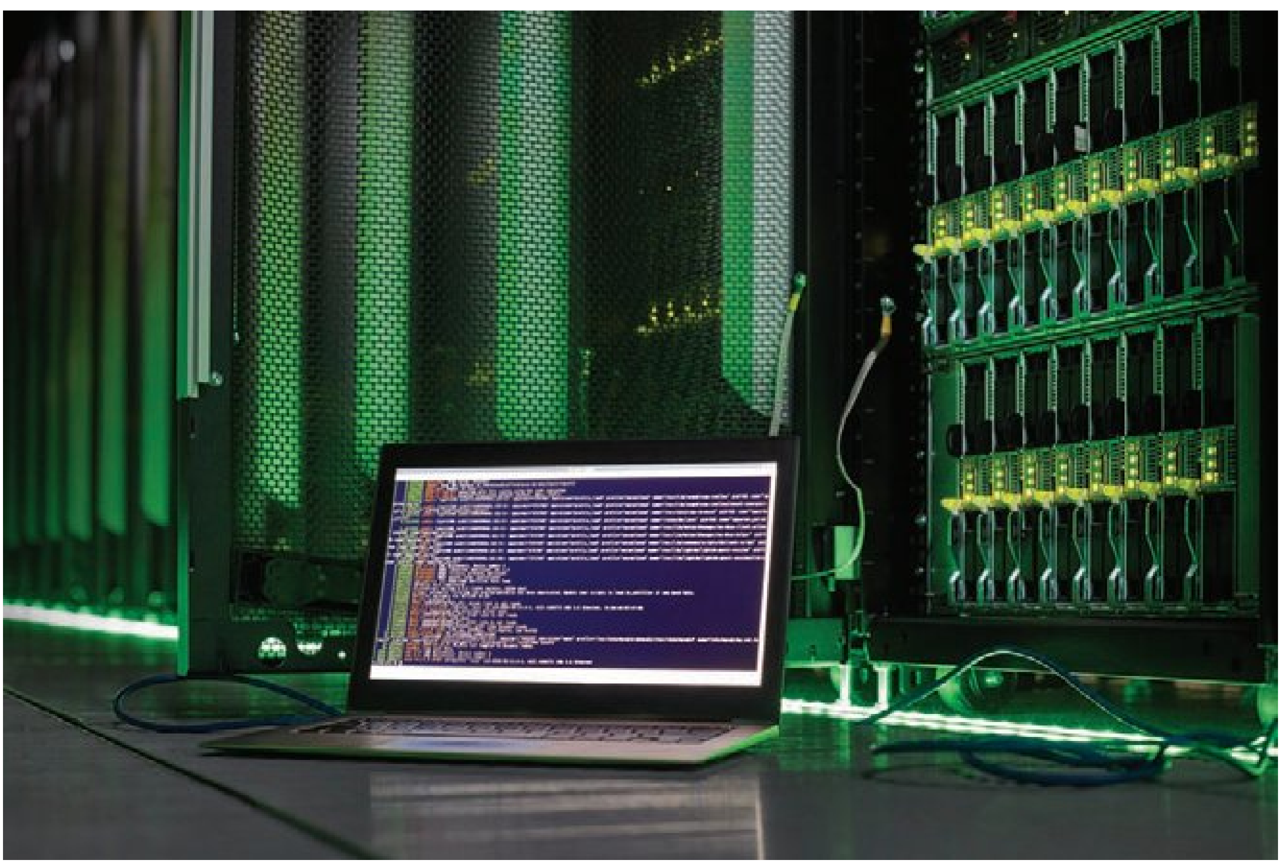
Algorithms occur in mathematics and computing, as well as in simple areas of daily life such as following a recipe. Algorithmic thinking is a type of thinking that involves designing algorithms to solve problems. The algorithms we design can then be written in a way that a computer program will understand, so that the computer does the hard computational work.

In the following activities you will carry out some algorithms as well as think about the design, analysis and implementation of your own algorithms.

The algorithms in these activities will be described through the use of spreadsheets, flow charts (a way of writing an algorithm in the form of a diagram), programming language and simulations. The following symbols will be used in the flow charts with arrows to connect each stage:



for input/output stages for process stages for decision stages



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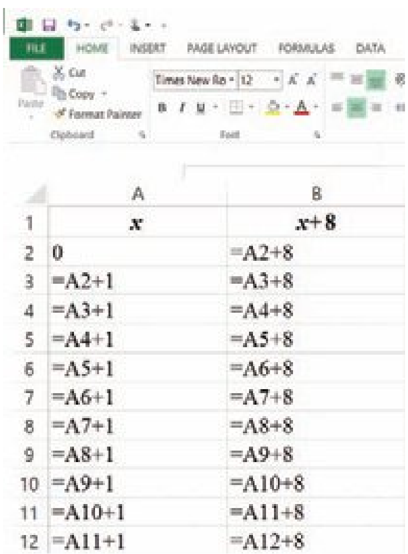
Activity 1: Solving equations numerically

Strand: NUMBER AND ALGEBRA

1.1 Solving for integer solutions

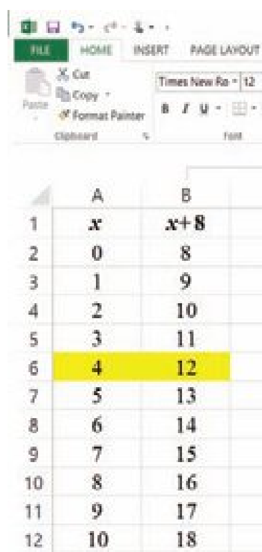
a Set up an Excel spreadsheet (or table on a calculator) to solve the equation $x + 8 = 12$ by following these steps.

1. Enter the columns, as shown below in Figure 1.
2. Start with $x = 0$ and then increase by 1 each time.
3. Continue to increase the value of x until $x + 8 = 12$. This x value is your solution to the equation. From Figure 2, $x = 4$ is the solution.



	A	B
1	x	x+8
2	0	=A2+8
3	=A2+1	=A3+8
4	=A3+1	=A4+8
5	=A4+1	=A5+8
6	=A5+1	=A6+8
7	=A6+1	=A7+8
8	=A7+1	=A8+8
9	=A8+1	=A9+8
10	=A9+1	=A10+8
11	=A10+1	=A11+8
12	=A11+1	=A12+8

Figure 1 Formulas in Excel



	A	B
1	x	x+8
2	0	8
3	1	9
4	2	10
5	3	11
6	4	12
7	5	13
8	6	14
9	7	15
10	8	16
11	9	17
12	10	18

Figure 2 Calculated values in Excel

b Solve the following equations using a similar process.

i $2x + 6 = 30$

ii $9 - 3x = 15$

c With part **a** in mind, set up an Excel spreadsheet to find the integer solutions of the following equations. Start by creating an x column and a column for each side of the equation. Fill down the columns until the two sides of the equation are equal.

i $3x + 7 = x - 5$

ii $2x - 22 = 8 - 3x$

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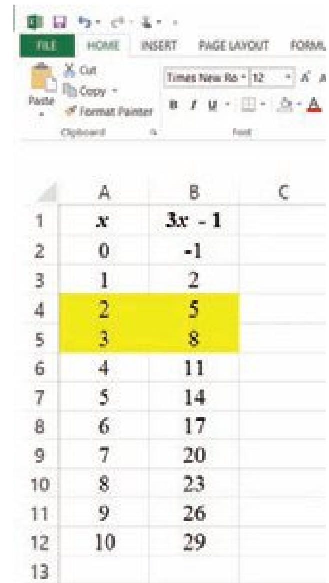
1.2 Solving for non-integer solutions

a To solve an equation such as $2x + 3 = 20$, the process above could be repeated using smaller increments, such as 0.5 or 0.1 (or even smaller), instead of 1. However, your increment choice may end up delivering a large amount of data to scroll through, which is not useful.

An alternative approach is to start with a relatively large increment until you get 'near' the solution and then adjust the increment to a smaller amount for that particular region.

For example, to solve $3x - 1 = 7$, use the process described in **1.1** and note which integer values of x make the value of $3x - 1$ change from less than 7 to greater than 7. In Figure 3, you can see this is $x = 2$ and $x = 3$.



	A	B	C
1	x	$3x$ - 1	
2	0	-1	
3	1	2	
4	2	5	
5	3	8	
6	4	11	
7	5	14	
8	6	17	
9	7	20	
10	8	23	
11	9	26	
12	10	29	
13			

Figure 3

i Now alter the spreadsheet to consider x values only between $x = 2$ and $x = 3$, using increments of 0.1. Fill down and find a solution for x, correct to one decimal place.

ii Now alter the spreadsheet to consider x values only between $x = 2$ and $x = 3$, using increments of 0.01. Fill down and find a solution for x, correct to two decimal places.

b A slightly different method to the above involves finding an average value of x between a pair of updated lower and upper bounds. The steps are:

1 Find a lower and upper bound that are two x values which contain the solution. For the equation $3x - 1 = 7$, the initial lower bound would be 2 and the upper bound would be 3.

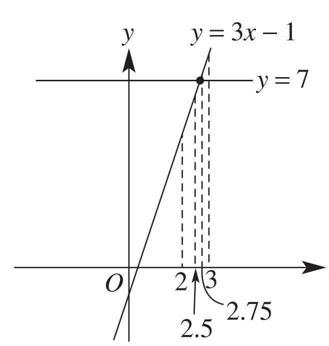
2 Now find the mean of your lower and upper bound. In this example, this would be 2.5.

3 Find the value of $3x - 1$ for your mean. In this example, this would be $3 \times 2.5 - 1 = 6.5$.

4 If the mean is on the same side of 7 as the lower bound, then make the mean the new lower bound. If the mean is on the same side of 7 as the upper bound, then make the mean the new upper bound. In this example, $6.5 < 7$, so the mean is on the same side as the lower bound so the new lower bound is 2.5 and the upper bound stays as 3.

5 Repeating the steps above will deliver smaller and smaller intervals that contain the solution. In this example, the next mean is 2.75 and this replaces the upper bound since $3 \times 2.75 - 1 = 7.25$ and this is greater than 7.

This graph shows the interval containing the solution to $3x - 1 = 7$ getting smaller and smaller.



i Work through the algorithm above and repeat until you are confident you have the solution to $3x - 1 = 7$, correct to three decimal places.

ii Now try this algorithm to solve $7x + 2 = 1$, correct to two decimal places.

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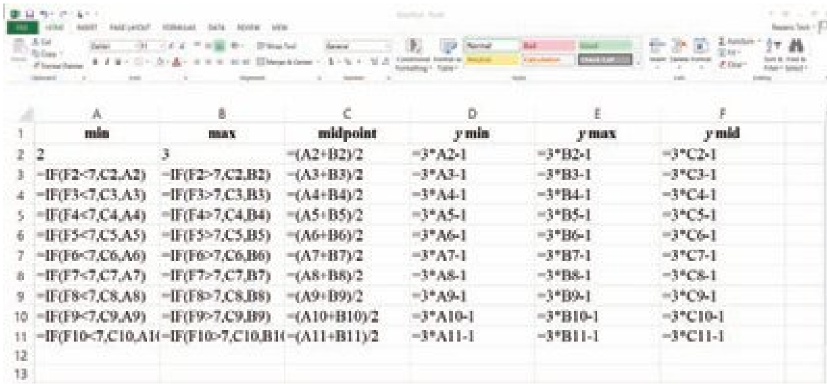
c Let's now consider a spreadsheet approach for the equation $3x - 1 = 7$.

i Set up the spreadsheet shown in Figure 4, where $y = 3x - 1$. Fill out the first three rows and then fill down.

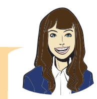
The 'IF' statements in columns A and B set the new lower or upper bounds of the interval. If the midpoint of the interval gives a y value less than 7 then it becomes the new lower bound, otherwise the current lower bound is maintained. If the midpoint gives a y value greater than 7, then it becomes the new upper bound of the interval.



	A	B	C	D	E	F
1	min	max	midpoint	y min	y max	y mid
2	2	3	=(A2+B2)/2	=3*A2-1	=3*B2-1	=3*C2-1
3	=IF(F2<7,C2,A2)	=IF(F2>7,C2,B2)	=(A3+B3)/2	=3*A3-1	=3*B3-1	=3*C3-1
4	=IF(F3<7,C3,A3)	=IF(F3>7,C3,B3)	=(A4+B4)/2	=3*A4-1	=3*B4-1	=3*C4-1
5	=IF(F4<7,C4,A4)	=IF(F4>7,C4,B4)	=(A5+B5)/2	=3*A5-1	=3*B5-1	=3*C5-1
6	=IF(F5<7,C5,A5)	=IF(F5>7,C5,B5)	=(A6+B6)/2	=3*A6-1	=3*B6-1	=3*C6-1
7	=IF(F6<7,C6,A6)	=IF(F6>7,C6,B6)	=(A7+B7)/2	=3*A7-1	=3*B7-1	=3*C7-1
8	=IF(F7<7,C7,A7)	=IF(F7>7,C7,B7)	=(A8+B8)/2	=3*A8-1	=3*B8-1	=3*C8-1
9	=IF(F8<7,C8,A8)	=IF(F8>7,C8,B8)	=(A9+B9)/2	=3*A9-1	=3*B9-1	=3*C9-1
10	=IF(F9<7,C9,A9)	=IF(F9>7,C9,B9)	=(A10+B10)/2	=3*A10-1	=3*B10-1	=3*C10-1
11	=IF(F10<7,C10,A1(	=IF(F10>7,C10,B1(	=(A11+B11)/2	=3*A11-1	=3*B11-1	=3*C11-1
12						
13						

Figure 4

Hint: If (F2 < 7, C2, A2) says if F2 < 7 then place value C2 in cell, otherwise (i.e. F2 ≥ 7), place value A2.

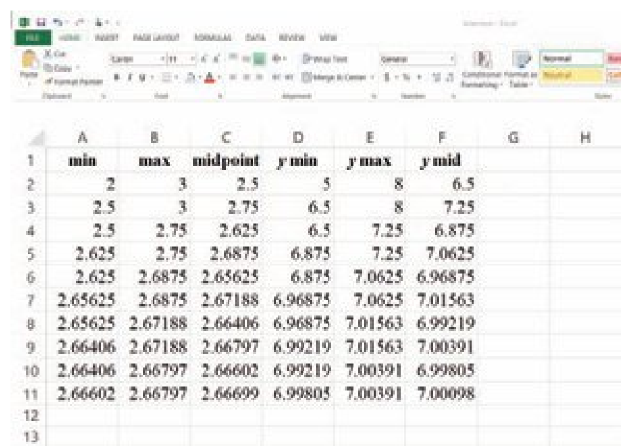


The result should be the spreadsheet values below, where the solution of $3x - 1 = 7$ (correct to two decimal places) can be seen as $x = 2.67$.

ii Try this process for yourself to solve $8x - 2 = 7$, correct to two decimal places.

iii Try once again to solve $9x + 4 = 11$, correct to three decimal places.

d *Extension*
Investigate the Excel function *Goal seek* in the *Data* menu under *What-if analysis*. How could this be used to complete the work above?



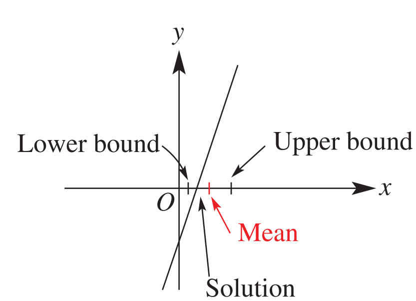
	A	B	C	D	E	F	G	H
1	min	max	midpoint	y min	y max	y mid		
2	2	3	2.5	5	8	6.5		
3	2.5	3	2.75	6.5	8	7.25		
4	2.5	2.75	2.625	6.5	7.25	6.875		
5	2.625	2.75	2.6875	6.875	7.25	7.0625		
6	2.625	2.6875	2.65625	6.875	7.0625	6.96875		
7	2.65625	2.6875	2.67188	6.96875	7.0625	7.01563		
8	2.65625	2.67188	2.66406	6.96875	7.01563	6.99219		
9	2.66406	2.67188	2.66797	6.99219	7.01563	7.00391		
10	2.66406	2.66797	2.66602	6.99219	7.00391	6.99805		
11	2.66602	2.66797	2.66699	6.99805	7.00391	7.00098		
12								
13								

1.3 Locating x-intercepts of linear graphs

Recall that the x-intercept of a linear graph with rule $y = mx + c$ is where $y = 0$. So finding the x-intercept for, say, $y = 3x - 1$ is the same as solving $3x - 1 = 0$. Use the process from parts **1.2b** and **c** above to locate the x-intercepts (correct to two decimal places) of the following graphs. This time you will be looking for the y value to change from positive to negative in your interval.

a $y = 7x - 2$

b $y = 12x - 4$



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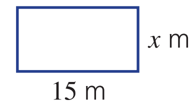
Activity 2: Measurement formulas and maximising areas

Strand: MEASUREMENT AND GEOMETRY

2.1 Measurement calculations using a spreadsheet

Consider a rectangular area that is fenced off to form a paddock. It has a fixed length of 15 m and a width of x m. Due to the limited amount of fencing wire available, the perimeter of the paddock is fixed at 48 m.



a Create a table in Excel like the one shown below that will find the value of the width, x m, that gives a perimeter of 48 m and also calculates the corresponding area. Enter the length into cell A2 and start with a width of 1 m in cell B2. Enter the perimeter formula in C2 and area formula in D2. Fill down cells B3, C3 and D3 to find the x value that gives the required perimeter of 48 m.

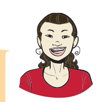
	A	B	C	D
1	Length (metres)	Width (x metres)	Perimeter (metres)	Area (m^2)
2	15	=1	=2*$A$2+2*B2	=$A$2*B2
3		=B2+1	=2*$A$2+2*B3	=$A$2*B3
4		.	.	.
5		.	.	.
6		.	.	.

b Alter your spreadsheet above to find the width (x m) and area if the perimeter is still 48 m but the length is 12 m.

c For case **a**, write a rule for the width x m for a length of 15 m and a perimeter of P m.

d We will now generalise the cases above in a bid to find the maximum rectangular area that can be formed from 60 m of fencing.

i For a width of x m and 60 m of fencing, explain why the length of the rectangular paddock is given by $(30 - x)$ m.



Hint:
x m
$P = 2x + 2l$
$60 = 2x + 2l$
Solve for l.

ii Create a table in Excel like the one shown below. It should include the changing integer width values (x m), the corresponding lengths, the perimeter (as a check, which should always be 60 m) and the corresponding area.

	A	B	C	D
1	Width (x metres)	Length (metres)	Perimeter (metres)	Area (m^2)
2	1	=30-A2	=2*A2+2*B2	=A2*B2
3	=A2+1	=30-A3	=2*A3+2*B3	=A3*B3
4				

Fill down the columns to $x = 30$ and determine the maximum area and the dimensions of the paddock that maximise the area.

iii Repeat part **ii** but for a perimeter of 100 m. You will need to change only the formula for the length of the paddock.

iv In parts **ii** and **iii**, what do you notice about the dimensions of your paddock that maximise the area for a given perimeter?

v *Extension*
Generalise your results for a perimeter of k m by finding the dimensions that generate the maximum area in terms of k and, hence, the maximum area in terms of k.

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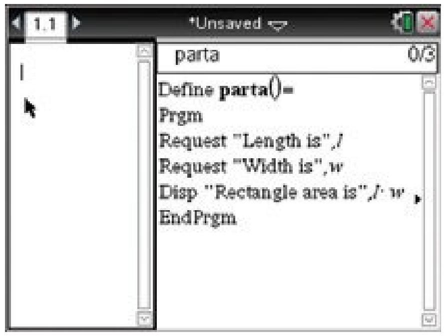
2.2 Measurement calculations using programming

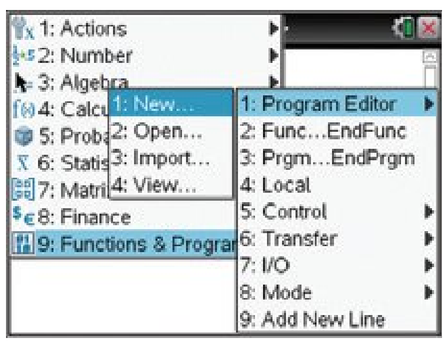
As you have seen, formulas are used in measurement to calculate perimeters, areas and more. Each of these formulas require a set of inputs (i.e. variable values) and produce an output (e.g. perimeter, area etc.).

a The program below is from a CAS calculator. Analyse the program and explain what you think it does.



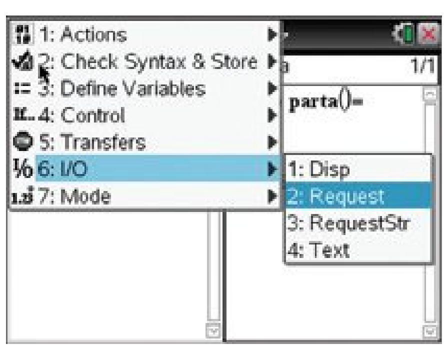
b Now follow these steps to enter the program into your calculator and run it.

1 Open a new document screen and select the *Functions & Programs* menu followed by *Program Editor* and *New*.



2 Give your program a name.

3 The program needs two inputs: length and width. Select *Menu* and choose *I/O* (Inputs and outputs menu) followed by *Request*.



Type the display text you want to use to prompt the user to input the length and width and choose a variable for each.

4 To display the result, from the *I/O* menu select *Disp* and type the display text you want to appear, followed by the formula for calculating the area of the rectangle; i.e. $l \times w$.

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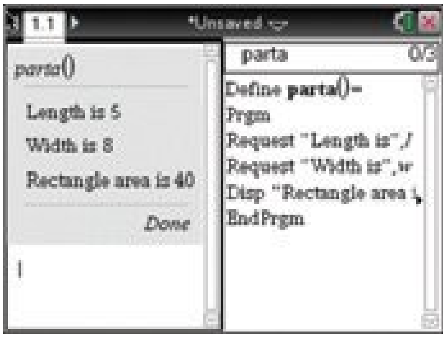
5 To save, from the menu select *Check syntax and store*.

6 To run the program, type the program name on the left-hand panel. You should be prompted for the inputs and the result will be displayed.



c Now design and run a program that calculates the area of a trapezium. Consider what inputs you will need and how to calculate the area.

d The circumference of a circle is given by $C = 2\pi r$. Design and run a program that calculates the radius of a circle when given its circumference.

In the design phase of a program, a flow chart may be used to consider the inputs, what the program needs to do and the output. An example is shown below for the area of a rectangle program (Figure 1). Figure 2 shows the same program but it has been modified so that it decides that if the length is equal to the width, then the rectangle is a square and the program displays this, otherwise it displays the area of a rectangle.

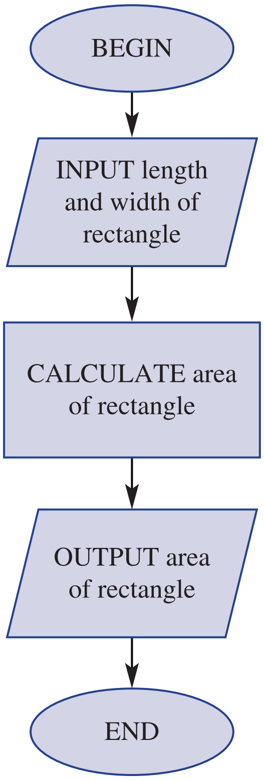


Figure 1

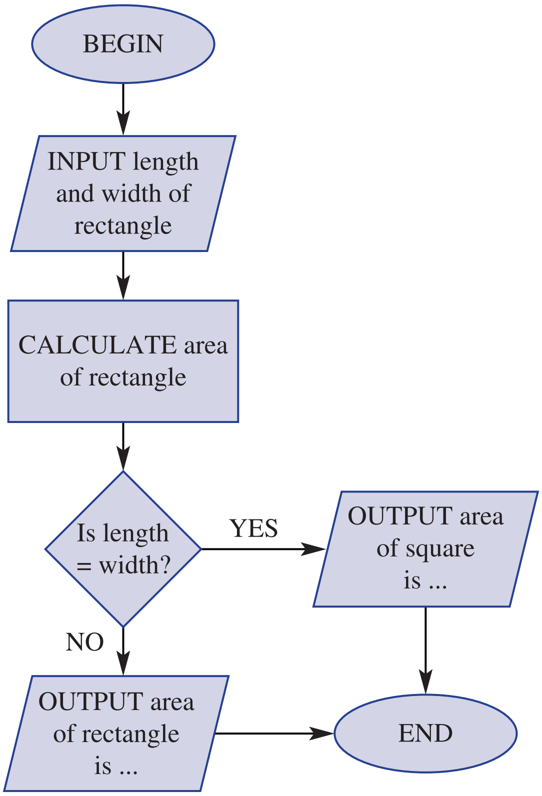


Figure 2

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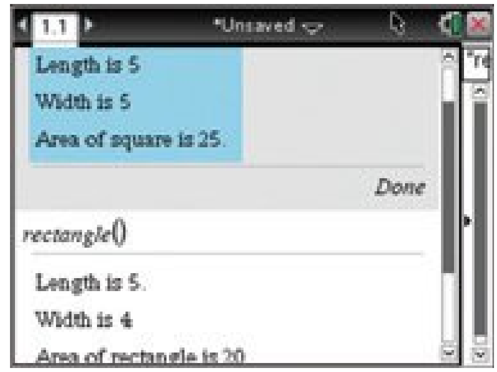
e Discover the use of the 'If. . . then. . . else' condition in the *Control* menu of the *Program Editor*. Use this to extend your program for the area of a rectangle to implement the modified program shown in Figure 2 on the previous page, which decides if the rectangle is a square or not and then displays its output as such.



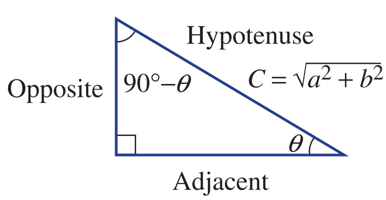
f This task involves designing a program that calculates the third side of a right-angled triangle, (when given the two shortest sides) and the two non-right angles of the triangle.

i Design your own flow chart for the program for the right-angled triangle sides and angles. Use the design shown below for the symbols; i.e. a parallelogram for input/output boxes, a rectangle for process boxes and a diamond for any decision boxes.

for input/output stages | for process stages | for decision stages

ii Now that you have completed the design, start a new program on your CAS calculator to find the third side length. Use Pythagoras' theorem and the other two angles, using the inverse tan function and complementary angles.
Recall that:

- $c = \sqrt{a^2 + b^2}$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ and $\theta = \tan^{-1}\left(\dfrac{\text{opposite}}{\text{adjacent}}\right)$.
- The two smaller angles in a right-angled triangle are complementary (i.e. sum to 90°).



iii Run your program and have it tested by a classmate. You can use the round() function to round your angles to two decimal places. Make sure you are in degrees mode and approximate mode.

iv Modify your program to also find the perimeter and area of the right-angled triangle.

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Activity 3: Walk the Plank

Strand: STATISTICS AND PROBABILITY

In the game of Walk the Plank, a numbered board represents the plank.

- Select a starting position somewhere along the plank.
- Toss a coin to determine whether you take one step forward (heads) or one step backwards (tails).
- Continue tossing the coin until you are either 'safe' and back on board the ship or 'overboard' and into the water.

On the plank, position 6 is considered 'safe' and position 0 is 'overboard'.

3.1 Playing the game

a Play the game 20 times, using a coin. Start from position 3, facing the deck. Count how many times you end up overboard.

b Calculate the proportion of times you end up overboard for the games you played in part **a**.

3.2 Walk the Plank as an algorithm

We will now think about Walk the Plank as an algorithm.

a Play the game once more. Think about the processes involved if you were going to write this game as an algorithm. Discuss your thoughts with a classmate.

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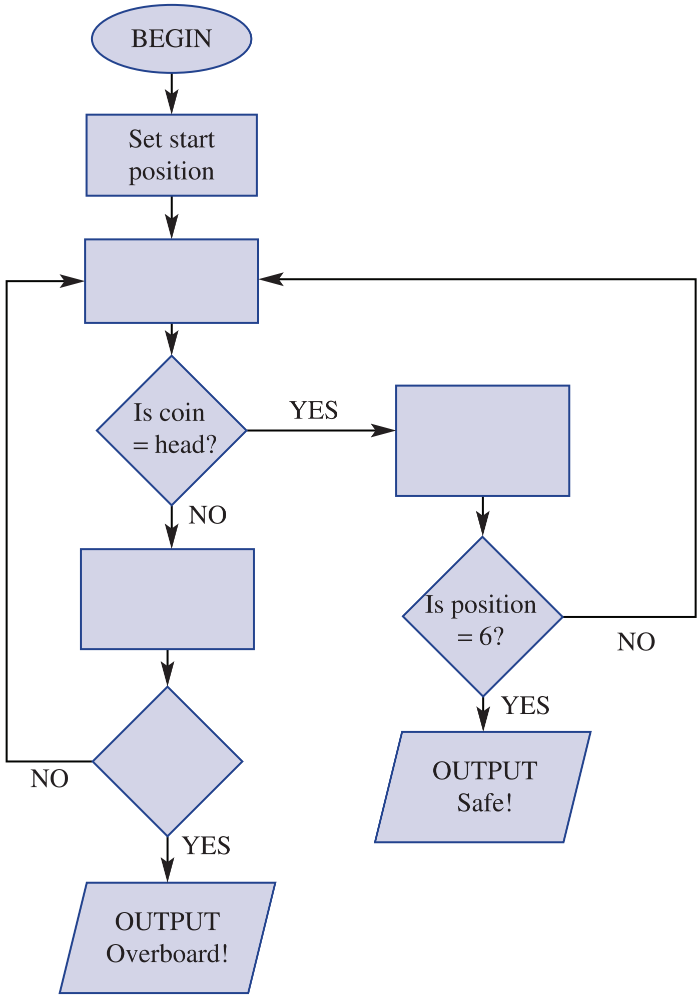
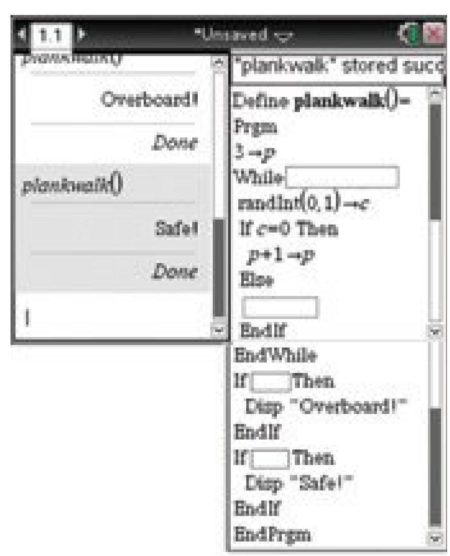
b Complete the flow chart below by filling in the empty boxes to describe one run-through of the game. The symbols are the same as those used in Task 2.2.



c By considering your flow chart, fill in the boxes in this program to complete it. The coin toss is simulated by generating the random numbers 0 or 1 and assigning 0 as heads and 1 as tails.

d Enter your program on your CAS calculator. The 'While' loop can be found in the *Control* menu. It ensures the program keeps running while a condition is still being met. p is the variable that stores the position on the 'plank'.



e Run your program 50 times and count the number of times you get the result 'Overboard'. Then calculate the proportion of times you end up overboard.

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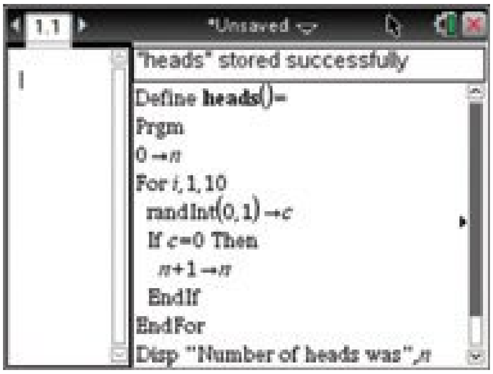
f Modify your program to cater for different start positions. Add an input (Request) so that the starting value of p can be chosen by the user instead of setting $p = 3$. For each start position, run your program 20 times and record (in a table like the one below) the number of times you get the result 'Overboard'.

Start position, p	Overboard tally	Overboard frequency
1		
2		
3		
4		
5		

g Construct a frequency histogram of your results. Comment on your results. What do you think would happen if you were to run the program 100 times for each starting position?

Rather than having to count the number of times overboard yourself and run the program 20 or 50 separate times, a 'For loop' can be used to run the simulation a set number of times. This can be found in the *Control* menu. The screen shown opposite shows a simple 'For loop' that simulates tossing a coin 10 times, counts the number of heads and then displays the result.



h Adjust your program to include a 'For loop' that runs the Walk the Plank simulation a number of times and counts the number of times the result is 'Overboard'.

i *Extension*
Consider how you could add more features to this game. For example, the plank could be longer and the process of moving along it could be more involved. You could toss a coin to determine if you are facing forwards or backwards and then roll a die to determine the number of steps you take, where rolling 1–3 could mean 1–3 steps forward and rolling 4–6 could mean 1–3 steps backwards.
Add your own ideas, design the algorithm and then write and run the program.

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A

Accuracy How true a measurement is compared to the real measurement

Acute Between 0 and 90 degrees

Adjacent (Trigonometry) The side next to the given angle that is not the hypotenuse

Algebraic fraction A fraction containing pronumerals as well as numbers

Algorithm A procedure involving a number of steps to find the answer of a problem

Alternate To sit across the transversal and equal in value

Angle of depression The angle of your line of sight from the horizontal when looking down at an object

Angle of elevation The angle of your line of sight from the horizontal when looking up at an object

Angle of reference The angle in a right-angled triangle that is used to find the opposite side and the adjacent side

Annual Occurring once per year

Area The number of square units needed to cover the space inside the boundaries of a 2D shape

Association A relationship between two variables

Asymptote A line whose distance to a curve approaches zero, and the curve never touches it

Average A summary statistic of a set of values, calculated by summing the set of values in a data set and dividing by the total number of values

Axis of symmetry A line through a shape so that each side is a mirror image

B

Backtracking A process for solving linear equations step by step

Base The number or pronumeral that is being raised to a power

Bimodal When a set of data has two modes

Binomial A two-term expression

Bivariate data Data that involves two variables

Borrow An amount of money that is lent by a bank or person with the intention of it being paid back

Box plot A diagram using rectangles and lines to show the spread of a set of data, using five important values

Budget A plan as to how money will be allocated to expenses

C

Capacity The volume of an object measured in litres or millilitres

Cartesian plane Formed by two perpendicular axes with points in the plane defined by a set of coordinates

Categorical data Data that is recorded in categories; e.g. favourite colour

Census Statistics collected from an entire population

Chance The likelihood of an event happening

Circle A shape with a centre and radius

Circumference The distance around the outside of a circle; the curved boundary

Clockwise In the same direction as hands on a clock

Coefficient A numeral placed before a pronumeral, showing that the pronumeral is multiplied by that factor

Cointerior To sit on the same side of a transversal and are supplementary

Column graph A graphical representation of a single category or type of data. Columns are used to show the frequency of scores

Commission Earnings of a salesperson based on a percentage of the value of goods or services sold

Common factor Factors that are same for two or more terms

Complement A set of outcomes containing the elements that are not in another given set

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period

Conditional probability A measure of the probability of an event occurring given that another event has occurred

Congruent (figures) Figures that are exactly the same size and shape

Constant of proportionality The constant value of the ratio of two proportional quantities x and y; usually written $y = kx$, where k is the constant of proportionality

Constant term The part of an equation or expression without any pronumerals

Continuous data Numerical data that can take on any value in a given range; e.g. time taken to run a marathon

Coordinates An ordered pair written in the form (x, y) that states the location of a point on the Cartesian plane

Correlation (or association) A measure of the relationship between two variables

Corresponding To be in similar positions and equal in value

Cosine (cos) The ratio of the length of the adjacent side to the length of the hypotenuse in a right-angled triangle

Cost price The price at which goods have been bought by a retailer

Cross-section The shape of a cut through a solid

Cylinder A solid with two parallel, congruent circular faces connected by a curved surface

D

Debit An amount of money removed from an account

Deductions Amounts of money taken from gross income

Define To give the meaning of a variable

Denominator The part of a fraction that sits below the dividing line

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Deposit A sum of money paid into an account or as the first instalment of a purchase

Diagonal A line segment joining opposite vertices in a shape

Diameter A line passing through the centre of a circle with its end points on the circumference

Difference of perfect squares (DOPS) When one square term is subtracted from another

Dilation A transformation where a curve is enlarged or reduced but the centre is not changed

Directly proportional When one quantity increases or decreases constantly with respect to another variable

Discount An amount subtracted from a price

Discrete data Numerical data that can only take a limited number of values; e.g. number of cars in a household

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

Dot plot A graph in which each dot represents one data value

Double time Pay earnt in overtime at twice the usual hourly rate

E

Elimination A method for solving simultaneous equations, where one equation is added to or subtracted from another to eliminate one of the variables

Employee The person who works for an employer

Employer The person or business who employs people to work for them

Equation A mathematical statement that states that two expressions have the same value

Equilateral triangle A triangle with all sides equal

Equivalent To have the same meaning or solution

Equivalent fraction A fraction that is of equal value to another

Evaluate To work out the answer

Event A situation involving chance or probability trials

Exact value A precise number with no rounding

Expand Remove grouping symbols (such as brackets)

Experimental probability Probability based on recording the outcomes of trials of an experiement

Exponent Another name for index or power that is the number of times a base factor is repeated under multiplication

Exponential decay Repeatedly decreasing a quantity by a constant percentage over time

Exponential growth Repeatedly increasing a quantity by a constant percentage over time

Exponential notation A way of representing repeated multiplication of the same number

Expression A group of mathematical terms containing no equals sign

Exterior angle theorem The theorem that, in a triangle, the exterior angle is equal to the sum of the two opposite interior angles

Extrapolation Determining information outside of the original data

F

Factor A term or value that divides, without leaving a remainder, into an expression

Factorise To write an expression as a product, often involving brackets

Fees A fixed price for a specific service

Five-figure summary A set of numbers that summarise a set of data: the minimum score, first quartile, median, third quartile and maximum score

Fixed expenses Expenses that are set and do not change during a particular time period

Formula A general rule for finding the value of one quantity given the values of others

Frequency table A table showing all possible scores in one column and the frequency of each score in another column

G

Gradient (m) The steepness of a slope

Gradient–intercept form The equation of a straight line, written with y as the subject of the equation

Gross income Total income before any deductions (e.g. income tax) are made

H

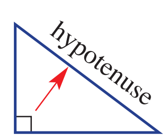
Highest common factor (HCF) The largest number that divides into all terms

Histogram A special type of column graph with no gaps between the columns; it illustrates the frequency of class intervals

Horizontal A flat line or plane

Horizontal axis The axis that runs parallel to the plane

Hypotenuse The longest side of a right-angled triangle (the side opposite the right angle)



I

Identify To establish key information from a problem

Independent events Two events that do not influence or are affected by each other

Index form A method of writing numbers that are multiplied by themselves

Index (power or exponent) The number of times a factor is repeated under multiplication (plural: indices)

Inequality An equation using an inequality symbol

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Inequality sign A symbol that compares the size of two or more expressions or numbers by pointing to the smaller one

Integer $\dots -3, -2, -1, 0, 1, 2, 3\dots$

Intercept The points where a curve cuts the x or y axes

Interest Additional money paid on a loan or an investment over time

Interpolation Reading information from within a graph

Interquartile range (IQR) A measure of spread giving the difference between the upper and lower quartiles

Intersection ($A \cap B$) The elements that are common to two or more sets of data

Intersection point The point where two or more lines meet

Inverse (Trigonometry) A function that reverses the sine, cosine and tangent functions

Inversely proportional The relationship between two variables such that when the value of one variable increases, the other decreases

Invest To put money into an account or other type of investment such as shares

Investment Money that is put into banks, shares or other financial scenarios with the aim of making a profit

Isosceles A triangle with one pair of equal length sides

K

Kite A quadrilateral with two adjacent pairs of equal sides

L

Length A measurement for distances

Levy An imposed fee

Like terms Terms with the same pronumerals and same powers

Linear Forms a straight line

Linear equation An equation whose pronumerals are always to the power of 1 and do not multiply or divide each other

Linear inequality An inequality that involves a linear function

Linear relationship The relationship between a variable and a constant term

Line of best fit A line that has the closest fit to a set of data points displayed in a scatter plot

Line segment A section of a straight line

Loan Money borrowed and then repaid, usually with interest

Long run proportion The ratio of favourable outcomes to the total number of trials in an experiment after conducting a very large number of trials

Lower fence The lower limit of a set of data. Any value below this is considered an outlier

Lower quartile The number above 25% of the ordered data

Lowest common denominator (LCD) The lowest common multiple of the denominators of fractions, used to compare, add or subtract fractions

M

Maximum (Graphing) A point which is the highest in a particular area

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Median The middle score when all the numbers in a set are arranged in order

Midpoint The point on an interval that is equidistant from the end points of the interval

Minimum (Graphing) A point which is the lowest in a particular area

Mode The score that appears most often in a set of numbers

Modelling The use of mathematics, equations and graphs, to describe a real-life situation

Monic quadratic A quadratic expression where the coefficient of the squared term is 1

Monic quadratic trinomial A 3-term quadratic with the coefficient of x^2 as 1

Multiple A number or expression multiplied by an integer greater than 1

Mutually exclusive Two events that cannot both occur at the same time

N

Net A 2-dimensional representation of the surfaces of a 3-dimensional solid

Net income Income remaining after deductions have been made from gross income

Nominal data Categorical data that has no order; e.g. colours

Non-linear An expression or graph that does not result in a straight line

Null factor law If two numbers multiply to give zero, then one or both of those numbers must be zero

Number plane A diagram on which two numbers can be used to locate any point

Numerator The part of a fraction that sits above the dividing line

Numerical data Data that is decribed with a number; e.g. the height of buildings

O

Obtuse Between 90 and 180 degrees

Opposite (Trigonometry) The side opposite the given angle

Ordinal data Categorical data that can be ordered; e.g. high, medium, low

Origin The point where the horizontal and vertical axes meet, it has coordinates $(0, 0)$

Outcome One of the possibilities from a chance experiment

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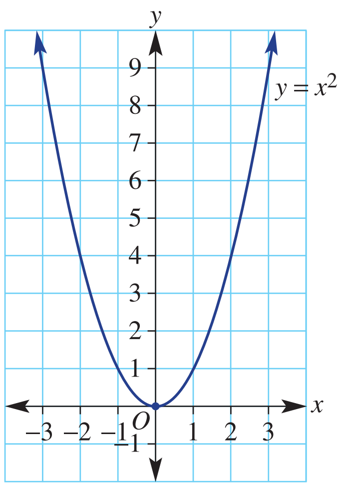
Outlier Any value that is much larger or much smaller than the rest of the data in a set

Overtime Time worked in addition to normal working hours

P

p.a. (per annum) Anually; that is, per year

Parabola A smooth U-shaped curve with the basic rule $y = x^2$



Parallel box plot Two box plots drawn on the same scale used to compare two related sets of data

Parallel lines Lines in the same plane that are the same distance apart and never intersect

Parallelogram A quadrilateral with two pairs of parallel sides

Percentage A convenient way of writing fractions with denominators of 100

Perfect square A quadratic trinomial that can be expressed as a single square

Perimeter The total distance (length) around the outside of a figure

Perpendicular To be at 90 degrees

Pi The number 3.14159… associated with circles

Plot Using points to help draw a graph

Point of intersection The point at which two lines cross each other and therefore have the same coordinates

Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Population A group of people or objects with something in common

Power Another name for index that is the number of times a base factor is repeated under multiplication

Precision The ability to obtain the same result many times

Principal (*P*) An amount of money invested in a financial institution or loaned to a person/business

Prism A solid with a uniform (constant) cross-section

Probability A measure of the likelihood that an event will occur.

Product The result of multiplying

Profit The amount of money made by selling an item or service for more than its cost

Pronumeral A letter representing a number

Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Q

Quadratic An expression of the form $ax^2 + bx + c$

Quadratic equation An equation with the highest power equal to 2; e.g. $x^2 - 3x + 1 = 0$

Quadrilateral A 4-sided shape

Quartiles The three values that separate the scores when a set of ordered data is divided into four equal parts

R

Radius The distance from the centre of a circle to its outside edge

Range The difference between the highest and lowest numbers in a set

Rate A measure of one quantity against another

Rate of interest (*r*) The annual percentage rate of interest paid on a loan or earned on an investment

Ratio A comparison of quantities usually written as a fraction or in the form $a : b$

Reciprocal One of a pair of numbers that when multipied together produce 1. The reciprocal of x is $1/x$

Rectangle A parallelogram with all angles 90 degrees

Reflection A transformation where a curve is flipped across a line on the number plane

Regular polygon A polygon with all sides equal and all angles equal

Repayment An amount paid to a financial institution at regular intervals to repay a loan, with interest included

Rhombus A parallelogram with all sides the same length

Right (Geometry) At 90 degrees

Rise The vertical change when calculating gradient

Rounding To approximate a number

Run The horizontal change when calculating gradient

S

Salary An employee's fixed agreed yearly income

Sample A group that has been chosen from a population

Sample space All the possible outcomes of an experiment

Scale factor The number you multiply each side length by to enlarge or reduce a shape

Scalene A triangle with all different side lengths

Scatter plot A diagram that uses coordinates to display values for two variables for a set of data

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Scientific notation A way to express very large and very small numbers using [a number between 1 and 10] $\times 10^{\text{power}}$

Sector A portion of a circle between two radii

Selling price The price that a retailer sells goods to a buyer for

Significant figure A digit that indicates how accurate a number is

Similar (triangles) Triangles whose corresponding angles are equal and whose corresponding sides are in the same ratio

Simple interest A type of interest that is paid on a loan or earned on an investment, which is always calculated on the principal amount loaned or invested

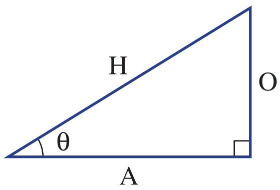
Simultaneous equation To find a solution that satisfies more than one equation

Sine (sin) The ratio of the length of the opposite side to the length of the hypotenuse in a right-angled triangle

Skewed data Data that is unevenly distributed either side of the mean or median

Slope The steepness of a line

SOH CAH TOA A way of remembering the trigonometric ratios: sine equals opposite over hypotenuse; cosine equals adjacent over hypotenuse; tangent equals opposite over adjacent



Solid A 3-dimensional shape

Solution The answer

Solve To find the value of an unknown quantity

Speed The rate at which something is moving

Square A parallelogram with all angles 90 degrees and all sides the same length

Square root A number whose square results in the number under the square root sign

Standard form (Quadratics) An expression of the form $ax^2 + bx + c$

Stationary An object not moving, that is at rest

Statistical data Information gathered by observation, survey or measurement

Stem-and-leaf plot A table that lists numbers in order, grouped in rows

Subject The pronumeral or variable that is alone on one side of an equation

Substitute To replace pronumerals with numerical values

Supplementary To add to 180 degrees

Surd A number with a root symbol

Survey A set of questions used to collect statistical data

Symmetrical data Data that is balanced on either side of the mean and median

T

Tangent (tan) The ratio of the length of the opposite side to the length of the adjacent side in a right-angled triangle

Taxable income The amount of a person's income to be taxed after deductions are subtracted

Taxation Money paid as tax to the government to pay for hospitals, schools, roads etc.

