Edrolo

MATHEMATICS

Year 7

2ND EDITION

Edrolo

YEAR 7 MATHEMATICS

2nd Edition

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Contents

Acknowledgements V
Guide to Edrolo Year 7 Mathematics VI

01 Whole numbers 1

Research summary 2
1A Relational thinking 7
1B Place value *(Revision)* 15
1C Adding and subtracting whole numbers 26
1D Multiplying whole numbers 34
1E Dividing whole numbers 42
1F Order of operations 49
1G Estimating and rounding *(Revision)* 56
Chapter 1 extended application 65
Chapter 1 review 66

02 Number theory 69

Research summary 70
2A Factors, multiples, and divisibility 73
2B Index notation 82
2C Prime and composite numbers 89
2D Prime factorisation 96
2E Lowest common multiples and highest common factors *(Revision)* 104
2F Perfect squares and square roots 111
2G Introduction to integers *(Revision)* 117
2H Adding and subtracting positive integers 124
2I Adding and subtracting negative integers 131
Chapter 2 extended application 139
Chapter 2 review 140

03 Fractions and ratios 143

Research summary 144
3A Introduction to fractions *(Revision)* 148
3B Proper fractions, improper fractions, and mixed numbers *(Revision)* 155
3C Equivalent fractions 163
3D Comparing fractions 170
3E Adding and subtracting fractions 178
3F Multiplying fractions 186
3G Dividing fractions 193
3H Ratios 200
Chapter 3 extended application 208
Chapter 3 review 210

04 Algebra 213

Research summary 214
4A Relational thinking with unknowns 217
4B Building algebraic expressions and equations 224
4C Substituting values into equations 232
4D Solving one-step equations 240
4E Solving two-step equations *(Extension)* 247
4F Solving multi-step equations *(Extension)* 255
4G Simplifying equations (collecting like terms) 264
4H Geometric patterns and numerical sequences 270
4I Creating rules from tables and sequences 279
Chapter 4 extended application 289
Chapter 4 review 290

05 Decimals and percentages 293

Research summary 294
5A Place value and comparing decimals *(Revision)* 298
5B Rounding decimals 309
5C Adding and subtracting with decimals *(Revision)* 316
5D Multiplying and dividing by powers of 10 324
5E Multiplying with decimals 332
5F Dividing with decimals 340
5G Introduction to percentages 348
5H Converting between fractions, decimals, and percentages 356
5I Finding percentages of a quantity 365
Chapter 5 extended application 375
Chapter 5 review 376

06 Angles and straight lines 379

Research summary 380
6A Introduction to angles *(Revision)* 384
6B Angles around a point 393
6C Properties of triangles 401
6D Properties of quadrilaterals 410
6E Parallel lines and transversals *(Extension)* 418
Skill builder 427
Chapter 6 extended application 429
Chapter 6 review 432

07 Statistics and probability 435

Research summary 436
7A Collecting and classifying data 441
7B Summary statistics 451
7C Dot plots and stem-and-leaf plots 459
7D Column graphs and pie charts 473
7E Line graphs 486
7F Introduction to probability 499
7G Probability of events *(Extension)* 508
Chapter 7 extended application 517
Chapter 7 review 519

08 Location and transformations 523

Research summary 524
8A Integers on the Cartesian plane 528
8B Symmetry and reflections 537
8C Translations 546
8D Rotations 554
Chapter 8 extended application 566
Chapter 8 review 568

09 2D shapes and 3D objects 573

Research summary 574
9A Units of length *(Revision)* 578
9B Area of rectangles and parallelograms 586
9C Area of triangles 595
9D Drawing 3D objects 605
9E Volume of prisms 616
Chapter 9 extended application 625
Chapter 9 review 627

Answers 631

Glossary 792

Acknowledgements

Images

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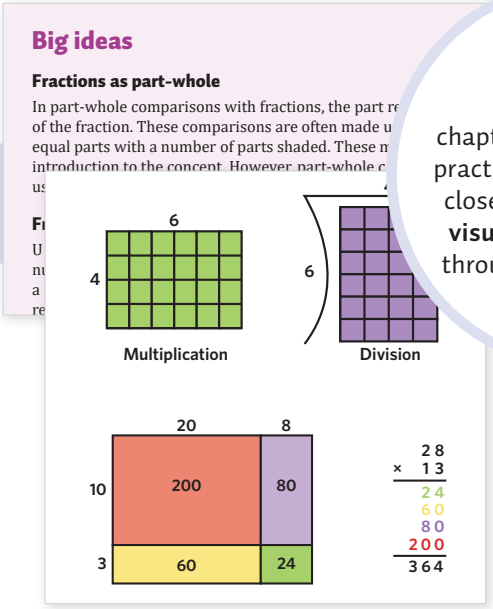
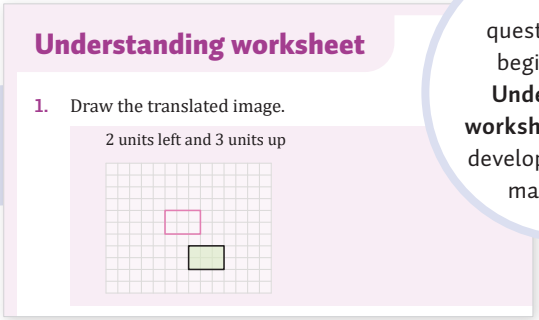
Guide to Edrolo Year 7 Mathematics

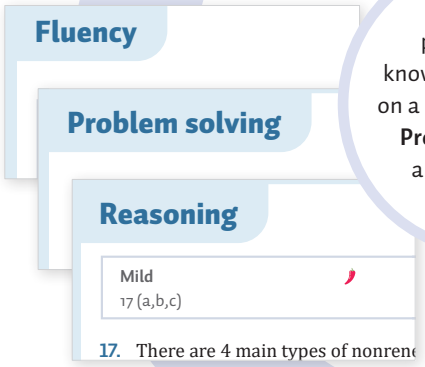
This resource deepens students' conceptual understanding to develop a strong foundation of knowledge through print and online video lessons.



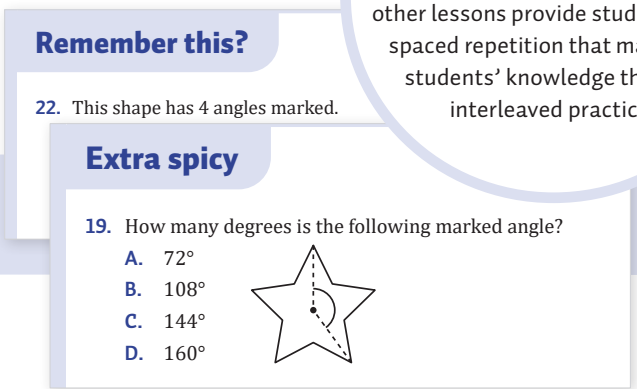
Big ideas are highlighted per chapter to guide teaching practice. Abstract math is closely intertwined with **visual representations** throughout each lesson.



Each question section begins with an **Understanding worksheet** to aid the development of core maths skills.



Students practice their knowledge and skills on a range of **Fluency**, **Problem solving**, and **Reasoning** questions.



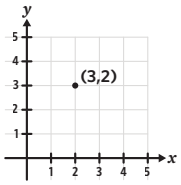
Extra spicy questions challenge students wanting to stretch themselves, and **Remember this?** questions from other lessons provide students with spaced repetition that maintains students' knowledge through interleaved practice.

Visit Edrolo.com.au to access all the digital materials.

Misconceptions

Students don't think that the order of the x and y values matter in an ordered pair.

Incorrect



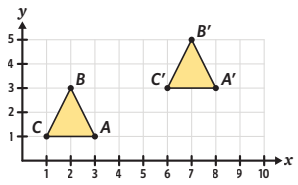
Misconceptions are actively highlighted, uncovered and addressed.

Worked examples

WE 1 **Describing a translation**

Describe each translation.

a.



Key terms and definitions

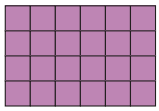
- A **metre (m)** is a standardise
- A **millimetre (mm)** is one-tho

New ideas and vocabulary used are highlighted at the beginning of each lesson

Key ideas

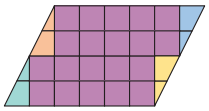
1 The area of a two-dimensional shape can be found by counting th a grid.

Rectangle



Area = 24 square units

Parallelogram



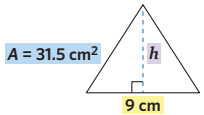
Area = 20 + (4 × 1) = 24 square units

14. Wilma recently travelled to Egypt and took a photo of the Kha Pyramid so that she could print it out as a poster. What is the height of the pyramid on Wilma's poster if the base is 9 cm ar the area is 31.5 cm^2?

Key points

- The pyramid has a base of 9 cm on the poster.
- The area of the pyramid is 31.5 cm^2.
- Calculate the height of the pyramid on the poster.

Explanation



The area of a triangle can be found using the form

Substitute $b = 9$ and $A = 31.5$ into the formula.

$$31.5 = \frac{9h}{2}$$

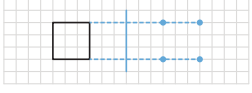
$$31.5 \times 2 = \frac{9h}{2} \times 2$$

$$63 = 9h$$

$$63 \div 9 = 9h \div 9$$

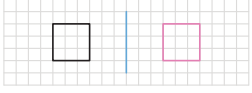
Explanation

Reflect the vertices of the shape over the line



Complete the reflected shape by matching th

Answer



Answers are given in the back of the book. Full worked problem solving solutions are stepped out in detail to understand the why behind the answer. Detailed solutions and videos are available online.

01

Whole numbers

Number and Algebra

Research summary

1A Relational thinking

1B Place value *(Revision)*

1C Adding and subtracting whole numbers

1D Multiplying whole numbers

1E Dividing whole numbers

1F Order of operations

1G Estimating and rounding *(Revision)*

Chapter 1 extended application

Chapter 1 review

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Research summary – Whole numbers

Big ideas

Relational thinking

Relational thinking is the ability to recognise that the equals sign expresses a relationship between identical values and understanding that it represents 'the same as'. Developing relational thinking in mathematics is vital and is the foundation of a solid understanding in all areas of mathematics. The equals sign can sometimes be interpreted in different ways with many often viewing it as an operator with a similar function as +, −, × and ÷. Some may also see it as representing 'the answer' or the 'next step' which is an incorrect interpretation of what it really means.

Place value

The idea of place value refers to the ability to 'unitise' which means that one can recognise that '10 ones is the same as 1 ten'. More commonly, the term 'place value' means the 'value' of a digit is dependent on its specific 'place' or position in the number. For example, if we take the number 465, the 6 does not have the value 6, but rather represents 60 because its 'place' or position is in the tens place. This is a key concept and is often misunderstood. For this reason it is really important to practice the partitioning of a range of numbers into their different place value parts.

Multiplicative thinking

Multiplicative thinking is a person's ability to work flexibly and efficiently with a large range of numbers. Contrary to the name, multiplicative thinking does not only involve multiplication but division as well. This is because multiplication and division are inverse operations and are directly related to one another. The development of multiplicative understanding is crucial for the success not just in multiplication and division of whole numbers, but in all abstract areas of maths such as fractions, decimals, percentages, rates and ratios.

Estimation

Estimation is an important skill in mathematics because it allows us to be able to judge the reasonableness of an answer. This means that when we are performing calculations, particularly difficult ones, we can check whether our answers make sense or are within a reasonable range. Estimation is also very common within our daily lives. We are more likely to use estimation when working with things like money, cooking or figuring out how much is in a collection of objects, than we are to calculate the exact amount.

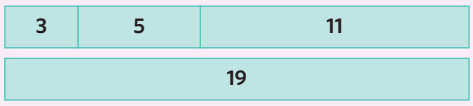
Visual representations

Balance scales

Just like the traditional pan balances, balance scales help develop relational thinking by drawing attention to the idea that in order for the scales to be balanced, both sides have to be the same or equal. For this to be true, whatever operation is applied to one side must also be applied to the other side.

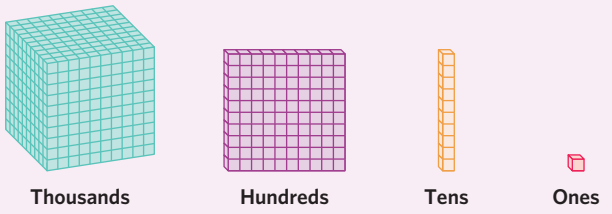
Bar models

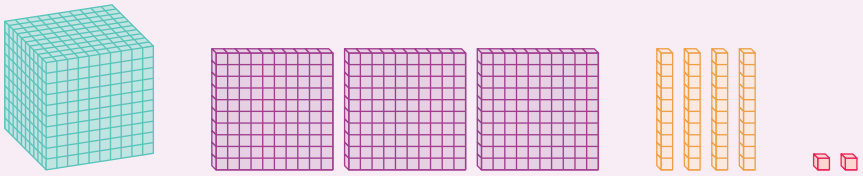
A bar model is a pictorial representation of a number or equation in the form of bars or boxes. They allow us to compare numbers and solve problems by using multiple bars of the same size that will represent the same values. Just like a balance scale, bar models are designed to allow us to work with equations more conceptually, but these models are also designed as a tool to help solve equations visually.



Base 10 blocks

Base 10 or Multi-base Arithmetic Blocks (MAB) are used in many areas of mathematics such as measurement, arithmetic and algebra, but they have a particular use in place value. This is because they provide a concrete or representational approach to regrouping, so that when we are working with larger numbers, we can more easily and accurately partition the numbers into their correct place value parts. This partitioning and regrouping is a crucial skill when performing addition, subtraction, multiplication and division with larger numbers.



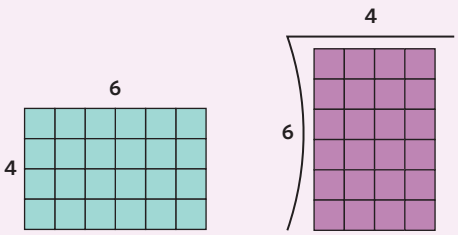


The number 1342 using base 10 blocks

Grid arrays

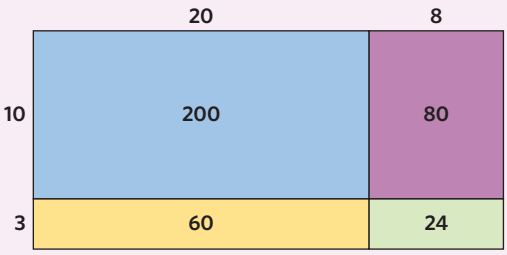
An array is formed by arranging objects into rows and columns so that they form a rectangle. Grid arrays are used to give meaning to multiplication and division because they are visual representations of these abstract concepts. When used frequently, they provide prolonged support in:

- encouraging multiplicative thinking.
- developing conceptual skills in multiplication, division, decimals, fractions and algebra.
- linking multiplication to area.
- demonstrating the commutative and associative property.
- demonstrating the distributive law.



Open arrays

Open arrays are useful for the multiplication of larger numbers. They follow the same principle as grid arrays but they do not show the units within the array. These types of arrays are encouraged once a deeper understanding of multiplicative thinking is established because they promote the use of multiplication strategies rather than counting. Open arrays are also more practical when using the distributive law for two and three digit multiplications as they lead nicely into the multiplication algorithm and provide a much deeper understanding of the steps involved with that method.



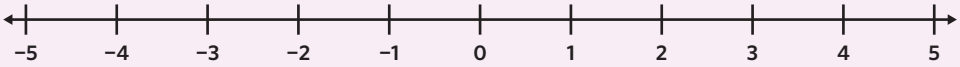
$28 \times 13 = 20 \times 10 + 20 \times 3 + 8 \times 10 + 8 \times 3$

$= 200 + 60 + 80 + 24$

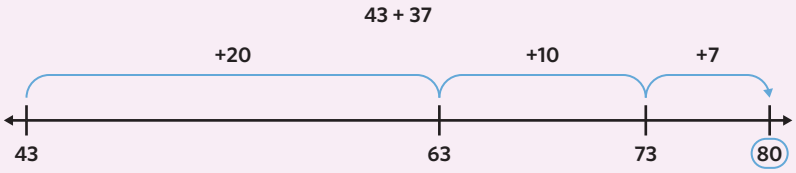
$= 364$

Number lines

A number line is a visual representation that shows the order and size of numbers and is very useful in a variety of mathematical concepts. The ordinal nature of number lines allows numbers to be seen along a continuum so that we can more easily arrange them in order of their size. They are particularly useful when working with fractions, decimals and negative numbers but can also allow us to work more conceptually with the arithmetic operations of addition, subtraction, multiplication and division. Traditionally we view a number line as a horizontal axis with incremental values increasing from left to right, but number lines can also be open without all of the ticks shown or can even be vertical.



Number line including negative numbers



Addition on an open number line

99 charts

A 99 chart is similar to a 1–100 chart but can provide a greater depth of understanding, with place value and rounding. This is because each row of a 99 chart is made up of its different place value parts. For example, all of the 'ones' are found in row 1, 'tens' in row 2, 'twenties' in row 3 etc. This also makes it much easier when rounding because when divided vertically down the middle the 99 chart allows us to clearly see what value a number is closest to.

0	1	2	3	4	5	6	7	8	9	10
10	11	12	13	14	15	16	17	18	19	20
20	21	22	23	24	25	26	27	28	29	30
30	31	32	33	34	35	36	37	38	39	40
40	41	42	43	44	45	46	47	48	49	50
50	51	52	53	54	55	56	57	58	59	60
60	61	62	63	64	65	66	67	68	69	70
70	71	72	73	74	75	76	77	78	79	80
80	81	82	83	84	85	86	87	88	89	90
90	91	92	93	94	95	96	97	98	99	100

Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students see the equals sign as 'the answer is', instead of 'the left-hand side has the same value as the right-hand side'.	For $5 + 10 = x + 7$ $5 + 10 = 15 + 7$	For $5 + 10 = x + 7$ $5 + 10 = 8 + 7$	1A
Students don't see that the commutative and associative properties can be applied to addition and multiplication and still make the result the same.	$8 + 15 \neq 15 + 8$ $3 + (4 + 7) \neq (3 + 4) + 7$ $5 \times 6 \neq 6 \times 5$ $(2 \times 11) \times 7 \neq 2 \times (11 \times 7)$	$8 + 15 = 15 + 8$ $3 + (4 + 7) = (3 + 4) + 7$ $5 \times 6 = 6 \times 5$ $(2 \times 11) \times 7 = 2 \times (11 \times 7)$	1A
Students don't include the value of the place for each digit when expressing values in expanded form.	1245 is written as $1245 = 1 + 2 + 4 + 5$ in expanded form.	1245 is written as $1245 = 1000 + 200 + 40 + 5$ in expanded form.	1B
In multidigit numbers, students don't understand that the position of the digit determines its value.	The value of the digit 2 in the number 1245 is 2.	The value of the digit 2 in the number 1245 is 200.	1B

	Incorrect	Correct	Exercise
Students think the space in a number means 'thousands' or 'new number'.	The number 4 526 738 is written as: • four thousand, five hundred twenty-six, seven hundred thirty-eight or, • four, five hundred twenty-six, seven hundred thirty-eight.	4 526 738 is written as four million, five hundred twenty-six thousand, seven hundred thirty-eight.	1B
Students think subtraction and division are commutative.	$20 - 5 = 5 - 20$ $15 \div 3 = 3 \div 15$	• $20 - 5 = 15$ and $5 - 20 = -15$ • $15 \div 3 = 5$ and $3 \div 15 = \frac{1}{5}$	1C 1E
Students add or subtract whole numbers without considering place value.	$2216 + 15 = 3716$ ✗ because $22 + 15 = 37$	$2216 + 15 = 2231$ ✓	1C
Students incorrectly subtract when using the partitioning strategy.	$57 - 44$ $= 50 + 7 - 40 + 4$ $= 50 - 40 + 7 + 4$ $= 10 + 11$ $= 21$ ✗	$57 - 44$ $= 50 + 7 - (40 + 4)$ $= 50 - 40 + 7 - 4$ $= 10 + 3$ $= 13$ ✓	1C
Students incorrectly apply the distributive law when simplifying multiplication calculations.	13×16 $= (10 + 3) \times (10 + 6)$ $= (10 \times 10) + (3 \times 6)$ $= 100 + 18$ $= 118$	13×16 $= (10 + 3) \times (10 + 6)$ $= 10 \times (10 + 6) + 3 \times (10 + 6)$ $= (10 \times 10) + (10 \times 6) +$ $(3 \times 10) + (3 \times 6)$ $= 100 + 60 + 30 + 18$ $= 208$	1D
Students use the process of algorithms without an understanding of place value.	$\begin{array}{r} 42 \\ \times\ 28 \\ \hline 336 \\ +\ 84 \\ \hline 420 \end{array}$	$\begin{array}{r} 42 \\ \times\ 28 \\ \hline 336 \\ +\ 840 \\ \hline 1176 \end{array}$	1D
Students don't understand the inverse operations.	$20 \div 5 = 4$ has no connection to $5 \times 4 = 20$	$20 \div 5 = 4$ can be rewritten as $20 = 4 \times 5$, because the inverse of a multiplication is division and vice versa. The inverse of an addition is subtraction and vice versa as well.	1E
Students don't understand that a remainder is a fraction of the divisor.	$100 \div 7 = 14\ r\ 2$ or 14.2	$100 \div 7 = 14\ r\ 2$ or $14\frac{2}{7}$	1E
Students don't consider the order of operations when performing calculations involving multiple operations.	Calculations are performed from left to right. $2 + 7 \times 5 - 3$ $= 9 \times 5 - 3$ $= 45 - 3$ $= 42$	Perform multiplication first, then work left to right. $2 + 7 \times 5 - 3$ $= 2 + 35 - 3$ $= 37 - 3$ $= 34$	1E
When using the order of operations, students don't work from left to right when working with inverse operations.	Working right to left. $16 \div 4 \times 2$ $= 16 \div 8$ $= 2$	Work from left to right. $16 \div 4 \times 2$ $= 4 \times 2$ $= 8$	1F
Students round the digit in the designated place but don't change the digits after it to zero.	128 998 rounded to the nearest thousand is 129 998.	128 998 rounded to the nearest thousand is 129 000.	1G
Students round 'down' by lowering the value of the digit in the designated place to the one below.	123 398 rounded to the nearest thousand is 122 000.	123 398 rounded to the nearest thousand is 123 000.	1G

Additional reading and resources

- Assessment for Common Misunderstandings: https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx
- Written algorithms in the primary years: Undoing the 'good work'? Doug Clarke, 2005.
- Developing the 'Big Ideas' in number, Dianne Siemon: https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/assessment/devbigideas.pdf
- Rocket rounding: https://www.topmarks.co.uk/maths-games/rocket-rounding

1A Relational thinking

The term 'relational thinking' in mathematics refers to the ability to recognise that when we have an equals sign, both the left-hand side and the right-hand side of the equation must have the same value regardless of what operations and numbers are being used. For example, $2 \times 3 = 5 + 1$ is a true statement because both sides have a value of 6 even though they don't use the same values or operations. Below are some examples where relational thinking can be applied.

- I bought 3 bags of 12 mixed lollies and gave away 4 so that I could have the same amount as my sister who had 4 bags of 8 mixed lollies.
- The distance from my home to school is 1500 metres. On the way to school, I pass a store 700 m from my house. On the way home, I decide to stop at the store. How far do I need to walk from school to get to the store?
- I always collect 2 dozen eggs from my chooks every week. By the second day of one week, I collected one full carton. On each of the third, fourth and fifth days of the week, I collected 3 eggs. How many eggs do I expect to collect on the sixth and seventh days in total?

Learning intentions

Students will be able to:

- recognise that both sides of an equals sign must have equivalent values
- identify the use of the commutative and associative laws
- apply the commutative and associative laws to solve problems efficiently
- use appropriate strategies to check that their answers are correct.

Key terms and definitions

- **Relational thinking** is the ability to recognise that the equals sign implies a relationship between both sides that means 'the same as'.
- A **number sentence** is an equation expressed using numbers and operators such as +, −, × and ÷.
- The **equals sign (=)** in an equation or number sentence indicates that the value on the left of the sign is the same value as on the right.
- The **not equals sign (≠)** in an equation or number sentence indicates that the value on the left of the sign is not the same value as on the right.
- In **equivalent** number sentences, both sides of an equals sign will have the same total value even when different numbers and operators are being used.
- An **operation** in maths is a mathematical process used to solve problems. The four basic operations in maths are addition, multiplication, subtraction and division.
- The **commutative law** means that the order of the numbers in an addition and multiplication calculation can change without affecting the result. E.g. $8 + 5 = 5 + 8$.
- The **associative law** means that how we group numbers in addition or multiplication can change without affecting the result. E.g. $(7 + 3) + 4 = 7 + (3 + 4)$.

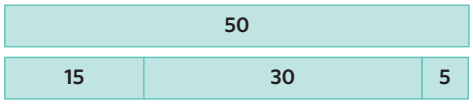
Key ideas

1 For two quantities to be equivalent, they must be equal in total value.

The balance scale represents the equivalent number sentence $8 + 6 + 2 = 11 + 5$. The scale is balanced because both sides equal 16.

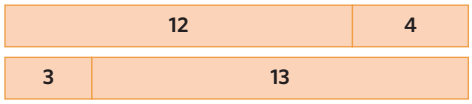
2 We can use a bar model to show equivalence between two quantities.

Example 1



50 = 15 + 30 + 5
50 = 50

Example 2

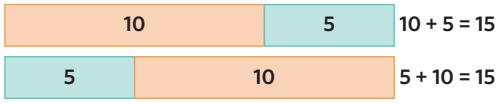


12 + 4 = 3 + 13
16 = 16

3 The commutative and associative laws apply to addition and multiplication but not subtraction and division.

The **commutative law** states that for any multiplication or addition we can reorder the numbers and still get the same result.

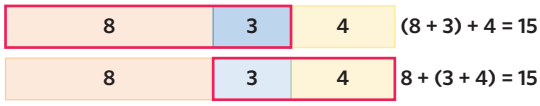
Commutative law for addition



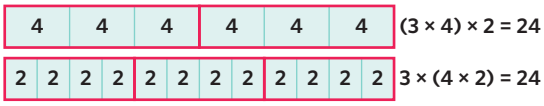
Commutative law for multiplication

The **associative law** states that for any multiplication and addition, we can regroup numbers and still get the same result.

Associative law for addition



Associative law for multiplication



The commutative and associative laws **do not** work for subtraction and division.

$6 - 4 \neq 4 - 6$ $\quad (11 - 8) - 3 \neq 11 - (8 - 3)$

$9 \div 3 \neq 3 \div 9$ $\quad (24 \div 6) \div 2 \neq 24 \div (6 \div 2)$

Worked examples

WE 1 Making equivalent number sentences

Make each number sentence **equivalent** by completing the missing value or operation.

a. $27 - 8 = \square + 2$

Working

LHS $= 27 - 8$

$= 19$

The RHS must also equal 19.

$17 + 2 = 19$

The missing value is 17.

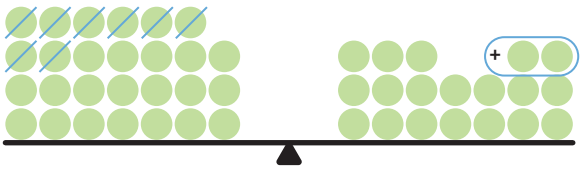
$27 - 8 = 17 + 2$

Thinking

Step 1: Evaluate the left-hand side (LHS).

Step 2: Determine the missing value that makes the number sentence equivalent.

Visual support



b. $34 \square 2 = 10 + 7$

Working

RHS $= 10 + 7$

$= 17$

The LHS must also equal 17.

$34 \div 2 = 17$

The missing operation is $\div$.

$34 \div 2 = 10 + 7$

Thinking

Step 1: Evaluate the right-hand side (RHS).

Step 2: Determine the missing operation that makes the number sentence equivalent.

Student practice

Make each number sentence **equivalent** by completing the missing value or operation.

a. $18 - 3 = \square + 6$ b. $4 \square 6 = 11 + 13$ c. $\square \times 3 = 33 - 9$ d. $21 - 9 = 6 \square 6$

WE 2 Determining commutative or associative property

Place an = (equals sign) or ≠ (not equal sign) in the box to make the equation true. If it is equal, state whether a commutative or associative property has been applied.

a. $12 + (7 + 4) \square (12 + 7) + 4$

Working

$12 + (7 + 4) = 12 + 11$

$= 23$

$(12 + 7) + 4 = 19 + 4$

$= 23$

LHS = RHS, so the equation is equivalent.

$12 + (7 + 4) = (12 + 7) + 4$

The commutative property involves changing the order of numbers and the associative property involves changing the grouping of numbers.

In this case, the grouping of the numbers has been changed without affecting the result.

An associative property has been applied.

Thinking

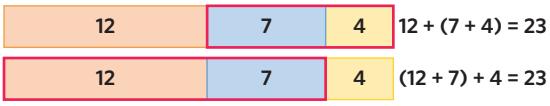
Step 1: Calculate the left-hand side (LHS).

Step 2: Calculate the right-hand side (RHS).

Step 3: Determine if the equation is equivalent.

Step 4: Determine whether a commutative or associative property has been applied.

Visual support



b. $18 \div 6 \square 6 \div 18$

Working

$18 \div 6 = 3$

$6 \div 18 = \frac{6}{18}$ or $\frac{1}{3}$

LHS ≠ RHS, so the equation is not equivalent. Division is not commutative.

$18 \div 6 \neq 6 \div 18$

Thinking

Step 1: Calculate the left-hand side (LHS).

Step 2: Calculate the right-hand side (RHS).

Step 3: Determine if the equation is equivalent.

Student practice

Place an = (equals sign) or ≠ (not equal sign) in the box to make the equation true. If it is equal, state whether a commutative or associative property has been applied.

a. $12 + (7 + 4) \square (12 + 7) + 4$ b. $5 \div 20 \square 20 \div 5$

c. $18 - 12 \square 12 - 18$ d. $2 \times 3 \times 6 \square 6 \times 3 \times 2$

WE 3 Checking number sentences

State whether each number sentence is true or false.

a. $3 + (7 + 8) = (3 + 7) + 8$

Working

The number sentence uses only addition, and all values are the same on each side of the equals sign. The values have been grouped differently on each side. This is the associative property.

Due to the associative property of addition, the equation is true.

LHS: $3 + (7 + 8) = 3 + 15$
$= 18$
RHS: $(3 + 7) + 8 = (10 + 8)$
$= 18$
LHS = RHS: $18 = 18$ ✓

Thinking

Step 1: Identify whether a commutative or associative property could be applied.

Step 2: Determine if the equation is true or false.

Step 3: Check to make sure your answer is correct.

b. $12 \div 4 \times 3 = 12 \times 3 \div 4$

Working

All values are the same on each side of the equals sign, but the number sentence uses a mix of division and multiplication. Neither the commutative nor the associative property applies.

As the commutative property does not apply, we need to check the equation by calculating both sides.

LHS: $12 \div 4 \times 3 = 3 \times 3$
$= 9$
RHS: $12 \times 3 \div 4 = 36 \div 4$
$= 9$

Since the values on either side of the equals sign are the same, the equation is true.

LHS = RHS: $9 = 9$ ✓

Thinking

Step 1: Identify whether a commutative or associative property could be applied.

Step 2: Determine if the equation is true or false.

Step 3: Check to make sure your answer is correct.

Student practice

State whether each number sentence is true or false.

a. $(16 + 14) + 9 = 16 + (14 + 9)$

b. $169 \div 13 = 13 \div 169$

c. $420 \div 7 + 2 = 420 \times 7 \div 7 + 2$

d. $(12 + 30) - 9 = 12 + (30 - 9)$

1A Activities and questions

STARTER TASKS

Image: SehrguteFotos/Shutterstock.com

Odd spot

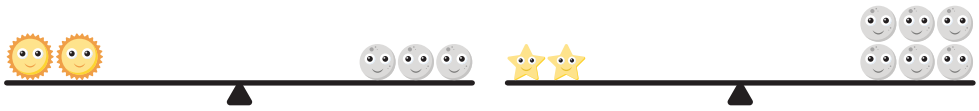
A group of researchers and scientists from the University of Auckland and the Leibniz Institute from Farm Biology in Germany have tried to toilet train 16 cows. They successfully trained 11 of them to use a 'mooloo'. The reason they tried to do this is because their urine and waste causes significant environmental damage to soil. This leads to land contamination and pollution of water systems.

Which equivalent number sentence shows the total number of cows that could not be toilet trained?

A. $16 - 11 = 11 - 16$ **B.** $16 - 11 = 5$

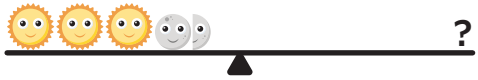
Puzzle

The balance scales show equivalent values.



a) How many  are required to balance the following scale?

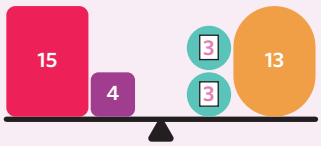
b) How many are required to balance the following scale?



Understanding worksheet

1. Fill in the boxes to keep the scales balanced and complete the number sentence that represents each image.

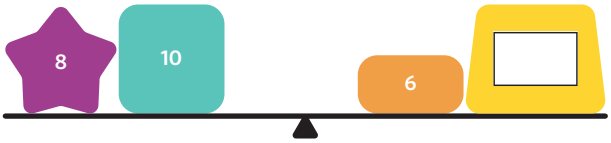
Example



Number sentence

$15 + 4 = \boxed{3} \times 2 + 13$

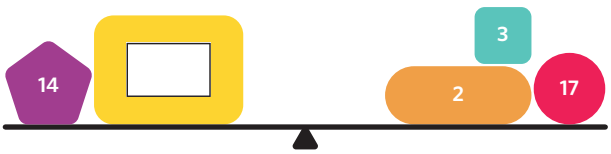
a.



Number sentence

$10 + 8 = 6 + \square$

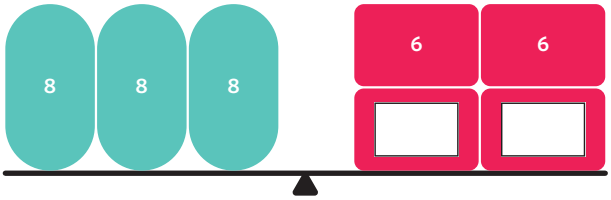
b.



Number sentence

$14 + \square = 2 + 3 + 17$

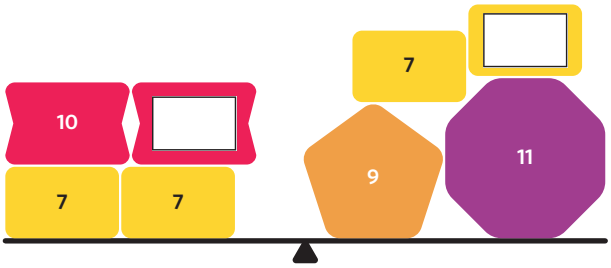
c.



Number sentence

$8 + 8 + 8 = 6 + 6 + \square + \square$

d.



Number sentence

$10 + \square + 7 + 7 = \square + 7 + 9 + 11$

2. The commutative or associative law has been used for each bar model. Place a tick next to the correct law used.

Example

9	18
18	9

Number sentence
$18 + 9 = 9 + 18$
Commutative ☑
Associative ☐

a.

9	4
4	9

Number sentence
$9 + 4 = 4 + 9$
Commutative ☐
Associative ☐

b.

3	3	
2	2	2

Number sentence
$3 \times 2 = 2 \times 3$
Commutative ☐
Associative ☐

c.

2	4	9
2	4	9

Number sentence
$(2 + 4) + 9 = 2 + (4 + 9)$
Commutative ☐
Associative ☐

d.

3	3	
2	2	2

Number sentence
$(3 \times 1) \times 2 = (2 \times 3) \times 1$
Commutative ☐
Associative ☐

3. Fill in the blanks by using the words provided.

commutative | division | addition | associative | equals

The ☐ sign in a number sentence or equation shows that the value on both sides is the same.

The number sentences $12 \times 3 = 3 \times 12$ and $5 + 7 = 7 + 5$ both demonstrate the ☐ property, while $(2 \times 3) \times 5 = 2 \times (3 \times 5)$ and $(1 + 4) + 6 = 1 + (4 + 6)$ demonstrate the ☐ property. The commutative and associative properties can only be applied to ☐ and multiplication but not to subtraction and ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h)

WE1 **4.** Make each number sentence **equivalent** by completing the missing value or operation.

a. $7 + 5 = 20 - \square$
b. $15 \times 2 = \square + 27$
c. $64 \div \square = 8 \times 8$
d. $25 \square 4 = 200 \div 2$
e. $\square - 7 + 2 = 40 + 5$
f. $16 - 4 + 4 + 20 = 13 - 5 + \square$
g. $130 \square 20 \square 10 = 120$
h. $25 \square 5 = 625 \square 5$

WE2 5. Place an = (equals sign) or ≠ (not equal sign) in the box to make the equation true. If it is equal, state whether a commutative or associative property has been applied.

a. $14 + 12 \square 12 + 14$
b. $14 - 12 \square 12 - 14$
c. $(3 - 2) + 7 \square 3 - (2 + 7)$
d. $3 + (2 + 7) \square (3 + 2) + 7$
e. $30 \div 5 \square 5 \div 30$
f. $30 \times 5 \square 5 \times 30$
g. $10 \times 5 \times 3 \square 3 \times 5 \times 10$
h. $6 \times 4 \div 8 \square 8 \times 4 \div 6$

WE3 6. State whether each number sentence is true or false.

a. $9 + (3 + 4) = (9 + 3) + 4$
b. $50 \div 5 = 5 \div 50$
c. $12 \times 2 \times 10 = 10 \times 12 \times 2$
d. $121 - 250 + 21 = 121 - 21 + 250$
e. $12 \times 6 \div 2 = 12 \times 2 \div 6$
f. $(16 + 14) - 18 = 16 + (14 - 18)$
g. $666 - 333 = 111 + 222$
h. $750 \div 25 = 30 \times 25 \div 25$

Problem solving

Mild	Medium	Spicy
7, 8, 9	8, 9, 10	9, 10, 11

7. Fi and Jim run the local football club and make raspberry lemonade for their team every weekend. Fi makes her raspberry lemonade with 50 mL of cordial and 220 mL of water while Jim only uses 45 mL of cordial to make his drink. How much water does Jim need to add to the cordial to make the same amount of raspberry lemonade as Fi?

8. Jon challenged Ben to a chicken nugget eating contest. Jon ate 7 packs of 12 nuggets plus another 8 nuggets while Ben ate 3 packs of 24 nuggets and another 15 nuggets. Are the judges correct if they claim that both boys came to a draw?

9. Fortuna is a big Formula1 fan and is explaining the rules to her friend. She says that in F1, each circuit can be made up of a different number of laps but will still cover the same total distance at the end of each race. Using the information from the table, determine if Fortuna is correct in her analysis of the F1 circuit rules.

Circuit	Number of laps	Distance per lap
Dutch Grand Prix	75	4 km
Singapore Grand Prix	60	5 km
Saudi Arabian Grand Prix	50	6 km

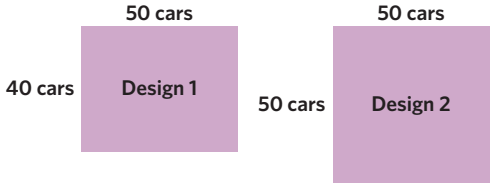
10. Eugene is a stamp collector and receives a bag of stamps and a stamp display book for his birthday. Eugene takes half of the stamps and spreads them over the first 8 pages. He puts 12 rows on each page, each row containing 5 stamps. He places the remaining stamps in 5 rows of 8 stamps. How many pages will his remaining stamps take up?

11. Ruby was organising her orchard for apple season and needed a total area of 2400 m^2 to plant her trees. She did not have one plot big enough so decided to plant them in two separate plots. What area does the second plot need to be if the first plot she planted was 60 metres by 30 metres?

Reasoning

Mild	Medium	Spicy
12 (a,b,c)	12 (a,b,c), 13 (a,b,c)	All

12. A local shopping centre is building a multi-level parking complex for their customers and need to decide on the number of parking levels and the dimensions of each level. They have been given 2 design options.



a. What is the minimum number of levels for each design to fit the same number of cars in the parking complex?
b. How many cars will each design in part **a** fit?
c. The designers realised that land space was a problem and the only dimensions that were possible would fit 20 rows, each with 100 cars. How many levels would be needed to fit the same number of cars as the original designs?
d. Do you think any of the car park designs are appropriate? How would you design a car park that was more adequate for a shopping centre?

13. Sandeep is working on a poster design to celebrate the year 7 graduation. Part of his design is to create a photo collage of all of the students in year 7 and frame it with some ribbon. Sandeep has only 96 centimetres of ribbon.
    a. Sandeep decided to make a rectangular collage. Using only whole numbers, what dimensions could he make the collage if he wanted to use all 96 cm of ribbon as a frame?
    b. After some initial design fails, Sandeep decided that a rectangular shape was too boring and thought about a regular hexagon instead. What would the side lengths of Sandeep's hexagonal design be?
    c. List the side lengths of Sandeep's designs if he was to choose an equilateral triangle, a square or a regular octagon.
    d. What design should Sandeep choose for his collage and why?

Extra spicy

14. Which one of the following is equivalent to $\frac{5 \times 6}{6 + 5}$?
    A. $\frac{6 + 5}{5 \times 6}$ B. $\frac{5 + 6}{6 \times 5}$ C. $\frac{6 \times 5}{5 + 6}$ D. $\frac{5}{6} \times \frac{6}{5}$

15. Which equation represents the associative law?
    A. $a + b = c + b$ B. $b + a = a + b$
    C. $a + b + c = a + b - c$ D. $(a + b) + c = a + (b + c)$

16. What is the value of n if $2^7 + 8^2 + 4 = 14^n$?

17. Select the expression equal to $\frac{y}{z} + \frac{x}{m}$.
    A. $\frac{z}{y} + \frac{m}{x}$ B. $\frac{m}{x} + \frac{y}{z}$ C. $\frac{xz + my}{mz}$ D. $\frac{z + m}{y + x}$

Remember this?

18. Which set is made up of prime numbers only?
    A. 2, 3, 7, 9 B. 3, 5, 7, 23 C. 2, 7, 11, 21 D. 3, 5, 13, 15

19. There are 2000 students at a school.
    One-quarter of the students are in primary school.
    How many students are in primary school?

20. Which one of the following is **not** equal to 30×45?
    A. $3 \times 10 \times 45$ B. $30 \times 1 \times 45$ C. 3×450 D. $30 \times 40 \times 5$

1B Place value

Place value is the value of each digit in a multi-digit number. The value of each digit depends on its position within the number. The base ten number system organises each place value in groups of ten. To understand the place value of each digit we can express numbers in numeric form, unit form, and expanded form. Understanding place value is useful in making everyday decisions relating to costs, weights, distances, and time. Below are some examples where place value can be applied.

- I want to buy the cheapest bag of coffee beans. Coffex costs $25 and Darkoffe costs $43. Which brand should I buy?
- My dog weighs 8 kilograms. Mini Schnauzers are considered healthy if their weight is between 5 and 9 kilograms. Is my dog considered healthy?
- I found $1 417 000 worth of treasure and my diving partner found $1 317 000 worth of treasure. How much more valuable is my treasure than my partner's?

Learning intentions

Students will be able to:

- determine the place value of a digit in a number
- regroup and partition numbers
- write and convert numbers in expanded form to numerals
- compare and order whole numbers.

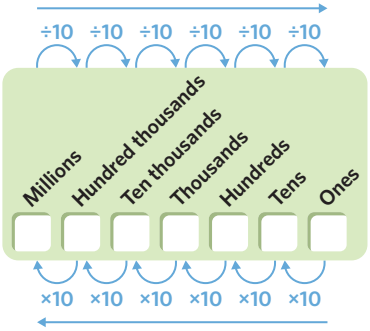
Key terms and definitions

- **Numeric form** is the combination of digits written together as a single number. E.g. 576.
- **Unit form** is when all of the place values of a number are written in words. E.g. 5 hundreds + 7 tens + 6 ones.
- **Expanded form** is a way of expressing a number as the sum of its place value parts or expressing index notation as repeated multiplications. E.g. 500 + 70 + 6.
- **Ascending** order means to arrange values from smallest to largest.
- **Descending** order means to arrange values from largest to smallest.

Key ideas

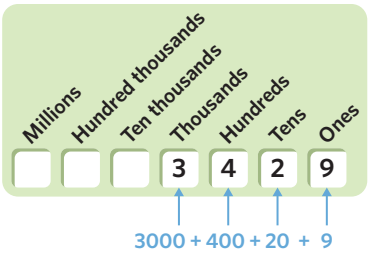
1 A number slider shows the place value of digits in a number.

Moving from left to right, the value of each place value column is divided by 10.



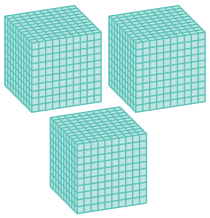
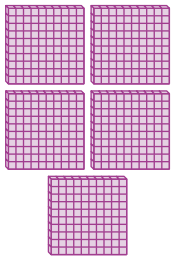
Moving from right to left, the value of each place value column is multiplied by 10.

For example 3429 can be shown in a number slider.



2 We can group numbers in groups of 10 using the base ten number system.

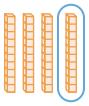
10 units = 1 rod
10 rods = 1 flat
10 flats = 1 cube

Name	Cube	Flat	Rod	Unit
Value	1000	100	10	1
3548				

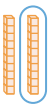
3548 = 3000 + 500 + 40 + 8

3 We can use standard decompositions or non-standard decompositions to understand the values of a whole number. Regrouping and partitioning is used in standard and non-standard decomposition.

Regrouping is used when 10 units can be combined by moving one place value to the left.

Tens	Ones	=	Tens	Ones
		=		
30	+ 15	=	40	+ 5
Non-standard decomposition			Standard decomposition	

Partitioning or breaking apart a group is used when a group of 10 is partitioned into 10 smaller units by moving one place to the right.

Tens	Ones	=	Tens	Ones
		=		
20	+ 6	=	10	+ 16
Standard decomposition			Non-standard decomposition	

Worked examples

WE 1 Finding the place value of numbers

Write the value of the highlighted digit.

a. 2346

Working

The '4' is in the second position indicating a place value of tens.

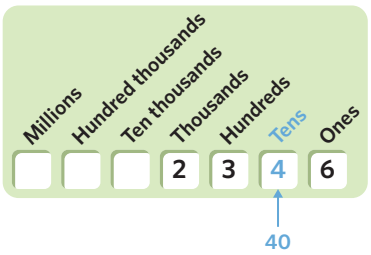
4 tens is 40.

Thinking

Step 1: Count the position of the highlighted digit from right to left.

Step 2: Write the value of the digit using its place value.

Visual support



b. 43 062 189

Working

The '6' is in the fifth position indicating a place value of ten thousands.

6 ten thousands is 60 000.

Thinking

Step 1: Count the position of the highlighted digit from right to left.

Step 2: Write the value of the digit using its place value.

Student practice

Write the value of the highlighted digit.

a. 4851 **b.** 9964 **c.** 24 036 497 **d.** 678 932 455

WE 2 Writing numbers in different forms

Express each value in the described form.

a. 4 thousands + 3 hundreds + 1 ten + 26 ones (numeric form)

Working

4 thousands + 3 hundreds + 1 ten + 26 ones

= 4000 + 300 + 10 + 26

= 4000 + 300 + 10 + 20 + 6

= 4000 + 300 + 30 + 6

= 4336

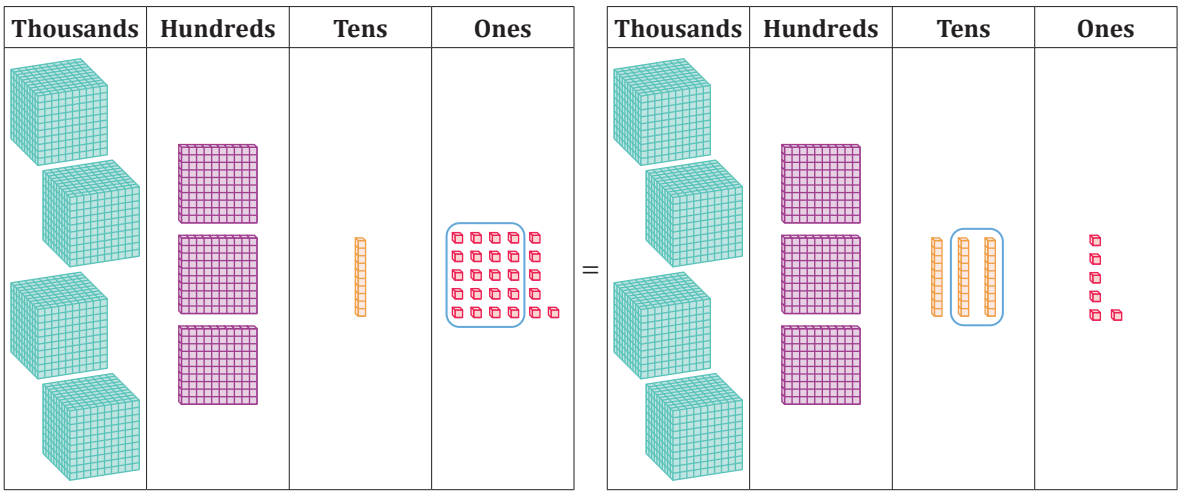
Thinking

Step 1: Represent the unit form in its expanded form.

Step 2: Regroup where necessary.

Step 3: Write in numeric form by finding the sum of all the values.

Visual support



b. 70 382 (expanded form)

Working

The '7' is in the fifth position indicating a place value of ten thousands.

The '3' is in the third position indicating a place value of hundreds.

The '8' is in the second position indicating a place value of tens.

The '2' is in the first position indicating a place value of ones.

70 000 + 300 + 80 + 2

Thinking

Step 1: Count the position of each digit from right to left and assign its place value.

Note: We don't need to assign a place value to a 0.

Step 2: Write in expanded form by expressing each digit's value using its place value.

Student practice

Express each value in the described form.

a. 7 thousands + 4 hundreds + 3 tens + 2 ones (numeric form)

b. 40 312 (expanded form)

c. 10 hundreds + 6 tens + 8 ones (numeric form)

d. 142 961 (expanded form)

WE 3 Comparing whole numbers in expanded form

Compare the whole numbers by placing the correct mathematical symbol (<, > or =) in the box.

a. 1000 + 200 + 50 + 4 ☐ 1000 + 200 + 30 + 4

Working

1000 + 200 + 50 + 4 = 1254

1000 + 200 + 30 + 4 = 1234

Thousands: 1254 ☐ 1234

Hundreds: 1254 ☐ 1234

Tens: 1254 ☐ 1234

5 > 3

1254 > 1234

1000 + 200 + 50 + 4 > 1000+ 200 + 30 + 4

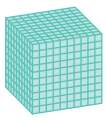
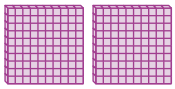
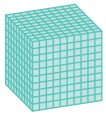
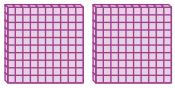
Thinking

Step 1: Represent the expanded form number in its numeric form.

Step 2: Compare the digits in each place value for each number starting from left to right. Stop when there is a different digit in the same place value.

Step 3: Compare the differing digits. The larger digit in the same place value indicates the larger whole number.

Visual support

Name	Cube	Flat	Rod	Unit
Value	1000	100	10	1
1254				
1254 =	1 thousand	2 hundreds	5 tens	4 ones
1234				
1234 =	1 thousand	2 hundreds	3 tens	4 ones

b. 1200 + 140 + 5 ☐ 1000 + 300 + 40 + 55

Working

1200 + 140 + 5 = 1345

1000 + 300 + 40 + 55 = 1395

Thousands: 1345 ☐ 1395

Hundreds: 1345 ☐ 1395

Tens: 1345 ☐ 1395

4 < 9

1345 < 1395

1200 + 140 + 5 < 1000 + 300 + 40 + 55

Thinking

Step 1: Represent the expanded form number in its numeric form.

Step 2: Compare the digits in each place value for each number starting from left to right. Stop when there is a different digit in the same place value.

Step 3: Compare the differing digits. The larger digit in the same place value indicates the larger whole number.

Student practice

Compare the whole numbers by placing the correct mathematical symbol (<, > or =) in the box.

a. 900 + 70 + 6 ☐ 900 + 70 + 9

b. 1000 + 800 + 60 + 12 ☐ 1000 + 800 + 70

c. 700 + 110 + 24 ☐ 700 + 90 + 44

d. 7800 + 200 + 110 + 27 ☐ 8000 + 100 + 20 + 14

WE 4 Ordering whole numbers

Place the values in the described order.

a. Descending: 123, 6, 453, 213, 89

Working

123, 453, 213 > 89 > 6

Hundreds: 453 > 213 > 123

453, 213, 123, 89, 6

Thinking

Step 1: Order the numbers according to the number of digits each has.

Step 2: Compare the digits in each place value of the numbers with the same number of digits starting from left to right. Order the numbers from largest to smallest.

Step 3: List the numbers from largest to smallest.

b. Ascending: 27 962, 7846, 7847, 27

Working

27 < 7846, 7847 < 27 962

Thousands: 7846 ☐ 7847

Hundreds: 7846 ☐ 7847

Tens: 7846 ☐ 7847

Ones: 7846 < 78447

27, 7846, 7847, 27 962

Thinking

Step 1: Order the numbers according to the number of digits each has.

Step 2: Compare the digits in each place value of the numbers with the same number of digits starting from left to right. Order the numbers from smallest to largest.

Step 3: List the numbers from smallest to largest.

Student practice

Place the values in the described order.

a. Descending: 9, 253, 900, 78, 189

b. Ascending: 6342, 1099, 3467, 6432

c. Descending: 1909, 9109, 1999, 1990

d. Ascending: 17 992, 117 000, 117 117, 117 719

1B Activities and questions

STARTER TASKS

Odd spot


Image: Justin Mcmanus/The Age

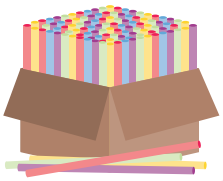
In 2020 and 2021, there was a shortage of fruit pickers working on Victorian farms. As a result a record amount of fruit went unpicked. Farmers who manage the fruit pickers found themselves working 18 hour days to harvest the unpicked fruit. Fredo and Reginald are both fruit pickers in the Goulburn Valley. They earn $10 an hour and receive a $100 bonus for every fruit bin that they fill.

Reginald works 60 hours and fills 15 bins per week. Fredo works 40 hours and fills 18 bins each week. Who earns more money per week and by how much?

A. Fredo earns $100 less than Reginald.

B. Fredo earns $100 more than Reginald.

Puzzle



A box contains between 10 000 and 12 000 straws.

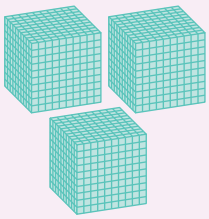
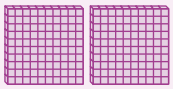
a) What are two different strategies that can be used to count the exact number of straws?

b) Explain which strategy might be 'easier' to use.

Understanding worksheet

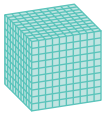
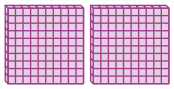
1. Write each number in expanded form.

Example

Value	1000	100	10	1
				
Number of units	3 thousands	2 hundreds	1 tens	8 ones

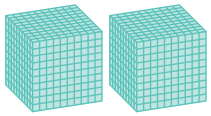
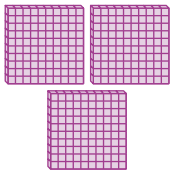
3218 = 3000 + 200 + 10 + 8

a.

Value	1000	100	10	1
				
Number of units	1 thousands	2 hundreds	2 tens	6 ones

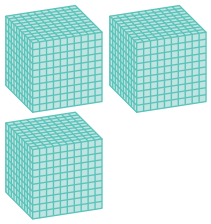
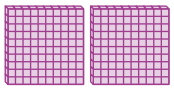
1226 = ☐ + ☐ + ☐ + ☐

b.

Value	1000	100	10	1
				
Number of units	2 thousands	3 hundreds	5 tens	2 ones

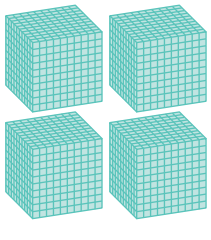
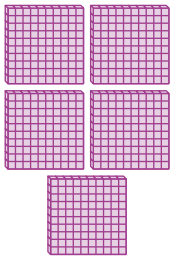
2352 = ☐ + ☐ + ☐ + ☐

c.

Value	1000	100	10	1
				
Number of units	3 thousands	2 hundreds	3 tens	6 ones

3236 = ☐ + ☐ + ☐ + ☐

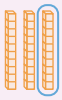
d.

Value	1000	100	10	1
				
Number of units	4 thousands	5 hundreds	5 tens	5 ones

4555 = ☐ + ☐ + ☐ + ☐

2. Write each number a different way based on the circled units that are being regrouped or partitioned.

Example

Tens	Ones		Tens	Ones
		=		

20 + 13 = 30 + 3

33 = 33

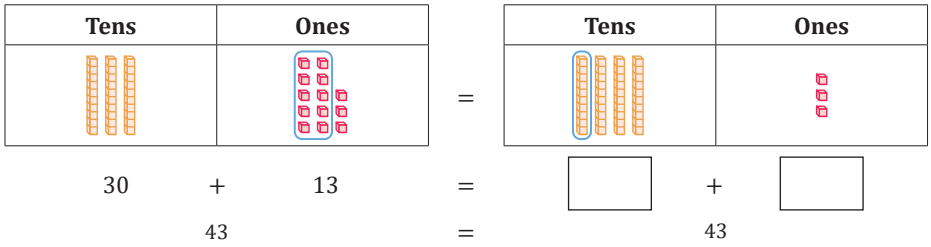
a.

Tens	Ones		Tens	Ones
		=		

10 + 12 = ☐ + ☐

22 = 22

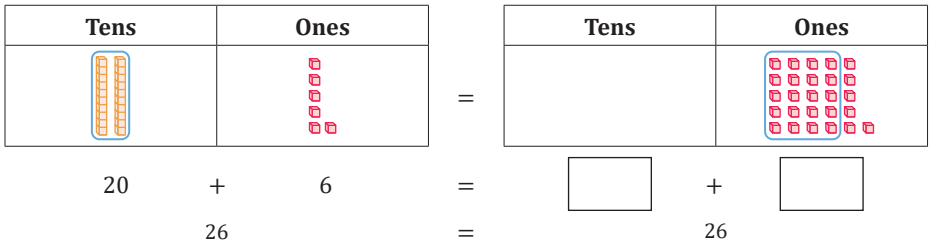
b.



30 + 13 = ☐ + ☐

43 = 43

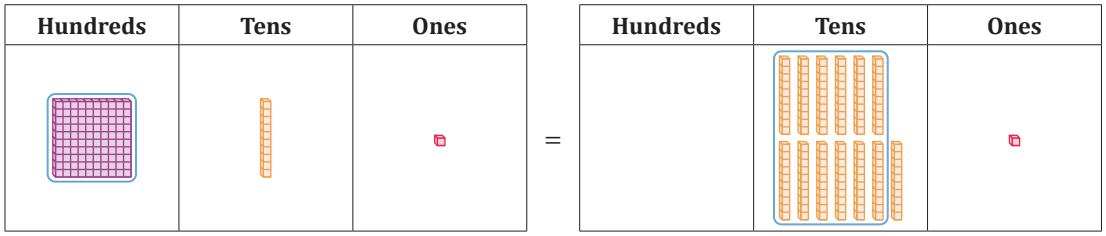
c.



20 + 6 = ☐ + ☐

26 = 26

d.



100 + 10 + 1 = ☐ + ☐ + ☐

111 = 111

3. Fill in the blanks by using the words provided.

compare | numeric | position | expanded

Place value describes the value of a digit based on its ☐ in a number. The number two thousand seven hundred and fifty-six is written in ☐ form as 2756. It can also be written in ☐ form as 2000 + 700 + 50 + 6. Understanding the value of a number is important when we need to ☐ numbers in everyday life.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 4. Write the value of the highlighted digit.

a. 37**8**6
b. 2**4**62
c. 18 **7**84
d. 29 **9**86
e. 1**2**9 945
f. 1 **8**76 222
g. **1**2 627 828
h. 222 **2**22 222

WE2a **5.** Write each number in numeric form.

a. 4 hundreds + 3 tens + 6 ones
b. 3 thousands + 8 hundreds + 7 tens + 9 ones
c. 1 ten thousand + 4 thousands + 2 hundreds + 2 ones
d. 2 hundred thousands + 2 ten thousands + 4 thousands + 8 hundreds + 6 tens + 4 ones
e. 8 hundreds + 15 tens + 17 ones
f. 125 thousands + 18 hundreds + 4 tens + 23 ones
g. 10 millions + 45 thousands + 60 tens + 32 ones
h. 2 hundred millions + 4 hundred thousands + 26 thousands + 734 hundreds

WE2b **6.** Write each number in expanded form.

a. 983 **b.** 7435 **c.** 12 276 **d.** 8868
e. 10 002 **f.** 203 070 **g.** 1 700 800 **h.** 937 021 809

WE3 **7.** Compare the numbers in expanded form using the symbols <, > or =.

a. 200 + 30 + 6 ☐ 200 + 30 + 8
b. 1000 + 300 + 80 + 2 ☐ 1000 + 300 + 8
c. 400 + 60 + 37 ☐ 400 + 70 + 24
d. 2000 + 100 + 40 ☐ 1200 + 940
e. 6200 + 240 + 5 ☐ 6200 + 200 + 70 + 15
f. 18 000 + 200 + 30 + 18 ☐ 10 000 + 8000 + 200 + 45
g. 40 000 + 35 000 + 2000 + 3000 + 4 ☐ 70 000 + 7000 + 2000 + 1000 + 4
h. 20 000 + 170 + 36 ☐ 12 000 + 7000 + 1120 + 80 + 9

WE4 **8.** Place the values in the described order.

a. Ascending: 127, 8764, 34, 900
b. Descending: 323, 2321, 461, 120
c. Ascending: 12, 21, 2221, 221,
d. Descending: 4672, 23 899, 21 873, 20 862
e. Ascending: 3462, 152 784, 9732, 3461
f. Descending: 128 874, 119 921, 118 992, 118 996
g. Ascending: 1 876 625, 1 900 000, 1 876 621, 1 876 532
h. Descending: 7 654 233, 7 642 984, 7 654 651, 7 654 221

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Mary has one $100 note and one $50 note and needs some change to pay for her guitar lessons. How many $10 notes will she receive if she exchanges all her money at the bank?

10. Antonia invited 34 people to her birthday party and wants to purchase party pies for her guests. She buys 3 packs of party pies where each pack contains 10 pies. Does she have enough pies so that everyone can have at least one?

11. The Oasis Deluxe Grazing Box costs $100. Hillery has a budget of $1215 to spend on gifts for her friends at her office. How many grazing boxes can she buy?

12. A box of donuts at Daniel's Donuts contains 10 donuts. How many boxes do they have available to sell if they have already made 12 boxes and have 32 donuts left to pack?

13. Roger wants to transfer a total of $12 500 dollars from his spending account into his savings account over a period of several months. He sets up a direct transfer of $1000 every month and then $100 a month when his spending account drops below a thousand dollars. How many months will it take Roger to transfer **all** of his money?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. The following table shows the viewership of the AFL finals series in 2017, 2018 and 2019.

	2017	2018	2019
Viewership (number of people who watched)	15 862 000	12 500 119	14 335 000

a. Did more people watch the AFL final series in 2018 or 2019?
b. Order the viewership of the 3 years from largest to smallest.
c. In 2019, the average number of viewers for each game in Sydney was 87 000 and the average number of viewers in Melbourne was 483 000. Do more people watch AFL in Melbourne or in Sydney?
d. What might explain why more people from one city watch the AFL finals than another?

15. The following table shows the top 10 highest paid male athletes and the sport they play.

Name	2020–2021 earnings in US dollars	Sport
Lebron James	96 500 000	NBA basketball
Kevin Durant	75 000 000	NBA basketball
Lionel Messi	130 000 000	Soccer
Cristiano Ronaldo	120 000 000	Soccer
Neymar	95 000 000	Soccer
Roger Federer	90 000 000	Tennis
Dak Prescott	107 500 000	NFL football
Tom Brady	76 000 000	NFL football
Conor McGregor	180 000 000	UFC
Lewis Hamilton	82 000 000	F1

a. Place the earnings of the top 10 athletes in descending order.
b. How much more did Lebron James make than Roger Federer?
c. How much less did Kevin Durant make than Conor McGregor?
d. Why might there be such a significant difference between what tennis players earn compared to soccer players?

Extra spicy

16. How many different numbers can be made by rearranging the digits shown by each die?

A. 21 B. 22 C. 23 D. 24

17. A palindromic number is a number that stays the same when the digits in each place value are reversed. How many palindromic numbers are in between 40 000 and 50 000?

A. 98 B. 99 C. 100 D. 101

18. What number is represented as 10^8?

A. Ten million B. One-hundred million C. One billion D. Ten billion

19. b and c are both five digit numbers. Together, they contain all ten digits from 0 to 9. What is the smallest possible difference between b and c?

A. 123 B. 247 C. 327 D. 372

Remember this?

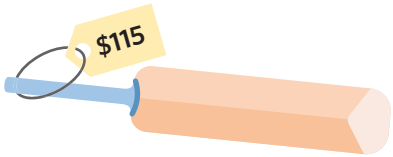
20. What is the area of a rectangle that is 100 cm wide and 3 cm long?

A. 206 cm **B.** 206 cm^2 **C.** 300 cm **D.** 300 cm^2

21. Sarah's car's petrol tank has a maximum capacity of 60 litres. While exploring South Australia her car used six full tanks and three-quarters of another tank. How much petrol did the car use in total?

A. 345 litres **B.** 375 litres **C.** 390 litres **D.** 405 litres

22. Millie has $69.



In order to buy the cricket bat how much more money does she need?

A. $44 **B.** $46 **C.** $54 **D.** $56

1C Adding and subtracting whole numbers

Whole numbers can be added and subtracted using different mental and written strategies. When adding or subtracting whole numbers, the place value of each digit needs to be considered. Adding and subtracting whole numbers efficiently is an important skill. Below are some examples where adding and subtracting whole numbers can be applied.

- I have $23 made up of $1 coins. How can I put the coins in groups to make them easier to count?
- I used two large nets when I went fishing. In one net I caught 47 fish and in the other I caught 58. How many fish did I catch altogether?
- There are 270 students in my year level. 47 students were unable to attend our graduation ceremony. How many students in my year attended the ceremony?

Learning intentions

Students will be able to:

- use the commutative and associative laws of addition
- add and subtract mentally using partitioning and compensation strategies
- use the vertical algorithm to add whole numbers
- use the vertical algorithm to subtract whole numbers.

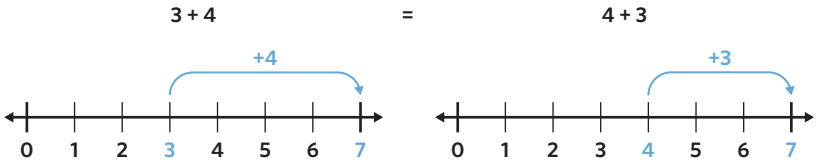
Key terms and definitions

- The **commutative law** means that the order of the numbers in an addition or multiplication calculation can change without affecting the result. E.g. $8 + 5 = 5 + 8$.
- The **associative law** means that how we group numbers in addition or multiplication can change without affecting the result. E.g. $(7 + 3) + 4 = 7 + (3 + 4)$.
- The **partitioning strategy** deconstructs larger numbers into smaller numbers so that calculations are simpler to solve. E.g. $13 + 25 = 10 + 3 + 20 + 5$.
- The **compensation strategy** uses multiples 10 and then adjustments to make calculations simpler to solve. E.g. $49 + 16 = 50 + 16 - 1$.

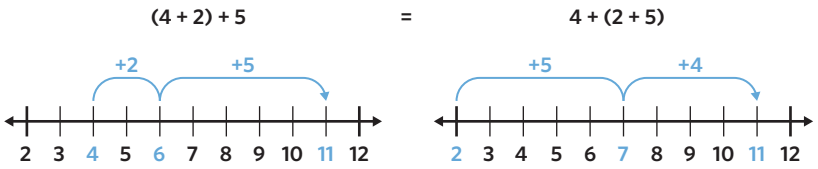
Key ideas

1 The commutative and associative laws of addition can be used when adding whole numbers.

The **commutative law** means that in addition, the order of the numbers does not change the result.



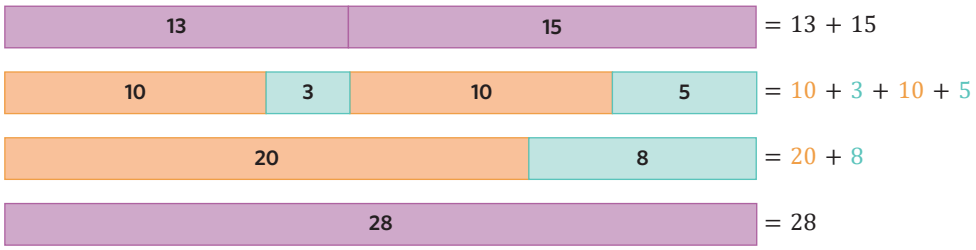
The **associative law** means that in addition, how we group the numbers does not change the result.



Note: In these number lines the calculation in the brackets is completed first.

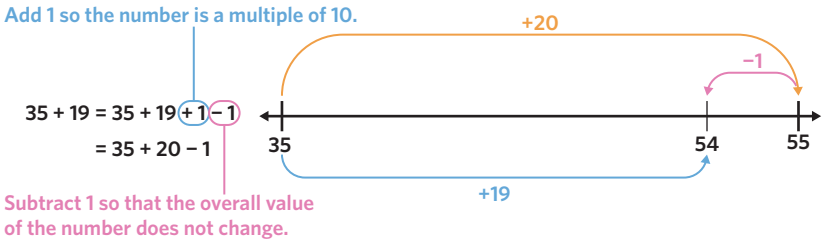
2 The partitioning strategy deconstructs larger numbers into smaller numbers. This is an efficient way to perform mental calculations.

Partition the number into its place values.



3 The compensation strategy involves making numbers that are multiple of 10 by adding some number, and then subtracting that number to keep the overall value the same.

Without changing the overall value of the numbers, adjust one of the numbers to make it easier to work with.



Worked examples

WE 1 Using the partitioning strategy

Use the partitioning strategy to complete each calculation mentally.

a. 24 + 37

Working

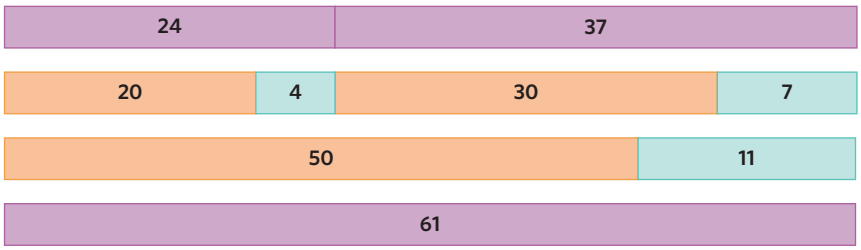
24 + 37 = 20 + 4 + 30 + 7
= 20 + 30 + 4 + 7
= 50 + 11
= 61

Thinking

Step 1: Write each number in expanded form.

Step 2: Regroup and simplify.

Visual support



b. 178 − 136

Working

178 − 136 = 100 + 70 + 8 − 100 − 30 − 6
= 100 − 100 + 70 − 30 + 8 − 6
= 40 + 2
= 42

Thinking

Step 1: Write each number in expanded form.

Step 2: Regroup and simplify.

Student practice

Use the partitioning strategy to complete each calculation mentally.

a. 52 + 23 b. 96 − 43 c. 86 + 95 d. 134 − 108

WE 2 Using the compensation strategy

Use the compensation strategy to complete each calculation mentally.

a. $27 + 19$

Working

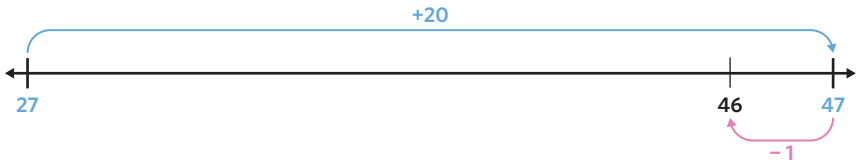
$27 + 19 = 27 + 19 + 1 - 1$
$= 27 + 20 - 1$
$= 47 - 1$
$= 46$

Thinking

Step 1: Adjust one of the numbers to make the calculation easier, without changing the overall value.

Step 2: Complete the calculation.

Visual support



b. $94 - 28$

Working

$94 - 28 = 94 - 28 - 2 + 2$
$= 94 - 30 + 2$
$= 64 + 2$
$= 66$

Thinking

Step 1: Adjust one of the numbers to make the calculation easier, without changing the overall value.

Step 2: Complete the calculation.

Student practice

Use the compensation strategy to complete each calculation mentally.

a. $36 + 29$ **b.** $73 - 38$ **c.** $299 + 64$ **d.** $78 - 39 - 18$

WE 3 Adding whole numbers using the vertical algorithm

Use the vertical algorithm to complete each addition.

a. $63 + 26$

Working

$$\begin{array}{r} 63 \\ +\ 26 \\ \hline \end{array}$$

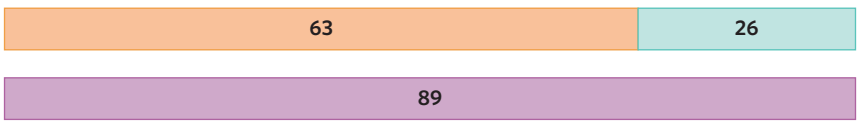
$$\begin{array}{r} 63 \\ +\ 26 \\ \hline 89 \end{array}$$

Thinking

Step 1: Write the numbers vertically, ensuring that the digits in each place value line up.

Step 2: From right to left, add the digits in each place value column.

Visual support



b. 127 + 254

Working

$$\begin{array}{r} 1\ 2\ 7 \\ +\ 2\ 5\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle +1} \\ 1\ 2\ 7 \\ +\ 2\ 5\ 4 \\ \hline 3\ 8\ 1 \end{array}$$

Thinking

Step 1: Write the numbers vertically, ensuring that the digits in each place value line up.

Step 2: From right to left, add the digits in each place value column. If the digits in a place sum to more than 9, regroup to the next place value column to the left.

Student practice

Use the vertical algorithm to complete each addition.

a. 32 + 57 **b.** 234 + 327 **c.** 43 + 24 + 281 **d.** 568 + 495

WE 4 Subtracting whole numbers using the vertical algorithm

Use the vertical algorithm to complete each subtraction.

a. 78 − 35

Working

$$\begin{array}{r} 78 \\ -\ 35 \\ \hline \end{array}$$

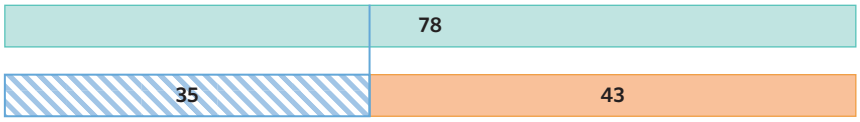
$$\begin{array}{r} 78 \\ -\ 35 \\ \hline 43 \end{array}$$

Thinking

Step 1: Write the numbers vertically, ensuring that the digits in each place value line up.

Step 2: From right to left, subtract the digit in the bottom row from the digit in the top row.

Visual support



b. 273 − 186

Working

$$\begin{array}{r} 2\ 7\ 3 \\ -\ 1\ 8\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 16} \\ 1\ \not{8}\ {\scriptstyle 13} \\ \not{2}\ \not{7}\ \not{3} \\ -\ 1\ 8\ 6 \\ \hline 8\ 7 \end{array}$$

Thinking

Step 1: Write the numbers vertically, ensuring that the digits in each place value line up.

Step 2: From right to left, subtract the digit in the bottom row from the digit in the top row. If the digit in the top row is smaller, regroup using the next place value column to the left.

Student practice

Use the vertical algorithm to complete each subtraction.

a. 84 − 41 **b.** 87 − 59 **c.** 387 − 298 **d.** 4532 − 3645

1C Activities and questions

STARTER TASKS

Odd spot

Cryptocurrencies are becoming more and more common. One of the most recognised cryptocurrencies is Bitcoin. Bitcoins are not physical money but rather a digital currency that is run by very powerful computers. The value of Bitcoin has been very volatile as many professionals in the finance industry question its long term value.

If one Bitcoin is worth $41 790 and it increased by $32, what is its new value?

A. $73 790 **B.** $41 822

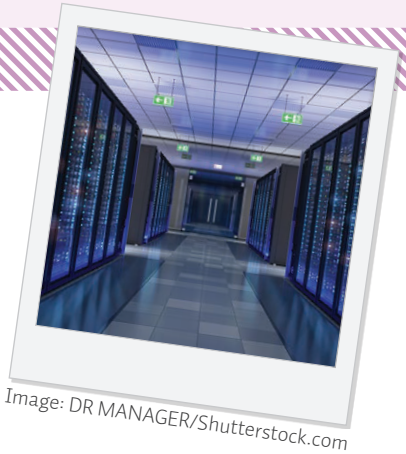


Image: DR MANAGER/Shutterstock.com

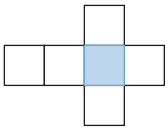
Puzzle

Use each of the following numbers once: 1, 2, 3, 4, 5, and 6.

a) Place the numbers so that the sum of the horizontal numbers is the same as the sum of the vertical numbers.

b) Find two more possible solutions to part a that have a different number in the blue box.

c) What do all the possible solutions have in common? Explain why.

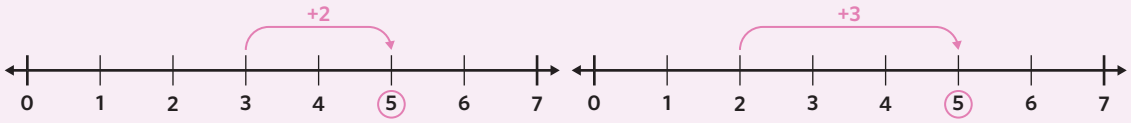


Understanding worksheet

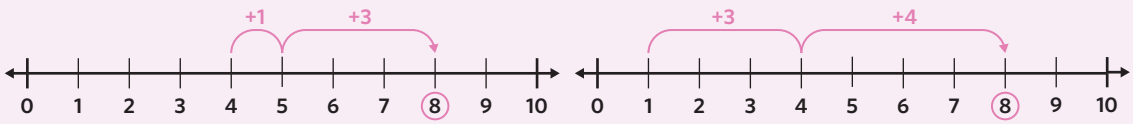
1. Show the commutative or associative law of addition by completing the 'jumps' on the number line.

Example

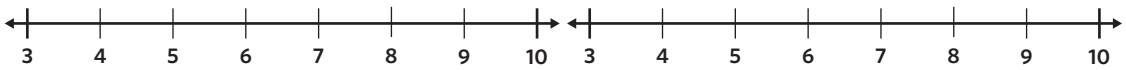
Commutative law: $3 + 2 = 2 + 3$



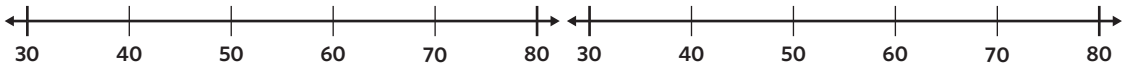
Associative law: $(4 + 1) + 3 = 4 + (1 + 3)$



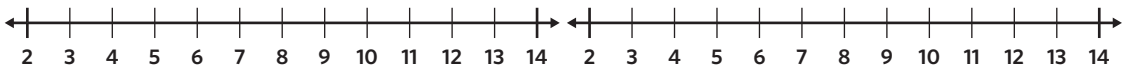
a. Commutative law: $3 + 7 = 7 + 3$



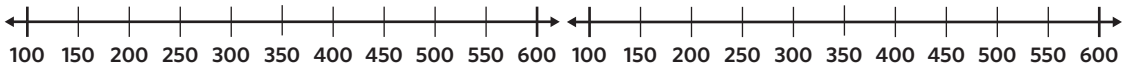
b. Commutative law: $40 + 30 = 30 + 40$



c. Associative law: $(5 + 2) + 6 = 5 + (2 + 6)$

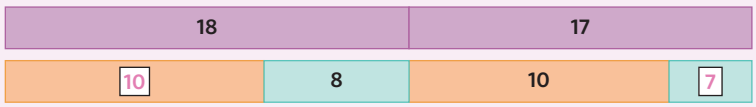


d. Associative law: $(200 + 300) + 50 = 200 + (300 + 50)$



2. Use the bar models and the partitioning strategy to deconstruct the numbers.

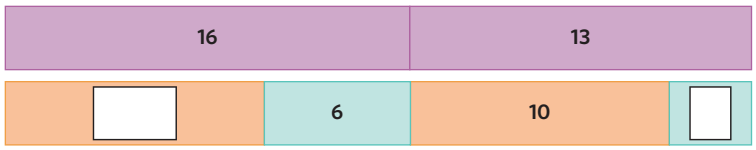
Example



18 + 17

10 + 8 + 10 + 7

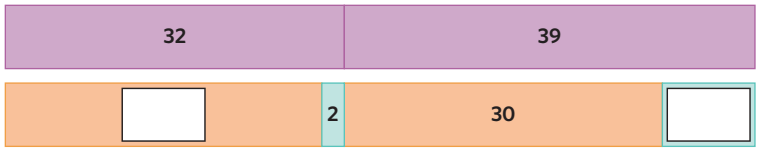
a.



16 + 13

□ + 6 + 10 + □

b.



32 + 39

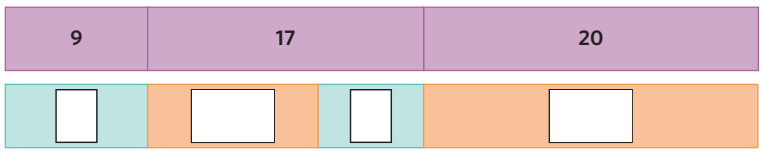
30 + □ + □ + 9

c.

43 + 19

□ + 3 + □ + □

d.



9 + 17 + 20

9 + □ + □ + □

3. Fill in the blanks by using the words provided.

addition | commutative | partitioning | compensation

The □ strategy involves deconstructing numbers into their place values in order to make □ and subtraction easier. The □ strategy involves adjusting one number in a calculation to make it easier. The □ law means that the order of the numbers in an addition or multiplication calculation does not change the result.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 **4.** Use the partitioning strategy to complete each calculation mentally.

a. 53 + 36 **b.** 85 − 21 **c.** 275 + 603 **d.** 458 − 432

e. 298 + 137 **f.** 85 + 73 + 52 **g.** 10 010 + 1001 **h.** 5317 − 4290

WE2

5. Use the compensation strategy to complete each calculation mentally.

a. 62 + 39 b. 84 − 48 c. 398 + 73 d. 253 + 45

e. 334 − 52 f. 676 − 78 g. 987 + 197 h. 8799 − 539

6. Complete each calculation.

a. $\begin{array}{r} 43 \\ +\ 32 \\ \hline \end{array}$ b. $\begin{array}{r} 327 \\ +\ 451 \\ \hline \end{array}$ c. $\begin{array}{r} 56 \\ -\ 35 \\ \hline \end{array}$ d. $\begin{array}{r} 469 \\ -\ 215 \\ \hline \end{array}$

e. $\begin{array}{r} 345 \\ -\ 216 \\ \hline \end{array}$ f. $\begin{array}{r} 6237 \\ +\ 856 \\ \hline \end{array}$ g. $\begin{array}{r} 6781 \\ -\ 5832 \\ \hline \end{array}$ h. $\begin{array}{r} 3297 \\ 832 \\ +\ 56 \\ \hline \end{array}$

WE3

7. Use the vertical algorithm to complete each addition.

a. 34 + 53 b. 148 + 521 c. 213 + 78 d. 783 + 427

e. 8245 + 687 f. 298 + 370 + 123 g. 1427 + 431 + 84 h. 10 789 + 893

WE4

8. Use the vertical algorithm to complete each subtraction.

a. 93 − 52 b. 278 − 143 c. 87 − 68 d. 964 − 692

e. 748 − 569 f. 903 − 547 g. 9234 − 5798 h. 10 007 − 3468

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. On a piano, there are 52 white keys and 36 black keys. How many keys are there in total?

10. Phil has 255 followers on TikTok. What will Phil's total number of followers be if, after one of his TikToks goes viral, he gains 1497 followers.

11. Steve has a 128 gigabyte iPhone. How much storage does Steve have on his phone for new content if he already has 18 gigabytes of photos and 56 gigabytes of apps stored on his phone?

12. The Tour de France is a 21 stage cycling race that is over 3000 kilometres long. What is the combined distance of the final three stages of the race if the given table shows the distance of each of the stages?

Stage	Distance (km)
19 – Mourenx to Libourne	227
20 – Libourne to Saint Emilion	81
21 – Chatou to Paris Champs-Elysées	109

13. Gina was born on the 30th of September 2004. Emma was born exactly twenty-seven months and one week before Gina. What date is Emma's birthday?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Usher is passionate about engineering and is curious about the tallest buildings in the world. As part of a project he has prepared a table to show the heights of some of his favourite structures.

Building (Country)	Height (metres)
Empire State Building (USA)	443
Eiffel Tower (France)	324
Burj Khalifa (UAE)	828
Eureka Tower (Australia)	301

a. How much taller is the Empire State Building than the Eiffel Tower?

b. The Jeddah Tower is currently under construction in Saudi Arabia. It will be 173 m taller than the Burj Khalifa in the United Arab Emirates. What is the planned height of the Jeddah Tower?

c. The 'Merdeka 118' is a hotel in Kuala Lumpur that is being constructed with 118 floors. How tall is the hotel if its planned height is 4 m shorter than double the height of the Eiffel Tower?

d. Usher would like to include other information about each tower in his project. What other number based facts about the towers might he want to include to make it more interesting?

15. The Beenleigh Line train in Brisbane travels from Beenleigh station to Ferny Grove station. Jasper uses the following train timetable to plan his journey to the office.

Train station	Arrival time	Arrival time	Arrival time
Beenleigh	7:17 am	7:35 am	8:07 am
Coopers Plains	8:04 am	8:22 am	8:54 am
Moorooka	8:10 am	8:28 am	9:00 am
Dutton Park	8:17 am	8:35 am	9:07 am
South Brisbane	8:22 am	8:40 am	9:12 am
Central	8:26 am	8:44 am	9:16 am

a. If Jasper travels on the 7:35 am train from Beenleigh station, what time does he arrive at Central station?

b. How much later will Jasper arrive at the central station if he travels on the 8:07 am train from Beenleigh instead of the 7:17 am train?

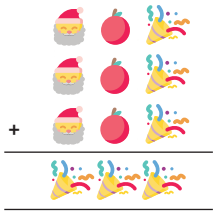
c. How many minutes does the journey from Beenleigh station to Central station take?

d. There is an express train that leaves Beenleigh at 7:32 am. It arrives one minute later to Central station than the 7:17 am train. How many minutes of travel time would Jasper save by travelling on the express train?

e. At times Jasper gets frustrated on the crowded trains particularly as the train gets closer to central station. Propose a way Jasper could travel to his office and avoid the crowded trains.

Extra spicy

16. Find three different digits to represent each emoji so that the calculation is correct.



17. When 10 011 is subtracted from 1 100 100, how many times does the digit 9 appear in the solution?

18. What is the time 290 hours after 7 am?

19. $320 - 319 + 318 - 317 + 316 - \ldots + 2 - 1 = ?$

A. 320 **B.** 300 **C.** 161 **D.** 160 **E.** 159

Remember this?

20. What time is shown by the clock?

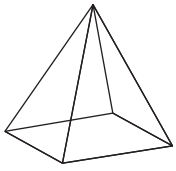
A. 4:36

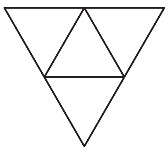
B. 7:23

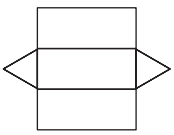
C. 8:23

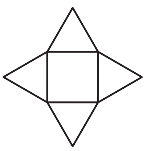
D. 23:07

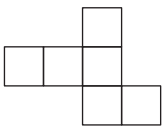
21. When folded and glued together, which cardboard cutout will create the following shape.



A. 

B. 

C. 

D. 

22. Which number shows forty-three thousands and eleven tens.

A. 4311 **B.** 43 000.11 **C.** 43 011 **D.** 43 110

1D Multiplying whole numbers

Multiplication of numbers is the same as a repeated addition. Multiplying two or more numbers together gives the product of these numbers. Multiplication allows us to calculate a series of repeated additions by performing these 'groups of' calculations much more efficiently. Below are some examples where multiplying whole numbers can be applied.

- My friends and I purchased 7 general admission tickets to an AFL match. If each ticket costs $25, what is the total cost of the tickets?
- I need ribbon for an arts and crafts project. How much ribbon do I have if I buy 8 packs of ribbon where each pack has 35 metres?
- In my school athletics tournament, I ran the 200 m sprint 6 times. What was the total distance that I ran in the tournament?

Learning intentions

Students will be able to:

- understand the associative and commutative law for multiplication
- apply the distributive law to complete multiplications
- use mental strategies to complete multiplications
- multiply whole numbers using the vertical algorithm.

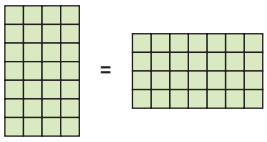
Key terms and definitions

- The **associative law** means that how we group numbers in addition or multiplication can change without affecting the result. E.g. $(7 \times 3) \times 4 = 7 \times (3 \times 4)$.
- The **commutative law** means that the order of the numbers in an addition and multiplication calculation can change without affecting the result. E.g. $8 \times 5 = 5 \times 8$.
- The **distributive law** for multiplication means that multiplying a given number by several other numbers is the same as multiplying the given number by the sum of the other numbers. E.g. $6 \times 17 = 6 \times (10 + 7) = (6 \times 10) + (6 \times 7)$.
- The **product** is the result when two or more values are multiplied together.

Key ideas

1 The commutative law for multiplication means the order in which we multiply does not affect the result.

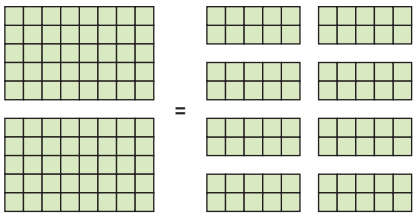
$a \times b = b \times a$



7 × 4 = 4 × 7

2 The associative law states that for multiplication, we can regroup numbers and not affect the result.

$a \times (b \times c) = (a \times b) \times c$

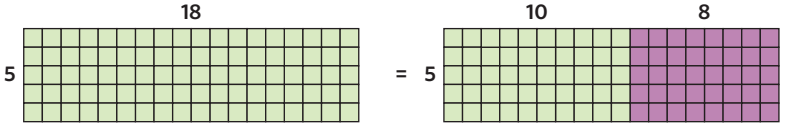


2 × (5 × 8) = (2 × 5) × 8

3 Different strategies can be applied to multiplication to make the calculations more efficient.

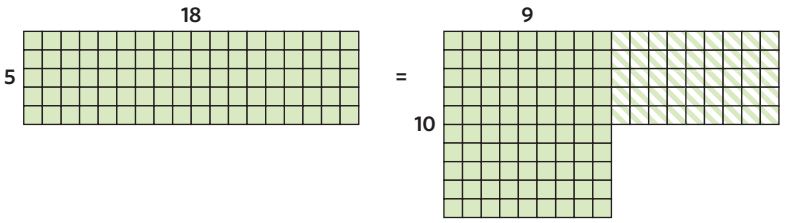
Distributive strategy

$a \times (b + c) = (a \times b) + (a \times c) = ab + ac$



5 × 18 = 5 × (10 + 8)
= 5 × 10 + 5 × 8
= 50 + 40
= 90

Halves and doubles strategy



5×18 = $(5 \times 2) \times (18 \div 2)$
$= 10 \times 9$
$= 90$

Worked examples

WE 1 Multiplying by powers of 10

Complete each calculation.

a. 33×10

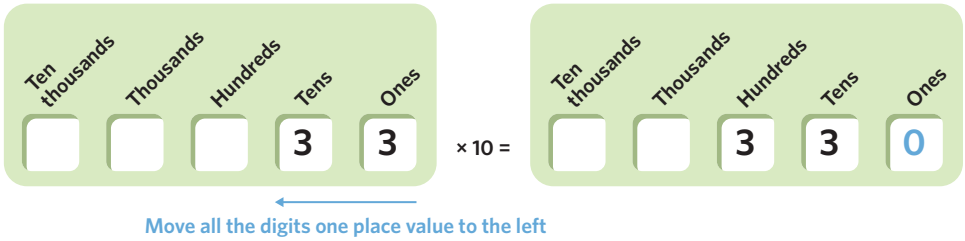
Working

$33 \times 10 = 330$

Thinking

We are multiplying by 10, so move each digit one place value to the left. Use zero as a place value holder in the ones place.

Visual support



b. 47×100

Working

$47 \times 100 = 4700$

Thinking

We are multiplying by 100, so move each digit two place values to the left. Use zero as a place value holder in the tens and ones place.

Student practice

Complete each calculation.

a. 17×10 **b.** 74×100 **c.** 1000×11 **d.** 100×100

WE 2 Multiplying using mental strategies

Evaluate each multiplication using the given mental strategy.

a. 7×24 (distributive strategy)

Working

$24 = 20 + 4$

$7 \times 24 = 7 \times (20 + 4)$

$7 \times 24 = (7 \times 20) + (7 \times 4)$

$= 140 + 28$

$= 168$

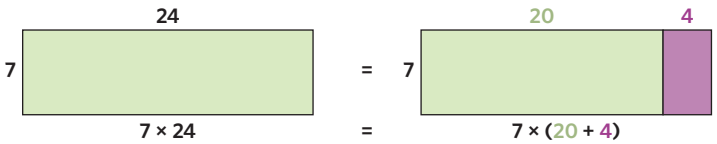
Thinking

Step 1: Partition one of the numbers into its place value parts.

Step 2: Apply the distributive law.

Step 3: Complete the calculation.

Visual support



b. 5×32 (halves and doubles strategy)

Working

$5 \times 32 = (5 \times 2) \times (32 \div 2)$

$= 10 \times 16$

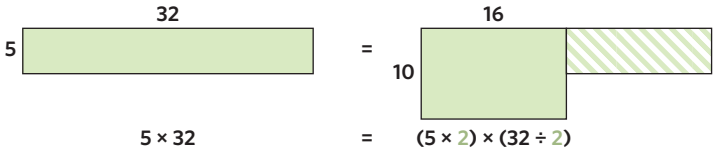
$= 160$

Thinking

Step 1: Identify which number to halve or double. Doubling 5 will give 10, which will make the multiplication easier. As 5 has doubled, halve 32.

Step 2: Complete the calculation.

Visual support



Student practice

Evaluate each multiplication using the given mental strategy.

a. 3×26 (distributive strategy)

b. 5×28 (halves and doubles strategy)

c. 20×22 (halves and doubles strategy)

d. 48×16 (distributive strategy)

WE 3 Multiplying whole numbers using a vertical algorithm

Complete the calculation using the vertical algorithm.

a. 123×4

Working

$$\begin{array}{r} 1\ 2\ 3 \\ \times\quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle +1} \\ 1\ 2\ 3 \\ \times\quad 4 \\ \hline 4\ 9\ 2 \end{array}$$

Thinking

Step 1: Line up the numbers vertically with the smaller number on the bottom.

Step 2: Multiply the digits in the ones place in the bottom number by each digit in the top number. If the product is larger than 9, regroup to the next place value column.

b. 103 × 19

Working

$$\begin{array}{rrr} & 1 & 0 & 3 \\ \times & & 1 & 9 \\ \hline \end{array}$$

$$\begin{array}{rrrr} & & +2 & \\ & 1 & 0 & 3 \\ \times & & 1 & 9 \\ \hline & 9 & 2 & 7 \end{array}$$

$$\begin{array}{rrrr} & & +2 & \\ & 1 & 0 & 3 \\ \times & & 1 & 9 \\ \hline & 9 & 2 & 7 \\ 1 & 0 & 3 & 0 \end{array}$$

$$\begin{array}{rrrr} & & +2 & \\ & 1 & 0 & 3 \\ \times & & 1 & 9 \\ \hline & 9 & 2 & 7 \\ + 1 & 0 & 3 & 0 \\ \hline 1 & 9 & 5 & 7 \end{array}$$

Thinking

Step 1: Line up the numbers vertically with the smaller number on the bottom.

Step 2: Multiply the digit in the ones place in the bottom number by each digit in the top number. If the product is larger than 9, regroup to the next place value column.

Step 3: Repeat step 2 for the digit in the tens place of the bottom number. We are multiplying by 10, so move all digits one place to the left using zero as a place value holder in the ones place.

Step 4: Add the two products.

Student practice

Complete the calculation using the vertical algorithm.

a. 132 × 6 **b.** 245 × 21 **c.** 378 × 22 **d.** 33 × 333

1D Activities and questions

STARTER TASKS

Odd spot

In-N-Out is a longstanding and iconic US fast food chain that has established a worldwide cult following. That is why when they travelled all the way down to Melbourne to open a pop up store, they completely sold out their 300 burgers in just 15 minutes. If the price of each burger was $10, how much did In-N-Out make from their pop up store that day?

A. $3000

B. $30 000

Image: Justin Mcmanus/The Age

Puzzle

The values in the table are found by multiplying the number from the left side column with the number from the top row.

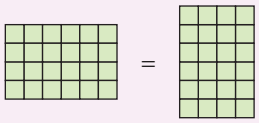
×		7		
	48			120
3		21		
5		35		50
	32		16	

Understanding worksheet

1. Fill in the boxes to show the commutative or associative laws for multiplication.

Example

Commutative law:



$4 \times 6 = 6 \times 4 = 24$

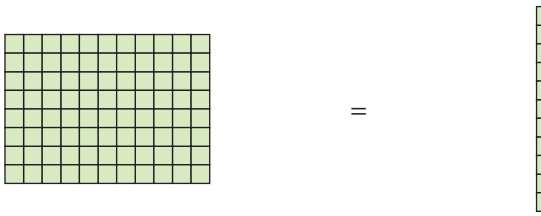
Associative law:

$2 \times (4 \times 3) = (2 \times 4) \times 3 = 24$

a. Commutative law:

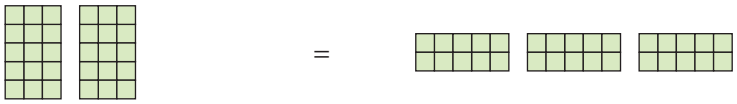
$\square = \square = \square$

b. Commutative law:



$\square = \square = \square$

c. Associative law:



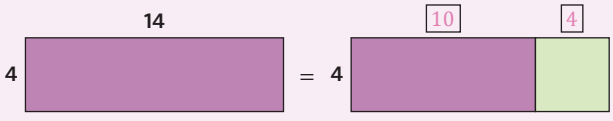
$\square = \square = \square$

d. Associative law:

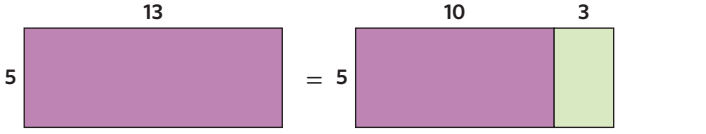
$\square = \square = \square$

2. Fill in the boxes using the open arrays to show the distributive strategy.

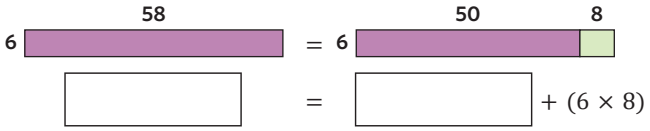
Example



a.

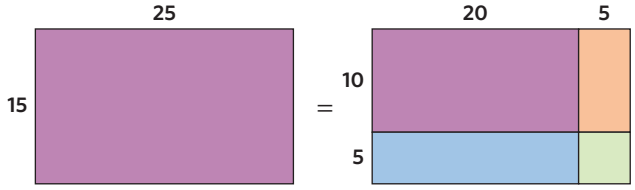


b.



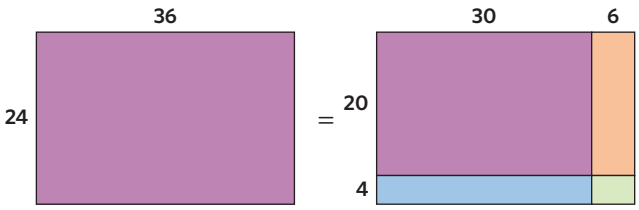
$\square = \square + (6 \times 8)$

c.



$\square = \square + (10 \times 5) + \square + (5 \times 5)$

d.



$\square = \square + (20 \times 6) + \square + (4 \times 6)$

3. Fill in the blanks by using the words provided.

algorithm | commutative | distributive | product

To find the ☐ of two numbers, we need to use multiplication. The ☐ law states that the order in which we multiply numbers does not affect the result. There are different ways to multiply whole numbers, these include using the ☐ law or the vertical ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Complete each calculation.

a. 7×10
b. 10×28
c. 100×6
d. 11×100
e. 71×1000
f. $10 \times 10 \times 10$
g. $100 \times 100 \times 100$
h. $1 \times 10 \times 100 \times 1000 \times 100 \times 10 \times 1$

WE2a **5.** Evaluate using the distributive strategy.

a. 22×3
b. 37×4
c. 7×54
d. 9×76
e. 133×8
f. 25×12
g. 61×22
h. 123×17

WE2b **6.** Evaluate using the halves and doubles strategy.

a. 24×5
b. 50×80
c. 20×43
d. 6×18
e. 4×62
f. 71×200
g. $28 \times 5 \times 10$
h. 2000×84

WE3 **7.** Complete the calculation using the vertical algorithm.

a. 41×7
b. 3×39
c. 67×4
d. 24×12
e. 77×50
f. 101×11
g. 145×27
h. 18×173

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Josh Giddey was a professional basketball player for the Adelaide 36ers. In his first season at the club, he scored 11 points per game. If he played 28 games altogether, what was the total number of points he scored in his first season?
9. How many kilometres did Sarah run last year if she ran 4 kilometres per day and last year was not a leap year?
10. Millie works as a lumberjack and can chop up 27 fire logs every hour. If she works 38 hours a week, how many fire logs does she chop up in a week?
11. Kathy's new car allows her to drive 18 kilometres per litre of fuel. Last month, Kathy used 133 litres of fuel. What distance did Kathy drive last month?
12. How many seconds are there in Aria's dancing lesson if her dancing lessons last an hour?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. The Optus Stadium is a relatively new multi-purpose stadium located in Perth. It is the third-largest stadium in Australia with a capacity of 61 266.
    a. In the summer, the Optus Stadium hosts cricket games and sells tickets at an average price of $10. For one game, there was an attendance of 27 383 people. What was the total ticket sales revenue for the cricket game?
    b. There are multiple food options for Optus Stadium patrons. One of the most popular items is the meat pie, which sells for $4. During an AFL match, 7208 fans purchased a meat pie. What was the total amount spent on meat pies for that match?
    c. Optus Stadium held the 2021 AFL grand final where there was a 60 000 capacity limit. The average ticket price was $250. How much money in total was spent on tickets by fans if the match was sold out.
    d. Is an average ticket price of $250 fair? Why or why not?
14. Fast fashion retailers are able to provide consumers with trendy and affordable clothing. While the prices of the clothes are low, the environmental and social costs are great. This issue has caused many brands to begin producing more sustainable clothing that reduces their environmental impact.
    a. An Australian consumer purchases an average of 27 kilograms of new clothing each year. How much clothing would 25 Australians purchase each year?
    b. Every 10 minutes an estimated 6 tonnes of textiles and clothing are dumped in Australian landfills. How many tonnes of textiles and clothing are dumped in landfills every day?
    c. Patagonia, an American clothing company, has a 'Worn Wear' initiative where they repair products so that they can be reused. Each repair reduces the company's carbon footprint by 7 kilograms. If they make 40 000 repairs in one year, what is the reduction in the company's carbon footprint per year?
    d. Should we buy more second-hand clothing to reduce our carbon footprint? Why or why not?

Extra spicy

15. A, B and C are digits in the number. What digit does B represent?

$$\begin{array}{r} 3ABC \\ \times \quad 7 \\ \hline 24178 \end{array}$$

A. 1 B. 3 C. 4 D. 5 E. 8

16. The product of two numbers is 324 and the difference between these two numbers is 0. What are the two numbers?
17. Using the distributive law, calculate 321×123.
18. How many minutes are there in a leap year?

Remember this?

19. Bella has a square prism and decides to cut it in half to create 2 smaller prisms.

What is the least number of faces that the smaller prisms could have?

A. 3 **B.** 4 **C.** 5 **D.** 6

20. Daniel has $1045 saved up for his new computer and his dad gave him another $465. How much does Daniel have now?

A. $1500 **B.** $1510 **C.** $1520 **D.** $1610

21. Sunray is selling $2 chocolate bars for a local fundraiser. She managed to sell 60 chocolate bars. Which equation represents the total money Sunray was able to raise?

A. $\textit{total money raised} = 60 + 2$

B. $\textit{total money raised} = 60 - 2$

C. $\textit{total money raised} = 60 \times 2$

D. $\textit{total money raised} = 60 \div 2$

1E Dividing whole numbers

The division of whole numbers can be thought of as finding how many equal groups are in a quantity or what is the size of given equal groups in a quantity. If there are parts remaining that cannot be divided into whole numbers equally, these are known as remainders and can be expressed as a fraction of the divisor. Below are some examples where dividing whole numbers can be applied.

- There are 18 players on a footy team. If there are 40 people trying out for two teams, how many players will be left out of a team?
- I have 240 pages to read in a novel for English class over the next fortnight. If I read the same number of pages each night, how many pages do I need to read each night to complete the book?
- There are 13 cookies in a pan and 4 people in my family want to share them equally. How many cookies does each person get?

Learning intentions

Students will be able to:

- use visual representations to divide whole numbers
- use mental strategies to divide efficiently
- divide whole numbers using the vertical algorithm
- divide whole numbers and express the remainder as a fraction.

Key terms and definitions

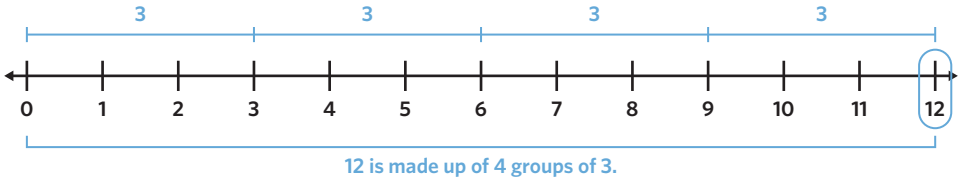
- A **dividend** is the first number in a division calculation. It is the number that is being divided by the second number.
- A **divisor** is the number by which a given value is divided.
- The **quotient** is the result or answer of a division calculation.
- A **remainder** is the leftover amount in a division calculation that is less than the divisor.
- An **open array** is an empty rectangle that is used to show multiplication and division using area.
- **Decomposition** is the process of breaking apart the original number into smaller parts.

Key ideas

1 Division is the same as 'is made up of how many groups?'.

$12 \div 3 = 4$

12 is made up of how many groups of 3? 4

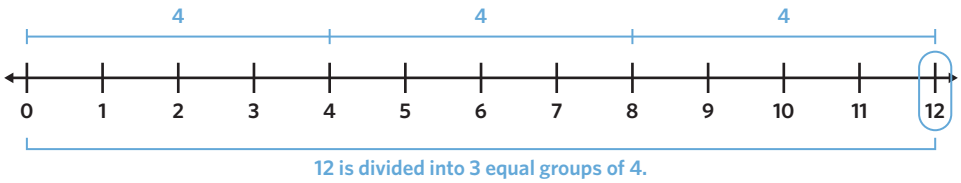


12 is made up of 4 groups of 3.

2 Division can be thought of as dividing into a given number of equally sized groups.

$12 \div 3 = 4$

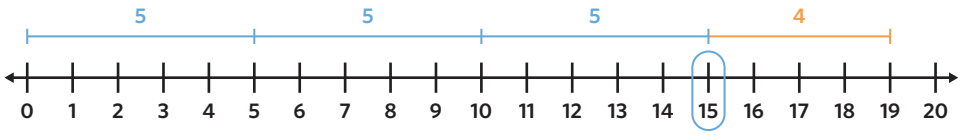
12 can be divided into 3 equal groups of 4.



12 is divided into 3 equal groups of 4.

3 Some divisions result in a remainder if a number cannot be divided completely in equally sized groups.

$19 \div 5 = 3\ r\ 4$ or $3\frac{4}{5}$



19 is divided into 3 equal groups of 5 with a remainder of 4.

Worked examples

WE 1 Dividing by powers of 10

Evaluate each expression.

a. 330 ÷ 10

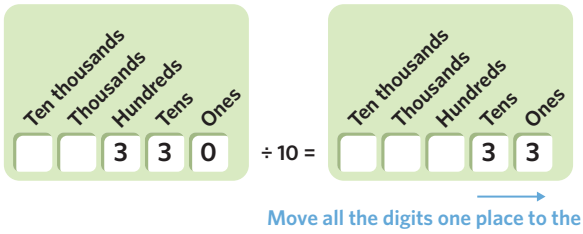
Working

330 ÷ 10 = 33

Thinking

Identify how many place values to move each digit based on the divisor. The divisor is 10, so each digit moves one place value to the right.

Visual support



b. 4800 ÷ 100

Working

4800 ÷ 100 = 48

Thinking

Identify how many place values to move each digit based on the divisor. The divisor is 100, so each digit moves two place values to the right.

Student practice

Evaluate each expression.

a. 240 ÷ 10 **b.** 4000 ÷ 100 **c.** 730 ÷ 10 **d.** 9900 ÷ 100

WE 2 Dividing using a vertical algorithm

Evaluate using short division.

a. 252 ÷ 12

Working

$12\overline{)252}$

$12\overline{)\overset{0}{2}\overset{2}{5}\ 2}$

$12\overline{)\overset{0}{2}\overset{2}{5}\overset{12}{2}}$

$12\overline{)\overset{0}{2}\overset{2}{5}\overset{12}{2}}\ \overset{1}{}$

252 ÷ 12 = 21

Thinking

Step 1: Set up the vertical algorithm with the divisor on the left and the dividend on the right, below the line.

Step 2: For 2 ÷ 12, the dividend is less than the divisor. So, we use 0 as a placeholder. Write the remainder 2 above the next digit 5.

Step 3: 25 ÷ 12 = 2 *remainder* 1 . We place the 2 on top of the division bar. Write the remainder 1 above the next digit 2.

Step 4: 12 ÷ 12 = 1 with no remainder. We place the 1 on top of the division bar.

Step 5: The final answer is above the division bar.

b. $332 \div 30$

Working

$30\overline{)332}$

$30\overline{)\overset{0}{3}\overset{3}{3}\overset{3}{3}\ 2}$

$30\overline{)\overset{0}{3}\overset{3}{3}\overset{3}{3}\overset{1}{2}}$

$30\overline{)\overset{0}{3}\overset{3}{3}\overset{3}{3}\overset{1}{2}}$ 0 1 1 *r* 2

$11\ r\ 2$ or $11\frac{2}{30}$

Thinking

Step 1: Set up the vertical algorithm with the divisor on the left and the dividend on the right, below the line.

Step 2: For $3 \div 30$, the dividend is less than the divisor. So, we use 0 as a placeholder. Write the remainder 3 above the next digit 3.

Step 3: $33 \div 30 = 1$ *remainder* 3. We place the 1 on top of the division bar. Write the remainder 3 above the next digit 2.

Step 4: $32 \div 30 = 1$ *remainder* 2. We place the 1 on top of the division bar. Write 2 as a remainder on top of the division bar.

Step 5: The final answer is above the division bar.

Student practice

Evaluate using short division.

a. $468 \div 9$ **b.** $3452 \div 15$ **c.** $2448 \div 12$ **d.** $18\,024 \div 30$

1E Activities and questions

STARTER TASKS

Odd spot

In 2001, Ken Edwards set the bizarre record for most live cockroaches eaten in 1 minute, when he ate 36 live cockroaches. Assuming he ate at a constant rate, how many cockroaches did Ken eat every 10 seconds?

A. 3.6 **B.** 6

Image: IrinaK/Shutterstock.com

Puzzle

Working from left to right and top to bottom, each row and column is a division problem.

a) What row or column should you complete first and why?

b) Would you start at the bottom right corner first?

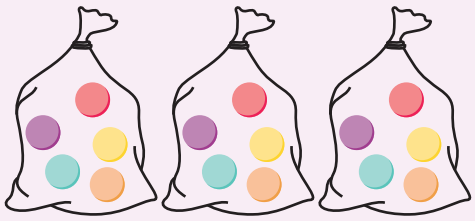
c) Solve for each missing square.

18	3	
6		2
	1	

Understanding worksheet

1. Complete the division using the visual representation.

Example



15 ÷ 3 = 3

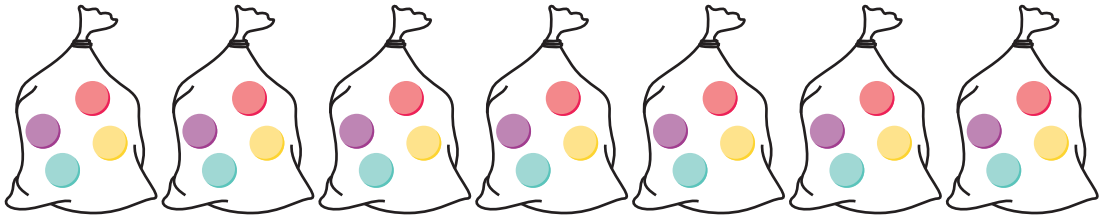
a.

12 ÷ 2 = ☐

b.

16 ÷ 4 = ☐

c.



28 ÷ 7 = ☐

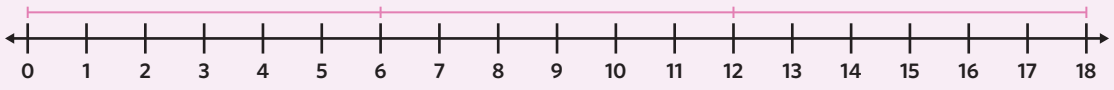
d.

30 ÷ 7 = ☐ *r* ☐

2. Complete the division by counting equal sized groups on the number line.

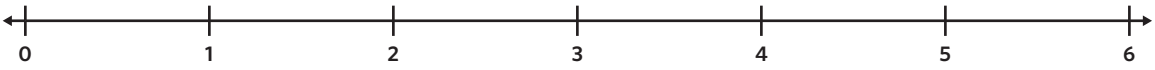
Example

18 contains 3 groups of size 6



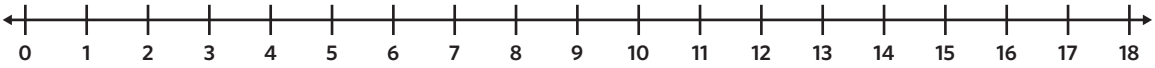
18 ÷ 6 = 3

a. 6 contains ☐ groups of size 3



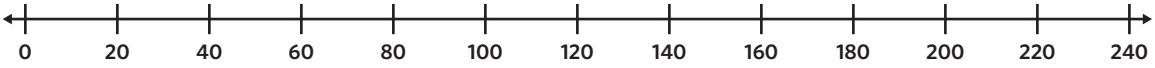
6 ÷ 3 = ☐

b. 18 contains ☐ groups of size 3



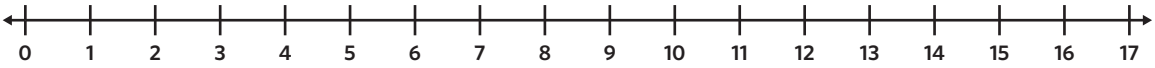
18 ÷ 3 = ☐

c. 240 contains ☐ groups of size 60



240 ÷ 60 = ☐

d. 17 contains ☐ groups of size 3 with a remainder of size ☐



17 ÷ 3 = ☐ *r* ☐

3. Fill in the blanks by using the words provided.

quotient | inverse | dividend | equal | divisor

Division can be thought of as the ☐ of multiplication. We can explain division as determining how many ☐ groups are in a total quantity or the size of equal groups in a total quantity. The total amount in a division is called the ☐ and is divided by the ☐ to find the ☐ or answer to the calculation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (b,c,d), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (d,e,f), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Evaluate each expression.

a. 20 ÷ 2 **b.** 36 ÷ 2 **c.** 21 ÷ 3 **d.** 45 ÷ 5

e. 88 ÷ 8 **f.** 56 ÷ 7

WE1 5. Evaluate each expression.

a. $30 \div 10$
b. $230 \div 10$
c. $400 \div 10$
d. $1100 \div 100$
e. $3000 \div 100$
f. $46\,000 \div 1000$
g. $52\,000 \div 100 \div 10$
h. $730\,000 \div 100 \div 10 \div 10$

WE2 6. Evaluate using short division.

a. $4\overline{)64}$
b. $8\overline{)97}$
c. $2\overline{)147}$
d. $4\overline{)3182}$
e. $7\overline{)42\,021}$
f. $15\overline{)3963}$
g. $12\overline{)2449}$
h. $24\overline{)29\,264}$

WE2 7. Evaluate using short devision.

a. $39 \div 3$
b. $84 \div 6$
c. $162 \div 9$
d. $112 \div 8$
e. $629 \div 18$
f. $1274 \div 13$
g. $2840 \div 22$
h. $7104 \div 13$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Rod's Fruit Market was having a sale on punnets of strawberries. Mary spent $30 on 10 punnets of strawberries. How much did each punnet cost?

9. Tyler works in construction and has 15 different work uniforms that he cycles through as his work gets quite dirty. His uniform consists of a pair of coveralls and boots. If he has 3 pairs of boots, how many different coveralls does he own?

10. Ellen's tree nursery has 3 separate seedling areas. How many rows of fruit trees does she have in her nursery in total if each seedling area contains 3500 fruit trees with 50 trees in a row?

11. Ryan has 3500 strawberry plants on his farm. How many strawberry plants does Calvin have on his farm if Ryan's farm has 7 times as many strawberry plants as Calvin?

12. Riley packs her strawberries so that there are 45 punnets of strawberries in a carton and 16 cartons on a pallet. If there were 5800 punnets in total, how many full pallets were there?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Paul works as a plumber and earns $1400 a week. He has been saving his money to take a 12 week campervan tour around Australia. Paul works 8 hours a day, Monday through Friday with no sick days off. Paul thinks he might even be able to take more than 12 weeks off because he has calculated that he can afford to give up $20 000 of pay while not working during his trip.
    a. How much does Paul earn per day?
    b. How much does Paul earn per hour?
    c. What is the maximum number of weeks Paul could take off from work?
    d. Is 12 weeks an appropriate amount of time to take off for a caravan trip around Australia?

14. Nancy keeps track of how many steps she walks while doing laps of her oval. She walks the same number of laps each day for a fortnight, walking a total of 168 000 steps. Nancy's average step length is 1 metre.
    a. How many steps around the oval does Nancy walk each day?
    b. If 1000 metres equals 1 kilometre, how many kilometres did Nancy walk each day?
    c. Nancy checked her fitbit and it showed that she had walked 396 km. If she continued walking the same distance each day, how many days did she walk to reach 396 km?
    d. It is recommended that the average person complete 10 000 steps per day. Suggest ways that you can achieve this amount every day.

Extra spicy

15. The value of $\frac{300 \times 6}{300 \div 6}$ is

A. 1 **B.** 6 **C.** 12 **D.** 36 **E.** 300

16. How many digits are there in the correct solution to the calculation $456\,456\,456 \div 456$?

A. 4 **B.** 5 **C.** 6 **D.** 7

17. True or false. $12\,224 \div 63 = (12\,224 \div 7) \div 9$

18. $0 \div 123\,421\,789 = ?$

Remember this?

19. John went hiking. In the morning it was -14 degrees celsius. By the afternoon it was 3 degrees celsius. What was the increase in temperature?

A. 15 degrees **B.** 18 degrees **C.** 12 degrees **D.** 17 degrees

20. A Disney cruise ship can hold 4000 people. 2325 were girls. How many boys are on the cruise ship?

A. 1625 **B.** 1380 **C.** 1675 **D.** 1475

21. Zellie has a bag of 12 marbles. 4 are red, 8 are blue. What is the chance she will pick a blue marble?

A. 3 out 12 **B.** 2 out 12 **C.** 8 out of 12 **D.** 6 out of 12

1F Order of operations

When calculations involving multiple operations are performed, the order that the operations are performed in is important. The order in which operations should be performed is as follows: brackets, indices, division, multiplication, addition, and subtraction. Below are some examples where order of operations can be applied.

- I bought 5 cans of tennis balls where each can had 4 balls. Over summer, I lost 3 balls. How many tennis balls do I have left?
- I bought 3 packets of caramel chocolate frogs to share with a group of 8 friends (9 including myself). If each packet has 12 pieces of chocolate, how many caramel chocolate frogs does each person receive?
- I collected a dozen eggs each day for three days and then 7 eggs each day for two days. On the sixth day I doubled the quantity of eggs that I collected over the first 5 days. Write an equation to show how many eggs I collected and then evaluate.

Learning intentions

Students will be able to:

- + solve mixed operation problems using visuals
- + understand and apply the correct order of operations to solve mixed operation problems
- + create equations to represent a contextual problem and solve.

Key terms and definitions

- **Brackets** are mathematical symbols that clarify the order that operations are completed in expressions. Brackets should always be evaluated first.
- An **operation** in maths is a mathematical process used to solve problems. The four basic operations in maths are addition, multiplication, subtraction, and division.
- **Inverse operations** are mathematical operations that undo each other. The inverse of multiplication is a division and the inverse of division is a multiplication; the inverse of addition is a subtraction and the inverse of subtraction is an addition.

Key ideas

1 When working with expressions with multiple operations we need to follow the appropriate order of operations.

The order of operations involve:

1. Brackets
2. Indices
3. Division and multiplication
4. Addition and subtraction

Correct ✓

Using the correct order of operations

1. Multiplication first
2. Addition and subtraction from left to right

$3 + 4 \times 2 - 7$	Multiplication
$= 3 + 8 - 7$	Addition
$= 11 - 7$	Subtraction
$= 4$	✓

Incorrect ✗

Ignoring the correct order of operations

$3 + 4 \times 2 - 7$	Addition
$= 7 \times 2 - 7$	Subtraction
$= 7 \times (-5)$	Multiplication
$= 4$	✗

2 When working with inverse operations, we must work from left to right. Multiplication and division are inverse operations. Addition and subtraction are inverse operations.

Correct ✓

$36 \div 2 \times 3$	Division
$= 18 \times 3$	Multiplication
$= 54$	✓

Incorrect ✗

$36 \div 2 \times 3$	Multiplication
$= 36 \div 6$	Division
$= 54$	✗

3 When solving an equation with multiple brackets, work from inner brackets to outer brackets.

$[(24 \div 8) + 4] \times 5$	Step 1: Simplify the inner bracket.
$= [3 + 4] \times 5$	Step 2: Simplify the outer bracket.
$= 7 \times 5$	
$= 35$	

Worked examples

WE 1 Completing order of operation calculations

Using the order of operations, complete each calculation.

a. $3 + 25 \div 5$

Working

According to the order of operations, division is calculated before addition.

$3 + 25 \div 5 = 3 + 5$

$= 8$

Thinking

Step 1: Identify the order that the operations will be applied in.

Step 2: Evaluate the division.

Step 3: Evaluate the addition.

Visual support

$3 + 25 \div 5$ Division

$= 3 + 5$ Addition

$= 8$

b. $50 - (22 - 10) \times 4$

Working

According to the order of operations, we will first evaluate the calculation inside the brackets, then the multiplication, then the subtraction.

$50 - (22 - 10) \times 4 = 50 - 12 \times 4$

$= 50 - 48$

$= 2$

Thinking

Step 1: Identify the order that the operations will be applied in.

Step 2: Evaluate the calculation inside the brackets.

Step 3: Evaluate the multiplication.

Step 4: Evaluate the subtraction.

Student practice

Using the order of operations, complete each calculation.

a. $1 + 36 \div 9$ **b.** $22 - (9 - 2) \times 3$ **c.** $(35 + 49) \div (3 + 4)$ **d.** $11 + 2 \times 3^2$

WE 2 Solving questions with multiple brackets

Complete each calculation.

a. $3 \times [5 \times (7 - 5) + 2]$

Working

$3 \times [5 \times (7 - 5) + 2] = 3 \times [5 \times 2 + 2]$

Inside the bracket, the multiplication is evaluated before the addition.

$= 3 \times [10 + 2]$

$= 3 \times 12$

$= 36$

Thinking

Step 1: Simplify the inner bracket.

Step 2: Simplify the outer bracket.

Step 3: Evaluate the multiplication.

b. $[56 \div (3 + 5) \times (33 \div 11)] + 7$

Working

$[56 \div (3 + 5) \times (33 \div 11)] + 7$
$= [56 \div 8 \times 3] + 7$

The division and multiplication will be calculated from left to right.

$= [7 \times 3] + 7$

$= 21 + 7$

$= 28$

Thinking

Step 1: Simplify each of the inner brackets.

Step 2: Simplify the outer bracket.

Step 3: Evaluate the addition.

Student practice

Complete each calculation.

a. $5 \times [8 \times (7 - 6) + 3]$

b. $96 \div [(5 - 3) \times (36 \div 6)] + 1$

c. $(4 - 2) \times [(9 + 11) - (9 \times 2)] - 3$

d. $12 + [88 \div (16 \div 4 + 7) \times (9 - 8)]$

WE 3 Solving worded order of operation questions

Write an expression for the worded sentence and solve.

a. Three plus the product of four and two.

Working

$3 + 4 \times 2$

According to the order of operations, the multiplication will be evaluated before the addition.

$3 + 4 \times 2 = 3 + 8$

$= 11$

Thinking

Step 1: Represent the worded problem as an expression.

Step 2: Identify the order that the operations will be applied in.

Step 3: Evaluate the multiplication.

Step 4: Evaluate the addition.

Visual support

$3 + 4 \times 2$ Multiplication
$= 3 + 8$ Addition
$= 11$

b. Sixty divided by the product of three and five.

Working

$60 \div (3 \times 5)$

According to the order of operations, we will evaluate the brackets first, then the division.

$60 \div (3 \times 5) = 60 \div 15$

$= 4$

Thinking

Step 1: Represent the worded problem as an expression.

Step 2: Identify the order that the operations will be applied in.

Step 3: Evaluate the brackets.

Step 4: Evaluate the division.

Student practice

Write an expression for the worded sentence and solve.

a. Two plus the product of four and two.

b. Forty-eight divided by the product of six and two.

c. Six multiplied by the sum of three, four and one.

d. The sum of fourteen and sixteen, divided by the product of five and two.

1F Activities and questions

STARTER TASKS

Odd spot

Over half of Victoria's litter is discarded in public areas such as the parks and at beaches. To combat this problem, the government is introducing a 'cash for cans' program to incentivise the correct recycling of drink containers. As part of the program, some reverse vending machines (one pictured) will be hosted at sites such as sporting clubs and charities.

Image: Chris Lane/The Age

A local netball team wants to raise $100 for a club barbeque by raising proceeds from recycling water bottles. The club was able to collect 780 water bottles where they could claim 10 cents per bottle. How much more does the club need to raise before hitting their $100 target?

A. $22 **B.** $78

Puzzle

□ + □ + □ = 12

□ × □ − ○ = 10

(○ + □) × △ = 30

□ × △ ÷ ○ = ⬟

a) What is the value of the square?
b) What is the value of the circle?
c) What is the value of the triangle?
d) What is the value of the pentagon?

Understanding worksheet

1. Circle the part that should be calculated first.

$2 + (4 \times 5) - 7 = 15$ *Example*

a. $4 \times 7 + 8 = 36$ **b.** $-4 + 9 \times 5 = 41$

c. $9 \times (2 + 3) - 5 = 40$ **d.** $2 \times [48 \div (4 \times 1) + 9] = 42$

2. The top question shows the correct equation. Will the equation still be correct if we remove the brackets?

$(3 + 3) \times 6 = 54$ *Example*

$3 + 3 \times 6 =$ 21

Yes ☐ No ☑

a. $2 + (1 \times 6) = 8$

$2 + 1 \times 6 =$ ☐

Yes ☐ No ☐

b. $4 + (2 \times 6) \div 3 = 8$

$4 + 2 \times 6 \div 3 =$ ☐

Yes ☐ No ☐

c. $7 \times (9 - 7) + 10 \div 5 = 16$

$7 \times 9 - 7 + 10 \div 5 =$ ☐

Yes ☐ No ☐

d. $5 + [2 \times (4 \times 3) + 6] \div 3 = 15$

$5 + 2 \times 4 \times 3 + 6 \div 3 =$ ☐

Yes ☐ No ☐

3. Fill in the blanks by using the words provided.

addition | brackets | multiplication | left | right

The order of operations is important when completing calculations. When expressions have multiple operations, we first complete _____, then indices. This is then followed by _____ and division and lastly _____ and subtraction. With all inverse operations, we work from _____ to _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f)

4. Identify which operation (addition, subtraction, multiplication, division) is completed first.

a. $7 + 3 - 9$
b. $5 - 7 \times 8 + 1$
c. $5 \times 7 - 8 \div 4$
d. $7 \times 8 + (24 - 3) - 9$
e. $3 \times [2 \times (5 - 3)]$
f. $[2 \times (22 + 3) - 2 \times 7 + 3] \div 2$
g. $[2 \times 5 + (14 \div 2) - (4 \times 4)] \times 3$
h. $7 \div [2^2 - (9 - 3) + 21 \div 7]$

WE1 **5.** Using the order of operations, complete each calculation.

a. $3 + 4 \times 3$
b. $4 \div 2 + 9$
c. $5 \times 7 - 20 \div 5$
d. $35 \div (13 - 8) \times 7$
e. $8 \times (4 + 2) \div (1 + 3)$
f. $4 \times (7 + 8) - (16 \div 2) \div 4$
g. $(18 + 3 \times 9) \div 9 + (4 - 4)$
h. $2^2 \times (5 + 8 \div 4 \times 3) - (3^2 \div 9) \times 2$

WE2 **6.** Complete each calculation.

a. $3 \times [(2 + 2) \div 1]$
b. $3 \times 2 + [7 - (10 \div 2)]$
c. $[2 \times (18 \div 3 \times 2)] \div 4$
d. $2 + [3 \times (5 - 3) \times 9] - 12$
e. $[6 \times (4 - 1) \div 9] \times 16$
f. $23 + [7 + 3 \times (1 + 2 \times 2)] - (5 \times 9)$
g. $25 \times 2 + \{30 \div [60 \times 2 \div (3 \times 4)]\}$
h. $107 + \{64 \times [(8 \div 2) \div (8^2 - 60)] \times 2^2\}$

WE3 **7.** Write an expression for the worded sentence and solve.

a. Three plus half of ten.
b. Thirty-three minus the product of five and four.
c. Three groups of fifteen divided by five.
d. The quotient of forty-four and eleven multiplied by the product of two and three.
e. The sum of three and seventeen multiplied by the difference of nine and four.
f. The square of four divided by the product of eight and two.

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Bobby is participating in a charity cycle where he has to ride 500 kilometres total over five weeks. He rode 120 kilometres per week for the first three weeks. How many kilometres does he have left to reach his 500 kilometre target?

9. Brock recently started playing Pokemon and has collected 15 cards that were in playing condition. His dad bought an additional 88 cards from an online seller. Upon arrival, Brock realised that only a quarter of these purchased cards were in playing condition. How many cards in playing condition does Brock now have?

10. How many points did Kai score in the state championship game? In the game, Kai made 4 free throws which are worth one point each, 5 two-pointers, and 4 three-pointers.

11. John and Louis are buying some pants at their favourite store. The pants they wish to purchase cost $145 but the store is running a promotion where 3 pairs cost $330. How much money did they save altogether if John and Louis bought 3 pairs of pants each?

12. Jessica owns an e-commerce business selling candles. Help Jessica calculate how much more profit she made this year than last year. It costs her $8 to make a candle and she sells each one for $17. Last year, Jessica sold 1100 candles and this year she tripled the amount she sold. (Note: profit = selling price − cost price)

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Terry and Linda are environmentally conscious citizens and decide to install 15 solar panels on their roof.
    a. The cost of each solar panel is $300 and there is a single installation fee of $350. If Terry and Linda get $1440 back from the government for their installation, how much would they pay for their solar panel system?
    b. Instead of a lump sum government payment, Terry and Linda could receive $100 back for every solar panel they purchase. How much would the solar panel installation cost under this method?
    c. Currently, Terry and Linda's solar panels produce 57 kW of energy per day. Every year, the solar panels' efficiency deteriorates and they produce 1 less kW per day. Determine the difference between the amount of energy currently produced over a two-week period, and the amount of energy produced over a two-week period in five years' time.
    d. Should every Australian household get solar panels? Why or why not?

14. Secondbite is one of Australia's largest food rescue organisations. They work with food growers, manufacturers and retailers to gather excess and unsold food and distribute it to local charities and not-for-profit organisations around Australia.
    a. One day, Secondbite rescued food from 8 markets in Victoria. They rescued 18 kg from each market. Upon sorting the food, they realised that 13 kg of the total amount of food was not edible. How much of the rescued food was edible?
    b. Every tonne of food that ends up in landfill emits 2 tonnes of greenhouse gases. In 2019, Secondbite was able to rescue and redistribute 19 000 000 kg of fresh food that would have otherwise gone to landfill. What impact did Secondbite have on greenhouse gas emissions in 2019? Note: 1 tonne = 1000 kg.
    c. Mary's class is running a fundraiser to support Secondbite. Every dollar donated to Secondbite allowed them to collect and distribute 3 kg of food. If Mary donated $17, her brother donated $20, her sister donated $10, and her parents each donated $55, how much food was Mary's family able to help Secondbite to distribute?
    d. What are some other things Mary's class can do to lower their environmental footprint?

Extra spicy

15. $6 \div -2(1 + 2 - 4)$ is equal to?
    A. -3 B. -2 C. $-\frac{3}{2}$ D. $\frac{3}{2}$ E. 3

16. $2^2[3^2(4^2)] \div 2^3$ is equal to?
    A. 12 B. 36 C. 48 D. 72 E. 216

17. The value of $5 \times 1234 - 25 \times 1234$ is?

18. The expression $[a(b - c) + b - (c \times a)]$ can be simplified to?
    A. $ab - c + b - ca$
    B. $ab - 2ac + b$
    C. $ab + b$
    D. $ab + c + b + ca$
    E. $ab + 2ac - b$

Remember this?

19. A local bubble tea shop sells their milk tea drinks in two sizes: regular and large. A large milk tea drink costs \$0.70 more than a regular milk tea drink. Toppings such as pearls cost an additional \$0.50.

Which of the equations represents the cost of a large milk tea drink with pearls?

A. Cost of a large milk tea drink with pearls $=$ cost of a regular milk tea drink $-$ \$1.20

B. Cost of a large milk tea drink with pearls $=$ cost of a regular milk tea drink $+$ \$0.50

C. Cost of a large milk tea drink with pearls $=$ cost of a regular milk tea drink $+$ 0.70

D. Cost of a large milk tea drink with pearls $=$ cost of a regular milk tea drink $+$ 1.20

20. Andrew is currently hiking in a north-easterly direction. Along his hike, his trail has a sharp quarter turn to the right. Which direction is he now moving in after the turn?

A. East **B.** South-easterly **C.** South **D.** South-westerly

21. A football match started at 3:15 pm and the match lasted 96 minutes. What time did the football match finish?

A. 4:41 pm **B.** 4:46 pm **C.** 4:51 pm **D.** 4:56 pm **E.** 5:01 pm

1G Estimating and rounding

There are some situations where estimates can be used instead of exact calculations or exact numbers. Making accurate estimates from exact numbers is crucial as sometimes it is not practical to work with exact numbers. This is why being able to accurately round numbers in estimation is such an important skill. Whole numbers can be rounded using different strategies and degrees of accuracy. Below are some examples where estimating and rounding can be applied.

- I have $215 dollars to spend on Christmas presents. Rounded to the nearest $100, how much money do I have to spend?
- I have collected 2130 coke cans over the last five years. Approximately how many cans do I collect each year?
- On the 1st of May I received 17 emails and on the 2nd I received 18. How many emails can I roughly expect to receive in May?

Learning intentions

Students will be able to:

- round numbers to the nearest 10, 100 and 1000
- estimate by rounding numbers
- identify overestimates and underestimates
- find reasonable estimates.

Key terms and definitions

- **Rounding** is the process of making a number simpler to use in calculations.
- The term **approximately equal to** is represented in calculations using the '≈' symbol.
- **Estimates** are approximate values that are close to the correct answer.
- **Lead digit rounding** rounds the number based on the first digit and replaces all other digits with zeros.
- **Overestimates** are estimates that are more than the exact calculation, number, quantity or size.
- **Underestimates** are estimates that are less than the exact calculation, number, quantity or size.
- A **critical digit** is the digit to the right of a digit being rounded. If the critical digit is 5 or above, add 1 to the digit being rounded. If the critical digit is less than 5, leave the digit being rounded the same.

Key ideas

1 A 99 chart can be used to round numbers to the nearest 10 or 100.

When rounding to the nearest 10 using the 99 chart, locate the number being rounded on the chart. Then identify which 10 it is closest to.

0	1	2	3	4	5	6	7	8	9	10
10	11	12	13	14	15	16	17	18	19	20
20	21	22	23	24	25	26	27	28	29	30
30	31	32	33	34	35	36	37	38	39	40
40	41	42	43	44	45	46	47	48	49	50
50	51	52	53	54	55	56	57	58	59	60
60	61	62	63	64	65	66	67	68	69	70
70	71	72	73	74	75	76	77	78	79	80
80	81	82	83	84	85	86	87	88	89	90
90	91	92	93	94	95	96	97	98	99	100

36 is between 30 and 40
36 is closer to 40
36 ≈ 40

74 is between 70 and 80
74 is closer to 70
74 ≈ 70

When rounding to the nearest 100 using the 99 chart, locate the number being rounded in the chart. Then identify which 100 it is closest to.

400	401	402	403	404	405	406	407	408	409
410	411	412	413	414	415	416	417	418	419
420	421	422	423	424	425	426	427	428	429
430	431	432	433	434	435	436	437	438	439
440	441	442	443	444	445	446	447	448	449
450	451	452	453	454	455	456	457	458	459
460	461	462	463	464	465	466	467	468	469
470	471	472	473	474	475	476	477	478	479
480	481	482	483	484	485	486	487	488	489
490	491	492	493	494	495	496	497	498	499

427 is between 400 and 500
427 is closer to 400
427 ≈ 400

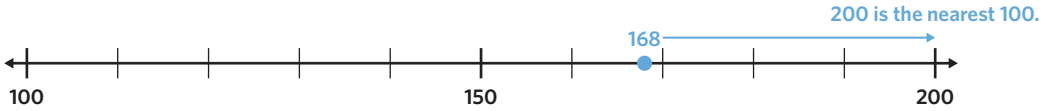
482 is between 400 and 500
482 is closer to 500
482 ≈ 500

500

2 Number lines can be used to show rounding.

Round 168 to the nearest 100:

168 is between 100 and 200. 150 is the middle number between 100 and 200, so we can see that 168 is rounded to 200 as it is greater than 150.



3 When rounding to a specific place value, consider the first digit to the **right** of the digit being rounded. This is the **critical digit**.

If the **critical digit** is **5 or greater**, increase the digit being rounded by 1 and all the digits in smaller place values change to zeros.

If the **critical digit** is **less than 5**, then the digit being rounded remains the same and all the digits in smaller place values change to zeros.

Round to the nearest 10:

Critical digit

$2945 \approx 2950$

Digit being rounded

Round to the nearest 100:

Critical digit

$2945 \approx 2900$

Digit being rounded

If the **digit being rounded** is **9**, increasing the digit being rounded by 1 is 10, so regroup to the next place value.

Round to the nearest 10:

Critical digit

$9796 \approx 9800$

Digit being rounded

Round to the nearest 1000:

Critical digit

$9796 \approx 10\,000$

Digit being rounded

4 How reasonable the estimate is depends on how much the numbers in a calculation are rounded. The smaller the difference between the original number and the rounded number, the more accurate an estimate is likely to be.

The table compares different estimates for the exact calculation $1672 + 1545 = 3217$.

	Estimate calculation	**Difference between estimate and actual**	**Overestimate/ underestimate**
Rounded to the nearest 1000	$2000 + 2000 = 4000$	$4000 - 3217 = 783$	Overestimate by 783
Rounded to the nearest 100	$1700 + 1500 = 3200$	$3200 - 3217 = -17$	Underestimate by 17
Rounded to the nearest 10	$1670 + 1550 = 3220$	$3220 - 3217 = 3$	Overestimate by 3

Worked examples

WE 1 Rounding whole numbers

Round each number to the specified place value.

a. 63 to the nearest 10

Working

63

$3 < 5$

$63 \approx 60$

Thinking

Step 1: Identify the digit being rounded and underline the critical digit.

Step 2: The critical digit is less than 5, so the digit being rounded stays the same. Replace all digits to the right of the digit being rounded with zeros.

Visual support

0	1	2	3	4	5	6	7	8	9	10
10	11	12	13	14	15	16	17	18	19	20
20	21	22	23	24	25	26	27	28	29	30
30	31	32	33	34	35	36	37	38	39	40
40	41	42	43	44	45	46	47	48	49	50
50	51	52	53	54	55	56	57	58	59	60
60	61	62	63	64	65	66	67	68	69	70
70	71	72	73	74	75	76	77	78	79	80
80	81	82	83	84	85	86	87	88	89	90
90	91	92	93	94	95	96	97	98	99	100

b. 976 to the nearest 100

Working

976

$7 > 5$

$976 \approx 1000$

Thinking

Step 1: Identify the digit being rounded and underline the critical digit.

Step 2: The critical digit is greater than 5, so increase the digit being rounded by 1. As the digit being rounded is 9, increasing it by 1 is 10, so regroup to the next place value. Replace all digits to the right of the digit being rounded with zeros.

Student practice

Round each number to the specified place value.

a. 54 to the nearest 10

b. 436 to the nearest 100

c. 2782 to the nearest 1000

d. 19 500 to the nearest 1000

WE 2 Estimating by rounding numbers

Estimate each calculation by first rounding both numbers to the specified place value.

a. 74 + 47 (nearest 10)

Working

74

47

74 ≈ 70

47 ≈ 50

70 + 50 = 120

The estimated sum is 120.

Thinking

Step 1: Identify the digit being rounded and underline the critical digit for each number.

Step 2: Increase the digit being rounded by 1 if the critical digit is greater or equal to 5. Keep the digit being rounded the same if the critical digit is less than 5. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the sum of the rounded numbers.

Visual support

b. 997 ÷ 456 (nearest 100)

Working

997

456

997 ≈ 1000

456 ≈ 500

1000 ÷ 500 = 2

The estimated result is 2.

Thinking

Step 1: Identify the digit being rounded and underline the critical digit for each number.

Step 2: Increase the digit being rounded by 1 if the critical digit is greater or equal to 5. If the digit being rounded is 9, increasing it by 1 is 10, so regroup to the next place value. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the quotient of the rounded numbers.

Student practice

Estimate each calculation by first rounding both numbers to the specified place value.

a. 38 + 83 (nearest 10)

b. 133 − 67 (nearest 10)

c. 1182 ÷ 197 (nearest 100)

d. 8359 × 2925 (nearest 1000)

WE 3 Estimating using lead digit rounding

Estimate each calculation by rounding each number using lead digit rounding.

a. 59 × 4

Working

59

4

59 ≈ 60

60 × 4 = 240

The estimated product is 240.

Thinking

Step 1: Identify the digit being rounded (the largest place value) and underline the critical digit for each number.

Step 2: Increase the digit being rounded by 1 if the critical digit is greater or equal to 5. If the number only consists of a digit in the ones place, leave it unrounded. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the product of the rounded numbers.

Visual support

b. $218 \div 48$

Working

218

48

$218 \approx 200$

$48 \approx 50$

$200 \div 50 = 4$

The estimated result is 4.

Thinking

Step 1: Identify the digit being rounded (the largest place value) and underline the critical digit for each number.

Step 2: Increase the digit being rounded by 1 if the critical digit is greater or equal to 5. Keep the digit being rounded the same if the critical digit is less than 5. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the quotient of the rounded numbers.

Student practice

Estimate each calculation by rounding each number using lead digit rounding.

a. 32×7 **b.** $482 \div 5$ **c.** $340 + 150$ **d.** 9643×3

1G Activities and questions

STARTER TASKS

Image: Andrey_Popov/Shutterstock.com

Odd spot

Australia has one of the highest rates of home ownership in the world. This is surprising given the incredible rise in house prices across the country. It's estimated that during 2020 the median house price in Melbourne increased by $3600 per week. Over the same period, the median house price in Sydney increased by $4200 per week.

When rounded to the nearest $1000, what is the difference between the increase in the median house price per week between Melbourne and Sydney.

A. $400 **B.** $0

Puzzle

Image: Christine Glade/Shutterstock.com

a) Estimate how many jelly beans are in the jar using at least two different strategies.

b) Explain which strategy might be more accurate.

Understanding worksheet

1. Round the number to the given place value.

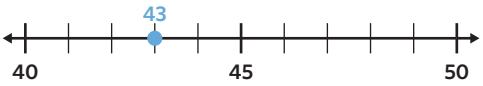
Example

Round to the nearest ten.

78 ≈ 80

a. Round to the nearest ten.

43 ≈ ☐

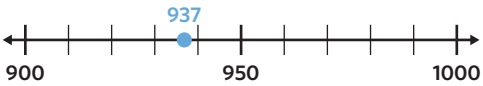


b. Round to the nearest hundred.

160 ≈ ☐

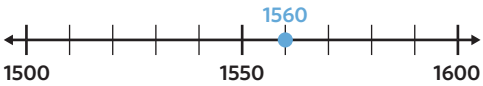
c. Round to the nearest hundred.

937 ≈ ☐



d. Round to the nearest hundred.

1560 ≈ ☐



2. The critical digit for each number is underlined. Circle the digit being rounded and to round each number as specified.

Example

Round to the nearest ten.

4 (7) 6 ≈ 480

a. Round to the nearest ten.

7 1 3 ≈ ☐

b. Round to the nearest 100.

3 6 2 ≈ ☐

c. Round to the nearest 1000.

7 5 9 9 ≈ ☐

d. Round to the nearest hundred.

9 9 5 0 ≈ ☐

3. Fill in the blanks by using the words provided.

zeros | underestimate | lead | rounding | estimates | overestimate

☐ are approximate calculations in which ☐ is used to make the numbers easier to work with. ☐ digit rounding rounds the first digit of a number and replaces all the digits to the right of the digit being rounded with ☐.

An ☐ is an estimate that is greater than the exact calculation. An ☐ is an estimate that is less than the exact calculation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 **4.** Round each number to the specified place value.

a. 78 to the nearest 10
b. 826 to the nearest 10
c. 697 to the nearest 100
d. 8492 to the nearest 1000
e. 968 to the nearest 100
f. 486 to the nearest 1000
g. 95 000 to the nearest 1000
h. 9995 to the nearest 10

WE2 **5.** Estimate each calculation by rounding both numbers to the specified place value.

a. $94 + 47$ (nearest 10)
b. $204 - 69$ (nearest 10)
c. 195×9 (nearest 10)
d. $268 + 175$ (nearest 100)
e. $1237 \div 649$ (nearest 100)
f. $1001 + 999$ (nearest 100)
g. $1666 - 777$ (nearest 1000)
h. 950×9950 (nearest 1000)

WE3 **6.** Estimate each calculation by rounding each number using lead digit rounding.

a. $19 \div 4$
b. 53×6
c. $236 + 43$
d. $186 \div 14$
e. $4820 - 921$
f. 199×28
g. $2399 - 1487$
h. $9999 + 999$

7. For each calculation:

i) Estimate the calculation by rounding each number using lead digit rounding.
ii) Find the exact answer, using a calculator if necessary.
iii) State whether the estimate is an overestimate or underestimate.

a. 43×7
b. $97 + 36$
c. $231 - 125$
d. 654×15
e. $580 \div 10$
f. $999 - 779$
g. $1199 + 10\,999$
h. $39 \times 12 + 592$

8. Without completing any of the calculations, predict whether estimate A or estimate B is the most accurate estimate for the exact calculation.

	Exact calculation	Estimate A	Estimate B
a.	67×5	70×5	70×10
b.	$465 + 270$	$500 + 300$	$470 + 270$
c.	$468 - 87$	$470 - 90$	$500 - 100$
d.	$966 \div 7$	$970 \div 10$	$1000 \div 7$
e.	439×65	400×70	440×70
f.	$3982 + 9573$	$4000 + 10\,000$	$4000 + 9600$
g.	$8793 - 7123$	$8800 - 7100$	$9000 - 7000$
h.	$2325 \div 25$	$2000 \div 30$	$2330 \div 30$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Over a busy weekend, 1137 sausages were sold at a Bunnings sausage sizzle. To the nearest thousand, how many sausages were sold?

10. Betty's address is 143 Bogabilla Road. 100 Bogabilla Road joins Toomelah Road, while 200 Bogabilla Road joins Dandenong Road. Does Betty live closer to Toomelah Road or Dandenong Road?

11. Richard and Julia bought a new two bedroom apartment. To furnish their apartment, they spent $935 in the first week, $5900 in the second week and $15 320 in the third week. Approximate how much Richard and Julia spent in total using lead digit rounding.

12. The year 7 and 8 cohorts at Bridge Academy have a hockey excursion together. Use lead digit rounding to find how many buses are required to transport all 393 students if 48 students fit in a bus.

13. What is the least and what is the greatest number of people that could live in Salisbury if its population is 37 000 when rounded to the nearest 1000?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Claudia and Jimmy are estimating the number of people that attended a cricket match at the Gabba. Claudia counted 904 people entering through gate 7 and Jimmy counted 2396 people entering through gate 2. They assume that approximately the same number of people enter each of the 10 gates in total.
    a. If Claudia rounded the number of people that entered at gate 7 to the nearest 100, how many people does Claudia think attended the match?
    b. If Jimmy rounded the number of people that entered at gate 2 to the nearest 1000 how many people does Jimmy think attended the match?
    c. The average number of entrees per gate is the number exactly halfway between 904 and 2396. Whose estimate is more accurate? Justify your answer.
    d. Propose another way that Jimmy and Claudia could improve the accuracy of their estimations.

15. The Doboy Ward is made up of four different regions. Each region is divided into smaller council sub-regions. The following table shows the number of votes two candidates received in an election ballot.

	Regions			
	Carina	**Hemmant**	**Murarrie**	**Tingalpa**
Candidate A	4480	9159	12 490	1485
Candidate B	3927	8756	12 520	1467

    a. Estimate how many votes candidate B received by rounding the votes from each region to the nearest 100.
    b. There are 3 equally sized council sub-regions in Carina. Round to the nearest 100 votes to approximate the average number of votes candidate A received from each council sub-region.
    c. If the number of votes in each area is rounded to the nearest thousand, how many votes did the more popular candidate win by?
    d. Candidate B suggests that when totalling the votes, the number of votes from each area should be rounded to the nearest thousand. Explain why rounding might not have been appropriate when counting votes in this election.

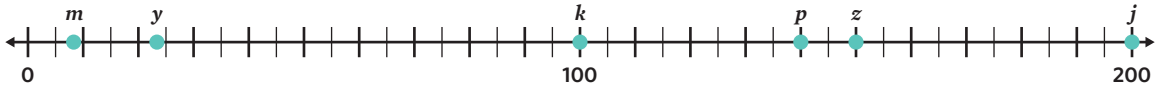
Extra spicy

16. A 160 cm person and a tree are shown to scale in the following diagram. Approximately how tall is the tree in metres?

17. Which value is closest to $\frac{5632 \times 95}{923}$?

    A. 50 B. 60 C. 500 D. 600

18. Each point on the number line represents a number. Which point approximately represents the product of the values of m and y?



    A. k B. j C. p D. z

19. A ship at a Sydney dock can transport 77 000 cubic metres of goods. A standard shipping container can hold 44 cubic metres of goods. A forklift driver can safely remove 5 shipping containers from a ship each hour. Use lead digit rounding to estimate how long it will take 4 drivers to empty a full ship.

Remember this?

20. Freya covers half of a star.

How many points does the star have?

A. 9 **B.** 10 **C.** 11 **D.** 12

21. Which number shows 4 thousands, 7 tens and 12 ones?

A. 4000.712 **B.** 4071.2 **C.** 4082 **D.** 4712

22. Mike calculated how much money he spent every week over a four week period on his new home gym.

Week 1: $6455

Week 2: $1092

Week 3: $567

Week 4: $13 299

How much did Mike spend in total on his home gym?

Chapter 1 extended application

1. Josiah and Leonard decide to sell food that they bake at the local Sunday market for 4 consecutive weeks. Josiah makes brownies and croissants, and Leonard makes doughnuts, cinnamon scrolls and cupcakes. They agree that they should both make the same number of baked goods.

   a. Josiah bakes 2 batches of 24 brownies and 1 batch of 18 croissants. Leonard makes 3 batches of 8 doughnuts and 2 batches of 16 cinnamon scrolls. How many cupcakes does Leonard need to make?

   b. If they bake the same amount of food for each of the 4 weeks, how many baked goods in total will they have made?

   c. Their food sold out really quickly in the first week, so, for the second week, they increased the food they made by the following amounts.

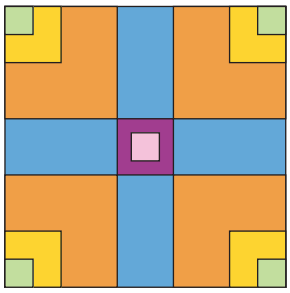
Food item	Week 2 increase
Brownies	+1 half-batch
Croissants	Double week 1
Doughnuts	+1 full batch
Cinnamon scrolls	+1 full batch
Cupcakes	+6

   How many baked goods in total did they make for their second week?

   d. For week 3, they decide to make the same amount of food as week 2. However, the weather is really bad and not many people attend the market. As a result, they made three times as much food as they sold. How many baked goods do they sell in total?

   e. How much food should they make in the 4^{th} week? Can you think of a strategy they should use to determine the right amount of food to make?

2. Sonia and Phillippe design a game to improve their throwing accuracy. They design a target where each colour hit corresponds to a certain number of points.



Colour	Points
Green	1000
Yellow	100
Orange	10
Blue	1
Purple	Score divided by 15
Pink	Score multiplied by 15

They each get 10 throws. For the first 9 throws, the purple and pink squares don't count. On the 10^{th} throw, only the purple and pink squares count. All other colours result in 0 points.

   a. In her first 9 throws, Sonia hits 1 green, 5 yellow, and 3 orange. How many points did she score?

   b. In his first 9 throws, Phillippe hits 1 yellow, 2 orange and 6 blue. How many points did he score?

   c. For her final throw, Sonia hits the purple square. What was her final score?

   d. For his final throw, Phillippe hits the pink square. What was his final score?

   e. What strategy would you use to get the most points?

3. Mark is a freelance plumber who charges his own rates for clients. For each client, he charges a fixed fee of $25 as well as $42 for each hour that the job takes.

   a. On a particular day, he has 2 clients. For the first client, the job takes 2 hours. For the second client, the job takes 3 hours, but he charges them half the hourly rate because they are family friends. How much money does Mark make that day?

   b. The following table shows the amount of money Mark makes on each day of the working week.

Day	Money made
Monday	$243
Tuesday	$109
Wednesday	$151
Thursday	$176
Friday	$302

   Round the amount of money earned each day to the nearest $10 to estimate how much money Mark made during the week.

   c. Use your estimate in part **b** to estimate how much money Mark makes in a year to the nearest $1000.

   Note: There are 52 weeks in a year.

   d. Think of some reasons why your answer in part **c** will not be an accurate estimate for how much money Mark makes per year.

Chapter 1 review

Multiple choice

1. Which option shows 7800 ÷ 100 correctly? 1E

A.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		7	8	0	0	÷ 100 =					7	8

B.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		7	8	0	0	÷ 100 =				7	8	0

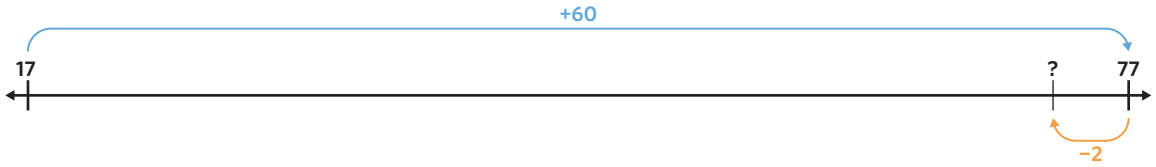
C.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		7	8	0	0	÷ 100 =		7	8	0	0	0

D.

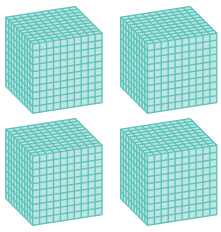
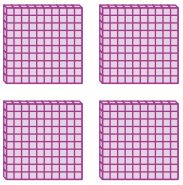
Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		7	8	0	0	÷ 100 =	7	8	0	0	0	0

2. Use the number line to solve 17 + 58. 1C



A. 65 B. 75 C. 79 D. 85

3. What is the value of the number shown in the table? 1B

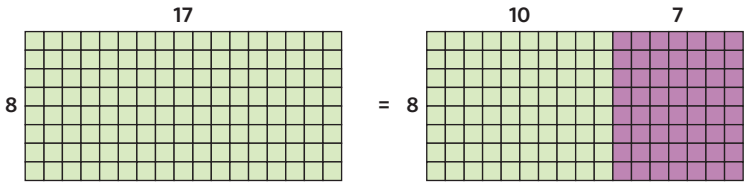
1000	100	10	1
			

A. 1246 B. 4412 C. 4502 D. 44 012

4. Which calculation should be done first to evaluate 3 + 4 × 3 − 8? 1F

A. 3 + 4 B. 4 × 3 C. 3 − 8 D. 3 × 3

5. Which of the following correctly shows the distributive strategy applied to the calculation 8 × 17? 1D



A. 8 × 17 B. 10 × (8 + 7) C. 7 × (10 + 8) D. 8 × (10 + 7)

Fluency

6. State whether each number sentence is true or false. 1A

a. 11 + (5 + 4) = (11 + 5) + 4
b. 87 − 7 + 41 = 87 − 47 + 1
c. (24 + 16) − 12 = 24 + (16 − 12)
d. 300 ÷ 25 = 12 × 25 ÷ 25

7. Write the value of the highlighted digit. 1B

a. 17**2**6 b. 12 **7**20 c. 8 **8**43 711 d. 444 4**4**4 456

8. Place the values in the described order. 1B

a. Ascending: 114, 1004, 97, 899
b. Descending: 4554, 20 026, 20 030, 20 100
c. Ascending: 114 231, 141 540, 108 992, 108 929
d. Descending: 54 831 275, 54 831 257, 54 813 572, 54 831 527

9. Evaluate each addition using the given mental strategy. 1C

a. 23 + 42 (partitioning strategy)
b. 58 + 73 (compensation strategy)
c. 164 + 632 (vertical algorithm)
d. 289 + 791 (vertical algorithm)

10. Evaluate each subtraction using the given mental strategy. 1C

a. 75 − 31 (partitioning strategy)
b. 296 − 89 (compensation strategy)
c. 662 − 540 (vertical algorithm)
d. 614 − 378 (vertical algorithm)

11. Evaluate each multiplication using the given mental strategy. 1D

a. 4 × 34 (distributive strategy)
b. 8 × 59 (vertical algorithm)
c. 74 × 5 (halves and doubles strategy)
d. 431 × 19 (vertical algorithm)

12. Evaluate each division. 1E

a. 112 ÷ 6 **b.** 165 ÷ 8 **c.** 315 ÷ 5 **d.** 419 ÷ 16

13. Identify which operation (addition, subtraction, multiplication, division) is completed first. 1F

a. 2 + 5 × 7
b. 1 − 8 × 12 + 7
c. 2 × [15 × (5 − 3)]
d. [2 × 8 + (36 ÷ 3) − (4 × 3)] × 74

14. Using the order of operations, evaluate each expression. 1F

a. 12 ÷ 4 + 19
b. 85 ÷ (17 − 12) × 2
c. 8 × [(4 + 8) ÷ 1]
d. [7 × (18 ÷ 6 × 3 + 4)]

15. Round each number to the specified place value. 1G

a. 158 to the nearest 10
b. 2457 to the nearest 100
c. 456 to the nearest 1000
d. 8985 to the nearest 10

16. Estimate each calculation by rounding each number using lead digit rounding. 1G

a. 39 ÷ 8 **b.** 256 ÷ 34 **c.** 199 × 28 **d.** 9999 + 999

Problem solving

17. Nick is aiming to run 42 kilometres in a single week. He ran 5 kilometres on each of the first 6 days. How many kilometres does he need to run on the last day? 1A

18. Cindy is buying banh mis for her friends at work. Banh mis cost $10 each and Cindy brought $115 to the office. If she has 11 coworkers, does she have enough money to buy banh mis for all of them, as well as herself? 1B

19. Zoe just uploaded a photo on Instagram and gained 245 followers. How many followers did she have before uploading the photo if she now has 2123 followers? 1C

20. Giannis Antetokounmpo won MVP in the 2018–19 NBA season, averaging 13 rebounds per game. How many rebounds did he secure in total if he played 72 games? 1D

21. James Harden came second in MVP voting in the 2018–19 NBA season. He attempted 1014 three-point shots in 78 games. How many three-point shots did he attempt, on average, per game? 1E

22. Paul George came third in MVP voting in the 2018–19 NBA season. He scored 28 points per game in 77 games. How many more points would he have needed to have scored 2500 points in total? 1F

23. It took Harry Styles 96 minutes to complete all the questions in lesson 1D. Round to the nearest 10 minutes to estimate how long it took Harry Styles, on average, to complete each of the 25 questions. 1G

Reasoning

24. The number of times some names are used in this textbook is shown in the following table.

Names	Number of times used
Duncan	46
Hannah	34
Rex	15
Ben	20
Jeremy	18
Anthony	53

a. List the names in descending order of the number of times they are used in this textbook.

b. How many more times would the name 'Hannah' have to be used to appear the same number of times as 'Ben' and 'Jeremy' combined?

c. Find the total number of times the names 'Duncan', 'Anthony' and 'Rex' are used in this textbook.

d. Use lead digit rounding to estimate the total number of times all of the names are used in this textbook.

e. Do you think it is important to include names that are uncommon?

25. The price of a house in Fitzroy is $1 305 000 for two bedrooms, $1 575 000 for three bedrooms and $2 200 000 for four bedrooms.

a. Amber, Kem, Chris and Olivia are looking to buy a four bedroom house in Fitzroy by each paying an equal amount of the cost. How much would they each need to pay?

b. Jack and Dani together save $124 000 per year. Would they be able to afford a three bedroom house in Fitzroy with the money they can save in 13 years?

c. Milly and Liam together save $96 000 per year. It will take them exactly 9 more years to save for a two bedroom house in Fitzroy. How much money do they already have saved?

d. How many years of working do you think is fair for someone to save enough money to buy a house?

02

Number theory

Number and Algebra

Research summary

2A Factors, multiples, and divisibility

2B Index notation

2C Prime and composite numbers

2D Prime factorisation

2E Lowest common multiples and highest common factors *(Revision)*

2F Perfect squares and square roots

2G Introduction to integers *(Revision)*

2H Adding and subtracting positive integers

2I Adding and subtracting negative integers

Chapter 2 extended application

Chapter 2 review

Image: Marish/Shutterstock.com

Research summary – Number theory

Big ideas

Number theory

Number theory is a branch of mathematics that explores **natural numbers** (1, 2, 3, 4, 5, ...) and **integers** (..., −3, −2, −1, 0, 1, 2, 3, ...). It has also been known as *arithmetic*. Young children use number theory when they begin to count using natural numbers and then move onto discovering patterns and connections between these numbers. This chapter examines several components of number theory and how they relate to each other. These include factors and multiples, prime numbers, prime factorisation, square numbers, and positive and negative integers.

Prime number theory

Prime numbers are one of the most important elements of multiplicative structures because they are the building blocks of natural numbers. This means that every positive whole number, including composite numbers, can be broken up into a set of prime factors and each set of factors will be unique to that number. This idea is important as it connects to number fluency and understanding the multiplicative nature of numbers. It aids facility in multiplication and division, finding factors and multiples, simplifying expressions, powers, roots, and fractions.

Factors and multiples

In order to better understand the structure and uniqueness of natural numbers, we can break them down into the product of their smaller components. This is called factorising or *'finding the factors'* of a number. When we break a number down into the products of its smallest components, this is called prime factorisation.

Factorising can be completed by repeatedly dividing whole numbers that give whole number results. Multiples on the other hand can be found through multiplying a number by different whole numbers.

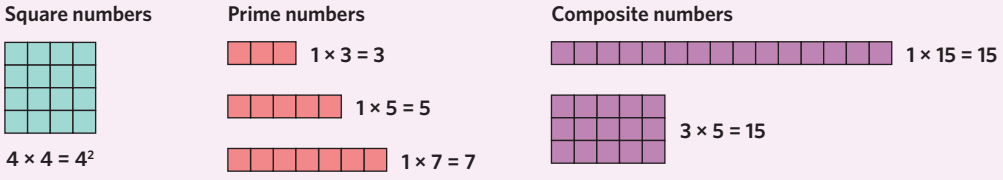
Negative numbers

Negative numbers are integers that extend the natural numbers below zero. It is common for difficulties to arise when working with negative numbers because they move beyond the perceptual understanding of positive whole numbers. It is more difficult to count with negative numbers as we can't physically see them as a collection. This can create problems when using them in arithmetic calculations. For this reason, it is critical to use conceptual models and concrete manipulatives when beginning the topic of integers and find ways of helping to visualise the abstraction of negative numbers.

Visual representations

Arrays

Arrays are versatile and can be used in a variety of ways. In this chapter, we will be using them to help identify square numbers, prime and composite numbers, and factors and multiples.



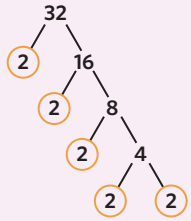
Divisibility tests

When a number is 'divisible' by another it means that when it is divided by that number the solution will be a whole number with no remainder. We can check whether a number is divisible by another using a divisibility test. There are several divisibility tests that can be performed which can often make it easier to determine some of the factors of a number.

The number is divisible by:	Conditions of divisibility	Divisible	Not Divisible
2	if its last digit is even.	6748	6745
3	if the sum of its digits is divisible by 3.	612	613
4	if its last 2 digits form a number that is divisible by 4.	10 516	10 522
5	if its last digit is 0 or 5.	5435	4438
6	if it is even and if the sum of its digits is divisible by 3.	72	140
7	*There is no simple test.*		
8	if its last 3 digits form a number that is divisible by 8.	1856	1862
9	if the sum of its digits is divisible by 9.	5697	18 775
10	if the last digit is 0.	18 770	18 775

Factor trees

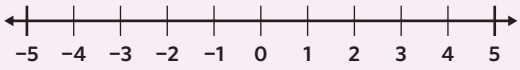
Factor trees are useful when trying to find the prime factors of any number. We can use them to find common factors between two or more numbers.



$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

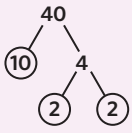
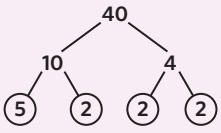
Number lines

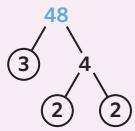
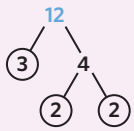
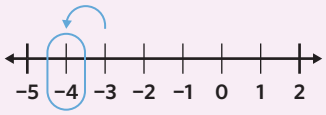
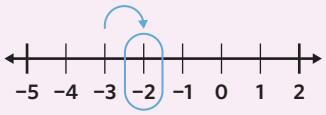
The ordinal nature of number lines allows numbers to be seen along a continuum so that we can more easily arrange them in order of their size. In this chapter we will be introduced to integers, and use number lines that extend in both the positive and negative directions. We will use number lines to help investigate the magnitude of numbers as well as perform operations with integers. Number lines are also an effective way to show multiples and to find the lowest common multiple of two different numbers.



Number line including negative numbers

Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students confuse factors and multiples.	Factors of 21: 21, 42, 63, 84	Factors of 21: 1, 3, 7, 21	2A
Students forget to include the number as its first multiple.	The first 5 multiples of 9: 18, 27, 36, 45, 54.	The first 5 multiples of 9: 9, 18, 27, 36, 45.	2A
Students do not consider the commutative law when finding factors and count factors more than once.	Factor pairs of 16: 1×16 2×8 4×4 8×2 16×1 16 has 10 factors.	Factor pairs of 16: 1×16 2×8 4×4 16 has 5 factors.	2A
Students multiply the base by the power.	$4^3 = 4 \times 3 = 12$	$4^3 = 4 \times 4 \times 4 = 64$	2B
Students add or subtract the base value and the index when completing calculations with indices.	$4^2 + 2^2 = 6^4$	$4^2 + 2^2 = 16 + 4 = 20$	2B
Students think that 1 is a prime number.	1 is prime because it has no factors other than 1 and itself.	1 is not a prime because it does not have two unique factors.	2C
Students think that some composite numbers are prime because they are not familiar working with them	51 is a prime number because it only has two factors itself and 1.	51 is a composite number because its factors are: 1, 3, 17, 51.	2C
Students think that all prime numbers are odd numbers.	All prime numbers are odd. E.g. 3, 5, 7, 11.	There is one prime number that is even: 2.	2C
Students do not complete the full prime factorisation.			2D
Students think that 'large' numbers must have a 'large' number of prime factors.	667 written as a product of its prime factors cannot be 23×29 because it is a large number and should be made up of more prime factors.	667 written as a product of its prime factors is 23×29, because 23 and 29 are prime numbers.	2D

	Incorrect	Correct	Exercise
Students multiply all the numbers shown on a factor tree to find the top composite number.			2D
When finding the lowest common multiple (LCM), students find the lowest number that can divide both numbers.	The LCM of 4 and 10 is 2.	The LCM of 4 and 10 is 20.	2E
Students multiply when trying to find the highest common factor (HCF) between two numbers.	The highest common factor of 12 and 30 is:$12 \times 30 = 360$	The highest common factor of 12 and 30 is: 12: 1, 2, 3, 4, 6, 12 30: 1, 2, 3, 5, 6, 10, 15, 30	2E
Students multiply when trying to find the lowest common multiple (LCM) between two numbers.	The lowest common multiple of 6 and 9 is: $6 \times 9 = 54$	The lowest common multiple of 6 and 9 is: 6: 6, 12, 18, 24. 9: 9, 18, 27	2E
Students calculate squares by doubling the base value	$5^2 = 5 \times 2 = 10$	$5^2 = 5 \times 5 = 25$	2F
Students halve the number in a square root calculation.	$\sqrt{36} = 18$	$\sqrt{36} = 6$	2F
Students see integers as two separate 'objects'. The sign ($-$ and $+$) and the number.	-12 means subtract 12.	-12 means negative 12.	2G
Students order a set of negative numbers by looking at the digit only and ignore the sign.	$-5, -1, -2, -4, -3$ are placed in ascending order as $-1, -2, -3, -4, -5$.	$-5, -1, -2, -4, -3$ are placed in ascending order as $-5, -4, -3, -2, -1$.	2G
Students only compare the whole number and ignore the sign.	$-120 > 30$	$-120 < 30$	2G
Students move in the wrong direction on the number line when adding or subtracting positive integers.	$-3 + 1 = -4$ 	$-3 + 1 = -2$ 	2H
Students disregard the negative signs when performing addition calculations.	$-2 + 3 = -5$	$-2 + 3 = 1$	2H
Students disregard the negative signs when performing subtraction calculations.	$2 - (-2) = 0$	$2 - (-2) = 4$	2I
Students think that 'two negatives must equal a positive'.	$-10 - (-5) = 15$	$-10 - (-5) = -5$	2I

Additional reading and resources

- NRICH: Adding and Subtracting Positive and Negative Numbers
- Prime Numbers as Building Blocks – Euclid's greatest discovery
- Factors, multiples and primes: https://www.transum.org/Maths/Activity/Prime.asp
- Factorisation forest: https://mrnussbaum.com/factorization-forest-online-game
- Pyramid Math: https://www.mathnook.com/math/pyramidmath.html

2A Factors, multiples, and divisibility

Factors, multiples, and divisibility are a large component of number theory. They allow us to better understand the relationship between multiplication and division. Below are some examples where factors, multiples, and divisibility can be applied.

- My classroom has 24 desks. How can they be arranged so that each row has the same number of desks?
- I collect 7 mushrooms each hour for a winter soup. If I collect mushrooms for 5 hours, how many will I have to put in my soup at the end of each hour?
- I have 41 pieces of Shane Warne memorabilia. Can I share all of them equally between 8 people in my family?

Learning intentions

Students will be able to:

- find all of the factors of a number
- list the multiples of a number
- use the divisibility tests to determine whether one number is divisible by another number.

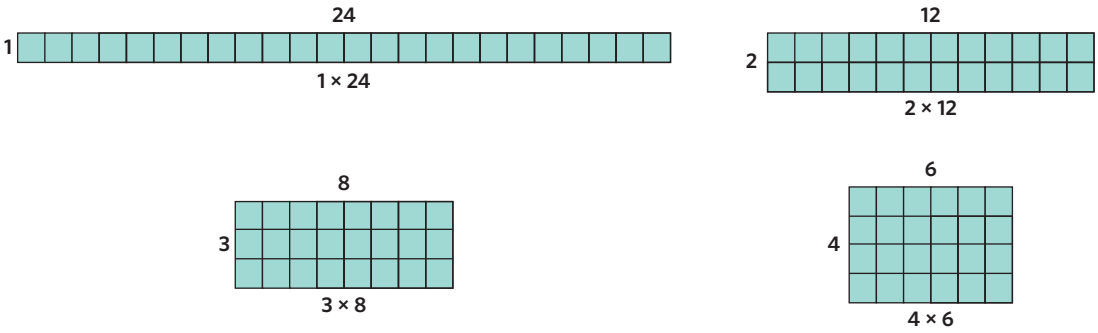
Key terms and definitions

- **Factors** are numbers that divide into another number with no remainder. E.g. factors of 6: 1, 2, 3, 6.
- **Factor pairs** are two numbers that multiply together to give another number. E.g. 27 is a factor pair of 14.
- A **multiple** is the product that we get when one number is multiplied by another number E.g. 12 is a multiple of 4 because $4 \times 3 = 12$.
- A number is **divisible** by another number when its quotient is a whole number. E.g. 18 is divisible by 6 because $18 \div 6 = 3$.

Key ideas

1 The factors of a number can be shown using a grid array.

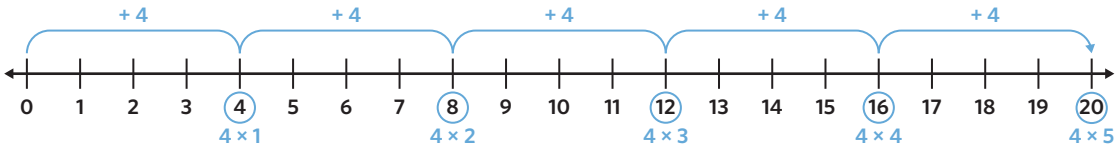
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.



Note: The commutative law shows that the order the factors are multiplied will give the same product.

2 The multiples of a number can be shown on a number line.

Multiples of 4: 4, 8, 12, 16, 20...



3 We can determine whether one number is divisible by another number using a range of divisibility tests. Using the tests means we can find the factors of numbers.

The number is divisible by:	Conditions of divisibility	Divisible	Not divisible
2	if its last digit is even.	6748	6745
3	if the sum of its digits is divisible by 3.	612	613
4	if its last 2 digits form a number that is divisible by 4.	10 516	10 522
5	if its last digit is 0 or 5.	5435	4438
6	if it is even and if the sum of its digits is divisible by 3.	72	140
7	*There is no simple test.*		
8	if its last 3 digits form a number that is divisible by 8.	1856	1862
9	if the sum of its digits is divisible by 9.	5697	5699
10	if its last digit is 0.	18 770	18 775

Worked examples

WE 1 Finding factors

List all the factors of each number in ascending order.

a. 21

Working

$1 \times 21 = 21$

$3 \times 7 = 21$

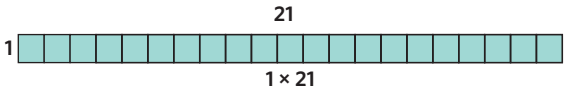
1, 3, 7, 21

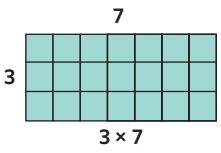
Thinking

Step 1: List the factor pairs as multiplications.

Step 2: List the numbers that form each factor pair in ascending order.

Visual support





b. 60

Working

$1 \times 60 = 60$

$2 \times 30 = 60$

$3 \times 20 = 60$

$4 \times 15 = 60$

$5 \times 12 = 60$

$6 \times 10 = 60$

1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Thinking

Step 1: List the factor pairs as multiplications. Stop once the next factor pair includes factors that have already been identified.

Step 2: List the numbers that form each factor pair in ascending order.

Student practice

List all the factors of each number in ascending order.

a. 14 **b.** 33 **c.** 100 **d.** 56

WE 2 Finding multiples

List the first five multiples of each number.

a. 7

Working

1st multiple: $1 \times 7 = 7$

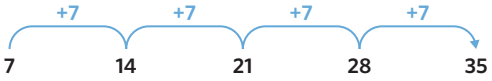
2nd multiple: $2 \times 7 = 14$

3rd multiple: $3 \times 7 = 21$

4th multiple: $4 \times 7 = 28$

5th multiple: $5 \times 7 = 35$

7, 14, 21, 28, 35



Thinking

Method 1

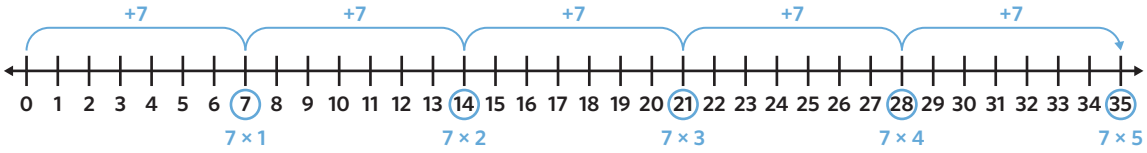
Step 1: Multiply 7 by 1, 2, 3, 4, and 5 to find the multiples.

Step 2: List each multiple in ascending order.

Method 2

Use repeated addition to list the multiples.

Visual support



b. 24

Working

1st multiple: $1 \times 24 = 24$

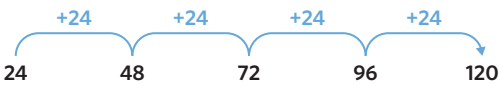
2nd multiple: $2 \times 24 = 48$

3rd multiple: $3 \times 24 = 72$

4th multiple: $4 \times 24 = 96$

5th multiple: $5 \times 24 = 120$

24, 48, 72, 96, 120



Thinking

Method 1

Step 1: Multiply 24 by 1 to 5 to find the multiples in each position.

Step 2: List each multiple in ascending order.

Method 2

Use repeated addition to list the multiples.

Student practice

List the first five multiples of each number.

a. 6 **b.** 12 **c.** 90 **d.** 17

WE 3 Testing divisibility

Complete the divisibility tests for each number and place ticks or crosses in the table.

a.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
5235								

Working

2, 4, 6, 8, 10 ✗

5 + 2 + 3 + 5 = 15 ✓

5235 ✓

5 + 2 + 3 + 5 = 15 ✗

Thinking

Step 1: Identify any numbers that can easily be ruled out. 5235 is an odd number so it is not divisible by any even numbers.

Step 2: Use the divisibility tests for the remaining numbers.

3: If the sum of the digits is divisible by 3.

5: If the last digit is 0 or 5.

9: If the sum of the digits is divisible by 9.

Step 3: Complete the table.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
5235	✗	✓	✗	✓	✗	✗	✗	✗

b.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
80 752								

Working

5, 10 ✗

80 572 ✓

8 + 0 + 5 + 7 + 2 = 22 ✗

6, 9 ✗

80 752 ✓

80 752 ✓

Thinking

Step 1: Identify any numbers that can easily be ruled out. The last digit is not a 0 or 5 so it is not divisible by 5 or 10.

Step 2: Use the divisibility tests for the remaining numbers.

2: If the last digit is even.

3: If the sum of the digits is divisible by 3.

Note: If a number is not divisible by 3, it is also not divisible by 6 or 9.

4: If the last two digits form a number that is divisible by 4.

8: If the last 3 digits form a number that is divisible by 8.

Step 3: Complete the table.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
80 752	✓	✗	✓	✗	✗	✓	✗	✗

Student practice

Use the divisibility tests to complete the table by placing a tick or a cross.

	Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
a.	6445								
b.	5124								
c.	75 990								
d.	436 852								

2A Activities and questions

STARTER TASKS

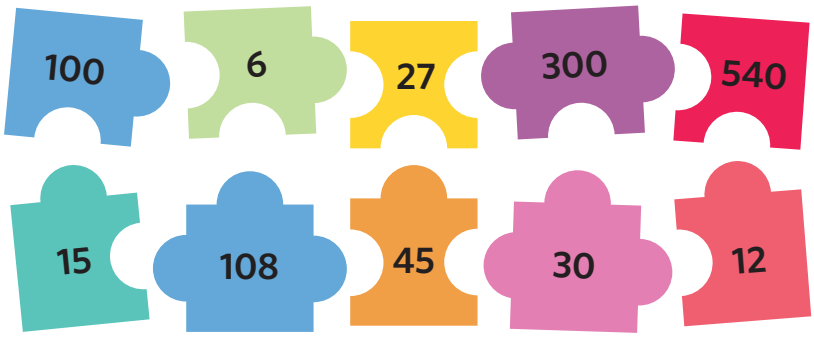
Odd spot

Tyson McMillan is the co-owner of Lumber Punks. Lumber Punks is just like a bowling alley, except patrons don't bowl bowling balls, they hurl small axes. Axe throwing has become so popular that Tyson has opened locations in Brisbane, Perth, and Melbourne. Players receive 8 points if the axe lands in the outer ring of the target. The score a player receives increases by multiples of 8 for each ring closer towards the bullseye. What are all the possible scores a player can receive for a throw that lands on a target that is made up of four rings?

A. 8, 16, 24, 32 **B.** 1, 2, 4, 8

Image: Vjacheslav Shishlov/ Shutterstock.com

Puzzle



a) Use every number exactly once and place it in its own cell in the below table.

b) There are two numbers missing in order to complete the table. Propose a different number to complete each box.

c) Is it possible to find a number that could be used to fill both of the missing boxes? If so, find it.

	Factors of 60	Multiples of 9	Multiples of 5
Odd Numbers			
Multiples of 3			
Even numbers			
Numbers divisible by 6			

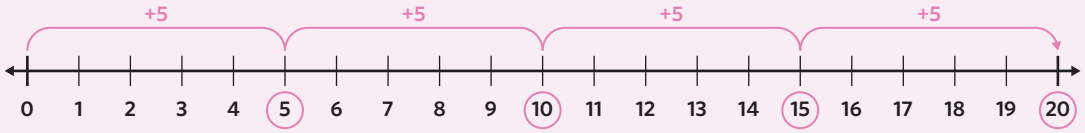
*There are multiple possible solutions.

Understanding worksheet

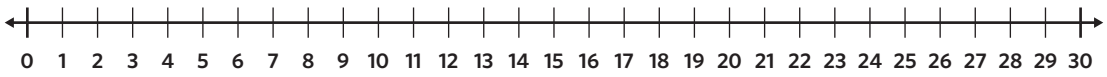
1. Perform the 'jumps' and circle the multiples of each number on the number line.

Example

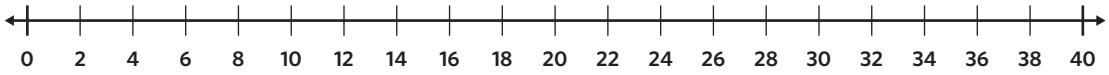
Multiples of 5.



a. Multiples of 6

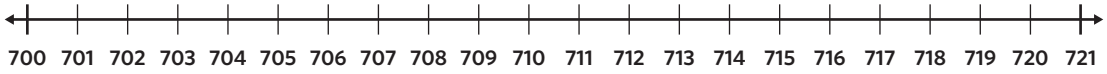


b. Multiples of 8



c. Multiples of 4

d. Multiples of 7



2. Circle the values in the table that the given number is divisible by.

Note: The divisibility tests are shown in Key idea 3.

Example

2420 is divisible by...				
(2)	3	(4)	(5)	6

a.

1650 is divisible by...				
2	3	4	5	6

b.

8748 is divisible by...				
3	4	5	9	10

c.

10 432 is divisible by...				
2	3	4	5	10

d.

51 840 is divisible by...				
3	4	6	8	10

3. Fill in the blanks by using the words provided.

multiples | divisibility | division | factors | divisible

_____ are numbers that divide into another number with no remainder. We can find the _____ of a number using repeated addition or multiplication. Factors and multiples are different, but are connected because they both relate to multiplication and _____.

_____ tests can be used to determine whether any number is _____ by another whole number.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (b,c,d), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7 (d,e,f), 8 (e,f,g,h)

WE1 **4.** List all the factors of each number in ascending order.

a. 8 **b.** 25 **c.** 22 **d.** 31
e. 45 **f.** 64 **g.** 70 **h.** 144

5. List the first five multiples of each number.

WE2 **a.** 10 **b.** 8 **c.** 9 **d.** 11
e. 25 **f.** 120 **g.** 19 **h.** 91

6. What number am I?

a. I am less than 20, I have four factors, and I am a multiple of 4.
b. I am less than 30, I have eight factors, and I am a multiple of 4.
c. I am less than 30, I have six factors, and I am a multiple of 9.
d. I am less than 50, I have three factors, and I am a multiple of 7.
e. I am less than 100, I have six factors, and one of them is 21.
f. I have ten factors, and I am a multiple of 6.

WE3 **7.** Use the divisibility tests to complete the table by placing a tick or a cross.

	Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
a.	6785								
b.	16 740								
c.	39 123								
d.	58 617								
e.	853 104								
f.	2 086 997								

8. State whether each calculation's solution is a whole number or a whole number with a remainder and explain why without performing the calculation.

a. 9642 ÷ 2 **b.** 10 493 ÷ 5 **c.** 408 920 ÷ 10 **d.** 89 748 ÷ 4
e. 91 793 ÷ 3 **f.** 806 202 ÷ 6 **g.** 34 376 ÷ 8 **h.** 1 206 118 ÷ 9

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Bailey is a greyhound and is competing in the Como Park greyhound race. How far has Bailey run at the completion of the first, second, third and fourth lap of a 600 m track?

10. There are 56 students in the Skinfield High's choir. The music teacher wants to arrange the students into a maximum of 7 rows. How many possible sets of dimensions can the music teacher use to arrange the choir if each row needs to have an equal number of students?

11. Carmen is training for an open water swimming race. In preparation, she completes laps between two buoys in the bay and each lap is 250 m. If her GPS watch lists the cumulative distance at the end of each lap, what are all the distances her watch displays after eight laps?

12. Mary wants to divide all her savings of $856 784 equally between her four children so that each child receives a whole dollar amount. Without performing any calculation, state whether this is possible and why.

13. Mel is designing a rectangular enclosure for reptiles with an area of 24 m^2 at the Territory Wildlife Park. She also wants the enclosure to have the smallest possible perimeter. What are the most suitable dimensions for the enclosure if each side needs to be a whole number of metres?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Sergio is a jewellery designer, specialising in diamond rings who is very well known in the jewellery industry for the beauty and symmetry of his designs.

- **a.** Juan asks Sergio to design a ring that includes 48 small diamonds, with no more than 6 rows of diamonds. List the dimensions of all the possible arrangements that match Juan's design preferences.
- **b.** Sergio does not set rings that have an odd number of diamonds in either the rows or columns as it affects the balance of the ring. Which possible options from part **a** do not meet Sergio's design criteria?
- **c.** Sergio's favourite array of diamonds is set as a square with an even number of diamonds in each row and column. What is the least number of Diamonds that Sergio can add to Juan's ring to achieve the square design?
- **d.** What other considerations might Juan make before buying an expensive piece of jewellery?

15. After graduating from secondary school Jack studied aviation. Since completing his degree, Jack has become an experienced pilot working in six out of the seven continents. The only continent that Jack is yet to visit is Antarctica.

- **a.** If Jack has completed 1150 commercial flights, is it possible that he has completed an equal number of flights in all the continents that he has visited?
- **b.** Jack recently finished a six week contract working in Europe completing 13 flights each week. Jack keeps a running total of the number of flights he has completed at the end of each week of his contract. What are the numbers on Jack's list?
- **c.** Antarctica is most easily accessible by plane or by boat from Ushuaia, Argentina. In December, a boat leaves the Ushuaia Port every 40 minutes. The first boat departs at 8:20 am, and the last boat departs at 5:00 pm. How many different boats does Jack have the option to travel on if he is only in Ushuaia for 24 hours?
- **d.** The Ushuaia Seaplane Company has five flights each day. The departure time of the first flight is 7:00 am and the final is 5:00 pm. If there is an even amount of time between all the flights, what are the departure times of the other three flights?
- **e.** Why might Jack prefer to travel by boat rather than plane?

Extra spicy

16. A palindromic number is a number that stays the same when the digits in each place value are reversed. For example, 676. What is the difference between the largest and smallest 4 digit palindromic numbers that are both multiples of 5?

A. 890 **B.** 900 **C.** 910 **D.** 990

17. Which of the following integers is not a multiple of 35?

A. 305 **B.** 490 **C.** 595 **D.** 735

18. Which of these calculations produces a factor of 128?

A. $3 \times 8 + 20$ **B.** $4 + 4 \times 4$ **C.** $12 - 8 \div 4$ **D.** $5 \times 4 + 24 \div 2$

19. The number _____ is divisible by 9.

- One thousand twenty-three
- Five hundred four
- Seven hundred five
- Eight hundred ninety-one
- One thousand eight

How many of the options can be placed in the gap to make the sentence true?

A. 2 **B.** 3 **C.** 4 **D.** 5

Remember this?

20. The table shows the weight of rice for different volumes.

Weight (tonnes)	Volume (cubic metres)
40	50
80	100
120	150

A transport container can store 95 cubic metres of rice.

How much rice can a full transport container store?

A. 76 tonnes **B.** 84 tonnes **C.** 120 tonnes **D.** 176 tonnes

21. Which expression results in the correct answer for 45×78 ?

A. $(40 \times 70) + (5 \times 8)$

B. $(40 \times 70) + (40 \times 8) + (5 \times 8)$

C. $(50 \times 80) - (5 \times 8)$

D. $(40 \times 78) + (5 \times 78)$

22. Mr. Anthony has less than 50 players at his netball academy. When he splits them into groups of 3, there is 1 player left. When he splits them into groups of 5, there are 2 players left. What is the most number of students that could attend the netball academy?

A. 13 **B.** 27 **C.** 37 **D.** 47

2B Index notation

Index notation refers to any value that is written with a base number and index number. We also commonly refer to a value expressed in index notation as a power. For example, 2^3 is the number eight expressed in index notation. 2^3 can also be expressed in its expanded form as $2 \times 2 \times 2$. Numbers written in index notation can be included in calculations – it is important to remember that index notation relates to multiplication. Below are some examples where index notation can be applied.

- The length of each side of my bedroom is 4 m. To calculate the area of my bedroom, I complete the multiplication 4×4. Express the calculation in index notation.
- I sent an email to 4 students at my school who each then forwarded the email to another 4 students at another school. Each of the four students from the other school then forwarded the email to another four students. How many students received my email altogether?
- The approximate distance of my flight from Melbourne to New Delhi was 10^4 km. As a whole number, how far did I roughly fly?

Learning intentions

Students will be able to:

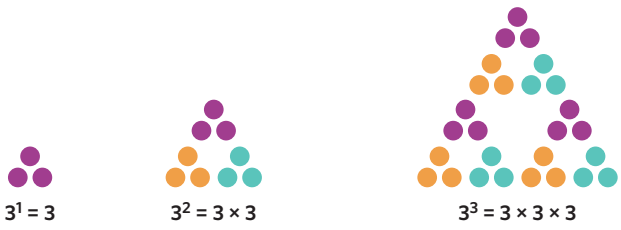
+ convert between expanded form and index notation
+ express powers as whole numbers
+ use order of operations with powers.

Key terms and definitions

- The **base number** in index notation is the number that is being multiplied by itself. E.g. for 8^3, 8 is the base number indicating that three 8's are multiplied together.
- The **index number** indicates how many times the base number is multiplied by itself . The index number can also be referred to as the power of the number. E.g. for 2^4, 4 is the index number indicating that four 2's are multiplied together.
- **Index notation** is a shorter way of representing repeated multiplications. E.g. 2^3 represents $2 \times 2 \times 2$.
- **Expanded form** is a way of expressing a number as the sum of its place value parts or expressing index notation as repeated multiplications. E.g. 2^4 in expanded form is $2 \times 2 \times 2 \times 2$.
- **Powers of a number** are the result of multiplying a number by itself a given number of times. E.g. powers of 10 are: 10, 100, 1000, 10 000 and 100 000.

Key ideas

1 Different powers of a number can be represented using an arrangement of dots to visualise how powers grow as the index number increases.



2 The index number indicates how many times to use the base number in the multiplication. Some whole numbers can be written in index notation, word form or in expanded form.

Index notation	Word form	Expanded form	Whole number value
2^2	Two to the power of two	2×2	4
2^3	Two to the power of three	$2 \times 2 \times 2$	8
2^4	Two to the power of four	$2 \times 2 \times 2 \times 2$	16
2^5	Two to the power of five	$2 \times 2 \times 2 \times 2 \times 2$	32

3 To find the value of a power, base numbers are multiplied by itself a certain number of times depending on the magnitude of the index number. Base numbers are **not** multiplied by the index number.

True	False
$4^2 = 4 \times 4$	$4^2 \neq 4 \times 2$
$3^4 = 3 \times 3 \times 3 \times 3$	$3^4 \neq 3 \times 4$

Worked examples

WE 1 Converting between index notation and expanded form

Write each expression in the described form.

a. 3×3 (index notation)

Working

Base number: 3

Index number: 2

3^2

Thinking

Step 1: Identify the base number which is the number being multiplied by itself.

Step 2: Identify how many times the base number appears in the multiplication, this is the index number.

Step 3: Write the number in index notation.

Visual support

Index notation	Word form	Expanded form	Whole number value
3^2	Three to the power of two	3×3	9

b. 6^3 (expanded form)

Working

Base number: 6

Index number: 3

$6 \times 6 \times 6$

Thinking

Step 1: Identify the base number which is the number being multiplied by itself repeatedly.

Step 2: Identify the index number which indicates how many times to use the base number in the multiplication.

Step 3: Write the number in expanded form.

Student practice

Write each expression in the described form.

a. 5×5 (index notation)

b. 8^4 (expanded form)

c. $2 \times 2 \times 2 \times 2$ (index notation)

d. $18^2 \times 9^3$ (expanded form)

WE 2 Evaluating powers

Evaluate each expression.

a. 4^3

Working

$4^3 = 4 \times 4 \times 4$

$= 16 \times 4$

$= 64$

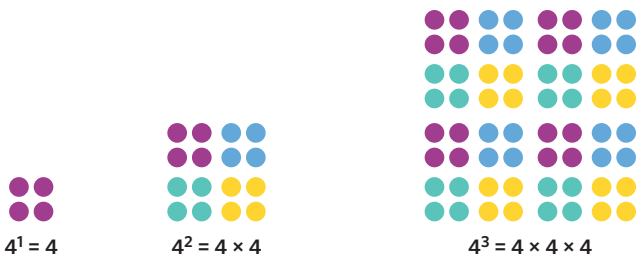
Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the first multiplication.

Step 3: Evaluate the second multiplication.

Visual support



b. $5^3 \times 2^2$

Working

$5^3 \times 2^2 = 5 \times 5 \times 5 \times 2 \times 2$

$= 125 \times 4$

$= 500$

Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplication.

Student practice

Evaluate each expression.

a. 8^2 **b.** 2^4 **c.** $2^3 \times 5^2$ **d.** $10^2 \times 10^3$

WE 3 Evaluating order of operations expressions with powers

Evaluate each expression.

a. $5^2 + 10^2$

Working

$5^2 + 10^2 = 5 \times 5 + 10 \times 10$

$= 25 + 100$

$= 125$

Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplications.

Step 3: Evaluate the addition.

b. $12^2 - 4^3 \times 2$

Working

$12^2 - 4^3 \times 2 = 12 \times 12 - 4 \times 4 \times 4 \times 2$

$= 144 - 64 \times 2$

$= 144 - 128$

$= 16$

Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplication.

Step 3: Evaluate the subtraction.

Student practice

Evaluate each expression.

a. $4^2 + 9^2$ **b.** $11^2 - 2^5$ **c.** $8^2 \times 2 - 5^3$ **d.** $6^2 \times 2 - 3^3 + 25$

2B Activities and questions

STARTER TASKS

Odd spot

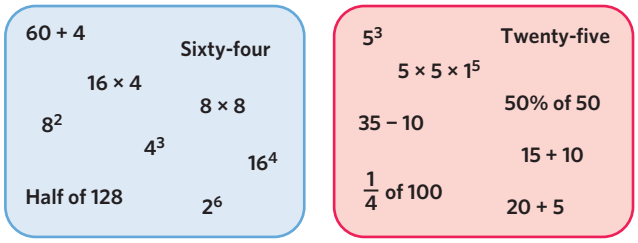
Centipedes are often recognised as the insect with 100 legs. Even though they are commonly known for having 100 legs, they typically don't have that exact number of legs. Most centipedes have between 15 pairs and 177 pairs of legs. Millipedes are an insect that can have even more legs, and can have between 40 and 400 pairs of legs.

If the number of legs a centipede has is represented as 3^4, how many legs does the centipede have?

A. 81 **B.** 12

Image: kooananoo7/Shutterstock.com

Puzzle



a) Select the 'odd one out' in the blue square.

b) Select the 'odd one out' in the red square.

c) Can you add a whole number to the red square that seems like it might belong, but is actually another 'odd one out'?

Understanding worksheet

1. Show the description in index notation.

Seven to the power of 3. *Example*

$7^{\square 3}$

a. Eleven to the power of five.

$11^{\square}$

b. Three to the power of four.

$\square^4$

c. Nine to the power of six.

$9^{\square}$

d. Sixteen to the power of three.

$\square^3$

2. Complete the missing values in the table.

Example

Index notation	Expanded form	Whole number value
4^3	$4 \times 4 \times 4$	64

	Index notation	Expanded form	Whole number value
a.	6^2		36
b.	3^4	$3 \times 3 \times 3 \times 3$	
c.		$2 \times 2 \times 2 \times 2 \times 2$	32
d.	5^3		125

3. Fill in the blanks by using the words provided.

index | index notation | base | expanded

_____ is used to make expressions more simple to understand. They are often used as a way to write very large or very small numbers. The _____ number is the number that is multiplied by itself and the _____ number tells us how many times the base number appears in the multiplication. Powers can be expressed in _____ form by using the multiplication operation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1a **4.** Write each expression in index notation.

a. 7×7
b. $3 \times 3 \times 3 \times 3$
c. $10 \times 10 \times 10$
d. $5 \times 5 \times 5 \times 5 \times 5 \times 5$
e. $200 \times 200 \times 200$
f. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
g. $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$
h. $100\,000 \times 100\,000$

WE1b **5.** Write each expression in expanded form.

a. 12^2
b. 8^3
c. 2^5
d. 80^4
e. $5^3 \times 10^2$
f. $4^4 \times 8^3$
g. $11^4 \times 3^5 \times 2^2$
h. $100^2 \times 15^3 \times 6^1$

6. State whether each equation is true or false.

a. $6 \times 6 = 6^2$
b. $8^3 = 8 \times 8 \times 8$
c. $5 \times 5 \times 3 = 5^3$
d. $12 \times 2 = 12^2$
e. $3^2 \times 2^4 = 3 \times 3 \times 2 \times 2 \times 2 \times 2$
f. $5 \times 3 \times 7 \times 4 = 5^3 \times 7^4$
g. $1^{100} = 100$
h. $9^4 \times 2^3 = 8 \times 9^2 \times 9^2$

WE2 **7.** Evaluate each expression.

a. 7^2
b. 3^3
c. 5^3
d. 10^4
e. 2^5
f. $4^2 \times 2^3$
g. $10^5 \times 8^2$
h. $11^2 \times 1^5 \times 2^3$

WE3 **8.** Evaluate each expression.

a. $12^2 - 3^2$
b. $4^3 + 2^4$
c. $5^3 - 11^2 + 6^2$
d. $10^2 \times 2^4 - 8^2$
e. $2^5 + 3^4 + 4^3 + 5^2$
f. $9^2 \times 3 - 12^2 + 25$
g. $10 + (2^3 \times 10^4) - 100$
h. $[(2^4 + 15) \times 1^{12}] - 31$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Hamlet is preparing his veggie garden for the pumpkin season and plants five pumpkin seeds. He expects that each seed will grow five large pumpkins. How many pumpkins does Hamlet expect to grow?

10. The equator is the imaginary line around the middle of planet Earth. Written as a whole number, what is the length of the earth's equator if it is equal to 4×10^4 kilometres?

11. Lady Macbeth is a famous fitness influencer. Lady Macbeth sold 250 tickets for a meet and greet. She also gave free tickets to 7 of her followers. How many people in total attended the meet and greet if each of the 7 followers invited 7 of their own friends too?

12. Romeo and Juliet had two children. Each of their children then had two children of their own. This pattern continued with each generation of the family. How many great-great-grandchildren do Romeo and Juliet have?

13. Lady Montague is preparing a family photo album. There are six people in her family and she plans to include six individual photos per family member. However, one of her sisters requested that Lady Montague only include 3 photos of her. What is the total number of photos in the album if it also includes 7 photos of the family dog?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Othello is working on his year 7 maths homework. He is given the following table that includes base numbers and index numbers. He can combine different values from the table to form powers. For example, he can use a 5 from the base number column and a 4 from the index number column to create 5^4.

Base number	Index number
2, 3, 4, 5, 8	2, 3, 4, 5, 6

   a. Using one number from the base number column and one number from the index number column, what is the largest power that Othello can create?
   b. Using one number from the base number column and one number from the index number column, what is the power with a value closest to 100 that Othello can make?
   c. Othello thinks that powers with different base numbers and different index numbers can never be equal. Give an example using numbers from the table to show that he is incorrect.
   d. Othello's favourite subject is maths and he often neglects his homework for other subjects. Suggest a way for Othello to make sure he completes his homework for all of his subjects.

15. The planet Mercury is the smallest recognised planet in the solar system. Mercury is the closest planet to the sun and does not have any moons or rings. The following table shows the distance between Mercury and the other planets in the solar system.

From Mercury to...	Approximate distance in km
Venus	503×10^5
Earth	917×10^5
Mars	17×10^7
Jupiter	7×10^8

   a. Written as a whole number, what is the approximate distance between Mercury and Mars?
   b. Is Mars or Jupiter further away from Mercury?
   c. How much closer is Venus to Mercury than Earth is to Mercury? Give your answer as a whole number.
   d. The approximate distance between Mercury and Uranus is roughly four times the distance between Mercury and Jupiter. Calculate the approximate distance between Mercury and Uranus.
   e. Commercial flights to space may start in the near future. What might a space company need to consider before sending civilians into space?

Extra spicy

16. A whole number cubed and a prime number have a sum of 125. What are the two numbers?

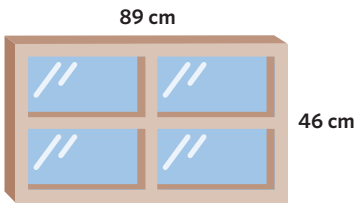
17. Which option has the greatest value?
   A. 18×88 B. 1^{888} C. 188^8 D. 188×8 E. $1^8 \times 8^8$

18. If 3^x is equal to a third of 3^{20}, what is x?

19. Select the true equation(s).
   A. $2 + 4^7 - 1 \times 8 = 16\,378$
   B. $2 + 4^7 - 1 \times 7 = 16\,377$
   C. $2 + 4^7 - 1 \times 6 = 16\,376$
   D. $2 + 4^7 - 1 \times 5 = 16\,375$

Remember this?

20. The length of each side of a window is shown in the following image.



What is the area of the window?

A. 135 cm

B. 890 cm^2

C. 4094 cm^2

D. 4904 cm^2

21. King Lear needs to print 8 homework booklets but does not have paper for printing.
Each of the booklets requires 70 pages.
He can buy printing paper in small packets of 50 pages.
How many small packets does King Lear need?

A. 10 **B.** 11 **C.** 12 **D.** 13

22. Which option represents 90 million?

A. 9×10^6 **B.** 9×10^7 **C.** 9×10^9 **D.** 90×10^5

2C Prime and composite numbers

The real number system consists of different types of numbers. Part of this number system are prime and composite numbers. Prime numbers are those that only have two factors; one and the number itself. Composite numbers have more than two factors. Below are some examples where prime and composite numbers can be used.

- I have less than 50 books. I cannot split them evenly into smaller groups of books. How many books might I have?
- I baked 36 cookies for the school bake sale and wanted to put them in packs so that two people could share a pack equally. How many cookies should I put into each pack?
- My best friend gave me 67 pokemon cards and asked me to share them equally among family and friends. How many equal groups can I make with the 67 cards?

Learning intentions

Students will be able to:

+ show prime and composite numbers using an array
+ recognise what makes numbers prime or composite
+ identify whether a number is composite using a divisibility test.

Key terms and definitions

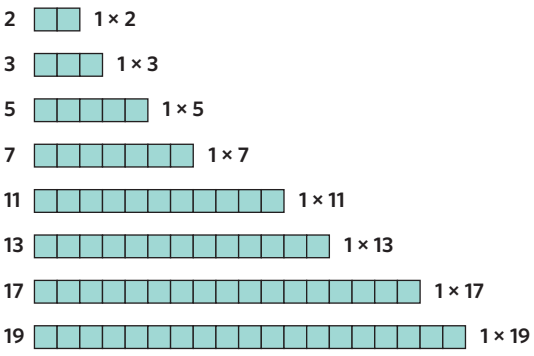
- A **prime number** is a number that only has two factors, 1 and itself.
- A **composite number** is a number that has 3 or more factors.
- A number is **divisible** by another number when its quotient is a whole number. E.g. 18 is divisible by 6 because $18 \div 6 = 3$.
- **Factors** are numbers that divide into another number with no remainder. E.g. factors of 6: 1, 2, 3, 6.
- **Factor pairs** are two numbers that multiply together to give another number. E.g. 2×7 is a factor pair of 14.
- **Common factors** are the same factors that occur between two or more numbers.
- Two numbers are **coprime** if they have no common factors other than one.

Key ideas

1 We can use arrays to determine whether a number is a prime number or a composite number.

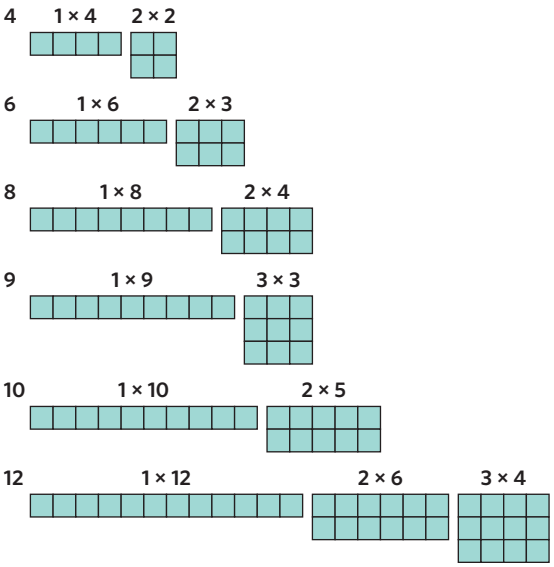
Prime numbers only have 2 factors and can only make single row arrays. This is because the two factors must be the prime number and 1.

Arrays for the first 8 prime numbers



Composite numbers have more than 2 factors and can form more than one array.

Arrays for the first 6 composite numbers



2 Two numbers are considered coprime when the only factor they have in common between them is the number 1. Some composite numbers are coprime with each other, but all prime numbers are coprime with all other numbers.

The numbers 6 and 25 are coprime because the only common factor they have is 1.

Factor pairs of 6

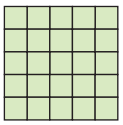

1 × 6


2 × 3

Factors: 1, 2, 3, 6

Factor pairs of 25


1 × 25


5 × 5

Factors: 1, 5, 25

3 We can determine whether a number is composite using divisibility tests.

Divisibility tests can be used as an efficient way of determining the factors of a number. If the number satisfies any of the divisibility conditions outlined in the tests, then it is a composite number.

The number is divisible by:	Conditions of divisibility	Divisible	Not divisible
2	if its last digit is even.	6748	6745
3	if the sum of its digits is divisible by 3.	612	613
4	if its last 2 digits form a number that is divisible by 4.	10 516	10 522
5	if its last digit is 0 or 5.	5435	4438
6	if it is even and if the sum of its digits is divisible by 3.	72	140
7	*There is no simple test.*		
8	if its last 3 digits form a number that is divisible by 8.	1856	1862
9	if the sum of its digits is divisible by 9.	5697	18 775
10	if the last digit is 0.	18 770	18 775

For example:

The number 69 is not a prime because it satisfies the divisibility test for 3.

$6 + 9 = 15$ and 15 is divisible by 3.

Worked examples

WE 1 Identifying prime and composite numbers

Identify whether each number is a prime or composite number.

a. 23

Working

$1 \times 23 = 23$

23 is a prime number.

Thinking

List the factor pairs as multiplications. If there is only one factor pair, the number is a prime number.

Visual support


1 × 23

b. 39

Working

$1 \times 39 = 39$

$3 \times 13 = 39$

39 is a composite number.

Thinking

List the factor pairs as multiplications. If there is more than one factor pair, the number is a composite number.

Student practice

Identify whether each number is a prime or composite number.

a. 12 b. 31 c. 41 d. 87

WE 2 Identifying coprime number pairs

Identify whether each of the pairs of numbers are coprime.

a. 15 and 22

Working

15: 1, 3, 5, 15

22: 1, 2, 11, 22

Common factor: 1

15 and 22 are coprime.

Thinking

Step 1: Find all the factors of the two numbers.

Step 2: Identify the common factors, if the only common factor is 1 then the numbers are coprime.

b. 18 and 27

Working

18: 1, 2, 3, 6, 9, 18

27: 1, 3, 9, 27

Common factors: 1 and 3

27 and 18 are **not** coprime.

Thinking

Step 1: Find all the factors of the two numbers.

Step 2: Identify the common factors, if the only common factor is 1 then the numbers are coprime.

Student practice

Identify whether each of the pairs of numbers are coprimes.

a. 2 and 4 b. 17 and 23 c. 21 and 44 d. 50 and 68

WE 3 Using divisibility to identify composite numbers

Use the divisibility tests to show that each number is composite.

a. 105

Working

2, 4, 6, 8, 10 ✗

5 ✓

3: 1 + 0 + 5 = 6 ✓

9: 1 + 0 + 5 = 6 ✗

105 is divisible by 3 and 5 therefore it is composite.

Thinking

Step 1: Identify any numbers that can easily be ruled out as factors. 105 is an odd number so it is not divisible by any even numbers.

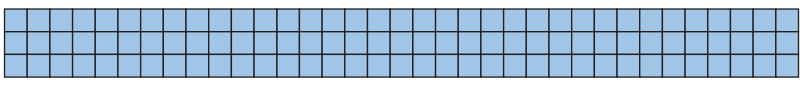
Step 2: Look at the last digit to see if it is a 5. 105 ends with a 5 so it is divisible by 5.

Step 3: Perform the divisibility tests for 3 and 9.

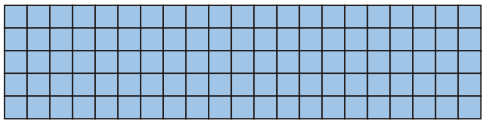
3: If the sum of the digits is divisible by 3.

9: If the sum of the digits is divisible by 9.

Visual support



3 × 35 = 105



5 × 21 = 105

b. 162

Working

5, 10 ✗

2 ✓

3: 1 + 6 + 2 = 9 ✓

4: 162 ✗

6: 1 + 6 + 2 = 9 ✓

8: 162 ✗

9: 1 + 6 + 2 = 9 ✓

162 is divisible by 2, 3, 6, and 9 therefore it is a composite number.

Thinking

Step 1: Identify any numbers that can easily be ruled out as factors. 162 ends in a 2 so it is not divisible by 5 or 10.

Step 2: Look at the last digit to see if it is even. 162 ends with a 2 so it is divisible by 2.

Step 3: Perform the divisibility tests for the remaining numbers to determine what other values it is divisible by.

3: If the sum of the digits is divisible by 3.

4: if its last 2 digits form a number that is divisible by 4.

6: if it is even and if the sum of its digits is divisible by 3.

8: if its last 3 digits form a number that is divisible by 8.

9: If the sum of the digits is divisible by 9.

Student practice

Use the divisibility tests to show that each number is composite.

a. 42 **b.** 142 **c.** 117 **d.** 297

2C Activities and questions

STARTER TASKS

Image: mark6mauno/flickr.com

Odd spot

Eratosthenese was a Greek Mathematician who was born in 276 BC. He was best known for being the first person to calculate the circumference of the Earth, but these days his name is most famous for being the founder of the Sieve of Eratosthenes which is a method of calculating all prime numbers from 1–100.

If Eratosthenese died in a prime number year, and his age was also a prime number when he died, in what year did he die?

A. 193 BC **B.** 181 BC

Puzzle

The Sieve of Eratosthenes

We can use the Sieve of Eratosthenes to find all the prime numbers from 1–100. Follow the steps and then complete the questions.

Step 1: On the hundreds chart, cross out the number 1.

Step 2: Circle the number 2, and then cross out all of the multiples of 2.

Step 3: Circle the next number that isn't crossed out, and then cross out all of the multiples of this number.

Step 4: Repeat this process with the next number that isn't crossed out. Keep repeating the process until there are no more numbers remaining to circle.

a) What do you notice about the circled numbers?

b) What do you notice about the crossed out numbers?

c) Which number has the greatest number of factors? What is a way that you could figure this out?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Understanding worksheet

1. Use the factor pairs of each number to circle whether the number is prime or composite.

Example

Factor pairs

1 × 23

23: (Prime) Composite

a. Factor pairs

1 × 14

2 × 7

14: Prime Composite

b. Factor pairs

1 × 29

29: Prime Composite

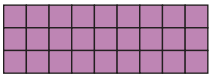
c. Factor pairs

1 × 17

17: Prime Composite

d. Factor pairs

1 × 27



3 × 9

27: Prime Composite

2. Using the divisibility tests, and the tick or a cross in the table to determine whether the given value is a prime or composite number.

Note: The divisibility tests are shown in Key idea 3.

Example

38

Divisible by								
2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**Prime or Composite**
✓	×	×	×	×	×	×	×	Composite

a. 45

Divisible by								
2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**Prime or Composite**
×	✓	×	✓	×	×	✓	×	

b. 57

Divisible by								
2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**Prime or Composite**
×	✓	×	×	×	×	×	×	

c. 79

Divisible by								
2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**Prime or Composite**
×	×	×	×	×	×	×	×	

d. 63

Divisible by								
2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**Prime or Composite**
✗	✓	✗	✗	✗	✗	✓	✗	

3. Fill in the blanks by using the words provided.

composite numbers | factors | prime number | coprime

Numbers that multiply together to form another number are called _____. When a number has two factors, 1 and the number itself, it is a _____. _____ have more than two factors. Two numbers are considered _____ when the only factor they have in common is the number 1.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Identify whether each number is a prime or composite number.

a. 2 **b.** 7 **c.** 11 **d.** 18
e. 27 **f.** 61 **g.** 49 **h.** 93

5. List all of the prime numbers **between** each pair.

a. 1 and 15 **b.** 23 and 33 **c.** 32 and 55 **d.** 50 and 62
e. 60 and 85 **f.** 80 and 110

WE2 **6.** Identify whether each of the pairs of numbers are coprime.

a. 3 and 5 **b.** 6 and 9 **c.** 16 and 29 **d.** 14 and 21
e. 23 and 47 **f.** 15 and 16 **g.** 28 and 35 **h.** 30 and 77

WE3 **7.** Use the divisibility tests to show that each number is composite.

a. 99 **b.** 122 **c.** 312 **d.** 975
e. 279 **f.** 632 **g.** 1161 **h.** 3081

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Elton has 25 Pokémon cards. How can he place them into four groups so that each group contains a prime number of cards?

9. Jesse is donating some of her clothes to charity. She has 83 pieces of clothing and plans to share them equally between a group of charities. Will she be able to share the clothes in the way that she plans?

10. Rae wanted to share her pack of Tim Tams between herself and 3 friends but noticed that each pack only contained 11 biscuits. How many packs will she need to make sure that each person receives an equal amount of Tim Tam biscuits?

11. Eileen baked 93 cookies for her school bake sale and wanted to pack them into equal cartons. How could Eileen package the cookies, so that there are no cookies left over?

12. Ronaldo plays soccer for his local club, he played in every match this season and scored a total of 51 goals. Is it possible that Ronaldo kicked the same number of goals in each match, if there are more than 4 and less than 25 matches in a season?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c,d)	All

13. Cicadas are large insects that spend much of their time hiding underground. In Australia, cicadas emerge from their underground hiding spots every 7 years to breed, but other species can stay underground for 13 or 17 years. Some scientists believe that this unusual cycle is an evolutionary method of avoiding predators.

- **a.** Are the emergence cycle of the different cicadas species prime or composite numbers?
- **b.** The last great emergence of cicadas in Australia was in 2017. What year are cicadas next expected to emerge in Australia?
- **c.** Birds are a common predator of the cicada. The population of birds peaks every 5 years and this is the most dangerous time for cicadas to emerge. North American cicadas emerge every 17 years. Assuming the population of birds peaks in the same year as the emergence of the North American cicadas, how often will the peak population of birds coincide with the cicadas' emergence?
- **d.** Other than to avoid predators, what alternative explanations could there be for a cicadas' unusual emergence cycle?

14. Tane and Emily were playing a maths game. They were each given the following cards along with a series of questions.

- **a.** The first question required Tane and Emily to form the smallest prime number by multiplying the values of two cards together. What two cards should they select to answer the first question?
- **b.** The second question required Tane and Emily to find all the cards that are not coprimes of 15. Which cards should they select to answer the second question?
- **c.** The third question required Tane and Emily to select two cards and any operation to create the smallest possible composite number. The teacher says that there is more than one solution. How many possible solutions are there?
- **d.** Tane thinks that the product of all the numbers on the cards is always an odd number because except for 1 they are all prime. Explain why Tane is incorrect.
- **e.** Propose other questions that could be asked relating to the 6 cards.

Extra spicy

15. Elsa will celebrate her 98^{th} birthday this week. The sum of her current age and age a year ago is a prime number. How many other times has this happened in Elsa's life?

16. Andrew picked two random prime numbers. Is it possible that their product is a prime number?

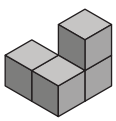
17. How many of the numbers from 1 to 20 can be represented as the sum of two different prime numbers?

18. 119 is a coprime number with:

A. 39 **B.** 51 **C.** 77 **D.** 102

Remember this?

19. This shape is made of 4 connected cubes.



If the shape was dipped in paint, how many faces would be painted?

A. 18 **B.** 19 **C.** 21 **D.** 24

20. Which expression has the same value as $49 \times 3 + 2$?

A. $150 - 6$ **B.** $100 + 49$ **C.** $50 \times 3 + 6$ **D.** 49×6

21. The Meyer comet passes the Earth every 4 years and the Catalina comet passes the Earth every 9 years. Suppose both comets passed Earth in 2021. How many times will the Meyer comet and Catalina comet pass Earth in the same year over the next 100 years?

2D Prime factorisation

Prime factorisation is the process of decomposing numbers into the product of their prime factors. Every composite number can be represented as a product of prime factors and these factors will be unique to that number. Knowing the primes that multiply to give us a number allows us to perform various calculations including finding all of the factors of a given number. Below are some examples where prime factorisation can be applied.

- I have a Rubik's Cube that is 216 cubic centimetres. What are the dimensions of my Rubik's Cube?
- I have 65 pieces of gum to share with my friends. If I give each of my friends more than 1 piece, what is the maximum number of friends I can equally share the gum between so I don't have any leftover pieces.

Learning intentions

Students will be able to:

+ understand that each composite number has a unique set of prime factors
+ use a factor tree to find the prime factors of a number
+ express numbers as a product of prime factors in expanded form and index notation.

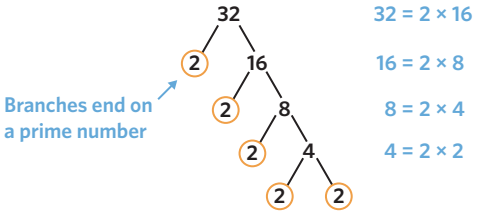
Key terms and definitions

- **Decomposition** is the process of breaking apart the original number into smaller parts. E.g. 14 can be decomposed as 2×7.
- A **factor tree** is a diagram used to find a number's prime factors.
- **Index notation** is a shorter way of representing repeated multiplications. E.g. 2^3 represents $2 \times 2 \times 2$.
- **Expanded form** is a way of expressing a number as the sum of its place value parts or expressing index notation as repeated multiplications. E.g. 2^4 in expanded form is $2 \times 2 \times 2 \times 2$.
- A **prime number** is a number that only has two factors, 1 and itself.
- A **composite number** is a number that has 3 or more factors.
- **Factors** are numbers that divide into another number with no remainder. E.g. factors of 6: 1, 2, 3, 6.
- **Prime factors** are prime numbers that can be multiplied together to give another number. E.g. 18 expressed as a product of its prime factors is $2 \times 3 \times 3$.

Key ideas

1 A factor tree can be used to find the prime factors of any number.

The branches that stem from each number represent two of its factors. A branch ends when the factor is a prime number.



Expanded form	Index notation	Number
$2 \times 2 \times 2 \times 2 \times 2$	2^5	32

2 Every composite number has a unique set of prime factors.

Prime factors of different composite numbers.

$20 = 2 \times 2 \times 5$
$21 = 3 \times 7$
$24 = 2 \times 2 \times 2 \times 3$
$25 = 5 \times 5$
$27 = 3 \times 3 \times 3$
$28 = 2 \times 2 \times 7$
$30 = 2 \times 3 \times 5$

3 We can use the prime factors of any composite number to determine all of its factors.

We can use the prime factors to form multiplications to find the composite factors of 36.

36 as a product of its prime factors.

$36 = 2 \times 2 \times 3 \times 3$

Factors of 36 that are prime numbers.

2 3

The composite factors of 36 that can be formed using two of its prime factors.

$2 \times 2 = 4$ $2 \times 3 = 6$ $3 \times 3 = 9$

The composite factors of 36 that can be formed using three of its prime factors.

$2 \times 2 \times 3 = 12$ $2 \times 3 \times 3 = 18$

All the factors of 36.

1, 2, 3, 4, 6, 9, 12, 18, 36

Worked examples

WE 1 Writing the product of primes in index form

Express each product of prime factors in index notation.

a. $2 \times 2 \times 3$

Working

$2 \times 2 = 2^2$

$3 = 3$

$2^2 \times 3$

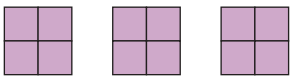
Thinking

Step 1: Identify how many times each factor appears in the calculation to find each factor's index number.

Step 2: Write the factors as a multiplication in index notation in ascending order.

Visual support

$2 \times 2 \times 3 = 2^2 \times 3$



b. $2 \times 2 \times 3 \times 5 \times 5 \times 5$

Working

$2 \times 2 = 2^2$

$3 = 3$

$5 \times 5 \times 5 = 5^3$

$2^2 \times 3 \times 5^3$

Thinking

Step 1: Identify how many times each factor appears in the calculation to find each factor's index number.

Step 2: Write the factors as a multiplication in index notation in ascending order.

Student practice

Express each product of prime factors in index notation.

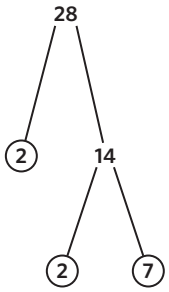
a. $2 \times 2 \times 7$ **b.** $2 \times 5 \times 5$ **c.** $3 \times 3 \times 3 \times 5$ **d.** $2 \times 2 \times 2 \times 5 \times 5$

WE 2 Using factor trees to factorise numbers

Using a factor tree, express each number as a product of prime factors in the described form.

a. 28 (expanded form)

Working



$28 = 2 \times 2 \times 7$

Thinking

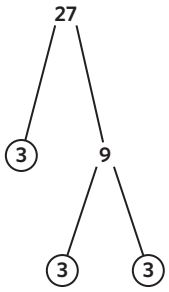
Step 1: Decompose 28 into two of its factors other than 1 and itself. 28 is even, so we can divide it by 2. Write the factors at the end of each branch.

Step 2: Circle any prime factors then decompose the other factor into two of its factors other than 1 and itself. Write the factors at the end of each branch.

Step 3: Circle the prime factors and write the prime factors in expanded form.

b. 27 (index notation)

Working



$27 = 3 \times 3 \times 3 = 3^3$

Thinking

Step 1: Decompose 27 into two of its factors other than 1 and itself. Write the factors at the end of each branch.

Step 2: Circle the prime factors, then decompose the other factor into two of its factors other than 1 and itself. Write the factors at the end of each branch.

Step 3: Circle the prime factors and write the prime factors in index notation.

Student practice

Using a factor tree, express each number as a product of prime factors in the described form.

a. 22 (expanded form) **b.** 16 (index notation) **c.** 56 (expanded form) **d.** 100 (index notation)

WE 3 Using prime factors to find factors

Use the prime factors to find all the factors of each number.

a. $30 = 2 \times 3 \times 5$

Working

$2 \times 3 = 6$

$2 \times 5 = 10$

$3 \times 5 = 15$

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Thinking

Step 1: Group the prime factors of 30 into unique products using two of its three prime factors to find composite factors.

Step 2: List the factors of 30 in ascending order including its prime factors, 1 and 30 itself.

Visual support

30 = 2 × 3 × 5

2 × 3 = 6 2 × 5 = 10 3 × 5 = 15

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

b. $56 = 2 \times 2 \times 2 \times 7$

Working

$2 \times 2 = 4$

$2 \times 7 = 14$

$2 \times 2 \times 2 = 8$

$2 \times 2 \times 7 = 28$

Factors of 56: 2, 4, 7, 8, 14, 28, 56

Thinking

Step 1: Group the prime factors of 56 into unique products using two of its four prime factors to find composite factors.

Step 2: Group the prime factors of 56 into unique products using three of its four prime factors to find composite factors.

Step 3: List the factors of 56 in ascending order including its prime factors, 1 and 56 itself.

Student practice

Use the prime factors to find all the factors of each number.

a. $52 = 2 \times 2 \times 13$ **b.** $99 = 3 \times 3 \times 11$ **c.** $42 = 2 \times 3 \times 7$ **d.** $84 = 2 \times 2 \times 3 \times 7$

2D Activities and questions

STARTER TASKS

Odd spot

In astrology, the term star sign refers to the sign of the zodiac. The zodiac is made up of 12 different constellations that appear in different sections of the sky. Depending on when someone is born, they are attributed a star sign. Each star sign is named after a figure, object or animal from Greek mythology. Some astrologers believe that luck can be calculated using the prime factors of 12, because it is the total number of constellations.

What is 12 expressed as a product of its prime factors?

A. $3 \times 2 \times 2$ **B.** 3×4

Image: WinWin artlab/Shutterstock.com

Puzzle

The colours in each of the images are represented by the following.

a) Using addition, find the number represented by each image.

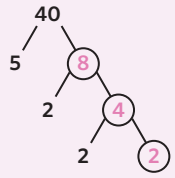
b) Using multiplication, find the number represented by each image.

c) What combination of colours represents the product of 140?

Understanding worksheet

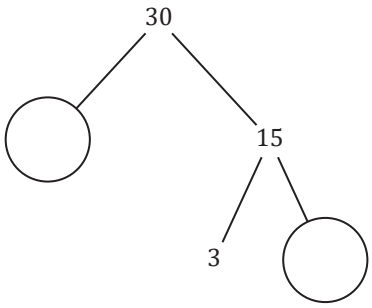
1. Complete the factor tree and express each number as a product of its prime factors.

Example



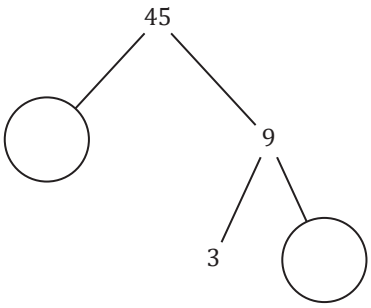
$40 = 5 \times 2 \times 2 \times 2$

a.



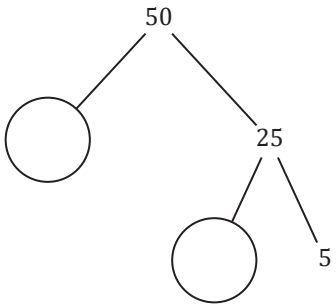
$30 = 2 \times 3 \times 5$

b.



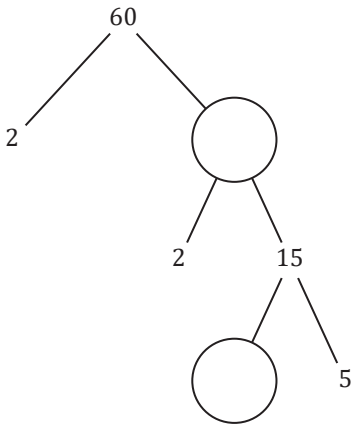
$45 = 5 \times 3 \times 3$

c.



$50 = 2 \times 5 \times 5$

d.



$60 = 2 \times 2 \times 3 \times 5$

2. Use the prime factors to find all factors of each number. List the factors in ascending order.

Example

$20 = 2 \times 2 \times 5$

The composite factors of 20 that can be formed using two of its prime factors.

$2 \times 2 = \boxed{4}$ $\quad 2 \times 5 = \boxed{10}$

The composite factors of 20 that can be formed using three of its prime factors.

$2 \times 2 \times 5 = \boxed{20}$

Factors of 20: 1, 2, 4, 5, 10, 20

a. $16 = 2 \times 2 \times 2 \times 2$

The composite factors of 16 that can be formed using two of its prime factors.

$2 \times 2 = \square$

The composite factors of 16 that can be formed using three of its prime factors.

$2 \times 2 \times 2 = \square$

The composite factors of 16 that can be formed using four of its prime factors.

$2 \times 2 \times 2 \times 2 = \square$

Factors of 16: 1, 2, 4, 8, 16

b. $27 = 3 \times 3 \times 3$

The composite factors of 27 that can be formed using two of its prime factors.

$3 \times 3 = \square$

The composite factors of 27 that can be formed using three of its prime factors.

$3 \times 3 \times 3 = \square$

Factors of 27: 1, 3, 9, 27

c. $12 = 2 \times 2 \times 3$

The composite factors of 12 that can be formed using two of its prime factors.

$2 \times 2 = \square$ $2 \times 3 = \square$

The composite factors of 12 that can be formed using three of its prime factors.

$2 \times 2 \times 3 = \square$

Factors of 12: 1, 2, 3, 4, 6, 12

d. $18 = 2 \times 3 \times 3$

The composite factors of 18 that can be formed using two of its prime factors.

$2 \times 3 = \square$ $3 \times 3 = \square$

The composite factors of 18 that can be formed using three of its prime factors.

$2 \times 3 \times 3 = \square$

Factors of 18: 1, 2, 3, 6, 9, 18

3. Fill in the blanks by using the words provided.

prime | composite | prime factorisation | factor tree

____ is the process of decomposing numbers into the product of their ____ factors. A ____ is a diagram that helps us to find the prime factors of any number. Every ____ number is made up of a unique product of prime factors.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c)	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d)	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f)

WE1 **4.** Express each product of prime factors in index notation.

a. $2 \times 2 \times 2$ **b.** $3 \times 3 \times 5$

c. $2 \times 2 \times 3 \times 3 \times 5$ **d.** $3 \times 5 \times 5 \times 5$

e. $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$ **f.** $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7$

g. $2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7 \times 11$ **h.** $3 \times 5 \times 7 \times 7 \times 7 \times 11 \times 11 \times 13 \times 13$

WE2a **5.** Using a factor tree, express each number as a product of its prime factors in expanded form.

a. 14 **b.** 15 **c.** 38 **d.** 81 **e.** 105 **f.** 72

WE2b **6.** Using a factor tree, express each number as a product of its prime factors in index form.

a. 8 **b.** 44 **c.** 24 **d.** 54 **e.** 60 **f.** 108

7. What composite number does each product of prime factors represent?
   a. $2 \times 2 \times 3$
   b. $5 \times 5 \times 5$
   c. $2^2 \times 3 \times 7$
   d. $2 \times 3 \times 5^2$
   e. $2^5 \times 7$
   f. $2 \times 3^2 \times 11$

WE3 8. Use the prime factors to find all the factors of each number.
   a. $28 = 2 \times 2 \times 7$
   b. $63 = 3 \times 3 \times 7$
   c. $105 = 3 \times 5 \times 7$
   d. $110 = 2 \times 5 \times 11$
   e. $90 = 2 \times 3 \times 3 \times 5$
   f. $250 = 2 \times 5 \times 5 \times 5$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Marcus and Lyndall are playing a game where they roll 5 dice. The numbers on the faces of each die are the first 6 prime numbers. What numbers did they roll, if the product of the faces is 48?

10. Carla has 95 lollies and would like to divide them into smaller bags to share them with her school friends. What is the maximum number of people she can share the lollies between, if she wants each person to receive an equal number of lollies?

11. Tommy is blowing up a giant inflatable cube. Each face of the cube is rainbow coloured and equally sized. What are the dimensions of the cube if its volume is 125 cubic metres?

12. Sienna is playing with 78 building blocks. What are the dimensions of her prism if she uses all the blocks to build a rectangular prism?

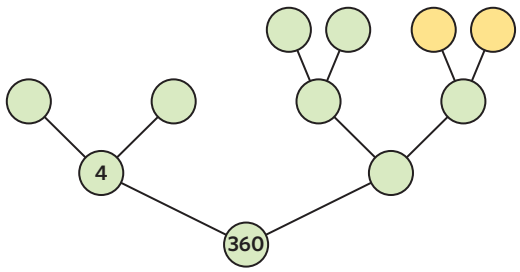
13. Charlotte is a baker and has 75 muffins. She wants to pack the muffins into cartons so that the muffins are arranged in a square, with the same number of muffins in each row and column. How can she pack the muffins so that no muffins remain unpacked?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Liza works at an electronics warehouse. She finds the job physically demanding as she has to pack the products onto a trolley in the warehouse and repack them on a truck. Liza uses prime factorisation as it helps her to plan how to layout boxes efficiently on her trolley.
   a. Liza is transporting 165 laptop boxes and needs to find the prime factors so that she can stack them easily. What would these factors be?
   b. While factorising, Liza got prime factors of 3 and 55. Explain how Liza may have thought that her factors are correct.
   c. Liza wants to transport all 165 laptops using a large trolley. She places the laptops in piles so that each pile has the same number of laptops. Liza needs to make at least 10 piles, otherwise the piles get too high and fall over. How many different ways can Liza pack the trolley?
   d. Propose a way for Liza to reduce the physical demands of her job?

15. Tyson loves to complete puzzles in his free time. Tyson is working on the following puzzle with his friend Maddie and is required to decompose the number 360 using an upside down factor tree. He has already put down 4 which he knows is a factor.



   a. The prime factors in the yellow circles are the same. Help Tyson to complete the factor tree by filling in each of the blank circles.
   b. State the prime factors of 360 in index form.
   c. Tyson's friend Maddie completed a different puzzle that also started from the number 360.
   What could Maddie's puzzle have looked like? There is more than one possible solution.
   d. Tyson sometimes finds it difficult to find the prime factors of large numbers. Propose a strategy that Tyson could use to complete the factor trees for large numbers.

Extra spicy

16. What is the sum of the prime factors of 23 100?

A. 33 **B.** 34 **C.** 35 **D.** 36

17. 2342 can be written as a product of prime factors. How many prime factors is it made up of?

A. 2 **B.** 3 **C.** 4 **D.** 5

18. The product of a number can be written as ★ × ★ × ★ × 3, where each ★ represents the same prime factor. What is the closest number to 1650 that ★ × ★ × ★ × 3 can represent?

19. Express 527 as a product of prime factors.

Remember this?

20. What number does the ? represent in this number sentence?

$? \times 73 = 73\,000$

A. 10 **B.** 100 **C.** 1000 **D.** 10 000

21. $(12 - 4 \times 2 + 3) + 8$ is equal to:

A. 0 **B.** 9 **C.** 15 **D.** 27

22. Dean puts $15 every week in his son Leo's bank account.

Leo's bank balance is:

A. Always even

B. Always odd

C. Never even

D. Sometime odd and sometimes even

2E Lowest common multiples and highest common factors

We use multiplication and division when determining the lowest common multiple and highest common factor of a set of numbers. Lowest common multiples are also used in fractions when finding the lowest common denominator. Highest common factors are used in Algebra and when simplifying fractions. Below are some examples where the lowest common multiple and highest common factors can be applied.

- I have 15 roses and 20 sunflowers. What is the greatest number of people I can share them with so that each person receives the same number of each type of flower and I have no leftover flowers?
- I have piano lessons every four days and guitar lessons every five days. How often do I have both piano and guitar lessons on the same day?
- I have 9 erasers and 27 pens. What is the greatest number of pencil cases I can fill so that I have no leftover stationery and that each pencil case has the same number of pens and the same number of erasers?

Learning intentions

Students will be able to:

+ find the lowest common multiple of a set of numbers
+ find the highest common factor of a set of numbers
+ use prime factors to find the lowest common multiple and highest common factor of a set of numbers.

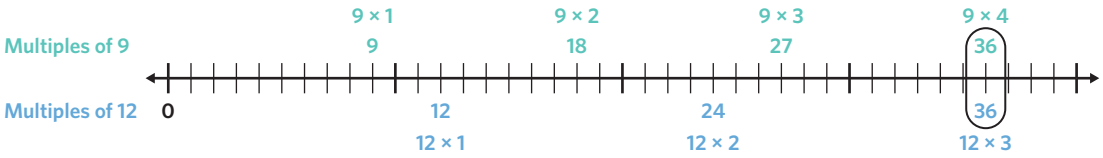
Key terms and definitions

- **Factor pairs** are two numbers that multiply together to give another number. E.g. 2×7 is a factor pair of 14.
- A **multiple** is the product that we get when one number is multiplied by another number. E.g. 12 is a multiple of 4 because $4 \times 3 = 12$.
- **Highest common factor (HCF)** is the largest number that is a factor of two or more numbers. E.g. 5 is the HCF of 5 and 15.
- **Lowest common multiple (LCM)** is the smallest number that is a multiple of two or more numbers. E.g. 12 is the LCM of 3 and 4.
- A **Venn diagram** is a mathematical representation showing the common and uncommon features between two or more sets of numbers.

Key ideas

1 The lowest common multiple of a pair of numbers can be shown on a number line.

The lowest common multiple is the first shared multiple on a number line.



2 We can find the lowest common multiple by listing multiples of a set of numbers. We can find the highest common factor by listing factors of a set of numbers.

Lowest common multiple of 6 and 15

Multiples of 6: 6, 12, 18, 24, 30, 36, ...
Multiples of 15: 15, 30, 45, ...

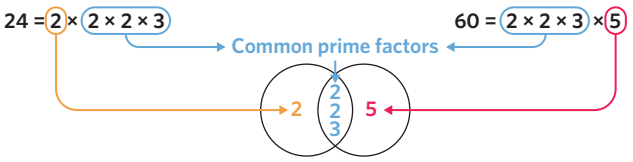
The LCM of 6 and 15 is 30.

Highest common factor of 6 and 15

Factors of 6: 1, 2, 3, 6
Factors of 15: 1, 3, 5, 15

The HCF of 6 and 15 is 3.

3 The HCF and LCM of two or more numbers can also be found using the products of prime factors.



The HCF of 24 and 60 is the product of their common prime factors:

HCF of 24 and 60 $= 2 \times 2 \times 3 = 12$

The LCM of 24 and 60 is the product of their common prime factors and the remaining factors:

LCM of 24 and 60 $= 2 \times 2 \times 2 \times 3 \times 5 = 120$

Worked examples

WE 1 Finding lowest common multiples

Find the lowest common multiple (LCM) of each set of numbers.

a. 6 and 9

Working

6: 6, 12, 18, 24

9: 9, 18, 27

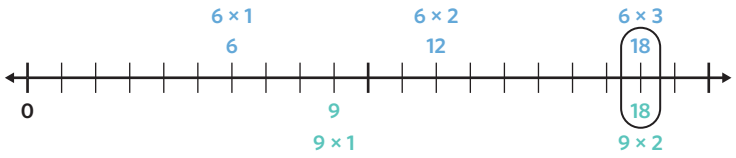
The LCM is 18.

Thinking

Step 1: List some of the multiples of 6 and 9.

Step 2: Identify the smallest number that appears in both sets of multiples.

Visual support



b. 3, 5 and 6

Working

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

5: 5, 10, 15, 20, 25, 30, 35, 40

6: 6, 12, 18, 24, 30, 36, 42

The LCM is 30.

Thinking

Step 1: List some of the multiples of 3, 5 and 6.

Step 2: Identify the smallest number that appears in all the sets of multiples.

Student practice

Find the lowest common multiple (LCM) of each set of numbers.

a. 4 and 7 **b.** 4 and 6 **c.** 4, 5 and 8 **d.** 20 and 70

WE 2 Finding highest common factors

Find the highest common factor (HCF) of each set of numbers.

a. 16 and 12

Working

16: 1, 2, 4, 8, 16

12: 1, 2, 3, 4, 6, 12

The HCF is 4.

Thinking

Step 1: List the factors of each number in ascending order.

Step 2: Identify the highest number that appears in both sets of factors.

b. 18, 24 and 36

Working

18: 1, 2, 3, 6, 9, 18

24: 1, 2, 3, 4, 6, 8, 12, 24

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

The HCF is 3.

Thinking

Step 1: List the factors of each number in ascending order.

Step 2: Identify the highest number that appears in all the sets of factors.

Student practice

Find the highest common factor (HCF) of each set of numbers.

a. 3 and 6 b. 8 and 10 c. 20 and 60 d. 12, 15 and 45

WE 3 Using prime factors to find highest common factor and lowest common multiple

Write each pair of numbers as a product of its prime factors then use the prime factors to find the:

i) highest common factor (HCF)

ii) lowest common multiple (LCM)

a. 6 and 12

Working

$6 = 2 \times 3$

$12 = 2 \times 2 \times 3$

i) $2 \times 3 = 6$

The HCF is 6.

ii) $2 \times 3 \times 2 = 12$

The LCM is 12.

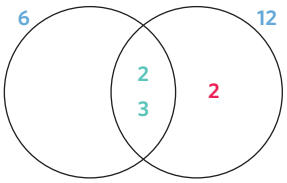
Thinking

Step 1: Write each of the numbers as a product of its prime factors.

Step 2: Multiply the common prime factors together to find the HCF.

Step 3: Multiply the common prime factors and the remaining uncommon factors to find the LCM.

Visual support



$6 = 2 \times 3$
$12 = 2 \times 3 \times 2$

HCF of 6 and 12: $2 \times 3 = 6$
LCM of 6 and 12: $2 \times 3 \times 2 = 12$

b. 21 and 35

Working

$21 = 3 \times 7$

$35 = 5 \times 7$

i) The HCF is 7.

ii) $3 \times 5 \times 7 = 105$

The LCM is 105.

Thinking

Step 1: Write each of the numbers as a product of its prime factors.

Step 2: Multiply the common prime factors together to find the HCF. If there is only one, this is the HCF.

Step 3: Multiply the common prime factors and the remaining uncommon factors to find the LCM.

Student practice

Write each pair of numbers as a product of its prime factors then use the prime factors to find the:

i) highest common factor

ii) lowest common multiple

a. 6 and 10 b. 15 and 25 c. 12 and 18 d. 100 and 150

2E Activities and questions

STARTER TASKS

Odd spot

Between 2019 and 2020 there were 17 915 accidental fire related house incidents reported across Australia. The majority of the incidents caused minor damage to the properties. Laws relating to smoke detectors in Australia changed in May 2017 where it is compulsory that all smoke detectors are less than 10 years old. Michelle uses two different brands of smoke detectors in her home. The LifeSaver brand detector requires a replacement every 10 years and the SmokeSmeller brand requires a replacement every 6 years.

Image: gopixa/Shutterstock.com

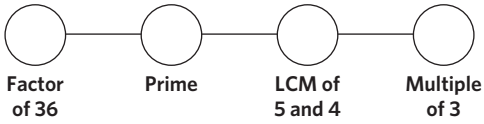
In 2020, Michelle fitted one LifeSaver smoke detector and one SmokeSmeller smoke detector in her home. What year will she next replace both detectors in the same year?

A. 2022 **B.** 2050

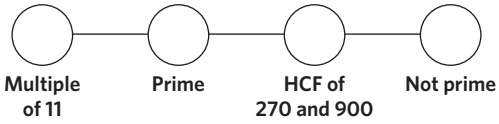
Puzzle

Use the clues to complete the sets of increasing consecutive numbers. All the numbers are less than 100.

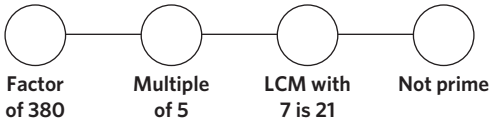
a)



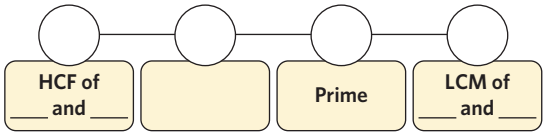
b)



c)



d) Complete the clues to create a set of increasing consecutive numbers.

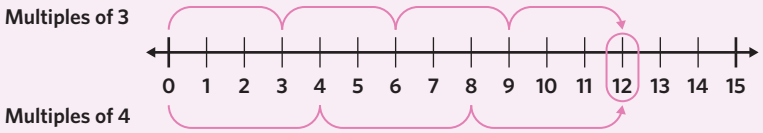


Understanding worksheet

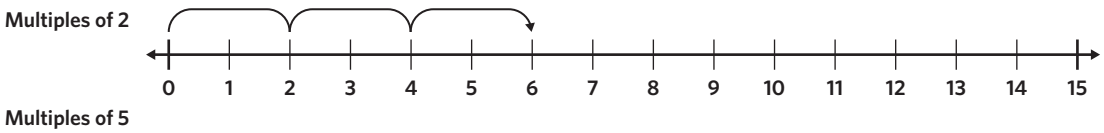
1. Perform the jumps on the number line and circle the lowest common multiple of the pair of numbers.

Example

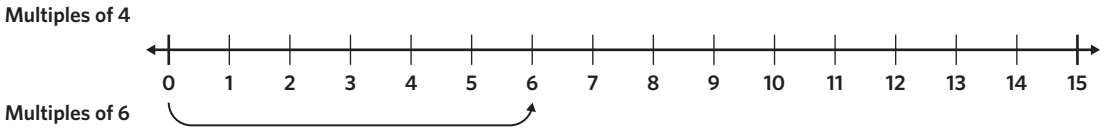
3 and 4



a. 2 and 5

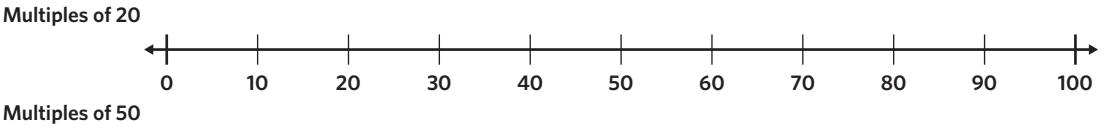


b. 4 and 6



c. 2 and 7

d. 20 and 50

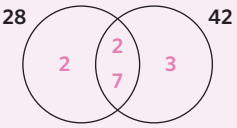


2. Place each number's prime factors in the Venn diagram.

Example

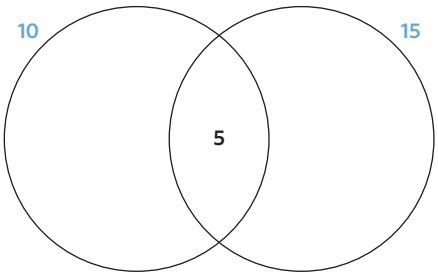
$28 = 2 \times 2 \times 7$

$42 = 2 \times 3 \times 7$



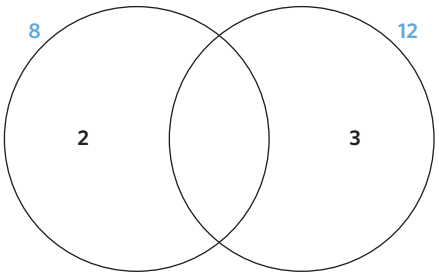
a. $10 = 2 \times 5$

$15 = 3 \times 5$



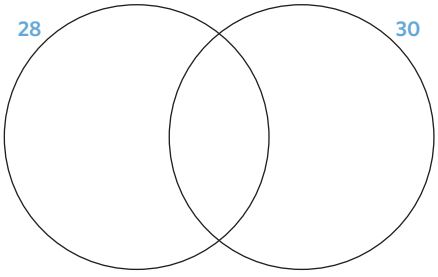
b. $8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$



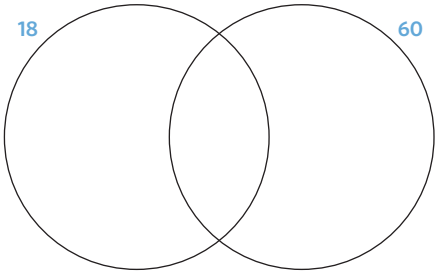
c. $28 = 2 \times 2 \times 7$

$30 = 2 \times 3 \times 5$



d. $18 = 2 \times 3 \times 3$

$60 = 2 \times 2 \times 3 \times 5$



3. Fill in the blanks by using the words provided.

prime | multiples | lowest | highest | factors

To find the _____ common multiple of a set of numbers we list the _____ of each number in the set. To find the _____ common factor of a set of numbers we list the _____ of each number in the set. Another way to find the lowest common multiple and highest common factor is to use _____ factorisation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f)

WE1a **4.** Find the lowest common multiple of each pair of numbers.

a. 4 and 5 **b.** 6 and 8 **c.** 8 and 12 **d.** 5 and 11

e. 20 and 25 **f.** 9 and 24

WE1b **5.** Find the lowest common multiple of each set of numbers.

a. 2, 3, 4 **b.** 2, 4, 5 **c.** 4, 6, 8 **d.** 5, 10, 15

e. 4, 5, 8 **f.** 6, 9, 12

WE2a **6.** Find the highest common factor of each pair of numbers.

a. 2 and 10 **b.** 5 and 7 **c.** 3 and 18 **d.** 12 and 20

e. 28 and 35 **f.** 42 and 54

WE2b **7.** Find the highest common factor of each set of numbers.

a. 2, 4, 6 **b.** 3, 9, 15 **c.** 10, 25, 35 **d.** 12, 18, 30

e. 24, 40, 72 **f.** 60, 80, 120

WE3 **8.** Write each pair of numbers as a product of its prime factors then use the prime factors to find the:

i) highest common factor

ii) lowest common multiple

a. 8 and 12 **b.** 15 and 18 **c.** 9 and 24 **d.** 20 and 30

e. 48 and 72 **f.** 36 and 55

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Ali and Jenny sometimes work at Sparkles Car Wash. Ali works every 5 days and Jenny works every 4 days. How often do they work a shift together?

10. Aviva has two different sized planks of wood. One is 96 cm in length and the other is 156 cm in length. What are the longest possible lengths of wood she can end up with if Aviva wants to cut both the planks into a number of equally sized pieces?

11. At Redbridge Secondary, 80 year 7 students and 64 year 8 students attended a school camp. The teachers need to split up the students into as few equal groups as possible, however each group can only contain year 7 students or year 8 students. How many students were in each group?

12. Loti the spaniel barks every 9 seconds and Sully the labrador barks every 12 seconds. When will they next bark at the same time if they last both barked at 2 seconds past 12:07 pm?

13. David, Ryan and Libby are all delivery drivers. David is paid $90 per delivery, Ryan earns $60 per delivery, and Libby receives $80 per delivery. Assuming all the drivers did more than one delivery, what is the minimum number of deliveries that each driver must complete so that they all get paid the same amount?

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c)	All

14. Three of the most popular intercity buses that depart from Brisbane Coach Terminal terminate at Noosa Heads, Byron Bay and Surfers Paradise. All three destinations have a bus that leaves Brisbane at 9.00 am. Buses to Noosa Heads depart every hour and a half. Buses to Byron Bay depart every 2 hours, and buses to Surfers Paradise depart every 4 hours.

a. What is the next time after 9.00 am that the buses to Byron Bay and Surfers Paradise depart at the same time.
b. What is the next time after 9.00 am that buses to all three locations depart at the same time?
c. Every Friday, all the drivers at the Brisbane Coach Terminal receive an identical snack pack. If there are 150 biscuits and 60 salmon bagels to share, what is the greatest number of snack packs they can be distributed without wasting any food?
d. How many biscuits and salmon bagels will each driver receive?
e. Some drivers work more than other drivers on a Friday. Propose a fairer way to share the snack packs amongst the drivers.

15. Borders bookshop was a popular bookseller in Australia in the early 2000s that sold books, DVDs, CDs, and stationery. As more people bought books online, Borders was forced to shut all of its stores. In 2005 the most popular book releases at Borders were Twilight by Stephenie Meyer and Harry Potter and the Half Blood Prince by J.K. Rowling.
a. Borders could order Twilight in batches of 300 books and Harry Potter and the Half Blood Prince in batches of 500 books. What is the smallest order they can make so that they receive the same number of each book.
b. Ten Borders stores sold out of both books after two days. Borders ordered an additional 20 batches of each book. How many books does each store receive if the order is distributed evenly between the ten stores and there are no remaining books from the order.
c. Borders have one batch of each book that they would like to donate to different community centres across Australia. If they want to donate a set number of the Twilight book and a set number of the Harry Potter book to each community centre and not have any remaining books, what is the maximum number of centres they can donate to?
d. Why might some people still prefer to read hard copy books even though digital books are cheaper?

Extra spicy

16. Find two numbers between 100 and 150 that have a highest common factor of 22.

17. Marge eats steak every second day. How often does she eat steak on a Tuesday?
A. Every week B. Once a fortnight C. Once a month D. Once a year

18. A pair of numbers have a highest common factor of 5 and a lowest common multiple of 495. If the sum of the numbers is 100, what is their difference?

19. If the numbering of the pages in Lion the Witch and the Spacecraft requires 822 digits in total, how many pages are there in Lion the Witch and the Spacecraft?
A. 272 B. 290 C. 310 D. 322

Remember this?

20. Which shape is not quadrilateral?
A. Rhombus B. Trapezium C. Pentagon D. Kite

21. Niall bought four new luxury yachts for a total of $64.8 million. If each yacht was the same price, what was the cost of each yacht?
A. $8.2 million B. $16.2 million C. $16.25 million D. $16.8 million

22. 2, 3, 5, 7, 11 are all prime numbers.
Select the statement that is true for all prime numbers.
A. All prime numbers are odd numbers.
B. Prime numbers only have one factor which is 1.
C. Prime numbers have two factors, 1 and itself.
D. All prime numbers are also square numbers.

2F Perfect squares and square roots

A perfect square, also known as a square number, is the result of a number multiplied by itself. This can be represented as a base number with an index number of 2. Finding the area of a square is the same as squaring a number. Finding the square root of a number is the inverse operation of squaring a number. Finding the side length of a square with a given area is the same as finding the square root. Below are some examples where perfect squares and square roots can be applied.

- I have four geese on my farm. If each goose lays 4 eggs per year, how many eggs do my geese lay altogether every year?
- The top of my bedside table is a square and has an area of 1600 cm^2. What is the length of one side of the bedside table?
- My square picture frame has a side length of 20 cm. What is the area of my picture frame?

Learning intentions

Students will be able to:

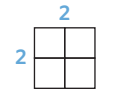
+ evaluate the square of a number
+ evaluate square roots
+ evaluate expressions with perfect squares and square roots.

Key terms and definitions

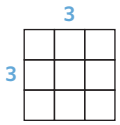
- A **perfect square** is the result of a number multiplied by itself. E.g. 4 is a perfect square because $2^2 = 2 \times 2 = 4$.
- The **square root** of a number is a value that, when multiplied by itself, gives the original number. E.g. $\sqrt{4} = 2$.

Key ideas

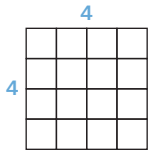
1 We can find a perfect square by multiplying a number by itself.



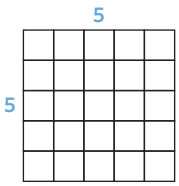
$2^2 = 2 \times 2 = 4$



$3^2 = 3 \times 3 = 9$



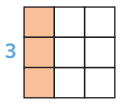
$4^2 = 4 \times 4 = 16$



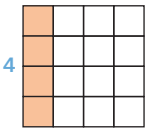
$5^2 = 5 \times 5 = 25$

2 The square root is the inverse operation of squaring a number. The square root of a number can be found by asking 'what number multiplied by itself equals' the number in the square root.

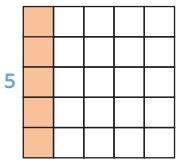
$\sqrt{4} = 2$



$\sqrt{9} = 3$



$\sqrt{16} = 4$



$\sqrt{25} = 5$

3 Perfect squares are unique because they always have an odd number of factors.

Perfect squares	Factors	Number of factors is odd
4	1, 2, 4	3 factors
9	1, 3, 9	3 factors
16	1, 2, 4, 8, 16	5 factors
25	1, 5, 25	3 factors
36	1, 2, 3, 4, 6, 9, 12, 18, 36	9 factors

Composite numbers	Factors	Number of factors is even
8	1, 2, 4, 8	4 factors
10	1, 2, 5, 10	4 factors
12	1, 2, 3, 4, 6, 12	6 factors
14	1, 2, 7, 12	4 factors
15	1, 3, 5, 15	4 factors

Worked examples

WE 1 Squaring numbers

Evaluate each squared number.

a. 5^2

Working

$5^2 = 5 \times 5$

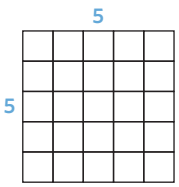
$= 25$

Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplication.

Visual support



$5^2 = 5 \times 5 = 25$

b. 11^2

Working

$11^2 = 11 \times 11$

$= 121$

Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplication.

Student practice

Evaluate each squared number.

a. 3^2 **b.** 7^2 **c.** 12^2 **d.** 19^2

WE 2 Evaluating square roots

Evaluate each square root.

a. $\sqrt{25}$

Working

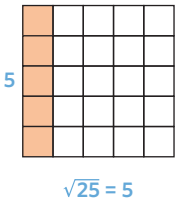
$25 = 5 \times 5$

$\sqrt{25} = 5$

Thinking

Identify which number multiplied by itself equals the number inside the square root.

Visual support



$\sqrt{25} = 5$

b. $\sqrt{400}$

Working

$\sqrt{400} = \sqrt{4} \times \sqrt{100}$

$4 = 2 \times 2$

$\sqrt{4} = 2$

$100 = 10 \times 10$

$\sqrt{100} = 10$

$\sqrt{400} = 2 \times 10$

$= 20$

Thinking

Step 1: Partition the square root into two familiar square roots.

Step 2: Identify which two numbers multiplied by itself equals the numbers inside the square roots.

Step 3: Complete the calculation to evaluate the square root.

Student practice

Evaluate each square root.

a. $\sqrt{9}$ **b.** $\sqrt{64}$ **c.** $\sqrt{1}$ **d.** $\sqrt{1600}$

WE 3 Evaluating expressions with squared numbers and square roots

Evaluate each expression.

a. $6^2 - 3^2$

Working

$6^2 - 3^2 = 6 \times 6 - 3 \times 3$

$= 36 - 9$

$= 27$

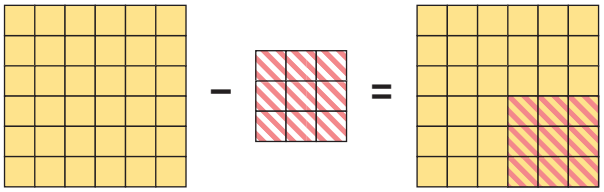
Thinking

Step 1: Write the expression in expanded form.

Step 2: Evaluate the multiplications.

Step 3: Evaluate the subtraction.

Visual support



b. $\sqrt{16} \times \sqrt{9}$

Working

$16 = 4 \times 4$

$\sqrt{16} = 4$

$9 = 3 \times 3$

$\sqrt{9} = 3$

$\sqrt{16} \times \sqrt{9} = 4 \times 3$

$= 12$

Thinking

Step 1: Evaluate the first square root by identifying which number multiplied by itself equals the number inside the square root.

Step 2: Evaluate the second square root by identifying which number multiplied by itself equals the number inside the square root.

Step 3: Evaluate the multiplication.

Student practice

Evaluate each expression.

a. $7^2 - 4^2$ **b.** $\sqrt{36} \times \sqrt{25}$ **c.** $\sqrt{0} \times 11^2 + 4^2$ **d.** $\sqrt{5^2} \times 5$

2F Activities and questions


Image: Joe Armao/The Age

STARTER TASKS

Odd spot

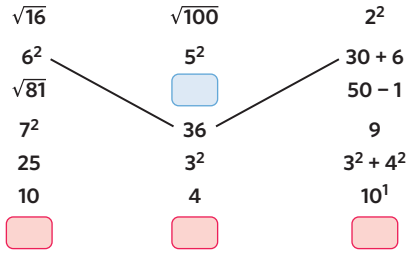
The University of Melbourne and the Australian Research Council Centre of Excellence for Dark Matter Particle Physics are building a new physics laboratory underground. The laboratory will be located in the Stawell gold mine (pictured) and is 1000 metres deep. The construction of the laboratory will be complete at the end of 2021 and it is the first laboratory of its kind in Australia.

What is the square root of the mine's depth?

A. 500 m **B.** 31.6 m

Puzzle

The values in each column can be matched to a value in each of the other columns. The black line shows one set of numbers that are matching.



a) Draw a line between each column to complete the matching process.

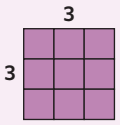
b) What number is missing from the blue box?

c) Using the red boxes, write three different values that match.

Understanding worksheet

1. Fill in the boxes to complete each calculation.

Example

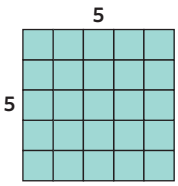


$\boxed{3}^2 = 3 \times 3 = 9$

a.

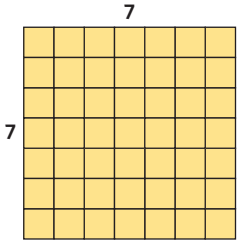
$\square^2 = 2 \times 2 = 4$

b.



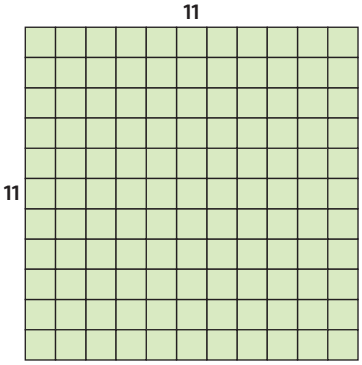
$\square^2 = 5 \times 5 = 25$

c.



$\square^2 = 7 \times 7 = 49$

d.



$\square^2 = 11 \times 11 = 121$

2. List the factors in ascending order to determine whether each number is a perfect square.

Example

Factors of 9: 1, [3], 9

Numbers of factors: (Odd) Even

Is 9 a perfect square? (Yes) No

a. Factors of 16: 1, 2, [ ], 8, 16

Number of factors: Odd Even

Is 16 a perfect square? Yes No

b. Factors of 20: 1, 2, 4, [ ], 10, 20

Number of factors: Odd Even

Is 20 a perfect square? Yes No

c. Factors of 77: 1, 7, [ ], 77

Number of factors: Odd Even

Is 77 a perfect square? Yes No

d. Factors of 100: 1, 2, 4, 5, [ ], 20, 25, 50, 100

Number of factors: Odd Even

Is 100 a perfect square? Yes No

3. Fill in the blanks by using the words provided.

odd | inverse | two | itself | base

Squaring a number is the same as multiplying a number by [ ]. Perfect squares can be represented as a [ ] number with an index number of [ ].

Finding the square root of a number is the [ ] operation of squaring a number.

Perfect squares are unique because they always have an [ ] number of factors.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7 (e,f,g,h)

WE1 **4.** Evaluate each squared number.

a. 1^2 **b.** 0^2 **c.** 6^2 **d.** 8^2

e. 11^2 **f.** 13^2 **g.** 24^2 **h.** 100^2

WE2a **5.** Evaluate each square root.

a. $\sqrt{4}$ **b.** $\sqrt{16}$ **c.** $\sqrt{1}$ **d.** $\sqrt{49}$

e. $\sqrt{100}$ **f.** $\sqrt{0}$ **g.** $\sqrt{121}$ **h.** $\sqrt{225}$

WE2b **6.** Evaluate each square root.

a. $\sqrt{900}$ **b.** $\sqrt{2500}$ **c.** $\sqrt{6400}$ **d.** $\sqrt{10\,000}$

e. $\sqrt{12\,100}$ **f.** $\sqrt{14\,400}$

WE3 **7.** Evaluate each expression.

a. 2×2^2 **b.** $5^2 + 6^2$ **c.** $\sqrt{9} \times 9$ **d.** $\sqrt{36} - \sqrt{9}$

e. $\sqrt{144} + 5^2$ **f.** $\sqrt{49} \times \sqrt{49}$ **g.** $10^2 \div 5^2 \div 2^2$ **h.** $\sqrt{13^2 - 12^2}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Chess is played with a square board. How many spaces are there on a chessboard in total if each side of a chessboard is made up of 8 alternating black and white spaces?

9. All 7 members of the Millstone family love their Kindle reader so much that each of them convinced 7 of their friends to also purchase a Kindle reader. How many people did the Millstone family convince to purchase a Kindle reader?

10. William is cutting up a large block of cheese. If each of the block's faces is a square with an area of 81 cm^2, what is the length of each edge of the cheese?

11. The Big Ben is a famous clock tower in London that was built in 1859. The clock's face is a circle and sits inside a square border. What is the perimeter of the square's border if its area is 144 m^2?

12. Farah is creating the Australian flag out of Lego pieces. The design is a rectangle made up of two side-by-side squares. How many pieces long is the flag if he uses 121 pieces to make each square?

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Brenda's job involves designing escape rooms. She is designing three layouts for a new escape room centre, Experience Escape. Experience Escape wants different sized rooms for different puzzles. All the rooms at Experience Escape are square shaped.
    a. The 'Vault Escape' room has a side length of 7 m. What is the area of the 'Vault Escape' room?
    b. The area of two 'Sherlock's Puzzle' rooms combined is 23 m^2 larger than the 'Vault Escape' room. What is the length of one of the sides of the 'Sherlock's Puzzle' room?
    c. Brenda wants to build a square 'Treasure Heist' room and for it to have the same total area as the 'Vault Escape' and two 'Sherlock's Puzzle' rooms combined. What is the perimeter of the 'Treasure Heist' room?
    d. If the 'Vault Escape' room, two 'Sherlock's Puzzle' rooms, and the 'Treasure Heist' room are built side by side, what is the total length from the start of the first room to the end of the last room?
    e. How would you design the layout of the four escape rooms differently?

14. The Palace is a new luxury dog day care centre. The Palace includes grooming services, specialist dog trainers and a world class doggie day spa.
    a. The Palace's doggie day spa is a small square room that includes a special dog sauna. The room's side length is 300 cm. What is the area of the doggie day spa room?
    b. Tony, the manager at The Palace, is preparing a new dog play centre. In order to do so, Tony will have to prepare 256 m^2 of square land. What is the side length of the play centre in cm?
    c. The Palace's new regional site is square-shaped and has an area of 498 m^2. The side length is a decimal value. Between which two whole numbers does this decimal lie?
    d. What should someone consider before sending their dogs to The Palace?

Extra spicy

15. Marge just celebrated her 121st birthday. Her age went from a multiple of 5 to a square number. Including her 121st birthday, how many times has this happened in Marge's life?
    A. 1 B. 2 C. 3 D. 4

16. How many of the expressions are perfect squares?
    $1^3 + 2^3$ $1^{10} + 2^3 + 4^2$ $3^3 - 2^3 - 3^1$ $5^3 + 4^3 + 3^3 + 2^3 + 1^5$

17. If ★ is a perfect square, which option can only represent an odd value?
    A. $\sqrt{★}$ B. $2 \times ★$ C. $4 \times ★ - 1$ D. $★^2$

18. $\sqrt{10} \times \sqrt{15} \times \sqrt{20}$ is closest to:
    A. 20 B. 30 C. 40 D. 50

Remember this?

19. Which number sentence is true when the ? is replaced with 10?
    A. $50 \div ? = 10$ B. $? + 15 = 35$ C. $15 - ? = 5$ D. $5 \times 3 = ?$

20. Jules went to sleep at 8:30 pm. She woke up 11 and a half hours later. What time did she wake up?
    A. 7:30 am B. 8:00 am C. 8:30 am D. 9:00 am

21. Malka is listing all the prime numbers between 40 and 50.
    She lists: 41 and 47.
    Which number is she missing?

2G Introduction to integers

Integers are similar to whole numbers but also include negative numbers. Integers do not include fractions or decimals. Positive integers are greater than zero and negative integers are less than zero. To denote a positive or negative number, we can use the + or − signs. Integers with no sign are positive. Below are some examples where integers can be applied.

- I am scuba diving and am currently at a depth of 15 m below sea level. Represent this as an integer.
- When I was in Sochi, the temperature was −4°C at 3 am and 10°C at 3 pm. What is the difference in temperature between 3 am and 3 pm?
- If I worked 39 hours in May and 41 hours in June, in which month did I work more hours?

Learning intentions

Students will be able to:

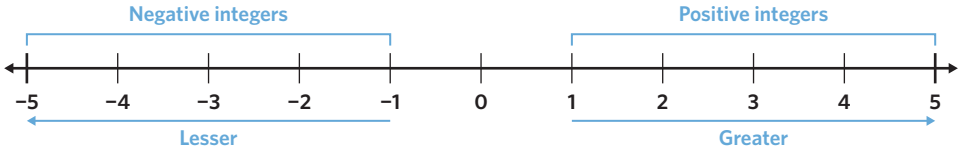
- understand positive and negative integers
- find the difference between integers
- compare two integers and order a set of integers in ascending or descending order.

Key terms and definitions

- An **integer** is a whole number that is either positive, negative or zero.
- **Positive numbers** are numbers with a value greater than zero.
- **Negative numbers** are numbers with a value that is less than zero.
- **Ascending** order means to arrange values from smallest to largest.
- **Descending** order means to arrange values from largest to smallest.
- **Opposite numbers** are numbers in the inverse position on the number line. The sum of opposite numbers is always zero.

Key ideas

1 Integers can be represented on a number line. Positive integers are numbers that are greater than zero. Negative integers are numbers that are less than zero.

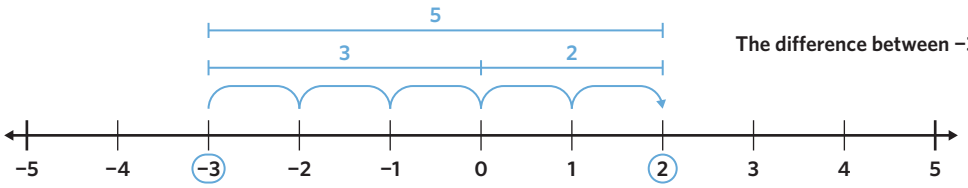


2 We can compare integers using < (less than) or > (greater than) symbols.

−5 < 5 because −5 is further to the left on the number line.

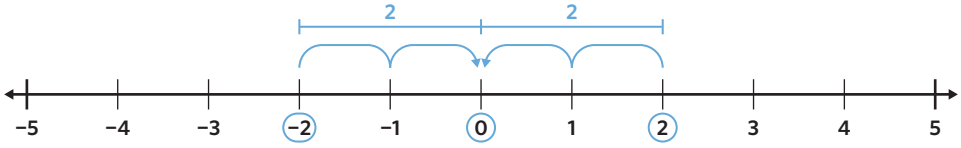
3 > −3 because 3 is further to the right on the number line

3 We can find the difference between integers using the number line. The difference between two integers is the distance between the numbers on the number line.



The difference between −3 and 2 is 5.

4 Opposite integers have the same distance to zero on the number line. The sum of opposite numbers is always zero.



Both −2 and 2 share the same distance to 0 on the number line, they are opposite numbers.

Worked examples

WE 1 Using integers to represent situations

State the integer that represents each description.

a. Jeffrey grew 7 cm this year.

Working

Jeffrey has grown. His height has increased so the integer will be positive.

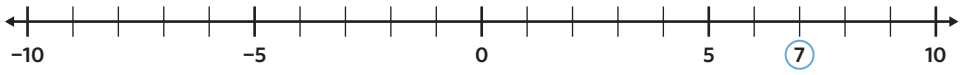
7

Thinking

Step 1: Identify whether the integer is positive or negative.

Step 2: State the integer.

Visual support



b. Susan is scuba diving 17 m below sea level.

Working

Susan is below sea level so the integer will be negative.

−17

Thinking

Step 1: Identify whether the integer is positive or negative.

Step 2: State the integer.

Student practice

State the integer that represents each description.

a. Grandma shrunk 2 cm this year.

b. The temperature is 8 degrees celsius.

c. James lives on the 14^{th} floor of the building.

d. I owe $55 to the bank.

WE 2 Finding the difference between integers

Find the difference between each pair of integers.

a. −1 and 4

Working

There is 1 unit between −1 and 0.

There are 4 units between 0 and 4.

1 + 4 = 5

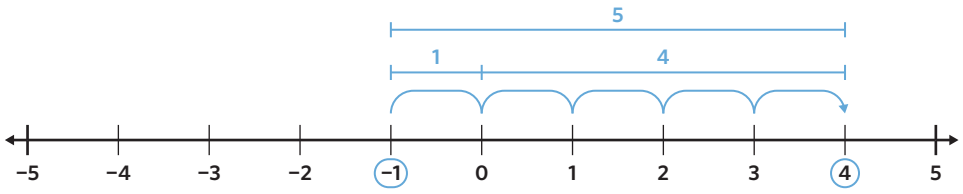
Thinking

Step 1: Count the units between −1 and 0.

Step 2: Count the units between 0 and 4.

Step 3: Sum the differences to find the total number of units between the two integers.

Visual support



b. −15 and −3

Working

There are 12 units between −15 and −3.

12

Thinking

Step 1: Count the units between −15 and −3.

Step 2: State the difference.

Student practice

Find the difference between each pair of integers.

a. −2 and 5 **b.** −10 and −1 **c.** 0 and 21 **d.** 4 and −13

WE 3 Comparing two integers

Place the correct mathematical symbol (<, >) in the box to compare the integers.

a. −5 ☐ 12

Working

−5 is negative and 12 is positive.

