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Australian Curriculum

MathsWorld 7

Australian Curriculum edition

Jill Vincent   Beth Price   Natalie Caruso
Allason McNamara   David Tynan

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Contents

Introduction

The ***MathsWorld Australian Curriculum editions*** have been rigorously developed to cover all the content and requirements of the Australian Curriculum. They are carefully written to provide comprehensive and accessible textbooks which cater for a range of ability levels. The textbooks are accompanied by a complete teacher resource package that provides extra resources, teacher notes and further support.

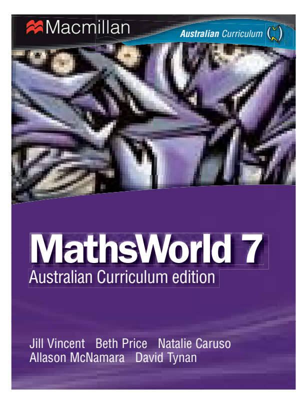
Access to Macmillan's innovative ebook platform, OneStopDigital, is included with purchase of the student textbook or is available as standalone digital access. As part of the teacher resource package, teachers have access to the ebook versions of both the student and teacher books.

OneStopDigital

Student book

Tossing a coin

Icons throughout the book indicate links to a wide range of worksheets, technology files, tests and quizzes. For example, the icon shown here appears on page 590 and links to an Excel file which simulates the tossing of a coin.



Theory and worked examples are structured into manageable sections and are designed to illustrate the main concepts and skills for each topic. Solutions to worked examples are structured into two columns, with a working column indicating what students should write and a reasoning column showing the thought process for each major step of working.

Sets of carefully graded sets of exercise questions occur at the end of each section. Exercise questions are cross-referenced to the relevant worked examples to reinforce learning and provide links where help is needed. Each exercise contains one or more clearly marked challenge questions.

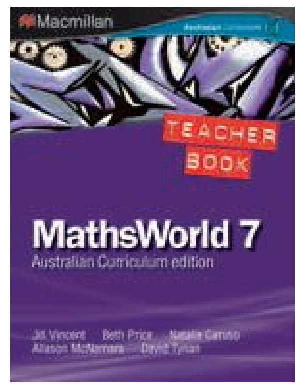
At the end of each chapter there is an analysis task which explores and ties together concepts covered in the chapter.

Each chapter ends with a thorough review section, consisting of a concise summary, a visual map task and sets of multiple-choice, short-answer and extended-response questions.

A practice quiz is included as a link from the ebook. The quiz can be used for revision at a later stage or could be used for individual student assignments.

Teacher book

The Teacher book provides a complete package of supplementary material and support with many time-saving and customisable resources available.



Pre-tests and answers in all chapters allow teachers to assess students' prior knowledge before commencing each chapter. These are also provided as PDF files linked from the student ebook.

Chapter warm-ups assist teachers with the introduction of the 'big idea' relevant to each topic. These are also provided as PDF files linked from the student ebook.

Teaching notes are provided for each chapter.

Additional worked examples are provided in the Teacher book—these can be used as teaching examples during the lesson.

Blackline masters and technology files are provided as links in the ebooks where appropriate.

The Teacher book also contains answers to all student exercises and all supplementary activities.

The answers to the Analysis tasks are provided in the Teacher book. Also provided are additional analysis tasks and answers.

Each chapter includes two tests, in editable Word format. Each chapter test includes multiple-choice, short-answer and extended-response questions and is provided with a marking scheme.

Curriculum links and planning documents, including a suggested teaching program, are included in the Teacher book.

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OTHER MATERIAL

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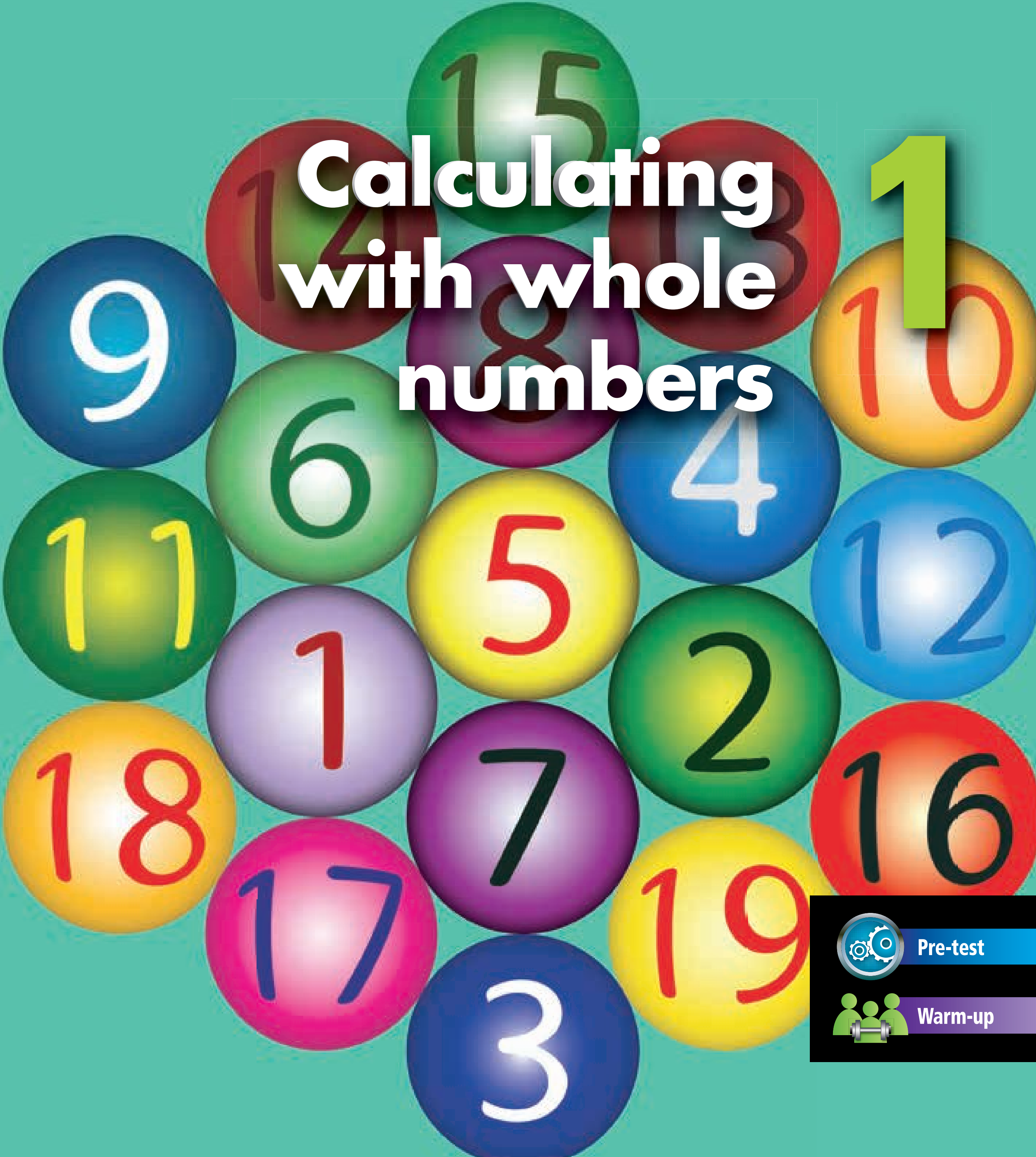
Calculating with whole numbers

1



Pre-test

Warm-up

There are many interesting number facts about this magic hexagon. It uses all the whole numbers from one to 19, but they are arranged in a special way. We can ask many questions about the numbers, for example, what is the total of all 19 numbers? What is the sum of the numbers on any straight line through the hexagon? Could any other number have been placed at the centre?

1.1 Positive integers and place value

Our number system, the Hindu–Arabic number system, originated in India over 2500 years ago and spread to Europe via Arab-Islamic scholars. You have met different sorts of numbers: whole numbers, fractions, and decimals. The word **integer** is used for whole numbers that may be positive or negative or zero. In this chapter we are looking at **positive integers**, that is, numbers such as 3, 14, 41 and 296.

When we write numbers, the position of each digit in the number determines its value. For example, when we write 13 or 156 or 1794 in our number system, the digit '1' has a different value in each number. In 13, 1 means 'one ten', in 156, 1 means 'one hundred' and in 1794, 1 means 'one thousand'.

The different place values for seven-digit numbers are summarised in the following table. The whole number 3 245 114 in words is three million, two hundred and forty-five thousand, one hundred and fourteen.

A zero is used to indicate an empty place value column. For example, the zero in 205 means that there are no tens.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	4	5	1	1	4

Example 1

Write the digits of the number 130 405 in the correct place value columns.

Working

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1	3	0	4	0	5

Reasoning

The value of 1 is 100 000.
The value of 3 is 30 000.
The value of 4 is 400.
The value of 5 is 5.
The thousands and tens place columns are empty.

Example 2

State the value of each of the digits in the number 4627.

Working

4 has the value 4000.
6 has the value 600.
2 has the value 20.
7 has the value 7.

Reasoning

Thousands	Hundreds	Tens	Ones
4	6	2	7

Example 3

What is the value of the digit 3 in each of these numbers?

a 63 917

b 4530

Working

a 3000

b 30

Reasoning

The digit 3 is in the thousands place value column.

The digit 3 is in the tens place value column.

Example 4

Write the following.

a 6 123 345 in words

b Nine thousand and forty-five as a number

Working

a Six million, one hundred and twenty-three thousand, three hundred and forty-five.

Reasoning

Identify the place value of each digit.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
6	1	2	3	3	4	5

continued

Example 4 continued

Working

b 9045

Reasoning

Thousands	Hundreds	Tens	Ones
9	0	4	5

To tell which is the larger number, we start by looking at the highest place value of each number. For example to compare 9476 and 34 815, we start by looking at the place value of the 9 in 9476 and the 3 in 34 815. In 9476, the place value of 9 is thousands. In 34 815, the place value of 3 is ten thousands. Ten thousands is bigger than thousands so 34 815 is larger than 9476.

Example 5

Insert > or < to make these statements true.

a 76 518 ___ 132 684

b 49 675 ___ 47 688

Working

a 76 518 < 132 684

Reasoning

The place value of 7 in 76 518 is ten thousands.
The place value of 1 in 132 684 is hundred thousands.
70 000 is less than 100 000.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
	7	6	5	1	8
1	3	2	6	8	4

continued

Example 5 continued

Working

b 49 675 > 47 688

Reasoning

The place value of 4 in both numbers is ten thousands.
Look at the next place value column.
The value of 9 in 49 675 is 9000.
The value of 7 in 47 688 is 7000.
9000 is greater than 7000 so 49 000 is greater than 47 000.

Ten thousands	Thousands	Hundreds	Tens	Ones
4	9	6	7	5
4	7	6	8	8

Example 6

Write these numbers in ascending order, that is, from smallest to largest.
1899, 20 345, 972, 2034, 19 989, 2304, 729, 2345, 1085, 927

Working	Reasoning
20 345, 19 989 1899, 2034, 2304, 2345, 1085 972, 729, 927	Sort into groups by the largest place value.
20 345, 19 989 2345, 2304, 2034, 1899, 1085 972, 927, 729	Sort each group by comparing the digits in the largest place value. If the digits in the largest place value are the same, sort by the next place value.
In ascending order, the numbers are 729, 927, 972, 1085, 1899, 2304, 2345, 19 989, 20 345.	Write the complete list from smallest to largest.

Place value notation

We can write numbers in place value notation to show the place value of each digit in the number.

Example 7

Write the following numbers in place value notation.

a 7030 **b** 45 716

Working

a 7030
$= 7 \times 1000 + 0 \times 100 + 3 \times 10 + 0$

Reasoning

The zeros are necessary to show that there is an empty hundreds place value column and an empty ones place value column.

Thousands	Hundreds	Tens	Ones
7	0	3	0

b 45 716
$= 4 \times 10000 + 5 \times 1000 + 7 \times 100 + 1 \times 10 + 6$

Ten thousands	Thousands	Hundreds	Tens	Ones
4	5	7	1	6

Powers of 10

When a number is multiplied by itself it can be written as a power. When we write 10^3 we know that 3 tens have been multiplied together. The 3 is called an **index**. The following table shows how we write powers of 10. The index for each power of 10 is the same as the number of tens that have been multiplied together. Notice that the index is also the same as the number of zeros in the place value number.

$$\underbrace{10 \times 10 \times 10 \times 10 \times 10}_{\text{5 tens multiplied together}}$$

10	10	10^1
100	10×10	10^2
1000	$10 \times 10 \times 10$	10^3
10 000	$10 \times 10 \times 10 \times 10$	10^4
100 000	$10 \times 10 \times 10 \times 10 \times 10$	10^5

Using the number 4627 as an example, the following table shows the values of each digit written using powers of 10.

Thousands	Hundreds	Tens	Ones
4000	600	20	7
4×1000	6×100	2×10	7
4×10^3	6×10^2	2×10^1	7

So 4627 can be written in place value notation as $4 \times 10^3 + 6 \times 10^2 + 2 \times 10^1 + 7$.

Example 8

Write the following numbers in place value notation.

a 35 847 **b** 4003

Working

a 35 847
$= 3 \times 10\,000 + 5 \times 1000 + 8 \times 100 + 4 \times 10 + 7$
$= 3 \times 10^4 + 5 \times 10^3 + 8 \times 10^2 + 4 \times 10^1 + 7$

Reasoning

The place value columns are ten thousands, thousands, hundreds, tens and ones.

b 4003
$= 4 \times 1000 + 0 \times 100 + 0 \times 10 + 3$
$= 4 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3$

4 has the value 4000.
The hundreds and tens place value columns are empty.
There are 3 ones.

Example 9

Write as ordinary decimal numbers.

a $3 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 8$
b $7 \times 10^5 + 0 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 2$

Working

a $3 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 8$
$= 3 \times 1000 + 5 \times 100 + 2 \times 10 + 8$
$= 3000 + 500 + 20 + 8$
$= 3528$

Reasoning

Evaluate each power of 10.
$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$

b $7 \times 10^5 + 0 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 2$
$= 7 \times 100\,000 + 0 \times 10\,000 + 3 \times 1000$
$+ 9 \times 100 + 0 \times 10 + 2$
$= 703\,902$

Evaluate each power of 10.
$10^5 = 100\,000$
$10^4 = 10\,000$
$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$

Powers of 10 are useful for writing large numbers. For example instead of writing 400 000,

$$400\,000 = 4 \times 100\,000$$
$$= 4 \times 10^5$$

Example 10

Write these numbers using powers of 10.

a 600 **b** 8 000 000 000 **c** 250 000

Working

a 600
$= 6 \times 100$
$= 6 \times 10^2$

b 8 000 000 000
$= 8 \times 1\,000\,000\,000$
$= 8 \times 10^9$

c 250 000
$= 25 \times 10\,000$
$= 25 \times 10^4$

Reasoning

$100 = \underbrace{10 \times 10}_{\text{2 tens multiplied together}}$

The index is 2.

1 000 000 000
$= \underbrace{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}_{\text{9 tens multiplied together}}$

The index is 9.

$10000 = \underbrace{10 \times 10 \times 10 \times 10}_{\text{4 tens multiplied together}}$

The index is 4.

exercise 1.1

LINKS TO Example 1

1 For each of the following numbers, write the digits in the correct place values.

		Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
a	4086							
b	12 792							
c	1 040 806							
d	390 600							
e	2 340 000							
f	964 005							

LINKS TO Example 2

What is the value of the digit in bold in the following numbers?

a 2**4**5 **b** **6**78 **c** **5**478 **d** 8**7**96
e 67 5**8**9 **f** 1**2**3 675 **g** 7 **8**79 706 **h** **2** 708 574

LINKS TO Example 3

What is the value of the digit 5 in each of these numbers?

a 250 **b** 5427 **c** 286 514 **d** 9 853 120
e 7985 **f** 16 529 **g** 7 245 381 **h** 596 800

LINKS TO Example 4a

Write the following numbers in words.

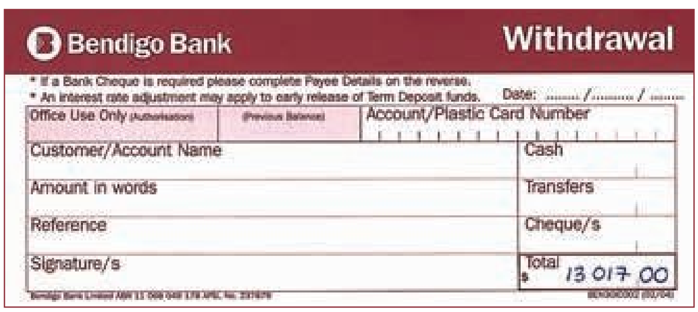
a 587 **b** 3476 **c** 75 896 **d** 134 876
e 200 413 **f** 7 897 784 **g** 6 009 500 **h** 2 333 333

LINKS TO Example 4b

Write the following as numbers.

a two hundred and fifty-seven
b one thousand, one hundred and forty-three
c eleven thousand, six hundred and twenty-nine
d one hundred and forty-five thousand and fifty-seven

Write in words the amount shown on the withdrawal form.


Bendigo Bank Withdrawal
* If a Bank Cheque is required please complete Payee Details on the reverse.
* An interest rate adjustment may apply to early release of Term Deposit funds.
Date: .... / .... / ....
Office Use Only (Authorisation) (Previous Balance)
Account/Plastic Card Number
Customer/Account Name
Cash
Amount in words
Transfers
Reference
Cheque/s
Signature/s
Total $ 13 017 00

Amina has been away from school and calls her friend Katie to find out what she missed in the mathematics class. Katie reads aloud a list of 12 numbers that Amina will need to complete her homework. Amina writes them down as numbers. The first six numbers Katie reads out are listed in words below. Write down each of the numbers as Amina would record them.

a one thousand, three hundred and forty-five
b two thousand and eighty
c twelve thousand, three hundred and eighty-two
d seven million, three hundred and fifty-seven thousand
e sixty-five thousand and twenty-one
f seven hundred and thirty-nine thousand

The next six numbers recorded by Amina are shown below. Write them in words.

g 3472 **h** 19 056 **i** 375 124
j 41 001 **k** 305 080 **l** 21 567 053

LINKS TO Example 5

Insert a $>$ (greater than) or $<$ (less than) sign to make the following true.

a 254 ____ 245 **b** 657 ____ 768 **c** 3453 ____ 2768
d 1786 ____ 985 **e** 1764 ____ 1674 **f** 2875 ____ 5782
g 8560 ____ 10 499 **h** 23 060 ____ 9896 **i** 11 000 ____ 10 003

LINKS TO Example 6

Sort these numbers into ascending order (from smallest to largest).

a 2009, 115, 2089, 2890, 1998, 511 **b** 728, 1506, 287, 516, 1560, 1800
c 80 100, 8900, 998, 10 800, 1800, 1098 **d** 5003, 5968, 5698, 53 000, 8965, 5010

Sort these numbers into descending order (from largest to smallest).

a 1700, 10 070, 1070, 710, 7100, 10 700 **b** 2030, 320, 2300, 302, 3020, 20 030
c 1090, 901, 10 090, 1009, 9001, 10 009 **d** 4800, 41 000, 804, 4080, 1400, 8004

LINKS TO Example 7

Write the following numbers in place value notation.

a 346 **b** 6478 **c** 8965 **d** 12 786
e 54 658 **f** 123 765 **g** 786 546 **h** 1 432 567

LINKS TO Example 8

Write these numbers in place value notation using powers of 10.

a 6439 **b** 396 **c** 2580 **d** 56 914
e 17 085 **f** 5092 **g** 24 305 **h** 675 854
i 20 800 **j** 13 005 **k** 7 654 205 **l** 6 035 002

LINKS TO Example 9

Write these numbers as ordinary decimal numbers.

a $5 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 4$
b $6 \times 10^4 + 3 \times 10^3 + 1 \times 10^2 + 8 \times 10^1 + 5$
c $8 \times 10^5 + 0 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 6$
d $9 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 1 \times 10^2 + 1 \times 10^1 + 5$
e $6 \times 10^5 + 7 \times 10^4 + 2 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 2$
f $9 \times 10^5 + 3 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 9$
g $5 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 0$
h $8 \times 10^5 + 0 \times 10^4 + 9 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 5$
i $7 \times 10^5 + 0 \times 10^4 + 0 \times 10^1 + 3 \times 10^3 + 9 \times 10^2 + 2$
j $6 \times 10^4 + 0 \times 10^3 + 9 \times 10^2 + 3 \times 10^1 + 8$
k $7 \times 10^8 + 0 \times 10^7 + 6 \times 10^6 + 3 \times 10^5 + 0 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 4$
l $2 \times 10^6 + 5 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 8 \times 10^2 + 9 \times 10^1 + 6$

LINKS TO Example 10

Write these numbers using powers of 10.

a 7000 **b** 900 **c** 40 000
d 600 000 **e** 3000 **f** 800 000
g 20 000 **h** 7 000 000 **i** 3 300 000
j 950 000 **k** 2 700 000 000 **l** 84 000 000 000 000 000

Write these numbers as ordinary numbers.

a 5×10^3 **b** 7×10^4 **c** 2×10^1 **d** 5×10^6
e 3×10^5 **f** 9×10^2 **g** 6×10^8 **h** 12×10^3
i 45×10^9 **j** 83×10^7 **k** 75×10^6 **l** 36×10^{12}

Write these numbers using powers of 10.

a Five thousand
b Six million
c Two hundred thousand
d Thirty thousand
e Seven thousand
f Two million
g Sixty-four thousand
h Five hundred and twenty thousand

exercise 1.1 challenge

For questions 17 and 18, assume that a whole number cannot start with zero.

Make the largest possible whole number using all of the following digits.

a 3, 8, 7, 8 and 5
b 5, 6, 2, 0, 8, 7 and 0

Make the smallest possible whole number using all of the following digits.

a 3, 5, 2 and 0
b 2, 1, 0, 0, 8, 3 and 9

Write in words as well as with digits.

a What is the largest seven-digit integer?
b What is the smallest seven-digit integer?

Write these numbers using powers of 10.

a Some stars have a temperature as high as 40 000 degrees Celsius.
b The speed of light is approximately 300 000 000 metres per second.
c The distance from Earth to the Sun is approximately 150 million kilometres.

1.2 Addition and subtraction

The result of adding two or more numbers is called the **sum**.

Example 11

Do these additions mentally.

a 17 + 8 **b** 24 + 16

Working

a 17 + 8 = 25

Reasoning

$$\begin{aligned} 17 + 8 &= 17 + 3 + 5 \\ &= 20 + 5 \\ &= 25 \end{aligned}$$

b 24 + 16 = 40

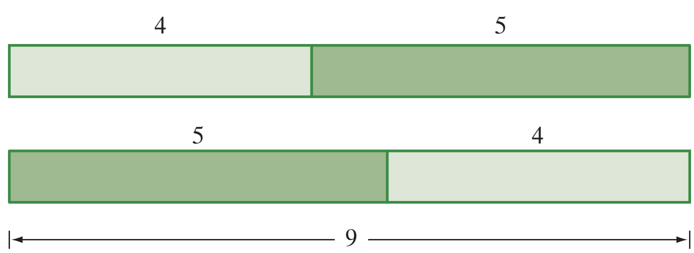
Reasoning

$$\begin{aligned} 24 + 16 &= (24 + 6) + 10 \\ &= 30 + 10 \\ &= 40 \end{aligned}$$

or,

$$\begin{aligned} 24 + 16 &= (20 + 10) + (4 + 6) \\ &= 30 + 10 \\ &= 40 \end{aligned}$$

When we add two numbers, *the order in which we add them does not matter*. For example, 4 + 5 = 9 and 5 + 4 = 9.



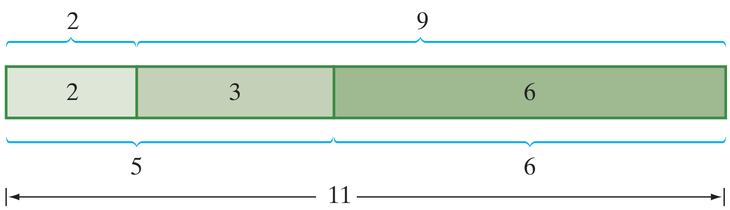
Mathematicians call this the **commutative law**. The word *commutative* has the same origin as the word *commuter*, who is someone who travels backwards and forwards to work each day.

(pronounced com-mew-ta-tive)

There are many things we do in everyday life where the order we do them in doesn't matter, for example, the order in which we put on our two shoes doesn't matter. Some things, though, must be done in a particular order—we would not put on our shoes before our socks. It might seem obvious to us that the addition of two numbers can be done in any order, but there are other mathematical operations that cannot be done in any order. For example, 12 − 4 is not the same as 4 − 12.

When we have several numbers to add, it does not matter which two we add first. For example,

$\underbrace{2 + 3} + 6$	and	$2 + \underbrace{3 + 6}$
$= 5 + 6$		$= 2 + 9$
$= 11$		$= 11$



Mathematicians call this the **associative law** because it doesn't matter how the numbers associate with each other when we add them.

These are important laws of arithmetic that apply to addition (but not to subtraction). They are very useful when we are adding several numbers mentally. We can look for pairs that are easy to add.

Example 12

Carry out these additions without a calculator.

a 23 + 46 + 17 + 34 **b** 281 + 350 + 29

Working	**Reasoning**
a 23 + 46 + 17 + 34 = 23 + 17 + 46 + 34 = 40 + 80 = 120	Look for pairs of numbers that can be added easily. 7 + 3 = 10 and 6 + 4 = 10 so pair 23 with 17 and 46 with 34.
b 281 + 350 + 29 = 281 + 29 + 350 = 290 + 20 +350 = 310 + 350 = 660	1 + 9 = 10 so pair 281 with 29.

For adding larger numbers or lists of numbers, we start by writing the numbers underneath each other, being careful to line up the place value columns. First we add the ones, carrying over any tens into the tens column. Then we add the tens, carrying over any hundreds into the hundreds column and so on.

Example 13

Carry out this addition.

$$\begin{array}{r} 3\,7\,3\,8 \\ 2\,1\,8\,9 \\ +\quad 1\,6\,7 \\ \hline \\ \hline \end{array}$$

Working

$$\begin{array}{r} 3\,7\,3\,8 \\ 2\,1\,8\,9 \\ +\quad {}_{1}1{}_{1}6{}_{2}7 \\ \hline 6\,0\,9\,4 \\ \hline \end{array}$$

The word 'sum' should be used only when we are talking about the result of addition.

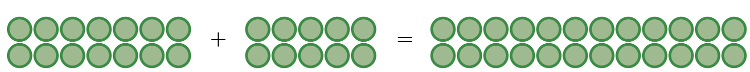
Reasoning

Add the ones: 8 + 9 + 7 = 24.
Write the 4 in the ones column.
Carry 2 tens to the tens column.
Add the tens: 3 + 8 + 6 + 2 = 19.
Write 9 in the tens column.
Carry 1 hundred to the hundreds column.
Add the hundreds: 7 + 1 + 1 + 1 = 10
Write 0 in the hundreds column.
Carry 1 thousand to the thousands column.
Add the thousands: 3 + 2 + 1 = 6.
Write 6 in the thousands column.

Odd and even numbers

An **even number** is a number that is exactly divisible by 2. When an **odd number** is divided by 2 there is always a remainder of 1.

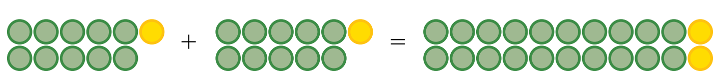
When two even numbers are added, the result is always even.



14 + 10 = 24

even + even = even

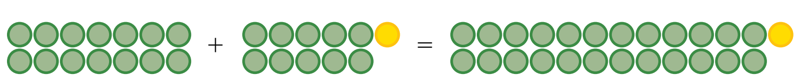
When two odd numbers are added, the result is always even.



11 + 11 = 22

odd + odd = even

When an even number and an odd number are added, the result is always odd.



14 + 11 = 25

even + odd = odd

Subtraction

We can see the relationship between addition and subtraction using a simple example such as $17 - 5 = 12$. This is equivalent to writing $12 + 5 = 17$. Any subtraction can be rewritten in this way as an addition.

When we subtract two numbers, the order in which we subtract them is important. For example, $17 - 5$ is not the same as $5 - 17$.

The result of subtracting one number from another is called the **difference**.

When we carry out subtractions mentally, we look for strategies that will make the subtraction easier, just as we did for addition. Look at the different methods that are used for the subtractions in example 15.

Example 14

Do these subtractions mentally, explaining the strategy.

a $67 - 28$ **b** $93 - 56$ **c** $126 - 49$

Working	Reasoning
a $67 - 28$ Starting at 28, 2 more will take us to 30 and another 37 will take us to 67, so altogether we have counted on 39. So $67 - 28 = 39$	We are really thinking $28 + ? = 67$.
b $93 - 56$ $93 - 50 = 43$ $43 - 6 = 37$ So $93 - 56 = 37$	Do the subtraction in two steps. $93 - 56 = 93 - 50 - 6$
c $126 - 49$ $126 - 50 = 76$ $76 + 1 = 77$ So $126 - 49 = 77$	Round 49 up to 50. Subtract 50 from 126. But this means we have subtracted too much so add 1. $126 - 49 = 126 - 50 + 1$

When subtracting larger numbers, it is easier to write the number you are subtracting underneath the other number, making sure that the place value columns are lined up.

There are two subtraction methods in common use. As long as you are confident with the method you are using, it does not matter which method you choose.

The two methods are shown below for 43 − 28.

Method 1	Method 2
We take 1 of the tens from the 4 tens of 43 and rename it as 10 ones so that we have 13 ones.	We add 10 ones to the 3 ones of 43, giving 13 ones. But to keep the result correct we must add 1 ten to 28.
$\begin{array}{r} ^{3}\not{4}\,^{1}3 \\ -\quad 2\,8 \\ \hline 1\,5 \\ \hline \end{array}$	$\begin{array}{r} 4\,^{1}3 \\ -\quad ^{3}\not{2}\,8 \\ \hline 1\,5 \\ \hline \end{array}$

Example 15

Carry out these subtractions.

a $\begin{array}{r} 5\,4\,1 \\ -\quad 3\,7\,6 \\ \hline \\ \hline \end{array}$

b $\begin{array}{r} 8\,0\,4\,1 \\ -\quad 1\,6\,5\,4 \\ \hline \\ \hline \end{array}$

Working

a **Method 1**

$$\begin{array}{r} ^{4}\not{5}\,^{13}\not{4}\,^{1}1 \\ -\quad 3\;\;7\,6 \\ \hline 1\;\;6\,5 \\ \hline \end{array}$$

Reasoning: Split 4 tens of 541 into 3 tens and 10 ones to make 11 ones. Split 5 hundreds of 541 into 4 hundreds and 10 tens to make 13 tens.

Method 2

$$\begin{array}{r} 5\,^{1}4\,^{1}1 \\ -\quad ^{4}\not{3}\,^{8}\not{7}\,6 \\ \hline 1\,6\,5 \\ \hline \end{array}$$

Reasoning: Add 10 ones to 541 to make 11 ones and balance by adding 1 ten to 376 to make 8 tens.
Add 10 tens to 541 to make 14 tens and balance by adding 1 hundred to 376 to make 4 hundreds.

b **Method 1**

$$\begin{array}{r} ^{7}\not{8}\,^{9}\not{0}\,^{13}\not{4}\,^{1}1 \\ -\quad 1\,6\,5\,4 \\ \hline 6\,3\,8\,7 \\ \hline \end{array}$$

Reasoning: Split 4 tens of 8041 into 3 tens and 10 ones to make 11 ones.
There are no hundreds in 8041, so split 8 thousands into 7 thousands and 10 hundreds.
Split the 10 hundreds into 9 hundreds and 10 tens to make 13 tens.

Method 2

$$\begin{array}{r} 8\,^{1}0\,^{1}4\,^{1}1 \\ -\quad ^{2}\not{1}\,^{7}\not{6}\,^{6}\not{5}\,4 \\ \hline 6\,3\,8\,7 \\ \hline \end{array}$$

Reasoning: Add 10 ones to 8041 to make 11 ones and balance by adding 1 ten to 1654 to make 6 tens.
Add 10 tens to 8041 to make 14 tens and balance by adding 1 hundred to 1654 to make 7 hundreds.
Add 10 hundreds to 8041 to make 10 hundreds and balance by adding 1 thousand to 1654 to make 2 thousands.

Tech tip

Addition and subtraction can be done on the TI-30XB MultiView calculator as follows.

Addition

23 + 34 = 57

Press [2][3][+][3][4][enter].

Subtraction

54 − 37 = 17

Press [5][4][−][3][7][enter].

exercise 1.2

LINKS TO Example 11

1 Complete these additions mentally.

a 6 + 14 b 23 + 7 c 8 + 12 d 11 + 19 e 5 + 26 f 17 + 13
g 17 + 14 h 14 + 16 i 14 + 17 j 18 + 12 k 18 + 15 l 23 + 29

LINKS TO Example 11

2 Complete these additions mentally.

a 29 + 71 b 48 + 9 c 67 + 8 d 98 + 102
e 34 + 17 f 156 + 34 g 83 + 128 h 74 + 76
i 53 + 98 j 117 + 42 k 65 + 76 l 48 + 65

LINKS TO Example 12

3 Complete these additions mentally by pairing numbers to make the addition easier.

a 21 + 19 + 35 + 25 b 34 + 16 + 42 + 58 c 17 + 28 + 43 + 52
d 11 + 47 + 29 + 23 e 18 + 23 + 62 + 57 f 21 + 24 + 46 + 79
g 32 + 77 + 8 + 23 h 25 + 19 + 41 + 85 i 13 + 46 + 87 + 54
j 14 + 28 + 46 + 13 k 17 + 37 + 23 + 64 l 22 + 47 + 63 + 40

LINKS TO Example 13

4 Complete the following additions without using a calculator.

a $\begin{array}{r} 98 \\ +\ 79 \\ \hline \\ \hline \end{array}$ b $\begin{array}{r} 87 \\ +\ 68 \\ \hline \\ \hline \end{array}$ c $\begin{array}{r} 56 \\ +\ 87 \\ \hline \\ \hline \end{array}$ d $\begin{array}{r} 32 \\ +\ 95 \\ \hline \\ \hline \end{array}$

e $\begin{array}{r} 285 \\ +\ 346 \\ \hline \\ \hline \end{array}$ f $\begin{array}{r} 394 \\ +\ 537 \\ \hline \\ \hline \end{array}$ g $\begin{array}{r} 951 \\ +\ 853 \\ \hline \\ \hline \end{array}$ h $\begin{array}{r} 886 \\ +\ 999 \\ \hline \\ \hline \end{array}$

i $\begin{array}{r} 1324 \\ +\ 2756 \\ \hline \\ \hline \end{array}$ j $\begin{array}{r} 7886 \\ +\ 9788 \\ \hline \\ \hline \end{array}$ k $\begin{array}{r} 3\,796\,104 \\ +\ 213\,896 \\ \hline \\ \hline \end{array}$ l $\begin{array}{r} 39\,327 \\ +\ 87\,743 \\ \hline \\ \hline \end{array}$

LINKS TO Example 14

5 Complete the following subtractions mentally. Compare your strategies with those of other students.

a 70 − 23 b 45 − 18 c 37 − 29 d 74 − 56
e 53 − 35 f 62 − 19 g 86 − 57 h 51 − 39
i 76 − 48 j 32 − 17 k 60 − 37 l 93 − 39

LINKS TO Example 15

Complete the following subtractions without using a calculator.

a $\begin{array}{r} 57 \\ -\ 43 \\ \hline \end{array}$ **b** $\begin{array}{r} 96 \\ -\ 58 \\ \hline \end{array}$ **c** $\begin{array}{r} 78 \\ -\ 69 \\ \hline \end{array}$

d $\begin{array}{r} 84 \\ -\ 66 \\ \hline \end{array}$ **e** $\begin{array}{r} 367 \\ -\ 243 \\ \hline \end{array}$ **f** $\begin{array}{r} 583 \\ -\ 497 \\ \hline \end{array}$

g $\begin{array}{r} 452 \\ -\ 375 \\ \hline \end{array}$ **h** $\begin{array}{r} 4763 \\ -\ 3676 \\ \hline \end{array}$ **i** $\begin{array}{r} 3507 \\ -\ 2688 \\ \hline \end{array}$

j $\begin{array}{r} 9001 \\ -\ 6789 \\ \hline \end{array}$ **k** $\begin{array}{r} 8709 \\ -\ 5789 \\ \hline \end{array}$ **l** $\begin{array}{r} 78\,746 \\ -\ 65\,879 \\ \hline \end{array}$

Complete the following subtractions.

a 92 − 53 **b** 85 − 26 **c** 345 − 288
d 878 − 799 **e** 2345 − 1767 **f** 8017 − 6349

The table shows the population of the Australian states and territories at the end of June 2010 and the area of each state in square kilometres. In each case the values are rounded to the nearest hundred.

a Calculate the total population.
b Calculate the total area.

State or Territory	Population at June 30, 2010	Area (km^2)
New South Wales	7 238 800	801 600
Queensland	4 516 400	1 727 200
South Australia	1 644 600	984 000
Tasmania	507 600	67 800
Victoria	5 547 500	227 600
Western Australia	2 296 400	2 525 500
Australian Capital Territory	358 900	2400
Northern Territory	229 700	1 346 200

1.2

Calculate the following.

a Samantha scored 98 for her Mathematics test and Eric scored 79 for his. What is the difference between their scores?

b Aidan owes Carlin \$87 for a Nintendo DS game minus \$19 for the lunch he bought Carlin. How much does Aidan owe Carlin now?

c A farmer has 1571 boxes of oranges. A delivery truck took 888 of the boxes to a factory outlet. How many boxes does the farmer still have?

d If Peter has 85 lollies and he eats 17 of them, how many lollies does he have left?

e In June 2009 the population of Australia to the nearest thousand was 21 950 000. If 11 020 000 were females, how many were males?

f Adult male walruses grow to about 1200 kg and females grow to about 850 kg. How much heavier are adult male walruses compared with female walruses?

Find the missing digits in the following additions.

a
```
   1 2 __
 + 2 4 3
 -------
   3 __ 1
```

b
```
   4 __ 8
 + __ 3 7
 --------
   6 3 __
```

c
```
   __ 4 __
 + 6 7 3
 --------
 1 2 __ 3
```

d
```
   __ 7 8
 + 9 9 __
 --------
 1 2 __ 5
```

Calculate the following by working from left to right.

a 8 + 4 − 12 **b** 13 + 7 − 7 **c** 24 + 12 − 14

d 19 + 8 − 18 **e** 5 + 26 − 5 **f** 14 − 8 − 6

g 11 + 17 − 7 **h** 28 + 12 − 2 **i** 16 + 9 − 19

j 17 + 11 − 8 **k** 9 + 15 − 9 **l** 12 + 45 − 12

A magic square is an array of numbers set in a square grid, in which each row, column and diagonal has the same sum. Complete the following magic squares.

a

8	9	
	7	
		6

b

20	17	8
22		

c

40		
7	31	55

d

		19
	21	23
23		

A hot air balloon can carry a maximum weight of 340 kilograms, including the pilot and passengers. The pilot weighs 71 kilograms and three passengers weigh 58 kilograms, 62 kilograms and 85 kilograms.

a Find the total weight of the pilot and three passengers.

b If another passenger goes on the flight, what can their maximum weight be?

Find the two numbers.

a the sum is 140 and their difference is 6

b the sum is 168 and their difference is 16

Two Australian rules football teams, the Mudlarks and the Galahs, were playing in the semi-final. Their points for each of the four quarters of the match were as follows.

	First quarter	Second quarter	Third quarter	Final quarter
Mudlarks	37	19	37	18
Galahs	18	23	40	27

a Calculate the total score for

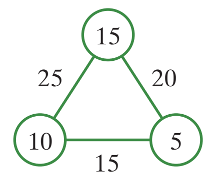
i the Mudlarks. **ii** the Galahs.

b Which team won, and by how many points?

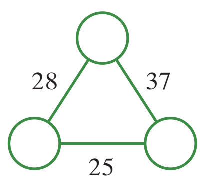
Josh borrows $150 000 from the Rural Bank. If he purchases a tractor for $75 500 and a harvester for $32 750, how much money will he have left over?

exercise 1.2 challenge

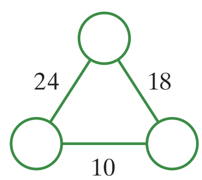
Complete the following arithmagons by inserting numbers into the circles. The numbers in the circles at the end of the line must sum to the number on the line like this example.



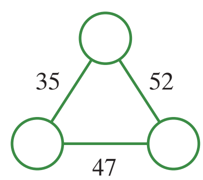
a



b

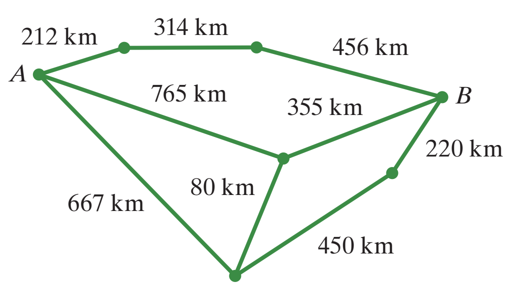


c

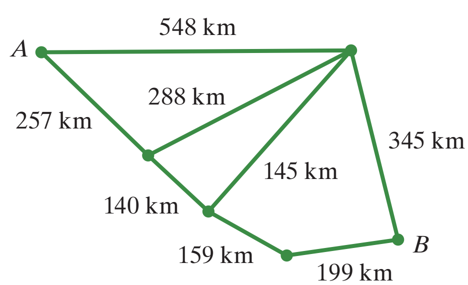


Find the shortest distance from town A to town B in each of the following.

a



b

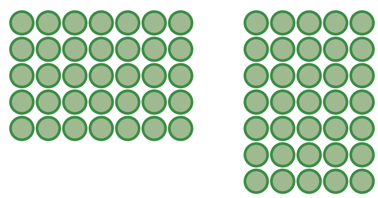


1.3 Multiplication

When we multiply two numbers, the result is called the product. As we shall see in this section there are many ways of finding the product of two numbers. The method we choose depends on the numbers that we are multiplying.

Multiplication can be represented as repeated addition. For example, $7 + 7 + 7 + 7 + 7$ is represented as a multiplication by 5×7.

The product of two numbers can be represented as an array. We can see that 5 rows of 7 is equal to 7 rows of 5, that is, $5 \times 7 = 7 \times 5$. Like addition, *the order does not matter for multiplication*.



If we have several numbers to multiply, it does not matter which two numbers we multiply first.

For example,

$$\underbrace{2 \times 4} \times 5 \qquad\qquad 2 \times \underbrace{4 \times 5}$$

$$= 8 \times 5 \qquad \text{and} \qquad = 2 \times 20$$

$$= 40 \qquad\qquad\qquad = 40$$

The commutative and associative laws apply to multiplication.

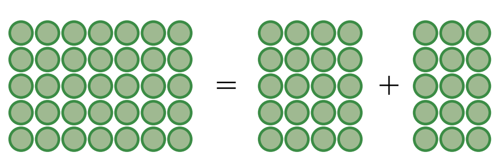
The distributive law

Distributive law

Notice that we can split the array into two parts and we still have the same number of dots. Here we have split 7 into $4 + 3$. We could also have used $5 + 2$ or $6 + 1$.

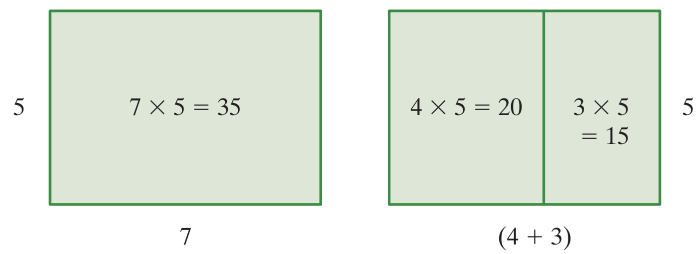
$$5 \times 7 = 5(4 + 3) = 5 \times 4 + 5 \times 3$$

Check: $35 = 20 + 15$



This number property is an example of the **distributive law**. It means that the number outside the brackets is distributed over the numbers inside the brackets. In the example below, we see that the 6 and the 7 inside the brackets are both multiplied by the 11 that is outside the brackets.

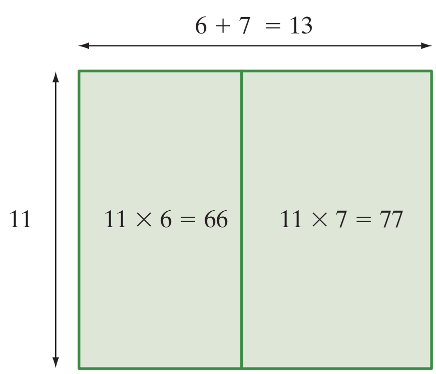
We can also use an area diagram to illustrate the distributive law.



Example 16

Draw an area diagram for $11 \times 13 = 11 \times 6 + 11 \times 7$ and hence find the product of 11 and 13.

Working



$$\begin{aligned} 11 \times 13 &= 11(6 + 7) \\ &= 11 \times 6 + 11 \times 7 \\ &= 66 + 77 \\ &= 143 \end{aligned}$$

Reasoning

The whole area is equal to the sum of the areas of the two smaller rectangles.

The distributive law also works if the bracket has a subtraction sign instead of an addition sign, for example, $4(12 - 7) = 4 \times 12 - 4 \times 7$.

Example 17

Evaluate these number expressions.

a $8 \times 23 + 8 \times 7$

b $29 \times 13 - 29 \times 3$

continued

Example 17 continued

Working | **Reasoning**

a $8 \times 23 + 8 \times 7$
$= 8(23 + 7)$
$= 8 \times 30$
$= 240$

The distributive law tells us that 8 times 23 plus 8 times 7 is equal to 8 times the sum of 23 and 7.

b $29 \times 13 - 29 \times 3$
$= 29(13 - 3)$
$= 29 \times 10$
$= 290$

The distributive law tells us that 29 times 13 minus 29 times 3 is equal to 29 times the difference of 13 and 3.

Example 18

Use the distributive law to find these products.

a 7×53 **b** 8×29

Working | **Reasoning**

a $7 \times 53 = 7(50 + 3)$
$= 7 \times 50 + 7 \times 3$
$= 350 + 21$
$= 371$

Split 53 into 50 and 3 so that multiplying by 7 is easier.
The distributive law tells us that we can write $7(50 + 3) = 7 \times 50 + 7 \times 3$.

b $8 \times 29 = 8(30 - 1)$
$= 8 \times 30 - 8 \times 1$
$= 240 - 8$
$= 232$

Write 29 as $30 - 1$ so that multiplying by 8 is easier.
The distributive law tells us that we can write $8(30 - 1) = 8 \times 30 - 8 \times 1$.

The distributive law is useful for finding quick mental strategies for multiplying.

Example 19

Calculate the products mentally.

a 9×19 **b** 8×73

Working | **Reasoning**

a $9 \times 19 = 9(20 - 1)$
$= 9 \times 20 - 9 \times 1$
$= 180 - 9$
$9 \times 19 = 171$

The distributive law tells us that 9 times 19 is equal to 9 times 20 minus 9 times 1.

b $8 \times 73 = 8(70 + 3)$
$= 8 \times 70 + 8 \times 3$
$= 560 + 24$
$8 \times 73 = 584$

The distributive law tells us that 8 times 73 is equal to 8 times 70 plus 8 times 3.

Multiplying by zero and one

The product of any number and zero is zero. For example, $6 \times 0 = 0$.

When a number is multiplied by 1 it is not changed. For example, $6 \times 1 = 6$.

Multiplying by powers of 10

Multiplying a number by 10 shifts all the digits up one place value so that the ones become tens, the tens become hundreds and so on. For example, 365 multiplied by 10 is 3650. Notice that we put a zero in the empty ones place column because there are no ones.

Thousands	Hundreds	Tens	Ones	
	3	6	5	× 10
3	6	5	0	

If we multiply by 100, each digit moves up two place values, if we multiply by 1000, each digit moves up three place values, and so on. We put zeros in the empty place values that are left when the digits move up.

Example 20

Carry out these multiplications.

a 46×1000

b 704×100

Working

a $46 \times 1000 = 46\,000$

Reasoning

Each digit moves up three place values.

Ten thousands	Thousands	Hundreds	Tens	Ones
			4	6
4	6	0	0	0

b $704 \times 100 = 70\,400$

Each digit moves up two place values.

Ten thousands	Thousands	Hundreds	Tens	Ones
		7	0	4
7	0	4	0	0

We have already seen that the order in which we multiply numbers does not matter. If we have three numbers to multiply, we can multiply any two numbers first and then multiply by the third number. For example, $2 \times 3 \times 5$ can be calculated in any of these ways:

$$\underbrace{2 \times 3} \times 5 = 6 \times 5 = 30 \qquad 2 \times \underbrace{3 \times 5} = 2 \times 15 = 30 \qquad \underbrace{2 \times 5} \times 3 = 10 \times 3 = 30$$

We can make use of this property when doing multiplications mentally by pairing the numbers to make the multiplication easier.

Example 21

Calculate $2 \times 13 \times 15$ mentally by pairing numbers as above.

Working

$2 \times 13 \times 15 = 2 \times 15 \times 13$
$= 30 \times 13$
$= 390$

Reasoning

By changing the order of the numbers in the multiplication, we can pair 2 and 15 to give 30. Multiplication of 13 by 3 and by 10 is then easy.

Short multiplication

The short multiplication method is used when we are multiplying a number by a single-digit number.

Example 22

Calculate the following.

$$\begin{array}{r} 8\,6\,5 \\ \times \quad 7 \\ \hline \\ \hline \end{array}$$

Working

$$\begin{array}{r} {}^{4}8\,{}^{3}6\,5 \\ \times \quad 7 \\ \hline 6\,0\,5\,5 \\ \hline \end{array}$$

or

865 7
$= 7\ (800 + 60 + 5)$
$= 7 \quad 800 + 7 \quad 60 + 7 \quad 5$
$= 5600 + 420 + 35$
$= 6055$

Reasoning

7 times 5 is 35. Put the 5 in the units column and carry the 3 tens.

7 times 6 is 42. Adding the carried 3 tens gives 45 tens. Put the 5 in the tens column and carry the 40 tens as 4 hundreds.

7 times 8 is 56. Adding the carried 4 hundreds gives 60 hundreds; that is, 6 thousands.

Put the 6 in the thousands column and the 0 in the hundreds column.

Two-step multiplications

Multiplying by two-digit numbers is easy if the two-digit number has simple factors. For example, if we are multiplying by 14, we can multiply by 2 then by 7 (or by 7 then by 2).

Example 23

Carry out these multiplications using two short multiplications.

a 18×14

b 320×24

Working

a 18×14
$= 18 \times 7 \times 2$
$= 126 \times 2$
$= 252$

Reasoning

Multiplying by 7 and then by 2 is easier than multiplying by 14.
Alternatively we could have multiplied by 2 first then by 7.
18×14
$= 18 \times 2 \times 7$
$= 36 \times 7$
$= 252$

b 320×24
$= 320 \times 4 \times 6$
$= 1280 \times 6$
$= 7680$

Alternatively we could have multiplied by 3 then by 8.
320×24
$= 320 \times 3 \times 8$
$= 960 \times 8$
$= 7680$

Example 24

Calculate the following.

a 213×300

b 1200×400

c $22\,000 \times 600$

Working

a 213×300
$= 213 \times 3 \times 100$
$= 639 \times 100$
$= 63900$

Reasoning

Write 300 as 3×100.
Multiply 213 by 3 then by 100.

b 1200×400
$= 12 \times 100 \times 4 \times 100$
$= 12 \times 4 \times 100 \times 100$
$= 48 \times 100 \times 100$
$= 4800 \times 100$
$= 480\,000$

Write 1200 as 12×100.
Write 400 as 4×100.
Multiply 12 by 4 then multiply by 100 and by 100 again.
Notice that there is a total of four zeros in 1200 and 400.
There are four zeros in 480 000.

continued

Example 24 continued

Working

c $22\,000 \times 600$
$= 22 \times 6 \times 1000 \times 100$
$= 132 \times 100\,000$
$= 13\,200\,000$

Reasoning

Write 2200 as 22×100.
Write 600 as 6×100.
Multiply 22 by 6 then multiply by 1000 and then by 100.
Notice that there is a total of five zeros in 22 000 and 600.
There are five zeros in 13 200 000.

Long multiplication

Many two-digit numbers cannot be written as the product of two simpler numbers as in example 24, so instead we use **long multiplication**. This method can also be used when multiplying by numbers with more than two digits. The long multiplication method is based on adding the products of two or more short multiplications.

Example 25

Find the product of 164 and 28 using long multiplication.

Working

$$\begin{array}{r} 1\,6\,4 \\ \times \quad 2\,8 \\ \hline 1\,3\,1\,2 \\ 3\,2\,8\,0 \\ \hline 4\,5\,9\,2 \\ \hline \end{array}$$

The zero is here because we are multiplying by 20.

The product of 164 and 228 is 4592.

Reasoning

Using the distributive law,
$164 \times 28 = 164(8 + 20)$
$= (164 \times 8) + (164 \times 20)$
$164 \times 8 = 1312$
$164 \times 20 = 3280$
So $(164 \times 8) + (164 \times 20)$
$= 1312 + 3280$
$= 4592$.

Example 26

Find the product of 396 and 207 using long multiplication.

Working

$$\begin{array}{r} 3\,9\,6 \\ \times \quad 2\,0\,7 \\ \hline 2\,7\,7\,2 \\ 7\,9\,2\,0\,0 \\ \hline 8\,1\,9\,7\,2 \\ \hline \end{array}$$

The two zeros are here because we are multiplying by 200.

The product of 395 and 207 is 81 972.

Reasoning

Using the distributive law,
$396(7 + 200)$
$= (396 \times 7) + (396 \times 200)$
$396 \times 7 = 2772$
$396 \times 200 = 79\,200$
So $(398 \times 7) + (396 \times 200)$
$= 2772 + 79\,200$
$= 81\,972$.

Tech tip

To multiply numbers using the TI-30XB MultiView calculator, for example, 12×28, type:

[1] [2] [×] [2] [8] [enter].

exercise 1.3

LINKS TO Example 16

Draw area diagrams and hence find each product.

a $7 \times 8 = 7(5 + 3)$

b $11 \times 17 = 11(9 + 8)$

c $9 \times 17 = 9(10 + 7)$

d $8 \times 14 = 8(7 + 7)$

e $13 \times 23 = 13(10 + 10 + 3)$

f $12 \times 34 = 12(10 + 20 + 4)$

g $11 \times 28 = 11(20 + 8)$

h $11 \times 28 = 11(10 + 9 + 9)$

LINKS TO Example 17

Evaluate these number expressions.

a $8 \times 23 + 8 \times 7$

b $29 \times 13 - 29 \times 3$

LINKS TO Example 17

Copy and complete these multiplications using the distributive law.

a $6 \times 43 = 6 \times 40 + 6 \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

b $8 \times 37 = 8 \times 30 + 8 \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

c $9 \times 62 = 9 \times \_\_ + 9 \times 2$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

d $4 \times 93 = 4 \times \_\_ + 4 \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

e $7 \times 79 = \_\_ \times 70 + \_\_ \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

f $8 \times 46 = \_\_ \times \_\_ + \_\_ \times 6$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

g $5 \times 93 = \_\_ \times \_\_ + 5 \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

h $7 \times 85 = \_\_ \times \_\_ + \_\_ \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

i $\_\_ \times \_\_ = 8 \times 60 + 8 \times 9$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

j $4 \times \_\_ = \_\_ \times 70 + \_\_ \times 9$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

k $9 \times 87 = \_\_ \times \_\_ + \_\_ \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

l $9 \times 58 = \_\_ \times \_\_ + \_\_ \times \_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

1.3

LINKS TO Example 17

Copy and complete these multiplications using the distributive law.

a $6 \times 49 = 6 \times 50 - 6 \times 1$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

b $8 \times 37 = 8 \times 40 - 8 \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

c $9 \times 58 = 9 \times \underline{\quad} - 9 \times 2$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

d $4 \times 79 = 4 \times \underline{\quad} - 4 \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

e $4 \times 58 = \underline{\quad} \times 60 - 4 \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

f $8 \times 38 = \underline{\quad} \times \underline{\quad} - \underline{\quad} \times 2$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

g $7 \times 49 = \underline{\quad} \times 50 - \underline{\quad} \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

h $5 \times 78 = \underline{\quad} \times \underline{\quad} - 5 \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

i $\underline{\quad} \times \underline{\quad} = 8 \times 70 - 8 \times 2$
$= \underline{\quad} + \underline{\quad}$
$= \underline{\quad}$

j $8 \times 89 = \underline{\quad} \times \underline{\quad} - \underline{\quad} \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

k $9 \times 99 = \underline{\quad} \times \underline{\quad} - \underline{\quad} \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

l $7 \times 79 = \underline{\quad} \times \underline{\quad} - \underline{\quad} \times \underline{\quad}$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

LINKS TO Example 18

Use the distributive law to find these products.

a 7×68 **b** 9×42 **c** 6×57 **d** 8×48
e 8×93 **f** 7×47 **g** 6×86 **h** 7×34

LINKS TO Example 19

Calculate these products mentally.

a 7×19 **b** 9×82 **c** 8×49 **d** 6×92
e 5×89 **f** 9×91 **g** 6×29 **h** 7×48
i 8×99 **j** 9×72 **k** 6×59 **l** 6×39

LINKS TO Example 20

Complete the following.

a Evaluate.
 i 19×10 **ii** 19×6 **iii** 19×16

b Describe a mental strategy you can use to multiply a number by 16

c Describe another strategy you could use to multiply 19×16

LINKS TO Example 20

Evaluate the following products.

a 10 10 **b** 50 100 **c** 66 10
d 98 1000 **e** 800 1000 **f** 345 1000
g 7860 1000 **h** 45 675 100 **i** 34 760 1000
j $267\,348 \times 10\,000$ **k** $123\,999 \times 100\,000$ **l** 4 898 210 100 000

LINKS TO Example 21

Calculate the following products mentally.

a 3 × 4 × 5 **b** 8 × 5 × 7 **c** 8 × 5 × 3
d 9 × 5 × 4 **e** 7 × 4 × 5 **f** 9 × 6 × 5
g 4 × 2 × 7 × 5 **h** 5 × 8 × 3 × 6 **i** 5 × 7 × 12 × 2
j 9 × 3 × 5 × 6 **k** 2 × 7 × 4 × 5 **l** 4 × 6 × 5 × 9

The order in which the numbers are multiplied doesn't matter.

LINKS TO Example 22

Find the following products without using a calculator.

a 45
3

b 92
7

c 869
7

d 693
9

e 4673
11

f 26 889
8

LINKS TO Example 23

Carry out these multiplications using two short multiplications.

a 16 × 12 **b** 18 × 24 **c** 17 × 18 **d** 28 × 20
e 53 × 16 **f** 38 × 14 **g** 36 × 15 **h** 37 × 18
i 29 × 22 **j** 35 × 35 **k** 62 × 45 **l** 41 × 42

LINKS TO Example 24

Find the answers to these without using a calculator.

a 9 80 **b** 30 60 **c** 50 70
d 60 300 **e** 800 90 **f** 30 700
g 900 600 **h** 800 500 **i** 7000 300
j 400 7000 **k** 50 15 000 **l** 120 700 000

Find the missing digits in the following multiplications.

a 3_6
3
97_

b 87_
_
6984

c 5_6
_
3822

d _53
_
7677

LINKS TO Example 25,26

Use long multiplication to find the answers to the following products. Check your answers with a calculator.

a 72
24

b 65
19

c 83
28

d 126
22

e 315
107

f 293
160

g 567
23

h 974
17

i 768
55

j 2356 89	**k** 8799 64	**l** 12 876 97
m 1036 28	**n** 224 132	**o** 1148 37

● Evaluate the following products.

a 34   23 **b** 55   46 **c** 245   21

d 789   37 **e** 7654   87 **f** 8796   59

g 675   531 **h** 878   786 **i** 10 657 × 76

j 109 333 × 99 **k** 3 345 425 × 85 **l** 767 242 × 154

● Calculate the following.

a Jack bought four shirts at $29 each and seven pairs of socks at $6 a pair. How much did he spend?

b Milly bought three T-shirts at $13 each and two pairs of shorts at $17 each. How much change did she receive out of $80?

● Nick has just received a parcel from ‘Wonder Seeds’. The parcel contains two packets of tomato seeds, two packets of pumpkin seeds and three packets of Thai green chilli seeds. There are 20 tomato seeds in a packet, 25 pumpkin seeds in a packet and 30 Thai green chilli seeds in a packet.

a How many seeds does Nick receive altogether?

b If Nick can plant 15 seeds in a row, how many rows does he have to prepare?

● Jiang has been running her ‘Happy Sounds Music School’ for 5 years. Each week Jiang conducts 12 violin lessons of 40 minutes duration, seven piano lessons of 37 minutes duration and two tuba lessons of 50 minutes duration.

a Calculate Jiang’s total teaching time per week in minutes.

b If students pay $25 per lesson, how much does Jiang earn each week?

● When Alice visited Wonderland she ate a piece of magic cake. She suddenly grew to double her previous height. She had always wanted to be taller, so she ate three more pieces and her height doubled each time. Alice was 148 cm before she ate any of the magic cake.

a How tall was Alice after eating the first piece of cake?

b How tall was Alice after eating the second piece of cake?

c How tall was Alice after eating four pieces of cake?

● Ten can be written as the product of two whole numbers in four different ways: 1   10, 10   1, 2   5 and 5   2.

a How many different ways can 20 be written as the product of two numbers?

b Write out the different ways 20 can be written as the product of three numbers.

● If I have 200 square tiles, how many different rectangular-shaped floor plans can be designed if all the tiles are used in each plan? Assume that only whole numbers are used.

exercise 1.3 challenge

● **a** The sum of two whole numbers is 70 and their product is 741. Find the whole numbers.

b The difference between two whole numbers is 12 and their product is 405. Find the whole numbers.

● Rosy keeps ants and spiders as pets. She counted all their heads and legs. Altogether there were 25 heads and 166 legs.

a How many animals were there altogether?

b If there were 12 ants and 13 spiders, how many legs would there be altogether?

c How many ants and how many spiders did Rosy have?

● Kerensa counted the heads and legs of her chickens and cows. Altogether there were 18 heads and 52 legs.

a How many animals were there altogether?

b If there were nine chickens and nine cows, how many legs would there be altogether?

c How many chickens and how many cows did Kerensa have?

1.4 Division

In section 1.3 we saw that when we multiply two numbers, the order does not matter. For example, 3×12 is equal to 12×3. However, with division, the order is important. For example, $12 \div 3$ is not the same as $3 \div 12$.

When considering division, it is useful to think of division as a sharing process.

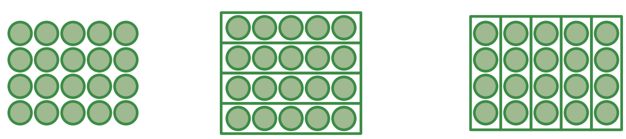
When the 20 dots below are divided into groups, there are two things we can see.

- the number of groups
- the number of dots in each group

If 20 is shared between 5 groups, we can write $20 \div 5 = 4$.

If 20 is shared between 4 groups, we can write $20 \div 4 = 5$.

Each of these is equivalent to writing $20 = 4 \times 5$.



	4 groups of 5	5 groups of 4
$20 = 4 \times 5$	$20 \div 4 = 5$	$20 \div 5 = 4$

Example 27

Rewrite these divisions as equivalent multiplications.

a $72 \div 9 = 8$

b $169 \div 13 = 13$

Working

a $72 \div 9 = 8$ is equivalent to $72 = 8 \times 9$

Reasoning

72 can be divided exactly into 9 groups of 8. Note that 72 can also be divided into 8 groups of 9.

b $169 \div 13 = 13$ is equivalent to $169 = 13 \times 13$

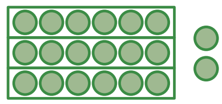
169 can be divided exactly into 13 groups of 13.

Quotient and remainder

It is not always possible to divide a number exactly into equal groups. If we divide 20 into 3 groups, there will be 6 in each group with 2 left over. We say that there is a **remainder** of 2.

$$20 \div 3 = 6 \text{ remainder } 2$$

The number we are dividing by is called the divisor. The number of groups we can make is the **quotient**. In this example, 3 is the divisor and 6 is the quotient.



$$20 \div 3 = 6 \text{ remainder } 2$$

$$20 = 6 \times 3 + 2$$

Example 28

For each of these divisions

- **i** write the division to show the quotient and remainder.
- **ii** write the equivalent multiplication.

a $23 \div 5$ **b** $58 \div 7$

Working	Reasoning
a **i** $23 \div 5 = 4$ remainder 3 **ii** $23 = (4 \times 5) + 3$	The divisor is 5. There are 5 groups of 4 with 3 left over. The quotient is 4.
b **i** $58 \div 7 = 8$ remainder 2 **ii** $58 = (8 \times 7) + 2$	The divisor is 7. There are 7 groups of 8 with 2 left over. The quotient is 8.

The distributive law with division

The distributive law works with division. For example,

$$\begin{aligned} 20 \div 4 &= (12 + 8) \div 4 \\ &= (12 \div 4) + (8 \div 4) \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

$$\begin{aligned} 32 \div 4 &= (40 - 8) \div 4 \\ &= (40 \div 4) - (8 \div 4) \\ &= 10 - 2 \\ &= 8 \end{aligned}$$

Example 29

Complete these number expressions.

a

$$\begin{aligned} 48 \div 4 &= (40 + 8) \div 4 \\ &= (\underline{\quad} \div 4) + (\underline{\quad} \div 4) \\ &= \underline{\quad} + \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

b

$$\begin{aligned} 35 \div 5 &= (40 - \underline{\quad}) \div 5 \\ &= (40 \div 5) - (\underline{\quad} \div 5) \\ &= \underline{\quad} - \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

continued

Example 29 continued

Working

a $48 \div 4 = (40 + 8) \div 4$
$= (40 \div 4) + (8 \div 4)$
$= 10 + 2$
$= 12$

Reasoning

When 40 and 8 are each divided by 4, the sum of the two quotients, 12, is the same as when 48 is divided by 4.

Working

b $35 \div 5 = (40 - 5) \div 5$
$= (40 \div 5) - (5 \div 5)$
$= 8 - 1$
$= 7$

Reasoning

When 40 and 5 are each divided by 5, the difference of the two quotients, 7, is the same as when 35 is divided by 5.

One and zero

When any number is divided by 1, the result is the number itself. Starting with the multiplication, $7 \times 1 = 7$, we can write the equivalent division $7 \div 7 = 1$.

However, if we start, for example, with the multiplication $0 = 6 \times 0$ we can see that it would not make sense to write $0 \div 0 = 6$. So dividing by 0 does not make sense.

We can divide 0 by any number, though. For example, $0 \div 6$ says 'How many 6s in zero?' There are none. So $0 \div 6 = 0$.

Short division

Short division is the method we normally use when dividing by numbers no larger than 12.

Example 30

Use short division to calculate $2142 \div 9$.

Working

$$\begin{array}{r} 2\ 3\ 8 \\ 9\ \overline{)\ 21^{3}4^{7}2} \end{array}$$

2142 divided by 9 is 238.

Reasoning

21 hundreds divided by 9 is 2 hundreds with remainder 3. Write 2 in the hundreds place at the top.
$21 = 9 \times 2 + 3$
3 hundreds = 30 tens, making 34 tens.
34 tens divided by 9 is 3 tens with remainder 7 tens. Write 3 in the tens place at the top.
$34 = 9 \times 3 + 7$
7 tens = 70 ones, making 72 ones.
72 ones divided by 9 is exactly 8 ones. Write 8 in the ones place at the top.

Divisions are sometimes written in other ways, for example, $72 \div 8$ can be written as $\frac{72}{8}$ and as $8\overline{)72}$.

Example 31

Calculate $\frac{3457}{12}$. Check your answer.

Working

$$\begin{array}{r} 2\ 8\ 8 \\ 12\overline{)\,34^{10}5^{9}7} \end{array} \text{ remainder } 1$$

3457   12 = 288 with remainder 1

or $288\frac{1}{12}$

288   12 + 1 = 3456 + 1
= 3457

Reasoning

34 hundreds divided by 12 is 2 hundreds with remainder 10 hundreds. Write 2 in the hundreds place at the top.
$34 = 12 \times 2 + 10$
10 hundreds = 100 tens, making 105 tens.
105 tens divided by 12 is 8 tens with remainder 9 tens. Write 8 in the tens place at the top.
$105 = 12 \times 8 + 9$
9 tens = 90 ones, making 97 ones.
97 ones divided by 12 is 8 ones with remainder 1. Write 8 in the ones place at the top.
Because we are dividing 3457 by 12, the remainder of 1 can be written as $\frac{1}{12}$.

Example 32

Calculate $180\,000 \div 200$.

Working

$180\,000 \div 200$

$= \frac{180\,000}{200}$

$= \frac{18}{2} \times \frac{10\,000}{100}$

$= 9 \times 100$

$= 900$

Reasoning

$180\,000 \div 200$ can be written as $\frac{180\,000}{200}$

$\frac{10\,0\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}} = 100$

We can take a short cut and cross out equal numbers of zeros on the top and the bottom

Two-step divisions

In section 1.3 we saw how we could carry out some multiplications in two steps and that the order of the two steps did not matter. In the same way we can carry out some divisions in two steps.

Example 33

Divide 288 by 16 in two steps.

Working

$288 \div 16$
$= 288 \div 2 \div 8$
$= 144 \div 8$
$= 18$

```
   1 8
8 )14⁶4
```

Reasoning

We could also have divided by 4 then by 4 again.

Long division

Example 34

Use long division to calculate $4560 \div 16$.

Working

```
      285
   16)4560
  −  32
     136
  −  128
       80
  −    80
        0
```

$1 \times 16 = 16$
$2 \times 16 = 32$
$3 \times 16 = 48$
$4 \times 16 = 64$
$5 \times 16 = 80$
$6 \times 16 = 96$
$7 \times 16 = 112$
$8 \times 16 = 128$
$9 \times 16 = 144$

$4560 \div 16 = 285$

Reasoning

Looking at the list of multiples 45 hundreds divided by 16 is 2 hundreds with a remainder of 13 hundreds.

$45 = 2 \times 16 + 13$

$2 \times 16 = 32$ so write 32 hundreds under 45 hundreds and subtract. Write 2 at the top in the hundreds place.

13 hundreds = 130 tens.

Bring down the 6 tens of 4560 to make 136 tens.

136 tens divided by 16 is 8 tens with remainder 8 tens.

$136 = 8 \times 16 + 8$

Write 8 at the top in the tens place.

$8 \times 16 = 128$ so write 128 tens under 136 tens and subtract.

continued

Example 34 continued

Working

Reasoning

8 tens = 80 ones.

Bring down the 0 ones of 4560. There are still 80 ones.

80 ones divided by 16 is 5 ones with no remainder.

$80 = 5 \times 16 + 0$

Write 5 at the top in the ones place.

$5 \times 16 = 80$ so write 80 ones under 80 ones and subtract.

Example 35

Use long division to calculate 43 457 ÷ 21.

Working

$$\begin{array}{rr} & 2069 \\ 21 & \overline{)43457} \\ - & \underline{42} \\ & 145 \\ - & \underline{126} \\ & 197 \\ - & \underline{189} \\ & 8 \end{array}$$

$1 \times 21 = 21$
$2 \times 21 = 42$
$3 \times 21 = 63$
$4 \times 21 = 84$
$5 \times 21 = 105$
$6 \times 21 = 126$
$7 \times 21 = 147$
$8 \times 21 = 168$
$9 \times 21 = 189$

43 457 ÷ 21 = 2069 remainder 8

or $2069\,\frac{8}{21}$

Reasoning

43 thousands divided by 21 is 2 thousands with a remainder of 1 thousand.

$43 = 2 \times 21 + 1$

Write 2 at the top in the thousands place.

$2 \times 21 = 42$ so write 42 thousands under 43 thousands and subtract.

1 thousand = 10 hundreds.

Bring down the 4 hundreds of 43 457 to make 14 hundreds.

We cannot divide 14 hundreds by 16, so write 0 at the top in the hundreds place.

14 hundreds = 140 tens.

Bring down the 5 tens of 43 457 to make 145 tens.

145 tens divided by 21 is 6 tens with remainder 19 tens.

$145 = 6 \times 21 + 19$

continued

Example 35 continued

Working

Reasoning

Write 6 at the top in the tens place.

$6 \times 21 = 126$ so write 126 tens under 145 tens and subtract.

19 tens = 190 ones.

Bring down the 7 ones of 43 457 to make 197 ones.

197 ones divided by 21 is 9 ones with 8 ones left over.

$197 = 9 \times 21 + 8$

Write 9 at the top in the ones place. $9 \times 21 = 189$ so write 189 ones under 197 ones and subtract. Because we are dividing by 21, the remainder of 8 can be written as $\frac{8}{21}$.

Tech tip

To divide numbers using the TI-30XB MultiView calculator, for example, $64 \div 14$, type:

[6] [4] [÷] [1] [4] [enter].

exercise 1.4

LINKS TO Example 27

1 Rewrite these divisions as equivalent multiplications.

a $88 \div 11 = 8$ **b** $144 \div 8 = 18$ **c** $91 \div 7 = 13$

d $57 \div 19 = 3$ **e** $64 \div 4 = 16$ **f** $156 \div 13 = 12$

g $84 \div 7 = 12$ **h** $256 \div 8 = 32$ **i** $85 \div 5 = 17$

LINKS TO Example 28

2 For each of these divisions

i write the division to show the quotient and remainder.

ii write the equivalent multiplication.

a $37 \div 6$ **b** $43 \div 8$ **c** $59 \div 11$ **d** $72 \div 10$

e $29 \div 8$ **f** $123 \div 12$ **g** $78 \div 9$ **h** $67 \div 7$

LINKS TO Example 29

Complete these number equations.

a $54 \div 6 = (48 + 6) \div 6$
$= \underline{\quad} \div 6 + \underline{\quad} \div 6$
$= \underline{\quad} + \underline{\quad}$
$= \underline{\quad}$

b $84 \div 12 = (96 - 12) \div 12$
$= 96 \div 12 - \underline{\quad} \div 12$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

c $63 \div 9 = (45 + 18) \div \underline{\quad}$
$= \underline{\quad} \div \underline{\quad} + \underline{\quad} \div \underline{\quad}$
$= \underline{\quad} + \underline{\quad}$
$= \underline{\quad}$

d $55 \div 11 = (88 - \underline{\quad}) \div 11$
$= \underline{\quad} \div 11 - \underline{\quad} \div 11$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

e $48 \div 6 = (12 + \underline{\quad}) \div 6$
$= \underline{\quad} \div 6 + \underline{\quad} \div 6$
$= \underline{\quad} + \underline{\quad}$
$= \underline{\quad}$

f $100 \div 5 = (150 - \underline{\quad}) \div 5$
$= \underline{\quad} \div 5 - \underline{\quad} \div 5$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

g $56 \div 7 = (70 - \underline{\quad}) \div 7$
$= \underline{\quad} \div 7 - \underline{\quad} \div 7$
$= \underline{\quad} - \underline{\quad}$
$= \underline{\quad}$

h $98 \div 7 = (63 + \underline{\quad}) \div 7$
$= 63 \div \underline{\quad} + \underline{\quad} \div \underline{\quad}$
$= \underline{\quad} + \underline{\quad}$
$= \underline{\quad}$

Complete the following divisions.

a 25 5 **b** 36 4 **c** 24 3 **d** 45 9
e 121 11 **f** 72 8 **g** 132 12 **h** 54 6

LINKS TO Example 30

Complete the following divisions.

a $5\overline{)435}$ **b** $8\overline{)656}$ **c** $6\overline{)5646}$ **d** $9\overline{)2565}$
e $12\overline{)65\,544}$ **f** $3\overline{)87\,654}$ **g** $2\overline{)567\,786}$ **h** $4\overline{)657\,668}$

Find the quotients in the following divisions.

a $\frac{458}{2}$ **b** $\frac{873}{9}$ **c** $\frac{7693}{7}$ **d** $\frac{6234}{6}$
e $\frac{12452}{11}$ **f** $\frac{34776}{12}$ **g** $\frac{145\,568}{8}$ **h** $\frac{2\,564\,228}{4}$

LINKS TO Example 31

Find the remainder in the following divisions.

a $7849 \div 8$ **b** $6574 \div 12$ **c** $\frac{7685}{3}$ **d** $\frac{8947}{9}$
e $9\overline{)12\,657}$ **f** $6\overline{)102\,359}$ **g** $\frac{1\,405\,345}{7}$ **h** $2\,465\,999 \div 11$

LINKS TO Example 32

Find the quotients in the following divisions.

a $\frac{3600}{900}$ **b** $\frac{27\,000}{9000}$ **c** $\frac{810\,000}{90\,000}$ **d** $\frac{450\,000}{9000}$
e $\frac{720\,000}{8000}$ **f** $\frac{144\,000\,000}{12\,000}$ **g** $\frac{550\,000}{500}$ **h** $\frac{2\,400\,000}{60}$

1.4

LINKS TO Example 33

Complete the following.

a 150 10 3 b 198 9 2 c 216 4 6 d 315 7 3
e 100 2 5 f 952 56 g 1242 54 h 1332 36

LINKS TO Example 34

Use long division to calculate each of these divisions.

a 435 ÷ 15 b 728 ÷ 13 c 522 ÷ 18 d 903 ÷ 21
e 1404 ÷ 18 f 1377 ÷ 17 g 4784 ÷ 23 h 3325 ÷ 19
i 4326 ÷ 14 j 9576 ÷ 18 k 15 376 ÷ 16 l 13 608 ÷ 21

LINKS TO Example 35

Use long division to calculate each of these divisions. There will be a remainder in each case.

a 547 ÷ 15 b 638 ÷ 19 c 1143 ÷ 21 d 1158 ÷ 17
e 1253 ÷ 13 f 1395 ÷ 16 g 2347 ÷ 23 h 4983 ÷ 24
i 4554 ÷ 14 j 9222 ÷ 18 k 16 256 ÷ 13 l 10 761 ÷ 21

The Great Barrier Reef stretches for 202 800 000 cm. The leatherback sea turtle is 185 cm in length.

a How many sea turtles of this size could be lined up, head to tail, along the Great Barrier Reef?
b Adding one more turtle to the line will make it longer than the Great Barrier Reef. How many centimetres longer than the Great Barrier Reef is the line of turtles?

Sam earns $42 120 per year.

a How much does Sam earn per month if he is paid the same amount each calendar month?
b How much does he earn per week?

Sally has to pack 1435 bonbons into boxes of six. She is allowed to keep any that are left over.

a How many boxes is she able to complete?
b How many bonbons is Sally able to keep?

Graham carts water in his 45 000 L water truck. How many 1500 L water tanks can he fill?

Find the missing digits.

a

```
        2_5
   13 3456
      2600
        _56
        _80
         7_
         _5
         11 remainder
```

b

```
        _9_
   14 5463
        _2
       126
       ___
        __ remainder
```

exercise 1.4 challenge

Luk Chai is a baby Asian elephant, born on July 4, 2009 in Sydney's Taronga Park Zoo. On July 6, two days after he was born Luk Chai weighed 96 kilograms.

- **a** On August 5, Luk Chai weighed 132 kilograms. How many kilograms per day does this represent? Give your answer correct to the nearest kilogam per day.
- **b** When fully grown, male Asian elephants can weigh about 5000 kilograms. How many tonnes is this?
- **c** If Luk Chai continued to grow at the rate you worked out in part a, how many days would it take for him to reach 5000 kg?
- **d** About how many years is this?

1331 is a palindromic number. (It reads the same forward as backwards.) When it is divided by a certain two-digit palindromic number, the result is a three-digit palindromic number. What are the two- and three-digit palindromic numbers?

What number am I?

- **a** When divided by 3 I have a quotient of 3456 and a remainder of 2.
- **b** When divided by 7 I have a quotient of 87 567 and a remainder of 5.

1.5 Mixed operations

Consider the following problem: Mae has 2 fruity bars and 5 packets each containing 6 fruity bars. How many fruity bars does she have altogether? We can write the total number of fruity bars as $2 + 5 \times 6$. In this problem, it is easy to see that we must do the multiplication first.

$$\begin{aligned}\text{Total number of fruity bars} &= 2 + 5 \times 6\\ &= 2 + 30\\ &= 32\end{aligned}$$

It would not make sense to add the 2 fruity bars to the 5 packets and then multiply by 6.

Many calculations involve more than one number operation, for example, addition, multiplication, and raising a number to a power. There is an order in which these operations must be carried out.

Anything in brackets is calculated first. If there is an 'of' we do that next. Then we complete divisions and multiplications, working from left to right. Finally we complete additions and subtractions, working from left to right.

1 Brackets, Powers and Roots
2 Of
3 { Divisions and Multiplications } are worked out from left to right in the order that they occur.
4 { Additions and Subtractions } are worked out from left to right in the order that they occur.

Calculators that are called scientific calculators use this order for doing calculations, but some very simple calculators, such as those in some mobile phones, just work from left to right.

Example 36

Do this calculation using a scientific calculator and a very simple calculator.

$5 + 4 \times 3$

a The scientific calculator gives the correct answer. What is the correct answer?
b What answer does a simple non-scientific calculator give? Why?
c What could you do to obtain the correct answer with the simple calculator?

Working	**Reasoning**
a The correct answer is 17.	Do the multiplication before addition. $5 + 4 \times 3 = 5 + 12 = 17$
b The simple calculator gives the incorrect answer 27.	The simple calculator does not follow the correct order of operations. It works from left to right, giving $5 + 4 \times 3 = 9 \times 3 = 27$
c Put brackets around the multiplication. $5 + (4 \times 3) = 5 + 12 = 17$	Brackets are worked out first.

Example 37

Evaluate each of the following.

a $2 \quad (5 + 6) + 3 \quad 8$ **b** $18 - 5 + 6 \div 3$ **c** $63 \div \frac{1}{2}$ of $14 + 4$

Working	Reasoning
a $2 \quad (5 + 6) + 3 \quad 8$	Do the brackets first.
$= 2 \quad 11 + 3 \quad 8$	Do the multiplications next.
$= 22 + 24$	Do the addition last.
$= 46$	
b $18 - 5 \quad 6 \div 3$	Do the multiplication and division before the subtraction. Work from left to right with the multiplication and division.
$= 18 - 30 \div 3$	
$= 18 - 10$	
$= 8$	
c $63 \div \frac{1}{2}$ of $14 + 4$	Think of $\frac{1}{2}$ of 14 as being in brackets so calculate this first.
$= 63 \div 7 + 4$	$63 \div \left(\frac{1}{2} \text{ of } 14\right) + 4$
$= 9 + 4$	Calculate division before addition, so divide 63 by 7.
$= 13$	

Example 38

Evaluate the following.

a $\dfrac{4 + 7 \times 2}{9}$ **b** $\dfrac{100}{3 + 2}$

Working	Reasoning
a $\dfrac{4 + 7 \times 2}{9}$	This is the same as $(4 + 7 \quad 2) \quad 9$.
$= \dfrac{4 + 14}{9}$	Simplify the numerator first. Do the multiplication first and then the addition.
$= \dfrac{18}{9}$	Finish with the division.
$= 2$	
b $\dfrac{100}{3 + 2}$	$\dfrac{100}{3 + 2}$ can be written as $100 \div (3 + 2)$.
$= \dfrac{100}{5}$	Simplify the denominator. Do the brackets first. In general, we simplify the numerator and denominator of a division before dividing.
$= 20$	

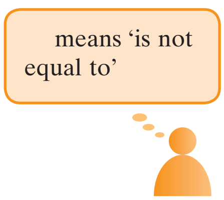
Example 39

Are the following statements true?

a 18 3 2 = 18 (3 2)

b 20 − 5 + 6 = 20 − (5 + 6)

Working	**Reasoning**
a 18 3 2 = 6 2 = 12	Work from left to right with the division and multiplication.
18 (3 2) = 18 6 = 3	The multiplication is done first because it is enclosed in brackets. Finish with the division. Note that this is the same as $\frac{18}{3 \times 2} = 3$
The statement is false. 18 3 2 18 (3 2)	Answer the question.
b 20 − 5 + 6 = 15 + 6 = 21	Work from left to right with the addition and subtraction.
20 − (5 + 6) = 20 − 11 = 9	Do the addition first because it is enclosed in brackets. Finish with the subtraction.
The statement is false. 20 − 5 + 6 20 − (5 + 6)	Answer the question.



Tech tip

Mixed operations can be done using the TI-30XB MultiView calculator.
For example, to calculate

$\frac{96 - 12}{6} = 12$, type:

(9 6 − 1 2) ÷ 6 enter.

exercise 1.5

LINKS TO Example 37

1 Evaluate the following without using a calculator.

a $4 \times 6 + 8$ **b** $6 \times 9 \div 2$ **c** $20 + \frac{1}{3}$ of 27

d $\frac{1}{2}$ of $\frac{100}{5}$ **e** $\frac{1}{2}$ of $\frac{72}{6}$ **f** 100 − 6 9

g $(50 - 26) \div 4$ **h** $50 - 28 \div 4$ **i** $(42 + 38) \times 6$

Which of the following does not equal 11?

A $\dfrac{132}{4 + 8}$ **B** $132 \quad (4 + 8)$ **C** $132 \quad 3 \quad 4$

D $132 \quad (3 \quad 4)$ **E** $\dfrac{132}{3 \times 4}$

Which of the following equals 24?

A $48 \div (4 \times 2)$ **B** $50 - 2 \div 2$ **C** $46 + 2 \div 2$

D $4 + 8 \div 2$ **E** $48 \div 4 \times 2$

$\dfrac{1464}{2 \times 6}$ cannot be expressed as

A $1464 \div (2 \times 6)$ **B** $1464 \div 12$ **C** $\dfrac{1}{2}$ of $1464 \times \dfrac{1}{6}$

D $1464 \div 2 \times 6$ **E** $\dfrac{1}{2} \times \dfrac{1464}{6}$

Evaluate the following without using a calculator.

a $5 \quad 7 - 11 \quad 2$ **b** $11 \quad 12 - 8 \quad 3$ **c** $(6 + 8) \quad 2 + 17$

d $(22 + 11) \quad 3 - 10$ **e** $\dfrac{1}{2}$ of $64 + 25 \quad 5$ **f** $23 + 24 \quad 3 \quad 7$

g $100 \div \dfrac{1}{2}$ of $40 + 3$ **h** $(96 \div 4 + 4) \div 7$ **i** $75 - 3 \times 5 + 40$

j $30 + 12 \div 3 + 3$ **k** $144 \div 3 - \dfrac{132}{5 + 6}$ **l** $54 \times 6 \div 3 \div 12$

m $10 - (4 + 5) \div 3 \times 2$ **n** $8 \times 9 - 6 \times 10 \div 5$ **o** $21 + 14 \div 7 + \dfrac{1}{2}$ of 64

p $56 \times (2 + 3) - 27 \div 3$ **q** $345 - (7 + 8) \times 3 - 200$ **r** $\dfrac{1}{4}$ of $384 \div 8 \times 7 \div 2$

LINKS TO Example 38

Evaluate the following without using a calculator.

a $\dfrac{3 \times 6 + 7}{5}$ **b** $\dfrac{81}{5 + 4}$

c $\dfrac{12 + 3}{5} + 8$ **d** $\dfrac{4 \times 2 + 5 \times 5}{9 + 2}$

e $\dfrac{5 \times 8 + 6 \div 3}{3 + 4} - 2 \times 3$ **f** $\dfrac{33 + 7 - 8}{4} + \dfrac{45 + 30}{3 \times 5}$

g $\dfrac{5 + 89}{2} - \dfrac{11 \times 3 + 7}{4 \times 10}$ **h** $\dfrac{8 \times 9 + 6}{30 + 9} + \dfrac{21 \div 7 \times 12}{9 \times 4}$

LINKS TO Example 39

Are the following statements true?

a $36 \div 3 \times 2 = 36 \div (3 \times 2)$ **b** $56 - (23 + 13) = 56 - 23 + 13$

c $96 \div 12 \times 8 = \dfrac{96 \times 8}{12}$ **d** $48 \div (4 + 2) = \dfrac{48}{4 + 2}$

e $30 - 4 - 3 = 30 - (4 + 3)$ **f** $4 \times 9 + 6 = 4 \times (9 + 6)$

g $\dfrac{11}{3 \times 22} \times \dfrac{3 + 4}{49} = \dfrac{11 \times (3 + 4)}{3 \times 22 \times 49}$ **h** $\dfrac{9 + 3}{6} + \dfrac{4 \times 3}{2} = \dfrac{9 + 3 + 4 \times 3}{6 + 2}$

Add brackets to the following to make the equations true.

a 5 + 4   9 = 81 **b** 9 + 6   3 = 5

c 11   8 + 3 = 121 **d** 12 + 8   3 + 2 = 52

e 24 + 16   8 + 16 = 21 **f** 32 + 64   4 + 4 = 40

Jane and Lexie both used their calculators to do the following calculation.

$$\frac{4 \times 15}{6 - 3}$$

Jane pressed the following sequence of keys on her calculator.

4		1	5		6	−	3	=

She obtained the answer 7. Lexie used a different sequence of keys and got the answer 20.

a Was Jane's answer correct? Explain.

b Was Lexie's answer correct? Explain.

c What sequence of keys might Lexie have used to obtain the answer 20?

In Australian rules football a goal is six points and a behind is one point. So if a team scores five goals and three behinds their score is a total of 5   6 + 3   1 = 33 points. Complete the missing data in the following table.

Team	Goals	Behinds	Score	Working
Brisbane Lions	18	5	___	$18 \times 6 + 5$
Geelong Cats	___	12	96	$\frac{96 - 12}{6}$
Sydney Swans	15	___	108	___
West Coast Eagles	___	16	88	___

exercise 1.5 challenge

Michael has 50 potato plants in each of 110 rows in one paddock and 66 potato plants in each of 130 rows in another paddock. There are approximately 15 potatoes on each plant.

a Insert the correct signs and the answer so that the following equation represents the approximate number of potatoes.

50 __ 110 __ 15 __ 66 __ 130 __ 15 = __

b Is there another way to represent the left side of the equation?

1.6 Estimation

In many everyday situations we do not need to know accurate answers to questions, so we use various strategies to find approximate answers. In all of the following examples, there is some sort of estimation or approximation occurring.

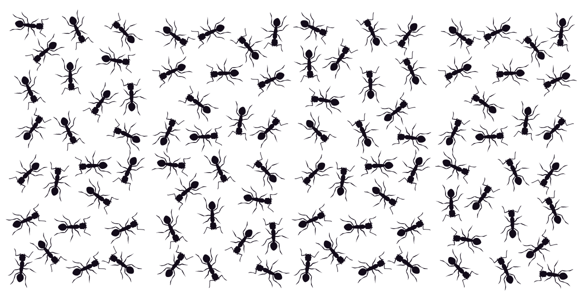
- A crowd of 75 000 sees the Galahs beat the Mudlarks.
- It's approximately 200 kilometres.
- I'll need about $50.
- I should be there by about 5:30.
- I reckon it will take about four litres of paint.

Estimation by sampling

Sometimes there is a need to make estimates of the size of groups of people or objects; for example, estimating the number of people in a crowd. Rather than trying to count all the people, we can count the people in a small section, or perhaps several small sections. We can then use those sample counts to make an estimate of the size of the whole crowd.

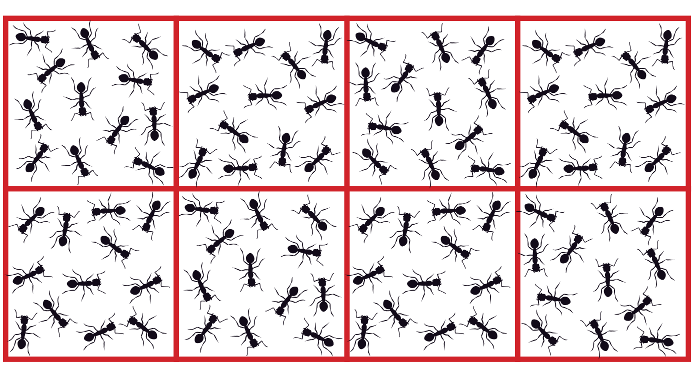
Example 40

Estimate the number of ants by sampling.



Working

There are approximately 12 ants in one sample region.
$8 \times 12 = 96$ ants
In the whole region there will be approximately 100 ants.



Reasoning

Divide the area into smaller regions and count the number of ants in a sample region.
In the 8 regions there will be approximately 96 ants.
Round to 100.

Estimating by rounding

If we are interested in obtaining only a rough idea of the size of a quantity, we often round numbers; for example, to the nearest whole number, to the nearest ten, or to the nearest thousand.

Example 41

Round

a 32, 65 and 97 to the nearest 10.

b 167, 745 and 850 to the nearest 100.

Working	Reasoning
a **3**2 ≈ 30 (32 is closer to 30 than to 40.)	Look at the ones digit, 2. 2 is less than 5. Round the number down: the tens digit stays as 3 and the ones digit becomes 0. (32 is closer to 30 than to 40.)
65 ≈ 70	Look at the ones digit, which is 5. 65 is halfway between 60 and 70, but it is the convention to round up to 70.
97 ≈ 100 (97 is closer to 100 than to 90.)	Look at the last digit, the ones digit, 7. 7 is greater than 5. Round up: the tens digit, 9, becomes 10 and the ones digit 0. (97 is closer to 100 than to 90.)
b **1**67 ≈ 200 (167 is closer to 200 than to 100.)	Look at the tens digit. 6 is greater than 5. Round up: the hundreds digit, 1, becomes 2 and the other digits 0. (167 is closer to 200 than to 100.)
745 ≈ 700 (745 is closer to 700 than to 800.)	Look at the tens digit. 4 is less than 5. Round down: the hundreds digit, 7, does not change and the other digits become 0. (745 is closer to 700 than to 800.)
850 ≈ 900	Look at the tens digit, which is 5. 850 is halfway between 800 and 900. The convention is to round it up to 900.

It is often useful to round to the **leading digit**. The leading digit is the first digit of the number, reading from left to right. The leading digit of 732 643 is 7. If we wanted to estimate a calculation involving 732 643, we could round 732 643 to 700 000.

Example 42

Round these numbers to the leading digit.

a 234 **b** 67 879 **c** 153 786

Working

a 234 ≈ 200

Reasoning

2 is the leading digit. Look at the number after the leading digit. Since 3 is less than 5, round down to 200.

b 67 879 ≈ 70 000

6 is the leading digit. The number after the leading digit is 7, which is greater than 5. Round up to 70 000.

c 153 786 ≈ 200 000

The leading digit is 1. The number after the leading digit is 5, so round up to 200 000.

If we are dividing, it is sometimes useful to round to the two leading digits. For example, if we were estimating 3162 divided by 8, we would round 3162 to 3200 rather than to 3000. The reason for this is that 3200   8 is much faster to calculate than 3000   8 because we know that 32   8 = 4, so 3200   8 = 400. When estimating, we are trying to make the calculation as easy as possible. Whether we choose just the leading digit or the first and second leading digits depends on the particular numbers.

Example 43

Estimate the answer to the following by rounding each number to one or two leading digits.

a 32 567 + 765 856 **b** 357 × 432 **c** 32 187 ÷ 418

Working

a 32 567 + 765 856
≈ 30 000 + 770 000
= 800 000

Reasoning

Round 32 567 to the leading digit.
Round 765 856 to the first two leading digits because 765 856 has hundred thousands. This means that both numbers are rounded to ten thousands.

b 357   432
≈ 400   400
= 160 000

Leading digits in both numbers are hundreds. Round 357 and 432 to the nearest hundred.

c 32 187   418
≈ 32 000   400
= 80

Round 32 187 to 32 000 and round 418 to 400. It is easy to divide 32 000 by 400 because we know that 32   4 = 8.

exercise 1.6

LINKS TO Example 40

1. Use sampling to estimate the number of people in this photograph.

LINKS TO Example 41

2. Round the following numbers to the nearest 10.

a 56 **b** 99 **c** 132 **d** 2347
e 7856 **f** 10 785 **g** 12 654 **h** 109 784

LINKS TO Example 41

3. Round the following numbers to the nearest 100.

a 56 **b** 350 **c** 428 **d** 989
e 1657 **f** 2387 **g** 34 736 **h** 897 802

LINKS TO Example 41

4. Round the following numbers to the nearest thousand.

a 3456 **b** 8543 **c** 12 345 **d** 98 768
e 678 456 **f** 878 535 **g** 1 234 324 **h** 5 555 555

LINKS TO Example 42

5. Round the following numbers to the leading digit.

a 2376 **b** 55 546 **c** 678 567
d 3 456 786 **e** 54 657 890 **f** 78 654 723

LINKS TO Example 43a

6. Estimate the answers to the following using leading digit rounding.

a 87 + 129 **b** 389 − 122 **c** 450 + 345 + 768
d 647 − 435 + 587 **e** 2 345 + 7 678 − 4 567 **f** 5 788 + 1 965 − 2 346

LINKS TO Example 43b, 43c

7. Estimate the answers to the following using one or two leading digit rounding.

a 4765   32 **b** 6793   43 **c** 81950   17
d 45 180   892 **e** $\frac{23\,670}{389}$ **f** $\frac{17\,732}{941}$

- 24 567 people attend a football match and pay $22 each.
  - **a** Estimate the total amount of money collected by rounding to the leading digit.
  - **b** Use your calculator to find an exact answer.
  - **c** Compare this with your estimate.
  - **d** Estimate the total amount collected by rounding the number of people to two leading digits.
  - **e** Which was the better estimate?
- Approximately 21 people enter a particular bank every minute.
  - **a** Estimate how many people enter the bank every hour.
  - **b** Estimate how many people would enter the bank in a day if the bank is open for six hours a day.
- A dairy farmer milks approximately 32 cows every 10 minutes in her rotary dairy.
  - **a** Estimate how many cows she would milk in an hour and a half.
  - **b** If each cow gives approximately 2.1 litres of milk per milking, approximately how much milk will the dairy farmer have in an hour and a half?
- A book contained 153 266 words and there were approximately 525 words per page.
  - **a** Use leading digit rounding to estimate the number of pages in the book.
  - **b** Estimate the number of pages by rounding the number of pages in the book to two leading digits.
  - **c** Which estimate was better? Explain.
- A school conducts three fundraising events to raise money to purchase new instruments for members of the orchestra. The total cost of the instruments is expected to be $9320. At the first fundraiser the school makes $2579, at the second they make $3321 and at the third they make $2876.
  - **a** Make an estimate of the total money raised by the school in the three fundraisers by rounding off to the nearest hundred.
  - **b** Use this estimate to determine whether or not there is enough money to purchase the instruments.
  - **c** Work out the exact total of money raised.

exercise 1.6 challenge

- An agricultural scientist counts the number of Paterson's Curse weeds in 2 m by 2 m squares in five places around a 50 m by 200 m paddock. The number of weeds were 11, 12, 8, 9 and 11.
  - **a** How big is the paddock?
  - **b** Approximately how many weeds are in a 2 m by 2 m square?
  - **c** How many 2 m by 2 m squares fit in the paddock?
  - **d** Estimate how many Paterson's Curse weeds are in the paddock.

Analysis task

Barcodes

Most products that you buy in Australia have a 13-digit barcode based on the European Article Numbering (EAN) code. The barcode identifies the product, its manufacturer and price. For books, the barcode shows the ISBN (International Standard Book Number).

Use these steps to find if this barcode number is valid.

9 321728 003021

Step 1: Find the sum of the 1st, 3rd, 5th, 7th, 9th, 11th and 13th digits.

Step 2: Find the sum of the 2nd, 4th, 6th, 8th, 10th and 12th digits then multiply this sum by 3. Add this to the sum from Step 1. The total should be exactly divisible by 10.

In fact, the last digit of the barcode number is the *check digit* which ensures that the sum calculated in this way is exactly divisible by 10.

For the barcode shown above,

$(9 + 2 + 7 + 8 + 0 + 0 + 1) + 3(3 + 1 + 2 + 0 + 3 + 2)$

$= 27 + 3 \times 11$

$= 27 + 33$

$= 60$ which is exactly divisible by 10.

a Show that 9 312136 823502 is not a valid barcode number.

b Check the barcode number on your mathematics textbook using the method described above.

c Choose three more 13-digit barcode numbers from books, food packets or other items at home and check them according to the method described above.

d Set up a spreadsheet to check 13-digit barcodes. Hint: enter the 13 digits of the barcode vertically in column A, then type an appropriate formula into column B to calculate the total according to the rule. Use your spreadsheet to test with five different barcode numbers.

e Before 2007, ISBNs had only 10 digits. Find out how the check digit was calculated.

Review Calculating with whole numbers

Summary

- The position of a digit in a number determines its place value. The first seven places are outlined below.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

Symbol	Meaning	Result
+	Addition	Sum or total
−	Subtraction	Difference
×	Multiplication	Product
÷	Division	Quotient

Symbol	Meaning
>	Greater than
<	Less than

- When adding or subtracting, the digits with the same place value should be aligned.
- Numbers can be written in place value notation using powers of 10, for example,

$$\begin{aligned}3906 &= 3 \times 1000 + 9 \times 100 + 0 \times 10 + 6\\ &= 3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 6.\end{aligned}$$

- When we add several numbers, the order in which we add them does not matter.
- When we multiply several numbers, the order in which we multiply them does not matter.
- The distributive law is useful when multiplying numbers mentally, for example,

$$59 \times 7 = 60 \times 7 - 1 \times 7.$$

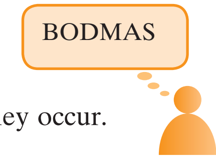
- Any number multiplied by zero is zero.
- Zero divided by any number is zero.
- Dividing a number by zero does not make sense.

1 Brackets, Powers and Roots

2 Of

3 { Divisions and Multiplications } are worked out from left to right in the order that they occur.

4 { Additions and Subtractions } are worked out from left to right in the order that they occur.



Estimating

When estimating by rounding, if the last digit is

- less than 5, round down.
- 5 or more, round up.

We can also round to the leading digit. The leading digit is the first digit in the number when reading from left to right.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

addition	divisor	minus	rounding
ascending order	equals	multiplication	sampling
denominator	equation	numerator	shared between
descending order	estimation	place value	subtraction
difference	greater than	power	sum
digit	integer	product	take away
distributive law	leading digit	quotient	times
division	less than	remainder	whole numbers

Revision

Multiple-choice questions

Answer the following without using a calculator.

● $24 + 12 \quad 4 + 2$ equals

A 6 **B** 11 **C** 26 **D** 27 **E** 29

(3 + 4)   6 − 4   2 is equivalent to

A (3 + 4)   2   2 **B** (3 + 4)   4 **C** 3 + 4   6 − 4   2

D 7   6 − 4   2 **E** $\frac{(3 + 4) \times 6 - 4}{2}$

A netball team bought a Tattslotto ticket for \$14 and they won \$1 563 072. If there are seven players in the team then each player is ahead by

A $\$\frac{1\,563\,072}{7}$ **B** $\$\frac{1\,563\,072}{7} - 14$ **C** $\$\frac{1\,563\,072 - 14}{7}$

D \$ 1 563 072   7 **E** \$ 1 563 072   7 − 14

The cows on a certain dairy farm are producing about 4 L of milk each per milking. If there are two milkings per day and 11 872 L is produced in a week then the number of cows the dairy farmer is milking is

A $\frac{11\,872}{4}$ **B** $\frac{11\,872}{8}$ **C** $\frac{11\,872}{4 \times 7}$

D $\frac{11\,872}{8 \times 7}$ **E** $\frac{11\,872 \times 7}{8}$

45 789 rounded to the leading digit is

A 45 790 **B** 45 800 **C** 46 000 **D** 40 000 **E** 50 000

Short-answer questions

What is the value of the digit 8 in each of these numbers?

a 8040 **b** 185 477

Write these numbers in place value notation using powers of 10.

a 24 592 **b** 180 045

c 84 326 **d** 7085

Write the following as ordinary numbers.

a $7 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 5 \times 10^2 + 1 \times 10^1 + 9$

b $8 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 5$

Write these numbers in place value form using powers of 10.

a 80 000 **b** three hundred thousand

c 7000 **d** 18 000 000

Complete these additions mentally.

a 23 + 17 + 65 + 35 **b** 52 + 18 + 41 + 59

c 18 + 24 + 46 + 72 **d** 19 + 43 + 41 + 37

Calculate.

a 15 + 9 − 5 **b** 23 + 21 − 11

Complete these subtractions.

a 2907 − 1864 **b** 3962 − 987

- Calculate.
  a 37 × 9 **b** 46 × 12
- Calculate these products by using two steps.
  a 26 × 18 **b** 32 × 15
- Find these products using long multiplication.
  a 356 × 18 **b** 750 × 105
- Complete these divisions.
  a 342 ÷ 9 **b** $\frac{1309}{7}$ **c** 6935 ÷ 19 **d** 10 358 ÷ 23
- Round these numbers to the nearest thousand.
  a 2369 **b** 37 452
- Estimate by rounding to one or two leading digits.
  a 320 × 890 **b** 17600 ÷ 8975
- Use sampling to estimate the number of logs in this stack of wood.

Extended-response questions

Grondark the goblin found the dragon's buried hoard of 1500 jewels. He managed to take half the jewels before the dragon woke. Grondark gave 110 of the jewels to each of his five goblin friends and kept the rest for himself. Write a mathematical expression to determine how many jewels Grondark kept for himself and then evaluate your expression.

Use this newspaper article to answer the questions.

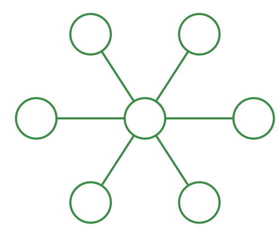
> **Mad on Mars**
>
> When 39-year-old Dave Smith dreamt that the recipe for Mars Bars was going to change, he was so afraid he wouldn't like the new bars that he stored 1200 in his freezer and another 200 in the fridge. Dave says he has eaten a Mars Bar every day for 30 years. 'I know it sounds a bit sad, but I couldn't sleep at night worrying about what would happen if they spoiled the taste.'

Practice quiz
Chapter 1

a Approximately how many Mars Bars had Dave eaten in the 30 years?

b For how many complete years would Dave's supply of Mars Bars last?

Place the numbers 11, 12, 13, 14, 15, 16 and 17 in the circles so that every line has the same sum.



2 Multiples, factors and indices

Pre-test

Warm-up

The numbers on these mailboxes can each be written as the product of two or more numbers. The number 7 is a prime number as it has only two factors, 1 and 7. It can be written as the product of its two factors as 1×7. There are nine more prime numbers in these mailbox numbers. Many numbers have more than two factors. These numbers are called composite numbers. Some of the numbers on these mailboxes have exactly three factors. One mailbox number has eight factors. In this chapter you will investigate factors, prime numbers and composite numbers.

2.1 Multiples

What are multiples?

Multiples of numbers are what we obtain when we skip count. For example, when we count by threes, we obtain multiples of 3, that is, 3, 6, 9, 12, 15, … We do this by multiplying 3 by 1, 2, 3, 4 and so on: 3 1 = 3, 3 2 = 6, 3 3 = 9, 3 4 = 12, 3 5 = 15, …

The symbol … written after 15 shows that we can keep on going to produce more and more multiples of 3.

When we count in fours, we obtain multiples of 4, that is, 4, 8, 12, 16, 20, … or 4 1 = 4, 4 2 = 8, 4 3 = 12, 4 4 = 16, 4 5 = 20, …

To make a list of multiples of any whole number, we multiply the whole number by 1, 2, 3, 4, 5, and so on.

Example 1

Write these multiples.

a the first six multiples of 7

b the fifth multiple of 7

c the first three multiples of 8

Working	Reasoning
a 7, 14, 21, 28, 35, and 42	Multiply 7 by 1, 2, … 6.
b 7 5 = 35	Either multiply 7 by 5 or list the multiples: 7, 14, 21, 28, 35.
c 8, 16, 24	Multiply 8 by 1, 2 and 3.

Finding if a whole number is a multiple of another whole number

To find out if a number is a multiple of another smaller number, we divide the larger number by the smaller number to see if we obtain a whole number. For example, to find out if 45 is a multiple of 5, divide 45 by 5. The result is 9 which is a whole number, so 45 is a multiple of 5. We could also count in fives to see if we come to 45, but this is a less efficient method.

Example 2

Find whether the number is a multiple of the number in brackets.

a 120 (15)

b 216 (22)

Working	Reasoning
a 120 ÷ 15 = 8 So 8 × 15 = 120 120 is a multiple of 15.	15 divides exactly into 120.
b 216 ÷ 22 = 9 remainder 18 216 is not a multiple of 22.	22 does not divide exactly into 216.

Lowest common multiple

If we consider the multiples of 3 and the multiples of 4, we see that there are many common multiples. The multiples that are common to both numbers are circled in red.

Multiples of 3	3, 6, 9, (12), 15, 18, 21, (24), 27, 30, 33, (36), …
Multiples of 4	4, 8, (12), 16, 20, (24), 28, 32, (36), …

We can see that 12, 24, 36 … are **common multiples** of 3 and of 4 because they are in both lists.

The smallest of these common multiples is 12 so we say that 12 is the **lowest common multiple** of 3 and 4.

Example 3

Find the following.

a the lowest common multiple of 3 and 5

b the lowest common multiple of 16, 24 and 36

Working	Reasoning
a 3, 6, 9, 12, (15), 18, 21 5, 10, (15), 20 The lowest common multiple of 3 and 5 is 15.	List the multiples of 3. List the multiples of 5 until you come to a multiple of 3. The multiple 15 is common to both lists.
b 16, 32, 48, 64, 80, 96, 112, 128, (144), … 24, 48, 72, 96, 120, (144), 168, … 36, 72, 108, (144), … The lowest common multiple of 16, 24 and 36 is 144.	List the multiples of 16. List the multiples of 24. List the multiples of 36 until you come to a multiple of 16 and 24. The multiple 144 is common to all lists.

exercise 2.1

1. Find the next three multiples in each of the following.
   - **a** 2, 4, 6, … **b** 5, 10, 15, … **c** 7, 14, 21, …
   - **d** 12, 24, 36, … **e** 50, 100, 150, … **f** 60, 120, 180, …
   - **g** 1000, 2000, 3000, … **h** 15, 30, 45, … **i** 17, 34, 51, …

2. Find the first four multiples of each of the following numbers.
   - **a** 3 **b** 4 **c** 8 **d** 11 **e** 9 **f** 13
   - **g** 17 **h** 21 **i** 25 **j** 32 **k** 101 **l** 687

LINKS TO Example 1

3. Write the following.
   - **a** the fifth multiple of 12 **b** the seventh multiple of 7
   - **c** the fourth multiple of 11 **d** the ninth multiple of 8
   - **e** the seventh multiple of 13 **f** the fifth multiple of 35
   - **g** the sixth multiple of 43 **h** the third multiple of 1956

LINKS TO Example 2

4. Find whether the number is a multiple of the number in brackets.
   - **a** 122 (11) **b** 91 (13) **c** 86 (14)
   - **d** 110 (22) **e** 217 (31) **f** 365 (45)

LINKS TO Example 3

5. Find the lowest common multiple of the following numbers.
   - **a** 4 and 8 **b** 2 and 5 **c** 7 and 5 **d** 6 and 8
   - **e** 6 and 15 **f** 3 and 11 **g** 4 and 6 **h** 8 and 12

6. Write the following.
   - **a** the first six multiples of 6
   - **b** the first four multiples of 9
   - **c** the first two common multiples of 6 and 9
   - **d** the lowest common multiple of 6 and 9
   - **e** the third common multiple of 6 and 9

7. Find the lowest common multiple of each of the following pairs of numbers and state whether it is equal to the product of the two numbers or less than the product of the two numbers.
   - **a** 7 and 12 **b** 3 and 11 **c** 4 and 14 **d** 8 and 14
   - **e** 10 and 15 **f** 12 and 18 **g** 5 and 13 **h** 3 and 19

8. Using your observations from question 7, explain when the lowest common multiple of two numbers is
   - **a** equal to the product of the two numbers.
   - **b** less than the product of the two numbers.

9. Find the first three common multiples of the following numbers.
   - **a** 3 and 5 **b** 4 and 6 **c** 7 and 21 **d** 5 and 7
   - **e** 14 and 12 **f** 15 and 20 **g** 20 and 21 **h** 16 and 24

2.1

- Find the lowest common multiple of the following sets of numbers.
  - **a** 2, 5 and 7 **b** 3, 7 and 10 **c** 6, 8 and 12 **d** 4, 6 and 18
  - **e** 5, 12 and 20 **f** 6, 9 and 12 **g** 11, 13 and 15 **h** 16, 20 and 24
- Peter drives an interstate delivery truck and takes six days to do a round trip from Melbourne to Perth. Kath works as a tour guide on 14-day outback camping tours.
  - **a** List the first ten multiples of 6.
  - **b** List the multiples of 14 until you reach one that is a multiple of 6 as well.
  - **c** If Peter and Kath both leave Melbourne at the same time, how many weeks will it be before they are back in Melbourne together?
- Trang and Grace are on holidays. They decide to watch their favourite DVDs on separate machines over and over again. Neither likes to be interrupted during their films.
  - **a** If Trang's DVD is an hour long and Grace's DVD is 90 minutes long, after how many minutes will the two girls be able to have a break together?
  - **b** How many times would each of them have seen their films?
- Aidan and Bridget are training for a marathon. It takes Aidan four minutes to run around the oval and Bridget six minutes. They both run at a constant speed.
  - **a** If both of them start together, after how many minutes will they pass the starting point together again?
  - **b** How many times would Aidan have passed Bridget?
  - **c** If they start running at 9.00 am list the times they would pass the starting point together within the next hour.
  - **d** At 10.00 am, their young brother David joins them. David takes seven minutes to run around the oval. If they all start running at 10.05 am, when will be the next time they pass the finishing line together?

exercise 2.1 challenge

- Class 7J's Maths teacher was playing 'Find my four-digit number' with the class. Read the following clues that she gave the class, then find the numbers.
  - **a** 'All my digits are different, but greater than one. None of my digits are multiples of each other. I am an ascending number (my digits are in increasing order from left to right) and the sum of the differences between my successive digits is 4. There is more than one answer.'
  - **b** 'I am a multiple of a palindromic number and I am a palindromic number myself. The lowest common multiple of my digits is 18. My first two digits are the age of my grandfather and my last two digits are the age of my father.'

2.2 Factors

If a whole number divides exactly into another whole number (that is, without any remainder) then it is said to be a **factor** of the other whole number. So 3 is a factor of 12 because 12 ÷ 3 = 4 with no remainder. This means 4 is also a factor of 12. We can write 12 as the product of these two factors: 3   4 = 12.

Factor pairs

One way to find the factors of a whole number is to find all the pairs of whole numbers that multiply together to give the original number.

For example, we can write

$1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$

so 1, 2, 3, 4, 6 and 12 are the factors of 12.

Remember: 1 is a factor of all whole numbers and the largest factor of a number is the number itself.

Example 4

Find all the factor pairs of the following numbers then list the numbers in ascending order (from smallest to largest).

a 24 **b** 125 **c** 225

Working	Reasoning
a 1 24 = 24 2 12 = 24 3 8 = 24 4 6 = 24 So the factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.	Find all pairs of numbers that multiply together to give 24.
b 1 125 = 125 5 25 = 125 1, 5, 25 and 125 are factors of 125.	Find all pairs of numbers which multiply together to give 125. State the answer in ascending order.
c 1 225 = 225 3 75 = 225 5 45 = 225 9 25 = 225 15 15 = 225 The factors of 225 are 1, 3, 5, 9, 15, 25, 45, 75 and 225.	Find all pairs of numbers that multiply together to give 225. Using a calculator can help. 225 3 = 75, so 3 75 = 225 225 5 = 45, so 5 45 = 225 225 9 = 25, so 9 25 = 225 225 15 = 15, so 15 15 = 225

Common factors

A **common factor** of a set of numbers is a factor which is common to all the numbers in the set.

Consider the numbers 12 and 16.

Factors of 12	(1), (2), 3, (4), 6, 12
Factors of 16	(1), (2), (4), 8, 16

The common factors of 12 and 16 are the factors that are in both lists. The **highest common factor** of 12 and 16 is 4 because it is the largest of the common factors.

Example 5

Find the highest common factor of 16, 24 and 32 by first finding all the factors of each number.

Working

The factors of 16 are 1, 2, 4, (8) and 16.
The factors of 24 are 1, 2, 3, 4, 6, (8), 12 and 24.
The factors of 32 are 1, 2, 4, (8), 16 and 32.

The highest common factor of 16, 24 and 32 is 8.

Reasoning

1 16 = 16, 2 8 = 16, 4 4 = 16
1 24 = 24, 2 12 = 24, 3 8 = 24,
4 6 = 24
1 32 = 32, 2 16 = 32, 4 8 = 32

The largest factor common to all three numbers is 8.

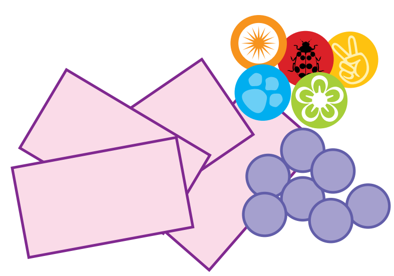
Many problems can be solved using common factors.

Example 6

A kindergarten teacher has 27 pieces of pink paper, 18 purple dots and 36 stickers to distribute among the groups of children.

a What is the largest number of groups the children can be divided into if each group is to have an identical set of paper, dots and stickers, with no materials left over?

b How many pieces of pink paper, purple dots and stickers will each group of children receive?



continued

Example 6 continued

Working	Reasoning
a 27 pieces of paper 18 dots 36 stickers	The teacher must be able to make the same number of groups from each of the three different types of items. So the problem can be solved by finding the highest common factor of 27, 18 and 36.
The factors of 27 are 1, 3, (9) and 27. The factors of 18 are 1, 2, 3, 6, (9) and 18. The factors of 36 are 1, 2, 3, 4, 6, (9), 12, 18 and 36.	List the factors of 27, 18 and 36.
The highest common factor of 18, 27 and 36 is 9. The kindergarten teacher should prepare 9 sets of art material.	Find the highest common factor.
b Each group of children would receive $\frac{27}{9} = 3$ pieces of pink paper, $\frac{18}{9} = 2$ purple dots and $\frac{36}{9} = 4$ stickers.	27 = 3 9 so 27 9 = 3 18 = 2 9 so 18 9 = 2 36 = 4 9 so 36 9 = 4

exercise 2.2

LINKS TO Example 3

Find all the factor pairs of each of the following numbers. List the factors in ascending order.

a 12 **b** 20 **c** 25 **d** 30
e 21 **f** 16 **g** 42 **h** 64

from smallest to largest

Find all the factor pairs of each of the following numbers. List the factors in descending order.

a 100 **b** 132 **c** 121 **d** 144
e 108 **f** 169 **g** 196 **h** 245

from largest to smallest

LINKS TO Example 4

Find the highest common factor of the following numbers.

a 12 and 15 **b** 24 and 36 **c** 25 and 35
d 42 and 48 **e** 66 and 99 **f** 32 and 80

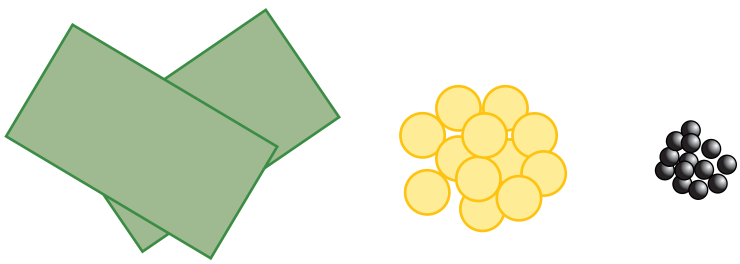
For each of the following list the factors of the first number, list the factors of the second number and find the highest common factor of both numbers.

a 33 and 99 **b** 60 and 125 **c** 72 and 98
d 144 and 152 **e** 150 and 225 **f** 230 and 300

- For each of the following list the factors of the first number, list the factors of the second number, list the factors of the third number and find the highest common factor of the numbers.
  a 121, 226 and 990 **b** 27, 81 and 135 **c** 132, 164 and 168
  d 21, 63 and 126 **e** 35, 105 and 210 **f** 44, 64 and 192

- A netball club has 126 under-13 members.
  a How many teams of seven players can be made?
  b If each team has two emergency players, how many teams are possible?

- Lee is preparing materials for the children in her kindergarten class to make cardboard dinosaurs. She has 72 pieces of green cardboard, 384 yellow dots and 96 black beads. Lee wants to make up sets of materials that all have the same number of pieces of card, yellow dots and black beads.



  a What is the maximum number of equal sets of material Lee can prepare if all the material is to be used?
  b How many pieces of green cardboard, yellow dots and black beads will be in each set?

- There are 132 students in Year 7. They are to be divided into equal-sized classes with no more than 25 students per class.
  a How many classes will there be?
  b How many students will be in each class?

LINKS TO Example 5

- 7M are making up lolly bags to sell on their social service stall. If they have 117 jelly snakes, 78 chocolate frogs and 195 mini-Pluto bars,
  a what is the maximum number of bags they can make up if they use all the lollies and each bag contains an identical set of lollies?
  b how many of each type of lolly will each bag contain?

exercise 2.2 challenge

- Ancient Greek mathematicians called certain numbers **perfect numbers**. A perfect number is a number whose factors, apart from the number itself, add up to itself. The smallest perfect number is 6. Apart from 6 itself, the factors of 6 are 1, 2 and 3. These factors add to 6: $1 + 2 + 3 = 6$. Show that 28 and 496 are perfect numbers.

- Two numbers are called amicable numbers if the factors of each number (excluding the number itself) add to the other number. Show that 220 and 284 are amicable numbers.

2.3 Divisibility

Divisibility tests

Divisibility tests provide a quick way to find factors of large numbers when a calculator is not available.

Divisibility tests for 2, 4 and 8

When testing for divisibility by 2, 4 and 8, remember the following points.

- A number is *divisible by 2* if it is even. (Even numbers end in 0, 2, 4, 6 or 8.)
- A number is *divisible by 4* if the last two digits of the number are divisible by 4.
- A number is *divisible by 8* if the last three digits are divisible by 8.
- If a number is divisible by 4, then it must be divisible by 2 because 2 is a factor of 4.
- If a number is divisible by 8, then it must be divisible by 2 and 4 because 2 and 4 are factors of 8.

Example 7

Use divisibility tests to answer the following questions.

a Is 3784 divisible by 2?

b Is 3784 divisible by 4?

c Is 3784 divisible by 8?

d Change the 7 in 3784 to another digit so that the new number is divisible by 4 but not 8.

Working	Reasoning
a 3784 is an even number. Hence, 3784 is divisible by 2.	The last digit is an even number. 3784 2 = 1892
b The last two digits, 84, are divisible by 4. So 3784 is divisible by 4.	Check to see if the last two digits are divisible by 4. 84 4 = 21
c The last three digits, 784, are divisible by 8. So 3784 is divisible by 8.	Check to see if the last three digits are divisible by 8. 784 8 = 98

continued

Example 7 continued

Working

d

	Divisible by 8
84	No
184	Yes, 184 8 = 23
284	No
384	Yes, 384 8 = 48
484	No
584	Yes, 584 8 = 73
684	No
884	No
984	Yes, 984 8 = 123

So the third digit could be 0, 2, 4, 6 or 8.

Reasoning

84 is divisible by 4 so any of the possibilities would be divisible by 4. 3_84 is not divisible by 8 if _84 is not divisible by 8. Try each of the digits 0 to 9. (We already know from part c that 3784 is divisible by 8.)

Divisibility tests for 3, 6 and 9

When testing for divisibility by 3, 6 and 9, remember the following points.

- A number is *divisible by 3* if the sum of its digits is divisible by 3.
- A number is *divisible by 6* if it is divisible by 2 and 3.
- A number is *divisible by 9* if the sum of its digits is divisible by 9.
- If a number is divisible by 6, then it must be divisible by 2 and 3 because 2 and 3 are factors of 6.
- If a number is divisible by 9, then it must also be divisible by 3 because 3 is a factor of 9.

Example 8

Use divisibility tests to answer the following questions.

a Is 463 716 divisible by 3?

b Is 463 716 divisible by 6?

c Is 463 716 divisible by 9?

continued

Example 8 continued

Working	Reasoning
a $4 + 6 + 3 + 7 + 1 + 6 = 27$ 27 is divisible by 3. So 463 716 is divisible by 3.	A number is divisible by 3 if the sum of its digits is divisible by 3.
b 463 716 is even and divisible by 3. So 463 716 is divisible by 6.	A number is divisible by 6 if it is divisible by 2 and by 3.
c 27 is divisible by 9. So 463 716 is divisible by 9.	A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility tests for 5 and 10

Divisibility tests for 5 and 10 are based on the following rules.

- A number is *divisible by 5* if it ends in 0 or 5.
- A number is *divisible by 10* if it ends in 0.
- If a number is divisible by 10, it is also divisible by 2 and 5 because 2 and 5 are factors of 10.

Example 9

Use the divisibility tests to answer the following questions.

a Is 1 267 765 divisible by 5?

b Is 1 267 765 divisible by 10?

Working	Reasoning
a 1 267 765 is divisible by 5.	A number is divisible by 5 as the last digit is 5.
b 1 267 765 is not divisible by 10.	The number is not divisible by 10 as the last digit is not zero.

Divisibility tests

There are divisibility tests for 7 and 11 but it is usually quicker to use short multiplication.

exercise 2.3

LINKS TO Example 7

Find if the following numbers are divisible by

a 2 **b** 4 **c** 8

32 66 260 232 1563 2768 12 346 108 556

LINKS TO Example 8

Find if the following numbers are divisible by

a 3 **b** 6 **c** 9

24 99 123 676 3248 7677 14 566 54 879

2.3

LINKS TO Example 9

Find if the following numbers are divisible by

a 5 **b** 10

25 30 124 765 870 1345 76 540 109 900

Which of the following numbers are divisible by 3?

a 345 **b** 768 **c** 7689 **d** 6779
e 12 678 **f** 76 898 **g** 109 786 **h** 657 777

Which of the following numbers are divisible by 4?

a 872 **b** 981 **c** 7892 **d** 8764
e 23 678 **f** 87 654 **g** 678 732 **h** 879 826

Which of the following numbers are divisible by 6?

a 889 **b** 780 **c** 1456 **d** 7684
e 54 786 **f** 65 967 **g** 203 654 **h** 456 562

Which of the following numbers are divisible by 8?

a 416 **b** 522 **c** 5080 **d** 8976
e 79 920 **f** 88 890 **g** 799 992 **h** 898 666

Which of the following numbers are divisible by 9?

a 315 **b** 342 **c** 4567 **d** 6120
e 34 678 **f** 79 992 **g** 231 201 **h** 567 647

Which of the following numbers are divisible by 7? Use short division.

a 252 **b** 799 **c** 4823 **d** 8790
e 48 993 **f** 91 234 **g** 657 888 **h** 485 716

Which of the following numbers are divisible by 11? Use short division.

a 999 **b** 759 **c** 2959 **d** 8245
e 73 326 **f** 89 998 **g** 733 326 **h** 678 666

Which of the following numbers are divisible by 3 but not by 9?

a 233 457 **b** 7 564 896 **c** 8 754 345 **d** 8 765 475

Which of the following numbers are divisible by 4 but not by 8?

a 145 672 **b** 2 654 688 **c** 3 647 812 **d** 4 576 104

Fill in the missing digits to make the following numbers divisible by the given number. State all possible answers.

a 124 56_ divisible by 2 **b** 124 56_ divisible by 6
c 1_7 643 divisible by 9 **d** 648_64 divisible by 8

Which of the following statements are true?

a 463 752 is divisible by 2, 4 and 8 **b** 165 516 is divisible by 3, 6 and 9
c 543 876 is divisible by 4 and 12 **d** 447 660 is divisible by 2, 3, 5 and 9

- The Year 7 Social Service Committee ran a competition to guess the number of Smarties in a jar. The prize for guessing the correct number was the jar of Smarties, to be shared if more than one person guessed the correct answer.
  - **a** If there were 2763 Smarties and nine people guessed correctly, would it be possible to share the Smarties exactly between the nine winners?
  - **b** How many Smarties would each of them get?
- 452 people are waiting to buy grand final tickets to the football. There are eight queues.
  - **a** Is it possible for each queue to have the same number of people?
  - **b** Would your answer to part a change if another thousand people showed up? Explain.
- What year am I?
  - **a** I am the first year in this century to be divisible by 3 and 6 but not by 9
  - **b** I am in the last year in this century to be divisible by 2 and 4 but not by 8
- What is the largest six-digit number that is divisible by 6 but not divisible by 9?
- When Tim asked his teacher how old she was, she replied: 'The sum of the digits of my age is divisible by 9. The first digit has three different factors.' How old was Tim's teacher?
- Find the missing digit in the following numbers, given the stated condition. Give all possible answers.
  - **a** 257 85_ One of my factors is 9
  - **b** 685 12_ One of my factors is 3 but not 9
  - **c** 457 _64 One of my factors is 6

exercise 2.3 challenge

- Find numbers that satisfy these conditions.
  - **a** the largest seven-digit number that is divisible by 6 and 9
  - **b** the smallest seven-digit number that is divisible by 4, 5 and 9
  - **c** the even number between 200 and 300 that is divisible by 5 and also by 9

2.4 Square numbers, square roots and cubes

Squares

A **square number** is formed when a whole number is multiplied by itself. When 3 is multiplied by itself, the product is 9, so 9 is a square number.

We can write $9 = 3 \quad 3 = 3^2$.

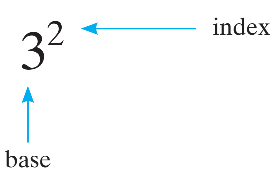
We read 3^2 as '3 squared'.

Similarly, $25 = 5 \quad 5 = 5^2$, so 25 is a square number.

When we write 9 as 3^2 we are writing it in **index notation**.

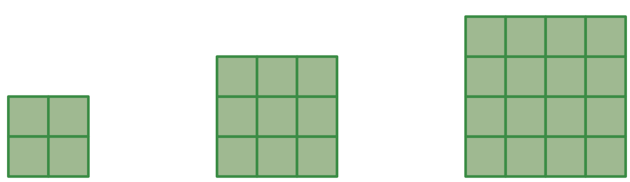
3 is called the base and 2 is called the index.

'Notation' simply means 'a way of writing'.



We call 3^2 a **power**. As well as reading 3^2 as '3 squared', we can also read it as '3 to the power 2'.

Square numbers can be represented as square arrays.



$2^2 = 2 \times 2 = 4$ $\quad$ $3^2 = 3 \times 3 = 9$ $\quad$ $4^2 = 4 \times 4 = 16$

Example 10

For the numbers 1 to 10
a square each number.
b write the squared numbers in index notation.

Working	Reasoning
a 1, 4, 9, 16, 25, 36, 49, 64, 81, 100	Work out $1 \times 1, 2 \times 2, 3 \times 3, \ldots$
b $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2$	$1 \times 1 = 1^2, 2 \times 2 = 2^2, 3 \times 3 = 3^2, \ldots$

Example 11

Use your calculator to find the following numbers.

a All the square numbers between 100 and 200.

b The number that when squared gives you 729.

Working	**Reasoning**
a 121, 144, 169, 196	$11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$
b $23 \quad 23 = 529$ 529 is too small.	Try guess and check to start. 729 ends in 9, so the number we want must end in 3 or 7 because $3 \quad 3 = 9$ and $7 \quad 7 = 49$. When we try 23 we find that $23^2 = 529$ is too small.
$27^2 = 729$ The number is 27.	Try 27. $27 \quad 27 = 729$

Tech tip

The TI-30XB MultiView calculator can be used to show the square of a number. To do this press the following keys shown here for 13^2.

[1] [3] [x^2] [enter]

Square roots

Finding the **square root** of a number is the opposite (inverse) of squaring the number; for example, the square root of 729 equals 27 because $27^2 = 729$. The symbol used for square root is $\sqrt{\ }$. So we can write $\sqrt{729} = 27$.

Example 12

Find the square root of each of these numbers. In part b use your calculator.

a 49 **b** 841

Working	**Reasoning**
a $\sqrt{49} = 7$	$7^2 = 49$
b $\sqrt{841} = 29$	Check: $29^2 = 841$

Tech tip

To show square roots on the TI-30XB MultiView calculator, for example, $\sqrt{169}$, type:

[2nd] [x^2] [1] [6] [9] [enter].

Note that [2nd] [x^2] gives [$\sqrt{\ }$].

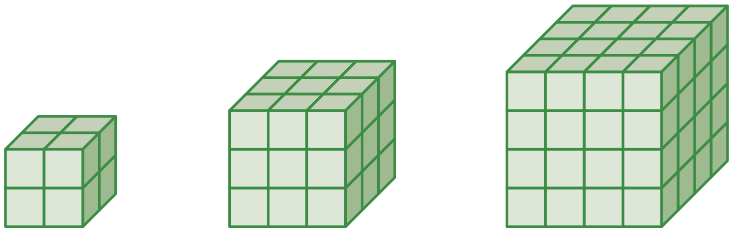
Cubes

When a number is raised to the power 3, we say that it is cubed.

$2 \times 2 \times 2 = 2^3 = 8$

We read 2^3 as '2 cubed'.

Numbers raised to the power 3 can be represented as an array that forms a cube.



$2^3 = 2 \times 2 \times 2 = 8$ $3^3 = 3 \times 3 \times 3 = 27$ $4^3 = 4 \times 4 \times 4 = 64$

Example 13

Write 5^3 as a product of its repeated factors then evaluate it.

Working

$5^3 = 5 \times 5 \times 5$
$= 125$

Reasoning

5^3 means that there are 3 fives multiplied together.

Tech tip

To raise a number to a power on the TI-30XB MultiView calculator, for example 13^3, type:

[1] [3] [^] [3] [▶] [enter].

exercise 2.4

LINKS TO Example 10

1. Copy and complete the following table of square numbers.

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
1	4								
11^2	12^2	13^2	14^2	15^2	16^2	17^2	18^2	19^2	20^2
121									

LINKS TO Example 11

Use your calculator to square the following numbers.

a 3	**b** 34	**c** 99	**d** 21
e 24	**f** 432	**g** 988	**h** 876
i 6789	**j** 8755	**k** 12 345	**l** 43 432

LINKS TO Example 11

Use your calculator to find all the square numbers between 400 and 500.