Tax bracket A range of incomes that are taxed at a given rate. This rate varies for different income ranges

Tax return A document completed and given to the tax office to calculate any extra tax they must pay or a refund they are owed

Term A group of numbers and/or pronumerals in an expression connected by only multiplication and division

Theoretical probability The expected probability of an event based on the number of favourable outcomes compared with the total possible outcomes

Time and a half A pay rate of overtime that is 1.5 times the normal hourly rate

Time-series data A set of data collected in sequence over a period of time

Total surface area (TSA) The total number of square units needed to cover the outside of a solid

Transformation A change in the position, orientation or size of an object

Translation A transformation where a curve is moved a certain distance on the number plane

Transversal A line that cuts two or more lines

Trapezium A 4-sided shape with at least one pair of parallel sides

Tree diagram A way of listing the sample space in a probability experiment involving two or more components

Trend A general direction that something is changing

Trend line Line of best fit

Trial One run of an experiment

Triangle A three sided shape

Trigonometry A branch of mathematics connecting angles and lengths

True bearing (°T) An angle that is measured clockwise from north

Turning point The highest or lowest point on a parabola

Two-step experiment A probability experiment that involves two actions to determine an outcome

Two-way table A table used to display two sets of outcomes

U

Uniform To be the same

Union ($A \cup B$) The combination of all elements from two or more sets of data

Unit A type of measurement; e.g. cm or litres

Upper fence The upper limit of a set of data. Any value above this is considered an outlier

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Upper quartile The number above 75% of the ordered data

V

Variable An unknown, which can take on any value

Variable expenses Expenses that may change during a particular period of time or over time

Venn diagram A diagram using circles to show the relationships between two or more sets of data

Verify To check or confirm

Vertex (Graphing) The turning point

Vertical At right angles to a horizontal line or plane

Vertical axis The axis that runs perpendicular to the horizontal axis

Vertically opposite To sit opposite when two lines cross. Vertically opposite angles are equal.

Volume The amount of three-dimensional space within an object

W

Wages Earnings paid to an employee based on an hourly rate

Without replacement An experiment where an item is not replaced before the next selection is made

With replacement An experiment where items are replaced before the next selection is made

X

x-axis The horizontal axis on a Cartesian plane

x-coordinate The first coordinate of an ordered pair

x-intercept The point at which a line or curve cuts the x-axis

Y

y-axis The vertical axis on a Cartesian plane

y-coordinate The second coordinate of an ordered pair

y-intercept The point at which a line or curve cuts the y-axis

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Answers

Chapter 1

Warm-up quiz

1 a Circle b Square c Parallelogram
d Triangle e Rectangle f Trapezium
g Semicircle h Rhombus
2 a 1000 b 100 c 10 d 1000
e 500 f 25
3 a 12 cm b 32 m c 5.9 mm
4 a 10 cm^2 b 70 m^2 c 36 km^2
5 a 4 cm^2 b 14 m^2 c 6 km^2
6 $C = 31.42$ m
$A = 78.54$ m^2

1A

Now you try

Example 1
a 460 cm b 3.2 km

Example 2
a 3.2 cm^2 b 240 m^2

Example 3
a 210 000 cm^3 b 94 cm^3

Exercise 1A

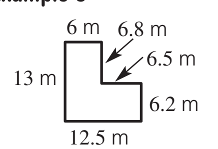
1 a 1000 b 10 c 100
2 a 100 b 10 000 c 1 000 000
3 a 1000 b 1 000 000 000 c 1 000 000
4 a 43.2 mm b 0.327 km c 8.34 m
d 96 mm e 0.2975 km f 1.27 cm
5 a 300 000 mm^2 b 5000 cm^2
c 5 000 000 m^2 d 29 800 cm^2
e 53 700 mm^2 f 230 cm^2
6 a 2000 mm^3 b 200 000 cm^3
c 5.7 cm^3 d 15 000 000 m^3
e 0.0283 km^3 f 0.762 m^3
7 5500 m
8 a 23.4 m b 22 m
9 a 118 mm b 147.3 cm c 453.258 km
d 15.5 cm^2 e 3251 cm^2 f 3739 m^2
g 484 500 mm^3 h 537 300 m^3
10 21.5 cm
11 For a high level of accuracy
12 a 8.85 km b 4.5 feet c 26.67 cm
d 1.243 miles e 57 000 m^2 f 0.247 L
g 8200 mL h 5 500 000 mL i 1000 sq feet
j 2000 L k 100 ha l 0.152 m^3

1B

Now you try

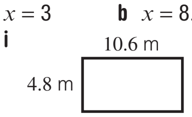
Example 4
a 19 m b 40.5 mm

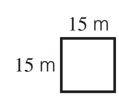
Example 5
9.9 cm

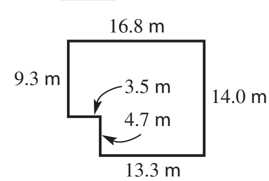
Example 6
a


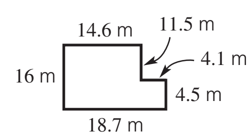
b 51 m

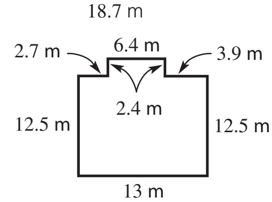
Exercise 1B

1 perimeter
2 a 6 b 7.1 c 4.3
3 a 12 cm b 23 m c 11 km
d 12 m e 19.2 cm f 10 m
4 a 6.7 cm b 65 mm c 18 m
d 810 m e 9.4 km f 180 cm
5 a $x = 4$ b $x = 2$ c $x = 6$
6 a $x = 3$ b $x = 8.8$ c $x = 0.1$
7 a i


ii 30.8 m
b i


ii 60 m
c i


ii 61.6 m
d i


ii 69.4 m
e i


ii 55.8 m

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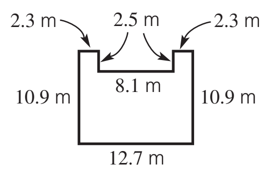
f i 

ii 52.2 m

8 20 m
9 15 cm
10 a $P = 4l$ **b** $P = 2l + 2w$ **c** $P = x + y + z$
d $P = a + 2b$ **e** $P = 4l$ **f** $P = 3s$
11 3
12 13

1C

Now you try

Example 7
a 31.42 m **b** 15.24 cm

Example 8
16.71 cm

Exercise 1C

1 a radius **b** diameter **c** circumference
2 a $C = \pi d$ **b** $C = 2\pi r$
3 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
4 a 18.85 m **b** 31.42 m **c** 31.42 km
d 113.10 cm **e** 61.07 mm **f** 3.36 km
5 a 27.42 m **b** 16.28 cm **c** 6.71 mm
d 12.22 cm **e** 14.71 m **f** 59.70 cm
6 a 9.42 m **b** \$423.90
7 a i 3.14 m **ii** 47.12 m
b 3.14 km
8 319 times
9 a 12.25 **b** 53.03 **c** 1.37
d 62.83 **e** 19.77 **f** 61.70
10 a $r = \frac{C}{2\pi}$
b i 5.57 cm **ii** 0.29 m **iii** 0.04 km
11 a 3.5 cm
b i 21.99 cm **ii** 65.97 cm **iii** 109.96 cm
c 12.73 cm

1D

Now you try

Example 9
a 9 m^2 **b** 40 cm^2 **c** 42 mm^2

Example 10
a 48.75 m^2 **b** \$536.25

Exercise 1D

1 a E **b** B **c** F
d C **e** D **f** A
2 a 2 cm **b** 4 m **c** 4.3 cm **d** 10 cm
3 a 4 m^2 **b** 21 cm^2 **c** 11.76 m^2
d 21 m^2 **e** 22.5 mm^2 **f** 2 m^2
4 a 25 cm^2 **b** 54.6 m^2 **c** 1.82 km^2
d 0.025 mm^2 **e** 1.1234 m^2 **f** 100 cm^2
5 a 0.96 m^2 **b** 9600 cm^2
6 21 m^2
7 a 13.6 m^2 **b** \$149.60
8 a 7.56 m^2 **b** \$491.40
9 1 and 24, 2 and 12, 3 and 8, 4 and 6
10 a 252.05 m^2 **b** 177.86 m^2
11 a $w = 2.88$ **b** $l = 14.35$ **c** $h = 1.44$
d $a = 1.05$ **e** $h = 1.87$ **f** $x = 8.89$
12 All answers = 3

1E

Now you try

Example 11
a 78.54 m^2 **b** 12.07 cm^2

Example 12
51.31 mm^2

Example 13
22.57 m^2

Exercise 1E

1 E
2 C
3 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d $\frac{1}{12}$ **e** $\frac{7}{12}$ **f** $\frac{5}{6}$
4 a 50.27 cm^2 **b** 201.06 m^2 **c** 72.38 m^2
d 38.48 m^2 **e** 0.82 mm^2 **f** 124.69 km^2
5 a 39.27 m^2 **b** 4.91 m^2 **c** 84.82 m^2
d 13.09 m^2 **e** 69.81 cm^2 **f** 8.03 m^2
6 157.1 cm^2
7 a 14.28 cm^2 **b** 178.54 m^2 **c** 32.14 mm^2
8 43.24 m^2
9 a 34.8 cm^2 **b** 63.5 m^2 **c** 8.7 m^2
d 103.3 mm^2 **e** 578.5 km^2 **f** 5.0 m^2
10 a 20 **b** 565.49 cm^2 **c** 154.51 cm^2 **d** 5

1F

Now you try

Example 14
118 cm^2

Example 15
480 mm^2

Exercise 1F

1 net
2 a 6 **b** 3
3 a 35 cm^2 **b** 21 cm^2 **c** 12 cm^2 **d** 96 cm^2
4 a 90 cm^2 **b** 34 mm^2 **c** 46 m^2
5 a 240 m^2 **b** 84 m^2 **c** 168 m^2 **d** 1176 cm^2
6 a 8.64 cm^2 **b** 96 mm^2 **c** 836.6 m^2
7 384 cm^2
8 3880 cm^2
9 3520 cm^2
10 a 5.116 L **b** 10.232 L
11 a Waterproof 13.76 L
Smooth paint 22.86 L
b \$553.46
12 a 105 cm^2 **b** 5 cm^2 **c** 16 m^2
13 Yes, only 5 L required

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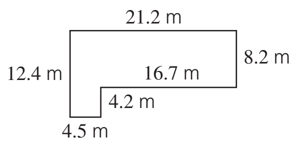
Progress quiz

1 a 9000 m b 45 000 cm^2
 c 145 000 mm^3
2 a 17 cm b 38 m
3 a



 b 67.2 m
4 a $C = 25.13$ cm, $A = 50.27$ cm^2
 b $C = 31.42$ m, $A = 78.54$ m^2
5 a 30 m^2 b 34 m^2 c 50 cm^2 d 33.5 cm^2
6 \$607.20
7 $A = 49.6$ m^2, $P = 37.9$ m
8 a 102 m^2 b 152 cm^2

1G

Now you try

Example 16
72.38 cm^2

Exercise 1G

1 a circle b $2\pi rh$
2 a i 4 m ii 7 m
 b 25.13 m
 c 276.46 m^2
3 a 100.53 m^2 b 376.99 cm^2
 c 1225.22 cm^2 d 74.64 mm^2
4 a 2557.3 cm^2 b 502.9 m^2
5 a 6.3 m^2 b 6283.2 cm^2
6 24.0 m^2
7 628.3 cm^2
8 75.4 m^2
9 a 173.9 cm^2 b 217.8 m^2
 c 31.6 m^2 d 52.9 cm^2
10 a 25.13 m^2 b 251.33 m^2 c 6.28 m
 d 159.15 times e 4000 m^2

1H

Now you try

Example 17
360 cm^3

Example 18
254.47 m^3

Example 19
72 L

Exercise 1H

1 a C b A c B
2 a 1000 b 1000
3 10 cm^3
4 a 240 m^3 b 480 cm^3 c 0.18 m^3
5 a 113.10 cm^3 b 19.63 m^3 c 4.83 mm^3
6 a 20 cm^2 b 90 cm^3
7 a 36 m^3 b 15 cm^3 c 0.572 mm^3
8 5890.49 cm^3
9 a 72 L b 2 L c 2 L
10 a 1583.36 m^3 b 30 km^3 c 196 cm^3
 d 30 m^3 e 10 cm^3 f 2.15 m^3
11 a 25 cm^2 b 4 cm
12 a 17 150 cm^3 b 384.85 cm^3 c 35
 d 3680.42 cm^3
13 a 83.3 m^3 b 1500 m^3 c 20.9 cm^3

1I

Now you try

Example 20
a 0.1 m or 10 cm b 0.01 km or 10 m

Example 21
a 35.5 cm to 36.5 cm b 15.05 m to 15.15 m

Exercise 1I

1 From 3.35 to under 3.45 (i.e. 3.43, 3.39 etc.)
2 a 347 cm b 3 m
3 6.65
4 a 1 cm b 0.1 mm
 c 1 m d 0.1 kg
 e 0.1 g f 1 m
 g 1 h h 0.01 m or 1 cm
 i 0.01 km or 10 m
5 a 4.5 m to 5.5 m b 7.5 cm to 8.5 cm
 c 77.5 mm to 78.5 mm d 4.5 ns to 5.5 ns
 e 1.5 km to 2.5 km f 34.15 cm to 34.25 cm
 g 3.85 kg to 3.95 kg h 19.35 kg to 19.45 kg
 i 457.85 t to 457.95 t j 18.645 m to 18.655 m
 k 7.875 km to 7.885 km l 5.045 s to 5.055 s
6 a 30 m b 145 g c 4.6 km
7 a 149.5 cm to 150.5 cm
 b 145 cm to 155 cm
 c 149.95 cm to 150.05 cm
8 a 24.5 cm to 25.5 cm
 b i 245 cm ii 255 cm
9 a 9.15 cm
 b 9.25 cm
 c 36.6 cm to 37 cm
10 a If they all choose a different level of accuracy, then they will have different answers. Also, human error plays a part.
 b Johan: nearest kg; Amy: nearest 100 g; Thomas: nearest 10 g.
 c Yes; however, the more decimal places being considered then the more accurate that the measurement will be when used in further calculations, if they are required.
11 a Distance between towns, cities, or airplane rides; length of major rivers
 b House plans, plumbing plans and building, in general
 c Mixing chemicals, administering cough mixture to children, matching paint colours, pay for petrol
 d Filling a swimming pool, describing the fuel tank of a car or plane
12 Compare your results with those of other students. Answers may vary.

Maths@Work: Bricklayer

1 a $l = 23$ cm $= 0.23$ m; $h = 11$ cm $= 0.11$ m
 b 253 cm^2; 0.0253 m^2
 c 1922.8 cm^3
 d i 2.39 m ii 23.99 m
 e 2990 mm
 f Approx. 237 bricks
2 a 36×23 bricks = 828 bricks b 42 bags c \$831
3 a 1 pack b 2 packs
4 a 8 layers
 b 159.6 cm
 c 64×40 bricks = 2560 bricks
 d \$3775

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5

Brick wall dimensions		Numbers of bricks used		
Length	Height	Per layer (row)	Number of layers	Total
5750	2150	24	18	432
21350	6230	89	52	4628

6

Bricks	Cost of pallets			
Number of bricks used	Decimal number of pallets	Number of whole pallets	Cost per pallet (including GST)	Total cost of pallets
432	0.432	1	\$1358.24	\$1358.24
4628	4.628	5	\$1541.56	\$7707.80

Cost of mortar				Total cost of mortar and pallets
Decimal number of mortar bags	Number of whole bags	Cost per bag (including GST)	Total cost of mortar	
21.6	22	\$7.95	\$174.90	\$1533.14
231.4	232	\$8.45	\$1960.40	\$9668.20

7 a \$3015.02 b \$10 711.17

Puzzles and games

1 PRISM 2 No 3 21.46% 4 11
5 $r = 2$ 6 9.42 cm^2 7 27 cm^3

Short-answers questions

1 a 5300 m b 2.7 m^2 c 40 mm^3
2 a 13 b 24 cm c 38 m
3 a 18.85 m b 28.27 m^2
4 a i 19.42 m ii 26.14 m^2
b i 19.50 m ii 21.87 m^2
c i 14.28 m ii 12.28 m^2
5 a 10 b 25.9 c 17.5
6 a 828 m^2 b 136 cm^2
7 376.99 m^2
8 a 40 cm^3 b 125.66 m^3 c 21 cm^3
9 a 5.5 cm to 6.5 cm b 4.15 kg to 4.25 kg
c 16.205 m to 16.215 m

Multiple-choice questions

1 E 2 B 3 C 4 A 5 A 6 B
7 E 8 D 9 B 10 C

Extended-response question

1 a i 23.21 m ii \$383
b 19.6 m^2
c 3
d 6.6 m^3
2 a 50.27 m^2 b 150.80 m^2
c 100.53 m^3 d 100 530.96 L

Chapter 2

Warm-up quiz

1 a \$99.62 b \$1612 c \$2742
d \$84.51 e \$1683.85
2 a $\frac{50}{100}$ b $\frac{75}{100}$ c $\frac{20}{100}$ d $\frac{68}{100}$ e $\frac{45}{100}$
3 a 0.5 b 0.25 c 0.2
d 0.4 e $0.\dot{3}$
4 a 16.79 b 7.35 c 45.34
d 6.84 e 102.90
5 $a = 4519.28$, $b = 50\,509.20$, $c = 29\,572$,
$d = 54\,878$, $e = 97\,903.46$
6 a \$65 312 b \$187 200 c \$49 686
d \$35 800
7 a \$40 b \$250 c \$2
d \$211 e \$2.56 f \$750
8 a \$20 b \$300 c \$64
9 $a = \$18$, $b = \$323$, $c = \$264$, $d = \$112$, $e = \$57.92$
10 a \$50 b \$56 c \$550

2A

Now you try

Example 1
a 24% b 43.75% c 15%

Example 2
a $\frac{23}{100}$ b $\frac{16}{25}$ c $\frac{21}{200}$

Example 3
a 0.28 b 0.03 c 0.6

Example 4
\$828

Exercise 2A

1 a multiply b divide c multiply d divide
2 a i 100 ii 23 100
b i 0.18 ii 0.05
3 a 10, 5 b 25, 412, 103 c 2, 1.2
4 a 50% b 20% c 25% d 10%
e 1% f 28% g 30% h 75%
i $62\frac{1}{2}\%$ j 76% k 99% l 94%
5 a 17% b 73% c 48% d 9%
e 6% f 13% g 113% h 101%
i 80% j 90% k 99% l 17.5%
6 a $\frac{71}{100}$ b $\frac{4}{5}$ c $\frac{1}{4}$ d $\frac{11}{20}$
e $\frac{2}{5}$ f $\frac{22}{25}$ g $\frac{3}{20}$ h $\frac{33}{200}$
i $\frac{7}{40}$ j $\frac{9}{400}$ k $\frac{21}{400}$ l $\frac{21}{40}$
7 a 0.61 b 0.83 c 0.75 d 0.45
e 0.09 f 0.9 g 0.5 h 0.165
i 0.073 j 2 k 4.3 l 0.005
8 a \$25 b \$150 c \$60 d \$90
e \$21.60 f 344 grams g \$50.40 h \$107.80
i 350 m
9 a $\frac{15}{300} = \frac{1}{20}$ b 5%
10 68 kg
11 47.52 minutes
12 16.67%
13 $11\frac{1}{9}\%$
14 \$205.97

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15 a 1.4 / 1.7 / 1.2 / 0.8 / 1.3 / 0.4 / 0.4 / 1.4 / 1.3
b i 32.0% ii 25.0% iii 10.9%
c NSW – 32%, Vic – 25%, Qld – 20%, SA – 7%, WA – 11%, Tas – 2%, NT – 1%, ACT – 2%
d 90°

2B

Now you try

Example 5
$728

Example 6
$2214

Example 7
a $109 b 12.27%

Example 8
a $41.60 b $192

Example 9
a $143 b $77

Exercise 2B

1 a 1.1 b 1.2 c 1.02 d 1.18
2 a 0.95 b 0.7 c 0.85 d 0.5
3 a selling price, cost price b selling price c selling price, cost price
4 a Profit : $5 b Profit : $2.50 c Loss : $16
5 a $94.50 b $440 c $66 d $529.20 e $56 f $7210 g $56.43 h $3.60
6 a $1425 b $360 c $376 d $68 e $412.50 f $47.03 g $101.58 h $35.80
7

a	$6	60%
b	$60	25%
c	$3	20%
d	$7.50	3%
e	$2325	75%
f	$0.99	18%

8 a $52.25 b $261.25
9 $1536
10 a $67.96 b $11.99
11 a $2140.25 b $305.75
12 a $221.40 b $147.60
13 a $1225 b $24.50
14 a $84.13 b $94.87
15 $104.71
16 a $106.15 b $37.15
17 a $2.20 b 122.22% c $66 d 122.22%
18 a $13 b $6.30 c $69.30 d 38.6%
19 a $1952.24 b $211.24 c 12.13% d $57.03

2C

Now you try

Example 10
a $962.50 b $716

Example 11
$2693.60

Example 12
$575.94

Exercise 2C

1 a D b C c A d B
2 $36 842
3 a $22.80 b $30.40
4 a $690.90 b $483.70
5

	Gross income	Net income
a	$570	$415
b	$984	$764
c	$604.90	$304.90
d	$3430	$2920
e	$930.15	$746.15

6 a $3558.72 b $92 526.72
7 a $1960 b $6960
8 $34 375.00
9 a $1480.77 b $150 c $1630.77 d $88 425
10 1.25%
11 $365.70
12 $2000
13 a Kuger Incorporated b Mathsville Credit Union, 00 754 031 c $72 454 d Fortnightly e $4420 f $26.45 g $600.60 h $16 016 tax, net = $49 793.90 i $34.71/h

2D

Now you try

Example 13
a $81 770 b $18 201.90 c $1635.40 d $19 837.30 e 24.3% f Not enough; must pay another $2392.30

Exercise 2D

1 Taxable income = gross income minus deductions
2 a True b False
3 37c
4 a $93 605 b $22 580.85 c $1872.10 d $24 452.95 e 26.1% f Refund of $796.05
5 a $2242 b $11 047 c $43 447 d $63 547
6 a $65 625 b $12 875.13 c $1312.50 d $14 187.63 e 21.6% (to 1 d.p.) f Not enough paid; owes $1117.63
7 $87 500
8 $6172.84
9 $95 000
10 If a person pays too much tax during the year they will receive a tax refund. If they do not pay enough tax during the year they will have a tax liability to pay.
11 They only pay 45 cents for every dollar over $180 000.
12 a The tax-free threshold has been increased from $6000 to $18 200. In the second tax bracket, the **marginal** rate has changed from 15c to 19c. In the third tax bracket, the **marginal** rate has changed from 30c to 32.5c.

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b

	2011/2012	2012/2013	
Ali	$0	$0	No change
Xi	$1350	$0	$1350 less tax to pay
Charlotte	$3600	$2242	$1358 less tax to pay
Diego	$8550	$7797	$753 less tax to pay

13

	Resident	Non-resident	
Ali	$0	$1625	Non-residents pay a lot more tax than residents.
Xi	$0	$4875	
Charlotte	$2242	$9750	
Diego	$24 947	$32 950	

14 a Answers will vary.
b i $17 547
ii $32.50, so this means that the $100 donation really costs you only $67.50.

2E

Now you try

Example 14
a $28 890 **b** $12 840 **c** Yes, approx $62 per week

Example 15
a $7182 **b** 8.8%

Example 16
The 4 double length rolls

Exercise 2E

1 a Fixed **b** Fixed **c** Variable **d** Variable
2 $124.28
3 $2162
4 a $41 250 **b** $33 750 **c** Yes, $144 per week
5 $106
6 a $33 068 **b** 73.5%
7 a $7756 **b** $3878 **c** $149.15
8 a $82 708 **b** $1590.54 **c** 24%
9 a 13.43% **b** 29.56 L
10 a $342.55 **b** $2137.51 **c** 11.68%
11 a

Food	$86.40
Recreation	$43.20
Transport	$56.16
Savings	$86.40
Taxation	$108.00
Clothing	$51.84

b $56.16 **c** 30% **d** $673.92 **e** 10%
12 200 tea bags
13 Daily
14 a Mon–Thurs – $87
Fri–Sat – $93.50
Weekly – $71.43
b Weekly
15 a 200 mL bottle $0.011 75, 500 mL bottle $0.01048
b 500 mL bottle **c** $2.10 **d** $0.25
e Cost of packaging
16 a $248 **b** $240, 6 containers

2F

Now you try

Example 17
a $56.25 **b** 2%

Example 18
a $7344 **b** $153

Exercise 2F

1 a interest **b** principal **c** rate of interest
d time
2 a $200 **b** $300
3 a $60 **b** 3%
4 a $140 **b** $420 **c** $192.50 **d** $46.88
e 3% p.a. **f** 4% p.a.
5 a $6650 **b** $184.72
6 a $5192.25 **b** $16 692.25 **c** $198.72
7 a $7600 **b** $17 600 **c** $366.67
8 $1008
9 a $228 **b** $684 **c** $4684
10 16%
11 12.5 years
12 $66 667
13 $7500
14 a $1250, $2500 **b** $1968.75, $1920.00
c $220.31, $331.11

Progress quiz

1 a 0.32 **b** $\frac{2}{25}$ **c** 44% **d** 31.25%
e 25.2% **f** $\frac{31}{200}$
2 $3840
3 a $285.20 **b** 194.4 mL **c** 132.6 cm
4 $194.35
5 40%
6 a $956 **b** $722.20
7 a $75 875 **b** $16 206.38 **c** $1517.50
d Cameron must pay $2468.88
8 a $31 256 **b** 38.1%
9 a $4720 **b** 4.5 years

2G

Now you try

Example 19
$3787.43

Example 20
a $n = 24, r = 0.25$ **b** $n = 8, r = 3.5$

Example 21
$7267.24

Exercise 2G

1 a $50 **b** $550 **c** $55 **d** $605
2 a $1200 **b** rate of interest **c** 3
3 a $700(1.08)^2$ **b** $1000(1.15)^6$
c $850(1.06)^4$
4 a $5105.13 **b** $11 946.33
c $13 652.22 **d** $9550.63

Essential Mathematics for the Victorian Curriculum CORE Year 10 ISBN 978-1-108-87859-3

5 a $106 000 b $112 360 c $119 101.60
d $133 822.56 e $179 084.77 f $239 655.82
6 a 6, 3% b 60, 1% c 52, 0.173%
d 14, 2.625% e 32, 3.75% f 120, 0.8%
7 $11 446.15
8 a $2254.32 b $87 960.39 c $1461.53
d $789.84 e $591.63
9 a $5075 b $5228.39 c $5386.42
10 a i $3239.42 ii $3348.15 iii $3446.15
iv $3461.88 v $3465.96
b $226.54
11 a $P = 300, n = 12, r = 7\%,$
$R = 14\%, t = 6$ years
b $P = 5000, n = 24, r = 2.5\%,$
$R = 30\%, t = 2$ years
c $P = 1000, n = 65, r = 0.036\%,$
$R = 0.936\%, t = 2.5$ years
d $P = 3500, n = 30, r = 0.0053\%,$
$R = 1.9345\%, t = 30$ days
e $P = 10\,000, n = 10, r = 7.8\%,$
$R = 7.8\%, t = 10$ years
12 5.3% compounded bi-annually
13 a i Approx. 6 years ii Approx. 12 years
iii Approx. 9 years iv Approx. 5 years
v Approx. 7 years vi Approx. 4 years
b Same answer as part **a**.
c Yes

2H

Now you try

Example 22

a $16 320 b $3320

Example 23

a $954 b $3294 c $114

Example 24

a $3.96 b $1.72 debited

Exercise 2H

1 a Repayment b Investment c Loan
2 a $2640 b $3960 c $13 200
3 $124.50
4 a $18 600 b $8600
5 a $5580 b $80
6 a 360 b $624 960 c $374 960
7 a $2550 b $10 620 c $13 170 d $420
8 a $4.50 b $1.26
9 a

May	June	July	August
$2.40	$3.00	$0.12	$5.00

September	October
$2.08	$0.73

b $13.33
10 a i $0 ii $0 iii $7500
b $6375
c $1125
11 a i $5250 ii $20 250 iii $281.25
b i $8400 ii $32 400 iii $270

12 a

Date	Deposit	Withdrawal	Balance
1 May			$3010
3 May	$490		$3500
5 May		$2300	$1200
17 May	$490		$1690
18 May		$150	$1540
20 May		$50	$1490
25 May		$218	$1272
31 May	$490		$1762

b $4.90 c $1759.55 d Wages
13 a i $73.13 ii $69.72 iii $75.17
b $1700 over 3 years
14 a $403.80 b $393.80 c 24 cents a day

2I

Now you try

Example 25

a $7299.92 b $7200

Exercise 2I

1 $P = 750, r = 7.5, n = 5$
2 $P = 300 \quad r = 3 \quad t = 25$
3 B
4 a i $7146.10 ii $6955.64 iii $6858.57
iv $7260 v $7916.37
b $6000 at 5.7% p.a. for 5 years
5 a i $7080 ii $7080 iii $7428 iv $7200
v $7710
b $6000 at 5.7% p.a. for 5 years
6 a i I $240, $240 II $480, $494.40
III $1200, $1352.90 IV $2400, $3163.39
ii I $240, $243.60 II $480, $502.04
III $1200, $1375.67 IV $2400, $3224.44
iii I $240, $246.71 II $480, $508.64
III $1200, $1395.40 IV $2400, $3277.59
b Compound interest
c Compound interest
7 a

Principal	Rate	Overall time	Interest	Amount
$7000	5%	5 years	$1750	$8750
$7000	10%	5 years	$3500	$10 500
$3300	10%	3 years	$990	$4290
$8000	10%	3 years	$2400	$10 400
$9000	8%	2 years	$1440	$10 440
$18 000	8%	2 years	$2880	$20 880

b i Interest is doubled ii No change
iii Interest is doubled
8

Principal	Rate	Period	Overall time
$7000	4.56%	annually	5 years
$7000	8.45%	annually	5 years
$9000	8%	fortnightly	2 years
$18 000	8%	fortnightly	2 years

Interest	Amount
$1750	$8750
$3500	$10 500
$1559.00	$10 559.00
$3118.01	$21 118.01

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9 a 8.45% **b** 8.19% **c** 8.12%
The more often interest is calculated, the lower the required rate.
10 a i 4.2% **ii** 8.7%
b It increases by more than this factor.

Maths@Work: Finance manager

1 a $108 474 **b** $110 974
c $108 474 **d** August by $2500
e The term deposit **f** 48.32%
g $70 417.33
2 a 92.07% **b** $1200 in August
c It may be billed only every 2 months. **d** $7200
3 a $135 459
b 137% increase of November's total.
The Payroll tax increased by $3300 and a performance review cost $75 000.
c i $4584 **ii** $55 008
d 62.05%
4 a

Starting balance	Scheduled payment	Interest due	Principal paid	Ending balance
$230 000.00	$5780.00	$1725.00	$4055.00	$225 945.00
$225 945.00	$5780.00	$1694.59	$4085.41	$221 859.59
$221 859.59	$5780.00	$1663.95	$4116.05	$217 743.53

b i $205 209.22 **ii** $1442.93
iii $347.37 **iv** $18 641.74; $50 718.26

Puzzles and games

1 commission, fortnightly, overtime, piecework, annual, gross, net, monthly, casual, salary
2 You take away his credit card.
3 7 years 4 months
4 59 games

Short-answer questions

1 $1440
2 a $1084.16 **b** $4557
3 a $11.40 **b** $3.80
4 $4200
5 $576.92
6 a $7400 **b** $616.67 **c** $142.31
7 a $536.06 **b** $80.41 **c** 11.57%
8 a $12 725 **b** 37%
9 a $1411.20 **b** $1080.70
10 a $21 247 **b** $1800
11 $35.55
12 a $1600 **b** $1166.67
13 a $624.32 **b** $1022.53 **c** $635.80
14 a $1050 **b** $12 000 **c** $6050

Multiple-choice questions

1 E **2** D **3** D **4** C **5** B
6 E **7** B **8** B **9** C **10** E

Extended-response questions

1 a $5624.32 **b** $624.32 **c** 4.16% **d** $636.36
2 a

Date	Deposit	Withdrawal	Balance
1st			217.63
7th		64.00	153.63
9th	140.00		293.63
11th		117.34	176.29
15th		0.51	175.78
20th	20.00	12.93	182.85
30th	140.00		322.85

b $153.63 **c** $0.08

Chapter 3

Warm-up quiz

1 a $3x$ **b** $a+1$ **c** $2m-5$ **d** $4(x+y)$
2 a 20 **b** 17 **c** 23 **d** 22
3 a No **b** Yes **c** Yes **d** No
4 a $8m$ **b** $5ab$ **c** $6x+8y$ **d** $8x$
e $15ab$ **f** $3y$
5 a $2x+10$ **b** $3y-6$ **c** $8x-12$ **d** $3x^2+x$
6 a 4 **b** 6 **c** $7a$ **d** $2x$
e x **f** $7xy$
7 a $\frac{31}{40}$ **b** $\frac{11}{21}$ **c** $\frac{2}{15}$ **d** $\frac{3}{2}$
8 a 7^4 **b** m^3 **c** x^2y^3 **d** 3^5a^5
9 a 49 **b** 27 **c** 16 **d** 64
10 a 3^9 **b** 3^2 **c** 3^{10} **d** 3^0
e 3^{-2}
11 a 38 **b** 2310 **c** 0.172 **d** 0.0018
e 1000 **f** 10 000

3A

Now you try

Example 1
a 4
b 1
c i 4 **ii** -2

Example 2
a $y+5$ **b** $3x-7$ **c** $\frac{a+b}{5}$ **d** xy^2

Example 3
a 36 **b** 38 **c** 24

Exercise 3A

1 a expression **b** constant term
c coefficient **d** term
2 a $x+3$ **b** $5y$ **c** $\frac{a}{5}$ **d** $2xy$
3 a 7 **b** 15 **c** 5 **d** 9 **e** 6
4 a i 3 **ii** 8 **iii** 5
b i 4 **ii** 2 **iii** -3
c i 3 **ii** -4 **iii** 1
5 a $x+2$ **b** $y-4$ **c** $ab+y$ **d** $2x-3$
e $5x$ **f** $2m$ **g** $3r$ **h** $\frac{1}{2}x$
i $\frac{3}{4}m$ **j** $\frac{x}{y}$ **k** $\frac{a+b}{4}$ **l** x^2y
6 a 12 **b** 3 **c** 9 **d** 10
e 10 **f** -2 **g** 1 **h** 4
i -6 **j** 10 **k** -2 **l** -9
7 a $5x$ c **b** $35y$ c **c** $\frac{\$500}{n}$
d $\frac{\$11}{m}$ **e** $11+x$
8 a $\$(3.40+2d)$
b i $23.40 **ii** $47.40
9 a i $2x$ **ii** $2x-3$ **iii** $3(2x-3)$
b 21
10 a $x+1$ and $x-1$
b No
c i $(x+1)(x-1)$ **ii** Less by one square metre.
11 a 21 sq. units **b** 1, 2, 3, 6, 9, 18

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12 a i $4x$ ii x^2
b i $2x+2y$ ii xy
c i $x+y+5$ ii $\frac{5x}{2}$
d i $4ab$ ii a^2b^2
e i $2a^2+2b$ ii a^2b
f i $mn+9$ ii $2mn$

3B

Now you try

Example 4

a $4a$ and $2a$
$5ab$ and $2ba$
b $-x^2y$ and $4x^2y$

Example 5

a $10x+2$ **b** $5a+2b$ **c** $3mn+5m^2n$

Example 6

a $20xw$ **b** $-12a^2c$

Example 7

a $5x$ **b** $\frac{3s}{5}$

Exercise 3B

1 a Y **b** Y **c** N **d** Y

2 a $10g$ **b** $5f$ **c** $8e$ **d** 0
e $6x$ **f** $17st$

3 a $6x$ **b** $12a$ **c** $10m$ **d** $-18y$

4 a $\frac{1}{2}$ **b** 4 **c** $\frac{2}{3}$ **d** $\frac{7}{3}$

5 a $3ac$ and $-2ac$ **b** $4pq$ and $3qp$
c $7x^2y$ and $4yx^2, -3xy^2$ and $2xy^2$ **d** $2r^2$ and $-r^2$
e $-2ab$ and $4ba$ **f** $3p^2q$ and $4qp^2$
g $\frac{1}{3}lm$ and $\frac{lm}{4}$ **h** x^2y and yx^2, $-xy$ and yx

6 a $7t+10$ **b** $4g+1$ **c** $7x-5$
d $m+2$ **e** $3x+3y$ **f** $2x+6y$
g $5a-2b$ **h** $-3m-2n$ **i** $5de+7de^2$
j $3kl-10k^2l$ **k** $7x^2y+xy^2$ **l** $3fg-fg^2$

7 a $6rs$ **b** $6hu$ **c** $16wh$ **d** $6r^2s$
e $-8es$ **f** $-10hv$ **g** $12cm^2$ **h** $35fl$
i $8x^2y$ **j** $24a^2b$ **k** $3xy^2$ **l** $-16a^2b$
m $-12m^2n^2$ **n** $20x^2y^2$ **o** $20a^2b^2$

8 a $3a$ **b** $\frac{x}{2}$ **c** $\frac{a}{3}$ **d** $\frac{ab}{4}$
e $2b$ **f** $3x$ **g** $\frac{y}{2}$ **h** $\frac{4a}{5}$
i $\frac{2x}{5}$ **j** $\frac{2xy}{3}$ **k** $3ab$ **l** $\frac{n}{3}$

9 a $8x$ **b** $3x^2$

10 a $5x$ **b** $8y$ **c** $4a$ **d** $-6x$
e $2x$ **f** $10a^2b$

11 a $P=4x+6, A=6x$ **b** $P=4y+16x, A=16xy$
c $P=20a, A=25a^2$

12 $3x$

13 a $6x$ **b** $8a+7$ **c** $4b$ **d** $11x$
e $3x^2$ **f** $15xy$ **g** $8x^2$ **h** $-29x^2$

3C

Now you try

Example 8

a $3x+12$ **b** $15x-10$ **c** $8a^2+20ab$

Example 9

a $-4x+20$ **b** $-6xy+12y^2$

Example 10

a $8a-1$ **b** $y+5$

Exercise 3C

1 brackets, outside

2 a $3(x+4)=3\times x+3\times 4$
$=3x+12$
b $2(x-5)=2\times x+2\times(-5)$
$=2x-10$
c $2(4x+3)=2\times 4x+2\times 3$
$=8x+6$
d $x(x-3)=x\times x+x\times(-3)$
$=x^2-3x$

3 a $2x+8$ **b** $3x+21$ **c** $4y-12$
d $5y-10$ **e** $6x+4$ **f** $8x+20$
g $9a-12$ **h** $14y-35$ **i** $10a+5b$
j $12a-9b$ **k** $2x^2+10x$ **l** $3x^2-12x$
m $6a^2+4ab$ **n** $6xy-8y^2$ **o** $6ab-15b^2$

4 a $-2x-6$ **b** $-5m-10$ **c** $-3w-12$
d $-4x+12$ **e** $-2m+14$ **f** $-7w+35$
g $-x-y$ **h** $-x+y$ **i** $-6x^2-8x$
j $-6x^2-15x$ **k** $-8x^2+8x$ **l** $-6y^2+27y$
m $-6x^2+10xy$ **n** $-9x^2-6xy$ **o** $-12xy-18y^2$

5 a $5x+17$ **b** $7x+17$ **c** $2x-1$ **d** $1-2x$
e $1-5x$ **f** $6x+1$ **g** $6x+11$ **h** $14-4x$
i $27-6x$ **j** $7x+18$ **k** $7p-11$ **l** $10x-4$
m $4s+5$ **n** 4 **o** $2x-7$

6 a 2 **b** 4 **c** $3x$ **d** $3x$
e $y, 1$ **f** $2x, 3y$

7 a $2x-10$ **b** x^2+3x
c $2x^2+8x$ **d** $6x^2-3x$

8 $2x^2+4x$

9 a $x-18\,200$ **b** $0.1x-1820$

10 a $x^2+7x+12$ **b** x^2+4x+3 **c** $x^2+7x+10$
d x^2-2x-8 **e** $x^2+3x-10$ **f** $2x^2+11x+12$
g $2x^2-x-6$ **h** x^2+x-12 **i** $4x^2+18x-10$

3D

Now you try

Example 11

a 5 **b** $7x$ **c** $3y$

Example 12

a $3(x+5)$ **b** $6(2m-3n)$

Example 13

a $3a(3+8b)$ **b** $5x(3x-7)$

Example 14

a $-4y(2y+9)$

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Exercise 3D

1 a 2 b 9 c 7 d 12

2 a T b F

3 a C b They have no common factor.

4 a 6 b 5 c 4 d 9
e $5a$ f $2m$ g $7x$ h $8a$
i $3a$ j $2x$ k $8y$ l $5xy$

5 a $3(x+3)$ b $4(x-2)$ c $10(y-2)$
d $6(a+5)$ e $5(x+y)$ f $4(3a+b)$
g $9(2m-3n)$ h $12(3x-4y)$ i $4(2x+11y)$
j $6(4a-3b)$ k $11(11m+5n)$ l $7(2k-9l)$

6 a $7x(2+3y)$ b $3b(2a-5)$ c $8y(4-5x)$
d $5x(x-1)$ e $x(x+7)$ f $2a(a+4)$
g $6a(2a+7b)$ h $9y(y-7)$ i $2x(3x+7)$
j $3x(3x-2)$ k $8y(2y+5)$ l $10m(1-4m)$

7 a $-2(x+3)$ b $-4(a+2)$ c $-3(x+2y)$
d $-7a(1+2b)$ e $-x(1+10y)$ f $-3b(1+4a)$
g $-x(x+7)$ h $-4x(x+3)$ i $-2y(y+5)$
j $-2x(4x+7)$ k $-4x(3x+2)$ l $-5a(3a+1)$

8 a $ab(7a+1)$ b $4a^2(b+5)$ c $xy(1-y)$
d $x^2y(1+4y)$ e $6mn(1+3n)$ f $5xy(x+2y)$
g $-y(y+8z)$ h $-3ab(a+2)$ i $-ab(b+a)$

9 a $4(x+5)$ b $8(x+2)$ c $2(3x+4)$

10 $(x+3)$ metres

11 a $2(x+2y+3z)$ b $3(x^2+4x+2)$
c $4(x^2+2xy+3)$ d $3x(2x+y-3)$
e $5x(2y-z+1)$ f $2y(2y-9+7x)$

12 a $(x+2)(4+x)$ b $(x+3)(x+2)$
c $(x+4)(x-7)$ d $(2x+1)(x-3)$
e $(y-3)(2x+4)$ f $(x-1)(2x-3)$

3E

Now you try

Example 15

a $\frac{9a}{4}$ b $\frac{1}{3}$

Example 16

a $2x-1$ b 3

Example 17

a $\frac{8}{5}$ b $\frac{16}{3}$

Example 18

a $\frac{x}{4}$ b 9

Exercise 3E

1 a $\frac{2}{3}$ b $\frac{3}{4}$ c $\frac{2x}{5}$ d $\frac{2x}{5}$

2 a $\frac{2}{3}$ b $\frac{3}{5x}$ c $\frac{1}{7}$ d $\frac{4}{x+3}$

3 a $\frac{2}{5}$ b $\frac{1}{12}$

4 a $\frac{y}{2}$ b $\frac{2a}{5}$ c $\frac{x}{5}$ d $5x$
e $x+1$ f $x-5$ g $\frac{x+1}{2}$ h 5
i 4 j $\frac{1}{2}$ k 3 l $\frac{3}{2}$

5 a $x+2$ b $a-5$ c $2y-3$ d $2b-3$
e 3 f 4 g 3 h 4
i $x+2$ j $x-5$ k $x+3$ l x
m x n $2x$ o $3x$

6 a $\frac{2}{3}$ b $\frac{3}{2}$ c $\frac{3}{4}$ d $\frac{5x}{3}$
e $\frac{3y}{2}$ f $\frac{5}{6}$ g $\frac{4}{9}$ h 10
i $\frac{15}{2}$ j $\frac{8}{5}$ k $\frac{4}{3x}$ l 12

7 a 3 b 6 c $8a$ d $\frac{x}{4}$
e $\frac{8}{15a}$ f $\frac{4}{x}$ g 3 h $\frac{5}{2}$
i $\frac{3}{4}$ j $\frac{1}{25}$ k 10 l $\frac{5}{4}$

8 a Must factorise first, $x+2$.
b Factorise first, $x+2$.
c Need to multiply by the reciprocal of the fraction after division sign, $\frac{6}{25}$.
d x is not a common factor, cannot cancel, $\frac{x+4}{5x}$.