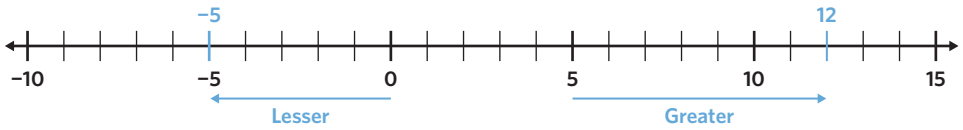
−5 < 12

Thinking

Step 1: Identify whether the integers are positive or negative.

Step 2: Compare the integers. All positive integers are greater than all negative integers.

Visual support



b. −13 ☐ −31

Working

Both of the integers are negative.

−13 > −31

Thinking

Step 1: Identify whether the integers are positive or negative.

Step 2: Identify the lesser value. Since they are both negative, the lesser value is the one further away from zero.

Student practice

Place the correct mathematical symbol in the box to compare the integers (<, >).

a. −10 ☐ 10 **b.** −9 ☐ −5 **c.** 23 ☐ 17 **d.** 5 ☐ −500

2G Activities and questions

STARTER TASKS

Odd spot

Kinetic is a Melbourne-based public transport company that operates a fleet of buses. They are shifting away from high emission vehicles, such as petrol-based buses to low or no emission vehicles, such as electric and hybrid vehicles.
By 2031, Kinetic will replace 341 out of the 537 high emission vehicles with electric or hybrid vehicles.

Select the option which represents the change in high emission vehicles.

A. -341 **B.** 341

Image: Joe Armao/The Age

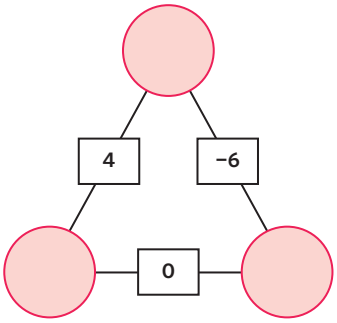
Puzzle

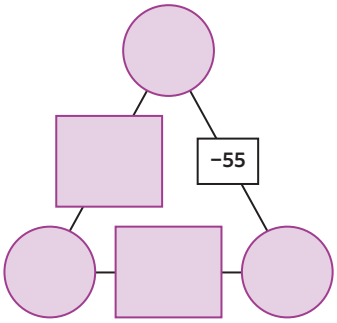
The rectangular boxes represent the number halfway between the two connecting circles.

a) Using the numbers -10, -2, and 10 complete the missing numbers in the red circles.

b) Using the numbers, -100, -75, -50, -30, -10 complete the purple circles and rectangles.

c) How can there be two possible solutions to part **b**?

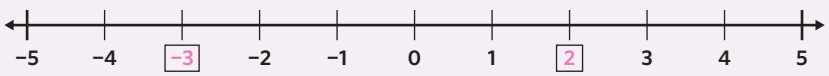




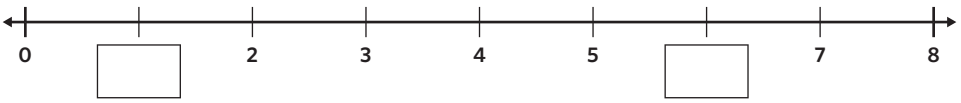
Understanding worksheet

1. Complete the number lines by placing the correct integer in each box.

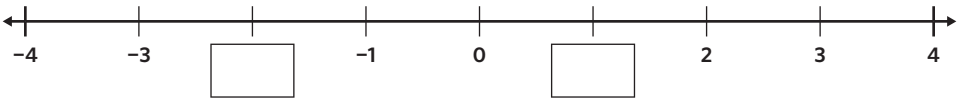
Example



a.

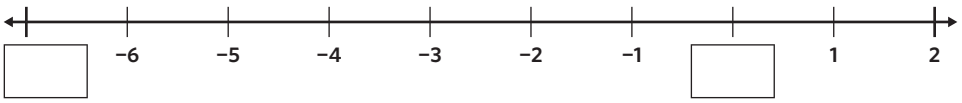


b.



c.

d.

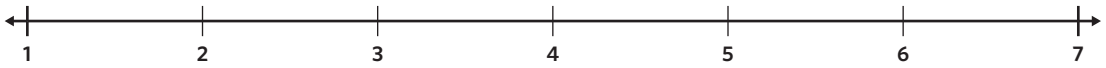


2. Circle the two integers on the number line and place the correct mathematical symbol in the box to compare them (< or >).

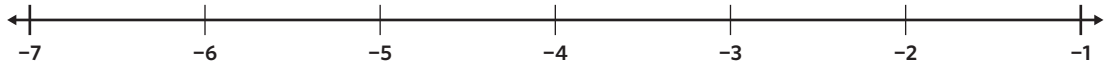
Example

$-2 \boxed{<} 2$

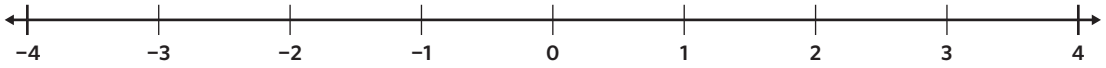
a. $5 \square 7$



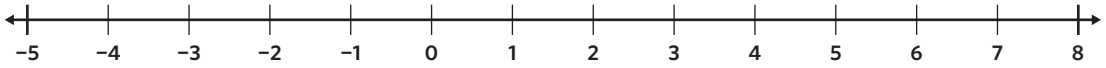
b. $-1 \square -7$



c. $4 \square 0$



d. $3 \square -4$



3. Fill in the blanks by using the words provided.

negative | less | greater | positive

Number lines can help us visualise whether a number is a _____ or negative integer. Numbers that are _____ than or to the right of zero on the number line are positive. Numbers that are _____ than or to the left of zero on the number line are _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c), 9 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f), 8 (b,c,d), 9 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h), 8 (d,e,f), 9 (d,e,f)

WE1 **4.** State the integer that represents each description.

a. The temperature is 23 degrees celsius.

b. Maria spent $10 dollars at the toy store.

c. It is negative 5 degrees celsius in Sapporo.

d. Camilla earned $500 at work this week.

e. A whale is swimming 1757 metres below the sea level.

f. Mount Buller has an elevation of 1805 metres.

5. Write the opposite integer for each integer.

a. 3 **b.** 21 **c.** −7 **d.** −33

e. 101 **f.** −1000

WE2 6. Find the difference between each pair of integers.

a. 2 and 10
b. −6 and −2
c. −1 and 1
d. −11 and −4
e. −10 and 6
f. 0 and −64
g. −25 and −9
h. −18 and 23

WE3 7. Place the correct mathematical symbol (<, >) in the box to compare the integers.

a. 7 ☐ 3
b. 21 ☐ 30
c. −4 ☐ −1
d. −11 ☐ 2
e. 4 ☐ −9
f. −8 ☐ 0
g. −121 ☐ −303
h. 0 ☐ −1001

8. Place each set of integers in ascending order.

a. 3, 1, 19, 11, 6
b. −17, −13, −2, −11, −3
c. −8, 0, 3, 5, −2
d. −3, 4, 3, −4, 0
e. 1, −11, 111, −1, −111
f. 99, 9, −999, 9999, 0

9. Place each set of integers in descending order.

a. 7, 29, 15, 3, 28
b. −17, −31, −28, −29, −18
c. −21, 17, 0, 3, −4
d. −2, 2, −1, 1, 0
e. 5, −15, 25, −5, −10
f. −2, 22, −222, −2222, 22 222

Problem solving

Mild	Medium	Spicy
10, 11, 12	11, 12, 13	12, 13, 14

10. Mariana Trench is the deepest natural point in the world. The bottom of the trench is 11 034 m below sea level. What is the integer that represents the depth of Mariana Trench?

11. The year 7 geography exam had 100 questions, each worth 1 mark. James got 11 questions incorrect, Zephyr got 10 questions incorrect, and Sophie got 88 questions correct. Who scored the lowest mark out of James, Zephyr and Sophie?

12. Order the following landmarks in ascending order based on their heights. Mount Everest has an elevation of 8848 m. Mount Fuji has an elevation of 3776 m. The bottom of the Dead Sea has an elevation of −430 m and Death Valley has an elevation of −86 m. The summit of El Capitan is 2307 m.

13. Melanie and Alex are both completing solo sailing trips. What is the distance between their anchoring locations if Melanie has anchored exactly 11 km north of Kangaroo Island and Alex has anchored exactly 23 km south of Kangaroo island?

14. Kendall is currently staying in Chicago for a business trip. On Monday it was −6°C, on Tuesday it was −15°C, on Wednesday it was 2°C, on Thursday it was 5°C and on Friday it was −9°C. What was the difference in temperature between the warmest day and the coldest day for that week?

Reasoning

Mild	Medium	Spicy
15 (a,b,c,d)	15 (a,b,c,d), 16 (a,b,c,d)	All

15. The following bank statement shows the balance of Andrea's credit account between the months of January and June 2021.

a. Identify the month in which Andrea ended with $35 in her account.
b. At the end of which month was Andrea's account balance at its lowest?
c. How much money did Andrea deposit into her account during the month of June? Assume that she spent no money from this account in June.
d. During which month did Andrea's account balance decrease the most?
e. Do you think it is important to save money? What are some strategies Andrea could use to help her save?

Month	Account balance (at end of month)
January	$35
February	−$38
March	$70
April	−$10
May	−$35
June	$53

16. Time zones are the local time of a region or a country and they help coordinate communities around the world. The most common time standard is the Coordinated Universal Time or UTC. UTC+10 means it is 10 hours ahead of the UTC time.

The following table shows the time zones in relation to UTC of different countries.

City	UTC
London	UTC+0
Melbourne	UTC+10
New York	UTC−5
Vancouver	UTC−8
Rome	UTC+1

a. On the following number line, label the cities according to their time zone in relation to UTC.

b. What is the time difference between Melbourne and New York?

c. If it is 10:00 am in London, what time is it in Melbourne?

d. If it is 1:30 pm in Rome, what time is it in Vancouver?

e. What might happen if every country in the world used a universal time rather than regional time zones (e.g. it was 12 pm everywhere in the world)?

Extra spicy

17. What is the difference between 3776 and −400?

18. Two consecutive even integers have a sum of 82. What are the two integers?

19. During a scuba dive, Smyrna descended 29 m, then ascended 11 m and finally descended 22 m. A single move that would get Smyrna to this same position would be to

A. descend 22 m. **B.** descend 39 m. **C.** descend 40 m. **D.** descend 51 m.

20. Which of the following is the furthest to the right on the number line?

A. −1599 **B.** 599 less than −900

C. 599 more than −2100 **D.** 2100 less than 599

Remember this?

21. Wendy is baking some brownies. She uses 75 grams of sugar per batch of brownies. If she made 5 batches of brownies, how many grams of sugar did she use altogether?

A. 150 grams **B.** 225 grams **C.** 350 grams **D.** 375 grams

22. Bobby's vintage car was manufactured in 1953. Approximately how old was Bobby's car in years when he purchased it in 2021?

A. 50 **B.** 60 **C.** 70 **D.** 80

23. Michael wants to purchase a laptop that is currently half price. He also has a $200 coupon that he can redeem. If the laptop was originally $1800, how much will he end up paying?

A. $600 **B.** $700 **C.** $800 **D.** $900

2H Adding and subtracting positive integers

Positive integers are added in the same way as whole numbers. Adding and subtracting positive integers can result in both positive and negative answers. A number line is a helpful way to understand how to add or subtract positive integers. Below are some examples where adding and subtracting positive integers are used.

- The temperature in my freezer is −10 degrees Celsius. I forget to close the door and the temperature rises 7 degrees Celsius. What is the temperature of my freezer now?
- I owe the school canteen $15. If I paid them $5 today, how much money do I still owe the canteen?

Learning intentions

Students will be able to:

+ use a number line to add and subtract positive integers
+ add a positive integer to any integer
+ subtract a positive integer from any integer.

Key terms and definitions

- An **integer** is a whole number that is either positive, negative or zero.
- **Negative numbers** are numbers with a value that is less than zero.
- **Positive numbers** are numbers with a value greater than zero.

Key ideas

1 We can use 0 as a benchmark number on the number line when adding and subtracting integers.

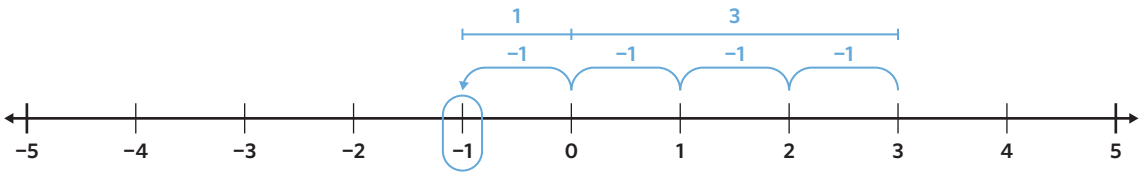
$3 - 4$

Move 3 units to the left to reach 0 on the number line.

Move the remaining 1 unit to the left to reach the final answer.

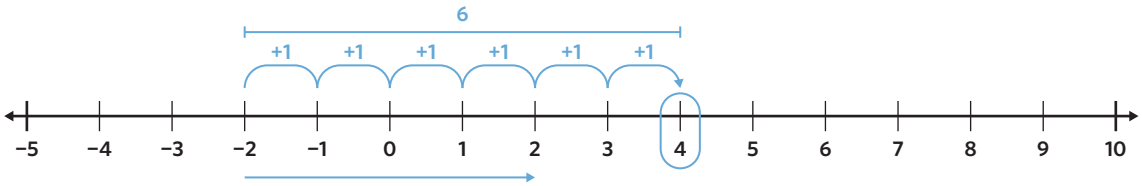
$3 - 4 = 3 - 3 - 1$

$= -1$



2 Adding positive integers to negative integers can be shown using a number line.

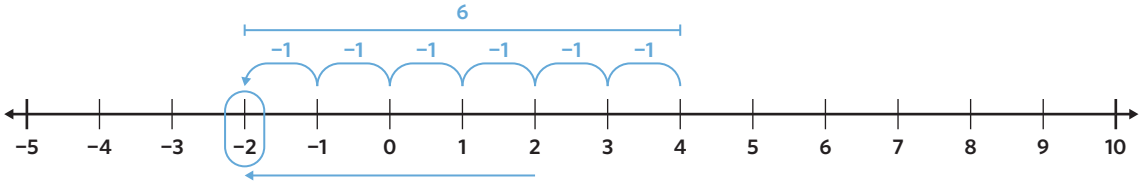
$-2 + 6 = 4$



Adding a positive integer increases the value of the number, so we move to the right on the number line.

3 Subtracting positive integers that result in negative number solutions can be shown using a number line.

$4 - 6 = -2$



Subtracting a positive integer decreases the value of the number, so we move to the left on the number line.

Worked examples

WE 1 Adding positive integers

Evaluate each expression.

a. $-5 + 7$

Working

$-5 + 7 = -5 + 5 + 2$

$= 0 + 2$

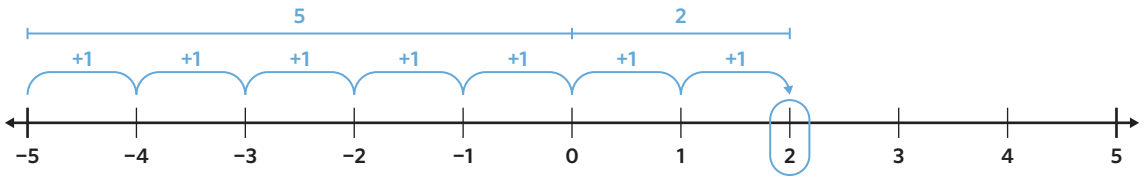
$= 2$

Thinking

Step 1: Partition 7 into 5 and 2 in order to use 0 as the benchmark number.

Step 2: Add 5 to -5 to give 0. Add the remaining 2 units to 0.

Visual support



b. $-12 + 4$

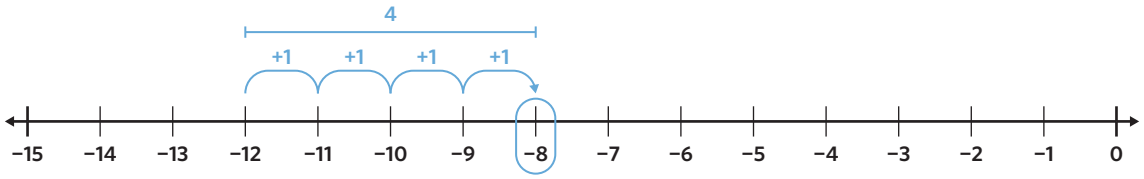
Working

$-12 + 4 = -8$

Thinking

4 is a being added to -12 so move in 4 units in the positive direction towards zero.

Visual support



Student practice

Evaluate each expression.

a. $-3 + 4$ **b.** $-10 + 3$ **c.** $-5 + 16$ **d.** $-100 + 40$

WE 2 Subtracting positive integers

Evaluate each expression.

a. $3 - 8$

Working

$3 - 8 = 3 - 3 - 5$

$= 0 - 5$

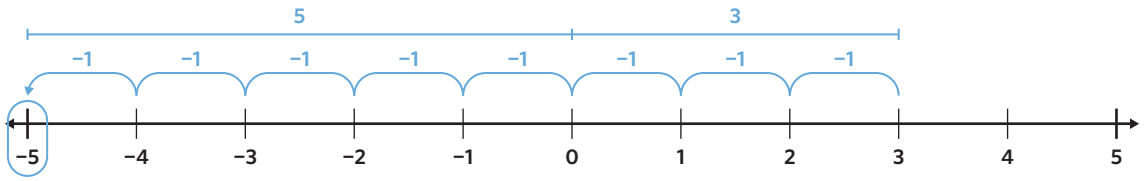
$= -5$

Thinking

Step 1: Partition 8 into 3 and 5 in order to use 0 as a benchmark number.

Step 2: Subtract 3 from 3 to give 0. Subtract the remaining 5 units from 0.

Visual support



b. $-4 - 7$

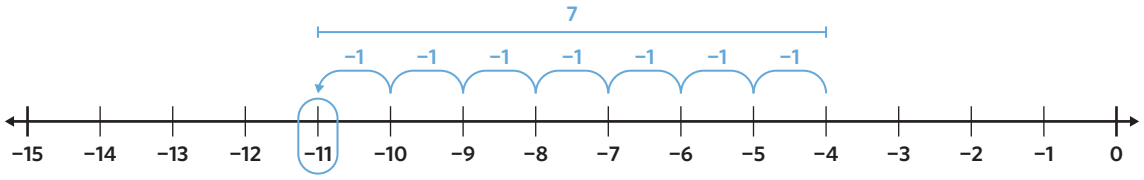
Working

$-4 - 7 = -11$

Thinking

7 is being subtracted from −4 so move 7 units in the negative direction, away from 0.

Visual support



Student practice

Evaluate each expression.

a. $7 - 15$ **b.** $-4 - 5$ **c.** $9 - 20$ **d.** $-5 - 15$

WE 3 Adding and subtracting multiple positive integers

Evaluate each expression.

a. $20 - 3 + 5 - 8$

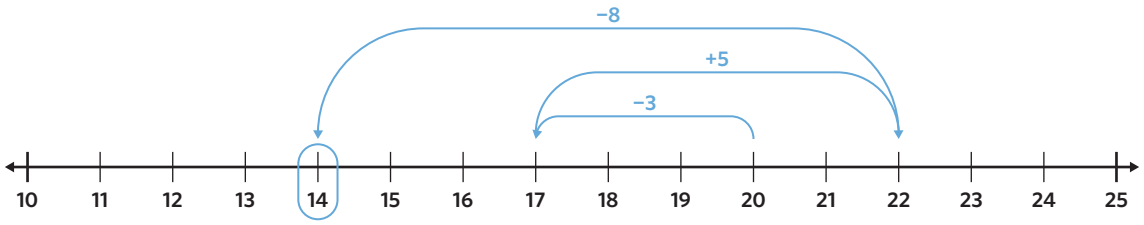
Working

$20 - 3 + 5 - 8 = 17 + 5 - 8$
$= 22 - 8$
$= 14$

Thinking

Complete each operation from left to right.

Visual support



b. $-4 + 16 - 7 + 3$

Working

$-4 + 16 - 7 + 3 = 12 - 7 + 3$
$= 5 + 3$
$= 8$

Thinking

Complete each operation from left to right.

Student practice

Evaluate each expression.

a. $18 - 7 + 3 - 2$ **b.** $-15 + 6 - 3 + 4$ **c.** $12 - 6 + 4 - 7$ **d.** $-15 + 2 - 6 + 14$

2H Activities and questions

STARTER TASKS

Image: Shin Okamoto/Shutterstock.com

Odd spot

Freedivers dive in the ocean as deep as possible without an oxygen tank. The Austrian Freediver Herbert Nitsch is the world record holder for the deepest freedive and is also known as 'the deepest man on Earth'. In 2006 he set the record by diving 183 metres below sea level. In 2007 he set a new record when he descended 31 metres deeper.

What was Herbert's new record in 2007?

A. 214 metres below sea level **B.** 152 metres below sea level

Puzzle

When adding a positive number, move to the left on the number line.	A negative number plus a positive number equals a negative number.
Any addition calculation gives a positive answer.	Any subtraction calculation gives a negative answer.
Adding a positive number is the same as subtracting a positive number.	Zero is always to the right of negative numbers on a number line.
Zero is always to the left of positive numbers on a number line.	A negative number plus a positive number equals a positive number.

a) For each statement, state whether it is always true, sometimes true or never true.

b) Form a statement relating to adding and subtracting positive integers that is always true.

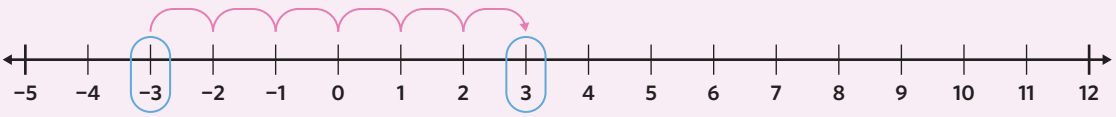
c) Form a statement relating to adding and subtracting positive integers that is sometimes true.

Understanding worksheet

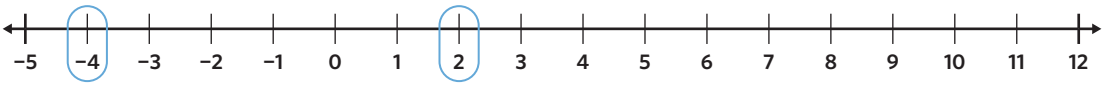
1. Complete the addition calculation by showing the 'jumps' on the number line.

Example

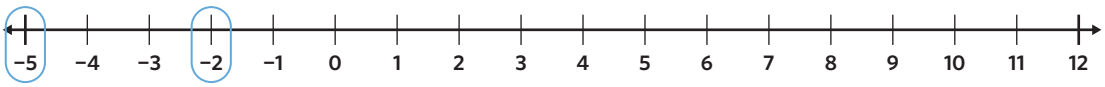
$-3 + 6 = 3$



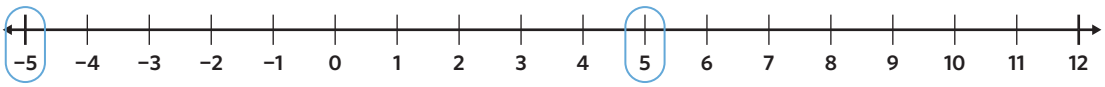
a. $-4 + 6 = 2$



b. $-5 + 3 = -2$



c. $-5 + 10 = 5$



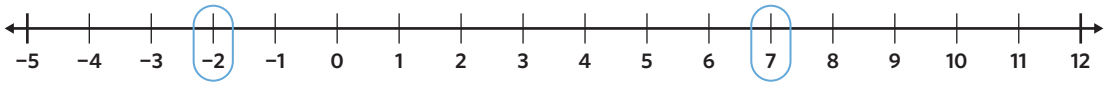
d. $-1 + 12 = 11$

2. Complete the subtraction calculation by showing the 'jumps' on the number line.

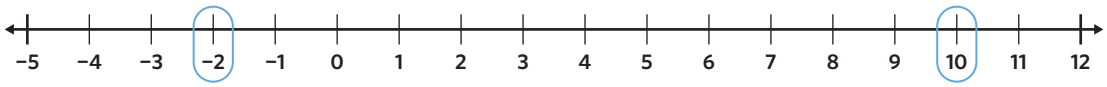
Example

$2 - 3 = -1$

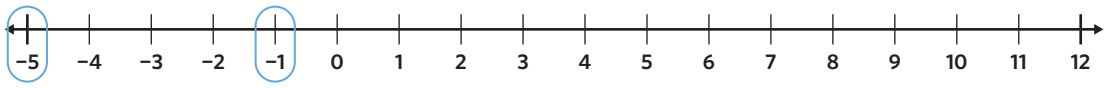
a. $7 - 9 = -2$



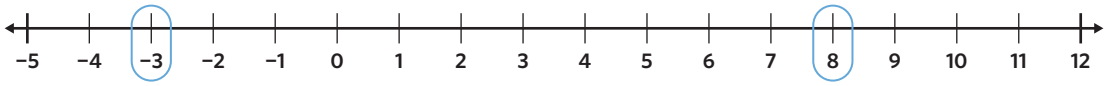
b. $10 - 12 = -2$



c. $-1 - 4 = -5$



d. $8 - 11 = -3$



3. Fill in the blanks by using the words provided.

negative | greater | positive | decreases

Adding _____ integers to a number makes the number _____. Subtracting a positive integer from a number _____ the value of the number. Adding and subtracting positive integers can result in a _____ answer.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (b,c,d), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7 (d,e,f), 8 (e,f,g,h)

WE1 **4.** Evaluate each expression.

a. $-2 + 3$ **b.** $-1 + 4$ **c.** $-1 + 7$ **d.** $-5 + 3$

e. $-11 + 7$ **f.** $-12 + 5$ **g.** $-40 + 20$ **h.** $-100 + 100$

WE2 **5.** Evaluate each expression.

a. $8 - 9$ **b.** $12 - 5$ **c.** $5 - 7$ **d.** $-2 - 5$

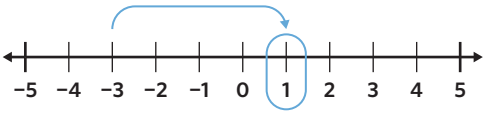
e. $-4 - 6$ **f.** $-8 - 3$ **g.** $12 - 15$ **h.** $-50 - 40$

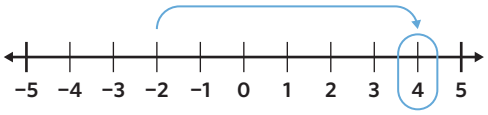
6. Without completing the calculation, state whether each expression will result in a positive or negative integer.

a. $-7 + 3$ **b.** $-2 + 5$ **c.** $-4 + 2$ **d.** $-6 + 3$

e. $-5 + 6$ **f.** $-9 + 10$

7. Write the number sentence that describes each calculation shown on the number line.

a.


b.


c.


d.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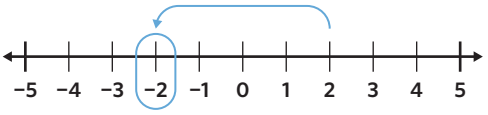

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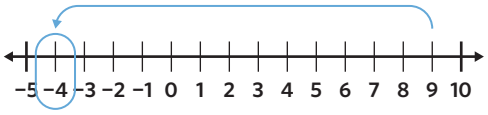


WE3 8. Evaluate each expression.

a. $12 + 3 - 4 - 2$ b. $19 - 4 + 6 - 1$ c. $-10 + 6 - 4 + 5$ d. $14 - 7 + 6 - 9$

e. $-12 + 5 - 3 + 11$ f. $17 - 4 + 12 - 6$ g. $-45 + 12 - 205 + 6$ h. $82 - 308 + 12 - 72$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. The temperature in the morning in Alice Springs is −5 degrees Celsius. The temperature is expected to rise by 35 degrees Celsius by the afternoon. What is the expected temperature in the afternoon?

10. What distance does an Olympic platform diver traverse if she jumps from a 10 metre high platform and touches the bottom of the pool that is 5 metres deep?

11. Talia parked her car three levels below the ground level of her apartment building. What level does she live on if she takes the elevator 14 levels up to from the car park to reach her apartment?

12. Anna used her credit card and has a bank balance of −$120. What is her balance after she repays $55?

13. Melbourne is known for having four seasons in a day. One day the temperature was 5 degrees Celsius in the morning and increased by 12 degrees Celsius by noon. In the afternoon the temperature dropped to 4 degrees Celsius and by midnight the temperature was −2 degrees Celsius. What was the difference between the day's high and low temperature?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. For a home science project, Kai was assigned to make ice cream in a bag. He received the following instructions from his teacher.

a. The temperature of the ice was −8 degrees Celsius when Kai removed it from the freezer. The temperature of the ice increases by 1 degree Celsius each minute it is outside the freezer. What is the temperature of the ice slush after 14 minutes?

b. After leaving the ice out of the freezer for 14 minutes, Kai adds the salt to the ice. The temperature of the ice slush decreases by 2 degrees Celsius every minute. How long will it take for the temperature of the ice slush to reach 0 degrees Celsius?

Science project
1. *Add to a medium size zip lock bag:*
   - *2 tablespoons of sugar*
   - *1 cup of cream*
   - *1 teaspoon vanilla extract*
2. *Seal the bag.*
3. *In a large size zip lock bag, add 3 cups of ice and a third of a cup of salt.*
4. *Place the medium size bag in the large bag and shake until the contents of the medium bag becomes solid ice cream.*

c. Once the temperature reaches 0 degrees Celsius, Kai places the medium bag into the larger bag with the ice slush. After 5 minutes of shaking the bag, the ice cream is ready. The ice has turned into water and the temperature of the water is −6 degrees Celsius. How much did the temperature of the ice vary from the beginning to the end of the experiment, adding together all the rise and falls in temperature?

d. If it is cheaper for Kai to buy the ingredients to make homemade ice cream, should Kai always make ice cream rather than buy it from a store?

15. Japan is an island country comprising a stratovolcanic archipelago of 6852 islands. Some of the highest and lowest altitudes are listed in the following table.

Location	Mount Fuji	Hachirogata	Hachinohe Mine
Altitude	3776 m	−4 m	−170 m

a. Hachirogata is the lowest natural point in Japan and Mount Fuji is the highest natural point. What is the vertical distance between these two extreme points?

b. Mount Fuji is known for its temperamental weather conditions which makes it a very difficult mountain to climb. In summer, the temperature at the base of Mount Fuji can reach 41 degrees Celsius while the temperature at the summit is −13 degrees Celsius. What is the potential difference in temperature between the base and summit?

c. Hachinohe Mine is a limestone quarry in the northern region of Japan. If there is a proposal to mine the quarry that would cause the altitude to decrease at a rate of 250 cm a year, what would the altitude of the mine be in 10 years?

d. Why might mining and quarrying be both beneficial and harmful to the environment?

Extra spicy

16. Complete with + or − signs to make the statement true.

$2 \square 4 \square 6 \square 3 = 10 - 5 - 3 - 1$

17. What are the next two numbers in the pattern?

406, 305, 203, 100, ...

18. What is the value of $3007 - 7003$?

19. What is the value of $[(2 - 2) - 2] - [2 - (2 - 2)]$?

A. −4 **B.** −2 **C.** 0 **D.** 4

Remember this?

20. One hundred and twenty-nine lies between which two whole numbers?

A. 128 and 130 **B.** 129 and 130 **C.** 127 and 128 **D.** 125 and 129

21. Which angle is closest to a right angle?

A. Angle A **B.** Angle B **C.** Angle C **D.** Angle D

22. Ben is booking tables at a restaurant for his family reunion. Each table can seat 6 people. If there are 126 people attending the family reunion, how many tables does Ben need to set up?

A. 12 **B.** 21 **C.** 22 **D.** 25

2I Adding and subtracting negative integers

Adding or subtracting a negative integer is different from adding and subtracting positive integers. Adding a negative integer is the same as subtracting a positive integer. Subtracting a negative integer is the same as adding a positive integer. Below are some examples where adding and subtracting a negative integer can be applied.

- One of my bank balances is −$10, and another is −$5. What is the total of my two bank balances?
- When I started skiing at noon the temperature in Mt Buller was −2 degrees. By 8 pm the temperature was −8 degrees. What is the difference between the temperature at noon and 8 pm?

Learning intentions

Students will be able to:

- use a number line to add and subtract negative integers
- know that adding a negative integer is equivalent to subtracting a positive integer
- know that subtracting a negative integer is equivalent to adding a positive integer
- add and subtract negative integers.

Key terms and definitions

- **Opposite numbers** are numbers in the inverse position on the number line. The sum of two opposite numbers is always zero.
- An **integer** is a whole number that is either positive, negative or zero.
- **Negative numbers** are numbers with a value that is less than zero.
- **Positive numbers** are numbers with a value greater than zero.

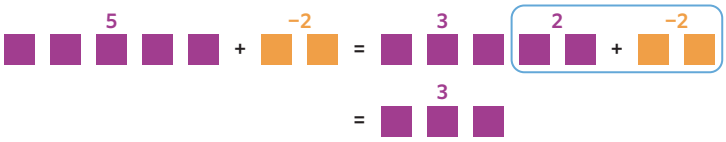
Key ideas

1 The sum of opposite numbers is always zero. We can use opposite numbers to make calculations easier.

Each purple tile is equal to +1, and each orange tile is equal to −1.

$5 + (-2) = 3 + 2 - 2$

$= 3$

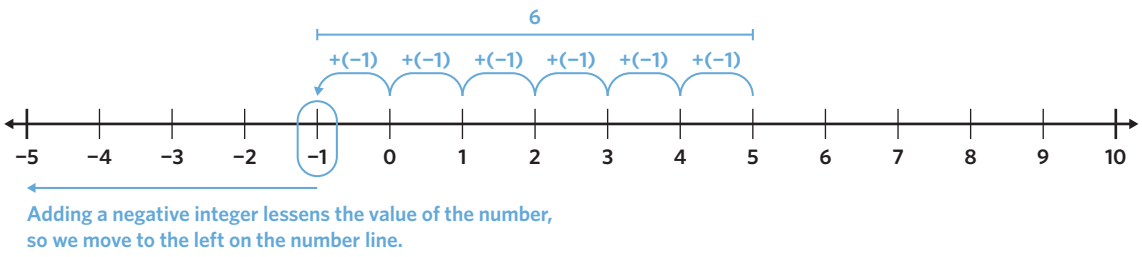


Note: Brackets are used around negative numbers to show that the '−' sign is not a subtraction.

2 Adding a negative integer is the same as subtracting a positive integer.

$5 + (-6) = 5 - 6$

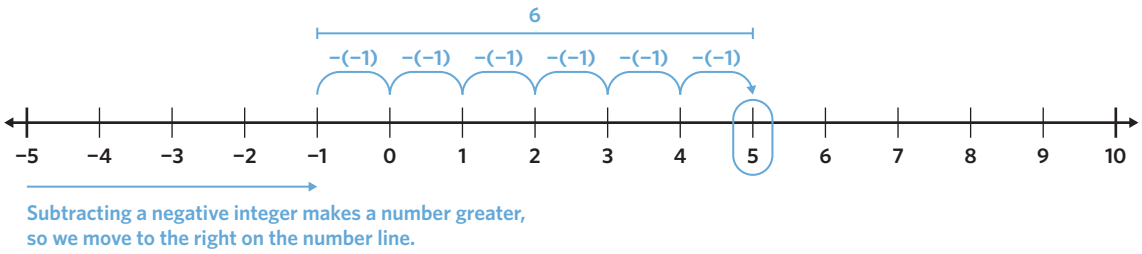
$= -1$



Note: Brackets are used around negative numbers to show that the '−' sign is not a subtraction.

3 Subtracting a negative integer is the same as adding a positive integer.

$-1 - (-6) = -1 + 6$
$= 5$



Subtracting a negative integer makes a number greater, so we move to the right on the number line.

Note: Brackets are used around negative numbers to show that the '−' sign is not a subtraction.

Worked examples

WE 1 Adding negative integers

Evaluate each expression.

a. $2 + (-4)$

Working

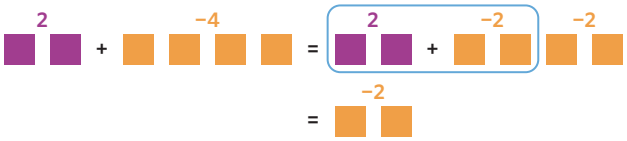
$2 + (-4) = 2 - 4$
$= 2 - 2 - 2$
$= 0 - 2$
$= -2$

Thinking

Step 1: Adding a negative integer is the same as subtracting a positive integer.

Step 2: Partition 4 into 2 and 2 in order to use 0 as a benchmark number then complete the subtraction.

Visual support



b. $-4 + (-3)$

Working

$-4 + (-3) = -4 - 3$
$= -7$

Thinking

Step 1: Adding a negative integer is the same as subtracting a positive integer.

Step 2: Move 3 units in the negative direction from −4.

Student practice

Evaluate each expression.

a. $5 + (-2)$ **b.** $1 + (-3)$ **c.** $-9 + (-9)$ **d.** $0 + (-10)$

WE 2 Subtracting negative integers

Evaluate each expression.

a. $5 - (-3)$

Working

$5 - (-3) = 5 + 3$

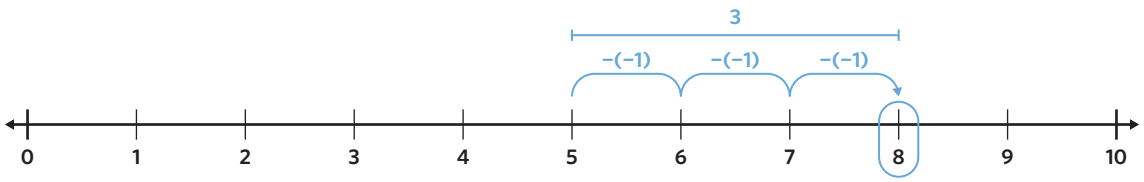
$= 8$

Thinking

Step 1: Subtracting a negative integer is the same as adding a positive integer.

Step 2: Complete the addition.

Visual support



b. $-8 - (-2)$

Working

$-8 - (-2) = -8 + 2$

$= -6$

Thinking

Step 1: Subtracting a negative integer is the same as adding a positive integer.

Step 2: Move 2 units in the positive direction from −8.

Student practice

Evaluate each expression.

a. $3 - (-5)$ **b.** $-4 - (-3)$ **c.** $7 - (-9)$ **d.** $-7 - (-9)$

WE 3 Adding and subtracting multiple negative integers

Evaluate each expression.

a. $-3 + (-2) - (-4)$

Working

$-3 + (-2) - (-4) = -3 - 2 - (-4)$

$= -3 - 2 + 4$

$= -5 + 4$

$= -1$

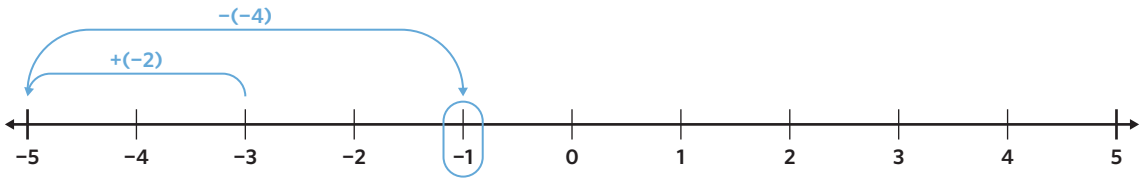
Thinking

Step 1: Adding a negative integer is the same as subtracting a positive integer.

Step 2: Subtracting a negative integer is the same as adding a positive integer.

Step 3: Complete each operation from left to right.

Visual support



b. $2 - (-5) + (-3) - (-6)$

Working

$2 - (-5) + (-3) - (-6) = 2 - (-5) - 3 - (-6)$

$= 2 + 5 - 3 + 6$

$= 7 - 3 + 6$

$= 4 + 6$

$= 10$

Thinking

Step 1: Adding a negative integer is the same as subtracting a positive integer.

Step 2: Subtracting a negative integer is the same as adding a positive integer.

Step 3: Complete each operation from left to right.

Student practice

Evaluate each expression.

a. $4 - (-1) + (-6)$

b. $-2 - (-6) + (-4) - (-1)$

c. $7 + (-2) - (-3)$

d. $-7 - (-4) + (-2) - (-9)$

2I Activities and questions

STARTER TASKS

Odd spot

For generations, young children have been told to drink milk because calcium makes bones stronger. However, there has been no definitive connection between milk consumption and bone strength. The T-score is a measure of bone density and strength that is determined using medical equipment. A T-score of 1 indicates normal bone density. A T-score of -3 indicates low bone density.

Which number sentence shows the difference between a normal bone density T-score and a low bone density T-score?

A. $1 - (-3) = -2$ **B.** $1 - (-3) = 4$

Image: paitoon/Shutterstock.com

Puzzle

2	1	−1	−5	3

a) What is the total value of all the cards added together?

b) If one positive integer card is removed, what happens to the total calculated in part **a**?

c) If one negative integer card is removed, what happens to the total calculated in part **a**?

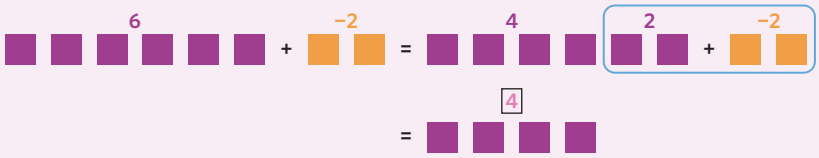
d) If the positive 1 card and negative 1 card are removed, what happens to the total calculated in part **a**?

Understanding worksheet

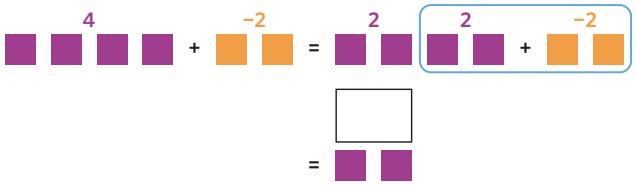
1. Fill in the box to complete the calculation.

Example

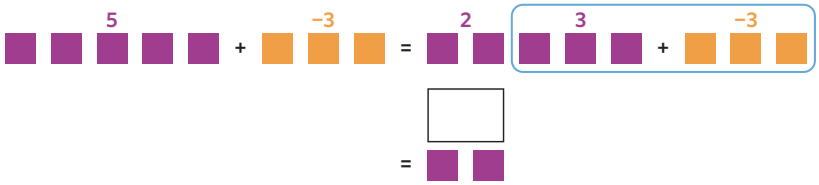
$6 + (-2)$



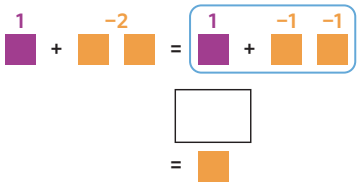
a. $4 + (-2)$



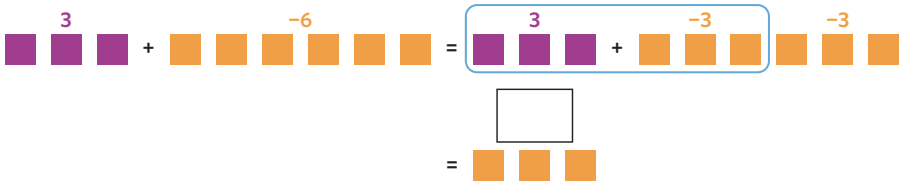
b. $5 + (-3)$



c. $1 + (-2)$



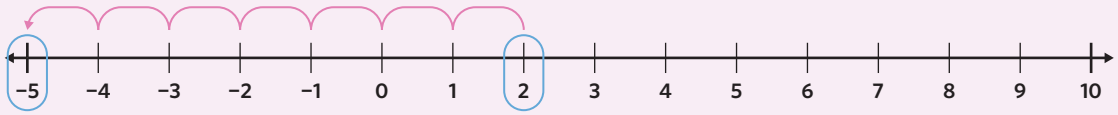
d. $3 + (-6)$



2. Show the 'jumps' on the number line to complete the calculation.

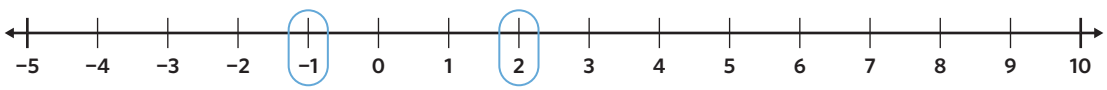
Example

$2 + (-7) = 2 - 7$



a. $2 + (-3) = 2 - 3$

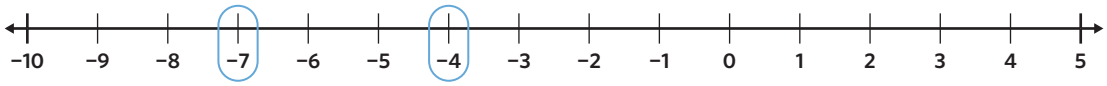
$= -1$



b. $7 + (-6) = 7 - 6$

$= 1$

c. $-7 - (-3) = -7 + 3$
$= -4$



d. $-3 - (-8) = -3 + 8$
$= 5$

3. Fill in the blanks by using the words provided.

zero | negative | adding | opposite

Finding the sum of ____ numbers always equals to ____. Adding a ____ integer is the same as subtracting a positive integer. Subtracting a negative integer is the same as ____ a positive integer.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (b,c,d), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7(d,e,f), 8 (e,f,g,h)

WE1 **4.** Evaluate each expression.

a. $4 + (-3)$ b. $8 + (-4)$ c. $-3 + (-5)$ d. $4 + (-7)$
e. $-2 + (-9)$ f. $-8 + (-8)$ g. $14 + (-5)$ h. $12 + (-14)$

WE2 **5.** Evaluate each expression.

a. $2 - (-4)$ b. $-4 - (-3)$ c. $-3 - (-6)$ d. $7 - (-3)$
e. $-5 - (-5)$ f. $-9 - (-4)$ g. $12 - (-13)$ h. $-15 - (-12)$

6. Fill in the missing operation to make both sides equivalent.

a. $6 - (-2) = 6 \square 2$ b. $3 + (-2) = 3 \square 2$
c. $-4 - (-6) = -4 \square 6$ d. $-7 + (-9) = -7 \square 9$
e. $9 - (-5) = 9 \square 5$ f. $-9 - (-5) = -9 \square 5$

7. Without completing the calculation, state whether each expression will result in a positive or negative integer.

a. $2 + (-4)$ b. $-5 + (-3)$ c. $7 + (-6)$ d. $-4 - (-6)$
e. $8 + (-9)$ f. $-12 - (-15)$

WE3 **8.** Evaluate each expression.

a. $-2 + (-4) + (-3)$ b. $-4 - (-3) - (-2)$
c. $3 + (-5) - (-6)$ d. $5 - (-3) + (-2) - (-4)$
e. $-7 + (-3) - (-2) + (-6)$ f. $12 + (-4) + (-5) + (-6)$
g. $-4 - (-8) - (-5) - (-7)$ h. $15 + (-12) - (-6) - (-6)$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. After the Stan family borrowed money to buy a new house, their bank balance was −$100 000. They then borrowed another $25 000 to renovate before they moved in. What was the family's bank balance after paying for the renovation?

10. Mickey is playing a card game with friends. The winners of each round receive 1 point and the losers of each round lose 2 points. What is Mickey's final score if she won three rounds and lost four rounds?

11. A vintage t-shirt shop is selling an original vintage t-shirt of The Bangles for $30. They also have a vintage Janet Jackson t-shirt for $40. What is Ella's credit card balance if she buys both t-shirts using her credit card which already had a balance of −$100?

12. A submarine descends in stages before it reaches its final depth. What is the final depth of a submarine that descends 75 metres in stage 1, descends another 70 metres in stage 2, rises 25 metres in stage 3, and then descends 85 metres in stage 4?

13. The South Pole is one of the coldest places in the world, reaching temperatures of −30 degrees Celsius. Yakutsk in Russia also descends to a temperature of −30 degrees Celsius. Bardufoss in Norway has a minimum temperature that is 5 degrees Celsius warmer than Yakutsk. What is the sum of the minimum temperatures in all three locations?

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c)	All

14. Deacon and Lachlie love playing golf together. They recently played a game and recorded their score in the following scorecard. In golf, the aim is to finish the course in the least amount of shots.

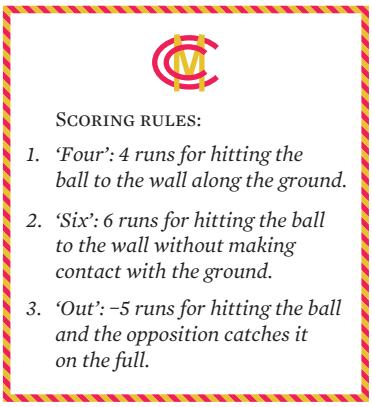
    Note: Par is the set number of shots expected for a player to get the ball in the hole. For example, if the par for a hole is 4, it is expected that a player will hit 4 shots to get the ball in the hole. If a player hits 5 shots, their score for the hole is +1. If a player hits 3 shots, their score for the hole is −1.

Hole number	1	2	3	4	5	6	7	8	9
Par	3	4	3	5	4	3	4	5	4
Deacon	+1	−1	0	+2	−1	0	−1	−2	−1
Lachlie	0	−1	−1	−1	+1	0	0	+1	0

    a. How many holes did each player score par?
    b. How many fewer shots did Deacon hit than Lachlie on the fifth hole?
    c. Who won and by how many points?
    d. If each player's best hole was removed from their score, who would have won and by how many points?
    e. Lachlie argues that the unpredictable weather in Melbourne makes it harder to play golf. How might the weather affect the game?

15. Lucy and Shae both play indoor cricket. Indoor cricket is less well known than outdoor cricket but is played in indoor sports facilities in Australia. The batter is the person who scores runs. They bat for 4 overs even if they have been given 'out'. One over in indoor cricket is 6 balls. The following instructions outline how to score in indoor cricket.

    a. What is the lowest potential score a batter can make in one over.
    b. What is Lucy's score if in one over she hits three 'fours', one 'six' and gets 'out' twice.
    c. Shae's batting score was −12 runs. However, on the last ball the umpire had made one mistake and said that Shae was 'out' when she actually scored a 'four'. What was Shae's correct score?
    d. Trying to score as many runs as possible in indoor cricket is risky as it increases the likelihood of going 'out'. Propose a strategy so that Lucy and Shae can maximise their scores.



SCORING RULES:

1. *'Four': 4 runs for hitting the ball to the wall along the ground.*
2. *'Six': 6 runs for hitting the ball to the wall without making contact with the ground.*
3. *'Out': −5 runs for hitting the ball and the opposition catches it on the full.*

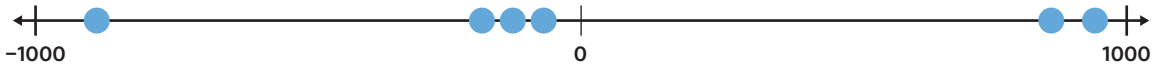
Extra spicy

16. Which option has the least value?

A. $-3999 - (-4999)$ **B.** $4999 - 3999$ **C.** $3999 + (-4999)$ **D.** $4999 + (-4999)$

17. What is the value of $1 + (-11) + (-111) + \ldots + (-1\,111\,111)$?

18. Each blue dot on the number line represents a value. Is the sum of the values that each dot represents greater or less than 0?



19. If ★ is a positive integer, then $★ + (-★) - ★ - (-★ - ★)$ is equal to:

A. 0 **B.** ★ **C.** $-★$ **D.** $★ + ★$

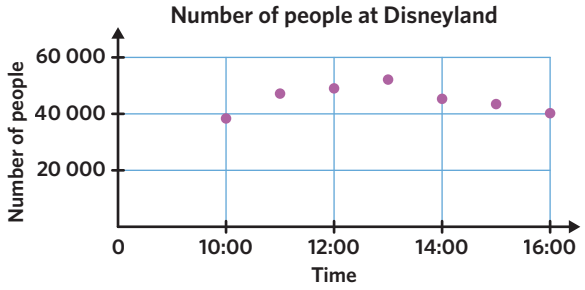
Remember this?

20. The spreadsheet shows 3 menu choices at a cafe. What menu item is in cell B4?

	A	B	C
1	Menu 1	Menu 2	Menu 3
2	Sandwich	Schnitzel	Salad
3	Apple	Orange	Pear
4	Granola	Yogurt	Muesli
5	Juice	Milk	Water

A. Granola **B.** Orange **C.** Yogurt **D.** Muesli

21. The graph shows the total number of people at Disneyland at different times during one day.



At what time did Disneyland have the most visitors during the day?

A. 11:00 **B.** 12:00 **C.** 13:00 **D.** 14:00

22. Dylan flew from Melbourne to Tokyo. His flight left at 9:30 pm Melbourne time and landed at 10:30 am the next day Melbourne time. How long was his flight?

A. 1 hour **B.** 10 hours **C.** 12 hours **D.** 13 hours

Chapter 2 extended application

1. Candace runs a bookstore and is looking to stock a new book that has come out. The orders come in packs of 13 books.
   a. Candace places 13 orders. How many books did she receive?
   b. In the first week she sold 121 books, and wants to display all the remaining books in a rectangular formation. List all the possible dimensions that Candace could use to display the books.
   c. The next day, she sold 5 more books and decided that, because the books are selling so quickly, she can afford to give the rest to no more than 10 different charities. Will Candace be able to evenly distribute the books to different charities?
   d. She identifies 8 charities to donate the books to. How many more books will she need to sell before she can donate the remaining books evenly?
   e. Should she give fewer books to more charities or more books to fewer charities? Provide a reason for your answer.

2. Marcus Bontempelli is a footballer for the Western Bulldogs. In 2021, he came second in the Brownlow medal, captained the Western Bulldogs to a grand final and averaged 105.7 AFL Fantasy points. The breakdown of how AFL Fantasy points are awarded is shown in the following table.

Match statistics	Fantasy points
Kick	3
Handball	2
Mark	3
Tackle	4
Free kick for	1
Free kick against	−3
Hitout	1
Goal	6

   a. In round 1, Marcus was awarded 1 free kick (free kick for) and gave away 3 free kicks (free kick against). Calculate how many fantasy points Marcus scored through free kicks.
   b. Marcus registered 4 tackles per game in 4 separate games throughout the season. Use index notation to represent the number of points he got from tackles in these games and calculate the result.
   c. In round 11, Marcus had 10 handballs and 5 marks at three-quarter time. What was the minimum number of handballs and marks he needed in the final quarter to get the same total number of points for both handballs and marks?
   d. In the grand final, he scored 108 points. 12 points came from marks, 18 points came from goals, 24 points came from tackles and −9 points came from free kicks against. Is it possible that the rest of his points came from kicks?
   e. Can you think of any reasons why this scoring system isn't an accurate reflection of the best players?

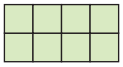
3. Conrad and Ginger are playing a game where they want to create the largest possible number. They fill 3 boxes with different coloured balls as shown to the right. Each ball is assigned either a 1 or a different prime number between 1 and 25.

   They randomly select a ball from each box. The number of the ball selected in box 1 is multiplied by the number of the ball selected in box 2. This is then multiplied by the number of the ball selected in box 3.
   a. On his first turn, Conrad scored 8. Which colour ball did he select from each box?
   b. On her first turn, Ginger scored 154. Which colour ball did she select from each box?
   c. On his second turn, Conrad claimed that he scored 125 points. Is this possible? Why or why not?
   d. Ginger tells Conrad that they forgot to include 2 prime numbers between 1 and 25. Which ones did they forget?
   e. Conrad said he doesn't like the game because it's based purely on luck. How could they modify the scoring so that it doesn't rely as much on chance?

Chapter 2 review

Multiple choice

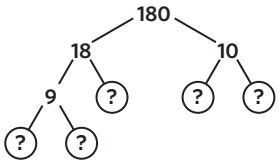
1. Which of the following is a prime number? 2C

A. 
B. 
C. 
D. 

2. Find the value of $6 + (-2)$. 2I

A. -4 B. 4 C. 8 D. 12

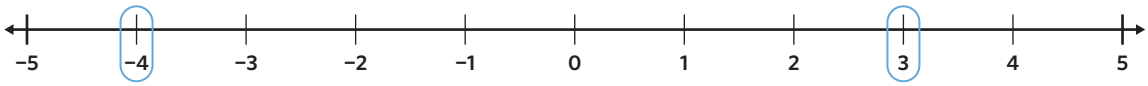
3. An incomplete factor tree of 180 is shown in the following diagram. 2D



Which of the following is not a prime factor of 180?

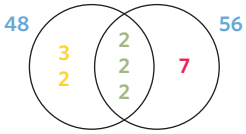
A. 7 B. 5 C. 3 D. 2

4. What is the difference between -4 and 3? 2G



A. 1 B. 3 C. 4 D. 7

5. Use the prime factors shown in the Venn diagram to find the lowest common multiple of 48 and 56. 2E



A. 8 B. 56 C. 336 D. 2688

Fluency

6. List all the factors of each number in ascending order. 2A

a. 16 b. 55 c. 48 d. 132

7. List the first five multiples of each number in ascending order. 2A

a. 6 b. 11 c. 17 d. 47

8. Evaluate each expression. 2B

a. 6^3 b. $4^3 - 5^2 + 6^2$
c. $9^2 \times 5 - 10^2$ d. $[(3^4 + 20) \times 1^{12}] - 8^2$

9. Identify whether each number is a prime or composite number. 2C

a. 11 b. 21 c. 49 d. 67

10. Using a factor tree, express each number as a product of its prime factors in expanded form. 2D

a. 24 b. 32 c. 85 d. 102

11. Find the lowest common multiple of each set of numbers. 2E

a. 3, 12 b. 9, 30 c. 4, 5, 6 d. 8, 12, 20

12. Find the highest common factor of each set of numbers. 2E

a. 24, 16 **b.** 64, 48 **c.** 40, 80, 100 **d.** 36, 56, 60

13. Evaluate each square root. 2F

a. $\sqrt{49}$ **b.** $\sqrt{121}$ **c.** $\sqrt{400}$ **d.** $\sqrt{8100}$

14. Place each set of integers in ascending order. 2G

a. $8, -2, 17, -13, 4$ **b.** $-20, -43, 2, -101, -3$

c. $13, -13, 133, -113, -133$ **d.** $455, 454, -445, -454, -45$

15. Evaluate each expression. 2H

a. $-4 + 11$ **b.** $-116 + 100$ **c.** $-2 - 9$ **d.** $14 - 22$

16. Evaluate each expression. 2I

a. $8 + (-7)$ **b.** $-10 + (-19)$ **c.** $6 - (-15)$ **d.** $-18 - (-29)$

17. Without completing the calculation, state whether each expression will result in a positive or negative integer. 2I

a. $10 + (-12)$ **b.** $-8 + (-6)$ **c.** $22 + (-17)$ **d.** $-50 - (-51)$

Problem solving

18. Ms Merakis is organising seats for a year 7 assembly. There are 160 year 7 students. Find the two possible formations of the seats so that each row seats an equal number of students, and each row has between 15 and 25 seats. 2A

19. On the 1st day of the COVID-19 pandemic there were three cases. It was estimated that the number of cases then tripled each day for the next month. How many cases were there estimated to be on the 5th day of the pandemic? 2B

20. Fletcher knows he has between 90 and 100 footy cards. He has tried to split his collection into equal groups but always has cards remaining. How many footy cards does Fletcher have? 2C

21. Sofia has made 45 pieces of rocky road. She wants to package them into equal groups of more than one piece. What is the least number of rocky road pieces Sofia can have in each group? 2D

22. Blaise works at Mr Miyagi every 4 days, while Louis works there every 6 days. On what day will they next work together if they just worked together on a Saturday night? 2E

23. The game of checkers is played on a square board with 64 squares. How many squares are there in each row? 2F

24. The highest point on Earth is Mount Everest, which has an altitude of 8848 metres. The lowest point on land is the Dead Sea, with an altitude of −413 metres. What is the difference in altitude between the top of Mount Everest and the bottom of the Dead Sea? 2G

25. Elise has driven to her office building and parked two levels underground. She travels up 5 levels in the elevator to the cafe, buys a coffee, and then travels up another 7 levels to the office. What floor is the office on? 2H

26. Emilie, Tallie and Matisse all use Afterpay to pay for their online shopping. Emilie is $75 in debt, Tallie is $135 in debt and Matisse is $120 in debt. How much are they in debt combined? 2I

Reasoning

27. Grace has a collection of 144 photos that she wants to organise on her bedroom wall. She can fit up to 50 columns of photos (horizontal space) and up to 20 rows of photos (vertical space).

a. Find all the **rectangular** (not including square) formations Grace can possibly make with her photos.

b. If Grace organised the photos in a square, how many photos would be in each row?

c. Grace chose a formation with a prime number of rows. What was the formation?

d. How many photos does Grace need to add to her collection to make a square that takes up all the vertical space on her wall?

e. Why might disposable cameras and printed photos remain popular despite there being quality cameras in most mobile phones?

28. Maria and Carlo love to scuba dive in their free time. Maria scuba dives every 9 months while Carlo scuba dives once every year at the same time. Recently, Maria and Carlo scuba dived together.

- **a.** Maria dived in first while Carlo was still in the boat. She reached an altitude of -34 metres, while Carlo was 2 metres above sea level. What was the vertical distance between them?
- **b.** Carlo dived into an underwater cave, reaching an altitude of -42 metres. He then rose 27 metres to reach a reef. What is the altitude of the reef?
- **c.** Maria joined Carlo at the reef before descending 18 metres into another cave. What was her altitude in the cave?
- **d.** How many years will it be until Maria and Carlo scuba dive together again?
- **e.** Do you think scuba diving is damaging to the environment, or could scuba diving help save ocean life?

03

Fractions and ratios

Number and Algebra

Research summary

3A Introduction to fractions *(Revision)*

3B Proper fractions, improper fractions, and mixed numbers *(Revision)*

3C Equivalent fractions

3D Comparing fractions

3E Adding and subtracting fractions

3F Multiplying fractions

3G Dividing fractions

3H Ratios

Chapter 3 extended application

Chapter 3 review

Image: Marish/Shutterstock.com

Research summary – Fractions and ratios

Big ideas

Fractions as part-whole

In part-whole comparisons with fractions, the **part** represents the numerator and the **whole** represents the denominator of the fraction. These comparisons are often made using an area model where a two dimensional shape is partitioned into equal parts with a number of parts shaded. These models are a very useful way of exploring fractions and provide a great introduction to the concept. However, part-whole comparisons with fractions should be made in a variety of other ways and using a number of other models in order to add rigour and avoid misconceptions.

Fractions as measures

Using fractions with number lines is different to other models because it involves the distance of certain points along the number line with respect to zero. In contrast to the area, discrete and volume models, a fraction as a measure doesn't require a segment but rather an increment. If not fully understood, sometimes each interval along the number line will be seen as representing a section rather than as counting units.

Fractions as operators

An operator is a set of instructions for carrying out a process. The mathematical operators are addition (+), subtraction (−), multiplication (×) and division (÷). When we find a fraction of another value, the word 'of' is an operator that instructs us to multiply the fraction by the value. Viewing the term 'fraction of' as an operator allows students to more simply perform calculations with fractions. This will also form an important part of their understanding when completing algebraic calculations.

Fractions as quotients

Another important interpretation of fractions is that they are actually the result of a division. When we first start learning about fractions, we may observe that $\frac{1}{4}$ of a pizza is one part out of four or two parts out of eight. However, as the level of mathematics increases, we will be required to apply fractions to more advanced concepts such as algebra and solving equations. It will become crucial at this point to recognise the fraction as the quotient or result of the division between the numerator and the denominator and not just as a part of a whole.

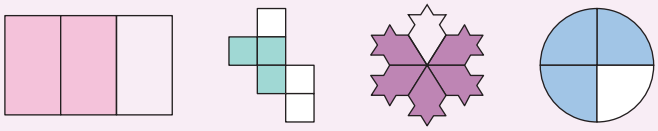
Fractions as ratios

A ratio is a comparison of two or more quantities which in many cases can be expressed as a fraction. However, when working with ratios, it is important to note that the arithmetic that works for fractions does not necessarily work for ratios and the interpretation of various situations may vary depending on the context. For example, a ratio may change depending on whether a part is being compared to another part or to the whole. It is also important to note that ratios, like fractions, are ordered pairs which means that $a : b$ is not the same as $b : a$.

Visual representations

Area model

Area models are visual tools that help us to understand the partitioning of a unit based on its area. This model represents a fraction as a two-dimensional shape that has been divided into equal parts and is one of the most common ways of working with fractions. However, a heavy reliance on area models can limit our understanding of fractions as rational numbers and can cause common misconceptions within the topic. It is therefore important to use the area model alongside other models and non-visual approaches when teaching and learning fractions.

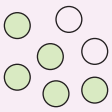


$\frac{2}{3}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{3}{4}$

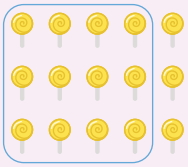
Discrete model

The discrete model of fractions requires representing the unit as a collection of objects, rather than one whole entity such as an area or number line. It is important to practise using fractions with discrete units because this contributes to the development of a 'growing whole' when comparing the size of fractions.

For example, 5 green objects out of a collection of 7 items represents $\frac{5}{7}$, but if the collection grew to 14 items, these same 5 green objects would only represent $\frac{5}{14}$. Finding fractions of quantities is also a common practice when comparing collections of items.



$\frac{5}{7}$ shaded green

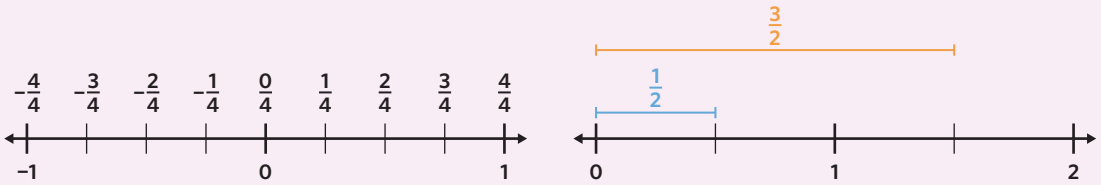


$\frac{4}{5}$ of the lollipops are circled

$\frac{3}{8}$ of the t-shirts are black

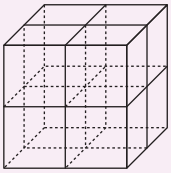
Number lines

When we refer to fractions as a measure we are referring to the measure assigned to some interval or region in one-dimensional space. Unlike a two-dimensional area, in one-dimensional space, we are required to work along a number line and recognise fractions as rational numbers that fall between integers. In number lines, the fraction is always an interval of length.

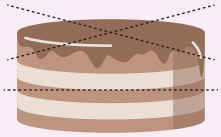


Volume model

The volume model allows us to practice fractions in three-dimensional space. This model is not extensively used but adding this third dimension into the practice of fractions provides an important step into the visualisation of fractions from abstract elements to tangible and contextual objects.

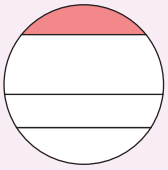
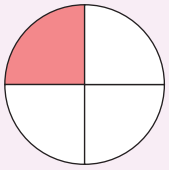
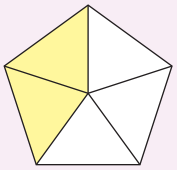
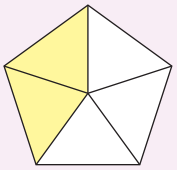


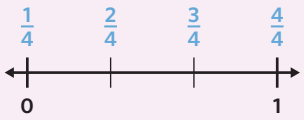
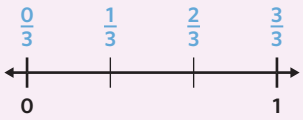
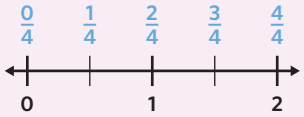
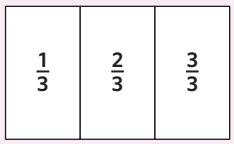
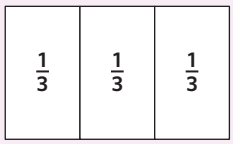
How many cubes make $\frac{3}{4}$?



Cut the cake into eighths using three slices.

Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students don't see that in order for a fraction to be correct, the parts must cover exactly the same amount of area.	 $\frac{1}{4}$	 $\frac{1}{4}$	3A
Students see fractions as part-part instead of part-whole.	 $= \frac{2}{3}$	 $= \frac{2}{5}$	3A
Students don't recognise that a fraction is the result of a division between the numerator and the denominator.	Not realising that $2 \div 3$ can be written as a fraction.	$2 \div 3 = \frac{2}{3}$	3A

	Incorrect	Correct	Exercise
Students count the ticks including the first one. For example, if they have a number line broken into 3 parts, they see this as broken into quarters, instead of thirds.	 4 ticks seen as quarters.	 3 intervals seen as thirds.	3A
Students don't see the magnitude of the number line as changing the value of the fraction. For example, in a number line from 0–2 with 4 intervals, students don't recognise that each interval will be $\frac{1}{2}$ not $\frac{1}{4}$.			3A
Students judge a mixed number as greater than an improper fraction because of the whole number in front.	$2\frac{3}{5} > \frac{13}{5}$ because $2\frac{3}{5}$ involves a whole number.	$2\frac{3}{5} = \frac{13}{5}$ because both involve the same number of $\frac{1}{5}$'s.	3B
Students think that a fraction with a larger denominator is greater.	$\frac{1}{4} > \frac{1}{3}$ because the number 4 is greater than the number 3.	$\frac{1}{4} < \frac{1}{3}$ because a third is larger than a quarter.	3C
When finding equivalent fractions, students only multiply the numerators but not the denominators.	$\frac{3}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$	$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$	3C
Students believe two fractions are the same because they both have the same difference between the numerator and denominator.	$\frac{9}{10} = \frac{99}{100}$ because both the numerators are 1 less than their respective denominators.	$\frac{90}{10} < \frac{99}{100}$ $\frac{9}{10} < \frac{99}{100}$	3D
Students label each section of the fraction using their ordinal parts. E.g. $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$.	 Suggesting there are $\frac{6}{3}$ in total.		3D
Students think that a smaller denominator means the fractional part is larger. They don't consider the numerator tells us how many parts of that fraction.	$\frac{2}{3} > \frac{3}{4}$ because thirds are larger than quarters.	$\frac{2}{3} < \frac{3}{4}$	3D
Students add fractions by adding the numerators and denominators together.	$\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$	$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	3E
Students find a common denominator but then add these as well as the numerators.	$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{8}$	$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$	3E
When multiplying a mixed number, students don't see the whole number as part of the fraction.	$2\frac{1}{2} \times \frac{1}{3} = 2\frac{1}{6}$ (just multiplying the fractional part and ignoring the whole number).	$2\frac{1}{2} \times \frac{1}{3} = \frac{5}{2} \times \frac{1}{3} = \frac{5}{6}$	3F
When multiplying a fraction by a whole number, students multiply both the numerator and denominator by the whole number.	$2 \times \frac{3}{4} = \frac{2 \times 3}{2 \times 4} = \frac{6}{8}$	$2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4}$ $= \frac{2 \times 3}{1 \times 4}$ $= \frac{6}{4}$	3F
Students only multiply the numerators when multiplying fractions.	$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5} = \frac{6}{5}$	$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$	3F

	Incorrect	Correct	Exercise
When multiplying values, students think that the result must always get larger.	$\frac{1}{2} \times \frac{1}{4} \neq \frac{1}{8}$ because $\frac{1}{8}$ is smaller than $\frac{1}{2}$ and $\frac{1}{4}$.	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ because when we take a fraction of another fraction, the result gets smaller.	3F
When dividing values, students think that the result will always get smaller.	$\frac{1}{2} \div \frac{1}{4} \neq 2$ because 2 is larger than $\frac{1}{2}$ and $\frac{1}{4}$.	$\frac{1}{2} \div \frac{1}{4} = 2$ because we are finding how many of one fraction fits into another.	3G
Students don't see ratios as ordered pairs where $a : b$ is not the same as $b : a$.	$a : b = b : a$ $3 : 4 = 4 : 3$	$a : b \neq b : a$ $3 : 4 \neq 4 : 3$ because, $\frac{3}{4} < \frac{4}{3}$	3H
Students don't recognise the difference between part-part and part-whole relationships when expressing ratios as fractions.	Ratio of apples to oranges is $2 : 3$. The fraction of fruits that are apples is $\frac{2}{3}$.	Ratio of apples to oranges is $2 : 3$. The fraction of fruits that are apples is $\frac{2}{5}$.	3H
Students believe that the 'best buy' is always the one that has a lower total cost.	Item 1: \$4 for a 500 g packet Item 2: \$6 for a 1 kg packet Item 1 is the best buy.	Item 1: \$4 for a 500 g packet Item 2: \$6 for a 1 kg packet Unit price of item 1: \$0.8 per 100 g Unit price of item 2: \$0.6 per 100g Item 2 is the best buy.	3H

Additional reading and resources

- Teaching fractions and ratios for understanding: Susan J. Lamon, Routledge 2012
- fractiontalks.com
- stevewyborney.com/2017/03/the-fraction-splat-series
- gdaymath.com/courses/fractions-are-hard

3A Introduction to fractions

Whole numbers are very useful, but there are many situations where we need to use values that are between whole numbers. One way of expressing these values is by using fractions. Below are some examples where fractions can be applied.

- I have 3 apples and 7 bananas. What fraction of all the fruit are apples?
- I bought two Mars bars and want to share them between three people. What fraction of a Mars bar will each person get?
- A large container of gelato has 5 flavours in equal amounts: lemon, coconut, chocolate, vanilla and bubblegum. What fraction of the container is coconut?

Learning intentions

Students will be able to:

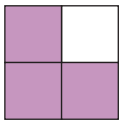
+ define and recognise numerators and denominators
+ write fractions to represent a proportion or division
+ represent fractions using objects and number lines.

Key terms and definitions

- A **fraction** is a value represented as a certain number of equal parts of a unit.
- The **denominator** is the value on the bottom of a fraction which tells us how many parts each unit is split into.
- The **numerator** is the value above the line in a fraction which tells us how many parts of a unit we are considering.

Key ideas

1 A fraction represents a certain number of parts of a whole.



$\frac{3}{4}$

This numerator tells us that we are considering 3 of the equal parts.

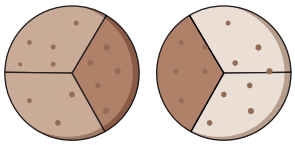
This denominator tells us to divide the whole into 4 equal parts.

2 Fractions are a way of representing division. When one number is divided by another, the result can be expressed as a fraction. The numerator tells us the quantity that is being divided and the denominator tells us how many groups to divide it into.

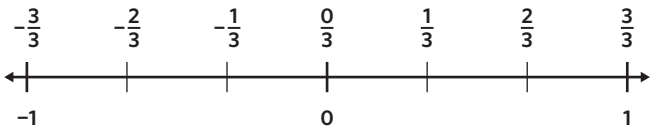
Divide 2 cookies between 3 people.

$= 2 \div 3$

$= \frac{2}{3}$ each



3 Fractions can be positive or negative values and can be placed on a number line. The intervals between whole numbers on a number line can be split into equal parts based on the denominator of the fraction we are working with.



Worked examples

WE 1 Understanding fractions

Write the fraction for each description.

a. 7 out of 10

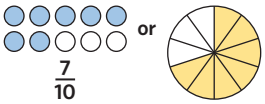
Working

$\frac{7}{10}$

Thinking

There are 7 parts (numerator) out of 10 parts in total (denominator).

Visual support



b. 2 divided by 5

Working

$\frac{2}{5}$

Thinking

The numerator 2 is divided by the denominator 5.

Student practice

Write the fraction for each description.

a. 8 out of 11

b. 5 divided by 12

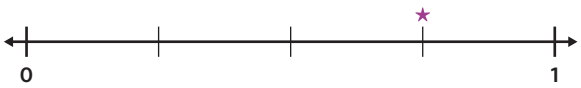
c. A denominator of 9 and numerator of 4

d. Negative six-seventeenths

WE 2 Reading fractions on a number line

Find the value of the ★.

a.



Working

$\frac{}{4}$

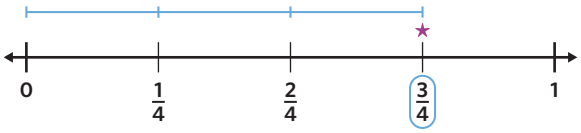
$\frac{3}{4}$

Thinking

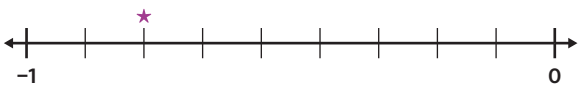
Step 1: Count the total number of intervals from 0 to 1. This is our denominator.

Step 2: The star is at the third interval from 0. This is our numerator.

Visual support



b.



Working

$-\frac{}{9}$

$-\frac{7}{9}$

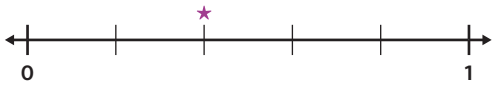
Thinking

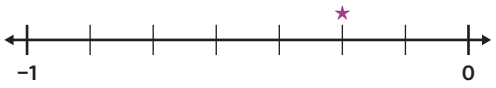
Step 1: Count the total number of intervals from −1 to 0. This is our denominator.

Step 2: The star is at the seventh interval from 0. This is our numerator.

Student practice

Find the value of the ★.

a.


b.


c.

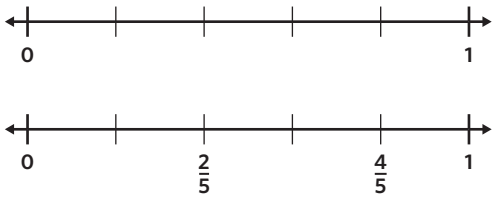

d.


WE 3 Representing fractions on a number line

Represent the fractions on a number line.

a. $\frac{2}{5}$ and $\frac{4}{5}$

Working



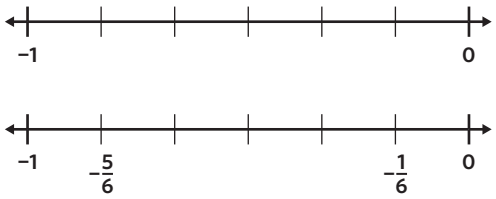
Thinking

Step 1: Draw a number line from 0 to 1 and split it into equal intervals based on the denominator. In this case, there are 5 intervals.

Step 2: Label the second interval from 0 as $\frac{2}{5}$ and the fourth interval from 0 as $\frac{4}{5}$.

b. $-\frac{1}{6}$ and $-\frac{5}{6}$

Working



Thinking

Step 1: Draw a number line from −1 to 0 and split it into equal intervals based on the denominator. In this case, there are 6 intervals.

Step 2: Label the first interval from 0 as $-\frac{1}{6}$ and the fifth interval from 0 as $-\frac{5}{6}$.

Student practice

Represent the fractions on a number line.

a. $\frac{1}{4}$ and $\frac{2}{4}$

b. $-\frac{3}{8}$ and $-\frac{5}{8}$

c. $-\frac{1}{3}$ and $\frac{2}{3}$

d. $-\frac{3}{5}$ and $\frac{4}{5}$

3A Activities and questions

STARTER TASKS

Image: Imagine Earth Photography/ Shutterstock.com

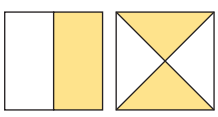
Odd spot

An adolescent blue whale weighs about 50 tonnes, but this is only $\frac{1}{3}$ of the weight that it will be once it is fully grown. How much does a fully grown blue whale weigh?

A. 150 tonnes **B.** 16.66 tonnes

Puzzle

Below shows two ways of shading exactly half of a square.



a) Draw three other ways that you can colour exactly half a square.

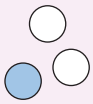
b) How do you know that you have coloured exactly half?

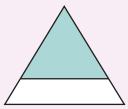
Understanding worksheet

1. Circle the diagram that represents the fraction of shaded parts.

Example

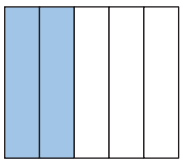
$\frac{1}{2}$

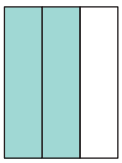
A. 

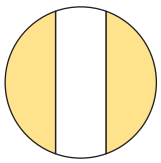
B. 

C. 

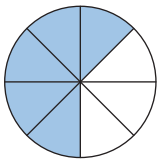
a. $\frac{2}{3}$

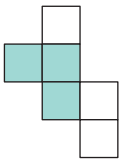
A. 

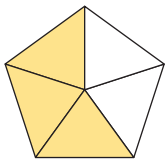
B. 

C. 

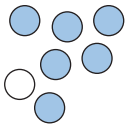
b. $\frac{3}{5}$

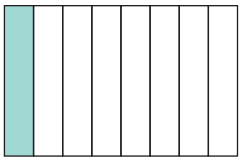
A. 

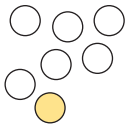
B. 

C. 

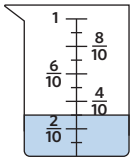
c. $\frac{1}{7}$

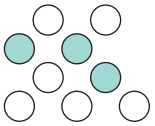
A. 

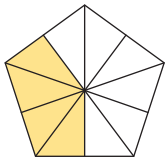
B. 

C. 

d. $\frac{3}{10}$

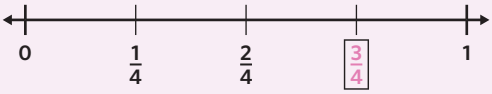
A. 

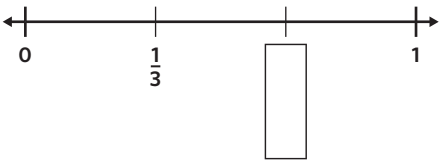
B. 

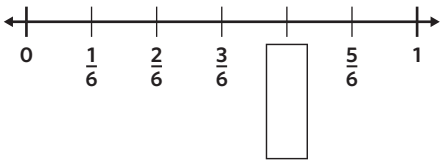
C. 

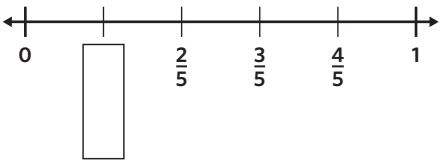
2. Write the missing fraction.

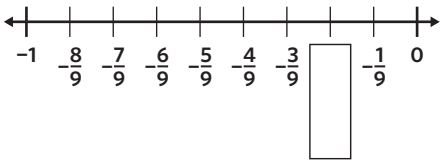
Example



a. 

b. 

c. 

d. 

3. Fill in the blanks by using the words provided.

parts | whole | numerator | denominator

The _____ is the value on the top of a fraction. It tells us how many _____ of a whole we have. The _____ is the value on the bottom of a fraction. It tells how many parts the _____ is split into.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h)

4. State the numerator and denominator of each fraction.

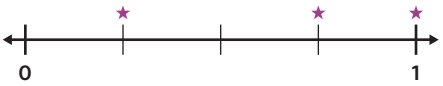
a. $\frac{1}{4}$ b. $\frac{2}{3}$ c. $\frac{5}{6}$ d. $\frac{7}{8}$

e. $\frac{1}{10}$ f. $\frac{93}{100}$

WE1 **5.** Write the fraction for each description.

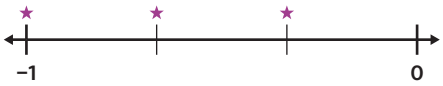
a. One-half
b. Three-quarters
c. Seven out of eight
d. 4 divided by 7
e. A numerator of 20 and denominator of 21
f. Eleven-thirteenths

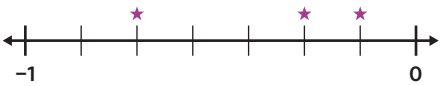
WE2 **6.** Write the fractions that correspond to each ★.

a.


b.


c.


d.


e.


f.


g.


h.


WE3 **7.** Represent the fractions on a number line.

a. $\frac{1}{2}$ and $\frac{2}{2}$ b. $\frac{1}{4}$ and $\frac{3}{4}$ c. $\frac{2}{3}$ and $\frac{3}{3}$ d. $\frac{0}{6}$ and $\frac{4}{6}$

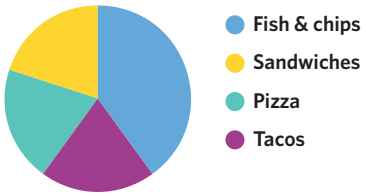
e. $-\frac{2}{3}$ and $-\frac{1}{3}$ f. $-\frac{1}{5}$ and $\frac{2}{5}$ g. $-\frac{3}{4}$, $-\frac{1}{4}$ and $\frac{2}{4}$ h. $-\frac{5}{7}$, $\frac{1}{7}$ and $\frac{6}{7}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Steph asked her class to vote on what food they should order for their end-of-year party. She used the data to draw this pie chart.

Which food did $\frac{2}{5}$ of the class vote for?



A. Fish & chips B. Sandwiches
C. Pizza D. Tacos

9. Mario wants to know what fraction of his money he spent at the bakery. He started with $7 and bought a cupcake for $3.

10. Alexia is riding home from school on her bike and has made it $\frac{2}{3}$ of the way home. How far has she ridden, if her house is 6 km from school?

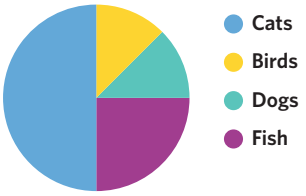
11. A baby slept for 17 hours of the day. What fraction of the day was the baby awake?

12. It takes Gaia 60 seconds to run a lap of the schoolyard. She ran $\frac{3}{4}$ of a lap before stopping to tie her shoelace. How many seconds will it take to finish the lap?

Reasoning

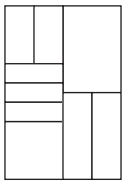
Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

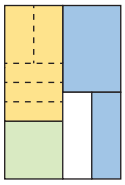
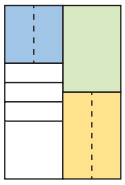
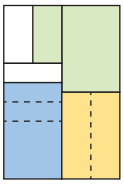
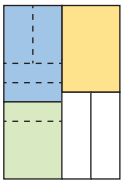
13. Maya drew a pie chart to show the type of pets owned by people in her class.



   a. What fraction of pets are birds?
      A. $\frac{1}{5}$ B. $\frac{2}{6}$
      C. $\frac{1}{3}$ D. $\frac{1}{8}$
   b. What fraction of pets are four-legged animals?
   c. All 24 students in Maya's class have one pet. How many students have a pet with four legs?
   d. Maya's classmate Tom says that the number of students that have fish is double the number of students that have dogs. Maya thinks that the number of people that have dogs is half the number of people that have fish. Who is correct?
   e. Some students in Maya's class are finding it difficult to understand the pie chart. Present the information from the pie chart in a different way.

14. Old McDonald has his farm split up into the paddocks shown. He wants to use $\frac{3}{8}$ of the area for sheep which he will colour in blue, $\frac{1}{6}$ of the area for chickens which he will colour in green, and $\frac{1}{3}$ of the area to grow wheat which he will colour in yellow.



   a. Select the design that Old McDonald should use to meet the requirements.
      A. 
      B. 
      C. 
      D. 
   b. The average Victorian farm has an area of 4000 hectares. If Old Mcdonald's farm has the same area as the average Victorian farm, what is the area of the sheep paddock?
   c. Old McDonald wants to split the remaining paddock equally between goats and pigs. Assuming the area of Old McDonald's farm is double the average Victorian farm, what fraction of the entire farm will the pigs have?
   d. Old McDonald would like another paddock design option that includes sections for the pigs and goats. Draw another design that meets the requirements.

Extra spicy

15. What fraction is halfway between $\frac{1}{4}$ and $-\frac{1}{2}$?

16. Lisa teaches a class of 27 students. $\frac{1}{3}$ of the class participated in a school swimming competition. Of those, $\frac{2}{3}$ won a medal. If $\frac{1}{3}$ of those students won a bronze medal, how many students won a silver or gold medal?

17. Which of the following values can be filled into the box to make a value in between $\frac{3}{10}$ and $\frac{1}{2}$?

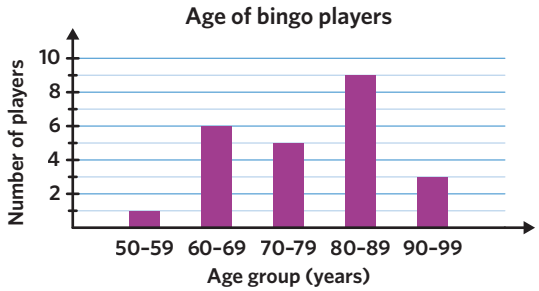
$$\frac{\square}{20}$$

A. 6 B. 8 C. 10 D. 12

18. If $\frac{1}{3}$ of a number is 6, then $\frac{4}{9}$ of the number is?

Remember this?

19. The following column graph shows the age of bingo players at a social club.



How many players are **under** the age of 70?

A. 7 B. 12 C. 17 D. 21

20. Which two odd numbers, greater than 1, have a product of 91?

21. A hummingbird flaps its wings 50 times every second. How many minutes will it take to flap its wings 6000 times?

3B Proper fractions, improper fractions, and mixed numbers

There are three types of fractions: proper fractions, improper fractions, and mixed numbers. All of these are different ways of representing fractions of whole numbers and are useful in different situations. Below are some examples of where proper fractions, improper fractions and mixed numbers can be applied.

- I divide a chocolate bar into 3 equal pieces and eat 1 piece. What fraction of the chocolate bar have I eaten?
- I watched a movie that was $1\frac{3}{4}$ hours long. How long was the movie in minutes?
- After one lap of an oval I have run $\frac{3}{5}$ of a kilometre. After two laps I have run $1\frac{2}{5}$ kilometres. After three laps, what distance have I run?

Learning intentions

Students will be able to:

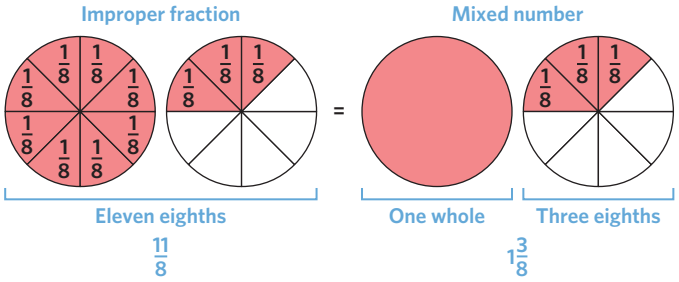
+ identify and compare proper fractions, improper fractions, and mixed numbers
+ convert between improper fractions and mixed numbers
+ continue a fractional sequence using proper and improper fractions and mixed numbers.

Key terms and definitions

- A **proper fraction** has a numerator that is less than its denominator. E.g. $\frac{3}{4}$.
- An **improper fraction** has a numerator greater than or equal to its denominator. E.g. $\frac{7}{5}$ or $\frac{3}{3}$.
- A **mixed number** is a combination of a whole number and a proper fraction. E.g. $2\frac{1}{3}$.
- A **sequence** is a collection of numbers in an order that follows a specific pattern or rule.

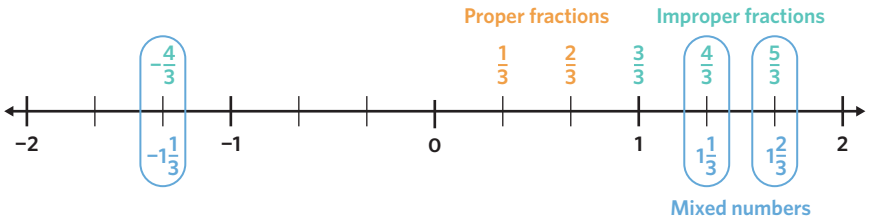
Key ideas

1 Values greater than 1 can be expressed as either improper fractions or mixed numbers and we can convert between the two. Both these notations will have identical values.



2 Proper fractions, improper fractions and mixed numbers can be represented on a number line.

Improper fractions and mixed numbers at the same point on the number line have the same value. E.g. $\frac{5}{3} = 1\frac{2}{3}$.



Worked examples

WE 1 Identifying fraction types

State whether each value is a proper fraction, improper fraction or mixed number.

a. $\frac{6}{5}$

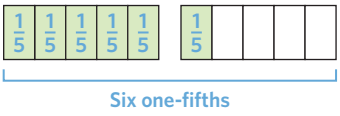
Working

Improper fraction

Thinking

The numerator is larger than the denominator so it is an improper fraction.

Visual support



b. $2\frac{1}{4}$

Working

Mixed number

Thinking

The value is made up of a whole number (2) and a proper fraction $\left(\frac{1}{4}\right)$ so it is a mixed number.

Student practice

State whether each value is a proper fraction, improper fraction or mixed number.

a. $\frac{17}{4}$ **b.** $5\frac{2}{3}$ **c.** $\frac{99}{99}$ **d.** $6\frac{7}{6}$

WE 2 Converting mixed numbers to improper fractions

Convert each mixed number to an improper fraction.

a. $2\frac{3}{4}$

Working

Method 1

$2 + \frac{3}{4} = 1 + 1 + \frac{3}{4}$

$= \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$

$= \frac{11}{4}$

Method 2

$2\frac{3}{4} = \frac{(2 \times 4 + 3)}{4}$

$= \frac{11}{4}$

Thinking

Method 1

Step 1: Separate the whole number (2) into single whole numbers.

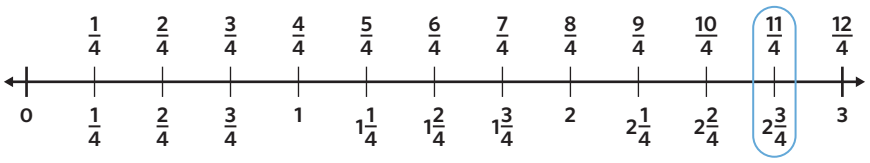
Step 2: Write each whole number as a fraction with a denominator of 4.

Step 3: Add the numerators together and keep the denominator of 4.

Method 2

Multiply the whole number (2) by the denominator (4) and then add the numerator (3). Write the result as the numerator and keep the same denominator (4).

Visual support



b. $4\frac{1}{3}$

Working

Method 1

$4 + \frac{1}{3} = 1 + 1 + 1 + 1 + \frac{1}{3}$

$= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$

$= \frac{13}{3}$

Method 2

$4\frac{1}{3} = \frac{(4 \times 3 + 1)}{3}$

$= \frac{13}{3}$

Thinking

Method 1

Step 1: Separate the whole number (4) into single whole numbers.

Step 2: Write each whole number as a fraction with a denominator of 3.

Step 3: Add the numerators together and keep the denominator of 3.

Method 2

Multiply the whole number (4) by the denominator (3) and then add the numerator (1). Write the result as the numerator and keep the same denominator (3).

Student practice

Convert each mixed number to an improper fraction.

a. $3\frac{4}{5}$ **b.** $10\frac{2}{7}$ **c.** $5\frac{11}{12}$ **d.** $14\frac{1}{23}$

WE 3 Converting improper fractions to mixed numbers

Convert each improper fraction to a mixed number.

a. $\frac{11}{3}$

Working

$11 \div 3 = 3$ remainder 2

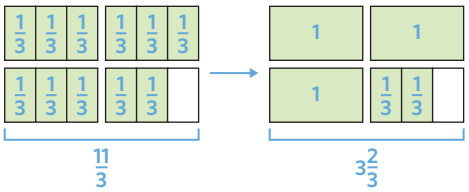
$= 3\frac{2}{3}$

Thinking

Step 1: Divide the numerator by the denominator.

Step 2: The remainder is how many parts of the whole are left over. Write the remainder as the numerator of the fraction using the denominator 3.

Visual support



b. $\frac{23}{5}$

Working

$23 \div 5 = 4$ remainder 3

$= 4\frac{3}{5}$

Thinking

Step 1: Divide the numerator by the denominator.

Step 2: The remainder is how many parts of the whole are left over. Write the remainder as the numerator of the fraction using the denominator 5.

Student practice

Convert each improper fraction to a mixed number.

a. $\frac{7}{4}$ **b.** $\frac{17}{6}$ **c.** $\frac{89}{12}$ **d.** $\frac{169}{13}$

WE 4 Completing sequences with fractions

Fill in the gaps to complete the sequence.

a. $\frac{3}{4}, ..., \frac{5}{4}, \frac{6}{4}, ..., \frac{8}{4}, ...$

Working

$+\frac{1}{4}$

$\frac{3}{4}, ..., \frac{5}{4}, \frac{6}{4}, ..., \frac{8}{4}, ...$

$+\frac{1}{4} \quad +\frac{1}{4} \quad +\frac{1}{4}$

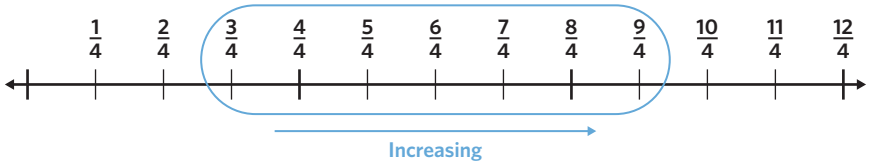
$\frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}$

Thinking

Step 1: The values are increasing from left to right. Find the increase by comparing two consecutive fractions. $\frac{5}{4}, \frac{6}{4}$ shows that each value in the sequence increases by $\frac{1}{4}$.

Step 2: Add $\frac{1}{4}$ to the values before each gap to complete the sequence using improper fractions.

Visual support



b. $0, ..., ..., -1, -1\frac{1}{3}, ..., -2$

Working

$-\frac{1}{3}$

$0, ..., ..., -1, -1\frac{1}{3}, ..., -2$

$-\frac{1}{3} \quad -\frac{1}{3} \quad -\frac{1}{3}$

$0, -\frac{1}{3}, -\frac{2}{3}, -1, -1\frac{1}{3}, -1\frac{2}{3}, -2$

Thinking

Step 1: The values are decreasing from left to right. Find the decrease by comparing two consecutive values. $-\frac{2}{3}, -1$ shows that each value in the sequence decreases by $\frac{1}{3}$.

Step 2: Subtract $\frac{1}{3}$ from the values before each gap to complete the sequence using mixed numbers.

Student practice

Fill in the gaps to complete each sequence.

a. $\frac{2}{5}, \frac{3}{5}, ..., ..., \frac{6}{5}, ...$

b. $4, ..., ..., 3\frac{4}{7}, 3\frac{3}{7}, ...$

c. $-\frac{11}{12}, ..., -\frac{3}{12}, \frac{1}{12}, ..., ...$

d. $3\frac{3}{13}, 1\frac{8}{13}, ..., ..., -3\frac{3}{13}, ...$

3B Activities and questions

STARTER TASKS

Odd spot

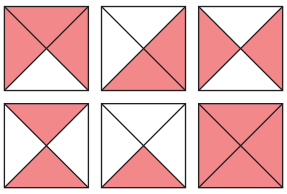
Fifty Ferrari-loving pastry chefs in the town of Oretano showed their love by using 40 000 cream pies to create an edible ferrari. The 5-metre replica of Michael Schumacher's formula one car was decorated with almond paste, custard cream and chocolate.

After $2\frac{1}{3}$ days the chefs had completed one replica. Assuming the pastry chefs take the same amount of time to complete each replica, how many days will it take to complete 5 replicas?

A. $11\frac{2}{3}$ days **B.** $10\frac{2}{3}$ days

Puzzle

The image shown is made up of 6 squares each broken up into 4 triangles (quarters).



a) In total, how many triangles are shaded red?

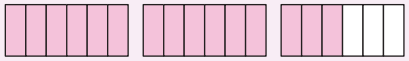
b) How many full squares can be made from the shaded triangles?

Understanding worksheet

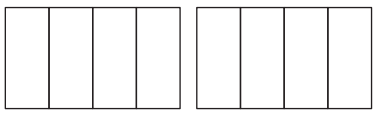
1. Shade each shape to represent the improper fraction or mixed number.

Example

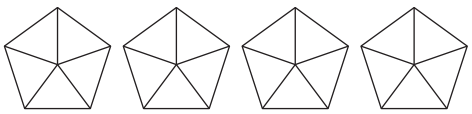
$2\frac{3}{6}$



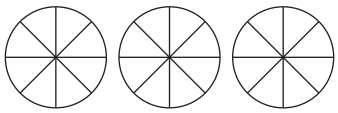
a. $1\frac{2}{4}$



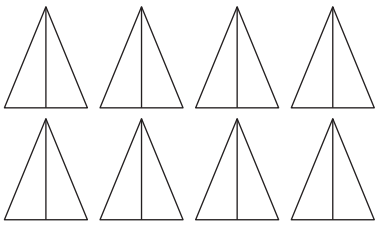
b. $\frac{17}{5}$



c. $2\frac{2}{8}$



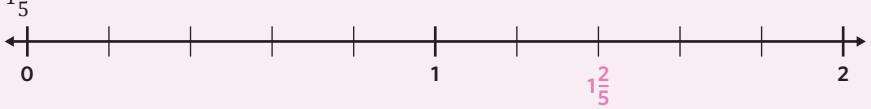
d. $\frac{13}{2}$



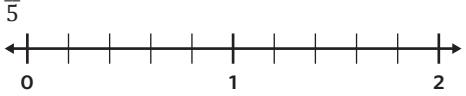
2. Write each value on the number line.

Example

$1\frac{2}{5}$

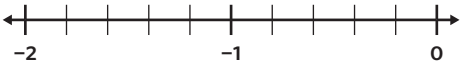


a. $\frac{9}{5}$

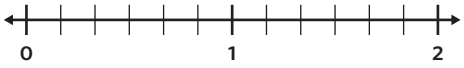


b. $-1\frac{1}{3}$

c. $-1\frac{3}{5}$



d. $\frac{7}{6}$



3. Fill in the blanks by using the words provided.

proper | improper | mixed number | less | greater | equal

A [ ] fraction has a numerator that is [ ] than its denominator.

An [ ] fraction has a numerator that is [ ] than or [ ] to its denominator. A [ ] is a combination of a whole number and a fraction.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 4. State whether each value is a proper fraction, improper fraction or mixed number.

a. $\frac{7}{5}$ b. $\frac{4}{9}$ c. $1\frac{7}{12}$ d. $-\frac{11}{20}$

e. $-1\frac{4}{7}$ f. $\frac{14}{5}$ g. $3\frac{8}{15}$ h. $-10\frac{1}{9}$

WE2 5. Convert each mixed number to an improper fraction.

a. $4\frac{1}{5}$ b. $3\frac{3}{7}$ c. $7\frac{6}{7}$ d. $10\frac{8}{11}$

e. $2\frac{3}{4}$ f. $5\frac{8}{9}$ g. $4\frac{1}{6}$ h. $3\frac{4}{12}$

WE3 6. Express each improper fraction as a mixed number.

a. $\frac{6}{4}$ b. $\frac{8}{3}$ c. $\frac{15}{4}$ d. $\frac{27}{5}$

e. $\frac{37}{10}$ f. $\frac{19}{6}$ g. $\frac{91}{12}$ h. $\frac{27}{4}$

WE4 7. Complete the following fractional sequences by filling in the gaps.

a. $\frac{1}{3}, 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3}, \ldots, \ldots, \ldots$ b. $4, 4\frac{1}{4}, 4\frac{2}{4}, 4\frac{3}{4}, \ldots, \ldots, \ldots$

c. $\ldots, \ldots, \ldots, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$ d. $-\frac{2}{7}, \ldots, 0, \frac{1}{7}, \frac{2}{7}, \ldots, \ldots$

e. $\ldots, 1\frac{2}{3}, 2\frac{1}{3}, 3, 3\frac{2}{3}, \ldots, \ldots$ f. $\ldots, 2\frac{6}{8}, 4\frac{7}{8}, 7, 9\frac{1}{8}, \ldots, \ldots$

g. $\ldots, 1\frac{2}{5}, 2, 2\frac{3}{5}, \ldots, 3\frac{4}{5}, \ldots$ h. $\ldots, -2\frac{4}{6}, \ldots, 0, \ldots, 2\frac{4}{6}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Koda had seven cookies and wanted to share them equally between herself and three friends. How many cookies will each person get?

9. Amira and her friends shared Komino's pizzas. How many slices of pizza did they eat? Each pizza was sliced into 8 and they finished $4\frac{3}{8}$ pizzas.

10. Ronald is running laps on a 400 m track. If he ran 5900 m in total, how many laps did he run?

11. What is the difference in years between Stephanie's two cats Jesse and James? Jesse is 156 months old and James is 125 months old. Express answer as a mixed fraction

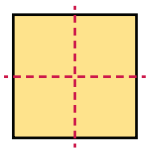
12. On the Overland Track there is a clean water station every $2\frac{2}{3}$ kilometres. Atif filled up his bottles at the start of the track. After how many kilometres did Atif refill, if he topped up his bottles at the third station?

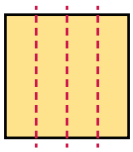
Reasoning

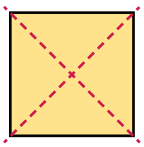
Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

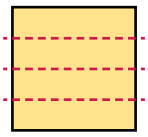
13. Eddie has 20 slices of bread and is making sandwiches for 35 people.

a. Eddie can make 10 sandwiches with his 20 slices of bread. He wants to cut the sandwiches into 4 equal pieces. Select all of the ways Eddie can cut the sandwiches.

A. 

B. 

C. 

D. 

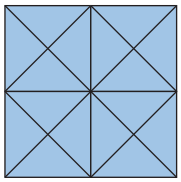
b. At the end of the meal, there are only 3 pieces left over. How many sandwiches were eaten? Write your answer as a mixed number.

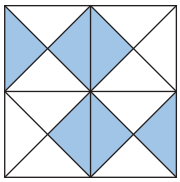
c. Eddie ordered an ice cream cake for dessert using UberEats. The ice cream shop is 12 km from his home. If the UberEats driver has driven $\frac{15}{3}$ km of the journey. How far away is the driver?

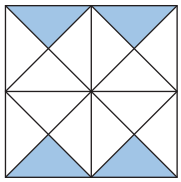
d. The UberEats driver collected the ice cream cake at 7:30 pm. Eddie checks his phone at 7:51 pm and sees that the driver has $1\frac{1}{2}$ km remaining of his 12 km journey. Assuming the driver maintains the same speed, at what time will the delivery arrive?

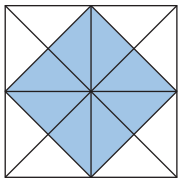
e. Eddie decides to make the sandwiches using large rectangular slices of bread. He would like to cut each sandwich into eight equal pieces. Draw different options to show that Eddie could cut the sandwiches in more than 4 different ways.

14. Anna's bathroom contains mosaic tiles with patterns. Three of Anna's tiles have missing pieces. Each white triangle represents a missing piece.


Full tile


Tile 1


Tile 2


Tile 3

a. What is the fraction of missing pieces in each tile?

i) Tile 1 ii) Tile 2 iii) Tile 3

b. How many new tiles will Anna need to buy to replace the missing pieces?

c. At the tile shop, Anna is given two options to replace the missing tiles. She can pay $30 per tile or $2 per missing piece. If Anna wants to select the cheaper option, which option should she select?

d. Using this style of mosaic tile, create a design for one of Anna's bathroom tiles.

Extra spicy

15. Three fifths of a number is 48. What is the number?

16. Which of the numbers 7, 8, 9, 10 or 11, when placed in the box below provides an improper fraction closest to 3?

$$\frac{25}{\square}$$

17. The value of $\frac{1007 + 13}{100}$ is:

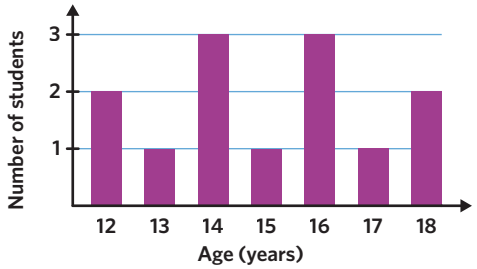
A. 1.02 B. 1.2 C. 102 D. 10.2 E. 10.02

18. Find the next value in the sequence.
$\frac{3}{2}, \frac{18}{8}, 3, \frac{15}{4}, \ldots$

Remember this?

19. Sophie had 33 lollipops. She gave an equal number of lollipops to 7 friends. She gave each of them as many as possible and kept the rest for herself. How many lollipops did Sophie keep for herself?

20. The age of students on a school swimming team is recorded in a graph.



How many students are in the swimming team?

21. This table shows the length and width of four rectangular rooms in metres.

Room	A	B	C	D
Length (m)	5	7	10	15
Width (m)	6	8	20	15

Which room has a perimeter of 30 m?

3C Equivalent fractions

Equivalent fractions represent the same quantity but with a different number of parts. The values of the fractions do not change but the numerators and denominators will increase or decrease by a certain factor. Below are some examples where equivalent fractions can be applied.

- On Monday I ate $\frac{1}{3}$ of a chocolate bar and on Tuesday I ate another $\frac{2}{6}$ of the chocolate bar. Did I eat a different amount of chocolate on each day?
- Yesterday I walked $10\frac{2}{5}$ kilometres. I wanted to walk the same distance today so I walked $\frac{104}{10}$ kilometres. Did I walk the same distance on both days?
- I baked 16 cupcakes and decided to give $\frac{1}{4}$ to my sister. My brother got jealous and wanted the same amount so I gave him $\frac{4}{16}$ of my cupcakes. Is my brother happy?

Learning intentions

Students will be able to:

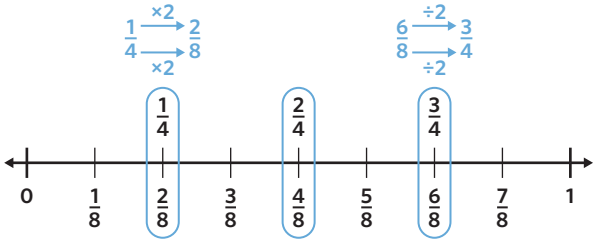
+ find equivalent fractions using multiplication and division
+ fully simplify fractions using the highest common factor between the numerator and denominator.

Key terms and definitions

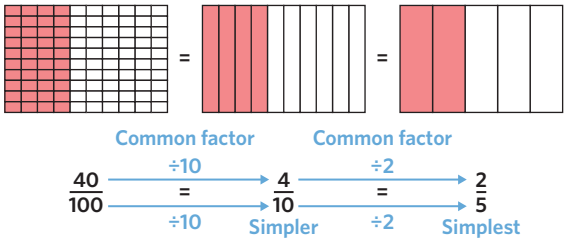
- **Equivalent fractions** are fractions which express the same quantity but with a different numerator and denominator. Eg. $\frac{4}{6}$ and $\frac{8}{12}$ are equivalent fractions.
- A fraction is in its **simplest form** when the numerator and denominator have no common factors other than 1. E.g. $\frac{5}{10}$ in the simplest form is $\frac{1}{2}$.
- **Simplifying fractions** is the process of expressing an equivalent fraction with a smaller numerator and smaller denominator.
- **Highest common factor (HCF)** is the largest number that is a factor of two or more numbers. E.g. 5 is the HCF of 5 and 15.

Key ideas

1 Fractions remain equivalent if the numerator and denominator are both multiplied or divided by the same value.

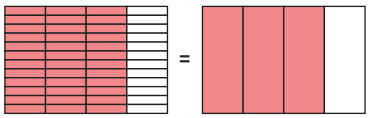


2 Finding an equivalent fraction with a smaller numerator and denominator is called simplifying a fraction.



3 To fully simplify a fraction, find the highest common factor (HCF) of the numerator and the denominator, then divide both the numerator and the denominator by the HCF.

The HCF of 36 and 48 is 12.



$\frac{36}{48} \xrightarrow{\div 12} \frac{3}{4}$

Worked examples

WE 1 Finding an equivalent fraction

Find the equivalent fractions by performing the appropriate multiplication or division.

a. $\frac{3}{5} = \frac{?}{10}$

Working

$\frac{3}{5} = \frac{?}{10}$ ($\times 2$)

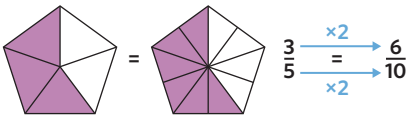
$\frac{3}{5} = \frac{6}{10}$ ($\times 2$)

Thinking

Step 1: Find the relationship between the two given denominators.

Step 2: Apply the change to the known numerator to find the equivalent fraction.

Visual support



$\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10}$

b. $2\frac{18}{42} = 2\frac{3}{?}$

Working

$2\frac{18}{42} = 2\frac{3}{?}$ ($\div 6$)

$2\frac{18}{42} = 2\frac{3}{7}$ ($\div 6$)

Thinking

Step 1: The whole numbers of the mixed numbers are the same, so find the relationship between the two given numerators.

Step 2: Apply the change to the known denominator to find the equivalent fraction.

Student practice

Find the equivalent fractions by performing the appropriate multiplication or division.

a. $\frac{4}{5} = \frac{16}{?}$

b. $3\frac{25}{35} = 3\frac{?}{7}$

c. $\frac{7}{9} = \frac{?}{108}$

d. $4\frac{28}{42} = 4\frac{2}{?}$

WE 2 Finding equivalent fractions

Find the missing values for all the equivalent fractions.

a. $\frac{1}{3} = \frac{?}{12} = \frac{12}{?} = \frac{?}{72} = \frac{120}{?}$

Working

$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

$\frac{1 \times 12}{3 \times 12} = \frac{12}{36}$

$\frac{1 \times 24}{3 \times 24} = \frac{24}{72}$

$\frac{1 \times 120}{3 \times 120} = \frac{120}{360}$

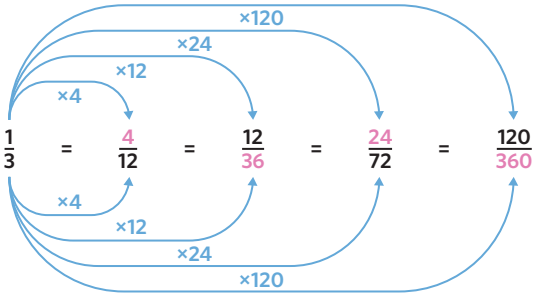
Thinking

Step 1: Find the relationship between the two known denominators in the first two fractions. Apply this change to find the unknown numerator in the second equivalent fraction.

Step 2: Find the relationship between the two known numerators in the first and third fraction. Apply this change to find the unknown denominator in the third equivalent fraction.

Step 3: Use the first fraction to repeat the process and find the remaining equivalent fractions.

Visual support



b. $\frac{60}{80} = \frac{?}{40} = \frac{15}{?} = \frac{?}{16} = \frac{3}{?}$

Working

$\frac{60 \div 2}{80 \div 2} = \frac{30}{40}$

$\frac{60 \div 4}{80 \div 4} = \frac{15}{20}$

$\frac{60 \div 5}{80 \div 5} = \frac{12}{16}$

$\frac{60 \div 20}{80 \div 20} = \frac{3}{4}$

Thinking

Step 1: Find the relationship between the two known denominators in the first two fractions. Apply this change to find the unknown numerator in the second equivalent fraction.

Step 2: Find the relationship between the two known numerators in the first and third fraction. Apply this change to find the unknown denominator in the third equivalent fraction.

Step 3: Use the first fraction to repeat the process and find the remaining equivalent fractions.

Student practice

Find the missing values for all the equivalent fractions.

a. $\frac{1}{2} = \frac{?}{6} = \frac{4}{?} = \frac{?}{12} = \frac{10}{?}$

b. $\frac{30}{60} = \frac{?}{30} = \frac{10}{?} = \frac{?}{10} = \frac{1}{?}$

c. $\frac{2}{3} = \frac{?}{33} = \frac{30}{?} = \frac{?}{60} = \frac{100}{?}$

d. $\frac{400}{500} = \frac{?}{250} = \frac{100}{?} = \frac{?}{20} = \frac{8}{?}$

WE 3 Converting fractions to simplest form

Express each fraction in its simplest form.

a. $\frac{8}{12}$

Working

Factors of 8: 1, 2, 4, 8

Factors of 12: 1, 2, 3, 4, 6, 12

The HCF is 4.

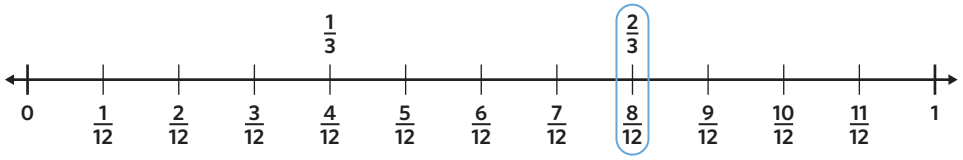
$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Thinking

Step 1: Find the highest common factor (HCF) of the numerator and denominator.

Step 2: Divide the numerator and denominator by the highest common factor.

Visual support



b. $-5\frac{9}{27}$

Working

Factors of 9: 1, 3, 9

Factors of 27: 1, 3, 9, 27

The HCF is 9.

$-5\frac{9 \div 9}{27 \div 9} = -5\frac{1}{3}$

Thinking

Step 1: Find the highest common factor (HCF) of the numerator and denominator.

Step 2: Divide the numerator and denominator by the highest common factor. Keep the whole number the same.

Student practice

Express each fraction in its simplest form.

a. $\frac{33}{39}$ **b.** $-2\frac{18}{24}$ **c.** $\frac{45}{60}$ **d.** $8\frac{8}{88}$

3C Activities and questions

STARTER TASKS

Image: Anna Kucera/The Age

Odd spot

Christopher Thé (right) is the creator of Black Star Pastry's Strawberry Watermelon Cake which is the world's most instagrammed cake. They started in Australia but now sell the cake around the world including in the US and Asia. It is estimated that they sell 16 000 slices of cake a week. Last week, Christopher helped bake 6000 slices of cake. What fraction of the total slices of cake did Christopher help bake last week? Express in the simplest form.

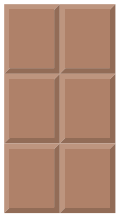
A. $\frac{3}{8}$ **B.** $\frac{3}{4}$

Puzzle

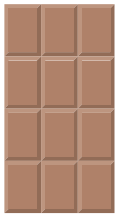
a) Would you prefer to have 2 pieces from chocolate bar A or 4 pieces from chocolate bar B?

b) How could you share these two chocolate bars equally between 3 people?

Chocolate bar A



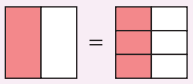
Chocolate bar B



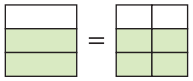
Understanding worksheet

1. Complete the equivalent fractions for each of these diagrams.

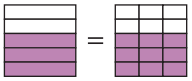
Example



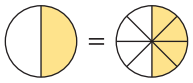
$\frac{1}{2} = \frac{3}{6}$

a. 

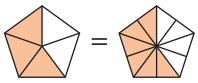
$\frac{2}{3} = \frac{\square}{6}$

b. 

$\frac{3}{\square} = \frac{\square}{15}$

c. 

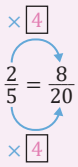
$\frac{\square}{\square} = \frac{\square}{\square}$

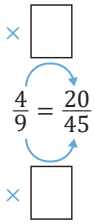
d. 

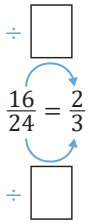
$\frac{\square}{\square} = \frac{\square}{\square}$

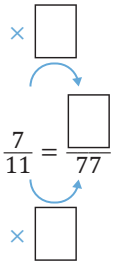
2. Fill in the missing boxes to show the change between the two fractions and complete the equivalent fractions.

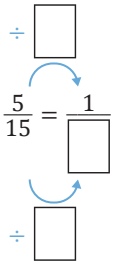
Example



a. 

b. 

c. 

d. 

3. Fill in the blanks by using the words provided.

highest | equivalent | value | common | simplify

To _____ a fraction, we divide the numerator and denominator by the ☐ ☐ factor. We can also use the highest common factor to simplify a fraction to find an ☐ fraction. Equivalent fractions represent the same ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7 (e,f,g,h)

WE1 **4.** Find the equivalent fraction by performing the appropriate multiplication or division.

a. $\frac{1}{3} = \frac{?}{6}$ **b.** $\frac{7}{6} = \frac{21}{?}$ **c.** $\frac{16}{24} = \frac{2}{?}$ **d.** $\frac{45}{36} = \frac{?}{4}$

e. $1\frac{3}{8} = 1\frac{?}{40}$ **f.** $3\frac{15}{21} = 3\frac{?}{7}$ **g.** $\frac{7}{3} = 2\frac{?}{21}$ **h.** $4\frac{6}{10} = \frac{?}{20}$

WE2 5. Find the missing values for all the equivalent fractions.

a. $\frac{1}{4} = \frac{?}{12} = \frac{6}{?} = \frac{?}{28} = \frac{40}{?}$

b. $\frac{5}{6} = \frac{?}{12} = \frac{30}{?} = \frac{?}{90} = \frac{150}{?}$

c. $\frac{8}{5} = \frac{?}{10} = \frac{64}{?} = \frac{?}{75} = \frac{136}{?}$

d. $\frac{9}{4} = \frac{?}{8} = \frac{72}{?} = \frac{?}{48} = \frac{144}{?}$

e. $\frac{40}{120} = \frac{?}{60} = \frac{10}{?} = \frac{?}{12} = \frac{1}{?}$

f. $\frac{200}{350} = \frac{?}{175} = \frac{40}{?} = \frac{?}{35} = \frac{4}{?}$

g. $\frac{480}{440} = \frac{?}{220} = \frac{120}{?} = \frac{?}{22} = \frac{12}{?}$

h. $\frac{260}{60} = \frac{?}{30} = \frac{65}{?} = \frac{?}{12} = \frac{13}{?}$

6. Find the highest common factor of the groups of numbers.

a. 4, 16 b. 5, 25 c. 6, 12, 15 d. 16, 18, 24

e. 16, 24, 52 f. 18, 27, 54

WE3 7. Express each fraction in its simplest form.

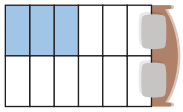
a. $\frac{15}{18}$ b. $\frac{8}{20}$ c. $-\frac{7}{28}$ d. $\frac{17}{51}$

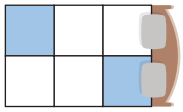
e. $\frac{36}{27}$ f. $-4\frac{15}{33}$ g. $\frac{84}{48}$ h. $-\frac{112}{60}$

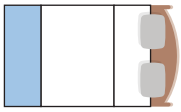
Problem solving

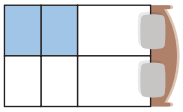
Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Which bedspread should Alice buy if she wants a bedspread that is $\frac{1}{3}$ blue?

A. 

B. 

C. 

D. 

9. Dave and Susie have the same size chocolate bars. Who ate more if Dave broke his chocolate bar into 18 pieces and ate 12 of them, whilst Susie broke her chocolate bar into 6 pieces and ate 4 of them?

10. What fraction of the Harry Potter movies has Lucy seen? There are eight movies and she has watched four of them. Express in simplest form.

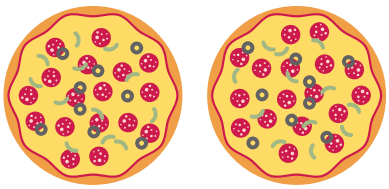
11. Steven has 80 mL left in his 400 mL bottle of juice. What fraction of the bottle has he drunk (in simplest form)?

12. Garfield divided a lasagna to share equally between 12 people. What fraction of people did not finish their serve if four people had leftovers? Express in simplest form.

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c,d)	All

13. Danny and Sophie order two identical pizzas. Danny cuts his pizza into 6 equal slices and Sophie cuts her pizza into 4 equal slices.



a. Danny was hungry and ate 3 slices of his pizza. What fraction of his pizza did he eat?

b. Sophie thought her pizza slices were too big so she cut each existing slice into thirds and ate 6 slices. Use equivalent fractions to determine how many slices she would've eaten if she kept the original sizes.

c. Did Danny and Sophie eat an equivalent fraction of their pizzas?

d. Danny and Sophie decide to take their remaining pizza home. How should Danny and Sophie fairly divide their leftovers?

14. Rebecca and Elaine are following a recipe that makes 35 cookies for their mum. The butter, eggs, milk and sugar are combined first, weighing 350 g altogether. Then 250 g of flour needs to be added to finish the mixture.

a. What fraction of the mixture is flour? Express in simplest form.

b. Elaine wants to make raisin cookies. If she adds raisins to $\frac{1}{5}$ of the cookies, how many raisin cookies did she make?

c. Rebecca wants to make chocolate chip cookies. If she adds chocolate chips to $\frac{4}{7}$ of the cookies, how many chocolate chip cookies did she make?

d. $\frac{8}{35}$ of the remaining cookies are plain and do not have any chocolate chips or raisins. Is there a way to simplify this fraction of cookies? Why or why not?

e. How should they split the cost of the ingredients?

Extra spicy

15. If a fraction's numerator and denominator are both prime numbers, can it be further simplified? Explain why.

16. What is an equivalent fraction of $\frac{6^2}{8^2}$?

A. $\frac{1}{2}$ B. $\frac{9}{16}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. $\frac{4}{3}$

17. At an animal shelter, $\frac{2}{5}$ of the animals are dogs and $\frac{3}{5}$ are cats. After 4 of the dogs are adopted, $\frac{1}{3}$ of the animals are dogs and $\frac{2}{3}$ are cats. How many cats are there at the animal shelter?

18. Find an equivalent fraction to $\frac{5}{9}$ where the sum of the numerator and denominator is 462.

Remember this?

19. Joel buys 39 sushi rolls for him and his guests to eat at his birthday party. Everyone can eat 3 sushi rolls each. Which number sentence shows the number of people that are at Joel's party?

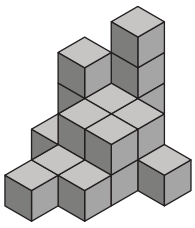
A. $39 + 3 = 42$ B. $39 - 3 = 36$ C. $39 \times 3 = 117$ D. $39 \div 3 = 13$

20. How many more people lived in Tokyo than Seoul in 2020?

A. 2 620 237 B. 2 793 239
C. 26 375 233 D. 27 431 867

City	Population in 2020
Tokyo	37 394 824
Seoul	9 962 957

21. This is a view of a solid made of 22 identical cubes.



How many cubes are completely hidden in this view?

A. 5 B. 7 C. 9 D. 15

3D Comparing fractions

Just like with whole numbers, we can compare two fractions to determine which is larger or smaller. We can also place a set of fractions in ascending or descending order. Below are some examples where comparing fractions can be applied.

- I kicked 3 out of 4 of my shots on goal and my brother kicked 4 out of 6 of his shots. Who kicked more accurately?
- $\frac{8}{9}$ of the students in my maths class completed their homework. $\frac{10}{12}$ of the students in my art class completed their homework. Which class had a smaller fraction of students complete homework?
- On Thursday I ran $\frac{3}{5}$ of a kilometre, on Friday $\frac{7}{12}$ of a kilometre and on Sunday $\frac{1}{2}$ of a kilometre. Compare how far I ran on each day.

Learning intentions

Students will be able to:

+ compare fractions with the same denominator or numerator
+ find the lowest common denominator for a set of fractions
+ compare fractions with different denominators and numerators.

Key terms and definitions

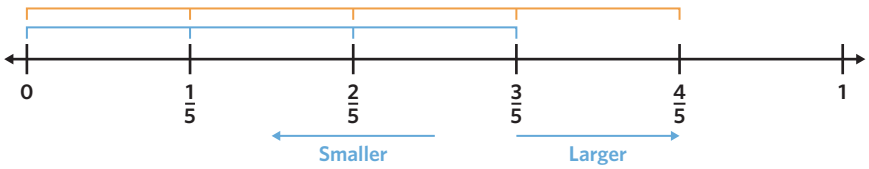
- The **lowest common denominator (LCD)** is the lowest number that is a multiple of all the given denominators.
- **Ascending** order means to arrange values from smallest to largest.
- **Descending** order means to arrange values from largest to smallest.
- **Inequality symbols** are used to show which number is larger or smaller. The pointy end faces the smaller value.
  - **>** means 'is larger or greater than'.
  - **<** means 'is smaller or less than'.

Key ideas

1 Fractions with the **same denominator** can be compared by looking at their numerators.

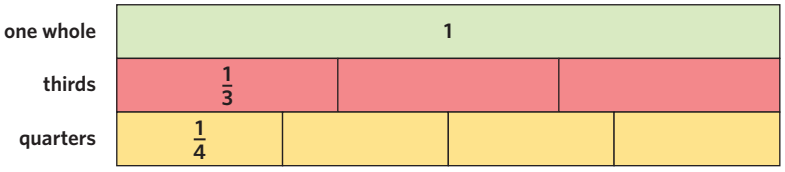
$\frac{3}{5} < \frac{4}{5}$ because $\frac{3}{5}$ has the same denominator as $\frac{4}{5}$ but has a smaller numerator.

$\frac{3}{5}$ is further to the left than $\frac{4}{5}$ on the number line meaning it is smaller.



2 Fractions with the **same numerator** can be compared by looking at their denominators.

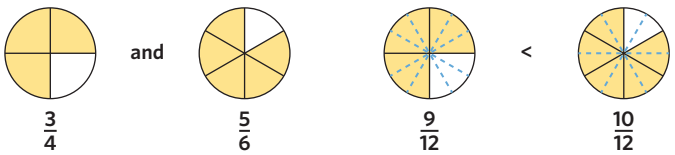
$\frac{1}{3} > \frac{1}{4}$ because $\frac{1}{3}$ has the same numerator as $\frac{1}{4}$ but has a smaller denominator.



A larger denominator means we are dividing the whole into more parts. When the whole is divided into more parts, each individual part is smaller.

3 When fractions have **different denominators and numerators**, determine the lowest common denominator to find equivalent fractions so that the numerators can be compared.

$\frac{3}{4}$ and $\frac{5}{6}$



The lowest common denominator for both fractions is 12.

The larger fraction is $\frac{10}{12}$ or $\frac{5}{6}$ because it has a greater number of shaded parts

Worked examples

WE 1 Comparing fractions with the same denominators

Place the correct mathematical symbol in the box to compare the fractions (<, >, =).

a. $\frac{5}{6} \square \frac{3}{6}$

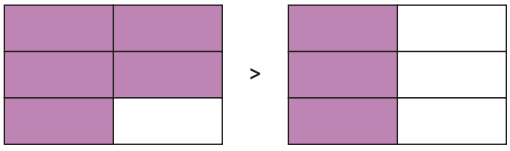
Working

$\frac{5}{6} > \frac{3}{6}$

Thinking

The denominators are the same so we can compare the numerators. 5 is larger than 3.

Visual support



$\frac{5}{6}$ > $\frac{3}{6}$

b. $\frac{13}{12} \square \frac{17}{12}$

Working

$\frac{13}{12} < \frac{17}{12}$

Thinking

The denominators are the same so we can compare the numerators. 13 is less than 17.

Student practice

Place the correct mathematical symbol in the box to compare the fractions (<, >, =).

a. $\frac{1}{8} \square \frac{3}{8}$ b. $\frac{17}{25} \square \frac{15}{25}$ c. $\frac{13}{9} \square \frac{19}{9}$ d. $\frac{23}{5} \square 4\frac{2}{5}$

WE 2 Comparing fractions with different denominators

Compare the fractions using the mathematical symbols <, > or =.

a. $\frac{3}{4}, \frac{5}{8}$

Working

Multiples of 4: 4, 8, 12

Multiples of 8: 8, 16, 24

The LCD is 8.

$\frac{3}{4} = \frac{6}{8}$

$\frac{6}{8} > \frac{5}{8}$

$\frac{3}{4} > \frac{5}{8}$

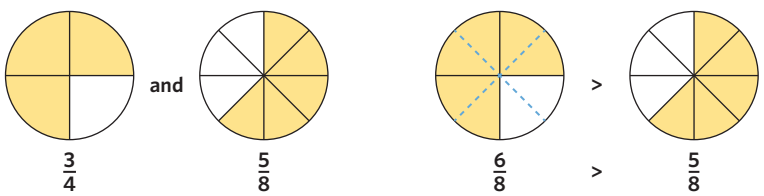
Thinking

Step 1: Find the lowest common denominator by finding the lowest common multiple of 4 and 8.

Step 2: Write equivalent fractions so that both fractions have the lowest common denominator 8.

Step 3: Now that the denominators are the same, compare the numerators.

Visual support



$\frac{3}{4}$ $\frac{5}{8}$ $\frac{6}{8}$ > $\frac{5}{8}$

b. $1\frac{3}{5}, 1\frac{5}{6}$

Working

Multiples of 5: 5, 10, 15, 20, 25, 30

Multiples of 6: 6, 12, 18, 24, 30

The LCD is 30.

$1\frac{3}{5} = 1\frac{18}{30}$

$1\frac{5}{6} = 1\frac{25}{30}$

$1\frac{18}{30} < 1\frac{25}{30}$

$1\frac{3}{5} < 1\frac{5}{6}$

Thinking

Step 1: Both mixed numbers have the same whole number, so we compare the fractions. Find the lowest common denominator by finding the lowest common multiple of 5 and 6.

Step 2: Write equivalent fractions for both $1\frac{3}{5}$ and $1\frac{5}{6}$ with the common denominator 30.

Step 3: Now that the denominators are the same, compare the numerators.

Student practice

Compare the fractions using the mathematical symbols <, > or =.

a. $\frac{4}{5}, \frac{7}{10}$ **b.** $2\frac{2}{3}, 2\frac{4}{5}$ **c.** $\frac{13}{5}, \frac{15}{6}$ **d.** $\frac{17}{6}, 2\frac{2}{15}$

WE 3 Ordering fractions

Place the values in the described order.

a. Ascending: $\frac{2}{3}, \frac{1}{2}, \frac{1}{6}, \frac{5}{6}$

Working

LCD is 6

$\frac{2}{3} = \frac{4}{6}$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{6} < \frac{3}{6} < \frac{4}{6} < \frac{5}{6}$

$\frac{1}{6} < \frac{1}{2} < \frac{2}{3} < \frac{5}{6}$

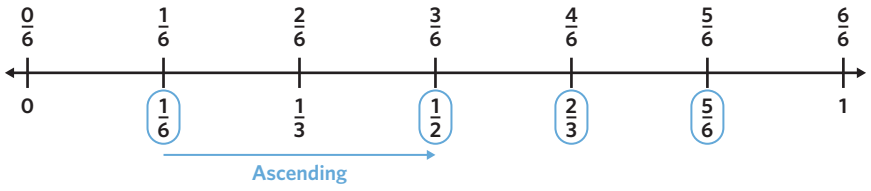
Thinking

Step 1: Find the lowest common denominator (LCD) by finding the lowest common multiple of 2, 3 and 6.

Step 2: Write equivalent fractions with the common denominator of 6 for all fractions.

Step 3: Now that all the denominators are the same, put the fractions in ascending order (smallest to largest) based on their numerators.

Visual support



b. Descending: $1\frac{4}{5}, -\frac{7}{2}, 3, \frac{12}{10}$

Working

LCD is 10

$1\frac{4}{5} = \frac{9}{5} = \frac{18}{10}$

$-\frac{7}{2} = -\frac{35}{10}$

$3 = \frac{3}{1} = \frac{30}{10}$

$-\frac{35}{10} > \frac{30}{10} > \frac{18}{10} > \frac{12}{10}$

$-\frac{7}{2} \quad > \quad 3 \quad > 1\frac{4}{5} > \frac{12}{10}$

Thinking

Step 1: Find the lowest common denominator (LCD) by finding the lowest common multiple of 5, 2 and 10.

Step 2: Express all values as improper fractions with the common denominator 10.

Step 3: Now that all the denominators are the same, put the fractions in descending order (largest to smallest) based on their numerators.

Student practice

Place the values in the described order.

a. Ascending: $\frac{4}{3}, \frac{3}{2}, \frac{6}{5}, \frac{22}{15}$

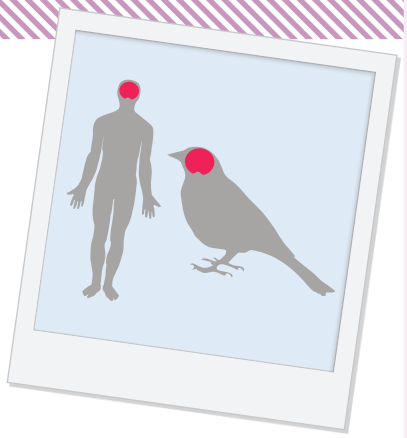
b. Descending: $4\frac{2}{4}, 5, \frac{37}{8}, \frac{11}{2}$

c. Ascending: $-\frac{6}{2}, \frac{17}{7}, -3\frac{3}{4}, 2\frac{10}{14}$

d. Descending: $-\frac{5}{6}, -2\frac{7}{8}, -\frac{25}{12}, -\frac{16}{18}$

3D Activities and questions

STARTER TASKS



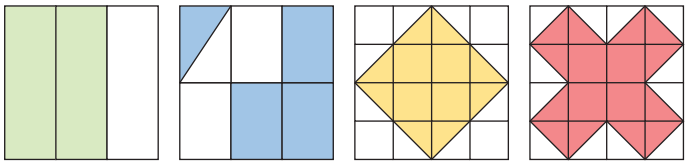
Odd spot

A 60 kg human has a brain that weighs about 1.5 kg. A 30 g sparrow has a brain that weighs about 1 g. Whose brain makes up a larger fraction of their body weight?

A. Human

B. Sparrow

Puzzle



Design A Design B Design C Design D

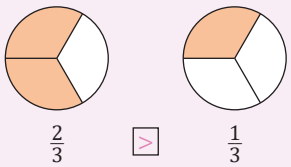
a) Which design has a larger coloured area, A or B?

b) Which design has a larger coloured area, C or D?

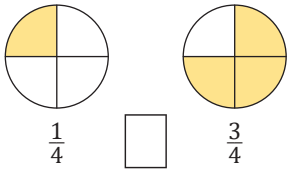
Understanding worksheet

1. Place the correct mathematical symbol in the box to compare the fractions (<, > or =).

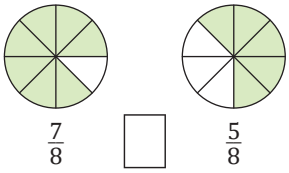
Example



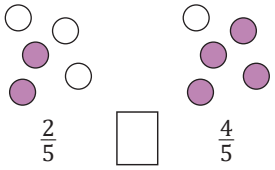
a.



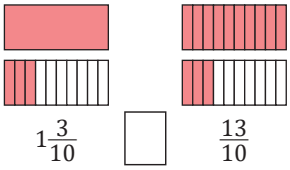
b.



c.

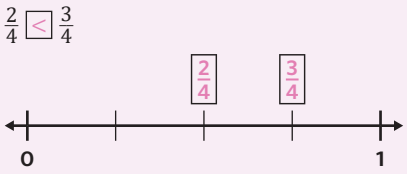


d.

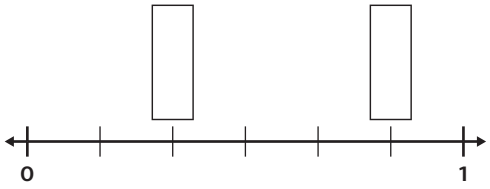


2. Place the correct mathematical symbol in the box to compare the fractions (<, >, =) and fill in the missing fractions on the number line.

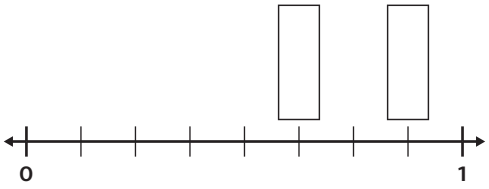
Example



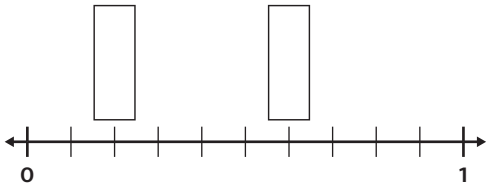
a. $\frac{2}{6} \square \frac{5}{6}$



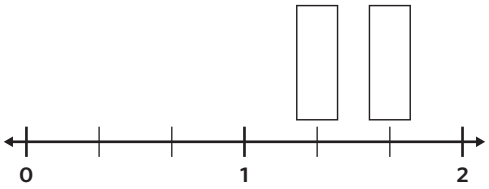
b. $\frac{5}{8} \square \frac{7}{8}$



c. $\frac{60}{100} \square \frac{20}{100}$



d. $1\frac{1}{3} \square 1\frac{2}{3}$



3. Fill in the blanks by using the words provided.

lowest | denominator | numerators | equivalent | common | different

When we compare fractions with common denominators, we can just compare the _____. When we compare fractions with _____ denominators, we need to first find the _____ _____ _____ and write _____ fractions.

Fluency

Question working paths

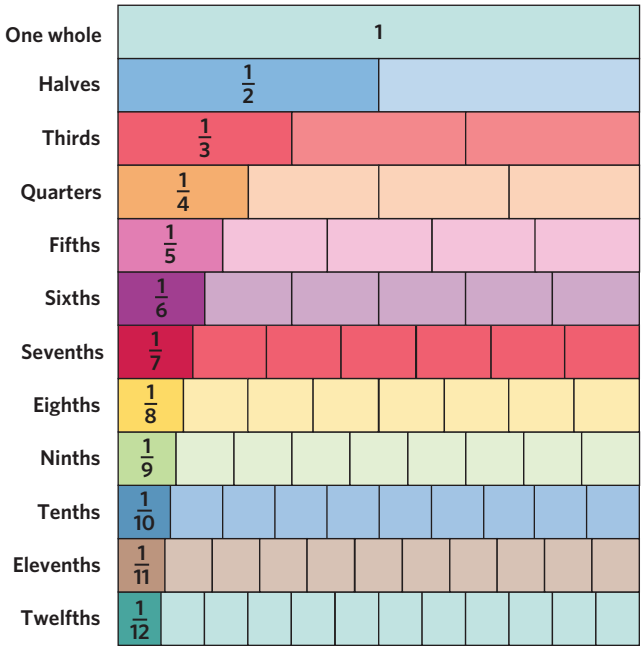
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d), 8 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f), 8 (d,e,f)

WE1 **4.** Place the correct mathematical symbol in the box to compare the fractions (<, >, =).

a. $\frac{4}{4} \square \frac{1}{4}$ b. $\frac{1}{6} \square \frac{5}{6}$ c. $\frac{7}{9} \square \frac{5}{9}$ d. $\frac{53}{100} \square \frac{77}{100}$

e. $\frac{8}{3} \square \frac{7}{3}$ f. $3\frac{1}{4} \square \frac{13}{4}$

5. Use the fraction wall to place the correct mathematical symbol in the box (< or >).



a. $\frac{1}{5} \square \frac{1}{4}$ b. $\frac{1}{12} \square \frac{1}{3}$ c. $\frac{2}{9} \square \frac{2}{10}$ d. $\frac{3}{10} \square \frac{3}{8}$

e. $\frac{5}{6} \square \frac{5}{8}$ f. $\frac{6}{11} \square \frac{6}{7}$

WE2 **6.** Compare the fractions using the mathematical symbols <, > or =.

a. $\frac{3}{6}, \frac{2}{3}$ b. $\frac{2}{4}, \frac{5}{12}$ c. $\frac{4}{5}, \frac{17}{25}$ d. $\frac{1}{3}, \frac{2}{5}$

e. $\frac{10}{4}, \frac{15}{6}$ f. $\frac{3}{7}, \frac{4}{11}$ g. $\frac{8}{12}, 1\frac{1}{8}$ h. $\frac{16}{4}, \frac{23}{9}$

WE3a **7.** Place the values in ascending order.

a. $\frac{1}{2}, \frac{3}{2}, \frac{3}{4}, \frac{5}{4}$ b. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, 1\frac{1}{6}$ c. $\frac{9}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{10}$ d. $\frac{5}{4}, \frac{3}{4}, \frac{5}{6}, \frac{5}{8}$

e. $-\frac{8}{4}, -\frac{1}{4}, -\frac{1}{2}, -\frac{2}{3}$ f. $-\frac{17}{8}, -\frac{2}{3}, -\frac{5}{4}, -\frac{5}{6}$

WE3b **8.** Place the values in descending order.

a. $\frac{1}{3}, \frac{8}{9}, \frac{5}{9}, \frac{2}{3}$ b. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{1}{2}$ c. $\frac{12}{10}, \frac{8}{5}, \frac{6}{2}, \frac{9}{10}$ d. $5, \frac{18}{4}, 5\frac{1}{2}, 4\frac{3}{4}$

e. $-\frac{31}{25}, -\frac{7}{4}, -2, -1\frac{2}{5}$ f. $3\frac{1}{3}, \frac{26}{9}, 3\frac{5}{6}, 4\frac{18}{27}$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Out of the four students who participated in the school spelling competition, who scored the highest? Annie got $\frac{8}{10}$ correct, Caitlyn got $\frac{7}{10}$ correct, Jax got $\frac{5}{10}$ correct and Milo got $\frac{9}{10}$ correct.

10. On Thursday night, Kelly and Finn's families both had takeaway pizza. Kelly and Finn are trying to figure out whose family ate more pizza. Kelly's family ate $1\frac{5}{8}$ pizzas and Finn's family ate 15 slices from pizzas that were sliced into 8.

11. Lee and Sunny each have a large identical piece of ribbon. Lee cut eight-elevenths of his ribbon to make a flower. Sunny cut five-sevenths of her ribbon to make a wreath. Who used less ribbon for their craft?

12. Which competition had lower student participation in Douglas' class of 24 students? Three-quarters of the class entered the swimming competition and a third of the class didn't enter the athletics competition.

13. Harry and Jim have both drunk some of their juice boxes. Harry has 60 mL left in his 240 mL juice box. Jim has 100 mL left in his 500 mL juice box. Who drank a greater fraction of their juice box?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Tom is trying to be more healthy and limit his sugar intake. However, he wants to eat one of his favourite doughnuts from Melbourne's two most famous doughnut vans. The 40 gram doughnut from American Doughnut Kitchen has 5 grams of white sugar and 5 grams of brown sugar. The 60 gram doughnut from Dandee Donuts, has 10 grams of white sugar and 3 grams of brown sugar.

Images: Eddie Jim/The Age

a. Which doughnut has the greatest fraction of sugar in total?

b. Assuming that the weight of the doughnut stays the same, how many grams of sugar does the maker of the less sweet doughnut need to add to their product to make it equally as sweet as their competitor?

c. Tom finds one doughnut to be too sweet while the other isn't sweet enough. Which of the following fractions could be the fraction of sugar that Tom thinks is the correct amount of sweetness?

A. $\frac{9}{30}$ B. $\frac{21}{120}$ C. $\frac{1}{20}$ D. $\frac{7}{30}$

d. Tom understands the importance of eating healthily, but plans to include some less healthy foods in his diet (like doughnuts). Why might he do this?

15. After his research into sugar, Tom decides to focus on the carbohydrates in his food. He comes up with a new rule for himself: don't eat any foods containing more than $\frac{2}{5}$ carbohydrates by weight. He compiles the following table of his favourite foods.

Food	Total Weight (g)	Carbohydrates (g)	Fibre (g)
Doughnut	200	100	4
Kiwi fruit	50	5	2
Doritos	30	18	1
Samosa	50	12	1

a. Based on his new rule, which foods from the list can Tom continue to eat?

b. If he also requires at least $\frac{1}{30}$ of the weight to be fibre, which options from his list of favourites can he eat?

c. Tom has two favourite brands of peanut butter. Using the labels, select which brand complies with Tom's carbohydrate rule?

Peanut Dream (% of weight)	
Fat	19%
Sodium	4%
Fibre	2%
Carbohydrates	42%
Sugar	20%
Protein	13%

Peanutter (per 20 g serve)	
Fat	4 g
Sodium	0.5 g
Fibre	2 g
Carbohydrates	7 g
Sugar	3 g
Protein	3.5 g

d. Tom's friend Charlie can understand the Peanut Dream label but is finding it difficult to understand the Peanutter label. Present the information from the Peanutter label in a way that is easier to understand.

Extra spicy

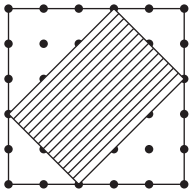
16. Which value is smaller and why? $1 + \dfrac{1}{\frac{1}{2} + \frac{1}{4}}$ or $1 + \dfrac{1}{\frac{1}{3} + \frac{1}{5}}$.

17. If $a = \frac{4}{7}$, $b = \frac{25}{42}$ and $c = \frac{2}{3}$, which of the following is true?

A. $a < b < c$ B. $a < c < b$ C. $b < a < c$ D. $b < c < a$

E. $c < a < b$ F. $c < b < a$

18. Compare the shaded and unshaded areas of the square.



19. Find the fraction that is equal to $\frac{1}{2}$ that can be made using all ten of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Remember this?

20. Together, Bill and Ted have collected 51 cans to recycle. If Bill collected 9 more cans than Ted, how many cans did Bill collect?

A. 21 B. 30 C. 34 D. 60

21. Jack is growing a beanstalk. He records the height of the beanstalk at the end of each week.

Week 1	Week 2	Week 3	Week 4	Week 5
2 cm	6 cm	18 cm	54 cm	?

If this pattern continues, how tall will the beanstalk be after 5 weeks?

A. 80 cm B. 108 cm C. 150 cm D. 162 cm

22. Hermione's school has 1992 students. These students are sorted equally into 4 houses. Which of these is closest to the number of students in each house?

A. 45 B. 50 C. 400 D. 500

3E Adding and subtracting fractions

Fractions can be added and subtracted just like whole numbers. When adding or subtracting fractions, the numerators and denominators need to be considered. Below are some examples where adding and subtracting fractions can be applied.

- I sharpened 12 pencils and broke $\frac{1}{3}$ of them on Monday, and another $\frac{1}{3}$ on Tuesday. How many pencils do I have left?
- I built a rectangular veggie patch. I planted $\frac{1}{2}$ of the veggie patch with carrots and $\frac{1}{4}$ with lettuce. What fraction of the patch has been planted?
- The distance from my home to school is 5 km. The supermarket is another $1\frac{2}{5}$ km from school. How many kilometres is it from my home to the supermarket?

Learning intentions

Students will be able to:

+ add and subtract fractions with common denominators
+ add and subtract fractions with different denominators
+ add and subtract mixed numbers and improper fractions.

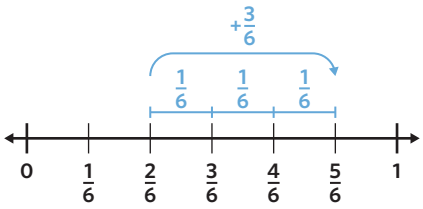
Key terms and definitions

- **Equivalent fractions** are fractions which express the same quantity but with a different numerator and denominator. E.g. $\frac{4}{6}$ and $\frac{8}{12}$ are equivalent fractions.
- Fractions have **common denominators** when their denominators are the same.
- The **lowest common denominator (LCD)** is the lowest number that is a multiple of all the given denominators.

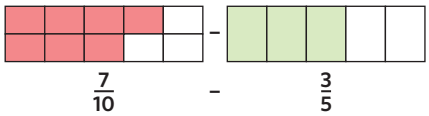
Key ideas

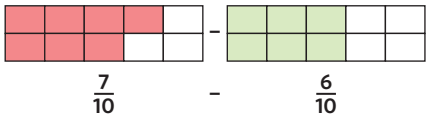
1 When adding or subtracting fractions with the same denominator, we add or subtract their numerators.

$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

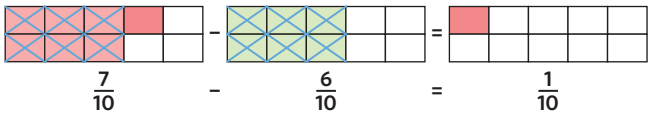


2 When adding or subtracting fractions with different denominators, we first convert the fractions so they have a common denominator.





Both fractions have a common denominator of 10.



Subtract the second fraction from the first fraction to find the difference.

Worked examples

WE 1 Adding and subtracting fractions with the same denominator

Calculate:

a. $\frac{5}{8} + \frac{2}{8}$

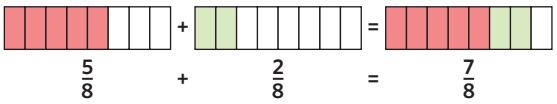
Working

$$\frac{5}{8} + \frac{2}{8} = \frac{5 + 2}{8}$$
$$= \frac{7}{8}$$

Thinking

Both fractions have the same denominator so we can add the numerators.

Visual support



b. $\frac{9}{10} - \frac{6}{10}$

Working

$$\frac{9}{10} - \frac{6}{10} = \frac{9 - 6}{10}$$
$$= \frac{3}{10}$$

Thinking

Both fractions have the same denominator so we can subtract the numerators.

Student practice

Calculate:

a. $\frac{4}{7} + \frac{2}{7}$ b. $\frac{12}{15} - \frac{10}{15}$ c. $\frac{19}{20} + \frac{19}{20}$ d. $\frac{13}{17} - \frac{9}{17}$

WE 2 Adding and subtracting fractions with different denominators

Calculate:

a. $\frac{1}{2} + \frac{1}{3}$

Working

2: 2, 4, 6

3: 3, 6

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$
$$= \frac{5}{6}$$

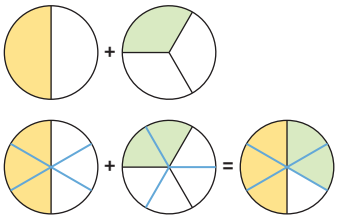
Thinking

Step 1: Find the lowest common denominator.

Step 2: Write the equivalent fractions with the lowest common denominator.

Step 3: Both fractions now have the same denominator so we can add the numerators.

Visual support



b. $\frac{3}{4} - \frac{4}{6}$

Working

4: 4, 8, 12

6: 6, 12

$\frac{3}{4} - \frac{4}{6} = \frac{9}{12} - \frac{8}{12}$

$= \frac{1}{12}$

Thinking

Step 1: Find the lowest common denominator.

Step 2: Write the equivalent fractions with the lowest common denominator.

Step 3: Both fractions now have the same denominator so we can subtract the numerators.

Student practice

Calculate:

a. $\frac{1}{3} + \frac{2}{4}$ **b.** $\frac{1}{2} - \frac{1}{3}$ **c.** $\frac{5}{6} + \frac{4}{9}$ **d.** $\frac{11}{12} - \frac{7}{8}$

WE 3 Adding and subtracting mixed numbers and improper fractions

Simplify:

a. $2\frac{2}{4} + 5\frac{3}{4}$

Working

Method 1

$2\frac{2}{4} + 5\frac{3}{4} = \frac{10}{4} + \frac{23}{4}$

$= \frac{33}{4}$

$= 8\frac{1}{4}$

Method 2

$2 + 5 + \frac{2}{4} + \frac{3}{4} = 7 + \frac{5}{4}$

$= 7 + 1\frac{1}{4}$

$= 8\frac{1}{4}$

Thinking

Method 1

Step 1: Convert both mixed numbers into improper fractions.

Step 2: Both fractions have the same denominator so we can add the numerators.

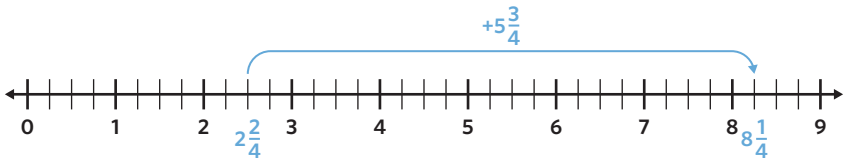
Step 3: Convert the improper fraction into a mixed number.

Method 2

Step 1: Add the whole numbers and add the fractions.

Step 2: Write $\frac{5}{4}$ as a mixed number and simplify it by adding the whole numbers together.

Visual support



b. $5\frac{1}{2} - \frac{6}{5}$

Working

$5\frac{1}{2} - \frac{6}{5} = \frac{11}{2} - \frac{6}{5}$

$= \frac{55}{10} - \frac{12}{10}$

$= \frac{43}{10}$

$= 4\frac{3}{10}$

Thinking

Step 1: Convert the mixed number into an improper fraction.

Step 2: The lowest common denominators (LCD) of 2 and 5 is 10. Write the equivalent fractions with the LCD.

Step 3: Both fractions have the same denominator so we can subtract the numerators.

Step 4: Write the answer as a mixed number.

Student practice

Simplify:

a. $1\frac{1}{4} + 3\frac{2}{4}$ b. $4\frac{2}{3} - \frac{7}{4}$ c. $2\frac{7}{9} + \frac{11}{4}$ d. $4\frac{11}{12} - \frac{14}{3}$

3E Activities and questions

STARTER TASKS

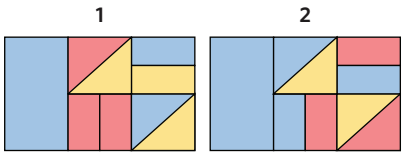
Odd spot

As a result of the COVID-19 pandemic, it is predicted that people will use public transport less. After the pandemic, it is estimated that only $\frac{1}{4}$ of the people who work in the city will use public transport. Before the pandemic, $\frac{1}{3}$ of city workers travelled into the city by car, $\frac{1}{10}$ walked or cycled, and the rest travelled by public transport. What fraction of people travelled by public transport?

A. $\frac{2}{13}$ B. $\frac{17}{30}$

Image: Eddie Jim/The Age

Puzzle



a) Can you create a full rectangle by combining the blue parts from 1 and 2?

b) By combining the red parts from both rectangles, what fraction of a full rectangle can you make?

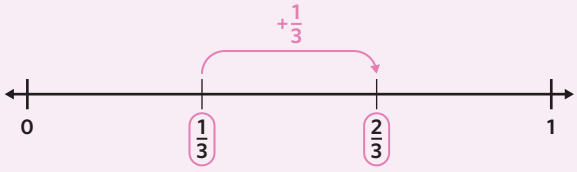
c) By combining the yellow parts from both rectangles, what fraction of a full rectangle can you make?

Understanding worksheet

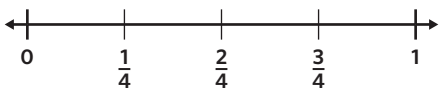
1. Represent each calculation on the number line.

Example

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$



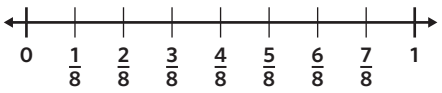
a. $\frac{1}{4} + \frac{3}{4} = 1$



b. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

c. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

d. $1 - \frac{5}{8} = \frac{3}{8}$



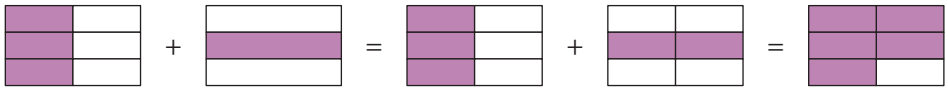
2. Fill in the missing boxes to complete each calculation.

Example

$\frac{3}{8} + \frac{1}{2}$

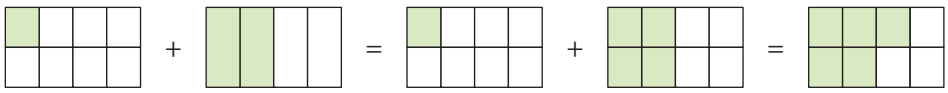
$\frac{3}{8} + \frac{\boxed{1}}{\boxed{2}} = \frac{3}{8} + \frac{\boxed{4}}{\boxed{8}} = \frac{\boxed{7}}{\boxed{8}}$

a. $\frac{3}{6} + \frac{1}{3}$



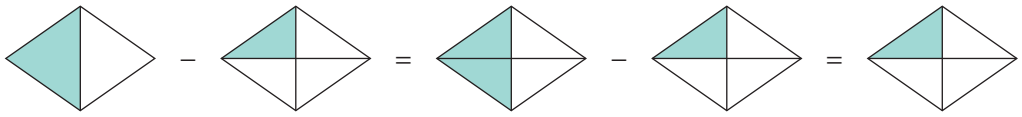
$\frac{3}{6} + \frac{1}{3} = \frac{3}{6} + \frac{\square}{6} = \frac{\square}{6}$

b. $\frac{1}{8} + \frac{2}{4}$



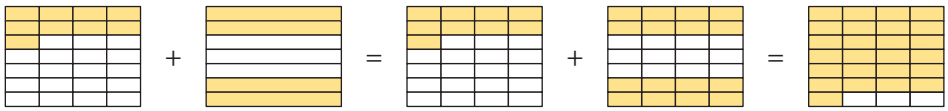
$\frac{1}{8} + \frac{2}{4} = \frac{1}{8} + \frac{\square}{8} = \frac{\square}{8}$

c. $\frac{1}{2} - \frac{1}{4}$



$\frac{\square}{2} - \frac{1}{4} = \frac{\square}{4} - \frac{1}{4} = \frac{\square}{4}$

d. $\frac{9}{28} + \frac{4}{7}$



$\frac{9}{28} + \frac{\square}{7} = \frac{9}{28} + \frac{\square}{28} = \frac{\square}{28}$

3. Fill in the blanks by using the words provided.

common | lowest | denominator | equivalent | same

When we add or subtract fractions, we want to have the _____ denominators. If the denominators are not the same, we can find _____ fractions by finding the _____ _____ _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Calculate:

a. $\frac{1}{5} + \frac{2}{5}$ b. $\frac{2}{4} + \frac{1}{4}$ c. $\frac{4}{10} + \frac{3}{10}$ d. $\frac{54}{100} + \frac{20}{100}$

e. $\frac{7}{8} - \frac{2}{8}$ f. $\frac{7}{12} - \frac{6}{12}$ g. $\frac{8}{7} - \frac{4}{7}$ h. $\frac{18}{13} - \frac{15}{13}$

WE2 **5.** Calculate and simplify.

a. $\frac{1}{3} + \frac{1}{6}$ b. $\frac{3}{4} - \frac{3}{8}$ c. $\frac{2}{3} + \frac{5}{9}$ d. $\frac{2}{3} - \frac{2}{5}$

e. $\frac{13}{14} + \frac{5}{4}$ f. $\frac{11}{15} - \frac{5}{9}$ g. $\frac{5}{12} + \frac{15}{16}$ h. $\frac{5}{14} + \frac{7}{10}$

WE3a **6.** Simplify:

a. $2\frac{1}{4} + 4\frac{2}{4}$ b. $6\frac{2}{5} - 2\frac{2}{5}$ c. $1\frac{3}{8} + 1\frac{4}{8}$ d. $3\frac{1}{7} - 2\frac{5}{7}$

e. $1\frac{3}{4} + 3\frac{3}{4}$ f. $2\frac{4}{5} + 2\frac{4}{5}$ g. $10\frac{2}{3} - 5\frac{1}{3}$ h. $62\frac{8}{10} - 3\frac{9}{10}$

WE3 **7.** Simplify:

a. $2\frac{3}{7} + 1\frac{2}{14}$ b. $3\frac{1}{3} - 1\frac{1}{4}$ c. $4\frac{3}{8} + 5\frac{5}{6}$ d. $8\frac{1}{2} - 3\frac{1}{9}$

e. $\frac{40}{10} - 1\frac{1}{8}$ f. $\frac{10}{4} - 1\frac{3}{10}$ g. $\frac{3}{8} + \frac{9}{7}$ h. $\frac{17}{12} + 3\frac{3}{16}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Mario used $3\frac{3}{4}$ cups of flour to make 60 cupcakes for his party. He wanted to make another 3 dozen cupcakes which required $2\frac{1}{4}$ more cups of flour. How much flour did Mario use altogether?

9. Jeremy is measuring wood for a DIY project. How much wood should he cut off? The length of the piece of wood he has is $11\frac{1}{3}$ cm but he only needs $8\frac{2}{3}$ cm of wood for his project.

10. Asha made two and one-quarter cups of kinetic sand. How many cups of sand did Asha give Kendra if after giving Kendra some sand, she had one and two-thirds cups of kinetic sand remaining?

11. How many whole bags of cement does Ronald need to buy? He plans to use one and a half bags to build a wall around his garden bed and another one and a third bags to repair his driveway. He likes to have an excess of at least a quarter bag in case he makes a mistake.

12. Leonardo is painting a portrait for a customer. This week, he completed four-sevenths of the portrait and has one-third more of the portrait to complete. How much of the portrait had Leonardo completed before commencing this week?

Reasoning

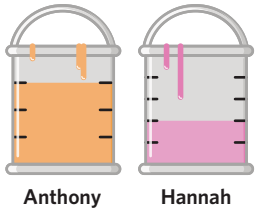
Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Anthony and Hannah have just finished painting their treehouse. The diagrams shows the amount of paint they each have leftover in their paint buckets.

a. What fraction of Anthony's and Hannah's buckets are filled with paint?

b. Altogether, how many buckets of paint do they have left?

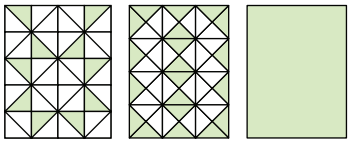
c. How many buckets of paint did it take to paint the treehouse?



d. Anthony and Hannah decided to use the leftover paint to paint each of their rooms. Anthony needs 6.5 litres of paint and Hannah needs 3.5 litres. If each of their buckets has a capacity of 8 litres, do they have enough leftover paint for both rooms? Explain.

e. Propose a fair way Anthony and Hannah could share the leftover paint.

14. Luigi has a single sheet of green glass. He used the sheet to create two stained glass windows.



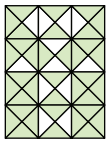
Design 1 Design 2 Green glass sheet

a. The original sheet of green glass was the same size as designs 1 and 2. After making design 1 and 2, how much of the sheet of green glass does Luigi have left?

A. $\frac{8}{12}$ B. $\frac{2}{3}$ C. $\frac{1}{3}$ D. $\frac{12}{3}$

b. Draw and shade how much of the original sheet of green glass is left over.

c. The third design will cover an area of 4 square metres. The original sheet of green glass was 6 square metres. Does Luigi have enough green glass to create design 3? Explain your answer.



Design 3

d. A friend Mario sees Luigi's stained glass windows and thinks they look great. Mario asks Luigi to create one for him. Should Luigi charge Mario to create a window for him?

Extra spicy

15. $\frac{2+4+6+8+10+12}{2+4+6+8+10} - \frac{2+4}{2+4+6} =$

A. $\frac{9}{10}$ B. $\frac{37}{30}$ C. $\frac{5}{4}$ D. $1\frac{9}{10}$ E. $11\frac{5}{6}$

16. What must be subtracted from $\frac{5}{7}$ to make $\frac{7}{5}$?

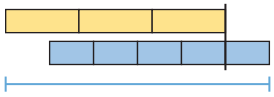
A. $\frac{5}{7}$ B. $-\frac{2}{7}$ C. $\frac{2}{5}$ D. $-\frac{24}{35}$ E. $\frac{24}{35}$

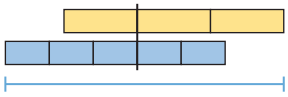
17. Using each number only once, create the largest possible fraction with 1, 2, 3, 4, 5, 6.

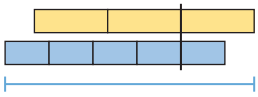
$\frac{?}{?} - \frac{?}{?} + \frac{?}{?} =$

18. Two strips of cardboard are both 60 cm long. The yellow cardboard is divided into thirds, and the blue cardboard is divided into fifths.

What is the total length of each arrangement

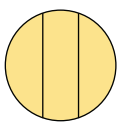
a. 

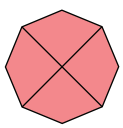
b. 

c. 

Remember this?

19. Which of these designs look identical after a three-quarter turn?

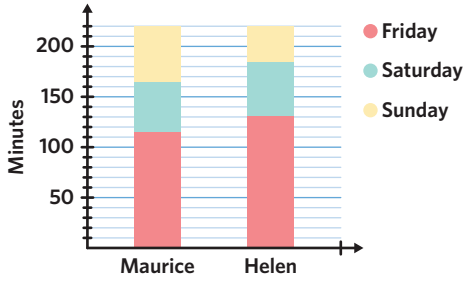
A. 

B. 

C. 

D. 

20. Maurice and Helen both make TikToks. On Friday, Saturday and Sunday, they each spent a total of 220 minutes making videos, as shown in the following graph.



How much more time than Maurice did Helen spend making TikToks on Saturday?

21. Which pattern is correct for the rule, 'triple the previous number and add 6'?

A. 1, 8, 22, 50

B. 2, 14, 44, 134

C. 2, 10, 26, 58

D. 2, 12, 42, 132

3F Multiplying fractions

It can be useful to be able to find a fraction of a quantity. This can be done by multiplying a whole number by a fraction. Below are some examples where multiplying fractions can be applied.

- I had 100 raffle tickets to sell for a fundraiser. If I sell $\frac{3}{4}$ of my tickets, how many have I sold?
- If I ate $\frac{1}{4}$ of one-half of a cake that I was given, how much of the whole cake have I eaten?
- I had $2\frac{1}{2}$ metres of rope and I wanted to cut my rope into quarters. How long is each piece of rope?

Learning intentions

Students will be able to:

- multiply a whole number by a fraction
- multiply a fraction by a fraction
- simplify fractions to make multiplication easier to perform.

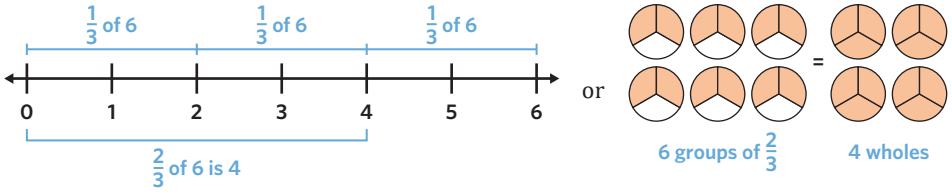
Key terms and definitions

- The **product** is the result when two or more values are multiplied together.
- A fraction is in its **simplest form** when the numerator and denominator have no common factors other than 1. E.g. $\frac{5}{10}$ in the simplest form is $\frac{1}{2}$.
- **Simplifying fractions** is the process of expressing an equivalent fraction with a smaller numerator and smaller denominator.

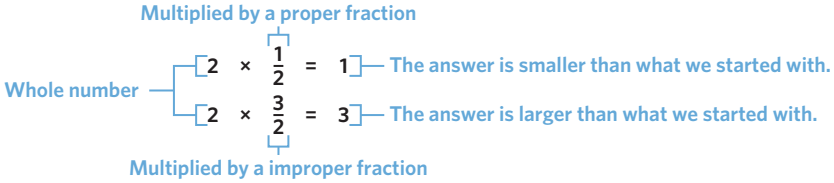
Key ideas

1 Multiplication (×) is the same as 'of' and 'groups of'.

$\frac{2}{3} \times 6$ can be thought of as $\frac{2}{3}$ of 6 or as 6 groups of $\frac{2}{3}$.

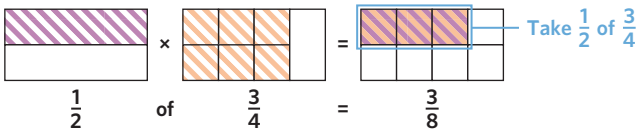


2 Multiplying any positive number by a proper fraction will result in a smaller value. Multiplying any positive number by an improper fraction (equal to or greater than 1) or by a mixed number will result in an equal or larger value.



3 To multiply fractions together, multiply the numerators and multiply the denominators.

$\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$



4 Before multiplying fractions we can simplify the multiplication by cancelling. This can be done by finding the highest common factor (HCF) of any numerator and denominator and then dividing the numerator and denominator to cancel.

The HCF of 3 and 6 is 3. Cancelling the numerator and denominator from the same fraction:

$$\frac{2}{5} \times \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$$

The HCF of 2 and 6 is 2. Cancelling the numerator and denominator from different fractions:

$$\frac{\cancel{2}^{1}}{5} \times \frac{3}{\cancel{6}_{3}}$$

Worked examples

WE 1 Multiplying a fraction and a whole number

Evaluate each calculation.

a. $\frac{2}{3} \times 12$

Working

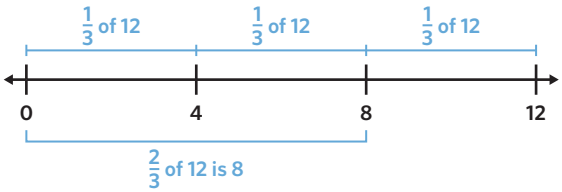
$$\frac{2}{3} \times \frac{12}{1} = \frac{2 \times 12}{3 \times 1}$$
$$= \frac{24}{3}$$
$$= 8$$

Thinking

Step 1: Rewrite the whole number (12) as a fraction with a denominator of 1 and multiply the numerators and the denominators.

Step 2: Simplify by completing the division $24 \div 3$.

Visual support



b. $7 \times \frac{3}{4}$

Working

$$\frac{7}{1} \times \frac{3}{4} = \frac{7 \times 3}{1 \times 4}$$
$$= \frac{21}{4}$$
$$= 5\frac{1}{4}$$

Thinking

Step 1: Rewrite the whole number (7) as a fraction with a denominator of 1 and multiply the numerators and the denominators.

Step 2: Simplify by completing the division $21 \div 4$.

Student practice

Evaluate each calculation.

a. $4 \times \frac{3}{5}$ b. $\frac{2}{6} \times 8$ c. $7 \times \frac{3}{14}$ d. $\frac{5}{6} \times 12$

WE 2 Multiplying fractions

Evaluate and simplify.

a. $\frac{1}{4} \times \frac{3}{5}$

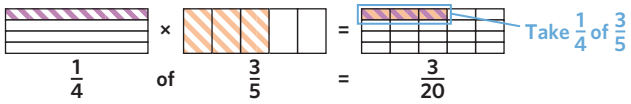
Working

$$\frac{1}{4} \times \frac{3}{5} = \frac{1 \times 3}{4 \times 5}$$
$$= \frac{3}{20}$$

Thinking

Multiply the numerators and multiply the denominators.

Visual support



b. $\frac{5}{2} \times 3\frac{1}{3}$

Working

$$\frac{5}{2} \times \frac{10}{3} = \frac{5 \times 10}{2 \times 3}$$
$$= \frac{50}{6}$$
$$= 8\frac{2}{6}$$
$$= 8\frac{1}{3}$$

Thinking

Step 1: Convert the mixed number into an improper fraction and then multiply numerators and multiply denominators.

Step 2: Simplify by completing the divisions $50 \div 6$.

Student practice

Evaluate and simplify.

a. $\frac{2}{3} \times \frac{4}{6}$ **b.** $\frac{9}{4} \times 2\frac{2}{5}$ **c.** $2\frac{1}{4} \times 5\frac{1}{7}$ **d.** $\frac{7}{6} \times 3\frac{9}{10}$

WE 3 Simplifying before multiplying fractions

Evaluate by simplifying first.

a. $\frac{2}{3} \times \frac{12}{5}$

Working

3 and 12 have a HCF of **3**.

$$\frac{2}{\cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{5} = \frac{2}{1} \times \frac{4}{5}$$
$$= \frac{8}{5} \text{ or } 1\frac{3}{5}$$

Thinking

Step 1: Find the highest common factor (HCF) of a numerator and denominator, then divide by the HCF to cancel.

Step 2: Multiply the numerators and the denominators.

b. $2\frac{2}{3} \times 1\frac{1}{4}$

Working

$$2\frac{2}{3} \times 1\frac{1}{4} = \frac{8}{3} \times \frac{5}{4}$$

8 and 4 have a HCF of **4**.

$$= \frac{\cancel{8}^{2}}{3} \times \frac{5}{\cancel{4}_{1}}$$
$$= \frac{2}{3} \times \frac{5}{1}$$
$$= \frac{10}{3} \text{ or } 3\frac{1}{3}$$

Thinking

Step 1: Convert the mixed numbers to improper fractions.

Step 2: Find the highest common factor (HCF) of a numerator and denominator, then divide by the HCF to cancel.

Step 3: Multiply the numerators and the denominators.

Student practice

Evaluate by simplifying first.

a. $\frac{16}{7} \times \frac{1}{8}$ **b.** $1\frac{2}{9} \times 2\frac{3}{4}$ **c.** $3\frac{1}{5} \times 10\frac{3}{8}$ **d.** $1\frac{2}{4} \times 3\frac{6}{12}$

3F Activities and questions

STARTER TASKS

Image: Steven Siewert/The Age

Odd spot

Chloe, Eliza and Bronte are identical triplets. Approximately $\frac{1}{30}$ of all pregnancies are twins, triplets or quadruplets. Of these types of pregnancies, about $\frac{1}{40}$ are triplets. What fraction of all pregnancies are triplets?

A. $\frac{7}{120}$ **B.** $\frac{1}{1200}$

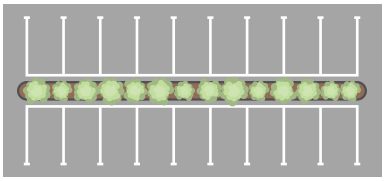
Puzzle

A parking lot has 18 spots. How many spots are taken if:

a) $\frac{1}{2}$ of 18 spots are taken.

b) $\frac{2}{3}$ of 18 spots are taken.

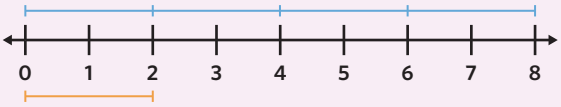
c) $\frac{4}{9}$ of 18 spots are taken.



Understanding worksheet

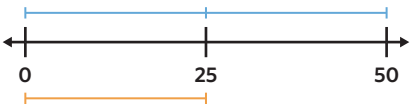
1. Complete the calculation on the number line.

Example



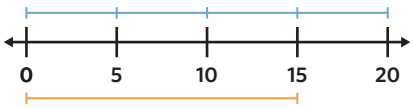
$\frac{1}{4}$ of 8 is $\boxed{2}$

a.



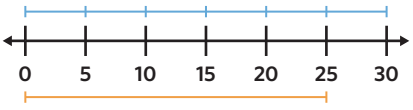
$\frac{1}{2}$ of 50 is $\square$

b.



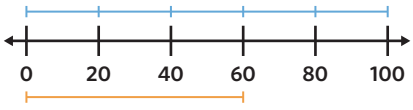
$\frac{3}{4}$ of 20 is $\square$

c.



$\frac{\square}{6}$ of 30 is $\square$

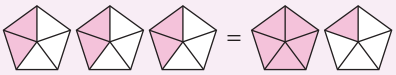
d.



$\frac{\square}{\square}$ of 100 is $\square$

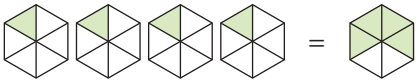
2. Complete the equation that the diagram represents.

Example



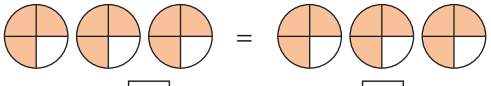
3 groups of $\frac{\boxed{2}}{5}$ = $\frac{\boxed{6}}{5}$

a.

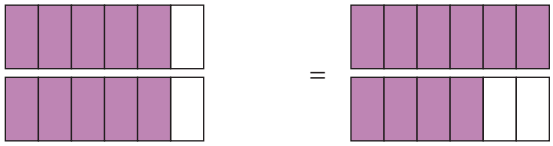


4 groups of $\frac{\square}{6}$ = $\frac{\square}{6}$

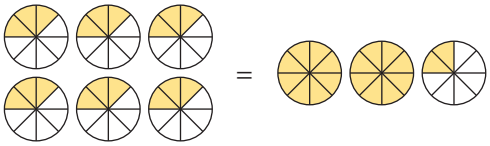
b.



3 groups of $\frac{\square}{4}$ = $\frac{\square}{4}$

c. 

$\square$ groups of $\frac{\square}{6}$ = $\frac{\square}{6}$

d. 

$\square$ groups of $\frac{3}{8}$ = $\frac{\square}{\square}$

3. Fill in the blanks by using the words provided.

common | factor | highest | product | smaller

When we multiply a whole number with a proper fraction, the [ ] will be [ ] than what we started with. When we cancel fractions, we want to find the [ ] [ ] [ ].

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

4. State whether the result of each multiplication will be larger, smaller or equal to 5.

a. $5 \times \frac{1}{2}$ b. $5 \times 5\frac{2}{5}$ c. $5 \times \frac{2}{3}$ d. $5 \times \frac{4}{6}$

e. $5 \times 2\frac{1}{4}$ f. $5 \times \frac{6}{2}$ g. $5 \times \frac{2}{2}$ h. $5 \times \frac{14}{10}$

WE1 **5.** Evaluate each calculation.

a. $\frac{1}{2}$ of 3 b. $\frac{3}{5}$ of 4 c. 6 groups of $\frac{7}{8}$ d. $\frac{2}{7}$ of 12

e. 9 groups of $\frac{3}{4}$ f. $\frac{2}{8}$ of 15 g. 7 groups of $\frac{4}{5}$ h. $\frac{3}{5}$ of 8

WE2 **6.** Evaluate and simplify.

a. $\frac{4}{7} \times \frac{1}{3}$ b. $\frac{3}{5} \times \frac{8}{5}$ c. $\frac{1}{2} \times \frac{11}{12}$ d. $\frac{6}{8} \times \frac{5}{7}$

e. $\frac{4}{10} \times \frac{8}{9}$ f. $\frac{5}{9} \times \frac{12}{13}$ g. $\frac{3}{4} \times \frac{10}{15}$ h. $\frac{11}{12} \times \frac{12}{13}$

WE3 **7.** Evaluate by simplifying the highlighted values first.

a. $\frac{5}{7} \times \frac{13}{15}$ b. $\frac{3}{8} \times \frac{11}{12}$ c. $\frac{12}{18} \times \frac{5}{7}$ d. $\frac{4}{16} \times \frac{2}{3}$

e. $\frac{3}{4} \times \frac{12}{48}$ f. $\frac{2}{5} \times \frac{15}{24}$ g. $\frac{8}{12} \times \frac{4}{64}$ h. $\frac{6}{56} \times \frac{7}{12}$

8. Evaluate each calculation.

a. $\frac{12}{11} \times \frac{2}{3}$ b. $\frac{77}{13} \times \frac{4}{11}$ c. $2\frac{1}{4} \times \frac{5}{7}$ d. $\frac{6}{17} \times \frac{9}{12}$

e. $2\frac{4}{5} \times 3\frac{1}{2}$ f. $\frac{9}{14} \times \frac{21}{3}$ g. $\frac{13}{18} \times 1\frac{1}{4}$ h. $4\frac{1}{3} \times 2\frac{5}{6}$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. There are 350 members of a local football club. How many people are members of both the local football club and soccer club if $\frac{3}{7}$ of the football club members are also members of the soccer club?

10. Two-sevenths of the skyscrapers in Melbourne are over 200 m tall while seven-sixteenths of these buildings are over 250 m tall. What fraction of the skyscrapers in Melbourne are over 250 m tall?

11. At the start of Rugby training Edith's 600 mL water bottle was five-sixths full. During training she drinks three-quarters of her bottle. How much water did Edith consume at training?

12. Max and Lucy drove to a music festival together. Max drove the first $\frac{3}{8}$ of the distance before he needed a break. Lucy took over and drove $\frac{3}{5}$ of the remaining distance. What fraction of the total trip did Lucy drive?

13. Mario is baking scones for his party. If Mario wants to make 36 scones, how many cups of flour does he need? The recipe he is using requires $2\frac{3}{4}$ cups of flour to make 16 scones.

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c,d)	All

14. Kalvin had a clearance sale at his clothing store and discounted everything to half its original price on the first day. On the second day, he halved the price of everything again.
    a. Choose all the correct statements.
        A. Since the price has been halved twice, everything must be free.
        B. The price on the second day is half of the price of the first day.
        C. The sale price is a quarter of the original price.
    b. If a shirt originally cost $80, what will the price be on the second day of the sale?
    c. If Kalvin starts the sale on a Friday and keeps halving the price every day, on what day will a $80 shirt sell for $5?
    d. If Kalvin keeps halving the price every day, will the shirt ever be free?
    e. For his next sale Kalvin wants to discount everything to half its original price but not continue to reduce the prices each day. Is this option better or worse? Explain why.

15. Sara is designing a flower garden for her new house. She has a square area that is $2\frac{2}{3}$ metres on one side.
    a. What is the total area of her garden?
    b. Sara wants to plant 4 columns of tulips. The width of each column is $\frac{3}{4}$ of a metre. Does she have enough space in her garden to plant all 4 columns?
    c. One tulip plant takes up $\frac{3}{16}$ of a square metre. Sara decides to plant 8 tulip plants and save the rest of her garden for other flowers. How much area does she have left for the other plants?
    d. How much area will Sara use for daffodils if she plans to plant them in $\frac{3}{4}$ of the remaining space?
    e. Design a layout for Sara's garden with tulips, daffodils and another flower.

Extra spicy

16. Evaluate $2 - \left(\frac{3}{4}\right)^2$.

17. Find the missing fraction so that the expression equals 24.
$\frac{3}{5} \times \frac{\square}{\square} \times 60$

18. What is the value of $\frac{1}{1000} \times 1011$?
    A. 0.1101 B. 1.101 C. 1.011 D. 101.1 E. 1011

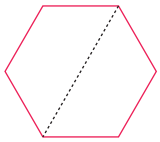
19. If $\frac{1}{a} \times \frac{1}{b} = \frac{1}{c}$, and a, b and c are all positive whole numbers, which of the following statement(s) are true?

A. $\frac{1}{a}$ is the largest fraction **B.** $\frac{1}{b}$ is the largest fraction **C.** $\frac{1}{c}$ is the largest fraction

D. $\frac{1}{a}$ is the smallest fraction **E.** $\frac{1}{b}$ is the smallest fraction **F.** $\frac{1}{c}$ is the smallest fraction

Remember this?

20. A regular hexagon is folded in half along the dotted line.



What type of shape is each half?

A. Triangle **B.** Rectangle **C.** Quadrilateral **D.** Pentagon

21. List all the factors of 42.

22. Taylen saw a movie at 7:30 pm. Before the movie started there were 30 minutes of previews. The movie then ran for $1\frac{3}{4}$ hours. What time did the movie end?

3G Dividing fractions

Unlike dividing with whole numbers where the result or quotient of the division is smaller than the dividend, dividing fractions often gives a result that is greater than the dividend. Below are some examples where dividing fractions can be applied.

- I have 2 whole cakes. If I cut the cake into quarters, how many pieces of cake will I get?
- I want to plant $\frac{1}{2}$ of my garden bed with different vegetable plants that each take up $\frac{1}{6}$ of the garden bed. How many different vegetable plants can I plant?
- The distance from my home to school is $2\frac{1}{2}$ kilometres. Every morning I walk $\frac{2}{5}$ of the distance and then catch the bus the rest of the way. After how many days will I have walked $2\frac{1}{2}$ kilometres?

Learning intentions

Students will be able to:

+ find the reciprocal of a whole number and a fraction
+ divide a whole number by a fraction
+ divide a fraction by another fraction.

Key terms and definitions

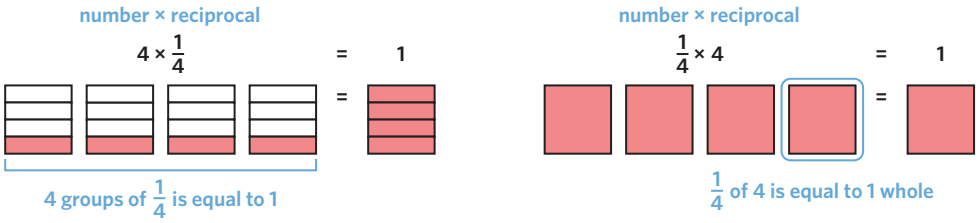
- A **dividend** is the first number in a division calculation. It is the number that is being divided by the second number.
- A **divisor** is the number by which a given value is divided.
- The **quotient** is the result or answer of a division calculation.
- The **reciprocal** of a number is 1 divided by that number. E.g. The reciprocal of a is $\frac{1}{a}$.

Dividend Divisor Quotient

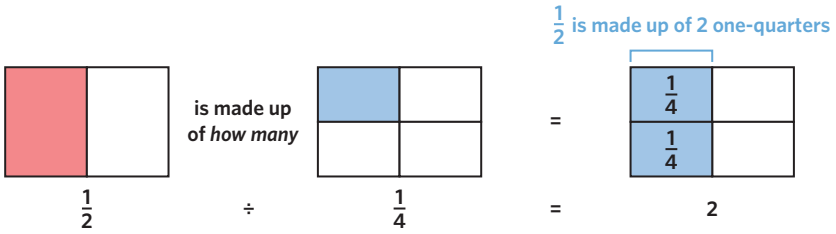
$3 \div \frac{1}{2} = 6$

Key ideas

1 Multiplying any number by its reciprocal equals 1.

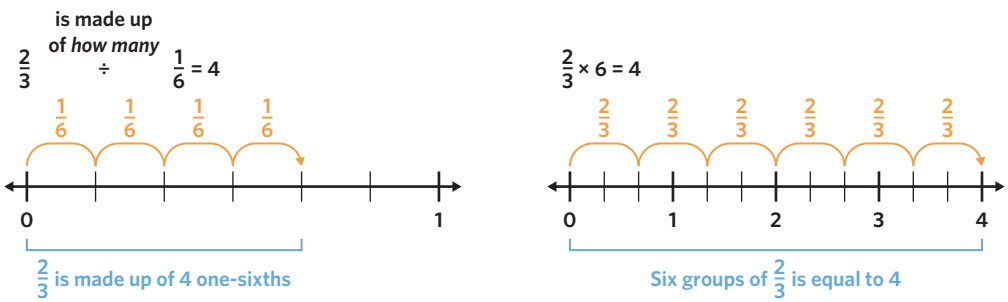


2 Division (÷) is the same as 'is made up of **how many**?'.



3 Dividing a fraction by a value is the same as multiplying a fraction by the reciprocal of that value.

$\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times 6$



Worked examples

WE 1 Finding reciprocals

Find the reciprocal of each value.

a. 8

Working

$8 = \frac{8}{1}$

$\frac{1}{8}$

Thinking

Step 1: Write the whole number with a denominator of 1.

Step 2: Swap the numerator and denominator.

b. $2\frac{2}{5}$

Working

$2\frac{2}{5} = \frac{10}{5} + \frac{2}{5}$

$= \frac{12}{5}$

$\frac{5}{12}$

Thinking

Step 1: Convert the mixed number to an improper fraction.

Step 2: Swap the numerator and denominator.

Student practice

Find the reciprocal of each value.

a. 11 **b.** $8\frac{1}{3}$ **c.** $\frac{15}{7}$ **d.** $9\frac{5}{8}$

WE 2 Dividing a whole number and a fraction

Calculate and simplify.

a. $4 \div \frac{1}{3}$

Working

$\frac{1}{3} \rightarrow \frac{3}{1}$

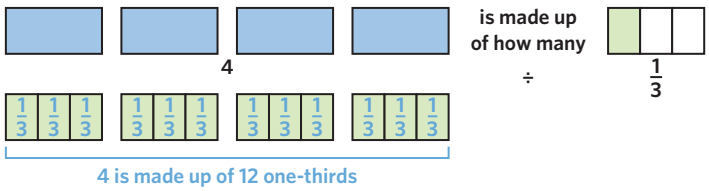
$4 \times 3 = 12$

Thinking

Step 1: Find the reciprocal of the divisor (second value).

Step 2: Multiply the dividend (first value) by the reciprocal of the divisor.

Visual support



b. $\frac{3}{8} \div 7$

Working

$7 \rightarrow \frac{1}{7}$

$\frac{3}{8} \times \frac{1}{7} = \frac{3 \times 1}{8 \times 7}$

$= \frac{3}{56}$

Thinking

Step 1: Find the reciprocal of the divisor (second value).

Step 2: Multiply the dividend (first value) by the reciprocal of the divisor by multiplying the numerators together and the denominators together.

Student practice

Calculate and simplify.

a. $5 \div \frac{3}{5}$ **b.** $\frac{2}{7} \div 10$ **c.** $12 \div \frac{18}{5}$ **d.** $6\frac{4}{6} \div 15$

WE 3 Dividing two fractions

Evaluate:

a. $\frac{4}{5} \div \frac{3}{10}$

Working

$\frac{3}{10} \rightarrow \frac{10}{3}$

$\frac{4}{5} \times \frac{10^{2}}{3} = \frac{8}{3}$ or $2\frac{2}{3}$

Thinking

Step 1: Find the reciprocal of the divisor (second value).

Step 2: Multiply the dividend (first value) by the reciprocal of the divisor (second value) by first simplifying using the highest common factors (HCF) between any numerators and denominators.

b. $2\frac{2}{7} \div 1\frac{1}{5}$

Working

$2\frac{2}{7} \div 1\frac{1}{5} = \frac{16}{7} \div \frac{6}{5}$

$= \frac{16}{7} \times \frac{5}{6_{3}}$

$= \frac{40}{21}$ or $1\frac{19}{21}$

Thinking

Step 1: Convert both mixed numbers to improper fractions.

Step 2: Multiply the dividend (first value) by the reciprocal of the divisor (second value) by first simplifying using the highest common factors (HCF) between any numerators and denominators.

Student practice

Evaluate:

a. $\frac{4}{7} \div \frac{3}{8}$ **b.** $3\frac{1}{6} \div 2\frac{2}{3}$ **c.** $\frac{12}{7} \div \frac{48}{21}$ **d.** $\frac{13}{12} \div 6\frac{3}{6}$

3G Activities and questions

STARTER TASKS

Odd spot

Kyle 'Bugha' Giersdorf won the first Fortnite World Cup, taking home $4.3 million AUD.

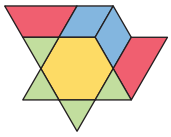
In the World Cup, competitors played 6 games of Fortnite for a total of $3\frac{3}{5}$ hours.

To the nearest dollar, how much money did Kyle 'Bugha' Giersdorf win per hour of playing?

A. $15 480 000 **B.** $1 194 444

Puzzle

The image shown is formed using pattern blocks made up of green equilateral triangles, red trapeziums, blue rhombuses and a yellow hexagon.



a) How many red trapeziums can fit into the yellow hexagon?

b) What fraction of the yellow hexagon is one blue rhombus?

c) What fraction of the yellow hexagon can be covered by one green triangle?

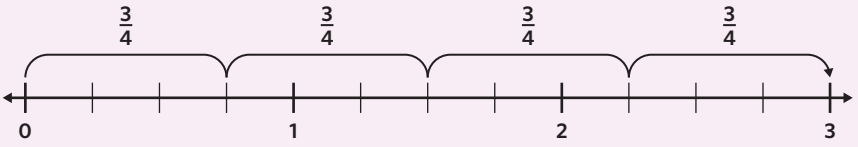
d) How many yellow hexagons can be made by combining all the other shapes?

Understanding worksheet

1. Show the division on the number line by completing the fractions.

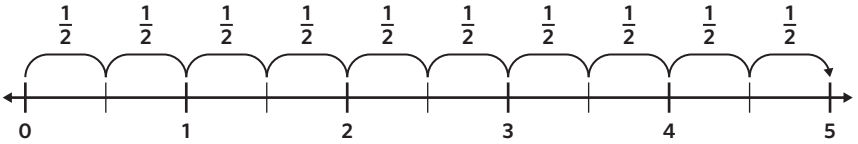
Example

3 is made up of how many $\frac{3}{4}$'s?



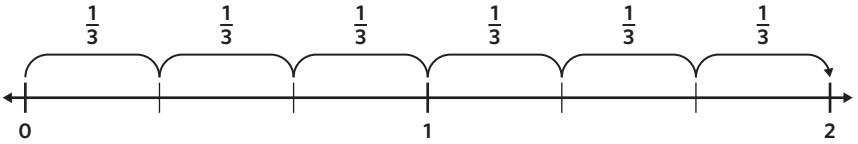
= [4] three-quarters

a. 5 is made up of how many $\frac{1}{2}$'s?



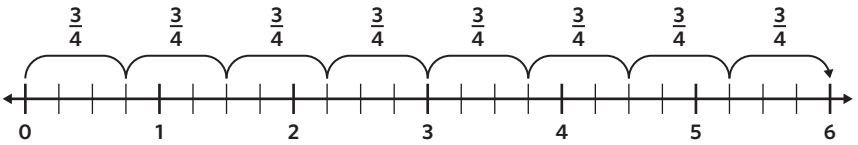
= ☐ one-halves

b. 2 is made up of how many $\frac{1}{3}$'s?



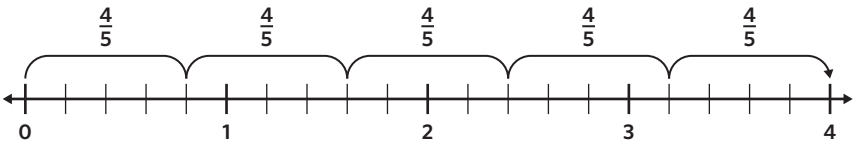
= ☐ one-thirds

c. 6 is made up of how many $\frac{3}{4}$'s?



= ☐ three-quarters

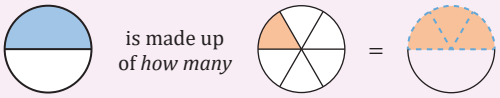
d. 4 is made up of how many $\frac{4}{5}$'s?



= ☐ four-fifths

2. Fill in the missing boxes to show the result of the division.

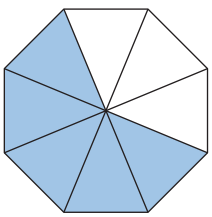
Example



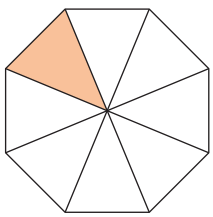
is made up of *how many*

$\frac{1}{2}$ ÷ $\frac{1}{6}$ = [3] one-sixths

a.



is made up of *how many*



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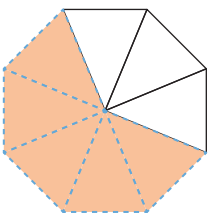
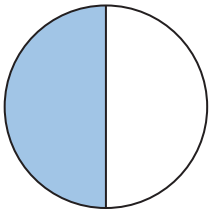
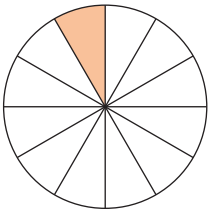


$\frac{5}{8}$ ÷ $\frac{1}{8}$ = ☐ one-eighths

b.



is made up of *how many*



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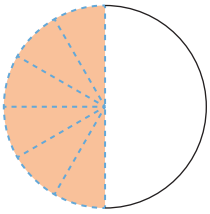
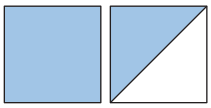


$\frac{1}{2}$ ÷ $\frac{1}{12}$ = ☐ one-twelfths

c.



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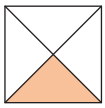

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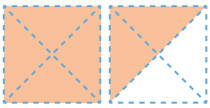
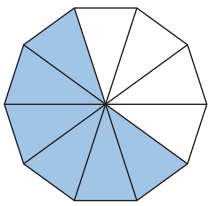
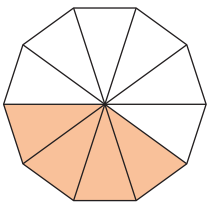


$1\frac{1}{2}$ ÷ $\frac{1}{4}$ = ☐ one-fourths

d.



is made up of *how many*



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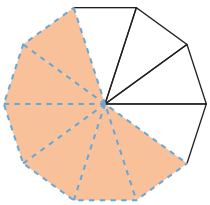


$\frac{6}{10}$ ÷ $\frac{4}{10}$ = ☐ four-sixths

3. Fill in the blanks by using the words provided.

multiply | reciprocal | whole | number | fraction

When a ☐ ☐ or ☐ is divided by another fraction, we can ☐ the dividend (first number) by the ☐ of the divisor (second number).

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Find the reciprocal of each value.

a. 5 **b.** $\frac{1}{3}$ **c.** $\frac{4}{7}$ **d.** 13

e. $\frac{9}{4}$ **f.** 1 **g.** $1\frac{4}{5}$ **h.** $3\frac{2}{11}$

WE2 **5.** Calculate and simplify.

a. $3 \div \frac{1}{4}$ **b.** $\frac{2}{3} \div 5$ **c.** $4 \div \frac{5}{6}$ **d.** $\frac{4}{10} \div 2$

e. $\frac{1}{3} \div 5$ **f.** $3 \div \frac{5}{7}$ **g.** $\frac{3}{5} \div 6$ **h.** $10 \div \frac{7}{9}$

WE3a **6.** Evaluate:

a. $\frac{2}{3} \div 4$ **b.** $\frac{1}{6} \div \frac{4}{5}$ **c.** $\frac{8}{3} \div \frac{11}{3}$ **d.** $\frac{2}{9} \div \frac{6}{5}$

e. $\frac{4}{3} \div \frac{14}{11}$ **f.** $\frac{5}{2} \div \frac{8}{13}$ **g.** $\frac{7}{12} \div \frac{10}{16}$ **h.** $\frac{8}{17} \div \frac{4}{34}$

WE3b **7.** Evaluate:

a. $\frac{6}{5} \div \frac{5}{2}$ **b.** $\frac{4}{3} \div \frac{5}{4}$ **c.** $2\frac{1}{2} \div \frac{7}{4}$ **d.** $3\frac{1}{4} \div 1\frac{2}{3}$

e. $\frac{9}{5} \div \frac{10}{4}$ **f.** $\frac{21}{16} \div \frac{7}{4}$ **g.** $6\frac{7}{8} \div 1\frac{1}{4}$ **h.** $3\frac{3}{12} \div 6\frac{2}{4}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Richard has $\frac{1}{3}$ of his patio left to tile. Each tile takes up $\frac{1}{20}$ of the entire patio. How many more tiles does he need to finish tiling the patio?

A. 7 **B.** 6 **C.** $\frac{1}{60}$ **D.** 20

9. Luke has a piece of wood that is $2\frac{5}{6}$ metres long that he wants to cut into smaller pieces. Luke wants to cut each piece into $\frac{1}{4}$ of a metre in length. How many $\frac{1}{4}$-metre pieces of wood will Luke end up with?

10. Bowser wants to know how many goodie bags he can make for all his party guests. He uses one-fifth of a bag of M&M's to make one goodie bag and has two and three-quarter bags of M&Ms to use.

11. Britt wants to exchange 90 Australian dollars (AUD) for United States dollars (USD). How much USD will Britt get after the exchange if the bank teller explains that she can buy 1 USD with $1\frac{1}{2}$ AUD?

12. On Monday morning, Issy made 1 cup of pancake batter and only used $\frac{2}{5}$ of the batter. That afternoon, she made a fresh cup of pancake batter and used $\frac{1}{5}$. The next day she combined all of the remaining batter. If $\frac{1}{3}$ of a cup of batter makes one pancake, how many whole pancakes can Issy make with the remaining batter?

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Helen's mum was planning meals for her family of 5 and decided to bake 3 cakes that were to be eaten for dessert over the next three days.

a. On the first day, Helen's mum used $\frac{1}{2}$ of the first cake to share equally between all 5 family members. What fraction of the first cake did each person get?

A. $\frac{1}{10}$ **B.** $\frac{1}{5}$ **C.** $\frac{1}{2}$ **D.** $\frac{5}{2}$

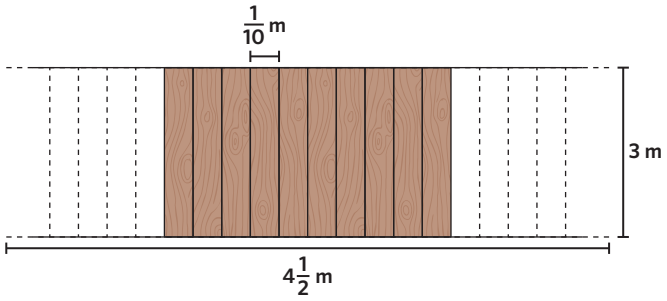
b. The next day Helen's mum decided that she would use the remaining $\frac{1}{2}$ of the first cake and another $\frac{1}{4}$ from the second cake. What fraction of a whole cake did each person get?

c. How many of the 3 cakes were left over for dessert on the third day?

d. Two of Helen's friends joined her family for dinner on the third day. If the remaining portion of cake was cut into equal sized pieces, what fraction of a whole cake did each person get for dessert?

e. Do you think that each person received a fair amount of cake in part **d**? How would you adjust the number of people at the dinner party to make each person's quantity of cake appropriate?

14. Rhea wants to design and build a feature wall by placing several wooden posts side by side. She needs the wall to be $4\frac{1}{2}$ metres long and 3 metres high. Each wooden post that she uses is $\frac{1}{10}$ of a metre wide.
Note: The diagram below is not to scale.



a. How many wooden posts does Rhea need to build a solid wall?

A. $\frac{2}{9}$ **B.** $\frac{9}{20}$ **C.** $13\frac{1}{2}$ **D.** 45

b. After seeing the design, Rhea decided that she wants to remove a few of the posts to create two gaps that are each $\frac{1}{2}$ m in width. How many posts does Rhea need to complete this new design?

c. Rhea can save some money by buying 5-metre long posts as she can cut them to her desired length and use the offcuts to complete the design. If the 5 m posts are the same width as the 3 m posts, how many will she need to complete the design?

d. Create a wall design with the wooden posts purchased in part **c**.

Extra spicy

15. What is the result of dividing $120\frac{3}{5}$ by $\frac{1}{6}$ and adding 20?

16. What is the value of $1 + \dfrac{2}{1 + \frac{2}{3}}$?

A. $2\frac{1}{5}$ **B.** $1\frac{2}{3}$ **C.** $4\frac{1}{3}$ **D.** $3\frac{1}{3}$ **E.** $1\frac{1}{5}$

17. What two digit number when divided by the sum of its two digits and then divided by $\frac{2}{3}$ gives 3?

18. If z is equal to two thirds of five-sixths and y is equal to three-quarters of four-fifths. What is the value of $\frac{z}{y}$?

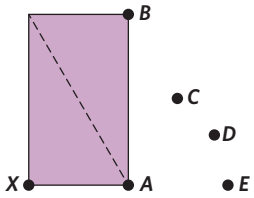
A. $\frac{27}{25}$ **B.** $\frac{25}{27}$ **C.** $\frac{9}{3}$ **D.** $\frac{3}{9}$ **E.** $\frac{5}{27}$

Remember this?

19. What is the value of 25^2?

A. 5 **B.** 50 **C.** 225 **D.** 625

20. The following rectangle is folded along the dotted line.



Which point does the corner labelled *X* move to?

21. Emilie arrived at Richmond station at 7:15 and caught the next train that went to Hawthorn.
What time did she arrive at Hawthorn station?

A. 7:21
B. 7:30
C. 7:36
D. 7:46

Station	Train 1	Train 2	Train 3	Train 4	Train 5
Richmond	7:11	7:20	7:28	7:36	7:45
East Richmond	7:13			7:38	
Burnley	7:16		7:31	7:41	
Hawthorn	7:21		7:36	7:46	
Glenferrie	7:25	7:30	7:40	7:50	7:55

3H Ratios

Like fractions, ratios describe a proportion and are expressed in the form of $a : b$. Ratios are an effective way to state what proportion quantities need to be in, without stating the specific amount. Below are some examples of where ratios can be applied.

- I competed in the school's spelling bee. I spelt 9 words correctly and 1 word incorrectly. Express the ratio of correctly spelt words : incorrectly spelt words.
- In my office car park there are 3 blue cars, 4 red cars and 5 black cars. What is the ratio of blue cars : total number of cars?
- I bought a 1 kg tub of yoghurt for $3.45 and my friend bought a 140 g pouch of yoghurt for $2.50. Which yoghurt is the best buy?

Learning intentions

Students will be able to:

- express fractions as a ratio
- simplify ratios
- determine best buys using the unitary method.

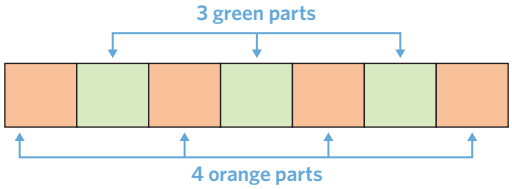
Key terms and definitions

- A **ratio** is a set of numbers in the form $a : b$ that expresses the relationship between two or more quantities or sizes.
- **Equivalent ratios** are two ratios that express the same relationship between parts. E.g. 1 : 2 and 2 : 4 are equivalent.
- **Best buy** refers to comparing the price of two or more goods when they have the same quantity
- The **unitary method** is a process of calculation where the first step is to determine the value of one unit.

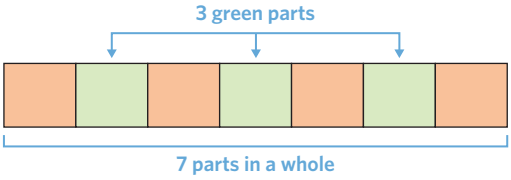
Key ideas

1 A ratio is written as $a : b$ which means 'a parts to b parts'. A ratio can also be written as $a : b$ which means 'a parts to b all parts in a whole'. A ratio does not require units because it is comparing quantities of the same unit.

The ratio of green parts to orange parts is 3 : 4.

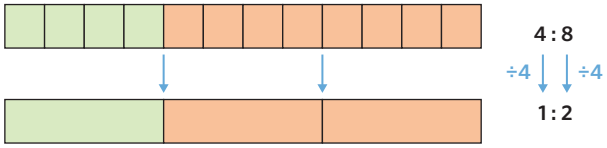


The ratio of green parts to all parts in a whole is 3 : 7.



2 A ratio can be simplified if the values within the ratio have a common factor other than 1. If a ratio has no common factors other than 1, it cannot be simplified further.

4 and 8 have a common factor of 4.



3 In the context of best buys, the unitary method calculates the price when the quantity is one unit. This is called the unit price. This allows prices to be compared when the quantities are different.

The unit price may refer to the price per 100 grams, per 100 mL or for a single item.

Worked examples

WE 1 Finding ratios with a fraction

Find the given ratio.

a. If $\frac{5}{7}$ of apples are green, what is the ratio of green apples : all apples?

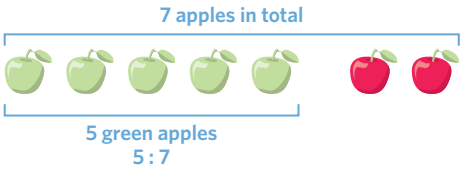
Working

5 : 7

Thinking

The denominator (7) represents the total number of parts and the numerator (5) represents the number of parts that are green apples.

Visual support



b. If $\frac{2}{11}$ of all the biscuits are Tim Tams, what is the ratio of Tim Tams : other biscuits?

Working

2 : 9

Thinking

The numerator (2) represents the number of parts that are Tim Tams. There are 11 parts in total so 9 parts of the biscuits are not Tim Tams.

Student practice

Find the given ratio.

a. If $\frac{3}{8}$ of nuts are peanuts, what is the ratio of peanuts : all nuts?

b. If $\frac{9}{13}$ of coins are gold, what is the ratio of gold coins : other coins?

c. If $\frac{7}{24}$ of the desks in a classroom are green, what is the ratio of green desks : all desks?

d. If $\frac{6}{30}$ of the chairs in a hall are blue, what is the ratio of blue chairs : other chairs?

WE 2 Finding ratios with multiple fractions

Find the given ratio.

a. If $\frac{1}{6}$ of animals are sheep, $\frac{2}{6}$ are cows and $\frac{3}{6}$ are chickens, what is the ratio of sheep : cows : chickens?

Working

$\frac{1}{6} : \frac{2}{6} : \frac{3}{6}$

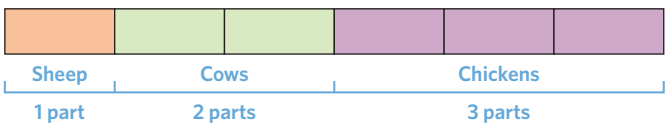
1 : 2 : 3

Thinking

Step 1: Write the fractions in the form of the ratio.

Step 2: As all the denominators are the same, use the numerators to write the ratio.

Visual support



b. If $\frac{2}{8}$ of cards are pink, $\frac{2}{6}$ are blue and the rest are grey, what is the ratio of grey cards : all cards?

Working

Fraction of grey cards $= 1 - \frac{2}{8} - \frac{2}{6}$

$= \frac{24}{24} - \frac{6}{24} - \frac{8}{24}$

$= \frac{10}{24}$

$= \frac{5}{12}$

5 : 12

Thinking

Step 1: Determine the fraction of grey cards by subtracting the fraction of pink and blue cards from 1.

Step 2: Use the lowest common denominator (24) to find equivalent fractions to complete the subtraction and simplify.

Step 3: The denominator (12) represents the total number of parts and the numerator (5) represents the number of parts that are grey cards.

Student practice

Find the given ratio.

a. If $\frac{4}{9}$ of drinks on a table are Sprite, $\frac{2}{9}$ are Coke and $\frac{3}{9}$ are Fanta, what is the ratio of Sprite : Coke : Fanta?

b. If $\frac{2}{7}$ of flowers in a garden are tulips, $\frac{1}{3}$ are sunflowers and the rest are roses, what is the ratio of roses : all flowers?

c. If $\frac{7}{60}$ ooshies are glow in the dark, $\frac{1}{15}$ are glitters and the rest are regular, what is the ratio of regular ooshies : other ooshies?

d. If $\frac{3}{7}$ of shoes in a store are leather, $\frac{2}{12}$ are polyester and the rest are nylon, what is the ratio of nylon shoes : all shoes?

WE 3 Simplifying a ratio

Simplify each ratio.

a. 20 : 35

Working

HCF is 5

20 ÷ 5 : 35 ÷ 5

4 : 7

Thinking

Step 1: Find the highest common factor (HCF) of 20 and 35.

Step 2: Divide both sides of the ratio by the HCF.

Visual support

20 : 35
÷5 ↓ ↓ ÷5
4 : 7

b. 700 : 2000

Working

HCF is 100

700 ÷ 100 : 2000 ÷ 100

7 : 20

Thinking

Step 1: Find the highest common factor (HCF) of 700 and 2000.

Step 2: Divide both sides of the ratio by the HCF.

Student practice

Simplify each ratio.

a. 21 : 14 **b.** 1000 : 4000 **c.** 48 : 204 **d.** 168 : 216

WE 4 Determining best buys

Determine the best buy for each pair of items using the unit in the brackets.

a. Four packets of biscuits for $20 or six packets of biscuits for $24 (1 packet)

Working

Four packets of biscuits for $20	Six packets of biscuits for $24
4 : 20	6 : 24
4 ÷ 4 : 20 ÷ 4	6 ÷ 6 : 24 ÷ 6
1 : 5	1 : 4
One packet costs $5	One packet costs $4

Six packets of biscuits for $24 is the best buy.

Thinking

Step 1: Write the ratio of the quantity to the total cost.

Step 2: Divide both sides of the ratio to calculate the cost for one unit.

Step 3: State which item is the best buy.

Visual support

Four packets for $20	Six packets for $24
4 : 20	6 : 24
÷4 ÷4	÷6 ÷6
1 : 5	1 : 4

b. A 600 mL bottle of energy drink for $3 or a 400 mL bottle of energy drink for $4 (100 mL)

Working

A 600 mL bottle of energy drink for $12	A 800 mL bottle of energy drink for $16
600 : 12	800 : 16
600 ÷ 6 : 12 ÷ 6	800 ÷ 8 : 16 ÷ 8
100 : 2	100 : 4
100 mL costs $2	100 mL costs $4

A 600 mL bottle of energy drink for $12 is the best buy.

Thinking

Step 1: Write the ratio of the quantity to the total cost.

Step 2: Divide both sides of the ratio to calculate the cost for one unit.

Step 3: State which item is the best buy.

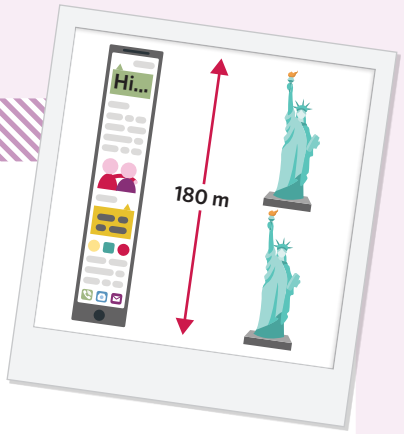
Student practice

Divide each quantity into the given ratio.

a. 30 sausages for $39 or 10 sausages for $20 (1 sausage)

b. 500 mL tub of hummus for $10 or 200 mL tub of hummus for $6 (100 mL)

3H Activities and questions



STARTER TASKS

Odd spot

The average person scrolls 180 m on their phone each day – that's almost twice the height of the Statue of Liberty in New York!

If each 3 cm flick of the thumb equates to a 10 cm scroll on a mobile phone, how far does the average person's thumb flick each day while scrolling?

A. 54 m **B.** 600 cm

Puzzle

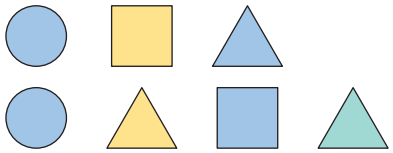
At a school sports carnival, a big batch of raspberry drink was made from 1 bottle of cordial and 4 bottles of water.

a) If a teacher tops up the raspberry drink with 1 bottle of cordial and 1 bottle of water, is the drink now stronger or weaker? Explain why.

b) Half of the raspberry drink has been consumed. If a teacher wants to top the drink up and maintain the original sweetness, how much cordial and water needs to be added to the drink?

Understanding worksheet

1. Write the ratio for each statement.



Example

Blue shapes to green shapes 4 : 1

a. Yellow shapes to blue shapes ☐ : ☐

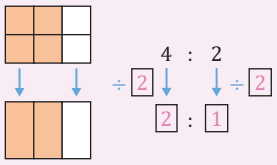
b. Squares to other shapes ☐ : ☐

c. Blue shapes to non-blue shapes ☐ : ☐

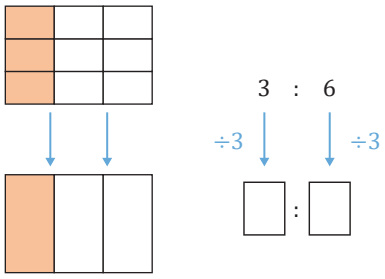
d. Triangles to all shapes ☐ : ☐

2. Simplify the ratios by filling in the gaps.

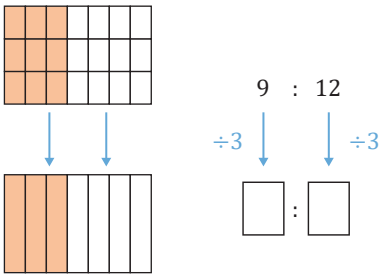
Example



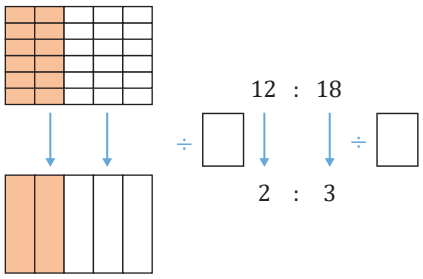
a.



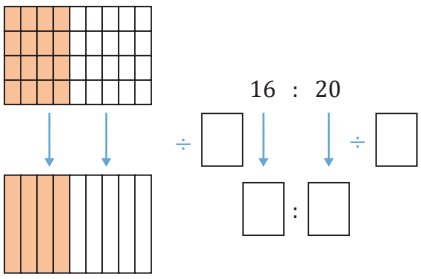
b.



c.



d.



3. Fill in the blanks by using the words provided.

simplifying | same | quantities | relative | units

A ratio is a set of numbers in the form $a : b$ that express the ☐ sizes of ☐ a and b. A ratio does not require ☐ because it is comparing two (or more quantities) of the ☐ unit. When ☐ ratios, if there are no common factors other than 1, then a ratio cannot be simplified further.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c)	4 (c,d,e,f), 5(c,d,e,f), 6 (c,d,e,f), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f)

WE1 **4.** Find the given ratio.

a. If $\frac{1}{5}$ of apples are red, what is the ratio of red apples : all apples?

b. If $\frac{5}{7}$ of shapes are circles, what is the ratio of circles : all shapes?

c. If $\frac{3}{4}$ of berries are red, what is the ratio of red berries : other berries?

d. If $\frac{2}{15}$ of all electronics sold at a store are iPads, what is the ratio of iPads : all electronics?

e. If $\frac{5}{9}$ of chocolate at Roles is Radbury, what is the ratio of Radbury chocolate : other chocolate?

f. If $\frac{5}{8}$ of all clothes on display are tshirts, what is the ratio of tshirts : all clothes?

g. If $\frac{2}{5}$ of the cutlery in a drawer are forks and the rest are spoons, what is the ratio of forks : spoons?

h. If $\frac{3}{6}$ of the vegetables in a stew are carrots and the rest are potatoes, what is the ratio of carrots : potatoes?

WE2 **5.** Find the given ratio.

a. If $\frac{3}{8}$ of books have hardcovers and $\frac{5}{8}$ have softcovers, what is the ratio of hardcovers : softcovers?

b. If $\frac{4}{9}$ of pens are blue, $\frac{2}{9}$ are black, and the rest are red, what is the ratio of blue : black : red?

c. If $\frac{5}{8}$ of the clothes are hats, $\frac{2}{8}$ are beanies and $\frac{1}{8}$ are scarves, what is the ratio of hats : beanies : scarves?

d. If $\frac{2}{7}$ of backpackers are Chinese, $\frac{4}{7}$ are Spanish and the rest are German, what is the ratio of Spanish backpackers : all other backpackers?

e. If $\frac{2}{5}$ of beds are single, $\frac{3}{10}$ are queen, and $\frac{6}{20}$ are king, what is the ratio of single : queen : king beds?

f. If $\frac{1}{6}$ of drinks ordered at a cafe are tea, $\frac{1}{9}$ are hot chocolate, and the rest are coffee, what is the ratio of coffee : all drinks?

g. If $\frac{1}{4}$ of fish are salmon, $\frac{2}{6}$ of fish are tuna, and the rest are halibut, what is the ratio of halibut : other fish?

h. If $\frac{4}{15}$ of the restaurants in chinatown are noodle restaurants, $\frac{1}{3}$ are hotpot restaurants and the rest are dumpling restaurants, what is the ratio of dumpling restaurants : all restaurants?

WE3 **6.** Simplify each ratio.

a. 15 : 25 **b.** 16 : 24 **c.** 21 : 18 **d.** 49 : 84

e. 14 : 63 **f.** 88 : 56 **g.** 63 : 108 : 81 **h.** 169 : 143 : 91

WE4 **7.** Determine the best buy for each pair of items using the unit in the brackets.

a. A box of 3 muesli bars for $9 or a box of 6 muesli bars for $12 (1 muesli bar)

b. A box of 48 rolls of premium toilet paper for $96 or a pack of 12 rolls for $36 (1 toilet roll)

c. A 200 mL bottle of juice for $6 or a 700 mL bottle of juice for $14 (100 mL)

d. A box of 36 Pokemon card packs for $180 or a box of 12 Yu-Gi-Oh card packs for $84 (1 card pack)

e. A 500 g box of cereal for $20 and a 1.5 kg box of cereal for $30 (100 g)

f. A 16 GB for $12 USB stick or 64 GB USB stick for $16 (4 GB)

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. From a bag of jelly beans, Luigi receives six jelly beans, Daisy receives four jelly beans and Peach receives five jelly beans. What is the ratio of jelly beans for Luigi to Daisy to Peach?

9. At the Melbourne Zoo, there are 75 monkeys, 25 elephants and 250 wombats. What is the ratio of monkeys to elephants to wombats in the simplest form?

10. Aamil has prepared some party bags for guests at his party. He knows that in each bag, $\frac{1}{8}$ of smarties are pink, $\frac{1}{4}$ are blue, $\frac{1}{2}$ are green. What is the ratio of yellow smarties to all smarties that each guest receives?

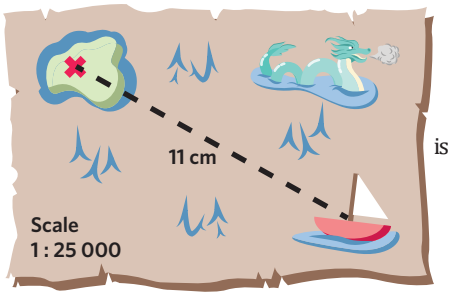
11. Rex is painting his house purple. He goes to the paint shop and is shown two different brands of can. Which can is the cheapest per litre if one can is 10 L and costs $150 and the other can is $36 for 2 L.

12. Lorraine is making brownies. The recipe contains flour, cocoa, and sugar in a ratio of 5 : 2 : 3. How much flour and cocoa will she need? Lorraine knows that she needs to use 375 grams of sugar but the quantities for the other ingredients are missing.

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. A scale of 1 : 100 means that every 1 cm length on the map represents 100 cm in real life. Captain Jack Sparrow is hunting for a treasure chest and is using a treasure map with a scale of 1 : 25 000.



   a. Jack calculated that he will have to sail his pirate ship for a length of 11 cm on the map to get to the island where the treasure chest is located. What is the distance in metres he will have to sail?
   b. Upon landing on the island, Jack then has to walk 2000 m to reach the treasure chest. What is this length in centimetres on the map?
   c. After hours of digging at the marked treasure chest location, they found the treasure chest! It contained 40 000 gold doubloons. Jack will receive $\frac{1}{10}$, his vice-captains will receive $\frac{2}{5}$ and the crewmen receive the rest. What is the ratio of doubloons received by Jack to his vice-captains to the crewmen?
   d. Jack is looking at buying a new sail for his ship. He is given two different types of sails to choose from: 3000 gold doubloons for 102 square metres and 5000 for 205 square metres. Which is the best buy per 1000 doubloons?
   e. Was the ratio to split the doubloons fair? Propose an alternative ratio for splitting the doubloons that is more fair for Jack, his vice-captains and the crewmen.

14. A normal deck of cards has 4 suits: clubs, diamonds, hearts and spades.
    Each suit contains 13 cards: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.
    For this question, give all ratios and fractions in their simplest form.
   a. What fraction of the deck are diamonds?
   b. What is the ratio of clubs : diamonds : hearts : spades?
   c. 13 random cards with a heart suit from another deck are added to the existing deck. What is the ratio of clubs : diamonds : hearts : spades after the cards have been added in?
   d. Should this modified deck be used to play card games? Justify your answer.

Extra spicy

15. The scale of a map to the actual distance is 200 : 10 000 000. If the actual distance between two points was 5 km, what was the length measured on the map?

16. The ratio of boys to girls in the school play is 3 : 5. There are 18 boys in the play. If 4 more boys decide to join, what will be the new ratio of boys to girls in the school play?

17. Express the ratio $4\frac{2}{3} : 2 : 0.5$ in its simplest integer form.

18. A 240 mL drink has a 2 : 3 ratio of cordial to water. If the amount of cordial does not change, how much water needs to be added to change the ratio of cordial to water to 3 : 5?

Remember this?

19. Which of the following number sentences is correct when $\bigstar = 7$?

A. $14 + \bigstar = 7$ **B.** $11 - 2 = \bigstar$ **C.** $\bigstar - 10 = -3$ **D.** $8 + \bigstar = 13$

20. Vincent owns more than 30 toys and is packing them away into boxes.
When he packs the toys equally into 6 boxes, there are 2 toys left over.
When he packs the toys equally into 5 boxes, there are 3 toys left over.
What is the minimum number of toys that Vincent could own?

21. Eunice decided to treat herself and bought two identical cheesecakes.
After she had eaten $\frac{1}{3}$ of one of the cheesecakes, she had a total of 625 grams of cheesecake left.
What is the mass of one full cheesecake, in grams?

Chapter 3 extended application

1. Tom is a baker and sets up his own cake stand. Tom can sell a whole cake for $30 or a slice of cake for $5. Each slice is $\frac{1}{8}$ of a whole cake.
   a. How much more money does Tom make from selling a whole cake 'by the slice' compared to selling it as a whole?
   b. Tom can cut a cake into 16 slices and sell each slice for $3. If he is aiming to make as much money as possible from each whole cake he sells, should Tom cut each cake into 16 slices and sell them for $3 or continue to sell an $\frac{1}{8}$ for $5?
   c. Why might Tom want to sell a whole cake at once for less than he could sell it for in slices?
   d. If you could cut the cake up into as many slices as you like and charge what you like, what would you do? Discuss this with a partner to reach an agreement on how you would sell the cake. (You can use an A4 piece of paper as a guide for the size of your cake).

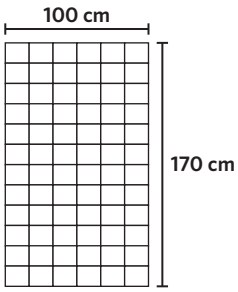
2. Stan is painting his cubby house because he wants to have the coolest cubby house in the neighbourhood.

   Stan is having some friends for a sleepover in 10 days.

   Stan thinks that he will be able to paint $\frac{1}{10}$ of the cubby house every day and is only planning on painting for 30 minutes each day over a 10 day period.
   a. After 3 days, Stan has painted $\frac{1}{5}$ of the cubby house in total. Is he ahead of schedule, behind schedule or on schedule? Justify your answer.
   b. If Stan continues painting at the same rate, how many days in total will it take him to complete painting the cubby house?
   c. With 7 days to go, Stan realises that he might not finish the cubby house in time. He has thought of two options to help manage his time so that he can complete the painting before the sleepover.
      - Option A – Stan can miss watching his favourite half-hour TV show and instead spend 1 hour painting for 6 of the 7 remaining nights.
      - Option B – Stan can do half an hour of painting and watch half an hour of TV each night BUT will need to paint for 3 hours the night before the sleepover.

      Which option would you recommend to Stan? Justify your recommendation.
   d. Come up with a different option for how Stan could paint the cubby house over the next 7 days. Write one advantage and/or one disadvantage of your option.

3. You have decided to install a solar powered system on your roof in order to reduce your carbon footprint and agree on a residential system which consists of 170 cm × 100 cm panels.



The solar panel company sends you this table containing information about your options:

Power output for your system	Roof area required	Number of solar panels required
3 kilowatts (3000 W)	15 – 20 m^2	9 × 330 W panels
		12 × 250 W panels
5 kilowatts (5000 W)	25.5 – 34 m^2	15 × 330 W panels
		20 × 250 W panels
7 kilowatts (7000 W)	36 – 48 m^2	21 × 330 W panels
		28 × 250 W panels

First, you need to consider how much electricity output you want from your system. You can have a 3 kW, 5 kW or 7 kW system.

a. An average household in Australia consumes 18 units of energy per day with 8 units of that being consumed in the daylight hours. What fraction of the daily energy consumption is required in the daylight hours?

b. Your most recent energy bills suggest that you use $\frac{3}{5}$ of your daily energy in daylight hours. Is this more or less than the average household?

You will then need to decide how much 'usable' roof space you have for the panels. Roof space is considered 'usable' if:

- it is north-facing.
- it is not in the shade.
- it is away from other structures and equipment.

c. The contractor hired to install your solar panels mentioned that on your roof, only $\frac{1}{5}$ of the 150 m^2 of roof space is 'usable'. How much usable roof area do you have to install the solar panels?

d. In Australia, it is suggested that a 5 kW system is enough to power the average home. Do you have enough roof space for a 5 kW system?

You can use either 330 W panels or 250 W panels. The 330 W panels are more expensive but can output more power.

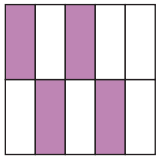
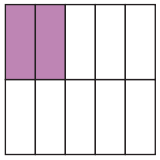
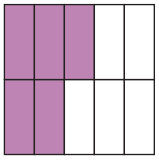
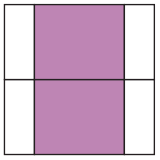
e. Based on the amount of 'usable' area on your roof, will you choose to use 330 W panels or the 250 W panels if both panels take up an area of $1\frac{2}{3}$ m^2?

f. What fraction of your **total roof area** will be taken up with solar panels?

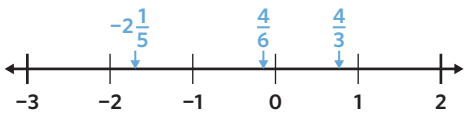
Chapter 3 review

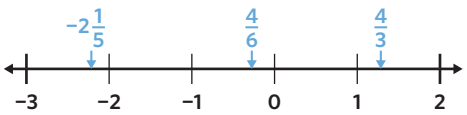
Multiple choice

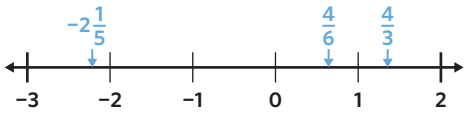
1. Which square has two-fifths shaded? 3A

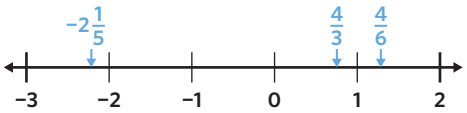
A. 
B. 
C. 
D. 

2. Which number line shows the correct locations of $-2\frac{1}{5}$, $\frac{4}{6}$ and $\frac{4}{3}$? 3D

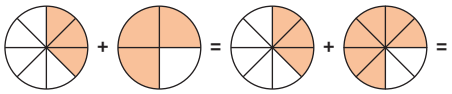
A. 

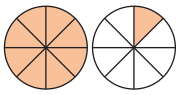
B. 

C. 

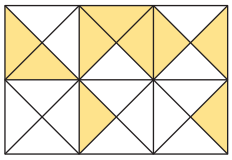
D. 

3. Complete the visual equation. 3E



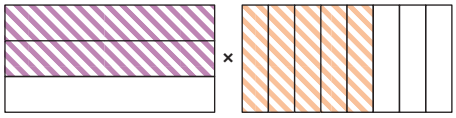
A. 
B. 
C. 
D. 

4. What is the simplified ratio of unshaded : shaded in the following diagram? 3H



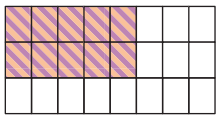
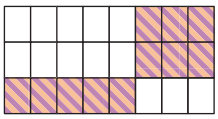
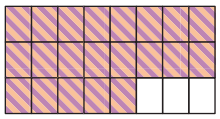
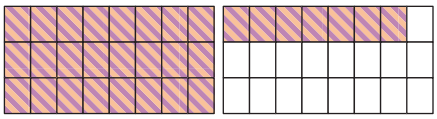
A. 3 : 5 B. 8 : 3 C. 5 : 3 D. 3 : 8

5. Consider the following multiplication. 3F



$\frac{2}{3}$ $\frac{5}{8}$

Which of the given shapes shows its product?

A. 
$\frac{5}{12}$
B. 
$\frac{11}{24}$
C. 
$\frac{7}{8}$
D. 
$\frac{31}{24}$ or $1\frac{7}{24}$

Fluency

6. Write a fraction for each of the following descriptions. 3A

a. Denominator of 7, numerator of 4 **b.** 3 divided by 5

c. Six-eighths **d.** 2 out of 9

7. Convert each mixed number to an improper fraction. 3B

a. $1\frac{5}{6}$ **b.** $4\frac{2}{7}$ **c.** $3\frac{1}{5}$ **d.** $2\frac{7}{12}$

8. Express each improper fraction as a mixed number. Simplify where possible. 3B

a. $\frac{9}{5}$ **b.** $\frac{11}{4}$ **c.** $\frac{13}{2}$ **d.** $\frac{26}{6}$

9. Complete the equivalent fractions. 3C

a. $\frac{1}{6} = \frac{\square}{12} = \frac{6}{\square} = \frac{\square}{60} = \frac{15}{\square}$ **b.** $\frac{3}{4} = \frac{\square}{8} = \frac{12}{\square} = \frac{\square}{40} = \frac{120}{\square}$

c. $\frac{120}{180} = \frac{\square}{120} = \frac{40}{\square} = \frac{\square}{9} = \frac{2}{\square}$ **d.** $\frac{100}{160} = \frac{\square}{80} = \frac{20}{\square} = \frac{\square}{16} = \frac{5}{\square}$

10. Fully simplify each fraction. 3C

a. $\frac{8}{12}$ **b.** $\frac{18}{30}$ **c.** $\frac{15}{35}$ **d.** $2\frac{2}{8}$

11. Compare the fractions using mathematical symbols (<, >, =). 3D

a. $\frac{1}{6} \square \frac{1}{7}$ **b.** $\frac{5}{3} \square \frac{15}{9}$ **c.** $\frac{5}{6} \square \frac{19}{24}$ **d.** $\frac{14}{9} \square \frac{10}{6}$

12. Put the list of values in ascending order (from smallest to largest). 3D

a. $\frac{5}{4}, \frac{9}{4}, 1\frac{3}{4}$ **b.** $\frac{7}{12}, \frac{3}{6}, \frac{2}{3}$ **c.** $\frac{2}{5}, \frac{1}{2}, \frac{6}{10}$ **d.** $\frac{17}{24}, \frac{13}{16}, \frac{9}{12}$

13. Evaluate the following (and simplify where possible). 3E

a. $\frac{3}{6} + \frac{2}{6}$ **b.** $\frac{14}{20} - \frac{3}{4}$ **c.** $\frac{6}{8} + \frac{4}{5}$ **d.** $\frac{28}{16} + 3\frac{15}{24}$

14. Evaluate: 3F

a. $\frac{1}{2} \times 6$ **b.** $\frac{3}{7} \times 3$ **c.** $\frac{6}{5} \times \frac{15}{18}$ **d.** $\frac{7}{54} \times \frac{9}{42}$

15. Find (and simplify where possible): 3F

a. $\frac{2}{7}$ of 14 **b.** $\frac{4}{5}$ of $\frac{3}{8}$ **c.** $\frac{58}{18}$ of $\frac{12}{11}$ **d.** $\frac{3}{4}$ of $2\frac{9}{10}$

16. Evaluate (and simplify where possible): 3G

a. $3 \div \frac{1}{5}$ **b.** $\frac{1}{8} \div \frac{6}{5}$ **c.** $\frac{14}{8} \div \frac{11}{6}$ **d.** $\frac{2}{7} \div \frac{3}{16}$

17. Fully simplify each ratio. 3H

a. 18 : 24 **b.** 7 : 42 **c.** 12 : 27 **d.** 36 : 30

18. Determine the best buy for each pair of items using the unit in the brackets. 3H

a. A box of 12 soft drink cans for $24 or a four-pack of soft drink cans for $12 (1 can)

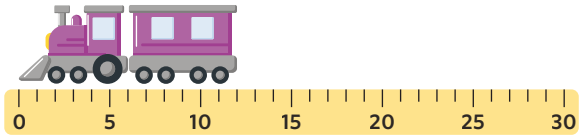
b. A can of 5 tennis balls for $20 or a large bag of 40 tennis balls for $80 (1 ball)

c. A large 500 g bag of chips for $10 or a small 200 g bag of chips for $2 (100 g)

d. A 600 mL tub of premium ice cream for $18 or a pack of four 50 mL ice creams for $8 (100 mL)

Problem solving

19. Henderson measures his model train against his 30 cm ruler. 3A



The train is 12 cm long.

What is the length of the train as a fraction of the total length of the ruler?

20. Issac is doing a 2.5 km time trial on a 400 m running track. How many laps does he need to run? Give your answer as a mixed number. 3B

21. Ciaran is a strength and conditioning coach and tells his clients that exactly $\frac{2}{7}$ of the time spent on a workout should be spent stretching. 3C

Select a correct amount of time spent doing cardio and stretching.

A. 70 minutes of cardio, 25 minutes of stretching.
B. 70 minutes of stretching, 20 minutes of cardio.
C. 105 minutes of cardio, 40 minutes of stretching.
D. 100 minutes of cardio, 40 minutes of stretching.

22. Britt and Timm are completing the same maths test during class. 3D

Britt completed $\frac{9}{10}$ of the test.

Timm completed more of the test than Britt.

What fraction of the test could Timm have completed?

A. $\frac{9}{11}$ **B.** $\frac{12}{13}$ **C.** $\frac{10}{12}$ **D.** $\frac{8}{9}$

23. Gerald is participating in the Oxfam 100 km trailwalk. He walks $23\frac{1}{4}$ km to one of the checkpoints and has a rest. 3E

He then walks another $22\frac{2}{3}$ km to another checkpoint before resting again. How many kilometres does Gerald have left to walk?

24. Samantha is mixing a glass of cordial. The glass is five-sixths full. One-fifth of the liquid is cordial mix, and the rest is water. What fraction of the glass is made up of cordial mix? 3F

25. Richie is trying to fit books in his bookshelf that is 45 cm in length. Each book is $3\frac{1}{3}$ cm thick. What is the maximum number of books Richie can fit in his bookshelf? 3G

26. Leonardo Da Vinci was 1.75 m tall. 3H

A statue of the artist was made with a statue : actual height scale of 2 : 1.

How tall is the statue?

A. 1750 cm **B.** 350 m **C.** 175 cm **D.** 350 cm

Reasoning

27. Malachy and Moo caught up at a dumpling restaurant. They both ordered a plate of the same amount of dumplings. After 30 minutes Malachy had eaten $\frac{11}{15}$ of his dumplings and Moo had eaten $\frac{4}{5}$ of her dumplings.

a. Altogether, how many plates of dumplings had Malachy and Moo eaten after 30 minutes? Give your answer as an improper fraction.
b. Convert this to a mixed number.
c. Who had eaten more dumplings after 30 minutes?
d. Malachy decided to eat one more of his dumplings. Each plate originally contained 15 dumplings. Have Malachy and Moo now eaten an equivalent fraction of their dumplings?
e. Is it an issue that Malachy and Moo did not finish their order? How could they limit their amount of food waste?

28. Malachy and Moo caught up again at a Korean BBQ restaurant. Moo ordered 6 wings and ate 4 of them. Malachy ate $\frac{3}{5}$ of the wings he ordered.

a. What fraction of her wings did Moo eat?
b. Of the total number of wings that were ordered by both Malachy and Moo, Moo ordered $\frac{3}{8}$ of them. How many wings did Malachy order?
c. How many wings did Malachy eat?
d. Between both Malachy and Moo there were 3 flavours of wings ordered; BBQ, honey soy and teriyaki. In a ratio of BBQ : honey soy : teriyaki, the amount of each flavour ordered was 4 : 3 : 1. How many honey soy wings were ordered?
e. Malachy offered to pay for the whole meal because he invited Moo to come. Moo insisted that she pay half. What do you think is fair?

04

Algebra

Number and Algebra

Research summary

4A Relational thinking with unknowns

4B Building algebraic expressions and equations

4C Substituting values into equations

4D Solving one-step equations

4E Solving two-step equations *(Extension)*

4F Solving multi-step equations *(Extension)*

4G Simplifying equations (collecting like terms)

4H Geometric patterns and numerical sequences

4I Creating rules from tables and sequences

Chapter 4 extended application

Chapter 4 review

Image: Marish/Shutterstock.com

Research summary – Algebra

Big ideas

Relational thinking

Relational thinking is the ability to recognise that the equals sign expresses a relationship between identical values which means, 'the same as'. Developing relational thinking in mathematics is the foundation of a solid understanding in algebra. The equals sign can sometimes be interpreted in different ways with it often being viewed as an operator with a similar function as +, −, × and ÷. Some may also see it as representing 'the answer' or the 'next step' which is an incorrect interpretation.

Symbolism

Assigning symbols in the form of letters or other mathematical objects to unknown values is a key component of algebra. It allows for the generalisation and reasoning of algebraic situations in order to find patterns, develop rules and relationships, and solve problems. In algebra, the letters x, y and z are commonly used to replace an unknown value. Other letters, images and symbols are also used. Once this use of symbols is effectively applied, the successful navigation of more complex areas of mathematics will be more likely.

Patterns

The ability to recognise patterns in mathematics is a vital skill when working with algebraic problems. Much of our world is composed of patterns and the ability to notice and interpret these patterns is a useful skill in many areas of work and beyond. Patterns can be geometrically repeating patterns or they can be numerical sequences that exist within a problem. Being able to work with these patterns will allow us to more easily solve problems and interpret situations in a real-world context.

Generalising

The idea of generalising is about finding patterns in a problem or situation and deriving a mathematical rule from it. This is done in order to repeat the pattern, extend upon a concept, or solve a problem. A generalised rule may be in the form of an equation or a function that can be used to show all possible outcomes of the problem. The idea of generalising does not only apply to algebra but to all areas of mathematics and beyond. Whenever we are developing a principle in order to solve a variety of problems, we are creating a generalisation or rule.

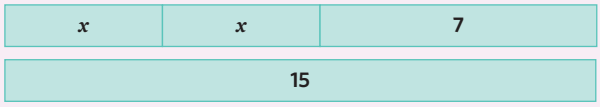
Visual representations

Balance scales

Just like the traditional pan balances, balance scales help develop relational thinking by drawing attention to the idea that in order for the scales to be balanced, both sides have to be the same or equal. In order for the scales to remain balanced, whatever operation is applied to one side we also need to apply to the other side.

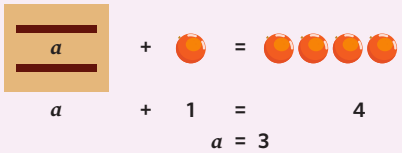
The bar model

A bar model is a pictorial representation of a number or equation in the form of bars or boxes. They encourage the comparison of two values using multiple bars of the same size that represent the same values, and help to solve problems visually.



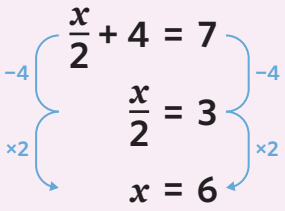
Symbols

Creating equations using images helps to develop a more conceptual understanding of the function of equations. In the image shown, a = the number of oranges in the box. When solving equations visually with symbols it can be much clearer to find the value of an unknown.



Flowcharts

Flowcharts can be used to help in solving one-step and multi-step equations. They provide scaffolding to show how equations are constructed when inverse operations are applied to solve for the unknown.



Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students see the equals sign as 'the answer is', instead of 'the left-hand side has the same value as the right-hand side'.	For $5 + 10 = x + 7$ $5 + 10 = 15 + 7$	For $5 + 10 = x + 7$ $5 + 10 = 8 + 7$	4A
Students possess the 'empty box' misconception where they think that the unknown 'x' could have two or more different values in the same number sentence.	$x + x + x = 12$ can be $5 + 5 + 2 = 12$.	A repeated unknown will have the same value for the whole number sentence: $x + x + x = 12$ must mean that $x = 4$, so $4 + 4 + 4 = 12$	4A
Students believe a different letter must mean a different value even when both sides of the equations are equal.	$a + b$ cannot equal $a + c$.	$a + b = a + c$ can be true if b and c are the same value.	4B
Students incorrectly interpret a letter as an object rather than a number (non-numerical thinking).	a stands for 'apples' because 'apples' begins with the letter a.	a represents 'the number of apples'.	4B
Students incorrectly identify the number of terms in an expanded form equation.	$2x \div 3y + 7$ has three terms.	$2x \div 3y + 7$ has two terms because, $2x \div 3y + 7 = \frac{2x}{3y} + 7$	4B
Students don't consider that the commutative/associative laws make an expression the same.	$b \times 5$ is not recognised to be the same as $5b$.	$b \times 5 = 5b$ $zxy = xyz$	4B
Students incorrectly believe that the value of a pronumeral is its position in the alphabet.	$a = 1, b = 2, c = 3$	a can take any value. E.g. $a = 20$.	4C
Students don't see the coefficient of a pronumeral as representing a multiplication.	$3x - 5 = 29$ when $x = 4$because $34 - 5 = 29$	$3x - 5 = 7$ when $x = 4$ because, $3 \times x - 5 = 3 \times 4 - 5$ $= 12 - 5$ $= 7$	4C
Students incorrectly believe pronumerals have the same fixed value for all problems.	If $d = 10$ in part a of the question, then d must equal 10 everywhere.	d can have a solution of $d = 10$ in one question and $d = 23$ in a different question.	4C
Students don't recognise that inverse operations are needed when solving equations.	Solving $x + 2 = 5$ for x by adding 2 to both sides.	Solving $x + 2 = 5$ for x by subtracting 2 from both sides.	4D
Students solve equations by applying inverse operations to one side only.	$x + 2 = 5$ $x + 2 - 2 = 5$ $x = 5$	$x + 2 = 5$ $x + 2 - 2 = 5 - 2$ $x = 3$	4D
Students perform inverse operations in an incorrect or inefficient order.	Solving $3x - 5 = 7$ by first dividing both sides by 3.	Solving $3x - 5 = 7$ by first adding 5 to both sides.	4E
Students don't understand the order of operations when solving equations with brackets.	Solving $3(x + 5) = 45$ by first subtracting 5 from both sides.	Solving $3(x + 5) = 45$ by first dividing both sides by 3.	4F
Students don't understand the order of operations when solving equations with algebraic fractions and constants.	Solving $\frac{2z}{5} + 4 = 10$ by first multiplying both sides by 5.	Solving $\frac{2z}{5} + 4 = 10$ by first subtracting 4 from both sides.	4F

	Incorrect	Correct	Exercise
Students incorrectly group together different terms because they contain similar pronumerals.	$2x + 3xy = 5xy$	$2x + 3xy$ is in simplest form because $2x$ and $3xy$ are not like terms.	4G
Students incorrectly simplify expressions by grouping different terms together in a similar way to a multiplication.	$3m + 5p = 15mp$	$3m + 5p$ is in its simplest form.	4G
Students think that n describes the number in each position of the pattern instead of the position itself.	2, 4, 6, 8 $n = 2$ for the first number. $n = 4$ for the second number. $n = 6$ for the third number. $n = 8$ for the fourth number.	2, 4, 6, 8 $n = 1$ for the first number. $n = 2$ for the second number. $n = 3$ for the third number. $n = 4$ for the fourth number.	4H
Students believe that the constant in the equation represents the change in a pattern, instead of the coefficient of n representing the change.	The equation $t = 3n + 5$ represents a pattern where each term increases by 5.	The equation $t = 3n + 5$ represents a pattern where each term increases by 3.	4I

Additional reading and resources

- Algebra Tiles Australia – Tiles and Manual, Lorraine Day.
- https://mathsbot.com/manipulatives/tiles
- https://brilliant.org/courses/pre-algebra/?tour=true
- https://phet.colorado.edu/en/simulations/filter?subjects=math&sort=alpha&view=grid
  - Expression Exchange.
  - Balancing Act.
  - Equality Explorer: Two Variables.

4A Relational thinking with unknowns

Relational thinking examines the properties of numbers, operations and the equals sign. It is the idea that in any mathematical equation or number sentence, the value on both sides of the equals sign must be the same. Below are some examples where relational thinking can be applied.

- If I picked 5 apples each day for five days, I could pack them in a 5 by 5 square box.
- I bought 15 muffins from the store. I filled two containers with six muffins each to save and I took 3 to work.
- I collected 6 eggs each morning for four days. This gave me the same amount of eggs as collecting 12 eggs a day for the next two days.

Learning intentions

Students will be able to:

+ understand that in an equation, both sides of the equals sign must be the same value
+ identify equivalent number sentences using all four operations
+ recognise and create equivalent algebraic expressions and equations.

Key terms and definitions

- **Relational thinking** is the ability to recognise that the equals sign implies a relationship between both sides that means 'the same as'.
- **Equivalent values** are two values, numbers or quantities which are the same.
- A **number sentence** is an equation expressed using numbers and operators such as +, −, × and ÷.
- An **equation** is another name for a number sentence. An equation can contain letters, numbers and operations, as well as an equals sign. Each side of an equation is equal.

Key ideas

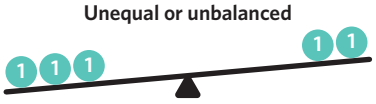
1 In an equation, the equals sign indicates that the total value on the left-hand side (LHS) of the equals sign is the same as the total value on the right-hand side (RHS) of the equals sign.

Equal	Not Equal
$2 + 7 = 7 + 2$	$2 - 7 \neq 7 - 2$
$20 + 9 - 9 = 9 - 9 + 20$	$20 + 9 - 9 \neq 9 - 9 - 20$
$10 \times 9 = 5 \times 18$	$10 \div 9 \neq 5 \div 18$

2 Balance scales can be used to show equivalence in equations. If both sides of the balance scale has the same value, the scale will be **equal** or **balanced**. If something is added or subtracted from one side only, then the balance scale will be **unequal** or **unbalanced**.

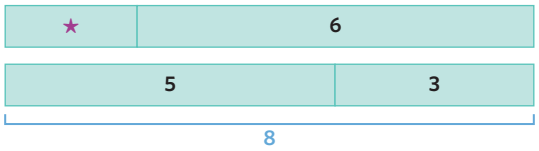
Equal or balanced

Unequal or unbalanced



3 Bar models can be used to show equivalence in equations.

The two bars in our bar model are the same size to show that both sides of our number sentence or equation are equal.



★ + 6 = 8
2 + 6 = 8
★ = 2

Worked examples

WE 1 Checking number sentences

State whether each number sentence is true or false.

a. $6 + 7 = 4 + 8$

Working

$6 + 7 = 4 + 8$

$6 + 7 = 13$

$4 + 8 = 12$

$13 \neq 12$ ✗

False

Thinking

Step 1: The equals sign in the equation means the left-hand side (LHS) is equivalent to the right-hand side (RHS).

Step 2: Calculate the LHS.

Step 3: Calculate the RHS.

Step 4: The two sides are not equal.

b. $3 + 3 + 3 + 3 + 2 = 3 \times 4 + 2$

Working

$3 + 3 + 3 + 3 + 2 = 3 \times 4 + 2$

$3 + 3 + 3 + 3 + 2 = 14$

$3 \times 4 + 2 = 14$

$14 = 14$ ✓

True

Thinking

Step 1: The equals sign in the equation means the left-hand side (LHS) is equivalent to the right-hand side (RHS).

Step 2: Calculate the LHS.

Step 3: Calculate the RHS.

Step 4: The two sides are equal.

Student practice

State whether each number sentence is true or false.

a. $12 + 5 = 8 + 9$

b. $2 + 2 + 2 + 4 = 2 \times 3 + 4$

c. $23 - 11 = 34 - 12$

d. $24 \div 4 + 5 = 24 - 6 - 6 - 6 + 5$

WE 2 Completing equations

Complete the equations to make them true.

a. $3 + 4 + \bigstar - 2 = 12$

Working

$3 + 4 + \bigstar - 2 = 12$

$3 + 4 - 2 + \bigstar = 12$

$5 + \bigstar = 12$

$\bigstar = 7$

$3 + 4 + 7 - 2 = 12$ ✓

Thinking

Step 1: The equals sign in the equation means the left-hand side (LHS) is equivalent to the right-hand side (RHS). The RHS equals 12.

Step 2: Use order of operations to simplify the LHS.

Step 3: Calculate the number that can replace the ★ to make the equation true.

Step 4: Check to make sure your answer is correct.

Visual support

b. $4 + 4 + 4 + 4 - 2 + \bigstar = 19$

Working

$4 + 4 + 4 + 4 - 2 + \bigstar = 19$

$4 + 4 + 4 + 4 - 2 + \bigstar = 19$

$14 + \bigstar = 19$

$\bigstar = 5$

$4 + 4 + 4 + 4 - 2 + 5 = 19$ ✓

Thinking

Step 1: The equals sign in the equation means the left-hand side (LHS) is equivalent to the right-hand side (RHS). The RHS equals 19.

Step 2: Use order of operations to simplify the LHS.

Step 3: Calculate the number that can replace the ★ to make the equation true.

Step 4: Check to make sure your answer is correct.

Student practice

Complete the equations to make them true.

a. $5 + 3 - 2 + \bigstar = 10$

b. $6 + 6 + 6 - 4 = 7 + \bigstar$

c. $16 - 8 - \bigstar + 4 = 11$

d. $9 + 9 + 9 + 9 - 21 = \bigstar + 9$

WE 3 Linking equations and situations

Create an equation to represent the situation.

a. Jenny and Sarah earn the same amount of money each week. Jenny earns $25 an hour for 4 hours of work and Sarah earns $50 an hour for 2 hours of work.

Working

amount Jenny earns = amount Sarah earns

$25 \times 4 = 100$

$50 \times 2 = 100$

$25 \times 4 = 50 \times 2$

Thinking

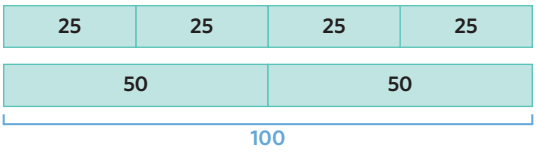
Step 1: Determine the relationship. Jenny and Sarah earn the same amount every week so their earnings are equal to each other.

Step 2: Write one of the equations. Jenny earns $25 an hour and works for 4 hours.

Step 3: Write the other equation. Sarah earns $50 an hour and works for 2 hours.

Step 4: Since they are both equal, write an equation to represent the equivalence.

Visual support



b. Create a situation to represent the equation: $4 + 4 + 4 = 3 \times 2 + 6$

Working	Thinking
Stacey and Archer both went to the carnival and spent the same amount of money.	**Step 1:** The left-hand side (LHS) has the same value as the right-hand side (RHS).
Stacey bought three rollercoaster tickets that cost $4 each.	**Step 2:** Create a situation for the LHS. $4 + 4 + 4$
Archer bought three bags of popcorn for $2 each and a burger for $6.	**Step 3:** Create a situation for the RHS. $3 \times 2 + 6$
Stacey and Archer both went to the carnival and spent the same amount of money. Stacey bought three rollercoaster tickets that cost $4 each. Archer bought three bags of popcorn for $2 and a burger for $6.	**Step 4:** Write all the parts together to create the full situation.
Note: Many situations are possible.	

Student practice

a. Write an equation to represent the situation: Louisa read 10 pages a day for 6 days to finish her book. Janice read 15 pages a day for 4 days to read the same book.

b. Create a situation to represent the equation: $5 \times 10 + 2 = 52$

c. Create an equation to represent the situation: Rachel watched 3 episodes of her favourite TV show. Each episode runs for 24 minutes. Monica also watched 3 episodes of a different TV show. Each episode runs for 21 minutes, and there were 9 minutes of ads.

d. Create a situation to represent the equation: $12 + 12 + 8 = 2 + 3 \times 10$

4A Activities and questions

STARTER TASKS

Image: Cole Bennetts/The Sydney Morning Herald

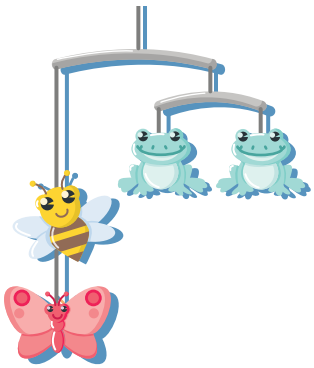
Odd spot

Australian bushfires in the summer of 2019–2020 burned around 12 million hectares of koala habitat. If 3 million hectares of koala habitat remains, how many hectares of koala habitat were there before the fires?

A. 15 million hectares **B.** 9 million hectares

Puzzle

The left side and the right side of this balanced mobile each have a value of 24.



a) What can be said about the value of the insects on the left and the frogs on the right in this image?

b) The right side of the mobile is also balanced. What must the value of each of the frogs be?

c) Can the frogs be any other value?

d) What could be some possible values for the bee and the butterfly?

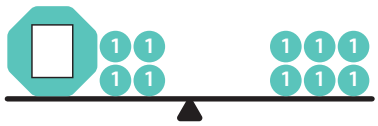
e) Can these two be the same value? Why or why not?

Understanding worksheet

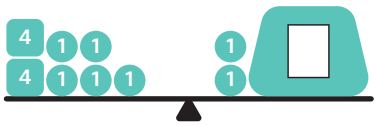
1. Complete the equation to balance the scale.

Example

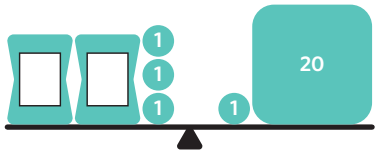
a.



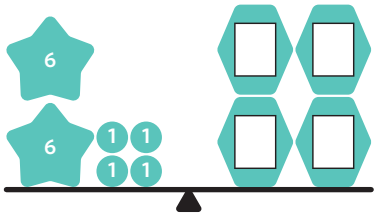
b.



c.



d.



2. Fill in the box to complete the number sentences.

Example

$3 \times 5 = 24 - \boxed{9}$

a. $\square + 4 - 4 = 13$

b. $16 - \square = 4 + 9$

c. $9 + 5 = 6 + \square$

d. $15 + 12 = \square - 12$

3. Fill in the blanks by using the words provided.

both | relational thinking | same | equation

_____ is the idea that in any mathematical _____ or number sentence, the value on _____ sides of the equals sign must be the _____ in order to make it true.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b), 7 (a,b), 8 (a,b)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c), 7 (b,c), 8 (b,c)	4 (e,f,g,h), 5 (e,f,g,h), 6 (c,d), 7 (c,d), 8 (c,d)

WE1 **4.** State whether each number sentence is true or false.

a. $5 + 7 = 12 + 5$

b. $3 + 4 - 4 = 4 - 3 + 4$

c. $18 - 6 + 6 = 10 + 2$

d. $6 \times 7 = 7 \times 6$

e. $2 \times 5 = 8 + 2$

f. $45 - 22 = 25 - 2$

g. $6 \times 7 - 2 \times 3 = 2 \times 3 - 6 \times 7$

h. $1 \times 2 \times 3 \times 4 \times 5 = 5 \times 1 \times 4 \times 2 \times 3$

WE2 **5.** Complete the equations to make them true.

a. $3 + 2 + 5 = \bigstar$

b. $4 - 2 + 7 = \bigstar + 6$

c. $2 + 2 + 2 = 2 \times \bigstar$

d. $5 \times 3 - 2 = 3 + 3 + 3 + 3 + 3 - \bigstar$

e. $6 + 4 - \bigstar + 3 = 12$

f. $3 \times \bigstar + 8 = 7 + 7$

g. $9 + 4 - 3 \times \bigstar = 7$

h. $8 - 5 + \bigstar \times \bigstar = 12$

6. Write the equation that is described.
   a. Two plus eight equals ten
   b. Three groups of seven equals twenty-one
   c. One group of fifteen minus six equals nine
   d. Eight equals twenty minus two groups of six

WE3a 7. Write an equation to represent the situation.
   a. Jake bought three chocolate bars and four bags of chips so he could have one snack each day over the next seven days.
   b. Miles and Gabriel went to an orchard and both took home the same amount of fruit. Miles picked 8 pears and 12 apples to take home. Gabriel picked 4 pears, 3 peaches, 5 apples and 8 pears.
   c. In a basketball game Ben shot 5 three-pointers and 6 free throws (one point each). He scored a total of 21 points in the game.
   d. Paul and Tao went on a shopping spree. Paul bought 9 t-shirts and 5 jackets, then ended up returning 2 of the t-shirts. Tao bought 4 pairs of pants and 8 jumpers. They each ended up with the same amount of clothing.

WE3b 8. Create a situation to represent the equation.
   a. $2 + 3 = 5$
   b. $4 + 4 + 4 = 4 \times 3$
   c. $2 \times 5 + 3 = 13$
   d. $3 \times 2 + 4 \times 5 = 26$

Problem solving

Mild ❜	Medium ❜❜	Spicy ❜❜❜
9, 10, 11	10, 11, 12	11, 12, 13

9. Shawn and Timmy have to shear 35 sheep between them. One morning, Shawn shears 9 sheep and Timmy shears 8 sheep. That afternoon, Shawn shears 7 sheep. How many sheep does Timmy have to shear to complete all 35 sheep?

10. Sarah has a collection of comic books. Half of these are Marvel and the other half are DC. Ron has the same amount of comic books as Sarah, but he only owns Marvel. How many Marvel comic books do they each have, if they have 240 comic books altogether?

11. Eva is 8 years old and is currently 3 years older than her sister Penny. In four years time, how old will both Eva and Penny be?

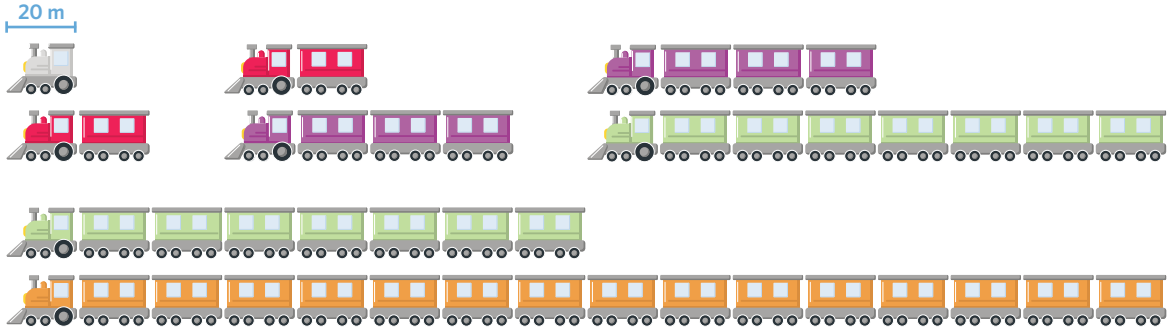
12. Holden's farm is planting crops for the new season. They plant a square of 12 rows of 12 corn stalks. Gemma's farm plants the same number of corn stalks but only has room for 8 rows. How many corn stalks are in each of Gemma's rows?

13. Farmer Fred is calculating the cost of an individual chicken. He bought 20 sheep, 30 ducks and 50 chickens. He paid $15 for each sheep and $10 for each duck. In total, he spent $850 on all 100 animals.

Reasoning

Mild ❜	Medium ❜❜	Spicy ❜❜❜
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. The lengths of several trains are shown below. The white train is 20 m in length. In each image, the top train is half as long as the bottom train.



   a. Write a number sentence to represent the length of the purple train and state the length of the purple train.
   b. How long is the orange train?
   c. Continue the pattern to find the length of the next train, a blue train.
   d. Is this pattern a good design for the train? Why or why not?

15. Bill and Ted own two equally-sized plots of land. Each one has an area of 3600 m^2.

a. Choose the responses that fit the dimensions of

i) Bill's plot of land

A. 60 m × 60 m

B. 120 m × 30 m

C. 40 m × 9 m

D. 180 m × 200 m

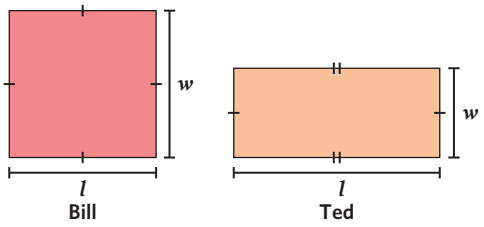
ii) Ted's plot of land.

A. 60 m × 60 m

B. 120 m × 30 m

C. 40 m × 9 m

D. 180 m × 200 m



b. Using the correct responses from part **a**, write a number sentence that represents the length and width of Bill's and Ted's plots of land in relation to their area.

c. If Ted adds 30 metres to the length of his plot, how much does Bill need to add to his length to make sure their plots of land have the same area?

d. Draw a scaled diagram of what their new plots of land will look like, labelling each of the side lengths.

e. Bill and Ted are trying to decide what to do with their plots of land. Bill thinks they can raise animals, Ted thinks they should plant crops. What factors could they consider when making their choice?

Extra spicy

16. There are a number of people riding on a train with 6 carriages. Three people get out of each carriage, and 6 people get on each carriage. Now there are 54 people on the train. How many people were on the train to begin with?

17. In year 7 maths, Class A have an average test score of 70, Class B have an average test score of 92 and Class C have an average test score of ★. Class A, Class B and Class C all have the same number of students. If the average of all three classes' test scores were 83, what were Class C's average test scores?

18. Misha and Kayla share some lollies with the ratio 7 : 3. Misha gives 3 lollies to Kayla and now the ratio is 5 : 3. How many lollies did Misha and Kayla originally have?

19. Write down a subtraction equation $a - b = c$ where each of the numbers a, b and c are made up of the digits 1–9 with each digit used only once in each number.

Remember this?

20. Pete's coins can be made into 2 or 3 equal stacks. If Pete divided his coins into 5 equal stacks and had 3 coins left over, how many coins could Pete have?

A. 12 **B.** 18 **C.** 24 **D.** 36

21. Which of the transformations, taken in order, have been applied to *figure 1* to make *figure 2* and then *figure 3*?

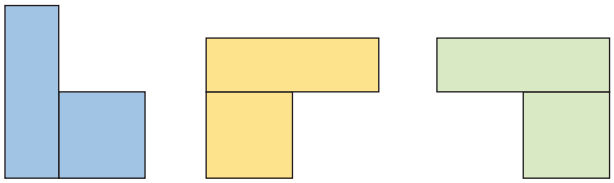


Figure 1 *Figure 2* *Figure 3*

A. Reflection and then 90-degree anticlockwise rotation

B. Reflection and then 90-degree clockwise rotation

C. 90-degree clockwise rotation and then reflection

D. 90-degree anticlockwise rotation and then reflection

22. Which set of fractions is placed in descending order?

A. $\frac{1}{2}, \frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{13}{24}$ **B.** $\frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{13}{24}, \frac{1}{2}$ **C.** $\frac{1}{2}, \frac{5}{8}, \frac{2}{3}, \frac{13}{24}, \frac{7}{12}$ **D.** $\frac{1}{2}, \frac{13}{24}, \frac{7}{12}, \frac{5}{8}, \frac{2}{3}$

4B Building algebraic expressions and equations

In this lesson, we will learn some of the common language of algebra including expressions, equations, pronumerals, terms, constants, and coefficients. We will also learn about different ways of representing equations. Below are some examples where algebraic equations can be applied.

- A box of b apples and another two more apples made a total of 5 apples. How many apples are in the box?
- If I had r raisin cookies and my friend had a Anzac biscuits and we decide to share them equally, how many cookies do we each get?
- Four truck loads of c cars plus another 3 cars makes 19 cars. How many cars are in each truckload?

Learning intentions

Students will be able to:

+ understand the key words: expression, equation, pronumeral, term, constant, and coefficient
+ write algebraic expressions in simplified form
+ create algebraic equations, using order of operations, from given information.

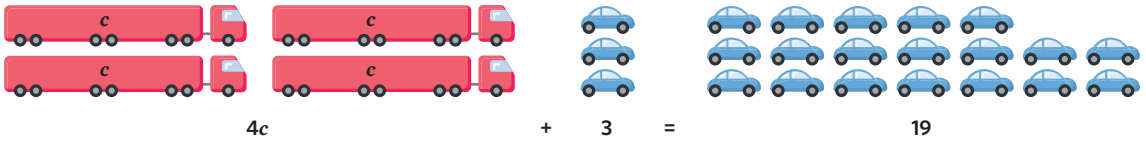
Key terms and definitions

- A **pronumeral** is a letter used to represent a value that is unknown or may vary. This is also known as a variable.
- A **term** is a part of an expression or equation made up of a number and/or pronumerals. Terms are separated by addition or subtraction. E.g. $5x$, $3ab$, 7, $\frac{2a}{3}$.
- An **expression** is a number of terms grouped together by operations. E.g. $5x + 3$ or $3ab - 4a + b$.Note: An expression does not contain an equals sign.
- An **equation** is another name for a number sentence. It is made up of an expression on each side of an equal side. Each side of an equation is equal.
- A **constant** is a number that can not change its value in an expression or equation. E.g. 5, $\frac{3}{4}$, 0.2.
- A **coefficient** is the number that a pronumeral is being multiplied by. E.g. For the term $5x$, the coefficient is 5. For the term $\frac{2a}{3}$, the coefficient is $\frac{2}{3}$. For the term y, the coefficient is 1.
- An **operator** is a symbol used to show that an action has been performed on a set of mathematical terms. The most common operators are $+$, $-$, $\times$ and $\div$.

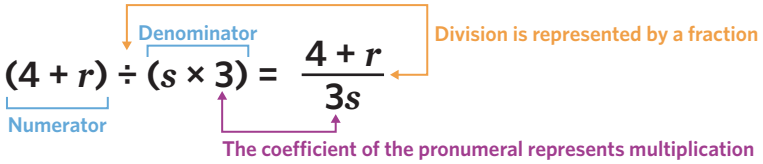
Key ideas

1 We can use a pictorial diagram to represent an algebraic equation.

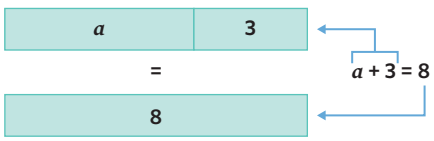
c = the number of cars in each truck



2 The order of operations must always apply when we are writing algebraic equations using multiple operations.



3 An equation can be used to represent any situation where two values are equal, even when some of those values are unknown.



Worked examples

WE 1 Identifying parts of an algebraic expression

For each expression, state:

i) the number of terms in the expression

ii) the coefficient of x

iii) the constant.

a. $5x + 3y - 10$

Working

i) There are 3 terms in this expression: $5x$, $3y$ and -10.

ii) The coefficient of x is 5.

iii) The constant is -10. $5x + 3y + (-10)$

Thinking

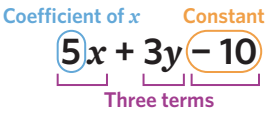
Step 1: Find all the terms. A term is a part of an expression made up of a number and/or pronumerals. Terms are separated by addition or subtraction.

Step 2: Identify the coefficient. The coefficient of x is the number the pronumeral is being multiplied by.

Step 3: Identify the constant. The constant is the whole number without a pronumeral.

Note: Subtraction is the same as adding a negative value.

Visual support



b. $2 \div k - 4x^2 + x$

Working

$\frac{2}{k} - 4x^2 + x$

i) There are 3 terms in this expression: $\frac{2}{k}$, $-4x^2$ and x.

ii) The coefficient of x is 1.

The pronumeral x also means $1x$.

Note: -4 is the coefficient of x^2.

iii) There is no constant in this expression.

Thinking

Step 1: $2 \div k$ can be written as the fraction $\frac{2}{k}$.

Step 2: Find all the terms. A term is a part of an expression made up of a number and/or pronumerals.

Step 3: Identify the coefficient. The coefficient of x is the number the pronumeral is being multiplied by.

Note: The coefficient of x^2 is different from the coefficient of x.

Step 4: Identify the constant. The constant is the whole number without a pronumeral.

Student practice

For each expression, state:

i) the number of terms in the expression

ii) the coefficient of x

iii) the constant.

a. $3x + 2ab - 7$

b. $6x + 2 - 4x^2$

c. $6xy + 6x - 2yz + 3$

d. $-4xz - 6yz + 3xy - 5 - 2x$

WE 2 Simplifying algebraic expressions

Simplify each expression using the correct algebraic notation.

a. $(3 + a) \div (b - 7)$

Working

$(3 + a)$ is grouped together.

$(b - 7)$ is grouped together.

$$\frac{3 + a}{b - 7}$$

Thinking

Step 1: Using order of operations, we know the brackets indicate values are grouped together.

Step 2: The division sign indicates a fraction where the denominator of the fraction becomes all the terms after the division sign.

b. $x \times 4 \times y \times 11 \times z \div t \times 4$

Working

$4 \times 11 \times 4 \times x \times y \times z \div t$

$176xyz \div t$

$$\frac{176xyz}{t}$$

Thinking

Step 1: Rearrange the values in the multiplications so that the numbers come before the pronumerals.

Step 2: Rewrite the terms without the multiplication signs.

Step 3: The division sign indicates a fraction where the denominator of the fraction becomes all the terms after the division sign.

Student practice

Simplify each expression using correct algebraic notation.

a. $(2 + r) \div (s - 5)$

b. $f \times 3 \times g \times 7 \div (15 \times h \times e)$

c. $2a \div 4 + 3 \times 4b$

d. $\frac{x}{3} + 7 + \frac{2x}{3}$

WE 3 Writing algebraic equations and statements

Answer each of the questions.

a. Write an equation to represent the statement: 7 less than twice n is 33.

Working

$2 \times n$ or $2n$

$2n - 7$

$2n - 7 = 33$

Thinking

Step 1: 'Twice n' means we multiply n by 2.

Step 2: '7 less than' means we subtract 7.

Step 3: 'Is 33' means that the equation equals 33.

b. Write a statement describing the equation $\frac{a}{3} + 2 = 5$.