LINKS TO Example 11

Use your calculator to find the numbers that when squared give you

a 784	**b** 841	**c** 1225	**d** 1024
e 1296	**f** 961	**g** 2025	**h** 4096

Square each of these numbers then look for a pattern in the number of zeros in the square number. Use your calculator to help you.

a 20	**b** 30	**c** 50	**d** 100
e 120	**f** 200	**g** 700	**h** 2000

LINKS TO Example 12

Evaluate the following, using your calculator when needed.

a $\sqrt{625}$	**b** $\sqrt{961}$	**c** $\sqrt{1089}$
d $\sqrt{441}$	**e** $\sqrt{1369}$	**f** $\sqrt{1}$
g $\sqrt{0}$	**h** $\sqrt{17^2}$	**i** $\sqrt{100}$
j $\sqrt{10000}$	**k** $\sqrt{6400}$	**l** $\sqrt{810000}$

LINKS TO Example 13

Write each of these cubes as a product of its repeated factors then evaluate. Use your calculator where you need to.

a 6^3	**b** 10^3	**c** 1^3	**d** 8^3
e 12^3	**f** 20^3	**g** 100^3	**h** 1000^3

exercise 2.4 challenge

Nadim's postcode is a four-digit number. The sum of the digits is even. The first 2 digits form a two-digit number that is divisible by 9. The first and third digits form a square number. The second and fourth digits form a multiple of 11. What is Nadim's postcode?

Numbers that are made up only of the digit 1 form some interesting patterns when they are squared.

a Use your calculator to evaluate the following squares.

i 11^2	**ii** 111^2	**iii** 1111^2	**iv** $11\,111^2$

b Use the pattern you observed in part a to predict the value of the following squares.

i $111\,111^2$	**ii** $1\,111\,111^2$	**iii** $11\,111\,111^2$	**iv** $111\,111\,111^2$

c Use a calculator, such as the Microsoft Windows calculator, (which can display 32 digits) to check your predictions.

d Predict what might happen in the case of $1\,111\,111\,111^2$, then use the computer calculator to check your prediction.

2.5 Numbers in index form

Index notation

We have seen in the previous section how square and cube numbers can be written in index form (or index notation). In the same way, any number that is the product of repeated factors can be written in index form. In 7^5, for example, the factor that is repeated is 7, so 7 is the base. There are 5 repeated factors, so the index is 5.

5 is the index

$$\underbrace{7 \times 7 \times 7 \times 7 \times 7}_{\text{5 factors}} = 7^5$$

7 is the base

We read 7^5 as '7 to the power 5'.

Example 14

Write these numbers in index notation and state which number is the base and which is the index.

a four to the power seven

b $5 \times 5 \times 5 \times 5 \times 5 \times 5$

Working

a four to the power seven $= 4^7$
4 is the base and 7 is the index.

b $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$
5 is the base.
6 is the index.

5 to the power 6

Reasoning

'to the power seven' tells us that the index is 7.
There are seven 4s multiplied together.

The repeated factor is 5. There are 6 factors multiplied together.

We can only write products of factors in index notation if the factors are the same. If a product includes two or more different repeated factors then we can write each of the repeated factors in index notation.

Example 15

Write $3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4 \times 7 \times 7$ in index notation.

Working

$3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4 \times 7 \times 7$
$= 3^3 \times 4^5 \times 7^2$

Reasoning

There are three 3s, five 4s and two 7s.

Expanded notation

When a number in index notation is written as a product of its factors, we say it is written in **expanded notation**. We write 7^5 in expanded notation as $7 \times 7 \times 7 \times 7 \times 7$.

Example 16

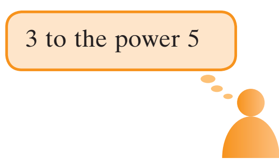
Write these numbers in expanded notation.

a 3^5

b $2^3 \times 3^4 \times 7^2$

Working

a $3^5 = 3 \times 3 \times 3 \times 3 \times 3$



b $2^3 \times 3^4 \times 7^2$
$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$

Reasoning

3 is the base and 5 is the index. There are five 3s multiplied together.

There are three 2s, four 3s and two 7s.

Example 17

Do repeated multiplication on a calculator to find the value of 8^5.

Working

$8^5 = 8 \times 8 \times 8 \times 8 \times 8$
$= 32768$

Reasoning

8 raised to the power 5 is equal to five 8s multiplied together.

Most calculators have a key for raising numbers to a power. Find out how to do this on your calculator.

Example 18

Use your calculator to evaluate $3^4 \times 2^5$.

Working

$3^4 \times 2^5 = 2592$

Reasoning

Check:

$3^4 \times 2^5 = 81 \times 32$
$= 2592$

Tech tip

To show repeated multiplication by the same factor on the TI-30XB MultiView calculator, for example $13 \times 13 \times 13 \times 13 \times 13 \times 13$, type:

1 3 × 1 3 enter × 1 3 enter enter enter .

To show the multiplication of two or more numbers each raised to a power on the TI-30XB MultiView calculator, for example $3 \times 2^4 \times 3^3$, type:

3 × 2 ^ 4 × 3 ^ 3 enter .

exercise 2.5

LINKS TO Example 14

1 Copy and complete this table.

Words	Index notation	Expanded notation	Value
Three to the power six			
Seven cubed			
		$5 \times 5 \times 5 \times 5$	
	6^5		
		13×13	
	3^8		
Eleven to the power four			
		$29 \times 29 \times 29$	
			32
	10^7		

LINKS TO Example 14

Copy and complete this table.

		Base	Index
a	2^8		
b	8^2		
c	17^3		
d	10^5		
e	1^6		
f	4^7		
g	3^{11}		
h	11^3		
i	15^2		
j	25^6		

LINKS TO Example 15

Write the following using index notation.

a 3 3 3 3
b 5 5 5 5 5 5
c 6 6 6
d 7 7 7 7 7 7 7 7
e $9 \times 9 \times 9 \times 9 \times 9$
f $12 \times 12 \times 12 \times 12 \times 12 \times 12$
g $31 \times 31 \times 31 \times 31 \times 31 \times 31 \times 31$
h $10 \times 10 \times 10 \times 10 \times 10$
i $57 \times 57 \times 57$
j $100 \times 100 \times 100 \times 100$
k 4 4 4 4 4 4 4 4
l 8 8 8 8

Write the following using index notation.

a 2 2 2 3
b 7 7 9 9 9 9
c 4 4 6 6 6
d 2 2 2 2 5 5 5 5
e 2 3 3 3 5 5
f 5 5 5 12 12 12
g 4 6 6 7 7
h 7 7 7 9 13 17 17 17
i $2 \times 2 \times 5 \times 5 \times 19 \times 19 \times 19$
j $11 \times 13 \times 13 \times 13 \times 15 \times 15 \times 15$
k 3 3 8 8 8 11 11
l 2 2 2 5 5 5 13 13

LINKS TO Example 16a

Write the following numbers in expanded form.

a 2^5
b 7^3
c 4^6
d 5^4
e 1^3
f 3^5
g 9^6
h 6^9
i 4^3
j 3^4
k 15^5
l 36^4

LINKS TO Example 16b

Write the following numbers in expanded form.

a $2^3 \quad 5^6$
b $3^2 \quad 6^5$
c $7^3 \quad 8^4$
d $4^5 \quad 9^3$
e $3^4 \times 4^3$
f $11^5 \times 7^4$
g $13^3 \times 8^6$
h $29^2 \times 31^4$
i $43^6 \times 57^2$
j $134^4 \times 234^3$
k $1243^2 \times 9899^5$
l $8788^3 \times 333^6$

6 6 6 5 5 5 3 3 3 is *not* equivalent to

A 6^3 5^3 3^3 **B** 6^3 3^5 3^3 **C** 30^3 3^3 **D** 6^3 15^3 **E** 90^3

LINKS TO Example 17

Evaluate the following using your calculator.

a 5^8 **b** 8^3 **c** 4^5 **d** 2^7

e 7^4 **f** 3^8 **g** 9^5 **h** 1^{11}

i 12^6 **j** 45^3 **k** 62^5 **l** 102^4

LINKS TO Example 18

Use your calculator to evaluate each of the following.

a $2^3 \times 3^5$ **b** $2^4 \times 5^2$ **c** $5^3 \times 7^2$ **d** $7^3 \times 2^4$

e $5^4 \times 2^5$ **f** $2^4 \times 3^4$ **g** $3^2 \times 2^3$ **h** $2^4 \times 3^6$

exercise 2.5 challenge

Kate sends letters to seven of her friends. Each of her friends sends letters to another seven friends, who in turn send letters to another seven friends. How many stamps were used?

Write 64 in index notation in three different ways, that is, with three different bases.

Find as many ways as you can of writing 144 in index notation.

2.6 Prime numbers

A **prime number** is a whole number that has only two factors, 1 and itself.

2 is a prime number because its only factors are 1 and 2.

5 is a prime number because its only factors are 1 and 5.

A **composite number** is a whole number that has more than two factors.

4 is a composite number because it has more than two factors: 1, 2 and 4.

12 is a composite number because it has more than two factors: 1, 2, 3, 4, 6, 12.

The number 1 is regarded as neither prime nor composite because it has only one factor, 1.

Prime numbers have fascinated mathematicians for thousands of years because no pattern has ever been found in sequences of prime numbers.

Eratosthenes was a Greek astronomer and mathematician who lived from about 270 BCE until 194 BCE. He devised a method for finding all the prime numbers between 1 and 100 by sifting out the numbers that he knew were not prime. Imagine a container with holes in it (a sieve) that allows the composite numbers to fall through the holes, but the prime numbers stay behind.

Sieve of Eratosthenes

When you complete the task *Sieve of Eratosthenes* in the student ebook, you will have a list of all the prime numbers from 2 to 100.

Example 19

Find these prime numbers.

a List the primes between 50 and 80.

b What is the largest prime less than 100?

Working

a 53, 59, 61, 67, 71, 73, 79

b 97 is the largest prime less than 100.

Example 20

State whether each of the following numbers is composite or prime.

a 34 **b** 57 **c** 83 **d** 47

continued

Example 20 continued

Working	Reasoning
a 34 is composite.	34 is divisible by 2. $34 = 2 \quad 17$
b 57 is composite.	57 is divisible by 3. $57 = 3 \quad 19$
c 83 is prime.	83 has only two factors: 1 and 83.
d 47 is prime.	47 has only two factors: 1 and 47.

To test if a number greater than 100 is prime, we test it for divisibility by prime numbers. If we find that the number is not divisible by 2, 3, 5, 7, and so on, we know that it is also not divisible by 4, 6, 8, 9 and so on. We only need to continue until we reach the largest prime before the square root of the number. If we have found that the number is not divisible by any prime numbers up to and including its square root, then it must be prime.

The reason we need only go as far as the square root is that factors of a number occur in pairs. For example, the square root of 20 is approximately 4.47. We can write 20 as the product of factor pairs 1×20, 2×10, or 4×5. Once we reach the square root, the factor pairs are the same but reversed in order, 5×4, 10×2, or 20×1 so there are no new factors.

Example 21

Is 223 a prime number?

Working	Reasoning
$\sqrt{223} \approx 14.9$	Use a calculator to find the square root of 223. Test for all the prime numbers as far as the last prime number before 14.9, that is, 13.
223 is not divisible by 2, 3 or 5.	223 is an odd number so it is not divisible by 2. This means that it is not divisible by 4 or 8 either. The sum of the digits is $2 + 2 + 3 = 7$, which is not divisible by 3, so 223 is not divisible by 3. This means it is not divisible by 6 or 9 either. 223 does not end in 0 or 5 so it is not divisible by 5.
223 is not divisible by 7, 11 or 13.	7, 11 and 13 are the next prime numbers after 5. Use a calculator to check for divisibility by these numbers.
223 is a prime number because its only factors are 1 and 223.	223 is not divisible by any prime numbers up as far as its square root.

Factors and primes

Example 22

Is 377 a prime number?

Working

$\sqrt{377}$ is approximately 19.4.
Test 377 for divisibility by all the primes up to and including 19.

377 is not divisible by 2, 3 or 5.

377 is not divisible by 7, 11.

377 is divisible by 13.

377 is not prime. 377 = 13   29

Reasoning

Use a calculator to find $\sqrt{377}$.
19 is a prime.
You need to test all primes less than or equal to 19.

Use divisibility tests to check.

Use a calculator to check.

Use a calculator: 377   13 = 29.

exercise 2.6

LINKS TO Example 19

1 The prime numbers from 1 to 100 plus one composite number are listed below.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97

a Which number is the composite number?

b Write down the factors of the composite number in ascending order.

c Complete the following table. The first row has been done for you.

Category	Prime numbers	Number of primes
Between 0 and 10	2, 3, 5, 7	4
Between 10 and 20		
Between 20 and 30		
Between 30 and 40		
Between 40 and 50		
Between 50 and 60		
Between 60 and 70		
Between 70 and 80		
Between 80 and 90		
Between 90 and 100		

d How many prime numbers less than 50 are there?

e How many odd prime numbers are there in the above list?

2.6

LINKS TO Example 19

Which of the following is not a prime number?

A 1 **B** 2 **C** 31 **D** 97 **E** 107

LINKS TO Example 20

State whether each of the following numbers is composite or prime.

a 71 **b** 51 **c** 39 **d** 29
e 91 **f** 97 **g** 49 **h** 41

LINKS TO Examples 21, 22

Use divisibility tests to find the next two prime numbers after 97.

LINKS TO Examples 21, 22

Use divisibility tests and your calculator to test if the following numbers are prime. If they are not prime, list the factors.

a 173 **b** 131 **c** 129 **d** 1071

True or false?

a A prime number has only two factors which are one and itself.
b 7 is a prime number.
c 15 is not a prime number.
d 1 is a prime number.
e There are 4 prime numbers between 30 and 40.
f There are no prime numbers which end in zero.
g 21 is a composite number.
h There are only two prime numbers that differ by 1.

If you reverse the digits of some two-digit prime numbers you get another prime number, for example, 13 and 31. Write down any other pairs of prime numbers between 13 and 100 in which the digits are reversed.

Each of the following questions relates to prime numbers. When answering these questions, show your working.

a Find the sum of the first five prime numbers.
b Find the difference between the largest prime number which is less than 50 and the smallest prime number which is greater than 20.
c Find the product of the only even prime number and the smallest prime number greater than 80.
d Find the sum of the two prime numbers closest to but greater than 40.
e Find the sum of the prime numbers between 10 and 20.

Pairs of prime numbers that differ by two are called twin primes. For example, 3 and 5 are twin primes. List all the other pairs of twin primes less than 100.

A set of cards are numbered 1 to 20. Mia takes out all of the cards divisible by 5, then Michael takes out all of the remaining cards divisible by 3, then Olivia takes out the cards remaining that are divisible by 2.

a List the cards that Mia took.
b List the cards that Michael took.
c List the cards that Olivia took.
d List the cards left after they have all selected their cards.

e If Olivia went first and selected the cards which are multiples of 2, then Michael took the multiples of 3, then Mia took the remaining multiples of 5, list the cards each would have.

f Is the set left over the same as before or different? Why?

- 'Square numbers cannot be prime numbers.' Is this statement true? Explain.

- The following questions relate to prime numbers.

a Find 3 three-digit palindromic numbers which are prime.

b What is the largest two-digit prime number whose digits are both prime?

c What is the largest three-digit prime number whose digits are all prime?

d Explain why 3, 5 and 7 are the only triple primes, that is, three prime numbers that are each two apart.

e Find all prime numbers between 1 and 200 that are one more than a perfect square.

f Find all prime numbers between 1 and 200 that are one less than a perfect square.

exercise 2.6 challenge

- In 1742, Goldbach made a conjecture about prime numbers.

'Every **even** number greater than two can be written as the sum of two prime numbers.'

This has never been proved, but no exceptions have ever been found either. Goldbach's claim is therefore called a **conjecture**.

Show how each of these even numbers can be written as the sum of two prime numbers. Try to find as many ways as you can.

a 4 b 26 c 38 d 46 e 96 f 98

- Another conjecture about prime numbers is that 'Every **odd** number greater than five can be written as the sum of three prime numbers'.

Show how each of these odd numbers can be written as the sum of three prime numbers. Find as many ways as you can.

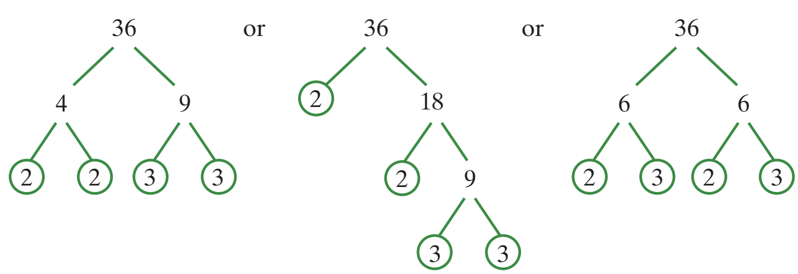
a 7 b 11 c 27 d 31 e 257 f 413

2.7 Finding prime factors

Prime factors of a composite number are the factors of the composite number that are prime numbers. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12, but the prime factors of 12 are just 2 and 3 because these are the only factors of 12 that are prime numbers.

Factor trees and factor ladders

A **factor tree** is one way of finding the prime factors of a composite number. Start with the number 36 and find a factor pair, say 4 and 9. It does not matter which pair you use as the starting pair. Factor trees for 36 are shown below, starting with 4 and 9 and starting with 2 and 18. In each case the prime factors of 36 are circled in green.



Find a factor pair.

Find another factor pair if the number is not prime.

The **prime factors** of 36 are 2 and 3. They are the only factors of 36 which are prime numbers. The other factors of 36 (1, 4, 6, 12, 18 and 36) are not prime numbers.

We can write 36 as the **product of its prime factors**: $36 = 2 \quad 2 \quad 3 \quad 3$.

There is a shorter way of writing this if we use indices: $36 = 2^2 \quad 3^2$. The breaking down of a number into its prime factors is called **prime decomposition**. When we write a number as a product of its prime factors, we write the factors in ascending order.

Indices is the plural of index.

Example 23

For each of these numbers

- **i** use a factor tree to find the prime factors.
- **ii** list the prime factors.
- **iii** write each number as a product of its prime factors.

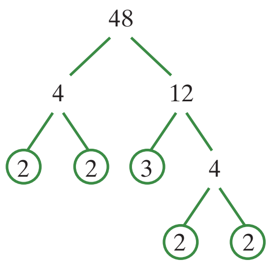
a 48 **b** 4725

continued

Example 23 continued

Working

a i



Find a pair of factors of 48.

Find a pair of factors for each of the composite numbers.

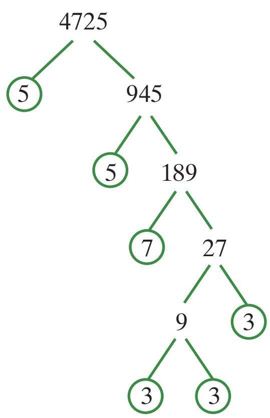
ii The prime factors of 48 are 2 and 3.

There are only two different prime factors: 2 and 3.

iii $48 = 2 \quad 2 \quad 2 \quad 2 \quad 3$
$= 2^4 \quad 3$

There are four branches that end with a 2 and one that ends with 3.

b i



Find a pair of factors of 4725.

Find a pair of factors for each of the composite numbers.

ii The prime factors of 4725 are 3, 5 and 7.

iii $4725 = 3 \quad 3 \quad 3 \quad 5 \quad 5 \quad 7$
$= 3^3 \quad 5^2 \quad 7$

There are 2 branches which end with a 5, one which ends with 7 and three which end with 3.

A much more efficient way of finding prime factors is to use a factor ladder.

When completing a factor ladder, we test the number for divisibility by 2, then by 3, then by 5 and so on. We continue dividing by prime numbers in order until we end up with 1. The left hand column then gives a neat list of all the prime factors from smallest to largest.

Example 24

For each of these numbers

i use the factor ladder method to find the prime factors.

ii list the prime factors.

iii write the number as the product of its prime factors in index notation.

a 312 **b** 2880

Working

a i

2	312
2	156
2	78
3	39
13	13
	1

Reasoning

Start with 2 as it is the smallest prime factor. Keep dividing by 2 until 2 is not a factor of the quotient. (As soon as the quotient is an odd number you know 2 is not a factor.)

Divide by 3, the next smallest prime factor. The next prime that is a factor of 39 is 13.

Dividing by 13 gives a quotient of 1.

ii The prime factors of 312 are 2, 3 and 13.

The left-hand column of the factor ladder gives us the prime factors.

iii 312 = 2 2 2 3 13
= 2^3 3 13

Write 2 2 2 as 2^3.

b i

2	2880
2	1440
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

Start with 2 and keep dividing by 2 until the quotient is not even.
Divide by 3, the next smallest prime factor, until the quotient is not divisible by 3.
Divide by 5. this gives a quotient of 1.

ii The prime factors of 2880 are 2, 3 and 5.

The left hand column gives us the prime factors.

iii 2880 = 2 2 2 2 2 2
3 3 5
= 2^6 3^2 5

Write $2 \times 2 \times 2 \times 2 \times 2 \times 2$ as 2^6 and 3×3 as 3^2.

Using index notation to find highest common factors

When two numbers are expressed in index notation, it is easy to find their highest common factor. The highest common factor is found by:

- Step 1: Look for the prime factors that are common to both numbers.
- Step 2: For each common prime factor, find the power that is common to both numbers, that is, the lower of the two powers.
- Step 3: Multiply the lowest of the two powers of each common prime factor.

If we consider the two numbers 108 and 96, for example, $108 = 2^2 \times 3^3$ and $96 = 2^5 \times 3$.

- Step 1: Prime factors common to both numbers are 2 and 3.
- Step 2: The power of 2 common to both numbers is 2^2 and the power of 3 common to both numbers is 3^1 (if we took a higher power it would not be common to both 108 and 96).
- Step 3: The highest common factor of 108 and 96 is $2^2 \times 3^1 = 12$.

Example 25

Find the highest common factor of each of these pairs of numbers by first writing each number in index notation.

a 18 and 48

b 588 and 168

Working	Reasoning
a $18 = 2 \times 3^2$ $48 = 2^4 \times 3$ The highest common factor of 18 and 48 is $2^1 \times 3^1$, that is, 6.	The power of 2 common to both numbers is 2^1. The power of 3 common to both numbers is 3^1.
b $588 = 2^2 \times 3 \times 7^2$ $168 = 2^3 \times 3 \times 7$ The highest common factor of 588 and 168 is $2^2 \times 3^1 \times 7^1$, that is, 84.	The power of 2 common to both numbers is 2^2. The power of 3 common to both numbers is 3^1. The power of 7 common to both numbers is 7^1.

Using index notation to find lowest common multiples

When two numbers are expressed in index notation, it is easy to find their lowest common multiple.

- Step 1: Look for all the prime factors that occur in the numbers.
- Step 2: For each prime factor choose the higher of two powers.
- Step 3: Multiply the higher of the powers of each prime factor occurring in either number.

If we consider the two numbers 48 and 90, for example, $48 = 2^4 \times 3$ and $90 = 2 \times 3^2 \times 5$:

- Step 1: Prime factors that occur in the numbers are 2, 3 and 5.
- Step 2: The highest powers are 2^4, 3^2 and 5^1. (The lowest common multiple must include 2^4 so that it is a multiple of 48 and it must include 3^2 and 5 so that it is a multiple of 90.)
- Step 3: The lowest common multiple of 48 and 90 is therefore $2^4 \times 3^2 \times 5 = 16 \times 9 \times 5 = 720$.

Example 26

Find the lowest common multiple of each of these pairs of numbers by first writing each number in index notation.

a 12 and 80

b 120 and 126

Working

a $12 = 2^2 \times 3$
$80 = 2^4 \times 5$
The prime factors are 2, 3 and 5.
The highest powers are 2^4, 3^1 and 5^1.
The lowest common multiple of 12 and 80 is $2^4 \times 3 \times 5$, that is, 240.

Reasoning

The lowest common multiple must include 2^4 as a factor so it is a multiple of 80. It must include 3 as a factor so that it is a multiple of 12 and it must include 5 as a factor so that it is a multiple of 80.

b $120 = 2^3 \times 3 \times 5$
$126 = 2 \times 3^2 \times 7$
The prime factors are 2, 3, 5 and 7
The highest powers are 2^3, 3^2, 5^1 and 7^1.
The lowest common multiple of 120 and 126 is $2^3 \times 3^2 \times 5 \times 7$, that is, 2520.

The lowest common multiple must include 2^3 as a factor so it is a multiple of 120. It must include 5 as a factor so that it is a multiple of 120. It must include 3^2 as a factor so it is a multiple of 126 and it must include 7 as a factor so that it is a multiple of 126.

Finding how many factors a number has

When we are dealing with small numbers such as 6, it is easy to find all the factors: 1, 2, 3 and 6. However, when we need to list all the factors of very large numbers, such as 3240, we may easily miss some of the factors. Writing a number as the product of its prime factors in index form gives us a very easy method of working out the total number of factors.

Let's look at the factors of 24. Written as the product of its prime factors $24 = 2^3 \times 3$. In expanded notation this is $2 \times 2 \times 2 \times 3$. To find all the factors of 24, we multiple all the possible combinations of these prime factors.

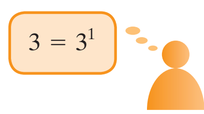
We can have:

2	2
2×2	4
$2 \times 2 \times 2$	8
3	3
2×3	6
$2 \times 2 \times 3$	12
$2 \times 2 \times 2 \times 3$	24

This gives 7 factors, but of course 1 is also a factor of every number, so 24 has a total of 8 factors. There is a shortcut way of working out this total number of factors.

> Write the number as the product of its prime factors in index notation, look at the indices of the prime factors, add 1 to each index and multiply these numbers together.

$$24 = 2^3 \times 3$$



The indices are 3 and 1. Adding 1 to each of these indices gives 4 and 2.

Multiply: $4 \times 2 = 8$, so there are 8 factors.

It can be shown that this method works for any number.

Example 27

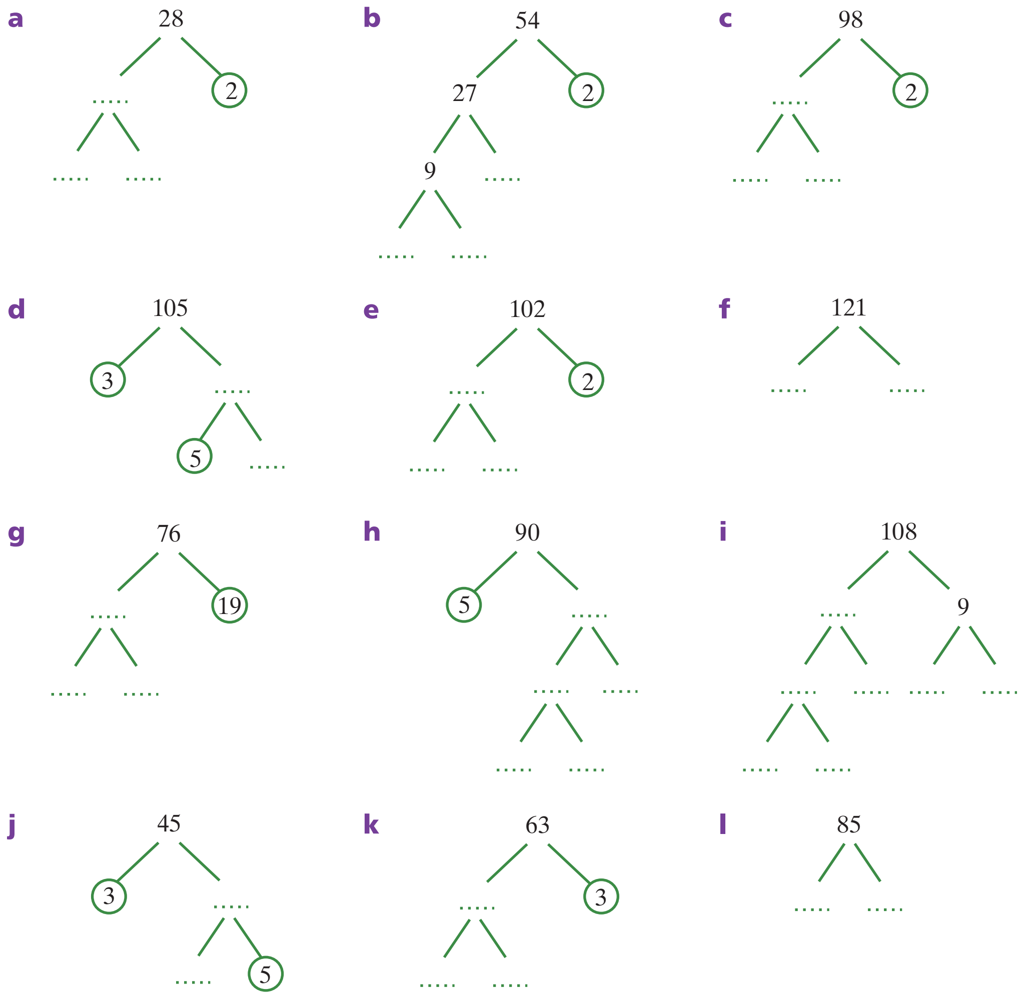
How many factors does 48 have?

Working	Reasoning
$48 = 2^4 \times 3$	Write 48 as the product of its prime factors in index notation.
The indices are 4 and 1. Add 1 to each index and multiply. $5 \times 2 = 10$ 48 has ten factors.	Add 1 to each index and multiply.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.	Check that there are 10 factors.

exercise 2.7

LINKS TO Example 23

Copy and complete the following factor trees. Circle the prime factors.



LINKS TO Example 23

For each of the following numbers

- **i** use a factor tree to find the prime factors.
- **ii** list the prime factors of the number.
- **iii** write each number as the product of its prime factors in index form.

a 27	**b** 36	**c** 45	**d** 12
e 96	**f** 81	**g** 99	**h** 68
i 132	**j** 264	**k** 312	**l** 245
m 144	**n** 567	**o** 623	**p** 754

LINKS TO Example 24

Complete the following factor ladders to find all the prime factors of each number. Then write the number as the product of its prime factors in index notation.

a

2	18
3	9

b

2	48
2	24

c

2	84
2	

d

2	42
3	

e

	1430

f

	7350

g

	1800

h

	2475

i

	1225

j

	3024

k

	79380

l

	89100

LINKS TO Example 24

For each of the following numbers

- **i** use a factor ladder to find the prime factors.
- **ii** list the prime factors of the number.
- **iii** write each number as the product of its prime factors in index form.

a 140 **b** 3060 **c** 10 584 **d** 1872

e 7200 **f** 39 600 **g** 72 765 **h** 4480

The prime factors of 345 are

A 5, 23 and 69 **B** 1, 5, 23 and 69 **C** 1, 5 and 23 **D** 5 and 23 **E** 3, 5 and 23

Which of the following is *not* a factor of 2482?

A 1 **B** 2 **C** 17 **D** 59 **E** 73

LINKS TO Example 24

For each of these pairs of numbers

i write each number in index form (use a factor tree to find the prime factors).

ii use the index form to find the highest common factor of the two numbers.

a 16 and 24 **b** 60 and 126 **c** 70 and 98
d 28 and 182 **e** 72 and 108 **f** 44 and 242
g 16 and 96 **h** 27 and 162 **i** 45 and 204
j 98 and 126 **k** 144 and 168 **l** 112 and 196

LINKS TO Example 25

For each of these pairs of numbers

i write each number in index form (use a factor tree to find the prime factor).

ii use the index form to find the lowest common multiple of the two numbers.

a 16 and 24 **b** 18 and 32 **c** 15 and 54
d 64 and 108 **e** 72 and 108 **f** 44 and 64
g 16 and 88 **h** 27 and 486 **i** 45 and 100
j 36 and 98 **k** 24 and 144 **l** 32 and 120

LINKS TO Example 26, 27

For each of the following numbers

i write the number as a product of its prime factors in index notation.

ii state how many factors the number has.

iii list all the factors.

a 81 **b** 100 **c** 32 **d** 2565

State whether the following are *true* or *false*.

a The prime factors of 1246 are 2, 7 and 89.

b The prime factors of 256 897 are 1, 31 and 8287.

c $6592 = 2^6 \times 103$

d 658 has 3 prime factors.

e 897 has 3 factors.

f $2^6 \times 3^4 \times 5^2$ has 105 factors.

exercise 2.7 challenge

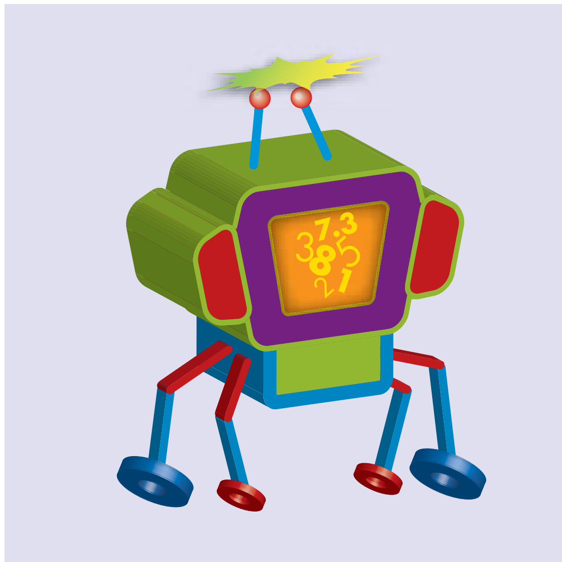
Jack's four-digit postcode has prime factors 3, 5 and 7. The postcode has a total of 18 factors. Jack lives in Victoria so his postcode starts with 3. What is his postcode?

Find all possible whole numbers that have only 2 and 3 as their prime factors and have a total of 12 factors.

Analysis task

Fleadles

The following problem is from the 1996 Junior Mathematics Challenge for Young Australians and is reproduced with the permission of the Australian Mathematics Trust.



At a factory where they make fleadles, they have four quality inspectors inspecting the fleadles as they come off the assembly line.

A batch of fleadles is numbered in order from 1 to 2500 and sent for inspection, each inspector inspecting one part.

- Every 5th fleadle has its screen inspected.
- Every 7th fleadle has its handle checked.
- Every 17th fleadle has its wheels tested.
- Every 23rd fleadle has its control panel wiring examined.

a Find the numbers of the first six fleadles that are checked by at least two inspectors.

b Explain why no fleadle is checked by all four inspectors.

c Make a list of the numbers of all the fleadles that are checked by three inspectors.

d Find a pair of consecutively numbered fleadles that are checked by two inspectors. Note: think how you might be able to use a spreadsheet to assist you with this problem.

Review Multiples, factors and indices

Summary

- Multiples of a number can be found by multiplying the number by a whole number; for example, the multiples of 7 are 7   1 = 7, 7   2 = 14, 7   3 = 21 …
- The lowest common multiple of a set of numbers is the smallest multiple that is common to all the numbers; for example, the lowest common multiple of 9 and 12 is 36.
- A factor of a whole number is a whole number that divides exactly into that number; for example, 4 is a factor of 20 because 4 divides exactly into 20.
- A common factor of a set of numbers is a factor which is common to all numbers in the set; for example, 7 is a common factor of 14, 21 and 35.
- The highest common factor of a set of numbers is the largest factor that is common to all the numbers in the set; for example, the common factors of 24 and 40 are 1, 2, 4 and 8, so the highest common factor of 24 and 40 is 8.
- A prime number is a natural number that has only two factors, 1 and itself; for example, 17 is a prime number.
- A composite number is a natural number that has more than two factors; for example, 12 is a composite number because it has factors 1, 2, 3, 4, 6 and 12.
- The number 1 is neither a prime number nor a composite number.
- The product of repeated numbers can be written in index notation; for example, 3   3   3   3   5   5 = 3^4 5^2.
- If a number is written in index notation, there is a base and index; for example, 2^3 has a base of 2 and index of 3.
- A composite number can be written as a product of its prime factors; for example, 72 = 2   2   2   3   3 = 2^3 3^2.
- A factor tree or ladder can be used to find the prime factors of a composite number.
- The number of factors of a number can be determined by finding the product of one more than the index of each of the prime factors; for example, 144 = 2^4 3^2 so 144 has (4 + 1)   (2 + 1) = 15 factors.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

ascending	factor	power
base	highest common factor	prime factors
common factors	index	prime numbers
common multiples	index notation	product
composite numbers	lowest common multiple	square numbers
cube	multiple	square root
descending		

Revision

Multiple-choice questions

- A multiple of 24 is
  A 1 **B** 4 **C** 6 **D** 12 **E** 48
- The highest common factor of 36 and 54 is
  A 2 **B** 6 **C** 9 **D** 18 **E** 36
- The lowest common multiple of 36 and 42 is
  A 1512 **B** 252 **C** 42 **D** 7 **E** 6
- A missing digit which makes 654 __56 divisible by 8 could be
  A 3 **B** 5 **C** 6 **D** 7 **E** 9
- Between 10 and 50 there are
  A 9 primes **B** 10 primes **C** 11 primes **D** 12 primes **E** 13 primes

Short-answer questions

- List the first three multiples of
  a 12 **b** 14 **c** 35
- List all the factors of
  a 21 **b** 100 **c** 132
- Find the lowest common multiple of
  a 5 and 17 **b** 45 and 63 **c** 6, 7 and 12

Find the highest common factor of

a 144 and 240 **b** 48 and 75 **c** 36, 54 and 81

Use divisibility tests to find out whether the following numbers are divisible by 4 and 8.

a 48 348 **b** 63 232 **c** 5 032 134 **d** 312 268

Use divisibility tests to see which of the following numbers are divisible by 3 and by 9.

a 1785 **b** 295 314 **c** 672 812 **d** 803 952

Evaluate without using your calculator.

a 4^2 **b** 2^5 **c** 2^3 3^2 **d** $\sqrt{49}$

Use your calculator to evaluate.

a 4^5 **b** 5^4 **c** 7^5 9^4 **d** 4^3 3^4

Simplify these products of powers (leaving them in index notation).

a $2^4 \times 2^5$ **b** $5^2 \times 5^4$ **c** $3^7 \times 3^1$ **d** $4^5 \times 4^3 \times 4^2$

A number can be written in index form as 2^5 3^2 5^3 7. How many factors does the number have?

Extended-response questions

Consider the number 576.

a Use a factor tree to find the prime factors of 576.

b Write 576 as a product of its prime factors.

c How many factors does 576 have?

Consider the number 27 720.

a Use a factor ladder to find the prime factors of 27 720.

b Write 27 720 as a product of its prime factors.

c How many factors does 27 720 have?

For each of these pairs of numbers

i use a factor ladder to find the prime factors of each number.

ii write each number in index form.

iii find the highest common factor of the two numbers.

a 32 and 108 **b** 70 and 392

For each of these pairs of numbers

i use a factor ladder to find the prime factors of each number.

ii write each number in index form.

iii use the index form to find the lowest common multiple of the two numbers.

a 28 and 36 **b** 16 and 18

A primary school bell rings every 30 minutes. At the secondary school next door, a buzzer sounds every 50 minutes. At 9 am, the bell rings and the buzzer sounds. List all the times when the bell and the buzzer will sound together before 4 pm.

Jess and Tom are making up food parcels. They have 91 packets of pasta, 156 tins of soup, 169 packets of noodles and 78 tins of baked beans.

a If each food parcel is to contain the same set of items, how many food parcels can be made?

b What will each food parcel contain?

Consider the following factor problems.

a List all the numbers less than 100 that have exactly three factors, including themselves and 1.

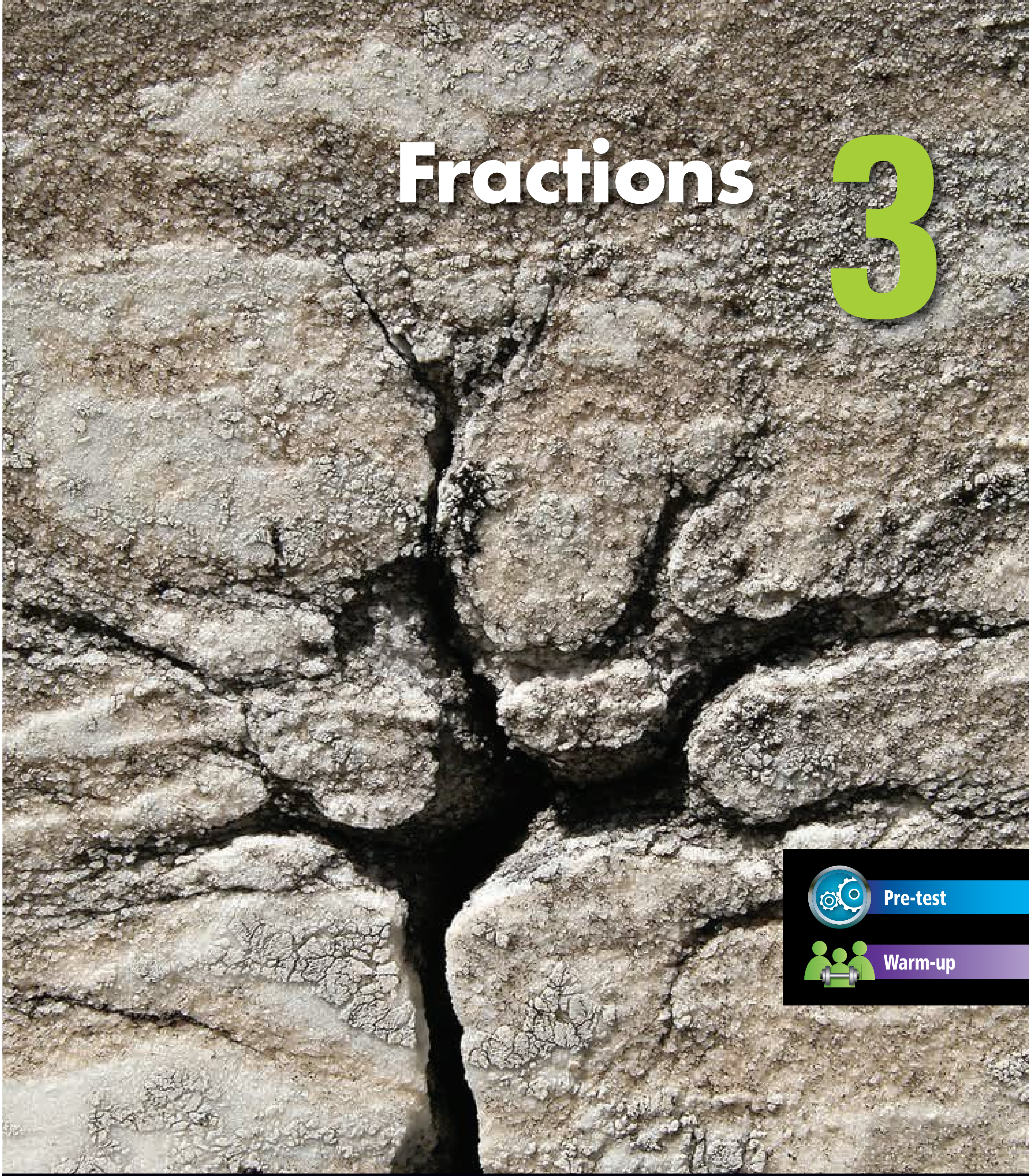
b Find the smallest five-digit number that has 9 and 2 as two of its factors.

c Find the largest four-digit number that has 5 and 6 as two of its factors.

d Find the smallest positive whole number that has among its factors 2, 3, 7, 10, 15, 20, 21 and 28.

e What number am I? I am a two-digit number. Both my digits are square numbers. I have three factors including 1 and myself.

Fractions 3



Pre-test

Warm-up

The word 'fraction' comes from the same Latin word as 'fracture'. If you fracture a bone, you break it into two or more parts. Fractions are associated with parts of wholes. The rock in the photograph has been fractured, perhaps by ice.

3.1 What is a fraction?

A fraction is written as one whole number over another whole number. Examples of fractions are $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ and $1\frac{2}{7}$.

- The whole number on the top is called the **numerator**.
- The whole number on the bottom is called the **denominator**.

$$\frac{3}{4} \begin{array}{l} \longleftarrow \text{numerator} \\ \longleftarrow \text{denominator} \end{array}$$

Meanings of fractions

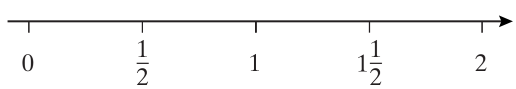
1 A fraction can mean some things out of a group. For example, I have 4 books and I have read 3 out of the 4 or $\frac{3}{4}$ of them. The denominator tells how many objects altogether. The numerator tells how many objects are being talked about.

2 A fraction can mean some of the equal parts in a whole. For example, I have cut my sandwich into 4 equal parts and eaten 3, so I have eaten $\frac{3}{4}$ of the sandwich. The denominator tells how many equal pieces the whole is split into. The numerator tells how many of those pieces are being talked about.

3 A fraction can mean that some objects are split into equal shares. For example, I have 3 chocolate bars and I share them equally between 4 people. Each gets $\frac{3}{4}$ of a chocolate bar. The numerator tells how many objects there are altogether. The denominator tells how many equal shares the objects are split into.

Number line fractions eighths

4 A fraction is a number that can be located on the number line. For example, $\frac{1}{2}$ is halfway between 0 and 1.



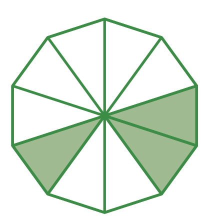
Number line fractions tenths

Number line fractions twelfths

Example 1

a What fraction of the stars are shaded?

b What fraction of this shape is shaded?



continued

Example 1 continued

Working

a $\frac{1}{4}$ of the stars are shaded.

b $\frac{3}{10}$ of the shape is shaded.

Reasoning

One out of the four stars is shaded. The numerator is 1 and the denominator is 4.

The whole shape is split into 10 equal parts. The denominator is 10.
Three parts are shaded. The numerator is 3.

Example 2

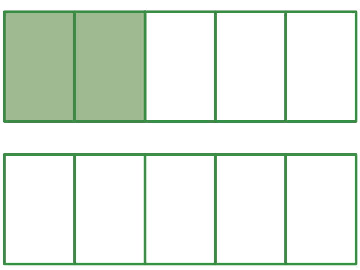
When two fruit bars are shared between five people how much does each person get? Draw a diagram to show this.

Working

One way to do this is to split each fruit bar into 5 pieces of equal length giving 10 pieces.

Each person gets 2 of these 10 pieces.

Each person gets $\frac{2}{5}$ of a fruit bar.



Reasoning

The denominator tells how many shares. It also tells how many parts to split each whole fruit bar into.

The numerator tells how many fruit bars are being shared.
It also tells how many pieces for each share.

The fraction tells how much of a fruit bar per share.
There are 5 lots of $\frac{2}{5}$ of a fruit bar in two fruit bars.

Example 3

Show these numbers on a number line.

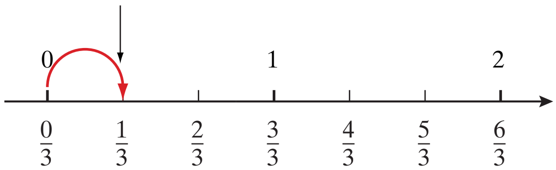
a $\frac{1}{3}$ **b** $\frac{3}{5}$ **c** $\frac{17}{10}$

continued

Example 3 continued

Working

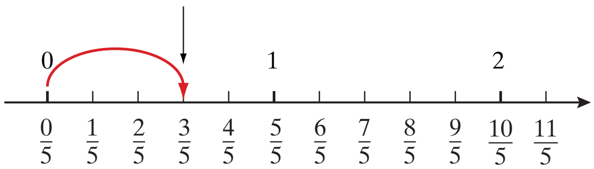
a



Reasoning

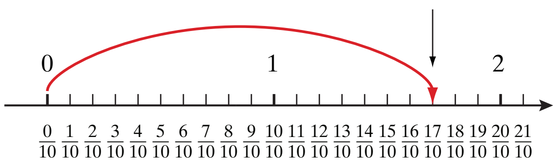
The denominator is 3, so mark 3 equal lengths between 0 and 1. Label the thirds. The fraction $\frac{1}{3}$ is one third of the distance from 0 to 1.

b



The denominator is 5, so mark 5 equal lengths between 0 and 1. Label the fifths. The fraction $\frac{3}{5}$ is three fifths of the way from 0 to 1.

c



The denominator is 10, so mark 10 equal lengths between 0 and 1 and between 1 and 2.
Label the tenths. The fraction $\frac{17}{10}$ is between 1 and 2.

It is 7 tenths past 1.

Whole numbers

When we look at the number lines in Example 3 we see that 1 has been shown as $\frac{3}{3}$ (that is, 3 thirds), as $\frac{5}{5}$ (5 fifths) and as $\frac{10}{10}$ (10 tenths). We could also write 1 as $\frac{1}{1}, \frac{2}{2}, \frac{4}{4}$ and so on.

Other whole numbers can also be written as fractions. For example, 2 can be written as $\frac{2}{1}, \frac{6}{3}, \frac{10}{5}, \frac{20}{10}$.

exercise 3.1

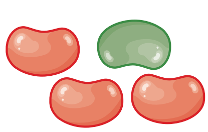
LINKS TO Example 1

1 For each group of jelly beans below

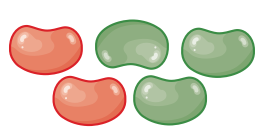
i what fraction of the jelly beans in each group are red?

ii what fraction are *not* red?

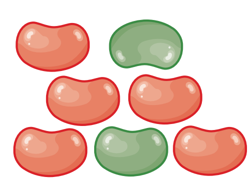
a



b



c



LINKS TO Example 2

In each of the following

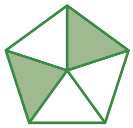
i what fraction of each large shape is shaded?

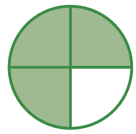
ii what fraction of each large shape is *not* shaded?

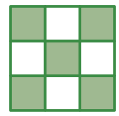
a 
rectangle

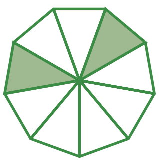
b 
rectangle

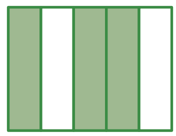
c 
rectangle

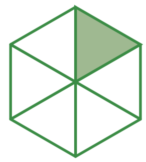
d 
pentagon

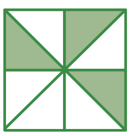
e 
circle

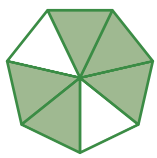
f 
square

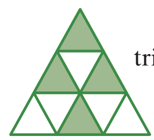
g 
nonagon

h 
rectangle

i 
hexagon

j 
square

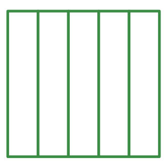
k 
heptagon

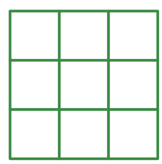
l 
triangle

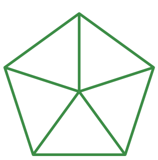
LINKS TO Example 2

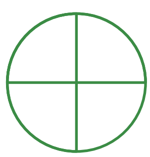
Copy these shapes and shade the indicated fraction of each shape.

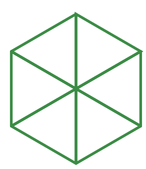
a 
$\frac{3}{8}$ of the rectangle

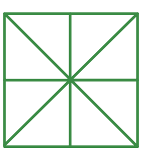
b 
$\frac{1}{5}$ of the rectangle

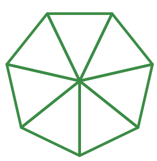
c 
$\frac{2}{9}$ of the square

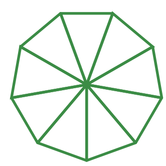
d 
$\frac{4}{5}$ of the pentagon

e 
$\frac{1}{4}$ of the circle

f 
$\frac{5}{6}$ of the hexagon

g 
$\frac{5}{8}$ of the square

h 
$\frac{4}{7}$ of the heptagon

i 
$\frac{4}{9}$ of the nonagon

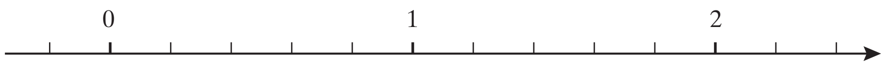
LINKS TO Example 3

Copy each number line into your workbook. Mark and label the position of each fraction.

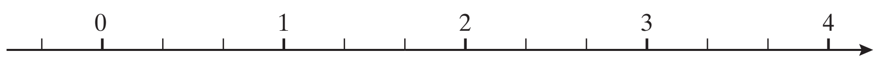
a $\frac{7}{10}$ $\frac{3}{10}$ $\frac{9}{10}$ $\frac{10}{10}$ $\frac{12}{10}$

b $\frac{5}{8}$ $\frac{8}{8}$ $\frac{12}{8}$ $\frac{2}{8}$ $\frac{11}{8}$

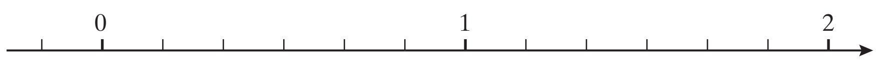
c $\frac{4}{5}$ $\frac{2}{5}$ $\frac{5}{5}$ $\frac{10}{5}$ $\frac{12}{5}$



d $\frac{1}{3}$ $\frac{0}{3}$ $\frac{3}{3}$ $\frac{7}{3}$ $\frac{11}{3}$

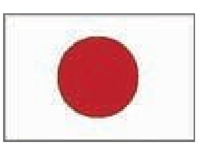


e $\frac{5}{6}$ $\frac{12}{6}$ $\frac{9}{6}$ $\frac{11}{6}$ $\frac{6}{6}$



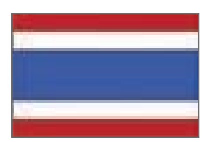
f $\frac{11}{4}$ $\frac{5}{4}$ $\frac{9}{4}$ $\frac{8}{4}$ $\frac{10}{4}$

The flags of seven Asian countries are shown below.


Japan


China


Indonesia


Thailand


Philippines


East Timor


Vietnam

What fraction of the flags

a has more than two colours?

b include blue in their colours?

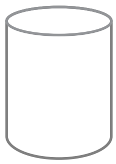
c include stars?

A shape may be divided into equal areas in different ways so that a fraction may look different but still be the same fraction of the whole shape.

a Draw a rectangle. Divide it into four sections of equal area. Now colour $\frac{3}{4}$ of the rectangle blue.

b Draw another rectangle the same size and shape as your first rectangle. Divide it in a different way into four sections of equal area. Now colour $\frac{3}{4}$ of this rectangle in red.

Use the following shape for questions 7–10.



Orange juice fills $\frac{5}{6}$ of this drinking glass. Copy the glass and shade it to show the orange juice filling $\frac{5}{6}$ of it.

Rice fills $\frac{2}{3}$ of a measuring cup. Draw a measuring cup and shade it to show the rice filling $\frac{2}{3}$ of it.

Sand fills $\frac{3}{8}$ of a drum. Draw a drum and shade it to show the sand filling $\frac{3}{8}$ of it.

Water fills $\frac{4}{5}$ of a tank. Draw a water tank and shade it to show the water filling $\frac{4}{5}$ of it.

LINKS TO Example 2

For each of the following sharing situations

i write a fraction to describe the amount that each person gets.

ii draw a diagram to show your reasoning.

a Five muesli bars are shared between three people. How much does each get? Draw a diagram to show your reasoning.

b Two home-made pizzas are shared equally between five people. What fraction of a pizza does each get?

c Three large pies are shared equally between eight people. How much pie does each get?

d Four apples are shared equally between three people. How much apple does each get?

e Five small tarts are shared equally between three people. How much tart does each get?

exercise 3.1 challenge

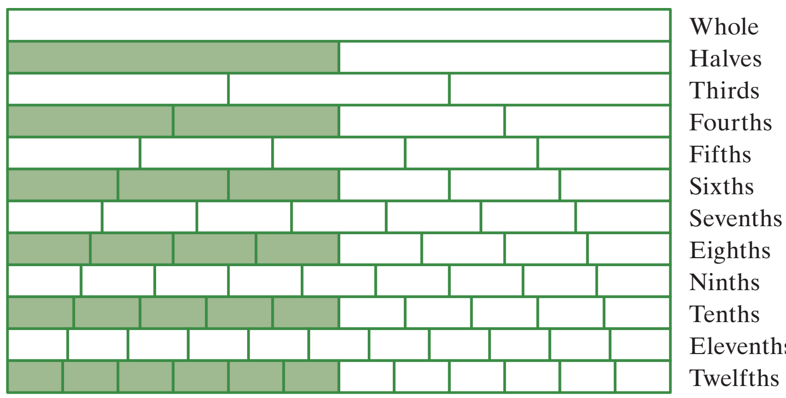
Natalie and Caleb have a box of 24 chocolates. Together they eat $\frac{2}{3}$ of the chocolates. Caleb eats three times as many as Natalie. How many chocolates does Caleb eat?

3.2 Equivalent fractions

BLM Fraction wall

Comparing fractions

In this fraction wall we can see that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$.

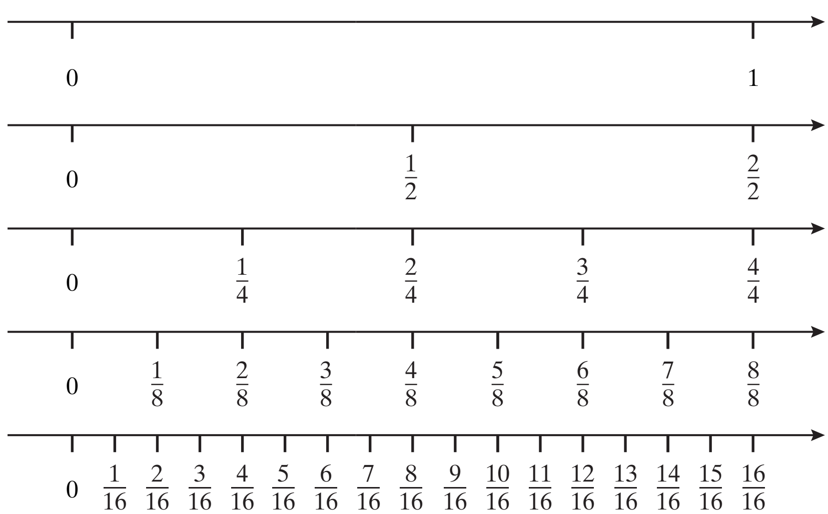


Equivalent fractions are located at the same position on the number line.

The fractions $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$ and $\frac{8}{16}$ are at the same position on the number line. They are equivalent fractions.

The fractions $\frac{1}{4}, \frac{2}{8}$ and $\frac{4}{16}$ are at the same position on the number line. They are equivalent fractions.

The fractions $\frac{3}{4}, \frac{6}{8}$ and $\frac{12}{16}$ are at the same position on the number line. They are equivalent fractions.



Making equivalent fractions

We can make equivalent fractions by multiplying the numerator and denominator of a fraction by the same number.

$\frac{1 \times 2}{4 \times 2} = \frac{2}{8}$ and $\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

The reason we can do this is that $\frac{2}{2} = 1$ and $\frac{4}{4} = 1$. We know from Chapter 1 that multiplying any number by 1 does not change its size.

Example 4

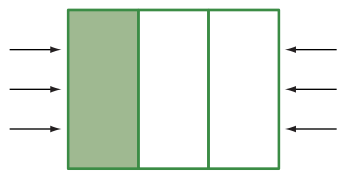
A large rectangle is shown at right.

a What fraction of the large rectangle is shaded?

b Join each pair of opposite arrows with a dotted line. Write the shaded fraction in a different way.

c By what number have the numerator and denominator of the first fraction been multiplied to make the second fraction?

d Make three more fractions equivalent to the first fraction.

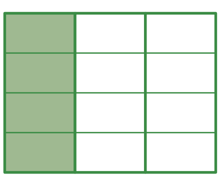


Working

a The shaded fraction is $\frac{1}{3}$ of the whole rectangle.

Reasoning: The rectangle is split into 3 equal sections. 1 of them is shaded.

b



The shaded fraction can now be written as $\frac{4}{12}$ of the whole rectangle.

Reasoning: There are now 12 parts. 4 of these are shaded.

c $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

The numerator and the denominator have both been multiplied by 4 to make a fraction equivalent to $\frac{1}{3}$.

Reasoning: Multiplying the numerator and denominator by 4 is equivalent to multiplying the fraction by $\frac{4}{4}$.

We can do this because $\frac{4}{4} = 1$

continued

Example 4 continued

d $\dfrac{1 \times 2}{3 \times 2} = \dfrac{2}{6}$

To convert a denominator of 3 into 6, both numerator and denominator must be multiplied by 2. We can do this because $\dfrac{2}{2} = 1$.

$\dfrac{1 \times 3}{3 \times 3} = \dfrac{3}{9}$

To convert a denominator of 3 into 9, both numerator and denominator must be multiplied by 3. $\left(\dfrac{3}{3} = 1\right)$

$\dfrac{1 \times 5}{3 \times 5} = \dfrac{5}{15}$

To convert a denominator of 3 into 15, both numerator and denominator must be multiplied by 5. $\left(\dfrac{5}{5} = 1\right)$

Three fractions equivalent to $\dfrac{1}{3}$ are $\dfrac{2}{6}$, $\dfrac{3}{9}$ and $\dfrac{5}{15}$.

If the same number can be divided exactly into the numerator and denominator of a fraction, then we can write a simpler equivalent fraction. This process is called **cancelling**.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

We can write this more simply as

$$\frac{8}{12} = \frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}} = \frac{2}{3}.$$

Example 5

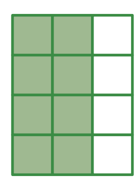
A rectangle is shown at right.

a Write the fraction of this rectangle that is shaded.

b Now write the fraction in a simpler way.

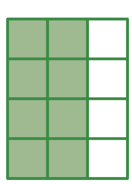
c Write these fractions in simplified, or cancelled, form.

i $\dfrac{8}{10}$ **ii** $\dfrac{9}{21}$



Working

a The fraction shaded is $\dfrac{8}{12}$.



Reasoning

There are 12 small equal squares. 8 of these are shaded.

continued

Example 5 continued

b $\dfrac{8}{12} = \dfrac{8 \div 4}{12 \div 4}$

$= \dfrac{2}{3}$

The fraction shaded is $\dfrac{2}{3}$.

4 is a factor of 8 and 12.

Dividing the numerator and the denominator by a common factor of 4 is equivalent to dividing the fraction by 1 so it does not change its value.

c i $\dfrac{8}{10} = \dfrac{8 \div 2}{10 \div 2}$

$= \dfrac{4}{5}$

2 is a factor of 8 and of 10.

ii $\dfrac{9}{21} = \dfrac{9 \div 3}{21 \div 3}$

$= \dfrac{3}{7}$

3 is a factor of 9 and of 21.

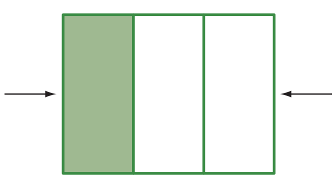
exercise 3.2

LINKS TO Example 4

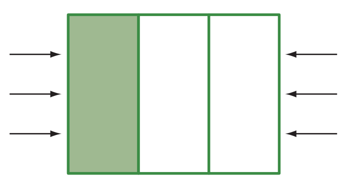
1 For each of the following

- **i** what fraction of each large rectangle is shaded?
- **ii** copy the diagram and join the opposite arrows with a dotted line.
- **iii** write the shaded fraction in a different way.

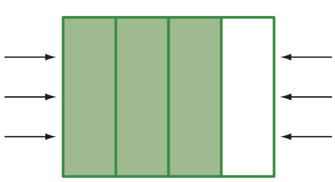
a



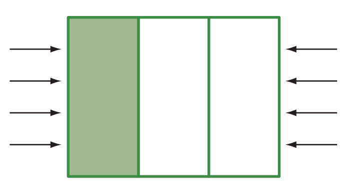
b



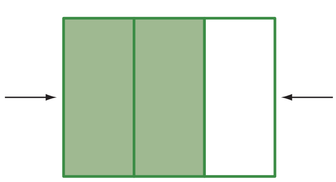
c



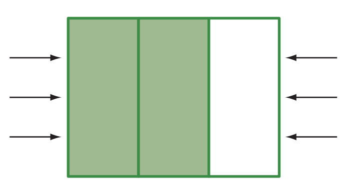
d



e



f



LINKS TO Example 5

Copy and complete.

a $\frac{2}{5} = \frac{4}{} = \frac{}{15} = \frac{22}{}$ **b** $\frac{4}{7} = \frac{}{14} = \frac{20}{} = \frac{}{49}$ **c** $\frac{1}{2} = \frac{}{4} = \frac{5}{} = \frac{}{20}$

d $\frac{2}{3} = \frac{4}{} = \frac{}{12} = \frac{10}{}$ **e** $\frac{3}{4} = \frac{}{12} = \frac{12}{} = \frac{}{24}$ **f** $\frac{1}{8} = \frac{3}{} = \frac{}{16} = \frac{5}{}$

g $\frac{4}{5} = \frac{}{15} = \frac{16}{} = \frac{}{30}$ **h** $\frac{5}{6} = \frac{10}{} = \frac{}{30} = \frac{55}{}$ **i** $\frac{2}{7} = \frac{}{14} = \frac{8}{} = \frac{}{56}$

j $\frac{3}{5} = \frac{12}{} = \frac{}{30} = \frac{36}{}$ **k** $\frac{9}{10} = \frac{}{20} = \frac{45}{} = \frac{}{80}$ **l** $\frac{5}{11} = \frac{15}{} = \frac{}{44} = \frac{40}{}$

Write each of these fractions as twelfths.

a $\frac{2}{3}$ **b** $\frac{1}{4}$ **c** $\frac{5}{6}$ **d** $\frac{1}{2}$

Write each of these fractions as twentieths.

a $\frac{1}{2}$ **b** $\frac{3}{10}$ **c** $\frac{2}{5}$ **d** $\frac{3}{4}$

Write each of these fractions as eighteenths.

a $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{5}{6}$ **d** $\frac{4}{9}$

Write each of these fractions as twenty-fourths.

a $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{3}{4}$ **d** $\frac{5}{8}$

Write each of these fractions as twenty-eighths.

a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{5}{7}$ **d** $\frac{3}{14}$

Write each of these fractions as thirtieths.

a $\frac{2}{3}$ **b** $\frac{3}{5}$ **c** $\frac{5}{6}$ **d** $\frac{7}{10}$

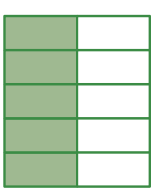
LINKS TO Example 5

For these shaded rectangles

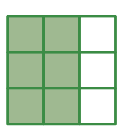
i write the fraction that is shaded.

ii write the fraction that is shaded in its simplest form.

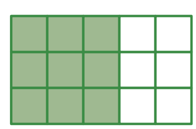
a



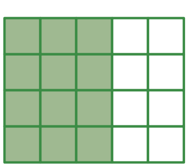
b



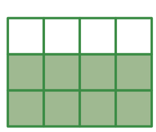
c



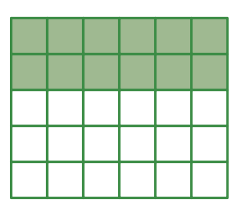
d

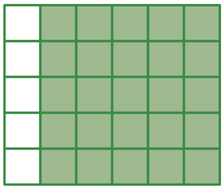
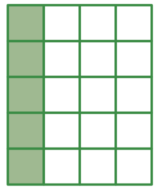
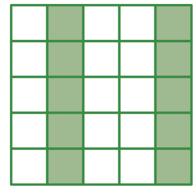


e



f



g  **h**  **i** 

LINKS TO Example 5

Write each of these fractions in simplest form by cancelling.

a $\frac{6}{8}$ **b** $\frac{3}{9}$ **c** $\frac{12}{14}$ **d** $\frac{10}{15}$

e $\frac{11}{44}$ **f** $\frac{12}{15}$ **g** $\frac{4}{16}$ **h** $\frac{70}{90}$

i $\frac{15}{20}$ **j** $\frac{6}{15}$ **k** $\frac{40}{70}$ **l** $\frac{55}{88}$

m $\frac{45}{50}$ **n** $\frac{16}{24}$ **o** $\frac{12}{30}$ **p** $\frac{24}{32}$

Match the fractions a to d with the equivalent fractions i to iv.

a $\frac{2}{3}$ **b** $\frac{5}{7}$ **c** $\frac{3}{4}$ **d** $\frac{4}{9}$

i $\frac{8}{18}$ **ii** $\frac{15}{20}$ **iii** $\frac{6}{9}$ **iv** $\frac{20}{28}$

Match the fractions a to d with the equivalent fractions i to iv.

a $\frac{2}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{8}$ **d** $\frac{5}{6}$

i $\frac{9}{24}$ **ii** $\frac{7}{21}$ **iii** $\frac{10}{12}$ **iv** $\frac{22}{55}$

Match the fractions a to d with the equivalent fractions i to iv.

a $\frac{1}{4}$ **b** $\frac{3}{5}$ **c** $\frac{2}{3}$ **d** $\frac{4}{7}$

i $\frac{6}{9}$ **ii** $\frac{12}{20}$ **iii** $\frac{8}{14}$ **iv** $\frac{7}{28}$

Match the fractions a to d with the equivalent fractions i to iv.

a $\frac{70}{100}$ **b** $\frac{15}{20}$ **c** $\frac{4}{14}$ **d** $\frac{30}{54}$

i $\frac{5}{9}$ **ii** $\frac{3}{4}$ **iii** $\frac{7}{10}$ **iv** $\frac{2}{7}$

Match the fractions a to d with the equivalent fractions i to iv.

a $\frac{55}{88}$ **b** $\frac{14}{21}$ **c** $\frac{5}{30}$ **d** $\frac{24}{40}$

i $\frac{2}{3}$ **ii** $\frac{5}{8}$ **iii** $\frac{3}{5}$ **iv** $\frac{1}{6}$

Always write fractions in their simplest form.

- An hour is the same as 60 minutes. What fraction of an hour is
  a 15 minutes? **b** 50 minutes? **c** 45 minutes? **d** 40 minutes?
- April has 30 days. What fraction of the month has passed at the end of
  a 15th April? **b** 20th April? **c** 22nd April? **d** 25th April?
- A bus seats 50 passengers. What fraction of the seats in the bus are taken if the number of passengers is
  a 5? **b** 10? **c** 15? **d** 22?
- A jigsaw puzzle has 500 pieces. What fraction of the puzzle is done if the number of pieces in place is
  a 150? **b** 200? **c** 350? **d** 360?

exercise 3.2 challenge

- The length of musical notes (the note values) form a series of fractions as shown in this table.

Name of note	Semibreve	Minim	Crotchet	Quaver	Semiquaver
		𝅗𝅥	♩	♪	
Note value	**Whole note**	**Half note**	**Quarter note**	**One-eighth note**	**One-sixteenth note**

a How many crotchets are equivalent to a semibreve?
b How many quavers are equivalent to a minim?
c How many semiquavers are equivalent to a crotchet?
d How many semiquavers are equivalent to a semibreve?
e The time signature written at the beginning of a piece of music tells us the value of the beat and how many beats are in each bar of music. The time signature $\begin{smallmatrix}3\\4\end{smallmatrix}$ tells us that there are three quarter-note beats to each bar; for example, ♩♩♩.
Write four more different combinations of notes that would be equivalent to three quarter-note beats.

3.3 Improper fractions and mixed numbers

If the numerator of a fraction is smaller than the denominator, the fraction is called a **proper fraction**. Examples of proper fractions are $\frac{1}{4}, \frac{2}{3}, \frac{5}{12}, \frac{5}{7}$. These proper fractions are all between 0 and 1 on the number line.

If the numerator of a fraction is larger than the denominator, the fraction is called an **improper fraction**. Examples of improper fractions are $\frac{7}{4}, \frac{13}{3}, \frac{13}{12}, \frac{9}{7}$. Improper fractions are all greater than 1.

Example 6

Sort these numbers into three groups: proper fractions, improper fractions and mixed numbers.

$\frac{17}{4} \quad \frac{3}{11} \quad 1\frac{3}{5} \quad \frac{9}{10} \quad \frac{13}{6} \quad 2\frac{4}{7}$

Working

Proper fractions	Improper fractions	Mixed numbers
$\frac{3}{11}, \frac{9}{10}$	$\frac{17}{4}, \frac{13}{6}$	$1\frac{3}{5}, 2\frac{4}{7}$

Reasoning

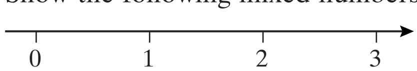
Proper fractions are less than 1.
The numerator is smaller than the denominator.
Improper fractions are greater than 1. The numerator is larger than the denominator.
Mixed numbers have a whole number part and a fraction part.

Mixed numbers

A **mixed number** is made up of a whole number and a fraction. Examples of mixed numbers are $2\frac{1}{2}, 1\frac{2}{5}, 4\frac{3}{4}$ and $1\frac{5}{6}$. Mixed numbers can all be written as improper fractions.

Example 7

Show the following mixed numbers on the number line below.



a $1\frac{3}{4}$

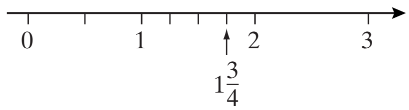
b $2\frac{3}{5}$

continued

Example 7 continued

Working

a

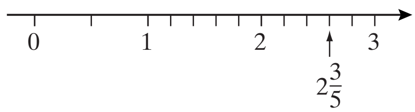


Reasoning

$1\frac{3}{4}$ is between 1 and 2.

The denominator is 4 so split the section between 1 and 2 into 4 equal parts. Count 3 parts past 1 and mark it.

b



$2\frac{3}{5}$ is between 2 and 3.

The denominator is 5 so split the section between 2 and 3 into 5 equal parts. Count 3 parts past 2 and mark it.

Example 8

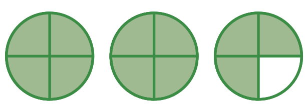
Change the mixed number $2\frac{3}{4}$ to an improper fraction.

Working

Method 1: using a diagram

Reasoning

The denominator is 4 so there are quarters.



$$2\frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$

$$= \frac{11}{4}$$

Write each whole number as 4 quarters. Add all of the quarters.

Method 2: calculation only

$$2\frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4}$$

$$= \frac{8 + 3}{4}$$

$$= \frac{11}{4}$$

Each whole number has 4 quarters so

$$2 = \frac{2}{1} = \frac{2 \times 4}{4}$$

This gives 8 quarters to add to the 3 quarters.

Example 9

Change the improper fraction $\frac{13}{5}$ to a mixed number.

continued

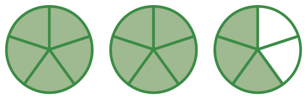
Example 9 continued

Working

Reasoning

Method 1: using a diagram

The denominator is 5 so we are dealing with fifths.
The numerator is 13 so there are 13 fifths.



$$\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5}$$

$$= 1 + 1 + \frac{3}{5}$$

It takes 5 fifths to make a whole.
There are two lots of 5 fifths and 3 fifths left over.

$$= 2\frac{3}{5}$$

Write the wholes and the left over pieces.

Method 2: calculation only

$$\frac{13}{5} = 13 \div 5$$

$\frac{13}{5}$ can mean 13 things split into 5 equal shares.

Each share is 13 ÷ 5.

$$= 2\frac{3}{5}$$

This gives 2 wholes for each share. The 3 remaining wholes are split into 5 equal shares so each share is $2\frac{3}{5}$.

exercise 3.3

LINKS TO Example 6

1. Make a table with three columns. Head the columns 'Proper fractions', 'Improper fractions', and 'Mixed numbers'. The table should have six rows in addition to the headings. Sort these fractions into the correct columns.

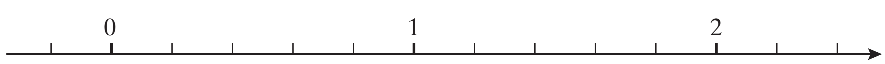
   $4\frac{2}{5}, \frac{5}{8}, \frac{9}{4}, \frac{1}{7}, 3\frac{3}{4}, 1\frac{1}{2}, \frac{12}{5}, \frac{3}{11}, \frac{4}{3}, 2\frac{4}{7}, \frac{4}{5}, \frac{15}{8}, \frac{3}{10}, 1\frac{5}{6}, 4\frac{1}{3}, \frac{10}{9}, \frac{8}{3}, \frac{5}{12}$

LINKS TO Example 7

2. Copy each number line into your workbook. Mark and label the position of each mixed number.

   a $1\frac{1}{2}, 2\frac{1}{4}, \frac{10}{4}, 2\frac{3}{4}, \frac{7}{4}$

   b $1\frac{4}{5}, 2\frac{1}{5}, \frac{7}{5}, \frac{12}{5}, 1\frac{1}{5}$

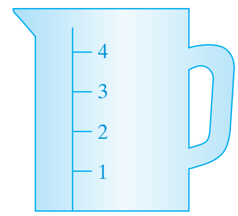


c $2\frac{2}{3}$, $\frac{10}{3}$, $\frac{12}{3}$, $1\frac{1}{3}$, $3\frac{2}{3}$

d $1\frac{2}{3}$, $1\frac{1}{2}$, $\frac{11}{6}$, $\frac{4}{3}$, $\frac{1}{3}$

LINKS TO Example 8

The measuring jug on the right is marked in litres. Write each mixed number as an improper fraction. Draw a similar jug then mark on each of these the volumes in litres.



a $3\frac{1}{2}$ **b** $2\frac{1}{3}$ **c** $2\frac{3}{4}$ **d** $1\frac{5}{6}$

LINKS TO Example 8

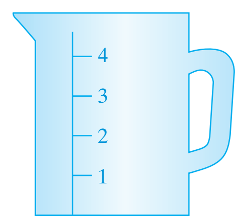
Each fraction gives the length of a pencil in centimetres. Change these mixed number lengths into improper fractions.

a $1\frac{2}{3}$ **b** $2\frac{1}{4}$ **c** $3\frac{5}{6}$ **d** $1\frac{1}{2}$ **e** $2\frac{3}{7}$ **f** $3\frac{1}{3}$

g $4\frac{1}{6}$ **h** $5\frac{3}{4}$ **i** $2\frac{3}{8}$ **j** $6\frac{1}{2}$ **k** $2\frac{4}{5}$ **l** $3\frac{2}{9}$

LINKS TO Example 9

The measuring jug on the right is marked in litres. Write each fraction as a mixed number then draw similar jugs to show each of these numbers of litres.



a $\frac{5}{2}$ **b** $\frac{8}{3}$ **c** $\frac{9}{4}$ **d** $\frac{9}{5}$

LINKS TO Example 9

Change these improper fractions into mixed numbers.

a $\frac{8}{3}$ **b** $\frac{11}{7}$ **c** $\frac{9}{4}$ **d** $\frac{15}{8}$ **e** $\frac{12}{5}$ **f** $\frac{7}{2}$

g $\frac{17}{6}$ **h** $\frac{17}{4}$ **i** $\frac{16}{9}$ **j** $\frac{13}{12}$ **k** $\frac{16}{7}$ **l** $\frac{18}{5}$

Write the next three numbers in each of these patterns in their simplest fraction form.

a

Fraction	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$			
Simplest form of fraction	$\frac{1}{6}$	$\frac{1}{3}$				

b

Fraction	$1\frac{4}{10}$	$1\frac{2}{10}$	1			
Simplest form of fraction	$1\frac{2}{5}$	$1\frac{1}{5}$				

c

Fraction	$1\frac{1}{8}$	$1\frac{6}{8}$	$2\frac{3}{8}$			
Simplest form of fraction	$1\frac{1}{8}$					

d

Fraction	$2\frac{1}{4}$	3	$3\frac{3}{4}$			
Simplest form of fraction	$2\frac{1}{4}$					

e

Fraction	$1\frac{5}{11}$	$1\frac{8}{11}$	2	$2\frac{3}{11}$		
Improper fraction	$\frac{16}{11}$	$\frac{19}{11}$	$\frac{22}{11}$			

f

Fraction	$1\frac{3}{6}$	$1\frac{4}{6}$	$1\frac{5}{6}$			
Improper fraction	$\frac{9}{6} = \frac{3}{2}$	$\frac{10}{6} = \frac{5}{3}$	$\frac{11}{6}$			

g

Fraction	$3\frac{1}{7}$	$3\frac{5}{7}$	$4\frac{2}{7}$			
Improper fraction in sevenths	$\frac{22}{7}$					

h

Fraction	$2\frac{1}{5}$	$2\frac{3}{5}$	3			
Improper fraction in fifths	$\frac{11}{5}$					

● Write a mixed number to describe each of these situations.

a Sandwiches were cut into quarters. Sally ate seven quarters.

b The family ate 11 pieces of pizza. Each pizza had been cut into sixths.

c A batch of biscuits takes a quarter of an hour to bake. Five batches are cooked one after the other.

Four students were following the same recipe. The recipe said to use one-third of a cup of rice.

a Write as an improper fraction the total amount of rice used by the four students.

b Express this as a mixed number.

For a large family party, sponge cakes were cut into 10 equal pieces. Each of 17 people had one piece.

a What fraction of a sponge cake is each slice?

b Write as an improper fraction of sponge cakes the total amount of cake eaten.

c Write the number of sponge cakes eaten as a mixed number.

This platform at London's Kings Cross Railway Station is well-known as the platform from which the Hogwarts Express train departs.

a Write the platform number as an improper fraction.

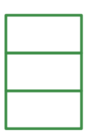
b Why is this number unusual for a platform number?

c What whole number is the platform number closest to?

exercise 3.3 challenge

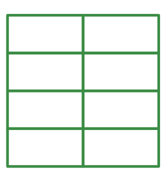
For each of the following, if the shape on the left represents one unit, how would you write fractions for shapes i and ii?

a

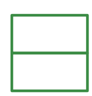


one unit

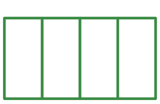
i



ii

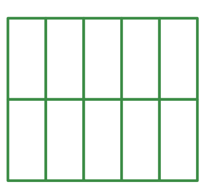


b

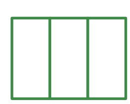


one unit

i



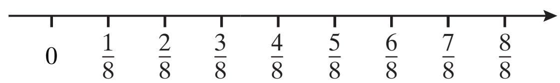
ii



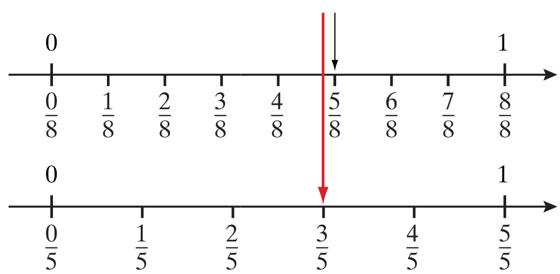
3.4 Common denominators and comparing fractions

Comparing fractions

The position of a fraction on a number line shows whether it is smaller than or larger than another fraction. The further to the right on the number line, the larger the number. All the fractions on the number line below have the same denominator so we can compare the size of the fractions by comparing their numerators. It is easy to see on the number line below that $\frac{5}{8}$ is larger than $\frac{4}{8}$, that is, $\frac{5}{8}$ is larger than $\frac{1}{2}$.



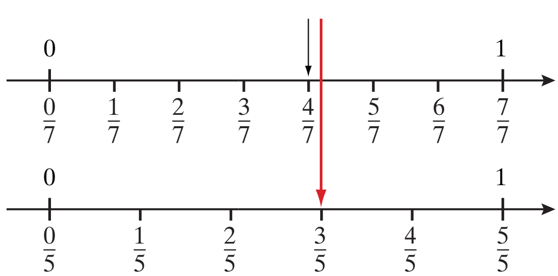
Fractions that have different denominators can also be compared on a number line. We can see by comparing the positions of the fractions on the two number lines that $\frac{5}{8}$ is larger than $\frac{3}{5}$.



Example 10

Which of these fractions is larger, $\frac{3}{5}$ or $\frac{4}{7}$?

Working



$\frac{3}{5} > \frac{4}{7}$

Reasoning

Both fractions are less than 1.
Draw two number lines.
Divide one line into 7 equal intervals and label the sevenths.
Divide the other line into 5 equal intervals and label the fifths.
Compare the positions of $\frac{3}{5}$ and $\frac{4}{7}$.
The number that is further to the right is larger so $\frac{3}{5}$ is larger than $\frac{4}{7}$.

Common denominators

Instead of drawing number lines, we can compare the size of two fractions by making equivalent fractions that have the same denominator. This is called finding a **common denominator**.

The **lowest common denominator** is the lowest common multiple of the two denominators.

To compare $\frac{5}{8}$ and $\frac{3}{5}$, we find the lowest (smallest) common multiple of 8 and 5.

5, 10, 15, 20 25, 30, 35, (40), …

8, 16, 24, 32, (40), …

The lowest common multiple is 40, so we find equivalent fractions to $\frac{5}{8}$ and $\frac{3}{5}$ with denominator 40.

$$\frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40} \qquad \frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40}$$

Now that both fractions are fortieths, we can compare them by looking at the numerators. 25 fortieths is greater than 24 fortieths. So $\frac{5}{8}$ is greater than $\frac{3}{5}$.

We write $\frac{5}{8} > \frac{3}{5}$.

Example 11

Find the lowest common denominator of $\frac{3}{5}$ and $\frac{4}{7}$.

Working	Reasoning
7, 14, 21, 28, (35), 42, …	Choose the largest denominator—here it is 7. Count by this number (which gives multiples). Find the first number in this list that is also a multiple of the smaller number. This is the lowest common denominator.
35 is the lowest common denominator.	Check: Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, … 35 is a multiple of both 7 and 5.

Example 12

Write $\frac{3}{5}$ and $\frac{4}{7}$ with the same denominator and hence find which fraction is larger.

Working	**Reasoning**
$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$	The lowest common denominator is 35.
$\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$	Compare the numerators.
$\frac{21}{35} \quad \frac{20}{35}$	21 is greater than 20.
so $\frac{3}{5} \quad \frac{4}{7}$	

When we are comparing improper fractions, it is easier to turn them into mixed numbers. If the whole number parts are different then it easy to tell which is larger.

Example 13

Write the fractions $\frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{3}{4}$ in ascending (increasing) order.

Working	**Reasoning**
The lowest common denominator is 24.	We need the lowest common multiple of 3, 8, 12 and 4. If we count in 12s until we find the first number that is a multiple of 3, 4 and 8, we obtain 24.
$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$	
$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$	
$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$	Write each fraction in 24ths.
$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$	
In ascending order:	
$\frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{18}{24}$	Now use the numerators to put the fractions in order.
$\frac{7}{12}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}$	Lastly, write the list using the starting fractions.

Example 14

Which of the fractions $\frac{18}{5}$ and $\frac{20}{7}$ is larger?

Working

$\frac{18}{5} = 3\frac{3}{5}$

$\frac{20}{7} = 2\frac{6}{7}$

$\frac{18}{5} = 18 \div 5 = 3\frac{3}{5}$

$\frac{20}{7} = 20 \div 7 = 2\frac{6}{7}$

$\frac{18}{5} \quad \frac{20}{7}$

Reasoning

Change both improper fractions into mixed numbers.

$3\frac{3}{5}$ is between 3 and 4.

$2\frac{6}{7}$ is between 2 and 3.

This means that $\frac{18}{5}$ is larger.

If two mixed numbers have the same whole number part, we convert the fraction parts into equivalent fractions with the same denominator to see which is larger.

Example 15

Which is larger, $2\frac{5}{6}$ or $2\frac{7}{10}$? Make a true statement with $<$ or $>$.

Working

$\frac{5}{6} = \frac{50}{60}$

$\frac{7}{10} = \frac{42}{60}$

$\frac{50}{60} > \frac{42}{60}$

$2\frac{5}{6} > 2\frac{7}{10}$

Reasoning

The whole number parts are the same.
Compare the fraction parts.
Write each fraction part as an equivalent fraction with the same denominator.
Compare the numerators.

Example 16

Write the numbers $\frac{11}{6}, 2\frac{1}{8}, \frac{7}{4}, 1\frac{2}{3}$ in descending (decreasing) order.

Working

$\frac{11}{6} = 1\frac{5}{6}$ $\qquad$ $\frac{7}{4} = 1\frac{3}{4}$

$2\frac{1}{8}$ is the larger.

Reasoning

Change any improper fractions into mixed numbers.

$2\frac{1}{8}$ is between 2 and 3.

continued

Example 16 continued

Working	Reasoning
$\frac{5}{6} = \frac{20}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{2}{3} = \frac{16}{24}$	$1\frac{2}{3}, 1\frac{5}{6}, 1\frac{3}{4}$ are between 1 and 2. Compare the fractional parts of $1\frac{2}{3}, 1\frac{5}{6}$ and $1\frac{3}{4}$.
In descending order: $2\frac{1}{8}, 1\frac{5}{6}, 1\frac{3}{4}, 1\frac{2}{3}$ that is, $2\frac{1}{8}, \frac{11}{6}, \frac{7}{4}, 1\frac{2}{3}$.	Change $\frac{2}{3}, \frac{5}{6}$, and $\frac{3}{4}$ to the same denominator (24). Put the numbers in order from largest to smallest. Write them the same way as in the question.

exercise 3.4

LINKS TO Example 10

To decide which fraction in each pair is larger show both numbers on a number line. Make a true statement with or .

a $\frac{1}{3}, \frac{1}{4}$ **b** $\frac{3}{4}, \frac{5}{8}$ **c** $\frac{1}{3}, \frac{2}{6}$ **d** $\frac{1}{4}, \frac{3}{8}$

e $\frac{1}{3}, \frac{4}{9}$ **f** $\frac{2}{3}, \frac{4}{5}$ **g** $\frac{3}{4}, \frac{3}{5}$ **h** $\frac{4}{5}, \frac{5}{8}$

LINKS TO Example 11a

Find the lowest common denominator for each pair of fractions.

a $\frac{3}{10}, \frac{1}{4}$ **b** $\frac{5}{12}, \frac{1}{3}$ **c** $\frac{5}{7}, \frac{4}{5}$ **d** $\frac{7}{8}, \frac{11}{12}$

e $\frac{3}{4}, \frac{5}{6}$ **f** $\frac{2}{9}, \frac{1}{3}$ **g** $\frac{5}{6}, \frac{4}{5}$ **h** $\frac{2}{3}, \frac{5}{7}$

LINKS TO Example 11b

Find the lowest common denominator for these sets of fractions.

a $\frac{2}{10}, \frac{3}{5}, \frac{1}{15}$ **b** $\frac{2}{3}, \frac{3}{7}, \frac{1}{2}$ **c** $\frac{3}{4}, \frac{1}{9}, \frac{5}{12}$ **d** $\frac{1}{8}, \frac{5}{12}, \frac{13}{24}, \frac{1}{3}$

LINKS TO Example 12

Write each pair of fractions with the same denominator. Make a true statement with or .

a $\frac{2}{3}, \frac{1}{2}$ **b** $\frac{1}{4}, \frac{3}{8}$ **c** $\frac{2}{3}, \frac{6}{9}$ **d** $\frac{7}{15}, \frac{3}{5}$ **e** $\frac{2}{9}, \frac{1}{3}$ **f** $\frac{2}{5}, \frac{3}{10}$

g $\frac{3}{5}, \frac{12}{20}$ **h** $\frac{2}{3}, \frac{7}{10}$ **i** $\frac{2}{3}, \frac{5}{7}$ **j** $\frac{5}{6}, \frac{7}{9}$ **k** $\frac{4}{15}, \frac{3}{10}$ **l** $\frac{7}{8}, \frac{11}{12}$

LINKS TO Example 13

Arrange each set of fractions in ascending order.

a $\frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{3}{4}$ **b** $\frac{2}{5}, \frac{7}{15}, \frac{3}{10}, \frac{13}{30}$ **c** $\frac{1}{2}, \frac{11}{20}, \frac{3}{5}, \frac{7}{10}$

d $\frac{4}{9}, \frac{5}{6}, \frac{1}{2}, \frac{2}{3}$ **e** $\frac{5}{6}, \frac{7}{9}, \frac{11}{12}, \frac{2}{3}$ **f** $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \frac{1}{2}$

g $\frac{1}{4}, \frac{3}{11}, \frac{1}{2}, \frac{7}{22}$ **h** $\frac{5}{14}, \frac{1}{4}, \frac{3}{7}, \frac{9}{28}$ **i** $\frac{1}{3}, \frac{5}{12}, \frac{3}{8}, \frac{1}{4}$

LINKS TO Examples 14, 15, 16

Arrange each set of numbers in descending order.

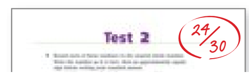
a $1\frac{5}{6}, \frac{5}{2}, \frac{5}{3}, 2\frac{1}{4}$ **b** $\frac{9}{8}, 1\frac{1}{4}, \frac{3}{2}, 1\frac{3}{8}$ **c** $3\frac{4}{5}, \frac{7}{2}, 2\frac{7}{10}, \frac{11}{5}$

d $2\frac{3}{4}, \frac{10}{3}, 1\frac{7}{12}, \frac{5}{2}$ **e** $2\frac{1}{9}, \frac{11}{6}, 1\frac{13}{18}, \frac{8}{3}$ **f** $\frac{31}{15}, 2\frac{3}{5}, \frac{7}{3}, 1\frac{14}{15}$

g $\frac{37}{20}, 1\frac{5}{8}, 2\frac{7}{10}, 1\frac{3}{4}$ **h** $\frac{7}{2}, 1\frac{1}{4}, 2\frac{7}{16}, \frac{15}{8}$ **i** $1\frac{2}{3}, \frac{7}{4}, 2\frac{1}{12}, \frac{13}{6}$

Which would be more, $\frac{3}{8}$ of a block of chocolate or $\frac{1}{3}$ of an identical block of chocolate? Explain your reasoning.

Tom did two tests of the same difficulty. His score on the first was 15 out of 20. On the second test he scored 24 out of 30.



a Write each fraction in simplest form.

b Find the lowest common denominator for the two fractions from part a.

c Express the two fractions from part a as fractions with the same denominator.

d Which was Tom's better result, Test 1 or Test 2?

Mel needed to make a cake for a friend on a low sugar diet. One recipe used $\frac{3}{5}$ of a cup of sugar. The other used $\frac{2}{3}$ of a cup of sugar.

a Find the lowest common denominator for the two fractions.

b Write each fraction with the same denominator.

c Which recipe should Mel use, the one with $\frac{3}{5}$ or the one with $\frac{2}{3}$ of a cup of sugar?

The usual way to make concrete is with a mix of $\frac{4}{7}$ gravel, $\frac{2}{7}$ sand and $\frac{1}{7}$ cement. Ryan's dad is trying a new mix with $\frac{5}{8}$ gravel. He makes up the same total amount of concrete as usual. Which mix has more gravel, the usual one or Ryan's dad's new mix?

Write three fractions between $\frac{3}{5}$ and $\frac{4}{5}$.

Write three fractions between $\frac{1}{3}$ and $\frac{2}{3}$.

exercise 3.4 challenge

a What happens to the value of a fraction if you subtract the same number from the numerator and denominator? Give three examples of this, all starting with the fraction $\frac{6}{11}$.

b What happens to the value of a fraction if you add the same number to the numerator and denominator? Give three examples of this, all starting with the fraction $\frac{6}{11}$.

3.5 Adding and subtracting proper fractions

Fractions with the same denominator

To add two fractions that have the same denominator, we keep that denominator and simply add the numerators together. For subtraction we do the same, but we take one numerator away from the other.

Example 17

Calculate.

a 1 quarter plus 2 quarters **b** $\frac{1}{5} + \frac{2}{5}$ **c** $\frac{7}{8} - \frac{3}{8}$

Working

Reasoning

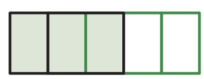
a 1 quarter plus 2 quarters is 3 quarters.

$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

Both fractions are quarters so we add the number of quarters.

b $\frac{1}{5} + \frac{2}{5}$

Keep the denominators the same. They tell the size of the pieces.



$= \frac{3}{5}$

Add the numerators. They tell how many pieces. One fifth plus 2 fifths is 3 fifths.

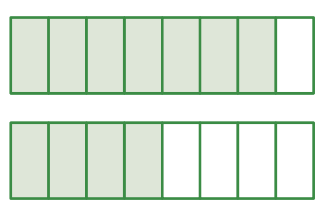
c $\frac{7}{8} - \frac{3}{8}$

$\frac{7}{8} - \frac{3}{8}$

$= \frac{4}{8}$

$= \frac{1}{2}$

Subtracting 3 eighths from 7 eighths leaves 4 eighths.



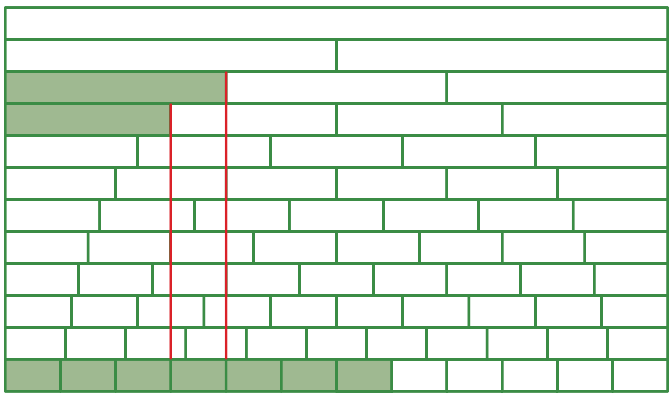
Simplify $\frac{4}{8}$.

Fractions with different denominators

If the denominators are different, we convert the fractions to equivalent fractions with the same denominator. We do this in the same way that we did when we compared fractions (see example 12).

To calculate $\frac{1}{3} + \frac{1}{4}$ we can use a fraction wall to find equivalent fractions with the same denominator. We can rewrite the addition as $\frac{4}{12} + \frac{3}{12}$ which is $\frac{7}{12}$.

We can also see from the fraction wall that $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.



Example 18

Calculate.

a $\frac{5}{6} + \frac{1}{10}$ **b** $\frac{3}{4} - \frac{1}{3}$ **c** $\frac{3}{4} + \frac{7}{8}$

Working	Reasoning
a $\frac{5}{6} + \frac{1}{10}$	Find the common denominator. 10 is the larger denominator. 10, 20, (30), 40, 50, … 30 is also a multiple of 6.
$= \frac{25}{30} + \frac{3}{30}$	Write both fractions in thirtieths.
$= \frac{28}{30}$	Add the numerators.
$= \frac{14}{15}$	Express the answer in simplest form.
b $\frac{3}{4} - \frac{1}{3}$	Find the lowest common denominator for these fractions. 4 is the larger denominator. 4, 8, (12), 16, 20, … 12 is also a multiple of 3. 12 is the lowest common denominator.
$= \frac{9}{12} - \frac{4}{12}$	Write both fractions in twelfths.
$= \frac{5}{12}$	Subtract the numerators.

continued

Example 18 continued

Working

c $\frac{3}{4} + \frac{7}{8}$

$= \frac{6}{8} + \frac{7}{8}$

$= \frac{13}{8}$

$= 1\frac{5}{8}$

Reasoning

8 is a multiple of 4, so the lowest common multiple is 8.
Write both fractions in eighths.
Add the numerators.
Write the improper fraction as a mixed number.

Fraction problems involving addition and subtraction

- Adding parts of the whole
- Finding the difference between parts of the whole
- Finding how much is left from the whole.

The three parts of example 19 show these three different types of problems.

Example 19

Li and Mai shared a pizza.

a Li ate $\frac{2}{5}$ of a pizza and Mai ate $\frac{1}{3}$. What fraction of the pizza did they eat altogether?

b How much more did Li eat than Mai?

c What fraction of the pizza is left?

Working

a $\frac{2}{5} + \frac{1}{3}$

$= \frac{6}{15} + \frac{5}{15}$

$= \frac{11}{15}$

Li and Mai ate $\frac{11}{15}$ of the pizza.

Reasoning

A total is required.
Add the fractions.

b $\frac{2}{5} - \frac{1}{3}$

$= \frac{6}{15} - \frac{5}{15}$

$= \frac{1}{15}$

Li ate $\frac{1}{15}$ more of the pizza than Mai ate.

A difference is required.
Subtract the fractions.

continued

Example 19 continued

Working

c $1 - \frac{11}{15}$

$= \frac{15}{15} - \frac{11}{15}$

$= \frac{4}{15}$

There is $\frac{4}{15}$ of the pizza left.

Reasoning

Subtract the total amount eaten from the whole.
The whole is 1, in this case, 15 fifteenths.

Tech tip

To add fractions using the TI-30XB Multiview calculator, for example, $\frac{3}{4} + \frac{7}{10}$ (example 18), type:

[3] [n/d] [4] [+] [7] [n/d] [1] [0] [enter].

To change the answer to a mixed number press [2nd] [n/d ◂ ▸ U n/d] [enter].

exercise 3.5

LINKS TO Example 17

Write down the answer to each of these.

a 1 fifth plus 2 fifths
b 5 sevenths minus 2 sevenths
c 3 ninths plus 4 ninths
d 6 tenths plus 3 tenths
e 6 eighths minus 1 eighth
f 4 fifths minus 2 fifths

LINKS TO Example 18

Add the following.

a $\frac{2}{7} + \frac{3}{7}$
b $\frac{1}{5} + \frac{2}{5}$
c $\frac{4}{11} + \frac{2}{11}$
d $\frac{3}{9} + \frac{4}{9}$
e $\frac{1}{3} + \frac{1}{6}$
f $\frac{2}{9} + \frac{1}{3}$
g $\frac{3}{8} + \frac{1}{2}$
h $\frac{1}{5} + \frac{7}{10}$
i $\frac{1}{3} + \frac{1}{4}$
j $\frac{2}{5} + \frac{1}{3}$
k $\frac{1}{6} + \frac{3}{4}$
l $\frac{1}{3} + \frac{1}{2}$
m $\frac{1}{6} + \frac{7}{10}$
n $\frac{2}{9} + \frac{1}{2}$
o $\frac{5}{9} + \frac{1}{6}$
p $\frac{2}{3} + \frac{1}{8}$

LINKS TO Example 18

Subtract the following.

a $\frac{4}{5} - \frac{1}{5}$
b $\frac{5}{9} - \frac{4}{9}$
c $\frac{6}{7} - \frac{4}{7}$
d $\frac{7}{11} - \frac{2}{11}$
e $\frac{5}{6} - \frac{1}{2}$
f $\frac{7}{8} - \frac{3}{4}$
g $\frac{3}{5} - \frac{3}{10}$
h $\frac{8}{9} - \frac{2}{3}$
i $\frac{5}{9} - \frac{1}{3}$
j $\frac{5}{6} - \frac{3}{4}$
k $\frac{1}{3} - \frac{1}{5}$
l $\frac{6}{7} - \frac{2}{3}$
m $\frac{7}{9} - \frac{1}{6}$
n $\frac{9}{10} - \frac{3}{4}$
o $\frac{5}{6} - \frac{3}{8}$
p $\frac{5}{7} - \frac{1}{4}$

- Add or subtract the following.

a $\frac{4}{5}+\frac{3}{5}$ **b** $\frac{8}{9}-\frac{5}{9}$ **c** $\frac{6}{7}+\frac{4}{7}$ **d** $\frac{9}{10}-\frac{7}{10}$

e $\frac{5}{6}-\frac{1}{3}$ **f** $\frac{7}{8}+\frac{3}{4}$ **g** $\frac{4}{5}-\frac{3}{10}$ **h** $\frac{8}{9}+\frac{2}{3}$

i $\frac{3}{4}+\frac{5}{6}$ **j** $\frac{7}{12}-\frac{1}{3}$ **k** $\frac{2}{3}+\frac{3}{5}$ **l** $\frac{5}{6}-\frac{1}{2}$

m $\frac{1}{2}-\frac{3}{10}$ **n** $\frac{5}{8}+\frac{5}{12}$ **o** $\frac{17}{20}-\frac{3}{4}$ **p** $\frac{7}{10}+\frac{8}{15}$

LINKS TO Example 19

- Visitors ate $\frac{1}{4}$ of a cake. Then Matthew ate $\frac{1}{6}$ of the cake.

a What fraction of the cake had been eaten altogether?

b What fraction remained uneaten?

- Three-eighths of the class walked to school. One-third of the class came to school by bus. The rest came by car.

a What fraction of the class walked or came by bus?

b What fraction of the class came by car?

- Rachel practised the piano for $\frac{3}{4}$ of an hour. Then she watched TV for $\frac{1}{2}$ an hour.

a How many hours did all this take?

b How many minutes did it take in total?

- At Ryan's house there are two large identical water tanks. One of them is $\frac{3}{5}$ full. The other is only $\frac{1}{4}$ full.

a Altogether what fraction of a tank of water is there?

b How much more is in the one which is $\frac{3}{5}$ full? Give your answer as a fraction of a tank.

- To get to soccer practice, Sam walked for $\frac{1}{6}$ of an hour and then sat on a bus for $\frac{1}{3}$ of an hour.

a Altogether, how long did it take Sam to get to soccer practice?

b On the way home, Sam sat on the bus for $\frac{1}{4}$ of an hour and then walked for $\frac{1}{3}$ of an hour. Altogether, how long did it take Sam to get home from soccer practice?

c How much longer did it take him to get home?

exercise 3.5 challenge

- A water container was $\frac{1}{4}$ full. Two litres of water were added. The container was then $\frac{1}{3}$ full. How much did the container hold?

3.6 Adding and subtracting mixed numbers

Addition

There are two methods we can use when adding mixed numbers.

Method 1	Add the whole number parts. Add the fraction parts (making the denominators the same if necessary). Then combine the two sums.
Method 2	Convert each mixed number into an improper fraction. Add the improper fractions (making the denominators the same if necessary). Then convert back to a mixed number.

Use the method that you prefer.

Example 20

Calculate $1\frac{2}{5} + 2\frac{1}{5}$.

Working

Method 1

$$1\frac{2}{5} + 2\frac{1}{5} = 1 + 2 + \frac{1}{5} + \frac{2}{5}$$

Add the whole numbers.

$$= 3 + \frac{3}{5}$$

Add the fractions: The denominators are the same. Add the numerators.

$$= 3\frac{3}{5}$$

Write as a mixed number.

Method 2

$$1\frac{2}{5} + 2\frac{1}{5} = \frac{7}{5} + \frac{11}{5}$$

Convert the mixed numbers into improper fractions. One whole is equal to 5 fifths, so $1\frac{2}{5} = \frac{7}{5}$. Two wholes are equal to 10 fifths, so $2\frac{1}{5} = \frac{11}{5}$.

$$= \frac{18}{5}$$

The denominators are already the same. Add the numerators.

$$= 3\frac{3}{5}$$

Convert the improper fraction into a mixed number.

Example 21

Calculate $1\frac{1}{4} + 1\frac{5}{6}$.

Working	Reasoning
Method 1	
$1\frac{1}{4} + 1\frac{5}{6} = 1 + 1 + \frac{1}{4} + \frac{5}{6}$	Add the whole numbers.
$= 2 + \frac{3}{12} + \frac{10}{12}$	Convert the fractions to equivalent fractions with the same denominator.
$= 2 + \frac{13}{12}$	Add the numerators.
$= 2 + 1\frac{1}{12}$	Convert the improper fraction to a mixed number.
$= 3\frac{1}{12}$	Add the whole numbers.
Method 2	
$1\frac{1}{4} + 1\frac{5}{6} = \frac{5}{4} + \frac{11}{6}$	Convert the mixed numbers to improper fractions.
$= \frac{15}{12} + \frac{22}{12}$	Rewrite the improper fractions with the same denominators.
$= \frac{37}{12}$	Add the numerators.
$= 3\frac{1}{12}$	Convert the improper fraction to a mixed number.

Example 22

Calculate $3\frac{1}{4} + 2\frac{5}{12}$.

Working	Reasoning
Method 1	
$3\frac{1}{4} + 2\frac{5}{12} = 3 + 2 + \frac{1}{4} + \frac{5}{12}$	Add the whole numbers.
$= 5 + \frac{3}{12} + \frac{5}{12}$	Convert the fractions to equivalent fractions with the same denominator.
$= 5 + \frac{8}{12}$	Add the numerators.
$= 5\frac{2}{3}$	Simplify the fraction. Write the fraction and whole number parts as a mixed number.

continued

Example 22 continued

Working

Method 2

$$3\frac{1}{4} + 2\frac{5}{12} = \frac{13}{4} + \frac{29}{12}$$

Convert the mixed numbers to improper fractions.

$$= \frac{39}{12} + \frac{29}{12}$$

Rewrite the improper fractions with the same denominators.

$$= \frac{68}{12}$$

Add the numerators.

$$= 5\frac{2}{3}$$

Convert the improper fraction to a mixed number.

When subtracting fractions, it is easier to convert them to improper fractions first.

Example 23

Calculate.

a $2\frac{3}{4} - 1\frac{1}{10}$

b $4\frac{1}{3} - 1\frac{5}{6}$

Working

a $2\frac{3}{4} - 1\frac{1}{10} = \frac{11}{4} - \frac{11}{10}$

Convert the mixed numbers into improper fractions.

$$= \frac{55}{20} - \frac{22}{20}$$

Work out the common denominator for these fractions by finding the lowest common multiple of 4 and 10.
Write both fractions as equivalent fractions with the same denominator.

$$= \frac{33}{20}$$

Subtract the numerators of the fraction.

$$= 1\frac{13}{20}$$

Convert the improper fraction into a mixed number.

b $4\frac{1}{3} - 1\frac{5}{6} = \frac{13}{3} - \frac{11}{6}$

Convert the mixed numbers into improper fractions.

$$= \frac{26}{6} - \frac{11}{6}$$

Work out the common denominator for these fractions. 6 is the lowest common multiple of 3 and 6.
Write both fractions as equivalent fractions with the same denominator.

$$= \frac{15}{6}$$

Subtract the numerators.

$$= 2\frac{3}{6}$$

Convert the improper fraction into a mixed number.

$$= 2\frac{1}{2}$$

If possible, simplify the answer.

Tech tip

To add mixed numbers using the TI-30XB MultiView calculator, for example, $1\frac{1}{4} + 1\frac{5}{6}$ (example 21), type:

[1] [2nd] [n/d] [1] [n/d] [4] [+] [1] [2nd] [n/d] [5] [n/d] [6] [enter].

To correct the answer to a mixed number, type:

[2nd] [n/d ◄ ► U n/d] [enter].

Note that [2nd] [n/d] gives [U n/d].

exercise 3.6

LINKS TO Examples 20, 21, 22

Evaluate each of the following, simplifying the result where possible.

a $1\frac{2}{3} + 2\frac{2}{3}$ **b** $1\frac{2}{7} + 2\frac{3}{7}$ **c** $1\frac{8}{11} + 1\frac{2}{11}$ **d** $1\frac{4}{9} + 2\frac{1}{9}$

e $1\frac{3}{4} + 2\frac{3}{4}$ **f** $1\frac{5}{7} + 2\frac{4}{7}$ **g** $2\frac{1}{2} + 1\frac{2}{3}$ **h** $1\frac{2}{3} + 1\frac{1}{6}$

i $2\frac{1}{2} + 1\frac{3}{4}$ **j** $2\frac{1}{3} + 1\frac{4}{9}$ **k** $2\frac{1}{2} + 1\frac{4}{5}$ **l** $1\frac{5}{6} + 1\frac{1}{9}$

LINKS TO Examples 20, 21, 22

Calculate the answer to each of the following, giving the result in simplest form.

a $3\frac{4}{5} - 1\frac{3}{5}$ **b** $2\frac{2}{9} - 1\frac{5}{9}$ **c** $3\frac{1}{12} - 1\frac{5}{12}$ **d** $4\frac{7}{10} - 1\frac{3}{10}$

e $3\frac{7}{8} - 1\frac{3}{8}$ **f** $3\frac{9}{10} - 1\frac{3}{10}$ **g** $2\frac{5}{6} - \frac{3}{4}$ **h** $1\frac{7}{10} - 1\frac{1}{4}$

i $3\frac{1}{3} - 1\frac{4}{5}$ **j** $2\frac{3}{4} - 1\frac{3}{5}$ **k** $3\frac{3}{4} - 2\frac{1}{8}$ **l** $4\frac{2}{3} - 1\frac{1}{2}$

LINKS TO Example 23

Evaluate each of the following, simplifying the result where possible.

a $2\frac{4}{5} - 1\frac{2}{5}$ **b** $\frac{6}{7} + 1\frac{4}{7}$ **c** $2\frac{1}{9} - 1\frac{7}{9}$ **d** $1\frac{3}{8} + 1\frac{5}{8}$

e $1\frac{7}{8} + 2\frac{1}{4}$ **f** $2\frac{2}{3} - 1\frac{2}{9}$ **g** $1\frac{5}{6} + 1\frac{1}{3}$ **h** $2\frac{7}{10} - 1\frac{1}{5}$

i $2\frac{1}{5} - 1\frac{2}{3}$ **j** $2\frac{3}{4} - 1\frac{5}{6}$ **k** $1\frac{1}{10} + 1\frac{1}{6}$ **l** $3\frac{2}{3} - 1\frac{1}{2}$

Manoj ran for $1\frac{1}{12}$ hours, then rode his bike for $2\frac{2}{3}$ hours. How long did it take altogether?

Joshua's football practice went for $1\frac{2}{5}$ hours. Of this time, $\frac{3}{4}$ of an hour was spent on general fitness, and the rest on skills. What fraction of an hour was spent on skills?

- At the year 7 camp, Stephen slept for $7\frac{1}{2}$ hours on the first night, and Nick slept for $5\frac{3}{4}$ hours. For how much longer did Stephen sleep?
- Lin travelled for $2\frac{3}{4}$ hours by train then another $1\frac{1}{2}$ hours by bus. For how many hours was Lin travelling?
- Ten pizzas were bought for a party. If $1\frac{3}{8}$ pizzas remained, how much was eaten?
- Tilly's age is $8\frac{2}{3}$ years and her brother Tommy's age is $5\frac{7}{12}$ years. How much older than Tommy is Tilly?
- Kim rode for $2\frac{1}{4}$ km before turning around and riding back towards home. If he had ridden $1\frac{2}{3}$ km back towards home, how far did he still have to ride to reach home?
- Amanda's mum bought a $4\frac{1}{2}$ kg bag of apples on Friday. On Sunday, the remaining apples weighed $1\frac{2}{3}$ kg. What weight of apples had been eaten?
- Tien bought eight bags of mulch. He put $2\frac{1}{4}$ bags of mulch on the flower bed, then $3\frac{2}{3}$ bags of mulch on the vegetable plants.
  - **a** How many bags of mulch did he use on the flowers and vegetables?
  - **b** How much mulch was left?
- Lauren's school bag weighed $4\frac{5}{8}$ kg. Amy's weighed $3\frac{1}{6}$ kg.
  - **a** How much heavier was Lauren's bag?
  - **b** How much did the two bags weigh together?

exercise 3.6 challenge

- The following sums of proper fractions follow a pattern.

  i $\frac{1}{3} + \frac{2}{3}$ **ii** $\frac{1}{4} + \frac{2}{4} + \frac{3}{4}$

  iii $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}$ **iv** $\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6}$

  - **a** Evaluate each of the following sums.
  - **b** Look for a pattern in your answers to part a. Use this pattern to predict the value of $\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}$.
  - **c** Calculate the value of $\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}$ and see if your prediction was correct.
- Write $\dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{3}}}$ as a single fraction.

3.7 Multiplying fractions

Multiplying a fraction by a whole number

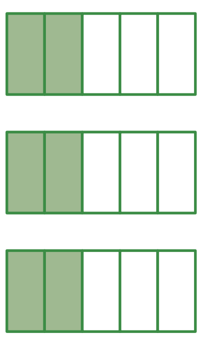
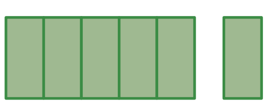
Multiplying a fraction by a whole number such as 4 is equivalent to '4 lots of' the fraction.

For example, four times one-third is four-thirds. We write this as 4 $\frac{1}{3} = \frac{4}{3}$.

Example 24

Calculate 3 $\frac{2}{5}$.

a Use a diagram to show 3 $\frac{2}{5}$. **b** Evaluate 3 $\frac{2}{5}$.

Working	Reasoning
a 3 $\frac{2}{5}$	We need 3 lots of $\frac{2}{5}$.
	This is $\frac{6}{5}$.
= 	It is the same as $1\frac{1}{5}$.
b 3 $\frac{2}{5}$	
$= \frac{3}{1} \quad \frac{2}{5}$	Write 3 as $\frac{3}{1}$.
$= \frac{3 \times 2}{1 \times 5}$	Multiply the numerators and multiply the denominators.
$= \frac{6}{5}$	3 lots of $\frac{2}{5}$ is $\frac{6}{5}$.
$= 1\frac{1}{5}$	As the answer is an improper fraction convert it to a mixed number.

Example 25

Calculate $\frac{3}{4} \times 7$, giving the product as a mixed number.

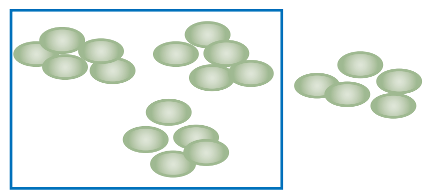
Working	Reasoning
$\frac{3}{4} \times 7$	
$= \frac{3}{4} \times \frac{7}{1}$	Write 7 as a fraction.
$= \frac{21}{4}$	7 times 3 quarters is 21 quarters.
$= 5\frac{1}{4}$	Convert the improper fraction into a mixed number.

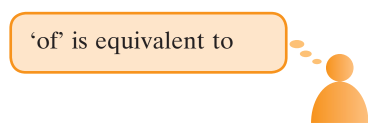
The meaning of 'of'

If we have 20 chocolate buttons and we take $\frac{3}{4}$ of them we can see that we will have 15 chocolate buttons.

We can write $\frac{3}{4}$ of $20 = 15$.

We obtain the same result if we write $\frac{3}{4} \times \frac{20}{1} = \frac{60}{4} = 15$.





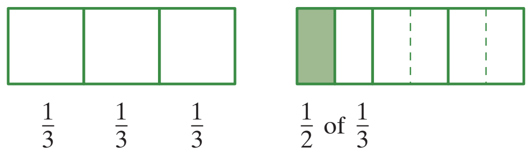
Example 26

Calculate $\frac{5}{6}$ of 3.

Working	Reasoning
$\frac{5}{6}$ of 3	
$= \frac{5}{6} \quad \frac{3}{1}$	
$= \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{1}$	If possible divide a numerator and a denominator by a common factor.
$= \frac{5 \times 1}{2 \times 1}$	Multiply the numerators and multiply the denominators.
$= \frac{5}{2}$	As the answer is an improper fraction convert it to a mixed number.
$= 2\frac{1}{2}$	

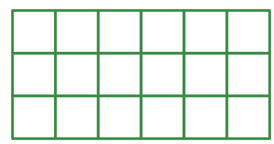
Multiplying proper fractions

Suppose we have a chocolate bar divided into three equal pieces and we take half of one of the pieces, that is, we have taken $\frac{1}{2}$ of $\frac{1}{3}$. We can see that $\frac{1}{2}$ of $\frac{1}{3}$ of the chocolate bar is $\frac{1}{6}$ of the chocolate bar. We obtain the same result if we write $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.



Example 27

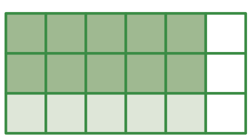
a Find $\frac{2}{3}$ of $\frac{5}{6}$ of this chocolate bar on the right.



b Calculate $\frac{2}{3}$ of $\frac{5}{6}$.

Working

a $\frac{2}{3}$ of $\frac{5}{6}$

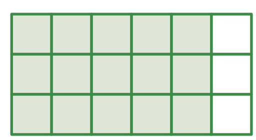


$= \frac{10}{18}$

$= \frac{5}{9}$

Reasoning

We need a fraction of a fraction. Here the whole chocolate bar is one unit.
It is divided into six columns and five of these, that is $\frac{5}{6}$, are coloured light green.



The bar is divided again into thirds with three rows, making eighteenths.

We want $\frac{2}{3}$ of the light green part, which is the dark green section. This is $\frac{10}{18}$ which can be simplified to $\frac{5}{9}$.

continued

Example 27 continued

Working

b $\frac{2}{3}$ of $\frac{5}{6}$

$= \frac{2}{3} \quad \frac{5}{6}$

$= \frac{\not{2}^{1}}{3} \times \frac{5}{\not{6}_{3}}$

$= \frac{1 \times 5}{3 \times 3}$

$= \frac{5}{9}$

Reasoning

Write 'of' as ' '.

If possible, divide a numerator and a denominator by a common factor.

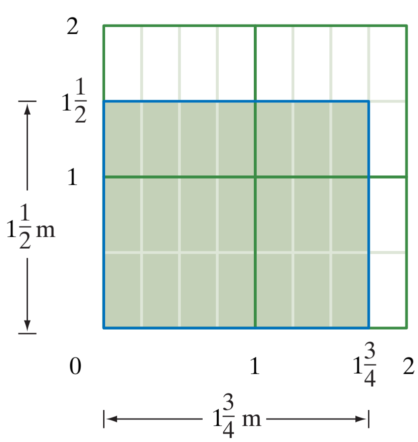
Multiply the numerators and multiply the denominators.

Multiplying mixed numbers

The area of a rectangle is calculated by multiplying the length by the width. This gives a way of representing what we mean by multiplying two mixed numbers.

Suppose a guinea pig enclosure is $1\frac{1}{2}$ m wide and $1\frac{3}{4}$ m long. The area is found by calculating $1\frac{3}{4} \times 1\frac{1}{2}$.

In the diagram below, the green shaded area is the guinea pig enclosure. Each square is 1 square metre. Each of the four squares is divided into 8 equal parts, so each of these parts is $\frac{1}{8}$ of a square metre. The area of the guinea pig enclosure is 21 of these eighths, that is, $\frac{21}{8}$ square metres.



$1\frac{3}{4} \times 1\frac{1}{2}$

$= \frac{7}{4} \times \frac{3}{2}$

$= \frac{21}{8}$

Example 28

Evaluate $1\frac{4}{5} \quad 2\frac{1}{3}$.

Working

$1\frac{4}{5} \quad 2\frac{1}{3} = \frac{9}{5} \quad \frac{7}{3}$

$= \frac{\overset{3}{\cancel{9}}}{5} \times \frac{7}{\underset{1}{\cancel{3}}}$

$= \frac{3 \times 7}{5 \times 1}$

$= \frac{21}{5}$

$= 4\frac{1}{5}$

Reasoning

Convert each mixed number into an improper fraction.

Cancel where it is possible to do so.

Multiply the numerators and multiply the denominators.

As the answer is an improper fraction make it into a mixed number.

Example 29

Calculate $2\frac{2}{5} \times 2\frac{2}{3} \times 1\frac{2}{3}$.

Working

$2\frac{2}{5} \times 2\frac{2}{3} \times 1\frac{2}{3}$

$= \frac{12}{5} \times \frac{8}{3} \times \frac{5}{3}$

$= \frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{8}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{5}}}{3}$

$= \frac{32}{3}$

$= 10\frac{2}{3}$

Reasoning

Convert each mixed number into an improper fraction.

Cancel where possible.
Multiply the numerators.
Multiply the denominators.

Convert the improper fraction into a mixed number.

Tech tip

To multiply mixed numbers using the TI-30XB MultiView calculator, for example, $1\frac{4}{5} \times 2\frac{1}{3}$ (example 28), type: [1] [2nd] [n/d] [4] [n/d] [5] [×] [2] [2nd] [n/d] [1] [n/d] [3] [enter].

To convert the answer to a mixed number, type: [2nd] [n/d ◂ ▸ U n/d] [enter].

exercise 3.7

LINKS TO Example 24

1. Evaluate each of the following.

a $2 \quad \frac{3}{7}$ **b** $4 \quad \frac{2}{3}$ **c** $5 \quad \frac{1}{4}$ **d** $3 \quad \frac{2}{5}$

e $4 \quad \frac{5}{6}$ **f** $5 \quad \frac{3}{10}$ **g** $14 \quad \frac{5}{7}$ **h** $12 \quad \frac{3}{4}$

i $2 \times \frac{4}{5}$ **j** $3 \times \frac{2}{7}$ **k** $5 \times \frac{2}{9}$ **l** $4 \times \frac{3}{5}$

m $6 \times \frac{2}{3}$ **n** $2 \times \frac{3}{4}$ **o** $3 \times \frac{5}{6}$ **p** $8 \times \frac{7}{8}$

LINKS TO Example 25,26

2. Calculate these fractions of whole numbers.

a $\frac{2}{3} \quad 8$ **b** $\frac{1}{5} \quad 6$ **c** $\frac{3}{8} \quad 10$ **d** $\frac{3}{7} \quad 14$

e $\frac{4}{5} \quad 3$ **f** $\frac{3}{4} \quad 2$ **g** $\frac{4}{9} \quad 6$ **h** $\frac{2}{5} \quad 10$

i $\frac{1}{4} \quad 7$ **j** $\frac{2}{3} \quad 4$ **k** $\frac{3}{5} \quad 2$ **l** $\frac{5}{6} \quad 8$

m $\frac{2}{7} \quad 14$ **n** $\frac{3}{4} \quad 6$ **o** $\frac{5}{9} \quad 3$ **p** $\frac{4}{5} \quad 10$

LINKS TO Example 27

3. For each part of this question draw the bar of chocolate described. Calculate the answer, then show the multiplication by shading the chocolate bar.

a three blocks long and two blocks wide. Calculate and show $\frac{1}{2}$ of $\frac{2}{3}$.

b four blocks long and three blocks wide. Calculate and show $\frac{2}{3}$ of $\frac{3}{4}$.

c five blocks long and three blocks wide. Calculate and show $\frac{2}{3}$ of $\frac{4}{5}$.

d six blocks long and four blocks wide. Calculate and show $\frac{3}{4}$ of $\frac{1}{6}$.

e seven blocks long and three blocks wide. Calculate and show $\frac{1}{3}$ of $\frac{3}{7}$.

f seven blocks long and four blocks wide. Calculate and show $\frac{3}{4}$ of $\frac{6}{7}$.

g eight blocks long and three blocks wide. Calculate and show $\frac{2}{3}$ of $\frac{5}{8}$.

h eight blocks long and four blocks wide. Calculate and show $\frac{3}{4}$ of $\frac{7}{8}$.

LINKS TO Example 27

Evaluate each of the following.

a $\frac{3}{5}$ of $\frac{1}{2}$ **b** $\frac{2}{5} \quad \frac{3}{4}$ **c** $\frac{3}{7}$ of $\frac{4}{9}$ **d** $\frac{1}{5} \quad \frac{3}{8}$

e $\frac{3}{4}$ of $\frac{6}{7}$ **f** $\frac{5}{9} \quad \frac{3}{5}$ **g** $\frac{4}{5}$ of $\frac{7}{8}$ **h** $\frac{3}{8} \quad \frac{4}{9}$

i $\frac{5}{6} \quad \frac{3}{8} \quad \frac{4}{5}$ **j** $\frac{2}{3} \quad \frac{3}{16} \quad \frac{8}{9}$ **k** $\frac{1}{4} \quad \frac{2}{9} \quad \frac{6}{7}$ **l** $\frac{14}{15} \quad \frac{10}{21} \quad \frac{9}{16}$

Are these multiplications correct? If not, explain what is incorrect and fix them.

a $\frac{\cancel{4}^{2}}{9} \times \frac{\cancel{2}^{1}}{5}$

$= \frac{2}{45}$

b $\frac{2}{3} \quad \frac{1}{2}$

$= \frac{4}{6} \quad \frac{3}{6}$

$= \frac{12}{6}$

$= 2$

LINKS TO Example 28

Carry out these multiplications of mixed numbers.

a $1\frac{1}{2} \quad 1\frac{1}{6}$ **b** $2\frac{2}{3} \quad 1\frac{1}{2}$ **c** $1\frac{3}{7} \quad 1\frac{1}{5}$ **d** $3\frac{1}{5} \quad 1\frac{3}{8}$

e $2\frac{5}{8} \quad 2\frac{2}{3}$ **f** $1\frac{1}{3} \quad 2\frac{2}{5}$ **g** $2\frac{4}{5} \quad 1\frac{1}{4}$ **h** $1\frac{5}{9} \quad 1\frac{2}{7}$

i $1\frac{1}{4} \times 1\frac{1}{5}$ **j** $1\frac{2}{5} \times 1\frac{3}{7}$ **k** $2\frac{2}{5} \times 1\frac{5}{6}$ **l** $1\frac{1}{5} \times 3\frac{1}{3}$

m $2\frac{2}{3} \times 1\frac{1}{8}$ **n** $1\frac{3}{5} \times 2\frac{1}{4}$ **o** $3\frac{1}{2} \times 1\frac{3}{7}$ **p** $2\frac{1}{4} \times 1\frac{5}{6}$

LINKS TO Example 29

Evaluate each of the following.

a $\frac{5}{6} \times \frac{3}{8} \times \frac{4}{5}$ **b** $\frac{2}{3} \times \frac{3}{16} \times \frac{8}{9}$ **c** $\frac{1}{4} \times \frac{2}{9} \times \frac{6}{7}$ **d** $\frac{14}{15} \times \frac{10}{21} \times \frac{9}{16}$

e $\frac{15}{16} \times \frac{4}{5} \times 1\frac{1}{3}$ **f** $\frac{5}{6} \times 1\frac{7}{8} \times \frac{4}{5}$ **g** $\frac{8}{11} \times 16\frac{1}{2} \times \frac{3}{4}$ **h** $\frac{14}{15} \times 6 \times 1\frac{1}{4}$

i $1\frac{1}{3} \times 3\frac{1}{2} \times 1\frac{1}{2}$ **j** $1\frac{3}{7} \times \frac{3}{4} \times 1\frac{2}{5}$ **k** $1\frac{1}{4} \times 2\frac{2}{5} \times 1\frac{1}{6}$ **l** $1\frac{1}{8} \times 3\frac{1}{3} \times 1\frac{1}{5}$

Two-sevenths of the population is left-handed. In a class of 28 students how many left-handed people would you expect?

Great Aunt Mary dies and leaves $24 000 in her will. Calculate how many thousands of dollars are inherited by

a Sarah, who gets $\frac{1}{6}$ of the money.

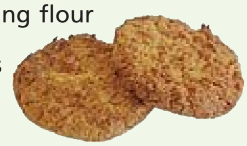
b Justin, who gets $\frac{1}{12}$ of the money.

c the cats' home which gets $\frac{3}{4}$ of the money.

- Becky and Bailey have a recipe for Anzac biscuits. It is shown at right.
  - **a** To double the number of biscuits, what would the amount of each ingredient be multiplied by?
  - **b** To triple the number of biscuits, what would the amount of each ingredient be multiplied by?
  - **c** The recipe uses $\frac{1}{8}$ kg of butter.
    - **i** How could they calculate the amount of butter needed to make six times the number of biscuits in the recipe?
    - **ii** How much butter is this?
  - **d** The recipe uses $\frac{3}{4}$ of a cup of coconut.
    - **i** How could they calculate the amount of coconut needed to make six times the number of biscuits in the recipe?
    - **ii** How much coconut is this?
  - **e** The recipe uses $1\frac{1}{4}$ cups of self-raising flour.
    - **i** How could they calculate the amount of self-raising flour needed to make six times the number of biscuits in the recipe?
    - **ii** How much self-raising flour is this?

Anzac biscuits

$\frac{3}{4}$ cup wholemeal flour
$1\frac{1}{4}$ cups self-raising flour
1 cup rolled oats
$\frac{3}{4}$ cup coconut
1 cup sugar (raw, brown or white)
$\frac{1}{2}$ teaspoon bicarbonate of soda
$\frac{1}{8}$ kg melted butter
2 tablespoons of boiling water
2 tablespoons golden syrup

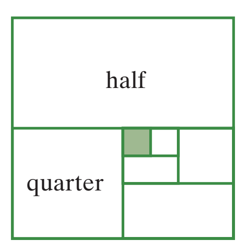


Mix all dry ingredients. Dissolve soda in boiling water and add to melted butter and golden syrup. Add to dry ingredients and mix well. Place in teaspoonfuls on greased biscuit tray. Bake in a moderate oven (160 °C) until golden brown.

- James likes to walk.
  - **a** If James walks three kilometres per hour for two hours, how far will he walk altogether?
  - **b** If James walks four kilometres per hour for three hours, how far will he walk altogether?
  - **c** How can the total distance walked be calculated from the speed and the time taken?
  - **d** If James walks $5\frac{1}{4}$ kilometres per hour for $2\frac{2}{3}$ hours, how far will he walk altogether?
- A school has the students divided into four houses for sporting and music competitions. Three-quarters of the students are not in Redvers House. One-sixth of those students are in Year 7. What fraction of the students in the school are the Year 7 students who are not in Redvers House?

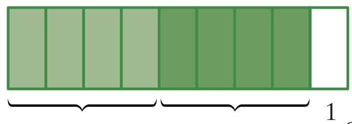
exercise 3.7 challenge

- Each rectangle is half the area of the remaining shape. What fraction of the whole square is unshaded?



3.8 Dividing fractions

When dividing whole numbers, we look for the number of groups of a given size that we can make from the starting number. For example, to find $9 \div 4$ we see how many groups of 4 can be made from 9.



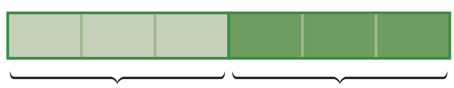
The diagram shows 9 rectangles in groups of 4. We see that $9 \div 4 = 2\frac{1}{4}$.

The answer to the question 'How many fours in 9?' is $2\frac{1}{4}$.

We can also write $2\frac{1}{4} \times 4 = 9$.

The same sort of thinking can help us to understand dividing with fractions.

Just as $9 \div 4$ can be read as 'How many fours in 9?', we can ask 'How many thirds in 2 wholes?' The diagram below shows that there are 6 thirds in 2 wholes.



So $2 \div \frac{1}{3} = 6$. We can also write $6 \times \frac{1}{3} = 2$

Notice that the result of dividing 2 by $\frac{1}{3}$ is the same result we get by multiplying 2 by $\frac{3}{1}$.

$2 \div \frac{1}{3} = 6$ and $2 \times \frac{3}{1} = 6$. The fraction $\frac{3}{1}$ is called the reciprocal of $\frac{1}{3}$.

The **reciprocal** of any fraction is the fraction we obtain by swapping the numerator and denominator.

Example 30

Write the reciprocal of these fractions.

a $\frac{3}{8}$ **b** $1\frac{3}{4}$

Working	Reasoning
a The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$.	Swap the numerator and denominator.

continued

Example 30 continued

Working

b $1\frac{3}{4} = \frac{7}{4}$

The reciprocal of $1\frac{3}{4}$ is $\frac{4}{7}$.

Reasoning

Convert the mixed number into an improper fraction.
Swap the numerator and denominator.

It can be shown that to divide any number by a fraction, we can simply multiply the number by the reciprocal of the fraction.

Example 31

Calculate the following.

a $5 \div \frac{2}{3}$

b $4 \div \frac{3}{5}$

Working

a $5 \div \frac{2}{3}$

$= 5 \times \frac{3}{2}$

$= \frac{5}{1} \times \frac{3}{2}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Reasoning

Multiply 5 by the reciprocal of $\frac{2}{3}$.

There are $5 \times 3 = 15$ thirds in 5.
There will be only half as many two-thirds in 5.

So there will be $15 \div 2 = 7\frac{1}{2}$ two-thirds in 5.