9 a $\frac{1+2a}{3}$ b $\frac{4}{5}$ c $\frac{x}{4}$
d $\frac{2}{5}$ e $\frac{7}{18}$ f $\frac{2x}{3}$
g $\frac{5}{4}$ h $\frac{x}{9}$ i $\frac{x+5}{9}$

10 a -3 b -2 c $-x$
d $\frac{x+2}{3}$ e $x-6$ f $2x-3$

11 a $\frac{x+1}{2}$ b $\frac{2(x+1)}{3}$ c $\frac{x-2}{2}$
d $\frac{x+2}{8}$ e $\frac{x-3}{12}$ f $\frac{5(2x+1)}{8}$

3F

Now you try

Example 19

a $\frac{9x}{20}$ b $\frac{19x}{14}$ c $\frac{3x+7}{9}$

Example 20

a $\frac{x-9}{30}$ b $\frac{9x-2}{20}$

Exercise 3F

1 a 20 b 12 c 30

2 a $3x$ b $6x$ c 5

3 a $\frac{x}{4}+\frac{x}{5}=\frac{5x}{20}+\frac{4x}{20}$
$=\frac{9x}{20}$
b $\frac{2x}{5}-\frac{x}{10}=\frac{4x}{10}-\frac{x}{10}$
$=\frac{3x}{10}$

4 a $\frac{7x}{12}$ b $\frac{7x}{10}$ c $\frac{2x}{9}$ d $\frac{2x}{35}$
e $\frac{13x}{15}$ f $\frac{7x}{6}$ g $\frac{7x}{18}$ h $\frac{13x}{40}$
i $\frac{-5x}{14}$ j $\frac{-3x}{10}$ k $\frac{-x}{30}$ l $\frac{-6x}{5}$

5 a $\frac{2x+3}{4}$ b $\frac{3x+10}{15}$ c $\frac{8x+21}{60}$
d $\frac{5x-8}{20}$ e $\frac{6x-5}{9}$ f $\frac{10-3x}{12}$

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6 a $\frac{5x+4}{6}$ b $\frac{13x+12}{15}$ c $\frac{5x-4}{8}$
d $\frac{x+8}{6}$ e $\frac{x+10}{10}$ f $\frac{3x+14}{24}$
g $\frac{9x+22}{20}$ h $\frac{11x+13}{14}$ i $\frac{5x+7}{12}$
j $\frac{11x+14}{12}$ k $\frac{3x-2}{10}$ l $\frac{14x-13}{24}$

7 a Need to multiply numerators also when getting common denominator, $\frac{17x}{12}$.
b Need common denominator before subtracting, $\frac{x}{10}$.
c $3(x+2) = 3x+6$ and $5(x+4) = 5x+20$, $\frac{8x+26}{15}$
d $-$ sign changed to $+$, $\frac{4x+9}{6}$

8 a $\frac{x+16}{30}$ b $\frac{2x+22}{15}$ c $\frac{x-23}{20}$ d $\frac{x+9}{4}$

9 a $\frac{12+2x}{3x}$ b $\frac{3x+8}{4x}$ c $\frac{2x+15}{5x}$
d $\frac{3x-14}{7x}$ e $\frac{x-20}{5x}$ f $\frac{24-5x}{8x}$

10 a $\frac{3x+10}{4x}$ b $\frac{x+15}{6x}$ c $\frac{6x-5}{20x}$
d $\frac{3x+5}{x^2}$ e $\frac{4x+1}{x^2}$ f $\frac{3-5x}{x^2}$
g $\frac{3x+4}{2x^2}$ h $\frac{12x+7}{3x^2}$

Progress quiz

1 a 4
b 5
c i -3 ii $\frac{1}{2}$

2 a 6 b 9 c 14
3 a $6x-3$ b $5x+5y$ c $2x^2y+7xy^2$
4 a $12r^2s$ b $\frac{x}{3}$ c $\frac{5n}{2}$
5 a $6x+9$ b $20x^2-8x$ c $-12x+18$
6 a $8x-6$ b $3x+32$
7 a $6(m+2)$ b $5a(3-4b)$ c $x(4y+1)$
d $2x(3x-5)$ e $-4(2x+5)$ f $-3y(y+2)$
8 a 2 b $3x-7$ c $2x$
9 a $\frac{3}{2}$ or $1\frac{1}{2}$ b $\frac{3x}{16}$
10 a $\frac{17x}{20}$ b $\frac{6x-4}{9}$
11 $\frac{5x-1}{12}$

3G

Now you try

Example 21
a 3^4 b 7^2y^3 c m^3n^2

Example 22
a x^8 b x^5y^5 c $10a^4b^6$

Example 23
a b^5 b $\frac{5a^4}{2}$ c $\frac{2a^2b}{3}$

Example 24
$\frac{2a^3b}{3}$

Exercise 3G

1 a base, index or power b power c index
d multiply e expanded
2 a $8\times8\times8$ b $7\times7\times7\times7\times7$
c $x\times x\times x\times x\times x\times x$ d $ab\times ab\times ab\times ab$
3 a $7\times7\times7\times7$, 7^7 b $5\times5\times5\times5$, 5^4
4 a add b subtract
5 a 9^4 b 3^6 c 15^3 d 5^2x^3
e 4^3a^4 f $7b^4$ g x^3y^2 h a^2b^4
i $3^3x^2y^3$ j $4^2x^2z^2$
6 a x^7 b p^7 c t^9 d d^5
e g^4 f f^3 g $2p^5$ h $3c^8$
i $6s^{11}$ j a^5b^8 k d^9f^5 l v^5z^8
m $15a^3b^6$ n $6x^3y^3$ o $18e^9r^3$ p $-8p^4c^3$
q $-10r^7s^8$ r $6d^6f^4$
7 a a^2 b d c r^2 d c^4
e l f b^3 g $4d^2$ h $\frac{1}{2}$
i $3n^3$ j $2p^2$ k $\frac{3m^4}{2}$ l $\frac{d^2}{3}$
m $4t^3r$ n $\frac{5h^4d}{3}$ o $2q^2$ p $\frac{xy^2}{2}$
q $\frac{r^2s}{3}$ r $\frac{2cd^5}{5}$ s $\frac{a^3y}{2}$ t $\frac{n}{2}$
u $-6x^2y^2$
8 a xy^2 b m c r^2s^4 d $2a^2b^2$
e $\frac{9x^8y^4}{2}$ f $4w^4$
9 Stuart hasn't put brackets around -2; i.e. $(-2)^4$.
10 a i 9 ii -9
b i is $-3\times(-3)$ and ii is $-(3\times3)$
c i -8 ii -8
d They are both -8 since $-2\times(-2)\times(-2) = -8$.
11 a 13 b 18 c 81 d 27
e 64 f 16

3H

Now you try

Example 25
a 0 b 4 c 4

Example 26
a y^{18} b $5m^8$

Example 27
a $27m^3$ b $a^{12}b^8$ c $\frac{y^4}{81}$

Example 28
a $2a$ b $\frac{27x^3}{y^6}$ c $25m^8+1$

Exercise 3H

1 a 1 b a^{mn} c $a^m\times b^n$ d $\frac{a^m}{b^n}$

2 a
$$(2a)^3 = 2a\times2a\times2a = 2\times2\times2\times a\times a\times a = 2^3a^3$$
b
$$\left(\frac{4}{7}\right)^4 = \frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7} = \frac{4\times4\times4\times4}{7\times7\times7\times7} = \frac{4^4}{7^4}$$

Answers

3G

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3 **a** 1 **b** 1 **c** 1 **d** 1 **e** 3 **f** 4
g 2 **h** 0 **i** 1 **j** 3 **k** 2 **l** 9
4 **a** b^{12} **b** f^{20} **c** k^{21} **d** $3x^6$
e $5c^{18}$ **f** $4s^{18}$
5 **a** $9x^2$ **b** $64m^3$ **c** $125y^3$ **d** $16x^{12}$
e $x^{10}y^5$ **f** $27a^9$ **g** $x^{24}y^{12}$ **h** a^6b^3
i $m^{12}n^{12}$ **j** $\frac{x^2}{25}$ **k** $\frac{y^4}{81}$ **l** $\frac{m^4}{16}$
m $\frac{x^6}{y^3}$ **n** $\frac{x^{12}}{y^8}$ **o** $\frac{x^3}{y^{15}}$
6 **a** m^6 **b** $\frac{y^5}{10}$ **c** $\frac{4b^4}{3}$ **d** $\frac{27c^4}{2}$
e $\frac{25r^4}{3}$ **f** $\frac{8p^5}{3}$ **g** $\frac{16s^8}{t^{12}}$ **h** $\frac{r^8}{625s^{12}}$
7 **a** 1 **b** 1 **c** 9 **d** 4
8 **a** $2p^3q^7$ **b** $108a^9b^6$ **c** $48r^9y^{12}$ **d** $2m^9n^4$
e $21s^3y^2$ **f** $\frac{3c^4d^{10}}{2}$ **g** $2r^6$ **h** y
9 **a** $a^{11}b^{10}$ **b** $2x^5y^{13}$ **c** $18p^{12}q^2$
d $4a^2b^3$ **e** $\frac{324r^5}{s}$ **f** $\frac{2x^3y^8}{s^2}$

3I

Now you try

Example 29

a $\frac{1}{b^4}$ **b** $\frac{3}{a^3}$ **c** $\frac{4y^4}{x^2}$

Example 30

a y^2 **b** $3a^4$ **c** $\frac{7x^2}{y^3}$

Example 31

a $\frac{y}{x^2}$ **b** $\frac{9}{xy^4}$

Exercise 3I

1 **a** $\frac{1}{a^m}$ **b** a^m
2 **a** 3 **b** m^2, m^2 **c** 4
3 **a** $\frac{1}{y^3}$ **b** $\frac{1}{x^4}$ **c** $\frac{1}{x^2}$ **d** $\frac{1}{a^5}$
e $\frac{3}{x^2}$ **f** $\frac{3}{b^3}$ **g** $\frac{4}{x}$ **h** $\frac{2}{m^9}$
i $\frac{2x^2}{y^3}$ **j** $\frac{3x}{y^4}$ **k** $\frac{3b^4}{a^2}$ **l** $\frac{5n^2}{m^3}$
4 **a** b^4 **b** x^7 **c** y
d $5m^3$ **e** $2y^2$ **f** $3x^4$
g $5a^2b^3$ **h** $\frac{4y^5}{x^2}$ **i** $\frac{10a^2}{b^4}$
5 **a** $\frac{4}{x^2y^3}$ **b** $\frac{1}{5a^2b^3}$ **c** $2a^3b^2$
d $\frac{a^4b^5}{3}$ **e** $\frac{x^3}{y^2}$ **f** $\frac{x^3}{y^4}$
6 **a** $\frac{b^4}{a^3}$ **b** $\frac{x^4}{y^3}$ **c** $\frac{1}{x^2y^4}$ **d** $\frac{b}{a^4}$
7 **a** $\frac{1}{x^8}$ **b** $\frac{1}{x^6}$ **c** 1
d $\frac{8}{y^6}$ **e** $\frac{a^2}{y^6}$ **f** $\frac{x^6}{16}$
g $\frac{3x}{4y^6}$ **h** $\frac{a^3}{b^{10}}$ **i** $\frac{n^5}{m^8}$

8 1.37 g
9 **a** $\frac{1}{4}$ **b** $\frac{1}{125}$ **c** 36 **d** 40
e $-\frac{3}{4}$ **f** $\frac{1}{36}$ **g** 64 **h** $\frac{1}{2}$
10 **a** **i** $\frac{3}{5}$ **ii** 4 **iii** $\frac{2}{x}$ **iv** $\frac{b}{a}$
b It results in the reciprocal of the fraction.
c **i** $\frac{4}{9}$ **ii** $\frac{25}{9}$ **iii** 27 **iv** $\frac{81}{16}$

3J

Now you try

Example 32

a 32 700 **b** 0.0012

Example 33

a 3.2×10^5 **b** 2×10^{-4}

Example 34

a 5.37×10^4 **b** 3.63×10^{-4}

Exercise 3J

1 **a** Y **b** N **c** N **d** Y
2 **a** Positive **b** Negative **c** Positive **d** Negative
3 **a** 4.87 **b** 4.872 **c** 4.9
4 **a** 3 120 **b** 54 293 **c** 710 500
d 8 213 000 **e** 59 500 **f** 800 200
g 10 120 **h** 9 990 000 **i** 210 500 000
j 0.0045 **k** 0.0272 **l** 0.000 308 5
m 0.007 83 **n** 0.000 092 **o** 0.265
p 0.000 100 2 **q** 0.000 006 235 **r** 0.98
5 **a** 4.3×10^4 **b** 7.12×10^5 **c** 9.012×10^5
d 1.001×10^4 **e** 2.39×10^4 **f** 7.03×10^8
g 7.8×10^{-4} **h** 1.01×10^{-3} **i** 3×10^{-5}
j 3.004×10^{-2} **k** 1.12×10^{-1} **l** 1.92×10^{-3}
6 **a** 6.24×10^3 **b** 5.73×10^5 **c** 3.02×10^4
d 4.24×10^5 **e** 1.01×10^4 **f** 3.50×10^7
g 7.25×10^4 **h** 3.56×10^5 **i** 1.10×10^8
j 2.42×10^{-3} **k** 1.88×10^{-2} **l** 1.25×10^{-4}
m 7.87×10^{-3} **n** 7.08×10^{-4} **o** 1.14×10^{-1}
p 6.40×10^{-6} **q** 7.89×10^{-5} **r** 1.30×10^{-4}
7 **a** 7.7×10^6 km^2 **b** 2.5×10^6 **c** 7.4×10^9 km
d 1×10^{-2} cm **e** 1.675×10^{-27} kg **f** 9.5×10^{-13} g
8 The numeral part has to be between 1 and 10, i.e. 3.8×10^8.
9 **a** 2.85×10^{-3} **b** 1.55×10^{-3} **c** 4.41×10^{-8}
d 6.38×10^{-3} **e** 8.00×10^7 **f** 3.63×10^8
g 1.80×10^{-3} **h** 3.42×10^{15}
10 328 minutes
11 **a** **i** 9×10^{17} J **ii** 2.34×10^{21} J
iii 2.7×10^{15} J **iv** 9×10^{11} J
b **i** 1.11×10^8 kg **ii** 4.22×10^{-1} kg
iii 9.69×10^{-13} kg **iv** 1.89×10^{-19} kg
c 5.4×10^{41} J

3K

Now you try

Example 35

a $P = 3000\,(1.02)^n$ **b** $V = 36\,000\,(0.94)^n$

Example 36

a $V = 20\,000\,(1.04)^n$
b **i** \$20 800 **ii** \$24 333.06
c 5.7 years

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Exercise 3K

1 a $1050 b $\frac{5}{100}$, 1.05 c $1215.51

2 a 4.9 kg b $\frac{2}{100}$, 0.98 c 4.52 kg

3 a Growth b Growth c Decay d Decay
e Growth f Decay

4 a A = amount of money at any time,
n = number of years of investment
$A = \$200\,000 \times 1.17^n$
b A = house value at any time,
n = number of years since initial valuation
$A = \$530\,000 \times 0.95^n$
c A = car value at any time,
n = number of years since purchase
$A = \$14\,200 \times 0.97^n$
d A = population at any time,
n = number of years since initial census
$A = 172\,500 \times 1.15^n$
e A = litres in tank at any time,
n = number of hours elapsed
$A = 1200 \times 0.9^n$
f A = cell size at any time,
n = number of minutes elapsed
$A = 0.01\text{ cm}^2 \times 2^n$
g A = size of oil spill at any time,
n = number of minutes elapsed
$A = 2\text{ m}^2 \times 1.05^n$
h A = mass of substance at any time,
n = number of hours elapsed
$A = 30\text{ g} \times 0.92^n$

5 a $A = 500\,000 \times 1.1^n$
b i $665 500 ii $1 296 871.23 iii $3 363 749.98
c After 7.3 years

6 a $A = 300\,000 \times 0.85^n$
b i $216 750 ii $96 173.13 iii $42 672.53
c 3.1 years

7 a $V = 15\,000 \times 0.94^n$
b i 12 459 L ii 9727 L
c 769.53 L
d 55.0 hours

8 a 3000
b i 3000 ii 20 280 iii 243 220
c 7 hours 46 minutes

9 a $D = 10 \times 0.875^t$, where t = number of 10 000 kms travelled
b Yes
c 90 000

10 a $T = 90°C \times 0.92^t$
b i 76.2°C ii 79.4°C
c 3 minutes 13 seconds

11 a i $1610.51 ii $2143.59 iii $4177.25
b i $1645.31 ii $2218.18 iii $4453.92

12 a $2805.10 b $2835.25 c $2837.47

Maths@Work: Electrical trades

1 a i $1\text{ C} = 6.24 \times 10^{18}$ e ii $2\text{ C} = 1.25 \times 10^{19}$ e
iii $3\text{ C} = 1.87 \times 10^{19}$ e iv $250\text{ C} = 1.56 \times 10^{21}$ e
v ½ C $= 3.12 \times 10^{18}$ e vi $12\text{ C} = 7.49 \times 10^{19}$ e
b $1\text{ e} = 1.6 \times 10^{-19}$ C
c i $1.24830195 \times 10^{19}$ e/s ii $6.24150975 \times 10^{19}$ e/s
iii $1.24830195 \times 10^{20}$ e/s iv $3.12075488 \times 10^{18}$ e/s
v $3.12075488 \times 10^{19}$ e/s

2 a 1.32×10^{-5} b 3.168×10^{-4} mm
c Any construction using steel needs to allow for the expansion of steel with a temperature increase. For example, gaps need to be left in a bridge to allow for steel expansion and, hence stop, the road buckling.

3

Conversion table between coulomb and electron charges			
Charge (coulombs, C)	Charge (number of electrons, e)	Charge (number of electrons, e)	Charge (coulombs, C)
1	6.24150975E+18	1	1.60217646E-19
10	6.24150975E+19	10	1.60217646E-18
100	6.24150975E+20	100	1.60217646E-17
1000	6.24150975E+21	1000	1.60217646E-16
10000	6.24150975E+22	10000	1.60217646E-15
100000	6.24150975E+23	100000	1.60217646E-14
1000000	6.24150975E+24	1000000	1.60217646E-13

i $6.24150975 \times 10^{20}$ e ii $1.60217646 \times 10^{-13}$ C
iii $6.23526824 \times 10^{24}$ e

4

Conversion table between amperes, time and electrical charge measured in coulombs and numbers of electrons			
Electrical current (in amps)	Time (in seconds)	Charge *Q* (in coulombs C)	Charge *Q* number of electrons, e)
0.5	1	5.000E-01	3.12075488E+18
	30	1.500E+01	9.36226463E+19
	60	3.000E+01	1.87245293E+20
	90	4.500E+01	2.80867939E+20
	120	6.000E+01	3.74490585E+20
	150	7.500E+01	4.68113231E+20
	180	9.000E+01	5.61735878E+20

a i 4.5×10^1 C ii $2.80867939 \times 10^{20}$ e
b i 2.25×10^2 C ii $1.40433969 \times 10^{21}$ e

Puzzles and games

1 Magic square sum $= 3x + 2y$

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$4y$	$x + y$	$2x - 3y$
$x - 2y$	$2x + 2y$	$2y$

2 3^{3n-3}
3 1c and then double each day
4 5
5 2^{24}
6 200
7 $\frac{5 - 2x}{30}$
8 n^2, 225

Short-answer questions

1 a 4 b 5 c i 4 ii −3

2 a $y + 3$ b $xy - 5$ c $\frac{a + b}{4}$

3 a 14 b −30 c 35

4 a $7x - 5$ b $13a - 2b$ c $xy - 3xy^2$
d $12mn$ e $-14x^2y$ f $\frac{2b}{3}$

5 a $10x + 20$ b $-6x + 8y$ c $6x^2 + 15xy$
d $4a + 15$ e $5y + 13$ f $8t + 11$

6 a $8(2x - 5)$ b $5xy(2x + 7y)$
c $2x(2x - 5)$ d $-2x(y + 9)$

7 a $\frac{14x}{15}$ b $\frac{6 - 7a}{14}$ c $\frac{9x + 8}{20}$

8 a $\frac{3}{8}$ b $4x$ c $3x - 1$ d $\frac{2}{3}$

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9 a $12x^7$ b $8x^4y^8$ c b^4 d $\frac{2a^2b^3}{3}$
10 a b^8 b $8m^6$ c $\frac{x^2}{49}$ d $\frac{64y^6}{z^{12}}$
11 a 1 b 4 c 6 d 1
12 a $\frac{4}{x^3}$ b $\frac{3r^4}{s^2}$ c $\frac{2y^4}{3x^3}$ d $4m^5$
13 a $\frac{5y^6}{4}$ b $\frac{25x^3y^2}{2}$ c $\frac{10}{x^3}$ d $\frac{x^8}{2y^3}$
14 a 4250 b 37 000 000
c 0.021 d 0.0000725
15 a 1.24×10^5 b 3.95×10^7
c 9.02×10^{-6} d 4.60×10^{-4}
16 a $P = 20(1.1)^n$ b $A = 100\,000\,(0.85)^n$

Multiple-choice questions

1 C 2 B 3 C 4 D 5 B
6 D 7 C 8 D 9 A 10 E
11 C 12 D

Extended-response questions

1 a $2(5x + 1)$ m b 32 m c $(5x^2 + 3x)$ m^2
d $1080
2 a $A = 2\,(1.09)^t$
b i 2.3762 m^2 ii 3.0772 m^2
c 37.4 weeks

Chapter 4

Warm-up quiz

1 a 11
b i $\frac{1}{11}$ ii $\frac{2}{11}$ iii $\frac{4}{11}$
iv $\frac{7}{11}$ v $\frac{3}{11}$ vi $\frac{8}{11}$
2 a $\frac{1}{8}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{4}$
e $\frac{5}{8}$ f $\frac{7}{8}$ g $\frac{1}{4}$
3 0, 1 in 5, 39%, 0.4, $\frac{1}{2}$, 0.62, 71%, $\frac{3}{4}$, $\frac{9}{10}$, 1
4 a i 14 ii 25 iii 11
b i $\frac{18}{25}$ ii $\frac{7}{25}$ iii $\frac{7}{25}$
5 a $\frac{7}{16}$ b $\frac{9}{16}$
6 a 16
b i $\frac{1}{16}$ ii $\frac{3}{16}$ iii $\frac{3}{8}$ iv $\frac{5}{8}$
v $\frac{13}{16}$ vi $\frac{3}{16}$
7 a 4
b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{2}$ iv $\frac{3}{4}$ v $\frac{1}{2}$ vi 1

4A

Now you try

Example 1

a 6
b i $\frac{1}{6}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{1}{2}$

Example 2

a 34
b 51
c i 0.15 ii 0.09 iii 0.49 iv 0.85

Exercise 4A

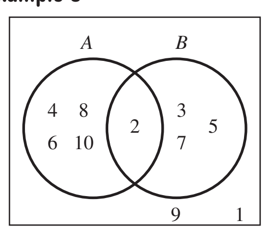
1 a sample space b 0, 1 c likely d outcome
2 C, A, B, D
3 a $\frac{1}{4}$ b $\frac{3}{8}$ c 0
4 a 7
b i $\frac{1}{7}$ ii $\frac{2}{7}$ iii $\frac{5}{7}$ iv $\frac{3}{7}$
5 a $\frac{3}{10}$ b $\frac{2}{5}$ c $\frac{3}{5}$ d $\frac{1}{2}$
6 a 43
b 47
c i 0.09 ii 0.43 iii 0.47 iv 0.91
7 a 0.62 b 0.03 c 0.97 d 0.38
8 a $\frac{1}{10}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{2}$
e $\frac{2}{5}$ f $\frac{1}{5}$ g $\frac{3}{10}$
9 a $\frac{1}{50}$ b $\frac{3}{10}$ c $\frac{49}{50}$
10 a $\frac{6}{25}$ b $\frac{1}{50}$ c $\frac{21}{25}$ d $\frac{2}{5}$
e $\frac{2}{25}$ f $\frac{4}{25}$
11 a 59 b 4, as $\frac{41}{100}$ of 10 is closest to 4.
c 8, as $\frac{41}{100}$ of 20 is closest to 8.
12 a $\frac{1}{4}$ b $\frac{1}{13}$ c $\frac{1}{52}$ d $\frac{1}{2}$
e $\frac{2}{13}$ f $\frac{4}{13}$ g $\frac{12}{13}$ h $\frac{9}{13}$

4B

Now you try

Example 3

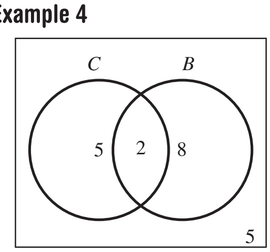
a



b i {2} ii {2, 3, 4, 5, 6, 7, 8, 10}
c i $\frac{1}{2}$ ii $\frac{1}{10}$ iii $\frac{4}{5}$
d They are not mutually exclusive as there is a number in A ∩ B.

Example 4

a



b i 5 ii 5
c i $\frac{7}{20}$ ii $\frac{1}{4}$ iii $\frac{1}{10}$

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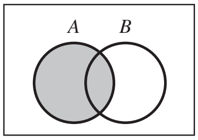
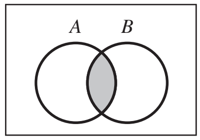
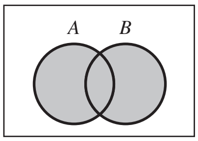
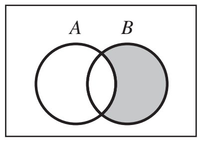
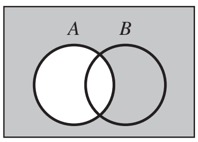
Exercise 4B

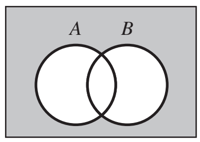
1 a D b C c B d A
2 a No b Yes c No
3 a


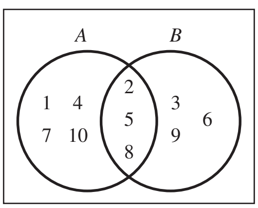
b


c


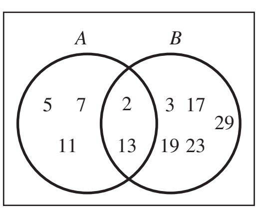
d


e


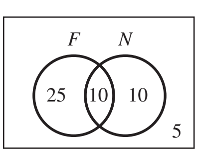
f


4 a


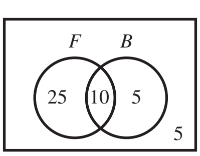
b i $A \cap B = \{2, 5, 8\}$
ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
c i $\frac{7}{10}$ ii $\frac{3}{10}$ iii 1
d No, there is at least one number in $A \cap B$.

5 a


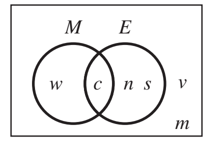
b i $A \cap B = \{2,\ 13\}$
ii $A \cup B = \{2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29\}$
c i $\frac{1}{2}$ ii $\frac{7}{10}$ iii $\frac{1}{5}$ iv 1

6 a


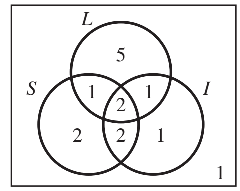
b i 25 ii 5
c i $\frac{2}{5}$ ii $\frac{1}{5}$ iii $\frac{1}{5}$

7 a


b i 25 ii 5
c i $\frac{7}{9}$ ii $\frac{2}{9}$ iii $\frac{8}{9}$ iv $\frac{2}{9}$ v $\frac{1}{9}$
8 3
9 5

10 a


b i $\frac{1}{3}$ ii $\frac{2}{3}$ iii $\frac{1}{6}$ iv $\frac{2}{3}$ v $\frac{1}{3}$

11 a


b 1
c i $\frac{3}{5}$ ii $\frac{1}{3}$ iii $\frac{13}{15}$ iv $\frac{1}{15}$

4C

Now you try

Example 5

a

	A	A′	
B	3	2	5
B′	6	4	10
	9	6	15

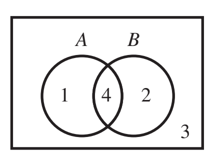
b i 3 ii 2 iii 6 iv 4
v 9 vi 10 vii 11
c i $\frac{1}{5}$ ii $\frac{2}{5}$ iii $\frac{2}{5}$

Exercise 4C

1 a B b A c D d C
2 a i 4 ii 6 iii 3 iv 1
v 10 vi 7 vii 4 viii 7
b 13
3 a

	A	A′	
B	2	6	8
B′	5	3	8
	7	9	16

b i 2 ii 6 iii 5 iv 3
v 7 vi 8 vii 13
c i $\frac{1}{8}$ ii $\frac{9}{16}$ iii $\frac{5}{16}$

4 a


b

	A	A′	
B	4	2	6
B′	1	3	4
	5	5	10

c i 2 ii 5 iii 4 iv 7
d i $\frac{3}{5}$ ii $\frac{2}{5}$ iii $\frac{1}{10}$ iv $\frac{2}{5}$ v $\frac{7}{10}$

5 a

	A	A′	
H	4	3	7
H′	4	1	5
	8	4	12

b i 3 ii 1
c i $\frac{11}{12}$ ii $\frac{1}{3}$

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6 a

	A	A'	
B	3	3	6
B'	4	1	5
	7	4	11

b

	A	A'	
B	2	7	9
B'	2	1	3
	4	8	12

7 a $\frac{1}{8}$ b $\frac{5}{24}$

8 a 0

b

	A	A'	
B	0	6	6
B'	10	2	12
	10	8	18

9 a $\frac{3}{8}$ b $\frac{5}{32}$

10 a $\frac{4}{13}$ b $\frac{4}{13}$ c $\frac{7}{13}$ d $\frac{7}{13}$

11 a 18 b 75

4D

Now you try

Example 6

a $\frac{8}{13}$ b $\frac{3}{13}$ c $\frac{3}{4}$ d $\frac{3}{8}$

Example 7

a

	A	A'	
B	5	5	10
B'	7	3	10
	12	8	20

b $\frac{1}{4}$ c $\frac{5}{12}$ d $\frac{1}{2}$

Exercise 4D

1 a Pr(A|B) b $A \cap B$, B

2 a $\frac{1}{2}$ b $\frac{1}{3}$

3 a $\frac{7}{10}$ b $\frac{7}{12}$

4 a i $\frac{9}{13}$ ii $\frac{3}{13}$ iii $\frac{3}{7}$ iv $\frac{1}{3}$

b i $\frac{14}{17}$ ii $\frac{4}{17}$ iii $\frac{4}{7}$ iv $\frac{2}{7}$

c i $\frac{3}{4}$ ii $\frac{5}{8}$ iii $\frac{5}{7}$ iv $\frac{5}{6}$

d i $\frac{7}{16}$ ii $\frac{1}{8}$ iii $\frac{1}{4}$ iv $\frac{2}{7}$

5 a

	A	A'	
B	9	6	15
B'	4	1	5
	13	7	20

b $\frac{1}{5}$ c $\frac{3}{5}$ d $\frac{9}{13}$

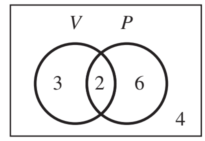
6 a i $\frac{7}{18}$ ii $\frac{1}{9}$ iii $\frac{1}{5}$ iv $\frac{2}{7}$

b i $\frac{4}{9}$ ii $\frac{1}{9}$ iii $\frac{1}{5}$ iv $\frac{1}{4}$

c i $\frac{8}{17}$ ii $\frac{7}{17}$ iii $\frac{7}{10}$ iv $\frac{7}{8}$

d i $\frac{3}{4}$ ii $\frac{1}{4}$ iii $\frac{2}{3}$ iv $\frac{1}{3}$

7 a

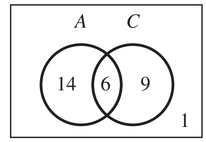


b 4 c $\frac{2}{5}$ d $\frac{1}{4}$

8 a $\frac{1}{13}$ b $\frac{1}{13}$

9 $\Pr(A \mid B) = \Pr(B \mid A) = 0$ as $\Pr(A \cap B) = 0$

10 a



	A	A'	
C	6	9	15
C'	14	1	15
	20	10	30

b i $\frac{3}{10}$ ii $\frac{7}{15}$

c $\frac{2}{5}$ d $\frac{3}{10}$

4E

Now you try

Example 8

a

		Spin 2		
		1	2	3
Spin 1	1	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b 9

c $\frac{1}{9}$

d i $\frac{1}{3}$ ii $\frac{4}{9}$ iii $\frac{8}{9}$

Example 9

a

		1st			
		T	R	E	E
2nd	T	X	(R, T)	(E, T)	(E, T)
	R	(T, R)	X	(E, R)	(E, R)
	E	(T, E)	(R, E)	X	(E, E)
	E	(T, E)	(R, E)	(E, E)	X

b i $\frac{1}{6}$ ii $\frac{1}{6}$ iii $\frac{1}{3}$

Exercise 4E

1 a without replacement b with replacement

2 a i

		1st		
		D	O	G
2nd	D	(D, D)	(O, D)	(G, D)
	O	(D, O)	(O, O)	(G, O)
	G	(D, G)	(O, G)	(G, G)

ii

		1st		
		D	O	G
2nd	D	×	(O, D)	(G, D)
	O	(D, O)	×	(G, O)
	G	(D, G)	(O, G)	×

b i 9 ii 6

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3 a 9 **b** 6

4 a

		1st roll			
		1	2	3	4
2nd roll	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

b 16 **c** $\frac{1}{16}$

d i $\frac{1}{4}$ **ii** $\frac{5}{8}$ **iii** $\frac{13}{16}$

5 a

		1st toss	
		H	T
2nd toss	H	(H, H)	(T, H)
	T	(H, T)	(T, T)

b 4
c $\frac{1}{4}$
d i $\frac{1}{2}$ **ii** $\frac{3}{4}$
e 250

6 a

		1st			
		S	E	T	S
2nd	S	×	(E, S)	(T, S)	(S, S)
	E	(S, E)	×	(T, E)	(S, E)
	T	(S, T)	(E , T)	×	(S, T)
	S	(S, S)	(E , S)	(T, S)	×

b i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{3}$ **v** 1

7 a

		1st				
		L	E	V	E	L
2nd	L	×	(E, L)	(V, L)	(E, L)	(L, L)
	E	(L, E)	×	(V, E)	(E, E)	(L, E)
	V	(L, V)	(E, V)	×	(E, V)	(L, V)
	E	(L, E)	(E, E)	(V, E)	×	(L, E)
	L	(L, L)	(E, L)	(V, L)	(E, L)	×

b 20
c i 8 **ii** 12 **iii** 12
d i $\frac{2}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{3}{5}$
e $\frac{1}{5}$

8 a

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b 36
c i 2 **ii** 6 **iii** 15
d i $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{35}{36}$ **iv** $\frac{1}{12}$
e $\frac{1}{6}$. Min's guess is wrong.

9 a $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{1}{2}$
10 a Without **b** With **c** With **d** Without

11 a

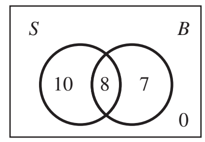
		1st			
		2.5	5	10	20
2nd	2.5	5	7.5	12.5	22.5
	5	7.5	10	15	25
	10	12.5	15	20	30
	20	22.5	25	30	40

b 16
c i 1 **ii** 8 **iii** 8
d i $\frac{1}{16}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$ **iv** $\frac{3}{16}$
e $\frac{7}{16}$

Progress quiz

1 a $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** $\frac{3}{4}$ **d** $\frac{3}{8}$
2 a 33
b i $\frac{21}{100}$ **ii** $\frac{67}{100}$ **iii** $\frac{79}{100}$

3 a



b i 10 **ii** 0
c i $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{8}{25}$ **iv** $\frac{2}{5}$ **v** $\frac{8}{15}$

4 a

	P	***P′***	
B	12	1	13
B′	4	3	7
	16	4	20

b i 4 **ii** 17
c i $\frac{4}{5}$ **ii** $\frac{1}{20}$ **iii** $\frac{3}{5}$ **iv** $\frac{12}{13}$

5 a

	1	2	3	4	5	6
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

b 12
c $\frac{1}{12}$
d i $\frac{1}{4}$ **ii** $\frac{5}{12}$

6 a

	R	R	B	G
R	×	(R, R)	(B, R)	(G, R)
R	(R, R)	×	(B, R)	(G, R)
B	(R, B)	(R, B)	×	(G, B)
G	(R, G)	(R, G)	(B, G)	×

12 outcomes
b i $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{3}$

4F

Now you try

Example 10

a $\frac{2}{5}$
b $\frac{1}{5}$

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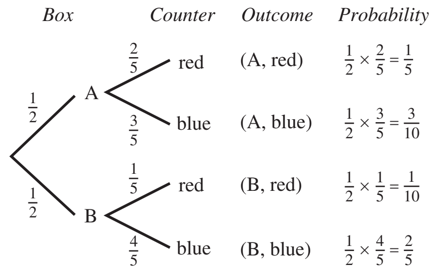
c

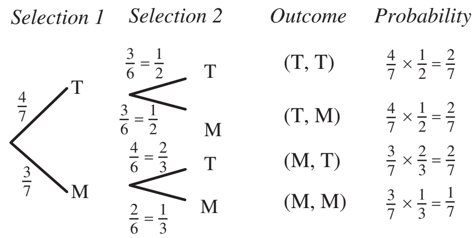


d $\frac{1}{10}$

e $\frac{3}{10}$

Example 11

a



b **i** $\frac{2}{7}$ **ii** $\frac{1}{7}$ **iii** $\frac{4}{7}$

c **i** $\frac{12}{49}$ **ii** $\frac{9}{49}$ **iii** $\frac{24}{49}$

Exercise 4F

1 **a** 8 **b** $\frac{1}{8}$

c **i** 3 **ii** 4

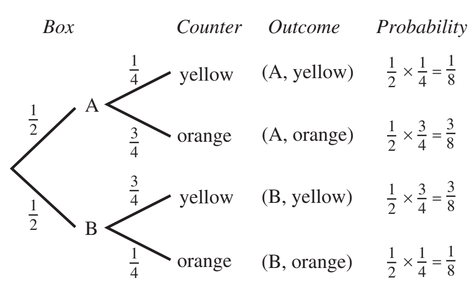
2 **a** **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$

b **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$

c **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

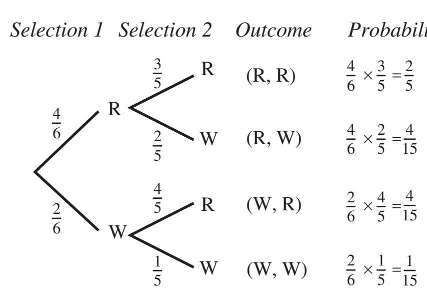
3 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$

c



d $\frac{3}{8}$ **e** $\frac{1}{2}$

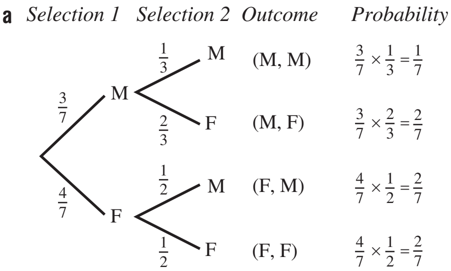
4 **a**



b **i** $\frac{4}{15}$ **ii** $\frac{2}{5}$ **iii** $\frac{8}{15}$

c **i** $\frac{2}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{4}{9}$

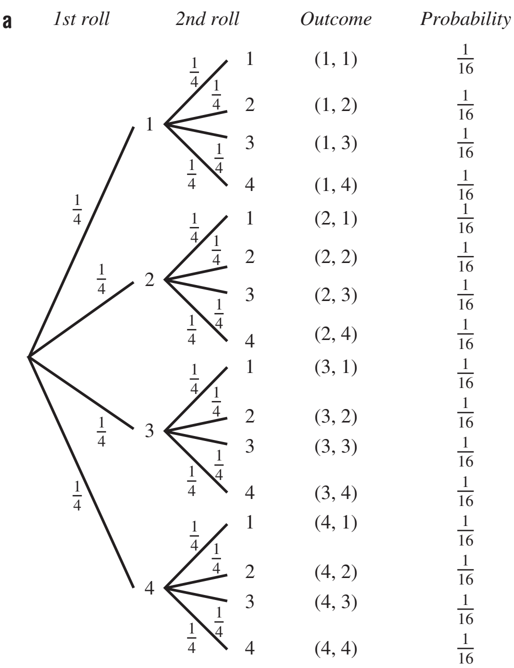
5 **a**



i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{3}{7}$

b **i** $\frac{9}{49}$ **ii** $\frac{16}{49}$ **iii** $\frac{24}{49}$ **iv** $\frac{25}{49}$

6 **a**

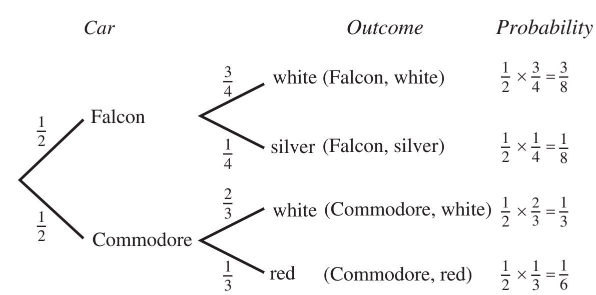


b 16

c **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$

d **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{5}{8}$

7 **a**



b **i** $\frac{3}{8}$ **ii** $\frac{1}{6}$ **iii** $\frac{17}{24}$ **iv** $\frac{7}{24}$

v $\frac{5}{6}$ **vi** $\frac{1}{3}$

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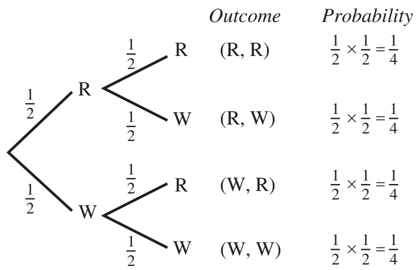
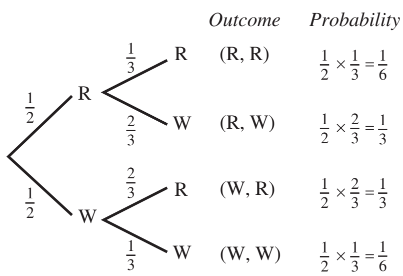
8 a



i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$ **iv** $\frac{3}{4}$

b



i $\frac{1}{6}$ **ii** $\frac{2}{3}$ **iii** $\frac{5}{6}$ **iv** $\frac{5}{6}$

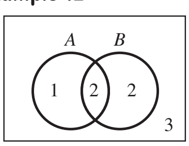
9 a i 0.17 **ii** 0.11 **iii** 0.83
b i 0.1445 **ii** 0.0965 **iii** 0.8555

4G

Now you try

Example 12

a



b i $\frac{3}{8}$ **ii** $\frac{1}{2}$

c The events A and B are not independent.

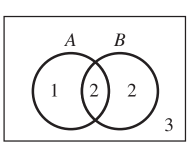
Exercise 4G

1 a no **b** yes

2 a i $\frac{3}{10}$ **ii** $\frac{1}{3}$
b no
c no

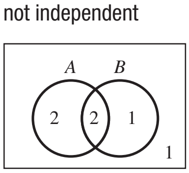
3 a with **b** without

4 a



b i $\frac{3}{8}$ **ii** $\frac{1}{2}$
c not independent

5 a



b i $\frac{2}{3}$ **ii** $\frac{2}{3}$ **c** independent

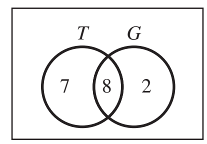
6 a i $\frac{3}{4}, \frac{1}{2}$ **ii** not independent
b i $\frac{1}{4}, \frac{1}{4}$ **ii** independent
c i $\frac{1}{3}, \frac{1}{3}$ **ii** independent
d i $\frac{2}{7}, 0$ **ii** not independent

7 a $\Pr(A) = \frac{1}{2}, \Pr(A \mid B) = \frac{1}{2}$, independent
b $\Pr(A) = \frac{3}{10}, \Pr(A \mid B) = \frac{1}{4}$, not independent
c $\Pr(A) = \frac{5}{12}, \Pr(A \mid B) = \frac{3}{20}$, not independent
d $\Pr(A) = \frac{1}{9}, \Pr(A \mid B) = \frac{1}{9}$, independent

8 False. $\Pr(A \mid B) = 0$, but $\Pr(A) = \frac{2}{9}$

9 a $\frac{1}{32}$ **b** $\frac{31}{32}$ **c** $\frac{31}{32}$

10 a



	T	**T′**	
G	8	2	10
G′	7	0	7
	15	2	17

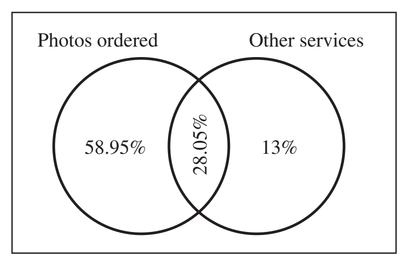
i $\frac{15}{17}$ **ii** $\frac{7}{17}$ **iii** $\frac{4}{5}$
b no

Maths@Work: Business analyst

1 a

	Other services	No other services	Total orders
Photos ordered	28.05%	58.95%	87%
No photos	13%	-	13%
Total	41.05%	58.95%	100%

b



c \$6264
d Advertising should focus on photo print services as 87% of the weekly orders are for photo prints.

2 a i 820 **ii** 160 **iii** 180 **iv** 20
b Business cards are ordered by 57% of customers with 42% ordering only business cards. This is greater than either newsletter printing (38%, 25% only newsletters) and promotional material (25%, 15% only promotional material). There is a strong market for business cards so the company must focus on offering a quality product with a competitive price.
c Greater percentages in newsletters and promotional material to increase the company's business over these product lines.

3 a *See table at top of next page.*
b i Running **ii** Yoga
c *See figure on next page.*
d Gym wear increased by 89 sales from Q3 to Q4. The advertising was successful as this is a much larger increase than the other increases of: Q1 to Q2, 7 sales; Q2 to Q3, 19 sales.
e *See figure on next page.*
f A 3.4% increase. The graph in part **e** shows the Quarter 4 column is twice as high as the Quarter 1 column. The reason why this is misleading is because the percentage scale does not start at 0. With the percentage scale starting at zero, comparisons are more realistic.
See figure on next page.