Working

One third of a...

Two more than one third of a...

Two more than one third of a equals five.

Note: There is more than one way to write this.

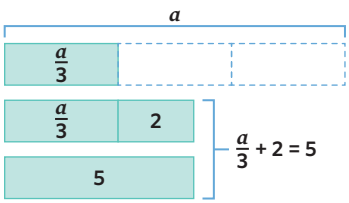
Thinking

Step 1: The first term is a fraction and involves a division. We can say that we have taken a third of a.

Step 2: The addition is two more than.

Step 3: The first two terms together have a value of 5.

Visual support



Student practice

a. Write an equation to represent the statement: 2 more than one fifth of p is 9.

b. Write an equation to represent the statement: The sum of 4 and 3 divided by the product of $2x$ and 5 equals 9.

c. Write a statement describing the equation $x^2 - 4 = 0$.

d. Write a statement describing the equation $3 + 2 - (4x \div 5) = 27$.

WE 4 Applying algebra to real world situations

A bag of lollies contains p lollies. Ryan bought 3 bags and then ate 8 of the lollies. He now has 19 lollies left. Write an equation to represent this scenario.

Working

$3p$

$3p - 8$

$3p - 8 = 19$

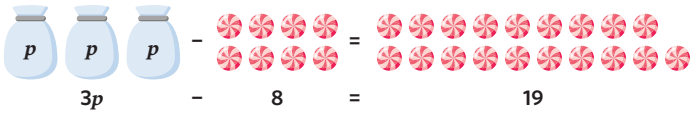
Thinking

Step 1: 3 bags of lollies gives us 3 groups of p.

Step 2: When Ryan eats 8 lollies that means he has 8 less than $3p$ lollies.

Step 3: This is then equal to 19.

Visual support



Student practice

Write an equation based on each scenario.

a. Veronica had 3 boxes of toy soldiers each containing s soldiers. She also had 11 displayed on her shelf, giving her a total of 47 soldiers.

b. Kitty's pet store had c cats and Doug's pet store had d dogs. Altogether they had 25 dogs and cats.

c. Elian spent 24 dollars on 3 slurpees (s) and 5 donuts (d) at the 7-Eleven after school.

d. Alex is 4 years older than Tom (t) and together their ages equal 74.

4B Activities and questions

STARTER TASKS

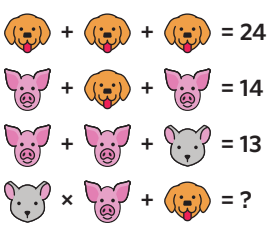
Odd spot

Cuban artist Felix Gonzales-Torres laid out 650 kg of chocolate caramels in London's Serpentine Gallery. Visitors can eat the exhibit named 'Untitled (Placebo)', which is refilled daily. If he buys chocolate caramels in 5 kg bags, what is an equation for the number of bags, b, he will need to set up the exhibition?

A. $5b = 650$ **B.** $\frac{5}{b} = 650$

Puzzle

Solve the following puzzle.

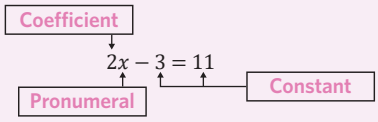


Understanding worksheet

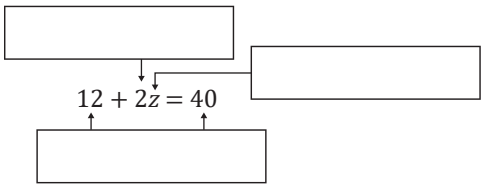
1. Fill in the boxes using the following words.

- Pronumeral
- Coefficient
- Constant

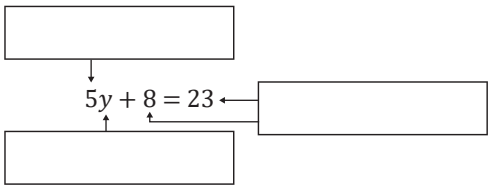
Example



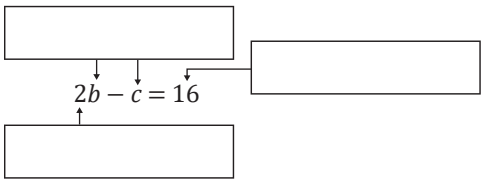
a.



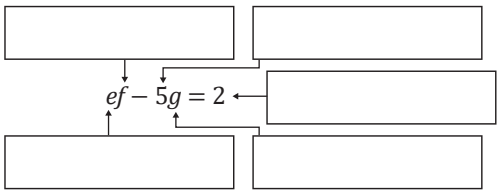
b.



c.



d.



2. If b represents the number of apples in each box, write an equation for each image.

Example

a.

b.

c.

d.

3. Fill in the blanks by using the words provided.

pronumeral | constants | equation | coefficient

In the _____ $2x + 4 = 12$, the x is the _____, the 2 is the _____. There are also two _____ in the equation. These are the numbers 4 and 12.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 **4.** For each expression, state:

i) the number of terms in the expression

ii) the coefficient of x

iii) the constant.

a. $3x + xy - 7$ **b.** $3x^2 - 2x$ **c.** $2w + x + 3$ **d.** $4f - 3x + 6$

e. $3 \div w + 2x$ **f.** $5u - 50 + x$ **g.** $4x - (d \div 4) + 8$ **h.** $3x - 2 \div 4y$

WE2 **5.** Simplify each expression using correct algebraic notation.

a. $c \times y$ **b.** $m \times b$

c. $5 + f \times g$ **d.** $a + (b \times 5) \times (c \times 4)$

e. $c \div h$ **f.** $x + 5 \div 15$

g. $(100 - b) \div (b \times 5 \times c)$ **h.** $a \times d \times 3 \times (2 \div p) \times e \times 6$

WE3a **6.** Write an equation to represent each of these statements.

a. 4 more than p is 11. **b.** 8 less than t is equal to 14.

c. Half of y is 13. **d.** 45 is 3 times greater than x.

e. The value of a is 4 times greater than the sum of 8 and 12. **f.** A quarter of z is 3 larger than 12.

g. Half the sum of b squared and 5 is equal to 9. **h.** 7 less than the product of 3 and c is equal to d cubed less than a tenth of 3.

WE3b **7.** Write a statement describing each of these equations.

a. $f + 3 = 7$ **b.** $m - 4 = 5$ **c.** $k + k + k = 90$ **d.** $2z = 12$

e. $\frac{a}{3} = 4$ **f.** $3s + 7 = 13$ **g.** $x^2 = 36$ **h.** $\frac{g + 2}{5} = 4$

WE4 **8.** Write an equation based on each scenario.

a. Each class has x students in it. There are six classes with a total of 132 students.

b. 12 students shared b books equally. Each student received 9 books.

c. Ronnie has a number of basketball cards (r). The number of basketball cards Mick has (m) is 4 more than Ronnie.

d. Nikiah has a collection of crystals (c). Rhaya's (r) collection is triple the size of Nikiah's.

e. There are p lollies in a bag. Gilbert bought 7 bags and ate 5 lollies. He then has 128 lollies left.

f. Simon had d stickers then collected 4 more before sharing them between 3 friends. Each friend got 9 stickers each.

g. Lola placed c cookies on a tray for baking. Charlie added three more cookies to her tray. They then tripled the number of trays to make 32 cookies.

h. Pete had r raffle tickets that he shared equally between himself and 4 friends. He then bought 6 more raffle tickets and ended up with 8 tickets.

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Julio is 4 and his brother Fletcher is 12. Which equation for Julio's age will still be correct in a year's time?

A. *Julio's age* $= 3 \times$ *Fletcher's age*

B. *Julio's age* $=$ *Fletcher's age* $\div 3$

C. *Julio's age* $=$ *Fletcher's age* $+ 8$

D. *Julio's age* $=$ *Fletcher's age* $- 8$

10. Sarah is given $\$d$ for her first week of pocket money. For another 4 weeks, she gets the same amount and ends up with exactly enough to buy the latest Aquaman comic for $20. Write an equation to represent this situation.

11. When Sarah goes into the comic book store to buy her $20 comic she sees a copy of a first edition Magnus Robot Fighter #40 from 1975. It is the last volume she needs to complete her collection, but notices that it is selling for $38 400. If the number of $20 comic books she needs to trade for the 'Magnus Robot Fighter' comic book is c, write an equation to represent this situation.

12. Tony pays 65 cents to make an international phone call, plus an extra 15 cents per minute. He paid a total of $2.45 for the call. Write an equation to represent this, using m as the length of the phone call in minutes.

13. The cost of a taxi is $c = 2d + 5$ where c is the total cost in dollars and d is the distance travelled in kilometres. How much does the cost change for each additional kilometre traveled?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Tashi works two part-time jobs. She earns $\$a$ per hour at the supermarket, and earns $\$b$ per hour babysitting. In one week her earnings were $5a + 3b = 86$ dollars.
    a. How many hours did she work in total?
    b. The next week she earned $3a + 5b$ dollars which worked out to be more than the previous week. How did Tashi earn more if the values of a and b haven't changed?
    c. If Tashi earns $5a + 3b = 86$ dollars per week in February and there are 4 weeks in February. Write an equation to show how much she would earn in February.
    d. If Tashi earns more money per hour babysitting, why might she choose to also work at the grocery store?

15. Nathalia was selling tickets for the school play.
    a. She sold 10 more adult tickets (a) than children tickets (c). Write an equation to represent the number of adult tickets sold.
    b. She sold twice as many senior tickets (s) as children tickets (c). Write an equation to represent the number of senior tickets sold.
    c. Write an equation for the total number of tickets (t) sold in terms of a, c, and s.
    d. Adult tickets cost $5, children's tickets cost $2 and senior tickets cost $3. Write an equation to represent the total amount of money Nathalia made from ticket sales (r).
    e. Do you think it's fair that adult tickets cost more than children tickets?

Extra spicy

16. A 3-digit number has the digit a in the hundreds position, the digit 7 in the tens position, and the digit b in the units position. If a is a whole number, which algebraic expression gives the value of the 3-digit number?
    A. $a \times 7 \times b$
    B. $7ab$
    C. $a + 7 + b$
    D. $100a + 7 + b$
    E. $100a + 70 + b$

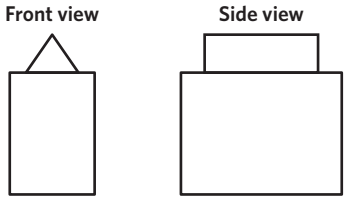
17. Kai says he's a magician and can read your mind. He asks the following:
    'Pick a number between 0 and 10 but this number cannot be 0 or 10. Multiply that number by 9. Add the digits of that number together. Now add 4. Your answer is 13.'
    Everyone is surprised that Kai can read minds. Explain how this 'magic' trick works.

18. The expression $4a + 3 + a$ is equivalent to $5a + 3$. Write four expressions with 3 terms that are also equivalent to $5a + 3$.

19. In the United States of America, temperature is measured in Fahrenheit (°F). In the rest of the world, temperature is measured in Celsius (°C). The equation to get from Celcius to Farhenheit is $C = \frac{5}{9}(F - 32)$. Find a temperature in °C where the Celsius reading is the same as Farhenheit.

Remember this?

20. Two blocks were stacked with the following front and side view:



Which of these shows the correct view from above?

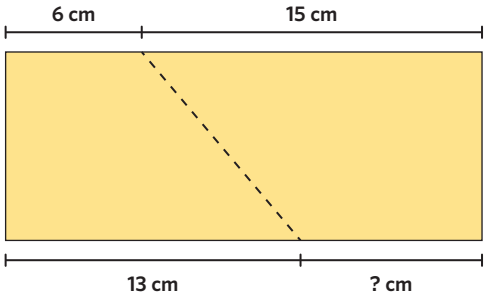
A. 

B. 

C. 

D. 

21. A rectangle has been separated into two pieces, as shown.



What is the missing length?

22. There were 5000 people at a carnival. Two-fifths of the people went on the Ferris wheel. How many people didn't go on the Ferris wheel?

4C Substituting values into equations

Pronumerals are used to represent an unknown value. We can replace pronumerals with different values in order to model situations or make calculations. Substitution often involves replacing pronumerals with given values. Below are some examples where substitution can be applied.

- A dose of cough syrup for dogs is calculated per weight in kilograms. The instructions say to drink 5 mL per kilogram of weight. If my dog weighs 7 kilograms, how many mL should I pour in one dose?
- When I deliver newspapers in the morning I get paid $0.50 for each paper plus a flat rate of $12. How much do I get paid when I deliver 200 newspapers?

Learning intentions

Students will be able to:

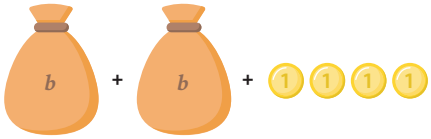
- substitute values into algebraic expressions and evaluate
- substitute values into algebraic equations to check if they are balanced
- use formulas by substituting in values.

Key terms and definitions

- **Substitution** is the process of replacing a pronumeral with a given value.
- **Evaluate** means to carry out mathematical operations in the correct order to find a value of an expression or equation.
- A **true equation** is when the left-hand side and right-hand side of an equation are equal.
- A **false equation** is when the left-hand side and right-hand side of an equation are not equal.
- A **formula** is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.
- The **order of operations** is a set of rules that tells us what calculations are performed first in a number sentence or equation. The order often follows BIDMAS, which stands for Brackets, Indices, Division, Multiplication, Addition and Subtraction.

Key ideas

1 Pronumerals are letters or symbols in an expression that represent a given value. When we replace these pronumerals with values we can evaluate the expression.



$b + b + 4$

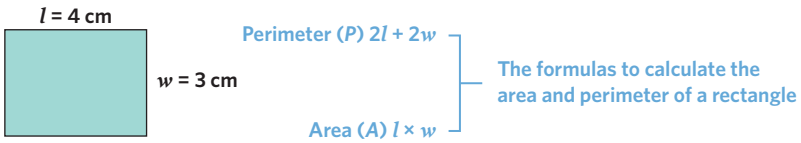
If $b = 3$:

$3 + 3 + 4 = 10$

2 We can check if the value of a pronumeral is correct by using substitution. The value of the pronumeral is correct when the substituted value results in both sides of the equation being equal. The value is incorrect when the substituted value results in both sides of the equation being unequal.

In the equation $2d + 3 = 11$, we can check if $d = 4$.	In the equation $2d + 3 = 11$, we can check if $d = 5$.
d | d | 3 = 11 ; $2d + 3 = 11$ Check by substituting 4 in for d 4 | 4 | 3 = 11 ; $4 + 4 + 3 = 11$, $11 = 11$	d | d | 3 = 11 ; $2d + 3 = 11$ Check by substituting 5 in for d 5 | 5 | 3 ≠ 11 ; $5 + 5 + 3 = 11$, $13 \neq 11$
d must equal to 4.	d cannot be 5.

3 Formulas are rules that are used to describe an equivalent mathematical situation. When we have a formula, we can use substitution of known quantities to find the value of unknown quantities in the formula.



$P = 2 \times l + 2 \times w$

Replace l with 4 and w with 3.

$P = 2 \times 4 + 2 \times 3$

$= 8 + 6$

$= 14$ cm

The perimeter of this rectangle is 14 cm.

$A = l \times w$

Replace l with 4 and w with 3.

$A = 4 \times 3$

$= 12$ cm^2

The area of this rectangle is 12 cm^2.

Worked examples

WE 1 Substituting one pronumeral

Substitute $a = 10$ to evaluate the expressions.

a. $a + 7$

Working

$a + 7 = 10 + 7$

$= 17$

Thinking

Replace the pronumeral a with 10 and simplify by performing all calculations.

Visual support

b. $4a - 5$

Working

$4a - 5 = 4 \times a - 5$

$= 4 \times 10 - 5$

$= 40 - 5$

$= 35$

Thinking

Step 1: Rewrite the expression $4a$ as $4 \times a$.

Step 2: Replace the pronumeral a with 10 and simplify using order of operations.

c. $\frac{a}{5} + 25$

Working

$\frac{a}{5} + 25 = \frac{10}{5} + 25$

$= 2 + 25$

$= 27$

Thinking

Replace the pronumeral a with 10 and simplify using order of operations.

Student practice

Substitute $c = 8$ to evaluate the expressions.

a. $c + 50$ **b.** $3c - 7$ **c.** $\frac{c}{4} + 9$ **d.** $\frac{5c}{2}$

WE 2 Substituting into a formula

a. The perimeter (P) of a rectangle can be found using the formula $P = 2l + 2w$, where l is the length and w is the width of a rectangle. Find the perimeter if $l = 5$ cm and $w = 2$ cm.

Working

$P = 2l + 2w$

$= 2 \times l + 2 \times w$

$= 2 \times 5 + 2 \times 2$

$= 10 + 4$

$= 14$ cm

Thinking

Step 1: Rewrite the formula $P = 2l + 2w$ as $P = 2 \times l + 2 \times w$.

Step 2: Replace the pronumeral l with 5 and w with 2, and simplify using order of operations. Remember to include units.

b. The area (A) of a triangle can be found using the formula $A = \frac{bh}{2}$, where b is the base length and h is the height of a triangle. Find the area of a triangle if $b = 4$ cm and $h = 6$ cm.

Working

$A = \frac{bh}{2}$

$= \frac{b \times h}{2}$

$= \frac{4 \times 6}{2}$

$= \frac{24}{2}$

$= 12 \text{ cm}^2$

Thinking

Step 1: Rewrite the formula $A = \frac{bh}{2}$ as $A = \frac{b \times h}{2}$.

Step 2: Replace the pronumeral b with 4 and h with 6, and simplify using order of operations. Remember to include units.

Student practice

The perimeter (P) of a rectangle can be found using the formula $P = 2l + 2w$, where l is the length and w is the width of a rectangle.

Find the perimeter if

a. $l = 6$ cm and $w = 3$ cm.

b. $l = 4\frac{1}{2}$ cm and $w = 3$ cm.

The area (A) of a triangle can be found using the formula $A = \frac{bh}{2}$, where b is the base length and h is the height of a triangle.

Find the area of a triangle if:

c. $b = 10$ cm and $h = 4$ cm.

d. $b = 9\frac{1}{3}$ cm and $h = 3$ cm.

WE 3 Substituting to check if equations balance

Substitute $j = 2$ and $k = 5$ and state whether each equation is true or false.

a. $3j + 4k = 26$

Working

$3j + 4k = 26$

$3 \times j + 4 \times k = 26$

LHS $= 3 \times 2 + 4 \times 5$

$= 6 + 20$

$= 26$

RHS $= 26$

The equation is true as both sides of the equation equal 26.

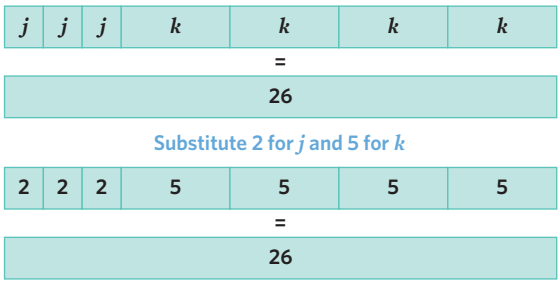
Thinking

Step 1: Rewrite the expression $3j$ as $3 \times j$ and the expression $4k$ as $4 \times k$.

Step 2: Replace the pronumeral j with 2 and k with 5 and calculate both the left-hand side (LHS) and right-hand side (RHS) of the equation.

Step 3: Compare both sides of the equations. If they are equal, then the equation is true.

Visual support



$3j + 4k = 26$

Substitute 2 for j and 5 for k

$3 \times 2 + 4 \times 5 = 26$
$26 = 26$

b. $2jk = 11j + 44$

Working

$2jk = 11j + 44$

$2 \times j \times k = 11 \times j + 44$

$\text{LHS} = 2 \times 2 \times 5$
$= 20$

$\text{RHS} = 11 \times 2 + 44$
$= 22 + 44$
$= 66$

The equation is false as both sides have different values.

Thinking

Step 1: Rewrite the expression $2jk$ as $2 \times j \times k$ and the expression $11j$ as $11 \times j$.

Step 2: Replace the pronumeral j with 2 and k with 5 and simplify both sides using order of operations.

Step 3: Compare both sides of the equations. If they are equal, then the equation is true.

Student practice

Substitute $x = 4$ and $y = 6$ and state whether each equation is true or false.

a. $4y + 3x = 30$ **b.** $y + x = y + x$ **c.** $3xy - 7 = 4x + 40$ **d.** $2yx = \frac{24x}{2}$

WE 4 Substituting multiple pronumerals

If $m = 2$, $v = 4$ and $t = 3$, evaluate:

a. $5v^2 + t$

Working

$5v^2 + t = 5 \times v \times v + t$

$= 5 \times 4 \times 4 + 3$
$= 5 \times 16 + 3$
$= 83$

Thinking

Step 1: Rewrite the expression $5v^2$ as $5 \times v \times v$. Remember that $v^2 = v \times v$.

Step 2: Replace the pronumeral v with 4 and t with 3 and solve using order of operations.

b. $m^3 - \sqrt{v} + t$

Working

$m^3 - \sqrt{v} + t = m \times m \times m - \sqrt{v} + t$

$= 2 \times 2 \times 2 - \sqrt{4} + 3$
$= 8 - 2 + 3$
$= 9$

Thinking

Step 1: Rewrite the expression m^3 as $m \times m \times m$.

Step 2: Replace the pronumerals m with 2, t with 3 and v with 4, and solve using order of operations.

Student practice

If $x = 1$, $y = 4$ and $z = 5$, evaluate:

a. $4z^2 - y$ **b.** $5xz + y$ **c.** $\sqrt{y} + z^2 - x$ **d.** $z^3 + \frac{25}{x} - \frac{3}{y}$

4C Activities and questions

STARTER TASKS

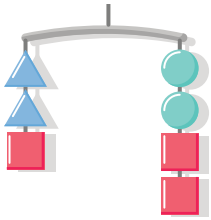
Odd spot

In China's Xinjiang region, government officials found that introducing 16 000 ducks to the habitat was faster and more environmentally friendly than using pesticides to fight a plague of locusts. If a duck can eat 200 locusts a day, how many locusts in total can be eaten by the 16 000 ducks in one day?

A. 80 **B.** 3.2 million

Puzzle

Consider the following mobile:



a) If the weight of each shape is a whole number of kilograms that is less than 5 kilograms, what is the weight for each shape that would keep the mobile balanced?

b) If the weight of each shape was still less than 5 kilograms, but did not need to be a whole number of kilograms, suggest some possible ways that we can now solve this puzzle.

c) If the weight of each shape is greater than or equal to 5 kilograms, what are some possible weights for each shape that would keep the mobile balanced?

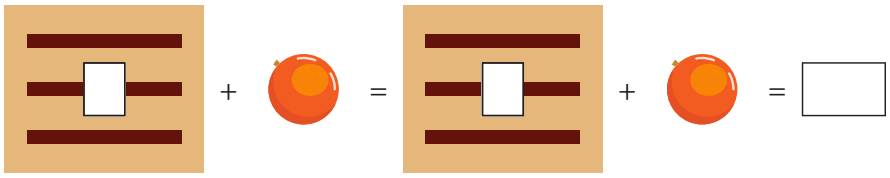
Understanding worksheet

1. For each part, fill in the missing boxes and use substitution to evaluate.

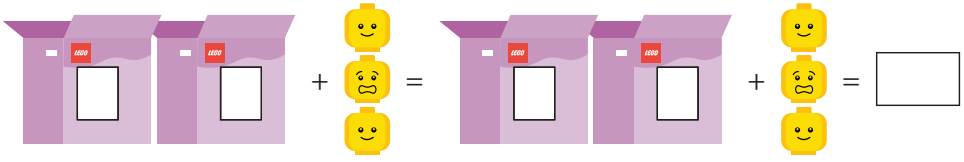
Example

The number of coins in each bag is c. How many coins are there altogether if $c = 10$?

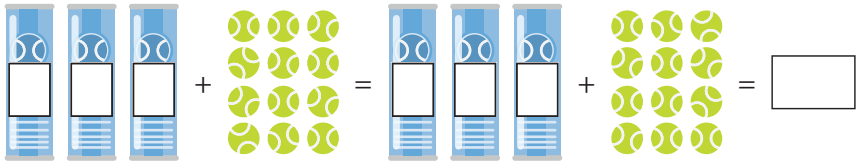
a. The number of oranges in each box is r. How many oranges are there altogether when $r = 5$?



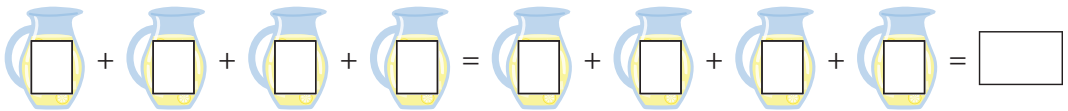
b. The number of lego pieces in each box is l. How many lego pieces are there altogether when $l = 6$?



c. The number of tennis balls in each tin is t. How many tennis balls are there altogether when $t = 4$?



d. The number of cups that each lemonade jug holds is c. How many cups of lemonade are there altogether if $c = 5$?



2. Substitute $f = 7$ and $b = 3$ to evaluate each bar model.

Example

$f + b + 2$

f	b	2
7	3	2
12		

a. $3b + 2$

b	b	b	2
		3	2

b. $2b + f + 1$

b	b	f	1
		7	1

c. $2f + b + 1$

f	f	b	1
			1

d. $3f + 2b$

f	f	f	b	b

3. Fill in the blanks by using the words provided.

value area expression formula substitution unknown

[ ] involves replacing a pronumeral in an [ ] or equation with a given [ ]. A [ ] is a rule containing more than one [ ] value. For example we can use formulas to find the _____ of different shapes.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 **4.** Substitute $p = 10$ to evaluate the expressions.

a. $p + 2$ **b.** $p + p + p$ **c.** $9p$ **d.** $5p - 6$

e. $100 - 4p$ **f.** $\frac{p}{5} + 3$ **g.** $4 - \frac{10}{p}$ **h.** $3p + 15 + p$

WE2a **5.** The perimeter (P) of a rectangle can be found using the formula $P = 2l + 2w$, where l is the length and w is the width of a rectangle. Find the perimeter if:

a. $l = 10$ cm and $w = 5$ cm **b.** $l = 8$ mm and $w = 3$ mm

c. $l = 16$ km and $w = 4$ km **d.** $l = 120$ m and $w = 210$ m

e. $l = 6.5$ m and $w = 3$ m **f.** $l = 20\frac{1}{3}$ m and $w = 5\frac{1}{3}$ m

WE2b **6.** The area of a triangle (A) can be found using the formula $A = \frac{bh}{2}$, where b is the base length and h is the height of a triangle. Find the area of a triangle if:

a. $b = 5$ cm and $h = 2$ cm
b. $b = 10$ cm and $h = 5$ cm
c. $b = 120$ m and $h = 60$ m
d. $b = 15$ mm and $h = 6$ mm
e. $b = 12\frac{1}{2}$ cm and $h = 6$ cm
f. $b = 4.5$ km and $h = 4$ km

WE3 **7.** Substitute $x = 3$ and $y = 5$ and state whether each equation is true or false.

a. $x + y + x = 11$
b. $y - x = 8$
c. $2x + 3y = 21$
d. $4xy = 65$
e. $3yx = 4y + 7$
f. $\frac{4y}{10} + 4x = 14$
g. $5x + 3y = 3x + 5y$
h. $\frac{8}{x + y} + 4x = \sqrt{144}$

WE4 **8.** If $r = 4$, $s = 5$ and $t = 6$, evaluate:

a. $3r^2$
b. $2rs - t$
c. $r^3 - 3s - t$
d. $t^2 + s^2 - \sqrt{r}$
e. $\frac{12}{t} + 3rs$
f. $\frac{28}{s + t - r}$
g. $2t^2 - \sqrt{r} - \frac{100}{s}$
h. $\frac{15 - s}{1} + \frac{t}{2} - \frac{2r}{4}$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Mr Goldenfold starts the school year with 7 pencils, and buys 5 boxes of new pencils. If there are 12 pencils in a box, how many pencils does Mr. Goldenfold have?

10. NBA staticians are trying to check that Yao Ming's height in inches and centimetres is listed correctly in NBA player records. Yao Ming is listed as 90 inches tall and 230 centimetres. Use the statisticians' formula $height\ (\text{cm}) = 2\frac{1}{2} \times height\ (\text{inches})$ to check that the NBA player records are correct.

11. If Maria wants to make the most money for 6 hours of babysitting, which job should she choose? Jack's parents pay a flat rate of \$10 plus \$5 for each hour of work and Russell's parents pay \$3 per hour plus an additional \$20.

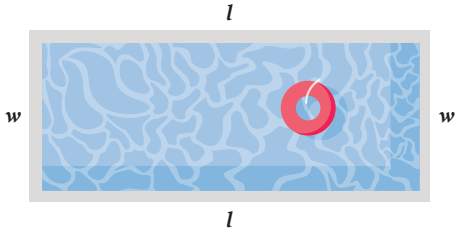
12. Ron and Sarah's comic books increase in value over time. The value of a comic can be calculated by using the formula $value\ (\$) = 20 + 2y$ where y is the age of the comic in years. Ron's oldest comic is 52 years old, while Sarah's oldest comic is 39 years old. What is the difference in value between Ron and Sarah's oldest comics?

13. Kathy is baking cookies and needs to set the oven to 200 degrees celsius (°C). Her oven's temperature can only be set using a fahrenheit (°F) scale. Her recipe book provides the formula $°F = \frac{9}{5}°C + 32$ for converting °C to °F. Use this formula to help Kathy calculate what temperature she should set her oven.

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c)	All

14. Lucy is designing a rectangular pool to put in her backyard.



a. If the planned length of the pool is 12 m and the width is 6 m, what is the perimeter of the pool?
b. Using the area of a rectangle formula $A = l \times w$, calculate the area of the backyard that Lucy's pool will take up.
c. Lucy's neighbour Phil has a square pool with a side length of 9 m. Lucy thinks that the perimeter of her pool is greater than Phil's pool. Phil thinks that his pool's perimeter is greater. Who is correct?
d. Lucy wants her pool to have an area of 108 m^2 while maintaining the side length of 12 m. By how much does she need to extend the width? Show your working.
e. If Lucy does not want to have a rectangular shaped swimming pool, what other designs could she have that would cover an area of 108 m^2? Explain whether your design is realistic or not?

15. Kathy is preparing a Thanksgiving dinner at her friend Bonnie's house. Bonnie is at work and sent Kathy a text message with instructions on how long to cook the roast turkey for.

    a. How long will it take to cook an 8 kilogram turkey on the fan forced oven setting?

    b. How much longer will the 8 kg turkey take to cook using the conventional setting?

    c. Kathy bought a 7 kg turkey and cooked it using the conventional setting. Before serving the turkey at 7:30 pm, she took the turkey out to rest for half an hour. What time did she put the turkey in the oven?

    d. Kathy struggled to understand how to calculate the cooking time using Kathy's formulas. Write a message that Bonnie could send to Kathy that is easier to understand.

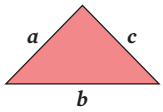
Extra spicy

16. For what value of m do both expressions equal the same whole number?
    $\frac{4 + 3m}{8}$ and $\frac{1 + 2m}{5}$

17. The sum of seven consecutive numbers is 98. If the smallest number is equal to n, the sequence of numbers can be written as $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6)$.
    What is the value of $n + 4$?

18. If $\frac{k^2 + 6}{5}$ is equal to a whole number, find all the possible values of k that are less than 15 and explain why these numbers work.

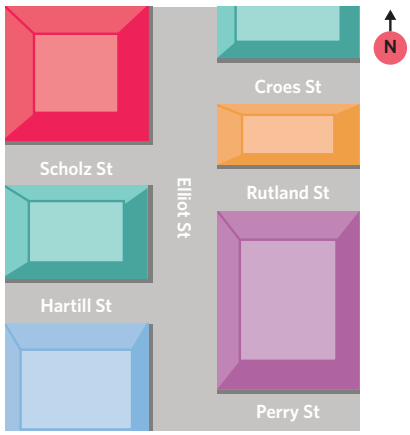
19. The formula to find the area of a triangle given all 3 side lengths is $A = \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$. Using these formulas, what is the area of a triangle where $a = 6$ m, $b = 8$ m and $c = 10$ m?



Remember this?

20. Denzel walked west along Croes St and turned left onto Elliot St. What was the second street he passed on his right?

    A. Scholz St
    B. Rutland St
    C. Hartill St
    D. Perry St



21. Which statement is true for the product of 95×0.1?

    A. The product is less than 95.
    B. The product is equal to 95.
    C. The product is greater than 95.

22. Max rode his bike 16 kilometres in 20 minutes.
    How long will it take Max to ride 20 kilometres at the same speed?

    A. 40 minutes
    B. 24 minutes
    C. 24 kilometres
    D. 25 minutes

4D Solving one-step equations

When using equations to find unknown values, we use the process of solving. To solve one-step equations, we apply an inverse operation to isolate the pronumeral and find its value. Below are some examples where solving one-step equations can be applied.

- I am completing a reading challenge that requires me to read 30 books. If I have read 7 books so far, how many more books (b) do I need to read in order to complete the challenge? $7 + b = 30$
- My sisters age (s) is twice my age. If I am 5 years old, how old is my sister? $\frac{s}{2} = 5$
- I bought 8 lollipops for $2 from the candy shop. How much did each lollipop cost (c)? $8c = 2$

Learning intentions

Students will be able to:

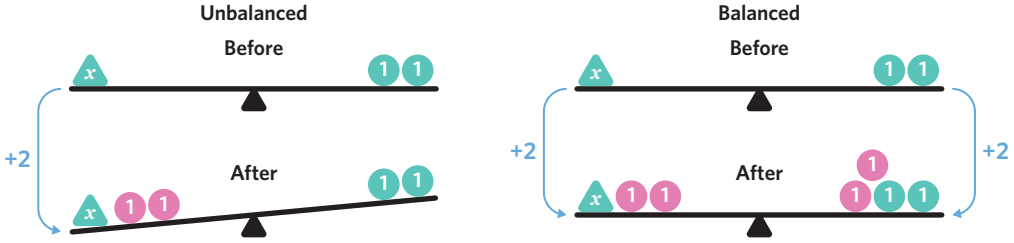
- keep equations balanced by performing the same operation on both sides
- identify inverse operations that can be used to solve equations
- solve one-step equations.

Key terms and definitions

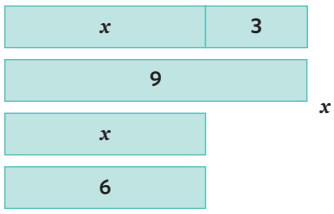
- **Solving equations** is a process used to find the value of the unknown by performing a series of inverse operations.
- **Inverse operations** are mathematical operations that undo each other. The inverse of multiplication is a division and the inverse of division is a multiplication; the inverse of addition is a subtraction and the inverse of subtraction is an addition.
- A **one-step equation** is an equation that can be solved by applying a single inverse operation.

Key ideas

1 To keep an equation balanced, any operation applied to one side of the equation (left-hand side or right-hand side) must also be applied on the other side of the equation.



2 We can use bar models to help us solve one-step equations.



$x + 3 = 9$

$x + 3 - 3 = 9 - 3$

$x = 6$

3 To isolate and find the value of the unknown in an equation we can use inverse operations.

The inverse of multiplication is division.

$7x = 56$ ÷7

$x = 8$ ÷7

To isolate x, divide both sides by 7.

The value of x is 8.

Worked examples

WE 1 Identifying the operation applied to the pronumeral

State the operation being applied to the pronumeral.

a. $x + 3$

Working

The operation being applied to x is $+3$.

Thinking

Identify the pronumeral (x) in this expression.
To get $x + 3$, we have added 3 to x.

Visual support

b. $\frac{k}{5}$

Working

The operation being applied to k is $\div 5$.

Thinking

Identify the pronumeral (k) in this expression.
To get $\frac{k}{5}$, we have divided k by 5.

Student practice

State the operation being applied to the pronumeral.

a. $x - 8$ **b.** $k \times 3$ **c.** $p \div 7$ **d.** $9 + z$

WE 2 Identifying the inverse operation used to isolate the pronumeral

Identify the operation being applied to the pronumeral and the inverse operation that could be used to isolate the pronumeral.

a. $x + 4 = 6$

Working

The operation being applied to x is $+4$.

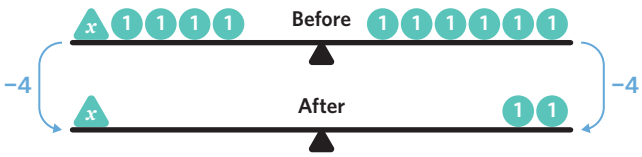
The inverse operation that could be used to isolate x is -4.

Thinking

Step 1: Identify the pronumeral (x) in the equation.
To get $x + 4$ we add 4 to x.

Step 2: The inverse of an addition is a subtraction.
The inverse of $+4$ is -4.

Visual support



b. $\frac{m}{100} = 18$

Working

The operation being applied to m is $\div 100$.

The inverse operation that could be used to isolate m is $\times 100$.

Thinking

Step 1: Identify the pronumeral (m) in the equation.
To get $\frac{m}{100}$ we divide m by 100.

Step 2: The inverse of a division is a multiplication.
The inverse of $\div 100$ is $\times 100$.

c. $9v = 72$

Working

The operation being applied to v is $\times 9$.

The inverse operation that could be used to isolate v is $\div 9$.

Thinking

Step 1: Identify the pronumeral (v) in the equation. To get $9v$ we multiply 9 by v.

Step 2: The inverse of a multiplication is division. The inverse of $\times 9$ is $\div 9$.

Student practice

Identify the operation being applied to the pronumeral and the inverse operation that could be used to isolate the pronumeral.

a. $x + 99 = 121$ **b.** $c - 11 = 76$ **c.** $8i = 64$ **d.** $\frac{y}{7} = 6$

WE 3 Solving one step equations

Solve the equations.

a. $x - 3 = 12$

Working

The operation is -3.

$x - 3 + 3 = 12 + 3$

$x = 12 + 3$

$x = 15$

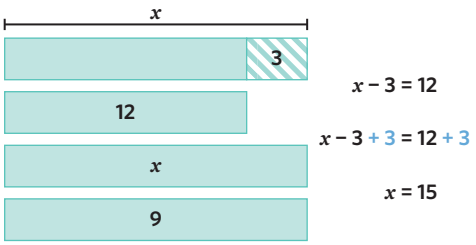
Thinking

Step 1: Identify the operation being applied to the pronumeral x.

Step 2: Isolate x by applying the inverse operation $+3$. To keep the equation true apply the same operation on both sides.

Step 3: Simplify both sides of the equation to find the solution for x.

Visual support



b. $\frac{c}{5} = 10$

Working

The operation is $\div 5$.

$\frac{c}{5} \times 5 = 10 \times 5$

$c = 10 \times 5$

$c = 50$

Thinking

Step 1: Identify the operation being applied to the pronumeral c.

Step 2: Isolate c by applying the inverse operation $\times 5$. To keep the equation true apply the same operation on both sides.

Step 3: Simplify both sides of the equation to find the solution for c.

c. $7p = 28$

Working

The operation is $\times 7$.

$7p \div 7 = 28 \div 7$

$p = 28 \div 7$

$p = 4$

Thinking

Step 1: Identify the operation being applied to the pronumeral p.

Step 2: Isolate p by applying the inverse operation $\div 7$. To keep the equation true apply the same operation on both sides.

Step 3: Simplify both sides of the equation to find the solution for p.

Student practice

Solve the equations.

a. $x + 5 = 22$ b. $5c = 10$ c. $p - 11 = 33$ d. $\frac{z}{3} = 15$

4D Activities and questions

STARTER TASKS

Odd spot

Fried insects have long been sold in markets in Asia. While they can be seen as a strange food, they are an excellent source of protein and nutrition. Skye Blackburn runs The Edible Bug Shop which is a store in Australia that sells edible insects.

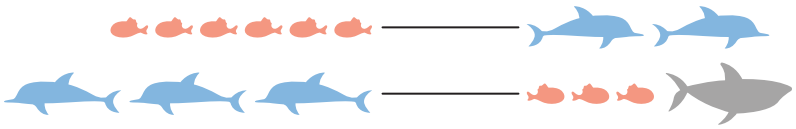
One of the items that The Edible Bug Shop sells is a 10 g packet of ant candy made from tyrant ants. If Andy wants to buy 500 g of Ant candy, how many packets does he buy?

A. 50 **B.** 5000

Image: Meredith O'Shea/The Age

Puzzle

The weight of 6 fish equals the weight of 2 dolphins. The weight of 3 dolphins equals the weight of 3 fish and a shark.



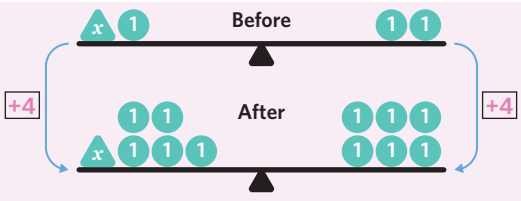
a) How many fish would equal the weight of a shark?

b) In the image below, which side is heavier?

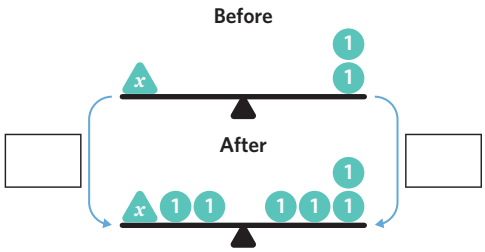
Understanding worksheet

1. Write the operation that has been applied to both sides of the scale to keep it balanced.

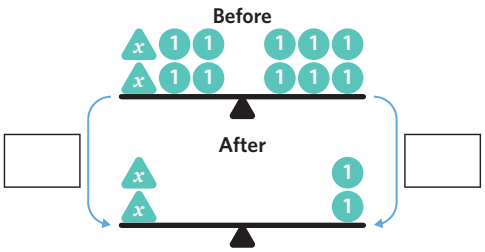
Example



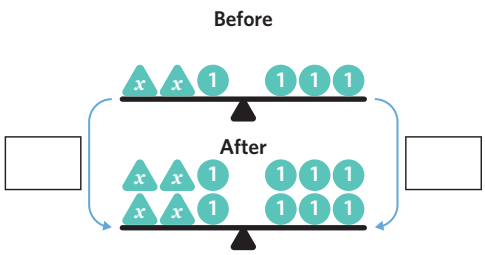
a.



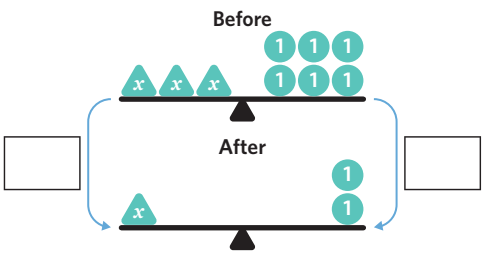
b.



c.

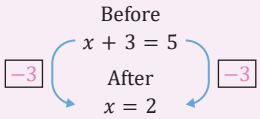


d.

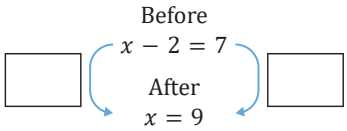


2. Write the operation that has been applied to both sides of the equation.

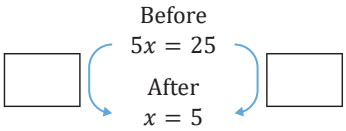
Example



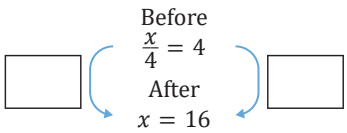
a.



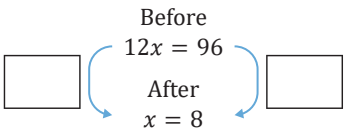
b.



c.



d.



3. Fill in the blanks by using the words provided.

division | addition | inverse | unknown | multiplication | subtraction

Algebraic one-step equations are equations that can be solved by applying one operation. To solve a one-step equation, we identify and isolate the _____ variable using _____ operations. The inverse of multiplication is _____ and the inverse of division is _____; the inverse of _____ is subtraction and the inverse of _____ is addition.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 4. State the operation being applied to the pronumeral.

a. $x + 10$ b. $x - 55$ c. $5x$ d. $\frac{k}{4}$

e. $\frac{c}{6}$ f. $m - 300$ g. $37 + k$ h. $2 \times c \times 3$

WE2 5. In each equation in the given table, identify the operation being applied to x and the inverse operation that could be used to isolate x.

	Equation	Operation being applied to x	Inverse operation used to isolate x
a.	$x + 3 = 5$		
b.	$5x = 10$		
c.	$\frac{x}{4} = 8$		
d.	$x - 12 = 7$		
e.	$3 + x = 1$		
f.	$7x = 21$		
g.	$\frac{x}{15} = 2$		
h.	$x^2 = 16$		

WE3 6. Solve each equation.

a. $y + 3 = 8$ b. $y - 7 = 15$ c. $24 = 2x$ d. $5x = 75$

e. $8 = \frac{x}{3}$ f. $\frac{c}{2} = 24$ g. $\frac{9}{3} = \frac{c}{3}$ h. $a^2 = 121$

7. For each of the given situations, write an equation and then solve for the unknown.

a. When I add 2 to a number (n), the answer is 9.

b. When I subtract 7 from a number (a), the answer is 16.

c. When I add 5 to a number (s), the answer is -4.

d. When I subtract 11 from a number (t), the answer is -11.

e. When I divide a number (c) by 9, the answer is 11.

f. When I multiply a number (p) by 4, the answer is 76.

g. When I multiply a number (r) by itself, the answer is 36.

h. When I find the square root of a number (q), the answer is 7.

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Zoe and Oscar are playing on a seesaw. Zoe weighs 48 kilograms and Oscar weighs 59 kilograms. Zoe is wearing a backpack. How much does the backpack need to weigh (b) for the seesaw to balance when Zoe and Oscar sit on it? Create an equation and solve for b.

9. Zach was given 3 packets of Pokémon cards for his birthday. How many Pokémon cards (c) are there in each packet if he now has 27 more cards in his deck? Create an equation and solve for c.

10. Marcus came third in his work footy tipping competition. How much was the total prize pool (t) if his winnings of \$65 was a fifth of the total prize pool? Create an equation and solve for t.

11. Ron wants two of Sarah's DC comics that are worth \$20 each. How many of his Marvel Comics (n) will Ron have to trade to get the two DC comics he wants? Ron's Marvel Comics are worth \$5 each. Create an equation and solve for n.

12. How much does Julie spend (c) per cup of coffee if she spent \$1460 buying a cup of coffee everyday for a year? Create an equation and solve for c.

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Joyce wants to buy some posters of the seven members of the K-pop group BTS so that she can stick them on the wall in her bedroom.

The website shows that all the posters have the same dimensions. The width of all seven posters next to each other is 294 cm and the height of each poster is 60 cm.

a. Write an algebraic equation to represent the width (p) of an individual poster.

b. Solve the equation in part **a** to find the width p of each poster.

c. Joyce realised that the store had provided the wrong width measurements for the posters. When she measured the posters, each poster had a width of 47 cm. What is the actual width of seven posters when they're placed next to each other?

d. The height of the wall in Joyce's bedroom is 240 cm. If she wants the posters to be placed exactly in the middle of the wall (i.e. the distance from the bottom of the posters to the floor is the same as the distance between the top of posters to the ceiling), how high off the ground should she stick the posters?

e. Is it a waste of paper to buy such a large poster? Should Joyce look for different posters that are smaller?

14. Charlie is in high school and is working part-time at a fast food restaurant. He is financially responsible and wants to start saving.
    a. Charlie earns $16 an hour and has a savings goal of $1200. Write an algebraic equation to represent the number of hours (h) he would have to work to reach his goal.
    b. How many hours will Charlie need to work to make $1200?
    c. Warren works at the same restaurant as Charlie but earns $4 more per hour. How many hours would Warren need to work to make $1200?
    d. Why might Warren earn more than Charlie despite working at the same restaurant?

Extra spicy

15. A number is added to a sixth of itself. The result is 42. What is the number?

16. $\frac{1}{7}$ of an unknown number is $\frac{4}{5}$. What is $\frac{1}{9}$ of the unknown number? Express as a fraction.

17. A jumper is currently on sale for 64% of the retail price. If I buy the item from the online store, I can get an extra 25% off the sale price. I bought the jumper online for $60. What was the original retail price of the jumper?

18. David is currently three times older than his brother Edward. In three years time, David will only be two times older than his brother. How old is Edward currently?

Remember this?

19. A farmer is ordering some fencing to build their new chicken coop.
    What unit would be the most appropriate to order lengths of chicken wire in?
    A. Kilometres B. Litres C. Metres D. Kilograms

20. What fraction is exactly halfway between $\frac{1}{2}$ and $\frac{6}{7}$?

21. This table shows how many lollies of each colour are in a bag of lollies.

Lolly colour	Blue	Green	Red	Orange
Number of lollies	20	6	5	9

    What is the chance of picking a red lolly?
    A. $\frac{1}{8}$ B. $\frac{6}{40}$ C. $\frac{1}{2}$ D. $\frac{4}{7}$

4E Solving two-step equations

Solving two-step equations is similar to solving one-step equations. However, two-step equations require two inverse operations to isolate the pronumeral instead of one. Below are some examples where solving two-step equations can be applied.

- My dad is triple my age (a) plus 3 more years. If my dad is 45 years old, how old am I? $3a + 3 = 45$
- I participated in my school's footy tipping competition. How much was the total jelly bean prize pool (t) if my winnings of 100 jelly beans was $\frac{5}{9}$ of the total jelly bean prize pool? $\frac{5t}{9} = 100$
- I went to the supermarket to buy 5 packs of gum (g). If I gave the clerk a \$20 note and received \$12 in change, how much did each pack of gum cost? $5g + 12 = 20$

Learning intentions

Students will be able to:

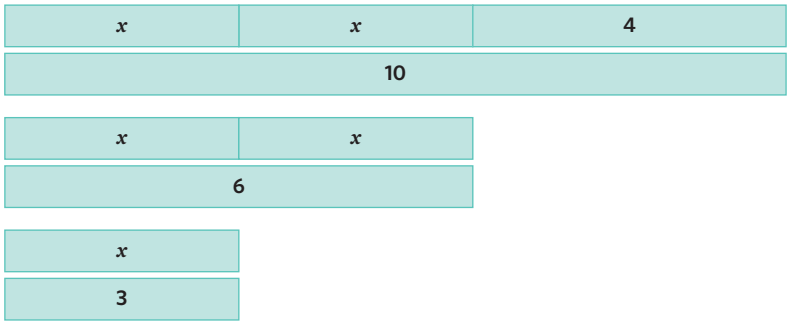
+ balance equations by performing the same operation on both sides
+ identify and apply inverse operations
+ solve two-step equations.

Key terms and definitions

- A **two-step equation** is an equation that can be solved by applying two inverse operations.

Key ideas

1 Two-step equations require two applications of inverse operations to isolate and find the value of the unknown.



$2x + 4 = 10$

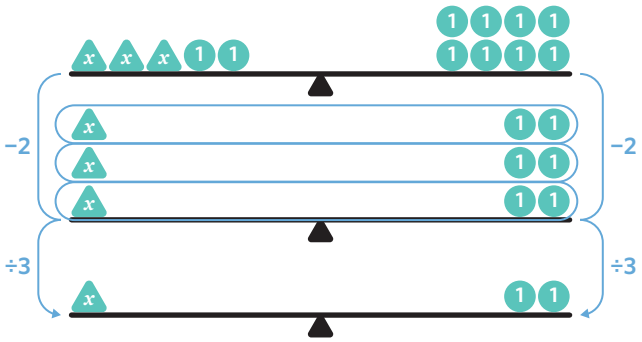
$2x + 4 - 4 = 10 - 4$

$2x = 6$

$2x \div 2 = 6 \div 2$

$x = 3$

2 We can think of division as equally sharing a quantity or collection among a group of the same pronumeral.



$3x + 2 = 8$

$3x = 6$

$x = 2$

3 We solve for unknowns by applying the inverse operation in the reverse order that an equation has been constructed.

Constructing the equation with respect to x

$x = 14$ (÷2 / ×2)

$\frac{x}{2} = 7$ (−4 / +4)

$\frac{x}{2} - 4 = 3$

Solving the equation for x

$\frac{x}{2} - 4 = 3$ (+4 / −4)

$\frac{x}{2} = 7$ (×2 / ÷2)

$x = 14$

Worked examples

WE 1 Identifying the first inverse operation to apply

Identify the first inverse operation that should be applied to solve the equation.

a. $6x - 5 = 25$

Working

$6x - 5 = 25$

$6x - 5 + 5 = 25 + 5$

The first inverse operation is +5.

Thinking

Step 1: Identify the pronumeral (x) in this expression. To get $6x - 5$, we multiply x by 6 and then subtract 5.

Step 2: The first inverse operation is opposite of the last operation used to construct the equation. The last operation is −5.

Step 3: The inverse of a subtraction is an addition. The inverse of −5 is +5.

Visual support

$$\overset{+5}{\curvearrowright} \quad 6x - 5 = 25 \quad \overset{+5}{\curvearrowleft}$$
$$6x = 30$$

b. $\frac{q + 4}{3} = 7$

Working

$\frac{q + 4}{3} = 7$

$\frac{q + 4}{3} \times 3 = 7 \times 3$

The first inverse operation is ×3.

Thinking

Step 1: Identify the pronumeral (q) in this expression. To get $\frac{q + 4}{3}$, we add 4 to q and then divide by 3.

Step 2: The first inverse operation is opposite of the last operation used to construct the equation. The last operation is ÷3.

Step 3: The inverse of a division is a multiplication. The inverse of ÷3 is ×3.

Student practice

Identify the first inverse operation that should be applied to solve the equation.

a. $7y + 3 = 24$ **b.** $\frac{q + 4}{5} = 8$ **c.** $5c - 3 = 32$ **d.** $\frac{f}{3} - 4 = 28$

WE 2 Solving two-step equations

a. Solve the equation $3q + 4 = 19$.

Working

$3q + 4 - 4 = 19 - 4$

$3q = 15$

$3q \div 3 = 15 \div 3$

$q = 5$

Thinking

Step 1: Identify the operation being applied to the pronumeral (q).

Step 2: Isolate $3q$ by applying the inverse operation −4. To keep the equation true, apply the same operation on both sides.

Step 3: Isolate q by applying the inverse operation ÷3. To keep the equation true, apply the same operation on both sides.

Step 4: Simplify both sides of the equation to find the solution for q.

Visual support

$$\begin{array}{rl} 3q + 4 &= 19 \quad (-4) \\ 3q &= 15 \quad (\div 3) \\ q &= 5 \end{array}$$

b. Solve the equation $\frac{k}{3} - 5 = 6$ and then check the solution by substituting the value into the equation.

Working

$\frac{k}{3} - 5 + 5 = 6 + 5$

$\frac{k}{3} = 11$

$\frac{k}{3} \times 3 = 11 \times 3$

$k = 33$

$\frac{33}{3} - 5 = 6$

$11 - 5 = 6$

$6 = 6$ ✓

Thinking

Step 1: Identify the operation being applied to the pronumeral (k).

Step 2: Isolate $\frac{k}{3}$ by applying the inverse operation +5. To keep the equation true, apply the same operation on both sides.

Step 3: Isolate k by applying the inverse operation 3. To keep the equation true, apply the same operation on both sides.

Step 4: Simplify both sides of the equation to find the solution for k.

Step 5: Replace the pronumeral k with 6 and simplify by performing all calculations.

Student practice

a. Solve the equation $2x + 8 = 16$.

b. Solve the equation $\frac{y}{3} + 5 = 13$ and then check the solution by substituting the value into the equation.

c. Solve the equation $5c - 15 = 90$.

d. Solve the equation $\frac{p}{7} - 11 = -3$ and then check the solution by substituting the value into the equation.

WE 3 Solving worded sentence equations.

Write an equation for the worded sentence and then solve for the unknown.

a. When I multiply a number (c) by 5 and then add 2, the result is 22.

Working

$5c + 2 = 22$

$5c + 2 - 2 = 22 - 2$

$5c = 20$

$5c \div 5 = 20 \div 5$

$c = 4$

Thinking

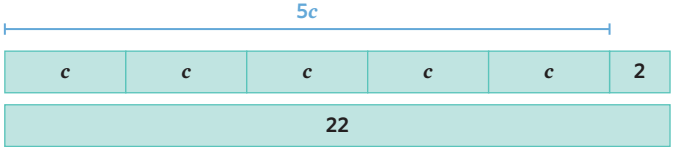
Step 1: Write the statement as an equation.

Step 2: Isolate $5c$ by applying the inverse operation -2 to both sides.

Step 3: Isolate c by applying the inverse operation $\div 5$ to both sides.

Step 4: Simplify both sides of the equation to find the value of c.

Visual support



b. When I divide a number (p) by 4 and then subtract 6, the result is 4.

Working

$$\frac{p}{4} - 6 = 4$$

$$\frac{p}{4} - 6 + 6 = 4 + 6$$

$$\frac{p}{4} = 10$$

$$\frac{p}{4} \times 4 = 10 \times 4$$

$$p = 40$$

Thinking

Step 1: Write the statement as an equation.

Step 2: Isolate $\frac{p}{4}$ by applying the inverse operation +6 to both sides.

Step 3: Isolate p by applying the inverse operation ×4 to both sides.

Step 4: Simplify both sides of the equation to find the solution for p.

Student practice

Write an equation for the worded sentence and then solve for the unknown.

a. When I multiply a number (x) by 3 and then add 7, the result is 37.

b. When I multiply a number (m) by 7 and then subtract 8, the result is 55.

c. When I subtract 2 from a number (z) and then divide by 12, the result is 6.

d. When I divide a number (j) by 4 and then multiply it by 7, the result is 14.

4E Activities and questions

STARTER TASKS

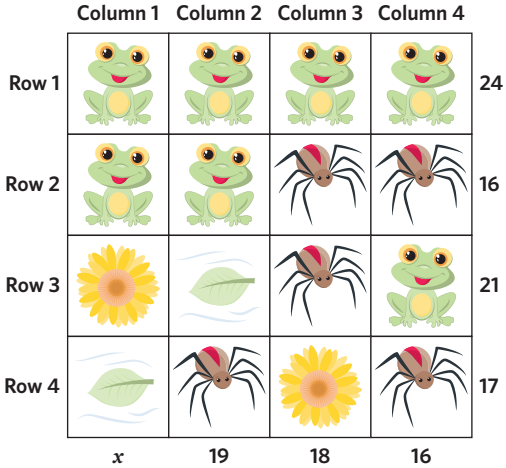
Odd spot

The Big Mango is a 10-metre high, 10-tonne, tourist attraction located in Bowen Queensland that was stolen in February of 2014. The robbers picked it up using a crane and it was later discovered, covered up in a nearby paddock. Fast food chicken chain Nandos later claimed responsibility for the theft, revealing that it was a marketing stunt for the release of their new sauce. If the height of the Big Mango is twice the width plus an additional 2 metres, how wide is the structure?

A. 4 metres **B.** 22 metres

Puzzle

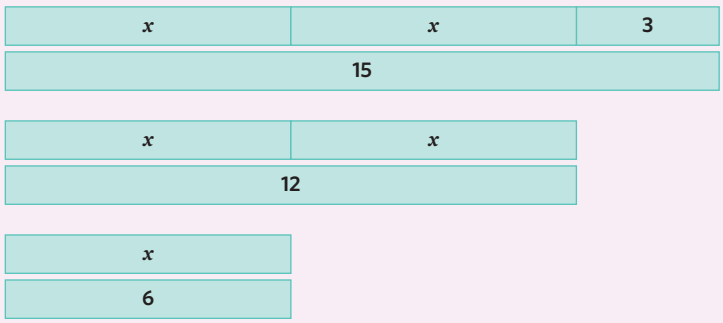
The sum of the images in each column and row is shown by the values in the grid. Use this information to find the value of x.



Understanding worksheet

1. Solve for x in each bar model by first completing the inverse operations in the boxes provided.

Example



$2x + 3 = 15$

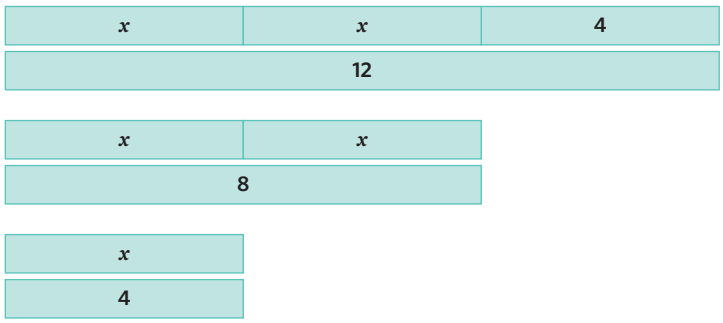
$2x + 3 \boxed{-3} = 15 \boxed{-3}$

$2x = 12$

$2x \boxed{\div 2} = 12 \boxed{\div 2}$

$x = \boxed{6}$

a.



$2x + 4 = 12$

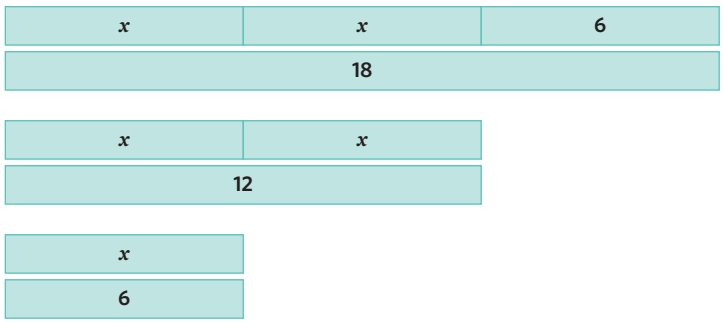
$2x + 4 \square = 12 \square$

$2x = 8$

$2x \square = 8 \square$

$x = 4$

b.



$2x + 6 = 18$

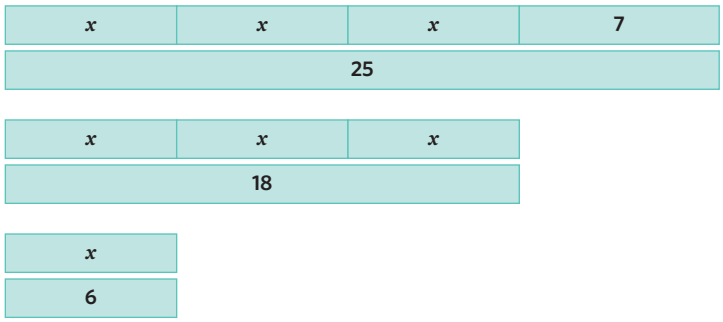
$2x + 6 \square = 18 \square$

$2x = 12$

$2x \square = 12 \square$

$x = 6$

c.



$3x + 7 = 25$

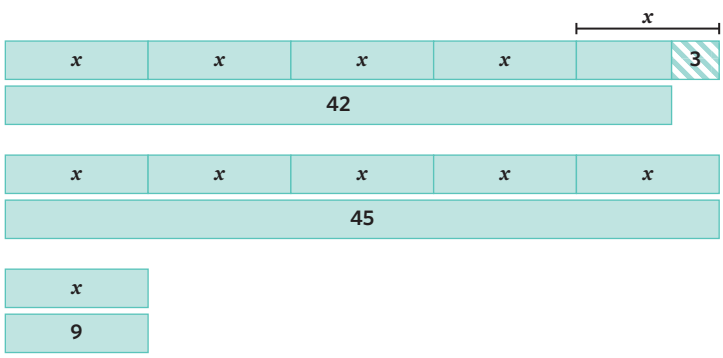
$3x + 7 \square = 25 \square$

$3x = 18$

$3x \square = 18 \square$

$x = 6$

d.



$5x - 3 = 42$

$5x - 3 \square = 42 \square$

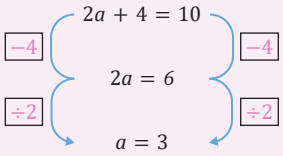
$5x = 45$

$5x \square = 45 \square$

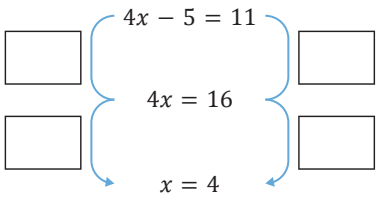
$x = 9$

2. Solve each equation by completing the boxes.

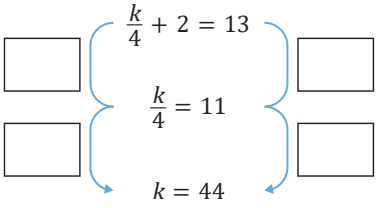
Example



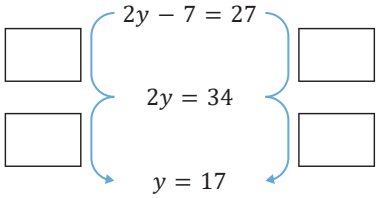
a.



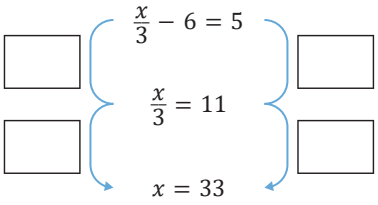
b.



c.



d.



3. Fill in the blanks by using the words provided.

simplify | inverse | unknown | order | solve

Solving two-step equations involves isolating the _____ variable using _____ operations. To efficiently _____ two-step equations, we must consider the _____ of the operations. Our first inverse operation should allow us to _____ our equation and reduce the number of terms we work with.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Identify the first inverse operation that should be applied to solve the equation.

a. $2x + 3 = 5$ **b.** $5x - 17 = 23$ **c.** $20 = \frac{y}{3} + 5$ **d.** $\frac{c}{7} - 3 = 18$

e. $28 = 2k + 4$ **f.** $\frac{x + 6}{5} = 41$ **g.** $-4 + 8x = 64$ **h.** $-6 = \frac{-x - 9}{7}$

WE2a **5.** Solve the equations.

a. $3x + 5 = 17$ **b.** $5y + 15 = 65$ **c.** $3 = 4m - 9$ **d.** $1 + 2z = 3$

e. $6 = \frac{3k}{4}$ **f.** $\frac{p + 8}{2} = 9$ **g.** $60 = -9 + 3c$ **h.** $-7 + a^2 = 42$

WE2b **6.** Solve the equations and then check the solution by substituting the value into the equation.

a. $2z + 4 = 6$ **b.** $25 = 3w - 5$ **c.** $\frac{u}{4} + 2 = 5$ **d.** $6i - 6 = 36$

e. $\frac{y + 6}{3} = 12$ **f.** $33 = -7 + 4v$ **g.** $3 = \frac{k - 3}{9}$ **h.** $-8 + n^2 = 73$

WE3 7. Write an equation and then solve for the unknown.

a. When I multiply a number (k) by 5 and then add 4, the result is 19.

b. When I multiply a number (c) by 4 and then subtract 7, the result is 17.

c. When I divide a number (a) by 3 and then add 5, the result is 8.

d. When I divide a number (z) by 7 and then add 2, the result is 9.

e. When I add 3 to a number (y) and then divide by 5, the result is 5.

f. When I subtract 4 from a number (k) and then divide by 8, the result is 4.

g. When I multiply a number (x) by 5 and then divide it by 3, the result is 15.

h. When I divide a number (p) by 9 and then multiply it by 2, the result is $\frac{12}{9}$.

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. An Uber ride charges \$4 per km travelled plus a \$6 service fee. If Beth paid \$54 for her ride, how many kilometres (d) did she travel?

9. Richard's backpack is three times as heavy as Mortimer's. He decided he wanted to remove a gadget that weighed 450 g from his backpack. How much does Mortimer's backpack (m) weigh if Richard's backpack weighs 3750 g after removing the gadget?

10. Rebecca owns a bakery and has to complete 42 cake orders in a week. If she baked 12 cakes on the first day, how many cakes (c) on average would she have to bake everyday for the remaining six days?

11. Shane received a \$25 voucher at his favourite sports shop for his birthday. He bought a tennis racquet that was on sale for 50% of the retail price. After using his voucher, Shane paid another \$175 for the racquet. How much does one tennis racquet cost at its retail price (t)?

12. Taylen bought a square block of land to add to his current land of 250 m^2. If the total area of his two blocks of land is now 394 m^2, what is the length (l) of his new block of land?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Jessica loves bubble soccer and is planning a birthday party with her friends. It costs \$500 dollars for unlimited play and an additional \$30 per bubble suit for each person.

Image: Mauro Rodrigues/Shutterstock.com

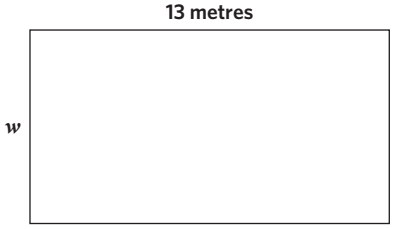
a. If Jessica has a budget of \$950 for her birthday party, write an algebraic equation to represent the number of people (n) she can have at her bubble soccer party.

b. Solve the equation in part **a** to find how many friends she can invite with her budget. Remember: Jessica also needs to attend her birthday party.

c. Jessica decided to pick up some extra shifts at work and increased her birthday party budget by \$300. How many friends can she now invite to her bubble soccer party?

d. Jessica found out that 60% of the friends that she invited prefer bowling over bubble soccer. Should Jessica change her birthday plans and organise a bowling party or stick with bubble soccer?

14. Molly was asked to decorate a rectangular room. She was sent the following floorplan of the room and was informed that the total perimeter of the room is 40 m.



a. Write an equation to represent the width (w) of the room that Molly is decorating.

b. Use the equation from part **a** to find the width of the room.

c. Molly wants to place a new square rug with an area of 49 m^2 in the center of her room and still have at least 1 metre between all four sides of the rug and the walls of the room. Will her new rug fit?

d. What would be some different rug dimensions that would work well with Molly's design? Give reasons.

Extra spicy

15. If $x = \frac{y + 4}{3}$, what is the value of y?

A. $\frac{x}{4} + 3$ B. $\frac{x - 4}{3}$ C. $\frac{x + 4}{3}$ D. $3x - 4$ E. $3x + 4$

16. Frederick is 31 years older than his son Barnaby. In four years, the sum of Frederick and Barnaby's ages will be 63 years. How old is Frederick currently?

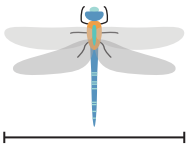
17. If $x + y = 87$ and $x - y = 51$ what is y?

18. Richard is investigating bacterial growth in his petri dish. On Monday, he observed 1000 bacteria in his dish. On Tuesday, there was an additional 1100 bacteria in his dish. The bacteria in the dish remarkably doubled on Wednesday, and doubled again on Thursday. What was the average daily bacterial growth for that period?

Remember this?

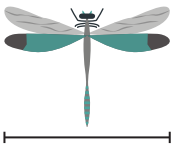
19. Crissy records the wingspans of the dragonflies in her collection. Which dragonfly has a wingspan closest to 5 cm?

1



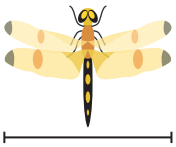
5.6 cm

2



5.15 cm

3



5.2 cm

4

4.90 cm

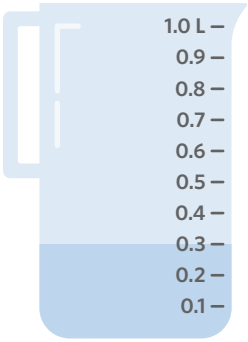
A. Dragonfly 1 B. Dragonfly 2 C. Dragonfly 3 D. Dragonfly 4

20. Chase wants to calculate $15 \times 2 + 15 \times 4$. Which expression can he use to work out his answer?

A. $15 \times 2 + 4$ B. $15 \times (2 + 4)$ C. $15 \times 17 \times 4$ D. $15 + 15 \times 2 \times 4$

21. This measuring jug holds 1 litre (L) of water. What fraction of the measuring jug is filled with water?

A. $\frac{1}{3}$
B. $\frac{3}{7}$
C. $\frac{3}{10}$
D. 0.3



4F Solving multi-step equations

Some equations include fractions and brackets. Fractions are used to represent division. Brackets help clarify the order of operations. To solve multi-step equations, more than one inverse operation needs to be applied to isolate the pronumeral. Below are some examples where solving multi-step equations can be applied.

- The cost of an adult ticket at the movies is \$3 more than a child's ticket (t). If four adult tickets cost me \$16, how much is one child's ticket? $4(t + 3) = 16$
- My partner and I hired a car for \$$c$ per day and spent an additional \$60 per day on petrol. The total cost for four days was \$300. If we split the cost between us, how much did I pay? $\frac{4(c + 60)}{2} = 300$
- The number of movies I watched this month is one-third of how many my older sister watched plus an additional 7 movies. If I watched 13 movies, how many did my sister watch (m)? $\frac{m}{3} + 7 = 13$

Learning intentions

Students will be able to:

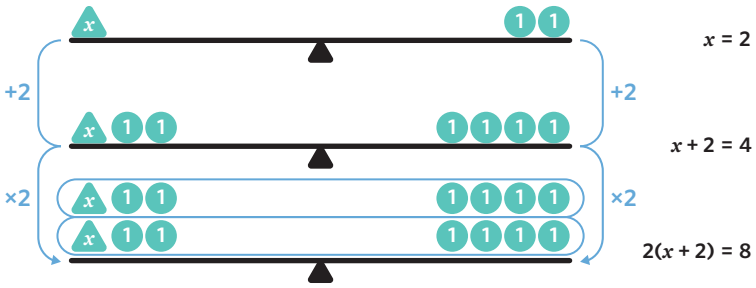
+ solve multi-step equations with fractions
+ solve multi-step equations with brackets
+ solve multi-step equations with fractions and brackets.

Key terms and definitions

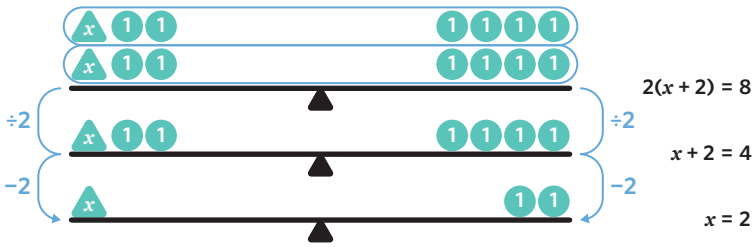
- **Brackets** are mathematical symbols that clarify the order that operations are completed in expressions. Brackets should always be evaluated first.
- A **multi-step equation** is an equation that can be solved by applying more than two inverse operations.

Key ideas

1 Before solving equations with brackets, it is important to understand the order of operations involved in constructing the equations first.



2 When there is a number next to the outside of the brackets, the contents of the bracket are being multiplied by that number. When solving the equation, we first apply the inverse operation of dividing both sides by the number.



3 A pronumeral may be part of a fraction. The appropriate first inverse operation depends on the construction of the equation.

In equation 1, the left-hand side is a fraction. We must first multiply both sides by the denominator.	In equation 2, the left-hand side is a fraction that has a constant subtracted from it. We must first apply the inverse operation to remove the constant.
Equation 1 $\frac{2c - 4}{3} = 6$ (×3, ×3) $2c - 4 = 18$	Equation 2 $\frac{2c}{3} - 4 = 6$ (+4, +4) $\frac{2c}{3} = 10$

Worked examples

WE 1 Solving multi-step equations with fractions

Solve each equation.

a. $\frac{5x - 2}{4} = 7$

Working

$\frac{5x - 2}{4} \times 4 = 7 \times 4$

$5x - 2 = 28$

$5x - 2 + 2 = 28 + 2$

$5x = 30$

$5x \div 5 = 30 \div 5$

$x = 6$

Thinking

Step 1: The left-hand side is a fraction. Multiply both sides by the denominator ($\times 4$) and simplify.

Step 2: Isolate $5x$ by applying the inverse operation $+2$ to both sides.

Step 3: Isolate x by applying the inverse operation $\div 5$ to both sides.

Visual support

$\frac{5x - 2}{4} = 7$ ($\times 4$)

$5x - 2 = 28$ ($+2$)

$5x = 30$ ($\div 5$)

$x = 6$

b. $\frac{4m}{5} + 7 = 15$

Working

$\frac{4m}{5} + 7 - 7 = 15 - 7$

$\frac{4m}{5} = 8$

$\frac{4m}{5} \times 5 = 8 \times 5$

$4m = 40$

$4m \div 4 = 40 \div 4$

$m = 10$

Thinking

Step 1: The left-hand side is a fraction that has a constant subtracted from it (-7). Apply the inverse operation ($+7$) to remove the constant.

Step 2: The left-hand side is now a fraction. Multiply both sides by the denominator ($\times 5$) and simplify.

Step 3: Isolate m by applying the inverse operation $\div 4$ to both sides.

Visual support

$\frac{4m}{5} + 7 = 15$ (-7)

$\frac{4m}{5} = 8$ ($\times 5$)

$4m = 40$ ($\div 4$)

$m = 10$

Student practice

Solve each equation.

a. $\frac{2x - 8}{6} = 1$

b. $\frac{2m}{4} + 4 = 16$

c. $20 = \frac{8m - 40}{10}$

c. $10 = \frac{x}{5} - 30$

WE 2 Solving multi-step equations with brackets

Isolate the brackets to solve each equation.

a. $3(q + 4) = 21$

Working

$3(q + 4) \div 3 = 21 \div 3$

$q + 4 = 7$

$q + 4 - 4 = 7 - 4$

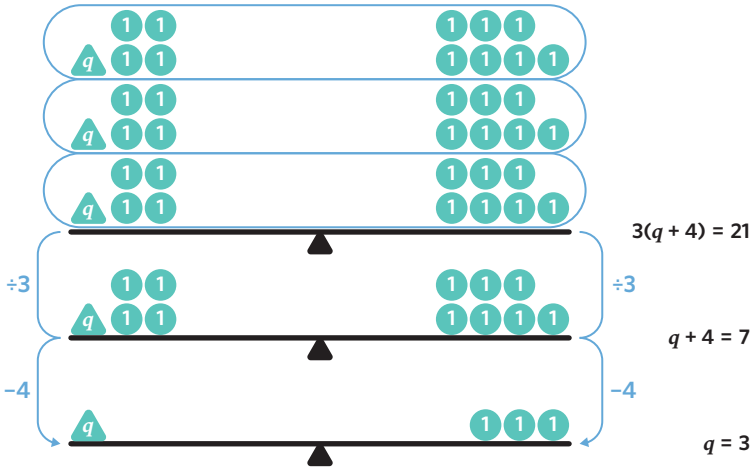
$q = 3$

Thinking

Step 1: The contents of the brackets are being multiplied by 3. Isolate the brackets by applying the inverse operation ÷3 to both sides of the equation.

Step 2: Isolate q by applying the inverse operation −4 to both sides.

Visual support



b. $9(2k - 6) = 18$

Working

$9(2k - 6) \div 9 = 18 \div 9$

$(2k - 6) = 2$

$2k - 6 + 6 = 2 + 6$

$2k = 8$

$2k \div 2 = 8 \div 2$

$k = 4$

Thinking

Step 1: The contents of the brackets are being multiplied by 9. Isolate the brackets by applying the inverse operation ÷9 to both sides of the equation.

Step 2: Isolate $2k$ by applying the inverse operation +6 to both sides.

Step 3: Isolate k by applying the inverse operation ÷2 to both sides.

Student practice

Isolate the brackets to solve each equation.

a. $2(x + 3) = 20$ **b.** $3(5y - 7) = 9$ **c.** $50 = 5(v - 3)$ **d.** $8(5 + m) = 24$

WE 3 Solving multi-step equations with fractions and brackets

Solve the following equations.

a. $\frac{3(m+1)}{4} = 6$

Working

$\frac{3(m+1)}{4} \times 4 = 6 \times 4$

$3(m+1) = 24$

$3(m+1) \div 3 = 24 \div 3$

$m + 1 = 8$

$m + 1 - 1 = 8 - 1$

$m = 7$

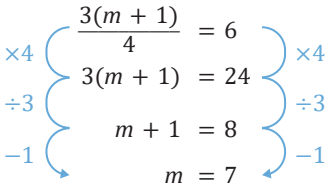
Thinking

Step 1: The left-hand side is a fraction. Multiply both sides by the denominator (×4) and simplify.

Step 2: The contents of the brackets are being multiplied by 3. Isolate the brackets by applying the inverse operation ÷3 to both sides of the equation.

Step 3: Isolate m by applying the inverse operation −1 to both sides.

Visual support



b. $10 = \frac{2(x-12)}{5}$

Working

$10 \times 5 = \frac{2(x-12)}{5} \times 5$

$50 = 2(x - 12)$

$50 \div 2 = 2(x - 12)$

$25 = x - 12$

$25 + 12 = x - 12 + 12$

$37 = x$

$x = 37$

Thinking

Step 1: The left-hand side is a fraction. Multiply both sides by the denominator (×5) and simplify.

Step 2: The contents of the brackets are being multiplied by 2. Isolate the brackets by applying the inverse operation ÷2 to both sides of the equation.

Step 3: Isolate x by applying the inverse operation +12 to both sides.

Student practice

Solve the following equations.

a. $\frac{2(m+5)}{3} = 4$ **b.** $9 = \frac{3(x-8)}{5}$ **c.** $\frac{5(g-2)}{7} = 2$ **d.** $\frac{10(m+6)}{3} = \frac{2}{3}$

4F Activities and questions

STARTER TASKS

Odd spot

New York artist Cosimo Cavallaro wants to coat the inside and outside walls of a house with 5000 kilograms of cheese.

Cosimo wants to use cheddar, brie, manchego and parmesan cheese to cover his house. He currently has 450 kg of each cheese. If he wants to use equal amounts of all four cheeses to cover his house, how much more of each cheese does he need to buy?

A. 1137.5 kg **B.** 800 kg

Image: Tasting Table/Shutterstock.com

Puzzle

Max is using a balance scale to compare the weight of four different objects.

a) Which is heavier?

b) Which is heavier?

Understanding worksheet

1. Complete the missing boxes to show how each equation is solved.

Example

$\frac{3x+4}{5} = 2$

$\times 5$ $\times 5$

$3x + 4 = 10$

-4 -4

$3x = 6$

$\div 3$ $\div 3$

$x = 2$

a.

$\frac{7j-12}{9} = 1$

$7j - 12 = 9$

$7j = 21$

$j = 3$

b.

$\frac{3r}{5} + 2 = 8$

$\frac{3r}{5} = 6$

$3r = 30$

$r = 10$

c.

$$2 = \frac{2x + 4}{7}$$
$$14 = 2x + 4$$
$$10 = 2x$$
$$5 = x$$

d.

$$8 = \frac{10y}{4} - 7$$
$$15 = \frac{10y}{4}$$
$$60 = 10y$$
$$6 = y$$

2. Complete the missing boxes to show how each equation is solved.

Example

$$\frac{5(x - 3)}{10} = 1$$
×10 / ×10
$$5(x - 3) = 10$$
÷5 / ÷5
$$x - 3 = 2$$
+3 / +3
$$x = 5$$

a.

$$\frac{3(c - 8)}{5} = 6$$
$$3(c - 8) = 30$$
$$c - 8 = 10$$
$$c = 18$$

b.

$$\frac{9(x + 1)}{3} = 12$$
$$9(x + 1) = 36$$
$$x + 1 = 4$$
$$x = 3$$

c.

$$10 = \frac{5(x - 4)}{8}$$
$$80 = 5(x - 4)$$
$$16 = x - 4$$
$$20 = x$$

d.

$$1 = \frac{25(x + 6)}{100}$$
$$100 = 25(x + 6)$$
$$4 = x + 6$$
$$-2 = x$$

3. Fill in the blanks by using the words provided.

numerator | divide | denominator | multiplied | inverse | brackets

In the equation $5(f - 6) = 20$, which contains _____, $f - 6$ is being _____ by 5. In order to solve this equation we must apply the _____ operation, which is to _____ by 5. In the equation $\frac{2m}{7} = 10$ we can isolate the _____ by multiplying both sides of the equation by the _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1a **4.** Solve each equation.

a. $\frac{3m+8}{5}=4$ b. $\frac{2z-1}{3}=5$ c. $5=\frac{10x+30}{12}$ d. $\frac{3x-3}{6}=4$

e. $7=\frac{q+22}{3}$ f. $\frac{4+c}{2}=5$ g. $8=\frac{5x+1}{7}$ h. $\frac{4+17x}{11}=5$

WE1b **5.** Solve for x.

a. $\frac{3x}{4}+4=10$ b. $\frac{2x}{5}-3=1$ c. $\frac{5x}{2}-1=9$ d. $3=\frac{8x}{7}+2$

e. $1=\frac{6x}{7}-5$ f. $7=2+\frac{5x}{6}$ g. $\frac{9x}{2}-14=-3$ h. $-20+\frac{3x}{4}=11$

WE2 **6.** Isolate the brackets to solve each equation.

a. $3(m-5)=18$ b. $4(y+2)=16$ c. $36=9(k-3)$ d. $2(x+8)=96$

e. $99=11(n+6)$ f. $2(d+3)=11$ g. $3(x-2)=10$ h. $13=4\left(a-1\frac{1}{2}\right)$

WE3 **7.** Solve the following equations.

a. $\frac{3(m+1)}{4}=6$ b. $\frac{2(a-5)}{3}=2$ c. $12=\frac{2(x-10)}{5}$ d. $8=\frac{4(y+1)}{10}$

e. $\frac{150(r-200)}{100}=3$ f. $\frac{3(t+60)}{10}=15$ g. $1=\frac{3(b+2)}{4}$ h. $\frac{2(p-7)}{3}=\frac{4}{5}$

8. Solve for x.

a. $\frac{2x}{3}-7=3$ b. $\frac{2x+10}{4}=3$ c. $8(x+2)=40$ d. $5=\frac{8x+13}{3}$

e. $\frac{3x}{8}-5=1$ f. $6=\frac{14(x-10)}{7}$ g. $\frac{4x}{5}+1=7$ h. $\frac{4(x-3)}{6}=\frac{2}{3}$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Each morning, for two days, Max ate c cashews and 5 almonds. Altogether he ate 20 nuts. Create and solve an equation to work out how many cashews (c) Max ate each day.

10. Ellie wanted to buy movie tickets for some of her friends. She invited d friends from her dance class plus 3 friends from her football team. The movie tickets cost $8 each. If Ellie paid $104 in total, how many friends did she invite from her dance class?

11. Rhonda collects ooshies. Before her birthday she had a collection of 90 ooshies. On her birthday, she had a party and her friends gave her 11 ooshies each. During the party, someone stole half of all her ooshies and she was left with 100 ooshies. How many friends (f) came to her party?

12. Ibrahim is making biscotti. The recipe says to set the oven to 200 degrees fahrenheit (°F) but his oven can only be set using degrees celsius (°C). Ibrihim knows that the formula for converting °C to °F is $°\text{F}=\frac{9°\text{C}}{5}+32$. What temperature should Ibrahim set for the oven?

13. Mick and Nathan have $68.50 to spend on an online shopping order and want to calculate how many pairs of boardshorts (b) they can buy. They have a promotion code, so they only pay one-quarter of the $23 marked price of boardshorts, and they also receive an additional $12 discount as part of the online sale.

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. The number of goals (g) a quidditch team score can be calculated using a special wizard and witch formula. The formula is $\textit{points scored} = \frac{12g + 20}{8}$. If a team catches the magic flying ball they get an extra 500 points, however, sometimes the magic flying ball is not caught.

- **a.** If the official scorecard says that Harold's team scored 25 points, how many goals did his team score?
- **b.** Ronnie thinks that the score is wrong and that Harold's team scored 8 goals less than the official score card suggests. What is the difference between the number of points scored on the official scorecard and what Ronnie thinks the score should be?
- **c.** In another game, Harold's team scored a total of 540 points, which included the 500 points for catching the magic ball. How many goals did the team score in the game?
- **d.** Harold and Ronnie both want to play in the same position. What factors should they consider to make the best decision for their team?

15. Divya and her extended family are going on a camping trip. Her extended family is made up of 15 people including herself. Some of Divya's cousins invited some friends along too.

- **a.** The cost of hiring a campsite is \$12 per person. How many friends of Divya's extended family went camping (f) if the total cost for hiring the campsite was \$264?
- **b.** Divya has 6 packets of new pegs and 6 extra pegs for the tents. She is responsible for dividing all the pegs equally between the five families. Based on her calculations, each family should receive 18 pegs. Using p to represent the number of pegs in one packet, write an equation that represents the situation and solve for p.
- **c.** Just before she is about to distribute the pegs, the family next to them on the campsite has three large bags of pegs and suggests they pool all of the pegs and share them between everyone. Divya now calculates that each of the 6 families (including the family next to them) will receive 25 pegs. Using b to represent the number of pegs in each of the large bags, write an equation to represent the situation and solve for b.
- **d.** Divya found it difficult to divide all the pegs equally between the 6 families without knowing exactly how many pegs she had to begin with. Suggest an efficient strategy that will allow Divya to divide the pegs equally between each family.

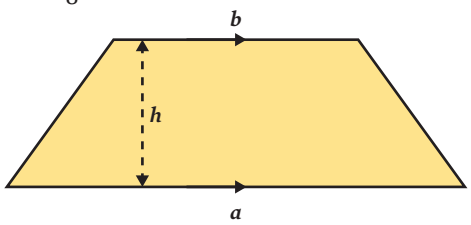
Extra spicy

16. An aeroplane with a full tank of petrol weighs 45 000 kilograms. The aeroplane weighs 44 850 kilograms after it has used one-quarter of its petrol tank. What is the weight of the aeroplane by itself?

17. An archery competition involves 12 shots. 16 points are awarded for a bullseye, 0 points for an arrow that misses the target and 6 points for hitting the target (not the bullseye). Natalia scored a total of 70 points in the competition. The largest number of arrows that missed the target she could have is:

A. 2 **B.** 16 **C.** 22 **D.** 4 **E.** 7

18. The area of a trapezium can be calculated using the formula $\textit{area} = \frac{a + b}{2} \times h$. What is the length of a if $b = 2\frac{1}{3}$ m, $h = 3\frac{3}{8}$ m, and $\textit{area} = 9\ \text{m}^2$?



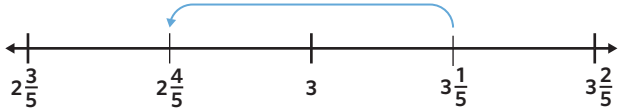
19. If $\frac{d}{d - 4t} = 6$, what does $\frac{d}{t}$ equal?

Remember this?

20. The Statue of Liberty is 93 metres tall. You can buy a 1:1500 scale model replica of the statue in a gift shop. Approximately how tall is the model statue?

A. 60 cm **B.** 6 cm **C.** 600 cm **D.** 72 cm

21. What number is subtracted from $3\frac{1}{5}$ to give $2\frac{4}{5}$?



A. $2\frac{3}{5}$ **B.** 3 **C.** $\frac{3}{5}$ **D.** $\frac{2}{5}$

22. Frankie bought 4 meat pies and 3 bottles of iced coffee.

The calculation he used to calculate the total cost was $3 \times 1.95 + 4 \times 4.50$.

What was the price of one meat pie?

A. \$1.95 **B.** \$3 **C.** \$4.50 **D.** \$4

4G Simplifying equations (collecting like terms)

In algebra we use different pronumerals to represent different values. We can also use the same pronumeral more than once in an equation. When terms have the same pronumerals, they are called 'like terms'. Like terms can be collected using addition or subtraction to make expressions and equations simpler. Below are some examples where collecting like terms can be applied.

- I bought 6 bags of apples on Monday, 3 bags of apples on Tuesday and 5 bags of apples on Wednesday. If there are a apples in a bag, what is the expression for the number of apples I have?
- The width of my bedroom is x metres and the length of my bedroom is $2x$ metres. What is the perimeter of my bedroom in terms of x?

Learning intentions

Students will be able to:

- identify like terms
- collect like terms to simplify expressions
- solve equations that contain like terms.

Key terms and definitions

- **Like terms** are algebraic terms that contain the same pronumeral or collection of pronumerals.
- **Collecting like terms** is the process of using addition or subtraction to group together algebraic terms that contain the same pronumeral or pronumerals.
- To **simplify** an algebraic expression means to rewrite it in a more concise and usable way. This is done by first collecting like terms.
- The **commutative law** means that the order of the numbers in an addition and multiplication calculation can change without affecting the result. E.g. $x + y = y + x$ and $xy = yx$.
- A **pronumeral** is a letter used to represent a value that is unknown or may vary. This is also known as a variable.

Key ideas

1 Like terms are terms which have the exact same pronumeral (or collection of pronumerals).

Like terms must have the same pronumerals. For example, the terms $3x$ and xy are not like terms because one contains the pronumeral x and the other the two pronumerals $x \times y$.

Like terms

$xy + 3y + 2xy - 5y$

Like terms

Not like terms

$3y + xy$

2 Like terms can be grouped together by adding or subtracting terms that have the exact same pronumeral (or pronumerals).

Once we have collected like terms, we can simplify the expression by completing all of the operations.

Like terms

Like terms

$3a - 6b + 4a + 2b + 2 + 1$

Like terms

$3a + 4a - 6b + 2b + 2 + 1 = 7a - 4b + 3$

3 When solving equations with like terms, simplify the equation by collecting like terms first.

$$\begin{aligned} 3g + 4g &= 35 \\ 7g &= 35 \\ g &= 5 \end{aligned}$$

Worked examples

WE 1 Identifying terms

Identify the like terms.

a. $3a - 5b + 4a$

Working

$3a$ and $4a$ are like terms.

Thinking

Identify terms with the same pronumeral(s). Terms that have the exact same pronumerals are like terms.

b. $2xy + 5x - 4yx$

Working

$2xy$ and $-4yx$ are like terms.

Thinking

Identify terms with the same pronumeral(s). Terms that have the exact same pronumerals are like terms.

Student practice

Identify the like terms.

a. $3j + 2 + j$ **b.** $4b - 2ab + 2b + 12$ **c.** $6rs + 2sr - 4r$ **d.** $3m - 4 + 2n + 4m + 7$

WE 2 Simplifying like terms

Simplify by collecting like terms.

a. $8b - 2b + 3b$

Working

$8b - 2b + 3b = 6b + 3b$

$= 9b$

Thinking

Identify like terms. Combine all the like terms by applying the operations.

b. $12 - 5 = 4a + 6b + a$

Working

$12 - 5 = 4a + a + 6b$

$7 = 5a + 6b$

Thinking

Identify like terms. Simplify the equation by combining all the like terms.

Note: a by itself means $1a$.

Student practice

Simplify by collecting like terms.

a. $4r + 3m - 2r$ **b.** $2 + 3x + 4 + 2y - 5$ **c.** $2b - b + 4c = 10 + 3$ **d.** $6kl - 3k + 2lk - 2k = 12$

WE 3 Simplifying and solving for x

Simplify and solve for x.

a. $3x - 2x + 9x = 7$

Working

$3x - 2x + 9x = 7$

$x + 9x = 7$

$10x = 7$

$10x \div 10 = 7 \div 10$

$x = \frac{7}{10}$

Thinking

Step 1: Simplify the equation by collecting like terms.

Step 2: Isolate x by applying the inverse of $\times 10$ which is $\div 10$. To keep the equation balanced, we apply the same operation on both sides.

Visual support

Like terms

$$\underbrace{3x - 2x + 9x} = 7$$

$$10x = 7$$

$\div 10$ $\qquad$ $\div 10$

$$x = \frac{7}{10}$$

b. $7 + x - 3 + 4x = 24$

Working

$$7 + x - 3 + 4x = 24$$
$$5x + 4 = 24$$
$$5x + 4 - 4 = 24 - 4$$
$$5x = 20$$
$$5x \div 5 = 20 \div 5$$
$$x = 4$$

Thinking

Step 1: Simplify the equation by collecting like terms.

Step 2: Isolate x by first applying the inverse of $+4$ which is -4 and then the inverse of $\times 5$ which is $\div 5$. To keep the equation balanced, we apply the same operation on both sides.

Student practice

Simplify and solve for x.

a. $2x + 3x - 4 = 6$

b. $4x + 3 + 5x = 12$

c. $5x + 7x + 13 - 4x = 1 + 14$

d. $5 - 2 + 4x - x = 10 - 2$

4G Activities and questions

STARTER TASKS

Odd spot

Michael Phelps is a swimmer who has won the most medals in Olympic history (28 medals, 23 gold). During the peak of his career, he was known for his huge volume of training and eating. Phelps would reportedly spend 5 hours eating per day, 9 hours sleeping and 4 hours travelling or spending time with his family. If he spent the rest of the time training, how many days would it take him to do 90 hours of training?

A. 6 days **B.** 15 days

Image: Petr Toman/Shutterstock.com

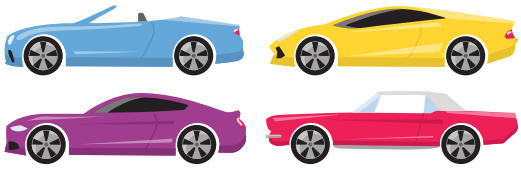
Puzzle

The blue car costs $20 000 more than the red car.

The yellow car costs twice as much as the red car.

The purple car costs $10 000 less than the red car.

If the cars cost $210 000 altogether, how much does the red car cost?



Understanding worksheet

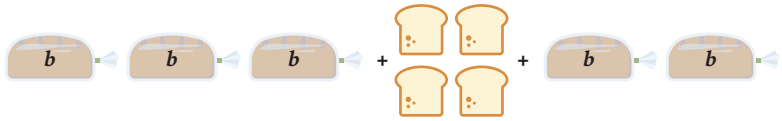
1. Each image represents an algebraic expression. Write the simplified expression using the given pronumerals.

s is the number of skittles in each box. *Example*

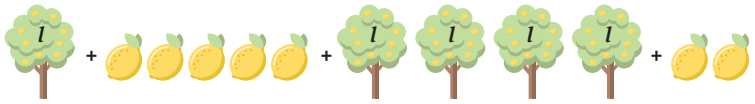
$5s + 4$

a. *c* is the number of coins in each bag.

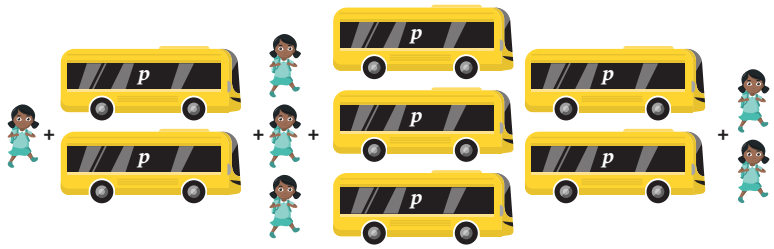
b. *b* is the number of slices of bread in each loaf.



c. *l* is the number of lemons on each tree.



d. *p* is the number of people on each bus.



2. Rearrange the expression to group like terms together.

$4w + 3 + 2w =$ $4w + 2w + 3$ *Example*

a. $2t + 4 + t + 8 =$

b. $3j + 10 + j + 6 + 4k =$

c. $8ab + 7a - 3b + 2a =$

d. $5xy - 12 + 3yx - y =$

3. Fill in the blanks by using the words provided.

algebraic | simplify | like terms | pronumeral(s)

To _____ an expression or equation we first need to collect _____.

To do this we need to group _____ terms within an expression that contain the same _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Identify the like terms.

a. $2x - 4y + 3x$
b. $4y + 7 - 3r$
c. $5 + 2 - 4a + 3a - 2a$
d. $3r - t + 5 + 4r - 2$
e. $6ab + 4a + 3b - 2ab$
f. $12t - 3m + 5 + 3m - 9t$
g. $5yx + 3xy - 4 + 3 - 2$
h. $\frac{2x}{3} - 4mn + 6nm - 3 + 7 + x$

WE2a **5.** Simplify the expressions by collecting like terms.

a. $2a + 3a$
b. $8g - g$
c. $9c + 4ab - 6c + 24 + 4c$
d. $j - 3j + 8j - 10 + 5$
e. $3b + 5 - 2b - 4 + 2ab$
f. $12f - ab + 6 - 7f$
g. $10 + 4cd - 7 + 6dc$
h. $9qpr + 5rqp + 6q - 2pq$

WE2b **6.** Simplify the equations by collecting like terms.

a. $4y - 2y = 24$
b. $20 = c + 3c$
c. $3r - r + 3s + 2r = 3 + 7$
d. $5j - 2k - 6j + 7k = 12 - 4 + 6$
e. $6y - 3yz - 10y + 7yz + 2z = 4 + 2$
f. $9qr - 4q + 6q - 6rq + 3q = 30 + 2$
g. $17 - 4 = 93b + b - 80c + 2b$
h. $2s + t + 6s + 3t - 4t = 28$

WE3 **7.** Simplify and solve for x.

a. $2x + 3x = 35$
b. $9x - 4x - 3x = 8$
c. $4x - 10 + 5x + 3 = 20$
d. $2x - 17 + 5x + 24 = 3 + 13 - 2$
e. $2x - 7x + 3 + 12 + 6x = 8 + 17 - 2$
f. $\frac{x}{3} + \frac{2x}{3} = 47$
g. $\frac{x}{2} + 2x = 25$
h. $12 + 6 + x + \frac{x}{2} = 34$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. There are s students going on a class trip. Each student needs to pay $12 for the bus ride and $19 entry for the zoo. Write a simplified expression (in terms of s) showing how much the entire class will pay for the trip.

9. The perimeter of a rectangular swimming pool is 24 m. What is the length and width (w) of the pool, if the length of the pool is twice its width?

10. Create and solve an equation to work out many books Ron should read during a read-a-thon with Sarah. They are aiming to read 72 books in total, and Sarah has promised she will read twice as many books as Ron.

11. Jasmine goes to an organic orchard with her friends and spends 50 dollars. She buys 5 apples and 10 oranges. If apples cost x dollars and oranges are $2x$ dollars, how much does each apple and orange cost?

12. Susie was delivering flyers every day in her neighbourhood. Write a simplified expression (in terms of x and y) to represent how many flyers Susie delivered in total. During the first week, she was able to deliver x flyers per day. During the next two weeks she delivered y flyers per day.

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c,d)	All

13. Phoebe has a sister, Paige, who was born exactly 1 year after her, and a sister Piper who was born exactly 2 years before her.

a. If Phoebe is x years old, write an expression for Paige's age.

b. If Phoebe is x years old, write an expression for Piper's age.

c. The three sisters' combined age is 73 years. Write an equation using x to represent this.

d. Use your equation from part **c** to find the age of all three sisters.

e. What do you imagine you will be doing in your life when you are Phoebe's age?

14. Tim is planning a ski trip and needs to hire some ski gear.

Item	Hire cost (per day)
Skis	$23
Jacket	$19
Snowboard	$28
Helmet	$10

a. Tim would like to ski for p number of days. He needs to hire skis, a jacket and a helmet. Write a simplified expression to represent the cost for Tim to hire the gear.

b. Tim's younger sister also needs a helmet and snowboard for t number of days. Write a simplified expression to represent the cost for Tim to hire all the items for himself and his sister.

c. Using the expression from part **b**, calculate the total cost of hire if Tim hires his gear for two days and his sister's for one day.

d. Assuming that Tim and his sister both hire gear for the same number of days (d), how many days can they ski if they have $500 to spend?

e. Why might the snowboard rental be more expensive than the ski rental?

Extra spicy

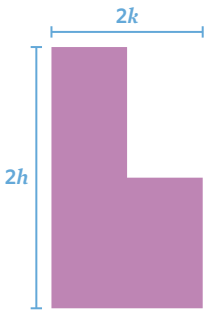
15. $3a - 4 - (4 - 3a) = ?$

A. -8 **B.** 0 **C.** $6a - 8$ **D.** $6a$ **E.** 8

16. Select another way to represent the expression $ab - ac$.

A. $c - (a + b)$ **B.** $(a - b) - c$ **C.** $a(b - c)$ **D.** $a(c - b)$ **E.** $b - a$

17. Find the perimeter.

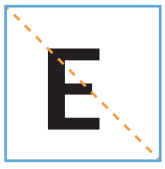


18. Write a simplified expression that represents the mean of the expressions $k + 3$, $7 - 2k$, $4k - 1$ and $k + 7$.

Remember this?

19. Kody arrived home from school at 4:45 pm. He left for school the next morning at 7:30 am. How long was Kody at home?

20. The letter E was drawn on a piece of paper and while the ink was still wet, the paper was folded in half along the dotted line.



When it was unfolded, what print would be seen?

A. 

B. 

C. 

D. 

21. On a school hike, every student received 3 muesli bars and 4 pieces of fruit. If 60 pieces of fruit were given out, how many muesli bars were given out?

4H Geometric patterns and numerical sequences

Patterns are an important part of algebraic thinking. Rules can be used to describe and explain different numerical sequences and geometric patterns. Rules can also be used to predict future values in a pattern or sequence. Below are some examples where rules for patterns and sequences can be applied.

- My newborn daughter weighed 3.5 kg when she was born. If she gains 200 grams a month, how heavy will she be when she is six-months old?
- I have $10 000 in my savings account. I save $500 a month. How much money will I have in my savings account after a year of saving? How much will I have after another year of saving?
- The cost of a taxi ride can be represented by the pricing formula: $\$c = 2k + \$9.50$, where k represents the kilometres travelled. If the distance from my home to the airport is 24 kilometres, how much would the taxi ride cost based on this formula?

Learning intentions

Students will be able to:

- describe and continue numerical sequences
- describe and continue geometric patterns
- represent numerical sequences in a table
- use a rule to find terms in a sequence.

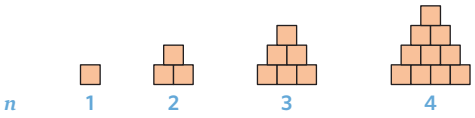
Key terms and definitions

- A **geometric pattern** is a sequence of geometric shapes that continues to increase or decrease in a consistent way.
- A **numerical sequence** is a collection of numbers that increase or decrease in a consistent way.
- A **rule** is a formula written using mathematical symbols and pronumerals to describe a pattern.
- **Consecutive terms** refers to values that directly follow each other within a pattern or a sequence.
- The **common difference** is the constant between consecutive terms in a sequence that determines how the sequence increases or decreases.

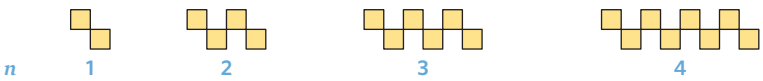
Key ideas

1 The pronumeral n refers to the **position** of each term in a geometric pattern or numerical sequence.

In the pattern shown, $n = 1$ describes the first position in the pattern, $n = 2$ the second position, $n = 3$ the third position and $n = 4$ the fourth position.



2 Tables can be used to represent numerical sequences and number patterns.



Position (n)	1	2	3	4
Number of squares (s)	2	4	6	8

3 A numerical sequence is where the values increase or decrease in a consistent way.

4 Rules can be used to predict future terms in numerical sequences and geometric patterns.

We can substitute position numbers (n values) in a rule to find the corresponding value in that position of the sequence or pattern.

The 100th term of the rule

$$t = 3n + 2$$
$$t = 3 \times 100 + 2$$
$$= 300 + 2$$
$$= 302$$

Worked examples

WE 1 Describing and continuing numerical sequences

Write the next term in the numerical sequence.

a. 1, 5, 9, 13, ...

Working

Each consecutive term increases by 4.

The common difference is +4

$13 + 4 = 17$

1, 5, 9, 13, 17

Thinking

Step 1: Identify if the sequence is increasing or decreasing by comparing consecutive terms. Find the common difference between consecutive terms.

Step 2: Apply the common difference to the last given term in the sequence to generate the next term in the sequence.

b. 36, 29, 22, 15, ...

Working

Each consecutive term decreases by 7.

The common difference is −7

$15 - 7 = 8$

36, 29, 22, 15, 8

Thinking

Step 1: Identify if the sequence is increasing or decreasing by comparing consecutive terms. Find the common difference between consecutive terms.

Step 2: Apply the common difference to the last given term in the sequence to generate the next term in the sequence.

Student practice

Write the next term in the sequence.

a. 3, 5, 7, 9, ...

b. 15, 12, 9, 6, ...

c. −8, −19, −30, −41, ...

d. $15, 21\frac{1}{2}, 28, 34\frac{1}{2}, \ldots$

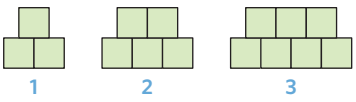
WE 2 Describing and continuing geometric patterns

For each pattern:

i) State how many squares there will be in the next position of the geometric pattern.

ii) Draw the next position to continue the geometric pattern.

a.



Working

i)

3, 5, 7

The common difference is +2.

$7 + 2 = 9$

There will be 9 squares in the next position of the geometric pattern.

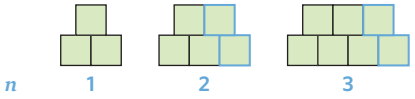
Thinking

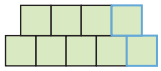
i)

Step 1: Count the number of squares in the first, second and third position and write them as a numerical sequence. Find the common difference between consecutive terms.

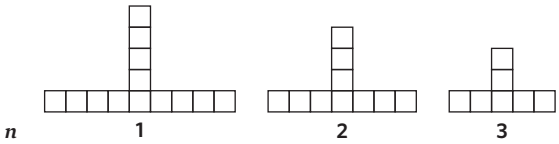
Step 2: Apply the common difference to the last given term in the sequence to find out how many squares there will be in the next position.

ii)





b.



Working

i)

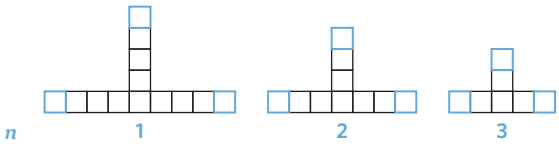
13, 10, 7

The common difference is -3.

$7 - 3 = 4$

There will be 4 squares in the next position of the geometric pattern.

ii)



Thinking

i)

Step 1: Count the number of squares in the first, second and third position and write them as a numerical sequence. Find the common difference between consecutive terms.

Step 2: Apply the common difference to the last given term in the sequence to find out how many squares there will be in the next position.

ii)

Step 1: Identify the difference of squares between each position. Each following position adds two squares, one to the top row and one to the bottom.

Step 2: Continue the geometric pattern by adding two squares in the appropriate locations to the pattern.

ii)

Step 1: Identify the difference of squares between each position. Each following position removes three squares, one from each end.

Step 2: Continue the geometric pattern by removing three squares in the appropriate locations from the pattern.

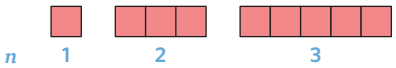
Student practice

For each pattern:

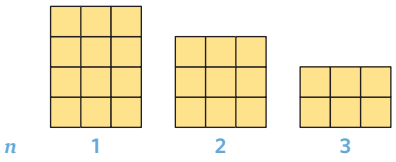
i) State how many squares there will be in the next position of the geometric pattern.

ii) Draw the next position to continue the geometric pattern.

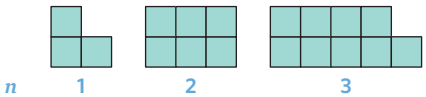
a.



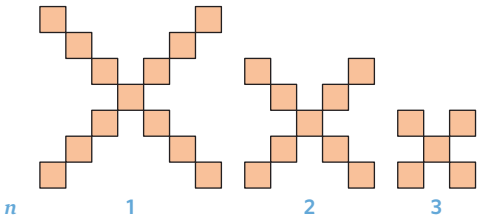
b.



c.



d.



WE 3 Using rules to complete numerical tables

Use the rule to complete each table of values.

a. $t = 3n + 2$

Position (n)	1	2	3	4	5
Term (t)					

Working

$t = 3n + 2$

$n = 1: 3 \times 1 + 2 = 5$

$n = 2: 3 \times 2 + 2 = 8$

$n = 3: 3 \times 3 + 2 = 11$

$n = 4: 3 \times 4 + 2 = 14$

$n = 5: 3 \times 5 + 2 = 17$

n	1	2	3	4	5
t	5	8	11	14	17

Thinking

Step 1: Substitute the respective n values into the rule to find the corresponding t term value.

Step 2: Complete the table by filling the corresponding values of t.

b. $t = 4n - 3$

Position (n)	1	3	5	7	9
Term (t)					

Working

$t = 4n - 3$

$n = 1: 4 \times 1 - 3 = 1$

$n = 3: 4 \times 3 - 3 = 9$

$n = 5: 4 \times 5 - 3 = 17$

$n = 7: 4 \times 7 - 3 = 25$

$n = 9: 4 \times 9 - 3 = 33$

n	1	3	5	7	9
t	1	9	17	25	33

Thinking

Step 1: Substitute the respective n values into the rule to find the corresponding t term value.

Step 2: Complete the table by filling the corresponding values of t.

Student practice

Use the rule to complete the table.

a. $t = 3n - 1$

n	1	2	3	4	5
t					

b. $t = 5n + 4$

n	1	2	3	4	5
t					

c. $t = 4n - 10$

n	3	4	5	6	7
t					

d. $t = \frac{n}{3} + 4$

n	6	9	12	15	18
t					

WE 4 Using rules to find terms

Using the given rule, find:

a. The 100^{th} term of $t = 2n + 5$.

Working

$t = 2 \times 100 + 5$
$= 200 + 5$
$= 205$

Thinking

Substitute 100 for n within the rule and solve for t to find the 100^{th} term value.

b. The 50^{th} term of $t = 5n - 40$.

Working

$t = 5 \times 50 - 40$
$= 250 - 40$
$= 210$

Thinking

Substitute 50 for n within the rule and solve for t to find the 50^{th} term value.

Student practice

Using the given rule for each, find:

a. The 10^{th} term of $t = 2n + 3$.

b. The 40^{th} term of $t = 8n - 70$.

c. The 85^{th} term of $t = n + \frac{2}{3}$.

d. The 96^{th} term of $t = \frac{n}{16} - 14$.

4H Activities and questions

STARTER TASKS

Odd spot

Some people know Sydney's Bourke St Bakery for its pork and fennel sausage rolls, however they also have some delicious sweets like their ginger brûlée tart. In 2012, one of their pastry chefs, Tim Cooper, recreated a classic Peach Melba in a challenge posed by *Good Living* to create the ultimate Australian dish.

Image: Marco Del Grande/The Sydney Morning Herald

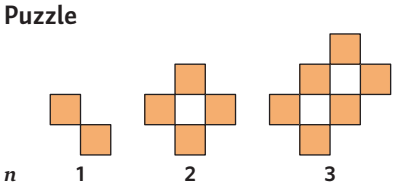
The following table shows the cups of cream required for the number of Peach Melba's made.

Peach Melbas	8	7	6
Cups of cream	$19\frac{1}{5}$	$16\frac{4}{5}$	$14\frac{2}{5}$

How many cups of cream will we need for 4 Peach Melbas?

A. $9\frac{3}{5}$ cups **B.** 4 cups

Puzzle



a) How many squares does each consecutive pattern grow by?

b) How many squares will there be in the next position of the geometric pattern?

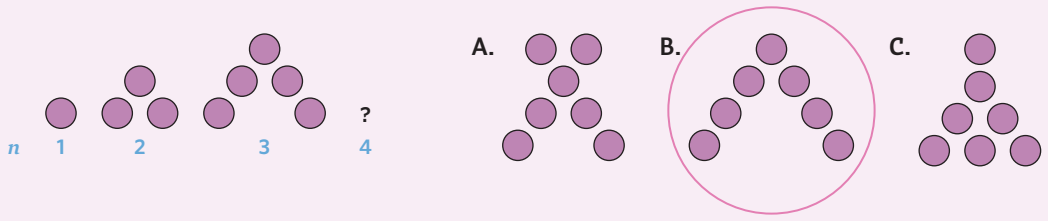
c) Draw the next figure in the geometric pattern.

d) How many squares will the 10^{th} position have?

Understanding worksheet

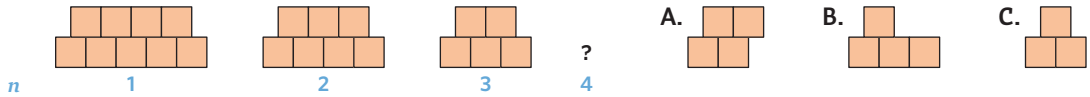
1. Circle the diagram that completes the geometric pattern.

Example

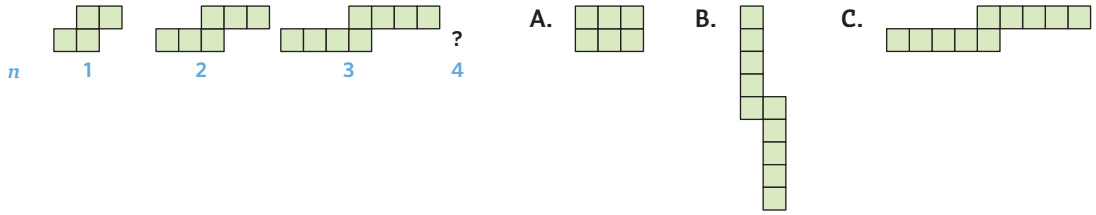


a.

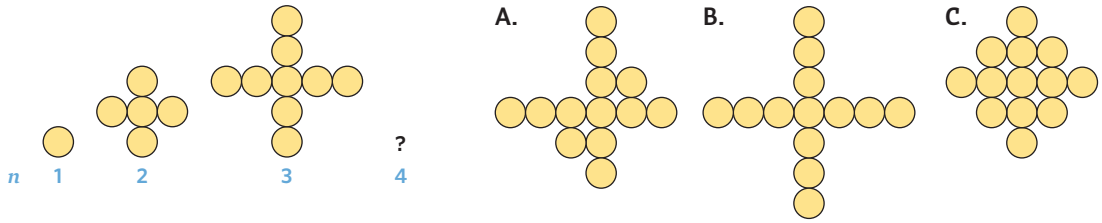
b.



c.

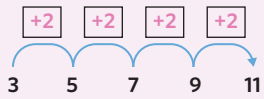


d.

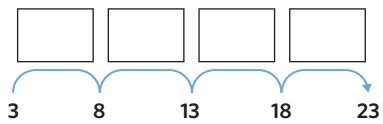


2. Identify the common difference between consecutive terms and then complete the numerical sequence.

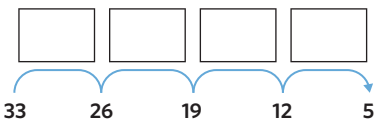
Example



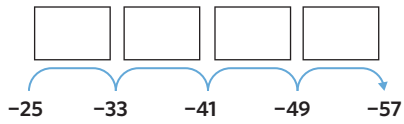
a.



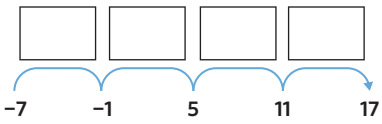
b.



c.



d.



3. Fill in the blanks by using the words provided.

rule | geometric | consistent | predict | numerical

A _____ pattern is made up of shapes that repeat in a _____ way.

Similarly, a _____ sequence is made up of numbers that increase or decrease in a consistent way.

A _____ can be used to describe geometric patterns and numerical sequences and allows us to _____ future terms.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c,d)	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (c,d,e,f)	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (e,f,g,h)

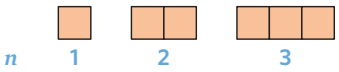
WE1 **4.** Write the next term in each numerical sequence.

a. 4, 7, 10, 13, ...
b. 20, 18, 16, 14, ...
c. 100, 115, 130, 145, ...
d. 96, 84, 72, 60, ...
e. $-40, -35, -30, -25, ...$
f. $21, 9, -3, -15, ...$
g. $-22, -16, -10, -4, ...$
h. $-9\frac{1}{2}, -6, -2\frac{1}{2}, 1, ...$

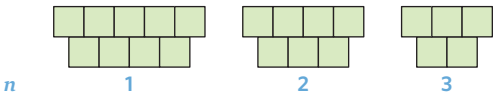
WE2 **5.** For each pattern:

i) State how many squares there will be in the next position of the geometric pattern.
ii) Draw the next position to continue the geometric pattern.

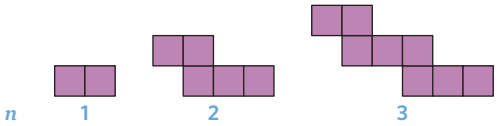
a.



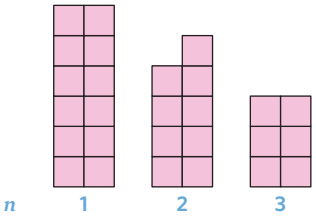
b.



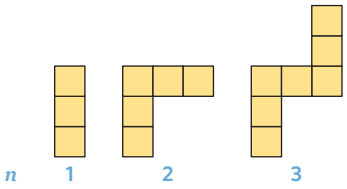
c.



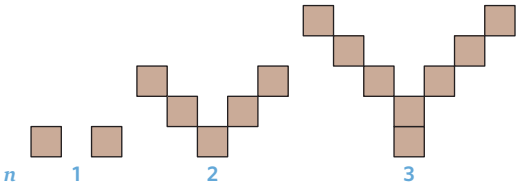
d.



e.



f.



WE3 **6.** Use the rule to complete each table of values.

a. $t = 5n - 3$

Position (n)	1	2	3	4	5
Term (t)					

b. $t = 7n + 3$

Position (n)	1	2	3	4	5
Term (t)					

c. $t = 2n + 10$

Position (n)	3	4	5	6	7
Term (t)					

d. $t = 4n - 20$

Position (n)	8	9	10	11	12
Term (t)					

e. $t = 50n - 150$

Position (n)	9	7	5	3	1
Term (t)					

f. $t = \frac{n}{4} + 6$

Position (n)	10	12	14	16	18
Term (t)					

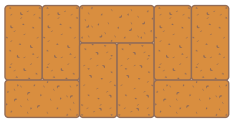
WE4 **7.** Using the given rule, find:

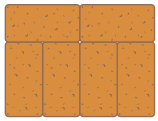
a. The 10^{th} term of $t = 6n + 3$
b. The 25^{th} term of $t = 3n + 17$
c. The 30^{th} term of $t = 10n - 70$
d. The 200^{th} term of $t = 2n$
e. The 56^{th} term of $t = \frac{n}{8} + 14$
f. The 17^{th} term of $t = 13n - 19$
g. The 40^{th} term of $t = 13n - 0.5n$
h. The 100^{th} term of $t = \frac{n}{3} - \frac{10}{9}$

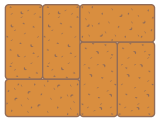
Problem solving

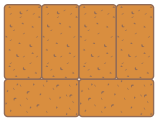
Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

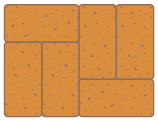
8. Hilda likes a brick wall design called the running and stack bond. It is a design that repeats itself in a consistent pattern. Which 6 brick design would continue the running and stack bond?



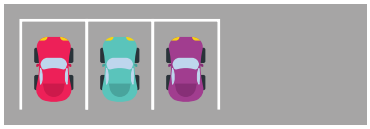
A. 

B. 

C. 

D. 

9. Magda is painting parking spots with straight lines. If she has completed 3 parking spots already, how many additional lines will she need to complete an additional 2 parking spots?



10. How much can Greg expect to pay for 5 hours of court hire if the cost of hiring for an additional hour is constant?

Hours (h)	1	2	3
Cost (c)	$20	$35	$50

11. Lenny is following a squat program which increases the recommended squat weight by the same amount each week. The program follows the rule $s = 5w + 35$, where s represents the squat weight in kilograms and w denotes the number of weeks. How much will Lenny be able to squat in the twelfth week?

12. Kishwar needs to work out how many pages she can read on her kindle in three quarters of an hour. Her kindle predicts the number of pages she can read based on the formula $p = \frac{3t}{5} + 3$, where p denotes the pages read and t represents the number of minutes spent reading.

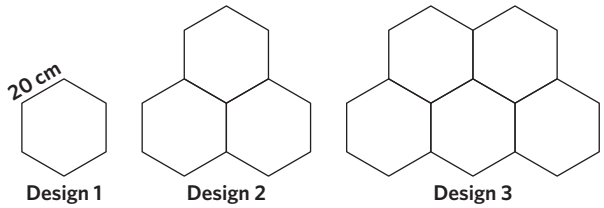
Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c,d)	All

13. Ms. Frankie and Mr. Jones need to mark the year 7 end of year maths exam papers. Ms. Frankie is able to mark 6 math exam papers per hour.

- **a.** Construct a table to show the total number of exams marked by Ms. Frankie in 1, 2, 3 and 4 hours.
- **b.** Mr. Jones is more familiar with the exam and is able to mark exams faster than Ms. Frankie. In the first hour he completes 8 exams, in the second hour 8 exams and in the third hour another 8 exams. How many exams would he expect to have marked after four hours?
- **c.** If Ms. Frankie and Mr. Jones both mark exams for 5 hours, what is the difference in the number of exams they mark?
- **d.** Is it realistic for Mr. Jones to continue this pattern for the sixth hour? Justify your answer.

14. A set of shelves is constructed by connecting 20 cm pieces of timber together to create hexagon sections. The sides of each section that are adjoining share the same piece of timber.



- **a.** Following the current geometric pattern, draw design 4.
- **b.** How many pieces of timber are added to design 3 to complete design 4?

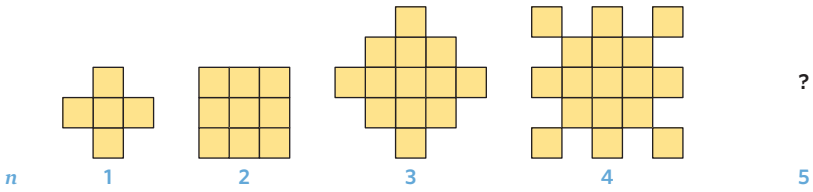
c. What will the total length of timber used to create design 4?

d. Barry wants to build design 4 and it takes him 40 minutes to complete a hexagon section. If he starts building at 2:10 pm, what time will he finish design 4?

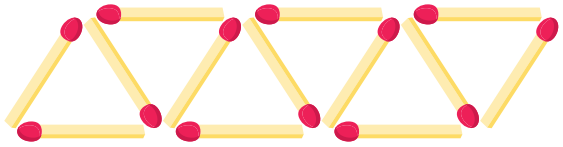
e. Barry's friend thinks the designs aren't suitable for shelves because they make storage difficult. Create some different designs with the same length of timber from design 4 that you think would be more suitable.

Extra spicy

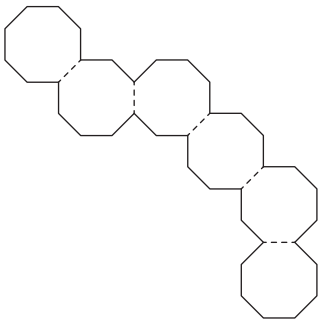
15. Draw the next figure that follows the geometric pattern.



16. If the given pattern was repeated using 127 matchsticks, how many triangles will be formed?



17. Casey is planning a new walking path using octagonal paving stones to form a continuous path across his front lawn. The path will be bordered by wood on each side of the paving stone that does not have a common edge. If 128 pieces of wood are used, how many octagonal paving stones make up the path?



18. Given the sequence of numbers ..., g, h, i, j, 0, 1, 1, 2, 3, 5, 8, ..., where g, h, i, j, are the first 4 numbers in the sequence, what would be the value of g?

Remember this?

19. Mia made some smiley face cookies and used 2 jellybeans and 5 skittles to make each face. If Mia used 60 skittles how many jellybeans did she use?

A. 24 B. 30 C. 48 D. 120

20. Tamah used a recipe that suggested he use 720 grams of flour to make 36 blueberry muffins but he only needed to make 24 muffins. How much flour would Tamah need to make 24 blueberry muffins?

A. 240 grams B. 360 grams C. 480 grams D. 620 grams

21. Together, Eunice, Gaby and Leah had 30 carnival ride tickets in total. Eunice had half as many tickets as Gaby and Leah had three times as many tickets as Eunice. How many tickets does Eunice have?

A. 5 B. 10 C. 15 D. 20

4I Creating rules from tables and sequences

Tables are an efficient way to represent different information from geometric patterns and numerical sequences. We can use the information from tables to create rules that describe these patterns or numerical sequences. In a similar way, we can create rules from numerical sequences that can be used to predict future numbers in the sequences. Below are some examples where creating rules from tables and sequences can be applied.

- The cost of my favourite frozen yoghurt can be calculated using the formula $cost\,(c) = 2 \times number\ of\ toppings + 3$. How much do I pay for each extra topping?
- I use a table to monitor the total number of bones eaten at my local kennel. What rule can I use to describe the total number of bones eaten in relation to time?

Days (d)	1 day	2 days	3 days
Total bones eaten (b)	7	14	21

- In January I joined a new book club. The number of books I own increases each month in the pattern; 27, 29, 31, 33. Write an equation to describe how my collection of books changes.

Learning intentions

Students will be able to:

+ understand how the coefficient of n in a rule affects the terms in a sequence
+ write a rule for a geometric pattern from a table
+ write a rule for a numerical sequence from a table
+ create a rule to describe a number sequence.

Key terms and definitions

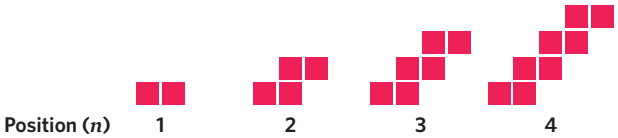
- A **rule** is a formula written using mathematical symbols and pronumerals to describe a pattern.
- A **coefficient** is the number that a pronumeral is being multiplied by. Eg. For the term $5x$ the coefficient is 5. For the term $\frac{2a}{3}$ the coefficient is $\frac{2}{3}$. For the term y the coefficient is 1.

Key ideas

1 Numerical sequences can be described using rules. The value that a pattern increases or decreases by is the coefficient of n in the rule.

Numerical sequence	**Change**	**Coefficient**	**Rule**
10, 20, 30, 40	+10	Each term **increases** by 10. The coefficient of n is 10.	$10n$
4, 9, 14, 19	+5	Each term **increases** by 5. The coefficient of n is 5.	$5n - 1$
18, 12, 6, 0	−6	Each term **decreases** by 6. The coefficient of n is −6.	$-6n + 24$

2 We can use the information from a geometric pattern to create tables that show the number of shapes in each position. Then, we can use the table to build a rule that describes the pattern.



Position (n)	1	2	3	4
Number of squares (s)	2	4	6	8

The rule for this pattern is: $s = 2n$

3 When working with numerical sequences, we refer to the n^{th} position. We can use the rule to find terms in different positions.

For the rule $term\ in\ the\ n^{th}\ position = 10n$

$term\ in\ the\ 3^{rd}\ position = 10 \times 3$

$= 30$

Position (n)	1	2	3	4
Number of squares (s)	10	20	30	40

Worked examples

WE 1 Understanding the coefficient of *n*

For each rule, state how each consecutive term will change.

a. *term in the* n^{th} *position* $= 7n - 3$

Working

The coefficient of n is 7.

The coefficient of n is positive.

Each term increases by 7.

Thinking

Step 1: Find the coefficient of n.

Step 2: State the change in each consecutive term by referring to the coefficient of n.

b. *term in the* n^{th} *position* $= -3n + 15$

Working

The coefficient of n is -3.

The coefficient of n is negative.

Each term decreases by 3.

Thinking

Step 1: Find the coefficient of n.

Step 2: State the change in each consecutive term by referring to the coefficient of n.

Student practice

For each rule, state how each consecutive term will change.

a. *term in the* n^{th} *position* $= 5n + 6$

b. *term in the* n^{th} *position* $= -8n - 4$

c. *term in the* n^{th} *position* $= \frac{2}{3}n$

d. *term in the* n^{th} *position* $= 3 - 6n$

WE 2 Writing a rule for a geometric pattern from a table

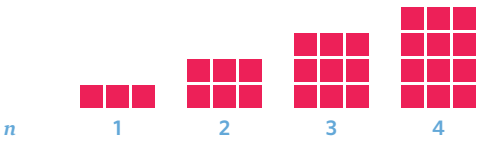
For each geometric pattern:

i) complete the table.

ii) write a rule to predict the number of shapes, s, in each position, n.

Position (n)	1	2	3	4
Number of squares (s)				

a.



Working

i)

Position (n)	1	2	3	4
Number of squares (s)	3	6	9	12

Thinking

i)

Count the number of shapes in the first, second, third and fourth position and complete the table.

ii)

The number of shapes increases by 3 each time.

$s = 3n + c$

Substitute in $n = 1$ and $s = 3$.

$3 = 3 \times 1 + c$

$3 = 3 + c$

$3 - 3 = 3 - 3 + c$

$0 = c$

$s = 3n$

ii)

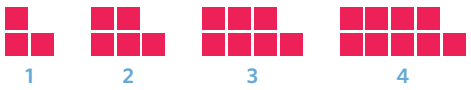
Step 1: The coefficient of n is the difference between the number of shapes in each position.

Step 2: Write an equation in the form $s = ? \times n + c$, where ? is the coefficient of n and c is the constant.

Step 3: Substitute any value of n and its corresponding value of s into the equation and solve for c.

Step 4: Write the rule.

b.



Working

i)

Position (n)	1	2	3	4
Number of squares (s)	3	5	7	9

ii)

The number of shapes increases by 2 each time.

$s = 2n + c$

Substitute in $n = 1$ and $s = 3$.

$3 = 2 \times 1 + c$

$3 = 2 + c$

$3 - 2 = 2 - 2 + c$

$1 = c$

$s = 2n + 1$

Thinking

i)

Count the number of shapes in the first, second, third and fourth position and complete the table.

ii)

Step 1: The coefficient of n is the difference between the number of shapes in each position.

Step 2: Write an equation in the form $s = ? \times n + c$, where ? is the coefficient of n and c is the constant.

Step 3: Substitute any value of n and its corresponding value of s into the equation and solve for c.

Step 4: Write the rule.

Student practice

For each geometric pattern:

i) complete the table.

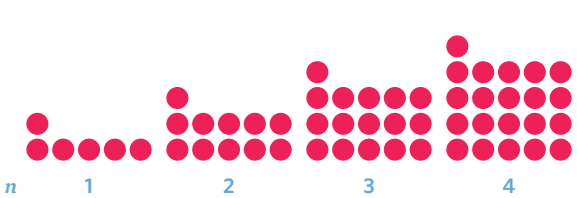
ii) write a rule to predict the number of shapes, s, in each position, n.

Position (n)	1	2	3	4
Number of squares (s)				

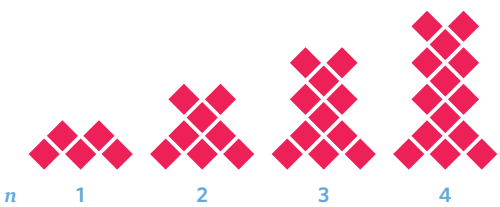
a.

b.

c.



d.



WE 3 Writing a rule for a numerical sequence from a table

For each numerical sequence:

i) complete the table.

Position (n)	1	2	3	4
Number of squares (s)				

ii) write a rule to predict the number of squares, s, in the n^{th} position, n.

a. 5, 10, 15, 20, ...

Working

i)

Position (n)	1	2	3	4
Number of squares (s)	5	10	15	20

ii)

Each term increases by 5.

$s = 5n + c$

Substitute in $n = 1$ and $t = 5$.

$5 = 5 \times 1 + c$

$5 = 5 + c$

$5 - 5 = 5 - 5 + c$

$0 = c$

$s = 5n$

Thinking

i)

Complete the table by writing the term in its corresponding position.

ii)

Step 1: The coefficient of n is the difference between each term.

Step 2: Write an equation in the form $s = ? \times n + c$, where ? is the coefficient of n and c is the constant.

Step 3: Substitute any value of n and its corresponding value of t into the equation and solve for c.

Step 4: Write the rule.

b. 4, 11, 18, 25, ...

Working

i)

Position (n)	1	2	3	4
Number of squares (s)	4	11	18	25

ii)

Each term increases by 7.

$s = 7n + c$

Substitute in $n = 1$ and $t = 4$.

$4 = 7 \times 1 + c$

$4 = 7 + c$

$4 - 7 = 7 - 7 + c$

$-3 = c$

$s = 7n - 3$

Thinking

i)

Complete the table by writing the term in its corresponding position.

ii)

Step 1: The coefficient of n is the difference between each term.

Step 2: Write an equation in the form $s = ? \times n + c$, where ? is the coefficient of n and c is the constant.

Step 3: Substitute any value of n and its corresponding value of t into the equation and solve for c.

Step 4: Write the rule.

Student practice

For each numerical sequence:

i) complete the table.

Position (n)	1	2	3	4
Number of squares (s)				

ii) write a rule to predict the number of squares, s, in the n^{th} position, n.

a. 12, 24, 36, 48, ... **b.** 3, 9, 15, 21, ... **c.** 22, 26, 30, 34, ... **d.** $-70, -60, -50, -40, ...$

4I Activities and questions

STARTER TASKS

Odd spot

The Indian Ocean Dipole (IOD) is a natural climate phenomenon that affects the rainfall patterns in locations near to the Indian Ocean. The IOD's positive phase reduces the chance of rain and can cause droughts, while the negative phase increases the chance of rainfall. Scientists have identified that due to climate change, a strong positive phase (drought) may now occur every 8 years. The IOD's last positive phase was in 2015.

Image: Alex Ellinghausen/The Sydney Morning Herald

Which rule describes which years each positive phase may now occur?

A. *positive phase* $= 2015n + 8$

B. *positive phase* $= 8n + 2007$

Puzzle

The number in each cell in the table follows a pattern.

	6	12
5	1	6
11	7	18
	8	

a) What number does represent?

b) What number does represent?

c) What number does represent?

Understanding worksheet

1. Use the geometric pattern to complete the table.

Example

Position (n)	1	2	3	4
Number of dots (s)	3	6	9	12
Change in number of dots		+3	+3	+3

a.

Position (n)	1	2	3	4
Number of dots (s)	3	5		
Change in number of dots		+2		

b.

Position (n)	1	2	3	4
Number of dots (s)	12	9		
Change in number of dots		−3		

c.

Position (n)	1	2	3	4
Number of dots (s)	1	5		
Change in number of dots				

d.

Position (n)	1	2	3	4
Number of dots (s)				
Change in number of dots				

2. Use the numerical sequence to complete the table.

Example

16, 22, 28, 34

Position (n)	1	2	3	4
Term in the n^{th} position	16	22	28	34
Change in n^{th} term		+6	+6	+6

a. 9, 16, 23, 30

Position (n)	1	2	3	4
Term in the n^{th} position	9			
Change in n^{th} term				

b. 1, 12, 23, 34

Position (n)	1	2	3	4
Term in the n^{th} position	1			
Change in n^{th} term				

c. 2, −6, −14, −22

Position (n)	1	2	3	4
Term in the n^{th} position	2			
Change in n^{th} term				

d. $-3, -1\frac{1}{2}, 0, 1\frac{1}{2}$

Position (n)	1	2	3	4
Term in the n^{th} position				
Change in n^{th} term				

3. Fill in the blanks by using the words provided.

n | sequences | position | coefficient | terms

Sequences are made up of _____ that repeat in a consistent way. We can write rules to describe _____. In the rule, *term in the* n^{th} *position* $= 4n - 2$, the pronumeral _____ refers to the _____ of a term. If a sequence is increasing, the _____ of *n* will be positive. If a sequence is decreasing, the coefficient of *n* will be negative.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f)

WE1 **4.** For each rule, state how each consecutive term will change.

a. *term in the* n^{th} *position* $= 10n - 2$

b. *term in the* n^{th} *position* $= -2n + 7$

c. *term in the* n^{th} *position* $= n + 28$

d. *term in the* n^{th} *position* $= \frac{3}{4}n$

e. *term in the* n^{th} *position* $= 7 + 8n$

f. *term in the* n^{th} *position* $= 15 - \frac{4}{5}n$

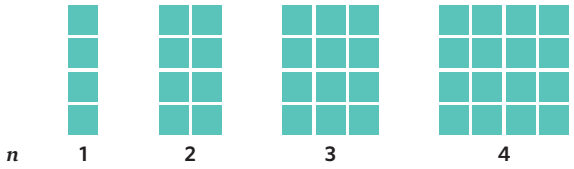
WE2 **5.** For each geometric pattern:

i) complete the table.

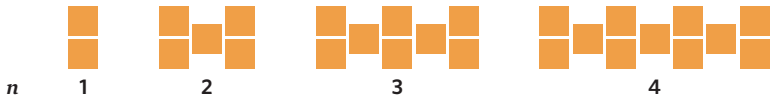
Position (*n*)	**1**	**2**	**3**	**4**
Number of shapes (*s*)				

ii) write a rule that represents the number of shapes, *s*, in each position, *n*.

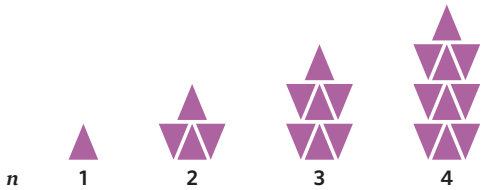
a.



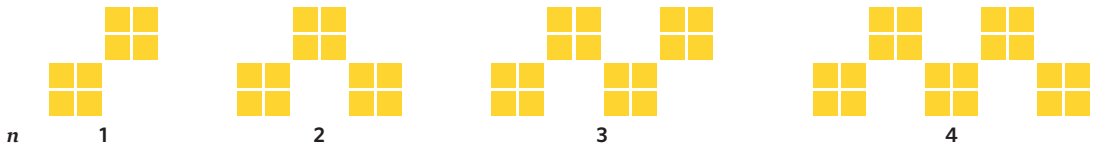
b.



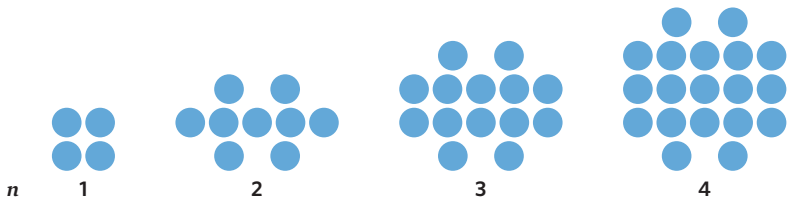
c.



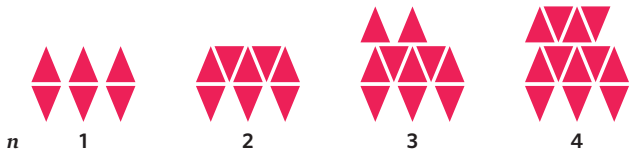
d.



e.



f.



WE3 6. For each numerical sequence:

i) complete the table.

Position (n)	1	2	3	4
Term in the n^{th} position (t)				

ii) write a rule to predict the term in the n^{th} position, t.

a. 15, 30, 45, 60, ...

b. 2, 8, 14, 20, ...

c. 16, 21, 26, 31, ...

d. $-6, -3, 0, 3, ...$

e. $-50, -43, -36, -29, ...$

f. $-10\frac{1}{3}, -6\frac{1}{3}, -2\frac{1}{3}, 1\frac{2}{3}, ...$

7. For each numerical sequence:

i) write a rule to predict the term in the n^{th} position, t.

ii) find the 50^{th} term.

a. 9, 18, 27, 36, ...

b. 12, 17, 22, 27, ...

c. 1, 13, 25, 27, ...

d. $-175, -150, -125, -100, ...$

e. $8\frac{1}{4}, 11\frac{3}{4}, 15\frac{1}{4}, 18\frac{3}{4}, ...$

f. $-6\frac{5}{6}, -4\frac{1}{3}, -1\frac{5}{6}, \frac{2}{3}, ...$

Problem solving

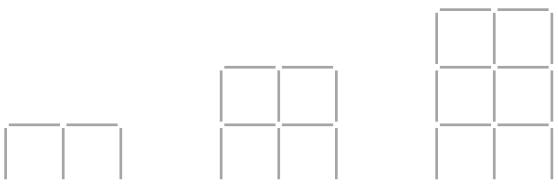
Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Frank uses the following rule to model the amount of water in litres, w, inside his bath n minutes after he turns on the tap: $w = 12n$. How much water is added to the bath each minute according to the rule?

9. On his first run, Drew ran for 7 minutes. After the first run, Drew extended the amount of time he ran in each consecutive session by 6 minutes. Fill in the table to show how many minutes Drew will run in the fifth, sixth and seventh session of his routine.

Session number (n)	5	6	7
Number of minutes (m)			

10. The number of concrete slabs needed to build apartment buildings increases constantly for each additional level of the building.



Write a rule to predict the number of concrete slabs required, s, from the number of levels, l, in an apartment building. Note: The concrete slabs refer to the lines in the diagram and not the boxes that are created.

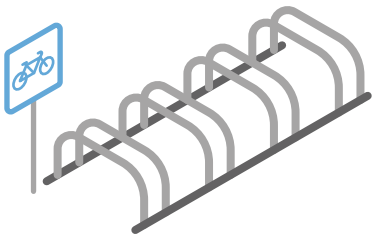
11. Tom is knitting a jumper. The number of stitches he uses to knit a size 1, 2 and 3 jumper is 56, 63 and 70 stitches respectively. If the pattern of number of the stitches he uses for each size continues, write a rule to describe the number of stitches (s) required for an n size jumper.

12. A boat leaves a port and travels 80 kilometres east per day. Where is it after the 4th day of its journey in relation to Fraser Island, if the departure port was 74 kilometres west of Fraser Island?

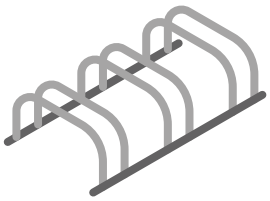
Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c,d)	All

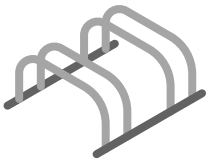
13. The Sunshine Coast Ironman is held annually. The organisers need to build an outdoor bike station where all the race participants can store their bikes on a bike rack. In order to prepare the required bike storage area, the organisers have the following images which show the length of each rack.



110 cm: 4 bikes



80 cm: 3 bikes



50 cm: 2 bikes

20 cm: 1 bike

a. Represent the information provided in a table to show the relationship between the number of bikes stored and the length of the bike rack.

b. Write a rule to show the relationship between the number of bikes stored (b) and the length, in cm, (l) of the storage rack.

c. Race organisers are unsure if 250 people will participate in the race or 300 people. How much shorter is a single rack for 250 bikes than a single rack for 300 bikes?

d. Due to good weather, 300 people will be completing the race. The organisers want to use 6 separate storage racks, each with the same number of bikes. How much shorter will the total length of the racks be than if they had used one long storage rack for all 300 bikes?

e. Some Ironman races have up to 1700 participants. Propose an efficient way to store the bikes that is fair and allows each participant to easily locate their bike.

14. The cost of having a sailing party at Leopold's Yacht Club varies depending on the number of people that are invited. The cost of a party for adults has an additional flat fee for an extra chef. A party for children has an extra flat fee for two qualified lifesavers.

The following table shows the cost of a party for adults and a party for children based on the number of attendees.

Number of attendees (a)	Cost in dollars (c) of a party for adults	Cost in dollars (c) of a party for children
1	190	305
2	245	335
3	300	365
4	355	395

a. Assuming that the cost increases in a consistent way, create two rules to describe the relationship between the number of attendees (a) and the cost in dollars (c) for a party for adults, and a party for children.

b. What is the difference between the cost of each additional adult at a party for adults and the cost of each additional child at a party for children?

c. What is the cost of one lifesaver at a party for children?

d. How many adults need to attend a party so that it is more expensive than a party for children with the same number of attendees?

e. Leopold's Yacht Club has received some complaints because people planning parties for more than 4 people need to continue the pattern from the table to calculate the cost of their parties. Suggest a way that the yacht club could better communicate the cost of their parties.

Extra spicy

15. Find the missing terms in the numerical sequence $\frac{1}{12}$, ..., $\frac{1}{4}$, ..., ..., and state the coefficient of n in its n^{th} term rule.

16. Write a general rule to find the n^{th} term, t, for the sequence $\frac{2}{3}, \frac{5}{6}, \frac{30}{30}, \frac{14}{12}, 1\frac{1}{3}$, ...

17. What value of x makes the following four expressions consecutive terms in a linear number sequence?

$11 + x \quad 4x + 3 \quad 6x \quad 37$

18. The first four terms of a linear sequence add to $25\frac{1}{3}$. If the first term of the sequence is $\frac{1}{3}$, what is the rule to find the n^{th} term, t, of the sequence?

Remember this?

19. What is 280 written correctly as a product of its prime number factors?

A. $3^2 \times 5 \times 7$ **B.** $2^3 \times 5^2 \times 7$ **C.** $2^3 \times 5 \times 7$ **D.** $2^3 \times 35$

20. The table shows the fraction of New South Wales students enrolled in different courses.

Course	Midwifery	Construction	Science	Business
Fraction of students	$\frac{3}{150}$	$\frac{3}{50}$	$\frac{3}{20}$	$\frac{3}{25}$

Which of the courses has the highest number of students enrolled?

A. Science **B.** Midwifery **C.** Construction **D.** Business

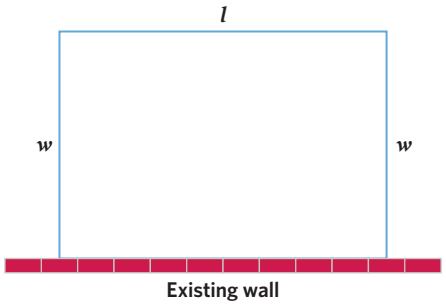
21. A fish and chips shop sells a serving of calamari for 2.5 times the price of a medium chips. 2 servings of calamari and 3 medium chips cost Mila $30.

What is the cost of one medium chips?

Chapter 4 extended application

1. You are travelling around Tasmania and need a form of transportation to get around. You consider hiring a car from one of two companies.
   a. Hiring a car from Buzz Rentals costs $100 plus another $25 per day. This follows the equation $cost = 100 + 25d$, where d is the number of days that a car is hired for. How much does it cost to rent a car for 5 days?
   b. Hiring a car from Woody Automotive costs $200 plus $15 per day. What equation could you use to find the cost of hiring a car from Woody Automotive?
   c. If you were to hire a car for two weeks, which company would you hire from and why?
   d. You then find out that Buzz Rentals contribute $10 per day to carbon offsets (planting trees to absorb pollution created by driving). Does this impact your decision at all? Why or why not?
   e. If you weren't able to hire a car, what other ways could you get around on your holiday in Tasmania?

2. A farmer needs to build a new fence around a paddock to keep his sheep enclosed. The farmer plans to use an existing wall as one side of the paddock. The following image shows a birds eye view of the planned sheep paddock, with the fence shown in blue.



   a. Choose 3 different sets of values for w and l that would use up exactly 100 m of fencing.
   b. Find the area for each of your chosen values in part **a** using the formula $A = l \times w$.
   c. Which of the three sets of dimensions would you use and why?
   d. If the farmer instead needs to create two identical paddocks using the same amount of fencing wire (100 m), how would you arrange them? Include measurements.
   e. How many sheep would you set as the limit to fit inside one of the farmer's paddocks? Why?
   f. With a partner, decide how you would fence in 100 sheep if each metre of fencing wire costs $50.

3. 1 Australian dollar ($1 AUD) is worth approximately 50 Indian rupees. A bank charges $5 AUD to convert any amount of Australian dollars into Indian rupees. The equation to exchange Australian dollars (A) for Indian rupees (R) is $R = 50(A - 5)$.
   a. How many Indian rupees would you receive from the bank in return for $100 AUD?
   b. How many Australian dollars would you need to exchange to recieve 2000 Indian rupees?
   c. In India, petrol costs around 70 rupees per litre, while in Australia it costs around $1.50 per litre. Can you explain why this might be?
   d. In Australia the median wage is $48 000. In India the median wage is 46 000 rupees. Which country do you think is the wealthier country and why?

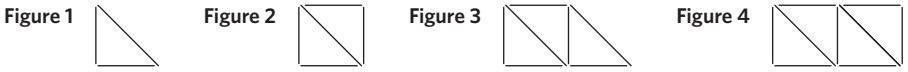
Chapter 4 review

Multiple choice

1. What number should go in the trapezium to balance the scales? 4A

A. 3 B. 4 C. 5 D. 6

2. How many sticks will there be in figure 6? 4H



A. 11 B. 13 C. 15 D. 16

3. Which flowchart shows how to solve the equation $8d = 56$? 4D

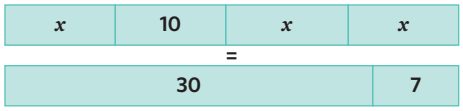
A. $8d = 56$ → (×8) → $d = 7$

B. $8d = 56$ → (×8) → $d = 8$

C. $8d = 56$ → (÷8) → $d = 7$

D. $8d = 56$ → (÷8) → $d = 8$

4. Find the value of x in the bar model. 4E



A. $x = 3$ B. $x = 9$ C. $x = \frac{37}{3}$ D. $x = 27$

5. The number of pencils in a box is p. 4C

How many pencils are there in total when $p = 12$?

A. 12 B. 19 C. 43 D. 55

Fluency

6. Complete the equations to make them true. 4A

a. $7 + 1 + 4 = \bigstar$ b. $7 - 6 + 8 = \bigstar + 6$

c. $3 + 3 + 3 = 3 \times \bigstar$ d. $8 \times 2 - 7 = 1 + 2 + 3 + 4 + 5 - \bigstar$

7. Simplify each expression using correct algebraic notation. 4B

a. $x \times y$ b. $9 + x \times y$

c. $x \div y$ d. $x \times 4 \times y \times (3 \div z)$

8. Write an equation to represent each situation. 4B

a. Tahlia planted t apple trees in her orchard. After a year, 5 new apple trees began growing, bringing the total to 41 trees.

b. Zephyr weighed w kg before he went on a trip overseas. When he came back he found that he had lost 6 kg over the course of his trip and now weighed 69 kg.

c. Sophie bought a pot plant for $\$p$. Two weeks later the pot plant had a new price which was triple the original price. The new price was $39.

d. Bartholomew invited x friends to his 21st birthday party. 19 friends came, but this was only a quarter of the number of friends that he invited.

9. Substitute $x = 4$ and $y = 6$ and state whether each equation is true or false. 4C

a. $x + y + x = 14$ **b.** $2x = 26 - 3y$ **c.** $\frac{5y}{10} + 4x = 13$ **d.** $5x + y = 3x + 2y$

10. The area of a triangle (A) can be found using the formula $A = \frac{bh}{2}$, where b is the base length and h is the height of a triangle. Find the area of a triangle if: 4C

a. $b = 8$ cm and $h = 2$ cm **b.** $b = 10$ km and $h = 3$ km

c. $b = 11$ m and $h = 70$ m **d.** $b = 19$ mm and $h = 4$ mm

11. Solve each equation. 4D

a. $y - 11 = 13$ **b.** $4m = 32$ **c.** $\frac{x}{5} = 40$ **d.** $\frac{c}{2} = \frac{11}{6}$

12. Solve each equation. 4E

a. $4x + 8 = 20$ **b.** $-4 + 6c = 38$ **c.** $\frac{y + 5}{4} = 12$ **d.** $6m - 9 = 11$

13. Write an equation and then solve for the unknown. 4E

a. When I multiply k by 4 and then add 8, the result is 24.

b. When I divide a by 9 and then add 5, the result is 12.

c. When I subtract 4 from z and then divide by 6, the result is 5.

d. When I multiply x by 7 and then divide it by 3, the result is 14.

14. Solve for x. 4F

a. $\frac{6x}{5} - 3 = 3$ **b.** $12(x + 7) = 132$ **c.** $6 = \frac{3(x - 5)}{9}$ **d.** $\frac{4(x - 113)}{64} = \frac{1}{8}$

15. Simplify the equations by collecting like terms. 4G

a. $8x - 5x = 24$ **b.** $11j - 4k - 7j + 9k = 12 - 16$

c. $9qr - 6q + 12q - 11rq + q = 20 + 6 - 1$ **d.** $-4s + t + 9s + 2t - 4t = 37$

16. Simplify and solve for x. 4G

a. $4x + x = 45$ **b.** $2x - 14 + 8x + 24 = 3 + 13 - 1$

c. $\frac{x}{4} + 5 + x = 10$ **d.** $11 - 6 + 2x + \frac{x}{2} = 30$

17. Determine the first three numbers ($n = 1, 2, 3$) in the sequence for each equation. 4H

a. $s = 4n + 1$ **b.** $s = 3n - 8$ **c.** $s = 6n + 17$ **d.** $s = 11n - 13$

18. Using the given rule, find: 4H

a. the 5^{th} term of $t = 2n + 3$ **b.** the 12^{th} term of $t = 5n - 11$

c. the 40^{th} term of $t = 11n - 3.5$ **d.** the 63^{rd} term of $t = \frac{n}{9} + \frac{4}{3}$

19. For each numerical sequence, write a rule to predict the term in the n^{th} position, t. 4I

a. 11, 14, 17, 20, ... **b.** $-200, -180, -160, -140, \ldots$

c. $3\frac{1}{4}, 4\frac{3}{4}, 6\frac{1}{4}, 7\frac{3}{4}, \ldots$ **d.** $-4\frac{1}{2}, -2, \frac{1}{2}, 3, \ldots$

Problem solving

20. Bobby is aiming to run 42 km over a single week. Between Monday and Saturday he ran 5 km per day. How many km does he need to run on Sunday? 4A

21. Horace is making a replica model of a farm. He needs 200 small wooden pieces, and 120 large wooden pieces. The small pieces are x cm long and the large pieces are y cm long. The total length of the wooden pieces is 3600 cm. Write an equation to represent this situation. 4B

22. Donald owns a patisserie. In a single day he sold 24 boxes of croissants and 43 single croissants. If there are 4 croissants in a box, how many croissants did Donald sell in total? 4C

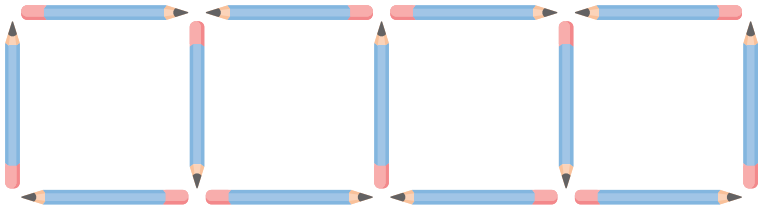
23. Rafael ordered an extra large Halal Snack Pack for him and 3 of his friends to share. It had a weight of 2400 g. Using w to represent the amount of grams each of them ate, create an equation that represents this situation and solve for w. 4D

24. In a game of AFL a goal is worth 6 points and a behind is worth 1 point. The Giants scored 13 behinds and ended on 97 points. Using g to represent the number of goals they kicked, create an equation that represents this situation and solve for g. 4E

25. James is planning his 22nd birthday party. He is planning on giving a gift to everyone he invites. He invites c friends from his cricket team and 2 friends from work. The gifts will cost \$20 each and he plans on spending \$260 in total. How many friends did he invite from his cricket team? 4F

26. Sid went to the store to buy cat food. He bought 3 boxes of biscuits, 4 cans of tuna and a packet of red meat. Each box of biscuits costs $\$3x$, each can of tuna costs $\$x$ and each packet of red meat costs $\$4x$. If the total price was \$34, how much does a packet of red meat cost? 4G

27. Roger uses pencils to create the following pattern with four squares. 4H

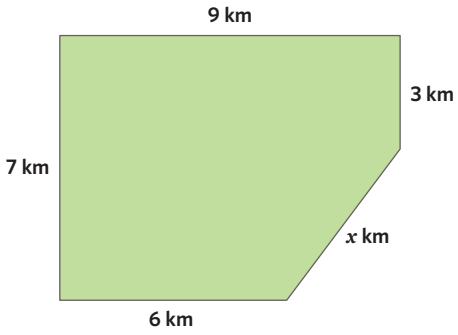


How many pencils in total are needed to make six squares?

28. Felix is a competitive swimmer and has made a 28-day training plan for an upcoming competition. On the first day of his training plan, he will swim 3750 metres. Each day, he will increase the distance he swims by 50 metres. How far will Felix swim on the last day of his training plan? 4I

Reasoning

29. Tommy just bought a plot of land. Its dimensions are shown in the following diagram.



a. What is the length of x if the perimeter is 30 km?

Note: The perimeter is the sum of all the side lengths.

b. Tommy buys another plot of land that has the same shape. However, each side length is m times as large. Create a simplified equation that represents the perimeter (P) of the new plot of land.

c. Each side length of the new plot of land is 3 times as long as in the original plot. Use your expression from part **b** to find the perimeter of the new plot.

d. Tommy finds that the number of fence posts (f) he needs for his electric fence follows the equation $\frac{f}{110} = P$, where P is the perimeter of the plot. How many fence posts would he need for the original plot?

e. Would you rather live in a city or in the country? Give a reason for your answer.

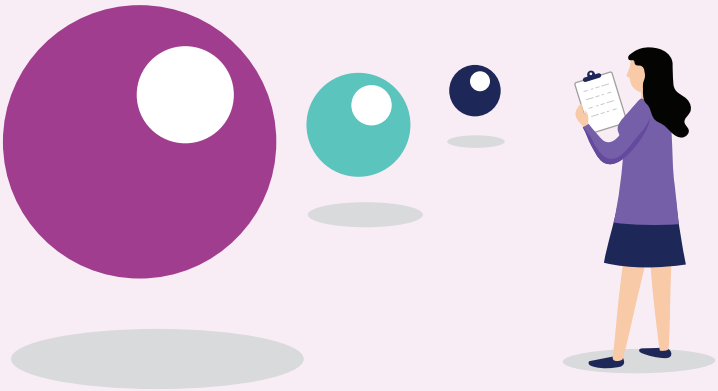
30. Arthur is completing a month-long push up challenge. On the first day he must do 10 push ups and then increase the number of push ups he does by 4 each day.

a. Write an equation to represent this pattern using p as the number of push ups and n as the day number.

b. On what day does he do 50 push ups?

c. After this, Arthur went on holiday for 2 days and forgot to do any push ups. This makes the equation $p = 4(n - 2) + 6$ for all the days after his holiday. On what day did he do 70 push ups?

d. Is it a good idea for Arthur to keep doing this challenge after the end of the month? Would you modify how many extra push ups Arthur does each day?



05

Decimals and percentages

Number and Algebra

Research summary

5A Place value and comparing decimals *(Revision)*

5B Rounding decimals

5C Adding and subtracting with decimals *(Revision)*

5D Multiplying and dividing by powers of 10

5E Multiplying with decimals

5F Dividing with decimals

5G Introduction to percentages

5H Converting between fractions, decimals, and percentages

5I Finding percentages of a quantity

Chapter 5 extended application

Chapter 5 review

Image: Marish/Shutterstock.com

Research summary – Decimals and percentages

Big ideas

The base 10 fractions

Despite being regularly exposed to decimals in the form of money, sporting statistics and on scales, decimals still remain a difficult concept for many. Part of the problem is that when first introduced to decimals, we are often taught to perceive the digits after the decimal point as whole values in an attempt to reduce their difficulty and increase our ability to manipulate them. However, working with decimal digits as whole numbers can cause many misconceptions. Instead, we should always view decimals as what they are, base 10 fractions, also referred to as decimal fractions. These are fractions with the denominators of 10, 100, 1000, and so on.

Decimal fractions and place value

By the time we begin learning about decimals, we would have spent time working with the place value system with whole numbers. Just as the base 10 structure of '10 makes 1' extends to digits in larger and larger place values, it also extends to smaller and smaller place values. Instead of regrouping tens or hundreds, we instead regroup one-tenths, one-hundredths and so on. This place value interpretation of decimals should always be used when working with decimals to make the connection between the decimal place and its fractional value. For example, the number 34.67 is also seen as $34 + \frac{67}{100}$ and $34 + \frac{6}{10} + \frac{7}{100}$. This concept is particularly important when operating with decimals, or performing computations with decimals such as ordering and rounding.

The decimal point

The decimal point represents the bridge between the whole values in a number and its fractional parts, and sits between the ones and the tenths place values. Often when we are multiplying or dividing decimals by factors of 10, we see this decimal point as moving to the right or the left depending on whether the number is getting bigger or smaller. However, the decimal point does not in fact move but remains fixed in its position between the ones and tenths. Instead, it is the value of the digits that when regrouped, move to a different place along the place value system. For example, when a number is multiplied by 10, the digits move into the larger place value on the left rather than the decimal point moving one place to the right. This is an important construct because it reinforces the establishment of the place value system for both decimals and whole numbers.

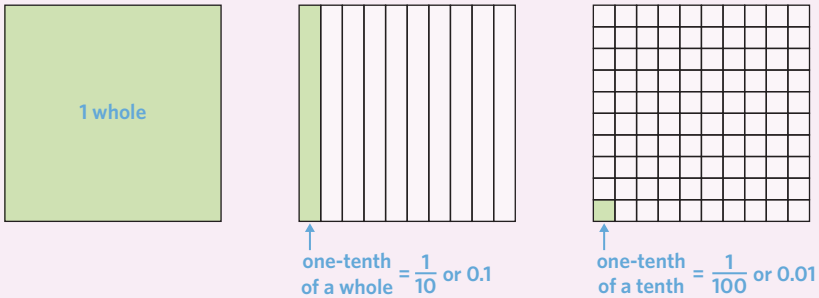
Making the connection with percentages

Percentages can be problematic for many people despite their common real world applications. This may be because they are often introduced using computational processes rather than using conceptual techniques. At its core, the term percent means per hundred and should be understood as a standard ratio where the denominator is 100. They can be represented using the percentage symbol '%'. When we start investigating percentages, the importance of working with decimal fractions becomes very apparent. This is because the term percent can be substituted for the term hundredths so that 37% is understood to be $\frac{37}{100}$ or 0.37 – highlighting the connection with decimal fractions. Once this conceptual understanding of percentages as hundredths has been established, working practically with percentages becomes much simpler.

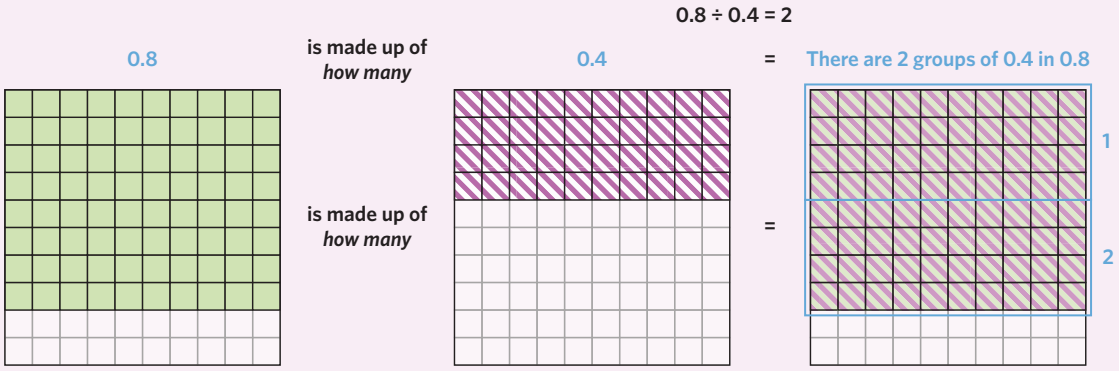
Visual representations

Hundreds grid

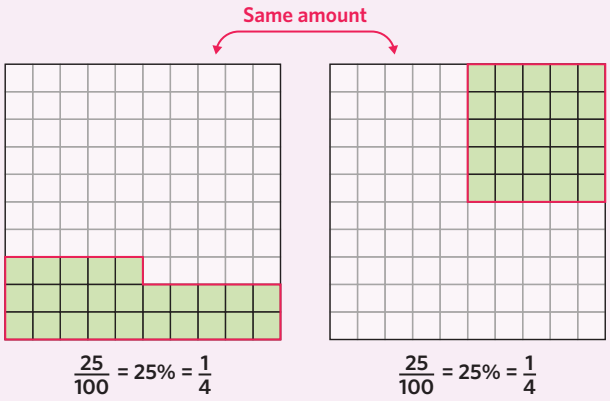
A hundreds grid is a visual representation that encourages the use of fractional language to describe decimals. The hundreds grid consists of a large square that represents one whole. This whole is then broken up into ten parts which represent the tenths. Each tenth is further broken into another ten parts, where each part represents hundredths. Shading different parts of the hundreds grid provides a conceptual way to make connections between decimals, fractions and percentages.



It is a useful way of connecting the multiplication and division of decimals to its abstract process.

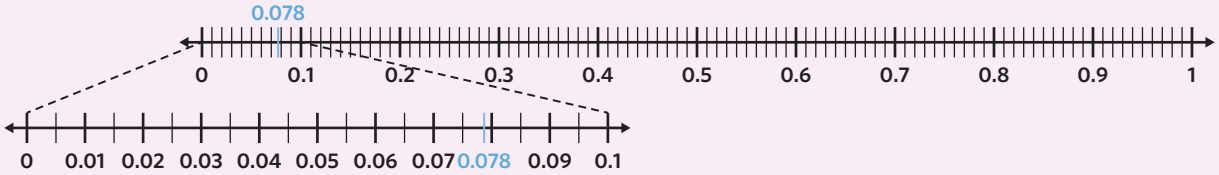


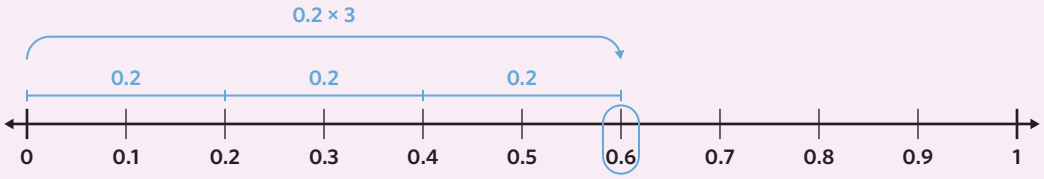
Hundredths grids, when used with percentages, can help build a deeper understanding of the link between fractions, decimals and percentages. They can also be used to provide a conceptual link when it comes to computation with percentages and can make working with these ratios simpler.



Number lines

Number lines are a great way of comparing fractions and decimals. They are also useful and very important when ordering and investigating the magnitude of decimals. A zoomed-in number line like the one shown can also help establish an understanding of decimals as smaller and smaller base 10 fractional parts. Number lines can also be used to show operations with decimals.





Number sliders

Number sliders can be used when multiplying and dividing decimals by powers of 10. They are most useful when it comes to addressing misconceptions around the decimal point and building knowledge around decimal place value.

Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students state decimals as whole numbers without considering the place value.	1.35 is 'one point thirty-five'.	1.35 is one and thirty-five hundredths.	5A
Students possess the 'longer is larger' misconception where they believe that a decimal with more decimal places makes it larger without considering place value.	$3.45 > 3.8$ because 3.45 has more digits which makes it larger.	$3.45 < 3.8$ because $3\frac{45}{100} < 3\frac{80}{100}$	5A
Students possess the 'shorter is larger' misconception where they believe that a decimal with less decimal places makes it larger without considering place value.	$3.45 < 3.2$ because tenths are larger than hundredths.	$3.45 > 3.2$ because $3\frac{45}{100} > 3\frac{2}{10}$	5A
Students ignore the decimal point and treat the decimal as a whole value.	$4.08 > 5.6$ because $408 > 56$	$4.08 < 5.6$ because $4\frac{8}{100} < 5\frac{60}{100}$	5A
Students don't see the zero as having any meaning as a place value holder.	$0.305 = \frac{35}{1000}$	$0.305 = \frac{305}{1000}$	5A
Students use the digit being rounded instead of the critical digit to round decimals.	2.37 to the nearest tenth is 2.3 because $3 < 5$.	2.37 to the nearest tenth is 2.4 because $7 > 5$.	5B
When the rounding digit is 9, students round to 10 instead of regrouping to the next place value.	Round 0.95 to the nearest whole number. $0.95 \approx 0.10$ ✗	Round 0.95 to the nearest whole number. $0.95 \approx 1.0$ ✓	5B
Students misrepresent place value when rounding.	246.72 to the nearest tenth is 247 because the tenth digit is used to round the decimal.	246.72 to the nearest tenth is 246.7.	5B
Rounding 'down' means the digit being rounded becomes one less.	5.53 rounded to the nearest tenth is 5.4.	5.53 rounded to the tenth is 5.5.	5B

	Incorrect	Correct	Exercise
When adding decimals, students add the whole numbers and decimal places separately.	$22.16 + 15.89 = 37.105$	$22.16 + 15.89 = 38.05$	5C
Students add or subtract decimals without considering place value.	$22.16 + 0.1 = 22.17$ and $15 - 0.1 = 14$	$22.16 + 0.1 = 22.26$ and $15 - 0.1 = 14.9$	5C
Students think in terms of whole numbers and apply the operation to both sides of the decimal point when multiplying by a power of 10.	$4.72 \times 10 = 40.720$	$4.72 \times 10 = 47.2$	5D
Students think in terms of whole numbers and apply the operation to both sides of the decimal point when dividing by a power of 10.	$30.50 \div 10 = 3.5$	$30.5 \div 10 = 3.050$ $= 3.05$	5D
Students use regrouping in the same way they do with whole numbers when multiplying decimals ignoring the decimal point.	$\begin{array}{r} {\scriptstyle 2} \\ 0.5 \\ \times\ 0.5 \\ \hline 2.5 \end{array}$	$0.5 \times 0.5 = ?$ $5 \times 5 = 25$ So $0.5 \times 0.5 = 0.25$ (with two decimal places).	5E
Students use the same method used in multiplication when dividing decimals, counting the number of decimal places.	$0.25 \div 0.5 = 0.005$ Because there are 3 decimal places in total and $25 \div 5 = 5$.	$0.25 \div 0.5 = 0.5$	5F
Students see the percent as the denominator when converting to fractions.	$15\% = \frac{1}{15}$ $25\% = \frac{1}{25}$	$15\% = \frac{15}{100}$ $25\% = \frac{25}{100}$	5G
Students use the incorrect method when converting fractions to percentages.	Expressing 20 out of 40 as a percentage is: $\frac{20}{100} \times 40 = 8\%$	$\frac{20}{40} \times 100 = 50\%$	5G
Students see the decimal as a whole number when converting it to a percentage.	0.2 expressed as a percentage: $0.2 = 2\%$ ✗	0.2 expressed as a percentage: $0.2 \times 100 = 20\%$ ✓	5H
Students add or subtract the percentage value to the amount when increasing or decreasing an amount by a given percentage.	Increasing 35 by $10\% = 45$	Increasing 35 by 10% $35 \times 1.1 = 38.5$	5I
Students think that a percentage of a number is fixed. They don't recognise this can change as the number changes.	If 25% of $40 = 10$ then, 25% of $60 = 10$.	25% of $60 = 15$	5I
Students see percentages as a value instead of a proportion.	25% always equals 0.25 so, 25% of $30 = 0.25$.	25% of 30 0.25×30 $= 7.5$	5I

Additional reading and resources

- Why is learning fraction and decimal arithmetic so difficult? Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Developmental Review, 38, 201–221.
- Decimats: Helping Students To Make Sense Of Decimal Place Value. Anne Roche. Australian Primary Mathematics Classroom vol. 15 no. 2, 4–11
- Decimal squares – Interactive Decimal grids: http://www.decimalsquares.com/
- Interactive hundredths grid – fractions, decimals percentages: https://www.visnos.com/demos/percentage-fraction-decimals-grid

5A Place value and comparing decimals

A decimal number is made up of a whole number part and a fractional part that are separated by a decimal point. The digits following the decimal points have a value less than 1. We most commonly see decimals expressed in decimal notation such as 2.134; where the whole number is represented by the 2 and the fractional parts by the three digits 1, 3 and 4 respectively after the decimal point. Below are some examples where decimals can be applied.

- The petrol station closest to me had their fuel advertised for 145.9 cents per litre and the one in the next suburb had it advertised for 145.90 cents per litre. Which station sells cheaper petrol?
- During triple jump practice I jumped the following distances: 10.129 metres, 10.2 metres and 10.12 metres. What is the order of my three jumps from least to greatest distance jumped?
- While watching the 2020 Olympics men's gymnastics finals, the top 4 results were: Great Britain: 255.760, Japan: 262.397, Russia: 262.50 and China 261.894. Who won the gold, silver and bronze medals?

Learning intentions

Students will be able to:

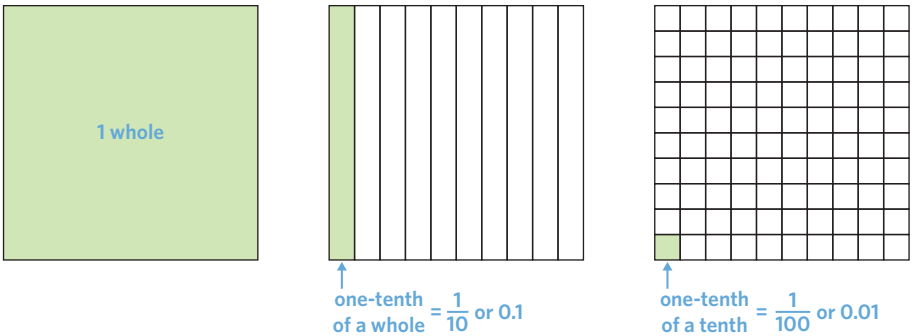
- use visuals to represent decimals and represent a decimal using a visual
- express decimals as decimal fractions and in expanded decimal form, expanded fraction form, decimal notation, and word form
- compare two decimals to determine which is larger or smaller and place a set of decimals in ascending order.

Key terms and definitions

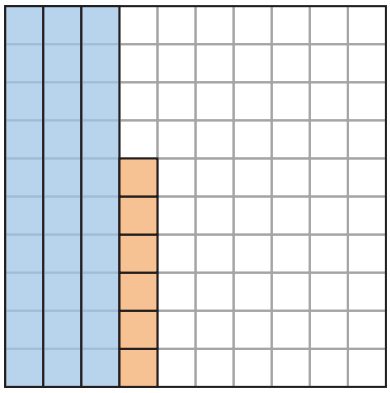
- A **digit** is a single numeral from 0–9 that is used to make a number. E.g. 57 is made up of the digits 5 and 7.
- **Decimal notation** is a number that is expressed using digits that are separated by a decimal point. E.g. 1.35 is a number expressed in decimal notation.
- A **decimal fraction** is a fraction where the denominator is a power of 10, such as 10, 100, 1000, 10 000. E.g. the decimal 0.25 written as a decimal fraction is $\frac{25}{100}$.
- A decimal expressed in **expanded decimal form** is written as a sum of all of its place value parts. The value of the digits to the right of the decimal point are expressed in decimal notation. E.g. 3.245 is written in expanded decimal form as $3 + 0.2 + 0.04 + 0.005$.
- A decimal expressed in **expanded fraction form** is written as a sum of all of its place value parts. The value of the digits to the right of the decimal point are expressed as decimal fractions. E.g. 3.245 is written in expanded fraction form as $3 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$.
- **Word form** means writing the number in words while ensuring that the decimal point is replaced by the word 'and'. E.g. 3.245 in word form is written as 'three and two hundred forty-five thousandths'.
- **Ascending** order means to arrange values from smallest to largest.

Key ideas

1 A hundreds grid is a visual way of showing how decimals can be connected to fractions of a whole.



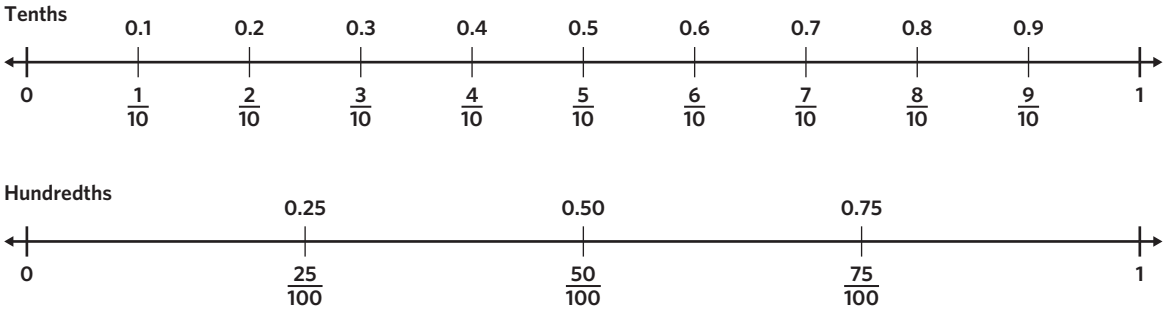
Place value	Decimal	Fraction	Is $\frac{1}{10}$ of...
Tenth	0.1	$\frac{1}{10}$	a whole
Hundredth	0.01	$\frac{1}{100}$	a tenth
Thousandth	0.001	$\frac{1}{1000}$	a hundredth



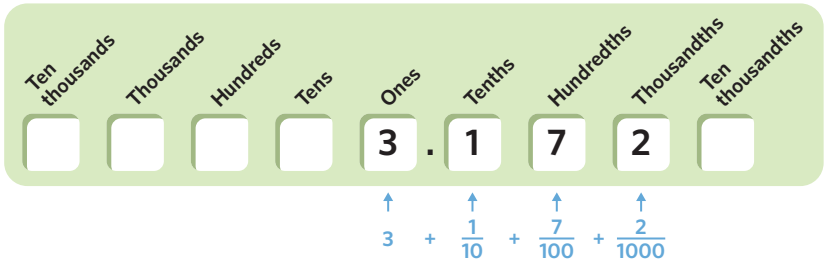
We can shade different fractions of the hundreds grid to represent a decimal.

$= \frac{3}{10} + \frac{6}{100} + 0.36$

2 Like fractions, we can also place decimals on a number line. Each decimal can be written as a fraction.



3 Decimals can be written as decimal fractions or in expanded decimal, expanded fraction, or word form.



Decimal notation	Decimal fraction	Expanded decimal form	Expanded fraction form	Word form
3.172	$3\frac{172}{1000}$ or $\frac{3172}{1000}$	$3 + 0.1 + 0.07 + 0.002$	$3 + \frac{1}{10} + \frac{7}{100} + \frac{2}{1000}$	Three, and one hundred seventy-two thousandths.

Worked examples

WE 1 Finding the place value as a fraction

Write the value of the highlighted digit(s) in each decimal as a fraction.

a. 1.4596

Working

The '9' is in the third decimal place indicating a place value of thousandths.

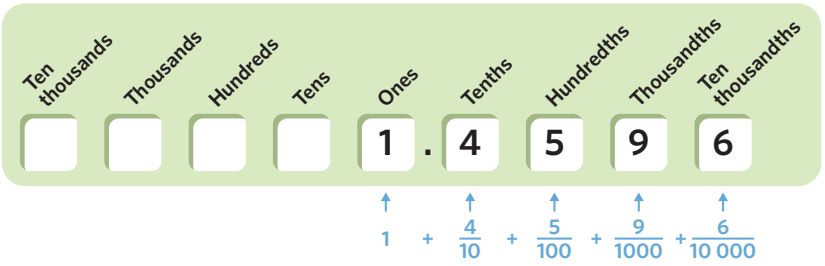
$\frac{9}{1000}$

Thinking

Step 1: Count the position to the right of the decimal point of the highlighted digit.

Step 2: Write a fraction with the highlighted digit as the numerator and the denominator based on the digit's place value.

Visual support



b. 12.765

Working

The '6' is in the second decimal place indicating a place value of hundredths.

The '5' is in the third decimal place indicating a place value of thousandths.

The '6' is $\frac{6}{100}$.

The '5' is $\frac{5}{1000}$.

$\frac{6}{100} + \frac{5}{1000}$

$= \frac{60}{1000} + \frac{5}{1000}$

$= \frac{65}{1000}$

Thinking

Step 1: Count the position to the right of the decimal point for each highlighted digit.

Step 2: Write a fraction for each highlighted digit with the digit as the numerator and the denominator based on the digits' place value.

Step 3: Add the fractions to find the total place value of the highlighted digits.

Student practice

Write the value of the highlighted digit(s) in each decimal as a fraction.

a. 0.4567 **b.** 19.05102 **c.** 15.2199 **d.** 40.334

WE 2 Writing in decimal notation

Write each of the following in decimal notation.

a. $0.6 + 0.05$

Working

0.6 is 6 tenths, so there is a 6 in the first decimal place.

0.05 is 5 hundredths, so there is a 5 in the second decimal place.

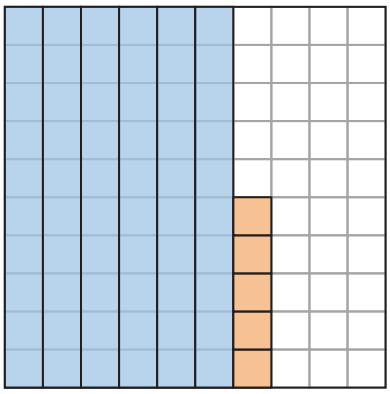
0.65

Thinking

Step 1: Find which decimal place each part will occupy.

Step 2: Write the decimal. As there is no whole number, there should be a 0 to the left of the decimal point.

Visual support



0.6 + 0.05 = 0.65

b. $300 + 40 + 5 + \frac{3}{10} + \frac{7}{1000}$

Working

$300 + 40 + 5 = 345$

$\frac{3}{10}$ is 3 tenths, so there is a 3 in the first decimal place.

$\frac{7}{1000}$ is 7 thousandths, so there is a 7 in the third decimal place.

0.307

$345 + 0.307 = 345.307$

Thinking

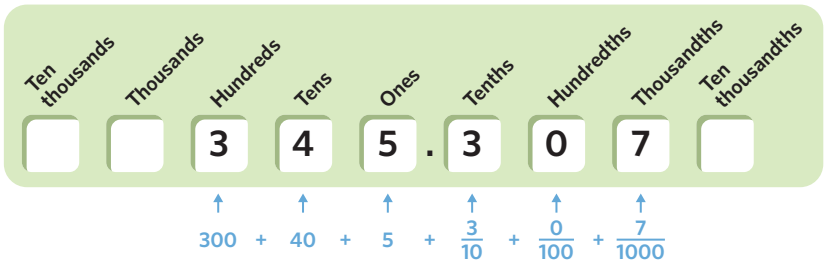
Step 1: Simplify the whole numbers by completing the addition.

Step 2: Find which decimal place each fraction will occupy.

Step 3: Write the decimal value. As there is no hundredth fraction, there should be a 0 in the second decimal place.

Step 4: Add the whole number and decimal together.

Visual support



c. $\frac{2347}{100}$

Working

$\frac{2347}{100} = 23\frac{47}{100}$

$= 23 + \frac{47}{100}$

$\frac{47}{100} = 0.47$

$23 + 0.47 = 23.47$

Thinking

Step 1: Convert the decimal fraction to a mixed number to find the whole number part of the decimal.

Step 2: Identify the decimal place of the fraction using the denominator. Identify the decimal value using the numerator.

Step 3: Add the whole number and decimal together.

d. Eighty-four and two hundred forty-five thousandths.

Working

'Eighty-four' is the whole number 84.

'and' represents the decimal position.

'Two hundred forty-five' is the numerator.

'Thousandths' indicates a denominator of 1000.

$= 84\frac{245}{1000}$

$\frac{245}{1000} = 0.245$

$84 + 0.245 = 84.245$

Thinking

Step 1: Write the sentence out as a mixed number.

Step 2: Identify the decimal place of the fraction using the denominator. Identify the decimal value using the numerator.

Step 3: Add the whole number and decimal together.

Student practice

Write each of the following in decimal notation.

a. $0.7 + 0.01 + 0.009$

b. $2000 + 500 + 7 + \frac{3}{10} + \frac{1}{100} + \frac{6}{1000}$

c. $\frac{1345}{10\,000}$

d. Twenty-seven and thirty-nine thousandths.

WE 3 Comparing decimal values

Compare the decimals by placing the correct mathematical symbol (<, > or =) in the box.

a. $5.2874 \square 5.2864$

Working

Whole numbers: 5.2874 □ 5.2864

Tenths: 5.2874 □ 5.2864

Hundredths: 5.2874 □ 5.2864

Thousandths: 5.2874 □ 5.2864

$7 > 6$

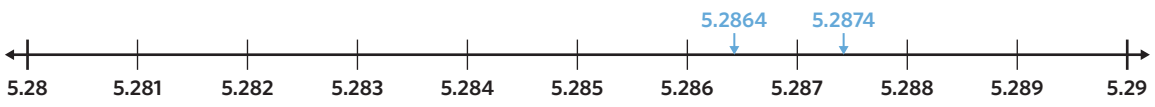
$5.2874 > 5.2864$

Thinking

Step 1: Identify the digits in each place value for each decimal starting from left to right. Stop when there is a different digit in the same place value.

Step 2: Compare the differing digits. The larger digit in the same place value will indicate the larger decimal.

Visual support



b. $10 + 3 + \frac{12}{1000} \square 13.12$

Working

$10 + 3 + \frac{12}{1000} = 13.012$

Whole numbers: 13.012 $\square$ 13.12

Tenths: 13.012 $\square$ 13.12

$0 < 1$

$13.012 < 13.12$

$10 + 3 + \frac{12}{1000} < 13.12$

Thinking

Step 1: Convert all values into decimal notation.

Step 2: Identify the digits in each place value for each decimal starting from left to right. Stop when there is a different digit in the same place value.

Step 3: Compare the differing digits. The larger digit in the same place value will indicate the larger decimal.

Student practice

Compare the fractions by placing the correct mathematical symbol (<, > or =) in the box.

a. 0.3216 $\square$ 0.322

b. 5.1900 $\square$ 5.19

c. $130.16 \square 100 + 30 + \frac{1}{10} + \frac{6}{1000}$

d. $18\frac{405}{1000} \square 10 + 8 + \frac{4}{10} + \frac{5}{100}$

WE 4 Ordering decimal values

Place each set of decimals in ascending order.
4.701, 4.7000, 4.8, 4.07, 4.68451

Working

Whole numbers: 4.701, 4.7000, 4.8, 4.07, 4.68451

Tenths: 4.07 < 4.68451 < 4.701 $\square$ 4.7000 < 4.8

Hundredths: 4.701 $\square$ 4.7000

Thousandths: 4.701 > 4.7000

4.07, 4.68451, 4.7000, 4.701, 4.8

Thinking

Step 1: Compare the digits in each place value for each decimal starting from left to right. Stop when there is a different digit in the same place.

Step 2: List the decimals from smallest to largest.

Student practice

Place each set of decimals in ascending order.

a. 1.43, 1.34, 1.86, 1.68, 1.6

b. 1.41, 1.40, 1.14, 1.15, 1.5

c. 5.9997, 5.99997, 5.0999997, 5.10, 5.8999

d. 100.1001, 101.0001, 101, 100.1101, 100.01010

5A Activities and questions

STARTER TASKS

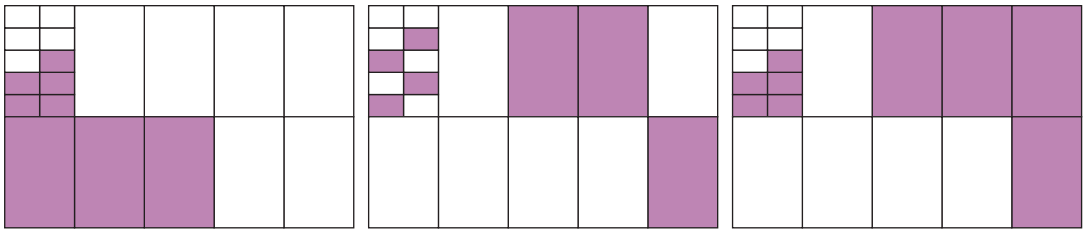
Odd spot

In 1966, Australia's currency changed from the British imperial system of pounds, shillings and pence to the decimal currency system of dollars and cents.
This image is a photo from 1966 showing how toys would have been priced in both currency systems. For the price of $7.55 what does the first 5 after the decimal point represent as a decimal fraction?

A. $\frac{50}{100}$ B. $\frac{5}{100}$

Image courtesy of the National Archives of Australia. NAA: A1200, L52585.

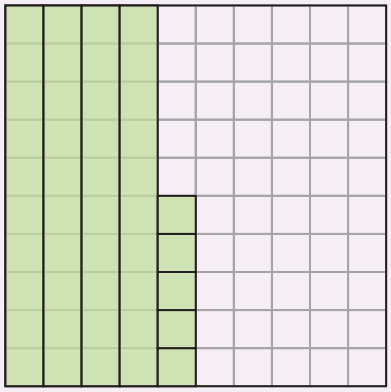
Puzzle



a) What do you notice about how the whole rectangle in each image is divided?
b) Which rectangle has the largest purple area? Which rectangle has the smallest purple area?
c) What fraction of the whole rectangle does the purple area represent in each rectangle? Be prepared to explain your thinking.

Understanding worksheet

1. In the boxes provided, state the value represented by the hundreds grid.

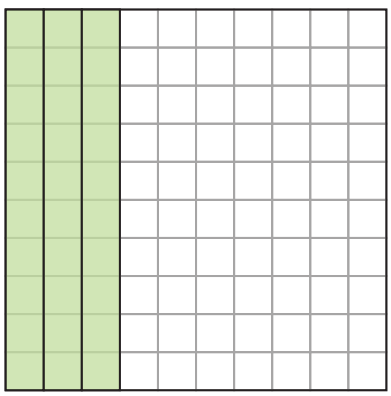


Example

Expanded decimal form: $0.4 + 0.05$ Decimal notation: 0.45

Expanded fraction form: $\frac{4}{10} + \frac{5}{100}$ Decimal fraction: $\frac{45}{100}$

a.



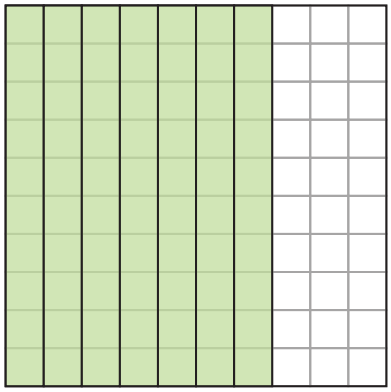
Expanded decimal form
0.3

Decimal notation

Expanded fraction form
$\frac{3}{10}$

Decimal fraction

b.



Expanded decimal form

Decimal notation

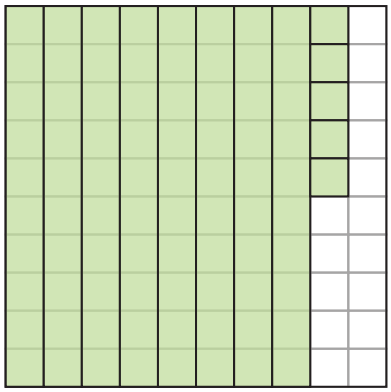
0.7

Expanded fraction form

Decimal fraction

$\frac{7}{10}$

c.



Expanded decimal form

$0.8 + 0.05$

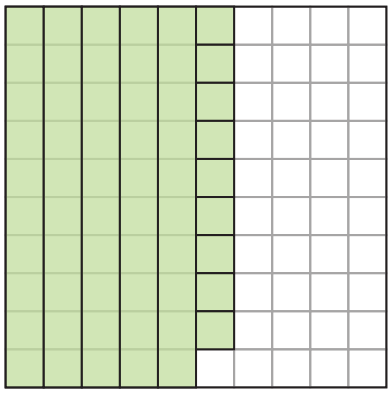
Decimal notation

Expanded fraction form

$\frac{8}{10} + \frac{5}{100}$

Decimal fraction

d.



Expanded decimal form

Decimal notation

0.59

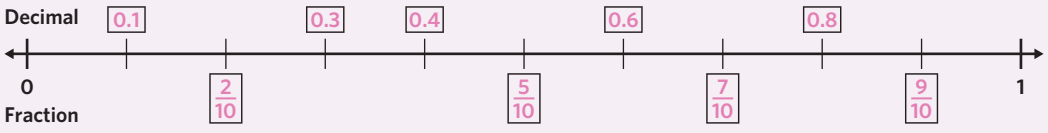
Expanded fraction form

Decimal fraction

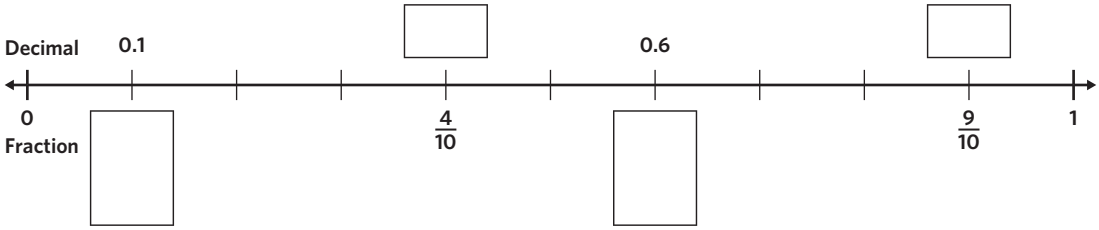
$\frac{59}{100}$

2. Complete the number lines by placing the correct decimal or fraction in each box.

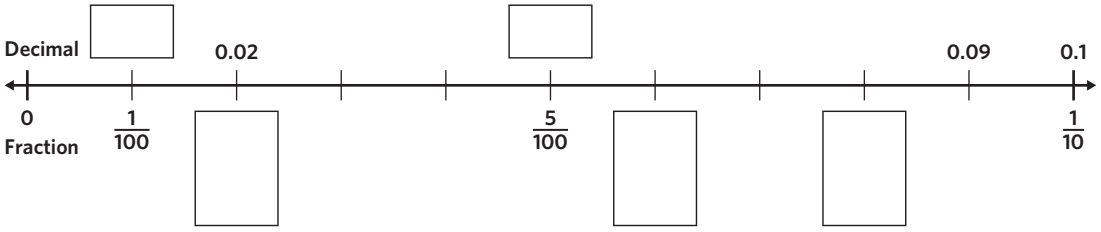
Example



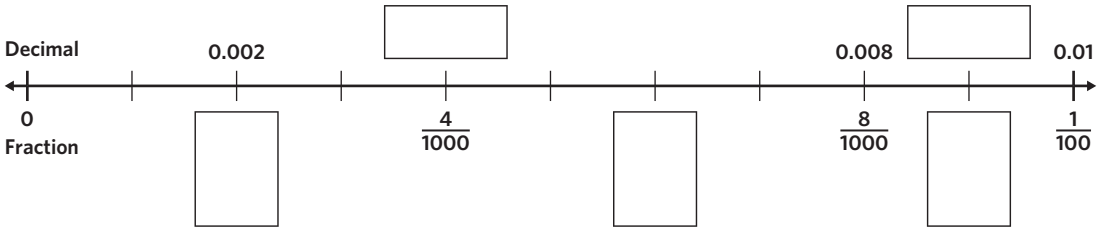
a.



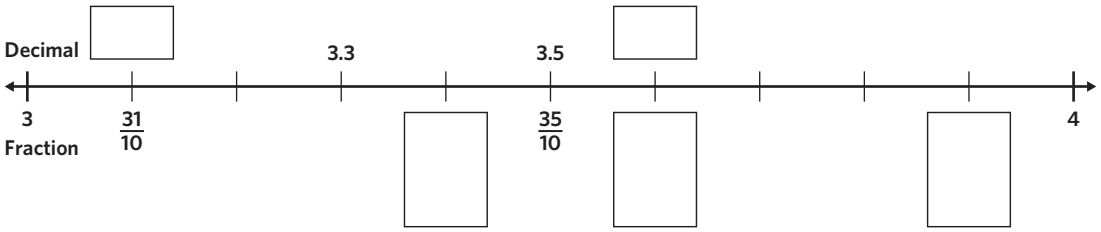
b.



c.



d.



3. Fill in the blanks by using the words provided.

digit | decimal notation | decimal fraction | expanded form

A single number that has a decimal point is said to be expressed in _____. Decimals can be written in _____ where the value of each _____ to the right of the decimal point is written as a decimal or _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b), 6 (a,b), 7 (a,b), 8 (a,b), 9 (a,b,c,d), 10 (a,b,c)	4 (c,d,e,f), 5 (b,c), 6 (b,c), 7 (b,c), 8 (b,c), 9 (c,d,e,f), 10 (b,c,d)	4 (e,f,g,h), 5 (c,d), 6 (c,d), 7 (c,d), 8 (c,d), 9 (e,f,g,h), 10 (d,e,f)

WE1 4. Write the value of the highlighted digit(s) in each decimal as a fraction.

a. 0.2347 b. 0.1543 c. 23.400 d. 1.701

e. 85.527 f. 124.601 g. 76.124 h. 13.0002

WE2a 5. Write each of the following in decimal notation.

a. 0.4 + 0.07 + 0.009

b. 7 + 0.2 + 0.08 + 0.008 + 0.0001

c. 20 + 2 + 0.4 + 0.02 + 0.0003

d. 79 + 0.4 + 0.03 + 0.008 + 0.0002

WE2b 6. Write each of the following in decimal notation.

a. $\frac{2}{10} + \frac{3}{100} + \frac{5}{1000}$

b. $3 + \frac{6}{10} + \frac{2}{100} + \frac{9}{1000}$

c. $48 + \frac{7}{100} + \frac{1}{10\,000}$

d. $120 + 14 + 7 + \frac{1}{10} + \frac{6}{1000} + \frac{3}{100\,000}$

WE2c 7. Write each of the following in decimal notation.

a. $\frac{5}{10}$

b. $\frac{159}{1000}$

c. $2\frac{75}{100}$

d. $\frac{497}{10}$

WE2d **8.** Write each of the following in decimal notation.

a. Three hundred twenty-seven thousandths.

b. One hundred sixty-three and forty-five hundredths.

c. Ten thousand five hundred and sixty-one thousandths.

d. Seventy-five thousand three hundred eighty-eight and two hundred forty-one thousandths.

WE3 **9.** Compare the fractions by placing the correct mathematical symbols (<, > or =) in the box.

a. $0.156 \square 0.2$

b. $23\frac{5}{10} \square 23.50$

c. $\frac{1}{10} + \frac{9}{100} \square \frac{1}{10} + \frac{9}{1000}$

d. $2.175 \square 2 + \frac{3}{10}$

e. $15.9 \square 15.9000$

f. $40 + 5 + \frac{3}{10} + \frac{1}{1000} \square 45.31$

g. $29\frac{9998}{10000} \square 29.998$

h. $1001.1001 \square 1001.1010$

WE4 **10.** Place each set of decimals in ascending order.

a. 0.125, 0.47, 0.12, 0.308, 0.501

b. 0.999, 0.23, 0.001, 0.105, 1.000

c. 1.0123, 1.123, 1.1023, 1.1203, 1.12

d. 3.10, 3.9, 2.9990, 3.0999, 2.9998

e. 6.245, 6.0452, 6.8, 6.79, 6.00001

f. 19.1113, 19.0009, 19.11113, 19.001, 19.1

Problem solving

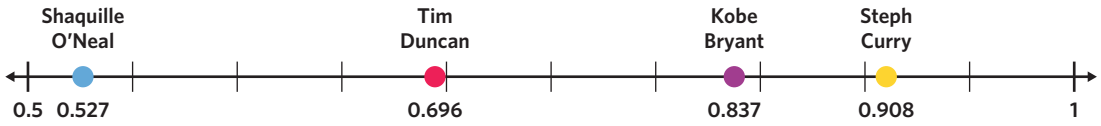
Mild	Medium	Spicy
11, 12, 13	12, 13, 14	13, 14, 15

11. Piper and Dorethea went on a hike together. After finishing their hike they checked their smart watches to see the total distance they had walked. Who walked further if Piper's watch displayed 6.357 km and Dorethea's watch displayed 6.36 km?

12. What distance does the '7' represent on Piper's smart watch reading of 6.357 km if there are 1000 metres in a kilometre?

13. Nate had 25 mL of his 1000 mL jug of milk left, so he picked up another 2 litres from the supermarket. How much milk does Nate now have altogether in litres?

14. The probability that LeBron James makes a free throw is 0.73. The probability that 4 other NBA stars make a free throw is shown on the number line.



How many of the four NBA stars is Lebron James better than at free throws?

15. Melanie wants to pick the drink that has the least amount of sugar. Use the table to determine which soft drink Melanie should choose.

Soft drink (375 mL)	Sugar (g/375 mL)
Coca-Cola	40.787
Dr Pepper	40.0999
Fanta	40.8
Solo	40.93

Reasoning

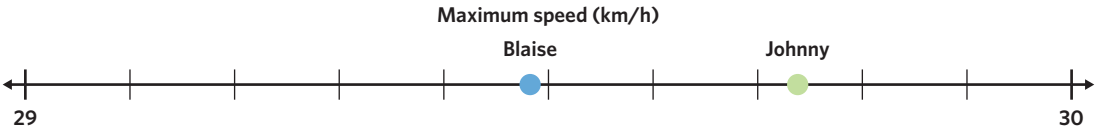
Mild	Medium	Spicy
16 (a,b,c,d)	16 (a,b,c,d), 17 (a,b,c,d)	All

16. Four track athletes tested their maximum sprinting speed. Their results are shown in the following table.

Name	Malachy	Blaise	Johnny	Bailey
Speed (km/h)	29.9	29.49	29.74	29.557

a. Who has the fastest maximum speed?

b. Copy the number line and add Malachy and Bailey to the following number line.



c. Write Blaise's speed as a decimal fraction.

d. Write Johnny's speed in expanded fraction form.

e. Do you think Malachy, Blaise, Johnny and Bailey's maximum speeds are reasonable? Explain why.

17. Max and Delilah are working together in their year 7 maths classroom. They are given the following six individual cards where one of the cards is a decimal point.

9 0 5 4 7 .

a. Using three cards only, what is the closest number to 500 that Max and Delilah can make?

b. Using two number cards and the decimal point card, what is the closest number to 5 that Max and Delilah can make?

c. Using four number cards and a decimal point card after the third digit, what number can Max and Delilah make that is the furthest from 450 on a number line?

d. Using all six cards what is the closest number to 85.6 that Max and Delilah can make?

e. Max and Delilah need to use all the cards to make the smallest and largest numbers possible. Propose how they could organise their time so they could complete the task efficiently.

Extra spicy

18. The value of $\frac{17}{25}$ as a decimal is:

A. 0.17 **B.** 0.68 **C.** 0.068 **D.** 6.8 **E.** 0.017

19. Write $\frac{0.8}{4}$ as a decimal fraction.

20. Simplify $2\frac{1}{4} + 3\frac{2}{5}$ and express the answer as a decimal.

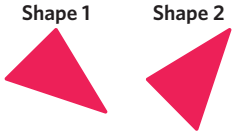
21. Use the digits 0–9 once each in the spaces below to create a true equation.

□.□□ + □.□□ = □□.□□

Remember this?

22. Which of these describes the translation needed to get from shape 1 to shape 2?

A. 45° anticlockwise
B. 45° clockwise
C. 90° anticlockwise
D. 90° clockwise



23. Which of these values for [?] makes the following number sentence false?

$7 \times [?] < 50$

A. 5 **B.** 6 **C.** 7 **D.** 8

24. The number 0.12 is between

A. 0.001 and 0.01. **B.** 0.01 and 0.1. **C.** 0.1 and 1. **D.** 1 and 10.

5B Rounding decimals

Decimal numbers can sometimes contain more decimal places than is needed. Decimals can be rounded to a value with fewer decimal places - this is useful to estimate an approximate answer. Rounding decimals is commonly used when working with money and measurement. Rounding decimals is also used in division calculations with remainders. Below are some examples where rounding decimals can be applied.

- My recipe for cinnamon scrolls requires 250 grams of flour. My digital scale shows 249.9875 grams. If I round to the nearest gram, do I have enough flour?
- My three fastest times for the 25 m freestyle are 16.176 seconds, 16.179 seconds and 16.174 seconds. What is my fastest time rounded to a hundredth of a second?
- I bought four items at a cafe for the following prices: $4.93, $5.67, $9.21, $8.17. How much did I approximately spend if I rounded the cost of each item to the nearest dollar?

Learning intentions

Students will be able to:

- round decimals to the nearest whole number
- round decimals up to 4 decimal places
- use rounding to estimate calculations with decimals.

Key terms and definitions

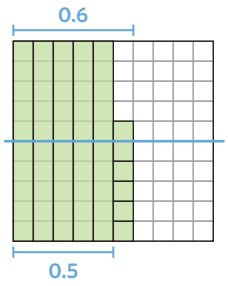
- **Rounding** is the process of making a number simpler to use in calculations.
- **Estimates** are approximate values that are close to the right answer.
- The term **approximately equal** to is represented in calculations using the '≈' sign.
- A **critical digit** is the digit to the right of a digit being rounded. If the critical digit is 5 or above, add 1 to the digit being rounded. If the critical digit is less than 5, leave the digit being rounded the same.
- **Benchmark decimals** are decimals that are recognisable, such as 0.25, 0.50 and 0.75. They help us to more accurately make estimates about the position of a number on a number line.

Key ideas

1 A hundreds grid can be used when rounding to the nearest tenth or whole number.

If a tenths column is **half or more** shaded, round to the next tenth.

0.56 rounded to the nearest tenth:

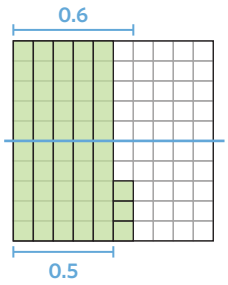


0.56 is between 0.5 and 0.6
0.56 is closer to 0.6

0.56 ≈ 0.6

If the tenths column is **less than half** shaded, then the digit being rounded remains the same.

0.53 rounded to the nearest tenth:



0.53 is between 0.5 and 0.6
0.53 is closer to 0.6

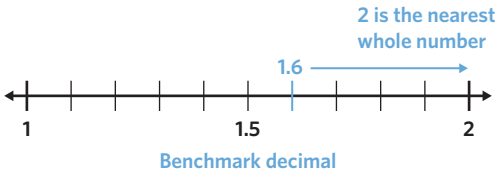
0.53 ≈ 0.5

2 A number line can be used to show how a decimal number is rounded. A benchmark decimal is used as a point of reference to help us round.

1.6 rounded to the nearest whole number:

1.6 is between 1 and 2.

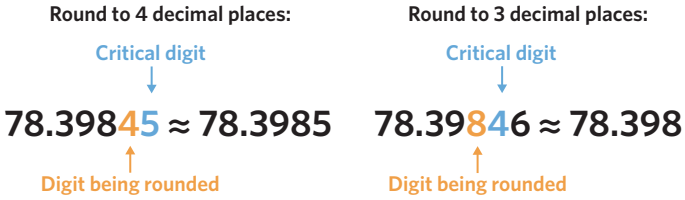
1.5 is the benchmark decimal halfway between 1 and 2, so we can see 1.6 is rounded to 2 as it is greater than 1.5.



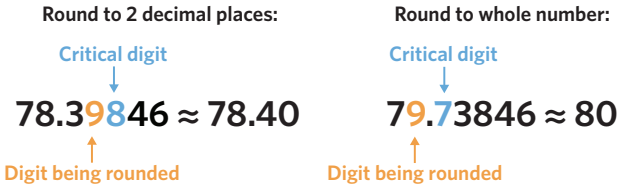
3 When rounding to a specific place value, consider the first digit to the **right** of the digit being rounded. This is the **critical digit.**

If the **critical digit** is **5 or greater**, increase the digit being rounded by 1 and all the digits in the smaller place values change to zeros.

If the **critical digit** is **less than 5**, then the digit being rounded remains the same and all the digits in smaller place values change to zeros.



4 If the **digit being rounded** is **9**, increasing the digit being rounded by 1 is 10, so regroup to the next place value.



Worked examples

WE 1 Rounding to a whole number

Round each decimal to the nearest whole number.

a. 2.357

Working

2.357

$3 < 5$

$2.357 \approx 2.000$

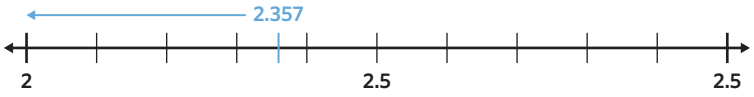
$2.357 \approx 2$

Thinking

Step 1: Identify the whole number and underline the critical digit.

Step 2: The critical digit is less than 5, so the digit being rounded stays the same. Replace all digits to the right of the digit being rounded with zeros.

Visual support



b. 9.962

Working

9.962

$9 \geq 5$

$9.962 \approx 10.000$

$9.962 \approx 10$

Thinking

Step 1: Identify the whole number and underline the critical digit.

Step 2: The critical digit is greater than 5, so increase the digit being rounded by 1. As the digit being rounded is 9, increasing it by 1 is 10, so regroup to the next place value. Replace all digits to the right of the digit being rounded with zeros.

Student practice

Round each decimal to the nearest whole number.

a. 4.215 **b.** 19.853 **c.** 87.47 **d.** 998.967

WE 2 Rounding to a different decimal place value

Round each decimal as specified.

a. 0.37 to the nearest tenth

Working

0.37

$7 \geq 5$

$0.37 \approx 0.40$

$0.37 \approx 0.4$ correct to 1 decimal place.

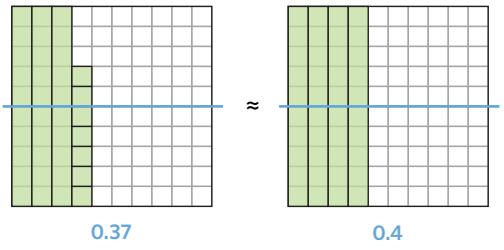
Thinking

Step 1: Identify the digit being rounded and underline the critical digit.

Step 2: The critical digit is greater than 5, so increase the digit being rounded by 1. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Write the answer correct to the specified place value.

Visual support



b. 12.37954 to 3 decimal places

Working

12.37954

$5 \geq 5$

$12.37954 \approx 12.38000$

$12.37954 \approx 12.380$ correct to 3 decimal places.

Thinking

Step 1: Identify the digit being rounded and underline the critical digit.

Step 2: The critical digit is equal to 5, so increase the digit being rounded by 1. As the digit being rounded is 9, increasing it by 1 is 10, so regroup to the next place value. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Write the answer correct to the specified place value.

Student practice

Round each decimal as specified.

a. 0.6387 to the nearest hundredth **b.** 2.3167 to 3 decimal places

c. 0.43265 to the nearest ten-thousandth **d.** 2.971 to 1 decimal place

WE 3 Estimating a sum or difference by rounding to whole numbers

Estimate each calculation by rounding each decimal to the nearest whole number.

a. 4.2763 + 5.746

Working

4.2763

5.746

4.2763 ≈ 4.0000 ≈ 4

5.746 ≈ 6.000 ≈ 6

4 + 6 = 10

The estimated sum is 10.

Thinking

Step 1: Identify the whole number and underline the critical digit for each decimal.

Step 2: Increase the whole number by 1 if the critical digit is greater or equal to 5. Keep the whole number as is if the critical digit is less than 5. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the sum of the rounded numbers.

b. 9.7125 − 2.942

Working

9.7125

2.942

9.7125 ≈ 10.0000 ≈ 10

2.942 ≈ 3.000 ≈ 3

10 − 3 = 7

The estimated difference is 7.

Thinking

Step 1: Identify the whole number and underline the critical digit for each decimal.

Step 2: Increase the digit being rounded by 1 if the critical digit is greater or equal to 5. If the digit being rounded is 9, increasing it by 1 is 10, so regroup to the next place value. Replace all digits to the right of the digit being rounded with zeros.

Step 3: Evaluate the difference of the rounded numbers.

Student practice

Estimate each calculation by rounding each decimal to the nearest whole number.

a. 3.2167 + 2.652

b. 4.617 − 1.873

c. 4.982 + 5.198

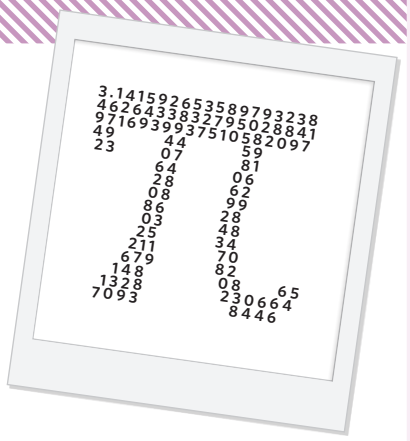
d. 9.942 − 6.462

5B Activities and questions

STARTER TASKS

Odd spot

Pi is a number with decimals that continues on forever without a pattern. Suresh Kumar Sharma has the world record for listing the most digits of pi. In 2015 he recalled 70 030 digits while blindfolded. If the 70 031st digit of pi is 2, did Suresh Kumar Sharma correctly state pi if he rounded pi to 70 030 decimal places?



A. Yes **B.** No

Puzzle

What number solves the riddle?
I am greater than 32 tenths.
If you round me to the nearest whole, I am 3.
My tenths digit is odd.
If you round me to the nearest tenths digit, the tenths digit remains the same.

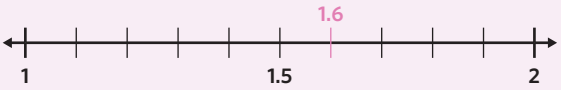
A. 3.38 **B.** 2.72 **C.** 4.01 **D.** 3.32

Understanding worksheet

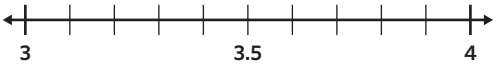
1. Place the decimal on the number line and round the decimal to the nearest whole number using the benchmark decimal as a guide.

Example

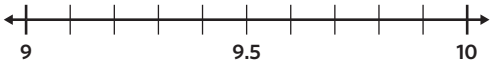
1.6 ≈ 2



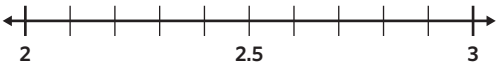
a. 3.2 ≈ ☐



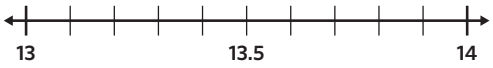
b. 9.9 ≈ ☐



c. 2.72 ≈ ☐



d. 13.46 ≈ ☐



2. Circle the digit being rounded and underline the critical digit to help you round the decimal number.

Example

Round to the nearest thousandth.

2.76438 ≈ 4.764

a. Round to the nearest hundredth.

3.276 ≈ ☐

b. Round to the nearest tenth.

9.871 ≈ ☐

c. Round to the nearest thousandth.

0.6372 ≈ ☐

d. Round to the nearest whole number.

9.8736 ≈ ☐

3. Fill in the blanks by using the words provided.

benchmark | round | estimating | critical | rounded

We ☐ decimals as it makes them simpler to use when ☐ or approximating numbers. We can use a ☐ decimal such as 0.25, 0.5 or 0.75 to help us more accurately identify where a decimal is positioned on a number line. The ☐ digit is the digit to the right of the digit being ☐. It helps us determine if we add one to the digit being rounded or if it remains the same.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 **4.** Round each decimal to the nearest whole number.

a. 0.674 **b.** 8.3 **c.** 27.556 **d.** 43.8753
e. 112.4 **f.** 237.323 **g.** 129.56 **h.** 999.79076

WE2a **5.** Round each decimal as specified.

a. 0.27 to the nearest tenth
b. 3.32913 to the nearest hundredth
c. 125.91247 to the nearest tenth
d. 0.484425 to the nearest thousandth
e. 321.4891 to the nearest hundredth
f. 19.9267 to the nearest tenth
g. 987.7983 to the nearest hundredth
h. 129.97154 to the nearest tenth

WE2b **6.** Round each decimal as specified.

a. 0.73 to 1 decimal place
b. 0.2985 to 1 decimal place
c. 0.67321 to 3 decimal places
d. 0.2714 to 2 decimal places
e. 3.982178 to 4 decimal places
f. 4.17638 to 3 decimal places
g. 2.987 to 1 decimal place
h. 9.995 to 2 decimal places

WE3a **7.** Estimate each calculation by rounding each decimal to the nearest whole number.

a. 2.174 + 0.876
b. 0.2300 + 3.58
c. 15.963 + 5.921
d. 99.532 + 0.225
e. 100.932 + 99.653
f. 19.999 + 21.125
g. 0.92763847 + 124.926
h. 999.256 + 1.2065

WE3b **8.** Estimate each calculation by rounding each decimal to the nearest whole number.

a. 3.96 − 2.88
b. 6.178 − 1.976
c. 9.91 − 3.21
d. 3.98628 − 1.5539
e. 25.1298 − 13.7623
f. 15.1872 − 14.97
g. 19.912 − 9.345
h. 99.8216 − 49.972

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Jamie goes to buy a watermelon. The clerk gives Jamie a watermelon that weighs 10.262 kg. What is the weight of the watermelon rounded to the nearest tenth of a kilogram?

10. James is ranked first on his cricket team with a batting average of 99.9387. Cricket batting averages are usually only calculated to two decimal places. What is James' batting average rounded to two decimal places?

11. In 2018, the life expectancy in Australia was 82.75 years. The life expectancy in America was 78.54 in the same year. Round to the nearest year to estimate how much longer the average Australian lives than the average American.

12. Kai and Eric are going to the grocery store with $15 to buy ingredients to make cupcakes. How much can they approximately expect to pay if they round the cost of each item to the nearest dollar?

SHOPPING LIST

Cake mix	*$4.90*
Caster sugar	*$3.50*
Butter	*$2.85*
Sprinkles	*$1.75*

13. Each of Jenny's school textbooks weigh 2.04 kg. When Jenny's backpack is full of textbooks it weighs 8.25 kg. The backpack without any textbooks in it weighs 1.87 kg. Estimate how many textbooks can fit in Jenny's backpack.

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Julia is planning to make a fruit salad for a party. She goes to Rod's Fruit and Vegetable shop at the South Melbourne Market where the following fruits are sold.

Fuji apples	$0.90 each
Granny Smith apples	$0.85 each
Bananas	$2.99 per box (2 kg)
Blueberries	$2.95 per punnet (125 g)
Dragon fruit	$6.75
Cantaloupe	$3.10 each

Julia wants to buy 3 Fuji apples, 3 Granny Smith apples, 2 kg of bananas, 375 grams of blueberries, 1 dragon fruit and 1 cantaloupe.

a. Considering the quantities that Julia plans to buy, which fruit will she spend the most money on if she rounds the price of each fruit to the nearest dollar?

b. Estimate how much Julia will spend on her fruit salad.

c. Julia's friend Vicky said that she will bring a cantaloupe to the party. Julia has $25 to spend on the rest of the fruit. Does she have enough money?

d. How much of each fruit would you buy if you wanted to make a fruit salad for around $20?

15. Anthony is training to run the Melbourne Marathon. The distance of a marathon is 42.195 kilometres and the average completion time is 4.15 hours. The average pace of a runner in kilometres per hour is calculated by dividing the total distance in kilometres by the total time in hours.

a. Round the distance and average time to the nearest whole number to estimate the average pace of runners in kilometres per hour.

b. In Anthony's training, he has been running 14.2 km in 1.091 hours. Round all numbers to the nearest whole number to estimate how long it would take Anthony to complete the Melbourne Marathon. Choose the best estimate below.

A. 2.5 hours B. 3 hours C. 3.5 hours D. 4 hours

c. The gold medalist in the Tokyo 2020 Olympic games completed the marathon in 2.13 hours. Estimate the average pace, in kilometres per hour, that Anthony would need to run in order to achieve a gold medal time.

d. Anthony wants to qualify for the Olympics. What could Anthony work on to help him reach his goal?

Extra spicy

16. Estimate the expression 63.499×48.501 by rounding both decimals to the nearest whole number.

17. If each of the following decimals is rounded to the nearest thousandth, what is the sum of the smallest and largest rounded numbers?

0.51396, 0.912782, 0.826566, 0.2899

18. Jessalyn was adding decimals on a calculator. Instead of adding 35.95 she added 35 095. What should Jessalyn do in order to correct her mistake with only one step?

19. What is 999.998999 rounded to 5 decimal places?

Remember this?

20. Hugo, Johnny and Lachie competed in an Easter egg hunt. Lachie found $\frac{14}{25}$ of the eggs and Hugo found $\frac{1}{25}$ more than Johnny. What fraction of the eggs did Johnny find?

A. $\frac{4}{25}$ B. $\frac{5}{25}$ C. $\frac{6}{25}$ D. $\frac{12}{25}$

21. Otis bought 3 surfboards for $426. Let the cost of each surfboard be $\$s$.

Which of the following statements is false?

A. $s = 426 \div 3$ B. $3 = 426 \div s$ C. $3 \times s = 426$ D. $3 \times 426 = s$

22. Which of these has a 7 in the ten thousandth place?

A. 12.30701 B. 14.37241 C. 18.34574 D. 24.02347

5C Adding and subtracting with decimals

Decimals are added and subtracted in the same way as whole numbers where we need to consider the place value of the digits being added or subtracted. We can add and subtract decimals using decimal fractions. A vertical algorithm can also be used to add and subtract decimals. Below are some examples where adding and subtracting decimals can be applied.

- I had two 1 litre (L) juice bottles. The first contained 0.65 L of apple juice and the second contained 0.825 L of orange juice. How many litres of juice did I have altogether?
- While running a race, I recorded the time taken to complete each lap and got the following times; 52.453 seconds, 51.85 seconds, 52.005 seconds and 51.9 seconds. What was the total time taken to complete the race?
- I bought three items from the local cafe and paid $4.55, $3.60 and $1.25. How much change will I receive from $20?

Learning intentions

Students will be able to:

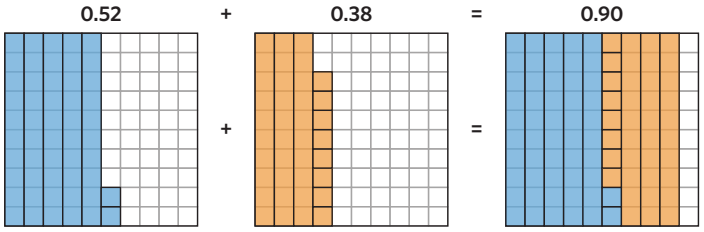
- use visual representations to add and subtract decimals
- add and subtract decimal fractions
- add and subtract decimals using the vertical algorithm
- use estimation to add and subtract decimals.

Key terms and definitions

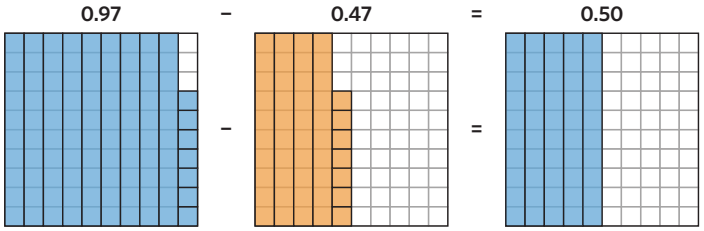
- A **decimal fraction** is a fraction where the denominator is a power of 10, such as 10, 100, 1000, 10 000.
  E.g. the decimal 0.25 written as a decimal fraction is $\frac{25}{100}$.
- A **vertical algorithm** is a method of adding and subtracting numbers where the place value of each digit is lined up and the results recorded underneath.

Key ideas

1 A hundreds grid can be used to show how to add and subtract decimals visually.



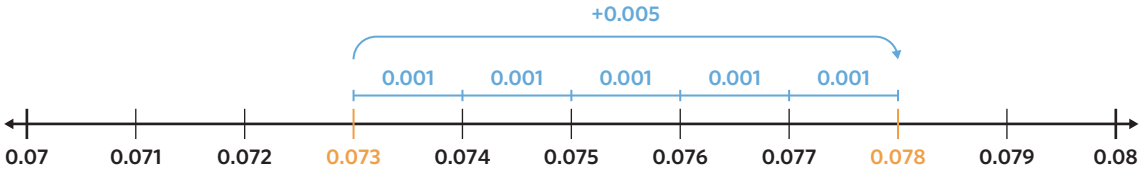
$$0.52 + 0.38 = \frac{52}{100} + \frac{38}{100} = \frac{90}{100} = 0.90$$



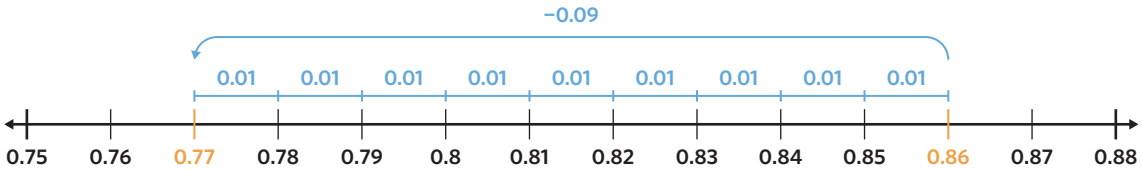
$$0.97 - 0.47 = \frac{97}{100} - \frac{47}{100} = \frac{50}{100} = 0.50$$

2 A number line can be used to help add and subtract decimals.

$0.073 + 0.005 = 0.078$



$0.86 - 0.09 = 0.77$



Worked examples

WE 1 Adding and subtracting decimals using decimal fractions

Use decimal fractions to perform each calculation. Express the answer in decimal notation.

a. $0.12 + 0.56$

Working

$$0.12 + 0.56 = \frac{12}{100} + \frac{56}{100}$$
$$= \frac{12 + 56}{100}$$
$$= \frac{68}{100}$$
$$= 0.68$$

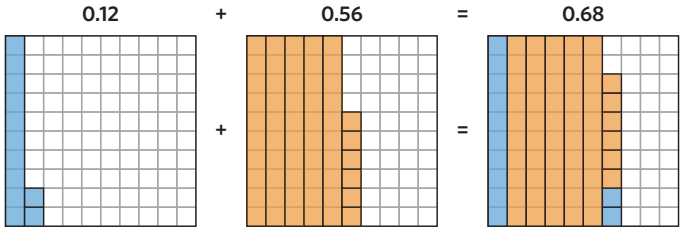
Thinking

Step 1: Express each decimal as a decimal fraction.

Step 2: Since the denominators are the same, perform the addition using the numerators.

Step 3: Convert back to a decimal.

Visual support



b. $3.875 - 1.76$

Working

$$3.875 - 1.76 = \frac{3875}{1000} - \frac{176}{100}$$
$$= \frac{3875}{1000} - \frac{1760}{1000}$$
$$= \frac{3875 - 1760}{1000}$$
$$= \frac{2115}{1000}$$
$$= 2.115$$

Thinking

Step 1: Express each decimal as a decimal fraction.

Step 2: Express both fractions with the same denominator.

Step 3: Perform the subtraction using the numerators.

Step 4: Convert back to a decimal.

Student practice

Use decimal fractions to perform each calculation. Express the answer in decimal notation.

a. $0.53 + 0.29$ **b.** $2.192 - 0.568$ **c.** $1.34 + 2.907$ **d.** $4.05 - 2.816$

WE 2 Adding and subtracting decimals using the vertical algorithm

Use a vertical algorithm to complete each calculation.

a. $2.54 + 3.15$

Working

$$\begin{array}{r} 2.54 \\ +\ 3.15 \\ \hline \end{array}$$

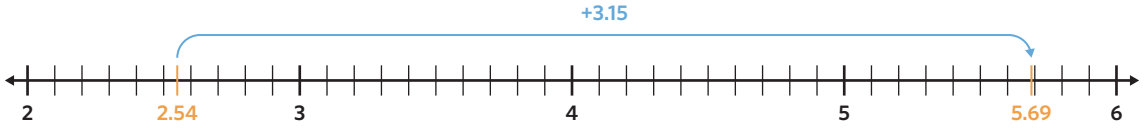
$$\begin{array}{r} 2.54 \\ +\ 3.15 \\ \hline 5.69 \end{array}$$

Thinking

Step 1: Write the decimals vertically, ensuring that the decimal points and the digits in each place value line up.

Step 2: Add the digits in each place value column, from right to left.

Visual support



b. 15.47 − 10.052

Working

$$\begin{array}{r} 15.47 \\ -\ 10.052 \\ \hline \end{array}$$

$$\begin{array}{r} 15.470 \\ -\ 10.052 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 6\ 10} \\ 15.4\not{7}\not{0} \\ -\ 10.052 \\ \hline 5.418 \end{array}$$

Thinking

Step 1: Write the decimals vertically, ensuring that the decimal points and the digits in each place value line up.

Step 2: Use zero as a placeholder if there is a place value digit missing.

Step 3: Subtract the digits in each place value column, from right to left, regrouping where necessary.

Student practice

Use a vertical algorithm to complete each calculation.

a. 4.08 + 1.35 **b.** 11.252 − 9.06 **c.** 14.47 + 3.085 **d.** 26.104 − 18.83

WE 3 Using rounding to estimate decimal addition and subtraction

Round each decimal to the place value indicated in the brackets to estimate each calculation.

a. 2.58 + 0.75 (tenth)

Working

2.58 rounded to the nearest tenth is 2.6.

0.75 rounded to the nearest tenth is 0.8.

Method 1: Decimal fractions

$$2.6 + 0.8 = \frac{26}{10} + \frac{8}{10}$$
$$= \frac{34}{10}$$
$$= 3.4$$

Method 2: Vertical algorithm

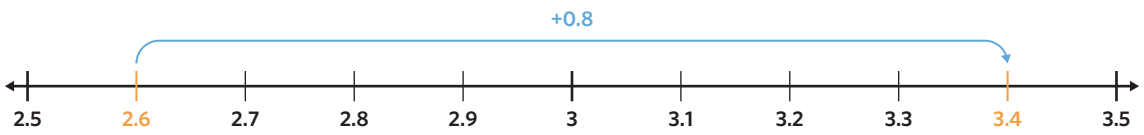
$$\begin{array}{r} {\scriptstyle +1} \\ 2.6 \\ +\ 0.8 \\ \hline 3.4 \end{array}$$

Thinking

Step 1: Round each decimal to the nearest tenth.

Step 2: Perform the addition of the rounded decimals using decimal fractions or a vertical algorithm.

Visual support



b. 8.176 − 4.296 (hundredth)

Working

8.176 rounded to the nearest hundredth is 8.18.

4.296 rounded to the nearest hundredth is 4.30.

Method 1: Decimal fractions

$$8.18 - 4.30 = \frac{818}{100} - \frac{430}{100}$$
$$= \frac{388}{100}$$
$$= 3.88$$

Method 2: Vertical algorithm

$$\begin{array}{r} {\scriptstyle 7\ \ 11} \\ 8.\not{1}\not{8} \\ -\ 4.30 \\ \hline 3.88 \end{array}$$

Thinking

Step 1: Round each decimal to the nearest hundredth.

Step 2: Perform the subtraction of the rounded decimals using decimal fractions or a vertical algorithm.

Student practice

Round each decimal to the place value indicated in the brackets to estimate each calculation.

a. 0.93 + 3.46 (tenth)

b. 9.595 − 7.813 (hundredth)

c. 21.945 + 8.667 (hundredth)

d. 15.1097 − 6.4454 (thousandth)

5C Activities and questions

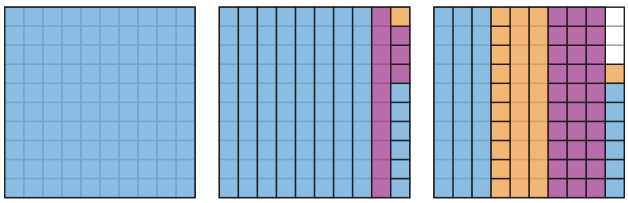
STARTER TASKS

Odd spot

At the Tokyo 2020 Olympics, Australian javelin thrower Kelsey-Lee Barber progressed to the final with a 62.59 metre throw. She won the bronze medal in the final, throwing a distance of 64.56 metres. How much further was her throw in the final compared to her previous throw?

A. 1.97 metres **B.** 2.03 metres

Puzzle



a) If the first blue square is one whole, what number is represented by the shaded parts of all 3 squares combined?

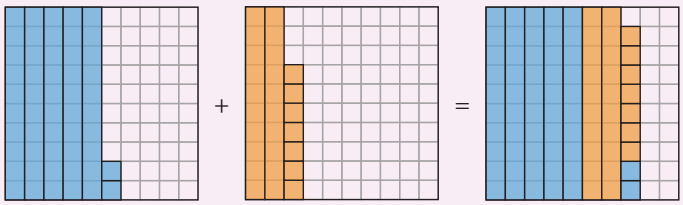
b) What number do the purple parts represent?

c) What number do the orange parts represent?

Understanding worksheet

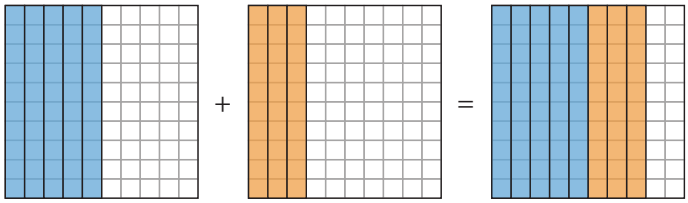
1. Use **decimal fractions** and the hundreds grid to help complete each calculation.

Example



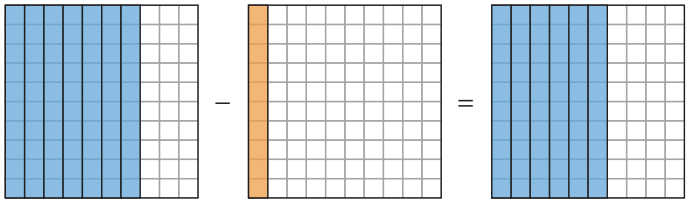
0.52 + 0.27 = 0.79

a.



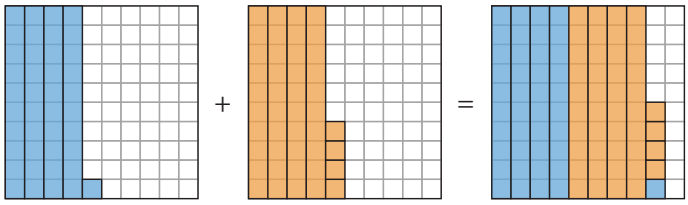
0.50 + 0.30 = ☐

b.



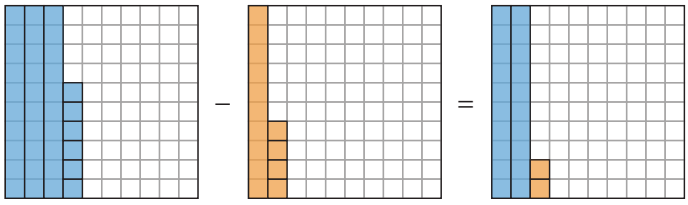
0.70 − 0.10 = ☐

c.



0.41 + 0.44 = ☐

d.



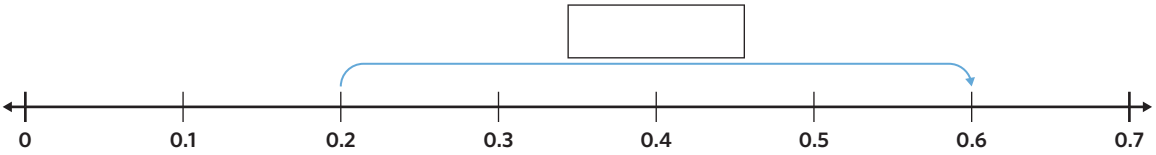
0.36 − 0.14 = ☐

2. Use the number line to complete each calculation.

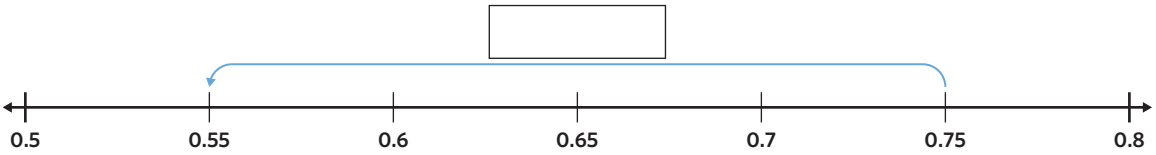
Example

2.4 + 1.5 = 3.9

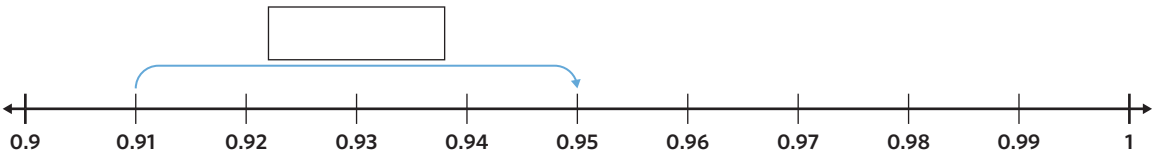
a. 0.2 + 0.4 = ☐



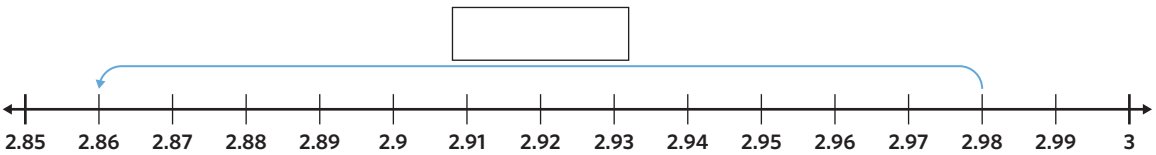
b. 0.75 − 0.2 = ☐



c. 0.91 + 0.04 = ☐



d. 2.98 − 0.12 = ☐



3. Fill in the blanks by using the words provided.

decimal | vertical | decimal points

When adding or subtracting decimals we can keep them in ☐ notation. If we are using the ☐ algorithm, we need to make sure that we line up the ☐ before adding or subtracting.

Fluency

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Use decimal fractions to perform each calculation. Express the answer in decimal notation.

a. 0.31 + 0.46 **b.** 1.5 + 4.2 **c.** 2.7 − 1.3 **d.** 12.45 + 8.46

e. 13.74 − 12.6 **f.** 6.157 − 5.92 **g.** 9.099 + 8.997 **h.** 3.005 − 1.990

WE2 5. Use a vertical algorithm to complete each calculation.

a. 0.22 + 0.45 b. 3.4 + 6.3 c. 7.99 − 4.67 d. 12.763 − 10.453
e. 16.09 + 14.947 f. 18.007 − 3.998 g. 12.14 + 0.997 h. 27.43 − 19.651

WE3 6. Round each decimal to the place value indicated in the brackets to estimate each calculation.

a. 1.24 + 0.78 (tenth) b. 7.891 + 4.349 (hundredth)
c. 91.456 − 50.42 (tenth) d. 49.5591 − 23.3287 (thousandth)
e. 22.005 + 4.909 (hundredth) f. 14.93 + 17.07 (tenth)
g. 33.999 − 22.999 (hundredth) h. 12.9909 − 12.0999 (thousandth)

7. Use any method to calculate each of the following. Express all answers as decimals.

a. 0.28 + 0.15 b. 7.932 − 6.05
c. 1.2 + 5.3 + 2.7 d. 5.64 + 8.44 + 3.75
e. 20.99 − 4.32 − 12.15 f. 4.22 + 3.05 − 0.27
g. 14.23 + 8.11 − 3.09 h. 16.191 − 14.321 + 3.777

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Captainglitterz needed more snacks for his Minecraft party. He ordered some Tim Tams for $40.74 and some Reeses Pieces for $27.95. How much did Captainglitterz spend on snacks for his Minecraft party?

9. A Year 7 science competition involved collecting plastic for recycling over a three-week period. Using the following table, which class collected the greatest amount of plastic at the end of the three weeks?

	Class 7A	Class 7B	Class 7C
Week 1	5.28 kg	4.32 kg	8.03 kg
Week 2	5.102 kg	5.007 kg	2.559 kg
Week 3	5.97 kg	6.87 kg	5.71 kg

10. Stan and his three teammates were running a 200-metre relay, and were trying to beat their record of 140 seconds. How much did Stan's team beat their record by, if the teammates' individual times were 27.46 seconds, 31.58 seconds, 28.11 seconds and 36.43 seconds?

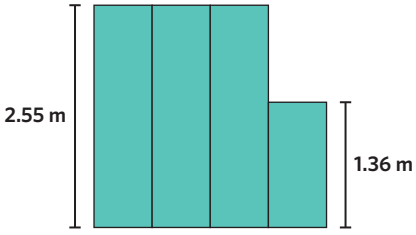
11. The largest insect, the giant weta, has a recorded weight of 71 grams and the smallest insect, the parasitic wasp, has a recorded weight of 0.000025 grams. What is the difference between the weights of these two insects?

12. Vlad, Frankie and Drac were donating blood at the local blood drive. How much blood did they donate altogether if Vlad donated 0.45 litres, Frankie donated 0.03 litres **less** than Vlad and Drac donated 0.02 litres **more** than Vlad?

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c,d)	All

13. Luna was laying a new lawn in her front yard using rolls of turf grass. Each roll was 2.55 metres in length and she lay them in her yard as shown in the image.



a. If Luna used 3 whole rolls of turf and 1.36 metres from a 4th roll, how many metres of turf grass did Luna lay in her front yard?
b. If Luna can only buy full rolls and each roll costs $24.50, how much did Luna pay for the turf grass that she needed?
c. Luna now needed to place a fence around her lawn but did not have a measuring tape long enough to do it accurately. She knew that each roll of grass was 0.65 metres wide. Use this information to help Luna determine the perimeter of her lawn.
d. If Luna could only purchase her turf grass in packs of 5 rolls, how many metres of grass did Luna have left after she completed her front lawn?
e. What could Luna do with the leftover turf grass?

14. The table shows the daily minimum and maximum temperatures recorded in Melbourne in the first week of July 2021.
Also shown is the difference between the recorded temperatures and the predicted temperatures for each day.
A negative value means that it was actually colder than predicted, and a positive value means that it was actually hotter than predicted.

Day	Minimum temperature (°C)	Difference from predicted min. temperature (°C)	Maximum temperature (°C)	Difference from predicted max. temperature (°C)
Thursday	10.5	+2.8	17.2	+2.7
Friday	9.7	+2.0	14.5	0.0
Saturday	6.6	−1.1	10.6	−3.9
Sunday	6.5	−1.2	13.1	−1.4
Monday	4.6	−3.1	13.1	−1.4
Tuesday	6.7	−1.0	13.2	−1.3
Wednesday	6.8	−0.9	13.1	−1.4

a. What was the highest temperature recorded and on what day was it recorded?
b. What is the difference between the highest recorded temperature and the lowest recorded temperature in that week?
c. Which day had the most inaccurate predicted maximum temperature?
d. What was the predicted maximum temperature on that day?
e. How do you think the temperatures and the temperature fluctuations would be different if the information collected in the table was for Queensland?

Extra spicy

15. Evaluate $3(4.67 - 3.9)$.

16. Find the sum of the greatest and the least of the numbers 0.32, 0.357, 0.558, 0.4201, 0.3001.

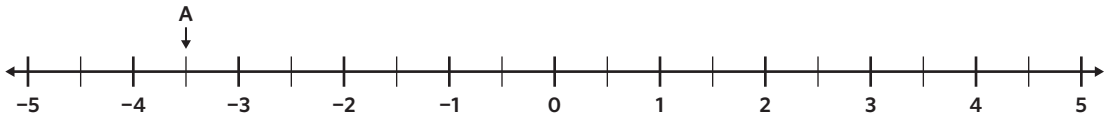
17. Find the 5^{th} decimal place when $\frac{2}{7}$ is expressed as a decimal.

18. If we know that the recurring decimals $\frac{1}{9} = 0.\overline{1}$ and $\frac{1}{11} = 0.\overline{09}$, what would $\frac{1}{9} + \frac{1}{11}$ equal as a recurring decimal?

Remember this?

19. Sue bought a new Tesla car. Choose the most appropriate unit of measurement to weigh the car.
A. Grams **B.** Kilometres **C.** Kilograms **D.** Metres

20. State the value of A shown on the number line as a mixed number in its simplest form.



21. Pierre wrote a list of supplies he needed for a project.

Supplies
Ribbon $3.75
Glue $4.68
Glitter $5.25
Cardboard $6.57

If he rounds the price of each item to the nearest dollar, what is the estimated total cost of Pierre's project?

5D Multiplying and dividing by powers of 10

Positive powers of ten refers to exponents of 10 such as: 10, 100, 1000 and 10 000. Multiplying and dividing by powers of 10 is an important skill to understand large and small numbers. It is important to have a good understanding of place value in order to work with decimals and powers of 10. Below are some examples where multiplying and dividing by powers of 10 can be applied.

- It costs me $3.87 to produce 1 litre of kombucha. How much will it cost to make 100 litres?
- My bag of 1000 screws weighs 5851 grams. How much does one screw weigh?
- I need 0.24 metres of blue ribbon and 0.18 metres of green ribbon to make a single scout's participation badge. How much ribbon in total would I need to make 10 badges?

Learning intentions

Students will be able to:

- multiply decimals by powers of 10
- divide decimals by powers of 10
- use order of operations with decimals and powers of 10.

Key terms and definitions

- **Powers of 10** are the result of multiplying ten by itself a certain number of times. E.g. 10, 100, 1000, 10 000 and 100 000 are all powers of 10.
- A **number slider** is a visual representation that can be used to explain how the digits in a number move a place value when multiplying or dividing by powers of 10.
- A **multiplier** is the number by which a given value is multiplied.
- A **divisor** is the number by which a given value is divided.

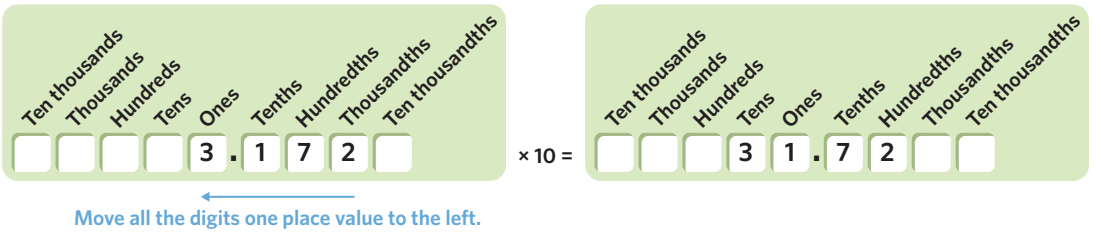
Key ideas

1 A power tells us how many times a base number is multiplied by itself. Powers of 10 can be written in index form (with a power), expanded form, or as a value.

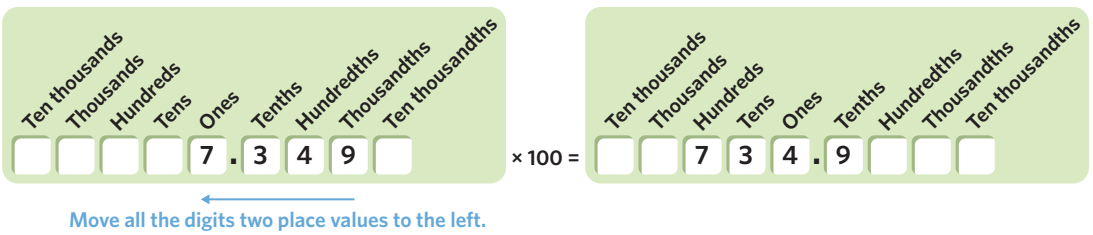
Index form (powers of 10)	Expanded form	Value
10^1	10	10
10^2	10×10	100
10^3	$10 \times 10 \times 10$	1000
10^4	$10 \times 10 \times 10 \times 10$	10 000

2 A number slider can be used to show how **multiplying** by powers of 10 affects the place value of the digits in that number.

A decimal number that is multiplied by 10, makes each digit 10 times its original value. This means that we move each digit one place value to the left.

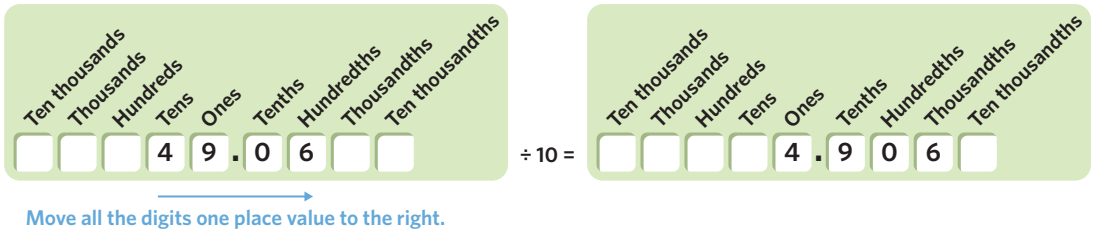


A decimal number that is multiplied by 100, makes each digit 100 times its original value. This means that we move each digit two place values to the left.



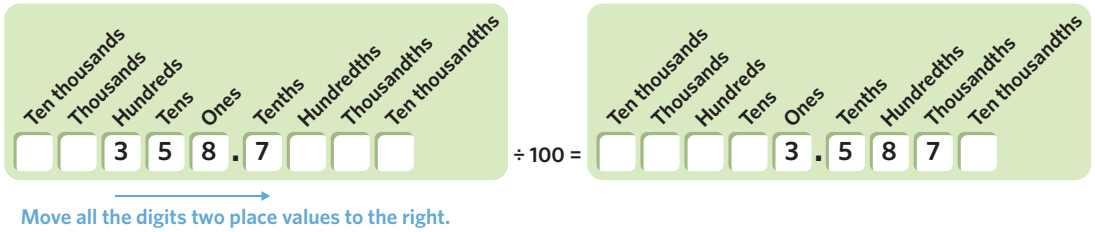
3 A number slider can be used to show how **dividing** by powers of 10 affects the place value of the digits in that number.

A decimal number that is divided by 10, makes each digit one-tenth of its original value. This means that we move each digit one place value to the right.



Move all the digits one place value to the right.

A decimal number that is divided by 100, makes each digit one-hundredth of its original value. This means that we move each digit two place values to the right.



Move all the digits two place values to the right.

Worked examples

WE 1 Multiplying by powers of 10

Complete each multiplication.

a. 8.297 × 100

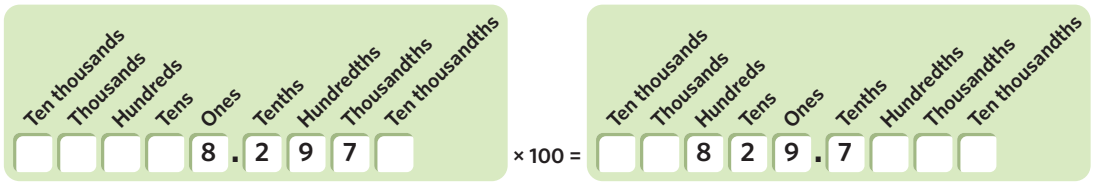
Working

8.297 × 100 = 829.7
= 829.7

Thinking

Identify how many place values to move each digit based on the multiplier. The multiplier is 100, so move each digit two place values to the left.

Visual support



Move all the digits two places value to the left.

b. 21.56 × 1000

Working

21.56 × 1000 = 21 560.
= 21 560

Thinking

Identify how many place values to move each digit based on the multiplier. The multiplier is 1000, so move each digit three place values to the left.

Note: When there is a place value digit missing, use zero as a placeholder.

Student practice

Complete each multiplication.

a. 3.451 × 100 b. 67.28 × 1000 c. 0.08 × 10 d. 9.2 × 10 000

WE 2 Dividing by powers of 10

Complete each division.

a. 348.92 ÷ 10

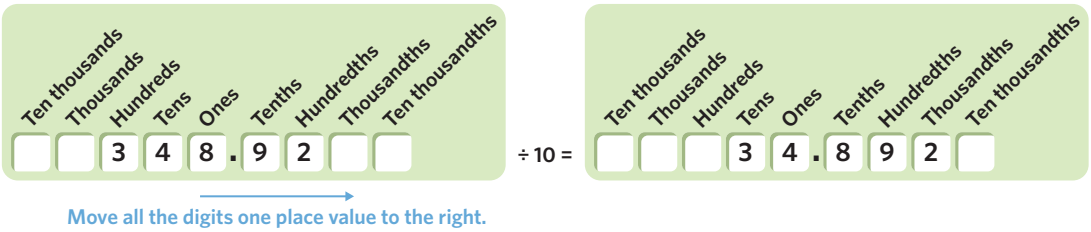
Working

348.92 ÷ 10 = 34.892
= 34.892

Thinking

Identify how many place values to move each digit based on the divisor. The divisor is 10, so each digit moves one place value to the right.

Visual support



b. 76.9 ÷ 1000

Working

76.9 ÷ 1000 = 0.0769
= 0.0769

Thinking

Identify how many place values to move each digit based on the divisor. The divisor is 1000, so each digit moves three place values to the right.

Note: When there is a place value digit missing, use zero as a placeholder.

Student practice

Complete each division.

a. 983.74 ÷ 10 **b.** 72.1 ÷ 1000 **c.** 0.06 ÷ 100 **d.** 52 ÷ 10 000

WE 3 Using order of operations with decimals and powers of 10

Use the order of operations to evaluate each expression.

a. 5.67 × 10 + 89.1 ÷ 100

Working

5.67 × 10 = 56.7
= 56.7

89.1 ÷ 100 = 0.891
= 0.891

$$\begin{array}{r} {\scriptstyle +1} \\ 56.700 \\ +\ \ 0.891 \\ \hline 57.591 \end{array}$$

Thinking

Step 1: Perform the multiplication. The multiplier is 10, so move each digit one place value to the left.

Step 2: Perform the division. The divisor is 100, so each digit moves two place values to the right.

Note: When there is a place value digit missing, use zero as a placeholder.

Step 3: Perform the addition.

b. $10(0.518 \times 100) - 3276 \div 10$

Working

$0.518 \times 100 = 51.8$

$10 \times 51.8 - 3276 \div 10$

$10 \times 51.8 = 518.$

$3276 \div 10 = 327.6$

$$\begin{array}{r} 4\ 11\ 7\ \ 10 \\ 5\ \not{1}\ \not{8}\ .\ \not{0} \\ -\ 3\ 2\ 7\ .\ 6 \\ \hline 1\ 9\ 0\ .\ 4 \end{array}$$

Thinking

Step 1: Perform the multiplication in the brackets. The multiplier is 100, so each digit moves two place values to the left.

Step 2: Write out the updated equation.

Perform the multiplication and then the division.

The multiplier is 10, so each digit moves one place value to the left.

The divisor is 10, so each digit moves one place value to the right.

Step 3: Perform the subtraction.

Student practice

Use the order of operations to evaluate each expression.

a. $4.38 \times 10 + 31.8 \div 100$

b. $10(0.7839 \times 100) - 5607 \div 10$

c. $71.7 \div 100 + 100(0.123)$

d. $13 \div 1000 - 15 \div 10\,000$

5D Activities and questions

STARTER TASKS

Odd spot

In July 2021, Jeff Bezos, the richest man in the world and founder of Amazon flew a rocket into space. Bezos is on a mission to make the cost of space travel more affordable for the general public. To reduce the cost of travel, Bezos is looking to reuse launched rockets. Bezos is aiming to reduce the cost of space travel to approximately $2105.20 per kilogram for each person.

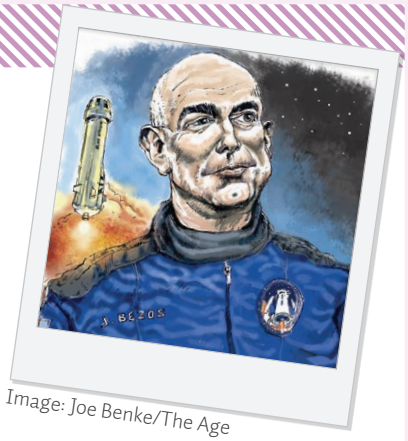


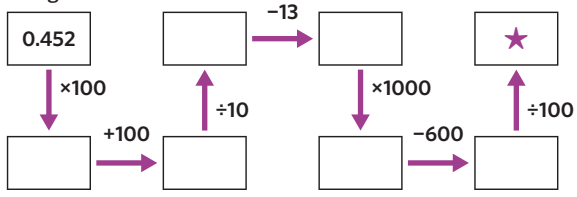
Image: Joe Benke/The Age

If Bezos achieves his goal, what is the approximate cost of a flight for a group of people with a total weight 1000 kg?

A. $21 050 000.20 **B.** $2 105 200.00

Puzzle

Starting value



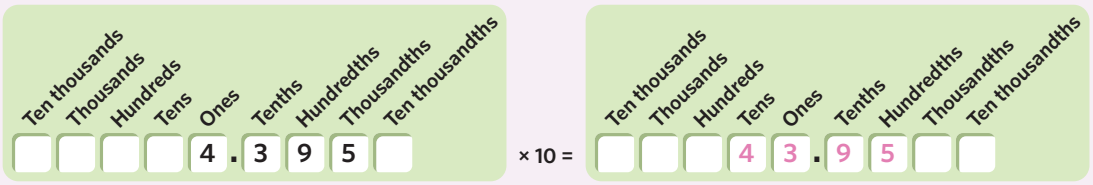
a) What is the value in the box with the ★?

b) What is the starting value if the ★ is 50?

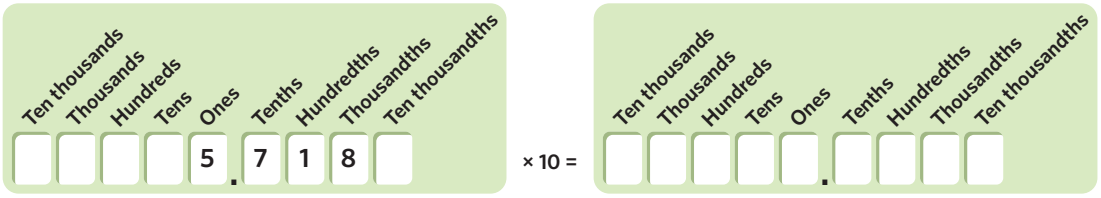
Understanding worksheet

1. Complete each multiplication using the number slider.

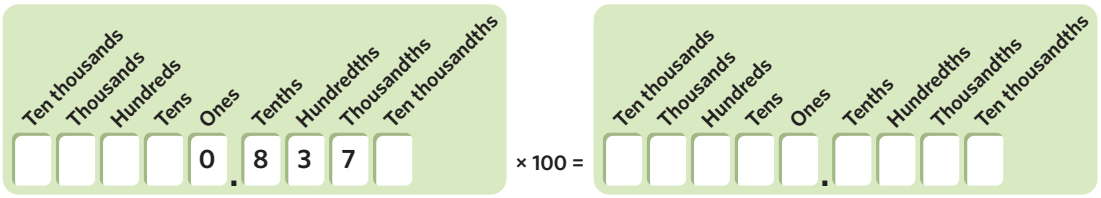
Example



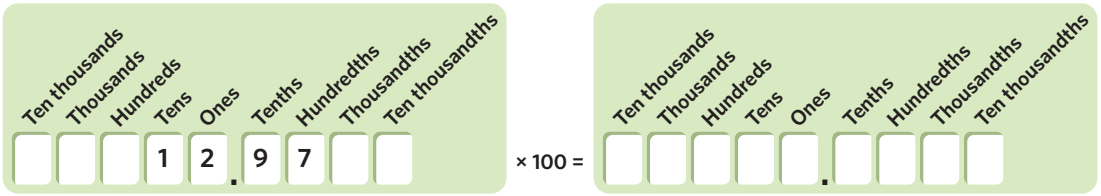
a.



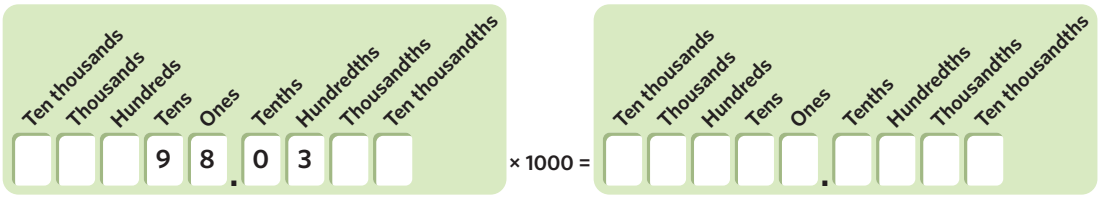
b.



c.

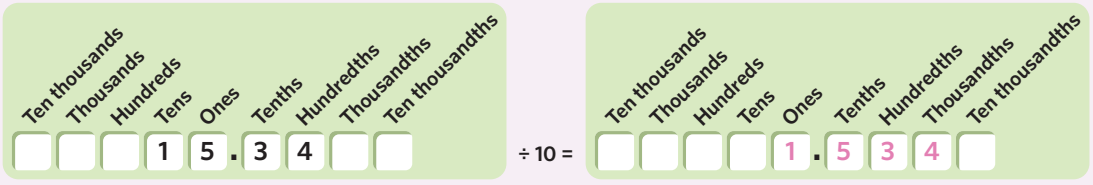


d.

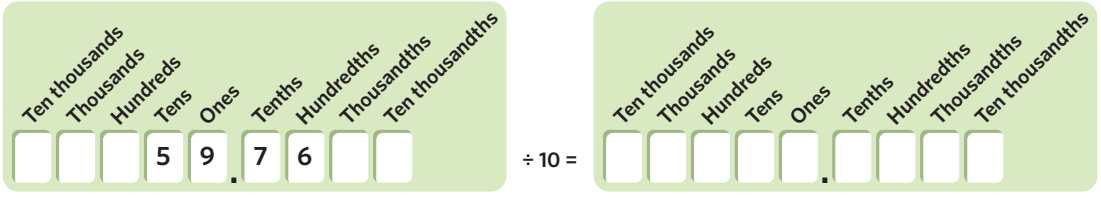


2. Complete each division using the number slider.

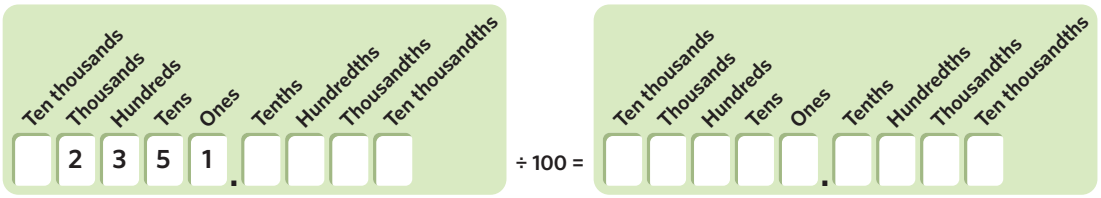
Example



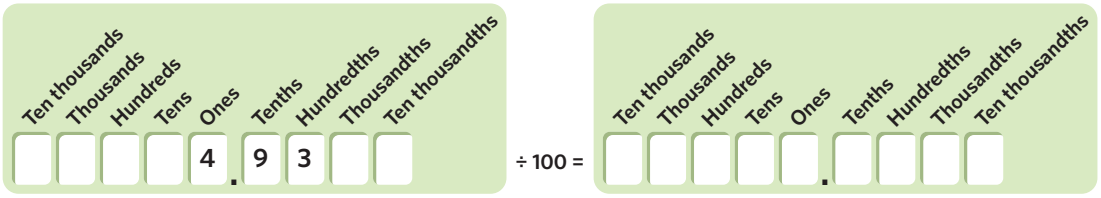
a.



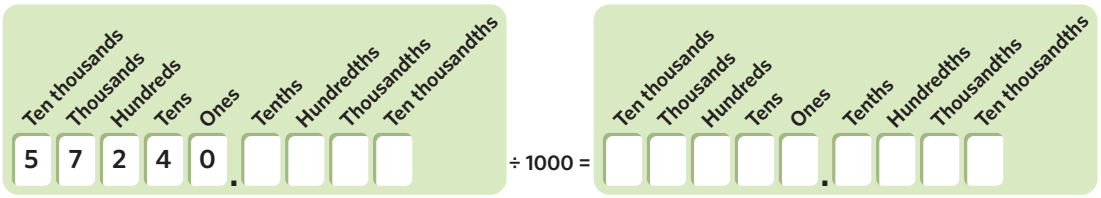
b.



c.



d.



3. Fill in the blanks by using the words provided.

order | left | right | operations | powers

When we divide decimals by _____ of 10 we move the digits' place values to the _____. However, when we multiply decimals by powers of 10 we move the digits to the _____. It is also important that we remember to apply the _____ of _____ when calculations include decimals, powers of 10 and different operations.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f)

WE1 **4.** Complete each multiplication.

a. 5.673×100 b. 10.3×10 c. 456.3×100 d. 67.28×1000

e. 2.7×1000 f. $0.07 \times 10\,000$

WE2 **5.** Complete each division.

a. $689.43 \div 10$ b. $34.2 \div 100$ c. $0.16 \div 100$ d. $0.09 \div 1000$

e. $35.6 \div 1000$ f. $19 \div 10\,000$

6. Evaluate:

a. 15.92×10 b. $35.8 \div 10$ c. 2.193×1000 d. 67.2×100

e. $92.45 \div 1000$ f. $3 \div 10\,000$ g. $12.009 \times 10\,000$ h. $4000.07 \div 100\,000$

WE3 **7.** Use the order of operations to evaluate each expression.

a. $3.97 \times 10 + 52.1 \div 100$ b. $10(0.6348 \times 100) - 532 \div 10$

c. $0.24 \times 1000 + 7344.1 \times 10$ d. $137 \div 100 - 178 \div 1000$

e. $345 \div 10\,000 + 100(0.123)$ f. $100(1.7) + 13.8 \div 100 - 23.9 \div 1000$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. The minimum wage for a 17 year-old in Australia is $14.76. How much will a 17 year-old, who is being paid minimum wage, earn for 10 hours of work?

9. Penny's dairy farm collects 28 500 litres of milk per day. How many litres of milk does each cow produce on average if she has 1000 cows on the farm?

10. In kilometres per hour, what is a peregrine falcon's top flying speed if it can fly 10 times faster than a green iguana can run? Green iguanas are able to run at a speed of 38.73 km/h.

11. Frank's house is on a 254.8 m^2 block of land. He bought the neighbouring 312 m^2 block of land. He plans to demolish his house and combine both properties to build his dream home. Frank wants a front garden with an area of 78.45 m^2 and a back garden that is 5.32 m long and 10 m wide. What area of land remains for Frank to build his house?

12. A 2.1 kg sack of rice can store approximately 100 000 grains of rice. How much does one grain of rice approximately weigh in grams?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c,d)	All

13. A brochure for tourists in Australia contains a table showing the population of cities across Australia in 2020.

Significant urban area	Sydney	Melbourne	Adelaide	Canberra	Launceston
Approximate population (100 000's)	49.67	49.69	13.358	4.649	0.8919

   a. Out of the Australian cities shown in the table, Melbourne's significant urban area has the largest population. What is the population of Melbourne?
   b. What is the combined population of Adelaide and Canberra?
   c. Only one-third of the people who live in Launceston are old enough to work; a third of whom work in agriculture. How many people in Launceston work in agriculture?
   d. Knowing the population of a city provides a 'snapshot' of what a city may be like. Suggest other information that could be included in this table to give someone who does not know anything about Australian cities a more accurate 'snapshot' of each city.

14. Nicola and her friend Tom are moving from London to Nicola's home city of Brisbane. The freight company Nicola and Tom are using to transport their belongings offer different shipping options. The table shown is provided on the freight company's website.

Items	Shipping type	Air	Sea
Non-fragile	Express	$32/kg	$9.30/kg
	General	$21.57/kg	$8/kg
Fragile	Express	$65.79/kg	$21.75/kg
	General	$47.09/kg	$15.05/kg
Discount	For any orders of general freight by sea of 1000 kg or more, we will reduce the price per kg by $1.55.		

   a. How much will it cost Nicola to send 100 kg of her non-fragile items express by sea?
   b. What method of freight did Tom use if he sent a 10 kg box and paid $215.70?
   c. Tom says that it's not that much more expensive to send belongings by air. How much more will it cost Nicola if she sends the 100 kg of non-fragile belongings express and by air instead of by sea?
   d. The following note shows how Nicola intends to send her belongings home. Using the information from the table, calculate the cost of sending all 1110 kg of Nicola's belongings home.

> 100kg: non-fragile, sea, express
> 1000kg: non-fragile, sea, general
> 10kg: fragile, air, express

   e. Why might the freight company offer a discount for large freight deliveries?

Extra spicy

15. A large pear weighs 170 grams. A small sized watermelon weighs ten times the weight of a large pear. What is the total weight of 100 large pears and 10 small watermelons?

A. 34 kilograms **B.** 3.4 kilograms **C.** 3400 grams **D.** 17 000 grams

16. If $10^{-1} = 0.1$ and $10^{-2} = 0.01$, what is the value of 10^{-5}?

17. Select the option that is less than 12.345×10^2.

A. 12340 **B.** 1.2345×10^3

C. $123\ 450 \div 10^4$ **D.** $0.12345 \times 10^2 \times 10^2$

18. The distance between Uranus and Venus is 2 765 350 000 km and can be written using scientific notation as 2.76535×10^9 km. The distance between Venus and Mars is 119 740 000 km. What is the difference in distance between Venus and Mars in scientific notation?

Remember this?

19. The teachers at Parkside High School record the amount of food they each waste annually.

- The school has 60 teachers.
- Each teacher wastes 300 kilograms of food annually.
- Every 1000 kilograms of food equates to 200 grocery bags of waste.

How many grocery bags in total do the teachers at Parkside High School waste annually?

20. The table shows the number of hot dogs eaten at a music festival.

Hour	1	2	3	4	6
Hotdogs eaten	50	65	80	95	110

Select the option that represents how to find the number of hot dogs that have been eaten.

A. number of hours plus 15 hotdogs **B.** number of hours multiplied by 15

C. 15 hotdogs for every hour plus an additional 35 hotdogs **D.** 35 hotdogs for every hour plus an additional 15 hotdogs

21. The table shows the US men's shoe sizes and length in inches.

US shoe size	8	8.5	9	9.5
Inches	9.94	10.12	10.3	10.48

How many inches are size 6.5 US men's shoes?

5E Multiplying with decimals

It can sometimes be easier to multiply decimals using decimal fractions so that the relationship between the place value of the numbers being multiplied is explicitly shown. Decimals can also be multiplied using the vertical algorithm. It is good practice to estimate answers before multiplying decimals so that the reasonableness of the calculation can be checked. Below are some examples where multiplying decimals can be applied.

- I pay $0.30 per 10 MB of data that I use in excess of my plan. If I use 50 extra MB one month, how much extra will I be charged?
- I read 23.5 pages a night for 7 days. How many pages did I read in total that week?
- I earn $22.75 per hour working at a cafe. How much will I earn for 10.25 hours of work?

Learning intentions

Students will be able to:

- use visual representations to multiply decimals
- multiply decimal fractions
- multiply decimals using the vertical algorithm
- use estimation to multiply decimals.

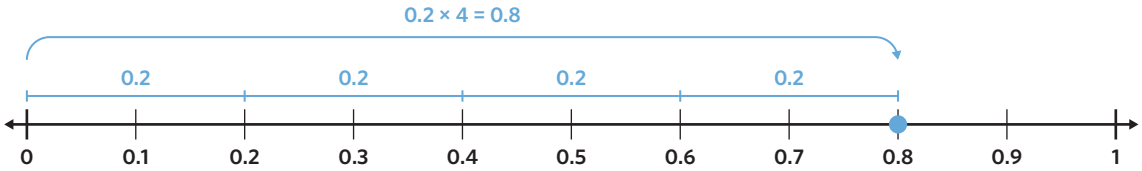
Key terms and definitions

- The **product** is the result when two or more values are multiplied together.
- **Factors** are numbers that can be multiplied together to calculate another number. E.g. $2 \times 3 = 6$, the 2 and 3 are factors of 6.

Key ideas

1 Multiplication is the same as repeated addition and can be shown on a number line.

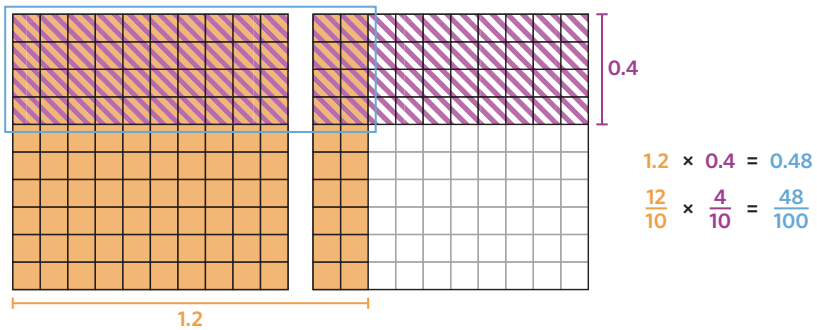
$0.2 \times 4 = 0.8$



2 The hundreds grid can be used to show how to multiply decimals visually.

The overlapping region represents the product of the multiplication.