Working

b $4 \div \frac{3}{5}$

$= 4 \times \frac{5}{3}$

$= \frac{4}{1} \times \frac{5}{3}$

$= \frac{20}{3}$

$= 6\frac{2}{3}$

Reasoning

Multiply 4 by the reciprocal of $\frac{3}{5}$.

There are $4 \times 5 = 20$ fifths in 4.
There will be only a third as many three-fifths in 4.

So there will be $20 \div 3 = 6\frac{2}{3}$ three-fifths in 4.

Example 32

Calculate $\frac{6}{7} \div \frac{3}{5}$.

Working

$\frac{6}{7} \div \frac{3}{5}$

$= \frac{\cancel{6}^{2}}{7} \times \frac{5}{\cancel{3}_{1}}$

$= \frac{10}{7}$

$= 1\frac{3}{7}$

Reasoning

Dividing by a fraction can be done by multiplying by its reciprocal. $\div \frac{3}{5}$ becomes $\times \frac{5}{3}$.

6 and 3 are both divisible by 3.

Multiply the numerators and multiply the denominators.
If the answer is improper, make it a mixed number.

Example 33

Calculate $4\frac{1}{2}$ $1\frac{3}{4}$.

Working

$4\frac{1}{2}$ $1\frac{3}{4}$

$= \frac{9}{2}$ $\frac{7}{4}$

$= \frac{9}{2}$ $\frac{4}{7}$

$= \frac{9}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{7}$

$= \frac{18}{7}$

$= 2\frac{4}{7}$

Reasoning

Convert mixed numbers into improper fractions.

Convert $\frac{7}{4}$ to $\frac{4}{7}$.

4 and 2 are both divisible by 2.
Multiply the numerators and denominators.

As the answer is an improper fraction convert it into a mixed number.

Tech tip

To divide mixed numbers using the TI-30XB MultiView calculator, for example, $4\frac{1}{2} \div 1\frac{3}{4}$ (example 33), type: [4] [2nd] [n/d] [1] [2] [÷] [1] [2nd] [n/d] [3] [4] [enter].

To convert the answer to a mixed number press [2nd] [n/d◄►Un/d] [enter].

Note: [2nd] [n/d] gives [Un/d].

exercise 3.8

LINKS TO Example 30

Write the reciprocal of each of the following fractions.

a $\frac{2}{3}$ **b** $\frac{1}{5}$ **c** $\frac{3}{8}$ **d** $\frac{3}{7}$

e $\frac{5}{2}$ **f** $\frac{7}{3}$ **g** $\frac{6}{5}$ **h** $\frac{11}{9}$

LINKS TO Example 31

Evaluate each of the following.

a $2 \quad \frac{1}{3}$ **b** $3 \quad \frac{1}{4}$ **c** $4 \quad \frac{2}{5}$ **d** $8 \quad \frac{4}{9}$

e $3 \quad \frac{2}{5}$ **f** $2 \quad \frac{3}{7}$ **g** $3 \quad \frac{5}{8}$ **h** $5 \quad \frac{2}{3}$

LINKS TO Example 32

Evaluate each of the following.

a $\frac{2}{5} \quad \frac{1}{2}$ **b** $\frac{1}{9} \quad \frac{2}{5}$ **c** $\frac{2}{8} \quad \frac{4}{9}$ **d** $\frac{7}{10} \quad \frac{3}{20}$

e $\frac{6}{7} \quad \frac{5}{14}$ **f** $\frac{7}{8} \quad \frac{3}{4}$ **g** $\frac{11}{12} \quad \frac{3}{8}$ **h** $\frac{14}{15} \quad \frac{7}{10}$

i $\frac{5}{6} \quad \frac{2}{5}$ **j** $\frac{4}{5} \quad \frac{3}{10}$ **k** $\frac{8}{9} \quad \frac{1}{6}$ **l** $\frac{21}{22} \quad \frac{2}{11}$

LINKS TO Example 33

Carry out these divisions of mixed numbers.

a $1\frac{1}{2} \quad \frac{2}{3}$ **b** $1\frac{2}{3} \quad \frac{3}{4}$ **c** $1\frac{3}{8} \quad 1\frac{1}{4}$ **d** $3\frac{1}{5} \quad 1\frac{2}{5}$

e $1\frac{1}{6} \quad 1\frac{5}{9}$ **f** $5\frac{1}{2} \quad 3\frac{2}{3}$ **g** $2\frac{1}{2} \quad 1\frac{7}{8}$ **h** $3\frac{1}{9} \quad 1\frac{1}{6}$

i $4\frac{1}{6} \quad 2\frac{1}{2}$ **j** $4\frac{1}{2} \quad 3\frac{3}{8}$ **k** $2\frac{14}{15} \quad 1\frac{1}{10}$ **l** $6\frac{2}{5} \quad 4\frac{4}{15}$

LINKS TO Example 33

How many $\frac{1}{8}$ kg cups of flour can be taken from a 2 kg packet of self-raising flour?

How many lots of $\frac{2}{3}$ cup of sugar can be taken from a 3 kg bag of sugar?

A bus driver drives for 7 hours in one day shift. On average it takes $1\frac{1}{4}$ hours for the bus to travel its entire route. How many times will she drive the bus route? (Assume another driver takes over at the end of the 7-hour shift.)

On a 55 minute CD, songs average $3\frac{2}{3}$ minutes each. How many songs would you expect on a CD?

exercise 3.8 challenge

There are $6\frac{2}{5}$ litres of petrol left in the tank of a car. This car uses $1\frac{1}{4}$ litres of petrol to travel 10 km. How many kilometres can the car travel on the petrol left in its tank?

3.9 Order of operations with fractions

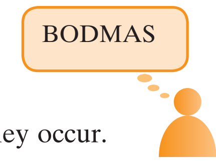
The same order of operations that you saw in Chapter 1 also applies to fractions. Although the operation 'of' with fractions is calculated as a multiplication, it must be done before multiplications and divisions. It is helpful to think of a calculation such as $\frac{3}{4}$ of $\frac{5}{9}$ as having brackets around it.

1 Brackets, Powers and Roots

2 Of

3 $\left\{\begin{array}{l}\text{Divisions and} \\ \text{Multiplications}\end{array}\right\}$ are worked out from left to right in the order that they occur.

4 $\left\{\begin{array}{l}\text{Additions and} \\ \text{Subtractions}\end{array}\right\}$ are worked out from left to right in the order that they occur.



Example 34

Calculate $\frac{4}{5} \div \frac{1}{2}$ of $\frac{8}{9}$.

Working	Reasoning
$\frac{4}{5} \div \frac{1}{2}$ of $\frac{8}{9}$	
$= \frac{4}{5} \div \left(\frac{1}{\cancel{2}_1} \times \frac{\cancel{8}^4}{9}\right)$	Put brackets as a reminder to calculate 'of' first.
$= \frac{4}{5} \div \frac{4}{9}$	Work out the multiplication.
$= \frac{\cancel{4}^1}{5} \times \frac{9}{\cancel{4}_1}$	Multiply by the reciprocal of $\frac{4}{9}$.
$= \frac{9}{5}$	Convert the improper fraction into a mixed number.
$= 1\frac{4}{5}$	

Example 35

Calculate $\frac{3}{4} + \frac{2}{3} \times \frac{1}{8}$.

Working	Reasoning
$\frac{3}{4} + \frac{2}{3} \times \frac{1}{8}$	Calculate multiplication before addition.
$= \frac{3}{4} + \frac{\not{2}^{1}}{3} \times \frac{1}{\not{8}_{4}}$	
$= \frac{3}{4} + \frac{1}{12}$	Write the fractions with the same denominator.
$= \frac{9}{12} + \frac{1}{12}$	Add the numerators
$= \frac{10}{12}$	Simplify the fraction.
$= \frac{5}{6}$	

Example 36

Calculate $\left(\frac{2}{3} + \frac{1}{2}\right) \times 2\frac{1}{2}$.

Working	Reasoning
$\left(\frac{2}{3} + \frac{1}{2}\right) \times 2\frac{1}{2}$	Evaluate the expression in the brackets first.
$\left(\frac{4}{6} + \frac{3}{6}\right) \times 2\frac{1}{2}$	Write the fractions with a common denominator.
$= \frac{7}{6} \times \frac{5}{2}$	Add the numerators. Convert $2\frac{1}{2}$ into an improper fraction.
$= \frac{35}{12}$	Multiply the numerators and multiply the denominators.
$= 2\frac{11}{12}$	Convert the improper fraction into a mixed number.

exercise 3.9

LINKS TO Examples 34, 35

Evaluate the following.

a $\frac{5}{6} - \frac{1}{2} \quad \frac{3}{4}$ **b** $3 \quad \frac{1}{2} \quad \frac{1}{4}$ **c** $3 \quad \frac{1}{2}$ of $\frac{1}{4}$

d $\frac{1}{2}$ of $\frac{1}{4} \quad 3$ **e** $\frac{1}{2} \quad \frac{1}{4} \quad 3$ **f** $\frac{3}{5}$ of $\frac{8}{9} - \frac{1}{3}$

g $\frac{9}{10} \quad \frac{5}{12} \quad \frac{3}{4}$ **h** $\frac{11}{12} - \frac{2}{3} + \frac{1}{4}$ **i** $\frac{3}{7} \quad \frac{1}{2}$ of $\frac{2}{3} + \frac{11}{15}$

LINKS TO Example 36

Evaluate the following.

a $3\frac{2}{3} - \frac{1}{2} \quad \frac{3}{5}$ **b** $3\frac{3}{4} \quad \frac{5}{8}$ of $2\frac{2}{3}$ **c** $3\frac{1}{3} + 1\frac{1}{2} - 2\frac{5}{6}$

d $\frac{1}{2}$ of $2\frac{2}{3} - \frac{5}{6}$ **e** $\frac{1}{2}$ of $2\frac{2}{3} - \frac{5}{6}$ **f** $3\frac{2}{3} - 1\frac{1}{6} \quad 1\frac{2}{5}$

g $1\frac{1}{4} + 3 \quad 1\frac{1}{2}$ **h** $3 \quad \frac{1}{5} - \frac{2}{5} \quad 1\frac{1}{2}$ **i** $7\frac{1}{2} \quad 1\frac{2}{3}$ of $2\frac{1}{5}$

Use your calculator to evaluate each of the following.

a $\frac{(3 + 1)}{5}$

b $(3 + 1) \quad 5$

c $3 + \frac{1}{5}$

d $3 + 1 \quad 5$

e Is $\frac{(3 + 1)}{5}$ the same as $(3 + 1) \quad 5$?

f Explain why the brackets around 3 + 1 in part b make the answer to part b different from the answer to part d.

exercise 3.9 challenge

Find the missing numerator.

$$\frac{1}{2}\left(\frac{7}{5} + \frac{\square}{3}\right) = 1\frac{8}{15}$$

Analysis task

Lotus flowers

This problem comes from a book called *Lilavati*, written in 1150 CE by the Hindu mathematician Bhaskara. The book was named after his daughter.

> Out of a heap of pure lotus flowers one-third, one-fifth and one-sixth were offered respectively to the gods Shiva, Vishnu and the Sun. One-quarter was presented to the goddess Parvati. The remaining six flowers were offered at the feet of the teacher. Tell me the number of all the flowers.


The Hindu goddess, Lakshmi, holding lotus flowers

a What common denominator would be needed for fractions of one-third, one-fifth, one-sixth and one-quarter?

b What total fraction of the flowers was given to Shiva, Vishnu, the Sun, and Parvati?

c What fraction was left for the teacher?

d If the teacher's share was six flowers, how many flowers were there altogether?

e How many lotus flowers were given to each of the gods Shiva, Vishnu, and the Sun?

f How many lotus flowers were given to the goddess Parvati?

g This similar problem also comes from the book *Lilavati*.

> A necklace broke.
> A row of pearls mislaid.
> One sixth fell to the floor.
> One fifth upon the bed.
> The young woman saved one third of them.
> One tenth were caught by her friend.
> If six pearls remained upon the string
> How many pearls were there altogether?

- **i** What common denominator would be needed for fractions of one-sixth, one-fifth, one-third and one-tenth?
- **ii** What fraction of the pearls were lost?
- **iii** What fraction of the pearls remained on the string?
- **iv** If six pearls were left on the string, how many pearls were there altogether?
- **v** How many pearls fell on the floor?

h Use the internet to look up more information about Bhaskara II. (For example, see www-history.mcs.st-and.ac.uk/Biographies/Bhaskara_II.html)

- **i** In what country was he born?
- **ii** What was his job?
- **iii** How many books on Mathematics did he write?

Review Fractions

Summary

Fractions

- Fractions are numbers that can be located on the number line.
- Fractions can be parts of a whole.
- Whole numbers can be written as fractions, for example, $5 = \frac{5}{1}$.
- Proper fraction: numerator $<$ denominator
- Improper fraction: numerator $>$ denominator
- Mixed numbers have a whole number part and a fraction part.
- Mixed numbers can be converted into improper fractions.
- Equivalent fractions are fractions that can be simplified to the same fraction, for example, $\frac{4}{12}$, $\frac{3}{9}$ and $\frac{2}{6}$ are equivalent fractions. They can all be simplified to $\frac{1}{3}$.
- Equivalent fractions are located at the same position on the number line.

Comparing and ordering fractions

- Convert to equivalent fractions with the same denominator then compare the numerators. If the denominators are the same, the larger the numerator, the larger the fraction.

Adding fractions

- Proper and improper fractions: convert to equivalent fractions with the same denominator, then add the numerators. If the sum is an improper fraction convert to a mixed number.
- Mixed numbers: add the whole number parts then add the fraction parts as above.

Subtracting fractions

- Convert mixed numbers to improper fractions. Convert to equivalent fractions with same denominators then subtract the numerators.

Multiplying fractions

- Convert mixed numbers to improper fractions. Cancel where possible. Multiply the numerators. Multiply the denominators. If the product is an improper fraction convert to a mixed number.

Dividing fractions

- Convert mixed numbers to improper fractions. Multiply by the reciprocal of the fraction that comes after the division sign. Then proceed as for multiplying fractions.

1 Brackets, Powers and Roots
2 Of
3 { Divisions and Multiplications } are worked out from left to right in the order that they occur.
4 { Additions and Subtractions } are worked out from left to right in the order that they occur.

Visual map

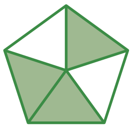
Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

add	inverse	proper fraction
cancel	lowest common denominator	reciprocal
common denominator	lowest common multiple	simplest form
denominator	mixed number	subtract
divide	multiply	square root
improper fraction	numerator	

Revision

Multiple-choice questions

● What fraction of this pentagon has been shaded?



A $\frac{2}{5}$ **B** $\frac{3}{5}$ **C** $\frac{2}{3}$ **D** $\frac{3}{2}$ **E** $\frac{5}{3}$

● As an improper fraction $2\frac{3}{4}$ becomes

A $\frac{5}{4}$ **B** $\frac{6}{4}$ **C** $\frac{11}{4}$ **D** $\frac{14}{4}$ **E** $\frac{3}{11}$

● Write $\frac{13}{6}$ as a mixed number.

A $\frac{6}{13}$ **B** $\frac{26}{12}$ **C** $1\frac{3}{6}$ **D** $2\frac{1}{6}$ **E** $3\frac{1}{6}$

● A fraction equivalent to $\frac{4}{5}$ is

A $\frac{16}{20}$ **B** $\frac{3}{4}$ **C** $\frac{18}{30}$ **D** $\frac{5}{4}$ **E** $\frac{5}{6}$

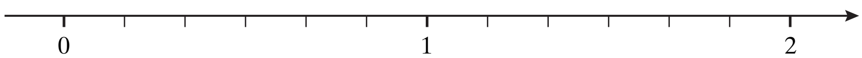
In simplest form $\frac{30}{42}$ is

A $\frac{10}{14}$ **B** $\frac{6}{7}$ **C** $\frac{3}{4}$ **D** $\frac{5}{7}$ **E** $\frac{15}{21}$

Short-answer questions

Copy each number line. Mark the position and label each set of fractions.

a $\frac{1}{2}, 1\frac{5}{6}, \frac{2}{3}, \frac{7}{6}, \frac{3}{2}$



b $\frac{3}{4}, 2\frac{1}{4}, \frac{7}{4}, \frac{5}{2}, \frac{5}{4}$

c $1\frac{2}{5}, \frac{5}{5}, \frac{3}{5}, \frac{12}{5}, 2\frac{1}{5}$

Make true statements using or .

a $\frac{2}{5}$ ____ $\frac{1}{3}$ **b** $\frac{7}{10}$ ____ $\frac{3}{4}$

Evaluate the following.

a $\frac{2}{9} + \frac{5}{9}$ **b** $\frac{1}{3} + \frac{4}{15}$ **c** $\frac{4}{5} + \frac{1}{2}$ **d** $2\frac{1}{4} + 1\frac{7}{8}$

Evaluate the following.

a $\frac{6}{7} - \frac{2}{7}$ **b** $\frac{5}{6} - \frac{1}{4}$ **c** $2\frac{7}{11} - 1\frac{3}{11}$ **d** $1\frac{9}{10} - 1\frac{3}{5}$ **e** $3\frac{1}{4} - 1\frac{5}{6}$

Cancel but do not multiply.

a $\frac{7}{18} \quad \frac{15}{14}$ **b** $\frac{14}{15} \quad \frac{11}{21} \quad \frac{5}{22}$

Evaluate the following.

a $\frac{1}{3} \quad 12$ **b** $6 \quad 1\frac{1}{2}$ **c** $\frac{2}{3} \quad \frac{1}{7}$

d $\frac{1}{5}$ $\frac{5}{6}$ **e** $\frac{9}{10}$ $\frac{5}{21}$ **f** $1\frac{5}{9}$ $2\frac{7}{10}$

Evaluate the following.

a $\frac{3}{7}$ $\frac{1}{2}$ **b** $\frac{4}{7}$ $\frac{3}{4}$ **c** $\frac{2}{3}$ $\frac{5}{6}$ **d** $1\frac{1}{14}$ $1\frac{3}{7}$

Goran likes to drink a special mixture of fruit juice for breakfast. The mixture is $\frac{1}{4}$ orange juice, $\frac{1}{5}$ apple juice, $\frac{3}{8}$ pineapple juice. The rest is grapefruit juice. What fraction of Goran's drink is grapefruit juice?

Sarah practised violin for $8\frac{3}{4}$ hours this week. If she practised for the same length of time each day, for how long did she practise each day?

Extended-response questions

There were 36 Smarties in a packet. Caitlin ate $\frac{1}{4}$ of them. Jack ate $\frac{2}{9}$ of them and Will ate the rest.

a How many Smarties did Caitlin eat?
b How many Smarties did Jack eat?
c How many Smarties did Will eat?
d What fraction of the Smarties did Will eat?
e Check that Caitlin's share, $\frac{1}{4}$, Jack's share, $\frac{2}{9}$ and Will's share, worked out in part d, add to 1, which is the whole lot.

As a fundraiser, a team from 7B is washing cars. They have worked out that on average it takes them $\frac{1}{4}$ hour to wash one car. They charge $6 per car.

a How long would it take them to wash seven cars?
b How many cars must they wash to make $100? The answer should be a whole number of cars.
c How long will it take them to earn $100?

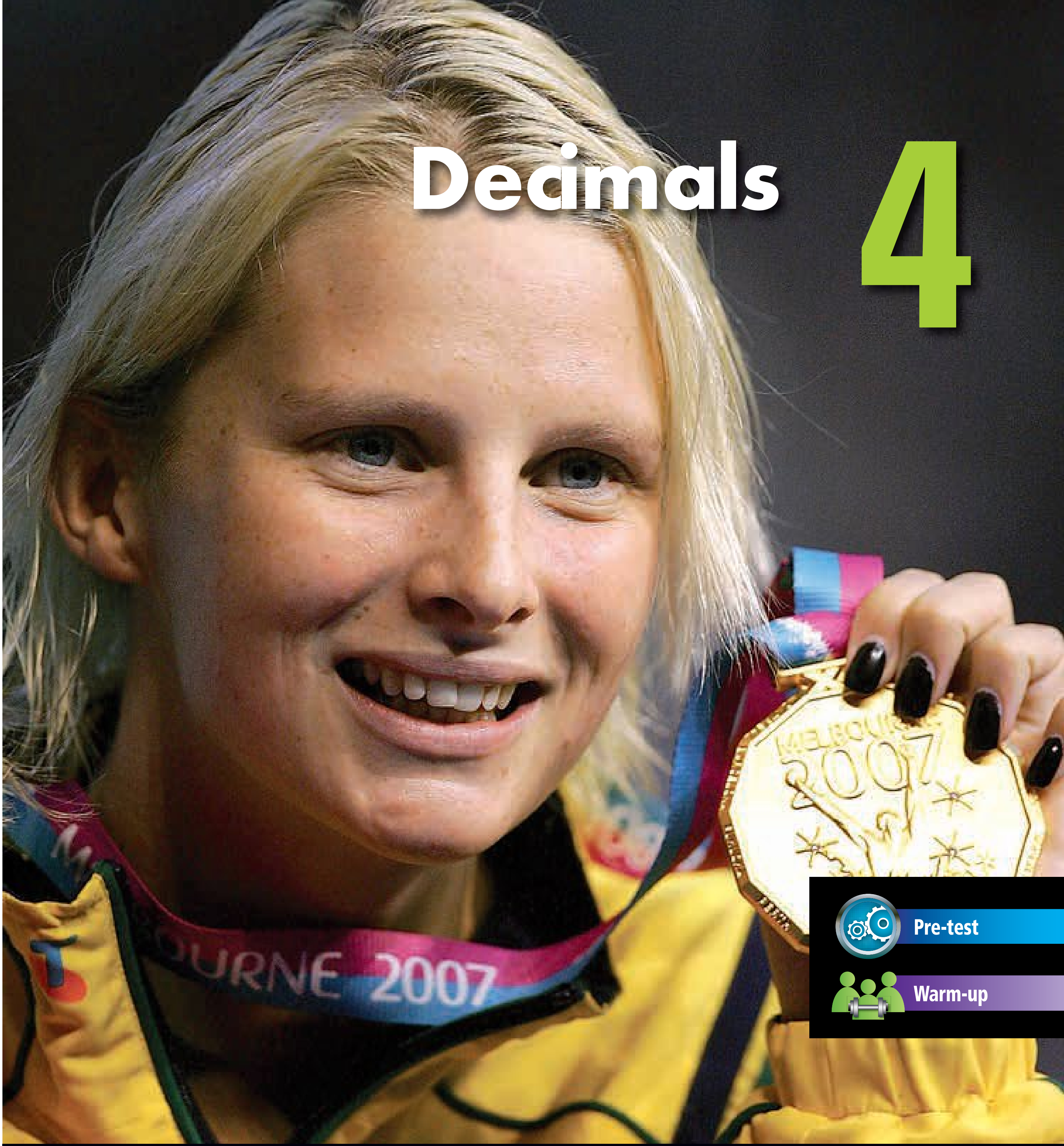
To make a particular green coloured paint, $\frac{3}{4}$ of a can of yellow paint was added to $\frac{1}{6}$ of a can of blue paint. To make it a lighter shade, $\frac{2}{3}$ of a can of white paint was added.

a How many cans of green paint would this make?
b What would these quantities need to be multiplied by to make $4\frac{3}{4}$ cans of the green paint?
c To make $4\frac{3}{4}$ cans of the same shade of green, how many cans of each colour should be mixed?

Practice quiz
Chapter 3

Decimals

4



Pre-test

Warm-up

Like fractions, decimals allow us to represent whole numbers as well as quantities that are not whole numbers, for example, 6.08. Decimals play an important part in our financial system and in metric measurements. Decimals are particularly important to swimmers and athletes. When a world record is broken, the new time may be only 0.01 seconds faster than the previous record time.

4.1 Place value

We saw in chapter 1 how the position of each digit in a whole number determines its value. For example, in the number 3807, the value of 3 is 3000, the value of 8 is 800, there are no tens and the value of 7 is 7.

Thousands	Hundreds	Tens	Ones
3	8	0	7

This place value system also extends to numbers that are parts of a whole. The place values are called tenths, hundredths, thousandths, and so on. The following table shows the place values for the number 1534.9204. The decimal point between the ones and the tenths separates the whole number part of the number from the fractional part.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten thousandths
1	5	3	4 •	9	2	0	4
1×1000	5×100	3×10	4	$\frac{9}{10}$	$\frac{2}{100}$	$\frac{0}{1000}$	$\frac{4}{10\,000}$
1000	500	30	4	0.9	0.02	0.000	0.0004

We refer to the number of digits to the right of the decimal point as **decimal places**. The number 51.91 has 2 decimal places.

Example 1

In the number 23.461

a what is the value of the 6?

b how many decimal places are there?

Working

a 6 hundredths

b 3 decimal places

Reasoning

On the right of the decimal point are tenths, hundredths and thousandths.

The number of decimal places is the number of digits to the right of the decimal point. This number has 3 digits to the right of the decimal point.

Example 2

Consider the number 23.824.

a Write the number in words as we would say it.

b Write the value of each digit in the number 23.824

i in numbers. **ii** in words.

Working	Reasoning
a Twenty-three point eight two four	To the right of the decimal point we read the digits individually. We do *not* say 'point eight hundred and twenty four'.
b i 23.824 is made up of 20, 3, $\frac{8}{10}$, $\frac{2}{100}$ and $\frac{4}{1000}$	The numbers to the left of the decimal point are the whole number values. Going left from the decimal point are ones, tens, hundreds and thousands.
ii 23.824 is made up of twenty, three, 8 tenths, 2 hundredths and 4 thousandths	The numbers to the right of the decimal point are the fractional parts. Going right from the decimal point are tenths, hundredths and thousandths.

Writing decimal numbers in place value notation

As in chapter 1, where we wrote whole numbers using place value notation, we can also do this with decimal numbers. The number 2534.9204, for example, can be written as

$$2000 + 500 + 30 + 4 + \frac{9}{10} + \frac{2}{100} + \frac{0}{1000} + \frac{4}{10\,000}.$$

We read this number as 'two thousand five hundred and thirty-four point nine two zero four'.

Example 3

Write the following.

a the number 947.513 in place value notation

b $800 + 50 + 2 + \frac{6}{10} + \frac{9}{1000}$ as a single number

Working	Reasoning
a $947.513 = 900 + 40 + 7 + \frac{5}{10} + \frac{1}{100} + \frac{3}{1000}$	Place value notation means to write each digit so that its value is clear.
b $800 + 50 + 2 + \frac{6}{10} + \frac{9}{1000} = 852.609$	The way each digit is written shows its place value. In this number there are no hundredths. A zero must be put in the hundredths place.

Example 4

Write $\frac{43}{1000}$ in decimal form.

Working

$\frac{43}{1000}$

$= \frac{40}{1000} + \frac{3}{1000}$

$= \frac{4}{100} + \frac{3}{1000}$

$= 0.043$

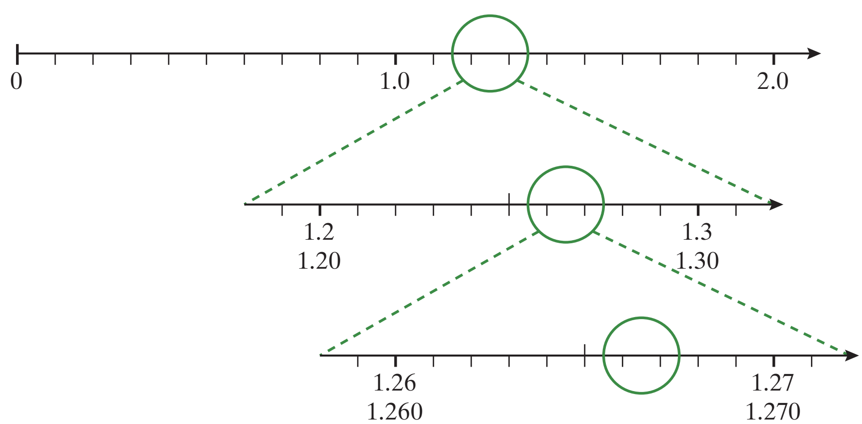
Reasoning

This is 43 thousandths.
It can be split into 40 thousandths and 3 thousandths.
40 thousandths is the same as 4 hundredths.
Fill the empty tenths place value column with zero.

Decimal numbers on the number line

Like whole numbers and fractions, decimal numbers can be shown on the number line.

The small scale marks on the top number line show tenths. The scale marks can be labelled as 0.1, 0.2, 0.3, and so on.



Number line tenths

Zooming in, the section in between each tenth mark is split into 10 smaller pieces called hundredths, as shown on the middle number line section above. Hundredths are tenths of tenths; that is, $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{10} \quad \frac{1}{10} = \frac{1}{100}$.

Number line hundredths

Zooming in again, the sections between the hundredths are split into 10 smaller parts called thousandths. Thousandths are tenths of hundredths; that is, $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{10} \quad \frac{1}{100} = \frac{1}{1000}$.

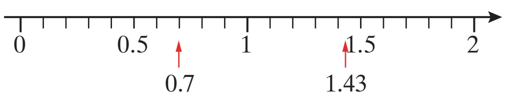
Notice that on a number line, larger numbers (e.g., 2, 30, 100) are further to the right. Smaller numbers (e.g., 0.5, 0.1, 0.002) are further to the left towards zero.

This must not be confused with the way we write numbers, where the larger place values (ones, tens, hundreds, etc.) are to the left of the decimal point and the smaller place values (tenths, hundredths, etc.) are to the right of the decimal point.

Example 5

Show both 0.7 and 1.43 on a number line.

Working



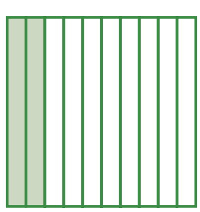
Reasoning

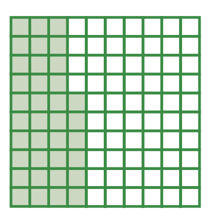
0.7 has no ones and 7 tenths.
It is between 0 and 1.
The number line has marks every tenth or 0.1.
0.7 is 2 tenths bigger than 0.5.
1.43 has 1 one, 4 tenths and 3 hundredths.
It is between 1 and 2.
It is between 1.4 and 1.5.
It is closer to 1.4 than 1.5.

Representing decimals as areas

The large squares in these diagrams each represent one whole. The square on the left has been divided into 10 equal parts, and 2 of these are shaded. This represents the fraction $\frac{2}{10}$ and the decimal number 0.2.

The square on the right is divided into 100 small squares and 36 of these are shaded. This represents the fraction $\frac{36}{100}$ and the decimal number 0.36.

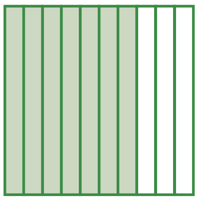




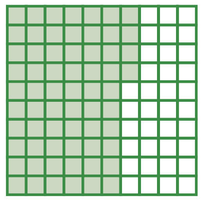
Example 6

What decimal numbers are represented by these diagrams?

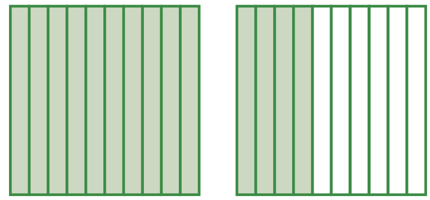
a



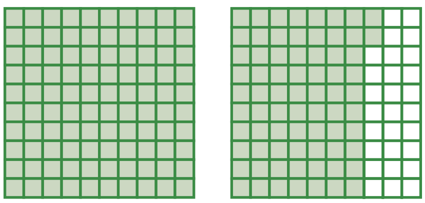
b



c



d



Working	**Reasoning**
a 0.7	7 tenths of the square are shaded.
b 0.64	64 hundredths of the square are shaded.
c 1.4	1 whole plus 4 tenths are shaded.
d 1.72	1 whole plus 72 hundredths are shaded.

exercise 4.1

LINKS TO Example 1

How many decimal places are there in each of these numbers?

a 3.435 **b** 51.7 **c** 8.3077 **d** 0.004235
e 4.65000 **f** 44 **g** 44.0 **h** 44.00

LINKS TO Example 1

For each of the following numbers

i what is the value of the digit 4?
ii how many decimal places are there?

a 45.3 **b** 90.4 **c** 24.7 **d** 3.74
e 496.1 **f** 3.214 **g** 0.541 **h** 19.468
i 354.82 **j** 76.543 **k** 2.0004 **l** 592.3841

LINKS TO Example 2

For each of the following numbers

i write the number in words as we would say it.
ii write the place value of each digit as a fraction.
iii write the place value of each digit in words.

a 12.6 **b** 1.74 **c** 24.8 **d** 0.736 **e** 308.2 **f** 7.093
g 0.294 **h** 30.408 **i** 7.106 **j** 20.089 **k** 301.065 **l** 7090.0041

LINKS TO Example 3

1. Write each of these numbers
   i in expanded notation.
   ii as a decimal number.

a 3 and 4 tenths
b 6 and 9 hundredths
c 5 units + 2 tenths + 6 hundredths
d 7 units + 4 tenths + 8 hundredths
e 2 hundredths + 9 thousandths
f 3 tenths + 1 ten-thousandth
g 8 tens + 3 tenths + 4 thousandths
h 5 units + 2 hundredths + 7 thousandths
i 6 tenths + 3 hundredths + 9 thousandths
j 2 hundreds + 1 hundredth + 3 thousandths
k 4 tens + 8 tenths + 2 ten thousandths
l 7 thousands + 6 units + 4 thousandths

LINKS TO Example 3

2. Change each of these numbers from place value form to decimal form.

a $3 + \frac{1}{10} + \frac{6}{100}$
b $10 + 2 + \frac{0}{10} + \frac{7}{100}$
c $40 + 7$
d $40 + 7 + \frac{2}{10}$
e $40 + 7 + \frac{2}{10} + \frac{9}{100}$
f $40 + 7 + \frac{2}{10} + \frac{9}{100} + \frac{1}{1000}$
g $50 + 6 + \frac{0}{10} + \frac{8}{100}$
h $70 + 4 + \frac{6}{10} + \frac{3}{1000}$
i $400 + 9 + \frac{1}{100} + \frac{8}{1000}$
j $20 + 5 + \frac{4}{100} + \frac{3}{10\,000}$
k $700 + 30 + 8 + \frac{2}{10} + \frac{9}{1000}$
l $9000 + 2 + \frac{6}{10} + \frac{8}{10\,000}$

3. Copy and complete.

a 8 units + 36 hundredths = 8 units + 3 tenths + 6 hundredths = __
b 2 units + 48 hundredths = 2 units + __ tenths + __ hundredths = 2.48
c 7 units + __ hundredths = 7 units + 6 tenths + 9 hundredths = 7.69
d __ units + 1 hundredth = 4 units + __ tenths + 1 hundredth = 4.01
e 7 units + 39 thousandths = 7 units+ __ tenths+ __ hundredths + 9 thousandths = 7.039
f 52 tenths + 1 hundredth = __ units+ 2 tenths + 1 hundredth = __
g 23 units + 14 hundredths = 2 __ + 3 units + 1 __ + 4 hundredths = __
h 81 tenths + 6 thousandths = 8 __ + 1 tenth + __ hundredths + 6 thousandths = __

Copy and complete this table.

Fraction (words)	Fraction	Decimal
	$\frac{7}{10}$	
		0.02
Forty-three hundredths		
	$\frac{3}{1000}$	

LINKS TO Example 4

Write each number in decimal form.

a $\frac{7}{100}$ **b** $\frac{4}{100}$ **c** $\frac{11}{100}$ **d** $\frac{27}{100}$ **e** $3\frac{6}{100}$ **f** $7\frac{1}{100}$

g $2\frac{39}{100}$ **h** $5\frac{44}{100}$ **i** $7\frac{3}{10}$ **j** 8 **k** $\frac{164}{100}$ **l** $\frac{53}{10}$

m $\frac{203}{100}$ **n** $\frac{457}{100}$ **o** $\frac{670}{100}$ **p** $\frac{810}{100}$ **q** $\frac{34}{10}$ **r** $\frac{58}{10}$

LINKS TO Example 5

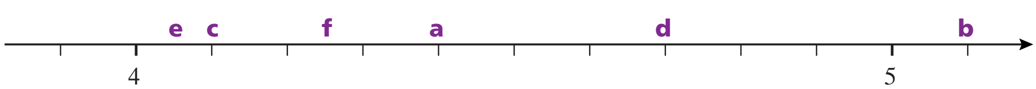
Copy this number line into your book. Make the scale marks 1 cm apart. On the number line mark each of the following decimals.

a 2.1 **b** 2.5 **c** 2.8 **d** 2.9 **e** 2.85 **f** 3.0

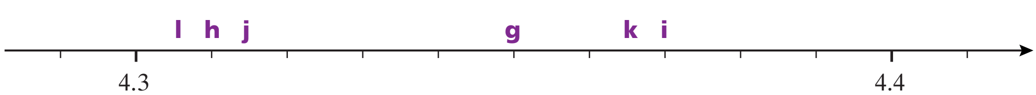
g 2.00 **h** 2.45 **i** 2.95 **j** 2.05 **k** 2.65 **l** 2.30

LINKS TO Example 5

Each letter on these number lines represents a decimal number.
For each letter, write the number that it shows to the nearest 0.05.



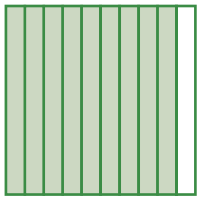
For each letter, write the number that it shows to the nearest 0.005.



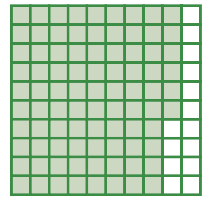
LINKS TO Example 6

Write the decimal numbers represented by each of these shaded areas.

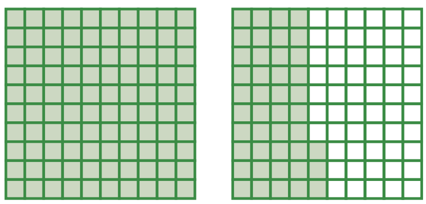
a



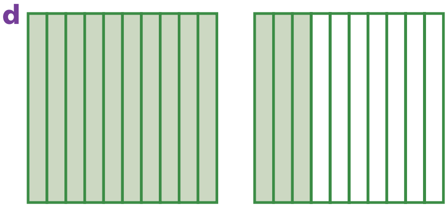
b



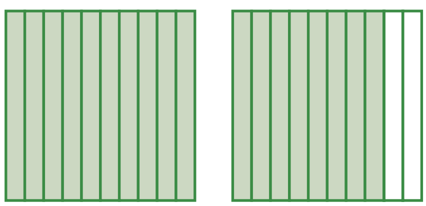
c



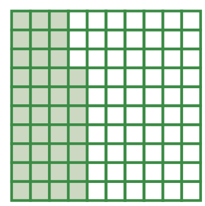
d



e



f



Copy and write three more numbers for each pattern.

a 0.2, 0.4, 0.6, ___, ___, ___, …

b 0.3, 0.6, 0.9, ___, ___, ___, …

c 2.1, 2.3, 2.5, ___, ___, ___, …

d 4.1, 4.4, 4.7, ___, ___, ___, …

e 0.56, 0.57, 0.58, ___, ___, ___, …

f 1.37, 1.38, 1.39, ___, ___, ___, …

Consider the following.

a Write down five numbers with 2 decimal places that are between 6.1 and 6.2.

b Which of your five numbers are closer to 6.1 than to 6.2?

c Which are closer to 6.2?

d Is 6.15 closer to 6.1 or 6.2?

exercise 4.1 challenge

When one whole number was divided by another whole number the answer was 5.125. If both numbers were less than 50, what were the two numbers?

4.2 Comparing decimals

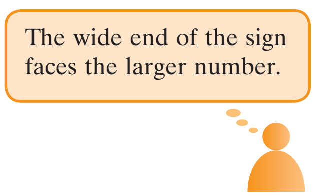
We saw in chapter 1 that whole numbers could be compared and put in order by looking at the place values of their digits. We can do the same with decimal numbers. The number line can also be used to compare decimal numbers. Numbers that are further to the right on the number line are larger than numbers to the left of them.

Example 7

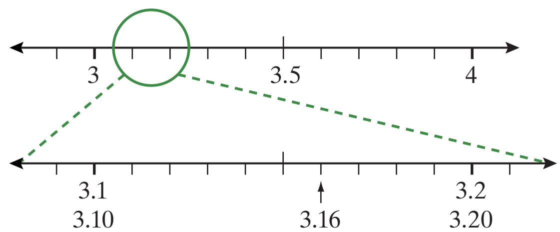
Use or to make a true statement: 3.2 ___ 3.16.

Working

3.2 ___ 3.16



Reasoning



The number line shows that 3.2 is larger than 3.16.

Without drawing a number line, we can compare the place values. Both numbers have 3 as the whole number part. 3.2 has 2 in the tenths place but 3.16 has 1 in the tenths place.

If we write 0 in the empty hundredths place of 3.2, it is easier to compare the 2 numbers. 3.2 has 20 hundredths, but 3.16 has only 16 hundredths.

3.20 > 3.16

To order decimal numbers, first compare the whole numbers, then the tenths, the hundredths and so on, as shown below.

Example 8

Sort these numbers into ascending order, that is, into order from smallest to largest.
4.102, 1.24, 4.21, 1.204, 4.201

Working

4.102, 1.24, 4.21, 1.204, 4.201

Reasoning

Compare whole numbers first.
The numbers that start with 1 are smaller than the numbers that start with 4.

continued

Example 8 continued

Working	Reasoning
Look at 1.24, 1.204	Now compare the tenths. 1.24 has 2 tenths and 1.204 also has 2 tenths, so we need to look further. Comparing the hundredths, 1.24 has 4 hundredths and 1.204 has no hundredths.
So far in order: 1.204, 1.24	1.204 is smallest than 1.24.
Look at 4.102, 4.21, 4.201.	Now for the numbers that start with 4. Compare the tenths. 4.102 has 1 tenth. 4.21 has 2 tenths and 4.201 also has 2 tenths. This means that 4.102 is smaller than 4.201 and 4.21. Compare the hundredths for the last 2 numbers. 4.21 has 1 hundredth and 4.201 has no hundredths.
So far in order: 4.102, 4.201, 4.21	4.201 is smaller than 4.21.
So, in ascending order: 1.204, 1.24, 4.102, 4.201, 4.21	

exercise 4.2

LINKS TO Example 7

Make each statement true by filling the gap with either or .

a 1.6 __ 1.4 **b** 4.08 __ 4.13 **c** 2.06 __ 2.6 **d** 7.24 __ 7.2
e 0.354 __ 0.435 **f** 2.45 __ 4.553 **g** 31.54 __ 31.276 **h** 7.2 __ 7.319
i 4.13 __ 4.138 **j** 9.3333 __ 9.3 **k** 0.299 __ 0.30 **l** 5.17 __ 5.017

Copy each pair of numbers and circle the larger number.

a 0.12, 0.104 **b** 3.7, 2.9 **c** 4.665, 4.66 **d** 0.3, 0.37
e 2.508, 2.085 **f** 0.261, 0.27 **g** 4.77, 4.771 **h** 6.238, 6.24
i 0.047, 0.47 **j** 1.9213, 1.9132 **k** 3.8, 3.79 **l** 6.228, 7.8

LINKS TO Example 8

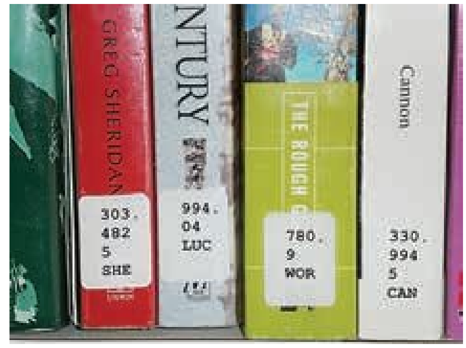
Arrange each of the following sets of numbers in ascending order.

a 6.12, 6.02, 6.21, 6.01 **b** 4.67, 7.64, 7.46, 4.76
c 3.03, 3.003, 3.33, 3.3 **d** 0.508, 0.85, 0.058, 0.58
e 2.115, 2.51, 2.15, 2.151 **f** 9.26, 9.6, 9.206, 9.026

Arrange each of the following sets of numbers in descending order.

a 1.54, 1.04, 1.05, 1.45 **b** 8.129, 8.921, 8.219, 8.192
c 4.73, 3.74, 3.47, 4.37 **d** 5.022, 5.2, 5.202, 5.22
e 7.646, 7.46, 7.464, 7.64 **f** 3.9, 3.909, 3.99, 3.099

Libraries use a decimal numbering system, called the Dewey system after its inventor, for classifying books into topics. Write the Dewey numbers of the books in the photo here in the order from left to right in which they would be placed on a library shelf. Use ascending order.



The five birds with the largest wingspan listed below are:

white pelican	3.1 m
marabou stork	3.3 m
lammergeier	2.74 m
wandering albatross	3.5 m
Andean condor	2.2 m

Write the names of these birds in order from the largest wingspan to the smallest. This is descending order of length.

The times in seconds for the 100 m women's butterfly final in the Beijing Olympics are shown below. Write the times of the swimmers in order from the fastest to the slowest.

Name	Country	Time (seconds)
Lisbeth Trickett	Australia	56.73
Jessicah Schipper	Australia	57.25
Gabriella Silva	Brazil	58.10
Jemma Lowe	Britain	58.06
Zhou Yafei	China	57.84
Inge Dekker	Netherlands	58.54
Li Tao	Singapore	57.99
Christine Magnuson	United States	57.10

In alphabetical order, these are the five largest islands in the world. Their areas are given in millions of square kilometres.

List these islands in descending order of area.

Australia	7.692
Borneo	0.737
Greenland	2.1756
Madagascar	0.587
New Guinea	0.79

● Listed below are the competitors in the final of the men's pole vault at the Beijing Olympics. List the first, second and third place getters and the heights they jumped.

Name	Country	Height (metres)
Steve Hooker	Australia	5.96
Jan Kudlicka	Czech Republic	5.45
Jerome Clavier	France	5.60
Danny Ecker	Germany	5.70
Raphael Holzdeppe	Germany	5.60
Przemyslaw Czerwinski	Poland	5.45
Evgeny Lukyanenko	Russia	5.85
Dmitry Starodubtsev	Russia	5.70
Igor Pavlov	Russia	5.60
Denys Yurchenko	Ukraine	5.70
Derek Miles	United States	5.70

● Gina and Tom are comparing the size of decimal numbers.

a Tom claimed that 0.24 is bigger than 0.3 because 24 is bigger than 3. How would you explain to Tom that his reasoning is incorrect?

b Gina claimed that 0.2 is bigger than 0.24 because 0.2 just has tenths and tenths are bigger than hundredths. How would you explain to Gina that her reasoning is incorrect?

exercise 4.2 challenge

● Consider the following numbers.
0.985, 1.064, 1.21, 0.90, 0.976, 1.014

a Arrange these numbers in ascending order.

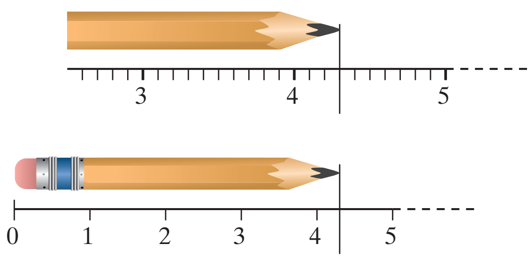
b Which number is closest to 1? Explain.

c Which number is closest to 0.95? Explain.

4.3 Rounding decimals

To estimate the answers to calculations it is often helpful to round the numbers involved. For example, the product of 6.145×1.932 will be close to 6×2 which is 12. Here, the numbers were rounded to the nearest whole number.

Lengths can be measured very accurately or less accurately. The pencil in the diagram can be measured accurately as 4.3 cm or less accurately as 4 cm (measured to the nearest whole centimetre).



To round a measurement we decide what two numbers (with the desired accuracy) the measurement lies between. For the pencil the length lies between 4 cm and 5 cm, measuring in whole centimetres. Then we decide which of these two it is closer to. For the pencil the length is closer to 4 cm. If it is right in the middle we round it up to the larger measurement. If the length of the pencil had been 4.5 cm we would have rounded it up to 5 cm.

Example 9

Round 37.289 to the nearest whole number.

Working

37.|289 is between 37 and 38.

> Put a line after the required rounding place, in this case, after the ones.

$37.289 \approx 37$

Reasoning

Whole numbers are the numbers to the left of the decimal point.
This number is a little larger than 37, so it is between 37 and 38.
Look at the digit in the next smallest place value column.
The tenths digit is 2 so the number 37.289 is less than halfway between 37 and 38.
So it is closer to 37.

Example 10

Round 13.5239 to the nearest hundredth.

Working	Reasoning
13.52|39 is between 13.52 and 13.53	Put a line after the required rounding place, in this case after the hundredths. We are rounding to the nearest hundredth. Decide which two numbers with hundredths 13.52|39 is between. Look at the third decimal place, that is, the thousandth digit. Is 13.523 closer to 13.520 or 13.530?
≈ 13.52	It is less than halfway between so it is closer to 13.520. Note: 13.52 is not the same as 13.5239 so an approximately equals sign is used, not an equals sign.

Example 11

Write 13.5239 correct to 3 decimal places.

Working	Reasoning
13.523|9 is between 13.523 and 13.524	Put a line after the required rounding place, which is 3 decimal places. Decide which two numbers with 3 decimal places 13.523|9 is between.
≈ 13.524	Now look at the fourth decimal place. 13.523|9 is between 13.5230 and 13.5240. It is more than halfway between so the number is closer to 13.524 than to 13.523.

Example 12

Round 4.972 to 1 decimal place.

Working	Reasoning
4.9|72 is between 4.9 and 5.0	Put a line after the required rounding place. Decide which two numbers with 1 decimal place 4.9|72 is between.
	Now look at the second decimal place. 4.9|7 is between 4.90 and 5.00.
≈ 5.0	It is more than halfway between so the number is closer to 5.0 than to 4.9.

Rounding decimal numbers, as in the next exercise, can be done using a scientific calculator.

Tech tip

To round numbers to a specified number of decimal places on the TI-30XB MultiView calculator, for example, 4.972 to 1 decimal place. Press [mode], arrow down to **FLOAT** and across to 1, press [enter] and [2nd] [mode]. Now type [4] [·] [9] [7] [2] [enter]. Note that [2nd] [mode] gives quit.

exercise 4.3

LINKS TO Example 9

Round each of these numbers to the nearest whole number. Write the number as it is here, then an approximately equals to sign before writing your answer.

a 16.9305	**b** 64.67	**c** 23.2	**d** 0.07
e 567.99	**f** 0.731	**g** 0.13	**h** 41.8
i 7.392	**j** 82.561	**k** 9.065	**l** 24.8
m 176.499	**n** 19.631	**o** 0.714	**p** 63.5

LINKS TO Example 10

Round each of these numbers to 2 decimal places, that is, to the nearest hundredth.

a 6.492	**b** 28.516	**c** 8.045	**d** 42.1834
e 281.396	**f** 7.698	**g** 0.312	**h** 36.2945
i 7.995	**j** 93. 631	**k** 4.098	**l** 71.8237
m 12.499	**n** 33.561	**o** 8.147	**p** 53.5918

LINKS TO Example 11

Round each of these numbers to the nearest thousandth, that is, to 3 decimal places.

a 7.7292	**b** 0.4789	**c** 4.75321	**d** 8.0027
e 19.1476	**f** 24.6984	**g** 63.5918	**h** 87.5637
i 0.17482	**j** 8.33116	**k** 4.8297	**l** 2.9998
m 21.4996	**n** 30.2495	**o** 26.3099	**p** 31.77273

LINKS TO Example 12

Round each of these numbers to the nearest tenth. Write the number as it is here, then an approximately equals to sign before writing your answer.

a 3.1489	**b** 98.9054	**c** 7.422446	**d** 131.5841
e 0.0084	**f** 3.829	**g** 224.973	**h** 52.96
i 9.77273	**j** 8.993	**k** 2.9582	**l** 56.9298
m 34.4996	**n** 1.9724	**o** 8.33116	**p** 7.17482

Copy and complete this table by rounding each number to the nearest whole number, tenth, hundredth and thousandth.

Number	Nearest whole number	Nearest tenth (1 decimal place)	Nearest hundredth (2 decimal places)	Nearest thousandth (3 decimal places)
23.5289				
0.72053				
3.6193				
28.4075				
7.158937				
0.63927				
7.1839				
0.99872				
36.2949				
8.9997				

For each of the following, write the two closest numbers with 2 decimal places which the given number is between. Then underline the one that is nearer to the given number. For example, 7.8461 is between 7.84 and <u>7.85</u>.

a 4.231 **b** 8.747 **c** 3.14289 **d** 5.6392 **e** 12.545
f 0.0074 **g** 50.296 **h** 26.19731 **i** 5.3974 **j** 8.993

Christopher used his calculator to work out the area of the playground. The result was 42.6457 square metres. Round this area to the nearest tenth of a square metre.

Igor rides his bike each day. At the end of a week he calculated his average daily distance to be 3.485 km. Round this to 1 decimal place.

The interest on Megan's bank account was calculated to be $21.4562. Round this to 2 decimal places, which is the same as to the nearest cent.

Jing's bank pays monthly interest on the average amount that has been in the account for the month. Jing's average amount was $672.24612. Round this to 2 decimal places.

Maria and her friends calculated their average height. The result was 162.34343434 cm. Round this to 1 decimal place.

When Harry worked out his average bike riding speed he got 17.48375 km/h. Round this to 3 decimal places.

The area of Australia has been quoted as 7.692 million square kilometres. Write the area of Australia

a if measured to the nearest million square kilometres?

b if measured to the nearest tenth of a million square kilometres?

c if measured to the nearest hundredth of a million square kilometres?

Give three numbers that can be rounded to 2.2 that have

a 2 decimal places. **b** 3 decimal places. **c** 4 decimal places.

When several steps are involved in a calculation, it is important to consider the effect that rounding will have on the final answer.

a Use your calculator to multiply 3.257 by 4.1681.

b Round your answer to 2 decimal places.

c Round 3.257 to 1 decimal place.

d Round 4.1681 to 1 decimal place.

e Multiply your rounded answers to parts c and d.

f Does it make a difference whether you round before multiplying as in part e or after multiplying as in part b? Explain why or why not.

When we calculate the cost of buying a number of items, we round the total to the nearest five cents.

a The purchases in three shops total \$37.21, \$46.43 and \$28.97. Round each amount to the nearest five cents.

b Write directions to explain to someone else how this should be done.

exercise 4.3 challenge

To calculate average speed, the distance travelled is divided by the time taken. The online route planner says that a trip across the city of 29.53 km should take 32.25 minutes.

a Change the time into hours by dividing it by 60. Round the answer to 3 decimal places.

b Calculate the average speed by dividing the distance in kilometres by the time in hours. Round the answer to the nearest whole number.

c Is this a reasonable average speed to be travelling in a city?

d The route given online uses a freeway for part of the journey. Why do you think that the average speed is so far below the freeway speed limit?

The number of decimal places needed depends on the situation. On a Qantas flight from Los Angeles to Melbourne the following lengths in kilometres are measured. How many decimal places would be needed for each of the following?

a The distance between Los Angeles and Melbourne.

b The length of the runway.

c The distance of the plane from the landing gate once the plane had landed.

4.4 Converting decimals to fractions

Number line tenths

Often it is helpful to change a decimal to a fraction. For some very common fractions the decimal equivalents are well known. For example, one-half is 0.5, one-quarter is 0.25 and three-quarters is 0.75. In this section we will learn to change less familiar decimals to fractions.

Number line hundredths

When converting a decimal number to a fraction, the digits of the decimal number indicate the numerator of the fraction. The number of decimal places indicates whether the denominator of the fraction is, for example, 10, 100 or 1000.

Number line eighths

Example 13

Convert each of these decimals to a fraction.

a 0.03 **b** 0.017 **c** 0.24

Working	**Reasoning**
a 0.03 $= \frac{3}{100}$	The digits after the decimal point, 03, become the numerator. The last digit, 3, is hundredths so the fraction is hundredths, that is, the denominator is 100. Check: 0.03 has 2 decimal places and the denominator has 2 zeros.
b 0.017 $= \frac{17}{1000}$	The digits after the decimal point, 017, become the numerator of the fraction (we leave out zero). The last digit, 7, is thousandths so the fraction is thousandths, that is, the denominator is 1000. Check: 0.017 has 3 decimal places and the denominator has 3 zeros.
c 0.24 $= \frac{24}{100}$ $= \frac{6}{25}$	The digits after the decimal point, 24, become the numerator. The last digit, 4, is hundredths so the fraction is hundredths, that is, the denominator is 100. Check: 0.24 has 2 decimal places and the denominator has 2 zeros. Simplify the fraction. Here, 4 is a factor of both numerator and denominator.

Example 14

Convert each of these decimals to a mixed number.

a 7.013 **b** 3.15 **c** 6.044

Working	Reasoning
a 7.013	Write the whole number, 7. The digits after the decimal point, 013, become the numerator of the fraction (leave out zero). The last digit, 3, is thousandths, so the fraction will be thousandths. Check: there were 3 decimal places and the denominator has 3 zeros.
$= 7\frac{13}{1000}$	The numerator and denominator have no common factors. This means that the fraction cannot be cancelled.
b 3.15	Write the whole number, 3.
$= 3\frac{15}{100}$	The digits after the decimal point, which are 15, become the numerator of the fraction. The last digit, 5, is hundredths, so the fraction will be hundredths. Check: there were 2 decimal places and the denominator has 2 zeros.
$= 3\frac{3}{20}$	5 is a factor of both 15 and 100. Simplify the fraction by dividing numerator and denominator by 5.
c 6.044	Write the whole number, 6.
$= 6\frac{44}{1000}$	The digits after the decimal point, which are 044, become the numerator of the fraction.
$= 6\frac{11}{250}$	The last digit, 4, is thousandths, so the denominator will be thousandths. Check: there were 3 decimal places and the denominator has 3 zeros. Simplify by dividing numerator and denominator by 4.

Tech tip

The TI-30XB MultiView calculator can change decimals to fractions.

To do this, first type in the decimal number.

Then press [2nd] [table] which gives [f◂▸d]. Lastly, press [enter].

The fraction will be given in simplest form.

If the decimal number is greater than 1, an improper fraction will be given.

To change it to a mixed number press [2nd] [×10ⁿ] which gives [n/d◂▸U n/d]. Then press [enter].

exercise 4.4

LINKS TO Example 13

Write the following numbers as fractions, without simplifying.

a 0.3	**b** 0.7	**c** 0.1	**d** 0.9
e 0.01	**f** 0.43	**g** 0.29	**h** 0.87
i 0.619	**j** 0.433	**k** 0.0367	**l** 0.0041
m 0.699	**n** 0.0917	**o** 0.0029	**p** 0.0561

LINKS TO Example 14

Write the following numbers as mixed numbers.

a 3.7	**b** 6.1	**c** 4.9	**d** 8.3
e 7.03	**f** 8.21	**g** 5.67	**h** 4.09
i 2.589	**j** 3.211	**k** 19.613	**l** 24.803
m 7.0391	**n** 82.0069	**o** 9.0607	**p** 63.0057

Write the following numbers as fractions or mixed numbers, simplifying where possible.

a 0.7	**b** 3.4	**c** 6.5	**d** 4.8
e 7.02	**f** 0.45	**g** 1.25	**h** 0.08
i 0.16	**j** 0.003	**k** 0.244	**l** 0.069
m 1.305	**n** 0.602	**o** 4.014	**p** 0.025

The best way to express 0.024 as a fraction is

A $\frac{6}{25}$ **B** $\frac{24}{100}$ **C** $\frac{6}{250}$ **D** $\frac{12}{500}$ **E** $\frac{24}{1000}$

exercise 4.4 challenge

As you know, many fractions form recurring decimals; for example, $\frac{1}{3} = 0.333\ldots$ (or $0.\overline{3}$). All recurring decimals can be converted to fractions.

To convert 0.333… into a fraction, we look at how many recurring digits there are. As there is only one recurring digit, we start by multiplying 0.333… by 10.

$$10 \quad 0.333\ldots = 3.333\ldots$$
$$1 \quad 0.333\ldots = 0.333\ldots$$

Subtracting, we obtain a whole number on the right-hand side.

$$9 \quad 0.333\ldots = 3$$

Dividing both sides by 9, we obtain

$$0.333\ldots = \frac{3}{9}$$
$$= \frac{1}{3}$$

If there are two recurring digits, as in 0.1818… we multiply by 100 instead of 10. If there are three recurring digits, we multiply by 1000.

Use the same method to convert each of these recurring decimals into fractions.

a 0.666…	**b** 0.222…	**c** 0.555…
d 0.3636…	**e** 0.8282…	**f** 0.213213…

4.5 Adding and subtracting decimals

To add or subtract whole numbers, the numbers are set out under each other so that the ones, tens, hundreds and thousands are all in their correct place value columns.

With decimals, the whole number parts are still set out in their usual columns. The decimal points are lined up under each other. The tenths, hundredths and thousandths are also lined up in their place value columns.

To add or subtract whole numbers we start in the right-hand column and work to the left. We do the same with decimals.

Example 15

Evaluate the following.

a 23.7 + 46.9 + 17.3

b 13.7 + 4.235 + 6.12 + 19.4

c 0.97 + 16.04 + 8.023

Working	Reasoning
a $\begin{array}{r} 23.7 \\ 46.9 \\ +\ 17.3 \\ \hline 87.9 \\ \hline \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. Add the digits starting from the right. Line up the decimal point in the answer.
b $\begin{array}{r} 13.7 \\ 4.235 \\ 6.12 \\ +\ 19.4 \\ \hline 43.455 \\ \hline \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. Add the digits starting from the right. Line up the decimal point and the place values in the answer.
c $\begin{array}{r} 0.97 \\ 16.04 \\ +\ 8.203 \\ \hline 25.213 \\ \hline \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. Add the digits starting from the right. Line up the decimal point in the answer.

Example 16

Evaluate the following.

a 0.735 − 0.234 **b** 25.98 − 2.76 **c** 24.8 − 1.78

Working	Reasoning
a $\begin{array}{r} 0.735 \\ -\ 0.234 \\ \hline 0.501 \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. Subtract the digits starting from the right. Line up the decimal point in the answer.
b $\begin{array}{r} 25.98 \\ -\ \ 2.76 \\ \hline 23.22 \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. Subtract as usual working from right to left. Line up the decimal point and the place values in the answer.
c $\begin{array}{r} 24.80 \\ -\ \ 1.78 \\ \hline \end{array}$	Write the numbers under each other in place value columns, lining up the decimal points. If the bottom number has more decimal places, write zeros in the top line.
$\begin{array}{r} 2\,4.{}^{7}\not{8}{}^{1}0 \\ -\ \ 1.\,7\,8 \\ \hline 2\,3.\,0\,2 \end{array}$	Subtract as usual from right to left. Line up the decimal point in the answer. Check the answer by estimating. $25 - 2 = 23$ so it is okay.

Tech tip

The TI-30XB MultiView calculator can be used to add and subtract decimals.

For example, to find 23.7 + 46.9 + 18.2, type:

[2] [3] [·] [7] [+] [4] [6] [·] [9] [+] [1] [8] [·] [2] [enter].

To calculate 0.735 − 0.234, type:

[0] [·] [7] [3] [5] [−] [0] [·] [2] [3] [4] [enter].

exercise 4.5

LINKS TO Example 15

1 Calculate each of the following.

a $\begin{array}{r} 31.7 \\ +\ 22.1 \\ \hline \end{array}$ **b** $\begin{array}{r} 4.74 \\ +\ 3.92 \\ \hline \end{array}$ **c** $\begin{array}{r} 215.2 \\ 37.81 \\ +\ 156.7 \\ \hline \end{array}$ **d** $\begin{array}{r} 64.5 \\ 57 \\ +\ 46.891 \\ \hline \end{array}$

e $\begin{array}{r} 4.316 \\ 1.8 \\ +\,17.42 \\ \hline \end{array}$ **f** $\begin{array}{r} 0.207 \\ 0.65 \\ +\ 0.391 \\ \hline \end{array}$ **g** $\begin{array}{r} 7.032 \\ 6.94 \\ +\ 14.308 \\ \hline \end{array}$ **h** $\begin{array}{r} 8.5 \\ 246 \\ +\ 314.4 \\ \hline \end{array}$

LINKS TO Example 15

Carry out the following additions by first lining up the digits in their columns.

a 2.43 + 7.51 **b** 26.2 +12.6 **c** 124.7 + 231.5 **d** 0.645 + 0.271
e 3.45 + 7.3 **f** 19.4 + 8.71 **g** 34 + 17.2 **h** 8.623 + 11.9

0.175 kg of sugar is added to 0.25 kg of butter. What is the total weight of the mixture?

Ravi rides 11.6 km, stops for lunch then rides 8.2 km more. How far has he ridden?

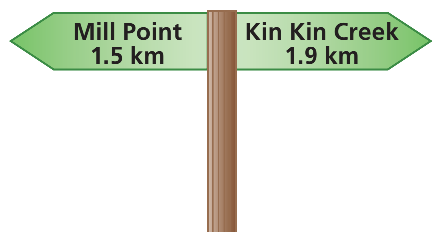
Becky adds 0.6 litres of white paint to 2.75 litres of purple paint. What volume of paint does she have now?

One track on a CD takes 3.16 minutes, the next takes 2.84 minutes and the one after takes 3.29 minutes. How long would it take to listen to all three tracks?

Choose the best answer for 1.7 + 0.52

A 0.69 **B** 1.22 **C** 1.59 **D** 2 **E** 2.22

Tom is walking in Cooloola National Park with his family. They have just walked 1.2 km from the camping ground to Mill Point, and then from Mill Point to a sign. The sign indicates that Kin Kin Creek is a further 1.9 km ahead, and that Mill Point is 1.5 km in the direction they have just come from. If they walk on to Kin Kin Creek, how far will they have walked altogether?



LINKS TO Example 16

Calculate each of the following. Where necessary include zeros to fill gaps in the decimal places.

a $\begin{array}{r} 0.69 \\ -\ 0.15 \\ \hline \end{array}$ **b** $\begin{array}{r} 64.5 \\ -\ 32.5 \\ \hline \end{array}$ **c** $\begin{array}{r} 29.85 \\ -\ 16.66 \\ \hline \end{array}$ **d** $\begin{array}{r} 0.852 \\ -\ 0.347 \\ \hline \end{array}$

e $\begin{array}{r} 0.706 \\ -\ 0.252 \\ \hline \end{array}$ **f** $\begin{array}{r} 0.913 \\ -\ 0.468 \\ \hline \end{array}$ **g** $\begin{array}{r} 0.57 \\ -\ 0.163 \\ \hline \end{array}$ **h** $\begin{array}{r} 0.2724 \\ -\ 0.1036 \\ \hline \end{array}$

LINKS TO Example 16

Carry out the following subtractions, first setting them out in place value columns.

a 46.8 − 15.2 **b** 0.785 − 0.283 **c** 0.792 − 0.481 **d** 35.47 − 14.31
e 4.092 − 3.94 **f** 31.205 − 29.31 **g** 11.7 − 9.25 **h** 14.72 − 6.315
i 0.936 − 0.272 **j** 7.038 − 3.725 **k** 0.219 − 0.087 **l** 29.16 − 16.43
m 24.71 − 19.662 **n** 93.8 − 51.523 **o** 4.27 − 3.729 **p** 42.9 − 23.803

Harry walked along a trail to a waterfall. At the start, a sign showed that it was 15.3 km to the waterfall. Later, another sign showed that it was now 9.2 km to the waterfall. How far had Harry walked when he reached the second sign?

- Hannah had $231.85 in her bank account. She withdrew $50 from the automatic teller. How much is in her account now?

- Chloe pays for tickets with a hundred dollar note. The cost of the tickets is $36.45. How much change should she get?

- In 2008 at the Beijing Olympic Games, the Australian women's 4 × 100 m medley relay team broke the world record. The times for each swimmer are listed in the table below.

Swimmer	Stroke	Time (seconds)
Emily Seebohm	Butterfly	59.33
Leisel Jones	Backstroke	64.58
Jessicah Schipper	Breaststroke	56.25
Lisbeth Trickett	Freestyle	52.53

a What was the difference between Emily Seebohm's time and Jessicah Schipper's time?
b Altogether, how long in seconds did it take the team to swim the four legs of the relay?
c What is this time in minutes and seconds?

exercise 4.5 challenge

- Copy these calculations and fill in the missing numbers.

a

```
   2 7 .   3
 − 1 6   . 7 4
   3   2 . 3
```

b

```
     2 .   3 1
     7 . 5
 +     . 1 0 4
     2 . 1 2
```

4.6 Multiplication and division by powers of 10

Multiplying by a power of 10

Multiplying and dividing by powers of 10

In chapter 1 we saw that multiplying a whole number by 10 shifts all the digits up 1 place value so that the ones become tens, the tens become hundreds and so on.

The diagram below shows how each of the digits of a decimal number moves up one place value when we multiply the number by 10. Each time we multiply by 10, the digits move up another place value. When we are left with empty place values, we fill these with 0.

Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	
				5 •	3	1	4	× 10
			5	3 •	1	4		× 10
		5	3	1 •	4			× 10
	5	3	1	4 •				× 10
5	3	1	4	0 •				

Multiplying by 10 twice is the same as multiplying by 100 (or 10^2) and the digits move up a total of two places.

Multiplying by 10 three times is the same as multiplying by 1000 (or 10^3) and the digits move up a total of three places.

Notice that the number of places the digits move up is the same as the number of zeros in the power of 10.

Although it is the digits that are moving to the left to higher place values, the effect on appearance of the number is that the decimal point moves to the right. As long as we understand why we move the decimal point, this is a simple way of multiplying a decimal number by a power of 10.

Example 17

Evaluate the following.

a 6.104 × 10

b 4.25 × 1000

Working

a 6.104 × 10
= 61.04

Reasoning

Tens	Ones	Tenths	Hundredths	Thousandths	
	6 •	1	0	4	× 10
6	1 •	0	4		

10 has 1 zero. The digits all move one place to the left (to the bigger place values).

Or, **6 • 1 • 0 4**

10 has 1 zero so move the point place to the right to make a bigger number.

Working

b 4.25 1000
= 4250

Reasoning

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	
			4 •	2	5	× 1000
4	2	5	0 •			

1000 has 3 zeros. The digits all move 3 places to the left (to the bigger place values).

There are no ones, so we fill the ones place with a zero. We don't need the decimal point because there are no decimal places.

Or, **4 • 2 5 0 •**

1000 has 3 zeros so move the point 3 places to the right to make a bigger number.
Fill the gap with zero.

Multiplying and dividing by powers of 10

Dividing by a power of 10

When we divide by powers of 10, the reverse occurs. The digits move to the right to smaller place values. The effect of this is that the decimal point appears to move to the left.

The diagram below shows how each of the digits of a decimal number moves down one place value when we divide the number by 10. Each time we divide by 10, the digits move down another place value. The effect on the appearance of the number is that the decimal point moves to the left.

Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths	
7	6 •	9				10
	7 •	6	9			10
	0 •	7	6	9		10
	0 •	0	7	6	9	10

Example 18

Evaluate the following.

a 23.7 ÷ 10

b 23 10000

Working

a 23.7 ÷ 10
= 2.37

Reasoning

Tens	Ones	Tenths	Hundredths	
2	3 •	7		10
	2 •	3	7	

10 has 1 zero. The digits all move one place to the right (to the smaller place values).

Or, **2 • 3 • 7**

continued

Example 18 continued

Working

b 23   10 000
= 0.0023

Reasoning

Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
2	3 •				
	0 •	0	0	2	3

10 000

10 000 has four zeros. The digits all move 4 places to the right (to the smaller place values).
There are no tenths or hundredths so we fill these places with zeros.
If there are no ones, tens, etc., we write a zero in the ones place.

Or,

exercise 4.6

LINKS TO Example 17

1. How many places would you shift the digits when multiplying by
   a 100? **b** 10 000? **c** 1 000 000? **d** 10 **e** 1000? **f** 100 000?

LINKS TO Example 17

2. Mentally calculate each of the following, and write the answer.

a 4 10	**b** 8.0 10	**c** 3.8 10	**d** 0.2 10
e 12.531 10	**f** 6.72 10	**g** 9.1 1000	**h** 24.33 100 000
i 60.74 10 000	**j** 98.241 100	**k** 5.2383 100	**l** 4.102 1000
m 4.7352 × 1000	**n** 23.6 × 10	**o** 21.33 × 1000	**p** 0.06 × 100
q 0.143 × 100	**r** 0.07 × 1000	**s** 0.0018 × 10	**t** 0.005 × 10 000

LINKS TO Example 18

3. How many places would you shift the digits when dividing by
   a 1000? **b** 100? **c** 10? **d** 100 000? **e** 1 000 000? **f** 10 000?

LINKS TO Example 18

4. Mentally calculate each of the following and write the answer.

a 30 ÷ 10	**b** 57 ÷ 10	**c** 41.6 ÷ 10	**d** 20.4 ÷ 10
e 46.169 ÷ 10	**f** 560.37 ÷ 10	**g** 34.21 ÷ 100	**h** 812.9 ÷ 100
i 3.0 ÷ 10	**j** 7 ÷ 10	**k** 4.1 ÷ 10	**l** 34.02 ÷ 100
m 4.73 ÷ 1000	**n** 23.6 ÷ 10	**o** 21.3 ÷ 1000	**p** 0.8 ÷ 10
q 0.25 ÷ 100	**r** 2.3 ÷ 100	**s** 0.0018 ÷ 10	**t** 7 ÷ 10 000

Sam's microscope enlarges objects so that they appear 100 times greater than they really are. How long would each of these objects appear, given their actual size below?

a a moth antenna, which is 0.09 mm long

b a pollen grain, which is 0.002 cm long

c the wing of a fly, which is 0.3 cm long

The water flea on the right has been photographed through a microscope. The magnification is 100.

a Measure the marked length on the photo in millimetres and record.

b Work out the actual length of the water flea.

exercise 4.6 challenge

When numbers are very large they are sometimes written as a decimal number between 1 and 10 multiplied by a power of 10. For example, 4560 can be written as 4.56×1000 or 4.56×10^3.

The distance of the Moon from the Earth is 3.84×10^5 kilometres. How many kilometres is this?

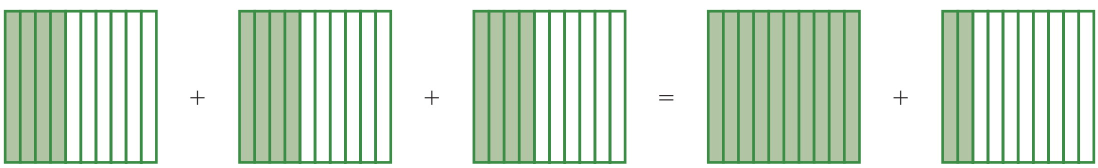
4.7 Multiplication with decimal numbers

We will start by looking at the multiplication of a simple decimal number by a whole number, 0.4×3.

In the diagram below each square represents 1. Each square is divided into 10 strips, so each thin strip represents one tenth, that is, 0.1. So the 4 green strips in each square represent 4 tenths, that is, 0.4. The diagram shows 3 lots of 4 tenths making a total of 12 tenths, that is 1 whole square plus an extra 2 tenths.

$$3 \times \frac{4}{10} = \frac{12}{10} = 1\frac{2}{10}$$

$$3 \times 0.4 = 1.2$$



By converting the decimal number to a fraction we can see where the decimal point goes in the product.

Example 19

Calculate the following.

a 0.4×7 **b** 0.006×12 **c** 7×1.85

Working	**Reasoning**
a 0.4×7 $= \frac{4}{10} \times \frac{7}{1}$ $= \frac{28}{10}$ $= 2.8$	Write 0.4 as 4 tenths. 4 tenths multiplied by 7 is 28 tenths. $28 \div 10 = 2.8$
b 0.006×12 $= \frac{6}{1000} \times \frac{12}{1}$ $= \frac{72}{1000}$ $= 0.072$	Write 0.006 as 6 thousandths. 6 thousandths multiplied by 12 is 72 thousandths. $72 \div 1000 = 0.072$

continued

Example 19 continued

Working

c 7×1.85

$= 7 \times \frac{185}{100}$

$= \frac{1295}{100}$

$= 12.95$

Reasoning

Write 1.85 as 185 hundredths.
185 hundredths multiplied by 7 is 1295 hundredths.
$1295 \div 100 = 12.95$

Multiplying a decimal number by a multiple of 10

Multiplying by a multiple of 10 can be done in two steps. For example, multiplying by 200 is equivalent to multiplying by 100 then by 2. Doing a simple multiplication by rounding is useful to check that we have the decimal point in the correct place.

Example 20

Evaluate the following.

a 5.17×30

b 3.184×200

Working

a 5.17×30

$= 5.17 \times 10 \times 3$

$= 51.7 \times 3$

$= 155.1$

Reasoning

Multiplying by 30 is equivalent to multiplying by 10 then by 3.
To multiply by 10, move the decimal point 1 place to the right to make the number 10 times larger.
$5 \times 30 = 150$ so we know where to place the decimal point.

Working

b 3.184×200

$= 3.184 \times 100 \times 2$

$= 318.4 \times 2$

$= 636.8$

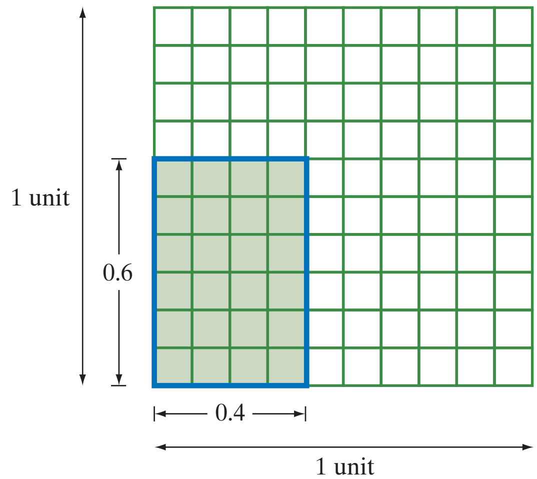
Reasoning

Multiplying by 200 is equivalent to multiplying by 100 then by 2.
To multiply by 100, move the decimal point 2 places to the right to make the number 100 times larger.
$3 \times 200 = 600$ so we know where to place the decimal point.

Multiplying a decimal number by another decimal number

To multiply a decimal number by another decimal number, we follow the same process as for multiplying whole numbers, but we then need to work out where to place the decimal point.

The diagram on the right shows how the multiplication of 2 decimal numbers can be represented as an area. The large square is 1 unit. The rectangle outlined in blue has dimensions $\frac{4}{10}$ by $\frac{6}{10}$; that is 0.4 by 0.6.



$$\begin{aligned}\text{Area of rectangle} &= 0.4 \quad 0.6\\ &= \frac{4}{10} \quad \frac{6}{10}\\ &= \frac{24}{100}\\ &= 0.24\end{aligned}$$

Each of the small squares is one-hundredth of the large square. We can see that the yellow rectangle consists of 24 of the small squares; that is, 24 hundredths or 0.24.

Example 21

Calculate the following.

a 0.4×0.6 **b** 0.03×1.72

Working	Reasoning
a 0.4×0.6	Convert each decimal number into a fraction.
$= \frac{4}{10} \times \frac{6}{10}$	
$= \frac{24}{100}$	Multiply the numerators. Multiply the denominators.
$= 0.24$	Convert the fraction back into a decimal number.
b 0.03×1.72	Convert each decimal number into a fraction.
$= \frac{3}{100} \times \frac{172}{100}$	Multiply the numerators. Multiply the denominators.
$= \frac{516}{10000}$	
$= 0.0516$	Convert the fraction back into a decimal number.

Multiplying decimal numbers that have two or more digits

Consider the multiplication 2.37 × 6.2. To complete this multiplication correctly we must know where to put the decimal point in the answer.

If we first write each number in fraction form the power of 10 in the denominator indicates where to put the decimal point. Long multiplication is used to multiply the numerators.

$$\begin{aligned} & 2.37 \times 6.2 \\ &= \frac{237}{100} \times \frac{62}{10} \\ &= \frac{237 \times 62}{1000} \end{aligned}$$

$$\begin{array}{r} 2\,3\,7 \\ \times \quad 6\,2 \\ \hline 4\,7\,4 \\ 1\,4\,2\,2\,0 \\ \hline 1\,4\,6\,9\,4 \\ \hline \end{array}$$

The denominator of 1000 tells us that we must divide 14 694 by 1000, that is, we move the decimal point 3 places to the left.

So 23.7 × 0.62 = 14.694

We should always check by rounding the numbers and doing a quick short multiplication.

2 × 6 = 12 so we know we have the decimal point in the correct position.

Example 22

Calculate these products.

a 34.2 and 1.8

b 2.85 and 5.4

Working

a

$$\begin{aligned} & 34.2 \times 1.8 \\ &= \frac{342}{10} \times \frac{18}{10} \\ &= \frac{342 \times 18}{100} \\ &= \frac{6156}{100} \\ &= 61.56 \end{aligned}$$

$$\begin{array}{r} 3\,4\,2 \\ \times \quad 1\,8 \\ \hline 2\,7\,3\,6 \\ 3\,4\,2\,0 \\ \hline 6\,1\,5\,6 \\ \hline \end{array}$$

Reasoning

Write each decimal number as a fraction.
Multiply 342 by 18 using long multiplication.
The denominator, 100, tells us that we must divide 2156 by 100.
Check: 30 × 2 = 60

b

$$\begin{aligned} & 2.85 \times 5.4 \\ &= \frac{285}{100} \times \frac{54}{10} \\ &= \frac{285 \times 54}{1000} \\ &= \frac{15390}{1000} \\ &= 15.39 \end{aligned}$$

$$\begin{array}{r} 2\,8\,5 \\ \times \quad 5\,4 \\ \hline 1\,1\,4\,0 \\ 1\,4\,2\,5\,0 \\ \hline 1\,5\,3\,9\,0 \\ \hline \end{array}$$

Write each decimal number as a fraction.
Multiply 285 by 54 using long multiplication.
The denominator, 1000, tells us that we must divide 15 390 by 1000.
Check: 3 × 5 = 15

exercise 4.7

LINKS TO Example 19

Calculate.

a 0.4 2 **b** 0.07 3 **c** 0.003 6 **d** 0.02 4
e 0.8 2 **f** 0.005 3 **g** 0.06 7 **h** 0.9 5

LINKS TO Example 19

Carry out the following multiplications.

a 6.27 3 **b** 17.06 2 **c** 411.7 5 **d** 8.092 6
e 29.31 7 **f** 363.9 4 **g** 6.193 8 **h** 0.616 7
i 0.218 × 4 **j** 0.73 × 6 **k** 0.496 × 2 **l** 0.703 × 9
m 2.308 × 7 **n** 140.6 × 3 **o** 3.07 × 8 **p** 60.07 × 5

LINKS TO Example 20

Rewrite these long multiplications as short multiplications. For example, 3.8 20 = 38 2.

a 3.71 40 = ___ 4 **b** 0.235 600 = ___ 6 **c** 63.2 300 = 6320 ___ **d** 9.025 80 = 90.25 ___
e 0.825 70 **f** 36.18 200 **g** 573.2 60 **h** 414.02 7000
i 42.6 × 70 **j** 0.76 × 300 **k** 0.793 × 50 **l** 0.0082 × 6000
m 3.05 × 40 **n** 40.6 × 900 **o** 0.0341 × 8000 **p** 20.09 × 2000

LINKS TO Example 20

Rewrite these long multiplications as short multiplications, then evaluate. For example, 3.8 20 = 38 2 = 76.

a 6.13 40 **b** 5.2 300 **c** 4.526 700 **d** 21.24 50
e 0.483 20 **f** 52.33 600 **g** 35.83 80 **h** 0.5223 4000
i 12.3 × 40 **j** 0.22 × 600 **k** 0.162 × 30 **l** 0.0052 × 4000
m 4.05 × 50 **n** 70.8 × 200 **o** 0.0412 × 6000 **p** 30.07 × 7000

LINKS TO Example 21

Calculate each of the following.

a 0.1 0.9 **b** 0.04 0.6 **c** 0.002 0.08
d 0.3 0.07 **e** 0.5 0.5 **f** 0.04 0.8
g 0.06 0.01 **h** 0.003 0.5 **i** 0.2 × 0.07
j 0.08 × 0.3 **k** 0.04 × 0.09 **l** 0.003 × 0.6
m 1.1 × 0.5 **n** 0.12 × 0.6 **o** 0.04 × 0.11

LINKS TO Example 21

Carry out the following multiplications.

a 3.1 0.5 **b** 6.4 0.03 **c** 0.37 0.2
d 9.2 0.7 **e** 0.04 4.1 **f** 25. 7 0.4
g 0.003 27.1 **h** 23.84 0.4 **i** 5.3 × 0.06
j 0.08 × 4.34 **k** 2.71 × 0.09 **l** 0.003 × 2.635
m 26.1 × 0.4 **n** 0.05 × 37.6 **o** 9.06 × 0.7

LINKS TO Example 22

Carry out each of the following calculations.

a 7.6 1.7 **b** 8.2 0.32 **c** 5.16 2.1
d 4.72 4.3 **e** 37.2 2.8 **f** 0.418 5.2
g 1.74 4.9 **h** 72.3 0.063 **i** 5.8 × 0.16

j 7.14 × 5.3 **k** 3.91 × 0.46 **l** 0.26 × 5.134
m 6.1 × 39.4 **n** 2.03 × 45.2 **o** 4.09 × 8.7

- Find the cost of three ice-creams if they cost $1.75 each.
- The distance to the local shop is 0.62 km. How far is it altogether to walk there and back twice?
- The cost of petrol is 115.3 cents per litre. How much will it cost to fill a 40 L tank? Give your answer in cents, and then in dollars and cents.
- What would be the cost of 30 party hats if they cost $2.05 each?
- The average weight of a Pink Lady apple is 0.13 kg. How much would you expect 600 of these apples to weigh?
- Daniel eats cereal for breakfast. On the side of the packet it says that the fat in one serve with half a cup of skim milk is 0.8 g. Each morning Daniel has 1.5 times as much as the recommended serve. How much fat would there be in Daniel's serve?
- Grace puts 32.5 L of petrol in her car to fill the petrol tank. Petrol costs $1.34 cents per litre. How much will this cost altogether?
- A cheetah runs at 76 km/h and a domestic cat runs at 20 km/h. Simon can run at 5.4 m/s.

a What would Simon's speed be in kilometres per hour? (To change a speed in metres per second into kilometres per hour, multiply by 3.6.)
b Put the speeds of the cheetah, the domestic cat and Simon in order from fastest to slowest.

- To change a mass in kilograms to the older measure of pounds, multiply by 2.2. What is the mass in pounds of a new baby weighing 3.74 kg?

exercise 4.7 challenge

When two *whole* numbers, except for 1, are multiplied, the answer is always larger than either number.

Carry out each of these calculations, and state whether the answer is larger than both numbers, between the two numbers or smaller than both numbers.

a 0.4 0.2
b 0.5 0.03
c 0.8 1.1
d 2.1 0.7
e 1.1 2.3
f 5.6 1.2
g When *decimals* are multiplied, when is the answer
  i larger than both numbers?
  ii between the two numbers?
  iii smaller than both numbers?

In competitive diving, dives are rated with a degree of difficulty. This is a number less than 5 given to 1 decimal place. Harder dives have bigger numbers. In club competitions each of three judges scores each dive out of 10. The three judges' scores are added. The total is multiplied by the degree of difficulty. This is done for each dive. A diver completes five dives in a competition.

a Olivia's program of dives on the 1 m springboard consisted of the dives shown in the table below. Calculate the overall score for each dive by multiplying the judges' score by the degree of difficulty.

Dive	Description	Degree of difficulty	Total of three judges' scores
i	Forward double somersault in the pike position	2.3	21.3
ii	Back double somersault in the pike position	2.2	23.5
iii	Reverse double somersault in the pike position	2.6	19.8
iv	Inward double somersault in the pike position	2.8	20.2
v	Forward twist in the pike position	1.9	26.4

b Calculate Olivia's score for the competition. Would she have scored more than her rival Emma whose score was 267.13?

4.8 Converting fractions to decimals

Some fractions can easily be converted to decimals by writing equivalent fractions with 10 or 100 as the denominator.

Example 23

Convert these fractions to decimals.

a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{9}{25}$

Working	Reasoning
a $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$	As 5 is a factor of 10, convert $\frac{3}{5}$ to an equivalent fraction with denominator 10.
b $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$	As 4 is a factor of 100, convert $\frac{3}{4}$ to an equivalent fraction with denominator 100.
c $\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100} = 0.36$	As 25 is a factor of 100, convert $\frac{9}{25}$ to an equivalent fraction with denominator 100.

We can also think of a fraction as a division. So we can convert fractions into decimals by dividing the numerator by the denominator.

Example 24

Change the following to decimals.

a $\frac{7}{8}$ **b** $2\frac{3}{4}$

continued

Example 24 continued

Working

a

$$\begin{array}{r} 0.875 \\ 8\overline{)7.{}^{7}0{}^{6}0{}^{4}0} \end{array}$$

$\frac{7}{8} = 0.875$

Reasoning

Divide the numerator by the denominator.
Write extra zeros as needed in the tenths, hundredths and thousandths places.
There is no remainder after 40 is divided by 8, so the division ends here.

b

$$\begin{array}{r} 0.75 \\ 4\overline{)3.{}^{3}0{}^{2}0} \end{array}$$

$2\frac{3}{4} = 2.75$

The whole number part stays the same.
Convert $\frac{3}{4}$ to a decimal by dividing the numerator by the denominator.

Terminating and recurring decimals

The decimal equivalents of the fractions in example 23 and example 24 are terminating decimals, that is, they end after a definite number of decimal places.

Fraction	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{9}{25}$
Decimal	0.6	0.75	0.36

Many fractions do not convert to terminating decimals.

Instead, digits occur in a repeating pattern that does not end or terminate. For example $\frac{1}{3} = 0.33333\ldots$ The decimal number 0.33333… is called a recurring decimal. In some recurring decimals, there may be more than one digit that repeats, for example, $\frac{3}{13} = 0.2307692307692\ldots$ We show that a number is a recurring decimal by writing a dot or bar over the repeating digit or sequence of digits.

$$\frac{1}{3} = 0.\overline{3} \text{ or } 0.\dot{3} \text{ and } \frac{3}{13} = 0.\overline{230769} \text{ or } 0.\dot{2}3076\dot{9}$$

Example 25

a Write 0.45454545… using a line to show which digits recur.

b Write 10 decimal places for $5.46\overline{123}$.

Working

a 0.45454545…
$= 0.\overline{45}$

Reasoning

The recurring digits are 45.

continued

Example 25 continued

Working	Reasoning
b $5.46\overline{123}$ $= 5.4612312312\ldots$	The recurring digits are 123. Count 10 digits to the right of the decimal point, then put three dots to show that the number goes on.

Example 26

Convert these fractions to decimals.

a $\frac{4}{9}$ **b** $\frac{5}{7}$

Working	Reasoning
a $\begin{array}{r} 0.\;4\;4\;4\ldots \\ 9\overline{)4.{}^{4}0{}^{4}0{}^{4}0\ldots} \end{array}$ $\frac{4}{9} = 0.\overline{4}$ or $0.\dot{4}$	Divide 4 by 9. Put zeros in the tenths, hundredths and thousandths places of 4. Continue until it is obvious that the quotient is a recurring decimal. There is a single recurring digit, 4. Write the decimal number with a bar (or dot) over the 4 to indicate that it is recurring.
b $\begin{array}{r} 0.\;7\;1\;4\;2\;8\;5\;7\;1\ldots \\ 7\overline{)5.0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0\ldots} \end{array}$ $\frac{5}{7} = 0.\overline{714285}$ or $0.\dot{7}1428\dot{5}$	Divide 5 by 7. Put zeros in the tenths, hundredths and thousandths places of 5. Continue until it is obvious that the quotient is a recurring decimal. There is a sequence of six recurring digits, 714 285, after the decimal point. Write the decimal number with a bar over the digits 71 428 to indicate that they are recurring. Alternatively, put a dot over the first and last digit in the recurring sequence.

Tech tip

To convert a fraction to a decimal using the TI-30XB MultiView calculator, for example, $\frac{2}{3}$, type:

[2] [n/d] [3] [2nd] [table] [enter].

To convert $4\frac{2}{3}$ to a decimal, type:

[4] [2nd] [n/d] [2] [n/d] [3] [2nd] [table] [enter].

exercise 4.8

LINKS TO Example 23

1. Convert these fractions or mixed numbers into tenths or hundredths. Then write each fraction as a decimal.

a $\frac{7}{10}$ **b** $\frac{4}{5}$ **c** $\frac{1}{2}$ **d** $\frac{9}{10}$ **e** $\frac{2}{5}$

f $\frac{41}{100}$ **g** $\frac{3}{100}$ **h** $\frac{7}{50}$ **i** $\frac{11}{20}$ **j** $1\frac{9}{50}$

k $4\frac{3}{10}$ **l** $2\frac{6}{25}$ **m** $6\frac{7}{20}$ **n** $3\frac{57}{100}$ **o** $5\frac{2}{5}$

LINKS TO Example 24

2. Convert each of these fractions into a decimal.

a $\frac{3}{8}$ **b** $\frac{5}{8}$ **c** $1\frac{3}{4}$ **d** $\frac{15}{8}$ **e** $4\frac{7}{8}$

f $2\frac{1}{4}$ **g** $2\frac{3}{8}$ **h** $4\frac{3}{4}$ **i** $\frac{17}{8}$ **j** $\frac{11}{4}$

LINKS TO Example 25

3. Rewrite these recurring decimals using a line to show the recurring pattern.

a 0.3333333333… **b** 0.666666666… **c** 4.1818181818… **d** 7.4545454545…

e 1.2173333333… **f** 8.4227422742… **g** 3.8888888888… **h** 7.3636363636…

i 2.1666666666… **j** 1.7272727272… **k** 5.4166666666… **l** 0.3076923076…

LINKS TO Example 26

4. Convert each of these fractions into a decimal.

a $\frac{3}{7}$ **b** $\frac{4}{9}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$ **e** $\frac{6}{11}$

f $\frac{5}{6}$ **g** $\frac{7}{9}$ **h** $\frac{11}{3}$ **i** $\frac{23}{6}$ **j** $\frac{12}{11}$

k $\frac{17}{3}$ **l** $1\frac{2}{3}$ **m** $7\frac{1}{6}$ **n** $2\frac{4}{11}$ **o** $5\frac{7}{12}$

exercise 4.8 challenge

1. Some fractions convert to recurring decimals and some convert to terminating decimals.

a Sort these fractions into two groups, those whose equivalent decimals recur and those that terminate.

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{4}{5}$ $\frac{1}{6}$ $\frac{4}{9}$ $\frac{9}{10}$ $\frac{2}{11}$ $\frac{7}{12}$

b Write the denominators of the fractions whose equivalent decimals terminate. What are their prime factors?

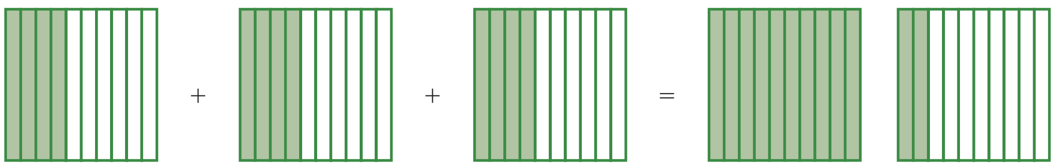
c Write the denominators of the fractions whose equivalent decimals recur. What are their prime factors?

d By looking at the prime factors of the denominators in the two lists, explain how to tell whether a fraction will convert to a terminating decimal or a recurring decimal.

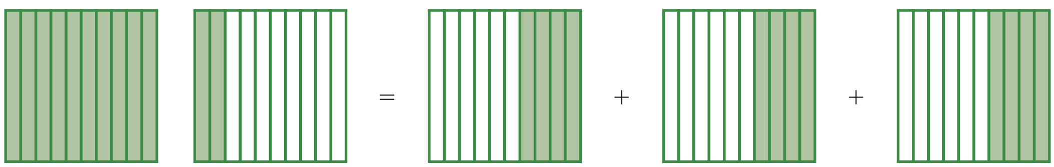
4.9 Division with decimal numbers

Division by a whole number

In section 4.7 we used the following diagram to show that $0.4 \times 3 = 1.2$.



By reversing the diagram, we can see that $1.2 \div 3 = 0.4$.



In words, 12 tenths divided by 3 is 4 tenths.

We can set this out as a short division.

$$\begin{array}{r} 0.4 \\ 3\overline{)1.2} \end{array}$$

Example 27

Evaluate the following.

a $23.16 \div 4$

b $7.11 \div 6$

Working

a

$$\begin{array}{r} 5.\ 7\ 9 \\ 4\overline{)23.{}^{3}1{}^{3}6} \end{array}$$

b

$$\begin{array}{r} 1.\ 1\ 8\ 5 \\ 6\overline{)7.{}^{1}1{}^{5}1{}^{3}0} \end{array}$$

Reasoning

The division is done in the same way as a division with whole numbers.
Line up the decimal point in 23.16 and in the quotient, 5.79.

Line up the decimal point in 7.11 and in the quotient, 1.185.
A zero is placed in the thousandths place of 7.11 so that the remainder, 3 hundredths becomes 30 thousandths.
The division can then be completed to give 5 thousandths in the quotient.

Often the whole number does not divide exactly into the decimal number. We saw in section 3.8 that fractions with denominators of 3, 6, 7 and 9 gave recurring decimals when the numerator of the fraction was divided by the denominator. When the division of a decimal number by a whole number gives a recurring decimal we can either round the quotient to a required number of decimal places or write it with a bar over the recurring digits.

Example 28

Evaluate 13.345 ÷ 7 correct to 2 decimal places.

Working

$$7\overline{)13.{}^{6}3{}^{0}4{}^{4}5\ldots}$$ quotient: 1. 9 0 6…

Correct to 2 decimal places, 13.345 ÷ 7 is 1.91.

Reasoning

Line up the decimal point in 13.345 and in the quotient, 1.906….
We need the quotient correct to 2 decimal places so we need only go as far as 3 decimal places in the quotient.
1.906 is closer to 1.91 than to 1.90.

Division of a decimal number by another decimal number

If we use the example of $3 \times 0.4 = 1.2$ again, we can rewrite this as the division

$1.2 \div 0.4 = 3$. In words we are saying: 'How many 4 tenths in 12 tenths? There are 3.'

To avoid dividing by a decimal number we can write the division as $\frac{1.2}{0.4}$ then multiply by $\frac{10}{10}$.

We know that $\frac{10}{10} = 1$ and that multiplying a number by 1 does not change its size.

$$\begin{aligned}\frac{1.2}{0.4} &= \frac{1.2}{0.4} \times \frac{10}{10}\\ &= \frac{12}{4}\\ &= 3\end{aligned}$$

Example 29

Evaluate the following.

a 34.5 ÷ 0.8

b 94.5 ÷ 0.06

continued

Example 29 continued

Working

a $34.5 \div 0.8 = \dfrac{35.4}{0.8}$

$= \dfrac{34.5}{0.8} \times \dfrac{10}{10}$

$= \dfrac{345}{8}$

$= 43.125$

$$\begin{array}{r} 4\,3.\,1\,2\,5 \\ 8\overline{)34^{2}5.^{1}0^{2}0^{4}0} \end{array}$$

Reasoning

Write the division as $\dfrac{34.5}{0.8}$

Multiply by $\dfrac{10}{10}$ to make the divisor a whole number.

Divide 345 by 8 by short division.

Put zeros in the tenths, hundredths and thousandths places of 354. to complete the division.

Working

b $94.5 \div 0.06 = \dfrac{94.5}{0.06}$

$= \dfrac{94.5}{0.06} \times \dfrac{100}{100}$

$= \dfrac{9450}{6}$

$= 1575$

$$\begin{array}{r} 1\,5\,7\,5 \\ 6\overline{)9^{3}4^{4}5^{3}0} \end{array}$$

Reasoning

Write the division as $\dfrac{94.5}{0.06}$

Multiply by $\dfrac{100}{100}$ to make the divisor a whole number.

Divide 9450 by 6 by short division.

Example 30

Calculate the following.

a $45.6 \div 0.7$ correct to 2 decimal places **b** $28.56 \div 0.09$

Working

a $45.6 \div 0.7 = \dfrac{45.6}{0.7}$

$= \dfrac{45.6}{0.7} \times \dfrac{10}{10}$

$= \dfrac{456}{7}$

$= 65.142\ldots$

$$\begin{array}{r} 6\,5.\,1\,4\,2\ldots \\ 7\overline{)45^{3}6.^{1}0^{3}0^{2}0\ldots} \end{array}$$

Correct to two decimal places, $45.6 \div 0.7$ is 65.14.

Reasoning

Write the division as $\dfrac{45.6}{0.7}$

Multiply by $\dfrac{10}{10}$ to make the divisor a whole number.

Divide 456 by 7 by short division.

Two decimal places are required so calculate to 3 decimal places.

Put zeros in the tenths, hundredths and thousandths places. to continue the division.

continued

Example 30 continued

Working

b $28.56 \div 0.09 = \dfrac{28.56}{0.09}$

$= \dfrac{28.56}{0.09} \times \dfrac{100}{100}$

$= \dfrac{2856}{9}$

$$9\overline{)28^{1}5^{6}6.^{3}0^{3}0^{3}0\ldots}$$
quotient: $3\,1\,7.\,3\,3\,3\ldots$

$= 317.333\ldots$

$= 317.\overline{3}$

Reasoning

Write the division as $\dfrac{28.56}{0.09}$

Multiply by $\dfrac{100}{100}$ to make the divisor a whole number.

Divide 2856 by 9 by short division.

Put zeros in the tenths, hundredths and thousandths places of 2856. to continue the division.

The quotient is a recurring decimal.

Put a dot or line over 3 in the tenths place to show that the digit 3 is recurring.

Tech tip

To divide a decimal by a decimal on the TI-30XB MultiView calculator, for example, 28.56 ÷ 0.09, type:

[2] [8] [·] [5] [6] [÷] [0] [·] [0] [9] [enter].

Value for money

Many supermarket items are sold in more than one size of container. If one container has exactly twice the amount of another, it is easy to compare prices. For example, if 250 g of cheese costs \$2.88 and 500 g of the same cheese costs \$5.38, then it is easy to see that the 500 g packet is better value because two 250 g packets would cost \$5.76. Often, it is not so easy to compare prices.

To compare prices when it is not easy to see which is better value, we find the cost per unit. For example, cost per gram, cost per litre, cost per millilitre.

From 1 December 2009, it became compulsory for supermarkets to display a unit price on labels and in advertising where a selling price is displayed. This makes it easy for shoppers to compare the price and value of similar types of items. Here is an example of how price labels on milk have now been labelled.

Full cream milk 600 mL	\$1.80 per L	\$1.08
Full cream milk 1 L	\$1.37 per L	\$1.37
Full cream milk 2 L	\$1.24 per L	\$2.47
Full cream milk 3 L	\$1.22 per L	\$3.66

Example 31

Calculate the better buy in each of these cases.

a a 450 g packet of Coco Pops costs \$5.66 and a 735 g packets costs \$8.18. Compare the costs in cents/g.

b \$4.34 for 300 g of rice bubbles or \$5.44 for 490 g of rice bubbles. Compare the price per 100 g. Compare the costs in \$/100 g.

Working	Reasoning
a \$5.66 for 450 g	Convert the cost to cents and divide by the number of grams to find the cost per gram.
$\text{Unit price} = \dfrac{566\text{ cents}}{450\text{ g}}$	
$\approx 1.26\text{ cents/g}$	
\$8.18 for 735 g	
$\text{Unit price} = \dfrac{818\text{ cents}}{735\text{ g}}$	
$\approx 1.11\text{ cents/g}$	
So the 735 g packet is better value because the cost per gram is less.	1.11 cents per gram is cheaper than 1.26 cents per gram.
b \$4.34 for 300 g	Divide the cost for 300 g by 3 to find the cost for 100 g.
$= \dfrac{\$4.34}{3}\text{/100 g}$	
$\approx \$1.45\text{ /100 g}$	
\$5.44 for 490 g	
$= \dfrac{\$5.44}{490}\text{/g}$	Divide the cost for 490 g by 490 to find the cost for 1 g.
$\approx \$0.0111/g$	
$= \$1.11\text{ /100 g}$	Multiply by 100 to find the cost per 100 g.
	\$1.11 /100 g is cheaper than \$1.45/100 g.

exercise 4.9

LINKS TO Example 27a

Calculate the following.

a 22.8   3 **b** 13.5   5 **c** 1.92   8 **d** 231.2   4

e 316.92   4 **f** 0.171   3 **g** 320.16   12 **h** 12.045   11

i 87.6 ÷ 6 **j** 392.4 ÷ 9 **k** 2.667 ÷ 7 **l** 0.2152 ÷ 4

m 9.18 ÷ 3 **n** 10.35 ÷ 5 **o** 33.648 ÷ 8 **p** 0.5621 ÷ 7

LINKS TO Example 27b

Calculate the following.

a 23.1 2 **b** 41.3 5 **c** 1.72 8 **d** 25.5 6
e 31.34 4 **f** 2.007 6 **g** 306.4 5 **h** 0.161 8
i 17.13 ÷ 5 **j** 7.134 ÷ 4 **k** 424.2 ÷ 6 **l** 0.363 ÷ 2
m 5.691 ÷ 6 **n** 0.0124 ÷ 8 **o** 9.006 ÷ 4 **p** 0.0526 ÷ 5

LINKS TO Example 28

Calculate the following. Give your answers correct to 2 decimal places.

a 46.7 3 **b** 30.8 7 **c** 5.36 6 **d** 21.5 9
e 31.37 8 **f** 2.05 6 **g** 428.9 12 **h** 0.351 11
i 3.521 ÷ 7 **j** 0.2303 ÷ 3 **k** 18.22 ÷ 6 **l** 1.406 ÷ 9
m 4.243 ÷ 6 **n** 561.7 ÷ 8 **o** 34.21 ÷ 3 **p** 0.0823 ÷ 7

LINKS TO Example 29

Evaluate.

a 3.16 ÷ 0.8 **b** 4.11 ÷ 0.3 **c** 8.106 ÷ 0.6 **d** 31.5 ÷ 0.9
e 54.1 ÷ 0.2 **f** 5.7 ÷ 0.8 **g** 23.1 ÷ 0.4 **h** 5.23 ÷ 0.5
i 4.752 ÷ 1.1 **j** 6.84 ÷ 0.5 **k** 90.06 ÷ 0.2 **l** 3.12 ÷ 0.4
m 8.64 ÷ 0.6 **n** 24.3 ÷ 0.9 **o** 0.126 ÷ 0.3 **p** 7.92 ÷ 1.1

LINKS TO Example 30

Evaluate.

a 5.4266 ÷ 0.02 **b** 0.04707 ÷ 0.03 **c** 22.5 ÷ 0.09 **d** 0.423 ÷ 0.06
e 1.308 ÷ 0.04 **f** 2.106 ÷ 0.12 **g** 0.2814 ÷ 0.07 **h** 2.431 ÷ 0.05
i 0.7344 ÷ 0.12 **j** 5.02 ÷ 0.11 **k** 6.106 ÷ 0.06 **l** 6.231 ÷ 0.08
m 6.13 ÷ 0.003 **n** 0.4101 ÷ 0.002 **o** 0.0613 ÷ 0.005 **p** 0.2936 ÷ 0.012

Which one of the divisions below can be simplified to give 5.41 ÷ 3?

A 5.41 ÷ 0.03 **B** 0.541 ÷ 0.3 **C** 0.0541 ÷ 0.3 **D** 54.1 ÷ 0.03

If a box of four ice-creams costs $6.60, how much does each ice-cream cost?

Sarah wants to split the bill for a meal equally between herself and her three friends. If the bill is $57.60, how much should each pay?

Nick is installing the hot water pipes in a new house. He has a length of copper pipe that is 4.6 m. He needs to cut it into three equal lengths. How long must each piece be correct to 2 decimal places?

LINKS TO Example 31

At the market, a 5 kg bag of oranges was sold for $7.45. What was the price per kilogram?

Which is better value for mixed lollies: $14.30 for 3 kg or $23.75 for 5 kg?

For each of the following, calculate the cost in cents per unit as shown in the last column of the table, and then decide which is the better buy. In each case, the two different sizes are the same type and brand.

Ice-cream	$5.94 for 1.25 L	$7.62 for 2 L	cents/L
Pineapple juice	$2.45 for 850 mL (0.85 L)	$3.80 for 2 L	cents/L
Cornflakes	$3.04 for 500 g	$4.57 for 800 g	$/100 g
Pasta sauce	$4.46 for 785 g	$3.48 for 580 g	$/100 g

- Compare the cost in cents per litre for each of these different sizes of chocolate flavoured milk.

  a \$4.99 for 2 L **b** \$2.92 for 600 mL (0.6 L)
  c \$1.69 for 300 mL (0.3 L)

- At a farmers' market, one stall sells five watermelons for \$12 and another sells a dozen watermelons for \$30. Which is the better buy?
- As part of a science experiment, the amount of food eaten by a pet mouse in a week was found to be 36.76 g. On average, how many grams was this per day? Give your answer correct to 2 decimal places.
- A hamburger contains 0.2 kg of meat. How many hamburgers could be made from 1.6 kg of meat?
- An ice-cream costs \$1.10. How many of them could you buy with \$50.00?
- To make a part of the costume for the Rock Eisteddfod, each person needs 0.9 m of red fabric. There are 26.01 m of fabric left on the roll. How many lengths of 0.9 m can be cut from it?

exercise 4.9 challenge

- Petrol costs about 134 cents or \$1.34 per litre.

  a How many litres will I get for \$30? Give your answer correct to two decimal places.
  b A small car in the city uses about 1 L to drive 10 km. How far can I travel for \$50 of petrol? Give your answer correct to the nearest kilometre.
  c I need to travel 84 km. Will I have enough petrol if I have \$10 of petrol?
  d If the answer is 'yes' how much further could I travel with the petrol I have? If the answer is 'no', how many more litres of petrol do I need?

Analysis task

FM radio

This table shows the frequencies of 18 Adelaide FM radio stations in megahertz.

ABC Classic FM	**103.9 MHz**
Adelaide Greek Radio	**87.6 MHz**
Coast FM	**88.7 MHz**
EBI	**103.1 MHz**
Fresh FM	**92.7 MHz**
MBS	**99.9 MHz**
MIX 102.3 FM	**102.3 MHz**
Nova 91.9	**91.9 MHz**
PBA-FM	**89.7 MHz**
Power FM	**100.3 MHz**
Radio Adelaide	**101.5 MHz**
SAFM	**107.1 MHz**
SBS Radio	**106.3 MHz**
Three D Radio	**93.7 MHz**
Triple B FM	**89.1 MHz**
Triple J	**105.5 MHz**
Triple M FM	**104.7 MHz**
WOW FM	**100.5 MHz**

a While it is useful to have the stations in alphabetical order to look up programs, to tune into a station it is more useful to have them in frequency order. List the 15 stations in increasing order of frequency.

b On a number line from 80 to 110 MHz, show where all of the stations would be.

c How many MHz are there between the highest and the lowest frequency station?

d Which two stations have the closest frequencies according to this list?

e What is the most common difference between frequencies next to each other?

f Find out the frequencies of FM radio in your region and sort the radio stations in frequency order.

Challenge

g Find out what 'bandwidth' is. How does it relate to the earlier parts of this question?

Review Decimals

Summary

Rounding and ordering

- The number of decimal places is the number of digits to the right of the decimal point.
- To order decimals, compare the whole number part, then the tenths, hundredths, thousandths, etc.
- To round to a given number of decimal places, look at the next decimal place. If it is less than halfway, round down. Otherwise, round up.

Converting between fractions and decimals

- To change a decimal to a fraction, first write any whole numbers, then put the digits to the right of the decimal point over a power of 10. The number of zeros in the denominator should be the same as the number of decimal places in the starting number. If it is possible, simplify the fraction.
- To change a fraction to a decimal, first write any whole numbers. If the fractional part has a denominator such as 2, 4, 5 or 8, convert the fraction to an equivalent fraction with a power of 10 (10, 100, 1000, etc.) in the denominator then write as a decimal. If the fraction cannot be changed like this, divide the numerator by the denominator, adding zeros as needed.

Addition and subtraction

- To add or subtract decimals, line up the digits under each other in place value columns then add or subtract as usual. For subtraction it is helpful to put zeros so that the two numbers have the same number of decimal places.

Multiplication and division

- To multiply a decimal by a power of 10 (10, 100, 1000, etc.), move the decimal point to the right to make a larger number. The number of places to move the point is the same as the number of zeros in the power of 10.
- To divide a decimal by a power of 10 (10, 100, 1000, etc.), move the decimal point to the left to make a smaller number. The number of places to move the point is the same as the number of zeros in the power of 10.
- To multiply decimals, rewrite as fractions with a power of 10 in the denominator. Multiply the numerators (using short or long multiplication) and multiply the denominators. The power of 10 in the denominator shows where to place the decimal point.
- To divide 1 decimal number by another, rewrite the two numbers over each other. Multiply by $\frac{10}{10}$, $\frac{100}{100}$, etc. (depending on the number of decimal places in the denominator) to convert the denominator to a whole number. Divide as for whole numbers, being careful to keep the digits of the quotient in correct place value columns.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

ascending order	difference	numerator	quotient
decimal	digit	place value column	recurring digit
denominator	divisor	power of 10	remainder
descending order	equivalent fraction	product	sum

Revision

Multiple-choice questions

$3 + \frac{4}{10} + \frac{7}{1000} + \frac{2}{10\,000}$ as a decimal is written as

A 0.34072 **B** 0.3472 **C** 3.4072 **D** 3.4702 **E** 3.472

5.038 in expanded notation is

A $5 + \frac{3}{100} + \frac{8}{1000}$ **B** $5 + \frac{3}{10} + \frac{8}{1000}$ **C** $5 + \frac{3}{10} + \frac{8}{100}$

D $\frac{5}{10} + \frac{3}{100} + \frac{8}{1000}$ **E** $5 + \frac{3}{10} + \frac{8}{10}$

The largest of the numbers below is

A 0.51 **B** 0.651 **C** 0.156 **D** 0.5 **E** 0.16

The smallest of the numbers below is

A 0.51 **B** 0.651 **C** 0.156 **D** 0.5 **E** 0.16

6.9714 rounded to 1 decimal place is

A 6.0 **B** 7.0 **C** 6.9 **D** 6 **E** 7

Short-answer questions

Evaluate 2.43 + 51.62 + 124.77

Evaluate.

a 534.2 − 52.12 **b** 36.5 − 18.147

Evaluate 62.53   4

Evaluate.

a 513.24   10 **b** 86.6   1000 **c** 274.31   10 **d** 7.846   1000

Evaluate 42.1   60

Evaluate 0.07   0.011

- Evaluate 223.56 3.2
- Evaluate 27.2 4
- Evaluate 7.313 0.5
- Write as fractions in simplest form.
  a 0.8 **b** 0.31 **c** 2.05 **d** 7.347
- Write these recurring decimals using dots or lines to show repeating digits.
  a 0.7777777777… **b** 0.5333333333… **c** 0.1818181818… **d** 0.3214214214…
- Change these fractions to decimals.
  a $\frac{5}{8}$ **b** $\frac{6}{11}$ **c** $1\frac{13}{20}$ **d** $3\frac{1}{4}$

Extended-response questions

- Ally decides to install three computer games onto her hard disk drive. They are listed in the table on the right.

Dragon Flyers	8.3 megabytes
Magic Arts	12.7 megabytes
Wizard Wonders	6.8 megabytes

  a How many megabytes of memory in total are needed for the three games?
  b Ally has 20.6 megabytes available on the hard disk drive. How many more megabytes are needed in order to install the three games?
  c Ally decides that only two of the games can be installed. Which two would be possible?

- Compare the cost in \$/100 g for each of these different sizes of Vegemite.
  a \$8.93 for 600 g **b** \$6.51 for 400 g
  c \$3.66 for 220 g **d** \$3.26 for 150 g

- Sam has measured that four of his paces total 2.4 m.
  a How long is Sam's pace in metres?
  b Sam counted his paces on the way to buy a carton of milk. He took 426 paces to get there. How far did he walk there and back?
  c Sam knows that he walks 0.4 km to the bus stop each day. How far is that in metres? How many paces would this be for him?

Practice quiz
Chapter 4

5 Algebra

Pre-test

Warm-up

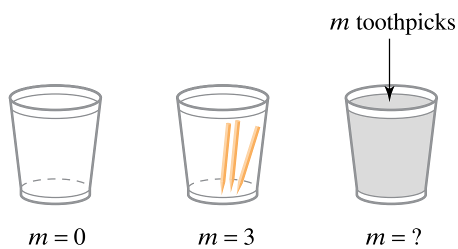
There are many numbers in our everyday lives that vary. In an Australian rules football match, for example, the number of goals and the number of behinds for each team changes during the match. In algebra we let letters stand for numbers that vary. The letters are called *pronumerals*, which means they stand for a number. We can choose any letter we like to stand for a number. We could let g be the number of goals and b be the number of behinds. In the scoreboard above, for the Swans b has the value 1 and g has the value 4. For the Bulldogs, b has the value 1 and g has the value 2.

5.1 Introduction to variables

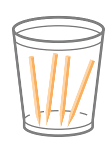
A teacher has some toothpicks and she uses cups as holders for the toothpicks.

She chooses the letter m to represent the number of toothpicks in the cup.

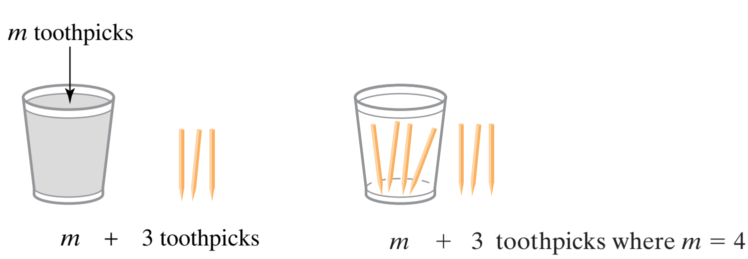
The letter m is a pronumeral. When a letter stands for a number it is called a **pronumeral**. The word 'pronumeral' means 'for number'. If there are no toothpicks in the cup, $m = 0$, as shown below. When there are three toothpicks in the cup, $m = 3$, as shown in the diagram below. In the third example below, we cannot see how many toothpicks are in the cup, so m is an unknown number.



Each student in the class is given his or her own value to use for m. The value of m is the same as the number of letters in their name. The teacher gives Nina the value 4, so for Nina $m = 4$ and she puts 4 toothpicks in her cup.



The teacher then asks each student to place 3 more toothpicks *beside* their cup. Each student now has $m + 3$ toothpicks, where the value of m is the number of letters in their name. For Nina $m = 4$, so altogether she has 7 toothpicks.



The pronumeral m is also called a **variable** in these examples because it can vary. This means that it can have different values.

Example 1

In the same classroom activity, Harry and Silvana are using toothpicks to represent the expression $m + 3$.

a Harry has the value $m = 5$. What will be the value of the expression $m + 3$?

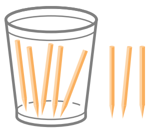
b Silvana has the value $m = 7$. What will be the value of the expression $m + 3$?

Working

a Harry
When $m = 5$
$m + 3$
$= 5 + 3$
$= 8$

Reasoning

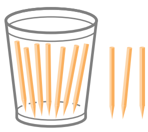
There are 5 toothpicks in the cup when $m = 5$.
The diagram shows $m + 3$ where $m = 5$.



Altogether there are 8 toothpicks.

b Silvana
When $m = 7$
$m + 3$
$= 7 + 3$
$= 10$

There are 7 toothpicks in the cup when $m = 7$.
The diagram shows $m + 3$ where $m = 7$.

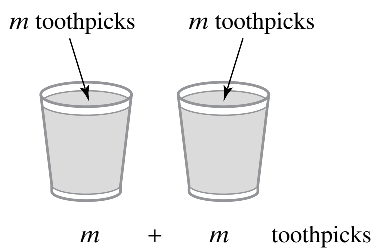


Altogether there are 10 toothpicks.

Writing multiplications in algebra

If there are 2 lots of m toothpicks, we can write this as $m + m$.

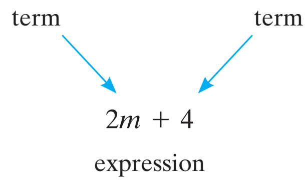
However, we can also write this as $2 \times m$. We know this is true because if Harry had 2 lots of 5 toothpicks, he would have $5 + 5$ toothpicks, but this is the same as 2×5.



In algebra we do not put in the multiplication sign between a number and a pronumeral. We write $2 \times m$ as $2m$. So $m + m = 2m$ and we know that this means $2 \times m$. The number that a pronumeral is multiplied by is called the **coefficient**. When we write $2m$, the coefficient of m is 2.

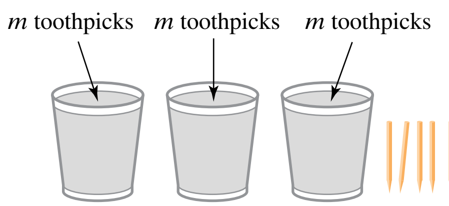
Expressions and terms

$2m + 4$ and $m + 3$ and are called **expressions**. Each part of the expression is called a **term**. So $2m$ is a term, 4 is a term, m is a term and 3 is a term.



Example 2

The diagram shows m toothpicks in each of 3 cups, with 5 extra toothpicks.



a Write an expression for the total number of toothpicks.

b What is the total number of toothpicks if $m = 4$?

Working

a $m + m + m + 5$
$= 3m + 5$

b $3 \times 4 + 5$
$= 12 + 5$
$= 17$
If $m = 4$, there are 17 toothpicks.

Reasoning

$m + m + m = 3 \times m$
In algebra we write $3m$ instead of $3 \times m$.

$3m$ means $3 \times m$.
We know that $4 + 4 + 4 = 3 \times 4$.
We do multiplication before addition.

Example 3

Tom and Nina are using toothpicks to represent the expression $2m + 4$.

a Tom has the value $m = 3$. What will be the value of the expression $2m + 4$?

b Nina has the value $m = 4$. What will be the value of the expression $2m + 4$?

continued

Example 3 continued

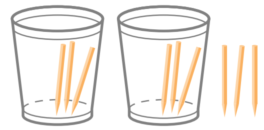
Working

a Tom
$2m + 4$
$= 2 \quad m + 4$
When $m = 3$
$= 2 \quad 3 + 4$
$= 10$

or
$2m + 4$
$= m + m + 4$
When $m = 3$
$= 3 + 3 + 4$
$= 10$

Reasoning

$m = 3$ means there are 3 toothpicks in each cup.



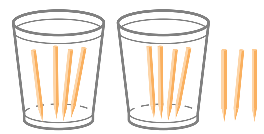
Working

b Nina
$2m + 4$
$= 2 \quad m + 4$
When $m = 4$
$= 2 \quad 4 + 4$
$= 12$

or
$2m + 4$
$= m + m + 4$
When $m = 4$
$= 4 + 4 + 4$
$= 12$

Reasoning

$m = 4$ means there are 4 toothpicks in each cup.



We could use any letter instead of m to represent the number of toothpicks.

The letter used for the variable is not important here, the important thing is to see that m can take a range of values, that is, m is a variable.

Example 4

The teacher puts m toothpicks in each of three cups. The total number of toothpicks is $m + m + m$. A shorter way to write this is $3m$.

Draw a separate diagram to represent

a $m = 2$. **b** $m = 4$.

For each case, work out the total number of toothpicks.

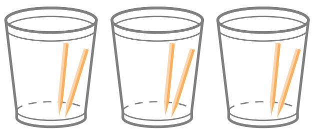
continued

Example 4 continued

Working

Reasoning

a

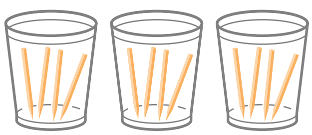


3*m* means 3 lots of *m* toothpicks.

$3m$
$= 3 \quad m$ (or $m + m + m$)
When $m = 2$
$= 3 \quad 2$ (or $2 + 2 + 2$)
$= 6$

When $m = 2$ there are 3 lots of 2 toothpicks.

b



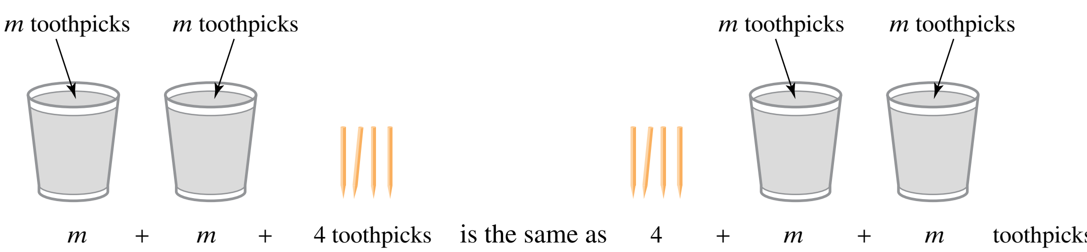
Again, 3*m* means 3 lots of *m* toothpicks.

$3m$
$= 3 \quad m$ (or $m + m + m$)
When $m = 4$
$= 3 \quad 4$ (or $4 + 4 + 4$)
$= 12$

When $m = 4$ there are 3 lots of 4 toothpicks.

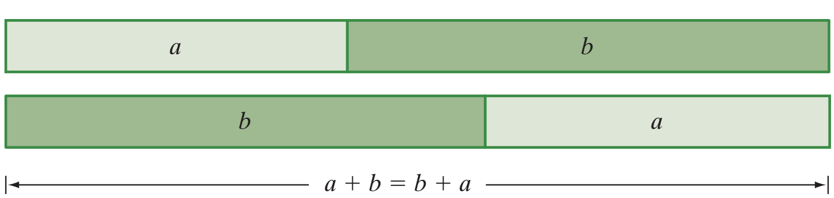
Order of terms

Notice that $2m + 4$ means the same as $4 + 2m$.



We know from our work with numbers that $4 + 7$ is equal to $7 + 4$.

Using algebra, we can write this any order rule for addition as: $a + b = b + a$ where a and b are two pronumerals that stand for numbers.



Just as for numbers, we saw in Chapter 1 that the any order rule also applies to multiplication. Using algebra, we can write $a \times b = b \times a$, or more simply as $ab = ba$.

When we write $2m$ as a short way of writing $2 \times m$, we could also write $m \times 2$. However, in algebra it is normal to write the coefficient first then the pronumeral, so we write $2m$ rather than $m2$.

Writing divisions in algebra

In Chapter 1 we wrote divisions in two different ways, as $12 \div 3$ and $\frac{12}{3}$. Divisions in algebra are written like the second of these ways. When m is divided by 4, for example, we write $\frac{m}{4}$. This means $m \div 4$.

Example 5

Translate these phrases into symbols. Use n to represent the certain number. Then find the value of each when $n = 6$.

a a certain number plus 4

b 3 times a certain number

c 2 less than a certain number

d the sum of twice a certain number and 11

e the product of a certain number and 10

f the quotient when a certain number is divided by 3

Working	**Reasoning**
a A certain number plus 4 is written as $n + 4$. When $n = 6$ $n + 4 = 6 + 4$ $= 10$	Replace 'a certain number' by n. Replace 'plus' by +.
b Three times a certain number is written as $3n$. When $n = 6$ $3n = 3 \quad 6$ $= 18$	Replace 'a certain number' by n. Replace 'times' by or just write $3n$.
c Two less than a certain number is written as $n - 2$. When $n = 6$ $n - 2 = 6 - 2$ $= 4$	Replace 'a certain number' by n. Replace 'two less than' by –2. Be careful. This is not the same as $2 - n$.

continued

Example 5 continued

Working

d The sum of twice a certain number and 11 is written as $2n + 11$.

When $n = 6$

$2n + 11 = 2 \quad 6 + 11$

$= 23$

Reasoning

Replace 'a certain number' by n.
Replace 'sum of' by $+$.
Replace 'twice' by 2 or just 2 at the front of the n.

Working

e The product of a certain number and 10 is written as $10n$.

When $n = 6$

$10n = 10 \quad 6$

$= 60$

Reasoning

Replace 'a certain number' by n.
Replace 'product of' by or just write $10n$.

Working

f When a certain number is divided by 3, the quotient is written as $\frac{n}{3}$.

When $n = 6$

$\frac{n}{3} = \frac{6}{3}$

$= 2$

Reasoning

Replace 'a certain number' by n.
Replace 'quotient of' by $\div$ or write $\frac{n}{3}$.

Replacing a pronumeral, or letter, in an expression with a number is called **substitution**.

Example 6

Substitute $m = 4$ in each of these expressions.

a $2m + 1$

b $5m - 2$

Working

a When $m = 4$,

$2m + 1 = 2 \times 4 + 1$

$= 8 + 1$

$= 9$

Reasoning

Put 4 in the place of m.
Use the correct order of operations to calculate the value.

Working

b When $m = 4$,

$5m - 2 = 5 \times 4 - 2$

$= 20 - 2$

$= 18$

Reasoning

Put 4 in the place of m.
Use the correct order of operations to calculate the value.

We have seen how numbers of toothpicks can be used to model algebraic expressions. We will now summarise the new words that have been used in this section.

Word	Meaning	Examples
Pronumeral	A letter that stands for a number	m or x or h
Variable	A pronumeral that can have a variety of values	m could be 1 or 4 or 5 or 0
Term	It may be a number, a single pronumeral or a pronumeral multiplied by a number.	$3m$ or 4 or $6x$ or $5h$ or 6 or h or 1
Coefficient	The number that a pronumeral is multiplied by	In the term $3m$, the coefficient of m is 3. In the term h, the coefficient of h is 1 because h is the same as h multiplied by 1.
Expression	Is made up of terms	$3m + 4$ or $6x$ or $5h + 6 + h + 1$
Substitute	To replace a pronumeral with a particular number	In the expression $3m + 4$ we could substitute 5 in place of m, so we would have $3 \times 5 + 4$.

Example 7

For the expression $3n + 5 + 7n + 6 + n$, write

a the number of terms and list them.

b the coefficient of the first term.

c the coefficient of the fifth term.

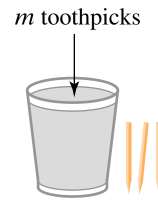
d the pronumeral in the expression.

Working	Reasoning
a There are five terms: $3n$, 5, $7n$, 6 and n.	A term is a number, a single pronumeral or a pronumeral multiplied by a number.
b The coefficient of the first term is 3.	The first term is $3n$. The pronumeral n is multiplied by 3.
c The coefficient of the fifth term is 1.	The fifth term is n. The pronumeral n is multiplied by 1.
d The pronumeral in the expression is n.	n is a letter that stands for a number in the expression.

exercise 5.1

LINKS TO Example 1

1 Each of the students below has m toothpicks in a cup (where m is the number of letters in their name) and 3 extra toothpicks beside the cup. For each student, draw a diagram and write the total number of toothpicks the student has.

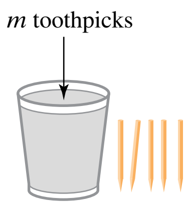


- **a** Yamal
- **b** Alexander
- **c** Jo
- **d** Giovannetta
- **e** Mae
- **f** Bartholomew
- **g** Jack
- **h** Jessica

LINKS TO Example 2

2 The diagrams show the number of toothpicks in each cup and the number of extra toothpicks beside the cups. For each diagram write an expression to show the total number of toothpicks. Check that you have written your expression in the shortest way.

a



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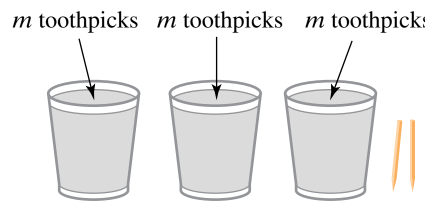
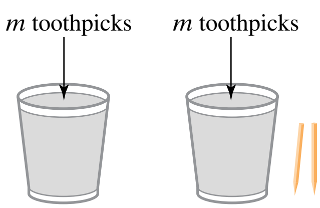
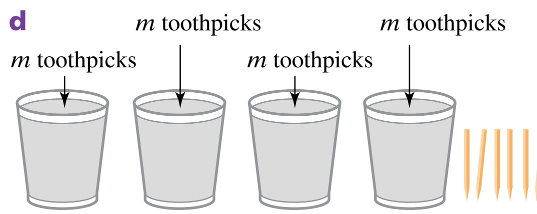


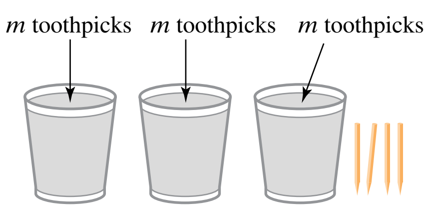
c



d



e



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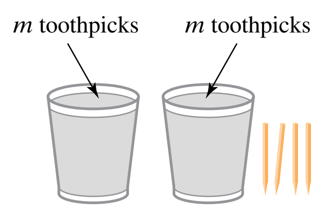
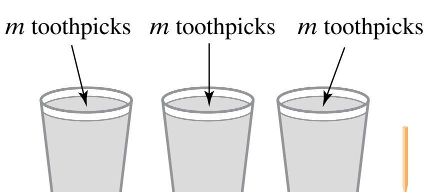
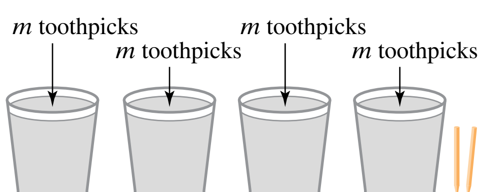


g



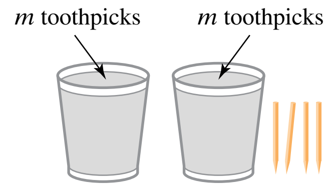
h



LINKS TO Example 3

Each of the following students has $2m + 4$ toothpicks (where m is the number of letters in their name). Find the total number of toothpicks each student will have.