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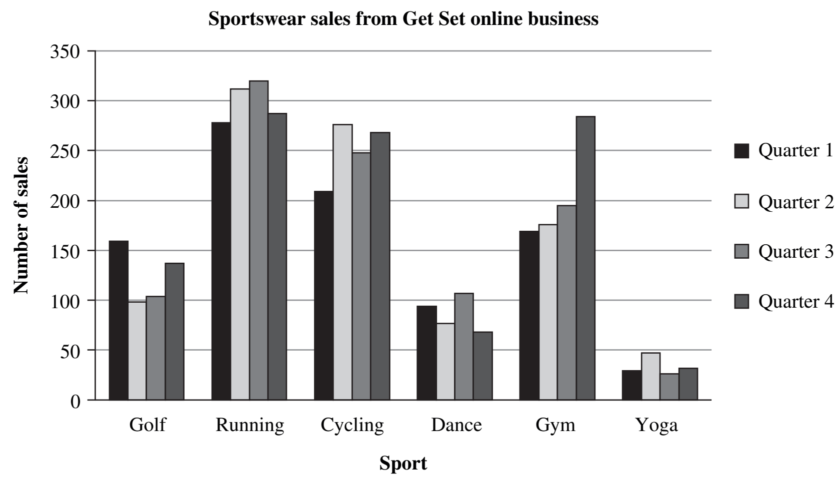
3a

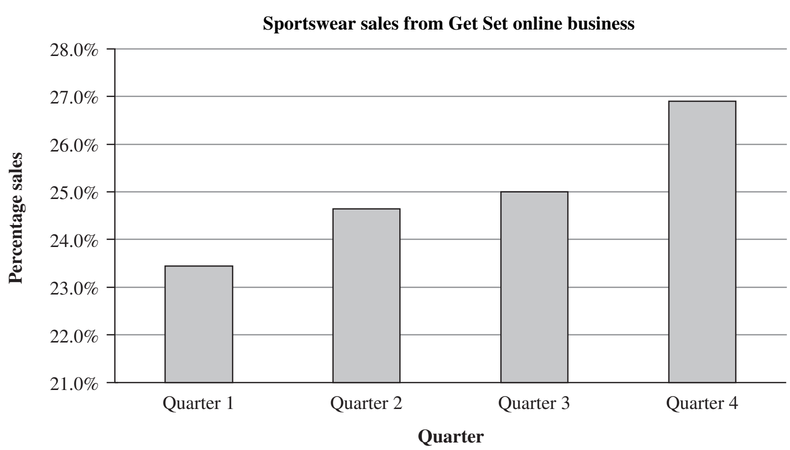
Sportswear sales from Get Set online business

Sport	QUARTER 1	QUARTER 2	QUARTER 3	QUARTER 4	Totals	Percentages
Golf	159	98	104	137	498	12.5%
Running	278	312	320	287	1197	29.9%
Cycling	209	276	248	268	1001	25.0%
Dance	94	77	107	68	346	8.7%
Gym	169	176	195	284	824	20.6%
Yoga	29	47	26	32	134	3.4%
Totals	938	986	1000	1076	4000	100.0%
Percentages	23.5%	24.7%	25.0%	26.9%	100.0%	

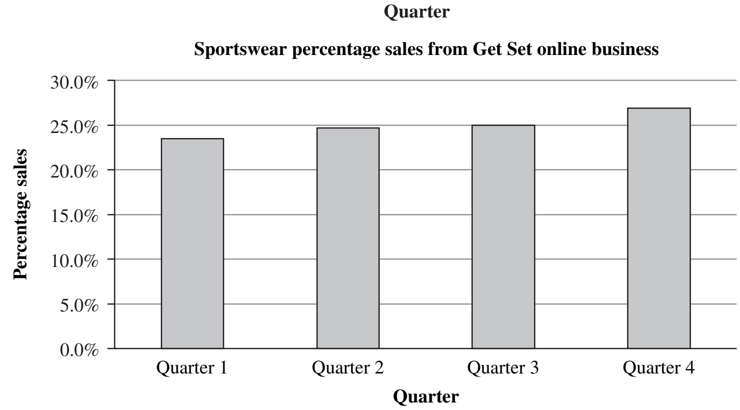
c



e



f



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Puzzles and games

1 MUTUALLY EXCLUSIVE

2 $\frac{1}{6}$

3 Results may vary.

4 **a** $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$

5 $\frac{15}{16}$

6 $\frac{3}{5}$

7 $\frac{1}{12}$

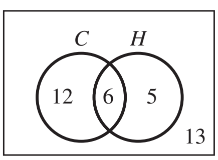
Short-answer questions

1 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$

2 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$
d $\frac{5}{8}$ **e** $\frac{1}{2}$

3 **a** **i** $\frac{2}{5}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{5}$
iv $\frac{1}{10}$ **v** $\frac{1}{20}$
b **i** $\frac{3}{5}$ **ii** $\frac{17}{20}$

4 **a**



b

	C	C'	
H	6	5	11
H'	12	13	25
	18	18	36

c 13
d **i** $\frac{1}{6}$ **ii** $\frac{5}{36}$ **iii** $\frac{1}{2}$

5 **a** 8 **b** $\frac{6}{13}$

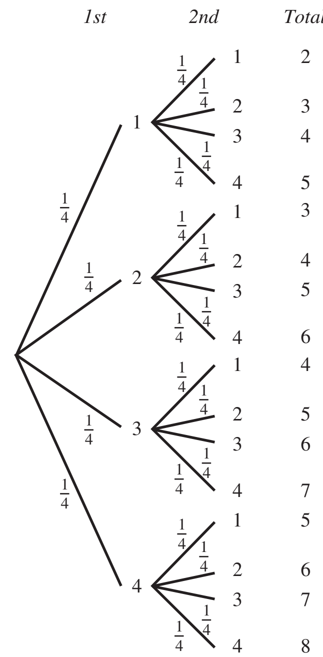
6 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$

7 **a**

		1st				
		H	A	P	P	Y
2nd	H	(H, H)	(A, H)	(P, H)	(P, H)	(Y, H)
	E	(H, E)	(A, E)	(P, E)	(P, E)	(Y, E)
	Y	(H, Y)	(A, Y)	(P, Y)	(P, Y)	(Y, Y)

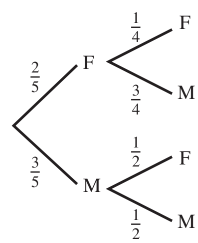
b 15
c **i** $\frac{1}{15}$ **ii** $\frac{2}{15}$ **iii** $\frac{13}{15}$

8 **a**



b **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** 0 **iv** 1

9



a $\frac{2}{5}$ **b** $\frac{3}{4}$ **c** $\frac{3}{10}$ **d** $\frac{3}{5}$ **e** $\frac{7}{10}$

10 **a** $\frac{7}{11}, \frac{2}{5}$, no **b** $\frac{1}{2}, \frac{1}{2}$, yes

Multiple-choice questions

1 C **2** C **3** A **4** D **5** C
6 B **7** B **8** E **9** B **10** E

Extended-response questions

1 **a** 8
b **i** $\frac{7}{15}$ **ii** $\frac{1}{15}$
c

	R	R'	
S	3	1	4
S'	3	8	11
	6	9	15

d **i** $\frac{1}{2}$ **ii** $\frac{3}{4}$

2 **a**

		1st		
		R	S	W
2nd	R	(R, R)	(S, R)	(W, R)
	S	(R, S)	(S, S)	(W, S)
	W	(R, W)	(S, W)	(W, W)

b **i** $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{5}{9}$ **iv** $\frac{4}{9}$
c 4
d $\frac{5}{9}$

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Chapter 5

Warm-up quiz

1 **a** 8 **b** 40 **c** 82 **d** 13
2 **a** 6
b **i** 19 **ii** 23
c 30
d 10%
3 **a** 8 **b** 8.5 **c** 5 **d** 8
4 **a** **i** Mean = 45 **ii** Mode = 41
iii Median = 41 **iv** Range = 20
b **i** Mean = 6 **ii** Mode = 2
iii Median = 6 **iv** Range = 10
5 **a** 15 **b** 111, 139 are most frequent.
c Min = 98 g, Max = 145 g **d** 47

5A

Now you try

Example 1

a Categorical and ordinal
b Numerical and discrete

Exercise 5A

1 Selection of people, structure of questions, unbiased, clear
2 **a** C **b** D **c** A
d B **e** F **f** E
3 **a** B **b** E **c** C
d D **e** F **f** A
4 C
5 D
6 **a** Numerical and discrete
b Numerical and discrete
c Categorical and nominal
d Numerical and continuous
e Categorical and ordinal
7 D
8 Answers will vary.
9 D
10 **a** Carrying out survey at a train station will create a very high proportion of train users in survey's results.
b Survey will reach only those people who use computers.
c Survey will access only people over 18 years of age.
11 Check with your teacher.
12 Check with your teacher.

5B

Now you try

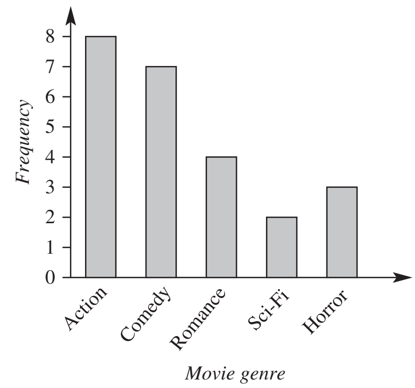
Example 2

a

Movie Category	Tally	Frequency
Action	卌 III	8
Comedy	卌 II	7
Romance	IIII	4
Sci-Fi	II	2
Horror	III	3
Total	**24**	**24**

b

Favourite movie genre



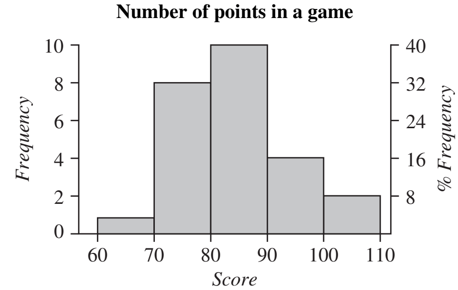
Example 3

a

Class interval	Tally	Frequency	Percentage frequency
60–	I	1	4%
70–	卌 III	8	32%
80–	卌 卌	10	40%
90–	IIII	4	16%
100–109	II	2	8%
Total	25	25	100%

b

Number of points in a game



c The 80 – interval
d 64%

Exercise 5B

1 **a** Histogram **b** Column graph
c Column graph **d** Histogram
2 **a**

Car colour	Tally	Frequency
Red	III	3
White	卌	5
Green	II	2
Silver	II	2
Total	**12**	**12**

b

Class interval	Frequency	Percentage frequency
80–	8	16%
85–	23	46%
90–	13	26%
95–100	6	12%
Total	**50**	**100%**

3 **a** 8
b 25
c **i** 8 **ii** 32%

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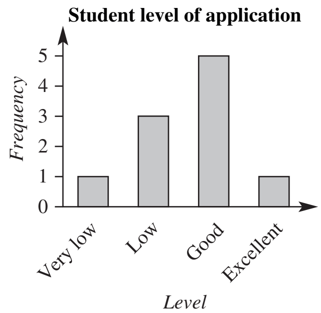
4 a i

Application	Tally	Frequency
Very low	|	1
Low	|||	3
Good	卌	5
Excellent	|	1
Total	**10**	**10**

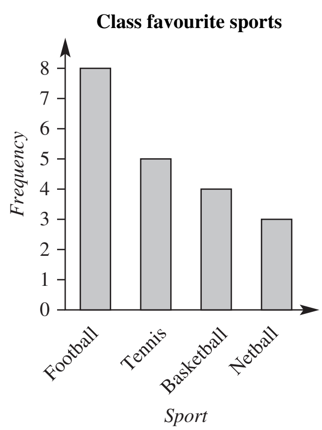
ii



b i

Favourite sport	Tally	Frequency
Football	卌 |||	8
Tennis	卌	5
Basketball	||||	4
Netball	|||	3
Total	**20**	**20**

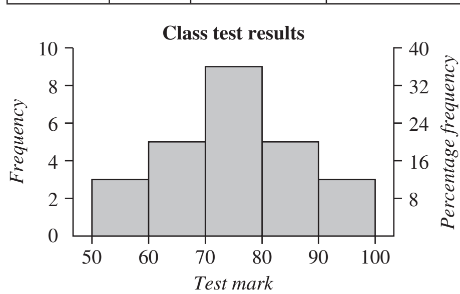
ii



5 a

Class interval	Tally	Frequency	Percentage frequency
50–	|||	3	12%
60–	卌	5	20%
70–	卌 ||||	9	36%
80–	卌	5	20%
90–100	|||	3	12%
Total	**25**	**25**	**100%**

b



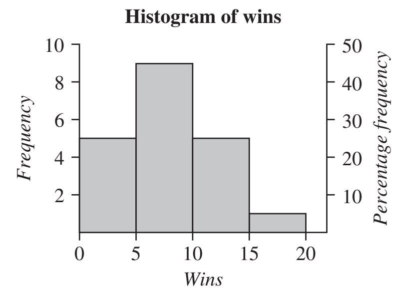
c The 70– interval

d 32%

6 a

Class interval	Tally	Frequency	Percentage frequency
0–	卌	5	25%
5–	卌 ||||	9	45%
10–	卌	5	25%
15–19	|	1	5%
Total		**20**	**100%**

b



c The 5– interval **d** 75%

7 a

Type of transport	Frequency	Percentage frequency
Car	16	40%
Train	6	15%
Tram	8	20%
Walking	5	12.5%
Bicycle	2	5%
Bus	3	7.5%
Total	**40**	**100%**

b i 6 **ii** Car **iii** 40% **iv** 17.5%
v 42.5%

8 a Skewed **b** Symmetrical

9 a

Mass	Frequency	Percentage frequency
10–	3	6%
15–	6	12%
20–	16	32%
25–	21	42%
30–34	4	8%
Total	**50**	**100%**

b 50 **c** 32% **d** At least 25 g but less than 30 g.
e 42% **f** 94%

10 a

Section	Frequency	Percentage frequency
String	21	52.5%
Woodwind	8	20%
Brass	7	17.5%
Percussion	4	10%
Total	**40**	**100%**

b 40 **c** 52.5% **d** 47.5% **e** 9.3%

11 Frequencies of individual scores are lost if the histogram displays only categories of scores.

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12 **a** Russia, ~ 14 years
b Pakistan
c In nearly all countries, the female life expectancy is more than that for males.
d Extreme poverty, lack of resources and prevalance of disease.

13 **a** Saturday and Sunday; vendor would expect greater sales at the weekend.
b May have been a particularly warm day or a public holiday.
c **i** Wednesday; $250 **ii** Thursday
d The graph does not help us to visualise the profit and loss.

5C

Now you try

Example 4

a 16
b 1
c 23
d One household had 5 pets but the data were generally skewed towards a lower number of pets.

Example 5

a

Stem	Leaf
1	0 1 4 5 8
2	2 4 4 6 8
3	1 2
4	0 4
5	4
6	3

3 | 1 means 31

b The distribution of the data is skewed.

Example 6

a

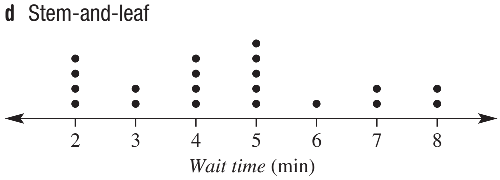
Friend 1 Leaf	Stem	Friend 2 Leaf
	1	6 8 9
2	2	2 4 8 9
6 2 0	3	0 2
8 6 4 1	4	1 4
7 4	5	2
6 2	6	

5 | 4 means 54

b Friend 1's spending is symmetrical whereas friend 2's spending is skewed.

Exercise 5C

1 **a** Histogram **b** Dot plot **c** Column graph
d Stem-and-leaf

2


3 **a** 32, 35, 41, 43, 47, 54, 54, 56, 60, 62, 71, 71
b 0.2, 0.3, 0.7, 1.4, 1.4, 1.8, 1.9, 2.3, 2.6, 2.6, 3.0, 3.5

4 **a** 11 **b** 1 **c** 16
d One family had 3 children but the data were generally symmetrical.

5 **a** 9
b 39
c He had one bad hole with 7 strokes, but generally the data were consistent, between 3 and 5 strokes.

6 **a** **i**

Stem	Leaf
1	5 5
2	0 1 2 4 5 6 6
3	1 7 7 8
4	6
5	2

1 | 5 means 15

ii Skewed

b **i**

Stem	Leaf
1	2 6
2	1 3 5 7
3	1 2 5 5 6 6 8
4	0 0 2 4 8
5	1 3 5

3 | 2 means 32

ii Symmetrical

c **i**

Stem	Leaf
11	6 7 8 9
12	1 4 5 7
13	3 5 7
14	5 7 9
15	3 8
16	0 2

13 | 5 means 135

ii Skewed

d **i**

Stem	Leaf
2	0
3	3 7 8
4	3 4 5 7 8 9
5	2 4 5 8
6	1 3 5
7	0

3 | 7 means 3.7

ii Symmetrical

7 **a** **i**

Set 1 Leaf	Stem	Set 2 Leaf
9 8	3	2 5
8 6 3 2 2 0	4	1 7 7
9 7 3 3	5	2 2 5 6
4 1	6	0 1 3 4 7

5 | 2 means 52

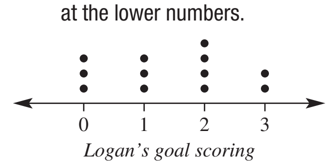
ii Set 1 is symmetrical and set 2 is skewed with more data at the higher end.

b **i**

Set 1 Leaf	Stem	Set 2 Leaf
8 6 4	16	0 3 3 5 7
6 5 2 1	17	0 1 1 4 8
8 7 7 5 1 0	18	2 2 6 7
9 5 2	19	0 4
8 1	20	

19 | 5 means 195

ii Set 1 is symmetrical and set 2 is skewed with most of the data at the lower numbers.

8 **a**


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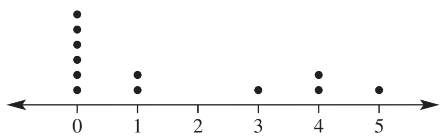
b



Max's goal scoring

c Well spread performance
d Irregular performance, skewed

9 a 4.1 minutes b ~ 22.5 minutes
c This would increase the average time.

10 a

Inner city Leaf	Stem	Outer suburb Leaf
9 6 4 3 1 1	0	3 4 9
9 4 2 0	1	2 8 8 9
4 1	2	1 3 4
	3	4
	4	1

2 | 1 means 21 km

b For the inner city, the data values are closer together and bunched around the lower distances. The outer suburb data values are more spread out.
c In the outer suburbs, students will be travelling farther distances to their school, whereas at inner city schools they are more likely to live close to the school.

11 a $a = 3, b = 9, c = 7$ or 8
b $a = 0$ or 1, $b = 0, 1, 2, 3, 4$ or 5

12 a The stem 1 is allocated the leaves 0–4 (included) and 1^* is allocated 5–9 (included).
b i 1 ii 0^*
c For city B, for example, most temperatures are in the 20s; splitting into 20–24 and 25–29 allows better analysis of the data and still means that a stem-and-leaf is an appropriate choice of graph.
d City A experienced cooler weather, with temperatures between 8°C and 18°C. City B had warmer weather and a wider range of temperatures, between 17°C and 31°C.
e The cities may have been experiencing different seasons; maybe winter and summer.

5D

Now you try

Example 7

a i 6.5 ii 10 iii 11
b i 18 ii 22 iii 22

Example 8

a 10
b 21

Example 9

a 27 b 39 c 35.7 d 38.5

Exercise 5D

1 a mode b mean c median
d bimodal e range

2 a 7 b 10 and 14

3 a 28 b 7 c 4

4 a i 5.4 ii 8 iii 6
b i 16.25 ii 10 iii 45
c i 70 ii 50, 90 iii 35
d i 25 ii no mode iii 18
e i 2.325 ii 1.9 iii 1
f i 1.6 ii no mode iii 1.2

5 a 7 b 4 c 11 d 75
e 7 f 5 g 33.5 h 3.15

6 a 7 b 6 c 7 d 6

7 a \$42 b \$17.50 c \$20.75
d Due to the \$50 value, which is much larger than the other amounts.

8 a i 28 ii 4 iii 17.2 iv 17
b i 24 ii no mode iii 110 iv 108
c i 3.2 ii 3.0, 5.3 iii 4.6 iv 4.9

9 a Mark:
i 83.6 ii 85 iii 31
Hugh:
i 76.4 ii 79 iii 20
b Mark's scores varied more greatly, with a higher range, whereas Hugh's results were more consistent. Mark had the higher mean and median though, as he had several high scores.

10 The median, since the mean is affected by the one large value (\$1 700 000).

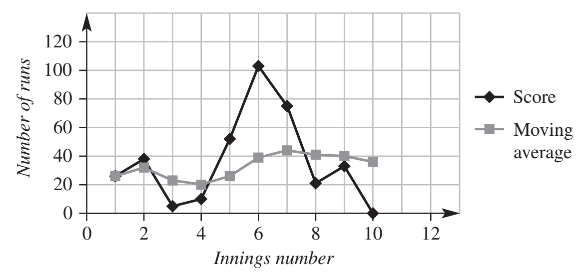
11 a 4
b 3.7
c i The median is unchanged in this case.
ii The mean is decreased.

12 a

Innings	1	2	3	4	5
Score	26	38	5	10	52
Moving average	26	32	23	20	26

Innings	6	7	8	9	10
Score	103	75	21	33	0
Moving average	39	44	41	40	36

b



c i The score fluctuates wildly.
ii The graph is fairly constant with small increases and decreases.
d The moving average graph follows the trend of the score graph but the fluctuations are much less significant.

5E

Now you try

Example 10

a $Q_1 = 10, Q_3 = 28$ b 18

Example 11

a $Q_1 = 14.4, Q_3 = 20.6$ b 6.2

Example 12

a i Min = 2, max = 14 ii 5.5
iii $Q_1 = 4$ $Q_3 = 7$ iv 3
b 14 is an outlier.
c Matches went late on a particular day.
d Range = 7
IQR = 3
Outlier affects the range significantly but does not overly affect the IQR.

Exercise 5E

1 a Min, lower quartile (Q_1), median (Q_2), upper quartile (Q_3), max
b Range is max − min; IQR is $Q_3 - Q_1$. Range is the spread of all the data, IQR is the spread of the middle 50% of data.
c An outlier is a data point (element) outside the vicinity of the rest of the data.

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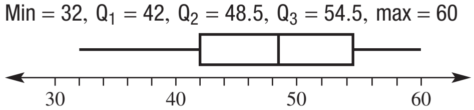
2 **a** 1.5
b **i** 1 **ii** 2
3 **a** 5 **b** −4.5 and 15.5 **c** An outlier
4 **a** **i** $Q_1 = 4, Q_3 = 8$ **ii** 4
b **i** $Q_1 = 11, Q_3 = 18$ **ii** 7
c **i** $Q_1 = 51, Q_3 = 62$ **ii** 11
d **i** $Q_1 = 1.8, Q_3 = 2.7$ **ii** 0.9
5 **a** **i** $Q_1 = 2, Q_3 = 11$ **ii** 9
b **i** $Q_1 = 2, Q_3 = 10$ **ii** 8
c **i** $Q_1 = 1.0, Q_3 = 1.6$ **ii** 0.6
d **i** $Q_1 = 10.5, Q_3 = 22.5$ **ii** 12
6 **a** **i** Min = 0, max = 17 **ii** Median = 13
iii $Q_1 = 10, Q_3 = 15$ **iv** IQR = 5
b 0 is an outlier.
c Road may have been closed that day.
d Range = 9
IQR = 5
Range is reduced but IQR is unaffected.
7 **a** **i** Min = 4, max = 14 **ii** 7.5
iii $Q_1 = 5, Q_3 = 9$ **iv** IQR = 4
v No outliers
b **i** Min = 16, max = 31 **ii** 25
iii $Q_1 = 21, Q_3 = 27$ **iv** IQR = 6
v No outliers
c **i** Min = 2, max = 24 **ii** 12
iii $Q_1 = 10, Q_3 = 13$ **iv** IQR = 3
v 24 and 2
d **i** Min = 1, max = 17 **ii** 7
iii $Q_1 = 4, Q_3 = 10$ **iv** IQR = 6
v No outliers
8 **a** **i** Min = 25, max = 128 **ii** 47
iii $Q_1 = 38, Q_3 = 52.5$ **iv** IQR = 14.5
v Yes, 128 **vi** 51.25
b A more advanced calculator was used.
c Median, as it is not affected dramatically by the outlier.
d The IQR, as it is not affected dramatically by the outlier.
9 2 bags; 30 and 31 will be checked
10 2 fridges; 350 and 1700 are outliers
11 **a** IQR = 10 **b** Yes, 1 is an outlier. **c** No
12 **a** 35 or more **b** 35–49 inclusive
13 Check with your teacher

5F

Now you try

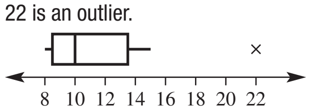
Example 13

a Min = 32, $Q_1 = 42$, $Q_2 = 48.5$, $Q_3 = 54.5$, max = 60
b



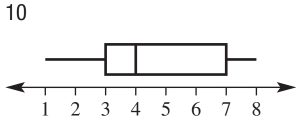
Example 14

a $Q_1 = 8.5$, $Q_2 = 10$, $Q_3 = 13.5$
b 22 is an outlier.
c

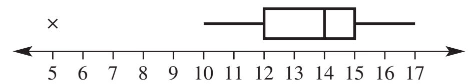


Exercise 5F

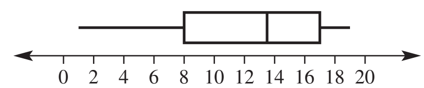
1 **a** Minimum **b** Q_1 **c** Q_2(median)
d Q_3 **e** Maximum **f** Whisker
g Box
2 **a** 15 **b** 5 **c** 25
d 20 **e** 10 **f** 20
g 10
3 **a**



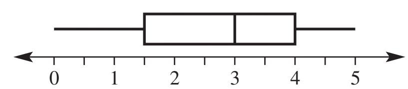
b



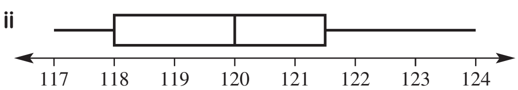
4 **a** Q_3 **b** Q_1, Q_3 **c** minimum, Q_1
d Q_3, maximum
5 **a** **i** Min = 1, $Q_1 = 8$, $Q_2 = 13.5$, $Q_3 = 17$, max = 19
ii



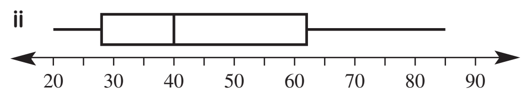
b **i** Min = 0, $Q_1 = 1.5$, $Q_2 = 3$, $Q_3 = 4$, max = 5
ii



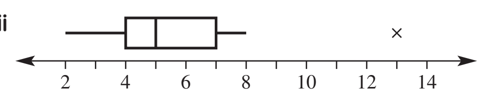
c **i** Min = 117, $Q_1 = 118$, $Q_2 = 120$, $Q_3 = 121.5$, max = 124
ii



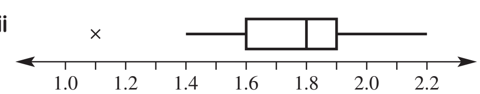
d **i** Min = 20, $Q_1 = 28$, $Q_2 = 40$, $Q_3 = 62$, max = 85
ii



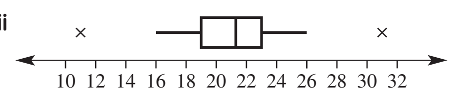
6 **a** **i** $Q_1 = 4$, $Q_2 = 5$, $Q_3 = 7$ **ii** Outlier is 13.
iii



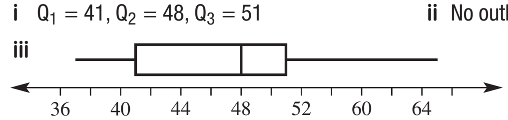
b **i** $Q_1 = 1.6$, $Q_2 = 1.8$, $Q_3 = 1.9$ **ii** Outlier is 1.1.
iii



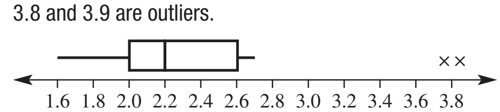
c **i** $Q_1 = 19$, $Q_2 = 21.5$, $Q_3 = 23$
ii Outliers are 11 and 31.
iii



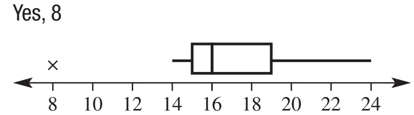
d **i** $Q_1 = 41$, $Q_2 = 48$, $Q_3 = 51$ **ii** No outliers
iii



7 **a** **i** 1.6 **ii** 2.0 **iii** 2.2
iv 2.6 **v** 3.9 **vi** 0.6
b 3.8 and 3.9 are outliers.
c



8 **a** **i** 8 **ii** 15 **iii** 16 **iv** 19
v 24 **vi** 4
b Yes, 8
c



9 **a** Same minimum of 1.
b B
c **i** 5 **ii** 10
d Data points for B are more evenly spread than those for A.

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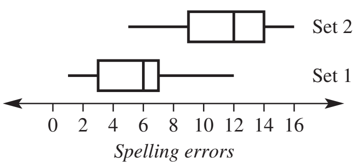
10 a



b Yes, examiner 2 found more errors.

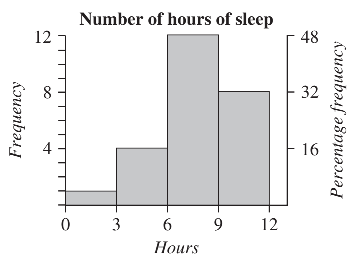
Progress quiz

1 a Categorical and nominal
b Numerical and discrete

2 a

Class Interval	Tally	Frequency	Percentage frequency
0–	|	1	4%
3–	||||	4	16%
6–	~~||||~~ ~~||||~~ ||	12	48%
9 – 11	~~||||~~ |||	8	32%
Total	25	25	100%

b



c 20% of those surveyed had fewer than 6 hours of sleep.
d The 6– interval is the most frequent.

3 a 20 b 5 days c 4.5 days d 4.15 days e 7

4 a

Stem	Leaf
0	3 7
1	1 5 6
2	1 2 4 5
3	0 0 3 4
4	0 2 5 6 7 8 9

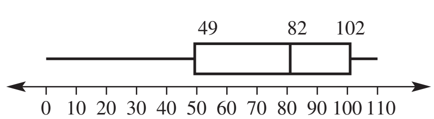
4|2 means 42

b Data is skewed.
c i 30 ii 30

5 a 19.375 b 17 c 39

6 a $Q_1 = 10.5, Q_3 = 19$ b 8.5

7 a $Q_1 = 49, Q_2 = 82$ and $Q_3 = 102$
b There are no outliers.
c

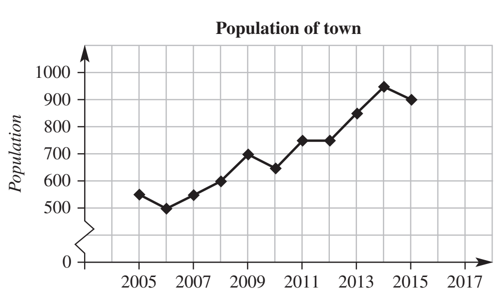


5G

Now you try

Example 15

a



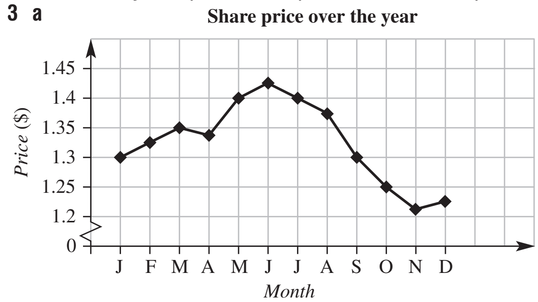
b Generally linear in a positive direction.

Exercise 5G

1 a Linear b No trend c Non-linear d Linear

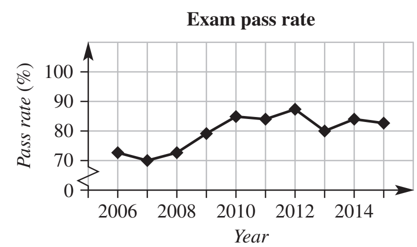
2 a i 20°C ii 30°C
b 36°C
c i Noon to 1 p.m. ii 3 to 4 p.m.
d Temperature is increasing from 8 a.m. to 3 p.m. in a generally linear way. At 3 p.m. the temperature starts to drop.

3 a



b The share price generally increased until it peaked in June and then continually decreased to a yearly low in November before trending upwards again in the final month.
c $0.21

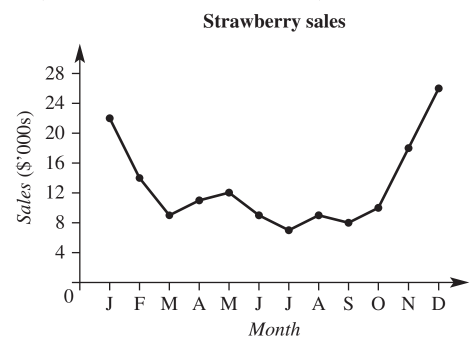
4 a



b The pass rate for the examination has increased marginally over the 10 years, with a peak in 2012.
c 2012
d 11%

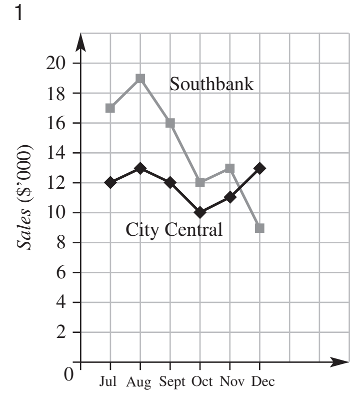
5 a Linear
b i $650 000 ii $750 000

6 a



b The sales start high and decrease during the middle of the year, before increasing again towards the end of the year.
c Strawberries are in season in the warmer months but not in the cooler winter months.

7 a i $6000 ii $4000
b 1
c



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d **i** The sales trend for City Central for the 6 months is fairly constant.

ii Sales for Southbank peaked in August before taking a downturn.

e About \$5000

8 **a** **i** 5.8 km **ii** 1.7 km

b **i** Blue Crest slowly gets closer to the machine.

ii Green Tail starts near the machine and gets farther from it.

c 8:30 p.m.

9 **a** Increases continually, rising more rapidly as the years progress.

b Compound interest – exponential growth

10 **a** Graphs may vary but it should decrease from room temperature to the temperature of the fridge.

b No. Drink cannot cool to a temperature *lower* than that of the internal environment of the fridge.

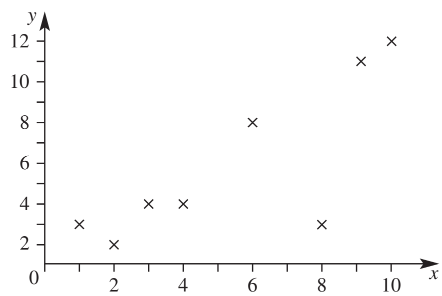
c Check with your teacher

5H

Now you try

Example 16

a



b Positive correlation

c Strong correlation

d Outlier is (8, 3)

Exercise 5H

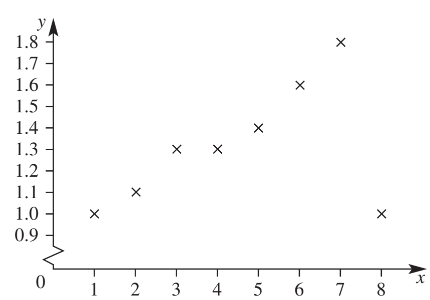
1 **a** Likely **b** Likely **c** Unlikely

d Unlikely **e** Likely

2 **a** increases **b** decreases

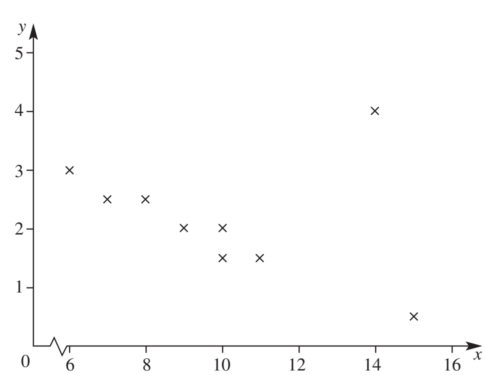
3 **a** Weak negative **b** Strong positive **c** Strong negative

4 **a**



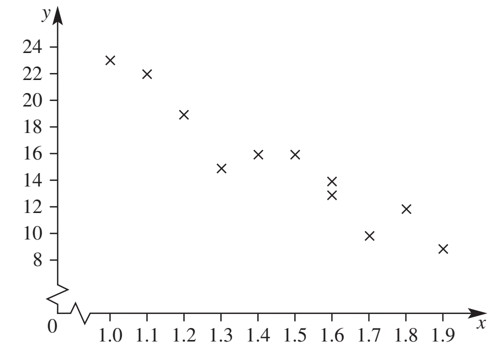
b Positive **c** Strong **d** (8, 1.0)

5 **a**

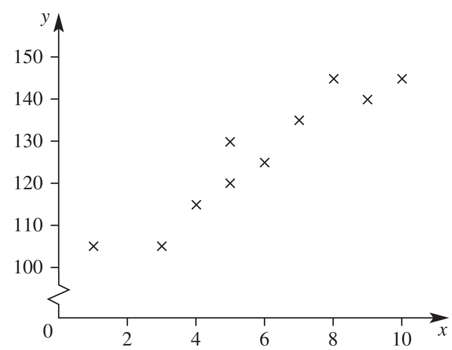


b Negative **c** Strong **d** (14, 4)

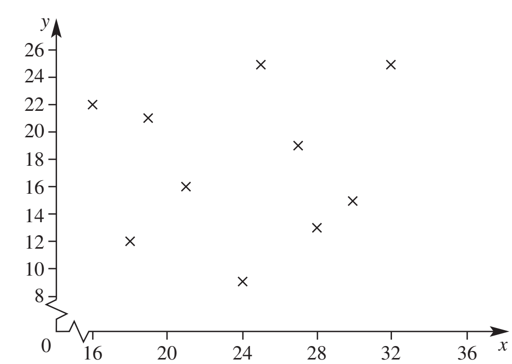
6 **a** Negative



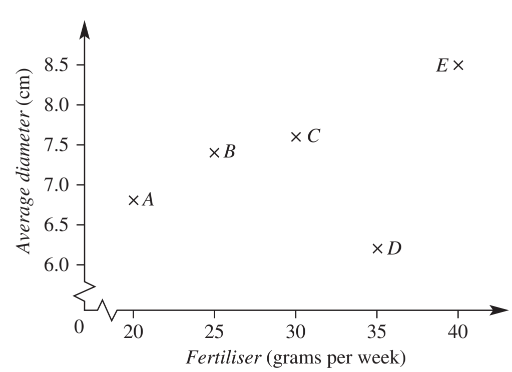
b Positive



c None



7 **a**



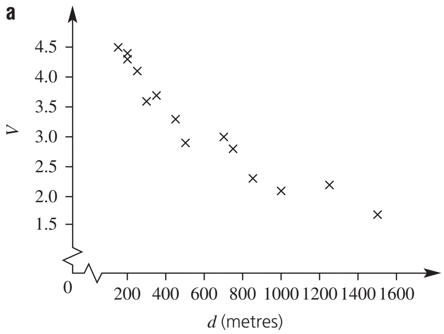
b D **c** Yes, although small sample size does lead to doubt.

8 **a** Yes

b Decrease

c **i** Yes **ii** Car H

9 a

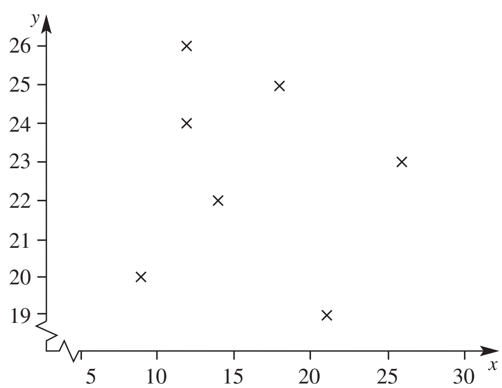


b Negative

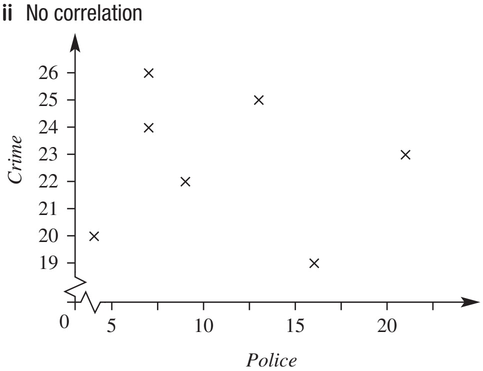
c As d increases, V decreases.

10 Each axis needs a better scale. All data are between 6 and 8 hours sleep and shows only a minimum change in exam marks. Also, there are only 4 observations presented on the plot, which is not enough to form strong conclusions.

11 a i Weak negative correlation



ii No correlation



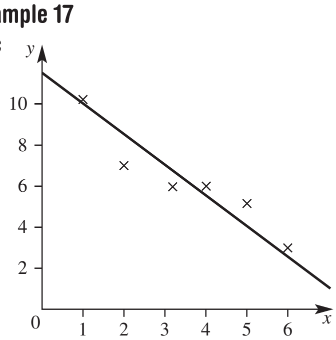
b Survey 1, as this shows an increase in the number of police has seen a decrease in the incidence of crime.

5I

Now you try

Example 17

a&c



b Negative correlation

d **i** $y \approx 4.8$ **ii** $y \approx 11.5$

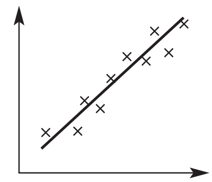
iii $x \approx 2.8$ **iv** $x \approx 7.5$

Exercise 5I

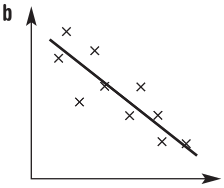
1 a When the data appear to fit on or near a straight line, it shows a definite linear trend.

b Balance points evenly either side of the line; ignore outliers when taking distance from the line into account.

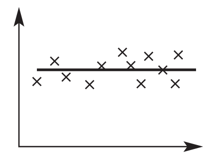
2 a



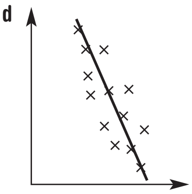
b



c

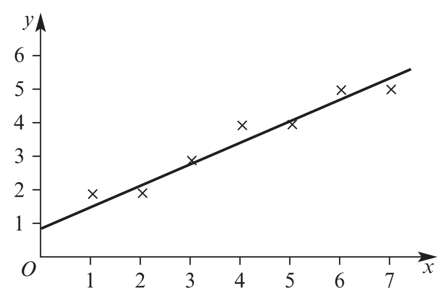


d



3 a interpolation **b** extrapolation

4 a



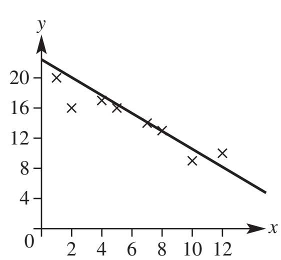
b Positive correlation

c As above

d All answers are approximate.

i 3.2 **ii** 0.9 **iii** 1.8 **iv** 7.4

5 a



b Negative

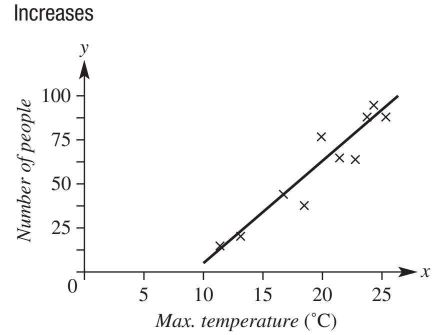
c As above

d **i** 13.5 **ii** 23 **iii** 9 **iv** 7

6 a ≈ 4.5 **b** ≈ 6 **c** ≈ 0.5 **d** ≈ 50

7 a Increases

b



c **i** 65 **ii** 15°C

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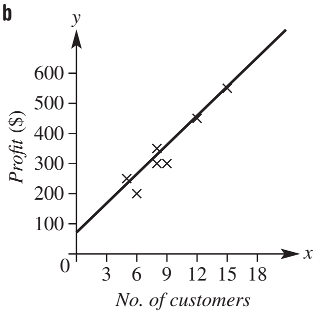
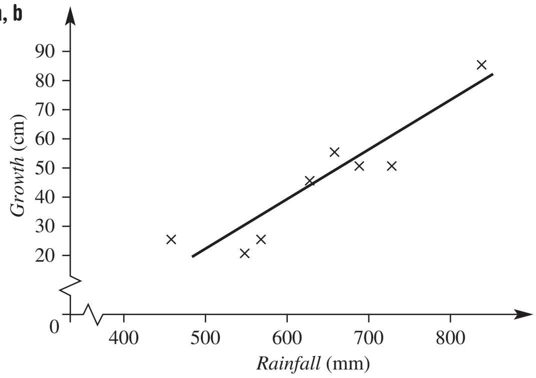
Cambridge University Press
Updated May 2023

8 a, b



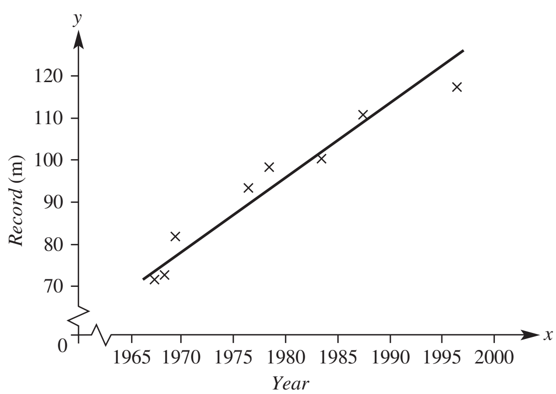
c \$600 **d** 2

9 a, b



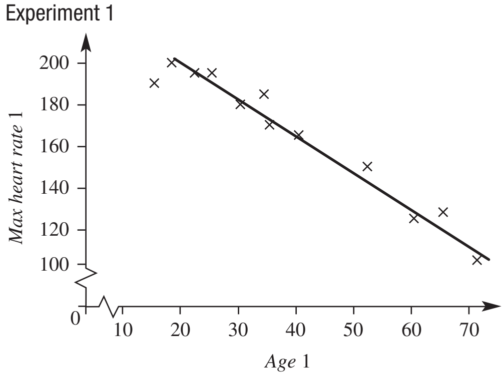
c i ≈ 25 cm **ii** ≈ 85 cm
d i ≈ 520 mm **ii** ≈ 720 mm

10 a, b

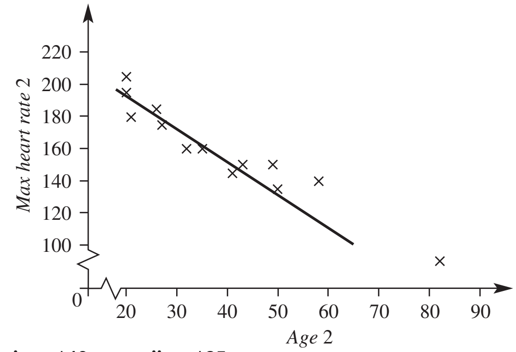


c i 130 m **ii** 170 m
d No, records are not likely to continue to increase at this rate.