$1.2 \times 0.4 = 0.48$



$1.2 \times 0.4 = 0.48$

$\frac{12}{10} \times \frac{4}{10} = \frac{48}{100}$

Worked examples

WE 1 Multiplying decimals using decimal fractions

Use decimal fractions to complete each calculation. Express the answer in decimal notation.

a. 0.2×0.4

Working

$$0.2 \times 0.4 = \frac{2}{10} \times \frac{4}{10}$$

$$= \frac{2 \times 4}{10 \times 10}$$

$$= \frac{8}{100}$$

$$= 0.08$$

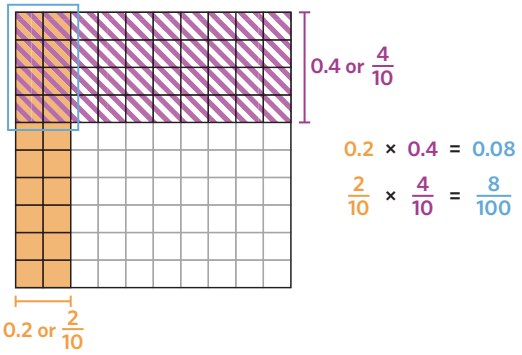
Thinking

Step 1: Represent the decimal multiplication as a fraction multiplication.

Step 2: Perform the multiplication.

Step 3: Convert back to a decimal.

Visual support



b. 0.5×0.02

Working

$$0.5 \times 0.02 = \frac{5}{10} \times \frac{2}{100}$$

$$= \frac{5 \times 2}{1000}$$

$$= \frac{10}{1000} = \frac{1}{100}$$

$$= 0.01$$

Thinking

Step 1: Represent the decimal multiplication as a fraction multiplication.

Step 2: Perform the multiplication. Simplify where possible.

Step 3: Convert back to a decimal.

Student practice

Use decimal fractions to complete each calculation.

a. 0.3×0.2 **b.** 0.4×0.03 **c.** 0.06×0.02 **d.** 0.04×1.3

WE 2 Multiplying decimals using a vertical algorithm

Use a vertical algorithm to complete each calculation.

a. 1.6×4

Working

1.6×4

1 decimal place in the factors means 1 decimal place in the answer.

$$\begin{array}{r} 16 \\ \times\ \ 4 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle +2} \\ 16 \\ \times\ \ 4 \\ \hline 64 \end{array}$$

$1.6 \times 4 = 6.4$ (with one decimal place)

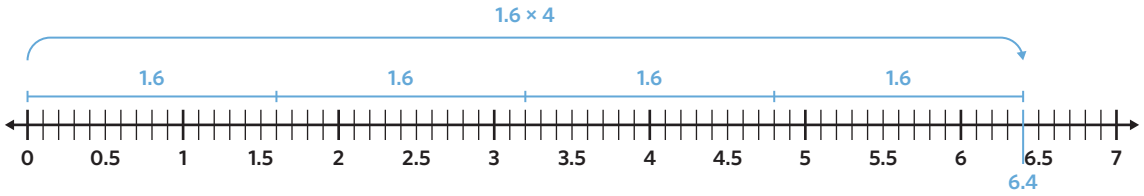
Thinking

Step 1: Determine how many decimal places are in the answer by counting the total number of decimal places in the factors.

Step 2: Set up the vertical algorithm as if multiplying whole numbers.

Step 3: Perform the multiplication and place the decimal point to give the correct number of decimal places.

Visual support



b. 12.3×0.21

Working

12.3×0.21

3 decimal places in the factors means 3 decimal places in the answer.

$$\begin{array}{r} 123 \\ \times\ \ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 123 \\ \times\ \ 21 \\ \hline 123 \\ +\ 2460 \\ \hline 2583 \end{array}$$

$12.3 \times 0.21 = 2.583$ (with three decimal places)

Thinking

Step 1: Determine how many decimal places are in the answer by counting the total number of decimal places in the factors.

Step 2: Set up the vertical algorithm as if multiplying whole numbers.

Step 3: Perform the multiplication and place the decimal point to give the correct number of decimal places.

Student practice

Use a vertical algorithm to complete each calculation.

a. 1.2×3 **b.** 12.4×0.32 **c.** 14.6×0.02 **d.** 24.22×0.003

WE 3 Using rounding to estimate decimal multiplication

Round each decimal to the place value indicated in the brackets to estimate the multiplication.

a. 0.983 × 2.678 (whole number)

Working

0.98 ≈ 1

2.678 ≈ 3

1 × 3 = 3

Thinking

Step 1: Round each number to the specified place value.

Step 2: Perform the multiplication of the rounded numbers.

b. 19.6323 × 0.4217 (tenth)

Working

19.6323 ≈ 19.6

0.4217 ≈ 0.4

19.6 × 0.4

2 decimal places in the factors means 2 decimal places in the answer.

```
  +3  +2
   1   9   6
×              4
-------------------
   7   8   4
```

19.6 × 0.4 = 7.84 (with two decimal places)

Thinking

Step 1: Round each number to the specified place value.

Step 2: Determine how many decimal places are in the answer by counting the total number of decimal places in the rounded factors.

Step 3: Perform the multiplication using the vertical algorithm as if multiplying whole numbers. Place the decimal to give the correct number of decimal places.

Student practice

Round each decimal to the place value indicated in the brackets to estimate the multiplication.

a. 0.9752 × 3.76512 (whole number)

b. 15.82096 × 0.3281 (tenth)

c. 19.99896 × 3.87 (whole number)

d. 29.976 × 0.98723 (tenth)

5E Activities and questions

STARTER TASKS

Image: Prostock-studio/Shutterstock.com

Odd spot

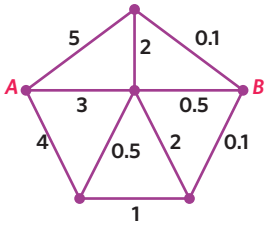
In the 1970s in Australia, a loaf of bread typically cost $0.53. This is much cheaper than the cost of bread now. Baker Bleu, the popular Melbourne bakery, sells a loaf of dark rye bread for 26.415 times the cost of bread in the 1970s. What is the selling price of a loaf of dark rye bread at Baker Bleu?

A. $14

B. $1.40

Puzzle

There are many paths from *A* to *B*. The value of the path is the product of each segment. For example one path from *A* to *B* could be 5 × 2 × 0.5. Each path can only use each segment once.



a) What path from *A* to *B* has the highest value?

b) What path from *A* to *B* has the lowest value?

c) What are some strategies that could be used to determine the path with the highest or lowest value without calculating the multiplication?

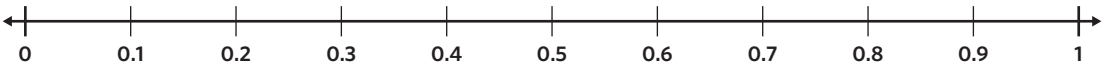
Understanding worksheet

1. Complete the calculation on the number line.

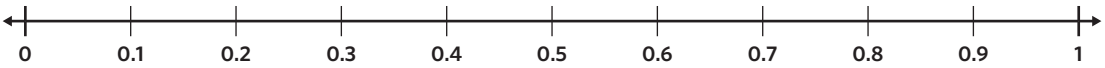
Example

0.2 × 3

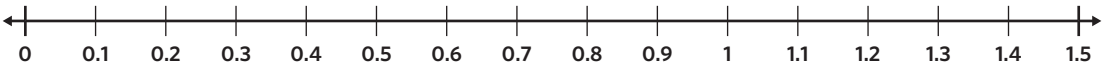
a. 0.3 × 3



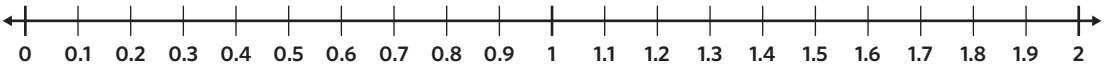
b. 2 × 0.2



c. 5 × 0.3



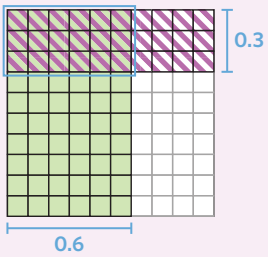
d. 0.4 × 4



2. Complete the multiplication using decimal fractions.

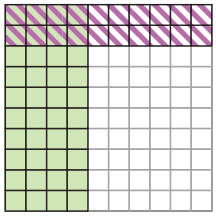
Example

0.6 × 0.3



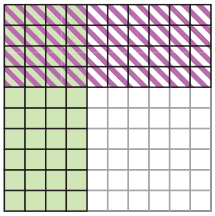
$\frac{\boxed{6}}{10} \times \frac{\boxed{3}}{10} = \frac{\boxed{18}}{100} = \boxed{0.18}$

a. 0.4 × 0.2



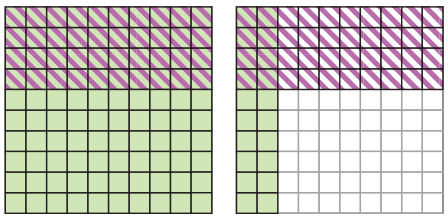
$\frac{4}{10} \times \frac{2}{10} = \frac{\square}{100} = \square$

b. 0.4 × 0.4



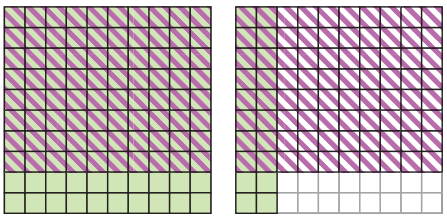
$\frac{4}{10} \times \frac{\square}{10} = \frac{\square}{100} = \square$

c. 1.2×0.4



$\frac{12}{10} \times \frac{\square}{10} = \frac{\square}{100} = \square$

d. 1.2×0.8



$\frac{\square}{10} \times \frac{\square}{10} = \frac{\square}{100} = \square$

3. Fill in the blanks by using the words provided.

product | fractions | reasonableness | estimation

When multiplying decimals, it can be easier to multiply decimal [ ] so we can understand how many decimal places are in the [ ]. For example $0.2 \times 0.3 = 0.06$ can be written in decimal fractions as $\frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$. It is good practice to use [ ] to check the [ ] of your answers.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h)

4. Complete each calculation. Express the answers in decimal notation.

a. 4×0.2 b. 6×0.5 c. 1.1×3 d. 4×2.3

e. 3.2×3 f. 4.01×2 g. 4×5.04 h. 3.11×6

WE1 WE2 **5.** Use decimal fractions or a vertical algorithm to complete each calculation. Express the answers in decimal notation.

a. 0.1×0.3 b. 0.2×0.5 c. 0.3×0.02 d. 0.12×0.02

e. 2.25×1.2 f. 12.2×4.6 g. 46.82×5.4 h. 0.342×0.242

WE3 **6.** Round each decimal to the place value indicated in the brackets to estimate the multiplication.

a. 3.3×4.4 (whole number)
b. 2.345×4.9231 (whole number)
c. 0.3786×4.2431 (tenth)
d. 2.125×3.5623 (tenth)
e. 12.234×6.528 (whole number)
f. 19.643×3.227 (whole number)
g. 0.28749×1.96749 (hundredth)
h. 6.982×5.12 (tenth)

7. Use any method to calculate the following.

a. 0.8×2 b. 0.2×0.65 c. 4.3×1.2 d. 12.45×0.2

e. 0.126×0.3 f. 4.56×3.2 g. 12.14×3.12 h. 15.75×6.75

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. A jar of pasta sauce weighs 0.4 kg. Justin wants to make a batch of spaghetti and meatballs as a celebration dinner for his basketball team's state championship win. He buys 6 jars to make enough sauce. What is the total weight of the jars of pasta sauce?

9. Sophie makes scrunchies and uses 0.7 m of material to make each one. How much material will she need to complete an order of 2 scrunchies?

10. Brooklyn wants to install a swimming pool in her back garden. What is the area of the pool if she wants the length to be 10.4 metres and the width to be 3.67 metres?

11. Desmond is a casual worker, answering phones and helping with office work at his cousin's dental office. If Desmond earns $20.25 per hour how much does earn in a week if he works 17.6 hours?

12. Sarah completes a lap of her neighbourhood every day by running and walking. According to her smartwatch, she completes an average of 5.24 kilometres per day. If she walks $\frac{4}{10}$ of this distance, how many kilometres does Sarah run on average every day?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Casey wants to start harvesting his own eggs in his back garden. He wants to make a standard home garden chicken coop. Most chicken experts recommend 1.2 square metres per chicken in the coop area.
    a. If Casey wants to have 4 chickens, how many square metres in his back garden will he need?
    b. Casey will need 45 pieces of timber each 2.15 metres long to build the coop. How many total metres of timber will he need?
    c. The cost for timber is $3.25 per metre. What will the total cost of timber be?
    d. Casey wants to build a chicken coop to save costs on buying eggs for his family. Do you think that it would be more cost efficient to raise your own chickens, or to buy eggs from the store every week? Explain why.

14. Roger and Shirley want to get out of the city of Melbourne so they are planning a road trip over the summer holiday. Their Kia Sportage drives an average of 9.709 km per 1 litre of petrol. The fuel tank can hold 62 litres of petrol. They are trying to work out when they need to stop for fuel and how many fuel stops they will need to make on their journey.
    a. Estimate how far Roger and Shirley can travel with a full tank of petrol by rounding the average distance travelled per 1 litre of petrol to the nearest whole number.
    b. They stopped for the night at Lakes Entrance after using 34 litres of petrol from the last fuel stop where they filled up the tank to full. How much further can Roger and Shirley drive before their tank is empty?
    c. It is 700 km from Lakes Entrance to Sydney, how many times would Roger and Shirley need to stop to refuel before they reached Sydney?
    d. As they are planning their road trip, Shirley thinks it will be better to buy an electric car, but Roger thinks a hybrid car is better than full electric. Help Roger or Shirley come up with reasons to determine which type of car is better.

Extra spicy

15. Evaluate $3.7 \times 1.12 - 8.1 \times 2.2$.

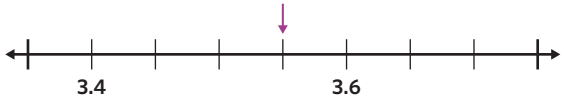
16. A farmer is putting a fence around his rectangular paddock. He uses 270.06 metres of fencing. If the width of his paddock is 44.9 metres, what is the area of his paddock?

17. What is 0.402^2?

18. Write a number with 4 decimal places that when multiplied by 345.62 will give an answer between 1 and 2.

Remember this?

19. What is the value indicated by the arrow?



A. 3.43 B. 3.5 C. 3.55 D. 3.7

20. Bertha keeps a record of the cost of food she buys for lunch each day.

- On Friday, she bought a sandwich, a bottle of juice, an apple and a cookie for $10.50.
- On Saturday, she bought a sandwich and a cookie for $6.
- On Sunday, she bought a sandwich and a bottle of juice for $7.50.

She noticed that the sandwich costs twice as much as the bottle of juice.

How much does the apple cost?

21. Connor has completed $\frac{1}{6}$ of his school project. He wants to have completed $\frac{2}{3}$ of the project by the end of the day.

How much more of the project does Connor have to complete today to reach his goal?

5F Dividing with decimals

Decimals are divided using the same methods as dividing with whole numbers. Just like multiplying with decimals we can also divide decimals using visual models, decimal fractions and algorithms. Below are some examples where dividing decimals can be applied.

- My car uses 8.5 litres of petrol per 100 km driven. How many litres of petrol are used for 1 km of travel?
- I have 560.5 grams of cupcake batter and each cupcake uses 112.1 grams of batter. How many cupcakes can I make?
- I earned $262.46 one week and worked 10.25 hours. How much do I make per hour of work?

Learning intentions

Students will be able to:

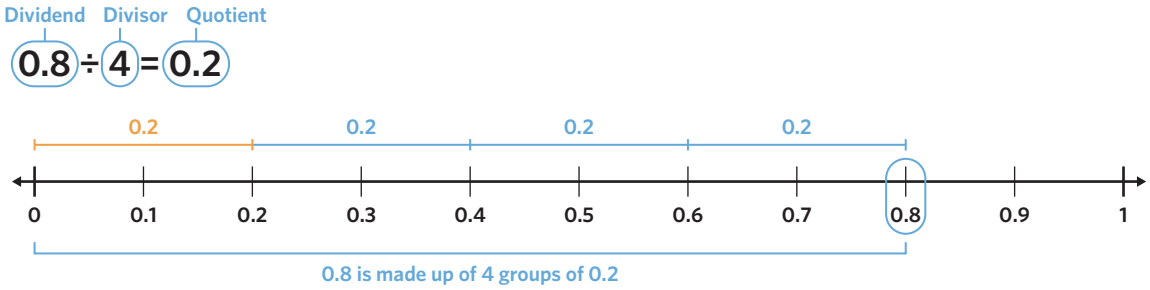
- use visual representations to divide decimals
- divide decimal fractions
- divide decimals using the vertical algorithm
- use rounding to answer division with remainders.

Key terms and definitions

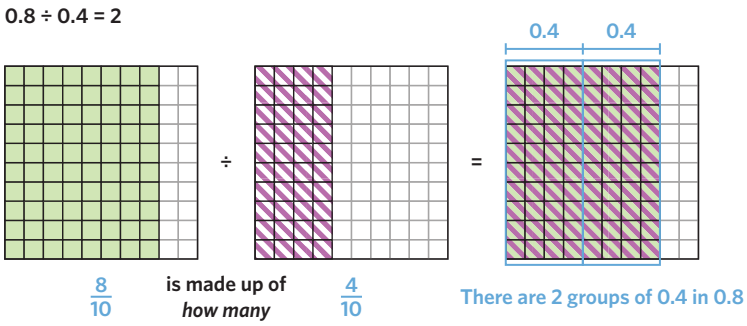
- A **dividend** is the first number in a division calculation. It is the number that is being divided by the second number.
- A **divisor** is the number by which a given value is divided.
- The **quotient** is the result or answer of a division calculation.

Key ideas

1 Decimal division can be represented on a number line showing equal groups of the divisor, making it useful when dividing a decimal by a whole number.



2 The hundreds grid can be used to show how to divide decimals visually.



Worked examples

WE 1 Dividing decimals using decimal fractions

Use decimal fractions to complete each calculation. Express the answer in decimal notation.

a. $0.8 \div 0.2$

Working

$$0.8 \div 0.2 = \frac{8}{10} \div \frac{2}{10}$$

$$= \frac{8}{\cancel{10}_{1}} \times \frac{\cancel{10}^{1}}{2}$$

$$= \frac{8}{2} = 4$$

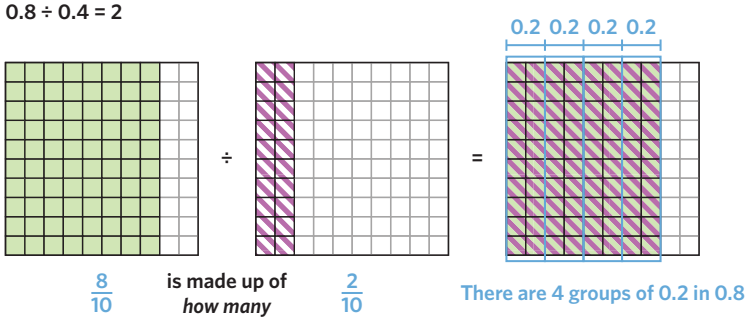
Thinking

Step 1: Represent the decimal division as a fraction division.

Step 2: Multiply the dividend (first number) by the reciprocal of the divisor (second number) and simplify.

Visual support

0.8 ÷ 0.4 = 2



b. $0.126 \div 3$

Working

$$0.126 \div 3 = \frac{126}{1000} \div \frac{3}{1}$$

$$= \frac{42\ \cancel{126}^{\cancel{42}}}{1000} \times \frac{1}{\cancel{3}_{1}}$$

$$= \frac{42}{1000}$$

$$= 0.042$$

Thinking

Step 1: Represent the decimal division as a fraction division.

Step 2: Multiply the dividend (first number) by the reciprocal of the divisor (second number) and simplify.

Step 3: Convert back to a decimal.

Student practice

Use decimal fractions to complete each calculation. Express the answer in decimal notation.

a. $0.4 \div 4$ **b.** $0.16 \div 0.2$ **c.** $1.8 \div 9$ **d.** $0.25 \div 0.5$

WE 2 Dividing decimals using a vertical algorithm

Use the vertical algorithm to complete each calculation.

a. $2.4 \div 6$

Working

$6\overline{)2.4}$

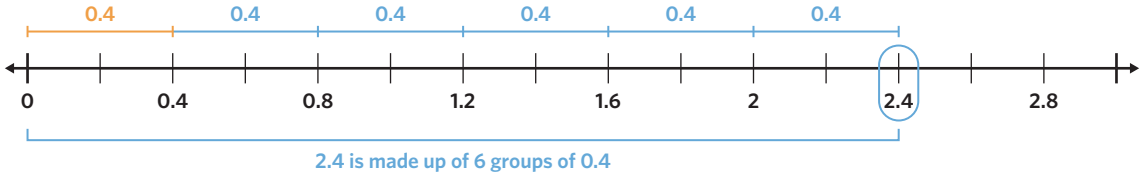
$6\overline{)2.4}^{0.4}$

Thinking

Step 1: Set up short division. Align the decimal point in the quotient with the decimal point in the dividend.

Step 2: Perform short division the same way as with whole numbers. If the dividend is not divisible by the divisor, use a 0 as the placeholder.

Visual support



b. $0.369 \div 0.3$

Working

$0.369 \times 10 = 3.69$

$0.3 \times 10 = 3$

$3.69 \div 3$

$3\overline{)3.69}$ 1.23

$0.369 \div 0.3 = 1.23$

Thinking

Step 1: Multiply the dividend and divisor by the same number so that the divisor is a whole number.

Step 2: Align the decimal point in the quotient with the decimal point in the dividend and divide using short division.

Student practice

Use the vertical algorithm to complete each calculation.

a. $6.3 \div 7$ **b.** $0.486 \div 0.3$ **c.** $13.125 \div 5$ **d.** $0.063 \div 0.003$

WE 3 Rounding decimal division calculations

Calculate and round the answer to 2 decimal places.

a. $4.127 \div 2$

Working

$2\overline{)4.127}$

$2\overline{)4\,.\,1\,2\,{}^{1}7\,{}^{1}0}$ $2\,.\,0\,6\,3\,5$

$2.0635 \approx 2.06$ correct to 2 decimal places.

Thinking

Step 1: Set up short division. Align the decimal point in the quotient with the decimal point in the dividend.

Step 2: Perform short division the same way as with whole numbers. If the dividend is not divisible by the divisor, use a 0 as the placeholder.

Step 3: Round the answer to the nearest hundredth.

b. $5.32 \div 0.03$

Working

$5.32 \times 100 = 532$

$0.03 \times 100 = 3$

$532 \div 3$

$3\overline{)5\,{}^{2}3\,{}^{2}2\,.\,{}^{1}0\,{}^{1}0\,{}^{1}0}$ $1\,7\,7\,.\,\overline{3\,3\,3}$

$177.\overline{333} \approx 177.33$ correct to 2 decimal places.

Thinking

Step 1: Multiply the dividend and divisor by the same number so that the divisor is a whole number.

Step 2: Align the decimal point in the quotient with the decimal point in the dividend and divide using short division.

Step 3: Round the answer to the nearest hundredth.

Student practice

Calculate and round the answer to 2 decimal places.

a. $13.12 \div 3$ **b.** $4.33 \div 0.6$ **c.** $17.4 \div 8$ **d.** $0.36 \div 0.07$

5F Activities and questions

STARTER TASKS

Odd spot

The US women's gymnastics team won Silver in the Team All-Around competition at the Tokyo 2020 Olympics. US gymnast Sunisa Lee won Gold in the Individual All-Around competition and was the first Hmong-American gymnast to compete in the Olympics.

The sum of the six US team members' heights is 920.7 cm. What is the average height of the gymnasts?

A. 153.45 cm **B.** 154.5 cm

Image: A.RICARDO/Shutterstock.com

Puzzle

11.28 ÷ 2.6

433846

a) Where would you put the decimal point in the quotient? Justify your answer.

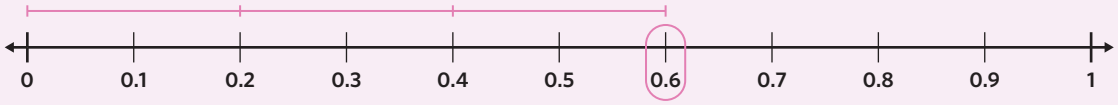
b) Would the answer be larger or smaller than the divisor? Why or why not?

Understanding worksheet

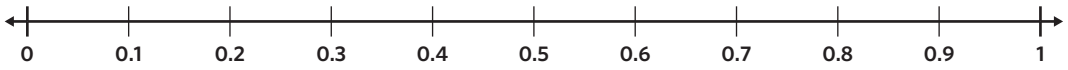
1. Complete the calculation on the number line.

Example

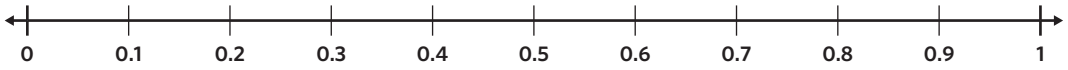
0.6 ÷ 3 = 0.2



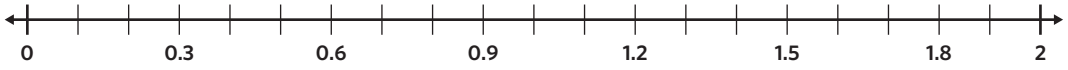
a. 0.8 ÷ 2 = ☐



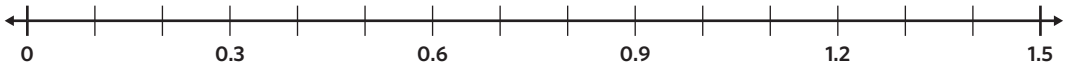
b. 0.5 ÷ 5 = ☐



c. 1.8 ÷ 3 = ☐



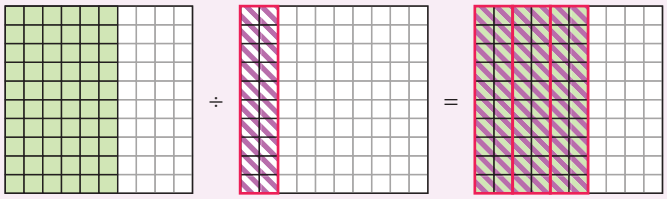
d. 1.2 ÷ 3 = ☐



2. Complete the division using the hundreds grid.

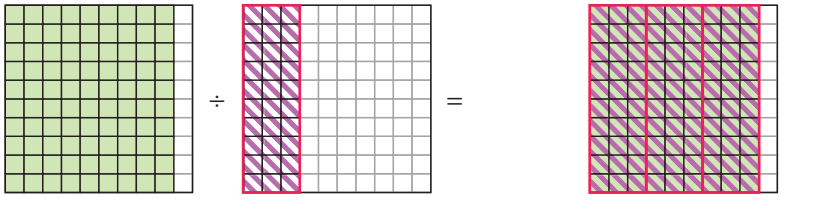
Example

$0.6 \div 0.2 = \boxed{3}$



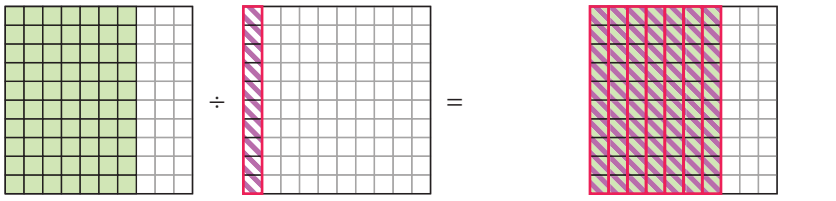
$\frac{6}{10} \div \frac{2}{10} =$ 3 groups of $\frac{2}{10}$

a. $0.9 \div 0.3 = \square$



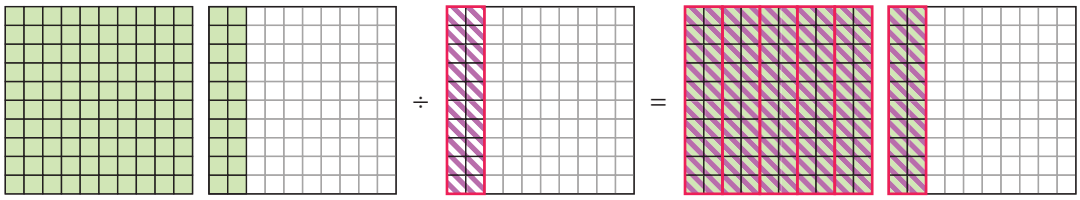
$\frac{9}{10} \div \frac{3}{10} = \square$

b. $0.7 \div 0.1 = \square$



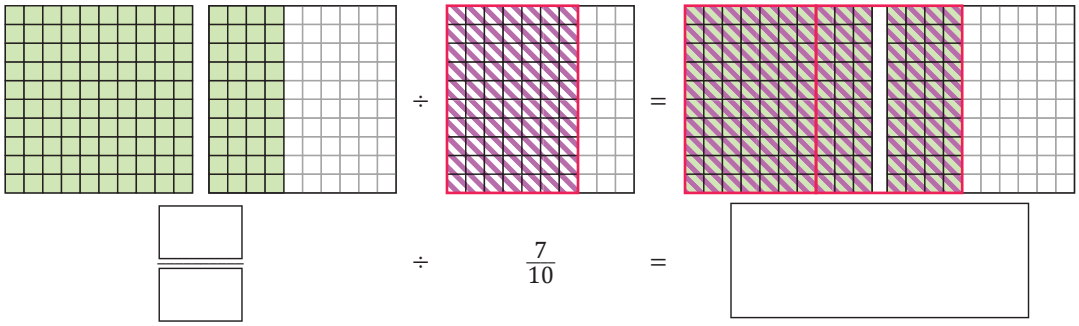
$\frac{7}{10} \div \frac{1}{10} = \square$

c. $1.2 \div 0.2 = \square$



$\frac{12}{10} \div \frac{\square}{\square} = \square$

d. $1.4 \div 0.7 =$ ☐



☐ ÷ $\frac{7}{10}$ = ☐

3. Fill in the blanks by using the words provided.

quotient divisor whole fractions

Division is finding how many equal groups of the ☐ are in the dividend. The result is the ☐. When dividing a decimal by another decimal, multiply both the divisor and the dividend by a factor that makes the divisor a ☐ number. Decimals can also be thought of as ☐ so decimal division can also be performed as fraction division.

Fluency

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1a **4.** Use decimal fractions to complete each calculation. Express the answer in decimal notation.

a. $1.5 \div 3$ **b.** $3.2 \div 4$ **c.** $2.7 \div 9$ **d.** $0.14 \div 2$
e. $0.125 \div 5$ **f.** $12.24 \div 6$ **g.** $12.24 \div 12$ **h.** $14.82 \div 6$

WE2a **5.** Use the vertical algorithm to complete each calculation.

a. $2.4 \div 3$ **b.** $46.15 \div 5$ **c.** $0.18 \div 3$ **d.** $2.156 \div 7$
e. $24.032 \div 8$ **f.** $2.0006 \div 4$ **g.** $0.00125 \div 4$ **h.** $124.42 \div 8$

WE1b **6.** Use decimal fractions to complete each calculation. Express the answer in decimal notation.

a. $2.4 \div 0.06$ **b.** $0.32 \div 0.8$ **c.** $1.25 \div 0.05$ **d.** $8.1 \div 0.9$
e. $1.16 \div 0.04$ **f.** $5.14 \div 0.02$ **g.** $0.369 \div 0.3$ **h.** $12.5 \div 0.005$

WE2b **7.** Use the vertical algorithm to complete each calculation.

a. $2.4 \div 0.6$ **b.** $18.32 \div 0.8$ **c.** $3.256 \div 0.04$ **d.** $0.3669 \div 0.003$
e. $0.6283 \div 0.004$ **f.** $4.21 \div 0.004$ **g.** $13.136 \div 0.08$ **h.** $23.457 \div 0.07$

WE3 **8.** Use any method to calculate each division and round your answer to 2 decimal places.

a. $7.1 \div 0.3$ **b.** $5.1 \div 4$ **c.** $2.54 \div 1.3$ **d.** $16.39 \div 6$
e. $14.536 \div 9$ **f.** $1.736 \div 0.9$ **g.** $0.0469 \div 0.3$ **h.** $17.365 \div 0.04$

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Thomas and 3 of his friends spend $426.16 on dinner at Komeyui Japanese restaurant in South Melbourne. If they split the bill equally amongst them, how much does each person pay?

10. What is the price per litre of petrol if Elise filled up 40 L of petrol and paid $72.25? Round to the nearest cent.

11. Kelsey works out that she needs 0.25 L of cordial for each bottle of lemon drink that she makes. How many bottles of lemon drink can she make with 5.2 L of cordial?

12. Daniel's parents own a picture framing shop. They just received a new shipment of timber to make their frames. How many whole frames can they make if they bought 326.8 metres of timber and an average frame uses 0.9 metres of timber?

13. In the Tokyo 2020 Olympics, the top three running times for the 100 m men's sprint were 9.80 seconds, 9.84 seconds, and 9.89 seconds. What was the average running time for these three runners, correct to two decimal places?
Note: To find an average, add all values together and divide by the number of values there are.

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Tina is surprising her parents and preparing dinner for them when they return home from work. She decides to roast a rack of lamb.
    a. Tina spends $52.50 on 0.75 kg of a rack of lamb from Hagen's organic butcher. How much does the lamb cost per kilogram?
    b. The cooking time for lamb is based on its weight. For 0.75 kg, the cooking time is 47 minutes. What would the cooking time be for a 1 kg rack of lamb? Give your answer in minutes, correct to two decimal places.
    c. If her parents usually get home at 6:15 pm from work, what time should Tina put her roast in the oven for it to be ready when her parents get home?
    d. Tina's brother wants to help out and bake a cake for dessert but they only have 1 oven in the house. How could they work out timing so they could make both the roast and the cake for their parents?

15. Jack is an avid basketball fan. He is comparing the average heights of point guards on the Brooklyn Nets team in the NBA to the point guards on the Melbourne United team in the NBL.
    a. There are 6 guards on the Melbourne United team. What is the average height of the point guards in centimetres, correct to two decimal places? Note: To find an average, add all values together and divide by the number of values.

Melbourne United point guard	Height (cm)
Chris Goulding	192.16
Matthew Dellavedova	193.24
Shea Ili	184.37
William Hickey	192.21
Zac Triplett	198.43
Dion Prewster	194.91

    b. The following heights for the four Brooklyn Nets point guards have been given in inches, an American unit of measurement. If 1 inch is equal to 2.54 centimetres, convert the heights from inches to centimetres and fill in the table.

Brooklyn Nets point guard	Height (in)	Height (cm)
Patty Mills	73	
Kyrie Irving	74	
Jevon Carter	73	
Mike James	73	

    c. What is the average height, in centimetres, of the 4 point guards for the Brooklyn Nets?

d. Which team has the taller average point guard?

e. Jack thinks that the height difference could be because of varying factors such as where and how the players grew up. What could be some factors that affect height?

Extra spicy

16. Write the decimal for $\frac{18}{369}$.

17. What is $201.4 \div 3.06$?

18. An airplane travelled 2781.1 miles in 6.4 hours. If 1 mile equals 1.6 kilometres, what is the average speed in kilometres per hour the airplane was travelling?

19. Jack is climbing Mt. Arapiles at a constant rate. At 2:00 pm he was $\frac{1}{6}$ of the way up. At 4:00 pm he was $\frac{3}{4}$ of the way up. At 3:00 pm what was the fraction he had climbed written as a decimal?

Remember this?

20. Sophie spends $185 per week on rent.
How much does she spend on rent over 52 weeks?

21. In the following table, each pair of numbers follows the rule $\square = 3 \times \triangle - 6$.

$\triangle$	2	5	12	60
$\square$	0	9	30	?

What is the missing number?

A. 22 **B.** 57 **C.** 174 **D.** 186

22. Halle had $15 and used it to buy 6 packs of gum.
Each pack of gum costs $1.70.
How much change should Halle receive?

A. $4.80 **B.** $5.40 **C.** $10.20 **D.** $13.30

5G Introduction to percentages

The term percent means out of one hundred and is represented with the percentage sign '%'. Fractions and percentages are closely related. Percentages can be expressed as a fraction with a denominator of 100. Percentages are used to represent a quantity of a whole. Below are some examples where percentages can be applied.

- If I got 94 questions correct in a 100 question test, what percentage of questions did I get correct?
- I've completed $\frac{3}{8}$ of my painting. What percentage of my painting have I completed?
- I purchased a laptop with a 25% discount. What fraction of the original price did I pay?

Learning intentions

Students will be able to:

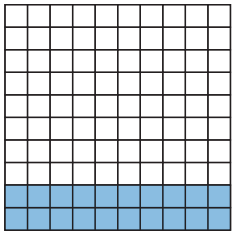
- understand that percentages are always out of 100
- use visual representations to show percentages
- find complementary percentages
- convert between fractions and percentages.

Key terms and definitions

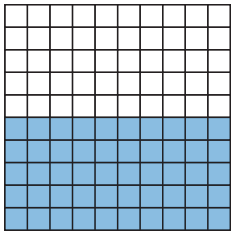
- A **percentage** represents a value as a part out of 100, where 100 represents a whole.
- **Complementary percentages** are two percentage values that add up to 100%, where 100% refers to the whole amount. E.g. 60% and 40% are complementary percentages as they add up to 100%.
- A **proportion** is a part that is considered in relation to the value of a whole.

Key ideas

1 Percentages are a proportion of 100 and can be represented as fractions with a denominator of 100.

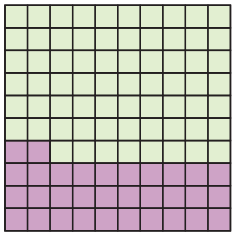


$\frac{20}{100} = 20\%$



$\frac{50}{100} = 50\%$

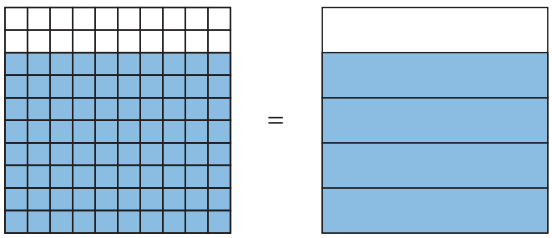
2 We can find complementary percentages by subtracting the known percentage from 100%.



$100\% - 68\% = 32\%$

32% and 68% are complementary percentages

3 We can convert percentages to fractions by representing the percentage as a fraction with a denominator of 100, and then simplifying.



80% or $\frac{80}{100}$ $=$ $\frac{4}{5}$

4 We can convert fractions to percentages using equivalent fractions or multiplying by 100.

Method 1: Equivalent fractions

$$\frac{1}{4} \overset{\times 25}{=} \frac{25}{100}$$

$\frac{1}{4} = 25\%$

Method 2: Multiplying by 100

$$\frac{1}{4} \times 100 = \frac{100}{4}$$
$$= 100 \div 4$$
$$= 25$$

$\frac{1}{4} = 25\%$

Worked examples

WE 1 Converting percentages to fractions

Express each percentage as a fraction in its simplest form.

a. 40%

Working

$40\% = \frac{40}{100}$

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

The HCF is 20.

$\frac{40 \div 20}{100 \div 20} = \frac{2}{5}$

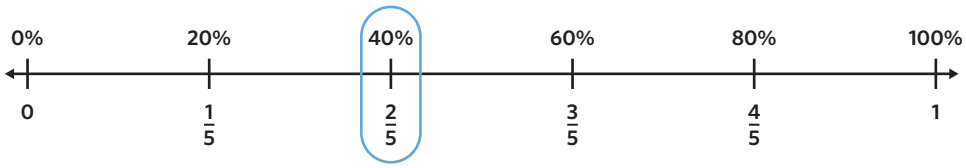
Thinking

Step 1: Represent the percentage as a fraction with the percent value as the numerator and 100 as the denominator.

Step 2: Identify the highest common factor (HCF) of the numerator and denominator.

Step 3: Divide the numerator and denominator by the HCF.

Visual support



b. 112%

Working

$112\% = \frac{112}{100}$

Factors of 112: 1, 2, 4, 7, 8, 14, 16, 28, 56, 112

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

The HCF is 4.

$\frac{112 \div 4}{100 \div 4} = \frac{28}{25}$ or $1\frac{3}{25}$

Thinking

Step 1: Represent the percentage as a fraction with the percent value as the numerator and 100 as the denominator.

Step 2: Identify the HCF of the numerator and denominator.

Step 3: Divide the numerator and denominator by the HCF.

Student practice

Express each percentage as a fraction in its simplest form.

a. 34% **b.** 108% **c.** −90% **d.** 350%

WE 2 Converting fractions to percentages using equivalent fractions

Use equivalent fractions to convert each fraction to a percentage.

a. $\frac{1}{5}$

Working

$$\frac{1}{5} \overset{\times 20}{=} \frac{20}{100}$$

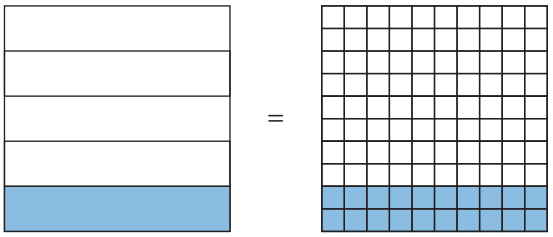
$$\frac{20}{100} = 20\%$$

Thinking

Step 1: Find a number that when multiplied by the denominator, gives a product of 100. Multiply the numerator and denominator by this number.

Step 2: Write the equivalent fraction as a percentage.

Visual support



$\frac{1}{5}$ $\frac{20}{100}$ or 20%

b. $\frac{31}{25}$

Working

$$\frac{31}{25} \overset{\times 4}{=} \frac{124}{100}$$

$$\frac{124}{100} = 124\%$$

Thinking

Step 1: Find a number that when multiplied by the denominator, gives a product of 100. Multiply the numerator and denominator by this number.

Step 2: Write the equivalent fraction as a percentage.

Student practice

Use equivalent fractions to convert each fraction to a percentage.

a. $\frac{2}{4}$ **b.** $\frac{13}{20}$ **c.** $\frac{250}{200}$ **d.** $3\frac{26}{50}$

WE 3 Converting fractions to percentages by multiplying by 100

Convert each fraction to a percentage by first multiplying the fraction by 100.

a. $\frac{7}{20}$

Working

$$\frac{7}{20} \times \frac{100}{1} = \frac{7}{1} \times \frac{5}{1}$$
$$= \frac{35}{1}$$
$$= 35$$

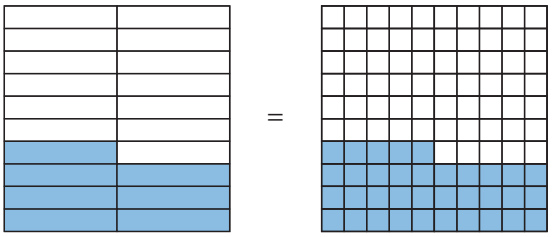
$$\frac{7}{20} = 35\%$$

Thinking

Step 1: Multiply the fraction by $\frac{100}{1}$ (another way to represent 100). Simplify where possible before completing the multiplication. The result will be the percent value.

Step 2: Add a percentage sign.

Visual support



$\frac{7}{20}$ = $\frac{3}{5}$ or 35%

b. $2\frac{3}{4}$

Working

$2\frac{3}{4} = \frac{2 \times 4 + 3}{4}$

$= \frac{11}{4}$

$\frac{11}{4} \times \frac{100}{1} = \frac{11}{1} \times \frac{25}{1}$

$= \frac{275}{1}$

$= 275$

$2\frac{3}{4} = 275\%$

Thinking

Step 1: Convert the improper fraction to a mixed number.

Step 2: Multiply the fraction by $\frac{100}{1}$ (another way to represent 100). Simplify where possible before completing the multiplication. The result will be the percent value.

Step 3: Add a percentage sign.

Student practice

Convert each fraction to a percentage by first multiplying the fraction by 100.

a. $\frac{15}{20}$ **b.** $3\frac{1}{4}$ **c.** $\frac{15}{75}$ **d.** $\frac{220}{80}$

5G Activities and questions

STARTER TASKS

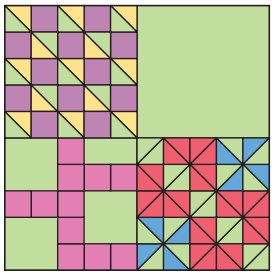
Odd spot

Alan Thomas is a long serving employee at the Everest Ice Cream Factory. Alan is often in the lab experimenting with new ingredients and creating new flavours. He has spent 32 years making ice cream, 12 of which was spent at Everest Ice Cream Factory. What percentage of Alan's ice cream making career was spent at the Everest Ice Cream Factory?

A. 37.5% **B.** 4.5%

Image: Justin McManus/The Age

Puzzle



a) What percentage of the image is shaded green?

b) How much of the image is NOT shaded green?

c) What percentage of the image is shaded blue?

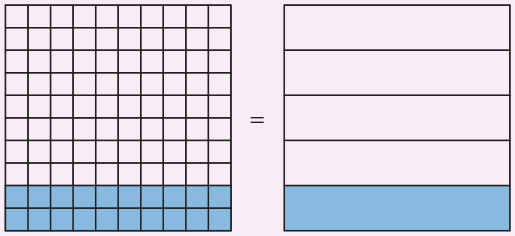
d) Which two colours shade an equivalent percentage of the entire image?

Understanding worksheet

1. Use the hundreds grid to identify the percentage and fraction shaded.

Example

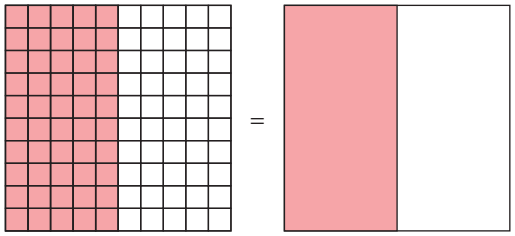
Percentage Fraction



20% = $\frac{1}{5}$

a.

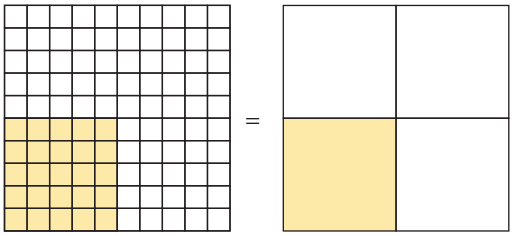
Percentage Fraction



□% = □

b.

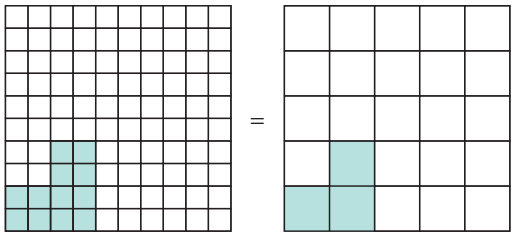
Percentage Fraction



□% = □

c.

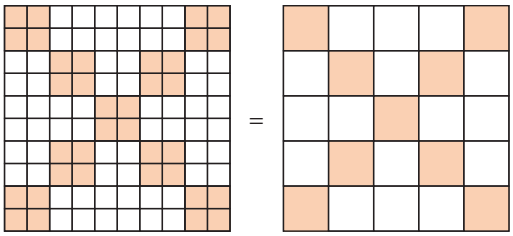
Percentage Fraction



□% = □

d.

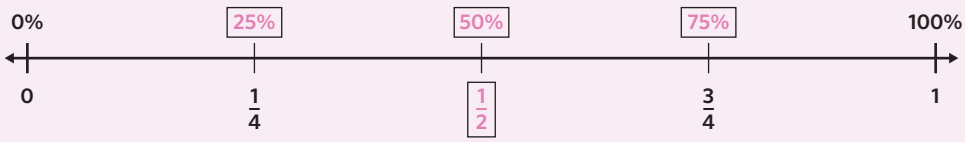
Percentage Fraction



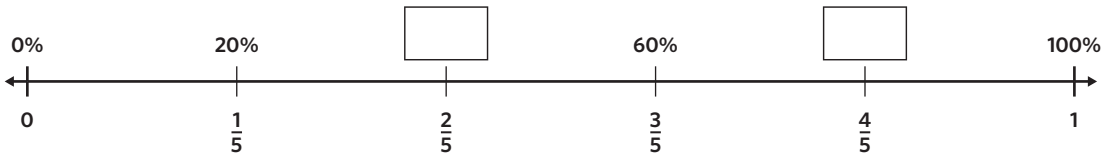
□% = □

2. Complete the number lines by placing the correct percentage or fraction in each box.

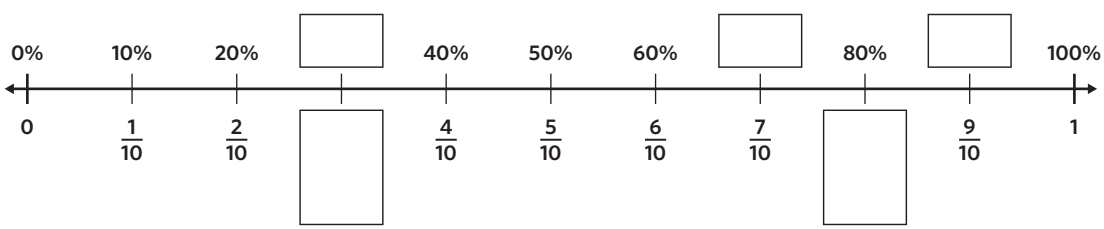
Example



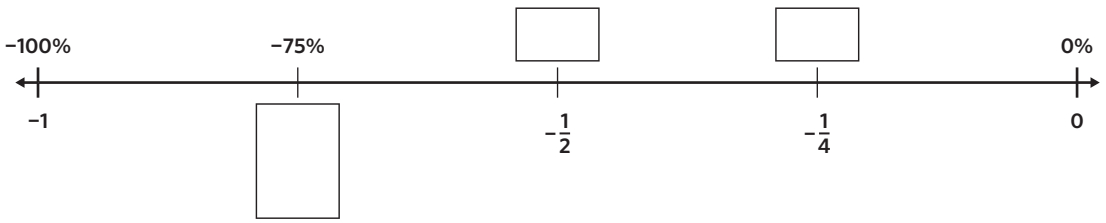
a.



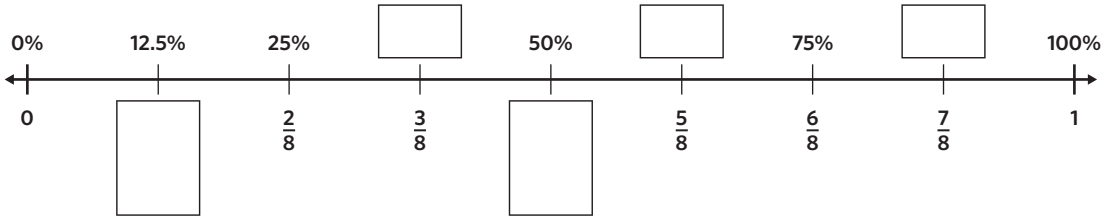
b.



c.



d.



3. Fill in the blanks by using the words provided.

fraction | multiply | one hundred | proportion | equivalent

The term percent means out of _____. A percentage is a _____ that shows the number of parts for every hundred and can be represented by a _____. To convert a fraction to a percentage, we can either find an _____ fraction with a denominator of 100 or _____ the fraction by 100 and add a percentage sign.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b), 6 (a,b,c,d), 7 (a,b,c,d)	4 (c,d,e,f), 5 (b,c), 6 (c,d,e,f), 7 (c,d,e,f)	4 (e,f,g,h), 5 (c,d), 6 (e,f,g,h), 7 (e,f,g,h)

WE1 **4.** Express each percentage as a fraction in its simplest form.

a. 10% **b.** 50% **c.** 80% **d.** 23%

e. 120% **f.** −45% **g.** −175% **h.** 37.5%

5. Find the complementary percentage for each percentage.

a. 60% **b.** 83% **c.** 7% **d.** 97%

WE2 **6.** Use equivalent fractions to convert each fraction to a percentage.

a. $\frac{1}{5}$ **b.** $\frac{7}{10}$ **c.** $\frac{4}{5}$ **d.** $\frac{17}{25}$

e. $\frac{11}{4}$ **f.** $1\frac{1}{2}$ **g.** $\frac{250}{125}$ **h.** $5\frac{30}{150}$

WE3 **7.** Convert each fraction to a percentage by first multiplying the fraction by 100.

a. $\frac{1}{4}$ **b.** $\frac{2}{5}$ **c.** $\frac{11}{20}$ **d.** $\frac{5}{8}$

e. $2\frac{15}{20}$ **f.** $\frac{23}{10}$ **g.** $3\frac{5}{20}$ **h.** $\frac{49}{7}$

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Lissandra cut her birthday cake into ten equal pieces to share with her friends and family. What percentage does each slice of cake represent as a proportion of the whole cake?

9. Rebecca wanted to bake some cookies for dessert. What fraction of the cookie dough did Rebecca use if she only used 40% and froze the rest for another day. Express your answer in the simplest form.

10. What percentage score did Zoro get on his science test if he got 17 out of a total of 20 marks correct?

11. Josh had a full bottle of water in the morning. By lunchtime, his bottle was 65% full. What fraction of his water bottle did Josh drink? Express in the simplest form.

12. Jessica is the star player on her basketball team. At three quarter time, Jessica had scored 19 out of her team's 44 points. In the fourth quarter, Jessica scored another 8 points out of her team's 16 points. What percentage of the team's total points were scored by Jessica?

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Samuel is representing his school at a national chess tournament. Each tournament has participants playing 7 games. A win results in 1 point, a stalemate results in 0.5 points and a loss results in no points. Samuel finished the tournament with 6.5 points.
    a. How many games did Samuel win?
    b. What fraction of games did Samuel stalemate?
    c. What percentage of games did Samuel lose?
    d. Samuel played a second tournament and won or stalemated in 71.4% of his games. What percentage of games did he lose?
    e. Samuel's friend Cassandra finished the national chess tournament (first tournament) with 5 points. Is Samuel a better chess player than Cassandra? Justify your answer.

14. The following table shows the marks obtained by four students for the year 7 Japanese exam. The exam consists of two sections that are worth 40 marks each. To pass the exam, students must score at least 70% in **each** section.

Student	Speaking (40 marks)	Writing (40 marks)	Total (80 marks)
Marko	34	36	70
Sylvia	26	40	66
Atif	38	22	60
Bashir	27	29	56

    a. How many of these four students passed the Japanese exam?
    b. What was each student's total grade as a percentage?
    c. Of the total number of sections that these four students completed, what percentage of these sections **did not** result in a pass grade?
    d. Is getting at least 70% on both sections a fair way to assess students? Justify your answer.

Extra spicy

15. What is $\frac{3}{8}$% expressed as a decimal?

    A. 0.00375 B. 0.0038 C. 0.0375 D. 0.375 E. 0.38

16. The number 72 is 75% of what number?

    A. 48 B. 56 C. 84 D. 96 E. 108

17. Chris runs an online e-commerce shop. Last year he sold $75 000 worth of products and this year he sold $300 000 worth of products. How much did his revenue increase by?

    A. 4% B. 100% C. 200% D. 300% E. 400%

18. Five years ago, the population of a town was approximately 200 000 people. Since then, the town's population has grown by a total of 340%. What is the population of the town today?

Remember this?

19. Samantha followed a rule to create a numerical sequence. To get to the next number of the sequence, you minus 2 and then triple. If Samanatha started the sequence with the number 4, what is the 5^{th} term of this sequence?

A. 6 B. 12 C. 30 D. 84

20. Laura wants to buy a pair of sunglasses. If she has $80 dollars saved up and the sunglasses cost $232, how much more money does she need to buy the sunglasses?

A. $132 B. $142 C. $152 D. $162

21. Ruka is running in a 10 km race. In the middle of his race he checked his running watch, which shows that he has already completed 6200 metres. What fraction of the race, in simplest form, has Ruka completed so far?

5H Converting between fractions, decimals, and percentages

Fractions, decimals, and percentages are often used in day-to-day language. Being able to convert between decimals, fractions, and percentages can help to address many different types of problems. An important part of understanding decimals and percentages is understanding how they are connected to fractions. Making this connection enables us to use percentages in different contexts. Below are some examples where converting fractions, decimals, and percentages can be applied.

- $\frac{1}{4}$ of my friends are fans of the Queensland Firebirds netball team. What percentage of my friends go for the Firebirds?
- The probability that I am accepted into university is 0.93. What is the percentage probability that I am accepted into university?
- I got 75% in my end of year Science exam. What proportion of questions did I get correct, as a decimal?

Learning intentions

Students will be able to:

- convert decimals to fractions in their simplest form
- convert fractions to decimals
- convert decimals to percentages
- convert percentages to decimals.

Key terms and definitions

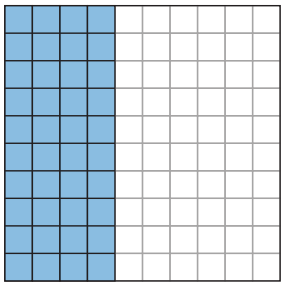
- A **percentage** represents a value as a part out of 100, where 100 represents a whole.
- A **decimal fraction** is a fraction where the denominator is a power of 10, such as 10, 100, 1000, 10 000. E.g. the decimal 0.25 written as a decimal fraction is $\frac{25}{100}$.
- **Converting** is the process of changing a value into a different but equal form. E.g. The fraction $\frac{1}{4}$ can be converted to 25%.

Key ideas

1 Decimals, decimal fractions, and percentages are different ways of representing the same value.

$$65\% = \frac{65}{100} = 0.65$$

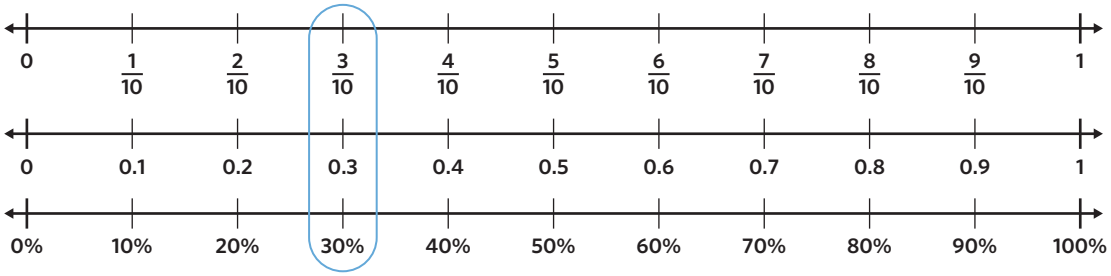
2 We can use hundreds grids to represent the same area in different ways and express the same number in different forms.



Decimal		Percentage		Fraction
0.4	=	40%	=	$\frac{4}{10}$

3 We can represent fractions, decimals, and percentages using number lines.

$\frac{3}{10} = 0.3 = 30\%$



Worked examples

WE 1 Converting percentages to decimals

Express each percentage as a decimal.

a. 29%

Working

29% = 29 ÷ 100

29 ÷ 100 = 0.29

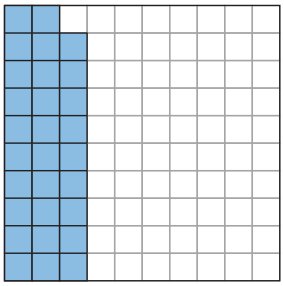
= 0.29

Thinking

Step 1: Express the percentage as the percent value divided by 100.

Step 2: Complete the division. As the divisor is 100, each **digit** moves two place values to the right.

Visual support



Percentage		Decimal
29%	=	0.29

b. 63.4%

Working

63.4 % = 63.4 ÷ 100

63.4 ÷ 100 = 0.634

= 0.634

Thinking

Step 1: Express the percentage as the percent value divided by 100.

Step 2: Complete the division. As the divisor is 100, each **digit** moves two place values to the right.

c. 0.92%

Working

0.92 % = 0.92 ÷ 100

0.92 ÷ 100 = 0.0092

= 0.0092

Thinking

Step 1: Express the percentage as the percent value divided by 100.

Step 2: Complete the division. As the divisor is 100, each **digit** moves two place values to the right.

Student practice

Express each percentage as a decimal.

a. 83% **b.** 12.8% **c.** 0.76% **d.** 274.6%

WE 2 Converting decimals to percentages

Express each decimal as a percentage.

a. 0.6

Working

$0.6 \times 100 = 60.0$

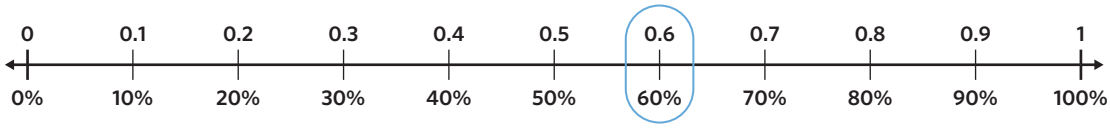
$0.6 = 60\%$

Thinking

Step 1: Multiply the decimal by 100.

Step 2: Complete the multiplication. As the multiplier is 100, each **digit** of the decimal moves two place values to the left.

Visual support



b. 0.56

Working

$0.56 \times 100 = 56.0$

$0.56 = 56\%$

Thinking

Step 1: Multiply the decimal by 100.

Step 2: Complete the multiplication. As the multiplier is 100, each **digit** of the decimal moves two place values to the left.

c. 0.281

Working

$0.281 \times 100 = 28.1$

$0.281 = 28.1\%$

Thinking

Step 1: Multiply the decimal by 100.

Step 2: Complete the multiplication. As the multiplier is 100, each **digit** of the decimal moves two place values to the left.

Student practice

Express each decimal as a percentage.

a. 0.2 **b.** 0.97 **c.** 0.863 **d.** 3.294

WE 3 Converting decimals to fractions

Express each decimal as a fraction in its simplest form.

a. 0.45

Working

$0.45 = \frac{45}{100}$

Factors of 45: 1, 3, 5, 9, 15, 45

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

HCF is 5.

$\frac{45 \div 5}{100 \div 5} = \frac{9}{20}$

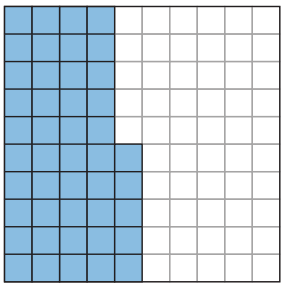
Thinking

Step 1: Convert the decimal into a decimal fraction with a denominator that is a power of 10.

Step 2: Find the highest common factor (HCF) of the numerator and denominator.

Step 3: Divide the numerator and denominator by the HCF.

Visual support



Decimal		Fraction
0.45	=	$\frac{9}{20}$

b. 6.25

Working

$6.25 = 6\frac{25}{100}$

Factors of 25: 1, 5, 25

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

HCF is 25.

$6\frac{25 \div 25}{100 \div 25} = 6\frac{1}{4}$

Thinking

Step 1: Convert the decimal into a decimal fraction with a denominator that is a power of 10.

Step 2: Find the highest common factor (HCF) of the numerator and denominator.

Step 3: Divide the numerator and denominator by the HCF.

Student practice

Express each decimal as a fraction in its simplest form.

a. 0.35 **b.** 0.8 **c.** 1.75 **d.** 7.06

WE 4 Converting fractions to decimals

Express each fraction as a decimal.

a. $\frac{2}{5}$

Working

$\frac{2}{5} \overset{\times 2}{=} \frac{4}{10}$

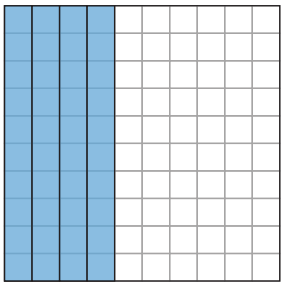
$\frac{4}{10} = 0.4$

Thinking

Step 1: Find an equivalent decimal fraction with a denominator that is a power of 10.

Step 2: Express the decimal fraction as a decimal.

Visual support



Fraction		Decimal
$\frac{2}{5}$	=	0.4

b. $\frac{3}{8}$

Working

$\frac{3}{8} = 3 \div 8$

$8\overline{)3}$

$8\overline{)3.000}$ (0.375)

$\frac{3}{8} = 0.375$

Thinking

Step 1: Express the fraction as a division. As this cannot be simplified easily, use short division.

Step 2: Align the decimal point in the quotient with the decimal point in the dividend.

Step 3: Perform short division the same way as with whole numbers.

If the dividend is not divisible by the divisor, use a zero as the placeholder.

Student practice

Express each fraction as a decimal.

a. $\frac{3}{5}$ **b.** $\frac{7}{8}$ **c.** $-4\frac{17}{20}$ **d.** $\frac{11}{16}$

5H Activities and questions

STARTER TASKS

Odd spot

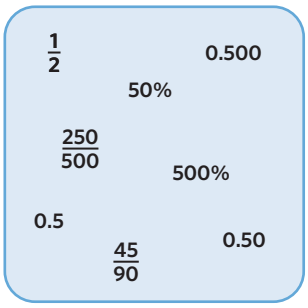
Many businesses and industries were affected by the COVID-19 pandemic. One of the most affected industries was aviation, as many countries shut their international borders. Alan Joyce, the CEO of Qantas, Australia's largest national airline, has made it mandatory for 100% of Qantas staff to be vaccinated. Many countries around the world are aiming to vaccinate up to 80% of their populations to resume international travel.

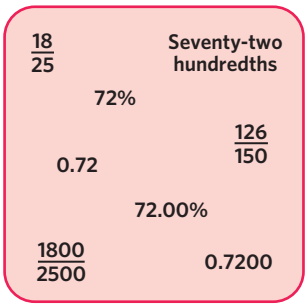
What fraction of the population are countries aiming to vaccinate?

A. $\frac{1}{80}$ **B.** $\frac{4}{5}$

Image: Penny Stephens/Western Health/The Age

Puzzle





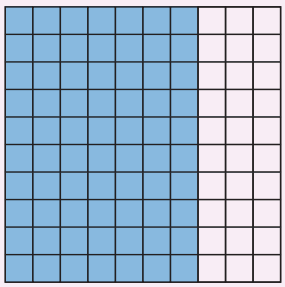
a) Select the 'odd one out' in the blue square.

b) Select the 'odd one out' in the red square.

c) Can you add a fraction and decimal to the red square that seems like it might belong, but is actually another 'odd one out'?

Understanding worksheet

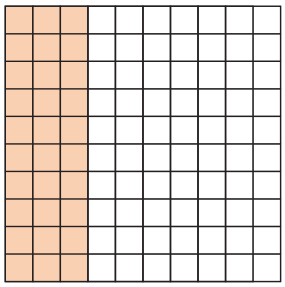
1. Write the missing value in each box.



Example

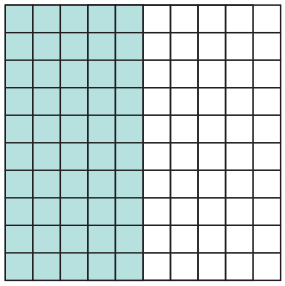
Decimal		Percentage		Fraction
0.7	=	70%	=	$\frac{7}{10}$

a.



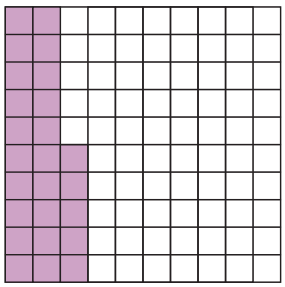
Decimal		Percentage		Fraction
0.3	=	30%	=	

b.



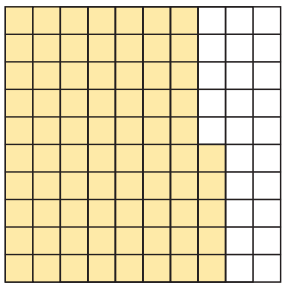
Decimal		Percentage		Fraction
0.5	=		=	$\frac{1}{2}$

c.



Decimal		Percentage		Fraction
	=	25%	=	$\frac{1}{4}$

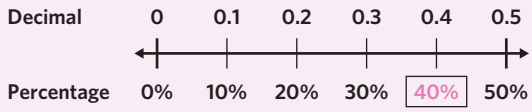
d.



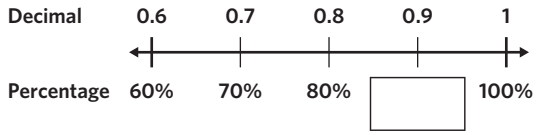
Decimal		Percentage		Fraction
0.75	=		=	$\frac{3}{4}$

2. Complete the number lines by placing the correct fraction, decimal, or percentage in each box.

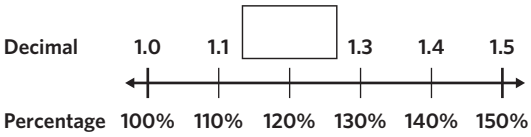
Example



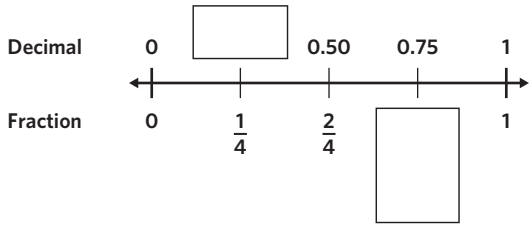
a.



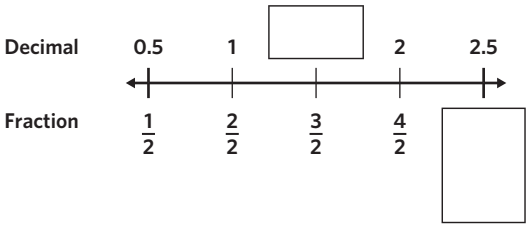
b.



c.



d.



3. Fill in the blanks by using the words provided.

percentages | decimals | power of 10 | simplest form | %

Decimals can be converted to _____ by multiplying the decimal number by 100 and adding a _____ sign. We can express _____ as fractions with a denominator that is a _____, and then reduce the fraction to its _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d), 8 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f), 8 (d,e,f)

WE1 **4.** Express each percentage as a decimal.

a. 23%	**b.** 96%	**c.** 150%	**d.** 18.5%
e. 0.39%	**f.** 9.71%	**g.** 436.3%	**h.** 0.084%

WE2 **5.** Express each decimal as a percentage.

a. 0.34	**b.** 0.7	**c.** 0.05	**d.** 0.592
e. 3.4	**f.** 2.75	**g.** 8.936	**h.** 0.002

WE3 **6.** Express each decimal as a fraction in its simplest form.

a. 0.4	**b.** 0.85	**c.** 7.6	**d.** 2.75
e. 8.13	**f.** 1.02	**g.** 13.125	**h.** 5.006

WE4a **7.** Express each fraction as a decimal by first finding an equivalent decimal fraction.

a. $\frac{1}{5}$	**b.** $\frac{21}{25}$	**c.** $-\frac{33}{50}$	**d.** $2\frac{19}{25}$
e. $\frac{38}{200}$	**f.** $7\frac{400}{500}$		

WE4b **8.** Express each fraction as a decimal by using short division.

a. $\frac{5}{8}$	**b.** $\frac{5}{16}$	**c.** $\frac{9}{40}$	**d.** $\frac{6}{75}$
e. $\frac{37}{40}$	**f.** $\frac{6}{32}$		

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. The probability of buying a winning raffle ticket at a charity fundraising event is 0.15. Expressed as a fraction in its simplest form, what is the probability of buying the winning ticket?

10. What is Campbell's age expressed as a decimal if he is twelve years and three months old?

11. In a survey, 47% of women between the age of 20 and 25 would rather live in a city than a country town. Express, as a decimal, the proportion of 20 to 25 year-old females that prefer to live in a country town.

12. Hillside High School conducted a student survey to determine what hot food to add to their canteen menu. Which item should be added to the menu if 31% of students voted for a kebab, 0.301 of the students voted for sushi and $\frac{8}{25}$ of the students voted for fish pies?

13. What mark did Melanie achieve in her motorcycle licence test? Melanie got 15 out of 24 questions correct. Express her mark as a percentage.

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. In order to purchase a property, Frank and Lucy enquire about borrowing money from their local bank. Frank and Lucy both have full-time jobs so are considered a dual-income household. They are instructed to apply for a mortgage. A mortgage is when the bank lends money to a borrower specifically for buying a property.
    a. 42% of all mortgage applications are rejected. Expressed in its simplest form, what fraction of mortgage applications are rejected?
    b. The bank worker says that the chance of a mortgage application being accepted for a dual income household is 0.93. What is the percentage chance that Frank and Lucy's application will be successful?
    c. The bank worker informs Frank and Lucy that the probability of a doctor's mortgage application being accepted is 1.12. Frank says that it is impossible. Express the probability as a percentage and justify why Frank is correct.
    d. Other than employment status, what other factors might a bank need to consider when approving or rejecting a mortgage?

15. Magda, Drew, and Aliya all love to make their own cereal mix. Although they have different preferences for the amount of each ingredient, they all like to use the same ingredients. The ingredients they use are: dried fruit, oats, mixed nuts, and yogurt.
    a. One-fifth of Magda's mix is made up of oats, and 28% is dried fruit. Expressed as a decimal, what proportion of Magda's mix is made up of mixed nuts and yogurt?
    b. In 200 grams of his cereal mix, Drew puts 50 grams of oats. 0.25 of Aliya's mix is made up of oats. Explain why they have the same proportion of oats.
    c. 0.5 of Magda's entire mix is yogurt whereas 42% of Drew's mix is yogurt. What fraction of Aliya's mix is yogurt, if the amount she uses is halfway between the fraction Drew and Magda use?
    d. Recommend a way that all three friends might be able to create a recipe to sell in a store.

Extra spicy

16. Place each value in descending order:
    1.212, 12%, $\frac{1212}{100}$, 1.212^2

17. π represents the ratio of the circumference of a circle to its diameter. Written as a decimal to fourteen decimal places, π is 3.14159265358979. Which value is the closest to π: $\frac{44}{14}$, 3.1416 or 3.1415%?

18. Cups A, B and C contain mixtures of salt and water.

- Cup A has a ratio of salt to water of 6 : 44
- 87.25% of Cup B is water
- One-eighth of Cup C is salt

Which cup has the highest proportion of salt?

19. Which one of the following contains a set of three values that are spaced evenly on a number line?

A. $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}$

B. $125\%, \frac{16}{10}, 1.75$

C. $0.5, \frac{5}{8}, 25\%$

D. $\frac{12}{15}, 0.95, 110\%$

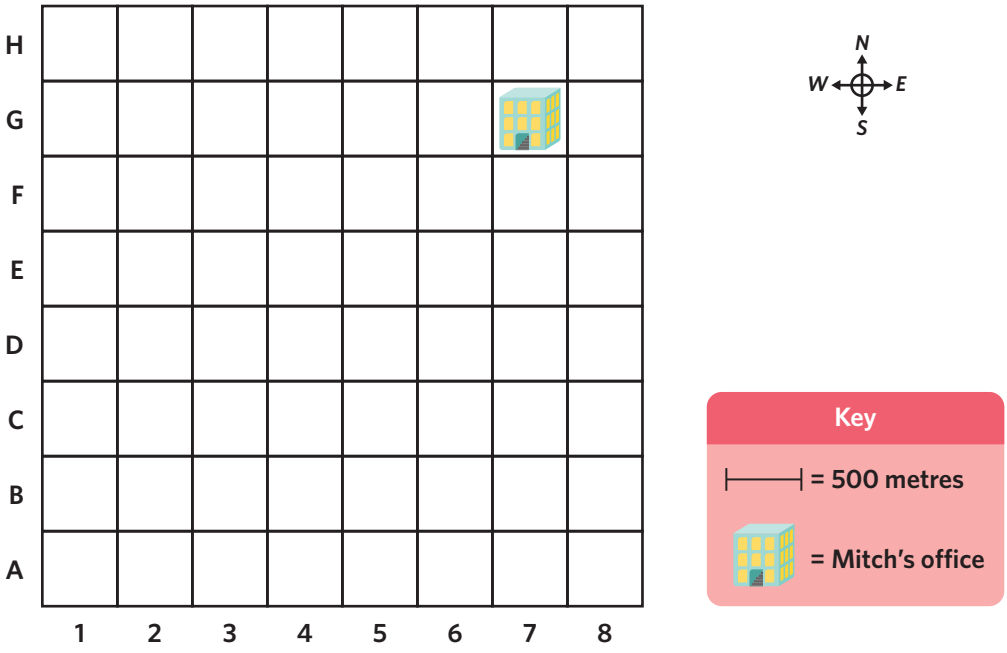
E. $400\%, \frac{3000}{100}, 2$

Remember this?

20. 73 people attended a halloween party. 19 people were dressed as superheroes. How many people at the party were not dressed as superheroes?

A. 19 **B.** 54 **C.** 64 **D.** 92

21. Mitch's office is pictured on the grid.



Mitch walks 2 kilometres south and 3 kilometres west from his office to his house. Where on the grid is Mitch's house?

A. A3 **B.** C1 **C.** D5 **D.** E4

22. Max had 2 kilograms of turkey and used $\frac{4}{5}$ of a kilogram for lunch. Select the amount that shows how much turkey Max has remaining for dinner?

A. 0.2 kg **B.** 0.8 kg **C.** 1.2 kg **D.** 1.8 kg

5I Finding percentages of a quantity

One of the most common uses of percentages is finding a percentage of a number. An important part of being able to work with percentages and quantities is understanding the connection between percentages and fractions. This is the foundation for being able to apply percentages to real life situations that involve percentage increases and decreases. Below are some examples where finding percentages of a quantity can be applied.

- 30% of the 200 houses in my neighbourhood have a swimming pool. How many houses in my neighbourhood have a pool?
- I am 160 cm tall and I expect to grow by 15% over the next two years. How tall am I expecting to be in two years' time?
- I used to receive $120 of pocket money per month. After getting in trouble at school, my mum reduced my monthly pocket money by 60%. How much pocket money do I now receive per month?

Learning intentions

Students will be able to:

+ calculate a percentage of a number or quantity
+ increase a quantity by a percentage and calculate a percentage increase
+ decrease a quantity by a percentage and calculate a percentage decrease.

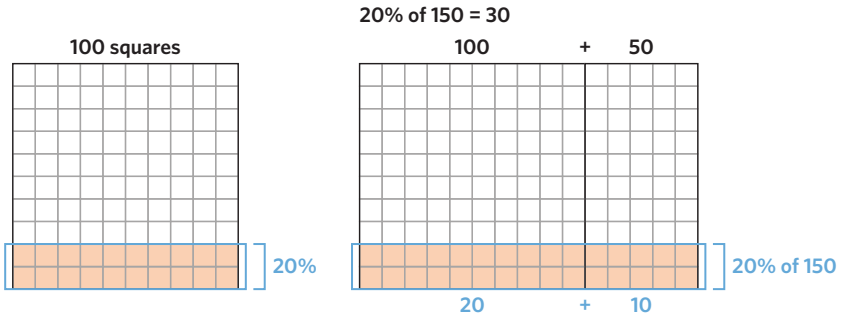
Key terms and definitions

- A **discount** is when a price is reduced by a given amount or by a percentage.
- A **proportion** is a part that is considered in relation to the value of a whole.

Key ideas

1 We can use a hundreds grid to find different percentages of a number.

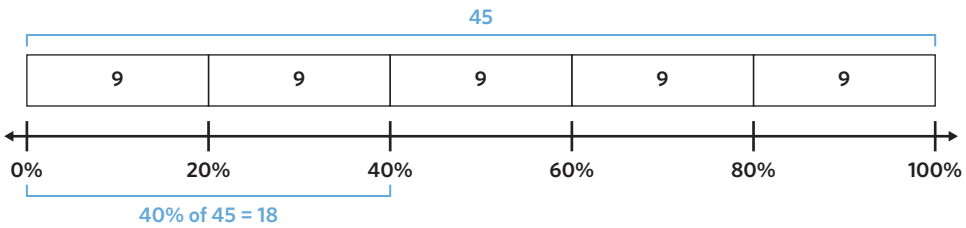
For example, we can find 20% of 150 by shading the same proportion.



2 A bar model and number line can be used together to find percentages of a number.

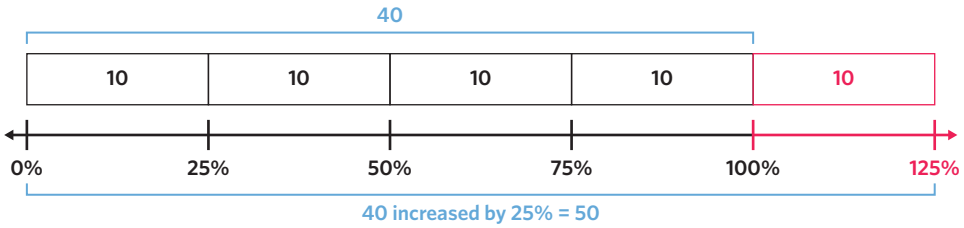
The bar model shows the quantity and the number line shows the percentage. Each section of the bar model represents its equivalent percentage on the number line.

For example, 40% of 45 can be shown as the following:

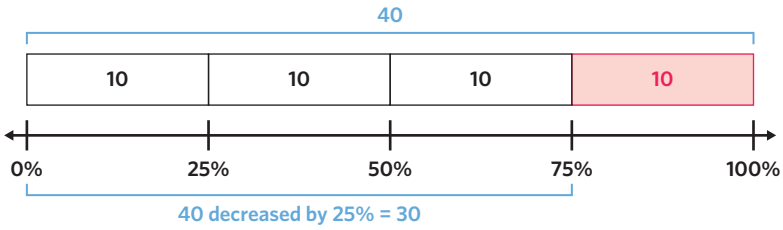


3 A bar model and number line can be used together to show how to increase or decrease a number by a given percentage.

40 increased by 25% can be shown as:



40 decreased by 25% can be shown as:



Worked examples

WE 1 Finding a percentage of a number

Calculate:

a. 10% of 50

Working

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$\frac{1}{10} \times \frac{50}{1} = \frac{1}{1} \times \frac{5}{1}$$

$$= \frac{5}{1}$$

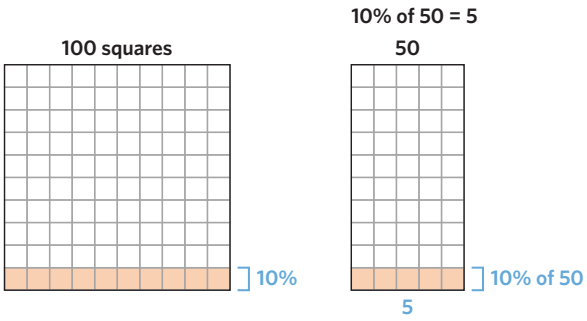
$$= 5$$

Thinking

Step 1: Convert the percentage to a fraction with a denominator of 100 and simplify.

Step 2: Express the whole number as a fraction then complete the multiplication. Simplify by cancelling.

Visual support



b. 35% of 200

Working

$$35\% = \frac{35}{100} = \frac{7}{20}$$

$$\frac{7}{\cancel{20}_{1}} \times \frac{\cancel{200}^{10}}{1} = \frac{7}{1} \times \frac{10}{1}$$
$$= \frac{70}{1}$$
$$= 70$$

Thinking

Step 1: Convert the percentage to a fraction with a denominator of 100 and simplify.

Step 2: Express the whole number as a fraction then complete the multiplication. Simplify by cancelling.

c. 120% of 60

Working

$$120\% = \frac{120}{100} = \frac{6}{5}$$

$$\frac{6}{\cancel{5}_{1}} \times \frac{\cancel{60}^{12}}{1} = \frac{6}{1} \times \frac{12}{1}$$
$$= \frac{72}{1}$$
$$= 72$$

Thinking

Step 1: Convert the percentage to a fraction with a denominator of 100 and simplify.

Step 2: Express the whole number as a fraction then complete the multiplication. Simplify by cancelling.

Student practice

Calculate:

a. 10% of 90 **b.** 45% of 300 **c.** 120% of 80 **d.** 11% of 50

WE 2 Increasing a number by a percentage

Increase each number by the given percentage.

a. 150 by 10%

Working

Method 1

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$\frac{1}{\cancel{10}_{1}} \times \frac{\cancel{150}^{15}}{1} = \frac{1}{1} \times \frac{15}{1}$$
$$= 15$$

$$150 + 15 = 165$$

Method 2

$$100\% + 10\% = 110\%$$

$$110\% \times 150 = \frac{\cancel{110}^{11}}{\cancel{100}_{10}} \times \frac{\cancel{150}^{15}}{1}$$
$$= \frac{11}{1} \times \frac{15}{1}$$
$$= \frac{165}{1}$$
$$= 165$$

Thinking

Method 1

Step 1: Find 10% of 150 by representing both values as a fraction multiplication. Simplify by cancelling.

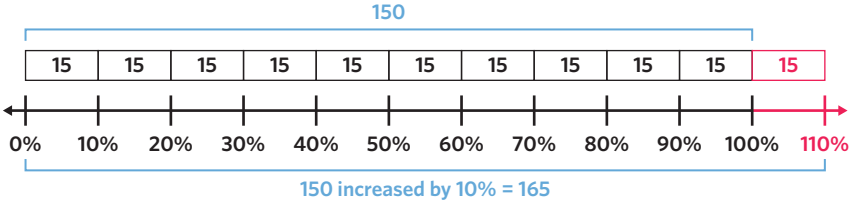
Step 2: Add 10% of 150 to 150.

Method 2

Step 1: Add the percentage increase to 100%.

Step 2: Find 110% of 150 by representing both values as a fraction multiplication. Simplify by cancelling.

Visual support



b. 200 by 55%

Working

Method 1

$$55\% = \frac{55}{100} = \frac{11}{20}$$

$$\frac{11}{\cancel{20}_{1}} \times \frac{\cancel{200}^{10}}{1} = \frac{11}{1} \times \frac{10}{1}$$

$$= 110$$

$$200 + 110 = 310$$

Thinking

Method 1

Step 1: Find 55% of 200 by representing both values as a fraction multiplication. Simplify by cancelling.

Step 2: Add 55% of 200 to 200.

Note: Method 2 can also be used.

c. 40 by 120%

Working

Method 1

$$120\% = \frac{120}{100} = \frac{6}{5}$$

$$\frac{6}{\cancel{5}_{1}} \times \frac{\cancel{40}^{8}}{1} = \frac{6}{1} \times \frac{8}{1}$$

$$= 48$$

$$40 + 48 = 88$$

Thinking

Method 1

Step 1: Find 120% of 40 by representing both values as a fraction multiplication. Simplify by cancelling.

Step 2: Add 120% of 40 to 40.

Note: Method 2 can also be used.

Student practice

Increase each number by the given percentage.

a. 250 by 10% **b.** 700 by 55% **c.** 4000 by 175% **d.** 60 by 12%

WE 3 Decreasing a number by a percentage

Decrease each number by the given percentage.

a. 80 by 20%

Working

Method 1

$$20\% = \frac{20}{100} = \frac{1}{5}$$

$$\frac{1}{\cancel{5}_{1}} \times \frac{\cancel{80}^{16}}{1} = \frac{1}{1} \times \frac{16}{1}$$

$$= 16$$

$$80 - 16 = 64$$

Method 2

$$100\% - 20\% = 80\%$$

$$80\% = \frac{80}{100} = \frac{8}{10}$$

$$\frac{8}{\cancel{10}_{1}} \times \frac{\cancel{80}^{8}}{1} = \frac{8}{1} \times \frac{8}{1}$$

$$= 64$$

Thinking

Method 1

Step 1: Find 20% of 80 by converting both values to a fraction multiplication. Simplify by cancelling.

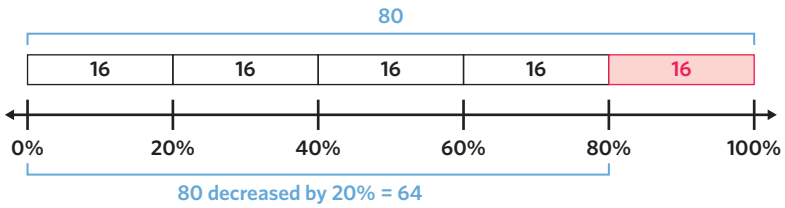
Step 2: Subtract 20% of 80 from 80.

Method 2

Step 1: Subtract the percentage decrease from 100%.

Step 2: Find 80% of 80 by representing both values as a fraction multiplication. Simplify by cancelling.

Visual support



b. 50 by 35%

Working

Method 1

$$35\% = \frac{35}{100} = \frac{7}{20}$$

$$\frac{7}{\cancel{20}_{2}} \times \frac{\cancel{50}^{5}}{1} = \frac{7}{2} \times \frac{5}{1}$$

$$= \frac{35}{2}$$

$$= 17.5$$

$$50 - 17.5 = 32.5$$

Thinking

Method 1

Step 1: Find 35% of 50 by converting both values to a fraction multiplication. Simplify by cancelling.

Step 2: Subtract 35% of 50 from 50.

Note: Method 2 can also be used.

Student practice

Decrease each number by the given percentage.

a. 60 by 20% **b.** 80 by 35% **c.** 200 by 72% **d.** 20 by 9.5%

5I Activities and questions

STARTER TASKS

Odd spot

Everyday for 20 years, Chris O'Neil took the train from Melbourne's outer suburbs to the city for work. Chris' routine has now changed, as many city based businesses are allowing their employees to work from home. The capacity of a regular Melbourne train is 900 people. Due to more businesses allowing employees to work from home, it is expected that trains will not exceed 50% of their maximum capacity.

Image: Luis Enrique Ascui/The Age

What is the new maximum number of expected commuters on a Melbourne train?

A. 0.50 **B.** 450 people

Puzzle

Each of the cards below has a matching pair with the same value. There is one card that does not have a matching pair.

50% of 20% of 40	25% of 100	33.3% of 51	Four-fifths of 45	70% of 150% of 10
Decrease 16 by 75%	0.25 × 100	17	450% of 8	10.5
32.5% of 100	32.5	One-half of 20 000	Increase 5000 by 100%	15

a) What is the one card that does not have a matching pair?

b) Create a matching card for the card that does not have a pair.

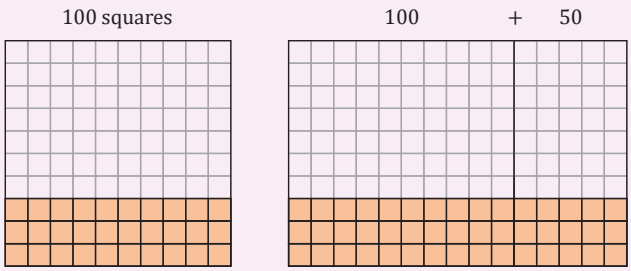
c) Create two new sets of matching pairs.

Understanding worksheet

1. Complete the box to find the given percentage of each number.

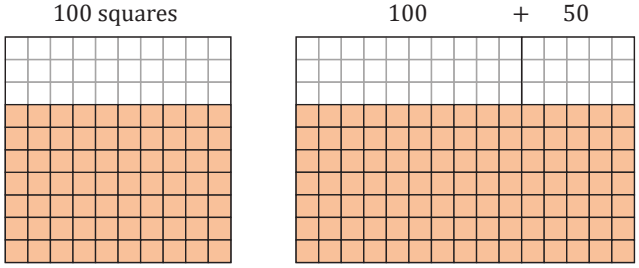
Example

30% of 150 = 45



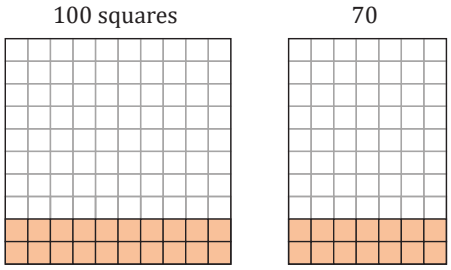
30% 30 + 15

a. 70% of 150 = ☐



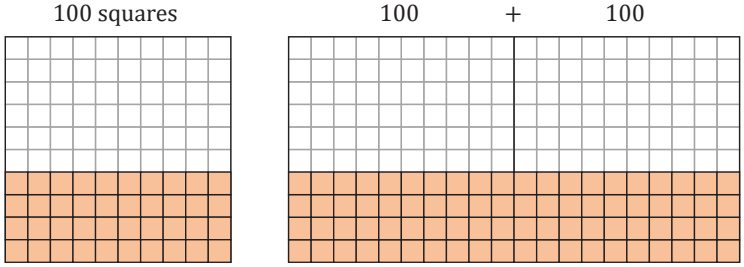
70% 70 + ☐

b. 20% of 70 = ☐



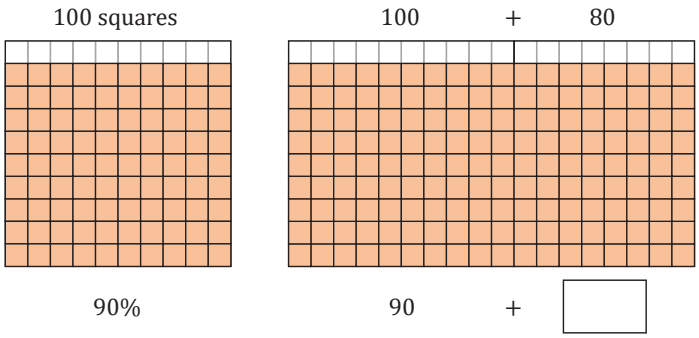
20% ☐

c. 40% of 200 = ☐



40% 40 + ☐

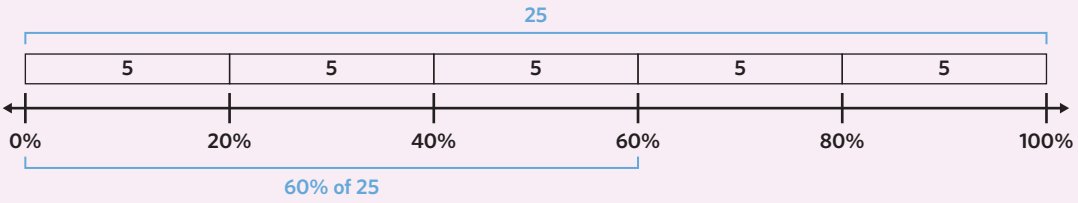
d. 90% of 180 = ☐



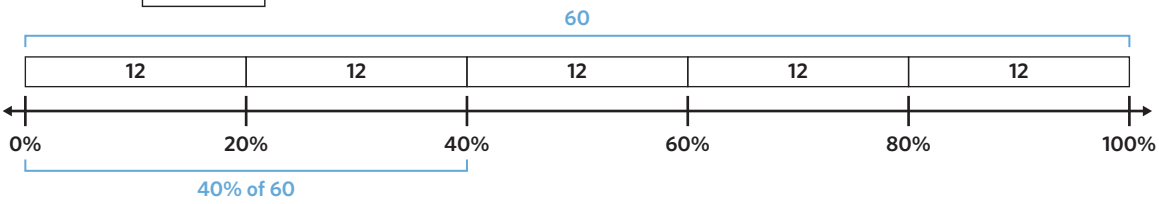
2. Fill in the missing box to complete each calculation.

Example

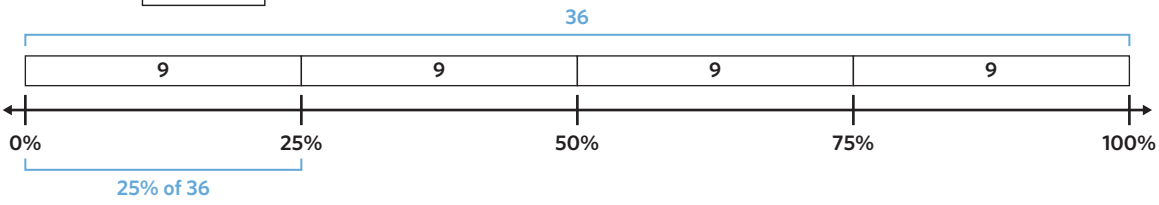
60% of 25 = 15



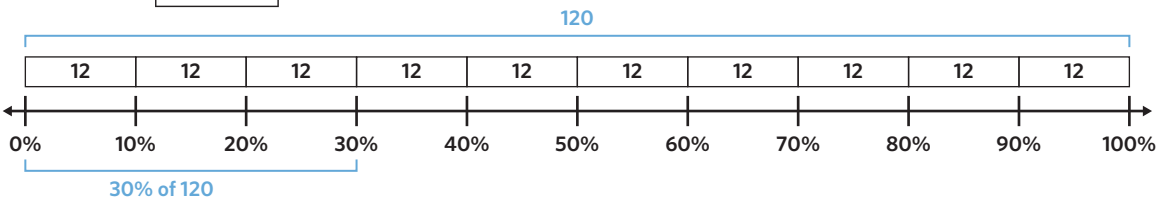
a. 40% of 60 = ☐



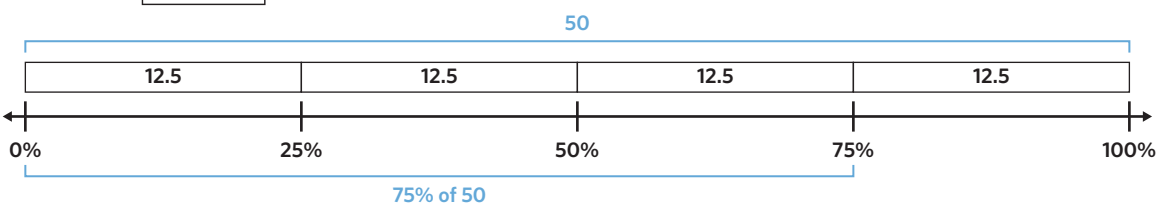
b. 25% of 36 = ☐



c. 30% of 120 = ☐



d. 75% of 50 = ☐



3. Fill in the blanks by using the words provided.

of | fraction | multiplication | decimal

When finding percentages [ ] numbers the word 'of' is related to [ ].

One way to find a percentage of a number is to convert the percentage to a [ ] in its simplest form and multiply it by the number. Another way to complete this process is to convert the percentage to a [ ] and then multiply it by the number.

Fluency

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE1 4. Calculate:

a. 10% of 70 b. 80% of 50 c. 5% of 160 d. 15% of 800
e. 10% of 8 f. 300% of 420 g. 9% of 600 h. 12% of 15

5. Compare the amounts by placing the correct mathematical symbol (<, > or =) in the box.

a. 40% of 50 ☐ 20% of 90 b. 60% of 30 ☐ 25% of 80
c. 5% of 300 ☐ 30% of 40 d. 95% of 200 ☐ 5% of 2000
e. 12% of 60 ☐ 9% of 80 f. 1% of 250 ☐ 3% of 125

WE2 6. Increase each number by the given percentage.

a. 350 by 10% b. 80 by 25% c. 60 by 40% d. 75 by 60%
e. 130 by 15% f. 3 by 150% g. 170 by 205% h. 70 by 13%

WE3 7. Decrease each number by the given percentage.

a. 90 by 30% b. 60 by 70% c. 200 by 25% d. 55 by 60%
e. 150 by 35% f. 70 by 15% g. 320 by 99% h. 90 by 47%

8. Calculate the following.

a. 10% of $400 b. Decrease 40 mL by 5%
c. Increase 500 kg by 15% d. 70% of 160 cm
e. Increase $50 by 25% f. Increase 7 m by 250%
g. Decrease 2 L by 83% h. 3% of 14 m

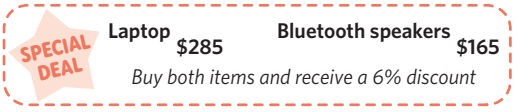
Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11,12 13

9. The Edrolo Year 7 maths textbook contains 60 lessons. 15% of all the lessons include questions about cooking. How many lessons in the textbook have questions about cooking?

10. How many female fans are in a stadium if there are 30 000 fans in attendance and 83% of them are male?

11. Ruby saw the following special deal in an electronics shop. How much would Ruby pay if she buys the laptop and the bluetooth speakers?



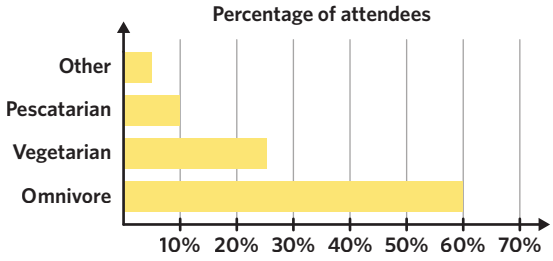
12. Oscar travelled from Brisbane to Cairns and back again by car. What was his average speed on the return journey, if it was 35% faster than the first trip? His average speed on the first trip was 60 km/h.

13. Mohammad is buying collectable Disney dolls. He has 10 out of the 20 dolls. Each one costs $16.25 and Mohammad wants to buy the remaining dolls in the collection. How much will Mohammad pay if he uses a coupon which lowers the price by 4%?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Anabelle works for a catering company and she is responsible for catering Edrolo's Christmas party. Anabelle is expecting 280 people to attend the party. She has compiled all the dietary requirements in a bar graph.



a. How many pescatarians are attending the party?

b. What is the difference in the number of guests that are omnivores and vegetarians?

c. The receptionist at Edrolo informed Anabelle that 30% of the people who work at Edrolo have invited their partners to come when dessert is going to be served. How many desserts in total does Anabelle need to cater for?

d. Present the information from the graph in a way so the chef knows exactly how many servings of each meal type to make.

15. Francis has just moved into her new house and needs to buy outdoor furniture. She finds two sets of outdoor furniture that she really likes but cannot decide which one to buy. The advertisements on the websites show each deal.

a. What is the price of the furniture set at Dream Furniture without the discount?

b. What is the final price of the furniture set at Furniture Palace?

c. After the discounts have been applied, how much money can Francis save by buying the cheaper furniture set?

d. Francis' partner thinks that they should always shop at Dream Furniture because the discount is larger. Explain why a larger discount does not always result in a lower price.

e. Redesign Dream Furniture's sign so that the price of the furniture set is more clear.

Extra spicy

16. Select all true statement(s).

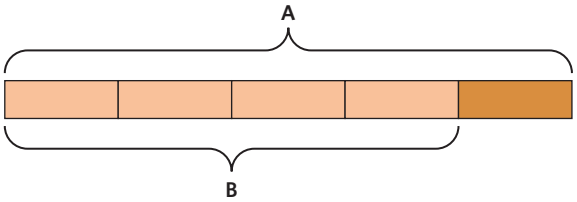
A. A is 20% more than B

B. A is 25% more than B

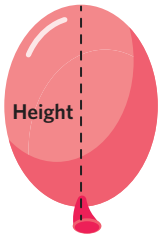
C. B is one-quarter less than A

D. B is 20% less than A

E. B is four-fifths of A



17. Each day, the height of a balloon is 60% of its height from the previous day. How long would a 90 cm balloon be after 3 days?



18. If I increase a number by 4% and then again by 3%, what is the total percentage increase of the number?

A. 0.12% B. 4% C. 7% D. 7.12% E. 12%

19. If I decrease a number by 2.4%, the result is 40.992. What was the original number?

A. 0.983 B. 1.008 C. 41.975 D. 42 E. 43.392

Remember this?

20. In 2020 a total of 916 968 vehicles were sold in Australia. Of these cars, 538 971 were second hand vehicles.
How many new vehicles were bought in Australia in 2020?

A. 377 991 **B.** 377 997 **C.** 388 917 **D.** 1 455 939

21. Nicole has a membership at her local footy club. She pays an annual fee of $78 to be a member and pays an additional $12 for every game she plays for the club.
Which expression shows how much Nicole will pay if she plays m games in a year.

A. $78m$ **B.** $12m$ **C.** $12m + 78$ **D.** $78m + 12$

22. Abdul had a 10 litre bottle of cooking oil. He used $\frac{3}{4}$ of it for frying fish and chips.
How much oil does Abdul have left?

A. 0.75 L **B.** 2.25 L **C.** 2.5 L **D.** 7.5 L

Chapter 5 extended application

1. Steve is a swimming coach and is looking at the following times of the swimmers who will be competing in the 4 × 100 mixed medley relay. Each time is represented in seconds.

	Backstroke	**Breaststroke**	**Butterfly**	**Freestyle**
Paula	67.55	69.24	58.42	52.69
Onyx	64.29	71.50	59.24	53.76
Edmund	61.41	70.38	61.50	52.26
Elena	64.92	69.23	57.76	53.91

a. Who is the fastest butterfly swimmer?

b. Who is a faster backstroke swimmer; Onyx or Elena?

c. Round each of Edmund's times to the nearest second and order his strokes from fastest to slowest.

d. How much faster is Paula at breaststroke than Onyx?

e. Steve is trying to allocate a stroke to each swimmer so that they can swim the fastest time possible in the medley relay. Edmund has the fastest freestyle time, but why might it be better for someone else to swim freestyle instead?

2. Greta is a painter and sells paintings from a studio at her house. The following table shows the cost of each painting according to the size of the canvas.

Canvas size	Small	Medium	Large
Cost ($)	$25.45	$36.50	$75.00

a. In March, Greta made $365 from selling medium sized paintings. How many medium sized paintings did she sell?

b. In the same month, she also sold 6 small paintings and 2 large paintings. How much total money did she make in March?

c. In April, Greta found a canvas supplier that allows her to buy large canvases at a cheaper cost. As a result, she reduces the cost of her large paintings by 15%. How much does she charge for a large painting now?

d. If she makes the exact same sales in April as she did in March, how much less money will she make?

e. Greta is finding that she's not making as much money from her painting business as she would like, but she's not sure whether to increase or decrease her prices. Suggest some ways Greta could make some more money from her painting business.

3. The average yearly rainfall in Melbourne over the last 160 years is 648.3 mm per year.

a. What is the average monthly rainfall in millimetres?

b. It rains an average of 100 days per year. What is the average rainfall, in millimetres, of each rainy day?

c. What proportion of days does it rain per year? Express your answer as a simplified fraction, assuming 365 days in a year.

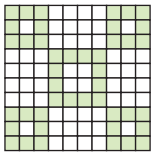
d. How many more days per year will it need to rain in order for there to be rainfall on 40% of the days?

e. Which months would you expect the most rainfall? Would this be the same if we were predicting the rainfall in Darwin?

Chapter 5 review

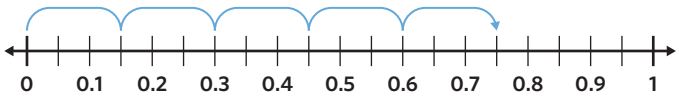
Multiple choice

1. What percentage of the hundredths grid is **not** shaded? 5G



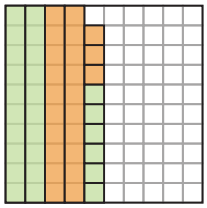
A. 44% B. 48% C. 56% D. 78%

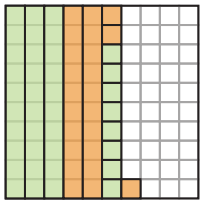
2. Which multiplication is shown on the number line? 5E

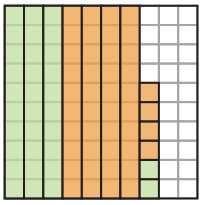


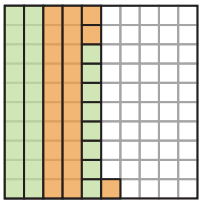
A. 0.15×3 B. 0.15×5 C. 0.25×3 D. 0.25×5

3. Which of the hundreds grids correctly shows the sum of 0.38 and 0.23? 5C

A. 

B. 

C. 

D. 

4. Which of the following shows $100 + 2 + \frac{3}{10} + \frac{4}{1000}$ written as a decimal? 5A

A.

Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten thousandths
		1	1	2	.	3	4	0	

B.

Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten thousandths
			1	2	.	3	4	5	

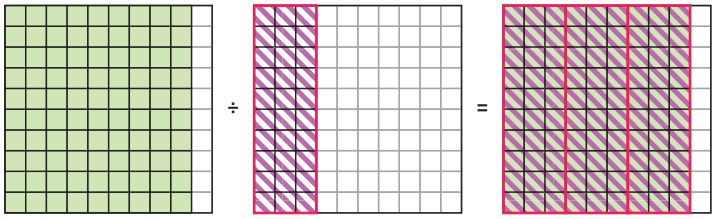
C.

Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten thousandths
		1	0	2	.	3	4	5	

D.

Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten thousandths
		1	0	2	.	3	0	4	

5. Which of the following divisions is shown below? 5F



A. $0.9 \div 0.3 = 3$ B. $0.9 \div 0.1 = 0.3$ C. $3 \div 0.3 = 0.9$ D. $3 \div 0.9 = 0.3$

Fluency

6. Express each of the following in decimal notation. 5A

a. $1 + 0.4 + 0.03 + 0.009$

b. $20 + 5 + \frac{7}{10} + \frac{5}{1000}$

c. Forty-one and sixty-three thousandths

d. $\frac{8134}{10\,000}$

7. Round each decimal as specified. 5B

a. 1.4472 to 2 decimal places
b. 0.804363 to 4 decimal places
c. 0.90995 to the nearest thousandth
d. 4.996 to the nearest hundredth

8. Evaluate each addition. 5C

a. $0.38 + 0.46$
b. $1.49 + 2.207$
c. $1.234 + 0.345 + 0.678$
d. $0.8 + 4.73 + 5.916$

9. Evaluate each subtraction. 5C

a. $1.76 - 0.55$
b. $2.43 - 1.96$
c. $5.4 - 2.821$
d. $10.63 - 4.7 - 2.809$

10. Evaluate: 5D

a. 3.456×100
b. $78.9 \div 100$
c. $0.003 \times 10\,000$
d. $15 \div 1000$

11. Use the order of operations to evaluate each expression. 5D

a. $0.0343 \times 100 + 56.1 \div 10$
b. $482 \div 100 - 10(153 \div 1000)$
c. $629.1 \div 10\,000 + 0.0045 \times 10$
d. $100(0.41) + 26.8 \div 100 - 903 \div 1000$

12. Evaluate each multiplication. 5E

a. 0.9×3
b. 5.7×1.2
c. 9.87×0.3
d. 11.25×2.13

13. Evaluate each division. 5F

a. $3.6 \div 0.9$
b. $48.1 \div 5$
c. $2.136 \div 0.08$
d. $6.003 \div 2$

14. Calculate the complementary percentages. 5G

a. 65%
b. 23%
c. 39%
d. 51%

15. Convert each fraction to a percentage. 5G

a. $\frac{2}{5}$
b. $\frac{3}{8}$
c. $\frac{37}{25}$
d. $2\frac{3}{20}$

16. Express each of the following in the given form. 5H

a. Express 3.405 as a percentage.
b. Express 3.25 as a fraction in its simplest form.
c. Express $\frac{72}{300}$ as a decimal.
d. Express $\frac{7}{80}$ as a decimal.

17. Calculate: 5I

a. 15% of 200
b. 24% of 350
c. 250% of 84
d. 6% of 750

18. Increase or decrease each number by the given percentage. 5I

a. Increase 45 by 20%
b. Decrease 300 by 15%
c. Increase 210 by 225%
d. Decrease 70 by 11%

Problem solving

19. It is recommended for males to have a blood hemoglobin level of above 13.2 g/dL. 5A
Which of the males in the table have hemoglobin lower than the recommended level?

	Andreas	Bai	Clarence	Dom	Elijah
Hemoglobin level (g/dL)	13.15	13.22	13.581	13.094	13.35

20. Yolanda is visiting her family in New Zealand for Christmas. She wants to know if her luggage is within the airline's weight limit of 20 kg. If she rounds each of the weights shown to the nearest kilogram, will her luggage be within the weight limit? 5B

suitcase	3.15 kg
clothes	9.47 kg
toiletries	4.53 kg
Christmas presents	3.68 kg

21. Sophie is buying ingredients to bake cinnamon scrolls. She buys butter for $4.25, cinnamon for $2.59, yeast for $2.40 and sugar for $1.96. How much change will she receive if she pays using a $20 note? 5C

22. A colony of 100 000 ants weighs 520 grams. On average, how much does one ant weigh in grams? 5D

23. Sophie's cinnamon scroll recipe requires 0.385 kg of flour. She wants to make two-and-a-half batches of the recipe. How many kilograms of flour will she need? 5E

24. Last week, Greg earned $195.50 at his part-time job. What is his hourly wage if he worked 8.5 hours last week? 5F

25. Jerome got 60 out of 75 marks on a recent maths test. Express his test result as a percentage. 5G

26. Lachie wants to buy a new video game with a retail price of $60. JB Hi-Fi and EB Games both currently have sales, as shown. 5H

Where should Lachie buy the video game from if he wants to save the most money?

27. Three years ago, Florence was furnishing her lounge and bought a vintage Tessa couch for $1750 and a mid-century coffee table for $325. Now, the value of the couch has increased by 36% and the value of the coffee table has increased by 30%. What is the current total value of the furniture? 5I

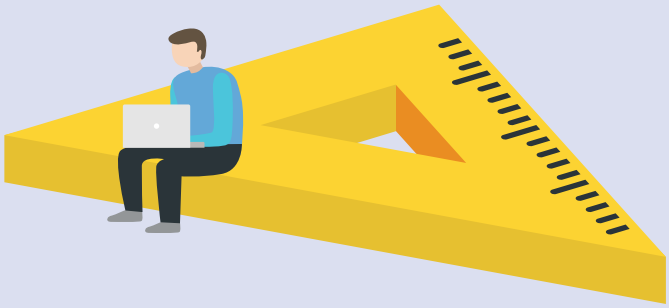
Reasoning

28. Members of a Year 7 basketball team measured their heights for a recent maths project.
    a. Three team members' heights are shown in the table. Who is the tallest out of these three?

Name	Kendall	Antony	Bernard
Height (metres)	1.56	1.5095	1.558

    b. What is the difference between Antony and Bernard's heights, in metres?
    c. There are 10 players on the team. If the sum of all of the heights was 15.629 metres, what is the average height of the players, in metres?
    d. Leonard measured his height to be 1.5525 metres. Express his height in centimetres, rounded to the nearest centimetre.
    e. For this project, each player measured their own height at home. What are some things they could do to minimise inaccuracy when measuring their heights?

29. Rabita is making traditional hot cross buns for Easter. The recipe she is using requires 0.375 litres of milk, but Rabita wants to make more hot cross buns than the recipe calls for, so she multiplies all the quantities by 4.
    a. How much milk does Rabita use in litres?
    b. Each individual hot cross bun contains 31.25 mL of milk. How many hot cross buns did Rabita make?
    Note: There are 1000 mL in 1 litre.
    c. Rabita brought the hot cross buns to a family lunch, and 42 of them were eaten. Express the number of hot cross buns eaten as a simplified fraction.
    d. What percentage of the hot cross buns weren't eaten?
    e. Some of Rabita's family said they didn't eat the hot cross buns because they prefer chocolate chip to traditional. Should Rabita change her recipe next year?



06

Angles and straight lines

Measurement and Geometry

Research summary

6A Introduction to angles *(Revision)*

6B Angles around a point

6C Properties of triangles

6D Properties of quadrilaterals

6E Parallel lines and transversals *(Extension)*

Skill builder

Chapter 6 extended application

Chapter 6 review

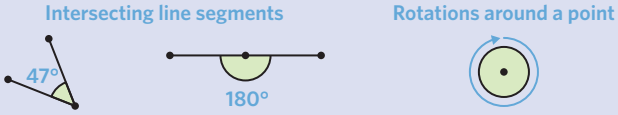
Image: Marish/Shutterstock.com

Research summary – Angles and straight lines

Big ideas

Angles

Learning angles can be a difficult concept for many students. This is because there are many distinctions between types of angles and definitions. This can lead to confusion about what constitutes an angle. When learning angles, we must focus on the amount of turn between two intersecting line segments or the amount of rotation around a point. The intersecting line segment variations are the most commonly taught and understood, and consist of two line segments, or rays, that meet at a single point. Rotations around a point don't contain any line segments or rays and are usually represented with wheels, door knobs and spinners.



Geometric reasoning

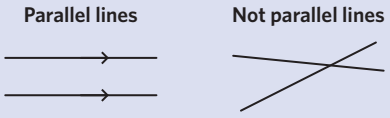
Geometric reasoning, also known as Euclidean geometry, is the study of 1-dimensional, 2-dimensional and 3-dimensional objects and their properties along a plane. It does not focus on the rote memorisation of principles but rather focuses on the mathematical thinking involved in identifying the length of a line or the size of an angle, based on their relationships with other lines and angles. Euclidean geometry is based on 5 postulates.

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. A circle can be constructed when a line segment is used for its radius, and one endpoint is used as the centre of the circle.
4. All right angles are equal.
5. If a line segment intersects two straight lines and the interior angles on one side sum to less than 180°, then the two straight lines are not parallel, and will eventually meet if extended on that side.

Using these five postulates, we can make deductions about angles and lines that allow us to arrive at answers without having to rely on protractors.

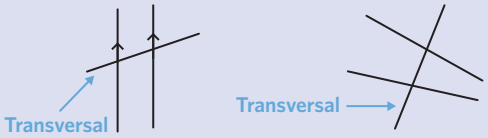
Parallel lines

Two lines are parallel when they extend indefinitely in both directions but never intersect one another. When parallel lines are crossed by another line, called a transversal line, certain properties arise that make it easy for us to use geometric reasoning to find unknown angles. These properties are useful when working with lines, but also with interior angles of regular polygons.



Transversal lines

A transversal is a line that intersects two or more straight lines. In real life, transversal lines can be used to determine whether a pair of lines are parallel, and are therefore used in a range of fields such as astronomy, engineering, architecture and general design. When used with parallel lines, transversal lines are useful for teaching students the properties of corresponding, co-interior and alternate angles. More broadly, it also allows students to demonstrate their understanding of supplementary angles and practice the use of deductive reasoning skills.

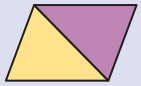


Interior angles of polygons

Polygons are closed shapes made up of at least 3 sides and three interior angles. The polygon with the least number of sides is a triangle and has interior angles that sum to 180°. Other types of polygons include quadrilaterals (4-sides), pentagons (5-sides), hexagons (6-sides), septagons (7-sides) and octagons (8-sides).

Every polygon is named according to how many sides it is made up of. Polygons with four or more sides are formed by combining smaller triangles. This means that the interior angle sum of any polygon can be calculated based on the number of triangles used to construct it. For example, a quadrilateral is made up of two triangles, which means that the sum of the interior angles is 360°. A pentagon is made up of three triangles, which means the sum of the interior angles is 540°. This pattern can be used to help us calculate unknown angles in polygons.

A quadrilateral is made up of two triangles.



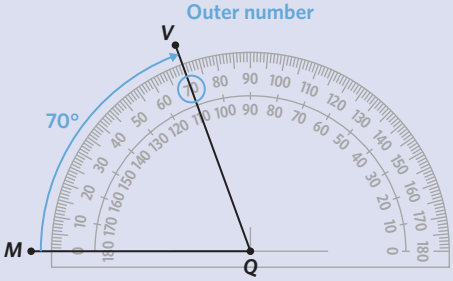
A pentagon is made up of three triangles.

Visual representations

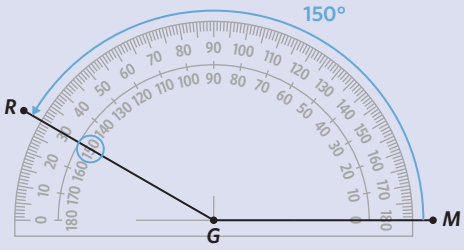
Protractors

Protractors are a useful visual tool to conceptualise angle sizes. A protractor allows us to apply a measuring tool to understand and draw angles of different sizes. This provides a simple and accessible way of measuring acute, obtuse and reflex angles. Protractors are a useful way to introduce students to the idea that an angle is a measure of turn. Depending on the way an angle is drawn, students will need to determine whether to use the inner circle or outer circle of numbers for measuring the angle. The following are examples of when to read the inner circle or outer circle of numbers for an angle measurement.

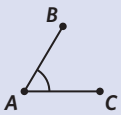
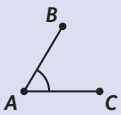
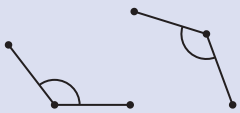
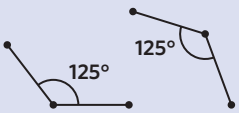
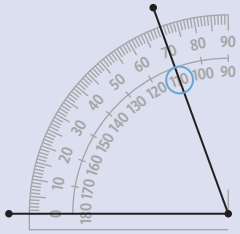
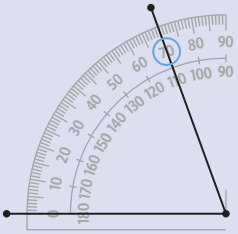
Outer number

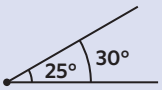
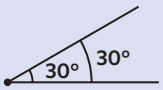
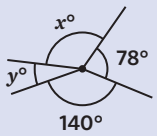
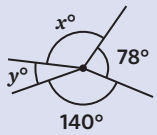
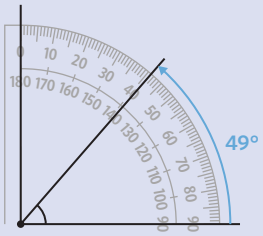
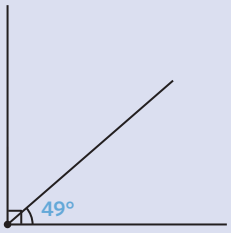
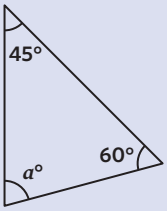
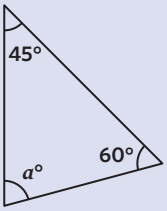
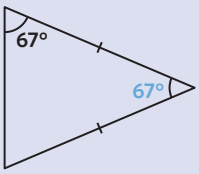
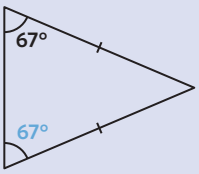
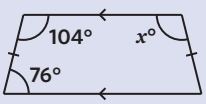
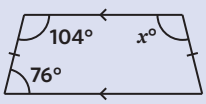


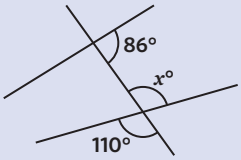
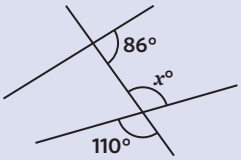
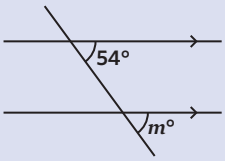
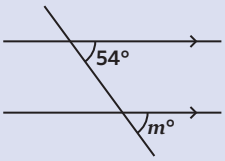
Inner number



Misconceptions

	Incorrect	Correct	Exercise
Students incorrectly label lines and angles with the letter at their point.	 $\angle A$	 $\angle BAC$ or $\angle CAB$	6A
Students don't recognise that angle rays with different orientations can have the same angle.	Two angles with different orientation cannot be the same. 	Two angles with different orientations can have the same angle. 	6A
Students misread the protractor by using the internal degree rather than the external.			6A

	Incorrect	Correct	Exercise
Students think that angles are only represented by the intersection of two lines and not straight lines and circles.	No angle shown on a straight line.	A straight line has an angle of 180°. 	6A
Students associate the size of an angle with the length of the arc.			6B
Students identify vertically opposite angles when four different straight lines meet at a point.	$x° = 140°$ and $y° = 78°$ 	x and y do not have vertically opposite angles. 	6B
Students confuse the size of complementary and supplementary angles.	Complementary angles sum to 180°, and supplementary angles sum to 90°.	Complementary angles sum to 90°, and supplementary angles sum to 180°.	6B
Students 'measure' angles with a protractor when instructed to 'calculate' an angle.	Find the missing angle. 	Find the missing angle. $90 - 49 = 41°$ 	6B
Students confuse the sum of the interior angles for triangles and quadrilaterals.	The interior angles of a triangle sum to 360°.	The interior angles of a triangle sum to 180°.	6C
Students identify the sizes of angles based on their appearance and not on their geometric properties.	$a° = 90$ because it looks like a right angle. 	$a° = 75°$ because the interior angles of a triangle sum to 180°. 	6C
Students mislabel the equal angles of an isosceles triangle when finding missing interior angles.			6C
Students think that the sum of interior angles of all shapes does not change with the number of sides.	The sum of interior angles of both triangles and quadrilaterals is 180°.	The sum of interior angles of triangles is 180° and quadrilaterals is 360°.	6D
Students believe that opposite angles in quadrilaterals are always equal because opposite angles in parallelograms are equal.	$x° = 76°$, because the angles are opposite to each other. 	$x° = 104°$, because they are adjacent angles. 	6D

	Incorrect	Correct	Exercise
Students apply the angle properties of parallel lines, even though the lines are not parallel.	$x = 94$ because $x°$ and 86° are co-interior angles so sum to 180°. 	$x = 110$ because $x°$ and 110° are vertically opposite angles so are equal. 	6E
Students think that alternate or corresponding angles in parallel lines sum to 180° like co-interior angles do.	$m = 126$ because $m°$ and 54° are corresponding angles, so sum to 180°. 	$m = 54$ because $m°$ and 54° are corresponding angles, so are equal. 	6E

6A Introduction to angles

An angle is a measure of turn that can be measured using a protractor. Students should understand that one full turn represents 360 degrees. Angles are formed when two rays meet at a common point, or vertex. When measuring different angles, they can range from 0 degrees to 360 degrees. These angles can be classified based on their size. Being able to classify different types of angles is an important part of geometric reasoning. Below are some examples where angles can be applied.

- My yoga teacher asks me to raise my legs to a right angle. How many degrees do I need to raise my legs?
- I lean a ladder on my house to clean the gutters. What type of angle does the ladder form where it leans on the wall?
- My dishwasher is left ajar and forms an acute angle at its hinge. How many degrees might this angle be?

Learning intentions

Students will be able to:

- label line segments, rays and angles with letter notation
- measure and draw angles with a protractor
- name angles according to their size.

Key terms and definitions

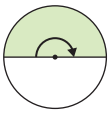
- An **angle** is a measure of turn that is given in degrees (°).
- A **line segment** has two fixed end points.
  E.g. A D.
- A **ray** is a line that has one fixed end point and no end in one direction. E.g. A D.
- An **arc** is the curve that joins two rays and it represents the angle.
- An **arm** is a line that forms an angle.
- A **vertex** is the point at which two rays meet to create an angle.
- An **acute angle** is any angle that is greater than 0° but less than 90°.
- A **right angle** is an angle that is exactly 90°.
- An **obtuse angle** is any angle greater than 90° but less than 180°.
- A **reflex angle** is any angle greater than 180° and less than 360°.

Key ideas

1 An angle is a measure of turn from one line to another line.

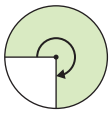
One quarter of a turn.

$\frac{1}{4} \times 360° = 90°$



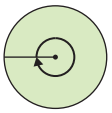
One half of a turn.

$\frac{1}{2} \times 360° = 180°$



Three quarters of a turn.

$\frac{3}{4} \times 360° = 270°$

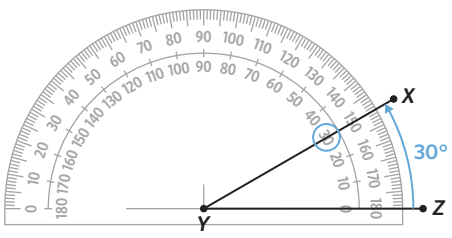


One full turn.

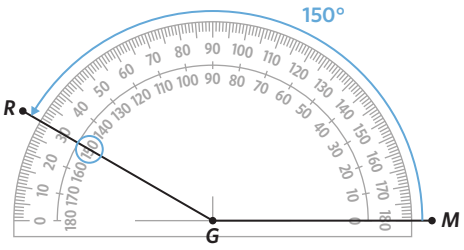
$1 \times 360° = 360°$

2 A protractor can be used to measure angles between two line segments. Letter notation is used to label angles.

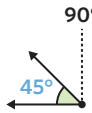
$\angle XYZ = 30°$ or $\angle ZYX = 30°$



$\angle MGR = 150°$ or $\angle RGM = 150°$

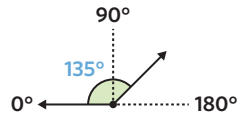


3 There are different types of angles and they can be named based on their size.

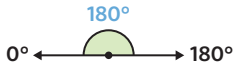


Acute angle
Any angle greater than 0° and less than 90°.

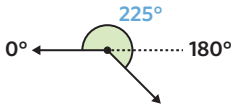
Right angle
Any angle that is exactly 90°.



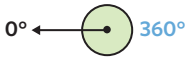
Obtuse angle
Any angle greater than 90° and less than 180°.



Straight line
An angle that is exactly 180°.



Reflex angle
Any angle greater than 180° and less than 360°.



Full turn
An angle that is exactly 360°.

Worked examples

WE 1 Naming line segments, rays, and angles

Use letter notation to name each line segment, ray, or angle.

a. 

Working

$\overline{CD}$ or $\overline{DC}$

Thinking

Identify the two endpoints (C and D) and express both points with a — to represent a line segment.

b. 

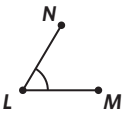
Working

$\overrightarrow{FJ}$

Thinking

Identify the endpoint (F). Express the endpoint first (F) followed by the other point (J) with a → to represent a ray.

Note: Rays can only be labelled one way.

c. 

Working

$\angle NLM$ or $\angle MLN$

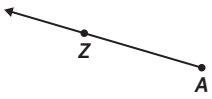
Thinking

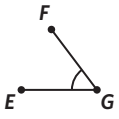
Identify the vertex (L), then write it in between the letters that represent the angle arms' end points (N and M). Use $\angle$ to represent an angle.

Student practice

Use letter notation to name each line segment, ray, or angle.

a. 

b. 

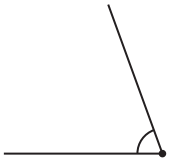
c. 

d. 

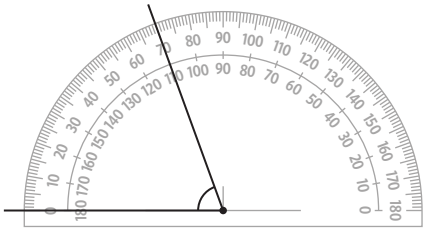
WE 2 Measuring and classifying angles

Measure each angle and state whether it is an acute, obtuse, reflex or right angle.

a.



Working



The angle is 70°.

It is an acute angle.

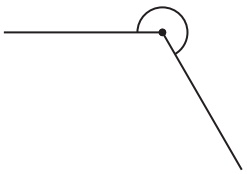
Thinking

Step 1: Place the centre of the protractor on the vertex and align the horizontal arm with 0°.

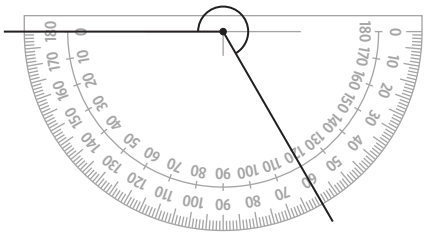
Step 2: Identify which number the other arm lines up with (70°), using the outer scale that starts from 0°.

Step 3: State the angle measurement and state what type it is based on its size (0° to 90° is acute).

b.



Working



$360 - 120 = 240$

The angle is 240°.

It is a reflex angle.

Thinking

Step 1: Place the centre of an upside down protractor on the vertex and align the horizontal arm with 0°.

Step 2: Identify which number the other arm lines up with (120°), using the inner scale that starts from 0°.

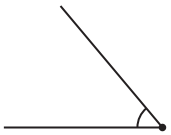
Step 3: Subtract the measured angle (120°) from 360° as a full turn is 360°.

Step 4: State the angle measurement and state what type it is based on its size (180° to 360° is reflex).

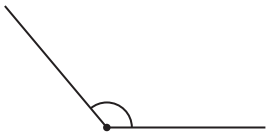
Student practice

Measure each angle and state whether it is an acute, obtuse, reflex or right angle.

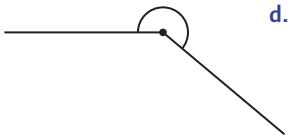
a.



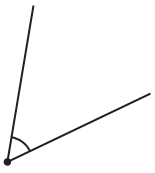
b.



c.



d.

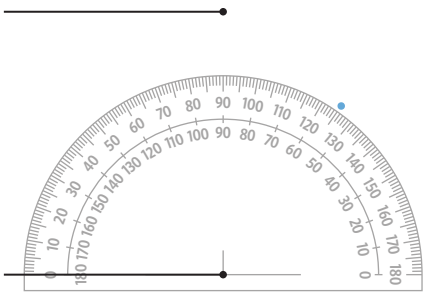


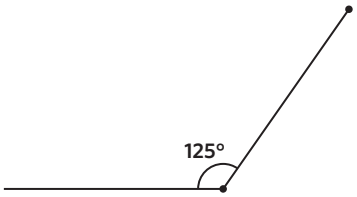
WE 3 Drawing angles

Use a protractor to draw each angle.

a. 125°

Working





Thinking

Step 1: Draw a line segment with one endpoint. This is the vertex of the angle.

Step 2: Place the centre of the protractor on the vertex and align the line with 0°.

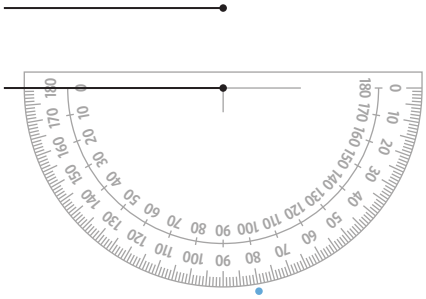
Step 3: Using the scale that starts from 0° (outer), find the angle (125°) and mark a dot at the edge of the protractor.

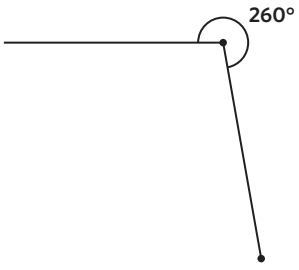
Step 4: Draw a line from the vertex to the marked dot and label the angle (125°).

b. 260°

Working

$360 - 260 = 100$





Thinking

Step 1: As the angle is greater than 180° subtract the required angle (260°) from 360°.

Step 2: Draw a line segment with one endpoint. This is the vertex of the angle.

Step 3: Place the centre of an upside down protractor on the vertex and align the horizontal arm with 0°.

Step 4: Using the scale that starts from 0° (inner), find the angle (100°) and mark a dot at the edge of the protractor.

Step 5: Draw a line from the vertex to the marked dot and label the reflex angle (260°).

Student practice

Use a protractor to draw each angle.

a. 30° **b.** 170° **c.** 330° **d.** 255°

6A Activities and questions

STARTER TASKS

Odd spot

The International Astronomical Union uses the position angle convention to measure the position of a star in the galaxy. The position angle of a star is found by drawing an imaginary line from a primary star to the north celestial star and another imaginary line to a secondary star. The angle between these two lines form the position angle of the secondary star.

If K represents the position of the north celestial pole and H represents the position of the primary star, what is the angle to describe a star that is in line with the point A?

A. $\angle H$ **B.** $\angle AHK$

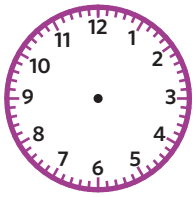
Image: AstroStar/Shutterstock.com

Puzzle

a) What time could it be if the minute hand is pointing to 12 o'clock and the hour hand points in a direction that forms a right angle with the minute hand?

b) What time could it be if the time is 30 minutes past the hour, and the hour hand points in a direction that forms an obtuse angle with the minute hand?

c) What are possible times with the minute hand pointing to 12 o'clock and the hour hand points in a direction that forms an acute angle with the minute hand?

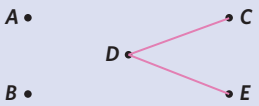


Understanding worksheet

1. Use the points to form the described line segment, ray or angle.

Example

$\angle CDE$



a. $\overline{BE}$

b. $\overrightarrow{CD}$

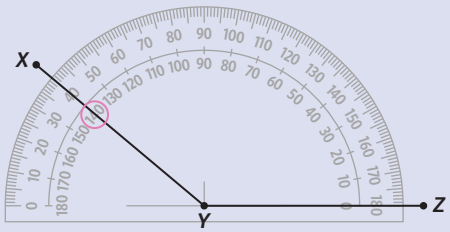
c. $\angle BEC$

d. $\angle CDB$

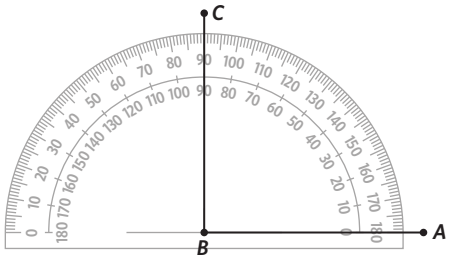
2. Use the given protractor to measure the angle and circle the measurement.

Example

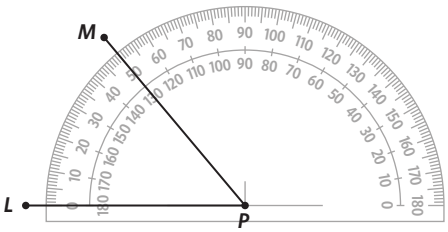
$\angle XYZ = \boxed{140°}$



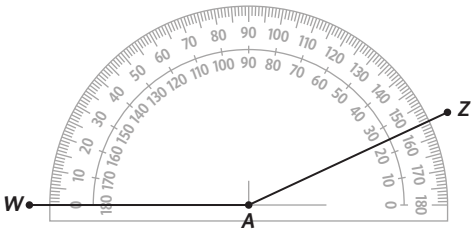
a. $\angle ABC =$ ☐



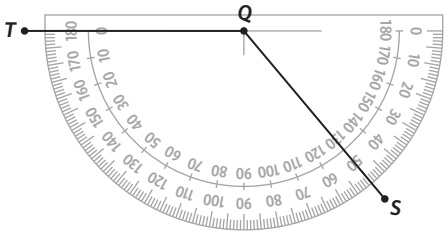
b. $\angle MPL =$ ☐



c. $\angle WAZ =$ ☐



d. $\angle TQS =$ ☐



3. Fill in the blanks by using the words provided.

90° | acute | 360° | reflex | protractor

An angle is a measure of turn. A ☐ is used to measure angles. Angles can be classified based on their size. One full turn is measured as ☐. An ☐ angle is between 0° and 90°. A right angle is exactly ☐. An obtuse angle is between 90° and 180°. A ☐ angle is between 180° and 360°.

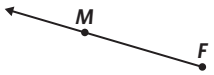
Fluency

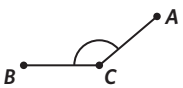
Question working paths

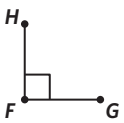
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c,d)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d), 8 (c,d,e,f)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f), 8 (e,f,g,h)

WE 1 **4.** Use letter notation to name each line segment, ray, or angle.

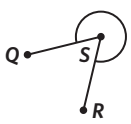
a. 

b. 

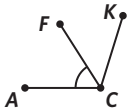
c. 

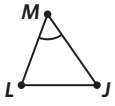
d. 

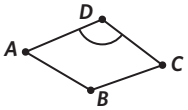
e. 

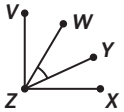
f. 

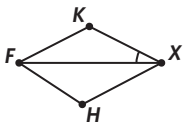
5. Use letter notation to name each angle arc.

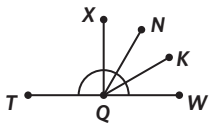
a. 

b. 

c. 

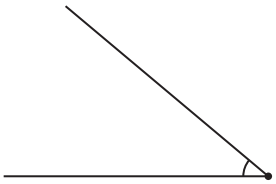
d. 

e. 

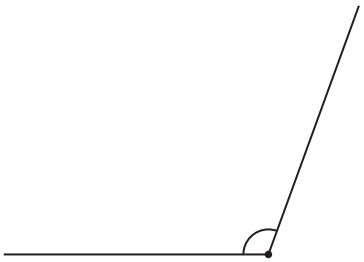
f. 

WE 2 **6.** Measure each angle and state whether it is an acute, obtuse, reflex or right angle.

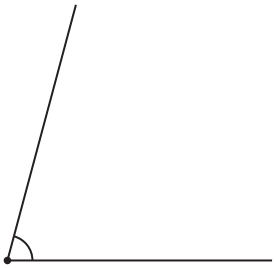
a.



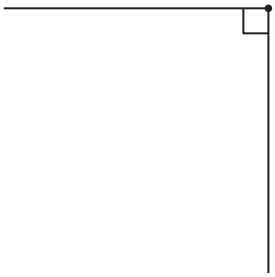
b.



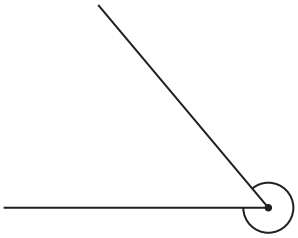
c.



d.

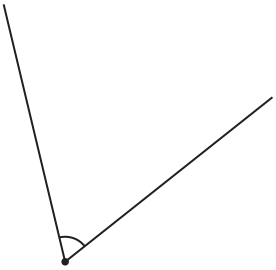


e.

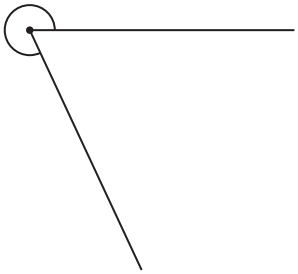


f.

g.



h.



7. State whether each measurement or angle is an acute, obtuse, reflex or right angle.

a. 87°

b.

c. 173.57°

d.

e. 0.2°

f.

8. Use a protractor to draw each angle.

a. 20°
b. 140°
c. 180°
d. 95°
e. 300°
f. 210°
g. 255°
h. 337°

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. A gymnastics balance beam is a narrow wooden apparatus that is 180°. What type of angle does a balance beam represent?

10. Tony Hawk was the first skateboarder to land the famous skateboard trick called a 900. How many full turns did Tony complete when he flipped 900° in mid-air?

11. How many degrees does the minute hand turn between 11 o'clock and quarter to 12 o'clock on a standard analogue clock?

Back to contents

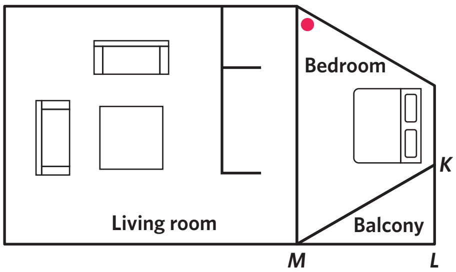
12. Mikalea was hiking and completed a 90° turn to her left and ended up facing directly south. What direction was she facing before completing the turn?

13. Felix is a carpenter and is building a wooden kitchen table. What type of angle will each corner form if the table is a regular octagon and Felix measures the angles inside the table?

Reasoning

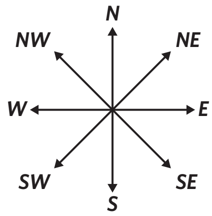
Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Jonti is working with her architect and is reviewing the floor plans for her new apartment. The following shows the plans for the upstairs living room, bathroom, bedroom and balcony.



- **a.** Jonti's balcony can be accessed from her bedroom. Using letter notation, label the balcony's corner that is at a right angle.
- **b.** Jonti is selecting some furniture to put in the corner of her room that is marked with a red dot. The furniture maker needs to know the internal angle of this corner. Jonti uses a protractor to measure the angle, how many degrees is the angle?
- **c.** The walls inside Jonti's bedroom connect to form different types of angles. How many of the angles formed by the walls inside her bedroom are obtuse?
- **d.** Jonti does not like the shape of her bedroom. Create an alternative design for a bedroom with a balcony that is more functional.

15. Ilham and Joe both love hiking. Rather than using an electronic GPS, they enjoy the challenge of navigating with a compass. The following diagram shows the face of the compass that Ilham and Joe use while hiking. *N* represents north and *NE* represents northeast.



- **a.** If Ilham is facing directly south, how many degrees does she need to turn so that she is facing directly north?
- **b.** How many degrees does Joe need to turn in the clockwise direction if he is facing north but needs to face directly towards southeast?
- **c.** If Joe and Ilham are facing directly west and turn 225° in the anticlockwise direction, what direction will they now be facing?
- **d.** At the end of the hike, Joe and Ilham are facing the same direction. Joe repositions himself and completes $\frac{7}{8}$ of a clockwise turn. How many degrees can Ilham turn in the anticlockwise direction, so that she will be facing the same direction as Joe.
- **e.** Hiking with only a compass can be dangerous. How might Ilham and Joe ensure that when they hike without a GPS they remain safe?

Extra spicy

16. What is the greatest difference between the sum of two different whole number reflex angles and the product of two different whole number acute angles?

17. The letters in the word FACE are all turned 90° anticlockwise.

Which option is the result?

A.  **B.**  **C.**  **D.**  **E.** 

18. What time will a clock show if at 12 o'clock, someone turns the hour hand 780 degrees clockwise and the minute hand 150 degrees clockwise?

A. 12:15 **B.** 2:15 **C.** 2:20 **D.** 2:25 **E.** 3:20

19. If a is a whole number, how many different obtuse angles that are less than 120° can $3a$ represent?

A. 7 **B.** 8 **C.** 9 **D.** 10 **E.** 11

Remember this?

20. The fraction $\frac{1}{4}$ expressed as a decimal is 0.25. What is the fraction $\frac{1}{8}$ expressed as a decimal?

21. The table shows the total number of hours worked by waiters at a restaurant.

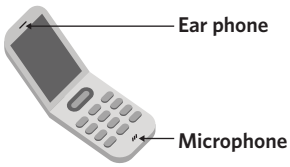
Day	Saturday	Sunday	Monday
Total number of hours worked	100	80	65

Each waiter earns $30 per hour.

What was the mean amount of money paid to the waiter per night?

A. $25.00 **B.** $81.66 **C.** $2400 **D.** $2450

22. A flip phone can be opened to form different types of angles.



Select the image which shows that the angle between the earphone and the microphone is an acute angle.

A. 

B. 

C. 

D. 

6B Angles around a point

When working with angles around a point, angle properties may exist. For example vertically opposite angles, complementary, and supplementary angles can be used to find missing angle measurements. Below are some examples of where angles around a point can be applied.

- My yoga teacher says I need to raise a leg to form a 90° angle. I have raised my leg about 30°, how much further do I need to raise my leg?
- I am turning around to face the opposite direction. I have turned 80°, about how much further do I need to turn?
- I am a ballerina and need to complete a full turn in 3 moves. I turn 60° in my first move and 120° in my second move. How many degrees is my third move?

Learning intentions

Students will be able to:

+ identify complementary, supplementary, adjacent and vertically opposite angles
+ calculate angles around a point
+ use algebra to solve for missing angles.

Key terms and definitions

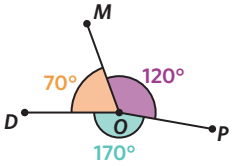
- **Adjacent angles** are angles that share a vertex and a common side.
- **Complementary angles** are two or more angles that form a right angle which is 90°.
- **Supplementary angles** are two or more angles that form a straight line which is 180°.
- **Vertically opposite angles** are two non-adjacent angles that are formed by two intersecting lines.
- A **revolution** is a full rotation of a circle, or a complete turn which is 360°.

Key ideas

1 The angles around a point sum to 360°.

$\angle MOP + \angle DOP + \angle MOD = 360°$

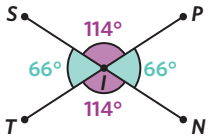
$120° + 170° + 70° = 360°$



2 Vertically opposite angles share the same vertex where two straight lines intersect and are equal.

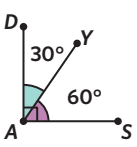
$\angle SIT = \angle PIN = 66°$

$\angle SIP = \angle TIN = 114°$



3 Complementary angles form a right angle and supplementary angles form a straight line.

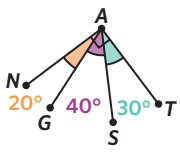
Complementary angles sum to 90°.



$\angle DAY = 30°$

$\angle SAY = 60°$

$30° + 60° = 90°$



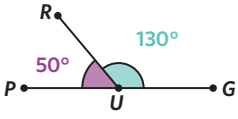
$\angle NAG = 20°$

$\angle GAS = 40°$

$\angle SAT = 30°$

$20° + 40° + 30° = 90°$

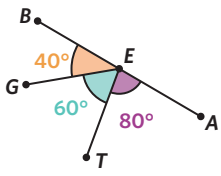
Supplementary angles sum to 180°.



$\angle PUR = 50°$

$\angle RUG = 130°$

$50° + 130° = 180°$



$\angle BEG = 40°$

$\angle GET = 60°$

$\angle TEA = 80°$

$40° + 60° + 80° = 180°$

Worked examples

WE 1 Finding vertically opposite angles

Determine the value of each pronumeral.

a.

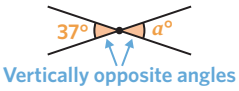
Working

$a = 37$

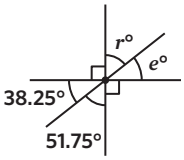
Thinking

Identify the vertically opposite angle to the missing angle ($a°$). Vertically opposite angles are equal.

Visual support



b.



Working

$r = 51.75$

$e = 38.25$

Thinking

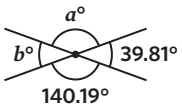
Identify the vertically opposite angle to each missing angle ($r°$ and $e°$). Vertically opposite angles are equal.

Student practice

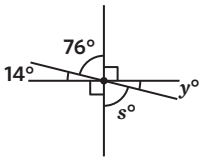
Determine the value of each pronumeral.

a.

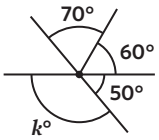
b.



c.



d.



WE 2 Solving for missing complementary and supplementary angles

Set up an equation and solve to find the value of each pronumeral.

a.

Working

$k° + 71° = 90°$

$k° + 71° - 71° = 90° - 71°$

$k = 19$

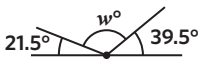
Thinking

Step 1: Write an equation to represent the complementary angles ($k°$ and $71°$) that sum to $90°$.

Step 2: Apply inverse operations to solve for the missing angle ($k°$).

Visual support

b.



Working

$w° + 21.5° + 39.5° = 180°$

$w° + 61° = 180°$

$w° + 61° - 61° = 180° - 61°$

$w = 119$

Thinking

Step 1: Write an equation to represent the supplementary angles ($w°$, 21.5° and 39.5°) that sum to 180°.

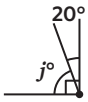
Step 2: Simplify the equation by adding like terms.

Step 3: Apply inverse operations to solve for the missing angle ($w°$).

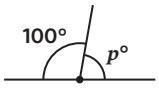
Student practice

Set up an equation and solve to find the value of each pronumeral.

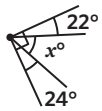
a.



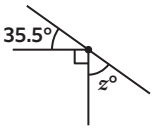
b.



c.



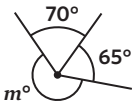
d.



WE 3 Solving for missing angles around a point

Set up an equation and solve to find the value of each pronumeral.

a.



Working

$m° + 70° + 65° = 360°$

$m° + 135° = 360°$

$m° + 135° - 135° = 360° - 135°$

$m = 225$

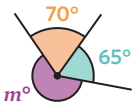
Thinking

Step 1: Write an equation to represent the angles around a point ($m°$, 70°, and 65°) that sum to 360°.

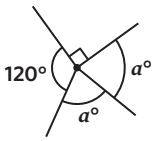
Step 2: Simplify the equation by adding like terms.

Step 3: Apply inverse operations to solve for the missing angle ($m°$).

Visual support



b.



Working

$a° + a° + 90° + 120° = 360°$

$(2a)° + 210° = 360°$

$(2a)° + 210° - 210° = 360° - 210°$

$2a = 150$

$2a \div 2 = 150 \div 2$

$a = 75$

Thinking

Step 1: Write an equation to represent the angles around a point ($a°$, $a°$, 90°, and 120°) that sum to 360°.

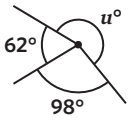
Step 2: Simplify the equation by adding like terms.

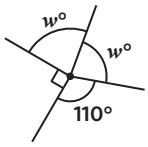
Step 3: Apply inverse operations to solve for the missing angle ($a°$).

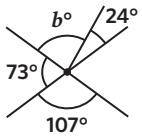
Student practice

Set up an equation and solve to find the value of each pronumeral.

a. 

b. 

c. 

d. 

6B Activities and questions

STARTER TASKS

Odd spot

The Melbourne Star was an observation wheel that opened in 2008. The Melbourne Star was like a large ferris wheel and was popular with visitors to Melbourne. It cost $100 million to build and was visited by approximately 300 000 people each year. The tourist attraction was shut down in 2021 and no longer operates.

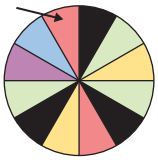
Image: Chethan Kumar HS/ Shutterstock.com

The angle between each section of the star can be measured close to the Stars' centre or near where the tourists sit. Are the angles marked m and r equal or not equal?

A. Equal **B.** Not equal

Puzzle

The following wheel is made up of 12 equally sized parts and can be spun around. The winning colour is the one that lands on the arrow.

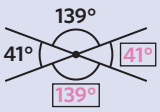


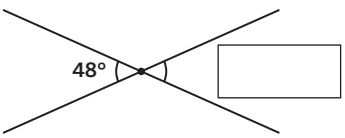
a) What is the combined angle measurement formed by all the yellow sections?
b) What is the combined angle measurement formed by all the black sections?
c) What colour(s) are least and most likely to land on the arrow after the wheel is spun?

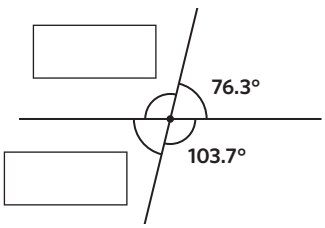
Understanding worksheet

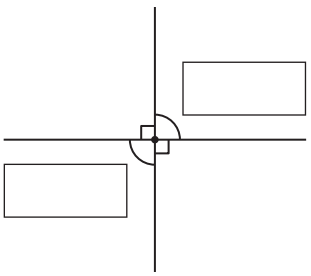
1. Fill in the angle measurements for each pair of vertical angles.

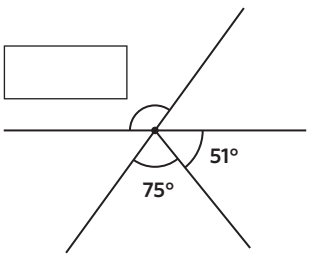
Example



a. 

b. 

c. 

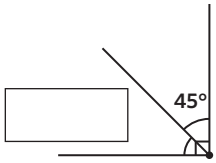
d. 

2. Fill in each missing angle and circle complementary or supplementary to describe the angle property used.

Example

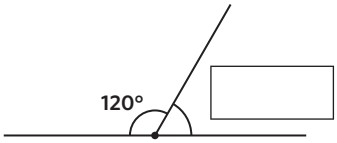
complementary supplementary

a.



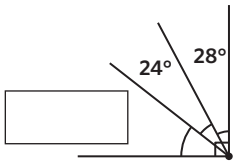
complementary supplementary

b.



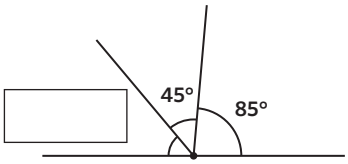
complementary supplementary

c.



complementary supplementary

d.



complementary supplementary

3. Fill in the blanks by using the words provided.

complementary | supplementary | 90° | 360°

Adjacent angles form special angle properties. When two adjacent angles meet to form a right angle, they are _____ and they sum to _____. When two adjacent angles meet to form a straight line, they are _____ angles and they sum to 180°. Adjacent angles that meet around a point sum to _____.

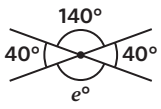
Fluency

Question working paths

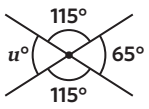
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h)

WE 1 **4.** Determine the value of each pronumeral.

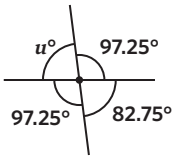
a.



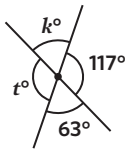
b.



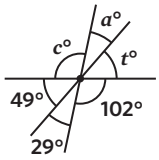
c.



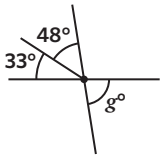
d.



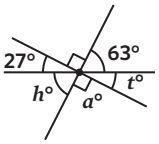
e.



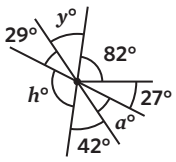
f.



g.



h.



5. State whether each set of angles is complementary, supplementary, a full revolution or neither.

a. 100° and 80°

b. 15° and 75°

c. 100° and 90°

d. 120°, 200° and 40°

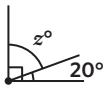
e. 45°, 100° and 45°

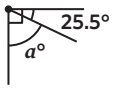
f. 103°, 84.5°, 46.5°, 76° and 50°

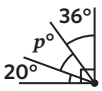
g. 15°, 25°, 10°, 11° and 29°

h. 153°, 53°, 54°, 20° and 26°

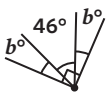
WE 2a **6.** Set up an equation and solve to find the value of each pronumeral.

a. 

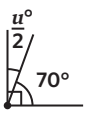
b. 

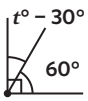
c. 

d. 

e. 

f. 

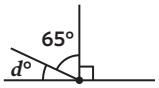
g. 

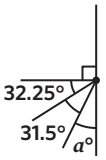
h. 

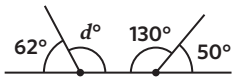
WE 2b **7.** Set up an equation and solve to find the value of each pronumeral.

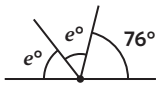
a. 

b. 

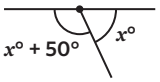
c. 

d. 

e. 

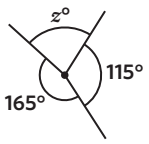
f. 

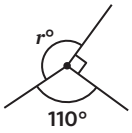
g. 

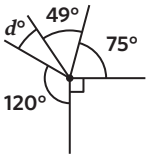
h. 

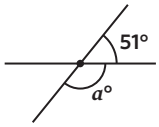
WE 3 **8.** Set up an equation and solve to find the value of each pronumeral.

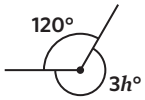
a. 

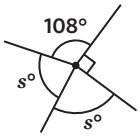
b. 

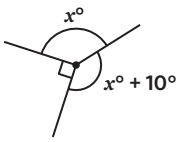
c. 

d. 

e. 

f. 

g. 

h. 

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Malia is cutting her hedges with large garden shearers. She opens them and the two blades form a 37° angle. What angle is formed by the handles on the opposite side?

10. How many degrees further does Daniella need to turn if she is completing a u-turn in her truck and has already turned 100°?

11. Raphael has a reclining chair with a backrest that can be adjusted. How many degrees does he need to lower the backrest to make the chair flat, if the angle between the seat and the backrest is 119°?

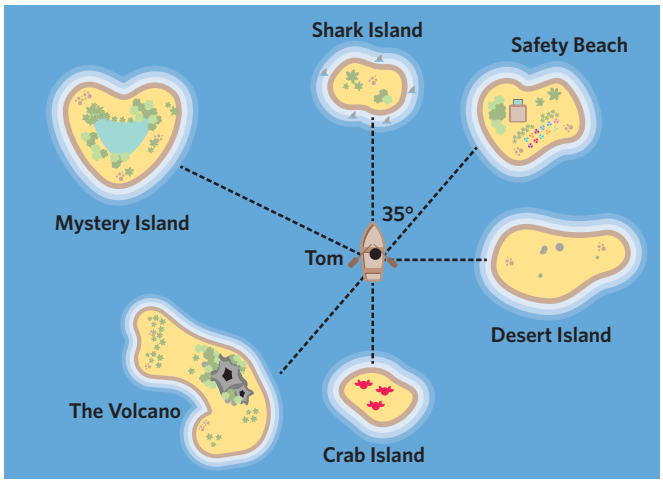
12. A ceiling fan has three blades that are all connected to a central bolt and spin in a circle. If the blades are equally spaced around a central bolt, how many degrees are between each blade?

13. Blake is cutting a circular cake into different pieces. He cuts the cake in half and then cuts each half into thirds. What is the angle measurement formed by each slice of cake?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Tom is a castaway and has been living on a small island by himself. Shark Island is directly north of his location and Crab Island is directly south. The angle formed by the imaginary lines from his position to Shark Island and Desert Island form a right angle. The following image is a bird's-eye view of Tom's position at sea.



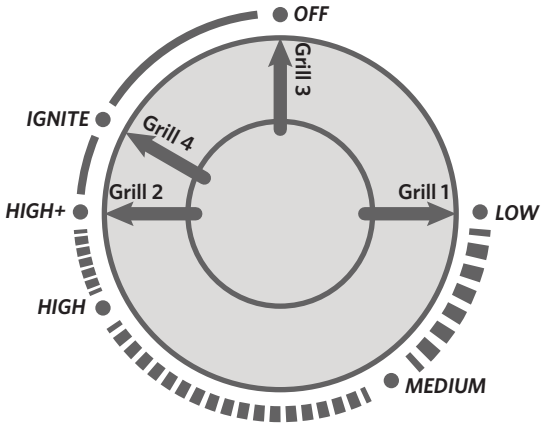
a. If Tom is facing directly towards Safety Beach, how many degrees does he need to turn to his right to face Desert Island?

b. Assume that the imaginary line from The Volcano and Safety Beach is a straight line. How many degrees to his left should Tom turn to face Crab Island if he is facing The Volcano?

c. Tom is trying to figure out some of the other angles formed by the imaginary lines from his position to the other islands. Based on the information Tom has, is it possible to find any more missing angles?

d. Tom is finding it difficult to decide which island to try and relocate to. Other than the degrees from his location what information might help him to make a more informed decision about which island to relocate to?

15. Becky's barbecue has four different grills. The heat on all the grills is controlled by a master knob that has different arrows that can be turned to increase or decrease each grill's temperature. The following is an image of the barbecue's master knob.



a. Which set of arrows form two adjacent angles that are complementary?

b. The LOW and HIGH+ temperatures are positioned directly opposite each other on the knob. If the turn from LOW to MEDIUM is a third of the turn from LOW to HIGH+, how many degrees does Becky need to turn the Grill 1 arrow to increase its temperature to MEDIUM?

c. After increasing Grill 1's temperature to MEDIUM, Becky would like to turn it OFF. How many degrees does Becky need to turn the arrow in the anticlockwise direction to turn the grill off?

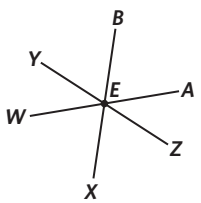
d. Becky thinks that the knob is not designed in an easy to use way. Propose a design for a knob that will make it easier to control the barbecue's temperature.

Extra spicy

16. How many degrees is the following marked angle?

A. 72°
B. 108°
C. 144°
D. 160°

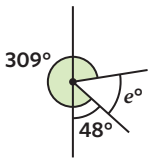
17. WA, XB and ZY are all straight lines and intersect at point E. If $\angle WEY = 48°$ and $\angle BEA = 83°$, what is the size of $\angle XEZ$?



A. 48°
B. 49°
C. 83°
D. 97°

18. Three different angles fit exactly around a point. The first angle is 20° less than the second angle. The third angle is triple the size of the second angle. What is the size of each of the three angles?

19. Solve for e.



Remember this?

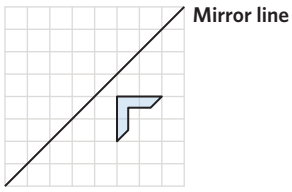
20. Bri measured the length of 21 wooden twigs that she collected from her garden. She recorded the lengths in a stem-and-leaf plot. The information she collected is shown in the following stem-and-leaf plot.

Stem	Leaf					
10	0	0	0	4	4	7
11	2	4	4	5		
12	4	5	5	5	7	
14	2	6	7	8		
15	1	5				

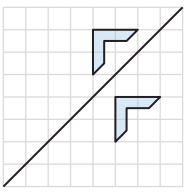
Key
10 | 10.3 cm

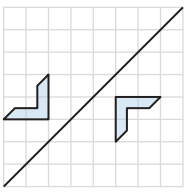
What is the median length of a twig?

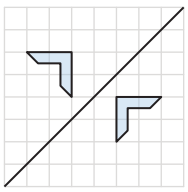
21. Dreak draws a design by reflecting a shape across a mirror line.

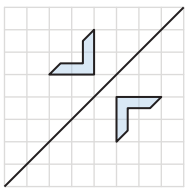


Which design shows the correct reflection?

A.


B.


C.


D.


22. During a leap, an ice skater made three full rotations of their body. How many degrees did the ice skater rotate their body in the leap?

A. 500° **B.** 720° **C.** 918° **D.** 1080°

6C Properties of triangles

There are different types of triangles. A triangle can be classified based on its side lengths or on the size of its interior angles. There are different types of triangles but all triangles have three sides and three interior angles that sum to 180°. Understanding the properties of the interior and exterior angles of a triangle helps students to improve their geometric reasoning. Below are some examples where properties of triangles can be used.

- I am building a bedside table that is an equilateral triangle. How long should I make each side?
- My triangular roof has two equal angles that are 75°. What is the third angle of my roof?
- I built a skateboard ramp with an incline of 30°. What is the exterior angle formed by my ramp and the ground?

Learning intentions

Students will be able to:

+ classify different types of triangles
+ find missing interior angles of triangles
+ find missing exterior angles of triangles.

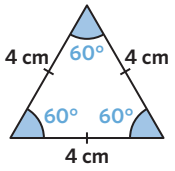
Key terms and definitions

- An **interior angle** is an angle measured inside a shape.
- An **equilateral triangle** contains three sides of equal length and three 60° interior angles.
- An **isosceles triangle** contains two sides of equal length and two angles of equal size.
- A **scalene triangle** contains three sides of different length and three interior angles of different size.
- A **right-angled triangle** contains one interior angle that is 90°.
- An **acute angle triangle** contains three interior angles that are between 0° and 90° in size.
- An **obtuse angle triangle** contains one interior angle that is between 90° and 180° in size.
- An **exterior angle** is the angle formed by the side of a shape and the imaginary line that is extended from the shape.

Key ideas

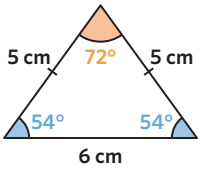
1 We can classify a triangle based on its side lengths and on its interior angles. Sides with the same markings have the same length. Sides with different markings have different lengths.

Equilateral triangle



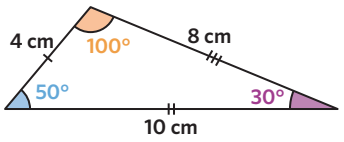
All three sides have an equal length.
All three angles are 60°.

Isosceles triangle



Two sides have an equal length.
Two angles are equal.

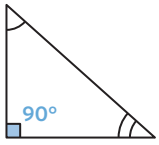
Scalene triangle



All three sides have different lengths.
All three angles are different sizes.

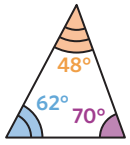
2 We can classify a triangle based on its interior angles. Angles with the same markings are the same size. Angles with different markings are different sizes.

Right-angled triangle



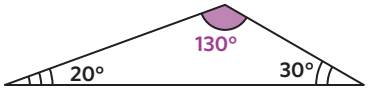
One angle is 90°.

Acute angle triangle



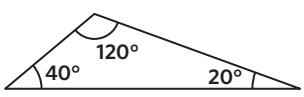
All three angles are acute angles.

Obtuse angle triangle



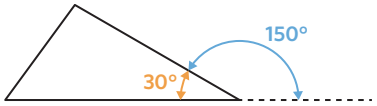
One angle is an obtuse angle.

3 The interior angles of a triangle sum to 180°.



40° + 120° + 20° = 180°

4 Adjacent exterior and interior angles that lie on a straight line form supplementary angles that sum to 180°.



Interior angle + Exterior angle = 180°
30° + 150° = 180°

Worked examples

WE 1 Classifying triangle types

Classify each triangle as one of the given options.

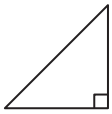
a.  (equilateral, isosceles or scalene)

Working

Isosceles triangle

Thinking

Compare the markings on each side. A triangle with two equal sides is an isosceles triangle.

b.  (right-angled, acute or obtuse)

Working

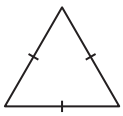
Right-angled triangle

Thinking

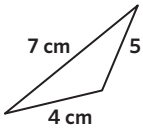
Identify the marked interior angle. A triangle that contains a right angle is a right-angled triangle.

Student practice

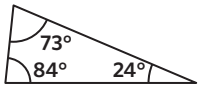
Classify each triangle as one of the given options.

a.  (equilateral, isosceles or scalene)

b.  (right-angled, acute or obtuse)

c. 

(equilateral, isosceles or scalene)

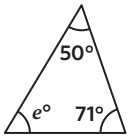
d. 

(equilateral, isosceles or scalene)

WE 2 Finding missing angles inside a triangle

Solve to find each pronumeral.

a.



Working

$e° + 50° + 71° = 180°$

$e° + 121° = 180°$

$e° + 121° - 121° = 180° - 121°$

$e = 59$

Thinking

Step 1: Write an equation to represent the interior angles (50°, 71° and e°) that sum to 180°.

Step 2: Simplify and apply inverse operations to solve for the pronumeral (e).

b.

Working

$x° + x° + 52° = 180°$

$2x° + 52° = 180°$

$2x° + 52° - 52° = 180° - 52°$

$2x° = 128°$

$2x° \div 2 = 128° \div 2$

$x = 64$

Thinking

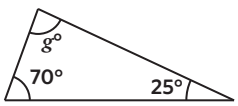
Step 1: Classify the triangle as an isosceles as it has two sides of equal length. The two base angles of an isosceles triangle are equal. Write an equation to represent the interior angles (52°, x° and x°) that sum to 180°.

Step 2: Simplify and apply inverse operations to solve for the pronumeral (x).

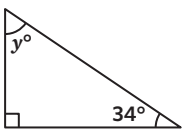
Student practice

Solve to find each pronumeral.

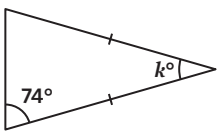
a.



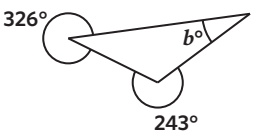
b.



c.



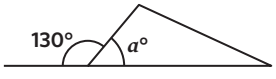
d.



WE 3 Finding missing exterior and interior angles

Solve to find each pronumeral.

a.



Working

$130° + a° = 180°$

$a° + 130° - 130° = 180° - 130°$

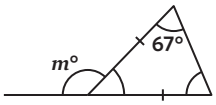
$a = 50$

Thinking

Step 1: Identify the adjacent exterior and interior angles. Write an equation to represent the angles (130° and a°) that sum to 180°.

Step 2: Simplify and apply inverse operations to solve for the pronumeral (a).

b.



Working

67°

$180° - 67° - 67° = 46°$

$m° + 46 = 180°$

$m° + 46° - 46° = 180° - 46°$

$m = 134$

Thinking

Step 1: Find the missing base angle inside the triangle. The two base angles of an isosceles triangle are equal.

Step 2: Subtract the two base angles from 180° to find the third interior angle.

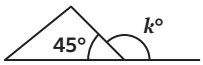
Step 3: Identify the adjacent exterior and interior angles. Write an equation to represent the angles (46° and m°) that sum to 180°.

Step 4: Simplify and apply inverse operations to solve for the pronumeral (m).

Student practice

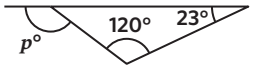
Solve to find each pronumeral.

a.

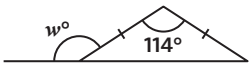


b.

c.



d.



6C Activities and questions

STARTER TASKS

Odd spot

The Tokyo Gate Bridge in Japan is a truss bridge. A truss bridge is typically composed of many connected triangles that are known as trusses. Triangles are regularly used in construction of long bridges as they are recognised as a strong structural support and are made up of a relatively small amount of material for how strong they are.

What is the sum of the interior angles of a triangular truss?

A. 360° **B.** 180°

Image: Sakarin Sawasdinaka/ Shutterstock.com

Puzzle

a) How many triangles are shown in the diagram?

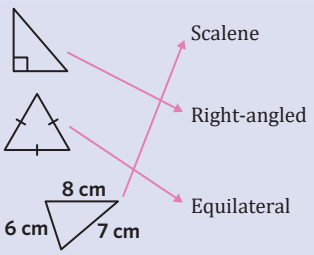
b) What do all the shown triangles have in common?

c) What is an efficient strategy to ensure all the triangles are found?

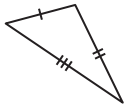
Understanding worksheet

1. For each set of triangles draw a line to its correct classification.

Example



a.

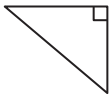


Equilateral

Scalene

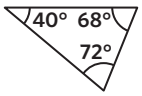
Isosceles

b.



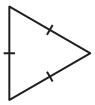
Acute angle

Obtuse angle

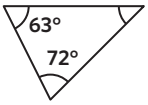


Right-angled

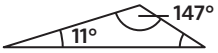
c.



Acute angle

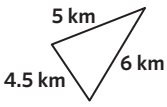


Obtuse angle

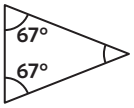


Equilateral

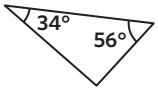
d.



Isosceles



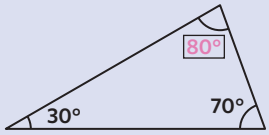
Scalene



Right-angled

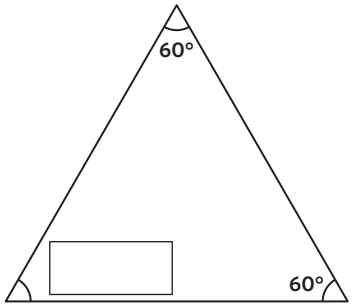
2. Fill in each of the missing angle measurements to make the equation true.

Example



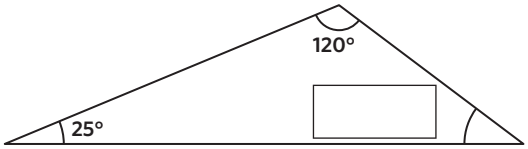
80° + 30° + 70° = 180°

a.



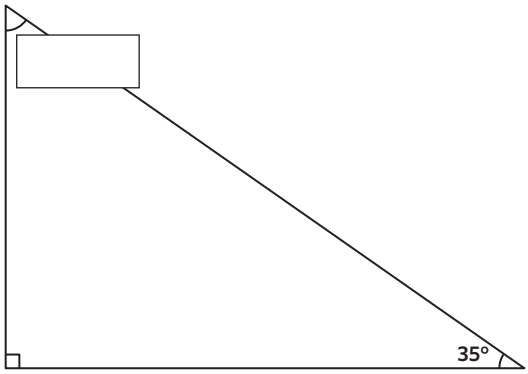
□ + 60° + 60° = 180°

b.



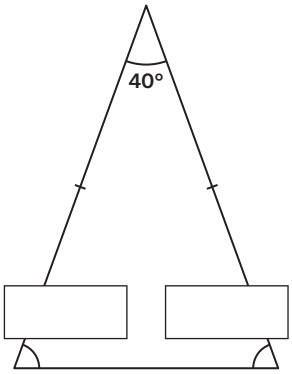
25° + □ + 120° = □

c.



□ + □ + 35° = □

d.



□ + □ + 40° = 180°

3. Fill in the blanks by using the words provided.

equilateral | 180° | size | length | three | 60°

All types of triangles have _____ sides and interior angles that sum to _____.

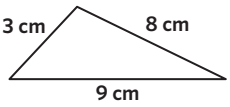
We can classify a triangle based on the _____ of its sides or by the _____ of its angles. For example, an _____ triangle has three sides of equal length and interior angles that are _____.

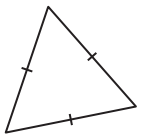
Fluency

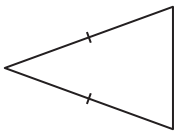
Question working paths

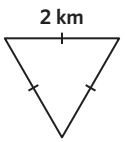
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f), 8 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h), 8 (d,e,f)

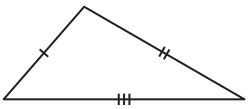
WE 1a **4.** Classify each triangle as an equilateral, isosceles or scalene triangle.

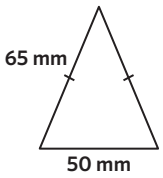
a.


b.


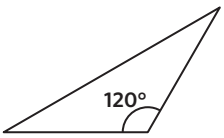
c.


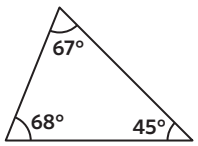
d.


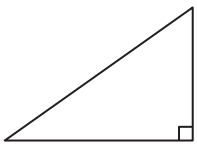
e.


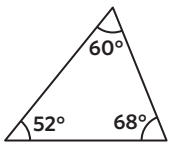
f.


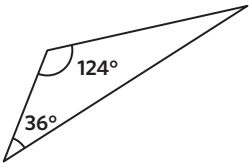
WE 1b **5.** Classify each triangle as an acute, obtuse or right-angled triangle.

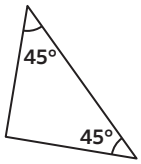
a.


b.


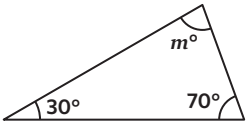
c.


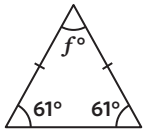
d.


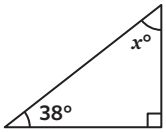
e.


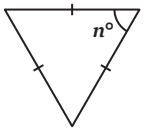
f.


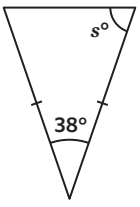
WE 2 **6.** Solve to find each pronumeral.

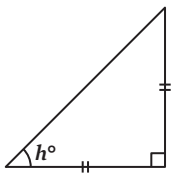
a.


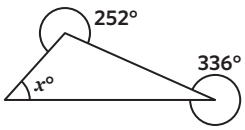
b.


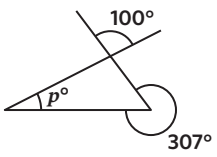
c.


d.


e.


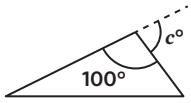
f.


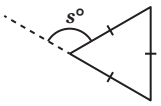
g.


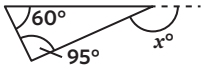
h.


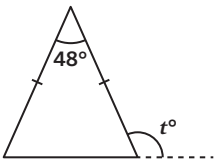
WE 3 **7.** Solve to find each pronumeral.

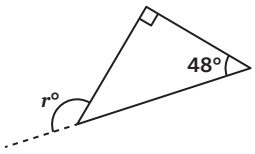
a.

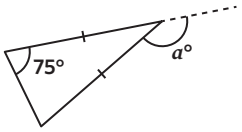

b.


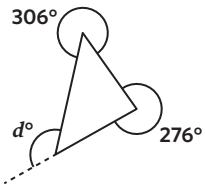
c.


d.


e.


f.


g.


h.


8. State whether each description describes a triangle or not.

a. Interior angles of 50°, 40° and 90°.

b. Interior angles of 85°, 15° and 70°.

c. Two right angles and an acute angle.

d. Exterior angles of 120°, 120° and 120°.

e. Interior angles of 43.5°, 27.25° and 109.25°.

f. Two obtuse interior angles and one acute.

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Mellissa's teepee has an entrance that is a triangle. The two vertical sides form the same angle at the entrance's base. What type of triangle does the teepee's entrance form?

10. Many standard Australian road signs are triangles. What size is each of the interior angles of a *GIVE WAY* sign if all three of its sides have the same length?

11. Pippa is making handmade corn chips using a stencil that is an acute triangle. What is the third angle of each chip if one of the interior angles is 65° and another is 75°?

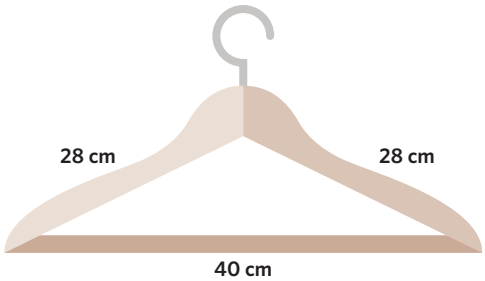
12. Lionel is building a long ramp at his parent's house. If the ramp has an incline of 12°, what is the exterior angle formed by the ramp and the floor?

13. Ricki is designing a triangular business logo that is asymmetric. What are all the different types of triangles that she can draw?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. Rupert is very organised and always keeps his wardrobe very neat. He only uses a specific wooden coat hanger to store his clothes. The brand he likes is called CoatHanga. The following image shows Rupert's favourite hanger.



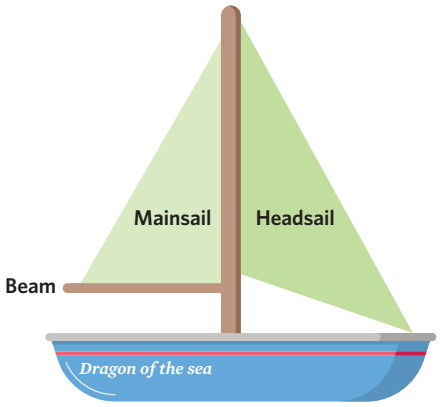
a. Based on the CoatHanga's side lengths, what type of triangle is it?

b. Two of the interior angles of the hanger are 25°. What is the size of the obtuse interior angle of the coat hanger?

c. Rupert is trying to create a new coat hanger with a metal wire. He creates an isosceles triangle with an interior angle that is 15° more than one of the equal angles. What is the size of each of the three interior angles?

d. Propose a design of a coat hanger that is balanced and label the interior angles.

15. Sally is an avid sailor and regularly explores the coast of Tasmania. The following image is a replica of her boat the Dragon of the sea.



a. The mainsail of a sailboat has three sides with different lengths. Based on its side lengths, what type of triangle is the mainsail?

b. The head sail of a sailboat is also a triangle. What is the sum of all the interior angles in both the mainsail and headsail?

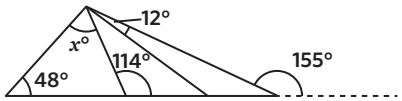
c. The mainsail has one right angle. The ratio between the two non-right angle angles in Sally's mainsail is 2 : 1. What are the two other interior angles of the mainsail?

d. The bottom of the mainsail is supported by a long straight beam. How many degrees is the exterior angle formed by the beam and the outside slanted edge of the mainsail?

e. Sally is considering sailing her boat from Tasmania to Sydney. What factors should she consider before making her decision to sail a large distance?

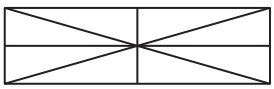
Extra spicy

16. What is the value of x?



A. 25°
B. 36°
C. 57°
D. 66°
E. 107°

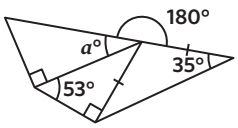
17. How many triangles are in the following image? Any size triangles should be counted.



A. 8
B. 12
C. 14
D. 16
E. 18

18. What is value of f?

19. What is the value of a?

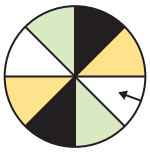


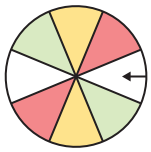
A. 33°
B. 35°
C. 37°
D. 53°
E. 110°

Remember this?

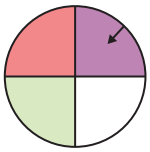
20. It costs a plastic fork manufacturer 0.021 cents to make one premium fork.
How much does it cost the manufacturer to make 10 000 premium forks?

21. A coloured spinning wheel has an arrow. The probability of the arrow landing on each colour is the same.
Which of the following spinning wheels does not meet the description?

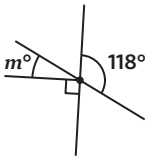
A. 

B. 

C. 

D. 

22. What is the value of m in the diagram below?



A. 28
B. 38
C. 45
D. 76

6D Properties of quadrilaterals

There are different types of quadrilaterals. A quadrilateral can be classified based on its side lengths, interior angles or by the number of pairs of parallel lines it is formed by. There are different types of quadrilaterals but all quadrilaterals have four sides and interior angles that sum to 360°. Below are some examples where properties of quadrilaterals can be used.

- My favourite table mat has four sides and each side has the same length as the opposing side. What type of quadrilateral is my table mat?
- My bedroom window is a quadrilateral and each interior angle of the window is equal. What is each interior angle of my window?
- My favourite deck of playing cards has a diamond that is a rhombus. Two of the interior angles are 65°. What is the size of the other two interior angles?

Learning intentions

Students will be able to:

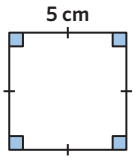
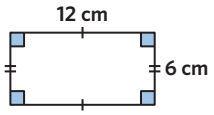
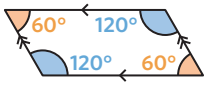
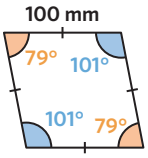
+ classify different types of quadrilaterals
+ find missing interior angles of quadrilaterals.

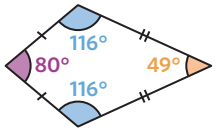
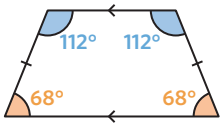
Key terms and definitions

- A **quadrilateral** is a four sided shape.
- **Parallel lines** never touch and are always the same distance apart.

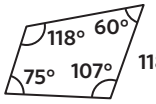
Key ideas

1 We can classify a quadrilateral based on its side lengths, interior angles or number of pairs of parallel lines formed by its sides.

	Square	**Rectangle**	**Parallelogram**	**Rhombus**
				
Side length	All sides are of equal length.	Opposite sides are of equal length.	Opposite sides are of equal length.	All sides are of equal length.
Angles	All four angles are 90°.	All four angles are 90°.	Opposite angles are equal in size. Adjacent angles sum to 180°.	Opposite angles are equal in size. Adjacent angles sum to 180°.
Parallel lines	Opposite sides are parallel.	Opposite sides are parallel.	Opposite sides are parallel.	Opposite sides are parallel.

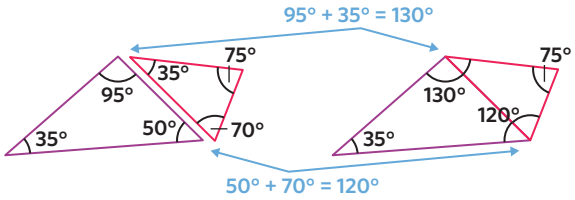
	Kite	**Trapezium (isosceles)**
		
Side length	There are two pairs of adjacent sides of equal length.	One pair of opposite sides are of equal length.
Angles	One pair of opposite angles are equal.	Adjacent angles that share a parallel side are equal in size.
Parallel lines	No pairs of parallel lines.	One pair of sides are parallel. Note: All trapeziums have a pair of parallel opposite sides but may not have two opposite sides of equal length.

2 The interior angles of a quadrilateral sum to 360°.



118° + 75° + 107° + 60° = 360°

3 All quadrilaterals can be formed by two triangles and thus the sum of all of the internal angles in a quadrilateral is 360°.

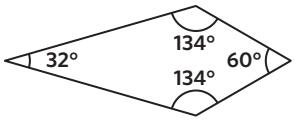


Worked examples

WE 1 Classifying quadrilateral types

Classify each quadrilateral as a square, rectangle, parallelogram, rhombus, kite or trapezium.

a.



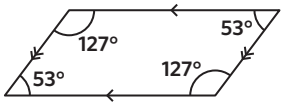
Working

Kite

Thinking

Identify pairs of interior angles. A quadrilateral with one pair of opposite angles of equal size is a kite.

b.



Working

Parallelogram

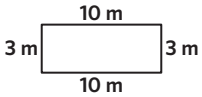
Thinking

Compare each side's markings and the interior angles. A quadrilateral with parallel opposite sides and equal opposite angles is a parallelogram.

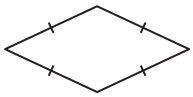
Student practice

Classify each quadrilateral as a square, rectangle, parallelogram, rhombus, kite or trapezium.

a.

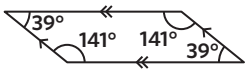


b.



c.

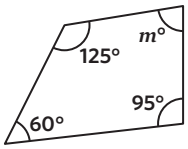
d.



WE 2 Finding missing angles inside a quadrilateral

Solve to find each pronumeral.

a.



Working

$m° + 125° + 60° + 95° = 360°$

$m° + 280° = 360°$

$m° + 280° - 280° = 360° - 280°$

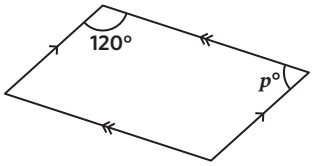
$m = 80$

Thinking

Step 1: Write an equation to represent the interior angles (125°, 60°, 95° and $m°$) that sum to 360°.

Step 2: Simplify and apply inverse operations to solve for the pronumeral (m).

b.



Working

$p° + 120° = 180°$

$p° + 120° = 180°$

$p° + 120° - 120° = 180° - 120°$

$p = 60$

Thinking

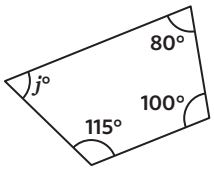
Step 1: Adjacent angles in parallelograms sum to 180°. Write an equation to represent the interior angles (120° and $p°$) that sum to 180°.

Step 2: Apply inverse operations to solve for the pronumeral (p).

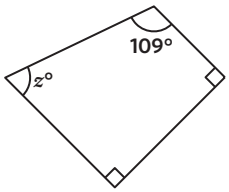
Student practice

Solve to find each pronumeral.

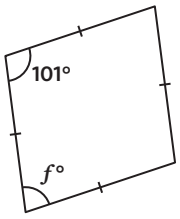
a.



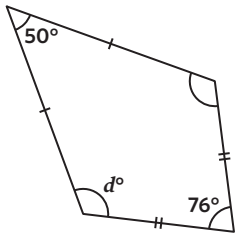
b.



c.



d.



6D Activities and questions

STARTER TASKS

Image: FPWing/Shutterstock.com

Odd spot

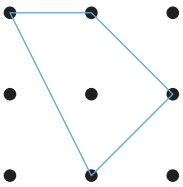
'Kite fighting' is a popular game in some Asian countries including, Pakistan, India, Nepal, and Afghanistan. 'Fighter kites' typically have four sides and are made out of lightweight materials. The aim of 'kite fighting' is to cut the opponent's line so that they can no longer fly their kite. The winner is the person who is able to keep their kite in the sky without their line being cut by others.

What is the sum of the interior angles of a typical kite?

A. 180° **B.** 360°

Puzzle

A grid of dots can be used to draw different types of quadrilaterals. The following drawing is an example of a quadrilateral drawn on a three by three dotted grid.



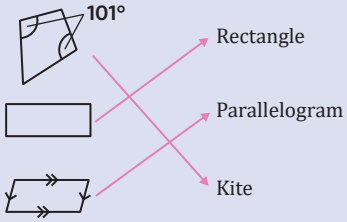
Draw a three by three dot grid to complete each question.

a. What is the smallest quadrilateral that can be drawn?
b. Draw a kite.
c. Draw a quadrilateral that contains a reflex angle.
d. How many different rectangles can be drawn?

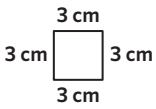
Understanding worksheet

1. For each set of quadrilaterals draw a line to its correct classification.

Example

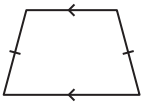


a.



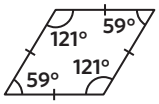
Rectangle

Square

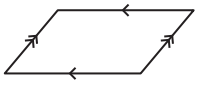


Trapezium

b.



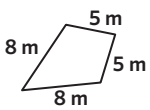
Rectangle



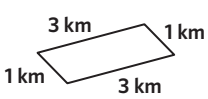
Parallelogram

Rhombus

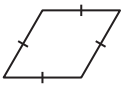
c.



Parallelogram



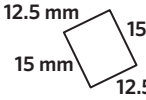
Kite



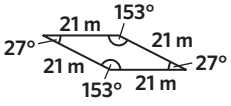
Rhombus

d.

Rhombus



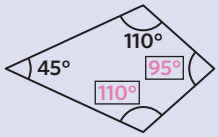
Irregular quadrilateral



Rectangle

2. Fill in each of the missing angle measurements to make the equation true.

Example

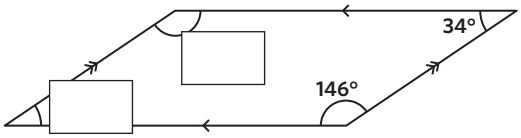


110° + 110° + 45° + 95° = 360°

a.

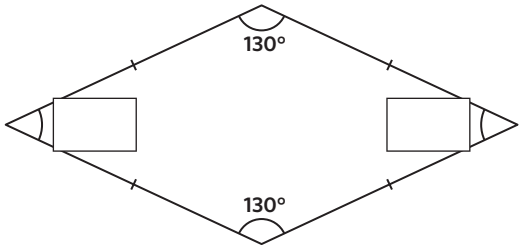
90° + 90° + ☐ + ☐ = 360°

b.



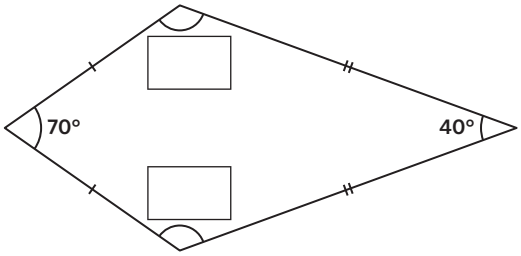
34° + ☐ + 146° + ☐ = ☐

c.



130° + ☐ + 130° + ☐ = ☐

d.



70° + 40° + ☐ + ☐ = ☐

3. Fill in the blanks by using the words provided.

rectangle | parallel | four | trapezium | 360°

All types of quadrilaterals have ☐ sides and interior angles that sum to ☐. We can classify some quadrilaterals based on their interior angles or the number of pairs of ☐ lines formed by its sides. For example, a ☐ has only one pair of parallel lines. A ☐ has four interior 90° angles.

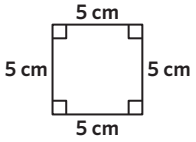
Fluency

Question working paths

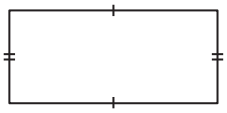
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f)

WE 1 **4.** Classify each quadrilateral as a square, rectangle, parallelogram, rhombus, kite or trapezium.

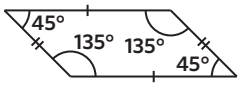
a.



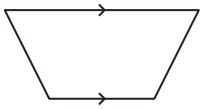
b.



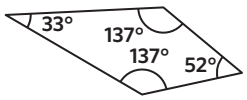
c.



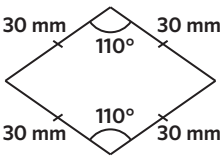
d.



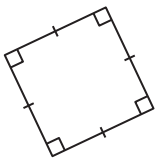
e.



f.



g.



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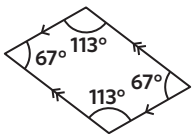
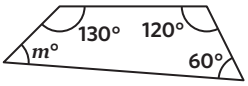
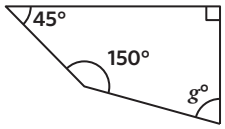
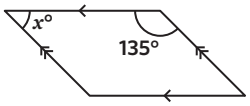
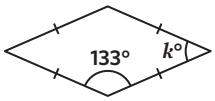


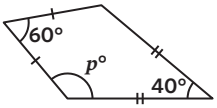
WE 2 **5.** Solve to find each pronumeral.

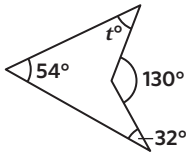
a. 

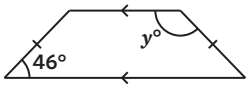
b. 

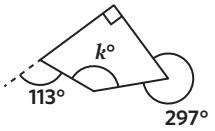
c. 

d. 

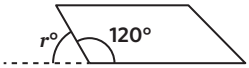
e. 

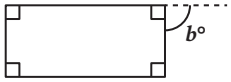
f. 

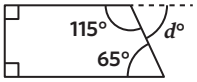
g. 

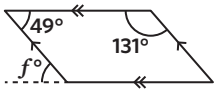
h. 

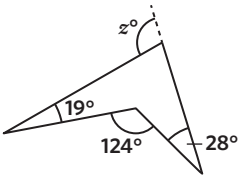
6. Solve to find each pronumeral.

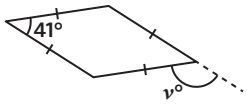
a. 

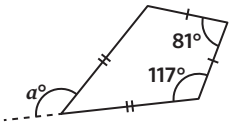
b. 

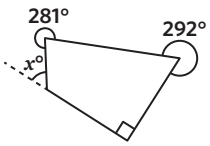
c. 

d. 

e. 

f. 

g. 

h. 

7. State whether each description could describe a quadrilateral or not.

- **a.** Interior angles of 120°, 80° and 90° and 60°.
- **b.** Four interior right angles.
- **c.** Only three interior angles.
- **d.** Interior angles of 85.5°, 102.5° and 72.75° and 99.25°.
- **e.** Two right interior angles and two obtuse interior angles.
- **f.** An acute interior angle, a right interior angle, an obtuse interior angle and a reflex interior angle.

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. A Monopoly $500 note has four interior right angles and a pair of sides that is 10 cm long and another pair of sides that is 5 cm wide. What type of quadrilateral does the note form?

9. Paige collects abstract pieces of street art. What type of quadrilateral is the frame of one of her pieces if it has two pairs of sides of equal length and only one pair of equal opposite angles?

10. Alexa is a carpenter and welds a quadrilateral shaped piece of iron. One of the interior angles is 48°, and the other three angles are equal. What is the size of the other angles?

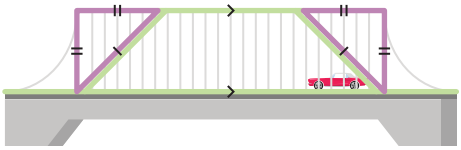
11. Mr Borg has trapezium-shaped tables in his classroom. Each table contains two pairs of equal angles. What is the size of each interior angle, if one pair of the equal angles sum to 148°?

12. Hamid measured two of the interior angles of his kite. One was 126° and the other was 34°. What is the size of the remaining interior angles if they are opposite angles?

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c,d)	All

13. Jill is designing a bridge for a primary school engineering challenge. Jill describes her bridge as a quadrilateral with two right-angled isosceles triangles on either side. She marks the equal lengths and parallel sides of the triangle's sides in the following model.



a. Jill needs to label the quadrilateral in her design so that she can accurately describe the bridge. What type of quadrilateral is the green shape that is included in her design?

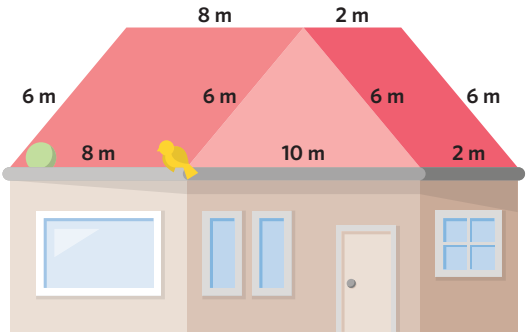
b. What is the size of each of the two equal angles inside of the triangles on either side of the bridge?

c. As part of the competition, Jill is required to list the size of all the interior angles of the green quadrilateral in her design. What are four angles that Jill will need to list?

d. Jill thinks that the sum of the interior angles of the two triangles is the same as the sum of the interior angles of the trapezium in her design. Is she correct? Show your working.

e. Design a bridge for the competition that includes quadrilaterals and triangles.

14. Bob is a builder and has recently completed the construction of his new family home. He has a partner and a young son. The following is a picture of his home with the lengths of different parts of the roof labelled.



a. Bob's bedroom has a square shaped window. After entering his house, is Bob's bedroom to the right or left?

b. From the front view of the house, Bob's roof is made up of two quadrilaterals on either side of a triangle. What type of quadrilateral is the roof on the right side of the triangular part?

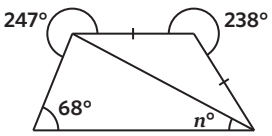
c. The triangular part of Bob's roof has a 70° angle at the top. What is the size of the interior angle where the bird is sitting?

d. Bob's son loves to play sports and threw a volleyball on the roof. What is the interior angle formed by the roof where the volleyball is stuck in the gutter?

e. Bob is considering adding an extension to his home. What factors should he consider before commencing the construction of the extension?

Extra spicy

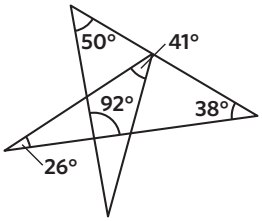
15. What is the value of n?



A. 28°
B. 52°
C. 60°
D. 61°
E. 68°

16. The angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. State all four interior angles of the quadrilateral.

17. How many of the unmarked interior angles are equal to 66°?



A. 1
B. 2
C. 3
D. 4
E. 5

18. One angle of a parallelogram is 15 less than 4 times the size of its adjacent angle. What is the size of all four interior angles of the parallelogram?

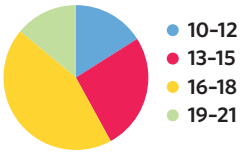
Remember this?

19. Lisa has 3.98 kilograms of dog food left in different containers. All the containers are full.
Each container comes in three different sizes. 1.5 kilograms, 400 grams and 240 grams.
How many containers does Lisa have?

A. 5
B. 6
C. 7
D. 8

20. Freida asked all the students at her ballet company their age and used the information to create a pie chart.
What is the age range of about a quarter of the students?

A. 10–12
B. 13–15
C. 16–18
D. 19–21



21. Which of the following types of triangles can be drawn?

A. scalene, reflex angle triangle
B. equilateral, obtuse angle triangle
C. right-angled scalene triangle
D. right-angled equilateral triangle

6E Parallel lines and transversals

Parallel lines are always the same distance apart and never intersect. A transversal line passes across two or more other lines. When a transversal line passes through a pair of parallel lines it forms different pairs of angles with specific properties. These pairs of angles can be classified as corresponding, alternate or co-interior. Below are some examples where parallel lines and transversal lines can be used.

- I travel on the number 16 tram to school. Are tram tracks parallel or not parallel?
- I place two straight planks of wood 50 cm apart to line a path. How far apart do I need to place the next planks to continue the path?
- My ladder has two parallel sides and a crooked horizontal step that form co-interior angles. One of the co-interior angles is 93°, what is the size of the other co-interior angle?

Learning intentions

Students will be able to:

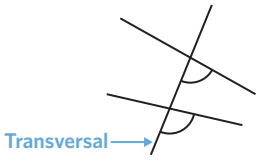
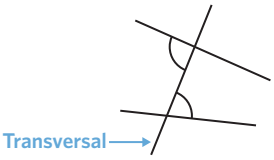
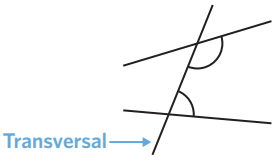
- identify corresponding, alternate and co-interior angles
- solve for missing angles when two parallel lines are crossed by a transversal
- state whether a pair of lines is parallel or not using angle facts.

Key terms and definitions

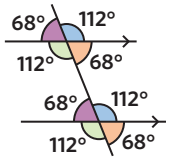
- **Parallel lines** never touch and are always the same distance apart.
- **Transversal lines** pass across two or more straight lines.

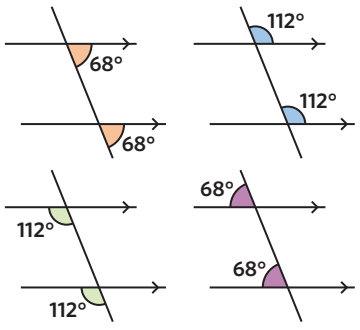
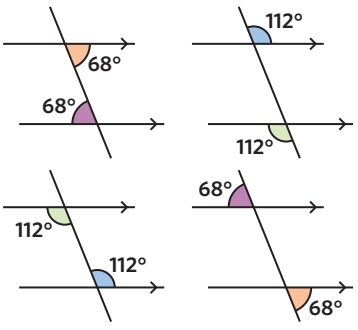
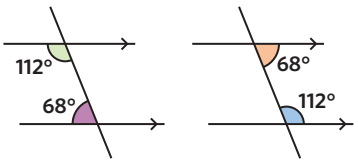
Key ideas

1 When a transversal line passes across two or more lines it forms pairs of angles called corresponding, alternate and co-interior angles.

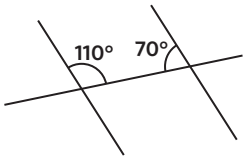
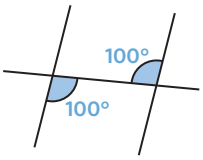
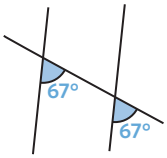
Corresponding angles	Alternate angles	Co-interior angles
		
Corresponding angles are in matching positions. They are on the same side of the transversal.	Alternate angles are on either side of the transversal. They are either both in between two straight lines, or both outside of two straight lines.	Co-interior angles are in between two straight lines and on the same side of the transversal.

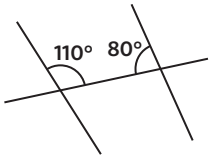
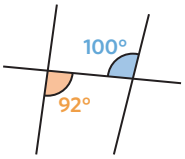
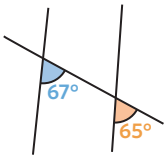
2 When a transversal line passes across two or more parallel lines, the pairs of corresponding and alternate angles formed are equal, and co-interior angles formed sum to 180°.



Corresponding angles	Alternate angles	Co-interior angles
Corresponding angles are equal. 	Alternate angles are equal. 	Co-interior angles sum to 180°. 

3 We can use angle facts to determine whether a pair of straight lines is parallel or not.

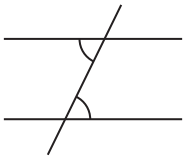
Parallel lines		
		
Co-interior angles sum to 180°. $110° + 70° = 180°$	Alternate angles are equal. $100° = 100°$	Corresponding angles are equal. $67° = 67°$

Not parallel lines		
		
Co-interior angles do not sum to 180°. $110° + 80° \neq 180°$	Alternate angles are not equal. $92° \neq 100°$	Corresponding angles are not equal. $67° \neq 65°$

Worked examples

WE 1 Classifying pairs of angles

Classify each pair of angle arcs as corresponding, alternate or co-interior angles.

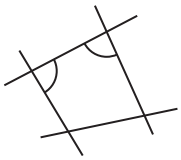
a. 

Working

Alternate angles

Thinking

Identify the position of the marked angles. They are on either side of a transversal. This pair of angles is alternate.

b. 

Working

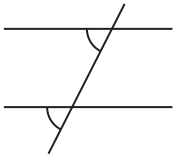
Co-interior angles

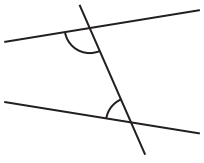
Thinking

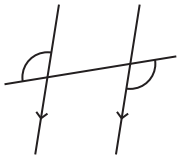
Identify the position of the marked angles. They are in between two straight lines and on the same side of a transversal. They are co-interior angles.

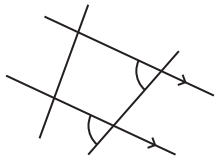
Student practice

Classify each pair of angle arcs as corresponding, alternate or co-interior angles.

a. 

b. 

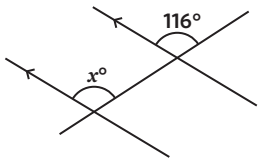
c. 

d. 

WE 2 Solving for missing angles in parallel lines

Solve for the pronumeral in each set of parallel and transversal lines. Give a reason for your answer.

a.



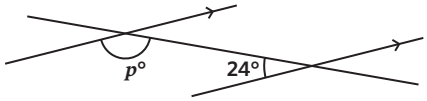
Working

$x = 116$ because $x°$ and $116°$ are corresponding angles.

Thinking

Identify the positions of the marked angle (116°) and the pronumeral (x). They are in matching positions on the same side of a transversal. They are corresponding angles.

b.



Working

$p = 156$ because $p°$ and $24°$ are co-interior angles.

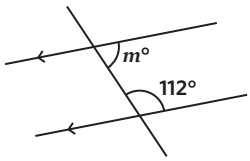
Thinking

Identify the position of the marked angles (24°) and the pronumeral (p). They are in between two parallel lines and on the same side of a transversal. They are co-interior angles.

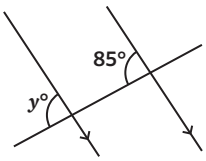
Student practice

Solve for the pronumeral in each set of parallel and transversal lines. Give a reason for your answer.

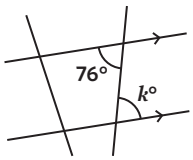
a.



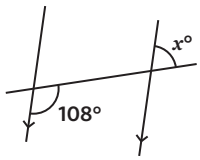
b.



c.



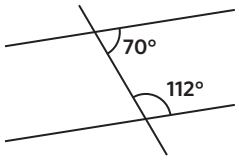
d.



WE 3 Identifying parallel and non-parallel lines

Identify whether each pair of lines that are crossed by a transversal are parallel or not. Give a reason for your answer.

a.



Working

$70° + 112° = 182°$

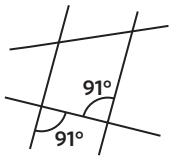
The lines are not parallel because the sum of co-interior angles is not 180°.

Thinking

Step 1: Identify the positions of the marked angles (70° and 112°). They are in between two straight lines and on the same side of a transversal. They are co-interior angles. Calculate the sum of the angles.

Step 2: State whether the lines are parallel or not parallel. The co-interior angles do not sum to 180°.

b.



Working

The lines are parallel as the alternate angles are equal.

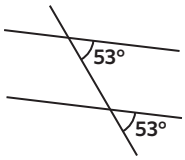
Thinking

Identify the position of the marked angles (91° and 91°). They are on either side of a transversal. They are alternate angles. A pair of parallel lines form equal alternate angles.

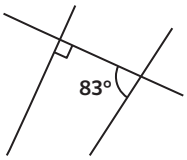
Student practice

Identify whether each pair of lines are parallel or not parallel. Give a reason for your answer.

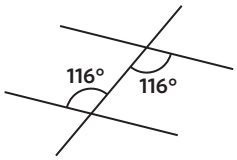
a.



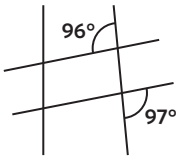
b.



c.



d.



6E Activities and questions

STARTER TASKS

Odd spot

Longitude and Latitude lines are imaginary lines drawn on maps so that precise location can be determined. Lines of Latitude are known as Parallels of Latitude as the lines are parallel to each other. Lines of Longitude are known as Meridians of Longitude and are not parallel to each other as they intersect at the North and South Poles.

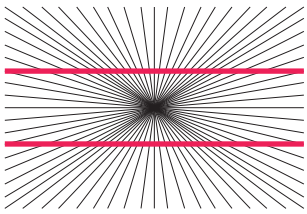
If a transversal line crosses two Meridians of Longitude above the equator will the co-interior angles sum to 180°?

A. Yes **B.** No

Image: Anton Shahrai/Shutterstock.com

Puzzle

The following image is made up of a number of different lines.



a) Do any of the black lines form a pair of parallel lines? Give a reason for your answer.

b) How can you check if the two red lines are straight or not?

c) Is it possible to determine from the image alone whether the pair of red lines are parallel or not? Give a reason for your answer.

Understanding worksheet

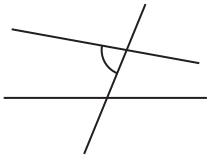
1. Draw the missing angle arc for the corresponding, alternate or co-interior angle.

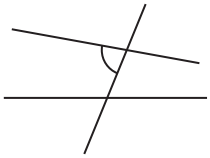
Example

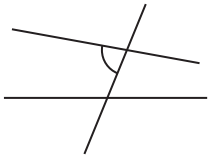
Corresponding 

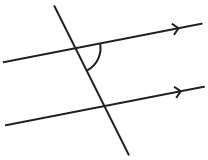
Alternate 

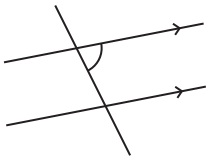
Co-interior 

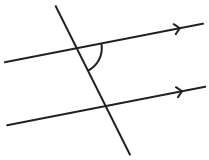
a. Corresponding 

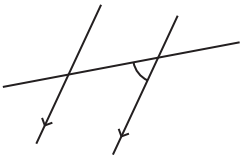
Alternate 

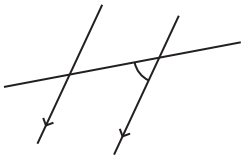
Co-interior 

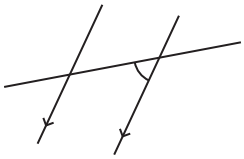
b. Corresponding 

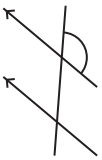
Alternate 

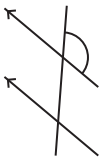
Co-interior 

c. Corresponding 

Alternate 

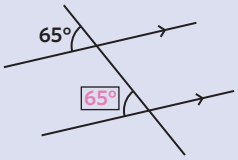
Co-interior 

d. Corresponding 

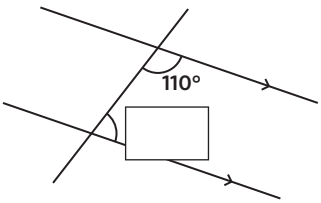
Alternate 

2. Fill in the missing angle and missing information to give a reason for your answer.

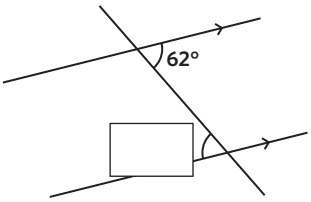
Example



Corresponding angles in parallel lines are equal.

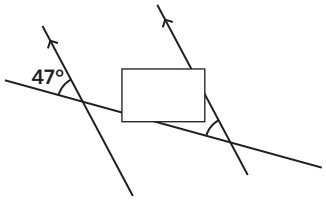
a. 

[ ] angles in between parallel lines sum to 180°.

b. 

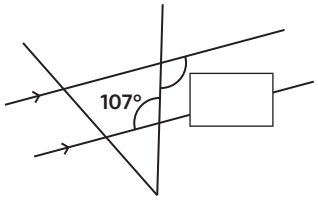
[ ] angles in between parallel lines are equal.

c.



[ ] angles in parallel lines are equal.

d.



[ ] angles in between parallel lines are equal.

3. Fill in the blanks by using the words provided.

alternate | parallel | co-interior | transversal

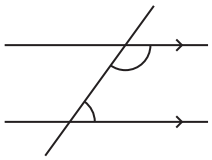
[ ] lines never touch and are always the same distance apart. A [ ] line passes across two or more straight lines. When transversal lines pass across pairs of straight lines it forms different angles. [ ] and corresponding angles in parallel lines are equal. [ ] angles in parallel lines sum to 180°.

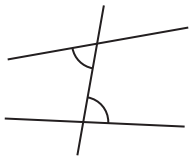
Fluency

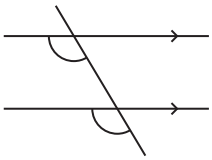
Question working paths

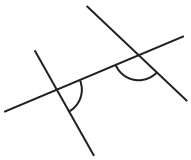
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f)

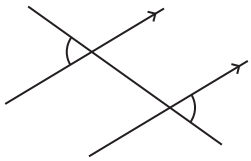
WE 1 **4.** Classify each pair of angle arcs as corresponding, alternate or co-interior angles.

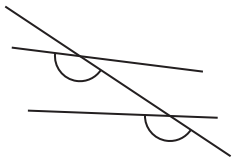
a. 

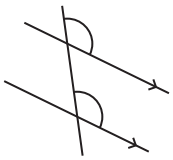
b. 

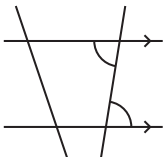
c. 

d. 

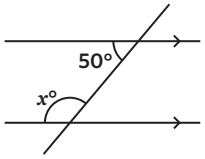
e. 

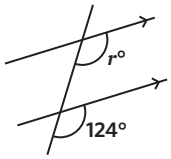
f. 

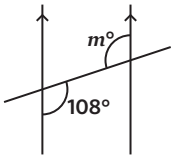
g. 

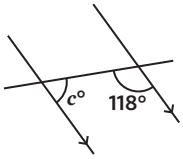
h. 

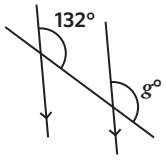
WE 2 **5.** Solve for the pronumeral in each set of parallel and transversal lines. Give a reason for your answer.

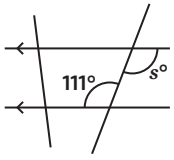
a. 

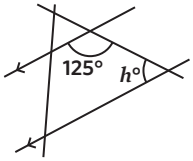
b. 

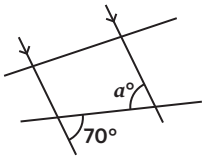
c. 

d. 

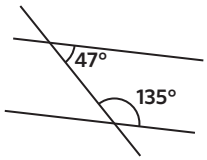
e. 

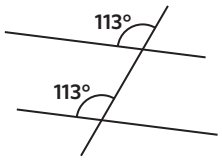
f. 

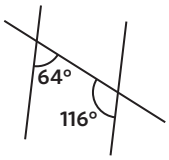
g. 

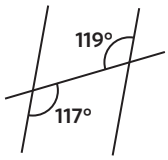
h. 

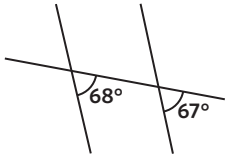
WE 3 **6.** Identify whether each pair of lines that are crossed by a transversal are parallel or not. Give a reason for your answer.

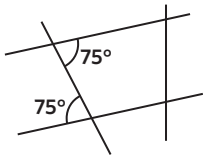
a.


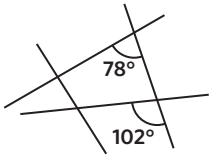
b.


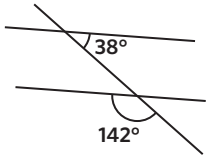
c.


d.


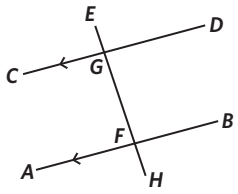
e.


f.


g.


h.


7. State whether each statement is true or false in relation to the following diagram.

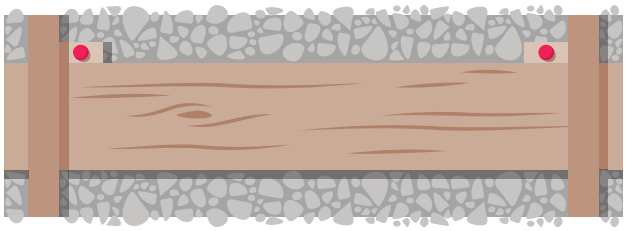


a. $\overline{CD}$ and $\overline{AB}$ are parallel lines.
b. $\angle CGF$ and $\angle AFG$ are co-interior angles.
c. $\angle EGD$ and $\angle AFG$ are corresponding angles.
d. $\angle DGF$ and $\angle AFH$ are alternate angles.
e. $\angle GFB$ and $\angle CGF$ are equal in size.
f. The sum of $\angle DGF$ and $\angle BFG$ is equal to the sum of $\angle CGF$ and $\angle BFH$.

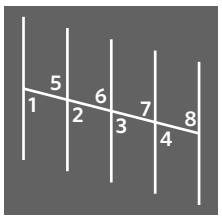
Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

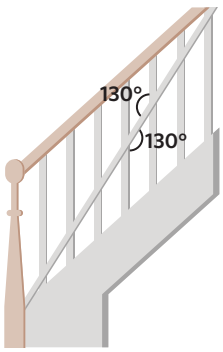
8. A rustic railroad track was built using two parallel rails that were bolted into wooden beams. What name is given to the pair of angles that are marked by the two bolts in the following image?



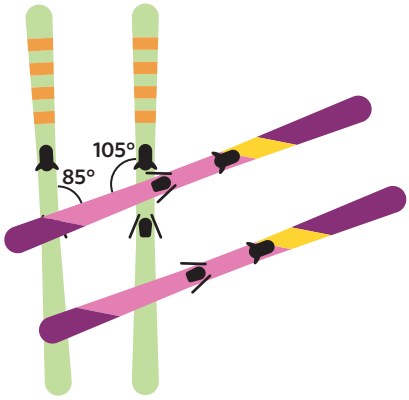
9. A set of parking bays is made up of parallel lines, a transversal line, and a parking spot number. What pair of angles are formed if an angle is measured at the number 2 and number 6?



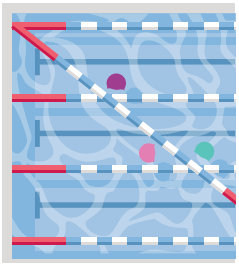
10. The following stair bannister is built with multiple vertical supports and one diagonal support. Based on the marked angles, state whether the vertical supports are parallel or not. Give a reason for your answer.



11. Lisa is an avid skier and has a collection of different skis. She tries to store each pair of her skis parallel to one another. Is the orange pair stored parallel or not in the following image? Give a reason for your answer.



12. The lane ropes in a standard swimming pool are parallel. An additional lane rope is connected diagonally across a pool. If the angle formed by the lane ropes at the floating purple cap is 137°, what is the size of the angles formed by the lane ropes at the green and pink caps? Give a reason for your answer.

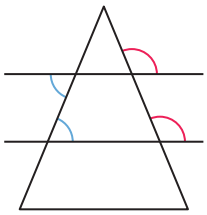


Reasoning

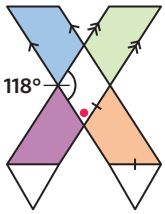
Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Stefanos is a graphic designer. One of his clients has requested a new logo. Stefanos uses a graphic design software that can mark the angles between intersecting straight lines. Stefanos took the following screenshot on his computer of each design option.

Design 1



Design 2



a. Stefanos has marked two blue angle arcs in the incomplete design 1. Classify this pair of angles as corresponding, alternate or co-interior angles.

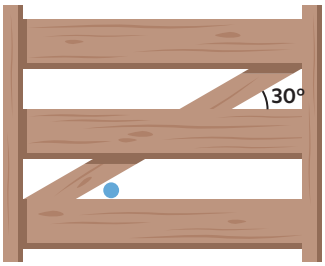
b. If all the horizontal lines in design 1 are parallel and one of the angles marked with a red angle arc is 110°, what is the size of the other angle marked with a red arc? Give a reason for your answer.

c. The parallel lines in design 2 are indicated and the angle measuring 118° is marked. What is the size of the angle marked with a red dot? Give a reason for your answer.

d. Assuming that the orange quadrilateral in design 2 is a trapezium with two marked sides of equal length, what is the size of all four interior angles of the trapezium?

e. Stefanos is not satisfied with both designs. Draw another business logo design that contains two sets of parallel lines.

14. Phillipa is a farmer and is in the process of building wooden gates for her entire farm. She is using a standard wooden gate that is made up of six pieces of wood. The following is a replica of Phillipa's farm gates including one marked angle.



a. Assuming that the horizontal pieces are parallel to each other, what is the size of the angle marked by the blue dot? Give a reason for your answer.

b. Phillipa says that the horizontal and vertical pieces form right angles. How many interior right angles are formed by the gate's pillars?

c. Phillipa believes that all of the gate's interior angles are multiples of 30°. List the multiples that Phillipa has found.

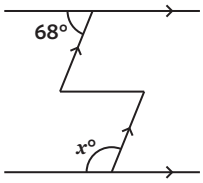
d. Phillipa needs more gates at her farm. Other than the design, what other factors might she need to consider before buying more gates?

Extra spicy

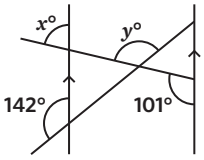
15. A pair of parallel lines is crossed by a transversal line.
Which statement(s) are true?

A. The co-interior angles sum to 180°.
B. Corresponding angles are equal.
C. Vertically opposite angles are equal.
D. Alternate angles are equal.
E. Eight angles are formed.

16. What is the value of x?



17. What is the value of x and y?

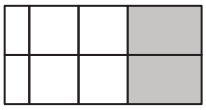
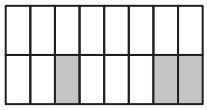
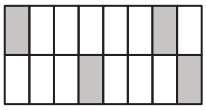
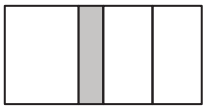


18. A pair of parallel lines is crossed by another pair of parallel lines.
Which statement(s) are false?

A. There are two different sized angles formed.
B. There are four different sized angles formed.
C. Each pair of co-interior angles sum to 180°.
D. There are two intersection points in the diagram.
E. There are four intersection points in the diagram.

Remember this?

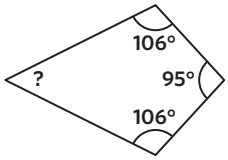
19. Which rectangle has a quarter shaded?

A. 
B. 
C. 
D. 

20. A 50 cent coin weighs 7.80 grams.
How much money does Tilda have if she has 249.6 grams of 50 cent coins?

A. $8.00 **B.** $12.00 **C.** $16.00 **D.** $32.00

21. Select the value of the ?.



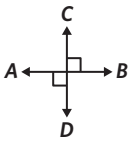
A. 27° **B.** 37° **C.** 53° **D.** 95°

Skill builder

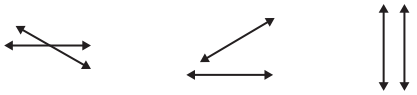
This skill builder task focuses on constructing perpendicular and parallel lines using only a compass and ruler, without measuring any angles or lengths. A compass is an instrument that can be used to accurately draw circles and arcs, and a ruler is a tool used to draw straight lines.

Key ideas

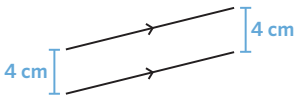
1 Perpendicular lines are lines that meet at a right angle. The symbol ⊥ means that two lines are perpendicular. For example, $AB \perp CD$ and $EF \perp GH$ are shown below.



Below are some examples of non-perpendicular lines.



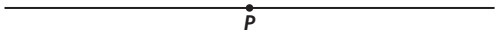
2 Parallel lines are lines that will never meet, and are the same distance from each other at every point.



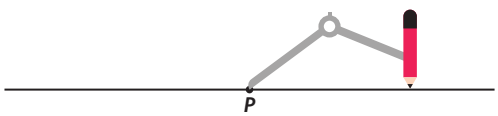
Using a compass

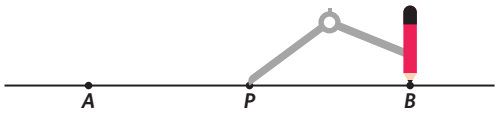
Constructing perpendicular lines

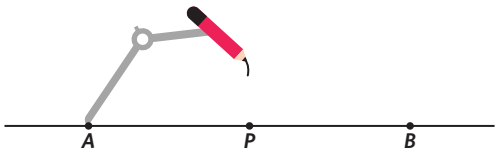
Draw a perpendicular line from point *P*.

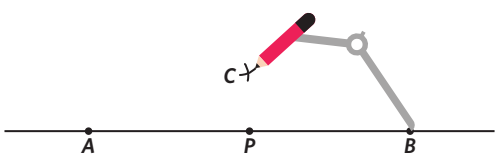


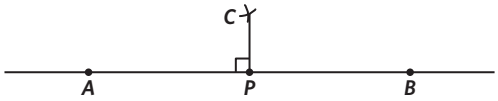
Working











Thinking

Step 1: Place the compass on point *P* and open its width so that it is close to the end of the line.

Step 2: Keeping the compass' width the same, mark a point an equal distance on either side of *P*. Label the points A and B.

Step 3: Make the compass's width larger, place the compass on point *A*, and mark an arc above point *P*.

Step 4: Keeping the same compass width from step 3, place the compass on point *B* and mark an arc above point *P* so that the two arcs cross. Label the point *C*.

Step 5: Using a ruler, draw a straight line from point *P* to where the arcs cross at *C*. Mark a right angle at point *P*.

Student practice

Draw a perpendicular line from point A.

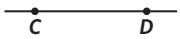
a.

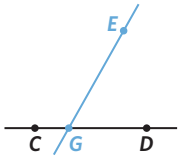
b.

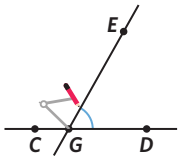
Constructing parallel lines

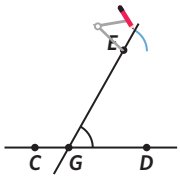
Construct a pair of parallel lines and label them $\overline{CD}$ and $\overline{EF}$.

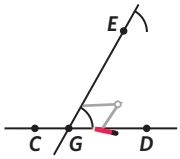
Working

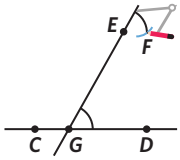


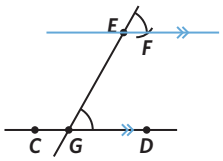












Thinking

Step 1: Draw a line and label points on the line C and D.

Step 2: Draw a line that crosses line $\overline{CD}$ with a ruler. This will be the transversal. Label the intersection G and a point on the transversal E.

Step 3: Place the compass on point G and draw an arc that passes through both lines.

Step 4: Keeping the same compass width as in step 3, place the compass on point E and mark an arc that crosses the line $\overline{EG}$ above point E.

Step 5: Adjust the compass's width so that it is the same as the lower arc from line to line.

Step 6: Place the compass above point E where the arc crosses the transversal. Draw an arc and label the intersection of the upper arcs point F.

Step 7: Draw a straight line with a ruler from point E to where the arcs cross at point F. Label $\overline{CD}$ and $\overline{EF}$ with arrows to show they are parallel.

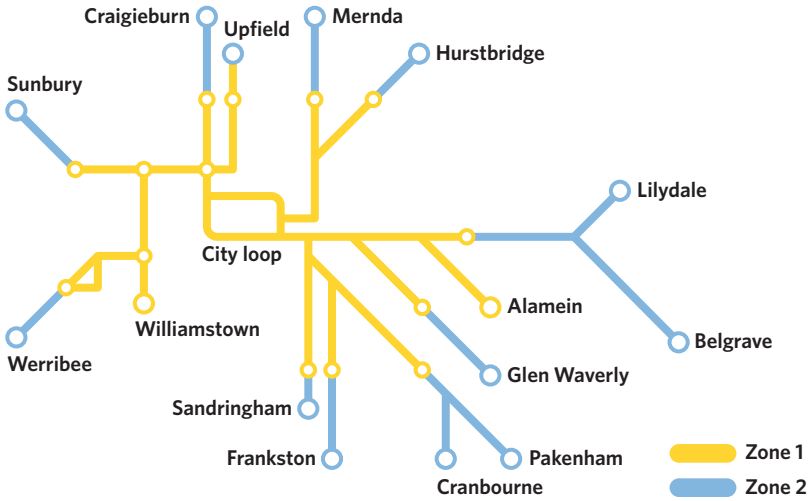
Student practice

a. Construct a pair of horizontal parallel lines and label them $\overline{AZ}$ and $\overline{PF}$.

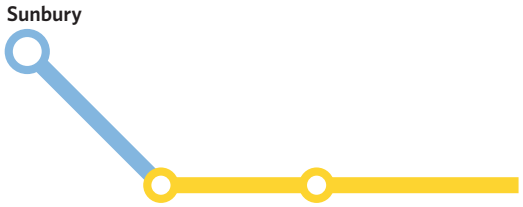
b. Construct a pair of vertical parallel lines and label them $\overline{MK}$ and $\overline{TY}$.

Chapter 6 extended application

1. The following image shows the metropolitan train lines in Melbourne and which zones they operate in.



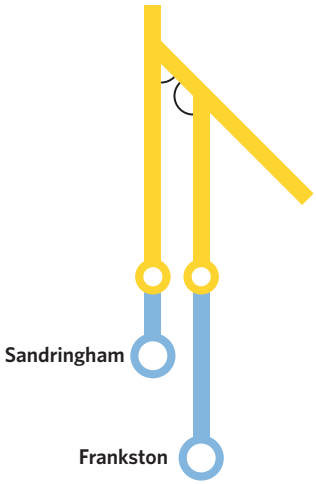
a. The Sunbury line includes zone 1 and zone 2 sections. The lesser angle at the vertex of zone 1 and zone 2 is 135°. Label the described angle on the train line map using an angle arc and state what type of angle this is.



b. Assume the triangle in the Werribee line has angles of 45°, 90° and 45°. Explain why this can be classified as both an isosceles triangle and a right-angled triangle.

c. At the point where the Cranbourne and Pakenham lines split off, the acute angle is 45°. State the size of the supplementary angle.

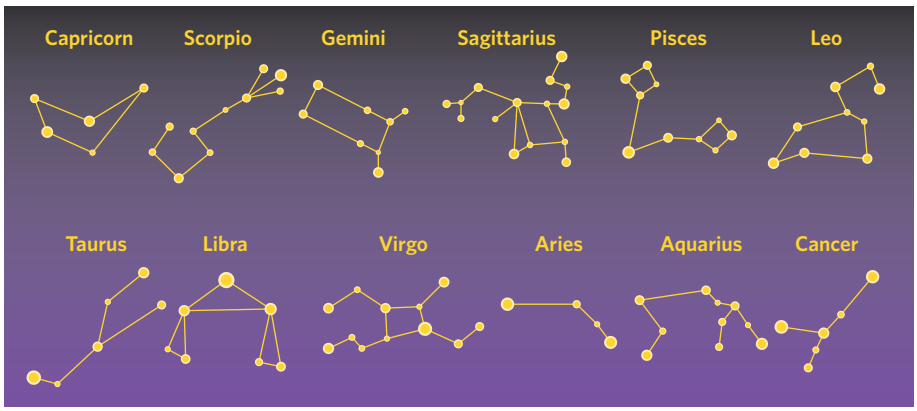
d. The Frankston line starts at the bottom of the map and joins the Pakenham line, and later the Sandringham line.



By looking at the map it is impossible to determine whether the two lines are parallel. How can you prove that the two lines are parallel?

e. Ella usually takes the train to the city for university. Due to level crossing construction work, there are bus replacements for a section of Ella's usual train line. What might Ella need to do differently, in order to get to university each day?

2. The following image shows the stars that form the twelve constellations of the Western zodiac.



a. How many line segments are in the Aries constellation? Assume that each star represents a fixed end point.

b. Assuming that there are exactly two right angles in the enclosed shape from the Gemini constellation, what type of quadrilateral would it be?

c. In the Libra constellation, one of the triangles has angles of 20° and 100°. Calculate the size of the third angle.

d. There is a pattern to the number of degrees that co-interior angles sum to. Identify the pattern and fill in the following table.

Number of sides	3	4	5	6	7	8
Sum of interior angles		360°	540°	720°		1080°

e. The golden ratio is when one object is a certain size or distance away from a second object. An example of this is the following image of a plant, where the golden ratio can be seen in the way that the leaves grow, and is said to make it more aesthetically pleasing (nice to look at).

Image: Africa Studio/Shutterstock.com

In geometry, the golden angle is found when two rays or line segments are at an angle of 137.5°. If golden angles make lines more aesthetically pleasing, determine whether Pisces or Leo is the better looking constellation.

f. Society is divided in their views on astrology and horoscopes, which is the use of a person's birth date to determine their character traits and provide predictions of life events. What is one argument for or against the use of astrology and horoscopes?

3. Daniel and Joyce buy the following four-bedroom house for themselves and their three children.

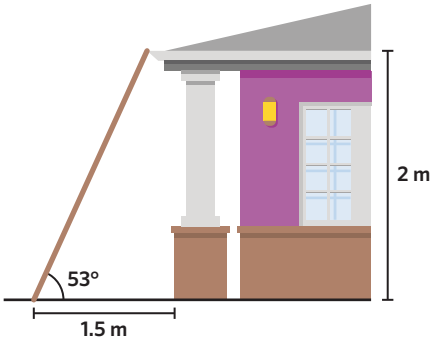
Image: Justin_Krug/Shutterstock.com

a. The house is made up of triangles and rectangles. Are there more triangles or quadrilaterals?

b. Squares and rectangles are two types of quadrilaterals that are part of the house's design. Explain why squares and rectangles are classified as quadrilaterals.

c. Daniel needs to use a ladder to clean the gutter of leaves in autumn. For safety purposes, the ladder must form a 60° angle to the ground, at a maximum. If the ladder is 60° to the ground, what is the size of the complementary angle?

d. Daniel decides that he wants to place the ladder 1.5 m away from the wall of the house, so that the angle is 53° and safer than if the ladder were more steep.



Using a formula called Pythagoras' Theorem, Daniel can calculate the length of the ladder that is needed.

One version of the formula for Pythagoras' Theorem is $c = \sqrt{a^2 + b^2}$. If a represents the height of the house, b represents the distance from the ladder to the house and c represents the length of the ladder, use substitution to calculate the length of the ladder.

e. Some houses have flat roofs.

Image: Traveller70/Shutterstock.com

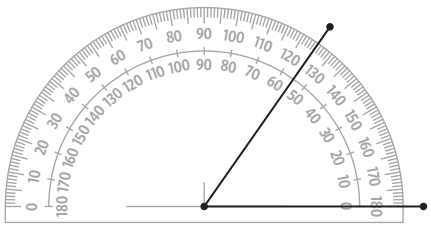
Why might an architect design a house with a flat roof instead of a sloped roof?

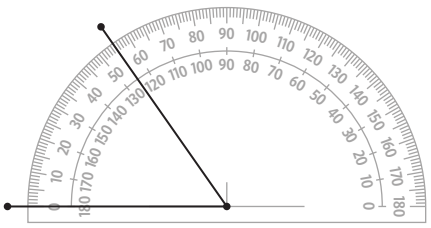
f. Design the outside of your dream home. Think about whether you would want flat or sloped rooves, the number of windows you might want and the shapes that you can use. Draw the view from the front.

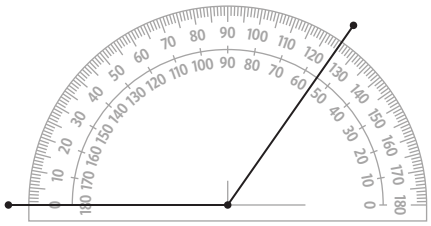
Chapter 6 review

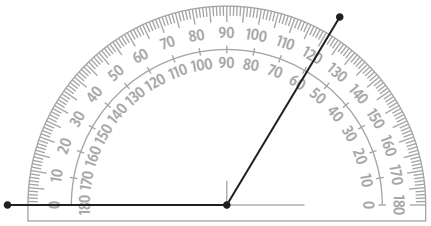
Multiple choice

1. The following protractors measure the given angles. Which angle has a size of 125°? 6A

A. 

B. 

C. 

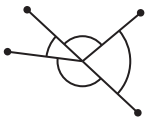
D. 

2. Which of the following pairs of angles is supplementary? 6B

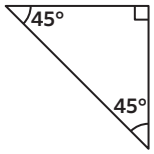
A. 

B. 

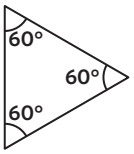
C. 

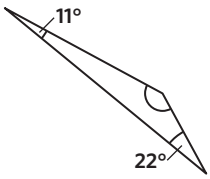
D. 

3. Which of the following triangles is a scalene triangle? 6C

A. 

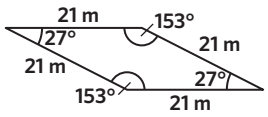
B. 

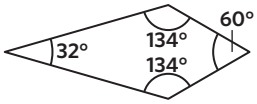
C. 

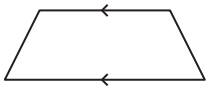
D. 

4. Which of the following quadrilaterals is a rhombus? 6D

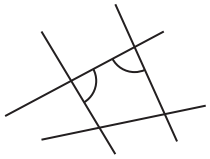
A. 

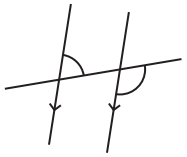
B. 

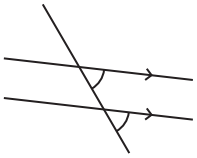
C. 

D. 

5. Which of the following pairs of marked angles are equal? 6E

A. 

B. 

C. 

D. 

Fluency

6. Use a protractor to draw each angle and label using the appropriate letter notation. 6A

a. $\angle AOD = 60°$ b. $\angle KLM = 105°$ c. $\angle CGB = 350°$ d. $\angle XYZ = 285°$

7. Measure each angle and state whether it is an acute, obtuse or reflex angle, or a straight line. 6A

a. 

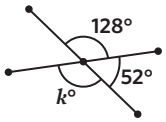
b. 

c. 

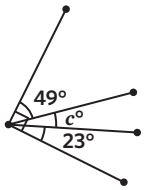
d. 

8. Determine the value of each pronumeral. 6B

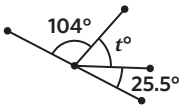
a.



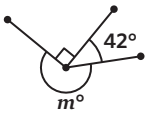
b.



c.

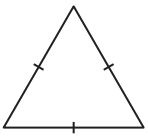


d.

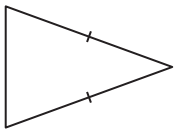


9. Classify each triangle as an equilateral, isosceles, scalene, right-angled, acute angle, or obtuse angle triangle. 6C

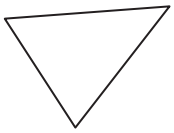
a.



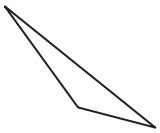
b.



c.

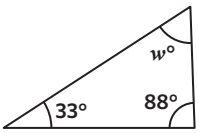


d.

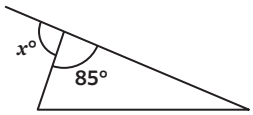


10. Solve to find each pronumeral. 6C

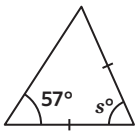
a.



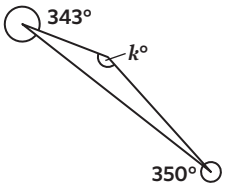
b.



c.

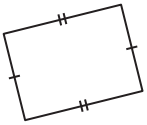


d.

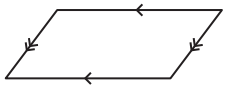


11. Classify each quadrilateral as a square, rectangle, parallelogram, rhombus, kite or trapezium. 6D

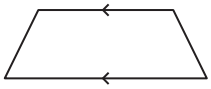
a.



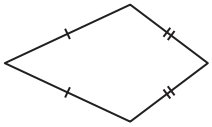
b.



c.

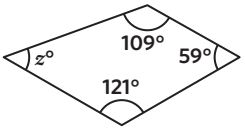


d.

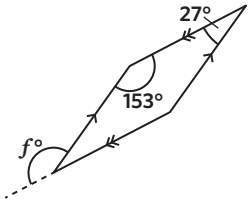


12. Solve to find each pronumeral. 6D

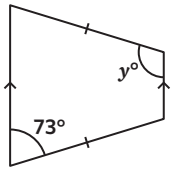
a.



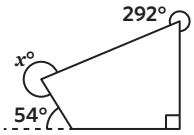
b.



c.

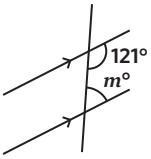


d.

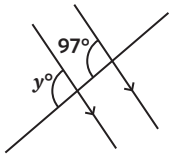


13. Solve for the pronumeral in each set of parallel and transversal lines. Give a reason for your answer. 6E

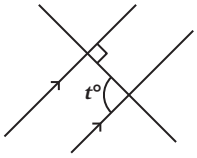
a.



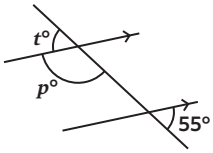
b.



c.

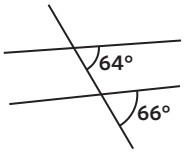


d.

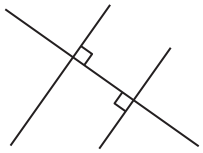


14. Identify whether each pair of lines that are crossed by a transversal are parallel or not. Give a reason for your answer. 6E

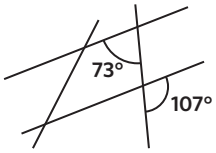
a.



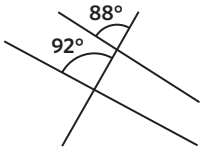
b.



c.



d.



Problem solving

15. Dan is a pizza lover. He buys one third of a margherita pizza and cuts it into two identical slices. What is the angle measurement formed by each slice of pizza? 6A

16. When Joey's classroom door is fully open it forms a straight line with the wall. If Joey opens the door of the classroom by 95°, how many degrees further does Joey need to open the door if she wants the door to be fully opened? 6B

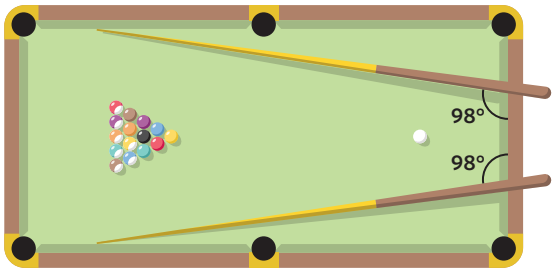
17. Alice buys a loaf of sliced bread and uses two slices to make a sandwich. Assuming each slice of bread is in the shape of a square, what type of triangle is formed if Alice cuts her sandwich diagonally in half? 6C

18. Qin's suitcase is shaped as a trapezium with two sides of equal length and two opposite parallel sides. One of the interior angles of the trapezium is measured to be 78°. What are the measurements of the other three interior angles? 6D

19. Freddy is investigating the pairs of angles formed by some capital letters. How many pairs of alternate angles does the letter H have? 6E

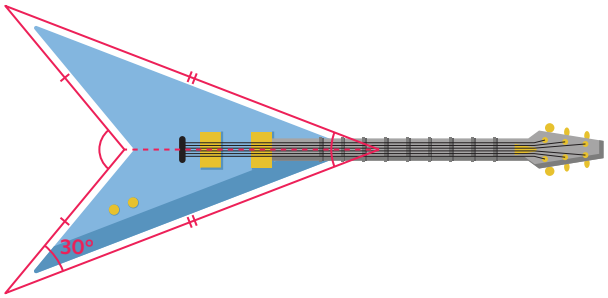
Reasoning

20. A standard snooker table is rectangular. Donni is an avid snooker player and often practices using two different cues. The following image shows Donni's set up before she starts a training session.



a. Four angles are formed at the four corners of the rectangular table. What type of angles are they?
b. The balls are contained by a triangular rack. What type of triangle is the rack?
c. What is the size of the triangular rack's interior angles?
d. Donni's two cues are overhanging one side of the table and form the two marked angles. Are the two cues in parallel positions? Give a reason for your answer.
e. Donni finds it difficult to get the balls into holes. How might Donni be able to improve her snooker skills?

21. Noelle is a guitar player. Her guitar is shaped as a quadrilateral. Some of the side lengths and angle measurements are labeled in the following image.



a. What is the sum of the interior angles formed by the red shape?
b. This quadrilateral shape of the guitar is formed by two identical triangles. What type of triangle are they? Give a reason for your answer.

   Note: There are two possible triangle classifications.
c. If the marked interior angle at the strings of the guitar is 55°, what is the measurement of the marked exterior angle at the strings?
d. Design a guitar with a different quadrilateral shape that is formed by two identical triangles.

07

Statistics and probability

Statistics and Probability

Research summary

7A Collecting and classifying data

7B Summary statistics

7C Dot plots and stem-and-leaf plots

7D Column graphs and pie charts

7E Line graphs

7F Introduction to probability

7G Probability of events *(Extension)*

Chapter 7 extended application

Chapter 7 review

Image: Marish/Shutterstock.com

Research summary – Statistics and probability

Big ideas – Statistics

Classification of data

The classification of data involves making decisions about the information that has been collected. There are two types of data, categorical and numerical, and each one can be classified and represented in different ways for interpretation and analysis.

Numerical data, also known as quantitative data, is data that can be measured or counted. Numerical data can be analysed using summary statistics (mean, median, range and mode). Numerical data is classified as being either **discrete** (limited values between a range are acceptable) or **continuous** (all values between a range are acceptable). A good rule of thumb is that discrete data is counted whilst continuous data is measured.

Categorical data, also known as qualitative data, is data that is collected and grouped as names or labels. Categorical data can only be analysed using the summary statistic mode, it is not possible to calculate the mean, median or range.

Sources of data

Data collection plays an important role in deciding on the accuracy and validity of the statistics we are recording. **Primary data** is collected directly by the researcher and publisher of the statistics we are reading. **Secondary data** is when we compile statistics from data collected or published by others.

Choosing appropriate representation

When representing a set of data as a visual display, there are various options that can be used depending on which one best suits the type of data collected. In this chapter we will focus on the visual representations shown below. **Numerical data** is represented using frequency tables, stem-and-leaf plots, dot plots, line graphs, and histograms. **Categorical data** is represented using frequency tables, column or bar charts, dot plots, and pie charts.

Visual representations – Statistics

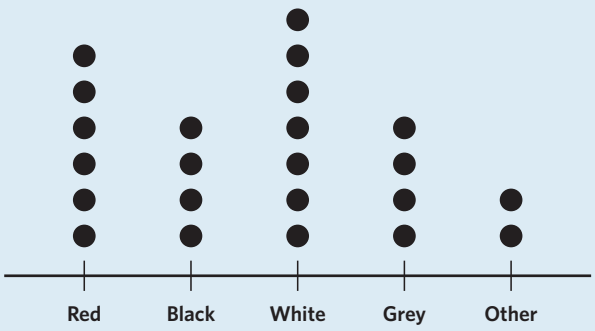
Frequency tables

A frequency table is used to summarise the tally of unique data in a data set. Some statisticians analyse very large sets of data. Frequency tables are a useful way to group these large data sets for easier analysis.

Colour	Tally	Frequency
Red	IIII	4
Green	𝍸 II	7
Orange	III	3
Blue	𝍸 IIII	9
Yellow	II	2
Total		25

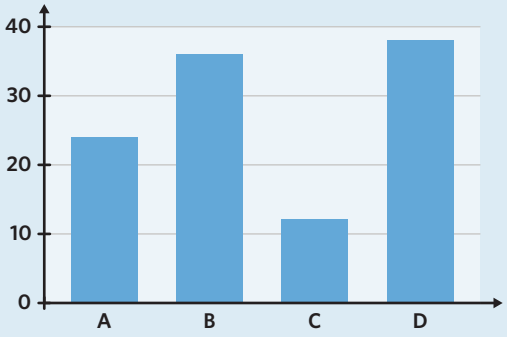
Dot plots

A dot plot is used to display a visual frequency by stacking equally sized and spaced dots to represent the frequency of each unique data in a data set. By keeping the dots equally sized and spaced, it is easy for us to determine the mode of the data by visual inspection of the tallest stack.



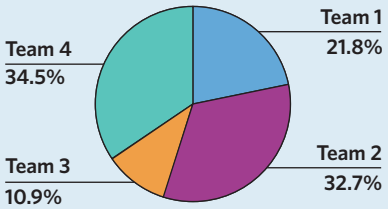
Column graphs

A column graph, similar to a dot plot, is used to display a visual frequency of unique data in a data set. In contrast to a dot plot, individual data points are not shown. Instead, all of the data from the same category is grouped together in one visual bar and the height of the bar corresponds with the data's frequency.



Pie charts

A pie chart is used to show the frequency of unique data in the form of a percentage.



Stem-and-leaf plots

A stem-and-leaf plot is used in a similar way to dot plots and column graphs. A stem-and-leaf plot allows visualisation of data in horizontal stacks, but also retains the integrity of each individual data value for future analysis. A key is always included with a stem-and-leaf plot.

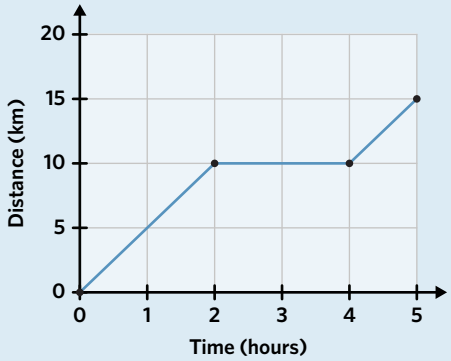
Stem	Leaf
0	8 8 9
1	0 1 1 2 5 7 9
2	4 4 5 7
3	0 4 5
4	1

Key

1 | 5 = 15

Line graphs

A line graph is useful when there are two continuous numerical variables and in particular, in showing change over time. A line graph allows for estimations and predictions, due to the fact that both data sets are continuous and numerical.



Big ideas – Probability

Theoretical probability

Theoretical probability is the hypothetical chance of something occurring based on the number of favourable outcomes compared to the number of total possible outcomes.

Experimental probability

Experimental probability is determined from the actual, real-life outcomes of multiple experiments (trials). The theoretical and experimental probability of the same event can sometimes be very different, but will draw closer together as the number of trials is increased.

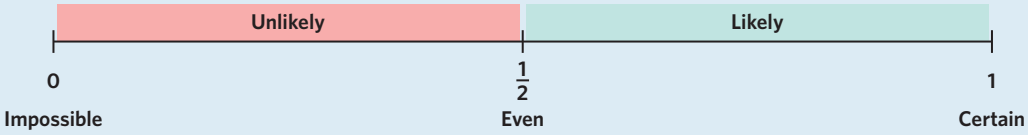
Fairness

In probability, an experiment is considered fair if all outcomes of the experiment are equally likely. For example, if we are talking about a 'fair' die, we are referring to the equal likelihood of each of the possible six outcomes. For a die to be considered 'unfair', it would need to be tampered with in some way that would increase the likelihood of some outcomes over others.

Visual representations – Probability

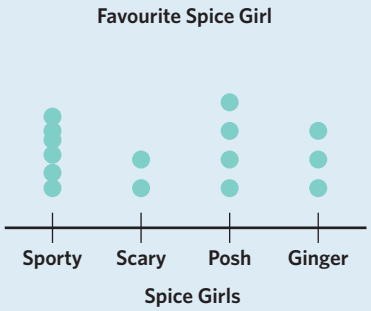
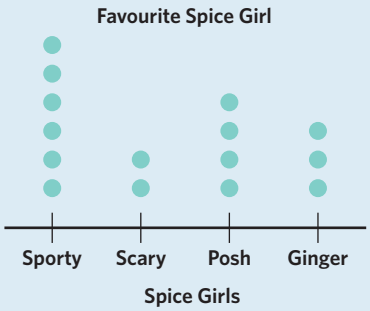
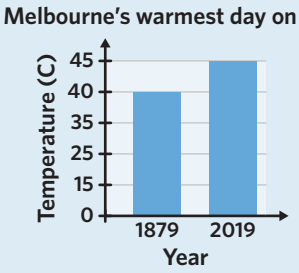
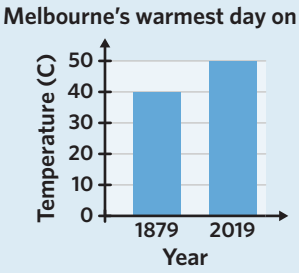
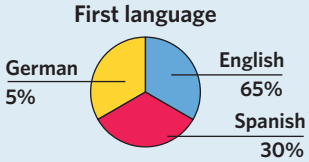
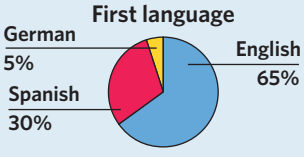
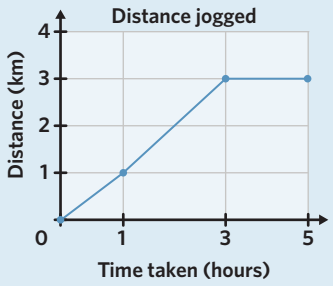
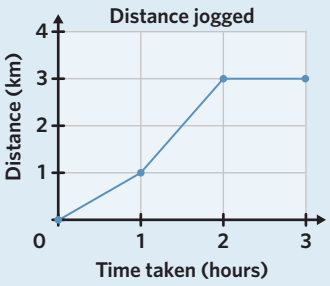
Number lines

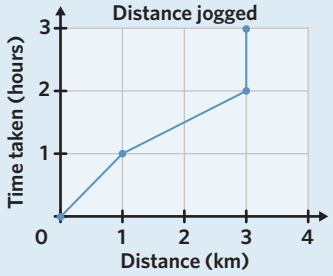
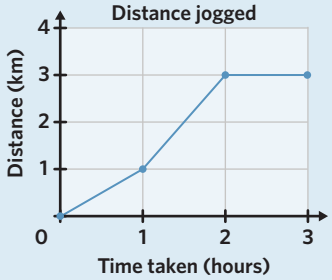
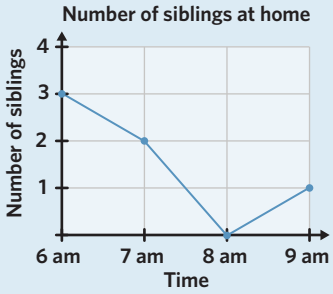
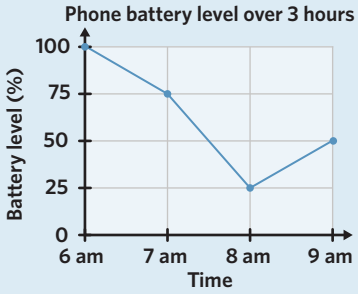
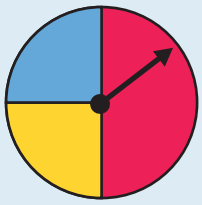
We can use a number line that ranges from 0 to 1 to show the probability of an event occurring. Placing an event at 0 means that this event is impossible. Placing an event at 1 means that this event is certain to occur. Other events can then be placed along the number line to show varying levels of probability.



Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students incorrectly classify all data that contains numbers as numerical data.	The heights of students in a class in cm (140–149, 150–159, 160+) is a numerical variable.	The heights of students in a class in cm (140–149, 150–159, 160+) have been categorised into 3 different groups, so it is a categorical variable.	7A
Students make generalisations from a sample, including biased samples.	The school captain wants to understand student opinion on school uniforms and collect data from 10 close friends who are all in year 12.	To understand the opinion of students in the school, a larger sample should be used and the sample should include a selection from different groups in terms of age, gender, and background.	7A
Students believe that primary and secondary sources are equally reliable.	A friend of a friend said that crime has gotten worse over time in Victoria, so it must be true.	Data from the Victoria Police website shows that crime rates have decreased over time.	7A
Students find the median of a data set without first ordering it.	Raw data: 5, 3, 8, 6, 1 *Median* $= 8$	Ordered data: 1, 3, 5, 6, 8 *Median* $= 5$	7B
Students apply statistical averages to categorical data.	*Mean* $= \frac{5+8+6+3+8}{5} = 6$ *Median* $= 6$ *Mode* $= 8$ **Favourite ice-cream flavours** / **Frequency** Vanilla: 5 Chocolate: 8 Strawberry: 6 Mint: 3 Bubblegum: 8	We cannot work out the mean or median for categorical data. The mean and median of Vanilla, Chocolate, Strawberry, Mint and Bubblegum is not 6. The mode is Chocolate and Bubblegum.	7B

	Incorrect	Correct	Exercise
Students do not order the leaves in a stem-and-leaf plot and incorrectly find the median from the unordered leaves.	**Stem** | **Leaf**: 1 | 4 7 6; 2 | 8 5 9; 3 | 5 1 2. **Key**: 1 | 2 = 12. The median is 25.	**Stem** | **Leaf**: 1 | 4 6 7; 2 | 5 8 9; 3 | 1 2 5. **Key**: 1 | 2 = 12. The median is 28.	7C
Students do not consider the stem when writing a list of data from a stem-and-leaf plot.	**Stem** | **Leaf**: 1 | 2 3 9; 2 | 1 6 3. **Key**: 1 | 7 = 17. The data set is: 2, 3, 9, 1, 6, 3.	**Stem** | **Leaf**: 1 | 2 3 9; 2 | 1 6 3. **Key**: 1 | 7 = 17. The data set is: 12, 13, 19, 21, 26, 23.	7C
Students use inconsistent spacing when constructing dot plots, which leads to incorrect interpretations of a set of data.	 Posh is the most popular.	 Sporty is the most popular.	7C
Students misrepresent a data set by inconsistently labelling the scale on an axis.	The vertical axis does not have a consistent scale, having spacings of 15, 10, and 5. 	The vertical axis has a consistent scale, with a spacing of 10 between each value. 	7D
Students split a pie chart into equal sized parts depending on the number of categories.	The sectors are equal in size indicating that the results are the same for each response. 	The size of each sector in the pie chart represents the percentage of each response. 	7D
Students think that the scale on the two axes do not need to be evenly spaced or to have a consistent incremental value.			7E

	Incorrect	Correct	Exercise
Students place the variables on the two axes in any order.	The graph has a time-based variable on the vertical axis. 	The graph has a time-based variable on the horizontal axis. 	7E
Students use line graphs for variables that are not continuous numerical variables.	One axis has a discrete numerical variable. 	Both axes have a continuous numerical variable. 	7E
When considering spinners, probability is determined by the number of sections rather than the size of angles or value in each section.	The probability of spinning a red is $\frac{1}{3}$ as there are 3 sections. 	The probability of spinning a red is $\frac{1}{2}$ as red takes up $\frac{1}{2}$ of the spinner. 	7F
Students believe earlier outcomes always affect later outcomes.	If three heads are tossed in a row, we are more likely to get tails on the next toss.	Tossing heads is always equally as likely as tossing tails.	7G
Students calculate theoretical probabilities based on the ratio of outcomes rather than the fraction of total outcomes.	In a bag of 3 red and 5 blue marbles: $Pr(red) = \frac{3}{5}$	In a bag of 3 red and 5 blue marbles: $Pr(red) = \frac{3}{8}$	7G
Students do not understand the difference between theoretical probability and experimental probability.	A coin is tossed 10 times and lands on heads 3 times. *theoretical probability* $= \frac{1}{2}$ *experimental probability* $= \frac{1}{2}$	A coin is tossed 10 times and lands on heads 3 times. *theoretical probability* $= \frac{1}{2}$ *experimental probability* $= \frac{3}{10}$	7G

7A Collecting and classifying data

Data can be classified as either discrete numerical, continuous numerical, or categorical. We collect data from populations using either census or sampling. Data can come from a primary or secondary source. Below are some examples where collecting and classifying data can be applied.

- I want to find out the number of people in my class who support the same football team as me. Is the data I collect discrete numerical, continuous numerical or categorical data?
- All the students in my school vote to elect a school captain. Has the result been collected using census or sampling?
- I use a Wikipedia page to get information for my project on volcanoes. Is this page a primary or secondary source?

Learning intentions

Students will be able to:

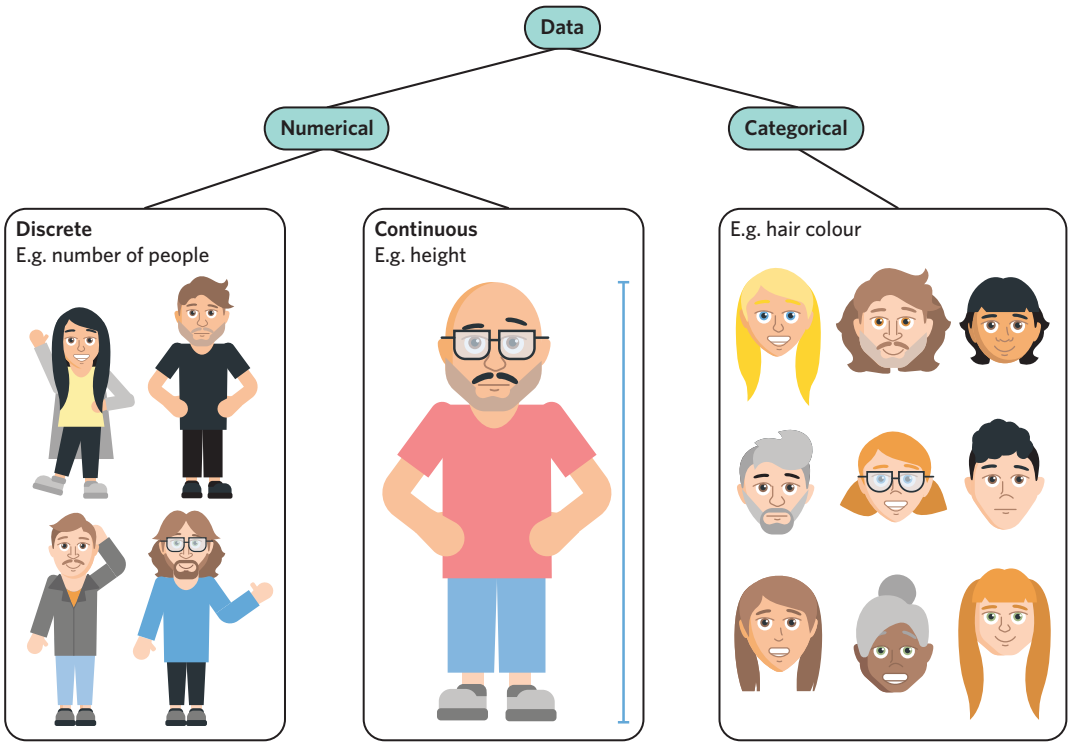
+ distinguish between discrete numerical, continuous numerical and categorical data
+ understand the difference between primary and secondary data
+ understand methods of collecting data using census and sampling
+ display data sets in frequency tables.

Key terms and definitions

- **Numerical data** is quantitative data that can either be discrete or continuous.
- **Discrete data** is counted and generally uses whole numbers. E.g. number of people.
- **Continuous data** is measured, often with decimals or fractions and a unit of measurement such as centimetres, degrees Celsius, litres, and dollars. E.g. height of a person.
- **Categorical data** is descriptive information and separated into groups. E.g. hair colour.
- A **population** refers to the entire group that the conclusions of the study are applied to as opposed to part of the group.
- A **sample**, or **sampling**, refers to the collection of data from a small group that is representative of the population.
- A **census** is when data is collected on the whole population.
- A **primary source** is a firsthand account of a specific event collected by the individual or group using the data.
- A **secondary source** uses and interprets primary and other secondary sources to present information instead of collecting the data firsthand.
- A **frequency table** lists categorical data and shows the number of times each category occurs.

Key ideas

1 Data sets can either be numerical or categorical. Numerical data can be classified as discrete or continuous data.



2 Primary data that is collected ourselves is generally a more reliable source than secondary data that others have collected.

Primary source

Image: AlexandrMusuc/Shutterstock.com

More reliable because we collect it ourselves.

E.g. Go to the garden and write down the number of plants and their names.

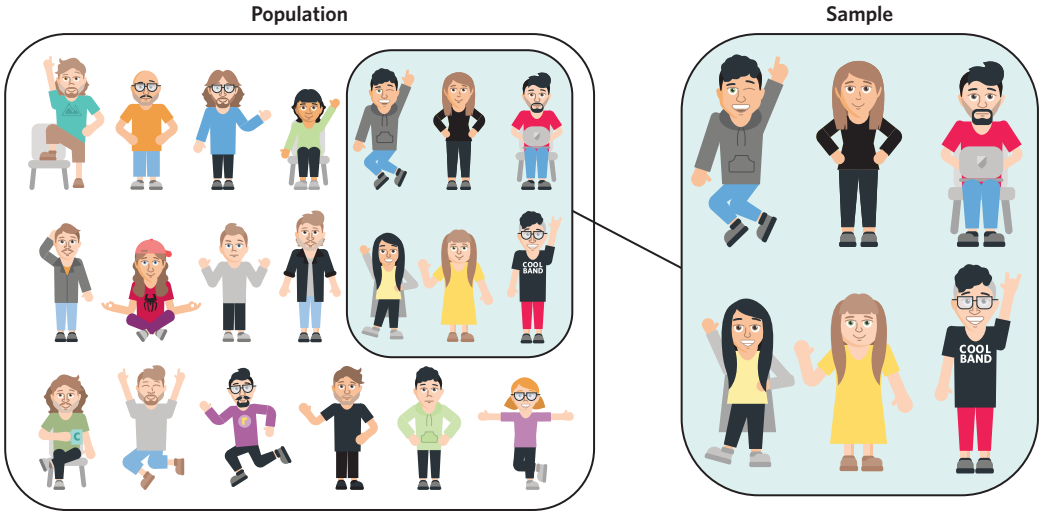
Secondary source

Image: Caftor/Shutterstock.com

Less reliable because it has passed through one or more others.

E.g. Ask classmates how many plants they have in their gardens and what their names are.

3 The more data that is collected, the more reliable it is. Even though population data is more reliable than sample data, it is sometimes more appropriate to use sample data.



Key strengths and weaknesses of population and sample data		
	Strengths	**Weaknesses**
Population	• Reliable • Represents entire group	• Time consuming • Costly • Impractical with large populations
Sample	• Faster • Cheaper • Better for large populations	• Less reliable • May not represent entire group

Worked examples

WE 1 Distinguishing between continuous numerical, discrete numerical and categorical data

Classify each variable as continuous numerical, discrete numerical or categorical.

a. Number of sheep

Working

Number of sheep is numerical.

Number of sheep is discrete numerical.

Thinking

Step 1: Determine whether the variable is numerical or categorical.

Step 2: Determine whether the variable is discrete numerical or continuous numerical. The variable is counted.

b. Favourite movie

Working

Favourite movie is categorical.

Thinking

Determine whether the variable is numerical or categorical. The variable is descriptive.

Student practice

Classify each variable as continuous numerical, discrete numerical or categorical.

a. Number of siblings

b. Favourite console

c. Time taken to complete a maths test

d. Laptop brand

WE 2 Distinguishing between primary and secondary sources

Classify each source as a primary or secondary source of data.

a. A school student collecting data on the heights of their family members for a school project.

Working

The data is a primary source.

Thinking

The data is collected firsthand and presented by the student.

b. A journalist ranking AFL players based on their previous season statistics.

Working

The data is a secondary source.

Thinking

The journalist is using and interpreting existing data that they have not collected.

Student practice

Classify each source of data as primary or secondary.

a. A news article based on the results of the recent Australian census.

b. A clothing company sharing data on their most popular products.

c. Footage of a volcano eruption.

d. A movie review.

WE 3 Distinguishing between census and sampling

Determine whether census or sampling is used in each data collection.

a. Eddie plans to get all the 20–30 year olds in Victoria to fill out a form to determine the average income in this age bracket.

Working

Eddie is using a census.

Thinking

The population is all 20–30 year olds in Victoria.

Eddie plans to collect data on the whole population.

b. A teacher times his year 7 students to determine how long it takes year 7s to complete a test.

Working

The teacher is using sampling.

Thinking

The population is all year 7 students.

The teacher collects data on the students in his class.

Student practice

Determine whether census or sampling is used in each data collection.

a. Peregrin watches 6 of his neighbours to determine the average time that the people on his street put their bins out.

b. A coach asks every person on a team to vote for who should be captain.

c. A school asks every student to complete a teacher feedback form.

d. Simone collects data on the number of bookings her restaurant has in one week to determine how many bookings she can expect in a year.

WE 4 Constructing frequency tables

Construct a frequency table for each data set.

a. Coin toss results: heads, heads, tails, heads, heads, tails, tails, heads, tails, heads

Working

Coin toss	Tally	Frequency
Heads		
Tails		
Total		

Coin toss	Tally	Frequency
Heads	𝍸 I	6
Tails	IIII	4
Total		

Coin toss	Tally	Frequency
Heads	𝍸 I	6
Tails	IIII	4
Total		**10**

Thinking

Step 1: Set up a table with an outcome column, a 'tally' column and a 'frequency' column. A 'total' row can also be added.

Step 2: Count the number of times each outcome occurs and fill in the 'tally' and 'frequency' columns.

Step 3: Add the numbers in the 'frequency' column to find the total. The total should match the number of pieces of data in the provided data set.

b. Dice roll results: 1, 6, 4, 2, 6, 4, 3, 3, 1, 5, 2, 3, 6, 3, 4, 2, 3, 3, 5, 6

Working

Number	Tally	Frequency
1		
2		
3		
4		
5		
6		
Total		

Number	Tally	Frequency
1	II	2
2	III	3
3	𝍸 I	6
4	III	3
5	II	2
6	IIII	4
Total		

Number	Tally	Frequency
1	II	2
2	III	3
3	𝍸 I	6
4	III	3
5	II	2
6	IIII	4
Total		**20**

Thinking

Step 1: Set up a table with an outcome/variable column, a 'tally' column and a 'frequency' column. A 'total' row can also be added.

Step 2: Count the number of times each outcome occurs and fill in the 'tally' and 'frequency' columns.

Step 3: Add the numbers in the 'frequency' column to find the total. The total should match the number of pieces of data in the provided data set.

Student practice

Construct a frequency table for each data set.

a. Eye colour: blue, blue, green, brown, blue, brown, grey, blue, brown

b. Number of siblings: 2, 1, 0, 3, 2, 2, 3, 1, 1

c. Favourite musical instrument: guitar, piano, violin, piano, cello, guitar, violin, trumpet, violin, guitar, guitar, trumpet, trombone, guitar, violin

d. Pieces of fruit eaten per day: 2, 2, 1, 2, 3, 4, 0, 1, 2, 2, 5, 0

7A Activities and questions

STARTER TASKS

Odd spot

Google collects data from internet users on the type of content they consume, in order to personalise their internet experience. In 2020, the Australian Competition and Consumer Commission (ACCC) took legal action against Google for misleading consumers after Google claimed they had consent to target its users with specific ads.

Image: Uladzik Kryhin/Shutterstock.com

Is the data collected by Google numerical or categorical data?

A. Categorical data

B. Numerical data

Puzzle

The following data sets can be classified as categorical, discrete numerical or continuous numerical.

Weight | 5-star rating | Height | Postcode | Victorian population

Number of siblings | Olympic 100 m freestyle record | Favourite colour | Do you agree? (Yes/No)

a) Using the data sets listed, complete the missing information in the following table.

Categorical	Discrete numerical	Continuous numerical

b) Create your own type of data to complete the missing information in the table from part **a**.

Understanding worksheet

1. Mark the correct classification for each variable.

Example

Variable	Categorical	Discrete numerical	Continuous numerical
Number of siblings	☐	☑	☐

	Variable	Categorical	Discrete numerical	Continuous numerical
a.	Number of pets	☐	☐	☐
b.	Hair colour	☐	☐	☐
c.	Number of students in a year level	☐	☐	☐
d.	Volume of a box	☐	☐	☐

2. Tick whether each source is primary or secondary data.

Example

Source	Primary	Secondary
Weather station rainfall	☑	☐

	Source	Primary	Secondary
a.	A diary entry by a war survivor.	☐	☐
b.	A newspaper article about the grand final.	☐	☐
c.	A project you completed on World War II.	☐	☐
d.	A letter from your friend about school.	☐	☐

3. Fill in the blanks by using the words provided.

primary secondary census population sample

A _____ is the entire group that you want to apply your conclusions to. When a part of the population is selected to represent the group as a whole, this is called a _____. When we collect data from an entire population, this is called a _____. Data that we collect ourselves is said to come from a _____ source and is generally more reliable. Data that uses and interprets other sources of information is said to be a _____ source.

Fluency

Question working paths

Mild 🌶	Medium 🌶🌶	Spicy 🌶🌶🌶
4 (a,b,c), 5 (a,b,c,d), 6 (a,b), 7 (a,b,c), 8 (a,b)	4 (b,c,d), 5 (c,d,e,f), 6 (b,c), 7 (b,c,d), 8 (b,c)	4 (d,e,f), 5 (e,f,g,h), 6 (c,d), 7 (d,e,f), 8 (c,d)

WE 1 **4.** Classify each variable as continuous numerical, discrete numerical or categorical.

a. Favourite food
b. Number of coffees sold
c. School suburb
d. Marathon time
e. Country of birth
f. Number of eggs in a cake

WE 2 **5.** Classify each source as a primary or secondary source of data.

a. A paper from a research group discussing the results from their survey on people's favourite watch brands.
b. A blog analysing statistics from a research paper.
c. Data on the average number of people in each household from the Australian Bureau of Statistics website.
d. A YouTube video discussing the rising price of houses in Sydney.
e. A newspaper article that offers different interpretations of census data.
f. A documentary by a group of individuals that discusses their own findings on experiments done in the North Pole.
g. An opinion piece in which an author presents their own results on an investigation into the popularity of movie genres.
h. A school project on average household water usage using data collected from the Australian Bureau of Statistics website.