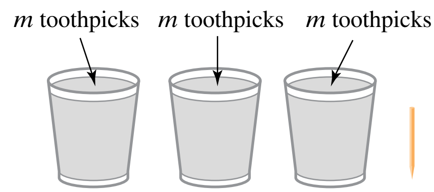
a Yamal **b** Alexander
c Jo **d** Giovannetta
e Mae **f** Bartholomew
g Jack **h** Jessica



LINKS TO Example 3

Each of these students has $3m + 1$ toothpicks (where m is the number of letters in their name). Find the total number of toothpicks each student will have.

a Samantha **b** Ben
c William **d** Rani
e Anastasia **f** Li
g Thomas **h** Jorja



Find the value of the expression $3m + 2$ for each of these values of m.

a $m = 2$ **b** $m = 5$ **c** $m = 11$ **d** $m = 7$
e $m = 20$ **f** $m = 15$ **g** $m = 32$ **h** $m = 25$

LINKS TO Example 4

Consider $5m + 2$ toothpicks.

a Draw a diagram.
b Work out the total number of toothpicks for each of the cases.
i $m = 1$ **ii** $m = 3$ **iii** $m = 4$

Draw a diagram for the following.

a $5m + 2$ toothpicks **b** $5m + 2$ toothpicks when $m = 3$
c $5m + 2$ toothpicks when $m = 7$

Consider $6 + m$ toothpicks.

a Draw a diagram.
b Work out the total number of toothpicks for each of the following cases.
i $m = 2$ **ii** $m = 3$ **iii** $m = 4$

Let k represent the number of sweets in a paper bag.

a Draw a diagram to represent $2k + 1$ sweets.
b Work out the total number of sweets for each of the following values of k.
i 2 **ii** 3 **iii** 4

h stands for the number of eggs in a bowl.

a Draw a diagram for $4h + 2$ eggs.
b Find the total number of eggs when h has the following values.
i 12 **ii** 9 **iii** 3

LINKS TO Example 5

For a certain number n, write down an algebraic expression for each of the following.

a 5 more than a certain number

b 16 less than a certain number

c 5 times a certain number

d the quotient of a certain number and 9

e 14 minus 3 times a certain number

f 2 more than the product of a certain number and 4

LINKS TO Example 5

Write each of these expressions in words.

a $3p$ **b** $x + 7$ **c** $5 - t$ **d** $m - 8$ **e** $2d - 6$ **f** $6 + \frac{r}{2}$

LINKS TO Example 6

Substitute $b = 5$ in each of these expressions.

a $4b$ **b** $3b + 7$ **c** $2b - 1$

d $b + 11$ **e** $2b + 6$ **f** $10b - 2$

LINKS TO Example 6

Substitute $h = 3$ in each of these expressions.

a $5h - 4$ **b** $\frac{2h}{3}$ **c** $6h + 5$

d $h + 8$ **e** $3h - 1$ **f** $4h + 3$

LINKS TO Example 7

For each of these expressions, state

i the pronumeral.

ii the number of terms and list the terms.

iii the coefficient of the second term.

a $3m + 4m + 11$ **b** $5 + x + 11 + 3x$

c $7k + 2k + 5 + 4k + 8$ **d** $6 + b + 9$

e $1 + 3y + 4y$ **f** $n + n + 4n + 7 + 6n + 11 + 3$

g $5 + 2h + 4h + 7 + 6h + h$ **h** $a + 5a + 2 + 3a + 6 + 5a + 3$

Demetrius places x apples in a basket and has one extra apple in his hand to eat. The total number of apples is given by $x + 1$. Find the total number of apples for each value of x.

a 5 **b** 7 **c** 14

Mai has some lollies. She places m lollies into each of 3 bags. She then has 2 lollies left over. The expression $3m + 2$ gives the total number of lollies. Find the total number of lollies if

a $m = 4$ **b** $m = 2$ **c** $m = 10$ **d** $m = 12$

Rohith is planting some tomato seedlings in his garden. He plants 10 rows, each containing t tomatoes and he has 4 seedlings left over.

a Write down an expression for the total number of seedlings.

b Find the total number of seedlings when

i $t = 5$ **ii** $t = 6$ **iii** $t = 8$

- Chen has a bag containing p marbles. He removes one marble from the bag. The number of marbles remaining in the bag is given by the expression $p - 1$. Find the number of marbles remaining in the bag if

  a $p = 4$ **b** $p = 7$ **c** $p = 12$

- At the start of the school year, Padma has 6 new folders, each containing s sheets of folder paper. In her first Maths class she removes 4 sheets of paper from one of the folders and gives them to a friend.

  a Write down an expression for the total number of sheets of paper that remain.

  b Find the total number of sheets Padma has left if

  i $s = 10$ **ii** $s = 15$ **iii** $s = 20$

exercise 5.1 challenge

- **a** Draw a diagram to show $2(m + 3)$ toothpicks, which means 2 lots of $m + 3$ toothpicks.

  b Find the value of the expression when

  i $m = 4$ **ii** $m = 6$ **iii** $m = 13$

- It is Anna's birthday today. She is b years old. She asks the ages of people at her party. Her cousin Marco is 8 years older than Anna. His age today is $b + 8$ years old.

  a Write the ages of the following using algebra.

  i Anna's sister Julia who is 5 years younger than Anna

  ii Anna's aunt Sofie who is 3 times Anna's age.

  b If Anna is 7 years old today, work out the ages of her relatives, and write their names in order from youngest to oldest.

  c If Anna is 13 years old today, work out the ages of her relatives, and write their names in order from youngest to oldest.

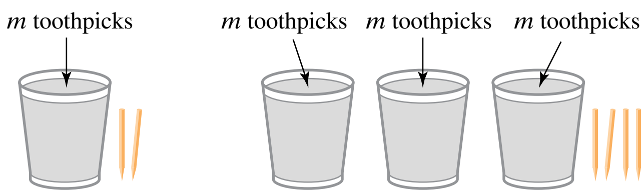
5.2 Combining expressions

Algebraic expressions can be written in different ways. For example, $m + m$ can be written as $2m$ and $m + m + m + m + m$ can be shortened to $5m$. In algebra, we try to write expressions as simply as we can. Sometimes we can combine two or more expressions to make one simpler expression.

We will use the toothpicks model again to show how this is done. Recall from the last section that the cups are simply there as holders for the toothpicks.

Example 8

This diagram represents $m + 2 + m + m + m + 4$ where m is the number of toothpicks in each cup.



a Find a shorter way to write this expression.

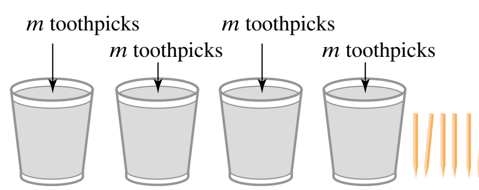
b Show the case where $m = 1$.

Working

a $m + 2 + m + m + m + 4$
$= m + 2 + 3m + 4$
$= m + 3m + 2 + 4$
$= 4m + 6$

Reasoning

We can redraw the diagram with the four lots of m toothpicks (in their cups) together, and the 6 extra toothpicks together.

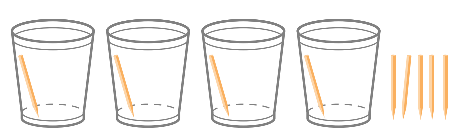


Working

b $m + 2 + m + m + m + 4$
$= 4m + 6$
When $m = 1$
$4m + 6$
$= 4 \quad m + 6$
$= 4 \quad 1 + 6$
$= 4 + 6$
$= 10$

Reasoning

To represent $m = 1$, there should be 1 toothpick in each cup.
There are 4 lots of 1 toothpick plus 6 extra toothpicks.



Sometimes brackets can be used to simplify expressions. An expression such as $m + 1 + m + 1$ can be written as $2(m + 1)$ which means two 'lots' of $m + 1$.

Distributive law algebra

We also know from the distributive law that

$$2(m + 1)$$
$$= 2 \times m + 2 \times 1$$
$$= 2m + 2$$

Example 9

Consider the expression $2(k + 4)$.

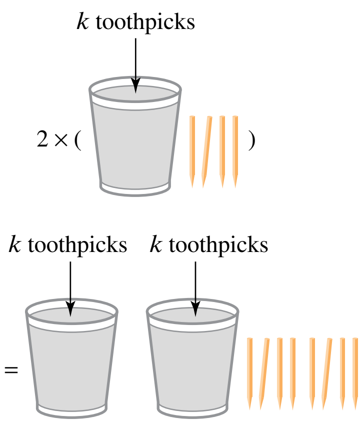
a Represent the expression using a diagram.

b Write $2(k + 4)$ without brackets.

c Find the value for the expression when $k = 3$.

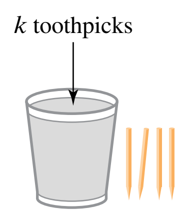
Working

a

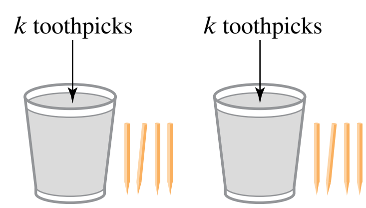


b $2(k + 4) = k + 4 + k + 4$
$= k + k + 4 + 4$
$= 2k + 8$

Reasoning



$k + 4$ toothpicks



2 lots of $(k + 4)$ toothpicks

This gives 2 of everything so you get 2 lots of k and 2 lots of 4.
We also know from the distributive law that
$2(k + 4)$
$= 2 \times k + 2 \times 4$
$= 2k + 8$
The order in which we add terms does not matter so we can rearrange the expression.

continued

Example 9 continued

Working

c When $k = 3$

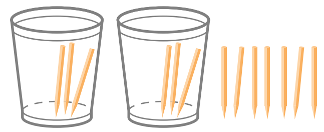
$2k + 8 = 2 \quad 3 + 8$

$= 6 + 8$

$= 14$

Reasoning

If $k = 3$ put 3 toothpicks in each cup.



When evaluating expressions

1. write the missing × signs, for example, $3m = 3 \times m$.
2. replace the pronumeral (or letter) by the value we are given.
3. calculate the value for the expression using the usual order of operations (brackets, divisions and multiplications left to right, and then additions and subtractions left to right).

Class activity
Algebra Grand Prix

Example 10

Find the value of each of the following when the pronumeral has the value given.

a $d - 5$ when $d = 12$

b $2k + 7$ when $k = 4$

c $\frac{y}{2} + 3$ when $y = 8$

Working

a When $d = 12$

$d - 5 = 12 - 5$

$= 7$

Reasoning

Substitute 12 for d.
Calculate the value.

b When $k = 4$

$2k + 7 = 2 \quad k + 7$

$= 2 \quad 4 + 7$

$= 15$

Put in the multiplication sign between 2 and k.
Substitute 4 for k.
Calculate the value. Do before +.

c When $y = 8$

$\frac{y}{2} + 3$

$= \frac{8}{2} + 3$

$= 4 + 3$

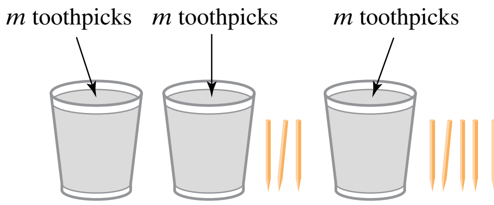
$= 7$

Substitute 8 for y.
Calculate the value. Do ÷ before +.

exercise 5.2

LINKS TO Example 8

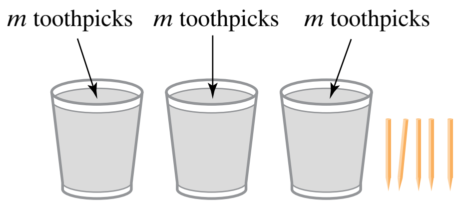
1. This diagram represents $m + m + 3 + m + 5$.



   a Find a shorter way to write this expression.
   b Show the case when $m = 2$.

LINKS TO Example 8

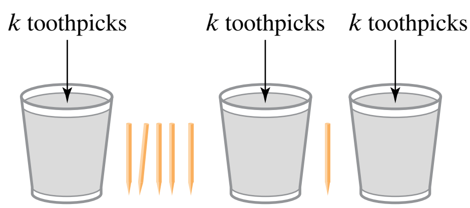
2. The expression $m + m + m + 5$ is shown in this diagram.



   a Find a shorter way to write $m + m + m + 5$.
   b Draw a diagram to represent the case when $m = 7$.

LINKS TO Example 8

3. $k + 2 + 3 + k + 1 + k$ is shown, where k is the number of toothpicks in each cup. Find a shorter way to write this expression.



4. Consider the expression $k + 1 + k + k + 3$.
   a Draw a diagram to represent this expression.
   b Find a shorter way to write the expression.

5. Consider the expression $p + 2 + 1 + p + p$.
   a Draw a diagram to represent this expression.
   b Find a shorter way to write the expression.

6. Consider the expression $3w + 4 + 2w + w$.
   a Draw a diagram to represent this expression.
   b Find a shorter way to write the expression.

Consider the expression $5 + 2x + 3x + 2x$.

a Draw a diagram to represent this expression.

b Find a shorter way to write the expression.

c Find the value of the expression when $x = 5$.

LINKS TO Example 9

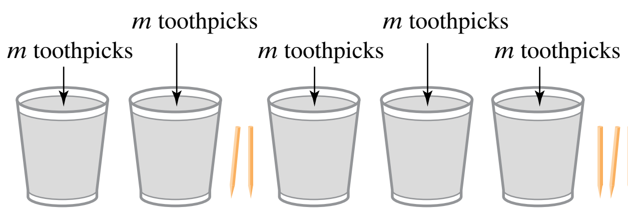
Consider the expression $3(y + 2)$.

a Represent this expression using a diagram.

b Write $3(y + 2)$ without brackets.

c Find the value of the expression when $y = 4$.

The expression $(m + m + 2) + (m + m + m + 3)$ is shown below, where m is the number of toothpicks in each cup.



a Find a shorter way to write $(m + m + 2) + (m + m + m + 3)$.

b Draw a diagram to represent the case when $m = 2$.

Consider the expression $2(4x + 3)$.

a Draw a diagram to represent this expression.

b Find the value of the expression when $x = 5$.

c Find the value of the expression when $x = 0$.

Consider the expression $2(y + 3) + 1$.

a Draw a diagram to represent this expression.

b Write this expression in a shorter way.

c Find the value of the expression when $y = 4$.

Narelle has $k + 9$ Smarties, Anna has $k + 6$ Smarties and Marco has $k + 7$ Smarties. Find an expression for the total number of Smarties that the three friends have, writing this answer in the simplest way possible.

Write each of the following expressions in the shortest way possible.

a $m + m + 5 + m + 2$

b $z + 7 + 2z + 4$

c $2p + 5 + p + 2 + p + p + 7$

d $2(d + 3) + 4$

e $4(2y + 1) + 5y$

LINKS TO Example 10

Find the value of the following expressions using the values of the pronumerals provided.

a $m + 5$ when $m = 4$ **b** $p - 8$ when $p = 12$ **c** $3b + 5$ when $b = 8$

d $5z - 7$ when $z = 3$ **e** $2 + 4r$ when $r = 5$ **f** $8q$ when $q = 0$

g $2(a + 4)$ when $a = 6$ **h** $5(2b - 6)$ when $b = 9$ **i** $3(4k + 2)$ when $k = 3$

j $\frac{m}{4}$ when $m = 28$ **k** $\frac{x}{3} + 5$ when $x = 9$ **l** $\frac{a}{6} - 1$ when $a = 72$

exercise 5.2 challenge

Consider the expression $6m + 8$.

a Write five different expressions which can be simplified to $6m + 8$.

b Check that these expressions mean the same thing by finding the value of each when $m = 3$.

Consider the expression $3m + 2(3 + m)$.

a Draw a diagram to represent this expression.

b Find a shorter way to write this expression.

c What is the value of the expression when $m = 2$?

5.3 Rules and tables

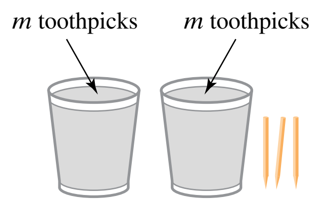
We have seen how

- a pronumeral standing for a number may have many different values.
- we can find the value of an expression by replacing the pronumeral with different numbers.

In this section we are going to see how

- expressions can be used to make rules.
- we can organise the different values of an expression into a table of values.

Suppose that each student in a Year 7 class has $2m + 3$ toothpicks, where m is the number of toothpicks in each cup.



We are now going to use another pronumeral, t, to stand for the number of toothpicks each student has. We can write $t = 2m + 3$.

This is a **rule** (or **formula**) for working out how many toothpicks a student has, if we know the value of m for that student. If $m = 3$ for Jay, then we can find out how many toothpicks Jay has altogether by substituting $m = 3$ in the rule.

When $m = 3$

$$
\begin{aligned}
t &= 2m + 3 \\
&= 2 \times 3 + 3 \\
&= 6 + 3 \\
t &= 9
\end{aligned}
$$

So Jay has nine toothpicks.

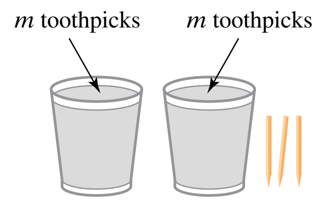
Note that

- the rule $t = 2m + 3$ has an equals sign in it. It tells us that t and $2m + 3$ have the same value, that is, they are equal to each other.
- we can also call the rule an **equation** because the expressions on each side are equal to each other.

We can then use the rule to make a **table of values** showing the different values for t.

Example 11

Each student in a Year 7 class has two empty cups as holders for their toothpicks. The total number of toothpicks that each student has is given by the rule $t = 2m + 3$, where m is the number of toothpicks in each cup.



a Jay puts 3 toothpicks in each cup, that is, $m = 3$. What is his total number of toothpicks, t?

b Nina puts 4 toothpicks in each cup, that is, $m = 4$. What is her total number of toothpicks, t?

c Alfred puts 6 toothpicks in each cup, that is, $m = 6$. What is his total number of toothpicks, t?

d Complete the table using the values of t for the 3 students.

m	3	4	6
t			

e Another student used a total of 19 toothpicks, that is $t = 19$ for this student. How many toothpicks did she put in each cup, that is, what was her value for m? Extend your table to include this.

f Explain what the rule $t = 2m + 3$ means.

Working / Reasoning

a Jay: $m = 3, t = 9$

$t = 2 \times 3 + 3$
$= 6 + 3$
$t = 9$

b Nina: $m = 4, t = 11$

$t = 2 \times 4 + 3$
$= 8 + 3$
$t = 11$

c Alfred: $m = 6, t = 15$

$t = 2 \times 6 + 3$
$= 12 + 3$
$t = 15$

d

m	3	4	6
t	9	11	15

Write the value for t for each student directly underneath their value for m.

e 8 toothpicks, so $m = 8$

m	3	4	6	8
t	9	11	15	19

3 of the 19 toothpicks would be the extra 3. That leaves 16 toothpicks to be divided equally between the 2 cups. So there would be 8 toothpicks in each cup.

f The total number of toothpicks is twice the number of toothpicks in each cup plus the extra 3.

$t = 2m + 3$ means $2 \times m$ plus another 3.

Example 12

Complete the table of values using the rule $y = 2x + 1$.

x	0	2	3	5	8	12
y						

Working

When $x = 0$
$2 \times 0 + 1$
$= 0 + 1$
$= 1$

When $x = 2$
$2 \times 2 + 1$
$= 4 + 1$
$= 5$

When $x = 3$
$2 \times 3 + 1$
$= 6 + 1$
$= 7$

When $x = 5$
$2 \times 5 + 1$
$= 10 + 1$
$= 11$

When $x = 8$
$2 \times 8 + 1$
$= 16 + 1$
$= 17$

When $x = 12$
$2 \times 12 + 1$
$= 24 + 1$
$= 25$

Reasoning

Substitute each of the x values into the rule, in turn, and calculate the matching y values.

The toothpicks model helped us to create a table of values. It summarised the total number of toothpicks for each person. In the following example we create tables of values without using the model.

Example 13

Complete the table of values using the rule $m = \frac{w}{2} + 5$.

w	2	8	16	20
m				

Working

When $w = 2$,

$$m = \frac{w}{2} + 5$$
$$= \frac{2}{2} + 5$$
$$= 1 + 5$$
$$m = 6$$

w	2	8	16	20
m	6			

Reasoning

Substitute 2 for w and work out the value of m.
Each value for m is written beneath its matching value for w.
Write 6 beneath the value 2.

continued

Example 13 continued

Working

When $w = 8$,

$$m = \frac{w}{2} + 5$$
$$= \frac{8}{2} + 5$$
$$= 4 + 5$$
$$m = 9$$

w	2	8	16	20
m	6	9		

Reasoning

Substitute 8 for w and work out the value of m.
Write 9 beneath the value 8.

Working

When $w = 16$, $m = 13$
When $w = 20$, $m = 15$

w	2	8	16	20
m	6	9	13	15

Reasoning

Calculate the other values for m in the same way.

Sometimes we may need to work backwards to find missing values in a table.

Example 14

Complete the following table of values using the rule $y = 2x + 3$.

x	0	2	3		
y				13	17

Working

When $x = 0$,
$2 \times 0 + 3$
$= 0 + 3$
$= 3$

When $x = 2$,
$2 \times 2 + 3$
$= 4 + 3$
$= 7$

When $x = 3$,
$2 \times 3 + 3$
$= 6 + 3$
$= 9$

x	0	2	3		
y	3	7	9	13	17

Reasoning

Substitute each of the x values into the rule, in turn, and calculate the matching y values.

continued

Example 14 continued

$2 \quad 5 + 3 = 13$
$2 \quad 7 + 3 = 17$

x	0	2	3	5	7
y	3	7	9	13	17

For $y = 13$ and $y = 17$, we have to work backwards to see what values for x would give these values for y.
Before 3 is added, $2x$ must have the values 10 and 14.
So x must have the values 5 and 7.

exercise 5.3

LINKS TO Example 11

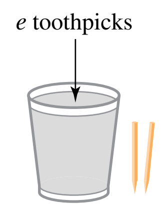
The rule $d = e + 3$ relates to this diagram.

a What does e stand for?

b What does d stand for?

c Use the rule to complete the table of values.

e	1	3	4	7		
d					12	20



LINKS TO Example 11

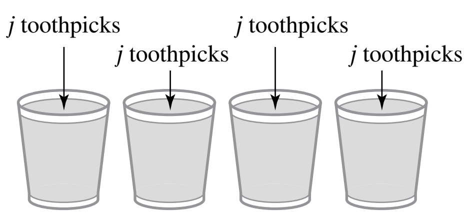
The rule $k = 4j$ relates to this diagram.

a What does j stand for?

b What does k stand for?

c Use the rule to complete the table of values.

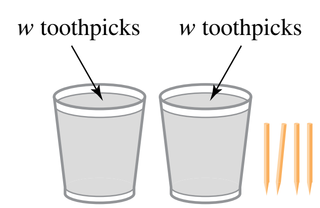
j	1	3	4	7		
k					36	44



LINKS TO Example 11

The rule that shows how to calculate output values for the picture here is $g = 2w + 5$, where w is the number of toothpicks in each cup and g is the total number of toothpicks. Copy and complete the following table of values.

w	2	5	8	11		
g					29	45



LINKS TO Example 11

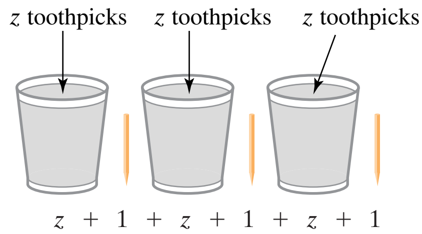
The rule $w = 3(z + 1)$ relates to the diagram.

a What does z stand for?

b What does w stand for?

c Use the rule to complete the table of values.

z	2	4	7	9		
w					33	60



LINKS TO Example 11

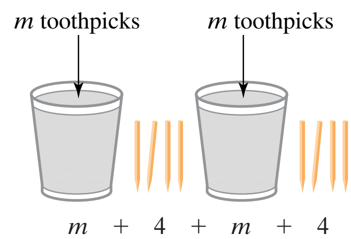
The rule $p = 2(m + 4)$ relates to this diagram.

a What does m stand for?

b What does p stand for?

c Use the rule to complete the table of values.

m	1	2	4	6		
p					24	40



LINKS TO Examples 12, 13

Complete each table of values using the rule given.

a $u = t + 6$

t	0	2	5	9
u				

b $c = 3b$

b	0	2	5	7
c				

c $k = 8 - j$

j	1	3	6	8
k				

d $e = 4t + 7$

t	0	2	3	12
e				

e $h = \frac{g}{5} + 3$

g	0	5	15	30
h				

f $r = 3(q + 1)$

q	2	3	4	5
r				

g $z = \frac{2y}{3}$ (Multiply by 2 then divide by 3.)

y	0	6	9	21
z				

h $x = \frac{w + 1}{4}$ (Add 1 then divide by 4.)

w	3	11	15	27
x				

i $q = 4(p - 2)$

p	2	5	7	12
q				

j $s = \frac{r - 3}{2}$

r	7	11	17	21
s				

LINKS TO Example 14

Complete each of the following tables of values using the rule given.

a $y = a + 5$

a	2	3	4	6		
y					14	20

b $p = 2w + 1$

w	0	2		5		10
p			9		15	

c $m = \frac{k}{4}$

k	0	4		16	20	
m			3			7

d $t = 3x - 1$

x	1	2	4		7	
t				14		23

e $c = \frac{b}{4}$

b	4	12	16		
c				6	10

f $y = 2x + 4$

x	1	2	3		
y				18	24

Copy and complete each of the following tables with the rule given.

a $d = e - 5$

e	d
6	
10	
14	
	12
	30

b $b = 6a$

a	b
0	
3	
5	
	48
	66

c $w = 2u + 7$

u	w
0	
2	
3	
	27
	33

d $n = 15 - 2m$

m	n
0	
2	
	7
	9
7	

e $y = 4(x + 3)$

x	y
0	
	16
3	
5	
	44

f $q = \dfrac{p + 3}{2}$

p	q
1	
7	
	6
11	
	10

● Each week, Emma, Chloe and Jade go to the market together to purchase their vegetables. One week they buy a box of oranges which they divide equally. There are two oranges left over.

a If each girl gets x oranges, write down a rule for the number n of oranges in the box.

b Use this rule to complete the table below.

x	7	8	10		15
n				38	

● Maddy has a pile of stickers. She shares the stickers with three friends. Use t to stand for the total number of stickers. Use m to stand for Maddy's share.

a Write a rule starting with '$m =$ ' for calculating Maddy's share from the total number of stickers t.

b Use the rule to complete this table.

t	20	28	36	400
m				

exercise 5.3 challenge

● Tim has bought a bag of sweets plus two extra sweets. He will share the bag of sweets with his brother, and keep the two extra sweets himself. Use h to represent the number of sweets in the bag. Use k to represent the number of sweets in Tim's share.

a Write a rule for calculating k when you know h.

b This table shows how many sweets there might have been in the bag. Use your rule to also show how many of these sweets are in Tim's share.

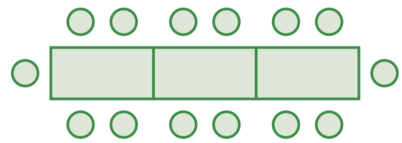
h	12	16	22	30
k				

5.4 Using algebra to solve problems

Rules, formulas and tables of values can help us solve problems.

Example 15

When there are many people for a meal, tables are sometimes put together as shown in the diagram.



a Choose pronumerals to represent the number of tables and the number of people who can be seated.

b Construct a table of values to show the number of people who can be seated at one, two and three tables.

c Write a rule to link the number of people who can be seated to the number of tables, and explain why it works.

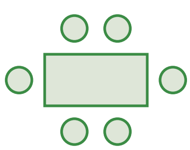
d Use the rule to predict how many people could be seated at eight tables.

Working

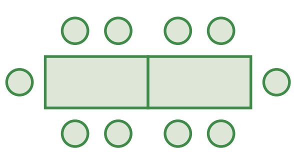
a Let T represent the number of tables.
Let P represent the number of people.

Reasoning: Any letters may be used. They do not need to be linked with tables or people. They may be lower or upper case.

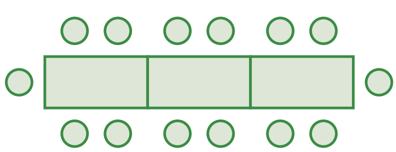
b



Reasoning: Draw diagrams to show one, two and three tables.



Reasoning: Count the number of people at each group of tables.



Reasoning: Make a table of values with the number of tables on the top line. The number of people who can be seated depends on the number of tables. So we usually write the number of tables in the top row and the number of people below.

T	1	2	3
P	6	10	14

Reasoning: Write in the number of people for each number of tables.

continued

Example 15 continued

Working

c The rule is $P = 4T + 2$. With each extra table, four more people can be seated. Plus, there is always one at each end.

Reasoning

Check: When $T = 2$,

$P = 4 \times 2 + 2$

$= 10$

Working

d $P = 4T + 2$

When $T = 8$,

$P = 4 \times 8 + 2$

$= 34$

There will be 34 people altogether when there are 8 tables.

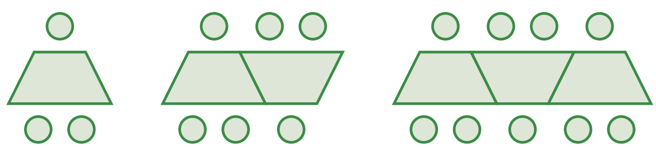
Reasoning

Give your answer in the words of the problem.

exercise 5.4

LINKS TO Example 15

In many classrooms the tables are trapezium-shaped. They can be placed in rows so that people sit like this.



Let n stand for the number of tables.
Let p stand for the number of people.

a Complete this table of values.

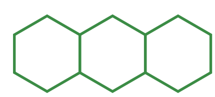
n	1	2	3
p			

b Write the rule connecting n and p.

c Explain why the rule works.

d If there were 10 tables in a row like this, how many people could be seated?

Year 7 students at Maytown Secondary College make a mathematics garden. In one part they build a chain of hexagon-shaped garden beds made from wooden edge pieces.



a How many edge pieces are needed for the chain of three hexagons?

b Draw chains of 1, 2, 3, 4 and 7 hexagons and for each chain count the number of edge pieces needed.

c Complete the following table, where H is the number of hexagons and P is the number of edge pieces.

H	1	2	3	4	7
P					

d Katie said that for each chain we need 5 times the number of hexagons plus 1 more. Is Katie correct? Check that it works for the values in your table.

e Turn Katie's sentence into an algebraic rule for calculating P if we know the value of H.

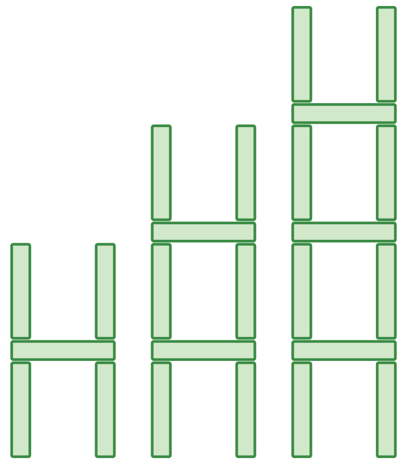
f Use your rule to find how many edge strips would be needed for 10 hexagons.

g If the students had 56 edge strips, how many hexagons could they have in their chain of hexagons?

Some prep-school children are making ladders on the floor using popsticks.

Let n be the number of rungs (or steps) in the ladder.

Let p be the number of popsticks needed.



a Copy and complete the table.

n	1	2	3	5	8
p					

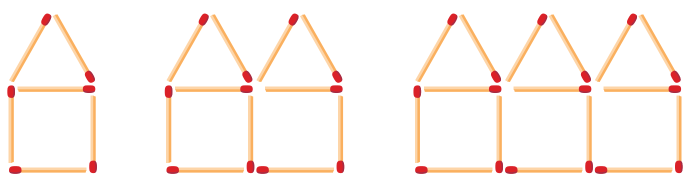
b Write in words how you would find the number of popsticks needed if you knew the number of rungs?

c Turn your sentence into an algebraic rule for calculating p if we know the value of n.

d Use your rule to find how many popsticks would be needed for a ladder with 13 rungs.

e Suppose Tim had 44 popsticks. How many rungs could he have in his ladder?

The following rows of houses are made from matches.



Let n be the number of houses in the row.

Let m be the number of matches needed.

a Draw the next three rows of houses in the pattern and count the number of matches needed for each.

b Copy and complete the table.

n	1	2	3	4	5	6
m						

c Write in words how you can find the number of matches needed if you know the number of houses in the row.

d Turn your sentence into an algebra rule for calculating m if we know the value of n.

e Use your rule to find how many matches would be needed for a row of 30 houses.

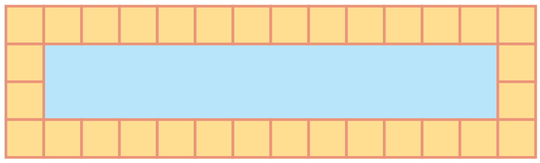
f How many houses could you make in the row if you had 61 matches?

Todd designs lap pools (for swimming laps). Each pool is two metres wide, but the length varies. He uses one-metre square paving blocks around the edges.

Let L be the number of metres in the length of the pool.

Let n be the number of paving blocks.

a This pool is 12 metres long. Count the number of paving blocks.



b Copy and complete the table.

L	10	11	12	13	14
n					

c Write in words how you can find the number of paving blocks needed if you know the length of the pool.

d Turn your sentence into an algebra rule for calculating n if we know the value of L.

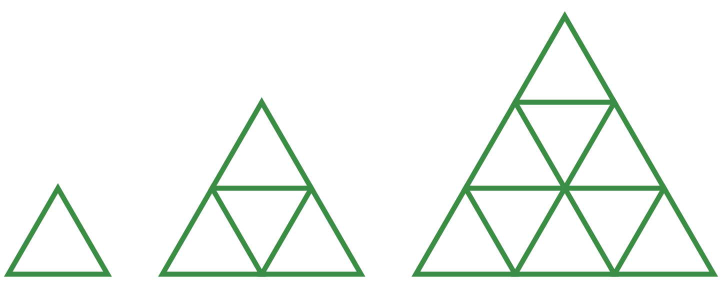
e Use your rule to find how many paving blocks would be needed for a 20 metre pool.

f How long could Todd make the pool if he had 62 paving blocks?

Patterns are made from triangles as shown.

Let n be the pattern number.

Let t be the number of small triangles in the pattern.



pattern number n: 1 2 3

a Copy and complete the table to show the number of small triangles in each pattern.

n	1	2	3	4	5	9	
t							144

b Write in words how you can find the number of small triangles if you know the pattern number.

c Turn your sentence into an algebraic rule for calculating t if we know the value of n.

d Use your rule to find how many small triangles there would be in pattern number 9.

e If there are 144 small triangles in a pattern, what is the pattern number?

exercise 5.4 challenge

Sally plans to take part in a charity fun run. Friends will give a total of $50 to the charity. Others will give $4 per kilometre that she runs.

Let k be the number of kilometres that Sally runs.

Let d be the total number of dollars that is given.

a If Sally runs two kilometres, how many dollars will be given altogether?

b If she runs five kilometres, how many dollars will be given altogether?

c If Sally doesn't complete any kilometres in the run, how many dollars will be given?

d Copy and complete the table.

k	0	1	2	3	4
d					

e Write in words how you can find the total number of dollars given if you know the number of kilometres that Sally runs.

f Turn your sentence into an algebraic rule for calculating d if we know the value of k.

g Use your rule to find how many dollars would be given if Sally runs eight kilometres.

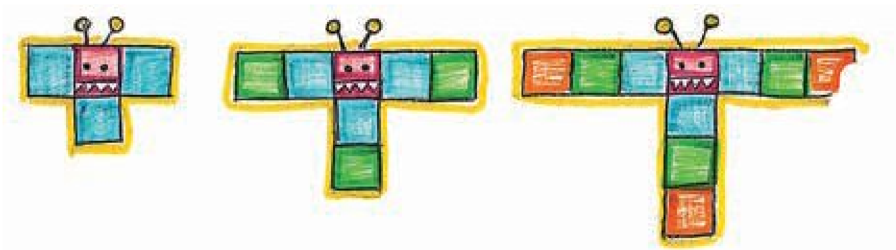
h If Sally collected a total of $74 how many kilometres did she run?

Analysis task

BLM Shape animals template

Shape animals

The square-legged tripus



A student used squares to design and draw a shape animal which she named a square-legged tripus. Each new animal she drew had one more square in each leg.

a Let L be the number of squares in each leg. Let N be the total number of squares needed to make each shape animal. Make a table of values to show L and N.

b Find the rule that tells how to get N from L. Write the rule in words. Write the rule in symbols.

c Explain why the rule works.

d Use the rule to work out how many squares you would need altogether to make a shape animal with legs 20 squares long.

Hexa-legged bipods

Another student drew hexa-legged bipods.

e Let L be the number of hexagons in each leg. Let N be the total number of hexagons needed to make each shape animal. Make a table of values to show L and N.

f Find the rule that tells how to get N from L. Write the rule in words. Write the rule in symbols.

g Explain why the rule works.

h Use the rule to work out how many hexagons you would need altogether to make a shape animal with legs 20 hexagons long.

Two more types of shape animal

i Design two more types of shape animal. Draw each type and name it.

j For each, let L be the number of shapes in each leg. Let N be the total number of shapes needed to make each shape animal. Make a table of values to show L and N.

k Find the rule that tells how to get N from L. Write the rule in words. Write the rule in symbols.

l Explain why the rule works.

m Use the rule to work out how many shapes you would need altogether to make a shape animal with legs 20 shapes long.

Review Algebra

Summary

- *Pronumerals* are letters that stand for numbers. If they can have a variety of values they are also called *variables*.
- An *expression* can be used to say how to calculate something. For example, the expression $3m + 2$ can stand for the total number of toothpicks. This also means $3 \quad m + 2$ and $m + m + m + 2$.
- The parts of an expression connected by plus or minus signs are called *terms*. For example, in the expression $3m + 2$, the terms are $3m$ and 2.
- A *coefficient* is the number multiplying the pronumeral. The term $3m$ has a coefficient of 3.
- An *equation* is a statement that two expressions are equal. An equation must have an = sign.
- A *rule or formula* is a special type of equation which starts with a pronumeral followed by the = sign. For example, $t = 3m + 2$ is a rule or formula.
- When a rule or formula is used to calculate a value, we *substitute* the value of the input pronumeral into the right side of the rule. The number goes in the place of the letter and any missing multiplication or division signs are written. For example, if $t = 3m + 2$, when $m = 4$, $t = 3 \quad 4 + 2 = 14$.
- Divisions are written like fractions; for example, e divided by 6 is written as $\frac{e}{6}$.
- Brackets are used when there are several lots of an expression; for example, $2(3m + 2)$ means two lots of $3m + 2$, which equals $3m + 2 + 3m + 2$, which equals $6m + 4$.
- When many different values of the pronumeral are used for calculations, a *table of values* is a useful way to show all of the results.
- To solve problems involving patterns, it can be helpful to collect information and organise it into a table of values. The rule for the table can then be found and used to solve the problem.
- The numbers in a table of values can be written as a list of ordered pairs.

Visual map

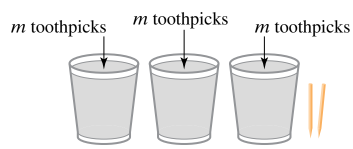
Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

coefficient	pronumeral	term
equation	rule	variable
expression	substitute	
formula	table of values	

Revision

Multiple-choice questions

- What is the value of $3m + 2$ when m has the value 5? The picture for this expression is shown here.



A 10 **B** 11 **C** 17
D 21 **E** 37

- The only expression below that does not mean the same as $x + x + x + 4 + x + 2$ is

A $3x + 4 + x + 2$ **B** $x + x + x + x + 4 + 2$ **C** $3x + x + 4 + 2$
D $6x + 4$ **E** $4x + 6$

- When '5 less than a number, n' is written using symbols it is

A $n + 5$ **B** $n - 5$ **C** $5 - n$ **D** $5n$ **E** $\frac{n}{5}$

- A number, n, is multiplied by 3 then 7 is subtracted. Written in symbols, this is

A $7n - 3$ **B** $7 - 3n$ **C** $3n - 7$ **D** $3 - 7n$ **E** $n - 3 \times 7$

- A rule for this table of values is

A $a = 3b + 6$ **B** $a = 6b + 3$
C $b = 3a + 6$ **D** $b = 6a + 3$
E $b = 9a + 3$

a	0	1	2	3	4
b	3	9	15	21	27

Short-answer questions

- Consider $5 + 2m$ toothpicks.
  a Draw a diagram.
  b Work out the total number of toothpicks for each of the following cases.
  i $m = 1$ **ii** $m = 3$ **iii** $m = 4$

Evaluate the following expressions when

i $n = 2$ ii $n = 4$

a $2n - 1$ **b** $3n + 7$ **c** $\frac{n}{2} + 3$ **d** $2(m + 5)$

Consider the expression $2(m + 5)$.

a Represent the expression using a diagram.

b Write $2(m + 5)$ in a simpler way without brackets.

c Find the value of the expression when $m = 3$.

Courtney has a total of T lollies. She shares them equally between five lolly bags for her sister's birthday party. Use b to stand for the number of lollies in one bag.

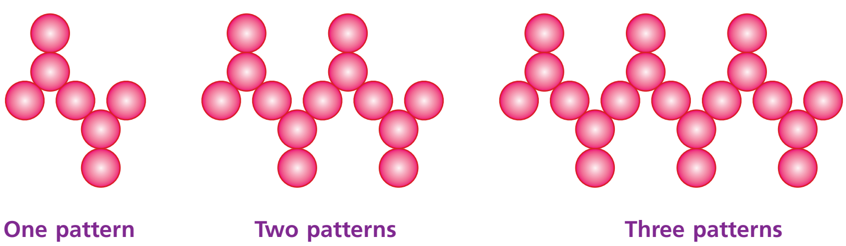
a Write a rule starting with $b =$ for calculating the number of lollies in each bag when there are a total of T lollies shared out.

b Use the rule to complete this table.

T	20	30	35	60
b				

Extended-response question

Stephanie is making a bead necklace. The picture below shows how it looks as each new repeating pattern of beads is added.



a Choose and define pronumerals for the number of repeating patterns Stephanie has included, and for the total number of beads used.

b Make a table of values to show the total number of beads for 1, 2, 3, 4, 5 and 6 repeating patterns.

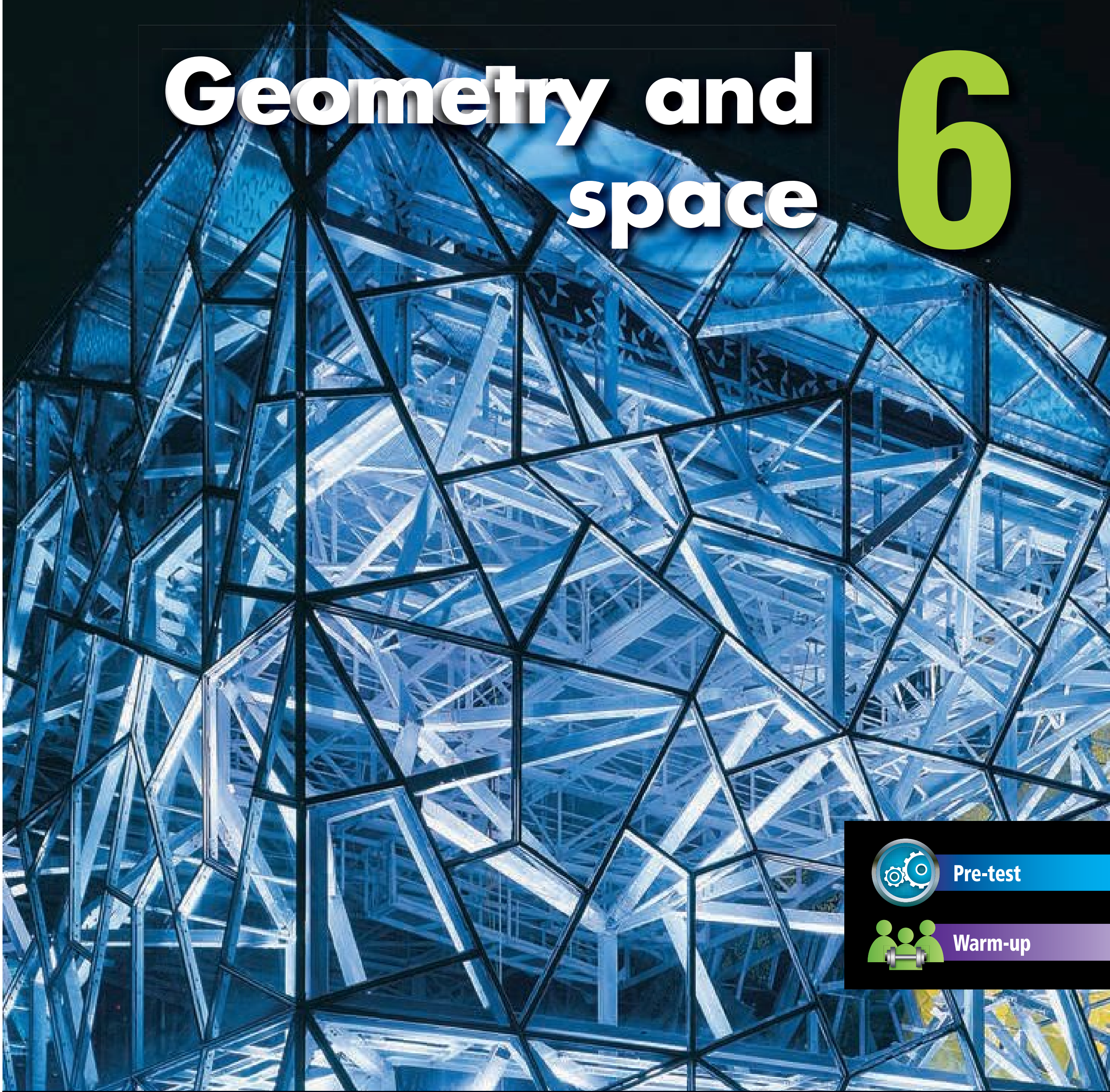
c Find the rule to describe the relationship between the number of repeating patterns and the total number of beads used, and write this rule in symbols.

d Use the rule to predict the number of beads needed for 10 repeating patterns.

e With 97 beads, how many repeating patterns can be made?

Practice quiz
Chapter 5

Geometry and space 6



Pre-test

Warm-up

'Geometry' literally means 'measuring the Earth'. In geometry we explore lines and angles and shapes such as triangles and quadrilaterals. Triangles are rigid shapes—a property that makes them useful in constructions such as roof frames and bridges. In this structure at Federation Square in Melbourne, triangles and quadrilaterals have been used to create a unique architectural design.

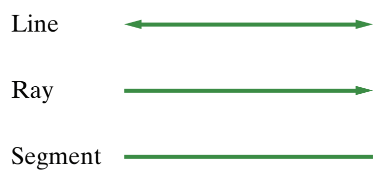
6.1 Lines, rays and segments

A **line** extends forever in both directions.

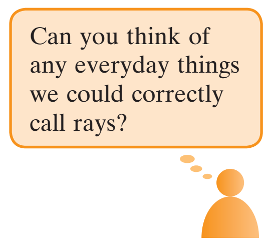
A **line segment** has a definite starting point and finishing point; that is, it is *part of a line*.

A **ray** has a definite starting point, but extends forever in one direction.

In the diagram below the arrows show the directions in which the line and ray extend forever.

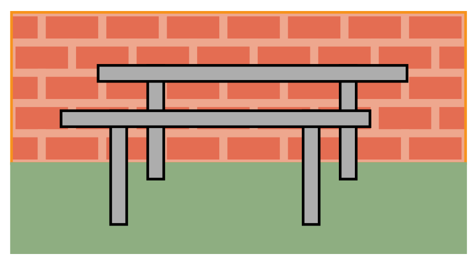


The word *line* is used in everyday language when we are actually talking about line segments, for example, the lines in an exercise book. In this chapter when we are referring to mathematical diagrams we shall use the words line segment and line correctly.



Parallel lines

The word *parallel* has a pair of parallel segments in it. Parallel bars are used in gymnastics. Notice how the two bars in the diagram to the right are the same distance apart at all positions.



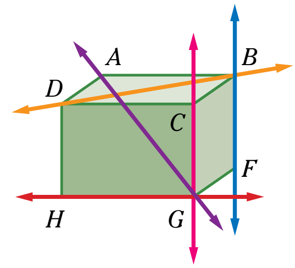
This is what we mean by **parallel**.

Parallel lines are always the same distance apart.

It is important that we measure the distance, d, between the lines in a direction at right angles to each of the lines, as shown in the diagram.

Sometimes parallel lines are described as lines that will never meet. The railway tracks shown below are parallel since they are always the same distance apart, as there is a fixed distance between the train wheels.

It is possible to have lines that will never meet even though they are not parallel. The edges *BF* and *CG* of the box in the diagram on the right are parallel because they are in the same plane and they will never meet, even if extended in both directions. They are always the same distance apart. However, although *BF* and *HG* will never meet, they are not parallel because they are not in the same plane.

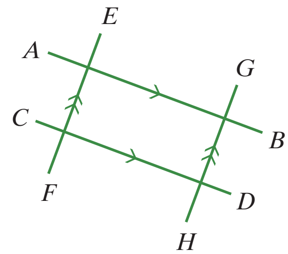


Lines or line segments that never meet are parallel only if they are in the same plane; that is, if they lie on the same flat surface. There are two ways of describing parallel lines:

- Lines or segments are parallel if they are *the same distance apart at all points along them*.
- Lines or segments are parallel if they are *in the same plane and they never meet*.

On a diagram, matching arrowheads are used to indicate that lines or line segments are parallel. For example, *AB* and *CD* are parallel and *EF* and *GH* are parallel. (Do not confuse these arrowheads with arrowheads that are put at the ends of lines or rays to indicate that the line or ray extends forever.)

In describing parallel lines, line segments or rays, we use the symbol . In the diagram at right, for example, *AB* *CD* and *EF* *GH*.

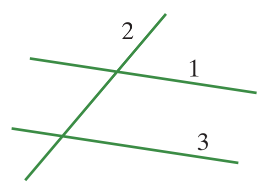


Example 1

The diagram at right shows line segments 1, 2 and 3. Two of them are parallel.

a Which line segments are parallel?

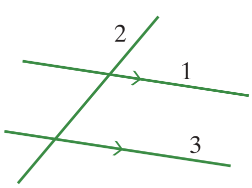
b Mark the diagram with symbols to show this.



Working

a Line segments 1 and 3 are parallel.

b



Reasoning

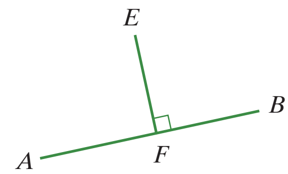
These two segments:

- are the same distance apart at all positions
- are in the same plane
- do not meet.

We use arrowheads to show that two lines or line segments are parallel.

Perpendicular lines

Lines or line segments that are at right angles to each other, as shown in this diagram, are said to be **perpendicular**.



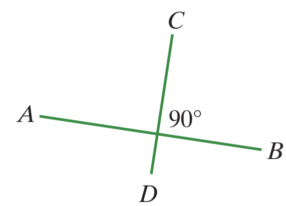
Segment *EF* is perpendicular to segment *AB*. Notice that the right angle symbol is used on a diagram to show that line segments or lines are perpendicular. We write *EF*   *AB*.

Example 2

The diagram at right shows two line segments, *AB* and *CD*.

a What can we say about line segments *AB* and *CD*?

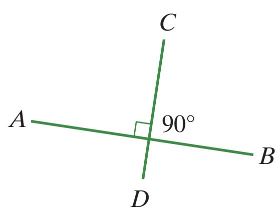
b Mark the diagram with a symbol to show this.



Working

a Line segments *AB* and *CD* are perpendicular.

b



Reasoning

Lines are perpendicular if there is an angle of 90° between them.

The right angle symbol shows that the segments are perpendicular.

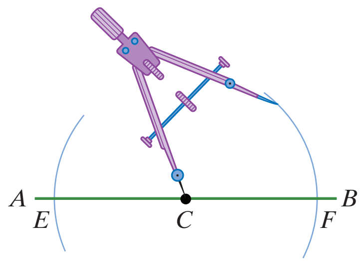
Constructing a line perpendicular to another line

A ruler, compass (often known as a *pair of compasses*) and a sharpened pencil can be used to construct a line perpendicular to a given line through a point on the line. In compass and ruler constructions, the compass is used as a measuring instrument. The ruler is simply used as a straight edge to rule straight lines, rather than to make measurements.

Example 3

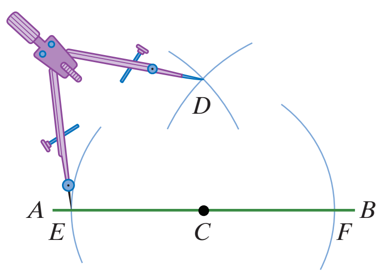
Construct a perpendicular to the line *AB* through the point *C*.

Working

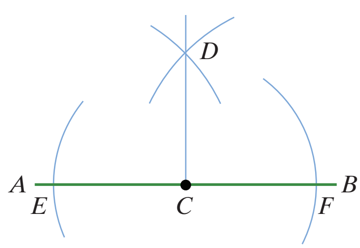


Reasoning

Place the compass point at *C* and draw arcs to cut *AB* as shown (at *E* and *F*). This has marked off equal distances *CE* and *CF*.



Open the compass wider so that the opening is greater than the distance *EC*. Place the point at *E* and draw an arc above *AB* as shown. Keeping the compass opening exactly the same, place the point at *F* and draw another arc to cut the other arc as shown (at point *D*).
This marks equal distances *ED* and *FD*.

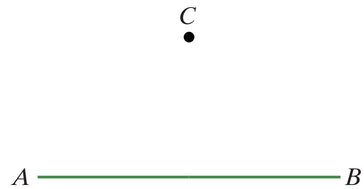


Rule a line through *C* and *D*. This line is perpendicular to *AB*. We can explain this in the following way.
We know that $EC = CF$; $ED = FD$, so we know that we have made two identical triangles $\triangle ECD$ and $\triangle FCD$ that share the side *CD*. So $\angle ECD = \angle FCD$. But *AB* is a straight line (180°) so each of the angles must be 90°, that is, *CD*   *AB*.

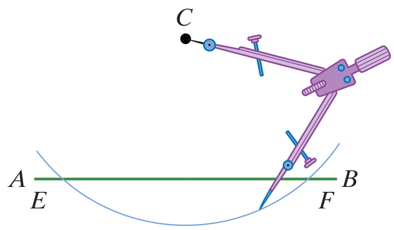
A similar method can be used to construct a line perpendicular to a given line through a point that is not on the line.

Example 4

Construct a perpendicular to the line *AB* through the point *C*.

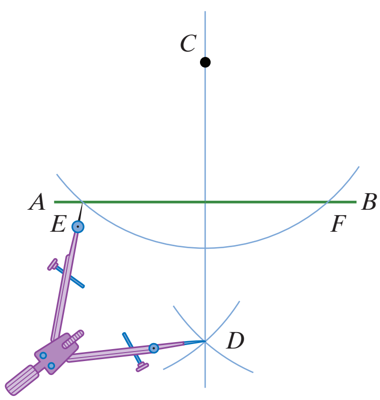


Working



Reasoning

Place the compass point at *C* and draw arcs to cut *AB* as shown (at *E* and *F*). This has marked off equal distances *CE* and *CF*.



Place the point at *E* and draw an arc below *AB* as shown. Keeping the compass opening exactly the same, place the point at *F* and draw another arc to cut the other arc as shown (at point *D*).
This marks equal distances *ED* and *FD*.
Rule a line through points *C* and *D*.
CD   *AB*.

Vertical and horizontal lines

Builders use a plumb line (a heavy weight on a string) to find the **vertical**. The direction of vertical is towards the centre of the Earth. A lamp post is normally vertical.

The direction of horizontal is at right angles to vertical. A spirit level is used to find the **horizontal**. A bubble of air in the spirit level moves as the spirit level is tilted. When the spirit level is horizontal, the bubble is centred between two markers. Some spirit levels can also be used to find the vertical. The floor of a building and the top of a table are normally horizontal.

'Plumbum' is the Latin name for lead. The chemical symbol for lead is Pb.

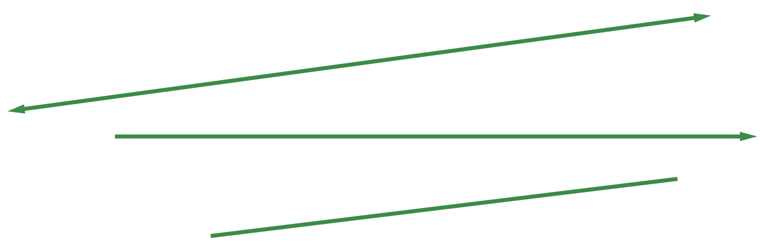
A plumb bob shows the direction of vertical

A builder uses a spirit level to check that bricks are horizontal

In everyday language we often use the word *vertical* to describe, for example, the side edge of a page resting on a table. Although the edge is perpendicular to the horizontal bottom edge of the page, it is not vertical in the correct sense of the word vertical.

exercise 6.1

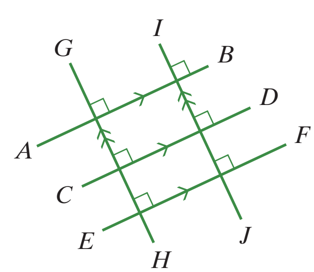
- Using your ruler and a sharp pencil, copy the following drawings and label them correctly as representing a line, a segment or a ray.



- Look back at the picture of the parallel bars on page 246.
As well as the bars, what other parallel segments can you see?

LINKS TO Example 1, 2

- Write your answers to the following questions using the symbols or .
  a Which segments are parallel to segment AB?
  b Which segments are perpendicular to segment GH?

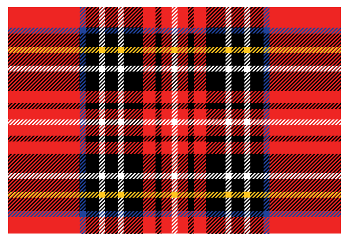


- Using your ruler, draw a line segment 8 cm long and label the ends *A* and *B*. Use your ruler to draw a line segment 6 cm long and parallel to *AB*. Label the ends of this segment *C* and *D*. Put arrowheads on segments *AB* and *CD* to show that they are parallel.

- Using your ruler, draw a line segment 4 cm long parallel to the left-hand edge of your page. Label the ends *P* and *Q*. Mark a point somewhere along *PQ* and label it *R*. Now draw a line segment, *RS*, 5 cm long perpendicular to *PQ* (use the lines of your page as a guide). Use the right angle symbol to show that your segments are perpendicular.

- Use the words *perpendicular* or *parallel* to complete the following sentences.
  In striped material, the stripes are ______________ to each other. In check material, the horizontal stripes are ______________ to the vertical stripes.

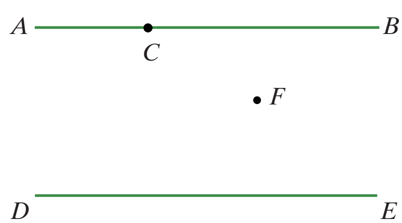
striped



check

LINKS TO Example 3, 4

- Use a ruler, compass and sharp pencil to construct each of the following.
  a A line perpendicular to *AB* passing through point *C*.
  b A line perpendicular to *DE* passing through point *F*.



- Copy and complete the sentences using the words *perpendicular*, *parallel*, *vertical* or *horizontal*.
  Chris is building a house with rectangular rooms. He has checked that the floor is ____________. The walls must be ____________ and ____________ to the floor. The walls on opposite sides of the room must be ____________ to each other. Two walls meeting at a corner of the room must be ____________ to each other.

- Match each of these words with one of the things below and sketch each of your answers: parallel, vertical, horizontal, perpendicular.
  a top and side of a book
  b a goal post
  c lines in an exercise book
  d surface of the water in a swimming pool

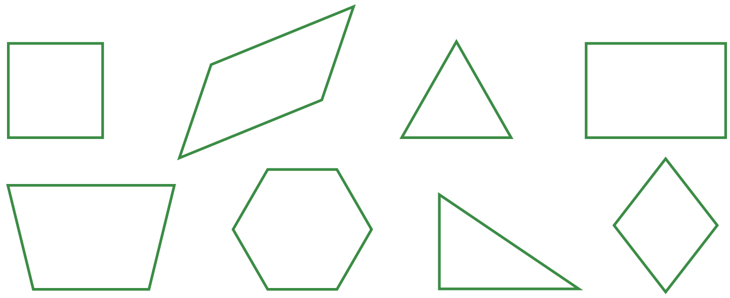
6.1

Complete the questions below.

a Copy the shapes that appear to have at least one pair of parallel sides, and use arrowheads to indicate the sides that are parallel. If the shape has two pairs of parallel sides, use double arrowheads for the second pair.

b Copy the shapes that appear to contain at least one pair of perpendicular segments and use the correct symbol to show the sides that are perpendicular.

c Copy any of these shapes that appear to contain parallel *and* perpendicular segments, and use appropriate symbols to show the parallel and perpendicular segments.



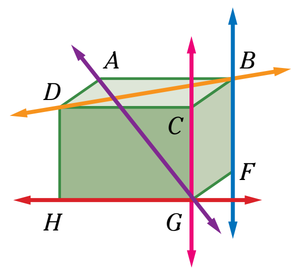
Using a cardboard box to help you, decide whether the pairs of lines will meet or are parallel.

a DB and BF

b DB and CG

c DB and HG

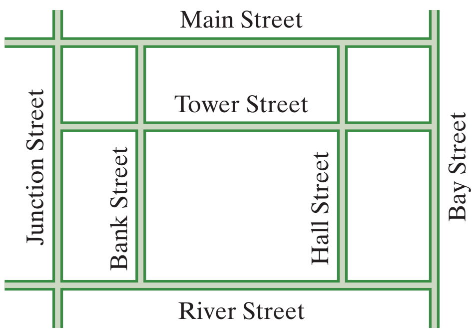
d AG and BF



exercise 6.1 challenge

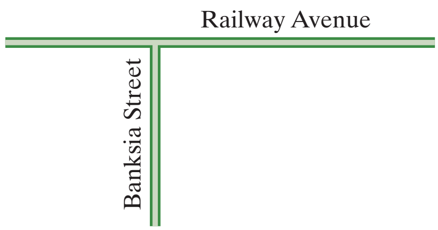
a Name two streets that are parallel to Hall Street.

b Name two streets that are perpendicular to Junction Street.



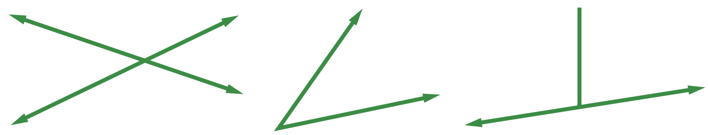
Tranh was explaining to his friend how to get to his house: 'It's in Grant Street, which is parallel to Railway Avenue and at right angles to Banksia Street'.

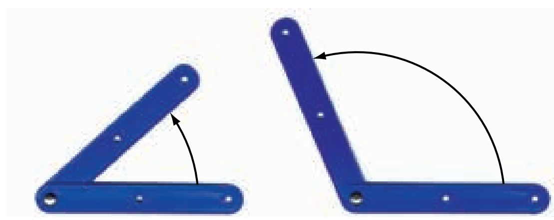
Copy the drawing then draw and label where you think Grant Street might be.



6.2 Angles

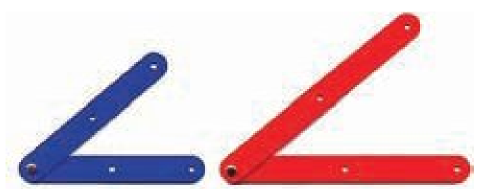
Angles are formed wherever rays, lines or line segments meet or intersect.



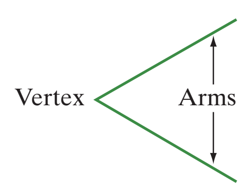


The size of an **angle** represents the amount of turning between the two rays, lines or segments.

The size of the angle depends only on the amount of turning, and does not depend on the length of the arms of the angle. The two angles shown here are the same size. We would see this if we placed one on top of the other.



The rays, lines or line segments which form the angle are called the **arms** of the angle. The point where the arms meet is called the **vertex** of the angle.

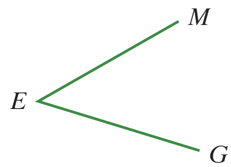


Naming and classifying angles

To name an angle we usually use three letters: the one at the end of each arm of the angle and the one at the vertex. The letter at the vertex of the angle is always written in the middle of the three letters. We use the symbol $\angle$ for angle; for example, $\angle ABC$.

Example 5

Name this angle.



Working

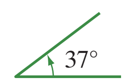
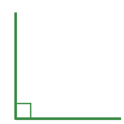
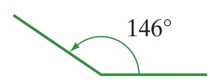
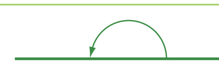
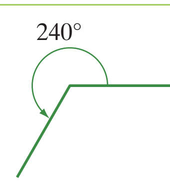
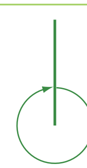
∠*MEG* or ∠*GEM*

Reasoning

The letter *E* is at the vertex of the angle so it goes in the middle of the three letters in the name.

The most common unit for measuring angles is the degree. We use the symbol ° for degrees, so that 180° means 180 degrees. The symbol for right angle is the same symbol as for perpendicular.

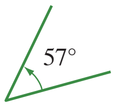
The following table summarises different types of angles. Copy the table into your book. Make up your own examples for acute, obtuse and reflex angles.

Size of angle	Name	Example
Less than 90°	**Acute angle**	
90° (a quarter turn)	**Right angle**	
Greater than 90° but less than 180°	**Obtuse angle**	
180° (a half turn)	**Straight angle**	
Greater than 180° but less than 360°	**Reflex angle**	
360° (a full turn)	**One revolution**	

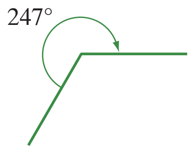
Example 6

Give the type of angle for each of the following angles.

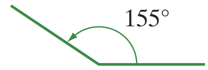
a



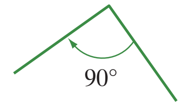
b



c



d

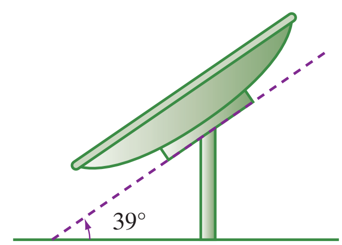


Working	Reasoning
a Acute angle	The angle is less than 90°.
b Reflex angle	The angle is more than a straight angle, so it is in between 180° and 360°.
c Obtuse angle	The angle is more than a right angle but less than a straight angle, so it is in between 90° and 180°.
d Right angle	A right angle is defined as ‘an angle of 90°’.

Measuring angles

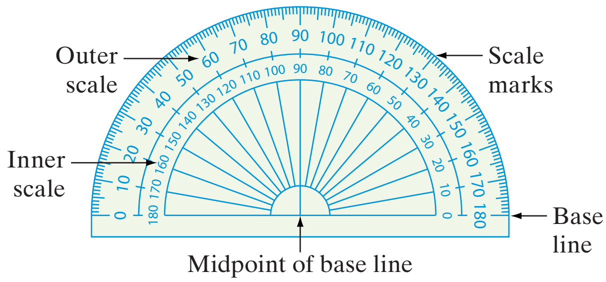
Builders, architects and graphic designers are some of the people who need to measure angles accurately. In this chapter you will meet some other situations where angles are important.

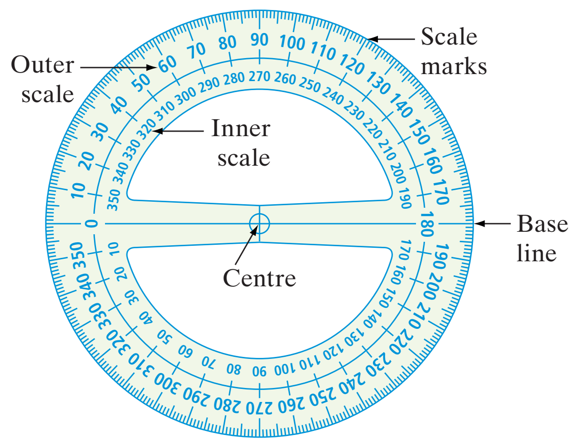
Joel works for a company that installs satellite dishes. Each dish must be placed at the correct angle to the horizontal so that it faces the required satellite. Joel is installing a satellite dish on a building in Melbourne so that it receives beams from a satellite TV channel. He knows that the dish must be tilted so that its frame makes an angle of 39° to the horizontal for this particular satellite.



Some different types of angle-measuring tools that Joel could use to get the correct angle for the satellite dish are shown below. Notice that one of them has an automatic digital display.

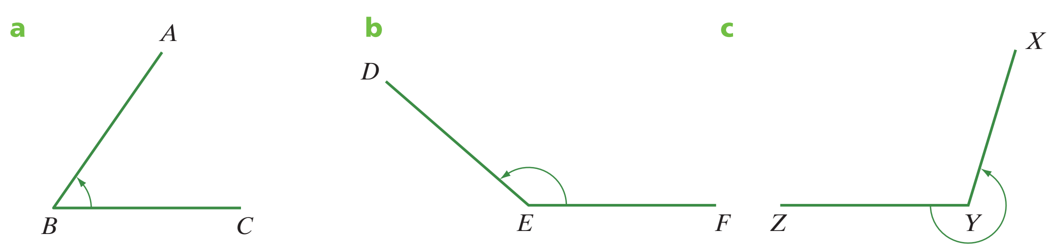
As shown in the illustrations below, protractors have two scales going in opposite directions. Before you start to measure an angle, it is important that you decide whether the angle is acute, obtuse or reflex, as this will help you to check that you are reading the angle size correctly.





Example 7

Use a protractor to find the sizes of the following marked angles.



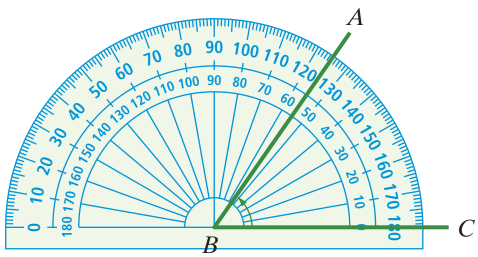
continued

Example 7 continued

Working

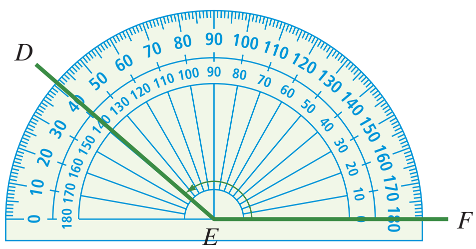
Reasoning

a $\angle ABC = 55°$.



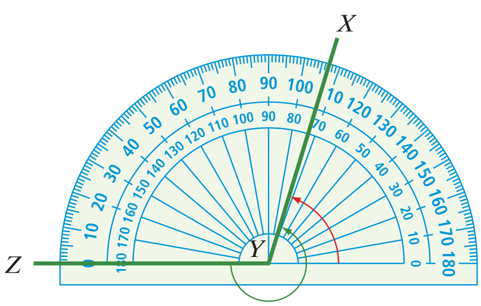
Place the protractor with the midpoint of its baseline at the vertex of the angle (at B). Measure around from zero on the inner scale.
Check: $\angle ABC$ is an acute angle so its measure must be less than 90°.

b $\angle DEF = 139°$.

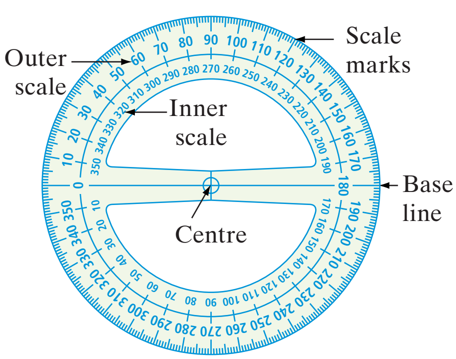


Place the protractor with the midpoint of its baseline at the vertex of the angle (at E). Measuring around from zero on the inner scale gives the angle size as 139°.
Check: $\angle DEF$ is an obtuse angle, so it must be between 90° and 180°.

c Obtuse $\angle XYZ = 107°$
so reflex $\angle XYZ = 360° - 107° = 253°$.



Place the protractor with the midpoint of its baseline at the vertex of the angle (at Y). Measure the obtuse angle XYZ from zero at Z on the outer scale. Subtract the obtuse angle $\angle XYZ$ from 360° to find the reflex angle $\angle XYZ$.
Check: reflex $\angle XYZ$ is between 180° and 270°.
Another way of finding reflex $\angle XYZ$ is to add 180° to the acute angle 73° (shown in red). $73° + 180° = 253°$



Measuring reflex angles is easier with a 360° protractor.

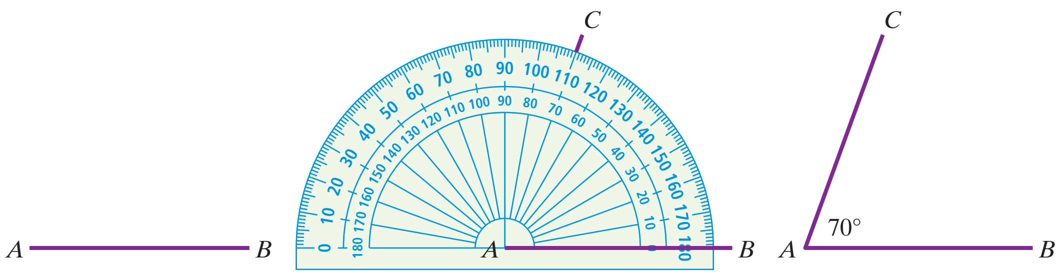
Constructing angles

Example 8

Use your ruler and protractor to draw an angle of 70°.

Working

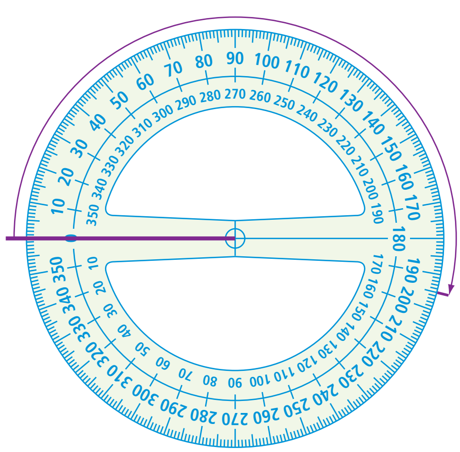
Rule a segment *AB* to form one arm of the angle. Then place your protractor with the midpoint of its baseline at the left-hand end of your segment (at *A*), making sure the base line of the protractor is exactly along your segment. Make a small mark on your page at the 70° mark (at *C*), remove the protractor and join the mark to the left-hand end of the segment (at *A*). The angle *CAB* is 70°.

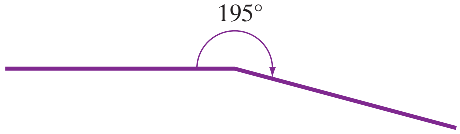


Example 9

Use a protractor to draw an angle of 195°.

Working





Reasoning

Rule a line segment and place the centre of the protractor at the right-hand end of the segment, with the zero half of the base line exactly along the segment. Measure around from zero to 195° and place a small mark. Remove the protractor and join the mark to the right-hand end of the line segment to form the angle.

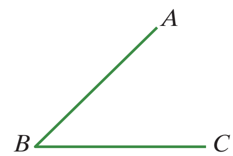
Note that 195° is a reflex angle.

Copying an angle with compass and ruler

An angle of any size can be copied using a compass and ruler using the following steps.

Example 10

Copy $\angle ABC$ so that $\angle LMN$ is exactly the same size.

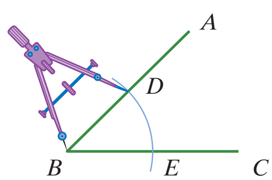


Working

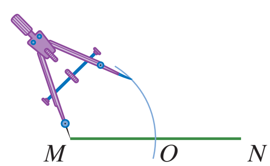
Reasoning

The compass is being used as a measuring instrument so that the same lengths can be copied.

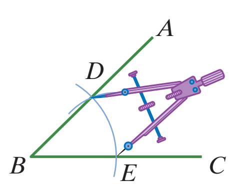
1 Using a ruler, draw the line segment MN to make one arm of $\angle LMN$.



2 With the compass point at B, draw an arc to cut AB and BC (at the points labelled D and E).

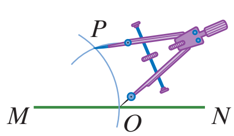


3 Keeping the compass open at exactly the same amount, place the compass point at M and draw an arc to cut MN (at the point labelled O).

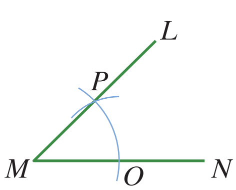


In this step, the compass is being used to measure the distance ED so that it can be copied.

4 Place the compass point at E and open the compass so that the pencil point is exactly at D.



5 Keeping the compass opening exactly the same, place the point at O and draw an arc to cut the first arc (at the point labelled P).



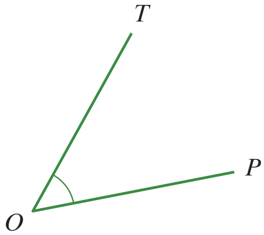
6 Using the ruler, draw a line segment from M through P so that it forms the other arm, MN of $\angle LMN$.

exercise 6.2

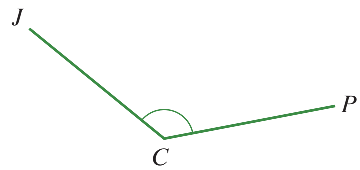
LINKS TO Example 5, 6

1. For each of the following angles
   - **i** name the angle—for example, ∠*ABC*.
   - **ii** state its type—for example, acute angle.

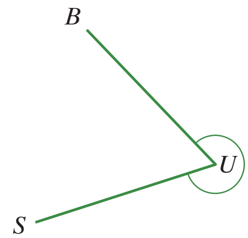
a



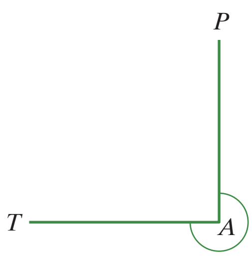
b



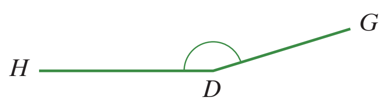
c



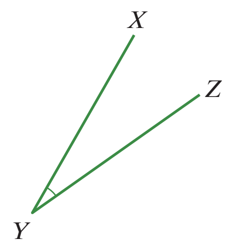
d



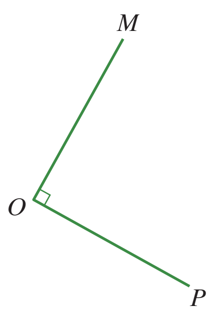
e



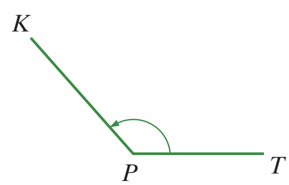
f



g



h



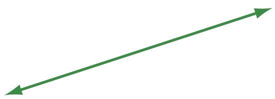
i

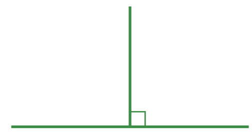
2. State whether each of the following angles is acute, obtuse, right, straight, reflex or a revolution.

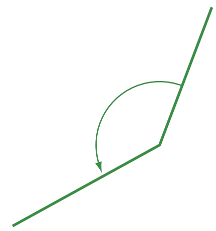
a 100°	**b** 20°	**c** 315°	**d** 90°
e 97°	**f** 242°	**g** 360°	**h** 180°

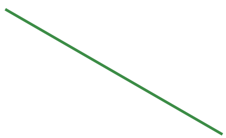
Match each diagram with a term from the list below.

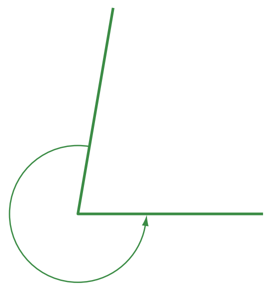
perpendicular	acute angle	reflex angle	parallel
ray	line segment	obtuse angle	line

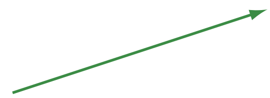
a


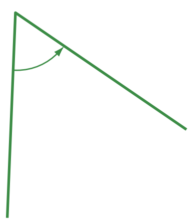
b


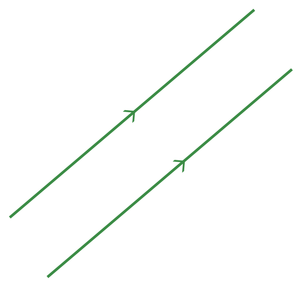
c


d


e


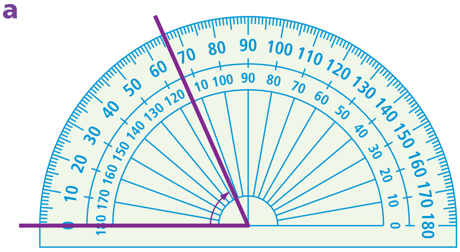
f


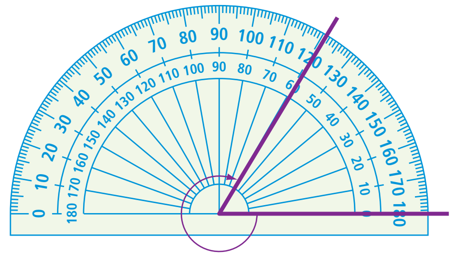
g


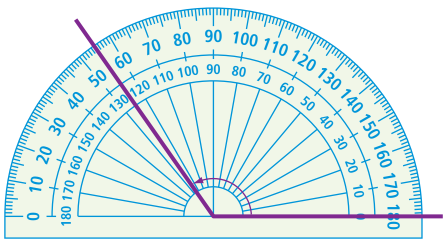
h


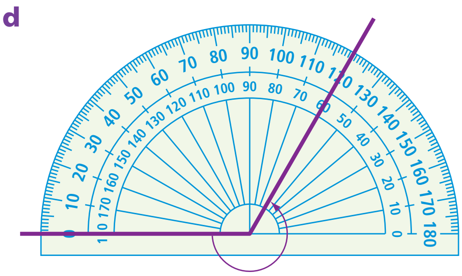
LINKS TO Example 7

Find the size of each of these angles and state whether it is acute, obtuse or reflex.

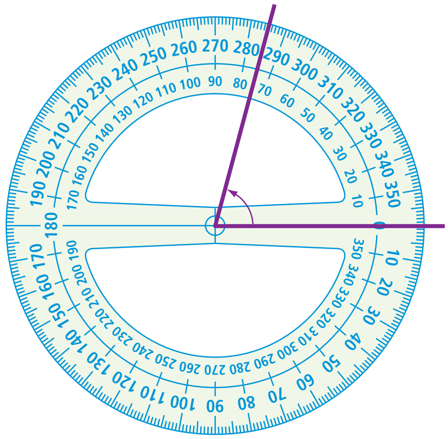
a


b


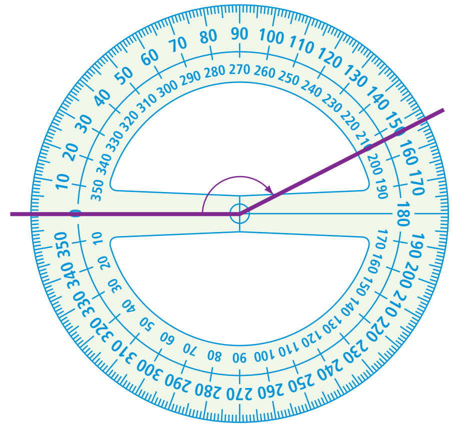
c


d


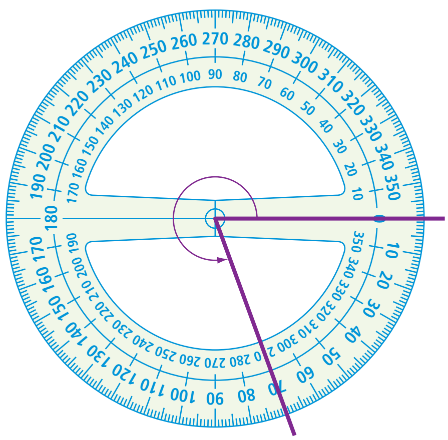
e



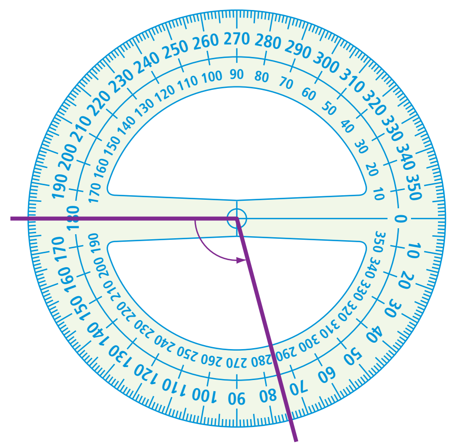
f



g



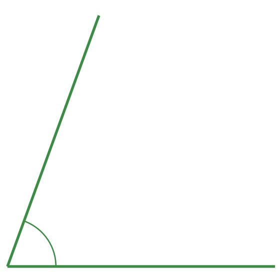
h



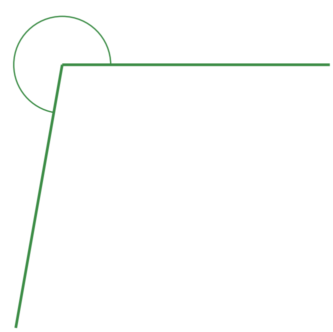
LINKS TO Example 7

Use your protractor to measure each of the following angles as accurately as you can.

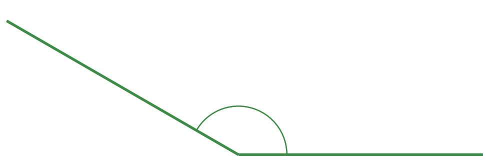
a



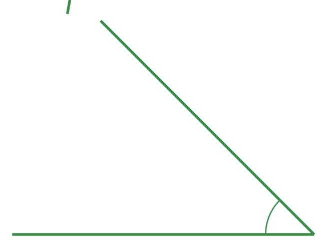
b



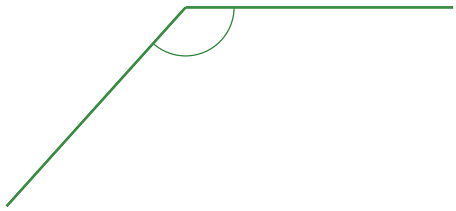
c



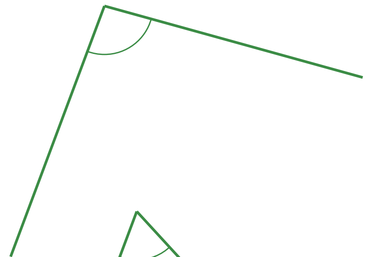
d



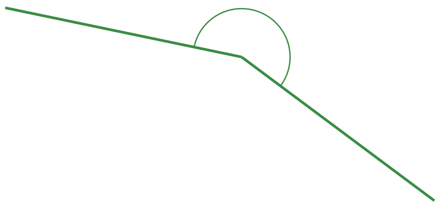
e



f



g



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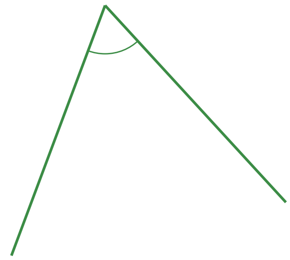
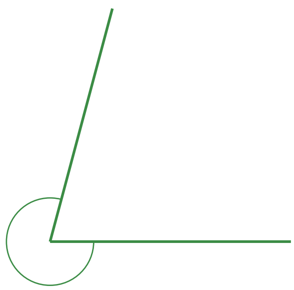
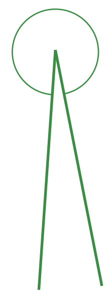


i



j



● The following questions refer to the playground equipment in this photograph.

a Use your protractor to measure the angle that the blue slide makes with the horizontal.

b What angle does the yellow rail of the climbing frame steps make with the horizontal?

6.2

● The photographs below show the 70 metre gateway to Melbourne on the Tullamarine Freeway.

a Which of the two photographs would allow you to more accurately estimate the angle that the yellow beam makes with the horizontal? Explain.

b Use your protractor to estimate the angle that the beam makes with the horizontal.

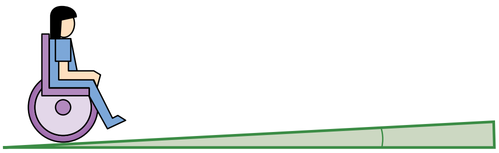
c Using your answer for part b, what angle does the beam make with the vertical?

● In each of the following pictures the angle of the slope is suitable for the activity shown. Angles greater than this would not normally be considered safe.

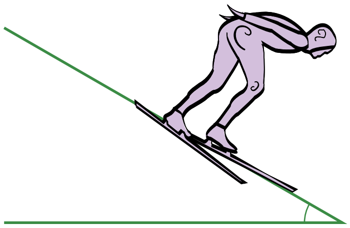
a Why do you think each of these angles is suitable for the particular activity shown?

b Measure each of the marked angles.

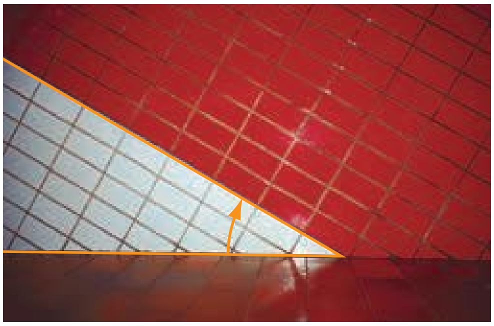
i wheelchair ramp



ii downhill ski slope



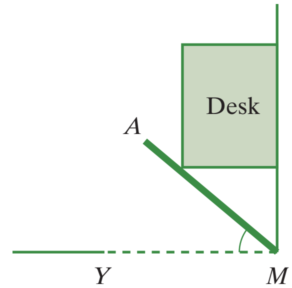
iii escalator (note: the red and white tiles make the same angle as the escalator)



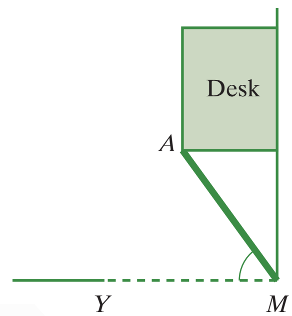
c In the photographs of the escalator, why are we unable to use the photograph on the left to find the angle that the escalator makes with the horizontal?

Amy is rearranging her bedroom. When she moves her desk she finds that she can't open her door fully.

a Use your protractor to find $\angle AMY$.



b Amy realises that she can't fit her new chest of drawers through the doorway unless the door will open through at least 50°, so she moves her desk further away from the door. Has she moved the desk enough?



Tom decided that angles in leaves would make an interesting project for his Mathematics Talent Quest entry. He collected lots of autumn leaves and used his protractor to carefully measure the angles between the main vein of the leaves and the veins each side, as shown on the leaves here.

Maple leaves

Plane leaves

a Use your protractor to measure the angles and hence the values of *a*–*j*.

b Can you suggest some ideas relating to leaf angles that Tom might try to investigate in his project? What are some of the factors that might affect the size of the angles?

LINKS TO Example 8, 9

Use your ruler, protractor and a sharpened pencil to draw angles with the following sizes. State whether each angle is acute, obtuse or reflex.

a 35° **b** 70° **c** 155° **d** 80°

e 25° **f** 270° **g** 138° **h** 313°

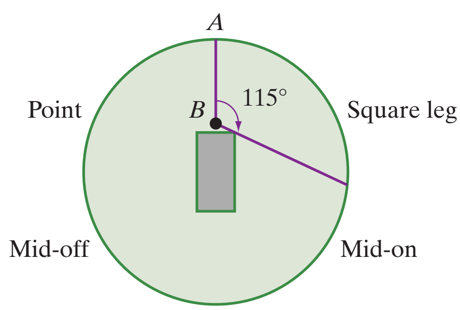
LINKS TO Example 10

Use a pair of compasses, ruler and pencil to copy each of the angles you have drawn for question 12.

exercise 6.2 challenge

BLM Question 13

In cricket, diagrams called 'wagon wheels' are used to display the directions where each batter hits the ball during an innings. In the diagram below, the black dot represents the batter. The angle for each ball is measured in a clockwise direction from the line segment labelled *AB*. Freddy Wackett hit the first ball of his innings at an angle of 115°.



The angles for Freddy's next 14 scoring strokes are given below.

61°, 175°, 110°, 194°, 168°, 172°, 264°, 153°, 165°, 138°, 185°, 283°, 180°, 142°

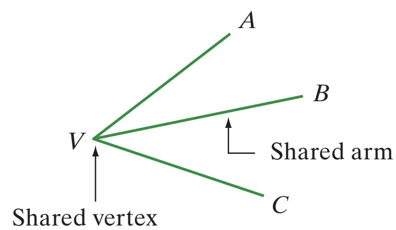
a Copy the diagram, making the diameter of your circle 10 cm and *AB* 6 cm, and use your protractor and ruler to carefully rule line segments to represent the direction of the ball for all fifteen angles.

b If you were the captain of the opposing team, where would you concentrate your fieldsmen while Freddy was batting?

6.3 Calculating angle sizes

Adjacent angles

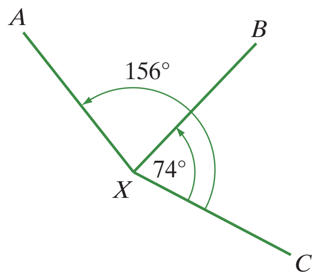
Adjacent means *next to*; for example, two next door houses in a street would be adjacent. Two angles that are next to each other, sharing the same vertex and having a shared arm, are called **adjacent angles**. For example, $\angle AVB$ and $\angle BVC$ are adjacent angles.



Note that $\angle AVB + \angle BVC = \angle AVC$.

Example 11

Find the size of $\angle AXB$.



Working

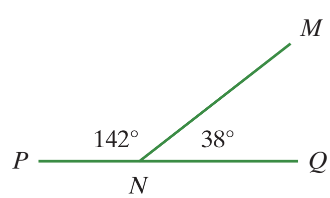
$$\angle AXB = 156° - 74°$$
$$= 82°$$

Reasoning

$$\angle AXB + \angle BXC = \angle AXC$$
$$\angle AXB + 74° = 156°$$

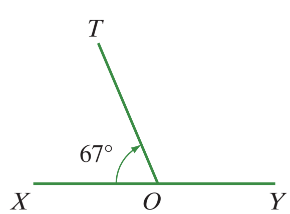
Angles that form a s traight line

Two adjacent angles that form a straight line are supplementary angles because a straight angle measures 180°. For example, $\angle MNP$ and $\angle MNQ$ are supplementary angles because together they make a straight line ($142° + 38° = 180°$).



Example 12

Find the size of $\angle TOY$.



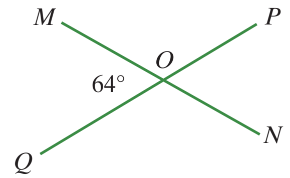
continued

Example 12 continued

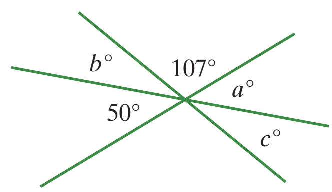
Working	Reasoning
$\angle TOY = 180° - 67°$ $= 113°$	$\angle TOY$ and $\angle TOX$ make a straight angle—that is, 180°.

Example 13

a Calculate the sizes of $\angle MOP$, $\angle PON$, and $\angle QON$ (without using a protractor).



b Find the values of a, b and c.



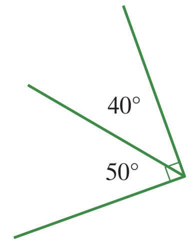
Working	Reasoning
a $\angle MOP + \angle MOQ = 180°$ $\angle MOP = 180° - 64°$ $= 116°$	Adjacent angles in a straight angle add to 180°C.
$\angle PON = \angle MOQ$ $\angle PON = 64°$ $\angle QON = \angle MOP$ $\angle QON = 116°$	$\angle MOQ$ and $\angle PON$ are vertically opposite. Vertically opposite angles are equal. $\angle MOP$ and $\angle QON$ are vertically opposite. Vertically opposite angles are equal.
b $a = 50$	Vertically opposite angles are equal. Notice that $a = 50$, not $a = 50°$, because the diagram already has the degrees sign after a. The letter a stands for the unknown number of degrees. A letter that stands for a number is called a pronumeral.
$50 + b + 107 = 180$ $157 + b = 180$ $b = 180 - 57$ $b = 123$	Adjacent angles making a straight angle add to 180°.
$c = 23$	Vertically opposite angles are equal.

Notice how finding the size of one angle leads to the next angle, and so on. Notice also that two methods have been used for labelling angles. In example 13 part a, the angles are named by referring to the letters at the vertices, for example, $\angle MOQ$. In example 13 part b, the unknown angles have been labelled with pronumerals to represent their sizes, for instance, $a°$.

Complementary and supplementary angles

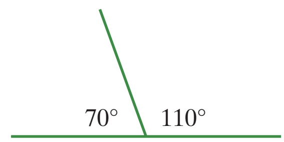
Complementary angles

Complementary angles are angles that add to 90°. The two angles 50° and 40° are complementary angles and we say that 40° is the **complement** of 50°.



Supplementary angles

Supplementary angles are angles that add to 180°. The two angles 110° and 70° are supplementary angles and we say that 70° is the **supplement** of 110°.



Example 14

Find the complement of the following angles.

a 23° **b** 89°

Find the supplement of the following angles.

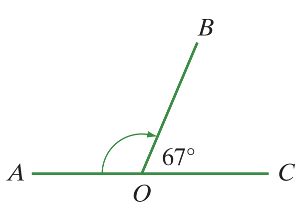
c 45° **d** 156°

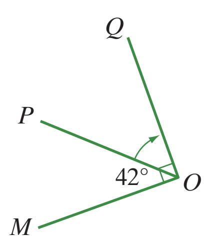
Working	Reasoning
a $90° - 23° = 67°$	Complementary angles add to 90°.
b $90° - 89° = 1°$	Complementary angles add to 90°.
c $180° - 45° = 135°$	Supplementary angles add to 180°.
d $180° - 156° = 24°$	Supplementary angles add to 180°.

Example 15

Find the size of the following angles.

a $\angle POQ$ **b** $\angle AOB$





continued

Example 15 continued

Working	Reasoning
a $\angle POQ = 90° - 42°$ $\angle POQ = 48°$	$\angle POQ$ and $\angle MOP$ are complementary angles—that is, they add to 90°.
b $\angle AOB = 180° - 67°$ $\angle AOB = 113°$	$\angle AOB$ and $\angle BOC$ are supplementary angles—that is, they add to 180°.

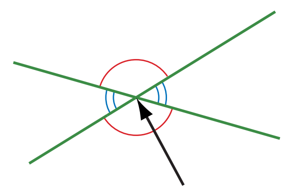
To remember which is complementary and which is supplementary, remember *C* comes before *S* in the alphabet and 90 comes before 180.

Vertically opposite angles

Vertically opposite angles

The point where two straight lines or segments intersect is called a vertex. Angles that are on the opposite side of a vertex are called **vertically opposite angles**.

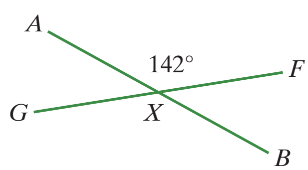
In the diagram below, the two angles marked in red are vertically opposite angles, and the two angles marked in blue are vertically opposite angles.



Example 16

Find the size of each of these angles.

a $\angle GXB$ **b** $\angle AXG$ **c** $\angle FXB$



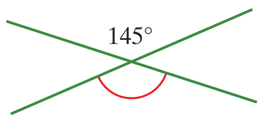
Working	Reasoning
a $\angle GXB = 142°$	$\angle GXB$ and $\angle AXF$ are vertically opposite angles and vertically opposite angles are equal.
b $\angle AXG = 180° - 142°$ $\angle AXG = 38°$	$\angle AXG$ and $\angle AXF$ together make a straight angle.
c $\angle FXB = 38°$	$\angle FXB$ and $\angle AXG$ are vertically opposite angles and vertically opposite angles are equal.

exercise 6.3

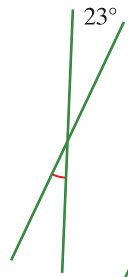
LINKS TO Example 11

Find the size of the vertically opposite angle, marked in red, in each of the following diagrams.

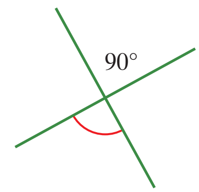
a



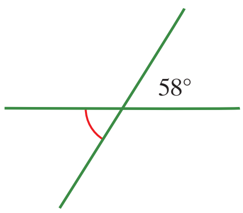
b



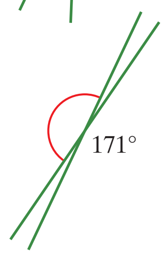
c



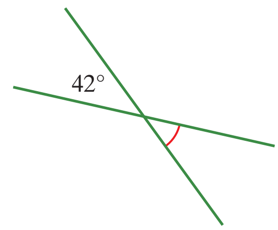
d



e



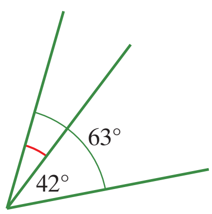
f



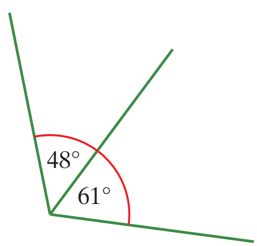
LINKS TO Example 12

Find the size of the angle marked in red in each of the following diagrams.

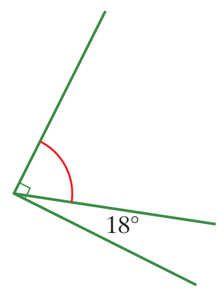
a



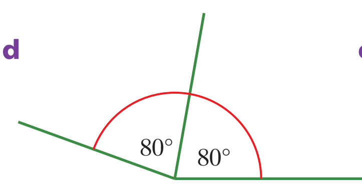
b



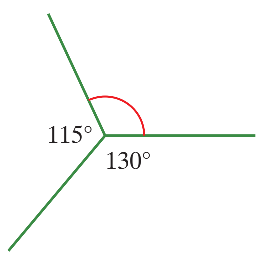
c



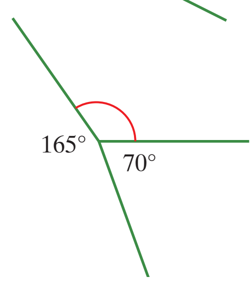
d



e



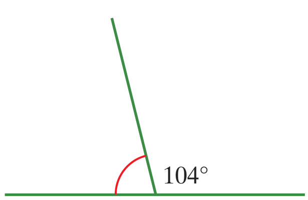
f



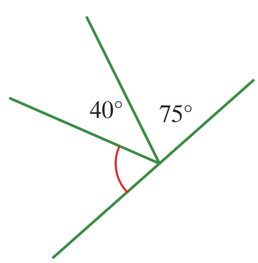
LINKS TO Example 13

Find the size of the angle marked in red in each of the following diagrams.

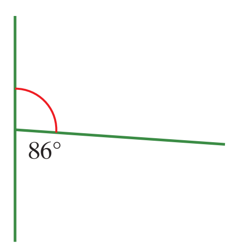
a



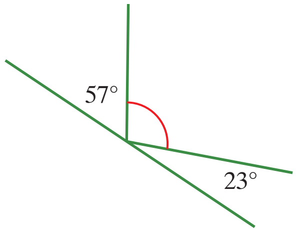
b



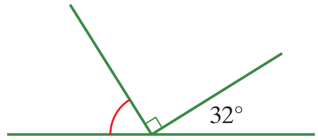
c



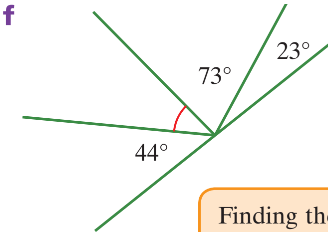
d



e



f

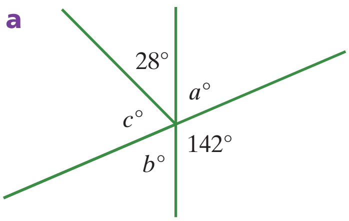


LINKS TO Example 14

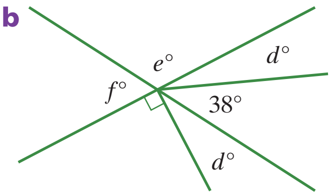
Find the value of the each of the pronumerals in the following diagrams.

> Finding the size of one angle leads us to the size of the next angle.

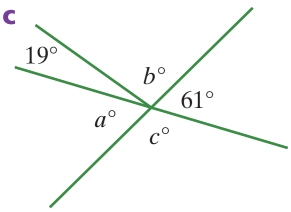
a



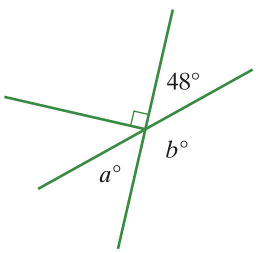
b



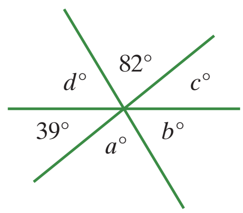
c



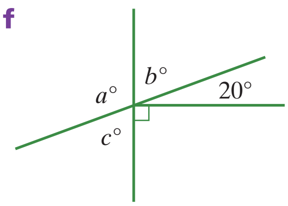
d



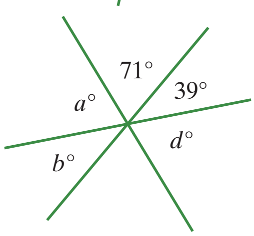
e



f



g



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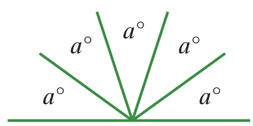
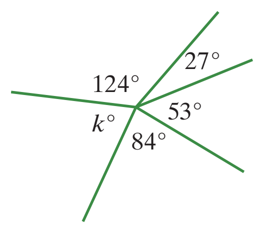


i



LINKS TO Example 15a

Copy and complete these tables of complementary angles.

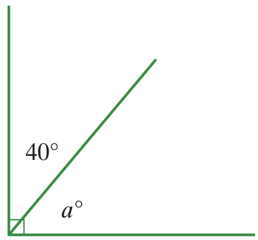
Angle	Complement
14°	
88°	
45°	
90°	

Angle	Complement
32°	
75°	
15°	
1°	

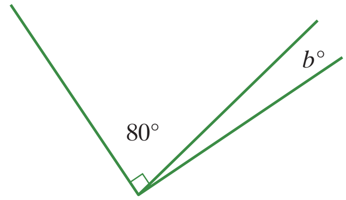
LINKS TO Example 16a

Find the value of the pronumeral in each of the following diagrams.

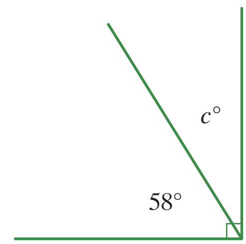
a



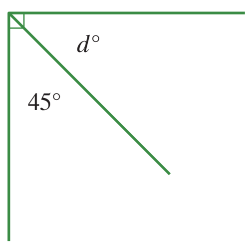
b



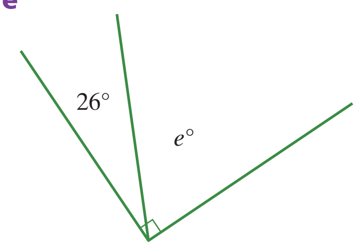
c



d



e



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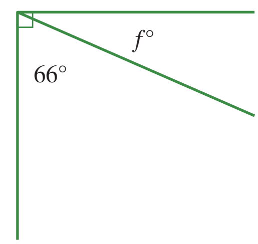


LINKS TO Example 15b

Copy and complete these tables of supplementary angles.

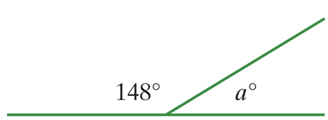
Angle	Supplement
160°	
23°	
91°	
100°	

Angle	Supplement
90°	
175°	
78°	
1°	

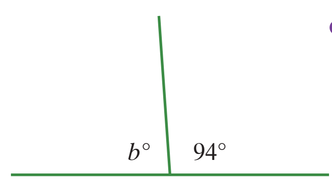
LINKS TO Example 16b

Find the size of the pronumeral in each of the following diagrams.

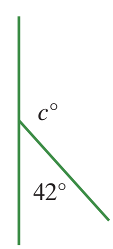
a



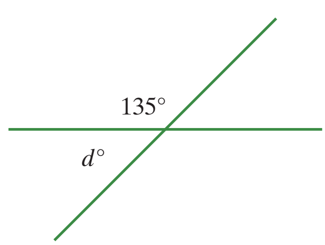
b



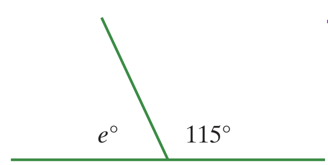
c



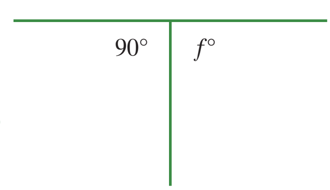
d



e



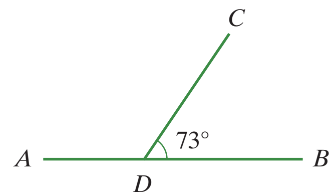
f



In this diagram $\angle BDC = 73°$.

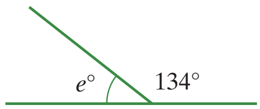
a Calculate the size of $\angle ADC$.

b Explain why $\angle ADC$ and $\angle BDC$ are supplementary angles.

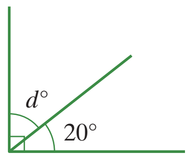


Find the values of the pronumerals in each of the following diagrams.

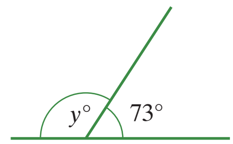
a



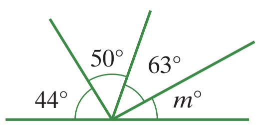
b



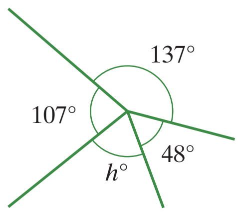
c



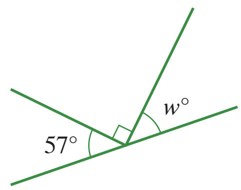
d



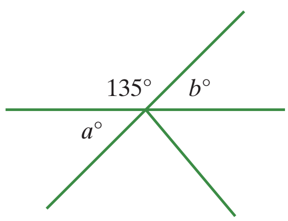
e



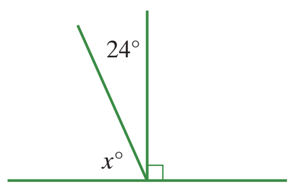
f



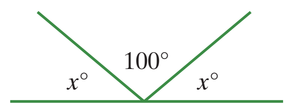
g



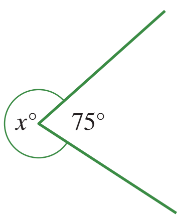
h



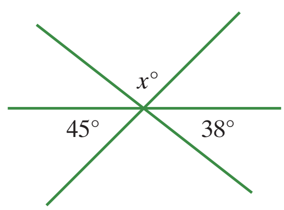
i



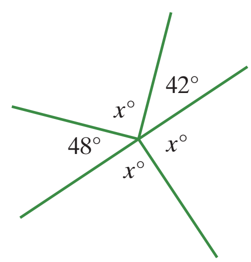
j



k

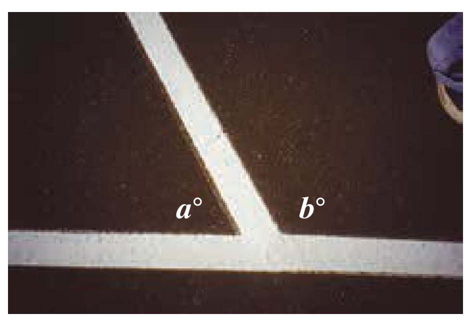


l

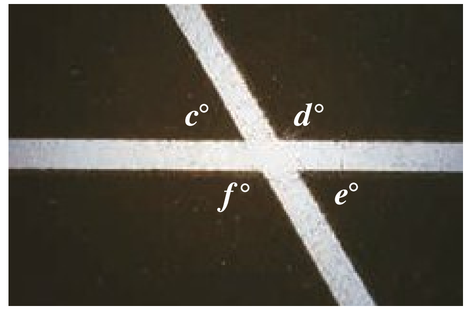


These photographs show the white lines marking out parking spaces in a car park.

a Use your protractor to measure the angle labelled $a°$. Without measuring it, find the size of the angle labelled $b°$.



b Use your protractor to measure the angle labelled $c°$. Without measuring them, find the sizes of the angles labelled $d°$, $e°$ and $f°$.



Use the map of part of the City of Melbourne and Carlton below to answer the questions.

a Use your protractor to measure the acute angle between La Trobe Street and Victoria Street at **A**.

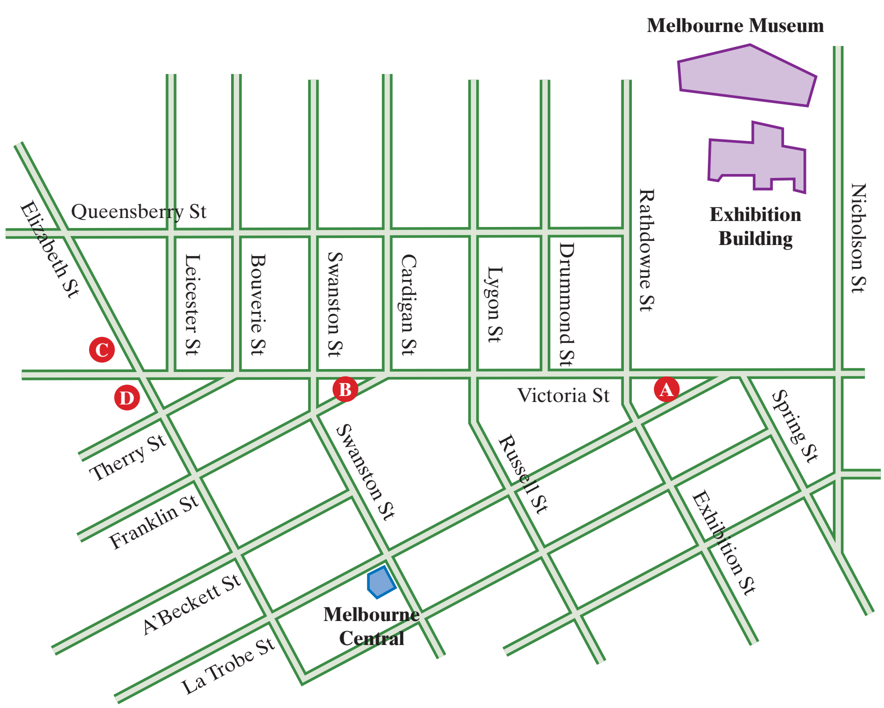
b Use your protractor to find the size of the acute angle that Franklin Street makes with Victoria Street at **B**. Compare it with the size of the acute angle that La Trobe Street makes with Victoria Street. What do you notice?

c Use your protractor to find the size of the acute angle between Elizabeth Street and Victoria Street at **C**.

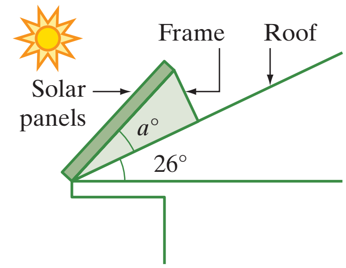
d Without using your protractor, what is the size of the obtuse angle at the junction of Elizabeth Street with Victoria Street at **D**? Explain your answer.

e Name a street that is parallel to Victoria Street.

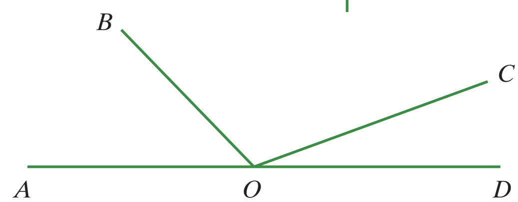
f Name two streets that are perpendicular to Franklin Street.



Daniel's roof makes an angle of 26° with the horizontal. The solar panels on the roof are tilted at an angle of 50° to the horizontal. What angle, $a°$, do the solar panels make with Daniel's roof?

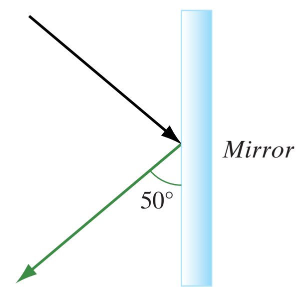


a Name an angle adjacent to $\angle AOB$.
b Name an angle supplementary to $\angle COD$.



Class activity
Pool angles

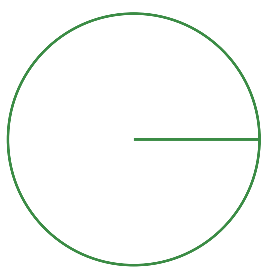
Light rays are reflected from a mirror at the same angle as they meet the mirror. If a light ray reflects off a mirror as shown, what angle does the incoming ray make with its reflection?



exercise 6.3 challenge

Bec cut a pizza into 10 equal pieces as shown.
a What is the angle size of each piece?
b Bec and Sarah each ate three pieces and Emma ate four pieces. For each of the three girls, find the total angle of the pieces eaten.

Zac was making a circular spinner for a game. He wanted to divide the spinner into four sectors so that one-quarter of the spinner was coloured blue, one-third green, one-fifth yellow and the remainder red. Zac drew a circle and marked a radius line as shown.
a Calculate the number of degrees for the blue, green and yellow sectors.
b Find the size of the angle for the red sector.
c Draw a circle and use your protractor to mark the correct angles for the four sectors of the spinner.



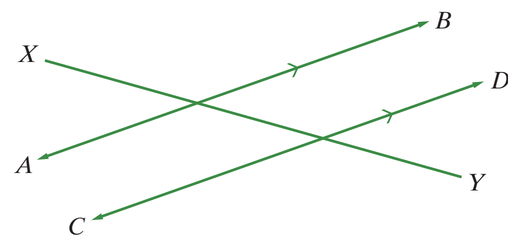
6.4 Angles and parallel lines

Parallel lines and transversals

Parallel lines

Lines and transversal

A line or line segment that cuts across a set of parallel lines is called a **transversal**. In this figure, the line segment XY is a transversal, cutting across the parallel lines AB and CD.

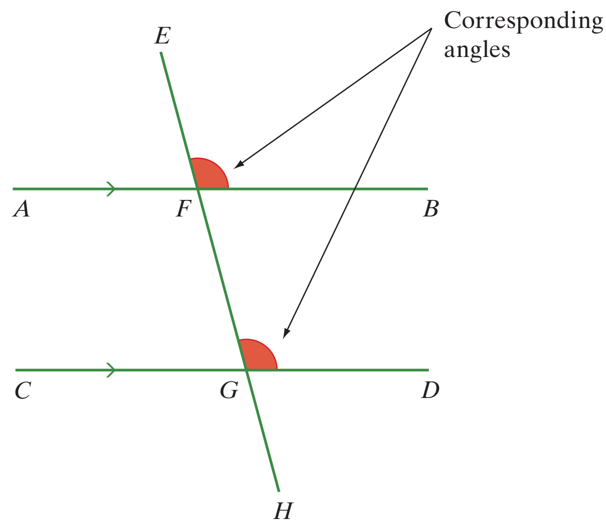


Class activity
Lines and transversals

Corresponding angles

When parallel lines are crossed by a transversal, **corresponding angles** are angles that are in corresponding positions on the same side of the transversal.

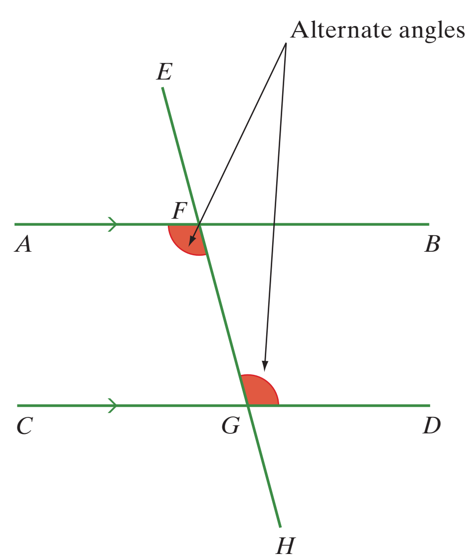
Corresponding angles are equal.



Alternate angles

Alternate angles are angles that are between a pair of parallel lines, but on opposite sides of the transversal.

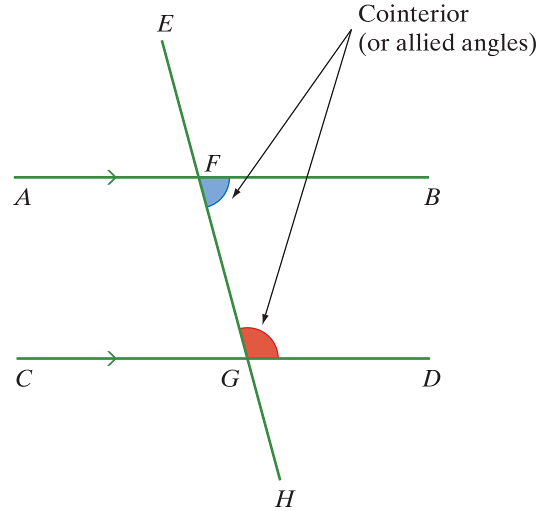
Alternate angles are equal.



Cointerior angles (or allied angles)

Cointerior angles are sometimes called allied angles. They are angles that are between a pair of parallel lines, but on the same side of the transversal.

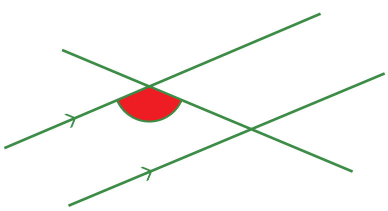
Cointerior angles are supplementary.



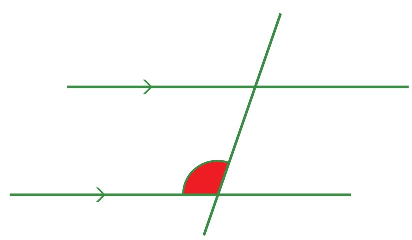
Example 17

Each of the following diagrams has one marked angle. Use a cross to mark each diagram with the angle indicated.

a alternate angle

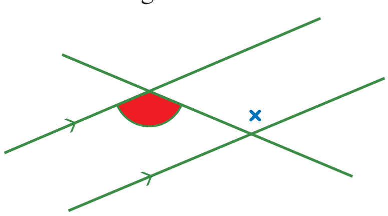


b cointerior angle



Working

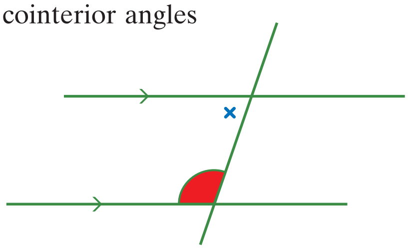
a alternate angles



Reasoning

Alternate angles are between the parallel lines and on opposite sides of the transversal.

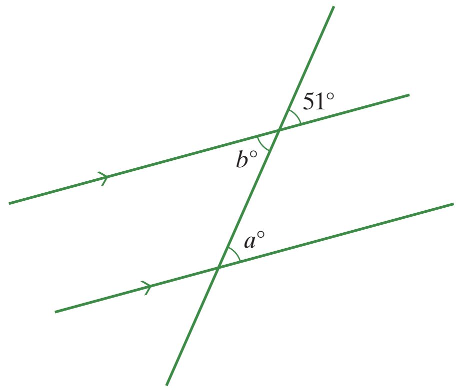
b cointerior angles



Cointerior angles are between the parallel lines and on the same side of the transversal.

Example 18

Find the values of the pronumerals and give reasons.



Working

$a = 51$ (Corresponding angles)

Reasoning

The angles marked $a°$ and 51° are corresponding angles. As the lines are parallel, the corresponding angles are equal.

$b = 51$ (Alternate angles)

The angles marked $a°$ and $b°$ are alternate angles. As the lines are parallel, the alternate angles are equal.

Or

$a = 51$ (Alternate angles)

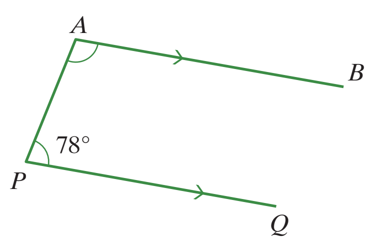
The angles marked $a°$ and $b°$ are alternate angles. As the lines are parallel, the alternate angles are equal.

$b = 51$ (Vertically opposite angles)

The angles marked $b°$ and 51° are vertically opposite angles, so they are equal.

Example 19

Find the size of $\angle PAB$.



Working

$\angle PAB = 180° - 78°$
$= 102°$ (cointerior angles)

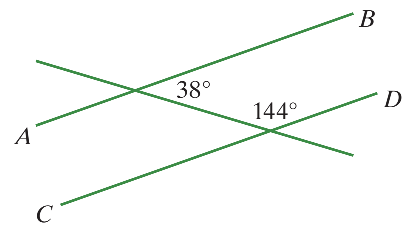
Reasoning

$\angle PAB$ and $\angle APQ$ are cointerior angles. As AB and PQ are parallel, the marked angles add to 180°.

When two lines are cut by a transversal and corresponding angles (or alternate angles) are equal, then the lines are parallel. Similarly, if the cointerior angles add to 180°, then the lines must be parallel.

Example 20

Are line segments *AB* and *CD* parallel?



Working

38° and 144° are cointerior angles.
The allied angles do not add to 180°.

Line segments *AB* and *CD* are not parallel.

Reasoning

38° and 144° are allied angles.
38 + 144 = 182°

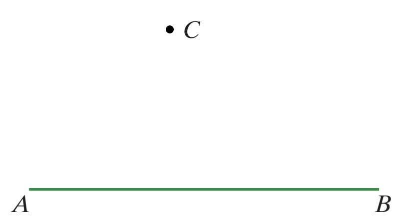
Cointerior angles formed by a transversal cutting across two line segments add to 180° if the line segments are parallel.

Constructing a line through a point parallel to a given line

A compass and ruler can be used to construct a line parallel to a given line through a given point.

Example 21

Construct a line *PQ* parallel to the line *AB* through the point *C*.

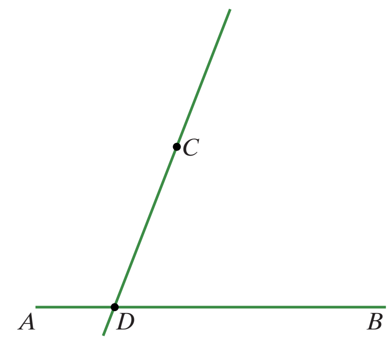


continued

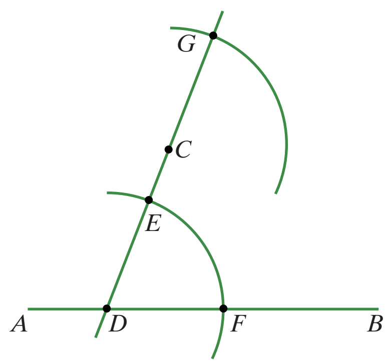
Example 21 continued

Working

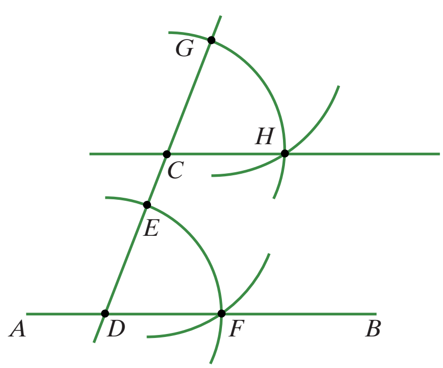
Reasoning



Choose a point D on the line AB. Rule a straight line through D and C.



Place the compass point at D and draw an arc to cut AB at F and DC at E. Keeping exactly the same compass opening, place the compass point at C and draw an arc to cut the line through DC at G.



Place the compass point at E and open so that the pencil exactly reaches F. Keeping exactly the same compass opening, place the compass point at G and draw an arc to cut the arc that goes through G. Label this point H.

Rule a line CH. This line is parallel to AB because corresponding angles GCH and EDF are equal.

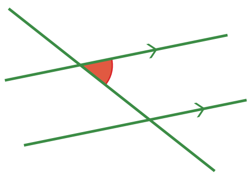
When a transversal cuts a pair of parallel lines, we know that the corresponding angles are equal. In the construction of a line parallel to another line in example 21, we have used the converse (opposite) argument: if corresponding angles are equal, then the lines must be parallel.

exercise 6.4

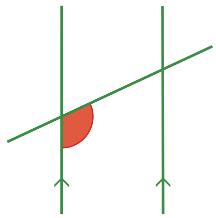
LINKS TO Example 17

Each of the following diagrams has one marked angle. Copy each diagram and mark with a cross the angle indicated, for example in part a, use a cross to mark the alternate angle to the marked angle.

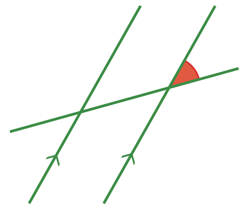
a alternate angle



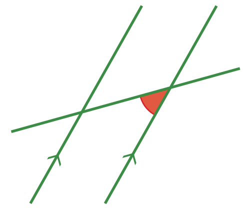
b cointerior angle



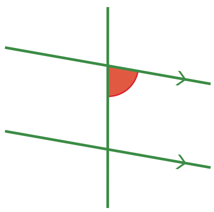
c vertically opposite angle



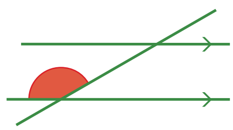
d alternate angle



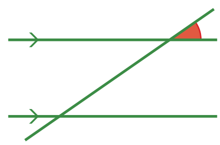
e corresponding angle



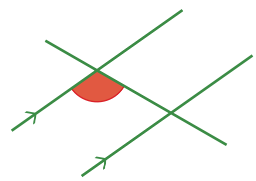
f alternate angle



g corresponding angle



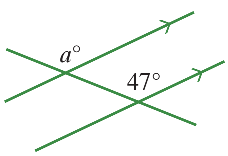
h cointerior angle



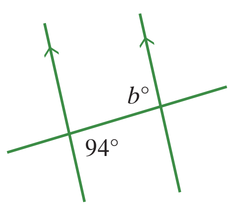
LINKS TO Example 18

Find the unknown angles in each of the following figures, giving reasons. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

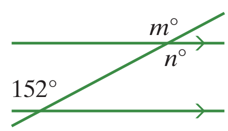
a



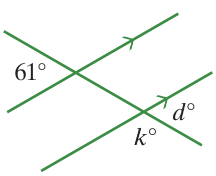
b



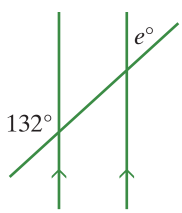
c



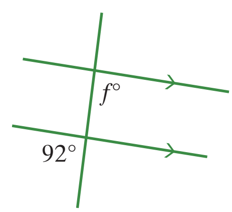
d



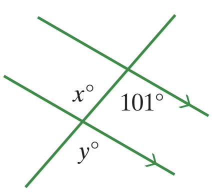
e



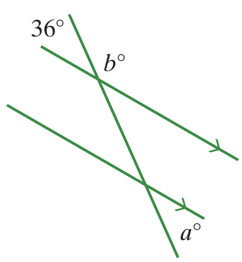
f



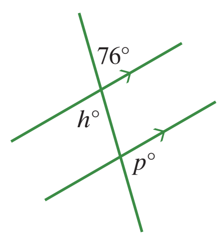
g



h



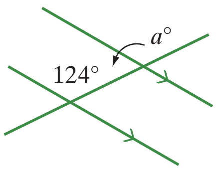
i



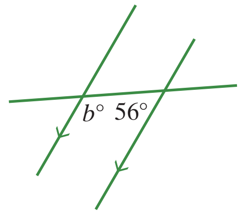
LINKS TO Example 19

Find the unknown angles in each of the following figures. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

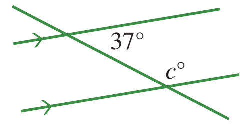
a



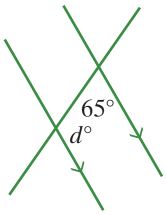
b



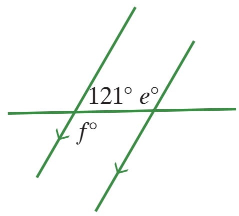
c



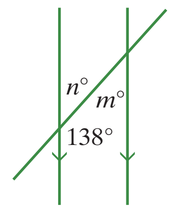
d



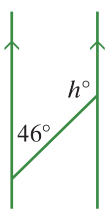
e



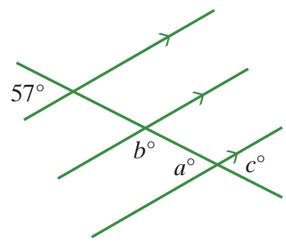
f



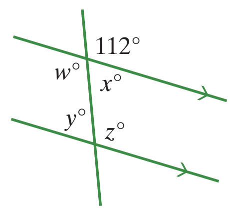
g



h



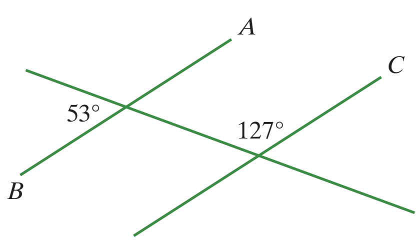
i



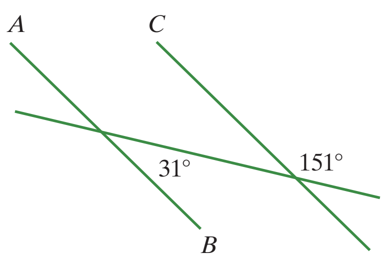
LINKS TO Example 20

State if the line segments AB and CD are parallel in each of the following figures. Justify each answer.

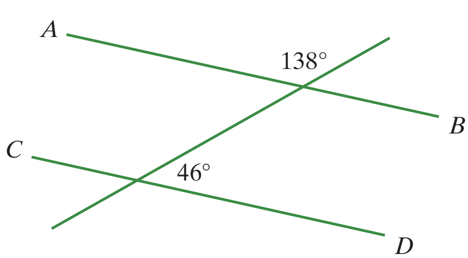
a



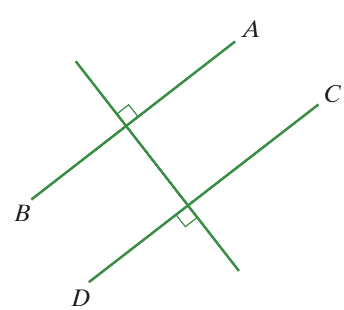
b



c

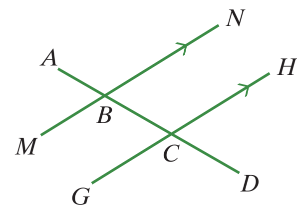


d

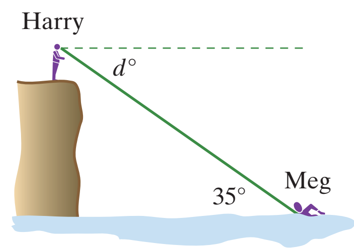


Name each of the following angles.

a the corresponding angle to $\angle ABN$
b the allied angle to $\angle MBC$
c the alternate angle to $\angle NBC$
d an angle that is adjacent to $\angle DCH$ and is the supplement of $\angle DCH$
e the corresponding angle to $\angle GCB$
f the allied angle to $\angle HCB$
g the angle vertically opposite $\angle GCD$
h all the angles that are the supplement of $\angle NBC$

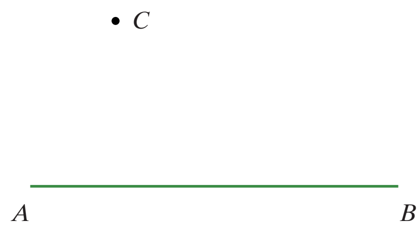


Harry is standing on top of a cliff, looking downwards at Meg who is swimming in the sea below. Meg looks up at Harry through an angle of 35°. Through what angle, d°, does Harry look down from the horizontal as he watches Meg? Justify your answer.