11 a Experiment 1



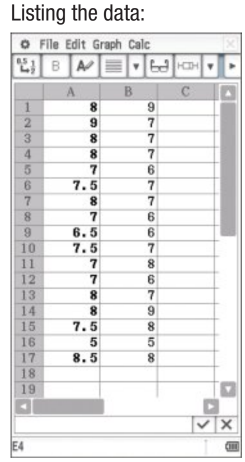
Experiment 2



b i ≈ 140 **ii** ≈ 125
c i ≈ 25 **ii** ≈ 22
d Experiment 2
e Check with your teacher

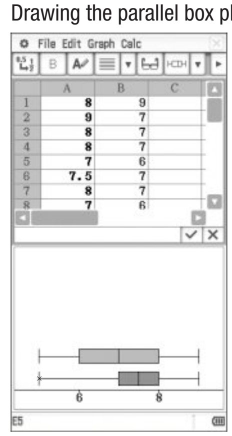
Maths@Work: Project manager on a building site

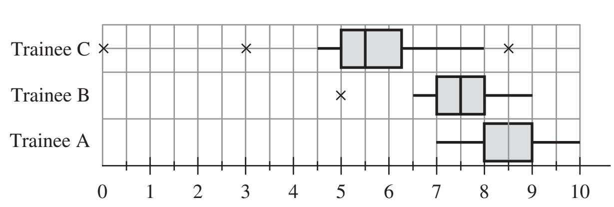
1 a 8.4 **b** 8 **c** 7, 8, 8, 9, 10
2 a 7.5 **b** 7.5 **c** 5, 7, 7.5, 8, 9
3 *See figure at bottom of page.*
4 a 8.4, 7.5, 5.3 **b** 8, 7.5, 5.5
c 7, 5, 0 **d** 10, 9, 8.5
e 1, 1, 1.25 **f** 3, 4, 8.5
5 a *Note that a Class Pad II is used for the CAS screenshots below.*
Listing the data:



	A	B	C
1	8	9	
2	9	7	
3	8	7	
4	8	7	
5	7	6	
6	7.5	7	
7	8	7	
8	7	6	
9	6.5	6	
10	7.5	7	
11	7	8	
12	7	6	
13	8	7	
14	8	9	
15	7.5	8	
16	5	5	
17	8.5	8	
18			
19			

Drawing the parallel box plots:





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b Trainee B: 5, 7, 7.5, 8, 9
Trainee D: 5, 6, 7, 8, 9

c Trainee B IQR = 1; trainee D IQR = 2
Trainee B has more consistent results as his/her IQR shows that the middle 50% of scores only vary by 1 unit but trainee D's middle 50% of scores vary by 2 units.

Puzzles and games

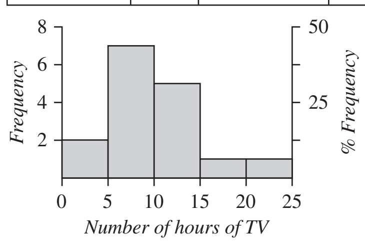
1 66 kg
2 88%
3 8
4 **a** Larger by 3 **b** Larger by 3
c No change
5 1.1
6 3, 3, 9, 11
7 19
8 **a** Mean ↑ by 10; median ↑ by 10; mode ↑ by 10
b Mean is × by 10; median is × by 10; mode is × by 10
9 494 mm

Short-answer questions

1 **a**

Class interval	Tally	Frequency	Percentage frequency
0–	||	2	12.5%
5–	~~||||~~ ||	7	43.75%
10–	~~||||~~	5	31.25%
15–	|	1	6.25%
20–24	|	1	6.25%
Total		**16**	**100%**

b



c Skewed

2 **a**

Stem	Leaf
1	5 9
2	0 4 8 9
3	2 4 7 8 8
4	2 9
5	0

3 | 2 means 32

b The data set is symmetrical about scores in the 30s.

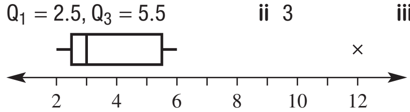
3 **a** **i** 5 **ii** 6 **iii** 5
b **i** 30.5 **ii** 57 **iii** 20
c **i** 1.6 **ii** 1.2 **iii** 1.5

4 **a** 43.2 years
b 38 years
c The mean is affected by the high ages 76 and 87.

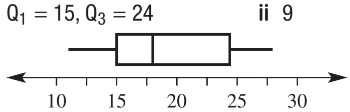
5 **a** $Q_1 = 8, Q_2 = 11, Q_3 = 15$
b $Q_1 = 8, Q_2 = 13, Q_3 = 15.5$

6 **a** Range = 19, IQR = 5.5
b Outlier = 21
c Range = 8, IQR = 6
Range is significantly reduced, the affect on the IQR is smaller.

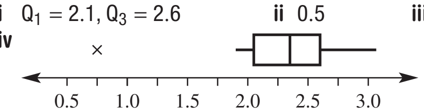
7 **a** **i** $Q_1 = 2.5, Q_3 = 5.5$ **ii** 3 **iii** 12
iv



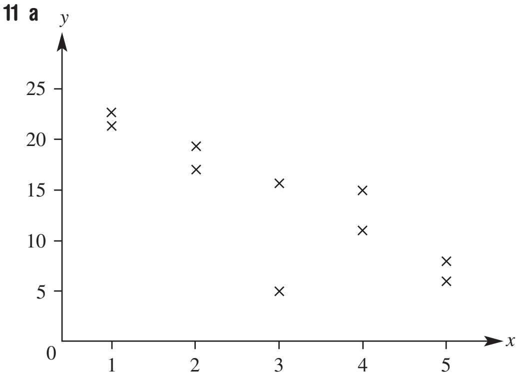
b **i** $Q_1 = 15, Q_3 = 24$ **ii** 9 **iii** none
iv



c **i** $Q_1 = 2.1, Q_3 = 2.6$ **ii** 0.5 **iii** 0.7
iv

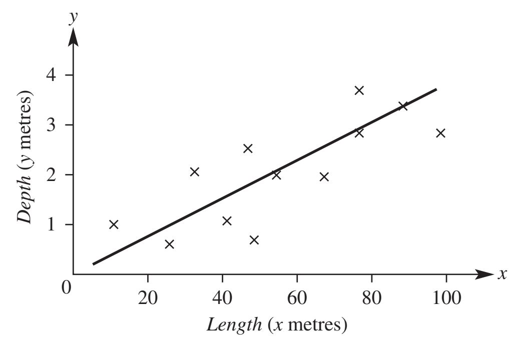


8 **a** False **b** True **c** True **d** True
9 **a** Linear **b** 10 a.m.–11 a.m.
10 **a** Negative **b** None **c** Positive
11 **a**



b Negative **c** Strong **d** (3, 5)

12 **a**



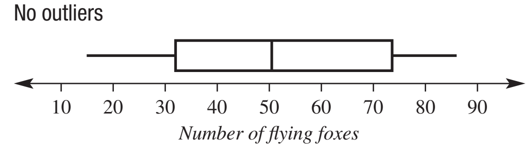
b 1.8 m

Multiple-choice questions

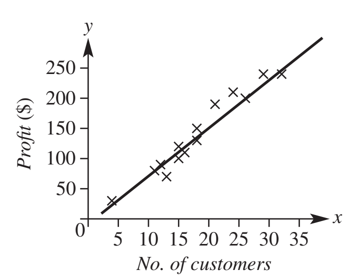
1 D **2** C **3** B **4** A **5** C **6** A
7 E **8** C **9** D **10** B

Extended-response questions

1 **a** 41
b No outliers
c



2 **a**



b **i** $80 **ii** $150 **iii** $240
c **i** 7 **ii** 16 **iii** 27

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Semester review 1

Measurement

Short-answer questions

1 **a** 43 cm **b** $320\ \text{cm}^2$ **c** $30\,000\ \text{cm}^3$
2 **a** 8 cm **b** 44 m **c** 9 m
3 **a** **i** 37.70 cm **ii** $113.10\ \text{cm}^2$
b **i** 14.28 cm **ii** $12.28\ \text{cm}^2$
c **i** 11.14 m **ii** $7.14\ \text{m}^2$
4 **a** $10.5\ \text{m}^2$ **b** $112\ \text{cm}^2$ **c** $8\ \text{m}^2$
5 **a** **i** $45\ \text{cm}^3$ **ii** $78\ \text{m}^2$
b **i** $30\ \text{m}^3$ **ii** $72\ \text{m}^2$
c **i** $942.48\ \text{mm}^3$ **ii** $534.07\ \text{mm}^2$

Multiple-choice questions

1 C **2** E **3** B **4** A **5** D

Extended-response question

a $7.85\ \text{m}^3$ **b** $31.42\ \text{m}^2$ **c** $1130

Consumer arithmetic

Short-answer questions

1 $170.40
2 **a** $82.80 **b** $110.40
c $2097.60 **d** $2346
3 $2128
4 $1550
5 $102
6 $5392
7 **a** $37 180 **b** $12 833.60
c $22 829.56 **d** $439.03
8 $8837.34
9 $16.20

Multiple-choice questions

1 C **2** A **3** C **4** D **5** A

Extended-response question

a $711.55
b $59.92
c **i** $149.80 **ii** $832 **iii** $83 **iv** 11.08%

Algebra and indices

Short-answer questions

1 **a** $7xy + 4x$ **b** $-21ab$ **c** $\frac{a}{2}$
2 **a** **i** $-4x + 12$ **ii** $15x^2 + 6x$ **iii** $13x - 6$
b **i** $6(3 - b)$ **ii** $3x(x + 2)$ **iii** $-4y(2x + 3)$
3 **a** $x + 3$ **b** $\frac{3}{4}$ **c** $\frac{9x}{10}$ **d** $\frac{2x - 3}{8}$
4 **a** $10x^6$ **b** $\frac{4x^2}{y^3}$ **c** $8m^{12}$ **d** 4
e $\frac{9a^2}{b^8}$ **f** $\frac{3b^2}{a^5}$ **g** $4t^5$ **h** $\frac{2y^2}{x^4}$
5 **a** **i** 473 000 **ii** 0.00521
b **i** 2.76×10^{-5} **ii** 8.71×10^6

Multiple-choice questions

1 C **2** D **3** A **4** D **5** B

Extended-response question

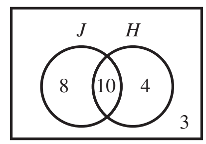
a $A = 3000(1.06)^n$
b **i** $3370.80 **ii** $4255.56
c 11.9 years

Probability

Short-answer questions

1 **a** $\frac{2}{5}$ **b** $\frac{11}{20}$ **c** $\frac{17}{20}$ **d** $\frac{9}{10}$

2 **a**



b

	H	H'	
J	10	8	18
J'	4	3	7
	14	11	25

c 3
d **i** $\frac{4}{25}$ **ii** $\frac{22}{25}$ **iii** $\frac{2}{5}$ **iv** $\frac{5}{9}$
3 **a** 36
b **i** $\frac{1}{9}$ **ii** $\frac{1}{6}$ **iii** $\frac{7}{12}$ **iv** $\frac{1}{6}$
4 **a** $\frac{1}{15}$ **b** $\frac{8}{15}$ **c** $\frac{14}{15}$
5 **a** Yes, $\Pr(A) = \Pr(A \mid B) = \frac{1}{3}$
b No, $\Pr(A) = \frac{5}{8} \neq \Pr(A \mid B) = \frac{3}{5}$

Multiple-choice questions

1 C **2** E **3** A **4** D **5** B

Extended-response question

a

	T	A
T	(T, T)	(A, T)
A	(T, A)	(A, A)

b **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$
c $\frac{1}{4}$

Statistics

Short-answer questions

1 **a**

Class interval	Frequency	Percentage frequency
0–	2	10%
5–	4	20%
10–	4	20%
15–	3	15%
20–	6	30%
25–	1	5%
Total	**20**	**100%**

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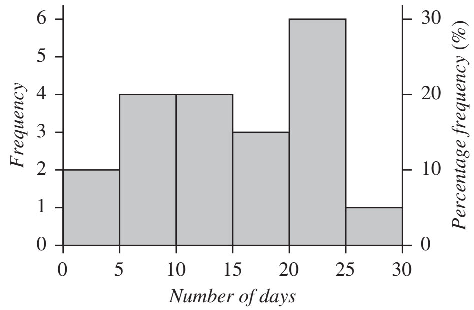
b



c **i** 14 **ii** 50%

iii 20 − 24 days, those that maybe catch public transport to work or school each week day.

2 **a**

Stem	Leaf
0	4 7 8 9
1	2 5 5 7 8
2	4 4 6
3	2 6
4	1

3 | 6 means 36

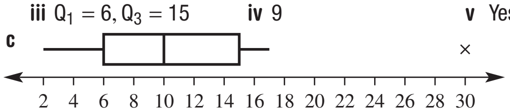
b Skewed

3 **a** 2 4 6 7 8 10 12 12 15 17 30

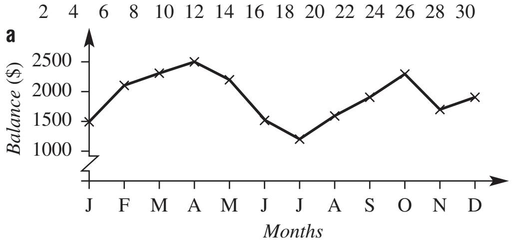
b **i** Min = 2, max = 30 **ii** 10

iii $Q_1 = 6, Q_3 = 15$ **iv** 9 **v** Yes, 30

c



4 **a**



b Balance fluctuated throughout the year but ended up with more money after 12 months.

c May and June

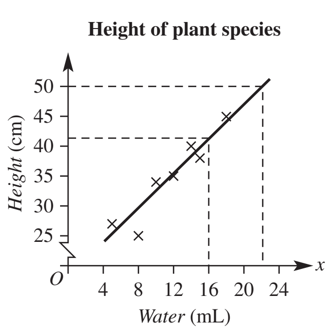
d Increase of $500.

Multiple-choice questions

1 A **2** C **3** E **4** B **5** C

Extended-response question

a, c



b Positive

d **i** 41 cm **ii** 22 mL

Chapter 6

Warm-up quiz

1 **a** $(3,\ 5)$ **b** $(4, -2)$ **c** $(-4, -4)$ **d** $(-3, 1)$
e $(2, -2)$ **f** $(2, 0)$

2 **a** G **b** D **c** B **d** S
e N **f** Q

3 **a** Square **b** Isosceles triangle **c** Hexagon

4 **a** 11 **b** 19 **c** 10 **d** 3.5
e 0 **f** −1 **g** 3.5 **h** −9

5 **a** 120 min **b** 200 km **c** 100 km/h

6 **a** 5 **b** 13 **c** 10 **d** 41
e 3.61 **f** 8.54

7 **a** 3, 4, 5, 6 **b** −2, −1, 0, 1 **c** 0, 2, 4 **d** 6, 5, 4

8 **a** 6 **b** 9 **c** 3 **d** 9
e 5 **f** 9

9 **a** Positive **b** Zero **c** Undefined **d** Negative

6A

Now you try

Example 1

a 240 km **b** 3 hours **c** 360 km

Example 2

a 50 **b** 20 **c** 30

Example 3

a 40 cm **b** 16 cm **a** 3.75 months

Example 4

a $500 **b** $1000 **c** 3 years

Exercise 6A

1 **a** 30 mins **b** 60 mins **c** 90 mins

2 **a** Interpolation **b** Extrapolation

3 **a** 360 km **b** 4 h **c** 540 km

4 **a** **i** $10 000 **ii** $40 000
b Increased
c $30 000

5 **a** 300 deer **b** 100 deer **c** 200 deer

6 **a** **i** 50 cm **ii** 45 kg
b **i** 10 **ii** 3

7 **a** 80 cm **b** 40 cm **c** Approx. $2\frac{1}{4}$ months

8 **a** 400 m **b** Approx. 250 m
c Approx. $3\frac{1}{2}$ days

9 **a** $10 000 **b** $0 **c** 12 years

10 **a** 200 g **b** 200 g **c** $2\frac{1}{4}$ h (2 h 15 min)

11 **a** $\frac{1}{2}$ h (30 min)
b **i** 1 p.m. **ii** 1:15 p.m.
iii Approx. 1:08 p.m. **iv** 1:30 p.m.
c **i** −120 m **ii** −80 m
d **i** 0 m **ii** −160 m **iii** −280 m
e **i** 12:45 p.m. **ii** 2 p.m. **iii** 2:45 p.m.

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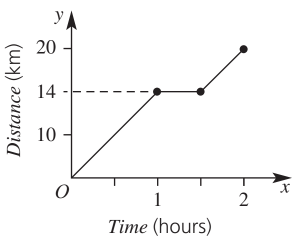
6B

Now you try

Example 5

a 3 km **b** 20 minutes **c** 2.5 km

Example 6



Exercise 6B

1 A, C, E – moving at a constant rate; B, D – at rest

2 **a** S **b** P **c** Q **d** R
e T **f** S

3 **a** 20 km **b** 2 h **c** approx. 17.5 km

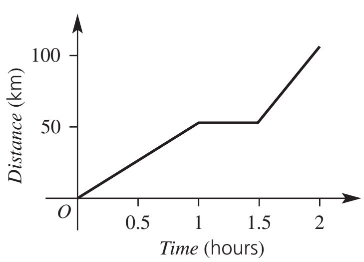
4 **a** **i** 40 kg **ii** 50 kg **iii** 80 kg
b $\frac{1}{2}$ h
c 1st hour

5 **a** 200 m
b 80 s
c **i** Approx. 31 m **ii** 62 m **iii** 150 m

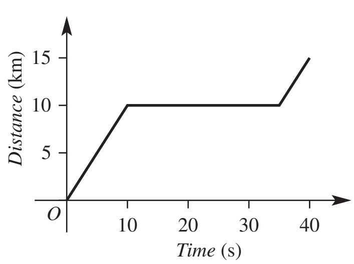
6 **a** **i** 3 m **ii** 2 m **iii** 2 m
b The lawn
c **i** 1.5 m **ii** 4 m **iii** 6.5 m

7 **a** 10 km **b** 20 km **c** 27 min **d** 9 min

8

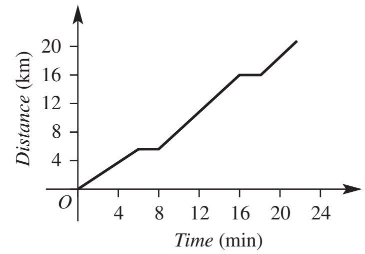


9

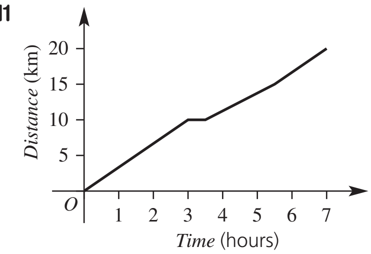


10 **a** 20 km **b** 22 min

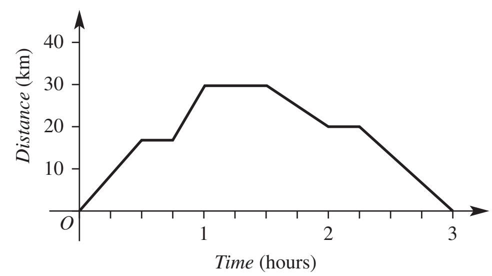
c



11



12 **a**



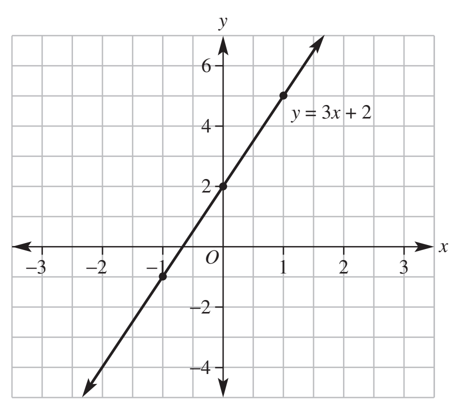
b 48 km/h **c** 20 km/h

6C

Now you try

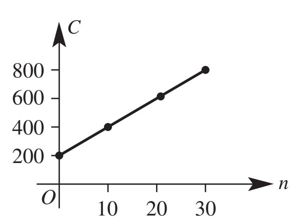
Example 7

x	−1	0	1
y	−1	2	5



Example 8

a



b **i** $700 **ii** 15

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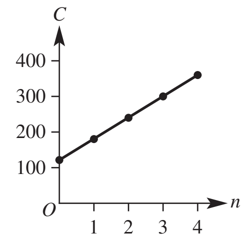
Example 9

a

No. of hours (n)	1	2	3	4
Cost (C)	180	240	300	360

b



c **i** \$270 **ii** 3.5 hours

Exercise 6C

1 **a** $A(2,1)$ $B(-3,3)$ $C(2,-3)$ $D(-4,0)$ $E(4,4)$ $F(0,-2)$ $G(3,0)$ $H(-3,-2)$ $I(1,4)$ $J(-2,-4)$ $K(-4,5)$

b D, O, G **c** O, F **d** (0, 0)

2 (−2, 1) (−1, −1) (0, −3) (1, −5) (2, −7)

3 **a** 6 + 4 = 10, (−3, 10)

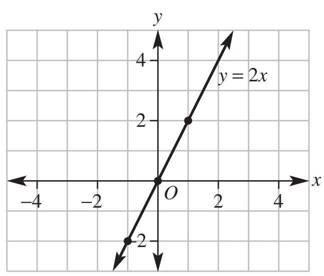
b 2 + 4 = 6, (−1, 6)

c 0 + 4 = 4, (0, 4)

d −4 + 4 = 0, (2, 0)

4

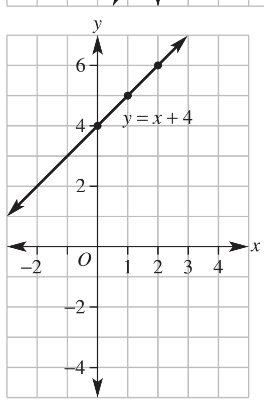
a



$y = 2x$

x	−1	0	1
y	−2	0	2

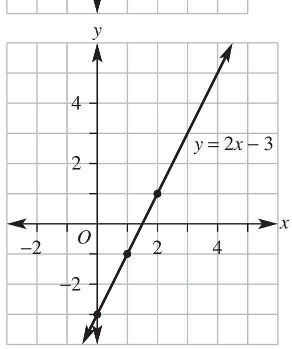
b



$y = x + 4$

x	0	1	2
y	4	5	6

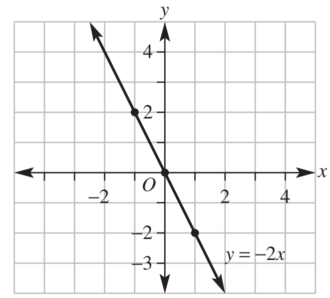
c



$y = 2x - 3$

x	0	1	2
y	−3	−1	1

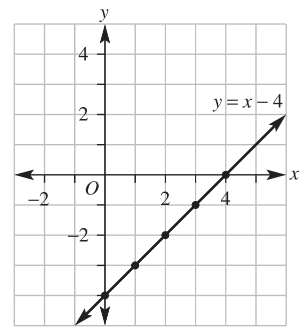
d



$y = -2x$

x	−1	0	1
y	2	0	−2

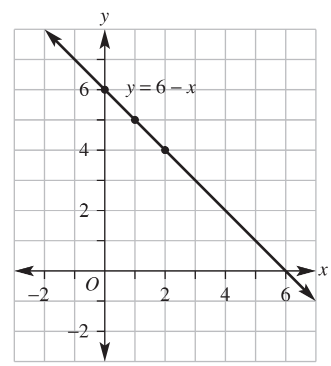
e



$y = x - 4$

x	1	2	3
y	−3	−2	−1

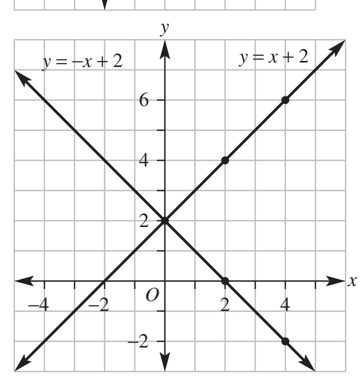
f



$y = 6 - x$

x	0	1	2
y	6	5	4

5 **a**



i

$y = x + 2$

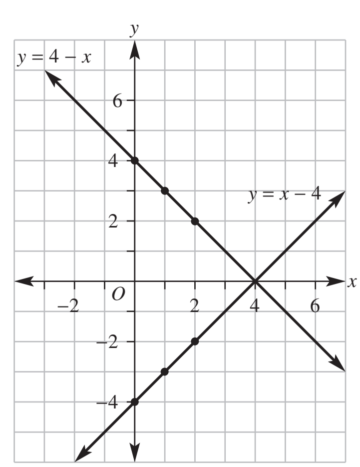
x	0	2	4
y	2	4	6

ii

$y = -x + 2$

x	0	2	4
y	2	0	−2

b



i

$y = x - 4$

x	0	1	2
y	−4	−3	−2

ii

$y = 4 - x$

x	0	1	2
y	4	3	2

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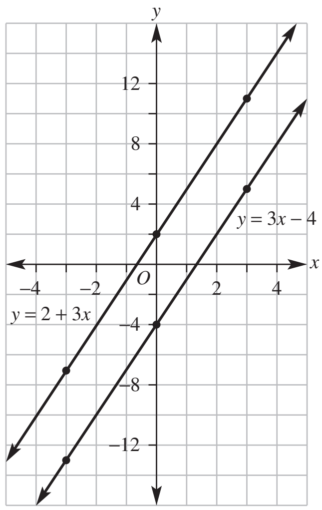
c



i

$y = 2 + 3x$

x	−3	0	3
y	−7	2	11

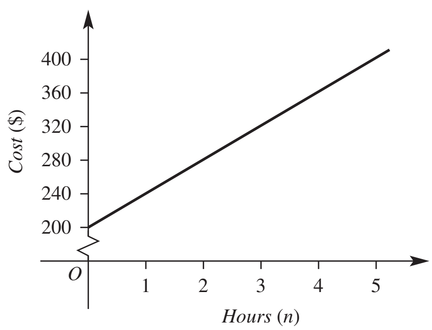
ii

$y = 3x - 4$

x	−3	0	3
y	−13	−4	5

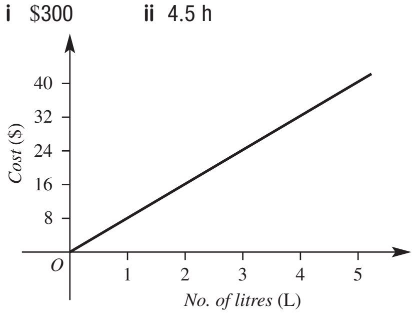
6 **a** $(0, 0)$ **b** $(1, 4)$ **c** $(-1, 3)$ **d** $(0, 2)$ **e** $(-1, -5)$

7 **a**



b **i** \$300 **ii** 4.5 h

8 **a**

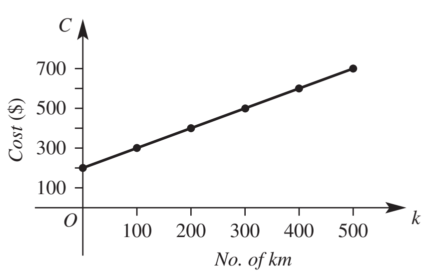


b **i** \$28 **ii** 2.5 litres

9 **a**

No. of km (k)	100	200	300	400	500
Cost (C)	300	400	500	600	700

b

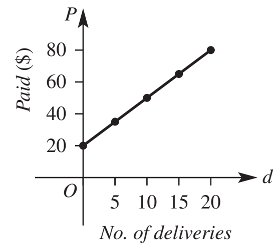


c **i** \$450 **ii** 450 km

10 **a**

No. of deliveries (d)	0	5	10	15	20
Pay (P)	20	35	50	65	80

b



c **i** \$56 **ii** 18

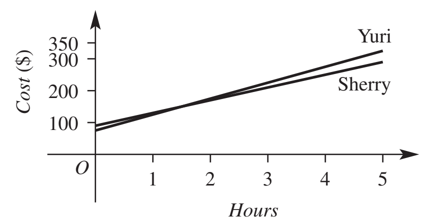
11 **a** Yuri

No. of hours work	0	1	2	3	4	5
Cost (C)	75	125	175	225	275	325

Sherry

No. of hours work	0	1	2	3	4	5
Cost (C)	90	130	170	210	250	290

b



c **i** \$250 **ii** \$150 **iii** 0.5 h **iv** 4.25 h **v** 1.5 h

d Yuri is cheaper only for 1.5 hours or less.

6D

Now you try

Example 10

(4, 7)

Example 11

(4.5, −2)

Example 12

$\sqrt{41}$

Example 13

$\sqrt{52}$

Exercise 6D

1 **a** midpoint **b** length

2 **a** $M = \left(\frac{1+3}{2}, \frac{4+8}{2}\right)$

$M = (2, 6)$

b $M = \left(\frac{-1+5}{2}, \frac{3+-3}{2}\right)$

$= (2, 0)$

3 **a** $AB^2 = 7^2 + 3^2$

$= 49 + 9$

$= 58$

$AB = \sqrt{58}$

4 **a** $(3.5, 5)$ **b** $(3, 4)$ **c** $(0, -1)$ **d** $\left(\frac{1}{2}, \frac{1}{2}\right)$

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5 **a** $(2, 5)$ **b** $(4, 8)$ **c** $(3, 5)$
d $(2.5, 4.5)$ **e** $(6, 2.5)$ **f** $(2.5, 4)$
g $(-1, -2)$ **h** $(-3, -4)$ **i** $(-4, -3)$
j $(1, 1)$ **k** $(-3, -4)$ **l** $(0, 0)$

6 **a** 5 **b** $\sqrt{41}$ **c** 5 **d** $\sqrt{74}$

7 **a** $\sqrt{13}$ **b** $\sqrt{18}$ **c** $\sqrt{29}$
d $\sqrt{29}$ **e** $\sqrt{13}$ **f** $\sqrt{25} = 5$

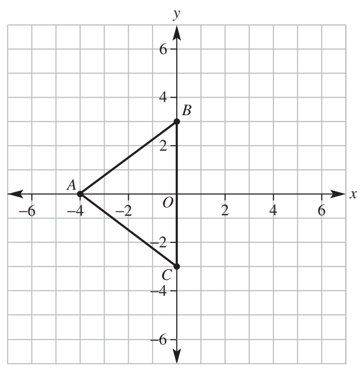
8 **a** 5 **b** 10 **c** 11.2
d 5.7 **e** 5 **f** 3.6

9 **a** $\sqrt{2}$ **b** $\sqrt{13}$ **c** $\sqrt{34}$
d $\sqrt{89}$ **e** $\sqrt{26}$ **f** $\sqrt{10}$

10 $(-2, -5)$

11 **a** $A(1, 4), B(6, 4), C(6, 1), D(1, 1)$
b $(3.5, 2.5)$ **c** $(3.5, 2.5)$
d The diagonals of a rectangle bisect (cut in half) each other.

12 **a, b**



c **i** 5 **ii** 5
d Isosceles
e $P = 16$ units, $A = 12$ units^2
f (4, 0)

13 **a** (3, 4) **b** $\sqrt{13}$
c $\sqrt{13}$; length of radius **d** 22.7 units
e 40.8 units^2

6E

Now you try

Example 14

a $\frac{4}{3}$ **b** $-\frac{5}{2}$

Example 15

a 3 **b** Undefined **c** $-\frac{1}{2}$ **d** 0

Example 16

a $\frac{1}{2}$ **b** $-\frac{7}{5}$

Exercise 6E

1 **a** zero **b** negative **c** positive **d** undefined

2 **a** + **b** − **c** +
d − **e** − **f** +

3 **a** 1 **b** $\frac{1}{4}$ **c** $-\frac{3}{5}$

4 **a** $-\frac{3}{8}$ **b** $\frac{1}{15}$ **c** -3

5 **a** 2 **b** 5 **c** -3 **d** -2 **e** $\frac{4}{3}$
f 0 **g** 0 **h** Undefined **i** Undefined

6 $EF\,\frac{2}{3}, GH\,\frac{2}{3}, DC\ 1, AB\,\frac{3}{2}$

7 **a** 3 **b** 2 **c** $-\frac{1}{2}$ **d** -1 **e** 0 **f** Undefined

8 **a**

Line segment	Rise	Run	Gradient
AB	1	2	$\frac{1}{2}$
AC	2	4	$\frac{1}{2}$
AD	3	6	$\frac{1}{2}$
BC	1	2	$\frac{1}{2}$
BD	2	4	$\frac{1}{2}$
CD	1	2	$\frac{1}{2}$

b They have the same gradient.

9 **a** 2 **b** 1 **c** -1
d -2 **e** 1 **f** $-\frac{2}{5}$

10 **a** 1 **b** 1 **c** $-\frac{3}{5}$ **d** 0
e 11 **f** $\frac{1}{3}$

11 Gradient = 0.344

12 **a** A, D **b** C, E, G **c** D
d B, F, H **e** G

6F

Now you try

Example 17

a **i** 10 km **ii** 10 km/h
b **i** 10 km/h **ii** 0 km/h **iii** 7.5 km/h

Example 18

a **i** 2 L **ii** 2 L
b **i** 0.4 L/s **ii** 0.2 L/s

Exercise 6F

1 **a** gradient **b** two **c** units
d km/h **e** L/min

2 **a** 90 km/h **b** 15 L/min

3 **a** **i** 60 km **ii** 60 km/h
b **i** 60 km/h **ii** 0 km/h **iii** 90 km/h

4 **a** **i** 15 km **ii** 15 km/h
b **i** 0 km/h **ii** 30 km/h

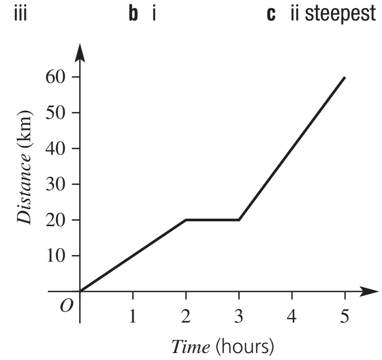
5 **a** **i** 2 L **ii** 0.5 L
b **i** 0.2 L/s **ii** 0.05 L/s **iii** 0.05 L/s

6 **a** **i** 1.5 L **ii** 0.5 L
b **i** 0.15 L/s **ii** 0.05 L/s **iii** 0.15 L/s

7 **a** 3 km
b 4 min
c **i** 0.5 km/min **ii** 0.75 km/min
iii 0.5 km/min **iv** 0.25 km/min

8 **a** iii **b** i **c** ii steepest

9 **a**



b 60 km

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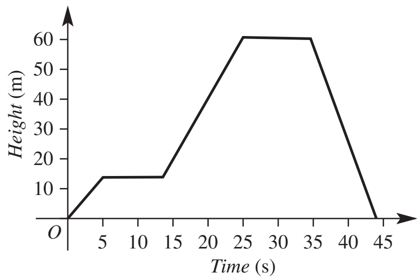
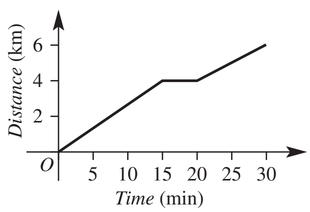
10 

Time is 44 seconds.

11 a 3
b i 6 km **ii** 14 km
c B, D, G
d E, H
e i 6 km/h **ii** 14 km/h **iii** 16 km/h
iv 6.4 km/h **v** 16 km/h
f E and H, same gradient
g $5\frac{1}{4}$ h
h 40 km
i 10 km/h

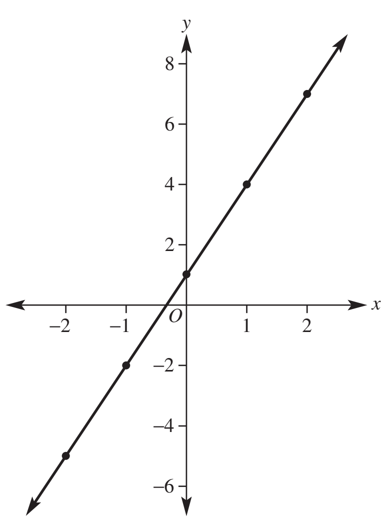
Progress quiz

1 a 10 cm **b** 30 cm **c** 30 cm **d** 50 cm
2 a 160 km **b** 80 km/h **c** 50 km/h

3 

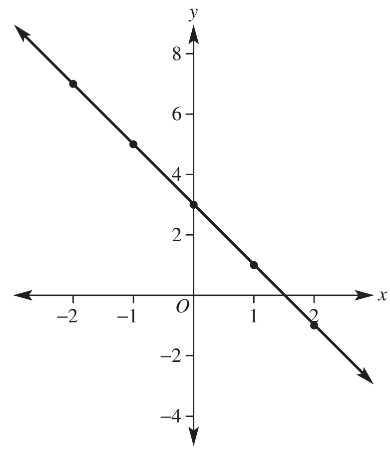
4 a $y = 3x + 1$

x	−2	−1	0	1	2
y	−5	−2	1	4	7



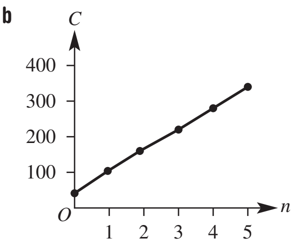
b $y = -2x + 3$

x	−2	−1	0	1	2
y	7	5	3	1	−1



5 a

No. of hours (n)	1	2	3	4	5
Cost ($$C$)	100	160	220	280	340

b 

c i $190 **ii** 3.5 hours
6 a Midpoint (3.5, 8), distance = 5 units
b Midpoint (1, 9.5), distance = $\sqrt{61}$
7 a 4 **b** $-\frac{1}{2}$ **c** 0 **d** Undefined
8 $\frac{4}{5}$

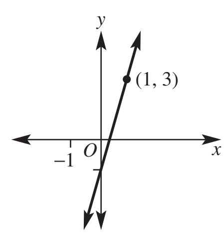
6G

Now you try

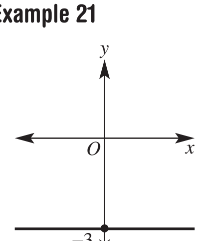
Example 19

a i 2 **ii** −5
b i $-\frac{1}{3}$ **ii** 2

Example 20



Example 21

a 

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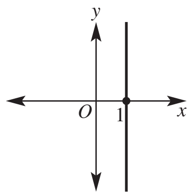
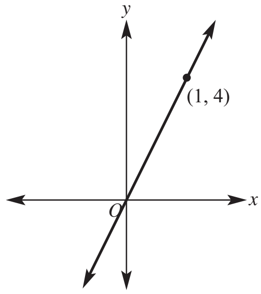
b



c



Example 22

No

Exercise 6G

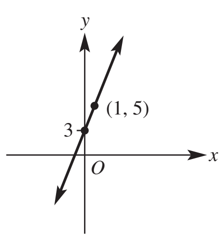
1 **a** gradient–intercept **b** gradient **c** coefficient
d y-intercept **e** zero

2 **a** $y = 3x - 1$ **b** $y = -\frac{3}{2}x + 2$ **c** $y = -x$

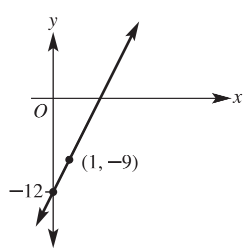
3 **a** Horizontal
b Vertical
c Passes through the origin
d Vertical and passes through the origin
e Passes through the origin

4 **a** **i** 2 **ii** 4
b **i** 6 **ii** −7
c **i** $-\frac{2}{3}$ **ii** 7
d **i** −7 **ii** −3
e **i** $\frac{3}{5}$ **ii** −8
f **i** 9 **ii** −5

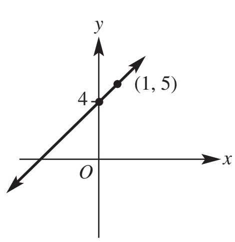
5 **a**



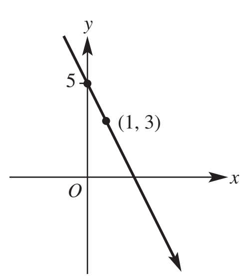
b



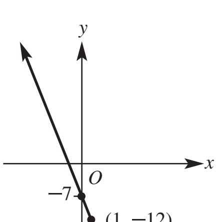
c



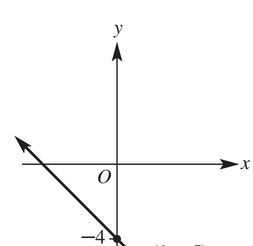
d



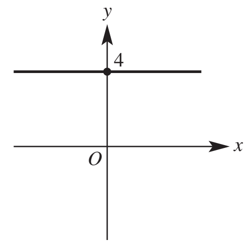
e



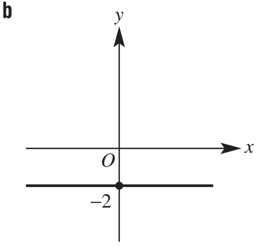
f



6 **a**



b



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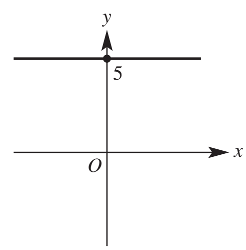

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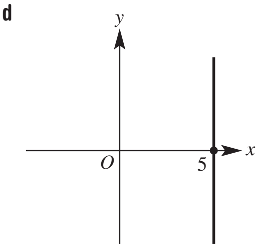

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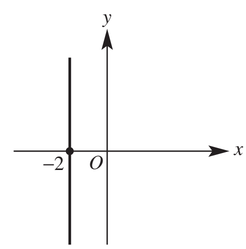

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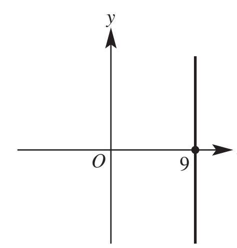

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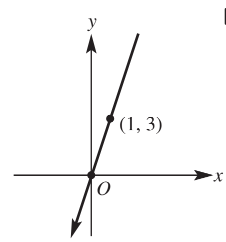
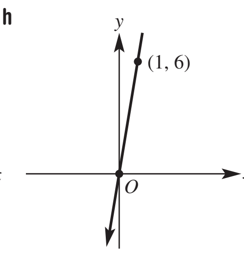
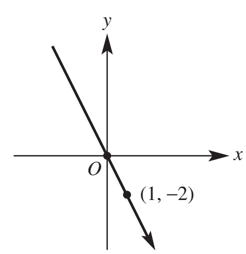


h



i



7 **a** 1, 0 **b** $-\frac{3}{2}$, 3 **c** $\frac{1}{2}$, 4.5 **d** 0, 4
e Undefined, none

8 **a** iv **b** ii **c** v **d** i **e** vi
f vii **g** iii **h** viii **i** ix

9 **a** $c = 5$ in each equation; e.g. $y = 2x + 5$, $y = -3x + 5$ etc.
b $c = -2$ in each equation; e.g. $y = 7x - 2$, $y = x - 2$ etc.

10 **a** $m = 3$ in each equation; e.g. $y = 3x - 1$, $y = 3x$, $y = 3x + 4$ etc.
b $m = -1$ in each equation;
e.g. $y = -x$, $y = -x + 7$, $y = -x - 3$ etc.
c $m = 0$ in each equation; e.g. $y = 4$, $y = -2$ etc.
d m is undefined in each equation; e.g. $x = -7$ etc.

11 **a** ii and iii **b** i and iv

12 **a** No **b** Yes **c** No **d** Yes **e** Yes

13 **a** $y = x + 4$ **b** $y = x - 1$
c $y = \frac{x}{2}$ **d** $y = 2x + 1$

14 Check with your teacher.

15 **a** They all have the same gradient.
b They all have a y-intercept at −1.
c Check with your teacher.

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6H

Now you try

Example 23

a Parallel **b** Perpendicular

Example 24

a $y = 2x + 4$ **b** $y = -\frac{1}{4}x + 3$

Example 25

a $y = 3x - 3$ **b** $y = 2x + 6$

Exercise 6H

1 a C, D **b** B, E **c** Yes **d** Yes **e** Yes

2 a 4 **b** −7 **c** $-\frac{3}{4}$ **d** $\frac{8}{7}$

3 a $-\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $-\frac{8}{7}$ **d** $\frac{9}{4}$

4 a Parallel **b** Perpendicular **c** Neither
d Neither **e** Parallel **f** Parallel
g Neither **h** Neither **i** Perpendicular
j Perpendicular

5 a $y = 4x + 2$ **b** $y = 2x + 4$
c $y = -x - 3$ **d** $y = -\frac{1}{2}x + 3$
e $y = -\frac{1}{3}x - 5$ **f** $y = -2x - 10$
g $y = -6x + 6$ **h** $y = 4x - 7$

6 a $y = x + 4$ **b** $y = -x + 8$
c $y = -4x - 1$ **d** $y = \frac{2}{3}x - 6$
e $y = -\frac{1}{2}x + 6$ **f** $y = \frac{1}{4}x - 2$
g $y = -\frac{3}{2}x + 5$ **h** $y = \frac{7}{2}x + 31$

7 a i 1 **ii** −1
b i $y = x$ **ii** $y = x - 2$ **iii** $y = x + 3$
iv $y = -x + 2$ **v** $y = -x$ **vi** $y = -x - 4$

8 a $y = 2x + 2$ **b** $y = 2x - 4$
c $y = -\frac{1}{2}x - 4$ **d** $x = -8$

9 a $y = -\frac{3}{2}x + 7$ **b** $y = -\frac{2}{3}x - 2$
c $y = \frac{5}{4}x + 2$ **d** $y = \frac{3}{2}x - \frac{7}{2}$

10 a Parallel **b** Parallel **c** Perpendicular
d Perpendicular

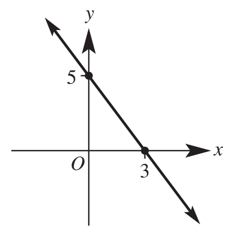
11 a i 1 **ii** 1 **iii** −1 **iv** −1
b Opposite sides are parallel; adjacent sides are perpendicular.
c Rectangle

12 a i $\frac{4}{3}$ **ii** $-\frac{3}{4}$ **iii** 0
b Right-angled triangle because AB is perpendicular to BC.