WE 3 6. Determine whether census or sampling is used in each data collection.

a. Randomly selecting 100 members of a school to fill out a questionnaire to determine the most popular subject in the school.

b. Using camera footage to count how many cars pass through a red light to determine the average number of cars that pass through a particular intersection.

c. Asking all employees at a company to rate their workplace satisfaction on a scale of 1–100.

d. Recording statistics at every match of a cricket season to determine the leading run-scorers and wicket-takers.

7. State whether a population or a sample is the most appropriate group for an individual to collect data for in each situation.

a. The average time it takes for Australians to make their breakfast.

b. The armspan of all the students in your class.

c. The average arm span of 13 year olds.

d. The number of year 7 students in the school who play musical instruments.

e. The preferred social media platform of all the members in your soccer team.

f. A company seeking feedback on a new clothing line.

WE 4 8. Construct a frequency table for each data set.

a. Favourite Australian holiday destination:

Noosa, Gold Coast, Perth, Gold Coast, Sydney, Noosa, Byron Bay, Gold Coast, Melbourne, Perth, Byron Bay, Byron Bay, Hobart, Gold Coast, Noosa, Melbourne, Sydney, Byron Bay, Broome

b. Number of hours per week spent doing homework:

5, 4, 6, 5, 5, 7, 3, 4, 6, 7, 5, 6, 7, 8, 8, 7, 7, 5, 6, 5, 7, 4, 3, 2

c. Favourite Harry Potter character:

Ron, Hermione, Hermione, Harry, Hagrid, Hermione, Harry, Dumbledore, Ron, Voldemort, Hermione, Ron, Hermione, Harry, Lockhart

d. Number of foreign countries visited:

4, 1, 0, 4, 12, 4, 2, 7, 1, 0, 1, 10, 3, 2, 0, 0, 1, 22

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Graham is completing a music project for school. He records the favourite music genre of every member of his extended family. What type of data did Graham collect?

10. Daphne enjoys fishing and always measures the weights of the fish she catches. What type of data is the weight of the fish, if the fish she caught on Sunday were 2.14 kg, 6.31 kg, and 3.95 kg?

11. Lance has been assigned a project on the history of the ANZACs. Did he collect information from a primary or secondary source, when he interviewed his great grandfather who fought in Gallipoli?

12. A research group is investigating whether libraries are becoming more or less popular in Parramatta. They randomly select 50 Parramatta residents and ask them if they have attended the local library in the last week. What is the population and sample of this experiment?

13. Baxter is looking for a primary source to help with his investigation into the effects of pollution on the Great Barrier Reef. Should he use a research paper from a group that conducted a year-long study on the effects of oil on marine wildlife, or a newspaper opinion article on how to help preserve the ocean?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c,d)	All

14. There are 4 main types of nonrenewable resources: oil, natural gas, coal and nuclear energy. In Australia, the largest energy resource is coal. At the end of 2019, Australia's recoverable coal resources totalled 149 293 million tonnes (Mt) of black and brown coal.

a. Are the resources of coal discrete numerical, continuous numerical or categorical?

b. The following frequency table shows the coal resources of 10 different countries.

Resources (Mt)	Frequency
0–<50 000	5
50 000–<100 000	2
100 000–<150 000	2
150 000–<200 000	0
200 000+	1

Is the data presented in the table discrete numerical, continuous numerical or categorical data?

c. An Australian scientist wants to attain the most accurate information possible on coal. Would they be better to use Australian Bureau of Statistics information, or data they found in an article posted on Facebook? Explain your answer.

d. Should Australia continue using nonrenewable resources like coal, or focus more on using renewable resources such as wind and solar energy? Explain your answer.

15. Donald attends a popular sports school, and wants to find out what the most popular sport is in his school. Donald is not sure how he plans on collecting data.

a. Explain whether a census or sampling will be the most reliable form of data collection in this situation.

b. If Donald decides to survey all the students in his school, is he using census or sampling?

c. State two limitations of using the data collection method identified in part **b**.

d. Donald then decides to ask his 10 closest friends what their favourite sport is. Explain whether this is an accurate representation of the population.

e. Suggest a better way Donald could have generated a sample.

Extra spicy

16. Data relating to the following five variables was collected from birds at the zoo.

- Colour
- Name of species
- Wingspan (in centimetres)
- Body length (in millimetres)
- Body weight (in milligrams)

The number of these variables that are discrete numerical variables is:

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

17. Matt collected data from the following sources to complete his science project on Melbourne weather.

- A friend's Facebook post
- A celebrity Instagram post
- Matt's experiment report
- A newspaper article
- The Bureau of Meteorology website

How many of the sources are secondary?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

18. At Thor's departure from Norway, he recorded the equipment he had on his ship in percentages. Upon arrival in England, he recorded the pieces of equipment he had in a frequency table.

Departure

Equipment	Percentage
Helmets	28%
Axes	16%
Swords	20%
Shields	36%

Arrival

Equipment	Frequency
Helmets	21
Axes	12
Swords	15
Shields	12

Assuming that no other pieces of equipment were lost, how many shields were lost on the journey?

A. 11 **B.** 15 **C.** 24 **D.** 36 **E.** 48

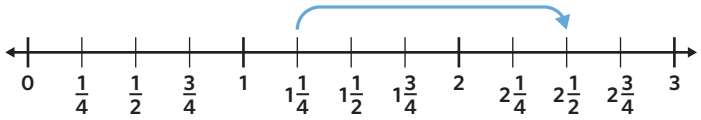
19. Data was collected in a study investigating the relationship between *frequency of nightmares* (low, high) and *snores* (no, yes). Some data is missing from the table.

Frequency of nightmares	***Snores***		**Total**
	No	**Yes**	
low			138
high			31
Total	99	70	169

There are twice as many people who do not snore and have a low frequency of nightmares than people who do snore and have a low frequency of nightmares. How many people from the study snored and had a high frequency of nightmares?

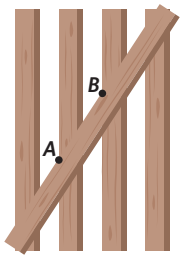
Remember this?

20. What number is added to $1\frac{1}{4}$ to equal $2\frac{1}{2}$?



21. Roy is a warehouse worker at a local paper supply company. Last week they shipped 5000 reams of paper. On 526 of these reams, they accidentally printed a watermark on pages that were supposed to be blank. Write the proportion of pages with a watermark as a decimal.

22. Brenda designs a gate to install at her house, shown in the diagram.



She measures the angle at point *A* and finds that it is 150°. If she knows that the vertical planks are parallel, what is the angle at point *B*?

7B Summary statistics

Data sets can sometimes be very large and this can make it difficult to interpret them. Summary statistics can be used to provide information about a data set. Summary statistics communicate information about a data set in a simple and easy to understand way. We can then use them to make generalisations about a data set, and to compare different data sets. The following are some examples where summary statistics can be used.

- I ran three kilometres. I ran the first kilometre in 3:35 minutes, the second in 4:05 minutes and the third in 4:30 minutes. What was my average pace per kilometre?
- My highest score in the diving competition was 9.87 and my lowest score was 5.23. What was the range of my scores?
- Students in my class wrote down their favourite subject. Which was the most popular subject?

Learning intentions

Students will be able to:

- calculate the mean, median, mode, and range for a set of data
- interpret mean, median, mode and range in the context of data
- use summary statistics to compare data sets.

Key terms and definitions

- A **data set** is a collection of values relating to a particular variable.
- A **measure of centre** gives an idea of what a typical value in the data set would be. The measures of centre are mean, median and mode.
- The **mean**, also known as the average, is calculated by taking the sum of all values in the data set, then dividing by the number of data points in the set.
- The **median** is the middle value when the data is sorted from lowest to highest.
- The **minimum** is the lowest value in the data set.
- The **maximum** is the highest value in the data set.
- The **mode**, also known as the **modal value**, is the value that occurs most frequently.
- The **range** measures the spread of data. It is the difference between the maximum and minimum values.

Key ideas

1 The mean, also known as the average, is the sum of all values divided by the number of data points, and it can be used as a measure of centre to summarise a data set.

Sum of values = 1 + 2 + 2 + 4 + 7 + 11 + 22
= 49

$$1, 2, 2, 4, 7, 11, 22 \longrightarrow \textit{Mean} = \frac{49}{7} = 7$$

Number of values = 7

2 The median is the middle value in an ordered data set, and it can be used as a measure of centre to summarise a data set.

If there is an odd number of values in the data set, the median is the middle value:

1, 2, 2, 4, 7, 11, 15

Median

If there is an even number of values in the data set, the median is the mean of the middle two values:

1, 2, 2, 4, 7, 11, 15, 15

$\textit{Median} = \frac{4+7}{2} = 5.5$

3 The mode is the most frequently occurring value or category in a data set. It can be found for both numerical and categorical data. There can be multiple modes, or sometimes no mode at all.

1, 2, 2, 4, 7, 11, 11, 15

The modes are 2 and 11.

rabbit	cat	fish	horse
dog	dog	dog	cat

The mode is dog.

4 The range is the difference between the maximum and minimum and is used to measure the spread of a data set.

Minimum value Maximum value

1, 2, 2, 4, 7, 11, 15

Range = maximum value − minimum value
Range = 15 − 1 = 14

Worked examples

WE 1 Calculating the mean

Calculate the mean of each data set.

a. 3, 4, 5, 6, 6, 9, 9

Working

$sum\ of\ values = 3 + 4 + 5 + 6 + 6 + 9 + 9$
$= 42$

$number\ of\ values = 7$

$\frac{42}{7} = 6$

The mean is 6.

Thinking

Step 1: Calculate the sum of the values.

Step 2: Count the number of values in the data set.

Step 3: Calculate the mean. The mean is the sum of the values divided by the number of values in the data set.

Visual support

Sum of values = 3 + 4 + 5 + 6 + 6 + 9 + 9
= 42

3, 4, 5, 6, 6, 9, 9 → *Mean* = $\frac{42}{7}$ = 6

Number of values = 7

b. 14, 11, 21, 7, 8, 10, 14, 13, 22, 17

Working

$sum\ of\ values = 14 + 11 + 21 + 7 + 8 + 10 + 14 + 13 + 22 + 17$
$= 137$

$number\ of\ values = 10$

$\frac{137}{10} = 13.7$

The mean is 13.7.

Thinking

Step 1: Calculate the sum of the values.

Step 2: Count the number of values in the data set.

Step 3: Calculate the mean. The mean is the sum of the values divided by the number of values in the data set.

Student practice

Calculate the mean of each data set.

a. 1, 3, 4, 4, 5, 7

b. 10, 4, 13, 9, 6, 15, 6

c. 20, 24, 24, 31, 38, 39, 44, 46

d. 33, 15, 67, 29, 1, 88, 54, 21

WE 2 Determining the median

Determine the median of each data set.

a. 1, 5, 7, 8, 8, 11, 13

Working

There are 7 values so the median is the 4^{th} value.

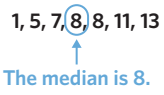
The median is 8.

Thinking

Step 1: Count the number of values in the data set to determine the position of the median.

Step 2: Locate the middle value.

Visual support



b. 21, 8, 13, 15, 2, 9, 18, 11, 7, 26

Working

2, 7, 8, 9, 11, 13, 15, 18, 21, 26

There are 10 values so the median is the mean of the 5^{th} and 6^{th} values.

$$median = \frac{11 + 13}{2}$$
$$= 12$$

The median is 12.

Thinking

Step 1: Order the values from smallest to largest.

Step 2: Count the number of values in the data set to determine the position of the median.

Step 3: Calculate the median.

Student practice

Determine the median of each data set.

a. 3, 9, 14, 15, 18

b. 45, 48, 55, 59, 63, 70

c. 25, 8, 19, 13, 10, 11, 22

d. 101, 68, 403, 180, 29, 555, 330, 212

WE 3 Finding the mode

State the mode(s) of each data set.

a. 13, 20, 20, 23, 26, 28, 34

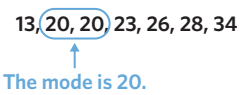
Working

The mode is 20.

Thinking

Identify the most frequently occurring value.
The only value that occurs more than once is 20.

Visual support



b. chocolate, vanilla, honeycomb, lemon, vanilla, honeycomb, strawberry, hazelnut

Working

The modes are vanilla and honeycomb.

Thinking

Identify the most frequently occurring category. The only categories that occur twice are vanilla and honeycomb.

Student practice

State the mode(s) of each data set.

a. 7, 8, 9, 11, 11, 13

b. margherita, capricciosa, hawaiian, meatlovers, hawaiian, margherita, pepperoni, margherita

c. 20, 40, 60, 30, 40, 50, 20, 10, 70

d. car, tram, bus, walk, bus, train, walk, car, walk, bus, train, bus

WE 4 Calculating the range

Calculate the range of each data set.

a. 2, 6, 10, 15, 16, 20, 23, 29

Working

$minimum = 2$

$maximum = 29$

$range = maximum - minimum$

$= 29 - 2$

$= 27$

The range is 27.

Thinking

Step 1: Identify the maximum and minimum values.

Step 2: Calculate the difference between the two values.

Visual support

Minimum value Maximum value

(2), 6, 10, 15, 16, 20, 23, (29)

b. 55, 21, 90, 103, 39, 58, 92, 66

Working

$minimum = 21$

$maximum = 103$

$range = maximum - minimum$

$= 103 - 21$

$= 82$

The range is 82.

Thinking

Step 1: Identify the maximum and minimum values.

Step 2: Calculate the difference between the two values.

Student practice

Calculate the range of each data set.

a. 1, 9, 11, 19, 21, 29

b. 21, 9, 34, 16, 42

c. 75, 25, 150, 35, 125, 85, 225, 45

d. 56, 103, 29, 87, 241, 98, 145, 38, 66

7B Activities and questions

STARTER TASKS

Odd spot

The first Olympic Games was held in Athens, Greece in 1896. Australia has competed in every Summer Olympics since its inauguration.

The number of medals won by Australian athletes in each Summer Olympics since 1988 is shown in the following table.

Year	1988	1992	1996	2000	2004	2008	2012	2016	2021
Number of medals	14	27	41	58	49	46	35	29	46

What is the median number of medals won by the Australian Olympic team since 1988?

A. 41 medals **B.** 49 medals

Image: fifg/Shutterstock.com

Puzzle

The following jumbled up set of numbers make up three different data sets.

4 1 4 2 10 2 5 7 6 11

Data set 1
3 numbers:
median = 5
range = 6

☐, ☐, ☐

Data set 2
3 numbers:
mean = 5
mode = 2

☐, ☐, ☐

Data set 3
4 numbers:
median = 5
mean = 6

☐, ☐, ☐, ☐

a) For data set 1, what missing number may it be best to complete first?

b) For data set 2, which two missing numbers might it be best to complete first?

c) Use the numbers from the purple box and the clues to recreate the three ordered data sets. All the numbers can only be used once.

Understanding worksheet

1. Circle the maximum and minimum of each data set and complete the missing information to calculate the range.

Example

16, 11, 15, (10) 12, (21) 15, 18, 20

Range = *Maximum* − *Minimum*

Range = [21] − [10] = [11]

a. 4, 7, 7, 10, 13, 16, 17

Range = *Maximum* − *Minimum*

Range = 17 − 4 = ☐

b. 15, 19, 25, 26, 38, 41, 47, 51, 53

Range = *Maximum* − *Minimum*

Range = 53 − ☐ = ☐

c. 20, 11, 6, 15, 23, 9

Range = *Maximum* − *Minimum*

Range = ☐ − ☐ = 17

d. 42, 30, 89, 25, 56, 13, 38, 19, 77

Range = *Maximum* − *Minimum*

Range = ☐ − ☐ = ☐

2. Reorder each set of data, then circle the middle value(s) and determine the median.

Example

9, 10, 7, 6

6, 7, 9, 10

$Median = \frac{7 + 9}{2} = 8$

a. 4, 9, 7, 2, 12

2, ☐, 7, 9, 12

$Median =$ ☐

b. 44, 25, 101, 100, 21

21, ☐, 44, 100, ☐

$Median =$ ☐

c. 3, 0, 8, 11, 6, 10,

0, 3, ☐, 8, ☐, 11

$Median = \frac{6 + \square}{2} =$ ☐

d. 24, 15, 73, 24, 101, 58,

15, 24, ☐, 58, ☐, 101

$Median = \frac{\square + \square}{2} =$ ☐

3. Fill in the blanks by using the words provided.

mean | median | mode | range | average

The ☐ shows the spread of data. The ☐ is the most frequently occurring value. The ☐ is calculated by summing the data values and then dividing by the number of values. It is also known as the ☐. The ☐ is the middle value in an ordered data set.

Fluency

Question working paths

Mild	Medium	Spicy
'	''	'''
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (b,c)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (c,d)

WE 1 **4.** Calculate the mean of each data set.

a. 6, 7, 8, 8, 11
b. 11, 11, 11, 12, 12, 13, 13, 13
c. 2, 4, 4, 7, 9, 11, 15, 18, 29
d. 101, 105, 108, 109, 121, 125
e. 5, 0, 12, 3, 10, 9, 7, 8
f. 76, 45, 82, 54, 76, 50, 50, 47, 0, 100
g. 1.9, 1.8, 1.9, 1.5, 2, 1.1, 1.7, 0.9
h. 982, 16, 29, 114, 650, 188, 203, 7, 97

WE 2 **5.** Determine the median of each data set.

a. 5, 9, 11, 14, 18
b. 23, 30, 42, 47, 49, 51, 65
c. 8, 13, 16, 20, 24, 25
d. 14, 17, 18, 19, 21, 28, 35, 36
e. 21, 0, 15, 23, 8, 13, 14
f. 956, 103, 85, 322, 49, 560, 12, 151, 673
g. 287, 25, 350, 95, 100, 210
h. 83, −101, 20, 16, 45, −41, 99, 47

WE 3 **6.** State the mode(s) of each data set, where possible.

a. 10, 100, 100, 1000, 10 000, 100 000
b. leopard, lion, tiger, tiger, lion, leopard, leopard, tiger, leopard
c. 45, 50, 53, 46, 45, 42, 53, 41, 59, 51
d. blonde, brown, red, black, black, red, brown, blonde, red, red, black, blonde
e. Africa, Europe, Australia, Asia, North America, Africa, South America, Europe, Asia, Australia, Antarctica, Europe, South America, Antarctica, Africa, North America
f. 5, 1, 4, 5, 3, 2, 3, 2, 1, 5, 4, 3, 2

g. Vegemite, Nutella, peanut butter, jam, jam, Vegemite, jam, peanut butter, Nutella, Vegemite, Nutella, peanut butter
h. 18, 21, 34, 9, 5, 30, 45, 2, 19, 26

WE 4 7. Calculate the range of each data set.

a. 9, 11, 11, 14, 18, 26
b. 12, 21, 23, 36, 47, 79, 84, 95, 101
c. 1000, 5000, 2000, 6000, 3000
d. 19, 36, 12, 32, 8, 14, 7
e. 25, 28, 27, 23, 26, 25, 29, 21, 22, 28, 24
f. 2000, 20, 200 000, 20 000, 20
g. 110, 250, 85, 0, 400, 620, 325
h. 380, 142, 861, 78, 220, 699, 1023, 473, 218, 943

8. Data set A: 20, 13, 7, 11, 12, 18, 13, 14, 21, 9, 10, 15, 19
Data set B: 24, 11, 12, 15, 14, 11, 20, 16, 17, 15, 11, 14
State which data set has a greater:

a. mode
b. range
c. median
d. mean

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Leo went to Port Douglas and collected shells on the beach every day for seven days. By the end of his holiday he had collected 84 shells. What was the mean number of shells Leo collected each day?

10. The 24 students in a Year 7 Maths class each noted their favourite social media platform, as shown.

Instagram TikTok Facebook TikTok
Snapchat Instagram Twitter Snapchat
TikTok YouTube Instagram TikTok
Instagram Reddit Twitter Snapchat
YouTube Facebook Instagram Snapchat
TikTok Instagram Facebook TikTok

Which social media platform(s) is/are the most popular amongst the students?

11. Kelly is the shortest in her family. She has listed the heights, in centimetres, of all the members of her family in the following list. 176, 165, 138, 159, 146, 179, 169, 185, 180. How much taller than Kelly is her brother Dexter if he is the tallest in the family?

12. In 2021, the mean weekly time spent on social media was 10.5 hours. Over five weeks Shareef spent: 13, 8, 5, 17, and 7 hours per week on social media and Jasmine spent: 7, 16, 10, 9, and 14 hours. Were Jasmine and Shareef above or below the weekly Australian mean?

13. Four friends, Fernanda, Penelope, Clove and Willa, have a median age of 13. How old is Willa if Penelope is the oldest, Clove is the youngest and Fernanda is 13?

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c)	All

14. Macy is considering moving from Melbourne to Darwin but is worried that she will not like the warmer climate. Over a two-week period, she recorded the daily maximum temperatures (°C) for Melbourne and Darwin in the following table.

Melbourne	17	24	23	26	17	16	19	22	24	24	27	32	34	23
Darwin	33	34	32	30	31	28	30	27	30	30	31	31	32	29

a. What is the modal temperature for each city?
b. Calculate the range in temperatures for each city. Using these values, state which of the cities has a more variable climate.
c. Determine the median temperature for each city and use them to explain which of the cities generally has a higher temperature.
d. What would the daily maximum temperature in Melbourne need to be on the 15^{th} day, in order for Melbourne's mean temperature to be exactly 24°C?
e. Other than temperature, what factors might Macy consider before moving from Melbourne to Darwin?

15. The time taken to get to school, in minutes, was recorded for students from two different schools; an urban school and a rural school. The data is shown.

Urban school: 15, 9, 23, 18, 34, 7, 11, 17

Rural school: 32, 9, 5, 48, 70, 29, 40, 54, 64

a. Calculate the median and mean time taken to get to each school.

b. Using your answers from part **a**, which school does it generally take longer for the students to travel to?

c. Jill attends the rural school and it takes her 149 minutes to get to school each day. If Jill's time was included in the data set, how much would the mean time taken to get to the rural school increase by?

d. Given that it takes some students a long time to get to school, do you think that school should start later and finish later? Give reasons for your answer.

Extra spicy

16. The median house price in Warrnambool in 2021 was $516 000. The prices of the last 5 properties that sold in 2022 are as follows: $575 000, $293 000, $365 000, $670 000, $290 000.

In order for the median to increase above $516 000, what is the minimum number of extra properties that need to be sold?

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

17. The ages, in years, of six relatives at a family reunion, are given as follows: 18, 44, 11, 47, 23, 31.

Geraldine arrives late to the reunion and causes the mean age to increase by 3 years. How old is Geraldine?

18. Mikey recorded the following goals that Manchester United scored in the English Premier League for the season: 1, 3, 2, 0, 3, 1, 1, 1, 3, 0, 4, 0, 0, 2, 1, 0, 2, 2, 1, 5.

What is the minimum number of additional games that Manchester United need to play such that the mode and the median are different?

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

19. The average weight of a bag of diamonds is tripled when another large diamond is added to a bag that already has 5 diamonds in it. What is the ratio of the weight of the large diamond to the total weight of the other five diamonds?

A. 5 : 1 **B.** 1 : 5 **C.** 13 : 1 **D.** 13 : 5 **E.** 26 : 5

Remember this?

20. Saoirse has 2000 potatoes to pack away into boxes. Each box can fit 8 layers of potatoes with 28 potatoes in each layer. Saoirse packs as many full boxes as she can. How many potatoes are left over?

A. 8 **B.** 16 **C.** 192 **D.** 208

21. Jisoo wants to buy a designer jumpsuit with a retail price of $210.

During Boxing Day sales, the jumpsuit goes on sale for 20% off the regular price.

What is the sale price of the jumpsuit?

A. $42 **B.** $168 **C.** $178 **D.** $252

22. The following table shows some of the tallest buildings in the world.

Building	Height (m)	Country
Burj Khalifa	828	United Arab Emirates
Shanghai Tower	632	China
KXJB-TV Tower	628	USA
Goldin Finance 117	596	China
Ping'an International Financial Center	555	China
Lotte World Tower	555	South Korea
Tianjin CTF Finance Centre	530	China
Guangzhou CTF Finance Centre	530	China

How much taller than South Korea's tallest building is China's tallest building?

A. 25 metres **B.** 77 metres **C.** 123 metres **D.** 273 metres

7C Dot plots and stem-and-leaf plots

Dot plots are visual displays for categorical and numerical data. Stem-and-leaf plots are visual displays for numerical data only. In this lesson we will learn how to construct both dot plots and stem-and-leaf plots, and how to interpret them. We will also learn to calculate summary statistics based on these visual representations, where they exist. Dot plots and stem-and leaf plots can be used in the following situations.

- My favourite actor is Jennifer Lawrence, my three sisters' favourite actor is Halle Berry and my mum and dad's is Maggie Smith. How can I show this information in a dot plot?
- I ask the students in my class how many Instagram followers they each have and put the data in a stem-and-leaf plot. What key can I use?
- I have listed the lengths of the 15 longest bridges in the world. Should I use a dot plot or stem-and-leaf plot to display my data?

Learning intentions

Students will be able to:

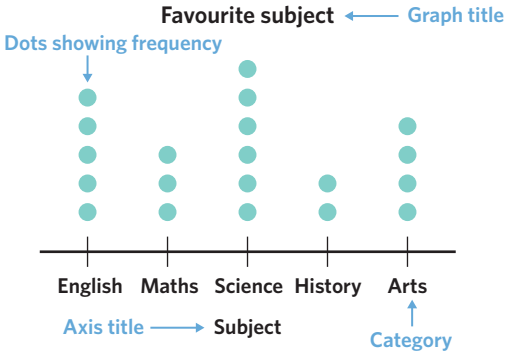
- + interpret dot plots and stem-and-leaf plots
- + construct dot plots and stem-and-leaf plots
- + understand what types of data sets are appropriate to be represented by dot plots and stem-and-leaf plots.

Key terms and definitions

- **Dot plots** are visual representations for categorical and numerical data, where each value is shown as a dot above an axis.
- **Stem-and-leaf plots** are visual representations for numerical data in which each data value is split into two parts, known as the 'stem' and 'leaf'.

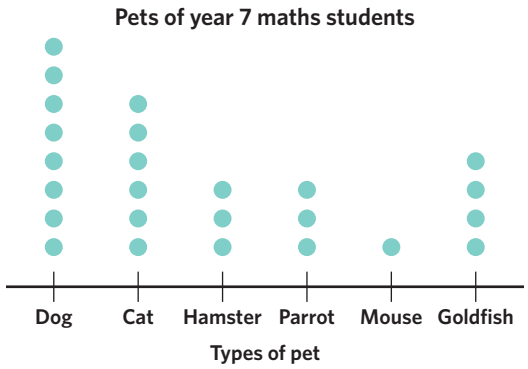
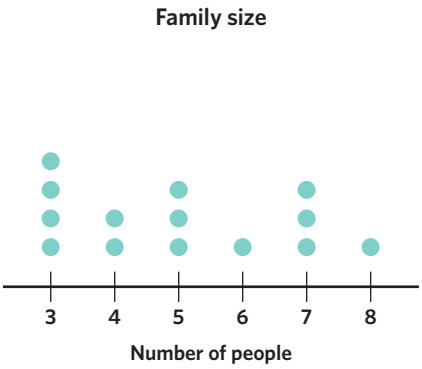
Key ideas

1 Dot plots and stem-and-leaf plots must be labelled so that they can be interpreted correctly.

Dot plot	Features
	• A graph title • A title for the axis • Equal space between each category • A label for each category

Stem-and-leaf plot	Features
Line separating 'stem' and 'leaf' Key included to show how to read the stem plot **Stem** | **Leaf** 1 | 0 2 4 7 7 2 | 1 4 5 6 8 9 3 | 0 3 3 5 6 4 | 5 6 9 5 | 6 | 0 **Key** 1 | 10 = 10 minutes 'Leaf' values ordered from smallest to largest in each row 'Stem' values	• A key • Labels above the stem and leaf sections • Stem values • Leaf values

2 Dot plots can be used to show categorical data or numerical data. For categorical data, the axis is labelled with worded categories. For numerical data, the axis is labelled with numbers.

Dot plot with categorical data	Dot plot with numerical data
	

3 Stem-and-leaf plots can be used to show numerical data.

For example, the following data set is represented in the stem-and-leaf plot below:

10, 14, 16, 23, 23, 27, 29, 31, 32, 35, 40, 40, 41, 47, 48

Stem	Leaf				
1	0	4	6		
2	3	3	7	9	
3	1	2	5		
4	0	0	1	7	8
5					
6	0				

1 | 0 is 10
2 | 7 is 27
3 | 2 is 32
4 | 8 is 48

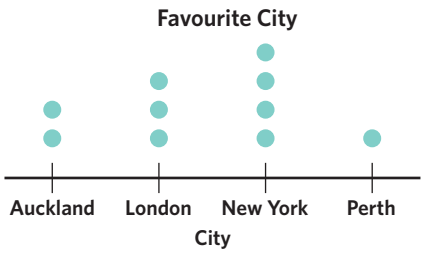
Key

1 | 3 = 13

Worked examples

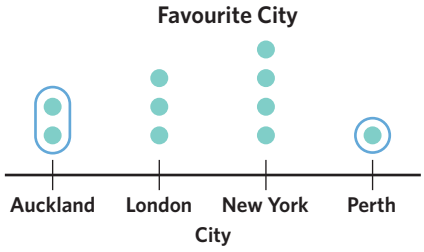
WE 1 Interpreting dot plots

The following dot plot shows the favourite city of 10 people.



a. How many people chose Auckland or Perth as their favourite city?

Working



Auckland: 2

Perth: 1

$2 + 1 = 3$

3 people favour Auckland or Perth.

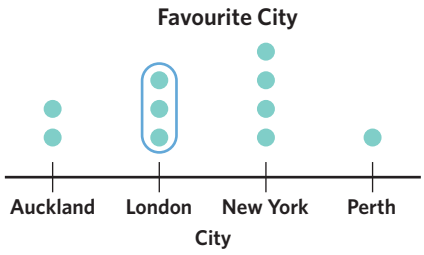
Thinking

Step 1: Determine the frequency of the specified categories (Auckland and Perth) by counting the dots.

Step 2: Sum the frequency of the specified categories.

b. What percentage of people listed London as their favourite city?

Working



3 people's favourite city is London.

$\frac{3}{10} \times 100 = 30\%$

30% of the people favour London.

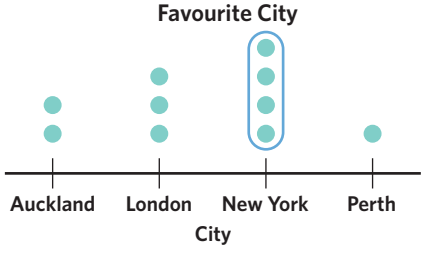
Thinking

Step 1: Determine the frequency of the specified category (London) by counting the dots.

Step 2: Divide the frequency of the specified category (London) by the sample size and convert it to a percentage.

c. What is the modal city?

Working



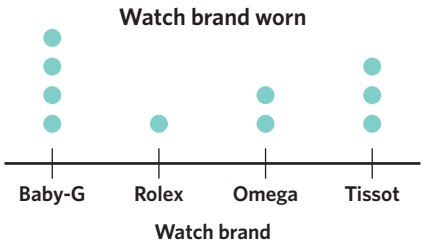
The modal city is New York.

Thinking

Identify the category with the highest frequency (the most number of dots).

Student practice

The following dot plot shows the watch brands worn by 10 people.



a. How many people wear a Rolex?

b. How many more people wear a Baby-G as opposed to a Tissot?

c. What percentage of people wear an Omega watch?

d. What is the modal watch?

WE 2 Interpreting stem-and-leaf plots

The following stem-and-leaf plot shows the weight, in kilograms, of 20 household pets.

Stem	Leaf					
0	2	4	6	7	7	8
1	1	3	4	4	5	9
2	2	2	3	5	6	
3	2					
4	1	8				

Key 0 | 2 = 2 kg

a. How many pets weigh between 20 kg and 30 kg?

Working

Stem	Leaf					
0	2	4	6	7	7	8
1	1	3	4	4	5	9
2	②	②	3	⑤	⑥	
3	2					
4	1	8				

Key 0 | 2 = 2 kg

5 pets weigh between 20 kg to 30 kg.

Thinking

Identify the corresponding stem values (2 and 3) in order to count the number of leaves within the specified range (pets that weigh between 20 kg to 30 kg).

b. What percentage of pets weigh less than 15 kilograms?

Working

Stem	Leaf					
0	②	④	⑥	⑦	⑦	⑧
1	①	③	④	④	5	9
2	2	2	3	5	6	
3	2					
4	1	8				

Key 0 | 2 = 2 kg

There are 10 pets that weigh less than 15 kilograms.

$\frac{10}{20} \times 100 = 50\%$

50% of pets weigh less than 15 kilograms.

Thinking

Step 1: Identify the corresponding stem value (1) in order to count the number of leaves to find the frequency of the specified range (pets that weigh less than 15).

Step 2: Divide the frequency of the specified range (pets that weigh less than 15 kg) by the sample size and convert it to a percentage.

c. What is the median?

Working

Stem	Leaf					
0	2	4	6	7	7	8
1	1	3	4	4	5	9
2	2	2	3	5	6	
3	2					
4	1	8				

Key
0 | 2 = 2 kg

Stem	Leaf					
0	2	4	6	7	7	8
1	1	3	4	4	5	9
2	2	2	3	5	6	
3	2					
4	1	8				

Key
0 | 2 = 2 kg

$\frac{14 + 15}{2} = 14\frac{1}{2} = 14.5$

The median weight is 14.5 kg.

Thinking

Step 1: Cross out the leaves of the smallest and largest values.

Step 2: Cross out the leaves of the next smallest and largest values. Repeat this process until one or two leaves are left.

Step 3: Sum the remaining two numbers and divide the sum by 2.

Student practice

The following stem-and-leaf plot shows the pocket money received, in dollars, by 20 students.

Stem	Leaf				
1	0	5	5	7	
2	0	5	7	7	8
3	0	0	2	5	5
4	0	7	9		
5	0	5	7		

Key
1 | 5 = $15

a. How many people receive $32?

b. How many people receive $50 or more?

c. What percentage of people receive more than $50?

d. What is the median pocket money amount?

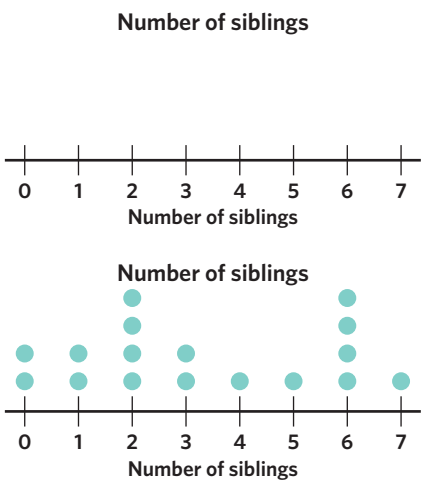
WE 3 Constructing dot plots and stem-and-leaf plots

Create the specified visual representation for each data set.

a. Dot plot

Number of siblings: 0, 5, 6, 3, 2, 2, 6, 1, 3, 6, 4, 2, 1, 0, 7, 6, 2

Working



Thinking

Step 1: Write the title (Number of siblings) then draw and label a horizontal axis (Number of siblings) where the scale ranges from the smallest to the largest values in the data set.

Step 2: Draw dots for each value by matching the number of dots with its frequency from the list.

b. Stem-and-leaf plot

Age of backgammon players: 12, 43, 56, 33, 21, 29, 18, 44, 51, 18, 42, 53, 37, 17, 29, 13, 25, 30

Working

12, 13, 17, 18, 18, 21, 25, 29, 29, 30, 33, 37, 42, 43, 44, 51, 53, 56

Key: 1 | 2 = 12 years old

Stem	Leaf

Key

1 | 2 = 12 years old

Stem	Leaf				
1	2	3	7	8	8
2	1	5	9	9	
3	0	3	7		
4	2	3	4		
5	1	3	6		

Key

1 | 2 = 12 years old

Thinking

Step 1: Order the data set from the smallest to largest.

Step 2: Create a key. The data set has two digit numbers. Use the digit in the tens place as the stem and the digit in the ones place as the leaf.

Step 3: Draw the stem-and-leaf line and label the key.

Step 4: Enter the stem values from smallest to largest. Enter the leaf values for each row, sorted from smallest to largest.

Student practice

Create the specified visual representation for each data set.

a. Dot plot
Outcomes from rolling a 6-sided die: 3, 5, 2, 5, 6, 3, 4, 6, 2, 3

b. Stem-and-leaf plot
Number of students per class: 14, 12, 21, 15, 23, 8, 11, 25, 12, 30, 19, 6, 24, 16, 8

c. Dot plot
Favourite painter: Vincent van Gogh, Claude Monet, Pablo Picasso, Leonardo da Vinci, Vincent van Gogh, Vincent van Gogh, Leonardo da Vinci, Pablo Picasso, Leonardo da Vinci, Vincent van Gogh

d. Stem-and-leaf plot
Number of songs in students' favourite Spotify playlist: 54, 48, 69, 32, 48, 52, 37, 56, 40, 35, 63, 70, 51, 57, 47

7C Activities and questions

STARTER TASKS

Odd spot

Beyond Meat is a vegan food manufacturer that makes different plant-based products. All of their products are meat-free. The company's mission is to positively impact people, the environment and the planet as a whole. Some of their meat-free products include plant-based burgers, sausages, chicken and meatballs.

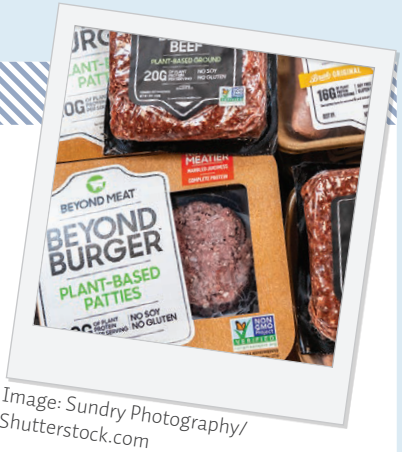
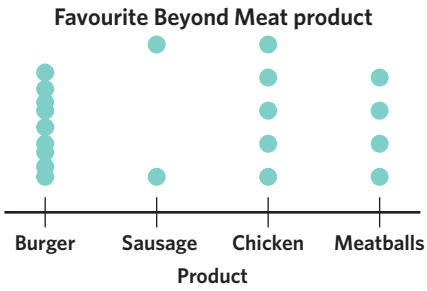


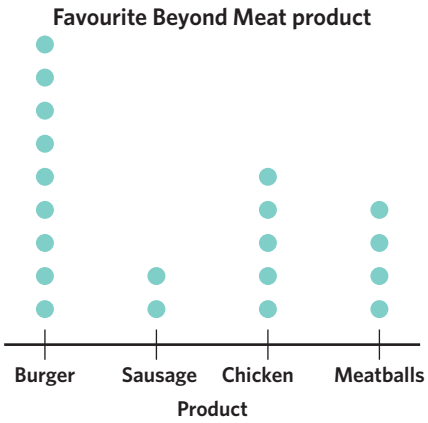
Image: Sundry Photography/ Shutterstock.com

The following dot plot shows the favourite Beyond Meat product of 20 customers. Which dot plot is a more accurate representation of the collected data?

A.

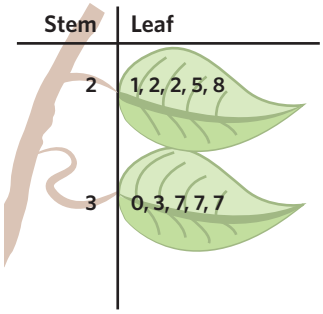


B.



Puzzle

The following stem-and-leaf plot can be used to visually represent different sets of data.



a) Draw a line to connect each description, with its matching key and data set.

Description	Key	Data set
Number of movies seen last year	2 | 1 = 21%	2.1, 2.2, 2.2, 2.5, 2.8, 3.0, 3.3, 3.7, 3.7
Weight of babies born before expected due date (kg)	2 | 1 = 21	$210, $220, $220, $250, $280, $300, $330, $370, $370
Cost of visit to the dentist	2 | 1 = 2.1	21, 22, 22, 25, 25, 28, 30, 33, 37, 37
Percentage of teachers who take public transport to school	2 | 1 = $210	21%, 22%, 22%, 25%, 25%, 28%, 30%, 33%, 37%, 37%

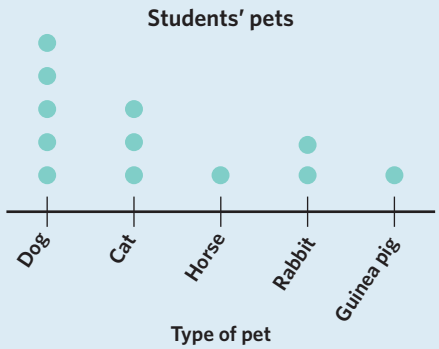
b) Create your own description, matching key and data set that could be suitable for this stem-and-leaf plot.

Understanding worksheet

1. Use each dot plot to complete the frequency table.

Example

Pet	Number of students
Dog	5
Cat	3
Horse	1
Rabbit	2
Guinea Pig	1
Total	12



a.

Shoe size	Number sold
5	1
6	
7	
8	6
9	8
10	
11	1
Total	28

b.

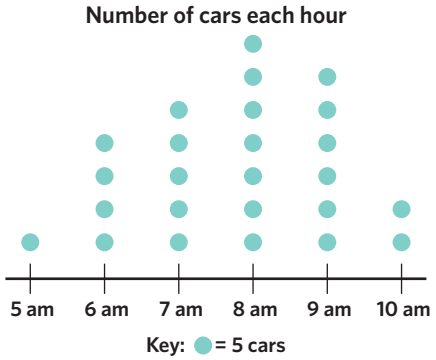
Colour	Number of players
Pink	3
Red	
Yellow	
Blue	1
Purple	
Total	10

c.

Gift requested	Number of children
Bicycle	4
Book	
X-Box	
PlayStation	3
	5
Total	

d.

Time	Number of cars
5 am	5
	20
7 am	
8 am	35
9 am	
	10
Total	125



2. Complete the missing information in the list from the stem-and-leaf plot.

Example

List of numbers: 1, 3, 3, 8, 11, 17, 19, 19, 20, 24, 27, 42

Stem	Leaf	Key
0	1 3 3 8	0 | 1 = 1
1	1 7 9 9	
2	0 4 7	
3		
4	2	

a. List of numbers: 5, 7, 8, ☐, 12, 17, 19, ☐, 33, 34, 36

Stem	Leaf	Key
0	5 7 8	1 | 5 = 15
1	0 2 7 9	
2	1	
3	3 4 6	

b. List of numbers: 12, 12, 17, ☐, 25, 26, 41, ☐, 44, 49, 58

Stem	Leaf	Key
1	2 2 7	1 | 2 = 12
2	3 5 6	
3		
4	1 1 4 9	
5	8	

c. List of numbers: 113, 115, ☐, 119, 120, ☐, 121, 133, 136, ☐, 145, 145, 145

Stem	Leaf	Key
11	3 5 7 9	11 | 3 = 113
12	0 1 1	
13	3 6 9	
14	5 5 5	

d. List of numbers: 100, ☐, 103, 105, ☐, 111, 132, 135, 135, ☐, 140, 144, ☐, 148, 149

Stem	Leaf				
10	0	0	3	5	8
11	1				
12					
13	2	5	5	7	
14	0	4	5	8	9

Key: 10 | 0 = 100

3. Fill in the blanks by using the words provided.

key | numerical | dot | categorical

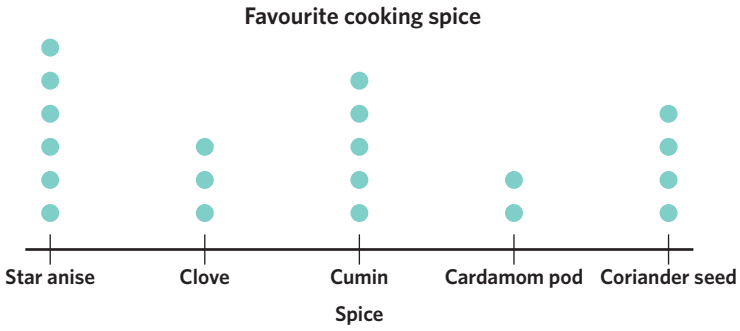
Dot plots are a way to represent ☐ and numerical data. A ☐ can be used to represent each separate value from a data set. Stem-and-leaf plots are a way to represent ☐ data only. Every stem-and-leaf plot must have a ☐.

Fluency

Question working paths

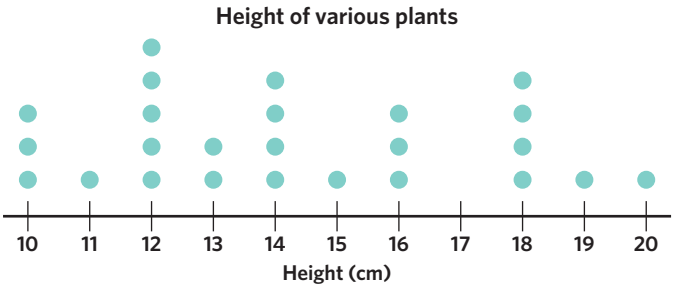
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b), 8 (a,b)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c), 8 (b,c)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f,g), 7 (c,d), 8 (c,d)

WE1 **4.** The following dot plot shows the favourite cooking spice of 20 Masterchef Junior contestants.



a. How many contestants favour cumin?
b. How many contestants favour coriander seed or cardamom pod?
c. How many more contestants favour star anise than clove?
d. How many fewer contestants favour cardamom pods than cumin?
e. What is the modal spice?
f. What percentage of contestants favour star anise or coriander seed?

WE1 **5.** The following dot plot shows the height, in centimetres, of 25 different plants.



a. How many plants have a height of 16 cm?
b. How many plants have a height of 17 cm?
c. How many plants have a height less than 13 cm?
d. How many more plants have a height of 18 cm than those with a height of 11 cm?
e. What percentage of plants have a height greater than or equal to 15 cm?
f. What is the median height?

WE 2 **6.** The following stem-and-leaf plot shows the number of customers a restaurant has each day in April.

Stem	Leaf
4	2 3 6 6 9
5	1 3 3 7 7 7
6	1 3 3 3 3 5 8 9
7	3 4 4 5 9
8	2 4 6 6 7
9	0

Key
4 | 2 = 42

a. For how many days does the restaurant have 86 customers?
b. For how many days does the restaurant have 70 customers?
c. For how many days does the restaurant have between 50 and 60 customers?
d. What percentage of days does the restaurant have 57 customers?
e. What percentage of days does the restaurant have more than 80 customers?
f. What is the mode?
g. What is the median?

WE 3a **7.** Create a dot plot for each set of data.

a. Favourite cuisine of 15 people:
French, Spanish, French, Chinese, Spanish, Italian, Chinese, Italian, French, Spanish, French, Chinese, Spanish, French, Italian
b. The number of times 17 students have gone bowling this year:
1, 7, 2, 5, 2, 1, 2, 3, 4, 3, 4, 4, 3, 4, 4, 2, 2
c. Favourite sport of 20 PE students:
basketball, footy, lacrosse, footy, soccer, swimming, soccer, soccer, tennis, basketball, swimming, soccer, basketball, footy, swimming, soccer, tennis, soccer, basketball, footy
d. The number of hours 18 students spent doing homework last week:
6, 4, 8, 7, 14, 11, 6, 5, 9, 11, 10, 11, 12, 8, 7, 4, 10, 9

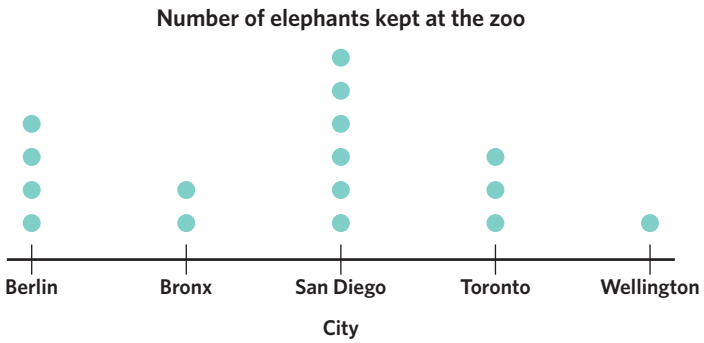
WE 3b **8.** Create a stem-and-leaf plot for each set of data.

a. The amount of money, in dollars, in red packets received from 12 relatives during Lunar New Year:
10, 10, 10, 15, 15, 18, 20, 24, 25, 25, 30, 31
b. The temperature, in degrees, of 20 summer nights in Melbourne:
8, 19, 24, 26, 27, 27, 28, 30, 28, 26, 31, 36, 30, 20, 25, 21, 18, 18, 25, 26
c. The IQ score of 25 students:
89, 112, 105, 76, 81, 95, 102, 125, 105, 120, 88, 112, 83, 91, 86, 78, 81, 110, 108, 104, 85, 83, 96, 78, 101
d. The time to eat a hot dog, in seconds, of 30 competitors:
3.3, 2.9, 2.9, 3.5, 3.9, 4.5, 4.6, 5.2, 5.5, 5.5, 6.1, 6.5, 7.2, 7.6, 8.1, 8.7, 9.3, 10.0, 10.4, 10.6, 11.3, 11.5, 12.4, 11.9, 11.5, 11.5, 10.8, 10.4, 9.9, 9.4

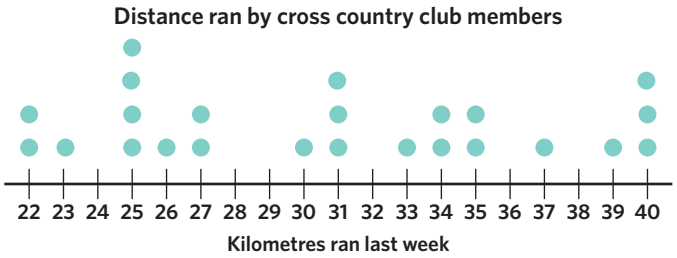
Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Victor has constructed the following dot plot for a school project on the number of elephants at different zoos around the world. Which zoos have three or more elephants?



10. Pearl runs at a cross country club and all the runners recorded the total distance they ran last week. How many runners ran more than 32 km, if each dot in the following dot plot represents two runners?



11. Trevor is a journalist and is tracking the number of times each of his tweets has been retweeted. What is the range of the data, if the following stem-and-leaf plot shows the number of times his last 10 tweets have been retweeted?

Stem	Leaf
0	4 4 6 8 9
1	2 3 4 6
2	1

Key
0 | 4 = 4 times

12. Axel recorded the number of tomatoes he harvested each week and used the following stem-and-leaf plot to display the data. He thinks a dot plot may be a better way to represent the data. Construct a dot plot to display Axel's data.

Stem	Leaf
0	4 4 5 6 6 8 9 9 9
1	0 0 0 0 1 3 3 3 4 4 5

Key
0 | 4 = 4 tomatoes

13. Aiko owns a Japanese restaurant and constructed the following stem-and-leaf plot to display the salaries of 12 of his employees. Aiko forgot to include his own salary of $99 000. If Aiko's salary was included, how much would the median salary increase by?

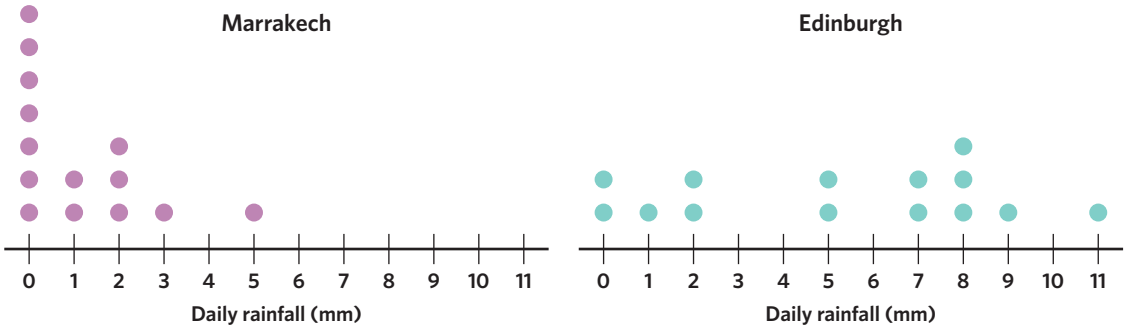
Stem	Leaf
2	3
3	0 4 4 4 9
4	1 2 5
5	2 2
6	0

Key
2 | 3 = $23 000

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c,d)	All

14. Deshan is completing a project on rainfall in Marrakech and Edinburgh. He created the following dot plots for the daily rainfall in Marrakech and in Edinburgh over some days in October.



a. Over how many days were the daily rainfalls recorded in the two cities?
b. What is the modal daily rainfall for each city?
c. How much greater is Edinburgh's range of daily rainfall than Marrakech's?
d. Deshan is interpreting the dot plots and says that it is more likely to have no rain than a little bit of rain in Marrakech. Explain why this is not true.
e. If you had to summarise the information in one sentence for a travel brochure, explain whether the mean, mode or range is the best summary statistic to use.

15. IQ, short for Intelligence Quotient, is a universal measure of a person's reasoning ability. In a study to investigate environmental effects on IQ, 40 people were chosen and were split evenly into two groups. Their IQs are shown in the following back-to-back stem-and-leaf plot.

Group 1	Stem	Group 2
9	7	2 8
8 3 2	8	1 4 5 7 9
7 6 3 3	9	0 2 2 2 3 8
9 8 6 4 4 0	10	1 4 4 5 7
9 5 5 3	11	0 6
2 1	12	

Key
7 | 2 = 72

a. Which group has a larger range of IQ scores?

b. Throughout the entire sample of people, what is the modal IQ?

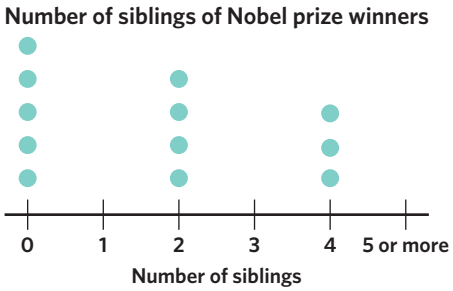
c. In the entire human population, the average IQ score is 100. What percentage of people in each group have an IQ over 100?

d. Calculate the median IQ of each group. Use them to determine which group had the higher IQ.

e. It has been argued that an IQ test may not be a fair assessment of a person's general intelligence as it is a specific type of test. What might be a fairer way to measure a person's general intelligence?

Extra spicy

16. Nobel prize winners were surveyed to discover how many siblings they each have. The following dot plot has missing information about the number of Nobel Prize winners who have 1, 3 or 5 or more siblings.



What is known is that the same number of winners have no siblings and 3 siblings, and the same number of winners have 1 sibling and 5 or more siblings. If a total of 21 winners were surveyed, how many winners have 5 or more siblings?

17. Yuri recorded the weight of 8 suitcases to help him plan how much to pack for his first trip to Korea. He recorded the weights in a stem-and-leaf plot, but one number was smudged and became illegible.

Stem	Leaf
2	7
3	5 8
4	1 2 6
5	[illegible] 9

Key
3 | 5 = 3.5 kg

Which of the following could be the average weight of the 8 suitcases?

A. 3.75 kg **B.** 4.15 kg **C.** 4.25 kg **D.** 5.5 kg **E.** 41.8 kg

18. What number could be added to the data in the following stem-and-leaf plot so that the mean and median are equal?

Stem	Leaf
4	8
5	2 7 8
6	5 6
7	0 4 8 8 8
8	0 1
9	5

Key
1 | 2 = 12

A. 54 **B.** 70 **C.** 74 **D.** 100 **E.** 150

19. Stephano caught 42 flies on Friday, 84 on Saturday, 63 on Sunday and 28 on Monday. If Stephano were to display the amount of flies caught each day in a dot plot, how many flies should each dot represent to use the least amount of dots?

A. 7 **B.** 8 **C.** 9 **D.** 10 **E.** 11

Remember this?

20. The Eiffel Tower is approximately 300 metres tall. Francisca bought a small model of the tower during a school trip to Paris. The model was built with a scale of 1:5000. What is the height of the model?

A. 6 cm **B.** 8 cm **C.** 16.7 cm **D.** 60 cm

21. Edward spent 50% of his savings on an original copy of Harry Potter and the Philosopher's Stone. The book cost him $4950. How much money did Edward have in his savings before purchasing the book?

22. The table shows the ice cream sales by an ice cream truck over 5 days.

Day	Ice cream sales ($)
Monday	120
Tuesday	145
Wednesday	105
Thursday	150
Friday	160

What was the mean daily sales amount?

7D Column graphs and pie charts

Column graphs are a visual display for categorical data. A pie chart is a visual display for only categorical data. It can be difficult to interpret categorical data that is only displayed in a frequency table. Column graphs and pie charts are used to make interpreting data more efficient. Below are some examples where column graphs and pie charts can be used.

- The number of gold, silver and bronze medals I won at a swimming carnival is shown on a column graph. How many gold medals did I win?
- I use a pie chart to show the percentage of people from different countries living in Sydney. What percentage is made up of Italians?
- I found a column graph that showed the results from a survey asking for people's favourite brand of chocolate. How many people said their favourite chocolate was Bueno?

Learning intentions

Students will be able to:

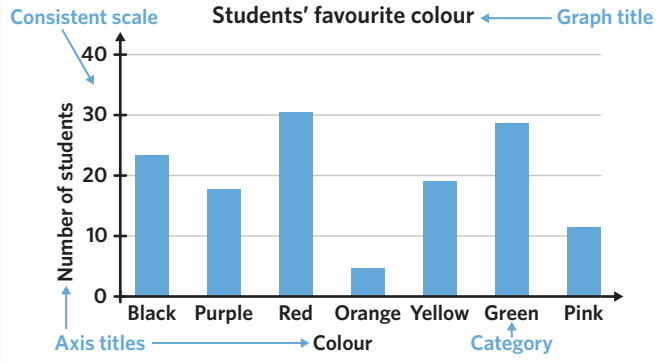
+ construct column graphs and pie charts
+ interpret column graphs and pie charts.

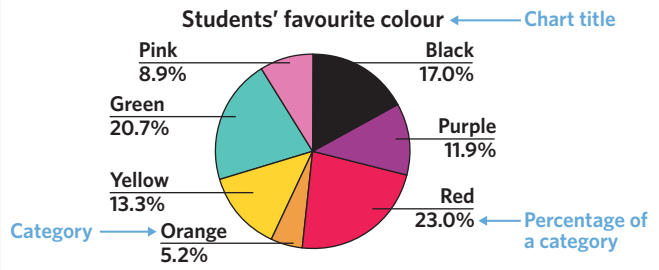
Key terms and definitions

- **Frequency** refers to how many times something occurs.
- A **frequency table** represents a set of data as categories and how many times each category occurs.
- **Column graphs** display categorical data, with the vertical height of each column representing the frequency of each category.
- **Bar charts** display categorical data, with the horizontal length of each bar representing the frequency of each category.
- **Pie charts** show categorical data with a circle split into sectors whose area represents the percentage of each category.
- A **sector** is the area of a circle enclosed by two straight lines from the centre of a circle to its edge.

Key ideas

1 Column graphs and pie charts must be labelled so that they can be interpreted correctly.

Column graph	Features
	• A graph title • A title for each axis • A consistent scale on the vertical axis • Spaces between each column category • A label for each column category

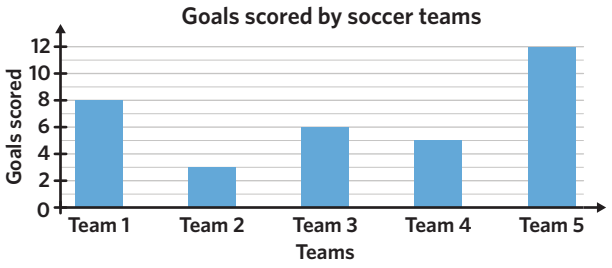
Pie chart	Features
	• A chart title • A label for each sector's category • A percentage (or frequency count) for each sector's category

2 A column graph and bar chart can represent the same categorical data set.

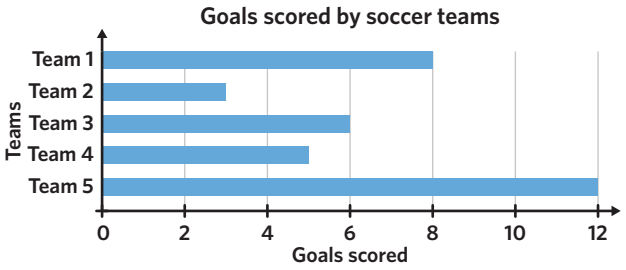
The following frequency table and column and bar graphs show the number of goals scored by five different soccer teams.

Teams	Goals scored
Team 1	8
Team 2	3
Team 3	6
Team 4	5
Team 5	12

Column graph



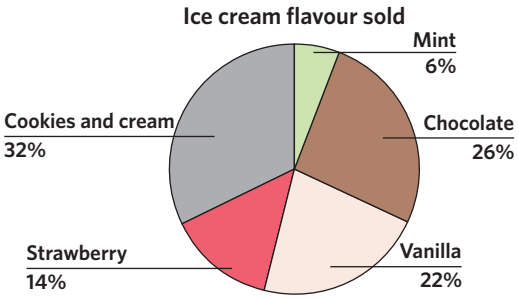
Bar chart



3 Pie charts show the percentage that each category makes up of a set of categorical data.

The following frequency table and pie chart shows the percentage of each ice cream flavour sold at a supermarket.

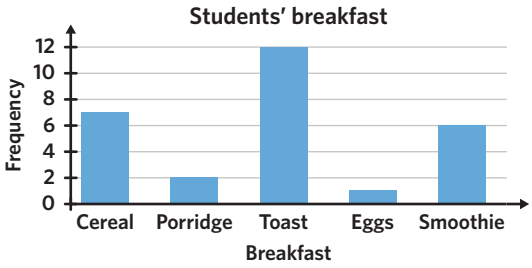
Ice cream flavour sold	Frequency	Percentage
Chocolate	13	$\frac{13}{50} = 26\%$
Vanilla	11	$\frac{11}{50} = 22\%$
Strawberry	7	$\frac{7}{50} = 14\%$
Cookies and cream	16	$\frac{16}{50} = 32\%$
Mint	3	$\frac{3}{50} = 6\%$



Worked examples

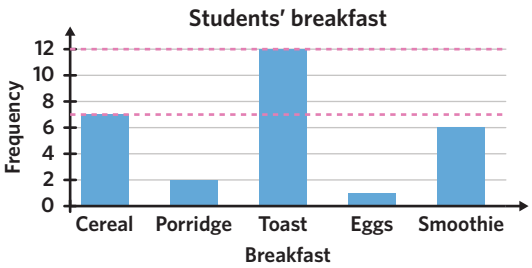
WE 1 Interpreting column graphs

Students were asked what they eat for breakfast. The results are shown in the following column graph.



a. How many students eat either cereal or toast for breakfast?

Working



Cereal: 7

Toast: 12

$7 + 12 = 19$

19 students eat either cereal or toast for breakfast.

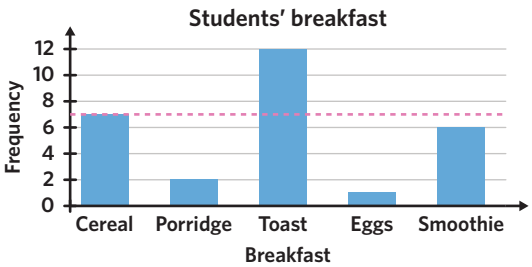
Thinking

Step 1: Find the frequency of the specified categories (cereal and toast) using the horizontal scale.

Step 2: Sum the frequency of the specified categories.

b. What percentage of students eat cereal for breakfast?

Working



Cereal: 7

$7 + 2 + 12 + 1 + 6 = 28$

$\frac{7}{28} = 0.25 = 25\%$

25% of students eat cereal for breakfast.

Thinking

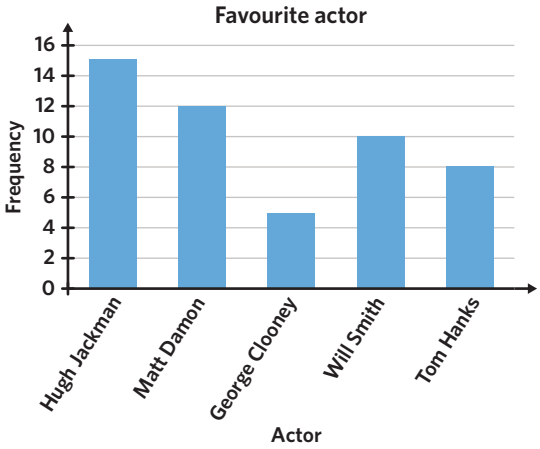
Step 1: Find the frequency of the specified category (cereal).

Step 2: Sum all the frequencies to find the sample size.

Step 3: Divide the specified category (cereal) by the sample size and convert it to a percentage.

Student practice

A group of people were asked who their favourite male actor was. The results are shown in the following column graph.



a. How many people's favourite actor is Matt Damon?

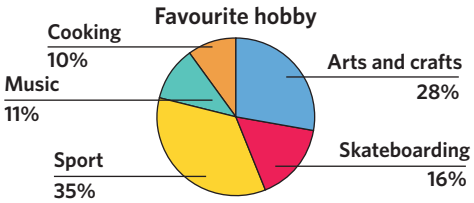
b. How many people's favourite actor is not George Clooney?

c. What percentage of people's favourite actor is Will Smith?

d. What percentage of people's favourite actor is Hugh Jackman or Tom Hanks?

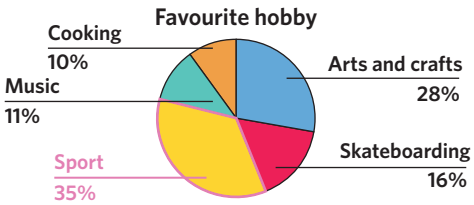
WE 2 Interpreting pie charts

100 students were asked what their favourite hobby is. The results are shown in the following pie chart.



a. What is the most popular hobby and what percentage of students picked it?

Working



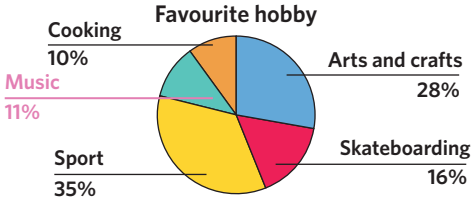
Sport is the most popular hobby. The percentage of students that picked sport was 35%.

Thinking

Find the hobby that makes up the largest sector of the pie chart. The percentage of students that picked that hobby can be read under its label.

b. How many students picked music as their favourite hobby?

Working



11% as a decimal is 0.11.

$0.11 \times 100 = 11$

11 students picked music as their favourite hobby.

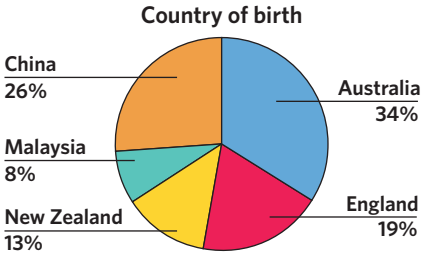
Thinking

Step 1: Find the percentage of students that picked music as their favourite hobby.

Step 2: Convert the percentage of students to the number of students by multiplying the percentage as a decimal by the number of students that were surveyed.

Student practice

100 students were asked where they were born. The results are shown in the following pie chart.



a. What percentage of students were born in Australia?

b. What percentage of students were born in England or New Zealand?

c. How many students were born in China?

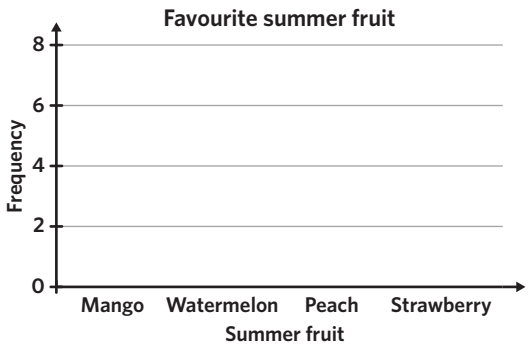
WE 3 Constructing column graphs and pie charts

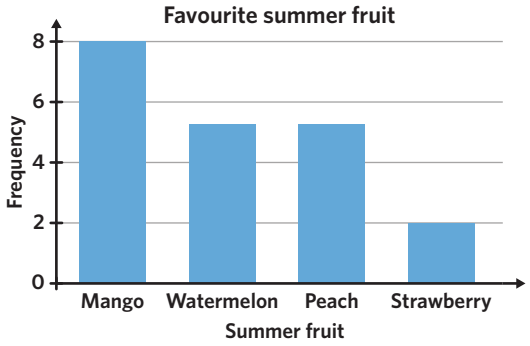
Create the described graph or chart for each set of data.

a. Column graph

Favourite summer fruit	Frequency
Mango	8
Watermelon	5
Peach	5
Strawberry	2

Working





Thinking

Step 1: Write the title (Favourite summer fruit) then draw and label horizontal (Summer fruit) and vertical (Frequency) axes. Mark a consistent scale to the largest frequency (8) from the table.

Step 2: Draw columns for each category by matching the height of each column with its frequency from the table.

b. Pie chart

Favourite soft drink	Frequency
Coke	4
Sprite	2
Pepsi	3
Fanta	1

Working

$Total\ frequency = 4 + 2 + 3 + 1$
$= 10$

Coke: $\frac{4}{10} = 40\%$

Sprite: $\frac{2}{10} = 20\%$

Pepsi: $\frac{3}{10} = 30\%$

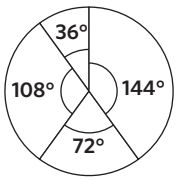
Fanta: $\frac{1}{10} = 10\%$

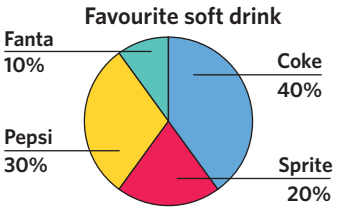
Coke: $\frac{4}{10} \times 360° = 144°$

Sprite: $\frac{2}{10} \times 360° = 72°$

Pepsi: $\frac{3}{10} \times 360° = 108°$

Fanta: $\frac{1}{10} \times 360° = 36°$





Thinking

Step 1: Sum all the frequencies to find the size of the sample.

Step 2: Write each frequency as a fraction of the total sample size. Convert each fraction into a percentage (to be used as labels on the completed pie chart).

Step 3: Multiply the fraction for each category by 360°. This is the size of the sector's angle for that category.

Step 4: Draw a circle with a vertical line from its centre to its edge. Measure the sector angle of each category and draw a line to form a sector.

Note: Do not mark the angle measurements.

Step 5: Label the chart's title, and each sector with its corresponding category and percentage.

Student practice

Create the described graph or chart for each set of data.

a. Column graph

Winter fruit	Frequency
Orange	10
Kiwi	4
Apple	6
Mandarin	5

b. Pie chart

Hair colour	Frequency
Blonde	1
Red	1
Brown	3
Black	5

c. Column graph

Weather type	Frequency
Rainy	30
Sunny	50
Cloudy	45
Foggy	10

d. Pie chart

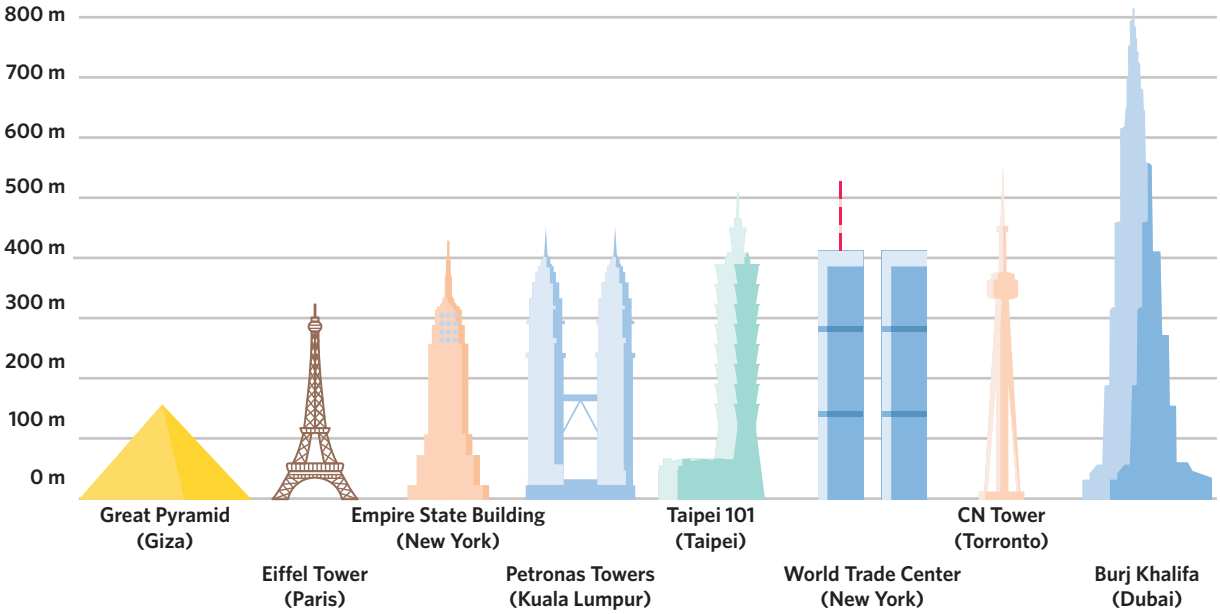
Laptop brand	Frequency
Apple	36
Dell	27
Lenovo	9
Chromebook	18

7D Activities and questions

STARTER TASKS

Odd spot

The Great Pyramid of Giza was the tallest structure in the world for almost 4000 years. The Burj Khalifa is currently the tallest man-made structure and is much taller than the Great Pyramid. The following diagram shows the structures that have been the tallest in the world at some point in time.



If you were to stack Great Pyramids, how many would it take to be **taller** than the Burj Khalifa?

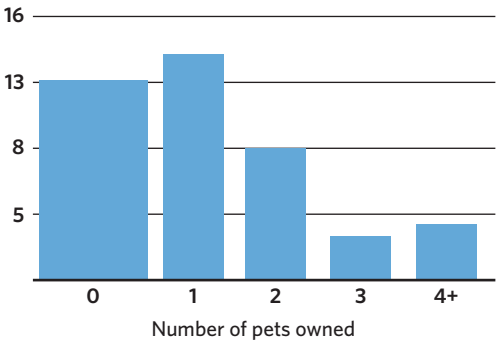
A. 5

B. 6

Puzzle

Spot the errors!

The following column graph shows the number of pets owned by a group of university students.



a) Identify the four errors in the column graph.

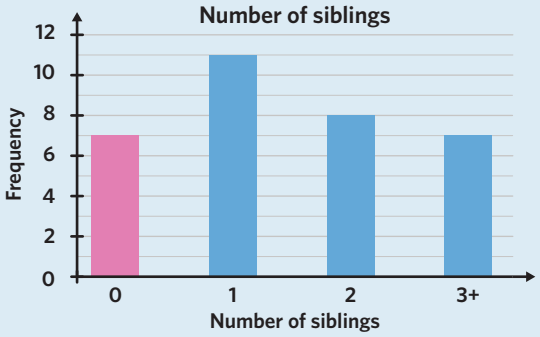
b) Why is it important to correctly complete column graphs?

Understanding worksheet

1. Draw the missing columns on the column graphs.

Example

Number of siblings	Frequency
0	7
1	11
2	8
3+	7

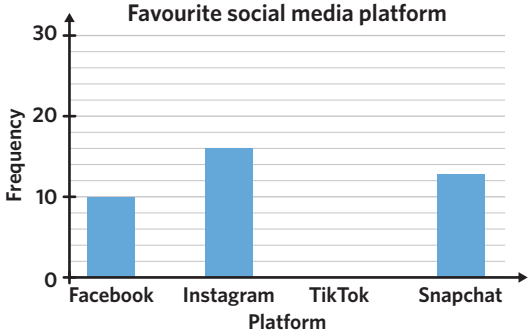


a.

Colour	Frequency
Black	6
Brown	8
Blonde	5
Red	2

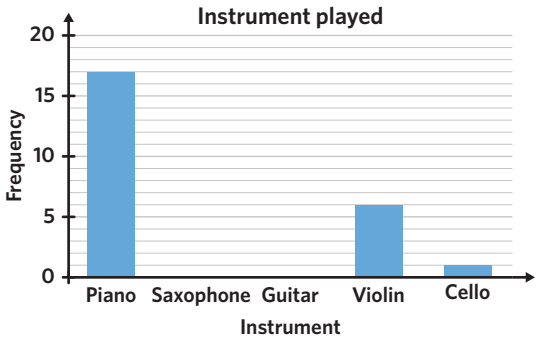
b.

Platform	Frequency
Facebook	10
Instagram	16
TikTok	27
Snapchat	13



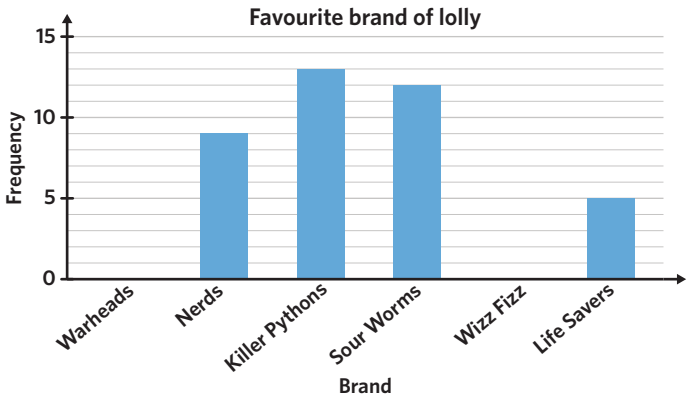
c.

Instrument	Frequency
Piano	17
Saxophone	3
Guitar	12
Violin	6
Cello	1



d.

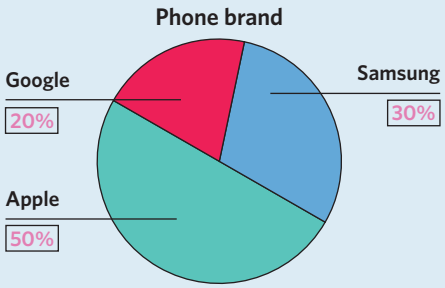
Brand	Frequency
Warheads	4
Nerds	9
Killer Pythons	13
Sour Worms	12
Wizz Fizz	7
Life Savers	5



2. Complete each pie chart using the frequency table.

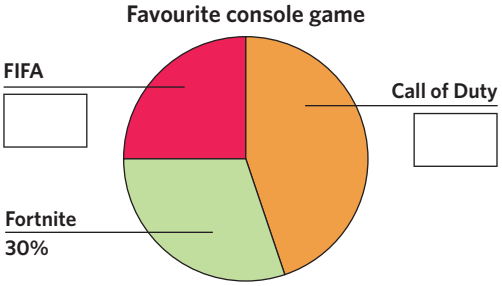
Example

Brand	Frequency	Percentage
Samsung	6	$\frac{6}{20} = 30\%$
Apple	10	$\frac{10}{20} = 50\%$
Google	4	$\frac{4}{20} = 20\%$



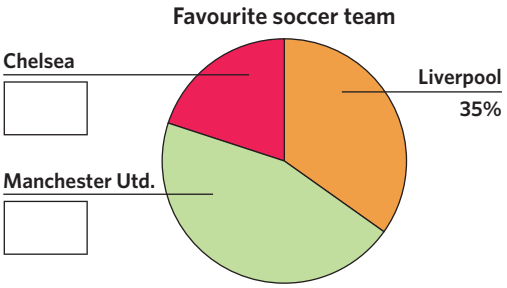
a.

Game	Frequency	Percentage
Call of Duty	9	$\frac{9}{20} = 45\%$
Fortnite	6	$\frac{6}{20} = 30\%$
FIFA	5	$\frac{5}{25} = 25\%$



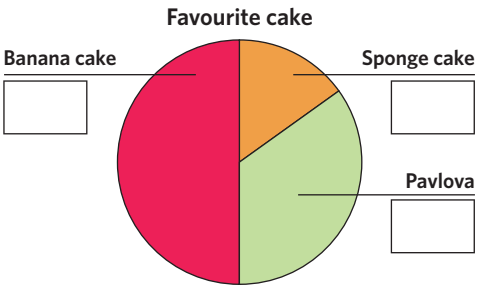
b.

Team	Frequency	Percentage
Liverpool	70	$\frac{70}{200} = 35\%$
Manchester United	90	$\frac{90}{200} = 45\%$
Chelsea	40	$\frac{40}{200} = 20\%$



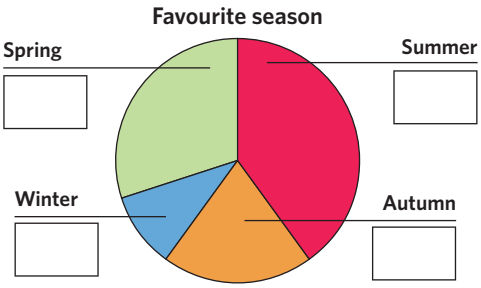
c.

Cake	Frequency	Percentage
Pavlova	7	$\frac{7}{20} = 35\%$
Banana cake	10	$\frac{10}{20} = 50\%$
Sponge cake	3	$\frac{3}{20} = 15\%$



d.

Season	Frequency	Percentage
Summer	4	$\frac{4}{10} = 40\%$
Autumn	2	$\frac{2}{10} = 20\%$
Winter	1	$\frac{1}{10} = 10\%$
Spring	3	$\frac{3}{10} = 30\%$



3. Fill in the blanks by using the words provided.

categorical | percentage | frequency | bar

Column graphs, _____ charts, and pie charts display _____ data.

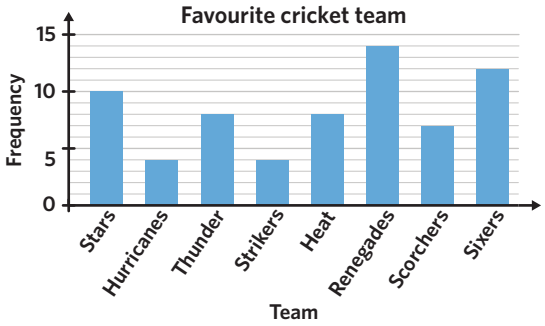
Column graphs and bar charts generally present the _____ of each category while pie charts present the _____ of each category.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b), 8 (a,b)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c), 8 (b,c)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (c,d), 8 (c,d)

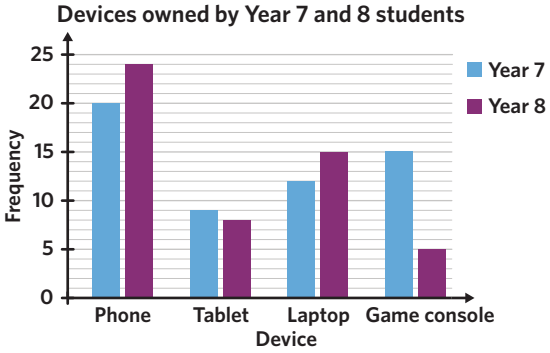
WE 1 **4.** A group of cricket fans were asked what team was their favourite. The results are shown in the following column graph.



a. How many fans' favourite team is the Thunder?
b. How many fans' favourite team is the Hurricanes?
c. What is the modal (most favoured) team?
d. How many more fans answered Sixers compared to Strikers?
e. How many more fans answered Stars compared to Heat?
f. What percentage of the fans answered Scorchers? Round to the nearest whole number.

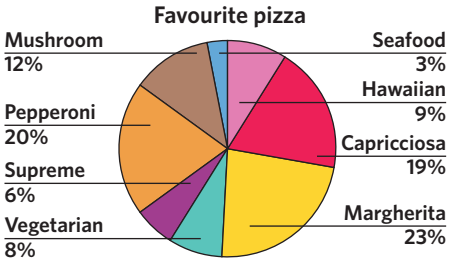
5. A survey is collected on the devices owned by 30 Year 7 students and 30 Year 8 students.

Identify whether each of the following statements are true or false.



a. There are exactly 20 Year 8 students with phones.
b. Less than 10 Year 7 students have tablets.
c. More Year 8 students have laptops than Year 7 students.
d. The Year 7 students own more tablets than laptops.
e. The number of Year 8 students with laptops is equal to the number of Year 7 students with game consoles.
f. There are 3 times as many Year 7 students with a game console than Year 8 students.

WE 2 **6.** 700 people were asked what their favourite pizza variety is. The results are shown in the following pie chart.



a. What percentage of people answered capricciosa?
b. What percentage of people answered hawaiian?
c. What is the most popular pizza variety?
d. How many people said their favourite pizza is pepperoni?
e. Vegetarian, mushroom and margherita all do not have meat. What percentage of people's favourite pizzas do not have meat?
f. How many people said their favourite pizza is vegetarian?

WE 3a **7.** Create a column graph for each set of data.

a. Winter sport played

Sport	Frequency
Soccer	4
Netball	5
Rugby	1
Aussie rules	10

b. Transport to school

Transport	Frequency
Train	20
Bus	60
Car	40
Bike	35
Walk	15

c. Favourite mobile game

Game	Frequency
Angry birds	100
Temple run	150
Candy crush	200
Fruit ninja	450
Subway surfers	300
Jetpack joyride	50

d. University course studied

Course	Frequency
Science	66
Law	21
Business	19
Arts	54
Medicine	24
Engineering	16

WE 3b **8.** Create a pie chart for each set of data.

a. Favourite food

Food	Frequency
Pizza	5
Sushi	8
Chicken wings	3
Burgers	4

b. Favourite dinosaur

Dinosaur	Frequency
Triceratops	12
Brachiosaurus	3
Tyrannosaurus Rex	30
Velociraptor	9
Stegosaurus	6

c. Number of siblings

Siblings	Frequency
None	54
One	90
Two	108
Three	72
Four+	36

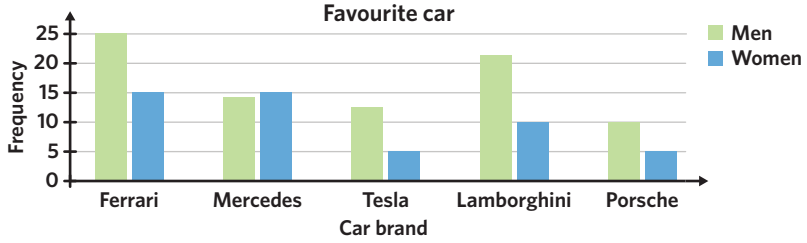
d. Favourite nail polish colour

Colour	Frequency
Red	24
Black	18
Dark green	6
Light blue	36
Purple	30
Brown	6

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

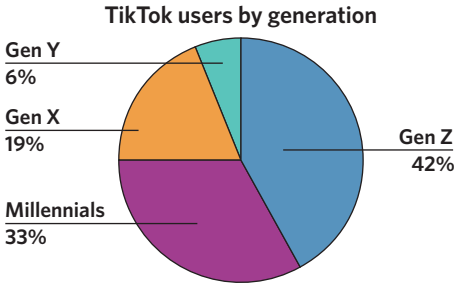
9. The following column graph shows survey results from men and women on their favourite cars.



How many more men favour Ferrari than favour Porsche?

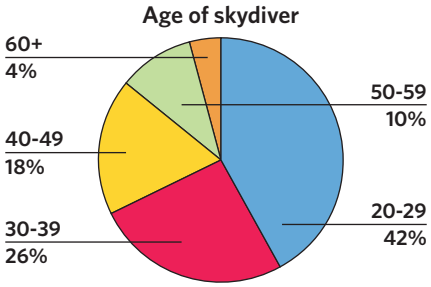
10. Using the same column graph from the previous question, what percentage of women said Tesla was their favourite car brand?

11. The following pie chart shows the percentage of TikTok users from each generation.



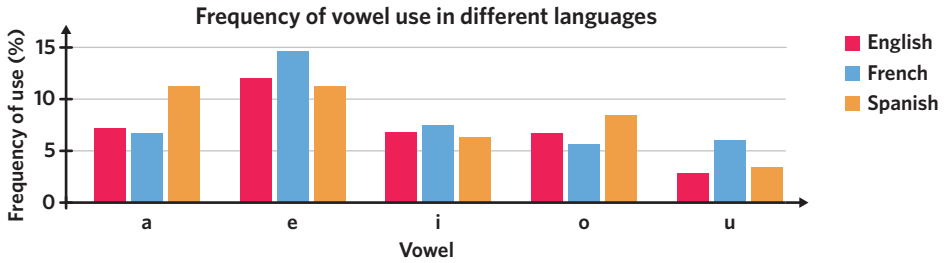
How many 'Gen Y' use TikTok if the app has 700 million users in total?

12. Michelle operates a skydiving business. She uses the following pie chart as a visual display to represent the age of her customers.



How many 60+ year olds does Michelle expect to skydive annually, if she predicts that 300 customers will skydive every six months?

13. The following column graph shows how frequently vowels are used in different European languages.

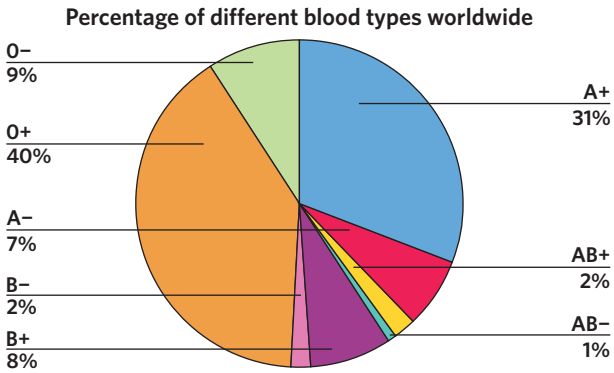


Which vowels are both used more frequently in Spanish than English and more frequently in English than French?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. The following pie chart shows the percentage of the worldwide population with each blood type.



- **a.** What is the most common blood type?
- **b.** People with O− blood type can donate blood to anyone. What percentage of people can donate blood to anyone?
- **c.** What percentage of people have a negative blood type?
- **d.** Which visual display, between pie graph and a column graph, do you think is most useful for showing data on blood types?

15. The percentages of different blood types in Australia is exactly the same as the worldwide percentages. Use the pie chart from question 17 to complete each question.

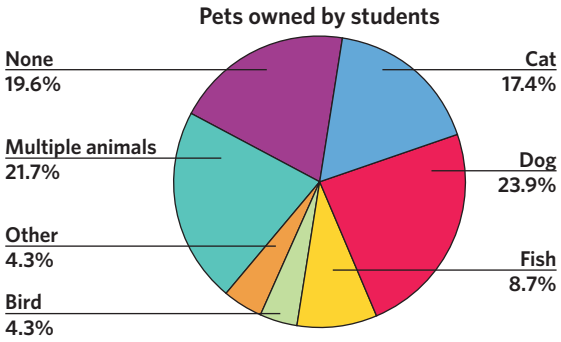
- **a.** Assume that there are 25 million people living in Australia. Create a column graph displaying the data seen in the pie chart.
- **b.** Only people with AB+ blood type can receive blood donations of any other blood type. How many Australians can receive blood donations from anyone?

c. People with A− blood type can only receive blood from people with O− and A−. How many Australians can donate to a person with A− blood type?

d. How would you persuade someone to donate blood?

Extra spicy

16. 46 students were surveyed on the pet they own. If they owned multiple types of animals, for example, a cat and a dog, it was recorded as 'multiple animals'.



What is the **maximum** number of students that could own a dog as a pet

A. 10 B. 11 C. 21 D. 22 E. 23

17. The data from the following table is used to create a pie chart. Rounded to the nearest hundredth degree, what is the size of the angle formed by New Zealand's sector?

Country	Gold medals won
USA	75
United Kingdom	44
Australia	52
New Zealand	21

18. A class of 39 students were surveyed on their favourite movie genre out of action, sci-fi, thriller and romance. Twice as many students favour action compared to sci-fi. Three times as many students favour thriller compared to romance. Half as many students favour thriller compared to action. Create a column graph showing the results of the survey.

Remember this?

19. $\frac{4}{9} + \frac{5}{6} = ?$

A. $1\frac{5}{18}$ B. $2\frac{1}{18}$ C. $1\frac{1}{2}$ D. $\frac{18}{19}$

20. There are 85 dogs entered into a dog show.

There are 13 more female dogs than male dogs.

How many male dogs are entered into the dog show?

A. 49 B. 43 C. 36 D. 32

21. The table below shows the number of tickets sold each weekday at a local cinema.

Day	Number of tickets sold
Monday	36
Tuesday	165
Wednesday	53
Thursday	45
Friday	129

What was the mean amount of money that the cinema collected each weekday if each ticket cost $15?

7E Line graphs

Line graphs can be used to understand the relationship between two numerical variables. Line graphs are often used to show changes over time. We can use a table of values to construct a line graph. This allows us to interpret and make estimations based on the information available. Below are some examples where line graphs can be used.

- The Ultraviolet (UV) index tracks the level of UV radiation from sunlight at different times of day. What times of day should I put on sunscreen?
- My ice cream shop tracks its sales over 12 months in a line graph. How much stock should I order for the winter months each year?
- I made a line graph to show how far I travelled each hour of a road trip. Approximately how far had I travelled after 2 hours and 45 minutes?

Learning intentions

Students will be able to:

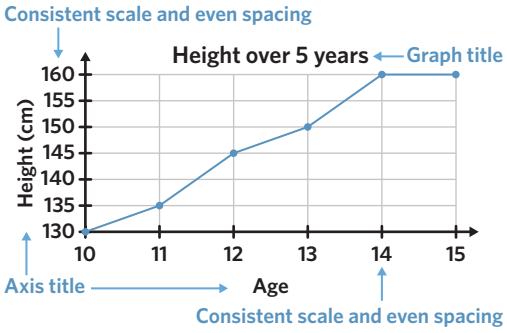
+ interpret line graphs
+ construct line graphs
+ estimate values within a line graph.

Key terms and definitions

- **Line graphs** contain points connected by straight lines. They are used to graph numerical data and show how a variable changes over time.
- Line graphs are used to represent **bivariate data**. This is data that has two numerical variables.
- A **coordinate** is usually made up of two numbers and describes the position of a point on a graph.
- A **variable** is a quantity that changes and can include qualities that are given numerical values. For example, time, temperature and sleep quality on a scale of 1 to 10.

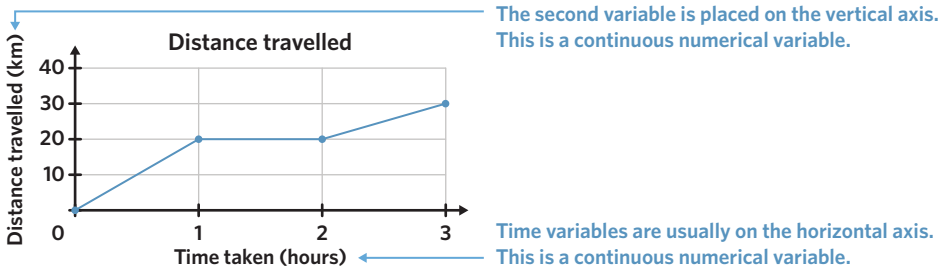
Key ideas

1 Line graphs must be labelled so that they can be interpreted correctly.

Line graph	Features
	• A graph title • A title for each axis • A consistent scale on the vertical and horizontal axis • Even spacing between each value on both axes

2 A line graph can be constructed from a table of values and is used when there are two continuous numerical variables.

Time taken (hours)	0	1	2	3
Distance travelled (km)	0	20	20	30



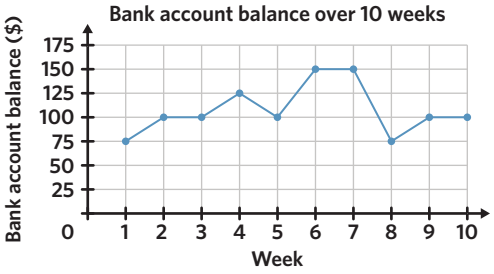
Some examples of variables on a line graph:

Description	Horizontal axis variable	Vertical axis variable
Height of a tomato plant over time	Time (weeks)	Height of tomato plant (cm)
Ice cream sales vs. temperature	Temperature (°C)	Ice cream sales ($)
Distance walked over time	Time (hours)	Distance walked (km)

Worked examples

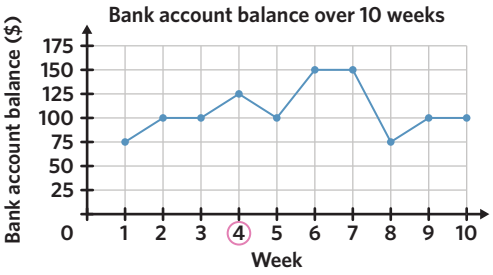
WE 1 Interpreting line graphs

The following line graph shows how much money Julien has in his bank account at the beginning of each week over a 10-week period.



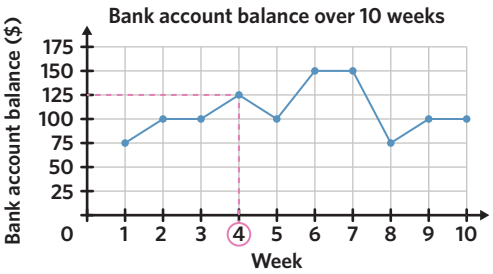
a. How much money does Julien have at the beginning of week 4?

Working



Thinking

Step 1: Identify the specified horizontal axis value (week 4).

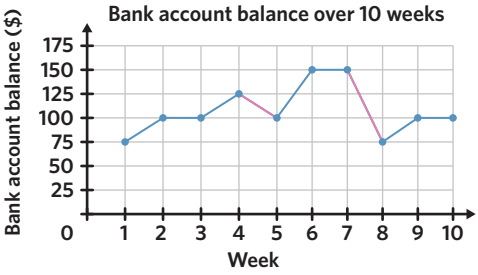


Step 2: Trace it to its corresponding vertical axis value.

Julien has $125 at the beginning of week 4.

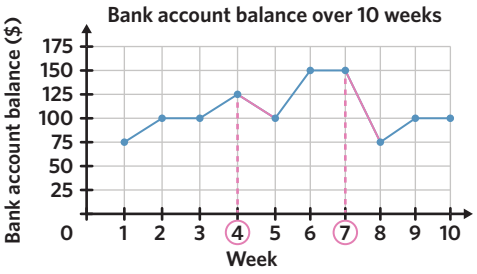
b. In which week(s) did Julien's bank account balance decrease?

Working



Thinking

Step 1: Identify the line segments that have a downwards slope.

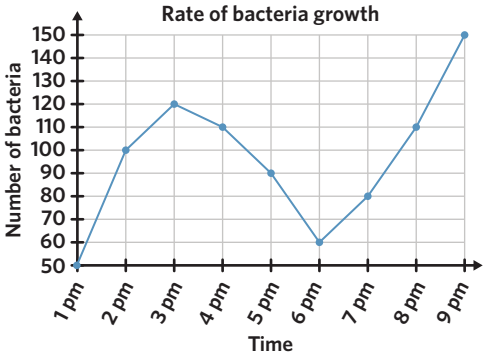


Step 2: Determine which horizontal axis values these correspond to.

Julien's bank balance decreased in weeks 4 and 7.

Student practice

The following line graph shows the rate of bacteria growth over 8 hours.



a. What is the number of bacteria at 2 pm?

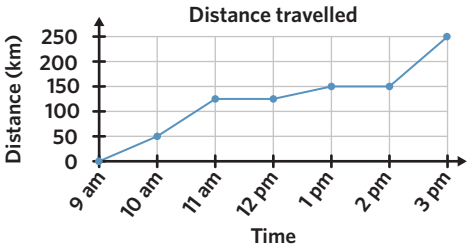
b. What is the number of bacteria at 5 pm?

c. At which hour is the number of bacteria at its lowest?

d. For how many hours is the number of bacteria declining?

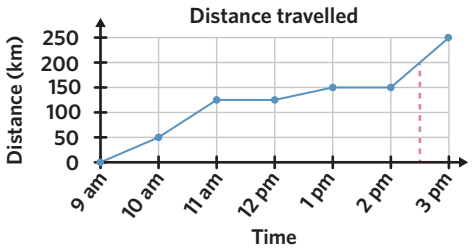
WE 2 Estimating values in a line graph

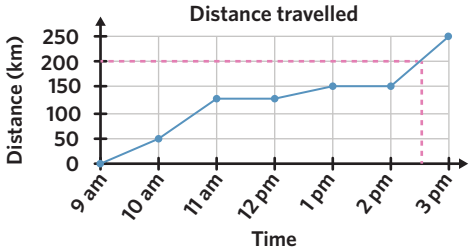
The following line graph shows the distance a car has travelled from 9 am to 3 pm.



a. How far has the car travelled by 2:30 pm?

Working





The car had travelled 200 km by 2:30 pm.

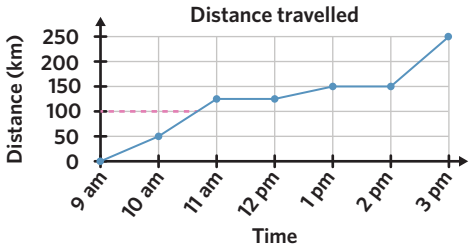
Thinking

Step 1: Draw a vertical line from the specified horizontal axis value (2:30 pm) until it reaches the graph.

Step 2: Trace the line horizontally across until it reaches the vertical axis.

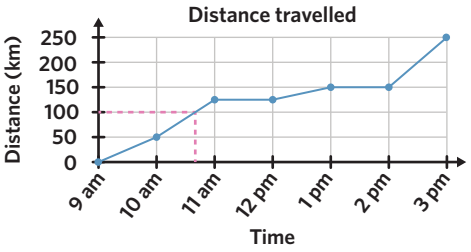
b. At what time will the car have travelled 100 km?

Working



Thinking

Step 1: Draw a horizontal line from the specified vertical axis value (100 km) until it reaches the graph.



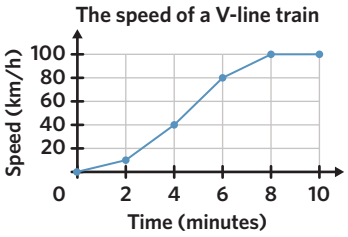
The car had travelled 100 km at approximately 10:40 am.

Note: Acceptable answer ranges from 10:38 am to 10:42 am.

Step 2: Trace the line vertically down until it reaches the horizontal axis. Estimate the horizontal axis value.

Student practice

The following line graph shows the speed of a V-line train.



a. How fast is the V-line train travelling 5 minutes after departing?

b. How fast is the V-line train travelling 7 minutes after departing?

c. How many minutes does it take for the train to reach 5 km/h?

d. How fast is the V-line train travelling 3 minutes after departing?

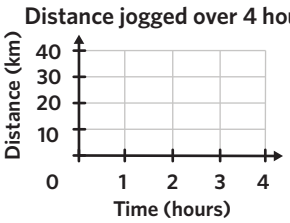
WE 3 Constructing line graphs

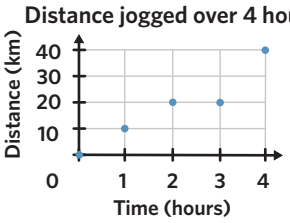
Create a line graph for each set of data.

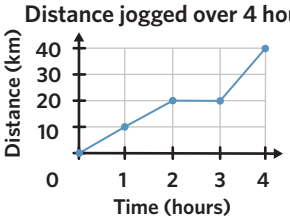
a. Distance jogged over 4 hours

Time (hours)	0	1	2	3	4
Distance (km)	0	10	20	20	40

Working







Thinking

Step 1: Write a title (Distance jogged over 4 hours) and draw the two axes. Label the axes with the appropriate variable and mark a consistent scale.

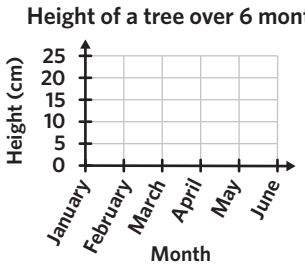
Step 2: Plot the coordinates presented in the table as points on the graph.

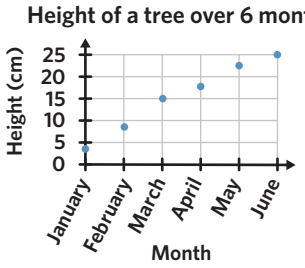
Step 3: Connect the points with straight lines.

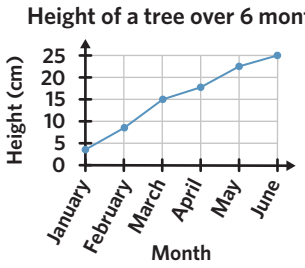
b. Height of a tree over 6 months

Month	January	February	March	April	May	June
Height (cm)	4	9	15	18	23	25

Working







Thinking

Step 1: Write a title (Height of a tree over 6 months) and draw the two axes. Label the axes with the appropriate variable and mark a consistent scale.

Step 2: Plot the coordinates presented in the table as points on the graph.

Step 3: Connect the points with straight lines.

Student practice

Create a line graph for each set of data.

a. Depth of water in a storage tank

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Depth (m)	2	4	3	0	3

b. The temperature of a winter afternoon in Melbourne

Time	1:00 pm	2:00 pm	3:00 pm	4:00 pm	5:00 pm
Temperature (°C)	15	16	13	12	8

c. The weight of a puppy

Week	1	2	3	4	5	6
Weight (kg)	2	2.5	2.5	3	4	4.5

d. The volume of water boiling in a pot

Time (minutes)	1	2	3	4	5	6
Volume (litres)	1	0.83	0.65	0.47	0.29	0

7E Activities and questions

STARTER TASKS

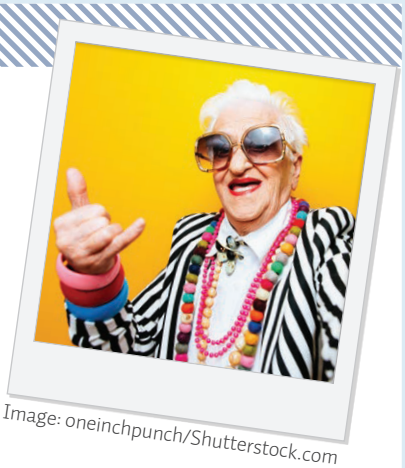


Image: oneinchpunch/Shutterstock.com

Odd spot

For thousands of years, humans have searched for ways to find happiness. Scientific evidence suggests that having strong social connections leads to happiness (life satisfaction) and can increase how long a person is expected to live (life expectancy).

A line graph can be used to track life satisfaction compared to age. If the two variables are age and life satisfaction, which variable should be on the vertical axis?

A. Life satisfaction

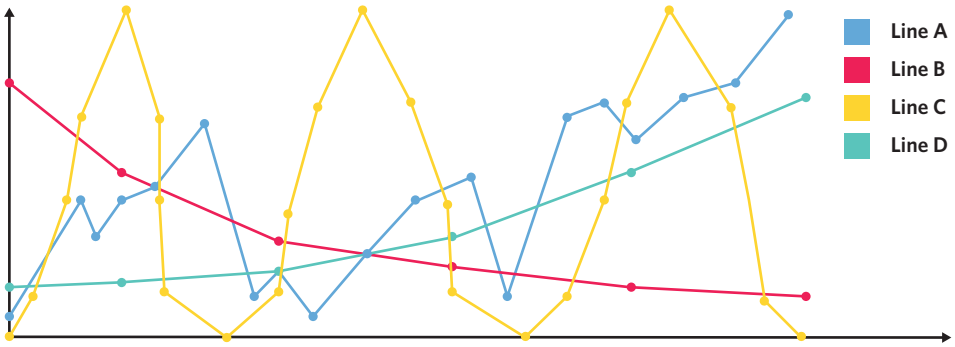
B. Age

Puzzle

Whose line is it anyway?

The lines shown on the graph represent the following variables.

- Height of a tree over time
- Height of the ocean tide over time
- Australian sharemarket price over time
- Value of a Toyota Corolla car over time



a) Match the line with the variable that makes the most sense, given the shape and direction of the line.

Line A	Height of a tree over time
Line B	Height of the ocean tide over time
Line C	Australian sharemarket price over time
Line D	Value of a Toyota Corolla car over time

b) State a different real life scenario that could be represented by line C.

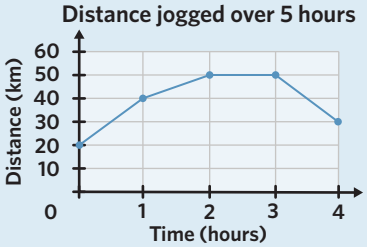
c) State a different real life scenario that could be represented by line D.

Understanding worksheet

1. Use the line graph to complete the missing information in each table.

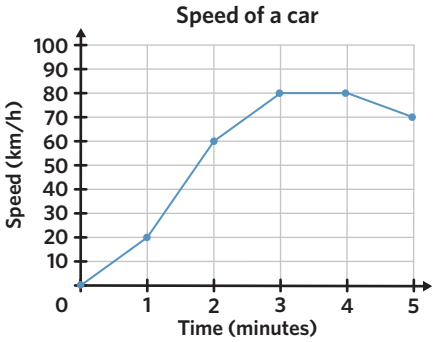
Example

Time (hours)	0	1	2	3	4
Distance (km)	20	40	50	50	30



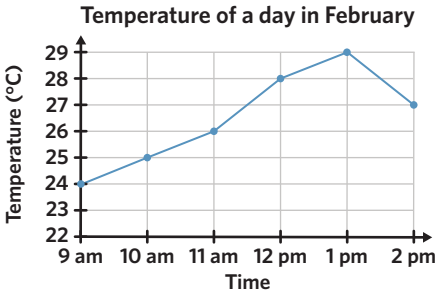
a.

Time (minutes)	0	1	2	3	4	5
Speed (km/h)	0		60		80	



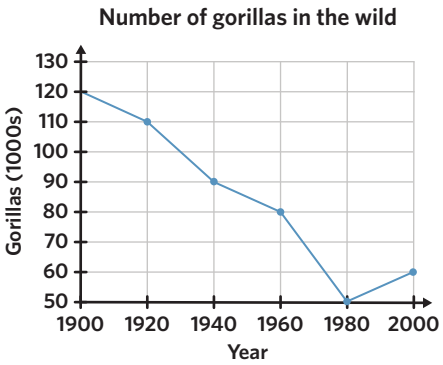
b.

Time	9 am	10 am	11 am	12 pm	1 pm	2 pm
Temperature (°C)	24				29	



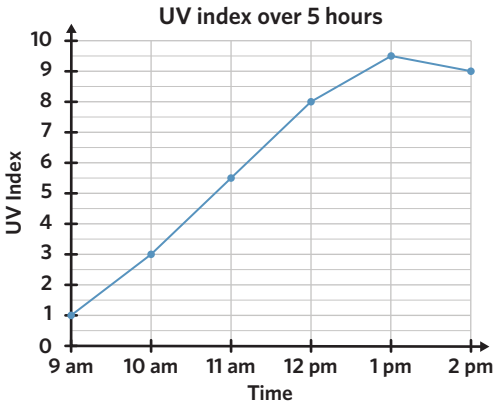
c.

Year		1920	1940			2000
Gorillas (1000's)	120			80	50	



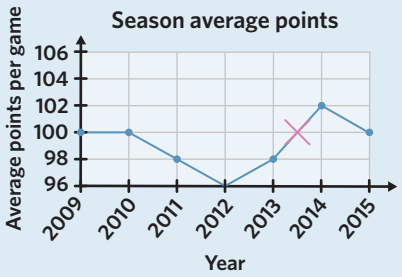
d.

Time		10 am		12 pm		2 pm
UV index	1	3			9.5	



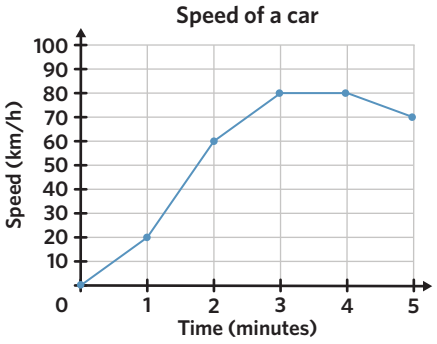
2. Mark the line graph and complete each estimation.

Example



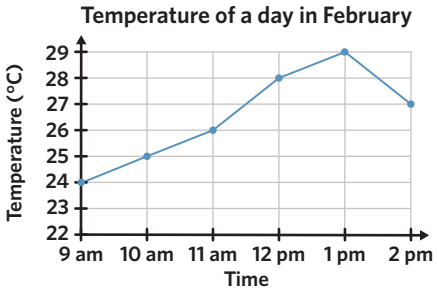
The average points per game in mid-2013 was 100.

a.



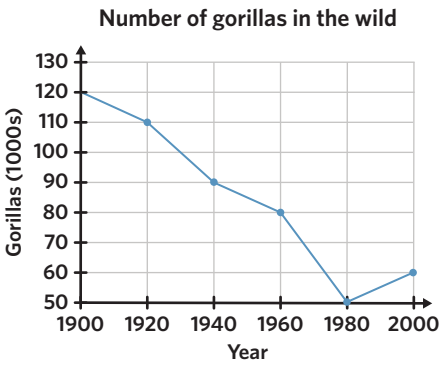
The speed of the car was ____ km/h at 2 minutes and 30 seconds.

b.



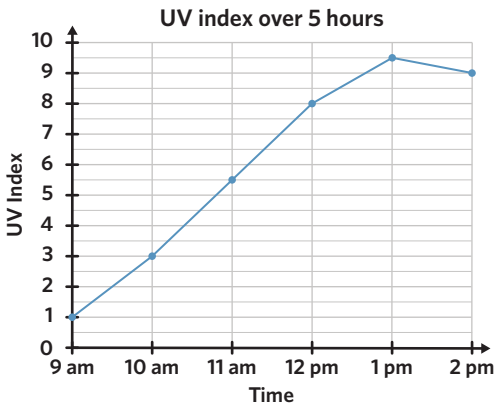
The temperature at 12:30pm was ____ degrees.

c.



In ____, there were 85 000 gorillas.

d.



At ____, the UV index was 2.

3. Fill in the blanks by using the words provided.

coordinates | bivariate | numerical | estimate | time

Line graphs are used to graph _____ data and usually show how a variable changes over _____. They require data that has two variables, also known as _____ data. _____ are used to locate points on a graph. We can also _____ the values along the line segments, between points.

Fluency

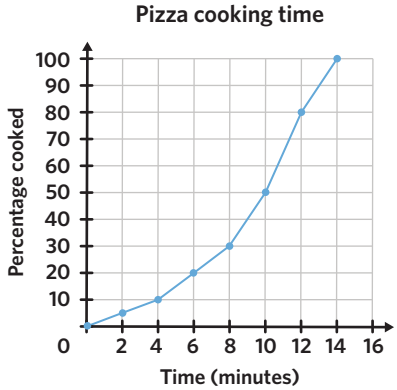
Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c)	4 (d,e,f), 5 (d,e,f), 6 (c,d,e), 7 (c,d)

WE 1 **4.** The following line graph shows the distance Jean walked over a 60-minute period.

a. How far has Jean walked after 20 minutes?
b. After how many minutes has Jean walked 2 km?
c. How long does Jean stop for breaks in total?
d. Excluding breaks, when is Jean walking the slowest?
e. In which time interval does Jean walk the same speed as the interval 30 to 40 minutes?
f. What was Jean's average speed in km/hour across the whole journey?

WE 2 **5.** The following line graph shows the time it takes to fully cook a pizza.



a. Approximately what percentage of the pizza had been cooked after 2 minutes?
b. Approximately what percentage of the pizza had been cooked after 7 minutes?
c. Approximately how many minutes does it take for the pizza to be 90% cooked?
d. Approximately how many minutes does it take for the pizza to be 15% cooked?
e. Approximately what percentage of the pizza was cooked from minutes 9 to 13?
f. Approximately how many minutes does it take for the pizza to increase from 40% to 80% cooked?

6. Determine which variable should be located on the horizontal axis and which should be located on the vertical axis.
   a. *distance travelled* and *time taken*
   b. *month* and *average daily temperature*
   c. *number of customers* and *time of day*
   d. *weight* and *age*
   e. *day of the week* and *money spent*

WE 3 7. Create a line graph for each set of data.
   a. Distance travelled

Time (hour)	0	1	2	3	4	5
Distance travelled (km)	0	5	15	30	45	45

   b. The average time people spend in a cafe

Time of the day	9:00 am	10:00 am	11:00 am	12:00 pm	1:00 pm	2:00 pm	3:00 pm
Average time (minutes)	30	38	42	65	31	22	18

   c. The weight of a baby over 5 weeks

Week	1	2	3	4	5
Weight (kg)	3.5	3.7	3.8	4	4.1

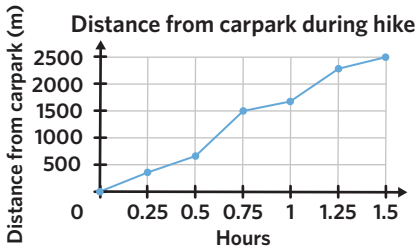
   d. Trunk circumference of a tree

Age of tree (years)	50	100	150	200	250
Trunk circumference (m)	2.20	3.55	4.10	4.95	5.65

Problem solving

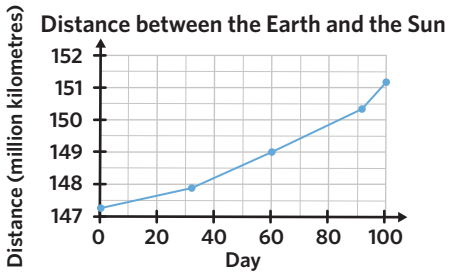
Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Max went hiking in the You Yangs mountain range. He created the following line graph to show how far he was from the carpark.



How far was Max from his car after 0.75 hours?

9. The following line graph shows the distance between the Sun and the Earth in million kilometres over 100 days.

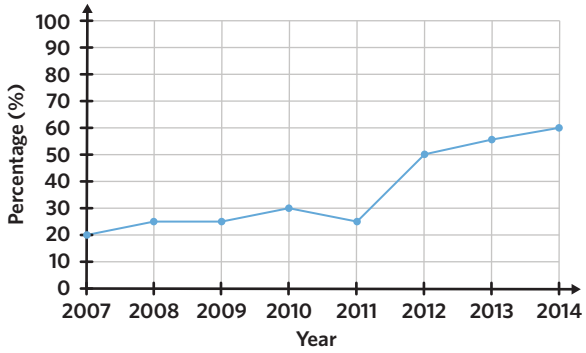


After approximately how many days does it take for the distance between the Sun and the Earth to exceed 151 million kilometres?

10. Construct a line graph to represent the following table, which shows the amount of petrol in a delivery driver's van at the beginning of each hour.

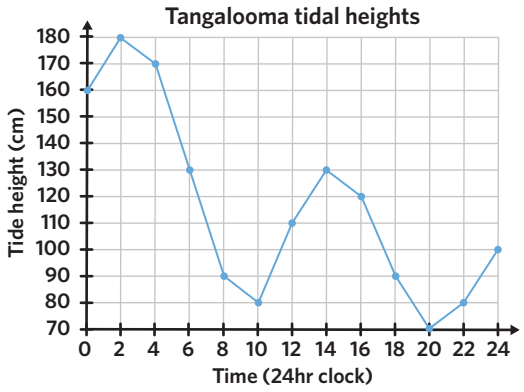
Time	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm
Petrol (L)	65	60	52	46	46	40	32

11. The following graph shows the percentage of water in a storage tank for a small town on the first day of each year, over a 7-year period.



For how many calendar years did the water storage percentage increase by approximately 5%?

12. Mikala created a line graph to show the change in tide height at her local beach in Tangalooma.

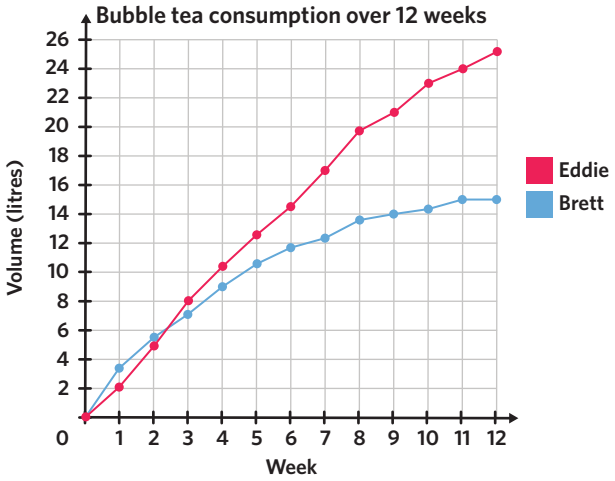


Between 5:00 am and 10:00 am, approximately how much did the tide change per hour? Round your answer to the nearest centimetre.

Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c,d)	All

13. Brett and Eddie are bubble tea fanatics. They recorded the total volume of bubble tea consumed over 12 weeks. The data is shown on the following graph.



a. How much bubble tea did Eddie consume in the first week?

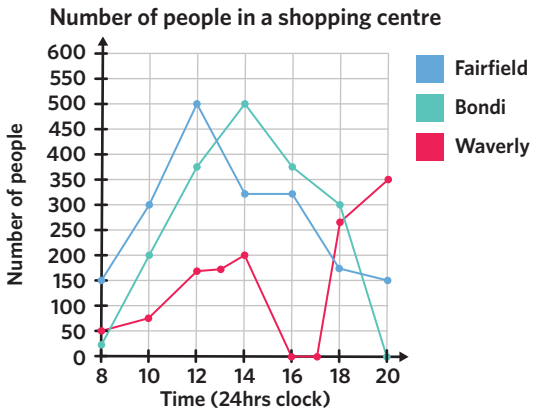
b. Who was more successful at cutting down on their bubble tea consumption?

c. Approximately how much bubble tea did Brett consume in the first 6 weeks?

d. By the end of week 12, approximately how much more bubble tea did Eddie consume compared to Brett? Round to the nearest litre.

e. Brett and Eddie want to spend less on bubble tea. Propose a strategy so that they spend less.

14. Mrs. Burd manages three small shopping centres. The following line graph shows the number of people in each shopping centre at different times on a particular day.



a. Which shopping centre had the most number of people at 10 am?

b. The fire alarm was activated at Waverly shopping centre at 2 pm and everyone was evacuated. How many hours did it take to evacuate the entire shopping centre?

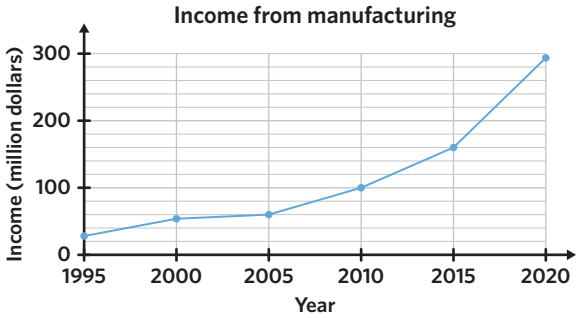
c. At approximately what time did the number of people in Bondi shopping centre first reach 350 people?

d. For approximately how long was the number of people in Bondi shopping centre greater than the number of people in Fairfield shopping centre? Round your answer to the nearest hour.

e. Mrs. Burd would like to sell one of her shopping centres. Other than the number of people that visit, what other factors should she consider when selecting which centre to sell?

Extra spicy

15. The following graph shows the growth of manufacturing from 1995–2020.



Over the course of this 25-year period, income from manufacturing grew to:

A. Approximately two times the original amount

B. Approximately three times the original amount

C. Approximately five times the original amount

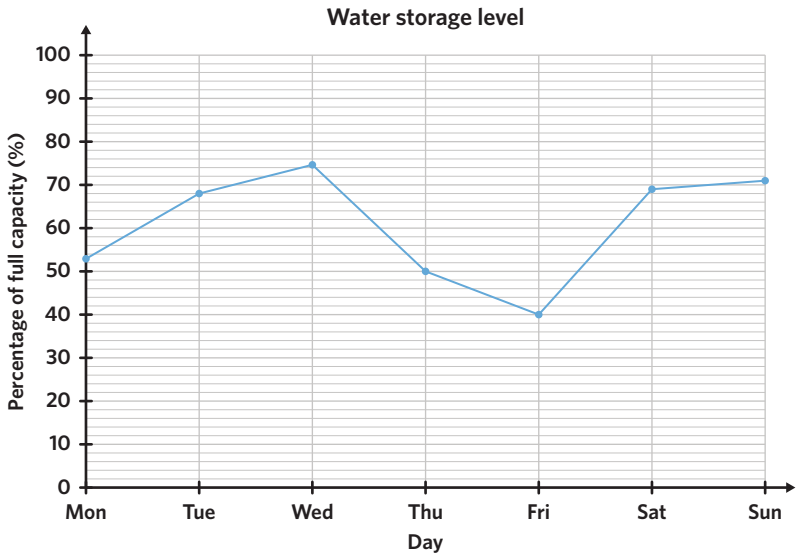
D. Approximately six times the original amount

E. Approximately nine times the original amount

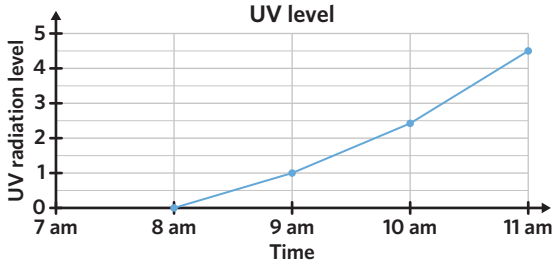
16. A school has a rainwater tank with a capacity of 2000 litres.

The following graph shows the percentage of water in the tank, measured each morning.

Calculate the change in water level for the tank from Thursday to Friday, providing your answer to the nearest litre.

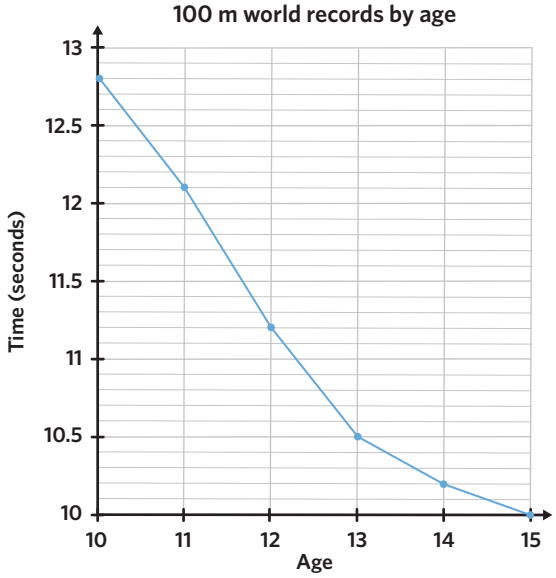


17. The following graph shows the Ultraviolet (UV) radiation level on a Melbourne autumn morning.



The UV level at 11 am is 40% of the daily maximum. Calculate the maximum UV level on this particular day.

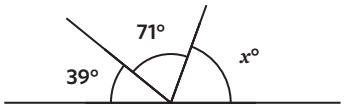
18. The following graph shows the world records for the 100 m sprint by age.



If *age* is represented as a continuous variable, how much time would it take a runner who is 11 years and 72 days old, to run the 100 m sprint? Give your answer to 1 decimal place.

Remember this?

19. Determine the value of x.

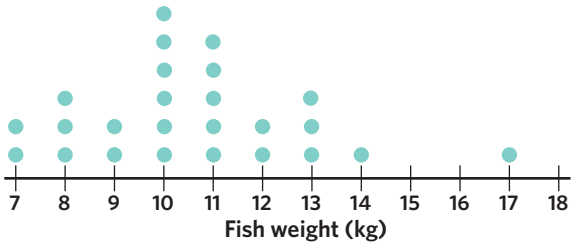


20. Sharelle lives 22.3 km from work.

In January, she rides to and from work 19 times.

How far did she ride in total in January?

21. Sandy enjoys fishing and catches 25 fish over the course of a week. She weighs the fish and makes a dot plot to show the weights of the fish she caught in kg.



What fraction of the fish weights are greater than 8 kg and less than 13 kg? Give your answer in its simplest form.

7F Introduction to probability

Probability is the likelihood of an event occurring and can be tested by conducting chance experiments. In this lesson we will look at some of the key terms relating to probability, including trials, outcomes, sample spaces and events. We will also learn how to describe the likelihood of specific events occurring in a chance experiment compared to other possible events. Below are some examples where probability can be applied.

- I want to describe how likely it is that Collingwood will win its next AFL game.
- I am asked for the sample space of a dice roll.
- I am attempting to create a spinner with an even chance of landing on red.

Learning intentions

Students will be able to:

+ use the terms chance experiment, event, sample space, probability, trial and outcome
+ identify whether events have an impossible, unlikely, even, likely or certain chance of occurring
+ construct and identify the outcomes of chance experiments.

Key terms and definitions

- A **chance experiment** is a test conducted to examine the results of chance activities, such as tossing a coin, rolling a die or picking a card out of a well-shuffled deck.
- A **trial** is a single test of the chance activity. E.g. tossing a coin once would be one trial.
- An **outcome** is the result of the trial. E.g. a coin landing on heads.
- The **sample space** refers to all the possible outcomes of an experiment. E.g. the sample space of rolling a 6-sided die is written as $S = \{1, 2, 3, 4, 5, 6\}$.
- An **event** refers to a single outcome or a grouped outcome of a chance experiment. Tossing a coin and landing a head is a single outcome event. Rolling a die and landing an even number is a grouped outcome event because there are multiple outcomes.
- **Probability** refers to the likelihood of an event occurring and is expressed as a number between 0 and 1 (inclusive). It is generally expressed as a fraction, decimal or percentage, or using words such as impossible, unlikely, even, likely or certain.

Key ideas

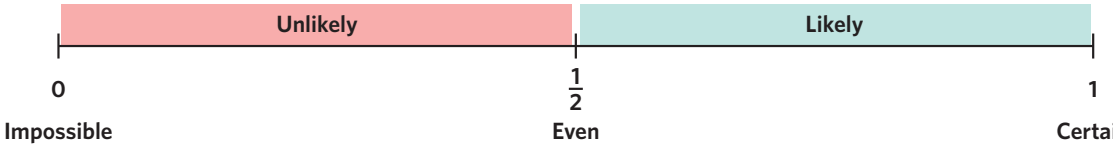
1 Events, sample spaces, probability, trials and outcomes are all involved in a chance experiment.

In the following chance experiment we are testing the probability of rolling a 3 or 5 with a standard 6-sided die.

Chance experiment	Sample space	Event	Probability	Trial	Outcome
E.g. rolling a die	$S = \{1, 2, 3, 4, 5, 6\}$	Rolling a 3 or 5	Rolling a 3 or 5: unlikely Not rolling a 3 or 5: likely	Rolling a die once	Rolled a 4: The event did not occur

2 The probability of events can be described as impossible, unlikely, even, likely or certain.

Classification	Description	Example
Impossible	No chance of occurring.	Your laptop transforming into a cat.
Unlikely	Possible, but occurs less than half of the time.	Winning the lottery.
Even chance	Equally as likely to occur as it is to not occur.	Picking a red card out of a shuffled deck of cards.
Likely	Occurs more than half of the time but is not guaranteed.	Rain falling at some point during Spring.
Certain	Guaranteed to occur.	Saturday coming after Friday.



3 We can compare the likelihood of different events to each other.

- Green and yellow both have two segments each. This means that the chances of landing on green and yellow are **equally likely.**
- Landing on green is **twice as likely** as landing on red. The same can be said for yellow compared to red.
- Landing on blue is also **three times as likely** as landing on red.

Worked examples

WE 1 Constructing and identifying the outcomes of chance experiments

Jimmy is testing probability by picking fruits and vegetables out of his fridge at random. In his fridge, he has an apple, carrot, lettuce, pumpkin, banana, broccoli, and watermelon.
He wants to test the probability of picking out a vegetable. He picks out an item and records what it is. In the first trial Jimmy picked out a pumpkin.

a. **i)** What is the chance experiment?

ii) What is the sample space?

Working

i) Picking a fruit or vegetable out of the fridge.

ii) $S = \{\text{apple, carrot, lettuce, pumpkin, banana, broccoli, watermelon}\}$

Thinking

Step 1: Describe the test that is being conducted.

Step 2: List all of the possible outcomes of the experiment in the form $S = \{.., ..\}$.

b. What is the event? Did the event occur in the outcome of the first trial?

Working

The event is a vegetable being picked out.

Yes. The event was the outcome of the first trial.

Thinking

Step 1: Describe the outcome or grouped outcome of the chance experiment whose probability is being tested.

Step 2: Determine whether the event was the outcome of the first trial. A pumpkin is a vegetable, so the event occurred.

Student practice

Leo is rolling a standard 8-sided die. He wants to test the probability of rolling either a 2, 4 or 8. He rolls the die and records the number. In the trial Leo rolled a 6.

a. What is the chance experiment?

b. What is the sample space?

c. What is the event?

d. What was the outcome of the first trial? Did the event occur?

WE 2 Describing probabilities

Consider a fair 6-sided die containing the numbers 1, 3, 4, 5, 5 and 8. Classify the likelihood of each event occurring as impossible, unlikely, even, likely or certain.

a. Rolling an odd number.

Working	Thinking
Odd numbers: 1, 3, 5, 5	**Step 1:** Determine the number of outcomes where the event does occur. The event is rolling an odd number.
Even numbers: 4, 8	**Step 2:** Determine the number of outcomes where the event does not occur.
It is likely that an odd number will be rolled.	**Step 3:** Compare the number of outcomes. There are a greater number of odd number outcomes than even number outcomes so it is likely that an odd number will be rolled.

b. Rolling a 5 compared to rolling a 1.

Working	Thinking
There are two 5's.	**Step 1:** Determine the number of outcomes where one event occurs. The event is rolling a 5.
There is one 1.	**Step 2:** Determine the number of outcomes where the other event occurs. The event is rolling a 1.
Rolling a 5 is twice as likely as rolling a 1.	**Step 3:** Compare the number of outcomes of where each event occurs.

Student practice

Consider the following spinner. Classify the likelihood of each event occurring as impossible, unlikely, even, likely or certain.

a. Landing on red.

b. Landing on purple.

c. Landing on green compared to yellow.

d. Landing on blue compared to red.

7F Activities and questions

STARTER TASKS


Image: Cloudsrest Images/ Shutterstock.com

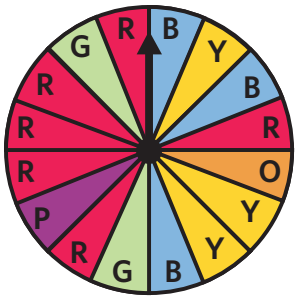
Odd spot

In June 2020, Western Australia experienced torrential storms over the course of one weekend. There were a predicted 22 000 lightning strikes in Perth, 44 000 strikes in Bunbury and 318 000 strikes in Mandurah. How much more likely were Perth residents to be struck by lightning compared to Bunbury residents?

A. Half as likely

B. Twice as likely

Puzzle



Order these events from least likely to most likely.

A. The spinner landing on red, purple or green.

B. The spinner landing on grey.

C. The spinner landing on any one of the colours shown on the spinner.

D. The spinner landing on yellow.

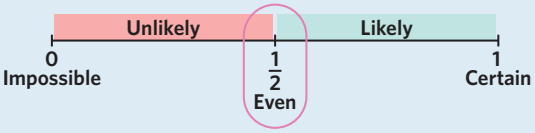
E. The spinner landing on red or blue

Understanding worksheet

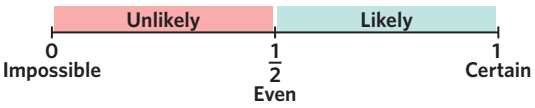
1. Circle impossible, unlikely, even, likely or certain to describe the probability of each event occurring.

Example

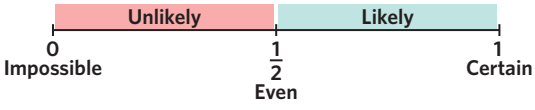
Picking whether someone has an object in their left or right hand.



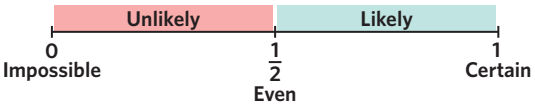
a. Throwing a dart at a dartboard with your eyes closed and hitting the bulls-eye.



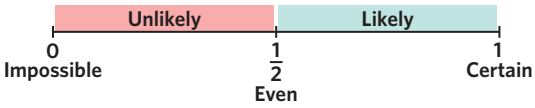
b. 60 seconds passing in one minute.



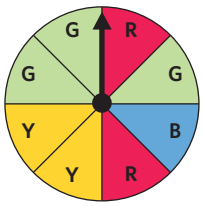
c. Tossing 6 heads in a row.



d. A person jumping over the moon.



2. Tick either true or false for the statements about the following spinner.



Example

	True	False
Yellow and red have an equal chance of occurring.	✓	

		True	False
a.	Green is three times as likely to occur as red.		
b.	Yellow and green have an equal chance of occurring.		
c.	Green has a likely chance of occurring.		
d.	Red is twice as likely to occur as blue.		

3. Fill in the blanks by using the words provided.

trials | outcome | probability | sample space

_____ measures the likelihood of an event occurring. It can be tested using a chance experiment, where repeated _____ are conducted and the _____ of each is recorded. The _____ is all the possible outcomes of an experiment.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c)	4 (a,b,c), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d)	4 (a,b,c,d), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f)

WE 1 **4.** Ms Swatt is testing probability by picking cards out of a hat. She wants to test the probability of picking out a face card (J, Q or K). She picks out a card and records what it is. In the first trial Ms Swatt picked out the 10.

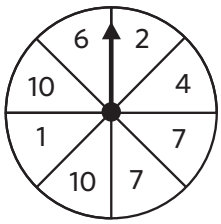
a. What is the chance experiment?
b. What is the sample space?
c. What is the event?
d. What is the outcome of the first trial? Did the event occur?

5. For each of the following chance experiments:

i) State the sample space.
ii) Identify the event.

a. Rolling an even number on a fair 12-sided die.
b. Randomly selecting the letter 'o' out of all the vowels in the alphabet.
c. Picking any coloured ball at random out of the bag in the following image.

d. Spinning the following spinner and landing a 4 or 10.



e. Picking an ace out of a shuffled deck of cards (ignoring the suit).

f. Picking an even numbered card out of a shuffled deck of cards (ignoring the suit).

WE 2a **6.** State whether the following outcomes have an impossible, unlikely, even, likely or certain chance of occurring.

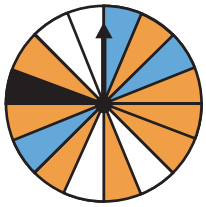
a. Rolling a fair 6-sided die and landing on a 4.

b. The time being 11:00 am one hour after 10:00 am.

c. All the students in one class getting the exact same result on a maths test.

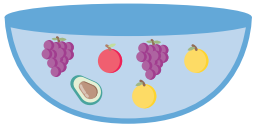
d. Getting 13 questions correct on a test containing 10 questions.

e. Landing orange on the following spinner.

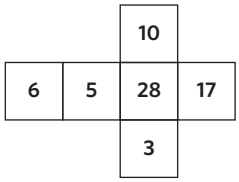


f. Selecting a teddy bear at random out of the following toy box.

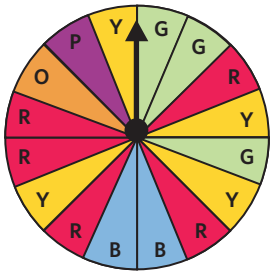
g. Picking a banana at random out of the following fruit bowl.



h. Rolling an odd number with a die created from the following net.



WE 2b **7.** Fill in the blanks for the statements related to the spinner.



a. Landing on green is _____ times more likely than landing on purple.

b. Landing on red is _____ times more likely than landing on orange.

c. Landing on _____ is two times more likely than landing on blue.

d. Landing on orange has the same chance as occurring as landing on _____.

e. Landing on either red or _____ has an even chance of occurring.

f. Landing on red has the same chance as occurring as landing on either green or _____.

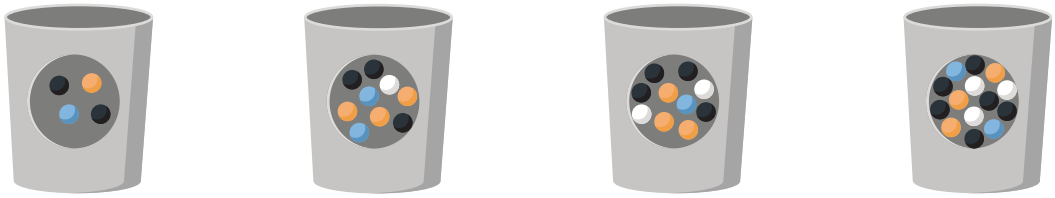
Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Isabelle uses a random number generator to choose a number from 1 to 20. She decides the event for her chance experiment is the random number generated being an even number. Is the event unlikely to occur, an even chance to occur, or likely to occur?

9. Mark is rolling a 12-sided die with the numbers 1, 1, 3, 4, 5, 5, 6, 8, 8, 10, 11, 12. What outcomes satisfy the conditions of the event if Mark is testing the probability of rolling a number greater than 6?

10. Nick randomly selects a ball from one of 4 buckets. From which bucket is Nick most likely to select a black ball?

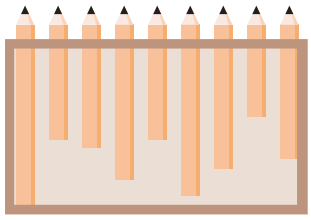


1 2 3 4

11. Jim has been asked to select one pencil out of the following pencil case. Only the tops are visible, so Jim doesn't know the length of each pencil.

He selects a pencil at random. Which of the following is least likely to happen?

A. A pencil longer than 10 cm is selected.
B. A 9 cm pencil is selected.
C. A 20 cm pencil is selected.
D. A pencil shorter than 6 cm is selected.



20 9 13 17 9 18 15 6 14
Pencil lengths (cm)

12. Construct a spinner with the following conditions:

- 8 equally sized segments
- 5 colours: red, blue, orange, purple, yellow
- An even chance of landing on purple.

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c,d)	All

13. 100 tickets are handed out at a raffle night. There are four different colours: black, blue, orange and green. Each colour has 25 tickets labelled 1–25. Johnny has bought every ticket from 1 to 5 from each colour. Ella has bought each of the remaining blue, orange and green tickets. One ticket is drawn out as the winner.

a. Describe the probability of Ella winning.

A. Unlikely **B.** Even chance **C.** Likely **D.** Certain

b. Describe the event in which Johnny wins as well as its probability of occurring.

c. Describe the event in which neither Johnny nor Ella wins as well as its probability of occurring.

d. The raffle night didn't draw a large crowd. How could the organisers increase attendance of the event?

14. A regular deck of cards has 4 suits: clubs, diamonds, hearts and spades. Each suit contains 13 cards: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Jazzy shuffles a regular deck and picks a card at random.

a. How many possible outcomes are there?

b. Describe the likelihood that a club or a diamond is selected.

c. Does the likelihood of selecting a club or diamond change if two regular decks are used instead of one? If it has changed, describe what it has changed to.

d. All of the spades are removed from the deck. Does the likelihood of selecting a club or diamond change? If it has changed, describe what it has changed to.

e. Jazzy's favourite cards are the J, Q and K and is very confident she can pick one of her favourite cards at random. Explain why Jazzy should not be so confident.

Extra spicy

15. Trey is randomly selecting a card out of a regular shuffled deck. What is the probability that a spade, king, queen, jack or ace is picked out?

A. Impossible **B.** Unlikely **C.** Even chance **D.** Likely **E.** Certain

16. A spinner has 13 blue sectors, 2 red sectors, 1 yellow sector, 4 orange sectors, 2 green sectors and an unknown number of purple sectors. What is the minimum number of purple sectors so that it is unlikely for the spinner to land on blue?

17. The following image shows the cafeteria menu at a school.

Menu (v) = Vegan

Lunch
Chicken sandwich: $8
Tofu salad (v): $9
Meat pie: $5
Sausage roll: $5
Vegan pizza (v): $10
Lamb curry: $11
Potato curry (v): $12
Spaghetti bolognese: $9
Eggplant lasagne(v): $10
Ham and cheese toastie: $4
Fruit salad (v): $6

Snacks
Brownie: $4
Jelly (v): $3
Hummus and carrot sticks (v): $4
Choc chip cookie: $2
Banana bread (v): $3

Drinks
Soft drink: $1.50
- Raspberry (v)
- Orange (v)
- Lemon (v)

Chocolate milk: $3
Strawberry milk: $2.50

Jason is a vegan and randomly selects a lunch item, a snack, and a drink for lunch. How many possible vegan combinations could he purchase?

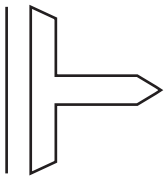
A. 5 **B.** 12 **C.** 15 **D.** 30 **E.** 45

18. A bag contains the following shapes: squares, circles, diamonds, triangles and hexagons. State the minimum number of each shape that could be in the bag so that a random selection of a shape satisfies the following conditions:

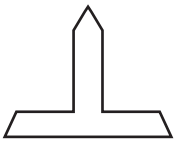
- There should be an even chance of selecting a circle.
- Selecting a circle should be 3 times as likely as selecting a square.
- Selecting a diamond should be equally likely as selecting a square.
- Selecting a square should be twice as likely as selecting a triangle.
- Selecting a triangle should be equally as likely as selecting a hexagon.

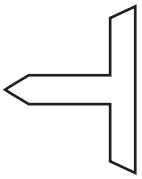
Remember this?

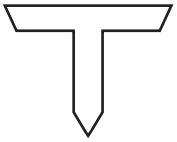
19. The following shape is reflected across the line and then rotated 90° anti-clockwise.

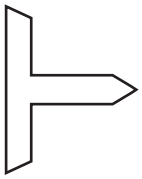


Which image shows the appearance of this shape after the transformations?

A. 

B. 

C. 

D. 

20. Dion has 3 less than 4 times the number of books that Toby has.

If b represents the number of books Toby has, which expression represents the number of books Dion has?

A. $3b - 4$ **B.** $4b + 3$ **C.** $3b + 4$ **D.** $4b - 3$

21. Peggy and Nancy both ran every day over a two-week period. The stem-and-leaf plots show the distance they travelled in each of their last 14 runs.

Peggy

Stem	Leaf
1	8 9
2	4 4 5 8
3	1 3 3 3 6
4	5 6
5	0

Nancy

Stem	Leaf
1	4 4 7
2	0 1 1 4
3	3 4 4 7
4	0 2

Key

1 | 8 = 1.8 km

Which one of the following statements about the data is false?

A. Peggy ran the longest single distance.

B. Peggy ran a distance greater than 3.3 km more times than Nancy.

C. The range of distances run by Peggy is greater than the range of distances run by Nancy.

D. The median distance run by Nancy is less than the median distance run by Peggy.

7G Probability of events

The probability of an event occurring can be represented by a number from 0 to 1. We can calculate two different kinds of probabilities; theoretical probability and experimental probability. Instead of using words such as likely and unlikely to describe probability, we will begin to calculate the percentage chance of an event occurring. Below are some examples where probability can be applied.

- I want to determine the probability of rolling an even number on a standard die.
- I want to know how many times I should pick a heart out of a standard deck if I pick a card 100 times at random.
- I have just flipped a coin and got heads 5 times in a row. What is the probability of the coin landing on heads on the next flip?

Learning intentions

Students will be able to:

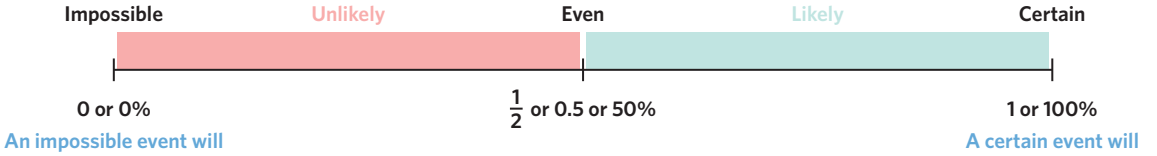
+ calculate theoretical probabilities of simple events
+ calculate the expected number of occurrences of an event
+ calculate the experimental probability of an event given the results of an experiment
+ understand how experimental probability relates to theoretical probability with a large number of trials.

Key terms and definitions

- The **theoretical probability** is a measurement of the likelihood that an event will occur.
- The **expected occurrence** of an event is the number of times an event is expected to take place in a given number of trials.
- The **experimental probability** of an event is the probability of an outcome that is obtained from the results of an experiment.

Key ideas

1 Both theoretical and experimental probability are expressed as a number ranging from 0 to 1. They can be in the form of a decimal, fraction or percentage.



An impossible event will have a probability of 0.

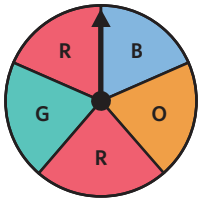
A certain event will have a probability of 1.

2 Theoretical probability measures the likelihood of an event occurring. It can be expressed as a decimal, fraction or percentage.

$$\textit{theoretical probability} = \frac{\textit{number of outcomes where the event occurs}}{\textit{total number of possible outcomes}}$$

Instead of writing 'the theoretical probability of an event is', we can write $Pr(event)$.

E.g. Determine the theoretical probability of landing on red on the following spinner.



Number of red segments: 2
Total number of outcomes: 5

$Pr(red) = \frac{2}{5} = 40\% = 0.4$

3 The expected occurrence is the number of times an event is expected to happen in a certain number of trials. The expected occurrence will not necessarily be the actual outcome.

$$\textit{expected occurrence} = \textit{theoretical probability} \times \textit{number of trials}$$

0.5 × 10 → 5 heads

The theoretical probability of a coin flip landing on heads is 50% or 0.5.

Imagine 10 trials are conducted.

It is expected that the coin will land on heads 5 times.

4 Experimental probability is an estimate of theoretical probability that is found using the results of an experiment. It can be expressed as a decimal, fraction or percentage.

$$\textit{experimental probability} = \frac{\textit{number of times the event occurred}}{\textit{total number of trials}}$$

E.g. The experimental probability of rolling a 4 on a fair 6-sided die is tested by rolling a die 10 times.

Number rolled							Total
Occurrences	2	1	0	3	1	3	10

$$\textit{experimental probability} = \frac{3}{10} = 30\% = 0.3$$

5 As the number of trials increases, the experimental probability should get closer to the theoretical probability.

E.g. A coin is flipped a differing number of times and the number of heads is noted.

Number of coin flips	Number of times 'heads' was flipped		Experimental probability
2	II	(2)	$\frac{2}{2} = 1$
10	𝍸 II	(7)	$\frac{7}{10} = 0.7$
20	𝍸 𝍸 II	(12)	$\frac{12}{20} = 0.6$
50	𝍸 𝍸 𝍸 𝍸 𝍸 II	(27)	$\frac{27}{50} = 0.54$

The experimental probability should get closer to the theoretical probability of 0.5 as the number of trials increases.

Worked examples

WE 1 Determining theoretical probabilities of simple events

State the theoretical probability of each event. Give your answer as a number.

a. Summer occurring after spring.

Working

The theoretical probability is 1.

Thinking

Determine how likely it is that the event will take place. Summer is certain to follow spring, therefore the probability is 1.

b. Rolling two standard 6-sided dice and the sum of the faces being 1.

Working

The theoretical probability is 0.

Thinking

Determine the likelihood of the event. The smallest number on a standard 6-sided die is 1, so the smallest sum of two rolls is 2. This means that the event is impossible.

Student practice

State the theoretical probability of each event.

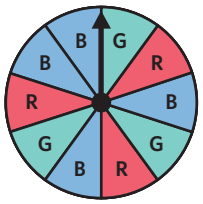
a. A week lasting 7 days.

b. Rolling a 7 on a standard 6-sided die.

c. Winning the lottery without buying a ticket.

d. 8 plus 10 equalling 18.

WE 2 Calculating theoretical probabilities and expected occurrences



a. Calculate the theoretical probability, as a decimal, of the spinner landing on red.

Working

Red segments: 3

Total segments: 10

$$Pr(red) = \frac{3}{10}$$
$$= 0.3$$

The theoretical probability of the spinner landing on red is 0.3.

Thinking

Step 1: Count the number of red segments and the total number of segments.

Step 2: Divide the number of red segments by the total number of segments and convert to the required form (decimal).

b. Calculate the expected number of occurrences of the spinner landing on blue from 20 spins.

Working

$$Pr(blue) = \frac{4}{10}$$

$$expected\ occurrence = \frac{4}{10} \times 20$$
$$= \frac{80}{10}$$
$$= 8$$

The expected occurrence of the spinner landing on blue after 20 trials is 8.

Thinking

Step 1: Divide the number of blue segments by the total number of segments to calculate the theoretical probability.

Step 2: Multiply the theoretical probability of landing on blue by the number of trials.

Student practice

A fair 10-sided die has faces with the numbers: 3, 3, 3, 5, 8, 8, 8, 9, 10, 10.

a. Calculate the theoretical probability, as a fraction, of rolling a 3.

b. Calculate the expected number of occurrences of rolling an 8 from 20 rolls.

c. Calculate the theoretical probability, as a decimal, of rolling an odd number.

d. Calculate the expected number of occurrences of rolling a number 8 or larger from 150 rolls.

WE 3 Calculating experimental probability

The letters in 'MATHS' were placed in a hat and picked out randomly and then put back in the hat. The results after 10 trials are shown in the following table.

Letter	M	A	T	H	S
Occurrences	3	0	2	1	4

a. Calculate the experimental probability, as a decimal, of picking out an 'S'.

Working

$$experimental\ probability = \frac{4}{10}$$
$$= 0.4$$

The experimental probability of picking out an 'S' is 0.4.

Thinking

Divide the occurrences of 'S' being picked by the total number of trials to determine the experimental probability.

b. Determine whether the experimental probability of picking out an 'S' is less than, equal to, or greater than the theoretical probability.

Working

$$\text{experimental probability} = \frac{4}{10}$$
$$= 0.4$$

$$\text{theoretical probability} = \frac{1}{5}$$
$$= 0.2$$

$0.4 > 0.2$

The experimental probability of picking out an 'S' is greater than the theoretical probability.

Thinking

Step 1: Determine the experimental probability of picking out an 'S' by dividing the occurrences of an 'S' by the total number of trials.

Step 2: Determine the theoretical probability of picking out an 'S'.

Step 3: Compare the experimental probability to the theoretical probability.

Student practice

The letters in 'GOLF' were placed in a hat and picked out randomly and then put back in the hat. The results after 20 trials are shown in the following table.

Letter	G	O	L	F
Occurrences	5	0	9	6

a. Calculate the experimental probability, as a fraction, of rolling an 'L'.

b. Determine whether the experimental probability of rolling a 'G' is less than, equal to, or greater than the theoretical probability.

c. Calculate the experimental probability, as a decimal, of rolling an 'O' or 'F'.

d. Determine whether the experimental probability of rolling a 'G' or 'F' is less than, equal to, or greater than the theoretical probability.

7G Activities and questions

STARTER TASKS

Image: Studio Romantic/Shutterstock.com

Odd spot

Roulette is a game commonly seen in casinos where a ball is dropped on a spinning wheel. There are two colours on the wheel, red and black, and the ball is equally likely to land on either.

In 1943 a streak of games was recorded where the ball landed on red 32 times in a row. What was the theoretical probability of the ball landing on red again on the next spin?

A. Less than 0.5 **B.** 0.5

Puzzle

The Monty Hall problem is a famous probability brain teaser that has tricked many mathematicians. See if you can answer it.

Imagine a game show where you have to pick between 3 identical doors. One of the doors has a brand new car behind it, while the other two have goats behind them. Once you have chosen a door, one of the two doors that you **did not** pick is opened to reveal a goat. You then get the choice to stick with the door you originally chose, or switch your choice to the other door that is still closed. Should you switch or does it make no difference?

Understanding worksheet

1. Complete the missing information and calculate the theoretical probability of each event.

Example

Selecting a black ball.

Black balls: 4 Total balls: 9

Fraction: $\frac{4}{9}$

a. Selecting a black ball.

Black balls: ☐ Total balls: 10

Fraction: $\frac{\square}{\square}$

b. Selecting a black card.

Black cards: 5 Total cards: ☐

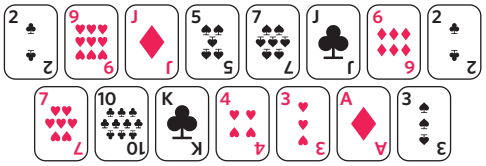
Fraction: $\frac{\square}{\square}$

c. Selecting an orange ball.

Orange balls: ☐ Total balls: ☐

Fraction: $\frac{\square}{\square}$

d. Selecting a diamond (♦) or a club (♣).



Diamonds or clubs: ☐ Total cards: ☐

Fraction: $\frac{\square}{\square}$

2. Calculate the expected number of occurrences of each event from the given number of trials.

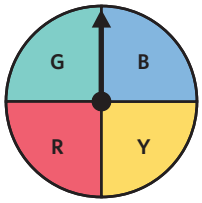
Example

Landing on blue after 10 spins.

Blue segments: 3 Total number of segments: 10

$Pr(blue) = \frac{3}{10}$ Expected occurrence: $10 \times \frac{3}{10} = 3$

a. Landing on red from 4 spins.



Red segments: 1 Total number of segments: 4

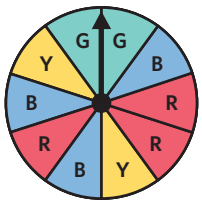
$Pr(red) = \frac{1}{4}$ Expected occurrence: $4 \times \frac{\square}{4} = \square$

b. Selecting a black ball from 5 selections.

Black balls: 3 Total number of balls: 5

$Pr(black) = \frac{3}{5}$ Expected occurrence: $5 \times \frac{\square}{5} = \square$

c. Landing on blue from 20 spins.



Blue segments: 3 Total number of segments: 10

$Pr(blue) = \frac{3}{10}$ Expected occurrence: $20 \times \frac{\square}{10} = \square$

d. Selecting a white ball from 18 selections.

White balls: 2 Total number of balls: 9

$Pr(white) = \frac{2}{9}$ Expected occurrence: $\square \times \frac{\square}{9} = \square$

3. Fill in the blanks by using the words provided.

zero | experimental | one | certain | theoretical

_____ probability is a number from zero to _____ that measures how likely it is that an event will occur. _____ probability is a number from 0 to 1 that is a representation of the proportion of times an event occurred in a chance experiment. An event with a probability of _____ is impossible. An event with a probability of one is _____.

Fluency

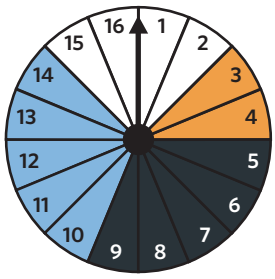
Question working paths

Mild	Medium	Spicy
'	''	'''
4 (a,b,c), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c)	4 (b,c,d), 5 (c,d,e,f), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d)	4 (d,e,f), 5 (e,f,g,h), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f)

WE 1 **4.** State the theoretical probability of each event. Give your answer as a number.

a. An apple growing on a lemon tree.
b. Christmas occurring on December 25th.
c. A person's birthday changing dates.
d. A new continent being discovered.
e. The Earth orbiting the sun once a year.
f. September lasting 30 days.

WE 2a **5.** Calculate the theoretical probability, as a fully simplified fraction, of each event relating to the spinner.



a. Landing on orange.
b. Landing on an even number.
c. Landing on white.
d. Landing on a number less than 7.
e. Landing on blue or black.
f. Landing on a colour that is **not** white.
g. Landing on blue or a number greater than 9.
h. Landing on a number that is a factor of 16.

WE 2b **6.** A fair 10-sided die has each face numbered from 1–10. Calculate the expected number of occurrences of each event from the given number of trials.

a. Rolling a 1 (30 trials).
b. Rolling an odd number (50 trials).
c. Rolling a 12 (100 trials).
d. Rolling a number 8 or greater (150 trials).
e. Rolling a 2, 4, 6 or 8 (200 trials).
f. Rolling a multiple of 3 (600 trials).

WE 3a **7.** Six chance experiments were performed where a coin was flipped a different number of times for each experiment. Calculate the experimental probability, as a decimal, of flipping heads for each set of results.

a. 10 trials

Outcome	Heads	Tails
Occurrences	8	2

b. 12 trials

Outcome	Heads	Tails
Occurrences	3	9

c. 20 trials

Outcome	Heads	Tails
Occurrences	14	6

d. 30 trials

Outcome	Heads	Tails
Occurrences	18	12

e. 50 trials

Outcome	Heads	Tails
Occurrences	22	28

f. 200 trials

Outcome	Heads	Tails
Occurrences	106	94

WE 3b **8.** A chance experiment is performed where a ball is picked out of a bucket and the colour is noted. The ball is replaced after each trial. The results after 20 trials are shown in the following table.

Colour	Black	White	Orange	Blue
Occurrences	5	8	1	6

Determine whether the experimental probability is less than, equal to, or greater than the theoretical probability for each event.

a. Picking a black ball.
b. Picking a white ball.
c. Picking a blue ball.
d. Picking an orange ball.
e. Picking a black or white ball.
f. Picking an orange or blue ball.

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Louis is about to pick the name of a horse running in the Melbourne Cup randomly out of a hat. There are 20 horses running in the race. What is the theoretical probability, as a percentage, that Louis picks the horse that wins the Melbourne Cup?

10. A standard dartboard has a total area of 1500 cm^2. The coloured areas cover 240 cm^2 of the board. Belle throws 150 darts at random on a standard dart board. How many times is it expected that a dart lands on a coloured area?

11. Use the following AFL ladder to calculate the experimental probability, as a decimal, of the Collingwood Magpies winning a game.

Position	**Team**	**Wins**	**Losses**	**Draws**
1	Port Adelaide Power	17	3	0
2	Brisbane Lions	16	3	1
3	Richmond Tigers	15	4	1
4	Geelong Cats	15	5	0
5	West Coast Eagles	15	5	0
6	St Kilda Saints	14	6	0
7	Western Bulldogs	12	8	0
8	Collingwood Magpies	11	8	1

12. The Brisbane Lions played 10 out of 20 games at their home ground. The theoretical probability of the Lions winning a game at home is 0.8 and the theoretical probability of the Lions winning a game away is 0.5.

Use the AFL ladder in the previous question to determine whether the Lions won more or less games than expected.

13. The local surf lifesaving club is hosting a raffle. There are 6 colours of tickets, each having tickets numbered from 1 to 40. Tom has bought 36 raffle tickets, 16 of which are blue and 20 of which are red. The winning ticket is blue. What is the theoretical probability, as a decimal, that Tom has won the raffle prize?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Belle is rolling a standard 8-sided die. The results of the first 50 trials are shown in the following table.

Number rolled	1	2	3	4	5	6	7	8
Occurrences	5	8	6	0	12	4	10	5

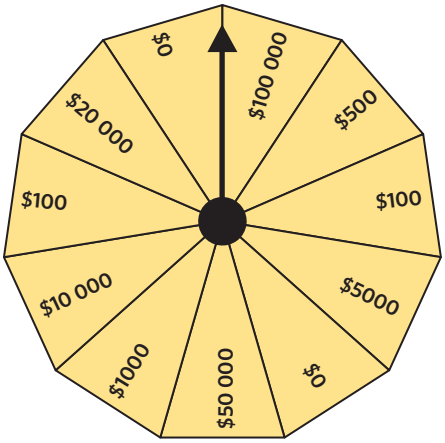
a. The experimental probability of rolling a 4 is zero. Does this mean it is impossible to roll a 4?

b. What is the experimental probability of rolling an odd number? Give your answer as a decimal.

c. Do you expect the experimental probability of rolling an odd number to increase or decrease with more trials?

d. Out of theoretical probability and experimental probability, which do you think is a more valuable measurement?

15. Wheel of Fortune is a quiz show where the winners get to spin a wheel to determine their prize. The spinner has an equal chance of landing on each segment.



a. What is the theoretical probability, as a fraction, that a winner gets a prize worth more than $1000?

b. In 110 spins of the Wheel of Fortune, how many winners are expected to receive a prize that is $5000 or less?

c. Out of the first 110 winners, 104 have received a prize that is $5000 or less. Is there reason to believe that the Wheel of Fortune show may be rigged?

d. Would you rather be given $1000 or spin the Wheel of Fortune?

Extra spicy

16. After playing 500 games, Tori's success rate at Solitaire is 49%. Assuming she wins every game from now on, how many extra games does she need to play in order for the success rate to increase to 50%?

A. 1 B. 2 C. 5 D. 10 E. 50

17. Two six-sided dice are thrown at random. What is the probability that the two numbers obtained sum to a perfect square?

A. $\frac{1}{9}$ B. $\frac{2}{9}$ C. $\frac{7}{36}$ D. $\frac{1}{4}$ E. $\frac{1}{3}$

18. Neil has a bag with 12 red and 8 white marbles. One marble is randomly selected. The selected marble is red and is removed altogether from the bag. Neil then places b blue marbles into the bag. When a second marble is chosen at random, what is the probability that the second marble is red?

A. $\frac{11}{b+19}$ B. $\frac{11}{b+20}$ C. $\frac{12}{b+20}$ D. $\frac{12}{b+19}$ E. $\frac{b+12}{b+20}$

19. There are three runners in a cross country race. Runner X is twice as likely to win as runner Y and four times as likely to win as runner Z. Calculate the probability, as a fraction, of runner Y winning the race.

Remember this?

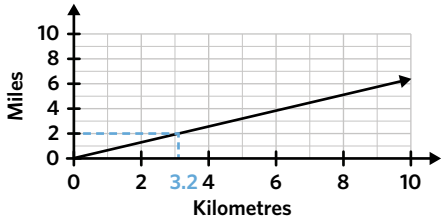
20. 32 020 people attended the Australian Open final. Three-quarters of the audience were supporting Daniil Medvedev. Which of these is closest to the amount of people that supported Daniil Medvedev?

A. 800 **B.** 2400 **C.** 8000 **D.** 24 000

21. A horned dung beetle is 0.001 grams and can carry balls of dung that are up to 1180 times its own weight. If four balls of dung are weighed, which one would be the heaviest ball that the horned dung beetle could carry?

A. 0.12 g **B.** 1.1 g **C.** 1.2 g **D.** 11 g

22. The following graph can be used to convert from kilometres to miles.

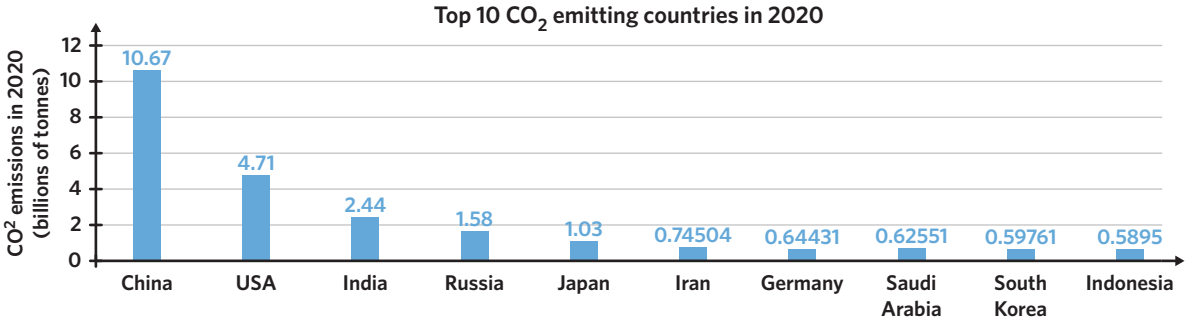


The blue dotted lines show how 2 miles can be converted to 3.2 kilometres and vice versa.

Use the graph to convert 8 kilometres to miles.

Chapter 7 extended application

1. Carbon dioxide gas (CO_2) is the byproduct of many processes, both natural and artificial. It is generally attributed to the burning of fossil fuels as well as natural processes, like decomposition. The following column chart shows the top 10 CO_2 emitters in the world, using data gathered in 2020.



a. Which countries in the chart emitted the most and least amount of CO_2 in 2020? Name them and find the difference in their emissions, in billions of tonnes.

b. How much CO_2 did the top 10 countries in the chart emit altogether, in billions of tonnes?

c. In 2020, the total amount of CO_2 emitted by the whole world was 34.81 billions of tonnes. As a percentage, how much of this amount can we attribute to the top 10 CO_2 emitters in the chart?

d. In 2020, China was recorded to have a population of 1.402 billion people. Calculate the per capita, or average, CO_2 emissions attributed to each individual in China in 2020, based on the figures given. This number can be obtained by dividing the total amount of CO_2 emitted by a country by the number of people living in the country at the time.

e. Overall in 2020, Asia was recorded to be the highest CO_2 emitting continent in the world. Using the information given in the graph, as well as your own reasoning, state whether this is a realistic statement and explain why.

2. In 1986, financial news magazine *The Economist* came up with the Big Mac Index. It is a way to compare the wealth of countries and the individuals living there. Here is a list of the top ten performing countries in 2022.

Country	Currency	Value of Big Mac (under/over valued)
Switzerland	Franc	+20.2%
Norway	Krone	+10%
USA	US dollar	Base currency = 0%
Sweden	Krona	−0.4%
Uruguay	Peso	−6.6%
Israel	Shekel	−7.9%
Canada	Canadian dollar	−8.4%
Venezuela	Bolivar	−12.9%
Denmark	Krone	−17.0%
Britain	British pound	−17.1%

The last column represents how much the equivalent of the cost of one Big Mac is valued at in each country, compared to the USA. For example, if a country's Big Mac index is shown to be −5%, then the value of its currency is 100% − 5% = 95% of the value of the US dollar. The lower the value of currency, the less an average person can afford in that country compared to the rest of the world.

a. Which country in the world has the most valuable currency, according to the above table? Write down by how much the Big Mac is overvalued there, compared to the USA.

b. According to the table, which countries' currencies are more valuable than that of the USA?

c. In which country is the currency worth 92.1% of US currency? Name this country and its currency.

d. Australia is not listed as one of the ten countries above as its Big Mac index is −22.4%. What is the value of the Australian dollar, compared to the US dollar, according to the Big Mac index?

e. Plot a line graph of the data in the table, using the horizontal axis to list the 10 countries and the vertical axis to represent the Big Mac index. Make sure that the USA's Big Mac index is plotted at 0% with all the other countries' values plotted in comparison to it.

f. The Big Mac index may appear as a lighthearted and somewhat fun way to look at world economics. However, it has proven to be useful and relevant over all this time, since its creation in 1986. Do you think that this is a good way to compare the wealth of countries? List some advantages and disadvantages of using the Big Mac index.

3. The table below lists the most common eye colours in the world along with the proportions of people who have them.

Eye colour	**Brown**	**Blue**	**Hazel**	**Amber**	**Grey**	**Green**	**Other**
Probability of occurrence	$\frac{75}{100}$	$\frac{8}{100}$	$\frac{5}{100}$	$\frac{5}{100}$	$\frac{3}{100}$	$\frac{2}{100}$	$\frac{2}{100}$

a. Convert each of the probabilities in the table to percentages.

b. A newborn baby is picked at random from anywhere in the world. Which eye colour is this baby most likely to develop as an adult and why?

c. Out of a group of 10 randomly chosen people, approximately how many of them would you expect to have blue eyes? Show working with your answer.

d. In Mr Venkmann's maths class of 29 people, 5 students have blue eyes and 2 have green eyes. There are 4 times as many people with brown eyes as there are blue, and the remaining students in the class have grey eyes. Complete the table below using the given information.

Eye colour of students in the class	**Brown**	**Blue**	**Hazel**	**Amber**	**Grey**	**Green**	**Other**	
Number of students								Total = 29

e. Using the table from part **d**, calculate the probabilities of eye colour occurrences in Mr Venkmann's maths class. Express these probabilities as percentages and compare them against the values you calculated in part **a**. Comment on the differences and similarities between these percentages and give possible reasons for any unexpected findings.

f. The iris of the eyes is said to be more unique than fingerprints, and this allows for biometric technology that can scan and identify individuals by their eyes. What kind of implications does this have for our society as a whole? List some advantages and disadvantages of having technology that is capable of this.

Chapter 7 review

Multiple choice

1. What is the modal eye colour in the following frequency table? 7B

Eye colour	Tally	Frequency
Blue	𝍸 𝍸 𝍸 ||	17
Hazel	𝍸 𝍸 𝍸 ||||	19
Brown	𝍸 𝍸 𝍸 𝍸 |	26
Green	𝍸 𝍸 𝍸	15

A. Blue **B.** Hazel **C.** Brown **D.** Green

2. In which bucket does an orange ball have a theoretical probability of 0.25 of being pulled out? 7G

A.  **B.**  **C.**  **D.** 

3. Twenty year 11 students were asked how many hours of driving they had completed last month. The results are shown on the following stem-and-leaf plot. 7C

Stem	Leaf
0	0 2 7
1	4 5 5 6 8 8
2	1 3 4
3	0 0 2 7 8
4	5 6
5	1

Key

0 \| 5 = 5 hours

How many students have over 30 hours of driving experience?

A. 5 **B.** 6 **C.** 8 **D.** 9

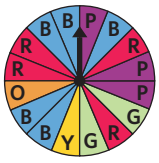
4. Which of the following statements about the spinner is true? 7F

A. Black is the most likely to occur.

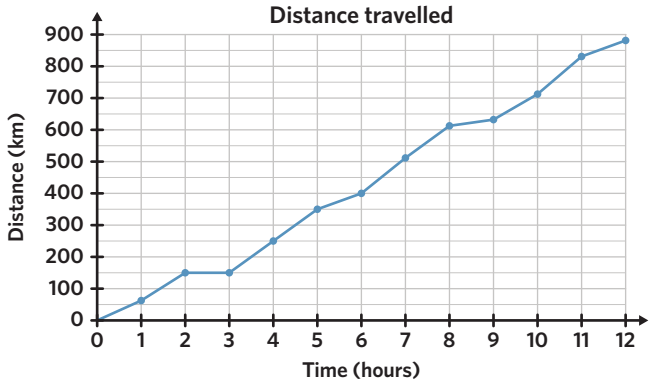
B. Yellow is the most likely to occur.

C. There is an even chance of landing on green, and on purple.

D. The probability of landing on red is $\frac{4}{16}$.



5. The following line graph shows the distance travelled by Frederica as she drove from Melbourne to Sydney. 7E



How far did Frederica travel during the 5th hour?

A. 50 km **B.** 100 km **C.** 350 km **D.** 400 km

Fluency

6. Classify each variable as continuous numerical, discrete numerical, or categorical. 7A

a. Number of Instagram followers
b. Favourite online shop
c. Distance from home to school
d. Temperature outside

7. Determine the median of each data set. 7B

a. 1, 3, 10, 18, 29, 29, 55
b. 103, 141, 155, 161, 193, 222
c. 531, 109, 203, 411, 98, 295, 165, 354, 306
d. 356, 209, 340, 296, 180, 311, 281, 303

8. Calculate the mean of each data set. 7B

a. 5, 8, 12, 13, 17
b. 3, 3, 5, 10, 11, 8, 5, 3
c. 47, 21, 59, 101, 29, 30, 69, 87, 88, 95
d. 3.3, 2.9, 10.6, 2.4, 7.8, 9.9, 7.7, 12.5, 4.1

9. The number of sheep in 12 different paddocks are given in the following data set. 7C
21, 60, 35, 29, 44, 68, 35, 49, 11, 63, 59, 30

a. Construct a stem-and-leaf plot to display the data.
b. How many paddocks contained over 40 sheep?
c. What is the modal number(s) of sheep per paddock?
d. What is the range?

10. The number of laser tag games 16 students have played is given in the following data set. 7C
1, 0, 2, 6, 5, 2, 1, 1, 0, 3, 1, 2, 6, 2, 1, 1

a. Construct a dot plot to display the data.
b. What is the modal number of laser tag games that students have played?
c. How many students have never played laser tag?
d. What percentage of the students have played laser tag at least twice?

11. 40 people were surveyed on their favourite musicals. 7D

Favourite musical	Hamilton	Les Misérables	Hairspray	Wicked	Total
Frequency	8	14	5	13	40

a. Create a column graph for the data.
b. What is the modal (most favoured) musical?
c. How many more people favour Wicked compared to Hairspray?
d. What percentage of the people surveyed answered Hamilton?

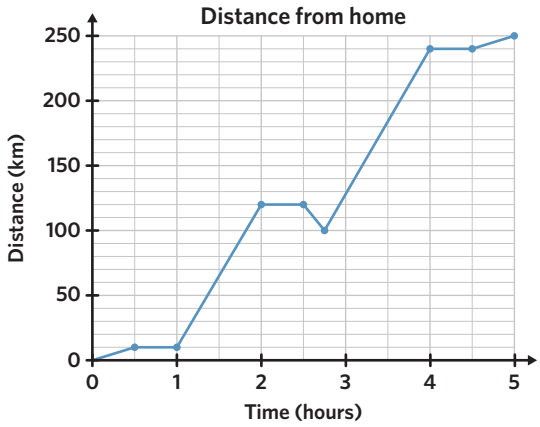
12. 50 people were surveyed on their favourite dog breeds. 7D

Favourite dog breed	Corgi	Border Collie	Whippet	Dalmatian	French Bulldog	Total
Frequency	10	15	8	12	5	50

a. Create a pie chart for the data.
b. What is the modal (most favoured) dog breed?
c. What percentage of people answered 'Dalmatian'?
d. What is the difference in percentage of people that answered 'Border Collie' compared to 'Whippet'?

13. The following line graph shows the distance Bailey was from home while driving to a campsite. He arrived after 5 hours. 7E

a. How many kilometres from home was Bailey after 2 hours?
b. How many kilometres from home was the campsite?
c. How many breaks from driving did Bailey take?
d. Two and a half hours into the trip Bailey took a wrong turn and headed back towards home. How many kilometres did he travel in the wrong direction before turning back?



14. State whether the following outcomes have an impossible, unlikely, even, likely or certain chance of occurring. 7F

a. An earthquake occurring in Melbourne today.
b. Tossing a coin and it landing on Tails.
c. Rolling a factor of 60 on a standard 6-sided die.
d. A randomly chosen year having 365 days.

15. A deck of cards has 52 cards made up of 4 suits (spades, clubs, diamonds, hearts), each having the cards A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Calculate the theoretical probability, as a fraction, of each event occurring. 7G

a. Picking out a diamond.
b. Picking out an ace.
c. Picking out a card with an even number.
d. Picking out a face card (J, Q, K) that is a diamond or a heart.

16. A chance experiment is performed with a spinner. The results after 40 trials are shown in the following table. 7G

Colour	Green	Purple	Red	Yellow	Blue
Occurences	2	9	20	8	1

Determine whether the experimental probability is less than, equal to, or greater than the theoretical probability for each event.

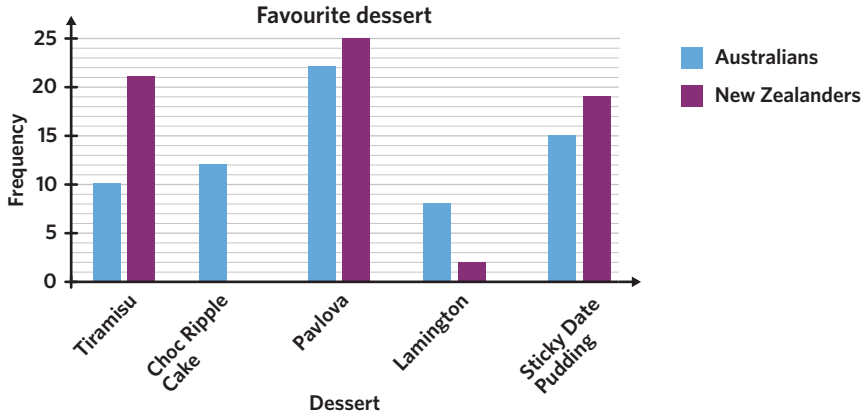
a. Landing on green.
b. Landing on red.
c. Landing on yellow.
d. Landing on blue or purple.

Problem solving

17. A group of scientists are researching the effectiveness of The Voice Australia in launching music careers for its contestants. They randomly select 20 past contestants from The Voice Australia to interview. What is the population and sample of this experiment? 7A

18. The mean height of the 8 players on a basketball team is 178 cm. By how much would the mean height increase if a new player with a height of 196 cm joined the team? 7B

19. The following column graph shows survey results from Australians and New Zealanders on their favourite desserts. 7D



How many more New Zealanders favour Pavlova compared to Australians?

20. Effy has recorded the number of novels she has read each year for the past 20 years. She displayed her data in the following stem-and-leaf plot before realising that a dot plot may be more appropriate. Construct a dot plot to display this data. 7C

Stem	Leaf
0	7 7 7 7 9 9 9
1	0 0 0 0 1 1 2 2 2 3 4 4 5

Key
0 | 7 = 7 novels

21. Luther and Herbert decided to take on a fitness challenge where they had to walk 150 km in one week. Luther completed the following table showing the total number of kilometres he had travelled by the end of each day. 7E

Day	1	2	3	4	5	6	7
Total distance travelled by Luther (km)	15	28	62	80	110	132	150

Use the information to construct a line graph showing Luther's progression through the fitness challenge.

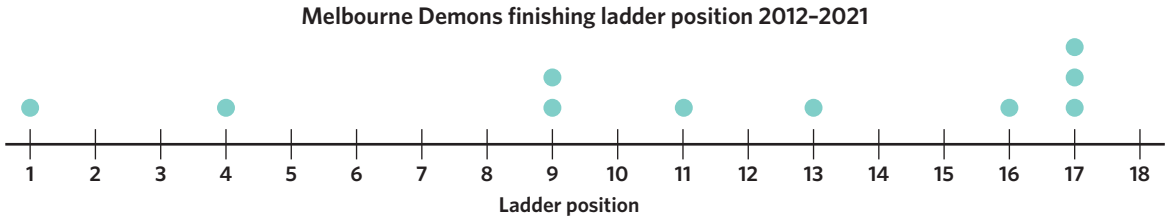
22. Construct a spinner with the following conditions: 7F

- 6 even segments
- 5 colours: red, blue, green, yellow and purple
- landing on blue is twice as likely as landing on green.

23. When Ben Simmons shoots a three-point shot in basketball, he has a 15% chance of it going in. If Ben Simmons shoots 400 three-point shots at training, how many are expected to go in? 7G

Reasoning

24. The finishing positions of the Melbourne Demons on the AFL ladder from 2012 to 2021 is shown on the following dot plot.



a. Only the top 8 teams on the AFL ladder at the end of the season make the finals. In how many of the seasons from 2012 to 2021 did the Melbourne Demons not make the finals?

b. Calculate the mean finishing position of the Melbourne Demons from 2012 to 2021. Round to the nearest whole number.

c. The following frequency table shows the number of Melbourne Demons players who live in different suburbs within Melbourne.

Suburb	**Frequency**
St Kilda	4
Brighton	9
Northcote	5
Richmond	5
Elwood	2
Total	25

Construct a pie chart using this data.

d. What are some factors that help sports teams improve from season to season?

25. Lorenzo is conducting a chance experiment where a ball is picked out of a bucket and the colour is noted. The ball is replaced after each trial. The results after 300 trials are shown in the following table.

Colour	**White**	**Black**	**Orange**	**Blue**
Occurences	60	150	50	40

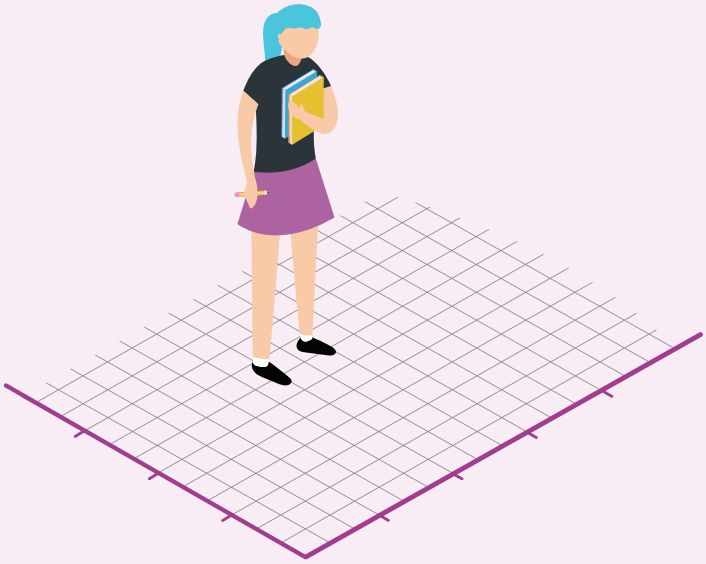
a. Is the data that Lorenzo collected numerical or categorical?

b. Construct a column graph displaying the occurrences of each colour in Lorenzo's experiment.

c. Using the theoretical probability, classify the likelihood of picking out a black ball as impossible, unlikely, even, likely or certain.

d. Is the experimental probability of picking out a blue ball larger than, equal to, or less than the theoretical probability?

e. How many trials do you think it takes for the experimental probability to be reliable?



08

Location and transformations

Number and Algebra

Research summary

8A Integers on the Cartesian plane

8B Symmetry and reflections

8C Translations

8D Rotations

Chapter 8 extended application

Chapter 8 review

Image: Marish/Shutterstock.com

Research summary – Location and transformations

Big ideas

Spatial sense

Spatial sense is an awareness or intuition about shapes and their relationships with other shapes. It is also an ability to visualise mental objects in 3-dimensions and turn them around in our minds in order to correctly observe them from different angles. It was typically believed that spatial sense was something you were born with, but just like number sense, it is now understood that spatial sense can be developed through continued and regular exposure to geometric experiences. Strong spatial sense has also been linked to a greater understanding of mathematical concepts beyond the scope of geometry due to its promotion of critical thinking.

Location

Location is an important part of our own spatial sense and position in the world. It is not only reading a map or identifying coordinate points on a Cartesian plane, but also involves understanding and describing the position of an object, or person, relative to other objects and people. Using tools such as location grids, with and without Cartesian coordinates, can enable us to map our surroundings more efficiently.

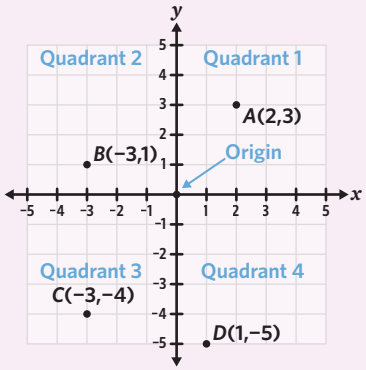
Rigid transformations

Rigid transformations are those that do not change the size or shape of the image being rearranged. These transformations include translations (slides), reflections (flips), and rotations (turns). The study of transformations is important because it provides opportunities to explore abstract concepts such as congruence and symmetry. A key understanding of transformations in geometry is to recognise them as mapping on the plane, not merely a motion across the plane. This is established through the use of Cartesian coordinate geometry.

Visual representations

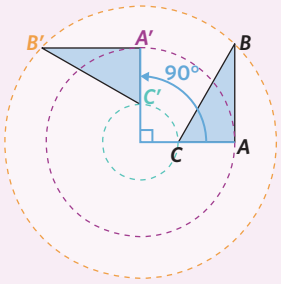
The Cartesian plane

Cartesian coordinates describe a plane with two intersecting perpendicular lines, representing each dimension of the plane, that form four quadrants. Points are located on the plane by their coordinates, which provide their location within the plane. Points can be connected to create figures that lie on the plane. Shapes can be plotted on a Cartesian plane to show the change in locations, based on specific transformations.



Full circle arc

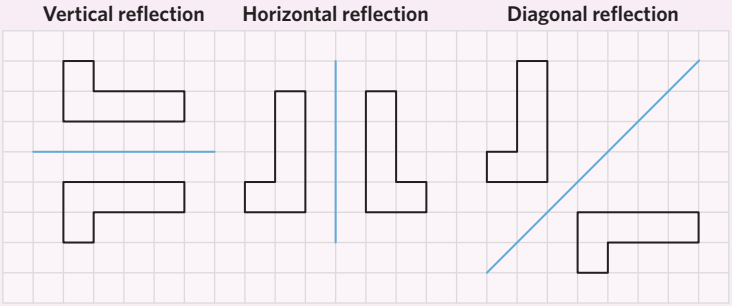
A full circle arc can be used to rotate images. A rotation occurs around a centre point, and points or figures can be rotated 360° around the point.



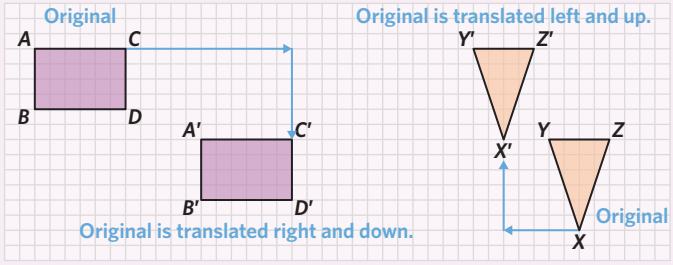
Rigid transformations

Rigid transformations are those that do not change the size or shape of the image being transformed. These transformations include translations (slides), reflections (flips), and rotations (turns), which are illustrated in the following diagrams.

Reflections

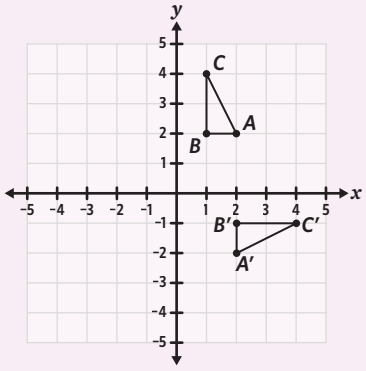


Translations

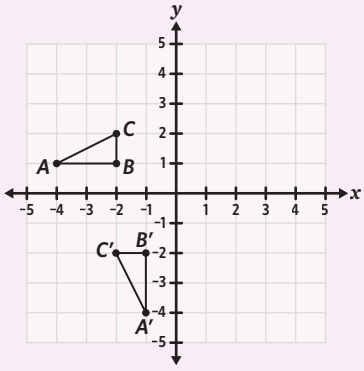


Rotations

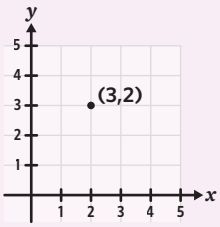
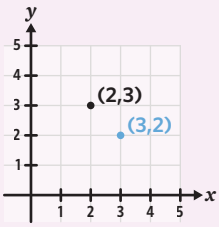
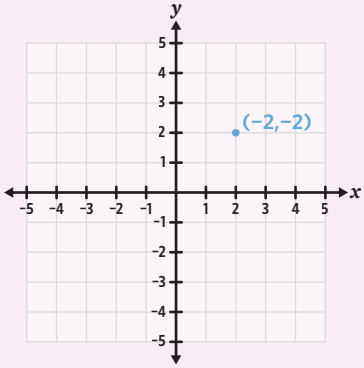
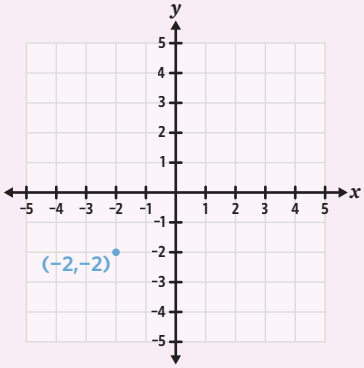
90° clockwise (−90°) or 270° anticlockwise (270°)

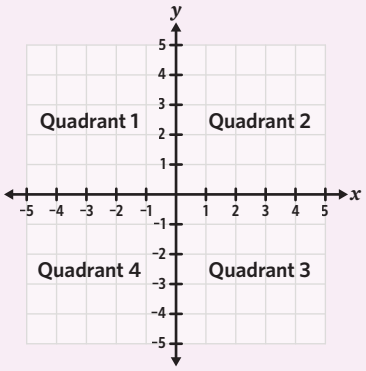
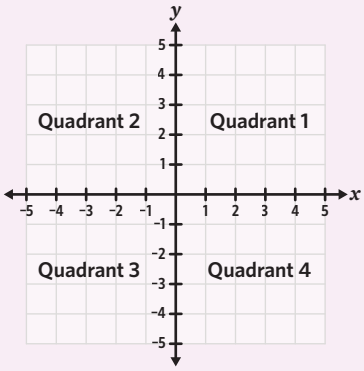
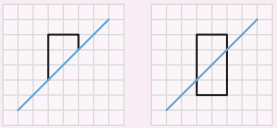
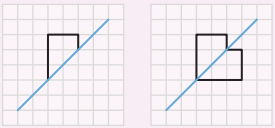
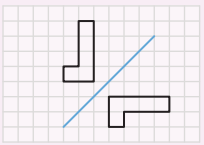
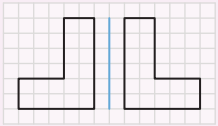
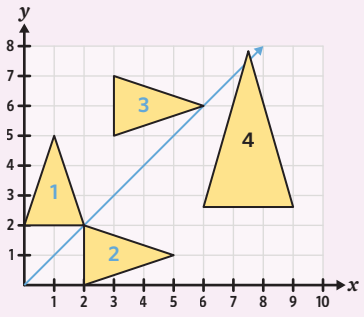
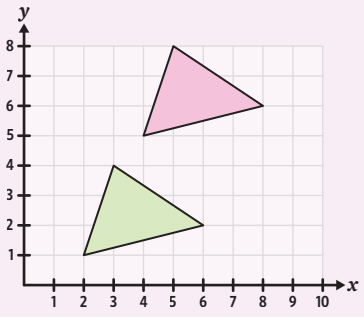


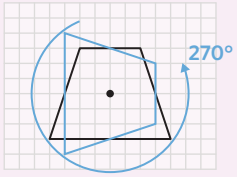
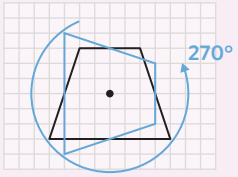
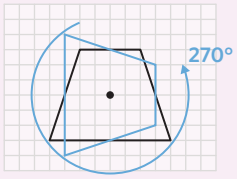
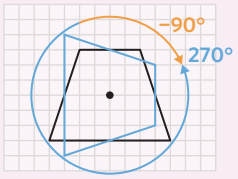
270° clockwise (−270°) or 90° anticlockwise (90°)



Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students don't think that the order of the x and y values matter in an ordered pair.			8A
Students think that negative integers cannot be plotted on the Cartesian plane.			8A

	Incorrect	Correct	Exercise
Students think that Quadrant 1 is located in the top left of the Cartesian plane and count the quadrants in the clockwise direction.			8A
Students incorrectly draw a reflection when the mirror line is not horizontal or vertical but the object is horizontal or vertical.			8B
Students incorrectly identify an object as symmetrical when the line of symmetry bisects a shape through two vertices.		 Asymmetrical	8B
Students think that rotational symmetry is when a shape is rotated a full 360 degrees and looks the same.		 Does not have rotational symmetry.	8B
Students think reflection lines can only be horizontal or vertical.		Diagonal reflection 	8B
Students think that reflection lines can only be adjacent to the side of the original image.		Horizontal reflection 	8B
Students misinterpret Cartesian plane coordinates and confuse horizontal and vertical movements of a translation.	[8,7] is 8 units up and 7 units right.	[8,7] is 8 units right and 7 units up.	8C
Students think a translation can include a change to the size and orientation of a shape.			8C

	Incorrect	Correct	Exercise
Students reverse clockwise and anticlockwise direction.	Clockwise 	Anticlockwise 	8D
Students think there is only one rotation size and angle for a given rotation.	270° anticlockwise is the only way to achieve this rotation. 	There are multiple ways to achieve this rotation. It could be achieved with either a 90° clockwise rotation or a 270° anticlockwise rotation. 	8D

8A Integers on the Cartesian plane

The Cartesian plane (also known as a number plane or coordinate plane) are two perpendicular and intersecting axes that extend indefinitely in positive and negative directions. Named after the French philosopher and mathematician Rene Descartes, the Cartesian plane allows us to describe any point using coordinates. Cartesian planes establish the bridge between geometry and algebra. Below are some examples where integers on the Cartesian plane can be applied.

- Air traffic controllers monitor aeroplane locations on a Cartesian plane to make sure they do not hit each other in the air. What is the location of a plane over the Pacific Ocean?
- A city planner is using a map to decide where to install a new playground. What are the coordinates of the location where the playground should be installed?
- Apple iPhone developers used Cartesian coordinates to program the touch screen. What are the coordinates to program in the code for the home button to work?

Learning intentions

Students will be able to:

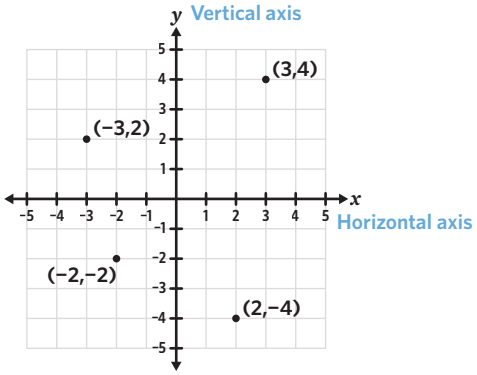
- label points on a Cartesian plane using x and y coordinates
- plot points using Cartesian coordinates
- identify the quadrant on a Cartesian plane of a given coordinate.

Key terms and definitions

- A **Cartesian plane** is a set of two perpendicular number lines that intersect at the origin.
- **Cartesian coordinates** are the numbers that indicate the location of a point on a Cartesian plane. The x-coordinate is written before the y-coordinate. E.g. (2,3).
- The ***x*-axis** is the horizontal axis that runs left to right through zero.
- The ***y*-axis** is the vertical axis that runs up and down through zero.
- The **origin** is the point (0,0) where the x-axis and y-axis intersect.

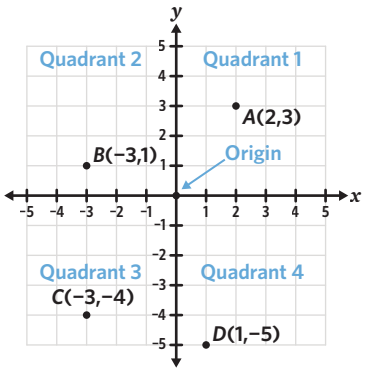
Key ideas

1 The x-axis is the horizontal line and the y-axis is the vertical line. The coordinates of a point shows where a point is located on the Cartesian plane.



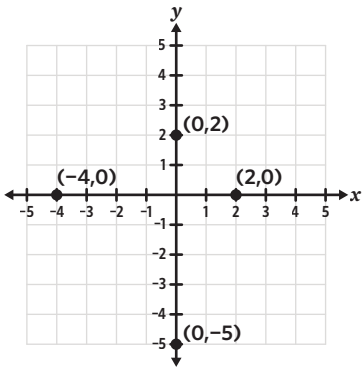
The x-coordinate indicates the horizontal position.
The y-coordinate indicates the vertical position.

2 The positive and negative signs of the coordinates indicate which quadrant the point is located in.



Quadrant	*x*-coordinate	*y*-coordinate	Example
Quadrant 1	Positive	Positive	(2,3)
Quadrant 2	Negative	Positive	(−3,1)
Quadrant 3	Negative	Negative	(−3,−4)
Quadrant 4	Positive	Negative	(1,−5)

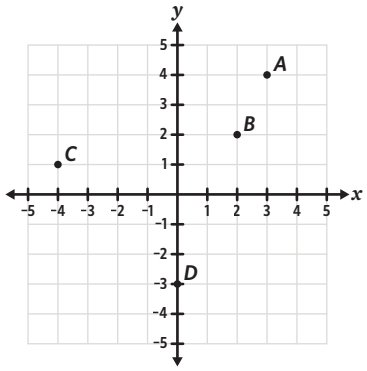
3 Any point on the x-axis will always have a y-coordinate that is zero. Any point on the y-axis will always have a x-coordinate that is zero.



Worked examples

WE 1 Finding the Cartesian coordinates of a point

State the Cartesian coordinates of each labelled point.



Working

A: (3,4)

B: (2,2)

C: (−4,1)

D: (0,−3)

Thinking

Step 1: Identify the horizontal and vertical position of point A. The point corresponds to 3 on the x-axis and 4 on the y-axis.

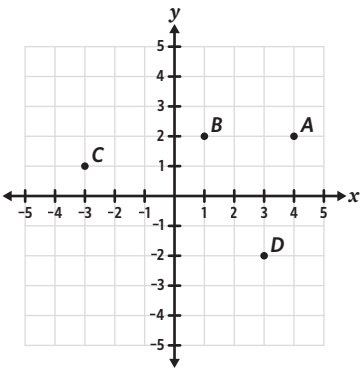
Step 2: Identify the horizontal and vertical position of point B. The point corresponds to 2 on the x-axis and 2 on the y-axis.

Step 3: Identify the horizontal and vertical position of point C. The point corresponds to −4 on the x-axis and 1 on the y-axis.

Step 4: Identify the horizontal and vertical position of point D. The point is on the y-axis at −3, so the x-coordinate is 0 and the y-coordinate is −3.

Student practice

State the Cartesian coordinates of each labelled point.

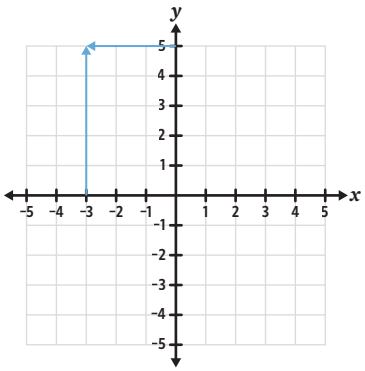


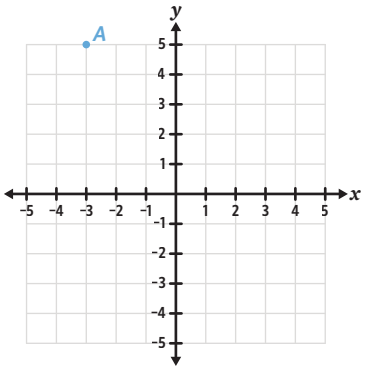
WE 2 Plotting points with Cartesian coordinates

Draw a Cartesian plane from −5 to 5 on both axes, then plot and label each point using the Cartesian coordinates.

a. $A(-3,5)$

Working



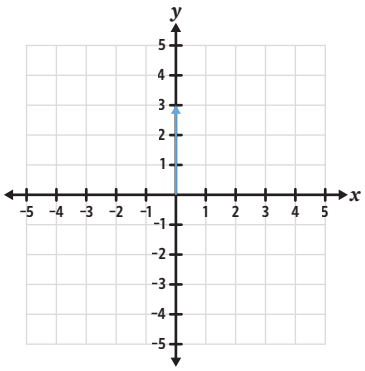


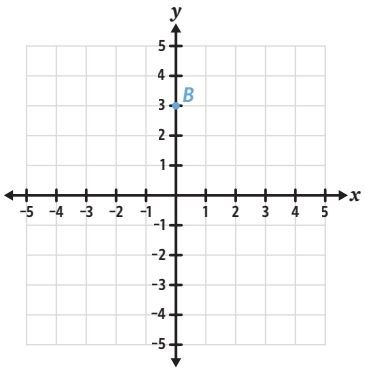
Thinking

The x-coordinate is the horizontal position of the point, so point A is 3 units to the left of the y-axis.
The y-coordinate is the vertical position of the point, so point A is 5 units above the x-axis.

b. $B(0,3)$

Working





Thinking

The x-coordinate is the horizontal position of the point. The x-coordinate of point B is 0, so point B is on the y-axis. The y-coordinate is the vertical position of the point, so point B is 3 units above the x-axis.

Student practice

Draw a Cartesian plane from −5 to 5 on both axes, then plot and label each point using their Cartesian coordinates.

a. Point $A(-2,1)$ **b.** Point $B(3,0)$ **c.** Point $C(-3,-1)$ **d.** Point $D(4,-2)$

WE 3 Identifying quadrants based on Cartesian coordinates

Identify the quadrant or axis in which each point is located on the Cartesian plane.

a. $A(-3,2)$

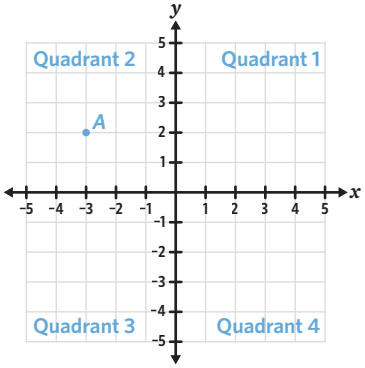
Working

Point A is located in quadrant 2.

Thinking

Identify the position of the x and y-coordinate in relation to the origin. The x-coordinate (-3) is negative, so it is to the left of the origin. The y-coordinate (2) is positive, so it is above the origin.

Visual support



b. $B(4,0)$

Working

Point B is located on the x-axis.

Thinking

Identify the position of the x and y-coordinate in relation to the origin. The x-coordinate (4) is positive, so it is to the right of the origin. The y-coordinate is 0, so it is on the x-axis.

Student practice

Identify the quadrant or axis in which each point is located on the Cartesian plane.

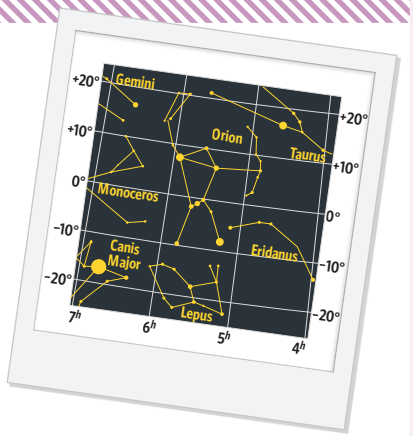
a. $(1,2)$ **b.** $(2,0)$ **c.** $(0,4)$ **d.** $(-5,-6)$

8A Activities and questions

STARTER TASKS

Odd spot

Constellation maps are divided into 88 sections that use stars to map the Earth's movement and rotation. There are 36 constellations in the northern sky and 52 constellations in the southern sky. As the Earth rotates on its axis, the placement of the constellations change. The movement of the constellations confirms that the Earth spins on its own axis and rotates around the sun.



From the map on the right, the Orion constellation can be seen between $(6^h,-10°)$ and $(5^h,10°)$. What is the name of the constellation located between $(6^h,-10°)$ and $(5^h,-20°)$?

A. Canis Major **B.** Lepus

Puzzle

a) The pumpkin is located at $(5,6)$, what are the coordinates of the watermelon, onion, chilli, and lemon?

b) What fruit or vegetable is the furthest up from the origin?

c) What fruit or vegetable is the furthest to the right of the origin?

d) What fruit or vegetable is on an axis?

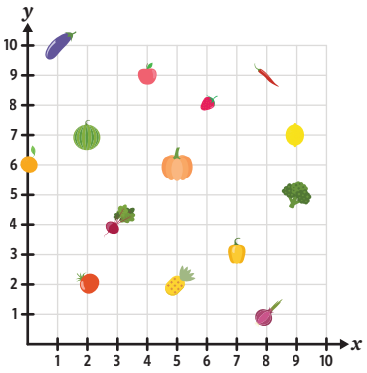
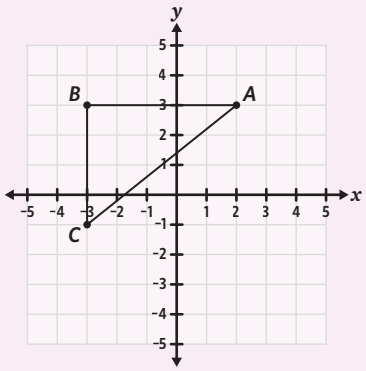


Image: light_s/Shutterstock.com

Understanding worksheet

1. Write the Cartesian coordinates of each point of the shape.

Example

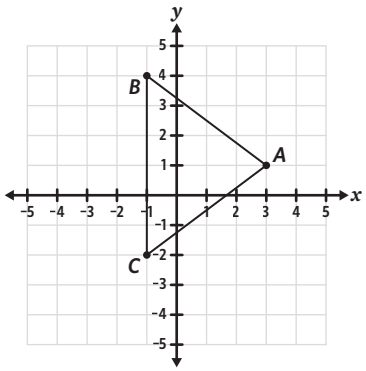


A(2,3)

B(−3,3)

C(−3,−1)

a.

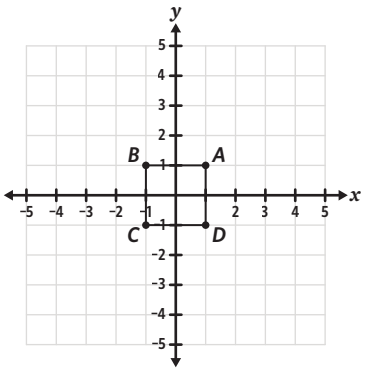


A(3 ,)

B(, 4)

C(,)

b.



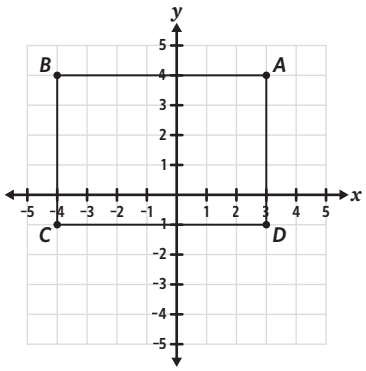
A(,)

B(,)

C(,)

D(,)

c.



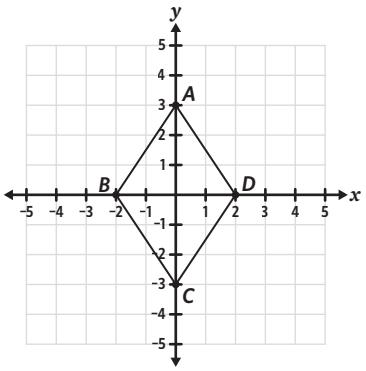
A(, 4)

B(−4 ,)

C(,)

D(,)

d.



A(,)

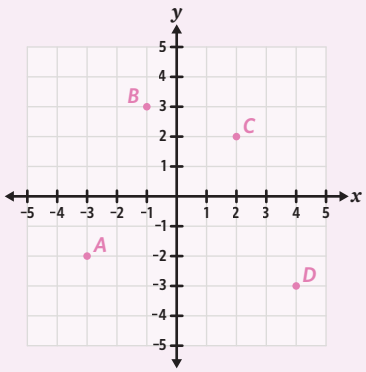
B(,)

C(,)

D(,)

2. Plot and label each point on the Cartesian plane and identify what quadrant the point lies in.

Example

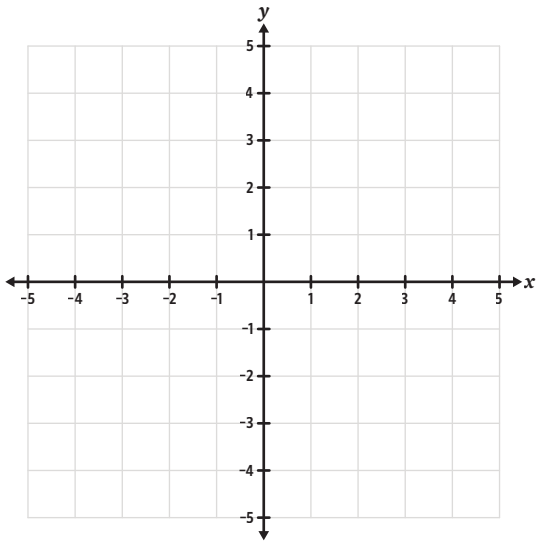


A(−3,−2)
Quadrant 3

B(−1,3)
Quadrant 2

C(2,2)
Quadrant 1

D(4,−3)
Quadrant 4



A(3,2)
Quadrant ☐

B(−4,−1)
Quadrant ☐

C(5,−2)
Quadrant ☐

A(−4,−5)
Quadrant ☐

3. Fill in the blanks by using the words provided.

Cartesian coordinates | vertical | number plane | horizontal | origin

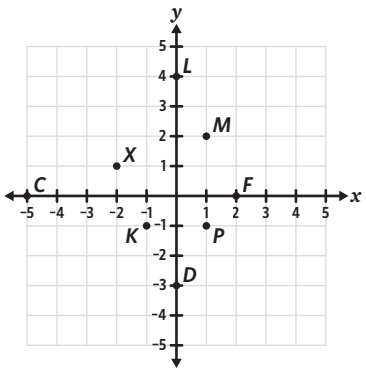
The Cartesian plane is also known as a ☐. The plane is made up of a ☐ x-axis and a ☐ y-axis that intersect at the ☐. The ☐ of the origin is (0,0). These coordinates are used to help locate a point on the Cartesian plane.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (d,e,f), 7 (d,e,f)

WE 1 **4.** State the Cartesian coordinates of each labelled point.



a. Point X
b. Point M
c. Point K
d. Point C
e. Point F
f. Point P
g. Point L
h. Point D

WE 2 **5.** Draw a Cartesian plane from −5 to 5 on both axes, then plot and label each point on the plane.

a. $A(3,2)$
b. $B(0,5)$
c. $C(-2,4)$
d. $D(3,-2)$
e. $E(2,0)$
f. $F(0,-1)$
g. $G(-1,-3)$
h. $H(-4,0)$

WE 3a **6.** Identify which quadrant each point lies on the Cartesian plane.

a. (2,2)
b. (3,−1)
c. (−2,1)
d. (−3,−3)
e. (1,−2)
f. (−2,−4)

WE 3b **7.** Identify which axis each point lies on the Cartesian plane.

a. (0,5)
b. (2,0)
c. (−3,0)
d. (0,−4)
e. (0,0)
f. (0,−1)

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

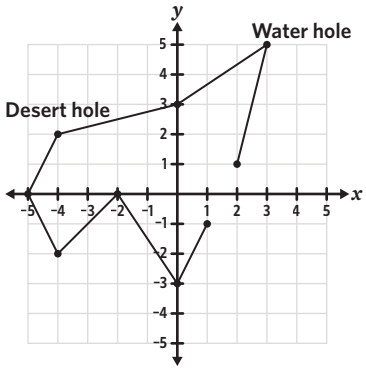
8. Hamish is plotting points on a Cartesian plane. He plots the following points: $A(-2,1)$, $B(2,4)$, $C(2,-1)$, $D(0,-2)$. Which point is the highest on the plane?

9. What are the Cartesian coordinates that answer Joanna's math riddle? The riddle says: 'You can find me 3 units to the left and 2 units above the intersection of the x and y axes'.

10. Stanley is drawing a shape on the Cartesian plane with both axes labelled from -5 to 5. He starts by drawing a vertical line up from the point $(5,-4)$. What are all the coordinates of points that lie on the line if he stops at $(5,5)$?

11. Sally draws a square with its centre at the origin of the Cartesian plane. If one of the vertices is situated at $(2,2)$, what are the coordinates of the other three vertices of the square?

12. Jacob is redesigning the Albert Park mini golf course. He uses the following Cartesian plane to create the new 9 hole layout.

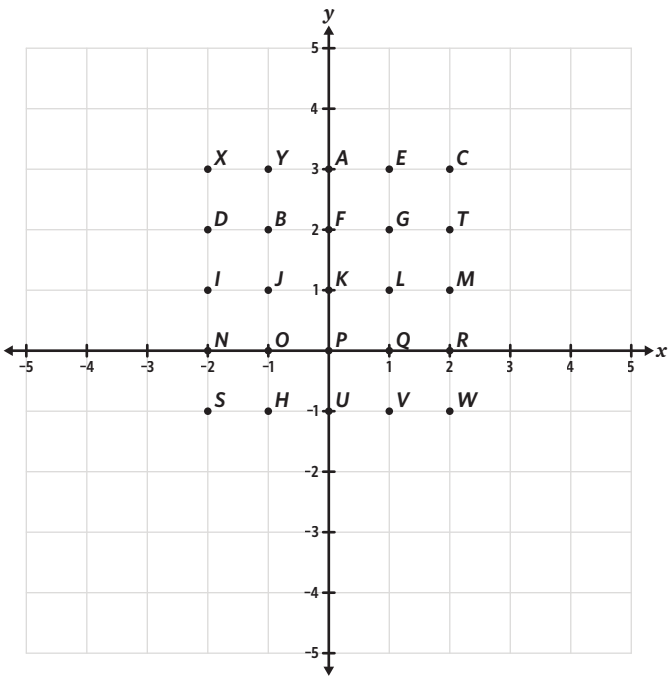


What are the coordinates of the desert hole and the water hole?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. In wars and battles, soldiers would often send encrypted messages to each other. One form of messaging was using Cartesian coordinates from Cartesian planes. The following is an example Cartesian plane that soldiers may have used.



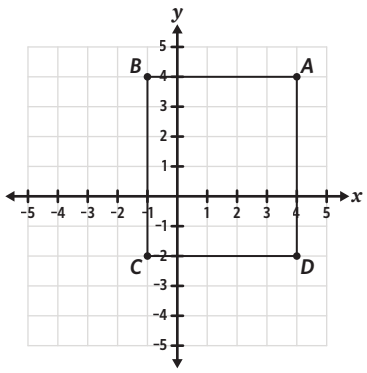
   a. What are all the letters that are marked on the x-axis?
   b. What is the message with the following coordinates: (0,3), (2,2), (2,2), (0,3), (2,3), (0,1).
   c. There are no spaces in code words, so the words are written altogether. What would the coordinate code be for the message: NEEDWATER?
   d. Another form of messaging used were SOS signals. SOS signals were a series of sounds that matched letters. Which method of messaging do you think is better for soldiers and why?

14. Jessica is creating a game with her family over the holidays. They set up their backyard into a grid that looks like a Cartesian plane. The distance between each coordinate is one step. After each instruction, the player has to plant a flag where they stand.
    a. In order to play the game, Jessica draws a Cartesian plane with both axes marked from -5 to 5. What does Jessica's Cartesian plane look like?
    b. The first instruction says to start at the origin, face the positive y direction, move 3 side steps to the right, and 2 steps forward. Draw a flag on the Cartesian plane to mark this position.
    c. The second instruction says to move 6 side steps to the left and plant a second flag there. In what quadrant is the second flag?
    d. Jessica realises that the directions are hard to follow as the instruction of 'forwards', 'backwards', 'right' and 'left' changes depending on which direction she is facing. What could be a better way to write the directions for the puzzle?

Extra spicy

15. What are the x and y coordinates of $(\sqrt{64}, \frac{3}{4} - \frac{30}{8})$?
    A. (3,2) B. $(8, 2\frac{3}{4})$ C. $(8,-3)$ D. (16,3) E. (12,2)

16. If each unit on the Cartesian plane is equal to $\frac{1}{3}$ cm, what is the perimeter of the shape formed by the points A, B, C, and D?



17. A rectangle is drawn on a Cartesian plane with vertices A, B, C, D. Point A is at (4,2). Point C is at $(-4,-2)$. Point A and C are diagonally opposite corners of the rectangle. What is the area of the rectangle if each unit on the Cartesian plane is 1 cm?

18. In how many different quadrants could the point (w, s) lie if $s > w$?
    A. 0 B. 1 C. 2 D. 3 E. 4

Remember this?

19. The height of a stack of books is 223.2 mm. If there are 9 books in the stack of books, how thick is each book?
    A. 24.8 mm B. 26.3 mm C. 27.2 mm D. 25.4 mm

20. It is recommended that 12 year-old children sleep between 9 and 11 hours every night. From the table below, who does not get enough sleep every night?

12 year old children	Hours of sleep per night
Rebecca	10
Timothy	9.5
Sara	7.75
Samantha	8.25

    A. Rebecca and Sara B. Sara and Timothy C. Timothy and Rebecca D. Sara and Samantha

21. A local pharmacy is open from 9 am to 9 pm from Monday to Friday. How many hours a week is the pharmacy open for?
    A. 45 B. 55 C. 60 D. 92

8B Symmetry and reflections

There are two types of symmetry, line symmetry and rotational symmetry. The line of symmetry is a line that can be drawn through a shape to divide it into two identical parts. Each part mirrors the other over the line of symmetry. This can be called reflectional symmetry as reflections also mirror each other over a line of symmetry/mirror line. Shapes can also have rotational symmetry if they can be rotated around a point and keep the same shape. A shape that only keeps the same shape if rotated a full 360 degrees is considered to not have rotational symmetry. Below are some examples where symmetry and reflections can be applied.

- One wing of a butterfly is a mirror image of the other. What is the mirror line of a butterfly?
- To write a message to be reflected in a mirror you must write the message in reverse order. How would you write hello as a mirror message?
- A star has rotational symmetry because it keeps the same shape when it is rotated. What are some other shapes with rotational symmetry?

Learning intentions

Students will be able to:

- identify the line of symmetry in a shape
- find the order of line symmetry of a shape
- find the order of rotational symmetry of a shape
- reflect points and images over a line of symmetry on the Cartesian plane.

Key terms and definitions

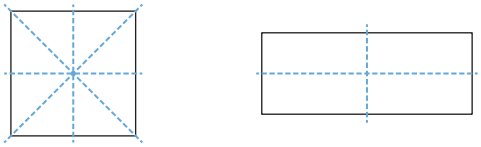
- A **line of symmetry** or axis divides a shape into two identical parts.
- **Order of line symmetry** is the number of lines of symmetry a shape has.
- **Order of rotational symmetry** is the number of times a shape can be rotated around a point (up to 360 degrees) and stay the same shape.
- A **reflection** uses a line of symmetry as a mirror line to change an image's position without changing its size or shape. A reflection point is written as A'.
- **Asymmetrical** shapes do not have any lines of symmetry.

Key ideas

1 Shapes can have no lines of symmetry, one line of symmetry or more than one line of symmetry. The order of line symmetry is the number of lines of symmetry in the shape.

Symetrical shape **Asymetrical shape**

Order of line symmetry = 4 **Order of line symmetry = 2**

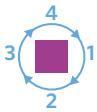


2 A shape has rotational symmetry if it can be turned around a point less than 360 degrees and appears the same.

A shape that has an order of rotational symmetry of 1 is considered to have no rotational symmetry.

Order of rotational symmetry is 1

A shape must have an order of rotational symmetry that is more than 1 to be considered to have rotational symmetry.

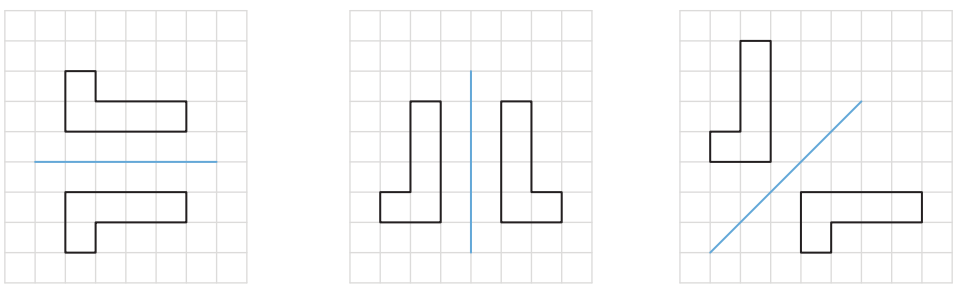


Order of rotational symmetry is 4

Order of rotational symmetry is 3

3 Shapes can be reflected over a mirror line to a new position. When shapes are reflected their position and orientation can change but not the shape or size.

Vertical reflection **Horizontal reflection** **Diagonal reflection**



Worked examples

WE 1 Finding the order of line symmetry and rotational symmetry

State the order of line symmetry and rotational symmetry for each image.

a.

Working

The order of line symmetry is 5.

The order of rotational symmetry is 5.

Thinking

Step 1: Find the number of lines that divide the shape into two identical parts.

Step 2: Find the number of times the shape looks the same when rotated 360 degrees around a point.

b.

Working

The order of line symmetry is 1.

The order of rotational symmetry is 1.

Thinking

Step 1: Find the number of lines that divide the shape into two identical parts.

Step 2: Find the number of times the shape looks the same when rotated 360 degrees around a point.

Student practice

State the order of line symmetry and rotational symmetry for each image.

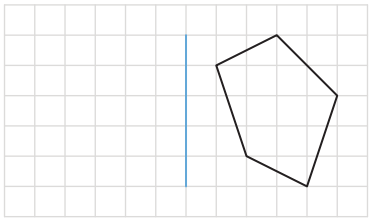
a.

b.

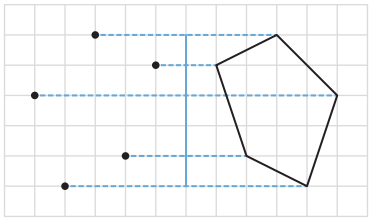
WE 2 Reflecting an image over a mirror line

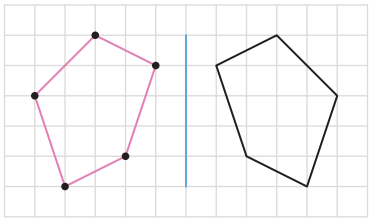
Reflect the image over the mirror line.

a.



Working



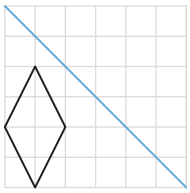


Thinking

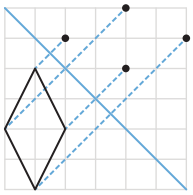
Step 1: Reflect the vertices of the shape by matching the perpendicular distance of each vertex to the line of reflection.

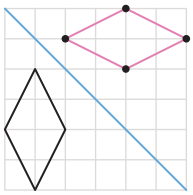
Step 2: Complete the reflected shape by matching the edges.

b.



Working





Thinking

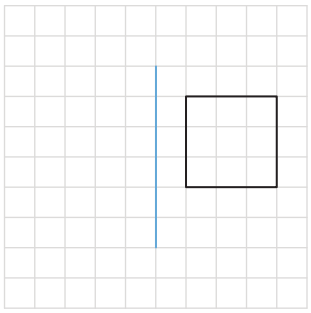
Step 1: Reflect the vertices of the shape by matching the perpendicular distance of each vertex to the line of reflection.

Step 2: Complete the reflected shape by matching the edges.

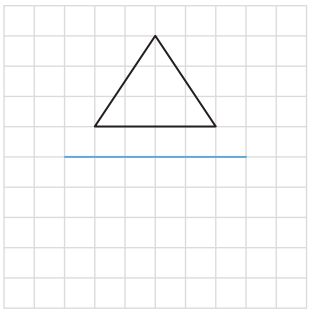
Student practice

Reflect the image over the mirror line.

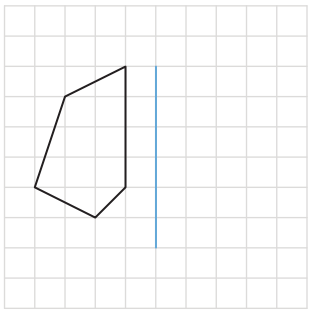
a.



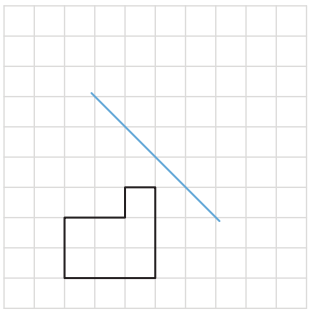
b.



c.



d.



WE 3 Finding the coordinates of a point reflected over the x or y-axis

Write the coordinates of the point reflected over the specified axis.

a. (4,3), x-axis

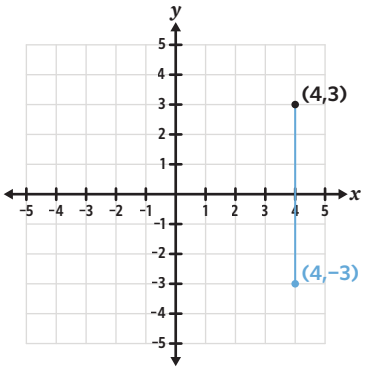
Working

(4,−3)

Thinking

Reflecting over the x-axis will change the positive y-coordinate to a negative y-coordinate. The x-coordinate will not change.

Visual support



b. (−1,−2), y-axis

Working

(1,−2)

Thinking

Reflecting over the y-axis will change the negative x-coordinate to a positive x-coordinate. The y-coordinate will not change.

Student practice

Write the coordinates of the points reflected over the specified axis.

a. (3,4), x-axis

b. (−5,1), y-axis

c. (4,2), x-axis

d. (−2,−4), y-axis

8B Activities and questions

STARTER TASKS

Odd spot

The Taj Mahal is located in the city of Agra in India. It was built in the 17th century and was commissioned by a Mughal emperor to house the tomb of his favourite wife. The building and its surrounding gardens were planned to be perfectly symmetrical. The following photo shows the Taj Mahal.

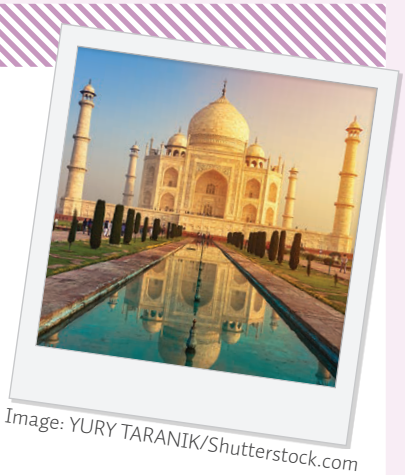


Image: YURY TARANIK/Shutterstock.com

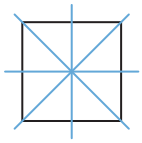
How many lines of symmetry does the Taj Mahal have?

A. 1 **B.** 2

Puzzle

When a shape is folded along a line of symmetry each side of the shape overlaps itself perfectly.

Square



Rectangle

a) Fold a square piece of paper to create four creases that represent its lines of symmetry.

b) Fold a rectangular piece of paper to create two creases that represent its lines of symmetry.

c) Create a piece of paper that is a parallelogram. How many lines of symmetry does it have?

Understanding worksheet

1. Draw the line(s) of symmetry for each shape.

Example

Order of line symmetry: 1

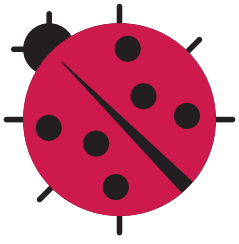
a.

Order of line symmetry:

b.

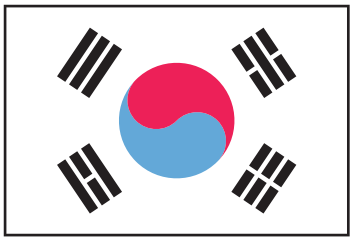
Order of line symmetry:

c.



Order of line symmetry:

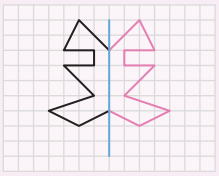
d.



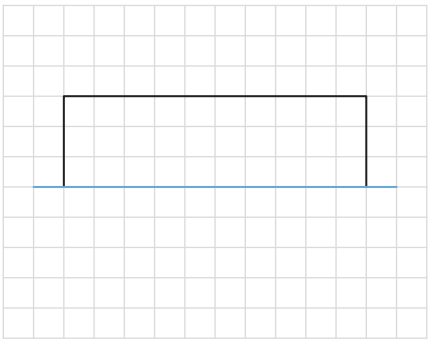
Order of line symmetry:

2. Reflect the image over the mirror line.

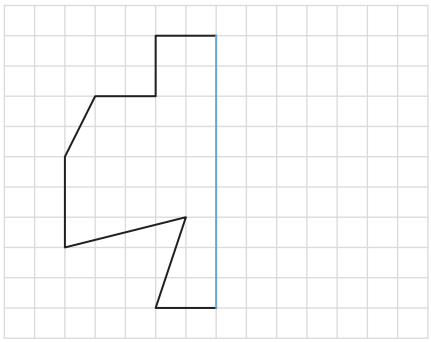
Example



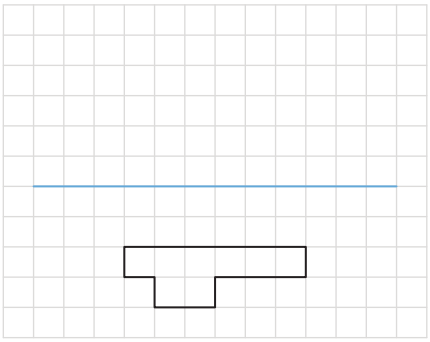
a.



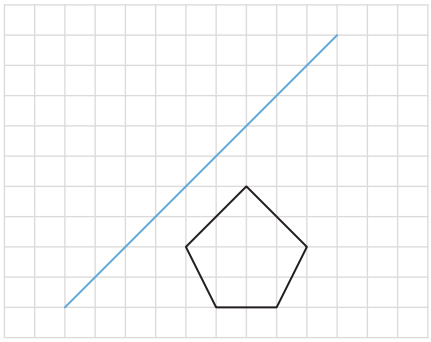
b.



c.



d.



3. Fill in the blanks by using the words provided.

symmetry | 360° | line | order

Shapes can have either or both _____ symmetry and rotational _____.

Rotational symmetry is when a shape can be turned less than _____ and look the same.

The _____ of line symmetry states how many lines of symmetry a shape has.

Fluency

Question working paths

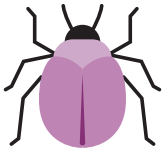
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,), 6 (a,b,c), 7 (a,b,c)	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d)	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f)

WE 1 **4.** State the order of line symmetry and rotational symmetry for each image.

a.

b.

c.

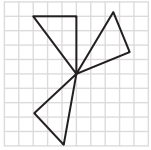


d.

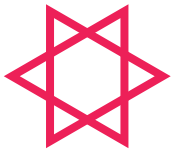
e.

f.

g.

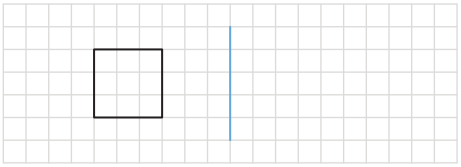


h.

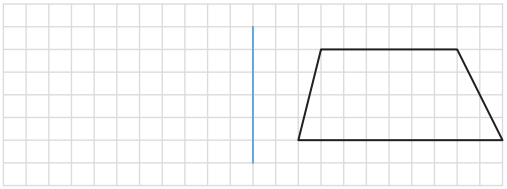


WE 2 5. Copy the shape onto grid paper and reflect the image over the mirror line.

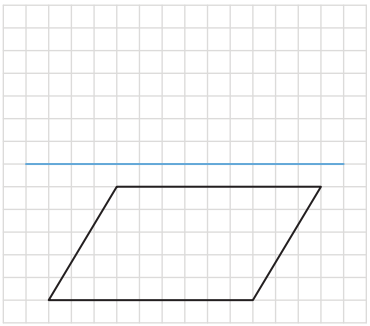
a.



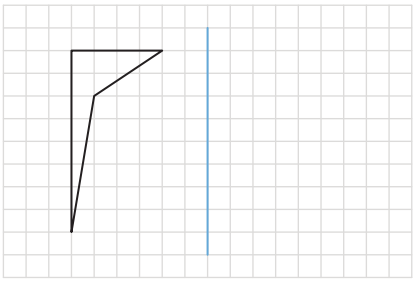
b.



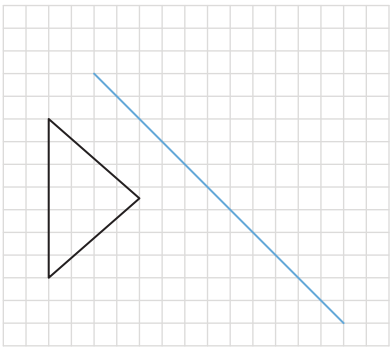
c.



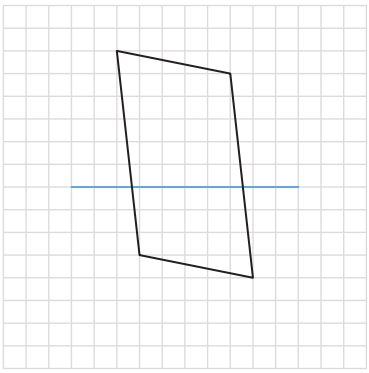
d.



e.



f.



WE 3a 6. Write the coordinates of the point reflected over the x-axis.

a. (3,1) b. (4,−1) c. (−2,2) d. (−1,4)

e. (−2,−4) f. (0,2)

WE 3b 7. Write the coordinates of the point reflected over the y-axis.

a. (2,4) b. (−3,1) c. (2,0) d. (−4,−5)

e. (1,−6) f. (0,2)

Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. The top of Cheryl's jewellery case is a plain solid gold regular hexagon. How many lines of symmetry does the top of the case have?

9. What is the order of rotational symmetry of Lucia's t-shirt if she has a standard t-shirt with a happy face emoji on the front?

10. What is the order of line symmetry and rotational symmetry of the top of a standard open umbrella with 8 sides?

11. Shawn is reading a secret message that has been reflected horizontally over a mirror line. How does the message appear before it is reflected if the message reads 'TODAY'?

12. Sierra is finding all the numbers from 0 to 9 that look the same when they are reflected over a vertical line. So far, she has the number 0. What is the other number that looks the same when reflected over a vertical line?

Reasoning

Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Stephen's class is investigating the characteristics of the letters of the alphabet. His teacher writes these letters on the board:

A B C D E F G H I J K L

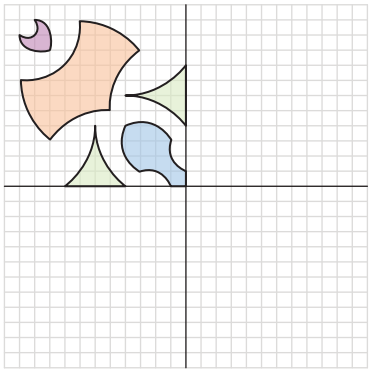
a. Which letters are asymmetrical?

b. Which letters have an order of line symmetry greater than 1?

c. Which letters have an order of rotational symmetry greater than 1?

d. A palindrome is a word that is spelled the same forwards and backwards. Anna is an example of a palindrome. What are some other palindromes you can think of?

14. A kaleidoscope image is created by reflecting an image over mirror lines. Nancy is building her own kaleidoscope image and is deciding between different design options. The image before being reflected is shown.



a. Nancy starts by only using a horizontal mirror line to create her design. Using her initial design, redraw what Nancy's design will look like with a vertical reflection.

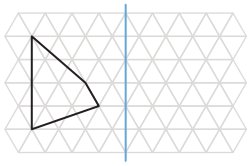
b. Nancy doesn't like the vertical reflection so recreates the design with only a horizontal reflection. Using her initial design, redraw what Nancy's design will look like with a horizontal reflection.

c. For a better effect, Nancy thinks an extra mirror line is needed. Redraw the design from part **a** with a horizontal reflection.

d. Create your own kaleidoscope image with 2 mirror lines.

Extra spicy

15. Reflect the shape over the mirror line.



16. After a how many degree turn will the image look like its original?

A. 90°

B. 120°

C. 180°

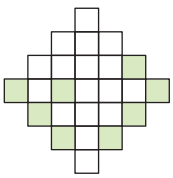
D. 270°

E. 290°

17. How many lines of symmetry does a circle have?

A. 0 **B.** 2 **C.** 4 **D.** 10 **E.** Infinite

18. What is the least number of squares that can be shaded to make this shape symmetrical?



A. 1

B. 2

C. 3

D. 4

E. 5

Remember this?

19. $4 + 2 \times ☮ = 16$

To make this a true number sentence, what is the value of ☮?

A. $☮ = 3$ **B.** $☮ = 5$ **C.** $☮ = 6$ **D.** $☮ = 8$

20. Janette loves to buy holiday wrapping paper in January because of the post holiday discounts. A pack of 3 rolls of wrapping paper is discounted by 20% after the holidays. If the pack usually costs $24, what is the sale price?

21. The point $(-3,2)$ is located in which quadrant of the Cartesian plane?

A. Quadrant 1 **B.** Quadrant 2 **C.** Quadrant 3 **D.** Quadrant 4

8C Translations

A translation is a type of transformation where a shape changes its position without being rotated and without changing size. A translation moves the position of a shape's vertices the same distance, in the same direction. All of the x and y coordinates of the shape will move the same number of units left, right, up or down. Translations can combine, moving a shape horizontally and vertically. Below are some examples of where translations can be used.

- I am playing checkers and double jump over my opponent. How many spaces have I moved from my original location?
- My desk is 3 metres from the front of the class. I move 2 more metres backwards. How far am I from the front?
- To walk home from school, I walk exactly 1 km directly south then 2 km directly east. How can I walk to school from home?

Learning intentions

Students will be able to:

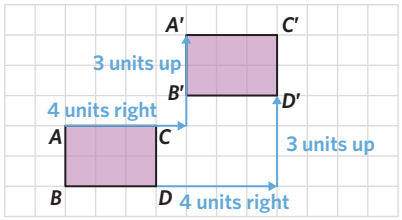
- move a shape left, right, up or down
- move a point of a shape in a given direction
- describe a translation from a set of coordinates
- determine an original point by reversing a translation.

Key terms and definitions

- A **transformation** changes the position of a point or set of points in an image based on a rule.
- The **original** refers to the point, shape, or image before it is moved.
- A **translation** is a type of transformation that shifts the location of the original but does not change the orientation or size of the original shape.
- A translation is one type of **isometric** transformation because the original image does not change in size or shape after the transformation.

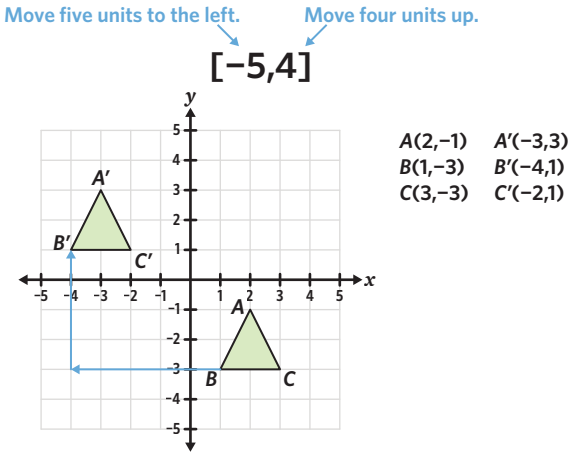
Key ideas

1 When a shape is translated, its points are moved the same number of units left, right, up or down. The size and orientation of the shape do not change.



For example, Point A is translated to Point A'.

2 Translations can be written as coordinates on the Cartesian plane in square brackets. Positive coordinates represent a translation in the positive direction (right or up). Negative coordinates represent a translation in the negative direction (left or down).



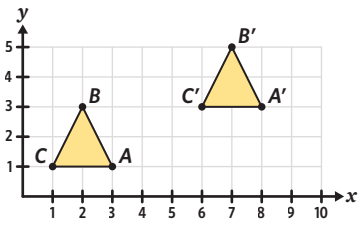
$A(2,-1)$ $A'(-3,3)$
$B(1,-3)$ $B'(-4,1)$
$C(3,-3)$ $C'(-2,1)$

Worked examples

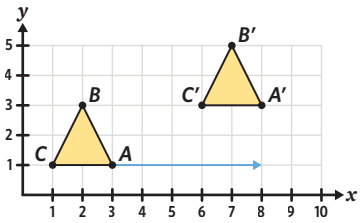
WE 1 Describing a translation

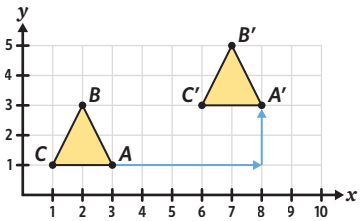
Describe each translation.

a.



Working





The object is translated 5 units right and 2 units up.

Thinking

Step 1: Describe the horizontal translation.
The object moves 5 units right.

Step 2: Describe the vertical translation.
The object moves 2 units up.

b. Point $B(-3,1)$ to $B'(-5,-2)$

Working

The point moves 2 units left.

The point moves 3 units down.

Point B is translated 2 units left and 3 units down.

Thinking

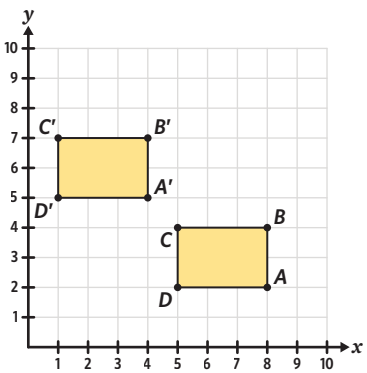
Step 1: Describe the horizontal translation.
The x-coordinate has changed from -3 to -5.

Step 2: Describe the vertical translation.
The y-coordinate has changed from 1 to -2.

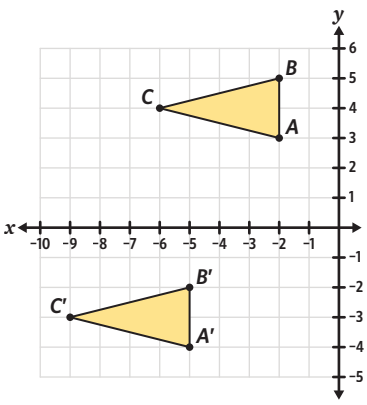
Student practice

Describe each translation.

a.



b.



c. Point $D(-2,-3)$ to $D'(2,5)$

d. Point $C(4,-2)$ to $C'(2,2)$

WE 2 Writing new coordinates of a translation

Write the coordinates for point A' when point $A(0,3)$ is moved by the given translation.

a. 5 units right and 1 unit up

Working

$0 + 5 = 5$

$3 + 1 = 4$

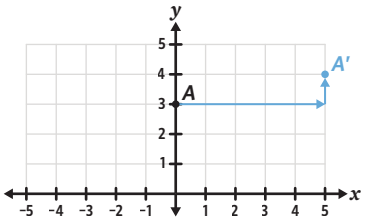
Point $A(0,3)$ moves to $A'(5,4)$.

Thinking

Step 1: Apply the horizontal translation. A translation of 5 units right will increase the x-coordinate of the point by 5.

Step 2: Apply the vertical translation. A translation of 1 unit up will increase the y-coordinate of the point by 1.

Visual support



b. $[-3,-2]$

Working

$0 - 3 = -3$

$3 - 2 = 1$

Point $A(0,3)$ moves to $A'(-3,1)$.

Thinking

Step 1: Apply the x-coordinate of the translation. The value is -3 so the x-coordinate of the point will decrease by 3.

Step 2: Apply the y-coordinate of the translation. The value is -2 so the y-coordinate of the point will decrease by 2.

Student practice

Write the coordinates for point E' when point $E(-4,3)$ is moved by the given translation.

a. 3 units right and 2 units down

b. 2 units left and 1 unit up

c. $[2,1]$

d. $[-3,2]$

WE 3 Reversing a translation

Reverse the given translation for point $A'(4,3)$ to find the original coordinates of point A.

a. 5 units right and 2 units up

Working

5 units left and 2 units down.

$4 - 5 = -1$

$3 - 2 = 1$

The coordinates of point A are $(-1,1)$.

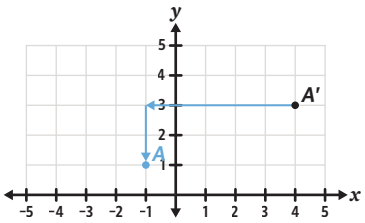
Thinking

Step 1: Reverse the translation. The reverse of 5 units right and 2 units up is 5 units left and 2 units down.

Step 2: Apply the reverse horizontal translation. A translation of 5 units left will decrease the x-coordinate of the point by 5.

Step 3: Apply the reverse vertical translation. A translation of 2 units down will decrease the y-coordinate of the point by 2.

Visual support



b. [4,3]

Working

[−4,−3]

$4 - 4 = 0$

$3 - 3 = 0$

The coordinates of point A are (0,0).

Thinking

Step 1: Reverse the translation coordinates.
The reverse of [4,3] is [−4,−3].

Step 2: Apply the x-coordinate of the reverse translation.
The value is −4 so the x-coordinate of the point will decrease by 4.

Step 3: Apply the y-coordinate of the reverse translation.
The value is −3 so the y-coordinate of the point will decrease by 3.

Student practice

Reverse the given translation for point $W'(2,5)$ to find the original coordinates of point W.

a. 2 units right and 1 unit down

b. 4 units left and 2 units up

c. [3,−2]

d. [5,3]

8C Activities and questions

STARTER TASKS

Image: Hi-Point/Shutterstock.com

Odd spot

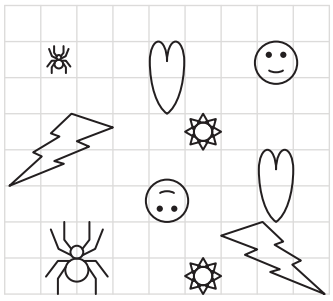
Tessellations are unique transformations of a shape that include translating a shape without any gaps or overlays. Tessellations have been used in art and architecture since ancient times to decorate walls, floors, and pavements of buildings.
One common tessellation pattern is shown below.

Describe the translation of the rhombus from A to B.

A. Moves down and to the right

B. Moves up and to the left

Puzzle



a) Which two images represent a translation of their original image?

b) Explain why the smiley face image cannot be classified as a translation.

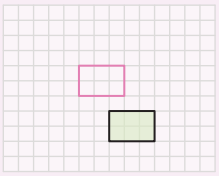
c) Assuming that the small spider is the original, describe the translation required so that it will be positioned below the sun.

Understanding worksheet

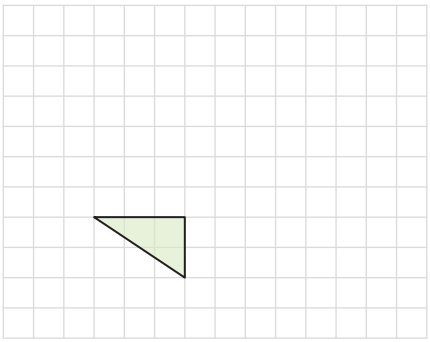
1. Draw the translated image.

Example

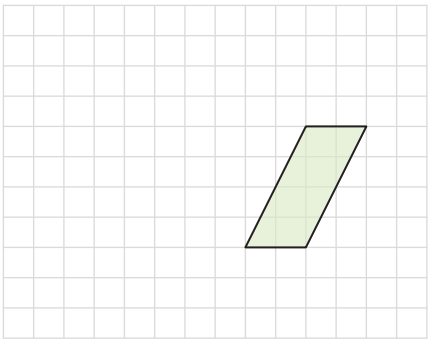
2 units left and 3 units up



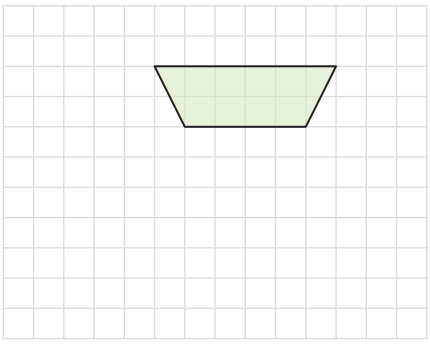
a. 3 units right and 4 units up



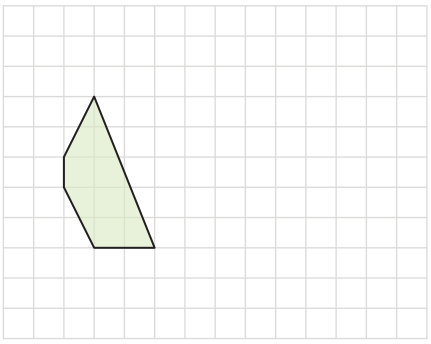
b. 2 units left and 2 units up



c. 3 units left and 3 units down



d. 4 units right and 2 units down



2. Fill in the blanks to describe each translation.

Example

Point $B(2,4)$ is translated to $B'(-1,2)$.

3 units left and 2 units down

a. Point $C(3,2)$ is translated to $C'(-2,-1)$.

____ units left and 3 units ____

b. Point $D(4,1)$ is translated to $D'(1,4)$.

____ units left and ____ units up

c. Point $E(3,2)$ is translated to $E'(-2,-2)$.

____ units left and 4 units ____

d. Point $G(-2,6)$ is translated to $G'(4,-1)$.

6 units ____ and 7 units ____

3. Fill in the blanks by using the words provided.

original translation size orientation

Transformations change a set of points based on a rule. Some transformations can change the ____ and orientation of a shape. A ____ is a transformation that only changes a shape's position. The position of the ____ shape changes, but not its size or ____.

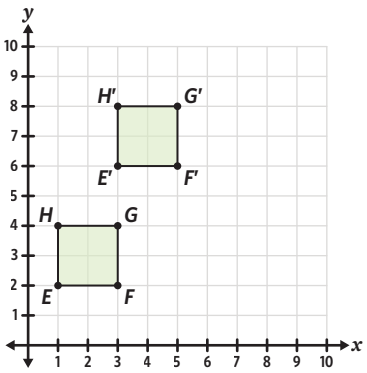
Fluency

Question working paths

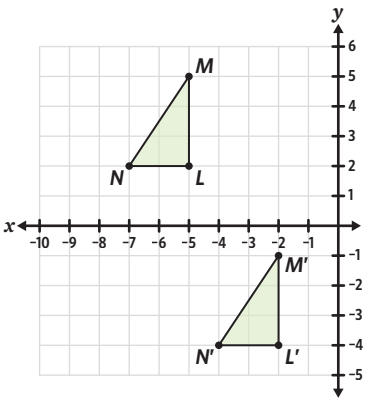
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c,d)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (c,d,e,f)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (e,f,g,h)

WE 1a **4.** Describe the translation of each shape.

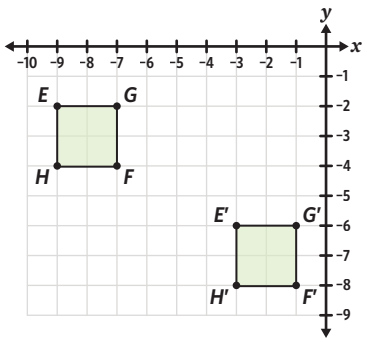
a.



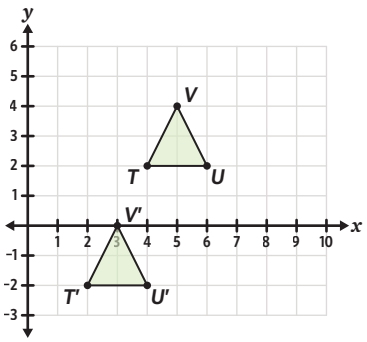
b.



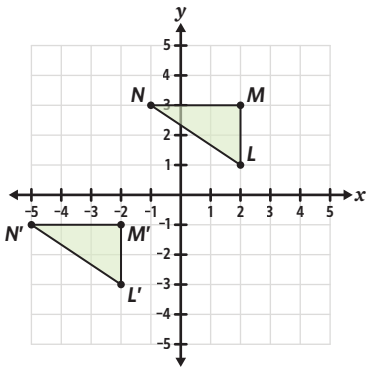
c.



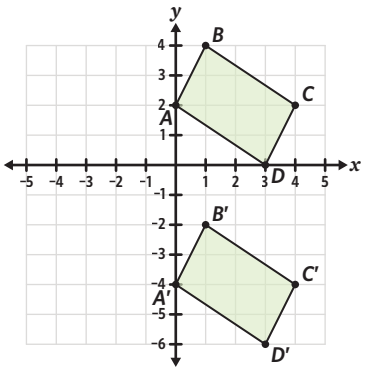
d.



e.



f.



WE 1b **5.** Describe the translation of each point.

a. Point $A(3,2)$ to $A'(1,-1)$

b. Point $B(2,-1)$ to $B'(4,2)$

c. Point $C(-3,3)$ to $C'(-1,1)$

d. Point $D(-2,-2)$ to $D'(-3,-5)$

e. Point $E(5,2)$ to $E'(0,0)$

f. Point $F(-4,-1)$ to $F'(0,4)$

WE 2 **6.** Write the coordinates for point A' when point $A(3,4)$ is moved by the given translation.

a. 2 units right and 1 unit up

b. [1,3]

c. 4 units left and 2 units down

d. 2 units right and 5 units up

e. [−2,2]

f. [3,−4]

g. 4 units left, 2 units up, 1 unit right, and 3 units down

h. 3 units right, 2 units down, 1 unit left, and 4 units up

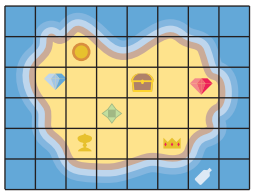
WE 3 7. Reverse the given translation for point $D'(4,3)$ to find the original coordinates of point D.

a. 1 unit right and 4 units up
b. [4,2]
c. [−2,−1]
d. 3 units left and 3 units up
e. [3,−1]
f. [−4,−3]
g. 3 units right, 2 units up, 5 units left, and 4 units down
h. 5 units left, 6 units up, 9 units right, and 2 units down

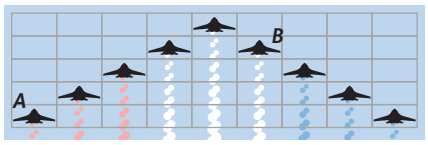
Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

8. Brennan found a treasure map and is trying to find all the treasure. Describe the translation if Brennan moves from the crown's location to the treasure chest.



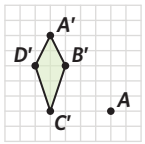
9. The planes in an airshow form a special formation and are positioned one plane width and one plane length apart. How many plane widths and lengths does plane A need to move to reposition to plane B's position?



10. Warren moves a knight from the bottom left of the chessboard two squares to the right and one square up. How should Warren move the knight so that it returns to its original position?

11. James is using a Cartesian plane to rearrange his room. What are the new coordinates of James's TV if his TV is currently at (2,2) on the Cartesian plane, and he moves it 6 units to the right and 1 unit down?

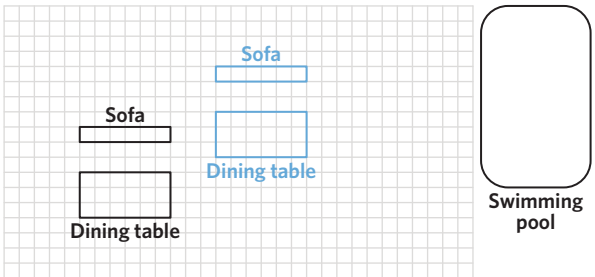
12. Stacey draws the following translated image. The original point A of the figure is also shown. Describe the translation Stacey performed.



Reasoning

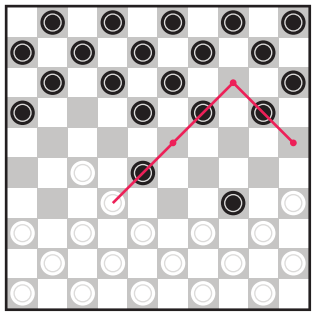
Mild	Medium	Spicy
13 (a,b,c)	13 (a,b,c), 14 (a,b,c)	All

13. Estelle added an extension to her living room to make an open plan dining room. She decided to move her dining table and sofa to a new position and maintain the same distance between each piece of furniture. Estelle used the following plan, where the length of each horizontal and vertical unit is one metre.



a. How many metres to the right and up did she move her dining table and sofa from the original position in black to the new position in blue?
b. Why is this move considered a translation?
c. She decides that the furniture is now too close to the swimming pool and plans to shift the dining table and sofa 3 metres to the left from its new position. What is the translation to get to this spot from her original position in black?
d. What is another possible translation for her dining table and sofa given Estelle's new space?

14. Carol and Jude are playing a game of draughts. The aim of the game is to eliminate all of the opponent's pieces. A piece is eliminated when it is 'jumped' over by the opposition's different colour piece. Carol is white and Jude is black. Carol can complete a 'triple jump' to eliminate three of Jude's pieces.



a. Describe the single translation of Carol's piece from its starting position to landing position after completing the 'triple jump'.

Note: Use the terms 'backward' and 'forward' to describe vertical movement.

b. Write the translation of Carol's 'triple jump' as Cartesian coordinates in square brackets.

c. Carol's final landing place after the triple jump is the combination of 3 translations each marked by a dot on the board. Write all three translations Carol makes as Cartesian coordinates in square brackets.

d. Is it a better strategy to make as many jumps as you can, or is it a better strategy to make less jumps but block your opponent from moving? Explain why.

Extra spicy

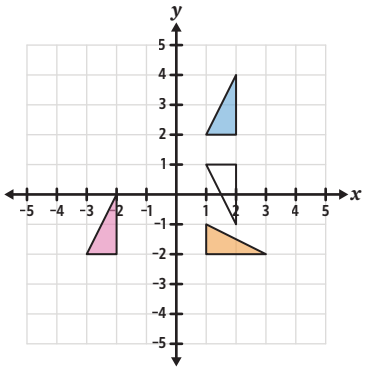
15. Reverse the following translation and write it as a single translation.

3 units left, 4 units right, 2 units up, and 6 units down.

16. Combine the following translations to write it as a single translation.

[2,1], [3,2], [−1,3], [−4,−2]

17. Which color triangle is a translation of the original blue triangle?



A. Pink **B.** White **C.** Pink and white **D.** All of the above

18. Point E is located on the Cartesian plane at (2,2). Which translation will find Point E' in the third Quadrant?

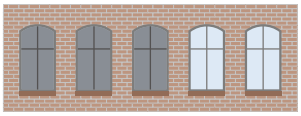
A. [2,−2] **B.** [−5,1] **C.** [−2,−2] **D.** [−4,−5] **E.** [2,−4]

Remember this?

19. An unknown number is added to 6. The result is multiplied by 4 to give an answer of 12. What is the unknown number?

A. −9 **B.** −3 **C.** 3 **D.** 9 **E.** 42

20. The following building has 5 windows of equal length and height.



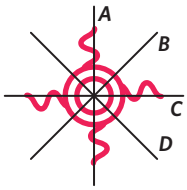
Three of the windows are tinted to shade the sunlight.

What percentage of windows are tinted?

A. 3% **B.** 20%

C. 40% **D.** 60%

21. Which are the lines of symmetry of this yam plant symbol?



A. A and B

B. A and C

C. B and D

D. A, B, C and D

8D Rotations

A rotation is a transformation where the shape is rotated around a centre point that does not move. The size and original shape do not change, but the orientation changes. How far the shape is rotated is decided by the size of the angle of rotation. These angles are commonly multiples of 30° or 45°. A rotation of 360° rotates the image a full circle ending at the original starting position. Below are some examples of rotations in real life.

- The earth rotates one full turn around the sun in a year. What is the angle of rotation in one year?
- The hour hand starts at 12 and rotates one full turn in 12 hours. What is the angle of rotation for the hour hand when it is 3:00?
- The steering wheel of my car rotates around its centre, how many degrees does it rotate when I make a right turn?

Learning intentions

Students will be able to:

- identify clockwise and anticlockwise rotations
- identify the size and angle of rotation
- rotate a shape around the centre of origin.

Key terms and definitions

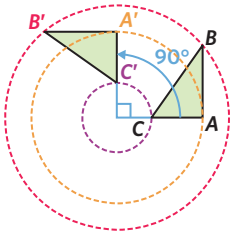
- **Clockwise** is the same direction that the hands of a clock move around.
- **Anticlockwise** is the opposite direction that the hands of a clock move around.
- The **orientation** refers to the position of a figure in a specific direction.
- The **angle of rotation** is the number of degrees an image is rotated.
- The **centre of rotation** is the fixed point an image or point is rotated around. This point is often labelled as O.

Key ideas

1 There are different types of rotations around a point. The point is called the centre of rotation and can be its centre, a part of the image, or outside the image.

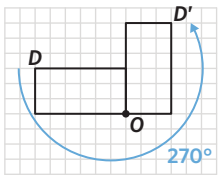
The vertices of triangle ABC are rotated 90° anticlockwise around point C, outside of the image, to become triangle $A'B'C'$.

In a rotation, the distance of each point from the centre of rotation is not changed as shown by the full circle arcs.

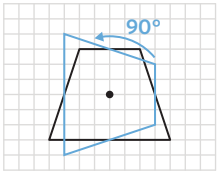


Other types of rotations can be:

Centre of rotation is part of the image.

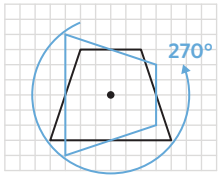


Centre of rotation is in the centre of the image.

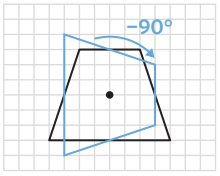


2 A rotation changes the position and orientation of an image but does not change its shape or size. An image can be rotated clockwise or anticlockwise.

Anticlockwise rotations are positive rotations.

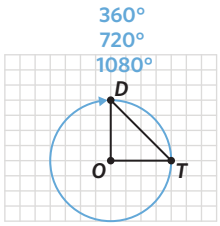


Clockwise rotations are negative rotations.



Clockwise and anticlockwise rotations that add up to 360° result in the same position. E.g. 90° anticlockwise is the same as 270° clockwise because $90 + 270 = 360$.

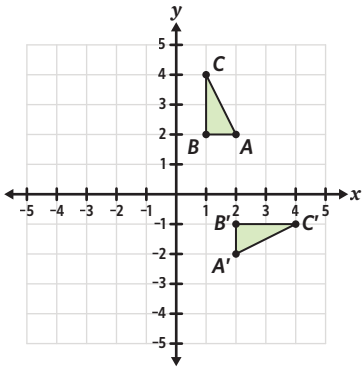
3 Multiples of 360° rotations around a point result in the same position as the original.



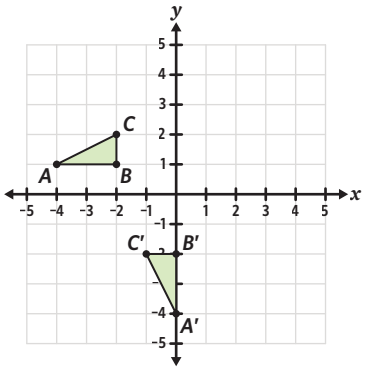
360° is one full rotation, 720° is two full rotations, and 1080° is three full rotations.
Multiples of 360° are all full rotations where the rotated image looks the same as the original.

4 Rotations on a Cartesian plane can be performed around the origin point $O(0,0)$.

90° clockwise (−90°) or 270° anticlockwise (270°)



270° clockwise (−270°) or 90° anticlockwise (90°)

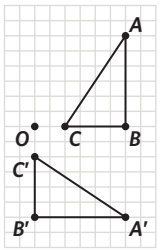


Worked examples

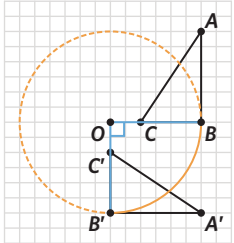
WE 1 Identifying the direction of a rotation

Identify whether each shape or point has been rotated clockwise or anticlockwise.

a. Triangle ABC is rotated 90° about point O.



Working



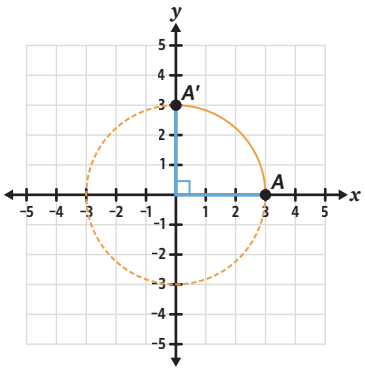
Triangle ABC is rotated clockwise.

Thinking

Find the direction of rotation of any vertex. As triangle ABC is rotated 90° to $A'B'C'$, points A, B and C are rotated clockwise.

b. Point $A(3,0)$ is rotated 90° about the origin to $A'(0,3)$.

Working



Point A is rotated anticlockwise.

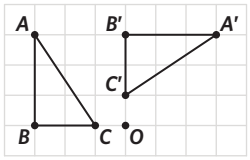
Thinking

Find the direction of rotation of the point. Point $A(3,0)$ is rotated 90° from the positive direction of the x-axis to the positive direction of the y-axis, so it is rotated anticlockwise.

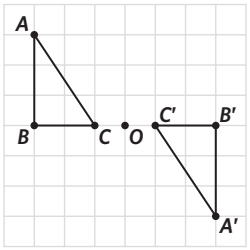
Student practice

Identify whether each shape or point has been rotated clockwise or anticlockwise.

a. Triangle ABC rotated 90° about point O.



b. Triangle ABC rotated 180° about point O.



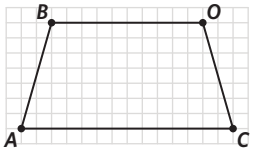
c. Point $C(0,2)$ is rotated 270° about the origin to $C'(2,0)$.

d. Point $D(2,2)$ is rotated 90° about the origin to $D'(2,-2)$.

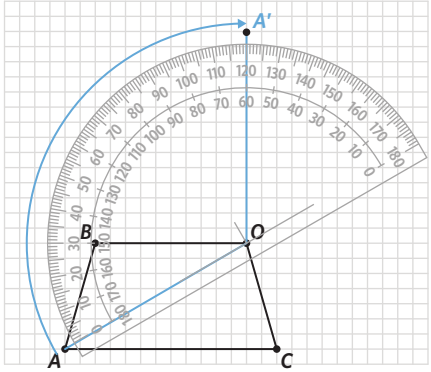
WE 2 Drawing rotations

Draw the given rotations about point O.

a. 120° clockwise

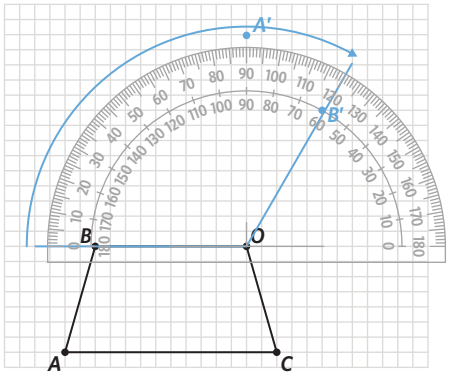


Working

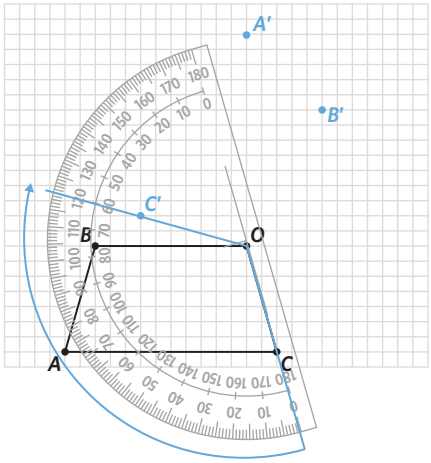


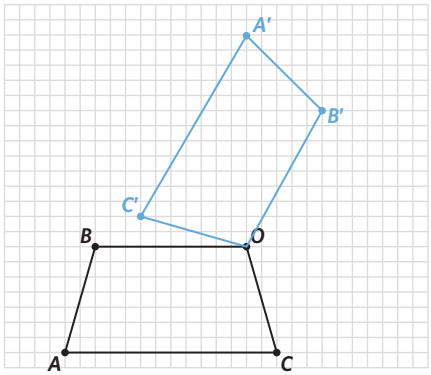
Thinking

Step 1: Draw a line from a chosen vertex to the point O. Align the base of the protractor to this line and measure a rotation of 120° in the clockwise direction to find the position of the rotated vertex.



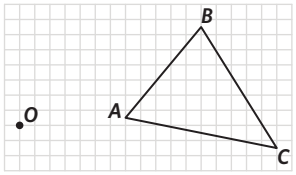
Step 2: Repeat the process for the remaining vertices. Ensure that the distance from the centre of rotation of the new point remains the same as the original.



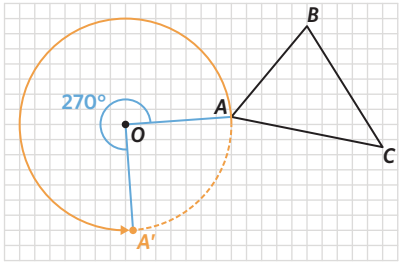


Step 3: Draw a straight edge to connect each vertex to complete the rotated shape. Vertex *O* is the centre of rotation so it will not change position.

b. 270° anticlockwise

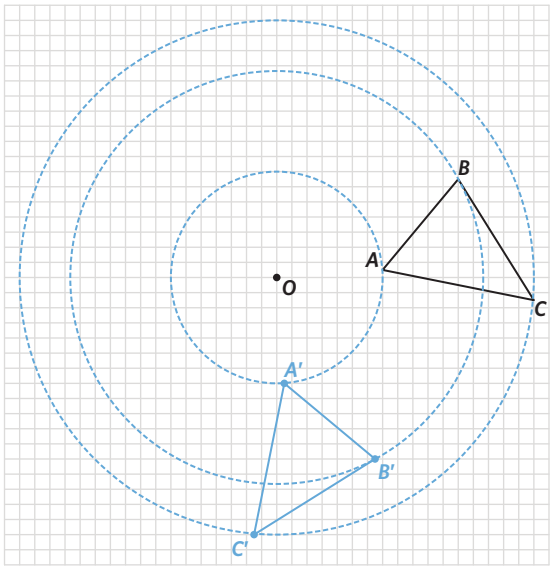


Working



Thinking

Step 1: Use a compass to draw a circle that has a centre point at point *O* and intersects with a chosen vertex. Use the circle to measure a rotation of 270° in the **anticlockwise** direction to find the position of the rotated vertex.

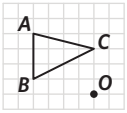


Step 2: Repeat the process for the remaining vertices. Draw a straight edge to connect each vertex to complete the rotated shape.

Note: A protractor could be used to complete the rotation by instead rotating the shape 90° in the clockwise direction.

Student practice

Copy the below image onto grid paper and draw the given rotations about point *O*.

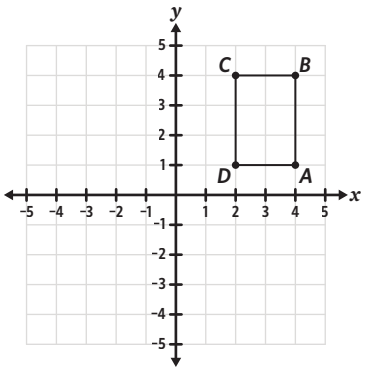


a. 90° anticlockwise **b.** 90° clockwise **c.** 180° **d.** −270°

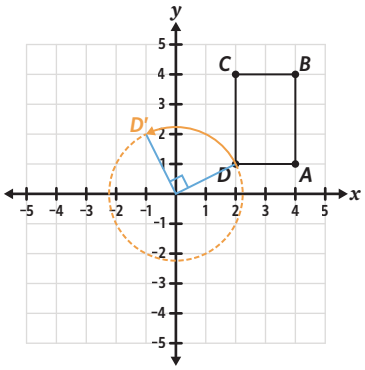
WE 3 Drawing rotations on the Cartesian plane

Write the Cartesian coordinates for the rotation.

a. Point *D* rotated 90° anticlockwise about the origin.



Working



Point *D′* has coordinates (−1,2).

Thinking

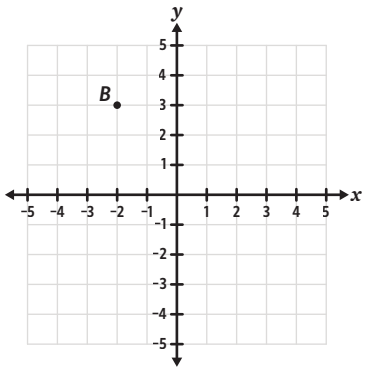
Step 1: Draw a circle that has a centre point at the origin and intersects with point *D*. Use the circle to measure a rotation of 90° in the anticlockwise direction to find the position of point *D′*.

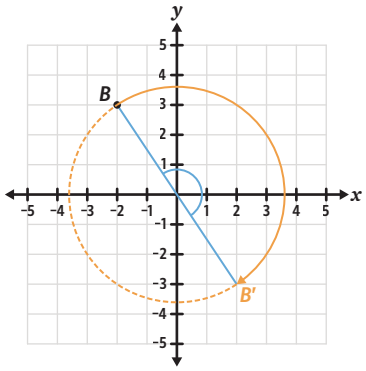
Note: In a 90° anticlockwise rotation about the origin, the *x*-coordinate of the rotated point is the negative of the *y*-coordinate of the original point. The *y*-coordinate of the rotated point is the *x*-coordinate of the original point.