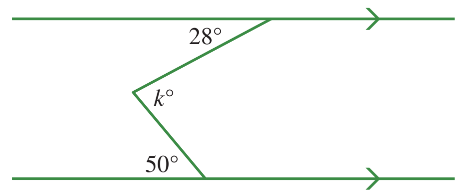
LINKS TO Example 21

Copy this diagram and construct a line parallel to AB through point C.



exercise 6.4 challenge

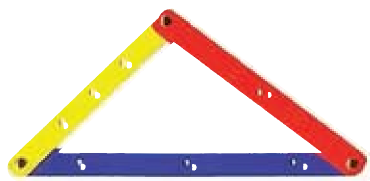
Work out the value of k. Try to find more than one method you could use.



6.5 Triangles

Class activity
Triangles in construction

Only one shape of triangle is possible when the three plastic strips shown here are joined. By contrast, when four strips are joined to form a quadrilateral, the shape can be changed by pushing on the sides. The quadrilateral can be made rigid by joining another strip (a diagonal) across the shape to form two triangles.

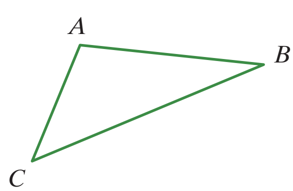


The triangle is a rigid shape.

The photographs below show roof trusses in a building construction, and a greenhouse in a plant nursery. In both cases, triangles have been used to make the structure strong.

Naming triangles

Each of the three corners of a triangle is called a **vertex**. Triangles are named according to letters at the three vertices (*vertices* is the plural of *vertex*). The symbol is used for the word triangle. We can start at any vertex, and go in either a clockwise direction or an anticlockwise direction. The triangle shown here could be named ABC, but we could also name it BCA or CAB or ACB or BAC or CBA.

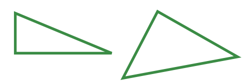
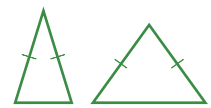
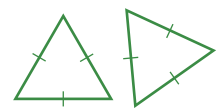


Triangles

Identifying triangles

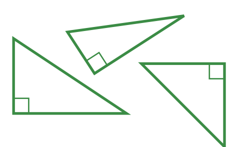
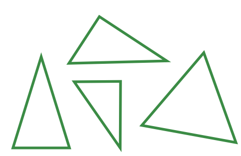
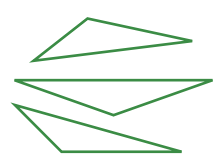
Triangles can be identified according to their side lengths, or according to the size of their angles.

Identifying triangles by the number of equal sides

Name	Number of equal sides	Example
Scalene	None	
Isosceles	Two	
Equilateral	Three	

Note: the same symbol shown on the sides indicates that the sides are equal.

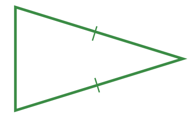
Identifying triangles by the sizes of their angles

Name	Type of angles	Example
Right-angled triangle	One angle is a right angle.	
Acute-angled triangle	All three angles are acute.	
Obtuse-angled triangle	One angle is obtuse.	

A triangle can be classified according to its side lengths and its angle sizes; for example, a right-angled scalene triangle, an acute-angled isosceles triangle.

Example 22

Describe the following triangle according to the length of its sides and according to the size of its angles.



Working

Acute-angled isosceles triangle.

Reasoning

All three angles are less than 90°, so the triangle is acute-angled.

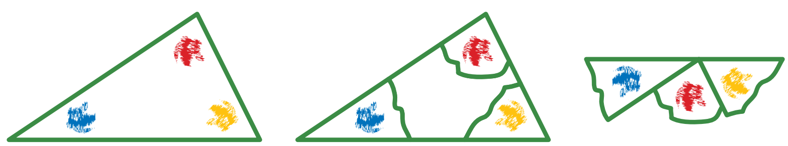
Two sides are equal so the triangle is an isosceles triangle.

The angles in a triangle

Class activity
Sum of the angles of a triangle

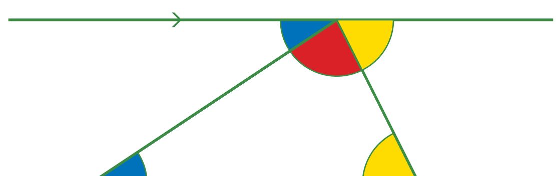
Angle sum of a triangle

If we draw a triangle and colour each of the three angles a different colour, we can tear off the three corners and arrange them together to form a straight line.



This suggests that the three angles of a triangle add to 180°. The diagram below shows why this is true.

- By drawing a line parallel to the base of the triangle, we can see that the blue angles are equal because they are alternate angles between the parallel lines.
- The yellow angles are also equal because they are alternate angles between parallel lines.
- We know that the sum of the red, blue and yellow angles at the top of the diagram is 180° because the three angles make a straight line.
- So we know that the three angles of the triangle must also add to 180°.



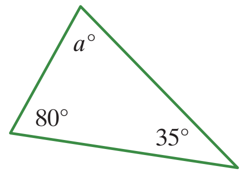
The three angles of all triangles add to 180°.

Finding unknown angles in triangles

Knowing that the three angles of a triangle add to 180°, if we are given two of the angles we can calculate the third angle.

Example 23

Find the value of a in this triangle.



Working

$80 + 35 + a = 180$

$115 + a = 180$

$a = 180 - 115$

$a = 65$

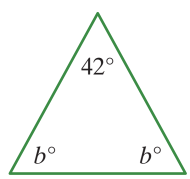
Reasoning

The three angles add to 180°.

We write $a = 65$, not $a = 65°$, because the angle label in the triangle already includes the degrees symbol.

Example 24

Find the value of b in this triangle.



Working

$180 - 42 = 138$

$138 \quad 2 = 69$

$b = 69$

Or

$b + b + 42 = 180$

$2b + 42 = 180$

$2b + 42 - 42 = 180 - 42$

$2b = 138$

$b = 69$

Reasoning

Subtract 42° from 180°.
Halve the difference.

The three angles add to 180°.

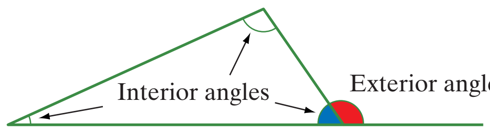
$b + b = 2b$

Divide both sides by 2.

Exterior angle of a triangle

Exterior angles of a triangle

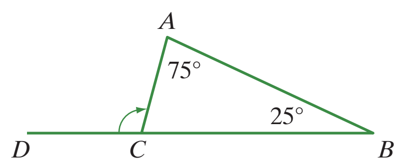
When the side of a triangle is extended, the angle that is formed outside the triangle is called an **exterior angle**. The three angles inside the triangle are **interior angles** of the triangle.



Notice that the exterior angle makes a straight line angle (180°) with the adjacent interior angle marked in blue.

Example 25

Find the size of the exterior angle $\angle ACD$.

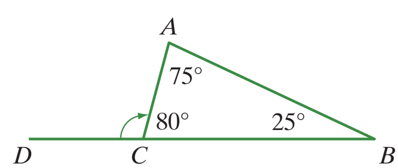


Working

$$\begin{aligned}\angle ACB + 75° + 25° &= 180°\\ \angle ACB + 100° &= 180°\\ \angle ACB &= 180° - 100°\\ \angle ACB &= 80°\end{aligned}$$

Reasoning

First find the adjacent interior angle, $\angle ACB$.
The three interior angles in a triangle add to 180°.



Straight angle

$\angle ACD + \angle ACB = 180°$

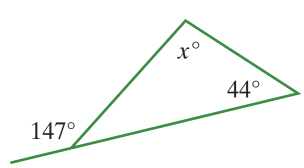
$\angle ACD + 80° = 180°$

$\angle ACD = 100°$

$\angle ACB = 80°$

Example 26

Find the value of the pronumeral.



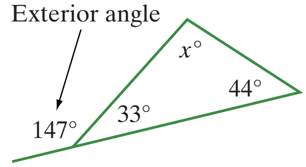
continued

Example 26 continued

Working

Adjacent interior angle to 147°
= 180° − 147°
= 33°

Reasoning

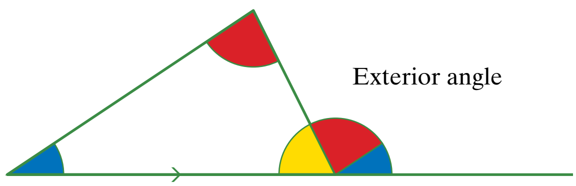


Exterior angle, 147°, and adjacent interior angle add to 180° because they form a straight line.

$x + 33 + 44 = 180$
$x + 77 = 180$
$x = 103$

Angles of a triangle add to 180°.

If we look at the torn-off corners and the matching diagram of the triangle again, we can see that the exterior angle of the triangle is equal to the sum of the red angle and the blue angle inside the triangle.

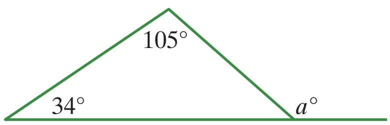


An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

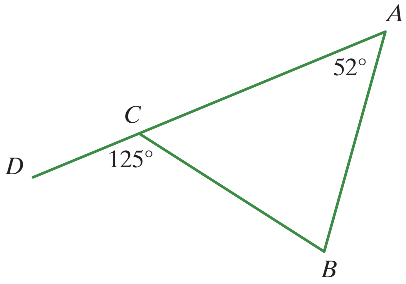
Example 27

Exterior angles are shown in each of these triangles.

a Find the value of a.



b Find the size of $\angle ABC$.



continued

Example 27 continued

Working	Reasoning
a $a = 105 + 34$ $a = 139$	An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
b $\angle ABC + 52° = 125°$	An exterior angle is equal to the sum of the two interior opposite angles.
$\angle ABC = 125° - 52°$	
$\angle ABC = 73°$	We write the degrees symbol because $\angle ABC$ is the angle, not a pronumeral.

Constructing triangles

Class activity Constructing special triangles

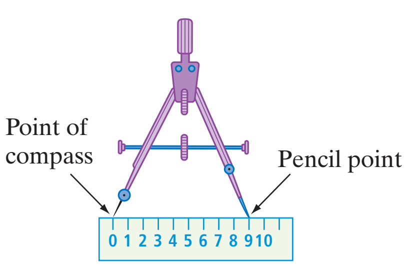
Triangles can be constructed using ruler, protractor and compass so that they have particular angle sizes or side lengths.

Example 28

Construct ABC so that side $AB = 10$ cm, $AC = 9$ cm and $BC = 7$ cm.

Working	Reasoning
Note: ABC means that the vertices of the triangle are A, B and C. **Step 1:** Using your ruler and pencil, carefully draw a line segment 10 cm long and label the ends A and B.	It does not matter which of the three sides we start with, but we have chosen here to start with the longest side.

Step 2: Place the point of your compass at 0 on your ruler and open the compass until the pencil point is at the 9 cm mark on the ruler.	By opening the compass to 9 cm we can use it to measure 9 cm from point A.

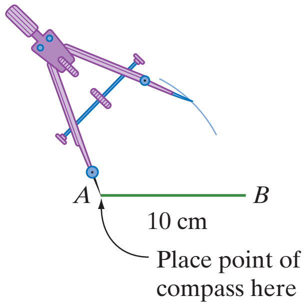


continued

Example 28 continued

Working

Step 3: Place the point of the compass at A and draw an arc (part of a circle) as shown.

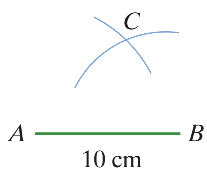


Reasoning

This arc represents a set of points that are all 9 cm away from A.

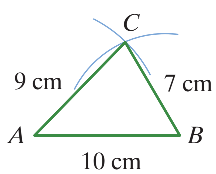
Step 4: Repeat step 2, but this time open the compass to 7 cm instead of 9 cm.

Step 5: Place the point of the compass at B and draw an arc that crosses your first arc.



By opening the compass to 7 cm we can use it to measure 7 cm from point B.

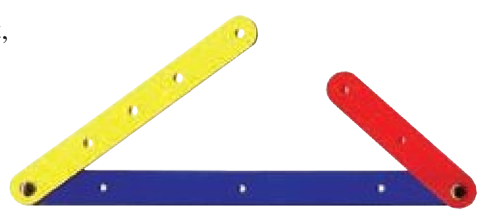
Step 6: Mark the point where the two arcs intersect and label it C. Using your ruler and pencil, join C to points A and B.



The second arc represents a set of points which are all 7 cm from B, so the point where the two arcs intersect is exactly 9 cm from A and 7 cm from B.

Can any three side lengths make a triangle?

As we can see from the three geometry strips at right, sometimes two sides are not long enough to meet. For example, a triangle cannot be constructed if the side lengths are 3 cm, 4 cm and 9 cm because the two shortest sides will not meet.



The sum of the two shortest sides of a triangle must be greater than the third side.

Example 29

For which of these sets of lengths is it possible to construct a triangle?

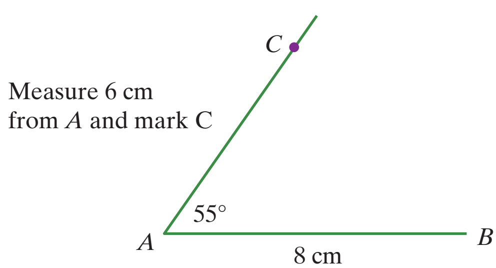
a 3 cm, 5 cm, 11 cm **b** 4 cm, 6 cm, 7 cm **c** 2.5 cm, 3 cm, 5.5 cm

Working	Reasoning
a 3 cm + 5 cm < 11 cm A triangle cannot be constructed.	The sum of the two shortest sides is less than the third side, so the sides will not meet.
b 4 cm + 6 cm > 7 cm A triangle can be constructed.	The sum of the two shortest sides is greater than the third side, so the sides will meet.
c 2.5 cm + 3 cm = 5.5 cm A triangle cannot be constructed.	The sum of the two shortest sides is equal to the third side. Although they meet to make the length of the third side, they cannot make a triangle.

Example 30

Construct $\triangle ABC$ so that side $AB = 8$ cm, $\angle CAB = 55°$ and $AC = 6$ cm.

Working	Reasoning
Step 1: Use ruler to draw a segment 8 cm long and label the ends A and B.	
Step 2: Place the centre mark of your protractor at A and construct an angle of 55°.	As the given angle is $\angle CAB$, the vertex of this angle is at A.

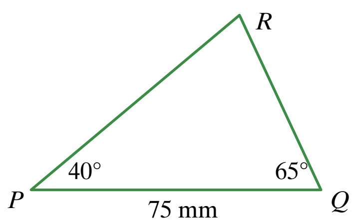


Working	Reasoning
Step 3: Measure 6 cm along the arm of the angle you have constructed and mark the point C.	As AC must be 6 cm.
Step 4: Join B and C to form the third side of the triangle. Erase the extra line beyond point C.	

Example 31

Construct and label a triangle PQR where $PQ = 75\,\text{mm}$, $\angle RPQ = 40°$ and $\angle RQP = 65°$.

Working	Reasoning
Step 1: Use a ruler to draw a segment 75 mm long and label the ends P and Q.	Start with the given side.
Step 2: Place the centre mark of your protractor at P and construct an angle of 40°.	Look at the letter in the centre of the three letters in the angle names. The 40° angle is $\angle RPQ$ so make the 40° angle at P.
Step 3: Place the centre mark of your protractor at Q and construct an angle of 65°.	The 65° angle is $\angle RQP$ so make the 65° angle at Q.
Step 4: If necessary, extend the angle arms from P and Q till they meet. This point is R. Erase the extra lines beyond R.	Check: $\angle PRQ$ should be $180° - (40° + 65°)$, that is, 75°.

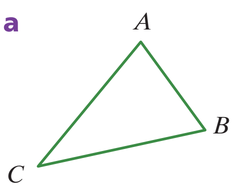


exercise 6.5

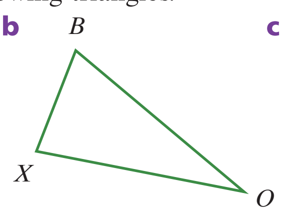
LINKS TO Example 22

Name each of the following triangles.

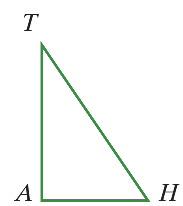
a



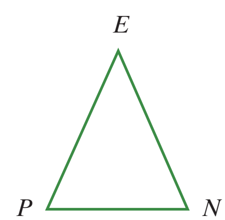
b



c



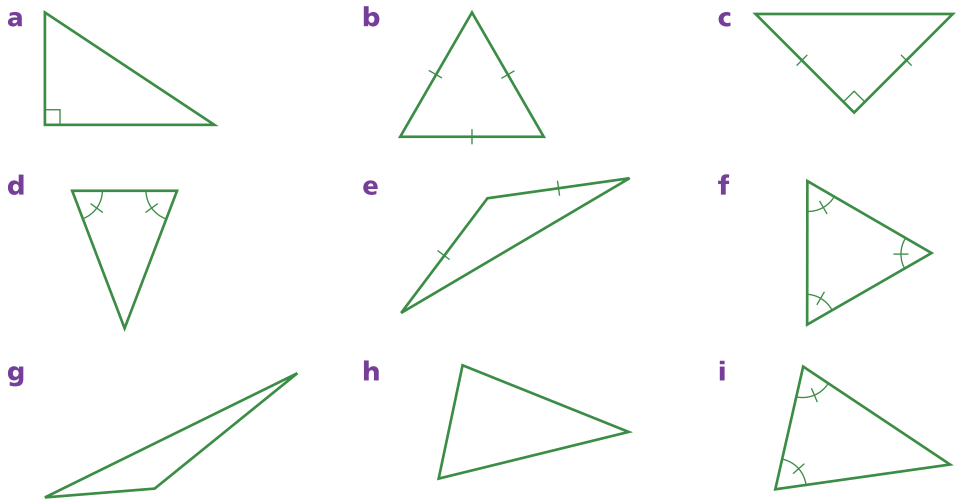
d



LINKS TO Example 22

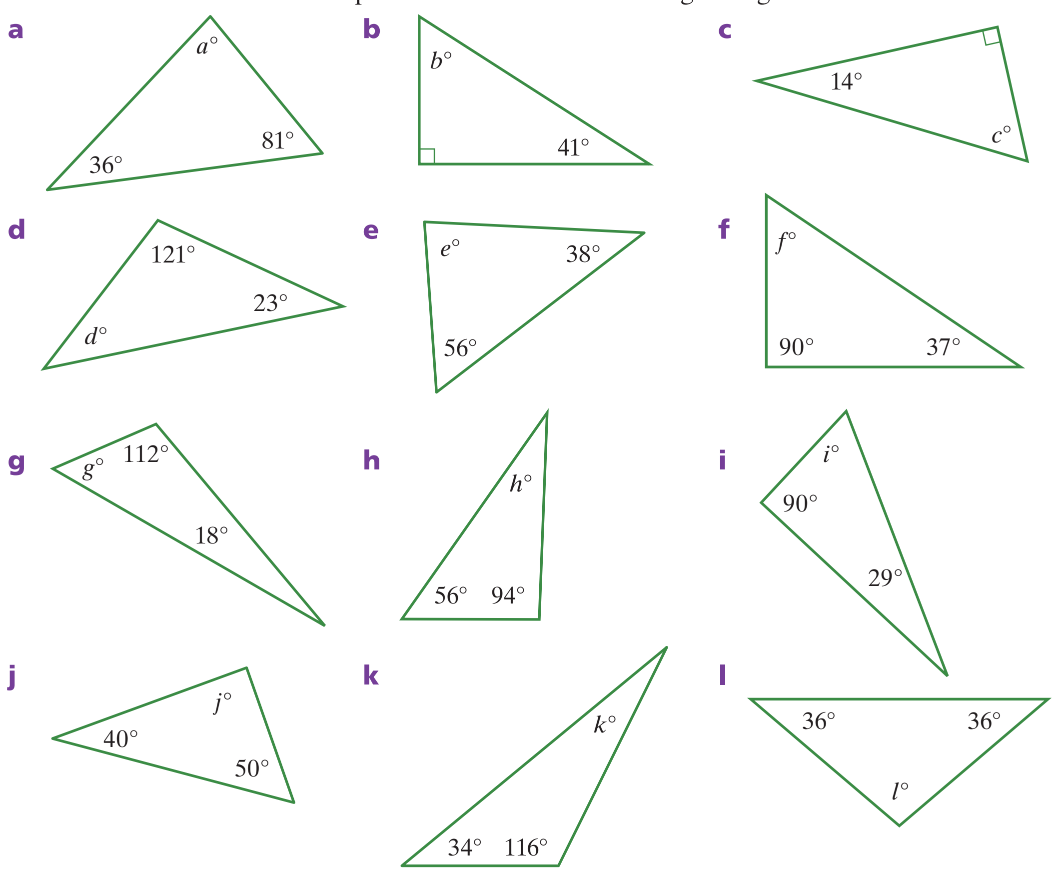
What are the special type of triangles formed by the supporting pillars and the floor in this building? Justify your answer.

Describe each of the following triangles according to the length of its sides and according to the size of its angles; for example, acute-angled isosceles triangle.



LINKS TO Example 23

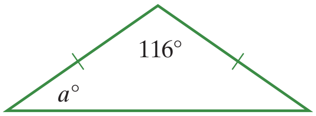
Find the value of each of the pronumerals in the following triangles.



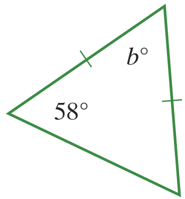
LINKS TO Example 24

Find the value of the pronumerals in each of the following triangles.

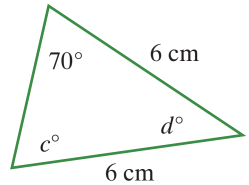
a



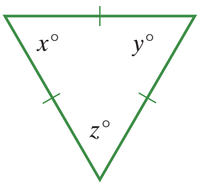
b



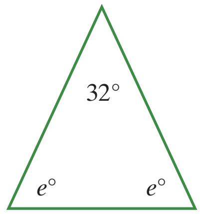
c



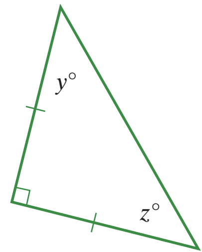
d



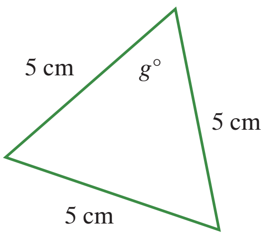
e



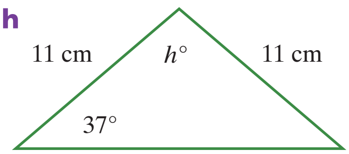
f



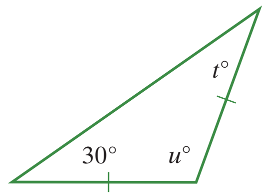
g



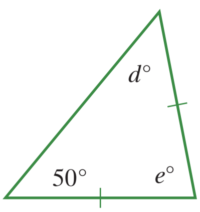
h



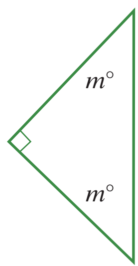
i



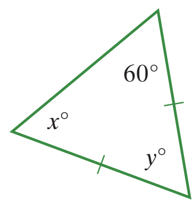
j



k



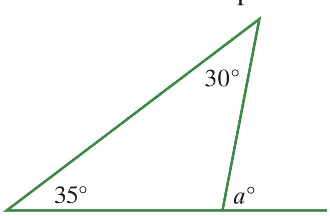
l



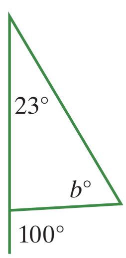
LINKS TO Examples 25, 26, 27

Find the value of the pronumerals in each of the following triangles.

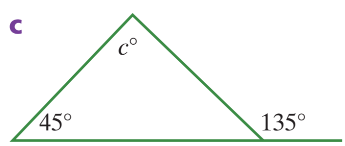
a



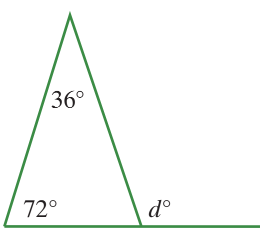
b



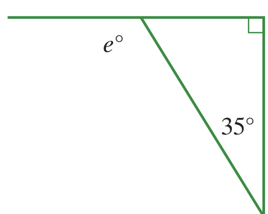
c



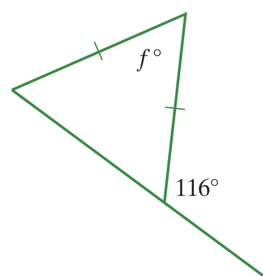
d



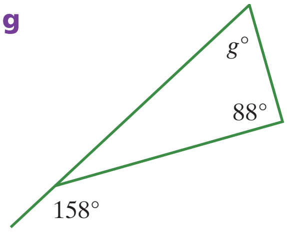
e



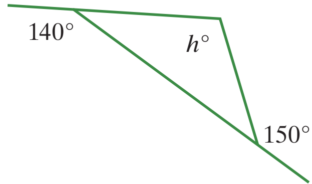
f



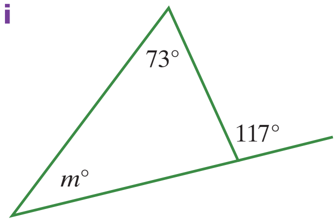
g



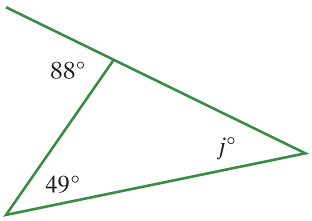
h



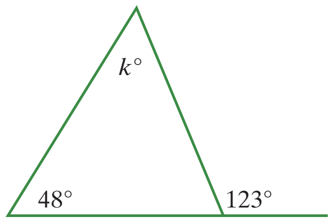
i



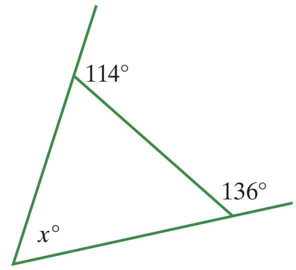
j



k



l

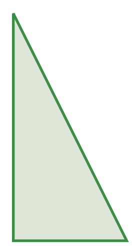


BLM Federation Square triangles

The design on the outside of the Federation Square buildings in Melbourne, shown in this photograph, is based on just one triangle shape.

a What sort of triangle is it? Classify it in terms of its sides and its angles.

b Copy the triangle shape and use it as a template to carefully cut out four identical triangles in coloured paper. Arrange two of the triangles to make an isosceles triangle. Now arrange the other two triangles to make a different isosceles triangle. Paste your two different isosceles triangles into your mathematics exercise book or onto a piece of paper.



LINKS TO Example 28

Use your compass, ruler and sharpened pencil to construct the following triangles. Label the vertices with the given letters. Label the given side measurements. Beside each triangle state the type of triangle

i according to the angles. **ii** according to the side lengths.

a $AB = 9\text{ cm}, AC = 8\text{ cm}, BC = 5\text{ cm}$

b $AB = 10\text{ cm}, AC = 6\text{ cm}, BC = 8\text{ cm}$

c $AB = 5\text{ cm}, AC = 5\text{ cm}, BC = 5\text{ cm}$

d $AB = 7\text{ cm}, AC = 7\text{ cm}, BC = 6\text{ cm}$

e $AB = 3\text{ cm}, AC = 4\text{ cm}, BC = 5\text{ cm}$

f $PQ = 8\text{ cm}, PR = 5\text{ cm}, QR = 5\text{ cm}$

g $PQ = 13\text{ cm}, PR = 12\text{ cm}, QR = 5\text{ cm}$

h $PQ = 8\text{ cm}, PR = 6\text{ cm}, QR = 6\text{ cm}$

LINKS TO Example 29

Is it possible to construct triangles with these side lengths? Explain.

a 5 cm, 3 cm, 9 cm
b 7.5 cm, 6 cm, 5 cm
c 4 cm, 2 cm, 2 cm
d 4 cm, 3 cm, 3 cm
e 5 cm, 4 cm, 9 cm
f 6 cm, 8 cm, 16 cm
g 10 cm, 3 cm, 11 cm
h 7 cm, 9 cm, 20 cm

LINKS TO Example 30

Using your protractor, ruler and pencil, accurately construct the following triangles. Label the vertices of each triangle with the given letters, and label the given angle and side measurements. Beside each triangle state the type of triangle

i according to the angles.
ii according to the side lengths.

a $BC = 6$ cm, $\angle ABC = 40°$, $AB = 7$ cm
b $BC = 7$ cm, $\angle ABC = 35°$, $AB = 7$ cm
c $BC = 8$ cm, $\angle ABC = 30°$, $AB = 8$ cm
d $BC = 6$ cm, $\angle ACB = 45°$, $AC = 4$ cm
e $AB = 9$ cm, $\angle CAB = 60°$, $AC = 4$ cm
f $AB = 10$ cm, $ABC = 55°$, $BC = 8$ cm
g $EF = 8$ cm, $EFG = FEG = 30°$
h $KL = 5$ cm, $KLM = 90°$, $LM = 5$ cm

LINKS TO Example 31

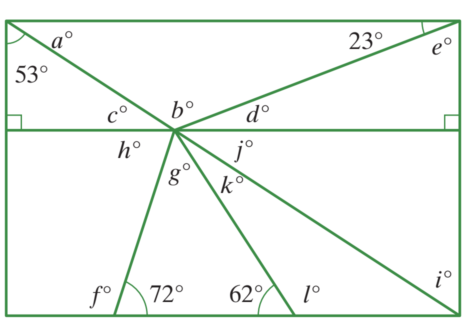
Using your protractor, ruler and sharpened pencil, construct and label the following triangles. Label the vertices with the given letters. Label the given side and angle measurements. Beside each triangle state the type of triangle

i according to the angles.
ii according to the side lengths.

a $BC = 8$ cm, $\angle ABC = 25°$ and $\angle ACB = 50°$
b $BC = 6$ cm, $\angle ABC = 40°$ and $\angle ACB = 50°$
c $BC = 6$ cm, $\angle ABC = 55°$ and $\angle ACB = 20°$
d $BC = 9$ cm, $\angle ABC = 50°$ and $\angle ACB = 40°$
e $AB = 5$ cm, $\angle CAB = 40°$ and $\angle CBA = 40°$
f $AB = 7$ cm, $\angle CAB = 70°$ and $\angle CBA = 20°$
g $PQ = 8$ cm, $\angle RPQ = 35°$ and $\angle RQP = 55°$
h $PQ = 10$ cm, $\angle RPQ = 40°$ and $\angle RQP = 40°$

exercise 6.5 challenge

The rectangle below is divided into a number of triangles. Find the values of all the pronumerals. Hint: start by finding a, then b, c, and so on.



Finding the size of one angle leads us to the next angle.

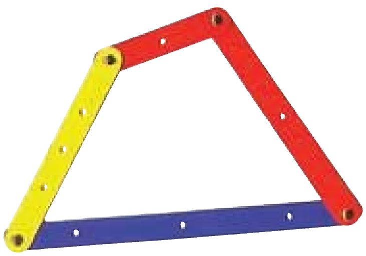
6.6 Quadrilaterals

Class activity
Sum of the angles of a quadrilateral

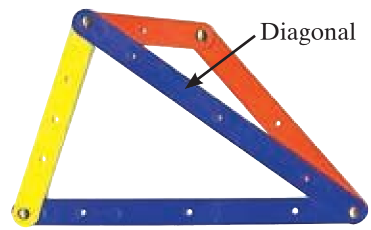
Angle sum of a quadrilateral

Plane (two-dimensional) figures with four straight sides are called **quadrilaterals**. 'Quadri' comes from the Latin word meaning 'four' and 'lateral' comes from the Latin word meaning 'side'.

Unlike triangles, quadrilaterals are not rigid shapes. When four plastic strips are joined, as shown below, the shape can be changed by pushing on the sides. Joining two vertices with a diagonal to make two triangles, as shown, makes the quadrilateral rigid.



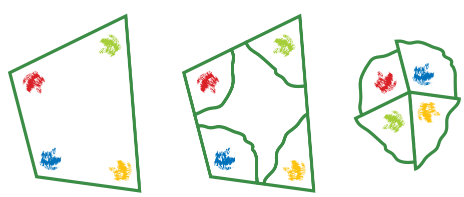
Quadrilaterals are not rigid.



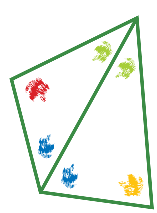
The diagonal makes the quadrilateral rigid.

Angles in quadrilaterals

When we tear the corners from a quadrilateral and rearrange them, we find that they fit around a point so that there are no gaps or overlaps. This suggests that the four angles of a quadrilateral add to 360°.



In the diagram, the quadrilateral is divided into two triangles. The angles of each triangle add to 180°, so altogether the angles add to 360°.



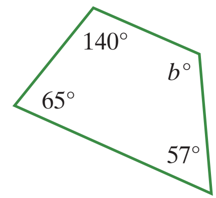
The four angles of all quadrilaterals add to 360°.

Finding unknown angles in quadrilaterals

If we know three of the angles of a quadrilateral, we can calculate the fourth angle.

Example 32

Calculate the value of b in this quadrilateral.



Working

$140 + 65 + 57 = 262$

$360 - 262 = 98$

So $b = 98$

Or

$140 + 65 + 57 + b = 360$

$262 + b = 360$

$b = 98$

Reasoning

The four angles must add to 360°.
Add the three given angles.
Subtract 262° from 360°.

The four angles must add to 360°.

Special quadrilaterals

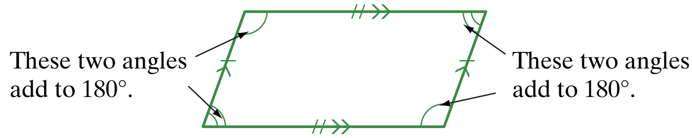
All quadrilaterals have certain properties in common—they all have four sides and their four angles add to 360°. However, there are several special groups of quadrilaterals which have extra features in common.

The parallelogram family

Parallelogram family

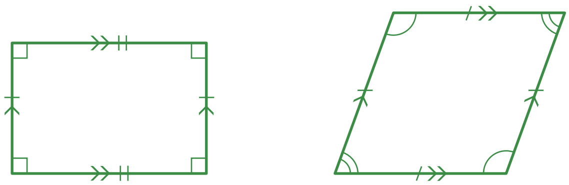
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. Parallelograms also have the following properties:

- both pairs of opposite sides are equal
- opposite angles are equal
- each pair of adjacent angles are supplementary (add up to 180°).



Rectangles, **rhombuses** and **squares** share the same properties as parallelograms, so they are all special members of the parallelogram family. However, they each have their own special properties as well.

- Rectangles have four right angles.
- Rhombuses have four equal sides.
- Squares have four equal sides and four right angles.



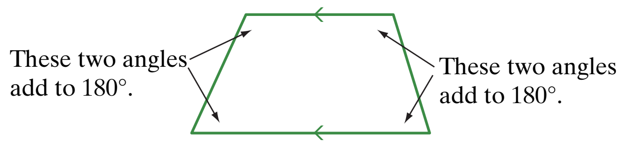
Rectangle   Rhombus   Square

Notice that squares are special rectangles and special rhombuses. We can think of a square as a rectangle with its four sides equal and we can think of a square as a rhombus with all its angles right-angles.

Trapeziums

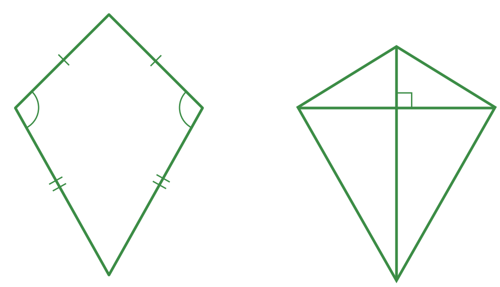
Trapeziums (or trapezia) have the following properties:

- one pair of opposite sides are parallel
- two pairs of supplementary angles.



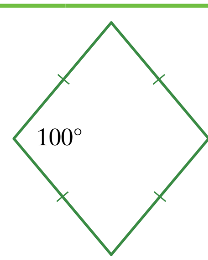
Kites

- Two pairs of adjacent (next to each other) sides are equal.
- One pair of opposite angles are equal.
- Diagonals are perpendicular.
- One diagonal bisects the unequal angles.



Example 33

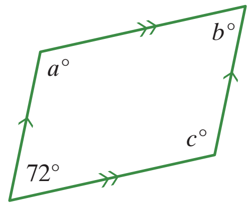
Identify this quadrilateral.



Working	Reasoning
	The quadrilateral has four equal sides so it could be a square or a rhombus.
The quadrilateral is a rhombus.	It does not have four right angles, so it is not a square.

Example 34

Find the values of a, b and c.



Working

$a = 180 - 72$
$= 108$

$b = 72$

$c = 108$

Reasoning

Adjacent angles of a parallelogram are supplementary.

Opposite angles of a parallelogram are equal.

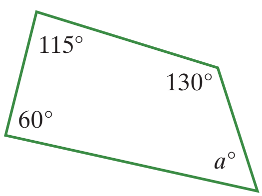
Opposite angles of a parallelogram are equal.

exercise 6.6

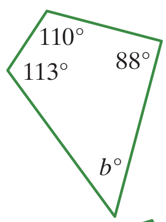
LINKS TO Example 32

1. For each of these quadrilaterals, calculate the value of the pronumeral.

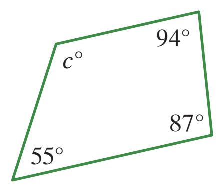
a



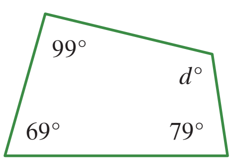
b



c



d



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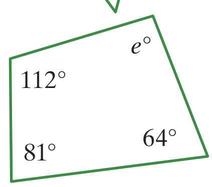
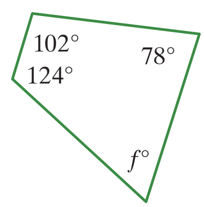


f



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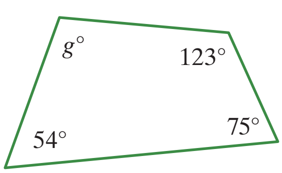

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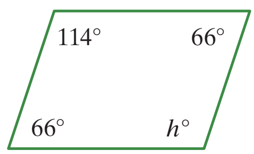
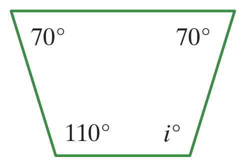
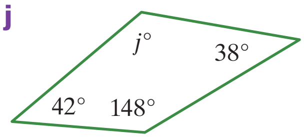


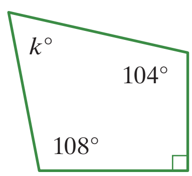
i



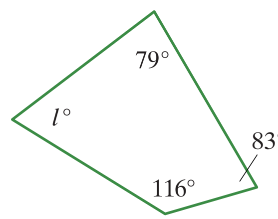
j



k



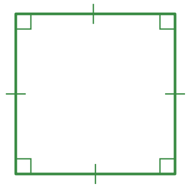
l



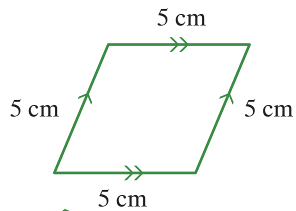
LINKS TO Example 33

Identify each of these quadrilaterals.

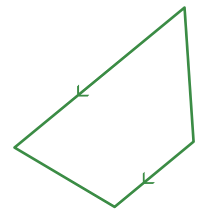
a



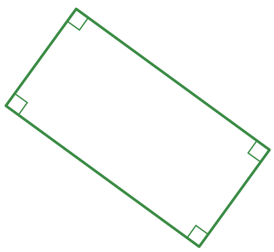
b



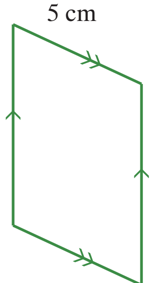
c



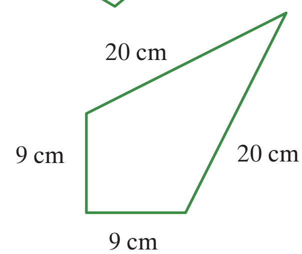
d



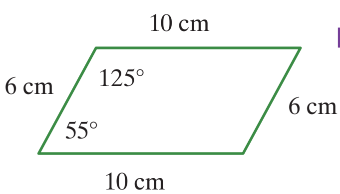
e



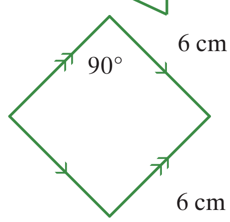
f



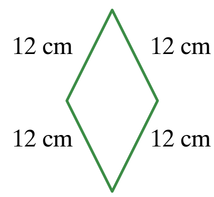
g



h



i

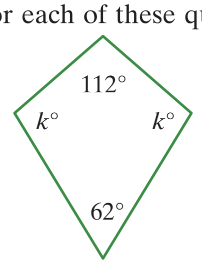


Name three special quadrilateral shapes in the paving bricks shown here.

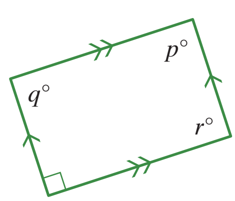
LINKS TO Example 34

For each of these quadrilaterals, calculate the value of each pronumeral.

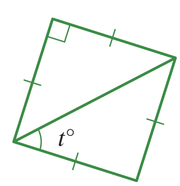
a



b



c



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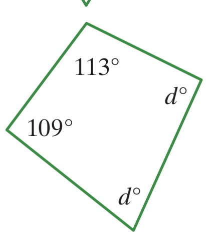

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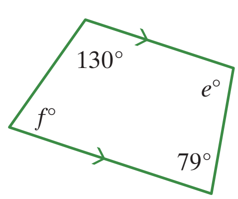

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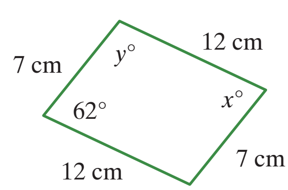

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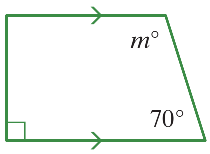

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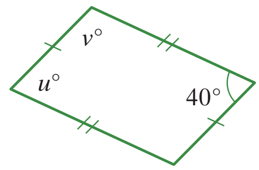
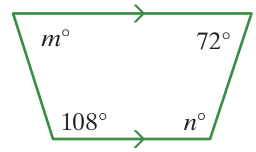
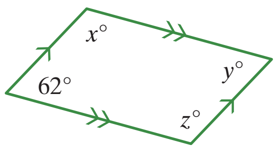


i



j



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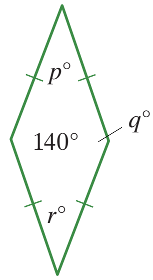
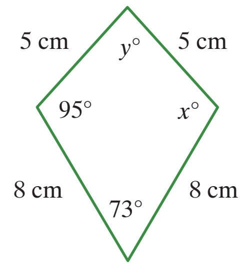
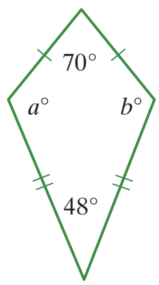


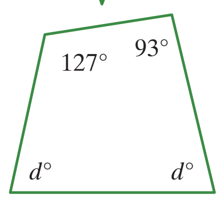
l



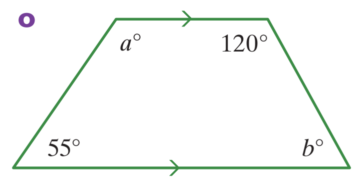
m



n

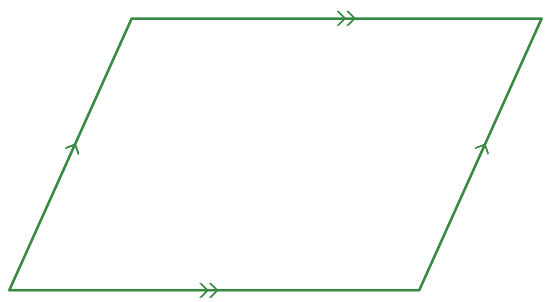


o



Class activity
Investigating special quadrilaterals

We define a parallelogram as a quadrilateral that has both pairs of opposite sides parallel. The following figure is a parallelogram because both pairs of opposite sides are parallel.

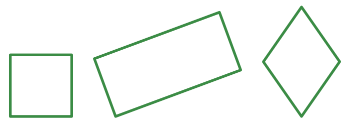


a Measure the lengths of the four sides and label them on a copy of the parallelogram in your workbook.

b Copy and complete the following sentence: Both pairs of opposite sides of a parallelogram are ______________ and ______________.

c Measure the four angles of the parallelogram and label the sizes on your copy. Copy and complete the following sentences: The opposite angles of a parallelogram are ______________. The adjacent angles of a parallelogram add to ______________°.

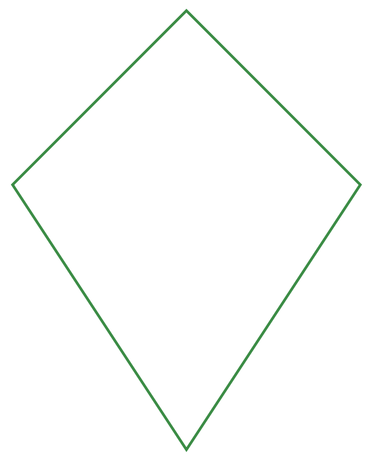
Alice drew three quadrilaterals—a square, a rectangle and a rhombus. She decided that the three special quadrilaterals all belonged to the family of parallelograms.



a Why did Alice decide this? Do all of Alice's shapes fit the definition of a parallelogram?

b Copy and complete these descriptions of the special members of the family of parallelograms: A ______________ is a parallelogram with four equal sides. A ______________ is a parallelogram with four right angles. A ______________ is a parallelogram with four equal sides and four right angles.

The following shape is a kite.

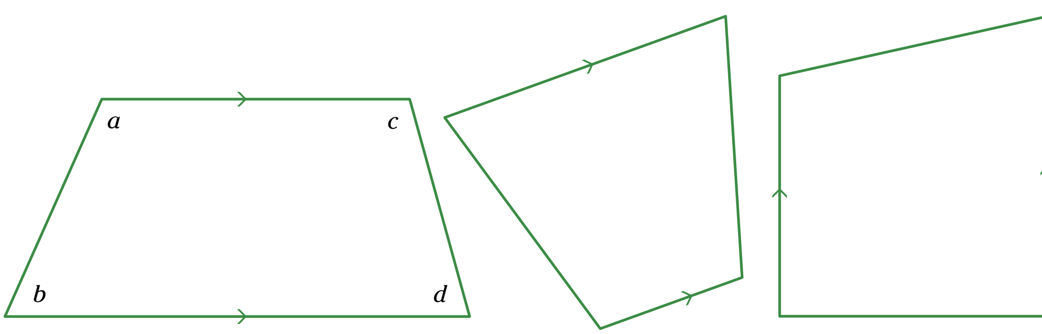


a Measure the four sides and the four angles of the kite. Label the lengths and angles on a copy of the diagram.

b Copy and complete the following sentences: The kite has two pairs of equal ______________ and one pair of equal ______________ .

c Notice how the right and left halves of the kite are identical; that is, the kite is a symmetrical shape. Rule a line on your diagram to show the axis of symmetry (that is, the line along which you could fold the kite into two identical halves).

The following shapes are all trapezia.



a Measure the size of the each of the angles in each trapezium and label the sizes on copies of the trapezia in your workbook.

b How many pairs of supplementary angles does each trapezium have?

6.6

Scissor lift

Umbrella

Car jack

Expanding toolbox

- The design of each of these objects is based on a special quadrilateral.

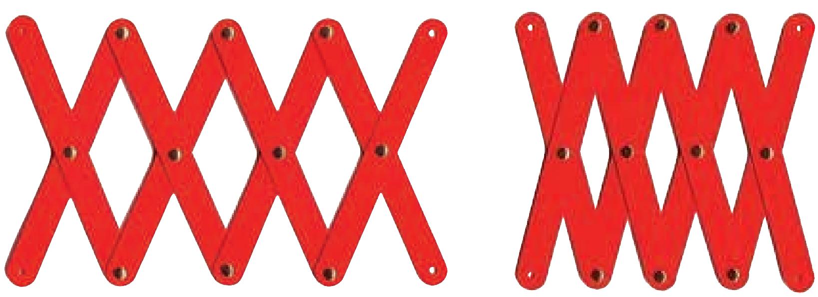
Bathroom mirror on expanding bracket

Scissor lift

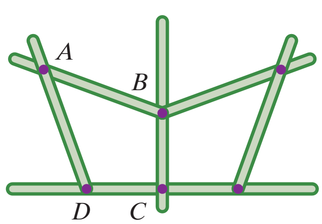
Class activity
Hinged quadrilaterals

a What is the special quadrilateral shape?

b Use this plastic geostrip model to help you work out why this special quadrilateral is useful in the design of the objects.



- This diagram shows a folding music stand. $AB = AD$ and $BC = DC$.



a What special quadrilateral shape is $ABCD$?

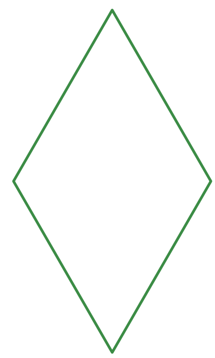
b When the music stand is in its fully opened position as shown, $ADC = 110°$. Find the size of DCB, CBA, and BAD.

The photograph below shows metal bike lockers for people to store their bikes at railway stations. The four lockers fit together in pairs, with the doors opening on alternate sides as shown in the floor plan diagram.

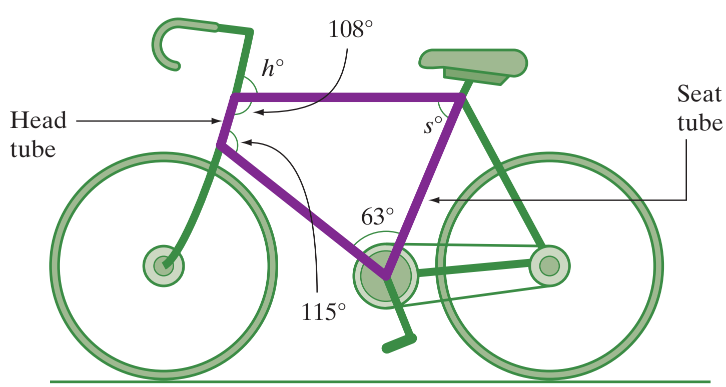
a What is the special quadrilateral shape of each locker shown in the floor plan diagram?

b Why do you think this shape has been used?

At the centre of this clock face there are six identical rhombuses. Calculate the sizes of the angles in the rhombuses.



Part of the frame of the bicycle shown below forms a quadrilateral. The angle, $h°$, which the head tube makes with the horizontal is called the *head angle*. The angle, $s°$, which the seat tube makes with the horizontal is called the *seat angle*. Calculate the size of h and s for this bicycle.



exercise 6.6 challenge

This question looks at how many right angles a quadrilateral can have.

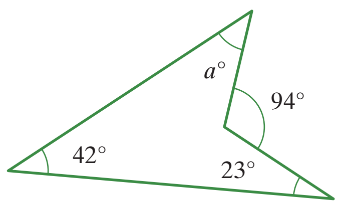
a Using your ruler and protractor, carefully draw each of the following quadrilaterals.

- **i** a quadrilateral with four right angles
- **ii** a quadrilateral that has exactly two right angles
- **iii** a quadrilateral that has exactly one right angle

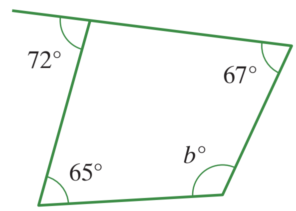
b Is it possible to draw a quadrilateral that has exactly three right angles?

Find the value of the pronumerals in each of the following figures.

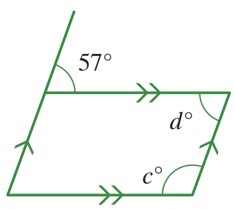
a



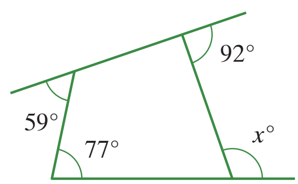
b



c



d



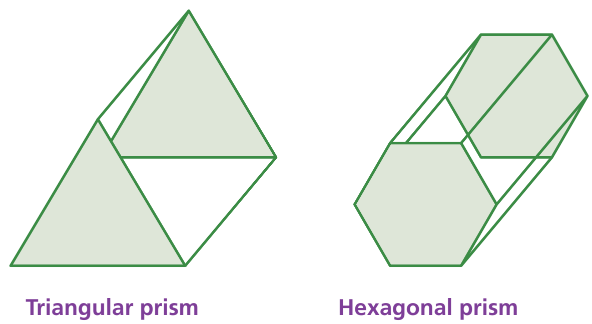
Josh and James were having a discussion about squares and rhombuses. Josh claimed that ‘A square is a rhombus so a rhombus is a square’ but James argued that ‘A square is a rhombus but a rhombus is not always a square’. What do you think? Use diagrams to explain your answer.

6.7 Representing three-dimensional objects in two dimensions

There are many different three-dimensional shapes, for example, prisms, cones, cylinders and pyramids.

Prisms

The two ends of a prism have an identical shape and size. The ends are joined by parallel straight edges that make up the rectangular faces of the prism. Prisms are named according to the shape of their ends.

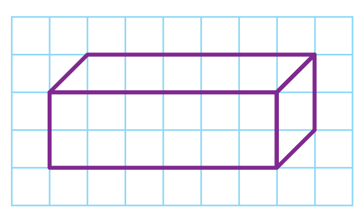


When we look at a three-dimensional shape, we see it in different ways depending on our position. If we are above the object it will look different from how it appears if we are looking at it from the side. The types of drawings we make of three-dimensional objects depends on the direction we are looking at the object and also on the purpose of the drawing. An artist may represent a shape in a very different way from how an architect would represent it.

In this section we look at several ways of representing three-dimensional objects.

Oblique drawings

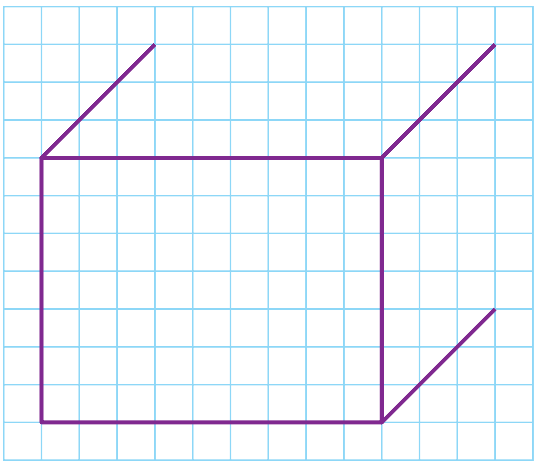
Oblique drawings can be made on a square grid. Horizontal lines on the object are drawn horizontally, vertical lines on the object are drawn vertically, and slanting lines are drawn at 45° to the horizontal.



Example 35

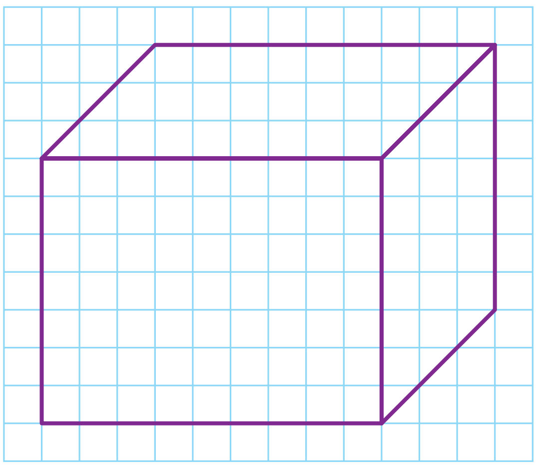
Use 5 mm graph paper to construct an oblique drawing of a box with a front face that is 45 mm by 35 mm.

Working



Reasoning

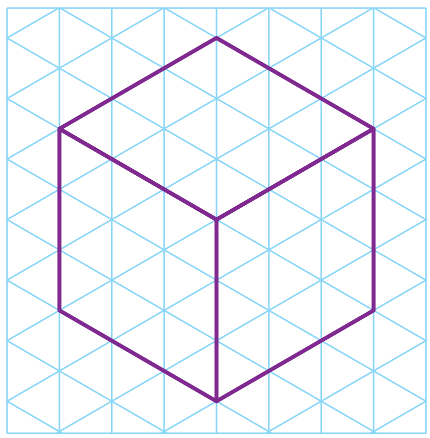
Draw a rectangle 45 mm by 35 mm (9 squares by 7 squares). Then draw line segments at 45° to the horizontal from three vertices of the rectangle. The length of these segments depends on how deep you wish the box to appear.



Draw the remaining horizontal and vertical line segments to complete the box.

Isometric drawings

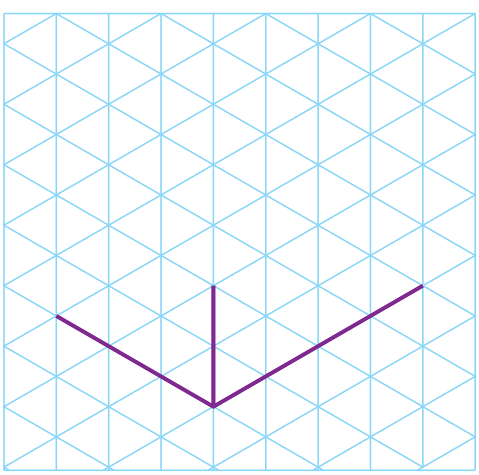
Isometric drawings are made on a grid of equilateral triangles. The advantage of an isometric drawing is that it can accurately show distances in the three directions. The disadvantage of an isometric drawing is that the drawing looks slightly distorted. This is because we are accustomed to seeing objects in perspective, with horizontal lines converging on the horizon. In this isometric drawing of a 3 cm cube on 1 cm isometric grid paper, notice how the length of each edge of the cube has been accurately represented.



Example 36

Use isometric grid paper to construct an isometric drawing of a box 20 mm high with a base 40 mm by 30 mm.

Working



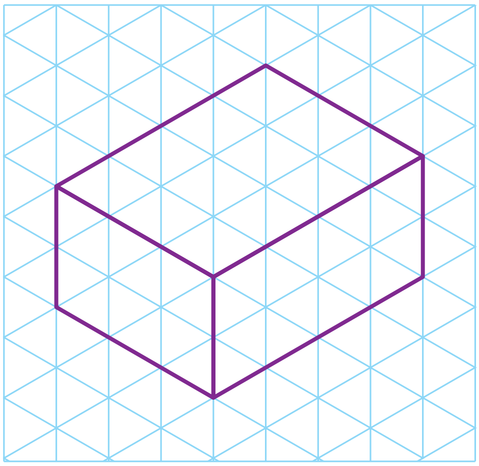
Reasoning

Draw a vertical line segment 2 units long to represent the height of the box. Draw segments 3 and 4 units long to represent two edges of the base of the box. Note that these two segments could have been drawn the other way around, with the 4-unit segment to the left and the 3-unit segment to the right.

continued

Example 36 continued

Working



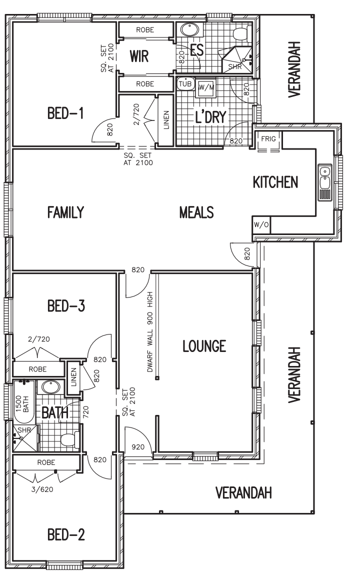
Reasoning

Draw the remaining line segments to complete the box, making sure the vertical edges are all 2 units long.

Plans and elevations

When architects design a building, they make drawings of the building as it would be seen from different positions. They may also construct a three-dimensional model of the building. Computer software now enables architects to construct screen images of different views of a building.

The **plan** of a solid object is a 'bird's eye' view—the view you would see if looking down on the object from above. An **elevation** is the view you see as you look directly at the object from a position in front of it or looking directly at the side of the object. In the diagram below, a model of a house is shown with a plan and elevations. The drawings below show the plan, the front and the side elevations of a house.



FRONT ELEVATION

SIDE ELEVATION

Source: Reprinted with permission by Hawkesbury Valley Homes.

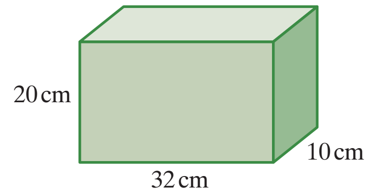
Example 37

For this rectangular prism, draw

a a plan.

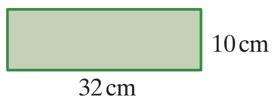
b a front elevation.

c a side elevation.

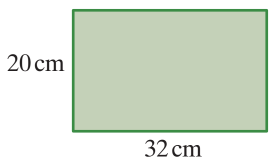


Working

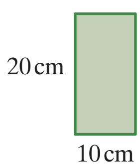
a Plan



b Front elevation



c Side elevation



Reasoning

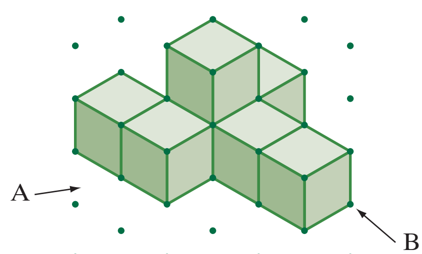
Example 38

This solid shape is made up of seven blocks.

a Draw a plan view.

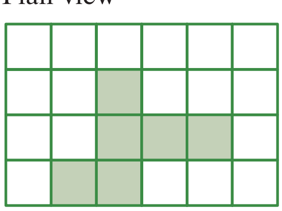
b Draw an elevation from direction A.

c Draw an elevation from direction B.



Working

a Plan view



Reasoning

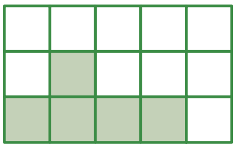
The plan view is what you would see if you looked down on top of the blocks. One of the seven blocks is hidden under the block in the second layer.

continued

Example 38 continued

Working

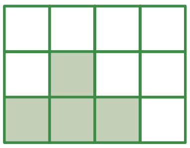
b Elevation from direction A



Reasoning

The elevation from direction A is what you would see if you were looking at the set of blocks from side A.

c Elevation from direction B



The elevation from direction B is what you would see if you were looking at the set of blocks from side B.

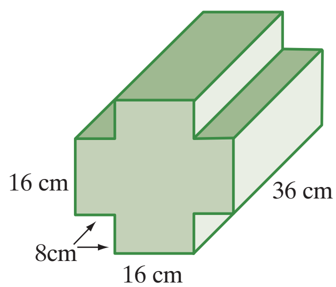
Example 39

For this steel bar, draw

a a plan.

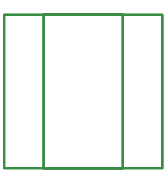
b a front elevation.

c a side elevation.

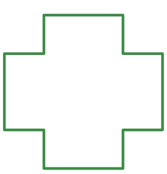


Working

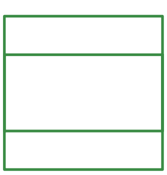
a Plan



b Front elevation

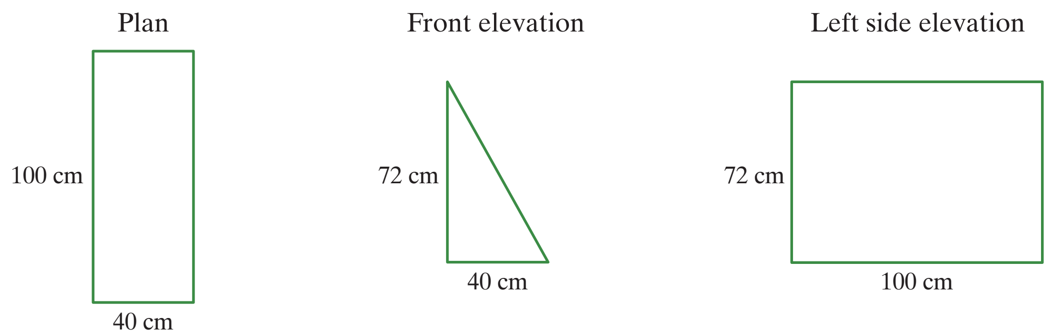


c Side elevation

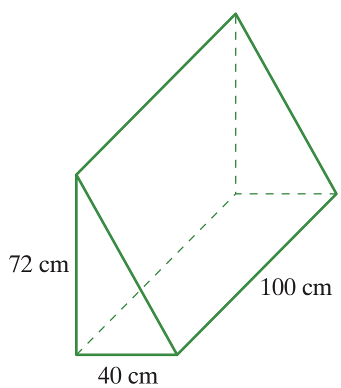


Example 40

Draw the three-dimensional shape represented by this plan, front elevation and left side elevation.



Working

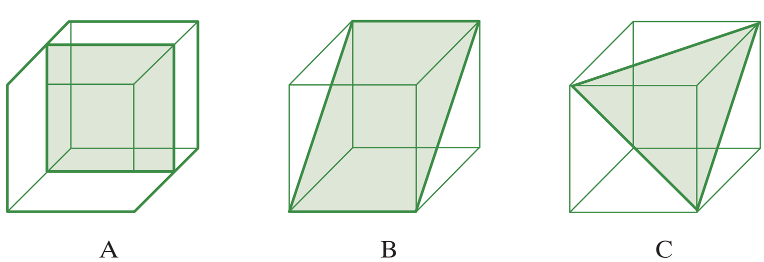


Reasoning

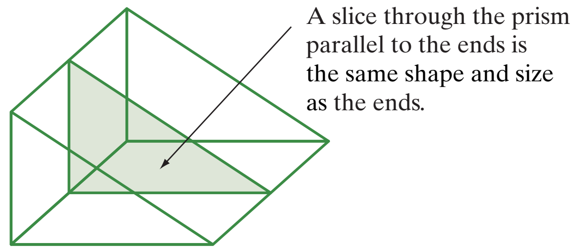
The shapes of the three drawings indicate a right-angled triangle prism.

Cross-sections

A cross-section of a three-dimensional solid is a slice through the solid. The shape of the cross-section depends on the direction we make the slice. A cross-section through a cube parallel to one of the square faces (cube A) is a square the same size as each face. A slanting cross-section through the cube as shown in cube B is a rectangle. A cross-section, cutting through the midpoints of three edges as shown in cube C, is an equilateral triangle.

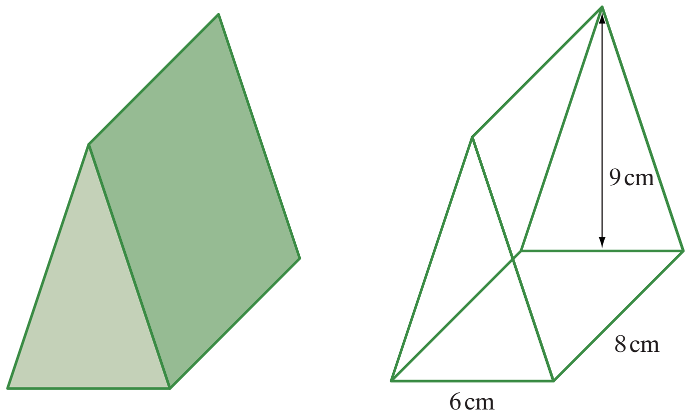


If we slice through a prism parallel to the ends, the shape of the cross-section will be the same shape and size as the ends.

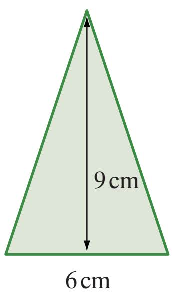


Example 41

Two different representations of the same triangular prism are shown here. Draw the cross-section shape of the prism.



Working



Reasoning

The cross-section is the same shape and size as the ends of the prism.

exercise 6.7

LINKS TO Example 35

Use squared paper to draw an oblique drawing of a cube with edges of length 4 cm.

LINKS TO Example 35

Use squared paper to draw an oblique drawing of a rectangular prism with width 4 cm, height 5 cm and appearing to have a depth of about 6 cm.

LINKS TO Example 36

Use isometric grid paper to make an isometric drawing of a cube with edges of length 4 cm.

LINKS TO Example 36

Use isometric grid paper to make an isometric drawing of a rectangular prism with width 4 cm, height 5 cm and depth 6 cm.

BLM 1 cm grid

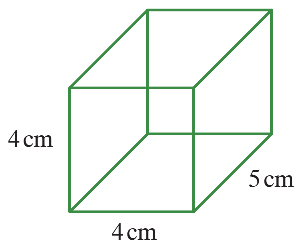
BLM Dotted paper

BLM Isometric grid

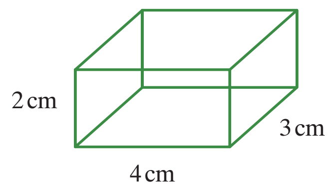
For each of these rectangular prisms, make accurate drawings of

i a plan. **ii** a front elevation. **iii** a side elevation.

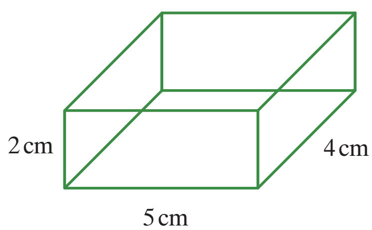
a



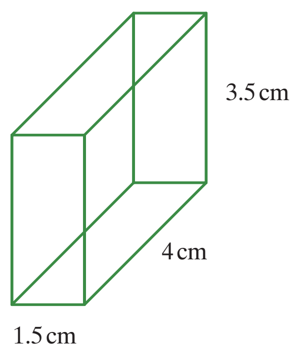
b



c



d



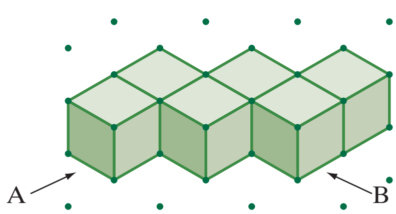
Using 1 cm squared paper

i draw a plan view for each of these solid shapes.

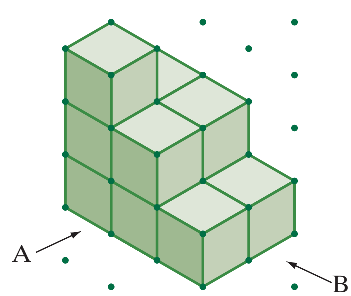
ii draw an elevation from direction A.

iii draw an elevation from direction B.

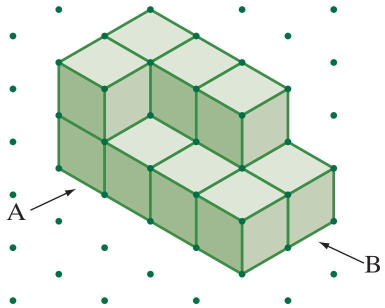
a



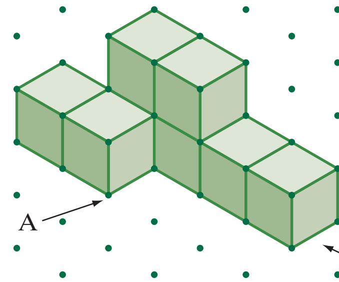
b



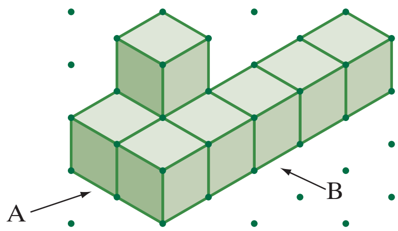
c



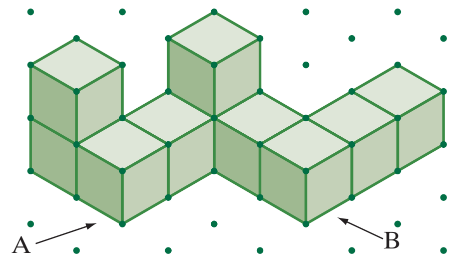
d



e



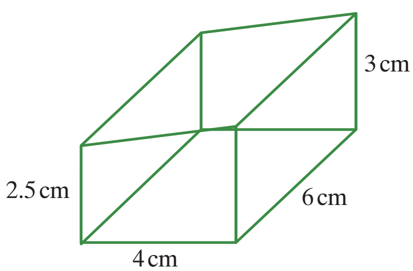
f



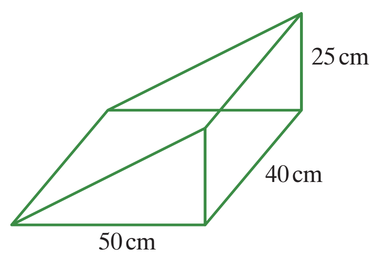
For each of these shapes, make accurate drawings of

- **i** a plan.
- **ii** a front elevation.
- **iii** a right side elevation.

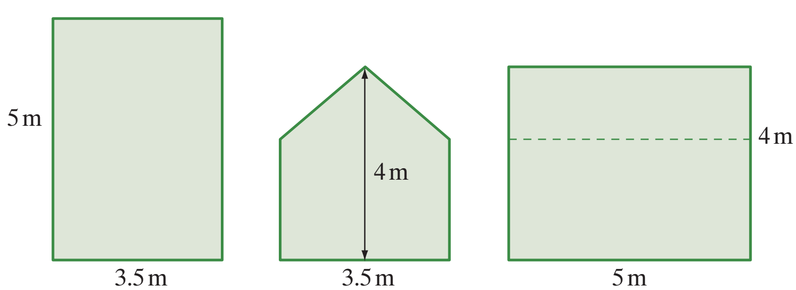
a



b

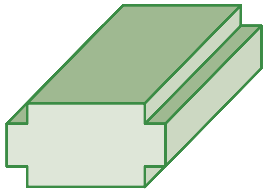


The diagrams below show the plan, front elevation and left side elevation of a building. Draw the building.

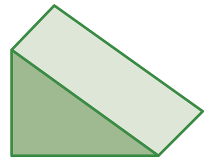


● Make a cross-section drawing for each of these prisms.

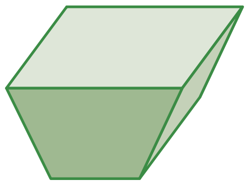
a



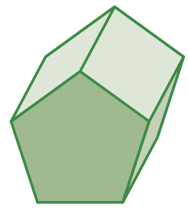
b



c



d

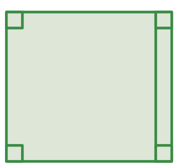


● Many chocolates and biscuits are sold in prism-shaped packets. Prism shapes are also used for decorative candles. Make a class collection of prisms with different cross-sections. For each prism, draw one of the rectangular faces and draw the cross-section shape.

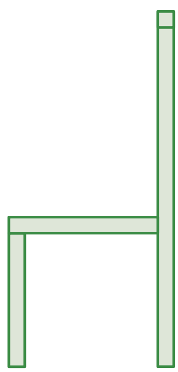
exercise 6.7 challenge

● Four different views of a chair are shown below. State the type of representation for each.

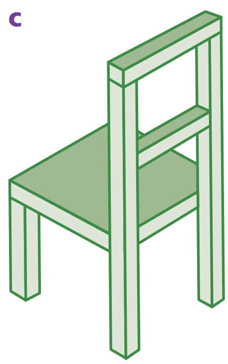
a



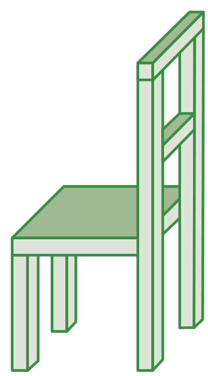
b



c



d



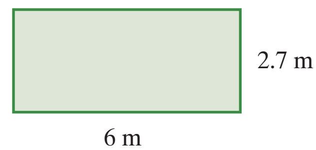
Analysis task

BLM A parking problem

A parking problem

In the town of Parkbourne there is a 40 m strip of roadway outside some of the shops. The local council is trying to decide between parallel parking and angle parking. If they choose angle parking, they then need to decide whether it should be 60° or 90° parking.

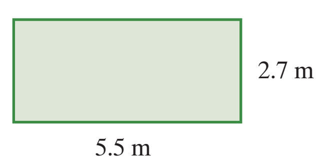
For parallel parking, the council recommends that each parking space is 2.7 m wide and 6 m long.



a On a piece of A3 paper, rule a line segment 40 cm long to represent the 40 m of kerb. Using a rectangle 2.7 cm by 6 cm to represent each parking space, carefully draw a diagram to show parallel parking along the kerbside. How many cars will fit along the 40 m strip?

b For 90° and 60° angle parking, the council recommends that each parking place is 2.7 m wide and 5.5 m long. Why do you think the parking spaces can be shorter for angle parking?

c Draw another 40 cm line segment and this time show the car parking spaces for 90° angle parking. Use your protractor and ruler to accurately draw the parking spaces. Remember to make each parking space 2.7 cm by 5.5 cm. How many cars fit along the kerbside this time?



d Repeat for 60° angle parking.

e What do you think are the advantages and disadvantages of the two types of parking?

f What are some of the factors the council would need to take into account in making their decision?

g Choose a parking strip in your town, for example, outside your school and investigate the type of parking.

Review Geometry and space

Summary

Lines, segments and rays

- A line extends forever in both directions.
- A line segment is part of a line. It has a definite start and finish.
- A ray has a definite starting point, but extends forever in one direction.

Angles

- An angle of 90° is called a right angle.
- There are 180° in a straight angle and 360° in one revolution.
- Acute angles are angles that are less than 90°.
- Obtuse angles are angles that are greater than 90° but less than 180°.
- Reflex angles are greater than 180° but less than 360°.
- Vertically opposite angles are equal.
- Adjacent angles are angles that are next to each other, sharing a common vertex and common arm.
- Complementary angles are two angles that add to 90°.
- Supplementary angles are two angles that add to 180°.
- Allied angles are angles between a pair of parallel lines and on the same side of the transversal. Allied angles add to 180°.
- Alternate angles are angles between parallel lines and on opposite sides of the transversal. Alternate angles are equal.
- Corresponding angles are angles that are in corresponding positions in relation to parallel lines and are on the same side of a transversal. Corresponding angles are equal.

Triangles

The three angles of all triangles add to 180°.

Classified by sides	
Scalene	All side lengths different
Isosceles	Two equal sides (and two equal angles)
Equilateral	Three equal sides (and all angles are 60°)

Classified by angles	
Right-angled	One angle is a right angle
Acute-angled	All three angles are acute
Obtuse-angled	One angle is an obtuse angle

The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

Quadrilaterals

The four angles of all quadrilaterals add to 360°.

Trapezium	One pair of opposite sides are parallel; two pairs of supplementary angles
Kite	Two pairs of adjacent sides are equal in length; one pair of opposite angles are equal
Parallelogram	Both pairs of opposite sides are parallel and opposite sides are equal in length; opposite angles are equal; adjacent angles add to 180° The special parallelograms also have the additional properties shown below. Rhombus—All four sides are equal in length. Rectangle—All angles are right angles. Square—All four sides are equal in length; all angles are right angles.

Visual map

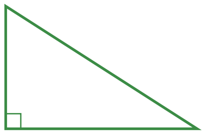
Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

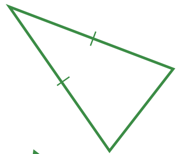
acute angle
acute-angled triangle
adjacent
adjacent angles
alternate angles
cointerior (allied) angles
complementary angles
corresponding angles
cross-section
diagonal
elevation
equilateral triangle
exterior angle
horizontal
interior angle
isosceles triangle
isometric drawing
kite
line
line segment
oblique drawing
obtuse angle
obtuse-angled triangle
parallel
parallelogram
perpendicular
plan
prism
quadrilateral
ray
rectangle
reflex angle
revolution
rhombus
right angle
right-angled triangle
scalene triangle
square
straight angle
supplementary angles
transversal
trapezium
vertical
vertically opposite angles

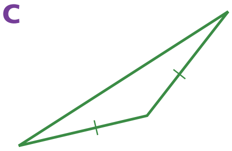
Revision

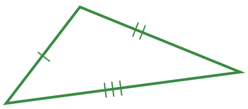
Multiple-choice questions

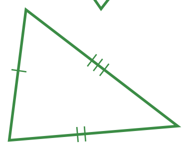
Which of the following triangles is an obtuse-angled isosceles triangle?

A


B


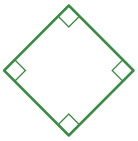
C


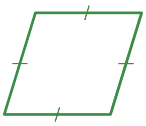
D


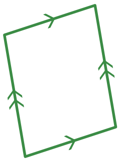
E


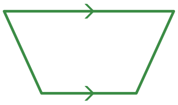
Which of the following figures is *not* a parallelogram?

A

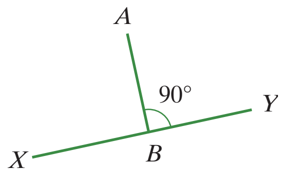

B


C


D


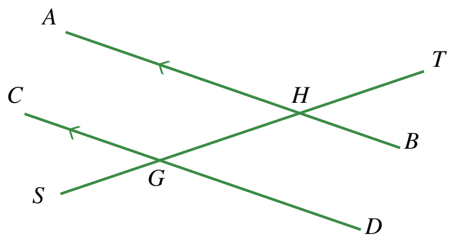
E


The line segments XY and AB are



A complementary.
B parallel.
C perpendicular.
D supplementary.
E vertical.

The angle that is alternate to $\angle AHG$ is



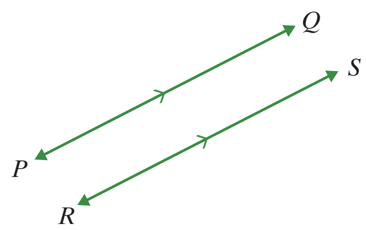
A $\angle BHG$
B $\angle DGH$
C $\angle AHT$
D $\angle CGH$
E $\angle BHT$

The angle that is corresponding to $\angle SGD$ is

A $\angle CGH$ **B** $\angle DGH$ **C** $\angle AHG$ **D** $\angle CGS$ **E** $\angle GHB$

Short-answer questions

a Explain what the diagram below shows.

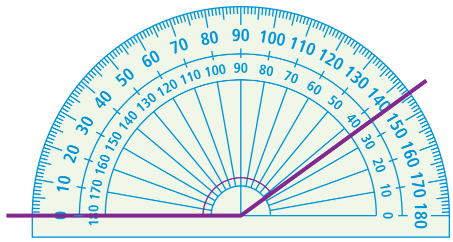


b Explain, in words and with a diagram, what each of the following means.

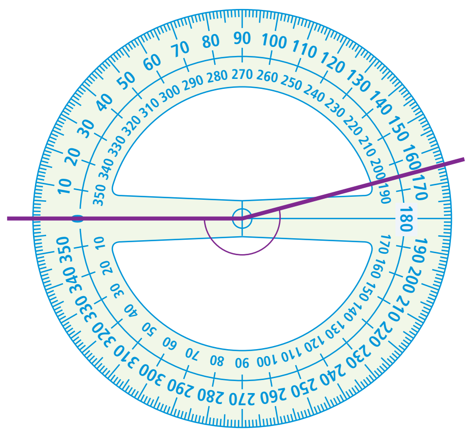
i $AB \quad CD$ **ii** $EF \quad GH$

Find the size of each of the marked angles.

a



b



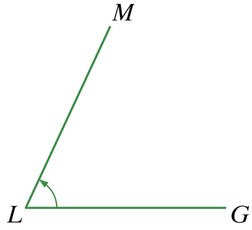
For each of the marked angles

i name the angle using the given letters.

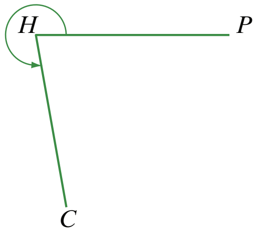
ii state the type of angle.

iii measure the angle.

a

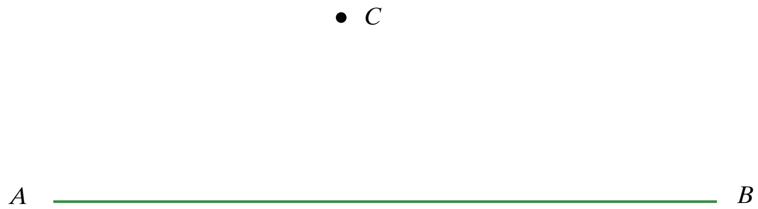


b



Use your ruler to draw a line segment 5 cm long. Label the ends A and B. Mark a point anywhere on AB and label it C. Use your compass and ruler to draw a line segment CD perpendicular to AB. Use the correct symbol to show that the line segments are perpendicular.

Copy the following diagram (it does not need to be an exact copy). Use your ruler, compass and pencil to construct a line segment through C parallel to AB. Use the correct symbols to show that your line segments are parallel.



a Which of the following angles are complementary?

127°   83°   63°   53°   37°

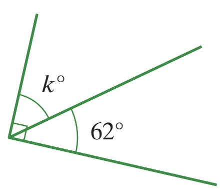
b Which of the following angles are supplementary?

221°   139°   49°   51°   41°

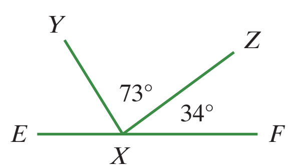
a Use your ruler and protractor to draw an angle of 135°. Label the angle CDE.

b What is the supplement of $\angle CDE$?

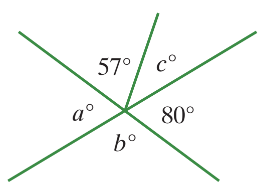
a Find the value of k.



b Find the size of $\angle EXY$.

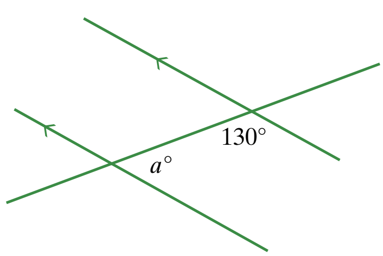


c Find the values of a, b, and c below.

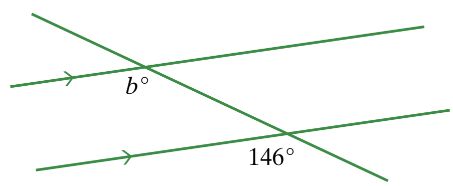


Find the value of each pronumeral. Give reasons.

a

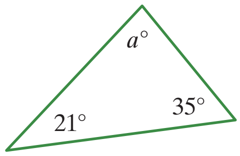


b

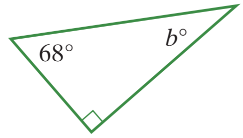


Find the value of the pronumerals in each of the following triangles.

a

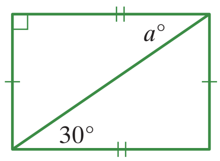


b

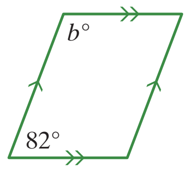


Name the type of each of the following special quadrilaterals and find the value of the pronumerals.

a



b



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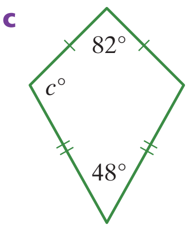

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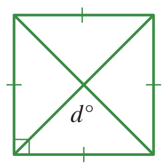

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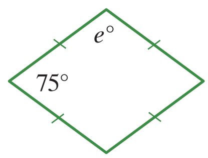
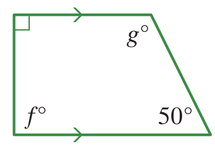


f

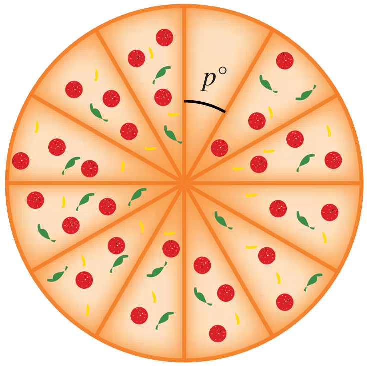


Explain whether a triangle could be constructed with each of the following sets of side lengths.

a 5 cm, 11 cm, 7 cm   **b** 6 cm, 4.5 cm, 3 cm   **c** 5 cm, 14 cm, 9 cm

Extended-response questions

Sergio divides a pizza into 12 equal sized pieces, with each piece making an angle of $p°$ at the centre of the pizza as shown.



a What is the value of p?

b Sergio eats five pieces of the pizza. What is the total angle of the five pieces?

c The pieces which Rosa eats have a total angle of 120°. How many pieces does Rosa eat?

d Anna eats the remaining pizza. What is the total angle of the pieces Anna eats?

a Use your compass, ruler and pencil to construct ABC with response sides $BC = 6$ cm, $AB = 5.5$ cm and $AC = 6$ cm.

b Classify ABC according to its side lengths.

c Classify ABC according to its angle sizes.

a Use your protractor and ruler to construct DEF with $EF = 5$ cm, $DE = 5$ cm and $DEF = 90°$.

b Classify DEF according to its side lengths.

c Classify DEF according to its angle sizes.

a Use your protractor and ruler to construct $\triangle JKL$ with $JK = 8$ cm, $\angle LJK = 30°$ and $\angle LKJ = 40°$.

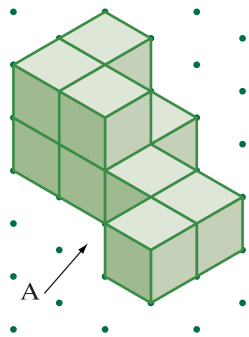
b Classify $\triangle JKL$ according to its side lengths.

c Classify $\triangle JKL$ according to its angle sizes.

Using 1 cm squared paper

i draw a plan view for this solid shape.

ii draw an elevation from direction A.



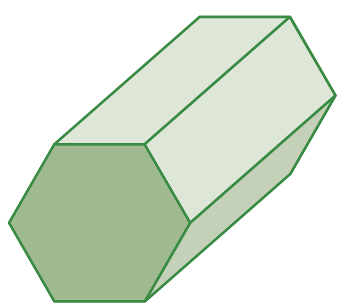
A rectangular prism has width 5 cm, height 3 cm and depth 6 m. Make the following drawings.

i an oblique drawing

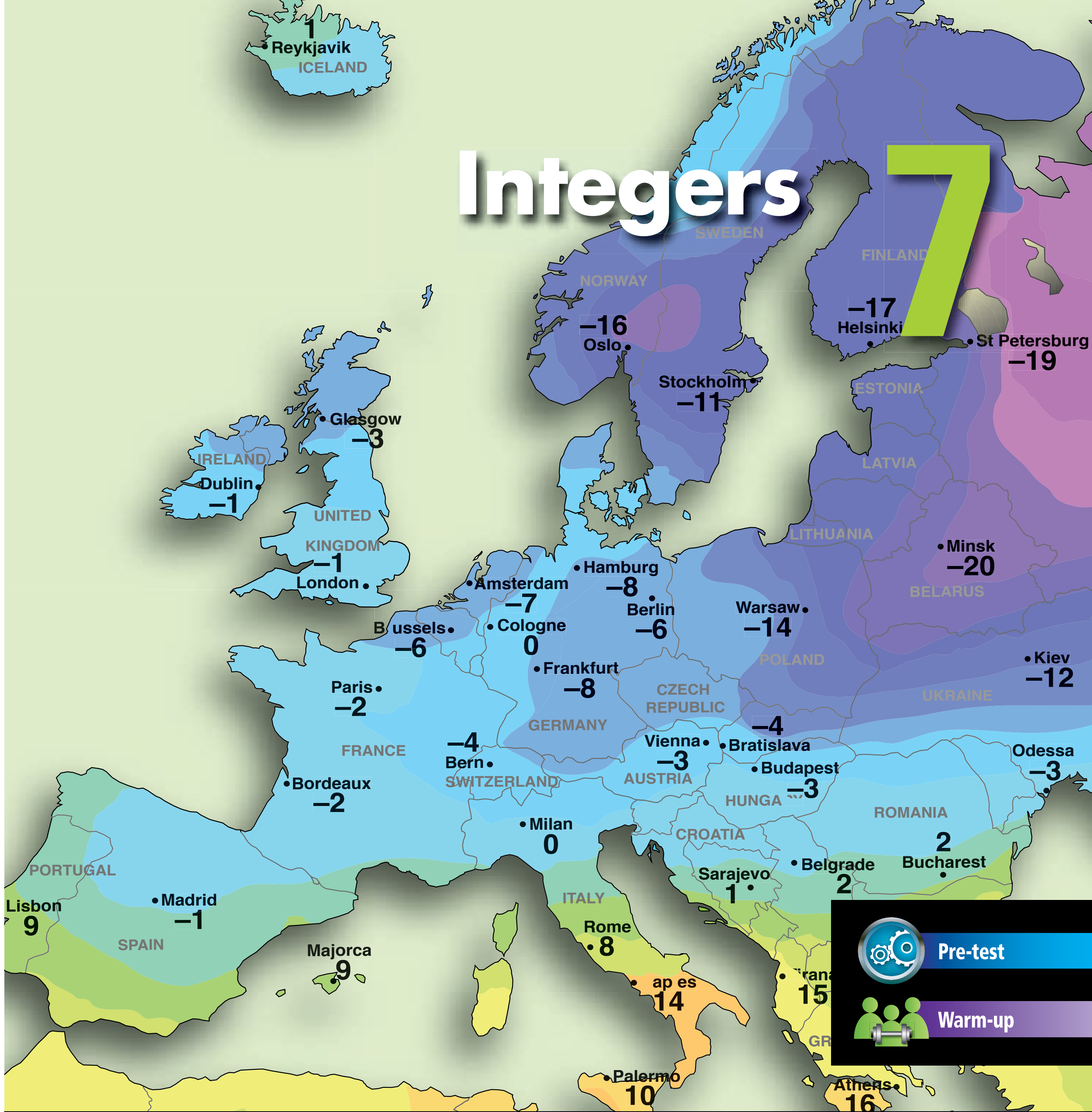
ii an isometric drawing

iii a plan view

Draw the cross-section of this prism.



Integers 7



Pre-test

Warm-up

If you have lived in or visited a place that is very cold in winter, you will be familiar with temperatures that are negative numbers. The lowest recorded temperature in Australia is −23.4°C, recorded at Charlotte Pass in New South Wales on June 29 1994. Temperatures as low as this are unusual in Australia and occur only in alpine areas. However, in many European countries, low temperatures occur quite often in winter. This map shows temperatures (in degrees Celsius) that are negative numbers in many cities in Europe. Negative numbers are used in other situations when we want to show opposites, for example, the car park floors below ground level in a tall city building may be labelled as −1, −2 and −3. Integers are whole numbers that may be positive, negative or zero.

7.1 What are integers?

In chapters 1 to 4, we considered numbers that were positive, that is, greater than zero. So that we can represent numbers on the other side of zero, we need negative numbers.

The word **integer** is used for *whole numbers* that are either *positive*, *negative* or *zero*.

Many of the things we deal with involve opposites. Some of these are

- temperatures above and below zero
- heights of land above and below sea level
- buildings with floors above and below ground
- the amount of money we have and the amount of money we owe
- the distance to the right and the distance to the left of some point
- the distance upward and distance downward from some point
- the number of degrees clockwise and the number of degrees anticlockwise for a rotation.

Example 1

For each of the following state the opposite.

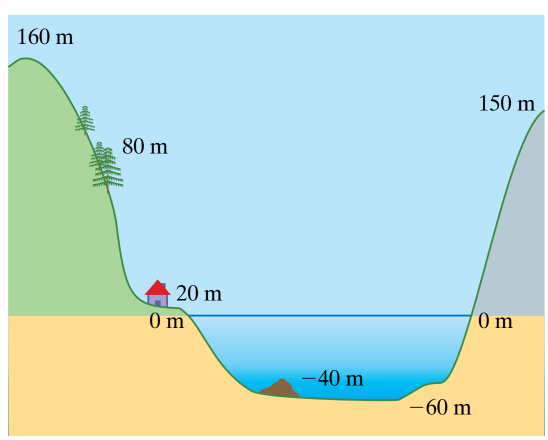
a 40 km east of a town

b a turn of 90° clockwise

c basement level 2 in a building

Working	Reasoning
a 40 km west of a town	The opposite of east is west.
b a turn of 90° anticlockwise	The opposite of anticlockwise is clockwise.
c the second floor in a building	The opposite of basement levels are the regular floors.

This diagram shows heights above and below water level. One way to show opposites is to use a positive sign in front of one quantity and a negative sign in front of its opposite. For example, if five metres above sea level is written as 5, then five metres below sea level can be written as −5.



We don't normally put a + sign in front of a number when it is positive. If there is no sign we assume the number is positive.

Example 2

Write an integer to represent each of the following.

a a temperature eight degrees below zero

b owing 20 dollars

c a height of six metres above sea level

d a height of 12 metres below sea level

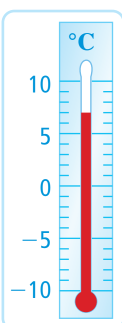
Working	Reasoning
a -8	The scale on a thermometer is like a vertical number line. Below zero are the negative temperatures.
b -20	Owing 20 dollars is the opposite of having 20 dollars.
c 6	Taking sea level as 0, above sea level is positive.
d -12	Taking sea level as 0, below sea level is negative.

Negative integers are the opposite of their matching positive integer and positive integers are the opposite of their matching negative integer. For example, -5 is the opposite of 5. This can be written $-5 = -(5)$. Also, 3 is the opposite of -3, which can be written $3 = -(-3)$.

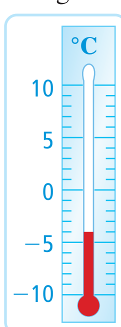
Example 3

Write the temperatures shown by the following thermometers.

a



b



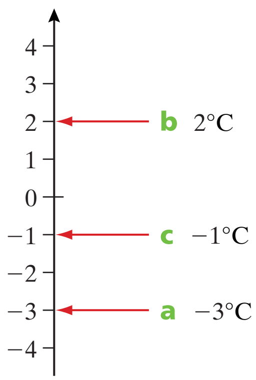
Working	Reasoning
a 7°C	This temperature is above 0°C so it is positive. This temperature is 2 scale marks above 5°C so it shows 7°C.
b -4°C	This temperature is below 0°C so it is negative. It is 4 scale markings below 0°C so it shows -4°C.

Example 4

Draw a vertical scale 8 cm long to represent a thermometer. Mark the scale from −4°C to 4°C. On the thermometer, use arrows to point to the following temperatures.

a −3°C **b** 2°C **c** −1°C

Working



Reasoning

−3°C is 3 degrees below 0°C.
Move down 3 spaces from 0°C. The arrow should point to the scale mark labelled −3.
2°C is 2 degrees above 0°C.
Move up 2 spaces from 0°C. The arrow should point to the scale mark labelled 2.
−1°C is 1 degree below 0°C.
Move down 3 spaces from 0°C. The arrow should point to the scale mark labelled −1.

Example 5

Evaluate the following.

a $-(-4)$ b $-(+4)$

Working

a $-(-4) = 4$

b $-(+4) = -4$

Reasoning

$-(-4)$ means the opposite of −4.

$-(+4)$ means the opposite of the opposite of −4.

Integers on the number line

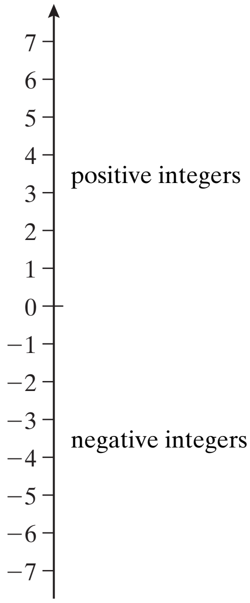
Just as we showed positive whole numbers, fractions and decimals on number lines in earlier chapters, we can also show negative integers on a number line. In between each of the negative integers there are negative fractions and decimals, but in Year 7 we will just work with negative integers.

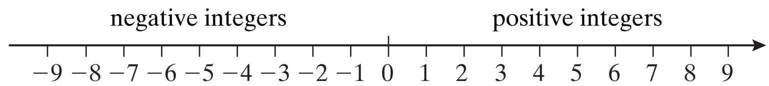
Number lines may be vertical or horizontal. On a vertical number line, positive integers are shown above zero and negative integers below zero.

Integers on the number line

Vertical number line

On a horizontal number line, positive integers are to the right of zero and negative integers are to the left of zero.





Example 6

Use number lines to show the following numbers

a on a vertical number line with a scale from −4 to 4.

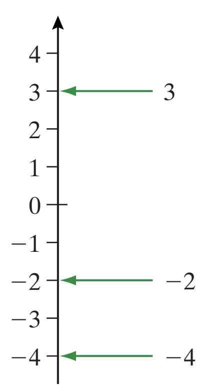
i −2 **ii** 3 **iii** −4

b on a horizontal number line with a scale from −4 to 4.

i 2 **ii** −3 **iii** −1

Working

a



Reasoning

First draw and label the number line.
Space the scale marks equal distances apart.

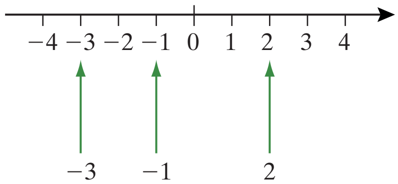
0 goes in the middle.
The positive numbers go up from 0.
The negative numbers go down from zero.

Then place the numbers next to the matching scale marks.

continued

Example 6 continued

Working



Reasoning

First draw and label the number line.
Again, 0 goes in the middle.
The positive numbers go right from 0.
The negative numbers go left from zero.

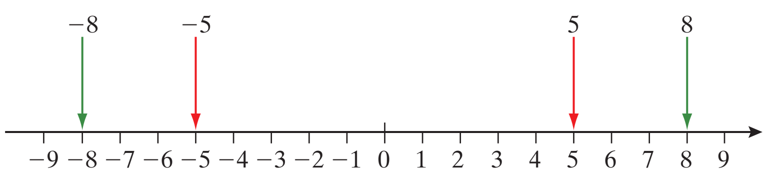
Then place the numbers below the matching scale marks.

Each positive integer has an opposite negative integer the same distance from zero.

Example 7

Show 5 and −8 and their opposites on the number line.

Working



Reasoning

The opposite of 5 is −5.
The opposite of −8 is 8.

exercise 7.1

LINKS TO Example 1

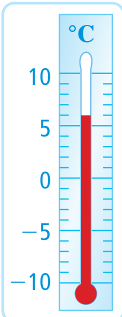
1. For each of the following state the opposite.
   - **a** 10 degrees below zero
   - **b** owing 80 dollars
   - **c** 2.6 metres above sea level
   - **d** three centimetres to the left of zero
   - **e** the fourth floor of a multistorey building with many basement levels
   - **f** 30 kilometres north of a town
   - **g** a turn of 45° anticlockwise
   - **h** 120 km west of a city

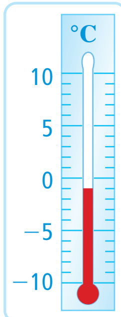
LINKS TO Example 2

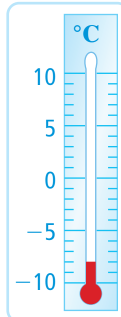
2. Write an integer to represent each of the following.
   - **a** a temperature 12 degrees above zero
   - **b** owing $15
   - **c** a depth of 3 cm
   - **d** 4 m above sea level
   - **e** two floors below ground level
   - **f** 6 m to the left of the starting point
   - **g** 5 km south of the starting point, if north is positive
   - **h** a turn of 15° anticlockwise, if clockwise is positive

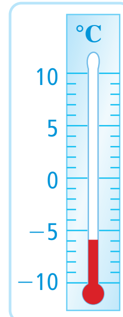
▶LINKS TO Example 3

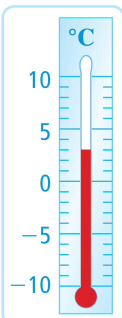
Write the temperatures shown by the following thermometers.

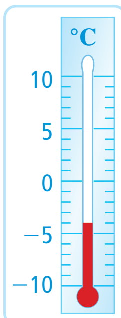
a 

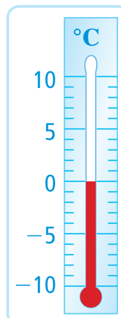
b 

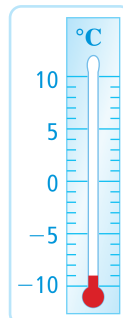
c 

d 

e 

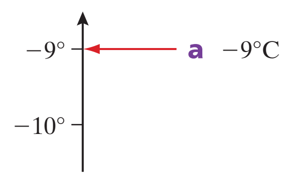
f 

g 

h 

▶LINKS TO Example 4

Draw a vertical line 20 cm long to represent a thermometer. Carefully mark the scale from −10°C to 10°C. On your thermometer use arrows to point to the following temperatures, as shown in this portion of the thermometer.



a −9°C **b** 7°C **c** −6°C **d** 1°C
e −1°C **f** −7°C **g** 5°C **h** −3°C

▶LINKS TO Example 5

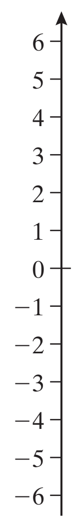
Evaluate.

a −(−6) **b** −(−8) **c** −(−1) **d** −(+3)

▶LINKS TO Example 6

A building has 3 floors above the ground floor and 3 floors below. We will call the ground floor 0, the floors above ground positive and the floors below ground negative. Write as an integer

a 2 floors above ground level. **b** 1 floor below ground level.
c 3 floors above ground level. **d** 3 floors below ground level.



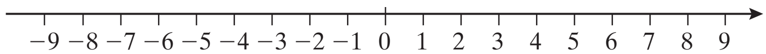
On a vertical number line with a scale from −6 to 6 mark these numbers.

a −5 **b** 4 **c** −1
d 0 **e** 3 **f** −4

On a horizontal number line with a scale from −6 to 6 mark these numbers.

a 5 **b** −3 **c** −2
d 0 **e** 1 **f** −6

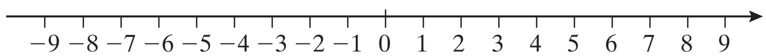
For each of these integers
- **i** write the integer.
- **ii** mark and label the integer on the number line.



a 4 units to the right of zero **b** 3 units to the left of zero
c 8 units to the left of zero **d** 7 units to the right of zero

Mark and label each of these integers on the number line then mark and label the opposite integer.

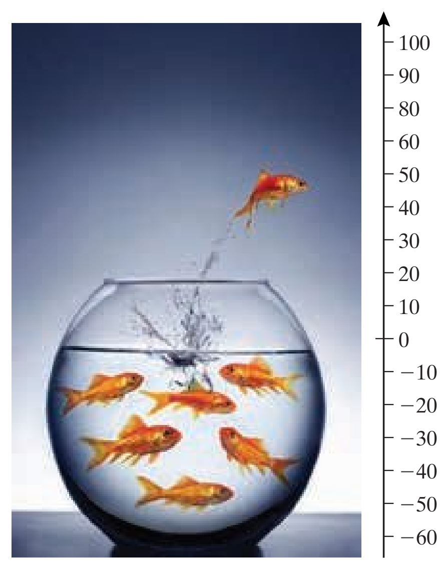
a 7 **b** −4 **c** −5 **d** 2



State the opposite of each of these integers.

a −8 **b** 5 **c** −2 **d** 9

The photo at right shows fish below (negative) and above (positive) the water level. Measurements are in centimetres.



In these questions the vertical position of the fish is its eye position. Answer to the nearest 10 cm.

a What is the position of the fish nearest the bottom?

b What is the position of the fish nearest the water level?

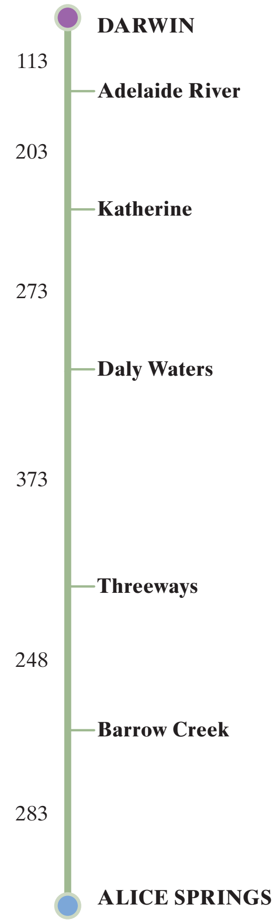
c How many fish are at −30 cm?

d What is the position of the fish that escaped the bowl?

e What is the position of the fish that is nearest the centre of the bowl?

exercise 7.1 challenge

1. The Yang family are on holiday in the Northern Territory. This strip map to the right shows the distances between towns on the route from Alice Springs to Darwin. (We can think of the strip map as a number line.) The Yang family sets the trip meter at zero as they leave Alice Springs so that the positions of places north of Alice Springs are positive.



a What is the position of
  i Daly Waters?
  ii Katherine?

When the Yangs reach Daly Waters they reset their trip meter to zero so that the positions of places south of Daly Waters are negative and the positions of places north of Daly Waters are positive.

b What is the position of
  i Katherine?
  ii Threeways?
  iii Barrow Creek?
  iv Adelaide River?

7.2 Comparing and ordering integers

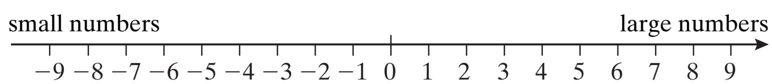
Most of our experience with negative numbers has been with temperatures. We will use this to help us get used to working with other negative numbers.

Which are colder—temperatures such as −3°C and −10°C, or temperatures like 3°C and 10°C? Of course, the negative temperatures are colder than the positive temperatures.

What does this tell us about other integers?

Which is smaller, −3 or 3? Which is smaller, −10 or 10?

Our experience with temperatures tells us that negative numbers are smaller than positive numbers.



Negative numbers that are further to the left of zero are smaller than those closer to zero.

It is the opposite for positive numbers. Positive numbers that are further to the right of zero are larger than those closer to zero.

Number lines are helpful for placing integers in order.

Example 8

Arrange the integers 4, −2, 0, 1, −5 in descending order.

Working

In descending order the numbers are 4, 1, 0, −2, −5.

Reasoning

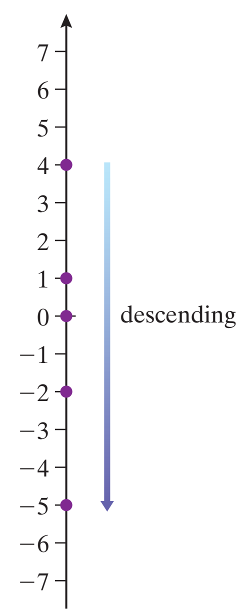
Look at the vertical number line.

'Descending' order means going from largest to smallest.

On the number line the large numbers are at the top.

The largest of these numbers is 4, then 1, 0, −2 and −5.

−5 is the lowest so it is the smallest of the five integers.



Example 9

For each of these pairs of integers

- **i** put a square around the larger number.
- **ii** use < or > to make a true statement.

a −4 or 3 **b** −4 or −7

Working	**Reasoning**
a i −4 or [3]	Positive numbers are larger than negative numbers. For example, 3 °C is 'warmer' than −4 °C.
ii −4 < 3	The wide end of the sign faces the larger number.
b i [−4] or −7	Positive numbers are larger than negative numbers.
ii −4 > −7	The wide end of the sign faces the larger number.

Example 10

In each of these patterns the integers are going up or down by a constant amount. Write the next two integers for each pattern.

a −11, −8, −5, __, __, …

b 7, 5, 3, __, __, …

Working

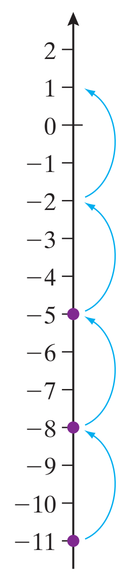
a The pattern with the next two integers is

−11, −8, −5, **2**, **1**, …

Reasoning

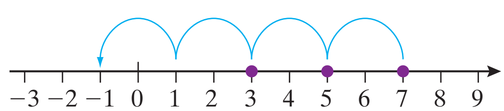
The numbers in this pattern are going up by threes.

Using a number line, the next two integers in the pattern are −2 and 1.



b The pattern with the next two integers is

7, 5, 3, **1**, **1**, …



The numbers in this pattern are going down by twos.

Using a number line, the next two integers in the pattern are 1 and −1.

exercise 7.2

LINKS TO Example 8

Arrange these sets of integers in ascending order.

a −2, −7, 1, −4, 6 **b** −7, −8, 7, −3, 0 **c** −5, 4, −4, 3, −1
d 3, −6, 9, −4, −3 **e** 1, −2, 0, 5, −6 **f** −8, 0, 7, −1, −5

LINKS TO Example 8

Arrange these sets of integers in descending order.

a −7, 7, −3, 0, −6 **b** −8, −5, 0, 7, −1 **c** 3, −1, −5, 4, −4
d −6, −8, 1, −2, 0 **e** −2, −7, 1, −4, 6 **f** 3, −4, 5, 9, −5

LINKS TO Example 8

The minimum temperatures for five cities on 7 March were:

Moscow −13°C Paris 2°C Rome 8°C Beijing −9°C Tokyo 1°C

Arrange these cities in ascending order of their minimum temperatures.

List all the integers between these integer pairs.

a −3 and 2 **b** −8 and −4 **c** 4 and −2 **d** 5 and 10
e −1 and −6 **f** 3 and −3 **g** −6 and 0 **h** 4 and −4

LINKS TO Example 9

Write the larger of each of the following integers.

a −6, 6 **b** −2, −5 **c** 7, −8 **d** 5, 10
e −3, −1 **f** 5, −10 **g** −9, 0 **h** −6, −10

LINKS TO Example 9

Copy these pairs of integers. Use $<$ or $>$ to make true statements.

a −2 1 **b** −7 −4 **c** 8 −1 **d** 3 9
e −3 −5 **f** 4 −8 **g** −6 0 **h** 5 −5

LINKS TO Example 10

Copy these number patterns and extend to the next two numbers.

a 6, 5, 4, __, __ **b** 1, 0, −1, __, __ **c** −7, −6, −5, __, __
d −2, −1, 0, __, __ **e** 6, 4, 2, __, __ **f** −5, −2, 1, __, __
g −7, −5, −3, __, __ **h** 10, 7, 4, __, __ **i** −18, −13, −8, __, __

If the lowest overnight temperature at Perisher Valley in NSW is −7°C, how many degrees will the temperature need to rise to get to the following?

a −3°C **b** 0°C **c** 5°C **d** 8°C

Answer the following. Use a number line to help you.

a Elly and Will share a piggy bank. Elly puts $5 into the bank, and later takes out $8. How much does she owe the bank?

b At noon the temperature is 15°C, but by midnight it has dropped 23°C. What is the temperature at midnight?

c A diver is 10 m below sea level and sees a treasure chest 7 m below him. What is the depth of the chest?

● Jo had a bank account with \$10 in it. We can write this as +10. For each of the following, write an integer for the new amount that Jo has in her account.

a Last month she withdrew \$8.

b She was charged \$2 in fees.

c She was paid \$1 in interest.

d She was charged another \$2 in fees.

e She deposited \$5 into her bank account.

exercise 7.2 challenge

● The building where Vicki parks her car each day has 4 basement levels below the ground floor for cars. The lowest of these is B4. There are also 5 floors of offices above the ground floor.

a If we call the office floors positive and the basement carpark floors negative, write the following as integers.

i the ground floor **ii** the third floor **iii** the carpark floor B2

b How many floors does Vicki travel up from the carpark floor B2 to the third floor?

c How many floors does Vicki travel down from the second floor to the carpark floor B1?

d What would be the greatest number of floors that she could travel up from a carpark floor to an office floor?

e What would be the least number of floors that she could travel up from a carpark floor to an office floor?

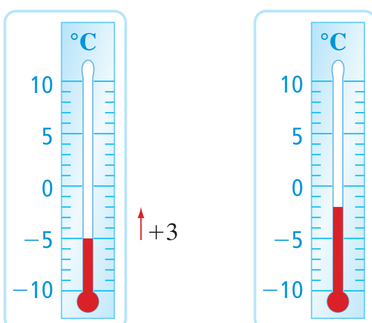
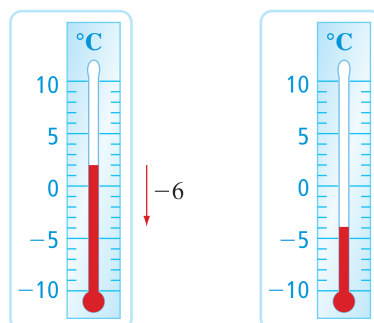
7.3 Adding and subtracting a positive integer

Adding and subtracting positive integers

We know that if we add a positive integer to another positive integer, the result will be a positive integer. For example, $6 + 5 = 11$. In this section we will see what happens when we add a positive integer to a negative integer.

Using a number line

If we think about temperature again we can see that an increase in temperature corresponds to adding. A decrease in temperature corresponds to subtracting.

If the temperature starts at -5°C and increases by 3 degrees, the new temperature is -2°C.	If the temperature starts at 2°C and decreases by 6 degrees, the new temperature is -4°C.
 $-5 + 3 = -2$	 $2 - 6 = -4$

Using a number line instead of a thermometer, we see that adding a positive integer takes us to larger numbers. Subtracting a positive integer takes us to smaller numbers.

Example 11

Write each of the following in symbols and then evaluate.

a The temperature is -2°C. It falls by 5°C. What is the temperature now?

b Fran's flat is on the 2nd floor. She takes the lift down 6 levels. Where is she now?

continued

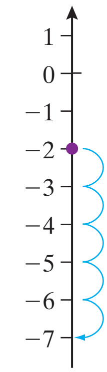
Example 11 continued

Working

a $-2 - 5 = -7$
The temperature is -7°C.

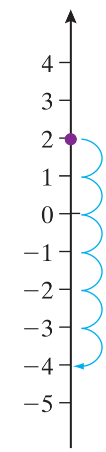
Reasoning

Write -2°C as -2.
Falling by 5 degrees is equivalent to subtracting 5.
Start at -2 on the number line.
Go down five spaces.
This ends at -7.



b $2 - 6 = -4$
She is on the 4th floor below ground level or Basement 4.

Write the 2nd floor as 2.
Going down 6 levels is equivalent to subtracting 6.
Start at 2 on the number line.
Go down six spaces.
Down 2 spaces takes you to 0, the ground floor. Then going down 4 more to -4.



Example 12

Write each of the following in symbols and then evaluate.

a A diver who is 6 metres below sea level rises 2 metres. What is the diver's new position?

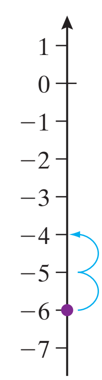
b Tony's car is in the Basement 4 carpark, 4 levels below the ground floor. He takes the lift up 7 levels. Where is he now?

Working

a $-6 + 2 = -4$
The diver is 4 metres below sea level.

Reasoning

Write 6 m below sea level as -6.
Going up 2 m is equivalent to adding 2.
Start at -6 on the number line.
Going up 2 takes us to -4.
-4 means 4 metres below sea level.



continued

Example 12 continued

Working

b $-4 + 7 = 3$

He is on the third floor.

Reasoning

Basement 4 is 4 levels below the ground floor.

Write it as -4.

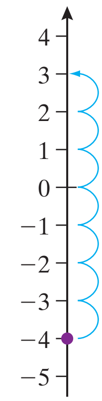
Going up 7 levels is equivalent to adding 7.

Start at -4 on the number line.

Going up 7 takes us to 3.

Going up 4 spaces takes you to 0, the ground floor. Then go up 3 more.

3 means the 3rd floor.



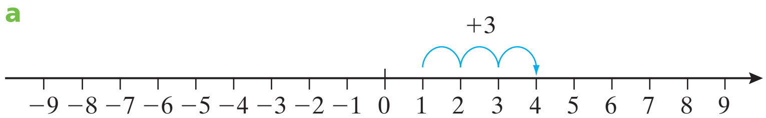
Example 13

Show the following additions on a horizontal number line then write the addition to show the result.

a $1 + 3$ **b** $-4 + 4$ **c** $-5 + 8$ **d** $-8 + 6$

Working

a

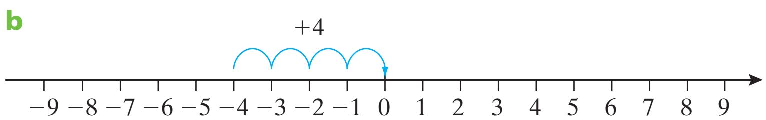


$1 + 3 = 4$

Reasoning

Start at 1. Adding a positive integer means moving to the right on the number line towards larger numbers.

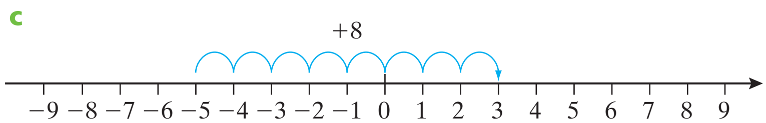
b



$-4 + 4 = 0$

Start at -4. Moving 4 to the right takes us to 0.

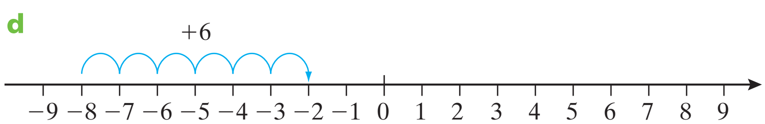
c



$-5 + 8 = 3$

Start at -5. Moving 5 to the right takes us to 0, then 3 more takes us to 3.

d



$-8 + 6 = -2$

Start at -8. Moving 6 to the right takes us to -2.

continued

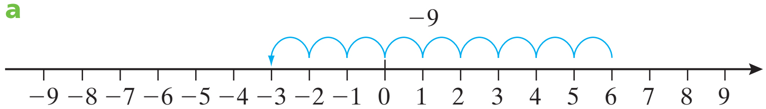
Example 14

Show the following subtractions on a horizontal number line then write the subtraction to show the result.

a $6 - 9$ **b** $-2 - 5$

Working

a

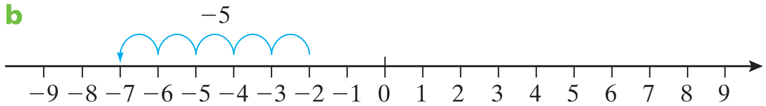


$6 - 9 = -3$

Reasoning

Start at 6. Subtracting a positive integer means moving to the left on the number line towards smaller numbers. Going back 6 takes us to 0 then back 3 more takes us to -3.

b



$-2 - 5 = -7$

Start at -2. Going back 5 takes us to -7.

We can also use the annihilation model to help us understand the subtraction of a larger integer from a smaller integer.

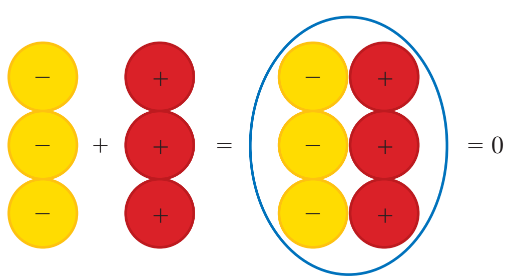
The annihilation model

Annihilation of integers: addition

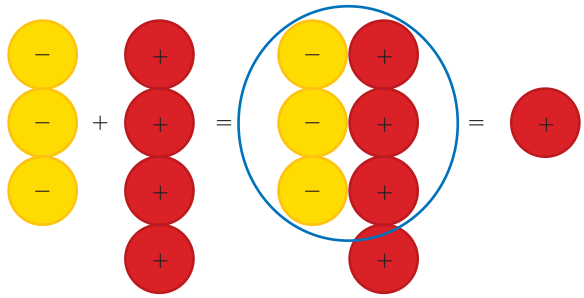
Another way of understanding the addition of a positive integer to a negative integer, is to look at what happens when a negative antiproton collides at high speed with a positive proton. Scientists have explored these collisions and found that the two particles annihilate each other (that is, cancel each other out), producing a burst of energy. In the diagram below, the red dots represent the protons and the yellow dots represent the antiprotons.

Annihilation of integers: subtraction

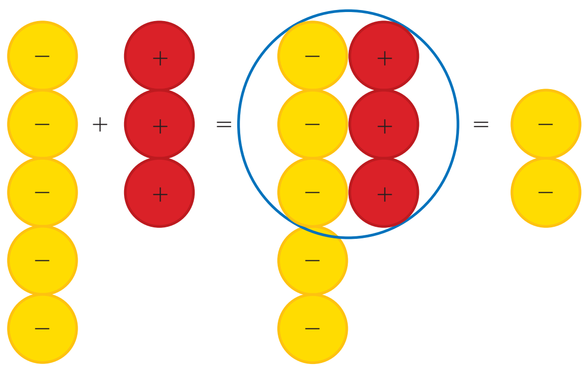
Three antiprotons represent the number -3 and three protons represent the number 3. They will annihilate each other, so $-3 + 3 = 0$.



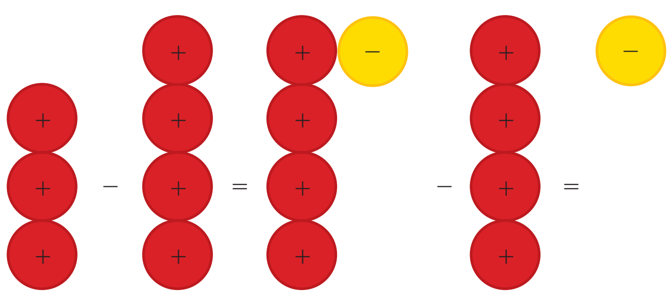
If there are 3 antiprotons but 4 protons, 3 of each will annihilate each other, leaving the extra proton, so $-3 + 4 = 1$.



If there are 5 antiprotons and 3 protons, 3 of each will annihilate each other, leaving 2 antiprotons, so $-5 + 3 = -2$.



We cannot take 4 protons from 3 protons. But we can add 1 antiproton and 1 proton to the 3 protons. We know that 1 proton and 1 antiproton is equivalent to 0 and we know that we can add 0 to a number without changing it. We can now subtract 4 protons, leaving 1 antiproton, so $3 - 4 = -1$.



Example 15

Use cancelling ('annihilation') of opposite pairs of integers to calculate these additions.

a $-3 + 4$

b $-5 + 2$

Working

a $-3 + 4 = -3 + \underbrace{3 + 1}_{4}$

$= \underbrace{-3 + 3}_{0} + 1$

$= 1$

Reasoning

If there are 3 antiprotons and 4 protons, 3 of each will annihilate each other, leaving one proton.

b $-5 + 2 = \underbrace{-3 - 2}_{-5} + 2$

$= -3 \underbrace{- 2 + 2}_{0}$

$= -3$

If there are 5 antiprotons and 2 protons, 2 of each will annihilate each other, leaving 3 antiprotons.

Counting on

Mentally counting on is another method you might use to work out the result of integer additions. Adding a positive integer means counting on to larger numbers (towards the right on the number line).

Example 16

Complete these additions by mentally counting on.

a $-2 + 9$

b $-7 + 4$

Working

a $-2 + 9 = 7$

Reasoning

Start at -2. Counting on two will take us to 0, then 7 more will take us to 7.

b $-7 + 4 = -3$

Start at -7. Counting on four takes us to -3: $-6, -5, -4, -3$

Counting back can be used to calculate the result of subtracting a positive integer. Subtracting a positive integer means counting back to smaller numbers (towards the left on the number line).

Example 17

Complete these subtractions by mentally counting back.

a $4 - 6$ **b** $-2 - 5$

Working	**Reasoning**
a $4 - 6 = -2$	Start at 4. Counting back four will take us to 0, then 2 more will take us to -2.
b $-2 - 5 = -7$	Start at -2. Counting back five takes us to -7: $-2, -3, -4, -5, -6, -7$.

Tech tip

The TI-30XB MultiView calculator can be used to add and subtract integers. For example, to find $-6 + 2$, type:

[(−)] [6] [+] [2] [enter].

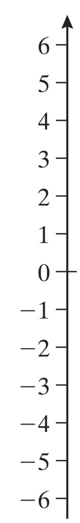
To find $-2 - 5$, type:

[(−)] [2] [−] [5] [enter].

Note: Be careful not to confuse negative [(−)] with minus [−]. They might look similar to us but the calculator sees them quite differently.

exercise 7.3

Use this vertical number line to help you with questions 1–4.



LINKS TO Example 11

For each of these temperature situations, use positive or negative integers to write what is happening, then work out the new temperature.

a The temperature at 6 am is -3°C. By noon it has risen by 8°C. What is the noon temperature?

b The lowest temperature in the morning was -5°C. The maximum temperature was twelve degrees higher. What was the maximum temperature?

c One night, the temperature was -4°C. It clouded over and the temperature rose two degrees. What was this later temperature?

d The temperature at 3 am is -2°C. By 3 pm it has risen by 20°C. What is the temperature now?

LINKS TO Example 12

For each of these temperature situations, use positive or negative integers to write what is happening, then work out the answer.

a The temperature at 3 pm is 5°C. By 11 pm it has dropped 9°C. What is the 11 pm temperature?

b The maximum temperature in the afternoon was 8°C. The minimum temperature was eleven degrees lower. What was the minimum temperature?

c One cloudy night, the temperature was 4°C. The clouds cleared and the temperature dropped ten degrees. What was this later temperature?

d The temperature at 2 pm is −4°C. By 2 am it has dropped by 3°C. What is the temperature now?

LINKS TO Example 12

Write each of the following using positive and negative integers. Then answer each question.

a The car is parked 2 floors below the ground floor. I take the lift up 5 floors. On which floor am I now?

b I am 3 floors below the ground floor. I take the lift up 2 floors. Where am I now?

c The storage space is 1 floor below ground level. Six floors higher is the family's flat. On what floor is their flat?

d Visitors park 2 floors below ground level. They take the lift up 14 floors to visit their cousins. On what floor do their cousins live?

LINKS TO Example 12

Write each of the following using positive and negative integers. Then answer each question.

a The family is staying on the tenth floor. They take the lift down twelve floors to collect their car. On which floor is their car?

b I am on the fourth floor. I take the lift down 7 floors. Where am I now?

c Sam's friend parked the car on the floor below the ground floor. Sam's own car was 2 floors lower. On what floor is Sam's car?

d Lily's office was on the seventh floor. She walked down 8 floors. Where was she then?

Use positive and negative integers to write each of the following and then evaluate.

a The diver was 15 metres below sea level. He rose 6 metres. What was his new position?

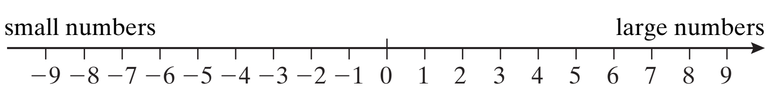
b The diver was 10 metres below sea level. He rose 8 metres. What was his new position?

c The deck of the boat was 1 metre above sea level. The diver dropped from the deck down 8 metres. What was his new position?

d The diver was 4 metres below sea level. He sank another 5 metres. What was his new position?

LINKS TO Example 13

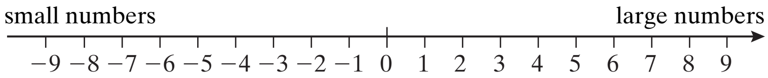
Evaluate. Use the horizontal number line to help you.



a −2 + 6	**b** −7 + 4	**c** −3 + 3	**d** −4 + 1
e −7 + 2	**f** −8 + 10	**g** −5 + 9	**h** −2 + 2
i −1 + 6	**j** −4 + 0	**k** −6 + 5	**l** −9 + 3
m −4 + 9	**n** −3 + 1	**o** −1 + 4	**p** −10 + 3
q −8 + 6	**r** −2 + 7	**s** −8 + 0	**t** −7 + 7

LINKS TO Example 14

Evaluate. Use the horizontal number line to help you.



a $-3 - 2$ **b** $8 - 3$ **c** $4 - 9$ **d** $-1 - 6$ **e** $8 - 8$
f $-2 - 4$ **g** $3 - 12$ **h** $9 - 7$ **i** $-6 - 1$ **j** $5 - 8$
k $-2 - 5$ **l** $7 - 5$ **m** $4 - 10$ **n** $-3 - 1$ **o** $6 - 8$
p $-2 - 6$ **q** $3 - 3$ **r** $4 - 5$ **s** $-7 - 0$ **t** $-3 - 3$

LINKS TO Examples 15, 16

Complete these additions by using either the annihilation method or counting on.

a $-4 + 5$ **b** $-8 + 3$ **c** $-1 + 1$ **d** $-3 + 6$ **e** $-9 + 2$
f $-2 + 7$ **g** $-6 + 4$ **h** $-1 + 9$ **i** $-11 + 2$ **j** $-2 + 12$
k $-13 + 1$ **l** $-1 + 20$ **m** $-16 + 3$ **n** $-4 + 18$ **o** $-12 + 8$
p $-7 + 11$ **q** $-13 + 4$ **r** $-18 + 14$ **s** $-20 + 20$ **t** $-13 + 17$

LINKS TO Example 17

Use counting back to help you calculate these subtractions.

a $-4 - 1$ **b** $2 - 2$ **c** $-1 - 2$ **d** $-5 - 3$ **e** $3 - 4$
f $-2 - 3$ **g** $-6 - 1$ **h** $7 - 3$ **i** $-9 - 2$ **j** $6 - 15$
k $-8 - 5$ **l** $2 - 14$ **m** $-10 - 5$ **n** $13 - 13$ **o** $-12 - 2$
p $13 - 7$ **q** $-11 - 3$ **r** $4 - 10$ **s** $20 - 3$ **t** $-8 - 3$

Calculate each of the following.

a $-5 + 3$ **b** $4 - 7$ **c** $-1 + 8$ **d** $-5 - 3$ **e** $4 - 5$
f $-2 + 4$ **g** $-5 - 2$ **h** $-7 + 4$ **i** $-8 - 2$ **j** $6 - 10$
k $-8 + 3$ **l** $-2 + 9$ **m** $-7 + 5$ **n** $-11 + 11$ **o** $-13 - 3$
p $12 - 8$ **q** $6 - 13$ **r** $-3 + 9$ **s** $14 - 20$ **t** $-12 + 5$

Evaluate each of the following, working from left to right.

a $-1 + 4 - 6$ **b** $5 - 7 + 3$ **c** $-2 - 4 + 1$ **d** $1 - 6 - 2$
e $5 + 1 - 7$ **f** $-4 + 4 - 9$ **g** $-9 + 1 + 5$ **h** $-3 - 6 + 4$
i $-8 + 3 - 2$ **j** $1 - 6 + 10$ **k** $7 - 4 - 1$ **l** $2 + 4 - 10$

The sea bird called a gannet dives for a fish. It then flies into the air to eat its catch. One gannet caught a fish 2 metres below sea level then flew up 10 metres. Its new position was

A 11 metres above sea level.
B 8 metres above sea level.
C 7 metres above sea level.
D 11 metres below sea level.
E 12 metres below sea level.