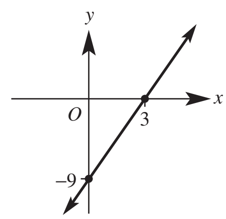
6I

Now you try

Example 26



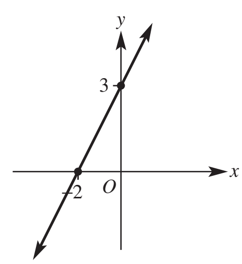
Example 27



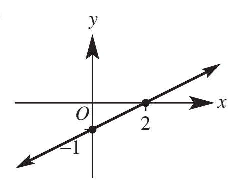
Exercise 6I

1 a The x-intercept is where $y = 0$.
b The y-intercept is where $x = 0$.

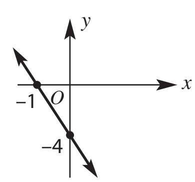
2 a



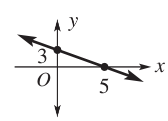
b



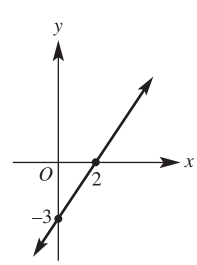
c



d



3 a



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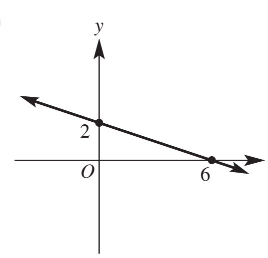
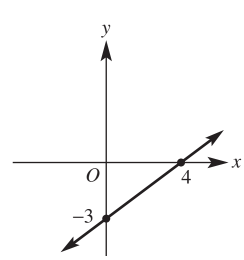
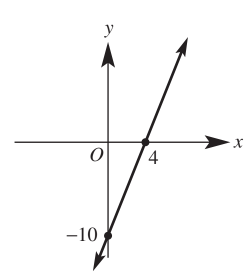


c



d



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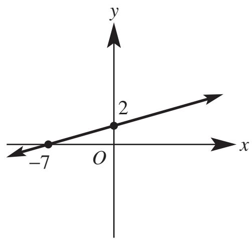
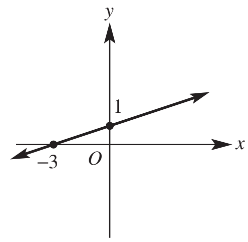
e



f



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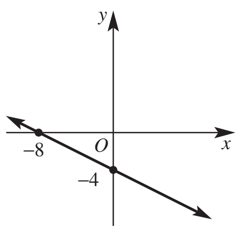

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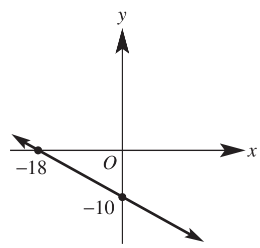
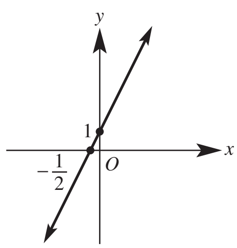


4 a ii **b** iii **c** i

5 a



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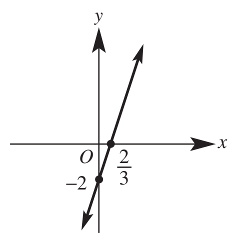

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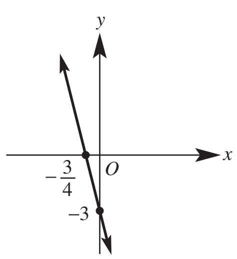
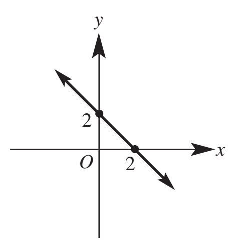


d



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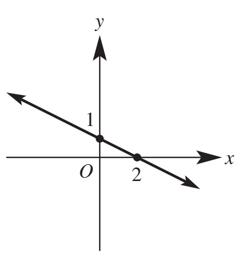
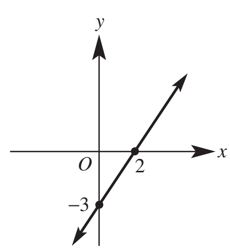


f



6 a iii **b** ii **c** i

7 a vi **b** ii **c** i
d v **e** iv **f** iii

8 a −2 **b** $\frac{1}{5}$ **c** 2 **d** $\frac{1}{2}$

9 a i 2, −4 **ii** $A = 4$ units^2
b i −2, 2 **ii** $A = 2$ units^2
c i −1.5, −3 **ii** $A = 2.25$ units^2
d i −4, 2 **ii** $A = 4$ units^2

10 a 90 m **b** $7\frac{1}{2}$ s

11 Many answers; e.g. $2x + y = 4, a = 2, b = 1, d = 4$

12 a 2, −4 **b** −5, −10 **c** 1, 1 **d** 2, 4
e −4, 6 **f** $-1, \frac{2}{3}$

6J

Now you try

Example 28

a $\frac{1}{2}$ **b** 2 **c** $y = \frac{1}{2}x + 2$

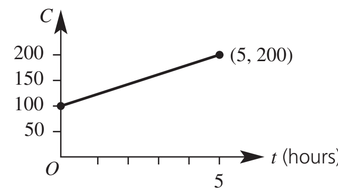
Example 29

a −3 **b** 21 **c** $y = -3x + 21$

Example 30

a $C = 20t + 100$

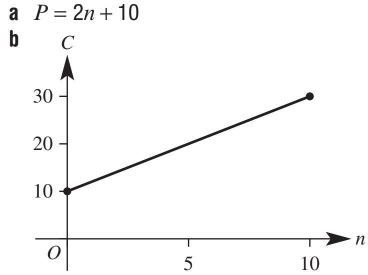
b



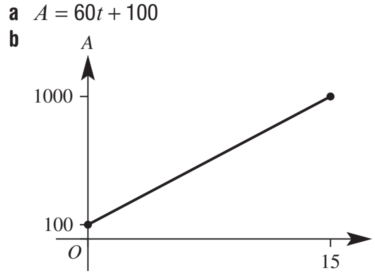
c i \$160 **ii** 8 hours

Exercise 6J

1 D
2 C
3 a i $m = 2$ **ii** 1 **iii** $y = 2x + 1$
b i $m = -3$ **ii** 2 **iii** $y = -3x + 2$
c i $m = 4$ **ii** −3 **iii** $y = 4x - 3$
d i $m = -4$ **ii** 0 **iii** $y = -4x$
e i $m = -1$ **ii** 2 **iii** $y = -x + 2$
f i $m = 2$ **ii** 0 **iii** $y = 2x$
4 a i 3 **ii** 1 **iii** $y = 3x + 1$
b i −1 **ii** 6 **iii** $y = -x + 6$
c i 5 **ii** 21 **iii** $y = 5x + 21$
d i 3 **ii** −16 **iii** $y = 3x - 16$
e i −3 **ii** 2 **iii** $y = -3x + 2$
f i $\frac{1}{2}$ **ii** $\frac{3}{2}$ **iii** $y = \frac{1}{2}x + \frac{3}{2}$
5 a $y = 4$ **b** $y = -1$ **c** $x = 3$
d $x = -2$ **e** $y = 2$ **f** $x = 7$
6 a $y = 2x$ **b** $y = 3x$ **c** $y = \frac{2}{3}x$
d $y = -3x$ **e** $y = -x$ **f** $y = -\frac{2}{5}x$
7 a i 7 **ii** $y = 7x$
b i $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
c i 1 **ii** $y = x + 2$
d i $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
e i 0 **ii** $y = 2$
f i Undefined **ii** $x = 3$
8 a $P = 2n + 10$
b



c i \$28 **ii** 23.5 kg
9 a $A = 60t + 100$
b



c i \$820 **ii** 10 h

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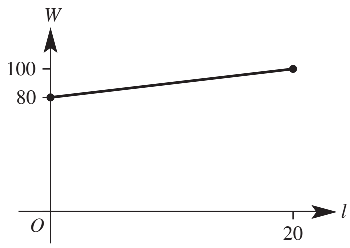
10 a $W = l + 80$

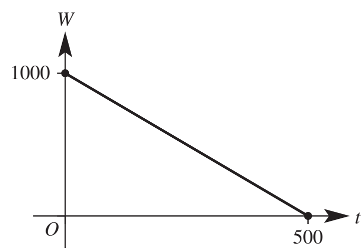
b



c i 87 kg **ii** 29 L

11 a −2, (0, 1000)

b



c 1000 L

d i 360 L **ii** 952 L **iii** 664 L

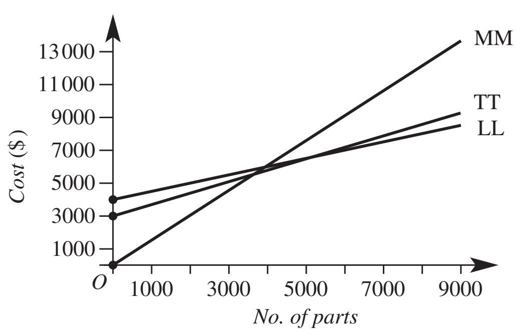
e i 350 h **ii** 407.5 h

12 a

p	0	1000	2000	3000	4000
Cost, MM	0	1400	2800	4200	5600
Cost, TT	3000	3700	4400	5100	5800
Cost, LL	4000	4500	5000	5500	6000

p	5000	6000	7000	8000	9000
Cost, MM	7000	8400	9800	11 200	12 600
Cost, TT	6500	7200	7900	8 600	9 300
Cost, LL	6500	7000	7500	8 000	8 500

b



c i $2100 **ii** $1400 **iii** $7250 **iv** $8750

d Mandy's is best for parts less than or equal to 4285. Terry's is best for between 4286 and 5000 parts and equal to Lenny's at 5000. Lenny's is best for parts > 5000.

6K

Now you try

Example 31

a $C = 1.38n$ **b** $66.24

Example 32

a $k = 12.5$ **b** $W = 12.5n$ **c** $75

d 19 hours

Example 33

a 50 cm **b** 24 inches

Exercise 6K

1 a decreases **b** increases

2 a ii and v

b i $k = 5$ **ii** $k = 12$ **iii** $k = 40$ **iv** $k = 22$

v $k = 7.5$

3 a $k = 4, y = 4x$ **b** $k = 1.2, y = 1.2x$

4 a $W = 11.5t$ **b** $431.25

5 a $V = 6t$ **b** 1440 L **c** $k = 8640,\ V = 8640t$

d 60 480 L

6 a $k = 244$ **b** $P = 244n$

c $33 184 **d** 1175 tonnes

7 a $k = 774$ **b** $d = 774t$

c 4.26 h **d** 619.2 km

8 a 48 cm **b** 10 inches **c** 30 cm

d 10 cm **e** 22 inches **f** 76 cm

g 63 cm (or 64 cm) **h** 100 cm

9 a 94 cm = 37 inches, $m = \frac{94}{37} = 2.54$ **b** 2.54 cm/inch

c $k = 2.54$ **d** $y = 2.54x$

e 127 cm

10 a i 98 NZD **ii** 62 EUR **iii** 51 GBP **iv** 41 AUD

v 41 AUD **vi** 41 AUD

b i 40 EUR = 50 AUD, $m = \frac{40}{50} = 0.8$ **ii** 0.8 EUR/AUD

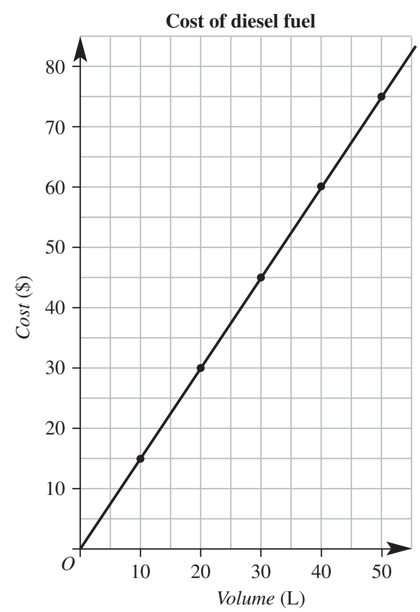
iii $k = 0.8$ EUR/AUD **iv** $y = 0.8x$

v 500 EUR

11 a

Volume (*V*) of diesel, in litres	0	10	20	30	40	50
Cost (*C*), in dollars	0	15	30	45	60	75

b



c $m = 1.5$ **d** Rate = $1.5/L

e $k = 1.5$

12 a Number of hours worked and wages earned are in direct proportion. If the number of hours worked doubles, then the wages earned also doubles.

b The cost of buying tomatoes and the number of kg are in direct proportion. If the number of kg doubles, then the cost also doubles.

c The speed and time taken are not in direct proportion. If the speed increases, then the time taken decreases. Alternatively, if the speed decreases, then the time taken increases.

d The size of the movie file and the time taken to download it are in direct proportion. If the size of the movie file is doubled, then the time to download will also double.

e The cost of a taxi ride and the distance travelled are not in direct proportion because the graph does not start at (0, 0). The y-intercept is the flag fall cost.

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13 a 15 cents/min **b** 40 cents/min
c 3 mL/s **d** 5 mL/s **e** 20 m/s **f** 5 m/s
g 1.5 cents/g **h** 3.2 cents/g
i 4.8 kg/year **j** 1.54 kg/week
14 a $d = 90t$ **b** $k = 25$, $d = 25t$
c 100 m
15 a Singapore dollar (SGD)
i SGD = 1.2 × AUD **ii** SGD \$288
iii AUD \$200
b Chinese yuan (CNY)
i CNY = 6.47 × AUD **ii** CNY 485.25
iii AUD \$11.59
c South African rand (ZAR)
i ZAR = 9.5 × AUD **ii** ZAR 475
iii AUD \$5.26

6L

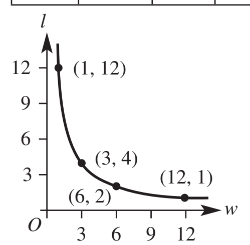
Now you try

Example 34

a

w	1	3	6	12
l	12	4	2	1

b



c 3 metres
d 4.8 metres

Example 35

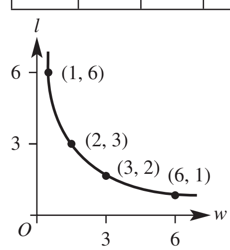
a $k = 8$ **b** 8

Exercise 6L

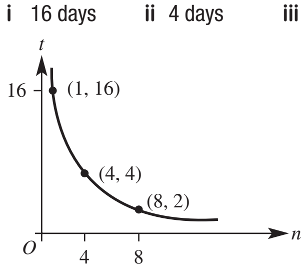
1 a Direct **b** Indirect **c** Direct **d** Direct **e** Indirect
2 a Yes **b** No **c** Yes
3 a 6 **b** 1
4 a

w	1	2	3	6
l	6	3	2	1

b



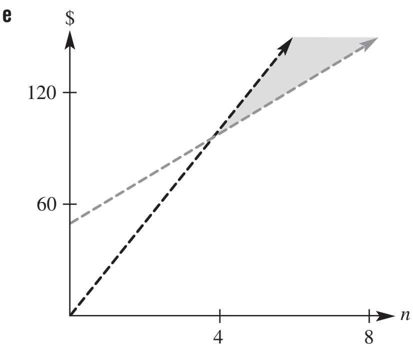
c 0.75 metres
d 4 metres
5 a i 16 days **ii** 4 days **iii** 2 days
b



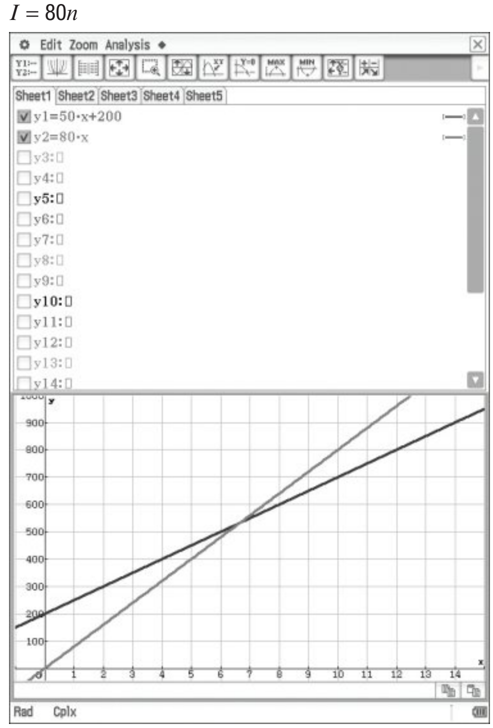
c 1.6 days
d 2 painters
6 a i 4 **ii** 2.5
b i 2 **ii** 0.5
7 a $k = 50$
b 25
8 a $k = 12$ and $\{4, 3\}$ **b** $k = 4$ and $\{4, 1\}$
9 a $k = 40$ **b** 10 cm^3 **c** 2.5 kg/cm^3
10 a i \$250 **ii** \$100
b $C = \dfrac{500}{n}$
c 8 people
11 a $I = \dfrac{240}{R}$ **b** 6 amps **c** 2 ohms **d** 100%

Maths@Work: Accountant or small business owner

1 a Income: blue line; costs: red line
b 0, 1, 2 or 3
c \$0, \$0
d 4
e



f $P = 13n - 50$
g i \$80 **ii** \$145 **iii** \$340 **iv** \$600
2 a 40; cost of advertisement = \$40
b \$2.67
c 6.67; \$6.67 retail price per glass
d 10
e i $C = 2.67n + 40$ **ii** $I = 6.67n$
iii $P = 4n - 40$
3 a $C = 200 + 50n$
b $I = 80n$
c



d 7 car parts
e \$760

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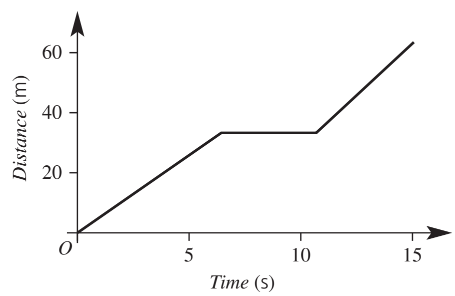
4 a i 500, \$5000 **ii** 375, \$4500 **iii** 300, \$4200
iv 250, \$4000
b $P = 500a - 5000$
c i \$0 **ii** \$1000 **iii** \$2000 **iv** \$3000
d People will stop buying parts if they are more expensive than the price from another business or if they are just too expensive for the customer to afford.

Puzzles and games

1 PLOTTING LINES!
3 Both 13 km apart
4 a 160 ice-creams for zero profit
b 493 ice-creams sold

Short-answer questions

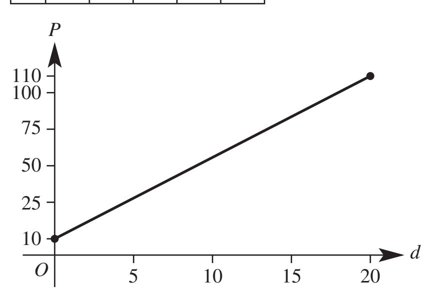
1 a 40 km **b** 2 hours **c** 60 km
2 a i \$6000 **ii** \$8000 **iii** \$9000
b i \$2000 **ii** \$4000 **iii** \$5000
c 10 years
3 a 15 km
b 3 h
c i 6 km **ii** 6 km **iii** 9 km **iv** 15 km
d $3\frac{1}{2}$ h
4



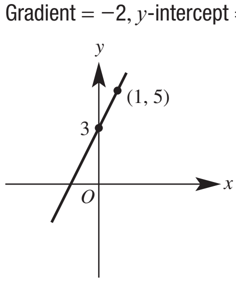
5 a

d	0	5	10	15	20
P	10	35	60	85	110

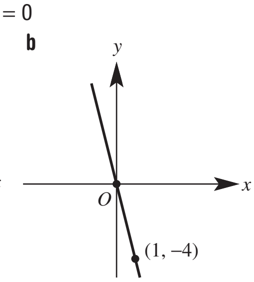
b



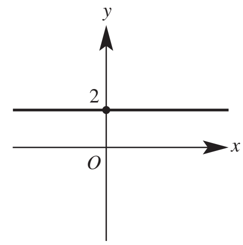
c i \$70 **ii** 17
6 a 1 **b** −1 **c** −2 **d** $\frac{1}{3}$
7 a (4, 2) **b** (2, 2.5) **c** (1.5, −1)
8 a $AB = 5$ **b** $PQ = \sqrt{50}$
9 a Gradient = 3, y-intercept = 4
b Gradient = −2, y-intercept = 0
10 a



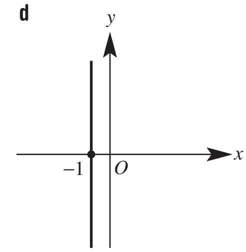
b



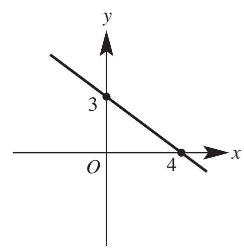
c



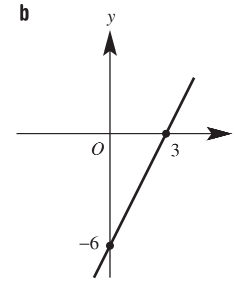
d



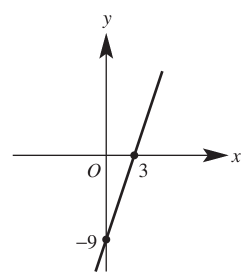
11 a



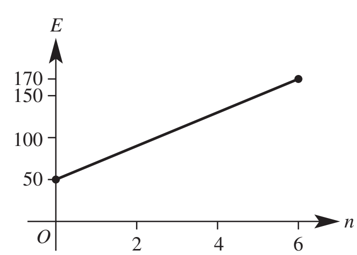
b



c



12 a i 2 **ii** 0 **iii** $y = 2x$
b i −4 **ii** 16 **iii** $y = -4x + 16$
13 a i
b iii
c vi
d ii
e iv
f v
14 a $E = 20n + 50$
b



c i \$130 **ii** 5.5 bins
15 a $y = 2x + 3$ **b** $y = \frac{1}{2}x - 1$ **c** $y = -x + 2$
d $y = -\frac{4}{3}x - 7$ **e** $y = -4x + 6$ **f** $y = -\frac{1}{2}x + 4$
16 a $k = \$16.50/\text{h}$
b $W = 16.5n$
c i \$132 **ii** 16 hours
17 a 8 km **b** 22 miles **c** $m = \frac{15}{24.14} = 0.621$
d 0.621 miles/km **e** $k = 0.621$ miles/km
f $y = 0.621x$ **g** 62.1 miles **h** 161 km
18 a Direct proportion. If the number of cricket balls increases, then the cost will also increase.
b Indirect proportion. If the number of people increases, then the cost per person decreases.

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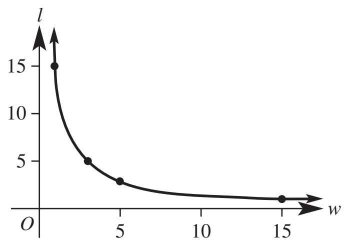
19 a $k = 15$

b

w	1	3	5	15
l	15	5	3	1

c



d 1.5 m
e 3.75 m

Multiple-choice questions

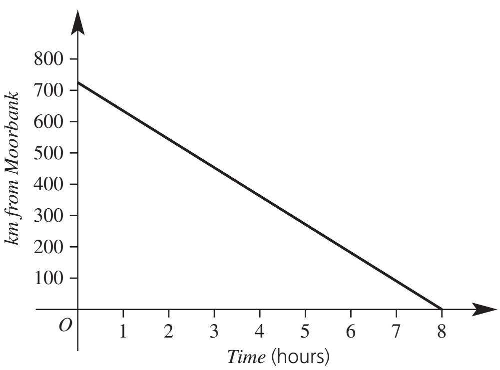
1 A	**2** E	**3** D	**4** A	**5** B	**6** E
7 C	**8** A	**9** B	**10** D	**11** D	**12** E
13 C	**14** D				

Extended-response questions

1 a

Time in hours (t)	0	2	4	6	8
km from Moorbank	720	540	360	180	0

b

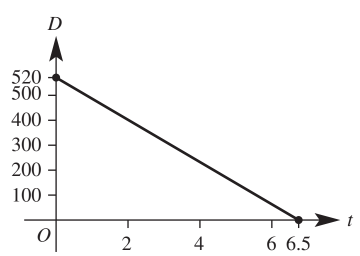


c 9 a.m. **d** 7 h **e** 8 h **f** 4 p.m.

2 a i 440 km **ii** 280 km

b i $D = -80t + 520$ or $D = 520 - 80t$

ii



c i 160 km **ii** 2.25 hours

Chapter 7

Warm-up quiz

1 a Acute **b** right **c** obtuse
d straight **e** reflex **f** revolution
g Complementary **h** Supplementary

2 a E **b** B **c** F
d A **e** C **f** D

3 a $a = 110$ **b** $b = 140$ **c** $c = 210$
d $d = 35$ **e** $e = 60$ **f** $f = 40$

4 a $a = 110, b = 70$ **b** $a = 105, b = 75$
c $a = 40, b = 140$

5 a 1 **b** 4 **c** 2
d 2 **e** 1 **f** 1

6 a $S = 360°, a = 130$ **b** $S = 540°, b = 120$
c $S = 720°, c = 120$

7 a D **b** C

8 $\triangle ABD$ and $\triangle CBD$

9 $\triangle ACD$ and $\triangle ABE$

7A

Now you try

Example 1

$a = 110$ (supplementary),
$b = 110$ (alternate)
$c = 70$ (corresponding or cointerior)
$d = 70$ (vertically opposite)

Example 2

a No – corresponding angles are not equal.
b Yes – cointerior angles are supplementary.

Exercise 7A

1 a 180°
b equal
c i equal **ii** equal **iii** supplementary

2 a Alternate **b** Vertically opposite
c Cointerior **d** Corresponding

3 $a = 20$ supplementary, $b = 20$ alternate,
$c = 160$ corresponding, $d = 160$ vertically opposite

4 $a = 100$ supplementary, $b = 100$ alternate,
$c = 80$ corresponding, $d = 80$ vertically opposite

5 a $x = 110,\ y = 110$ **b** $x = 40,\ y = 140$
c $x = 75,\ y = 105$ **d** $x = 120,\ y = 120$
e $x = 110,\ y = 70$ **f** $x = 105,\ y = 75$

6 a Yes, corresponding angles are equal.
b No, alternate angles are not equal.
c No, cointerior angles are not supplementary.
d Yes, cointerior angles are supplementary.
e No, corresponding angles are not equal.
f Yes, alternate angles are equal.

7 a 60 **b** 20 **c** 100
d 115 **e** 50 **f** 30

8 a $a = 90,\ b = 90,\ c = 90$ **b** $a = 90,\ b = 90,\ c = 90$
c $a = 135,\ b = 45,\ c = 135$ **d** $a = 50,\ b = 130,\ c = 50$
e $a = 90,\ b = 130$ **f** $a = 110,\ b = 120$

9 a $(a, e), (d, f), (b, h), (c, g)$
b $(d, h), (c, e)$ **c** $(c, h), (d, e)$
d $(a, c), (b, d), (e, g), (f, h)$

10 a 90 **b** 75 **c** 10
d 30 **e** 36 **f** 30

11 a 12 **b** 14 **c** 10

7B

Now you try

Example 3

$x = 20$

Example 4

$x = 75$

Example 5

a $x = 130$ **b** $x = 45$

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Exercise 7B

1 a 180° b isosceles c 60°
d obtuse e scalene f 90°
2 C
3 a $c = 120$ b $x = 60$ c $x = 25 + 35$
4 a 70 b 10 c 25
d 58 e 50 f 29
5 a 65 b 80 c 40
d 20 e 112 f 32
6 a 145 b 144 c 45
d 60 e 60 f 47
7 a Yes b Yes c No
d Yes e Yes f Yes
g No h Yes i Yes
8 a 65° b 115°
9 a $a = 70$ b $a = 120$ c $a = 70,\ b = 70$
d $a = 35,\ b = 105$ e $a = 115,\ b = 115,\ c = 45,\ d = 20$
f $a = 40,\ b = 100,\ c = 40,\ d = 40$
10 a a°, alternate b b°, alternate c Sum to 180°
11 a i 50° ii 130°
b i 20° ii 160°
c i 0° ii 180°

7C

Now you try

Example 6

120°

Example 7

a $x = 100, y = 145$ b $x = 90$

Exercise 7C

1 a 360° b trapezium and kite
2 a Parallelograms include squares, rectangles and rhombuses.
3 a Square, rectangle, rhombus, parallelogram
b Rectangle, square, parallelogram, rhombus, kite
c Rhombus, square, parallelogram, rectangle
d Trapezium e Kite f Square, rectangle
g Square, rectangle h Square, rhombus, kite
4 a $x = 20$ b $x = 110$ c $x = 240$
d $x = 125$ e $x = 30$ f $x = 55$
5 a $x = 60,\ y = 120,\ z = 60$
b $x = 110,\ y = 110,\ z = 70$
c $x = 30,\ y = 150,\ z = 30$ d $x = 45$
e $x = 100$ f $x = 25$ g $x = y = z = 90$
h $x = 100,\ y = 140$ i $x = 75,\ y = 20$
6 a 115 b 60 c 30
d 50 e 90 f 140
7 a It has two equal side lengths.
b i 120 ii 40
c It has two equal side lengths and two pairs of equal angles and one pair of parallel sides.
8 b 65°
9 a 125 b 118 c 115 d 110
10 a $a = 30,\ b = 120,\ c = 60,\ d = 60,\ e = 30$
b $a = 80,\ b = 100,\ c = 80,\ d = 50$
c $a = 40,\ b = 20,\ c = 50,\ d = 110$
d $b = 50,\ c = 70$
e $a = 60,\ b = 20$
f $a = 10,\ b = 80$

7D

Now you try

Example 8

$x = 125$

Example 9

a 35 cm b 900° c 128.57°

Exercise 7D

1 a 4 b 8 c 10 d 7
e 9 f 6 g 5 h 12
2 a 540° b 720° c 900°
d 1080° e 1260° f 1440°
3 All sides and all angles equal.
4 a 720°, 110 b 540°, 130
c 540°, 30 d 900°, 105
e 720°, 30 f 360°, 30
5 a 24 cm b 720° c 120°
6 a 28 cm b 1080° c 135°
7 a 120° b 240°
8 a 1620° b 3240°
9 a 144° b 165.6°
10 a 120° b 240°
11 a 72° b 252°
12 a $x = 108,\ y = 72$ b $x = 60,\ y = 60$
c $x = 45,\ y = 225$
13 a *See table at top of next page.*
b i $S = 180° \times (n - 2)$ ii $A = \dfrac{180° \times (n-2)}{n}$

7E

Now you try

Example 10

a $\triangle ABC \equiv \triangle DEF$ (SSS) b $\triangle MNO \equiv \triangle EFG$ (RHS)

Example 11

a $\angle ABC = \angle DEF$ (A)
$\angle ACB = \angle DFE$ (A)
$AB = DE$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)
b $XY = ST$ (S)
$\angle XYZ = \angle STU$ (A)
$YZ = TU$ (S)
$\therefore \triangle XYZ \equiv \triangle STU$ (SAS)

Exercise 7E

1 a False b False c True d True
2 SSS, SAS, AAS, RHS
3 a E b AC c $\angle EDF$
4 a $\triangle ABC \equiv \triangle DEF$ (SSS)
b $\triangle ABC \equiv \triangle DEF$ (SAS)
c $\triangle XYZ \equiv \triangle STU$ (RHS)
d $\triangle XYZ \equiv \triangle STU$ (SSS)
e $\triangle ABC \equiv \triangle DEF$ (AAS)
f $\triangle MNO \equiv \triangle PQR$ (AAS)
5 a $AB = DE$ (S)
$BC = EF$ (S)
$AC = DF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SSS)
b $AC = DF$ (S)
$AB = DE$ (S)
$BC = EF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SSS)
c $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SAS)

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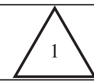
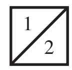
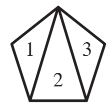
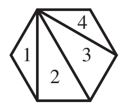
Polygon	No. of sides	Diagram	No. of triangles	Total angle sum (S)	Regular polygon internal angle (A)
Triangle	3		1	180°	60°
Quadrilateral	4		2	360°	90°
Pentagon	5		3	540°	108°
Hexagon	6		4	720°	120°
…					
n-gon	n		$n - 2$	$180° \times (n - 2)$	$\frac{180° \times (n-2)}{n}$

d $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SAS)

e $\angle CAB = \angle FDE$ (A)
$\angle CBA = \angle FED$ (A)
$BC = EF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)

f $\angle ABC = \angle DEF$ (A)
$\angle BAC = \angle EDF$ (A)
$AB = DE$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)

g $\angle BAC = \angle EDF = 90°$ (R)
$BC = EF$ (H)
$AB = DE$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (RHS)

6 d and g, c and e

7 **a** 25°, 75° **b** Yes, AAS

8 **a** $a = 4$ **b** $x = 3$, $y = 5$ **c** $a = 60$, $b = 7$
d $x = 55$, $y = 4$ **e** $x = 6$ **f** $a = 70$, $b = 7$
g $c = 3$, $d = 4$ **h** $a = 30$, $b = 5$ **i** $a = 20$, $b = 70$

9 **a** $AB = ED$ (S)
$BC = DF$ (H)
$\angle BAC = \angle DEF = 90°$ (R)
$\triangle ABC \equiv \triangle EDF$ (RHS)
b 6 m
c **i** 37° **ii** 53°

Progress quiz

1 **a** $a = 70$ (vertically opposite), $b = 70$ (alternate angles in parallel lines)
b $c = 70$ (cointerior in parallel lines), $d = 110$ (corresponding angles in parallel lines or supplementary)

2 **a** Yes, cointerior angles sum to 180°.
b No, alternate angles are not equal.

3 **a** $x = 30$ **b** $a = 68$ **c** $b = 100$
d $x = 128$, $y = 52$

4 $z = 85$

5 **a** Sum = 720°, $m = 150$ **b** Sum = 540°, $a = 6$, $x = 108$

6 **a** $BC = EF$ (S)
$\angle ABC = \angle DEF$ (A)
$BA = ED$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SAS)
b $\angle ABC = \angle FED = 90°$ (R)
$AC = FD$ (H)
$AB = FE$ (S)
$\therefore \triangle ABC \equiv \triangle FED$ (RHS)

7F

Now you try

Example 12

a $\frac{AB}{DE} = \frac{5}{2} = 2.5$ (S)
$\angle BAC = \angle EDF$ (A)
$\frac{AC}{DF} = \frac{2.5}{1} = 2.5$ (S)
$\therefore \triangle ABC \mathrel{|||} \triangle DEF$ (SAS)

b $\angle ACB = \angle DFE$ (A)
$\angle BAC = \angle EDF$ (A)
$\angle ABC = \angle DEF$ (A)
$\therefore \triangle ABC \mathrel{|||} \triangle DEF$ (AAA)

Example 13

Scale factor = 1.5

$x = 4.5$

Exercise 7F

1 SSA

2 If two pairs of angles are corresponding and equal, the third pair must be equal due to the angle sum of a triangle (180°).

3 **a** 1.5 **b** 1.5, the same **c** 1.5 **d** SAS

4 **a** $\angle BAC = \angle EDF$ (A)
$\angle ABC = \angle DEF$ (A)
$\angle ACB = \angle DFE$ (A)
$\therefore \triangle ABC \mathrel{|||} \triangle DEF$ (AAA)

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b $\angle BAC = \angle EDF$ (A)
$\angle ACB = \angle DFE$ (A)
$\angle ABC = \angle DEF$ (A)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (AAA)

c $\frac{DE}{AB} = 2$(S)
$\frac{DF}{AC} = 2$(S)
$\frac{EF}{BC} = 2$(S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (SSS)

d $\frac{DE}{AB} = 3$(S)
$\frac{DF}{AC} = 3$(S)
$\frac{EF}{BC} = 3$(S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (SSS)

e $\frac{AB}{DE} = 1.5$ (S)
$\angle BAC = \angle EDF$ (A)
$\frac{AC}{DF} = 1.5$ (S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (SAS)

f $\frac{DE}{AB} = 1.5$ (S)
$\angle BAC = \angle EDF$ (A)
$\frac{DF}{AC} = 1.5$ (S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (SAS)

g $\angle CAB = \angle FDE = 90°$ (R)
$\frac{EF}{BC} = 1.5$ (H)
$\frac{DE}{AB} = 1.5$ (S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (RHS)

h $\angle CAB = \angle FDE = 90°$ (R)
$\frac{BC}{EF} = 2$ (H)
$\frac{AB}{DE} = 2$ (S)
$\therefore \triangle ABC \parallel\!| \triangle DEF$ (RHS)

i $\angle BAC = \angle EFD$ (A)
$\angle ABC = \angle FED$ (A)
$\angle ACB = \angle FDE$ (A)
$\therefore \triangle ABC \parallel\!| \triangle FED$ (AAA)

5 a $x = 8$ b $x = 21$ c $x = 4$ d $x = 1.5$
e $x = 4.5$

6 a $\triangle ABC, \triangle ADE$ b AAA
c 2.5 d 3.75 m

7 a AAA, $x = 1.5$ b AAA, $x = 9$
c AAA, $x = 2.2$ d AAA, $x = 8$

8 a $\angle BAC = \angle DEC$ (alternate), $\angle ABC = \angle EDC$ (alternate), $\angle ACB = \angle ECD$ (vertically opposite)
b i $DC = 6$ cm ii $AC = 6$ cm

7G

Now you try

Example 14

a AAA b 2 c 1.6 m

Example 15

a $\triangle ABC$ and $\triangle DEC$ (AAA)
b Scale factor = 1.5
CD = 15 m

Exercise 7G

1 a AAA b AAA
2 a AAA b 2 c 3
3 a AAA b 2 c 30 cm
4 a AAA b 1.5 c 4.5 m
5 a $\frac{88}{5} = 17.6$ b 2
6 a $\triangle ABC, \triangle DEC$; AAA b 1.25 m
7 a AAA b 5 c 5 m
8 a $\triangle ABD, \triangle CBE$; AAA b $\frac{6}{5} = 1.2$ c 13.2 m
9 1.90 m
10 Answers will vary.
11 a AAA b 1.5 c 20 m
d Let $AE = x$
$1.5x = x + 10$
$\therefore x = 20$

7H

Now you try

Example 16

a 2.6 cm b 500 c 4 cm d 15 m

Example 17

Area ratio = 4 : 9
Area = 11.25 m^2

Example 18

Volume ratio = 8 : 125
Volume = 3.2 m^3

Exercise 7H

1 a 4 : 25 b 8 : 125
2 a 2 : 5 b 32 cm^2, 200 cm^2 c 4 : 25 (= $2^2 : 5^2$)
d 12 cm^3, 187.5 cm^3 e 8 : 125 (= $2^3 : 5^3$)
3 a 2 cm b 1500 c 5 cm d 75 m
4 a 5 cm b 1500 c 4 cm d 60 m
5 a 16 cm^2 b 45 m^2
6 a 3 mm^2 b 3 cm^2
7 a 2 cm^3 b 8 m^3
8 a 108 cm^3 b 156.25 m^3
9 a i 1 km ii 3 km
b i 10 cm ii 1 cm
c 2 km
10 a i 3 : 4 ii 9 : 16
b 450 cm^2
11 a 1 : 8 b 240 cm^3
12 a i 2 : 3 ii 4 : 9 iii 8 : 27
b 90 cm^2
c 54 cm^3
13 a 1 : 27 000 000 b 54 000 000 m^3
c 9 m^2

Maths@Work: Pool builder

1 a 7.05 m b 1 m c 60 cm
d 3 m^2 e 82° f 10
2 a Hexagon (6 sides)
b 720°
c 720°
d Drawing should be the same shape but 4 times larger.

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3 a Octagon and square (a quadrilateral)
b The octagon is not but the square is
c Octagon 135° and square 90°
d If the square was not used, there would be gaps and hence no tessellation.
e They should all join up with no gaps.

Puzzles and games

1 30
2 CONGRUENCE
3 130°
4 a 7 b 11
5 20 m

Short-answer questions

1 a $x = 70, y = 110$ b $x = 120, y = 120$
c $x = 65, y = 115$ d $x = 30, y = 150$
e $x = 90, y = 120$ f $x = 45$

2 a 20 b 30 c 77 d 20 e 60
f 30 g 130 h 70 i 160

3 a Square, rectangle, rhombus, parallelogram
b Parallelogram, square, rhombus, rectangle
c Kite d Square, rhombus, kite

4 a $a = 90$ b $a = 40$ c $a = 110, b = 30$
d $a = 90, b = 130$ e $a = 40, b = 140$
f $a = 110$

5 a 540°, 60 b 720°, 100
c 900°, 120

6 a 10 m b 540° c 108°

7 a $\angle BAC = \angle EDF = 90°$ (R)
$BC = EF$ (H)
$AB = DE$ (S)
$\therefore \triangle ABC = \triangle DEF$ (RHS)
b $AB = DE$ (S)
$AC = DF$ (S)
$BC = EF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SSS)
c $\angle ACB = \angle DFE$ (A)
$\angle CAB = \angle FDE$ (A)
$AB = DE$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)
d $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \triangle ABC \equiv \triangle DEF$ (SAS)

8 a $\angle BAC = \angle EDF$ (A)
$\angle ABC = \angle DEF$ (A)
$\angle ACB = \angle DFE$ (A)
$\therefore \triangle ABC \ ||| \ \triangle DEF$ (AAA)
b $\frac{AB}{DE} = 2$ (S)
$\frac{AC}{DF} = 2$ (S)
$\frac{BC}{EF} = 2$ (S)
$\therefore \triangle ABC \ ||| \ \triangle DEF$ (SSS)
c $\frac{DE}{AB} = 1.5$ (S)
$\angle EDF = \angle BAC$ (A)
$\frac{DF}{AC} = 1.5$ (S)
$\therefore \triangle ABC \ ||| \ \triangle DEF$ (SAS)
d $\angle BAC = \angle EDF = 90°$ (R)
$\frac{EF}{BC} = 2.5$ (H)
$\frac{DE}{AB} = 2.5$ (S)
$\therefore \triangle ABC \ ||| \ \triangle DEF$ (RHS)

9 a $\triangle ABE, \triangle ACD$ b AAA c 2.5 d 7.5 m

10 a AAA b 6.25 c 187.5 cm

11 a 2 cm b 5000 c 5 cm d 250 m

12 a i 2 : 3 ii 4 : 9 iii 8 : 27
b 32 cm^3

Multiple-choice questions

1 B 2 A 3 D 4 E 5 E
6 D 7 B 8 A 9 C 10 B

Extended-response questions

1 a 60, 60, 30 b AAA c 2.5
d 8 cm e 4 : 25 f 6.25
2 a 2 : 125 b 281.25 cm
c 4 : 15 625 d 0.00128 L

Chapter 8

Warm-up quiz

1 a 9 b −18 c 9 d 0
2 a 14 b 6 c −16 d 32
3 a $3a$ b $3m$ c $8p$ d 0
e $-5m$ f $7x + y$ g $9p$ h $6m$
4 a $15x$ b $16p$ c $32xy$ d $-30a$
e ab f x g 1 h $2a$
i 3
5 a 5 b 3 c 5 d 1
e 4 f 2 g 3 h 2
i 4 j 3 k 6 l 10
6 a $x + 3$ b $n + 6$ c $2w$ d $\frac{x}{2}$
e $2x + 6$ f $x - 7$ g $2(x + 3)$ h $3x + 1$
7 a $8x$ b $4x + 16$ c $3x + 4$
8 a F b T c F d F
e T f T g T h F
i T
9 a $m = 7$ b $y = 19$ c $x = 5$ d $b = 12$
10 a T b T c F

8A

Now you try

Example 1
a $x = 16$ b $x = 18$ c $x = 9$ d $x = -12$

Example 2
$x = 4$

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Example 3

$x = 28$

Example 4

$x = 7$

Example 5

a $x + 3 = 9, x = 6$ **b** $\frac{x}{3} - 7 = 0, x = 21$

c $\frac{x-6}{3} = 10, x = 36$

Exercise 8A

1 a equals **b** one

2 a

x	0	1	2	3
$2x + 3$	3	5	7	9

b $x = 2$

3 a Yes **b** No **c** No **d** No **e** Yes **f** Yes

4 a $t = 3$ **b** $m = 6$ **c** $x = 6$ **d** $m = 32$ **e** $a = -6$ **f** $m = 15$ **g** $x = 6$ **h** $x = 9$ **i** $x = 17$ **j** $x = 3$ **k** $x = -6$ **l** $x = 12$

5 a $p = 3$ **b** $c = 6$ **c** $d = 9$ **d** $p = 1$ **e** $m = -7$ **f** $p = -\frac{1}{2}$ **g** $x = 50$ **h** $m = 21$ **i** $a = -12$ **j** $z = 0$ **k** $w = \frac{3}{2}$ **l** $m = \frac{1}{2}$

6 a $x = 3$ **b** $x = 9$ **c** $x = -11$ **d** $x = 10$ **e** $x = 14$ **f** $x = 10$ **g** $x = 3$ **h** $x = 4$ **i** $x = 50$ **j** $x = 20$ **k** $x = 21$ **l** $x = 7$

7 a $x = 1$ **b** $x = 3$ **c** $x = 3$ **d** $x = -2$ **e** $x = -1$ **f** $x = -9$ **g** $x = 4$ **h** $x = 8$ **i** $x = 8$ **j** $x = -2$ **k** $x = -3$ **l** $x = 5$

8 a $x = 9$ **b** $x = 0$ **c** $x = 56$ **d** $x = 20$ **e** $x = 35$ **f** $x = 90$ **g** $x = -32$ **h** $x = -20$ **i** $x = -18$

9 a $m = 5$ **b** $a = 7$ **c** $x = 1$ **d** $x = 1$ **e** $n = 9$ **f** $m = 22$ **g** $w = -7$ **h** $m = 7$ **i** $w = 27$ **j** $a = 1$ **k** $a = -37$ **l** $m = -5$

10 a $x + 4 = 6, x = 2$ **b** $x + 12 = 8, x = -4$ **c** $x - 5 = 5, x = 10$ **d** $\frac{x}{3} + 2 = 8, x = 18$ **e** $2x + 3 = 9, x = 3$ **f** $\frac{x-3}{5} = 6, x = 33$ **g** $3x + 4 = 16, x = 4$

11 a 13 cm **b** 12 mm

12 a 3 **b** 5 **c** 28 **d** 42 **e** 82

13 a 11, 12 **b** 6 **c** 3 **d** 25, 44 **e** 8 m

14 a $C = 60h + 40$ **b** $280 **c** $1120 **d** 6 h

15 a $v = 6t + 5$ **b** 3 min **c** 11 min

8B

Now you try

Example 6

$x = 4$

Example 7

$x = 2$

Example 8

$x = -2$

Example 9

$x = 2.75$

Example 10

$x = 3$

Example 11

$x = 10$

Exercise 8B

1 a expand **b** Collect **c** one

2 C

3 E

4 a $x = 1$ **b** $x = 5$ **c** $x = -1$ **d** $a = 5$ **e** $a = 1$ **f** $x = 15$ **g** $m = 4$ **h** $d = 1$ **i** $a = 10$ **j** $a = 0$ **k** $x = 0$ **l** $a = 3$

5 a $x = 1$ **b** $x = 2$ **c** $x = 3$ **d** $x = 2$ **e** $x = 3$ **f** $x = 2$ **g** $x = -2$ **h** $x = -1$

6 a $x = 1$ **b** $x = 1$ **c** $x = 0$ **d** $x = 0$ **e** $x = 3$ **f** $x = 1$ **g** $x = 3$ **h** $x = -2$

7 a $x = 2$ **b** $x = 12$ **c** $x = -3$ **d** $x = 20$ **e** $x = 4$ **f** $x = 8$ **g** $x = 4$ **h** $x = -4$ **i** $x = -1$ **j** $x = 11$ **k** $x = 1$ **l** $x = -1$

8 a $x = 13$ **b** $x = 6$ **c** $x = 13$ **d** $x = 11$ **e** $x = 10$ **f** $x = 5$ **g** $x = 6$ **h** $x = 8$ **i** $x = -2$

9 a $x = 12$ **b** $x = 18$ **c** $x = 60$ **d** $x = 9\frac{1}{3}$ **e** $x = 5$ **f** $x = -6$

10 a $x = 10$ **b** $x = \frac{5}{3}$ **c** $x = 45$

11 a $x = 2$ **b** $x = 0$ **c** $x = \frac{3}{2}$

12 a $x = 3$ **b** $x = 5$ **c** $x = 10$ **d** $x = 30$ **e** $x = 7$

13 a $x = 7$ **b** $x = 3$ **c** $x = 5$ **d** $x = 2$ **e** $x = 5$ **f** $x = 6$ **g** $x = 11$

14 a $C = 850 + 156h$ **b** 7 h **c** 8:15 p.m.