Step 2: Identify the coordinates of point *D′*.

b. Point $B(-2,3)$ rotated 180° clockwise about the origin.

Working





Point B' has coordinates $(2,-3)$.

Thinking

Step 1: Locate point $B(-2,3)$ on a Cartesian plane.

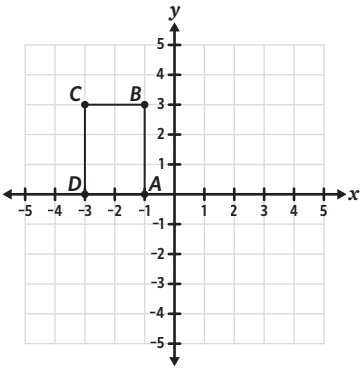
Step 2: Draw a circle that has a centre point at the origin and intersects with point B. Use the circle to measure a rotation of 180° to find the position of point B'.

Note: In a 180° rotation about the origin, the x-coordinate of the rotated point is the negative of the x-coordinate of the original point. The y-coordinate of the rotated point is the negative y-coordinate of the original point.

Step 3: Identify the coordinates of point B'.

Student practice

Write the Cartesian coordinates for the given rotation.



a. Point D in the diagram rotated 90° anticlockwise about the origin.

b. Point A in the diagram rotated 90° clockwise about the origin.

c. Point $F(-2,3)$ rotated 90° clockwise about the origin.

d. Point $G(0,-2)$ rotated 270° anticlockwise about the origin.

8D Activities and questions

STARTER TASKS

Odd spot

The Zodiac can be represented in many ways, either pictures or symbols. This is one representation of the Zodiac symbol for pisces. The symbol is based on the representation of two fish swimming head to tail. Assuming that the red dot in the following image is in the centre of the representation, how many degrees can the symbol be rotated to look the same as the original?

A. 180° **B.** 90°

Puzzle

The following image is created by putting together four different smaller images.

Image: New Africa/Shutterstock.com

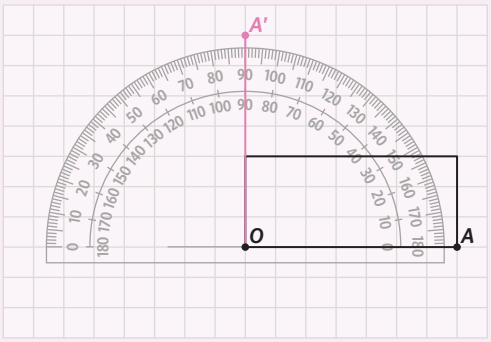
a) Which image is not in its correct position?
b) Describe a rotation to the image identified in part **a** so that the large image is completed correctly.
c) How can you rotate each smaller image so that the entire large image is upside down?

Understanding worksheet

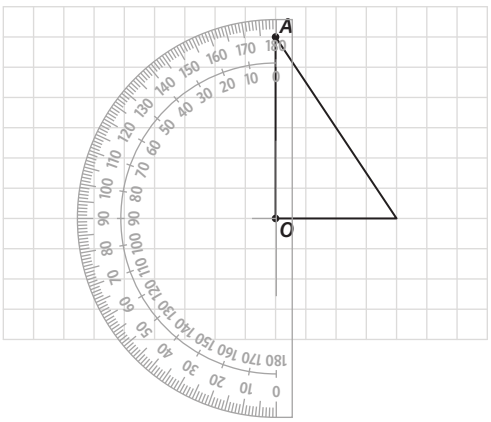
1. Use the given protractor to rotate the point.

Example

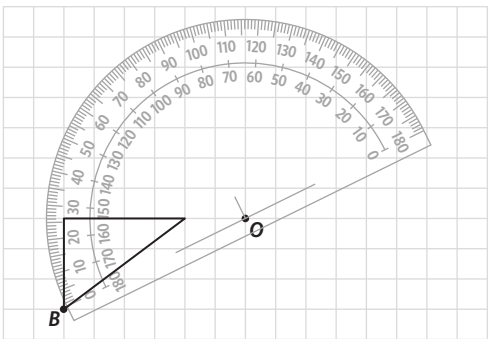
Rotate point A 90° anticlockwise about point O.



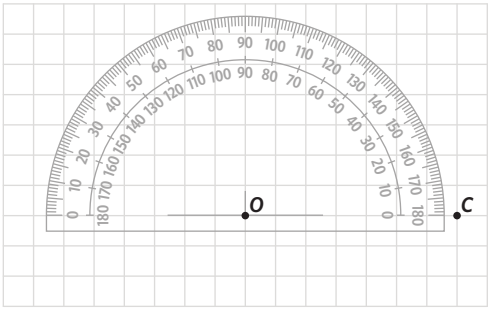
a. Rotate point A 90° anticlockwise about point O.



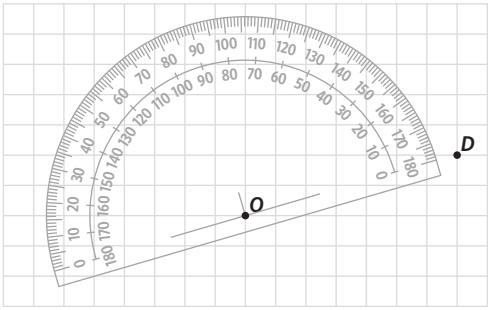
b. Rotate point B 180° clockwise about point O.



c. Rotate point C 120° anticlockwise about point O.



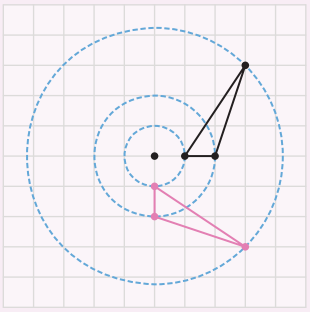
d. Rotate point D 45° anticlockwise about point O.



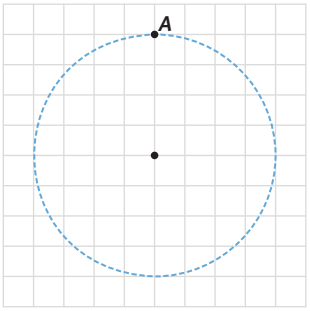
2. Complete the rotation using the full circle arc.

Example

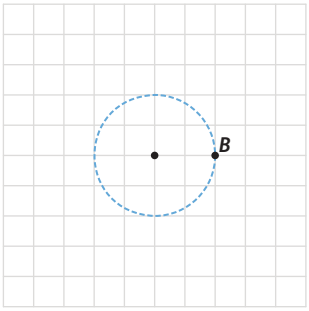
Rotate the shape 90° clockwise.



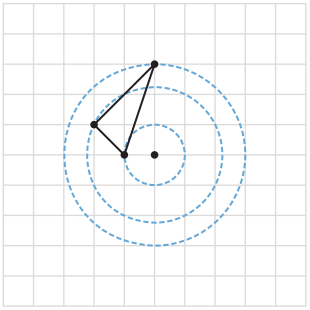
a. Rotate point A 90° clockwise.



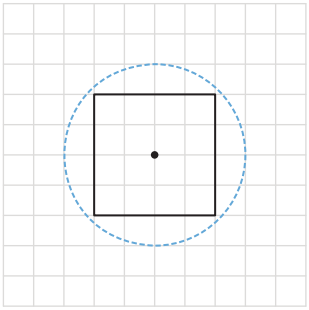
b. Rotate point B 180° anticlockwise.



c. Rotate the shape 90° anticlockwise.



d. Rotate the shape 135° clockwise.



3. Fill in the blanks by using the words provided.

angle | centre | anticlockwise | rotation | orientation

A _____ is a transformation that changes the position and _____ of an image but does not change the shape or size of the original image. An image can be rotated in a clockwise or _____ direction and the _____ of rotation tells you how many degrees to rotate the image. An image is rotated about a fixed point that is the _____ of rotation. This point can lie inside or outside the shape.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d), 8 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f), 8 (d,e,f)

4. Fill in the blank with the correct angle of rotation.

a. 180° clockwise is the same as _____ anticlockwise.

b. 90° clockwise is the same as _____ anticlockwise.

c. _____ clockwise is the same as 120° anticlockwise.

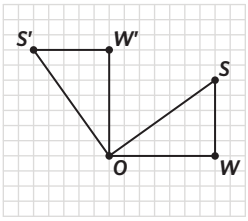
d. _____ clockwise is the same as 45° anticlockwise.

e. 105° clockwise is the same as _____ anticlockwise.

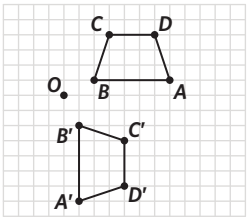
f. _____ clockwise is the same as 163° anticlockwise.

WE 1 **5.** Identify whether each shape or point has been rotated clockwise or anticlockwise.

a. *SWO* rotated 90° about point *O*.



b. *ABCD* rotated 90° about point *O*.



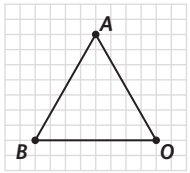
c. $A(2,0)$ rotated 180° about the origin to $A'(-2,0)$.

d. $T(-2,-3)$ rotated 180° about the origin to $T'(2,3)$.

e. $B(-1,0)$ rotated 90° about the origin to $B'(0,-1)$.

f. $R(0,-6)$ rotated 270° about the origin to $R'(6,0)$.

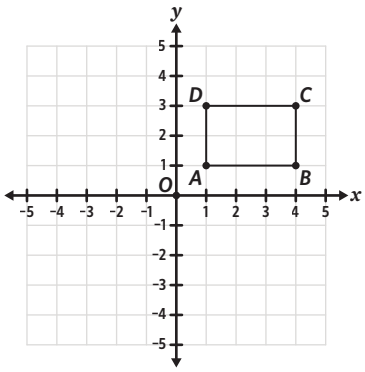
WE 2 **6.** Draw the given rotation of triangle *AOB* around point *O*.



a. 90° anticlockwise **b.** 180° clockwise **c.** 270° anticlockwise **d.** 360°

e. −90° **f.** 720° **g.** 45° **h.** −135°

WE 3a **7.** Write the Cartesian coordinates for the given rotation around the origin.



a. *A* rotated 180° clockwise

b. *A* rotated 270° anticlockwise

c. *B* rotated 90° anticlockwise

d. *C* rotated 360°

e. *C* rotated −90°

f. *D* rotated 540°

WE 3b **8.** Write the Cartesian coordinates for the given rotation around the origin.

a. $A(-2,3)$ rotated 90° clockwise

b. $B(1,4)$ rotated 180° anticlockwise

c. $C(-2,-3)$ rotated 270° clockwise

d. $D(-1,-4)$ rotated 90° clockwise

e. $E(4,2)$ rotated 270° anticlockwise

f. $F(-3,1)$ rotated 450° clockwise

Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Priyanka is completing a computer simulated driving test. In order to turn left does she need to turn the steering wheel clockwise or anticlockwise?

10. Bonnie loves collecting old records and playing them on her record player. If she started playing the album 'Abbey Road' by the Beatles, how much of a rotation would the record have completed for the title 'Abbey Road' to be perfectly upside down?

11. Sheldon is doing a school project where he is learning about flags of different countries. He has picked the following three flags. Which flags do not look like the original when rotated 270° anticlockwise?


Jamaica


Israel


Australia

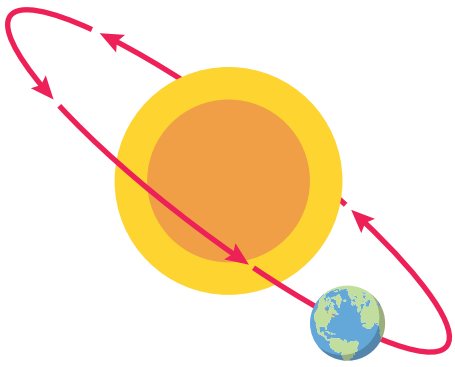
12. Bentley found a starfish while working on the beach. The starfish had five equally spaced rays. What would be the smallest angle of rotation that Bentley could rotate the starfish so that it looks the same?

13. On his way to the bathroom for a shower, Lance walks past a clock that reads exactly three o'clock. On his way back to his bedroom the clock reads three twenty-five. What was the angle of rotation of the minute hand of the clock during Lance's shower?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Every 365 days the earth completes one complete rotation around the sun, which is known as a calendar year. The earth's rotating position means that the northern and southern hemisphere experience each season at different times of the year.



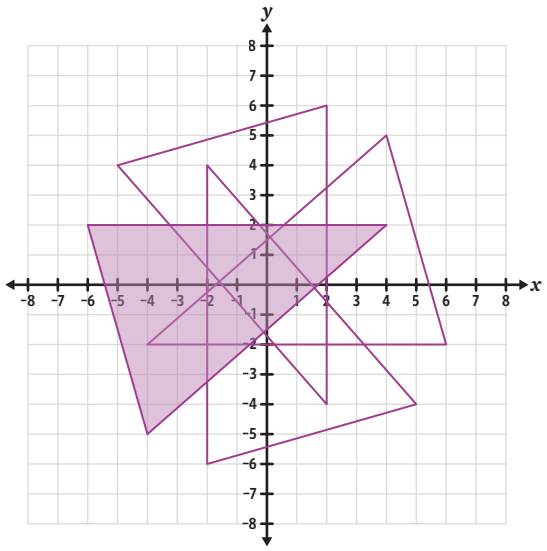
   a. What is the angle of rotation of the earth around the sun over two calendar years?

   b. There are four seasons in a year that are determined by the earth's rotation around the sun. Assuming the four seasons have an equal number of days, what is the angle of rotation after one season?

   c. In the southern hemisphere the calendar year begins in summer. What season will Australia be in after the earth completes a 270° rotation around the sun from the beginning of a calendar year.

   d. Summer begins at the end of a calendar year and overlaps the beginning of the following calendar year. Why do you think the calendar was not created to be in line with the start and end of seasons?

15. Jesse is in art school and created a pattern based on the rotation of one triangle around the origin of a Cartesian plane. The original triangle is shaded.



a. Jesse rotates the original shaded triangle 18 times to complete one full circle. What is the angle of each rotation?

b. If Jesse creates a different pattern by repeatedly rotating the original image 15° clockwise until it completes one full turn, how many triangles will make up the final pattern?

c. If the original triangle is not shaded, what is the maximum angle of rotation around the origin of the entire pattern before it looks like Jesse's starting pattern?

d. Create your own pattern by completing rotations of equal size around the origin of a Cartesian plane.

Extra spicy

16. The centre of the square $TUVW$ is located at the origin. Write the coordinates of the vertex $T(2,2)$ after the square has been rotated 90° anticlockwise around the origin.

17. A point is rotated 375° clockwise. This is the same as rotating the point:

A. 15° **B.** −15° **C.** −195° **D.** −345° **E.** 375°

18. What will the following square look like after 3 consecutive 90° rotations in the clockwise direction?

A. 

B. 

C. 

D. 

E. 

19. A shape is rotated 9145°. What is an equivalent angle of rotation between 0° and 360°?

Remember this?

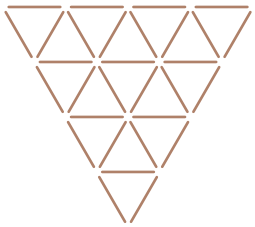
20. Which number will make this number sentence true?

$3.5 = \square \times 2.5$

A. $\frac{4}{25}$ **B.** $1\frac{4}{25}$ **C.** $1\frac{2}{5}$ **D.** $1\frac{4}{5}$

21. Jacinda is using sticks to make a pattern.

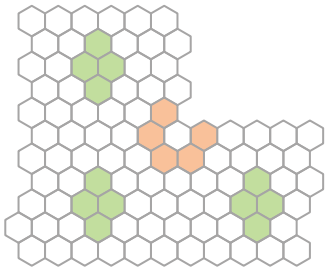
She starts with 3 sticks for stage 1.

Stage 1	Stage 2	Stage 3	Stage 4
			
3 sticks	9 sticks	18 sticks	30 sticks

How many sticks does she need for stage 6?

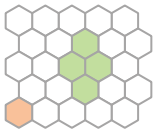
A. 32 **B.** 45 **C.** 54 **D.** 63

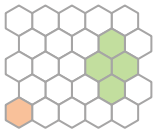
22. Some tiles are missing from this pattern of tiles.

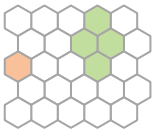


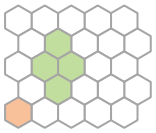
When complete, the design has two lines of symmetry.

Which of these completes the pattern?

A. 

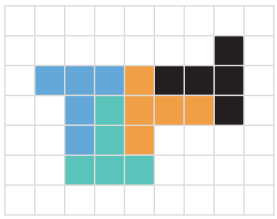
B. 

C. 

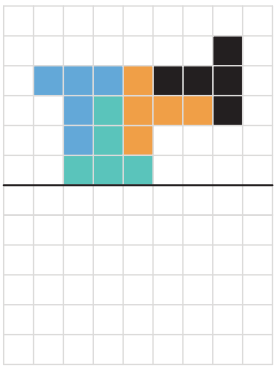
D. 

Chapter 8 extended application

1. Tori is designing a video game and she would like to use the following **original image** to create a pattern. In order to do this, she needs to investigate its properties using her knowledge of transformations.



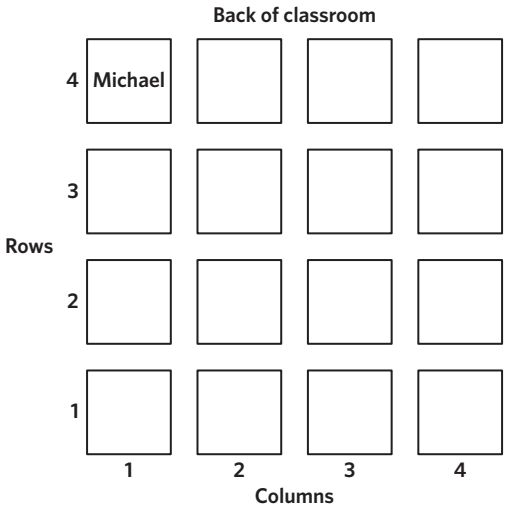
   a. Copy the outline of the blue T-shape on grid paper, using the same number of squares as in the original image.
   b. Show the transformation of the blue T-shape to the orange T-shape using your drawing from part **a**. Include any angle measurements and other important points.
   c. Tori would like to use the original image and repeat it over to create a larger pattern. Continue the pattern by translating the blue T-shape by $[4,-2]$ and positioning the green, orange, and black shapes around it in exactly the same way as the original image. Do all 4 original T-shapes undergo the same translation?
   d. Tori is also experimenting with recreating the pattern by reflecting the entire original image. Using the following grid paper, reflect the entire original image vertically over the blue line.



   e. Use a digital technology of your choice to recreate the pattern you drew in part c and fit a third copy of the original image around it.
   f. Good programmers are more efficient with their coding and can achieve the same result with fewer commands. In a similar way, there are usually many ways to describe the same transformation using different combinations of rotations, translations, and reflections. Suggest some ways that Tori could ensure that her patterns always repeat in a consistent way.

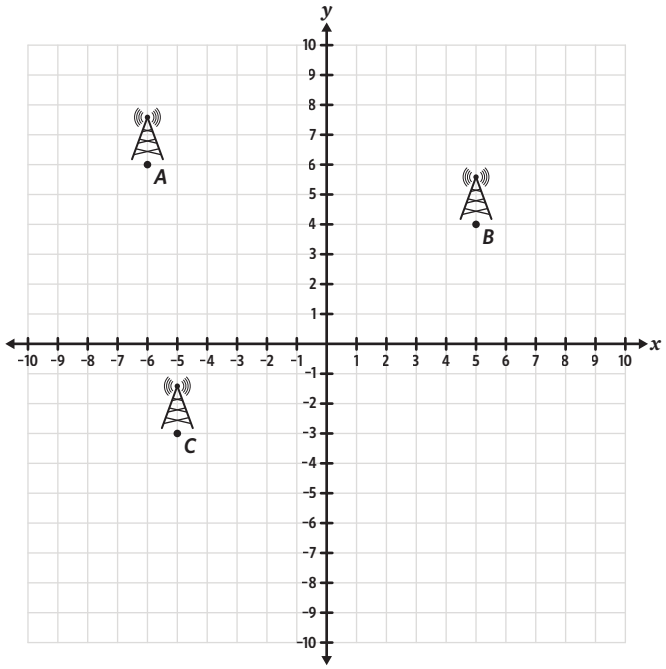
2. Ms Hammil is coming up with a new seating plan for her year 7 advanced maths class. There are 16 students altogether and she uses the following diagram of the desks to help her.

   To make it easier to remember the seat positions, she will use Cartesian coordinates to represent each one, where the x-coordinate represents the column number and the y-coordinate represents the row number. The first row is at the front of the classroom.



   a. Michael's seat position is labelled in the diagram. Write the position of his seat as (x-coordinate, y-coordinate).
   b. Jim has to sit as far away as possible from Michael. State the position of Jim's seat and label it with his name on the diagram.
   c. Harvey and Sally join the class. Harvey has to sit 3 seats to the right and 1 seat down from Michael and Sally sits directly next to him. State the coordinates of Harvey's and Sally's seats and label them on the diagram.
   d. Anita gets distracted by both Michael and Jim. She needs to sit as far away as possible from both of them. Emma wants to sit next to Anita, but is not allowed to sit in the back row. Give the coordinates of Anita's and Emma's seats and label them on the diagram.
   e. Ms Hammil likes to move the students around every 2 weeks but wants the students to work more collaboratively. Suggest a possible seating plan that promotes effective group work.

3. Kerby is very good at computer programming, but tends to lose his phone often. For this reason, he has developed an application that uses triangulation to locate a mobile phone, as long as it is turned on. This means that Kerby can narrow down the location of his lost phone to the triangular region formed by the three closest phone towers around it.



a. State the coordinates and quadrant locations of each of the towers *A*, *B*, and *C*.

b. Copy the diagram and label the points *A*, *B* and *C*. Draw the triangle formed by the three towers, and determine in which of the quadrants Kerby's phone is most likely located.

c. According to the triangulation map, in which of the quadrants is the phone definitely not located?

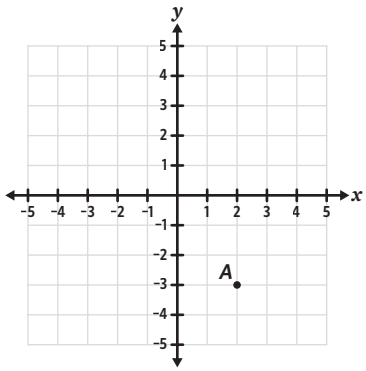
d. Kerby and a couple of his friends plan to distribute their search efforts in the most efficient way possible. List the quadrants on the map in order of search priority and explain your reasoning.

e. Kerby is particularly forgetful and has a tendency to lose some of his non-traceable items he keeps in his pockets like his keys and wallet. Propose ways that Kerby can ensure he does not lose his pocket items in the future.

Chapter 8 review

Multiple choice

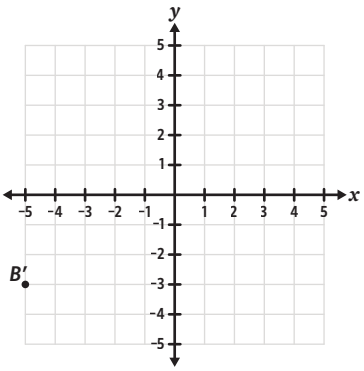
1. Which quadrant is point A in? 8A



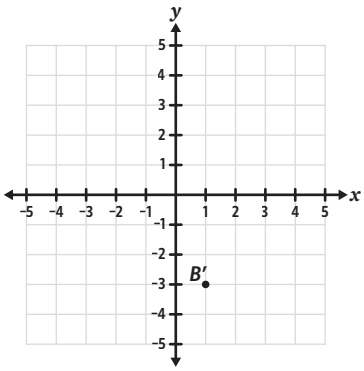
A. Quadrant 1 B. Quadrant 2 C. Quadrant 3 D. Quadrant 4

2. What is the correct position of point B' on a Cartesian plane when point $B(-2,4)$ is moved by the translation $[3,-7]$? 8C

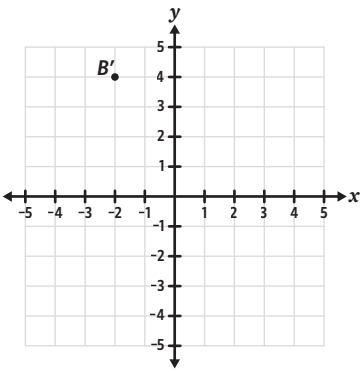
A.



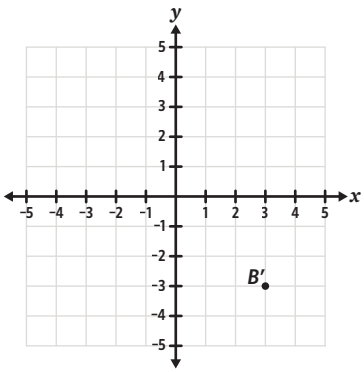
B.



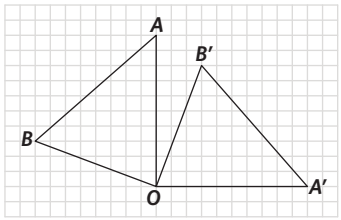
C.



D.

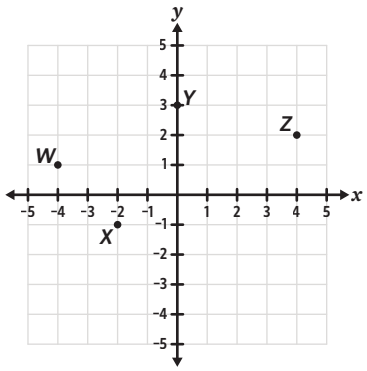


3. Select a possible rotation from triangle AOB to $A'OB'$ about point O. 8D



A. 45° clockwise B. 90° anticlockwise C. 180° clockwise D. 270° anticlockwise

4. Which point has the greatest y-coordinate value? 8A



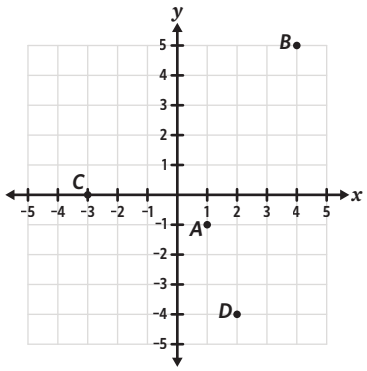
A. W B. X C. Y D. Z

5. How many lines of symmetry does the following image have? 8B

A. 0 B. 1 C. 2 D. 4

Fluency

6. State the Cartesian coordinates of each labelled point. 8A

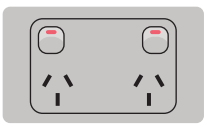


a. Point A b. Point B c. Point C d. Point D

7. Identify the quadrant or axis in which each point is located on the Cartesian plane. 8A

a. $(-2,5)$ b. $(8,0)$ c. $(-3,-8)$ d. $(0,-13)$

8. State the order of line symmetry and rotational symmetry for each image. 8B

a. 

b. 

c. 

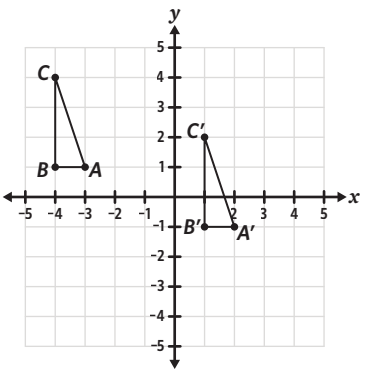
d. 

9. Write the coordinates of the point reflected over the specified axis. 8B

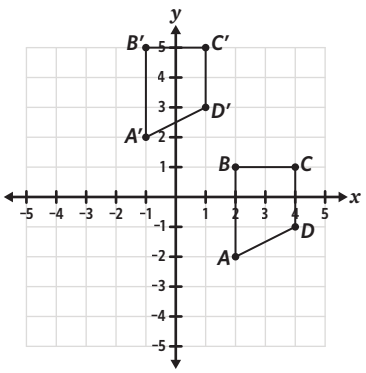
a. $(5,8)$, x-axis b. $(3,12)$, y-axis c. $(-4,6)$, x-axis d. $(-15,-2)$, y-axis

10. Describe the translation of each shape. 8C

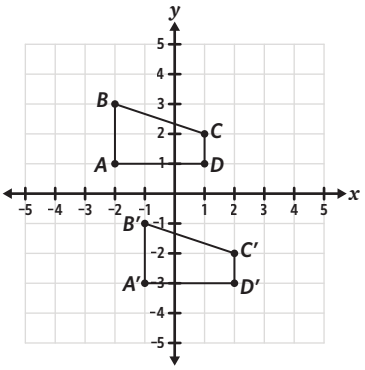
a.



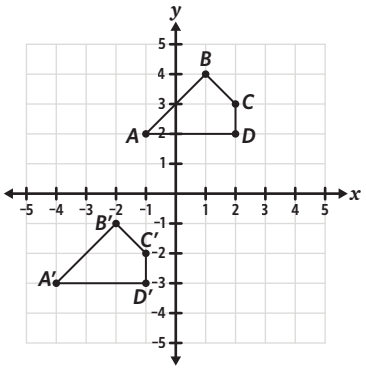
b.



c.



d.



11. Reverse the given translation for point $A'(3,1)$ to find the original point A coordinates. 8C

a. 3 units right and 2 units up

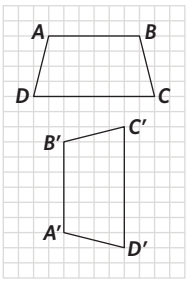
b. $[-5,4]$

c. 4 units right and -5 units up

d. $[11,-8]$

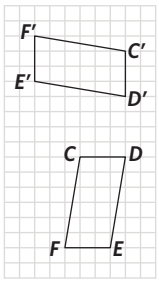
12. Identify whether each shape or point has been rotated clockwise or anticlockwise. 8D

a. Trapezium $ABCD$ is rotated 90°.



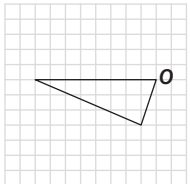
b. Point $B(-4,0)$ is rotated 90° around the origin to $B'(0,4)$.

c. Parallelogram $CDEF$ is rotated 270°.



d. Point $D(0,7)$ is rotated 270° around the origin to $D'(7,0)$.

13. Draw the shape after each rotation around point O. 8D



a. 90° clockwise

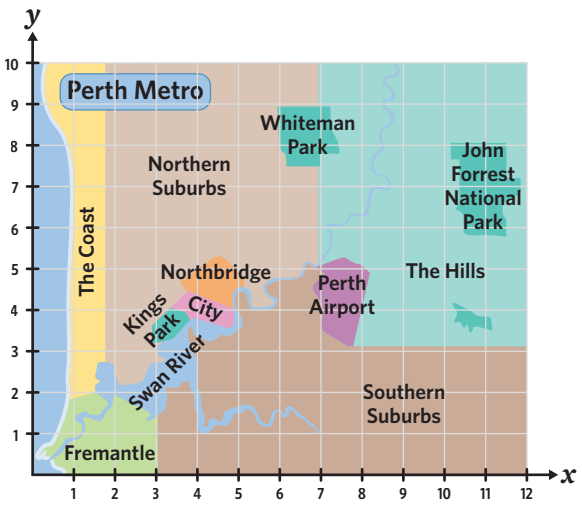
b. 180° anticlockwise

c. 45° clockwise

d. 135° anticlockwise

Problem solving

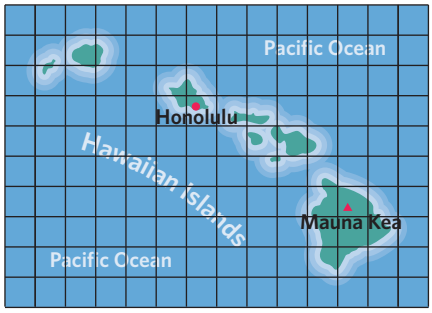
14. What are the coordinates of the city of Perth on the following map? 8A



15. Harry wants to write a secret message. He hides the message by reflecting it vertically over a mirror line. How does the hidden message appear if the true message reads 'STYLES'? 8B

16. Scott is travelling around Hawaii. He first landed in Honolulu before catching a boat to the largest island, and hiking up Mauna Kea. 8C

Describe the translation of Scott's movement from Honolulu to Mauna Kea.

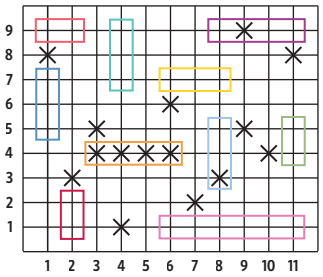


17. The London Eye is a Ferris wheel located in London. It takes 30 minutes to take a complete rotation around the wheel. Sunny got in a carriage at 2:45 pm. What angle of rotation of the London Eye had Sunny completed by 2:50 pm? 8D

Reasoning

18. Zoe and Vutha are playing Battleship. The game is won when one player sinks all the battleships of the other player by guessing coordinates each turn.

The following image shows Vutha's board. The X's show Zoe's guesses of Vutha's battleships. She has already sunk Vutha's orange battleship as all the coordinates of the ship have been guessed.



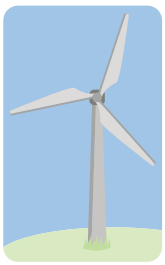
a. List the coordinates Zoe has to guess to sink Vutha's purple battleship.

b. Of all the positions with an x-coordinate of 8, which are **not** covered by one of Vutha's battleships?

c. Vutha thinks that Zoe will find it easy to sink the pink battleship in its current position and decides to move it while Zoe isn't looking. There is only one position on the board that the pink battleship can fit that has no coordinates that have been guessed yet. Describe the translation to this position.

d. Do you think it is a better strategy to put all your battleships close together or further apart?

19. Joe has a windmill on his farm. It has 3 identical blades that are separated equally and makes a full rotation every 12 seconds. When Joe looked at the windmill, the blades were in the position shown on the right.

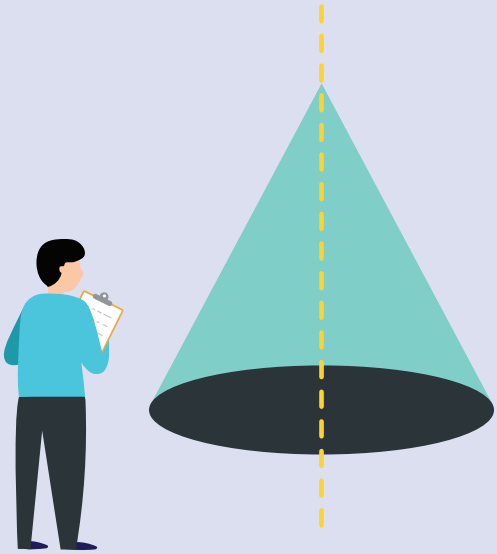


a. What is the smallest angle of rotation of the blades it would take for the windmill to look the same as when Joe looked at it?

b. How many times did the windmill look the same in the following minute after Joe looked at it?

c. What is the angle of rotation of the windmill in one minute?

d. How would you convince other farmers to allow windmills on their farms?



09

2D shapes and 3D objects

Measurement and Geometry

Research summary

9A Units of length *(Revision)*

9B Area of rectangles and parallelograms

9C Area of triangles

9D Drawing 3D objects

9E Volume of prisms

Chapter 9 extended application

Chapter 9 review

Image: Marish/Shutterstock.com

Research summary – 2D shapes and 3D objects

Big ideas

Recognising an object's attributes

In measurement, the '**attribute**' of an object is the property or trait of the object that is being measured. Attributes such as length, area, and volume are spatial attributes. Other attributes include physical ones such as mass and temperature, as well as attributes that have no physical features such as time. In this chapter, we will be focussing on the three spatial attributes of length, area, and volume.

Selecting units of measurement

Units of measurement enable us to measure different objects and compare one object to another. We must use a unit of measurement appropriate for the attribute of the object being measured. For example, if we are measuring the perimeter of a shape we must use units of length such as cm, m, or km rather than units of area (cm^2, m^2, km^2) or units of volume (cm^3, m^3, km^3).

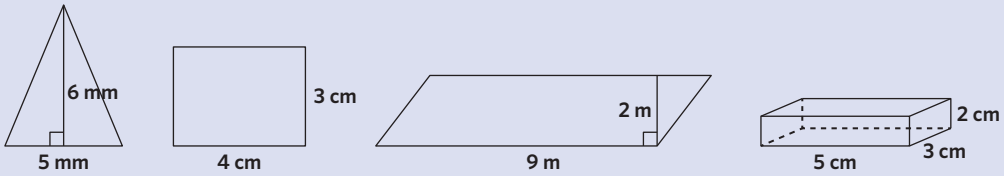
Comparing attributes and units

It is possible to use a range of units to measure the same attribute. For example, we could measure a specific length in either kilometres, metres, or centimetres. The most appropriate metric may vary based on the context of the measurement. When measuring the attribute of an object by itself, we can choose any appropriate unit, but when comparing an attribute between two or more objects, the units must be the same.

Visual representations

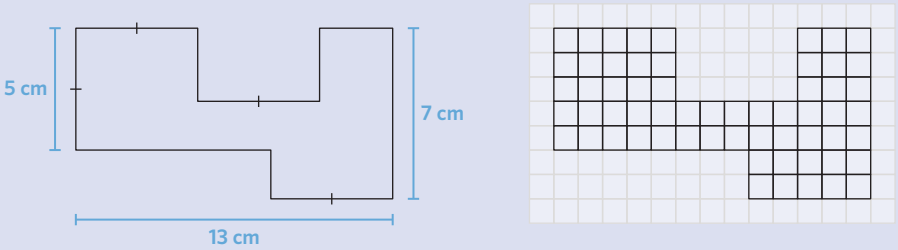
Open shapes and objects

Open shapes and objects require the use of formulas for calculations. In these shapes and objects, we need to know the value of certain lengths in order to be able to successfully calculate perimeter, area, and volume.



Grids

Grids can be a useful visual tool for the introduction of perimeter, area, and volume. They allow us to apply counting techniques rather than multiplication and formulas, and can provide a simple way of carrying out complex calculations.



The two images above have identical areas and perimeters. However, it can be simpler to count the squares of the grid on the right to work out both the perimeter and the area than it is to use the dimensions of the open composite shape on the left.

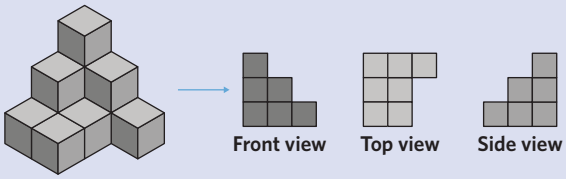
Nets

The net of an object is a two-dimensional representation that, when folded up, will recreate the 3D form of that object. An object may have more than one net depending on the number of faces and the shapes that make up each face. For example, a cube has 11 different possible nets. Three of these are shown below.

A net is a useful tool as it not only allows us to more easily calculate the surface area of different objects, but it also helps to improve our spatial understanding and awareness of all solids.

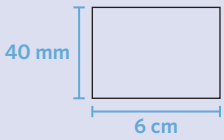
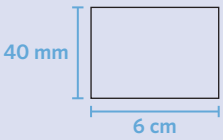
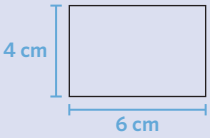
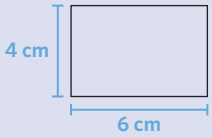
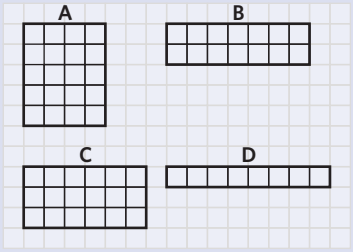
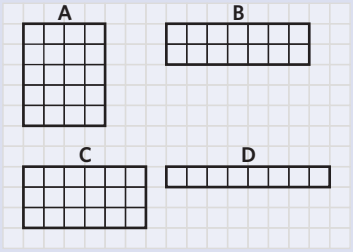
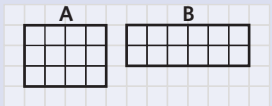
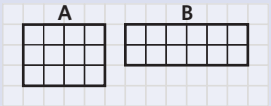
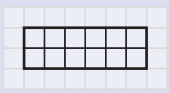
Isometric drawings

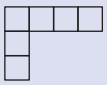
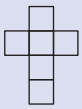
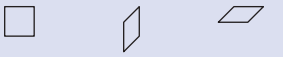
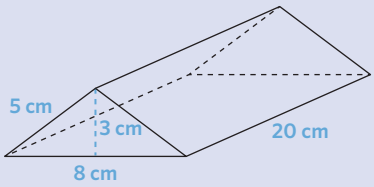
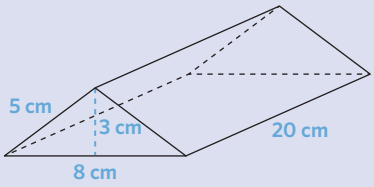
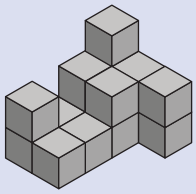
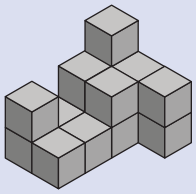
Isometric drawings are useful tools when creating 2D representations of 3D objects. They are particularly useful when drawing objects that are made up of more than one solid. Isometric drawings are used extensively in technical and engineering diagrams to create perspective in art and design. Part of an isometric drawing's usefulness is its ability to allow the different views of the solid in question to then be generated.

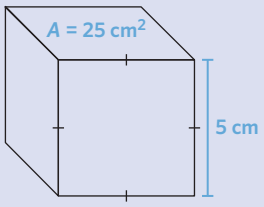
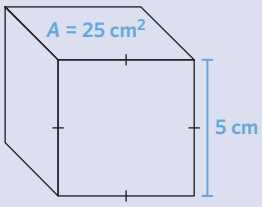
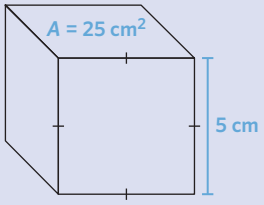
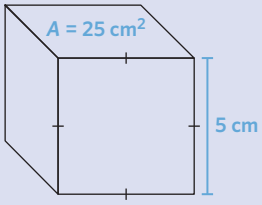


The above images show an isometric drawing along with how the object will look when viewed directly from the front, top, and side.

Misconceptions

Misconceptions	Incorrect	Correct	Exercise
Students do not convert all units of length to the same unit when calculating the perimeter of a shape.	Perimeter $P = (2 \times 40) + (2 \times 6) = 92$ cm 	Perimeter $P = (2 \times 4) + (2 \times 6) = 20$ cm 	9A
Students calculate the area when intending to calculate the perimeter of a shape.	Perimeter $P = 4 \times 6 = 24$ cm 	Perimeter $P = (2 \times 4) + (2 \times 6) = 20$ cm 	9A
Students believe that shapes with the same perimeter will have the same area.	All 4 shapes below have a perimeter of 18 square units meaning that their areas are also all the same. 	By counting the squares inside the perimeter of each shape it shows that the areas of shapes **A**, **B**, **C** and **D** are all different. 	9B
Students believe that shapes with the same area will have the same perimeter.	The two rectangles have the same perimeters because their areas are both 12 square units. 	Rectangle A has a perimeter $P = 3 + 3 + 4 + 4 = 14$ Rectangle B has a perimeter $P = 2 + 2 + 6 + 6 = 16$ 	9B
Students believe that the value of an area is always larger than the value of the perimeter.	Perimeter involves addition and area involves multiplication so therefore area must always be larger than the perimeter.	When the difference between the length and the width increases in a rectangle the area decreases making the value of the area less than the value of the perimeter. 	9B

	Incorrect	Correct	Exercise
Students believe that the height of a triangle is always equal to the side length of a triangle.	The height of a triangle always equals its side length.  *side length = height* *side length = height*	The height of a triangle does not always have to equal its side length.  *side length = height* *side length ≠ height*	9C
Students miscount the number of edges or vertices on a 3D object.	 A cube has 9 edges and 7 vertices.	 A cube has 12 edges and 8 vertices.	9D
Students construct the incorrect net to represent a 3D object.	A net with the correct faces and number of faces but the net will not fold up to form the 3D object A net for a cube can be: 	The net can fold up to represent the object. A net for a cube can be:  Note: There are other nets of cubes.	9D
Students draw the plan views of a 3D object with a front-on perspective only.	 **Front view** **Side view** **Top view** 	 **Front view** **Side view** **Top view** 	9D
Students only apply the formula $V = l \times w \times h$ to calculate all 3D objects.	The triangular prism has a volume of: $V = 8 \times 5 \times 20 = 800\ \text{cm}^3$  	The volume of any prism is the area of the base shape multiplied by the height. $V = A_{base} \times h$. $V = 12 \times 20 = 240\ \text{cm}^3$  	9E
Students only count the visible cubes when calculating the volume of cubic solids.	 If each cube is 1 cm^3, the volume of this object is 12 cm^3.	 If each cube is 1 cm^3, the volume of this object is 16 cm^3.	9E

	Incorrect	Correct	Exercise
Students calculate volume by multiplying the area of the base shape by the number of visible faces.	 $V = A_{base} \times \#\ of\ faces$ $V = 25\ \text{cm}^2 \times 3\ \text{cm}$ $= 75\ \text{cm}^3$	 $V = A_{base} \times height$ $V = 25\ \text{cm}^2 \times 5\ \text{cm}$ $= 125\ \text{cm}^3$	9E
Students calculate volume by multiplying the area of the base shape by the total number of faces.	 $V = A_{base} \times \#\ of\ faces$ $V = 25\ \text{cm}^2 \times 6\ \text{cm}$ $= 150\ \text{cm}^3$	 $V = A_{base} \times height$ $V = 25\ \text{cm}^2 \times 5\ \text{cm}$ $= 125\ \text{cm}^3$	9E

Additional reading and resources

- Teaching Measurement: Processes in the Primary School, Associate Professor Tom Cooper, Queensland University of Technology.
- https://calculate.org.au/measurement-and-geometry/

9A Units of length

Most countries in the world use the metric system to measure lengths and distances. The base unit for length used in the metric system is metres. Different units of length are based on a metre, such as millimetres, centimetres and kilometres. We can relate these different units to a metre by multiplying or dividing by powers of 10. Below are some examples where units of length can be applied.

- I am 1650 mm tall. How tall am I in cm?
- To get to school, I have to bus 1.9 km to the train station, take a train for 5 km and then walk another 850 m. How far do I travel to get to school in km?
- I'm drawing on a square piece of paper that has a side length of 14 cm. What is the perimeter of this piece of paper?

Learning intentions

Students will be able to:

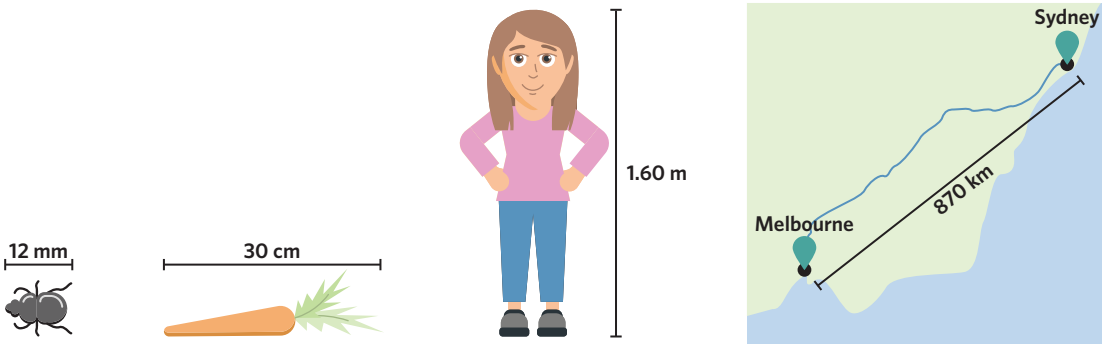
+ select appropriate units to measure length
+ convert between the different units of length
+ find the perimeter of shapes measured in common or different units.

Key terms and definitions

- A **metre (m)** is a standardised unit measuring length.
- A **millimetre (mm)** is one-thousandth of a metre.
- A **centimetre (cm)** is one-hundredth of a metre.
- A **kilometre (km)** is one thousand metres.
- The **perimeter** is the total length of the edge around a closed 2D shape.

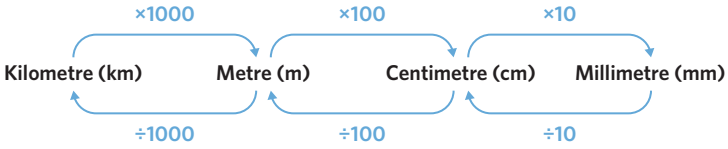
Key ideas

1 Different units of length are appropriate for measuring different lengths or distances. The most common units of length are millimetre, centimetre, metre and kilometre.



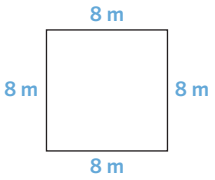
Using the appropriate units of length is important to avoid excessively large or small units.

2 We can convert to different units of length by multiplying or dividing by powers of 10.



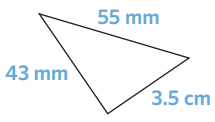
3 We can calculate the perimeter of a shape by adding the side lengths of the shape together.

Same unit of measurement



Perimeter = 8 m + 8 m + 8 m + 8 m
= 32 m

Different unit of measurement

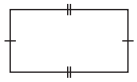


Perimeter = 43 mm + 55 mm + 3.5 cm
= 43 mm + 55 mm + 35 mm
= 133 mm

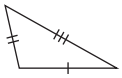
Note: Convert all units of length to the same unit to calculate the perimeter.

4 Sides with the same markings have the same length.

The side lengths are all equal.



All the widths are the same size.
All the lengths are the same size.
The widths and lengths are different sizes.



None of the side lengths are equal.

Worked examples

WE 1 Converting units of length

Convert each length to the specified units.

a. 40 cm to m

Working

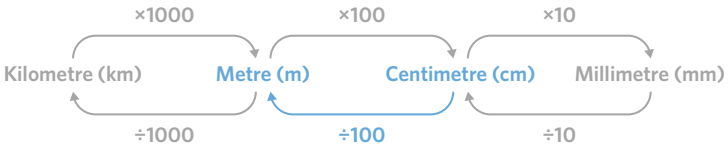
$40 \div 100 = 0.4$ m

Thinking

There are 100 cm in 1 m.

Divide by 100 to convert from cm to m.

Visual support



b. 2 km to cm

Working

$2 \times 1000 = 2000$ m

$2000 \times 100 = 200\,000$ cm

Thinking

Step 1: There are 1000 m in 1 km.
Multiply by 1000 to convert from km to m.

Step 2: There are 100 cm in 1 m.
Multiply by 100 to convert from m to cm.

Student practice

Convert each length to the specified units.

a. 80 cm to m **b.** 4 km to cm **c.** 100 mm to cm **d.** 3 m to mm

WE 2 Combining different units of length

Express each length in the described form.

a. 5 km + 300 m (km)

Working

$300 \div 1000 = 0.3$ km

$5 + 0.3 = 5.3$ km

Thinking

Step 1: Convert 300 m to km. There are 1000 m in 1 km, so divide by 1000.

Step 2: Calculate the total length.

Visual support

b. 400 m − 4000 cm (m)

Working

$4000 \div 100 = 40$ m

$400 - 40 = 360$ m

Thinking

Step 1: Convert 4000 cm to m. There are 100 cm in 1 m, so divide by 100.

Step 2: Calculate the difference in length.

Student practice

Express each length in the described form.

a. 3 km + 100 m (km)

b. 750 m − 300 cm (m)

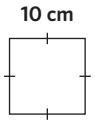
c. 10 mm + 80 cm (m)

d. 1400 m − 17 000 cm (km)

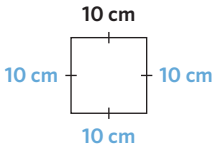
WE 3 Calculating the perimeter of shapes

Calculate the perimeter for each shape.

a.



Working



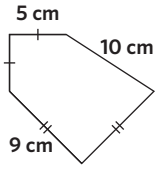
$10 + 10 + 10 + 10 = 40$ cm

Thinking

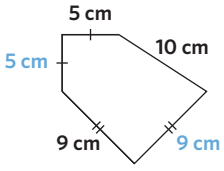
Step 1: Label all the side lengths. All of the sides have the same marking, so the side lengths must be equal.

Step 2: Calculate the perimeter by adding together the side lengths.

b.



Working



$5 + 10 + 9 + 9 + 5 = 38$ cm

Thinking

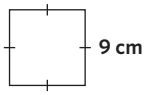
Step 1: Label all the side lengths. Sides with the same marking have equal lengths.

Step 2: Calculate the perimeter by adding together the side lengths.

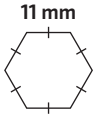
Student practice

Calculate the perimeter for each shape.

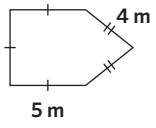
a.



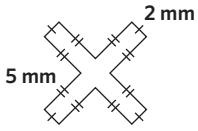
b.



c.



d.



9A Activities and questions

STARTER TASKS

Odd spot

Rod Laver Arena is one of the world's premier tennis stadiums and is the main venue for the Australian Open, a Grand Slam tournament. The arena was named to honour Rod Laver, an Australian tennis player who is recognised as one of the greatest tennis players of all time. The court inside Rod Laver Arena has a width of 10.973 m and a length of 23.77 m. What is the perimeter of the court?

A. 260.828 m

B. 69.486 m

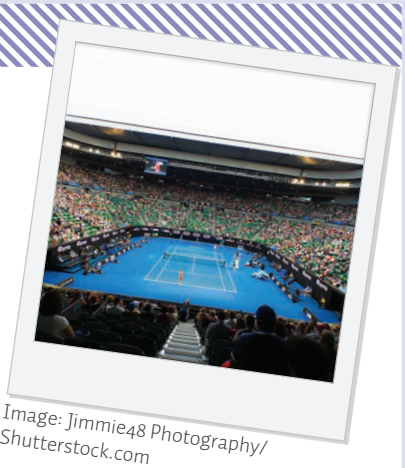


Image: Jimmie48 Photography/ Shutterstock.com

Puzzle

State all the units of length that are equivalent to 100 mm.

0.00001 km | 0.01 m | 1 m | 1 cm | 0.001 km | 0.1 cm | 0.1 m | 10 cm | 0.0001 km

Understanding worksheet

1. Select the unit of length that is most appropriate to measure each length.

Example

	Millimetre	Centimetre	Metre	Kilometre
The distance between Melbourne and Brisbane:	☐	☐	☐	☑

a.

	Millimetre	Centimetre	Metre	Kilometre
The length of a swimming pool:	☐	☐	☐	☐

b.

	Millimetre	Centimetre	Metre	Kilometre
The length of a ruler:	☐	☐	☐	☐

c.

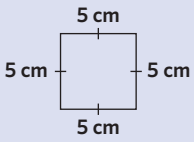
	Millimetre	Centimetre	Metre	Kilometre
The length of an ant:	☐	☐	☐	☐

d.

	Millimetre	Centimetre	Metre	Kilometre
The height of an apartment building:	☐	☐	☐	☐

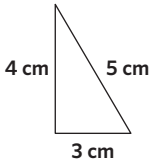
2. Complete the missing information to calculate the perimeter of each shape.

Example



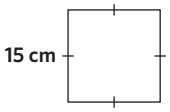
$\boxed{5}$ cm + $\boxed{5}$ cm + $\boxed{5}$ cm + $\boxed{5}$ cm = $\boxed{20}$ cm

a.



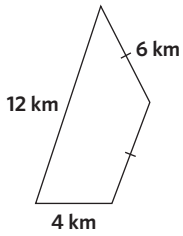
☐ cm + 4 cm + ☐ cm = ☐ cm

b.



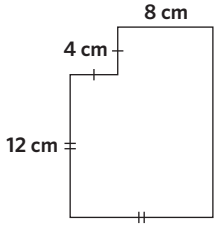
15 cm + ☐ cm + ☐ cm + ☐ cm = ☐ cm

c.



6 km + ☐ km + 4 km + 12 km = ☐ km

d.



☐ cm + 4 cm + ☐ cm + ☐ cm + ☐ cm + 12 cm = ☐ cm

3. Fill in the blanks by using the words provided.

different | same | measure | large | appropriate

Different units of length help us ☐ different lengths or distances. It is important to use the ☐ unit of length to avoid excessively ☐ or small numbers. When calculating perimeters with ☐ units of length, it is important to convert the different units of length to the ☐ unit in order to perform the correct perimeter calculation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c,d), 8 (a,b,c)	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (c,d,e,f), 8 (b,c,d)	4 (e,f,g,h), 5 (d,e,f), 6 (d,e,f), 7 (e,f,g,h), 8 (d,e,f)

WE 1 **4.** Convert each length to the specified units.

a. 150 cm to m **b.** 25 mm to cm **c.** 1.65 km to m **d.** 85 cm to mm
e. 12 m to cm **f.** 150 cm to km **g.** 175 000 mm to km **h.** 11.3 km to mm

WE 2a **5.** Express each length in the described form.

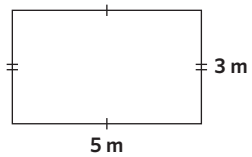
a. 5 m + 30 cm (m) **b.** 430 cm + 400 mm (cm)
c. 37 mm + 3 m (m) **d.** 831 cm + 2 km (km)
e. 0.87 km + 230 mm (m) **f.** 3000 cm + 40 000 mm (km)

WE 2b **6.** Express each length in the described form.

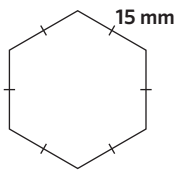
a. 19 cm − 50 mm (mm) **b.** 574 cm − 40 mm (cm)
c. 18 m − 370 mm (m) **d.** 10 km − 3380 cm (km)
e. 0.091 km − 777 cm (m) **f.** 40 000 cm − 0.02 km (m)

WE 3 **7.** Calculate the perimeter for each shape in the described unit.

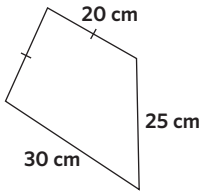
a. (m)



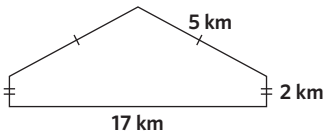
b. (mm)



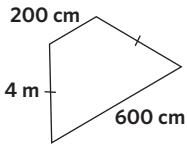
c. (cm)



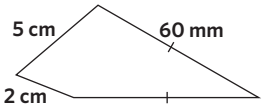
d. (km)



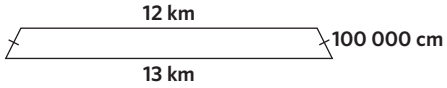
e. (m, cm)



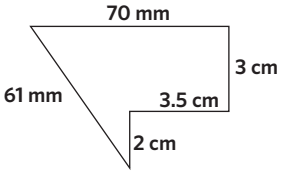
f. (cm, mm)



g. (km, cm)

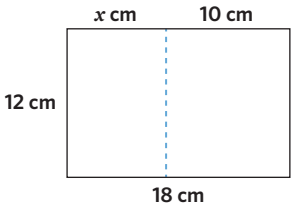


h. (cm, mm)

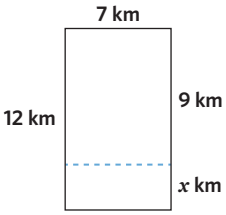


8. Calculate the value of x.

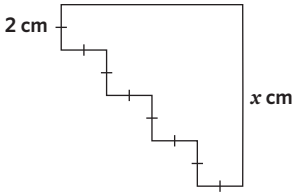
a.



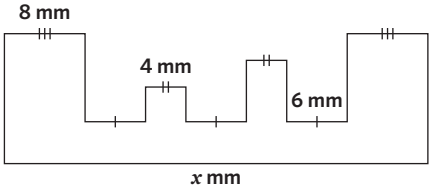
b.



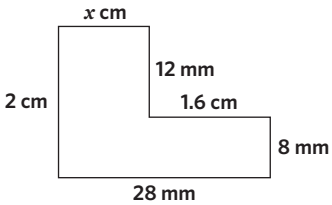
c.



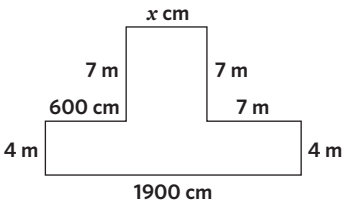
d.



e.



f.



Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Barry printed a photo that has a length of 105 mm and a width of 8.4 cm. How many centimetres is the perimeter of the photo?

10. Elwood wants to build a wooden fence around his square block of land. A carpenter agreed to build Elwood's fence for $80 per metre. If Elwood's land has a side length of 20 metres, how much would it cost to install the fence around his block of land?

11. Damien needs 5 different strings for a model pirate ship he is making. Which length of string is the longest if the length of strings he needs are: 75 millimetres, 10 centimetres, 550 centimetres, 0.4 metres and 3.5 metres?

12. Bill is 156 cm tall. His older sister Penelope is 1.7 m tall. When Bill puts his shoes on, he is 35 millimetres taller. How much taller is Penelope than Bill when he has his shoes on?

13. The Australian fifty-cent coin is dodecagonal shaped and has 12 sides. How many millimeters is each side of the coin if its perimeter is 10.56 cm?

Reasoning

Mild	Medium	Spicy
14 (a,b,c,d)	14 (a,b,c,d), 15 (a,b,c)	All

14. Jackson and Audrey are planning a hike in the You Yangs Regional Park using a map which details the regional park's geography. Each centimetre on the map represents a distance of 250 m in real life.

- **a.** How many millimetres on the map would represent 1 km?
- **b.** Audrey measured that a return trip on the map will equal 13 cm, how many kilometres is this?
- **c.** After covering 80 mm on the map, Jackson started feeling tired and wanted to take a break. How far had Jackson and Audrey hiked before they took a break?
- **d.** After finishing their hike, Jackson and Audrey planned their drive home using Google Maps. 2 mm on Google Maps represents one kilometre of driving. How many centimetres was the distance on Google Maps if they drove 75 km?
- **e.** Is driving to hike a good use of time? Why or why not?

15. Daniel is planning to build a basketball court in his new mansion so that he can host casual games with his friends.

- **a.** Write an algebraic equation to present the perimeter of Daniel's basketball court, using p to represent the perimeter, w to represent the width and l to represent the length.
- **b.** If the perimeter of Daniel's basketball court is 86 m and the width is 15 m, calculate the length of the court.
- **c.** Daniel decides to extend the length of his court so that he can build benches for people to rest. If he increases the length by 4 m, how much does the perimeter increase by?

  A. 4 m **B.** 8 m **C.** 12 m **D.** 16 m

- **d.** Is it a good idea to build a basketball court in your home? Why or why not?

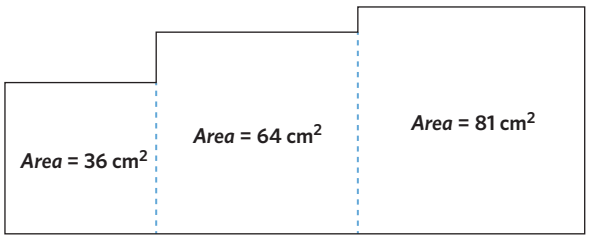
Extra spicy

16. There are other uncommon units of length. For example, 1000 micrometres make up a millimetre. How many micrometres are there in 0.0025 metres?

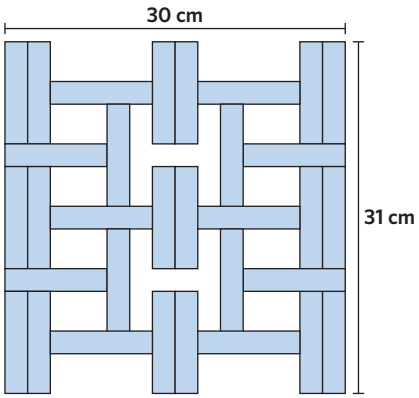
A. 250 micrometres **B.** 2500 micrometres
C. 25 000 micrometres **D.** 250 000 micrometres
E. 2 500 000 micrometres

17. The perimeter of a decagon is twice the perimeter of a regular octagon. If the perimeter of the decagon is 240 cm, what is the length of one side of the regular octagon in centimetres?

18. Three squares are joined together to form a shape. Calculate the perimeter of the following shape.



19. Consider the following diagram made out of identical rectangles.



Calculate the length and width of each of the small rectangles (the length and width are whole numbers).

Remember this?

20. Nelson likes to listen to podcasts when he goes for a walk. The following diagram shows how much of the podcast he has listened to as well as the total length of the podcast in hours, minutes, and seconds.

What percentage of the podcast remains for Nelson to listen to?

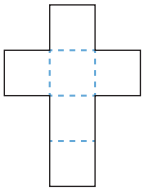
A. 40% **B.** 45% **C.** 55% **D.** 60%

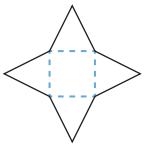
21. Donovan kept track of the number of disposals he had each week over the first 8 weeks of the footy season.

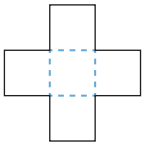
32, 24, 27, 36, 21, 14, 21, 25.

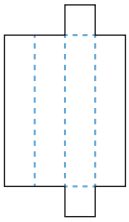
What is the mean number of disposals Donovan had per week?

22. Which of these nets will fold to make a cube?

A. 

B. 

C. 

D. 

9B Area of rectangles and parallelograms

Areas of shapes are important because they tell us how much space is contained within a boundary. This helps us develop a better understanding of size and magnitude – how big something is. There are multiple ways of calculating the area of a shape. One method to calculate area can be performed by placing a shape onto a grid paper and then counting or estimating the number of squares the shape occupies. Another way is to apply the dimensions of a shape in a formula. Below are some examples where areas of rectangles and parallelograms are applied.

- My A4 maths textbook is 297 mm long and 210 mm wide. What is the area of my textbook in square millimetres?
- I visited the Emirates Stadium in London and the soccer pitch size is 105 metres by 68 metres. What is the area of the pitch in square metres?
- I own a farm that covers 3 hectares. What is the size of my farm if I convert it to square metres?

Learning intentions

Students will be able to:

- understand the area of two-dimensional shapes
- apply formulas to find the area of rectangles and parallelograms
- convert between the different units of areas.

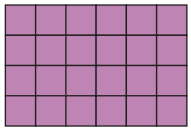
Key terms and definitions

- The **area** is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.
- A **hectare** is a metric unit of area equal to 10 000 square metres.
- A **parallelogram** is a quadrilateral (4-sided shape) with two opposite pairs of parallel sides.

Key ideas

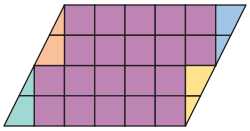
1 The area of a two-dimensional shape can be found by counting the number of unit squares the shape covers on a grid.

Rectangle



Area = 24 square units

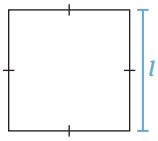
Parallelogram



Area = 20 + (4 × 1) = 24 square units

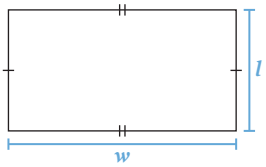
2 We can apply formulas to find the area of squares and rectangles.

Area of a square



$A = l \times l = l^2$

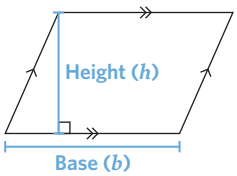
Area of a rectangle



$A = l \times w$

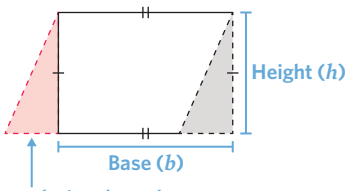
3 Parallelograms share similar properties with rectangles – we can alter parallelograms to look like rectangles. The area of the parallelogram can be found by multiplying the base by the height. The height of the parallelogram is always perpendicular to the base.

Parallelogram



$A = b \times h$

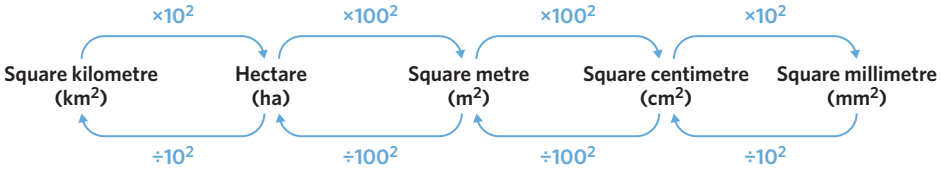
Rectangle



Shifting the dotted triangle to the right-hand side makes a rectangle.

Note: Sides with the same arrow markings are parallel.

4 We can convert between different units of area by multiplying or dividing by powers of 10.



Finding the area of a square with a side length of 2 cm, in mm^2:

Converting to mm first:	**Converting to mm^2 after:**
$A = 2 \times 10 \times 2 \times 10$	$A = 2 \times 2$
$= 2 \times 2 \times 10^2$	$= 4\ cm^2$
$= 4 \times 100$	$= 4 \times 10^2\ mm^2$
$= 400\ mm^2$	$= 400\ mm^2$

Worked examples

WE 1 Converting between units of area

Convert each area to the specified units.

a. 20 m^2 to cm^2

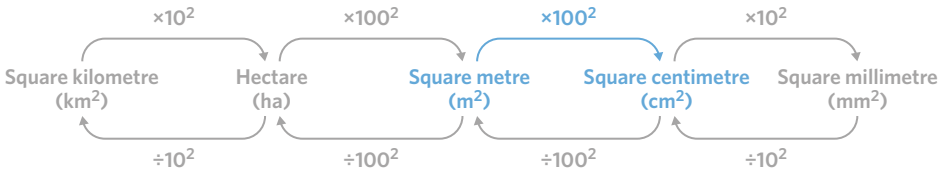
Working

$20 \times 100^2 = 20 \times 100 \times 100$
$= 20 \times 10\,000$
$= 200\,000\ cm^2$

Thinking

There are 100 cm in 1 m. So, there are 100^2 cm^2 in 1 m^2. Multiply 20 by 100^2 to convert from m^2 to cm^2.

Visual support



b. 5 ha to m^2

Working

$5 \times 100^2 = 5 \times 100 \times 100$
$= 5 \times 10\,000$
$= 50\,000\ m^2$

Thinking

There are 100^2 m^2 in 1 ha. Multiply 5 by 100^2 to convert from ha to m^2.

Student practice

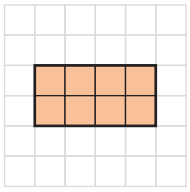
Convert each area to the specified units.

a. 400 cm^2 to m^2 **b.** 12 km^2 to ha **c.** 230 000 mm^2 to m^2 **d.** 2.8 ha to cm^2

WE 2 Identifying the area of shapes on centimetre grid paper

Find the area of each shape by counting on the centimetre grid.

a.



Working

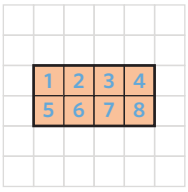
$Area = 2 \times 4$

$= 8 \text{ cm}^2$

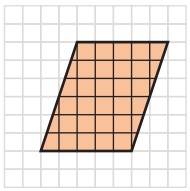
Thinking

This rectangle has 2 rows and 4 columns. We can either count all of the squares inside, or multiply the number of rows by the number of columns.

Visual support



b.



Working

There are 24 full squares.

There are 12 partial squares that make up 6 full squares.

$A = 24 + 6$

$= 30 \text{ cm}^2$

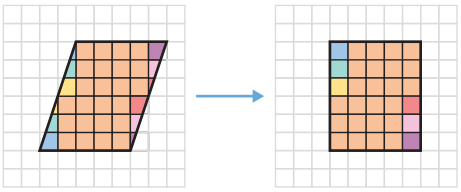
Thinking

Step 1: Count all of the full squares inside the parallelogram.

Step 2: Count the partial squares, combining them to make up full squares.

Step 3: Find the total area.

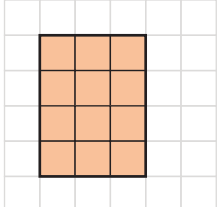
Visual support



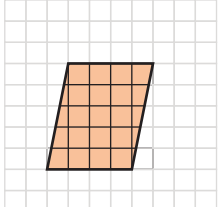
Student practice

Find the area of each shape drawn on the centimetre grid paper.

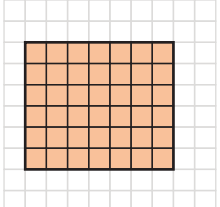
a.



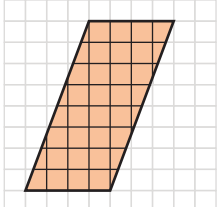
b.



c.



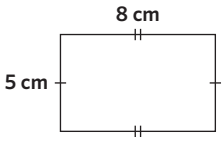
d.



WE 3 Calculating the area of shapes with area formulas

Find the area of each shape using a formula.

a.



Working

$A = l \times w$

$A = 5 \times 8$

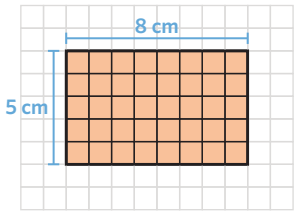
$= 40 \text{ cm}^2$

Thinking

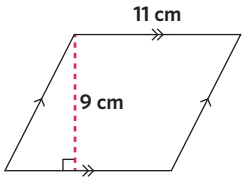
Step 1: Recall the formula for the area of a rectangle.

Step 2: Substitute $l = 5$ and $w = 8$ into the formula and simplify.

Visual support



b.



Working

$A = b \times h$

$A = 11 \times 9$

$= 99 \text{ cm}^2$

Thinking

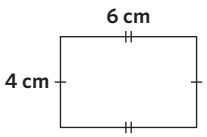
Step 1: Recall the formula for the area of a parallelogram.

Step 2: Substitute $b = 11$ and $h = 9$ into the formula and simplify.

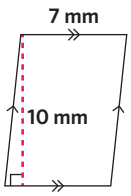
Student practice

Find the area of each shape using the area formulas.

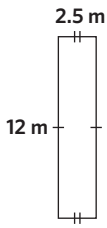
a.



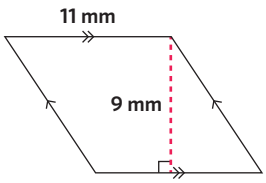
b.



c.



d.



9B Activities and questions

STARTER TASKS


Image: Daniel-Froehlich/Shutterstock.com

Odd spot

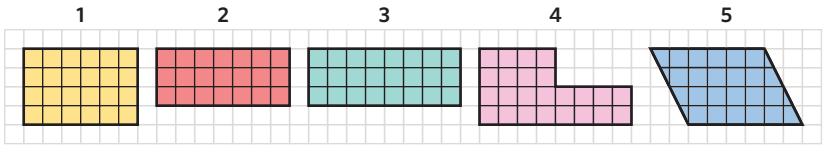
The Dockland office building in Hamburg, Germany is one of the most unique buildings in the world as it is shaped like a parallelogram. The building was inspired by its dock heritage and was constructed on a 66° incline and resembles a ship. The base of all windows for one floor of the building is 70 metres wide and 3.5 metres tall. If the perimeter of windows for this floor is 147 m, what is the area?

A. 147 m^2

B. 245 m^2

Puzzle

The following set of shapes may share some features.



a) Why might shape number 4 be considered the odd one out?

b) Why might shape number 2 be considered the odd one out?

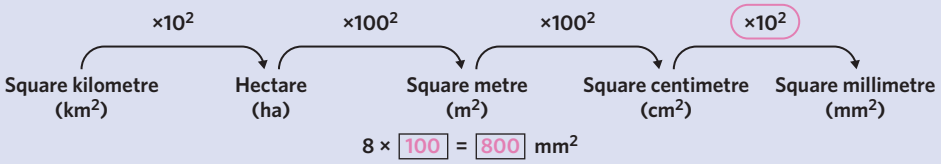
c) Can two shapes have the same area but different perimeters?

Understanding worksheet

1. Circle the required calculation(s) and then complete each unit conversion.

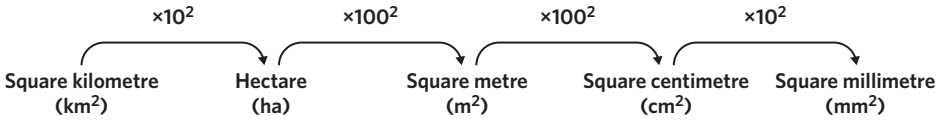
Example

8 cm^2 to mm^2



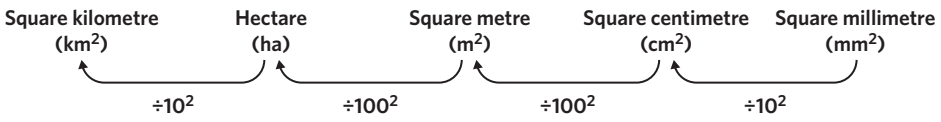
8 × 100 = 800 mm^2

a. 13cm^2 to mm^2



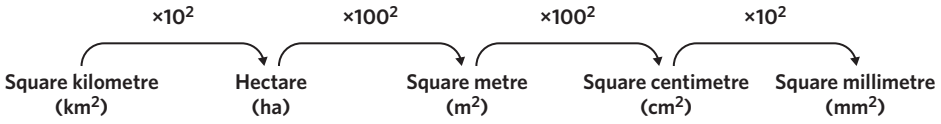
13 × ☐ = ☐ mm^2

b. 15 ha^2 to km^2



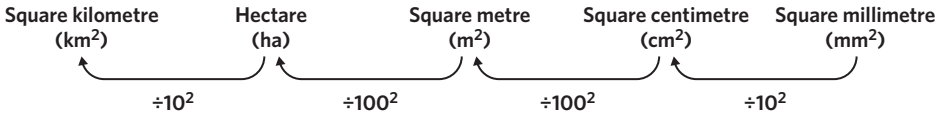
15 ÷ ☐ = ☐ km^2

c. 27 ha^2 to m^2



27 × ☐ = ☐ cm^2

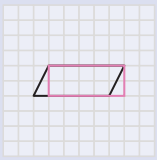
d. 80 000 mm^2 to m^2



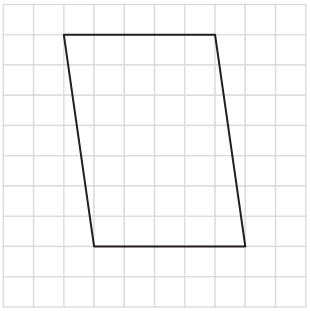
80 000 ÷ ☐ = ☐ m^2

2. Draw a rectangle with the same area over the parallelogram.

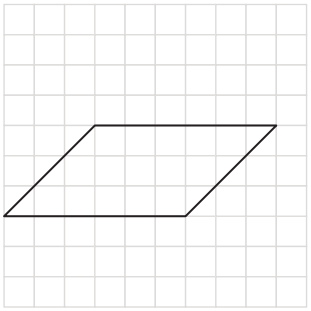
Example



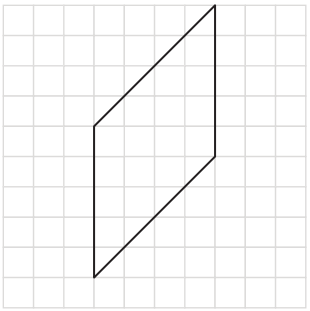
a.



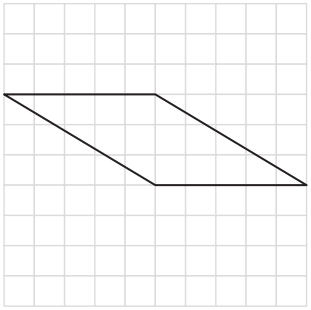
b.



c.



d.



3. Fill in the blanks by using the words provided.

around | space | grid | formula | square

Perimeter refers to the distance ☐ the boundary of a 2D shape whereas the area refers to the ☐ inside the boundary of a 2D shape. Area is measured in ☐ units and can help us understand space. We can measure the area of rectangles and parallelograms by counting the amount of space it occupies on a ☐ or by applying a ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c)	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d)	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (d,e,f)

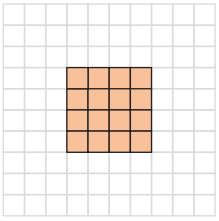
WE 1 **4.** Convert each area to the specified units.

a. 100 cm^2 to mm^2 **b.** 37 m^2 to cm^2 **c.** 1 ha to km^2 **d.** 88 000 mm^2 to cm^2

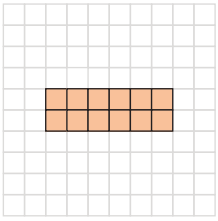
e. 10 ha to cm^2 **f.** 5 km^2 to cm^2 **g.** 5000 m^2 to km^2 **h.** 3 000 000 mm^2 to ha

WE 2 **5.** Find the area of each shape by counting on the centimetre grid.

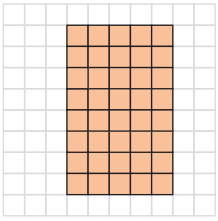
a.



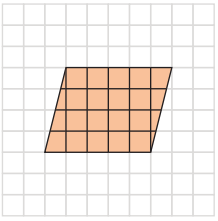
b.



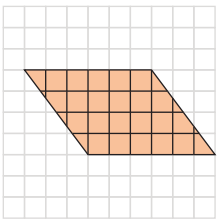
c.



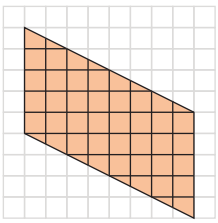
d.



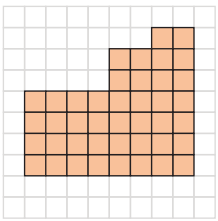
e.



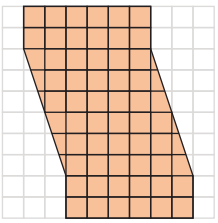
f.



g.

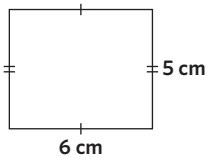


h.

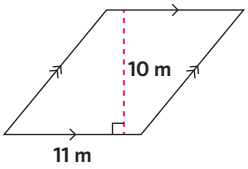


WE 3 **6.** Find the area of each shape using formulas for rectangles and parallelograms.

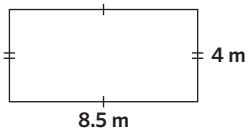
a.



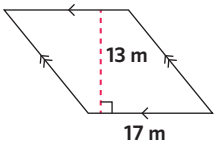
b.



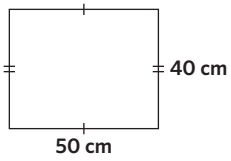
c.



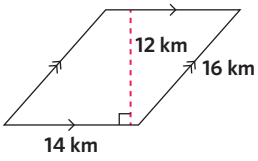
d.



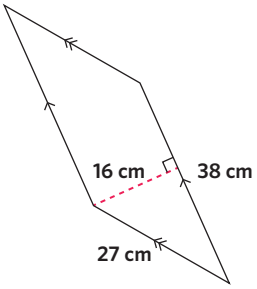
e.



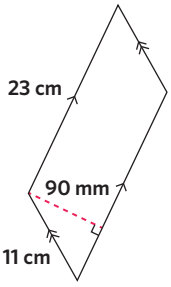
f.



g.

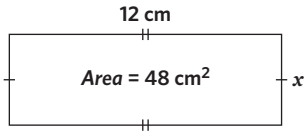


h.

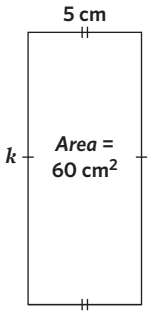


7. Find the missing value for each pronumeral.

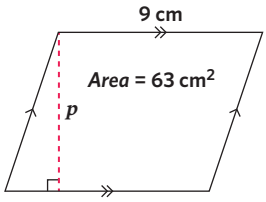
a.



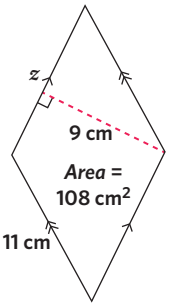
b.



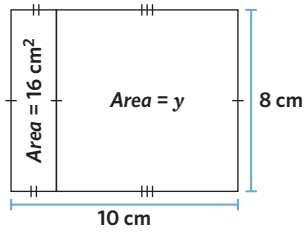
c.



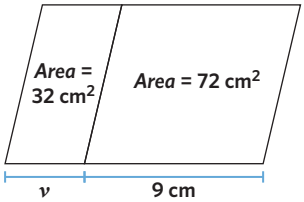
d.



e.



f.



Problem solving

Mild	Medium	Spicy
8, 9, 10	9, 10, 11	10, 11, 12

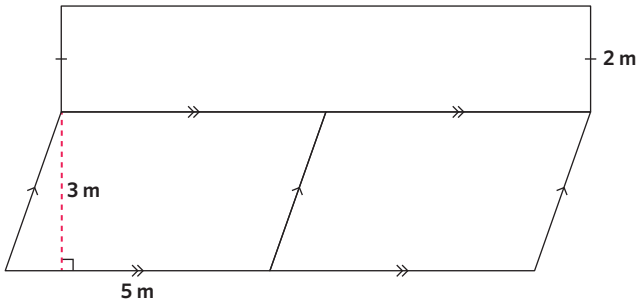
8. What is the area of the top of Angela's bedside table if it is 35 centimetres long and 50 centimetres wide?

9. Tony works at the local city council and wants to redevelop 3 hectares of grassland into a multi-use park. How many square metres is 3 hectares?

10. Cedric wants to replace his bathroom wall with parallelogram tiles. What is the area of 10 tiles if each tile has a base of 25 cm and a height of 21 cm?

11. Siobhan paid $3520 to install carpet in her rectangular shaped living room. If the length of her living room is 5.5 metres, what is the width of the living room? She was charged $80 per square metre of carpet.

12. Jett owns a pressure washing business where he washes mould, grime and dirt off surfaces. One of his customers has a unique driveway with the design illustrated below. If Jeff charges $15 per square metre he washes, how much should Jett charge to clean this driveway?



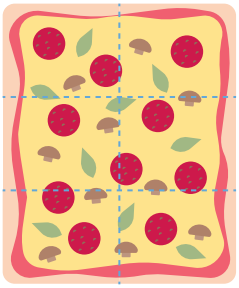
Reasoning

Mild	Medium	Spicy
13 (a,b,c,d)	13 (a,b,c,d), 14 (a,b,c)	All

13. Sofia recently moved to Melbourne from Rome. She is a pizza enthusiast and was surprised that not many shops sell Sicilian pizzas (rectangular pizzas). Seeing the gap in the market, she decided to open her own pizza shop and to sell Sicilian pizzas. Customers can purchase pizza by the slice for $5 or they can buy a whole pizza (6 slices) for $30.

Single slice

Whole pie



a. A single slice of pizza has a length of 12.5 centimetres and a width of 10 centimetres. What is the perimeter of a whole pizza in centimetres?

b. What is the area of a single slice of pizza in square centimetres?

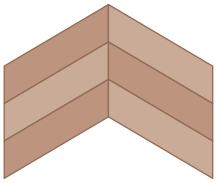
c. What is the area of a whole pizza in square centimetres?

d. How much would it cost to buy a square metre worth of Sicilian pizza from Sophia?

e. Sophia is thinking about increasing the price of her single slice pizzas. Is this a good idea, why or why not?

14. Derek is renovating his home and wants to replace his kitchen and dining room floors with a herringbone design illustrated below. Derek decided to go with a timber floor and wants to use two different colours. Each timber board is the same size.

6 boards of timber



a. Write an algebraic equation to represent the total area covered by the 6 boards of timber, using T to represent the total area, b to represent the base of one board and h to represent the height of one board.

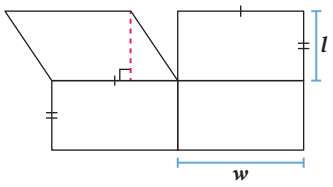
b. The area of the 6 boards of timber is 1470 cm^2 and the base of one board is 35 cm. What is the height of a single board of timber?

c. To completely renovate the kitchen and dining room, Derek needs exactly 1500 boards of timber. What is the area of Derek's kitchen and dining room in square metres?

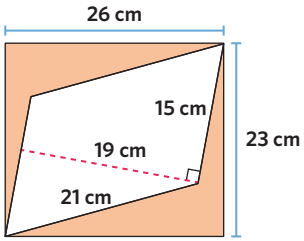
d. What might be some disadvantages of using parallelogram-shaped boards for flooring? Why might people typically prefer to use rectangular floorboards?

Extra spicy

15. Write an algebraic expression to represent the area of this shape where T is the total area, w is the width of one rectangle and l is the length of one rectangle.

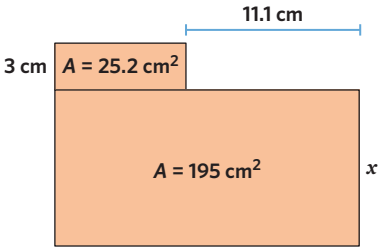


16. Calculate the area of the shaded shape.



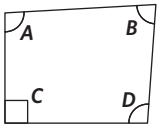
17. Jeremiah owns 1.2 hectares of farmland. His farmhouse covers 765 m^2 of the land. Jeremiah wants to plant flowers over the remaining land. If each flower plant covers 535 cm^2, how many plants does Jeremiah need?

18. Solve for the pronumeral x.



Remember this?

19. This shape has 4 angles marked.

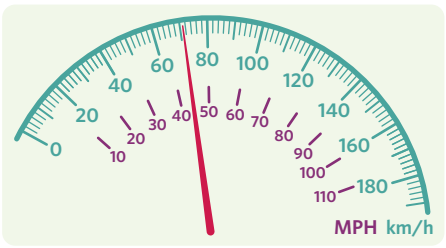


Which of these angles is less than 90°?

20. The following picture shows a car's speedometer.

At what speed, in km/h, is the car travelling at?

A. 44
B. 66
C. 71
D. 76



21. This jug can hold 1 L of water.

How much water is currently in the jug?

A. 800 mL
B. 600 mL
C. 400 mL
D. 300 mL

9C Area of triangles

Triangles are related to rectangles and parallelograms. Two identical triangles can be combined to form a rectangle or parallelogram. This is why a triangle can also be thought of as half of a rectangle or parallelogram. Because of this relationship, the formula for calculating the area of a triangle is the same as half of the area formula for a rectangle or parallelogram. Below are some examples where areas of triangles are applied.

- I ate a triangular corn chip with a base of 5 cm and a height of 5 cm. What is the area of this corn chip?
- I knitted an equilateral triangle patch with a side length of 6cm and a height of 5.2 cm. What is the area of the patch I knitted?
- I made a rectangular sandwich and cut the sandwich diagonally into two identical pieces. If the area of one piece is 30 cm^2 and the base length is 10 cm, what is the height of my sandwich?

Learning intentions

Students will be able to:

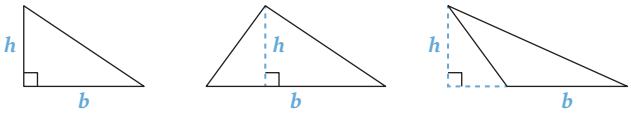
- identify the base and height of triangles
- connect the relationship between triangles and rectangles and parallelograms
- find the area of triangles.

Key terms and definitions

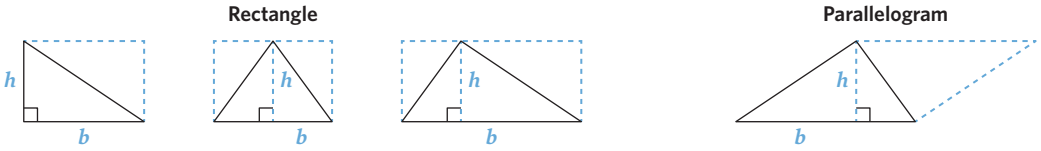
- The **area** is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.
- The **base** refers to any side of a triangle.
- The **height** is the perpendicular distance from the line that passes through the base to the opposite vertex.
- A **composite shape** is a geometric figure that is made up of two or more basic geometric shapes.

Key ideas

1 The base and height of a triangle is always perpendicular to each other.

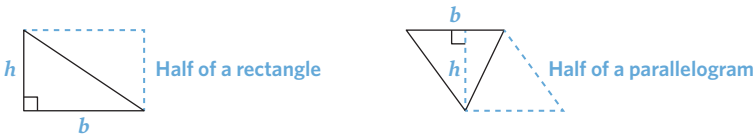


2 The area of a triangle is equal to **half** the area of a rectangle or parallelogram if they share the same base and height.



Note: Each rectangle and parallelogram is constructed with two identical triangles.

3 We can apply formulas to find the area of triangles.



$$\text{Area of triangle} = b \times h \times \frac{1}{2}$$

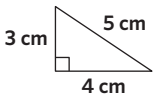
$$= \frac{bh}{2}$$

Worked examples

WE 1 Identifying the base and height of triangles

State the base and height for each triangle.

a.



Working

The base is 4 cm.

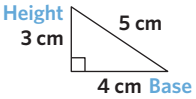
The height is 3 cm.

Thinking

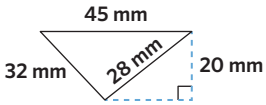
Step 1: The base is the bottom side of the triangle.

Step 2: The height is perpendicular to the base.

Visual support



b.



Working

The base is 45 mm.

The height is 20 mm.

Thinking

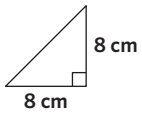
Step 1: The base can be any side of the triangle.

Step 2: The height is perpendicular to the base.

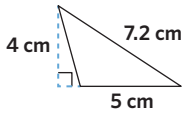
Student practice

State the base and height for each triangle.

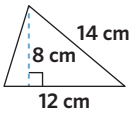
a.



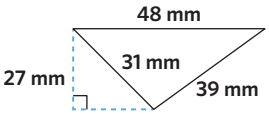
b.



c.



d.



WE 2 Finding the area of triangles

Find the area of each triangle using the area formula.

a.

Working

The base is 7 km and the height is 5 km.

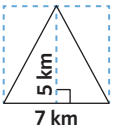
$$A = \frac{bh}{2}$$
$$= \frac{7 \times 5}{2}$$
$$= \frac{35}{2}$$
$$= 17.5 \text{ km}^2$$

Thinking

Step 1: State the base and height.

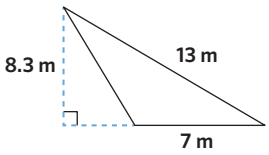
Step 2: Substitute $b = 7$ and $h = 5$ into $A = \frac{bh}{2}$.

Visual support



The area of the triangle is half of the area of a rectangle with the same base and height.

b.



Working

The base is 7 m and the height is 8.3 m.

$$A = \frac{bh}{2}$$
$$= \frac{7 \times 8.3}{2}$$
$$= \frac{58.1}{2}$$
$$= 29.05 \text{ m}^2$$

Thinking

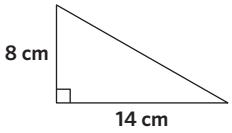
Step 1: State the base and height.

Step 2: Substitute $b = 7$ and $h = 8.3$ into $A = \frac{bh}{2}$.

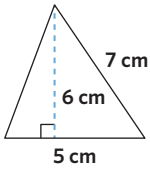
Student practice

Find the area of each triangle using the area formula.

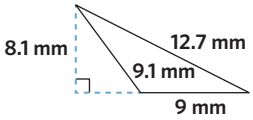
a.



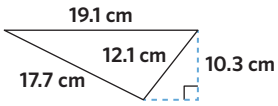
b.



c.



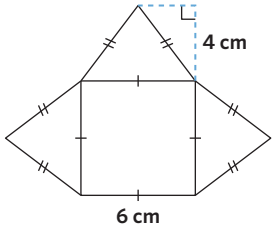
d.



WE 3 Finding the area of composite shapes

Calculate the area of each composite shape.

a.



Working

$$\text{Area of triangle} = \frac{6 \times 4}{2}$$
$$= \frac{24}{2}$$
$$= 12 \text{ cm}^2$$

$$\text{Area of square} = 6^2$$
$$= 36 \text{ cm}^2$$

$$\text{Total area} = 12 \times 3 + 36$$
$$= 36 + 36$$
$$= 72 \text{ cm}^2$$

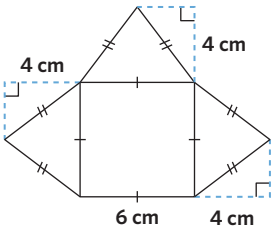
Thinking

Step 1: Find the area of one triangle by substituting $b = 6$ and $h = 4$ into $A = \frac{bh}{2}$.

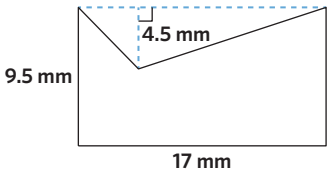
Step 2: Calculate the area of the square.

Step 3: Calculate the total area.

Visual support



b.



Working

$$\text{Area of rectangle} = l \times w$$
$$= 17 \times 9.5$$
$$= 161.5 \text{ mm}^2$$

$$\text{Area of triangle} = \frac{17 \times 4.5}{2}$$
$$= \frac{76.5}{2}$$
$$= 38.25 \text{ mm}^2$$

$$\text{Total area} = 161.5 - 38.25$$
$$= 123.25 \text{ mm}^2$$

Thinking

Step 1: Calculate the area of the rectangle.

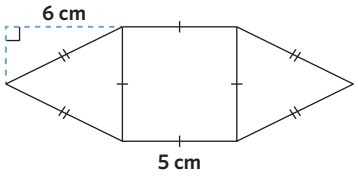
Step 2: Calculate the area of the triangle by substituting $b = 17$ and $h = 4.5$ into $A = \frac{bh}{2}$.

Step 3: Subtract the area of the triangle from the area of the rectangle.

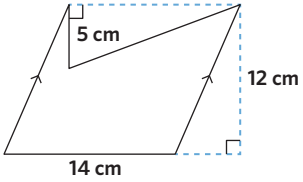
Student practice

Calculate the area of each composite shape.

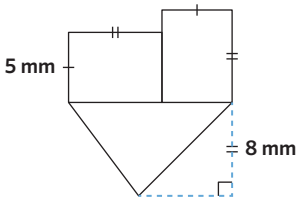
a.



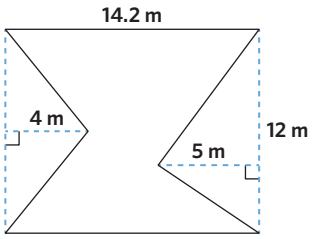
b.



c.



d.

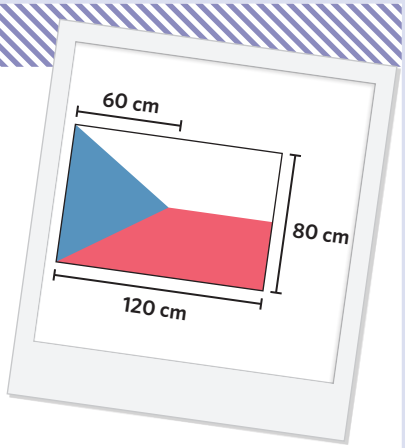


9C Activities and questions

STARTER TASKS

Odd spot

The Czech Republic is a landlocked country located in Central Europe. The flag of the Czech Republic is one of the eight flags in the world that has a triangle in its design. Tereza purchased a flag to help her decorate her home. The dimensions are specified below.

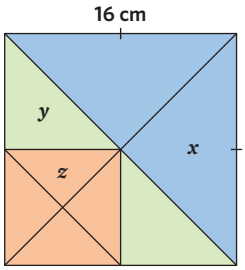


What is the height of the Czech Republic flag that Tereza purchased?

A. 60 cm **B.** 80 cm

Puzzle

The following square is made up of different triangles.



a) Calculate the area of the square.

b) Calculate the area of the blue triangle marked with a x.

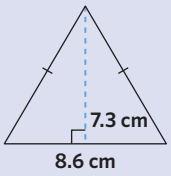
c) Calculate the area of the green triangle marked with a y.

d) Calculate the area of the orange triangle marked with a z.

Understanding worksheet

1. Identify the base and height of each triangle.

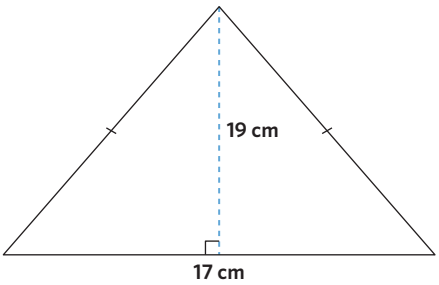
Example



Base of triangle: 8.6 cm

Height of triangle: 7.3 cm

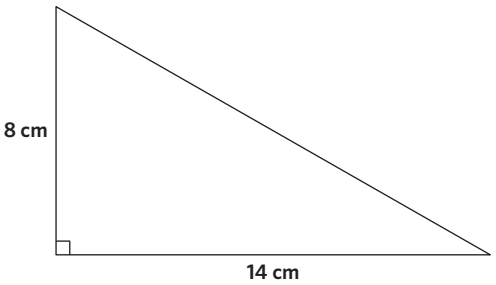
a.



Base of triangle: ____ cm

Height of triangle: ____ cm

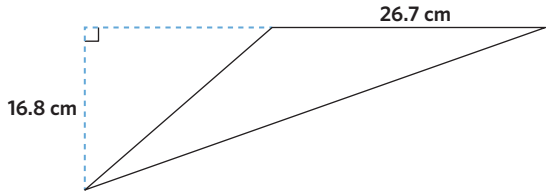
b.



Base of triangle: ____ cm

Height of triangle: ____ cm

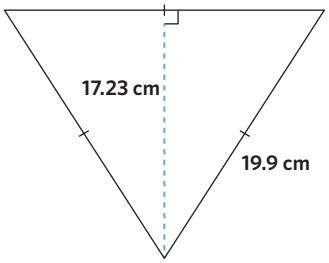
c.



Base of triangle: ☐ cm

Height of triangle: ☐ cm

d.

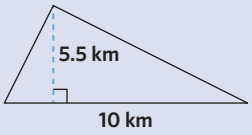


Base of triangle: ☐ cm

Height of triangle: ☐ cm

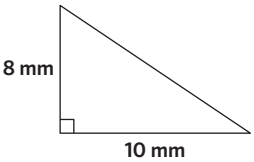
2. Fill in the boxes and calculate the area of each triangle.

Example



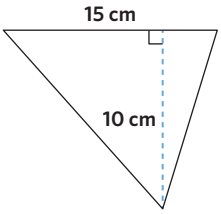
$\frac{1}{2} \times \boxed{10}$ km $\times$ 5.5 km $= \boxed{55}$ km^2

a.



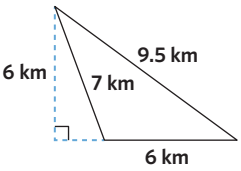
$\frac{1}{2} \times$ ☐ mm $\times$ 8 mm $=$ ☐ mm^2

b.



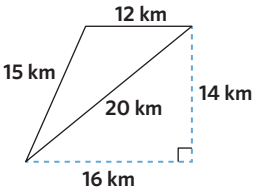
$\frac{1}{2} \times$ 15 cm $\times$ ☐ cm $=$ ☐ cm^2

c.



$\frac{1}{2} \times$ ☐ km $\times$ ☐ km $=$ ☐ km^2

d.



$\frac{1}{2} \times$ ☐ km $\times$ ☐ km $=$ ☐ km^2

3. Fill in the blanks by using the words provided.

base | height | combined | half

Triangles share a unique relationship with rectangles and parallelograms – two identical triangles can be ☐ to make parallelograms and rectangles. Because of this relationship, the area of a triangle is also ☐ of the area of a parallelogram or rectangle if both shapes have the same ☐ and ☐ lengths.

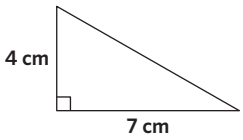
Fluency

Question working paths

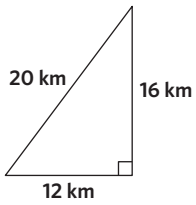
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c,d)	4 (b,c,d), 5 (b,c,d), 6 (c,d,e,f), 7 (b,c,d), 8 (c,d,e,f)	4 (d,e,f), 5 (d,e,f), 6 (e,f,g,h), 7 (d,e,f), 8 (e,f,g,h)

WE 1 **4.** State the base and height for each triangle.

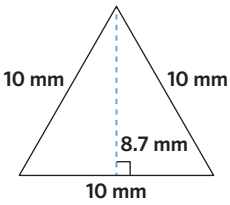
a.



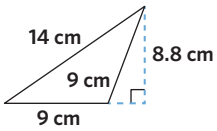
b.



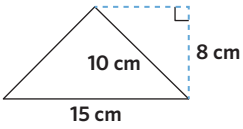
c.



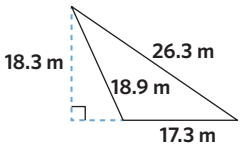
d.



e.

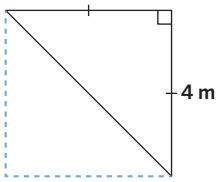


f.

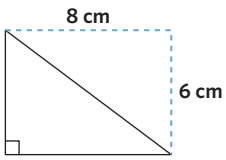


5. Calculate the area of the rectangle or parallelogram and use this to calculate the area of the triangle.

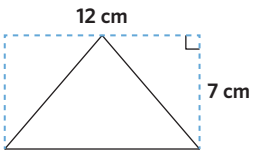
a.



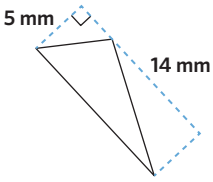
b.



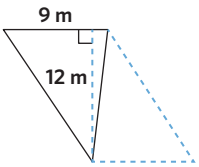
c.



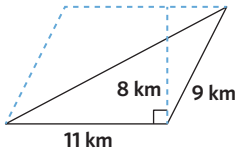
d.



e.

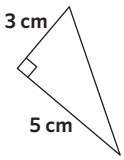


f.

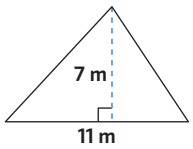


WE 2 **6.** Find the area of each triangle using the area formula.

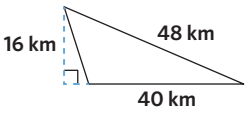
a.



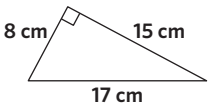
b.



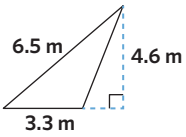
c.



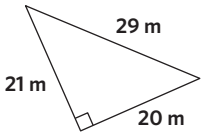
d.



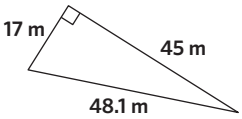
e.



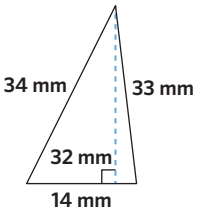
f.



g.

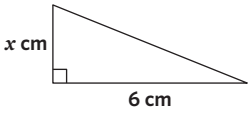


h.

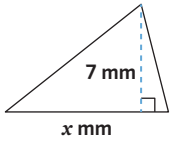


7. Find the missing value for each pronumeral.

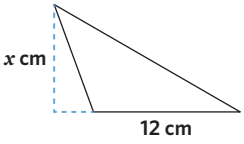
a. *Area* = 9 cm^2



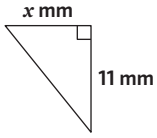
b. *Area* = 35 mm^2



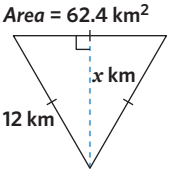
c. *Area* = 54 cm^2



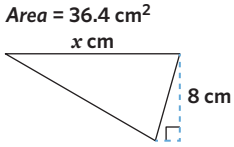
d. *Area* = 27.5 mm^2



e. *Area* = 62.4 km^2

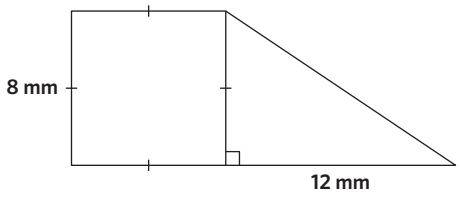


f. *Area* = 36.4 cm^2

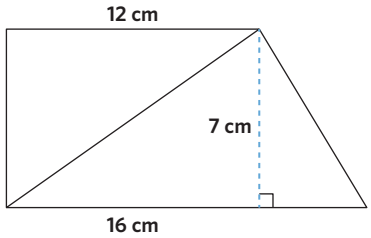


WE 3 **8.** Find the area of each composite shape.

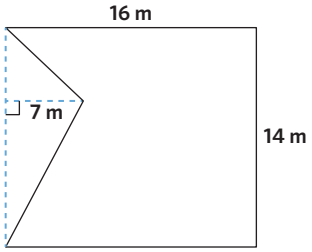
a.



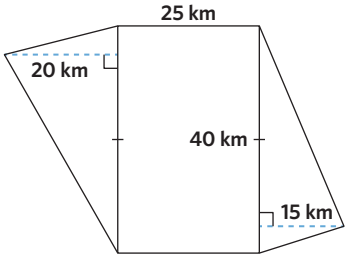
b.



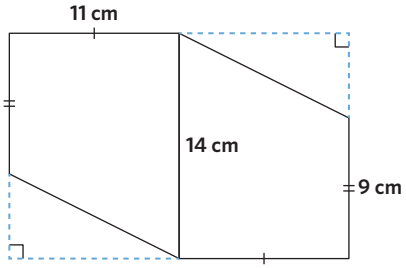
c.



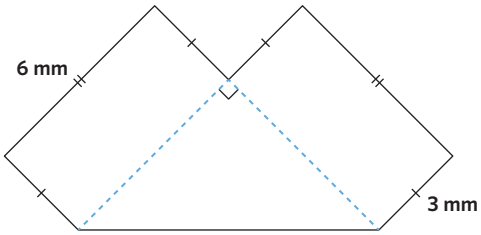
d.



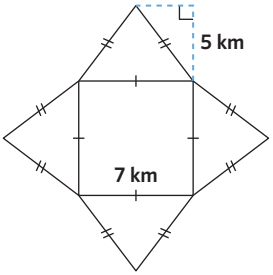
e.



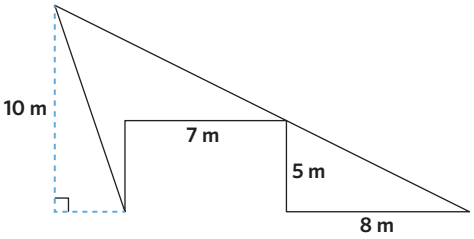
f.



g.



h.



Problem solving

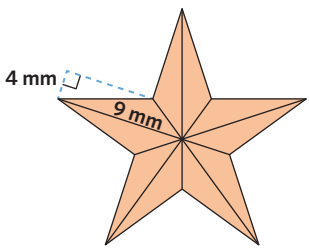
Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Wanda volunteers as a lollipop lady at her local primary school. She carries a triangular stop sign attached to a pole to help students cross safely. If the triangular sign has a base of 60 cm and a height of 50 cm, what is the area of this sign?

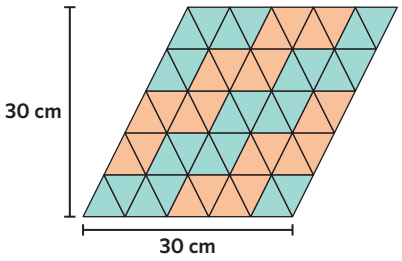
10. Alfredo has a triangular veggie patch in his garden which he uses to grow lettuce. What is the area of this veggie patch if the base is 2.4 metres and the height is 1.5 metres?

11. Wilma recently travelled to Egypt and took a photo of the Khafre Pyramid so that she could print it out as a poster. What is the height of the pyramid on Wilma's poster if the base is 9 cm and the area is 31.5 cm^2?

12. Amber recently purchased a star stamp that she plans to use to decorate her notebooks. The star design is made up of 10 identical scalene triangles. What is the area of one stamp imprint?



13. Jeremiah made a mosaic using isosceles triangular tiles.

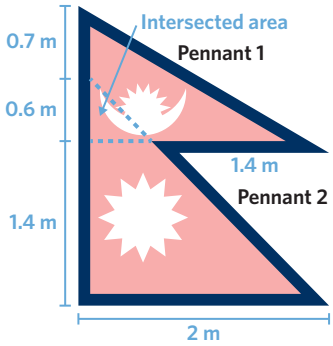


What is the total area of the orange tiles in the mosaic?

Reasoning

Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Nepal is the only country in the world that does not have a quadrilateral flag as the country has maintained tradition. The flag is characterised by two pennant (triangular) shapes and is crimson coloured, representing a symbol for bravery.



a. What is the base of pennant 1?

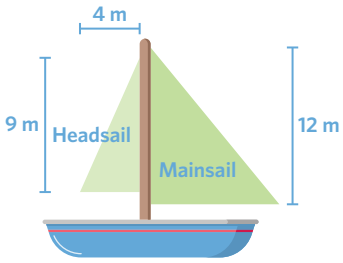
A. 0.6 m **B.** 0.7 m **C.** 1.4 m **D.** 2 m

b. What is the area of the **intersected area** between pennant 1 and pennant 2?

c. Use the given dimensions to find the area of the flag.

d. Should Nepal shift away from having a non-quadrilateral shaped flag? Why or why not?

15. Porter recently purchased a sailboat as he wanted to spend more time at sea. His sailboat has two sails, a headsail and a mainsail. The dimensions of Porter's sailboat can be found below.



a. What is the area of the headsail?

b. If the area of the mainsail is 66 m^2, what is the length of the base for the mainsail?

c. Porter wants to increase the size of his headsail and mainsails so he can sail faster. If he bought a new set of sails that extended the base of both sales by 3 m and their heights by 0.5 m, what would be the total area increase of his sails?

d. Why might sailing be such an expensive hobby?

Extra spicy

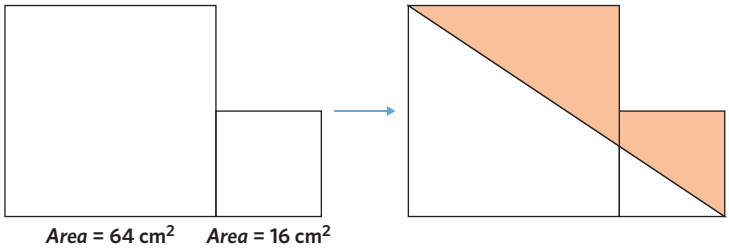
16. Jason folds a square diagonally once and then folds it in half 4 times to form a smaller triangle. How many of these smaller triangles would fit into Jason's original square?

A. 4 **B.** 8 **C.** 16 **D.** 32 **E.** 64

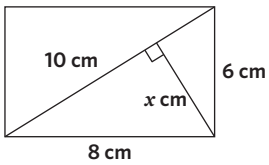
17. A triangle has a base of 13 cm and an area of 87.75 cm^2. What is its height?

A. 11 cm **B.** 11.5 cm **C.** 12.5 cm **D.** 13 cm **E.** 13.5 cm

18. What is the area of the shaded section?



19. What is the value of x?



Remember this?

20. Stanley had $6.75 and bought a second-hand jumper from Savers for $4.90.
How much money does he have left?

21. Lorraine randomly takes one cupcake from a plate.
Which of these plates gives Lorraine the best chance of choosing a chocolate cupcake?

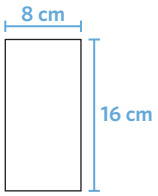
A. 

B. 

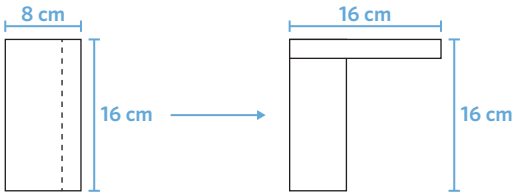
C. 

D. 

22. Vince has a rectangular piece of scrap paper, as shown.



He decides to cut a quarter of the rectangle off, and places it over the remaining paper to make a new shape, as shown.



What is the perimeter of this new shape?

9D Drawing 3D objects

Most things and objects we interact with day-to-day are not two-dimensional (2D) but three-dimensional (3D). We can represent 3D objects on a 2D plane through perspective drawings or isometric drawings. Nets can also be used to give us a 2D understanding of all the faces that make up a 3D object. Below are some examples where drawing 3D objects can be applied.

- Where I'm planning to put my desk in my bedroom.
- When I'm stacking toy blocks on top of each other to construct a building.
- When I'm designing a model for a diorama.

Learning intentions

Students will be able to:

- recognise and draw 3D objects and isometric drawings
- draw a net for common 3D objects
- understand and draw plan views for 3D objects and isometric drawings.

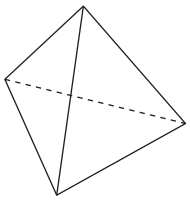
Key terms and definitions

- A **prism** is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.
- The **apex** is the highest point of a 3-dimensional shape.
- A **pyramid** is a 3-dimensional object with a polygon base and 3 or more triangular faces which all meet at a point.
- A **net** is a 2-dimensional representation showing all the faces of a 3-dimensional object. It could be cut out and folded up to create the 3-dimensional object that it represents.
- A **cone** is a 3-dimensional object that tapers from a circular base to an apex point.
- A **cylinder** is a 3-dimensional object that has 2 flat, circular bases which are connected by a curved surface.
- A **regular prism** is a prism where the base shape side lengths are all equal.

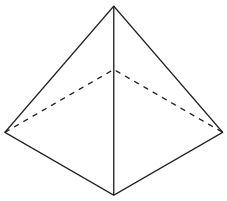
Key ideas

1 Prisms and pyramids are named according to the shape of their base.

Triangular pyramid

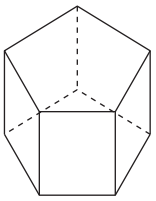


Square pyramid

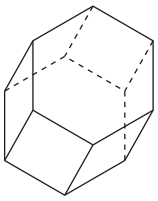


A pyramid has a flat base and triangular faces that meet at its apex.

Pentagonal prism



Hexagonal prism

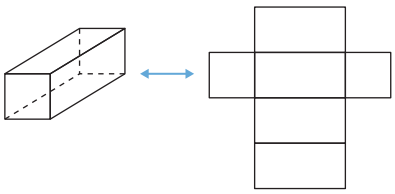


A prism has two identical opposite faces. All other faces are rectangular.

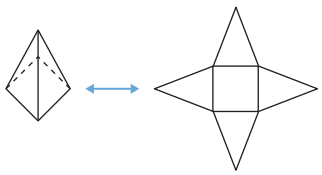
Name of prism/pyramid	Base shape	Number of sides of base shapes
Triangular	Triangle	3
Rectangular	Rectangle	4
Pentagonal	Pentagon	5
Hexagonal	Hexagon	6
Heptagonal	Heptagon	7
Octagonal	Octagon	8

2 We can use a net to represent a 3-dimensional object with all of its faces unfolded and flattened out.

Rectangular prism

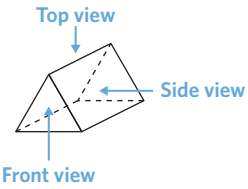


Square pyramid

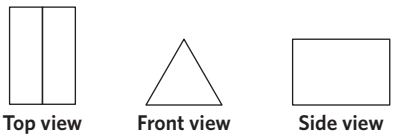


3 We can represent the front, side and top views of a 3-dimensional object as a 2-dimensional image. This is also known as plan views.

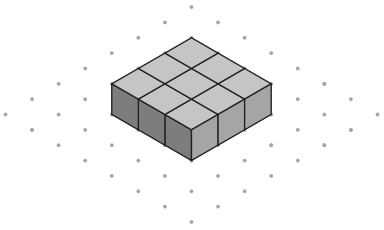
Triangular prism



Plan views



4 We can use isometric dot paper to represent 3-dimensional objects in 2-dimensional form.



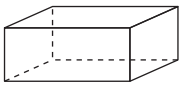
There are 9 squares in this isometric drawing.

Worked examples

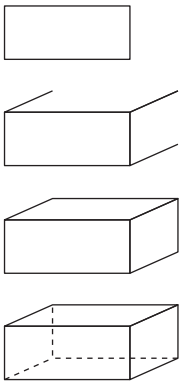
WE 1 Drawing 3D objects

Copy each 3D object as specified.

a. Rectangular prism



Working



Thinking

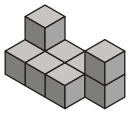
Step 1: Draw the base shape of the prism.

Step 2: Draw the slanted lines from the vertices of the base shape, depending on where the 3D object is facing.

Step 3: Connect the slanted lines.

Step 4: Draw the dotted lines that are hidden in the 3D object. Make sure opposite sides' lengths are parallel and equal.

b. This drawing on isometric dot paper.



Working

Thinking

Step 1: Working from front to back, draw the bottom layer of the solid.

Step 2: Use the squares of the bottom layer as a guide to draw the layer above.

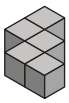
Step 3: Erase any lines that would be hidden from adding a layer.

Student practice

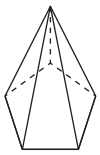
Copy each 3D object as specified.

a. Cube

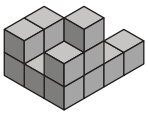
b. This drawing on isometric dot paper.



c. Regular pentagonal pyramid



d. This drawing on isometric dot paper.

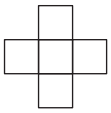


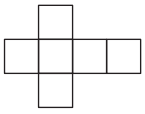
WE 2 Representing 3D objects as nets

Draw a net for each 3D object.

a. Cube

Working





Thinking

Step 1: Identify and draw the base shape. The base shape is a square.

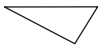
Step 2: Identify the sides that would fold up from the base. All four connecting sides are squares. Draw four squares connecting to the base.

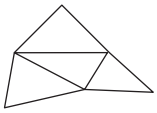
Step 3: Identify the other base shape. The other base shape is a square. Add the base shape to the net.

Note: There are other possible answers.

b. Triangular pyramid

Working





Thinking

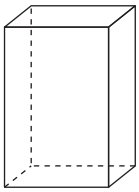
Step 1: Identify and draw the base shape. The base is a scalene triangle.

Step 2: Identify the sides. This is a pyramid. The sides are triangles that meet at an apex. The triangles are isosceles triangles with equal heights.

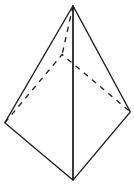
Student practice

Draw a net for each 3D object.

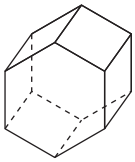
a. Rectangular prism



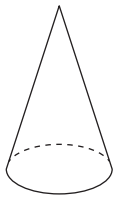
b. Square pyramid



c. Regular hexagonal prism



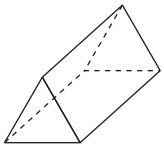
d. Cone



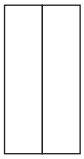
WE 3 Interpreting plan views of a 3D object

Draw the front, side and top views for each 3D object.

a.

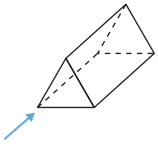


Working

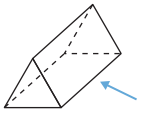


Thinking

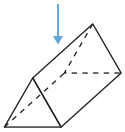
Step 1: Identify the shape of the 3D object from the front. It is an equilateral triangle.



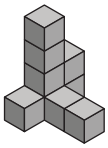
Step 2: Identify the shape of the 3D object from the side. It is a rectangle.



Step 3: Identify the shape of the 3D object from the top. This is a rectangle with a line in the middle.

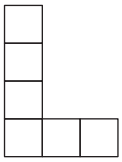


b.

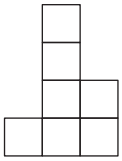


Working

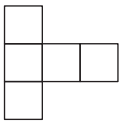
Front view:



Side view:

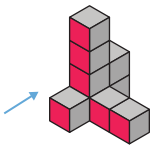


Top view:

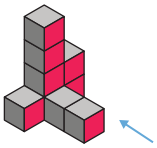


Thinking

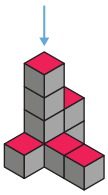
Step 1: Identify the shape of the 3D object from the front. Only the highlighted surface would be visible.



Step 2: Identify the shape of the 3D object from the side. Only the highlighted surface would be visible.



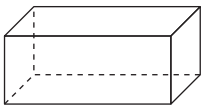
Step 3: Identify the shape of the 3D object from above. Only the highlighted surface would be visible.



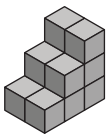
Student practice

Draw the front, side and top views for each 3D object.

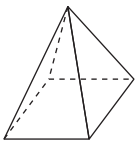
a.



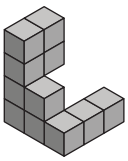
b.



c.



d.



9D Activities and questions

STARTER TASKS

Odd spot

The Eiffel Tower is a global cultural icon and one of the most visited structures in the world. The Eiffel Tower was the tallest man-made structure in the world at one point until the Chrysler Building took the title 41 years later. The Eiffel Tower is symmetrical on all 4 sides and resembles the geometric shape of a square pyramid.

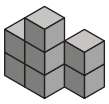
As the building is symmetrical, is the Eiffel Tower's front plan view and side plan view the same?

A. Yes **B.** No

Image: MarinaD_37/Shutterstock.com

Puzzle

The following isometric drawing is made up of a number of equally sized cubes.



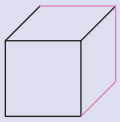
How many cubes could potentially be hidden in this isometric drawing?

Understanding worksheet

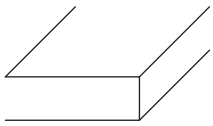
1. Complete each unfinished 3D object to show the visible edges and vertices.

Example

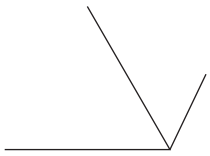
Cube



a. Rectangular prism

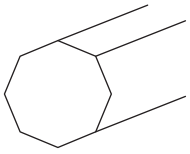


b. Square pyramid



c. Triangular prism

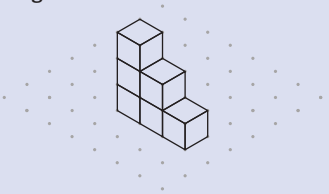
d. Regular octagonal prism



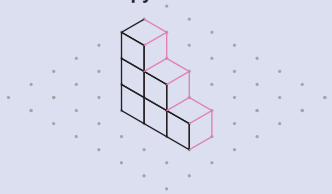
2. Complete the copy of the unfinished isometric drawing.

Example

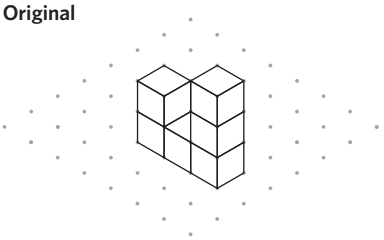
Original



Unfinished copy



a. **Original**



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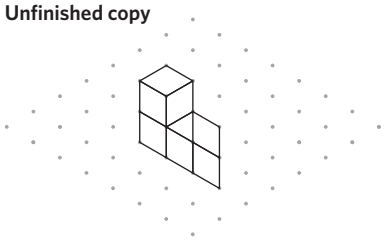
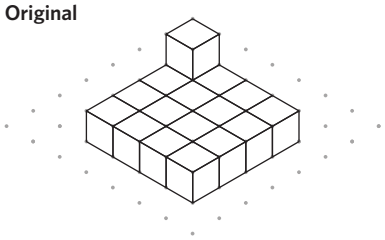


b. Original



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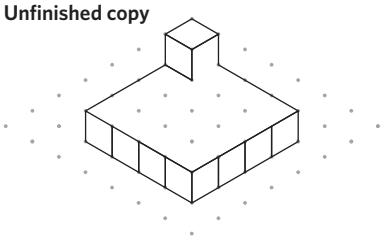
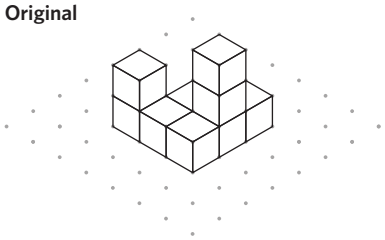


c. Original



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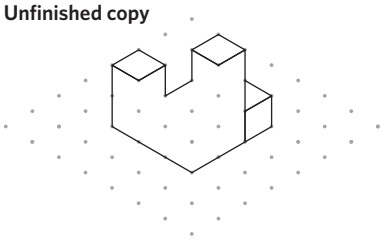
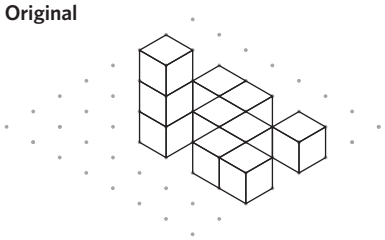


d. Original



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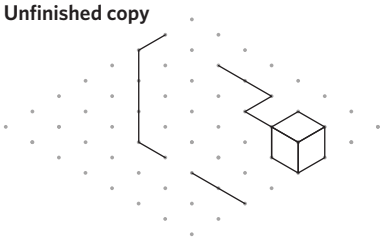


3. Fill in the blanks by using the words provided.

2D plane | different | 3D object | flattened | faces

Nets allow us to represent 3D objects on a _____. A net illustrates a _____ with all of its _____ unfolded and _____ out. 3D objects can have _____ nets to represent the same solid. For example, there are eleven different nets that can form a cube.

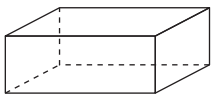
Fluency

Question working paths

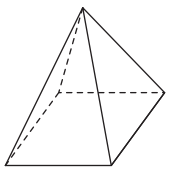
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c,d), 8 (a,b,c), 9 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (c,d,e,f), 8 (b,c,d), 9 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (e,f,g,h), 8 (d,e,f), 9 (d,e,f)

4. State whether each 3D object is a prism or a pyramid.

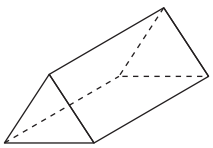
a.



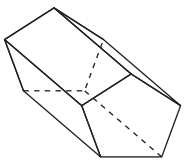
b.



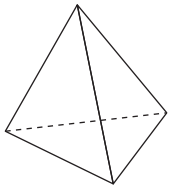
c.



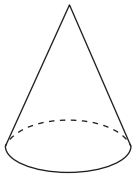
d.



e.

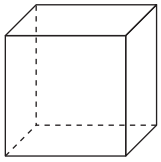


f.

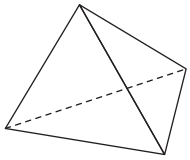


WE 1a **5.** Copy each 3D object as specified.

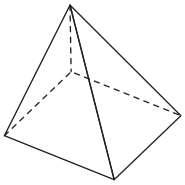
a. Cube



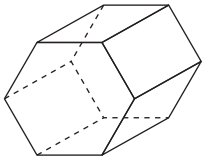
b. Triangular pyramid



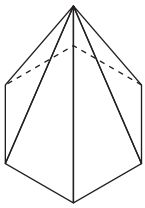
c. Regular square pyramid



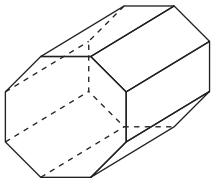
d. Regular hexagonal prism



e. Regular hexagonal pyramid

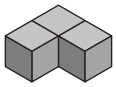


f. Regular octagonal prism

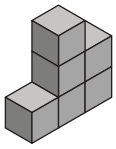


WE 1b **6.** Draw each drawing on isometric dot paper.

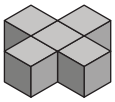
a.



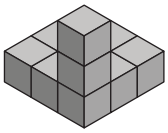
b.



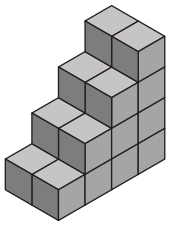
c.



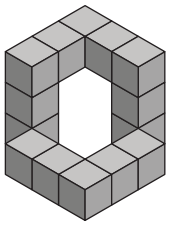
d.



e.

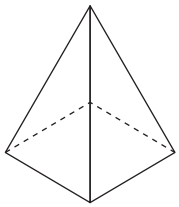


f.

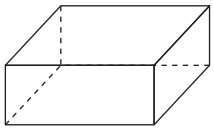


WE 2 **7.** Draw a net for each 3D object.

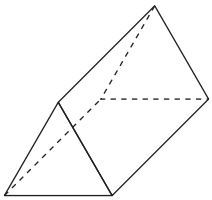
a.



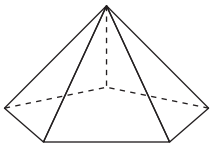
b.



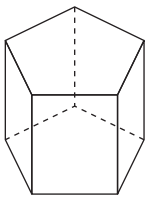
c.



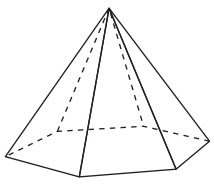
d.



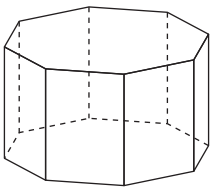
e.



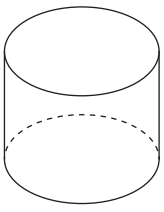
f.



g.

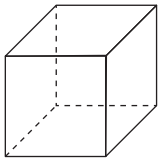


h.

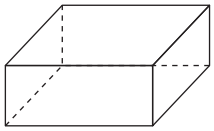


WE 3a **8.** Draw the front, side and top view for each 3D object.

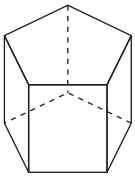
a.



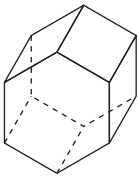
b.



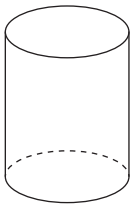
c.



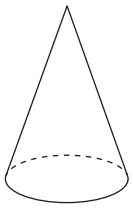
d.



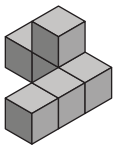
e.

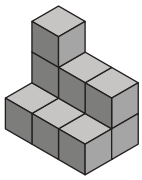


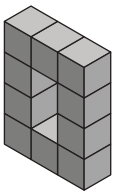
f.

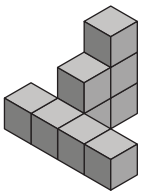


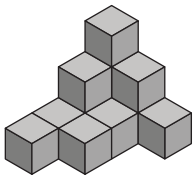
WE 3b **9.** Draw the front, side and top view for each isometric drawing.

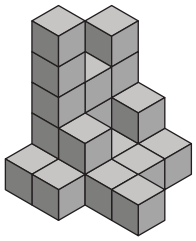
a. 

b. 

c. 

d. 

e. 

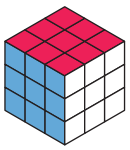
f. 

Problem solving

Mild	Medium	Spicy
10, 11, 12	11, 12, 13	12, 13, 14

10. Hariharan is an architecture student at university. As an assignment, he needs to create a 3D model of his room. He decides to use a rectangular prism to represent his work station. Draw a rectangular prism where its length and width are both greater than its height.

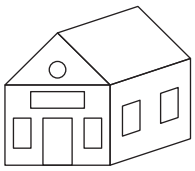
11. Ruby saw everyone else in her class play with a 3 × 3 Rubik's cube so she bought one herself. Copy Ruby's Rubik's cube on isometric dot paper.



12. Natasha recently purchased a pair of shoes that were packaged in a rectangular prism shaped cardboard box. Natasha realised that if she unfolded the box and laid the faces flat, she could create a net for a rectangular prism. Draw the net of a rectangular prism.

13. Chloe was stacking her wooden toy blocks and constructed a flight of 5 stairs that were 4 blocks wide. Draw the stairs that Chloe constructed on isometric dot paper.

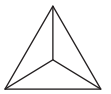
14. Harry used to live in a house before he moved to an apartment closer to his work. He was feeling nostalgic and searched his old house up on Google Earth. Draw the plan views (front, side and top views) of Harry's old unit.



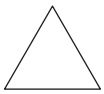
Reasoning

Mild	Medium	Spicy
15 (a,b,c)	15 (a,b,c), 16 (a,b,c)	All

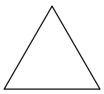
15. Carlin is building a structure for a design assignment.



Top view



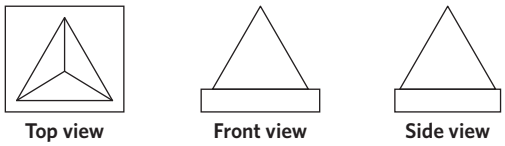
Front view



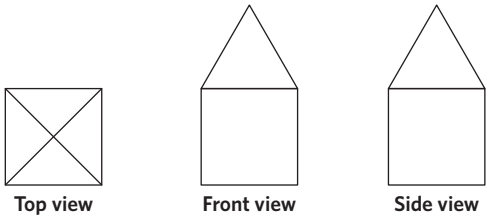
Side view

a. Based on the top, front and side views shown above, what 3D object has Carlin built?

b. Carlin completed his assignment by placing another object beneath the object from part **a**. Carlin took another set of photos shown below. What 3D object did he add to his structure?



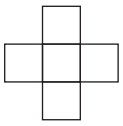
c. Carlin's friend Franka is also completing a design. Based on the top, front and side views shown below, what 3D object did Franka build?

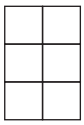


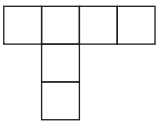
d. If you were an architect designing a home, why might you have a pyramid shaped roof instead of a prism shaped roof?

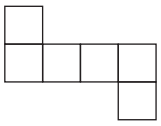
16. Sarah loves stacking and building different 3D objects with her wooden cube blocks.

a. Which of the following will fold up into a cube?

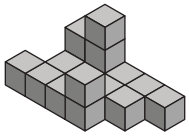
A. 

B. 

C. 

D. 

b. Sarah built the following 3D object with her wooden blocks. Copy the 3D object on isometric dot paper.

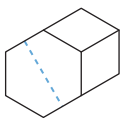


c. Draw the front, side and top view of the 3D object from part **b**.

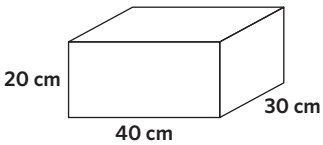
d. Sara's mum was originally going to buy plastic cube blocks but decided to buy wooden cube blocks instead. Why might wooden cube blocks be better than plastic cube blocks?

Extra spicy

17. A hexagonal prism is cut in half in the following manner. How many vertices will there be in one hexagonal prism half?

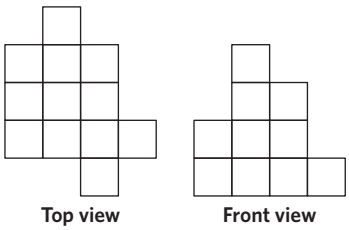


18. A large marble slab is made up of smaller rectangular prisms with the following dimensions.



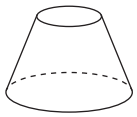
If the large marble slab has a length of 1.2 m, width of 0.6 m and height of 1.4 m, draw a top, front and side view of the large marble slab.

19. A stack of cubes has the following top and front view.



What is the minimum number of cubes required to make this object?

20. A truncated cone is a cone with its tip straight cut off. What are the plan views (front view, side view and top view) of a truncated cone?



Remember this?

21. $3 \times p = 57$

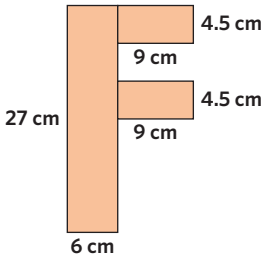
What number does p represent?

A. 9 **B.** 16 **C.** 17 **D.** 19

22. A standard six-sided dice is rolled once. What is the probability that the number on the top face is a multiple of 2?

A. $\frac{1}{3}$ **B.** $\frac{1}{2}$ **C.** $\frac{2}{3}$ **D.** $\frac{1}{6}$

23. Francisca made a letter 'F' to put on her bedroom door. She made it by sticking 3 rectangles together.



What is the perimeter of the letter in centimetres?

A. 93 cm **B.** 102 cm **C.** 111 cm **D.** 120 cm

9E Volume of prisms

When talking about area, we refer to the space between the boundaries of 2D shapes. Volume is an extension to the concept of area – it is the total amount of space that exists within a 3D object. As with area, we use different volume metric units to appropriately measure how large an object is. The most common units of volume are the cubic metre and litre. Below are some examples where the volume of prisms can be applied.

- I bought a 500 gram block of cheese with a height of 5 cm, a width of 8 cm and length of 15 cm. What is the volume of this block of cheese?
- I bought a circular cheesecake for my friend's birthday. If the area of the circular face is 50 cm^2 and the height of the cake is 7 cm, what is the volume of this cheesecake?
- The volume of my swimming pool is 126 m^3. If the length is 9 metres and the width is 7 metres, what is the depth of my swimming pool?

Learning intentions

Students will be able to:

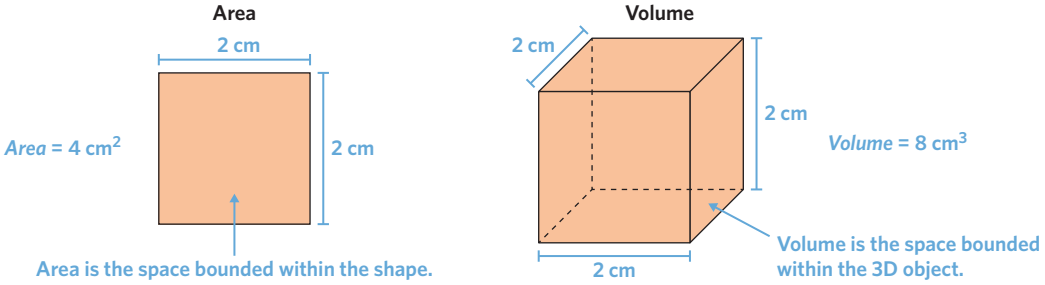
+ understand volume and the different metric units used to measure volume
+ calculate the volume of prisms
+ calculate the volume of composite 3D objects.

Key terms and definitions

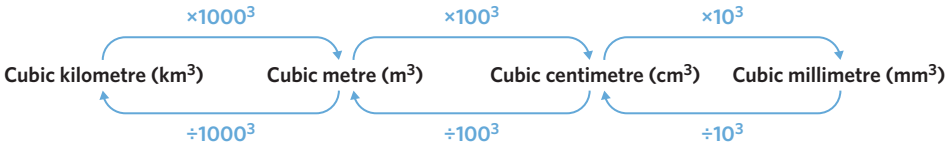
- The **volume** is the amount of space that is contained within a 3D object.
- A **cubic millimetre** (mm^3) is the space contained by the cube with edges of 1 millimetre in length.
- A **cubic centimetre** (cm^3) is space contained by a cube with edges of 1 centimetre in length.
- A **cubic metre** (m^3) is the space contained by a cube with edges of 1 metre in length.
- A **cubic kilometre** (km^3) is space contained by a cube with edges of 1 kilometre in length.

Key ideas

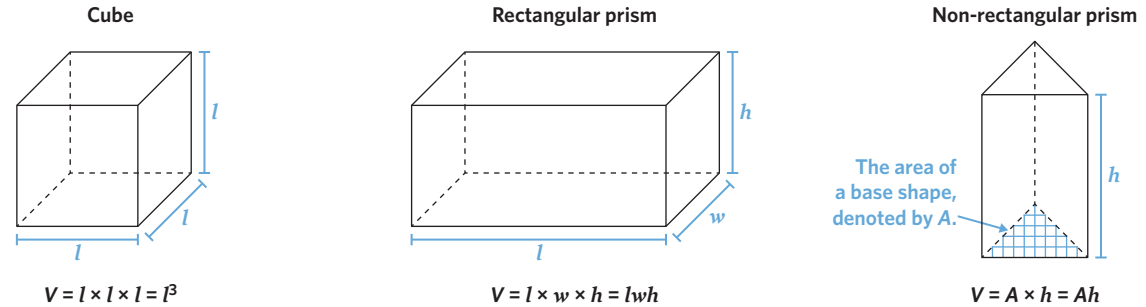
1 Volume is the amount of space that is contained within a 3D object and is measured in cubic units.



2 We can convert to different units of volume by multiplying or dividing by powers of 10.



3 The volume of a prism is the area of the base shape multiplied by the height of the prism. We can apply formulas to find the volume of prisms.

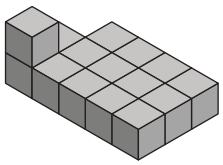


Worked examples

WE 1 Finding the volume of isometric objects

Find the volume of each object (each cube is 1 cm^3).

a.



Working

1

14

$14 + 1 = 15\ cm^3$

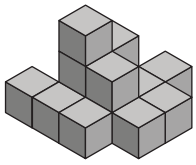
Thinking

Step 1: Count the number of visible cubes on the top layer.

Step 2: Count the number of visible cubes on the layer below.

Step 3: Add all the layers together to calculate the volume.

b.



Working

1

3

$7 + 3 = 10$

$1 + 3 + 10 = 14\ cm^3$

Thinking

Step 1: Count the number of visible cubes on the top layer.

Step 2: Count the number of visible cubes on the layer below.

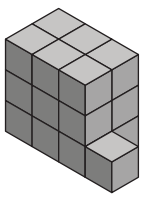
Step 3: Count the number of visible cubes on the bottom layer, then add the number of implied cubes (3 cubes = number of cubes from the layer above).

Step 4: Add all the layers together to calculate the volume.

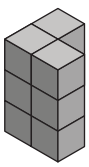
Student practice

Find the volume of each object (each cube is 1 cm^3).

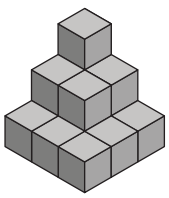
a.



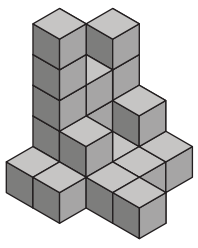
b.



c.



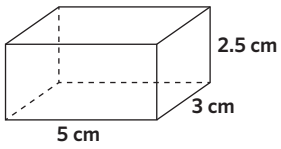
d.



WE 2 Calculating the volume of 3D objects with formulas

Find the volume of each object using a formula.

a.



Working

$V = l \times w \times h$

$V = 5 \times 3 \times 2.5$

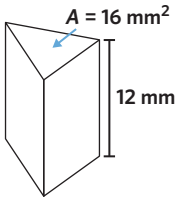
$= 37.5 \text{ cm}^3$

Thinking

Step 1: Recall the formula for the volume of a rectangular prism.

Step 2: Substitute $l = 5$, $w = 3$ and $h = 2.5$ into the formula and simplify.

b.



Working

$V = A_{base} \times h.$

$V = 16 \times 12$

$= 192 \text{ mm}^3$

Thinking

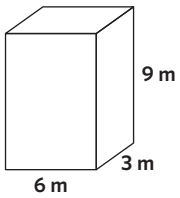
Step 1: Recall the formula for the volume of a triangular prism.

Step 2: Substitute $A = 16$ and $h = 12$ into the formula and simplify.

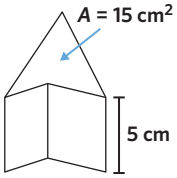
Student practice

Find the volume of each object using a formula.

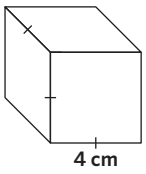
a.



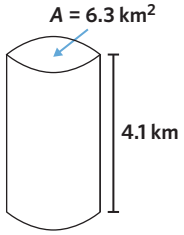
b.



c.



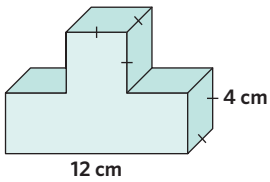
d.



WE 3 Finding the volume of composite objects

Calculate the volume of each composite object.

a.



Working

$V_{composite} = V_{small\ cube} + V_{rectangular\ prism}$

$= (4 \times 4 \times 4) + (12 \times 4 \times 4)$

$= 64 + 192$

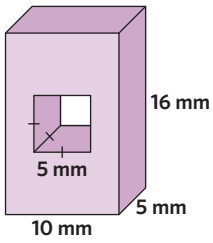
$= 256 \text{ cm}^3$

Thinking

Step 1: Identify the volume as the sum of volume of a rectangular prism and a cube.

Step 2: Calculate the volume of each 3D object and then sum the two volumes.

b.



Working

$V_{composite} = V_{prism} - V_{cube}$

$= (10 \times 5 \times 16) - (5 \times 5 \times 5)$

$= 800 - 125$

$= 675 \text{ mm}^3$

Thinking

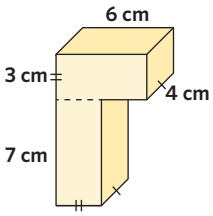
Step 1: Identify the volume as the difference in volume between the big rectangular prism and the small cube.

Step 2: Calculate the volume of the rectangular prism, subtract the volume of the cube from it, and simplify.

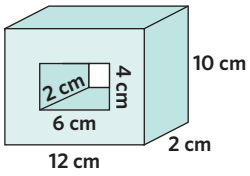
Student practice

Calculate the volume of each composite object.

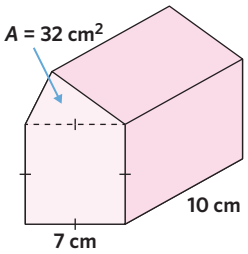
a.



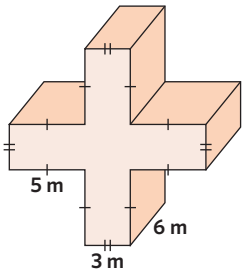
b.



c.



d.



9E Activities and questions

STARTER TASKS

Odd spot

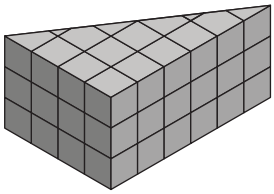
The Beijing National Aquatics Centre was an aquatics centre constructed for the 2008 Summer Olympics and Paralympics. It's informally known as the Water Cube or Ice Cube as the shape of the structure resembles a cuboid. The length of the Beijing National Aquatics Centre is 177 m, the width is 177 m and the height is 31 m. What is the correct method to calculate the volume of the Beijing National Aquatics Centre?

A. $177 \times 177 \times 31$ **B.** $177 \times 177 \times 2$

Image: TonyV3112/Shutterstock.com

Puzzle

A rectangular 3D object was cut in half diagonally to form a triangular prism.



a) If each cube has a volume of 1 cm^3, what was the volume of the rectangular prism before it was cut in half?

b) If each cube has a volume of 1 cm^3, what is the volume of the triangular prism?

Understanding worksheet

1. Select the unit of volume that is most appropriate to measure the volume of each object.

Example

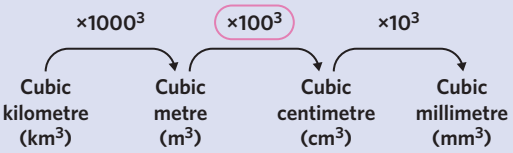
	Cubic kilometre	Cubic metre	Cubic centimetre	Cubic millimetre
The volume of a swimming pool:	☐	☑	☐	☐

		Cubic kilometre	Cubic metre	Cubic centimetre	Cubic millimetre
a.	The volume of a shoebox:	☐	☐	☐	☐
b.	The volume of a rice grain:	☐	☐	☐	☐
c.	The volume of the South Pacific Ocean:	☐	☐	☐	☐
d.	The volume of a bus:	☐	☐	☐	☐

2. Circle the required calculation(s) and then complete each unit conversion.

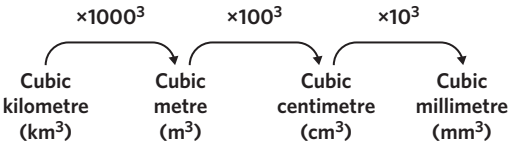
Example

$2\ m^3$ to cm^3



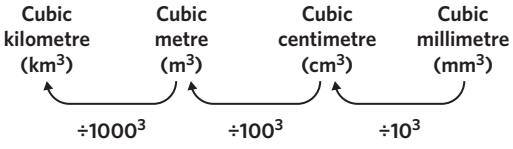
$2 \times \boxed{1\,000\,000} = 2\,000\,000\ cm^3$

a. $8\ cm^3$ to mm^3



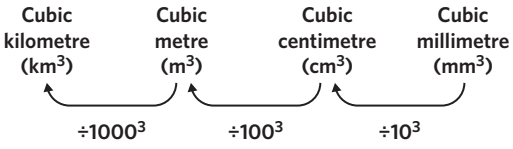
$8 \times \square = 8000\ mm^3$

b. $3000\ mm^3$ to cm^3



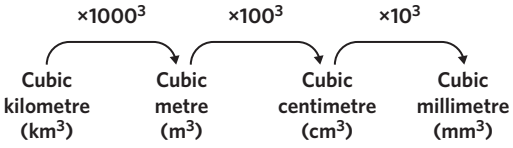
$3000 \div \square = 3\ cm^3$

c. $90\,000\,000\ cm^3$ to m^3



$90\,000\,000 \div \square = 90\ m^3$

d. $18\ m^3$ to cm^3



$18 \times \square = 18\,000\,000\ cm^3$

3. Fill in the blanks by using the words provided.

magnitude | 3D object | cubic metres | bigger | space

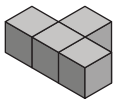
When we measure volume, we measure how much ☐ is contained within a ☐. Volume allows us to understand the ☐ of an object's size – the larger the volume, the ☐ it is. The two most common measurements of volume are litres and ☐, the former measuring capacity and the latter measuring volume.

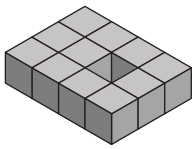
Fluency

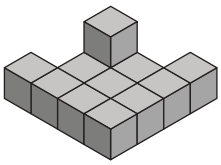
Question working paths

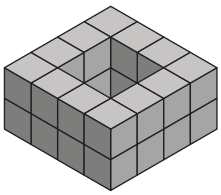
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b,c)	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c,d)	4 (d,e,f), 5 (d,e,f), 6 (d,e,f), 7 (d,e,f), 8 (d,e,f)

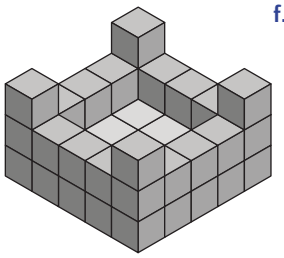
WE 1 **4.** Find the volume of each object (each cube is 1 cm^3).

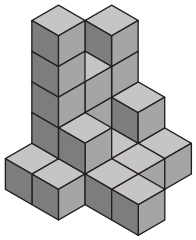
a. 

b. 

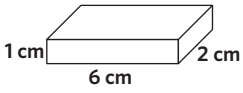
c. 

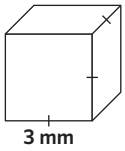
d. 

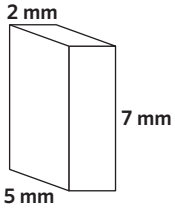
e. 

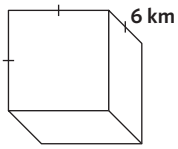
f. 

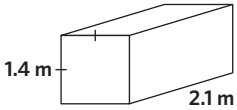
WE 2a **5.** Find the volume of each object using a formula.

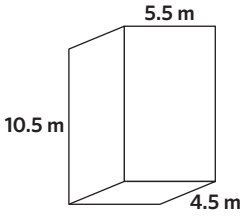
a. 

b. 

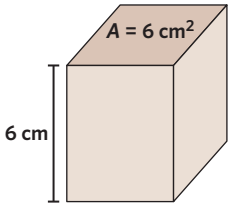
c. 

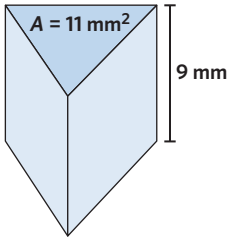
d. 

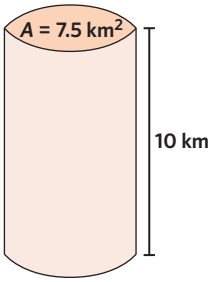
e. 

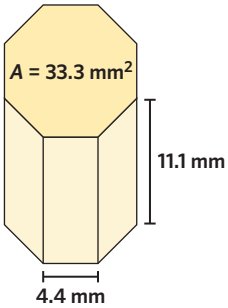
f. 

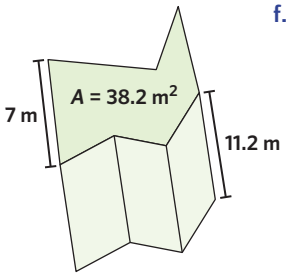
WE 2b **6.** Find the volume of each object using a formula.

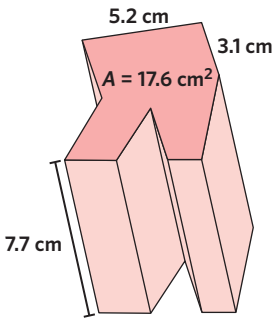
a. 

b. 

c. 

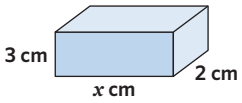
d. 

e. 

f. 

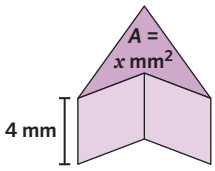
7. Find the missing value for each pronumeral.

a.



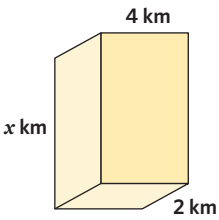
Volume = 48 cm^3

b.



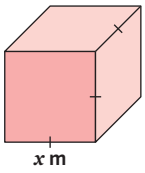
Volume = 48 mm^3

c.



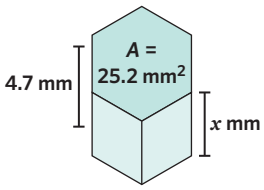
Volume = 72 km^3

d.



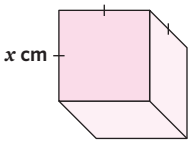
Volume = 27 m^3

e.



Volume = 126 mm^3

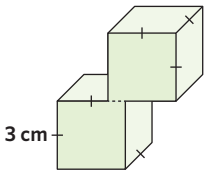
f.



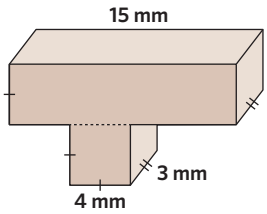
Volume = 216 cm^3

WE 3 **8.** Calculate the volume of each composite object.

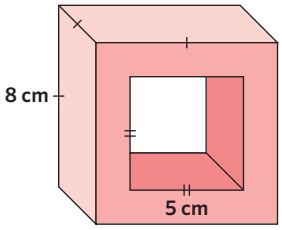
a.



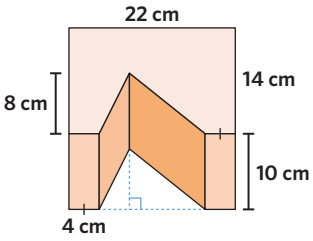
b.



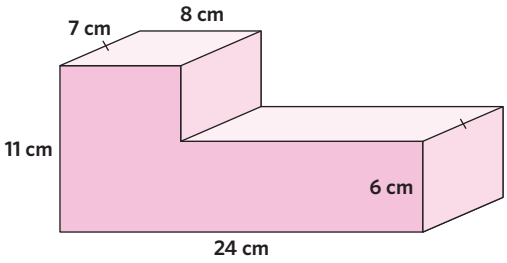
c.



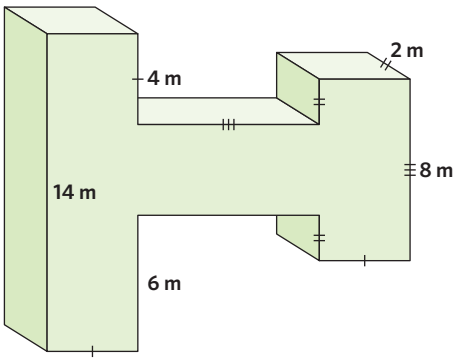
d.



e.



f.



Problem solving

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. Freddy found Sydney summers too hot and decided to install a pool in his backyard. The dimensions of Freddy's pool are 6 m × 4 m × 2.1 m. What is the volume of Freddy's pool?

10. Michael recently purchased a unique triangular prism shaped block of cheese at his local cheesemonger. What is the volume of this cheese block if the base of the cheese has an area of 14.8 cm^2 and the height of the cheese block is 5.2 cm?

11. What is the volume of Sarah's phone in cubic centimetres if the volume of her phone is 96 072.37 cubic millimetres?

12. Kenji's bedroom is rectangular and has a volume of 40.8m^3. If the length of the room is 4 m, and the width of the room is 3 m, what is the height of his room?

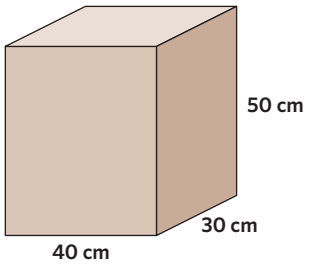
13. Natalia loves DIY projects and wants to build a bookshelf. She decides to use two pieces of wood – a short piece and a long piece. The short piece has dimensions of 4 cm × 10 cm × 25 cm and the long piece has dimensions of 50 cm × 10 cm × 5 cm. If she uses 3 short pieces and 2 long pieces of wood, what is the total volume of wood that she used for her project?

Reasoning

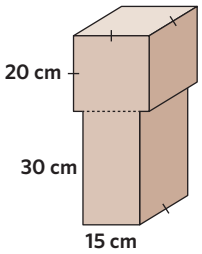
Mild	Medium	Spicy
14 (a,b,c)	14 (a,b,c), 15 (a,b,c)	All

14. Mr Johansson is an established woodwork teacher at his local high school. He is passionate about woodwork and helps students bridge the gap between design theory and real-world application.

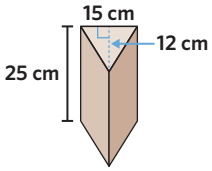
a. Mr Johansson is using a block of wood for his class today. What is the volume of this block of wood?



b. Mr Johanasson drew an outline on the original block from part **a** and then used a hacksaw to create the outline of his sculpture. What is the volume of wood that Mr Johanasson sawed off from his original block of wood?



c. Mr Johanasson used a coping saw to complete his structure. What is the volume of the finished sculpture?



d. Why might Mr Johanasson first create an outline before completing his sculpture?

15. Frederick works as a fleet manager for a global shipping company. He focuses on the road freight side of the business and ensures that the fleet of trucks are well utilised (capacity).

a. Frederick manages a fleet of trucks that carries a rectangular container with a volume of 87.5 m^3. If the width and height are both 2.5 m, what is the length of a container?

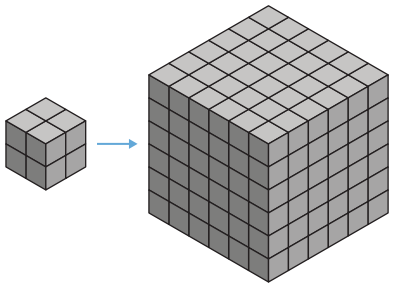
b. Innotech, a technology manufacturing company, wants to ship a parcel that has dimensions of 1 m × 0.5 m × 0.5 m. How many parcels can fit in one of the rectangular containers?

c. Frederick decided to renew and upgrade his fleet to 18-wheeler trucks. As a result, the trucks can now carry containers with dimensions of 13.6 m × 2.45 m × 2.8 m. How much more volume can the new containers load compared to the previous containers?

d. What are some things Frederick can do to minimise the environmental footprint of his fleet?

Extra spicy

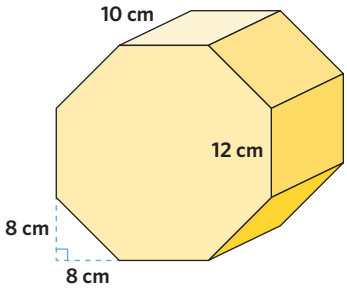
16. How many of the smaller cubes can fit inside the larger cube?



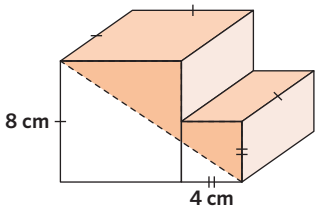
A. 3 **B.** 9 **C.** 18 **D.** 27 **E.** 36

17. A rectangular prism's length is 3 times its width and 1.5 times its height. If the volume of the prism is 162 mm^3, what is the width of the prism?

18. Calculate the volume of this regular octagonal prism.



19. Calculate the shaded volume.



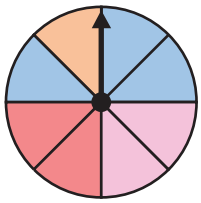
Remember this?

20. Fillipo is baking 8 batches of cookies for a fundraiser and has 2000 grams of chocolate. If each batch requires 220 grams, how much chocolate will he have left over?

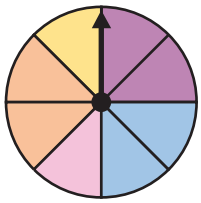
A. 24 g **B.** 176 g **C.** 240 g **D.** 1760 g

21. Which spinner shows a 50-50 chance of landing on blue?

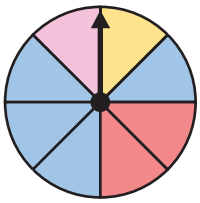
A.



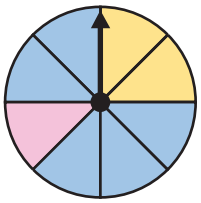
B.



C.

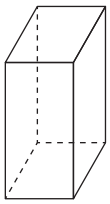


D.

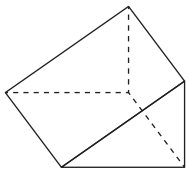


22. Which object is not a prism?

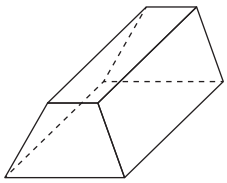
A.



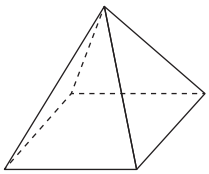
B.



C.



D.



Chapter 9 extended application

1. Jasmine has been tasked with designing packaging for a new chocolate brand called TriChoc for Winky Choc, a chocolate products manufacturer. William, the manager of Winky Choc, is obsessed with triangles and wants the chocolate bar shaped as a triangular prism, as shown below.

The prism has an equilateral triangle face. William has proposed that the box will have a width of 5 cm, and a length of 20 cm. He initially wants to sell chocolate bars weighing 400 g.

a. Given 100 g of chocolate has a volume of 140 cm^3, what is the volume of a 400 g chocolate bar?

b. The equilateral triangle faces have a height that is 0.86 times the length of its base. What is the area of each triangle face?

c. Jasmine is trying to figure out the amount of packaging material for each block of chocolate. Draw the net of the triangular prism, and determine the area of cardboard needed for the packaging.

d. What is the capacity of the planned triangular prism created by the proposed box? Is this big enough to hold the chocolate bar?

Jasmine wants to propose other designs with different shapes for the chocolate bar.

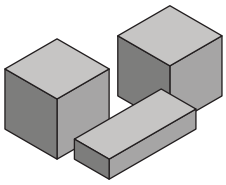
e. Draw two other box shapes of any shape, including dimensions, that would contain exactly the volume required for a 400 g chocolate bar.

f. What other things need to be considered when determining the shape and design of the box?

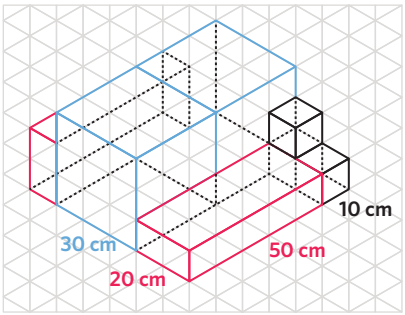
2. Patrick is stacking boxes inside of his storage unit. He has different sized boxes, shown in the table below.

Box type	Dimensions	Price per box
Rectangular box	10 cm × 20 cm × 50 cm	$3
Large cubic box	30 cm × 30 cm × 30 cm	$6
Small cubic box	10 cm × 10 cm × 10 cm	$17

He initially has two rectangular boxes, two large cubic boxes and three small cubic boxes. He is stacking them inside his storage unit that has dimensions of 60 cm × 60 cm × 60 cm.



a. Using isometric paper draw each of the types of boxes and use the scale of one isometric unit being equal to 10 cm.



b. Patrick uses an isometric drawing to demonstrate how he will pack them inside his storage space. Draw as many side or top views as you can. Colour the rectangular boxes red, the large cubes blue and the small cubes black.

c. Patrick wants to fill the remaining space in the storage unit with small boxes only. How many of the small boxes will he be able to fit in the remaining space?

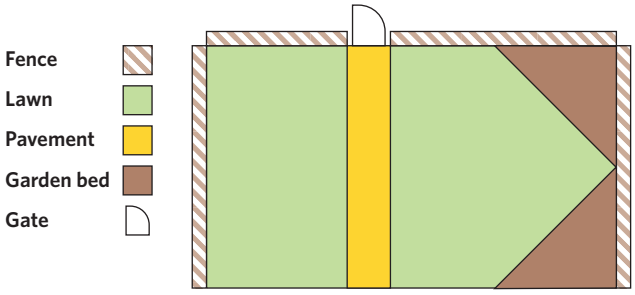
d. What is the total area of the surfaces of the boxes in the stack in part **c**?

e. Patrick has realised that using so many different boxes is very expensive. He is planning on getting another storage unit for some other boxes. How many of each box should Patrick use so that he is able to utilise all the space in the storage unit while minimising the cost of boxes used.

f. Patrick is concerned with the environmental impact of using so many boxes. Propose other ways Patrick can store his items so that he reduces the environmental impact of his actions.

3. Alanna is planning to redesign her garden. The garden will consist of a grass lawn, a stone pavement and two garden beds. It is surrounded by a wooden fence.

The design of the garden is shown below:



The width of the pavement is 80 cm, and it leads to a gate at the back of the garden. The triangular garden beds are identical and the lengths of the base and the height are the same.

The length and width of the whole garden area (not including the fences) are 10 m and 6 m.

a. Allana is filling out an online request for wooden fencing and needs to submit the length of fencing she requires. Calculate the length of fencing that Alanna needs to submit.

b. Grass lawn costs $10 per square metre. Assuming that Alanna can only buy grass lawn as whole square metres, how much will Alanna need to spend on grass?

c. Alanna is considering adding another veggie patch in addition to the garden beds to grow her own vegetables. She wants the new veggie patch to have an area of 12 m^2. State the dimensions and the perimeter of a veggie patch that will fit on the grass lawn, and ensure that the side lengths are integers.

d. Alanna is deciding whether to hire a contractor to complete the landscaping project or do it herself.

She contacts a contractor, Maria, who gives her the following quote:

- Labour costs of $50 per hour
- Purchase of materials where Maria receives a 5% discount off the original price
- Maria estimates the work will take one week, working 8 hours a day for 5 days.

If she chose to do it herself, Alanna estimates she would need to take 3 hours off work each day for three weeks in order to complete her garden. She earns $45 per hour in her regular job.

She has approximated the costs of the other materials, shown in the table below.

Item	Cost
Hardwood for fencing	$80 per metre
Soil (to a depth of 30 cm) for garden beds	$69 per cubic metre
Sandstone tiles for pavement	$25 per square metre

Alanna wants to complete her garden and save as much money as possible. State what is the chapter option and how much money Alanna can save by selecting the cheaper option.

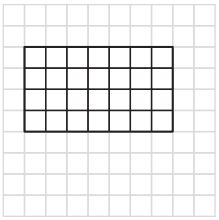
e. Alanna wants to plant vegetables in her garden that she can use in her cooking. Her favourite vegetables are cucumber, broccoli and carrot.

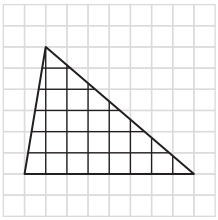
Devise a timeline of when Alanna should plant and harvest each vegetable throughout the year.

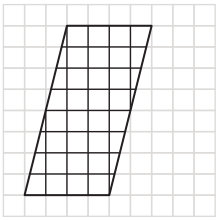
Chapter 9 review

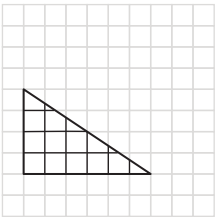
Multiple choice

1. The following four shapes are drawn on a centimetre grid. Which has an area of 24 cm^2? 9B 9C

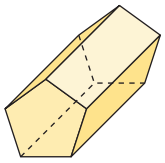
A. 

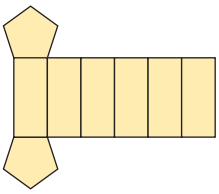
B. 

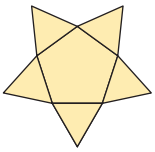
C. 

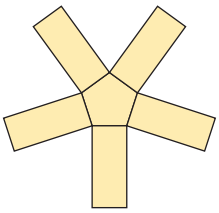
D. 

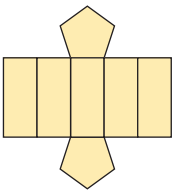
2. Which of the following is a net for the closed object below? 9D



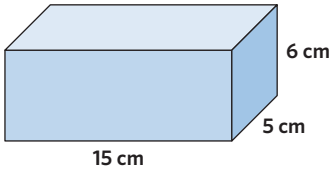
A. 

B. 

C. 

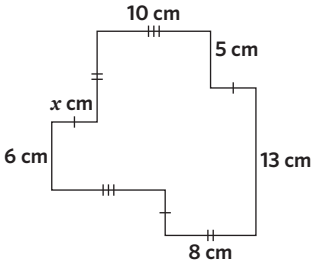
D. 

3. Calculate the volume of the following rectangular prism. 9E



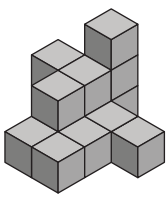
A. 27 cm^3 **B.** 75 cm^3 **C.** 390 cm^3 **D.** 450 cm^3

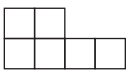
4. The following shape has a perimeter of 72 cm. State the value of x. 9A

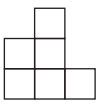


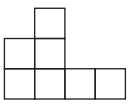
A. $x = 2$ **B.** $x = 3$ **C.** $x = 4$ **D.** $x = 5$

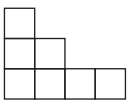
5. What is the front view of the following isometric drawing? 9D



A. 

B. 

C. 

D. 

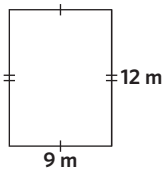
Fluency

6. Express each length in the described form. 9A

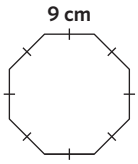
a. 95 cm + 3 km (km)
b. 0.47 km + 31 m (cm)
c. 1.28 km − 940 m (cm)
d. 650 000 mm − 0.017 km (m)

7. Calculate the perimeter of each shape in the described form. 9A

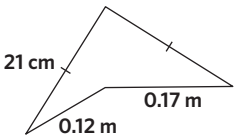
a. (m)



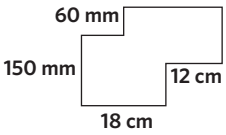
b. (cm)



c. (cm, m)

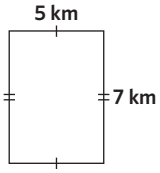


d. (mm, cm)

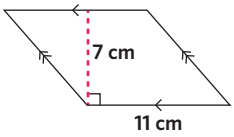


8. Calculate the area of each shape using formulas for rectangles and parallelograms. 9B

a.

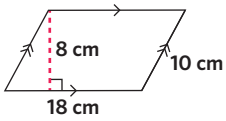


b.



c.

d.

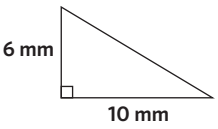


9. Convert each area to the specified units. 9B

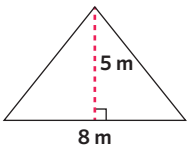
a. 39 cm^2 to mm^2
b. 530 000 m^2 to km^2
c. 42 100 000 000 cm^2 to km^2
d. 3 ha to mm^2

10. Calculate the area of each triangle. 9C

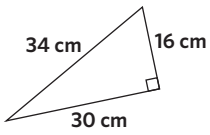
a.



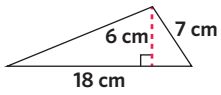
b.



c.

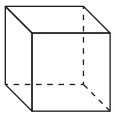


d.

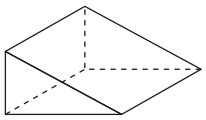


11. Draw a net for each 3D object. 9D

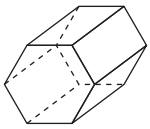
a.



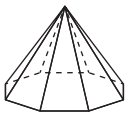
b.



c.

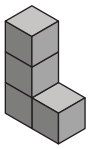


d.

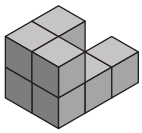


12. Draw the front, side and top view for each isometric drawing. 9D

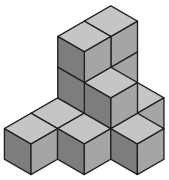
a.



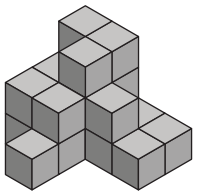
b.



c.

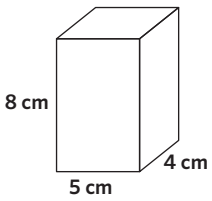


d.

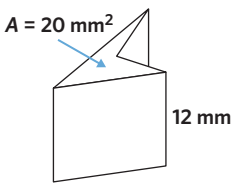


13. Calculate the volume of each object. 9E

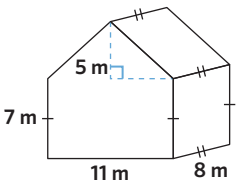
a.



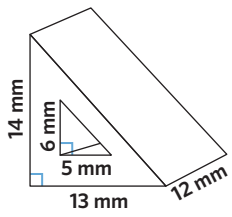
b.



c.



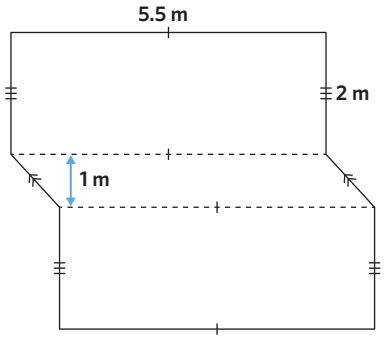
d.



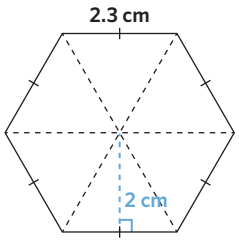
Problem solving

14. Felix and Troye competed in the long jump at their school's athletics carnival. The boys each completed two jumps. Felix's first jump measured 2.78 metres, and his second jump was 55 mm further than his first. Troye's best jump measured 284 cm. Who won? 9A

15. Leona wants to lay carpet on her bedroom floor. Her bedroom is made up of two rectangles and one parallelogram, as shown. How much will she spend in total if the carpet costs $30 per square metre? 9B



16. Sof's favourite flavour of Shapes is barbeque. The barbeque shape is made up of 6 identical triangles, as shown. Using the following diagram, calculate the area of the barbeque Shape. 9C

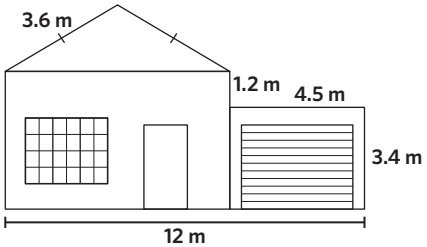


17. Milan purchased a jumbo Toblerone, which was packaged in a triangular prism shaped box. In order to recycle the box, Milan unfolded it and was left with the net for a triangular prism. Draw both the original and the unfolded Toblerone box. 9D

18. Damien's birthday cake is cylindrical. The area of the top of the cake is 300 cm^2 and the cake is 12 cm tall. If the cake is cut into 8 equal pieces, what is the volume of each piece of cake? 9E

Reasoning

19. Garret drew the front view of his house and his garage. The front view shows the main section of the house, its triangular roof and an adjoining garage.



a. Calculate the height of the front wall of the main section of the house in centimetres.

b. What is the area of the front view of the garage?

c. The top of the roof is 7.3 m above the ground. What is the area of the entire front view?

d. Garret is not sure whether it is more useful to have the unit of length as cm or m. Should construction measurements be given in cm or m? Justify your answer.

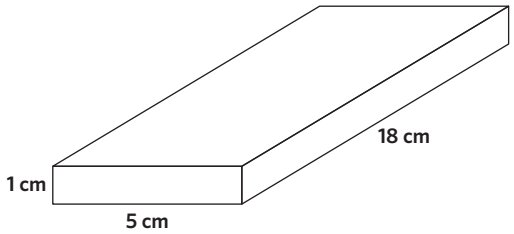
20. Claudio buys a packet of 6 pencils. Each unsharpened pencil is a hexagonal prism that is 170 mm long and the edges of its hexagonal sides are 4.8 mm in length.

a. Draw and label the lengths of one of the unsharpened pencils.

b. The area of the hexagonal face of each pencil is 60 mm^2. What is the volume of one pencil in mm^3?

c. Express the total volume of all 6 pencils in cm^3.

d. The pencils come in a rectangular prism box. When all 6 pencils are inside, how much empty space is there inside the box? Give your answer in cm^3.



e. Claudio thinks that the size and shape of the box is not appropriate. Draw a design for a different shaped box that is more suitable.

Answers

Image: Marish/Shutterstock.com

1A Relational thinking

Student practice

Worked example 1.

a. 9 b. × c. 8 d. +

Worked example 2.

a. =, Associative b. ≠

c. ≠ d. =, Commutative

Worked example 3.

a. True b. False c. False d. True

Understanding worksheet

1. a. 12 b. 8 c. 6, 6 d. 10, 7

2. a. Commutative b. Commutative
   c. Associative d. Associative

3. equals; commutative; associative; addition; division

Fluency

4. a. 8 b. 3 c. 1 d. ×
   e. 50 f. 28 g. −, + h. ×, ÷

5. a. =, Commutative b. ≠
   c. ≠ d. =, Associative
   e. ≠ f. =, Commutative
   g. =, Commutative h. ≠

6. a. True b. False c. True d. False
   e. False f. True g. True h. True

Problem solving

7. Fi and Jim run the local football club and make raspberry lemonade for their team every weekend. Fi makes her raspberry lemonade with 50 mL of cordial and 220 mL of water while Jim only uses 45 mL of cordial to make his drink. How much water does Jim need to add to the cordial to make the same amount of raspberry lemonade as Fi?

Key points

- Fi uses 50 mL of cordial in her raspberry lemonade.
- Fi also adds 220 mL of water.
- Jim uses 45 mL of cordial.
- To make the same volume of lemonade as Fi, how much water should Jim add?

Explanation

Fi:	50 mL	220 mL
Jim:	45 mL	225 mL

Fi makes 50 + 220 = 270 mL of raspberry lemonade.

Jim wants to make the same amount of lemonade, using 45 mL of cordial.

45 + □ = 270, where the missing value is the amount of water Jim needs to add.

45 + 225 = 270

Answer

Jim needs to add 225 mL of water.

8. Jon challenged Ben to a chicken nugget eating contest. Jon ate 7 packs of 12 nuggets plus another 8 nuggets while Ben ate 3 packs of 24 nuggets and another 15 nuggets. Are the judges correct if they claim that both boys came to a draw?

Key points

- Jon ate 7 packs of 12 nuggets plus another 8 nuggets.
- Ben ate 3 packs of 24 nuggets and another 15 nuggets.
- The judges claim that the boys drew. Are they right?

Explanation

Jon:	12	12	12	12	12	12	12	8	= 92
Ben:	24		24		24		15		= 87

Jon: 7 × 12 + 8 = 84 + 8
= 92

Ben: 3 × 24 + 15 = 72 + 15
= 87

92 ≠ 87

7 × 12 + 8 ≠ 3 × 24 + 15

Answer

No, the judges are not correct.

9. Fortuna is a big Formula1 fan and is explaining the rules to her friend. She says that in F1, each circuit can be made up of a different number of laps but will still cover the same total distance at the end of each race. Using the information from the table, determine if Fortuna is correct in her analysis of the F1 circuit rules.

Circuit	Number of laps	Distance per lap
Dutch Grand Prix	75	4 km
Singapore Grand Prix	60	5 km
Saudi Arabian Grand Prix	50	6 km

Key points

- Fortuna believes that all of the circuits cover the same total distance.
- The Dutch Grand Prix is 75 laps, each 4 km.
- The Singapore Grand Prix is 60 laps, each 5 km.
- The Saudi Arabian Grand Prix is 50 laps, each 6 km.
- Is Fortuna correct?

Explanation

Dutch	75	75	75	75			75 × 4 km = 300 km
Singapore	60	60	60	60	60		60 × 5 km = 300 km
Saudi Arabian	50	50	50	50	50	50	50 × 6 km = 300 km

The total distance covered by each circuit is *number of laps* × *distance of each lap*.

Dutch: 75 × 4 km

Singapore: 60 × 5 km

Saudi Arabian: 50 × 6 km

Fortuna believes that 75 × 4 = 60 × 5 = 50 × 6.

300 = 300 = 300

Answer

Yes, Fortuna is correct in her analysis of the F1 circuit rules.

10. Eugene is a stamp collector and receives a bag of stamps and a stamp display book for his birthday. Eugene takes half of the stamps and spreads them over the first 8 pages. He puts 12 rows on each page, each row containing 5 stamps. He places the remaining stamps into 5 rows of 8 stamps per page. How many pages will his remaining stamps take up?

Key points

- Eugene places half of the stamps into 12 rows of 5 stamps per page, over 8 pages.
- Eugene places the remaining stamps into 5 rows of 8 stamps per page.
- How many pages will Eugene use for the rest of stamps?

Explanation

60 stamps per page, 12 pages

$$(12 \times 5) \times 8 = 12 \times (5 \times 8)$$

8 pages, 40 stamps per page

The first half contains 12×5 stamps per page, over 8 pages.

This is a total of $(12 \times 5) \times 8$ stamps.

The second half contains 5×8 stamps per page.

Since the second half must contain the same number of stamps as the first half, there must be 12 of these pages.

The associative law states that changing the way numbers are grouped during multiplication will not change the result.

$(12 \times 5) \times 8 = 12 \times (5 \times 8)$

$480 = 480$

Answer

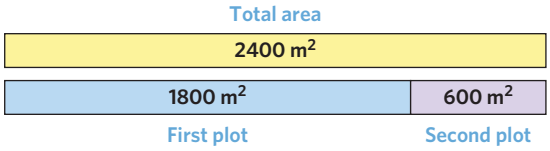
The remaining stamps will take up 12 pages.

11. Ruby was organising her orchard for apple season and needed a total area of 2400 m^2 to plant her trees. She did not have one plot big enough so decided to plant them in two separate plots. What area does the second plot need to be if the first plot she planted was 60 metres by 30 metres?

Key points

- Ruby needed 2400 m^2 to plant her trees.
- The first of two plots was 60 metres by 30 metres.
- What is the area of the second plot?

Explanation



The first plot has an area of $60 \times 30 = 1800$ m^2.

The total area Ruby needs for her trees is 2400 m^2.

This means that the total area is $1800 + \square = 2400$ m^2, where the missing value is the area of the second plot.

$1800 + 600 = 2400$.

Answer

The second plot must have an area of 600 m^2.

Reasoning

12. a. Design 1 would have 5 levels and Design 2 would have 4 levels.

b. Each design will fit 10 000 cars.

c. The parking complex will need to have 5 levels.

d. Suggested option 1: It may be more appropriate to spread the car parking locations around the shopping centre so customers do not have to walk as far from their car.

Suggested option 2: It may be more appropriate to have smaller car parks so it is easier for people to keep track of their car.

Note: There are other possible options.

13. a. The collage could have a length of 20 cm and width of 28 cm.

Note: There are many possible answers. Your answer is correct if the sum of the length and width is 48 cm.

b. The side lengths of the hexagonal design are 16 cm.

c. Equilateral triangle: Each side length would be 32 cm.

Square: Each side length would be 24 cm.

Regular octagon: Each side length would be 12 cm.

d. Suggested option 1: Sandeep should make the collage hexagonal because it is the most interesting shape.

Suggested option 2: Sandeep should make the collage rectangular because it would be more practical.

Note: There are other possible options.

Extra spicy

14. C 15. D 16. $n = 2$ 17. C

Remember this?

18. B

19. 500 students are in primary school.

20. D

1B Place value

Student practice

Worked example 1.

a. 800 b. 9000

c. 6000 d. 70 000 000

Worked example 2.

a. 7432

b. 40 000 + 300 + 10 + 2

c. 1068

d. 100 000 + 40 000 + 2000 + 900 + 60 + 1

Worked example 3.

a. < b. > c. = d. >

Worked example 4.

a. 900, 253, 189, 78, 9

b. 1099, 3467, 6342, 6432

c. 9109, 1999, 1990, 1909

d. 17 992, 117 000, 117 117, 117 719

Understanding worksheet

1. a. 1226 = 1000 + 200 + 20 + 6
   b. 2352 = 2000 + 300 + 50 + 2
   c. 3236 = 3000 + 200 + 30 + 6
   d. 4555 = 4000 + 500 + 50 + 5

2. a. 10 + 12 = 20 + 2
   b. 30 + 13 = 40 + 3
   c. 20 + 6 = 0 + 26
   d. 100 + 10 + 1 = 0 + 110 + 1

3. position; numeric; expanded; compare

Fluency

4. a. 6
   b. 2000
   c. 80
   d. 900
   e. 20 000
   f. 800 000
   g. 10 000 000
   h. 200 000

5. a. 436
   b. 3879
   c. 14 202
   d. 224 864
   e. 967
   f. 126 863
   g. 10 045 632
   h. 200 499 400

6. a. 900 + 80 + 3
   b. 7000 + 400 + 30 + 5
   c. 10 000 + 2000 + 200 + 70 + 6
   d. 8000 + 800 + 60 + 8
   e. 10 000 + 2
   f. 200 000 + 3000 + 70
   g. 1 000 000 + 700 000 + 800
   h. 900 000 000 + 30 000 000 + 7 000 000 + 20 000 + 1000 + 800 + 9

7. a. <
   b. >
   c. >
   d. =
   e. <
   f. >
   g. =
   h. <

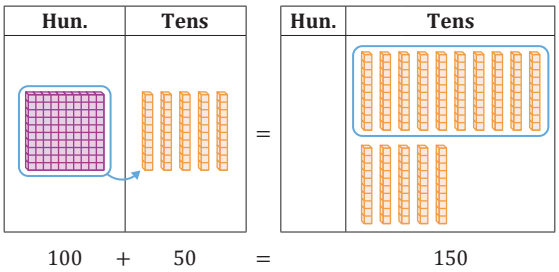
8. a. 34, 127, 900, 8764
   b. 2321, 461, 323, 120
   c. 12, 21, 221, 2221
   d. 23 899, 21 873, 20 862, 4672
   e. 3461, 3462, 9732, 152 784
   f. 128 874, 119 921, 118 996, 118 992
   g. 1 876 532, 1 876 621, 1 876 625, 1 900 000
   h. 7 654 651, 7 654 233, 7 654 221, 7 642 984

Problem solving

9. Mary has one $100 note and one $50 note and needs some change to pay for her guitar lessons. How many $10 notes will she receive if she exchanges all her money at the bank?

Key points
- Mary has one $100 note.
- She also has one $50 note.
- How many $10 notes will equal the same amount of money?

Explanation



100 + 50 = 150

Mary is exchanging the two notes for $10 notes. Find how many tens can be exchanged for each note.

100 is 10 tens.

50 is 5 tens.

10 + 5 = 15 tens.

Answer

Mary will receive fifteen $10 notes.

10. Antonia invited 34 people to her birthday party and wants to purchase party pies for her guests. She buys 3 packs of party pies where each pack contains 10 pies. Does she have enough pies so that everyone can have at least one?

Key points
- Antonia invited 34 people to her birthday party.
- She buys 3 ten packs of party pies.
- Will this be enough for all the people she invites?

Explanation

Number of people invited

Tens	Ones

>

Number of party pies

Tens	Ones

30 + 4 > 30

Antonia buys 3 ten packs, or 3 tens worth of party pies. This is equal to 30.

She invites 34 people. 30 is less than 34 so she does not have enough party pies.

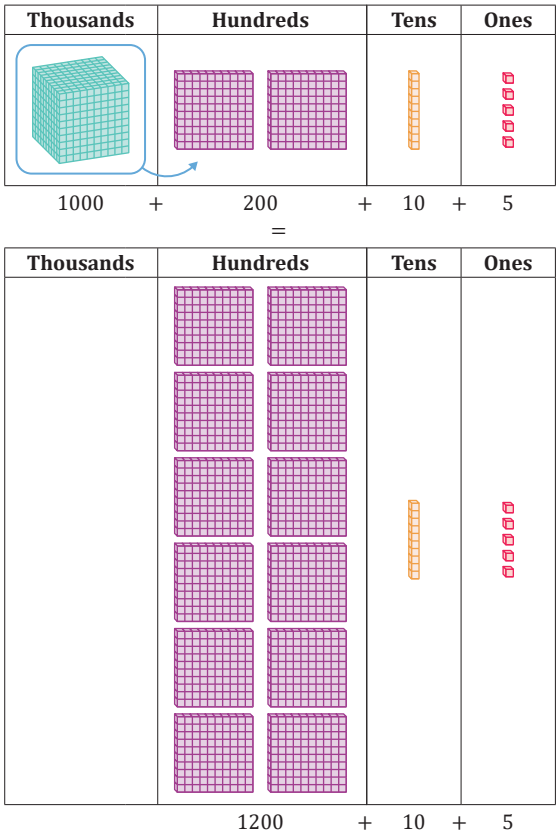
Answer

No, Antonia does not have enough pies for everyone.

11. The Oasis Deluxe Grazing Box costs $100. Hillery has a budget of $1215 to spend on gifts for her friends at her office. How many grazing boxes can she buy?

Key points
- The Grazing Box costs $100.
- Hillery has a budget of $1215.
- How many hundreds are in 1215?

Explanation



1215 contains 1 thousand and 2 hundreds.

In each thousand, there are 10 hundreds.

10 hundreds + 2 hundreds is 12 hundreds.

Answer

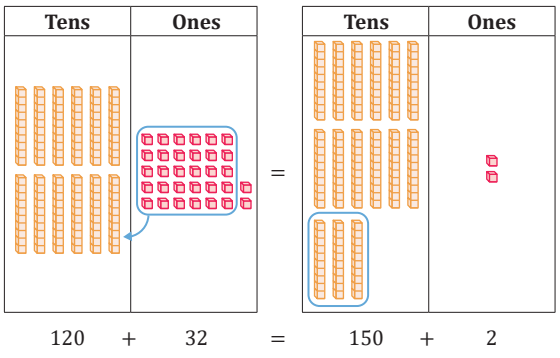
Hillery can buy 12 grazing boxes.

12. A box of donuts at Daniel's Donuts contains 10 donuts. How many boxes do they have available to sell if they have already made 12 boxes and have 32 donuts left to pack?

Key points

- The donuts are packed in boxes of 10.
- 12 boxes have been made so far.
- 32 donuts are left to pack.
- How many boxes do they have to sell in total?

Explanation



Find the number of tens in 32.

32 in its expanded form is 30 + 2.

As 30 is 3 tens, they can make 3 more full boxes with the remaining donuts.

They have already made 12 boxes, so they can sell 12 + 3 = 15 in total.

Answer

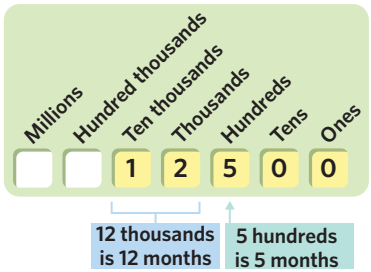
Daniel's donuts can sell 15 boxes.

13. Roger wants to transfer a total of $12 500 dollars from his spending account into his savings account over a period of several months. He sets up a direct transfer of $1000 every month and then $100 a month when his spending account drops below a thousand dollars. How many months will it take Roger to transfer **all** of his money?

Key points

- Roger wants to transfer $12 500.
- He transfers $1000 every month.
- Once the balance is below $1000, he transfers $100 every month.
- How many months will it be until Roger has transferred the full amount?

Explanation



Find how many thousands are in $12 500 to determine how many months he transfers $1000 for.

12 500 contains 1 ten thousand and 2 thousands.

10 + 2 is 12 thousands in total.

So, Roger transfers $1000 for 12 months and there will be $500 remaining.

Roger now transfers $100 every month.

500 contains 5 hundreds, so Roger transfers $100 for 5 months.

12 + 5 = 17 months.

Answer

It will take Roger 17 months to transfer all his money.

Reasoning

14. a. More people watched the AFL finals series in 2019.

b. 15 862 000, 14 335 000, 12 500 119

c. More people watch AFL in Melbourne.

d. Suggested option 1: More people might watch AFL in Melbourne because there are more Melbourne based AFL teams than Sydney based teams.

Suggested option 2: More people might watch AFL in Melbourne because people in Sydney prefer to watch rugby.

Note: There are other possible options.

15. a. 180 000 000, 130 000 000, 120 000 000, 107 000 000, 96 500 000, 95 000 000, 90 000 000, 82 000 000, 76 000 000, 75 000 000

b. Lebron James earned $6 500 000 more than Roger Federer.

c. Kevin Durant earned $105 000 000 less than Conor McGregor.

d. Suggested option 1: Soccer players might earn more because more people might watch soccer than tennis.

Suggested option 2: Soccer players might earn more because sponsors think soccer players are more marketable than tennis players.

Note: There are other possible options.

Extra spicy

16. D 17. C 18. B 19. B

Remember this?

20. D 21. D 22. B

1C Adding and subtracting whole numbers

Student practice

Worked example 1.

a. 75 b. 53 c. 181 d. 26

Worked example 2.

a. 65 b. 35 c. 363 d. 21

Worked example 3.

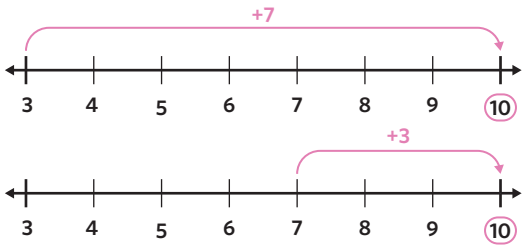
a. 89 b. 561 c. 348 d. 1063

Worked example 4.

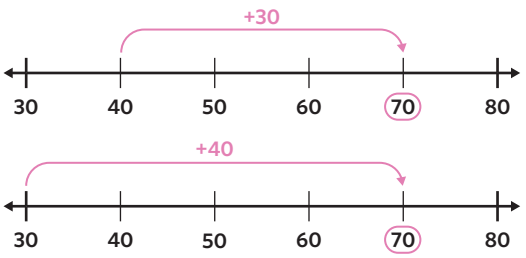
a. 43 b. 28 c. 89 d. 887

Understanding worksheet

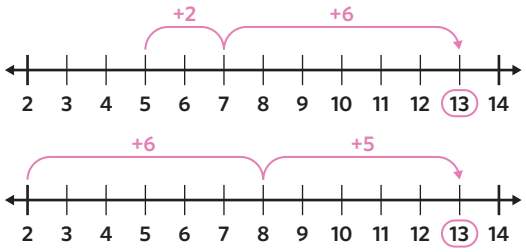
1. a.



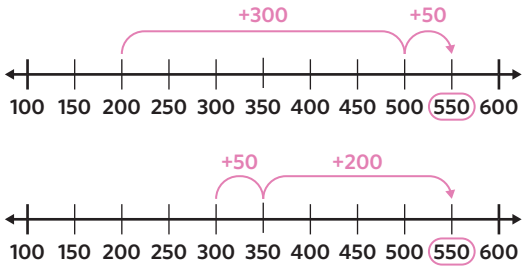
b.



c.



d.



2. a.

16		13	
10	6	10	3

16 + 13
10 + 6 + 10 + 3

b.

32		39	
30	2	30	9

32 + 39
30 + 2 + 30 + 9

c.

43		19	
40	3	10	9

43 + 19
40 + 3 + 10 + 9

d.

9	17		20
9	10	7	20

9 + 17 + 20
9 + 10 + 7 + 20

3. partitioning; addition; compensation; commutative

Fluency

4. a. 89 b. 64 c. 878 d. 26
e. 435 f. 210 g. 11 011 h. 1027

5. a. 101 b. 36 c. 471 d. 298
e. 282 f. 598 g. 1184 h. 8260

6. a. 75 b. 778 c. 21 d. 254
e. 129 f. 7093 g. 949 h. 4185

7. a. 87 b. 669 c. 291 d. 1210
e. 8932 f. 791 g. 1942 h. 11 682

8. a. 41 b. 135 c. 19 d. 272
e. 179 f. 356 g. 3436 h. 6539

Problem solving

9. On a piano, there are 52 white keys and 36 black keys.
How many keys are there in total?

Key points

- A piano has 52 white keys.
- A piano has 36 black keys.
- What is 52 + 36?

Explanation

Write each number in expanded form.

52 + 36 = 50 + 2 + 30 + 6

Regroup and simplify.

= 50 + 30 + 2 + 6
= 80 + 8
= 88

52		36	
50	2	30	6
80			8
88			

Answer

There are 88 keys in total on a piano.

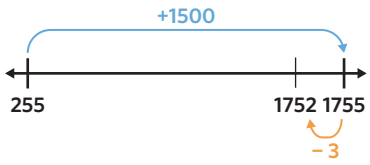
10. Phil has 255 followers on TikTok. What will Phil's total number of followers be if, after one of his TikToks goes viral, he gains 1497 followers.

Key points
- Phil has 255 followers on TikTok.
- He gains another 1497 followers.
- What is 255 + 1497?

Explanation

Adjust one of the numbers to make the calculation easier.

$$\begin{aligned} 255 + 1497 &= 255 + 1497 + 3 - 3 \\ &= 255 + 1500 - 3 \\ &= 1755 - 3 \\ &= 1752 \end{aligned}$$



Answer

Phil will have 1752 followers.

11. Steve has a 128 gigabyte iPhone. How much storage does Steve have on his phone for new content if he already has 18 gigabytes of photos and 56 gigabytes of apps stored on his phone?

Key points
- Steve has a 128 gigabyte iPhone.
- 18 gigabytes of this is taken up by photos and 56 gigabytes is taken up by apps.
- How much remaining storage does Steve have?

Explanation

Subtract 18 and 56 from 128 using the vertical algorithm. If the top digit is smaller than the sum of the bottom digits, regroup using the next place value column to the left.

$$\begin{array}{rccc} & & {\scriptstyle 11} & \\ & {\scriptstyle 0} & {\scriptstyle 1} & {\scriptstyle 18} \\ & \not{1} & \not{2} & \not{8} \\ & & 1 & 8 \\ - & & 5 & 6 \\ \hline & & 5 & 4 \end{array}$$

Answer

Steve has 54 gigabytes left on his phone.

12. The Tour de France is a 21 stage cycling race that is over 3000 kilometres long. What is the combined distance of the final three stages of the race if the given table shows the distance of each of the stages?

Stage	Distance (km)
19 – Mourenx to Libourne	227
20 – Libourne to Saint Emilion	81
21 – Chatou to Paris Champs-Elysées	109

Key points
- The 19th stage is 227 km.
- The 20th stage is 81 km.
- The 21st stage is 109 km.
- What is 227 + 81 + 109?

Explanation

We can use the vertical algorithm to sum the three distances to determine the combined distance. If the digits in a place sum to more than 9, regroup to the next place value column to the left.

Note: You can reorder the numbers in the vertical algorithm to make it easier to read.

$$\begin{array}{rccc} & & {\scriptstyle +1} & \\ & {\scriptstyle +1} & 8 & 1 \\ & 2 & 2 & 7 \\ + & 1 & 0 & 9 \\ \hline & 4 & 1 & 7 \end{array}$$

Answer

The combined distance of the final 3 stages is 417 km.

13. Gina was born on the 30th of September 2004. Emma was born exactly twenty-seven months and one week before Gina. What date is Emma's birthday?

Key points
- Gina was born on the 30th of September 2004.
- Emma was born exactly twenty-seven months and one week before Gina.
- Subtract twenty-seven months and one week from Gina's birthday to find Emma's birthday.

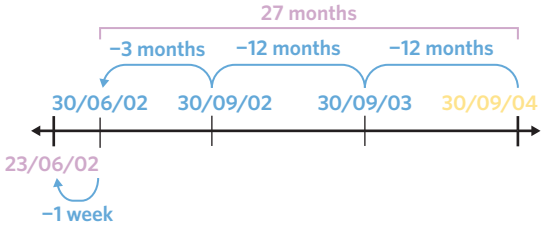
Explanation

There are 12 months in a year, so 27 months is 2 years and 3 months.

Subtracting 2 years from the 30th of September 2004 is the 30th of September 2002.

Subtracting 3 months from this is the 30th of June 2002.

Subtracting 1 week (7 days) from this is the 23rd of June 2002.



Answer

Emma's birthday is the 23rd of June 2002.

Reasoning

14. a. The Empire State Building is 119 m taller than the Eiffel Tower.
    b. The planned height of the Jeddah tower is 1001 m.
    c. The planned height for the 'Merdeka 118' is 644 m.
    d. Suggested option 1: Usher could include how much money it cost to construct each building.

    Suggested option 2: Usher could include how long it took to construct each building.

    Note: There are other possible options.

15. a. Jasper arrives at Central station at 8:44 am.
    b. Jasper will arrive 50 minutes later.
    c. The journey from Beenleigh station to Central station takes 69 minutes.
    d. Jasper would save 14 minutes by travelling on the express train.

e. Suggested option 1: Jasper could get off at an earlier station and walk to his office.

Suggested option 2: Jasper could ride a bike into work instead.

Note: There are other possible options.

Extra spicy

16.  = 1  = 8  = 5

17. 2 times 18. 9 am 19. D

Remember this?

20. B 21. C 22. D

1D Multiplying whole numbers

Student practice

Worked example 1.

a. 170 b. 7400 c. 11 000 d. 10 000

Worked example 2.

a. 78 b. 135 c. 440 d. 768

Worked example 3.

a. 792 b. 5145 c. 8316 d. 10 989

Understanding worksheet

1. a. $2 \times 7 = 7 \times 2 = 14$
   b. $8 \times 11 = 11 \times 8 = 88$
   c. $2 \times (5 \times 3) = (2 \times 5) \times 3 = 30$
   d. $4 \times (6 \times 2) = (4 \times 6) \times 2 = 48$

2. a. $5 \times 13 = (5 \times 10) + (5 \times 3)$
   b. $6 \times 58 = (6 \times 50) + (6 \times 8)$
   c. $15 \times 25 = (10 \times 20) + (10 \times 5) + (5 \times 20) + (5 \times 5)$
   d. $24 \times 36 = (20 \times 30) + (20 \times 6) + (4 \times 30) + (4 \times 6)$

3. product; commutative; distributive; algorithm

Fluency

4. a. 70 b. 280
   c. 600 d. 1100
   e. 71 000 f. 1000
   g. 1 000 000 h. 1 000 000 000

5. a. 66 b. 148 c. 378 d. 684
   e. 1064 f. 300 g. 1342 h. 2091

6. a. 120 b. 4000
   c. 860 d. 108
   e. 248 f. 14 200
   g. 1400 h. 168 000

7. a. 287 b. 117 c. 268 d. 288
   e. 3850 f. 1111 g. 3915 h. 3114

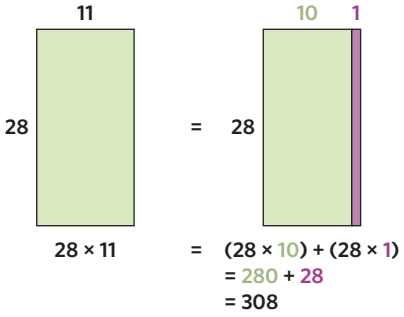
Problem solving

8. Josh Giddey was a professional basketball player for the Adelaide 36ers. In his first season at the club, he scored 11 points per game. If he played 28 games altogether, what was the total number of points he scored in his first season?

Key points
- Josh Giddey scored 11 points per game.
- He played 28 games altogether.
- How many points did he score in total?

Explanation

Multiply points per game by the number of games played.



Answer

Josh Giddey scored 308 points in his first season.

9. How many kilometres did Sarah run last year if she ran 4 kilometres per day and last year was not a leap year?

Key points
- Sarah ran 4 kilometres per day.
- Last year had 365 days.
- How many kilometres did Sarah run last year?

Explanation

Multiply 4 by 365 to find the total number of kilometres Sarah ran last year.

```
   +2 +2
    3  6  5
×         4
-----------
1  4  6  0
```

Answer

Sarah ran 1460 kilometres last year.

10. Millie works as a lumberjack and can chop up 27 fire logs every hour. If she works 38 hours a week, how many fire logs does she chop up in a week?

Key points
- Millie can chop up 27 fire logs every hour.
- She works 38 hours a week.
- How many fire logs does she chop up in a week?

Explanation

Multiply the number of logs chopped per hour by the number of hours worked in a week.

```
  +1
  +5
   38
×  27
-----
 +1
  266
+ 760
------
 1026
```

Answer
Millie can chop up 1026 logs in a week.

11. Kathy's new car allows her to drive 18 kilometres per litre of fuel. Last month, Kathy used 133 litres of fuel. What distance did Kathy drive last month?

Key points
- Kathy can drive 18 kilometres per litre of fuel.
- Kathy used 133 litres of fuel last month.
- Find how many kilometres Kathy drove last month.

Explanation
Multiply kilometres per litre of fuel by the number of litres of fuel used.

$$\begin{array}{rcccc} & & {\scriptstyle +2} & {\scriptstyle +2} & \\ & & 1 & 3 & 3 \\ \times & & & 1 & 8 \\ \hline & 1 & 0 & 6 & 4 \\ + & 1 & 3 & 3 & 0 \\ \hline & 2 & 3 & 9 & 4 \end{array}$$

Answer
Kathy drove 2394 kilometres last month.

12. How many seconds are there in Aria's dancing lesson if her dancing lessons last an hour?

Key points
- Aria's dancing lessons last an hour.
- How many seconds are in an hour?

Explanation
There are 60 minutes in an hour and 60 seconds in a minute. The number of seconds in an hour will be 60 multiplied by 60.

Method 1:

$$\begin{aligned} 60 \times 60 &= (6 \times 10) \times (6 \times 10) \\ &= (6 \times 6) \times (10 \times 10) \\ &= 36 \times 100 \\ &= 3600 \end{aligned}$$

Method 2:

$$\begin{array}{rr} & 60 \\ \times & 60 \\ \hline & 00 \\ + & 3600 \\ \hline & 3600 \end{array}$$

Answer
There are 3600 seconds in Aria's dancing lesson.

Reasoning

13. a. The ticket sales revenue was $273 830.
b. $28 832 was spent on meat pies.
c. The tickets sold for a total of $15 000 000.
d. Suggested option 1: Yes it is fair. The price is high because the grand final is a popular event.
Suggested option 2: No it is not fair. Many fans won't be able to afford the price to see their favourite team.
Note: There are other possible options.

14. a. 25 Australians would buy 675 kilograms of clothing each year.
b. 864 tonnes of textiles and clothing are dumped every day.
c. The 'Worn Wear' initiative reduces the carbon footprint by 280 000 kilograms.
d. Suggested option 1: Yes, wearing second hand clothing means less clothing is dumped in landfills.
Suggested option 2: Not necessarily, some new clothing is made environmentally friendly.
Note: There are other possible options.

Extra spicy

15. D 16. 18 and 18 17. 39 483 18. 527 040

Remember this?

19. C 20. B 21. C

1E Dividing whole numbers

Student practice

Worked example 1.

a. 24 b. 40 c. 73 d. 99

Worked example 2.

a. 52 b. 230 r 2 or $230\frac{2}{15}$
c. 204 d. 600 r 24 or $600\frac{24}{30}$

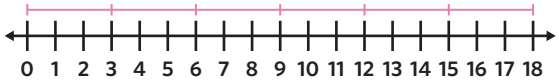
Understanding worksheet

1. a. $12 \div 2 = 6$ b. $16 \div 4 = 4$
c. $28 \div 7 = 4$ d. $30 \div 7 = 4\ r\ 2$

2. a. 6 contains 2 groups of size 3

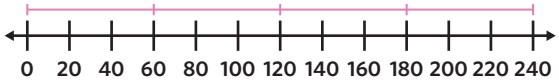
$6 \div 3 = 2$

b. 18 contains 6 groups of size 3



$18 \div 3 = 6$

c. 240 contains 4 groups of size 60



$240 \div 60 = 4$

d. 17 contains 5 groups of size 3 with a remainder of size 2

$17 \div 3 = 4\ r\ 2$

3. inverse; equal; dividend; divisor; quotient

Fluency

4. a. 10 b. 18 c. 7 d. 9
   e. 11 f. 8

5. a. 3 b. 23 c. 40 d. 11
   e. 30 f. 46 g. 52 h. 73

6. a. 16 b. 12 r 1 or $12\frac{1}{8}$
   c. 73 r 1 or $73\frac{1}{2}$ d. 795 r 2 or $795\frac{2}{4}$
   e. 6003 f. 264 r 3 or $264\frac{3}{15}$
   g. 204 r 1 or $204\frac{1}{12}$ h. 1219 r 8 or $1219\frac{8}{24}$

7. a. 13 b. 14
   c. 18 d. 14
   e. 34 r 17 f. 98
   g. 129 r 2 h. 546 r 6

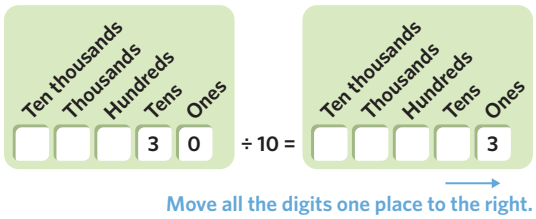
Problem solving

8. Rod's Fruit Market was having a sale on punnets of strawberries. Mary spent $30 on 10 punnets of strawberries. How much did each punnet cost?

Key points
- Mary spent $30 on 10 punnets of strawberries.
- What is 30 ÷ 10?

Explanation

We are dividing by 10, so move each digit one place value to the right.



30 ÷ 10 = 3

Answer

Each punnet costs $3.

9. Tyler works in construction and has 15 different work uniforms that he cycles through as his work gets quite dirty. His uniform consists of a pair of coveralls and boots. If he has 3 pairs of boots, how many different coveralls does he own?

Key points
- Tyler has 15 different work uniforms.
- His uniform consists of a pair of coveralls and boots.
- He has 3 pairs of boots.
- How many pairs of coveralls results in 15 different uniform combinations?

Explanation

Tyler has 15 different combinations of coveralls and boots. If he has 3 pairs of boots and 15 possible uniform combinations, then he must have 15 ÷ 3 pairs of coveralls to pair with each set of boots.

15 ÷ 3 = 5

Answer

Tyler owns 5 pairs of coveralls.

10. Ellen's tree nursery has 3 separate seedling areas. How many rows of fruit trees does she have in her nursery in total if each seedling area contains 3500 fruit trees with 50 trees in a row?

Key points
- Ellen's tree nursery has 3 areas.
- Each area contains 3500 fruit trees.
- The trees are planted in rows of 50.
- How many of these rows does Ellen have across all 3 areas?

Explanation

One seedling area has 3500 fruit trees in rows of 50. We can divide 3500 by 50 to give us the number of rows.

We can use the vertical algorithm to perform the division. If the dividend is not divisible by the divisor, use a 0 as the placeholder and divide the next 2 digits by the divisor.

$$50\overline{)3500}\quad 0070$$

3500 ÷ 50 = 70

There are three seedling areas in total, so we multiply the number of trees in each area by 3.

70 × 3 = 210

Answer

Ellen has 210 rows of fruit trees on her farm.

11. Ryan has 3500 strawberry plants on his farm. How many strawberry plants does Calvin have on his farm if Ryan's farm has 7 times as many strawberry plants as Calvin?

Key points
- Ryan's farm has 3500 strawberry plants.
- Ryan's farm has 7 times as many strawberry plants as Calvin's farm.
- What is 3500 ÷ 7?

Calculation

We can use the vertical algorithm to perform the division. If the dividend is not divisible by the divisor, use a 0 as the placeholder and divide the next 2 digits by the divisor.

$$7\overline{)3500}\quad 0500$$

3500 ÷ 7 = 500

Answer

Calvin has 500 strawberry plants on his farm.

12. Riley packs his strawberries so that there are 45 punnets of strawberries in a carton and 16 cartons on a pallet. If there were 5800 punnets in total, how many full pallets were there?

Key points
- There are 45 punnets in 1 carton.
- There are 16 cartons in 1 pallet.
- There were 5800 punnets in total.
- How many pallets is 5800 punnets?

Explanation

There are 45 punnets in 1 carton and 16 cartons on a pallet, so multiplying 45 by 16 will determine the number of punnets per pallet.

45 × 16 = 720

So there are 720 punnets of strawberries per pallet.

Riley has 5800 punnets in total so we need to divide this by the number of punnets per pallet (720) to determine how many pallets we would have in total.

We can use the vertical algorithm to perform the division.

$$720\overline{)5800}\quad 8\ r\ 40$$

$5800 \div 720 = 8\ r\ 40$.

We only want full pallets so we can ignore the remaining 40 punnets.

Answer

There were 8 full pallets.

Reasoning

13. a. Paul earns $280 per day.
    b. Paul earns $35 per hour.
    c. 14 is the maximum number of weeks Paul can take off.
    d. Suggested option 1: Yes, if you select the number of destinations and how many days you want to spend at each destination.

    Suggested option 2: No, if you plan to see more destinations or spend more days in each destination.

    Note: There are other possible options.

14. a. Nancy walked 12 000 steps around the oval each day.
    b. Nancy walked 12 km each day.
    c. She walked for 33 days.
    d. Suggested option 1: You could take public transportation and walk to the stations instead of driving.

    Suggested option 2: You could take regular walking breaks during the day for 5 or 10 minutes.

    Note: There are other possible options.

Extra spicy

15. D 16. D 17. True 18. 0

Remember this?

19. D 20. C 21. C

1F Order of operations

Student practice

Worked example 1.

a. 5 b. 1 c. 12 d. 29

Worked example 2.

a. 55 b. 9 c. 1 d. 20

Worked example 3.

a. $2 + 4 \times 2 = 10$ b. $48 \div (6 \times 2) = 4$

c. $6 \times (3 + 4 + 1) = 48$ d. $(14 + 16) \div (5 \times 2) = 3$

Understanding worksheet

1. a. $(4 \times 7) + 8 = 36$
   b. $-4 + (9 \times 5) = 41$
   c. $9 \times (2 + 3) - 5 = 40$
   d. $2 \times [48 \div (4 \times 1) + 9] = 42$

2. a. 8, yes b. 8, yes c. 58, no d. 31, no

3. brackets; multiplication; addition; left; right

Fluency

4. a. Addition b. Multiplication
   c. Multiplication d. Subtraction
   e. Subtraction f. Addition
   g. Division or multiplication h. Subtraction

5. a. 15 b. 11 c. 31 d. 49
   e. 12 f. 58 g. 5 h. 42

6. a. 12 b. 8 c. 6 d. 44
   e. 32 f. 0 g. 53 h. 363

7. a. $3 + 10 \div 2 = 8$ or $3 + 10 \times \frac{1}{2} = 8$
   b. $33 - 4 \times 5 = 13$
   c. $3 \times 15 \div 5 = 9$
   d. $(44 \div 11) \times (2 \times 3) = 24$
   e. $(3 + 17) \times (9 - 4) = 100$
   f. $4^2 \div (8 \times 2) = 1$

Problem solving

8. Bobby is participating in a charity cycle where he has to ride 500 kilometres total over five weeks. He rode 120 kilometres per week for the first three weeks. How many kilometres does he have left to reach his 500 kilometre target?

Key points

- Bobby wants to ride 500 km in five weeks.
- He rode 120 km per week for the first 3 weeks.
- How far does Bobby still have to ride?

Explanation

Bobby has already ridden 3×120 km. He wants to ride 500 km in total. This means that the distance left to ride is $500 - 3 \times 120$ km.

Evaluate the multiplication first.

$500 - 3 \times 120 = 500 - 360$

Evaluate the subtraction.

$= 140$

Answer

Bobby has 140 km left to reach his 500 km target.

9. Brock recently started playing Pokemon and has collected 15 cards that were in playing condition. His dad bought an additional 88 cards from an online seller. Upon arrival, Brock realised that only a quarter of these purchased cards were in playing condition. How many cards in playing condition does Brock now have?

Key points

- Brock has collected 15 cards in playing condition?
- Brock's dad bought 88 more cards online.
- A quarter of the cards bought online were in playing condition.
- How many of Brock's cards are in playing condition?

Explanation

Brock already has 15 cards in playing condition. 88 ÷ 4 cards bought online are also in playing condition. This means that the total number of cards in playing condition is 15 + 88 ÷ 4.

Evaluate the division first.

$15 + 88 \div 4 = 15 + 22$

Evaluate the addition.

$= 37$

Answer

Brock now has 37 cards in playing condition.

10. How many points did Kai score in the state championship game? In the game, Kai made 4 free throws which are worth one point each, 5 two-pointers, and 4 three-pointers.

Key points

- Kai made 4 free throws, worth one point each.
- Kai made 5 two-pointers.
- Kai made 4 three-pointers.
- What is the total number of points Kai scored?

Explanation

Kai scored 4×1 points from free throws, 5×2 points from two-pointers, and 4×3 points from three-pointers. This means that the total number of points Kai scored is $4 \times 1 + 5 \times 2 + 4 \times 3$.

Evaluate each of the multiplications first.

$4 \times 1 + 5 \times 2 + 4 \times 3 = 4 + 10 + 12$

Evaluate the additions.

$= 26$

Answer

Kai scored 26 points in the state championship game.

11. John and Louis are buying some pants at their favourite store. The pants they wish to purchase cost $145 but the store is running a promotion where 3 pairs cost $330. How much money did they save altogether if John and Louis bought 3 pairs of pants each.

Key points

- John and Louis want to buy pants that originally cost $145.
- 3 pairs of pants cost $330.
- John and Louis each buy 3 pairs of pants.
- In total, how much money did John and Louis save from the promotion?

Explanation

3 pairs of pants without the promotion would cost 3×145 dollars. With the promotion, John and Louis will each save $3 \times 145 - 330$ dollars.

The total savings between them is equal to $(3 \times 145 - 330) \times 2$ in dollars.

Evaluate the multiplication inside the brackets first.

$(3 \times 145 - 330) \times 2 = (435 - 330) \times 2$

Evaluate the subtraction inside the brackets.

$= 105 \times 2$

Evaluate the multiplication outside of the brackets.

$= 210$

Answer

John and Louis saved $210 altogether.

12. Jessica owns an e-commerce business selling candles. Help Jessica calculate how much more profit she made this year than last year. It costs her $8 to make a candle and she sells each one for $17. Last year, Jessica sold 1100 candles and this year she tripled the amount she sold. (Note: profit = selling price − cost price)

Key points

- Each candle costs $8 to make and sells for $17.
- Jessica sold 1100 candles last year.
- This year, Jessica sold three times as many candles as last year.
- How much more profit did Jessica make this year compared to last year?

Explanation

The profit made from each candle sale is $17 - 8$ dollars. Last year, Jessica made $1100 \times (17 - 8)$ dollars in profit. This year, she made three times this: $3 \times 1100 \times (17 - 8)$ dollars in profit.

The difference in profit between the two years is $3 \times 1100 \times (17 - 8) - 1100 \times (17 - 8)$ dollars.

Evaluate the subtractions inside the brackets first.

$3 \times 1100 \times (17 - 8) - 1100 \times (17 - 8)$
$= 3 \times 1100 \times 9 - 1100 \times 9$

Evaluate the multiplications.

$= 29\,700 - 9900$

Evaluate the subtraction.

$= 19\,800$

Answer

Jessica made $19 800 more profit this year than last year.

Reasoning

13. a. Terry and Linda would pay $3410 for their solar panel system.
b. The solar panel system would cost $3350 under this method.
c. The amount of energy currently produced over a two-week period is 70 kW more than the amount of energy produced over a two-week period in 5 years' time.
d. Suggested option 1: Yes, is important for Australians to consider renewable energy sources due to the ongoing climate crisis.

Suggested option 2: No, solar power can be more unreliable than energy from fossil fuels.

Note: There are other possible options.

14. a. 131 kg of the rescued food was edible.
b. Secondbite reduced greenhouse gas emissions by 38 000 tonnes in 2019.
c. Mary's family's donations helped to distribute 471 kg of food.
d. Suggested option 1: The class can make sure not to purchase too much food in order to minimise food waste.

Suggested option 2: The class can cycle or walk to school instead of driving.

Note: There are other possible options.

Extra spicy

15. E 16. D 17. −24 680 18. B

Remember this?

19. D 20. B 21. C

1G Estimating and rounding

Student practice

Worked example 1.

a. 50 b. 400 c. 3000 d. 20 000

Worked example 2.

a. 120 b. 60

c. 6 d. 24 000 000

Worked example 3.

a. 210 b. 100 c. 500 d. 30 000

Understanding worksheet

1. a. 40 b. 200 c. 900 d. 1600

2. a. 7①3 ≈ 710 b. ③62 ≈ 400
   c. ⑦599 ≈ 8000 d. ⑨950 ≈ 10 000

3. estimates; rounding; lead; zeros; overestimate; underestimate

Fluency

4. a. 80 b. 830 c. 700 d. 8000
   e. 1000 f. 0 g. 95 000 h. 10 000

5. a. 140 b. 130
   c. 2000 d. 500
   e. 2 f. 2000
   g. 1000 h. 10 000 000

6. a. 5 b. 300 c. 240 d. 20
   e. 4100 f. 6000 g. 1000 h. 11 000

7. a. i) 280
   ii) 301
   iii) Underestimate
   b. i) 140
   ii) 133
   iii) Overestimate
   c. i) 100
   ii) 106
   iii) Underestimate
   d. i) 14 000
   ii) 9810
   iii) Overestimate
   e. i) 60
   ii) 58
   iii) Overestimate
   f. i) 200
   ii) 220
   iii) Underestimate
   g. i) 11 000
   ii) 12 198
   iii) Underestimate
   h. i) 1000
   ii) 1060
   iii) Underestimate

8. a. A b. B c. A d. B
   e. B f. B g. A h. B

Problem solving

9. Over a busy weekend, 1137 sausages were sold at a Bunnings sausage sizzle. To the nearest thousand, how many sausages were sold?

Key points
- 1137 sausages were sold.
- Round the number of sausages sold to the nearest 1000.

Explanation

Digit being rounded: 1

Critical digit: 1

1 < 5, so the digit being rounded should not change.

All smaller place values should be zero.

1137 ≈ 1000

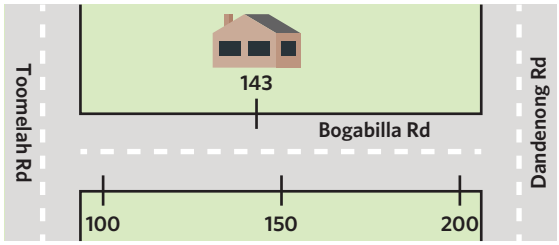
Answer

Rounded to the nearest thousand, the number of sausages sold was 1000.

10. Betty's address is 143 Bogabilla Road. 100 Bogabilla Road joins Toomelah Road, while 200 Bogabilla Road joins Dandenong Road. Does Betty live closer to Toomelah Road or Dandenong Road?

Key points
- Betty lives at 143 Bogabilla Road.
- 100 Bogabilla Road joins Toomelah Road.
- 200 Bogabilla Road joins Dandenong Road.
- Which road, out of Toomelah Road or Dandenong Road, is Betty's house closer to?

Explanation



Round 143 to the nearest 100.

Digit being rounded: 1

Critical digit: 4

4 < 5, so the digit being rounded should not change.

All smaller place values should be zero.

143 ≈ 100

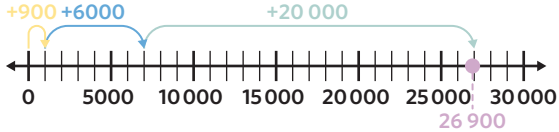
Answer

Betty lives closer to Toomelah Road.

11. Richard and Julia bought a new two bedroom apartment. To furnish their apartment, they spent $935 in the first week, $5900 in the second week and $15 320 in the third week. Approximate how much Richard and Julia spent in total using lead digit rounding.

Key points
- $935 was spent in the first week.
- $5900 was spent in the second week.
- $15 320 was spent in the third week.
- Approximately how much will Richard and Julia spend?

Explanation



First week:
Lead digit: 9
Critical digit: 3
935 ≈ $900

Second week:
Lead digit: 5
Critical digit: 9
5900 ≈ $6000

Third week:
Lead digit: 1
Critical digit: 5
15 320 ≈ $20 000
900 + 6000 + 20 000 = $26 900

Answer

Using lead digit rounding, Richard and Julia spent $26 900 in total.

12. The year 7 and 8 cohorts at Bridge Academy have a hockey excursion together. Use lead digit rounding to find how many buses are required to transport all 393 students if 48 students fit in a bus.

Key points

- 393 students need to be transported by bus.
- 48 students fit in a bus.
- How many buses are required?

Explanation

Students:
Lead digit: 3
Critical digit: 9
393 ≈ 400 students

Number of students that fit in a bus:
Lead digit: 4
Critical digit: 8
48 ≈ 50 students

Number of buses required:
400 ÷ 50 = 8

Answer

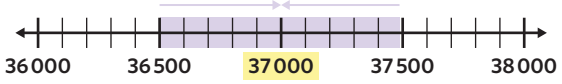
8 buses are required to transport all the students.

13. What is the least and what is the greatest number of people that could live in Salisbury if its population is 37 000 when rounded to the nearest 1000?

Key points

- Rounding the population of Salisbury to the nearest 1000 gives 37 000.
- What is the lowest population of Salisbury possible?
- What is the highest population of Salisbury possible?

Explanation



Lowest population:

When rounding the population, the thousands place would increase by 1 to become 7, so it must be 6.

The hundreds place would be the critical digit. The lowest critical digit that causes the digit being rounded to increase is 5.

The tens and ones place must be 0 for the population to be the lowest possible.

36 500 ≈ 37 000 ✓

Highest population:

When rounding the population, the thousands place would stay the same (7).

The hundreds place would be the critical digit. The highest critical digit that does not cause the digit being rounded to increase is 4.

The tens and ones place must be 9 for the population to be the highest possible.

37 499 ≈ 37 000 ✓

Answer

Lowest: 36 500

Greatest: 37 499

Reasoning

14. a. Claudia thinks 9000 people attended the match.
b. Jimmy thinks 20 000 people attended the match.
c. Jimmy's estimate is more accurate. His rounding makes his estimate more accurate whilst Claudia's rounding makes her estimate less accurate.
d. Suggested option 1: Claudia and Jimmy could have worked together and used the average of their results.
Suggested option 2: Claudia and Jimmy could have asked a friend to count at every gate.
Note: There are other possible options.

15. a. Candidate B received approximately 26 700 votes.
b. Candidate A received an average of 1500 votes in the council sub-regions in Carina.
c. Candidate B was the most popular candidate and won by 1000 votes.
d. Suggested option 1: Candidate A won in 3 of the 4 regions, yet lost the vote when rounding to the nearest 1000.
Suggested option 2: Candidate A won by a large number of votes in Carina and Hemmant but this did not count as a win when rounding to the nearest 1000.
Note: There are other possible options.

Extra spicy

16. 480 cm
Note: Answers between 450 and 500 cm are also acceptable.

17. D 18. B 19. 100 hours

Remember this?

20. C 21. C 22. $21 413

Chapter 1 extended application

1. a. Leonard needs to make 10 cupcakes.
b. Josiah and Leonard will have made 528 baked goods.
c. Josiah and Leonard 192 baked goods for the second week.
d. Josiah and Leonard sold 64 baked goods.
e. Suggested option 1: If the weather forecast is good, they should make the same amount of food as week 2.
Suggested option 2: If the weather forecast is bad, they should make the same amount of food as they sold in week 3.
Note: There are other possible options.

2. a. Sonia scored 1530 points.
b. Phillippe scored 126 points.
c. Sonia's final score is 102.
d. Phillippe's final score is 1890.

e. Suggested option 1: Aim for the green boxes as they are worth the most number of points.

Suggested option 2: Aim for the yellow boxes as they are worth the second most number of points, are bigger than the green boxes, and are further from the edge.

Note: There are other possible options.

3. a. Mark makes $197 that day.
   b. Mark makes approximately $980 per week.
   c. Mark makes approximately $52 000 per year.
   d. Suggested option 1: It might not be accurate because Mark could take some weeks off work instead of working every week of the year.

   Suggested option 2: It might not be accurate because Mark might have a lot of variation in the amount he earns each week.

   Note: There are other possible options.

Chapter 1 review

Multiple choice

1. A 2. B 3. B 4. B 5. D

Fluency

6. a. True b. False c. True d. True

7. a. 20 b. 700 c. 800 000 d. 40 000

8. a. 97, 114, 899, 1004
   b. 20 100, 20 030, 20 026, 4554
   c. 108 929, 108 992, 114 231, 141 540
   d. 54 831 527, 54 831 275, 54 831 257, 54 813 572

9. a. 65 b. 131 c. 796 d. 1080

10. a. 44 b. 207 c. 122 d. 236

11. a. 136 b. 472 c. 370 d. 8189

12. a. 18 r 4 or $18\frac{4}{6}$ or $18\frac{2}{3}$ b. 20 r 5 or $20\frac{5}{8}$
    c. 63 d. 26 r 3 or $26\frac{3}{16}$

13. a. Multiplication
    b. Multiplication
    c. Subtraction
    d. Division or multiplication

14. a. 22 b. 34 c. 96 d. 91

15. a. 160 b. 2500 c. 0 d. 8990

16. a. 5 b. 10 c. 6000 d. 11 000

Problem solving

17. Nick is aiming to run 42 kilometres in a single week. He ran 5 kilometres on each of the first 6 days. How many kilometres does he need to run on the last day?

Key points
- Nick is aiming to run 42 km in 7 days.
- Nick ran 5 kilometres per day for the first 6 days.
- How many kilometres does he have left to run on the last day?

Explanation

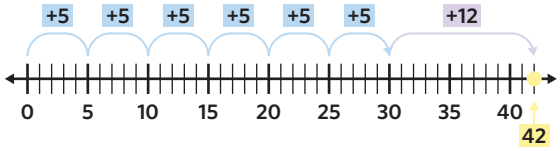
Distance run in first 6 days:

$5 \times 6 = 30$ km

Nick wants to run 42 km in total.

$30 + \square = 42$

$30 + 12 = 42$



Answer

Nick needs to run 12 km on the last day.

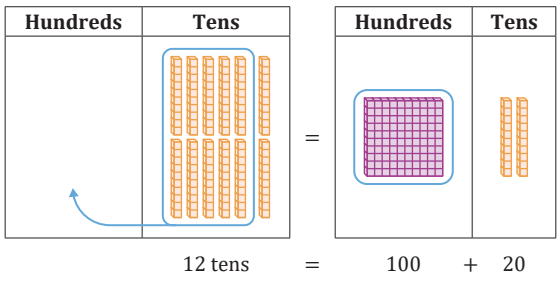
18. Cindy is buying banh mis for her friends at work. Banh mis cost $10 each and Cindy brought $115 to the office. If she has 11 coworkers, does she have enough money to buy banh mis for all of them, as well as herself?

Key points
- A banh mi costs $10.
- Cindy wants to buy banh mis for herself and 11 of her coworkers.
- Cindy brought $115 into the office.
- Does Cindy have enough money?

Explanation

Cindy wants to buy 12 banh mis in total, each of which cost $10.

12 tens can be regrouped to 1 hundred and 2 tens.



The banh mis will cost Cindy $120.

Cindy only has $115 so she does not have enough money.

Answer

No, Cindy does not have enough money.

19. Zoe just uploaded a photo on Instagram and gained 245 followers. How many followers did she have before uploading the photo if she now has 2123 followers?

Key points
- Zoe just gained 245 followers.
- She now has 2123 followers.
- How many followers did she have before she gained the followers?

Explanation

Subtract 245 from 2123 using the vertical algorithm. If the top digit is smaller than the bottom digits, regroup using the next place value column to the left.

$$\begin{array}{rcccc} & & 10 & 11 & \\ & 1 & \cancel{0} & \cancel{1} & 13 \\ & \cancel{2} & \cancel{1} & \cancel{2} & \cancel{3} \\ - & & 2 & 4 & 5 \\ \hline & 1 & 8 & 7 & 8 \end{array}$$

Answer

Zoe had 1878 followers before uploading the photo.

20. Giannis Antetokounmpo won MVP in the 2018–19 NBA season, averaging 13 rebounds per game. How many rebounds did he secure in total if he played 72 games?

Key points

- Giannis Antetokounmpo averaged 13 rebounds per game.
- He played 72 games.
- How many rebounds did he get in total?

Explanation

Multiply the number of rebounds per game by the number of games.

$$\begin{array}{rr} & 72 \\ \times & 13 \\ \hline & 216 \\ + & 720 \\ \hline & 936 \end{array}$$

Answer

Giannis Antetokounmpo secured 936 rebounds in total.

21. James Harden came second in MVP voting in the 2018–19 NBA season. He attempted a total of 1014 three-point shots in 78 games. How many three-point shots did he attempt, on average, per game?

Key points

- James Harden attempted 1014 three-point shots.
- He played 78 games.
- How many three-point shots did he attempt per game?

Explanation

Divide the total number of three-point attempts by the number of games using short division.

$$\begin{array}{r} 0\ \ 0\ \ 1\ \ 3 \\ 78\overline{)1\ \ 0\ \ 1^{23}4} \end{array}$$

Answer

James Harden attempted 13 three-point shots per game.

22. Paul George came third in MVP voting in the 2018–19 NBA season. He scored 28 points per game in 77 games. How many more points would he have needed to have scored 2500 points in total?

Key points

- Paul George averaged 28 points per game.
- He played 77 games.
- For him to have scored 2500 points in total, how many additional points would he have had to score?

Explanation

Paul George scored a total of 28×77 points. Subtract this amount from 2500 to find how many additional points he would have needed to score to score 2500 points total.

$2500 - 28 \times 77 = ?$

Evaluate the multiplication.

$$\begin{array}{rr} & {\scriptstyle +1} \\ & {\scriptstyle +5} \\ & 7\ 7 \\ \times & 2\ 8 \\ \hline {\scriptstyle +1} & \\ & 6\ 1\ 6 \\ + & 1\ 5\ 4\ 0 \\ \hline & 2\ 1\ 5\ 6 \end{array}$$

$2500 - 28 \times 77 = 2500 - 2156$

Evaluate the subtraction.

$$\begin{array}{rcccc} & & 4 & 9 & 10 \\ & 2 & \cancel{5} & \cancel{0} & \cancel{0} \\ - & 2 & 1 & 5 & 6 \\ \hline & & 3 & 4 & 4 \end{array}$$

$2500 - 28 \times 77 = 344$

Answer

Paul George would have needed to score 344 more points.

23. It took Harry Styles 96 minutes to complete all the questions in lesson 1D. Round to the nearest 10 minutes to estimate how long it took Harry Styles, on average, to complete each of the 25 questions.

Key points

- Harry Styles took 96 minutes to complete lesson 1D.
- There are 25 questions in lesson 1D.
- Round to the nearest 10 minutes to find the average time it took Harry Styles to complete each question.

Explanation

To find the average time, divide the rounded total time by the number of questions.

Total time:

Lead digit: 9

Critical digit: 6

$96 \approx 100$ minutes.

Average time:

$100 \div 25 = 4$

Answer

It took Harry Styles an average of 4 minutes to complete each question in lesson 1D.

Reasoning

24. a. Anthony, Duncan, Hannah, Ben, Jeremy, Rex

b. 'Hannah' would have to be used 4 more times.

c. 'Duncan', 'Anthony' and 'Rex' are used 114 times.

d. These names are used in the book approximately 190 times.

e. Suggested option 1: Yes, the names in this textbook should be inclusive of everyone.

Suggested option 2: No, the names do not affect the maths.

Note: There are other possible options.

25. a. They would each need to pay $550 000.

b. Yes, Jack and Dani will be able to afford a three bedroom house in Fitzroy.

c. Milly and Liam already have $441 000 saved.

d. Suggested option 1: 3 years

Suggested option 2: 10 years

Note: There are other possible options.

2A Factors, multiples, and divisibility

Student practice

Worked example 1.

a. 1, 2, 7, 14

b. 1, 3, 11, 33

c. 1, 2, 4, 5, 10, 20, 25, 50, 100

d. 1, 2, 4, 7, 8, 14, 28, 56

Worked example 2.

a. 6, 12, 18, 24, 30

b. 12, 24, 36, 48, 60

c. 90, 180, 270, 360, 450

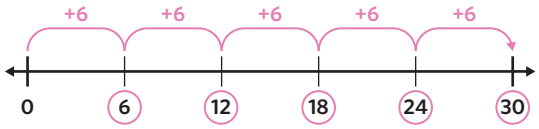
d. 17, 34, 51, 68, 85

Worked example 3.

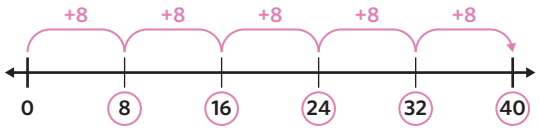
		Divisible by							
	Number	2	3	4	5	6	8	9	10
a.	6445	✗	✗	✗	✓	✗	✗	✗	✗
b.	5124	✓	✓	✓	✗	✓	✗	✗	✗
c.	75 990	✓	✓	✗	✓	✓	✗	✗	✓
d.	436 852	✓	✗	✓	✗	✗	✗	✗	✗

Understanding worksheet

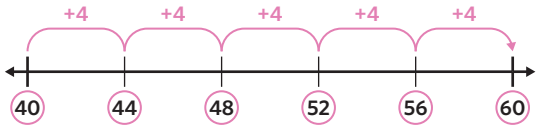
1. a.



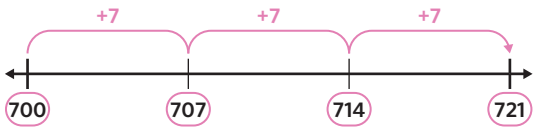
b.



c.



d.



2. a.

1650 is divisible by...				
(2)	(3)	4	(5)	(6)

b.

8748 is divisible by...				
(3)	(4)	5	(9)	10

c.

10 432 is divisible by...				
(2)	3	(4)	5	10

d.

51 840 is divisible by...				
(3)	(4)	(6)	(8)	(10)

3. factors; multiples; division; divisibility; divisible

Fluency

4. a. 1, 2, 4, 8
   b. 1, 5, 25
   c. 1, 2, 11, 22
   d. 1, 31
   e. 1, 3, 5, 9, 15, 45
   f. 1, 2, 4, 8, 16, 32, 64
   g. 1, 2, 5, 7, 10, 14, 35, 70
   h. 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

5. a. 10, 20, 30, 40, 50
   b. 8, 16, 24, 32, 40
   c. 9, 18, 27, 36, 45
   d. 11, 22, 33, 44, 55
   e. 25, 50, 75, 100, 125
   f. 120, 240, 360, 480, 600
   g. 19, 38, 57, 76, 95
   h. 91, 182, 273, 364, 455

6. a. 8 b. 24 c. 18 d. 49
   e. 63 f. 48

7.

		Divisible by							
		2	3	4	5	6	8	9	10
a.	6785	✗	✗	✗	✓	✗	✗	✗	✗
b.	16 740	✓	✓	✓	✓	✓	✗	✓	✓
c.	39 123	✗	✓	✗	✗	✗	✗	✓	✗
d.	58 617	✗	✓	✗	✗	✗	✗	✓	✗
e.	853 104	✓	✓	✓	✗	✓	✓	✗	✗
f.	2 086 997	✗	✗	✗	✗	✗	✗	✗	✗

8. a. Whole number, because the last digit is even.
   b. Whole number with a remainder, because the last digit is not equal to 0 or 5.
   c. Whole number, because the last digit is 0.
   d. Whole number, because the last 2 digits are divisible by 4.
   e. Whole number with a remainder, because the sum of the digits is not divisible by 3.
   f. Whole number, because it is even and the sum of the digits is divisible by 3.
   g. Whole number, because the last 3 digits are divisible by 8.
   h. Whole number with a remainder, because the sum of the digits is not divisible by 9.

Problem solving

9. Bailey is a greyhound and is competing in the Como Park greyhound race. How far has Bailey run at the completion of the first, second, third and fourth lap of a 600 m track?

Key points

- The track is 600 m.
- What are the first 4 multiples of 600?

Explanation

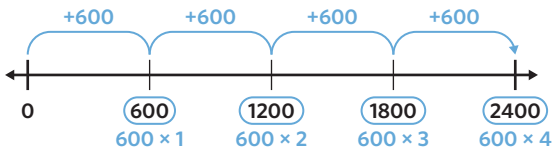
Multiply 600 by 1 to 4 to find the total distance Bailey has run after each lap.

Lap 1: $600 \times 1 = 600$

Lap 2: $600 \times 2 = 1200$

Lap 3: $600 \times 3 = 1800$

Lap 4: $600 \times 4 = 2400$



Answer

Lap 1: 600 m

Lap 2: 1200 m

Lap 3: 1800 m

Lap 4: 2400 m

10. There are 56 students in the Skinfield High's choir. The music teacher wants to arrange the students into a maximum of 7 rows. How many possible sets of dimensions can the music teacher use to arrange the choir if each row needs to have an equal number of students?

Key points

- There are 56 students in the choir.
- The maximum number of rows is 7.
- What are all the factor pairs that equal 56?

Explanation

Order the formations in *rows* × *columns*.

Stop when the next formation has more than 7 rows.

$1 \times 56 = 56$

$2 \times 28 = 56$

$4 \times 14 = 56$

$7 \times 8 = 56$

Answer

There are 4 possible formations.

11. Carmen is training for an open water swimming race. In preparation, she completes laps between two buoys in the bay and each lap is 250 m. If her GPS watch lists the cumulative distance at the end of each lap, what are all the distances her watch displays after eight laps?

Key points

- Each lap is 250 m.
- Her watch lists the cumulative distance at the end of each lap.
- What are the first 8 multiples of 250?

Explanation

Multiply 250 by 1 to 8 to find the distance Carmen has swum after each lap.

Lap 1: $250 \times 1 = 250$

Lap 2: $250 \times 2 = 500$

Lap 3: $250 \times 3 = 750$

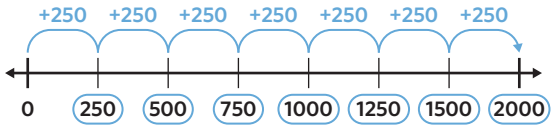
Lap 4: $250 \times 4 = 1000$

Lap 5: $250 \times 5 = 1250$

Lap 6: $250 \times 6 = 1500$

Lap 7: $250 \times 7 = 1750$

Lap 8: $250 \times 8 = 2000$



Answer

The distances displayed by Carmen's watch after each lap are: 250 m, 500 m, 750 m, 1000 m, 1250 m, 1500 m, 1750 m, 2000 m.

12. Mary wants to divide all her savings of $856 784 equally between her four children so that each child receives a whole dollar amount. Without performing any calculation, state whether this is possible and why.

Key points

- Mary has savings of $856 784.
- She is dividing it equally between her four children.
- She wants each child to receive a whole dollar amount.
- Is $856 784 divisible by 4 into a whole number?

Explanation

Use the divisibility test for 4.

A number is divisible by 4 if the last 2 digits form a number that is divisible by 4.

856 784

84 is divisible by 4, so Mary can divide her savings equally between her four children.

Answer

Mary can split her savings equally between her children because the last two digits form a number that is divisible by 4.

13. Mel is designing a rectangular enclosure for reptiles with an area of 24 m^2 at the Territory Wildlife Park. She also wants the enclosure to have the smallest possible perimeter. What are the most suitable dimensions for the enclosure if each side needs to be a whole number of metres?

Key points

- The reptile enclosure is 24 m^2.
- She wants the enclosure to have the smallest possible perimeter.
- Which factor pair of 24 has the smallest sum?

Explanation

Write the factor pairs in the form of length × width.

Stop once the next factor pair includes factors that have already been identified.

$1 \times 24 = 24$

$2 \times 12 = 24$

$3 \times 8 = 24$

$4 \times 6 = 24$

Calculate the perimeter for each factor pair. The formula for perimeter is Perimeter = 2 × length + 2 × width.

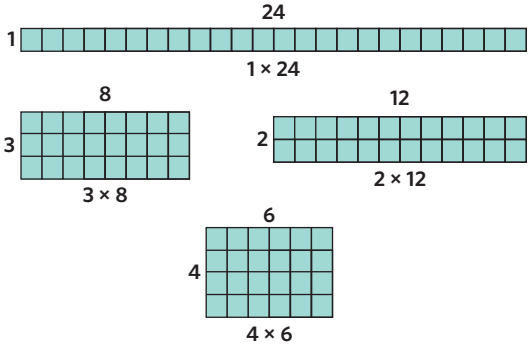
$(2 \times 1) + (2 \times 24) = 2 + 48 = 50$ m

$(2 \times 2) + (2 \times 12) = 4 + 24 = 28$ m

$(2 \times 3) + (2 \times 8) = 6 + 16 = 22$ m

$(2 \times 4) + (2 \times 6) = 8 + 12 = 20$ m

The smallest perimeter is 20 m, with dimensions 4 m × 6 m.



Answer

The most suitable dimension is 4 m × 6 m.

Reasoning

14. a. Possible dimensions: 1 × 48, 2 × 24, 3 × 16, 4 × 12, 6 × 8
    b. The 1 × 48 and 3 × 16 options do not meet Sergio's design criteria.
    c. 16 diamonds.
    d. Suggested option 1: Juan might want to consider how durable the ring will be.
    Suggested option 2: Juan might want to consider if the ring comes with a warranty.
    Note: There are other possible options.

15. a. No
    b. The numbers on Jack's list are: 13, 26, 39, 52, 65, and 78.
    c. Jack can travel on 14 different boats.
    d. 9:30 am, 12:00 am, 2:30 pm
    e. Suggested option 1: Jack might prefer the view he gets travelling by boat.
    Suggested option 2: It might be cheaper to travel by boat than plane.
    Note: There are other possible options.

Extra spicy

16. D 17. A 18. D 19. B

Remember this?

20. A 21. D 22. C

2B Index notation

Student practice

Worked example 1.

a. 5^2 b. $8 \times 8 \times 8 \times 8$
c. 2^4 d. $18 \times 18 \times 9 \times 9 \times 9$

Worked example 2.

a. 64 b. 16 c. 200 d. 100 000

Worked example 3.

a. 97 b. 89 c. 3 d. 70

Understanding worksheet

1. a. 5 b. 3 c. 6 d. 16

2. a. 6×6 b. 81
   c. 2^5 d. $5 \times 5 \times 5$

3. index notation; base; index; expanded

Fluency

4. a. 7^2 b. 3^4 c. 10^3 d. 5^6
   e. 200^3 f. 2^7 g. 1^8 h. $100\,000^2$

5. a. 12×12
   b. $8 \times 8 \times 8$
   c. $2 \times 2 \times 2 \times 2 \times 2$
   d. $80 \times 80 \times 80 \times 80$
   e. $5 \times 5 \times 5 \times 10 \times 10$
   f. $4 \times 4 \times 4 \times 4 \times 8 \times 8 \times 8$
   g. $11 \times 11 \times 11 \times 11 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2$
   h. $100 \times 100 \times 15 \times 15 \times 15 \times 6$

6. a. True b. True c. False d. False
   e. True f. False g. False h. True

7. a. 49 b. 27 c. 125 d. 10 000
   e. 32 f. 128 g. 6 400 000 h. 968

8. a. 135 b. 80 c. 40 d. 1536
   e. 202 f. 124 g. 79 910 h. 0

Problem solving

9. Hamlet is preparing his veggie garden for the pumpkin season and plants five pumpkin seeds. He expects that each seed will grow five large pumpkins. How many pumpkins does Hamlet expect to grow?

Key points
- Hamlet plants 5 pumpkin seeds.
- Each seed is expected to grow 5 pumpkins.
- How many pumpkins will Hamlet have in total?

Explanation

Seed 1 Seed 2 Seed 3 Seed 4 Seed 5

Each of the 5 seeds will produce 5 pumpkins.
There will be 5^2 pumpkins in total.
$5^2 = 5 \times 5$
$= 25$

Answer
Hamlet can expect to grow 25 pumpkins.

10. The equator is the imaginary line around the middle of planet Earth. Written as a whole number, what is the length of the earth's equator if it is equal to 4×10^4 kilometres?

Key points
- The equator has a length of 4×10^4 kilometres.
- What is 4×10^4 expressed as a whole number?

Explanation
Write the power in expanded form.
$4 \times 10^4 = 4 \times (10 \times 10 \times 10 \times 10)$
Evaluate the multiplications inside the brackets.
$= 4 \times 10\,000$
Evaluate the final multiplication.
$= 40\,000$

Answer
The equator has a length of 40 000 km.

11. Lady Macbeth is a famous fitness influencer. Lady Macbeth sold 250 tickets for a meet and greet. She also gave free tickets to 7 of her followers. How many people in total attended the meet and greet if each of the 7 followers invited 7 of their own friends too?

Key points
- Lady Macbeth sold 250 tickets.
- Lady Macbeth gave free tickets to 7 of her followers.
- Each of the 7 followers also brought along 7 of their friends.
- How many people went to the meet and greet?

Explanation

Represent the situation as an expression.

Lady Macbeth sold 250 tickets: 250

She gave away 7 free tickets to her followers: $250 + 7$

Each of the 7 followers also brought 7 friends: $250 + 7 + 7^2$

Express the power in expanded form.

$= 250 + 7 + 7 \times 7$

Evaluate the multiplication.

$= 250 + 7 + 49$

Evaluate the addition.

$= 306$

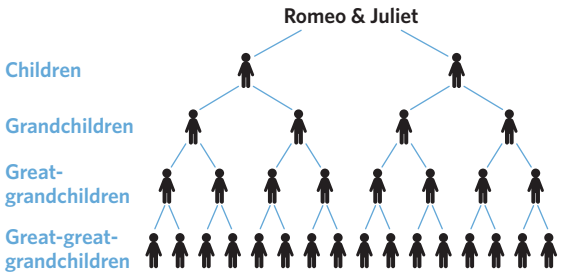
Answer

306 people attended the meet and greet.

12. Romeo and Juliet had two children. Each of their children then had two children of their own. This pattern continued with each generation of the family. How many great-great-grandchildren do Romeo and Juliet have?

Key points
- Romeo and Juliet had two children.
- Their children each had two children of their own.
- This pattern continued.
- What is the total number of their great-great-grandchildren?

Explanation



Romeo and Juliet have:

2 children

2^2 grandchildren

2^3 great-grandchildren

2^4 great-great-grandchildren

$2^4 = 2 \times 2 \times 2 \times 2$

$= 16$

Answer

Romeo and Juliet have 16 great-great-grandchildren.

13. Lady Montague is preparing a family photo album. There are six people in her family and she plans to include six individual photos per family member. However, one of her sisters requested that Lady Montague only include 3 photos of her. What is the total number of photos in the album if it also includes 7 photos of the family dog?

Key points
- There are six people in the family and each person should have 6 photos in the photo album.
- Lady Montague's sister has requested to only have 3 photos.
- There are also 7 photos of the family dog.
- How many photos are there in the album?

Explanation

Represent the situation as an expression.

Six people have six photos each: 6^2

Lady Montague's sister has 3 less photos: $6^2 - 3$

There are also 7 photos of the dog: $6^2 - 3 + 7$

Express the power in expanded form.

$6^2 - 3 + 7 = 6 \times 6 - 3 + 7$

Evaluate the multiplication.

$= 36 - 3 + 7$

Evaluate the addition and subtraction.

$= 40$

Answer

There are 40 photos in the album.

Reasoning

14. a. The largest power Othello can create is 8^6.

b. The power that Othello can make with a value closest to 100 is 3^4.

c. 2^4 and 4^2 are equal.

Note: There are other possible options.

d. Othello could create a schedule so he spends the same amount of time on each subject.

Othello could complete his maths homework on Tuesdays and Thursdays and complete his other homework on Mondays, Wednesdays and Fridays.

Note: There are other possible options.

15. a. The approximate distance between Mercury and Mars is 170 000 000 km.

b. Jupiter is further away from Mercury.

c. Venus is 41 400 000 km closer to Mercury than Earth is to Mercury.

d. The approximate distance between Mercury and Uranus is 2 800 000 000 km.

e. Suggested option 1: They would need to consider the safety of the aircraft.

Suggested option 2: They may need to consider their effect on the environment.

Note: There are other possible options.

Extra spicy

16. 4 and 61 17. C 18. $x = 19$ 19. A

Remember this?

20. C 21. C 22. B

2C Prime and composite numbers

Student practice

Worked example 1.

a. Composite
b. Prime
c. Prime
d. Composite

Worked example 2.

a. Not coprime
b. Coprime
c. Coprime
d. Not coprime

Worked example 3.

a. 42 is divisible by: 2, 3 and 6
b. 142 is divisible by: 2
c. 117 is divisible by: 3 and 9
d. 279 is divisible by: 3 and 9

Understanding worksheet

1. a. Composite
b. Prime
c. Prime
d. Composite

2. a. Composite
b. Composite
c. Prime
d. Composite

3. factors; prime number; composite numbers; coprime

Fluency

4. a. Prime
b. Prime
c. Prime
d. Composite
e. Composite
f. Prime
g. Composite
h. Composite

5. a. 2, 3, 5, 7, 11, 13
b. 29, 31
c. 37, 41, 43, 47, 53
d. 53, 59, 61
e. 61, 67, 71, 73, 79, 83
f. 83, 89, 97, 101, 103, 107, 109

6. a. Coprime
b. Not coprime
c. Coprime
d. Not coprime
e. Coprime
f. Coprime
g. Not coprime
h. Coprime

7. a. 99 is divisible by 3 and 9 therefore it is a composite number.
b. 122 is divisible by 2 therefore it is a composite number.
c. 312 is divisible by 2, 3, 4, 6, 8 therefore it is a composite number.
d. 975 is divisible by 3 and 5 therefore it is a composite number.
e. 279 is divisible by 3 and 9 therefore it is a composite number.
f. 632 is divisible by 2, 4, 8 therefore it is a composite number.
g. 1161 is divisible by 3 and 9 therefore it is a composite number.
h. 3081 is divisible by 3 therefore it is a composite number.

Note: There are other possible solutions.

Problem solving

8. Elton has 25 Pokémon cards. How can he place them into four groups so that each group contains a prime number of cards?

Key points
- Elton has 25 Pokémon cards.
- He is partitioning the cards into 4 groups.
- How can he partition the cards into 4 groups with a prime number of cards in each group?

Explanation

List the prime numbers from 1–25.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Select 4 numbers from the list that sum to 25.

$2 + 3 + 7 + 13 = 25$

Answer

Elton can split the cards into groups of 2, 3, 7 and 13.

Note: There are other possible solutions.

9. Jesse is donating some of her clothes to charity. She has 83 pieces of clothing and plans to share them equally between a group of charities. Will she be able to share the clothes in the way that she plans?

Key points
- Jesse has 83 pieces of clothing.
- She wants to share them equally between a group of charities.
- Can she split 83 pieces of clothing into equal groups?

Explanation

Jesse can only share 83 pieces of clothing equally if it is a composite number.

83 is not divisible by any number from 1–10.

83 only has two factors, 1 and itself.

List the factor pairs of 83.

$1 \times 83 = 83$

83 only has two factors: it is a prime number.

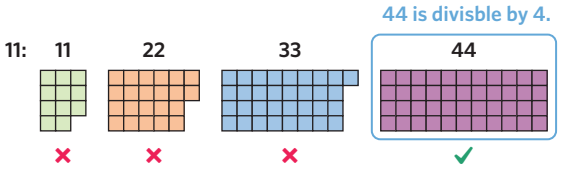
Answer

No, Jesse cannot split the clothing into equal groups.

10. Rae wanted to share her pack of Tim Tams between herself and 3 friends but noticed that each pack only contained 11 biscuits. How many packs will she need to make sure that each person receives an equal amount of TimTam biscuits?

Key points
- Rae wanted to share a pack of Tim Tams between 4 people.
- Each pack of Tim Tams contains 11 biscuits.
- Each person will receive the same number of Tim Tams.
- How many packs does she need?

Explanation



List the multiples of 11, to find the total number of Tim Tams Rae will have as she adds each additional packet.

11, 22, 33, 44, 55.

Use the divisibility test for 4.

A number is divisible by 4 if the last 2 digits form a number that is divisible by 4.

11 is divisible by 4. ✗

22 is divisible by 4. ✗

33 is divisible by 4. ✗

44 is divisible by 4. ✓

44 is the fourth multiple of 11.

Answer

Rae needs 4 packets of Tim Tams to be able to share the biscuits equally.

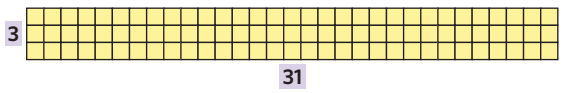
11. Eileen baked 93 cookies for her school bake sale and wanted to pack them into equal bags. How could Eileen package the cookies, so that there are no cookies left over?

Key points

- Eileen has 93 cookies to package.
- She has to share them into equal packs.
- What are 2 factors of 93 other than 1 and 93?

Explanation

$3 \times 31 = 93$



List the factor pairs and multiplications.

$1 \times 93 = 93$

$3 \times 31 = 93$

Due to the commutative law 3×31 can make 3 groups of 31, or 31 groups of 3.

Answer

Eileen could share the cookies into 3 bags of 31 cookies, or 31 bags of 3 cookies.

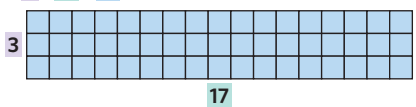
12. Ronaldo plays soccer for his local club, he played in every match this season and scored a total of 51 goals. Is it possible that Ronaldo kicked the same number of goals in each match, if there are more than 4 and less than 25 matches in a season?

Key points

- Ronaldo played every match this soccer season.
- He scored 51 goals.
- There are more than 4 games and less than 25 in a season.
- Is it possible he kicked an equal number of goals in each match that he played?

Explanation

$3 \times 17 = 51$



Identify any numbers that can easily be ruled out.

51 is odd so it is not divisible by any even numbers.

2, 4, 6, 8, 10 ✗

Use the divisibility tests for the remaining numbers.

3: If the sum of the digits is divisible by 3.

$5 + 1 = 6$ ✓

Divide 51 by 3 to find if it is between 4 and 25 matches.

$51 \div 3 = 17$

Answer

Yes it is possible that Ronaldo scored an equal number of goals in each match.

Reasoning

13. a. The emergence cycle of cicadas (7, 13 and 17) are all prime numbers.
    b. In Australia cicadas are next expected to emerge in 2024.
    c. The population of birds peaks at the same time as the emergence of North American cicadas every 85 years.
    d. Suggested option 1: They may need to consider specific weather cycles.

    Suggested option 2: Fluctuations in soil temperature.

    Note: There are other possible options.

14. a. Tane and Emily should select the 1 and 2 cards.
    b. Tane and Emily should select the 3 and 5 cards.
    c. There are 4 possible solutions.
    d. Tane is incorrect because the product of any number and 2 is an even number.
    e. Suggested option 1: Is the sum of all the cards a prime or composite number?

    Suggested option 2: Why is the product of any two cards not a composite number?

    Note: There are other possible options.

Extra spicy

15. 42 16. No 17. 13 18. A

Remember this?

19. A 20. B 21. 2 times

2D Prime factorisation

Student practice

Worked example 1.

a. $2^2 \times 7$ b. 2×5^2 c. $3^3 \times 5$ d. $2^3 \times 5^2$

Worked example 2.

a. 2×11 b. 2^4

c. $2 \times 2 \times 2 \times 7$ d. $2^2 \times 5^2$

Worked example 3.

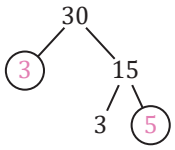
a. 1, 2, 4, 13, 26, 52

b. 1, 3, 9, 11, 33, 99

c. 1, 2, 3, 6, 7, 14, 21, 42

d. 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

Understanding worksheet

1. a.


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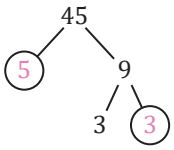
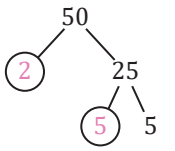
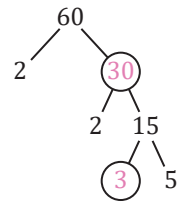


c.


d.


2. a. The composite factors of 16 that can be formed using two of its prime factors.

$2 \times 2 = 4$

The composite factors of 16 that can be formed using three of its prime factors.

$2 \times 2 \times 2 = 8$

The composite factors of 16 that can be formed using four of its prime factors.

$2 \times 2 \times 2 \times 2 = 16$

b. The composite factors of 27 that can be formed using two of its prime factors.

$3 \times 3 = 9$

The composite factors of 27 that can be formed using three of its prime factors.

$3 \times 3 \times 3 = 27$

c. The composite factors of 12 that can be formed using two of its prime factors.

$2 \times 2 = 4$ $2 \times 3 = 6$

The composite factors of 12 that can be formed using three of its prime factors.

$2 \times 2 \times 3 = 12$

d. The composite factors of 18 that can be formed using two of its prime factors.

$2 \times 3 = 6$ $3 \times 3 = 9$

The composite factors of 18 that can be formed using three of its prime factors.

$2 \times 3 \times 3 = 18$

3. prime factorisation; prime; factor tree; composite

Fluency

4. a. 2^3
b. $3^2 \times 5$
c. $2^2 \times 3^2 \times 5$
d. 3×5^3
e. $2^2 \times 3^3 \times 5^2$
f. $2^4 \times 3 \times 5^2 \times 7$
g. $2^2 \times 3 \times 5^3 \times 7 \times 11$
h. $3 \times 5 \times 7^3 \times 11^2 \times 13^2$

5. a. 2×7 b. 3×5
c. 2×19 d. $3 \times 3 \times 3 \times 3$
e. $3 \times 5 \times 7$ f. $2 \times 2 \times 2 \times 3 \times 3$

6. a. 2^3 b. $2^2 \times 11$
c. $2^3 \times 3$ d. 2×3^3
e. $2^2 \times 3 \times 5$ f. $2^2 \times 3^3$

7. a. 12 b. 125 c. 84 d. 150
e. 224 f. 198

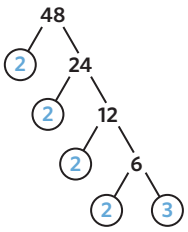
8. a. 1, 2, 4, 7, 14, 28
b. 1, 3, 7, 9, 21, 63
c. 1, 3, 5, 7, 15, 21, 35, 105
d. 1, 2, 5, 10, 11, 22, 55, 110
e. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
f. 1, 2, 5, 10, 25, 50, 125, 250

Problem solving

9. Marcus and Lyndall are playing a game where they roll 5 dice. The numbers on the faces of each die are the first 6 prime numbers. What numbers did they roll, if the product of the faces is 48?

Key points
- They roll 5 dice.
- The faces of each die are the first 6 prime numbers.
- The product of the faces is 48.
- What numbers did they roll?

Explanation



The first 6 prime numbers are 2, 3, 5, 7, 9 and 11.
These are the numbers on each of the dice.

Write a multiplication calculation using the numbers from each die that is equal to 48.

$2 \times 2 \times 2 \times 2 \times 3 = 48$

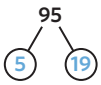
Answer

Marcus and Lyndall rolled four 2's and a 3.

10. Carla has 95 lollies and would like to divide them into smaller bags to share them with her school friends. What is the maximum number of people she can share the lollies between, if she wants each person to receive an equal number of lollies?

Key points
- Carla has 95 lollies.
- Each person gets the same amount.
- What is the maximum number of people she can share the lollies between?

Explanation



Express 95 as a product of its prime factors.

$95 = 5 \times 19$

Carla can divide her lollies into 5 equal groups of 19, or 19 equal groups of 5.

Answer

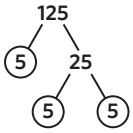
Carla can share the lollies between a maximum of 19 people.

11. Tommy is blowing up a giant inflatable cube. Each face of the cube is rainbow coloured and equally sized. What are the dimensions of the cube if its volume is 125 cubic metres?

Key points

- The inflatable cube has equally sized faces.
- The cube's volume is 125 cubic metres.
- What are the dimensions of the cube?

Explanation



Express 125 as a product of its prime factors.

$125 = 5 \times 5 \times 5$

Answer

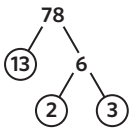
The dimensions of the cube are 5 m × 5 m × 5 m.

12. Sienna is playing with 78 building blocks. What are the dimensions of her prism if she uses all the blocks to build a rectangular prism?

Key points

- Sienna uses 78 blocks.
- Sienna makes a rectangular prism.
- What are the dimensions of a rectangular prism with 78 blocks?

Explanation



Express 78 as a product of its prime factors.

$78 = 2 \times 3 \times 13$

Answer

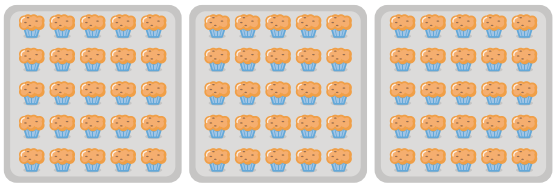
The dimensions of a rectangular prism are 2 blocks by 3 blocks by 13 blocks.

13. Charlotte is a baker and has 75 muffins. She wants to pack the muffins into cartons so that the muffins are arranged in a square, with the same number of muffins in each row and column. How can she pack the muffins so that none of the muffins remain unpacked?

Key points

- Charlotte has 75 muffins.
- The muffins are packed into cartons with the same number of rows and columns.
- How can she pack them so that she has no leftover muffins?

Explanation



75 expressed a product of its prime factors is $3 \times 5 \times 5$.

We can use the prime factors to see if Charlotte can pack the muffins in a square arrangement.

5×5 indicates 5 muffins in a row and 5 muffins in a column.

The 3 indicates that there are 3 cartons of muffins.

Answer

Charlotte can pack the muffins in three cartons with 5 muffins in a row and 5 muffins in a column.

Reasoning

14. a. The prime factors of 165 are 3, 5, and 11.

b. Liza might think she is correct because$3 \times 55 = 165$, however 55 is a composite number so it is not a prime factor.

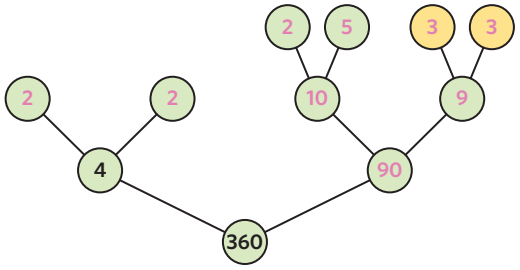
c. There are four ways that Liza can pile the computers onto the trolley.

d. Suggested option 1: Liza can see if other team members are available for help.

Suggested option 2: Liza can invest in a back brace and good shoes to help with the heavy lifting.

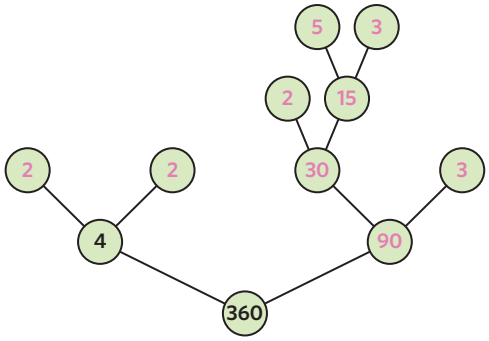
Note: There are other possible options.

15. a.



b. $2^3 \times 3^2 \times 5$

c.



Note: There are other possible options.

d. Suggested option 1: Try dividing the number by each prime in order from smallest to largest.

Suggested option 2: Use divisibility tests to find some initial factors to get the factor tree started.

Note: There are other possible options.

Extra spicy

16. C 17. A 18. 1029

19. $527 = 17 \times 31$

Remember this?

20. C 21. C 22. D

2E Lowest common multiples and highest common factors

Student practice

Worked example 1.

a. 28 b. 12 c. 40 d. 140

Worked example 2.

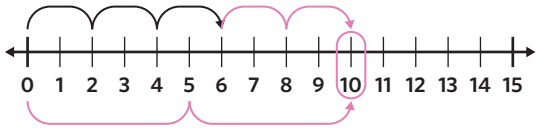
a. 3 b. 2 c. 20 d. 3

Worked example 3.

a. i) 2 ii) 30 b. i) 5 ii) 75 c. i) 6 ii) 36 d. i) 50 ii) 300

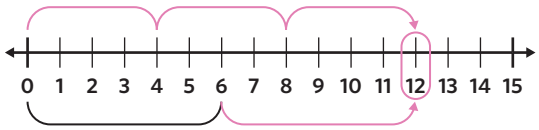
Understanding worksheet

1. a. Multiples of 2



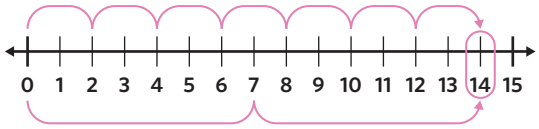
Multiples of 5

b. Multiples of 4



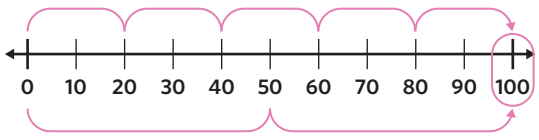
Multiples of 6

c. Multiples of 2



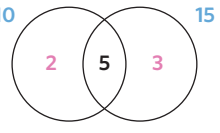
Multiples of 7

d. Multiples of 20

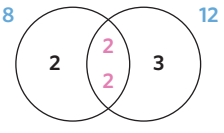


Multiples of 50

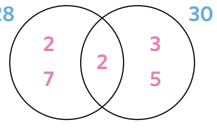
2. a.



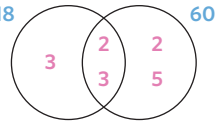
b.



c.



d.



3. lowest; multiples; highest; factors; prime

Fluency

4. a. 20 b. 24 c. 24 d. 55
e. 100 f. 72

5. a. 12 b. 20 c. 24 d. 30
e. 40 f. 36

6. a. 2 b. 1 c. 3 d. 4
e. 7 f. 6

7. a. 2 b. 3 c. 5 d. 6
e. 8 f. 20

8. a. i) 4 ii) 24 b. i) 3 ii) 90 c. i) 3 ii) 72 d. i) 10 ii) 60
e. i) 24 ii) 144 f. i) 1 ii) 1980

Problem solving

9. Ali and Jenny sometimes work at Sparkles Car Wash. Ali works every 5 days and Jenny works every 4 days. How often do they work a shift together?

Key points
- Ali works every 5 days.
- Jenny works every 4 days.
- What is the lowest common multiple of 4 and 5?

Explanation

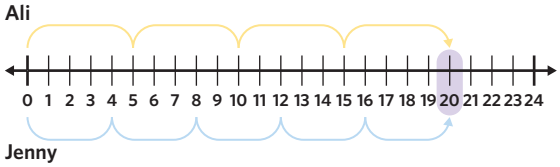
Find the LCM of 4 and 5.

List multiples:

4: 4, 8, 12, 16, 20

5: 5, 10, 15, 20, 25

Ali



Jenny

Answer

Ali and Jenny work together every 20 days.

10. Aviva has two different sized planks of wood. One is 96 cm in length and the other is 156 cm in length. What are the longest possible lengths of wood she can end up with if Aviva wants to cut both the planks into a number of equally sized pieces?

Key points
- One plank is 96 cm long.
- The other plank is 156 cm long.
- Find the highest common factor of 96 and 156.

Explanation

Find the prime factors of 96 and 156.

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$156 = 2 \times 2 \times 3 \times 13$

Common factors: 2, 2, 3

HCF: $2 \times 2 \times 3 = 12$

Answer

The longest length of equally sized pieces of wood that can be cut is 12 cm.