- For each of these temperature situations, use positive or negative integers to write what is happening, then work out the final temperature.
  - **a** The temperature at 6 pm was 4°C. By 11 pm it had dropped by 7°C.
  - **b** The temperature at midnight was −4°C. By 7 am it was 6°C higher.
  - **c** The temperature at 5 pm was 5°C. The clouds cleared and the temperature dropped ten degrees. Then it clouded over again and the temperature increased by 3°C.
  - **d** The temperature at noon was −2°C. By 3 pm it had increased by 4°C. By 6 pm it had dropped by 7°C.
- Write each of the following using positive and negative integers. Then work out the final floor for each person.
  - **a** Amanda enters the lift on the eighth floor. The lift goes down 6 floors. Then it goes down another 5 floors.
  - **b** Medhat works on the fourth floor. He walks down 5 floors of stairs. Then he climbs up 2 floors of stairs.
  - **c** Nikky parked her car on the floor below the ground floor. She took the lift up 6 floors. Later she walked down 2 floors of stairs.
  - **d** Mike visited a dentist on the seventh floor. He walked down 1 floor to the bathroom. Then he took the lift down 8 floors to get his car.

exercise 7.3 challenge

- Troy is scoring for a game of cards. He keeps a total for each player. Represent each of the following using positive and negative integers then calculate each player's score.
  - **a** Chris loses 480, then wins 20, then wins 100, then wins 240.
  - **b** Tai loses 260, then wins 140, then wins 100, then wins 60.
  - **c** Kate loses 480, then wins 140, then wins 10, then wins 60.
  - **d** Jane loses 260, then wins 40, then wins 30, then wins 240.
  - **e** Troy loses 320, then wins 10, then wins 10, then wins 200.
  - **f** Write the players' names in score order, starting with the winner.
- Let the height of the water in a garden tank at the end of Sunday be 0 cm. Positive is above this height and negative is below. Write each change using integers, including the height at the end of the day.
  - **a** On Monday it rained. The water rose 5 cm. No water was used.
  - **b** On Tuesday some plants were watered. The water level in the tank fell by 10 cm.
  - **c** On Wednesday it rained. The water level rose 12 cm.
  - **d** On Thursday part of the garden was watered. The water level dropped 25 cm. Then it rained a little and the water level went up 6 cm.

7.4 Adding and subtracting a negative integer

Look at this pattern of additions.

$$4 + 2 = 6$$
$$4 + 1 = 5$$
$$4 + 0 = 4$$
$$4 + (-1) =$$
$$4 + (-2) =$$

Following the pattern we see that $4 + (-1) = 3$ and $4 + (-2) = 2$. This means that

$$4 + (-1) = 4 - 1 = 3$$
$$4 + (-2) = 4 - 2 = 2$$

Adding a negative number is the same as subtracting the opposite (positive) number. For example, $4 + (-2) = 4 - 2 = 2$.

$$4 - 2 = 2$$
$$4 - 1 = 3$$
$$4 - 0 = 4$$
$$4 - (-1) =$$
$$4 - (-2) =$$

Following the pattern we see that $4 - (-1) = 5$ and $4 - (-2) = 6$. This means that

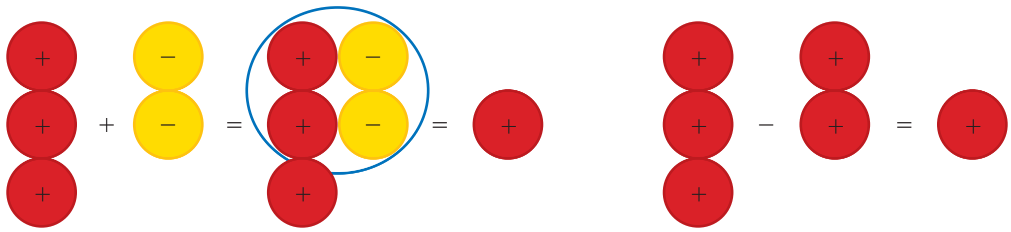
$$4 - (-1) = 4 + 1 = 5$$
$$4 - (-2) = 4 + 2 = 6$$

Subtracting a negative number is the same as adding the opposite (positive) number. For example, $4 - (-2) = 4 + 2 = 6$.

The cancelling ('annihilation') model is useful to help us understand the addition and subtraction of negative integers.

Suppose we add 2 antiprotons to 3 protons. $3 + (-2) = 1$. Notice that we get the same result if we subtract 2 protons from 3 protons: $3 - 2 = 1$.

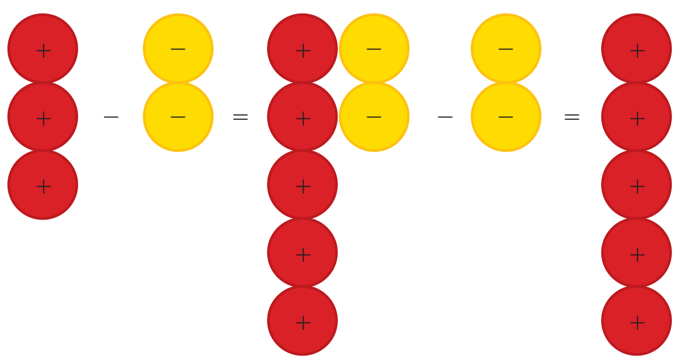
So $3 + (-2) = 3 - 2$.



We cannot subtract 2 antiprotons from 3 protons, but we can add two proton-antiproton pairs to the 3 protons. We can do this because we know that each proton-antiproton pair is equivalent to 0. Now we can subtract 2 antiprotons, leaving 5 protons: $3 - (-2) = 5$.

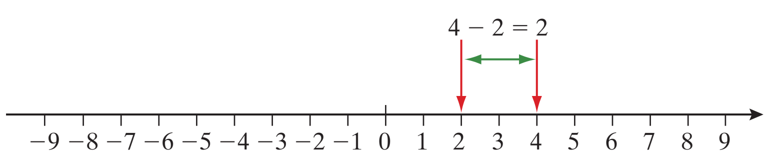
Notice that we get the same result if we add 2 protons to 3 protons: $3 + 2 = 5$.

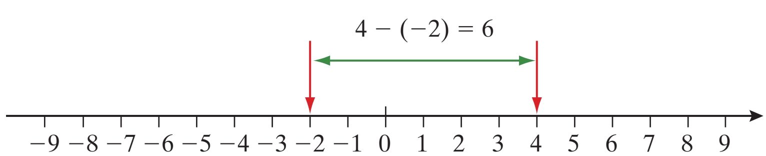
So $3 - (-2) = 3 + 2$.



Subtraction as finding the difference between two numbers

This makes sense if we think of subtracting as finding the difference between two numbers.





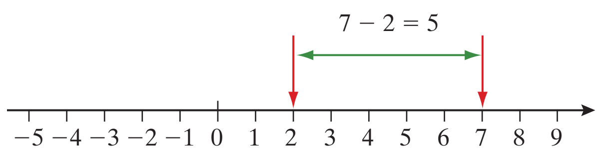
Example 18

Use a number line to show these differences.

a $7 - 2$ **b** $7 - (-2)$

Working

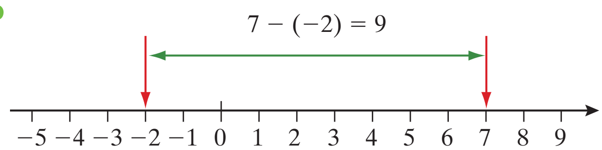
a



Reasoning

The difference between 7 and 2 is the distance between the 2 numbers on the number line.

b



The distance between 7 and −2 is the distance between the 2 numbers on the number line.

Adding and subtracting negative integers

Example 19

Calculate by first replacing the two signs between the numbers with a single sign.

a $3 + (-5)$ **b** $-3 + (-2)$ **c** $3 - (-4)$ **d** $-7 - (-2)$

Working	Reasoning
a $3 + (-5) = 3 - 5$ $= -2$	Adding a negative integer is the same as subtracting its opposite.
b $-3 + (-2) = -3 - 2$ $= -5$	Adding a negative integer is the same as subtracting its opposite.
c $3 - (-4) = 3 + 4$ $= 7$	Subtracting a negative integer is the same as adding its opposite.
d $-7 - (-2) = -7 + 2$ $= -5$	Subtracting a negative integer is the same as adding its opposite.

Tech tip

To find $-3 + (-2)$ (example 19 part b) using the TI-30XB MultiView calculator, type in the question from left to right like this:

[(−)] [3] [+] [(−)] [2] [enter].

To find $3 - (-4)$ (example 19 part c) using the TI-30XB MultiView calculator, type in the question from left to right like this:

[3] [−] [(−)] [4] [enter].

exercise 7.4

LINKS TO Example 18

- Use a number line to show these differences.

  a $7 - 2$ **b** $4 - (-3)$ **c** $2 - (-8)$ **d** $6 - (-7)$

LINKS TO Examples 19a and b

- Evaluate by first replacing the two signs between the numbers with a single sign.

  a $5 + (-3)$ **b** $4 + (-4)$ **c** $-4 + (-4)$ **d** $8 + (-2)$
  e $7 + (-2)$ **f** $-7 + (-3)$ **g** $-3 + (-1)$ **h** $9 + (-1)$
  i $-1 + (-3)$ **j** $-10 + (-4)$ **k** $10 + (-4)$ **l** $-4 + (-10)$
  m $4 + (-10)$ **n** $-2 + (-7)$ **o** $8 + (-8)$ **p** $-9 + (-3)$

LINKS TO Examples 19c and d

- Evaluate by first replacing the two signs between the numbers with a single sign.

  a $5 - (-3)$ **b** $4 - (-4)$ **c** $-4 - (-4)$ **d** $5 - (-2)$
  e $-5 - (-2)$ **f** $7 - (-3)$ **g** $3 - (-2)$ **h** $6 - (-1)$
  i $-1 - (-2)$ **j** $-7 - (-4)$ **k** $-2 - (-7)$ **l** $-4 - (-10)$
  m $-1 - (-2)$ **n** $-4 - (-7)$ **o** $-8 - (-8)$ **p** $-9 - (-3)$

- Evaluate by first replacing the two signs between the numbers with a single sign.

  a $7 + (-3)$ **b** $3 - (-2)$ **c** $-11 - (-11)$ **d** $9 + (-7)$
  e $-5 - (-8)$ **f** $12 + (-13)$ **g** $-8 + (-10)$ **h** $4 - (-5)$
  i $-1 + (-5)$ **j** $-7 + (-2)$ **k** $-12 - (-7)$ **l** $-12 + (-7)$
  m $-4 - (-7)$ **n** $-4 + (-7)$ **o** $-9 + (-9)$ **p** $-9 - (-9)$

- For each space in the table, add the number at the top of the column to the number at the left of the row. For example, -5 was obtained by adding -2 to -3.

  Example

  $-3 + (-2)$
  $= -3 - 2$
  $= -5$

+	−2	4	−7	2
−3	−5	1		
7				
−6				
−1				1

- For each space in the table, subtract the number at the top of the column from the number at the left of the row. For example, -5 was obtained by subtracting -1 from -6.

  Example

  $-6 - (-1)$
  $= -6 + 1$
  $= -5$

−	−1	5	−3	4
−6	−5			
2				
−7				
3				−1

- The expression −2 − (−5) will give the same answer as
  A 2 − (−5) **B** −2 + (−5) **C** 2 + (−5) **D** −2 + 5 **E** 2 + 5

- When evaluated, −5 + (−3) =
  A 8 **B** 5 **C** 2 **D** −2 **E** −8

LINKS TO Example 20

- Find the difference between these maximum and minimum temperatures to find the temperature range for the day.
  a maximum −3°C and minimum −8°C **b** maximum 6°C and minimum −2°C
  c maximum 4°C and minimum −6°C **d** maximum −8°C and minimum −23°C

- The following minimum and maximum temperatures were recorded for two weeks in Reykjavik in Iceland in December 2010.

December	14th	15th	16th	17th	18th	19th	20th	21st	22nd	23rd	24th	25th
Minimum (°C)	6	0	−4	−3	1	−2	−3	−8	−10	−4	0	0
Maximum (°C)	8	6	−1	−1	3	0	1	−4	−6	1	3	2

a What was the lowest minimum temperature and on what date did this occur?
b What was the lowest maximum temperature and on what date did this occur?
c How many degrees higher was the minimum temperature on December 23 than on December 22?
d How many degrees higher was the maximum temperature on December 23 than on December 22?
e How many degrees difference was there between the maximum temperature and the minimum temperature on December 16?
f How many degrees difference was there between the maximum temperature and the minimum temperature on December 23?
g How much lower was the minimum temperature on December 22 than on December 14?
h How much lower was the maximum temperature on December 22 than on December 14?

exercise 7.4 challenge

- In a magic square, the total of every row, every column and the two diagonals are all the same. Consider this magic square.

−3		
	0	
	−2	3

a What is the total for the diagonal that is filled in?
b Complete this magic square.

A new magic square can be made by adding or subtracting the same number to or from each number in the square.

c Make a new magic square by subtracting 2 from each number.
d What is the new total for each row, column and the two diagonals?
e Why has this new total occurred?

7.5 Multiplying integers

By looking at number patterns we can see what happens when we multiply by negative integers.

We start by looking at $5 \times 4 = 20$, $5 \times 3 = 15$ and so on. As we multiply 5 by 4, 3, 2, 1 and 0 we notice that the product is decreasing by 5 each time: 20, 15, 10, 5, 0. We can extend this to multiplying 5 by negative integers. The pattern will continue $-5, -10, -15, -20$. So we see that, logically, if a positive integer is multiplied by a negative integer, the product is negative.

We know also that the order in which we multiply two numbers does not matter, for example, $5 \times 4 = 4 \times 5 = 20$. So if $5 \times -4 = -20$, then $-4 \times 5 = -20$.

We now look at the pattern $-5 \times 4 = -20$, $-5 \times 3 = -15$ and so on. This time we notice that the products are *increasing* by 5 each time: $-20, -15, -10, -5, 0$ so logically the next products in the sequence are 5, 10, 15, 20. So the product of two negative integers is positive.

$5 \times 4 = 20$
$5 \times 3 = 15$
$5 \times 2 = 10$
$5 \times 1 = 5$
$5 \times 0 = 0$
$5 \times (-1) = -5$
$5 \times (-2) = -10$
$5 \times (-3) = -15$
$5 \times (-4) = -20$

$-5 \times 4 = -20$
$-5 \times 3 = -15$
$-5 \times 2 = -10$
$-5 \times 1 = -5$
$-5 \times 0 = 0$
$-5 \times (-1) = 5$
$-5 \times (-2) = 10$
$-5 \times (-3) = 15$
$-5 \times (-4) = 20$

If the signs of two integers are the *same* (both positive or both negative) the product is *positive*.

If the signs of two integers are *different* (one positive and the other negative) the product is *negative*.

Example 20

Carry out the following multiplications.

a -7×6 **b** 3×8 **c** $-2 \times (-9)$
d $4 \times (-5)$ **e** $-3 \times (-2) \times (-5)$ **f** $-4 \times 7 \times (-10)$

Working	Reasoning
a -7×6 $= -42$	-7 and 6 have different signs so the answer is negative.
b 3×8 $= 24$	3 and 8 have the same sign, both positive, so the answer is positive.
c $-2 \times (-9)$ $= 18$	-2 and -9 have the same sign, both negative, so the answer is positive.

continued

Example 20 continued

Working	Reasoning
d $4 \times (-5)$ $= -20$	4 and −5 have different signs so the answer is negative.
e $-3 \times (-2) \times (-5)$ $= 6 \times (-5)$ $= -30$	−3 and −2 have the same sign, both negative, so the answer to the first multiplication is positive. 6 and −5 have different signs so the final answer is negative.
f $-4 \times 7 \times (-10)$ $= -28 \times (-10)$ $= 280$	−4 and 7 have different signs so the answer to the first multiplication is negative. −28 and −10 have the same sign, both negative, so the final answer is positive.

Squares of negative integers

When we square a number we multiply it by itself. So the square of −2 is $-2 \times (-2)$ which is 4.

Brackets are important when we square negative numbers. Notice the difference between $(-2)^2$ and -2^2.

$$(-2)^2 = (-2) \times (-2) = 4 \qquad -2^2 = -(2 \times 2) = -4$$

Example 21

Calculate each of the following.

a $(+6)^2$ **b** $(-7)^2$ **c** -7^2

Working	Reasoning
a $(+6)^2$ $= 6^2$ $= 6 \times 6$ $= 36$	It is not necessary to write positive signs. When a number is squared, it is multiplied by itself.
b $(-7)^2$ $= -7 \times -7$ $= 49$	When a negative number is squared, it is necessary to place brackets around it. (On a calculator, if brackets are not used, the digit 7 is squared first and then the negative of the answer is found. This will give an incorrect answer of −49.)
c -7^2 $= -(7 \times 7)$ $= -49$	The negative sign is not part of the squaring process.

Multiplying integers can be done with a calculator. However, with small numbers it is quicker to work mentally.

Tech tip

The TI-30XB MultiView calculator can be used to multiply by a negative integer. For example, to find $-3 \times (-2)$, type:

[(−)] [3] [×] [(−)] [2] [enter].

To find $7 \times (-4)$, type:

[7] [×] [(−)] [4] [enter].

The TI-30XB MultiView calculator can be used to square integers. For example, to find $(-6)^2$, type:

[(] [−] [6] [)] [x^2] [enter].

To find -6^2, type:

[−] [6] [x^2] [enter].

Note: -6^2 does not mean the same as $(-6)^2$.

exercise 7.5

LINKS TO Example 21

1. Evaluate.

a 4×3	**b** $4 \times (-3)$	**c** -4×3	**d** $-4 \times (-3)$
e 11×2	**f** $11 \times (-2)$	**g** -11×2	**h** $-11 \times (-2)$
i 5×6	**j** $5 \times (-6)$	**k** -5×6	**l** $-5 \times (-6)$

2. **a** Complete this table of multiplications.

$5 \times 3 =$	$3 \times 3 =$	$-4 \times 3 =$	$-10 \times 3 =$
$5 \times 2 =$	$3 \times 2 =$	$-4 \times 2 =$	$-10 \times 2 =$
$5 \times 1 =$	$3 \times 1 =$	$-4 \times 1 =$	$-10 \times 1 =$
$5 \times 0 =$	$3 \times 0 =$	$-4 \times 0 =$	$-10 \times 0 =$
$5 \times (-1) =$	$3 \times (-1) =$	$-4 \times (-1) =$	$-10 \times (-1) =$
$5 \times (-2) =$	$3 \times (-2) =$	$-4 \times (-2) =$	$-10 \times (-2) =$
$5 \times (-3) =$	$3 \times (-3) =$	$-4 \times (-3) =$	$-10 \times (-3) =$

b Complete this table to summarise what happens to the signs when multiplying integers.

×	−	+
+		
−		

LINKS TO Example 21

Multiply each of the pairs of numbers below.

a 2×9 **b** -5×8 **c** $-3 \times (-7)$ **d** $6 \times (-4)$

e $10 \times (-3)$ **f** $-1 \times (-7)$ **g** 11×9 **h** -4×12

i $-5 \times (-4)$ **j** $3 \times (-6)$ **k** -2×7 **l** 10×5

m -7×5 **n** 4×9 **o** $8 \times (-10)$ **p** $-11 \times (-3)$

LINKS TO Example 21

Evaluate each of the following. Do not use your calculator.

a $2 \times (-5) \times 3$ **b** $3 \times (-1) \times (-4)$ **c** $2 \times 3 \times (-3)$

d $-2 \times 3 \times (-5)$ **e** $2 \times 3 \times (-2)$ **f** $-5 \times 3 \times (-2)$

g $-3 \times (-1) \times (-7)$ **h** $2 \times (-4) \times 3$ **i** $-7 \times 2 \times (-1)$

j $4 \times (-3) \times (-3)$ **k** $-2 \times (-2) \times -10$ **l** $-6 \times (-3) \times (-1)$

m $-5 \times (-4) \times 3$ **n** $3 \times (-2) \times 3$ **o** $-2 \times 4 \times (-2)$

The expression $-3 \times (-4)$ will give the same answer as

A $3 \times (-4)$ **B** -2×6 **C** $2 \times (-6)$ **D** -3×4 **E** 2×6

When evaluated, $-5 \times (-3) \times 2 =$

A 60 **B** 30 **C** 15 **D** -15 **E** -30

LINKS TO Example 22

Evaluate these squares.

a 7^2 **b** $(-7)^2$ **c** 1^2 **d** $(-1)^2$

e -2^2 **f** $(-2)^2$ **g** -5^2 **h** $(-5)^2$

Complete each of the following tables of values to show the input and output from a number machine.

a output = 3 × input

x	−2	−1	0	1	2
y					

b output = −4 × input

x	−2	−1	0	1	2
y					

c output = 7 × input

x	−2	−1	0	1	2
y					

d output = −5 × input

x	−2	−1	0	1	2
y					

exercise 7.5 challenge

Copy and complete each of these multiplication puzzles. The integers at the left of each row must be multiplied by the integers at the top of each column to give the integers in the white squares.

a

	−5		
		−21	
	30		−24
		−6	8

b

×		−6	
		−66	55
	−12		
	−21		−35

7.6 Dividing integers

Every multiplication can be rewritten as two divisions. For example:

$-5 \times (-3) = 15$ can be rewritten as $15 \div (-5) = -3$ and $15 \div (-3) = -5$.

$-5 \times 3 = -15$ can be rewritten as $-15 \div (-5) = 3$ and $-15 \div 3 = -5$.

> If numbers with the *same sign* are divided, the answer will be a *positive* number.
>
> If numbers with *different signs* are divided, the answer will be a *negative* number.

Example 22

Carry out the following divisions.

a $-24 \div 6$ **b** $-18 \div (-2)$ **c** $\frac{15}{5}$

d $\frac{20}{-4}$ **e** $-30 \div (-2) \div (-5)$ **f** $18 \div (-3) \div (-2)$

Working	Reasoning
a $-24 \div 6$ $= -4$	-24 and 6 have different signs so the answer is negative.
b $-18 \div (-2)$ $= 9$	-18 and -2 have the same sign, both negative, so the answer is positive.
c $\frac{15}{5}$ $= 15 \div 5$ $= 3$	15 and 5 have the same sign, both positive, so the answer is positive.
d $\frac{20}{-4}$ $= 20 \div (-4)$ $= -5$	20 and -4 have different signs so the answer is negative.
e $-30 \div (-2) \div (-5)$ $= 15 \div (-5)$ $= -3$	-30 and -2 have the same sign, both negative, so the answer to the first division is positive. 15 and -5 have different signs so the final answer is negative.
f $18 \div (-3) \div (-2)$ $= -6 \div (-2)$ $= 3$	18 and -3 have different signs so the answer to the first division is negative. -6 and -2 have the same sign, both negative, so the final answer is positive.

Order of operations

The order of operations that applies to the numbers that we have worked with before also applies to integers.

- Powers and roots or anything in brackets are worked first.
- Multiplication or divisions are worked from left to right in the order that they occur.
- Additions or subtractions are then worked from left to right in the order that they occur.

Example 23

Evaluate each of the following:

a $5 - (-24) \div (-6) \times 2 + 1$

b $\dfrac{-2 - 10}{-4}$

Working

a $5 - (-24) \div (-6) \times 2 + 1$
$= 5 - 4 \times 2 + 1$
$= 5 - 8 + 1$
$= -2$

Reasoning

Work the multiplications and divisions from left to right. $-24 \div (-6) = 4$ then $4 \times 2 = 8$.
Then work the additions and subtractions from left to right.

Working

b $\dfrac{-2 - 10}{-4}$
$= \dfrac{(-2 - 10)}{-4}$
$= \dfrac{-12}{-4}$
$= 3$

Reasoning

When there is an addition or subtraction in the numerator or denominator of a fraction, put brackets around it.
Here, since the numerator is divided by -4, the numerator needs to be worked out first. We put brackets around the numerator to remind us to work it out first.
There are no powers or roots and no 'of's.
Work any multiplications and divisions from left to right.
There are no additions or subtractions.

Tech tip

The TI-30XB MultiView calculator can be used to divide by a negative integer.
For example, to find $-6 \div (-2)$, type:

[(−)] [6] [÷] [(−)] [2] [enter].

To find $8 \div (-4)$, type:

[8] [÷] [(−)] [4] [enter].

exercise 7.6

LINKS TO Example 22

● Divide each of the pairs of numbers below.

a $30 \div (-3)$ **b** $-7 \div (-7)$ **c** $99 \div 9$ **d** $-48 \div 12$
e $18 \div 9$ **f** $-40 \div 8$ **g** $-21 \div (-7)$ **h** $24 \div (-4)$
i $-35 \div 5$ **j** $36 \div 9$ **k** $80 \div (-10)$ **l** $-33 \div (-3)$
m $-20 \div (-4)$ **n** $18 \div (-6)$ **o** $-14 \div 7$ **p** $50 \div 5$
q $-8 \div (-2)$ **r** $15 \div (-5)$ **s** $10 \div (-1)$ **t** $-121 \div (-11)$
u $-9 \div 3$ **v** $-32 \div (-8)$ **w** $-54 \div 9$ **x** $0 \div (-6)$

LINKS TO Example 22

● Carry out each of the following divisions.

a $\frac{-30}{-5}$ **b** $\frac{14}{-7}$ **c** $\frac{-36}{9}$ **d** $\frac{88}{11}$ **e** $\frac{36}{3}$ **f** $\frac{-25}{-5}$
g $\frac{-21}{7}$ **h** $\frac{8}{-4}$ **i** $\frac{32}{-8}$ **j** $\frac{-18}{-2}$ **k** $\frac{45}{9}$ **l** $\frac{-70}{10}$
m $\frac{-33}{3}$ **n** $\frac{24}{6}$ **o** $\frac{60}{-12}$ **p** $\frac{48}{-8}$ **q** $\frac{-30}{6}$ **r** $\frac{-32}{-4}$
s $\frac{-60}{20}$ **t** $\frac{-60}{-12}$ **u** $\frac{-28}{7}$ **v** $\frac{63}{-9}$ **w** $\frac{-72}{-9}$ **x** $\frac{0}{-5}$

● Find the missing factor for each multiplication. Then rewrite each multiplication as a division in two different ways.

a $5 \times \_\_ = -15$ **b** $-4 \times \_\_ = -24$ **c** $\_\_ \times -7 = 14$ **d** $\_\_ \times 11 = 44$
e $\_\_ \times 3 = -18$ **f** $-11 \times \_\_ = 33$ **g** $\_\_ \times (-8) = -48$ **h** $5 \times \_\_ = -25$

● The expression $\frac{-36}{-4}$ will give the same answer as

A $\frac{36}{-4}$ **B** $\frac{-36}{4}$ **C** $\frac{18}{-2}$ **D** $\frac{-18}{2}$ **E** $\frac{-18}{-2}$

● When evaluated, $-60 \div (-6) \div 2 =$

A -20 **B** -5 **C** 5 **D** 10 **E** 20

● Find the quotient for each of these divisions.

a 130 divided by -10 **b** -400 divided by -80 **c** -150 divided by 5

LINKS TO Example 23

● Carry out each of the following calculations.

a $-40 \div 2 \div (-5)$ **b** $-48 \div 6 \div 2$
c $90 \div (-9) \div (-5)$ **d** $-28 \div (-7) \div (-4)$
e $36 \div 3 \div (-4)$ **f** $30 \div (-3) \div (-2)$
g $-200 \div (-10) \div (-2)$ **h** $24 \div (-3) \div (-1)$
i $-50 \div (-2) \div (-5)$ **j** $88 \div (-11) \div 2$
k $-120 \div 10 \div (-3)$ **l** $-18 \div (-3) \div (-3)$
m $-90 \div 3 \div (-5)$ **n** $-36 \div (-9) \div (-2)$
o $-150 \div (-10) \div (-5)$ **p** $140 \div (-7) \div (-2)$

LINKS TO Example 23

Evaluate.

a $3 + (-14) \div (-7)$
b $(14 - (-2)) \div 4$
c $-2 \times (-4) + 3 \times (-3)$
d $-30 \div 5 \times (-2)$
e $11 - (9 + 6) \div (-5) \times 3$
f $-20 \div (-4) - 21 \div (-3)$
g $-6 \times 2 \div (-4) + 7$
h $6 - (-12) \div 6 \times 2$

Evaluate. Where there is more than one number in the numerator or denominator of a fraction, remember to bracket those numbers, and work them out first.

a $\frac{7 + 5}{3}$
b $\frac{9 - 1}{-5 + 3}$
c $\frac{-28}{3 - 7}$
d $\frac{4 - 13}{13 - 4}$
e $\frac{2 \times (-4) - 1}{3}$
f $\frac{-24}{9 - 2 \times 6}$
g $\frac{-2 + 22}{3 - 8}$
h $\frac{8 - 2 \times 4}{-5 - 2}$

The expression $\frac{6 - 3^2}{3}$ is equal to

A -1 **B** 1 **C** 3 **D** 4 **E** 6

Evaluate.

a $\frac{3 - 9}{2}$
b $\frac{-15}{3} + 4$
c $\frac{-42}{3 + 4}$
d $17 - \frac{-8}{2}$

exercise 7.6 challenge

Complete each of these tables of values using the given rule.

a $y = \frac{x}{3}$

x	-12	-6	0	6	12
y					

b $y = \frac{x}{-2}$

x	-12	-6	0	6	12
y					

c $y = \frac{x}{6}$

x	-12	-6	0	6	12
y					

d $y = \frac{x}{-1}$

x	-12	-6	0	6	12
y					

The minimum temperatures (in degrees Celsius) for five days in a row on Mount Buffalo are $-2, -1, 2, -3, -1$.

a Find the average minimum temperature for the five days.

b The minimum temperature on the next day is -7. Find the average minimum for the six days.

Analysis task

1, 2, 3, 4, what numbers are we heading for?

By using the operations of addition, subtraction, multiplication and division with the correct order of operations we can use the integers 1, 2, 3 and 4 to make other integers, including negative integers.

a Calculate $1 + 2 \times 3 - 4$

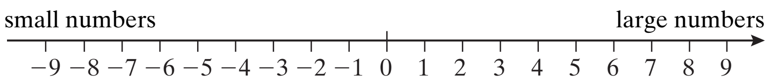
b Calculate $1 \times 3 - 4 \times 2$

c Use the numbers 1, 2, 3 and 4 in any order, and any of the operations of addition, subtraction, multiplication and division to write calculations that have as answers each of the integers from 20 to -20, including 0.

Review Integers

Summary

- The numbers below zero on a vertical number line (or to the left on a horizontal number line) are called **negative** numbers.
- Integers are whole numbers. They can be positive, negative and zero.
- To add and subtract positive integers, one way to calculate is to use a number line. To add, go towards the large numbers (usually up or to the right). To subtract, go towards the small numbers (usually down or to the left).



- Another way to add positive integers is to count on, and another way to subtract positive integers is to count back.
- The annihilation model can also help us with adding and subtracting numbers.
- Adding a negative integer gives the same result as subtracting the positive integer. Subtracting a negative integer gives the same result as adding the negative integer.
- To multiply or divide (but not to add or subtract) two numbers, if the signs of the numbers are the same, the result will be positive. If the signs of the numbers are different, the result will be negative.
- The same order of operations applies to integers as to any other numbers. (Brackets, Powers and Roots, Of, Divisions and Multiplications, Additions and Subtractions.)

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

ascending order	integer	number line	positive
descending order	negative	order of operations	squares

Revision

Multiple-choice questions

1. Arranged in ascending order 2, −5, 9, −1, −3 is

   A −1, 2, −3, −5, 9 **B** 9, −5, −3, 2, −1 **C** −5, −3, −1, 2, 9
   D 9, 2, −1, −3, −5 **E** −1, −3, −5, 2, 9

2. −2 + 6 is equal to

   A −12 **B** −8 **C** −4 **D** 4 **E** 8

3. −11 − (−3) is equal to

   A −33 **B** −14 **C** −8 **D** 8 **E** 14

4. −2 × 5 × (−3) × (−2) is equal to

   A −60 **B** −30 **C** −2 **D** 2 **E** 60

5. Evaluate $\frac{6 + 15 \div (-3) - 9}{3 - 5}$

   A −4 **B** 8 **C** −1 **D** −8 **E** 4

Short-answer questions

1. Carry out each of the following calculations.

   a −1 + 7 **b** −9 + 5 **c** −4 − 4 **d** −6 + 6

2. Evaluate each of the following.

   a −6 + (−4) **b** −9 − (+2) **c** 3 − (−2) **d** −7 + 4

3. Multiply each of the pairs of numbers below.

   a 6 × 3 **b** −4 × 7 **c** −8 × (−5) **d** 10 × (−2)

4. Divide each of the pairs of numbers below.

   a −15 ÷ 3 **b** $\frac{-18}{2}$ **c** 24 ÷ (−4) **d** $\frac{-55}{-5}$

5. Evaluate each of the following.

   a 4 − (−36) ÷ (−6) × 2 + 9 **b** $\frac{-3 + 21}{-6}$

   c $-3 \times (-4 + 2)^2$ **d** −5 × (−3) + 2 × (−6)

Extended-response questions

Professor Stuffitup is working on a new flavour of ice-cream. He mixes the ingredients then sets the thermostat.

- **a** He sets it at 9 degrees below zero. Write this as an integer.
- **b** The ice-cream develops ice crystals at this temperature so he sets the next batch 8 degrees higher.
  - **i** Write a calculation to show this.
  - **ii** What temperature is set for the second batch?
- **c** This does not freeze properly so he sets a third batch 5 degrees lower than the second batch.
  - **i** Write a calculation to show this.
  - **ii** What temperature is set for the third batch?

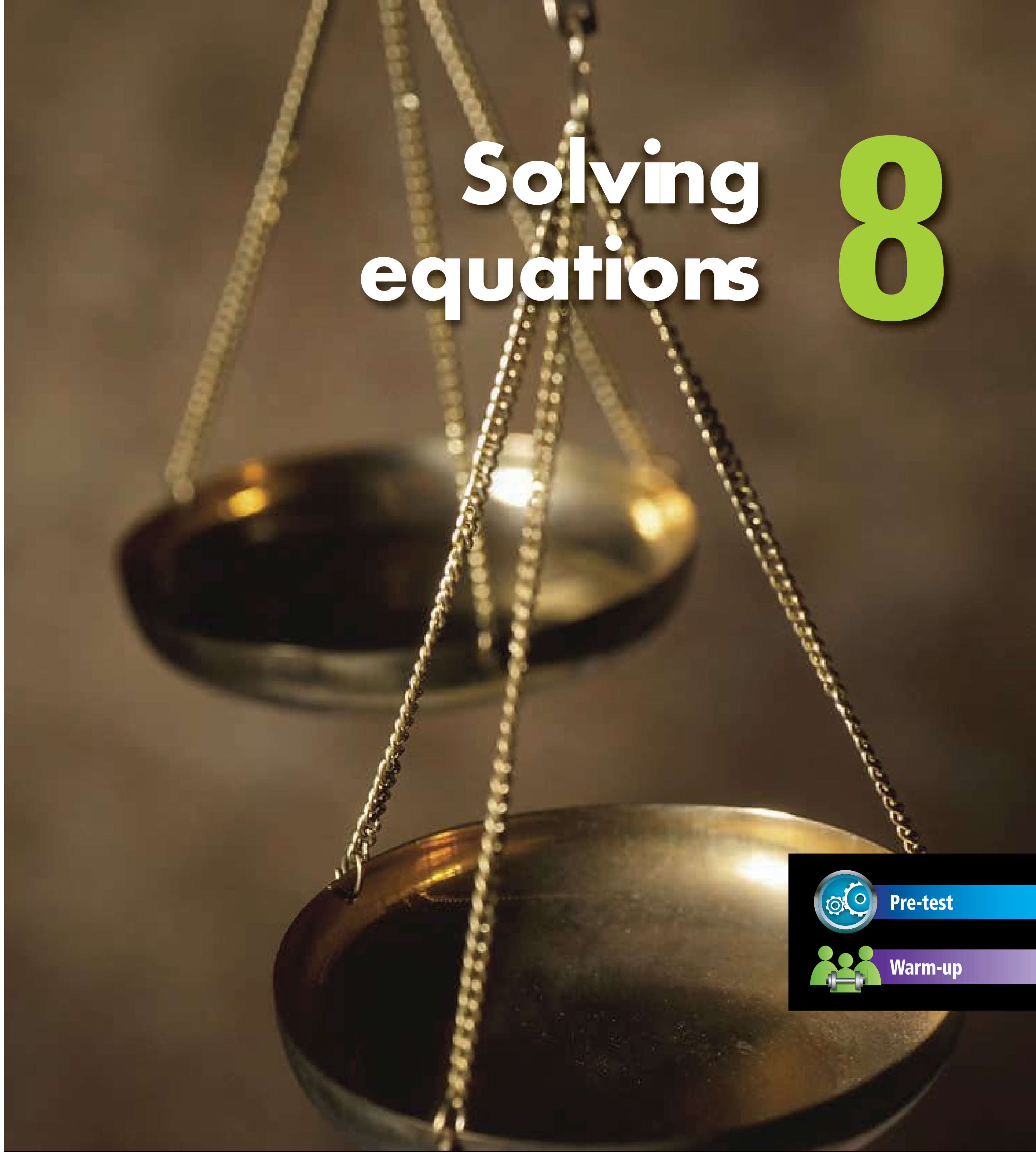
The third batch is the most successful. He wants to make another batch in a second freezer.

- **d** The second freezer is set at 0°C. He wants to lower the temperature in three equal jumps so that it reaches −6°C. How big should each jump be?

Artemis and Zorba are in a boat preparing for their first diving lesson.

- **a** They put on their air tanks on the deck of the boat, which is one metre above sea level. Write their position as an integer. (Make above sea level positive, and below sea level negative.)
- **b** They go to the diving platform at the back of the boat, which is on a level with the water. Write their position as an integer.
- **c** With their instructor they descend slowly for two metres.
  - **i** Write their location as an integer.
  - **ii** Find the difference between their location now and their location at the start.
- **d** They descend another 4 metres.
  - **i** Write the integer calculation to work out their location now.
  - **ii** What is that location?
- **e** They now descend at 3 metres per minute for four minutes.
  - **i** Write this as a multiplication of two integers.
  - **ii** Calculate the answer for part i and explain what this tells us about how their location has changed.
  - **iii** What is their depth now?
- **f** Artemis sees a clam shell at −8.1 m while Zorba sees a sea slug at −7.5 m. Which is deeper, the clam shell or the sea slug?

Practice quiz
Chapter 7



8 Solving equations

Pre-test

Warm-up

We can think of an algebraic equation as being like a set of scales. The two sides of the equation are equal, so the scales are balanced. If we add something to one side of the scales without adding something to the other side, the scales will no longer be balanced. We can solve equations by doing the same to both sides.

8.1 What is an equation?

Comparing the values of expressions

In mathematics, when two expressions have the same value we use an equals symbol ('=') to show this. For example, we know that 3   4 and 6 + 6 each has the value 12, so we can write 3   4 = 6 + 6. This type of number statement is called an **equation** because it states that the expressions on each side of the equation are equal in value. We can also describe such a statement as **true**.

Example 1

In each of the following equations, insert a number on the blank line to make the equation true.

a ___ + 5 = 17

b 4 × ___ = 32

Working	Reasoning
a ___ + 5 = 17 12 + 5 = 17	If the missing number is 12, the equation will be true.
b 4 × ___ = 32 4 × 8 = 32	If the missing number is 8, the equation will be true.

If two expressions are *not* equal in value, then mathematicians may use the symbol '   ' to show this. The statement 3   7 = 4   5 is an example of this. Since the two expressions are not equal in value, the statement is **false** (not true) and the '   ' should be used instead of the '=' symbol 3   7   4   5.

The following symbols are often used to compare the value of two expressions.

Symbol	Meaning	Example
=	is equal to	3 − 1 = 2 1
	is not equal to	3 − 1 4 1
	is greater than	3 − 1 1 1
	is less than	3 − 1 4 1

Example 2

State whether each of the following statements is true or false.

a 5 + 2 = 2 × 4 + 1

b 3 × 4 = 5 × 2 + 2

continued

Example 2 continued

Working	Reasoning
a $5 + 2 = 2 \times 4 + 1$ $LS = 5 + 2 = 7$ $RS = 2 \times 4 + 1$ $= 8 + 1$ $= 9$ $LS \neq RS$ so the statement is false.	If the statement is true, the right side and the left side must be equal. Here they are not equal so the statement is not true.
b $3 \times 4 = 5 \times 2 + 2$ $LS = 3 \times 4$ $= 12$ $RS = 5 \times 2 + 2$ $= 10 + 2$ $= 12$ $LS = RS$ so the statement is true.	The right side and the left side are equal so the statement is true.

Example 3

By changing only the expression on the right side of the equation $3 \quad 4 = 6 + 6$, find three other expressions which make this statement true.

Working	Reasoning
$3 \times 4 = 9 + 3$	$3 \times 4 = 12$ $9 + 3 = 12$ Both expressions have the same value.
$3 \times 4 = 24 \div 2$	$3 \times 4 = 12$ $24 \div 2 = 12$ Both expressions have the same value.
$3 \times 4 = 18 - 6$	$3 \times 4 = 12$ $18 - 6 = 12$ Both expressions have the same value.

exercise 8.1

LINKS TO Example 1

1 In each of the following equations, insert a number on the blank line to make the equation true.

a $\_\_ + 7 = 12$

b $15 - \_\_ = 9$

c $3 \times \_\_ = 48$

d $9 + \_\_ = 6 + 8$

e $2 \times \_\_ + 3 \times 4 = 8 \times 2$

f $7 \times 9 - \_\_ \times 9 = 4 \times 9$

g $6 \times (3 + \_\_) = 6 \times 5$

h $5 + \_\_ \times 3 = 17$

i $(5 - \_\_) \times 6 = 18$

j $6 \times 5 - 24 \div \_\_ = 22$

LINKS TO Example 2

State whether each of the following statements is true or false.

a $3 + 4 = 4 - 2 + 5$

b $4 - 3 = 7 \quad 1$

c $2 \quad 6 = 4 \quad 3 - 2$

d $\frac{12}{2} - 1 = 7 - 2$

e $16 \quad 8 + 3 = 5 + 0$

f $2 \quad 1 \quad 3 = 6 \quad 2 + 2$

g $12 - 3 \times 2 = 15 + 3$

h $10 + 14 \div 2 = 30 \div 2 + 2$

i $24 \div 3 + 1 = 72 \div 9 + 1$

j $2 \times 8 \div 4 = 20 \div 5$

k $3 \times (4 + 2) = 10 + 4 \times 2$

l $10 \div (3 - 1) = 42 \div 7$

LINKS TO Example 3

The equation $10 \times 4 = 32 + 8$ is true. Which one of the following could replace the expression on the right side of the equation whilst still making it a true statement?

A $6 + 2 \times 5$

B $5 \times 5 + 3$

C $5 \times 10 - 1$

D $5 \times 2 \times 3 + 1$

E $30 + 5 \times 2$

LINKS TO Example 3

The statement $4 \times (3 + 2) = 28 - 8$ will become false if the left side is replaced with:

A $3 \times 8 - 4$

B $(10 + 2) \times 5$

C $36 - 4^2$

D $4 + 8 \times 2$

E $25 - 10 \div 2$

Rani says that if a statement such as $19 - 2 \quad 3 \quad 10$ is true, then it is also correct that $19 - 2 \quad 3 \quad 10$. Is she right? Explain your answer.

exercise 8.1 challenge

LINKS TO Example 3

Change a single number in each of the following to turn the statement into an equation. Then replace the $<$, $>$ or $\neq$ with $=$.

a $2 \times 7 \neq 2 \times 5 + 2$

b $9 - 4 + 3 \neq 3 \times 5 + 3$

c $7 \times 3 - 5 \times 4 \neq 3 + 8$

d $18 - 6 \div 2 \neq 4 \times 3$

e $(18 - 9) \neq 20 \div 2 - 4$

f $3 + 3 > 10 - 7 + 1$

g $30 - 5 - 10 \div 2 < 24 - 3$

h $50 \div 10 - 4 < 20 \div 4 - 3$

i $25 - 3 \times 4 > 32 \div 8 + 5$

j $(12 - 8) \div 4 < 16 \div 4 - 1$

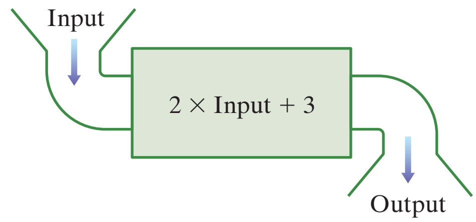
k $(18 - 6) \div 3 \times 5 > (5 + 1) \times 4$

l $16 \div 2 \div 8 \neq 30 \div 5 \div 2$

8.2 Input and output machines

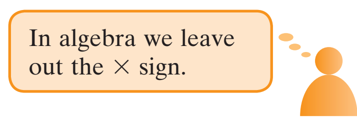
The equations considered so far have only contained numbers, and so you can tell that they are true statements. In an equation involving pronumerals as well as numbers, the statement may be true only for certain values of the pronumerals.

Consider a number machine that takes in numbers, doubles them and then adds 3.



$2 \times \text{Input} + 3 = \text{Output}$

Supposing the output number is 11. If the pronumeral n represents the input number, we can write $2 \times n + 3 = 11$ or simply, $2n + 3 = 11$.



$2n + 3 = 11$ is an equation. This means that the left side must equal the right side. There is only one value of n that makes this equation true. We can see that if the input number was 4, then the output number would be $2 \times 4 + 3$, which equals 11.

Finding a value of the pronumeral that makes an equation true is called **solving** the equation. A value of the pronumeral that makes an equation true is called a **solution** for the equation.

The solution to the equation $2n + 3 = 11$ is $n = 4$.

In example 4 the number machine processes are turned into algebraic expressions and then an equation is written. In later sections of this chapter we will look at ways of solving equations.

When translating a written statement into an algebraic expression there are some key words that indicate the operations involved in the expression. Some examples are provided in the following table. Notice that it does not matter which pronumeral we choose for the unknown number.

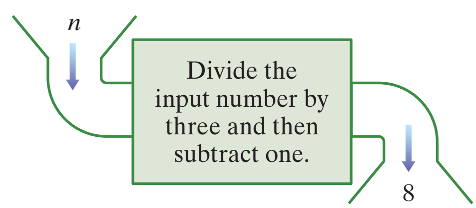
Addition		Subtraction	
Statement	**Expression**	**Statement**	**Expression**
A number plus two	$a + 2$	A number minus four	$d - 4$
The sum of a number and five	$b + 5$	The difference between ten and a number	$10 - e$
Four more than a number	$c + 4$	Six less than a number	$f - 6$

Multiplication		Division	
Statement	**Expression**	**Statement**	**Expression**
Four times a number	$4p$	A number divided by six	$\frac{m}{6}$
The product of three and a number	$3q$	The quotient of a number and eight	$\frac{n}{8}$
A number multiplied by five	$5r$	One fifth of a number	$\frac{t}{5}$

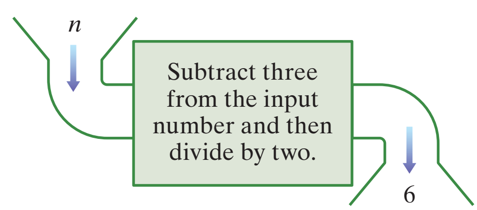
Example 4

Write an equation which represents the following number machines.

a



b



Working

a $\frac{n}{3} - 1 = 8$

b $\frac{n - 3}{2} = 6$

Reasoning

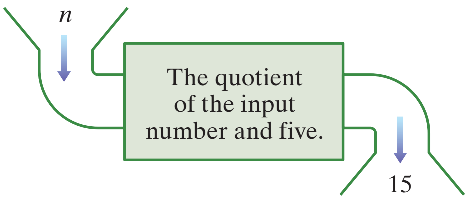
Divide the input number by 3 and then subtract 1. Put this expression equal to the output number.

Subtract 3 from the input number and then divide by 2. Put this expression equal to the output number.

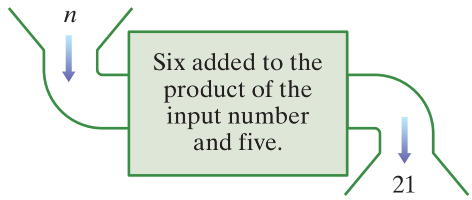
Example 5

Write an equation which represents the following number machines.

a



b



Working

a $\dfrac{n}{5} = 15$

Reasoning

Divide the input number by 5. Put this expression equal to the output number.

Working

b $5n + 6 = 21$

Reasoning

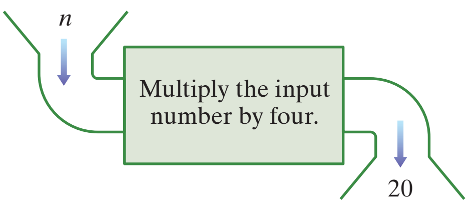
Multiply the input number by 5 and add 6. Put this expression equal to the output number.

exercise 8.2

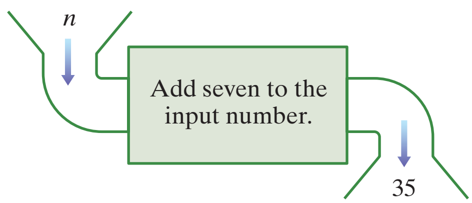
LINKS TO Example 4

● Write an equation which represents the following number machines.

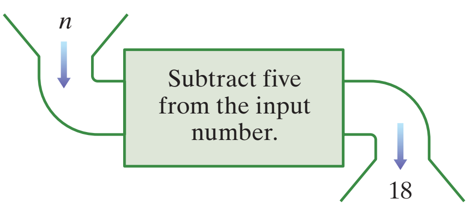
a



b



c



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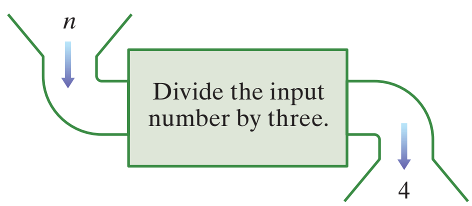

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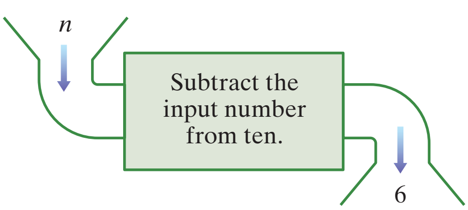

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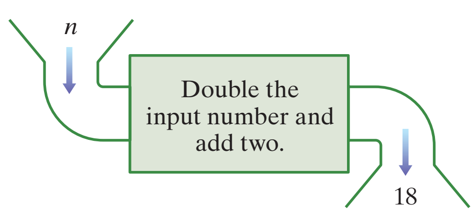

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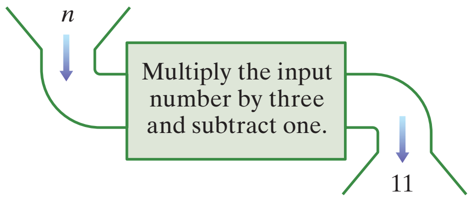

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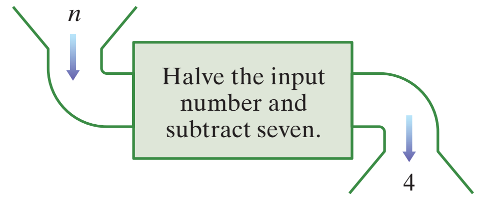

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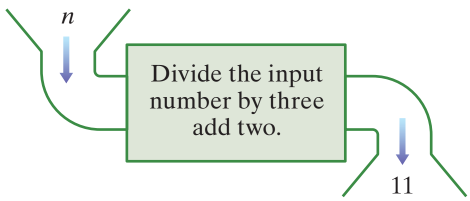

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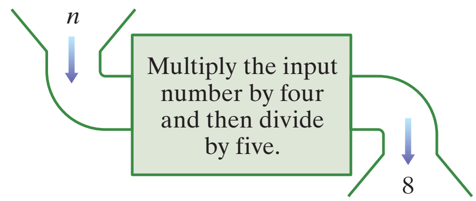

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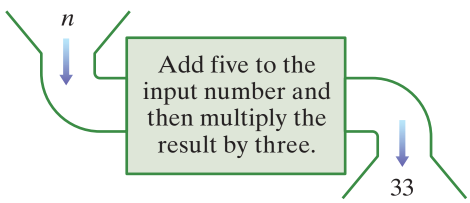

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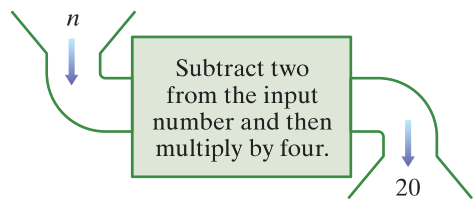
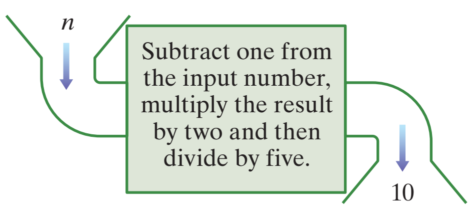
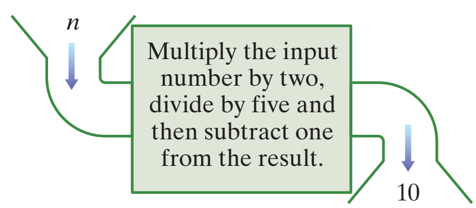


● Which one of the following number machines is represented by the equation $\frac{2x - 1}{5} = 10$?

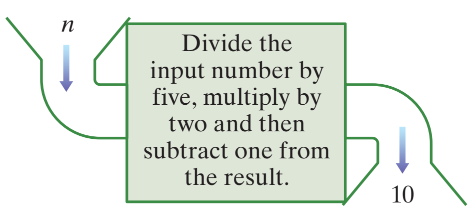
A



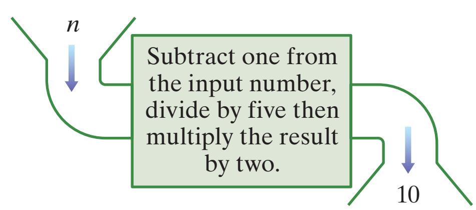
B



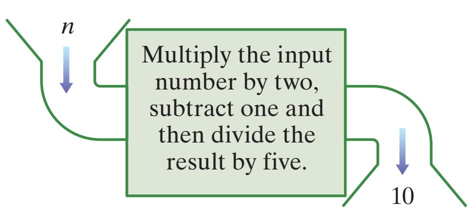
C



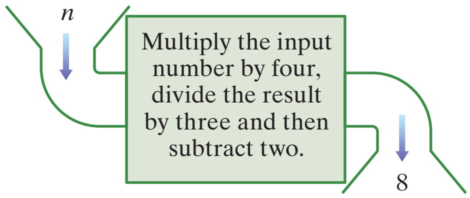
D



E



Which one of the following equations represents the number machine?



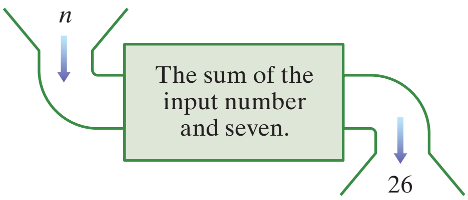
A $\dfrac{4(n-2)}{3} = 8$ **B** $\dfrac{4n-2}{3} = 8$ **C** $\dfrac{4n}{3} - 2 = 8$

D $4n - \dfrac{2}{3} = 8$ **E** $4\left(n - \dfrac{2}{3}\right) = 8$

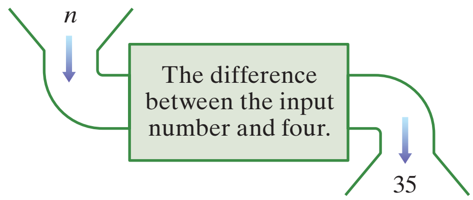
LINKS TO Example 5

Write an equation that represents each of the following number machines.

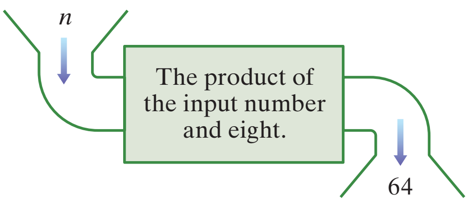
a



b



c



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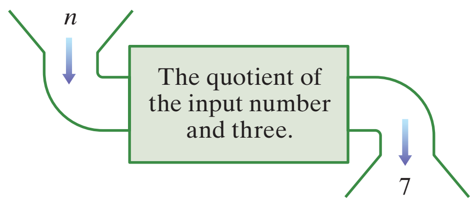
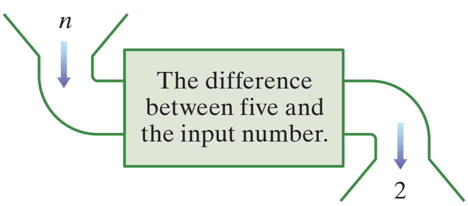
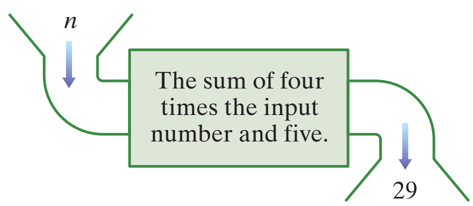


e



f



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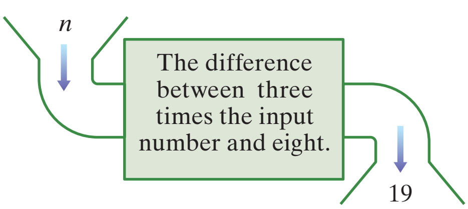

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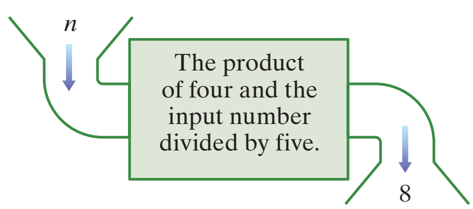

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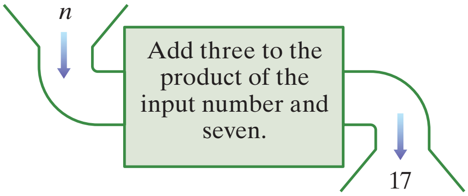

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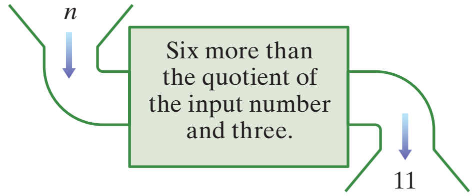

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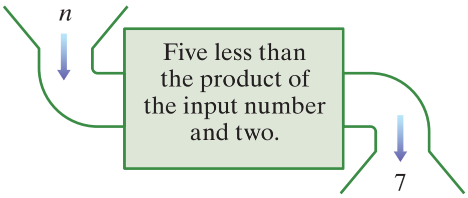

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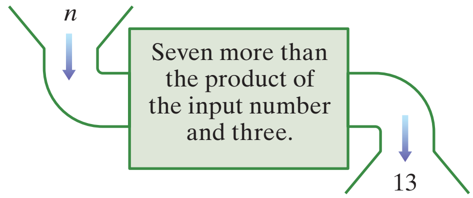
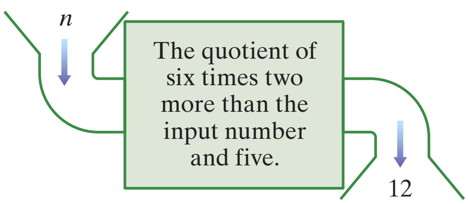


exercise 8.2 challenge

Which one of the following equations represents the number machine below?



A $\frac{6n + 2}{5} = 12$ **B** $\frac{6n}{5} + 2 = 12$ **C** $6n + \frac{2}{5} = 12$

D $\frac{6(n + 2)}{5} = 12$ **E** $\frac{5}{6(n + 2)} = 12$

Adrienne uses a two-step number machine. The outputs for three input values are shown in the table below.

Input	4	8	10
Output	10	22	28

a If the input number is 5, what is the output number?
b If the output number is 31, what is the input number?
c If the output number is x, what is the output number?

8.3 Solving equations: arithmetic strategies

Finding the value(s) of the pronumeral that makes an equation true is called solving the equation. A value of the pronumeral that makes an equation true is called a solution for the equation. This section considers some ways of solving equations using arithmetic rather than algebra.

Substitution

In section 8.2 we saw that the equation $2n + 3 = 11$ was true if $n = 4$.

If we substitute other values for n we find that the right side (RS) is not equal to the left side (LS).

n	$2n$ 3	Value of LS	Value of RS	LS RS?
0	2 0 + 3	3	11	False
1	2 1 + 3	5	11	False
2	2 2 + 3	7	11	False
3	2 3 + 3	9	11	False
4	2 4 + 3	11	11	True
5	2 5 + 3	13	11	False
6	2 6 + 3	15	11	False

By substituting a value for the pronumeral in an equation, we can find if that value makes the equation true.

Example 6

Use substitution to determine if the number given in brackets is a solution to the equation.

a $3x - 7 = 11$ (6)

b $\frac{2p}{3} - 1 = 4$ (9)

Working	Reasoning
a $3x - 7$	Work with the LS of the equation.
$= 3 \times 6 - 7$	Substitute the value for x.
$= 18 - 7$	Simplify.
$= 11$	
$x = 6$ is a solution to the equation.	LS = RS

continued

Example 6 continued

Working	Reasoning
b $\dfrac{2p}{3} - 1$	Work with the LS of the equation.
$= \dfrac{2 \times 9}{3} - 1$	Substitute the value for p.
$= \dfrac{18}{3} - 1$ $= 6 - 1$ $= 5$	Simplify. LS $= 5$, RS $= 4$
$p = 9$ is not a solution to the equation.	LS $\neq$ RS

Observation

The solution to an equation may be obvious just from looking at the equation. For example, we can see that the solution to the equation $x + 3 = 12$ is $x = 9$.

Example 7

Solve the following equations by observation.

a $3x = 15$ **b** $\dfrac{s}{4} = 6$ **c** $m - 7 = 9$

Working	Reasoning
a $3x = 15$ $3 \times 5 = 15$ $x = 5$	Write down the equation. Think of a number that can be multiplied by 3 to give 15. Try 5. Write down the solution to the equation.
b $\dfrac{s}{4} = 6$ $\dfrac{24}{4} = 6$ $s = 24$	Write down the equation. Think of a number that can be divided by 4 to give 6. Try 24. Write down the solution to the equation.
c $m - 7 = 9$ $16 - 7 = 9$ $m = 16$	Write down the equation. Think of a number from which you can subtract 7 and get 9. Try 16. Write down the solution to the equation.

Table of values

We can find the value of the pronumeral that makes an equation true by constructing a table of values for the left side.

Example 8

Use the table of values provided to find the value of x that makes the equation $2x + 7 = 19$ true.

x	$2x$ 7
1	9
2	11
3	13
4	15
5	17
6	19
7	21

Working

$x = 6$

Reasoning

When $x = 6$, $2x + 7$ has the value 19. The right side of the equation is 19.

x	$2x$ 7
6	19

Guess, check and improve

Another useful technique is *guess, check and improve*—try a value for the variable, and then check whether this value makes the equation a true statement. You can repeat this with other values until you get a value which does make the LS = RS.

Example 9

Find the value of x that will make the equation $2x - 3 = 9$ a true statement.

Working

$2x - 3$
$= 2 \times 10 - 3$
$= 20 - 3$
$= 17$

$17 \neq 9$ so the equation is not true for $x = 10$

Reasoning

Work with the LS of the equation.
Substitute a value for x. Make a guess that $x = 10$ might work.
Simplify.

LS ≠ RS

$x = 10$ was too big so try a smaller value.

Working

$2x - 3$
$= 2 \times 6 - 3$
$= 12 - 3$
$= 9$

$9 = 9$ so the equation is true for $x = 6$

Reasoning

Work with the LS of the equation
Substitute a value for x. This time, make a guess that $x = 6$ might work.
Simplify.

LS = RS

exercise 8.3

LINKS TO Example 6

Use substitution to determine if the number given in brackets is a solution to the equation.

a $5x = 15; (3)$ **b** $\frac{y}{4} = 5; (28)$ **c** $p - 1 = 7; (6)$

d $2x - 5 = 15; (10)$ **e** $5x - 4 = 7; (2.2)$ **f** $\frac{m}{9} + 9 = 12; (15)$

g $3(x - 4) = 6; (6)$ **h** $\frac{2y}{3} - 3 = 2; (6)$ **i** $4(2z + 3) = 20; (2)$

j $\frac{2(m - 5)}{3} = 6; (13)$ **k** $3x - 0.2 = 12.1; (4.1)$ **l** $\frac{2(4m + 3)}{5} = 5; \left(\frac{1}{2}\right)$

If $p = 3$, which of the following are true statements?

a $3p = 9$ **b** $p + 4 = 12$ **c** $p - 1 = 2$

d $\frac{4p - 2}{5} = 3$ **e** $3p + 1 = 10$ **f** $\frac{p}{3} - 1 = 1$

LINKS TO Example 7

In each of the following, use the table of values provided to find the value of the variable that makes the equation true.

a $3x + 1 = 13$

x	$3x$ 1
1	4
2	7
3	10
4	13
5	16

b $2y - 4 = 12$

y	$2y$ 4
4	4
6	8
8	12
10	16
12	20

c $2x + 1 = 9$

x	$2x$ 1
1	1
2	3
3	5
4	7
5	9

LINKS TO Example 7

Complete each of the following tables to find the value of the variable that makes the equation true.

a $3x - 1 = 14$

x	$3x$ 1
1	
2	
3	
4	
5	

b $5x + 7 = 27$

x	$5x$ 7
0	
1	
2	
3	
4	

c $2x - 3 = 9$

x	$2x$ 3
4	
5	
6	
7	
8	

d $4p + 1 = 13$

p	$4p \quad 1$
0	
1	
2	
3	
4	

e $5x - 1 = 44$

x	$5x \quad 1$
6	
7	
8	
9	
10	

f $\frac{m}{2} - 5 = 1$

m	$\frac{m}{2} \quad 5$
10	
12	
14	
16	
18	

LINKS TO Example 8

Solve the following equations by observation.

a $a + 2 = 6$ **b** $b + 4 = 7$ **c** $c - 1 = 12$ **d** $d - 4 = 15$

e $e + 8 = 23$ **f** $f - 6 = 21$ **g** $2g = 12$ **h** $5h = 40$

i $7i = 77$ **j** $\frac{j}{3} = 12$ **k** $\frac{k}{5} = 4$ **l** $\frac{l}{7} = 6$

LINKS TO Example 9

Use 'guess, check and improve' to find solutions to these equations.

a $2x + 19 = 51$ **b** $4x - 11 = 25$ **c** $3x + 7 = 34$ **d** $5x - 17 = 18$

exercise 8.3 challenge

LINKS TO Example 9

Use 'guess, check and improve' to find solutions to these equations.

a $3x + 5 = 71$ **b** $11x - 7 = 202$ **c** $17x + 23 = 159$ **d** $13x - 6 = 137$

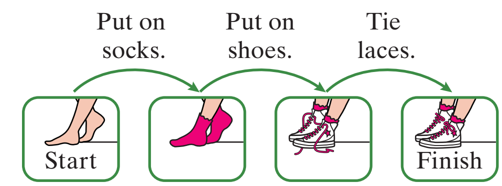
8.4 Forward tracking and backtracking

A flow chart provides a way of visualising a sequence of steps.

For instance, before going out at the start of the day, Sandy

- puts on her socks
- then puts on her shoes
- and finishes by tying her shoelaces.

The flow chart shows the order of the steps. This is called **forward tracking**.

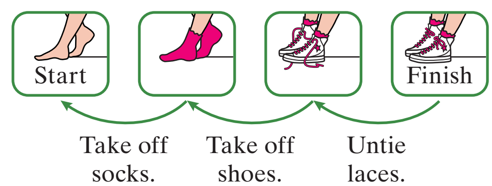


Sandy puts her shoes on—'forward' tracking!

At the end of the day, Sandy reverses these steps and

- unties her shoelaces
- takes off her shoes
- then takes off her socks.

This reverse process is called **backtracking**.



Sandy takes her shoes off—backtracking!

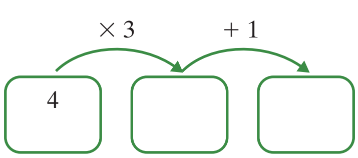
Forward tracking with numbers

In a number flow chart, there is a sequence of operations on the input number. Each operation leads us to working out the next number in the flow chart.

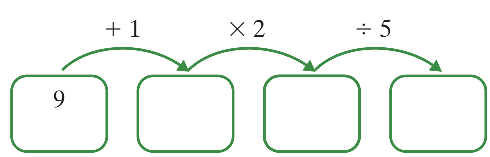
Example 10

Complete the following flow charts and find the output number.

a

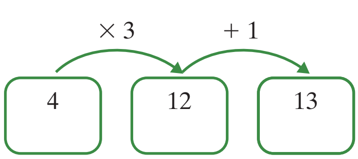


b



Working

a



The output number is 13.

Reasoning

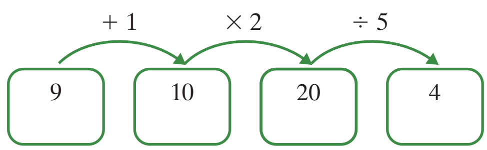
Perform each operation as you move from left to right through the flow chart.

continued

Example 10 continued

Working

b



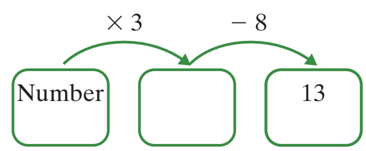
The output number is 4.

Reasoning

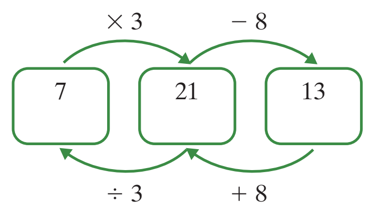
Perform each operation as you move from left to right through the flow chart.

Backtracking with numbers

Consider the following puzzle. Trish thinks of a number, multiplies it by three then subtracts eight and the result is thirteen. The puzzle can be written as a flow chart, shown at right.



To find Trish's number, we work our way backwards through the flow chart doing the opposite. This is **backtracking**.



So Trish's number is 7.

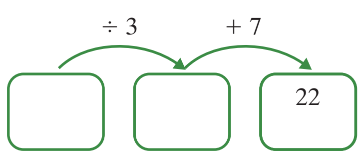
When we backtrack with numbers, we must undo what has been done to the input number. We do this by working backwards in the flow chart, carrying out the opposite or **inverse** number operation; for example, if 8 has been added, then we undo this by subtracting 8.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

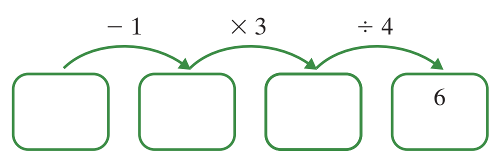
Example 11

Use backtracking to find the input number for each of the following flow charts.

a



b

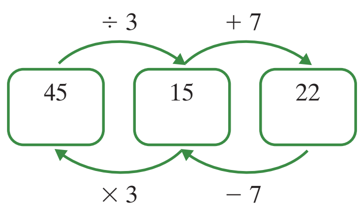


continued

Example 11 continued

Working

a

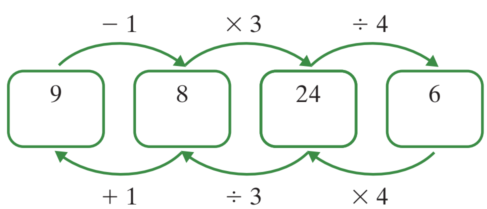


The input number is 45.

Reasoning

Working from right to left, the inverse of +7 is −7 so 22 − 7 is 15.
The inverse of ÷ 3 is × 3 so 15 × 3 is 45.

b

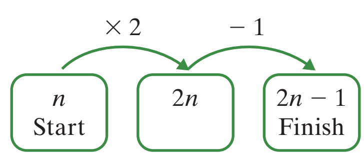


The input number is 9.

Working from right to left, the inverse of ÷ 4 is × 4 so 6 × 4 is 24.
The inverse of × 3 is ÷ 3 so 24 ÷ 3 is 8.
The inverse of −1 is +1 so 8 + 1 is 9.

Using forward tracking to build algebraic expressions

Using flow charts we can build up algebraic expressions. Instead of starting with a known number, we start with a pronumeral. At each step we write the new expression.

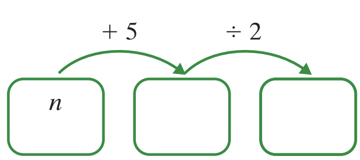


Supposing we start with a number n then multiply it by 2. This gives the expression $2n$. Next we subtract 1, so we now have the expression $2n - 1$.

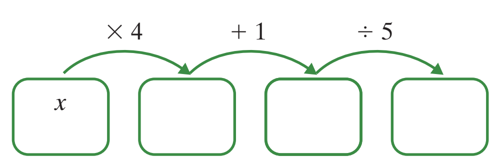
Example 12

Write an algebraic expression that represents each of the following flow charts.

a

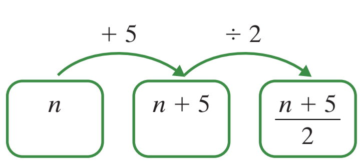


b



Working

a



Reasoning

First add 5 to n. This gives $n + 5$.

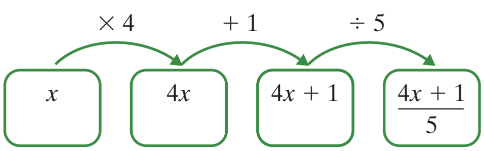
Then divide by 2. This gives $\frac{n+5}{2}$.

continued

Example 12 continued

Working

b



Reasoning

First multiply x by 4. This gives $4x$.

Then add 1. This gives $4x + 1$.

Finally divide by 5. This gives $\frac{4x + 1}{5}$.

We can work out how an algebraic expression has been built up by thinking about the usual order of operations with numbers. In the expression $3n + 1$ for example, multiplication would be done before addition. So n is multiplied by 3 and then 1 is added.

Example 13

In each of these expressions, what is the first operation that has been carried out on n?

a $2n + 5$ **b** $\frac{n}{3} + 4$ **c** $2(n + 4)$ **d** $\frac{n - 3}{2}$

Working

a n is multiplied by 2

b n is divided by 3

c 4 is added to n

d 3 is subtracted from n

Reasoning

n is multiplied by 2 then 5 is added to the result.

n is divided by 3 then 4 is added to the result.

Brackets are worked out before multiplication.
4 is added to n then the result is multiplied by 2.

We can think of $\frac{n - 3}{2}$ as $\frac{(n - 3)}{2}$.

3 is subtracted from n then the result is divided by 2.

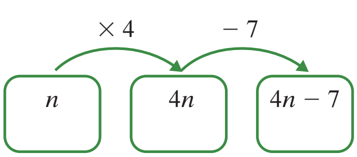
Example 14

Construct a flow chart for each of the following algebraic expressions.

a $4n - 7$ **b** $\frac{2p}{3} + 5$

Working

a



Reasoning

Let the input number be n.
Multiply n by 4. This gives $4n$.
Then subtract 7. This gives $4n - 7$.

continued

Example 14 continued

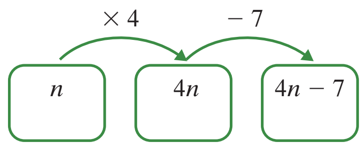
Construct a flow chart for each of the following algebraic expressions.

a $4n - 7$

b $\frac{2p}{3} + 5$

Working

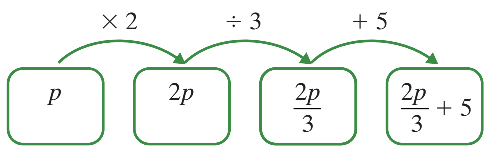
a



Reasoning

Let the input number be n.
Multiply n by 4. This gives $4n$.
Then subtract 7. This gives $4n - 7$.

b



Let the input number be p.
Multiply p by 2. This gives $2p$.
Then divide by 3. This gives $\frac{2p}{3}$.
Finally add 5. This gives $\frac{2p}{3} + 5$.

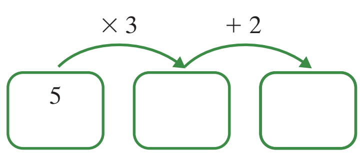
exercise 8.4

LINKS TO Example 10

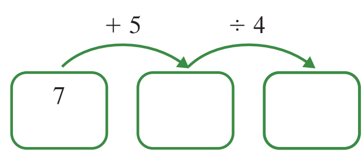
BLM Flow charts

1 Copy and complete the following flow charts and find the output number.

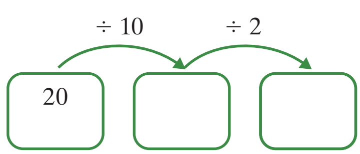
a



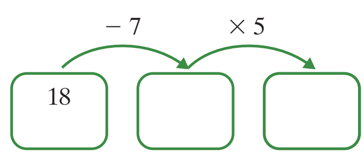
b



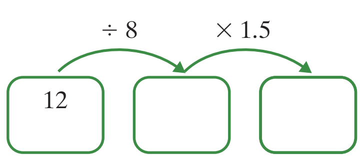
c



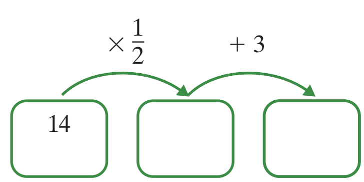
d



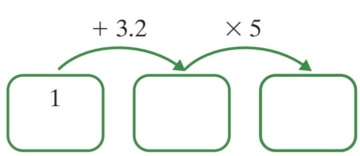
e



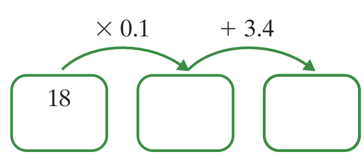
f



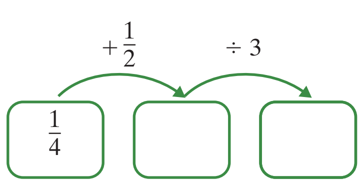
g



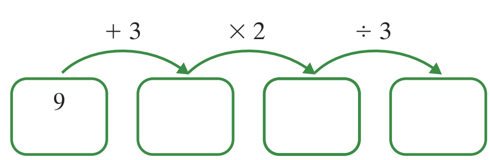
h



i



j



k 3 → + 5 → ☐ → × 2 → ☐ → + 6 → ☐

l 18 → ÷ 3 → ☐ → + 1 → ☐ → × 4 → ☐

m 30 → ÷ 5 → ☐ → ÷ 3 → ☐ → × 2 → ☐

n 8 → × 9 → ☐ → ÷ 12 → ☐ → + 4 → ☐

● For parts a and b, copy and complete the flow chart and use it to find the output number.

a Kristopher starts with the number 12, divides by 3 then adds 9.

☐ → ☐ → ☐

b Toby starts with the number 12, adds 9 and then divides by 3.

☐ → ☐ → ☐

c Do Kristopher and Toby obtain the same output number? Why or why not?

LINKS TO Example 11

● Use backtracking to find the input number for each of the following flow charts.

a ☐ → × 2 → ☐ → − 1 → 15

b ☐ → × 2 → ☐ → + 7 → 25

c ☐ → ÷ 3 → ☐ → × 2 → 16

d ☐ → − 5 → ☐ → × 4 → 20

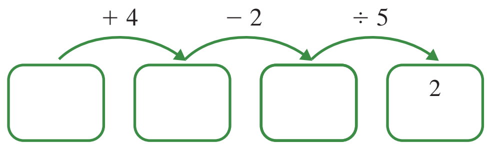
e ☐ → ÷ 4 → ☐ → − 3 → 1

f ☐ → × 7 → ☐ → − 5 → 51

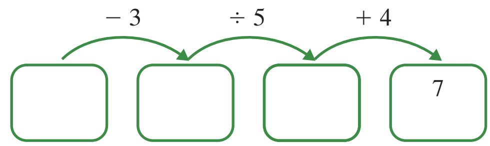
g ☐ → × 2 → ☐ → + 3 → 9

h ☐ → + 4 → ☐ → × 3 → 60

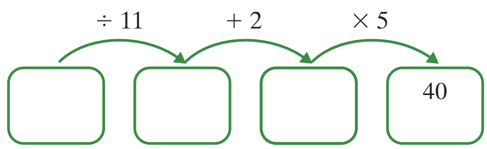
i



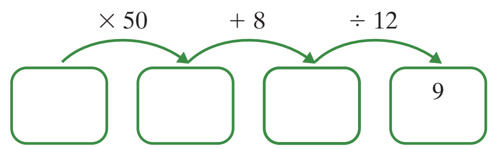
j



k



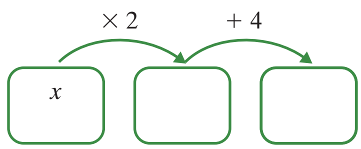
l



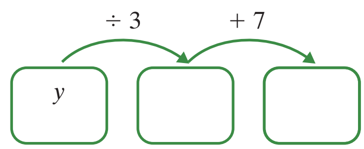
LINKS TO Example 12

Copy and complete these flow charts. Write the algebraic expression which represents each flow chart.

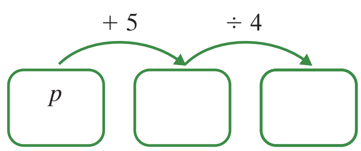
a



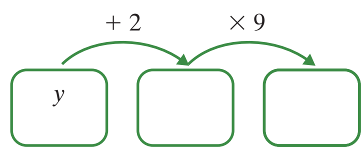
b



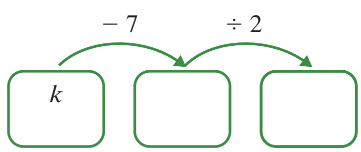
c



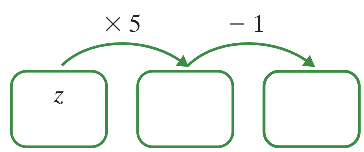
d



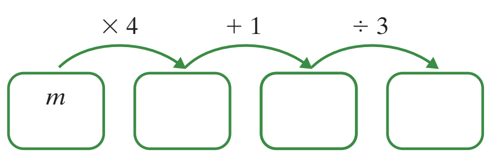
e



f



g



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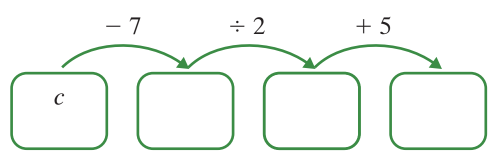
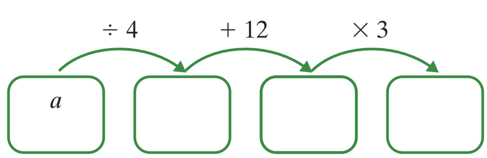
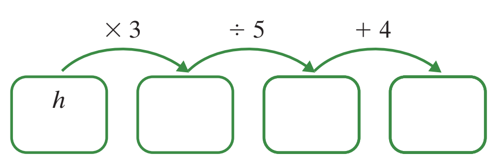


i



j



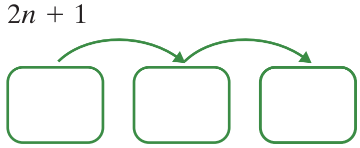
LINKS TO Examples 13, 14

For each of the following

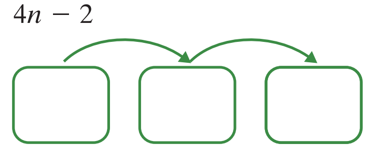
i state the first operation that is carried out on n.

ii copy and complete the flow chart.

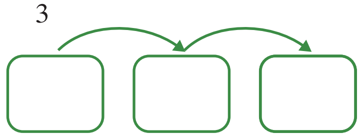
a $2n + 1$



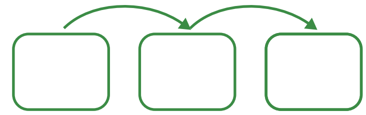
b $4n - 2$



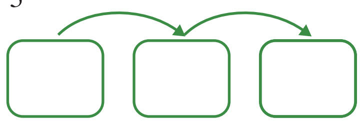
c $\dfrac{n-2}{3}$



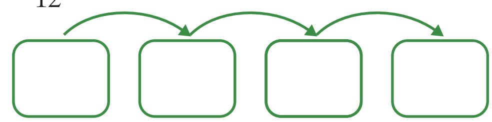
d $2(n-4)$



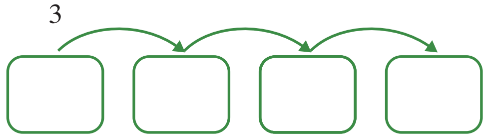
e $\dfrac{n}{3}+4$



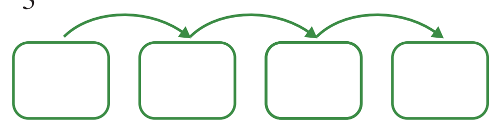
f $\dfrac{n-7}{12}+3$



g $\dfrac{7n-19}{3}$



h $\dfrac{2n}{3}+12$



- Thuy thinks of a number, then adds 11, doubles the result and finally subtracts 4. If the starting number was n.
  - **a** Draw a flow chart.
  - **b** Write an expression which represents the final number for this puzzle.
  - **c** What would the finishing number be if the starting number was 7?
  - **d** What would the starting number be if the finishing number was 22?
- Hannibal thinks of a number, then adds 1, halves the result and finally adds 4. If the starting number was n.
  - **a** Draw a flow chart.
  - **b** Write an expression which represents the final number for this puzzle.
  - **c** What would the finishing number be if the starting number is 5?
  - **d** What would the starting number be if the finishing number is 7?

exercise 8.4 challenge

- Thom thinks of a number, then multiplies it by 4 and then adds 6. He halves the result, then subtracts the number he first thought of. Finally, Thom subtracts 3.
  - **a** Draw a flow chart to represent this number puzzle.
  - **b** Write an expression for the final number in terms of a starting number n.
  - **c** What is the final number if the starting number is
    - **i** 1? **ii** 2? **iii** 3?
  - **d** What do you notice about your answers from part c? Explain why this has occurred.

8.5 Solving equations by backtracking

We saw in section 8.4 how flow charts are useful for

- forward tracking to find the finishing number if we know the starting number.
- reversing the steps to find a starting number if we know the finishing number.
- building up algebraic expressions.

In this section we will use backtracking to solve equations. This means finding the starting number that makes the equation true.

Example 15

For each of these number operations

i write an equation.

ii draw a flow chart and use backtracking to find the number.

a 4 is added to a number, n, and the result is 13

b a number, x, is multiplied by 5 and the result is 35

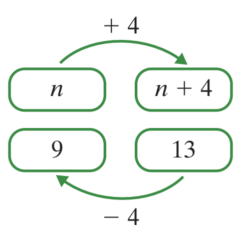
c 7 is subtracted from a number, a, and the result is 11

d a number, b, is divided by 8 and the result is 6

Working

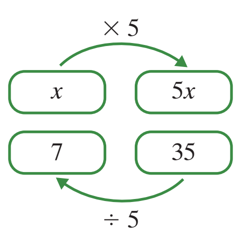
a i $n + 4 = 13$

ii



b i $5x = 35$

ii



$x = 7$

Reasoning

'4 is added to n' means $n + 4$.
Put this expression equal to 13.

Undo $+ 4$ by subtracting 4.
$13 - 4 = 9$

'x is multiplied by 5' means $5 \times x$. We write this as $5x$.
Put this expression equal to 35.

Undo $\times 5$ by dividing by 5.
$35 \div 5 = 7$

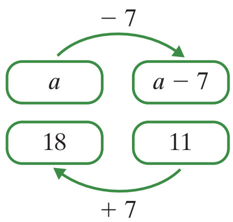
continued

Example 15 continued

Working

c i $a - 7 = 11$

ii



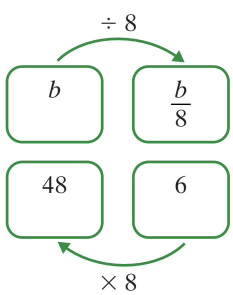
Reasoning

'7 is subtracted from a number a' means $a - 7$. Put this expression equal to 11.

Undo -7 by adding 7.
$11 + 7 = 18$

d i $\frac{b}{8} = 6$

ii



'b is divided by 8' means $b \div 8$. We write this as $\frac{b}{8}$. Put this expression equal to 6.

Undo $\div 8$ by multiplying by 8.
$6 \times 8 = 48$

Example 16

Rebus thinks of a number, multiplies it by 2, and then adds 3, giving a finishing number of 15.

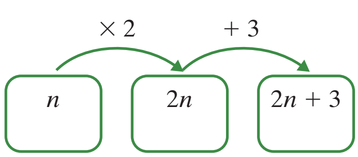
a Using n for Rebus' number, draw a flow chart to represent this 'two-step' process.

b Use the flow chart to write an equation.

c Use backtracking to solve the equation, that is, find the starting number.

Working

a



Reasoning

The first step of the forward tracking process is to multiply by 2, and the second step is to add 3.

b $2n + 3 = 15$

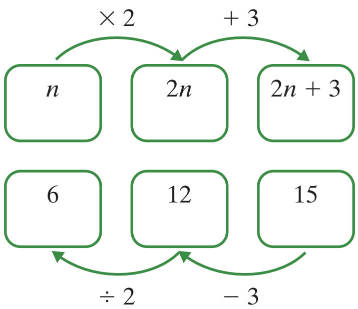
The finishing number is 15.

continued

Example 16 continued

Working

c



The starting number is 6.

$n = 6$

Check:

$LS = 2 \times 6 + 3$

$= 15$

$RS = 15$

Reasoning

The first step of the backtracking process is to subtract 3 from 15, and the second step is to divide 12 by 2. Subtracting 3 is the opposite (inverse) of adding 3. Dividing by 2 is the opposite of multiplying by 3.

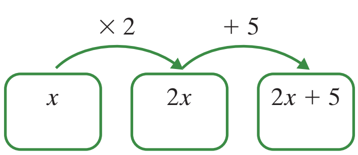
Example 17

Construct a flow chart to represent the left side of each of the following equations and then solve each equation using backtracking.

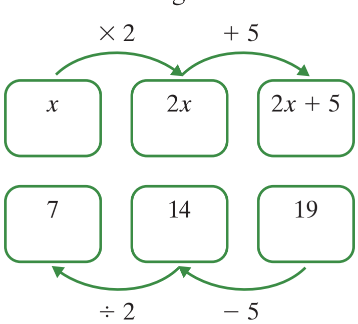
a $2x + 5 = 19$ **b** $\frac{x}{2} - 5 = 7$ **c** $3(x + 4) = 18$

Working

a As a flow chart $2x + 5 = 19$ can be represented as



So backtracking to find the starting number



Reasoning

Start with x, multiply by 2, then add 5.

The result or output is 19. Use backtracking to find x.

The opposite of $+5$ is -5 so subtract 5 from 19, which gives 12.

The opposite of $\times 2$ is $\div 2$ so divide 12 by 2, which gives 6.

Write the solution.

continued

Example 17 continued

Working

$x = 6$

Check:

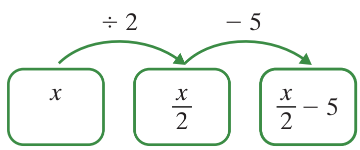
$LS = 2 \times 6 + 5$

$= 19$

$RS = 19$

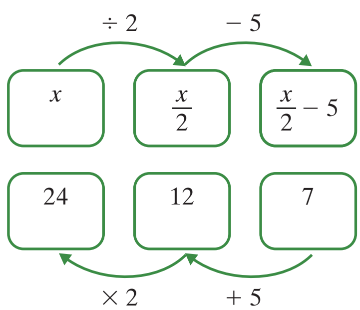
Reasoning

b As a flow chart $\frac{x}{2} - 5$ can be represented as



Construct a flow chart. Start with x, divide by 2 and then subtract 5.

So backtracking to find the starting number



$x = 24$

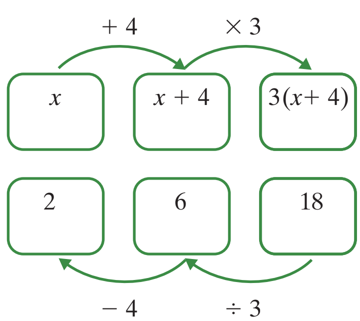
The result or output is 7. Use backtracking to find x.

The opposite to -5 is $+5$ so add 5 to 7 which gives 12.

The opposite to $\div 2$ is $\times 2$ so multiply 12 by 2 which gives 24.

Write the solution.

c



$x = 2$

Start with x, add 4 then multiply the result by 3. The brackets are necessary to show that the whole expression, $x + 4$, is multiplied by 3.

Backtrack to find the starting number. Dividing by 3 is the opposite of multiplying by 3.

Subtracting 4 is the opposite of adding 4.

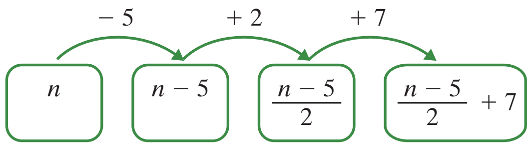
Write the solution.

Example 18

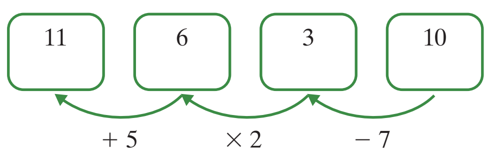
Construct a flow chart for this equation and use it to solve the equation.

$$\frac{n-5}{2} + 7 = 10$$

Working



So backtracking to find the starting number



$n = 11$

Reasoning

This is a 'three-step' process.
The first thing that is done to x is that 5 is subtracted from it. The result is then divided by 2. Finally 7 is added.

Use backtracking to find n.

The opposite of $+7$ is -7 so subtract 7 from 10, which gives 3.
The opposite of $\div 2$ is $\times 2$ so multiply 3 by 2, which gives 6. The opposite of -5 is $+5$, so add 5 to 6, which gives 11.

exercise 8.5

LINKS TO Example 15

For each of the following one-step equations
- **i** draw a flow chart.
- **ii** use backtracking to find the value of n.

a $n + 2 = 18$ **b** $n - 3 = 11$ **c** $5n = 35$

d $\frac{n}{2} = 24$ **e** $n + 4 = 13$ **f** $n - 4 = 12$

g $n + 8 = 17$ **h** $\frac{n}{11} = 5$ **i** $8n = 40$

LINKS TO Example 16

For each of the following two-step equations
- **i** draw a flow chart.
- **ii** use backtracking to find the value of n.

a $2n - 1 = 15$ **b** $\frac{n}{2} + 7 = 16$ **c** $2n + 1 = 15$

d $3n - 7 = 8$ **e** $7(n + 3) = 28$ **f** $\frac{4n}{3} = 32$

g $\frac{3n}{2} = 36$ **h** $\frac{n-2}{5} = 1$ **i** $\frac{n+2}{3} = 7$

j $\frac{n}{3} + 1 = 12$ **k** $\frac{n-5}{2} = 4$ **l** $\frac{n}{2} - 5 = 2$

m $4n + 3 = 23$ **n** $7n - 4 = 24$ **o** $\frac{n}{2} + 1 = 5$

p $\frac{n}{4} - 3 = 7$ **q** $\frac{n-7}{2} = 4$ **r** $\frac{n+5}{3} = 6$

LINKS TO Example 17

For each of the following three-step equations

i draw a flow chart.

ii use backtracking to find the value of n.

a $\frac{2n+4}{4} = 9$ **b** $3(2n - 7) = 27$ **c** $2(n + 3) - 5 = 19$

d $\frac{n-3}{3} - 3 = 1$ **e** $3(n + 9) - 7 = 20$ **f** $\frac{n-2}{5} - 7 = 1$

g $4(2n - 1) = 12$ **h** $4(n + 1) - 5 = 27$ **i** $3\left(\frac{n}{4} - 1\right) = 15$

LINKS TO Example 18

In each of the following, complete a flow chart and use backtracking to find the starting number.

a Cara thinks of a number, divides by 2 and adds 5 to the result. She gets 30 as her answer.

b Viki thinks of a number, adds 5, multiplies the result by 4 and then subtracts 2. She gets 42 as her answer.

c Garth thinks of a number, subtracts 8, divides the result by 3 and then adds 2. His answer is 5.

Use a flow chart and backtracking to solve each of the following equations.

a $3(a + 1) = 12$ **b** $2(b - 5) = 20$ **c** $\frac{2a}{3} + 1 = 5$

d $\frac{3s}{4} - 1 = 11$ **e** $\frac{2a+1}{5} = 7$ **f** $2(3m + 1) = 14$

g $4(5k - 2) = 52$ **h** $3(n - 2) + 2 = 11$ **i** $\frac{3n-5}{5} - 3 = 2$

exercise 8.5 challenge

For each of the following four-step equations

i draw a flow chart.

ii use backtracking to find the value of n.

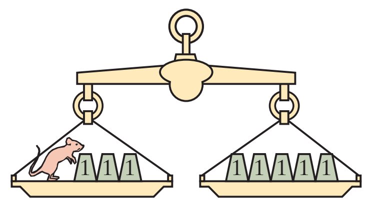
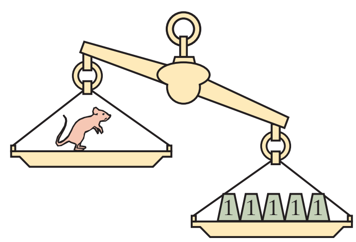
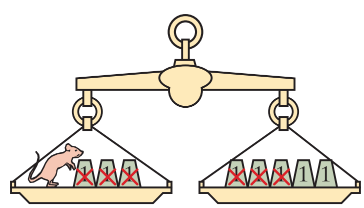
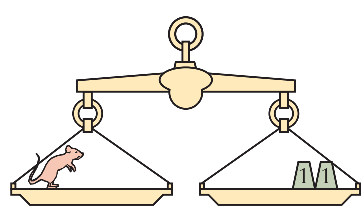
a $\frac{4(n+2)}{3} + 2 = 10$ **b** $4\left(\frac{n-1}{3} + 5\right) = 36$ **c** $5\left(\frac{n}{3} + 1\right) - 4 = 11$

d $\frac{2n+4}{2} + 3 = 7$ **e** $\frac{5(n+1)}{2} + 3 = 18$ **f** $4\left(\frac{n-1}{3} + 5\right) = 32$

8.6 Solving equations: balance scales

One way to think about the solving process is to consider an equation as a balance scale where the two sides of the balance must be kept level.

Consider the equation $x + 3 = 5$ where x kg is the mass of a toy mouse. By writing a series of equivalent equations we can solve the equation and find that $x = 2$.

Equivalent equations	Balance scales diagram
($x + 3 = 5$) The scale is balanced because the two sides are equal.	(Balanced) 
($x + 3 - 3 \neq 5$) By taking 3 kg from the left side, the scale is no longer balanced.	(Unbalanced) 
($x + 3 - 3 = 5 - 3$) By taking 3 kg from the right side as well, the scale is balanced again.	(Balanced) 
($x = 2$) The mass of the toy mouse is 2 kg.	(Balanced) 

We can write the solution process that we used with the scales above as follows.

$x + 3 = 5$

$x + 3 - 3 = 5 - 3$

$x = 2$ (doing the same to both sides).

Example 19

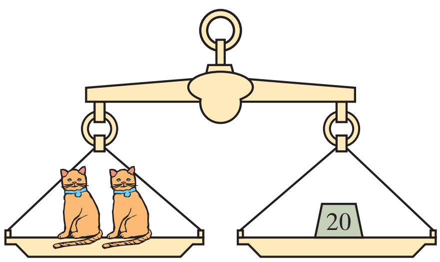
Consider the equation $2x = 20$. Where x kg represents the mass of a cat.

a Construct a balance scales diagram to represent the equation $2x = 20$, and then use diagrams and equivalent statements to solve this equation for x.

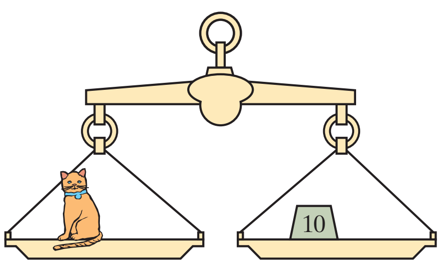
b Check your answer by substituting the answer in to the left side (LS) of the original equation to see whether it equals the right side (RS) of the equation.

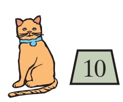
Working

a



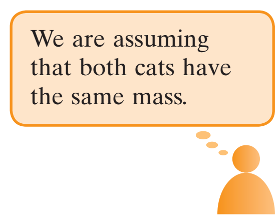
$2x = 20$





$$\frac{2x}{2} = \frac{20}{2}$$

Reasoning

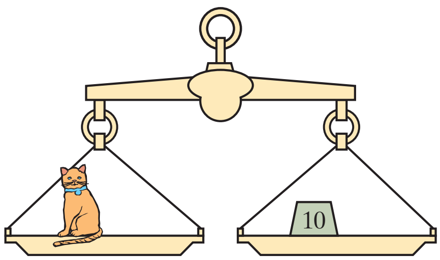


Divide each side by two. This is the same as halving each side.

continued

Example 19 continued

Working



$x = 10$

Reasoning

The cat on the left side balances the 10 kg mass on the right side.

b Check mentally:

$$\begin{aligned} LS &= 2x \\ &= 2 \quad 10 \\ &= 20 \\ &= RS \end{aligned}$$

Substituting $x = 10$ into the LS of the original equation shows that it is equal to the RS. If the solution is correct then you should get the same result on the LS and RS of the equation.

To make equivalent equations easier to read, it is common practice in algebra to line up the equals signs vertically, as in example 19. This makes it easier to identify the left side (LS) and right side (RS) of each equation.

The balance scales model can be used to solve more difficult equations; as long as we **do the same thing to both sides**, the balance scales will remain level. Each time you do the 'same to both sides' you are finding an equivalent equation to the step before.

Example 20

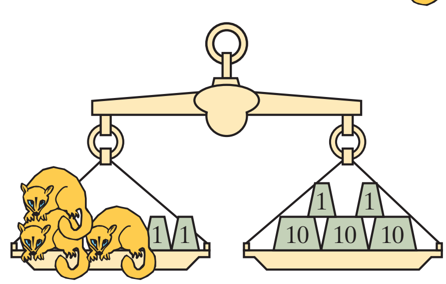
y kg represents the mass of a possum. Solve for y if $3y + 2 = 32$. Check your answer by substituting your solution into the original equation.

Working

$3y + 2 = 32$

Reasoning

In the example y kg represents the mass of a possum.



Working

$$\begin{aligned} 3y + 2 \color{red}{- 2} &= 32 \color{red}{- 2} \\ 3y &= 30 \end{aligned}$$

Reasoning

Subtracting 2 from both sides keeps the equation balanced.

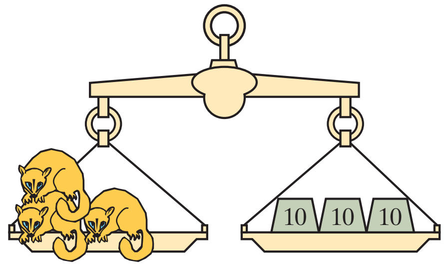
continued

Example 20 continued

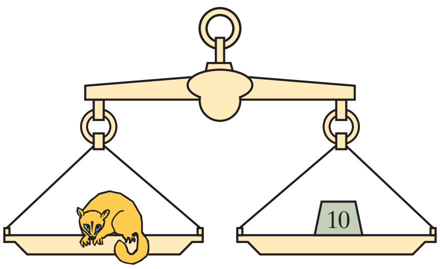
Working

$$\frac{3y}{3} = \frac{30}{3}$$

Reasoning



Divide both sides by 3. This is the same as sharing the 30 kg equally between the three possums.



$y = 10$

One possum has a mass of 10 kg, so $y = 10$.

Substituting $y = 10$ into the original equation

Substitute $y = 10$ into the left side of the equation.

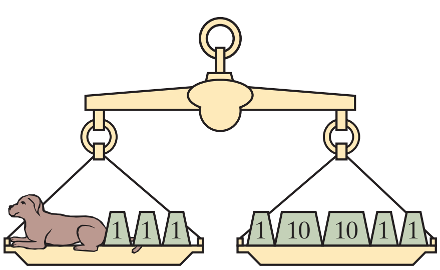
$$\begin{aligned} \text{LS} &= 3y + 2 \\ &= 3(10) + 2 \\ &= 32 \\ &= \text{RS} \end{aligned}$$

If LS = RS, the solution is correct.

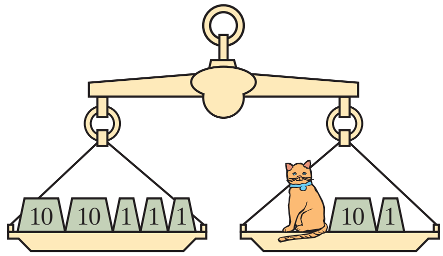
exercise 8.6

LINKS TO Examples 19, 20

● The diagram below shows Ruffy the dog on a set of balance scales. Draw an equivalent scale balance diagram which shows Ruffy's mass.



The diagram below shows Mojo the cat on a set of balance scales. Draw an equivalent balance scales diagram which shows Mojo's mass.

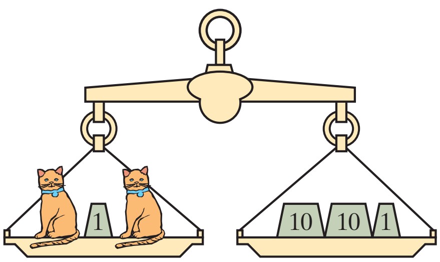


LINKS TO Example 20

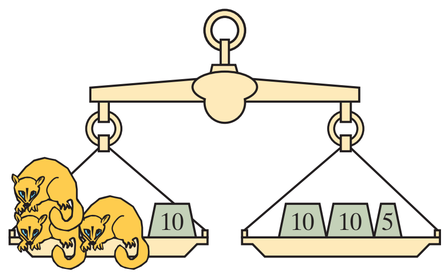
For each of the following balance scales

- **i** write an equation that represents the situation shown, using x kg to represent the mass of each toy animal.
- **ii** use equivalent equations to find the mass of the toy animals.

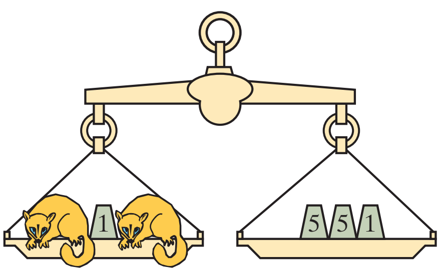
a



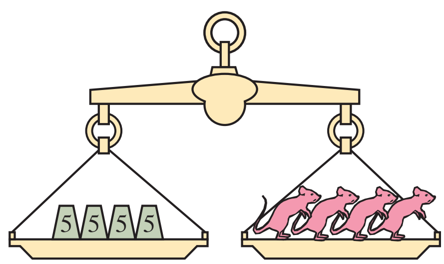
b



c



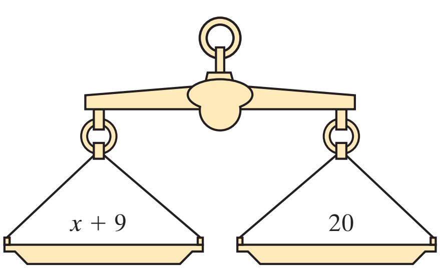
d



For each of the following balance scales

- **i** write the equation represented.
- **ii** describe in words how to get x on its own whilst keeping the scales balanced.
- **iii** find the solution to the equation.

a



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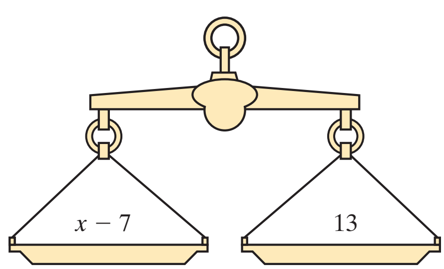

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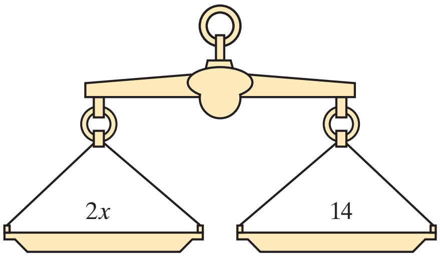

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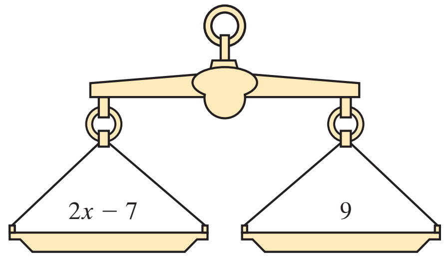

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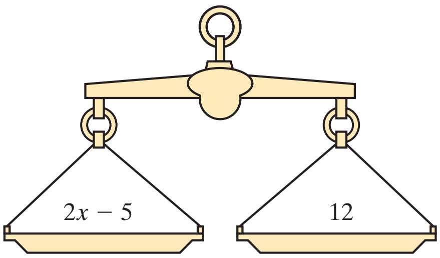

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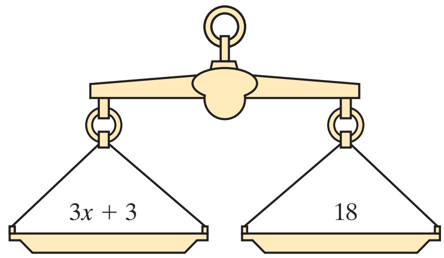

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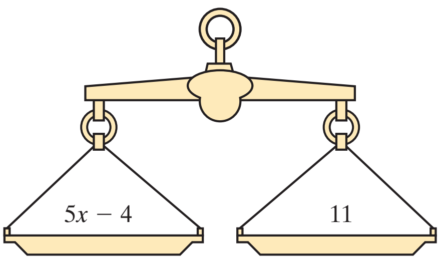

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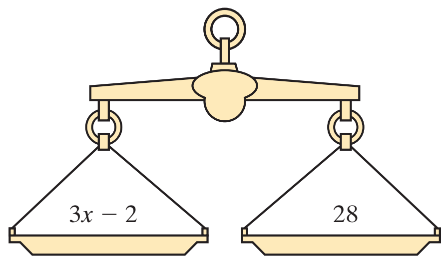
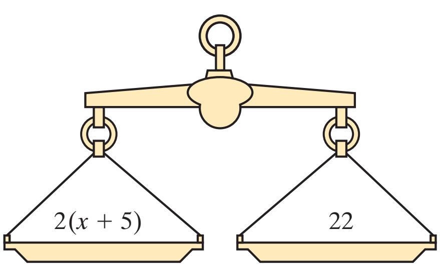


exercise 8.6 challenge

For each of the following balance scales

- **i** write the equation represented.
- **ii** describe in words how to get x on its own whilst keeping the scales balanced.
- **iii** find the solution to the equation.

a



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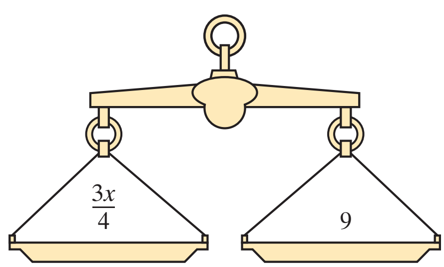


8.7 Solving equations: doing the same to both sides

So far in this chapter we have looked at several ways of solving equations.

- In section 8.3 we used arithmetic strategies: tables of values, inspection and guess, check and improve. When dealing with more complicated equations, it may not be possible to solve by inspection. It may not be efficient to use guess, check and improve.
- In section 8.5 we used backtracking in flow charts. Like arithmetic strategies, backtracking may not be efficient or useful when dealing with more complicated equations.
- In section 8.6 we used the scale balance approach of keeping the two sides balanced by doing the same to both sides. This ensured that we were always making equivalent equations. We obviously don't want to draw a scale balance every time we solve an equation, but the method of doing the same to both sides is an efficient method for solving equations.

In this section we will focus on algebraic solving of equations by doing the same to both sides to make equivalent equations. We will pay particular attention to careful setting out, even with very simple equations that you would be able to solve mentally.

Example 21

Solve each of the following equations. In each case, check your solution by substitution.

a $m - 7 = 12$ **b** $n + 6 = 14$

Working	Reasoning
a $m - 7 = 12$ $m - 7 + 7 = 12 + 7$ $m = 19$	m has had 7 subtracted from it. Add 7 to both sides.
Check: LS $= m - 7$ $= 19 - 7$ $= 12$ $=$ RS	Substituting $m = 19$ into the LS of the equation gives the same value as the RS. So, $m = 19$ is correct.
b $n + 6 = 14$ $n + 6 - 6 = 14 - 6$ $n = 8$	n has had 6 added to it. Subtract 6 from both sides.
Check: LS $= n + 6$ $= 8 + 6$ $= 14$ $=$ RS	Substituting $n = 8$ into the LS of the equation gives the same value as the RS. So, $n = 8$ is correct.

Example 22

Solve each of the following equations. In each case, check your solution by substitution.

a $7a = 42$ **b** $\frac{x}{5} = 3$

Working	Reasoning
a $7a = 42$	a has been multiplied by 7.
$7a \div 7 = 42 \div 7$	Divide both sides by 7.
$a = 6$	
Check:	Substitute $a = 6$ in the LS of the equation.
$\text{LS} = 7a$	
$= 7 \times 6$	
$= 42$	
$= \text{RS}$	
b $\frac{x}{5} = 3$	x has been divided by 5.
$\frac{x}{5} \times 5 = 3 \times 5$	Multiply both sides by 5.
$x = 15$	
Check:	
$\text{LS} = \frac{x}{5}$	Substitute $x = 15$ in the LS of the equation.
$= \frac{15}{5}$	
$= 3$	
$= \text{RS}$	

Example 23

Solve each of the following equations. In each case, check your solution by substitution.

a $10 = 2d + 4$ **b** $3b - 3 = 13$

Working	Reasoning
a $10 = 2d + 4$	We want to get d on one side of the equals sign by itself. In the expression $2d + 4$, d is first multiplied by 2 and then 4 is added. We need to undo the operations in the reverse order. First subtract 4 from both sides. Then divide both sides by 2.
$10 - 4 = 2d + 4 - 4$	
$6 = 2d$	
$\frac{6}{2} = \frac{2d}{2}$	
$3 = d$	
Check:	
$\text{RS} = 2d + 4$	Substitute $d = 3$ in the RS of the equation.
$= 2 \times 3 + 4$	
$= 6 + 4$	
$= 10$	
$= \text{LS}$	

continued

Example 23 continued

Working

b

$$3b - 3 = 13$$
$$3b - 3 + 3 = 13 + 3$$
$$3b = 16$$
$$\frac{3b}{3} = \frac{16}{3}$$
$$b = \frac{16}{3}$$
$$b = 5\frac{1}{3}$$

Check:

$$\begin{aligned} LS &= 3b - 3 \\ &= 3 \times \frac{16}{3} - 3 \\ &= 16 - 3 \\ &= 13 \\ &= RS \end{aligned}$$

Reasoning

We want to get b on one side of the equals sign on its own. In the expression $3b - 3$, b is first multiplied by 3 and then 3 is subtracted. First add 3 to both sides. Then divide both sides by 3.

Substitute $b = 5\frac{1}{3}$ in the LS of the equation.

Example 24

Solve the equation $\frac{t}{5} + 4 = 7$. Check your solution by substitution.

Working

d

$$\frac{t}{5} + 4 = 7$$
$$\frac{t}{5} + 4 - 4 = 7 - 4$$
$$\frac{t}{5} = 3$$
$$\frac{t}{5} \times 5 = 3 \times 5$$
$$t = 15$$

Check:

$$\begin{aligned} LS &= \frac{t}{5} + 4 \\ &= \frac{15}{5} + 4 \\ &= 3 + 4 \\ &= 7 \\ &= RS \end{aligned}$$

Reasoning

We want to get t on one side of the equals sign on its own. In the expression $\frac{t}{5} + 4$, t is first divided by 5 and then 4 is added.
First subtract 4 from both sides. Then multiply both sides by 5.

Substitute $t = 15$ in the LS of the equation.

8.7

exercise 8.7

▶LINKS TO Example 21

1 Solve the following equations by doing the same to both sides.

a $x + 7 = 11$ **b** $x - 7 = 11$ **c** $a + 6 = 9$
d $6 + a = 9$ **e** $b + 13 = 20$ **f** $b - 13 = 20$
g $a - 5 = 16$ **h** $d + 14 = 23$ **i** $d - 8 = 32$
j $m - 15 = 8$ **k** $m - 2.7 = 3.1$ **l** $p - 13.5 = 21.2$
m $x + 6.5 = 14.8$ **n** $x - 1.3 = 1.2$ **o** $7 + t = 35$

▶LINKS TO Example 22a

2 Solve the following equations by doing the same to both sides.

a $7m = 49$ **b** $12b = 60$ **c** $8m = 56$
d $5x = 65$ **e** $9m = 63$ **f** $2.4h = 24$
g $1.5k = 6$ **h** $23a = 115$ **i** $2x = 6.4$
j $5a = 12$ **k** $4b = 14$ **l** $8h = 84$

▶LINKS TO Example 22b

3 Solve the following equations by doing the same to both sides.

a $\frac{x}{7} = 2$ **b** $\frac{b}{9} = 7$ **c** $\frac{k}{5} = 20$
d $\frac{n}{2} = 25$ **e** $\frac{a}{8} = 12$ **f** $\frac{d}{4} = 2.5$
g $\frac{x}{9} = 15$ **h** $\frac{m}{11} = 6$ **i** $\frac{m}{4} = 1.2$
j $\frac{y}{3} = 2.5$ **k** $\frac{b}{1.5} = 3$ **l** $\frac{a}{2.2} = 7$

▶LINKS TO Example 23

4 Matthew wants to solve the equation $4x + 5 = 21$ for x. To do this he needs to

A add 5 to both sides and then multiply both sides by 4.
B subtract 5 from both sides and then multiply both sides by 4.
C add 5 to both sides and then divide both sides by 4.
D subtract 5 from both sides and then divide both sides by 4.
E divide both sides by 4 and then subtract 5 from the result.

▶LINKS TO Example 23

5 Solve each of these equations for a by doing the same to both sides.

a $3a + 4 = 34$ **b** $2a - 4 = 12$ **c** $3a - 7 = 23$
d $3a - 14 = 10$ **e** $6a + 7 = 31$ **f** $11a + 6 = 83$
g $2a + 6 = 9$ **h** $7a + 13 = 55$ **i** $3a + 14 = 59$
j $4a + 13 = 45$ **k** $4a - 13 = 19$ **l** $8a + 5 = 61$
m $3.2a + 1 = 7.4$ **n** $3a - 4.2 = 1.8$ **o** $2a - 5.6 = 1.2$
p $7a + 4.1 = 25.1$ **q** $2a + 1.3 = 5.7$ **r** $10a - 6.4 = 2.6$

6 The solution to the equation $\frac{m}{2} + 6 = 24$ is

A $y = 9$ **B** $y = 18$ **C** $y = 30$ **D** $y = 36$ **E** $y = 60$

LINKS TO Example 24

Solve each of these equations for n by doing the same to both sides.

a $\frac{n}{4} + 5 = 7$ **b** $\frac{n}{4} - 5 = 7$ **c** $\frac{n}{3} - 4 = 8$

d $\frac{n}{3} + 4 = 8$ **e** $\frac{n}{6} + 8 = 15$ **f** $\frac{n}{4} - 3 = 9$

g $\frac{n}{8} - 1 = 7$ **h** $\frac{n}{9} + 7 = 15$ **i** $\frac{n}{6} - 11 = 9$

j $\frac{n}{8} + 3 = 10$ **k** $\frac{n}{6} - 11 = 29$ **l** $\frac{n}{10} - 5.6 = 4.2$

m $\frac{n}{11} + 2 = 11$ **n** $\frac{n}{1.2} + 8 = 10$ **o** $\frac{n}{7} + 3.4 = 3.9$

p $\frac{n}{3} + 1.4 = 2.9$ **q** $\frac{n}{8} + 0.9 = 1.3$ **r** $\frac{n}{7} - 8 = 15$

Solve each of these equations.

a $d - 11 = 43$ **b** $\frac{x}{3} - 9 = 14$ **c** $5y = 18$

d $\frac{x}{12} = 9$ **e** $\frac{b}{7} - 4 = 7$ **f** $5a + 18 = 43$

g $m - 11.2 = 38.4$ **h** $\frac{k}{6} + 1 = 13$ **i** $4x = 8.4$

j $n + 7.3 = 15.9$ **k** $\frac{b}{5} - 2.4 = 1.6$ **l** $1.6a + 2.1 = 5.3$

m $\frac{b}{8} = 2.2$ **n** $a - 16.8 = 13.7$ **o** $3a + 12 = 69$

p $4a - 8.2 = 1.8$ **q** $\frac{h}{7} + 1.2 = 2.1$ **r** $13n = 143$

Hana thinks of a number, triples it and subtracts 4, which leaves her with 5. Write an equation which describes this number puzzle. Then solve this equation to find what the starting number must have been.

exercise 8.7 challenge

Solve for p. Give your answer correct to 2 decimal places.

a $31 = 3p - 16$ **b** $3.56p + 2.42 = 7.38$ **c** $7.645 = \frac{p}{4} + 7$

d $14 = 3.4p - 1$ **e** $\frac{p}{1.3} + 3 = 7$ **f** $10.41 = 2.23(p - 2)$

Solve for the unknown in the following equations.

a $3 + \frac{v}{5} = \frac{17}{5}$ **b** $5(a - 4.5) - 2 = 0.5$ **c** $\frac{q}{4} + \frac{3}{2} = \frac{7}{4}$

d $4a - \frac{3}{10} = \frac{1}{2}$ **e** $\frac{4}{3} = \frac{a}{3} + \frac{1}{6}$ **f** $\frac{k}{4} - \frac{3}{11} = \frac{5}{11}$

8.8 Further equations to solve by doing the same to both sides

Equations with brackets

When an equation includes an expression in brackets, we may be able to simplify the left side by dividing both sides of the equation by a common factor.

Example 25

Solve the equation $2(a + 3) = 12$, checking your solution by substitution.

Working	Reasoning
$2(a + 3) = 12$	$(a + 3)$ has been multiplied by 2.
$\frac{2(a + 3)}{2} = \frac{12}{2}$	There is a common factor of 2 on each side of the equation. Divide both sides by 2
$a + 3 = 6$	
$a + 3 - 3 = 6 - 3$	a has 3 added to it. Subtract 3 from both sides.
$a = 3$	
Check:	
$\text{LS} = 2(a + 3)$	Substitute $a = 3$ in the LS of the equation.
$= 2(3 + 3)$	
$= 2 \times 6$	
$= 12$	
$= \text{RS}$	

If there is no common factor on the two sides, we normally expand the brackets using the distributive law. For example,

$$2(a + 5) = 2 \times a + 2 \times 5$$

$$= 2a + 10$$

Example 26

Solve the equation $2(a + 3) = 11$, checking your solution by substitution.

continued

Example 26 continued

Working

$$2(a + 3) = 11$$
$$2 \times a + 2 \times 3 = 11$$
$$2a + 6 = 11$$
$$2a + 6 - 6 = 11 - 6$$
$$2a = 5$$
$$2a \div 2 = 5 \div 2$$
$$a = 2\frac{1}{2}$$

Check:

$$\begin{aligned} \text{LS} &= 2(a + 3) \\ &= 2\left(2\frac{1}{2} + 3\right) \\ &= 2 \times 5\frac{1}{2} \\ &= 11 \\ &= \text{RS} \end{aligned}$$

Reasoning

$(a + 3)$ has been multiplied by 2. Multiply each term inside the brackets by 2.

$2a$ has 6 added to it. Subtract 6 from both sides.

a is multiplied by 2. Divide both sides by 2.

Substitute $a = 2\frac{1}{2}$ in the LS of the equation.

Example 27

Solve $5(3 + a) + 2 = 20$ for a. Check your answer using substitution.

Working

$$5(3 + a) + 2 = 20$$
$$5(3 + a) + 2 - 2 = 20 - 2$$
$$5(3 + a) = 18$$
$$\frac{5(3 + a)}{5} = \frac{18}{5}$$
$$3 + a = 3.6$$
$$3 + a - 3 = 3.6 - 3$$
$$a = 0.6$$

Check:

$$\begin{aligned} \text{LS} &= 5(3 + a) + 2 \\ &= 5(3 + 0.6) + 2 \\ &= 5 \quad 3.6 + 2 \\ &= 18 + 2 \\ &= 20 \\ &= \text{RS} \end{aligned}$$

Reasoning

Subtract 2 from both sides of the equation.

Divide both sides of the equation by 5.

Subtract 3 from both sides of the equation.

Substitute $a = 0.6$ into the LS of the equation.

An algebraic fraction with an expression such as $x + 7$ in the numerator should be treated as if it were in brackets.

After expanding brackets, it may be possible to simplify the left side before doing the same to both sides.

Example 28

Solve the $\frac{x+7}{3} = 8$. Check your solution by substitution.

Working

b $\frac{x+7}{3} = 8$

$\frac{(x+7)}{3} \times 3 = 8 \times 3$

$x + 7 = 24$

$x + 7 - 7 = 24 - 7$

$x = 17$

Check:

$LS = \frac{x+7}{3}$

$= \frac{17+7}{3}$

$= \frac{24}{3}$

$= 8$

$= RS$

Reasoning

$(x + 7)$ has been divided by 3. Undo by multiplying both sides by 3.

x has 7 added to it. Subtract 7 from both sides.

Substitute $x = 17$ in the LS of the equation.

Equations with negative numbers

During the solving of some equations, the pronumeral may end up on its own on the left side with a negative coefficient. Multiplying both sides of the equation by -1 will change the coefficient of the pronumeral to a positive number.

Example 29

Solve the equation $12 - x = 3$. Check your solution by substitution.

Working

Method 1

$12 - x = 3$

$12 - 12 - x = 3 - 12$

$-x = -9$

$-x \times (-1) = -9 \times (-1)$

$x = 9$

Reasoning

Subtract 12 from both sides.

$3 - 12 = -9$

The coefficient of x is negative so multiply both sides by -1.

$-x \times (-1) = +x$ and $-9 \times (-1) = +9$

continued

Example 29 continued

Working

Method 2

$$12 - x = 3$$
$$12 - x + x = 3 + x$$
$$12 = 3 + x$$
$$12 - 3 = 3 - 3 + x$$
$$9 = x$$

Check:

$$\begin{aligned} LS &= 12 - x \\ &= 12 - 9 \\ &= 3 \\ &= RS \end{aligned}$$

Reasoning

x has been subtracted from 12.
Add x to both sides.

Subtract 3 from both sides.

Substituting $n = 8$ into the LS of the equation gives the same value as the RS.

Example 30

Solve these equations, checking the solution by substitution.

a $a + 11 = 4$

b $7b = -63$

Working

a

$$a + 11 = 4$$
$$a + 11 - 11 = 4 - 11$$
$$a = -7$$

Check:

$$\begin{aligned} LS &= a + 11 \\ &= -7 + 11 \\ &= 4 \\ &= RS \end{aligned}$$

b

$$7b = -63$$
$$\frac{7b}{7} = -\frac{63}{7}$$
$$b = -9$$

Check:

$$\begin{aligned} LS &= 7b \\ &= 7 \times (-9) \\ &= -63 \\ &= RS \end{aligned}$$

Reasoning

a Subtract 11 from both sides.
$4 - 11 = -7$

Substitute $a = -7$ in the LS.

b Divide both sides by 7.
A negative number divided by a positive number is negative.

Substitute $b = -9$ in the LS.

Example 31

Solve the equation $3x + 2.7 = 0.3$, checking the solution by substitution.

Working

$$3x + 2.7 = 0.3$$
$$3x + 2.7 - 2.7 = 0.3 - 2.7$$
$$3x = -2.4$$
$$\frac{3x}{3} = -\frac{2.4}{3}$$
$$x = -0.8$$

Check:

$$\begin{aligned} LS &= 3x + 2.7 \\ &= 3 \times -0.8 + 2.7 \\ &= -2.4 + 2.7 \\ &= 0.3 \\ &= RS \end{aligned}$$

Reasoning

Subtract 2.7 from both sides.
$0.3 - 2.7 = -2.4$

Divide both sides by 3.
A negative number divided by a positive number is negative.

Substitute $x = -0.8$ in the LS.

Equations with the pronumeral on both sides

In some equations the pronumeral occurs on both sides of the equation. We subtract the term containing the pronumeral from one side of the equation.

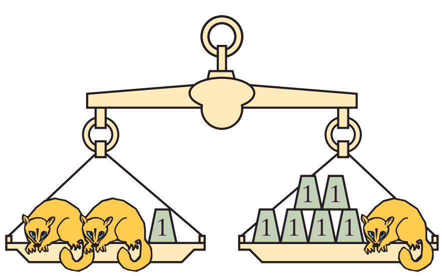
Example 32

If x represents the mass in kilograms of one toy possum in this balance scales diagram,

a write an equation to represent these balance scales.

b solve the equation to find the value of x.

c check the solution.



Working

a $2x + 1 = x + 6$

Reasoning

Mass in kilograms of LS $= 2 \times x + 1$
Mass in kilograms of RS $= x + 6$

continued

Example 32 continued

Working

b

$$2x + 1 = x + 6$$
$$2x + 1 - x = x + 6 - x$$
$$x + 1 = 6$$
$$x + 1 - 1 = 6 - 1$$
$$x = 5$$

Check:

$$\begin{aligned} LS &= 2x + 1 \\ &= 2 \times 5 + 1 \\ &= 11 \\ RS &= x + 6 \\ &= 5 + 6 \\ &= 11 \\ LS &= RS \end{aligned}$$

Reasoning

Take one possum from each side.
There is now one possum on the left side and none on the right side.
Subtract x from both sides so that x is removed from the right side.
Subtract 1 from both sides.

Substitute $x = 5$ into the LS and evaluate.
Substitute $x = 5$ into the RS and evaluate.

exercise 8.8

1. LINKS TO Example 25

Solve the following equations.

a $5(p + 3) = 25$ **b** $24 = 3(c + 1)$ **c** $2(m - 2) = 16$
d $2(x - 5) = 20$ **e** $21 = 3(m + 7)$ **f** $36 = 4(k + 1)$
g $3(z - 2) = 12$ **h** $14 = 7(x - 5)$ **i** $9(4x + 3) = 63$

2. LINKS TO Example 26

Solve the following equations, giving the values of m in fraction form.

a $2(m + 4) = 11$ **b** $4(m + 4) = 30$ **c** $2(m - 3) = 17$
d $5(m - 7) = 2$ **e** $3(m + 1) = 10$ **f** $5(m + 8) = 56$
g $2(m - 13) = 5$ **h** $4(m - 2) = 7$ **i** $3(m + 3) = 11$

3. LINKS TO Example 27

Solve the following equations, giving the value of a in decimal form where appropriate.

a $2(a + 3) + 7 = 25$ **b** $3(a + 7) - 13 = 32$ **c** $2(a + 4) + 11 = 25$
d $4(a + 5) - 7 = 19$ **e** $5(a + 2) - 13 = 9$ **f** $2(a - 3) + 7 = 25$
g $3(a + 4) + 2 = 14$ **h** $2(a + 7) - 9 = 5$ **i** $10(a + 3) - 17 = 41$

4. LINKS TO Example 28

Solve the following equations.

a $\frac{x + 4}{3} = 2$ **b** $\frac{x + 11}{5} = 7$ **c** $\frac{x - 3}{6} = 5$

d $\frac{x - 12}{5} = 4$ **e** $\frac{x + 1}{8} = 2$ **f** $\frac{x + 16}{4} = 4$

g $\frac{x - 13}{2} = 11$ **h** $\frac{x + 4.6}{3} = 7.2$ **i** $\frac{x - 3.8}{5} = 1.8$

8.8

▶LINKS TO Example 29

Solve the following equations.

a $6 - b = 4$ **b** $13 - b = 8$ **c** $21 - b = 16$

d $9 - b = 1$ **e** $17 - 2b = 9$ **f** $25 - 3b = 10$

g $26 - 5b = 11$ **h** $41 - 8b = 9$ **i** $17 - 5b = 6$

▶LINKS TO Example 30a

Solve the following equations.

a $x + 11 = 6$ **b** $x + 14 = 3$ **c** $x + 20 = 8$

d $x - 5 = -7$ **e** $x - 8 = -10$ **f** $x + 11 = -2$

g $x - 5 = -5$ **h** $x + 7 = -13$ **i** $x - 8 = -12$

▶LINKS TO Example 30b

Solve the following equations.

a $3m = -18$ **b** $2m = -22$ **c** $5m = -35$

d $-3m = 12$ **e** $-2m = 8$ **f** $-3m = -21$

g $\frac{m}{2} = -9$ **h** $\frac{m}{6} = -11$ **i** $\frac{m}{4} = -7$

j $-\frac{m}{7} = 5$ **k** $-\frac{m}{4} = -3$ **l** $-\frac{m}{3} = -5$

▶LINKS TO Example 31

Solve the following equations.

a $2n - 19 = -37$ **b** $5n + 6 = 31$ **c** $2n + 21 = 15$

d $5 - 2n = 13$ **e** $8 - 3n = -16$ **f** $-5n + 3 = 13$

g $7n + 17 = -39$ **h** $-3n - 8 = 4$ **i** $3n + 14 = -22$

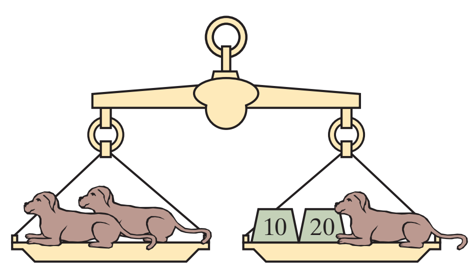
For each of these balance scales use x to represent the mass in kilograms of one toy animal.

i Write an equation to represent these balance scales.

ii Solve the equation to find the value of x.

iii Check the solution.

a



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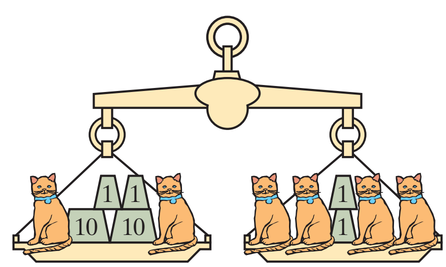

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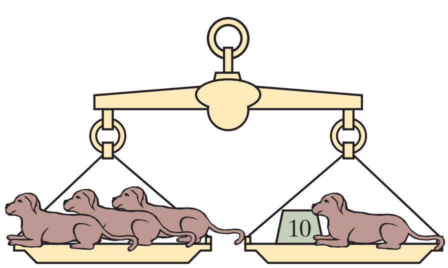

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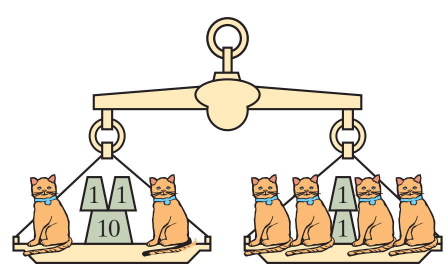

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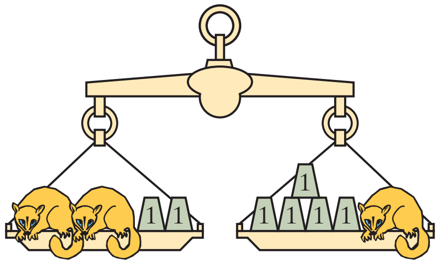

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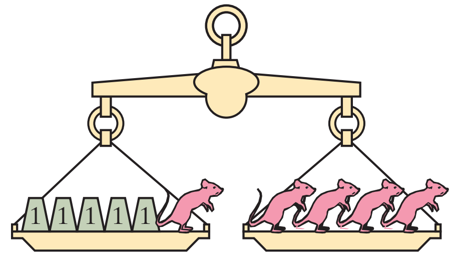


LINKS TO Example 32

Solve these equations for x.

a $4x + 7 = 3x + 8$

b $7x - 11 = 2x + 9$

c $3x - 13 = x + 15$

d $6x - 11 = -3x + 7$

e $8x - 15 = 4x - 7$

f $3.4x + 7.1 = 2.5x + 8$

g $2x + 22 = -3x + 7$

h $13x - 5 = 11x - 8$

Gemma starts with a number and then divides by 3, and subtracts 2 from the result. If she then doubles this, she ends up with the number 8.

a Write an equation which represents this number puzzle.

b Solve this equation to find what the starting number must have been.