15 a $x = 1$ **b** $x = -3$ **c** $x = 2\frac{2}{5}$ **d** $x = 8$ **e** $x = 3$ **f** $x = -\frac{8}{17}$

8C

Now you try

Example 12

$y = 6$

Exercise 8C

1 a I **b** F **c** V **d** A **e** c **f** P

2 a $m = 60$ **b** $A = 48$ **c** $A = 36$ **d** $v = 14.3$ **e** $m = 3.7$

3 a $t = 4$ **b** $t = 4$ **c** $t = 10$ **d** $t = 8$

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4 a $b = 10$ b $b = 9$ c $b = 17$ d $b = 1.3$
5 a $h = 5$ b $h = 12$ c $h = 3$ d $h = 7$
6 a $b = 15$ b $b = 16$ c $b = 12$ d $b = 32$
7 a $h = 8$ b $h = 8$ c $h = 12$ d $h = 28$
8 a $m = 4$ b $m = 40$ c $m = 72$ d $m = 4$
9 a $h = 5.7$ b $h = 5.1$ c $h = 5.7$ d $h = 16.0$
10 a 86°F b −1.1°C c 212°F d −17.8°C
11 a $32 b 60 km
12 a $P = 750$ b $t = 3.125$ c $r = 7.5$
13 a 1.5 tablets b 1250 mg
14 a 75 mL/h b $\frac{1}{3}$ h = 20 min
15 a Number of hours b 7.5 h
16 $P = 27.32$ cm, $A = 28.87$ cm^2
17 Iqra is now 17 and Urek is 7.

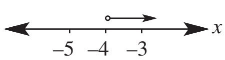
8D

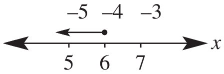
Now you try

Example 13

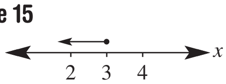
a $x \geq -3$ b $x < 9$

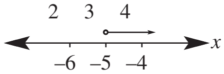
Example 14

a $x > -4$ 

b $x \leq 6$ 

Example 15

a $x \leq 3$ 

b $x > -5$ 

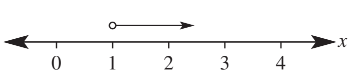
Example 16

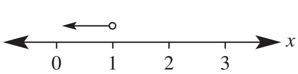
$x > -6$

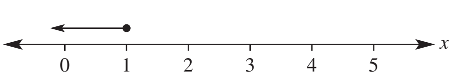
Exercise 8D

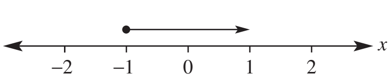
1 a C b B c A d E e D
2 a C b D c B d A
3 a $x \geq 1$ b $x < 7$ c $x \leq 4$
d $x > -9$ e $x \leq 1$ f $x > 8$
g $x < -7$ h $x \geq 1\frac{1}{2}$ i $x < 1$
j $x > 0$

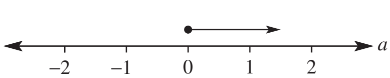
4 a 

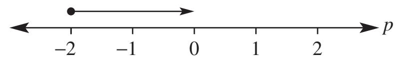
b 

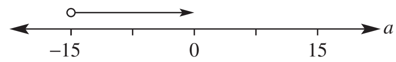
c 

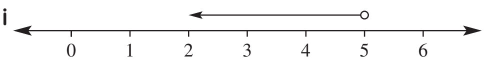
d 

e 

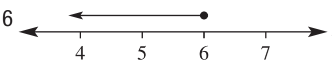
f 

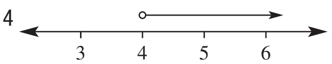
g 

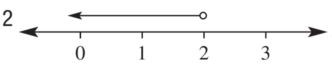
h 

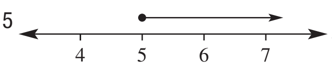
i 

5 a $x \geq -1$ b $x < 1$ c $a < 6$ d $a \geq -2$
e $w \leq 5$ f $w > -3$ g $x \leq 3$ h $x > -3.5$

6 a $x \leq 6$ 

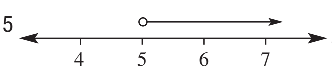
b $x > 4$ 

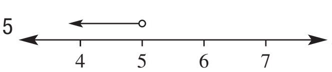
c $x < 2$ 

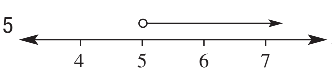
d $x \geq 5$ 

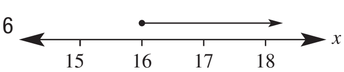
7 a $n > 100\,000$ b $n \geq 50$ c $n > 3$
d $n > 100$ e $n \leq 30$

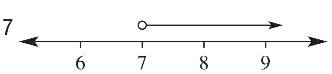
8 a $x < 0$; $-6, -2, -\frac{1}{2}$ b $x > 10$; 15, 24
c $x \geq 10$; 10, 15, 24 d $x \leq 0$; $-6, -2, -\frac{1}{2}, 0$
e $x \geq -1$; $-\frac{1}{2}, 0, 2, 5, 7, 10, 15, 24$
f $x < 10$; $-6, -2, -\frac{1}{2}, 0, 2, 5, 7$

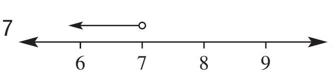
9 a $x > 5$ 

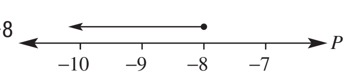
b $x < 5$ 

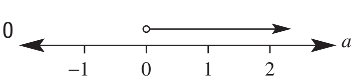
c $x > 5$ 

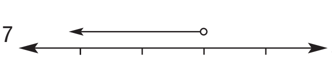
d $x \geq 16$ 

e $x > 7$ 

f $x < 7$ 

g $p \leq -8$ 

h $a > 0$ 

i $x < 7$ 

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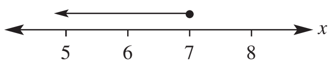
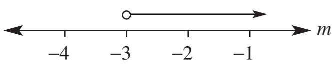
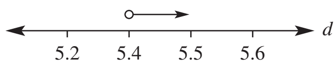
j $x \le 7$



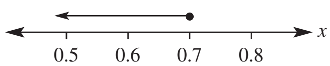
k $m > -3$



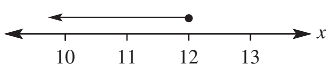
l $d > 5.4$



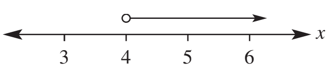
m $x \le 0.7$



n $x \le 12$



o $x > 4$



10 a $a \le 2$ **b** $y > 3$ **c** $p > 5$
d $x \ge 4$ **e** $x < 1$ **f** $w \ge \frac{3}{2}$
g $x < 1$ **h** $x \ge 1$ **i** $p < 1$

11 a $x \le 2$ **b** $a \le 1$ **c** $x \ge 28$
d $x > -15$ **e** $x < 4$ **f** $x < 30$
g $x > \frac{19}{2}$ **h** $x < -\frac{1}{2}$ **i** $x \ge -\frac{8}{3}$
j $x > 1$ **k** $x \ge \frac{3}{2}$ **l** $x < -\frac{9}{2}$

12 a $3x < 9, x < 3$ **b** $\frac{3x}{4} < 6, x < 8$
c $2x + 15 > 20, x > \frac{5}{2}$ **d** $x + x + 4 < 24, x < 10$
e $4x + 7 \le 27, x \le 5$

13 a $x > -1$ **b** $x \le -3$ **c** $p \ge -7$ **d** $a > 19$
e $w \le -6$ **f** $p > -6$ **g** $x > -1$ **h** $a \le -\frac{6}{7}$

14 a

4	6	4 < 6	T or F
4 + 3	6 + 3	7 < 9	T
4 − 3	6 − 3	1 < 3	T
4 × 2	6 × 2	8 < 12	T
4 ÷ 2	6 ÷ 2	2 < 3	T
4 × (−2)	6 × (−2)	−8 < −12	F
4 ÷ (−2)	6 ÷ (−2)	−2 < −3	F

b subtract, divide, positive, inequality, multiply, divide, number, true

8E

Now you try

Example 17

(1, −1)

Exercise 8E

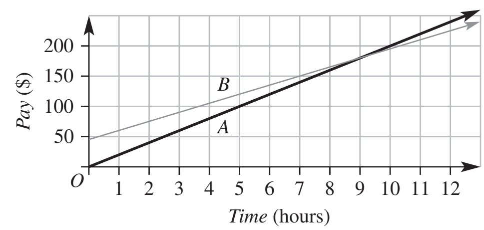
1 a 0 **b** 1
2 a (0, 3) **b** (1, 2) **c** No point of intersection
d (2, 8) **e** No point of intersection **f** (4, 3)
g (2, 5)
3 a (2, 3) **b** (−1, −4)
4 a (2, 3) **b** (2, 8) **c** (0, 9) **d** (−3, 8)
5 a (2, 3) **b** (3, −2)
6 a (3, 9) **b** (1, −3)
7 a (1, −4) **b** (1, 1)
8 a 200 km **b** $1400 **c** Best removals, $100 cheaper
d A+ removals, $200 cheaper

9 a

Time in hours	0	1	2	3	4	5	6
Pay of company A	$0	$20	$40	$60	$80	$100	$120
Time in hours	7	8	9	10	11	12	
Pay of company A	$140	$160	$180	$200	$220	$240	

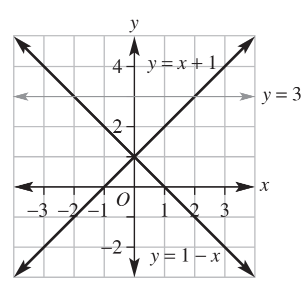
Time in hours	0	1	2	3	4	5	6
Pay of company B	$45	$60	$75	$90	$105	$120	$135
Time in hours	7	8	9	10	11	12	
Pay of company B	$150	$165	$180	$195	$210	$225	

b



c 9 h
d $180

10 a



b (−2, 3)(0, 1)(2, 3) **c** 4 units, $\sqrt{8}$ units, $\sqrt{8}$ units
d Isosceles right-angled triangle

11 a

Time in years	0	2	4	6
Value of luxury sports coupe	$70 000	$60 000	$50 000	$40 000
Time in years	8	10	12	
Value of luxury sports coupe	$30 000	$20 000	$10 000	

Time in years	0	2	4	6
Value of sedan	$50 000	$44 000	$38 000	$32 000
Time in years	8	10	12	
Value of sedan	$26 000	$20 000	$14 000	

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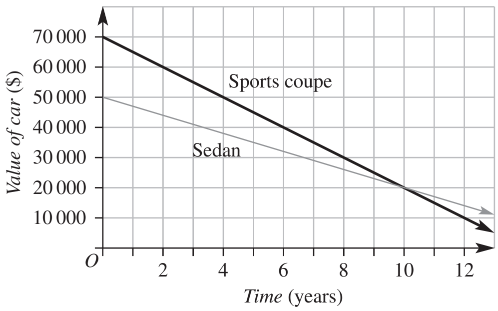
b



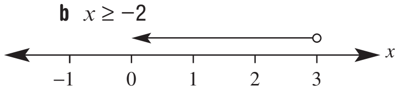
c 10 years
d \$20 000

12 a No **b** Parallel lines
d $(-6, -12)$, $(-5, -9)$, $(-4, -6)$, $(-3, -3)$

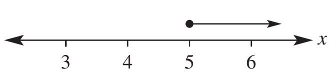
13 a Yes, $(0, -1)$ **b** Same y-intercept
d $(-2, -3)$, parallel, $(2, 5)$, $(1, 3)$

Progress quiz

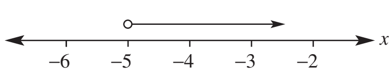
1 a $x = 6$ **b** $y = 17$ **c** $m = 7$ **d** $x = -10$
2 a $x = 9$ **b** $y = -4$ **c** $x = 9$ **d** $x = 6$
e $y = 9$ **f** $x = -5$
3 a $x = 3$ **b** $a = -6$ **c** $y = 3$ **d** $x = 4$
4 a $2x + 3 = 25$; $x = 11$ **b** $\frac{x-3}{2} = 6$; $x = 15$
c $3(x - 2) = 12$; $x = 6$ **d** $\frac{4(x+2)}{2} = 16$; $x = 6$
5 a $x = -2$ **b** $x = 2$ **c** $x = 4$
6 a $b = 4$ **b** $u = 5$ **c** $h = 2.4$
7 a $x < 1$ **b** $x \geq -2$
8 a $x < 3$;



b $x \geq 5$;



c $x > -5$;



9 $(1, 3)$

8F

Now you try

Example 18
$(1, 7)$

Example 19
$(-2, -3)$

Example 20
The width is 6.5 cm and the length is 9.5 cm.

Exercise 8F

1 a Simultaneous **b** intersection
c substituted **d** substituted
e substitution
2 a A **b** C
3 a $(2, 10)$ **b** $(-5, -15)$ **c** $(-4, -8)$
d $(1, 4)$ **e** $(2, 2)$ **f** $(2, 12)$
4 a $(2, 5)$ **b** $(1, 3)$ **c** $(2, 1)$ **d** $(4, 3)$
e $(-5, -3)$ **f** $(1, 6)$ **g** $(4, 1)$ **h** $(-1, -5)$
5 a $(-1, 2)$ **b** $(3, -1)$ **c** $(4, 4)$
6 a $(2, 4)$ **b** $(-3, 2)$ **c** $(7, -1)$
7 a $(1, 5)$ **b** $(4, 10)$ **c** $(2, 1)$ **d** $(2, 9)$
8 Kye: 43 years old, Viviana: 38 years old
9 Length = 18 cm, width = 6 cm
10 Vanilla thick shake: \$5, fruity twirl: \$3
11 Carlos: 37 years old, Ella: 17 years old
12 a i Joe's: \$60, Paul's \$150
ii Joe's: 20 c/km, Paul's 10 c/km
iii Joe's $C = 0.2k + 60$, Paul's $C = 0.1k + 150$
iv 900 km
b Joe's if you are travelling less than 900 km and Paul's for more than 900 km.

8G

Now you try

Example 21
$(4, -1)$

Example 22
$(1, 3)$

Example 23
$(-1, 2)$

Example 24
The two numbers are 42 and 55.

Example 25
Buckets of chips cost \$4 and drinks cost \$2.50.

Exercise 8G

1 a − **b** + **c** +
2 a $6x - 4y = -2$ **b** $9x - 6y = -3$
c $12x - 8y = -4$
3 a B **b** E
4 a $(6, 1)$ **b** $(3, 4)$ **c** $(2, 7)$
5 a $(7, 2)$ **b** $(3, 8)$ **c** $(4, 1)$
6 a $(2, 5)$ **b** $(2, 3)$ **c** $(4, 2)$ **d** $(2, 2)$
7 a $(1, 1)$ **b** $(2, 1)$ **c** $(2, -1)$
8 a $(4, 1)$ **b** $(1, 3)$ **c** $(5, 1)$
9 a $(4, -3)$ **b** $(1, 1)$ **c** $(2, 0)$ **d** $(1, 1)$
10 a $(1, 1)$ **b** $(4, 2)$ **c** $(3, 4)$
11 a $(3, 1)$ **b** $(1, -10)$ **c** $(-2, -3)$ **d** $(0, 1)$
e $(-1, 2)$ **f** $(5, -2)$
12 Ayden: 36 years old, Tamara: 20 years old
13 a Let a be the number of apples and m be the number of mangoes.
$10a + 5m = 1250$,
$15a + 4m = 1350$
b Apples: 50c, mangoes:\$1.50
c \$8.00
14 Children: 2500, adults: 2500
15 a Let m be the number of multiple-choice and s be the number of short-answer questions.
b $2m + 3s = 50$, $m + s = 22$
c 16 multiple-choice questions
16 a $x + y = 16$, $x - y = 2$, $x = 9$, $y = 7$
b $x + y = 30$, $x - y = 10$, $x = 20$, $y = 10$
c $2x + y = 12$, $x + y = 7$, $x = 5$, $y = 2$
17 a $x = 1$, $y = 2$ **b** $x = 2$, $y = 2$
18 a i $g + 30$ **ii** $d + 30$
b $g = d + 31$, $g + 30 = 2(d + 30)$
c Gordon is aged 32 and his daughter is aged 1.
19 a $(4, -3)$ **b** $(1, 1)$ **c** $(3, 4)$ **d** $(2, 2)$
e $\left(\frac{1}{2}, -1\right)$ **f** $\left(-3, \frac{1}{3}\right)$

Essential Mathematics for the Victorian Curriculum CORE Year 10 ISBN 978-1-108-87859-3

Maths@Work: Nurse

1 **a** 1.5 mL **b** 0.6 mL **c** 1 mL
d 2 capsules **e** 5 tablets
2 **a** 114 mg **b** 107 mg
c 27 mg **d** 471 mg
3 **a** 41.7 drops per minute
b 9:30 p.m. Tuesday
c 75 min or 1 h 15 min
4

Starting time	Time in minutes for IV	Time in hours and minutes for IV	Ending time
4:15 p.m.	315	5:15	9:30 p.m.
10 a.m.	222.2222	3:42	1:42 p.m.
1:25 p.m.	640	10:40	12:05 a.m.
6 p.m.	750	12:30	6:30 a.m.
2 a.m.	977.7778	16:18	6:18 p.m.

Puzzles and games

1 Each row, column and diagonal adds to 6.

9	−5	−4	6
−2	4	3	1
2	0	−1	5
−3	7	8	−6

2 **a** 64 **b** 8 **c** 29.3 **d** 18 years old
3 Many possible equations; e.g. $3x + 2 = -4$; $\frac{5x}{2} = -5$; $2(x + 5) = 6$.
4 Many possible simultaneous equations; e.g. $x + y = 1, 2x - y = 8$.
5 Toowoomba
6 **a** 4.5 **b** 117 km
7 Emily was jogging at 11.5 km/h and cycling at 23 km/h.

Short-answer questions

1 **a** $a = 8$ **b** $m = -30$ **c** $x = -8$
d $x = 8$ **e** $m = 0$ **f** $w = 15$
g $m = -0.2$ **h** $w = 4$ **i** $r = 6$
2 **a** $m = 2$ **b** $w = 8$ **c** $m = 10$
d $w = 8$ **e** $m = 14$ **f** $m = \frac{4}{3} = 1\frac{1}{3}$
g $a = \frac{3}{2} = 1\frac{1}{2}$ **h** $x = 1$ **i** $x = 3$
3 **a** $m = 3$ **b** $a = 7$ **c** $x = 4$
d $x = \frac{3}{2} = 1\frac{1}{2}$ **e** $m = 2\frac{1}{2}$ **f** $x = \frac{7}{8}$
g $x = 2$ **h** $x = 8$
4 **a** $p = 4$ **b** $p = 4$ **c** $p = -9$
d $p = -2$ **e** $p = -5$ **f** $p = 2$
5 **a** $6x = 420$, the number is 70.
b $x + 8 = 5$, the number is −3.
c $\frac{a}{9} = 12$, the number is 108.
d $x + 7 = 3$, the number is −4.
e $x + 2.3 = 7$, the number is 4.7.
6 **a** $b = 8$ **b** $w = 3.5$ **c** $x = 2.4$
d $m = 10$ **e** $c = 35$
7 **a** $m > -2$ **b** $n \le 0.5$ **c** $x \ge -1$
d $x > 0$ **e** $x < 15$
8 **a** $x \ge -18$ **b** $m < 3.5$ **c** $x > 2$
d $x < 3$ **e** $x \le 8$ **f** $m \ge 10$
9 **a** $x \ge -2$ **b** $x \ge -2$ **c** $x < 0$
10 **a** (1, 2) **b** (2, 3) **c** No point of intersection
11 **a** (1, 6) **b** (3, 2) **c** (0, 0)
12 **a** (3, 2) **b** (5, −1)
13 **a** (−2, −3) **b** (2, 1)
14 4 and 11
15 **a** x: number of 20-cent coins, y: number of 50-cent coins;
$x + y = 160$, $20x + 50y = 5000$
b 100 20-cent coins and 60 50-cent coins
16 50 children

Multiple-choice questions

1 C **2** B **3** C **4** D **5** B **6** C
7 D **8** B **9** C **10** D **11** A **12** E
13 B

Extended-response questions

1 **a** $P = 6x - 4$
b **i** $x = 22$ **ii** 20 cm, 27 cm, 37 cm, 7 cm, 30 cm
c **i** $x = 26$; 24 cm, 31 cm, 45cm, 7 cm, 38 cm
ii $x = 38$; 36 cm, 43 cm, 69 cm, 7 cm, 62 cm
2 **a** **i** $C = 70t + 50$ **ii** $C = 100 + 60t$
b 3 h
c (5, 400)

Chapter 9

Warm-up quiz

1 **a** 15.84 **b** 164.87 **c** 0.87
d 0.58 **e** 0.17 **f** 0.71
g 12.99 **h** 14.30
2 **a** 25 **b** 46.24 **c** 361
d 225 **e** 43.25 **f** 81
3 **a** 2.8 **b** 2.6 **c** 3.9
d 3.2 **e** 3.6 **f** 3.0
g 1.9 **h** 14.1
4 **a** c **b** p **c** y
d PQ **e** BC **f** XY
5 **a** $x = 3$ **b** $x = 4$ **c** $x = 35$ **d** $x = 9$
6 **a** $m = 3.65$ **b** $m = 1.2$ **c** $m = 5.2$ **d** $m = 5.724$
7 **a** $x = 0.6$ **b** $x = 0.2$ **c** $x = 2.1$ **d** $x = 0.4$
e $x = 2.4$ **f** $x = 9.2$
8 **a** $x = 4$ **b** $x = 20$ **c** $x = 13$ **d** $x = 4$
9 **a** 30° **b** 50° **c** 54°
d 90° **e** 45° **f** 30°
g 82° **h** 62° **i** 35°

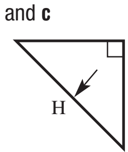
9A

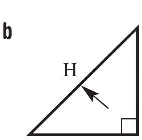
Now you try

Example 1
26

Example 2
12.5 m

Exercise 9A

1 **b** and **c**
2 **a**


b


3 **a** $z^2 = x^2 + y^2$ **b** $t^2 = m^2 + n^2$ **c** $s^2 = p^2 + r^2$
4 **a** 25 **b** 34 **c** 45

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5 **a** 5 **b** 17 **c** 13
d 25 **e** 41 **f** 15

6 **a** 5.8 cm **b** 7.3 mm **c** 18.0 m
d 11.2 km **e** 8.6 cm **f** 12.8 cm

7 **a** 8.91 cm **b** 3.62 m **c** 5.02 km
d 13.95 mm **e** 17.49 cm **f** 24.21 cm

8 181.5 cm

9 35.3 m

10 13 m

11 5.28 m

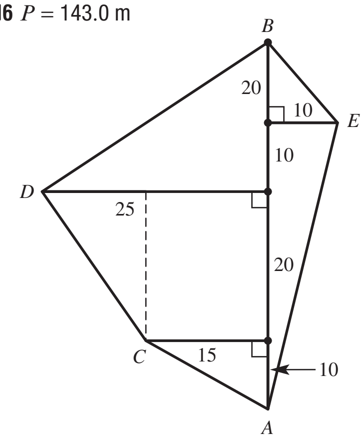
12 2.64 m

13 494.97 m

14 **a** No **b** No **c** Yes
d Yes **e** No **f** Yes

15 **a** **i** $AC = 11.2$ **ii** $BC = 30.4$
iii $DB = 21.2$ **iv** $AD = 29.2$
b 92 m
c 400 m^2

16 $P = 143.0$ m



9B

Now you try

Example 3
3

Example 4
17.0

Exercise 9B

1 **a** 10 **b** 24 **c** 41 **d** 1.5

2 **a** $w^2 = 10^2 - 6^2$ **b** $x^2 = 13^2 - 5^2$
c $p^2 = 30^2 - 18^2$

3 **a** $17^2 = 8^2 + w^2$ **b** $13^2 = m^2 + 5^2$
c $15^2 = x^2 + 9^2$

4 **a** 4 **b** 7 **c** 8
d 5 **e** 14 **f** 4.8

5 **a** 7.1 **b** 13.3 **c** 12.3
d 6.2 **e** 6.6 **f** 16.2

6 **a** 7.14 **b** 13.90 **c** 3.87
d 133.84 **e** 17.89 **f** 39.19

7 5.66 m

8 2.2 m

9 3.2 m

10 **a** **i** 3 cm **ii** 5.2 cm
b 15.6 cm^2

11 7.4 cm

12 **a** 7.1 **b** 4.5 **c** 5.2

9C

Now you try

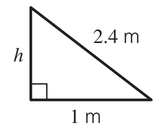
Example 5
5 m

Example 6
36.1 km

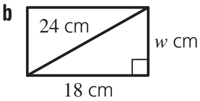
Example 7
9.22 mm

Exercise 9C

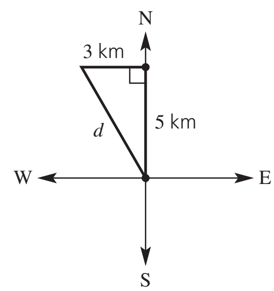
1 **a**



b



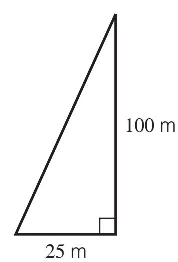
c



2 **a** $\triangle ADE$ **b** $\triangle PRO$ and $\triangle RQO$ **c** $\triangle AOB$

3 **a** AB **b** AB **c** FH
d BG **e** RT **f** EB

4 **a** 100 m
b



c 103.08 m

5 **a** 8.6 m **b** 13 m **c** 10 m

6 6.9 km

7 **a** 10 km **b** 41 km **c** 17 km

8 **a** 10.6 **b** 10.8 **c** 15.8 **d** 4.2

9 25.04 m

10 **a** 11.18 m **b** 12.2 m

11 Horizontal = 45.25 cm
Vertical = 71.66 cm

12 **a** 4.77 m **b** 4.90 cm

13 **a** 11.2 m **b** 8.1 cm

14 **a** 1.41 cm **b** 1.73 cm

15
- $\triangle PQR$ is a right-angled isosceles triangle
- With $\angle P = \angle R = 45°$
- The hypotenuse = $\sqrt{200}$ cm
- $PQ = QR = 10$ cm
- Perimeter = 34.1 cm
- Area = 50 cm^2

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9D

Now you try

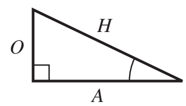
Example 8

a $\frac{15}{17}$ b $\frac{8}{17}$ c $\frac{15}{8}$

Example 9

$\frac{7}{\sqrt{58}}$

Exercise 9D

1 **a**


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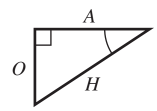

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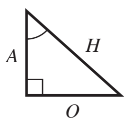

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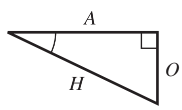

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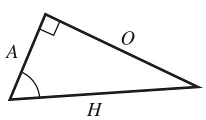
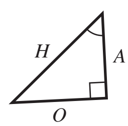


f


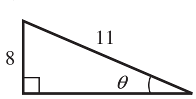
2 **a** PR **b** TP **c** TP
d PR **e** TR **f** $\angle T$

3 **a** BC **b** CA **c** BA **d** BA

4 **a** $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{4}{5}$

5 **a** **i** $\frac{b}{a}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{c}$
b **i** $\frac{n}{p}$ **ii** $\frac{m}{p}$ **iii** $\frac{n}{m}$
c **i** $\frac{y}{z}$ **ii** $\frac{x}{z}$ **iii** $\frac{y}{x}$
d **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$
e **i** $\frac{24}{26}=\frac{12}{13}$ **ii** $\frac{10}{26}=\frac{5}{13}$ **iii** $\frac{24}{10}=\frac{12}{5}$
f **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

6 **a** $\frac{a}{c}$ **b** $\frac{y}{z}$ **c** $\frac{b}{a}$ **d** $\frac{6}{10}=\frac{3}{5}$
e $\frac{40}{41}$ **f** $\frac{1}{\sqrt{3}}$ **g** $\frac{8}{6}=\frac{4}{3}$ **h** $\frac{9}{41}$
i $\sqrt{3}$

7 **a**


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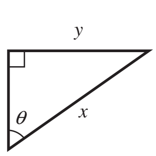
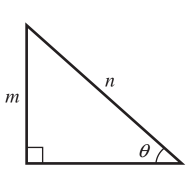
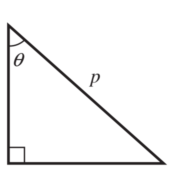


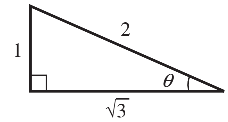
c


d


8 **a** $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{z}{x}$ **d** $\frac{y}{x}$
e $\frac{y}{z}$ **f** $\frac{z}{y}$

9 **a** $\sin\theta$ **b** $\tan\theta$ **c** $\sin\theta$ **d** $\cos\theta$
e $\cos\theta$ **f** $\tan\theta$

10 **a** $QR = 13$ m **b** $\sin\theta = \frac{5}{13}$

11 **a**


b $\sqrt{3}$
c **i** $\cos\theta = \frac{\sqrt{3}}{2}$ **ii** $\tan\theta = \frac{1}{\sqrt{3}}$

12

Angle (θ)	$\sin\theta$	$\cos\theta$
0°	0	1
5°	0.087	0.996
10°	0.174	0.985
15°	0.259	0.966
20°	0.342	0.940
25°	0.423	0.906
30°	0.5	0.866
35°	0.574	0.819
40°	0.643	0.766
45°	0.707	0.707
50°	0.766	0.643
55°	0.819	0.574
60°	0.866	0.5
65°	0.906	0.423
70°	0.940	0.342
75°	0.966	0.259
80°	0.985	0.174
85°	0.996	0.087
90°	1	0

a 45°
b **i** 85 **ii** 80 **iii** 30 **iv** 0
c If angles θ and α sum to 90°, $\sin\theta = \cos\alpha$.
d It's the same as the complement of sine.

9E

Now you try

Example 10

3.86

Example 11

a 1.03 b 8.78 c 14.18

Exercise 9E

1 **a** 0.1736 **b** 0.9848 **c** 0.1763 **d** 0.5774
e 0.7660 **f** 0.9397 **g** 0.1736 **h** 0.8391
i 0.9848

2 **a** 2.12 **b** 5.07 **c** 31.18 **d** 46.43
e 8 **f** 18.79

3 **a** sin **b** cos **c** tan
d cos **e** tan **f** sin

4 **a** $x = 1.37$ **b** $x = 5.12$
c $x = 91.44$ **d** $x = 13.86$
e $x = 9.19$ **f** $x = 9.19$

5 **a** 0.39 **b** 4.50 **c** 2.60 **d** 11.15
e 16.80 **f** 5.75 **g** 7.83 **h** 13.49
i 1

6 **a** 2.11 **b** 4.02 **c** 1.88

7 **a** 5.36 **b** 1.27 **c** 0.52

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8 **a** 5.49 **b** 8.51 **c** 9.23
9 **a** 3.76 **b** 2.12 **c** 2.80
d 4.94 **e** 4.14 **f** 0.75
10 **a** 26.33 m **b** 52.66 m
11 6.96 m
12 **a** $w = 1.27, l = 2.72$ **b** $w = 0.68, l = 1.88$
c $w = 3.06, l = 2.57$
13 **a** $a = 3.5, b = 3.2, x = 1.4$
b $a = 3.464, b = 3.139, x = 1.327$
c It is better not to round off during the process as sometimes it can change the final answer.

9F

Now you try

Example 12
a 4.99 **b** 10.44 **c** 3.54

Exercise 9F

1 **a** 17.32 **b** 13.86 **c** 106.73
d 19.84 **e** 24.69 **f** 13.20
2 **a** $\frac{10}{x}$ **b** $\frac{1.4}{m}$ **c** $\frac{19}{x}$ **d** $\frac{2.8}{w}$
3 C
4 **a** 8.77 **b** 9.44 **c** 8.49
5 **a** 4.62 **b** 23.39 **c** 2.86
6 **a** 5.96 **b** 1.62 **c** 1.72
7 **a** 4.73 **b** 6.19 **c** 6.14 **d** 3.00
e 26.08 **f** 27.82
8 2.54 m
9 13.9 m
10 **a** $AB = 42.89$ cm, $BC = 20$ cm
b $AB = 5.32$ m, $BC = 1.82$ m
c $AB = 14.62$ cm, $BC = 13.74$ cm
11 **a** 7.464 m **b** 7.727 m
12 **a** 30.5 m **b** 17.5 m
13 **a** 17.16 **b** 30 **c** 4.01 **d** 59.78
e 51.13 **f** 38.09

Progress quiz

1 **a** 26 m **b** 10.3 m
2 **a** $x = 6$ **b** $x = 7.1$
3 4.0 inches
4 4.47 m
5 12.6 cm
6 **a** $\cos\theta = \frac{40}{41}$ **b** $\sin\theta = \frac{9}{41}$ **c** $\tan\theta = \frac{9}{40}$
7 **a** $x = 10.9$ **b** $x = 6.3$
8 **a** $x = 8.7$ **b** $x = 6.7$
9 **a, b** 15.6 cm^2

9G

Now you try

Example 13
a 46° **b** 68° **c** 53°

Example 14
a 36.87° **b** 27.49° **c** 73.95°

Exercise 9G

1 $\sin 30° = 0.5$ and $\sin^{-1}(0.5) = 30$
2 **a** 45° **b** 30° **c** 58°
3 **a** $\cos\theta = \frac{5}{12}$ **b** $\sin\theta = \frac{7}{10}$ **c** $\tan\theta = \frac{4}{3}$
4 **a** 30° **b** 53° **c** 61° **d** 45°
e 41° **f** 53° **g** 48° **h** 6°
i 37° **j** 81° **k** 73° **l** 60°
m 42° **n** 48° **o** 34°
5 **a** 60° **b** 45° **c** 64.16° **d** 48.59°
e 53.13° **f** 68.20°
6 **a** 42° **b** 56° **c** 64° **d** 49°
e 54° **f** 67° **g** 53° **h** 50°
i 40° **j** 77° **k** 43° **l** 56°
7 10.62°
8 15.95°
9 46.66°
10 **a** 36.9°, 53.1° **b** 34.8°, 55.2°
c 36.9°, 53.1° **d** 26.6°, 63.4°
e 68.2°, 21.8°
11 Pitch $A = 47°$, $B = 43°$
12 **a** 5.54 m **b** 5.97 m
13 **a** 12° **b** Yes **c** 286.4 cm

9H

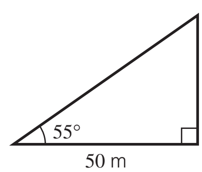
Now you try

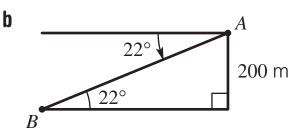
Example 15
28 m

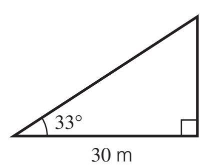
Example 16
137.2 m

Example 17
8°

Exercise 9H

1 **a** depression **b** elevation **c** horizontal
2 **a** 30° **b** 30° **c** 60°
3 **a**


b


c


4 **a** 21.88 m **b** 43.5 m **c** 23.41 m
d 6.06 m **e** 536.29 m **f** 38.97 m
5 **a** 112.0 **b** 49 m **c** 86 m
d 105 m **e** 9260 m
6 **a** 26.6° **b** 29.7° **c** 5.7°
d 47.1° **e** 48.3°
7 38.35 m
8 1.25 km
9 280.04 m
10 **a** 30° **b** 30° **c** Equal due to parallel lines
d 1.7
11 Answers will vary.

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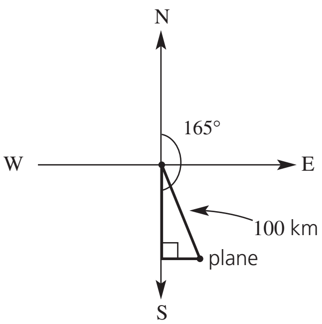
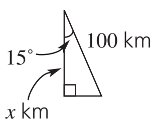
9I

Now you try

Example 18



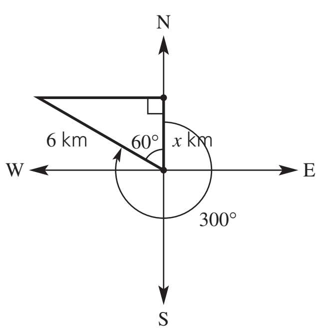


Example 19

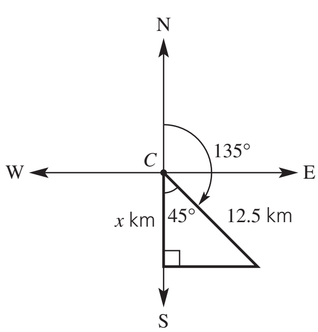
187.9 km

Exercise 9I

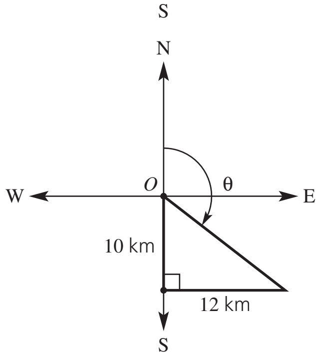
1 **a** S **b** W **c** N **d** SW
2 **a** v **b** iv **c** i
d iii **e** vi **f** ii
3 **a** 130° **b** 340° **c** 090°
d 128° **e** 313° **f** 355°
4 **a** north **b** east **c** west **d** 060°
e 240°
5 **a**



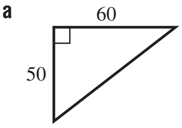
b



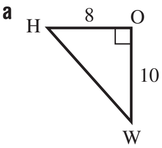
c



6 6.9 km
7 3.86 km
8 93.97 km
9 **a** 51.42 km **b** 61.28 km **c** 320°
10 **a**



b 78.10 km
c 50° **d** 050°
11 **a**



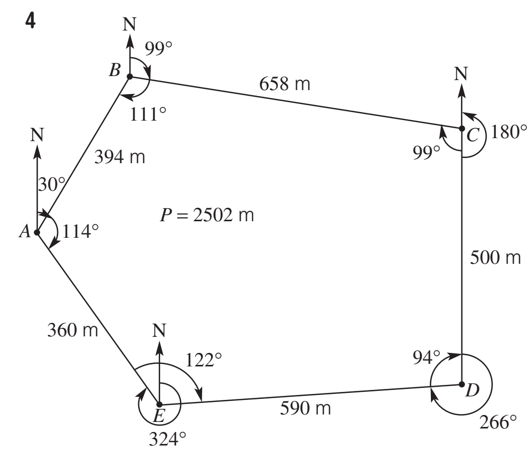
b 12.8 km
c 39° **d** 321°
12 3.25 km
13 115 km
14 **a** 18.03 km **b** 146°

Maths@Work: Surveyor

1 **a** 10.94 m **b** 8.03 m **c** 51.15 m
d 38.55 m **e** 145.90 m **f** 1793.37 m
2 **a** 1300 m **b** 1174.87 m **c** 1835.79 m
d 1081.67 m **e** 7342.33 m **f** 320°
g 265°
3 **a**

Marker	True bearing	Angle	Distance east	Distance north
A	055°	145°	80.28	56.21
B	082°	207°	124.14	17.45
C	051°	149°	122.63	99.31
D	078°	207°	102.14	21.71
E	-	192°	-	-

b 429.19 m
c 194.68 m
d 066°
4



Puzzles and games

1 **a** West **b** SW **c** NW **d** 198° **e** 120°
2 63.29 m
3 Answers will vary.