11. At Redbridge Secondary, 80 year 7 students and 64 year 8 students attended school camp. The teachers need to split up the students into as few equal groups as possible, however each group can only contain year 7 students OR year 8 students. How many students were in each group?

Key points
- There are 80 year 7 students.
- There are 64 year 8 students.
- Find the highest common factor of 80 and 64.

Explanation

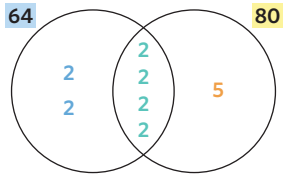
Find the prime factors of 80 and 64.

$80 = 2 \times 2 \times 2 \times 2 \times 5$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Common prime factors: 2, 2, 2, 2

HCF: $2 \times 2 \times 2 \times 2 = 16$



Answer

There are 16 students in each group.

12. Loti the spaniel barks every 9 seconds and Sully the labrador barks every 12 seconds. When will they next bark at the same time if they last both barked at 2 seconds past 12:07 pm.

Key points
- Loti the spaniel barks every 9 seconds.
- Sully the labrador barks every 12 seconds.
- Loti and Sully both barked at 2 seconds past 12:07 pm.
- At what time will they both bark next?

Explanation

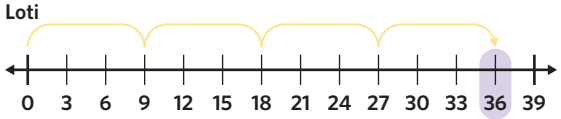
Find the LCM of 9 and 12.

List multiples:

9: 9, 18, 27, 36, 45

12: 12, 24, 36, 48, 60

Loti



Sally

Loti and Sally bark at the same time every 36 seconds.

36 seconds after 2 seconds past 12:07 pm is 38 seconds past 12:07 pm.

Answer

Loti and Sally will next bark at the same time 38 seconds past 12:07 pm.

13. David, Ryan and Libby are all delivery drivers. David is paid $90 per delivery, Ryan earns $60 per delivery, and Libby receives $80 per delivery. Assuming all the drivers did more than one delivery, what is the minimum number of deliveries that each driver must complete so that they all get paid the same amount?

Key points
- David gets paid $90 per delivery.
- Ryan earns $60 per delivery.
- Libby receives $80 per delivery.
- If they all earned the same amount of money, what is the least number of deliveries that each of them completed?

Explanation

Find the HCF of 90, 60 and 80.

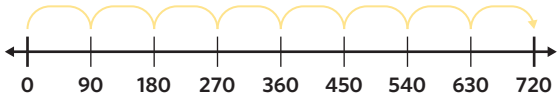
List multiples:

90: 90, 180, 270, 360, 450, 540, 630, 720, 810, 900, 990, 1080

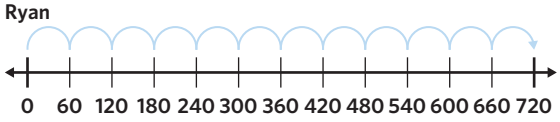
60: 60, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720

80: 80, 160, 240, 320, 400, 480, 560, 640, 720, 800, 880, 960

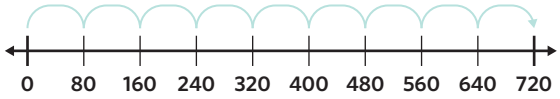
David



Ryan



Libby



The HCF is 720, so David, Ryan and Libby each earn $720.

David: $720 \div 90 = 8$ deliveries

Ryan: $720 \div 60 = 12$ deliveries

Libby: $720 \div 80 = 9$ deliveries

Answer

David can complete 8 deliveries, Ryan can complete 12 deliveries and Libby can complete 9 deliveries.

Reasoning

14. a. The next time a bus to both destinations leaves at the same time is 1.00 pm.

b. The next time a bus to all three destinations leaves at the same time is 9.00 pm.

c. 30 snack packs can be distributed.

d. 5 biscuits and 2 salmon bagels are in each snack pack.

e. Suggested option 1: Snack packs are bigger or smaller based on the amount of driving each driver will do that day.

Suggested option 2: Snack packs are stored so that a driver can grab it when needed.

Note: There are other possible options.

15. a. The smallest order is 5 batches of Twilight and 3 batches of Harry Potter and the Half Blood Prince.

b. Each store will receive 1600 books.

c. Borders can donate to 100 community centres.

d. Suggested option 1: It can be difficult to read digital books when it is bright outside.

Suggested option 2: Hard copy books don't need to be charged.

Note: There are other possible options.

Extra spicy

16. 110 and 132 17. B 18. 10 19. C

Remember this?

20. C 21. B 22. C

2F Perfect squares and square roots

Student practice

Worked example 1.

a. 9 b. 49 c. 144 d. 361

Worked example 2.

a. 3 b. 8 c. 1 d. 40

Worked example 3.

a. 33 b. 30 c. 16 d. 25

Understanding worksheet

1. a. 2 b. 5 c. 7 d. 11

2. a. 4, odd, yes b. 5, even, no
c. 11, even, no d. 10, odd, yes

3. itself; base; two; inverse; odd

Fluency

4. a. 1 b. 0 c. 36 d. 64
e. 121 f. 169 g. 576 h. 10 000

5. a. 2 b. 4 c. 1 d. 7
e. 10 f. 0 g. 11 h. 15

6. a. 30 b. 50 c. 80 d. 100
e. 110 f. 120

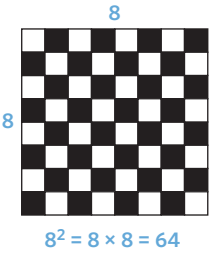
7. a. 8 b. 61 c. 27 d. 3
e. 37 f. 49 g. 1 h. 5

Problem solving

8. Chess is played with a square board. How many spaces are there on a chessboard in total if each side of a chessboard is made up of 8 alternating black and white spaces?

Key points
- Chess is played with a square board.
- Each side has 8 spaces.
- What is 8^2?

Explanation



Write the expression in expanded form and evaluate the multiplication.

$8^2 = 8 \times 8$

$= 64$

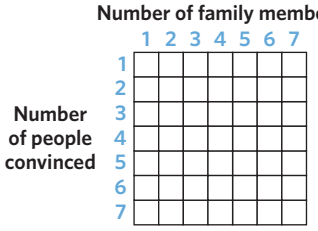
Answer

There are 64 spaces on a chessboard.

9. All 7 members of the Millstone family love their Kindle reader so much that each of them convinced 7 of their friends to also purchase a Kindle reader. How many people did the Millstone family convince to purchase a Kindle reader?

Key points
- All 7 members of the family bought a Kindle.
- They each convinced a further 7 people to purchase a Kindle.
- What is 7^2?

Explanation



$7 \times 7 = 49$

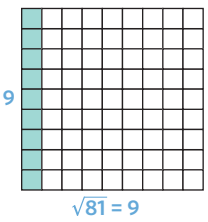
Answer

49 people were convinced to buy a Kindle.

10. William is cutting up a large block of cheese. If each of the block's faces is a square with an area of 81 cm^2, what is the length of each edge of the cheese?

Key points
- Each face is a square.
- The face has an area of 81 cm^2.
- What is the square root of 81?

Explanation



Identify which number multiplied by itself equals the number inside the square root.

$81 = 9 \times 9$

$\sqrt{81} = 9$

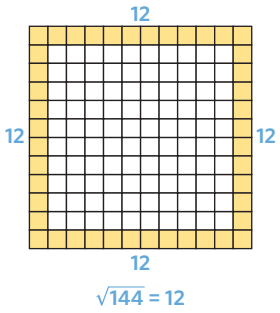
Answer

Each edge of cheese is 9 cm long.

11. The Big Ben is a famous clock tower in London that was built in 1859. The clock's face is a circle and sits inside a square border. What is the perimeter of the square's border if its area is 144 m^2?

Key points
- The clock sits inside a square border.
- The area of the square is 144 m^2.
- What is the perimeter of the border?

Explanation



$\sqrt{144} = 12$

Identify which number multiplied by itself equals the number inside the square root.

$144 = 12 \times 12$

$\sqrt{144} = 12$

To find the perimeter of a square, multiply the side length by 4.

$12 \times 4 = 48$

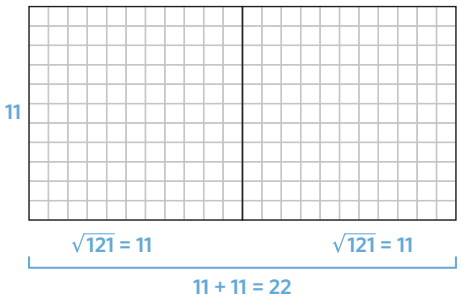
Answer

The perimeter of the square's border is 48 m.

12. Farah is creating the Australian flag out of Lego pieces. The design is a rectangle made up of two side-by-side squares. How many pieces long is the flag if he uses 121 pieces to make each square?

Key points
- The design is a rectangle made up of two squares.
- Each square uses 121 pieces.
- What is the length of the rectangle?

Explanation



Identify which number multiplied by itself equals the number inside the square root.

$121 = 11 \times 11$

$\sqrt{121} = 11$

Each square is 11 pieces long. This means the rectangle is $11 + 11 = 22$ pieces long.

Answer

The flag is 22 pieces long.

Reasoning

13. a. The area of the 'Vault Escape' room is 49 m^2.
b. The length of one of the sides of the 'Sherlock's Puzzle' room is 6 m.
c. The perimeter of the 'Treasure Heist' room is 44 m.
d. The total length would be 30 m.
e. Suggested option 1: The rooms could be designed in rectangles so there are no gaps.
Suggested option 2: The rooms could all be the same size so they use the space more efficiently.
Note: There are other possible options.

14. a. The area of the doggie day spa room is 90 000 cm^2.
b. The side length of the play centre is 1600 cm.
c. 22 and 23.
d. Suggested option 1: The activity level of the dog might change how much space it needs.
Suggestion option 2: Some might also consider the grooming services the dog needs.
Note: There are other possible options.

Extra spicy

15. D 16. 4 17. C 18. D

Remember this?

19. C 20. B 21. 43

2G Introduction to integers

Student practice

Worked example 1.

a. −2 b. 8 c. 14 d. −55

Worked example 2.

a. 7 b. 9 c. 21 d. 17

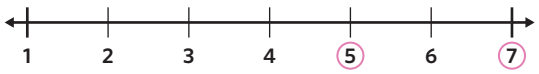
Worked example 3.

a. $-10 < 10$ b. $-9 < -5$ c. $23 > 17$ d. $5 > -500$

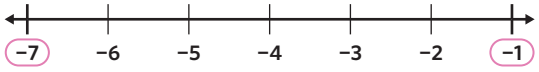
Understanding worksheet

1. a. 1,6 b. −2, 1 c. −12, −5 d. −7,0

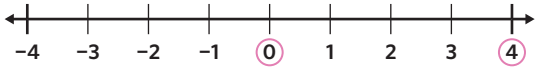
2. a. $5 < 7$



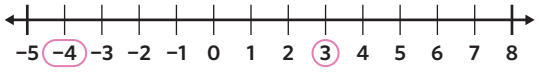
b. $-1 > -7$



c. $4 > 0$



d. $3 > -4$



3. positive; greater; less; negative

Fluency

4. a. 23 b. −10 c. −5 d. 500
   e. −1757 f. 1805

5. a. −3 b. −21 c. 7 d. 33
   e. −101 f. 1000

6. a. 8 b. 4 c. 2 d. 7
   e. 16 f. 64 g. 16 h. 41

7. a. $7 > 3$ b. $21 < 30$
   c. $-4 < -1$ d. $-11 < -2$
   e. $4 > -9$ f. $-8 < 0$
   g. $-121 > -303$ h. $0 > -1001$

8. a. 1, 3, 6, 11, 19 b. −17, −13, −11, −3, −2
   c. −8, −2, 0, 3, 5 d. −4, −3, 0, 3, 4
   e. −111, −11, −1, 1, 111 f. −999, 0, 9, 99, 9999

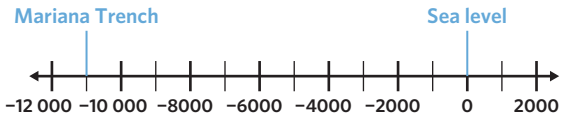
9. a. 29, 28, 15, 7, 3
   b. −17, −18, −28, −29, −31
   c. 17, 3, 0, −4, −21
   d. 2, 1, 0, −1, −2
   e. 25, 5, −5, −10, −15
   f. 22 222, 22, −2, −222, −2222

Problem solving

10. Mariana Trench is the deepest natural point in the world. The bottom of the trench is 11 034 m below sea level. What is the integer that represents the depth of Mariana Trench?

Key points
- The bottom of Mariana Trench is 11 034 m below sea level.
- Express this as an integer.

Explanation



The bottom of the trench is below sea level so the integer will be negative.

Answer

The depth of Mariana Trench can be represented by the integer −11 034.

11. The year 7 geography exam had 100 questions, each worth 1 mark. James got 11 questions incorrect, Zephyr got 10 questions incorrect, and Sophie got 88 questions correct. Who scored the lowest mark out of James, Zephyr and Sophie?

Key points
- The exam had 100 questions, each worth 1 mark.
- James got 11 questions incorrect.
- Zephyr got 10 questions incorrect.
- Sophie got 88 questions correct.
- Which student got the least questions correct?

Explanation

James got 11 questions incorrect. This means that he got $100 - 11 = 89$ questions correct.

Zephyr got 10 questions incorrect. This means that he got $100 - 10 = 90$ questions correct.

Sophie got 88 questions correct.

$88 < 89 < 90$

Sophie < James < Zephyr

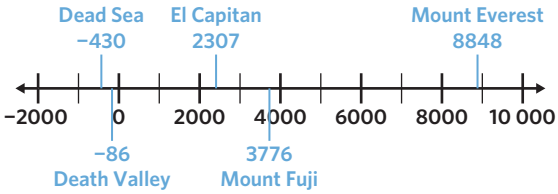
Answer

Sophie scored the lowest mark.

12. Order the following landmarks in ascending order based on their heights. Mount Everest has an elevation of 8848 m. Mount Fuji has an elevation of 3776 m. The bottom of the Dead Sea has an elevation of −430 m and Death Valley has an elevation of −86 m. The summit of El Capitan is 2307 m.

Key points
- Mount Everest has an elevation of 8848 m.
- Mount Fuji has an elevation of 3776 m.
- The bottom of the Dead Sea has an elevation of −430 m.
- Death Valley has an elevation of −86 m.
- The summit of El Capitan is 2307 m.
- Order the landmarks' heights from lowest to highest.

Explanation



We need to order the heights 8848 m, 3776 m, −430 m, −86 m, and 2307 m in ascending order.

The negative integers will be less than all the positive integers.

−430 ☐ −86 < 8848 ☐ 3776 ☐ 2307

Order the positive integers.

−430 ☐ −86 < 2307 < 3776 < 8848

Order the negative integers. The lesser negative integer is the one further away from zero.

−430 < −86 < 2307 < 3776 < 8848

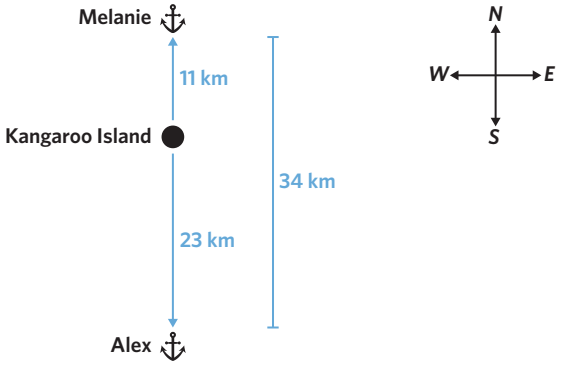
Answer

The Dead Sea, Death Valley, El Capitan, Mount Fuji, Mount Everest

13. Melanie and Alex are both completing solo sailing trips. What is the distance between their anchoring locations if Melanie has anchored exactly 11 km north of Kangaroo Island and Alex has anchored exactly 23 km south of Kangaroo island?

Key points
- Melanie is 11 km north of Kangaroo Island.
- Alex is 23 km south of Kangaroo Island.
- How far away is Melanie from Alex?

Explanation



The distance between Melanie and Kangaroo Island is 11 km.

The distance between Kangaroo Island and Alex is 23 km.

$11 + 23 = 34$

Answer

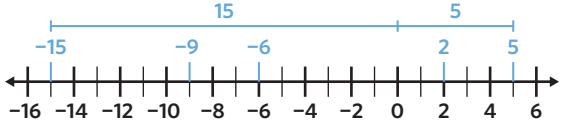
The distance between the anchoring locations is 34 km.

14. Kendall is currently staying in Chicago for a business trip. On Monday it was −6°C, on Tuesday it was −15°C, on Wednesday it was 2°C, on Thursday it was 5°C and on Friday it was −9°C. What was the difference in temperature between the warmest day and the coldest day for that week?

Key points

- The daily temperatures were −6°C, −15°C, 2°C, 5°C, and −9°C.
- How much hotter than the coldest day was the warmest day?

Explanation



First, find the highest and lowest of the daily temperatures.

$-15 < -9 < -6 < 2 < 5$

The lowest temperature was −15°C and the highest temperature was 5°C.

There are 15 units between −15 and 0. There are 5 units between 0 and 5.

$15 + 5 = 20$

Answer

The difference in temperatures was 20°C.

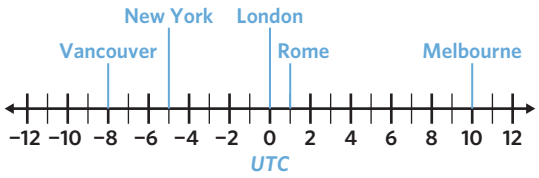
Reasoning

15. a. Andrea has $35 in her account at the end of January.
    b. Andrea's account balance was at its lowest at the end of February.
    c. Andrea deposited $88 in June.
    d. Andrea's account balance decreased the most during April.
    e. Suggested option 1: It is important to save money for emergencies or sudden expenses.

    Suggested option 2: Andrea could create a budget in order to spend less.

    Note: There are other possible options.

16. a.



    b. The time difference between Melbourne and New York is 15 hours.
    c. It is 8:00 pm in Melbourne.
    d. It is 4:30 am in Vancouver.
    e. Suggested option 1: It would be dark outside at 12 pm in some countries.

    Suggested option 2: In some countries 12 pm would be the middle of the night rather than the middle of the day.

    Note: There are other possible options.

Extra spicy

17. 4176 18. 40 and 42 19. C 20. B

Remember this?

21. D 22. C 23. B

2H Adding and subtracting positive integers

Student practice

Worked example 1.

a. 1 b. −7 c. 11 d. −60

Worked example 2.

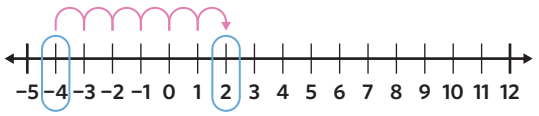
a. −8 b. −9 c. −11 d. −20

Worked example 3.

a. 12 b. −8 c. 3 d. −5

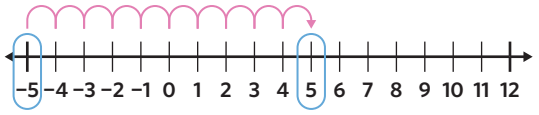
Understanding worksheet

1. a.

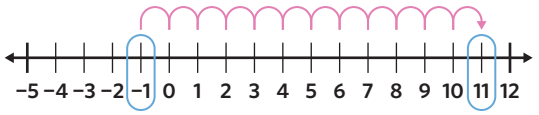


    b.

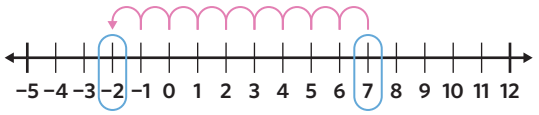
    c.



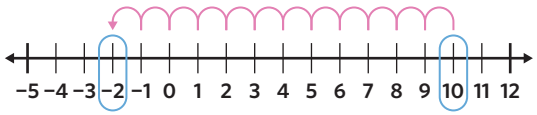
    d.



2. a.

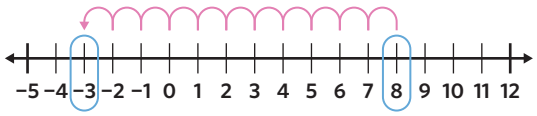


b.



c.

d.



3. positive; greater; decreases; negative

Fluency

4. a. 1 b. 3 c. 6 d. −2
   e. −4 f. −7 g. −20 h. 0

5. a. −1 b. 7 c. −2 d. −7
   e. −10 f. −11 g. −3 h. −90

6. a. Negative b. Positive
   c. Negative d. Negative
   e. Positive f. Positive

7. a. $-3 + 4 = 1$ b. $-2 + 6 = 4$
   c. $1 - 4 = -3$ d. $-2 + 10 = 8$
   e. $2 - 4 = -2$ f. $9 - 13 = -4$

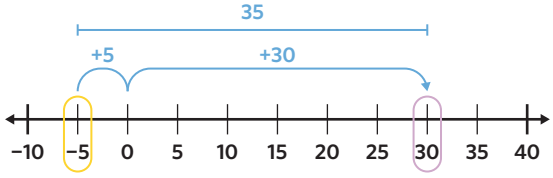
8. a. 9 b. 20 c. −3 d. 4
   e. 1 f. 19 g. −232 h. −286

Problem solving

9. The temperature in the morning in Alice Springs is −5 degrees Celsius. The temperature is expected to rise by 35 degrees Celsius by the afternoon. What is the expected temperature in the afternoon?

Key points
- The temperature in the morning in Alice Springs is −5 degrees Celsius.
- The temperature is expected to increase by 35 degrees Celsius.
- What temperature is it expected to increase to?

Explanation



Partition 35 into 5 and 30 to use 0 as a benchmark number.

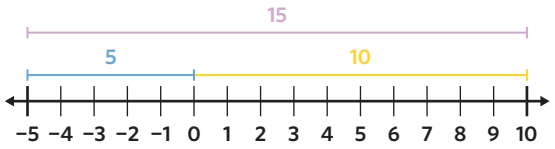
$-5 + 35 = -5 + 5 + 30$
$= 0 + 30$
$= 30$ degrees Celsius

Answer
The expected temperature in the afternoon is 30 degrees Celsius.

10. What distance does an Olympic platform diver traverse if she jumps from a 10 metre high platform and touches the bottom of the pool that is 5 metres deep?

Key points
- The Olympic diver jumps from a 10 metre platform.
- The Olympic diver touches the bottom of the pool, which is 5 metres deep.
- How far does the Olympic diver travel?

Explanation



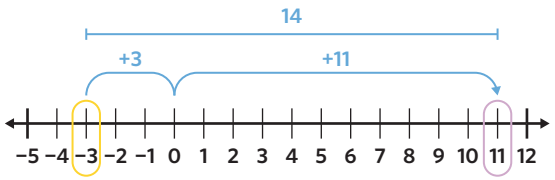
The distance from the diving platform to the pool is 10 metres.
The distance from the pool to the bottom of the pool is 5 metres.
The total distance traversed will be the sum of 10 and 5.
$10 + 5 = 15$ m

Answer
The Olympic diver traverses 15 m.

11. Talia parked her car three levels below the ground level of her apartment building. What level does she live on if she takes the elevator 14 levels up to from the car park to reach her apartment?

Key points
- Tahlia parked her car 3 levels below ground level.
- She takes the elevator 14 levels up to get to her apartment.
- What level of the apartment complex does she live on?

Explanation



Partition 14 into 3 and 11 to use 0 as a benchmark number.

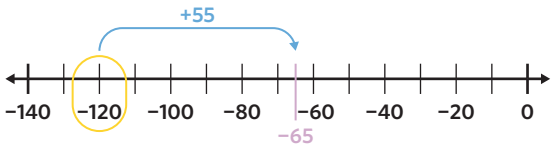
$-3 + 14 = -3 + 3 + 11$
$= 0 + 11$
$= 11$ floors

Answer
Tahlia lives on level 11.

12. Anna used her credit card and has a bank balance of −$120. What is her balance after she repays $55?

Key points
- Anna has a balance of −$120.
- What will her balance be after repaying $55?

Explanation



When repaying her balance, Anna moves her total balance in the positive direction. As Anna's negative balance is larger than how much she repays, it will still be negative.

$-120 + 55 = -\$65$

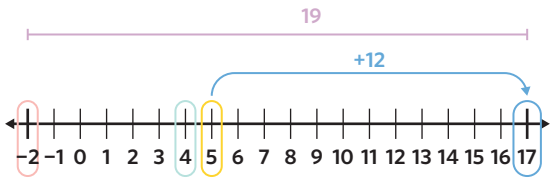
Answer
Anna's balance is −$65.

13. Melbourne is known for having four seasons in a day. One day the temperature was 5 degrees Celsius in the morning and increased by 12 degrees Celsius by noon. In the afternoon the temperature dropped to 4 degrees Celsius and by midnight the temperature was −2 degrees Celsius. What was the difference between the day's high and low temperature?

Key points

- The temperature was 5 degrees Celsius in the morning.
- The temperature increased by 12 degrees Celsius by noon.
- In the afternoon the temperature was 4 degrees Celsius.
- At midnight the temperature was −2 degrees Celsius.
- Find the difference between the highest temperature and the lowest temperature.

Explanation



Temperature at noon:

5 + 12 = 17 degrees Celsius

17 degrees Celsius is the maximum temperature. The minimum temperature is −2 degrees Celsius. To get from −2 to 17 we add 2 to get to 0 and then add 17. This is a difference of 19 degrees Celsius.

Answer

The difference between the highest temperature and the lowest temperature is 19 degrees Celsius.

Reasoning

14. a. The temperature of the ice slush is 6 degrees Celsius.
    b. It will take 3 minutes for the ice to reach 0 degrees Celsius.
    c. The temperature of the ice varied by 26 degrees Celsius from the beginning to the end of the experiment.
    d. Suggested option 1: Yes, it would save his parents money.
    Suggested option 2: No, it is too much effort to make ice cream from scratch.
    Note: There are other possible options.

15. a. The vertical distance between Mount Fuji and Hachirogata is 3780 m.
    b. The difference in potential temperatures is 54 degrees Celsius.
    c. Under the proposal, the Hachinohe Mine will have an altitude of −195 m in 10 years.
    d. Suggested option 1: Benefit; quarrying provides jobs and helps the local economy.
    Suggested option 2: Disadvantage; quarrying destroys habitats and causes pollution.
    Note: There are other possible options.

Extra spicy

16. 2 − 4 + 6 − 3 = 10 − 5 − 3 − 1
17. −4 and −109
18. −3996
19. A

Remember this?

20. A 21. A 22. B

2I Adding and subtracting negative integers

Student practice

Worked example 1.

a. 3 b. −2 c. −18 d. −10

Worked example 2.

a. 8 b. −1 c. 16 d. 2

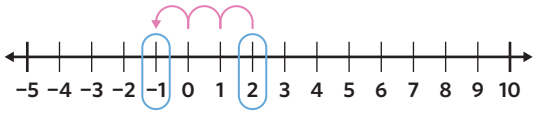
Worked example 3.

a. −1 b. 1 c. 8 d. 4

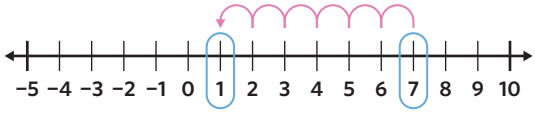
Understanding worksheet

1. a. 2 b. 2 c. −1 d. −3

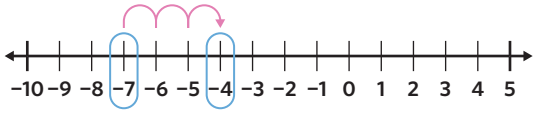
2. a.



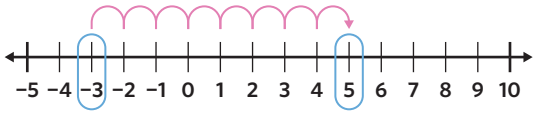
b.



c.



d.



3. opposite; zero; negative; adding

Fluency

4. a. 1 b. 4 c. −8 d. −3
   e. −11 f. −16 g. 9 h. −2

5. a. 6 b. −1 c. 3 d. 10
   e. 0 f. −5 g. 25 h. −3

6. a. + b. − c. + d. −
   e. + f. +

7. a. Negative b. Negative
   c. Positive d. Positive
   e. Negative f. Positive

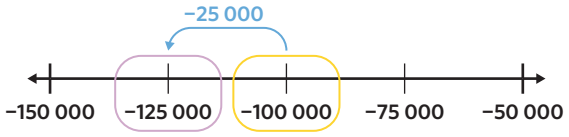
8. a. −9 b. 1 c. 4 d. 10
   e. −14 f. −3 g. 16 h. 15

Problem solving

9. After the Stan family borrowed money to buy a new house, their bank balance was −$100 000. They then borrowed another $25 000 to renovate before they moved in. What was the family's bank balance after paying for the renovation?

Key points
- The Stan family's bank balance was −$100 000.
- They then borrowed another $25 000.
- What was their bank balance after borrowing the extra $25 000?

Explanation



Bank balance: $-100\,000 + (-25\,000)$

$-100\,000 + (-25\,000) = -100\,000 - 25\,000$

$= -\$125\,000$

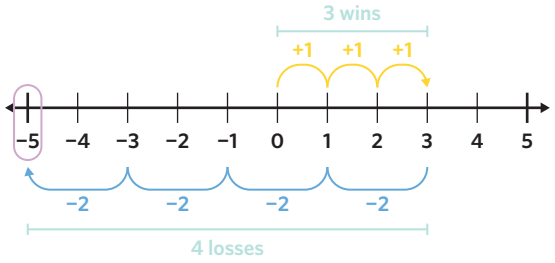
Answer

The Stan family's bank balance is −$125 000 after the renovation.

10. Mickey is playing a card game with friends. The winners of each round receive 1 point and the losers of each round lose 2 points. What is Mickey's final score if she won three rounds and lost four rounds?

Key points
- The winners of each round get +1 points.
- The losers of each round get −2 points.
- Mickey won 3 rounds and lost 4 rounds.
- What was her total score?

Explanation



3 wins and 4 losses: $1 + 1 + 1 + (-2) + (-2) + (-2) + (-2)$

$1 + 1 + 1 + (-2) + (-2) + (-2) + (-2)$

$= 1 + 1 + 1 - 2 - 2 - 2 - 2$

$= 3 - 8$

$= -5$

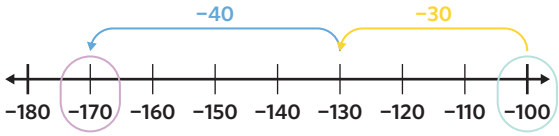
Answer

Mickey's final score is −5.

11. A vintage t-shirt shop is selling an original vintage t-shirt of The Bangles for $30. They also have a vintage Janet Jackson t-shirt for $40. What is Ella's credit card balance if she buys both t-shirts using her credit card which already had a balance of −$100?

Key points
- A vintage t-shirt shop is selling a t-shirt of The Bangles for $30.
- It is also selling a Janet Jackson t-shirt for $40.
- Ella's credit card balance is −$100.
- If she buys both t-shirts, what is her credit card balance?

Explanation



Credit card balance: $-100 + (-30) + (-40)$

$-100 + (-30) + (-40) = -100 - 30 - 40$

$= -130 - 40$

$= -\$170$

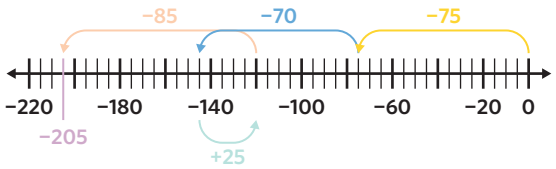
Answer

Ella's credit card balance is −$170 after buying both t-shirts.

12. A submarine descends in stages before it reaches its final depth. What is the final depth of a submarine that descends 75 metres in stage 1, descends another 70 metres in stage 2, rises 25 metres in stage 3, and then descends 85 metres in stage 4?

Key points
- A submarine first descends 75 m.
- It then descends a further 70 m.
- It then rises 25 m.
- It then descends another 85 m.
- After starting at sea level, what is the final depth of the submarine?

Explanation



Depth: $0 + (-75) + (-70) + 25 + (-85)$

$= 0 - 75 - 70 + 25 - 85$

$= -75 - 70 + 25 - 85$

$= -145 + 25 - 85$

$= -120 - 85$

$= -205$ m

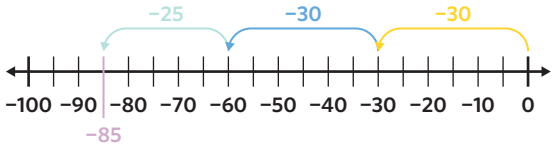
Answer

The final depth of the submarine is 205 m.

13. The South Pole is one of the coldest places in the world, reaching temperatures of −30 degrees Celsius. Yakutsk in Russia also descends to a temperature of −30 degrees Celsius. Bardufoss in Norway has a minimum temperature that is 5 degrees Celsius warmer than Yakutsk. What is the sum of the minimum temperatures in all three locations?

Key points
- The minimum temperature at the South Pole is −30 degrees Celsius.
- The minimum temperature in Yakutsk is −30 degrees Celsius.
- The minimum temperature in Bardufoss is 5 degrees Celsius warmer than Yakutsk.
- Add the three minimum temperatures together.

Explanation



South Pole and Yakutsk: −30 degrees Celsius

Bardufoss: −30 + 5 = −25 degrees Celsius

Sum:

$$\begin{aligned}-30 + (-30) + (-25) &= -30 - 30 - 25\\ &= -60 - 25\\ &= -85 \text{ degrees Celsius}\end{aligned}$$

Answer

The sum of the minimum temperatures of the three locations is −85 degrees Celsius.

Reasoning

14. a. Deacon scored par on 2 holes and Lachlie scored par on 4 holes.
    b. Deacon hit 2 shots less than Lachlie on the fifth hole.
    c. Deacon won by 2 shots.
    d. Without counting the best hole of each player, Deacon won by 1 shot.
    e. Suggested option 1: Wind could affect the flight of the ball.
    Suggested option 2: Rain could affect how the ball travels across the grass.
    Note: There are other possible options.

15. a. The lowest potential score a batter can make in an over is −30 runs.
    b. Lucy's score is 8 runs.
    c. Shae's correct score is −3 runs.
    d. Suggested option 1: Lucy and Shae should try to hit the ball on the ground so it doesn't get caught.
    Suggested option 2: Lucy and Shae should just try to hit 'sixes as they are worth the most runs.
    Note: There are other possible options.

Extra spicy

16. C
17. −1 234 565
18. Greater than 0
19. B

Remember this?

20. C
21. C
22. D

Chapter 2 extended application

1. a. Candace received 169 books.
   b. 1 × 48, 2 × 24, 3 × 16, 4 × 12, 6 × 8, 8 × 6, 12 × 4, 16 × 3, 24 × 2, 48 × 1
   c. No
   d. She will need to sell 3 books.
   e. Suggested option 1: She should give fewer books to more charities to help as many as she can.
   Suggested option 2: She should give more books to fewer charities who need them more.
   Note: There are other possible options.

2. a. Marcus scored −8 points through free kicks.
   b. Marcus scored 4^3 or 64 points from tackles in these games.
   c. Marcus needs 2 handballs and 3 marks in the final quarter.
   d. Yes, it is possible that the rest of Marcus' points came from kicks.
   e. Suggested option 1: It doesn't account for important moments during a match.
   Suggested option 2: There might be other important statistics that don't get players any AFL Fantasy points.
   Note: There are other possible options.

3. a. Box 1: Green
   Box 2: Green
   Box 3: Green
   b. Box 1: Green
   Box 2: Blue
   Box 3: Pink
   c. No, because there is no yellow ball in box 1.
   d. 3 and 23.
   e. Suggested option 1: They could get bonus points for how quickly they can calculate their total score.
   Suggested option 2: Points could be awarded based on whether they calculate the final result correctly or not.
   Note: There are other possible options.

Chapter 2 review

Multiple choice

1. B
2. B
3. A
4. D
5. C

Fluency

6. a. 1, 2, 4, 8, 16
   b. 1, 5, 11, 55
   c. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
   d. 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132

7. a. 6, 12, 18, 24, 30
   b. 11, 22, 33, 44, 55
   c. 17, 34, 51, 68, 85
   d. 47, 94, 141, 188, 235

8. a. 216
   b. 75
   c. 305
   d. 37

9. a. Prime
   b. Composite
   c. Composite
   d. Prime

10. a. 2 × 2 × 2 × 3
    b. 2 × 2 × 2 × 2 × 2
    c. 5 × 17
    d. 2 × 3 × 17

11. a. 12
    b. 90
    c. 60
    d. 120

12. a. 8
    b. 16
    c. 20
    d. 4

13. a. 7
    b. 11
    c. 20
    d. 90

14. a. −13, −2, 4, 8, 17
    b. −101, −43, −20, −3, 2
    c. −133, −113, −13, 13, 133
    d. −454, −445, −45, 454, 455

15. a. 7 c. −11 d. −8
b. −16

16. a. 1 b. −29 c. 21 d. 11

17. a. Negative b. Negative
c. Positive d. Positive

Problem solving

18. Ms Merakis is organising seats for a year 7 assembly. There are 160 year 7 students. Find the two possible formations of the seats so that each row seats an equal number of students, and each row has between 15 and 25 seats.

Key points
- There are 160 year 7 students.
- Each row seats an equal number of students.
- Each row has between 15 and 25 seats.
- Find the factor pairs that satisfy these conditions.

Explanation

Identify the factor pairs of 160. The correct two pairs will contain 1 factor between 15 and 25.

$1 \times 160 = 160$ ✗

$2 \times 80 = 160$ ✗

$4 \times 40 = 160$ ✗

$5 \times 32 = 160$ ✗

$8 \times 20 = 160$ ✓

$10 \times 16 = 160$ ✓

16 and 20 are both between 15 and 25.

Answer

8 rows of 20 seats or 10 rows of 16 seats.

19. On the 1st day of the COVID-19 pandemic there were three cases. It was estimated that the number of cases then tripled each day for the next month. How many cases were there estimated to be on the 5th day of the pandemic?

Key points
- There were 3 cases on the first day.
- The number of cases tripled each day for the next month.
- How many cases were there after it had tripled 5 times?

Explanation

On the 2nd day, there were 3×3 or 3^2 cases.

On the 3rd day, there were $3^2 \times 3$ or 3^3 cases.

So on the 5th day, there were 3^5 cases.

Express the power in expanded form.

$3 \times 3 \times 3 \times 3 \times 3$

Evaluate the multiplication.

$= 9 \times 9 \times 3$

$= 81 \times 3$

$= 243$

Answer

There were an estimated 243 cases on the 5th day.

20. Fletcher knows he has between 90 and 100 footy cards. He has tried to split his collection into equal groups but always has cards remaining. How many footy cards does Fletcher have?

Key points
- Fletcher has between 90 and 100 footy cards.
- He has tried to split his collection into equal groups but always has cards remaining.
- What is the prime number between 90 and 100?

Explanation

We are looking for a prime number because Fletcher can't split the cards evenly.

We can rule out 92, 94, 96 and 98 because they are even numbers and are divisible by 2.

We can also rule out 95 because it ends with a 5 so is divisible by 5.

Perform divisibility tests for the remaining numbers.

91 is divisible by 7. $70 + 21 = 91$ and both 70 and 21 are multiples of 7. ✗

93 is divisible by 3 because the sum of its digits is divisible by 3. ✗

$9 + 3 = 12$

The only factor pair for 97 is 97 and 1. ✓

99 is divisible by 3 and 9 because the sum of its digits is divisible by 9. ✗

$9 + 9 = 18$

Answer

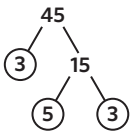
Fletcher has 97 footy cards.

21. Sofia has made 45 pieces of rocky road. She wants to package them into equal groups of more than one piece. What is the least number of rocky road pieces Sofia can have in each group?

Key points
- Sofia has made 45 pieces of rocky road.
- She wants to package them into equal groups of more than one piece.
- What is the smallest prime factor of 45?

Explanation

Express 45 as a product of its prime factors.



$45 = 3 \times 3 \times 5$

The smallest prime factor is 3.

Answer

The least number of rocky road pieces Sofia can have in each group is 3.

22. Blaise works at Mr Miyagi every 4 days, while Louis works there every 6 days. On what day will they next work together if they just worked together on a Saturday night?

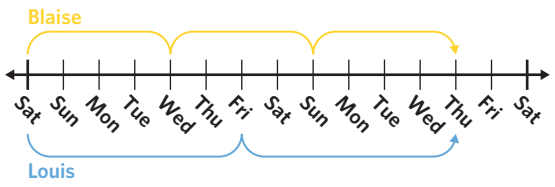
Key points
- Blaise works every 4 days.
- Louis works every 6 days.
- They just worked together on a Saturday night.
- What day will it be when they next work together?

Explanation

Find the LCM of 4 and 6 to determine how many days it will be until they work together again.

4: 4, 8, 12

6: 6, 12



14 days from Saturday will be Saturday, so 12 days from Saturday is Thursday.

Answer

They will next work together on a Thursday.

23. The game of checkers is played on a square board with 64 squares. How many squares are there in each row?

Key points

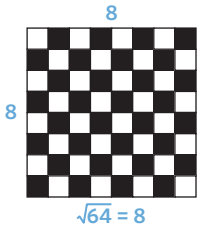
- The game of checkers is played on a square board.
- There are 64 squares.
- What is $\sqrt{64}$?

Explanation

Identify which number multiplied by itself equals the number inside the square root.

$64 = 8 \times 8$

$\sqrt{64} = 8$



Answer

There are 8 squares in each row.

24. The highest point on Earth is Mount Everest, which has an altitude of 8848 metres. The lowest point on land is the Dead Sea, with an altitude of −413 metres. What is the difference in altitude between the top of Mount Everest and the bottom of the Dead Sea?

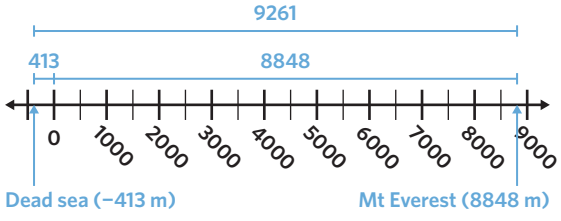
Key points

- Mount Everest has an altitude of 8848 metres.
- The Dead Sea has an altitude of −413 metres.
- What is the difference between 8848 and −413?

Explanation

There is 413 m between −413 m and 0 m.
There are 8848 m between 0 m and 8848 m.

$413 + 8848 = 9261$



There are 9261 m between −413 m and 8848 m.

Answer

The difference in altitude is 9261 m.

25. Elise has driven to her office building and parked two levels underground. She travels up 5 levels in the elevator to the cafe, buys a coffee, and then travels up another 7 levels to the office. What floor is the office on?

Key points

- Elise parks 2 levels underground.
- She travels up 5 levels.
- She then travels up another 7 levels.
- What is $-2 + 5 + 7$?

Explanation

Elise starts at −2. Partition 5 into 2 and 3 to use 0 as a benchmark number.

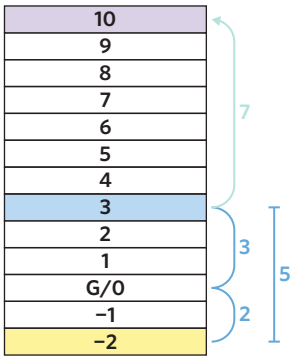
$-2 + 5 = -2 + 2 + 3$

$= 0 + 3$

$= 3$

She travels up another 7 levels.

$3 + 7 = 10$



Answer

The office is on the 10th floor.

26. Emilie, Tallie and Matisse all use Afterpay to pay for their online shopping. Emilie is $75 in debt, Tallie is $135 in debt and Matisse is $120 in debt. How much are they in debt combined?

Key points

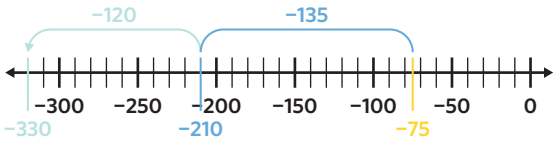
- Emilie is $75 in debt.
- Tallie is $135 in debt.
- Matisse is $120 in debt.
- What is $-75 + (-135) + (-120)$?

Explanation

$-75 + (-135) + (-120) = -75 - 135 - 120$

$= -210 - 120$

$= -330$



Answer

They are $330 in debt combined.

Reasoning

27. a. $3 \times 48, 4 \times 36, 6 \times 24, 8 \times 18, 9 \times 16, 16 \times 9, 18 \times 8$

b. There would be 12 photos in each row.

c. The formation was made up of 3 rows and 48 columns.

d. Grace needs to add 256 photos to her collection.

e. Suggested option 1: People look at their photos more if they're printed.

Suggested option 2: Photos that only exist in printed form might have more sentimental value than photos that permanently exist on a phone.

Note: There are other possible options.

28. a. There was 36 metres between them.

b. The reef has an altitude of −15 metres.

c. In the cave, Maria was at an altitude of −33 metres.

d. It will be 3 years until they scuba dive together again.

e. Suggested option 1: It is bad for the environment if divers are touching the coral reefs and sea life.

Suggested option 2: It could help save ocean life if divers are helping to clear waste and pollution.

Note: There are other possible options.

3A Introduction to fractions

Student practice

Worked example 1.

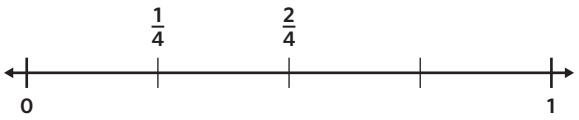
a. $\frac{8}{11}$ b. $\frac{5}{12}$ c. $\frac{4}{9}$ d. $-\frac{6}{17}$

Worked example 2.

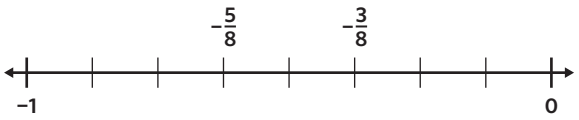
a. $\frac{2}{5}$ b. $-\frac{2}{7}$ c. 0 d. $-\frac{1}{6}$

Worked example 3.

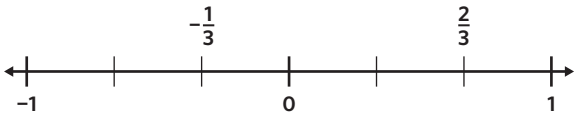
a.



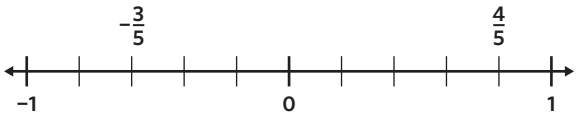
b.



c.



d.



Understanding worksheet

1. a. B b. C c. C d. A

2. a. $\frac{2}{3}$ b. $\frac{4}{6}$ c. $\frac{1}{5}$ d. $-\frac{2}{9}$

3. numerator; parts; denominator; whole

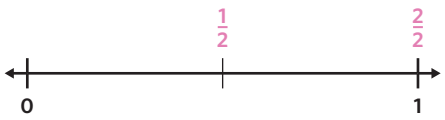
Fluency

4. a. Numerator: 1, Denominator: 4
   b. Numerator: 2, Denominator: 3
   c. Numerator: 5, Denominator: 6
   d. Numerator: 7, Denominator: 8
   e. Numerator: 1, Denominator: 10
   f. Numerator: 93, Denominator: 100

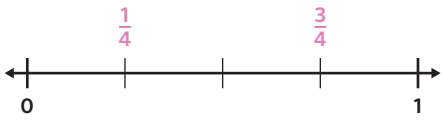
5. a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. $\frac{7}{8}$ d. $\frac{4}{7}$
   e. $\frac{20}{21}$ f. $\frac{11}{13}$

6. a. $\frac{1}{4}, \frac{3}{4}, \frac{4}{4}$ b. $\frac{0}{6}, \frac{3}{6}, \frac{5}{6}$
   c. $\frac{2}{10}, \frac{4}{10}, \frac{7}{10}$ d. $-\frac{3}{3}, -\frac{2}{3}, -\frac{1}{3}$
   e. $-\frac{5}{7}, -\frac{2}{7}, -\frac{1}{7}$ f. $-\frac{1}{3}, \frac{0}{3}, \frac{2}{3}$
   g. $-\frac{3}{5}, \frac{1}{5}, \frac{3}{5}$ h. $-\frac{6}{8}, -\frac{1}{8}, \frac{4}{8}$

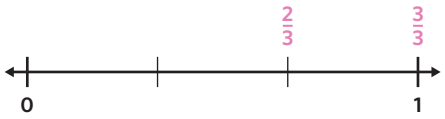
7. a.



b.



c.

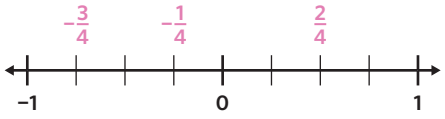


d.

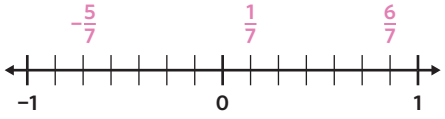
e.

f.

g.



h.



Problem solving

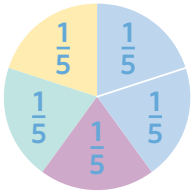
8. Steph asked her class to vote on what food they should order for their end-of-year party. She used the data to draw this pie chart.

Which food did $\frac{2}{5}$ of the class vote for?

Key points

- One of the shaded parts in the pie charts is $\frac{2}{5}$.
- Which of the shaded parts in the pie chart is $\frac{2}{5}$?

Explanation



Sandwiches: $\frac{1}{5}$

Pizza: $\frac{1}{5}$

Tacos: $\frac{1}{5}$

Fish and chips: $\frac{2}{5}$

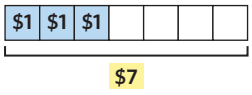
Answer

A

9. Mario wants to know what fraction of his money he spent at the bakery. He started with $7 and bought a cupcake for $3.

Key points
- Mario has $7 in total.
- A cupcake is $3.
- What fraction of the total ($7) was spent on a cupcake ($3)?

Explanation



$$\frac{\textit{the amount spent}}{\textit{the total amount of money}} = \frac{3}{7}$$

Answer

Mario spent $\frac{3}{7}$ of his money at the bakery.

10. Alexia is riding home from school on her bike and has made it $\frac{2}{3}$ of the way home. How far has she ridden, if her house is 6 km from school?

Key points
- Alexia has ridden $\frac{2}{3}$ of the way home.
- The distance from her school to home is 6 km.
- How far has she ridden?

Explanation

$\frac{1}{3}$ of 6 km is 2 km.

So, $\frac{2}{3}$ of 6 km is 4 km.

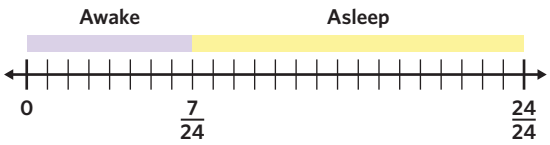
Answer

Alexia has ridden 4 km.

11. A baby slept for 17 hours of the day. What fraction of the day was the baby awake?

Key points
- 17 hours of sleep.
- 24 hours in total in a day.
- What fraction of the day was the baby awake?

Explanation



$24 - 17 = 7$ hours awake

$$\frac{\textit{hours baby was awake}}{\textit{total hours in a day}} = \frac{7}{24}$$

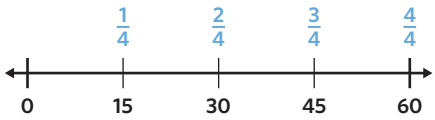
Answer

The baby was awake for $\frac{7}{24}$ of the day.

12. It takes Gaia 60 seconds to run a lap of the schoolyard. She ran $\frac{3}{4}$ of a lap before stopping to tie her shoelace. How many seconds will it take to finish the lap?

Key points
- It takes 60 seconds to run an entire lap.
- Gaia has run $\frac{3}{4}$ of the lap.
- How many seconds will it take to finish the lap?

Explanation



Gaia has run $\frac{3}{4}$ of the lap so she has $\frac{1}{4}$ left.

$\frac{1}{4}$ of 60 seconds is 15 seconds.

Answer

It will take Gaia 15 seconds to finish the lap.

Reasoning

13. a. D
    b. $\frac{5}{8}$ of the pets are four-legged animals.
    c. 15 students have four-legged pets.
    d. Both Tom and Maya are correct.
    e. Suggested option 1: the information could be shown as a bar chart.

    Suggested option 2: the information could be shown as a table.

    Note: There are other possible options.

14. a. A
    b. The area of the sheep paddock is 1500 hectares.
    c. The pigs will have $\frac{1}{16}$ of the farm.
    d. Suggested option 1: design the farm by dividing it into the largest fraction first.

    Suggested option 2: design the farm by dividing it into the smallest fraction first.

    Note: There are other possible options.

Extra spicy

15. $-\frac{1}{8}$ 16. 4 students 17. B 18. 8

Remember this?

19. A 20. 7 and 13 21. 2 minutes

3B Proper fractions, improper fractions, and mixed numbers

Student practice

Worked example 1.

a. Improper fraction
b. Mixed number
c. Improper fraction
d. Mixed number

Worked example 2.

a. $\frac{19}{5}$ b. $\frac{72}{7}$ c. $\frac{71}{12}$ d. $\frac{323}{23}$

Worked example 3.

a. $1\frac{3}{4}$ b. $2\frac{5}{6}$ c. $7\frac{5}{12}$ d. 13

Worked example 4.

a. $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}, \frac{7}{5}$

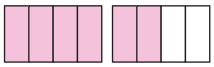
b. $4, 3\frac{6}{7}, 3\frac{5}{7}, 3\frac{4}{7}, 3\frac{3}{7}, 3\frac{2}{7}$

c. $-\frac{11}{12}, -\frac{7}{12}, -\frac{3}{12}, \frac{1}{12}, \frac{5}{12}, \frac{9}{12}$

d. $3\frac{3}{13}, 1\frac{8}{13}, 0, -1\frac{8}{13}, -3\frac{3}{13}, -4\frac{11}{13}$

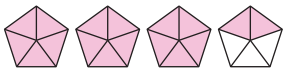
Understanding worksheet

1. a.

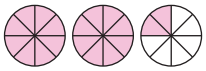


Note: Different parts may be shaded, as long as the total number of shaded parts is the same as the answer.

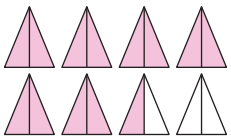
b.



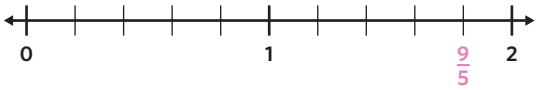
c.



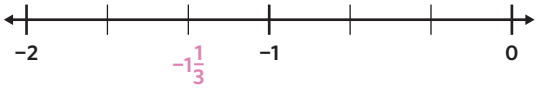
d.



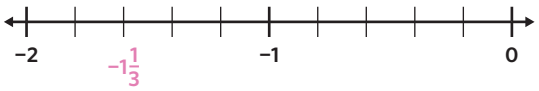
2. a.



b.



c.



d.

3. proper; less; improper; greater; equal; mixed number

Fluency

4. a. Improper fraction b. Proper fraction
   c. Mixed number d. Proper fraction
   e. Mixed number f. Improper fraction
   g. Mixed number h. Mixed number

5. a. $\frac{21}{5}$ b. $\frac{24}{7}$ c. $\frac{55}{7}$ d. $\frac{118}{11}$
   e. $\frac{11}{4}$ f. $\frac{53}{9}$ g. $\frac{25}{6}$ h. $\frac{40}{12}$

6. a. $1\frac{2}{4}$ b. $2\frac{2}{3}$ c. $3\frac{3}{4}$ d. $5\frac{2}{5}$
   e. $3\frac{7}{10}$ f. $3\frac{1}{6}$ g. $7\frac{7}{12}$ h. $6\frac{3}{4}$

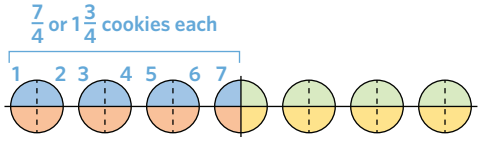
7. a. $\frac{1}{3}, 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3}, 4\frac{1}{3}, 5\frac{1}{3}, 6\frac{1}{3}$
   b. $4, 4\frac{1}{4}, 4\frac{2}{4}, 4\frac{3}{4}, 5, 5\frac{1}{4}, 5\frac{2}{4}$
   c. $\frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$
   d. $-\frac{2}{7}, -\frac{1}{7}, 0, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}$
   e. $1, 1\frac{2}{3}, 2\frac{1}{3}, 3, 3\frac{2}{3}, 4\frac{1}{3}, 5$
   f. $\frac{5}{8}, 2\frac{6}{8}, 4\frac{7}{8}, 7, 9\frac{1}{8}, 11\frac{2}{8}, 13\frac{3}{8}$
   g. $\frac{4}{5}, 1\frac{2}{5}, 2, 2\frac{3}{5}, 3\frac{1}{5}, 3\frac{4}{5}, 4\frac{2}{5}$
   h. $-4, -2\frac{4}{6}, -1\frac{2}{6}, 0, 1\frac{2}{6}, 2\frac{4}{6}$

Problem solving

8. Koda had seven cookies and wanted to share them equally between herself and three friends. How many cookies will each person get?

Key points
- Koda had seven cookies.
- Koda wanted to share them equally between herself and her three friends.
- How can Koda split 7 cookies equally between 4 people?

Explanation



Divide the total number of cookies by the number of people.

$\frac{7}{4} = 1$ remainder 3

$= 1\frac{3}{4}$

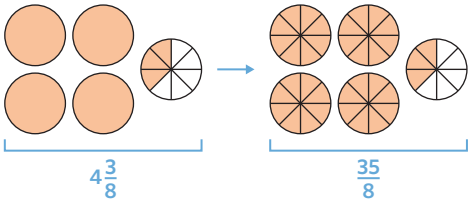
Answer

Each person gets $1\frac{3}{4}$ cookies.

9. Amira and her friends shared Komino's pizzas. How many slices of pizza did they eat? Each pizza was sliced into 8 and they finished $4\frac{3}{8}$ pizzas.

Key points
- The pizza was cut into 8 pieces.
- They ate $4\frac{3}{8}$ pizzas.
- How many slices of pizza were eaten in total?

Explanation



Multiply the whole number (4) by the denominator (8) and then add the numerator (3).

$4 \times 8 + 3 = 32 + 3$

$= 35$

Answer

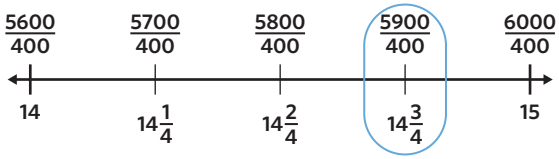
Amira and her friends ate 35 slices of pizza.

10. Ronald is running laps on a 400 m track. If he ran 5900 m in total, how many laps did he run?

Key points

- Ronald is running on a 400 m track.
- Ronald ran 5900 m in total.
- How many 400 m laps did Ronald run to make up 5900 m? Write the answer as a mixed number.

Explanation



Divide the distance he ran by the length of the track to find the total laps run.

$\frac{5900}{400} = 14$ remainder 3

$= 14\frac{3}{4}$

Answer

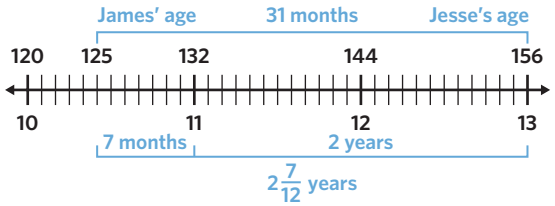
Ronald ran $14\frac{3}{4}$ laps.

11. What is the difference in years between Stephanie's two cats Jesse and James? Jesse is 156 months old and James is 125 months old. Express answer as a mixed fraction.

Key points

- Jesse is 156 months old.
- James is 125 months old.
- How much younger in years is James than Jesse? How much older in years is Jesse than James? Write the answer as a mixed number.

Explanation



Find the difference in age in months.

$156 - 125 = 31$ months

Convert to a mixed number to represent years.

$\frac{31}{12} = 2$ remainder 7

$= 2\frac{7}{12}$

Answer

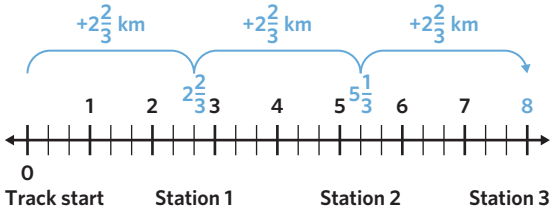
There is a $2\frac{7}{12}$ years difference between Jesse and James.

12. On the Overland Track there is a clean water station every $2\frac{2}{3}$ kilometres. Atif filled up his bottles at the start of the track. After how many kilometres did Atif refill, if he topped up his bottles at the third station?

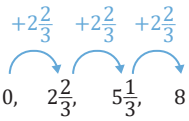
Key points

- There is a water station every $2\frac{2}{3}$ kilometres.
- Atif filled up at the beginning of the track.
- Atif refilled at the third station.
- After what distance did Atif refill?

Explanation



Find the fourth value of a sequence that starts at 0 and increases by $2\frac{2}{3}$.



Answer

Atif fills up after 8 kilometres.

Reasoning

13. a. A, B, C and D.

b. $9\frac{1}{4}$ sandwiches were eaten.

c. The UberEats driver is 7 km away.

d. The driver will arrive at 7:54 pm.

e. Suggested option 1: divide the rectangle by using only vertical or only horizontal lines.

Suggested option 2: divide the rectangle by using both vertical and horizontal lines.

Note: There are other possible options.

14. a. i) $\frac{10}{16}$ or $\frac{5}{8}$ of tile 1 is missing.

i) $\frac{12}{16}$ or $\frac{3}{4}$ of tile 2 is missing.

ii) $\frac{8}{16}$ or $\frac{1}{2}$ of tile 3 is missing.

b. Anna will need 2 full tiles to replace the missing pieces.

c. Anna can buy either option as they are both $60.

d. Suggested option 1: design the mosaic by colouring more than half the tiles.

Suggested option 2: design the mosaic by using 4 different colours in each quarter of the tile.

Note: There are other possible options.

Extra spicy

15. 80 16. 8 17. D 18. $\frac{18}{4}$ or $\frac{9}{2}$

Remember this?

19. Sophie kept 5 lollipops for herself.

20. There are 13 students on the team.

21. Room B has a perimeter of 30 m.

3C Equivalent fractions

Student practice

Worked example 1.

a. $\frac{4}{5} = \frac{16}{20}$

b. $3\frac{25}{35} = 3\frac{5}{7}$

c. $\frac{7}{9} = \frac{84}{108}$

d. $4\frac{28}{42} = 4\frac{2}{3}$

Worked example 2.

a. $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12} = \frac{10}{20}$

b. $\frac{30}{60} = \frac{15}{30} = \frac{10}{20} = \frac{5}{10} = \frac{1}{2}$

c. $\frac{2}{3} = \frac{22}{33} = \frac{30}{45} = \frac{40}{60} = \frac{100}{150}$

d. $\frac{400}{500} = \frac{200}{250} = \frac{100}{125} = \frac{16}{20} = \frac{8}{10}$

Worked example 3.

a. $\frac{11}{13}$ b. $-2\frac{3}{4}$ c. $\frac{3}{4}$ d. $8\frac{1}{11}$

Understanding worksheet

1. a. $\frac{2}{3} = \frac{4}{6}$ b. $\frac{3}{5} = \frac{9}{15}$ c. $\frac{1}{2} = \frac{4}{8}$ d. $\frac{3}{5} = \frac{6}{10}$

2. a. $\frac{4}{9} \overset{\times 5}{=} \frac{20}{45}$ b. $\frac{16}{24} \overset{\div 8}{=} \frac{2}{3}$ c. $\frac{7}{11} \overset{\times 7}{=} \frac{49}{77}$ d. $\frac{5}{15} \overset{\div 5}{=} \frac{1}{3}$

3. simplify; highest; common; equivalent; value

Fluency

4. a. $\frac{1}{3} = \frac{2}{6}$ b. $\frac{7}{6} = \frac{21}{18}$
   c. $\frac{16}{24} = \frac{2}{3}$ d. $\frac{45}{36} = \frac{5}{4}$
   e. $1\frac{3}{8} = 1\frac{15}{40}$ f. $3\frac{15}{21} = 3\frac{5}{7}$
   g. $\frac{7}{3} = 2\frac{7}{21}$ h. $4\frac{6}{10} = \frac{92}{20}$

5. a. $\frac{1}{4} = \frac{3}{12} = \frac{6}{24} = \frac{7}{28} = \frac{40}{160}$
   b. $\frac{5}{6} = \frac{10}{12} = \frac{30}{36} = \frac{75}{90} = \frac{150}{180}$
   c. $\frac{8}{5} = \frac{16}{10} = \frac{64}{40} = \frac{120}{75} = \frac{136}{85}$
   d. $\frac{9}{4} = \frac{18}{8} = \frac{72}{32} = \frac{108}{48} = \frac{144}{64}$
   e. $\frac{40}{120} = \frac{20}{60} = \frac{10}{30} = \frac{4}{12} = \frac{1}{3}$
   f. $\frac{200}{350} = \frac{100}{175} = \frac{40}{70} = \frac{20}{35} = \frac{4}{7}$
   g. $\frac{480}{440} = \frac{240}{220} = \frac{120}{110} = \frac{24}{22} = \frac{12}{11}$
   h. $\frac{260}{60} = \frac{130}{30} = \frac{65}{15} = \frac{52}{12} = \frac{13}{3}$

6. a. 4 b. 5 c. 3 d. 2
   e. 4 f. 9

7. a. $\frac{5}{6}$ b. $\frac{2}{5}$ c. $-\frac{1}{4}$ d. $\frac{1}{3}$
   e. $\frac{4}{3}$ f. $-4\frac{5}{11}$ g. $\frac{7}{4}$ h. $-\frac{28}{15}$

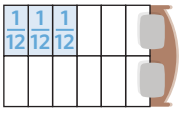
Problem solving

8. Which bedspread should Alice buy if she wants a bedspread that is $\frac{1}{3}$ blue?

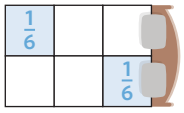
Key points

- Alice wants a bedspread that is $\frac{1}{3}$ blue.
- Which of the bedspreads has $\frac{1}{3}$ shaded blue?

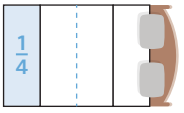
Explanation

A. 

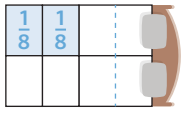
$\frac{3}{12}$ or $\frac{1}{4}$

B. 

$\frac{2}{6}$ or $\frac{1}{3}$

C. 

$\frac{1}{4}$

D. 

$\frac{2}{8}$ or $\frac{1}{4}$

A. $\frac{3}{12}$ is blue.
HCF is 3.
$\frac{3 \div 3}{12 \div 3} = \frac{1}{4}$ ✗

B. $\frac{2}{6}$ is blue.
HCF is 2.
$\frac{2 \div 2}{6 \div 2} = \frac{1}{3}$ ✓

C. $\frac{1}{4}$ is blue. ✗

D. $\frac{2}{8}$ is blue.
HCF is 2.
$\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$ ✗

Answer

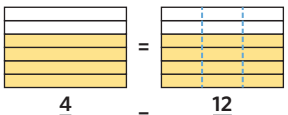
B

9. Dave and Susie have the same size chocolate bars. Who ate more if Dave broke his chocolate bar into 18 pieces and ate 12 of them, whilst Susie broke her chocolate bar into 6 pieces and ate 4 of them.

Key points

- Dave ate 12 out of 18 pieces.
- Susie ate 4 out of 6 pieces.
- Who ate a larger amount?

Explanation



$\frac{4}{6}$ = $\frac{12}{18}$

$\frac{4 \times 3}{6 \times 3} = \frac{12}{18}$

Both the numerator and denominator have been multiplied by the same factor (3), meaning the fractions are equivalent.

Answer

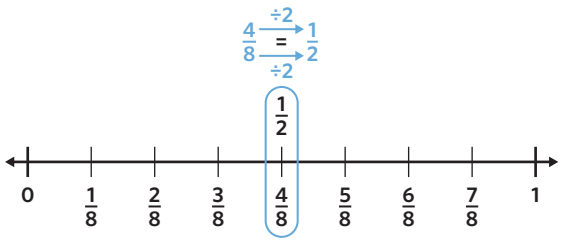
Dave and Susie ate the same amount of chocolate.

10. What fraction of the Harry Potter movies has Lucy seen? There are eight movies and she has watched four of them. Express in simplest form.

Key points

- Lucy has watched 4 out of 8 movies.
- What fraction (in simplest form) has she seen?

Explanation



Divide the numerator and denominator by the HCF of 10 and 12.

HCF is 2.

$\frac{4 \div 2}{8 \div 2} = \frac{1}{2}$

Answer

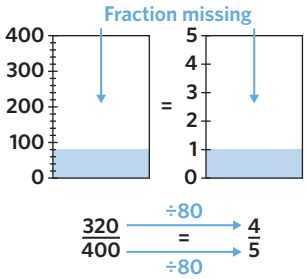
Lucy has seen $\frac{1}{2}$ of the Harry Potter movies.

11. Steven has 80 mL left in his 400 mL bottle of juice. What fraction of the bottle has he drunk (in simplest form)?

Key points

- There is 80 mL remaining in the bottle.
- The bottle contains 400 mL.
- What fraction of the juice is missing?

Explanation



Find the amount Steven has drunk (in mL).

$400 - 80 = 320$

Express this as a fraction of the entire bottle.

$\frac{320}{400}$

Fully simplify by dividing the numerator and denominator by the HCF.

HCF is 80

$\frac{320 \div 80}{400 \div 80} = \frac{4}{5}$

Answer

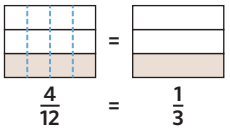
Steven has drunk $\frac{4}{5}$ of his bottle of juice.

12. Garfield divided a lasagna to share equally between 12 people. What fraction of people did not finish their serves if four people had leftovers?

Key points

- Garfield shares his lasagna equally between 12 people.
- 4 people did not finish.
- What is 4 out of 12 as a simplified fraction?

Explanation



Divide the numerator and denominator by the HCF of 4 and 12.

HCF is 4.

$\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$

Answer

$\frac{1}{3}$ of people did not finish their lasagna.

Reasoning

13. a. Danny ate $\frac{3}{6}$ or $\frac{1}{2}$ of his pizza.

b. Sophie would've eaten 2 of her original slices.

c. Yes, Danny and Sophie ate an equivalent fraction of their pizzas. They both ate $\frac{1}{2}$.

d. Suggested option 1: They could equally divide the leftover pizza.

Suggested option 2: If one of them wanted to take all the leftovers, they could pay the other person an equivalent amount.

Note: There are other possible options.

14. a. $\frac{5}{12}$ of the mixture is flour.

b. Elaine makes 7 raisin cookies.

c. Rebecca makes 20 chocolate chip cookies.

d. There is no way to simplify this fraction of cookies because the highest common factor is 1.

e. Suggested option 1: They can split the cost equally.

Suggested option 2: They can split the cost based on how many of their flavour cookies were baked.

Note: There are other possible options.

Extra spicy

15. Yes, but only if both the numerators and denominators are the same prime number.

16. B

17. There are 24 cats at the animal shelter.

18. $\frac{165}{297}$

Remember this?

19. D 20. D 21. B

3D Comparing fractions

Student practice

Worked example 1.

a. $\frac{1}{8} < \frac{3}{8}$ b. $\frac{17}{25} > \frac{15}{25}$ c. $\frac{13}{9} < \frac{19}{9}$ d. $\frac{23}{5} > 4\frac{2}{5}$

Worked example 2.

a. $\frac{4}{5} > \frac{7}{10}$ b. $2\frac{2}{3} < 2\frac{4}{5}$ c. $\frac{13}{5} > \frac{15}{6}$ d. $\frac{17}{6} > 2\frac{2}{15}$

Worked example 3.

a. $\frac{6}{5}, \frac{4}{3}, \frac{22}{15}, \frac{3}{2}$ b. $\frac{11}{2}, 5, \frac{37}{8}, 4\frac{2}{4}$

c. $-3\frac{3}{4}, -\frac{6}{2}, \frac{17}{7}, 2\frac{10}{14}$ d. $-\frac{5}{6}, -\frac{16}{18}, -\frac{25}{12}, -2\frac{7}{8}$

Understanding worksheet

1. a. < b. > c. < d. =

2. a. $\frac{2}{6} < \frac{5}{6}$

b. $\frac{5}{8} < \frac{7}{8}$

c. $\frac{60}{100} > \frac{20}{100}$

d. $1\frac{1}{3} < 1\frac{2}{3}$

3. numerators; different; lowest; common; denominator; equivalent

Fluency

4. a. > b. < c. > d. <
   e. > f. =

5. a. < b. < c. > d. <
   e. > f. <

6. a. $\frac{3}{6} < \frac{2}{3}$ b. $\frac{2}{4} > \frac{5}{12}$
   c. $\frac{4}{5} > \frac{17}{25}$ d. $\frac{1}{3} < \frac{2}{5}$
   e. $\frac{10}{4} = \frac{15}{6}$ f. $\frac{3}{7} > \frac{4}{11}$
   g. $\frac{8}{12} < 1\frac{1}{8}$ h. $\frac{16}{4} > \frac{23}{9}$

7. a. $\frac{1}{2} < \frac{3}{4} < \frac{5}{4} < \frac{3}{2}$ b. $\frac{1}{6} < \frac{1}{3} < \frac{2}{3} < 1\frac{1}{6}$
   c. $\frac{3}{10} < \frac{2}{5} < \frac{1}{2} < \frac{9}{10}$ d. $\frac{5}{8} < \frac{3}{4} < \frac{5}{6} < \frac{5}{4}$
   e. $-\frac{8}{4} < -\frac{2}{3} < -\frac{1}{2} < -\frac{1}{4}$ f. $-\frac{17}{8} < -\frac{5}{4} < -\frac{5}{6} < -\frac{2}{3}$

8. a. $\frac{8}{9} > \frac{2}{3} > \frac{5}{9} > \frac{1}{3}$ b. $\frac{5}{6} > \frac{3}{4} > \frac{2}{3} > \frac{1}{2}$
   c. $\frac{6}{2} > \frac{8}{5} > \frac{12}{10} > \frac{9}{10}$ d. $5\frac{1}{2} > 5 > 4\frac{3}{4} > \frac{18}{4}$
   e. $-\frac{31}{25} > -1\frac{2}{5} > -\frac{7}{4} > -2$ f. $4\frac{18}{27} > 3\frac{5}{6} > 3\frac{1}{3} > \frac{26}{9}$

Problem solving

9. Out of the four students who participated in the school spelling competition, who scored the highest? Annie got $\frac{8}{10}$ correct, Caitlyn got $\frac{7}{10}$ correct, Jax got $\frac{5}{10}$ correct and Milo got $\frac{9}{10}$ correct.

Key points
- Annie got $\frac{8}{10}$ correct.
- Caitlyn got $\frac{7}{10}$ correct.
- Jax got $\frac{5}{10}$ correct.
- Milo got $\frac{9}{10}$ correct.
- Who had the largest fraction of correct responses?

Explanation
The fractions have the same denominator so compare the numerators.
$\frac{5}{10} < \frac{7}{10} < \frac{8}{10} < \frac{9}{10}$
Jax < Caitlyn < Annie < Milo

Answer
Milo scored the highest in the spelling competition.

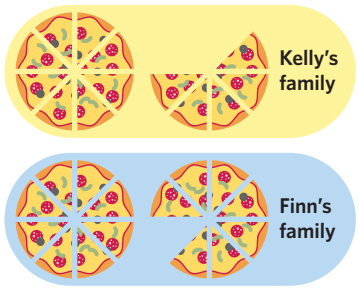
10. On Thursday night, Kelly and Finn's families both had takeaway pizza. Kelly and Finn are trying to figure out whose family ate more pizza. Kelly's family ate $1\frac{5}{8}$ pizzas and Finn's family ate 15 slices from pizzas that were sliced into 8.

Key points
- Kelly's family ate $1\frac{5}{8}$ pizzas.
- Finn's family ate $\frac{15}{8}$ pizzas.
- Which family ate more pizza?

Explanation
Kelly's family:
$1\frac{5}{8} = \frac{8}{8} + \frac{5}{8}$
$= \frac{13}{8}$

Finn's Family: $\frac{15}{8}$

$\frac{15}{8} > \frac{13}{8}$



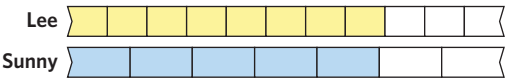
Answer
Finn's family ate more pizza than Kelly's family.

11. Lee and Sunny each have a large identical piece of ribbon. Lee cut eight-elevenths of his ribbon to make a flower. Sunny cut five-sevenths of her ribbon to make a wreath. Who used less ribbon for their craft?

Key points

- Lee cut $\frac{8}{11}$ of his ribbon to make a flower.
- Sunny cut $\frac{5}{7}$ of her ribbon to make a wreath.
- Who cut the shorter piece of ribbon?

Explanation



Express the fractions with the LCD of 77.

Ribbon Lee used: $\frac{8}{11} = \frac{56}{77}$

Ribbon Sunny used: $\frac{5}{7} = \frac{55}{77}$

$\frac{56}{77} > \frac{55}{77}$

$\frac{8}{11} > \frac{5}{7}$

Answer

Sunny used less ribbon than Lee.

12. Which competition had lower student participation in Douglas' class of 24 students? Three-quarters of the class entered the swimming competition and a third of the class didn't enter the athletics competition.

Key points

- $\frac{3}{4}$ of the class entered the swimming competition.
- $\frac{1}{3}$ of the class didn't enter the athletics competition.
- Which competition had a smaller fraction of student participation?

Explanation

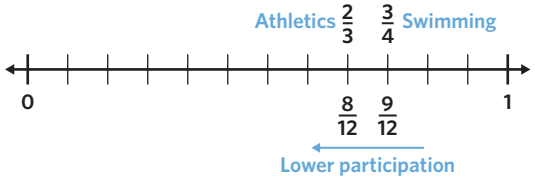
Express the fractions with the LCD of 12.

Swimming competition: $\frac{3}{4} = \frac{9}{12}$

Athletics competition: $\frac{2}{3} = \frac{8}{12}$

$\frac{9}{12} > \frac{8}{12}$

$\frac{3}{4} > \frac{2}{3}$



Answer

The athletics competition had lower student participation.

13. Harry and Jim have both drank some of their juice boxes. Harry has 60 mL left in his 240 mL juice box. Jim has 100mL left in his 500 mL juice box. Who drank a greater fraction of their juice box?

Key points

- Harry has 60 mL left from his 240 mL juice box.
- Jim has 100mL left from his 500 mL juice box.
- Who drank a greater fraction of their juice box?

Explanation

Harry drank $240 - 60 = 180$ mL.

$\frac{\textit{juice drunk}}{\textit{juice box capacity}} = \frac{180}{240} = \frac{3}{4}$

Jim drank $500 - 100 = 400$ mL.

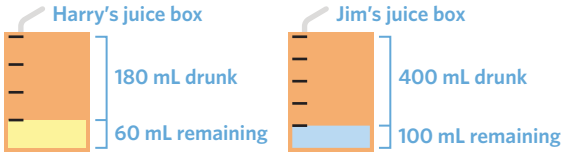
$\frac{\textit{juice drunk}}{\textit{juice box capacity}} = \frac{400}{500} = \frac{4}{5}$

LCD is 20.

Harry: $\frac{3}{4} = \frac{15}{20}$

Jim: $\frac{4}{5} = \frac{16}{20}$

$\frac{15}{20} < \frac{16}{20}$



Answer

Jim drank a greater fraction of his juice box.

Reasoning

14. a. The American Doughnut Kitchen doughnut has a greater fraction of sugar.

b. Dandee Donuts needs to add 2 grams of sugar to their recipe.

c. D

d. Suggested option 1: Tom recognises that he can still include less healthy foods that he enjoys eating as part of a balanced diet.

Suggested option 2: Tom thinks he is less likely to eat lots of unhealthy food if he includes them regularly in small amounts.

Note: There are other possible options.

15. a. Tom should eat kiwi fruits and samosas to follow his rule.

b. Tom can only eat the kiwi fruit according to his requirements.

c. Tom should buy the Peanutter brand.

d. Suggested option 1: the information could be shown in a table with percentages.

Suggested option 2: the information could be shown as a bar chart.

Note: There are other possible options.

Extra spicy

16. $1 + \frac{1}{\frac{1}{2} + \frac{1}{4}}$ is a smaller value than $1 + \frac{1}{\frac{1}{3} + \frac{1}{5}}$ because it has the same numerator but its denominator is larger.

17. A

18. Unshaded area $= \frac{13}{25}$

Shaded area $= \frac{12}{25}$

$\frac{13}{25} > \frac{12}{25}$

19. $\frac{45138}{90276}$

Remember this?

20. B 21. D 22. D

3E Adding and subtracting fractions

Student practice

Worked example 1.

a. $\frac{6}{7}$ b. $\frac{2}{15}$ c. $\frac{19}{10}$ or $1\frac{9}{10}$ d. $\frac{4}{17}$

Worked example 2.

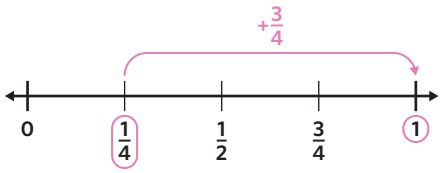
a. $\frac{5}{6}$ b. $\frac{1}{6}$ c. $\frac{23}{18}$ or $1\frac{5}{18}$ d. $\frac{1}{24}$

Worked example 3.

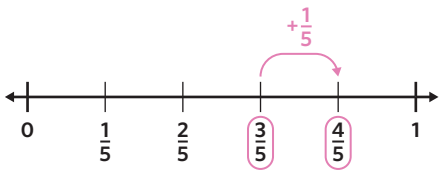
a. $4\frac{3}{4}$ b. $\frac{35}{12}$ or $2\frac{11}{12}$ c. $\frac{199}{36}$ or $5\frac{19}{36}$ d. $\frac{1}{4}$

Understanding worksheet

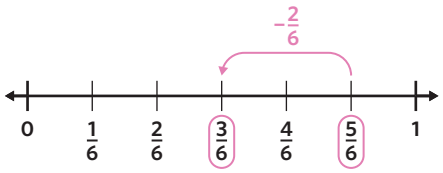
1. a.



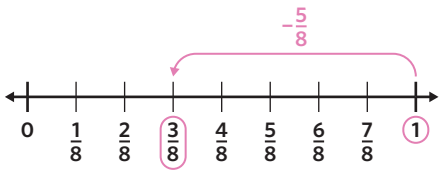
b.



c.



d.



2. a. $\frac{3}{6} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

b. $\frac{1}{8} + \frac{2}{4} = \frac{1}{8} + \frac{4}{8} = \frac{5}{8}$

c. $\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

d. $\frac{9}{28} + \frac{4}{7} = \frac{9}{28} + \frac{16}{28} = \frac{25}{28}$

3. same; equivalent; lowest; common; denominator

Fluency

4. a. $\frac{3}{5}$ b. $\frac{3}{4}$ c. $\frac{7}{10}$ d. $\frac{74}{100}$
e. $\frac{5}{8}$ f. $\frac{1}{12}$ g. $\frac{4}{7}$ h. $\frac{3}{13}$

5. a. $\frac{1}{2}$ b. $\frac{3}{8}$
c. $\frac{11}{9}$ or $1\frac{2}{9}$ d. $\frac{4}{15}$
e. $\frac{61}{28}$ or $2\frac{5}{28}$ f. $\frac{8}{45}$
g. $\frac{65}{48}$ or $1\frac{17}{48}$ h. $\frac{37}{35}$ or $1\frac{2}{35}$

6. a. $6\frac{3}{4}$ b. 4 c. $2\frac{7}{8}$ d. $\frac{3}{7}$
e. $5\frac{1}{2}$ f. $5\frac{3}{5}$ g. $5\frac{1}{3}$ h. $58\frac{9}{10}$

7. a. $3\frac{4}{7}$ b. $2\frac{1}{12}$ c. $10\frac{5}{24}$ d. $5\frac{7}{18}$
e. $2\frac{7}{8}$ f. $1\frac{1}{5}$ g. $1\frac{37}{56}$ h. $4\frac{29}{48}$

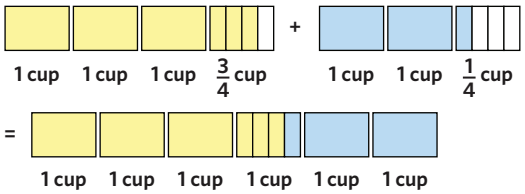
Problem solving

8. Mario used $3\frac{3}{4}$ cups of flour to make 60 cupcakes for his party. He wanted to make another 3 dozen cupcakes which required $2\frac{1}{4}$ more cups of flour. How much flour did Mario use altogether?

Key points

- Mario used $3\frac{3}{4}$ cups of flour to make 60 cupcakes for his party.
- Mario wanted to make another 3 dozen cupcakes which required $2\frac{1}{4}$ more cups of flour.
- How many cups of flour did Mario use to make 96 cupcakes?

Explanation



$3\frac{3}{4} + 2\frac{1}{4} = 3 + 2 + \frac{3}{4} + \frac{1}{4}$

$= 5 + \frac{4}{4}$

$= 6$

Answer

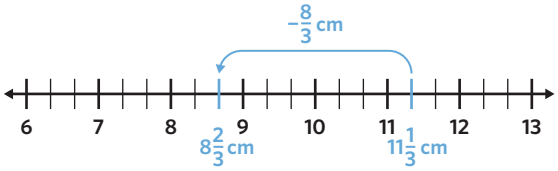
Mario uses 6 cups of flour.

9. Jeremy is measuring wood for a DIY project. How much wood should he cut off? The length of the piece of wood he has is $11\frac{1}{3}$ cm but he only needs $8\frac{2}{3}$ cm of wood for his project.

Key points

- Jeremy's initial piece of wood was $11\frac{1}{3}$ cm.
- Jeremy only needed $8\frac{2}{3}$ cm of wood for his project.
- How much wood will Jeremy need to cut off to meet the measurements of his project?

Explanation



$11\frac{1}{3} - 8\frac{2}{3} = \frac{34}{3} - \frac{26}{3}$

$= \frac{8}{3}$

$= 2\frac{2}{3}$

Answer

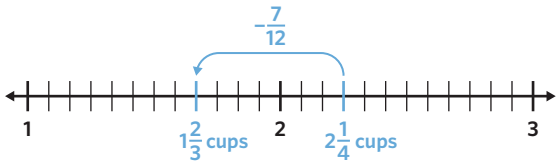
Jeremy cut $2\frac{2}{3}$ cm off his original piece.

10. Asha made two and one-quarter cups of kinetic sand. How many cups of sand did Asha give Kendra if after giving Kendra some sand, she had one and two-thirds cups of kinetic sand remaining?

Key points

- Asha made $2\frac{1}{4}$ cups of kinetic sand.
- After giving Kendra some sand, she had $1\frac{2}{3}$ cups of kinetic sand remaining.
- How many cups of sand did Asha give Kendra?

Explanation



LCD: 12

$$2\frac{1}{4} - 1\frac{2}{3} = 2\frac{3}{12} - 1\frac{8}{12}$$
$$= \frac{27}{12} - \frac{20}{12}$$
$$= \frac{7}{12}$$

Answer

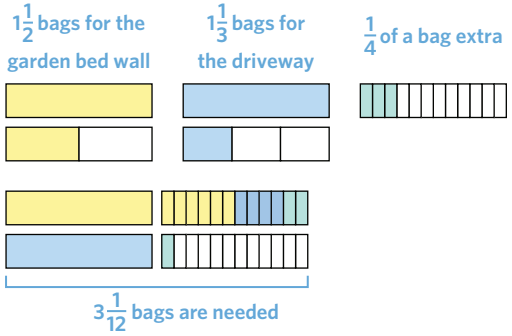
Asha gave Kendra $\frac{7}{12}$ cups of kinetic sand.

11. How many whole bags of cement does Ronald need to buy? He plans to use one and a half bags to build a wall around his garden bed and another one and a third bags to repair his driveway. He likes to have an excess of at least a quarter bag in case he makes a mistake.

Key points

- He plans to use $1\frac{1}{2}$ bags to build a wall around his garden bed.
- He plans to use another $1\frac{1}{3}$ bags to repair his driveway.
- He needs $\frac{1}{4}$ of a bag extra as spare.
- How many bags of cement should Ronald buy, rounded up to a whole bag?

Explanation



LCD: 12

$$1\frac{1}{2} + 1\frac{1}{3} + \frac{1}{4} = \frac{3}{2} + \frac{4}{3} + \frac{1}{4}$$
$$= \frac{18}{12} + \frac{16}{12} + \frac{3}{12}$$
$$= \frac{37}{12}$$
$$= 3\frac{1}{12}$$

Answer

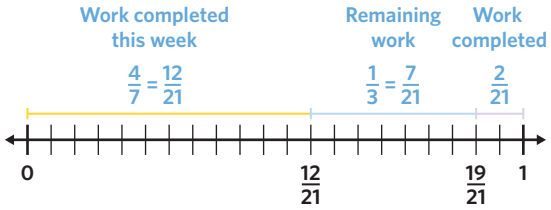
Ronald needs 4 bags of cement mix.

12. Leonardo is painting a portrait for a customer. This week, he completed four-sevenths of the portrait and has one-third more of the portrait to complete. How much of the portrait had Leonardo completed before commencing this week?

Key points

- Leonardo completed $\frac{4}{7}$ of the portrait this week.
- Leonardo has $\frac{1}{3}$ of the portrait remaining to complete after this week.
- How much of the portrait did Leonardo complete prior to his work this week?

Explanation



$1 = \textit{completed portrait}$

$$1 - \frac{4}{7} - \frac{1}{3}$$

LCD: 21

$$= \frac{21}{21} - \frac{12}{21} - \frac{7}{21}$$
$$= \frac{2}{21}$$

Answer

Leonardo completed $\frac{2}{21}$ of the portrait before commencing work this week.

Reasoning

13. a. Anthony: $\frac{3}{4}$, Hannah: $\frac{2}{5}$

b. There is $1\frac{3}{20}$ buckets left.

c. Anthony and Hannah used $\frac{17}{20}$ buckets to paint the treehouse.

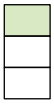
d. They do not have enough leftover paint for both rooms. They need $\frac{25}{20}$ buckets of paint and only have $\frac{23}{20}$ buckets leftover.

e. Suggested option 1: they could decide to completely paint one room.

Suggested option 2: they could decide to evenly split the leftover paint.

Note: There are other possible options.

14. a. C

b. 

Answers may vary.

c. No, Luigi does not have enough green glass left to create design 3. He only has 2 m^2 of green glass left over but he needs 3 m^2.

d. Suggested option 1: Luigi should not charge Mario to make a window for him because they are friends.

Suggested option 2: Mario should pay for the cost of the materials Luigi needs to make the window as well as buy him lunch.

Note: There are other possible options.

Extra spicy

15. A

16. D

17. $\frac{31}{4}$ or $7\frac{3}{4}$

18. a. 72 cm b. 76 cm c. 68 cm

Remember this?

19. B

20. Helen spent 5 more minutes making TikToks than Maurice on Saturday.

21. D

3F Multiplying fractions

Student practice

Worked example 1.

a. $\frac{12}{5}$ or $2\frac{2}{5}$ b. $\frac{8}{3}$ or $2\frac{2}{3}$ c. $\frac{3}{2}$ or $1\frac{1}{2}$ d. $\frac{60}{6}$ or 10

Worked example 2.

a. $\frac{8}{18}$ or $\frac{4}{9}$ b. $\frac{27}{5}$ or $5\frac{2}{5}$ c. $\frac{81}{7}$ or $11\frac{4}{7}$ d. $\frac{91}{20}$ or $4\frac{11}{20}$

Worked example 3.

a. $\frac{2}{7}$ b. $\frac{121}{36}$ or $3\frac{13}{36}$ c. $\frac{166}{5}$ or $33\frac{1}{5}$ d. $\frac{21}{4}$ or $5\frac{1}{4}$

Understanding worksheet

1. a. $\frac{1}{2}$ of 50 is 25 b. $\frac{3}{4}$ of 20 is 15
   c. $\frac{5}{6}$ of 30 is 25 d. $\frac{3}{5}$ of 100 is 60

2. a. 4 groups of $\frac{1}{6} = \frac{4}{6}$ b. 3 groups of $\frac{3}{4} = \frac{9}{4}$
   c. 2 groups of $\frac{5}{6} = \frac{10}{6}$ d. 6 groups of $\frac{3}{8} = \frac{18}{8}$

3. product; smaller; highest; common; factor

Fluency

4. a. Smaller b. Larger
   c. Smaller d. Smaller
   e. Larger f. Larger
   g. Equal to h. Larger

5. a. $1\frac{1}{2}$ or $\frac{3}{2}$ b. $2\frac{2}{5}$ or $\frac{12}{5}$
   c. $5\frac{1}{4}$ or $\frac{21}{4}$ d. $3\frac{3}{7}$ or $\frac{24}{7}$
   e. $6\frac{3}{4}$ or $\frac{27}{4}$ f. $3\frac{3}{4}$ or $\frac{15}{4}$
   g. $5\frac{3}{5}$ or $\frac{28}{5}$ h. $4\frac{4}{5}$ or $\frac{24}{5}$

6. a. $\frac{4}{21}$ b. $\frac{24}{25}$ c. $\frac{11}{24}$ d. $\frac{15}{28}$
   e. $\frac{16}{45}$ f. $\frac{20}{39}$ g. $\frac{1}{2}$ h. $\frac{11}{13}$

7. a. $\frac{13}{21}$ b. $\frac{11}{32}$ c. $\frac{10}{21}$ d. $\frac{1}{6}$
   e. $\frac{3}{16}$ f. $\frac{1}{4}$ g. $\frac{1}{24}$ h. $\frac{1}{16}$

8. a. $\frac{8}{11}$ b. $2\frac{2}{13}$ or $\frac{28}{13}$
   c. $1\frac{17}{28}$ or $\frac{45}{28}$ d. $\frac{9}{34}$
   e. $9\frac{4}{5}$ or $\frac{49}{5}$ f. $4\frac{1}{2}$ or $\frac{9}{2}$
   g. $\frac{65}{72}$ h. $12\frac{5}{18}$ or $\frac{221}{18}$

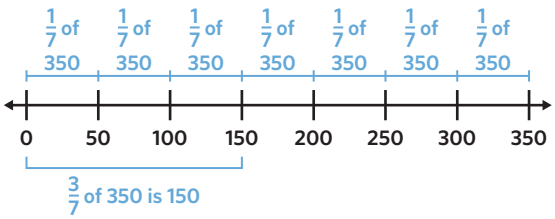
Problem solving

9. There are 350 members of a local football club. How many people are members of both the local football club and soccer club if $\frac{3}{7}$ of the football club members are also members of the soccer club.

Key points

- 350 members of a local football club.
- $\frac{3}{7}$ are also members of a local soccer club.
- How many people are in both the local football club and local soccer club?

Explanation



$\frac{3}{7}$ of $350 = \frac{3}{7} \times 350$

350 and 7 have a HCF of 7.

$\frac{3}{7} \times \frac{350}{1} = \frac{3}{1} \times \frac{50}{1}$

$= 150$

Answer

150 people are members of both the football and soccer club.

10. Two-sevenths of the skyscrapers in Melbourne are over 200 m tall while seven-sixteenths of these buildings are over 250 m tall. What fraction of the skyscrapers in Melbourne are over 250 m tall?

Key points

- $\frac{2}{7}$ of the skyscrapers in Melbourne are over 200 m tall.
- $\frac{7}{16}$ of this amount are over 250 m tall.
- What fraction of skyscrapers in Melbourne are more than 250 m tall?

Explanation

$\frac{2}{7}$ of $\frac{7}{16} = \frac{2}{7} \times \frac{7}{16}$

2 and 16 have a HCF of 2.

7 and 7 have a HCF of 7.

$\frac{2^{1}}{7_{1}} \times \frac{7^{1}}{16_{8}} = \frac{1}{1} \times \frac{1}{8}$

$= \frac{1}{8}$

Answer

$\frac{1}{8}$ of skyscrapers in Melbourne are over 250 m tall.

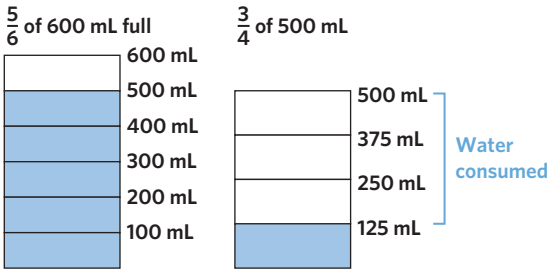
11. At the start of Rugby training Edith's 600 mL water bottle was five-sixths full. During training she drinks three-quarters of her bottle. How much water did Edith consume at training?

Key points

- Edith's has a 600 mL water bottle.
- At the start of training the bottle was $\frac{5}{6}$ full.
- She drinks $\frac{3}{4}$ of her bottle.
- How much water did Edith drink during training?

Explanation

Start of training: **Edith consumed:**



$\frac{5}{6}$ of 600 mL $= \frac{5}{6} \times 600$

$= 500$ mL

Edith started training with 500 mL.

$\frac{3}{4}$ of 500 mL $= \frac{3}{4} \times 500$

$= \frac{1500}{4}$

$= 375$ mL

Answer

Edith drank 375 mL during rugby training.

12. Max and Lucy drove to a music festival together. Max drove the first $\frac{3}{8}$ of the distance before he needed a break. Lucy took over and drove $\frac{3}{5}$ of the remaining distance. What fraction of the total trip did Lucy drive?

Key points

- Max drove $\frac{3}{8}$ of the trip.
- Lucy drove $\frac{3}{5}$ of the remaining trip.
- How much of the total trip did Lucy drive?

Explanation

1 − fraction of total distance driven = remaining distance (fraction)

$1 - \frac{3}{8} = \frac{5}{8}$

Find $\frac{3}{5}$ of $\frac{5}{8}$.

5 and 5 have a HCF of 5.

$\frac{3}{\cancel{5}^{1}} \times \frac{\cancel{5}^{1}}{8} = \frac{3}{1} \times \frac{1}{8}$

$= \frac{3}{8}$

Answer

Lucy drove $\frac{3}{8}$ of the total trip.

13. Mario is baking scones for his party. If Mario wants to make 36 scones, how many cups of flour does he need? The recipe he is using requires $2\frac{3}{4}$ cups of flour to make 16 scones.

Key points

- Mario wants to make 36 scones.
- Find how many times more 36 scones is than 16 scones.
- Mario's recipe requires $2\frac{3}{4}$ cups of flour to make 16 scones.
- How many cups of flour does Mario need to make 36 scones?

Explanation

Divide 36 by 16 to find how many times Mario needs to multiply the recipe.

$\frac{36}{16} = \frac{9}{4} = 2\frac{1}{4}$

Multiply the cups of flour needed for 16 scones by $2\frac{1}{4}$.

$2\frac{3}{4} \times 2\frac{1}{4} = \frac{11}{4} \times \frac{9}{4}$

$= \frac{99}{16}$

$= 6\frac{3}{16}$

Answer

Mario needs $6\frac{3}{16}$ cups of flour to make 36 scones.

Reasoning

14. a. B and C are correct.
    b. The price will be \$20.
    c. The shirt will be \$5 on Monday.
    d. No, the shirt will never reach \$0 exactly.
    e. Suggested option 1: It is better because Kalvin can sell items without large reductions in prices.

    Suggested option 2: It is worse because Kalvin may not attract as many people to the sale.

    Note: There are other possible options.

15. a. Sara's garden is $\frac{64}{9}$ or $7\frac{1}{9}$ square metres.
    b. Sara does not have enough space for 4 columns of tulips.
    c. Sara has $5\frac{11}{18}$ square metres for the other flowers.
    d. The daffodils will take up $4\frac{5}{24}$ square metres.
    e. Suggested option 1: design the garden by dividing it into 3 equal parts.

    Suggested option 2: design the garden by dividing it into 3 unequal parts.

    Note: There are other possible options.

Extra spicy

16. $1\frac{7}{16}$ or $\frac{23}{16}$ 17. $\frac{2}{3}$ 18. C 19. F

Remember this?

20. C
21. 1, 2, 3, 6, 7, 14, 21, 42
22. The movie ended at 9:45 pm.

3G Dividing fractions

Student practice

Worked example 1.

a. $\frac{1}{11}$ b. $\frac{3}{25}$ c. $\frac{7}{15}$ d. $\frac{8}{77}$

Worked example 2.

a. $\frac{25}{3}$ or $8\frac{1}{3}$ b. $\frac{1}{35}$ c. $\frac{10}{3}$ or $3\frac{1}{3}$ d. $\frac{4}{9}$

Worked example 3.

a. $\frac{32}{21}$ or $1\frac{11}{21}$ b. $\frac{19}{16}$ or $1\frac{3}{16}$ c. $\frac{3}{4}$ d. $\frac{1}{6}$

Understanding worksheet

1. a. 10 b. 6 c. 8 d. 5

2. a. 5 b. 6 c. 6 d. $1\frac{1}{2}$

3. whole; number; fraction; multiply; reciprocal

Fluency

4. a. $\frac{1}{5}$ b. 3 c. $\frac{7}{4}$ d. $\frac{1}{13}$
   e. $\frac{4}{9}$ f. 1 g. $\frac{5}{9}$ h. $\frac{11}{35}$

5. a. 12 b. $\frac{2}{15}$
   c. $\frac{24}{5}$ or $4\frac{4}{5}$ d. $\frac{1}{5}$
   e. $\frac{1}{15}$ f. $\frac{21}{5}$ or $4\frac{1}{5}$
   g. $\frac{1}{10}$ h. $\frac{90}{7}$ or $12\frac{6}{7}$

6. a. $\frac{1}{6}$ b. $\frac{5}{24}$
   c. $\frac{8}{11}$ d. $\frac{5}{27}$
   e. $\frac{22}{21}$ or $1\frac{1}{21}$ f. $\frac{65}{16}$ or $4\frac{1}{16}$
   g. $\frac{14}{15}$ h. 4

7. a. $\frac{12}{25}$ b. $\frac{16}{15}$ or $1\frac{1}{15}$
   c. $\frac{10}{7}$ or $1\frac{3}{7}$ d. $\frac{39}{20}$ or $1\frac{19}{20}$
   e. $\frac{18}{25}$ f. $\frac{3}{4}$
   g. $\frac{11}{2}$ or $5\frac{1}{2}$ h. $\frac{1}{2}$

Problem solving

8. Richard has $\frac{1}{3}$ of his patio left to tile. Each tile takes up $\frac{1}{20}$ of the entire patio. How many more tiles does he need to finish tiling the patio?

Key points

- Richard has $\frac{1}{3}$ of the patio left to tile.
- Each new tile is $\frac{1}{20}$ of the patio.
- How many more tiles does he need to finish tiling?

Explanation

Divide $\frac{1}{3}$ by $\frac{1}{20}$ to find how many more tiles Richard needs. Round the answer up as he can't buy a fraction of a tile.

$$\frac{1}{3} \div \frac{1}{20} = \frac{1}{3} \times \frac{20}{1}$$
$$= \frac{20}{3}$$
$$= 6\frac{2}{3}$$
$$= 7 \text{ full tiles (rounded up)}$$

Answer

A

9. Luke has a piece of wood that is $2\frac{5}{6}$ metres long that he wants to cut into smaller pieces. Luke wants to cut each piece into $\frac{1}{4}$ of a metre in length. How many $\frac{1}{4}$-metre pieces of wood will Luke end up with?

Key points

- Luke has a piece of wood that is $2\frac{5}{6}$ metres long.
- Luke wants to cut each piece into $\frac{1}{4}$ of a metre in length.
- How many pieces of wood will Luke end up with that are $\frac{1}{4}$ m in length?

Explanation

Divide $2\frac{5}{6}$ by $\frac{1}{4}$ to find how many pieces of wood Luke will end up with. Round the answer down as he can't have a fraction of a piece.

Divide the total length of wood by the length of each piece.

$$2\frac{5}{6} \div \frac{1}{4} = \frac{17}{6} \div \frac{1}{4}$$
$$= \frac{17}{6} \times \frac{4}{1}$$
$$= \frac{17}{3} \times \frac{2}{1}$$
$$= \frac{34}{3}$$
$$= 11\frac{1}{3}$$

Answer

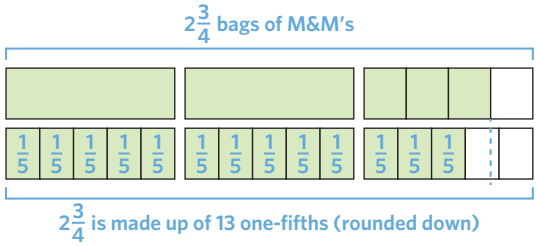
Luke will end up with 11 pieces of wood that are $\frac{1}{4}$ of a metre in length.

10. Bowser wants to know how many goodie bags he can make for all his party guests. He uses one-fifth of a bag of M&M's to make one goodie bag and has two and three-quarter bags of M&Ms to use.

Key points

- Bowser uses $\frac{1}{5}$ of a bag of M&M's to make 1 goodie bag.
- He has $2\frac{3}{4}$ bags of M&M's.
- How many whole goodie bags can Bowser make if each includes $\frac{1}{5}$ of a bag of M&M's?

Explanation



Divide the number of bags of M&M's by the fraction of a bag in each goodies bag.

$$2\frac{3}{4} \div \frac{1}{5} = \frac{11}{4} \div \frac{1}{5}$$
$$= \frac{11}{4} \times \frac{5}{1}$$
$$= \frac{55}{4}$$
$$= 13\frac{3}{4}$$
$$= 13 \text{ (rounded down)}$$

Answer

Bowser can make 13 full goodie bags.

11. Britt wants to exchange 90 Australian dollars (AUD) for United States dollars (USD). How much USD will Britt get after the exchange if the bank teller explains that she can buy 1 USD with $1\frac{1}{2}$ AUD?

Key points

- Britt has 90 AUD to exchange.
- 1 USD = $1\frac{1}{2}$ AUD.
- How much USD can she buy with 90 AUD?

Explanation

amount exchanged (AUD) ÷ exchange rate
(value of 1 AUD in USD) = USD received

As Britt is exchanging AUD for USD, we know that the amount in USD must be less than the AUD exchanged.

Divide the amount exchanged (90 AUD) by the exchange rate $\left(1\frac{1}{2}\right)$.

$$90 \div 1\frac{1}{2} = \frac{90}{1} \div \frac{3}{2}$$
$$= \frac{90}{1} \times \frac{2}{3}$$
$$= \frac{30}{1} \times \frac{2}{1}$$
$$= 60$$

Answer

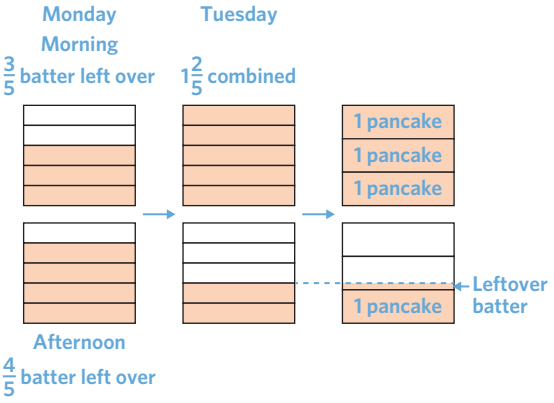
Britt can exchange 90 Australian dollars for 60 US dollars.

12. On Monday morning, Issy made 1 cup of pancake batter and only used $\frac{2}{5}$ of the batter. That afternoon, she made a fresh cup of pancake batter and used $\frac{1}{5}$. The next day she combined all of the remaining batter. If $\frac{1}{3}$ of a cup of batter makes one pancake, how many whole pancakes can Issy make with the remaining batter?

Key points

- Issy made 1 cup of pancake batter and used $\frac{2}{5}$.
- She then made a new cup of pancake batter and used $\frac{1}{5}$.
- She then combined the remaining batter.
- $\frac{1}{3}$ of a cup makes 1 pancake.
- Once Issy combines the remaining batter from her first 2 cups, how many pancakes can she make?

Explanation



batter made − batter used = remaining batter

Monday morning: $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$

Monday afternoon: $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

Total remaining (combined): $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$ or $1\frac{2}{5}$

Divide the combined batter by $\frac{1}{3}$ to find the number of pancakes.

$$\frac{7}{5} \div \frac{1}{3} = \frac{7}{5} \times \frac{3}{1}$$
$$= \frac{21}{5}$$
$$= 4\frac{1}{5}$$
$$= 4 \text{ (rounded down)}$$

Answer

Issy can make 4 whole pancakes.

Reasoning

13. a. A

b. Each person received $\frac{3}{20}$ of a whole cake.

c. $1\frac{3}{4}$ of the three cakes were left over for dessert on the third day.

d. Each person will get $\frac{1}{4}$ of a whole cake.

e. Suggested option 1: This is a fair amount of cake as it is shared equally.

Suggested option 2: Helen should invite more people because $\frac{1}{4}$ of a cake is more than they had on any other day.

Note: There are other possible options.

14. a. D

b. Rhea would need 35 posts for her new design.

c. Rhea needs 21 posts.

d. Suggested option 1: a design could be created with gaps like Rhea had planned.

Suggested option 2: a simple design could be created without any gaps.

Note: There are other possible options.

Extra spicy

15. $743\frac{3}{5}$ 16. A 17. 18 18. B

Remember this?

19. D

20. Corner *X* moves to C.

21. C

3H Ratios

Student practice

Worked example 1.

a. 3 : 8 b. 9 : 4 c. 7 : 24 d. 6 : 24

Worked example 2.

a. 4 : 2 : 3 b. 8 : 21 c. 49 : 11 d. 17 : 42

Worked example 3.

a. 3 : 2 b. 1 : 4 c. 4 : 17 d. 7 : 9

Worked example 4.

a. 10 sausages for $20 b. 500 mL tub of hummus

Understanding worksheet

1. a. 2 : 4 b. 2 : 5 c. 4 : 3 d. 3 : 7

2. a. 3 : 6 → ÷3, ÷3 → 1 : 2
   b. 9 : 12 → ÷3, ÷3 → 3 : 4
   c. 12 : 18 → ÷6, ÷6 → 2 : 3
   d. 16 : 20 → ÷4, ÷4 → 4 : 5

3. relative; quantities; units; same; simplifying

Fluency

4. a. 1 : 5 b. 5 : 7
   c. 3 : 1 d. 2 : 15
   e. 5 : 4 f. 5 : 8
   g. 2 : 3 h. 3 : 3 or 1 : 1

5. a. 3 : 5 b. 4 : 2 : 3
   c. 5 : 2 : 1 d. 4 : 3
   e. 8 : 6 : 6 or 4 : 3 : 3 f. 13 : 18
   g. 5 : 7 h. 6 : 15 or 2 : 5

6. a. 3 : 5 b. 2 : 3
   c. 7 : 6 d. 7 : 12
   e. 2 : 9 f. 11 : 7
   g. 7 : 12 : 9 h. 13 : 11 : 7

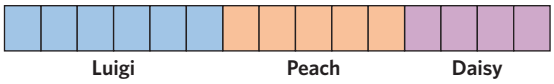
7. a. A box of 6 muesli bars for $12.
   b. A box of 48 rolls of premium toilet paper for $96.
   c. A 700 mL bottle of juice for $14.
   d. A box of 36 Pokemon card parks for $180.
   e. A 1.5 kg box of cereal for $30.
   f. A 64 gb USB stick for $16.

Problem solving

8. From a bag of jelly beans, Luigi receives six jelly beans, Daisy receives four jelly beans and Peach receives five jelly beans. What is the ratio of jelly beans for Luigi to Daisy to Peach?

Key points
- Luigi receives six jelly beans, Daisy receives four jelly beans and Peach receives five jelly beans.
- What is the ratio of jelly beans for Luigi to Daisy to Peach?.

Explanation



Write the ratio in the order Luigi : Daisy : Peach

6 : 4 : 5

Answer

6 : 4 : 5

9. At the Melbourne Zoo, there are 75 monkeys, 25 elephants and 250 wombats. What is the ratio of monkeys to elephants to wombats in the simplest form?

Key points
- There are 75 monkeys, 25 elephants and 250 wombats.
- Find the ratio of animals in the form monkeys : elephants : wombats.
- Express in the simplest form.

Explanation

To simplify, divide each value in the ratio by the HCF 25.

75 : 25 : 250 → ÷25, ÷25, ÷25 → 3 : 1 : 10

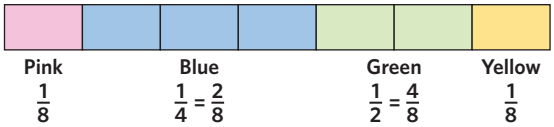
Answer

The simplest ratio of monkeys to elephants to wombats is 3 : 1 : 10.

10. Aamil has prepared some party bags for guests at his party. He knows that in each bag, $\frac{1}{8}$ of smarties are pink, $\frac{1}{4}$ are blue, $\frac{1}{2}$ are green. What is the ratio of yellow smarties to all smarties that each guest receives?

Key points
- $\frac{1}{8}$ of smarties are pink, $\frac{1}{4}$ are blue, $\frac{1}{2}$ are green.
- What is the ratio of yellow smarties to all smarties?

Explanation



Fraction of yellow smarties $= 1 - \frac{1}{8} - \frac{1}{4} - \frac{1}{2}$

$= \frac{8}{8} - \frac{1}{8} - \frac{2}{8} - \frac{4}{8}$

$= \frac{1}{8}$

8 represents the total number of parts, and 1 represents the number of parts that are yellow.

1 : 8

Answer

The ratio of yellow smarties to all smarties is 1 : 8

11. Rex is painting his house purple. He goes to the paint shop and is shown two different brands of can. Which can is the cheapest per litre if one can is 10 L and costs $150 and the other can is $36 for 2 L.

Key points
- One can costs $150 for 10 L.
- The other can costs $36 for 2 L.
- Which can is the best buy per litre?

Explanation

10 L for $150 : 10 : 150 ÷10 ↓ ↓ ÷10 : 1 : 15 — 1 L costs $15

2 L for $36 : 2 : 36 ÷2 ↓ ↓ ÷2 : 1 : 18 — 1 L costs $18

10 L for $150 is the best buy.

Answer

10 L for $150 is the cheapest per litre.

12. Lorraine is making brownies. The recipe contains flour, cocoa, and sugar in a ratio of 5 : 2 : 3. How much flour and cocoa will she need? Lorraine knows that she needs to use 375 grams of sugar but the quantities for the other ingredients are missing.

Key points
- Lorraine's brownie recipe uses flour, cocoa, and sugar in a ratio of 5 : 2 : 3.
- She needs to use 375 grams of sugar.
- Write the amount of flour and cocoa she will need.

Explanation

375 g of sugar represents 3 parts of the ratio.

$375 \div 3 = 125$ g per part

Multiply each share in the ratio by 125 to find the quantities of flour and cocoa.

5 : 2 : 3
↓×125 ↓×125 ↓×125
625 : 250 : 375

Answer

Lorraine will need 625 g of flour and 250 g of cocoa.

Reasoning

13. a. Jack will have to sail a distance of 2750 metres.
b. The length Jack travelled on foot is represented by 8 cm on the map.
c. The ratio of doubloons received by Jack, his vice-captains and the crewmen is 1 : 4 : 5.
d. 3000 gold doubloons for 102 square metres is the best buy.
e. Suggested option 1: Jack should get a greater proportion of the doubloons for being the captain.
Suggested option 2: The crewmen should get a greater proportion of the doubloons because they do the most physical labour.
Note: There are other possible options.

14. a. $\frac{1}{4}$ of the cards in a deck have a diamond suit.
b. The ratio of clubs : diamonds : hearts : spades is 1 : 1 : 1 : 1.
c. After adding the cards the ratio of clubs : diamonds : hearts : spades is 1 : 1 : 2 : 1.
d. Suggested option 1: this deck can still be used to play card games because not all card games involve the suits.
Suggested option 2: this deck shouldn't be used to play card games because some card games require the suits to be balanced equally.
Note: There are other possible options.

Extra spicy

15. 10 cm 16. 11 : 15 17. 28 : 12 : 3 18. 16 mL

Remember this?

19. C
20. The minimum number of toys Vincent could own is 38.
21. The mass of one full cheesecake is 375 grams.

Chapter 3 extended application

1. a. Tom makes $10 more by selling a whole cake in $\frac{1}{8}$ slices.
b. To make as much money as possible, Tom should sell the cake for $3 per $\frac{1}{16}$.
c. Suggested option 1: It could be harder to find 8 different customers instead of finding just one.
Suggested option 2: Time and effort is required to cut, package, and sell 8 individual pieces.
Note: There are other possible options.
d. Suggested option 1: Having more slices could allow you to sell at a lower cost.
Suggested option 2: A whole cake could be sold for special occasions, and the price could be adjusted to suit this.
Note: There are other possible options.

2. a. Stan is behind schedule because $\frac{1}{5}$ is less than the amount he expected to have painted $\left(\frac{3}{10}\right)$.
b. It will take Stan 15 days.
c. Suggested option 1: Stan should pick option A if he prefers to work for shorter chunks of time.
Suggested option 2: Stan should pick Option B if he doesn't want to miss his favourite TV show.
Note: There are other possible options.
d. Suggested option 1: Stan could ask his friends to help him with the painting. This would get the job done quicker, but his friends might not be so impressed when they come for the sleepover.
Suggested option 2: Stan can choose not to complete the cubby house before the sleepover. Stan might not like this idea because instead of having the coolest cubby house, Stan might upset his friends!
Note: There are other possible options.

3. a. $\frac{4}{9}$ of daily energy consumption is during daylight hours.
b. You use a larger fraction of your energy during daylight hours compared to the average.
c. 30 m^2 of roof space is usable.
d. Yes, there is enough roof space for a 5kW system.
e. The roof does not have enough roof space for the 250 W solar panels so the only option will be to choose the 330 W solar roof panels.
f. $\frac{1}{6}$ of the roof will be covered by the 330 W solar panels.

Chapter 3 review

Multiple choice

1. A 2. C 3. D 4. C 5. A

Fluency

6. a. $\frac{4}{7}$ b. $\frac{3}{5}$ c. $\frac{6}{8}$ d. $\frac{2}{9}$

7. a. $\frac{11}{6}$ b. $\frac{30}{7}$ c. $\frac{16}{5}$ d. $\frac{31}{12}$

8. a. $1\frac{4}{5}$ b. $2\frac{3}{4}$ c. $6\frac{1}{2}$ d. $4\frac{1}{3}$

9. a. $\frac{1}{6} = \frac{2}{12} = \frac{6}{36} = \frac{10}{60} = \frac{15}{90}$
   b. $\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{30}{40} = \frac{120}{160}$
   c. $\frac{120}{180} = \frac{80}{120} = \frac{40}{60} = \frac{6}{9} = \frac{2}{3}$
   d. $\frac{100}{160} = \frac{50}{80} = \frac{20}{32} = \frac{10}{16} = \frac{5}{8}$

10. a. $\frac{2}{3}$ b. $\frac{3}{5}$ c. $\frac{3}{7}$ d. $2\frac{1}{4}$

11. a. > b. = c. > d. <

12. a. $\frac{5}{4} < 1\frac{3}{4} < \frac{9}{4}$ b. $\frac{3}{6} < \frac{7}{12} < \frac{2}{3}$
    c. $\frac{2}{5} < \frac{1}{2} < \frac{6}{10}$ d. $\frac{17}{24} < \frac{9}{12} < \frac{13}{16}$

13. a. $\frac{5}{6}$ b. $-\frac{1}{20}$
    c. $1\frac{11}{20}$ or $\frac{31}{20}$ d. $5\frac{3}{8}$ or $\frac{43}{8}$

14. a. 3 b. $1\frac{2}{7}$ or $\frac{9}{7}$ c. 1 d. $\frac{1}{36}$

15. a. 4 b. $\frac{3}{10}$
    c. $3\frac{17}{33}$ or $\frac{116}{33}$ d. $2\frac{7}{40}$ or $\frac{87}{40}$

16. a. 15 b. $\frac{5}{48}$
    c. $\frac{21}{22}$ d. $1\frac{11}{21}$ or $\frac{32}{21}$

17. a. 3 : 4 b. 1 : 6 c. 4 : 9 d. 6 : 5

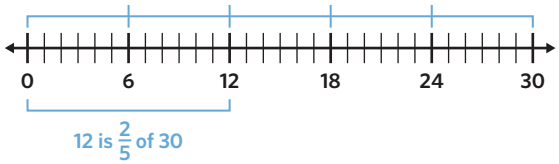
18. a. A box of 12 soft drink cans for $24.
    b. A large bag of 40 tennis balls for $80.
    c. A small 200 g bag of chips for $2.
    d. A 600 mL tub of premium ice cream for $18.

Problem solving

19. Henderson measures his model train against his 30 cm ruler. The train is 12 cm long. What is the length of the train as a fraction of the total length of the ruler?

Key points
- Henderson's ruler is 30 cm long.
- Henderson's train is 12 cm long.
- Find the fraction $\frac{\textit{length of train}}{\textit{length of ruler}}$.

Explanation



$\frac{\textit{length of train}}{\textit{length of ruler}} = \frac{12}{30}$

$= \frac{2}{5}$

Answer
The length of the train is $\frac{2}{5}$ of the length of the ruler.

20. Issac is doing a 2.5 km time trial on a 400 m running track. How many laps does he need to run? Give your answer as a mixed number.

Key points
- Issac must run 2.5 km.
- The running track is 400 m long.
- How many 400 m laps make up 2.5 km?

Explanation
2.5 km is equal to 2500 m.

To find the number of laps, divide the total distance by the length of the track.

$\frac{2500}{400} = 6$ remainder 1

$= 6\frac{1}{4}$

Answer
Issac needs to run $6\frac{1}{4}$ laps.

21. Ciaran is a strength and conditioning coach and tells his clients that exactly $\frac{2}{7}$ of the time spent on a workout should be spent stretching. Select a correct amount of time spent doing cardio and stretching.

Key points
- $\frac{2}{7}$ of time spent exercising should be spent stretching.
- Which of the options includes the correct fraction of time spent stretching?

Explanation
The fraction of time spent stretching is $\frac{\textit{time spent stretching}}{\textit{total time}}$.

We are looking for this to equal $\frac{2}{7}$.

A. $\frac{25}{95} = \frac{5}{19}$ ✗

B. $\frac{70}{90} = \frac{7}{9}$ ✗

C. $\frac{40}{145} = \frac{8}{29}$ ✗

D. $\frac{40}{140} = \frac{2}{7}$ ✓

Answer
D

22. Britt and Timm are completing the same maths test during class.

Britt completed $\frac{9}{10}$ of the test.

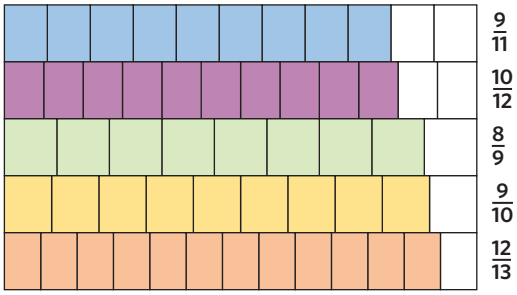
Timm completed more of the test than Britt.

What fraction of the test could Timm have completed?

Key points

- Britt completed $\frac{9}{10}$ of the test.
- Timm completed more of the test than Britt.
- Which of the given fractions is greater than $\frac{9}{10}$?

Explanation



A. $\frac{9}{10}$ and $\frac{9}{11}$ have the same numerator.

$\frac{9}{11} < \frac{9}{10}$ because it has a larger denominator. ✗

B. The LCD is 130.

$\frac{12}{13} = \frac{12 \times 10}{13 \times 10} = \frac{120}{130}$

$\frac{9}{10} = \frac{9 \times 13}{10 \times 13} = \frac{117}{130}$

$\frac{120}{130} > \frac{117}{130}$ ✓

C. The LCD is 60.

$\frac{10}{12} = \frac{10 \times 5}{12 \times 5} = \frac{50}{60}$

$\frac{9}{10} = \frac{9 \times 6}{10 \times 6} = \frac{54}{60}$

$\frac{50}{60} < \frac{54}{60}$ ✗

D. The LCD is 90.

$\frac{8}{9} = \frac{8 \times 10}{9 \times 10} = \frac{80}{90}$

$\frac{9}{10} = \frac{9 \times 9}{10 \times 9} = \frac{81}{90}$

$\frac{80}{90} < \frac{81}{90}$ ✗

Answer

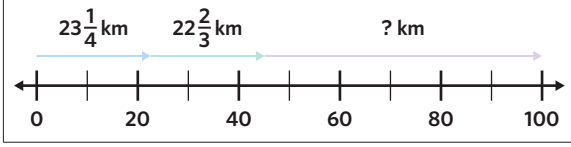
B

23. Gerald is participating in the Oxfam 100 km trailwalk. He walks $23\frac{1}{4}$ km to one of the checkpoints and has a rest. He then walks another $22\frac{2}{3}$ km to another checkpoint before resting again. How many kilometres does Gerald have left to walk?

Key points

- The Oxfam trailwalk is 100 km long.
- Gerald walks $23\frac{1}{4}$ km and has a rest.
- He then walks $22\frac{2}{3}$ more km.
- How many kilometres are left in the trailwalk?

Explanation



Represent the fractions with the LCD of 12.

$23\frac{1 \times 3}{4 \times 3} = 23\frac{3}{12}$

$22\frac{2 \times 4}{3 \times 4} = 22\frac{8}{12}$

Add the fractions:

$23\frac{3}{12} + 22\frac{8}{12} = 45\frac{11}{12}$

Subtract this from 100:

$100 - 45\frac{11}{12} = 54\frac{1}{12}$

Answer

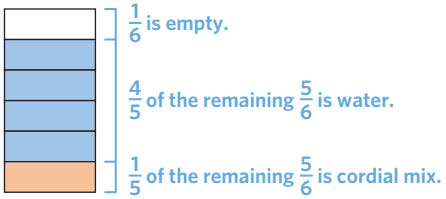
Gerald has $54\frac{1}{12}$ km left to walk.

24. Samantha is mixing a glass of cordial. The glass is five-sixths full. One-fifth of the liquid is cordial mix, and the rest is water. What fraction of the glass is made up of cordial mix?

Key points

- The glass is five-sixths full.
- One-fifth of the liquid is cordial mix.
- The rest of the liquid is water.
- What fraction of the entire glass does the cordial mix take up?

Explanation



$\frac{5}{6}$ of the glass is filled with liquid.

$\frac{1}{5}$ of this liquid is cordial mix.

Multiply $\frac{5}{6}$ by $\frac{1}{5}$ to find the fraction of the glass that is cordial mix.

$\frac{\cancel{5}^{1}}{6} \times \frac{1}{\cancel{5}_{1}} = \frac{1}{6} \times \frac{1}{1}$

$= \frac{1}{6}$

Answer

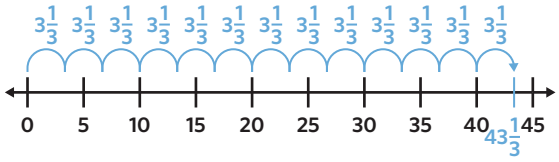
$\frac{1}{6}$ of the glass is made up of cordial mix.

25. Richie is trying to fit books in his bookshelf that is 45 cm in length. Each book is $3\frac{1}{3}$ cm thick. What is the maximum number of books Richie can fit in his bookshelf?

Key points

- The bookshelf is 45 cm long.
- Each book is $3\frac{1}{3}$ cm thick.
- How many $3\frac{1}{3}$ cm thick books will fit on a 45 cm shelf?

Explanation



To find the number of books he can fit, divide the length of the shelf by the width of each book.

Each book is $3\frac{1}{3} = \frac{10}{3}$ cm thick.

$$45 \div \frac{10}{3} = 45 \times \frac{3}{10}$$
$$= \frac{45 \times 3}{1 \times 10}$$
$$= \frac{135}{10}$$
$$= 13\frac{1}{2}$$

Answer

Richie can fit 13 books in his bookshelf.

---

26. Leonardo Da Vinci was 1.75 m tall.

A statue of the artist was made with a statue : actual height scale of 2 : 1.

How tall is the statue?

Key points

- Leonardo Da Vinci was 1.75 m tall.
- The statue : actual height scale is 2 : 1.
- What is the height of the statue?

Explanation

	2	:	1	
×175	↓		↓	×175
	350	:	175	

1.75 m is equal to 175 cm.

The statue is twice as tall as Da Vinci's actual height.

$175 \times 2 = 350$ cm

Answer

D

Reasoning

27. a. They had eaten $\frac{23}{15}$ plates of dumplings after 30 minutes.

b. They had eaten $1\frac{8}{15}$ plates of dumplings after 30 minutes.

c. Moo had eaten more dumplings after 30 minutes.

d. Yes, Malachy and Moo have now eaten an equivalent fraction of their dumplings.

e. Suggested option 1: Malachy and Moo could take their leftover food home with them to avoid it going to waste.

Suggested option 2: Malachy and Moo could order less food next time so there won't be any left over.

Note: There are other possible options.

---

28. a. Moo ate $\frac{2}{3}$ of her wings.

b. Malachy ordered 10 wings.

c. Malachy ate 6 wings.

d. 6 honey soy wings were ordered.

e. Suggested option 1: Malachy could pay for this meal and Moo could pay for their next meal together.

Suggested option 2: They could each pay for their own food.

Note: There are other possible options.

4A Relational thinking with unknowns

Student practice

Worked example 1.

a. True b. True c. False d. True

Worked example 2.

a. 4 b. 7 c. 1 d. 6

Worked example 3.

a. $10 \times 6 = 15 \times 4$

b. Joey fed his pet duck 10 treats for 5 days, and 2 treats on the 6^{th} day. He gave 52 treats to his pet duck.

Note: Many situations are possible.

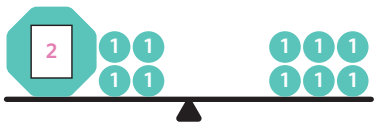
c. $24 \times 3 = 21 \times 3 + 9$

d. Phoebe and Mike have the same number of lollies. Phoebe has 12 red skittles, 12 green skittles and 8 yellow skittles. Mike has 2 lollipops and 3 packets of gummy bears. Each packet has 10 gummy bears.

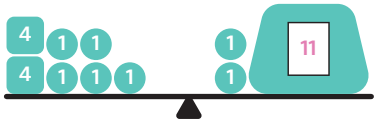
Note: Many situations are possible.

Understanding worksheet

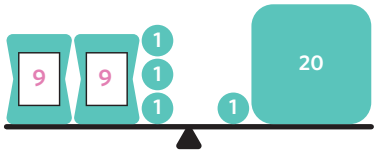
1. a.



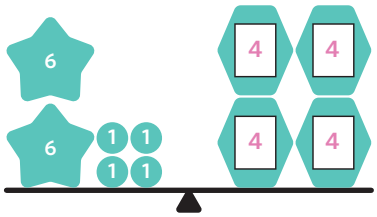
b.



c.



d.



2. a. 13 b. 3 c. 8 d. 39

3. relational thinking; equation; both; same

Fluency

4. a. False b. False c. False d. True
e. True f. True g. False h. True

5. a. $3 + 2 + 5 = 10$
b. $4 - 2 + 7 = 3 + 6$
c. $2 + 2 + 2 = 2 \times 3$
d. $5 \times 3 - 2 = 3 + 3 + 3 + 3 + 3 - 2$
e. $6 + 4 - 1 + 3 = 12$
f. $3 \times 2 + 8 = 7 + 7$
g. $9 + 4 - 3 \times 2 = 7$
h. $8 - 5 + 3 \times 3 = 12$

6. a. $2 + 8 = 10$ b. $3 \times 7 = 21$
c. $1 \times 15 - 6 = 9$ d. $8 = 20 - 2 \times 6$

7. a. $3 + 4 = 7$
b. $8 + 12 = 4 + 3 + 5 + 8$
c. $5 \times 3 + 6 \times 1 = 21$ or $5 \times 3 + 6 = 21$
d. $9 + 5 - 2 = 4 + 8$

8. a. D'angelo has 2 parents and 3 siblings. He has a total of 5 immediate family members.
b. Apollo and Blaize buy an equal number of fruit for their school lunches. Apollo buys 4 apples, 4 pears, and 4 oranges. Blaize buys 3 bunches of bananas with each bunch containing 4 bananas.
c. Warren buys two bags of 5 chocolate muffins and 3 more apple muffins to share with his friends. He has 13 muffins in total.
d. A class is split into three groups of 2 and four groups of 5. There are 26 students in the class in total.

Note: There are many possible scenarios for each question.

Problem solving

9. Shawn and Timmy have to shear 35 sheep between them. One morning, Shawn shears 9 sheep and Timmy shears 8 sheep. That afternoon, Shawn shears 7 sheep. How many sheep does Timmy have to shear to complete all 35 sheep?

Key points

- Shawn and Timmy have 35 sheep in total.
- Shawn shears 9 sheep and 7 more.
- Timmy shears 8 sheep.
- How many more of the 35 sheep are left for Timmy to shear?

Explanation

9	7	8	? = 11
35			

Express the situation as an equation.

$9 + 7 + 8 + ? = 35$

Simplify the LHS.

$24 + ? = 35$

$24 + 11 = 35$ ✓

The missing number of sheep must be 11 for the equation to be true.

Answer

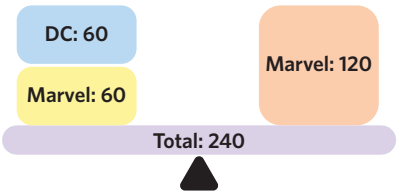
There are 11 sheep left for Timmy to shear.

10. Sarah has a collection of comic books. Half of these are Marvel and the other half are DC. Ron has the same amount of comic books as Sarah, but he only owns Marvel. How many Marvel comic books do they each have, if they have 240 comic books altogether?

Key points

- Sarah and Ron each have the same number of comic books.
- Half of Sarah's comic books are Marvel and half are DC.
- All of Ron's comic books are Marvel.
- They have 240 comic books altogether.
- How many of each of their comic books are Marvel?

Explanation



Number of comic books each: $240 \div 2 = 120$

Ron has 120 Marvel comic books.

Sarah's number of Marvel comic books: $120 \div 2 = 60$

Answer

Sarah has 60 Marvel comic books and Ron has 120 Marvel comic books.

11. Eva is 8 years old and is 3 years older than her sister Penny. In four years time, how old will both Eva and Penny be?

Key points

- Eva is 8 years old.
- Eva is three years older than Penny.
- What will Eva's age and Penny's age be in four years?

Explanation

We can first express Eva's age in terms of Penny's age.

Eva's age $=$ *Penny's age* $+ 3$

We can then make the equation true to find Penny's current age.

$8 = ? + 3$

$8 = 5 + 3$ ✓

Lastly, we can add 4 to each of their current ages to find their ages in four years' time.

Eva's age in four years: $8 + 4 = 12$

Penny's age in four years: $5 + 4 = 9$

Answer

Eva will be 12 years old.

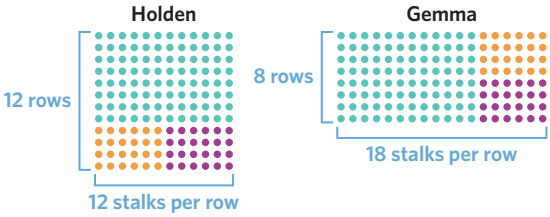
Penny will be 9 years old.

12. Holden's farm is planting crops for the new season. They plant a square of 12 rows of 12 corn stalks. Gemma's farm plants the same number of corn stalks but only has room for 8 rows. How many corn stalks are in each of Gemma's rows?

Key points

- Holden's farm plants 12 rows with 12 corn stalks in each row.
- Gemma's farm plants only 8 rows of corn stalks.
- How many corn stalks would Gemma need in each row for her to have the same number of stalks as Holden's farm?

Explanation



The number of corn stalks planted in Holden's farm is 12×12.

This is equal to the number of corn stalks in Gemma's farm, $8 \times ?$, where the number of stalks per row is unknown.

$12 \times 12 = 8 \times ?$

$144 = 8 \times ?$

$144 = 8 \times 18$ ✓

The missing number must be 18 for the equation to be true.

Answer

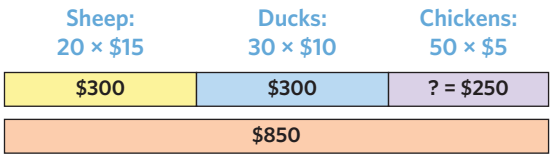
There are 18 corn stalks in each of Gemma's rows.

13. Farmer Fred is calculating the cost of an individual chicken. He bought 20 sheep, 30 ducks and 50 chickens. He paid $15 for each sheep and $10 for each duck. In total, he spent $850 on all 100 animals.

Key points

- Fred bought 20 sheep for $15 each.
- He bought 30 ducks for $10 each.
- He also bought 50 chickens.
- Fred spent $850 in total.
- How much did Fred spend on each chicken?

Explanation



Express the situation as an equation.

$20 \times 15 + 30 \times 10 + 50 \times ? = 850$

Simplify the LHS.

$600 + 50 \times ? = 850$

$600 + 50 \times 5 = 850$ ✓

The price of each chicken must be $5 for the equation to be true.

Answer

Each chicken cost $5.

Reasoning

14. a. $20 \times 2 \times 2 =$ ★

Note: There are other possible number sentences.

The length of the purple train is 80 m.

b. The orange train is 320 metres long.

c. The length of the blue train is 640 metres.

d. Suggested option 1: The pattern may be problematic because the trains become very long as the pattern goes on and the one engine might not have enough power to pull all the carriages.

Suggested option 2: The pattern is a good design because it allows for trains of a variety of different lengths.

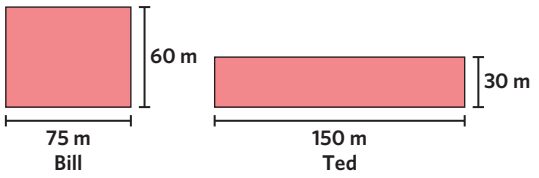
Note: There are other possible options.

15. a. i) A ii) B

b. Bill: $60 \times 60 = 3600$

Ted: $120 \times 30 = 3600$

c. Bill needs to add 15 metres to the length of his plot of land.

d. 

e. Suggested option 1: Bill and Ted could also consider the climate and the environment that their plots of land are situated on before deciding whether to raise animals or plant crops.

Suggested option 2: Bill and Ted could consider their personal ethics and what they would do with the animals or plants that they house on their land.

Note: There are other possible options.

Extra spicy

16. There were 36 people on the train to begin with.

17. Class C's average test score was 87.

18. Initially, Misha had 28 lollies and Kayla had 12 lollies.

19. $987\,654\,321 - 123\,456\,789 = 864\,197\,532$

Note: There are other possible answers.

Remember this?

20. B 21. C 22. B

4B Building algebraic expressions and equations

Student practice

Worked example 1.

a. i) 3 terms
ii) 3
iii) -7

b. i) 3 terms
ii) 6
iii) 2

c. i) 4 terms
ii) 6
iii) 3

d. i) 5 terms
ii) -2
iii) -5

Worked example 2.

a. $\frac{2+r}{s-5}$ b. $\frac{7fg}{5eh}$ c. $\frac{a}{2} + 12b$ d. $x + 7$

Worked example 3.

a. $\frac{p}{5} + 2 = 9$ b. $\frac{7}{10x} = 9$

c. 4 less than x squared is 0.

d. The sum of 3 and 2 minus one fifth of $4x$ gives 27.

Note: For c and d, there are other possible statements.

Worked example 4.

a. $3s + 11 = 47$ b. $c + d = 25$

c. $3s + 5d = 24$ d. $2t + 4 = 74$

Understanding worksheet

1. a. Coefficient, Pronumeral, Constant: $12 + 2z = 40$

b. Coefficient, Pronumeral, Constant: $5y + 8 = 23$

c. Pronumeral, Constant, Coefficient: $2b - c = 16$

d. Pronumeral, Coefficient, Constant, Pronumeral, Pronumeral: $ef - 5g = 2$

2. a. $2b + 6 = 10$ b. $b - 3 = 2$
c. $3b + 4 = 7$ d. $2b - 2 = b + 1$

3. equation; pronumeral; coefficient; constants

Fluency

4. a. i) 3 ii) 3 iii) -7
b. i) 2 ii) -2 iii) None
c. i) 3 ii) 1 iii) 3
d. i) 3 ii) -3 iii) 6
e. i) 2 ii) 2 iii) None
f. i) 3 ii) 1 iii) -50
g. i) 3 ii) 4 iii) 8
h. i) 2 ii) 3 iii) None

5. a. cy b. mb
c. $5 + fg$ d. $a + 20bc$
e. $\frac{c}{h}$ f. $x + \frac{1}{3}$
g. $\frac{100 - b}{5bc}$ h. $\frac{36ade}{p}$

6. a. $p + 4 = 11$ b. $t - 8 = 14$
c. $\frac{y}{2} = 13$ d. $3x = 45$
e. $a = 4 \times (8 + 12)$ f. $\frac{z}{4} = 3 + 12$
g. $\frac{b^2 + 5}{2} = 9$ h. $3c - 7 = \frac{3}{10} - d^3$

7. a. 3 more than f is 7.
b. 4 less than m is 5.
c. 90 is 3 times greater than k.
d. 12 is twice as large as z.
e. A third of a is 4.
f. 7 more than 3 of s is 13.
g. x squared is 36.
h. A fifth of the sum of g and 2 is 4.

8. a. $6x = 132$
   b. $\frac{b}{12} = 9$
   c. $r + 4 = m$
   d. $3c = r$
   e. $7p - 5 = 128$
   f. $\frac{d + 4}{3} = 9$
   g. $3(c + 3) = 32$
   h. $\frac{r}{5} + 6 = 8$

Problem solving

9. Julio is 4 and his brother Fletcher is 12. Which equation for Julio's age will still be correct in a year's time?

   A. *Julio's age* $= 3 \times$ *Fletcher's age*
   B. *Julio's age* $=$ *Fletcher's age* $\div 3$
   C. *Julio's age* $=$ *Fletcher's age* $+ 8$
   D. *Julio's age* $=$ *Fletcher's age* $- 8$

Key points
- Julio is 4 years old.
- Fletcher is 12 years old.
- Which of the options is an equation that correctly shows Julio's age now and in one year?

Explanation

Julio is currently 8 years younger than Fletcher.

As both Julio and Fletcher will be one year older in a year, Julio will still be 8 years younger.

This means that Julio's age will always be 8 less than Fletcher's age.

$4 = 12 - 8$

$5 = 13 - 8$

Julio's age $=$ *Fletcher's age* $- 8$

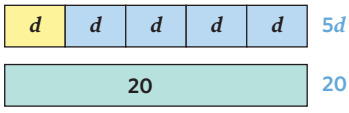
Answer

D

10. Sarah is given $\$d$ for her first week of pocket money. For another 4 weeks, she gets the same amount and ends up with exactly enough to buy the latest Aquaman comic for $20. Write an equation to represent this situation.

Key points
- Sarah is given $\$d$ in her first week.
- Sarah gets the same amount for another 4 weeks.
- Sarah ends up with exactly $20.
- What is an equation that represents this situation?

Explanation



Sarah is paid $\$d$ and then paid $\$d$ for another 4 weeks, so this is a total of 5 weeks.

5 of $\$d$ is equal to $20.

$5 \times d = 20$

Answer

$5d = 20$

11. When Sarah goes into the comic book store to buy her $20 comic she sees a copy of a first edition Magnus Robot Fighter #40 from 1975. It is the last volume she needs to complete her collection, but notices that it is selling for $38 400. If the number of $20 comic books she needs to trade for the 'Magnus Robot Fighter' comic book is c, write an equation to represent this situation.

Key points
- The Magnus Robot Fighter #40 comic book costs $38 400.
- c is the number of $20 comic books Sarah would have to trade for the Magnus Robot Fighter #40 comic book.
- What is an equation that represents this situation?

Explanation

$20 multiplied by the number of comic books traded in equals the price of the Magnus Robot Fighter #40 comic book.

The number of comic books traded in is c.

$20 \times c = 38\,400$

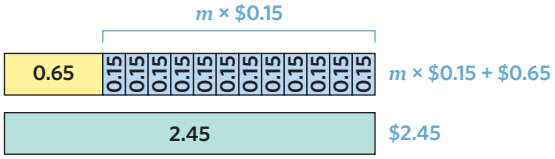
Answer

$20c = 38\,400$

12. Tony pays 65 cents to make an international phone call, plus an extra 15 cents per minute. He paid a total of $2.45 for the call. Write an equation to represent this, using m as the length of the phone call in minutes.

Key points
- It costs 65 cents to start an international phone call.
- It costs an additional 15 cents per minute.
- Tony paid a total of $2.45 for the call.
- m as the length of the phone call in minutes.
- What is an equation that represents this situation?

Explanation



The cost of an international phone call is 15 cents multiplied by the number of minutes spent on the call plus an additional 65 cents.

The number of minutes Tony spent on the call was m.

The cost of Tony's call was $2.45.

$0.15 \times m + 0.65 = 2.45$

Answer

$0.15m + 0.65 = 2.45$

13. The cost of a taxi is $c = 2d + 5$ where c is the total cost in dollars and d is the distance travelled in kilometres. How much does the cost change for each additional kilometre traveled?

Key points
- The formula for the cost of a taxi is $c = 2d + 5$.
- c is the total cost in dollars.
- d is the distance in kilometres.
- What is the cost for every additional kilometre?

Calculation

The cost of a taxi is twice the distance travelled (km) plus 5.

When you first enter the taxi the distance travelled is 0, so the cost is $2 \times 0 + 5 = \$5$.

After 1 kilometre the cost is $2 \times 1 + 5 = \$7$.

$\$7 - \$5 = \$2$ for each additional kilometre.

Answer

The cost of a taxi increases by $2 for each additional kilometre travelled.

Reasoning

14. a. 8 hours

    b. Tashi earned more this week because she spent more time babysitting instead of working at the supermarket, and she earns more per hour babysitting compared to working at the supermarket.

    c. $20a + 12b = 344$

    d. Suggested option 1: Tashi might think the supermarket job will look good on a resumé.

    Suggested option 2: Tashi might have friends at the supermarket that she enjoys spending time with.

    Note: There are other possible options.

15. a. $a = c + 10$

    b. $s = 2c$

    c. $t = a + c + s$

    d. $r = 5a + 2c + 3s$

    e. Suggested option 1: It is unfair because both children and adults are watching the same play.

    Suggested option 2: It is fair because adults usually have more money than children.

    Note: There are other possible options.

Extra spicy

16. E

17. When you add the digits of numbers that are the product of 9 and any number from 1 to 9, it always equals 9. So, adding 4 will always equal 13.

18. $2a + 3 + 3a$

    $1 + 5a + 2$

    $10a + 3 - 5a$

    $60 + 5a - 57$

    Note: There are many possible answers.

19. −40°C

Remember this?

20. B

21. 8 cm

22. 3000 people

4C Substituting values into equations

Student practice

Worked example 1.

a. 58 b. 17 c. 11 d. 20

Worked example 2.

a. 18 cm b. 15 m c. 20 cm^2 d. 14 cm^2

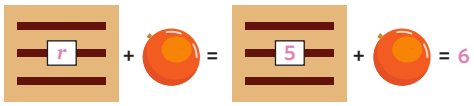
Worked example 3.

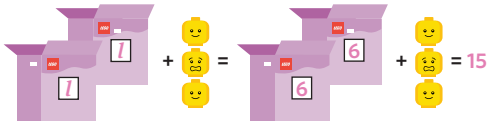
a. False b. True c. False d. True

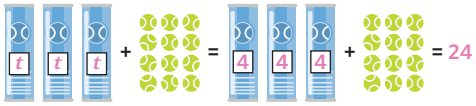
Worked example 4.

a. 96 b. 29 c. 26 d. $149\frac{1}{4}$

Understanding worksheet

1. a. 

   b. 

   c. 

   d. 

2. a.

b	*b*	*b*	2
3	3	3	2
11			

   b.

b	*b*	*f*	1
3	3	7	1
14			

   c.

f	*f*	*b*	1
7	7	3	1
18			

   d.

f	*f*	*f*	*b*	*b*
7	7	7	3	3
27				

3. substitution; expression; value; formula; unknown; area

Fluency

4. a. 12 b. 30 c. 90 d. 44
   e. 60 f. 5 g. 3 h. 55

5. a. 30 cm b. 22 mm c. 40 km d. 660 m
   e. 19 m f. $51\frac{1}{3}$ m

6. a. = 5 cm^2 b. 25 cm^2
   c. 3600 m^2 d. 45 mm^2
   e. $37\frac{1}{2}$ cm^2 f. 9 km^2

7. a. True b. False c. True d. False
   e. False f. True g. False h. False

8. a. 48 b. 34 c. 43 d. 59
   e. 62 f. 4 g. 50 h. 11

Problem solving

9. Mr Goldenfold starts the school year with 7 pencils, and buys 5 boxes of new pencils. If there are 12 pencils in a box, how many pencils does Mr. Goldenfold have?

Key points

- Mr Goldenfold started with 7 pencils.
- He bought 5 boxes of new pencils.
- Each box has 12 pencils.
- How many pencils does Mr Goldenfold have altogether?

Explanation

Mr Goldenfold has 7 pencils plus 5 boxes of pencils.

$p = 7 + 5 \times b$

There are 12 pencils in a box so replace b with 12.

$p = 7 + 5 \times 12$

$= 7 + 60$

$= 67$

Answer

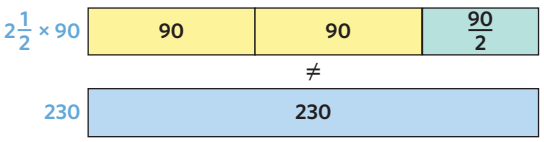
Mr Goldenfold has 67 pencils.

10. NBA staticians are trying to check that Yao Ming's height in inches and centimetres is listed correctly in NBA player records. Yao Ming is listed as 90 inches tall and 230 centimetres. Use the statisticians' formula *height* (cm) $= 2\frac{1}{2} \times$ *height* (inches) to check that the NBA player records are correct.

Key points

- Yao Ming is listed as 90 inches tall.
- Yao Ming is listed as 230 centimetres tall.
- The formula to convert inches to centimetres is *height* (cm) $= 2\frac{1}{2} \times$ *height* (inches).
- Is the NBA's conversion correct?

Explanation



Replace *height* (*cm*) with 230 and *height* (*inches*) with 90.

LHS $= 230$

RHS $= 2\frac{1}{2} \times 90$

$= 2 \times 90 + \frac{1}{2} \times 90$

$= 180 + 45$

$= 225$

$230 \neq 225$ so RHS $\neq$ LHS so the equation is not true.

Answer

The NBA incorrectly converted Yao Ming's height from inches to centimetres.

11. If Maria wants to make the most money for 6 hours of babysitting, which job should she choose? Jack's parents pay a flat rate of $10 plus $5 for each hour of work and Russell's parents pay $3 per hour plus an additional $20.

Key points

- Jack's parents pay the babysitter $10 and $5 more per hour worked.
- Russell's parents pay the babysitter $20 and $3 more per hour worked.
- For 6 hours of work, who pays more?

Explanation

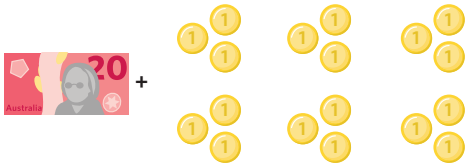
Jack's parents:

$10 + 5h = 10 + 5 \times 6$

$= 10 + 30$

$= \$40$

Russell's parents:



$20 + 3h = 20 + 3 \times 6$

$= 20 + 18$

$= \$38$

Answer

Maria should choose Jack's parents.

12. Ron and Sarah's comic books increase in value over time. The value of a comic can be calculated by using the formula *value* ($) $= 20 + 2y$ where y is the age of the comic in years. Ron's oldest comic is 52 years old, while Sarah's oldest comic is 39 years old. What is the difference in value between Ron and Sarah's oldest comics?

Key points

- The formula *value* ($) $= 20 + 2y$ can be used to find the value of the comics.
- Ron's comic is 52 years old.
- Sarah's comic is 39 years old.
- How much more valuable is Ron's comic?

Explanation

The dollar value of a comic book is $20 plus twice its age. Find the dollar value of both Ron and Sarah's oldest comics. Then subtract the value of Sarah's comic from the value of Ron's comic to find the difference.

Ron's comic:

Replace y with 52.

value ($) $= 20 + 2 \times 52$

$= 20 + 104$

$= \$124$

Sarah's comic:

Replace y with 39.

value ($) $= 20 + 2 \times 39$

$= 20 + 78$

$= \$98$

$\$124 - \$98 = \$26$

Answer

There is a $26 difference in value between Ron and Sarah's comics.

13. Kathy is baking cookies and needs to set the oven to 200 degrees celsius (°C). Her oven's temperature can only be set using a fahrenheit (°F) scale. Her recipe book provides the formula $°F = \frac{9}{5}°C + 32$ for converting °C to °F. Use this formula to help Kathy calculate what temperature she should set her oven.

Key points

- The oven needs to be set to 200°C.
- The formula to convert from °C to °F is $°F = \frac{9}{5}°C + 32$.
- What temperature does Kathy need to set the oven to in fahrenheit (°F)?

Explanation

C = 200 degrees celsius

$\frac{9}{5} \times °C$	32	$(\frac{9}{5} \times 200) + 32$
=		
360	32	360 + 32
=		
392°F		392

$°F = \frac{9}{5} \times °C + 32$

Replace °C with 200.

$°F = \frac{9}{5} \times 200 + 32$

$= 360 + 32$

$= 392°F$

Answer

Kathy needs to set the oven to 392°F.

Reasoning

14. a. The perimeter of the pool is 36 m.
    b. The area that the pool takes up is 72 m^2.
    c. Neither Lucy nor Phil is correct because Phil's pool has the same perimeter as Lucy's pool.
    d. Lucy needs to extend the width of the pool by 3 m.
    e. Suggested option 1: Lucy could pick a circle. However, this might not fit in her backyard if her backyard is thin.

    Suggested option 2: Lucy could pick a star. However, this might not be an enjoyable pool to swim in because of the sharp edges.

    Note: There are other possible options.

15. a. It will take 150 minutes to cook an 8 kg chicken using the fan forced setting.
    b. It will take 150 minutes longer to cook an 8 kg turkey using the conventional setting.
    c. Kathy put the turkey in the oven at 2:30 pm.
    d. Suggested option 1: If you use the conventional setting, it will take 30 minutes for each kilogram the turkey weighs plus an additional hour. Using the fan forced setting will take 12.5 minutes for each kilogram the turkey weighs plus an additional 50 minutes.

    Suggested option 2: If you use the conventional setting it will take one hour plus an additional half hour for each kilogram the turkey weighs. If you use the fan forced setting, it will take 50 minutes plus 12 minutes and 30 seconds for each kilogram the turkey weighs.

    Note: There are other possible options.

Extra spicy

16. $m = 12$
17. 15
18. $k = 2, 3, 7, 8, 12$ or 13.

    These numbers work because when we square each of these numbers they have the digit 4 or 9 in the ones place. When we add 6 to a number that has a 4 or 9 in the ones place, the solution will be a multiple of 5.
19. 24 m^2

Remember this?

20. C 21. A 22. D

4D Solving one-step equations

Student practice

Worked example 1.

a. −8 b. ×3 c. ÷7 d. +9

Worked example 2.

a. Operation: +99
   Inverse operation: −99
b. Operation: −11
   Inverse operation: +11
c. Operation: ×8
   Inverse operation: ÷8
d. Operation: ÷7
   Inverse operation: ×7

Worked example 3.

a. $x = 17$ b. $c = 2$ c. $p = 44$ d. $z = 45$

Understanding worksheet

1. a. +2, +2 b. −4, −4 c. ×2, ×2 d. ÷3, ÷3

2. a. +2, +2 b. ÷5, ÷5 c. ×4, ×4 d. ÷12, ÷12

3. unknown; inverse; division; multiplication; addition; subtraction

Fluency

4. a. +10 b. −55 c. ×5 d. ÷4
   e. ÷6 f. −300 g. +37 h. ×6

5. a. Operation: +3
      Inverse operation: −3
   b. Operation: ×5
      Inverse operation: ÷5
   c. Operation: ÷4
      Inverse operation: ×4
   d. Operation: −12
      Inverse operation: +12
   e. Operation: +3
      Inverse operation: −3
   f. Operation: ×7
      Inverse operation: ÷7
   g. Operation: ÷15
      Inverse operation: ×15
   h. Operation: Squared
      Inverse operation: Square root

6. a. $y = 5$ b. $y = 22$ c. $x = 12$ d. $x = 15$
   e. $x = 24$ f. $c = 48$ g. $c = 9$ h. $a = 11$

7. a. $n + 2 = 9$
      $n = 7$
   b. $a - 7 = 16$
      $a = 23$
   c. $s + 5 = -4$
      $s = -9$
   d. $t - 11 = -11$
      $t = 0$

e. $\frac{c}{9} = 11$
$c = 99$

f. $4p = 76$
$p = 19$

g. $r^2 = 36$
$r = 6$

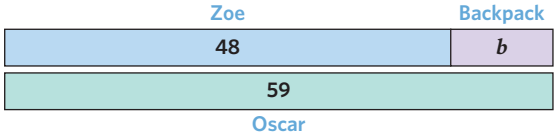
h. $\sqrt{q} = 7$
$q = 49$

Problem solving

8. Zoe and Oscar are playing on a seesaw. Zoe weighs 48 kilograms and Oscar weighs 59 kilograms. Zoe is wearing a backpack. How much does the backpack need to weigh (b) for the seesaw to balance when Zoe and Oscar sit on it? Create an equation and solve for b.

Key points
- Zoe and Oscar are on a seesaw.
- Zoe weighs 48 kilograms and is wearing a backpack.
- Oscar weighs 59 kilograms.
- If the weight on each side is the same, what is the weight of Zoe's backpack?

Explanation



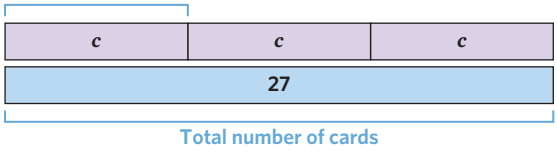
$48 + b = 59$
$48 + b - 48 = 59 - 48$
$b = 59 - 48$
$b = 11$ kg

Answer
The backpack must weigh 11 kg.

9. Zach was given 3 packets of Pokémon cards for his birthday. How many Pokémon cards (c) are there in each packet if he now has 27 more cards in his deck? Create an equation and solve for c.

Key points
- Zach was given 3 packets of Pokémon cards.
- These contained 27 cards in total.
- How many cards are in one packet?

Explanation



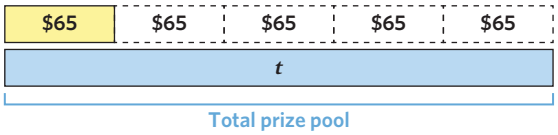
$3c = 27$
$3c \div 3 = 27 \div 3$
$c = 27 \div 3$
$c = 9$

Answer
There are 9 cards in each packet.

10. Marcus came third in his work footy tipping competition. How much was the total prize pool (t) if his winnings of $65 was a fifth of the total prize pool? Create an equation and solve for t.

Key points
- Marcus won $65.
- His winnings were a fifth of the total prize pool.
- What was the total amount of money given away as prizes?

Explanation



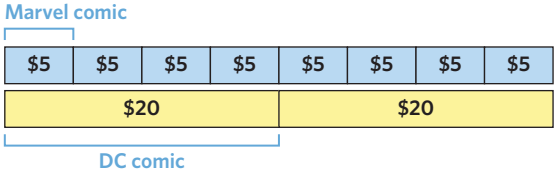
$\frac{t}{5} = 65$
$\frac{t}{5} \times 5 = 65 \times 5$
$t = 65 \times 5$
$t = \$325$

Answer
The total prize pool was $325.

11. Ron wants two of Sarah's DC comics that are worth $20 each. How many of his Marvel Comics (n) will Ron have to trade to get the two DC comics he wants? Ron's Marvel Comics are worth $5 each. Create an equation and solve for n.

Key points
- Ron wants two DC comics each worth $20.
- He will trade Marvel comics each worth $5.
- How many $5 Marvel comics have the same value as two $20 DC comics?

Explanation



$5n = 2 \times 20$
$5n = 40$
$5n \div 5 = 40 \div 5$
$n = 40 \div 5$
$n = 8$

Answer
Ron will have to trade 8 Marvel comics to get 2 DC comics.

12. How much does Julie spend (c) per cup of coffee if she spent $1460 buying a cup of coffee everyday for a year? Create an equation and solve for c.

Key points
- Julie bought a cup of coffee everyday for a year.
- She spent $1460 in total.
- How much does one cup of coffee cost?

Explanation

$365c = 1460$ ($\div 365$ / $\div 365$) $c = 5$

There are 365 days in a year and Julie bought a coffee everyday for a year.

$365c = 1460$
$365c \div 365 = 1460 \div 365$
$c = 1460 \div 365$
$c = \$4$

Answer
Julie spends $4 per cup of coffee.

Reasoning

13. a. $7p = 294$

b. The width of each poster is 42 cm.

c. The actual width of the seven posters is 329 cm.

d. Joyce should stick the posters 90 cm off the ground.

e. Suggested option 1: It is Joyce's decision whether she should buy such large posters, however she should consider the environmental impacts of doing so and how long she is likely to keep the posters.

Suggested option 2: Joyce may be able to sell these posters once she no longer wants them, so it may be better for her to buy large, good quality posters initially.

Note: There are other possible options.

14. a. $16h = 1200$

b. Charlie will need to work 75 hours to make $1200.

c. Warren would need to work 60 hours to make $1200.

d. Suggested option 1: Warren may have more relevant experience or may be older than Charlie, which would likely give him a higher hourly wage.

Suggested option 2: Warren and Charlie may have different job descriptions with Warren likely having more responsibility, allowing him to have a higher hourly wage.

Note: There are other possible options.

Extra spicy

15. 36 16. $\frac{28}{45}$ 17. $125 18. 3 years old

Remember this?

19. C

20. $\frac{19}{28}$ is halfway between $\frac{1}{2}$ and $\frac{6}{7}$.

21. A

4E Solving two-step equations

Student practice

Worked example 1.

a. -3 b. $\times 5$ c. $+3$ d. $+4$

Worked example 2.

a. $x = 4$ b. $y = 24$ c. $c = 21$ d. $p = 56$

Worked example 3.

a. $3x + 7 = 37$
$x = 10$

b. $7m - 8 = 55$
$m = 9$

c. $\frac{z - 2}{12} = 6$
$z = 74$

d. $\frac{7j}{4} = 14$
$j = 8$

Understanding worksheet

1. a. $2x + 4 = 12$
$2x + 4 - 4 = 12 - 4$
$2x = 8$
$2x \div 2 = 8 \div 2$
$x = 4$

b. $2x + 6 = 18$
$2x + 6 - 6 = 18 - 6$
$2x = 12$
$2x \div 2 = 12 \div 2$
$x = 6$

c. $3x + 7 = 25$
$3x + 7 - 7 = 25 - 7$
$3x = 18$
$3x \div 3 = 18 \div 3$
$x = 6$

d. $5x - 3 = 42$
$5x - 3 + 3 = 42 + 3$
$5x = 45$
$5x \div 5 = 45 \div 5$
$x = 9$

2. a. $4x - 5 = 11$ (+5, ÷4) → $4x = 16$ → $x = 4$

b. $\frac{k}{4} + 2 = 13$ (−2, ×4) → $\frac{k}{4} = 11$ → $k = 44$

c. $2y - 7 = 27$ (+7, ÷2) → $2y = 34$ → $y = 17$

d. $\frac{x}{3} - 6 = 5$ (+6, ×3) → $\frac{x}{3} = 11$ → $x = 3$

3. unknown; inverse; solve; order; simplify

Fluency

4. a. -3 b. $+17$ c. -5 d. $+3$
e. -4 f. $\times 5$ g. $+4$ h. $\times 7$

5. a. $x = 4$ b. $y = 10$ c. $m = 3$ d. $z = 1$
e. $k = 8$ f. $p = 10$ g. $c = 23$ h. $a = 7$

6. a. $z = 1$ b. $w = 10$ c. $u = 12$ d. $i = 7$
e. $y = 30$ f. $v = 10$ g. $k = 30$ h. $n = 9$

7. a. $5k + 4 = 19$
$k = 3$

b. $4c - 7 = 17$
$c = 6$

c. $\frac{a}{3} + 5 = 8$
$a = 9$

d. $\frac{z}{7} + 2 = 9$
$z = 49$

e. $\frac{y + 3}{5} = 5$
$y = 22$

f. $\frac{k - 4}{8} = 4$
$k = 36$

g. $\frac{5x}{3} = 15$
$x = 9$

h. $\frac{2p}{9} = \frac{12}{9}$
$p = 6$

Problem solving

8. An Uber ride charges $4 per km travelled plus a $6 service fee. If Beth paid $54 for her ride, how many kilometres (d) did she travel?

Key points

- Uber charges a fixed service fee of $6.
- Uber also charges an additional $4 per kilometre travelled.
- Beth's ride cost $54.
- How far (d) was Beth's Uber ride?

Explanation

d	d	d	d	6
54				

Cost of distance travelled is $4d$.

Total cost is $4d + 6 = 54$.

$$\begin{aligned} 4d + 6 &= 54 \\ 4d + 6 - 6 &= 54 - 6 \\ 4d &= 48 \\ 4d \div 4 &= 48 \div 4 \\ d &= 12 \text{ km} \end{aligned}$$

Answer

Beth travelled 12 km.

9. Richard's backpack is three times as heavy as Mortimer's. He decided he wanted to remove a gadget that weighed 450 g from his backpack. How much does Mortimer's backpack (m) weigh if Richard's backpack weighs 3750 g after removing the gadget?

Key points

- Initially, Richard's backpack is three times as heavy as Mortimer's.
- Richard removed 450 g from his backpack.
- Richard's backpack then weighs 3750 g.
- What is the weight of Mortimer's backpack (m)?

Explanation

$$\begin{aligned} 3m - 450 &= 3750 \quad (+450,\ \div 3) \\ 3m &= 4200 \\ m &= 1400 \end{aligned}$$

Richard's initial backpack weight is $3m$.

Richard's final backpack weight is $3m - 450 = 3750$.

$$\begin{aligned} 3m - 450 &= 3750 \\ 3m - 450 + 450 &= 3750 + 450 \\ 3m &= 4200 \\ 3m \div 3 &= 4200 \div 3 \\ m &= 1400 \text{ g} \end{aligned}$$

Answer

Mortimer's backpack weighs 1400 g.

10. Rebecca owns a bakery and has to complete 42 cake orders in a week. If she baked 12 cakes on the first day, how many cakes (c) on average would she have to bake everyday for the remaining six days?

Key points

- Rebecca needs to bake 42 cakes this week.
- She baked 12 cakes on the first day.
- She has 6 more days to bake an average of c cakes each day.
- How many cakes does Rebecca need to bake each day?

Explanation

c	c	c	c	c	c	12
42						

Rebecca needs to bake $6c$ more cakes.

Rebecca will end up with $6c + 12 = 42$ cakes.

$$\begin{aligned} 6c + 12 &= 42 \\ 6c + 12 - 12 &= 42 - 12 \\ 6c &= 30 \\ c \div 6 &= 30 \div 6 \\ c &= 5 \end{aligned}$$

Answer

On average, Rebecca must bake 5 cakes each day for the remaining six days.

11. Shane received a \$25 voucher at his favourite sports shop for his birthday. He bought a tennis racquet that was on sale for 50% of the retail price. After using his voucher, Shane paid another \$175 for the racquet. How much does one tennis racquet cost at its retail price (t)?

Key points

- Shane had a \$25 voucher.
- He bought a tennis racquet at half price.
- He spent his voucher and \$175 on the tennis racquet.
- What is the normal retail price (t) of the tennis racquet?

Explanation

$$\begin{aligned} \frac{t}{2} - 25 &= 175 \quad (+25,\ \times 2) \\ \frac{t}{2} &= 200 \\ t &= 400 \end{aligned}$$

The sale price of the tennis racquet is $\frac{t}{2}$ dollars.

Shane spent $\frac{t}{2} - 25 = 175$ dollars.

$$\begin{aligned} \frac{t}{2} - 25 &= 175 \\ \frac{t}{2} - 25 + 25 &= 175 + 25 \\ \frac{t}{2} &= 200 \\ \frac{t}{2} \times 2 &= 200 \times 2 \\ t &= 400 \end{aligned}$$

Answer

A tennis racquet costs \$400 at its retail price.

12. Taylen bought a square block of land to add to his current land of 250 m^2. If the total area of his two blocks of land is now 394 m^2, what is the length (l) of his new block of land?

Key points

- Taylen bought a square block of land.
- Taylen already has 250 m^2 of land.
- The total area of Taylen's land is 394 m^2.
- What are the side lengths (l) of Taylen's square block of land?

Explanation

$$\begin{aligned} l^2 + 250 &= 394 \quad (-250,\ \sqrt{}) \\ l^2 &= 144 \\ l &= 12 \end{aligned}$$

The area of the square block of land is l^2 m^2.

The total area of Taylen's land is $l^2 + 250 = 394$ m^2.

$$\begin{aligned} l^2 + 250 &= 394 \\ l^2 + 250 - 250 &= 394 - 250 \\ l^2 &= 144 \\ \sqrt{l^2} &= \sqrt{144} \\ l &= 12 \text{ m} \end{aligned}$$

Answer
The length of Taylen's new block of land is 12 metres.

Reasoning

13. a. $30n + 500 = 950$
    b. Jessica can invite 14 friends.
    c. Jessica can now invite 24 friends.
    d. Suggested option 1: Jessica could consider the price of bowling versus the price of bubble soccer and go with whichever is cheaper.
    Suggested option 2: Jessica may prefer bubble soccer and should be able to do what she wants since it's her birthday.
    Note: There are other possible options.

14. a. $2w + 26 = 40$
    b. The width of the room is 7 metres.
    c. No, the rug will not fit.
    d. Suggested option 1: The rug could be 5 m × 11 m. Molly would then have exactly 1 metre between all four sides of the rug and the walls.
    Suggested option 2: The rug could be 4 m × 10 m. Molly would then have more than 1 metre between all four sides of the rug and the walls.
    Note: There are other possible options.

Extra spicy

15. D
16. 43 years old
17. $y = 18$
18. 2467 bacteria per day

Remember this?

19. D
20. B
21. C

4F Solving multi-step equations

Student practice

Worked example 1.

a. $x = 7$ b. $m = 24$ c. $m = 30$ d. $x = 200$

Worked example 2.

a. $x = 7$ b. $y = 2$ c. v = 13 d. m = −2

Worked example 3.

a. $m = 1$
b. $x = 23$
c. $g = \frac{24}{5}$ or $g = 4\frac{4}{5}$
d. $m = -\frac{29}{5}$ or $m = -5\frac{4}{5}$

Understanding worksheet

1. a.
$\frac{7j - 12}{9} = 1$ (×9, +12, ÷7)
$7j - 12 = 9$
$7j = 21$
$j = 3$

b.
$\frac{3r}{5} + 2 = 8$ (−2, ×5, ÷3)
$\frac{3r}{5} = 6$
$3r = 30$
$r = 10$

c.
$2 = \frac{2x + 4}{7}$ (×7, −4, ÷2)
$14 = 2x + 4$
$10 = 2x$
$5 = x$

d.
$8 = \frac{10y}{4} - 7$ (+7, ×4, ÷10)
$15 = \frac{10y}{4}$
$60 = 10y$
$6 = y$

2. a.
$\frac{3(c - 8)}{5} = 6$ (×5, ÷3, +8)
$3(c - 8) = 30$
$c - 8 = 10$
$c = 18$

b.
$\frac{9(x + 1)}{3} = 12$ (×3, ÷9, −1)
$9(x + 1) = 36$
$x + 1 = 4$
$x = 3$

c.
$10 = \frac{5(x - 4)}{8}$ (×8, ÷5, +4)
$80 = 5(x - 4)$
$16 = x - 4$
$20 = x$

d.
$1 = \frac{25(x + 6)}{100}$ (×100, ÷25, −6)
$100 = 25(x + 6)$
$4 = x + 6$
$-2 = x$

3. brackets; multiplied; inverse; divide; numerator; denominator

Fluency

4. a. $m = 4$ b. $z = 8$ c. $x = 3$ d. $x = 9$
   e. $q = -1$ f. $c = 6$ g. $x = 11$ h. $x = 3$

5. a. $x = 8$
   b. $x = 10$
   c. $x = 4$
   d. $x = \frac{7}{8}$
   e. $x = 7$
   f. $x = 6$
   g. $x = \frac{22}{9}$ or $x = 2\frac{4}{9}$
   h. $x = \frac{124}{3}$ or $x = 41\frac{1}{3}$

6. a. $m = 11$
   b. $y = 2$
   c. $k = 7$
   d. $x = 40$
   e. $n = 3$
   f. $x = \frac{5}{2}$ or $x = 2\frac{1}{2}$
   g. $x = \frac{16}{3}$ or $x = 5\frac{1}{3}$
   h. $a = \frac{19}{4}$ or $a = 4\frac{3}{4}$

7. a. $m = 7$ b. $a = 8$
   c. $x = 40$ d. $y = 19$
   e. $r = 202$ f. $t = -10$
   g. $b = -\frac{2}{3}$ h. $p = \frac{41}{5}$ or $p = 8\frac{1}{5}$

8. a. $x = 15$ b. $x = 1$
   c. $x = 3$ d. $x = \frac{1}{4}$
   e. $x = 16$ f. $x = 13$
   g. $x = \frac{15}{2}$ or $7\frac{1}{2}$ h. $x = 4$

Problem solving

9. Each morning, for two days, Max ate c cashews and 5 almonds. Altogether he ate 20 nuts. Create and solve an equation to work out how many cashews (c) Max ate each day.

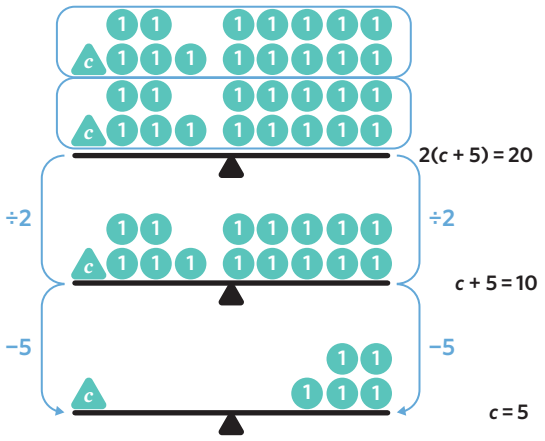
Key points
- Each day Max ate c cashews and 5 almonds.
- Max did this for 2 days.
- Over both days he ate 20 nuts.
- How many cashews did he eat each day?

Explanation

The number of nuts eaten each day is $c + 5$.

The equation for the total number of nuts eaten is $2(c + 5) = 20$.

$$2(c + 5) = 20$$
$$2(c + 5) \div 2 = 20 \div 2$$
$$c + 5 = 10$$
$$c + 5 - 5 = 10 - 5$$
$$c = 5$$



Answer

Max ate 5 cashews each day.

10. Ellie wanted to buy movie tickets for some of her friends. She invited d friends from her dance class plus 3 friends from her football team. The movie tickets cost $8 each. If Ellie paid $104 in total, how many friends did she invite from her dance class?

Key points
- Ellie invited d friends from her dance class.
- Ellie invited 3 friends from her football team.
- A ticket to the cinema costs $8.
- It cost Ellie $104 in total.
- How many friends from Ellie's dance class were invited?

Explanation

The number of friends Ellie invited is $d + 3$.

The equation for the total amount Ellie spent is $8(d + 3) = 104$.

$$8(d + 3) = 104$$
$$8(d + 3) \div 8 = 104 \div 8$$
$$d + 3 = 13$$
$$d + 3 - 3 = 13 - 3$$
$$d = 10$$

÷8 / −3: $8(d + 3) = 104$ → $d + 3 = 13$ → $d = 10$ ÷8 / −3

Answer

Ellie invited 10 friends from her dance class.

11. Rhonda collects ooshies. Before her birthday she had a collection of 90 ooshies. On her birthday, she had a party and her friends gave her 11 ooshies each. During the party, someone stole half of all her ooshies and she was left with 100 ooshies. How many friends (f) came to her party?

Key points
- Rhonda had 90 ooshies before her birthday.
- Her friends gave her 11 ooshies each for her birthday.
- After half of her ooshies were stolen she had 100 left.
- How many friends came to her party?

Explanation

Ooshies given to Rhonda for her birthday: $11f$

Ooshies before half were stolen : $11f + 90$

The equation for ooshies after half were stolen: $\frac{11f + 90}{2} = 100$

$$\frac{11f + 90}{2} = 100$$
$$\frac{11f + 90}{2} \times 2 = 100 \times 2$$
$$11f + 90 = 200$$
$$11f + 90 - 90 = 200 - 90$$
$$11f = 110$$
$$11f \div 11 = 110 \div 11$$
$$f = 10$$

×2 / −90 / ÷11: $\frac{11f + 90}{2} = 100$ → $11f + 90 = 200$ → $11f = 110$ → $f = 10$ ×2 / −90 / ÷11

Answer

10 friends came to Rhonda's party.

12. Ibrahim is making biscotti. The recipe says to set the oven to 200 degrees fahrenheit (°F) but his oven can only be set using degrees celsius (°C). Ibrihim knows that the formula for converting °C to °F is $°F = \frac{9°C}{5} + 32$. What temperature should Ibrahim set for the oven?

Key points
- Ibrahim needs to set his oven to 200°F.
- Ibrahim's oven can only be set in °C.
- $°F = \frac{9°C}{5} + 32$
- What temperature (°C) should Ibrahim set his oven to?

Explanation

Replace °F with 200 in the formula as the oven should be set to 200°F.

$$200 = \frac{9℃}{5} + 32$$
$$200 - 32 = \frac{9℃}{5} + 32 - 32$$
$$168 = \frac{9℃}{5}$$
$$168 \times 5 = \frac{9℃}{5} \times 5$$
$$840 = 9℃$$
$$840 \div 9 = 9℃ \div 9$$
$$93\frac{1}{3} = ℃$$

$200 = \frac{9℃}{5} + 32$ (−32 / −32)
$168 = \frac{9℃}{5}$ (×5 / ×5)
$840 = 9℃$ (÷9 / ÷9)
$93\frac{1}{3} = ℃$

Answer

Ibrahim should set his oven to $93\frac{1}{3}$°C.

13. Mick and Nathan have $68.50 to spend on an online shopping order and want to calculate how many pairs of boardshorts (b) they can buy. They have a promotion code, so they only pay one-quarter of the $23 marked price of boardshorts, and they also receive an additional $12 discount as part of the online sale.

Key points

- Mick and Nathan have $68.50 to spend on board shorts.
- They have a promotional code that means they pay a quarter of the $23 marked price.
- They also get a $12 discount.
- How many boardshorts can Mick and Nathan buy?

Explanation

The price of b boardshorts is usually $23b$.

The price after the promotional code is $\frac{23b}{4}$.

The price after the discount codeis $\frac{23b}{4} - 12 = 68.5$.

$$\frac{23b}{4} - 12 = 68.5$$
$$\frac{23b}{4} - 12 + 12 = 68.5 + 12$$
$$\frac{23b}{4} = 80.5$$
$$\frac{23b}{4} \times 4 = 80.5 \times 4$$
$$23b = 322$$
$$23b \div 23 = 322 \div 23$$
$$b = 14$$

$\frac{23b}{4} - 12 = 68.5$ (+12 / +12)
$\frac{23b}{4} = 80.5$ (×4 / ×4)
$23b = 322$ (÷23 / ÷23)
$b = 14$

Answer

Nathan and Mick can buy 14 board shorts.

Reasoning

14. a. Harold's team scored 15 goals.

b. The difference between the official scorecard and what Ronnie thinks the score should be is 12 points.

c. The team scored 25 goals.

d. Suggested option 1: Whoever is better at the position should play there as that would help the team more.

Suggested option 2: Maybe Harold plays that position more so he should let Ronnie have a go.

Note: There are other possible options.

15. a. 7 friends went camping.

b. $\frac{6p + 6}{5} = 18$

$p = 14$

c. $\frac{90 + 3b}{6} = 25$

$b = 20$

d. Suggested option 1: Divya could give each of the 6 families one of the regular bags, one of the spare pegs, and half of a large bag.

Suggested option 2: Divide all the pegs into groups of 10 and give each family the same number of groups of pegs. Any spare pegs can then be split into 6.

Note: There are other possible options.

Extra spicy

16. 44 400 17. E 18. 3 m 19. $\frac{24}{5}$

Remember this?

20. B 21. D 22. C

4G Simplifying equations (collecting like terms)

Student practice

Worked example 1.

a. $3j, j$ b. $4b, 2b$

c. $6rs, 2sr$ d. $3m, 4m$ and $-4, 7$

Worked example 2.

a. $3m + 2r$ b. $3x + 2y + 1$

c. $b + 4c = 13$ d. $8kl - 5k = 12$

Worked example 3.

a. $x = 2$ b. $x = 1$ c. $x = \frac{1}{4}$ d. $x = \frac{5}{3}$

Understanding worksheet

1. a. $3c + 3$ b. $5b + 4$ c. $5l + 7$ d. $7p + 6$

2. a. $2t + t + 4 + 8$ b. $3j + j + 10 + 6 + 4k$

c. $8ab + 7a + 2a - 3b$ d. $5xy + 3yx - y - 12$

3. simplify; like terms; algebraic; pronumeral(s)

Fluency

4. a. $2x, 3x$
   b. There are no like terms.
   c. $5, 2$
   $-4a, 3a, -2a$
   d. $3r, 4r$
   $5, -2$
   e. $6ab, -2ab$
   f. $-3m, 3m$
   $12t, -9t$
   g. $5yx, 3xy$
   $-4, 3, -2$
   h. $\frac{2x}{3}, x$
   $-4mn, 6nm$
   $-3, 7$

5. a. $5a$
   b. $7g$
   c. $7c + 4ab + 24$
   d. $6j - 5$
   e. $b + 2ab + 1$
   f. $5f - ab + 6$
   g. $10cd + 3$
   h. $14pqr + 6q - 2pq$

6. a. $2y = 24$
   b. $20 = 4c$
   c. $4r + 3s = 10$
   d. $-j + 5k = 14$
   e. $-4y + 2z + 4yz = 6$
   f. $5q + 3qr = 32$
   g. $13 = 96b - 80c$
   h. $8s = 28$

7. a. $x = 7$
   b. $x = 4$
   c. $x = 3$
   d. $x = 1$
   e. $x = 8$
   f. $x = 47$
   g. $x = 10$
   h. $x = \frac{32}{3}$

Problem solving

8. There are s students going on a class trip. Each student needs to pay \$12 for the bus ride and \$19 entry for the zoo. Write a simplified expression (in terms of s) showing how much the entire class will pay for the trip.

Key points
- s is the number of students going on a class trip.
- Each student pays \$12 for the bus ride.
- Each student also pays \$19 entry for the zoo.
- What is an expression that shows the entire amount the class pays for the trip?

Explanation

Each student pays \$12 for the bus ride and \$19 for entry to the zoo, so each student pays \$31 for the entire trip.

As there are s students, the total amount of money spent by the class will be $31 \times s$ dollars.

Total spent by class:

$12s + 19s = 31s$

Answer

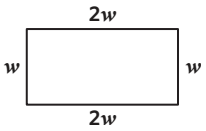
$31s$

9. The perimeter of a rectangular swimming pool is 24 m. What is the length and width (w) of the pool, if the length of the pool is twice its width?

Key points
- The perimeter of the rectangular pool is 24 m.
- The length of the rectangle is twice the width of the rectangle.
- What is the length and width of the rectangle?

Explanation

The perimeter of a rectangle is twice the length plus twice the width.



Perimeter = Length + Length + Width + Width

The perimeter is 24 and the length is twice the width.

$24 = 2w + 2w + w + w$

$24 = 6w$

$24 \div 6 = 6w \div 6$

$4 = w$

If the width is 4 m then the length is $4 \times 2 = 8$ m.

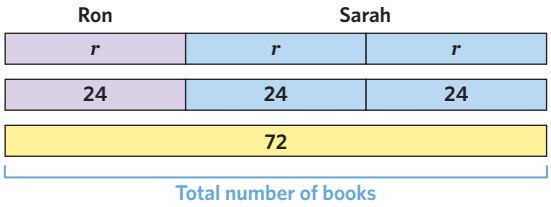
Answer

The length of the pool is 8 m and the width is 4 m.

10. Create and solve an equation to work out how many books Ron should read during a read-a-thon with Sarah. They are aiming to read 72 books in total, and Sarah has promised she will read twice as many books as Ron.

Key points
- Ron and Sarah will read a total of 72 books.
- Sarah will read twice as many books as Ron.
- How many books should Ron read?

Explanation



Let r represent the amount of books Ron read.

Sarah will read twice as many books as Ron so she read $2 \times r$ books.

$72 = r + 2r$

$72 = 3r$

$72 \div 3 = 3r \div 3$

$24 = r$

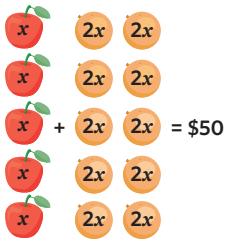
Answer

Ron should read 24 books.

11. Jasmine goes to an organic orchard with her friends and spends 50 dollars. She buys 5 apples and 10 oranges. If apples cost x dollars and oranges are $2x$ dollars, how much does each apple and orange cost?

Key points
- Jasmine spends a total of 50 dollars.
- Jasmine buys 5 apples.
- Jasmine buys 10 oranges.
- Apples cost x dollars.
- Oranges cost $2x$ dollars.
- How much does each apple and orange cost?

Explanation



$5x + 20x = \$50$

The amount spent on apples is $5 \times x$.

The amount spent on oranges is $10 \times 2x$.

The total amount spent is \$50.

$5x + 20x = 50$

$25x = 50$

$25x \div 25 = 50 \div 25$

$x = 2$

Apples cost \$2.

$2x = 2 \times 2$

$= 4$

Oranges cost \$4.

Answer

Each apple costs \$2 and each orange costs \$4.

12. Susie was delivering flyers every day in her neighbourhood. Write a simplified expression (in terms of x and y) to represent how many flyers Susie delivered in total. During the first week, she was able to deliver x flyers per day. During the next two weeks she delivered y flyers per day.

Key points

- Susie delivered x flyers per day in the first week.
- Susie delivered y flyers per day for the next 2 weeks.
- What is an expression that shows the total amount of flyers Susie delivered?

Explanation

In week 1 Susie delivered $7 \times x$ flyers.

In week 2 Susie delivered $7 \times y$ flyers.

In week 3 Susie delivered $7 \times y$ flyers.

$(7 \times x) + (7 \times y) + (7 \times y) = 7x + 7y + 7y$

$= 7x + 14y$

Answer

$7x + 14y$

Reasoning

13. a. $x - 1$
    b. $x + 2$
    c. $3x + 1 = 73$
    d. Phoebe: 24 years old
    Paige: 23 years old
    Piper: 26 years old
    e. Suggested option 1: Maybe you want to continue your education past high school and get a university degree.
    Suggested option 2: Maybe you want to start working full time as early as possible.
    Note: There are other possible options.

14. a. $52p$
    b. $52p + 38t$
    c. The total cost is \$142.
    d. Tim and his sister can ski for 5 days.
    e. Suggested option 1: Snowboarding might be more popular, so the rental company can afford to make the price higher.
    Suggested option 2: A snowboard is more expensive than skis so the rental company needs to make the price higher.
    Note: There are other possible options.

Extra spicy

15. C 16. C 17. $4h + 4k$ 18. $k + 4$

Remember this?

19. 14 hours and 45 minutes
20. A
21. 45 muesli bars

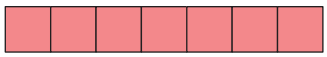
4H Geometric patterns and numerical sequences

Student practice

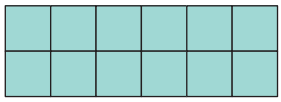
Worked example 1.

a. 11 b. 3 c. -52 d. 41

Worked example 2.

a. i) 7
ii) 

b. i) 3
ii) 

c. i) 12
ii) 

d. i) 1
ii) 

Worked example 3.

a.

n	1	2	3	4	5
t	2	5	8	11	14

b.

n	1	2	3	4	5
t	9	14	19	24	29

c.

n	3	4	5	6	7
t	2	6	10	14	18

d.

n	6	9	12	15	18
t	6	7	8	9	10

Worked example 4.

a. 23 b. 250 c. $85\frac{2}{3}$ d. -8

Understanding worksheet

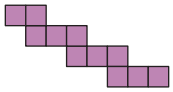
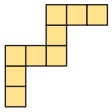
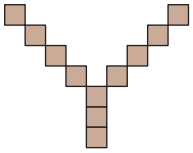
1. a. A b. C c. C d. B

2. a. +5 +5 +5 +5
   3 8 13 18 23
   b. −7 −7 −7 −7
   33 26 19 12 5
   c. −8 −8 −8 −8
   −25 −33 −41 −49 −57
   d. +6 +6 +6 +6
   −7 −1 5 11 17

3. geometric; consistent; numerical; rule; predict

Fluency

4. a. 16 b. 12 c. 160 d. 48
   e. −20 f. −27 g. 2 h. $4\frac{1}{2}$

5. a. i) 4 ii) 
   b. i) 3 ii) 
   c. i) 11 ii) 
   d. i) 3 ii) 
   e. i) 9 ii) 
   f. i) 11 ii) 

6. a.

Position (n)	1	2	3	4	5
Term (t)	2	7	12	17	22

b.

Position (n)	1	2	3	4	5
Term (t)	10	17	24	31	38

c.

Position (n)	3	4	5	6	7
Term (t)	16	18	20	22	24

d.

Position (n)	8	9	10	11	12
Term (t)	12	16	20	24	28

e.

Position (n)	9	7	5	3	1
Term (t)	300	200	100	0	−100

f.

Position (n)	10	12	14	16	18
Term (t)	$8\frac{1}{2}$	9	$9\frac{1}{2}$	10	$10\frac{1}{2}$

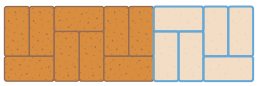
7. a. 63 b. 92
   c. 230 d. 400
   e. 21 f. 202
   g. 500 h. $32\frac{2}{9}$ or $\frac{290}{9}$

Problem solving

8. Hilda likes a brick wall design called the running and stack bond. It is a design that repeats itself in a consistent pattern. Which 6 brick design would continue the running and stack bond?

Key points
- The design repeats itself in a consistent pattern.
- Which 6 brick design continues the same pattern?

Explanation



The pattern includes groups of 3 bricks:
1 horizontal and 2 vertical.

With each group of 3 bricks, the horizontal brick alternates between being at the top and being at the bottom.

Answer

D

9. Magda is painting parking spots with straight lines. If she has completed 3 parking spots already, how many additional lines will she need to complete an additional 2 parking spots?

Key points
- Magda has completed 3 parking spots.
- She wants to add 2 more parking spots.
- How many lines does she require to add these spots?

Explanation

The number of lines required for each existing spot are 3, 5 and 7 as a sequence.

The common difference is +2.

This means that she needs 2 more lines for each additional spot.

As she needs 2 spots, she requires $2 \times 2 = 4$ more lines.

Answer

Magda will need 4 more lines to complete 2 additional parking spots.

10. How much can Greg expect to pay for 5 hours of court hire if the cost of hiring for an additional hour is constant?

Key points
- Greg wants to hire the court for 5 hours.
- The cost of hiring for an additional hour is constant.
- How much will Greg pay?

Explanation

Compare the consecutive costs to determine the common difference.

The common difference between 20, 35 and 50 is +15.
Apply the common difference to the cost for every extra hour.

Cost after 4 hours: $50 + 15 = 65$

Cost after 5 hours: $65 + 15 = 80$

Answer
Greg can expect to pay $80 for 5 hours of court hire.

11. Lenny is following a squat program which increases the recommended squat weight by the same amount each week. The program follows the rule $s = 5w + 35$, where s represents the squat weight in kilograms and w denotes the number of weeks. How much will Lenny be able to squat in the twelfth week?

Key points
- The program follows the rule $s = 5w + 35$.
- We want to find how much he will be able to squat in the twelfth week.
- What is the value of s when $w = 12$?

Explanation
Substitute $w = 12$ into the rule and solve for s.

$s = 5 \times 12 + 35$
$= 60 + 35$
$= 95$

Answer
Lenny will be able to squat 95 kg in the twelfth week.

12. Kishwar needs to work out how many pages she can read on her kindle in three quarters of an hour. Her kindle predicts the number of pages she can read based on the formula $p = \frac{3t}{5} + 3$, where p denotes the pages read and t represents the number of minutes spent reading.

Key points
- Kishwar reads for three quarters of an hour (45 minutes).
- Her kindle predicts the number of pages she can read based on the formula $p = \frac{3t}{5} + 3$.
- What is the value of p when $t = 45$?

Explanation
Substitute $t = 45$ into the rule and solve for p.

$p = \frac{3 \times 45}{5} + 3$
$= \frac{135}{5} + 3$
$= 27 + 3$
$= 30$

Answer
Kishwar can read 30 pages in three quarters of an hour.

Reasoning

13. a.

Hours	1	2	3	4
Total exams marked	6	12	18	24

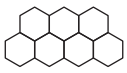
b. Mr. Jones would expect to have marked 32 exams.

c. 10 exams

d. Suggested option 1: Yes. Mr. Jones is experienced and has already been marking at the same rate for 5 hours.

Suggested option 2: No. Mr. Jones has been marking for a while and will likely get tired and slow down at some point.

Note: There are other possible options.

14. a. 

b. 8 pieces of timber are added to create design 4.

c. Barry will use 620 cm of timber for design 4.

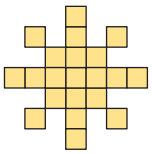
d. Barry will finish design 4 at 6:50 pm.

e. Suggested option 1: Barry could use different shapes that are better for storage purposes.

Suggested option 2: Barry could still use hexagons to create the shelves but rotate them so they sit flat.

Note: There are other possible options.

Extra spicy

15. 

16. 63 triangles

17. 21 paving stones are used.

18. $g = -3$

Remember this?

19. A 20. C 21. A

4I Creating rules from tables and sequences

Student practice

Worked example 1.

a. Each term increases by 5.

b. Each term decreases by 8.

c. Each term increases by $\frac{2}{3}$.

d. Each term decreases by 6.

Worked example 2.

a. i)

Position (n)	1	2	3	4
Number of shapes (s)	2	4	6	8

ii) $s = 2n$

b. i)

Position (n)	1	2	3	4
Number of shapes (s)	1	3	5	7

ii) $s = 2n - 1$

c. i)

Position (n)	1	2	3	4
Number of shapes (s)	6	11	16	21

ii) $s = 5n + 1$

d. i)

Position (n)	1	2	3	4
Number of shapes (s)	5	8	11	14

ii) $s = 3n + 2$

Worked example 3.

a. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	12	24	36	48

ii) $t = 12n$

b. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	3	9	15	21

ii) $t = 6n - 3$

c. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	22	26	30	34

ii) $t = 4n + 18$

d. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	−70	−60	−50	−40

ii) $t = 10n - 80$

Understanding worksheet

1. a.

Position (n)	1	2	3	4
Number of dots (s)	3	5	7	9
Change in number of dots		+2	+2	+2

b.

Position (n)	1	2	3	4
Number of dots (s)	12	9	6	3
Change in number of dots		−3	−3	−3

c.

Position (n)	1	2	3	4
Number of dots (s)	1	5	9	13
Change in number of dots		+4	+4	+4

d.

Position (n)	1	2	3	4
Number of dots (s)	6	12	18	24
Change in number of dots		+6	+6	+6

2. a.

Position (n)	1	2	3	4
Term in the n^{th} position	9	16	23	30
Change in n^{th} term		+7	+7	+7

b.

Position (n)	1	2	3	4
Term in the n^{th} position	1	12	23	34
Change in n^{th} term		+11	+11	+11

c.

Position (n)	1	2	3	4
Term in the n^{th} position	2	−6	−14	−22
Change in n^{th} term		−8	−8	−8

d.

Position (n)	1	2	3	4
Term in the n^{th} position	−3	$-1\frac{1}{2}$	0	$1\frac{1}{2}$
Change in n^{th} term		$+1\frac{1}{2}$	$+1\frac{1}{2}$	$+1\frac{1}{2}$

3. terms; sequences; n; position; coefficient

Fluency

4. a. Each term increases by 10.
b. Each term decreases by 2.
c. Each term increases by 1.
d. Each term increases by $\frac{3}{4}$.
e. Each term increases by 8.
f. Each term decreases by $\frac{4}{5}$.

5. a. i)

Position (n)	1	2	3	4
Number of shapes (s)	4	8	12	16

ii) $s = 4n$

b. i)

Position (n)	1	2	3	4
Number of shapes (s)	2	5	8	11

ii) $s = 3n - 1$

c. i)

Position (n)	1	2	3	4
Number of shapes (s)	1	4	7	10

ii) $s = 3n - 2$

d. i)

Position (n)	1	2	3	4
Number of shapes (s)	8	12	16	20

ii) $s = 4n + 4$

e. i)

Position (n)	1	2	3	4
Number of shapes (s)	4	9	14	19

ii) $s = 5n - 1$

f. i)

Position (n)	1	2	3	4
Number of shapes (s)	6	8	10	12

ii) $s = 2n + 4$

6. a. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	15	30	45	60

ii) $t = 15n$

b. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	2	8	14	20

ii) $t = 6n - 4$

c. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	16	21	26	31

ii) $t = 5n + 11$

d. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	−6	−3	0	3

ii) $t = 3n - 9$

e. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	−50	−43	−36	−29

ii) $t = 7n - 57$

f. i)

Position (n)	1	2	3	4
Term in the n^{th} position (t)	$-10\frac{1}{3}$	$-6\frac{1}{3}$	$-2\frac{1}{3}$	$1\frac{2}{3}$

ii) $t = 4n - 14\frac{1}{3}$

7. a. i) $t = 9n$ ii) $t = 450$

b. i) $t = 5n + 7$ ii) $t = 257$

c. i) $t = 12n - 11$ ii) $t = 589$

d. i) $t = 25n - 200$ ii) $t = 1050$

e. i) $t = 3\frac{1}{2}n + 4\frac{3}{4}$ ii) $t = 179\frac{3}{4}$

f. i) $t = 2\frac{1}{2}n - 9\frac{1}{3}$ ii) $t = 115\frac{2}{3}$

Problem solving

8. Frank uses the following rule to model the amount of water in litres, w, inside his bath n minutes after he turns on the tap: $w = 12n$. How much water is added to the bath each minute according to the rule?

Key points
- The amount of water in the bath (L), w, after n minutes is $w = 12n$.
- How many additional litres of water are in the bath after each minute?

Explanation

In the rule, n has a coefficient of 12.

This means that for every additional minute after the tap is turned on, 12 litres of water are added to the bath.

Answer

12 litres of water are added to the bath each minute.

9. On his first run, Drew ran for 7 minutes. After the first run, Drew extended the amount of time he ran in each consecutive session by 6 minutes. Fill in the table to show how many minutes Drew will run in the fifth, sixth and seventh session of his routine.

Key points
- Drew's first run lasted 7 minutes.
- Each of the next runs was 6 minutes longer than the one before.
- For how long did Drew run on his fifth, sixth, and seventh runs?

Explanation

Continue the pattern, starting at 7, and increasing by 6 each time.

Run 1: 7 mins

Run 2: $7 + 6 = 13$ mins

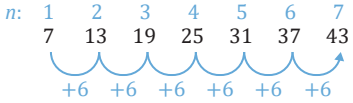
Run 3: $13 + 6 = 19$ mins

Run 4: $19 + 6 = 25$ mins

Run 5: $25 + 6 = 31$ mins

Run 6: $31 + 6 = 37$ mins

Run 7: $37 + 6 = 43$ mins



Answer

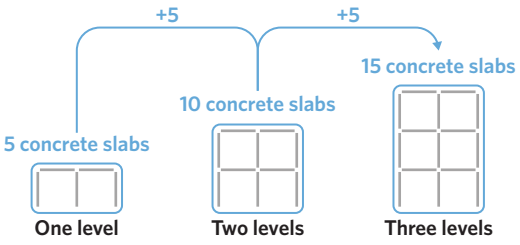
Session number (n)	5	6	7
Number of minutes (m)	31	37	43

10. The number of concrete slabs needed to build apartment buildings increases constantly for each additional level of the building.

Write a rule to predict the number of concrete slabs required, s, from the number of levels, l, in an apartment building.

Key points
- The number of concrete slabs needed increases constantly for each additional level.
- The pattern is shown in the diagram.
- What is an equation that can describe the number of concrete slabs, s, needed for an apartment building with l levels.

Explanation



The number of concrete slabs increases by 5 for each additional level.

This means that the coefficient of l is 5.

Subtract the coefficient of l from the number of slabs needed for a one-level building to find the constant.

$5 - 5 = 0$

There is no constant in the rule.

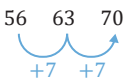
Answer

$s = 5l$

11. Tom is knitting a jumper. The number of stitches he uses to knit a size 1, 2 and 3 jumper is 56, 63 and 70 stitches respectively. If the pattern of number of the stitches he uses for each size continues, write a rule to describe the number of stitches (s) required for an n size jumper.

Key points
- The number of stitches used to knit a size 1, 2 and 3 jumper is 56, 63 and 70 stitches respectively.
- The pattern for the number of stitches continues.
- What is an equation that can predict the number of stitches (s) used for an n size jumper?

Explanation



The number of stitches increases by 7 for each increase in size.

This means that the coefficient of n is 7.

Substitute $n = 1$ and $s = 56$ into $s = 7n + c$ and solve for c.

$s = 7n + c$

$56 = 7 \times 1 + c$

$56 = 7 + c$

$56 - 7 = 7 - 7 + c$

$49 = c$

Substitute $c = 49$ into the rule.

$s = 7n + c$

$s = 7n + 49$

Answer

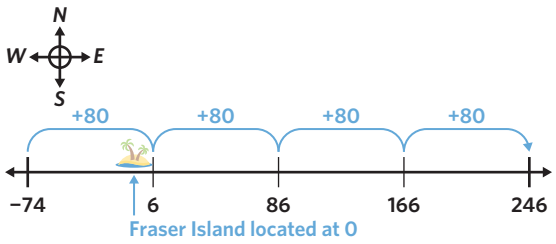
$s = 7n + 49$

12. A boat leaves a port and travels 80 kilometres east per day. Where is it after the 4th day of its journey in relation to Fraser Island, if the departure port was 74 kilometres west of Fraser Island?

Key points

- A boat travels 80 km east per day from the port.
- The departure port was 74 kilometres west of Fraser Island.
- What is the difference in locations between Fraser Island and the boat after 4 days?

Explanation



Let d be the distance east from Fraser Island after n days.

The boat travels 80 km each day. This means that the coefficient of n is 80.

The boat starts 74 km west of Fraser Island. This is -74 km east of Fraser Island when $n = 0$.

Substitute $n = 0$ and $d = -74$ into $d = 80n + c$ and solve for c.

$d = 80n + c$

$-74 = 80 \times 0 + c$

$-74 = 0 + c$

$-74 = c$

The rule is $d = 80n - 74$.

Substitute $n = 4$ into $d = 80n - 74$ to find the distance from Fraser Island after the 4th day.

$d = 80n - 74$

$d = 80 \times 4 - 74$

$d = 320 - 74$

$d = 246$

Answer

The boat is 246 km east of Fraser Island after the 4th day.

Reasoning

13. a.

Number of bikes stored (b)	1	2	3	4
Length of storage rack, in cm (l)	20	50	80	110

b. $l = 30b - 10$

c. The rack for 250 bikes is 1500 cm shorter than the rack for 300 bikes.

d. The total length of the racks will be 50 cm shorter than one 300-bike rack.

e. Suggested option 1: The storage racks can be organised into multiple separate rows, rather than just one long rack.

Suggested option 2: The bikes could be stored in a specific order based on the competitors' predicted finishing times so they are easily accessible for the athletes.

Note: There are other possible options.

14. a. Cost of a party for adults: $c = 55a + 135$

Cost of a party for children: $c = 30a + 275$

b. There is a $25 cost difference between an additional adult and an additional child.

c. One lifesaver at a party for children costs $137.50.

d. 6 adults need to attend a party so that it is more expensive than a children's party of the same size.

e. Suggested option 1: Leopold's Yacht Club could give customers the rules to calculate the cost of their party based on their number of guests.

Suggested option 2: Leopold's Yacht Club could extend the table further so customers do not have to calculate it for themselves.

Note: There are other possible options.

Extra spicy

15. $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}$

The coefficient of n is $\frac{1}{12}$.

16. $t = \frac{1}{6}n + \frac{1}{2}$

17. $x = 5$

18. $t = 4n - 3\frac{2}{3}$

Remember this?

19. C 20. A

21. One medium chips costs $3.75.

Chapter 4 extended application

1. a. It costs $225 to rent a car for 5 days.

b. $cost = 200 + 15d$

c. Woody Automotive because it is cheaper than Buzz Rentals.

d. Suggested option 1: Yes, because this will reduce your personal environmental impact.

Suggested option 2: Yes, because you want to support businesses that care about the environment.

Note: There are other possible options.

e. Suggested option 1: You could take flights between the cities with airports.

Suggested option 2: You could ride a bike around.

Note: There are other possible options.

2. a. $w = 10$ m, $l = 80$ m

$w = 25$ m, $l = 50$ m

$w = 35$ m, $l = 30$ m

Note: There are many possible sets of values.

b. 800 m^2, 1250 m^2, 1050 m^2

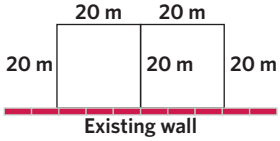
Note: Other answers are possible depending on your sets of values for part **a**.

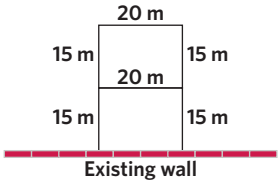
c. Suggested option 1: Use the option with the largest area.

Suggested option 2: Use the option that best fits the size and area of land available.

Note: There are other possible options.

d.





e. Suggested option 1: Follow the regulations about the amount of space required per sheep.

Suggested option 2: Work out how much space you think each sheep should have and divide the total area of the paddock by this amount of space.

Note: There are other possible options.

f. Suggested option 1: Use the least amount of fencing for the area you calculate that you need.

Suggested option 2: Create a paddock that fits the dimensions of the land that is available.

Note: There are other possible options.

3. a. You would receive 4750 rupees.

b. You would need $45 AUD.

c. The exchange rate means that the cost of petrol is very similar. It may be slightly cheaper in India for a number of reasons such as taxes and wages.

Note: There are other possible options.

d. Suggested option 1: Australia could be considered wealthier because 46 000 rupees is a much lower median wage than $48 000.

Suggested option 2: India could be considered wealthier if you compare the total amount of money earned by all people within each country.

Note: There are other possible options.

Chapter 4 review

Multiple choice

1. D 2. B 3. C 4. B 5. D

Fluency

6. a. $7 + 1 + 4 = 12$

b. $7 - 6 + 8 = 3 + 6$

c. $3 + 3 + 3 = 3 \times 3$

d. $8 \times 2 - 4 = 1 + 2 + 3 + 4 + 5 - 6$

7. a. xy b. $xy + 9$ c. $\frac{x}{y}$ d. $\frac{12xy}{z}$

8. a. $t + 5 = 41$ b. $w - 6 = 69$

c. $3p = 39$ d. $\frac{x}{4} = 19$

9. a. True b. True c. False d. False

10. a. 8 cm^2 b. 15 km^2 c. 385 m^2 d. 38 mm^2

11. a. $y = 24$ b. $m = 8$

c. $x = 200$ d. $c = \frac{11}{3}$ or $c = 3\frac{2}{3}$

12. a. $x = 3$ b. $c = 7$

c. $y = 43$ d. $m = \frac{10}{3}$ or $m = 3\frac{1}{3}$

13. a. $4k + 8 = 24$

$k = 4$

b. $\frac{a}{9} + 5 = 12$

$a = 63$

c. $\frac{z - 4}{6} = 5$

$z = 34$

d. $\frac{7x}{3} = 14$

$x = 6$

14. a. $x = 5$ b. $x = 4$ c. $x = 23$ d. $x = 115$

15. a. $3x = 24$ b. $4j + 5k = -4$

c. $7q - 2qr = 25$ d. $5s - t = 37$

16. a. $x = 9$ b. $x = \frac{1}{2}$ c. $x = 4$ d. $x = 10$

17. a. 5, 9, 13 b. −5, −2, 1 c. 23, 29, 35 d. −2, 9, 20

18. a. 13 b. 49 c. 436.5 d. $8\frac{1}{3}$

19. a. $t = 3n + 8$ b. $t = 20n - 220$

c. $t = 1\frac{1}{2}n + 1\frac{3}{4}$ d. $t = 2\frac{1}{2}n - 7$

Problem solving

20. Bobby is aiming to run 42 km over a single week. Between Monday and Saturday he ran 5 km per day. How many km does he need to run on Sunday?

Key points

- Bobby wants to run 42 km in a week.
- From Monday to Saturday, he ran 5 km per day.
- How many km does Bobby still need to run to reach 42 km?

Explanation

Bobby already ran 5 km per day for 6 days.

He has already run $5 \times 6 = 30$ km.

He wants to run 42 km in total.

5 km	5 km	5 km	5 km	5 km	5 km	?

42 km

$5 \times 6 + ? = 42$

$30 + ? = 42$

The unknown number must be 12 for the equation to be true.

$30 + 12 = 42$ ✓

Answer

Bobby needs to run 12 km on Sunday.

21. Horace is making a replica model of a farm. He needs 200 small wooden pieces, and 120 large wooden pieces. The small pieces are x cm long and the large pieces are y cm long. The total length of the wooden pieces is 3600 cm. Write an equation to represent this situation.

Key points

- Jake needs 200 small pieces that are each x cm long.
- He also needs 120 large pieces that are each y cm long.
- The total length of the pieces is 3600 cm.
- Represent this situation in an equation.

Explanation

200 small pieces that are each x cm long: $200x$

120 large pieces that are each y cm long: $120y$

The total length of both the small and large pieces is 3600 cm.

$200x + 120y = 3600$

Answer

$200x + 120y = 3600$

22. Donald owns a patisserie. In a single day he sold 24 boxes of croissants and 43 single croissants. If there are 4 croissants in a box, how many croissants did Donald sell in total?

Key points

- Donald sold 24 boxes of croissants.
- He also sold 43 single croissants.
- There are 4 croissants in each box.
- What was the total number of croissants sold?

Explanation

Donald sold 24 boxes of c croissants as well as 43 single croissants.

$total = 24c + 43$

Each box contains 4 croissants so replace c with 4.

$$\begin{aligned} total &= 24 \times 4 + 43 \\ &= 96 + 43 \\ &= 139 \end{aligned}$$

Answer

Donald sold 139 croissants.

23. Rafael ordered an extra large Halal Snack Pack for him and 3 of his friends to share. It had a weight of 2400 g. Using w to represent the amount of grams each of them ate, create an equation that represents this situation and solve for w.

Key points

- Rafael and 3 of his friends split a Halal Snack Pack.
- It had a total weight of 2400 g.
- Represent this as an equation and solve for w (the amount eaten by each person).

Explanation

w	w	w	w

2400

$$\begin{aligned} 4w &= 2400 \\ 4w \div 4 &= 2400 \div 4 \\ w &= 2400 \div 4 \\ w &= 600 \end{aligned}$$

Answer

Each person ate 600 g of the Halal Snack Pack.

24. In a game of AFL a goal is worth 6 points and a behind is worth 1 point. The Giants scored 13 behinds and ended on 97 points. Using g to represent the number of goals they kicked, create an equation that represents this situation and solve for g.

Key points

- A goal is worth 6 points and a behind is worth 1 point.
- The Giants scored 13 behinds.
- They ended on 97 points.
- Represent this as an equation and solve for g (the number of goals kicked).

Explanation

The points they gained from scoring behinds is 13.

The points they gained from kicking goals is $6g$.

In total, they scored 97 points: $6g + 13 = 97$

$$\begin{aligned} 6g + 13 &= 97 \\ 6g + 13 - 13 &= 97 - 13 \\ 6g &= 84 \\ 6g \div 6 &= 84 \div 6 \\ g &= 14 \end{aligned}$$

$$\begin{aligned} &6g + 13 = 97 \\ -13 \quad &\quad -13 \\ &6g = 84 \\ \div 6 \quad &\quad \div 6 \\ &g = 14 \end{aligned}$$

Answer

The Giants kicked 14 goals.

25. James is planning his 22nd birthday party. He is planning on giving a gift to everyone he invites. He invites c friends from his cricket team and 2 friends from work. The gifts will cost $20 each and he plans on spending $260 in total. How many friends did he invite from his cricket team?

Key points

- James invites c friends from his cricket team and 2 friends from work.
- He gives his guests gifts that cost $20 each.
- He spent $260 on the gifts.
- Create an equation and solve for c.

Explanation

The number of guests is $c + 2$.

The amount of money spent on gifts is $20(c + 2) = 260$.

$$\begin{aligned} 20(c + 2) &= 260 \\ 20(c + 2) \div 20 &= 260 \div 20 \\ c + 2 &= 13 \\ c + 2 - 2 &= 13 - 2 \\ c &= 11 \end{aligned}$$

$$\begin{aligned} &20(c + 2) = 260 \\ \div 20 \quad &\quad \div 20 \\ &c + 2 = 13 \\ -2 \quad &\quad -2 \\ &c = 11 \end{aligned}$$

Answer

James invited 11 friends from his cricket team.

26. Sid went to the store to buy cat food. He bought 3 boxes of biscuits, 4 cans of tuna and a packet of red meat. Each box of biscuits costs $\$3x$, each can of tuna costs $\$x$ and each packet of red meat costs $\$4x$. If the total price was $34, how much does a packet of red meat cost?

Key points

- Sid bought 3 boxes of biscuits, 4 cans of tuna and a packet of red meat.
- The price of a box of biscuits was $\$3x$.
- The price of a can of tuna was $\$x$.
- The price of a packet of red meat was $\$4x$.
- The total price was $34.
- Find the value of $4x$.

Explanation

Amount spent on biscuits: $3 \times 3x$

Amount spent on tuna: $4 \times x$

Amount spent on red meat: $4x$

Total amount spent: $3 \times 3x + 4 \times x + 4x = 34$

$$3 \times 3x + 4 \times x + 4x = 34$$
$$9x + 4x + 4x = 34$$
$$17x = 34$$
$$17x \div 17 = 34 \div 17$$
$$x = 2$$

Each packet of red meat costs $\$4x$.

$$4x = 4 \times 2$$
$$= \$8$$

Answer

A packet of red meat costs $8.

27. Roger uses pencils to create the following pattern with four squares. How many pencils in total are needed to make six squares?

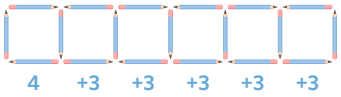
Key points

- The diagram shows the number of pencils required to form each number of squares.
- How many pencils would it take to build six squares?

Explanation

It takes 4 pencils to make one square, 7 pencils to make two squares, 10 pencils to make three squares and 13 pencils to make four squares.

The common difference is +3, meaning 3 more pencils are required for each additional square.



5 squares: $13 + 3 = 16$

6 squares: $16 + 3 = 19$

Answer

19 pencils are needed to make six squares.

28. Felix is a competitive swimmer and has made a 28-day training plan for an upcoming competition. On the first day of his training plan, he will swim 3750 metres. Each day, he will increase the distance he swims by 50 metres. How far will Felix swim on the last day of his training plan?

Key points

- On the first day, Felix will swim 3750 metres.
- Each day, Felix will swim 50 metres more than the day before.
- How many metres will Felix swim on the 28th day?

Explanation

Let s be the distance Felix swims on the n^{th} day of the training plan.

Felix increases the distance by 50 m each day. This means that the coefficient of n is 50.

Felix swims 3750 m on day 1. This means that $s = 3750$ when $n = 1$.

Substitute $n = 1$ and $s = 3750$ into $s = 50n + c$ and solve for c.

$$s = 50n + c$$
$$3750 = 50 \times 1 + c$$
$$3750 = 50 + c$$
$$3750 - 50 = 50 - 50 + c$$
$$3700 = c$$

The rule is $s = 50n + 3700$.

Substitute $n = 28$ into $s = 50n + 3700$ to find the distance Felix will swim on the 28th day.

$s = 50n + 3700$

$s = 50 \times 28 + 3700$

$d = 1400 + 3700$

$d = 5100$

Answer

Felix will swim 5100 metres on the last day of his training plan.

Reasoning

29. a. x is 5 km.

b. $P = 30m$

c. The perimeter of the new plot of land is 90 km.

d. Tommy would need 3300 fence posts for the original plot of land.

e. Suggested option 1: I would rather live in the city because it is easier to get around by public transport.

Suggested option 2: I would rather live in the country to be closer to nature and fresh air.

Note: There are other possible options.

30. a. $p = 4n + 6$

b. Arthur will do 50 push ups on day 11.

c. Arthur will do 70 push ups on day 18.

d. Suggested option 1: No, the number of push ups will continue to increase and will become ridiculous if he continues the challenge indefinitely.

Suggested option 2: He could start to decrease the number of additional push ups each day so it is still manageable.

Note: There are other possible options.

5A Place value and comparing decimals

Student practice

Worked example 1.

a. $\frac{5}{100}$ b. $\frac{1}{1000}$ c. $\frac{21}{100}$ d. $\frac{40\,334}{1000}$

Worked example 2.

a. 0.719 b. 2507.316 c. 0.1345 d. 27.039

Worked example 3.

a. $0.3216 < 0.322$

b. $5.1900 = 5.19$

c. $130.16 > 100 + 30 + \frac{1}{10} + \frac{6}{1000}$

d. $18\frac{405}{1000} < 10 + 8 + \frac{4}{10} + \frac{5}{100}$

Worked example 4.

a. 1.34, 1.43, 1.6, 1.68, 1.86

b. 1.14, 1.15, 1.40, 1.41, 1.5

c. 5.0999997, 5.10, 5.8999, 5.9997, 5.99997

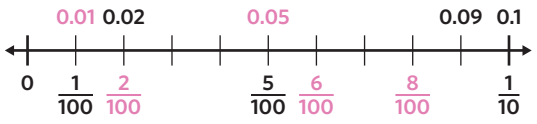
d. 100.01010, 100.1001, 100.1101, 101, 101.0001

Understanding worksheet

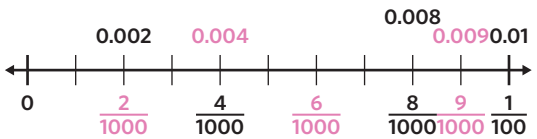
1. a. Decimal notation: 0.3
   Decimal fraction: $\frac{3}{10}$
   b. Expanded decimal form: 0.7
   Expanded fraction form: $\frac{7}{10}$
   c. Decimal notation: 0.85
   Decimal fraction: $\frac{85}{100}$
   d. Expanded decimal form: 0.5 + 0.09
   Expanded fraction form: $\frac{5}{10} + \frac{9}{100}$

2. a.

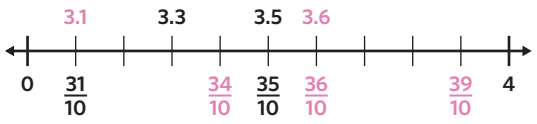
b.



c.



d.



3. decimal notation; expanded form; digit; decimal fraction

Fluency

4. a. $\frac{2}{10}$ b. $\frac{5}{100}$
   c. $\frac{4}{10}$ d. $\frac{0}{100}$ or 0
   e. $\frac{27}{1000}$ f. $\frac{1}{1000}$
   g. $\frac{124}{1000}$ h. $\frac{130\,002}{10\,000}$

5. a. 0.479 b. 7.2881
   c. 22.4203 d. 79.4382

6. a. 0.235 b. 3.629
   c. 48.0701 d. 141.10603

7. a. 0.5 b. 0.159 c. 2.75 d. 49.7

8. a. 0.327 b. 163.45
   c. 10 500.061 d. 75 388.241

9. a. $0.156 < 0.2$
   b. $23\frac{5}{10} = 23.50$
   c. $\frac{1}{10} + \frac{9}{100} > \frac{1}{10} + \frac{9}{1000}$
   d. $2.175 < 2 + \frac{3}{10}$
   e. $15.9 = 15.9000$
   f. $40 + 5 + \frac{3}{10} + \frac{1}{1000} < 45.31$
   g. $29\frac{9998}{10\,000} > 29.998$
   h. $1001.1001 < 1001.1010$

10. a. 0.12, 0.125, 0.308, 0.47, 0.501
    b. 0.001, 0.105, 0.23, 0.999, 1.000
    c. 1.0123, 1.1023, 1.12, 1.1203, 1.123
    d. 2.9990, 2.9998, 3.0999, 3.10, 3.9
    e. 6.00001, 6.0452, 6.245, 6.79, 6.8
    f. 19.0009, 19.001, 19.1, 19.11113, 1.91113

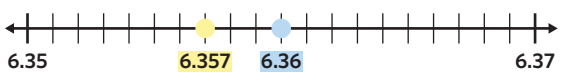
Problem solving

11. Piper and Dorethea went on a hike together. After finishing their hike they checked their smart watches to see the total distance they had walked.Who walked further if Piper's watch displayed 6.357 km and Dorethea's watch displayed 6.36 km?

Key points

- Piper walked 6.357 km.
- Dorethea walked 6.36 km.
- Who walked further?

Explanation



Compare the digits in each place value starting from left to right.

Whole numbers: 6.357 ☐ 6.36

Tenths: 6.357 ☐ 6.36

Hundredths: $6.357 < 6.36$

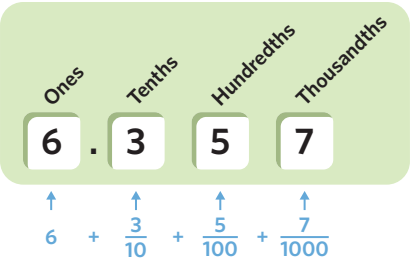
Answer

Dorethea walked further than Piper.

12. What distance does the '7' represent on Piper's smart watch reading of 6.357 km if there are 1000 metres in a kilometre?

Key points
- There are 1000 metres in a kilometre.
- What does the '7' represent in 6.357 km?

Explanation



$6.357 = 6 + \frac{3}{10} + \frac{5}{100} + \frac{7}{1000}$

The '7' represents $\frac{7}{1000}$ or 7 thousandths of a km.

A metre is a thousandth of a kilometre, so 7 thousandths of a kilometre is 7 metres.

Answer

The '7' represents 7 metres.

13. Nate had 25 mL of his 1000 mL jug of milk left, so he picked up another 2 litres from the supermarket. How much milk does Nate now have altogether in litres?

Key points
- Nate has $\frac{25}{1000}$ litres in his jug of milk.
- He bought another 2 litres of milk.
- How much milk does he now have in total?

Explanation

25 mL out of 1000 mL can be written as the fraction $\frac{25}{1000}$.

$2 + \frac{25}{1000} = 2 + 0.025$

$= 2.025$

Answer

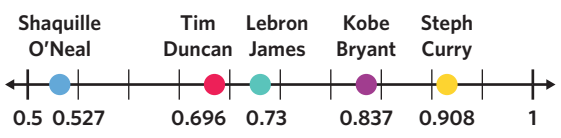
Nate has 2.025 litres of milk.

14. The probability that LeBron James makes a free throw is 0.73. The probability that 4 other NBA stars make a free throw is shown on the number line.

How many of the four NBA stars is Lebron James better than at free throws?

Key points
- The probability that Lebron James makes a free throw is 0.73.
- How many of the NBA stars is a free throw probability of 0.73 better than?

Explanation



Compare the digits in each place value for each decimal starting from left to right.

Tenths: $0.527 < 0.696 < 0.73 < 0.837 < 0.908$

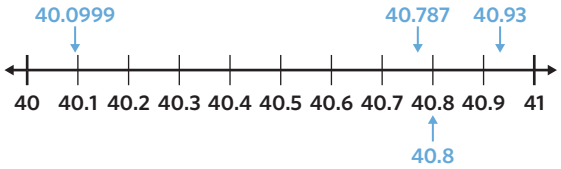
Answer

Lebron James is better than 2 of the 4 NBA stars at free throws.

15. Melanie wants to pick the drink that has the least amount of sugar. Use the table to determine which soft drink Melanie should choose.

Key points
- Melanie wants to pick the drink that has the least amount of sugar.
- Which drink has the least amount of sugar?

Explanation



Compare the digits in each place value starting from left to right.

Whole numbers: 40.787, 40.8, 40.93, 40.0999

Tenths: $40.0999 < 40.787 < 40.8 < 40.93$

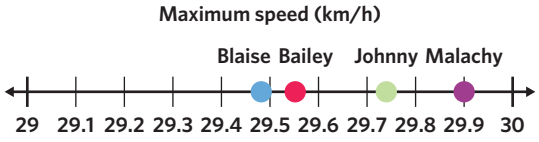
Answer

Melanie should choose Dr Pepper.

Reasoning

16. a. Malachy has the fastest maximum speed.

b.



c. $29\frac{49}{100}$ km/h or $\frac{2949}{100}$ km/h

d. $29 + \frac{7}{10} + \frac{4}{100}$ km/h

e. Suggested option 1: Yes, running at about 29.5 km/h would mean they would finish a 100 m sprint in 12.2 seconds, which seems reasonable.

Suggested option 2: No, maximum speeds should be higher for track athletes.

Note: There are other possible options.

17. a. 497

b. 5.0

c. 975.4

d. 90.457

e. Suggested option 1: Max could work on making the largest number while Delilah could work on making the smallest possible number.

Suggested option 2: Max and Delilah could work on each task together.

Note: There are other possible options.

Extra spicy

18. B 19. $\frac{2}{10}$ 20. 5.65

21. $3.25 + 7.64 = 10.89$

Note: There are many possible solutions.

Remember this?

22. C 23. D 24. C

5B Rounding decimals

Student practice

Worked example 1.

a. 4 b. 20 c. 87 d. 999

Worked example 2.

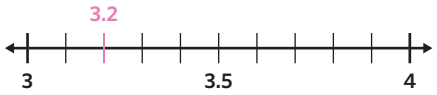
a. 0.64 b. 2.317 c. 0.4327 d. 3.0

Worked example 3.

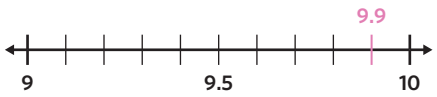
a. 6 b. 3 c. 10 d. 4

Understanding worksheet

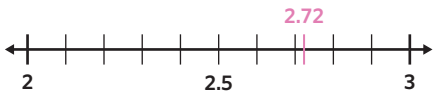
1. a. $3.2 \approx 3$



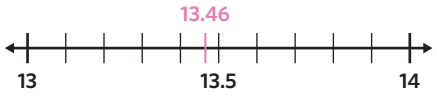
b. $9.9 \approx 10$



c. $2.72 \approx 3$



d. $13.46 \approx 13$



2. a. $3.2\underline{7}6 \approx 3.28$ b. $9.\underline{8}71 \approx 9.9$
   c. $0.63\underline{7}2 \approx 0.637$ d. $9\underline{8}736 \approx 10$

3. round; estimating; benchmark; critical; rounded

Fluency

4. a. 1 b. 8 c. 28 d. 44
   e. 112 f. 237 g. 130 h. 1000

5. a. 0.3 b. 3.33 c. 125.9 d. 0.484
   e. 321.49 f. 19.9 g. 987.80 h. 130.0

6. a. 0.7 b. 0.3 c. 0.673 d. 0.27
   e. 3.9822 f. 4.176 g. 3.0 h. 10.00

7. a. 3 b. 4 c. 22 d. 100
   e. 201 f. 41 g. 126 h. 1000

8. a. 1 b. 4 c. 7 d. 2
   e. 11 f. 0 g. 11 h. 50

Problem solving

9. Jamie goes to buy a watermelon. The clerk gives Jamie a watermelon that weighs 10.262 kg. What is the weight of the watermelon rounded to the nearest tenth of a kilogram?

Key points
- The watermelon weighs 10.262 kg.
- Find the weight of the watermelon to the nearest tenth of a kilogram.

Explanation

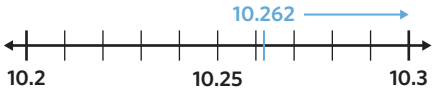
The watermelon weighs 10.262 kg.

Digit being rounded: 2

Critical digit: 6

$6 \geq 5$, so the digit being rounded should increase by 1.

$10.262 \approx 10.3$ kg



Answer

Rounded to the nearest tenth the watermelon's weight is 10.3 kg.

10. James is ranked first on his cricket team with a batting average of 99.9387. Cricket batting averages are usually only calculated to two decimal places. What is James' batting average rounded to two decimal places?

Key points
- James' batting average is 99.9387.
- What is 99.9387 rounded to two decimal places?

Explanation

Digit being rounded: 3

Critical digit: 8

$8 \geq 5$ so the digit being rounded should increase by 1.

99.9387 to 2 decimal places is 99.94.

Critical digit

$99.93\underline{8}7 \approx 99.94$

Digit being rounded

Answer

James' batting average to 2 decimal places is 99.94.

11. In 2018, the life expectancy in Australia was 82.75 years. The life expectancy in America was 78.54 in the same year. Round to the nearest year to estimate how much longer the average Australian lives than the average American.

Key points
- The average Australian life expectancy is 82.75 years.
- The average American life expectancy in 2018 is 78.54.
- Use rounding to estimate how much longer the average Australian lives than the average American.

Explanation

Australia:
- Whole number: 82
- Critical digit: 7
- $82.75 \approx 83$ years

America:

- Whole number: 78
- Critical digit: 5
- 78.54 ≈ 79 years

83 − 79 = 4 years

82.75 ≈ 83

Digit being rounded Critical digit

78.54 ≈ 79

Answer

The average Australian lived 4 years longer than the average American in 2018.

12. Kai and Eric are going to the grocery store with $15 to buy ingredients to make cupcakes. How much can they approximately expect to pay if they round the cost of each item to the nearest dollar?

SHOPPING LIST	
Cake mix	$4.90
Caster sugar	$3.50
Butter	$2.85
Sprinkles	$1.75

Key points

- Cake mix costs $4.90, caster sugar costs $3.50, butter costs $2.85 and sprinkles cost $1.75.
- Use rounding to estimate how much Kai and Eric will pay.

Explanation

Cake mix:

- Whole number: 4
- Critical digit: 9
- 4.90 ≈ $5

Caster sugar:

- Whole number: 3
- Critical digit: 5
- 3.50 ≈ $4

Butter:

- Whole number: 2
- Critical digit: 8
- 2.85 ≈ $3

Sprinkles:

- Whole number: 1
- Critical digit: 7
- 1.75 ≈ $2

5 + 4 + 3 + 2 = $14

Answer

Kai and Eric can expect to pay approximately $14.

13. Each of Jenny's school textbooks weigh 2.04 kg. When Jenny's backpack is full of textbooks it weighs 8.25 kg. The backpack without any textbooks in it weighs 1.87 kg. Estimate how many textbooks can fit in Jenny's backpack.

Key points

- Jenny's textbooks weigh 2.04 kg.
- Jenny's backpack with textbooks weighs 8.25 kg.
- Jenny's backpack without textbooks weighs 1.87 kg.
- Use rounding to estimate how many textbooks are in Jenny's backpack.

Explanation

Backpack with textbooks:

- Whole number: 8
- Critical digit: 2
- 8.25 ≈ 8 kg

Backpack without textbooks:

- Whole number: 1
- Critical digit: 8
- 1.87 ≈ 2 kg

Total weight of textbooks:

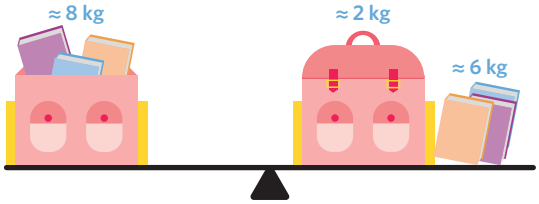
8 − 2 = 6 kg

Weight of one textbook:

- Whole number: 2
- Critical digit: 0
- 2.04 ≈ 2 kg

Number of textbooks:

6 ÷ 2 = 3



Answer

There are 3 textbooks in Jenny's backpack.

Reasoning

14. a. Blueberries will cost Julia the most money.
    b. Julia will spend approximately $28 on her fruit salad.
    c. Julia does have enough money.
    d. Suggested option 1: 2 Fuji apples, 2 Granny Smith apples, 2 kg of bananas, 125 g of blueberries, 1 dragon fruit and 1 cantaloupe.

    Suggested option 2: 3 Fuji apples, 1 Granny Smith apple, 2 kg of bananas, 250 g of blueberries and 1 dragon fruit.

    Note: There are other possible options.

15. a. The average pace is 10.5 km/h.
    b. B
    c. Anthony would need to run approximately 21 km/h in order to achieve a gold medal time.
    d. Suggested option 1: Anthony should work on his stamina so he can keep up his pace for longer.

    Suggested option 2: Anthony should work on his speed so he can increase his pace.

    Note: There are other possible options.

Extra spicy

16. Approximately 3087
17. 1.203
18. Subtract 35 059.05
19. 999.99900

Remember this?

20. B 21. D 22. C

5C Adding and subtracting with decimals

Student practice

Worked example 1.

a. 0.82 b. 1.624 c. 4.247 d. 1.234

Worked example 2.

a. 5.43 b. 2.192 c. 17.555 d. 7.274

Worked example 3.

a. 4.4 b. 1.79 c. 30.62 d. 8.665

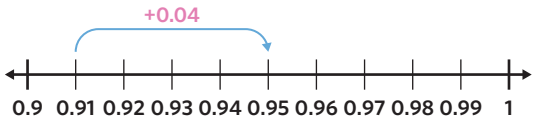
Understanding worksheet

1. a. 0.80 b. 0.60 c. 0.85 d. 0.22

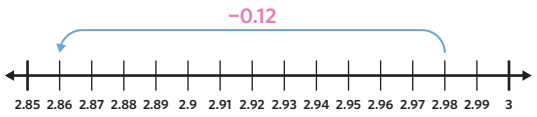
2. a. $0.2 + 0.4 = 0.6$

b. $0.75 - 0.2 = 0.55$

c. $0.91 + 0.04 = 0.95$



d. $2.98 - 0.12 = 2.86$



3. decimal; vertical; decimal points

Fluency

4. a. 0.77 b. 5.7 c. 1.4 d. 20.91
   e. 1.14 f. 0.237 g. 18.096 h. 1.015

5. a. 0.67 b. 9.7 c. 3.32 d. 2.31
   e. 31.037 f. 14.009 g. 13.137 h. 7.779

6. a. 2 b. 12.24 c. 41.1 d. 26.23
   e. 26.92 f. 32 g. 11 h. 0.891

7. a. 0.43 b. 1.882 c. 9.2 d. 17.83
   e. 4.52 f. 7 g. 19.25 h. 5.647

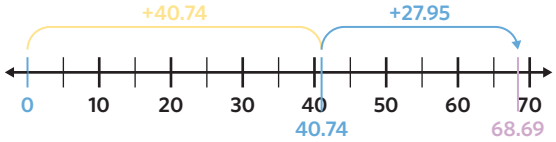
Problem solving

8. Captainglitterz needed more snacks for his Minecraft party. He ordered some Tim Tams for $40.74 and some Reeses Pieces for $27.95. How much did Captainglitterz spend on snacks for his Minecraft party?

Key points

- Captainglitterz ordered Tim Tams for $40.74.
- He ordered Reeses Pieces for $27.95.
- How much money did he spend in total?

Explanation



$$40.74 + 27.95 = \frac{4074}{100} + \frac{2795}{100}$$
$$= \frac{6869}{100}$$
$$= 68.69$$

Answer

Captainglitterz spent $68.69 on snacks for his party.

9. A Year 7 science competition involved collecting plastic for recycling over a three-week period. Looking at the following table, which class collected the greatest amount of plastic at the end of the three weeks?

Key points

- Three classes are collecting plastic for recycling.
- The amount of plastic collected each week for each class is shown in the table.
- Which of the classes had the greatest total weight of plastic?

Explanation

Calculate the total weight of plastic collected by each class by finding the sum of each column in the table. We are looking for the greatest weight of plastic.

Class 7A:

		+1		+1
	5	.	2	80
	5	.	1	02
+	5	.	9	70
	16	.	3	52

Class 7B:

		+1		
	4	.	3	20
	5	.	0	07
+	6	.	8	70
	16	.	1	97

Class 7C:

		+1		
	8	.	0	30
	2	.	5	59
+	5	.	7	10
	16	.	2	99

Answer

Class 7A collected the greatest amount of plastic.

10. Stan and his three teammates were running a 200-metre relay, and were trying to beat their record of 140 seconds. How much did Stan's team beat their record by, if the teammates' individual times were 27.46 seconds, 31.58 seconds, 28.11 seconds and 36.43 seconds?

Key points

- Stan and his teammates are trying to beat their record of 140 seconds.
- Each of their times are 27.46 secs, 31.58 secs, 28.11 secs and 36.43 secs.
- How much faster were they than their record of 140 seconds?

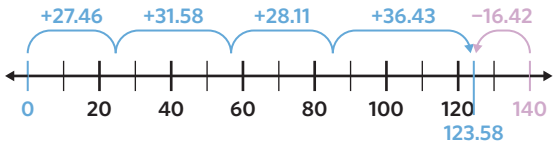
Explanation

Calculate the sum of the teammates' individual times.

	+2	+1		+1	
	2	7	.	4	6
	3	1	.	5	8
	2	8	.	1	1
+	3	6	.	4	3
1	2	3	.	5	8

Subtract the total time from the record of 140 seconds.

$$140 - 123.58 = \frac{14\,000}{100} - \frac{12\,358}{100}$$
$$= \frac{1642}{100}$$
$$= 16.42$$



Answer

They beat their record by 16.42 seconds.

11. The largest insect, the giant weta, has a recorded weight of 71 grams and the smallest insect, the parasitic wasp, has a recorded weight of 0.000025 grams. What is the difference between the weights of these two insects?

Key points

- The giant weta weighs 71 grams.
- The parasitic wasp weighs 0.000025 grams.
- How much heavier than the parasitic wasp is the giant weta?

Explanation

Subtract 0.000025 from 71.

$$71 - 0.000025 = 71 - \frac{25}{1000000}$$
$$= \frac{71000000}{1000000} - \frac{25}{1000000}$$
$$= \frac{70999975}{1000000}$$
$$= 70.999975 \text{ g}$$

Answer

The difference between the weights is 70.999975 grams.

12. Vlad, Frankie and Drac were donating blood at the local blood drive. How much blood did they donate altogether if Vlad donated 0.45 litres, Frankie donated 0.03 litres less than Vlad and Drac donated 0.02 litres **more** than Vlad?

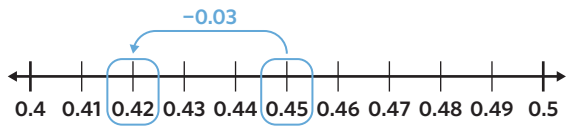
Key points

- Vlad donated 0.45 L of blood.
- Frankie donated 0.03 L less than Vlad.
- Drac donated 0.02 litres more than Vlad.
- What was the total amount of blood donated?

Explanation

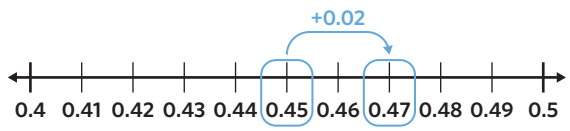
Frankie donated:

$$0.45 - 0.03 = \frac{45}{100} - \frac{3}{100}$$
$$= \frac{42}{100}$$
$$= 0.42$$



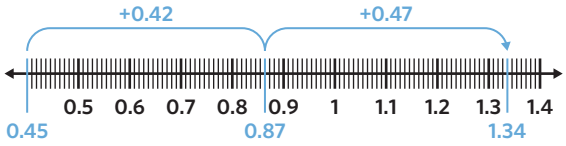
Drac donated:

$$0.45 + 0.02 = \frac{45}{100} + \frac{2}{100}$$
$$= \frac{47}{100}$$
$$= 0.47$$



Total amount donated:

$$0.45 + 0.42 + 0.47 = \frac{45}{100} + \frac{42}{100} + \frac{47}{100}$$
$$= \frac{134}{1000}$$
$$= 1.34 \text{ L}$$



Answer

They donated 1.34 L of blood altogether.

Reasoning

13. a. Luna used 9.01 metres of turf.
    b. Luna paid $98.00 for the turf.
    c. The perimeter of the lawn is 10.3 metres.
    d. Luna had 3.74 metres of turf left over.
    e. Suggested option 1: Luna could give the turf to a friend or neighbour who wants it.

    Suggested option 2: Luna could save the turf for a future project.

    Note: There are other possible options.

14. a. The highest temperature, 17.2°C, was recorded on Thursday.
    b. The difference between the highest and lowest temperatures is 12.6°C.
    c. Saturday had the most inaccurate predicted maximum temperature.
    d. The predicted maximum temperature on Saturday was 14.5°C.
    e. Suggested option 1: The temperatures would probably be higher because Queensland is generally warmer than Melbourne.

    Suggested option 2: The recorded temperatures might be less variable from the predicted temperatures because the weather in Queensland is more stable.

    Note: There are other possible options.

Extra spicy

15. 2.31 16. 0.8581 17. 1 18. $0.\overline{20}$

Remember this?

19. C 20. $-3\frac{1}{2}$ 21. $21

5D Multiplying and dividing by powers of 10

Student practice

Worked example 1.

a. 345.1 b. 67 280 c. 0.8 d. 92 000

Worked example 2.

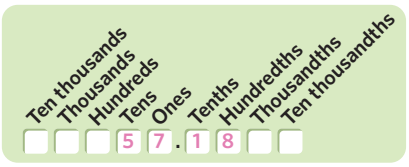
a. 98.374 b. 0.0721 c. 0.0006 d. 0.0052

Worked example 3.

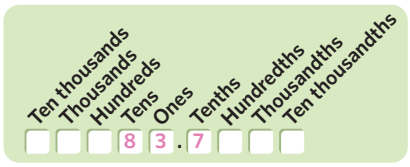
a. 44.118 b. 223.2 c. 13.017 d. 0.0115

Understanding worksheet

1. a.



b.



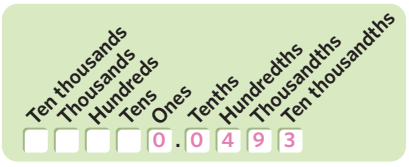
c.

d.

2. a.

b.

c.



d.

3. powers; right; left; order; operations

Fluency

4. a. 567.3 b. 103 c. 45 630 d. 67 280
   e. 2700 f. 700

5. a. 68.943 b. 0.342 c. 0.0016 d. 0.00009
   e. 0.0356 f. 0.0019

6. a. 159.2 b. 3.58
   c. 2193 d. 6720
   e. 0.09245 f. 0.0003
   g. 120 090 h. 0.0400007

7. a. 40.221 b. 581.6
   c. 73 681 d. 1.192
   e. 12.3345 f. 170.1141

Problem solving

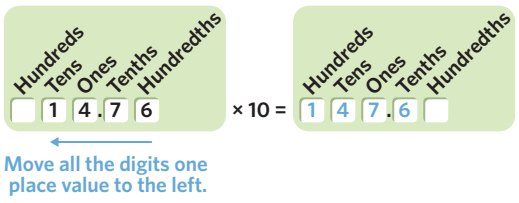
8. The minimum wage for a 17 year-old in Australia is $14.76. How much will a 17 year-old, who is being paid minimum wage, earn for 10 hours of work?

Key points
- The minimum wage for a 17 year-old is $14.76.
- A 17 year-old earning minimum wage works for 10 hours.
- What is 14.76 × 10?

Explanation

The multiplier is 10, so move each digit one place value to the left.

$$14.76 \times 10 = 147.6$$
$$= 147.6$$



Answer

A 17 year-old getting paid minimum wage would earn $147.60 for 10 hours of work.

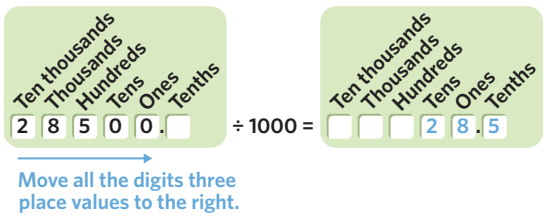
9. Penny's dairy farm collects 28 500 litres of milk per day. How many litres of milk does a cow produce on average, if she has 1000 cows on the farm?

Key points
- Penny's dairy farm collects 28 500 litres of milk per day.
- She has 1000 cows on the farm.
- What is 28 500 ÷ 1000?

Explanation

The divisor is 1000, so move each digit three place values to the right.

$$28\,500 \div 1000 = 28.500$$
$$= 28.5$$



Answer

Each cow on Penny's farm produces 28.5 litres of milk per day.

10. In kilometres per hour, what is a peregrine falcon's top flying speed if it can fly 10 times faster than a green iguana can run? Green iguanas are able to run at a speed of 38.73 km/h.

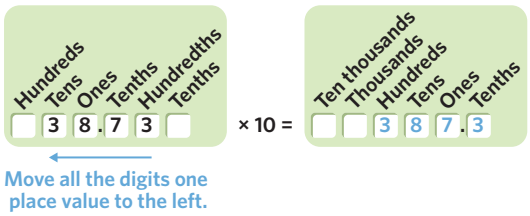
Key points

- A peregrine falcon can fly 10 times faster than a green iguana can run.
- Green iguanas are able to run at a speed of 38.73 km/h.
- What is 38.73 × 10?

Explanation

The multiplier is 10, so move each digit one place value to the left.

$$38.73 \times 10 = 387.3$$
$$= 387.3$$



Answer

A peregrine falcon's top flying speed is 387.3 km/h.

11. Frank's house is on a 254.8 m^2 block of land. He bought the neighbouring 312 m^2 block of land. He plans to demolish his house and combine both properties to build his dream home. Frank wants a front garden with an area 78.45 m^2 and a back garden that is 5.32 m long and 10 m wide. What area of land remains for Frank to build his house?

Key points

- Frank's land is 254.8 m^2.
- He bought his neighbouring 312 m^2 land.
- Frank wants a front garden with an area of 78.45 m^2 and the back garden that is 5.32 m long and 10 m wide.
- How much total land will Frank have once he uses the land required for his front and back garden?

Explanation

Total land in m^2: 254.8 + 312

Total garden area in m^2: 78.45 + 5.32 × 10

Land remaining in m^2
= *total land* − *total garden area*

Land remaining
= 254.8 + 312 − (78.45 + 5.32 × 10)

First, evaluate the brackets. The multiplier is 10, so move each digit one place value to the left.

$$5.32 \times 10 = 53.2$$
$$53.2$$

Total garden area in m^2: 78.45 + 53.20

$$\begin{array}{r} {\scriptstyle +1} \\ 78.45 \\ +\ 53.20 \\ \hline 131.65 \end{array}$$

Land remaining = 254.8 + 312 − 131.65

$$\begin{array}{r} 254.8 \\ +\ 312.0 \\ \hline 566.8 \end{array}$$

Land remaining = 566.8 − 131.65

$$\begin{array}{r} {\scriptstyle 7\ 10} \\ 5\,6\,6.\,\not{8}\,\not{0} \\ -\ 1\,3\,1.\,6\,5 \\ \hline 4\,3\,5.\,1\,5 \end{array}$$

Answer

Frank has 435.15 m^2 remaining to build his house.

12. A 2.1 kg sack of rice can store approximately 100 000 grains of rice. How much does one grain of rice approximately weigh in grams?

Key points

- A 2.1 kg sack of rice can store approximately 100 000 grains of rice.
- What is the approximate weight of one grain of rice in grams?

Explanation

There are 1000 grams in 1 kg. Multiply 2.1 by 1000 to find how many grams each sack weighs.

Divide this result by 100 000 to find the weight of each grain of rice in grams.

The multiplier is 1000, so each digit moves 3 place values to the left.

$$2.1 \times 1000 = 2100.$$
$$= 2100$$

The divisor is 100 000, so each digit moves 5 place values to the right.

$$2100 \div 100\,000 = 0.021$$
$$= 0.021$$

Answer

One grain of rice weighs approximately 0.021 g.

Reasoning

13. a. Melbourne's approximate population is 4 969 000.
    b. 1 800 700 people live in Adelaide and Canberra.
    c. 9910 people in Launceston work in agriculture.
    d. Suggested option 1: Most popular tourist destinations.
    Suggested option 2: Average temperatures throughout the year.
    Note: There are other possible options.

14. a. It will cost Nicola $930.
    b. Tom used the non-fragile general by air shipping method.
    c. It will cost $2270 more for Nicola to send her belongings by air.
    d. It will cost Nicola $8037.90 to send her belongings home.
    e. Suggested option 1: They might want to encourage people to send heavier deliveries and pay a higher total cost.
    Suggested option 2: They might want to offer a cheaper price than their competitors.
    Note: There are other possible options.

Extra spicy

15. A
16. 0.00001
17. C
18. 1.1974×10^8

Remember this?

19. The teachers waste 3600 grocery bags worth of food annually.

20. C

21. Size 6.5 US men's shoes are 9.4 inches long.

5E Multiplying with decimals

Student practice

Worked example 1.

a. 0.06 b. 0.012 c. 0.0012 d. 0.052

Worked example 2.

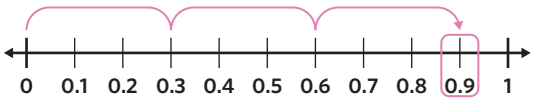
a. 3.6 b. 3.968 c. 0.292 d. 0.07266

Worked example 3.

a. 4 b. 4.74 c. 80 d. 30

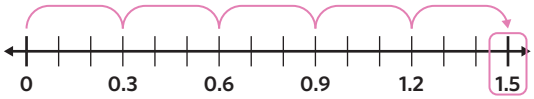
Understanding worksheet

1. a.

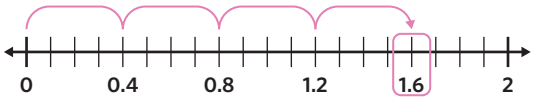


b.

c.



d.



2. a. $\frac{4}{10} \times \frac{2}{10} = \frac{8}{100} = 0.08$ b. $\frac{4}{10} \times \frac{4}{10} = \frac{16}{100} = 0.16$

c. $\frac{12}{10} \times \frac{4}{10} = \frac{48}{100} = 0.48$ d. $\frac{12}{10} \times \frac{8}{10} = \frac{96}{100} = 0.96$

3. fractions; product; estimation; reasonableness

Fluency

4. a. 0.8 b. 3.0 c. 3.3 d. 9.2
   e. 9.6 f. 8.02 g. 20.16 h. 18.66

5. a. 0.03 b. 0.1 c. 0.006 d. 0.0024
   e. 2.7 f. 56.12 g. 252.828 h. 0.082764

6. a. 12 b. 10 c. 1.68 d. 7.56
   e. 84 f. 60 g. 0.5713 h. 35.7

7. a. 1.6 b. 0.13
   c. 5.16 d. 2.49
   e. 0.0378 f. 14.592
   g. 37.8768 h. 106.3125

Problem solving

8. A jar of pasta sauce weighs 0.4 kg. Justin wants to make a batch of spaghetti and meatballs as a celebration dinner for his basketball team's state championship win. He buys 6 jars to make enough sauce. What is the total weight of the jars of pasta sauce?

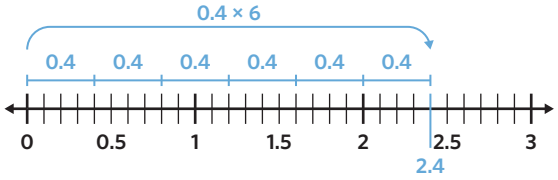
Key points
- A jar of pasta sauce weighs 0.4 kg.
- Justin has 6 jars.
- What is 0.4×6?

Explanation

$0.4 \times 6 = \frac{4}{10} \times \frac{6}{1}$

$\frac{4 \times 6}{10 \times 1} = \frac{24}{10}$

$= 2.4$



Answer

The total weight of the jars is 2.4 kg.

9. Sophie makes scrunchies and uses 0.7 m of material to make each one. How much material will she need to complete an order of 2 scrunchies?

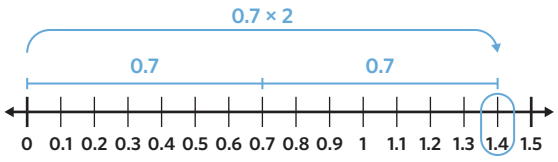
Key points
- Sophie uses 0.7 m to make one scrunchie.
- She is completing an order of 2 scrunchies.
- What is 0.7×2?

Explanation

$0.7 \times 2 = \frac{7}{10} \times \frac{2}{1}$

$\frac{7 \times 2}{10 \times 1} = \frac{14}{10}$

$= 1.4$



Answer

Sophie needs 1.4 m of material.

10. Brooklyn wants to install a swimming pool in her back garden. What is the area of the pool if she wants the length to be 10.4 metres and the width to be 3.67 metres?

Key points
- Brooklyn measures a space of 10.4×3.67 metres for her pool.
- What is 10.4×3.67?

Explanation

Determine how many decimal places are in the answer by counting the total number of decimal places in the factors. 10.4×3.67 has 3 decimal places.

Multiply using the vertical algorithm as if multiplying whole numbers and place the decimal point to give the correct number of decimal places.

$$\begin{array}{rr} & \begin{matrix}+1\\+2\\+2\end{matrix}\\ & 104\\ \times & 367\\ \hline & 728\\ +1 & 6240\\ + & 31200\\ \hline & 38168 \end{array}$$

$10.4 \times 3.67 = 38.168$ (with 3 decimal places)

Answer

Brooklyn will have 38.168 m^2 for her pool.

11. Desmond is a casual worker, answering phones and helping with office work at his cousin's dental office. If Desmond earns $20.25 per hour how much does he earn in week if he works 17.6 hours?

Key points

- Desmond earns $20.25 per hour.
- He works 17.6 hours per week.
- What is 20.25×17.60?

Explanation

Determine how many decimal places are in the answer by counting the total number of decimal places in the factors. 20.25 × 17.6 has 3 decimal places.

Multiply using the vertical algorithm as if multiplying whole numbers and place the decimal point to give the correct number of decimal places.

$$\begin{array}{rrrrrr} & & & \begin{matrix}+1&+3\\+1&+3\end{matrix} & & \\ & & 2 & 0 & 2 & 5\\ \times & & & 1 & 7 & 6\\ \hline & & & \begin{matrix}+1&+1\end{matrix} & & \\ & 1 & 2 & 1 & 5 & 0\\ 1 & 4 & 1 & 7 & 5 & 0\\ + & 2 & 0 & 2 & 5 & 0 & 0\\ \hline & 3 & 5 & 6 & 4 & 0 & 0 \end{array}$$

$20.25 \times 17.6 = 356.400$ (with 3 decimal places)

Answer

Desmond makes $356.40 per week.

12. Sarah completes a lap of her neighbourhood every day by running and walking. According to her smartwatch, she completes an average of 5.24 kilometres per day. If she walks $\frac{4}{10}$ of this distance, how many kilometres does Sarah run on average every day?

Key points

- Sarah completes the lap by running and walking.
- She runs/walks 5.24 kilometres per day.
- She spends $\frac{4}{10}$ of the distance walking.
- How much of the 5.24 kilometres does she run?

Explanation

If she spends $\frac{4}{10}$ of her time walking, she spends $\frac{6}{10}$ of her time running.

$\frac{6}{10} = 0.6$

To find how much she runs, find the value of 5.24×0.6.

Multiply using the vertical algorithm as if multiplying whole numbers and place the decimal point to give the correct number of decimal places.

5.24×0.6 will have 3 decimal places in the answer.

$$\begin{array}{rrrrr} & & \begin{matrix}+1&+2\end{matrix} & & \\ & 5 & 2 & 4 & \\ \times & & & 6 & \\ \hline & 3 & 1 & 4 & 4 \end{array}$$

$5.24 \times 0.6 = 3.144$ (with 3 decimal places)

Answer

Sarah runs 3.144 km every day.

Reasoning

13. a. Casey will need 4.8 square metres.
    b. Casey will need 96.75 metres of timber.
    c. The total cost of the timber is $314.44.
    d. Suggested option 1: Yes, because if you previously bought lots of eggs then the amount you save will eventually outweigh the cost of building the coop.

    Suggested option 2: No, because if you don't buy many eggs then the cost to build the coop is greater than the amount you would spend on eggs anyway.

    Note: There are other possible options.

14. a. Roger and Shirley can travel approximately 620 km before their tank is empty.
    b. Roger and Shirley can drive another 271.852 km before their tank is empty.
    c. Roger and Shirley will need to stop once to refuel.
    d. Suggested option 1: An electric car is better because it's more environmentally friendly and doesn't use any petrol.

    Suggested option 2: A hybrid car is better because there aren't currently enough charging stations in Australia for purely electric cars.

    Note: There are other possible options.

Extra spicy

15. −13.676
16. 4046.837 m^2
17. 0.161604
18. There are many possible options between 0.0029 and 0.0057. E.g. 0.0031.

Remember this?

19. C
20. The apple costs $2.
21. Connor has to complete $\frac{3}{6}$ or $\frac{1}{2}$ of his project today to reach his goal.

5F Dividing with decimals

Student practice

Worked example 1.

a. 0.1 b. 0.8 c. 0.2 d. 0.5

Worked example 2.

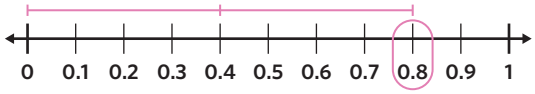
a. 0.9 b. 1.62 c. 2.625 d. 21

Worked example 3.

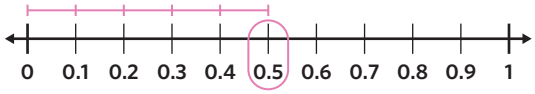
a. 4.37 b. 7.22 c. 2.18 d. 5.14

Understanding worksheet

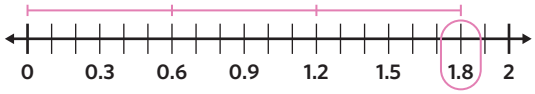
1. a. $0.8 \div 2 = 0.4$



b. $0.5 \div 5 = 0.1$



c. $1.8 \div 3 = 0.6$



d. $1.2 \div 3 = 0.4$

2. a. $0.9 \div 0.3 = 3$

$\frac{9}{10} \div \frac{3}{10} = 3$ groups of $\frac{3}{10}$

b. $0.7 \div 0.1 = 7$

$\frac{7}{10} \div \frac{1}{10} = 7$ groups of $\frac{1}{10}$

c. $1.2 \div 0.2 = 6$

$\frac{12}{10} \div \frac{2}{10} = 6$ groups of $\frac{2}{10}$

d. $1.4 \div 0.7 = 2$

$\frac{14}{10} \div \frac{7}{10} = 2$ groups of $\frac{7}{10}$

3. divisor; quotient; whole; fraction

Fluency

4. a. 0.5 b. 0.8 c. 0.3 d. 0.07
e. 0.025 f. 2.04 g. 1.02 h. 2.47

5. a. 0.8 b. 9.23
c. 0.06 d. 0.308
e. 3.004 f. 0.50015
g. 0.0003125 h. 15.5525

6. a. 40 b. 0.4 c. 25 d. 9
e. 29 f. 257 g. 1.23 h. 2500

7. a. 4 b. 22.9 c. 81.4 d. 122.3
e. 157.075 f. 1052.5 g. 164.2 h. 335.1

8. a. 23.67 b. 1.28 c. 1.95 d. 2.73
e. 1.62 f. 1.93 g. 0.16 h. 434.13

Problem solving

9. Thomas and 3 of his friends spend $426.16 on dinner at Komeyui Japanese restaurant in South Melbourne. If they split the bill equally amongst them, how much does each person pay?

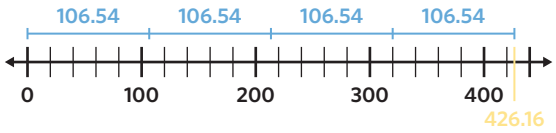
Key points
- Thomas and 3 friends spend $426.16 on dinner.
- They split the bill evenly between the 4 of them.
- What is the cost for each person?

Explanation

The price per person can be found using short division.

Align the decimal point in the quotient with the decimal point in the dividend.

$$4\overline{)4\ 2\ 6\ .\ 2^{2}1^{1}6}\quad 1\ 0\ 6\ .\ 5\ 4$$



Answer

Each person pays $106.54.

10. What is the price per litre of petrol if Elise filled up 40 L of petrol and paid $72.25? Round to the nearest cent.

Key points
- Elise purchased 40 L of petrol.
- The petrol cost $72.25.
- Round to the nearest cent.
- Find the price of 1 litre of petrol.

Explanation

The price per litre can be found by dividing the total cost by the total number of litres: $72.25 \div 40$

$$40\overline{)7\ 2\ .\ 3^{32}2^{2}5\ 0^{10}0^{20}0}\quad 0\ 1\ .\ 8\ 0\ 6\ 2\ 5$$

To round to the nearest cent, we round to 2 decimal places.

$\$1.80625 \approx \$1.81$

Answer

The price per litre of petrol rounded to the nearest cent is $1.81.

11. Kelsey works out that she needs 0.25 L of cordial for each bottle of lemon drink that she makes. How many bottles of lemon drink can she make with 5.2 L of cordial?

Key points
- Kelsey needs 0.25 L of cordial for each bottle of lemon drink.
- She has 5.2 L of cordial.
- How much lemon drink can she make with 5.2 L of cordial?

Explanation

Kelsey can make $5.2 \div 0.25$ bottles of lemon drink.

To divide, first multiply both the dividend and divisor by 100 so that the divisor is a whole number.

$5.2 \times 100 = 520$

$0.25 \times 100 = 25$

We can then calculate using short division.

$$25\overline{)5\ 2^{2}0\ .\ 0}\quad 0\ 2\ 0\ .\ 8$$

Answer

Kelsey can make 20.8 bottles of lemon drink.

12. Daniel's parents own a picture framing shop. They just received a new shipment of timber to make their frames. How many whole frames can they make if they bought 326.8 metres of timber and an average frame uses 0.9 metres of timber?

Key points

- Daniel's parents bought 326.8 metres of timber.
- On average, each frame uses 0.9 metres of timber.
- What is the total number of frames they can make using 326.8 m of timber?

Explanation

Daniel's parents will be able to make 326.8 ÷ 0.9 frames.

To divide, first multiply both the dividend and divisor by 10 so that the divisor is a whole number.

$326.8 \times 10 = 3268$

$0.9 \times 10 = 9$

We can then calculate using short division.

$$9\overline{)\,3\;2^{5}6^{2}8\;.^{1}0\ldots}\quad 0\;3\;6\;3\;.\;1\ldots$$

Since we are only looking for the number of **whole** frames, we can stop after one decimal place.

363.1... correct to the nearest whole number is 363.

Answer

Daniel's parents can make 363 whole frames.

13. In the Tokyo 2020 Olympics, the top three running times for the 100 m men's sprint were 9.80 seconds, 9.84 seconds, and 9.89 seconds. What was the average running time for these three runners, correct to two decimal places? Note: To find an average, add all values together and divide by the number of values there are.

Key points

- The top 3 running times were 9.80 s, 9.84 s, and 9.89 s.
- Find the average, correct to two decimal places, out of the top 3 times.

Explanation

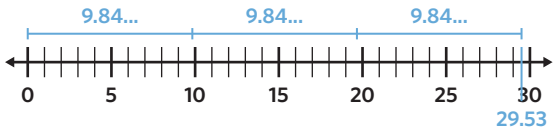
First, calculate the sum of the three times.

$9.80 + 9.84 + 9.89 = 29.53$

The average time will be 29.53 ÷ 3 seconds.

$$3\overline{)\,2\;9\;.^{2}5^{1}3^{1}0\ldots}\quad 0\;9\;.\;8\;4\;3\ldots$$

9.843... correct to the nearest hundredth is 9.84.



Answer

The average time for these three runners was 9.84 seconds, correct to two decimal places.

Reasoning

14. a. The lamb costs $70 per kilogram.

b. The cooking time for a 1 kg rack of lamb would be 62.67 minutes, correct to two decimal places.

c. Tina should put her roast in the oven at 5:28 pm.

d. Suggested option 1: Tina and her brother could cook both the cake and the roast at the same time if they require the same oven temperature.

Suggested option 2: Tina's brother could put the cake in the oven while their parents are eating the roast so it is fresh for dessert.

Note: There are other possible options.

15. a. The average height of the Melbourne United point guards is 192.55 cm, correct to two decimal places.

b.

Brooklyn Nets point guard	Height (in)	Height (cm)
Patty Mills	73	185.42
Kyrie Irving	74	187.96
Jevon Carter	73	185.42
Mike James	73	185.42

c. The average height of the Brooklyn Nets point guards is 186.055 cm.

d. Melbourne United have the taller average point guard.

e. Suggested option 1: Height is mostly genetic, so would be affected by the player's DNA.

Suggested option 2: During development, nutrition and hormones can affect growth.

Note: There are other possible options.

Extra spicy

16. 0.0487804878

17. 65.8169934641

18. 695.275 km/h

19. $0.458\overline{333}$

Remember this?

20. $9620

21. C

22. A

5G Introduction to percentages

Student practice

Worked example 1.

a. $\frac{17}{50}$ b. $\frac{27}{25}$ or $1\frac{2}{25}$ c. $-\frac{9}{10}$ d. $\frac{7}{2}$

Worked example 2.

a. 50% b. 65% c. 125% d. 352%

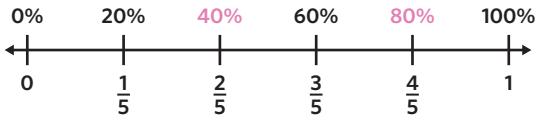
Worked example 3.

a. 75% b. 325% c. 20% d. 275%

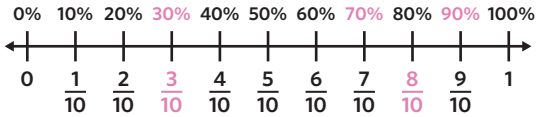
Understanding worksheet

1. a. $50, \frac{1}{2}$ b. $25, \frac{1}{4}$ c. $12, \frac{3}{25}$ d. $36, \frac{9}{25}$

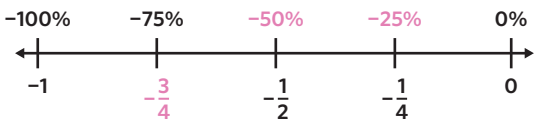
2. a.



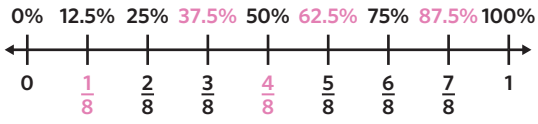
b.



c.



d.



3. one hundred; proportion; fraction; equivalent; multiply

Fluency

4. a. $\frac{1}{10}$ b. $\frac{1}{2}$
   c. $\frac{4}{5}$ d. $\frac{23}{100}$
   e. $\frac{6}{5}$ or $1\frac{1}{5}$ f. $-\frac{9}{20}$
   g. $-\frac{7}{4}$ or $-1\frac{3}{4}$ h. $\frac{3}{8}$

5. a. 40% b. 17% c. 93% d. 3%

6. a. 20% b. 70% c. 80% d. 68%
   e. 275% f. 150% g. 200% h. 520%

7. a. 25% b. 40% c. 55% d. 62.5%
   e. 275% f. 230% g. 325% h. 700%

Problem solving

8. Lissandra cut her birthday cake into ten equal pieces to share with her friends and family. What percentage does each slice of cake represent as a proportion of the whole cake?

Key points
- Lissandra cut her birthday cake into 10 equal slices.
- What percentage does each slice represent as part of the whole cake?

Explanation

1 out of 10 equal slices is $\frac{1}{10}$ of the cake.

Find an equivalent fraction with a denominator of 100.

$\frac{1}{10} \overset{\times 10}{=} \frac{10}{100}$

$\frac{10}{100} = 10\%$

Answer

Each slice of the cake represents 10% of the whole cake.

9. Rebecca wanted to bake some cookies for dessert. What fraction of the cookie dough did Rebecca use if she only used 40% and froze the rest for another day. Express your answer in the simplest form.

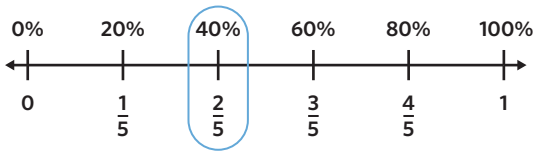
Key points
- Rebecca used 40% of the cookie dough.
- What fraction, in simplest form, of the cookie dough did Recebba use?

Explanation

$40\% = \frac{40}{100}$

The HCF is 20.

$\frac{40 \div 20}{100 \div 20} = \frac{2}{5}$



Answer

Rebecca used $\frac{2}{5}$ of the cookie dough.

10. What percentage score did Zoro get on his science test if he got 17 out of a total of 20 marks correct?

Key points
- Zoro got 17 out of a total of 20 marks correct.
- What percentage of questions did Zoro get correct?

Explanation

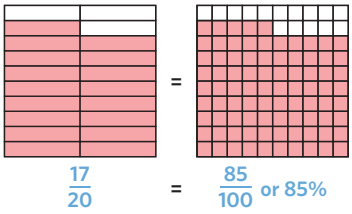
17 marks out of 20 is $\frac{17}{20}$.

$\frac{17}{20} \times \frac{100}{1} = \frac{17}{1} \times \frac{5}{1}$

$= \frac{85}{1}$

$= 85$

$\frac{17}{20} = 85\%$



Answer

Zoro got a score of 85% on his science test.

11. Josh had a full bottle of water in the morning. By lunchtime, his bottle was 65% full. What fraction of his water bottle did Josh drink? Express in the simplest form.

Key points
- Josh's water bottle was full in the morning.
- By lunchtime it was 65% full.
- How much did Josh drink as a fraction of his water bottle?

Explanation

The percentage of the water bottle Josh drank is the complementary percentage of 65%.

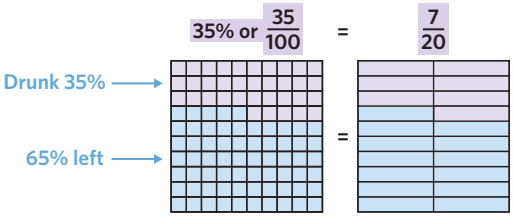
$100\% - 65\% = 35\%$

As a fraction:

$35\% = \frac{35}{100}$

The HCF is 5.

$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$



Answer

Josh drank $\frac{7}{20}$ of his water bottle.

12. Jessica is the star player on her basketball team. At three quarter time, Jessica had scored 19 out of her team's 44 points. In the fourth quarter, Jessica scored another 8 points out of her team's 16 points. What percentage of the team's total points were scored by Jessica?

Key points
- In the first 3 quarters Jessica scored 19 out of her team's 44 points.
- In the 4th quarter Jessica scored 5 out of her team's 16 points.
- Find Jessica's points as a percentage out of the team's total points.

Explanation

Total scored by Jessica: $19 + 8 = 27$

Total scored by the team: $44 + 16 = 60$

Jessage scored $\frac{27}{60}$ of her team's points.

$$\frac{27}{60} \times \frac{100}{1} = \frac{27}{3} \times \frac{5}{1}$$
$$= 9 \times 5$$
$$= 45$$
$$\frac{27}{60} = 45\%$$

Answer

Jessica scored 45% of her team's points.

Reasoning

13. a. Samuel won 6 games.
    b. Samuel stalemated $\frac{1}{7}$ of his games.
    c. Samuel lost 0% of his games.
    d. Samuel lost 28.6% of his games.
    e. Suggested option 1: Samuel is a better chess player as he scored more points than Cassandra.

    Suggested option 2: Cassandra may be better as she could have had a bad day and scored fewer points than usual.

    Note: There are other possible options.

14. a. 1 student passed the Japanese exam.
    b. Marko: 87.5%, Sylvia: 82.5%, Atif: 75%, Bashir: 70%
    c. The number of exam sections that did not result in a pass grade was 37.5%.
    d. Suggested option 1: Yes it is fair as students should be good at both speaking and writing.

    Suggested option 2: No it is not fair. Sylvia, for example, scored 82.5% overall but did not pass.

    Note: There are other possible options.

Extra spicy

15. A 16. D 17. D 18. 880 000

Remember this?

19. D 20. C 21. $\frac{31}{50}$

5H Converting between fractions, decimals, and percentages

Student practice

Worked example 1.

a. 0.83 b. 0.128 c. 0.0076 d. 2.746

Worked example 2.

a. 20% b. 97% c. 86.3% d. 329.4%

Worked example 3.

a. $\frac{7}{20}$ b. $\frac{4}{5}$ c. $1\frac{3}{4}$ d. $7\frac{3}{50}$

Worked example 4.

a. 0.6 b. 0.875 c. −4.85 d. 0.6875

Understanding worksheet

1. a. $\frac{3}{10}$ b. 50% c. 0.25 d. 75%

2. a. 90% b. 1.2 c. 0.25, $\frac{3}{4}$ d. 1.5, $\frac{5}{2}$

3. percentages; %; decimals; power of 10; simplest form

Fluency

4. a. 0.23 b. 0.96 c. 1.5 d. 0.185
   e. 0.0039 f. 0.0971 g. 4.363 h. 0.00084

5. a. 34% b. 70% c. 5% d. 59.2%
   e. 340% f. 275% g. 893.6% h. 0.2%

6. a. $\frac{2}{5}$ b. $\frac{17}{20}$ c. $7\frac{3}{5}$ d. $2\frac{3}{4}$
   e. $8\frac{13}{100}$ f. $1\frac{1}{50}$ g. $13\frac{1}{8}$ h. $5\frac{3}{500}$

7. a. 0.2 b. 0.84 c. −0.66 d. 2.76
   e. 0.19 f. 7.8

8. a. 0.625 b. 0.3125 c. 0.225 d. 0.08
   e. 0.925 f. 0.1875

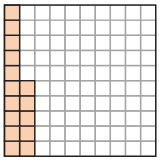
Problem solving

9. The probability of buying a winning raffle ticket at a charity fundraising event is 0.15. Expressed as a fraction in its simplest form, what is the probability of buying the winning ticket?

Key points

- The probability of winning a raffle ticket is 0.15.
- What is 0.15 as a fraction?

Explanation



Decimal	Percentage	Fraction
0.15	= 15%	= $\frac{3}{20}$

$0.15 = \frac{15}{100}$

HCF is 5.

$\frac{15 \div 5}{100 \div 5} = \frac{3}{20}$

Answer

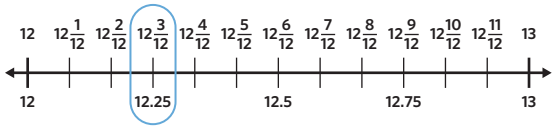
The probability of buying the winning ticket is $\frac{3}{20}$.

10. What is Campbell's age expressed as a decimal if he is twelve years and three months old?

Key points

- Campbell is 12 years and 3 months old.
- What is his age as a decimal?