Radha starts with a number and then subtracts 3, and divides this answer by 5 then adds 9 and the result is 11.

a Write an equation which represents this number puzzle.

b Solve this equation to find what the starting number must have been.

Grant thinks of a number, adds 3, multiplies the result by 4, then subtracts 2. His final number is 38. What is the original number?

exercise 8.8 challenge

Find the unknown value in each case.

a $\frac{2(p-1)}{5} - 2 = 2$

b $2\left(\frac{m}{4} + 1\right) + 3 = 15$

c $3\left(\frac{2m}{5} - 1\right) = 9$

d $4\left(\frac{k}{6} - 2\right) - 3 = 1$

e $5\left(\frac{a}{3} - 1\right) - 2 = 3$

f $4 + 2(a - 5) = 7$

g $\frac{4(x-1)+2}{3} = 6$

h $3\left(\frac{m}{4} - 2\right) - 1 = 5$

i $\frac{3(x-4)}{5} + 2 = 11$

8.9 Solving problems with equations

Many worded problems, including those with diagrams, can be solved using algebra. The process of translating the 'words' of a problem into algebra is referred to as **formulation**.

As a starting point, it is important to read the question carefully and identify the unknown, and then try to construct an equation that can be solved.

A four-step strategy exists for setting up and solving worded problems with algebra. It can be summarised as follows.

1. **Translate the words into algebra.** Decide on the unknown and give it a pronumeral, then formulate an equation using this pronumeral.
2. **Solve the equation.** Solve by 'doing the same to both sides'.
3. **Check the solution.** Substitute your solution back into the original equation to check that the LS = RS.
4. **Translate the algebra back into words.** Express your solution in terms of the problem wording.

Example 33

Katy has brought home some baby chicks from school. Her sister Rachel brings home another four, so they have a total of 11 chicks.

a Formulate an equation that represents the number of chicks in the home.

b Solve your equation to find out how many chicks Katy brought home.

Working

a Let c be the number of chicks that Katy brought home.

$c + 4 = 11$

Reasoning

Total chicks = number of chicks Katy has brought home + number of chicks Rachel brought home

So total chicks = $c + 4$

If the total number of chicks is 11, this means that $c + 4 = 11$

$c + 4 = 11$ is an equation that expresses the number of chicks.

Working

b $c = 7$, so Katy brought home seven chicks.

Reasoning

By inspection, it can be seen that $c = 7$.

Equations can be used to solve problems relating to **consecutive numbers**. Consecutive numbers are whole numbers that follow one another. For example, 8, 9, 10 are consecutive numbers. If we let n represent a whole number, then the next consecutive number is one more than n, that is, $n + 1$. The next consecutive number is $n + 2$.

Example 34

The sum of two consecutive whole numbers is 11. Find the numbers.

Working

Let n be the first number, and $n + 1$ be the next number.

$$n + (n + 1) = 11$$
$$2n + 1 = 11$$
$$2n + 1 - 1 = 11 - 1$$
$$2n = 10$$
$$\frac{2n}{2} = \frac{10}{2}$$
$$n = 5$$

Reasoning

Step 1: Words into algebra
Deciding on the variable is the first step, and then writing an equation that uses this variable is the next step.

Step 2: Solve the equation
Subtract 1 from both sides.
Divide both sides by 2.

Working

Substitute $n = 5$ into the original equation.

$$\begin{aligned} \text{LS} &= n + (n + 1) \\ &= 5 + (5 + 1) \\ &= 5 + 6 \\ &= 11 = \text{RS} \end{aligned}$$

Reasoning

Step 3: Check the solution
Substituting the solution $n = 5$ into the LS.

Working

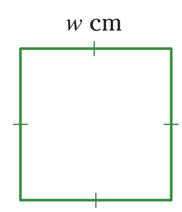
The two consecutive numbers are 5 and 6.

Reasoning

Step 4: Algebra into words
The first number is n, which is 5. The second number is $n + 1$, which is 6. Express your answer in terms of the problem.

Example 35

Formulate an equation which shows the perimeter of this shape in terms of the length of each side, w cm, given that the perimeter is known to be 15 cm. Then solve it for w.



continued

Example 35 continued

Working

If w is the side length, and the shape is a square, then an equation representing this is

$$w + w + w + w = 15$$
$$4w = 15$$
$$\frac{4w}{4} = \frac{15}{4}$$
$$w = 3\frac{3}{4} \text{ or } 3.75$$

Check:

If $w = 3.75$,

$$\begin{aligned} \text{LS} &= 4w \\ &= 4 \times 3.75 \\ &= 15 \\ &= \text{RS} \end{aligned}$$

Reasoning

The unknown variable w (side length) is given.

The sides are of equal length, and the total perimeter is 15 cm. The expression $4w$ has the same value as 15.

Divide both sides by 4.

Example 36

Josef works in a supermarket on the weekend. He earns \$8.50 an hour. If Josef earns \$105.75 one week but that amount includes a \$25 bonus for working an extra shift, use the four-step approach above to find out how many hours he worked in that week.

Working

Let h = the number of hours Josef works.
The amount of money Josef will earn is $8.5h + 25$

$$8.5h + 25 = 105.75$$
$$8.5h + 25 - 25 = 105.75 - 25$$
$$8.5h = 80.75$$
$$\frac{8.5h}{8.5} = \frac{80.75}{8.5}$$
$$h = 9.5$$

If $h = 9.5$, substitute this into the original equation $8.5h + 25 = 105.75$

$$\begin{aligned} \text{LS} &= 8.5h + 25 \\ &= 8.5 \quad 9.5 + 25 \\ &= 80.75 + 25 \\ &= 105.75 \\ &= \text{RS} \end{aligned}$$

Reasoning

Step 1: Words into algebra
Choose a pronumeral for the unknown.
Write an equation.

Step 2: Solve the equation
Subtract 25 from both sides.
Divide both sides by 8.5. h is a number so we write $h = 9.5$, not $h = 9.5$ hours.

Step 3: Check the solution
Substitute the solution $h = 9.5$ into the RS.

continued

Example 36 continued

Working	Reasoning
Josef works for $9\frac{1}{2}$ hours in that week.	**Step 4: Algebra into words** Express your answer in terms of the problem.

exercise 8.9

In this exercise, use the four steps for solving algebra word problems.

LINKS TO Example 33

1. Pearlie has been given a handful of jelly beans, and is then given seven more.
   a Let b represents the number of jelly beans that Pearlie started with. Write an expression for the number of jelly beans that Pearlie now has.
   b If she ends up with 23 jelly beans, write an equation which represents this situation.
   c Solve this equation to find the value of b.
   d How many jelly beans did Pearlie have in her hand to start with?

LINKS TO Example 33

2. There are 71 chocolate buttons in two piles. The second pile has 13 more than the first pile.
   a If there are c chocolate buttons in the first pile, write an expression for the number of chocolate buttons in the second pile.
   b Write an equation to describe this situation.
   c Solve the equation to find the value of c.
   d How many chocolate buttons are in each pile?

LINKS TO Example 33

3. Aniela has a bag of banana lollies that she shares with three friends. Each person receives k lollies and there are three left over.
   a Write an expression for the total number of lollies in the bag.
   b If there are 23 lollies in the bag, write an equation to find how many lollies each person receives.
   c Solve the equation to find the value of k.
   d How many lollies does each person receive?

LINKS TO Example 33

4. Graph paper is sold in packets containing x sheets. Narelle's folder holds six packets of graph paper. She has used five sheets from the folder.
   a Write an expression for the number of sheets of graph paper in Narelle's folder,
   b If there are 175 sheets of graph paper in Narelle's folder, write an equation to find out how many sheets come in a packet.
   c Solve the equation to find the value of x.
   d How many sheets come in a packet?

Two children are trying to work out the age of their two grandparents, Sarah and William. Here is what their grandmother told them.

'William is 10 years older than I am. If you add together our two ages you get 154 years.'

a Write an equation to represent this situation using x to represent Sarah's age in years.

b How old are the grandparents?

A teacher asked her class to write down an equation for the following sentence:

'If you add 6 to an unknown number and then divide the result by 3,
the answer is 5.'

The responses of two students are shown below. Which student do you think is correct? For the student who is incorrect, what is the error?

Denise	Michael
$n + \frac{6}{3} = 5$	$\frac{n + 6}{3} = 5$

LINKS TO Example 34

The sum of two consecutive whole numbers is 29.

a Let n be the smallest of the two numbers. Write an equation to represent this.

b Solve the equation to find the value of n.

c What are the two numbers?

LINKS TO Example 34

The sum of three consecutive numbers is 27.

a Let n be the smallest of the three numbers. Write an equation.

b Solve the equation to find the value of n.

c What are the three numbers?

A square garden bed has side length x metres.

a Write down an expression for the perimeter of the garden bed in terms of x.

b If the perimeter of the garden bed is 76 metres, write an equation to find the value of x.

c Solve the equation to find the value of x.

LINKS TO Example 35

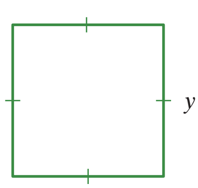
For each of these shapes, the perimeter is known, but there are unknown side lengths. For each shape

i write an equation using the given information. Do not include 'cm' in your equation.

ii solve the equation to find the value of the pronumeral.

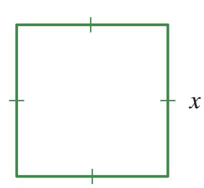
iii list all the side lengths.

a



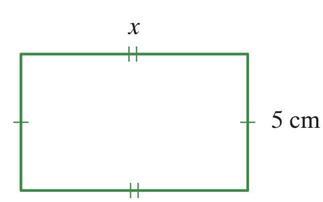
Perimeter = 64 cm

b



Perimeter = 24.8 cm

c



Perimeter = 26 cm

If you multiply a decimal number by 6 and then subtract 4, the result is 4.1.

a Using n to stand for the unknown number, write an equation.

b Solve the equation to find the unknown number.

Ali plants petunias in 12 rows. There are p petunias in each row and 132 petunias in total.

a Write this information as an equation.

b How many petunias are in each row?

LINKS TO Example 36

Alex works in a supermarket on the weekend. He earns $7.50 an hour.

a If Alex works for n hours, write an expression for the amount of money he earns. Do not include the $ sign in your expression.

b If Alex earns $37.50 one week, write an equation to work out how many hours he worked.

c Solve the equation and write down how many hours Alex worked.

LINKS TO Example 36

Madeleine works as a waitress at weekends and earns $16.50 per hour. One weekend she earned $265 which included $34 in tips.

a Let n be the number of hours Madeleine worked. Write an equation using the given information. Do not include the $ sign in your equation.

b Solve your equation.

c How many hours did Madeleine work?

A nurse worked a total of 48 hours in a week. She worked four normal shifts and 12 hours of overtime.

a Let the length of her normal shift be n hours. Write an equation using the given information. Do not include 'hours' in your equation.

b Solve your equation.

c How long is her normal shift?

exercise 8.9 challenge

Lucy is 3 years older than Peter and she is 6 years older than Dominic. If you add together the ages of the three children you get 30 years. Let d stand for Dominic's age.

a Write an expression for Lucy's age in terms of d.

b Write an expression for Peter's age in terms of d.

c Write an equation to represent the sum of the three ages.

d Solve the equation for d.

e List the ages of the three children.

Analysis task

Kath and Kim

Kath has \$70 and saves \$20 per week. Kim has \$510, but spends \$35 per week.

a Write an expression for how much money Kath will have after n weeks.

b How much money will Kath have after three weeks?

c After how many weeks will Kath have \$150?

d Write an expression for how much money Kim will have after n weeks.

e How much will Kim have after three weeks?

f How much will Kim have after 14 weeks?

g Use a table of values to find the value of n when Kath and Kim will have the same amount of money.

h Write an equation which can be used to find out when Kath and Kim will have the same amount of money.

i Solve the equation for n. Do you get the same value for n as in part g?

Review Solving equations

Summary

- An equation is a statement that two expressions have the same value.
- Solving is the process of finding which values of the variable will make the equation a true statement.
- Some equations can be solved by using mental strategies, or a guess, check and improve strategy.
- Forward tracking through a flow chart can be used to build up expressions.
- Backtracking involves using inverse operations to move backwards through a flow chart from the finishing number to find what the starting number must have been.
- Equations can be solved by using inverse operations and doing the same to both sides.
- A four-step process of solving algebraic word problems is below.

1. **Translate the words into algebra.** Decide on the unknown and give it a pronumeral, then formulate an equation using this pronumeral.
2. **Solve the equation.** Solve by 'doing the same to both sides'.
3. **Check the solution.** Substitute your solution back into the original equation to check that the LS = RS.
4. **Translate the algebra back into words.** Express your solution in terms of the problem wording.

Visual map

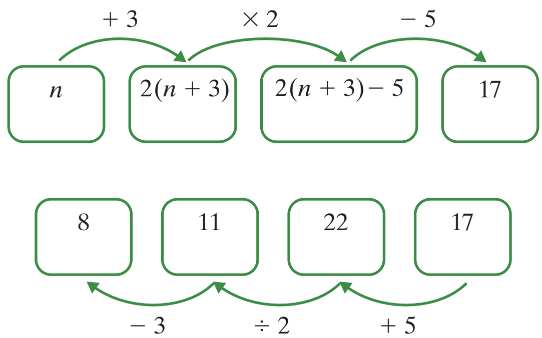
Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

backtracking	equivalent equation	solve
consecutive	expression	true statement
doing the same to both sides	product	unknown value
equation	solution	variable

Revision

Multiple-choice questions

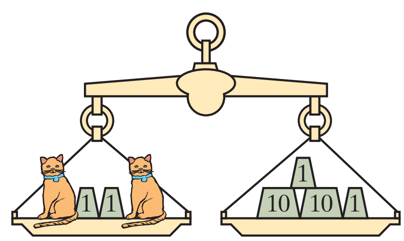
The flow chart below represents the solution of which equation?



A $2n + 3 - 5 = 17$
B $n + 3 \times 2 - 5 = 17$
C $3n \times 2 - 5 = 17$
D $(n + 3) \times 2 - 5 = 17$
E $n + (3 \times 2 - 5) = 17$

In this balance scales diagram, the mass of each cat is x kg. The equation represented in the diagram is $2x + 2 = 22$. The value of x is

A 10
B 11
C 12
D 20
E 24



The solution to the equation $4x - 13 = 27$ is

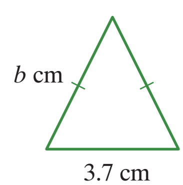
A $x = 1$ **B** $x = 3$ **C** $x = 10$ **D** $x = 12$ **E** $x = 13$

The solution to the equation $\frac{m}{5} - 3 = 11$ is

A $m = 40$ **B** $m = 14$ **C** $m = 70$ **D** $m = 8$ **E** $m = 26$

In the diagram, the value of b is

A 5.7
B 6
C 5
D 11.4
E 18.1



Perimeter = 15.1 cm

Short-answer questions

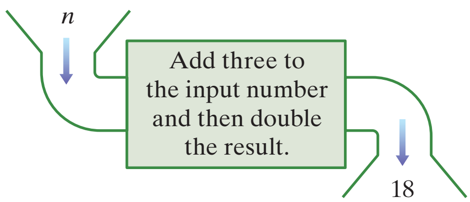
Insert the symbol , or = in each of the following to make a true statement.

a $2 + 3 \quad 5$ __ $5 \quad 3 - 2$ **b** $3 + 3 \quad 1$ __ $4 \quad 3 - 6$

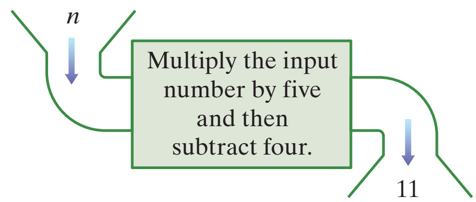
c $\frac{3 \times 6}{2}$ __ $3 \quad 3 - \frac{0}{2}$ **d** $\frac{7}{2} + \frac{4}{2}$ __ 11

Write an equation which represents the following number machines.

a



b

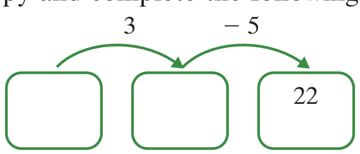


Use an arithmetic strategy to find the value of the unknown.

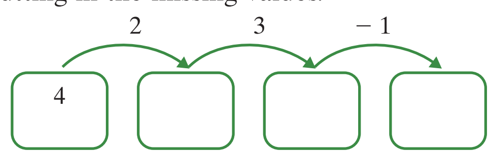
a $1 + 5h = 16$ **b** $7k - 12 = 16$ **c** $\frac{x}{3} + 1 = 10$

Copy and complete the following flow charts, putting in the missing values.

a



b



Use a flow chart and backtracking to work out the starting number for each of the following.

a $3n + 5 = 23$ **b** $3(n - 2) + 2 = 23$

Solve each of the following equations by doing the same to both sides.

a $m - 8 = 9$ **b** $b + 13 = 24$ **c** $7x = 84$ **d** $\frac{y}{2.4} = 5$

e $3d + 5 = 32$ **f** $\frac{n - 3}{5} = 6$ **g** $\frac{k}{5} + 1 = 6$ **h** $4n - 20 = 10$

i $\frac{p + 2}{3} = 4$ **j** $\frac{3y}{2} = 12$ **k** $1.2a + 1.1 = 4.7$

Extended-response questions

There are 103 corn chips in two piles. The second pile has 11 more than the first pile.

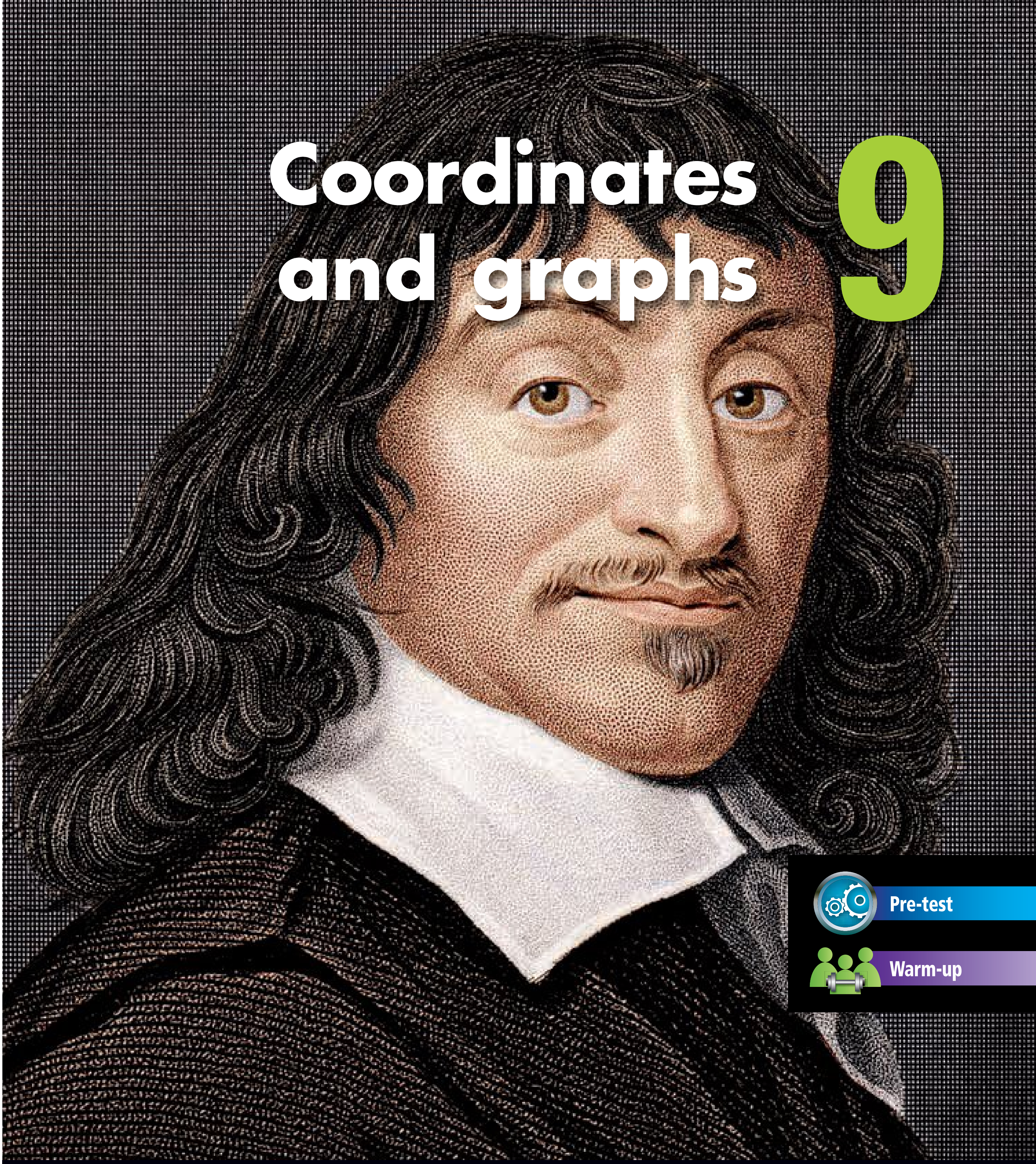
a Write an expression for the number, n, of corn chips in the first pile.
b Write an expression for the number of corn chips in the other pile.
c Write an equation to describe the situation.
d Solve the equation to find out the value of n.
e How many corn chips are in each pile?

The sum of two consecutive numbers is 121. Let n be the smaller number.

a Write an equation to represent the sum of the two numbers.
b Solve the equation to find n.
c Find the values of the consecutive numbers.

Practice quiz
Chapter 8

Coordinates and graphs 9



Pre-test

Warm-up

René Descartes was a French mathematician and philosopher who lived in the first half of the 17th century. Descartes developed the idea of two perpendicular axes and a pair of numbers (called coordinates) to describe the position of a point on a plane. His work led to the system of Cartesian coordinates.

9.1 The Cartesian plane

Cartesian coordinates

When we use grid references on a map, we specify which square a place is in, but the grid reference usually does not tell us the exact location of the place in the square. In mathematics we often need to be more precise and specify the exact position of a point on a plane surface.

Class activity
Scoring points

The idea of specifying points by two numbers, one in a horizontal direction and the other in a vertical direction, was first introduced by the 17th century French mathematician René Descartes. The numbers are now known as **Cartesian coordinates** and the grid on which points are located is referred to as the **Cartesian plane**. To specify the position of a point on the Cartesian plane, we need two number lines—a horizontal number line, which we call the ***x*-axis**, and a vertical number line which we call the ***y*-axis**.

The horizontal position of a point on the Cartesian plane is called the ***x*-coordinate** of the point and the vertical position of the point is called the ***y*-coordinate**.

The first number in the brackets is the *x*-coordinate, and the second number is the *y*-coordinate.

(3, 10)

x-coordinate   *y*-coordinate

Together, the *x*-coordinate and the *y*-coordinate of a point are called an **ordered pair**. The order of coordinates is easy to remember because the coordinates are in alphabetical order; that is *x*, then *y*.

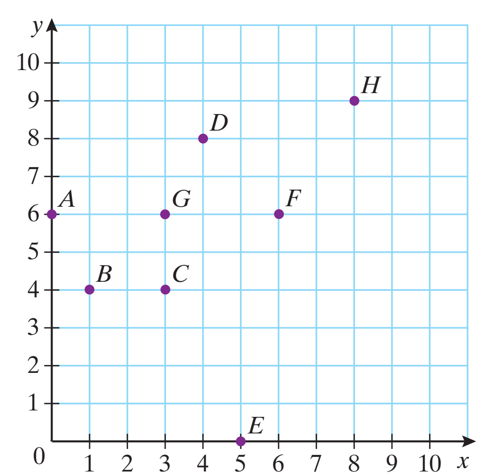
The *x*-axis and *y*-axis include negative numbers as well as positive numbers. The *x*-axis and the *y*-axis cross at the point (0, 0). This point is called the **origin**.

The two axes divide the Cartesian plane into four sections called **quadrants**. Example 1 includes only the first quadrant, where both *x* and *y* coordinates are positive.

Example 1

Write the coordinates of the points labelled *A* to *H* as ordered pairs.

Working	Reasoning
A (0, 6)	Across 0, up 6
B (1, 4)	Across 1, up 4
C (3, 4)	Across 3, up 4
D (4, 8)	Across 4, up 8
E (5, 0)	Across 5, up 0
F (6, 6)	Across 6, up 6
G (6, 3)	Across 6, up 3
H (8, 9)	Across 8, up 9

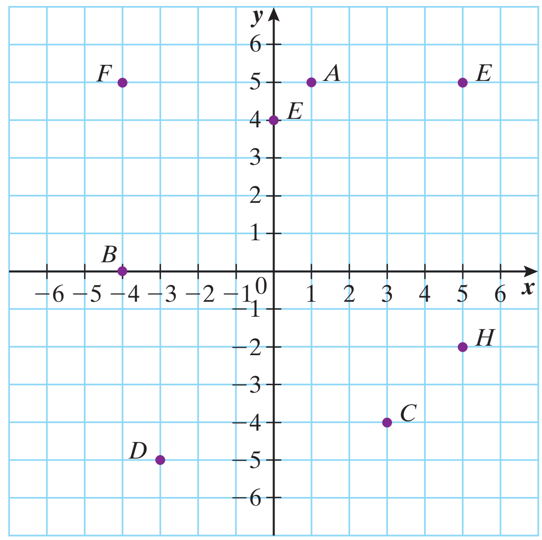


Cartesian coordinates specify an *actual point*, not a region as map references do. The x- and y-axes are like number lines, where the numbers mark a *particular value on the number line*. This is different from map references, where the numbers and letters are labels for the *intervals between the grid lines*.

Example 2

Write the coordinates of the points labelled A to H as ordered pairs.

Working	Reasoning
$A(1, 5)$	Right 1, up 5
$B(-4, 0)$	Left 4, on the x-axis
$C(3, -4)$	Right 3, down 4
$D(-3, -5)$	Left 3, down 5
$E(6, 5)$	Right 6, up 5
$F(-4, 5)$	Left 4, up 5
$G(0, 4)$	On the y-axis, up 4
$H(5, -2)$	Right 5, down 2

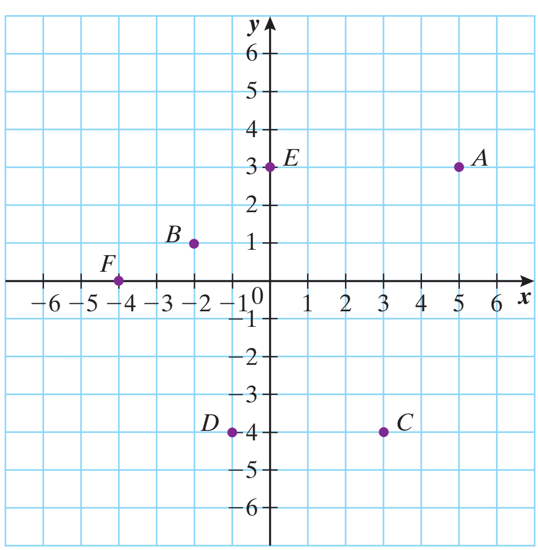


Example 3

Plot the following ordered pairs as points on the Cartesian plane.

$A\ (5, 3)$ $B\ (-2, 1)$ $C\ (3, -4)$ $D\ (-1, -4)$ $E\ (0, 3)$ $F\ (-4, 0)$

Working



Reasoning

A: 5 units to the right, 3 units up
B: 2 units to the left, 1 unit up
C: 3 units to the right, 4 units down
D: 1 unit to the left, 4 units down
E: on the y-axis, 3 units up
F: 4 units to the left on the x-axis

Example 4

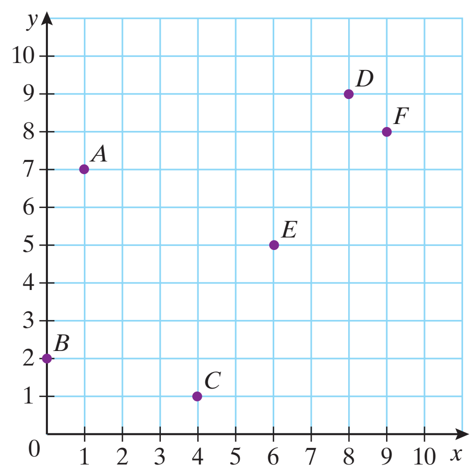
Which of the following points are on the x-axis: (0, 3), (5, 0), (4, 2)?

Working	Reasoning
(0, 3) is on the y-axis, not on the x-axis.	The point with coordinates (0, 3) is 0 units in the x-direction, and 3 units up, that is, in the positive y-direction.
(5, 0) is on the x-axis.	The point with coordinates (5, 0) is 5 units to the right in the x-direction, and 0 units in the y-direction.
(4, 2) is not on either axis.	(4, 2) is 4 units to the right, that is, in the positive x-direction, and 2 units up, that is, in the positive y-direction.

exercise 9.1

LINKS TO Example 1

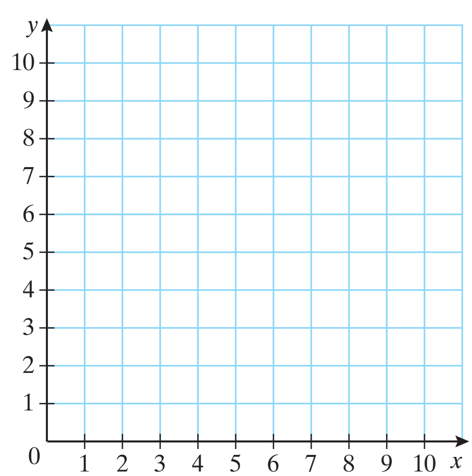
Give the coordinates for each of the points A, B, C, D, E, and F.



LINKS TO Example 1

Copy the coordinate axes on the right onto 1 cm graph paper. Mark and label the following points.

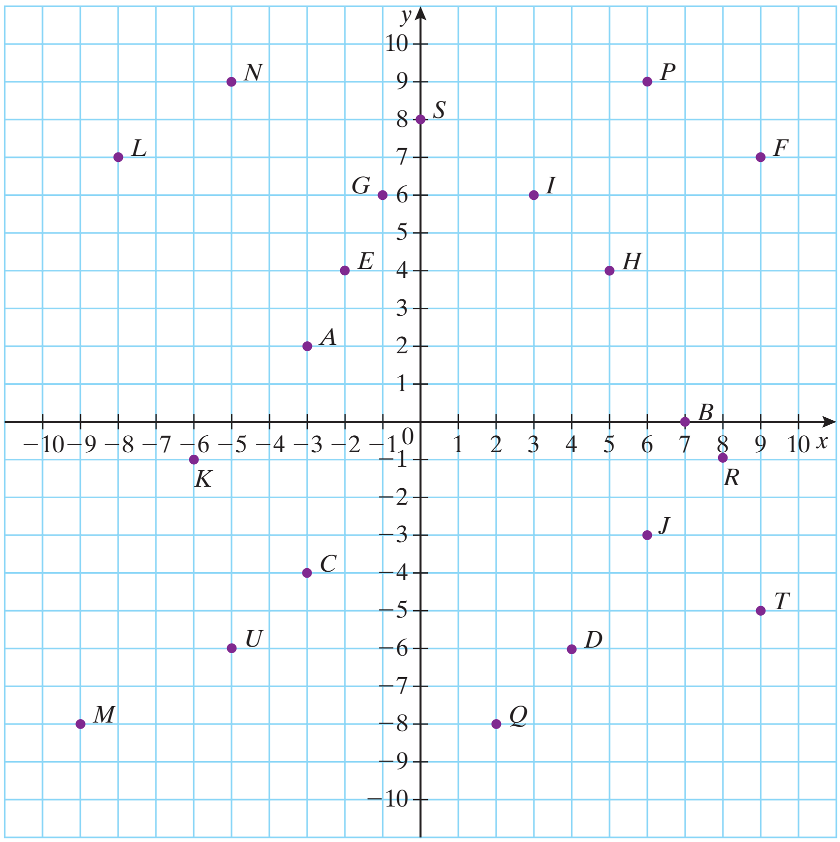
a (4, 1) **b** (0, 3) **c** (1, 0)
d (5, 2) **e** (3, 7) **f** (0, 5)
g (9, 8) **h** (4, 6) **i** (9, 0)



9.1

LINKS TO Example 2

Write the coordinates of each of these points.



LINKS TO Example 3

Plot these points on the Cartesian plane using x and y axes from −10 to 10.

A (4, 1)	B (6, 0)	C (−9, 0)	D (−4, −6)	E (−6, 6)
F (0, −8)	G (−4, −7)	H (7, −4)	I (5, −4)	J (0, 5)
K (−6, −8)	L (−5, 0)	M (−3, 4)	N (3, −9)	O (0, −2)
P (4, 6)	Q (−7, 2)	R (9, 0)	S (3, 0)	T (−8, −2)

LINKS TO Example 3

The point (−6, 5) is

A 6 units down and 5 units to the right.
B 6 units to the left and 5 units up.
C 6 units up and 5 units to the left.
D 6 units down and 5 units to the left.
E 6 units to the right and 5 units up.

LINKS TO Example 3

Which of these ordered pairs is 7 units to the left and 8 units down?

A (7, 8) **B** (8, −7) **C** (−8, −7) **D** (−7, −8) **E** (7, −8)

LINKS TO Example 4

The following ordered pairs tell us the position of points on the Cartesian plane.

A (5, 4)	B (0, 3)	C (5, 5)	D (2, 0)	E (0, 4)
F (−2, 7)	G (1, 5)	H (6, 0)	I (−1, 4)	J (−2, 6)

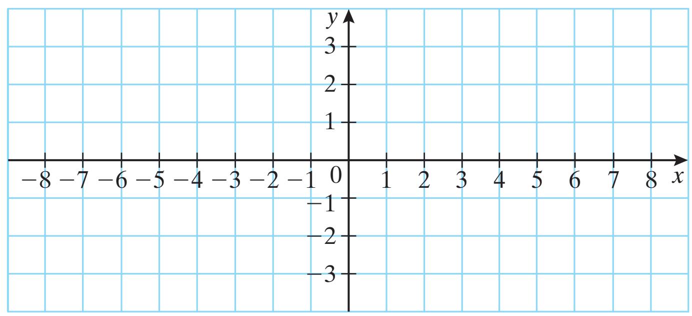
a Which points are on the x-axis?

b Which points are on the y-axis?

c Which point has the same y-coordinate as point C?

d Which point has the same x-coordinate as point F?

Using 1 cm graph paper, draw these axes and follow the steps.



1 Starting at (6, 2), plot each of the following points, using your ruler to join the points in the given order.
(6, 2), (7, 2), (8, 1), (6, −1), (−2, −1), (1, 2), (1,1), (5, 2), (6, −1)

2 Now place a dot at (0, 0).

BLM Coordinate picture grid

Using the *Coordinate picture* grid in the student ebook, plot the following points and join them in order.

1 Start at (−13, 7), then join to (−11, 5), (−9, 5), (−7, 7), (−7, 1), (−9, −1), (−11, −1), (−13, 1), (−13, 7) .

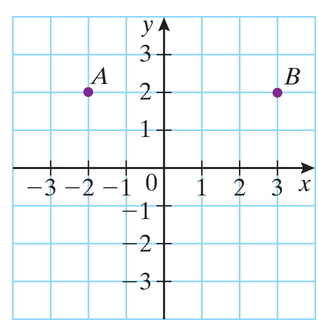
2 Start again at (−7, 5), then join to (5, 5), (9, 1), (9, −5), (−9, −5), (−9, −3), (−11, −5), (−13, −5), (−11, −3), (−11, −1).

3 Start again at (−7, −5) then join to (−7, −3), (7, −3).

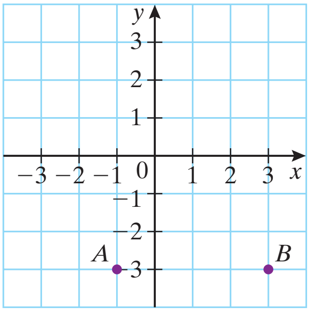
Consider points A and B and follow these steps.

a Copy the axes and points A and B onto 1 cm graph paper. Place two more points, C and D, so that $ABCD$ is a square. Join A–B, B–C, C–D, and D–A.

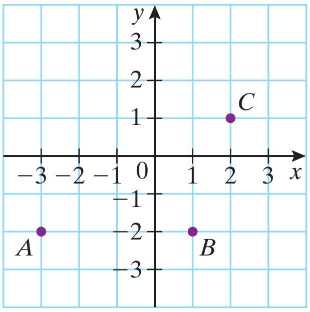
b What are the coordinates of C and of D?



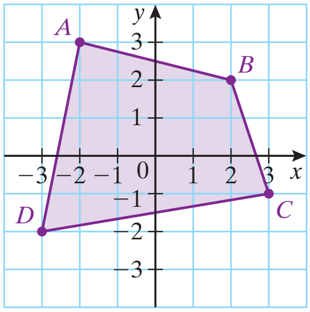
- Consider points A and B and follow these steps.
  - **a** Copy the axes and points A and B onto 1 cm graph paper. Place a point C so that ABC is an isosceles triangle with AB as its base, and a height of 5 cm. Join A–B, B–C, and C–A.
  - **b** What are the coordinates of C?



- Consider points A, B and C.
  - **a** Copy the axes and points A, B and C onto 1 cm graph paper. Place another point D so that $ABCD$ is a parallelogram. Join A–B, B–C, C–D, and D–A.
  - **b** What are the coordinates of D?



- $ABCD$ is a quadrilateral.
  - **a** List the coordinates of points A, B, C and D.
  - **b** Which of the following points are inside the quadrilateral?
    (2, 3), (0, 0), (−3, 1), (2, 1), (−2, −1), (2, −3)



- Consider these points.
  - **a** Is the point (7, 0) on the x-axis or the y-axis?
  - **b** Is the point (0, 3) on the x-axis or the y-axis?
  - **c** Which one of these points is not on one of the axes?
    (4, 0), (0, 5), (1, 2), (8, 0), (0, 1)

- Plot the following points on 1 cm graph paper and join them in the order shown.
  (−3, 1), (−1, 3), (1, 1), (−1, −3), (−3, 1)
  - **a** What shape is formed?
  - **b** Draw the two diagonals. What are the coordinates of the point where the diagonals intersect?

exercise 9.1 challenge

- Create your own coordinate picture. List the coordinates in the sequence they are to be joined and swap with another student.

9.2 Graphing linear relationships

Five different ways of expressing the same information

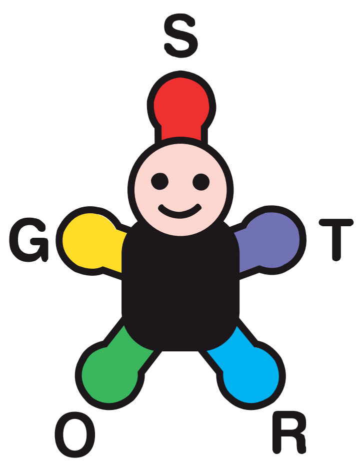
In section 5.4 we saw that

- a simple **story** (or **situation**) involving numbers could be written as an algebraic **rule** using pronumerals instead of numbers.
- the same information could be represented as a **table of values**.

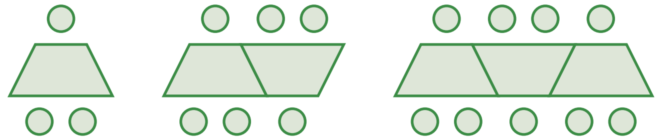
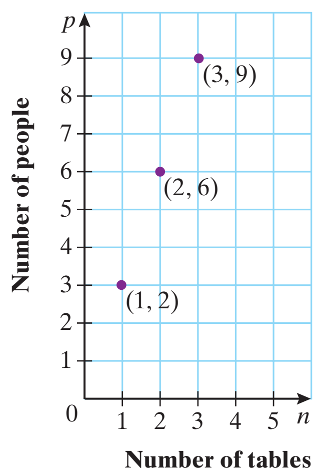
In this section we will represent the numbers in a table of values as a list of **ordered pairs** and as a **graph**.

STROG reminds us of these five different ways of saying the same thing.

- **S**tory or situation
- **T**able of values
- **R**ule
- **O**rdered pairs
- **G**raph



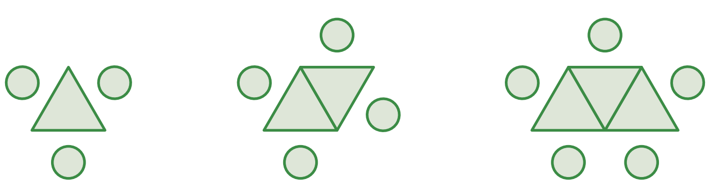
To illustrate the five different ways of representing relationships between two variables, we use the example of the number, p, of people who can be seated at n tables.

<table>
<tr><td>Story</td><td>A classroom has trapezium shaped tables.

Let n be the number of tables.
Let p be the number of people who can sit at n tables.</td></tr>
<tr><td>Table of values</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p</td><td>3</td><td>6</td><td>9</td></tr></table></td></tr>
<tr><td>Rule</td><td>$p = 3n$</td></tr>
<tr><td>Ordered pairs</td><td>(1, 3), (2, 6), (3, 9)</td></tr>
<tr><td>Graph</td><td>
</td></tr>
</table>

In the next example look at how STROG reminds us of the five different ways of saying the same thing.

Example 5

A kindergarten has tables in the shape of equilateral triangles. One child can sit at each edge of a table, so that when two tables are put together, four children can sit at the two tables, as shown.



continued

Example 5 continued

Working

a

x	1	2	3	4
y	3	4	5	6

Reasoning: Three children can sit at one table. When two tables are placed together, four children can sit at the two tables.

b The number of children is always two more than the number of tables.

Reasoning: Each time another table is added two extra places are made.

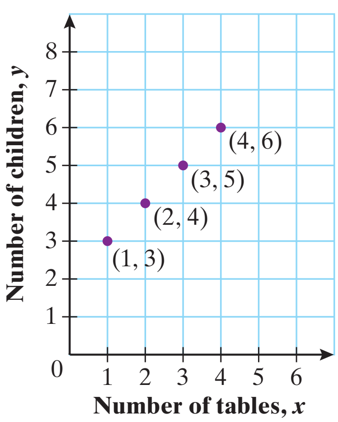
c $y = x + 2$

Reasoning: x is the number of tables, y is the number of children. The y value is always two more than the x value.

d (1, 3), (2, 4), (3, 5), (4, 6)

Reasoning: In an ordered pair the x-coordinate is first, followed by the y-coordinate.

e



Reasoning: The x-axis is the horizontal (across) axis. The y-axis is the vertical (up and down) axis.

Example 6

When Sophie cooks rice she adds two cups of water for each cup of rice.

Let x be the number of cups of rice. Let y be the number of cups of water.

a What is the rule connecting x and y?

b Complete the table of values.

x					
y					

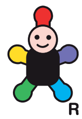
continued

Example 6 continued

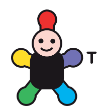
c Write the pairs of x and y values as a list of ordered pairs.

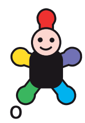
d Plot the points on 1 cm graph paper, labelling the axes and each ordered pair.

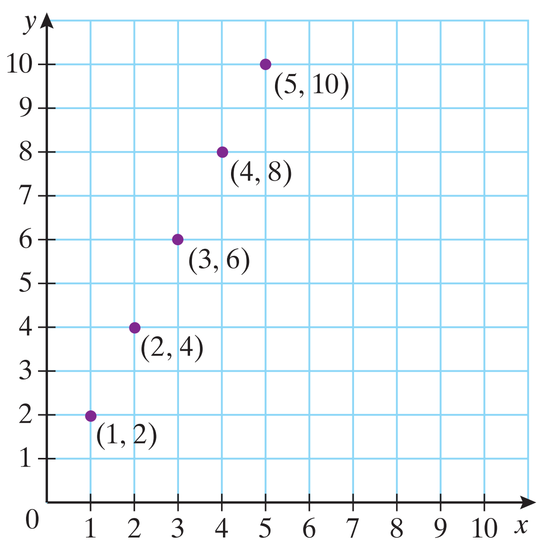
Working	Reasoning
a $y = 2x$	There are two cups of water for each cup of rice.
b (see table below)	The value of y is always twice the value of x.
c (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)	The x value goes first in each ordered pair.
d (see graph below)	The x-axis is the horizontal (across) axis. The y-axis is the vertical (up and down) axis.



x	1	2	3	4	5
y	2	4	6	8	10







exercise 9.2

LINKS TO Example 5

BLM 1 cm grid

BLM Cartesian grid

1. People who make tea in a teapot with loose tea leaves instead of tea bags often follow a rule: 'One teaspoon per person plus one for the pot'.
   Let x be the number of people who are having a cup of tea.
   Let y be the number of teaspoons of tea.

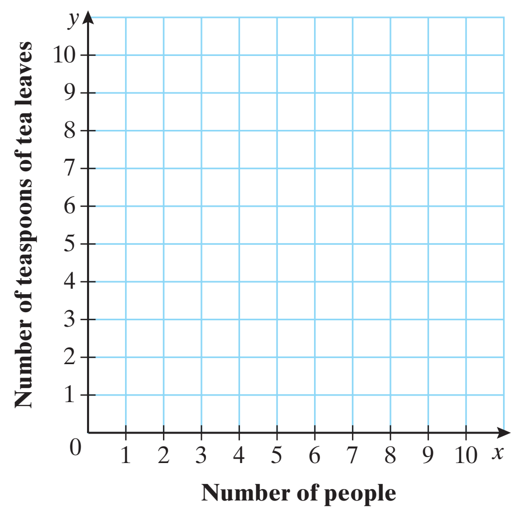
a Complete the table of values below to show the number of teaspoons of tea leaves for different numbers of people.

Number of people (x)	1	2	3	4	5	6
Number of teaspoons of tea leaves (y)						

b Write the values as a list of ordered pairs.

c Write this as a rule using the pronumerals x and y, where x is the number of people and y is the number of teaspoons of tea leaves.

d Copy the coordinate grid and axes onto 1 cm graph paper, then carefully plot and label the six ordered pairs. Label the axes.



LINKS TO Example 6

Most ordinary cars carry five people.
Let x stand for the number of cars.
Let y stand for the number of people that can travel in them.

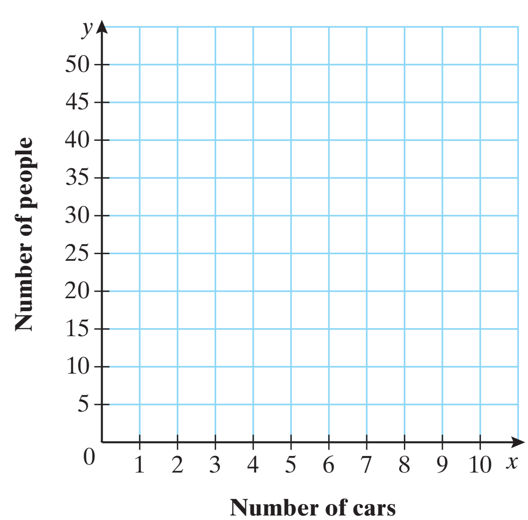
a Complete the following table of values.

Number of cars (x)	1	2	3	4	5	6
Number of people (y)						

b Write an algebra rule for calculating y if we know the value of x.

c Write the x and y values as a list of ordered pairs.

d Copy the coordinate grid and axes onto 1 cm graph paper, then carefully plot and label the six ordered pairs.



LINKS TO Example 6

A shop sells packets of pens. Each packet contains three pens.
Let x be the number of packets.
Let y be the number of pens.

a Complete the following table of values.

x	1	2	3	4	5	6
y						

b Write the values as a list of ordered pairs.

c Using 1 cm graph paper, draw and label the x and y axes, then carefully plot and label the six ordered pairs.

d Describe in words the relationship between the number of pens and the number of packets.

e Write this as a rule using the pronumerals x and y.

LINKS TO Example 6

Mugs are packed in boxes of four.
Let x be the number of boxes.
Let y be the number of mugs.

a Complete the following table of values.

x	1	2	3	4	5	6
y						

b Write the values as a list of ordered pairs.

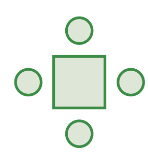
c Describe in words the relationship between the number of boxes and the number of mugs.

d Use the pronumerals x and y to write a rule that we can use to calculate the number of mugs if we know the number of boxes.

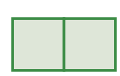
e Using 1 cm graph paper, draw and label the x and y axes, then carefully plot and label the six ordered pairs.

LINKS TO Example 5

A restaurant has square tables which seat one person on each of the four sides as shown.



a If two tables are placed together how many people can be seated?

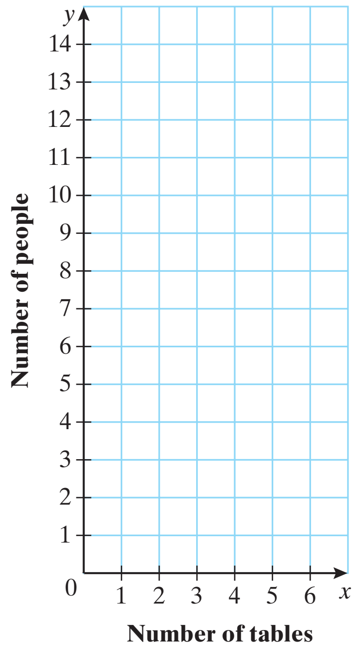


b In the following table, x stands for the number of tables and y stands for the number of people who can be seated. Copy and complete the table.

x	1	2	3	4	5
y	4				

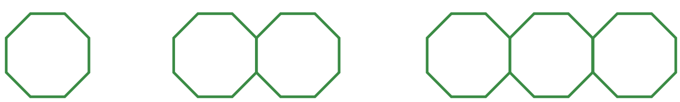
c Write the x and y values as a list of ordered pairs.

d Plot a graph of the ordered pairs.



exercise 9.2 challenge

A landscape gardener is making octagonal flower beds with wooden edging as shown. If she makes one octagonal flower bed, eight edging strips are needed.



a Copy and complete the following table to show how many edging strips are needed for 2, 3 and 4 joined octagonal flower beds.

Number of flower beds (x)	1	2	3	4
Number of edging strips (y)	8			

b Write each of the pairs of x- and y-coordinates as an ordered pair.

c On 1 cm graph paper rule and label a set of axes. How long will the x-axis need to be? How long will the y-axis need to be? Mark each of the ordered pairs as a point on your Cartesian plane. Label each point (1, 8), etc.

d Predict how many edging strips would be needed if the gardener had five octagonal flower beds.

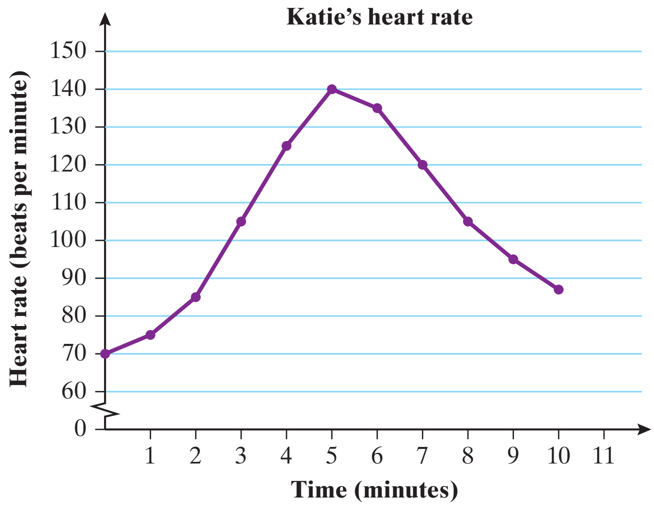
e Write an algebra rule to show the relationship between the number of flower beds, x, and the number of edging strips, y.

9.3 Line graphs

Line graphs are used to display numerical data, usually relating to a variable that changes with time. Line graphs are useful because they clearly show any general patterns (trends) in the data. Line graphs can also be used to help us make predictions about the future. The points are joined with straight lines or curves. In this section we use straight lines to join the points.

Example 7

This line graph shows Katie's heart rate over a 10 minute period during which she rides an exercise bike as part of an overall fitness assessment.



a Katie's resting heart rate is measured before she starts pedalling. What is Katie's resting heart rate?

b What is her heart rate after 4 minutes?

i What is her maximum heart rate?

ii How many minutes after the start did this occur?

Working	Reasoning
a Katie's resting heart rate is 70 beats per minute.	Go to the first point on the graph and read the value off the vertical axis.
b After 4 minutes, Katie's heart rate is 125 beats per minute.	Read across the horizontal axis and find the time of 4. Go up to the corresponding point on the graph then read the heart rate from the vertical axis.

continued

Example 7 continued

Working

i Katie's maximum heart rate is 140 beats per minute.

Reasoning

Locate the highest point on the graph then read the value corresponding to this point from the vertical axis.

Working

ii Her maximum heart rate occurs 5 minutes after the start.

Reasoning

Locate the highest point on the graph then read the value corresponding to this point from the horizontal axis.

Example 8

Katy conducts a science experiment where she slowly heats some liquid in a beaker and records the temperature of the liquid every 10 minutes. Her data is shown in the table below.

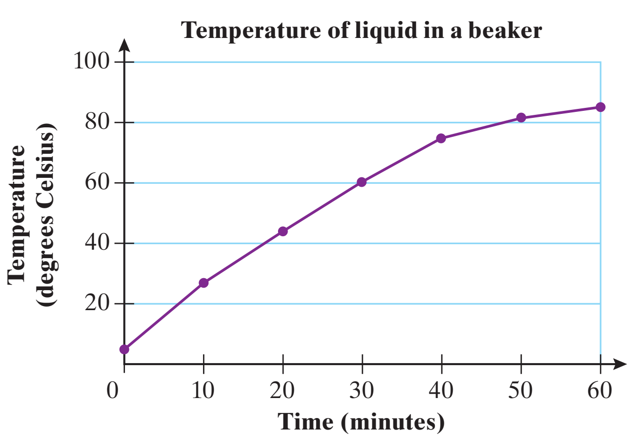
a Draw a line graph to represent this information.

b Estimate the temperature of the liquid in the beaker after 35 minutes of heating.

Time (minutes)	Temperature (°C)
0	4
10	27
20	44
30	60
40	75
50	81
60	85

Working

a



Reasoning

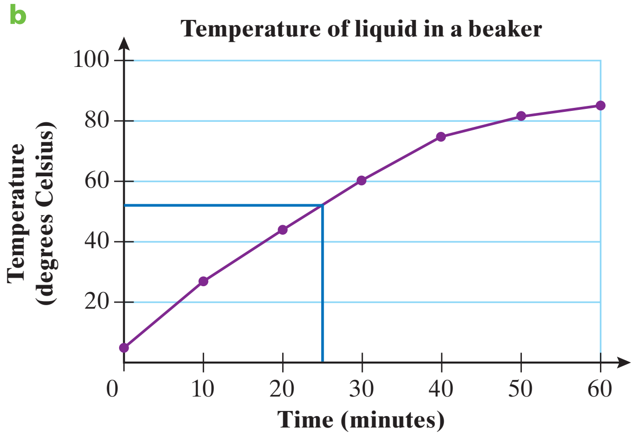
The horizontal axis represents the time variable, in this case time in minutes
The vertical axis represents temperature in °C.
Choose a suitable scale for each axis.
Plot a point to represent each time and corresponding temperature
Join the points with a line.

continued

Example 8 continued

Working

b



After 25 minutes the temperature of the liquid is approximately 55°C.

Reasoning

Read the temperature of the liquid from the graph at time 25 minutes.

Example 9

For people who suffer from hayfever and asthma, the daily pollen count, particularly grass pollen, is useful information. The number of pollen grains in the air depends on the flowering seasons of plants as well as daily fluctuations in temperature, wind, humidity and rain. The School of Botany at the University of Melbourne collects and analyses pollen samples from October 1 to January 31 each year by catching pollen grains in a sticky trap. The following data show the average daily grass pollen count for Melbourne for December 11–31, 2010. High allergy days are when the average daily grass pollen count is over 50 grains per cubic metre of air and extreme days are when the count is over 100 grains per cubic metre of air.

a Use a spreadsheet to construct a line graph of the data.

b On which dates was the average daily grass pollen count greater than 50 grains/m^3 of air?

c On which dates was the average daily grass pollen count less than 20 grains/m^3 of air?

d Which dates would have been the most difficult for hayfever and asthma sufferers?

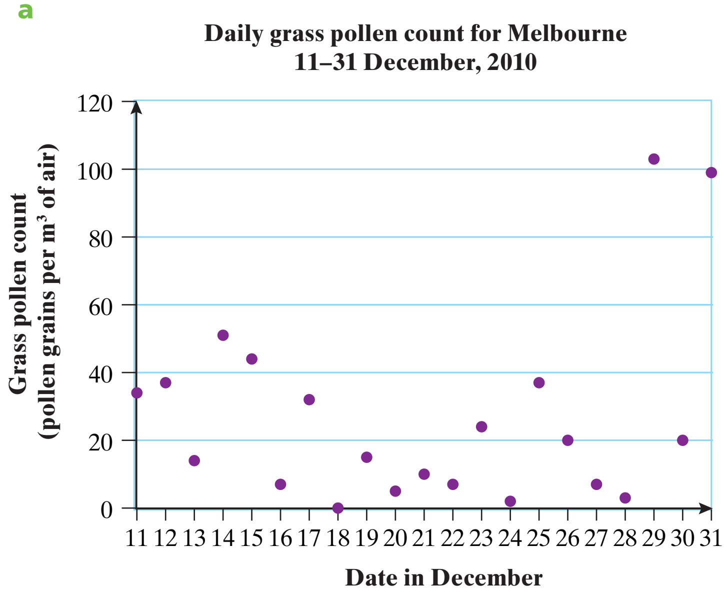
Date	Grass pollen count (grains/m^3 of air)
11	34
12	37
13	14
14	51
15	44
16	7
17	32
18	0
19	15
20	5
21	10
22	7
23	24
24	2
25	37
26	20
27	7
28	3
29	103
30	20
31	99

continued

Example 9 continued

Working

a



Reasoning

Enter the data in a spreadsheet.

Construct the XY(Scatter) graph. Label the axes and give the graph a title.

Set scale on x-axis to go from 11 to 31.

b The grass pollen count was greater than 50 grains/m^3 of air on December 14, 29 and 31.

c The grass pollen count was less than 20 grains/m^3 of air on December 13, 16, 18–22, 24, 27 and 28.

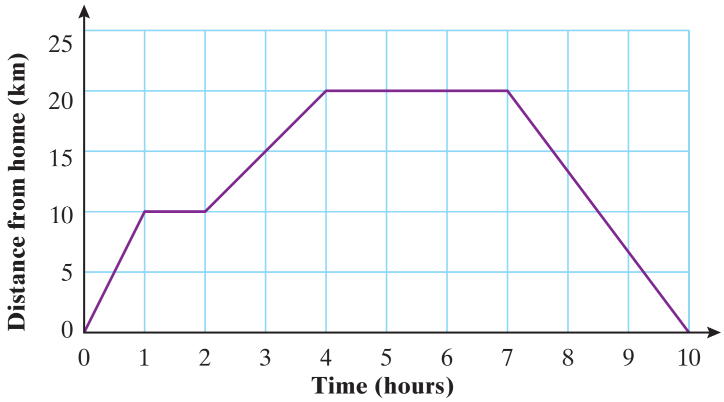
d December 29 and 31 would have been the most difficult days for hay fever and asthma sufferers.

The grass pollen counts were 103 grains/m^3 of air on December 29 and 99 grains/m^3 of air on December 31.

A travel graph is a line graph which shows time on the horizontal axis and the distance from a starting point on the vertical axis.

Example 10

Jamie and Sam cycled to the beach, leaving home at 8 am (Time 0). The following travel graph shows their distance from home through the day.



a How far did the boys cycle in the first hour?
b How far did they travel in the second hour?
c How far did they travel in the third hour?
d What does this tell you about their speed in the first hour and the third hour?
e How far from home was the beach?
f How long did the boys spend at the beach?
g How long did it take them to get home?
h At what time did they arrive home?

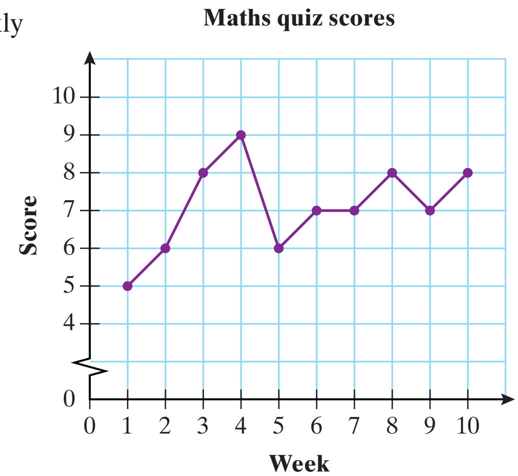
Working	Reasoning
a The boys cycled 10 km in the first hour.	At time 1 hour, the distance from home was 10 km.
b The distance from home during the second hour did not change. The boys must have stopped for an hour.	During the second hour, the distance from home stayed at 10 km.
c The boys cycled 5 km in the third hour.	In the third hour, the distance from home increased from 10 km to 15 km.
d They were travelling faster in the first hour than in the third hour.	They travelled 10 km in the first hour but only 5 km in the third hour. They were travelling twice as fast in the first hour.
e The beach was 20 km from home.	The boys stopped for 3 hours after 20 km.
f The boys spent 3 hours at the beach.	The boys stopped for 3 hours from 4 hours to 7 hours after leaving hone.
g It took them 3 hours to get home.	The distance from home was decreasing from 7 hours to 10 hours after leaving home.
h They arrived home at 6 pm.	The boys reached home 10 hours after setting out at 8 am and 10 hours after 8 am is 6 pm.

Notice in the travel graph in example 10 that the steepness of the graph indicates how fast the boys were cycling. The graph is steeper between 0 and 1 hour than between 2 and 4 hours.

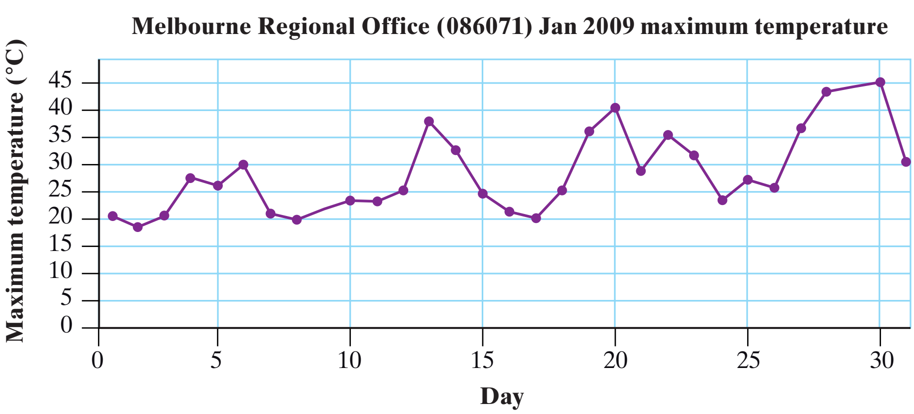
exercise 9.3

The graph shows Stefan's scores for his weekly maths quiz for one term. Which of the following is a true statement?

A His lowest score was in week 9.
B His score improved the most between week 4 and week 5.
C His highest score was 4.
D His score was the same in week 6 and week 7.
E He got a score of 8 four times.



This line graph shows the maximum temperature over 24 hours in Melbourne for each day in January 2009.



Source: Climate Data Online, Bureau of Meteorology
Copyright © Commonwealth of Australia, 2011

a How many days were over 40 °C?
b What was the lowest temperature and on what date did this occur?
c By how many degrees did the maximum temperature fall from January 30 to January 31?
d Collect data for your town or city for a month and construct a line graph of the data.

BLM Oslo temperature grid

The following data represents the average maximum and minimum temperatures for Oslo (the capital of Norway) for December 2010. The temperatures have been rounded to the nearest integer value. It was the coldest December on record in Oslo.

a Using the grid provided, plot the minimum daily temperatures for each day of December. Join the points with straight lines.

b Using a different colour, plot the maximum daily temperatures. Join the points with straight lines.

c What was the lowest minimum temperature?

d What was the lowest maximum temperature?

e What was the highest minimum temperature?

f On which date did the greatest difference between minimum and maximum temperatures occur?

g For how many days was the maximum temperature below −10°C?

h On what date was the maximum temperature 7 degrees higher than the minimum temperature?

i On 26 December, how much higher was the maximum temperature than the minimum temperature?

Date	Minimum (C)	Maximum (C)
1	−19	−11
2	−16	−11
3	−18	−11
4	−17	−15
5	−18	−8
6	−11	−10
7	−17	−14
8	−24	−16
9	−16	−8
10	−16	−11
11	−12	−4
12	−11	−5
13	−11	−8
14	−15	−7
15	−10	−8
16	−9	−1
17	−16	−8
18	−8	−4
19	−10	−10
20	−11	−10
21	−18	−16
22	−22	−17
23	−18	−13
24	−20	−12
25	−19	−13
26	−16	−5
27	−8	−6
28	−9	−9
29	−18	−16
30	−20	−12
31	−15	−2

The following data shows the average mass in grams of eggs laid by chickens at different ages (measured in weeks).

a Choosing a suitable scale for each axis, plot a line graph of mass of eggs against age of chickens. Join the points with straight lines. Label the axes and give your graph a title.

b Why do you think the data starts at 18 weeks?

c Describe how the average mass of eggs changes between 18 weeks and 24 weeks.

d Describe what happens after 24 weeks.

Age (weeks)	Average mass of egg (g)
18	43
20	48
22	54
24	58
26	60
28	60
30	60
32	61
34	62
36	62
38	62
40	63
42	63
44	63
46	64
48	64
50	64

The data in the table is a record of Grace's height, measured correct to the nearest centimetre. Her height was recorded on the day she was born and then every birthday until her 13th birthday.

a Construct a line graph to display this data.

b Use this graph to estimate Grace's height when she was 6 months old.

Age (years)	Height (cm)
0	51
1	76
2	86
3	95
4	103
5	110
6	116
7	122
8	127
9	132
10	138
11	140
12	147
13	153

● There is a lot of variation in how many words children know at different ages. Some children will be very advanced in their language development and will know many more words than other children. The table below shows the *average* numbers of words known at different ages for a sample of children.

Age (years)	No. of words
2	250
3	900
4	1500
5	2000
6	2500

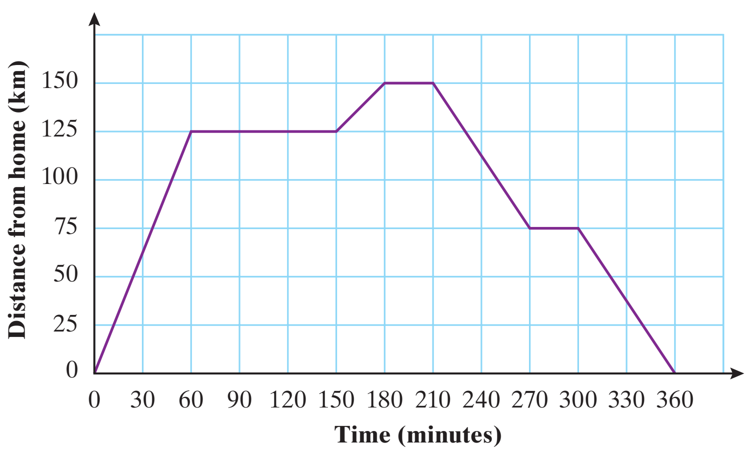
a Choosing a suitable scale for each axis, draw and label the two axes. Construct a line graph of the data.
b Use your graph to predict how many words an average seven-year-old might know.
c Would you be confident to use the graph to predict the vocabulary of a 20-year-old? Explain.
d Draw a sketch to show how you think the shape of the graph might be from the age of six to adult life.

● The table below shows baby Annie's weight for the first 10 weeks of her life.

Age (weeks)	0	1	2	3	4	5	6	7	8	9	10
Weight (kg)	3.08	2.94	3.06	3.31	3.42	3.60	3.76	3.92	4.10	4.33	4.53

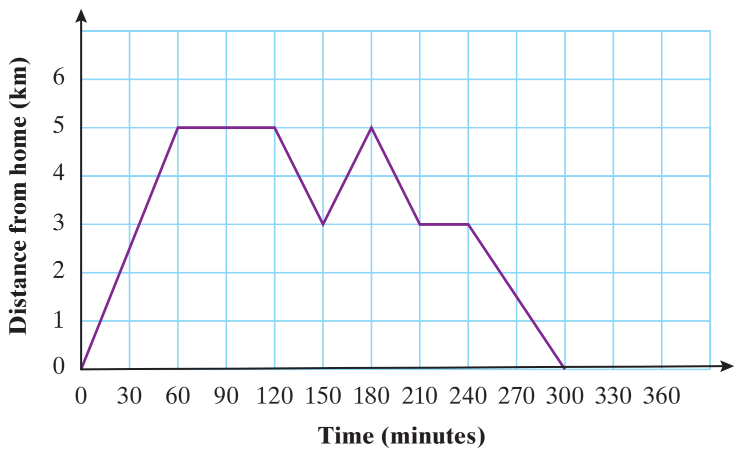
a Use a spreadsheet to represent this data using a line graph.
b Use your graph to describe Annie's weight for the first 10 weeks after she was born.

● The following travel graph shows the distance of the Lee family from home on a day out in the countryside. Their first stop is at the Magic Maze. Their next stop is at a picnic ground for lunch. Before returning home in the afternoon they stop at an alpaca farm.



a How long does it take the Lee family to reach the Magic Maze?
b How far from their home is the Magic Maze?
c How long do they stay at the Magic Maze?
d How far is the picnic ground from the Magic Maze?
e How long do they stay at the picnic ground?
f How far is it from the picnic ground to the alpaca farm?
g How long do they spend at the alpaca farm?
h How long does it take the Lee family to reach home after leaving the alpaca farm?
i How many hours after the Lee family left home in the morning do they arrive home again?

Marnie and Jay walk from their campsite into the nearby town of Seaville. They have shopping to do and a letter to post in Seaville. They intend finding a shady place for a rest and lunch on the way back.

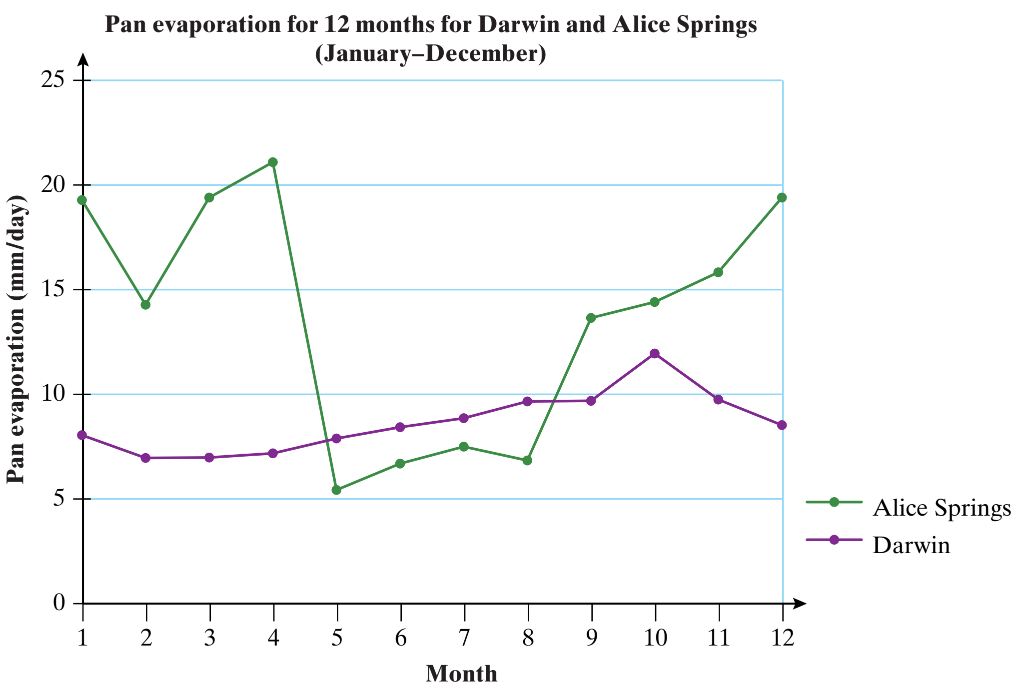


a How long to Marnie and Jay take to get to Seaville?
b How far is Seaville from the campsite?
c How long do they spend in Seaville?
d Suggest what might be happening between 150 minutes and 180 minutes after they left the campsite.
e In what section of their walk do you think Marnie and Jay are walking fastest? Explain your reasoning.
f What is the total time that Marnie and Jay are walking (not counting the time in Seaville and their rest stop)?

exercise 9.3 challenge

1 Loss of stored water from water storage dams and reservoirs is an important issue. An evaporation pan is a device for measuring how much evaporation occurs. By measuring changes in the water level, the amount of evaporation can be measured daily.

The following graph shows the average amounts of pan evaporation in mm/day at different times of the year for Darwin and Alice Springs during one year.



a In which month was pan evaporation greatest in Alice Springs?

b For how many months in Alice Springs was the pan evaporation over 15 mm per day?

c In which month was pan evaporation least in Alice Springs?

d For how many months in Alice Springs was the pan evaporation below 10 mm per day?

e In which month was pan evaporation greatest in Darwin?

f For how many months in Darwin was the pan evaporation over 15 mm per day?

g In which month was pan evaporation least in Darwin?

h For how many months in Darwin was the pan evaporation below 10 mm per day?

i Think about the factors that influence evaporation. Explain the differences between evaporation in Alice Springs and in Darwin.

The following data shows the average amounts of pan evaporation in mm/day at different times of the year for Ceduna in South Australia.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Month number	1	2	3	4	5	6	7	8	9	10	11	12
Evaporation (mm/day) Ceduna	10	8	7	5	4	3	3	4	5	8	9	10
Evaporation (mm/day) Mount Gambier	7	6	5	3	2	1	1	2	3	4	5	6

a Plot a line graph of the data with month number on the horizontal axis and the pan evaporation (in mm/day) on the vertical axis.

b In which months is evaporation greatest? Why do you think this is the case?

c In which months is evaporation the lowest?

d Why is this pattern of evaporation a problem for farmers?

e Compare the evaporation rates for Ceduna and Mount Gambier. How could the difference be explained?

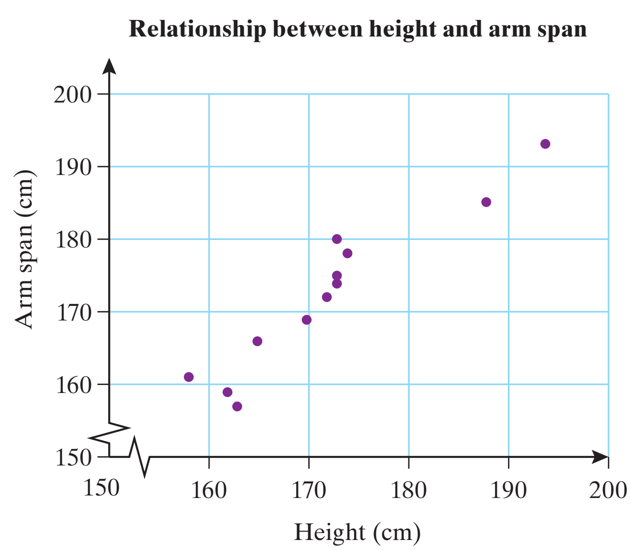
9.4 Exploring data with scatter plots

When we are analysing data, we are often looking at a single variable, for example, the number of children in families. Statistics such as the mean and the range are useful in comparing different sets of data for a single variable.

Sometimes, however, we are interested in the relationship between two variables. For example, we may be interested in the relationship between arm span and height. The following table shows the heights and corresponding arm spans of 12 people.

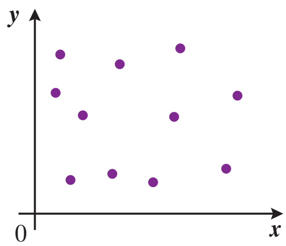
Height (cm)	163	162	174	173	188	172	194	158	173	170	165	173
Arm span (cm)	157	159	178	175	185	172	193	161	174	169	166	180

A graph of the data, called a **scatter plot**, shows that taller people tend to have greater arm spans. Notice that this doesn't mean that every tall person will have a large arm span or that every short person will have a small arm span. The data simply shows that there is a trend in the data.

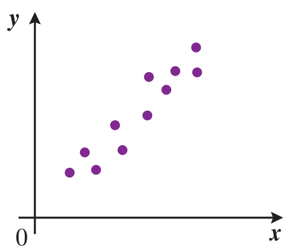


Sometimes we do not know whether there is a relationship between two variables. Constructing a scatter graph is a simple way of finding if there appears to be a relationship.

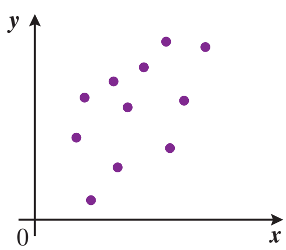
In all of the examples below, as x increases in value, y also increases in value. For example, as height increases, there is a tendency towards greater arm spans. We say that there is a positive relationship between the two variables.



There seems to be no relationship between the two variables, x and y.

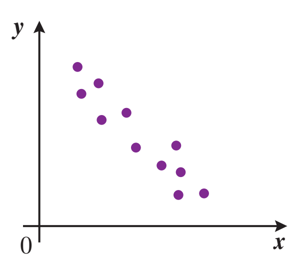


There seems to be quite a strong positive relationship between the two variables, x and y. As the x values increase, the y values also increase.



There seems to be a weak positive relationship between the two variables, x and y.

Sometimes, though, one variable might decrease as the other increases. The scatter plot would then look like this. For example, the number of hot drinks sold might increase as the temperature drops. We say that there is a negative relationship between the two variables.



We must always be careful in drawing conclusions from a scatter plot. Although two variables might appear to be related, it may be that there is some third variable that is affecting them both.

Example 11

The results of a Mathematics test and a Science test for a group of Year 10 students are shown in the table below.

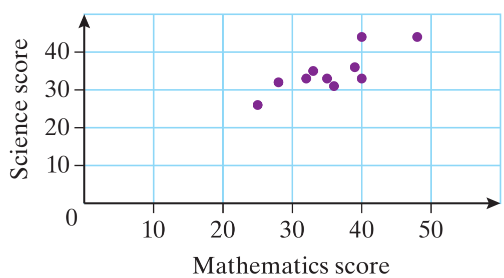
Mathematics	33	40	39	40	25	28	35	48	36	32
Science	35	33	36	44	26	32	33	44	31	33

a Draw a scatter plot for this set of data, with Mathematics on the horizontal axis and Science on the vertical axis.

b What can we say about the relationship between the two sets of test results?

Working

Science score vs Mathematics score

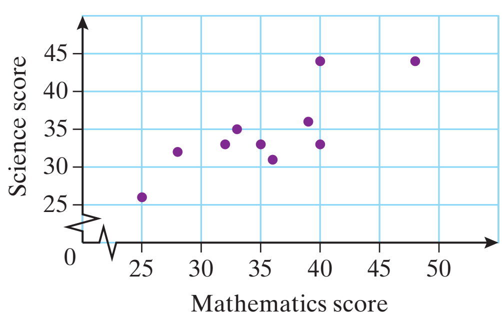


Reasoning

Each pair of test scores is represented by a point on the scatter plot.

In this scatter plot, each of the axes begins from 0. This means that there is a lot of 'wasted' space.

Science score vs Mathematics score



The axes need not start at 0. Using 'broken' axes allows us to 'zoom in' and see the association between the two variables more clearly.

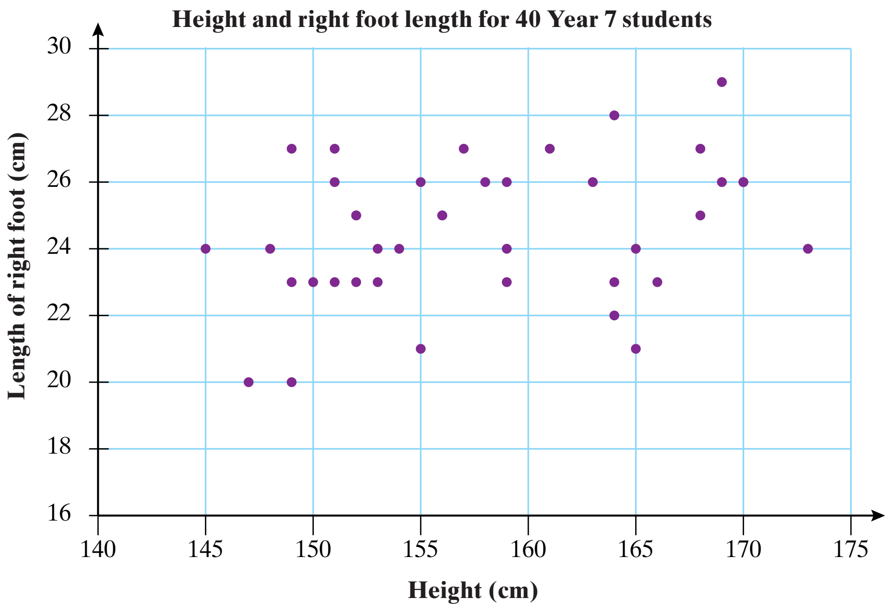
Broken axes allow a better view of this data.

b The students with higher Mathematics scores generally have higher Science scores.

There seems to be a moderately strong positive relationship between Mathematics and Science scores.

exercise 9.4

The following scatter plot shows the height and the right foot length for a sample of 40 Year 7 students.



a Does there seem to be a relationship between foot length and height?

b Do you think the length of a person's foot would be a good indicator of how tall the person was?

LINKS TO Example 11

During summer, the manager of a swimming pool investigated the relationship between the maximum daily temperature and attendance at the pool. She recorded this information on 10 randomly selected days.

a Use a spreadsheet to construct a scatter graph.

b Comment on the relationship between the two variables.

Temperature (C)	Attendance
21	65
21	78
25	100
28	135
31	220
38	380
35	390
23	120
18	22
20	56

9.4

During summer, the manager of a milk bar noticed a relationship between the maximum daily temperature and the number of drink cans sold. He recorded this information on seven randomly selected days.

Temperature (C)	Number of cans sold
25	259
28	314
29	325
31	403
34	480
36	498
37	526

a Construct a scatter plot of the data.

b Comment on the relationship between the two variables.

The following table shows the hand span and height for 27 Year 7 students.

Hand span (cm)	Height (cm)	Hand span (cm)	Height (cm)	Hand span (cm)	Height (cm)
19.5	160	18.9	162	19.0	154
18.5	162	20.1	165	20.6	168
20.1	163	16.5	157	22.5	166
21.0	170	20.5	156	15.5	152
19.0	160	20.5	170	19.8	163
20.0	150	18.5	160	18.0	165
20.1	164	19.2	149	18.5	162
19.5	162	19.5	161	16.7	158
18.5	156	22.0	155	16.0	156

a Use a spreadsheet to construct a scatter plot of the data, starting each axis at an appropriate value.

b Describe the association.

c Do you think hand span would be a good predictor of height?

A Maths test was given to 12 students. The foot length of each student was also measured. It was suggested that if a strong association between the foot lengths and test results was found, then measuring foot length would be an easier way of assessing Maths students than giving tests. This table shows the resulting data.

Foot length (*x* cm)	21	25	21	24	16	16	17	18	20	17	21	21
Maths score (*y*%)	65	74	69	77	46	52	53	48	65	53	67	55

a Use Excel to make a scatter plot of the data, starting each axis at zero.

b Describe the association between foot length and Maths score.

c Suggest a possible reason for the association in the collected data. What does the data *not* tell us?

d What can we learn from this question?

exercise 9.4 challenge

The following data shows the outside temperature at different altitudes as a plane descended to land at Sydney Airport at 5 am in August.

Altitude (*x* m)	Temperature (*y* C)
7900	−36
6200	−28
4500	−16
3000	−8
2900	−7
2500	−4
1850	0

a Use a spreadsheet to draw a scatter plot of the data, putting altitude on the horizontal axis and temperature on the vertical axis. We draw the graph like this because temperature depends on altitude, not the other way around. Choose the scatter plot where the points are not joined. Start the x-axis at 0.

b Print your graph and place your ruler along the points. Rule a straight line that seems to fit the points.

c Extend the line backwards to meet the y-axis.

d Use your graph to predict the temperature at ground level that day.

e Use your graph to predict the temperature at 1000 m.

When the wealth of people in a country is estimated, the total value of all the goods and services provided by the country is divided by the number of people in the country. Because it costs more to live in some countries, each person's share is adjusted so that we can make comparisons between different countries. The table below shows 2009 estimates for 20 countries of each person's share of the country's wealth (in $US) and the number of deaths of children under one year per 1000 live births.

a Use a spreadsheet to draw a scatter plot of infant deaths per 1000 births against the average wealth per person in $US.

Country	Wealth per person ($US)	Infant deaths per 1000 live births
Afghanistan	900	152
Australia	39 900	5
Botswana	12 700	12
Burundi	300	63
China	6700	16
Eritrea	700	42
Greece	31 000	5
Haiti	1200	77
India	3200	49
Indonesia	4000	29
Iran	10 900	43
Italy	29 900	5
Japan	32 600	3
Papua New Guinea	2300	45
Singapore	53 900	2
Somalia	600	107
South Africa	10 300	44
Sudan	2200	72
United States	46 000	6
Vietnam	2900	22

b In general, what can you say about the relationship between wealth and infant death rate?

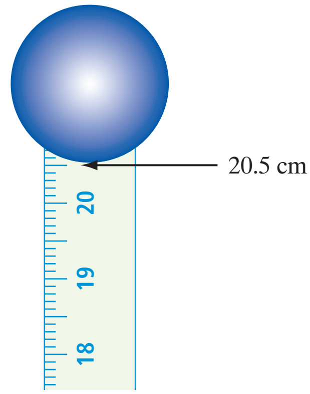
c Were there any countries where the infant death rate was higher or lower than you might have expected when compared with wealth?

d In countries where there is less wealth per person, what are some of the factors that might contribute to high infant death rates?

Analysis task

How high will it bounce?

For this investigation you will need a high-bounce ball and a one metre ruler. Measuring from the bottom of the ball, drop the ball from a height of 100 cm and record the height to which it bounces (again measuring to the bottom of the ball). Repeat twice more and calculate the mean of the three measurements. Continue for the other heights in the table (90 cm, 80 cm, and so on), taking three measurements each time. It is easier if you work in groups of three with one person dropping the ball, one doing the measuring and the third person to watch and check.



Record your results in a table.

Height from which ball is dropped (x cm)	Rebound height (y cm)			
	1	2	3	Average
0				
10				
20				
30				
40				
50				
60				
70				
80				
90				
100				

a Why were three measurements made for each height from which the ball was dropped?

b The two variables in this investigation are the height from which the ball is dropped and the rebound height.

c Using 1 cm graph paper, draw and label the x- and y-axes, choosing a suitable scale on each axis. Plot the mean rebound height (y cm) against the height (x cm) from which the ball was dropped.

d Do your points lie approximately along a straight line? Place your ruler along the points and rule a straight line that fits your points.

e Why would you expect the line to pass though the point (0, 0)?

f Work out the rule to show the connection between y and x.

g Use your rule to predict how high the ball would bounce if dropped from a height of 85 cm.

h Use your graph to check your prediction.

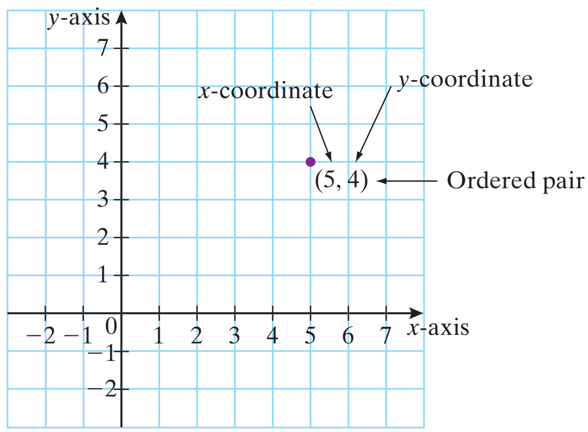
i Use the rule to predict how high the ball would bounce if dropped from a height of 10 metres.

j How reliable do you think this prediction might be?

Review Coordinates and graphs

Summary

- Points on the Cartesian plane are described by an *x*-coordinate (horizontal) and a *y*-coordinate (vertical).
- The coordinates of a point on the Cartesian plane are written as an ordered pair, for example, (5, 4).



- Line graphs are useful for showing changes in variables over time.
- Scatter plots are useful when we want to investigate the relationship between two variables.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

axis	negative	scale
Cartesian plane	ordered pair	scatter plot
coordinates	origin	*x*-axis
linear graph	positive	*y*-axis
line graph	rule	

Revision

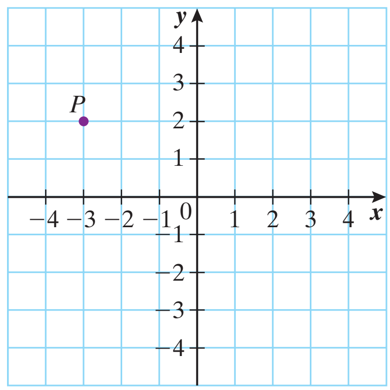
Multiple-choice questions

Which of these points is on the x-axis?

A (5, 1) **B** (4, 4) **C** (−2, 0) **D** (0, 6) **E** (1, 1)

P is the point

A (2, 3)
B (−2, −3)
C (−3, 2)
D (−2, 3)
E (2, −3)



To find the point (−1, 6) on the Cartesian plane, start at the origin then

A move 1 to the right and 6 down.
B move 6 to the right and 1 down.
C move 1 to the right and 6 down.
D move 1 to the left and 6 up.
E move 6 to the left and 1 down.

To find the point (−5, 0) on the Cartesian plane, start at the origin then

A move 5 to the left and 0 down.
B move 0 to the right and 5 down.
C move 5 to the right and 0 down.
D move 0 to the right and 5 down.
E move 5 to the right and 0 down.

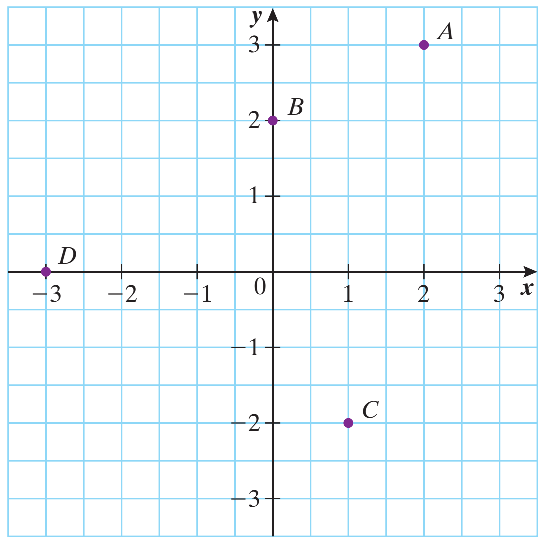
Which of these rules matches the ordered pairs in the table?

x	1	2	3	4	5
y	7	10	13	16	19

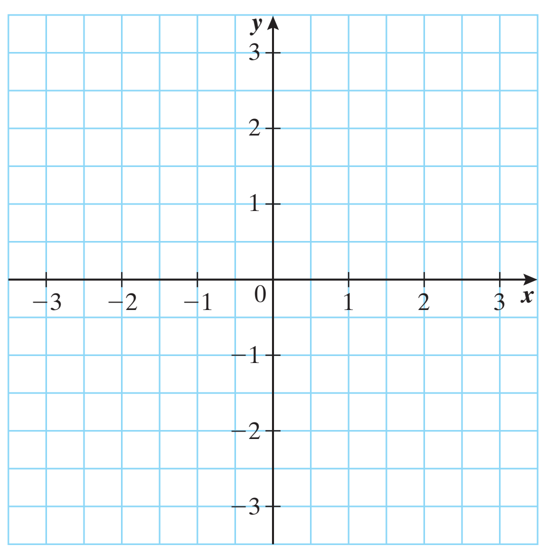
A $y = x + 6$ **B** $y = 4x$ **C** $y = 5x$ **D** $y = 3x + 4$ **E** $y = 2x + 5$

Short-answer questions

- In the ordered pair (3, 7) which number is the x-coordinate and which is the y-coordinate?
- What are the coordinates of the labelled points?



- Copy the coordinate plane onto 1 cm graph paper, then plot and label the following points: A $(-1, 2)$, B $(3, 1)$, C $(0, 3)$, D $(3, -2)$.



- **a** Is the point (7, 0) on the x-axis or the y-axis?
  b Is the point (0, 3) on the x-axis or the y-axis?
  c Which one of these points is not on one of the axes?
  (4, 0), (0, 5), (1, 2), (8, 0), (0, 1)

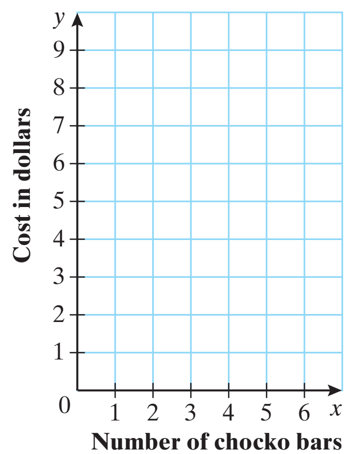
- Find the coordinates of the following points.
  a Point A is 3 units to the left of the y-axis and two units above the x-axis.
  b Point B is 4 units above the x-axis and its y-coordinate is twice its x-coordinate.

Extended-response questions

- Chocko bars are $2 each. In the following table, x represents the number of chocko bars and y is the cost in dollars.
  a Copy and complete the table.

x	1	2	3	4	5
y	2				

  b Write each of the pairs of x and y values as an ordered pair.
  c Copy the coordinate plane and carefully plot each of the ordered pairs.



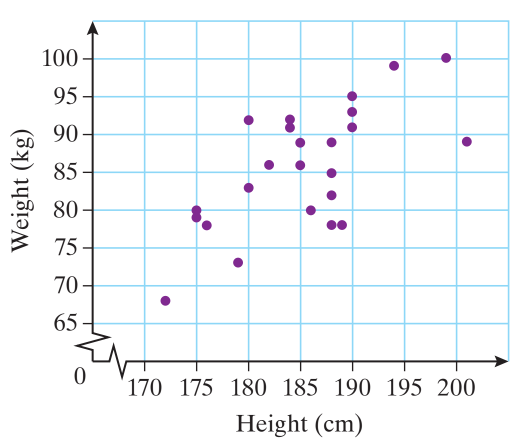
- A conservation group has recorded the height of a tree every 2 years since it was planted. Their results are shown in this table.

Age of tree (years)	2	4	6	8	10
Height (centimetres)	68	126	184	216	248

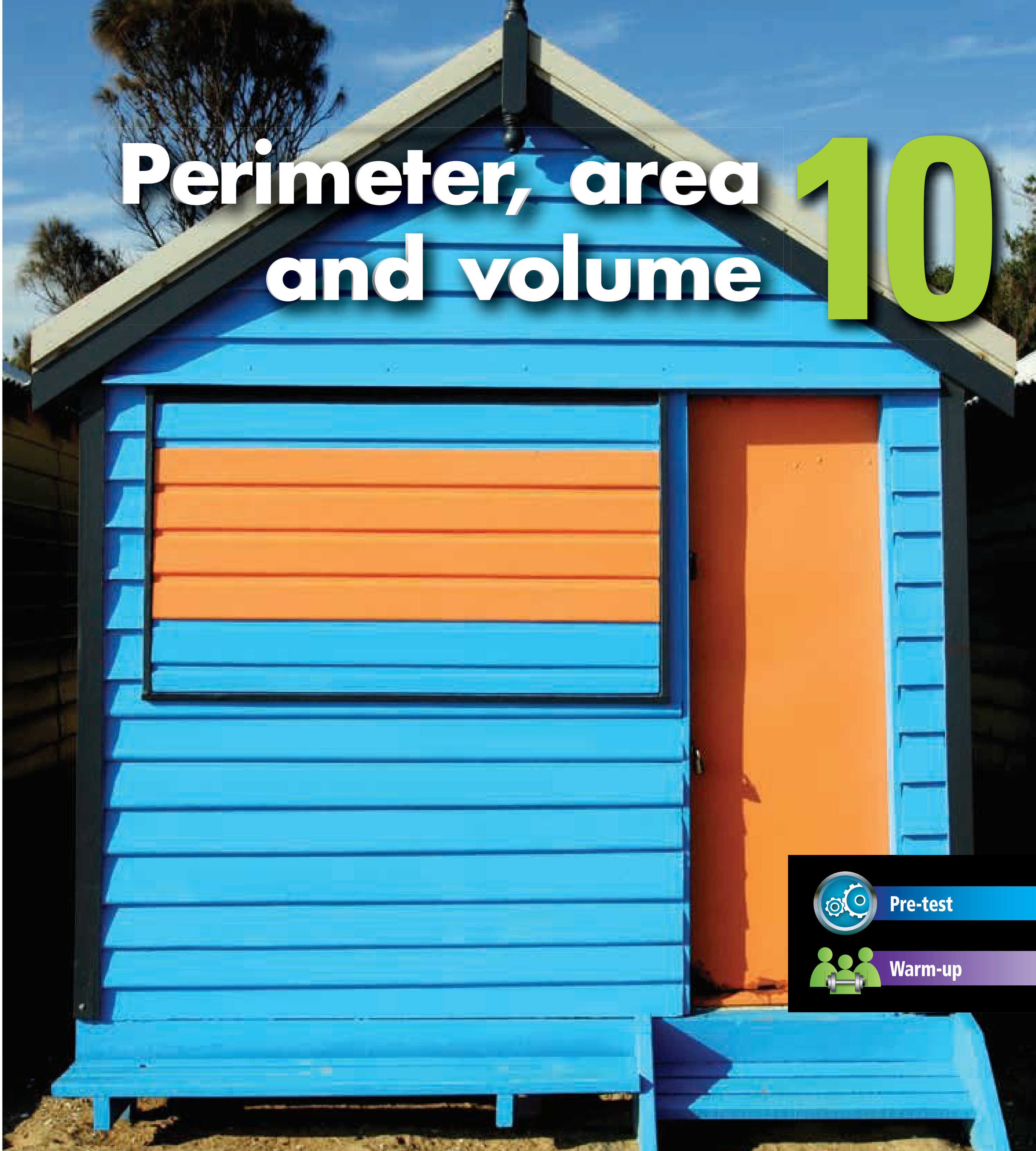
  a Construct a line graph representing this data.
  b Use the line graph to estimate the height of the tree after 5 years.
  c Estimate when the tree had a height of 2 metres.
  d Explain why your results in b and c are just estimates.
  e Use your graph to estimate the height of the tree when it is 12 years old.

- This scatter plot to the right shows the heights and weights (masses) of 24 players from a football team.
  Describe the relationship between the heights and weights of the footballers.

Heights and weights of 24 football players



10 Perimeter, area and volume

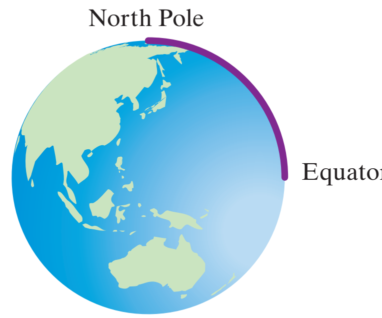


Pre-test

Warm-up

This beach hut has been recently painted. Estimating the amount of paint needed involves first calculating the area to be painted. What measurements would need to be made to find the area? If the label on the tin suggests that one litre of paint covers 15 square metres, how would you calculate the volume of paint required?

10.1 Units of length



The metre was originally defined as $\frac{1}{10\,000\,000}$ of the distance around the Earth from the North Pole to the equator.

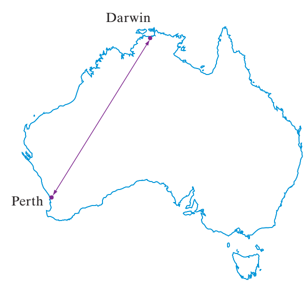
Class activity
Why do we need standard units?

There is now an International System of Units. An international organisation of approximately 50 countries ensures that the definitions of units keep up with advances in science and technology.

The metre is now defined in terms of the speed of light: one metre is the distance travelled by light in a vacuum during a time interval of $\frac{1}{29\,979\,245}$ of a second.

Choosing suitable length units

When we measure length or distance, it is important to choose a suitable unit. Some examples are shown below.

Distance from Perth to Darwin	Kilometres	
Length of a swimming pool	Metres	

Length of a lizard	Centimetres	
Length of grains of rice	Millimetres	

Some examples of length measurement

Object	Approximate measurements
Thickness of a spider's web	0.005 mm
Length of a standard school ruler	30 cm
Height of a door	2 m
Direct distance from Sydney to Melbourne	730 km
Length of the Murray–Darling River	3750 km

Example 1

Choose the most suitable metric unit for measuring each of the following.

a the height of an old tree

b the length of a new pencil

Working

a We would measure the height of a tree in metres.

b We would measure the length of a pencil in centimetres.

Reasoning

The height of an old tree could range from about three metres to over 50 metres.

The length of a new pencil would be about 15 centimetres.

Metric system prefixes

The following table shows the meanings of prefixes used in the metric system.

Nano	1 thousand-millionth
Milli	1 thousandth
Centi	1 hundredth
Deci	1 tenth
Kilo	1 thousand
Mega	1 million
Giga	1 thousand million

The prefixes milli, centi and kilo are also used in other metric units; for example, milligram, kilogram, millilitre, and megalitre.

The prefixes kilo, mega and giga are used for the size of computer files; for example, kilobyte, megabyte and gigabyte. However, 1 kilobyte = 1024 bytes, rather than 1000 bytes, 1 megabyte =1024 kilobytes and 1 gigabyte = 1024 megabytes.

Example 2

Rewrite

a 'thousand metres' as a single word by using a prefix.

b 'three millimetres' in words using metres.

Working	Reasoning
a kilometre	The prefix 'kilo' means one thousand.
b three thousandths of a metre	The prefix 'milli' means a thousandth, so a millimetre is a thousandth of a metre.

There is a symbol for each of the metric length units. These are symbols, not abbreviations, so we never write 'cms' or 'kms'.

Unit	Symbol
Kilometre	km
Metre	m
Centimetre	cm
Millimetre	mm

Example 3

Write the following in symbols or words.

a five centimetres in symbols

b 60 mm in words

Working	Reasoning
a 5 cm	The symbol for centimetre is cm. Note that we do not write '5 cms'.
b Sixty millimetres	mm is the symbol for millimetres.

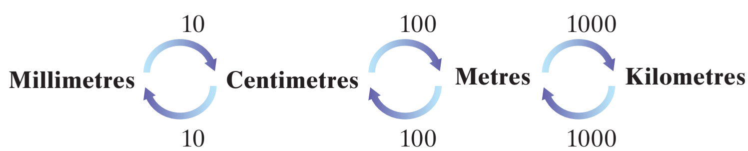
Converting units of length

We often want to compare lengths that have been given in different units. The metric system makes it easy to convert units.

10 mm = 1 cm

100 cm = 1 m

1000 m = 1 km



Example 4

Convert each of these measurements into the unit shown in brackets.

a 16.4 m (mm) **b** 2480 cm (m) **c** 1.08 km (m) **d** 48 mm (cm)

Working	Reasoning
a 16.4 m = 16.4 × 1000 mm = 16 400 mm	1 m = 100 × 10 mm = 1000 mm Convert the larger unit to a smaller unit, so that there are more of them. Multiply.
b 1.08 km = 1.08 × 1000 m = 1080 m	1 km = 1000 m Convert the larger unit to a smaller unit, so that there are more of them. Multiply.
c 2480 cm = 2480 ÷ 100 m = 24.8 m	100 cm = 1 m Convert the smaller unit to a larger unit, so that there are fewer of them. Divide.
d 48 mm = 48 ÷ 10 cm = 4.8 cm	10 mm = 1 cm Convert the smaller unit to a larger unit, so that there are fewer of them. Divide.

When length measurements are added, they must be in the same unit.

Example 5

a Find the sum of 4 km and 367 m

i in metres. **ii** in kilometres.

b Find the sum of 4.5 cm and 28 mm

i in millimetres. **ii** in centimetres.

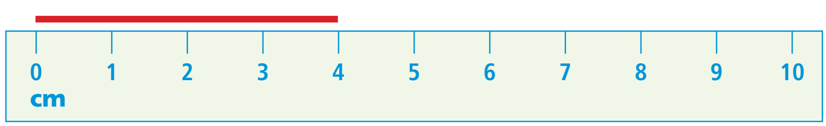
Working	Reasoning
a i 4 km + 367 m = 4000 m + 367 m = 4367 m	1 km = 1000 m so 4 km = 4000 m
ii 4367 m = 4.367 km	1000 m = 1 km
b i 4.5 cm + 28 mm = 45 mm + 28 mm = 73 mm	1 cm = 10 mm so 4.5 cm = 45 mm
ii 73 mm = 7.3 cm	10 mm = 1 cm

Measuring lengths

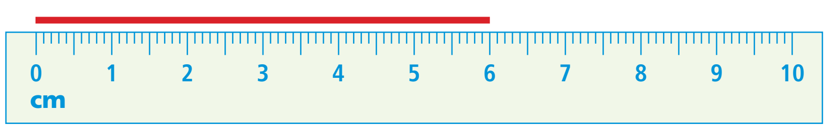
Example 6

Give the length of each of the following red strips

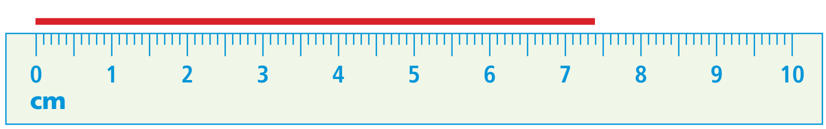
a in centimetres.



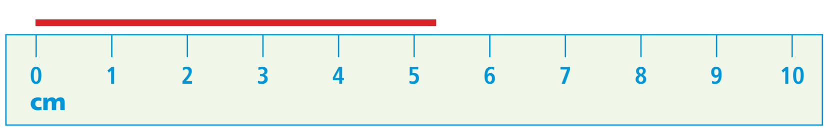
b **i** in millimetres. **ii** in centimetres.



c **i** in millimetres. **ii** in centimetres.



d in centimetres.



continued

Example 6 continued

Working	Reasoning
a 4 cm	Counting the 1 cm intervals, there are four centimetres.
b **i** 60 mm	Counting the 1 mm intervals, there are sixty millimetres.
ii 6 cm	Each 10 mm makes 1 cm.
c **i** 74 mm	There are 7 lots of 10 millimetres plus 4 more.
ii 7.4 cm	70 mm is 7 cm. The extra 4 mm is 0.4 cm.
d 5.3 cm	When there are no millimetre marks, we estimate the length as accurately as we can.

exercise 10.1

LINKS TO Example 1

1. Complete the table by choosing the most appropriate unit (millimetre, centimetre, metre or kilometre) to match each measurement of length or distance shown in the table.

The length of a netball court	
The length of a small ant	
The distance from Adelaide to Perth	
The height of your maths textbook	

LINKS TO Example 1

2. Suggest the most appropriate unit for measuring each of the following.

a length of a bus
b distance of a marathon race
c length of a flea
d length of a mobile phone
e distance across a football ground
f height of a flagpole
g thickness of a 20 cent coin
h height of a table
i width of a computer screen
j thickness of a hamburger
k distance between two airports
l your height

LINKS TO Example 1

3. For each of the following units, give two examples of something that you would measure in this unit.

a millimetres
b centimetres
c metres
d kilometres

LINKS TO Example 2

4. Rewrite each of the following as a single word by using a prefix.

a hundredth of a metre
b tenth of a metre
c thousandth of a metre
d thousand-millionth of a metre
e tenth of a centimetre

LINKS TO Example 2

Rewrite each of the following in words using metres.

a 5 kilometres
b 2 centimetres
c 6 millimetres
d 8 kilometres
e 3 decimetres
f 11 millimetres
g 12 centimetres
h 9 decimetres
i 2 centimetres
j 11 kilometres
k 8 decimetres
l 5 millimetres

LINKS TO Example 2

Rewrite each of the following in words using a more appropriate unit.

a 500 centimetres
b 2000 millimetres
c 300 centimetres
d 8000 metres
e 7000 millimetres
f 4000 metres
g 30 decimetres
h 2000 million millimetres
i 5000 millimetres
j 80 decimetres
k 7000 centimetres
l 12 hundred centimetres

In the word 'centimetre', the prefix 'centi' means 100. Give two other words that start with the prefix 'cent' or 'centi' and explain the connection of each word with 100.

LINKS TO Example 3

Rewrite each of the following in symbols.

a 5 metres
b 20 centimetres
c 18 millimetres
d 3 kilometres
e 20 metres
f 3.6 millimetres
g 4.5 centimetres
h 6.3 metres
i 25.6 kilometres
j 11 centimetres
k 7 millimetres
l 2.5 kilometres

LINKS TO Example 3

Rewrite each of the following in words.

a 6 cm
b 9 m
c 1 mm
d 8 km
e 3 mm
f 7 m
g 2 km
h 10 cm

LINKS TO Examples 4a, 4b

Convert each of these measurements into the unit shown in brackets. In each case you are converting from a larger unit (e.g., km) to a smaller unit (e.g., m).

a 11 km (m)
b 43 m (cm)
c 18 cm (mm)
d 0.34 km (m)
e 3.5 m (mm)
f 240 cm (mm)
g 23.8 cm (mm)
h 1.4 km (m)
i 18.4 m (mm)
j 140 m (cm)
k 0.8 m (cm)
l 0.2 km (cm)
m 4.86 cm (mm)
n 0.45 m (mm)
o 0.045 km (m)

LINKS TO Examples 4c, 4d

Convert each of these measurements into the unit shown in brackets. In each case you are converting from a smaller unit (e.g., mm) to a larger unit (e.g., cm).

a 724 mm (cm)
b 4500 m (km)
c 15 cm (m)
d 3200 m (km)
e 25 mm (cm)
f 580 mm (m)
g 16.8 mm (cm)
h 1.4 mm (cm)
i 2400 m (km)
j 18.4 mm (cm)
k 6500 mm (cm)
l 180 m (km)
m 13 mm (cm)
n 0.6 cm (m)
o 23 500 mm (m)

LINKS TO Example 4

Convert each of these measurements into the unit shown in brackets.

a 18.9 cm (mm) **b** 37 750 mm (m) **c** 65 m (km)
d 0.004 m (mm) **e** 0.04 cm (mm) **f** 1870 mm (m)
g 0.75 m (cm) **h** 0.025 km (m) **i** 958 mm (cm)
j 1638 mm (m) **k** 36 840 mm (m) **l** 38.65 m (mm)
m 1800 m (km) **n** 0.06 km (m) **o** 19 485 mm (m)

LINKS TO Example 5

Express each of these lengths as a single number in the unit shown in brackets.

a 5 centimetres and 8 millimetres (millimetre)
b 14 centimetres and 3 millimetres (centimetre)
c 12 centimetres and 9 millimetres (millimetre)
d 2 metres and 46 centimetres (metre)
e 3 metres and 5 centimetres (centimetre)
f 1 metre and 32 centimetres (millimetre)
g 67 centimetres and 9 millimetres (metre)
h 2 kilometres and 570 metres (metre)
i 3 metres and 275 millimetres (metre)
j 1 kilometre and 24 metres (kilometre)

5600 mm is the same as

A 0.56 m **B** 5.6 km **C** 56 m **D** 560 cm **E** 56 000 cm

If the lengths 240 000 cm, 0.3 km, 250 000 mm, and 1120 m are put into order from shortest to longest, the correct order is

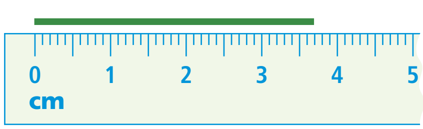
A 250 000 mm, 0.3 km, 1120 m, 240 000 cm
B 1120 m, 240 000 cm, 250 000 mm, 0.3 km
C 240 000 cm, 250 000 mm, 1120 m, 0.3 km
D 240 000 cm, 1120 m, 0.3 km, 250 000 mm
E 0.3 km, 240 000 cm, 1120 m, 250 000 mm

LINKS TO Example 6

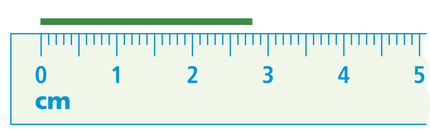
Estimate as accurately as you can the length of each of these pieces of tape in

i centimetres. **ii** millimetres.

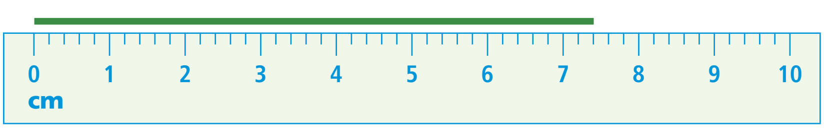
a



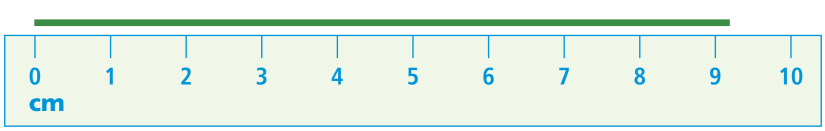
b



c



d



● What is the length of the key shown below in

a millimetres?

b centimetres?

● The sewing thread used in clothing factories for machining clothes comes on large reels of 5000 m.

a Express this length in

i centimetres. **ii** kilometres.

b If 65 m of thread is needed to make a pair of jeans, how many pairs of jeans could be stitched with one 5000 m reel? Round your answer to the nearest five pairs of jeans.

● Builders and carpenters usually work with millimetres rather than centimetres or metres.

a The length, width and height of a toolbox in a hardware catalogue are 550 mm, 235 mm and 245 mm, respectively. Convert these measurements to centimetres.

b The size of a door advertised in a hardware catalogue is 2040 mm by 820 mm. Convert these measurements to metres.

● Amy swims 23 laps of a 50 m pool.

a How many metres does she swim? How many kilometres is this?

b If Amy wanted to swim 2.5 km, how many laps would she need to complete?

● Carpenters usually measure in millimetres. Give the following measurements in millimetres.

a a post 5.3 cm thick **b** a bench 60 cm wide **c** shelves 2.2 m long

● Copy each of the following, using =, < (less than) or > (greater than) to make each statement true.

a 8000 m ___ 8 km **b** 2 m ___ 100 cm **c** 6 m ___ 6000 mm **d** 40 m ___ 1 km

e 300 cm ___ 3 m **f** 5 km ___ 500 m **g** 1000 mm ___ 3 m **h** 150 m ___ 1 km

i 1 m ___ 500 mm **j** 2 m ___ 200 cm **k** 7 km ___ 7000 m **l** 90 cm ___ 1 m

Zena rides her bike 2.3 km to the shops. She then rides 600 m to the park where she stops to have a drink. After her drink, Zena visits her friend Nadia. Nadia's house is 1.9 km from the park. Zena then rides 2.8 km home. How far did Zena ride

a in kilometres? **b** in metres?

exercise 10.1 challenge

Consider these heights and depths.

a The height above sea level of Australia's highest mountain, Mt Kosciuszko, is 2228 m. Convert this to kilometres.

b The highest mountain in the world is Mt Everest whose summit is 8848 m above sea level. Approximately how many times higher is this than the height of Mt Kosciuszko?

c The deepest part of the ocean is the Mariana Trench to the east of the Philippines and north of New Guinea. The Mariana Trench has a depth of 11 032 m. Convert this to kilometres.

Millipedes stop train

Millipedes stopped a train in Japan as they covered a length of rail track after an enormous hatching. The millipedes, each measuring 3 cm to 6 cm, were spread along about 400 m of track. To prevent the train from skidding, railway staff had to sweep the millipedes off the rails before the train could continue its journey.

Using your ruler, carefully draw two line segments with lengths 3 cm and 6 cm.

a Assume the millipedes had an average length of 4 cm, and suppose they were head to tail along both rails. How many millipedes would there have been in the 400 m of track?

b Why do you think millipedes are called millipedes? Is it an accurate name for them?

Will is making some biscuit slices. He has mixed the biscuit dough and is now reading the instructions in the recipe book:

Roll the dough into a 28 cm square. Cut into 60 slices 7 cm by 2 cm and place on trays to cook.

If Will follows these instructions, will he really get 60 slices?

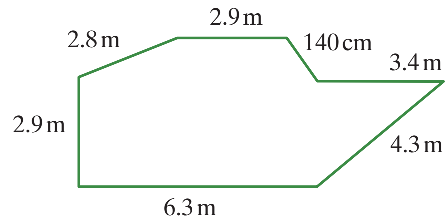
10.2 Perimeter

Class activity
Perimeter formulas

The distance around the outside of a plane (flat) figure is called the perimeter. To find the perimeter of a shape with straight sides, we add the lengths of each of the sides.

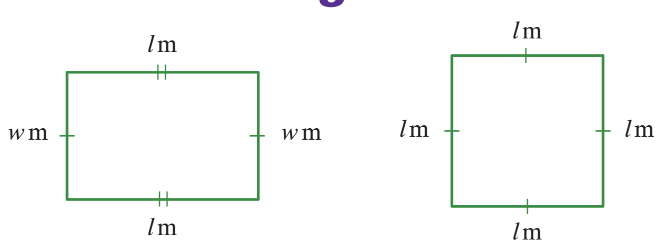
Example 7

Find the perimeter of the figure shown.



Working	Reasoning
140 cm = 1.4 m	All measures must be converted to the same units. 100 cm = 1 m
2.9 + 6.3 + 4.3 + 3.4 + 1.4 + 2.9 + 2.8 = 24.0 Perimeter = 24.0 m	Perimeter is the total distance around the shape.

Perimeter of a rectangle



Note: Sides marked | are equal in length and sides marked || are equal in length.

Adding the lengths of the sides, we obtain

$$\text{Perimeter } (P) = l + w + l + w.$$

This can be written in a simpler way as

$$P = 2l + 2w \text{ or } P = 2\,(l + w).$$

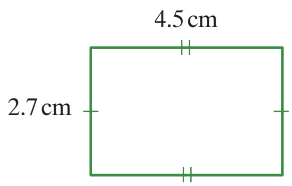
A square is a special rectangle in which the length and width are equal.

The perimeter of a square is given by $P = 4l$.

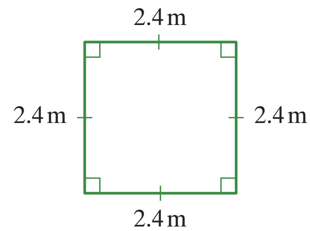
Example 8

Find the perimeter of

a this rectangle.



b this square.



Working

a $P = 2\,(l + w)$
$= 2 \quad (2.7 + 4.5)$
$= 2 \quad 7.2$
$= 14.4$ cm
The perimeter is 14.4 cm.

Reasoning

The perimeter of a rectangle is twice the sum of the length and width.

b $P = 4\,l$
$= 4 \times 2.4$ m
$= 9.6$ m
The perimeter is 9.6 m.

The perimeter of a square is four times the length of one side.

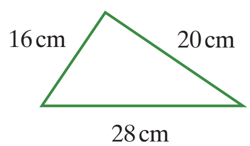
Perimeter of other shapes

When finding the perimeter of other shapes, we add the lengths of all the sides. But sometimes if there are two or more sides with the same length, we can shortcut the calculation.

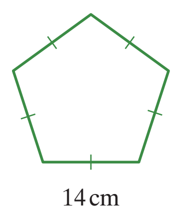
Example 9

Find the perimeter of each of these shapes.

a



b



Working

a Perimeter $= 16 + 20 + 28$ cm
$= 64$ cm

The perimeter is 64 cm.

Reasoning

Add the three side lengths.

continued

Example 9 continued

Working

b Perimeter $= 5 \times 14 = 70$ m

The perimeter is 70 m.

Reasoning

The five sides are equal in length, so multiply 14 m by 5.

Using perimeter to find lengths

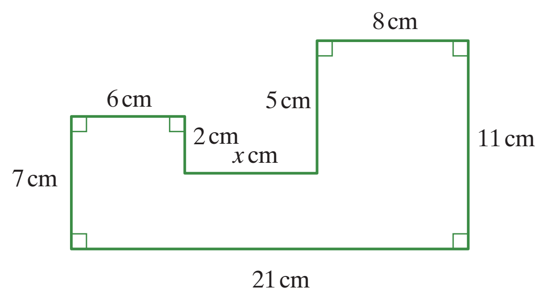
If we know the perimeter of a shape we can often work backwards to find a missing side length.

Example 10

Consider the shape below.

a Find the unknown side length marked x cm.

b Find the perimeter.



Working

a The width of the shape is 21 cm.

$$6 + x + 8 = 21$$
$$14 + x = 21$$
$$x = 7$$

The unknown length is 7 cm.

Reasoning

The three sides marked 6 cm, x cm and 8 cm must add to the total width of the shape, 21 cm.

Working

b $P = 6 + 2 + 7 + 5 + 8 + 11 + 21 + 7$

$P = 67$

The perimeter is 67 cm.

Reasoning

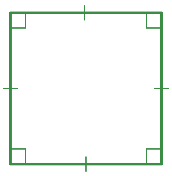
Add the lengths of all the sides. Start at one side and go clockwise or anticlockwise around the shape so no sides are missed.

10.2

Example 11

A square has a perimeter of 65.2 m. Find the length of the sides.

Working



$P = 4l$

$l = P \quad 4$

$l = 65.2 \quad 4$

$= 16.3\,\text{m}$

The length of each side is 16.3 m.

Reasoning

Draw a diagram.

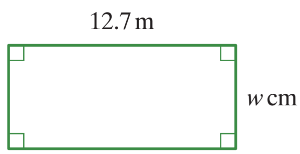
The four sides of a square are equal in length.

Example 12

A rectangular garden has a perimeter of 34.8 m. If the length of the garden is 12.7 m, what is the width of the garden?

Working

Let the width of the garden be w metres.



$$2(l + w) = P$$

$$2 \quad (12.7 + w) = 34.8$$

$$\frac{2 \times (12.7 + w)}{2} = \frac{34.8}{2}$$

$$12.7 + w = 17.4$$

$$12.7 - 12.7 + w = 17.4 - 12.7$$

$$w = 4.7\,\text{m}$$

The width of the garden is 4.7 metres.

Reasoning

Drawing a diagram helps us to see the given information and what we have been asked to find.

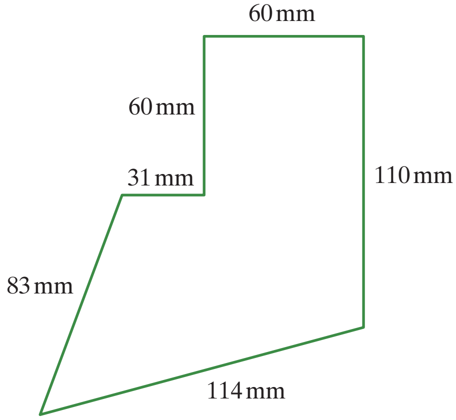
The perimeter of a rectangle is twice the sum of the length and width.
Solve the equation to find w.
Divide both sides of the equation by 2.
Subtract 12.7 from both sides.

exercise 10.2

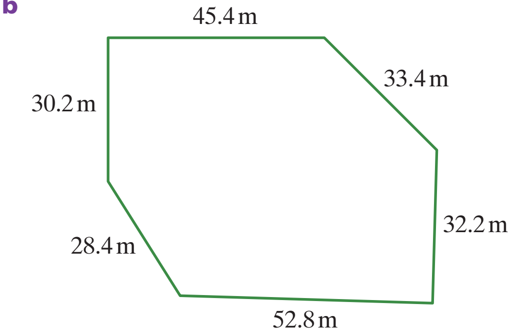
LINKS TO Example 7

Calculate the perimeter of each of these shapes.

a



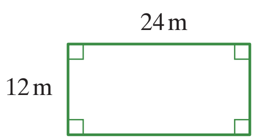
b



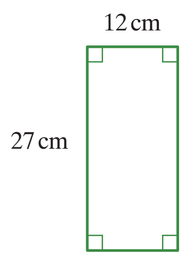
LINKS TO Example 8

Find the perimeter of each of these shapes.

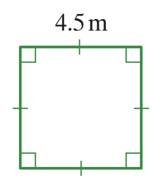
a



b



c



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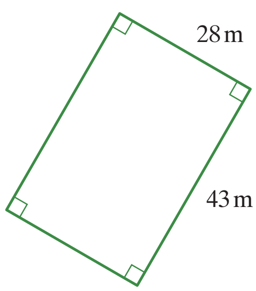
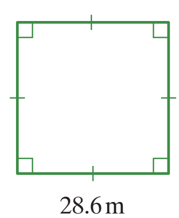


e



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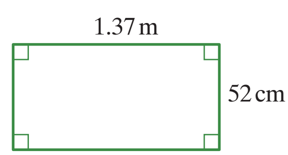

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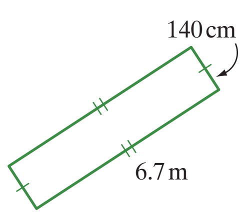

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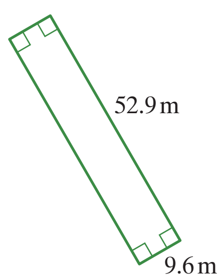

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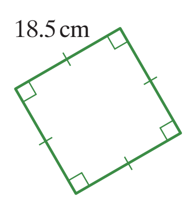

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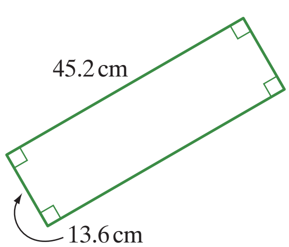

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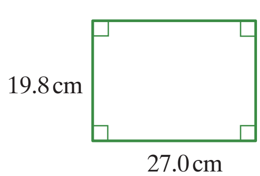
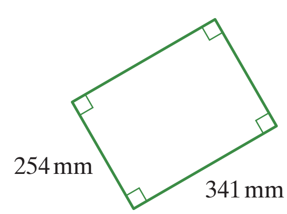
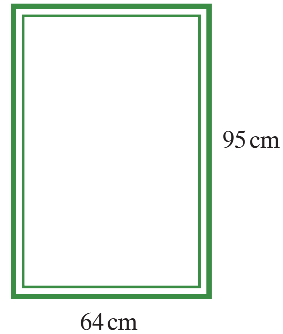


l



Maria, Steve and Anna are finding perimeters.

a Maria is replacing the door seal around her refrigerator door. The strip of seal must go right around the edge of the door shown. What length of door seal does she need?



b Steve is putting a border of green plastic edging around a rectangular garden which measures 280 cm by 180 cm. How many metres of edging will he need?

c Anna is making a rectangular tablecloth and is sewing tape around the edge. If the cloth is 172 cm by 126 cm, how many metres of tape will Anna need?

A square room has walls of length 4.2 m.

a Draw a labelled diagram to show the square shape and the length of the walls.

b What is the perimeter of the room?

A vegetable garden is a rectangular shape with sides of length 3.5 m and 1.8 m.

a Draw a labelled diagram to show the rectangle shape and the length of the sides.

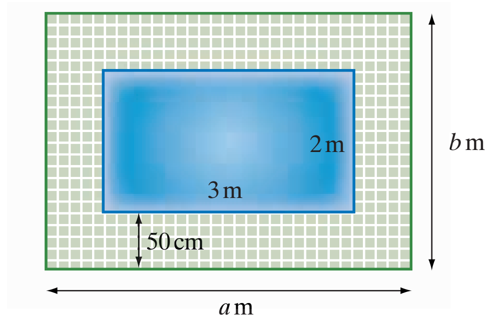
b What length of edging would be needed to make a border around the garden?

A rectangular fishpond measures 3 m by 2 m. There is a 50 cm wide path all around it.

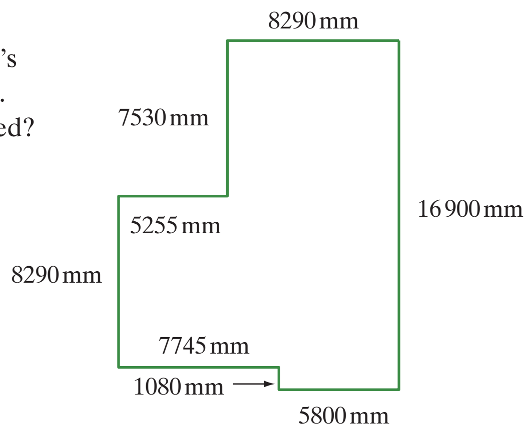
a What is the perimeter of the fishpond?

b What are the length and width of the outside edges of the path (labelled a m and b m)?

c What is the perimeter of the outside of the path?



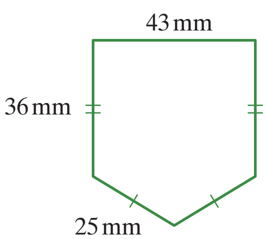
A house has guttering (spouting) around the entire perimeter of the roof. The builder's plan shows all the dimensions in millimetres. How many metres of guttering will be needed? Give your answer to the nearest metre.



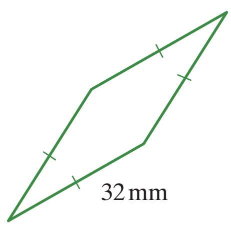
LINKS TO Example 9

Find the perimeter of each of these shapes.

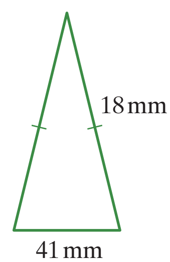
a



b



c



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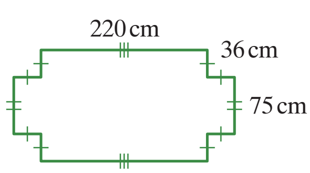
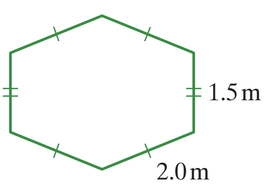


e



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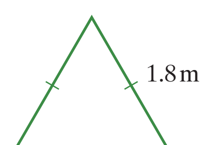

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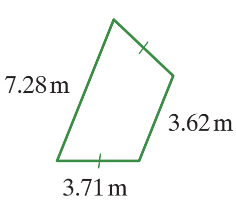

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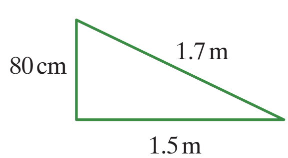
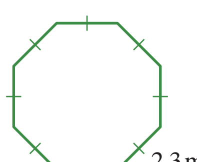


i



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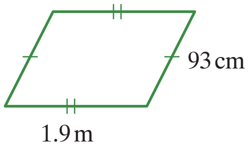

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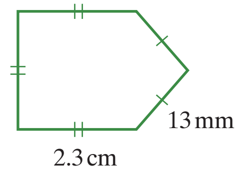
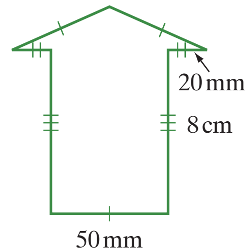


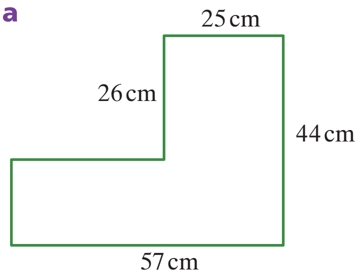
l



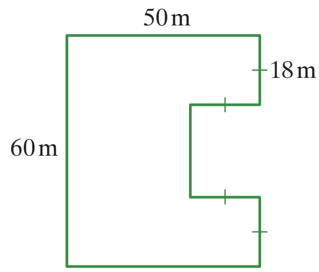
LINKS TO Example 10

Find the perimeter of each of these shapes. You may need to determine missing lengths first.