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4

S														H
I	D	E	P	R	E	S	S	I	O	N				Y
N		E												P
E			G			S								O
				R		I	A	D	J	A	C	E	N	T
					E	D				A	N	G	L	E
						E							O	N
		T	R	I	A	N	G	L	E				P	U
	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
		C	G	B						A	A		O	E
		O	H		E					N	T		S	
		S	T			A				G	I		I	
		I					R			E	O		T	
		N						I		N			E	
E	L	E	V	A	T	I	O	N	N	T				
					P	Y	T	H	A	G	O	R	A	S

5 068.8° T

Short-answer questions

1 No, $8^2 \neq 4^2 + 6^2$
2 a 15.17 m b 5.27 cm c 5.66 mm
3 a i BC ii AB iii AC
b i AC ii AB iii BC
4 a 0.57 b 0.96 c 8.14
5 a 4.50 b 1.5 c 0.93
6 a 6.31 b 12.94 c 4.16
7 a 3.63 m b 1.69 m
8 567.13 m
9 a 41.81° b 59.00° c 72.26°
10 a 45° b 42.4 m
11 a 2.3 km b 1.9 km c 040° T

Multiple-choice questions

1 B 2 E 3 C 4 B 5 C
6 C 7 D 8 E 9 A 10 C

Extended-response questions

1 a 54.46° b 130.25 m c 187 m
2 a 539 km b 68° c 248°

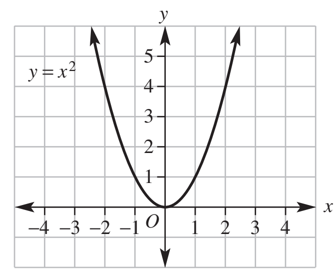
Chapter 10

Warm-up quiz

1 a 3 b 2 c 5
2 a $4x + 2y$ b $2xy - x$ c $2ab$
3 a $2a$ b $-2m$ c $18a^2$
d $-6x^2y$ e $-\frac{1}{3}$ f $2x$
4 a $4m + 4n$ b $-6x + 12$ c $6x^2 + 2x$
d $4a - 8a^2$ e $3x - 7$ f $-2x + 1$
5 a $7(x + 1)$ b $-9x(1 + 3x)$ c $a(a + b)$
6 a $-\frac{1}{2}$ b 3 c 5
7 a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b B



8 a T b F c T d T e F

10A

Now you try

Example 1
a $-2x - 6$ b $-15x + 5$

Example 2
a $x^2 + 6x - 7$ b $12x^2 - 11x + 2$

Example 3
a $x^2 + 6x + 9$ b $4x^2 - 4x + 1$

Example 4
a $x^2 - 25$ b $9x^2 - 4$

Exercise 10A

1 a PS b DOPS c DOPS d PS
2 a $ab + ac$ b $ab - ac$
c $ac + ad + bc + bd$ d $a^2 - b^2$
3 a $x^2 + 2x$ b $x^2 + 4x + 3$ c $x^2 + 8x + 16$
4 a $2x + 10$ b $3x - 12$ c $-5x - 15$
d $-4x + 8$ e $6x - 3$ f $12x + 4$
g $-10x + 6$ h $-20x - 15$ i $2x^2 + 5x$
j $3x^2 - x$ k $2x - 2x^2$ l $6x - 3x^2$
m $-6x^2 - 4x$ n $-18x^2 + 6x$ o $-10x + 10x^2$
5 a $x^2 + 10x + 16$ b $x^2 + 7x + 12$
c $x^2 + 12x + 35$ d $x^2 + 5x - 24$
e $x^2 + x - 30$ f $x^2 + x - 6$
g $x^2 - 4x - 21$ h $x^2 - 10x + 24$
i $x^2 - 13x + 40$ j $6x^2 + 13x + 5$
k $12x^2 + 23x + 10$ l $10x^2 + 41x + 21$
m $9x^2 - 9x - 10$ n $20x^2 + 2x - 6$
o $6x^2 + 5x - 25$
6 a $x^2 + 10x + 25$ b $x^2 + 14x + 49$
c $x^2 + 12x + 36$ d $x^2 - 6x + 9$
e $x^2 - 16x + 64$ f $x^2 - 20x + 100$
g $4x^2 + 20x + 25$ h $25x^2 + 60x + 36$
i $49x^2 - 14x + 1$
7 a $x^2 - 16$ b $x^2 - 81$
c $x^2 - 64$ d $9x^2 - 16$
e $4x^2 - 9$ f $64x^2 - 49$
g $16x^2 - 25$ h $4x^2 - 81$
i $25x^2 - 49$ j $49x^2 - 121$
8 a 6 b 7 c 16 d 4 e 4 f 6
9 a 3 b 3 c 3 d 8 e 1 f 2
10 a $(x^2 - 12x + 36)$ cm^2 b $(x^2 + 10x - 200)$ cm^2
11 a $-x^2 + 7x$ b $10a - 28$
c $x^2 - 81$ d $4x^2 + 12x + 9$
12 a i $x^2 + 5x$ ii $x^2 + 13x + 36$ iii $8x + 36$
b i 36 m^2 ii 68 m^2

10B

Now you try

Example 5
a $-4(x + 9)$ b $7b(2b - 3)$

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Example 6

a $(x-10)(x+10)$ b $(4a-9b)(4a+9b)$

Example 7

$4(a+3)(a-3)$

Exercise 10B

1 a $2(x+3)$ b $-4(x-1)$
c $(x+2)(x-2)$ d $(3x+2)(3x-2)$
2 a 7 b 6 c 8
d -5 e $2a$ f $3a$
g $-5a$ h $-3xy$ i $-2ab$
3 a $3(x-6)$ b $4(x+5)$
c $7(a+b)$ d $3(3a-5)$
e $-5(x+6)$ f $-2(2y+1)$
g $-3(4a+1)$ h $-b(2a+c)$
i $x(4x+1)$ j $x(5x-2)$
k $6b(b-3)$ l $7a(2a-3)$
m $5a(2-a)$ n $6x(2-5x)$
o $-x(2+x)$
4 a $(x+3)(x-3)$ b $(x+5)(x-5)$
c $(y+7)(y-7)$ d $(y+1)(y-1)$
e $(a+4)(a-4)$ f $(b+6)(b-6)$
g $(y+12)(y-12)$ h $(z+20)(z-20)$
i $(2x-3)(2x+3)$ j $(6a-5)(6a+5)$
k $(1+9y)(1-9y)$ l $(10-3x)(10+3x)$
m $(5x-2y)(5x+2y)$ n $(8x-5y)(8x+5y)$
o $(3a+7b)(3a-7b)$
5 a $(2+x)(2-x)$ b $(3+y)(3-y)$
c $(6+a)(6-a)$ d $(10+3x)(10-3x)$
e $(b+a)(b-a)$ f $(20+5a)(20-5a)$
g $(2a+3b)(2a-3b)$ h $(4y+11x)(4y-11x)$
6 a $2(x+4)(x-4)$ b $5(x+3)(x-3)$
c $6(y+2)(y-2)$ d $3(y+4)(y-4)$
e $3(x+5y)(x-5y)$ f $3(a+10b)(a-10b)$
g $3(2x+3y)(2x-3y)$ h $7(3a+4b)(3a-4b)$
i $3(6x-7y)(6x+7y)$
7 a i 100 m ii 96 m iii 36 m
b $(10+t)(10-t)$
c i 100 m ii 96 m iii 36 m
d 10 seconds
8 a 60 b 35 c 69
d 104 e 64 f 40
g 153 h 1260
9 a i x^2 cm^2 ii $(1600-x^2)$ cm^2
b $(40+x)(40-x)$ cm^2
c i 1200 cm^2 ii 1500 cm^2
d $x=30$

10C

Now you try

Example 8

a $(x+9)(x+4)$ b $(x+8)(x-1)$
c $(x-10)(x+2)$

Example 9

$(x-5)^2$

Example 10

$3(x-5)(x-1)$

Exercise 10C

1 a monic b constant, coefficient
2 a 9, 2 b 10, 2 c 5, −3
d 4, −3 e −8, 3 f −10, 3
g −2, −5 h −12, −3 i 8, −1
3 a i 5, 3 ii $(x+5)(x+3)$
b i 5, −2 ii $(x+5)(x-2)$
c i −4, −2 ii $(x-4)(x-2)$
4 a $(x+6)(x+1)$ b $(x+3)(x+2)$
c $(x+3)^2$ d $(x+5)(x+2)$
e $(x+4)(x+3)$ f $(x+9)(x+2)$
g $(x-1)(x+6)$ h $(x+3)(x-2)$
i $(x+4)(x-2)$ j $(x-1)(x+4)$
k $(x+10)(x-3)$ l $(x+11)(x-2)$
m $(x-2)(x-5)$ n $(x-4)(x-2)$
o $(x-4)(x-3)$ p $(x-1)^2$
q $(x-6)(x-3)$ r $(x-2)(x-9)$
s $(x-6)(x+2)$ t $(x-5)(x+4)$
u $(x-7)(x+2)$ v $(x-4)(x+3)$
w $(x+8)(x-4)$ x $(x-5)(x+2)$
5 a $(x-2)^2$ b $(x+3)^2$
c $(x+6)^2$ d $(x-7)^2$
e $(x-9)^2$ f $(x-10)^2$
g $(x+4)^2$ h $(x+10)^2$
6 a $2(x+5)(x+2)$ b $3(x+4)(x+3)$
c $2(x+9)(x+2)$ d $5(x-2)(x+1)$
e $4(x-5)(x+1)$ f $3(x-5)(x+2)$
g $-2(x+4)(x+3)$ h $-3(x-2)(x-1)$
i $-2(x-7)(x+2)$ j $-4(x-2)(x+1)$
k $-5(x+3)(x+1)$ l $-7(x-6)(x-1)$
7 a $2(x+11)^2$ b $3(x-4)^2$
c $5(x-5)^2$ d $-3(x-6)^2$
e $-2(x-7)^2$ f $-4(x+9)^2$
8 a i (x^2+3x) m^2 ii $(x^2+3x-10)$ m^2
b $(x+5)(x-2)$ m^2
c i 30 m^2 ii 60 m^2
9 a $x+6$ b $x-3$ c $x-3$
d $\frac{1}{x+7}$ e $\frac{1}{x-5}$ f $\frac{1}{x-6}$
g $x-2$ h $x+1$ i $x-8$

10D

Now you try

Example 11

a $x=\pm 8$ b $x=\pm 3$
c No solution

Example 12

a $x=\pm\sqrt{29}$ b $x=\pm\sqrt{11}\approx\pm 3.3$

Example 13

a $x=\pm 5$ b No solution
c $x=\sqrt{15}$ d $x=-3$

Exercise 10D

1 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
2 a 16 b 81 c 49 d 9 e 64
f 1
3 a 6 b −7 c −10 d 0 e 20
f 40
4 a $c>0$ b $c=0$ c $c<0$
5 a $x=\pm 5$ b $x=\pm 9$ c $x=\pm 6$
d $x=\pm 7$ e No solutions f No solutions
g $x=\pm 20$ h $x=\pm 12$ i $x=\pm 5$
j $x=\pm 4$ k $x=\pm 1$ l No solutions
m $x=\pm 8$ n $x=\pm 9$ o $x=\pm 6$
6 a i $x=\pm\sqrt{14}$ ii $x=\pm\sqrt{22}$ iii $x=\pm\sqrt{17}$
iv No solutions v $x=\pm\sqrt{5}$ vi $x=\pm\sqrt{6}$
vii $x=\pm\sqrt{21}$ viii $x=\pm\sqrt{6}$ ix $\pm\sqrt{15}$
b i $x=\pm 3.5$ ii $x=\pm 5.9$ iii $x=\pm 5.5$
iv $x=\pm 7.9$
7 a $x=\pm 2$ b $x=\pm 9$ c No solutions
d No solutions e $x=\pm 3$ f $x=\pm\sqrt{15}$
g $x=\sqrt{3}$ h $x=\pm 1$ i $x=\sqrt{2}$
j $x=-\sqrt{7}$ k No solutions l $x=\sqrt{5}$
8 a $x=\pm 4$ b $x=\pm 3$ c $x=\pm 3$
d $x=\pm 5$ e $x=\pm 2$ f No solutions

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9 24 m
10 4 seconds
11 a $x=\sqrt{29}$ b $x=8$ c $x=\sqrt{17}$ d $x=\sqrt{2}$
12 2.5 m
13 a $r=\pm 2.8$
b Since, in this practical scenario, r is the radius, then r must be greater than 0; i.e. $r=2.8$.
14 a $b>0$ b $b=0$ c $b<0$
15 a $b<0$ b $b=0$ c $b>0$
16 a $x=\pm 5$
b i $-5<x<5$ ii $x>5$ or $x<-5$
17 a $x=-4, 2$ b $x=-10, 4$ c $x=0, 4$ d $x=1, 9$
e $x=-3, 2$ f $x=0, 1$ g $x=0, \frac{4}{3}$ h $x=-\frac{3}{2}, 3$

10E

Now you try

Example 14

a $x=0, -7$ b $x=-6, \frac{2}{3}$

Example 15

$x=0, -6$

Example 16

$x=\pm 9$

Example 17

a $x=-5, -6$ b $x=4$

Exercise 10E

1 a $a=0, b=0$ b common factor, common factor.
2 a

x	−3	−2	−1	0	1	2
$(x+2)(x-1)$	4	0	−2	−2	0	4

b 1, −2
c −3, 2
3 a $x-2, 2$ b $x+4, 1, -4$
c $x+6, 2x-7, -6, 7, \frac{7}{2}$
4 a $x=0, -1$ b $x=0, 5$
c $x=0, 4$ d $x=3, -2$
e $x=-5, 4$ f $x=-1, 1$
g $x=2, -1$ h $x=\frac{2}{3}, 7$
i $x=0, -\frac{5}{4}$ j $x=\frac{1}{2}, -\frac{7}{3}$
k $x=\frac{5}{4}, -\frac{2}{5}$ l $x=-\frac{3}{8}, -\frac{3}{4}$
5 a $x=0, 4$ b $x=0, 3$
c $x=0, -2$ d $x=0, 4$
e $x=0, 5$ f $x=0, -2$
6 a $x=-5, 5$ b $x=6, -6$
c $x=10, -10$ d $x=\frac{3}{2}, -\frac{3}{2}$
e $x=\frac{4}{3}, -\frac{4}{3}$ f $x=\frac{9}{7}, -\frac{9}{7}$
7 a $x=-2, -1$ b $x=-3, -2$ c $x=2, 4$ d $x=5, 2$
e $x=-6, 2$ f $x=-5, 3$ g $x=5, -4$
h $x=8, -3$ i $x=4, 8$ j $x=-2$ k $x=-5$
l $x=4$ m $x=7$ n $x=12$
8 a 2 b 2 c 1 d 1 e 2
f 2 g 1 h 1 i 1
9 a i 3.2 m ii 4.8 m
b 0, 10
c 10 s
10 a $x=-2, -6$ b $x=-1, 11$
c $x=3$ d $x=2$
11 a 150 cm^2
b i $x(x+5)$ cm^2 $=(x^2+5x)$ cm^2
ii $(x^2+5x-150)$ cm^2
c $(x+15)(x-10)$ cm^2
d 10
e 20

10F

Now you try

Example 18

Width = 6 m, length = 12 m

Exercise 10F

1 a $x+4$ b $x+10$ c $x-7$
d $x-2$ e $x+6$
2 a $x^2+2x-3=0$ b $x^2-3x-5=0$
c $x^2+7x-4=0$
3 B, D, A, C
4 b $(x+5)$ m
c $x(x+5)=24$
d $x^2+5x-24=0, x=-8, 3$
e Width = 3 m, length = 8 m
5 a Width = 6 m, length = 10 m
b Width = 9 m, length = 7 m
c Width = 14 mm, length = 11 mm
6 a $A=\frac{1}{2}x(x+2)$ b $\frac{1}{2}x(x+2)=4$
c $x^2+2x-8=0$ d $x=2, h=4$
7 Height = 2 m, base = 7 m
8 a $x^2+x-132=0$
b −12, 11
c −12, −11 and 11, 12
9 8 and 9 or −9 and −8
10 15
11 1 m
12 a 3.75 m
b $t=1$ s, 3 s
c The ball will reach this height both on the way up and on the way down.
d $t=0$ s, 4 s
e $t=2$ s
f The ball reaches a maximum height of 4 m.
g No, 4 m is the maximum height. If $h=5$, there is no solution.
13 a $x=0, 100$
b The rocket starts at the launching site; i.e. at ground level, and hits the ground again 100 m from the launching site.
c 2 m or 98 m

Progress quiz

1 a $x^2+8x+12$ b $6x^2-x-15$
c $6x^2-17x+5$
2 a $x^2+12x+36$ b $4x^2-12x+9$
c x^2-25 d $9x^2-16$
3 a $(x-2)(x+2)$ b $(y-9)(y+9)$
c $(3x-5)(3x+5)$ d $(4x-7y)(4x+7y)$
4 a $(x+7)(x+2)$ b $(x-5)(x+3)$
c $(x-4)(x-1)$ d $(x+5)^2$
5 a $2(x-6)(x+6)$ b $3(2y-3)(2y+3)$
c $2(x+7)(x-2)$

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6 **a** $x = \pm 5$ **b** $x = \pm 6$
c $x = \pm\sqrt{14}$ **d** No real solution
e $x = \pm\sqrt{11}$ **f** $x = -2$

7 **a** $x = 0$ or $x = -2$ **b** $x = 3$ or $x = -3$
c $x = \frac{1}{2}$ or $x = -4$

8 **a** $x = 0$ or $x = 4$ **b** $x = 6$ or $x = -6$
c $x = 5$ or $x = -8$ **d** $x = 0$ or $x = 2$
e $x = \frac{1}{2}$ or $x = -\frac{1}{2}$ **f** $x = 4$

9 Length = 7 m, width = 3 m

10G

Now you try

Example 19

a **i** Minimum $(-2, -9)$ **ii** $x = -2$ **iii** $x = -5, 1$
iv $y = -5$

b **i** Maximum $(1, 0)$ **ii** $x = 1$ **iii** $x = 1$
iv $y = -2$

Example 20
See table at bottom of page.

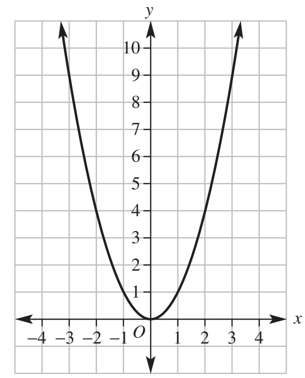
Example 21
See table at bottom of page.

Example 22
See table at bottom of page.

Exercise 10G

1

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



2 **a** maximum **b** $(-2, 9)$ **c** 5
d −5, 1 **e** $x = -2$

3 **a** **i** (2, −5.4), minimum **ii** $x = 2$ **iii** −1, 5
iv −3

b **i** (2, 0), maximum **ii** $x = 2$ **iii** 2
iv −1

c **i** (2, 5), minimum **ii** $x = 2$ **iii** No x-intercepts
iv 7

d **i** (−3, 0), minimum **ii** $x = -3$ **iii** −3
iv 4

e **i** (2, −2), minimum **ii** $x = 2$ **iii** 1, 3
iv 6

f **i** (0, 3), maximum **ii** $x = 0$ **iii** −3, 3
iv 3

4 *See table at bottom of page.*

5 *See table at top of next page.*

6 *See table at top of next page.*

7 **a** vi **b** iii **c** v **d** iv **e** i **f** ii

Example 20

	Formula	Max or min	Ref in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = \frac{1}{2}x^2$	Min	No	(0, 0)	$y = \frac{1}{2}$	Wider
b	$y = -3x^2$	Max	Yes	(0, 0)	$y = -3$	Narrower

Example 21

	Formula	Max or min	Ref in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x + 1)^2$	Min	No	(−1, 0)	$y = 4$	Same
b	$y = (x - 3)^2$	Min	No	(3, 0)	$y = 4$	Same

Example 22

	Formula	Max or min	Ref in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 - 2$	Min	No	(0, −2)	$y = -1$	Same
b	$y = x^2 + 1$	Min	No	(0, 1)	$y = 2$	Same

4

	Formula	Max or min	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$	Min	No	(0, 0)	$y = 3$	Narrower
b	$y = \frac{1}{2}x^2$	Min	No	(0, 0)	$y = \frac{1}{2}$	Wider
c	$y = 2x^2$	Min	No	(0, 0)	$y = 2$	Narrower
d	$y = -4x^2$	Max	Yes	(0, 0)	$y = -4$	Narrower
e	$y = -\frac{1}{3}x^2$	Max	Yes	(0, 0)	$y = -\frac{1}{3}$	Wider
f	$y = -2x^2$	Max	Yes	(0, 0)	$y = -2$	Narrower

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5

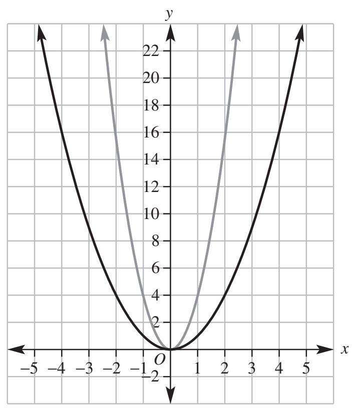
	Formula	Turning point	Axis of symmetry	y-intercept ($x = 0$)	x-intercept
a	$y = (x+3)^2$	$(-3, 0)$	$x = -3$	9	−3
b	$y = (x-1)^2$	$(1, 0)$	$x = 1$	1	1
c	$y = (x-2)^2$	$(2, 0)$	$x = 2$	4	2
d	$y = (x+4)^2$	$(-4, 0)$	$x = -4$	16	−4

6

	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$	$(0, 3)$	3	$y = 4$
b	$y = x^2 - 1$	$(0, -1)$	−1	$y = 0$
c	$y = x^2 + 2$	$(0, 2)$	2	$y = 3$
d	$y = x^2 - 4$	$(0, -4)$	−4	$y = -3$

8 **a** $y = x^2 + 2$ **b** $y = -x^2$
c $y = (x+1)^2$ **d** $y = (x-5)^2$

9 **a** **i**



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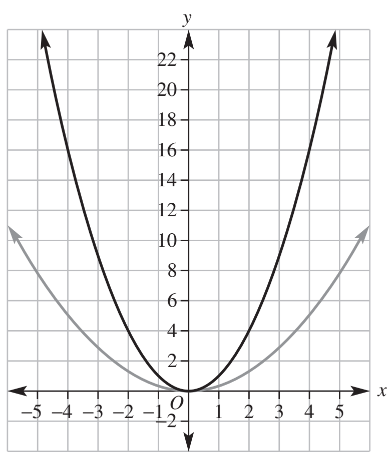

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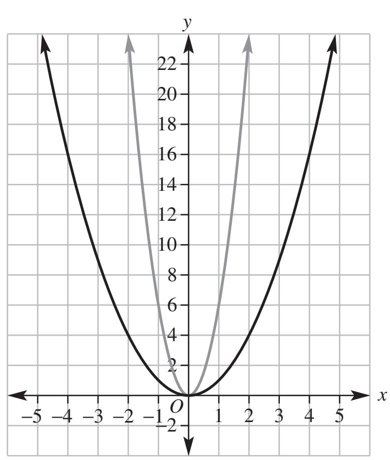

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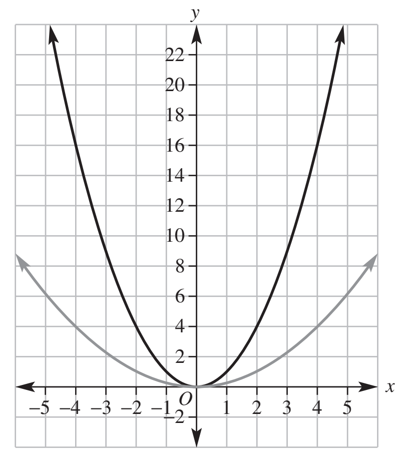

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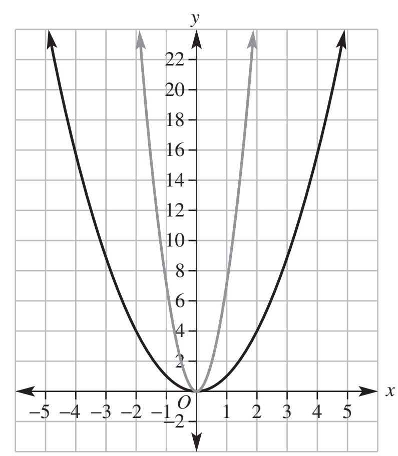

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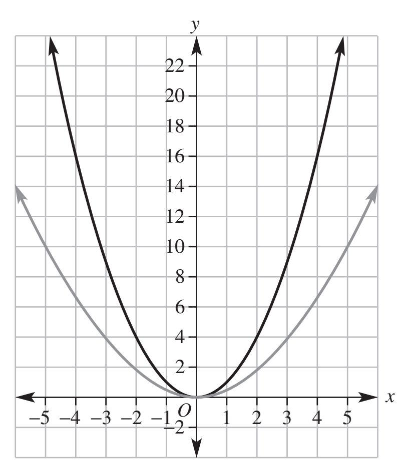

b The constant a determines the narrowness of the graph.

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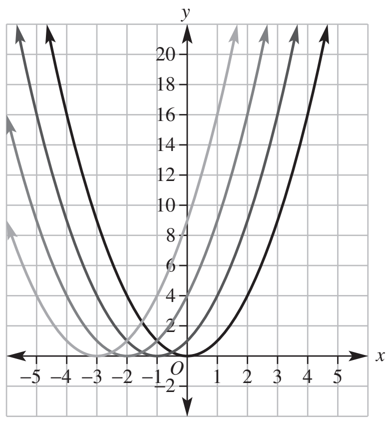
10 a i



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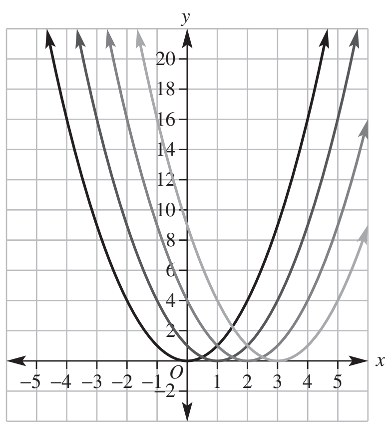
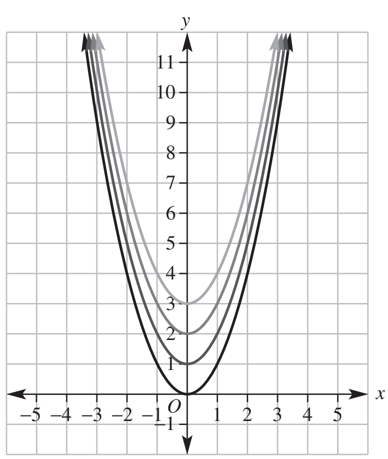


b The constant h determines whether the graph moves left or right from $y = x^2$.

11 a i



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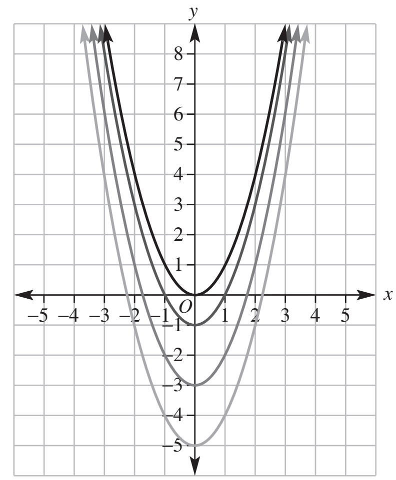


b The constant k determines whether the graph moves up or down from $y = x^2$.

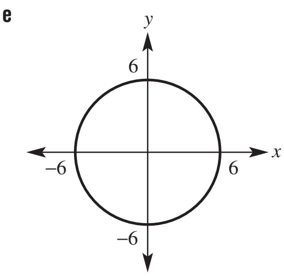
10H

Now you try

Example 23

a (0, 0) **b** 6 **c** ± 5.9 **d** ± 6

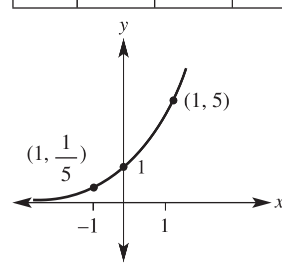
e



Example 24

a

x	−2	−1	0	1	2
y	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25



Exercise 10H

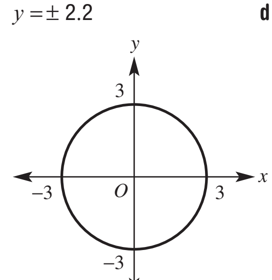
1 a $(0, 0), r = 3$ **b** $(0, 0), r = 6$
c $(-1, 0), r = 2$

2 B

3 C

4 a $(0, 0)$ **b** r

5 a $(0, 0)$ **b** $r = 3$
c $y = \pm 2.2$ **d** $x = \pm 3$
e



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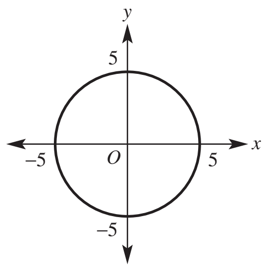
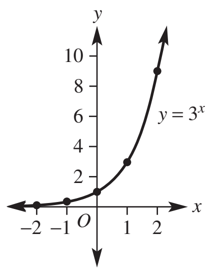
6 a (0, 0) **b** $r = 5$
c $y = \pm 3$ **d** $x = \pm 5$
e



7 a

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

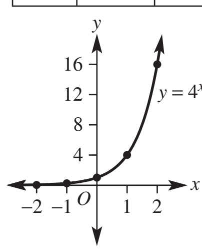
b



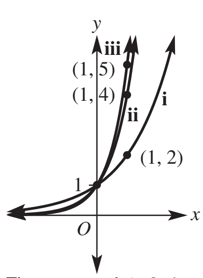
8 a

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

b



9 a



b The same point (0, 1).
c Makes it rise more quickly.

10 a $r = 6$ **b** $r = 9$ **c** $r = 12$
d $r = \sqrt{5}$ **e** $r = \sqrt{14}$ **f** $r = \sqrt{20}$

11 $x^2 + y^2 = 49$

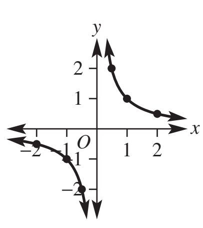
12 a C **b** A **c** B

13 a 1000
b i 2000 **ii** 8000
c i 2 years **ii** 4 years

14 a

x	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	−1	−2	2	1	$\frac{1}{2}$

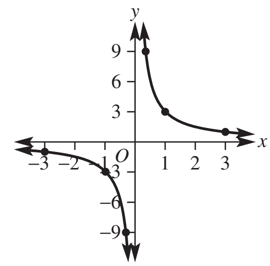
b



15 a

x	−3	−1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3
y	−1	−3	−9	9	3	1

b



Maths@Work: Driving instructor

1 a $\frac{1000}{3600} = \frac{5}{18}$ **b** $\frac{3600}{1000} = \frac{18}{5}$

2 a 11.11 m/s **b** 13.89 m/s **c** 16.67 m/s
d 19.44 m/s **e** 22.22 m/s **f** 27.78 m/s

3 $R = 0.7 \times s$

4 a 7.78 m **b** 9.72 m **c** 11.67 m
d 13.61 m **e** 15.56 m **f** 19.44 m

5 a 6.17 m **b** 9.65 m **c** 13.89 m
d 18.90 m **e** 24.69 m **f** 38.58 m

6 a $D = 0.7s + \frac{s^2}{20}$
b A quadratic equation.
c The total stopping distance increases by larger and larger amounts as the speed increases.

7 a

Speed in km/h	Speed in m/s	Reaction distance in m	Braking distance in m	Total stopping distance in m
0	0	0	0	0
10	2.78	1.94	0.39	2.33
20	5.56	3.89	1.54	5.43
30	8.33	5.83	3.47	9.31
40	11.11	7.78	6.17	13.95
50	13.89	9.72	9.65	19.37
60	16.67	11.67	13.89	25.56
70	19.44	13.61	18.90	32.52
80	22.22	15.56	24.69	40.25
90	25.00	17.50	31.25	48.75
100	27.78	19.44	38.58	58.02
110	30.56	21.39	46.68	68.07

b 50 km/h; 19.37 m
c i 50 km/h **ii** 90 km/h
d i 21% increase in braking distance. **ii** 44%; 69%
e *See figure at top of next page.*
f i 50 km/h **ii** 70 km/h
iii 90 km/h **iv** 100 km/h

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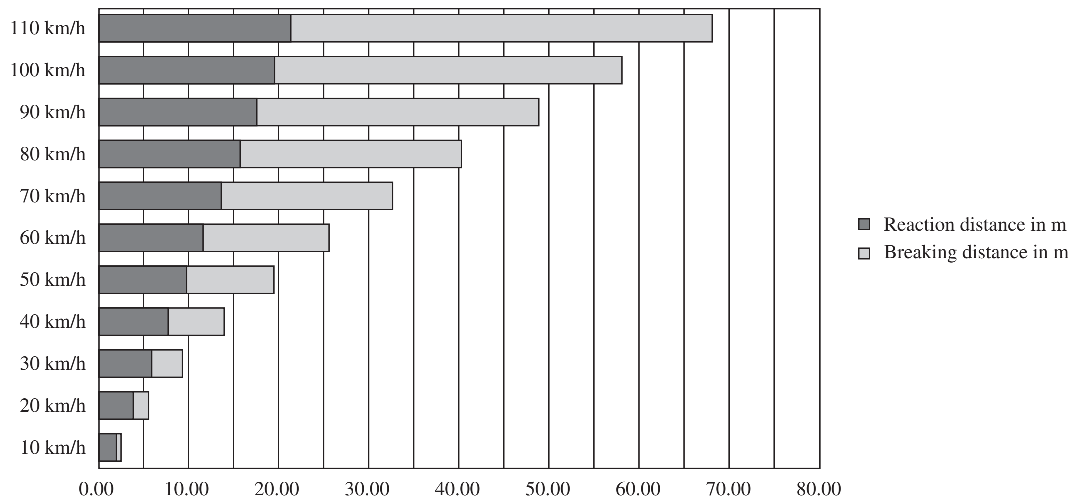
Stopping distances at various speeds



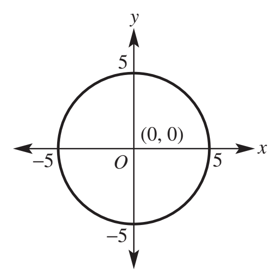
Puzzles and games

1 PARABOLA
2 a $(x+4)^2$ m^2 = $(x^2+8x+16)$ m^2 **b** 6
3 14 cm by 20 cm
4 12, 14 or −14, −12
5 64 and 8
6 5 cm
7 a $7x-6$ **b** $-4x$ **c** $x-3$
8 25 km/h

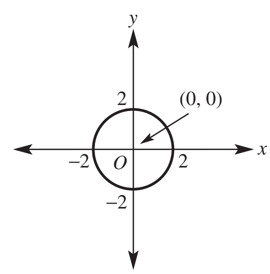
Short-answer questions

1 a $-2x-2$ **b** x^2+3x
c x^2+x-2 **d** $3x^2+11x-20$
e x^2-16 **f** $25x^2-4$
g x^2+4x+4 **h** $x^2-12x+36$
i $12x^2-23x+10$
2 a x^2+3x **b** x^2+4x+4
c $4x^2+18x$
3 a $3(x-3)$ **b** $-4(x+4)$
c $x(x+2)$ **d** $b(a-1)$
e $7x(1-2x)$ **f** $-ab(a+6)$
4 a $(x+7)(x-7)$ **b** $(3x+4)(3x-4)$
c $(2x+1)(2x-1)$ **d** $3(x+5)(x-5)$
e $2(x+3)(x-3)$ **f** $(2x+9)(2x-9)$
5 a $(x+2)(x+3)$ **b** $(x-3)(x+2)$
c $(x-6)(x-2)$ **d** $(x+12)(x-2)$
e $(x+10)(x-5)$ **f** $(x-8)(x-4)$
g $(x-3)^2$ **h** $(x+10)^2$
i $(x+20)^2$
6 a $\pm\sqrt{11}$ **b** $\pm\sqrt{3}$ **c** $\pm\sqrt{5}$
7 a $x=-1, 2$ **b** $x=3, -7$
c $x=\frac{1}{2}, -4$ **d** $x=0, 3$
e $x=0, -6$ **f** $x=0, \frac{5}{2}$
8 a $x=0, -4$ **b** $x=0, 3$
c $x=5, -5$ **d** $x=\pm\frac{4}{3}$
e $x=-3, -5$ **f** $x=3, 7$
g $x=4$ **h** $x=-5$ **i** $x=-9, 4$
9 Length = 8 m, width = 6 m
10 a Minimum at (1, −4) **b** $x=1$ **c** −1 and 3
d −3
11 *See table at top of next page.*

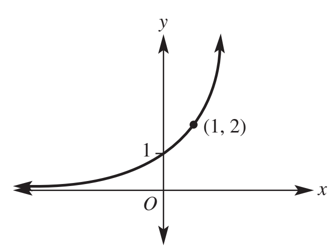
12 a



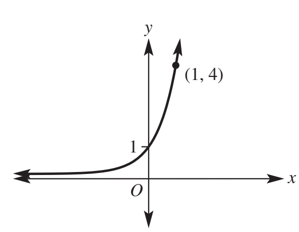
b



13 a



b



Multiple-choice questions

1 E	**2** D	**3** B	**4** C	**5** C
6 B	**7** B	**8** D	**9** E	**10** C

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	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 2x^2$	Min	No	$(0, 0)$	2	Narrower
b	$y = -\frac{1}{2}x^2$	Max	Yes	$(0, 0)$	$-\frac{1}{2}$	Wider
c	$y = (x - 2)^2$	Min	No	$(2, 0)$	1	Same
d	$y = x^2 - 2$	Min	No	$(0, -2)$	−1	Same

Extended-response questions

1 **a** **i** x^2 m^2 **ii** $(100 - x^2)$ m^2
b $(10 + x)(10 - x)$ m^2
c **i** 96 m^2 **ii** 84 m^2
d $x = 5$
2 **a** **i** $(16 + 2x)$ m **ii** $(12 + 2x)$ m
b Area = $(4x^2 + 56x + 192)$ m^2
c Trench = $(4x^2 + 56x)$ m^2
d Minimum width is 2 m.

Semester review 2

Straight-line graphs

Short-answer questions

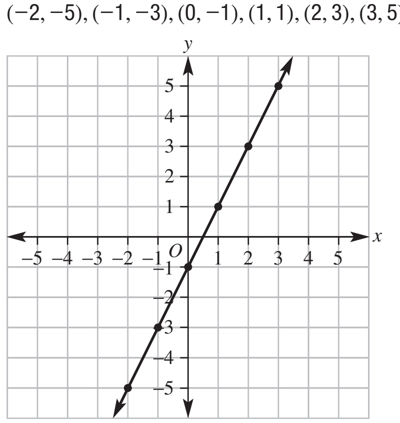
1 **a** \$6000 **b** 8 years **c** \$500

2 **a** **i** 16 km **ii** 24 km **iii** 32 km
b 16 km/h **c** 1.5 hours **d** 45 km
e 2.5 hours **f** 18 km/h **g** 90 km

3 **a**

x	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5

b $(-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)$
c



4 **a** AB midpoint $(1, 2)$, length = $\sqrt{68}$, gradient $m = \frac{1}{4}$
b PQ midpoint $(-1, -1)$, length = $\sqrt{72}$, gradient $m = -1$
c ST midpoint $(2, 0)$, length = 4, gradient undefined

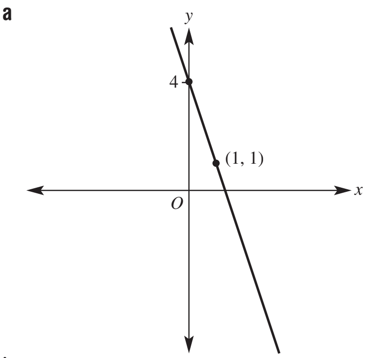
5 **a** $m = 2$ **b** $m = -3$

6 **a** PQ length = $\sqrt{41}$ **b** RM length = $\sqrt{89}$

7 **a** TK midpoint $(4, 8)$ **b** FG midpoint $(1, -2)$

8 **a** Line J: y-intercept = −2, $m = 2$, $y = 2x - 2$
Line K: y-intercept = 6, $m = -\frac{2}{3}$, $y = -\frac{2}{3}x + 6$
b $(3, 4)$

9 **a**



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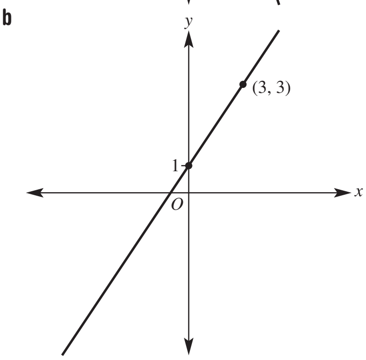

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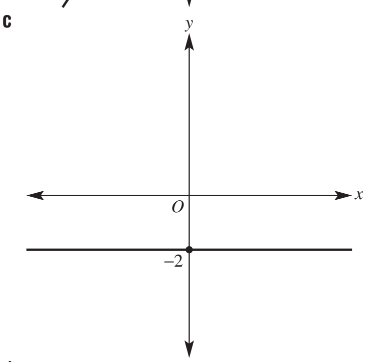

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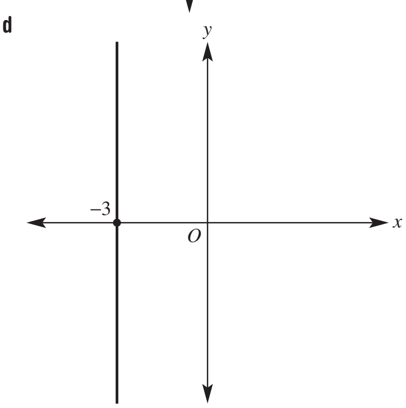

10 **a** x-intercept $(3, 0)$, y-intercept $(0, -6)$
b x-intercept $(6, 0)$, y-intercept $(0, -4)$

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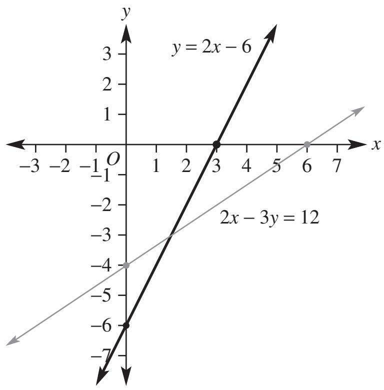


Multiple-choice questions

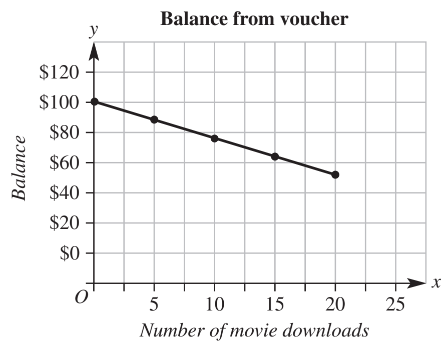
1 D **2** E **3** B **4** A **5** D

Extended-response question

a $B = 100 - 2.4n$
b **i** \$76 **ii** 30 movies
c

Number of movies, n	0	5	10	15	20
Balance, B	\$100	\$88	\$76	\$64	\$52

d



e 41 movies, \$1.60 remaining on voucher

Geometry

Short-answer questions

1 **a** 39 **b** 61 **c** 127
d 75 **e** 70 **f** 117
g 71 **h** 84 **i** 110
j 120 **k** 135 **l** 50
2 **a** $\triangle ABC \equiv \triangle DEF$(SAS)
b $\triangle STU \equiv \triangle MNO$(AAS)
3 **a** Yes, (RHS) **b** Yes, (AAA) **c** Yes, (SSS)
4 $x = 1.8, y = 5$

Multiple-choice questions

1 C **2** E **3** C **4** A **5** D

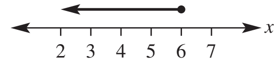
Extended-response question

a AAA **b** 2.5 **c** 5 m

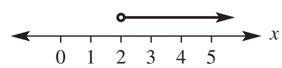
Equations

Short-answer questions

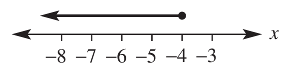
1 **a** $x = -9$ **b** $x = 26$ **c** $m = 32$
d $a = 4$ **e** $k = -6$ **f** $x = -8$
2 **a** $p = 2$ **b** $a = 4$ **c** $x = 12$ **d** $x = 20$
3 **a** $x = 8$ **b** $k = 3$ **c** $m = 9$
d $x = 5$ **e** $a = -4$ **f** $x = 4$
4 **a** $x - 5 = 8; x = 13$
b $4x + 8 = 20; x = 3$
c $2(3x - 6) = 18; x = 5$
5 **a** $b = 10$
b $P = 400$
6 **a** $x \le 4$ **b** $x > 8$
7 **a** $x \le 6$



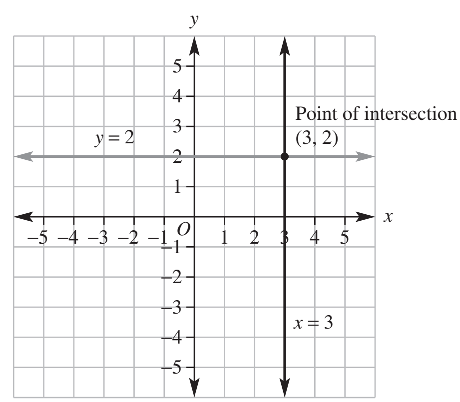
b $x > 2$



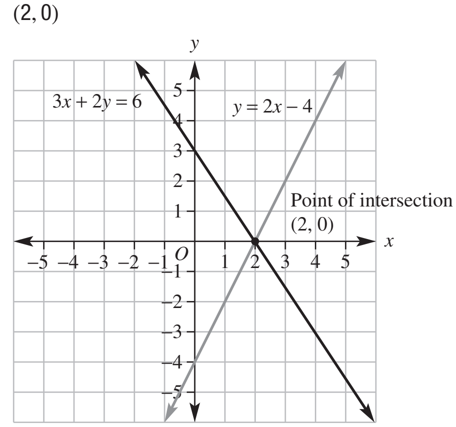
c $x \le -4$



8 **a** (3, 2)



b (2, 0)



9 **a** (1, 2) **b** (3, 9) **c** (−1, 4)
10 **a** (1, 1) **b** (2, 4) **c** (3, 1)
11 **a** Let a = Oliver's age, b = Ruby's age (any pronumeral selection is correct)
b $a - b = 7, a + b = 45$
c Oliver is 26 years old, Ruby is 19 years old

Multiple-choice questions

1 C **2** D **3** A **4** B **5** E

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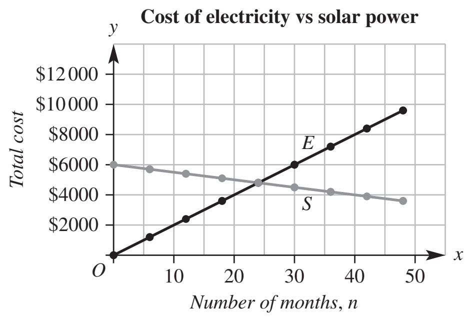
Extended-response question

a

n	E	S
0	\$0	\$6000
6	\$1200	\$5700
12	\$2400	\$5400
18	\$3600	\$5100
24	\$4800	\$4800
30	\$6000	\$4500
36	\$7200	\$4200
42	\$8400	\$3900
48	\$9600	\$3600

b



c $n = 24$ months, $E = S = \$4800$ **d** $E = 200n$, $S = 6000 - 50n$

e 24 months **f** \$6000 **g** 27 months

Pythagoras' theorem and trigonometry

Short-answer questions

1 a PR **b** QR **c** PQ

2 a $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{y}{z}$

3 a 0.64 **b** 0.63 **c** 0.32 **d** 7.50
e 31.25 **f** 4.59 **g** 36.88 **h** 32.33
i 9.77

4 a 11.40 **b** 0.83 **c** 6.02 **d** 14.69

5 a 10.2 **b** 27.0 **c** 18.6 **d** 21.2

6 14 m **7** 8.95 m

8 a 60° **b** 37° **c** 77°

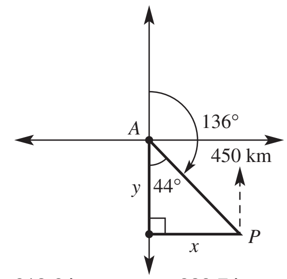
9 177.9 m **10** 259.8 m

Multiple-choice questions

1 E **2** C **3** A **4** B **5** D

Extended-response question

a



b 312.6 km **c** 323.7 km **d** 316°

Quadratics and non-linear graphs

Short-answer questions

1 a $-2x + 2$ **b** $x^2 - x - 6$
c $2x^2 - x - 21$ **d** $x^2 - 4$ **e** $x^2 - 6x + 9$
f $4x^2 + 4x + 1$

2 a $3(x - 4)$ **b** $-x(2 + x)$
c $(x + 5)(x - 5)$ **d** $(3x + 10)(3x - 10)$
e $(x + 3)(x + 4)$ **f** $(x - 3)(x + 2)$
g $(x + 4)(x - 2)$ **h** $(x - 4)^2$
i $(x + 3)^2$

3 a $x = 0, 3$ **b** $x = 0, -2$
c $x = -2, 2$ **d** $x = -\frac{3}{2}, \frac{3}{2}$
e $x = 3, \frac{1}{2}$ **f** $x = 5, -4$
g $x = -3, -7$ **h** $x = -4$ **i** $x = 7$

4 a $(-1, -1)$, minimum **b** $x = -1$
c $-2, 0$ **d** 0

5 *See table at bottom of page.*

Multiple-choice questions

1 B **2** C **3** E **4** A **5** D

Extended-response question

a i $x^2\,\text{m}^2$ **ii** $(400 - x^2)\,\text{m}^2$

b $(20 + x)(20 - x)\,\text{m}^2$

c i $384\,\text{m}^2$ **ii** $319\,\text{m}^2$

d i 15 **ii** 10

	Formula	Maximum or minimum	Reflected in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$	Minimum	No	$(0, 0)$	3	Narrower
b	$y = -\frac{1}{2}x^2$	Maximum	Yes	$(0, 0)$	$-\frac{1}{2}$	Wider
c	$y = (x + 2)^2$	Minimum	No	$(-2, 0)$	9	Same
d	$y = x^2 - 3$	Minimum	No	$(0, -3)$	-2	Same

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