Explanation



$\frac{3}{12} = \frac{1}{4}$

3 months is $\frac{1}{4}$ of a year, so Campbell is $12\frac{1}{4}$ years old.

Find an equivalent decimal fraction.

$12\frac{1}{4} = 12\frac{25}{100}$

$= 12.25$

Answer

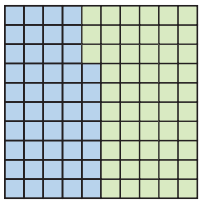
Campbell is 12.25 years old.

11. In a survey, 47% of women between the age of 20 and 25 would rather live in a city than a country town. Express, as a decimal, the proportion of 20 to 25 year-old females that prefer to live in a country town.

Key points

- 47% of women between the age of 20 and 25 would rather live in a city than a country town.
- As a decimal, what proportion of 20 to 25 year-old females would rather live in a country town?

Explanation



City Country town

$100\,\% - 47\,\% = 53\%$ would rather live in a country town.

$53\% = 53 \div 100$

$53 \div 100 = 0.53$

$= 0.53$

Answer

The proportion of 20 to 25 year-old females that would rather live in a country town is 0.53.

12. Hillside High School conducted a student survey to determine what hot food to add to their canteen menu. Which item should be added to the menu if 31% of students voted for a kebab, 0.301 of the students voted for sushi and $\frac{8}{25}$ of the students voted for fish pies?

Key points

- 31% of students voted for a kebab.
- 0.301 of the students voted for sushi.
- $\frac{8}{25}$ of the students voted for fish pies.
- Which item got the most votes?

Explanation

Sushi (as a percentage):

Multiply the decimal by 100.

$0.301 \times 100 = 30.1$

30.1% voted for sushi.

Fish pies (as a percentage):

Find an equivalent decimal fraction.

$\frac{8}{25} \overset{\times 4}{=} \frac{32}{100}$

$\frac{32}{100} = 32\%$

32% voted for fish pies.

Compare the percentages.

$30.1\% < 31\% < 32\%$

Answer

Fish pies should be added to the menu.

13. What mark did Melanie achieve in her motorcycle licence test? Melanie got 15 out of 24 questions correct. Express her mark as a percentage.

Key points

- Melanie got 15 out of 24 questions correct.
- What was Melanie's score as a percentage?

Explanation

Melanie's mark as a fraction is $\frac{15}{24}$.

HCF is 3.

$\frac{15 \div 3}{24 \div 3} = \frac{5}{8}$

Use short division to convert to a decimal.

$8\overline{)5.^{5}0^{2}0^{4}0}$ → 0.625

$\frac{5}{8} = 0.625$

Multiply the decimal by 100.

$0.625 \times 100 = 62.5$

$0.625 = 62.5\%$

Answer

Melanie scored 62.5% on her motorcycle licence test.

Reasoning

14. a. $\frac{21}{50}$ mortgage applications are rejected.

b. Frank and Lucy's application has a 93% chance of being successful.

c. $1.12 = 112\%$

An event cannot be more than 100% likely to occur.

d. Suggested option 1: Any outstanding debt.

Suggested option 2: The total income of the couple.

Note: There are other possible options.

15. a. 0.52 of Magda's mix is made up of mixed nuts and yogurt.

b. 50 grams out of 200 grams is equivalent to a proportion of 0.25.

c. $\frac{23}{50}$ of Aliya's mix is yogurt.

d. Suggested option 1: They could vote on which sample tastes the best.

Suggested option 2: They could mix all of their samples together.

Note: There are other possible options.

Extra spicy

16. $\frac{1212}{100}$, $1{,}212^2$, 1.212, 12%

17. 3.1416

18. Cup B

19. D

Remember this?

20. B

21. B

22. C

5I Finding percentages of a quantity

Student practice

Worked example 1.

a. 9 b. 135 c. 96 d. 5.5

Worked example 2.

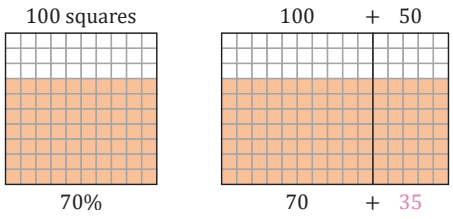
a. 275 b. 1085 c. 11 000 d. 67.2

Worked example 3.

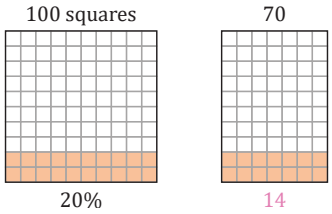
a. 48 b. 52 c. 56 d. 18.1

Understanding worksheet

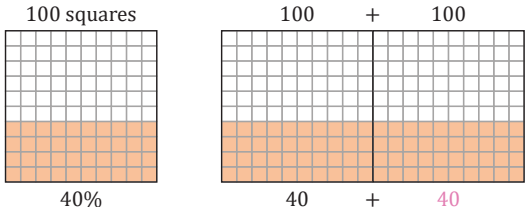
1. a. 70% of 150 = 105



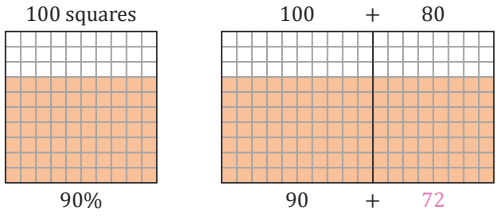
b. 20% of 70 = 14



c. 40% of 200 = 80



d. 90% of 180 = 162



2. a. 24 b. 9 c. 36 d. 37.5

3. of; multiplication; fraction; decimal

Fluency

4. a. 7 b. 40 c. 8 d. 120
   e. 0.8 f. 1260 g. 54 h. 1.8

5. a. > b. < c. > d. >
   e. = f. <

6. a. 385 b. 100 c. 84 d. 120
   e. 149.5 f. 7.5 g. 518.5 h. 79.1

7. a. 63 b. 18 c. 150 d. 22
   e. 97.5 f. 59.5 g. 3.2 h. 47.7

8. a. $40 b. 38 mL c. 575 kg d. 112 cm
   e. $62.50 f. 24.5 m g. 0.34 L h. 0.42 m

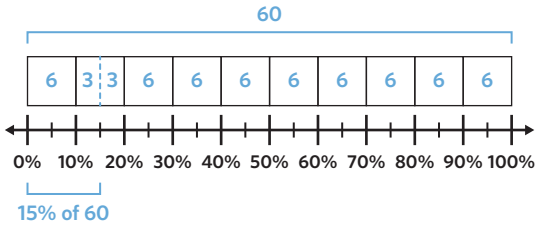
Problem solving

9. The Edrolo Year 7 maths textbook contains 60 lessons. 15% of all the lessons include questions about cooking. How many lessons in the textbook have questions about cooking?

Key points

- The textbook contains 60 lessons.
- 15% of these lessons include questions about cooking.
- What is 15% of 60?

Explanation



Convert the percentage to a fraction with a denominator of 100 and simplify.

$15\% = \frac{15}{100} = \frac{3}{20}$

Complete the multiplication.

$\frac{3}{\cancel{20}_{1}} \times \frac{\cancel{60}^{3}}{1} = \frac{3}{1} \times \frac{3}{1}$

$= 9$

Answer

9 lessons have questions about cooking.

10. How many female fans are in a stadium if there are 30 000 fans in attendance and 83% of them are male?

Key points

- There are 30 000 fans in a stadium.
- 83% of them are male.
- How many of the 30 000 fans are females?

Explanation

If 83% of the fans are male, then $100\% - 83 = 17\%$ of the fans are female.

Find 17% of 30 000.

Convert the percentage to a fraction with a denominator of 100 and simplify.

$17\% = \frac{17}{100}$

Complete the multiplication.

$\frac{17}{\cancel{100}_{1}} \times \frac{\cancel{30\,000}^{300}}{1} = \frac{17}{1} \times \frac{300}{1}$

$= 17 \times 300$

$= 5100$

Answer

There are 5100 female fans in the stadium.

11. Ruby saw the following special deal in an electronics shop. How much would Ruby pay if she buys the laptop and the bluetooth speakers?

Key points

- The laptop costs $285 and the speakers cost $165.
- If Ruby buys both items the total cost will be reduced by 6%.
- What is the total cost of the items decreased by 6%?

Explanation

The total cost of the items is $285 + 165 = \$450$.

Find the value of 450 decreased by 6%.

Find 6% of 450 by representing both values as a fraction multiplication.

$$6\% = \frac{6}{100} = \frac{3}{50}$$

$$\frac{3}{\cancel{50}_{1}} \times \frac{\cancel{450}^{9}}{1} = \frac{3}{1} \times \frac{9}{1}$$

$$= 27$$

Subtract this value from 450.

$450 - 27 = 423$

Answer

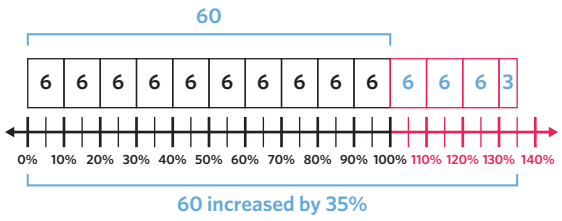
Ruby will pay $423 if she buys both items.

12. Oscar travelled from Brisbane to Cairns and back again by car. What was his average speed on the return journey, if it was 35% faster than the first trip? His average speed on the first trip was 60 km/h.

Key points

- Oscar's return journey was 35% faster than the first trip.
- His average speed on the first trip was 60 km/h.
- What is 60 km/h increased by 35%?

Explanation



Add the percentage increase to 100%.

$100\% + 35\% = 135\%$

Find 135% of 150 by representing both values as a fraction multiplication.

$$135\% = \frac{135}{100} = \frac{27}{20}$$

$$\frac{27}{\cancel{20}_{1}} \times \frac{\cancel{60}^{3}}{1} = \frac{27}{1} \times \frac{3}{1}$$

$$= 81$$

Answer

His average speed on the return journey was 81 km/h.

13. Mohammad is buying collectable Disney dolls. He has 10 out of the 20 dolls. Each one costs $16.25 and Mohammad wants to buy the remaining dolls in the collection. How much will Mohammad pay if he uses a coupon which lowers the price by 4%?

Key points

- Mohammad has 10 out of 20 dolls.
- He wants to buy the remaining dolls for $16.25 each.
- He uses a coupon which reduces the price by 4%.
- What is the total cost of the dolls decreased by 4%?

Explanation

Mohammad has 10 out of the 20 dolls, so he has 10 dolls remaining to buy.

Each doll costs $16.25, so the total cost is $10 \times 16.25 = 162.5$.

Find 162.5 decreased by 4%.

Find 4% of 162.5 by converting both values to a fraction multiplication.

$$4\% = \frac{4}{100} = \frac{1}{25}$$

$$\frac{1}{\cancel{25}_{1}} \times \frac{\cancel{1625}^{65}}{10} = \frac{1}{1} \times \frac{65}{10}$$

$$= 6.5$$

Subtract 6.5 from 162.5.

$162.5 - 6.5 = 156$

Answer

Mohammad will pay $156.

Reasoning

14. a. 28 pescatarians are attending the party.
    b. The difference in the number of guests that are omnivores and vegetarians is 98.
    c. Anabelle needs to cater for 364 desserts.
    d. Suggested option 1: Present the bar graph with the number of people for each dietary requirement.

    Suggested option 2: Present a table with the number of people for each dietary requirement.

    Note: There are other possible options.

15. a. The price at Dream Furniture without the discount is $1540.
    b. The final price at Furniture Palace is $1440.
    c. Francis will save $23 by buying the cheaper furniture set.
    d. Francis' partner is incorrect because the total discount depends on the listed price.
    e. Suggested option 1: Include GST in the price.

    Suggested option 2: Only show the final price after GST and the discount.

    Note: There are other possible options.

Extra spicy

16. B, D and E. 17. 19.44 cm 18. D 19. D

Remember this?

20. B 21. C 22. C

Chapter 5 extended application

1. a. Elena is the fastest butterfly swimmer.
   b. Onyx is a faster backstroke swimmer.
   c. From fastest to slowest:

   Freestyle at 52 seconds, backstroke at 61 seconds, butterfly at 62 seconds, breaststroke at 70 seconds
   d. Paula is 2.26 seconds faster at breaststroke than Onyx.

e. Suggested option 1: Edmund has the fastest backstroke time by far, so he should swim backstroke.

Suggested option 2: Paula is only 0.43 seconds slower than Edmund, so she could swim freestyle instead.

Note: There are other possible options.

2. a. Greta sold 10 medium sized paintings in March.
   b. In total, Greta made $667.70 in March.
   c. The cost of a large painting is now $63.75.
   d. Greta will make $22.50 less in April than in March.
   e. Suggested option 1: Greta could invest in some marketing to attract more customers to purchase her paintings.

   Suggested option 2: Greta could increase her prices to make a larger profit from each sale.

   Note: There are other possible options.

3. a. The average monthly rainfall is 54.025 mm.
   b. The average rainfall of each rainy day is 6.483 mm.
   c. It rains $\frac{20}{73}$ days per year.
   d. It will need to rain on 46 more days.
   e. Suggested option 1: I would expect the most rainfall during the winter and spring months in Melbourne.

   Suggested option 2: I would expect the most rainfall during the summer months in Darwin due to its tropical climate.

   Note: There are other possible options.

Chapter 5 review

Multiple choice

1. C 2. B 3. B 4. D 5. A

Fluency

6. a. 1.439	b. 25.705	c. 41.063	d. 0.8134
7. a. 1.45	b. 0.8044	c. 0.910	d. 5.00
8. a. 0.84	b. 3.697	c. 2.257	d. 11.446
9. a. 1.21	b. 0.47	c. 2.579	d. 3.121
10. a. 345.6	b. 0.789	c. 30	d. 0.015
11. a. 9.04	b. 3.29	c. 0.10791	d. 40.365
12. a. 2.7	b. 6.84	c. 2.961	d. 23.9625
13. a. 4	b. 9.62	c. 26.7	d. 3.0015
14. a. 35%	b. 77%	c. 61%	d. 49%
15. a. 40%	b. 37.5%	c. 148%	d. 215%
16. a. 340.5%	b. $3\frac{1}{4}$ or $\frac{13}{4}$	c. 0.24	d. 0.0875
17. a. 30	b. 84	c. 210	d. 45
18. a. 54	b. 255	c. 682.5	d. 62.3

Problem solving

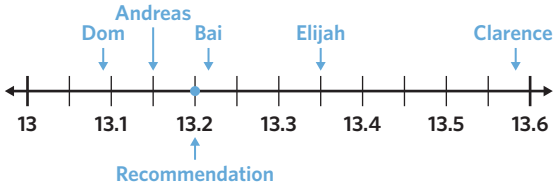
19. It is recommended for males to have a blood hemoglobin level of above 13.2 g/dL.

Which of the males in the table have hemoglobin lower than the recommended level?

Key points

- Males should have a blood hemoglobin level of above 13.2 g/dL.
- The hemoglobin levels of five males are given in the table.
- Which people do not have enough hemoglobin?

Explanation



Compare the digits in each place value for each decimal starting from left to right, including the recommended level (13.2).

Tenths:

13.094 < 13.15 < 13.2 < 13.22 < 13.35 < 13.581

Dom < Andreas < recommended < Bai < Elijah < Clarence

Answer

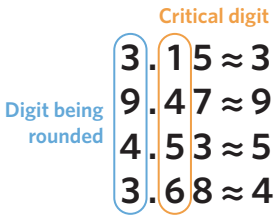
Andreas and Dom

20. Yolanda is visiting her family in New Zealand for Christmas. She wants to know if her luggage is within the airline's weight limit of 20 kg. If she rounds each of the weights shown to the nearest kilogram, will her luggage be within the weight limit?

Key points

- The airline has a weight limit of 20 kg.
- The weight of each item is shown in the diagram.
- Will the rounded values add up to more or less than 20 kg?

Explanation



Suitcase:
Whole number: 3
Critical digit: 1
3.15 ≈ 3

Clothes:
Whole number: 9
Critical digit: 4
9.47 ≈ 9

Toiletries:
Whole number: 4
Critical digit: 5
4.53 ≈ 5

Christmas presents:
Whole number: 3
Critical digit: 6
3.68 ≈ 4

3 + 9 + 5 + 4 = 21 kg

Answer

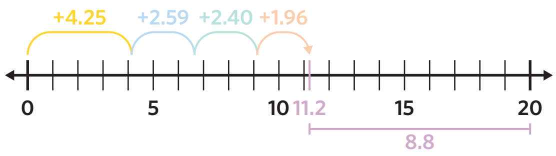
Yolanda will not be within the weight limit.

21. Sophie is buying ingredients to bake cinnamon scrolls. She buys butter for $4.25, cinnamon for $2.59, yeast for $2.40 and sugar for $1.96. How much change will she receive if she pays using a $20 note?

Key points

- Butter costs $4.25.
- Cinnamon costs $2.59.
- Yeast costs $2.40.
- Sugar costs $1.96.
- If Sophie pays for these ingredients with a $20 note, how much change will she receive?

Explanation



Add the cost of all of the ingredients together.

$4.25 + 2.59 + 2.40 + 1.96 = 11.2$

Subtract the total cost of the ingredients from $20.

```
     9
  1 10   10
  2  0 .  0
− 1  1 .  2
  ─────────
     8 .  8
```

8.8 is the same as 8.80.

Answer

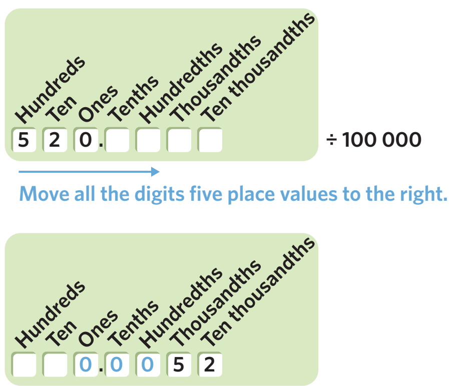
Sophie will receive $8.80 in change.

22. A colony of 100 000 ants weighs 520 grams. On average, how much does one ant weigh in grams?

Key points

- The colony has 100 000 ants.
- The total weight is 520 grams.
- What is 520 ÷ 100 000?

Explanation



The divisor is 100 000, so move each digit five place values to the right.

520.

$520 \div 100\,000 = 0.0052$

$= 0.0052$ g

Note: When there is a place value digit missing, use zero as a placeholder.

Answer

One ant weighs 0.0052 grams on average.

23. Sophie's cinnamon scroll recipe requires 0.385 kg of flour. She wants to make two-and-a-half batches of the recipe. How many kilograms of flour will she need?

Key points

- The recipe requires 0.385 kg of flour.
- She wants to make two-and-a-half batches of the recipe.
- What is 0.385 × 2.5?

Explanation

Determine how many decimal places are in the answer by counting the total number of decimal places in the factors.

0.385 × 2.5 has 4 decimal places. Multiply using the vertical algorithm as if multiplying whole numbers and place the decimal point to give the correct number of decimal places.

```
       +1 +1
       +4 +2
     3  8  5
×       2  5
────────────
  +1
  1  9  2  5
+ 7  7  0  0
────────────
  9  6  2  5
```

$0.385 \times 2.5 = 0.9625$ (with 4 decimal places).

Answer

Sophie will need 0.9625 kg of flour.

24. Last week, Greg earned $195.50 at his part-time job. What is his hourly wage if he worked 8.5 hours last week?

Key points

- Greg earned $195.50 at his part-time job.
- He worked 8.5 hours.
- What is 195.5 ÷ 8.5?

Explanation

To divide, first multiply both the dividend and divisor by 10 so that the divisor is a whole number.

$195.5 \times 10 = 1955$

$8.5 \times 10 = 85$

Short division:

```
     0  0  2  3
85)  1  9  5 ²⁵5
```

Answer

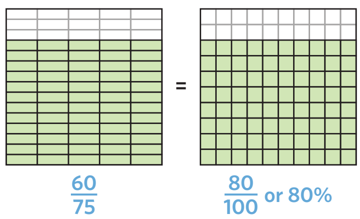
Greg earns $23.00 per hour.

25. Jerome got 60 out of 75 marks on a recent maths test. Express his test result as a percentage.

Key points

- Jerome got 60 out of 75 marks.
- Convert 60 out of 75 to a percentage.

Explanation



60 out of 75 is $\frac{60}{75}$ as a fraction.

Multiply this by $\frac{100}{1}$ to convert to a percentage.

$$\frac{60}{75} \times \frac{100}{1} = \frac{60}{3} \times \frac{4}{1}$$
$$= 20 \times 4$$
$$= 80$$
$$\frac{60}{75} = 80\%$$

Answer

Jerome got 80% on his test.

26. Lachie wants to buy a new video game with a retail price of $60. JB Hi-Fi and EB Games both currently have sales, as shown.

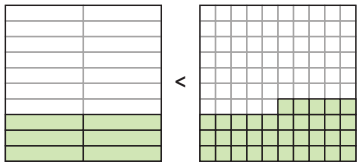
Where should Lachie buy the video game from if he wants to save the most money?

Key points

- At JB Hi-Fi, you pay $\frac{14}{20}$ of the retail price.
- At EB Games, the retail price is discounted by 35%.
- Which offer provides the greatest discount?

Explanation

JB HI-FI discount EB Games discount



$\frac{6}{20}$ or 30% 35%

At JB Hi-Fi, you pay $\frac{14}{20}$ of the retail price, which means it is discounted by $1 - \frac{14}{20} = \frac{6}{20}$.

Multiply this by $\frac{100}{1}$ to convert to a percentage and compare to the EB Games discount.

$$\frac{6}{20} \times \frac{100}{1} = \frac{6}{1} \times \frac{5}{1}$$
$$= 6 \times 5$$
$$= 30$$

$30\% < 35\%$, so the discount is greater at EB Games.

Answer

Lachie should buy the video game from EB Games.

27. Three years ago, Florence was furnishing her lounge and bought a vintage Tessa couch for $1750 and a mid-century coffee table for $325. Now, the value of the couch has increased by 36% and the value of the coffee table has increased by 30%. What is the current total value of the furniture?

Key points

- Florence bought a couch for $1750.
- Florence bought a coffee table for $325.
- The couch has increased in value by 36%.
- The table has increased in value by 30%.
- What is the sum of $1750 increased by 36% and $325 increased by 30%?

Explanation

To find the value of each furniture item, convert both the dollar value and the percentage to a fraction multiplication, and add this to the original value of each item.

Add the new value of each furniture item to find the current total value of the furniture.

Couch:

$$36\% = \frac{36}{100} = \frac{9}{25}$$
$$\frac{9}{25} \times \frac{1750}{1} = \frac{9}{1} \times \frac{70}{1}$$
$$= 9 \times 70$$
$$= \$630$$
$$1750 + 630 = \$2380$$

Table:

$$30\% = \frac{30}{100} = \frac{3}{10}$$
$$\frac{3}{10} \times \frac{325}{1} = \frac{3}{2} \times \frac{65}{1}$$
$$= \frac{195}{2}$$
$$= \$97.5$$
$$325 + 97.5 = \$422.5$$

Total:

$$2380 + 422.5 = \$2802.5$$

Answer

The current total value of the furniture is $2802.50.

Reasoning

28. a. Kendall is the tallest out of the three.

b. The difference between Antony and Bernard's heights is 0.0485 metres.

c. The average height of the players is 1.5629 metres.

d. Leonard's height is 155 cm, rounded to the nearest centimetre.

e. Suggested option 1: The players should get their parents to measure them.

Suggested option 2: The players should make sure they are wearing no shoes when their height is measured.

Note: There are other possible options.

29. a. Rabita uses 1.5 litres of milk.

b. Rabita made 48 hot cross buns.

c. $\frac{7}{8}$ of the hot cross buns were eaten.

d. 12.5% of the hot cross buns were not eaten.

e. Suggested option 1: Rabita should change her recipe to make her family happy.

Suggested option 2: Rabita should stick to her recipe to keep the Easter tradition going.

Note: There are other possible options.

6A Introduction to angles

Worked example 1.

a. $\overline{PQ}$ or $\overline{QP}$

b. $\overrightarrow{AZ}$

c. $\angle FGE$ or $\angle EGF$

d. $\angle XYZ$ or $\angle ZYX$

Worked example 2.

a. 50°, acute angle

b. 130°, obtuse angle

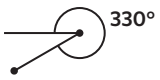
c. 220°, reflex angle

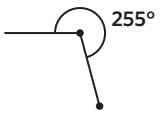
d. 55°, acute angle

Worked example 3.

a. 

b. 

c. 

d. 

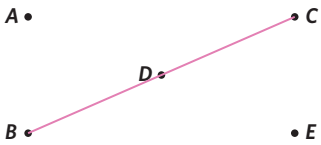
Note: There are other possible orientations.

Understanding worksheet

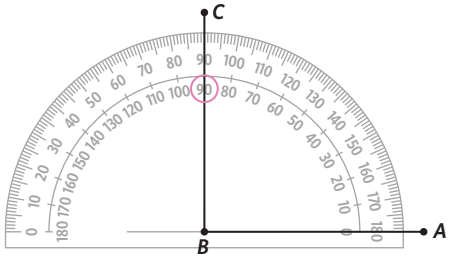
1. a. 

b. 

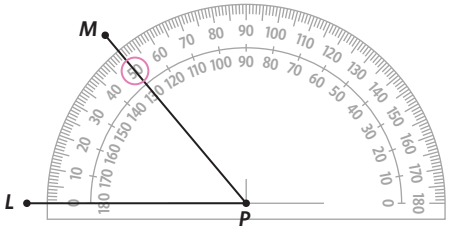
c. 

d. 

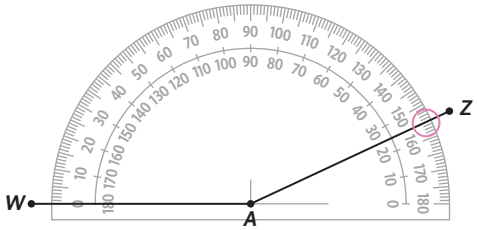
2. a. $\angle ABC = 90°$



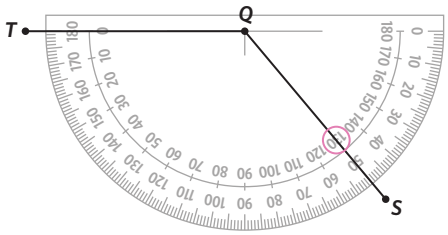
b. $\angle MPL = 50°$



c. $\angle WAZ = 155°$



d. $\angle TQS = 130°$



3. protractor; 360°; acute; 90°; reflex

Fluency

4. a. $\overline{XY}$ or $\overline{YX}$

b. $\overrightarrow{FM}$

c. $\angle BCA$ or $\angle ACB$

d. $\angle HFG$ or $\angle GFH$

e. $\angle VTS$ or $\angle STV$

f. $\angle QSR$ or $\angle RSQ$

5. a. $\angle ACF$ or $\angle FCA$

b. $\angle LMJ$ or $\angle JML$

c. $\angle ADC$ or $\angle CDA$

d. $\angle WZY$ or $\angle YZW$

e. $\angle FXK$ or $\angle KXF$

f. $\angle TQW$ or $\angle WQT$

6. a. 40°, acute angle

b. 110°, obtuse angle

c. 75°, acute angle

d. 90°, right angle

e. 310°, reflex angle

f. 175°, obtuse angle

g. 65°, acute angle

h. 295°, reflex angle

7. a. Acute angle

b. Reflex angle

c. Obtuse angle

d. Right angle

e. Acute angle

f. Obtuse angle

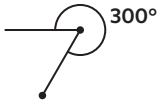
8. a. 

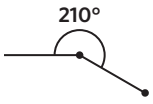
Note: There are other possible options.

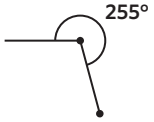
b. 

c. 

d. 

e. 

f. 

g. 

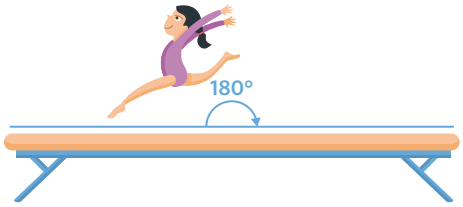
h. 

Problem solving

9. A gymnastics balance beam is a narrow wooden apparatus that is 180°. What type of angle does a balance beam represent?

Key points
- A gymnastics balance beam is a narrow wooden beam that is 180°.
- What type of angle does an 180°balance beam represent?

Explanation
A straight line is an angle that is exactly 180°.

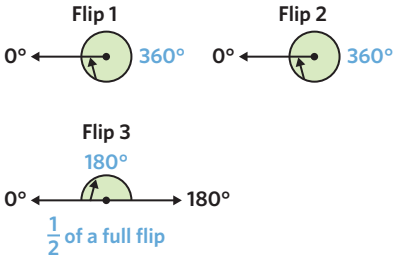


Answer
A balance beam represents a straight line.

10. Tony Hawk was the first skateboarder to land the famous skateboard trick called a 900. How many full turns did Tony complete when he flipped 900° in mid-air?

Key points
- Tony flipped 900° in mid-air.
- How many full turns (360°) did Tony complete?

Explanation
One full flip is 360°.



Divide 900 by 360 to find how many full flips he completed.

$900 \div 360$

$360\overline{)900}$ $2\ r\ 180$

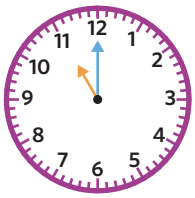
Answer
Tony completed two full turns.

11. How many degrees does the minute hand turn between 11 o'clock and quarter to 12 on a standard analogue clock?

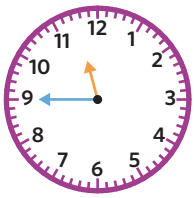
Key points
- The changes from 11:00 to 11:45.
- How many degrees does the minute hand turn?

Explanation

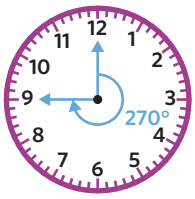
Clock at 11:00



Clock at 11:45



Change in minute hands position between 11:00 and 11:45



The difference between 11:00 and 11:45 is 3 quarters of an hour.

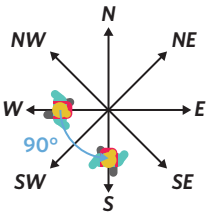
3 quarters of a full turn is 270°.

Answer
The minute hand turns 270° between 11 o'clock and quarter to 12.

12. Mikalea was hiking and completed a 90° turn to her left and ended up facing directly south. What direction was she facing before completing the turn?

Key points
- Mikaela completed a 90° turn to her left.
- She ended up facing directly south.
- What direction was Mikaela facing before turning?

Explanation
Mikaela completed a 90° turn to her left and ended up facing directly south.



Complete a 90° turn to the right to find the direction she was facing before the turn.

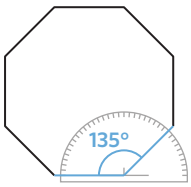
A 90° turn to the right from south is west.

Answer
Mikaela was facing west before she turned.

13. Felix is a carpenter and is building a wooden kitchen table. What type of angle will each corner form if the table is a regular octagon and Felix measures the angles inside the table.

Key points
- The table is a regular octagon.
- Felix measures the angles inside the table.
- What type of angle will each corner form?

Explanation
The internal angle of a regular octagon is 135°. This is an obtuse angle.



Answer
Each inside corner will form an obtuse angle.

Reasoning

14. a. $\angle MLK$ or $\angle KLM$
    b. The angle of the walls by the red dot is 60°
    c. Two angles formed by the walls inside her bedroom are obtuse.
    d. Suggested option 1: Jonti could consider changing the shape of the room to a rectangle.
    Suggested option 2: Jonti could consider adding another balcony on the other side.
    Note: There are other possible options.

15. a. 180°
    b. 135°
    c. Northeast
    d. 45°
    e. Suggested option 1: They can stay close to the trails and follow the signs.
    Suggested option 2: They can ensure that they bring their phone in case of an emergency.
    Note: There are other possible options.

Extra spicy

16. 715° 17. D 18. D 19. C

Remember this?

20. 0.125 21. D 22. C

6B Angles around a point

Worked example 1.

a. $w = 50$ b. $a = 140.19, b = 39.81$
c. $s = 76, y = 14$ d. $k = 130$

Worked example 2.

a. $j = 70$ b. $p = 80$ c. $x = 44$ d. $z = 54.5$

Worked example 3.

a. $c = 330$ b. $u = 200$ c. $w = 80$ d. $b = 83$

Understanding worksheet

1. a. 48° b. 103.7°, 76.3°
   c. 90°, 90° d. 126°

2. a. 45°, complementary b. 60°, supplementary
   c. 38°, complementary d. 50°, supplementary

3. Complementary; 90°; supplementary; 360°

Fluency

4. a. $e = 140°$
   b. $u = 65°$
   c. $u = 82.75°$
   d. $k = 63°, t = 117°$
   e. $a = 29°, c = 102°, t = 49°$
   f. $g = 81°$
   g. $a = 90°, h = 63°, t = 27°$
   h. $a = 29°, h = 109°, y = 42°$

5. a. Supplementary b. Complementary
   c. Neither d. A full revolution
   e. Neither f. A full revolution
   g. Complementary h. Neither

6. a. $z = 70°$ b. $a = 64.5°$ c. $p = 34°$ d. $a = 34°$
   e. $b = 22°$ f. $u = 30°$ g. $u = 40°$ h. $t = 60°$

7. a. $b = 60°$ b. $a = 30°$ c. $d = 25°$ d. $a = 26.25°$
   e. $d = 118°$ f. $e = 52°$ g. $t = 45°$ h. $x = 65°$

8. a. $g = 315°$ b. $z = 80°$ c. $r = 160°$ d. $d = 26°$
   e. $a = 129°$ f. $h = 80°$ g. $s = 81°$ h. $x = 130°$

Problem solving

9. Malia is cutting her hedges with large garden shearers. She opens them and the two blades form a 37° angle. What angle is formed by the handles on the opposite side?

Key points
- The two blades of the garden shearer form a 37° angle.
- What angle is formed by the handles?

Explanation
The two angles formed by both ends of the garden shearer are vertically opposite.

Vertically opposite angles are always equal, so the angle that is formed on the opposite side of the handles is 37°.

Answer
A 37° angle is formed by the handles.

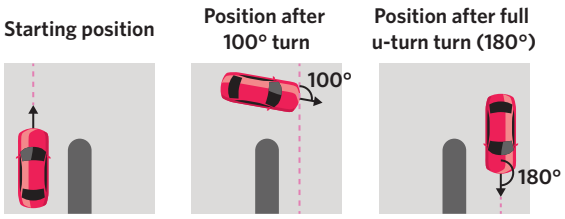
10. How many degrees further does Daniella need to turn if she is completing a u-turn in her truck and has already turned 100°?

Key points
- Daniella is completing a u-turn in her truck which will be a 180° turn.
- She has already turned 100°.
- How many degrees further does Daniella need to turn to complete the 180° turn?

Explanation

Subtract 100° from 180° to find how many more degrees Daniella needs to turn.

$180° - 100° = 80°$



Answer

Daniella needs to turn a further 80°.

11. Raphael has a reclining chair with a backrest that can be adjusted. How many degrees does he need to lower the backrest to make the chair flat, if the angle between the seat and the backrest is 119°?

Key points

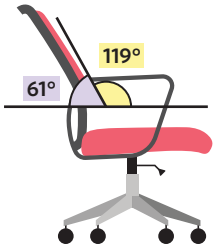
- The angle between the seat and the backrest is 119°.
- How many degrees does he need to lower the backrest to make the chair flat?

Explanation

A flat chair forms a straight line that is 180°.

Subtract 119° from 180° to find how many more degrees Raphael can lower the backrest.

$180° - 119° = 61°$



Answer

Raphael needs to lower the backrest 61° to make the chair flat.

12. A ceiling fan has three blades that are all connected to a central bolt and spin in a circle. If the blades are equally spaced around a central bolt, how many degrees are between each blade?

Key points

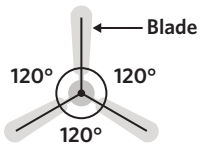
- A ceiling fan has three blades that are all connected to a central bolt and spin in a circle.
- The blades are equally spaced around a central bolt.
- How many degrees are there between each blade?

Explanation

Angles around a point sum to 360°. The blades are equally spaced around the central bolt.

Divide 360° by 3 to find the angle between each blade.

$360° \div 3 = 120°$



Answer

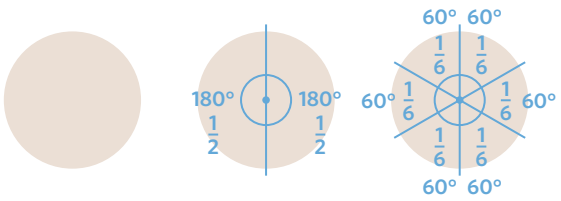
There is 120° between each blade of the ceiling fan.

13. Blake is cutting a circular cake into different pieces. He cuts the cake in half and then cuts each half into thirds. What is the angle measurement formed by each slice of cake?

Key points

- Blake is cutting a circular cake into different pieces.
- He cuts the cake in half and then cuts each half into thirds.
- What is the angle measurement formed by each slice of cake?

Explanation



A full circular cake is 360°.

Blake cuts the cake in half. Divide 360° by 2.

$360° \div 2 = 180°$

Then Blake cuts each half into thirds. Divide 180° by 3.

$180° \div 3 = 60°$

Answer

Each slice of cake forms an angle of 60°.

Reasoning

14. a. Tom needs to turn 55° to his right to face Desert Island.

b. Tom should turn 35° to his left to face Crab Island.

c. No, it is not possible for Tom to find any more missing angles.

d. Suggested option 1: Tom could consider the safety of each island.

Suggested option 2: Tom could consider which island he is most likely to be found on.

Note: There are other possible options.

15. a. The Grill 2, Grill 4 and Grill 3 arrows form two adjacent angles that are complementary.

b. Becky needs to turn 60° to increase Grill 1's temperature to MEDIUM.

c. Becky needs to turn 150° in an anticlockwise direction for Grill 1 to be turned off.

d. Suggested option 1: Becky could consider having individual knobs for each grill.

Suggested option 2: Becky could consider using a knob that has equal spacing between LOW, MEDIUM, HIGH and HIGH+.

Note: There are other possible options.

Extra spicy

16. C

17. B

18. The first angle is 56°.

The second angle is 76°.

The third angle is 228°.

19. $e = 51°$

Remember this?

20. 12.4 cm

21. D

22. D

6C Properties of triangles

Worked example 1.

a. Equilateral triangle
b. Acute triangle
c. Scalene triangle
d. Scalene triangle

Worked example 2.

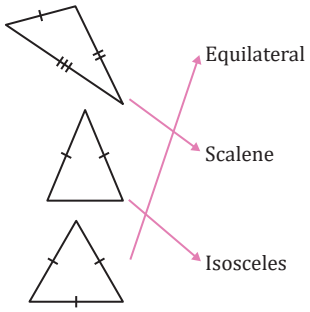
a. $g = 85°$ b. $y = 56°$ c. $k = 32°$ d. $b = 29°$

Worked example 3.

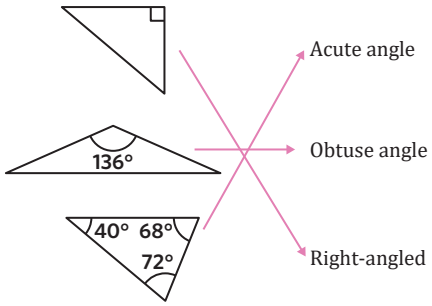
a. $k = 135°$ b. $x = 68°$ c. $p = 143°$ d. $w = 147°$

Understanding worksheet

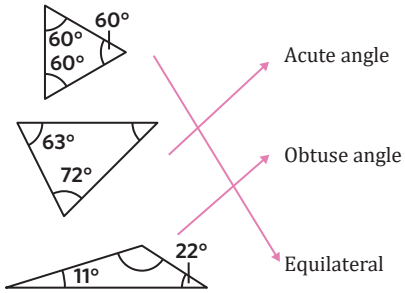
1. a.



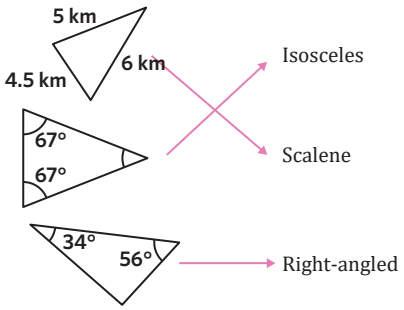
b.



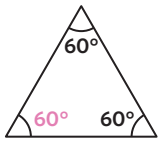
c.



d.



2. a.

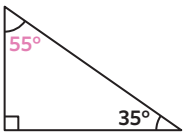


$60° + 60° + 60° = 180°$

b.

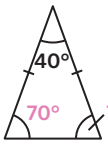
$25° + 35° + 120° = 180°$

c.



$90° + 55° + 35° = 180°$

d.



$70° + 70° + 40° = 180°$

3. three; 180°; length; size; equilateral; 60°

Fluency

4. a. Scalene triangle
b. Equilateral triangle
c. Isosceles triangle
d. Equilateral triangle
e. Scalene triangle
f. Isosceles triangle

5. a. Obtuse angle triangle
b. Acute angle triangle
c. Right-angled triangle
d. Acute angle triangle
e. Obtuse angle triangle
f. Right-angled triangle

6. a. $m = 80$ b. $f = 58$ c. $x = 52$ d. $n = 60$
e. $s = 71$ f. $h = 45$ g. $x = 48$ h. $p = 27$

7. a. $v = 110$ b. $c = 80$ c. $s = 120$ d. $x = 155$
e. $t = 114$ f. $r = 138$ g. $a = 150$ h. $d = 138$

8. a. A triangle
b. Not a triangle
c. Not a triangle
d. A triangle
e. A triangle
f. Not a triangle

Problem solving

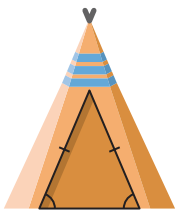
9. Mellissa's teepee has an entrance that is a triangle. The two vertical sides form the same angle at the entrance's base. What type of triangle does the teepee's entrance form?

Key points

- Mellissa's teepee has an entrance that is a triangle.
- The two vertical sides form the same angle at the entrance's base.
- What type of triangle is this?

Explanation

A triangle with two sides forming the same base angles is an isosceles triangle.



Answer

The teepee's entrance forms an isosceles triangle.

10. Many standard Australian road signs are triangles. What size is each of the interior angles of a *GIVE WAY* sign if all three of its sides have the same length?

Key points
- A *GIVE WAY* sign is a triangle that has three identical side lengths.
- What is the size of each of the interior angles of a *GIVE WAY* sign?

Explanation

A triangle that has three equal side lengths is an equilateral triangle.

The interior angles of an equilateral triangle are all 60°.

Answer

The size of each of the interior angles of a *GIVE WAY* sign is 60°.

11. Pippa is making handmade corn chips using a stencil that is an acute triangle. What is the third angle of each chip if one of the interior angles is 65° and another is 75°?

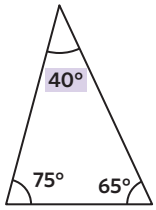
Key points
- Pippa is making corn chips in the shape of an acute triangle.
- One of the interior angles is 65° and another is 75°.
- What is the size of the third angle?

Explanation

The three interior angles of a triangle sum to 180°.

Find the missing angle by subtracting the known angles from 180°.

$180° - 65° - 75° = 40°$



Answer

The third angle of each chip is 40°.

12. Lionel is building a long ramp at his parent's house. If the ramp has an incline of 12°, what is the exterior angle formed by the ramp and the floor?

Key points
- Lionel is building a ramp that will form the shape of a triangle with the ground.
- One of the base angles in the triangle is 12°.
- What is the exterior angle that is adjacent to the 12° angle?

Explanation

Identify the adjacent exterior and interior angles that sum to 180°.

Subtract the interior angle to find the size of the exterior angle.

$180° - 12° = 168°$

Answer

The exterior angle formed by the ramp and the floor is 168°.

13. Ricki is designing a triangular business logo that is asymmetric. What are all the different types of triangles that she can draw?

Key points
- Ricki's business logo design is a triangle.
- The logo is asymmetric.
- What are all the types of triangle that Ricki can draw?

Explanation

An asymmetric triangle is a triangle with no lines of symmetry.

The following are examples of the types of triangles that contain no lines of symmetry.

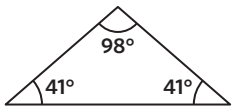
Answer

Ricki can draw a scalene triangle, right-angled triangle, obtuse angle triangle or acute angle triangle.

Reasoning

14. a. CoatHanga is an isosceles triangle.
    b. The size of the obtuse interior angle of the coat hanger is 130°.
    c. The three interior angles are 55°, 55° and 70°.
    d. Suggested option 1:

Suggested option 2:



Note: There are other possible options.

15. a. The mainsail is in the shape of a scalene triangle.
    b. The sum of the interior angles of the mainsail and headsail is 360°.
    c. The two other interior angles of the mainsail are 30° and 60°.
    d. The angle formed by the slanted edge and beam is 120°.
    e. Suggested option 1: Sally should consider the time it takes and prepare enough supplies.

    Suggested option 2: Sally should also carefully examine the weather forecast, that is the wind and ocean currents.

    Note: There are other possible options.

Extra spicy

16. D 17. D 18. 74° 19. A

Remember this?

20. 210 cents or $2.10
21. C
22. A

6D Properties of quadrilaterals

Worked example 1.

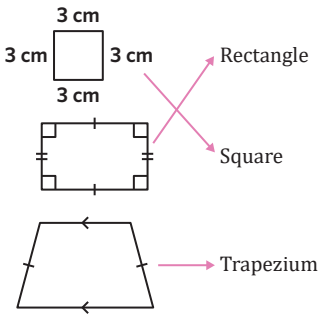
a. Rectangle
b. Rhombus
c. Trapezium
d. Parallelogram

Worked example 2.

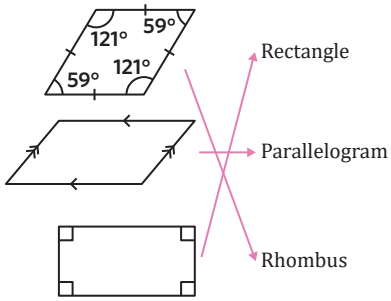
a. $j = 65$ b. $z = 71$ c. $f = 79$ d. $d = 117$

Understanding worksheet

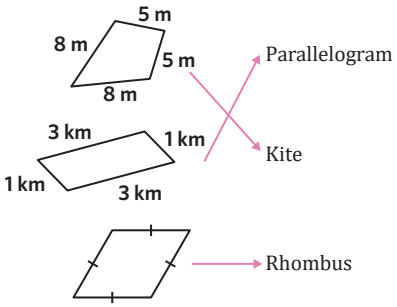
1. a.



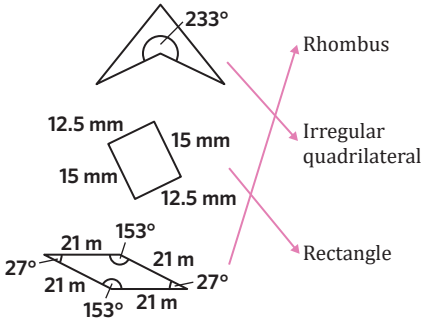
b.



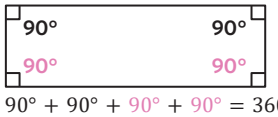
c.



d.

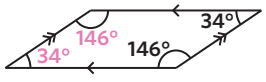


2. a.



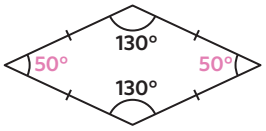
$90° + 90° + 90° + 90° = 360°$

b.



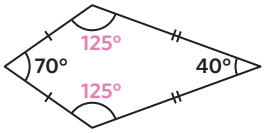
$34° + 34° + 146° + 146° = 360°$

c.



$130° + 50° + 130° + 50° = 360°$

d.



$70° + 40° + 125° + 125° = 360°$

3. four; 360°; parallel; trapezium; rectangle

Fluency

4. a. Square
b. Rectangle
c. Parallelogram
d. Trapezium
e. Kite
f. Rhombus
g. Square
h. Parallelogram

5. a. $m = 50$ b. $g = 75$ c. $x = 45$ d. $k = 47$
e. $p = 130$ f. $t = 44$ g. $y = 134$ h. $k = 140$

6. a. $r = 60$ b. $b = 90$ c. $d = 65$ d. $f = 49$
e. $z = 103$ f. $v = 139$ g. $a = 135$ h. $x = 57$

7. a. Not a quadrilateral
b. A quadrilateral
c. Not a quadrilateral
d. A quadrilateral
e. Not a quadrilateral
f. Not a quadrilateral

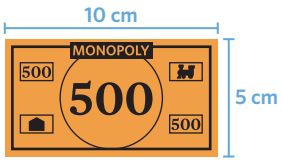
Problem solving

8. A Monopoly $500 note has four interior right angles and a pair of sides that is 10 cm long and another pair of sides that is 5 cm wide. What type of quadrilateral does the note form?

Key points

- A note has four interior right angles.
- Two sides that are 10 cm long and two sides that are 5 cm wide.
- What type of quadrilateral does the note form?

Explanation



The note forms a rectangle.

Answer

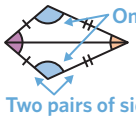
The Monopoly $500 note forms a rectangle.

9. Paige collects abstract pieces of street art. What type of quadrilateral is the frame of one of her pieces if it has two pairs of sides of equal length and only one pair of equal opposite angles?

Key points

- Paige's frame is a quadrilateral with two pairs of sides of equal length.
- The frame contains only one pair of equal opposite angles.
- What type of quadrilateral does the frame form?

Explanation



This is a kite.

Answer

Paige's frame forms a kite.

10. Alexa is a carpenter and welds a quadrilateral shaped piece of iron. One of the interior angles is 48°, and the other three angles are equal. What is the size of the other angles?

Key points

- One of the interior angles of Alexa's piece of iron is 48°.
- The other three angles are equal.
- What is the size of the other angles?

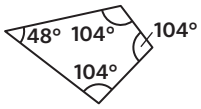
Explanation

The interior angles of a quadrilateral sum to 360°.

The size of the three identical interior angles $= \frac{360° - 48°}{3}$

$= \frac{312°}{3}$

$= 104°$



Answer

The size of the other angles is 104°.

11. Mr Borg has trapezium-shaped tables in his classroom. Each table contains two pairs of equal angles. What is the size of each interior angle, if one pair of the equal angles sum to 148°?

Key points

- Mr Borg's table forms a trapezium.
- Each table contains two pairs of equal angles.
- One equal pair of the angles sum to 148°.
- What is the size of all four of the interior angles?

Explanation

One equal pair of the angles sum to 148°.

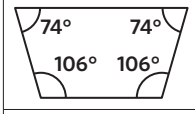
Divide 148° by 2 to find the size of one of the angles.

$148° \div 2 = 74°$

The other equal pair of the angles sum to $360° - 148° = 212°$.

Divide 212° by 2 to find the size of one of the angles.

$212° \div 2 = 106°$



Answer

The size of each angle is 74°, 74°, 106° and 106°.

12. Hamid measured two of the interior angles of his kite. One was 126° and the other was 34°. What is the size of the remaining interior angles if they are opposite angles?

Key points

- Two of the interior angles of Hamid's kite are 126° and 34° respectively.
- What is the size of the remaining interior angles?

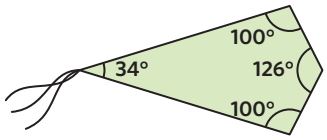
Explanation

Subtract 126° and 34° from 360 to find the sum of the remaining opposite angles.

$360° - 126° - 34° = 200°$

Divide 200° by 2 to find the size of one of the opposite angles.

$200° \div 2 = 100°$



Answer

Each of the remaining interior angles is 100°.

Reasoning

13. a. The green quadrilateral is a trapezium.
    b. The size of the two equal interior angles in each triangle is 45°.
    c. 45°, 45°, 135°, 135°
    d. Jill is correct.
    e. Suggested option 1: Jill could consider using obtuse triangles inside a trapezium.

    Suggested option 2: Jill could consider using a quadrilateral consisting of two triangles.

    Note: There are other possible options.

14. a. Bob's bedroom is to the right of the door.
    b. Bob's roof on the right of the triangular part is a parallelogram.
    c. The size of the interior angle where the bird is sitting is 125°.
    d. The interior angle formed by the roof where the volleyball is stuck in the gutter is 55°.
    e. Suggested option 1: Bob could consider the cost of materials for the extension.

    Suggested option 2: Bob could consider the area of the land.

    Note: There are other possible options.

Extra spicy

15. A
16. 40°, 80°, 100°, 140°
17. B
18. 39°, 141°, 39°, 141°

Remember this?

19. D
20. B
21. C

6E Parallel lines and transversals

Worked example 1.

a. Corresponding angles
b. Co-interior angles
c. Alternate angles
d. Corresponding angles

Worked example 2.

a. $m = 68$ because $m°$ and 112° are co-interior angles.
b. $y = 85$ because $y°$ and 85° are corresponding angles.
c. $k = 76$ because $k°$ and 76° are alternate angles.
d. $x = 72$ because $x°$ and 108° are supplementary angles.

Worked example 3.

a. The lines are parallel as the corresponding angles are equal.

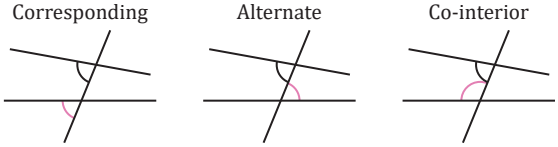
b. The lines are not parallel as the co-interior angles do not sum to 180°.

c. The lines are parallel as the alternate angles are equal.

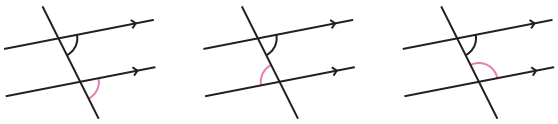
d. The lines are not parallel as the alternate angles are not equal.

Understanding worksheet

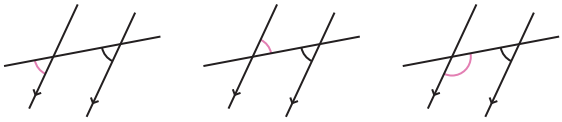
1. a. Corresponding Alternate Co-interior



b. Corresponding Alternate Co-interior

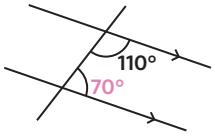


c. Corresponding Alternate Co-interior



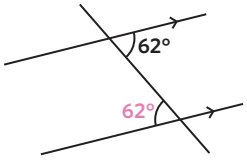
d. Corresponding Alternate

2. a.



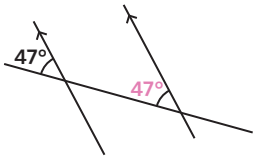
Co-interior angles in between parallel lines sum to 180°.

b.



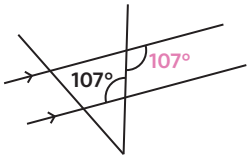
Alternate angles in between parallel lines are equal.

c.



Corresponding angles in parallel lines are equal.

d.



Alternate angles in between parallel lines are equal.

3. parallel; transversal; alternate; co-interior

Fluency

4. a. Co-interior angles
b. Alternate angles
c. Corresponding angles
d. Co-interior angles
e. Alternate angles
f. Corresponding angles
g. Corresponding angles
h. Alternate angles

5. a. $x = 130$ because x and 50° are co-interior angles.
b. $r = 124$ because $r°$ and 124° are corresponding angles.
c. $m = 108$ because $m°$ and 108° are alternate angles.
d. $c = 62$ because $c°$ and 118° are co-interior angles.
e. $g = 132$ because $g°$ and 132° are corresponding angles.
f. $s = 111$ because $s°$ and 111° are alternate angles.
g. $h = 55$ because $h°$ and 125° are co-interior angles.
h. $a = 70$ because $a°$ and 70° are alternate angles.

6. a. Not parallel; Co-interior angles do not sum to 180°.
b. Parallel; Corresponding angles are equal.
c. Parallel; Co-interior angles sum to 180°.
d. Not parallel; Alternate angles are not equal.
e. Not parallel; Corresponding angles are not equal.
f. Parallel; Alternate angles are equal.
g. Not parallel; Corresponding angles are not equal.
h. Parallel; The angles are supplementary.

7. a. True b. True c. False d. False
e. True f. True

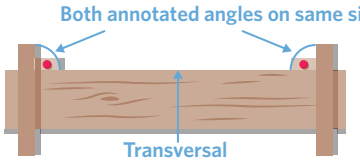
Problem solving

8. A rustic railroad track was built using two parallel rails that were bolted into wooden beams. What name is given to the pair of angles that are marked by the two bolts in the following image?

Key points

- A rustic railroad track was built using two parallel rails that were bolted into wooden beams.
- What name is given to the pair of angles that are marked by the two bolts in the following image?

Explanation



Identify the position of the marked angles. They are in between two straight lines and on the same side of the transversal. This pair of angles is co-interior.

Answer

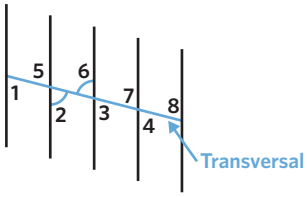
The two bolts in the image mark a pair of co-interior angles.

9. A set of parking bays is made up of parallel lines, a transversal line, and a parking spot number. What pair of angles are formed if an angle is measured at the number 2 and number 6?

Key points

- A set of parking bays is made up of parallel lines, a transversal line, and a parking spot number.
- An angle is measured at the number 2 and number 6.
- What pair of angles are formed?

Explanation



Identify the position of the marked angles number 2 and 6. They are on either side of a transversal. This pair of angles is alternate.

Answer

A pair of alternate angles are formed by the angles at the number 2 and number 6.

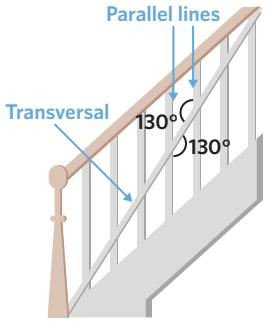
10. The following stair bannister is built with multiple vertical supports and one diagonal support. Based on the marked angles, state whether the vertical supports are parallel or not. Give a reason for your answer.

Key points

- The following stair bannister is built with multiple vertical supports and one diagonal support.
- Based on the marked angles, state whether the vertical supports are parallel or not. Give a reason for your answer.

Explanation

Identify the position of the marked angles (130° and 130°). They are on either side of a transversal. This pair of angles is alternate. A pair of parallel lines produce equal alternate angles.



Answer

The vertical supports are parallel as the alternate angles are equal.

11. Lisa is an avid skier and has a collection of different skis. She tries to store each pair of her skis parallel to one another. Is the orange pair stored parallel or not in the following image? Give a reason for your answer.

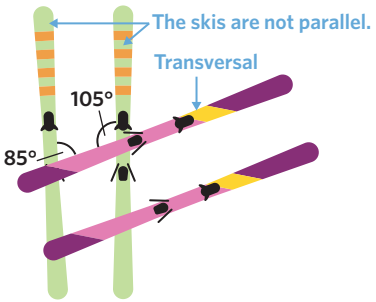
Key points

- Each pair of Lisa's skis should be parallel to one another in the image.
- State whether the orange pair is stored parallel or not parallel. Give a reason for your answer.

Explanation

Identify the position of the marked angles (85° and 105°). They are in between two straight lines and on the same side of a transversal. This pair of angles is co-interior.

The sum of the co-interior angles is $85° + 105° \neq 180°$.



Answer

Lisa's orange pair of skis are not stored parallel because the sum of the co-interior angles is not 180°.

12. The lane ropes in a standard swimming pool are parallel. An additional lane rope is connected diagonally across a pool. If the angle formed by the lane ropes at the floating purple cap is 137°, what is the size of the angles formed by the lane ropes at the green and pink caps? Give a reason for your answer.

Key points

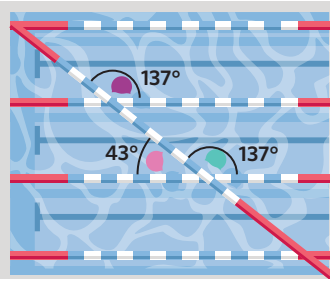
- The lane ropes in a standard swimming pool are parallel.
- An additional lane rope is connected diagonally across a pool.
- The angle formed by the lane ropes at the floating purple cap is 137°.
- What is the size of the angles formed by the lane ropes at the green and pink caps? Give a reason for your answer.

Explanation

Identify the position of the marked angles at purple and green caps. They are in matching positions on the same side of a transversal passing through two parallel lines. This pair of angles is corresponding.

The angle formed by lane ropes at the green cap is 137° because it forms corresponding angles with the purple cap.

The angle formed by lane ropes at the pink cap is $180° - 137° = 43°$ as it forms supplementary angles with the green cap.



Answer

The angle formed by lane ropes at the green cap is 137° because it forms corresponding angles with the purple cap.

The angle formed by lane ropes at the pink cap is 43° as it forms supplementary angles with the green cap.

Reasoning

13. a. The pair of angles that Stefanos marked is alternate.

b. The other angle marked with a red arc is 110° because the two angles are corresponding angles.

c. The angle marked with a red dot is 62° because it forms co-interior angles with the angle measuring 118°.

d. The four interior angles of the orange trapezium are 62°, 62°, 118°, and 118°.

e. Suggested option 1: Stefanos could consider designing two sets of parallel lines that only have three intersections.

Suggested option 2: Stefanos could consider colouring different parts of design 2.

Note: There are other possible options.

14. a. The angle marked with a blue dot is 30° because it forms corresponding angles with the angle measuring 30°.

b. Eight interior right angles are formed by the gate's pillars.

c. Phillipa has found multiples of 30° including 30°, 60°, 90° and 150°.

d. Suggested option 1: Phillipa could consider the cost of the gates.

Suggested option 2: Phillipa could consider what material is used to construct the gates.

Note: There are other possible options.

Extra spicy

15. A, B, C, D, E

16. $x = 112$

17. $x = 79, y = 117$

18. B, D

Remember this?

19. C

20. C

21. C

Chapter 6 extended application

1. a. This angle is an obtuse angle.

b. The triangle is an isosceles triangle because it has two angles of equal size (45°) and is a right-angled triangle because it has a right interior angle (90°).

c. 135°

d. If the two marked angles sum to 180°, the lines are parallel.

e. Suggested option 1: Ella could use the bus replacements provided on her usual train line.

Suggested option 2: Ella could drive to another train station and use a different train line.

Note: there are other possible options.

2. a. There are 3 line segments in the Aries constellation.

b. The quadrilateral would be a rectangle.

c. 60°

d. Pattern: +180°

Number of sides	3	4	5	6	7	8
Sum of interior angles	180°	360°	540°	720°	900°	1080°

e. The Leo constellation is the better looking constellation.

f. Suggested option 1: Astrology has no scientific evidence supporting it.

Suggested option 2: Astrological predictions often occur just by chance.

Note: there are other possible options

3. a. There are more quadrilaterals.

b. Squares have 4 sides of equal length and interior angles of 90°.

Rectangles have 2 pairs of opposite equal and parallel sides and interior angles of 90°.

c. 30°

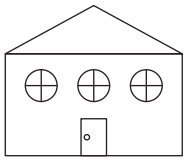
d. The length of the ladder is 2.5 m.

e. Suggested option 1: A flat roof would require less material which is cheaper.

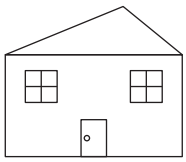
Suggested option 2: A flat roof is easier to construct as it would not require support beams.

Note: there are other possible options.

f. Suggested option 1: You could draw circle windows.



Suggested option 2: You could draw an asymmetric roof.



Note: there are other possible options.

Chapter 6 review

Multiple choice

1. C

2. A

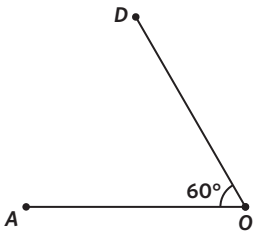
3. D

4. B

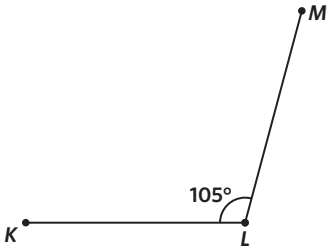
5. C

Fluency

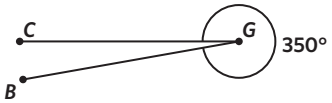
6. a.



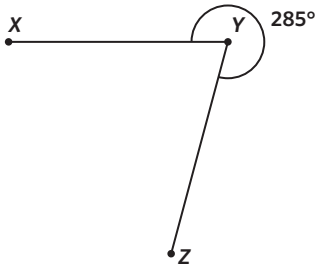
b.



c.



d.



7. a. 180°, straight line
b. 53°; acute angle
c. 130°; obtuse angle
d. 260°; reflex angle

8. a. $k = 128°$
b. $e = 18°$
c. $t = 50.5°$
d. $m = 228°$

9. a. Equilateral triangle; acute angle triangle
b. Isosceles triangle
c. Scalene triangle; acute angle triangle
d. Scalene triangle; obtuse

10. a. $w = 59°$
b. $x = 95°$
c. $s = 66°$
d. $k = 153°$

11. a. Rectangle
b. Parallelogram
c. Trapezium
d. Kite

12. a. $z = 71°$
b. $f = 153°$
c. $y = 107°$
d. $x = 284°$

13. a. $m = 59°$ because $m°$ and 121° are co-interior angles.
b. $y = 97°$ because $y°$ and 97° are corresponding angles.
c. $t = 90°$ because $t°$ and 90° are alternate angles.
d. $t = 55°$ because $t°$ and 55° are alternate angles.
$p = 125°$ because $t°$ and $p°$ are supplementary angles.

14. a. Not parallel; corresponding angles are not equal.
b. Parallel; alternate angles are equal.
c. Parallel; co-interior angles sum to 180°.
d. Not parallel; corresponding angles are not equal.

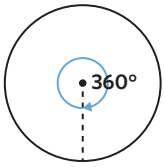
Problem solving

15. Dan is a pizza lover. He buys one third of a margherita pizza and cuts it into two identical slices. What is the angle measurement formed by each slice of pizza?

Key points
- Dan cuts one third of a slice into two identical slices.
- What is the angle measurement formed by one slice?

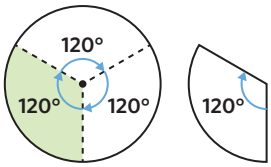
Explanation

A circular pizza represents a full revolution, so the angle measurement formed by the whole pizza is 360°.



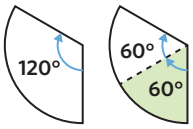
So the angle measurement of one third of a circular pizza is one third of 360°.

$360° \div 3 = 120°$



This third of a pizza is cut into two identical slices. The angle measurement of one of these slices is one half of 120°.

$120° \div 2 = 60°$



So each slice of pizza has an angle measurement of 60°.

Answer

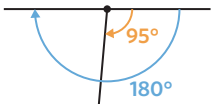
The angle measurement formed by each slice of pizza is 60°.

16. When Joey's classroom door is fully open it forms a straight line with the wall. If Joey opens the door of the classroom by 95°, how many degrees further does Joey need to open the door if she wants the door to be fully opened?

Key points
- The door has been opened to 95°.
- When the door is fully opened, it forms a straight line with the wall.
- How many degrees further does Joey need to open the door for the door to be fully opened?

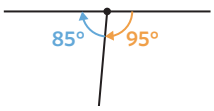
Explanation

A straight line forms a 180° angle. So when the door is fully open, it forms a 180° angle.



The angle that Joey needs to open the door further will be the difference between 180° and 95°.

$180° - 95° = 85°$ further



Answer

85° further

17. Alice buys a loaf of sliced bread and uses two slices to make a sandwich. Assuming each slice of bread is in the shape of a square, what type of triangle is formed if Alice cuts her sandwich diagonally in half?

Key points

- Each slice of bread is a square.
- Each slice is cut diagonally in half to form two triangles.
- What type of triangle is formed?

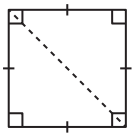
Explanation

A square has four right angles and four equal sides.

Cutting the square diagonally in half will create a right angled triangle with two equal sides.

A triangle with two equal side lengths is an isosceles triangle.

A triangle with a right angle is a right-angled triangle.



Answer

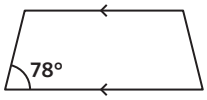
Alice forms an isosceles triangle; or Alice forms a right angled triangle.

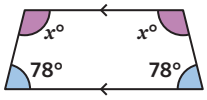
18. Qin's suitcase is shaped as a trapezium with two sides of equal length and two opposite parallel sides. One of the interior angles of the trapezium is measured to be 78°. What are the measurements of the other three interior angles?

Key points

- The suitcase is a trapezium with two sides of equal length and two opposite parallel sides.
- One interior angle is 78°.
- What are the measurements of the other three interior angles?

Explanation





Adjacent angles that share a parallel side are equal.
Represent the equal unknown angles with the pronumeral x.

The interior angles of a quadrilateral sum to 360°.

$$78° + 78° + x° + x° = 360°$$
$$156° + 2x° = 360°$$
$$156° + 2x° - 156° = 360° - 156°$$
$$2x° = 204°$$
$$2x° \div 2 = 204° \div 2$$
$$x = 102$$

So the other angle measurements are 102°, 102° and 78°.

Answer

The three other angle measurements are 102°, 102° and 78°.

19. Freddy is investigating the pairs of angles formed by some capital letters. How many pairs of alternate angles does the letter H have?

Key points

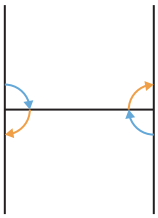
- How many pairs of alternate angles does the capital letter 'H' have?

Explanation

The letter 'H' is made up of two parallel vertical lines, and one horizontal line. The horizontal line is a transversal.

Alternate angles are in between two straight lines and on either side of a transversal.

The transversal does not extend beyond the two parallel lines, so only two pairs of alternate angles are formed.



Answer

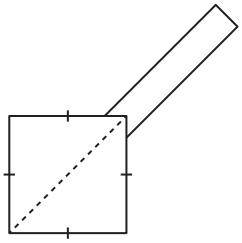
The capital letter H has 2 pairs of alternate angles.

Reasoning

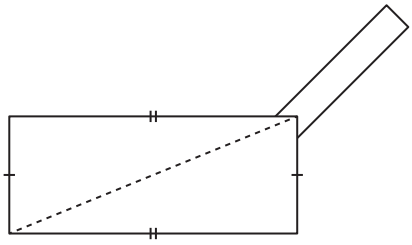
20. a. They are right angles
    b. The rack is an equilateral triangle.
    c. 60°
    d. They are not parallel, as the two co-interior angles do not sum to 180°.
    e. Suggested option 1: Donni could practice often to improve her skills over time.

    Suggested option 2: Donni could do some research on snooker strategy to improve her technique.

21. a. The sum of the interior angles is 360°.
    b. Scalene or obtuse angle triangle, as each side of the triangle is a different length and one of the interior angles is obtuse.
    c. The exterior angle is 115°.
    d. Suggested option 1:



Suggested option 2:



7A Collecting and classifying data

Worked example 1.

a. Discrete numerical
b. Categorical
c. Continuous numerical
d. Categorical

Worked example 2.

a. Secondary b. Primary c. Primary d. Secondary

Worked example 3.

a. Sampling b. Census c. Census d. Sampling

Worked example 4.

a.

Eye colour	Tally	Frequency
Blue	IIII	4
Green	I	1
Brown	III	3
Grey	I	1
Total		**9**

b.

Number of siblings	Tally	Frequency
0	I	1
1	III	3
2	III	3
3	II	2
Total		**9**

c.

Favourite musical instrument	Tally	Frequency
Guitar	𝍸	5
Piano	II	2
Violin	IIII	4
Cello	I	1
Trumpet	II	2
Trombone	I	1
Total		**15**

d.

Pieces of fruit	Tally	Frequency
0	II	2
1	II	2
2	𝍸	5
3	I	1
4	I	1
5	I	1
Total		**12**

Understanding worksheet

1. a. Discrete numerical
   b. Categorical
   c. Discrete numerical
   d. Continuous numerical

2. a. Primary
   b. Secondary
   c. Secondary
   d. Primary

3. population; sample; census; primary; secondary

Fluency

4. a. Categorical
   b. Discrete numerical
   c. Categorical
   d. Continuous numerical
   e. Categorical
   f. Discrete numerical

5. a. Primary
   b. Secondary
   c. Primary
   d. Secondary
   e. Secondary
   f. Primary
   g. Primary
   h. Secondary

6. a. Sampling b. Sampling c. Census d. Census

7. a. Sample
   b. Population
   c. Sample
   d. Population
   e. Population
   f. Sample

8. a.

Destination	Tally	Frequency
Noosa	III	3
Gold Coast	IIII	4
Perth	II	2
Sydney	II	2
Byron Bay	IIII	4
Melbourne	II	2
Hobart	I	1
Broome	I	1
Total		**19**

b.

Hours	Tally	Frequency
2	I	1
3	II	2
4	III	3
5	𝍸 I	6
6	IIII	4
7	𝍸 I	6
8	II	2
Total		**24**

c.

Character	Tally	Frequency
Ron	III	3
Hermione	𝍸	5
Harry	III	3
Hagrid	I	1
Dumbledore	I	1
Voldemort	I	1
Lockhart	I	1
Total		**15**

d.

Countries	Tally	Frequency
0	IIII	4
1	IIII	4
2	II	2
3	I	1
4	III	3
7	I	1
10	I	1
12	I	1
22	I	1
Total		**18**

Problem solving

9. Graham is completing a music project for school. He records the favourite music genre of every member of his extended family. What type of data did Graham collect?

Key points
- Graham collects data on music genres.
- Is this discrete numerical, continuous numerical or categorical data?

Explanation
A music genre is descriptive, and isn't counted or measured.

Answer
Graham has collected categorical data.

10. Daphne enjoys fishing and always measures the weights of the fish she catches. What type of data is the weight of the fish if the fish she caught on Sunday were, 2.14 kg, 6.31 kg and 3.95 kg?

Key points
- The fish Daphne caught weighed 2.14 kg, 6.31 kg and 3.95 kg.
- Is this discrete numerical, continuous numerical or categorical data?

Explanation
Weight is measured, not counted. It exists on an endless scale.

Answer
The weight of fish is continuous numerical data.

11. Lance has been assigned a project on the history of ANZAC. Did he collect information from a primary or secondary source, when he interviewed his great grandfather who fought in Gallipoli?

Key points
- Lance interviewed his great grandfather who fought in Gallipoli.
- Is Lance's interview a primary or secondary source?

Explanation
Lance's great grandfather fought in Gallipoli. This means he is providing a firsthand account of his experiences at war.

Answer
Lance collected information from a primary source.

12. A research group is investigating whether libraries are becoming more or less popular in Parramatta. They randomly select 50 Parramatta residents and ask them if they have attended the local library in the last week. What is the population and sample of this experiment?

Key points
- The research group is investigating libraries in Parramatta.
- They randomly select 50 residents of Parramatta.
- What is the population and sample of the study?

Explanation
A population is the group you want to apply your conclusions to.

A sample is a smaller group within the population.

Answer
Population: All Parramatta residents

Sample: 50 randomly selected Parramatta residents

13. Baxter is looking for a primary source to help with his investigation into the effects of pollution on the Great Barrier Reef. Should he use a research paper from a group that conducted a year-long study on the effects of oil on marine wildlife, or a newspaper opinion article on how to help preserve the ocean?

Key points
- Baxter needs to find a primary source.
- He has a research paper from a group that conducted a year-long study.
- He also has an article about ocean preservation.
- Which of these is a firsthand account collected by an individual or group?

Explanation
The year-long study has been collected and presented by a research group. This is a primary source.

The newspaper opinion article doesn't present original data. This is a secondary source.

Answer
Baxter should use the research paper.

Reasoning

14. a. Continuous numerical data

b. Categorical data

c. The scientist should use information from the Australian Bureau of Statistics because it is a primary source.

d. Suggested option 1: Australia should switch as soon as possible to using 100% renewable resources to avoid the production of fossil fuels.

Suggested option 2: Australia should gradually try to use more renewable resources and less nonrenewables as it will be a long and difficult process.

Note: There are other possible options.

15. a. A census would be the most reliable form of data collection.
    b. Donald is planning on using a census.
    c. Conducting a census can often be too time consuming and too expensive.
    d. Donald's sample is not an accurate representation of the population as it would not take into account the diverse range of ages, interests and genders.
    e. Suggested option 1: Donald could randomly select 100 students across the whole school.
    Suggested option 2: Donald could randomly select 5 students from each year level.
    Note: There are other possible options.

Extra spicy

16. A 17. D 18. B 19. 24

Remember this?

20. $1\frac{1}{4}$ 21. 0.1052 22. 150°

7B Summary statistics

Worked example 1.

a. 4 b. 9 c. 33.25 d. 38.5

Worked example 2.

a. 14 b. 57 c. 13 d. 196

Worked example 3.

a. 11 b. margherita c. 20, 40 d. bus

Worked example 4.

a. 28 b. 33 c. 200 d. 212

Understanding worksheet

1. a. (4), 7, 7, 10, 13, 16, (17)
Range = Maximum − Minimum
Range = 17 − 4 = 13

b. (15), 19, 25, 26, 38, 41, 47, 51, (53)
Range = Maximum − Minimum
Range = 53 − 15 = 38

c. 20, 11, (6), 15, (23), 9
Range = Maximum − Minimum
Range = 23 − 6 = 17

d. 42, 30, (89), 25, 56, (13), 38, 19, 77
Range = Maximum − Minimum
Range = 89 − 13 = 76

2. a. 2, 4, (7), 9, 12
Median = 7

b. 21, 25, (44), 100, 101
Median = 44

c. 0, 3, (6, 8), 10, 11
$Median = \frac{6+8}{2} = 7$

d. 15, 24, (24, 58), 73, 101
$Median = \frac{24+58}{2} = 41$

3. range; mode; mean; average; median

Fluency

4. a. 8 b. 12 c. 11 d. 111.5
   e. 6.75 f. 58 g. 1.6 h. 254

5. a. 11 b. 47 c. 18 d. 20
   e. 14 f. 151 g. 155 h. 32.5

6. a. 100 b. Leopard
   c. 45, 53 d. Red
   e. Africa, Europe f. 2, 3, 5
   g. No mode h. No mode

7. a. 17 b. 89 c. 5000 d. 29
   e. 8 f. 199 980 g. 620 h. 945

8. a. Data set A b. Data set A
   c. Data set B d. Data set B

Problem solving

9. Leo went to Port Douglas and collected shells on the beach every day for seven days. By the end of his holiday he had collected 84 shells. What was the mean number of shells Leo collected each day?

Key points
- Leo collected shells daily for 7 days.
- He collected 84 shells in total.
- On average, how many shells did Leo collect each day?

Explanation
The mean is given by $mean = \frac{sum\ of\ values}{number\ of\ values}$.
The sum of the values is the total number of shells collected; 84.
The number of values is the number of days that Leo collected shells; 7.
$mean = \frac{84}{7}$
$= 12$

Answer
Leo collected a mean of 12 shells per day.

10. The 24 students in a Year 7 Maths class each noted their favourite social media platform, as shown. Which social media platform(s) is/are the most popular amongst the students?

Key points
- 24 students wrote down their favourite social media platform.
- Find the modal social media platform.

Explanation
We can represent the data using a frequency table.

Social media platform	Tally	Frequency
Instagram	𝍸 I	6
TikTok	𝍸 I	6
Facebook	III	3
Snapchat	IIII	4
Twitter	II	2
YouTube	II	2
Reddit	I	1

The mode is the category with the highest frequency.
Instagram and TikTok both have a frequency of 6.

Answer
Instagram and TikTok are the most popular platforms amongst the students.

11. Kelly is the shortest in her family. She has listed the heights, in centimetres, of all the members of her family in the following list. 176, 165, 138, 159, 146, 179, 169, 185, 180. How much taller than Kelly is her brother Dexter if he is the tallest in the family?

Key points

- Kelly is the shortest in the family.
- Dexter is the tallest in the family.
- The family's heights, in cm, are: 176, 165, 138, 159, 146, 179, 169, 185, 180
- What is the height difference between Kelly and Dexter?

Explanation

Kelly is the shortest of the family. This means that her height is the minimum value.

Dexter is the tallest of the family. This means that his height is the maximum value.

Minimum Maximum

176, 165, (138), 159, 146, 179, 169, (185), 180

The difference between these heights is equal to the range.

$Range = maximum - minimum$

$= 185 - 138$

$= 47$

Answer

Dexter is 47 cm taller than Kelly.

12. In 2021, the mean weekly time spent on social media was 10.5 hours. Over five weeks Shareef spent: 13, 8, 5, 17 and 7 hours per week on social media and Jasmine spent: 7, 16, 10, 9, and 14 hours. Were Jasmine and Shareef above or below the weekly Australian mean?

Key points

- In 2021, the mean time spent on social media was 10.5 hours per week.
- Shareef spent 13, 8, 5, 17 and 7 hours per week over 5 weeks.
- Jasmine spent 7, 16, 10, 9, and 14 hours per week over 5 weeks.
- Calculate the mean time spent per week on social media for Shareef and Jasmine and compare this to the Australian mean.

Explanation

$mean = \frac{sum\ of\ data\ values}{number\ of\ values}$

Shareef:

$mean = \frac{13 + 8 + 5 + 17 + 7}{5}$

$= \frac{50}{5}$

$= 10$

$10 < 10.5$

Jasmine:

$mean = \frac{7 + 16 + 10 + 9 + 14}{5}$

$= \frac{56}{5}$

$= 11.2$

$11.2 > 10.5$

Answer

Shareef was below and Jasmine was above the weekly Australian mean.

13. Four friends, Fernanda, Penelope, Clove and Willa, have a median age of 13. How old is Willa if Penelope is the oldest, Clove is the youngest and Fernanda is 13?

Key points

- Fernanda, Penelope, Clove, and Willa have a median age of 13.
- Penelope is the oldest.
- Clove is the youngest.
- Fernanda is 13.
- What is Willa's age?

Explanation

When working with the median, the data must first be ordered.

Fernanda and Willa will be the 2nd and 3rd values in the ordered data set.

Since there are 4 friends, the median age is the mean of the 2nd and 3rd values.

Clove, (Fernanda 13, Willa ☐), Penelope

$median = \frac{Fernanda's\ age + Willa's\ age}{2}$

or

Clove, (Willa ☐, Fernanda 13), Penelope

$median = \frac{Fernanda's\ age + Willa's\ age}{2}$

The median is 13 and Fernanda is 13.

$13 = \frac{13 + Willa's\ age}{2}$

Willa's age must also be 13 for this equation to be true.

Answer

Willa is 13 years old.

Reasoning

14. a. The modal temperature in Melbourne is 24°C.

The modal temperature in Darwin is 30°C.

b. The range in temperatures for Melbourne is 18°C. The range in temperatures for Darwin is 7°C. Therefore, Melbourne has a more variable climate.

c. The median temperature in Melbourne was 23.5°C.
The median temperature in Darwin was 30.5°C. Therefore, Darwin generally has a higher temperature than Melbourne.

d. It would need to be 32°C in Melbourne on the 15th day in order for the mean temperature to be 24°C.

e. Suggested option 1: Macy might consider the cost of living in each city.

Suggested option 2: Macy might consider her future job prospects in each city.

Note: There are other possible options.

15. a. Urban school: The median is 16 minutes and the mean is 16.75 minutes.

Rural school: The median is 40 minutes and the mean is 39 minutes.

b. It generally takes longer for the students to travel to the rural school than the urban school.

c. If Jill's time was included, the mean time taken to get to the rural school would increase by 11 minutes.

d. Suggested option 1: Yes, school should start later so students can sleep in.

Suggested option 2: No, school shouldn't start later because there would then be less time for after-school activities.

Note: There are other possible options.

Extra spicy

16. B 17. 50 years old 18. B 19. D

Remember this?

20. D 21. B 22. B

7C Dot plots and stem-and-leaf plots

Worked example 1.

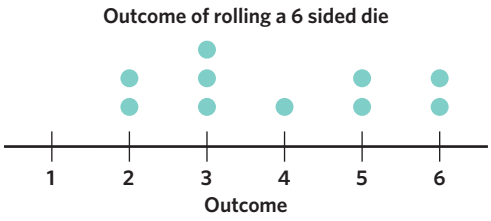
a. 1 person b. 1 person c. 20% d. Baby-G

Worked example 2.

a. 1 person b. 3 people c. 10% d. $30

Worked example 3.

a.

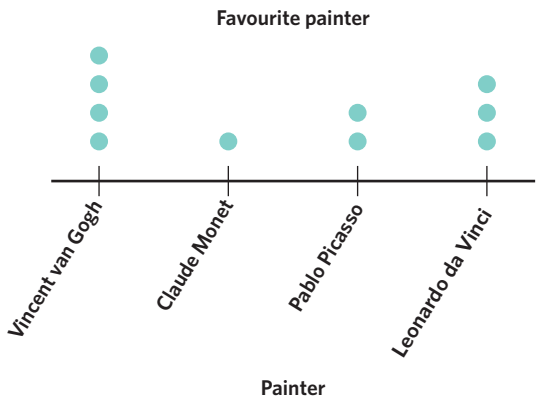


b.

Stem	Leaf
0	6 8 8
1	1 2 2 4 5 6 9
2	1 3 4 5
3	0

Key: 0 | 6 = 6 students

c.



d.

Stem	Leaf
3	2 5 7
4	0 7 8 8
5	1 2 4 6 7
6	3 9
7	0

Key: 0 | 6 = 6 students

Understanding worksheet

1. a.

Shoe size	Number sold
5	1
6	2
7	6
8	6
9	8
10	4
11	1
Total	28

b.

Colour	Number of players
Pink	3
Red	3
Yellow	2
Blue	1
Purple	1
Total	10

c.

Gift requested	Number of children
Bicycle	4
Book	3
X-Box	7
PlayStation	3
Board Game	5
Total	22

d.

Time	Number of cars
5 am	5
6 am	20
7 am	25
8 am	35
9 am	30
10 am	10
Total	125

2. a. List of numbers: 5, 7, 8, 10, 12, 17, 19, 21, 33, 34, 36

b. List of numbers: 12, 12, 17, 23, 25, 26, 41, 41, 44, 49, 58

c. List of numbers: 113, 115, 117, 119, 120, 121, 121, 133, 136, 139, 145, 145, 145

d. List of numbers: 100, 100, 103, 105, 108, 111, 132, 135, 135, 137, 140, 144, 145, 148, 149

3. categorical; dot; numerical; key

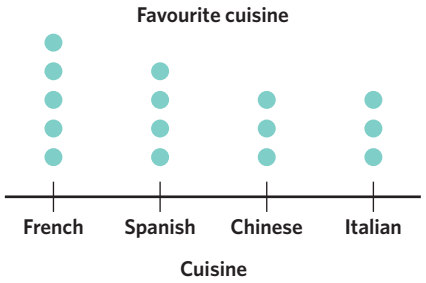
Fluency

4. a. 5 contestants
   b. 6 contestants
   c. 3 contestants
   d. 3 contestants
   e. Star anise
   f. 50%

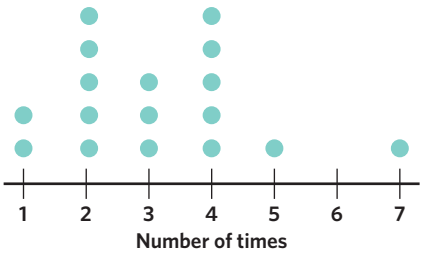
5. a. 3 plants
   b. 0 plants
   c. 9 plants
   d. 3 plants
   e. 40%
   f. 14 cm

6. a. 2 days
   b. 0 days
   c. 6 days
   d. 10%
   e. 20%
   f. 63 customers
   g. 63 customers

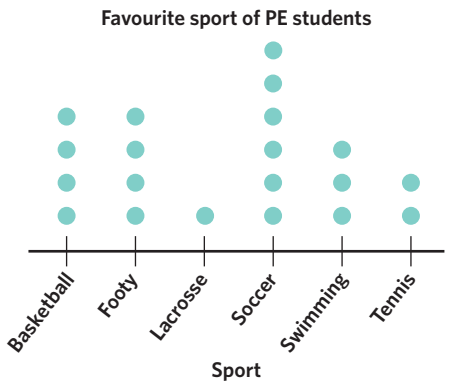
7. a.



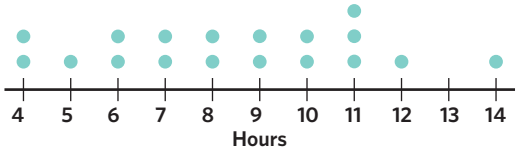
b.



c.



d.



8. a.

Stem	Leaf
1	0 0 0 5 5 8
2	0 4 5 5
3	0 1

Key: 1 | 0 = $10

b.

Stem	Leaf
0	8
1	8 8 9
2	0 1 4 5 5 6 6 6 7 7 8 8
3	0 0 1 6

Key: 0 | 8 = 8°C

c.

Stem	Leaf
7	6 8 8
8	1 1 3 3 5 6 8 9
9	1 5 6
10	1 2 4 5 5 8
11	0 2 2
12	0 5

Key: 7 | 6 = 76

d.

Stem	Leaf
2	9 9
3	3 5 9
4	5 6
5	2 5 5
6	1 5
7	2 6
8	1 7
9	3 4 9
10	0 4 4 6 8
11	3 5 5 5 9
12	4

Key: 2 | 9 = 2.9 seconds

Problem solving

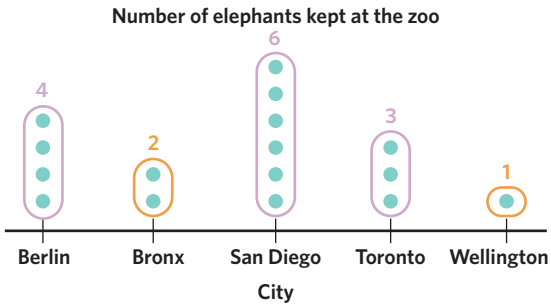
9. Victor has completed the following dot plot for a school project on the number of elephants at different zoos around the world. Which zoos have three or more elephants?

Key points

- Victor created a dot plot to record the number of elephants at zoos from different cities.
- Identify the cities with three or more dots.

Explanation

Count the number of dots from each zoo.



Answer

Zoos from Berlin, San Diego and Toronto have three or more elephants.

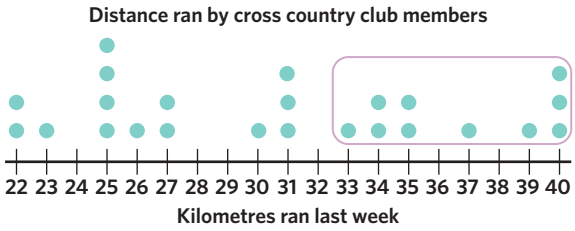
10. Pearl runs at a cross country club and all the runners recorded the total distance they ran last week. How many runners ran more than 32 km, if each dot in the following dot plot represents two runners?

Key points

- The dot plot records the total distance each member of the cross country club runs.
- Each dot on the graph represents two runners.
- How many runners ran more than 32 km?

Explanation

There are 10 dots that are greater than 32 km.



Each dot represents two runners.

$10 \times 2 = 20$

Answer

20 runners ran more than 32 km.

11. Trevor is a journalist and is tracking the number of times each of his tweets has been retweeted. What is the range of the data, if the following stem-and-leaf plot shows the number of times his last 10 tweets have been retweeted?

Key points

- The stem-and-leaf plot shows the number of times Trevor's tweets have been retweeted.
- Calculate the range of the data set.

Explanation

The range is the difference between the largest and the smallest value in a data set.

Stem	Leaf	Key
0	(4) 4 6 8 9	0 | 4 = 4 times
1	2 3 4 6	
2	(1)	

Smallest value: 4

Largest value: 21

Range $= 21 - 4$

$= 17$

Answer

The range of the data is 17.

12. Axel recorded the number of tomatoes he harvested each week and used the following stem-and-leaf plot to display the data. He thinks a dot plot may be a better way to represent the data. Construct a dot plot to display Axel's data.

Key points

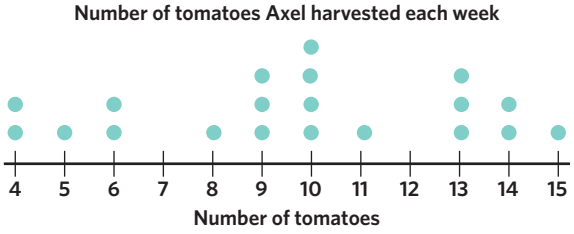
- Axel used a stem-and-leaf plot to display the data of the number of tomatoes he harvested each week.
- Construct a dot plot using the data.

Explanation

Identify an appropriate scale. The smallest number is 4, and the largest is 15.

Count the frequency of each number of the stem-and-leaf plot. This is the number of dots for each value

Answer



13. Aiko owns a Japanese restaurant and constructed the following stem-and-leaf plot to display the salaries of 12 of his employees. Aiko forgot to include his own salary of $99 000. If Aiko's salary was included, how much would the median salary increase by?

Key points

- Aiko uses a stem-and-leaf plot to display the salaries of 12 of his employees.
- His own salary is $99 000.
- How much higher would the new median be if Aiko includes his salary?

Explanation

Determine the current median.

Stem	Leaf	Key
2	3	2 | 3 = $23 000
3	0 4 4 4 9	
4	1 2 5	
5	2 2	
6	0	

$\frac{39 + 41}{2} = 40$

Median $=$ $40 000

Construct a new stem-and-leaf plot which includes Aiko's salary and find the new median.

Stem	Leaf	Key
2	3	2 | 3 = $23 000
3	0 4 4 4 9	
4	1 2 5	
5	2 2	
6	0	
9	9	

New median $=$ $41 000

41 000 − 40 000 = $1000

Answer

The median salary would increase by $1000.

Reasoning

14. a. The amount of rainfall was recorded over 14 days.

b. The modal daily rainfall for Marrakech is 0 mm and for Edinburgh is 8 mm

c. Edinburgh's range of daily rainfall is 6 mm greater than Marrakech's.

d. As there is an equal number of rainy days and non-rainy days, it is equally likely to rain and to not rain in Marrakech.

e. Suggested option 1: The range is the best summary statistic because it gives travellers an idea of what the maximum and minimum rainfall amounts are, which may help them plan activities accordingly.

Suggested option 2: The mean is the best summary statistic because it provides the most realistic rainfall for travellers and is a statistic that most people are familiar with.

Note: There are other possible options.

15. a. Group 2 has a larger range of IQ scores.

b. The modal IQ is 104.

c. The percentage of people with an IQ over 100 is 55% in group 1 and 35% in group 2.

d. The IQ of group 1 is higher, because it has the highest median (104) compared to group 2 (92).

e. Suggested option 1: Observe the person solving different types of real life problems.

Suggested option 2: Combine the IQ test with an Emotional Quotient (EQ) test.

Note: There are other possible options.

Extra spicy

16. 2 17. C 18. B 19. A

Remember this?

20. A 21. $9900 22. $136

7D Column graphs and pie charts

Worked example 1.

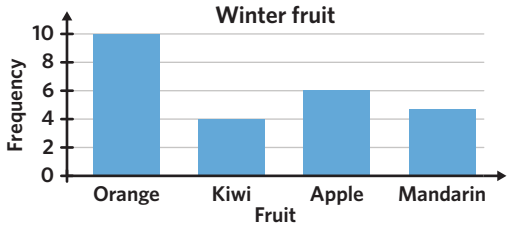
a. 12 people b. 45 people c. 20% d. 46%

Worked example 2.

a. 34% b. 32% c. 26 students

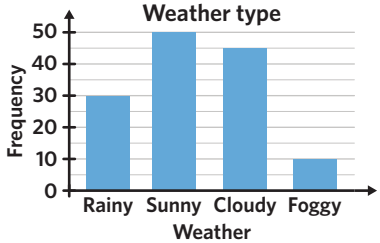
Worked example 3.

a.

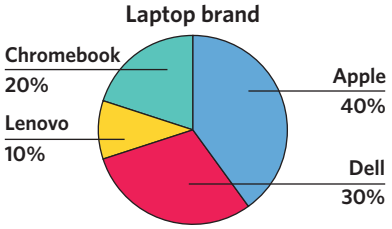


b.

c.

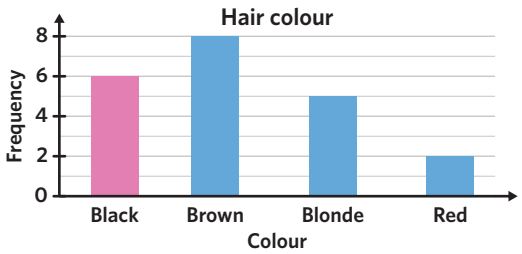


d.

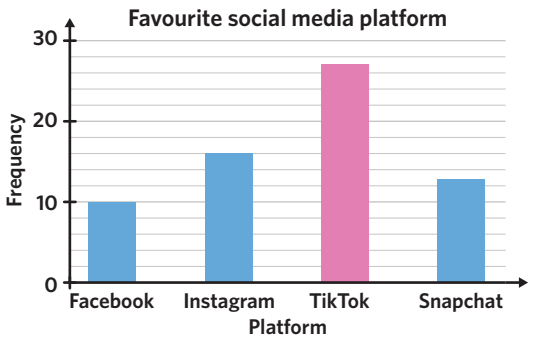


Understanding worksheet

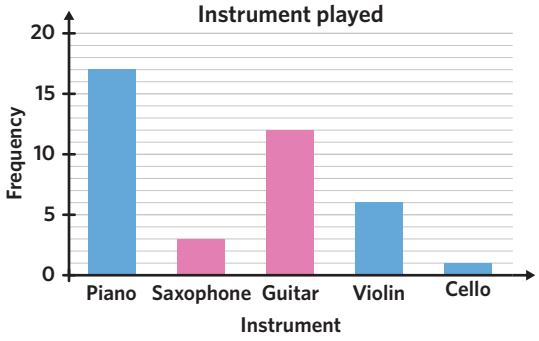
1. a.



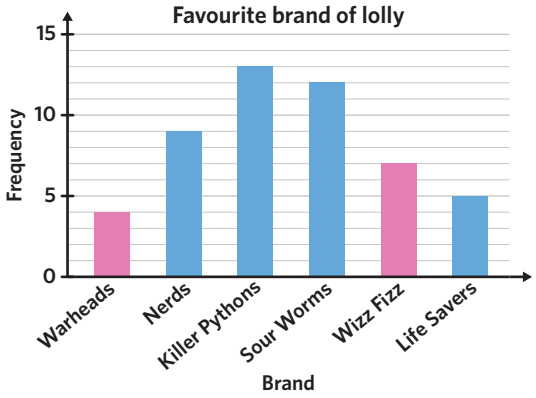
b.



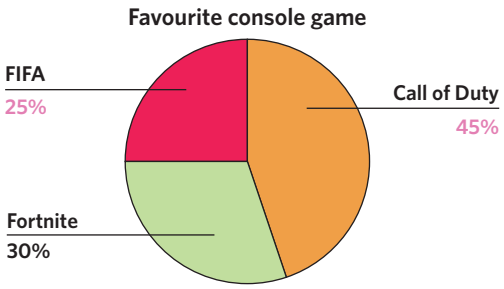
c.



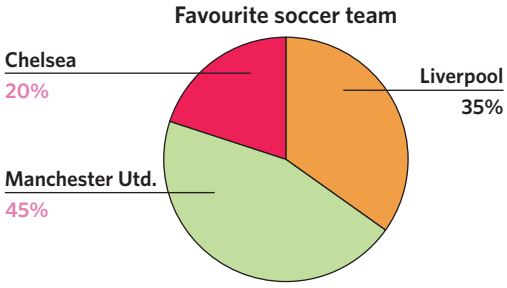
d.



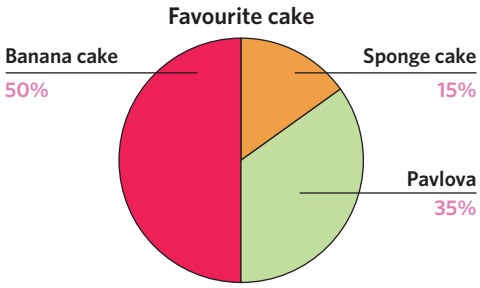
2. a.



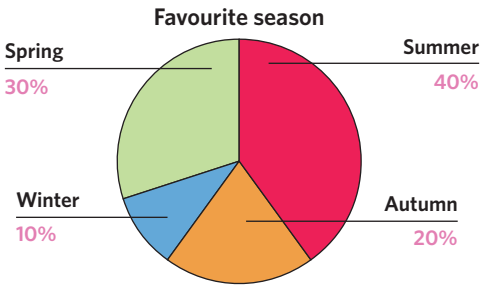
b.



c.



d.



3. bar; categorical; frequency; percentage

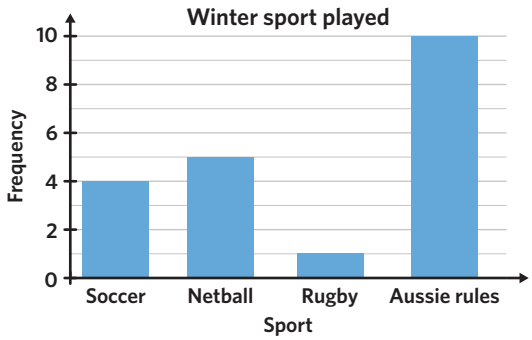
Fluency

4. a. 8 fans
b. 4 fans
c. Renegades
d. 8 fans
e. 2 fans
f. 10%

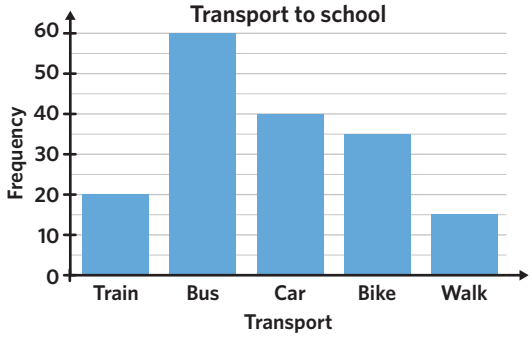
5. a. False
b. True
c. True
d. False
e. True
f. True

6. a. 19%
b. 9%
c. Margherita
d. 140 people
e. 43%
f. 56 people

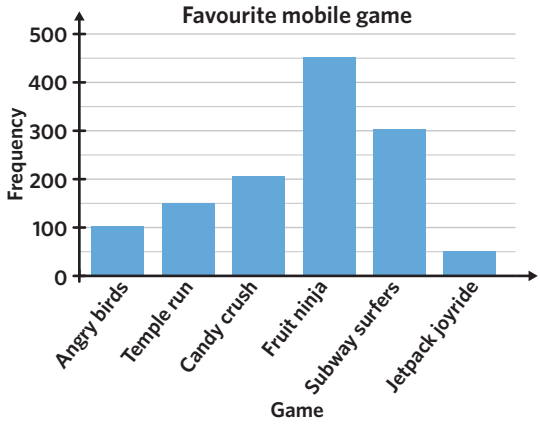
7. a.



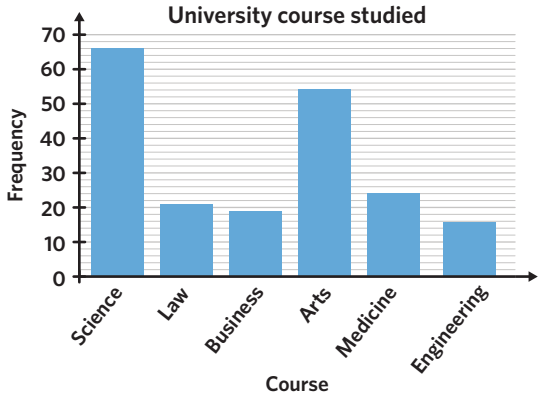
b.



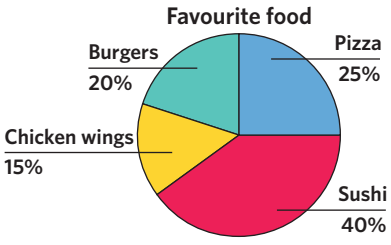
c.



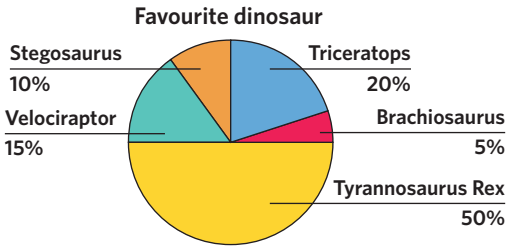
d.



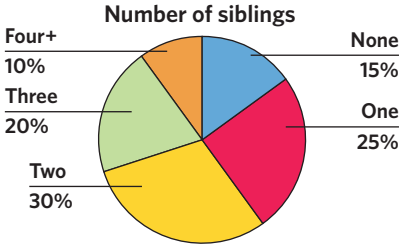
8. a.



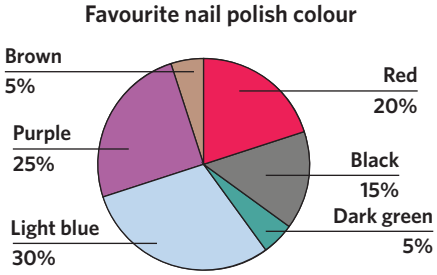
b.



c.



d.



Problem solving

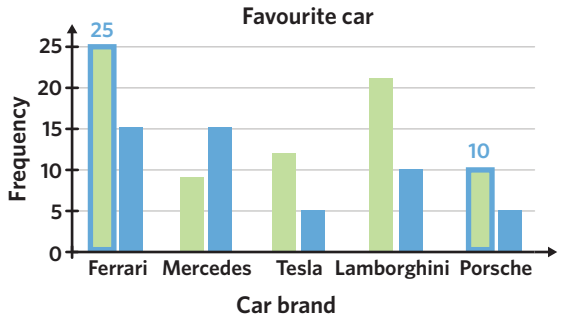
9. The following column graph shows survey results from men and women on their favourite cars. How many more men favour Ferrari than favour Porsche?

Key points

- The column graph shows how many men and how many women favour each of 5 different car brands.
- What is the difference between the number of men that favour Ferrari and the number of men that favour Porsche?

Explanation

25 men favour Ferrari and 10 men favour Porsche.



Subtract 10 from 25 to find the difference.

$25 - 10 = 15$

Answer

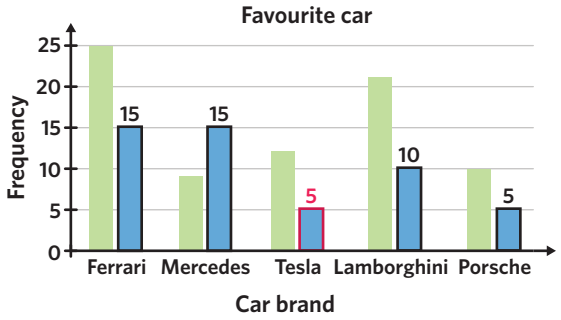
15 more men favour Ferrari than Porsche.

10. Using the same column graph from the previous question, what percentage of women said Tesla was their favourite car brand?

Key points

- The column graph shows how many men and how many women favour each of 5 different car brands.
- What percentage of women favour Tesla over other car brands?

Explanation



Total number of women:

$15 + 15 + 5 + 10 + 5 = 50$

5 women favour Tesla.

$$\textit{percentage frequency} = \frac{5}{50} \times 100$$
$$= \frac{1}{10} \times 100$$
$$= 10\%$$

Answer

10% of women favour Tesla.

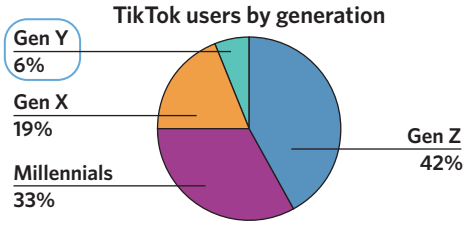
11. The following pie chart shows the percentage of TikTok users from each generation. How many 'Gen Y' use TikTok if the app has 700 million users in total?

Key points

- TikTok has 700 million users.
- The graph shows the percentage of TikTok users that each generation makes up.
- How many users are 'Gen Y'?

Explanation

6% of TikTok users are 'gen Y'.



Find 6% of 700 million.

$6\% = \frac{6}{100}$

$\frac{6}{100} \times 700\,000\,000 = 6 \times 7\,000\,000$

$= 42\,000\,000$

Answer

There are 42 million 'Gen Y' that use TikTok.

12. Michelle operates a skydiving business. She uses the following pie chart as a visual display to represent the age of her customers. How many 60+ year olds does Michelle expect to skydive annually, if she predicts that 300 customers will skydive every six months?

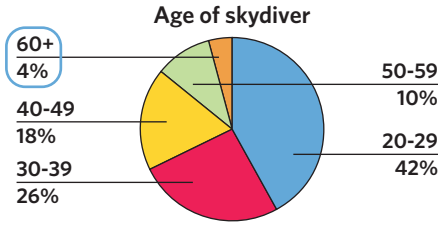
Key points

- Michelle expects 300 skydiving customers every half year.
- The pie chart shows the percentage of different skydiver age groups.
- How many 60+ year-old skydivers are there every year?

Explanation

Find the number of 60+ year-old skydivers every 6 months:

4% of skydivers are 60+ years old.



Find 4% of 300.

$4\% = \frac{4}{100}$

$\frac{4}{100} \times 300 = 4 \times 3$

$= 12$

Multiply 12 by 2 to find the number of 60+ year-old skydivers every year:

$12 \times 2 = 24$

Answer

Michelle expects 24 60+ year-old skydivers annually.

13. The following column graph shows how frequently vowels are used in different European languages. Which vowels are both used more frequently in Spanish than English and more frequently in English than French?

Key points

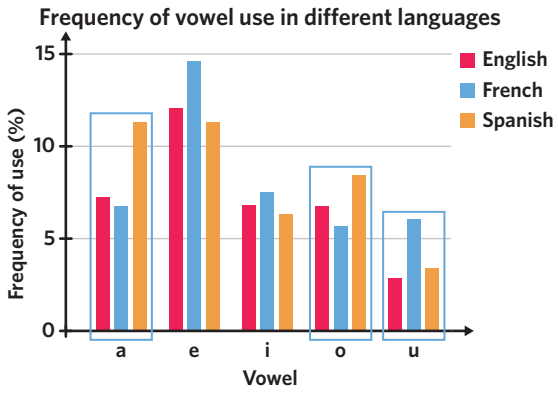
- The graph shows how frequently each of the five vowels are used in English, French and Spanish.
- List the vowels that have a higher frequency of usage in Spanish than in English as well as a higher frequency of usage in English than in French.

Explanation

Vowels that are used more in Spanish than in English:

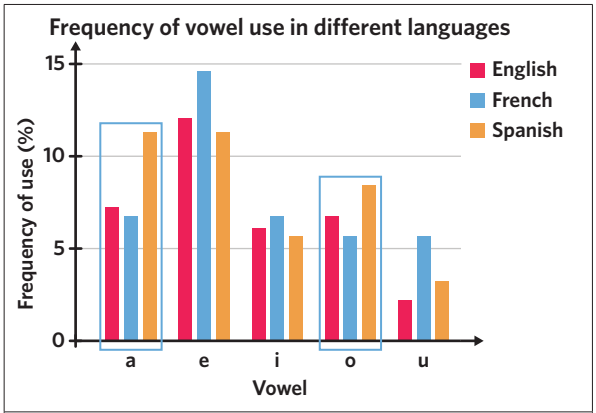
We are looking for the vowels that have an orange column higher than their red column.

These vowels are 'a', 'o', and 'u'.



Vowels that are used more in English than in French:

We are looking for the vowels (out of 'a', 'o' and 'u') that have a red column higher than their blue column.



Answer

'a' and 'o' are used more frequently in Spanish than English and more frequently in English than French.

Reasoning

14. a. The most common blood type is O+.

b. 9% of people can donate blood to anyone.

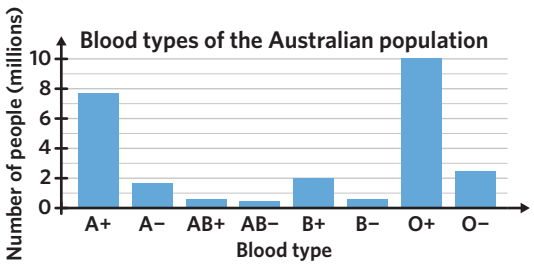
c. 19% of people have a negative blood type.

d. Suggested option 1: A pie chart is most useful as it allows us to see the percentage of one blood type relative to the other.

Suggested option 2: A column graph is most useful as it shows us the total number of people that have each blood type.

Note: There are other possible options.

15. a.



b. 500 000 Australians can receive blood donations from anyone.

c. 4 000 000 Australians can donate to a person with the blood type A−.

d. Suggested option 1: You can make an emotional appeal on how donating blood could save lives.

Suggested option 2: You can inform the person of the medical advancements that result from blood sample research.

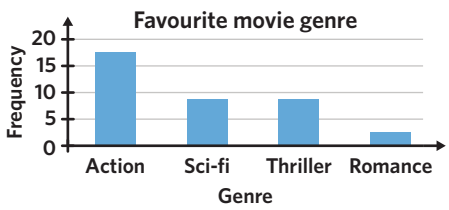
Note: There are other possible options.

Extra spicy

16. C

17. 39.38°

18.



Remember this?

19. A

20. C

21. $1284

7E Line graphs

Worked example 1.

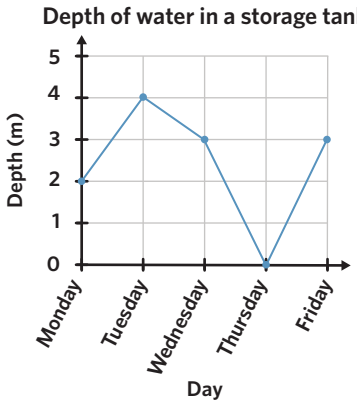
a. 100 b. 90 c. 1:00 pm d. 3 hours

Worked example 2.

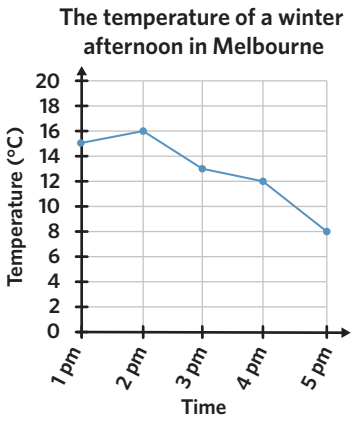
a. 60 km/h b. 90 km/h c. 1 minute d. 25 km/h

Worked example 3.

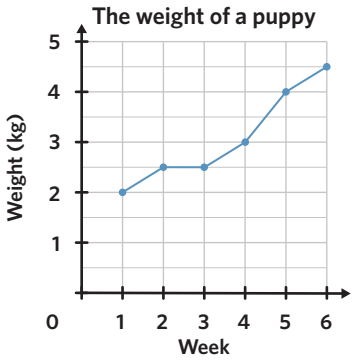
a.



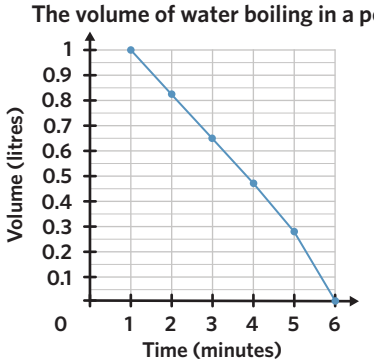
b.



c.



d.



Understanding worksheet

1. a.

Time (minutes)	0	1	2	3	4	5
Speed (km/h)	0	20	60	80	80	70

b.

Time	9 am	10 am	11 am	12 pm	1 pm	2 pm
Temperature (°C)	24	25	26	28	29	27

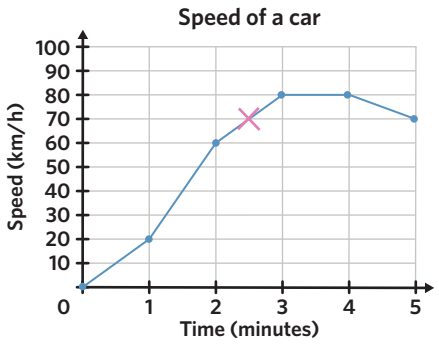
c.

Year	1900	1920	1940	1960	1980	2000
Gorillas (1000's)	120	110	90	80	50	60

d.

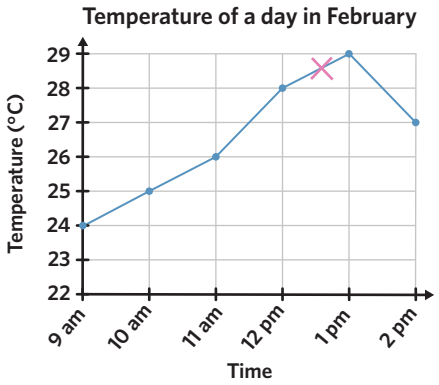
Time	9 am	10 am	11 am	12 pm	1 pm	2 pm
UV index	1	3	5.5	8	9.5	9

2. a.



The speed of the car was 70 km/h at 2 minutes and 30 seconds.

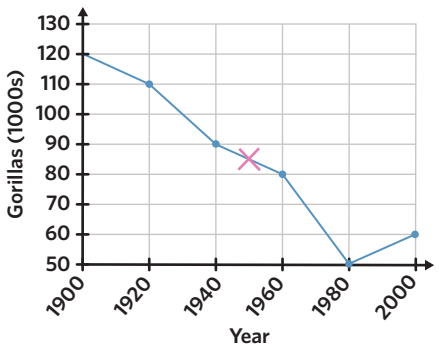
b.



The temperature at 12:30 pm was 28.5 degrees.

Note: Acceptable answer ranges from 28.4 degrees to 28.6 degrees.

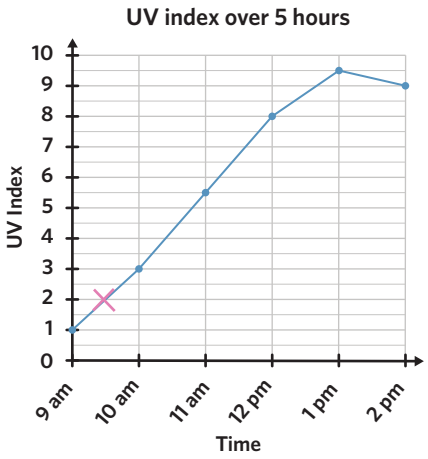
c.



In 1950, there were 85 000 gorillas.

Note: Acceptable answer ranges from 1948 to 1952.

d.



At 9:30 am, the UV index was 2.

Note: Acceptable answer ranges from 10:18 am to 10:22 am.

3. numerical; time; bivariate; coordinates; estimate

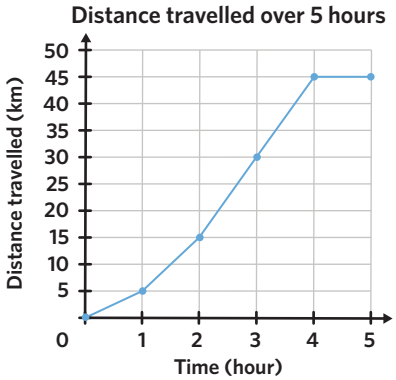
Fluency

4. a. 1 km
   b. 40 minutes
   c. 20 minutes
   d. The first 20 minutes or from 0 to 20 minutes
   e. 50 to 60 minutes
   f. 3 km/h

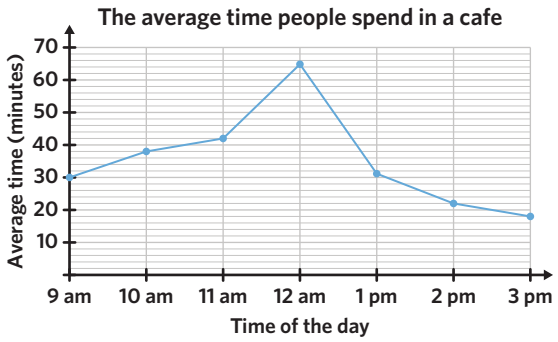
5. a. 5%
   b. 25%
   c. 13 minutes
   d. 5 minutes
   e. 50%
   f. 3 minutes

6. a. *distance travelled:* vertical axis
   time taken: horizontal axis
   b. *average daily temperature:* vertical axis
   month: horizontal axis
   c. *number of customers:* vertical axis
   time of day: horizontal axis
   d. *weight:* vertical axis
   age: horizontal axis
   e. *money spent:* vertical axis
   day of the week: horizontal axis

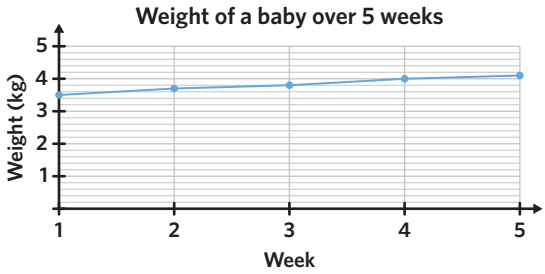
7. a.



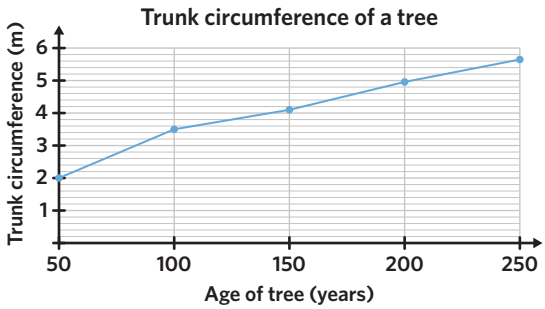
b.



c.



d.



Problem solving

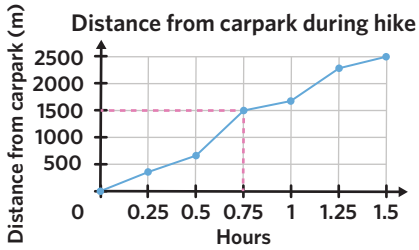
8. Max went hiking in the You Yangs mountain range. He created the following line graph to show how far he was from the carpark. How far was Max from his car after 0.75 hours?

Key points

- The line graph displays Max's distance from the carpark.
- What is the distance between Max and the carpark after he has hiked for 0.75 hours?

Explanation

Trace 0.75 hours to its corresponding distance.



Answer

Max was 1500 m from his car after 0.75 hours.

9. The following line graph shows the distance between the Sun and the Earth in million kilometres over 100 days.

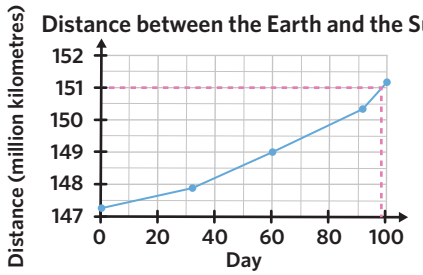
After approximately how many days does it take for the distance between the Sun and the Earth to exceed 151 million kilometres?

Key points

- The line graph displays the distance between the Sun and the Earth in a period of 100 days.
- Estimate the time it takes for the distance to reach 151 million kilometres.

Explanation

Trace 151 million kilometres to its corresponding day.



Answer

It takes approximately 98 days for the distance between the Sun and the Earth to exceed 151 million kilometres.

Note: Acceptable answer ranges from 97 days to 99 days.

10. Construct a line graph to represent the following table, which shows the amount of petrol in a delivery driver's van at the beginning of each hour.

Key points

- The table displays the data of the volume of petrol in a van.
- Construct a line graph using the data.

Explanation

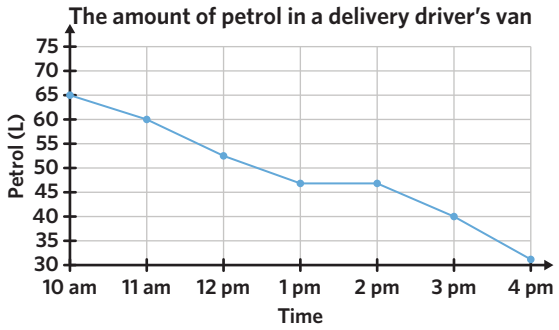
Time should be on the horizontal axis and petrol should be on the vertical axis.

The highest vertical axis value in the table is 75, so the scale on the vertical axis must go up to at least 75. An appropriate increment is 5.

Plot the points presented in the table on the graph and connect the points with straight lines.

Make sure to include a title and label the axes.

Answer



Note: There are other possible scales for the vertical axis.

11. The following graph shows the percentage of water in a storage tank for a small town on the first day of each year over a 7-year period.

For how many calendar years did the water storage percentage increase by approximately 5%?

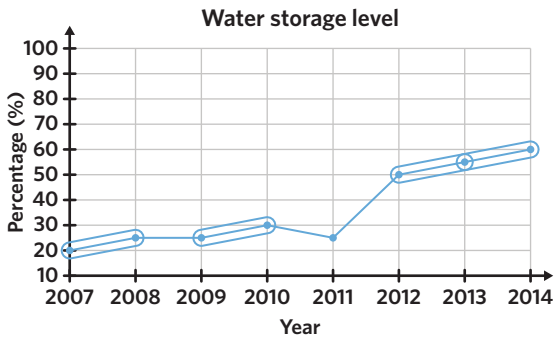
Key points

- The line graph displays the percentage of water capacity at the start of each year.
- Estimate the number of years where water capacity increases by 5%.

Explanation

We can first estimate the percentage of water available (the vertical axis value) for each of the years.

We can use these values to identify in which of the years the change was 5%.



Answer

The water storage percentage increased by approximately 5% in 4 calendar years.

12. Mikala created a line graph to show the change in tide height at her local beach in Tangalooma.

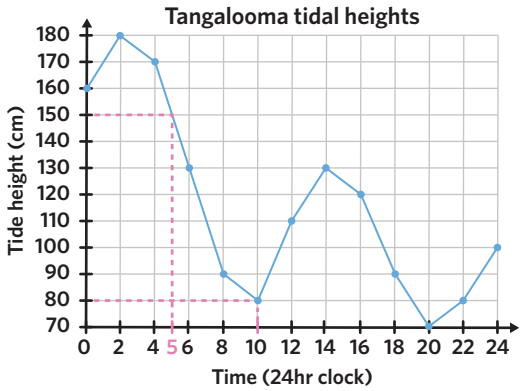
Between 5:00 am and 10:00 am, approximately how many centimetres did the tide change per hour?

Key points

- The line graph shows the change in tide height over time.
- Estimate the change in tide height between 5:00 am and 10:00 am.

Explanation

Trace to estimate the tide heights at 5:00 am and 10:00 am.



The tide heights at 5:00 am and 10:00 am are approximately 150 cm and 80 cm respectively.

Calculate the difference between the tide heights:

$150 - 80 = 70$ cm

This is the change in tide height in 5 hours.

Tide change per hour $=$ *total tide change* $\div$ *total number of hours*

$= 70 \div 5$

≈ 14 cm/h

Answer

Between 5:00 am and 10:00 am, the tide change is approximately 14 centimetres per hour.

Reasoning

13. a. Eddie consumed 2 litres of bubble tea in the first week.
    b. Brett is more successful at cutting down on his bubble tea consumption.
    c. Brett consumed approximately 11.7 litres in the first 6 weeks.
    Note: Acceptable answers range from 11.6 litres to 11.9 litres.
    d. By the end of week 12, Eddie consumed approximately 10 litres more than Brett.
    e. Suggested option 1: Brett and Eddie can consider making bubble tea themselves.
    Suggested option 2: Brett and Eddie can consider limiting the number of bubble teas they purchase each week.
    Note: There are other possible options.

14. a. Fairfield shopping centre had the most number of people at 10 am.
    b. It took 2 hours to evacuate the entire shopping centre.
    c. The number of people in Bondi shopping centre first reached 350 people at approximately 11:40 am.
    Note: Acceptable answer ranges from 11:35 am to 11:45 am.
    d. The number of people in Bondi shopping centre was greater than that of Fairfield for approximately 6 hours.
    e. Suggested option 1: Mrs Burd could consider how much profit each shopping centre makes.
    Suggested option 2: Mrs Burd could consider the location of each shopping centre and choose the one that is the furthest away from her, so that she can reduce her travel time.
    Note: There are other possible options.

Extra spicy

15. E 16. 200 L 17. 11.25 18. 11.9 seconds

Remember this?

19. 70° 20. 847.4 km 21. $\frac{3}{5}$

7F Introduction to probability

Worked example 1.

a. Rolling an 8-sided die
b. $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$
c. Rolling a 2, 4 or 8
d. Outcome: 6. The event did not occur

Worked example 2.

a. It is unlikely the spinner lands on red.
b. It is impossible for the spinner to land on purple.
c. The spinner landing on green is equally as likely as it is landing on yellow.
d. The spinner landing on blue is one-third as likely as it is landing on red.

Understanding worksheet

1. a. Unlikely
   b. Certain
   c. Unlikely
   d. Impossible

2. a. False b. False c. False d. True

3. probability; trials; outcome; sample space

Fluency

4. a. Picking a card out of a hat.
   b. $S = \{8, 9, 10, J, Q, K\}$
   c. Picking a J, Q or K
   d. Outcome: 10. The event did not occur.

5. a. i) $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
      ii) Rolling a 2, 4, 6, 8, 10 or 12
   b. i) $S = \{a, e, i, o, u\}$
      ii) Selecting o
   c. i) $S = \{red, green, blue, orange, pink, yellow\}$
      ii) Picking red, green, blue, orange, pink or yellow
   d. i) $S = \{1, 2, 4, 6, 7, 10\}$
      ii) Spinning 4 or 10
   e. i) $S = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$
      ii) Picking A
   f. i) $S = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$
      ii) Picking 2, 4, 6, 8 or 10

6. a. Unlikely b. Certain
   c. Unlikely d. Impossible
   e. Even chance f. Likely
   g. Impossible h. Even chance

7. a. Three b. Five c. Yellow d. Purple
   e. Green f. Blue

Problem solving

8. Isabelle uses a random number generator to choose a number from 1 to 20. She decides the event for her chance experiment is the random number generated being an even number. Is the event unlikely to occur, an even chance to occur, or likely to occur?

Key points
- Isabelle performs a chance experiment choosing a number from 1 and 20 using a random number generator.
- The event is even numbers.
- What is the likelihood of picking an even number?

Explanation

1 (2) 3 (4) 5 (6) 7 (8) 9 (10)
11 (12) 13 (14) 15 (16) 17 (18) 19 (20)

Numbers that satisfy the conditions of the event: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Numbers that do not satisfy the the conditions of the event: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

There are an equal amount of numbers that satisfy the conditions of the event and do not satisfy the conditions of the event.

Answer
The event has an even chance.

9. Mark is rolling a 12-sided die with the numbers 1, 1, 3, 4, 5, 5, 6, 8, 8, 10, 11, 12. What outcomes satisfy the conditions of the event if Mark is testing the probability of rolling a number greater than 6?

Key points

- Mark performs a chance experiment rolling a 12-sided die that has some repeated sides.
- The event is rolling a number greater than 6.
- Which outcomes would satisfy the condition of this event?

Explanation

Any number greater than 6 would satisfy the conditions of the event. Sides that have numbers greater than 6: 8, 10, 11, 12.

Answer

The outcomes that would satisfy the conditions of the event are 8, 10, 11, or 12.

10. Nick randomly selects a ball from one of 4 buckets. From which bucket is Nick most likely to select a black ball?

Key points

- Nick performs a chance experiment choosing a ball from four different buckets.
- Each bucket contains a different number of balls with varying colours.
- Which of the four buckets will give Nick the highest chance to pick a black ball?

Explanation

Bucket 1:
There are 2 black balls and 2 balls that are not black.
There is an even chance of picking out a black ball.

Bucket 2:
There are 3 black balls and 6 balls that are not black.
There is an unlikely chance of picking out a black ball.

Bucket 3:
There are 4 black balls and 6 balls that are not black.
There is an unlikely chance of picking out a black ball.

Bucket 4:
There are 6 black balls and 8 balls that are not black.
There is an unlikely chance of picking out a black ball.

Bucket 1 is the most likely as it has an even chance, while the rest are unlikely.

Answer

Nick will most likely select a black ball from bucket 1.

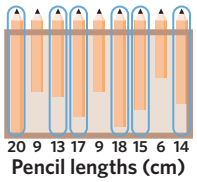
11. Jim has been asked to select one pencil out of the following pencil case. Only the tops are visible, so Jim doesn't know the length of each pencil. He selects a pencil at random. Which of the following is least likely to happen?

Key points

- Jim performs a chance experiment picking one pencil out of nine from a pencil case.
- Which of the multiple choice options has the lowest likelihood?

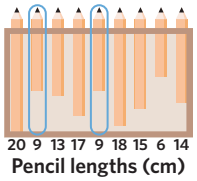
Explanation

A.



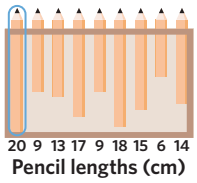
6 of the 9 pencils are longer than 10 cm, so this is likely.

B.



2 of the 9 pencils are 9 cm, so this is unlikely.

C.



1 of the 9 pencils is 20 cm, so this is unlikely.

D. There are no pencils shorter than 6 cm, so this is impossible.

Answer

D

12. Construct a spinner with the following conditions:

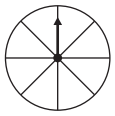
- 8 equally sized segments
- 5 colours: red, blue, orange, purple, yellow
- An even chance of landing on purple.

Key points

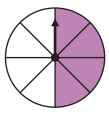
- The spinner has 8 equally sized segments.
- The spinner has 5 colours: red, blue, orange, purple, yellow.
- The spinner has an even chance of landing on purple.
- Create the spinner.

Explanation

Draw a spinner without any colours and split it into 8 equal segments.



There is an even chance of landing on purple. As there are 8 segments, there must be 4 purple segments and 4 segments that are not purple.



There are 4 colours that are not purple that need to also be on the spinner, so each will have 1 segment.

Answer

Note: The colours can be arranged differently.

Reasoning

13. a. C

b. The event in which Johnny wins occurs if any ticket is drawn that is labelled from 1 to 5.

It is unlikely that Johnny will win.

c. The event where neither Johnny nor Ella wins occurs if a black ticket that is labelled from 6 to 25 is drawn.

It is unlikely that neither Johnny nor Ella wins.

d. Suggested option 1: The organisers could hand out brochures and paste posters at public venues to advertise the event.

Suggested option 2: The organisers could advertise on social media platforms.

Note: There are other possible options.

---

14. a. There are 52 possible outcomes.

b. Even chance

c. The likelihood does not change.

d. The likelihood has changed from even to likely.

e. Suggested option 1: There are more cards that are not J, Q or K.

Suggested option 2: Even though the J, Q and K are Jazzy's favourite cards, it does not make them more likely to be selected.

Note: There are other possible options.

Extra spicy

15. B 16. 5 17. E

18. squares: 2; circles: 6; diamonds: 2; triangles: 1; hexagons: 1.

Remember this?

19. C 20. D 21. B

7G Probability of events

Worked example 1.

a. 1 b. 0 c. 0 d. 1

Worked example 2.

a. $\frac{3}{10}$ b. 6 c. 0.5 d. 90

Worked example 3.

a. $\frac{9}{20}$ b. Equal to c. 0.3 d. Greater than

Understanding worksheet

1. a. Black balls: 3 Total balls: 10

Fraction: $\frac{3}{10}$

b. Black cards: 5 Total cards: 8

Fraction: $\frac{5}{8}$

c. Orange balls: 3 Total balls: 11

Fraction: $\frac{3}{11}$

d. Diamond or clubs: 8 Total cards: 15

Fraction: $\frac{8}{15}$

2. a. Expected occurrence: $4 \times \frac{1}{4} = 1$

b. Expected occurrence: $5 \times \frac{3}{5} = 3$

c. Expected occurrence: $20 \times \frac{3}{10} = 6$

d. Expected occurrence: $18 \times \frac{2}{9} = 4$

---

3. theoretical; one; experimental; zero; certain

Fluency

4. a. 0 b. 1 c. 0 d. 0
e. 1 f. 1

---

5. a. $\frac{1}{8}$ b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. $\frac{3}{8}$
e. $\frac{5}{8}$ f. $\frac{3}{4}$ g. $\frac{7}{16}$ h. $\frac{5}{16}$

---

6. a. 3 b. 25 c. 0 d. 45
e. 80 f. 180

---

7. a. 0.8 b. 0.25 c. 0.7 d. 0.6
e. 0.44 f. 0.53

---

8. a. Greater than b. Equal to
c. Equal to d. Less than
e. Greater than f. Less than

Problem solving

9. Louis is about to pick the name of a horse running in the Melbourne Cup randomly out of a hat. There are 20 horses running in the race. What is the theoretical probability, as a percentage, that Louis picks the horse that wins the Melbourne Cup?

Key points

- Louis picks the name of one horse out of a hat.
- There are 20 horses in the race.
- Determine the theoretical probability of Louis picking the winning horse.

Explanation

Louis is picking randomly out of a hat, so he has an equal chance of picking each horse in the race.

Horses picked: 1

Horses running: 20

$Pr(win) = \frac{1}{20}$

$= 5\%$

Answer

The theoretical probability of Louis picking the winning horse is 5%.

---

10. A standard dartboard has a total area of 1500 cm^2. The coloured areas cover 240 cm^2 of the board. Belle throws 150 darts at random on a standard dart board. How many times is it expected that a dart lands on a coloured area?

Key points

- A standard dartboard has a total area of 1500 cm^2.
- The coloured areas cover 240 cm^2 of the board.
- Belle throws 150 darts at random on a standard dart board.
- What is the expected occurrence of landing on a coloured area?

Explanation

Determine the theoretical probability of a dart landing on a coloured area. As Bella throws the darts at random, there is an equal chance of each dart landing on any point on the board.

Coloured area: 240 cm^2

Total area: 1500 cm^2

$Pr(coloured) = \frac{240}{1500}$

$= \frac{24}{150}$

$= \frac{4}{25}$

Calculate the expected occurrence.

$150 \times \frac{4}{25} = \frac{600}{25}$

$= 24$

Answer

It is expected that 24 darts will land on a coloured area.

11. Use the following AFL ladder to calculate the experimental probability, as a decimal, of the Collingwood Magpies winning a game.

Position	Team	Wins	Losses	Draws
1	Port Adelaide Power	17	3	0
2	Brisbane Lions	16	3	1
3	Richmond Tigers	15	4	1
4	Geelong Cats	15	5	0
5	West Coast Eagles	15	5	0
6	St Kilda Saints	14	6	0
7	Western Bulldogs	12	8	0
8	Collingwood Magpies	11	8	1

Key points

- An AFL ladder is shown.
- Collingwood won 11 games, lost 8 games, and had 1 draw.
- What is the experimental probability of Collingwood winning a game?

Explanation

Find the number of games Collingwood won and the number of games Collingwood played.

Wins: 11

Games played: $11 + 8 + 1 = 20$

$experimental\ probability = \frac{11}{20}$

$= 0.55$

Answer

The experimental probability of the Collingwood Magpies winning a game is 0.55.

12. The Brisbane Lions played 10 out of their 20 games at their home ground. The theoretical probability of the Lions winning a game at home is 0.8 and the theoretical probability of the Lions winning a game away is 0.5.

Use the AFL ladder in the previous question to determine whether the Lions won more or less games than expected.

Key points

- An AFL ladder is shown.
- The Brisbane Lions played 10 out of their 20 games at home.
- The theoretical probability of the Lions winning a game at home is 0.8.
- The theoretical probability of the Lions winning a game away is 0.5.
- Determine whether the Lions won more or less games than expected.

Explanation

Expected wins at home:

$10 \times 0.8 = 8$

Expected wins away:

$10 \times 0.5 = 5$

Total expected wins:

$8 + 5 = 13$

The Brisbane Lions won 16 games, which is greater than the 13 expected wins.

Answer

The Brisbane Lions won more games than expected.

13. The local surf lifesaving club is hosting a raffle. There are 6 colours of tickets, each having tickets numbered from 1 to 40. Tom has bought 36 raffle tickets, 16 of which are blue and 20 of which are red. The winning ticket is blue. What is the theoretical probability that Tom has won the raffle prize?

Key points

- There are 40 raffle tickets for each colour.
- Tom has bought 16 blue raffle tickets.
- The winning ticket is blue.
- What is the theoretical probability of Tom winning?

Explanation

The winning ticket is blue, so Tom's red tickets can be ignored.

Blue tickets bought: 16

Total number of blue tickets: 40

$Pr(win) = \frac{16}{40}$

$= \frac{4}{10}$

$= 0.4$

Answer

The theoretical probability of Tom winning the raffle prize is 0.4.

Reasoning

14. a. No, an experimental probability of 0 does not mean the event is impossible.

b. The experimental probability of rolling an odd number is 0.66.

c. The experimental probability should decrease.

d. Suggested option 1: Theoretical probability is a better measurement as it is the true probability of an event occurring.

Suggested option 2: Experimental probability is a better measurement as it is based on data that has been collected.

Note: There are other possible options.

15. a. The theoretical probability that a winner gets a prize worth more than \$1000 is $\frac{5}{11}$.

b. In 110 spins, 70 winners are expected to receive a prize worth \$5000 or less.

c. Yes, there is reason to believe that the Wheel of Fortune is rigged.

d. Suggested option 1: I would rather $1000 as I am not certain to win more money by spinning the Wheel of Fortune.

Suggested option 2: I would rather spin the Wheel of Fortune as some of the prizes are worth much more than $1000.

Note: There are other possible options.

Extra spicy

16. D 17. C 18. A 19. $\frac{2}{7}$

Remember this?

20. D 21. B 22. 5 miles

Chapter 7 extended application

1. a. China emitted the most amount of CO_2 and Indonesia emitted the least amount of CO_2.

The difference in their emission is 10.0805 billions of tonnes.

b. All top 10 countries in the chart emit altogether 23.63197 billions of tonnes of CO_2 in 2020.

c. We can attribute 67.89% of the whole world's emission to the top 10 CO_2 emitters in the chart.

d. China's per capita or average CO_2 emissions attributed to each individual in China in 2020 is 7.61 tonnes.

e. Suggested option 1: Yes, this is a realistic statement because about 60% of the world population live in Asia.

Suggested option 2: No, this is not a realistic statement because non-asian countries such as the United States are also top contributors of CO_2 emissions in the world.

Note: There are other possible options.

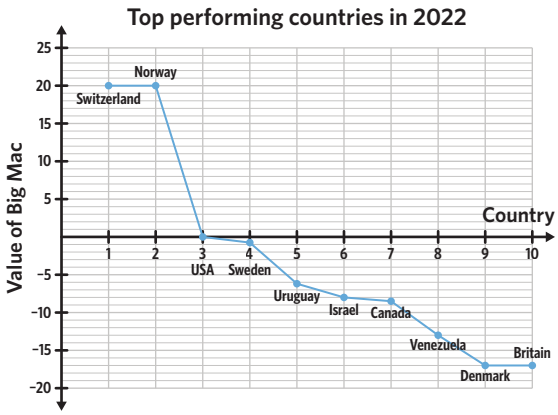
2. a. Switzerland has the most valuable currency. The Big Mac is 20.2% overvalued there compared to the USA.

b. Switzerland and Norway's currencies are more valuable than that of the USA.

c. Israel has the currency worth 92.1% of US currency, and its currency is −7.9%.

d. The value of the Australian dollar is 0.776 compared to the US dollar, according to the Big Mac index.

e.



f. Suggested option 1: Yes, it is a good way to compare the wealth of countries because almost every country has Big Mac which makes the data comparable.

Suggested option 2: No, it is not a good way to compare the wealth of countries because Big Mac is only one of many data available that indicates the wealth of a country. We might get a biased result by using only one specific data from Big Mac.

Note: There are other possible options.

3. a. The below table shows the converted percentage for the probability of each eye colour.

Eye colour	Brown	Blue	Hazel	Amber	Grey	Green	Other
Probability of occurrence	75%	8%	5%	5%	3%	2%	2%

b. This baby is most likely to develop brown eyes as an adult because it is the eye colour that has the highest percentage.

c. Out of a group of 10 randomly chosen people, approximately 1 person is expected to have blue eyes.

Eye colour of students in the class	Brown	Blue	Hazel	Amber	Grey	Green	Other
Number of students	20	5	0	0	2	2	0

d. The probability of each eye colour in Mr Venkmann's maths class.

Eye colour of students in the class	Brown	Blue	Hazel	Amber	Grey	Green	Other
Number of students %	68.97	17.24	0	0	6.90	6.90	0

The differences between the two sets of data are that there are more percentages of blue eyed, grey eyed, and green eyed people than the universal data. There are less percentages of brown eyed, hazel eyed, amber eyed and other eye colours in Mr Venkmann's maths class compared to the universal data.

The similarities are that brown-eyed people are the majority in both cohorts, followed by blue-eyed people. The rare eye colours in both sets are hazel, amber, grey, green and other colours.

Some reasons for the discrepancy in data could be that the demographic in Mr Venkmann's maths class is not representative of the universal demographic. In addition, the difference in data could also be chance as it is only one set of data.

e. Suggested option 1: An advantage of this technology is that it allows more accurate and more secure identification than fingerprints.

Suggested option 2: A disadvantage of this technology is that it could have implications regarding personal privacy from data collection and identification.

Note: There are other possible options.

Chapter 7 review

Multiple choice

1. C 2. A 3. B 4. D 5. A

Fluency

6. a. Discrete numerical b. Categorical
c. Continuous numerical d. Continuous numerical

7. a. 18 b. 158 c. 295 d. 299.5

8. a. 11 b. 6 c. 62.6 d. 6.8

9. a.

Stem	Leaf
1	1
2	1 9
3	0 5 5
4	4 9
5	9
6	0 3 8

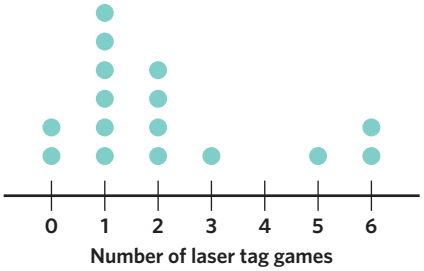
Key: 1 | 0 = 10 sheep

b. 6 paddocks

c. 35

d. 57

10. a. Number of times students have played laser tag

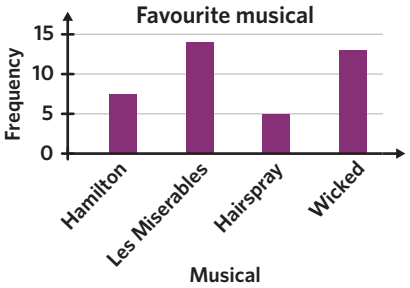


b. 1 game

c. 2 students

d. 50%

11. a.

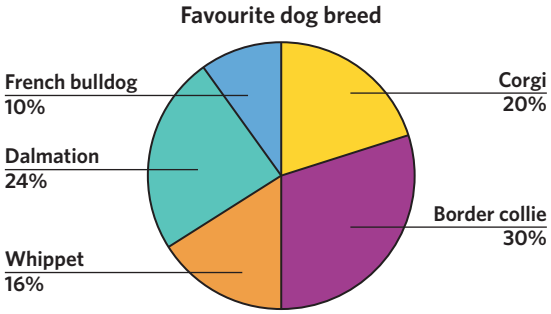


b. Les Misérables

c. 8

d. 20%

12. a.



b. Border collie

c. 24%

d. 14%

13. a. 120 km b. 250 km c. 3 breaks d. 20 km

14. a. Unlikely b. Even c. Certain d. Likely

15. a. $\frac{1}{4}$ b. $\frac{1}{13}$ c. $\frac{5}{13}$ d. $\frac{3}{26}$

16. a. Less than b. Greater than
c. Less than d. Equal to

Problem solving

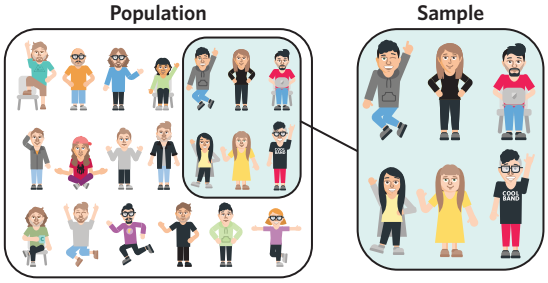
17. A group of scientists are researching the effectiveness of The Voice Australia in launching music careers for its contestants. They randomly select 20 past contestants from The Voice Australia to interview. What is the population and sample of this experiment?

Key points
- 20 past contestants are randomly selected to interview.
- What is the population and sample of this experiment?

Explanation

A population is the group you want to apply your conclusions to.

A sample is a smaller group within the population.



Answer

The population is all past contestants from The Voice Australia.

The sample is the 20 randomly chosen past contestants.

18. The mean height of the 8 players on a basketball team is 178 cm. By how much would the mean height increase if a new player with a height of 196 cm joined the team?

Key points
- The current mean height of the 8 players is 178 cm.
- A new player with a height of 196 cm joined the team.
- By how much would the mean height increase?

Explanation

$mean = \frac{sum\ of\ data\ values}{number\ of\ values}$

The original mean was 178 cm and there are 8 players (values). Substitute these into the equation to calculate the original sum of data values.

$178 = \frac{sum\ of\ data\ values}{8}$

$178 \times 8 = \frac{sum\ of\ data\ values}{8} \times 8$

$sum\ of\ data\ values = 1424$

When the new player joins the new sum of data values is $1424 + 196 = 1620$.

There are now 9 players so there are 9 data values.

$mean = \frac{1620}{9} = 180$

The mean is 180.

$Increase\ in\ mean = 180 - 178$

$= 2$ cm

Answer

The mean height would increase by 2 cm.

19. The following column graph shows survey results from Australians and New Zealanders on their favourite desserts.

How many more New Zealanders favour Pavlova compared to Australians?

Key points

- Column graph shows survey results from Australian and New Zealanders on their favourite desserts.
- How many more New Zealanders favour Pavlova compared to Australians?

Explanation

The column representing New Zealanders who favour Pavlova has a height of 22.

The column representing Australians who favour Pavlova has a height of 25.

$25 - 22 = 3$

Therefore 3 more New Zealanders than Australians favour Pavlova.

Answer

3 more New Zealanders favour Pavlova compared to Australians.

20. Effy has recorded the number of novels she has read each year for the past 20 years. She displayed her data in the following stem-and-leaf plot before realising that a dot plot may be more appropriate. Construct a dot plot to display this data.

Key points

- Effy recorded the number of novels she has read each year for the past 20 years.
- The data is displayed in a stem-and-leaf plot.
- Construct a dot plot to display this data.

Explanation

First count the frequency of each number of novels.

7 novels: 4

9 novels: 3

10 novels: 4

11 novels: 2

12 novels: 3

13 novels: 1

14 novels: 2

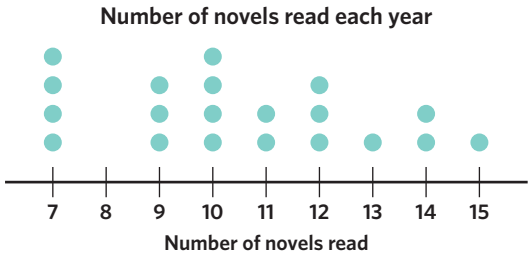
15 novels: 1

Draw a horizontal line and label the numbers from 7 to 15.

The number of dots in each category is equal to its frequency.

Make sure to include a title and label the horizontal axis.

Answer



21. Luther and Herbert decided to take on a fitness challenge where they had to walk 150 km in one week. Luther completed the following table showing the total number of kilometres he had travelled by the end of each day.

Use the information to construct a line graph showing Luther's progression through the fitness challenge.

Key points

- The table shows the total number of kilometres travelled by Luther by the end of each day.
- Construct a line graph.

Explanation

'Day' should be on the horizontal axis and 'Total distance travelled by Luther' should be on the vertical axis.

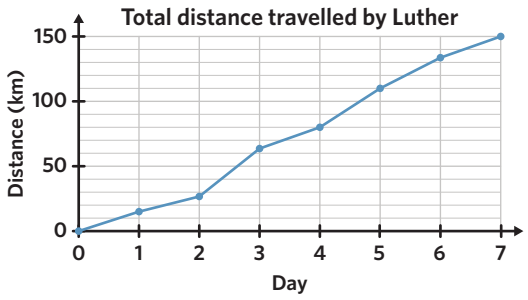
The highest vertical axis value in the table is 150, so the scale on the vertical axis must go up to at least 150.

An appropriate increment is 10.

Plot the points presented in the table on the graph and connect the points with straight lines.

Make sure to include a title and label the axes.

Answer



22. Construct a spinner with the following conditions:

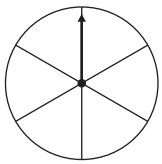
- 6 even segments
- 5 colours: red, blue, green, yellow and purple
- landing on blue is twice as likely as landing on green.

Key points

- There are 6 equal segments on the spinner.
- There are 5 colours: red, blue, green, yellow and purple.
- Blue is twice as likely as green.

Explanation

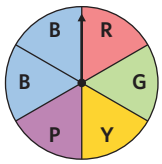
Draw a spinner without any colours, and split it into 8 equal segments.



Blue is twice as likely as green. This means there must be 2 times as many blue segments as green.

Since there are only 6 segments, there will be 2 blue segments and 1 segment for each other colour.

Answer



23. When Ben Simmons shoots a three-point shot in basketball, he has a 15% chance of it going in. If Ben Simmons shoots 400 three-point shots at training, how many are expected to go in?

Key points

- Ben Simmons has a 15% chance of the basketball going in when he shoots a three-point shot.
- Ben shoots 400 three-point shots at training.
- How many are expected to go in?

Explanation

The theoretical probability of it going in is 15%.

Calculate the expected occurrence of it going in if Ben Simmons shoots 400 three-point shots.

$15\% \times 400 = \frac{15}{100} \times 400 = 60$

Answer

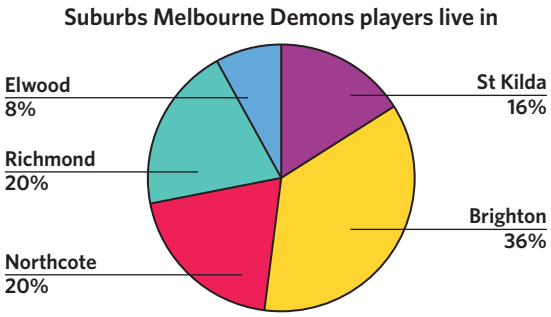
Ben Simmons is expected to make 60 three-point shots.

Reasoning

24. a. 8 seasons

b. 11^{th}

c.



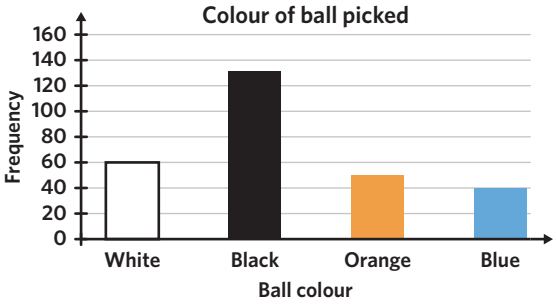
d. Suggested option 1: Drafting talented young players.

Suggested option 2: Training in the offseason.

Note: There are other possible options.

25. a. Categorical

b.



c. Unlikely

d. Less than

e. Suggested option 1: Experimental probability should be reliable after 100 trials.

Suggested option 2: Experimental probability should be reliable after 1000 trials.

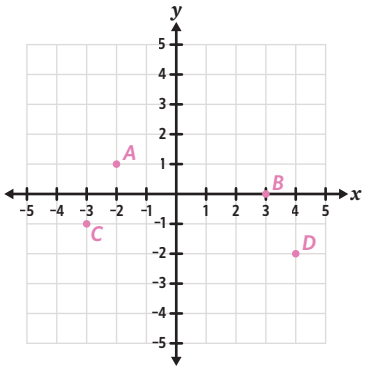
Note: There are other possible options.

8A Integers on the Cartesian plane

Worked example 1.

A: (4,2) *B*: (1,2) *C*: (−3,1) *D*: (3,−2)

Worked example 2.



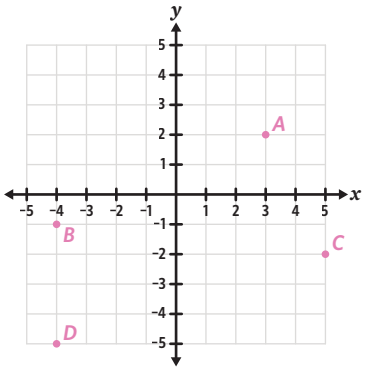
Worked example 3.

a. Quadrant 1 b. *x*-axis c. *y*-axis d. Quadrant 3

Understanding worksheet

1. a. $A(3,1)$; $B(-1,4)$; $C(-1,-2)$
   b. $A(1,1)$; $B(-1,1)$; $C(-1,-1)$; $D(1,-1)$
   c. $A(3,4)$; $B(-4,4)$; $C(-4,-1)$; $D(3,-1)$
   d. $A(0,3)$; $B(-2,0)$; $C(0,-3)$; $D(2,0)$

2.



A(3,2)
Quadrant 1

B(−4,−1)
Quadrant 3

C(5,−2)
Quadrant 4

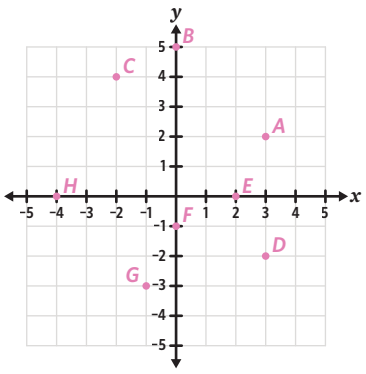
D(−4,−5)
Quadrant 3

3. number plane; horizontal; vertical; origin; Cartesian coordinates

Fluency

4. a. (−2,1) b. (1,2) c. (−1,−1) d. (−5,0)
   e. (2,0) f. (1,−1) g. (0,4) h. (0,−3)

5.



6. a. Quadrant 1 b. Quadrant 4 c. Quadrant 2
   d. Quadrant 3 e. Quadrant 4 f. Quadrant 3

7. a. *y*-axis b. *x*-axis c. *x*-axis
   d. *y*-axis e. Origin f. *y*-axis

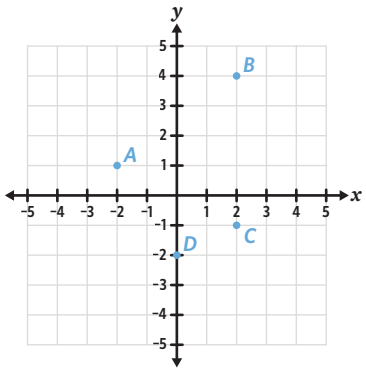
Problem solving

8. Hamish is plotting points on a Cartesian plane. He plots the following points: $A(-2,1)$, $B(2,4)$, $C(2,-1)$, $D(0,-2)$. Which point is the highest on the plane?

Key points

- Hamish plots the points $A(-2,1)$, $B(2,4)$, $C(2,-1)$, $D(0,-2)$.
- Which point is the highest on the plane?

Explanation



The 'height' of a point on a plane is the value of the *y*-coordinate. The largest *y*-coordinate in the points is 4, which is point *B*.

Answer

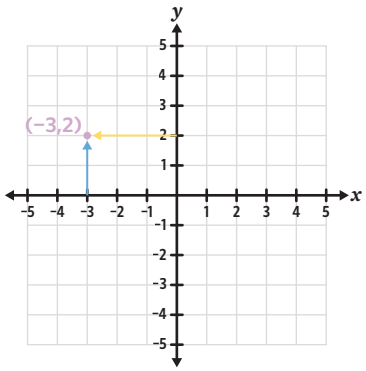
The highest point on the plane is point *B*.

9. What are the Cartesian coordinates that answer Joanna's math riddle? The riddle says: 'You can find me 3 units to the left and 2 units above the intersection of the *x* and *y* axes'.

Key points

- The point is 3 units to the left of the origin.
- The point is 2 units above the origin.
- Find the Cartesian coordinates of the point.

Explanation



The 'intersection of the two axes' is the origin.

'3 units to the left' of the origin means that the point has an *x*-coordinate of −3.

'2 units to above' the origin means that the point has an *y*-coordinate of 2.

Answer

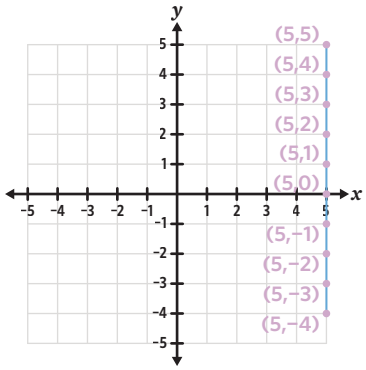
The Cartesian coordinates that answer Joanna's math riddle are (−3,2).

10. Stanley is drawing a shape on the Cartesian plane with both axes labelled from −5 to 5. He starts by drawing a vertical line up from the point (5,−4). What are all the coordinates of points that lie on the line?

Key points

- Stanley is drawing on a Cartesian plane with both axes labelled from −5 to 5.
- He draws a vertical line.
- The lowest point of the line is (5,−4).
- Find all the coordinates of the points on the line.

Explanation



As the line is vertical, all the points will have the same x-coordinate, and only the y-coordinate will change.

The line starts at the point (5,−4), so all the points will have an x-coordinate of 5.

The line is drawn up, so all the points on the line will have y-coordinates from −4 to 5.

Answer

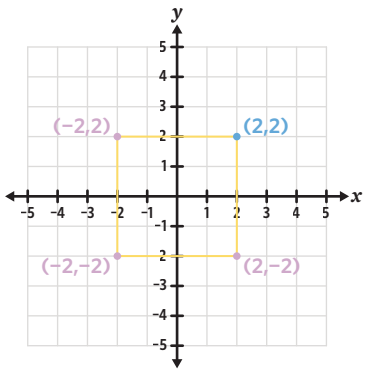
The coordinates of the points on the line are: (5,−4), (5,−3), (5,−2), (5,−1), (5,0), (5,1), (5,2), (5,3), (5,4), (5,5).

11. Sally draws a square with its centre at the origin of the Cartesian plane. If one of the vertices is situated at (2,2), what are the coordinates of the other three vertices of the square?

Key points

- Sally draws a square with its centre at the origin of the Cartesian plane.
- One of the vertices is situated at (2,2).
- What are the coordinates of the other 3 corners?

Explanation



As the square is centred around the origin, all the vertices will have x and y-coordinates that are different combinations of 2 and −2.

The four coordinates are: (2,2), (−2,2), (2,−2), (−2,−2).

Answer

The coordinates of the three other vertices are (−2,2), (2,−2) and (−2,−2).

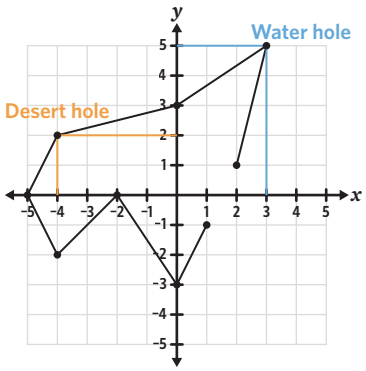
12. Jacob is redesigning the Albert Park mini golf course. He uses the following Cartesian plane to create the new 9 hole layout.

What are the coordinates of the desert hole and the water hole?

Key points

- The 9 hole layout is shown on the Cartesian plane.
- Find the coordinates of the desert hole and the water hole.

Explanation



The desert hole corresponds to −4 on the x-axis and 2 on the y-axis.

The water hole corresponds to 3 on the x-axis and 5 on the y-axis.

Answer

The coordinates of the desert hole are (−4,2) and the coordinates of the water hole are (3,5).

Reasoning

13. a. The letters on the x-axis are N, O, P, Q and R.

b. ATTACK

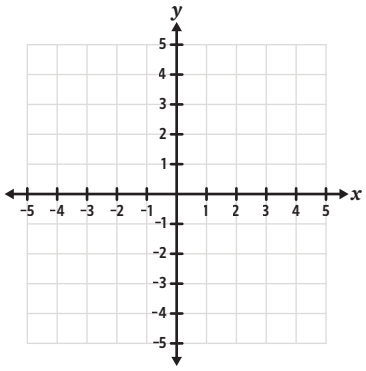
c. (−2,0), (1,3), (1,3), (−2,2), (2,−1), (0,3), (2,2), (1,3), (2,0)

d. Suggested option 1: Using the Cartesian coordinates is better as the code can be sent using letters.

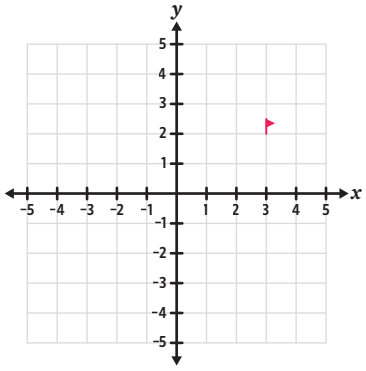
Suggested option 2: Using the SOS signals is better as it is harder to be intercepted by enemies.

Note: There are other possible options.

14. a.



b.



c. Quadrant 2

d. Suggested option 1: The directions could be written in terms of 'north', 'south', 'east' and 'west'.

Suggested option 2: The directions can still be written using 'forwards', 'backwards', 'right' and 'left', but they need to specify which way to face.

Note: There are other possible options.

Extra spicy

15. C 16. $\frac{22}{3}$ cm 17. 32 cm^2 18. D

Remember this?

19. A 20. D 21. C

8B Symmetry and reflections

Worked example 1.

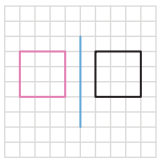
a. Order of line symmetry: 3

Order of rotational symmetry: 3

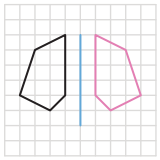
b. Order of line symmetry: 1

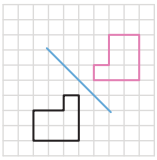
Order of rotational symmetry: 1

Worked example 2.

a. 

b. 

c. 

d. 

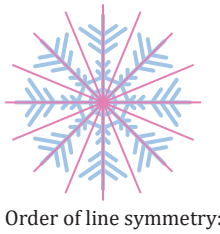
Worked example 3.

a. (3,−4) b. (5,1) c. (4,−2) d. (2,−4)

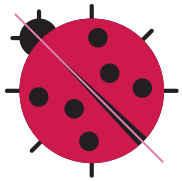
Understanding worksheet

1. a. 

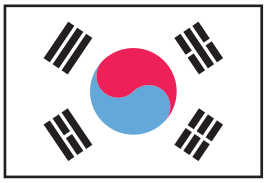
Order of line symmetry: 1

b. 

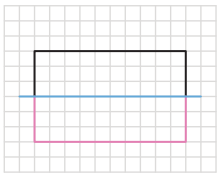
Order of line symmetry: 8

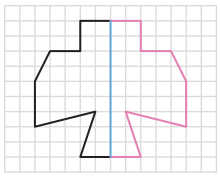
c. 

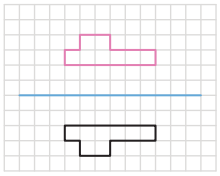
Order of line symmetry: 1

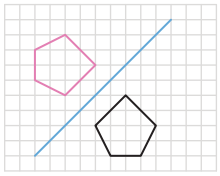
d. 

Order of line symmetry: 0

2. a. 

b. 

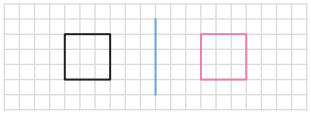
c. 

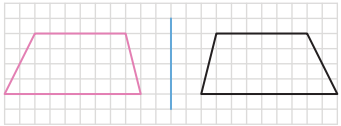
d. 

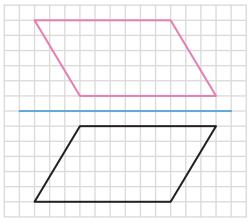
3. line; symmetry; 360°; order

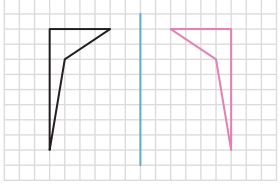
Fluency

4. a. Order of line symmetry: 6; Order of rotational symmetry: 6
   b. Order of line symmetry: 0; Order of rotational symmetry: 1
   c. Order of line symmetry: 1; Order of rotational symmetry: 1
   d. Order of line symmetry: 0; Order of rotational symmetry: 1
   e. Order of line symmetry: 8; Order of rotational symmetry: 8
   f. Order of line symmetry: 2; Order of rotational symmetry: 2
   g. Order of line symmetry: 0; Order of rotational symmetry: 1
   h. Order of line symmetry: 6; Order of rotational symmetry: 6

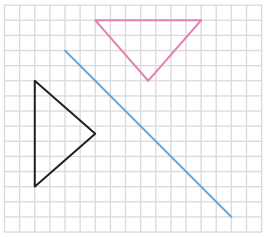
5. a. 

b. 

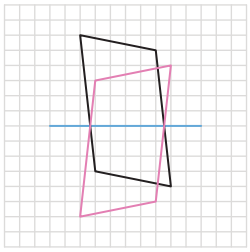
c. 

d. 

e.



f.



6. a. (3,−1) b. (4,1) c. (−2,−2) d. (−1,−4)
e. (−2,4) f. (0,−2)

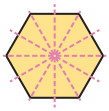
7. a. (−2,4) b. (3,1) c. (−2,0) d. (4,−5)
e. (−1,−6) f. (0,2)

Problem solving

8. The top of Cheryl's jewellery case is a plain solid gold regular hexagon. How many lines of symmetry does the top of the case have?

Key points
- The top of Cheryl's jewellery case is shaped as a regular hexagon.
- What is the order of line symmetry of a regular hexagon?

Explanation
Find the number of lines that divide the shape into two identical parts.



Answer
The top of Cheryl's jewellery case has 6 lines of symmetry.

9. What is the order of rotational symmetry of Lucia's t-shirt if she has a standard t-shirt with a happy face emoji on the front?

Key points
- Lucia has a standard t-shirt with a happy face emoji on the front.
- Find the order of rotational symmetry of the t-shirt.

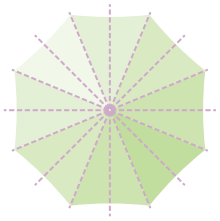
Explanation
Find the number of times the t-shirt looks the same when it is rotated 360 degrees.

Answer
The order of rotational symmetry of Lucia's t-shirt is 1.

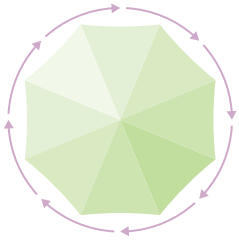
10. What is the order of line symmetry and rotational symmetry of the top of a standard open umbrella with 8 sides?

Key points
- The umbrella has 8 sides of equal length.
- Find the order of line symmetry and rotational symmetry of the top of the umbrella.

Explanation
Find the number of lines that divide the shape into two identical parts.



Find the number of times the top of the umbrella looks the same when it is rotated 360 degrees.



Answer
Both the order of line symmetry and rotational symmetry of the umbrella is 8.

11. Shawn is reading a secret message that has been reflected horizontally over a mirror line. How does the message appear before it is reflected if the message reads 'TODAY'?

Key points
- A message has been reflected horizontally over a mirror line.
- The reflected message reads 'TODAY'.
- What did the message look like before it was reflected?

Explanation
Reflecting 'TODAY' horizontally will reverse the order of the letters. Each of the individual letters will also appear reflected, however this will only impact the letter 'D'.

YAOOT | TODAY

Answer
YAOOT

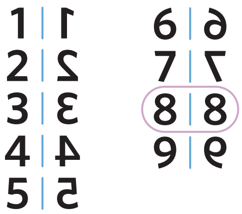
12. Sierra is finding all the numbers from 0 to 9 that look the same when they are reflected over a vertical line. So far, she has the number 0. What is the other number that looks the same when reflected over a vertical line?

Key points
- 0 looks the same when reflected over a vertical line.
- Which other digit also looks the same?

Back to contents

Explanation

Reflecting over a vertical line is the same as reflecting horizontally.



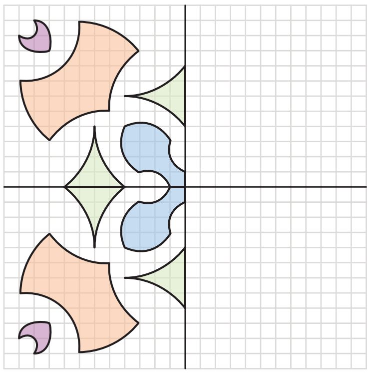
Answer

8 also looks the same when reflected over a vertical line.

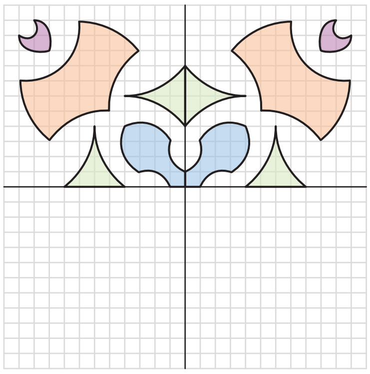
Reasoning

13. a. F, G, J, K and L are asymmetrical.
    b. H and I have an order of line symmetry greater than 1.
    c. H and I have an order of rotational symmetry greater than 1.
    d. Suggested option 1: Radar.
       Suggested option 2: Kayak.
       Note: There are other possible options.

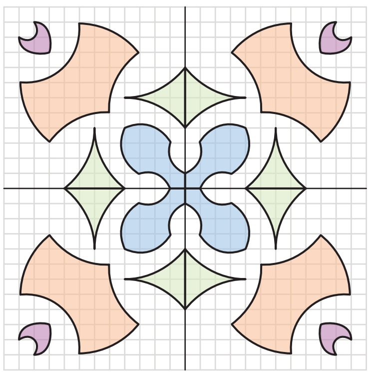
14. a.



b.



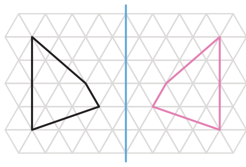
c.



d. Any shape or image that is reflected vertically and then horizontally (or vice versa) is a kaleidoscope image.

Extra spicy

15.



16. B

17. E

18. B

Remember this?

19. C

20. $19.20

21. B

8C Translations

Worked example 1.

a. 4 units left and 3 units up
b. 3 units left and 7 units down
c. 4 units right and 8 units up
d. 2 units left and 4 units up

Worked example 2.

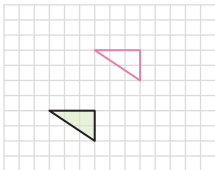
a. (−1,1) b. (−6,4) c. (−2,4) d. (−7,5)

Worked example 3.

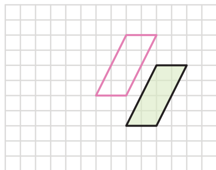
a. (0,6) b. (6,3) c. (−1,7) d. (−3,2)

Understanding worksheet

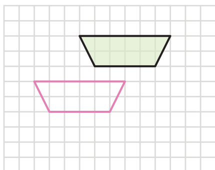
1. a.



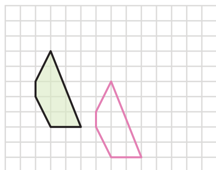
b.



c.



d.



2. a. 5 units left and 3 units down
   b. 3 units left and 3 units up
   c. 5 units left and 4 units down
   d. 6 units right and 7 units down

3. size; translation; original, orientation

Fluency

4. a. 2 units right and 4 units up
   b. 3 units right and 6 units down
   c. 6 units right and 4 units down
   d. 2 units left and 4 units down
   e. 4 units left and 4 units down
   f. 6 units down

5. a. 2 units left and 3 units down
   b. 2 units right and 3 units up
   c. 2 units right and 2 units down
   d. 1 unit left and 3 units down
   e. 5 units left and 2 units down
   f. 4 units right and 5 units up

6. a. (5,5) b. (4,7) c. (−1,2) d. (5,9)
e. (1,6) f. (6,0) g. (0,3) h. (5,6)

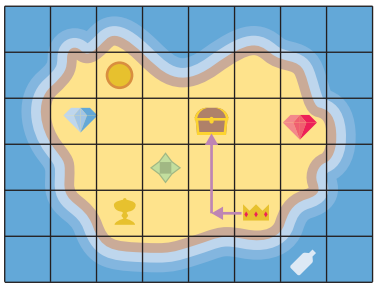
7. a. (3,−1) b. (0,1) c. (6,4) d. (7,0)
e. (1,4) f. (8,6) g. (6,5) h. (0,−1)

Problem solving

8. Brennan found a treasure map and is trying to find all the treasure. Describe the translation if Brennan moves from the crown's location to the treasure chest.

Key points
- A treasure map is shown.
- Brennan moves from the crown's location to the treasure chest.
- Describe the translation with each square on the map being one unit.

Explanation



Horizontal translation: 1 unit left

Vertical translation: 2 units up

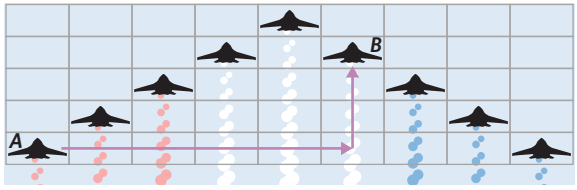
Answer

The translation is 1 unit left and 2 units up.

9. The planes in an airshow form a special formation and are positioned one plane width and one plane length apart. How many plane widths and lengths does plane *A* need to move to reposition to plane *B*'s position?

Key points
- An image of the airshow is shown.
- The planes are positioned one plane width and one plane length apart.
- Describe the movements for plane *A* to get to plane *B* using plane lengths and widths.

Explanation



Horizontal translation: 5 plane widths right

Vertical translation: 3 plane lengths up

Answer

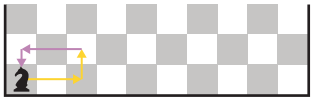
Plane *A* needs to move 5 plane widths right and 3 plane lengths up to reposition to plane *B*'s position.

10. Warren moves a knight from the bottom left of the chessboard two squares to the right and one square up. How should Warren move the knight so that it returns to its original position?

Key points
- Warren moves a knight two squares to the right and one square up.
- Describe the translation back to the original position.

Explanation

The reverse translation of 2 units right and 1 unit up is 2 units left and 1 unit down.



Answer

Warren should move the knight 2 units to the left and 1 unit down.

11. James is using a Cartesian plane to rearrange his room. What are the new coordinates of James's TV if his TV is currently at (2,2) on the Cartesian plane, and he moves it 6 units to the right and 1 unit down?

Key points
- James's TV is currently at (2,2) on the Cartesian plane.
- He moves it 6 units to the right and 1 unit down.
- Find the translated coordinates of the TV.

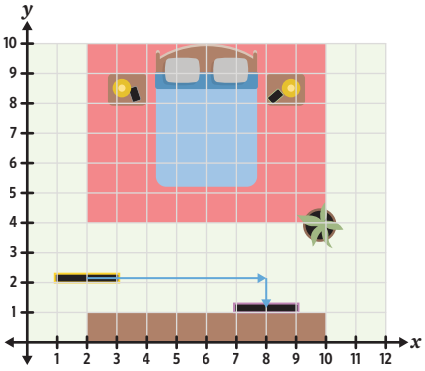
Explanation

A translation of 6 units right will increase the x-coordinate of the TV by 6.

$2 + 6 = 8$

A translation of 1 unit down will decrease the y-coordinate of the TV by 1.

$2 - 1 = 1$



Answer

The new coordinates of James's TV are (8,1).

12. Stacey draws the following translated image. The original point *A* of the figure is also shown. Describe the translation Stacey performed.

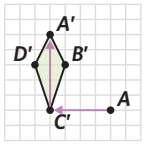
Key points
- A translated image is shown as well as one of the original points.
- Describe the translation from the original point to the translated point.

Explanation

To get from point A to point A':

Horizontal translation: 4 units left

Vertical translation: 5 units up



Answer

Stacey performed a translation of 4 units left and 5 units up.

Reasoning

13. a. The translation is 9 metres to the right and 4 metres up.

    b. The move is considered a translation because the shape and size of the pieces of furniture has not changed and it still has the same orientation in relation to the room because it hasn't been rotated.

    c. The translation is 6 metres to the right and 4 metres up.

    d. Suggested option 1: Estelle could move the dining table and sofa to the top left corner using the translation [−5,8].

    Suggested option 2: Estelle could move the dining table and sofa to the middle of the room using the translation [4,2].

    Note: There are other possible options.

14. a. The translation is 6 squares right and 2 squares forward.

    b. [6,2]

    c. First jump: [2,2]

    Second jump: [2,2]

    Third jump: [2,−2]

    d. Suggested option 1: It's a better strategy to make as many jumps as possible as the opponent will have less draughts as a result.

    Suggested option 2: It's a better strategy to block the opponent as it will stop them from advancing.

    Note: There are other possible options.

Extra spicy

15. 1 unit left and 4 units up [−1,4]

16. [0,4]

17. A

18. D

Remember this?

19. B

20. D

21. C

8D Rotations

Worked example 1.

a. Clockwise

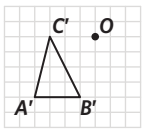
b. Clockwise or anticlockwise

c. Anticlockwise

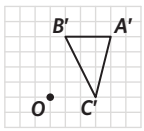
d. Clockwise

Worked example 2.

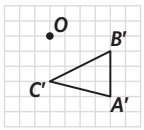
a.



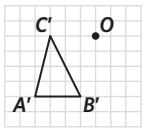
b.



c.



d.



Worked example 3.

a. Point $D'(0,-3)$

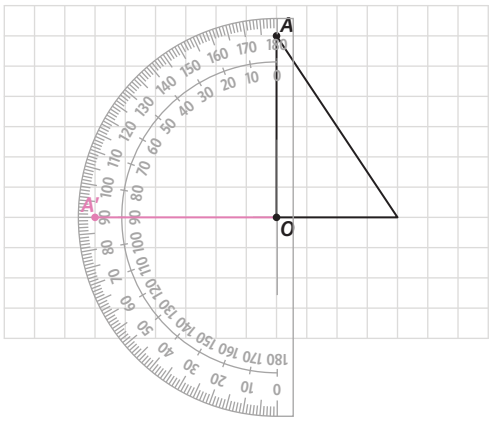
b. Point $A'(1,0)$

c. Point $F'(3,2)$

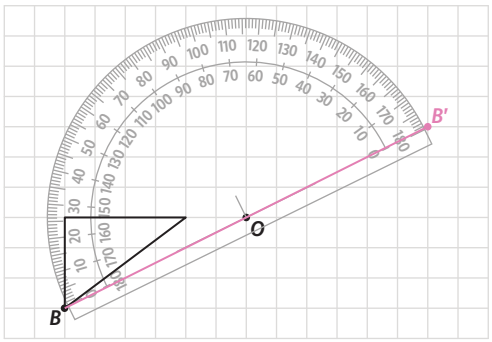
d. Point $G'(-2,0)$

Understanding worksheet

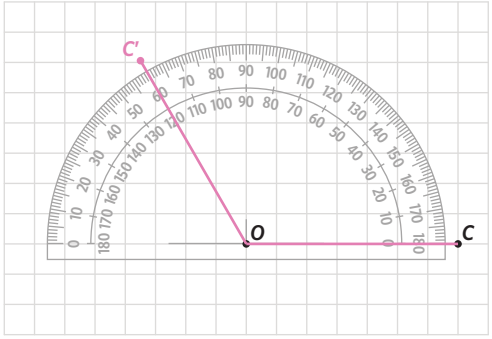
1. a.



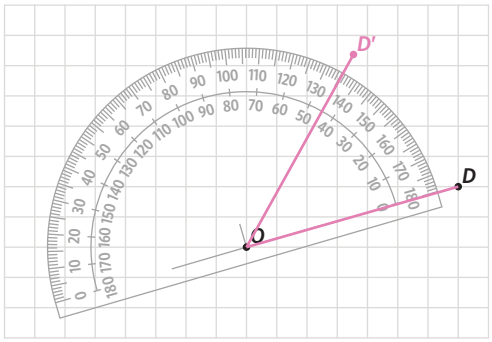
b.



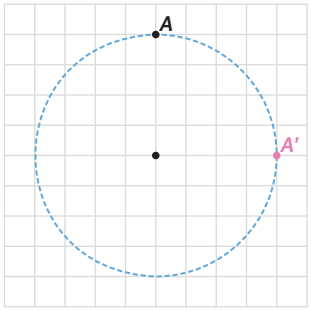
c.



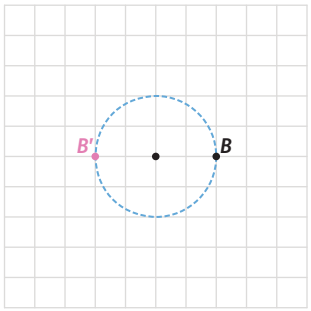
d.



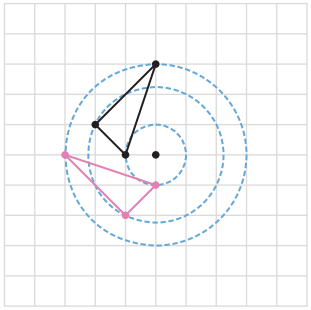
2. a.



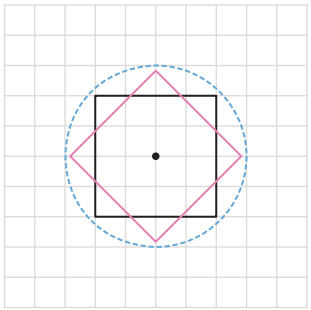
b.



c.



d.



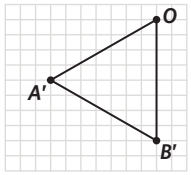
3. rotation; orientation; anticlockwise; angle; centre

Fluency

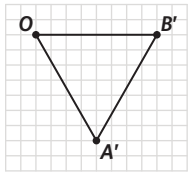
4. a. 180° b. 270° c. 240° d. 315°
   e. 255° f. 197°

5. a. Anticlockwise
   b. Clockwise
   c. Either clockwise or anticlockwise
   d. Either clockwise or anticlockwise
   e. Anticlockwise
   f. Clockwise

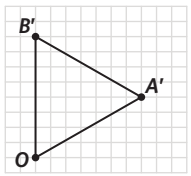
6. a.



b.



c.



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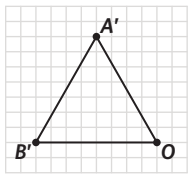
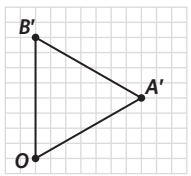
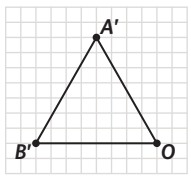


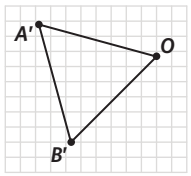
e.



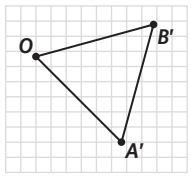
f.



g.



h.



7. a. (−1,−1) b. (1,−1) c. (−1,4) d. (4,3)
   e. (3,−4) f. (−1,−3)

8. a. (3,2) b. (−1,−4) c. (3,−2) d. (−4,1)
   e. (2,−4) f. (1,3)

Problem solving

9. Priyanka is completing a computer simulated driving test.
In order to turn left does she need to turn the steering wheel clockwise or anticlockwise?

Key points
- Priyanka wants to turn left when driving.
- Is the rotation of the steering wheel clockwise or anticlockwise?

Explanation

The steering wheel must be turned in the opposite of how the hands of a clock rotate.

Answer
Priyanka needs to turn the steering wheel anticlockwise.

10. Bonnie loves collecting old records and playing them on her record player. If she started playing the album 'Abbey Road' by the Beatles, how much of a rotation would the record have completed for the title 'Abbey Road' to be perfectly upside down?

Key points

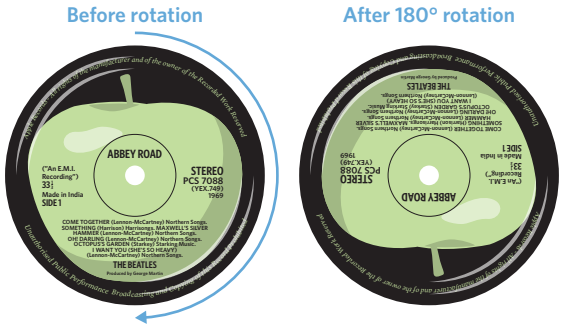
- Bonnie played the album 'Abbey Road'.
- What rotation will it take for the title to be upside down?

Explanation

For the album to be upside down a half rotation is needed.

Divide 360° by 2 to find the angle of rotation after $\frac{1}{2}$ of a rotation.

$360 \div 2 = 180°$



Answer

A 180° rotation is needed for the record title to be upside down.

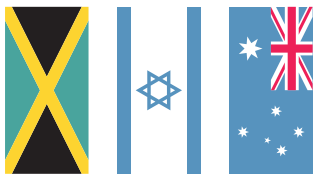
11. Sheldon is doing a school project where he is learning about flags of different countries. He has picked the following three flags. Which flags do not look like the original when rotated 270° anticlockwise?

Key point

- Which of the three flags change when rotated 270° anticlockwise?

Explanation

Rotate each flag 270° anticlockwise and check whether the flag has changed.



Answer

All three of the flags do not look like the original when rotated 270° anticlockwise.

12. Bentley found a starfish while working on the beach. The starfish had five equally spaced rays. What would be the smallest angle of rotation that Bentley could rotate the starfish so that it looks the same?

Key points

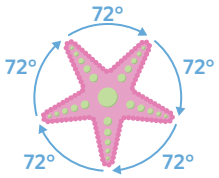
- The starfish Bentley found had 5 equally spaced rays.
- What angle does Bentley have to rotate the starfish to make it look the same?

Explanation

The order of rotational symmetry is 5. Bentley will need to rotate the starfish $\frac{1}{5}$ of a full rotation before it looks the same.

$\frac{1}{5} \times 360 = 360 \div 5$

$$5\overline{)3\,6^{1}0} \quad 0\ 7\ 2$$



Answer

The smallest angle of rotation that Bentley can rotate the starfish so that it looks the same is 72°.

13. On his way to the bathroom for a shower, Lance walks past a clock that reads exactly three o'clock. On his way back to his bedroom the clock reads three twenty-five. What was the angle of rotation of the minute hand of the clock during Lance's shower?

Key points

- On his way to the bathroom the clock reads three o'clock.
- On his way back to his bedroom the clock reads three twenty-five.
- Find the angle of rotation of the minute hand.

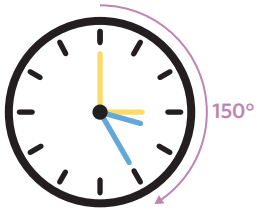
Explanation

There are 12 numbers on a clock.

$360 \div 12 = 30°$ per number.

The minute hand points to 12 at 3:00 and 5 at 3:25.
So, the minute hand moves 5 numbers clockwise between 3:00 and 3:25.

$5 \times 30 = 150°$



Answer

The minute hand rotated 150° during Lance's shower.

Reasoning

14. a. The angle of rotation is 720°.
    b. The angle of rotation is 90°.
    c. The season will be spring.
    d. Suggested option 1: The calendar year may have been created to start when it is hottest/coldest.

    Suggested option 2: The calendar year may have been created as it is for cultural/religious reasons.

    Note: There are other possible options.

15. a. The angle of rotation is 20°.
    b. There will be 24 triangles in the new pattern.
    c. The largest angle of rotation is 89°.
    d. Any pattern that was created by rotating a shape at equal intervals around the origin is correct.

Extra spicy

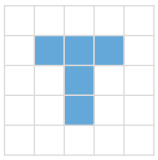
16. (−2,2) 17. B 18. C 19. 145°

Remember this?

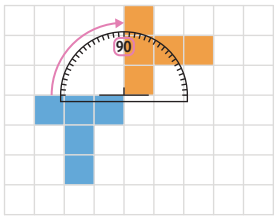
20. C 21. D 22. A

Chapter 8 extended application

1. a.

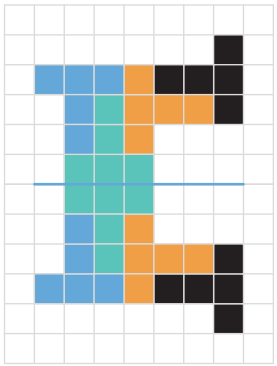


b.



c. All 4 original T-shapes undergo the same translation.

d.



e. Suggested option 1: You can use a CAS graphing calculator to recreate the pattern you drew in part c, and fit a third copy of the original image on top of the recreated image.

Suggested option 2: You can use a Desmos online graphing calculator to recreate the pattern you drew in part c, and fit a third copy of the original image to the right of the recreated image.

Note: There are other possible options.

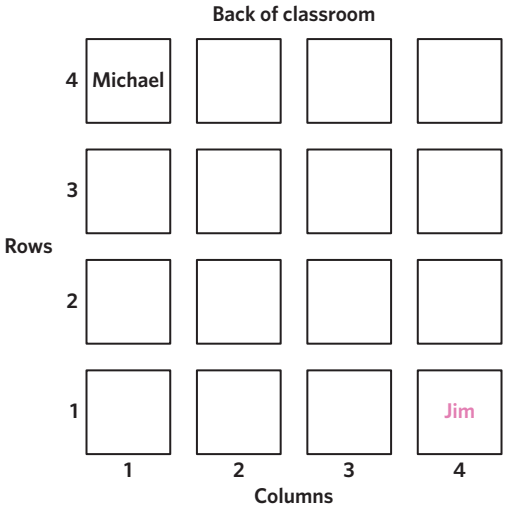
f. Suggested option 1: Tori could use a coding software to repeat a loop of commands including a specific rotation, translation and rotation to ensure that her patterns always repeat in a consistent way.

Suggested option 2: Tori could define a specific series of commands of transformations as f1 on her graphing calculator and repeat the command of f1 each time to ensure that her patterns always repeat in a consistent way.

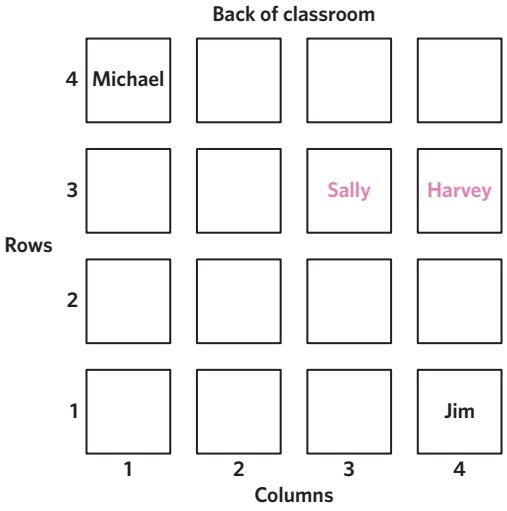
Note: There are other possible options.

2. a. Michael's seat is at (1,4).

b.



c.



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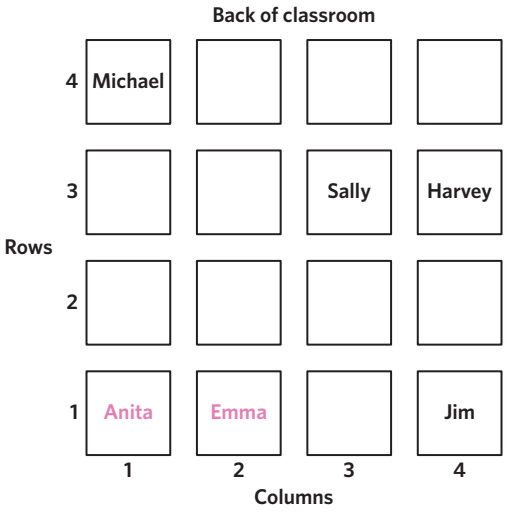


e. Suggested option 1: Ms Hammil could change the shape of the desks. If each desk is shaped like a sector and six desks form a circle, then students would sit face to face with each other which would promote effective group work.

Suggested option 2: Ms Hammil could join four square tables together to form a larger square so students can face each other during class, which would promote effective group work.

Back to contents

3. a. $A(-6,6)$, quadrant 2
   $B(6,4)$, quadrant 1
   $C(-5,-3)$, quadrant 3
   b. Kerby's phone is most likely located in quadrant 2.
   c. The phone is definitely not located in quadrant 4.
   d. The search priority should be quadrant 2 > quadrant 1 > quadrant 3. The more area the triangle emcompasses in each quadrant, the higher the search priority it has because the phone is more likely in that quadrant.
   e. Suggested option 1: Kerby could retrace his footsteps to check where he has been and where he might have lost something.
   Suggested option 2: Kerby should tie a string around his neck to hold his keys so he wouldn't lose it.
   Note: There are other possible options.

Chapter 8 review

Multiple choice

1. D 2. B 3. D 4. C 5. A

Fluency

6. a. (1,–1) b. (4,5) c. (–3,0) d. (2,–4)

7. a. Quadrant 2 b. x-axis
   c. Quadrant 3 d. y-axis

8. a. Order of line symmetry: 1; Order of rotational symmetry: 1
   b. Order of line symmetry: 2; Order of rotational symmetry: 2
   c. Order of line symmetry: 0; Order of rotational symmetry: 1
   d. Order of line symmetry: 12; Order of rotational symmetry: 12

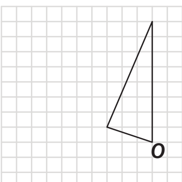
9. a. (–5,–8) b. (–3,12)
   c. (–4,–6) d. (15,–2)

10. a. The object is translated 5 units right and 2 units down.
    b. The object is translated 3 units left and 4 units up.
    c. The object is translated 1 unit right and 4 units down.
    d. The object is translated 3 units left and 5 units down.

11. a. (0,–1) b. (8,–3) c. (–1,6) d. (–8,9)

12. a. Anticlockwise b. Anticlockwise
    c. Clockwise d. Anticlockwise

13. a.



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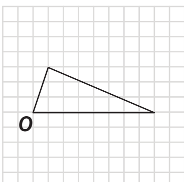

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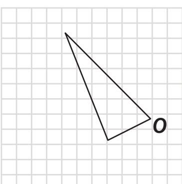

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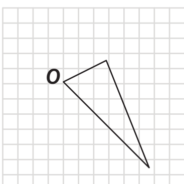


Problem solving

14. What are the coordinates of the city of Perth on the following map?

Key points
- The map is shown.
- Find the coordinates of the city of Perth.

Explanation
Identify the horizontal coordinate of the city of Perth,
The x-coordinate is 4.
Identify the vertical coordinate of the city of Perth,
The y-coordinate is 4.

Answer
The coordinates of the City of Perth are (4,4).

15. Harry wants to write a secret message. He hides the message by reflecting it vertically over a mirror line. How does the hidden message appear if the true message reads 'STYLES'?

Key points
- The true message reads 'STYLES'.
- The message is reflected vertically.
- What is the reflected message?

Explanation
Reflecting 'STYLE' vertically will reverse the order of the letters. Each of the individual letters will also appear upside down.

Answer
Ƨꓕ⅄ΓEƧ

16. Scott is travelling around Hawaii. He first landed in Honolulu before catching a boat to the largest island, and hiking up Mauna Kea.
Describe the translation of Scott's movement from Honolulu to Mauna Kea.

Key points
- The position of Honolulu and Manua Kea is shown on the map.
- Describe the translation from Honolulu to Mauna Kea.

Explanation
Count the number of units that the point of Honolulu needs to move in order to overlap Mauna Kea.
This is a horizontal translation of 5 units to the right and a vertical translation of 3 units down.

Answer
Scott moved 5 units to the right and 3 units down.

17. The London Eye is a Ferris wheel located in London. It takes 30 minutes to take a complete rotation around the wheel. Sunny got in a carriage at 2:45 pm. What angle of rotation of the London Eye had Sunny completed by 2:50 pm?

Key points
- A complete rotation takes 30 minutes.
- Sunny got in a carriage at 2:45 pm and was completed by 2:50pm.
- What angle of rotation did Sunny complete?

Explanation
There are 5 minutes between 2:45pm to 2:50pm. Sunny was on the wheel for 5 minutes.
It takes 30 minutes to complete a 360° full rotation.
5 minutes would be a $360° \div 6 = 60°$.

Answer
Sunny completed an angle of rotation of 60°.

Reasoning

18. a. Zoe has to guess the coordinates (8,9), (10,9), (11,9).
    b. The coordinates (8,2), (8,6) and (8,8) are not covered by one of Vutha's battleships.
    c. The translation to this position is 1 units left and 7 units up.
    d. Suggested option 1: Place the ships far apart, since an opponent who successfully guesses one set of coordinates will likely guess the surrounding coordinates to find the rest of the ship. Using this method, the opponent will likely find nearby ships.

    Suggested option 2: Place the ships close together so that the opponent's guesses are most likely to be in an area well away from your ships.

    Note: There are other possible options.

19. a. The smallest possible angle of rotation is 120°.
    b. The windmill looked the same 15 times in the following minute.
    c. The angle of rotation is 1800°.
    d. Suggested option 1: Wind turbines generate electricity, which reduces the need for fossil fuels.

    Suggested option 2: Farmers would receive payments from the developer in return for the lease of their land.

    Note: There are other possible options.

9A Units of length

Worked example 1.

a. 0.8 m b. 400 000 cm c. 10 cm d. 3000 mm

Worked example 2.

a. 3.1 km b. 747 m c. 0.81 m d. 1.23 km

Worked example 3.

a. 36 cm b. 66 mm c. 23 m d. 48 mm

Understanding worksheet

1. a. Metre b. Centimetre
   c. Millimetre d. Metre

2. a. 3 cm + 4 cm + 5 cm = 12 cm
   b. 15 cm + 15 cm + 15 cm + 15 cm = 60 cm
   c. 6 km + 6 km + 4 km + 12 km = 28 km
   d. 4 cm + 4 cm + 8 cm + 16 cm + 12 cm + 12 cm = 56 cm

3. measure; appropriate; large; different; same

Fluency

4. a. 1.5 m b. 2.5 cm
   c. 1650 m d. 850 mm
   e. 1200 cm f. 0.0015 km
   g. 0.175 km h. 11 300 000 mm

5. a. 5.3 m b. 470 cm
   c. 3.037 m d. 2.00831 km
   e. 870.23 m f. 0.07 km

6. a. 140 mm b. 570 cm
   c. 17.63 m d. 9.9662 km
   e. 83.23 m f. 380 m

7. a. 16 m b. 90 mm
   c. 95 cm d. 31 km
   e. 1600 cm or 16 m f. 190 mm or 19 cm
   g. 2 700 000 cm or 27 km h. 216 mm or 21.6 cm

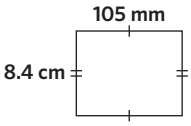
8. a. 8 cm b. 3 km c. 8 cm d. 42 mm
   e. 1.2 cm f. 600 cm

Problem solving

9. Barry printed a photo that has a length of 105 mm and a width of 8.4 cm. How many centimetres is the perimeter of the photo?

Key points
- The length of the photo is 105 mm.
- The width of the photo is 8.4 cm.
- Calculate the perimeter of the photo in cm.

Explanation



Express 105 mm in cm.

There are 10 mm in 1 cm, so divide by 10.

105 ÷ 10 = 10.5 cm

The perimeter is the sum of all side lengths.

10.5 + 10.5 + 8.4 + 8.4 = 37.8 cm

Answer

The perimeter of the photo is 37.8 cm.

10. Elwood wants to build a wooden fence around his square block of land. A carpenter agreed to build Elwood's fence for $80 per metre. If Elwood's land has a side length of 20 metres, how much would it cost to install the fence around his block of land?

Key points
- The fence costs $80 per metre.
- The square block of land has a side length of 20 m.
- What is the total cost of the fence?

Explanation

The perimeter is the sum of all side lengths.

The block of land is a square with a side length of 20 m.

perimeter = 4 × 20
= 80 m

The fence costs $80 per metre.

total cost = $80 × 80
= $6400

Answer

It would cost $6400 to install the fence.

11. Damien needs 5 different strings for a model pirate ship he is making. Which length of string is the longest if the length of strings he needs are: 75 millimetres, 10 centimetres, 550 centimetres, 0.4 metres and 3.5 metres?

Key points
- Damien needs 5 strings with lengths of 75 mm, 10 cm, 550 cm, 0.4 m and 3.5 m.
- Which is the longest length?

Explanation

We can express all of the lengths in cm.

Express 75 mm in cm.

There are 10 mm in 1 cm, so divide by 10.

75 ÷ 10 = 7.5 cm

Express 0.4 m and 3.5 m in cm.

There are 100 cm in 1 m, so multiply by 100.

0.4 × 100 = 40 cm

3.5 × 100 = 350 cm

Now that the lengths are all in cm, we can order them from smallest to largest.

7.5 < 10 < 40 < 350 < 550

Answer
The longest string is 550 centimetres long.

12. Bill is 156 cm tall. His older sister Penelope is 1.7 m tall. When Bill puts his shoes on, he is 35 millimetres taller. How much taller is Penelope than Bill when he has his shoes on?

Key points
- Bill is 156 cm tall, and his shoes add 35 mm to his height.
- Penelope is 1.7 m tall.
- When Bill has his shoes on, how much taller is Penelope?

Explanation

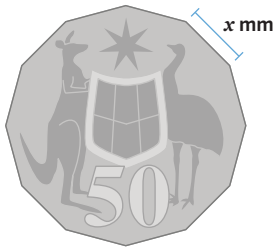
Express 35 mm in cm.
There are 10 mm in 1 cm, so divide by 10.
$35 \div 10 = 3.5$ cm
Express 1.7 m in cm.
There are 100 cm in 1 m, so multiply by 100.
$1.7 \times 100 = 170$ cm
With shoes on, Bill is $156 + 3.5 = 159.5$ cm tall.
Penelope is $170 - 159.5 = 10.5$ cm taller.

Answer
Penelope is 10.5 cm taller than Bill when he has his shoes on.

13. The Australian fifty-cent coin is dodecagonal shaped and has 12 sides. How many millimeters is each side of the coin if its perimeter is 10.56 cm?

Key points
- The Australian fifty-cent coin has 12 sides.
- The perimeter of the coin is 10.56 cm.
- Calculate the side length of one side of the coin, in mm.

Explanation



The coin has 12 sides of length.
The coin has a perimeter of 10.56 cm.
The length of one side is the perimeter divided by the number of sides.
$10.56 \div 12 = 0.88$ cm.
Express 0.88 cm in mm.
There are 10 mm in 1 cm, so multiply by 10.
$0.88 \times 10 = 8.8$ mm

Answer
The length of each side of the fifty-cent coin is 8.8 mm.

Reasoning

14. a. 40 mm on the map represents a distance of 1 km.
b. The return trip will be 3.25 km.
c. They hiked 2000 m, or 2 km, before taking a break.
d. The distance on Google Maps was 15 cm.
e. Suggested option 1: Yes, driving further away allows you to experience a different environment.
Suggested option 2: No, it is better to spend more time hiking at a closer location.
Note: There are other possible options.

15. a. $p = 2l + 2w$
b. The length of the court is 28 m.
c. B
d. Suggested option 1: Yes, if you play basketball often it would be very convenient.
Suggested option 2: No, it would take up a lot of space.
Note: There are other possible options.

Extra spicy

16. B
17. 15 cm
18. 64 cm
19. 9 cm and 2 cm

Remember this?

20. B
21. Donovan had a mean of 25 disposals per week.
22. A

9B Area of rectangles and parallelograms

Worked example 1.

a. 0.04 m^2
b. 1200 ha
c. 0.23 m^2
d. 280 000 000 cm^2

Worked example 2.

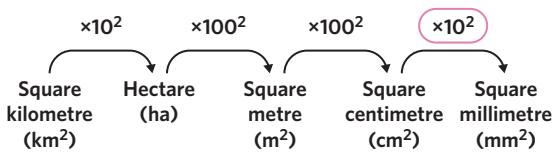
a. 12 cm^2
b. 20 cm^2
c. 42 cm^2
d. 32 cm^2

Worked example 3.

a. 24 cm^2
b. 70 mm^2
c. 30 m^2
d. 99 mm^2

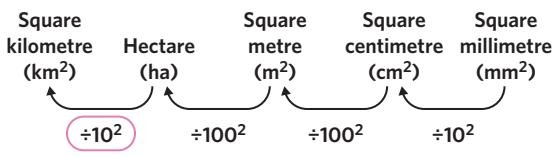
Understanding worksheet

1. a.



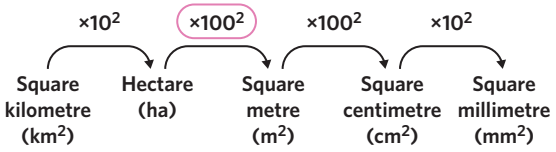
$13 \times 100 = 1300$ mm^2

b.



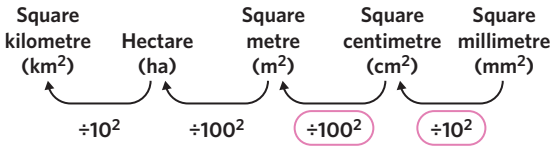
$15 \div 100 = 0.15$ km^2

c.



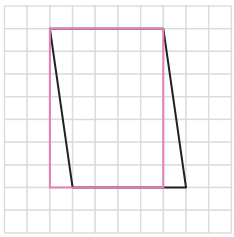
27 × 10 000 = 270 000 cm²

d.



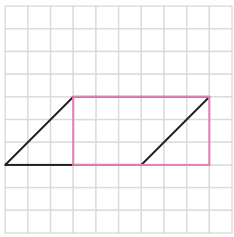
80 000 ÷ 1 000 000 = 0.08 m²

2. a.



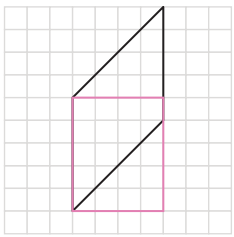
Note: There are other possible options.

b.



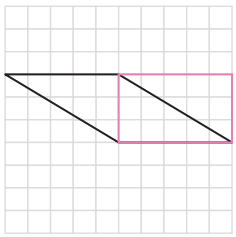
Note: There are other possible options.

c.



Note: There are other possible options.

d.



Note: There are other possible options.

3. around; space; square; grid; formula

Fluency

4. a. 10 000 mm²
b. 370 000 cm²
c. 0.01 km²
d. 880 cm²
e. 1 000 000 000 cm²
f. 50 000 000 000 cm²
g. 0.005 km²
h. 0.0003 ha

5. a. 16 cm²
b. 12 cm²
c. 40 cm²
d. 20 cm²
e. 24 cm²
f. 40 cm²
g. 42 cm²
h. 60 cm²

6. a. 30 cm²
b. 110 m²
c. 34 m²
d. 221 m²
e. 2000 cm²
f. 168 km²
g. 608 cm²
h. 20 700 mm² or 207 cm²

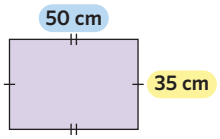
7. a. $x = 4$ cm
b. $k = 12$ cm
c. $p = 7$ cm
d. $z = 12$ cm
e. $y = 64$ cm²
f. $v = 4$ cm

Problem solving

8. What is the area of the top of Angela's bedside table if it is 35 centimetres long and 50 centimetres wide?

Key points
- The top of the bedside table has a length of 35 cm.
- The top of the bedside table has a width of 50 cm.
- Find the area.

Explanation



The formula for the area of a rectangle is $A = l \times w$.

Substitute $l = 35$ and $w = 50$ into the formula.

$A = l \times w$
$= 35 \times 50$
$= 1750$ cm²

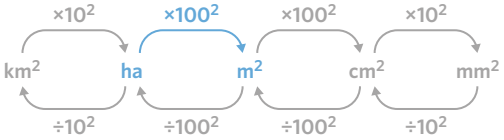
Answer

The top of the bedside table has an area of 1750 cm².

9. Tony works at the local city council and wants to redevelop 3 hectares of grassland into a multi-use park. How many square metres is 3 hectares?

Key points
- Tony wants to redevelop 3 hectares of grassland.
- Express 3 hectares in m².

Explanation



There are 100^2 m² in 1 ha.

Multiply by 100^2 to convert from ha to m².

$3 \times 100^2 = 3 \times 100 \times 100$
$= 3 \times 10\,000$
$= 30\,000$ m²

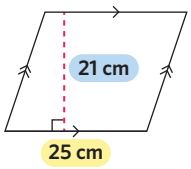
Answer

3 hectares is 30 000 m².

10. Cedric wants to replace his bathroom wall with parallelogram tiles. What is the area of 10 tiles if each tile has a base of 25 cm and a height of 21 cm?

Key points
- Cedric has parallelogram tiles with a base of 25 cm.
- Each tile has a height of 21 cm.
- Find the area of 10 tiles.

Explanation



The formula for the area of a parallelogram is $A = b \times h$.

Substitute $b = 25$ and $h = 21$ into the formula.

$A = b \times h$

$= 25 \times 21$

$= 525 \text{ cm}^2$

Each tile has an area of 525 cm^2.

The area of 10 tiles is $525 \times 10 = 5250 \text{ cm}^2$.

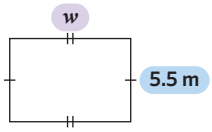
Answer

The area of 10 tiles is 5250 cm^2.

11. Siobhan paid $3520 to install carpet in her rectangular shaped living room. If the length of her living room is 5.5 metres, what is the width of the living room? She was charged $80 per square metre of carpet.

Key points
- Sophie paid $3520 for her living room carpet.
- Her living room is 5.5 m long.
- The carpet cost $80 per m^2.
- Find the width of her living room.

Explanation



The carpet was $80 per m^2, and Siobhan spent $3520 in total.

So the area of the living room floor must be $3520 \div 80 = 44 \text{ m}^2$.

The formula for the area of a rectangle is $A = l \times w$.

Substitute $l = 5.5$ and $A = 44$ into the formula and solve for w.

$A = l \times w$

$44 = 5.5 \times w$

$44 \div 5.5 = 5.5 \times w \div 5.5$

$8 = w$

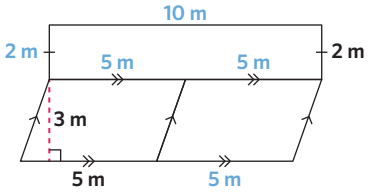
Answer

The width of the living room is 8 m.

12. Jett owns a pressure washing business where he washes mold, grime and dirt off surfaces. One of his customers has a unique driveway with the design illustrated below. If Jeff charges $15 per square metre he washes, how much should Jett charge to clean this driveway?

Key points
- The driveway is shown in the diagram.
- Jett charges $15 per square metre.
- What is the cost to clean this driveway?

Explanation



The driveway is made up of one rectangle and two identical parallelograms.

The formula for the area of a rectangle is $A = l \times w$.

Substitute $l = 2$ and $w = 10$ into the formula.

$A = l \times w$

$= 2 \times 10$

$= 20 \text{ m}^2$

The formula for area of a parallelogram is $A = b \times h$.

Substitute $b = 5$ and $h = 3$ into the formula.

$A = b \times h$

$= 5 \times 3$

$= 15 \text{ m}^2$

Total area:

$A = 20 + 2 \times 15$

$= 50 \text{ m}^2$

Total cost:

$\$15 \times 50 = \$750$

Answer

Jett should charge $750 to clean this driveway.

Reasoning

13. a. The perimeter of a whole pizza is 110 cm.

b. The area of one slice of pizza is 125 cm^2.

c. The area of a whole pizza is 750 cm^2.

d. It would cost $400 to buy a square metre worth of Sicilian pizza from Sophia.

e. Suggested option 1: Yes, as this could encourage customers to buy a whole pizza.

Suggested option 2: No, because she may lose customers if the price is too high.

Note: There are other possible options.

14. a. An algebraic equation for the total area is $T = 6bh$.

b. The height of a single board is 7 cm.

c. The area of Derek's kitchen and dining room is 36.75 m^2.

d. Suggested option 1: There may be a lot of the floorboards being wasted around the corners of rooms.

Suggested option 2: Rooms are typically rectangular in shape so rectangular floorboards would fit better.

Note: There are other possible options.

Extra spicy

15. $T = 4lw$
16. 313 cm^2
17. 210 000 plants
18. $x = 10$ cm

Remember this?

19. B
20. C
21. B

9C Area of triangles

Worked example 1.

a. Base: 8 cm; Height: 8 cm
b. Base: 5 cm; Height: 4 cm
c. Base: 12 cm; Height: 8 cm
d. Base: 48 mm; Height: 27 mm

Worked example 2.

a. 56 cm^2
b. 15 cm^2
c. 36.45 mm^2
d. 98.365 cm^2

Worked example 3.

a. 55 cm^2
b. 133 cm^2
c. 132 mm^2
d. 116.4 m^2

Understanding worksheet

1. a. Base of triangle: 17 cm
   Height of triangle: 19 cm
   b. Base of triangle: 14 cm
   Height of triangle: 8 cm
   c. Base of triangle: 26.7 cm
   Height of triangle: 16.8 cm
   d. Base of triangle: 19.9 cm
   Height of triangle: 17.23 cm

2. a. $\frac{1}{2} \times 10 \text{ mm} \times 8 \text{ mm} = 40 \text{ mm}^2$
   b. $\frac{1}{2} \times 15 \text{ cm} \times 10 \text{ cm} = 75 \text{ cm}^2$
   c. $\frac{1}{2} \times 6 \text{ km} \times 6 \text{ km} = 18 \text{ km}^2$
   d. $\frac{1}{2} \times 12 \text{ km} \times 14 \text{ km} = 84 \text{ km}^2$

3. combined; half; base; height

Fluency

4. a. Base: 7 cm; Height: 4 cm
   b. Base: 12 km; Height: 16 km
   c. Base: 10 mm; Height: 8.7 mm
   d. Base: 9 cm; Height: 8.8 cm
   e. Base: 15 cm; Height: 8 cm
   f. Base: 17.3 m; Height: 18.3 m

5. a. Area of rectangle: 16 m^2; Area of triangle: 8 m^2
   b. Area of rectangle: 48 cm^2; Area of triangle: 24 cm^2
   c. Area of rectangle: 84 cm^2; Area of triangle: 42 cm^2
   d. Area of rectangle: 70 mm^2; Area of triangle: 35 mm^2
   e. Area of parallelogram: 108 m^2; Area of triangle: 54 m^2
   f. Area of parallelogram: 88 km^2; Area of triangle: 44 km^2

6. a. 7.5 cm^2
   b. 38.5 m^2
   c. 320 km^2
   d. 60 cm^2
   e. 7.59 m^2
   f. 210 m^2
   g. 382.5 m^2
   h. 224 mm^2

7. a. $x = 3$
   b. $x = 10$
   c. $x = 9$
   d. $x = 5$
   e. $x = 10.4$
   f. $x = 9.1$

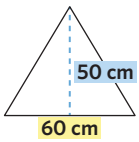
8. a. 112 mm^2
   b. 98 cm^2
   c. 175 m^2
   d. 1700 km^2
   e. 253 cm^2
   f. 54 mm^2
   g. 119 km^2
   h. 40 m^2

Problem solving

9. Wanda volunteers as a lollipop lady at her local primary school. She carries a triangular stop sign attached to a pole to help students cross safely. If the triangular sign has a base of 60 cm and a height of 50 cm, what is the area of this sign?

Key points
- The triangular sign has a base of 60 cm.
- The sign has a height of 50 cm.
- Calculate the area of the sign.

Explanation



The area of a triangle can be found using the formula $A = \frac{bh}{2}$.

Substitute $b = 60$ and $h = 50$ into the formula.

$$A = \frac{bh}{2}$$
$$= \frac{60 \times 50}{2}$$
$$= \frac{3000}{2}$$
$$= 1500 \text{ cm}^2$$

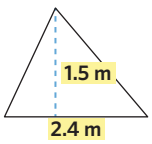
Answer

The sign has an area of 1500 cm^2.

10. Alfredo has a triangular veggie patch in his garden which he uses to grow lettuce. What is the area of this veggie patch if the base is 2.4 metres and the height is 1.5 metres?

Key points
- The triangular veggie patch has a base of 2.4 m and height of 1.5 m.
- Calculate the area of the triangular veggie patch.

Explanation



The veggie patch is triangular, with a base of 2.4 m and height of 1.5 m.

Substitute $b = 2.4$ and $h = 1.5$ into the formula $A = \frac{bh}{2}$.

$$A = \frac{bh}{2}$$
$$= \frac{2.4 \times 1.5}{2}$$
$$= \frac{3.6}{2}$$
$$= 1.8 \text{ m}^2$$

Answer

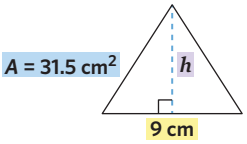
The area of the veggie patch is 1.8 m^2.

11. Wilma recently travelled to Egypt and took a photo of the Khafre Pyramid so that she could print it out as a poster. What is the height of the pyramid on Wilma's poster if the base is 9 cm and the area is 31.5 cm^2?

Key points

- The pyramid has a base of 9 cm on the poster.
- The area of the pyramid is 31.5 cm^2.
- Calculate the height of the pyramid on the poster.

Explanation



The area of a triangle can be found using the formula $A = \frac{bh}{2}$.

Substitute $b = 9$ and $A = 31.5$ into the formula.

$$31.5 = \frac{9h}{2}$$

$$31.5 \times 2 = \frac{9h}{2} \times 2$$

$$63 = 9h$$

$$63 \div 9 = 9h \div 9$$

$$h = 7 \text{ cm}$$

Answer

The height of the pyramid on Wilma's poster is 7 cm.

12. Amber recently purchased a star stamp that she plans to use to decorate her notebooks. The star design is made up of 10 identical scalene triangles. What is the area of one stamp imprint?

Key points

- Amber's stamp is made up of 10 identical scalene triangles.
- Find the area covered by the stamp.

Explanation

Each triangle has a base of 9 mm and height of 4 mm.

Substitute $b = 9$ and $h = 4$ into $A = \frac{bh}{2}$.

$$A = \frac{9 \times 4}{2}$$

$$= \frac{36}{2}$$

$$= 18 \text{ mm}^2$$

Each triangle has an area of 18 mm^2.

The star is made up of 10 triangles.

$10 \times 18 = 180$ mm^2

Answer

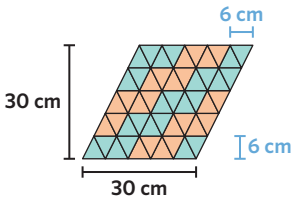
One imprint of the stamp is 180 mm^2.

13. Jeremiah made a mosaic using isosceles triangular tiles. What is the total area of the orange tiles in the mosaic?

Key points

- Jeremaiah's mosaic is made of isosceles triangular tiles.
- Find the area covered by orange tiles.

Explanation



The mosaic is 5 tiles wide and 5 tiles high. This means that 5 tiles must have a base and height of 30 cm. Therefore, the base and height of one tile must be $30 \div 5 = 6$ cm.

Substitute $b = 6$ and $h = 6$ into $A = \frac{bh}{2}$.

$$A = \frac{6 \times 6}{2}$$

$$= \frac{36}{2}$$

$$= 18 \text{ cm}^2$$

Each tile has an area of 18 cm^2.

There are 24 orange tiles within the mosaic.

$24 \times 18 = 432$ cm^2

Answer

The total area of the orange tiles is 432 cm^2.

Reasoning

14. a. D

b. The area of the intersected area between pennant 1 and pennant 2 is 0.18 m^2.

c. The area of the flag is 3.12 m^2.

d. Suggested option 1: Nepal should not shift away from having a non-quadrilateral shaped flag as it maintains the country's heritage and tradition.

Suggested option 2: Nepal should shift to a quadrilateral shaped flag so that it can be uniform with the flags of other nations.

Note: There are other possible options.

15. a. The area of the headsail is 18 m^2.

b. The length of the base for the mainsail is 11 m.

c. The total area increase of Porter's sails is 36.75 m^2.

d. Suggested option 1: Sailboats are quite expensive.

Suggested option 2: Sailboats require a high cost of maintenance.

Note: There are other possible options.

Extra spicy

16. D 17. E 18. $A = 32$ cm^2 19. $x = 4.8$ cm

Remember this?

20. Stanley has $1.85 left.

21. B

22. $P = 64$ cm

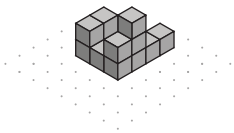
9D Drawing 3D objects

Worked example 1.

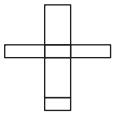
a. Refer to the question.

b. 

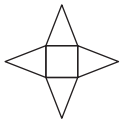
c. Refer to the question.

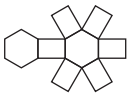
d. 

Worked example 2.

a. 

Note: There are other possible options.

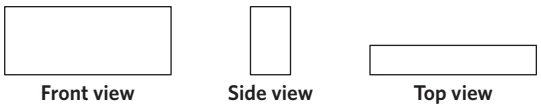
b. 

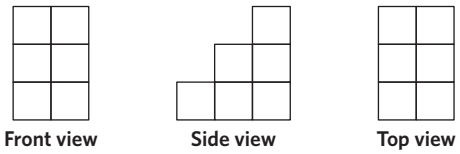
c. 

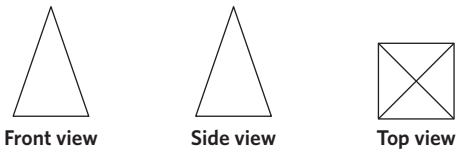
d. 

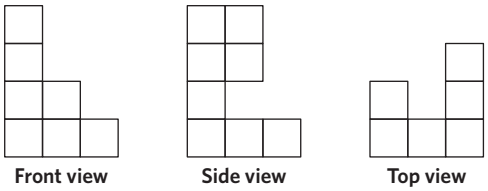
Note: There are other possible options.

Worked example 3.

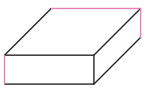
a. 

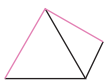
b. 

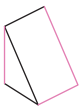
c. 

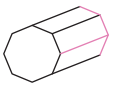
d. 

Understanding worksheet

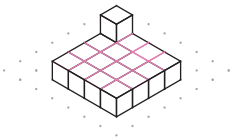
1. a. 

b. 

c. 

d. 

2. a. 

b. 

c. 

d. 

3. 2D plane; 3D object; faces; flattened; different

Fluency

4. a. Prism b. Pyramid c. Prism d. Prism
e. Pyramid f. Pyramid

5. a. Refer to question. b. Refer to the question.
c. Refer to the question. d. Refer to the question.
e. Refer to the question. f. Refer to the question.

6. a. 

b. 

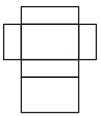
c. 

d. 

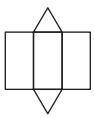
e. 

f. 

7. a. 

b. 

Note: There are other possible options.

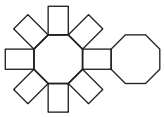
c. 

d. 

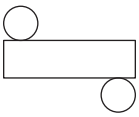
e. 

Note: There are other possible options.

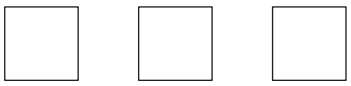
f. 

g. 

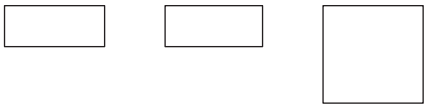
Note: There are other possible options.

h. 

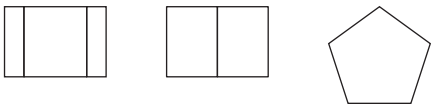
8. a. Front view: Side view: Top view:



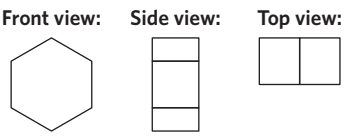
b. Front view: Side view: Top view:



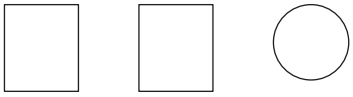
c. Front view: Side view: Top view:



d. Front view: Side view: Top view:

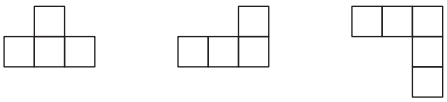


e. Front view: Side view: Top view:

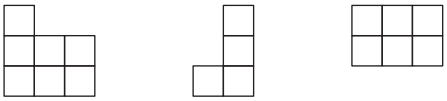


f. Front view: Side view: Top view:

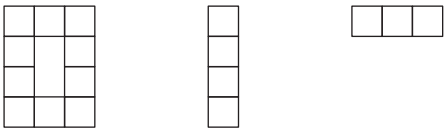
9. a. Front view: Side view: Top view:



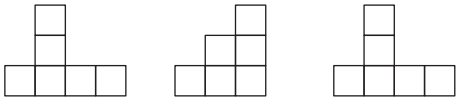
b. Front view: Side view: Top view:



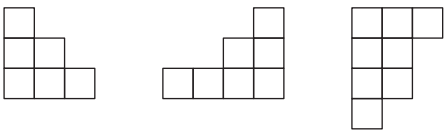
c. Front view: Side view: Top view:



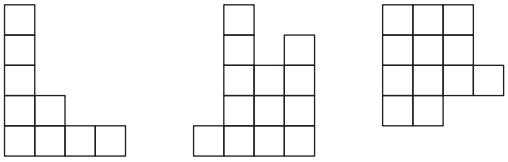
d. Front view: Side view: Top view:



e. Front view: Side view: Top view:



f. Front view: Side view: Top view:



Problem solving

10. Hariharan is an architecture student at university. As an assignment, he needs to create a 3D model of his room. He decides to use a rectangular prism to represent his work station. Draw a rectangular prism where its length and width are both greater than its height.

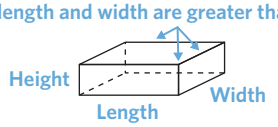
Key points

- Hariharan's work station is modeled using a rectangular prism.
- The length and width are both greater than its height.
- Draw this rectangular prism.

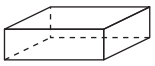
Explanation

Draw a rectangular prism where the length and width are greater than the height.

The length and width are greater than the height.



Answer



Note: There are other possible options.

11. Ruby saw everyone else in her class play with a 3 × 3 Rubik's cube so she bought one herself. Copy Ruby's Rubik's cube on isometric dot paper.

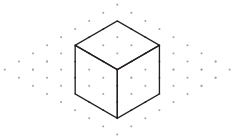
Key points

- Ruby bought a 3 × 3 Rubik's cube.
- Draw a 3 × 3 cube on an isometric dot paper.

Explanation

To start, draw a vertical line of three units. From the bottom of the vertical line, draw a line three units to the left and three units to the right.

Complete the cube



Fill in the missing lines to make the cube into a 3 × 3 Rubik's cube.

Answer

12. Natasha recently purchased a pair of shoes that were packaged in a rectangular prism shaped cardboard box. Natasha realised that if she unfolded the box and laid the faces flat, she could create a net for a rectangular prism. Draw the net of a rectangular prism.

Key points

- Natasha's shoebox is in the shape of a rectangular prism.
- Natasha unfolds her shoebox into a net.
- Draw a possible net that Natasha would get.

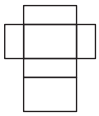
Explanation

The base of a rectangular prism is a rectangle.

The sides that would fold up from the base are four rectangles all with the same height.

Draw the other rectangular base.

Answer



Note: There are other possible options.

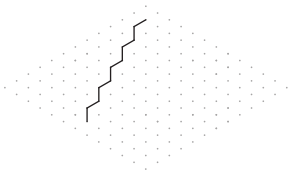
13. Chloe was stacking her wooden toy blocks and constructed a flight of 5 stairs that were 4 blocks wide. Draw the stairs that Chloe constructed on isometric dot paper.

Key points

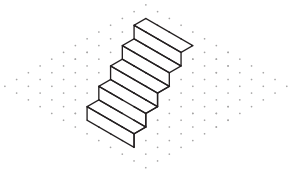
- Chloe's toy is a flight of 5 stairs.
- The stairs are 4 blocks wide.
- Draw the stairs on an isometric dot paper.

Explanation

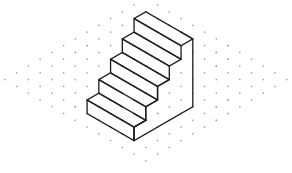
Outline a flight of 5 stairs.



On each corner and end point, draw a line four units long to the right. Then close it with the outline.



Complete the outline to join the top stairs with the bottom.



Fill in the missing lines to break the stairs into individual blocks.

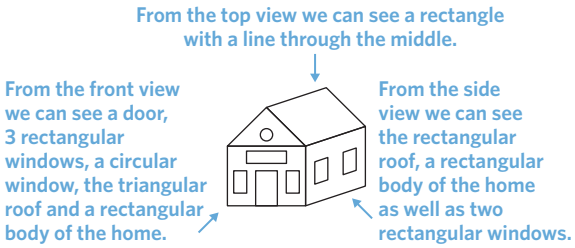
Answer

14. Harry used to live in a house before he moved to an apartment closer to his work. He was feeling nostalgic and searched his old house up on Google Earth. Draw the plan views (front, side and top views) of Harry's old unit.

Key point

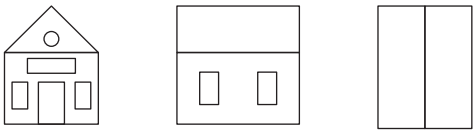
- Draw the front, side and top view of Harry's old unit.

Explanation



Answer

Front view: Side view: Top view:



Reasoning

15. a. Based on the top, front and side views, Carlin built a triangular pyramid.

b. Based on the top, front and side views, Carlin added a rectangular prism to his structure.

c. Based on the top, front and side views, Franka used a square pyramid and a cube.

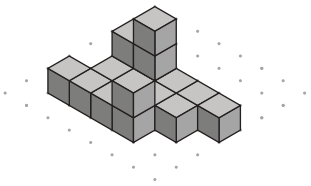
d. Suggested option 1: Having a pyramid shaped roof with a slope helps direct water and snow away from the house.

Suggested option 2: Pyramid shaped roofs are cheaper to maintain than flat roofs as they're less likely to be structurally damaged by hail and heavy rain.

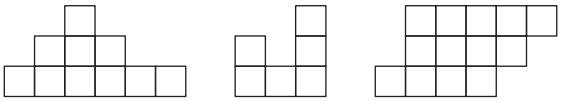
Note: There are other possible options.

16. a. D

b.



c. Front view: Side view: Top view:



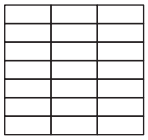
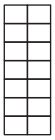
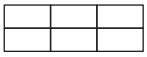
d. Suggested option 1: Wood is much more durable than plastic so the toy blocks are less likely to break.

Suggested option 2: Wood based items are generally better for the environment than plastic based items.

Note: There are other possible options.

Extra spicy

17. 10

18. Front view:  Side view:  Top view: 

19. 18 cubes.

20. Front view:  Side view:  Top view: 

Remember this?

21. D 22. B 23. B

9E Volume of prisms

Worked example 1.

a. 19 cm^3 b. 9 cm^3 c. 14 cm^3 d. 23 cm^3

Worked example 2.

a. 162 m^3 b. 75 cm^3 c. 64 cm^3 d. 25.83 km^3

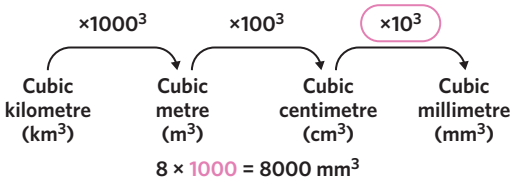
Worked example 3.

a. 156 cm^3 b. 192 cm^3 c. 810 cm^3 d. 414 m^3

Understanding worksheet

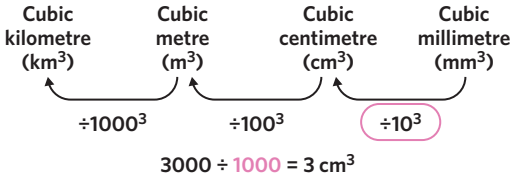
1. a. Cubic centimetre b. Cubic millimetre
   c. Cubic kilometre d. Cubic metre

2. a.



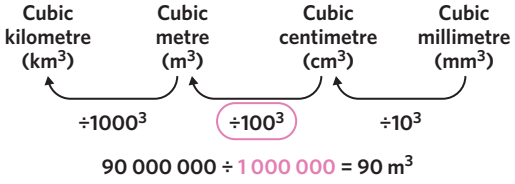
$8 \times 1000 = 8000$ mm^3

b.



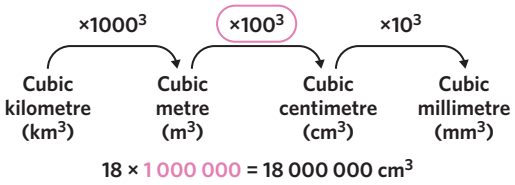
$3000 \div 1000 = 3$ cm^3

c.



$90\,000\,000 \div 1\,000\,000 = 90$ m^3

d.



$18 \times 1\,000\,000 = 18\,000\,000$ cm^3

3. space; 3D object; magnitude; bigger; cubic metres

Fluency

4. a. 4 cm^3 b. 11 cm^3 c. 12 cm^3 d. 24 cm^3
   e. 45 cm^3 f. 23 cm^3

5. a. 12 cm^3 b. 27 mm^3
   c. 70 mm^3 d. 216 km^3
   e. 4.116 m^3 f. 259.875 m^3

6. a. 36 cm^3 b. 99 mm^3
   c. 75 km^3 d. 369.63 mm^3
   e. 427.84 m^3 f. 135.52 cm^3

7. a. $x = 8$ b. $x = 12$ c. $x = 9$ d. $x = 3$
   e. $x = 5$ f. $x = 6$

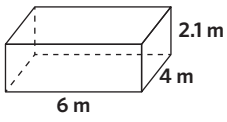
8. a. 54 cm^3 b. 228 mm^3
   c. 312 cm^3 d. 2520 cm^3
   e. 1288 cm^3 f. 240 m^3

Problem solving

9. Freddy found Sydney summers too hot and decided to install a pool in his backyard. The dimensions of Freddy's pool are 6 m × 4 m × 2.1 m. What is the volume of Freddy's pool?

Key points
- The dimensions of Freddy's pool are 6 m × 4 m × 2.1 m.
- What is the volume of Freddy's pool?

Explanation



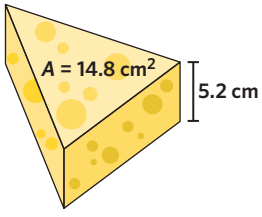
$V = l \times w \times h$
$= 6 \times 4 \times 2.1$
$= 50.4$ m^3

Answer
The volume of Freddy's pool is 50.4 m^3.

10. Michael recently purchased a unique triangular prism shaped block of cheese at his local cheesemonger. What is the volume of this cheese block if the base of the cheese has an area of 14.8 cm^2 and the height of the cheese block is 5.2 cm?

Key points
- The base of the cheese has an area of 14.8 cm^2.
- The height of the cheese block is 5.2 cm.
- What is the volume of this cheese block?

Explanation



$V = A_{base} \times h$
$= 14.8 \times 5.2$
$= 76.96$ cm^3

Answer
The volume of this cheese block is 76.96 cm^3.

11. What is the volume of Sarah's phone in cubic centimetres if the volume of her phone is 96 072.37 cubic millimetres?

Key points
- The volume of her phone is 96 072.37 cubic millimetres.
- What is the volume of Sarah's phone in cubic centimetres?

Explanation

Cubic centimetre (cm^3) ← ÷10^3 — Cubic millimetre (mm^3)

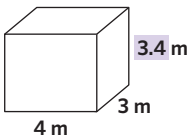
96 072.37 mm^3 to cm^3 : 96 072.37 ÷ 1000 = 96.07237 cm^3

Answer
The volume of Sarah's phone is 96.07237 cm^3.

12. Kenji's bedroom is rectangular and has a volume of 40.8 m^3. If the length of the room is 4 m, and the width of the room is 3 m, what is the height of his room?

Key points
- Kenji's bedroom is rectangular and has a volume of 40.8 m^3.
- The length of the room is 4 m.
- The width of the room is 3 m.
- What is the height of his room?

Explanation



Length: 4 m
Width: 3 m
Height: 3.4 m

Volume = 4 m × 3 m × 3.4 m = 40.8 m^3

$V = l \times w \times h$
$40.8 = 4 \times 3 \times h$
$40.8 = 12 \times h$
$h = 3.4$ m

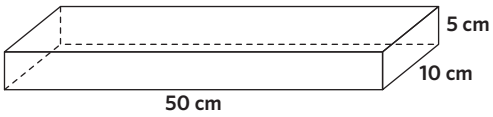
Answer
The height of Kenji's room is 3.4 m.

13. Natalia loves DIY projects and wants to build a bookshelf. She decides to use two pieces of wood – a short piece and a long piece. The short piece has dimensions of 4 cm × 10 cm × 25 cm and the long piece has dimensions of 50 cm × 10 cm × 5 cm. If she uses 3 short pieces and 2 long pieces of wood, what is the total volume of wood that she used for her project?

Key points
- Natalia uses 3 short pieces and 2 long pieces of wood to build the bookshelf.
- The short piece has dimensions of 4 cm × 10 cm × 25 cm.
- The long piece has dimensions of 50 cm × 10 cm × 5 cm.
- What is the total volume of wood that she used for her project?

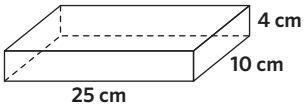
Explanation

Long piece



$V_{long} = l \times w \times h$
$= 50 \times 10 \times 5$
$= 2500$ cm^3

Short piece



$V_{short} = l \times w \times h$
$= 25 \times 10 \times 4$
$= 1000$ cm^3

$V_{total} = V_{short} \times 3 + V_{long} \times 2$
$= 1000 \times 3 + 2500 \times 2$
$= 3000 + 5000$
$= 8000$ cm^3

Answer
The total volume of wood that Natalia used for her project is 8000 cm^3.

Reasoning

14. a. The volume of this block of wood is 60 000 cm^3.
b. The volume sawed off is 43 000 cm^3.
c. The volume of the finished sculpture is 2250 cm^3.
d. Suggested option 1: Mr Johanasson might first create an outline before completing his sculpture as it'll allow him to be more accurate. He can use the certain saws to make his work more precise and look smoother.
Suggested option 2: Mr Johanasson might create an outline first to allow him to be more flexible in his design in case he wants to construct something else.
Note: There are other possible options.

15. a. The length of a container is 14 m.
b. 350 parcels can fit in one of the rectangular containers.
c. The new containers can load 5.796 m^3 more volume compared to the previous containers.
d. Suggested option 1: Upgrade the fleet to electric or hybrid vehicles to use less fossil fuels.
Suggested option 2: Plan and optimise the utilisation of container capacity so that fewer trips are required.
Note: There are other possible options.

Extra spicy

16. D 17. 3 mm 18. 6560 cm^3 19. 256 cm^3

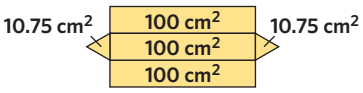
Remember this?

20. C 21. C 22. D

Chapter 9 extended application

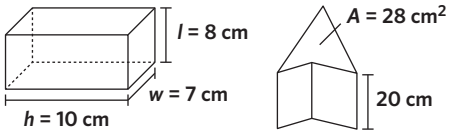
1. a. 400 g of chocolate has a volume of 560 cm^3.

   b. The area of each triangle face is 10.75 cm^2.

   c. 

   The area of cardboard needed for the packaging is 321.5 cm^2.

   d. The capacity of the proposed box is 215 cm^3. The proposed box is not big enough to hold the chocolate bar.

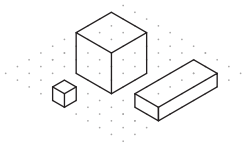
   e. 

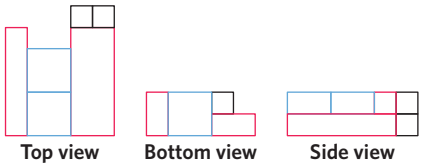
   Note: There are other possible options.

   f. Suggested option 1: Jasmine can consider the shape of the chocolate bar and design a box that would exactly fit 400g of chocolate bars with no space wasted.

   Suggested option 2: Jasmine can consider the relationship between the material and the shape of the box. For example, If Jasmine is using cardboard paper for the box, she can consider an octagonal shape box for it to be the strongest.

   Note: There are other possible options.

2. a. 

   b. 

   c. Patrick will be able to fit 139 small boxes in the remaining space.

   d. The total area of the surfaces of all the boxes in the stack in part **c** is 102 800 cm^2.

   e. To minimise the cost, Patrick should use 8 large cubic boxes.

   f. Suggested option 1: Patrick should consider using recycled card boxes to reduce the environmental impact.

   Suggested option 2: Patrick should only use reusable boxes so he does not need to purchase any new boxes.

   Note: There are other possible options.

3. a. The length of fencing that Alanna needs to submit is 21.2 m.

   b. Alana will need to spend $470 on grass.

   c. One possible dimensions of the veggie patch is 6 m × 2 m. The perimeter is 16 cm.

   Note: There are other possible options.

   d. It is cheaper for Alanna to hire Maria and she can save $170.35 by hiring Maria for this project.

   e. Suggested option 1: Alanna should check the back of seed packets of cucumber, broccoli and carrot to determine when she should plant each of the vegetables.

   Suggested option 2: Alanna should check the weather forecast and select the season that has the most number of sunny days to plant her vegetables.

   Note: There are other possible options.

Chapter 9 review

Multiple choice

1. B 2. D 3. D 4. C 5. C

Fluency

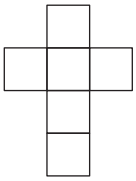
6. a. 3.00095 km b. 50 100 cm
   c. 34 000 cm d. 633 m

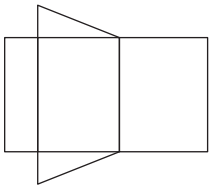
7. a. 42 m b. 72 cm
   c. 0.71 m or 71 cm d. 102 cm or 1020 mm

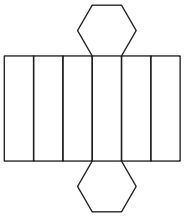
8. a. 35 km^2 b. 77 cm^2
   c. 60 mm^2 d. 144 cm^2

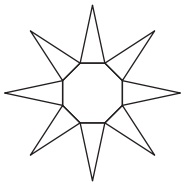
9. a. 3900 mm^2 b. 0.53 km^2
   c. 4.21 km^2 d. 30 000 000 000 mm^2

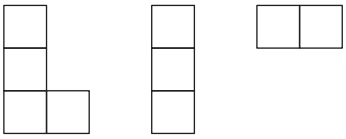
10. a. 30 mm^2 b. 20 m^2
    c. 240 cm^2 d. 54 cm^2

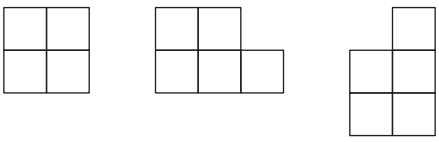
11. a. 
    Note: There are multiple possible nets.

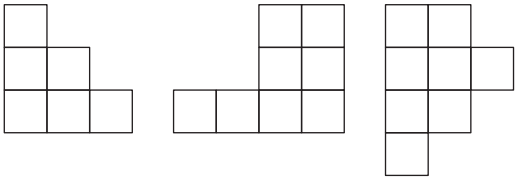
    b. 
    Note: There are multiple possible nets.

    c. 
    Note: There are multiple possible nets.

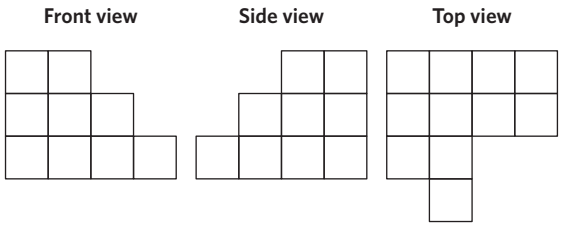
    d. 
    Note: There are multiple possible nets.

12. a. Front view Side view Top view
    

    b. Front view Side view Top view
    

    c. Front view Side view Top view
    

d.



13. a. 160 cm^3 b. 240 mm^3
c. 836 m^3 d. 912 mm^3

Problem solving

14. Felix and Troye competed in the long jump at their school's athletics carnival. The boys each completed two jumps. Felix's first jump measured 2.78 metres, and his second jump was 55 mm further than his first. Troye's best jump measured 284 cm. Who won?

Key points
- Felix's first jump was 2.78 m.
- Felix's second jump was 55 mm greater than his first.
- Troye's best jump was 284 cm.
- Who completed the best jump overall?

Explanation

Calculate the distances of Felix's best jump and Troye's best jump in metres, then compare to determine the longest jump.

Calculate Felix's best jump. Convert 55 mm to m, then add to 2.78 m to calculate the distance of his second jump.

Convert 55 mm to m:

There are 10 mm in 1 cm, so divide by 10.

$55 \div 10 = 5.5$ cm.

There are 100 cm in 1 m, so divide by 100.

$5.5 \div 100 = 0.055$ m.

$2.78 + 0.055 = 2.835$ m, so Felix's best jump is 2.835 m.

Troye's best jump was 284 cm. Convert 284 cm to m.

There are 100 cm in 1 m, so divide by 100.

$284 \div 100 = 2.84$ m.

Then compare the two distances.

$2.835 \text{ m} < 2.84$ m, so Troye's best jump is longer than Felix's.

Answer

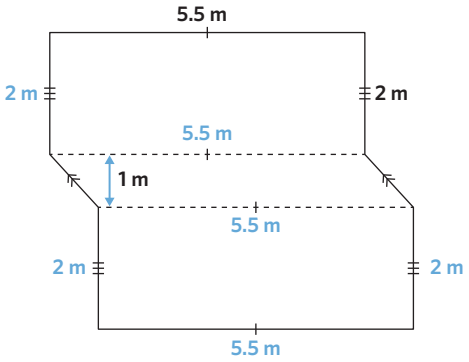
Troye won the long jump competition.

15. Leona wants to lay carpet on her bedroom floor. Her bedroom is made up of two rectangles and one parallelogram, as shown. How much will she spend in total if the carpet costs $30 per square metre?

Key points
- The bedroom floor is shown in the diagram.
- Carpet costs $30 per square metre.
- How much will Leona spend?

Explanation

Calculate the area of the two rectangles and parallelogram, then add together to determine the total area in square metres. Lastly, multiply the total area by the cost per square metre to calculate the total cost.



So we have two identical rectangles with length 5.5 m and width 2 m.

The formula for the area of a rectangle is $A = l \times w$.

Substitute $l = 5.5$ and $w = 2$ into the formula.

$A = l \times w$

$= 5.5 \times 2$

$= 11 \text{ m}^2$

The formula for area of a parallelogram is $A = b \times h$.

Substitute $b = 5.5$ and $h = 1$ into the formula.

$A = b \times h$

$= 5.5 \times 1$

$= 5.5 \text{ m}^2$

Total area:

$A = 2 \times 11 + 5.5$

$= 27.5 \text{ m}^2$

Total cost:

$\$30 \times 27.5 = \$825$

Answer

Leona will spend $825 on carpet for her bedroom.

16. Sof's favourite flavour of Shapes is barbeque. The barbeque shape is made up of 6 identical triangles, as shown. Using the following diagram, calculate the area of the barbeque Shape.

Key points
- The barbeque Shape is shown in the diagram.
- The barbeque Shape is a hexagon made up of 6 identical triangles.
- Calculate the area of one barbeque Shape.

Explanation

Calculate the area of one of the 6 identical triangles, then multiply by 6 to determine the total area.

The formula for the area of a triangle is $A = \frac{bh}{2}$.

Substitute $b = 2.3$ and $h = 2$ into the formula $A = \frac{bh}{2}$.

$= \frac{2.3 \times 2}{2}$

$= \frac{4.6}{2}$

$= 2.3 \text{ cm}^2$

Total area:

$A = 2.3 \times 6 = 13.8 \text{ cm}^2$

Answer

The area of one barbeque Shape is 13.8 cm^2.

17. Milan purchased a jumbo Toblerone, which was packaged in a triangular prism shaped box. In order to recycle the box, Milan unfolded it and was left with the net for a triangular prism. Draw both the original and the unfolded Toblerone box.

Key points

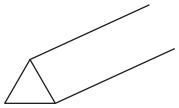
- Draw a triangular prism.
- Draw a net for this triangular prism.

Explanation

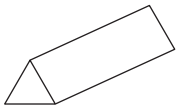
Draw the triangular prism:

Draw the base shape of the triangular prism, which is a triangle.

Draw two identical slanted lines from the vertices of the triangle.

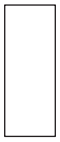


Connect the slanted lines to complete the drawing.



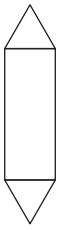
Draw the net:

Draw the rectangular side at the bottom of the prism.

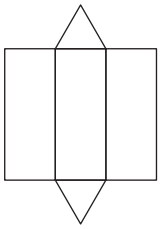


Identify the sides that would fold up from this rectangle.

There are two identical triangles connected to the short edges of this rectangle.



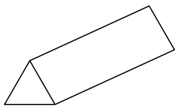
There are two identical rectangles connected to the long edges of this rectangle. The short edges of these two identical rectangles should be the same length as the two disconnected sides of the triangles.



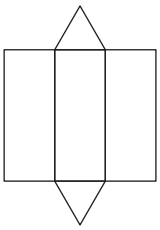
Note: There are other possible options.

Answer

Triangular prism



Net



18. Damien's birthday cake is cylindrical. The area of the top of the cake is 300 cm^2 and the cake is 12 cm tall. If the cake is cut into 8 equal pieces, what is the volume of each piece of cake?

Key points

- The cake is cylindrical.
- The area of the top of the cake is 300 cm^2.
- The cake is 12 cm tall.
- What is the volume of one eighth of the cake?

Explanation

Calculate the volume of the cylinder, then divide by eight to determine the volume of one piece.

The formula for the volume of a cylinder is $V = A_{base} \times h$.

Substitute $A_{base} = 300\ cm^2$ and $h = 12$ cm.

$V_{whole} = 300 \times 12$

$= 3600\ cm^3$

Divide by 8.

$V_{piece} = V_{whole} \div 8$

$= 3600 \div 8$

$= 450\ cm^3$

Answer

The volume of one piece of cake is 450 cm^3.

Reasoning

19. a. The height of the front wall of the main section of the house is 460 cm.

b. The area of the front view of the garage is 15.3 m^2.

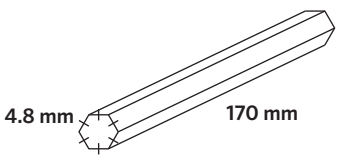
c. The area of the entire front view is 59.925 m^2.

d. Suggested option 1: It is better to measure in metres, because it is the unit of measurement closest to the object being measured.

Suggested option 2: It is better to measure in centimetres, because it allows for more precision when considering objects that might fit inside the house.

Note: There are other possible options.

20. a.

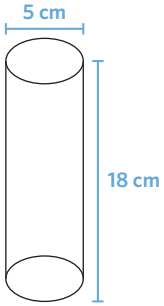


b. 10 200 mm^3

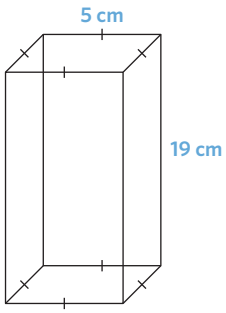
c. The total volume of all 6 pencils is 61.2 cm^3.

d. 28.8 cm^3

e. Suggested option 1:



Suggested option 2:



Note: There are many possible answers.

Glossary

A

Acute angle triangle An acute angle triangle contains three interior angles that are between 0° and 90° in size.

Acute angle An acute angle is any angle that is greater than 0° but less than 90°.

Adjacent angles Adjacent angles are angles that share a vertex and a common side.

Angle of rotation The angle of rotation is the number of degrees an image is rotated.

Angle An angle is a measure of turn that is given in degrees (°).

Anticlockwise Anticlockwise is the opposite direction that the hands of a clock move around.

Apex The apex is the highest point of a 3-dimensional shape.

Approximately equal to The term approximately equal to is represented in calculations using the '≈' symbol.

Arc An arc is the curve that joins two rays and it represents the angle.

Area The area is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.

Arm An arm is a line that forms an angle.

Ascending Ascending order means to arrange values from smallest to largest.

Associative law The associative law means that how we group numbers in addition or multiplication can change without affecting the result.

Asymmetrical Asymmetrical shapes do not have any lines of symmetry.

B

Bar chart Bar charts display categorical data with the horizontal length of each column representing the frequency of each category.

Base The base refers to any side of a triangle.

Base number The base number in index notation is the number that is being multiplied by itself. E.g. for 8^3, 8 is the base number indicating that three 8's are multiplied together.

Benchmark decimals Benchmark decimals are decimals that are recognisable, such as 0.25, 0.50 and 0.75. They help us to more accurately make estimates about the position of a number on a number line.

Bivariate data Line graphs are used to represent bivariate data. This is data that has two numerical variables.

Brackets Brackets are mathematical symbols that clarify the order that operations are completed in expressions. Brackets should always be evaluated first.

C

Cartesian coordinates Cartesian coordinates are the numbers that indicate the location of a point on a Cartesian plane. The x-coordinate is written before the y-coordinate. E.g. (2,3).

Cartesian plane A Cartesian plane is a set of two perpendicular number lines that intersect at the origin.

Centimetre A centimetre (cm) is one-hundredth of a metre.

Centre of rotation The centre of rotation is the fixed point an image or point is rotated around. This point is often labelled as O.

Chance experiment A chance experiment is a test conducted to examine the results of chance activities, such as tossing a coin, rolling a die or picking a card out of a well-shuffled deck.

Clockwise Clockwise is the same direction that the hands of a clock move around.

Coefficient A coefficient is the number that a pronumeral is being multiplied by. E.g. for the term $5x$, the coefficient is 5. For the term $\frac{2a}{3}$, the coefficient is $\frac{2}{3}$. For the term y, the coefficient is 1.

Collecting like terms Collecting like terms is the process of using addition or subtraction to group together algebraic terms that contain the same pronumeral or pronumerals.

Column graph Column graphs display categorical data with the vertical height of each column representing the frequency of each category.

Common denominators Fractions have common denominators when their denominators are the same.

Common difference The common difference is the constant between consecutive terms in a sequence that determines how the sequence increases or decreases.

Common factors Common factors are the same factors that occur between two or more numbers.

Commutative law The commutative law means that the order of the numbers in an addition and multiplication calculation can change without affecting the result.

Compensation strategy The compensation strategy uses multiples of 10 and then adjustments to make calculations simpler to solve. E.g. $49 + 16 = 50 + 16 - 1$.

Complementary angles Complementary angles are when two or more angles form a right angle which is 90°.

Complementary percentages Complementary percentages are two percentage values that add up to 100%, where 100% refers to the whole amount. E.g. 60% and 40% are complementary percentages as they add up to 100%.

Composite number A composite number is a number that has 3 or more factors.

Composite shape A composite shape is a geometric figure that is made up of two or more basic geometric shapes.

Cone A cone is a 3-dimensional object that tapers from a circular base to an apex point.

Consecutive terms Consecutive terms refers to values that directly follow each other within a pattern or a sequence.

Constant A constant is a number that can not change its value in an expression or equation. E.g. 5, 34, 0.2.

Converting Converting is the process of changing a value into a different but equal form. E.g. the fraction $\frac{1}{4}$ can be converted to 25%.

Coordinate A coordinate is usually made up of two numbers and describes the position of a point on a graph.

Coprime Two numbers are coprime if they have no common factors other than one.

Critical digit A critical digit is the digit to the right of a digit being rounded. If the critical digit is 5 or above, add 1 to the digit being rounded. If the critical digit is less than 5, leave the digit being rounded the same.

Cubic centimetre A cubic centimetre (cm^3) is space contained by a cube with edges of 1 centimetre in length.

Cubic kilometre A cubic kilometre (km^3) is space contained by a cube with edges of 1 kilometre in length.

Cubic metre A cubic metre is the space contained by a cube with edges of 1 metre in length.

Cubic millimetre A cubic millimetre (mm3) is the space contained by the cube with edges of 1 millimetre in length.

Cylinder A cylinder is a 3-dimensional object that has 2 flat, circular bases which are connected by a curved surface.

D

Data set A data set is a collection of values relating to a particular variable.

Decimal fraction A decimal fraction is a fraction where the denominator is a power of 10, such as 10, 100, 1000, 10 000. E.g. the decimal 0.25 written as a decimal fraction is $\frac{25}{100}$.

Decimal notation Decimal notation is a number that is expressed using digits that are separated by a decimal point. E.g. 1.35 is a number expressed in decimal notation.

Decomposition Decomposition is the process of breaking apart the original number into smaller parts.

Denominator The denominator is the value on the bottom of a fraction which tells us how many parts each unit is split into.

Descending Descending order means to arrange values from largest to smallest.

Digit A digit is a single numeral from 0–9 that is used to make a number. E.g. 57 is made up of the digits 5 and 7.

Discount A discount is when a price is reduced by a given amount or by a percentage.

Distributive law The distributive law for multiplication means that multiplying a given number by several other numbers is the same as multiplying the given number by the sum of the other numbers.
E.g. $6 \times 17 = 6 \times (10 + 7) = (6 \times 10) + (6 \times 7)$.

Dividend A dividend is the first number in a division calculation. It is the number that is being divided by the second number.

Divisible A number is divisible by another number when its quotient is a whole number. E.g. 18 is divisible by 6 because $18 \div 6 = 3$.

Divisor A divisor is the number by which a given value is divided.

E

Equal sign The equals sign (=) in an equation or number sentence indicates that the value on the left of the sign is the same value as on the right.

Equation An equation is another name for a number sentence. An equation can contain letters, numbers and operations, as well as an equals sign. Each side of an equation is equal.

Equilateral triangle An equilateral triangle contains three sides of equal length and three 60° interior angles.

Equivalent fractions Equivalent fractions are fractions which express the same quantity but with a different numerator and denominator. E.g. $\frac{4}{6}$ and $\frac{8}{12}$ are equivalent fractions.

Equivalent number sentence In equivalent number sentences, both sides of an equals sign will have the same total value even when different numbers and operators are being used.

Equivalent ratio Equivalent ratios are two ratios that express the same relationship between parts. E.g. 1 : 2 and 2 : 4 are equivalent.

Equivalent values Equivalent values are two values, numbers or quantities which are the same.

Estimates Estimates are approximate values that are close to the correct answer.

Evaluate Evaluate means to carry out mathematical operations in the correct order to find a value of an expression or equation.

Event An event refers to a single outcome or a grouped outcome of a chance experiment. Tossing a coin and landing a head is a single outcome event. Rolling a die and landing an even number is a grouped outcome event because there are multiple outcomes.

Expanded decimal form A decimal expressed in expanded decimal form is written as a sum of all of its place value parts. The value of the digits to the right of the decimal point are expressed in decimal notation. E.g. 3.245 is written in expanded decimal form as $3 + 0.2 + 0.04 + 0.005$.

Expanded form Expanded form is a way of expressing a number as the sum of its place value parts or expressing index notation as repeated multiplications. E.g. 2^4 in expanded form is $2 \times 2 \times 2 \times 2$.

Expanded fraction form A decimal expressed in expanded fraction form is written as a sum of all of its place value parts. The value of the digits to the right of the decimal point are expressed as decimal fractions. E.g. 3.245 is written in expanded fraction form as $3 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$.

Expected occurrence The expected occurrence of an event is the number of times an event is expected to take place in a given number of trials.

Experimental probability The experimental probability of an event is the probability of an outcome that is obtained from the results of an experiment.

Expression An expression is a number of terms grouped together by operations. E.g. $5x + 3$ or $3ab - 4a + b$. Note: An expression does not contain an equals sign.

Exterior angle An exterior angle is the angle formed by the side of a shape and the imaginary line that is extended from the shape.

F

Factor pairs Factor pairs are two numbers that multiply together to give another number. E.g. 2×7 is a factor pair of 14.

Factor tree A factor tree is a diagram used to find a number's prime factors.

Factors Factors are numbers that divide into another number with no remainder. E.g. factors of 6: 1, 2, 3, 6.

False equation A false equation is when the left-hand side and right-hand side of an equation are not equal.

Formula A formula is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.

Fraction A fraction is a value represented as a certain number of equal parts of a unit.

Frequency Frequency refers to how many times something occurs.

Frequency table A frequency table represents a set of data into categories and how many times each category occurs.

G

Geometric pattern A geometric pattern is a sequence of geometric shapes that continues to increase or decrease in a consistent way.

H

Hectare A hectare is a metric unit of area equal to 10 000 square metres.

Height The height is the perpendicular distance from the line that passes through the base to the opposite vertex.

Highest common factor (HCF) Highest common factor (HCF) is the largest number that is a factor of two or more numbers. E.g. 5 is the HCF of 5 and 15.

I

Improper fraction An improper fraction has a numerator greater than or equal to its denominator. E.g. $\frac{7}{5}$ or $\frac{3}{3}$.

Index notation Index notation is a shorter way of representing repeated multiplications. E.g. 5^3 represents $2 \times 2 \times 2$.

Index number The index number indicates how many times the base number is multiplied by itself. The index number can also be referred to as the power of the number. E.g. for 2^4, 4 is the index number indicating that four 2's are multiplied together.

Inequality symbols Inequality symbols are used to show which number is larger or smaller. The pointy end faces the smaller value. $>$ means 'is larger or greater than'. $<$ means 'is smaller or less than'.

Integer An integer is a whole number that is either positive, negative or zero.

Interior angle An interior angle is an angle measured inside a shape.

Inverse operation Inverse operations are mathematical operations that undo each other. The inverse of multiplication is a division and the inverse of division is a multiplication; the inverse of addition is a subtraction and the inverse of subtraction is an addition.

Isometric A translation is one type of isometric transformation because the original image does not change in size or shape after the transformation.

Isosceles triangle An isosceles triangle contains two sides of equal length and two angles of equal size.

K

Kilometre A kilometre (km) is one thousand metres.

L

Lead digit rounding Lead digit rounding rounds the number based on the first digit and replaces all other digits with zeros.

Like terms Like terms are algebraic terms that contain the same pronumeral or collection of pronumerals.

Line graphs Line graphs contain points connected by straight lines. They are used to graph numerical data and show how a variable changes over time.

Line of symmetry A line of symmetry or axis divides a shape into two identical parts.

Line segment A line segment has two fixed end points. E.g. 

.

Lowest common denominator (LCD) The lowest common denominator (LCD) is the lowest number that is a multiple of all the given denominators.

Lowest common multiple (LCM) Lowest common multiple (LCM) is the smallest number that is a multiple of two or more numbers. E.g. 12 is the LCM of 3 and 4.

M

Maximum The maximum is the highest value in the data set.

Mean The mean, also known as the average, is calculated by taking the sum of all values in the data set, then dividing by the number of data points in the set.

Measure of centre A measure of centre gives an idea of what a typical value in the data set would be. The measures of centre are mean, median and mode.

Median The median is the middle value when the data is sorted from lowest to highest.

Metre A metre (m) is a standardised unit measuring length.

Millimetre A millimetre (mm) is one-thousandth of a metre.

Minimum The minimum is the lowest value in the data set.

Mixed number A mixed number is a combination of a whole number and a proper fraction. E.g. $2\frac{1}{3}$.

Modal value The mode, also known as the modal value, is the value that occurs most frequently.

Mode The mode, also known as the modal value, is the value that occurs most frequently.

Multi-step equation A multi-step equation is an equation that can be solved by applying more than two inverse operations.

Multiple A multiple is the product that we get when one number is multiplied by another number E.g. 12 is a multiple of 4 because $4 \times 3 = 12$.

Multiplier A multiplier is the number by which a given value is multiplied.

N

Negative numbers Negative numbers are numbers with a value that is less than zero.

Net A net is a 2-dimensional representation showing all the faces of a 3-dimensional object. It could be cut out and folded up to create the 3-dimensional object that it represents.

Not equals sign The not equals sign ($\neq$) in an equation or number sentence indicates that the value on the left of the sign is not the same value as on the right.

Number sentence A number sentence is an equation expressed using numbers and operators such as $+$, $-$, $\times$ and $\div$.

Number slider A number slider is a visual representation that can be used to explain how the digits in a number move a place value when multiplying or dividing by powers of 10.

Numerator The numerator is the value above the line in a fraction which tells us how many parts of a unit we are considering.

Numeric form Numeric form is the combination of digits written together as a single number. E.g. 576.

Numerical sequence A numerical sequence is a collection of numbers that increase or decrease in a consistent way.

O

Obtuse angle triangle An obtuse angle triangle contains one interior angle that are between 90° and 180° in size.

Obtuse angle An obtuse angle is any angle greater than 90° but less than 180°.

One-step equation A one-step equation is an equation that can be solved by applying a single inverse operation.

Open array An open array is an empty rectangle that is used to show multiplication and division using area.

Operation An operation in maths is a mathematical process used to solve problems. The four basic operations in maths are addition, multiplication, subtraction and division.

Operator An operator is a symbol used to show that an action has been performed on a set of mathematical terms. The most common operators are $+$, $-$, $\times$ and $\div$.

Opposite numbers Opposite numbers are numbers in the inverse position on the number line. The sum of opposite numbers is always zero.

Order of line symmetry Order of line symmetry is the number of lines of symmetry a shape has.

Order of operations The order of operations is a set of rules that tells us what calculations are performed first in a number sentence or equation. The order often follows BIDMAS, which stands for Brackets, Indices, Division, Multiplication, Addition and Subtraction.

Order of rotational symmetry Order of rotational symmetry is the number of times a shape can be rotated around a point (up to 360 degrees) and stay the same shape.

Orientation The orientation refers to the position of a figure in a specific direction.

Origin The origin is the point (0,0) where the x-axis and y-axis intersect.

Original The original refers to the point, shape, or image before it is moved.

Outcome An outcome is the result of the trial. E.g. a coin landing on heads.

Overestimates Overestimates are estimates that are more than the exact calculation, number, quantity or size.

P

Parallel lines Parallel lines never touch and are always the same distance apart.

Parallelogram A parallelogram is a quadrilateral (4-sided shape) with two opposite pairs of parallel sides.

Partitioning strategy The partitioning strategy deconstructs larger numbers into smaller numbers so that calculations are simpler to solve. E.g. $13 + 25 = 10 + 3 + 20 + 5$.

Percentage A percentage represents a value as a part out of 100, where 100 represents a whole.

Perfect square A perfect square is the result of a number multiplied by itself. E.g. $2^2 = 2 \times 2 = 4$.

Perimeter The perimeter is the total length of the edge around a closed 2D shape.

Pie chart Pie charts show categorical data with a circle split into sectors whose area represents the proportion of each category.

Positive numbers Positive numbers are numbers with a value greater than zero.

Powers of 10 Powers of 10 are the result of multiplying ten by itself a certain number of times. E.g. 10, 100, 1000, 10 000 and 100 000 are all powers of 10.

Powers of a number Powers of a number are the result of multiplying a number by itself a given number of times. E.g. powers of 10 are: 10, 100, 1000, 10 000 and 100 000.

Prime factors Prime factors are prime numbers that can be multiplied together to give another number. E.g. 18 expressed as a product of it prime factors is $2 \times 3 \times 3$.

Prime number A prime number is a number that only has two factors, 1 and itself.

Prism A prism is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.

Probability Probability refers to the likelihood of an event occurring and is expressed as a number between 0 and 1 (inclusive). It is generally expressed as a fraction, decimal or percentage, or using words such as impossible, unlikely, even, likely or certain.

Product The product is the result when two or more values are multiplied together.

Pronumerals A pronumeral is a letter used to represent a value that is unknown or may vary. This is also known as a variable.

Proper fraction A proper fraction has a numerator that is less than its denominator. E.g. $\frac{3}{4}$.

Proportion A proportion is a part that is considered in relation to the value of a whole.

Pyramid A pyramid is a 3-dimensional object with a polygon base and 3 or more triangular faces which all meet at a point.

Q

Quadrilateral A quadrilateral is a four sided shape.

Quotient The quotient is the result or answer of a division calculation.

R

Range The range measures the spread of data. It is the difference between the maximum and minimum values.

Ratio A ratio is a set of numbers in the form $a : b$ that expresses the relationship between two or more quantities or sizes.

Ray A ray is a line that has one fixed end point and no end in one direction. E.g. 

.

Reciprocal The reciprocal of a number is 1 divided by that number. E.g. the reciprocal of a is $\frac{1}{a}$.

Reflection A reflection uses a line of symmetry as a mirror line to change an image's position without changing its size or shape. A reflection point is written as A'.

Reflex angle A reflex angle is any angle greater than 180° and less than 360°

Regular prism A regular prism is a prism where the base shape side lengths are all equal.

Relational thinking Relational thinking is the ability to recognise that the equals sign indicates a relationship between both sides that means 'the same as'.

Remainder A remainder is the leftover amount in a division calculation that is less than the divisor.

Revolution A revolution is a full rotation of a circle, or a complete turn which is 360°.

Right angle A right angle is an angle that is exactly 90°.

Right-angled triangle A right-angled triangle contains one interior angle that is 90°.

Rounding Rounding is the process of making a number simpler to use in calculations.

Rule A rule is a formula written using mathematical symbols and pronumerals to describe a pattern.

S

Sample space The sample space refers to all the possible outcomes of an experiment. E.g. the sample space of rolling a 6-sided die is written as $S = \{1, 2, 3, 4, 5, 6\}$.

Scalene triangle A scalene triangle contains three sides of different length and three interior angles of different size.

Sequence A sequence is a collection of numbers in an order that follows a specific pattern or rule.

Simplest form A fraction is in its simplest form when the numerator and denominator have no common factors other than 1. E.g. $\frac{5}{10}$ in the simplest form is $\frac{1}{2}$.

Simplify To simplify an algebraic expression means to rewrite it in a more concise and usable way. This is done by first collecting like terms.

Simplifying fractions Simplifying fractions is the process of expressing an equivalent fraction with a smaller numerator and smaller denominator.

Solving equations Solving equations is a process used to find the value of the unknown by performing a series of inverse operations.

Square root The square root of a number is a value that, when multiplied by itself, gives the original number. E.g. $\sqrt{4} = 2$.

Substitution Substitution is the process of replacing a pronumeral with a given value.

Supplementary angles Supplementary angles are when two or more angles form a straight line which is 180°.

T

Term A term is a part of an expression or equation made up of a number and/or pronumerals. Terms are separated by addition or subtraction. E.g. $5x$, $3ab$, 7, $\frac{2a}{3}$.

Theoretical probability The theoretical probability is a measurement of the likelihood that an event will occur.

Transformation A transformation changes the position of a point or set of points in an image based on a rule.

Translation A translation is a type of transformation that shifts the location of the original but does not change the orientation or size of the original shape.

Transversal lines Transversal lines pass across two or more straight lines.

Trial A trial is a single test of the chance activity.
E.g. tossing a coin once would be one trial.

True equation A true equation is when the left-hand side and right-hand side of an equation are equal.

Two-step equation A two-step equation is an equation that can be solved by applying two inverse operations.

U

Underestimates Underestimates are estimates that are less than the exact calculation, number, quantity or size.

Unit form Unit form is when all of the place values of a number are written in words.
E.g. 5 hundreds + 7 tens + 6 ones.

V

Variable A variable is a quantity that changes and can include qualities that are given numerical values. For example, time, temperature and sleep quality on a scale of 1 to 10.

Venn diagram A Venn diagram is a mathematical representation showing the common and uncommon features between two or more sets of numbers.

Vertex A vertex is the point at which two rays meet to create an angle.

Vertical algorithm A vertical algorithm is a method of adding and subtracting numbers where the place value of each digit is lined up and the results recorded underneath.

Vertically opposite angles Vertically opposite angles are two non-adjacent angles that are formed by two intersecting lines.

Volume The volume is the amount of space that is contained within a 3D object.

W

Word form Word form means writing the number in words while ensuring that the decimal point is replaced by the word 'and'. E.g. 3.245 in word form is written as 'three and two hundred forty-five thousandths'.

X

x-axis The x-axis is the horizontal axis that runs left to right through zero.

Y

y-axis The y-axis is the vertical axis that runs up and down through zero.