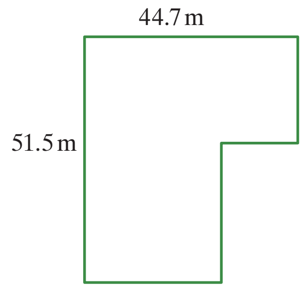
a



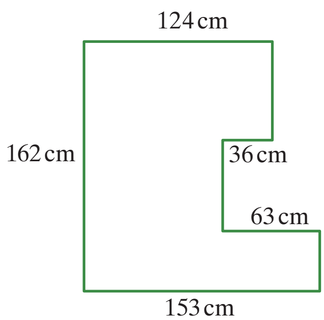
b



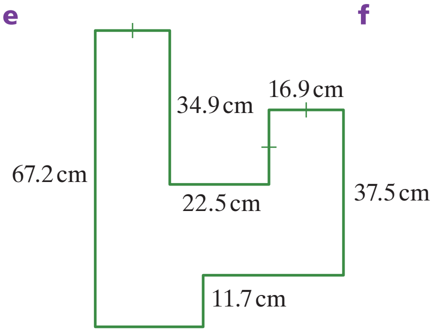
c



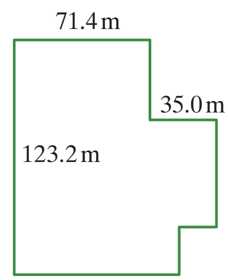
d



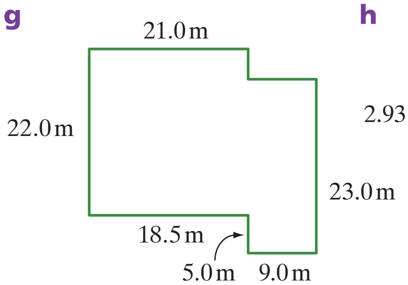
e



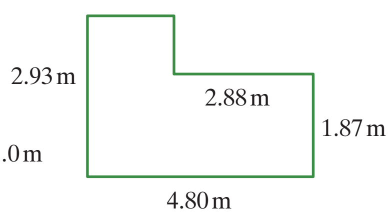
f



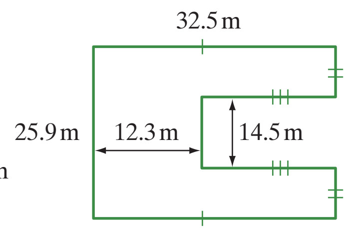
g



h

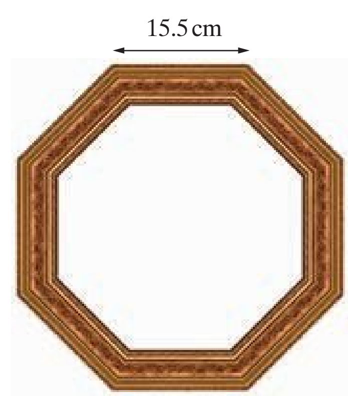


i



Geri is making a picture frame in the shape of a regular octagon. She first needs to cut eight equal pieces of framing strip. What total length of framing strip will she need?

A regular polygon has all its sides of equal length.

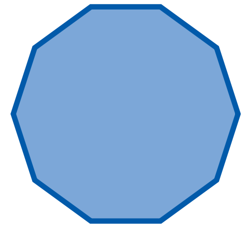


A border is to be placed around this ornamental pool which is in the shape of a regular decagon. What length of border is required? Give your answer

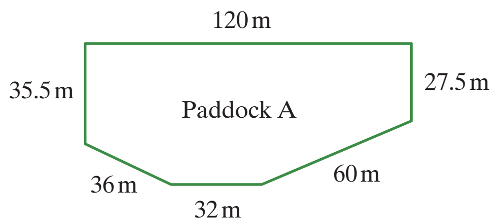
a in centimetres.

b in metres.

112 cm



Bruce is replacing some of his farm fences after a bushfire burnt through part of his farm. Paddock A has the shape and dimensions shown below and paddock B is a rectangle with length 150 m and width 45 m. Each paddock has a two-metre wide gate, which, of course, the fencing will not go across. Assume that the paddocks are not adjoining, that is, they don't share a fence.

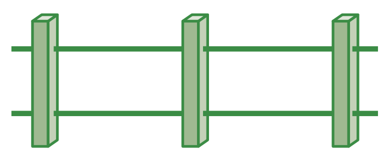


a Calculate the length of fencing required for paddock A. (Don't forget to allow for the gateway.)

b Draw a diagram of paddock B, labelling the dimensions.

c Calculate the length of fencing required for paddock B.

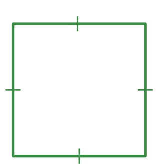
d The fencing has two rows of wire as shown. Calculate the total length of wire Bruce will need for the two paddocks.



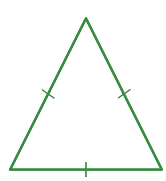
LINKS TO Example 11

Find the side length of each of the following shapes.

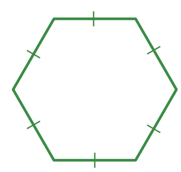
a Perimeter = 28 cm



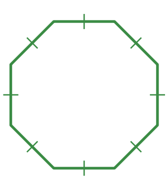
b Perimeter = 105 cm



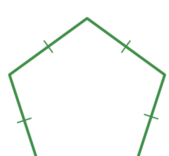
c Perimeter = 57 m



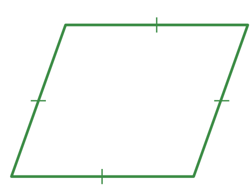
d Perimeter = 96 m



e Perimeter = 106 mm



f Perimeter = 106 mm

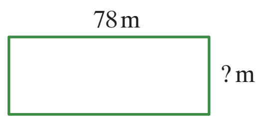


10.2

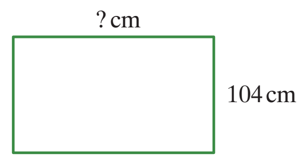
LINKS TO Example 12

Find the missing side length in each of these rectangles.

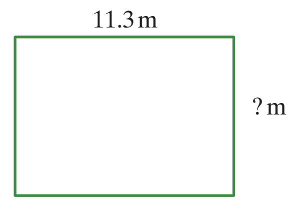
a Perimeter = 214 m



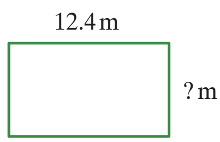
b Perimeter = 586 cm



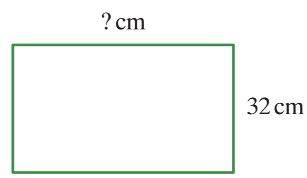
c Perimeter = 38.8 m



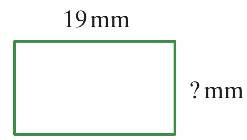
d Perimeter = 39.6 m



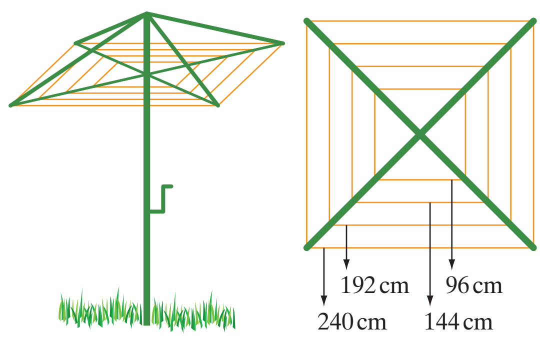
e Perimeter = 1.6 m



f Perimeter = 6.0 cm



Chris is replacing the plastic cord on a clothes line. There are 4 squares of cord as shown by the orange lines on the diagram. Chris will need an extra 15 cm for each of the 4 squares for tying the ends together. What is the total length of plastic cord needed? Give your answer in metres.

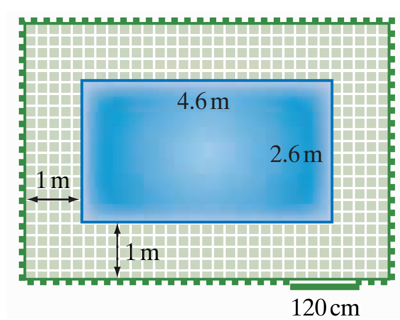


A swimming pool is 4.6 m long and 2.6 m wide, with a 1 m wide path along each edge of the pool as shown in the diagram on the right.

a What is the perimeter of the pool?

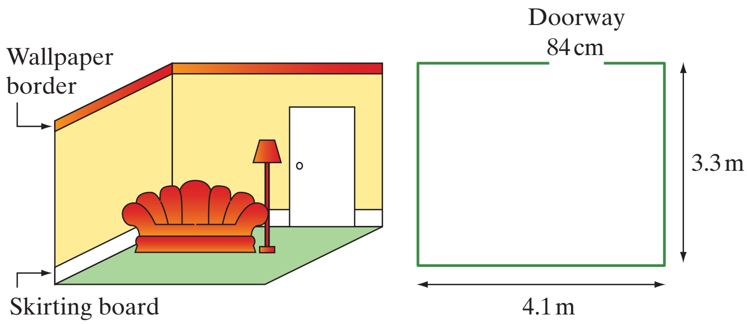
b A safety fence is constructed around the outside of the path as shown in the diagram. What are the length and width of the fence?

c There is a 120 cm gate that, of course, does not have fencing across it. What is the total length of the fence, excluding the gate?



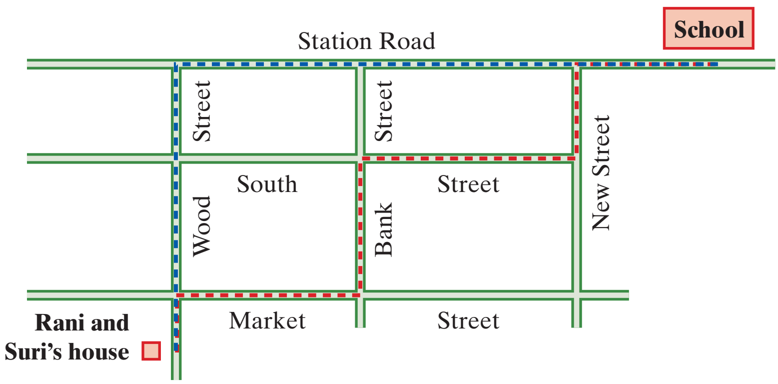
Meg is decorating her living room.

a She is putting a wallpaper border around the top of the four walls just below the ceiling. How many metres of wallpaper border will she need?



b Meg is replacing the skirting board (the white board at the bottom of the walls) around the four sides of the room. How many metres of skirting board are needed? (The skirting board doesn't go across the doorway, of course!)

Rani and Suri live in Wood Street and walk to their school, which is in Station Road. Rani likes to walk along the route shown in red—along Wood Street, turning right into Market Street, left into Bank Street, right into South Street, and then left into New Street until she reaches Station Road. Suri says it is much shorter to go straight along Wood Street and then turn left into Station Road (the route shown in blue). What do you think and why?



exercise 10.2 challenge

Sam is planting a border of daisy plants around the edge of a rectangular garden which has dimensions 2.4 m by 1.6 m.

a What is the perimeter of the garden?

b If Sam starts at one corner and plants the daisies 16 cm apart, how many daisy plants will he need?

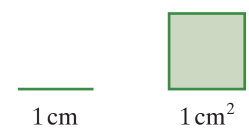
10.3 Units of area

The **area** of a plane (two-dimensional) shape is the amount of space enclosed by its boundary. We can also think of area as the amount of surface that a shape covers.

For example, we can think of the area of the whiteboard in the classroom as the amount of space enclosed by the edges of the whiteboard, or we can think of the area as the amount of wall covered by the whiteboard.

Units of area

Area is measured in square units; for example, square centimetres (cm^2), square metres (m^2), square kilometres (km^2).



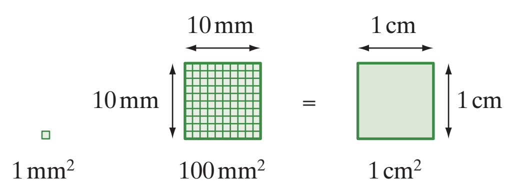
The superscript 2, as in cm^2, reminds us that there are 2 dimensions: length and width.

The relationships between square millimetres, square centimetres and square kilometres are shown below.

$$100\,mm^2 = 1\,cm^2$$

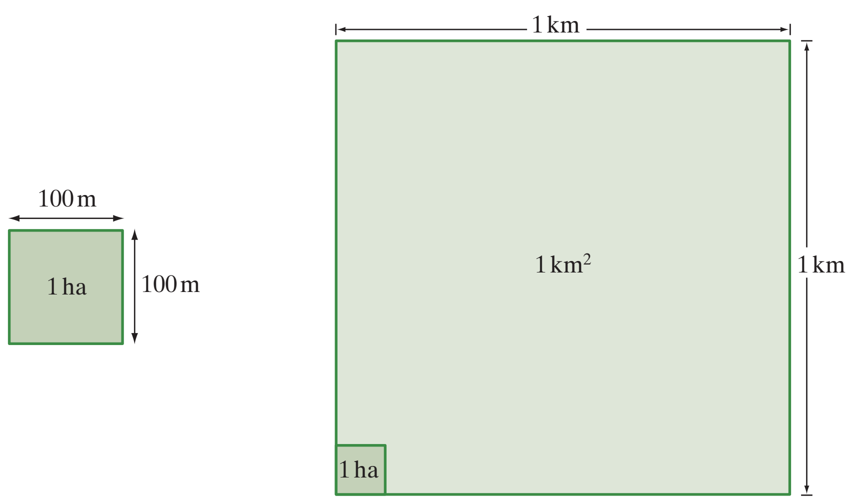
$$10\,000\,cm^2 = 1\,m^2$$

$$1\,000\,000\,m^2 = 1\,km^2$$



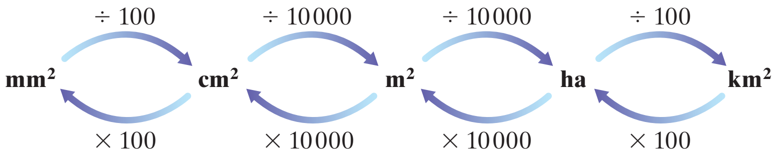
Areas of farms and parks are often measured in hectares. The symbol for hectare is ha. A square paddock 100 m by 100 m would have an area of one hectare.

$1\,ha = 10\,000\,m^2$



$100\,ha = 1\,km^2$

Converting units of area



Example 13

Convert the following.

a $0.85\,m^2$ into square centimetres

b $0.54\,km^2$ into square metres

Working

a $0.85\,m^2 = 0.85 \times 100 \times 100\,cm^2$
$= 8500\,cm^2$

b $0.54\,km^2 = 0.54 \times 1000 \times 1000\,m^2$
$= 540\,000\,m^2$

Reasoning

$1\,m = 100\,cm$
$1\,m^2 = 100 \times 100\,cm^2$

$1\,km = 1000\,m$
$1\,km^2 = 1000 \times 1000\,m^2$

Example 14

Convert the following.

a $36\,000\,m^2$ into square kilometres

b $180\,mm^2$ into square centimetres

Working

a $36\,000\,m^2 = 36\,000 \div 1000 \div 1000\,km^2$
$= 0.036\,km^2$

b $180\,mm^2 = 180 \div 10 \div 10\,cm^2$
$= 1.8\,cm^2$

Reasoning

$1000\,m = 1\,km$
$1000 \times 1000\,m^2 = 1\,km^2$

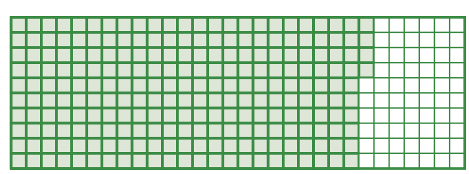
$10\,mm = 1\,cm$
$10 \times 10\,mm^2 = 1\,cm^2$

Example 15

Find the area of the shaded region in

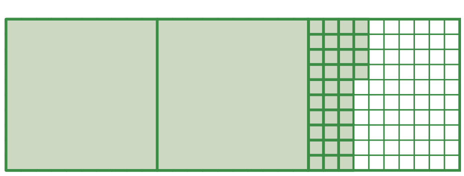
a square millimetres.

b square centimetres.



Working

a The area of the shaded region is $234\,mm^2$.



Reasoning

Two whole square centimetres $= 200\,mm^2$.

Another $34\,mm^2$ makes a total area of $234\,mm^2$.

continued

Example 15 continued

Working

b The area of the shaded region is $2.34\,cm^2$.

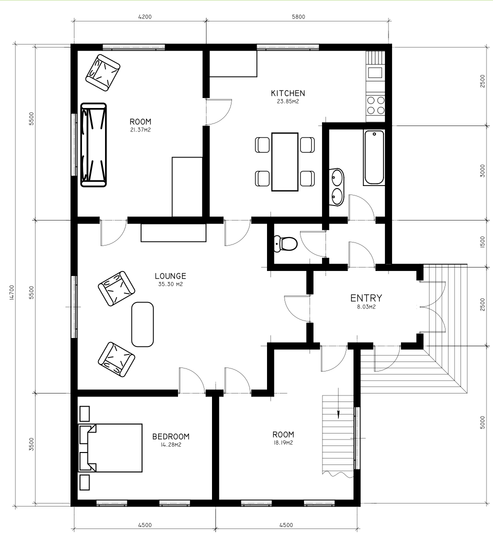
Reasoning

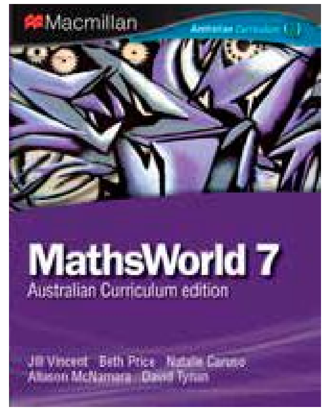
There are 2 whole square centimetres and 34 hundredths of a square centimetre. Alternatively, since each millimetre is one-tenth of a centimetre, to convert mm^2 to cm^2 we must divide by 10 and then by 10 again; that is, we divide by 100.

234 10 10
= 234 100
so area = $2.34\,cm^2$

Choosing suitable area units

The area unit we use depends on what we are measuring. Some examples are shown here.

Simpson Desert	Square kilometres	
Floor plan of a house	Square metres	

Cover of a book	Square centimetres	
Area of a bee's wing	Square millimetres	

exercise 10.3

LINKS TO Example 18

Convert each of these measurements into the unit shown in brackets. In each case you are converting from a larger unit (e.g., km^2) to a smaller unit (e.g., m^2).

a $2.5\,km^2$ (m^2) **b** $0.5\,m^2$ (cm^2) **c** $18\,cm^2$ (mm^2) **d** $0.56\,km^2$ (m^2)
e $0.08\,m^2$ (mm^2) **f** $240\,cm^2$ (mm^2) **g** $23.8\,m^2$ (cm^2) **h** $1.4\,km^2$ (m^2)
i $1.3\,m^2$ (mm^2) **j** $24\,m^2$ (cm^2) **k** $0.8\,m^2$ (cm^2) **l** $0.02\,km^2$ (cm^2)

An area of $0.054\,m^2$ is equal to

A $0.54\,cm^2$ **B** $5.4\,cm^2$ **C** $54\,cm^2$ **D** $540\,cm^2$ **E** $5400\,cm^2$

An area of $18\,000\,cm^2$ is equal to

A $1.8\,mm^2$
B $18\,mm^2$
C $180\,mm^2$
D $1\,800\,000\,mm^2$
E $18\,000\,000\,mm^2$

LINKS TO Example 14

Convert each of these measurements into the unit shown in brackets. In each case you are converting from a smaller unit (e.g., mm^2) to a larger unit (e.g., cm^2).

a $724\,mm^2$ (cm^2) **b** $4\,500\,000\,m^2$ (km^2) **c** $15\,000\,cm^2$ (m^2)
d $3200\,mm^2$ (cm^2) **e** $25\,mm^2$ (cm^2) **f** $580\,000\,mm^2$ (m^2)
g $16\,000\,mm^2$ (cm^2) **h** $140\,000\,m^2$ (km^2) **i** $2400\,m^2$ (km^2)
j $18.4\,mm^2$ (cm^2) **k** $6500\,mm^2$ (cm^2) **l** $180\,000\,000\,m^2$ (km^2)

An area of 350 cm^2 is equal to

A 0.035 m^2 **B** 3.5 m^2 **C** 35 m^2 **D** 3500 m^2 **E** 3 500 000 mm^2

An area of 75 000 000 mm^2 is equal to

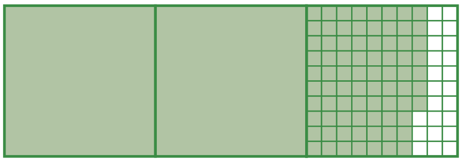
A 75 m^2 **B** 7500 m^2 **C** 750 000 cm^2 **D** 75 000 m^2 **E** 75 cm^2

LINKS TO Example 15

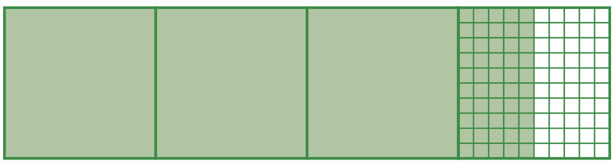
Find the area of each of the shaded regions in

i square millimetres. **ii** square centimetres.

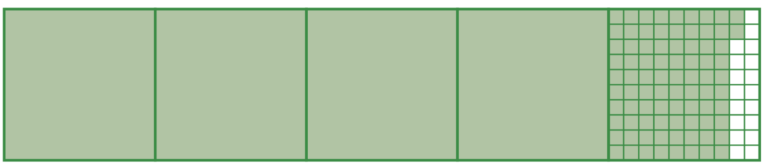
a



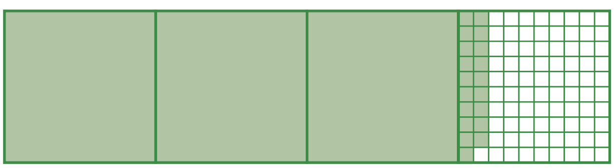
b



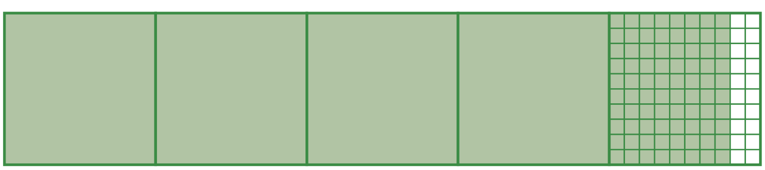
c



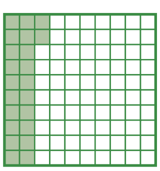
d



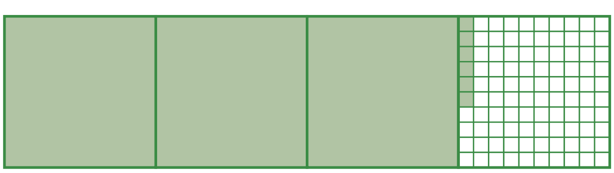
e



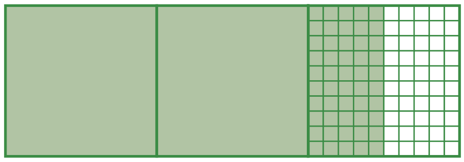
f



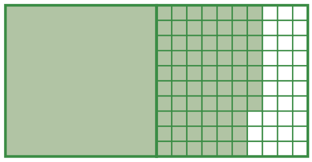
g



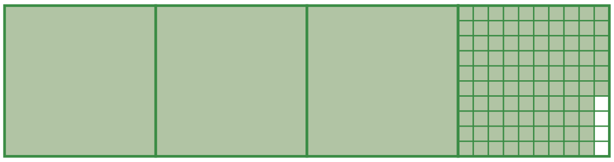
h



i



j



Suggest the most suitable unit for measuring each of the following areas.

a a classroom floor

b a letter envelope

c the finger nail on your little finger

d the area of South Australia

e a garage door

f the front of a cornflakes packet

A factory produces 80 million square metres per year of aluminium foil. How many square kilometres is this?

Tom reads that each seedling he is planting needs an area of 150 square centimetres of garden. Tom has 80 seedlings to plant. What is the total area of garden the seedlings will need? Give the area in

a square centimetres.

b square metres.

exercise 10.3 challenge

The area of Australia is approximately 7 700 000 square kilometres. The population of Australia is about 22 million people.

a If the area of Australia was divided equally between these 22 million people, how much area would each person have?

b How many square metres is this?

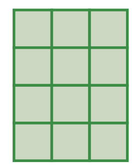
10.4 Area: rectangles

Finding area by counting squares

One way to find the area of a shape is to count the number of square centimetres that it covers. We can count whole squares then estimate to how many whole squares the part squares are equal.

Example 16

The following rectangle has been divided into 1 cm squares.
What is the area of the rectangle?



Working

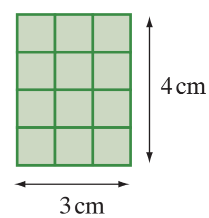
Four rows of three 1 cm squares.

$$\text{Area} = 4 \quad 3$$
$$= 12\,\text{cm}^2$$

Reasoning

Area is the number of square units enclosed by the shape.

In the example above, it can be seen that the length of the rectangle is 4 cm and the width is 3 cm.



So a shortcut method for finding the area of a rectangle is to multiply the length by the width. We must make sure, of course, that the length and width are in the same units.

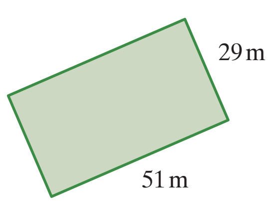
The area of a rectangle is given by $A = l \times w$ where l is the length and w is the width. The length and width must be in the same unit.

The area of a square is given by $A = l^2$ where l is the length of each side.

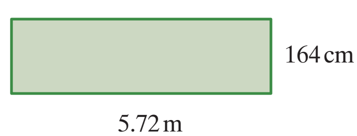
Example 17

Find the area of these rectangles. In part b give your answer in square metres correct to 2 decimal places.

a



b



continued

Example 17 continued

Working

a

$$A = lw$$
$$A = 29 \quad 51$$
$$\text{Area} = 1479\,\text{m}^2$$

Reasoning

The area of a rectangle may be calculated by multiplying the length by the width.

Working

b

$$164\,\text{cm} = 1.64\,\text{m}$$
$$A = l \quad w$$
$$= 5.72 \quad 1.64$$
$$= 9.3808\,\text{m}^2$$
$$\text{Area} = 9.38\,\text{m}^2 \text{ to 2 decimal places}$$

Reasoning

Convert length and width to the same units.

Round to 2 decimal places.

Example 18

Find the area of a square with sides of length 4.5 cm.

Working

$$A = l^2$$
$$= 4.5^2$$
$$\text{Area} = 20.25\,\text{cm}^2$$

Reasoning

Length and width are equal for a square, so area is $4.5 \times 4.5\,\text{cm}^2$.

If we know the area of a shape we can work backwards to find a missing dimension.

Example 19

Find the following.

a The area of a rectangle is $24\,\text{m}^2$. If the length of the rectangle is 6 m, what is the width?

b The area of a square is $64\,\text{cm}^2$. What is the length of each side?

Working

a

$$A = l \quad w$$
$$24 = 6 \quad w$$
$$w = \frac{24}{6}$$
$$= 4$$

The width is 4 m.

Reasoning

Width = area length

Working

b

$$A = l^2$$
$$64 = l^2$$
$$l = \sqrt{64}$$
$$l = 8$$

The length of each side is 8 cm.

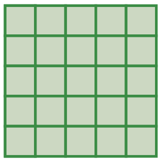
Reasoning

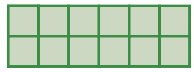
The length of each side is the square root of the area.

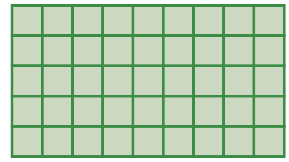
exercise 10.4

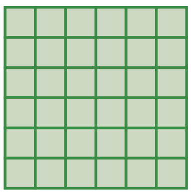
LINKS TO Example 16

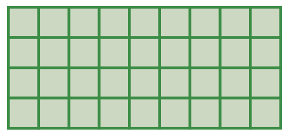
Find the area of each of the following rectangles if each small square is 1 square centimetre.

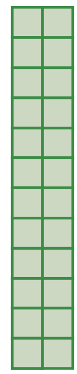
a


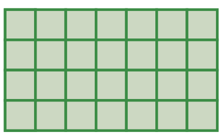
b


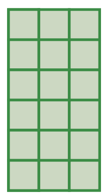
c


d


e


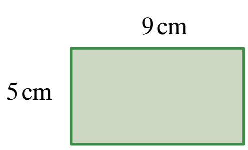
f


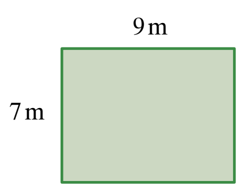
g


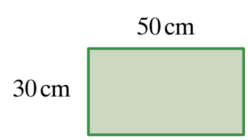
h


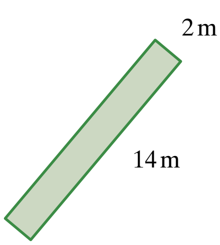
LINKS TO Examples 17, 18

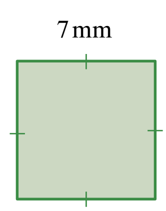
Calculate the area of each of these rectangles. In part i convert the length and width to the same unit.

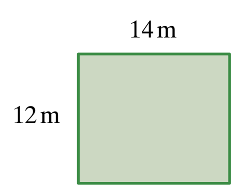
a


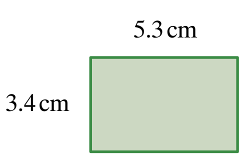
b


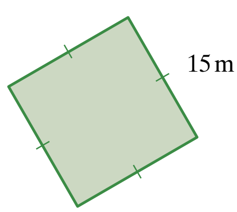
c


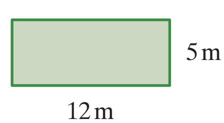
d


e


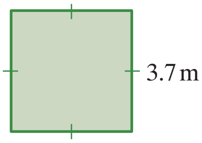
f


g


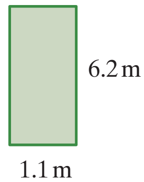
h


i


j



k

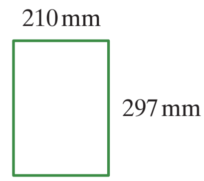


l

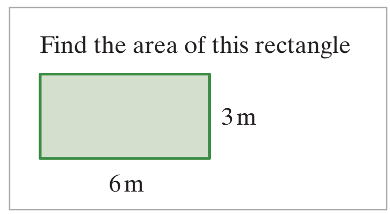
A4 paper measures 210 mm by 297 mm.

a Calculate the area of a sheet of A4 paper in square millimetres.

b Convert the length and width to centimetres, then calculate the area in square centimetres.



The following question was on a Year 7 test.



Here are two students' answers:

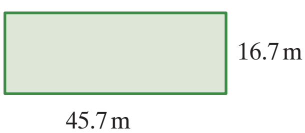
Jay's answer

Kay's answer

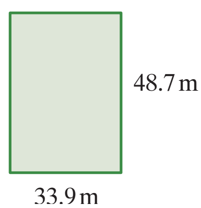
If the maximum mark for the question was 2, what mark would you give to Jay and to Kay? Justify your answer.

An estate agent is advertising two rectangular blocks of land for sale. The advertisements include the area of each block. Calculate the area of each of the blocks to the nearest square metre.

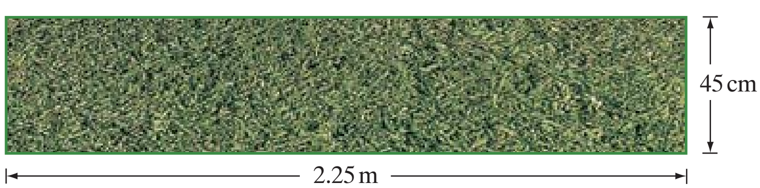
a



b



Instant lawn is sold as rectangular strips 2.25 m by 45 cm which are rolled up so they are easy to store and deliver.



a Calculate the area of a strip of instant lawn. Give your answer in square metres correct to one decimal place.

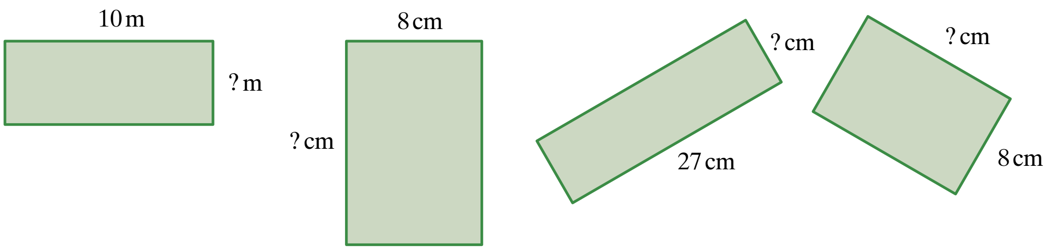
b Leni wants to buy instant lawn for part of her garden that measures 15 m by 8 m. What is the area of this part of Leni's garden?

c Using your answer to part a, how many strips of instant lawn would Leni need?

LINKS TO Example 19

For each of these rectangles you are given the area and one of the dimensions (length or width). Find the other dimension.

a Area = $40\,m^2$ **b** Area = $96\,cm^2$ **c** Area = $216\,cm^2$ **d** Area = $244\,cm^2$

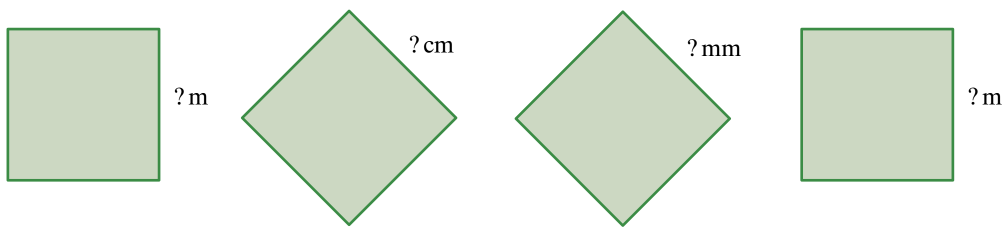


e Area = $864\,cm^2$, width = 24 cm
f Area = $486\,m^2$, length = 18 m
g Area = $2250\,mm^2$, length = 50 mm
h Area = $391\,km^2$, width = 17 km

LINKS TO Example 19

For each of these squares you are given the area. Find the length of the sides.

a Area = $81\,m^2$ **b** Area = $289\,cm^2$ **c** Area = $4900\,mm^2$ **d** Area = $1024\,m^2$



e Area = $484\,m^2$
f Area = $1369\,mm^2$
g Area = $7\,840\,000\,km^2$
h Area = $30625\,mm^2$

The area of a rectangular table top is $4.32\,m^2$ and the width is 1.2 m. How long is the table?

A farm is in the shape of a rectangle. The area is $0.56\,km^2$ and the length is 0.8 km. What is the width of the farm

a in kilometres? **b** in metres?

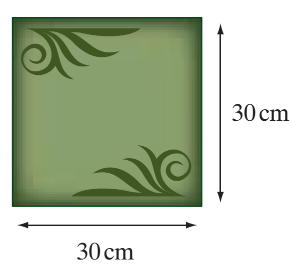
The area of a square garden is $12.25\,m^2$.
How long are the sides of the garden?

exercise 10.4 challenge

The area of a kitchen floor is $8.91\,m^2$.

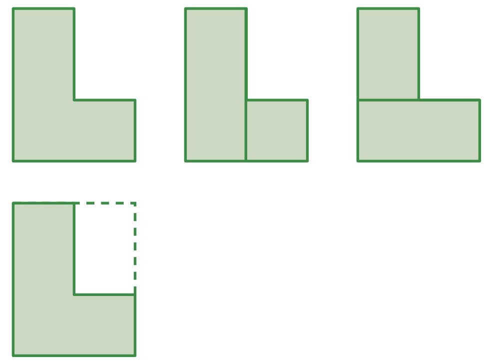
a If the length of the kitchen is 3.3 m, find the width.

b If the floor of the kitchen is to be covered with square tiles whose sides are 30 cm long, how many tiles would be needed?



10.5 Adding and subtracting rectangle areas

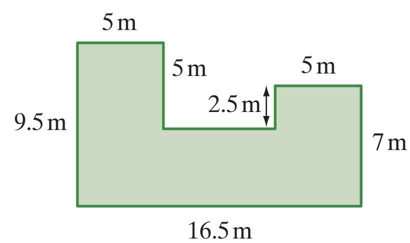
Many everyday shapes are based on rectangles. The floor plans of houses, for example, are sometimes L-shaped. The L-shape shown here can be divided into two rectangles in two different ways. In each case we find the total area by adding the areas of the two rectangles.



Instead of thinking of the shape as two rectangles added together, we could think of it as a rectangle with another rectangle cut out of one corner. We would then subtract the area of the cut-out rectangle from the larger rectangle.

Example 20

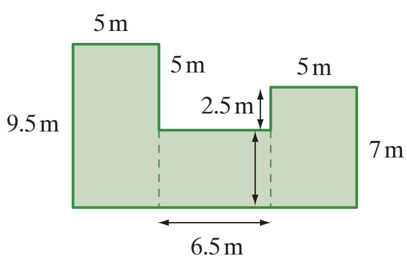
Find the area covered by this house.



Working

Dimensions of rectangle B:

$$16.5 - (5 + 5) = 6.5\,\text{m}$$
$$7 - 2.5 = 4.5\,\text{m}$$



Reasoning

The shape can be divided into three rectangles, A, B and C.

From the given information the dimensions of rectangle B can be found.

continued

Example 20 continued

Working

Area of rectangle A $= 5 \quad 9.5$
$= 47.5\,\text{m}^2$
Area of rectangle B $= 4.5 \quad 6.5$
$= 29.25\,\text{m}^2$
Area of rectangle C $= 5 \quad 7$
$= 35\,\text{m}^2$

Total area $= 47.5 + 29.25 + 35$
$= 111.75\,\text{m}^2$

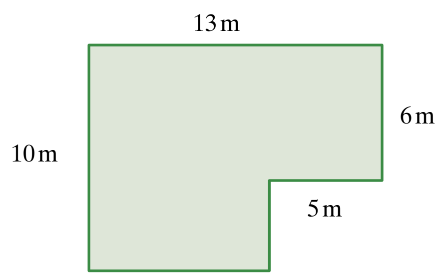
Reasoning

Calculate the area of each rectangle.

Add the three areas: A + B + C.

Example 21

Calculate the area of this shape.

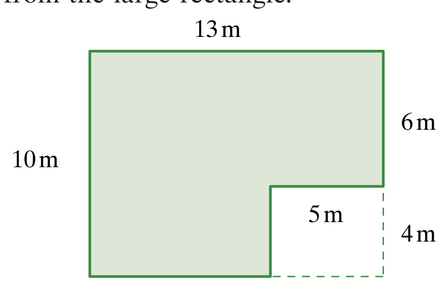


Working

$A = 13 \times 10 - 5 \times 4$
$= 130 - 20$
$A = 110$
The area is $110\,\text{m}^2$.

Reasoning

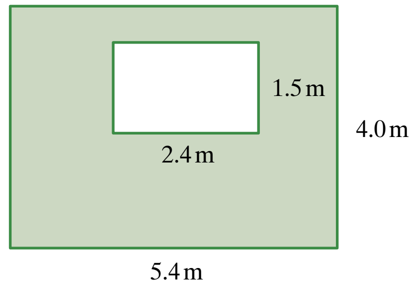
Subtract the area of the cut-out rectangle from the large rectangle.



Sometimes the required area is an area surrounding other areas that are not required—one or more regions appear as 'holes' within an outer region. In these cases we subtract the 'holes' from the outer region.

Example 22

Find the area of the shaded region.



Working

Area of large rectangle $= 4.0 \times 5.4\ \text{m}^2$
$= 21.6\ \text{m}^2$

Area of small rectangle $= 1.5 \times 2.4\ \text{m}^2$
$= 3.6\ \text{m}^2$

Shaded area $= 21.6 - 3.6\ \text{m}^2$
$= 18.0\ \text{m}^2$

The shaded area is $18.0\ \text{m}^2$.

Reasoning

Area of rectangle = length × width

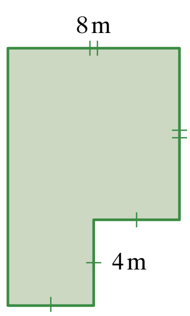
Shaded area
= area of the large rectangle − unshaded area

exercise 10.5

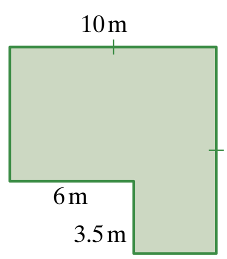
LINKS TO Examples 20, 21

1 Each of the following shapes can be divided into two or more rectangles in more than one way. Choose a way of dividing each shape and calculate the area.

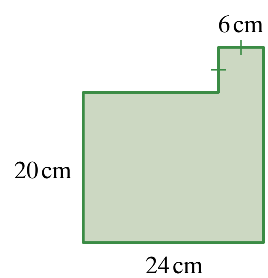
a



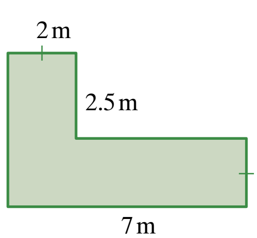
b



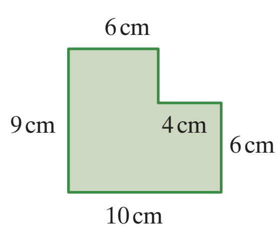
c



d



e



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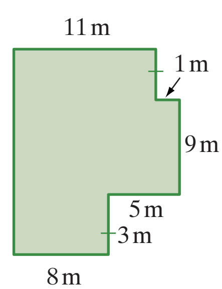
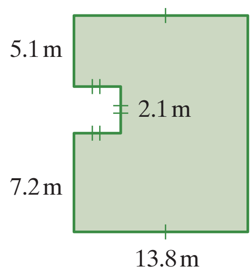
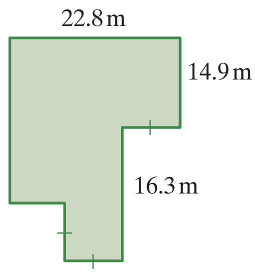


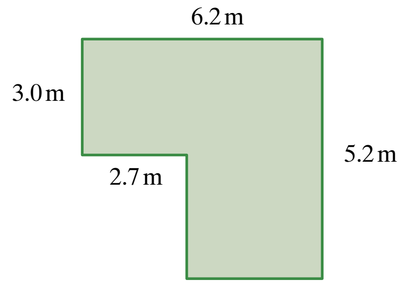
g



h

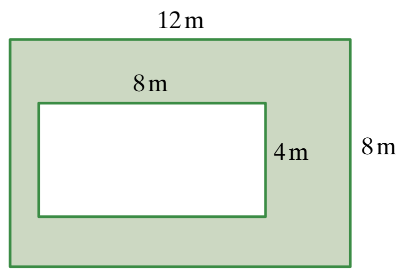


A flat has an L-shaped living room as shown at right. What is the area of the room?



Calculate the shaded area in each of these shapes.

a



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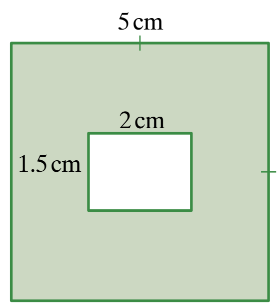

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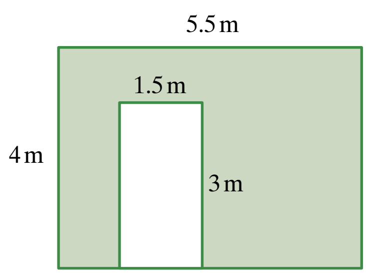

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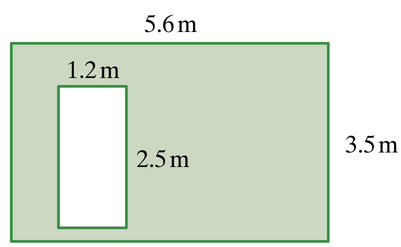

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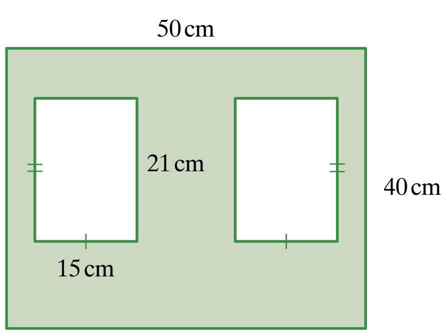

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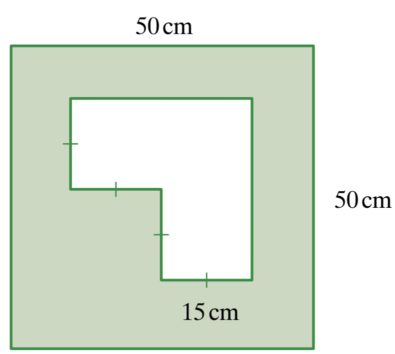
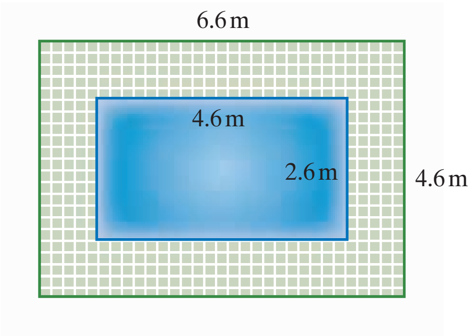


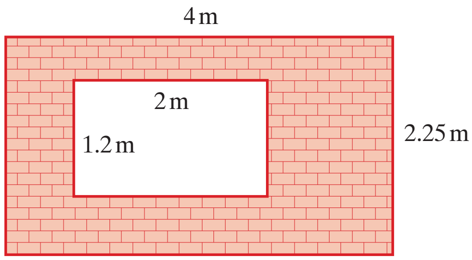
LINKS TO Example 22

Calculate the area of the path around the swimming pool below.



Consider the wall below.

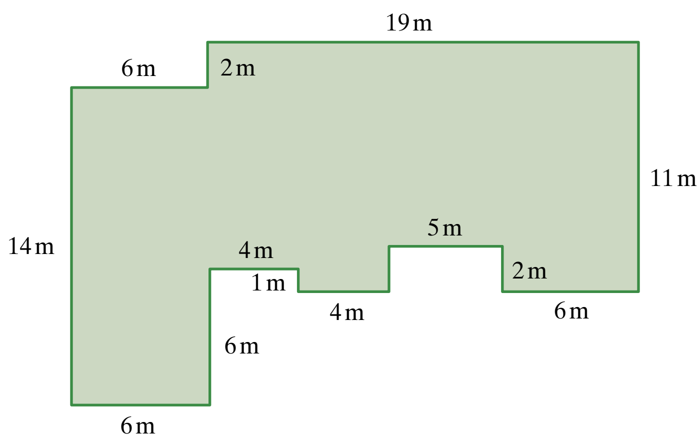
a Calculate the area covered by bricks.



b If there were approximately 50 bricks per square metre, about how many bricks would be needed to build this wall?

exercise 10.5 challenge

The diagram shows the plan of a house. Calculate the area covered by the house.



10.6 Area: parallelograms and triangles

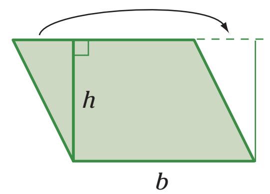
Parallelograms

Parallelogram area

Parallelogram area

Class activity
Parallelogram area

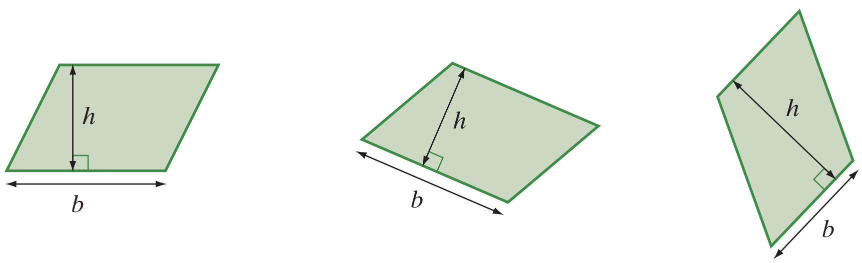
This diagram shows that we could move a triangle from one end of the parallelogram and place it at the other end to make a rectangle. So the parallelogram must have the same area as the rectangle. We know that the area of the rectangle is length × width, that is, base × perpendicular height.



Area of parallelogram = base × perpendicular height

$$A = b \times h$$

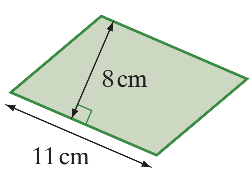
The base and height must be in the same unit.



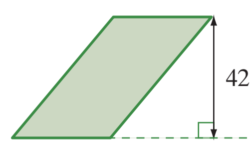
Example 23

Find the area of these parallelograms.

a



b



Working

a $A = b \quad h$

$= 11 \quad 8$

$= 88\,\text{cm}^2$

b $A = b \times h$

$= 35 \times 42$

$= 1470\,\text{cm}^2$

Reasoning

Base = 11 cm, height = 8 cm because the 8 cm measurement is perpendicular to the base.

Base = 35 cm

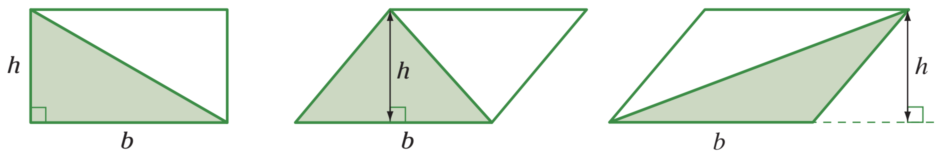
Perpendicular height = 42 cm

Area of a triangle

Area of a triangle

Triangles

We can think of a triangle as half a rectangle or half a parallelogram.



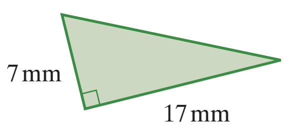
$$\text{Area of parallelogram} = \text{base} \quad \text{height}$$
$$= b \quad h$$

$$\text{Area of triangle} = \frac{1}{2}(\text{base} \quad \text{height})$$
$$= \frac{1}{2}(b \quad h) \text{ or } \frac{bh}{2}$$

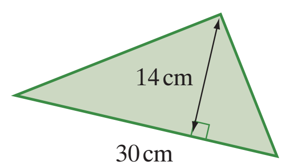
Example 24

Find the area of each of these triangles.

a



b



Working

a $\text{Area} = \frac{1}{2}(\text{base} \quad \text{height})$

$= \frac{1}{2}(17 \quad 7)$

$= 59.5\,\text{mm}^2$

b $\text{Area} = \frac{1}{2}(\text{base} \quad \text{height})$

$= \frac{1}{2}(30 \quad 14)$

$= 210\,\text{cm}^2$

Reasoning

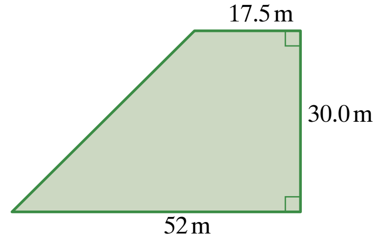
Base = 17 mm, height = 7 mm

Base = 30 cm, height = 14 cm

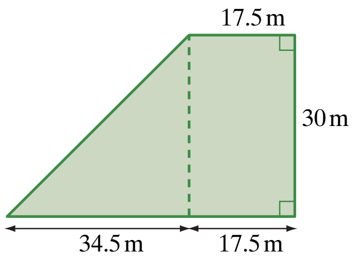
Some shapes are made up of a composite (or a combination) of other shapes. We can calculate the area of these shapes by cutting them into simpler shapes. We find the total area by adding the areas of all the pieces.

Example 25

Find the area of this shape.



Working



Reasoning

Divide the shape into a rectangle and a right-angled triangle.
Base of the triangle $= 52 - 17.5\,\text{m}$
$= 34.5\,\text{m}$

Area of rectangle $= l \times w$
$= 30 \times 17.5\ \text{m}^2$
$= 525\ \text{m}^2$

Rectangle:
Multiply length by width.

Area of triangle $= \frac{1}{2}(b \times h)$
$= \frac{1}{2}(34.5 \times 30)\ \text{m}^2$
$= 517.5\ \text{m}^2$

Triangle:
Half base times perpendicular height.

Total area $= 525 + 517.5\ \text{m}^2$
$= 1042.5\ \text{m}^2$

The area of the shape is $1042.5\,\text{m}^2$.

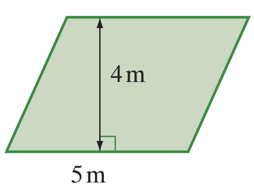
Add the area of the rectangle and the area of the triangle.

exercise 10.6

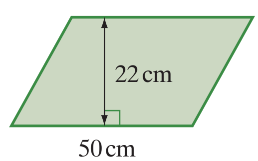
LINKS TO Example 23

1 Find the area of these parallelograms.

a



b



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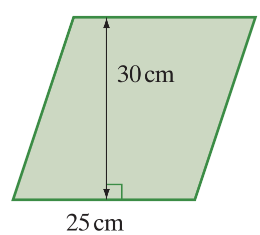

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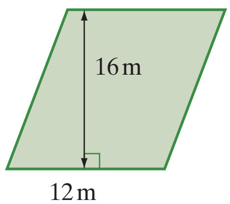

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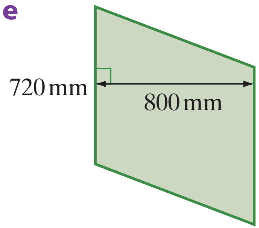

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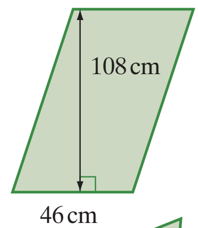

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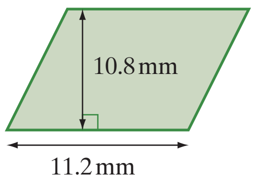

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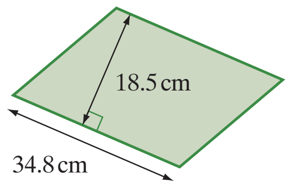

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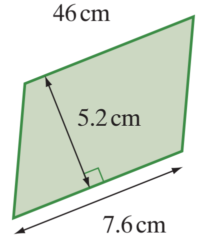
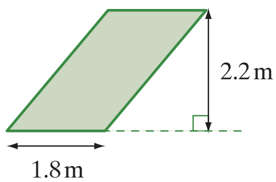


j



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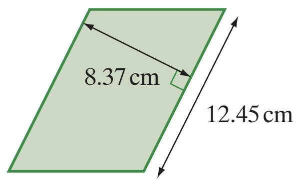
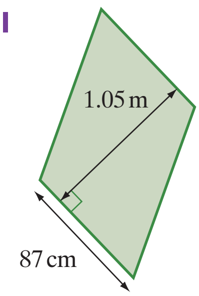


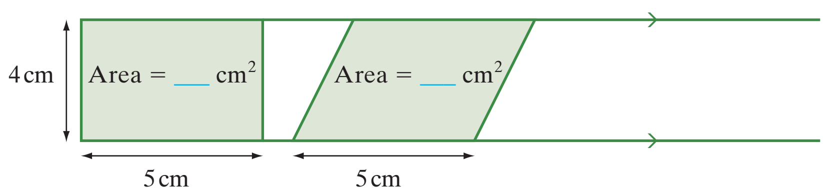
l



Using your ruler and pencil, draw a rectangle exactly 4 cm high and 5 cm wide at the left side of your page.

- Label the dimensions of the rectangle on your drawing and write its area inside the rectangle.
- Continue the top and bottom of the rectangle so that you have two parallel lines exactly 4 cm apart across your page.

- Alongside the rectangle draw a parallelogram that also has a base of 5 cm. Write the area inside it.



- Draw another parallelogram beside the first, again with a base of exactly 5 cm.
- What can you say about the areas of the rectangle and the two parallelograms? Explain.

The area of a parallelogram is 280 cm^2. The length of the base is 20 cm.

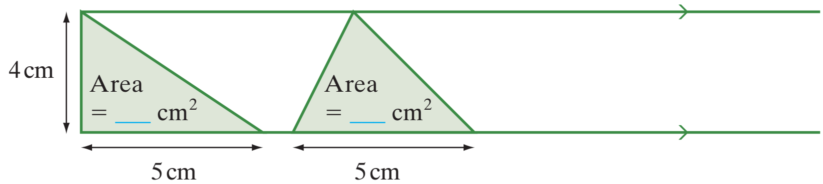
a Draw a parallelogram and label the length of the base. Draw a line on your diagram to represent the perpendicular height.

b Calculate the perpendicular height of the parallelogram.

Using your ruler and pencil, draw a right-angled triangle with a base exactly 5 cm and height 4 cm.

- Label the dimensions of the triangle on your drawing and write its area inside it.
- Rule a line through the top of the triangle parallel to the base so that you have two parallel lines exactly 4 cm apart across your page.
- Alongside the first triangle draw two more triangles (not right-angled) that also have bases of 5 cm. Write the area inside each triangle.

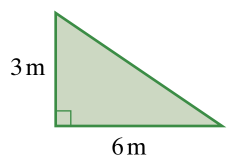
a What can you say about the areas of the three triangles? Explain.



LINKS TO Example 24

Calculate the area of these triangles (to 2 decimal places where necessary).

a



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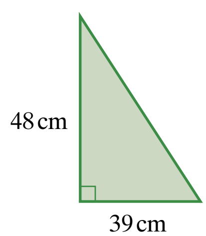

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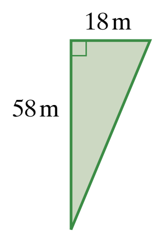

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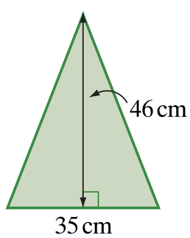

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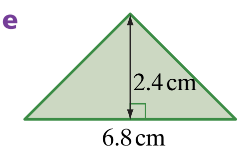

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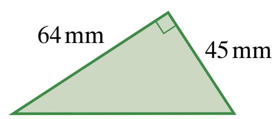

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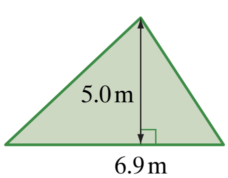

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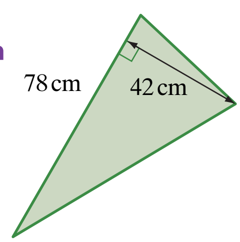

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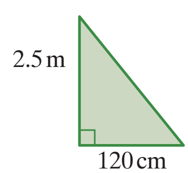
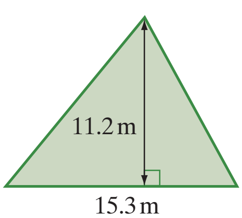


j



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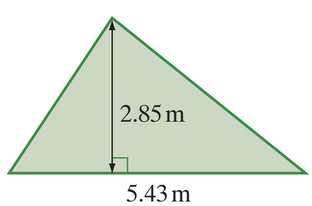

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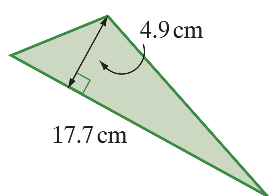

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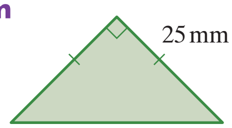
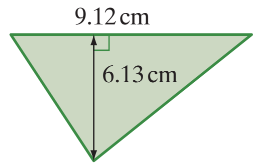
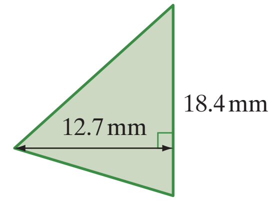


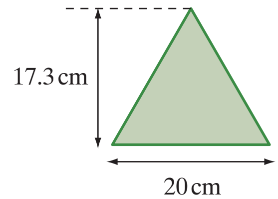
n



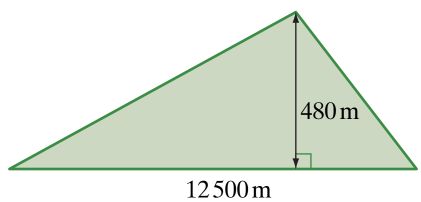
o



- The top of a chocolate box is in the shape of an equilateral triangle. Calculate the area of the top.



- A national park is in the shape of a triangle, with base 12 500 m and perpendicular height 480 m.

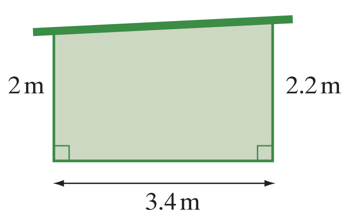


What is the area of the land

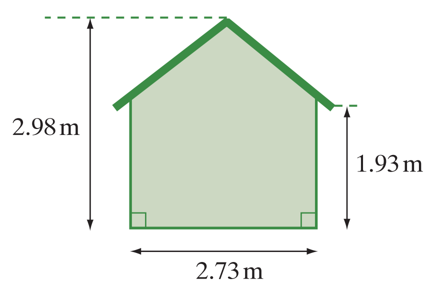
a in square metres? **b** in square kilometres?

- Calculate the area of the end of each of these sheds.

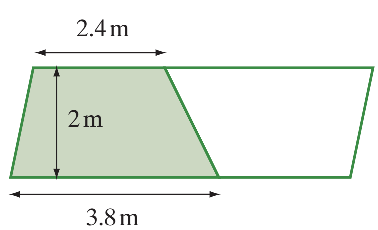
a



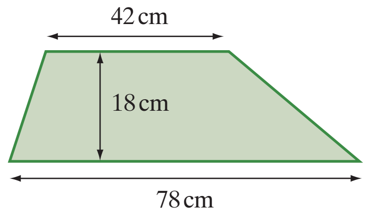
b



- Two identical trapezia fit together to make a parallelogram. Find the area of each trapezium.

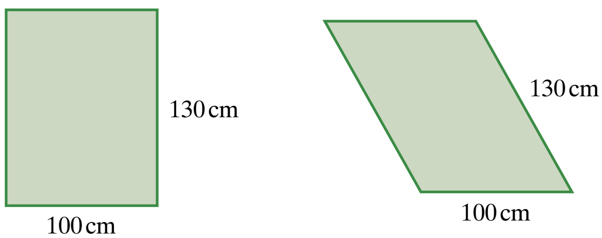


- Find the area of this trapezium by dividing it into two triangles.

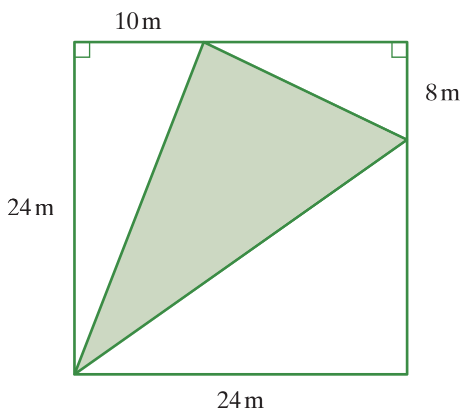


exercise 10.6 challenge

- William made a rectangular frame for a gate as shown in the diagram below. However, he didn't make sure that the corners stayed right angles and the frame got pushed sideways into a parallelogram shape. Is the area enclosed by the parallelogram frame the same as the area enclosed by the rectangle frame? Explain.



- Find the shaded area by first calculating the area of the white sections.

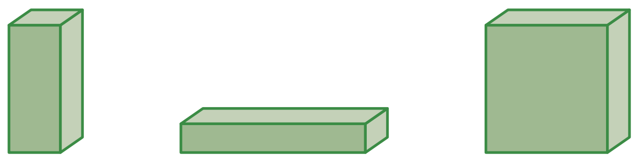


10.7 Units of volume

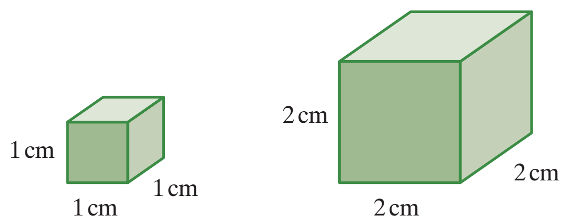
The volume of a three-dimensional object is a measure of how much space it takes up.

A solid object whose faces are rectangles is called a **rectangular prism** (or **cuboid**).

A **cube** is a special rectangular prism for which all the faces are squares.



Some examples of rectangular prisms or cuboids



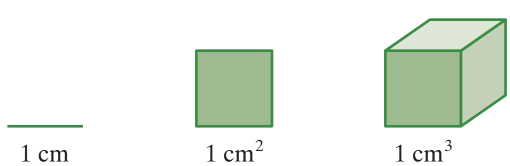
Some examples of cubes

Volume is measured in cubic units; for example cubic millimetres, cubic centimetres, and cubic metres.

Symbols for volume units

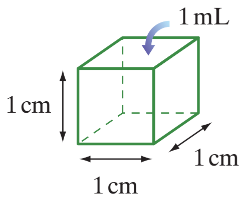
Cubic millimetres	Cubic centimetres	Cubic metres
mm^3	cm^3	m^3

The diagram below compares length, area and volume units.

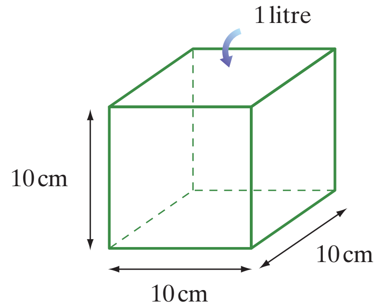


The superscript '3', as in cm^3, reminds us that there are three dimensions: length, width and height. Note that cm^2 = cm   cm and cm^3 = cm   cm   cm.

Volume is also measured in litres (L) and related units, for example, millilitre (mL), kilolitre (kL), megalitre (ML), gigalitre (GL). These units are usually used for liquids and gases or to describe how much a container will hold. This is also known as the **capacity** of a container.



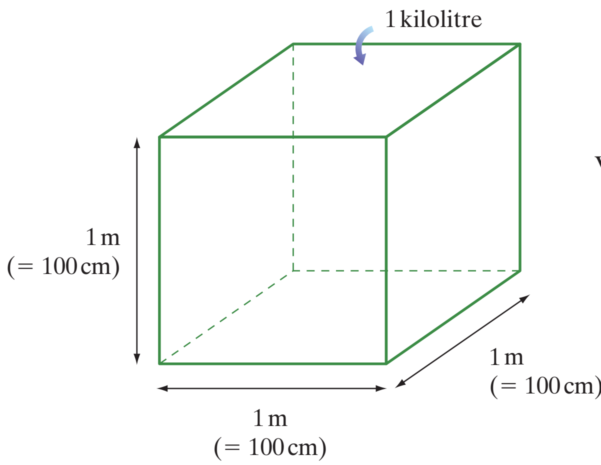
$1\ \text{cm}^3 = 1\ \text{mL}$



$$\begin{aligned} 1000\ \text{cm}^3 &= 1000\ \text{mL} \\ &= 1\ \text{L} \end{aligned}$$

$$\begin{aligned} 1\ \text{m}^3 &= 1\,000\,000\ \text{cm}^3 \\ &= 1\,000\,000\ \text{mL} \\ &= 1000\ \text{L} \\ &= 1\ \text{kL} \end{aligned}$$

Since $1\ \text{cm}^3$ is equivalent to 1 mL, $1\ \text{m}^3$ is equivalent to 1000 L or 1 kL.

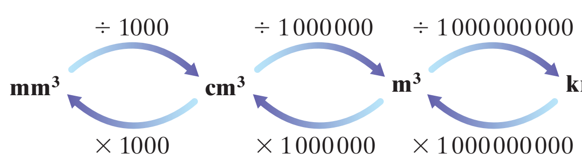


Volume = $1\ \text{m}^3$
($= 1\,000\,000\ \text{cm}^3$)

Converting units of volume

$$\begin{aligned} 1\ \text{cm}^3 &= 10\ \text{mm} \times 10\ \text{mm} \times 10\ \text{mm} \\ &= 1000\ \text{mm}^3 \end{aligned}$$

$$\begin{aligned} 1\ \text{m}^3 &= 100\ \text{cm} \times 100\ \text{cm} \times 100\ \text{cm} \\ &= 1\,000\,000\ \text{cm}^3 \end{aligned}$$



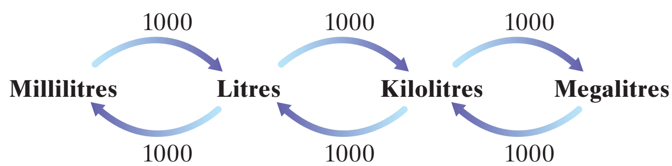
Example 26

Convert the following.

a $45\,cm^3$ to cubic millimetres
b $6.8\,m^3$ to cubic centimetres
c $1\,460\,000\,cm^3$ to cubic metres
d $180\,mm^3$ to cubic centimetres

Working	Reasoning
a $45\,cm^3 = 45 \times 10 \times 10 \times 10\,mm^3$ $= 45\,000\,mm^3$	$1\,cm = 10\,mm$ $1\,cm^3 = 10 \times 10 \times 10\,mm^3$
b $6.8\,m^3 = 6.8 \times 100 \times 100 \times 100\,cm^3$ $= 6\,800\,000\,cm^3$	$1\,m = 100\,cm$ $1\,m^3 = 100 \times 100 \times 100\,cm^3$
c $1\,460\,000\,cm^3$ $= 1\,460\,000 \div 100 \div 100 \div 100\,m^3$ $= 1.46\,m^3$	$100 \times 100 \times 100\,cm^3 = 1\,m^3$
d $180\,mm^3$ $= 180 \div 10 \div 10 \div 10\,cm^3$ $= 0.18\,cm^3$	$10 \times 10 \times 10\,mm^3 = 1\,cm^3$

1000 millilitres = 1 litre
1000 litres = 1 kilolitre
1000 kilolitre = 1 megalitre
1000 megalitres = 1 gigalitre



Example 27

Convert each of these measurements into the unit shown in brackets.

a 2.9 kL (L)
b 184 mL (L)
c $1500\,cm^3$ (L)

Working	Reasoning
a $2.9\,kL = 2.9 \times 1000\,L$ $= 2900\,L$	$1\,kL = 1000\,L$
b $184\,mL = 184 \div 1000\,L$ $= 0.184\,L$	$1000\,mL = 1\,L$
c $1500\,cm^3 = 1500\,mL$ $= 1.5\,L$	$1\,cm^3 = 1\,mL$ $1000\,mL = 1\,L$

Choosing suitable volume units

The volume unit we use depends on what we are measuring. The volumes of solid objects are usually given in cubic centimetres or cubic metres. Volumes of liquids and gases are often measured in litres (or the units based on the litre). Some examples of suitable volume units are shown here.

A sugar cube	Cubic millimetres	
Brick	Cubic centimetres	
Juice container	Millilitres	
Milk carton	Litres	

Shipping container	Cubic metres	
Underground water tank	Kilolitres	
Swimming pool	Megalitres	

Example 28

Choose the most appropriate unit for stating the volume of each of the following.

a a cornflakes packet

b a bedroom

Working

a cubic centimetres

Reasoning

The dimensions of the object give an indication of the most suitable unit of volume. A large cornflakes packet is approximately 10 cm × 25 cm × 40 cm.

continued

Example 28 continued

Working	Reasoning
b cubic metres	A typical bedroom would be approximately $3\,\text{m} \times 4\,\text{m} \times 4\,\text{m}$.

Example 29

Choose the most appropriate unit for measuring each of the following.

a the amount of water that a kitchen sink can hold

b the amount of milk on cereal at breakfast time

Working	Reasoning
a litre	A typical kitchen sink would hold about 25 litres.
b millilitre	It is unlikely that anyone would put a whole litre of milk on their breakfast!

exercise 10.7

LINKS TO Example 26

1. Convert each of these volumes into the unit shown in brackets.

a $25\,\text{cm}^3$ (mm^3)	**b** $300\,\text{cm}^3$ (m^3)	**c** $300\,\text{mm}^3$ (cm^3)
d $450\,000\,\text{mm}^3$ (cm^3)	**e** $1.8\,\text{m}^3$ (cm^3)	**f** $70\,\text{cm}^3$ (mm^3)
g $50\,000\,000\,\text{cm}^3$ (m^3)	**h** $1.2\,\text{m}^3$ (cm^3)	**i** $8\,\text{cm}^3$ (mm^3)
j $2400\,\text{cm}^3$ (m^3)	**k** $2000\,\text{mm}^3$ (cm^3)	**l** $350\,000\,\text{mm}^3$ (cm^3)
m $1.5\,\text{m}^3$ (mm^3)	**n** $4\,\text{cm}^3$ (mm^3)	**o** $1.8\,\text{m}^3$ (mm^3)

2. $186\,000\,\text{cm}^3$ is equal to

A $186\,\text{m}^3$	**B** $1.86\,\text{m}^3$	**C** $186\,000\,\text{mm}^3$
D $0.0186\,\text{m}^3$	**E** $1\,860\,000\,\text{mm}^3$	

LINKS TO Example 27

3. Convert each of these quantities into the unit shown in brackets.

a 58 L (mL)	**b** 12 000 000 mL (kL)
c 2.4 mL (kL)	**d** 48 000 L (kL)
e 3.75 mL (L)	**f** 0.9 kL (L)
g 720 L (kL)	**h** 237 000 mL (L)
i 18 mL (L)	**j** 15 000 mL (kL)
k 670 000 000 L (kL)	**l** 1.4 kL (L)
m 0.18 L (kL)	**n** 5820 mL (L)
o 17 500 000 mL (kL)	**p** 4.5 mL (kL)

LINKS TO Example 27

Convert each of these volumes into the unit shown in brackets.

a $25\,cm^3$ (mL) **b** 1.4 L (cm^3)
c $40\,m^3$ (kL) **d** $2000\,cm^3$ (L)
e $5000\,mm^3$ (mL) **f** 60 kL (m^3)
g 8 L (cm^3) **h** 7.5 mL (mm^3)
i $3.6\,cm^3$ (mL) **j** $240\,cm^3$ (mL)

LINKS TO Examples 28, 29

State the most suitable unit for measuring the volume of each of these objects.

a a cornflakes packet
b a backyard swimming pool
c a classroom
d a petrol tank in a car
e a school locker
f a wooden crate for transporting a car
g a small carton of orange juice
h a SIM (Subscriber Identity Module) card for a mobile phone

A plastic bucket has a capacity of 9500 mL. How many litres is this?

The sizes of car engines are generally stated in litres, whereas motorbike engines are sometimes given in cubic centimetres. The size of a motorbike engine is stated as 500 cc (cubic centimetres). What is this in

a millilitres? **b** litres?

exercise 10.7 challenge

This photograph of a stack of smart cars was taken in Canberra. The smart car engine size is 600 cc. The engine size of the Nissan Patrol is equivalent to the total size of the five smart car engines. What is the size in litres of the Nissan Patrol engine?

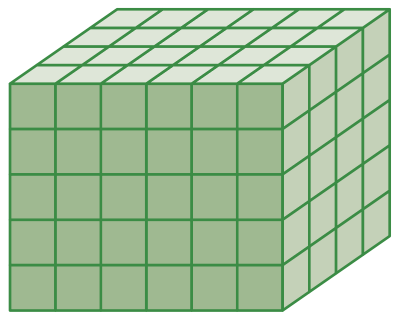
10.8 Volume: rectangular prisms

Class activity
Sugar cubes

We can find the volume of a rectangular prism by counting the number of 1 cm cubes.

Example 30

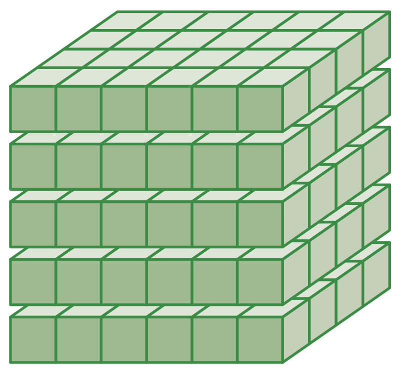
Find the volume of this rectangular prism.



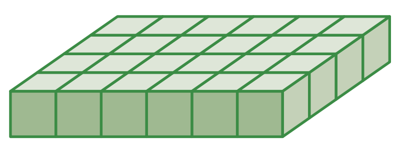
Working

The rectangular prism is made up of five layers.

Reasoning



Each layer is made up of four rows of six 1 cm cubes. The volume of each layer is
$4 \times 6 \text{ cm}^3$
$= 24 \text{ cm}^3$



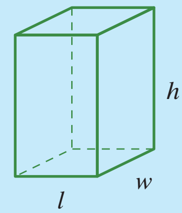
The unit is cm^3 because we are counting how many cubic centimetres.

The volume of the five layers is
$5 \times 4 \times 6 \text{ cm}^3$
$= 5 \times 24 \text{ cm}^3$
$= 120 \text{ cm}^3$

The volume of the rectangular prism is 120 cm^3.

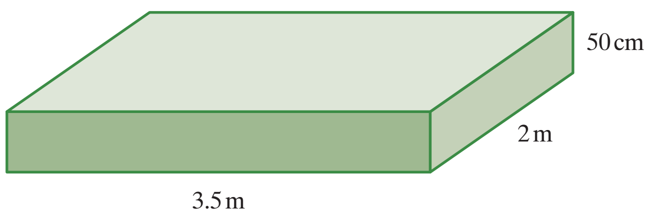
Notice that the volume of the rectangular prism can be calculated by simply multiplying the length by the width by the height.

The volume, V, of a rectangular prism is given by $V = l \times w \times h$ where l is the length of the base, w is the width of the base and h is the height. The dimensions must all be in the same unit.



Example 31

Calculate the volume of this rectangular prism.



Working

$V = lwh$

Volume = 3.5 m   2 m   0.5 m
= $3.5\,\text{m}^3$

Reasoning

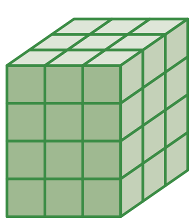
Convert 50 cm to metres.
The dimensions are in metres so the volume is in cubic metres.

exercise 10.8

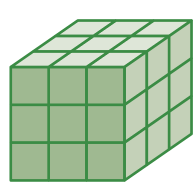
LINKS TO Example 30

1. In each of these rectangular prisms, the small cubes are cubic centimetre blocks. State
   - **i** the number of cubic centimetres in the top layer.
   - **ii** the number of layers.
   - **iii** the volume of the shape in cm^3.

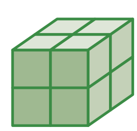
a



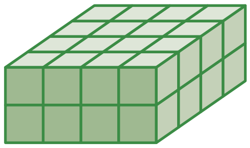
b



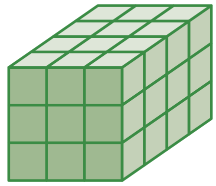
c



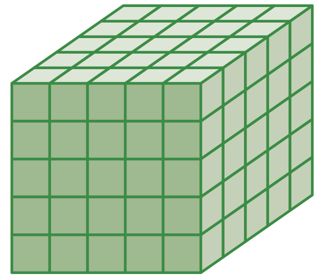
d



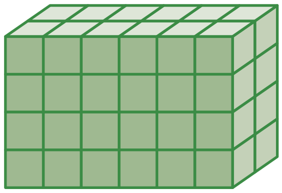
e



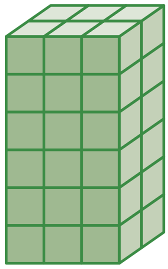
f



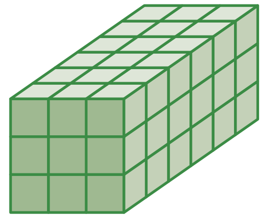
g



h



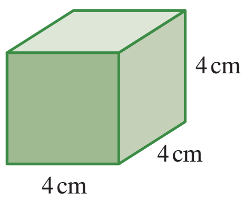
i



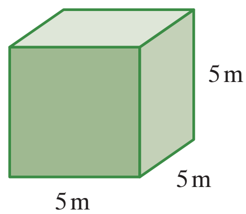
LINKS TO Example 31

● Calculate the volume of each of these rectangular prisms.

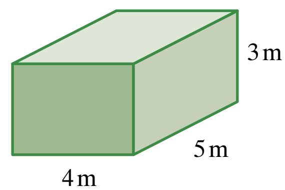
a



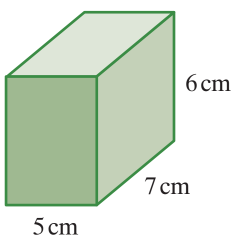
b



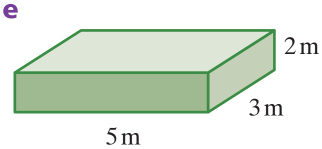
c



d



e



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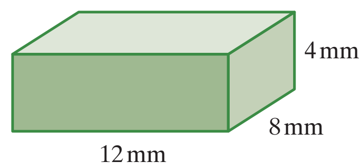
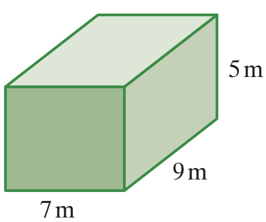
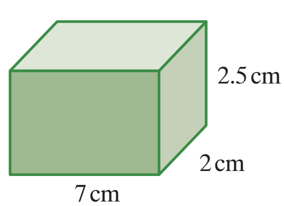


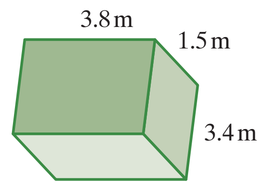
g



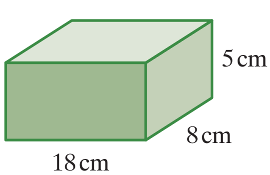
h



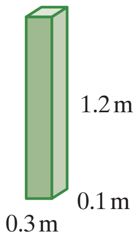
i



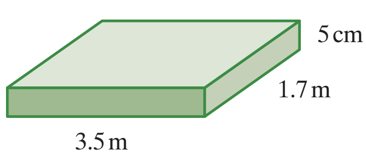
j



k

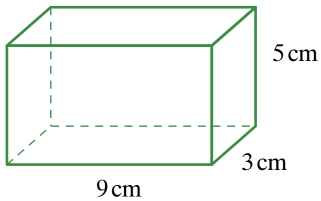


l

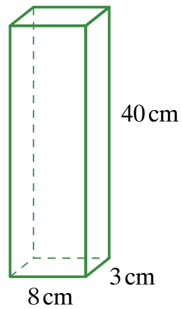


Find the volume of the following containers in
i cubic centimetres.
ii millilitres.

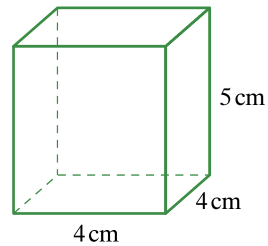
a



b



c



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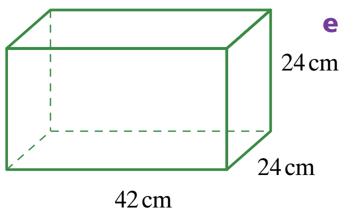

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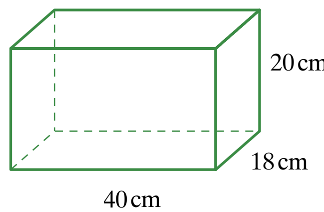

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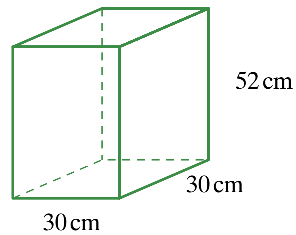
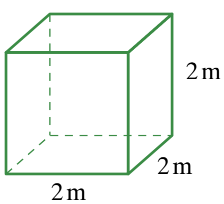


Find the volume of the following containers in
i cubic metres.
ii kilolitres.

a



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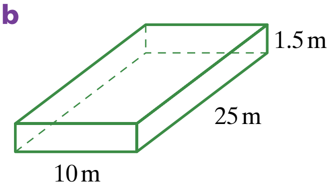

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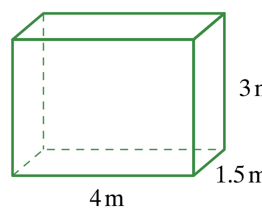

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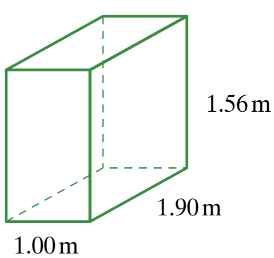
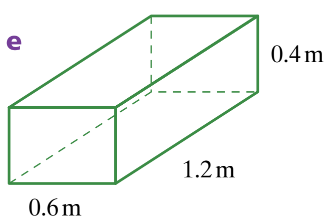


e



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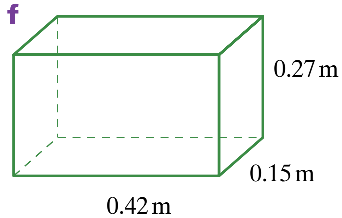
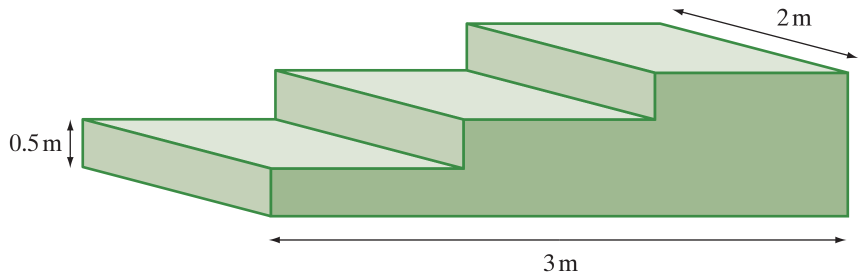


A box has a volume of $120\,cm^3$. List four possible sets of measurements for its length, width and height.

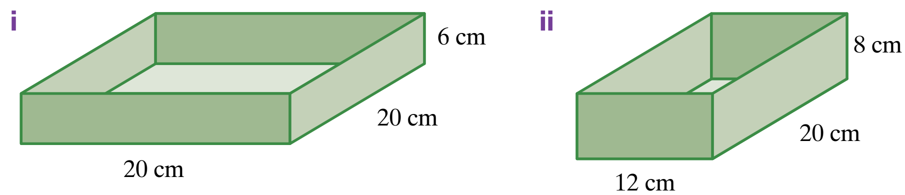
Find the volume of concrete required to build this flight of steps. Each step is 1 m wide.



Two cake tins are shown below.

a Calculate the volume of each of these cake tins.

b Which tin has the greatest volume?

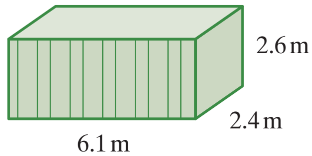


This carton has dimensions 30 cm by 50 cm by 40 cm. What is the volume of the carton?

It is important to know the volume of a room when designing heating or cooling. What is the volume of a room that is 6.5 m long, 4.8 m wide and 3.2 m high?

A hole is being excavated for a backyard swimming pool. If the hole is 8 m long, 4 m wide and 1.3 m deep, how many cubic metres of earth must be removed?

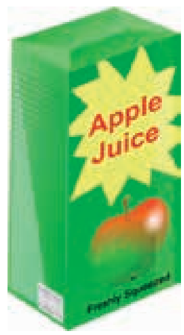
A shipping container is 6.1 m long, 2.4 m wide and 2.6 m high. Calculate the volume of the container.



A drink carton in the shape of a rectangular prism has dimensions 4 cm by 6.25 cm by 10 cm.

a What is the volume of the drink carton in cubic centimetres?

b How many millilitres will it hold?

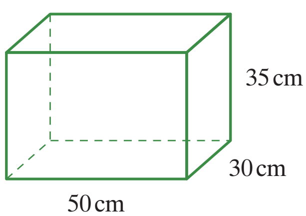


For each of the fish tanks below, calculate the volume

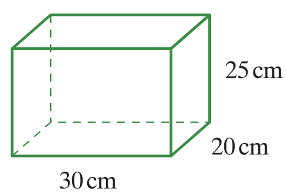
i in cubic centimetres.

ii in litres.

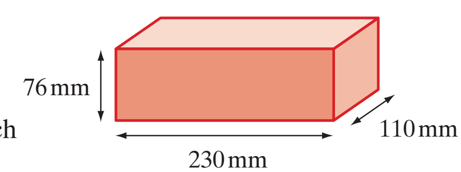
a



b



In order to save water, it has been suggested that householders could place a brick in the toilet cistern to reduce the amount of water that the cistern can hold.



a Calculate the volume of the brick in cubic centimetres. Round your answer to the nearest whole number.

b How many litres of water would be saved each time the toilet cistern fills after flushing the toilet? Give your answer to the nearest litre.

c Estimate how much water your household could save by placing a brick in the toilet cistern

i per day.

ii per year (assuming it is not a leap year).

exercise 10.8 challenge

A rectangular swimming pool is 5 m long, 3.5 m wide and has an average depth of water of 1 m.

- **a** Calculate the volume of water in cubic metres.
- **b** Remembering that $1\,\text{m}^3 = 100 \quad 100 \quad 100\,\text{cm}^3$, what is the volume of water in millilitres?
- **c** Convert the volume into litres.
- **d** Convert the volume into kilolitres.

Find a 1 L milk or juice carton.

- **a** Carefully measure the dimensions (to the nearest tenth of a centimetre) of the rectangular prism part of the carton. Using your ruler and pencil make a drawing of the rectangular prism part and label the dimensions.
- **b** Calculate the volume of the rectangular prism.
- **c** Is the volume of the rectangular prism exactly 1 L? Can you explain your findings?

The shape of an iceberg was found to be approximately a rectangular prism. The dimensions of the iceberg above sea level were 34 km by 53 km and its height above the water was approximately 100 m.

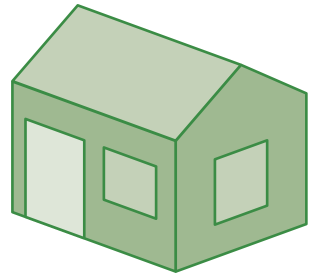
- **a** Calculate the volume of the iceberg showing above sea level. Give your answer to the nearest cubic kilometre.
- **b** Icebergs float so that only about one-tenth of their volume is above sea level. The other nine-tenths are below the sea. What is the total volume of the iceberg in part a?

How many 20 cm cube-shaped boxes could be packed in a carton 80 cm long, 40 cm wide and 60 cm high?

Analysis task

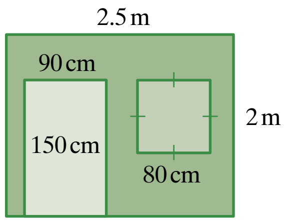
Painting the cubby house

Jake is going to paint a cubby house. He calculates the area he is going to paint so he can work out how much paint to buy.

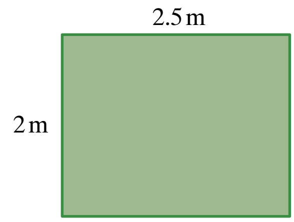


The dimensions of the walls are shown.

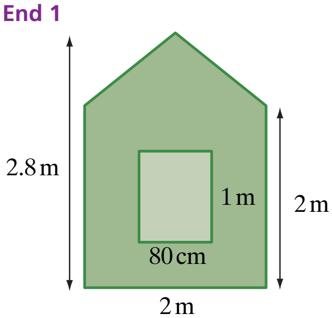
Front



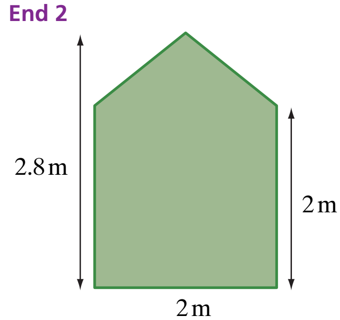
Back



End 1



End 2



a Find the area to be painted for each of the walls.

i front **ii** back **iii** end 1 **iv** end 2

b Find the total area of walls to be painted.

c 1 L of paint covers an area of 14 m^2. Jake is giving the cubby house two coats of paint. How much paint will he need?

d The paint is sold in 1L, 2 L or 4 L tins. The prices are:

1 L	2 L	4 L
\$16.50	\$31.25	\$42.50

Which should Jake buy?

e Jake is going to paint the outside of the door a different colour from the walls. What is the area of the door?

f He can buy a small tin of yellow paint that will cover 1.5 m^2. How many of these tins will he need?

Review *Perimeter, area and volume*

Summary

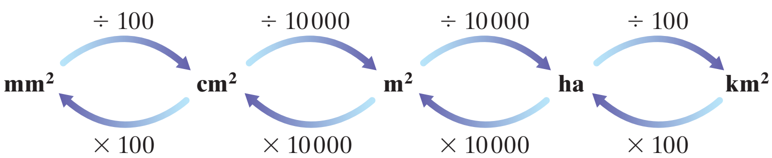
Perimeter

- Perimeter of a plane figure is the distance around it.
- Units: millimetres (mm), centimetres (cm), metres (m), kilometres (km).

Rectangle	$P = 2(l + w)$
Square	$P = 4l$

Area

- Area of a plane figure is the amount of space enclosed by its boundary.
- Units: square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2).



Rectangle	$A = l \quad w$
Square	$A = l^2$
Parallelogram	$A = b \quad h$
Triangle	$A = \frac{1}{2}(b \times h)$ or $A = \frac{bh}{2}$

Volume

- Volume of a three-dimensional shape is the amount of space it occupies.
- Units: cubic millimetres (mm^3), cubic centimetres (cm^3), cubic metres (m^3), millilitres (mL), litres (L), kilolitres (kL), megalitres (ML).

Rectangular prism	$V = l \times w \times h$

- Units: $1\,cm^3 = 1\,mL$

$mm^3 \xrightarrow{\div 1000} cm^3 \xrightarrow{\div 1\,000\,000} m^3 \xrightarrow{\div 1\,000\,000\,000} km^3$

$km^3 \xrightarrow{\times 1\,000\,000\,000} m^3 \xrightarrow{\times 1\,000\,000} cm^3 \xrightarrow{\times 1000} mm^3$

Millilitres (mL) → 1000 → **Litres (L)** → 1000 → **Kilolitres (kL)** → 1000 → **Megalitres (ML)**

Megalitres (ML) → 1000 → **Kilolitres (kL)** → 1000 → **Litres (L)** → 1000 → **Millilitres (mL)**

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

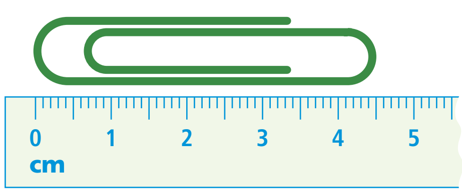
area	perimeter	square
base	perpendicular height	triangle
cube	prism	volume
dimensions	rectangle	
parallelogram	rectangular prism	

Revision

Multiple-choice questions

The length of this paper clip is closest to

A 4 mm **B** 4.5 mm **C** 40.5 mm **D** 45 mm **E** 45 cm



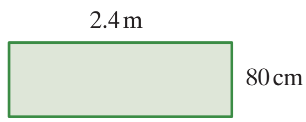
The lengths 78 000 cm, 0.4 km, 1 500 000 mm, and 890 m are put in order from shortest to longest. The correct order is

A 0.4 km, 78 000 cm, 890 m, 1 500 000 mm　**B** 890 m, 78 000 cm, 1 500 000 mm, 0.4 km
C 78 000 cm, 890 m, 1 500 000 mm, 0.4 km　**D** 0.4 km, 890 m, 78 000 cm, 1 500 000 mm
E 1 500 000 mm, 78 000 cm, 890 m, 0.4 km

A rectangular room has length 3.4 m and width 2.9 m. The perimeter of the room is

A 6.3 m　**B** 9.2 m　**C** 9.7 m　**D** 9.86 m　**E** 12.6 m

The area of this rectangle is



A 1.92 m^2　**B** 3.2 m^2　**C** 6.4 m^2　**D** 164.8 m^2　**E** 192 m^2

A rectangular prism has a square base with side length 6 cm and a height of 11 cm. The volume of the prism is

A 66 cm^3　**B** 132 cm^3　**C** 396 cm^3　**D** 726 cm^3　**E** 4356 cm^3

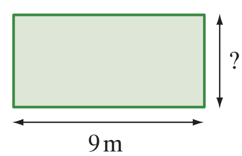
Short-answer questions

Calculate.

a What length of braid is needed to go around the edges of a rectangular rug which is 2.6 m long and 1.8 m wide?

b A rectangular wall is to be covered with square tiles with sides 15 cm. If the wall is 2.1 m by 2.4 m, how many tiles will be needed?

Calculate.

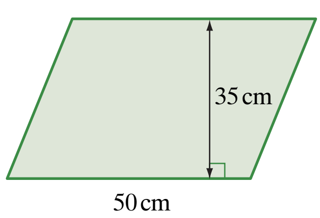


a If the perimeter of this rectangle is 28.4 m and the length is 9 m, what is its width?

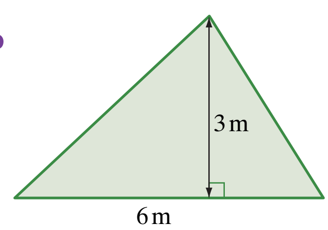
b If the area of a square is 36 cm^2, what is the perimeter of the square?

Find the area of these shapes.

a

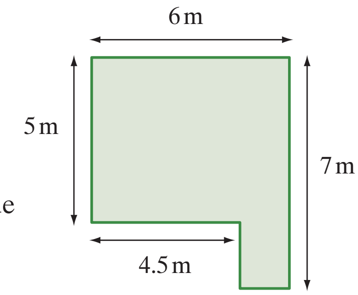


b

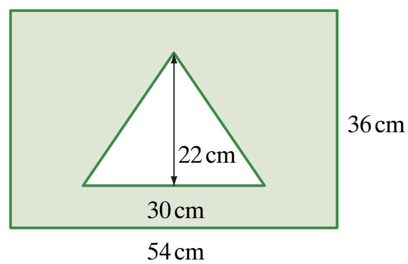


The diagram at right shows part of a garden that is to be planted with instant lawn.

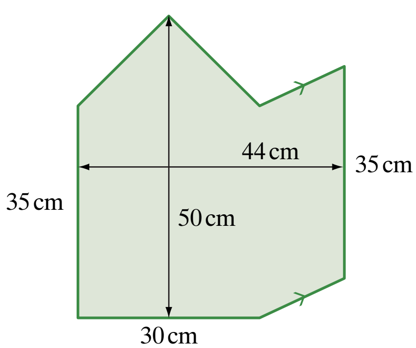
- **a** Calculate the area of instant lawn needed.
- **b** If instant lawn costs $7 per square metre, what will it cost for the lawn?
- **c** Garden edging is to be placed around the outside of the lawn. What length of edging is required?



- **a** Calculate the area of the shaded region.



- **b** Calculate the area of this shape.



Extended-response questions

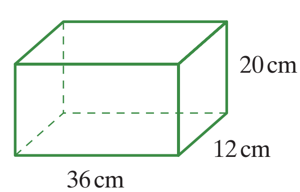
A rectangular swimming pool is 7.5 m long and 3.5 m wide.

- **a** What is the area of the pool?
- **b** A path 1 m wide surrounds the pool. Draw a diagram and label it to show the dimensions of the pool and the path.
- **c** What is the area of the path?

Practice quiz
Chapter 10

The interior of a fish tank is 36 cm long, 12 cm wide and 20 cm high. What is the volume of the tank

- **a** in cubic centimetres?
- **b** in litres?



Percentages and ratios 11

Pre-test

Warm-up

Comparing quantities as percentages and as ratios is important in many everyday situations. The ratio of colours required to make particular shades of colour is carefully controlled when paint is mixed in a paint-mixing machine. In the tins of paint that you can see in this photograph, what is the ratio of white paint to red paint? What percentage of the tins contain red paint?

11.1 What is a percentage?

Percentages are fractions where the denominator is 100. 'Per cent' means 'out of 100'.

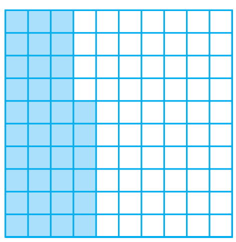
The whole amount is called 100 per cent, written as 100%.

Percentages are widely used. Because they all have a denominator of 100, percentages are easier to compare than fractions with different denominators.

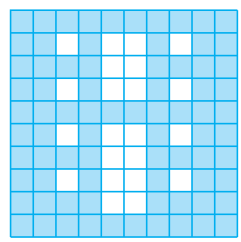
Example 1

What percentage of each of these grids of 100 squares is shaded, and what percentage is unshaded?

a



b



Working	Reasoning
a 36% of the grid is shaded.	By counting, 36 out of the 100 squares are shaded. This is 36%. By counting, 64 out of the 100 squares are not shaded.
64% of the grid is not shaded.	The total number of squares is 100. 100 − 36 are not shaded.
b 78% of the grid is shaded.	By counting, 78 out of the 100 squares are shaded. This is 78%. By counting, 22 out of the 100 squares are not shaded.
22% of the grid is not shaded.	The total number of squares is 100. 100 − 78 are not shaded.

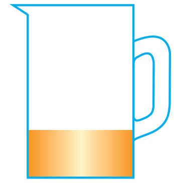
Example 2

In a plane, seats can be in first class, business class or economy class. If 5% of seats are first class and 80% are in economy class, what percentage of seats is in business class?

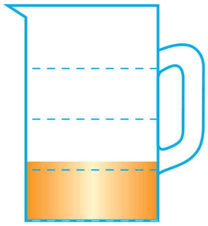
Working	Reasoning
Percentage in business class = 100% − 5% − 80% = 15%	The total number of seats is 100%. To find the remaining part, subtract the percentages for the parts that are known.

Example 3

Estimate the percentage of this container that is filled with orange juice.



Working



Estimate: This container is 30% full.

Reasoning

Mark on the container where one-half, or 50% is.

Also mark one-quarter, which is 25%, and three-quarters, which is 75%.

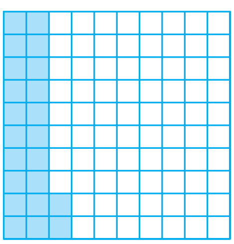
Here, a little more than 25% is filled.

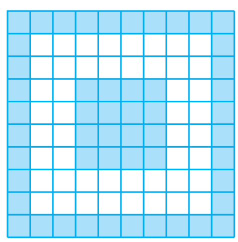
exercise 11.1

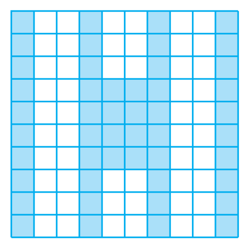
LINKS TO Example 1

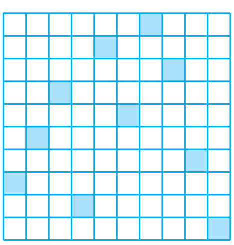
1. For each of the following grids split into one hundred equal squares, find

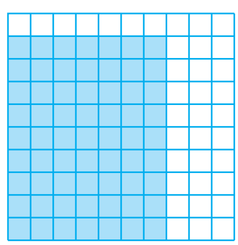
   i the percentage shaded. **ii** the percentage not shaded.

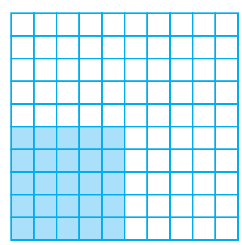
a 

b 

c 

d 

e 

f 

LINKS TO Example 2

Marco's Pizza Shop sells small, medium and large pizzas. For the first six months last year their sales were split as shown in the table. Copy and complete this table.

Month	Small	Medium	Large
January	20%		30%
February	10%	45%	
March		35%	25%
April	18%		33%
May	22%	31%	
June		43%	36%

LINKS TO Example 2

There have been many different models of the Boeing Airbus. In each, the percentages of first, business and economy class seats is slightly different.

a Complete the table below.

b Which class has the highest percentage of seats?

c Why do you think this is so?

Model	First class seats	Business class seats	Economy class seats
A 330–200	5%	14%	
A 330–300	4%		82%
A 340–200		10%	85%
A 340–300		14%	81%
A 340–500	4%		83%
A 340–600	3%	14%	

LINKS TO Example 2

Students in one high school were surveyed about their favourite subject. This table shows the results.

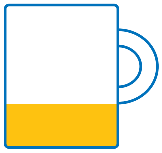
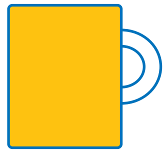
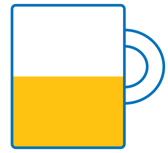
Favourite subject	Percentage choosing it
Mathematics or Science	22%
History, Geography, English or a language	19%
Music	13%
Art, Home economics or something creative	36%
Sport	

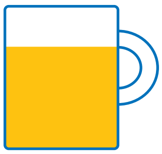
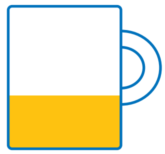
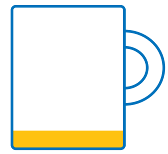
The percentage for whom sport was the favourite subject was

A 8% **B** 10% **C** 13% **D** 14% **E** 20%

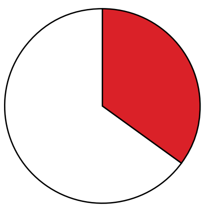
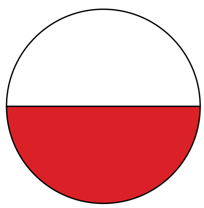
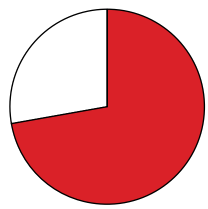
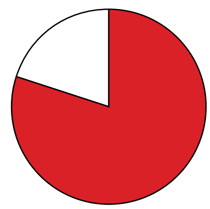
LINKS TO Example 3

Estimate the percentage of each container filled with orange juice.

a  **b**  **c** 

d  **e**  **f** 

Estimate the percentage of each circle that has been shaded red.

a  **b**  **c**  **d** 

When computer games are being installed, the percentage of the game that has loaded so far is shown on a shaded bar. For each of the following percentages, draw a bar and show the percentage. (If you draw the bar 10 cm long, then each 1% will be 1 mm long.)

a 50% **b** 75% **c** 40% **d** 15% **e** 90% **f** 35%

exercise 11.1 challenge

Explain what you think is meant by 100% in each of these signs.

a

b

11.2 Percentages, fractions and decimals

Decimal fraction percentage

Fractions, decimals and percentages can all be used to represent parts of a whole. You saw in chapter 4 how we convert between fractions and decimals. In this section we look at conversion between fractions, decimals and percentages.

Because a percentage is 'out of 100', it is very easy to convert a percentage to a fraction. We write the percentage as a fraction with a denominator of 100. Notice that this is the same as dividing the percentage by 100. We then simplify the fraction if possible.

Example 4

Convert these percentages to fractions.

a 37% **b** 60% **c** 48% **d** 125%

Working	Reasoning
a $37\% = \frac{37}{100}$	Divide the percentage by 100 by writing it as a fraction with a denominator of 100.
b $60\% = \frac{60}{100}$	Divide the percentage by 100 by writing it as a fraction with a denominator of 100.
$= \frac{3}{5}$	Simplify.
c $48\% = \frac{48}{100}$	Divide the percentage by 100 by writing it as a fraction with a denominator of 100.
$= \frac{12}{25}$	Simplify.
d $125\% = \frac{125}{100}$	Divide the percentage by 100 by writing it as a fraction with a denominator of 100.
$= \frac{5}{4}$	Simplify.
$= 1\frac{1}{4}$	A percentage greater than 100% means that it is greater than one whole, so its fraction equivalent will be a mixed number.

For converting a mixed number percentage to a fraction, we first convert the mixed number to an improper fraction then divide by 100 and simplify.

Example 5

Convert these percentages to fractions.

a $33\frac{1}{3}\%$ **b** $6\frac{1}{4}\%$

Working | **Reasoning**

a $33\frac{1}{3}\% = \frac{100}{3}\%$ — Convert the mixed number to an improper fraction.

$= \frac{100}{3} \div 100$ — Dividing by 100 is the same as multiplying by $\frac{1}{100}$.

$= \frac{\cancel{100}^{1}}{3} \times \frac{1}{\cancel{100}_{1}}$

$= \frac{1}{3}$ — Simplify.

b $6\frac{1}{4}\% = \frac{25}{4}\%$ — Convert the mixed number to an improper fraction.

$= \frac{25}{4} \div 100$ — Dividing by 100 is the same as multiplying by $\frac{1}{100}$.

$= \frac{\cancel{25}^{1}}{4} \times \frac{1}{\cancel{100}_{4}}$

$= \frac{1}{16}$ — Simplify.

To convert a percentage to a decimal, we start by writing the percentage as a fraction with a denominator of 100, just as we do for converting a percentage to a fraction. Dividing by 100 to convert to a decimal is then very easy.

Example 6

Convert these percentages to decimals.

a 56% **b** 4% **c** $6\frac{1}{2}\%$ **d** 150%

Working | **Reasoning**

a $56\% = \frac{56}{100}$

$= 0.56$ — Omit the % sign and divide by 100. To divide by 100, move the decimal point two places to the left so that the digits are in smaller place values.

b $4\% = \frac{4}{100}$

$= 0.04$ — Omit the % sign and divide by 100. To divide by 100, move the decimal point two places to the left so that the digits are in smaller place values.

continued

Example 6 continued

Working	Reasoning
c $6\frac{1}{2}\% = 6.5\%$ $= \frac{6.5}{100}$ $= 0.065$	Convert the mixed number to a decimal. Omit the % sign and divide by 100. To divide by 100, move the decimal point two places to the left so that the digits are in smaller place values.
d $150\% = \frac{150}{100}$ $= 1.50$	Omit the % sign and divide by 100. To divide by 100, move the decimal point two places to the left so that the digits are in smaller place values. A percentage that is greater than 100% converts to a decimal number that is greater than one whole.

Example 7 compares the methods for converting the same percentage to a fraction and to a decimal.

Example 7

Change 35% to

a a fraction.

b a decimal.

Working	Reasoning
a 35%	35% means 35 out of 100.
$= \frac{35}{100}$	This is 35 hundredths.
$= \frac{7}{20}$	Simplify if possible.
b 35%	35% means 35 out of 100.
$= \frac{35}{100}$	This is 35 hundredths.
$= 0.35$	Write the number in decimal form.

We have seen that to turn a percentage into a fraction or decimal, we *divide* by 100. To convert a fraction or a decimal to a percentage, we do the opposite of what we did to turn a percentage into a fraction or decimal. We *multiply* the fraction or decimal by 100%. For example, we can think of the fraction $\frac{2}{5}$ as meaning $\frac{2}{5}$ of one whole, which is the same as saying $\frac{2}{5}$ of 100% or $\frac{2}{5} \times 100\%$.

Example 8

Convert these fractions to percentages.

a $\frac{23}{100}$ **b** $\frac{4}{5}$ **c** $\frac{3}{8}$ **d** $\frac{13}{25}$ **e** $1\frac{3}{4}$

Working	Reasoning
a $\frac{23}{100} = 23\%$	$\frac{23}{100}$ already has 100 as the denominator so we know that it is 23%.
b $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1}\%$ $= \frac{4}{\cancel{5}_1} \times \frac{\cancel{100}^{20}}{1}\%$ $= 80\%$	Multiply the fraction by 100% and simplify. Alternatively, $\frac{4}{5} = 0.8 = 80\%$
c $\frac{3}{8} = \frac{3}{8} \times \frac{100}{1}\%$ $= \frac{3}{\cancel{8}_2} \times \frac{\cancel{100}^{25}}{1}\%$ $= \frac{3 \times 25}{2}\%$ $= \frac{75}{2}\%$ $= 37\frac{1}{2}\%$	Multiply the fraction by 100% and simplify. Alternatively, $\frac{3}{8} = 0.375 = 37.5\%$
d $\frac{13}{25} = \frac{13}{\cancel{25}_1} \times \frac{\cancel{100}^{4}}{1}\%$ $= 13 \times 4\%$ $= 52\%$	Multiply the fraction by 100% and simplify.
e $1\frac{3}{4} = \frac{7}{4}$ $= \frac{7}{\cancel{4}_1} \times \frac{\cancel{100}^{25}}{1}\%$ $= 7 \times 25\%$ $= 175\%$	Convert the mixed number to an improper fraction, multiply by 100% and simplify. A mixed number is greater than one whole, so its percentage equivalent will be greater than 100%.

Multiplying by 100% to convert a decimal to a percentage is easy. We move the decimal point two places to the right so that the digits are in larger place values.

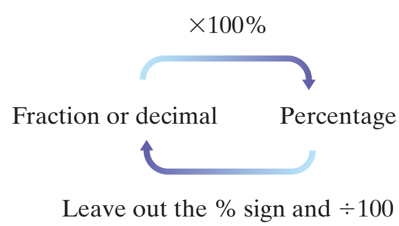
Example 9

Convert these decimals to percentages.

a 0.72 **b** 0.03 **c** 0.0075 **d** 2.14

Working	Reasoning
a $0.72 = 0.72 \times 100\%$ $= 72\%$	Multiply the decimal by 100%. To do this, move the decimal point two places to the right so that the digits are in larger place values. $0.72 \times 100 = 72$
b $0.03 = 0.03 \times 100\%$ $= 3\%$	Multiply the decimal by 100%. To do this, move the decimal point two places to the right so that the digits are in larger place values. $0.03 \times 100 = 3$
c $0.0075 = 0.0075 \times 100\%$ $= 0.75\%$	Multiply the decimal by 100%. To do this, move the decimal point two places to the right so that the digits are in larger place values. $0.0075 \times 100 = 0.75$
d $2.14 = 2.14 \times 100\%$ $= 214\%$	Multiply the decimal by 100%. To do this, move the decimal point two places to the right so that the digits are in larger place values. $2.14 \times 100 = 214$ A decimal number greater than one means that it is greater than one whole, so its percentage equivalent will be greater than 100%.

This diagram summarises how we convert between fractions, decimals and percentages.



The following example shows the calculation of fraction, decimal and percentage equivalents. Recall from chapter 4 that to convert a fraction to a decimal we divide the numerator by the denominator.

Example 10

Complete this table of fractions, decimals and percentages.

Fraction	Decimal	Percentage
$\frac{3}{4}$		
	0.48	
		54%
$1\frac{5}{8}$		
		$33\frac{1}{3}\%$

Working and Reasoning

Fraction	Decimal	Percentage
$\frac{3}{4}$	$3 \div 4 = 0.75$	$0.75 = 0.75 \times 100\% = 75\%$
$0.48 = \frac{48}{100} = \frac{12}{25}$	0.48	$0.48 = 0.48 \times 100\% = 48\%$
$54\% = \frac{54}{100} = \frac{27}{50}$	$54\% = \frac{54}{100} = 0.54$	54%
$1\frac{5}{8}$	$1\frac{5}{8} = \frac{13}{8} = 1.625$	$1\frac{5}{8} = 1.625 \times 100\% = 162.5\%$
$33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{300} = \frac{1}{3}$	$\frac{1}{3} = 0.\overline{3}$	$33\frac{1}{3}\%$

It is useful to memorise some common percentages and their fraction equivalents.

Fraction	Decimal	Percentage
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$
$\frac{1}{4}$	0.25	25%
$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{1}{2}$	0.5	50%

continued

Fraction	Decimal	Percentage
$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{3}{4}$	0.75	75%
$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%

Tech tip

The TI-30XB MultiView calculator can be used to convert a fraction to a percentage. For example, to convert $\frac{5}{8}$ to a percentage, type:

[5] [n/d] [8] [2nd] [▶] [enter].

To convert 0.875 to a percentage, type:

[0] [·] [8] [7] [5] [2nd] [▶] [enter].

Note that [2nd] [▶] gives [▶%].

exercise 11.2

LINKS TO Example 4

1. Change each of the following percentages to a fraction or mixed number, simplifying where possible.

a 17% **b** 29% **c** 30% **d** 24%

e 80% **f** 96% **g** 150% **h** 5%

i 15% **j** 36% **k** 38% **l** 240%

m 65% **n** 75% **o** 175% **p** 4%

11.2

LINKS TO Example 5

Change each of the following percentages to a fraction, simplifying where possible.

a $2\frac{1}{2}\%$ b $6\frac{1}{4}\%$ c $3\frac{1}{2}\%$ d $37\frac{1}{2}\%$

e $33\frac{1}{3}\%$ f $66\frac{2}{3}\%$ g $62\frac{1}{2}\%$ h $1\frac{1}{4}\%$

LINKS TO Example 6

Convert each of these percentages to a decimal.

a 35% b 70% c 25% d 92%

e 144% f 30% g 3% h 80%

i 8% j 23% k 230% l 2.3%

m 85% n 8.5% o 115% p 12%

LINKS TO Example 7

Convert each of these percentages

i to a fraction.

ii to a decimal.

a 20% b 70% c 75% d 15%

e 45% f 54% g 64% h 72%

LINKS TO Example 8a

Convert each of these fractions to a percentage.

a $\frac{37}{100}$ b $\frac{81}{10}$ c $\frac{3}{10}$ d $\frac{7}{10}$

e $\frac{9}{10}$ f $\frac{79}{100}$ g $\frac{127}{1000}$ h $\frac{375}{1000}$

LINKS TO Examples 8b, c, d

Convert each of these fractions to a percentage.

a $\frac{11}{20}$ b $\frac{14}{50}$ c $\frac{16}{25}$ d $\frac{32}{50}$

e $\frac{3}{4}$ f $\frac{19}{20}$ g $\frac{46}{50}$ h $\frac{5}{8}$

i $\frac{19}{50}$ j $\frac{21}{25}$ k $\frac{27}{50}$ l $\frac{7}{8}$

m $\frac{4}{25}$ n $\frac{3}{8}$ o $\frac{5}{16}$ p $\frac{15}{16}$

LINKS TO Example 8e

Convert each of these mixed numbers to a percentage.

a $1\frac{2}{5}$ b $2\frac{1}{2}$ c $1\frac{1}{4}$ d $1\frac{3}{10}$

e $3\frac{7}{10}$ f $1\frac{3}{4}$ g $2\frac{1}{4}$ h $1\frac{2}{5}$

LINKS TO Example 9

Convert each decimal to a percentage.

a 0.85 b 0.12 c 0.48 d 0.4

e 0.04 f 0.99 g 0.7 h 0.07

i 0.007 j 0.65 k 0.065 l 1.4

m 1.65 n 2.25 o 1.0 p 3.0

LINKS TO Example 10

Complete the table and try to memorise these common fraction/decimal/percentage equivalents.

Fraction	Decimal	Percentage
	0.5	
$\frac{1}{4}$		
	0.75	
		$33\frac{1}{3}\%$
$\frac{2}{3}$		
	0.1	
		$12\frac{1}{2}\%$

LINKS TO Example 10

Copy and complete this table.

Fraction	Decimal	Percentage
	0.7	
		8%
$\frac{2}{5}$		
		34%
	0.15	
$\frac{6}{25}$		
		6%
$\frac{3}{10}$		
	0.8	
		12%

0.74 as a percentage is

A 0.074% **B** 0.74% **C** 7.4% **D** 74% **E** 740%

45% as a fraction is

A $\frac{4}{9}$ **B** $\frac{5}{9}$ **C** $\frac{5}{20}$ **D** $\frac{45}{1}$ **E** $\frac{9}{20}$

$3\frac{1}{4}\%$ as a decimal is

A 3.25 **B** 32.50 **C** 325 **D** 0.325 **E** 0.0325

When a tennis ball is dropped, it bounces back to 0.3 of the height it was dropped.

a Express 0.3 as a percentage.

b For a pingpong ball the bounce is 0.42 of the dropping height. Express this as a percentage.

c A superball will bounce to 0.67 of the dropping height. Express this as a percentage.

Iron makes up about 0.05 of the mass of the earth's crust. Write this decimal as

a a fraction. **b** a percentage.

exercise 11.2 challenge

$\frac{8}{9}$ of an iceberg is below the surface of the water.

a Write this fraction

i as a recurring decimal.

ii as a percentage.

b What fraction of the iceberg is above the surface of the water?

c What percentage of the iceberg is above the surface of the water?

11.3 Expressing one quantity as a percentage of another

Problems involving percentages can be of several different sorts. It is important to understand what needs to be calculated. Compare the three different problems shown below. In each case a double number line has been drawn showing percentages on the left and the actual numbers on the right. The double number line helps us to see what is missing, that is, what we need to calculate.

1 Finding what percentage a part is of a whole

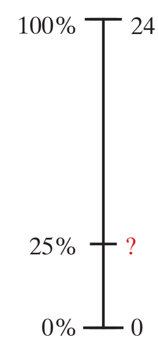
In a class, 6 students out of 24 play a musical instrument. What percentage of the students play a musical instrument?

What percentage is 6 of 24?

2 Finding the size of a part if the percentage is known

In a class of 24, 25% of students play a musical instrument. How many students play a musical instrument?

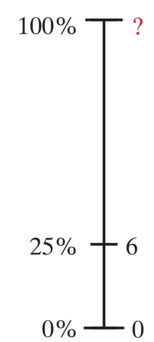
What number is 25% of 24?



3 Finding the whole

In a class, 25% of students play a musical instrument. If 6 students play a musical instrument, how many students are there in the class?

What is the whole if 6 is 25% of the whole?



In this section we will look at problems of the first type. Sections 11.4 and 11.5 deal with the other two types of problems.

To calculate one quantity as a percentage of another, we start by writing the first quantity as a fraction of the other. We then convert the fraction to a percentage in the usual way by multiplying by 100%. To calculate 6 as a percentage of 24, we write:

$$\frac{6}{24} = \frac{\cancel{6}^{1}}{\cancel{24}_{\cancel{4}\,1}} \times \frac{\cancel{100}^{25}\%}{1}$$

$$= 25\%$$

So 6 is 25% of 24.

Example 11

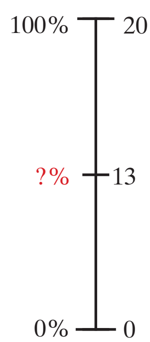
Express these as percentages.

a 13 out of 20 **b** 64 out of 72

Working

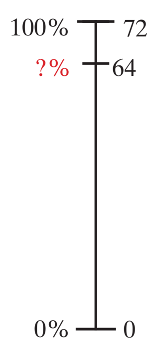
a
$$\frac{13}{\cancel{20}_{1}} \times \frac{\cancel{100}^{5}\%}{1}$$
$$= 13 \times 5\%$$
$$= 65\%$$

Reasoning



Write 13 out of 20 as a fraction. Multiply by 100%.

b
$$\frac{\cancel{64}^{8}}{\cancel{72}_{9}} \times \frac{100\%}{1}$$
$$= \frac{800}{9}\%$$
$$= 88.\overline{8}\%$$



Write 64 out of 72 as a fraction. Multiply by 100%.

Convert the improper fraction to a mixed number.

Example 12

Converting test scores into percentages makes it easier to compare results for different tests. Lou obtained 64 out of 85 for a test. Convert this to a percentage to the nearest whole number percent.

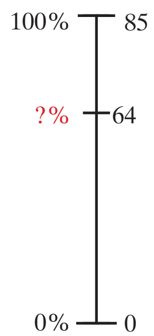
Working

$\frac{64}{85} \times \frac{100\%}{1}$

$\approx 75.29\ldots\%$

Lou obtained 75% for the test.

Reasoning



Write 64 as the numerator of a fraction with 85 in the denominator.
Multiply by 100%.
Calculator:
$64 \div 85 \times 100$
Round to the nearest whole number percent.

When one quantity is expressed as a percentage of another, the two quantities must be in the same unit.

Example 13

Express 27 minutes as a percentage of 3 hours.

Working

3 hours = 180 minutes.

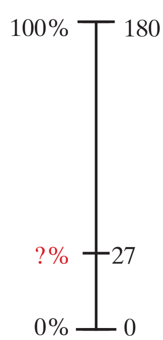
$\frac{27}{180} \times \frac{100\%}{1}$

$= \frac{\cancel{27}^{3}}{\cancel{180}_{\cancel{20}_{1}}} \times \frac{\cancel{100}^{5}\%}{1}$

$= 15\%$

27 minutes is 15% of 3 hours.

Reasoning



Convert 3 hours to minutes.
Write 27 as the numerator of a fraction with 180 in the denominator.
Multiply by 100%.
Simplify and evaluate.

Tech tip

The TI-30XB MultiView calculator can be used to express one quantity as a percentage of another.

For example, to express 13 as a percentage of 20 (example 11a), type:

[1] [3] [n/d] [2] [0] [2nd] [▶] [enter].

Note that [2nd] [▶] gives [▶%].

exercise 11.3

LINKS TO Example 1

- Express each of these as a percentage without using a calculator.

  a 45 out of 100 **b** 62 out of 100 **c** 80 out of 200 **d** 6 out of 10
  e 9 out of 10 **f** 12 out of 50 **g** 140 out of 200 **h** 43 out of 50

LINKS TO Example 11

- Express each of these as a percentage without using a calculator.

  a 12 out of 40 **b** 9 out of 25 **c** 60 out of 150 **d** 36 out of 80
  e 29 out of 50 **f** 18 out of 30 **g** 36 out of 60 **h** 75 out of 125
  i 16 out of 40 **j** 7 out of 20 **k** 120 out of 250 **l** 72 out of 180

LINKS TO Example 12

- Use a calculator to express each of these as a percentage correct to the nearest whole number percentage.

  a 12 out of 45 **b** 7 out of 18 **c** 16 out of 28 **d** 37 out of 70
  e 23 out of 65 **f** 8 out of 92 **g** 35 out of 120 **h** 18 out of 64
  i 13 out of 68 **j** 3 out of 22 **k** 85 out of 240 **l** 94 out of 165

- Fifteen students in class 7B were on an excursion. The other eight were at school. The percentage of the class on the excursion was closest to

  A 53% **B** 47% **C** 65% **D** 35% **E** 15%

- In a school, 48 of the Year 7 students learn Indonesian and 36 learn Italian. The percentage of students learning Italian is closest to

  A 75% **B** 57% **C** 43% **D** 25% **E** 33%

- There are 100 students in Year 7. Twenty-three of them belong to a hockey club.

  a What fraction of the Year 7 students belong to a hockey club?
  b What percentage of the Year 7 students belong to a hockey club?

- There are 25 students in 7F. Nine of these play basketball.

  a What percentage of 7F students play basketball?
  b What percentage of 7F students do not play basketball?
  c One basketball student leaves the school. The new student who comes does not play basketball. What percentage of 7F students play basketball now?

On a plane with 200 passengers, 30 passengers fly first class, 40 passengers fly business class and the rest fly economy class.

a What percentage of the passengers fly first class?

b What percentage of the passengers fly business class?

c What percentage of the passengers fly economy class?

LINKS TO Example 13

Express the first quantity as a percentage of the second quantity.

a 40 minutes as a percentage of 2 hours 40 minutes

b 160 g as a percentage of 0.8 kg

c 50 cm as a percentage of 2.5 m

d 75 cents as a percentage of $4

exercise 11.3 challenge

There are 50 students altogether in 7A and 7B. Twenty-eight of these come to school by bus.

a What percentage of 7A and 7B students come to school by bus?

b What percentage of 7A and 7B students do not come to school by bus?

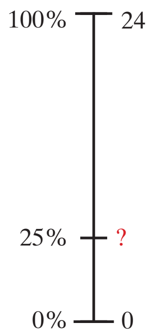
c Twins who normally come by bus are driven to school by their mother on Mondays. How does that change both percentages on Mondays?

11.4 Finding a percentage of a quantity

In this section we look at the second type of problem, where we know how many make up the whole but we want to know how many in a certain percentage of the whole.

25% of students in a class of 24 play a musical instrument. How many students play a musical instrument?

What number is 25% of 24?



To calculate 25% of 24, we write:

$$\begin{aligned} 25\% \text{ of } 24 &= \frac{25}{100} \times \frac{24}{1} \\ &= \frac{\cancel{25}^{\,1}}{\cancel{100}_{\cancel{4}\,1}} \times \frac{\cancel{24}^{\,6}}{1} \\ &= 6 \end{aligned}$$

So, six students in the class play a musical instrument.

In this particular problem, you may have noticed that 25% is equal to $\frac{1}{4}$ and $\frac{1}{4}$ of 24 is 6. We can often calculate percentages of quantities mentally.

Example 14

Calculate these amounts mentally.

a 6% of 400 **b** 8% of 2000

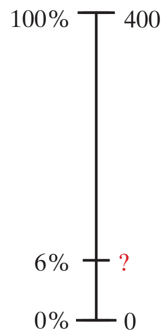
continued

Example 14 continued

Working

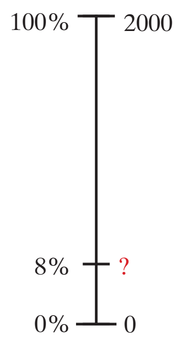
Reasoning

a 6% of 400 $= 6 \times 4$
$= 24$



6% means 6 out of every 100. There are 4 hundreds.

b 8% of 2000 $= 8 \times 20$
$= 160$



8% means 8 out of every 100. There are 20 hundreds.

If there is a simple fraction equivalent of a percentage, this can be used to calculate the percentage of a quantity without a calculator.

Example 15

Use known fraction-percentage equivalents to calculate this amount.

20% of 60

Working

Reasoning

20% of 60 $= \frac{1}{5}$ of 60
$= 12$

We could also calculate 10% of 60 then multiply by 2 because 20% is twice 10%.

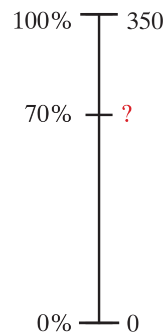
Example 16

Calculate 70% of \$350 without using a calculator.

Working

$$70\% \text{ of } 350 = \frac{70}{100} \times \frac{350}{1}$$
$$= 0.7 \times 350$$
$$= 7 \times 35$$
$$= \$245$$

Reasoning



Change the percentage to a fraction.
Replace 'of' with $\times$.
Simplify by cancelling.

Example 17

Use a calculator to find the following.

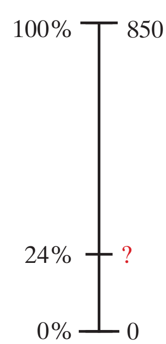
a 24% of 850

b 17.5% of \$435

Working

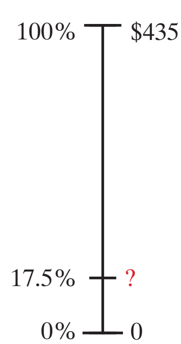
a $$24\% \text{ of } 850 = \frac{24}{100} \times 850$$
$$= 0.24 \times 850$$
$$= 204$$

Reasoning



On the calculator:
$24 \div 100 \times 850$

b $$17.5\% \text{ of } \$435 = \frac{17.5}{100} \times \$435$$
$$= 0.175 \times \$435$$
$$= \$76.13$$



On the calculator:
$17.5 \div 100 \times 435$
Round 76.125 to two decimal places which is equivalent to rounding to the nearest cent.

Shops often have sales where they offer a percentage discount on goods. For example, a sporting goods shop may give 20% discount on all cricket bats at the end of the cricket season. This means that you would pay only 80% (that is, 100% − 20%) of the normal price.

Example 18

A sports equipment shop advertises 20% discount on all goods. The normal price for a cricket bat is $150.

a By how much would the cricket bat be reduced?

b How much would the cricket bat cost?

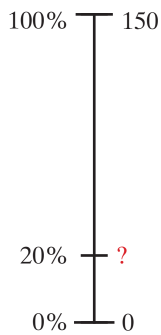
Working / Reasoning

a Discount = 20%

$$= \frac{20}{100} \times \$150$$

$$= 0.2 \times \$150$$

$$= \$30$$



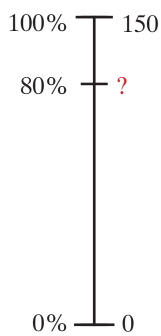
Calculate 20% of the original price.

b Reduced price

= original price − discount

= $150 − $30

= $120



To find the reduced price, subtract the discount from the original price.

Alternatively:

$$\text{Reduced price} = 80\% \text{ of } \$150$$

$$= \frac{80}{100} \times \frac{\$150}{1}$$

$$= \$120$$

Tech tip

The TI-30XB MultiView calculator can find a percentage of a quantity. For example, to find 70% of $350, type:

[7] [0] [2nd] [(] [×] [3] [5] [0] [enter].

Note that [2nd] [(] gives [%].

exercise 11.4

LINKS TO Example 14

Calculate the following amounts without a calculator.

a 10% of 240 **b** 50% of 800 **c** 25% of 16 **d** 5% of 200
e 20% of 50 **f** 100% of 180 **g** 25% of 24 000 **h** 10% of 130

LINKS TO Example 15

Use fraction equivalents to calculate each of these amounts.

a 25% of 84 **b** 20% of 250 **c** 40% of 500 **d** 75% of 400
e $33\frac{1}{3}$% of 240 **f** $66\frac{2}{3}$% of 180 **g** $37\frac{1}{2}$% of 8000 **h** $12\frac{1}{2}$% of 800

LINKS TO Example 16

Calculate the following amounts without a calculator.

a 48% of 75 **b** 56% of 150 **c** 7% of 2400 **d** 36% of 800
e 75% of 960 **f** 18% of 3000 **g** 25% of 384 **h** 12% of 350
i 125% of 200 **j** 240% of 500 **k** 64% of 700 **l** 120% of 900

LINKS TO Example 17

Use a calculator to calculate these amounts. Round to the nearest whole number.

a 24% of 168 **b** 57% of 380 **c** 11% of 270 **d** 15.6% of 6400
e 125% of 560 000 **f** 15% of 904 **g** 14.5% of 1600 **h** 17.5% of 2500
i 0.4% of 254 500 **j** 65% of 840 **k** 7% of 220 **l** 13% of 350
m 72% of 6500 **n** 17.5% of 264 **o** 12.4% of 2200 **p** 18.6% of 5600

A packet of crackers claims to be 96% fat-free.

a What percentage of the crackers is fat?
b How many grams of fat are in the crackers if the total mass of crackers is 250 g?

At Seabank College 47% of students are girls.

a If there are 785 students at the school, how many are girls? Round to the nearest whole number.
b What is the percentage of boys?

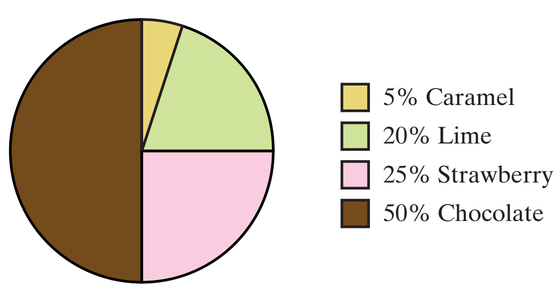
Forty-eight per cent of a 7.5 MB file has been downloaded.

a How many megabytes have been downloaded?
b How many megabytes remain to be downloaded?

Shaun earns $450 per week. He pays 15% tax on his earnings. How much tax does Shaun pay each week?

One hundred and twenty people were asked which of the four ice-cream flavours they liked best. Calculate the number of people who chose each flavour.

Favourite ice-cream flavours



LINKS TO Example 18

A sign on the window says that clothing is to be sold at 10% off the marked price.

a What fraction of the regular price is the discount?

b What fraction of the regular price will be paid after the discount is taken off?

c What will be the discounted price of a hoodie marked at $60?

LINKS TO Example 18

An electronics store advertises discounts of 20% on all games.

a What fraction of the regular price is the discount?

b What fraction of the regular price will be paid after the discount is taken off?

c What will be the discounted price of a game marked at $85?

An advertisement says that sporting equipment is to be sold at $33\frac{1}{3}\%$ off the regular price.

a What fraction of the regular price is the discount?

b What fraction of the regular price will be paid after the discount is taken off?

c How much will a tennis racquet marked at $114 cost in this sale?

The following table shows the 2006 estimated number and percentages of the population of each Australian state or territory living in the capital city. For each state and territory, calculate the number of people living in the capital city. Round each number to the nearest hundred people.

Australian state or territory	Population (2006)	Percentage of population living in the capital city
Australian Capital Territory	344 200	99.6
New South Wales	6 967 200	63
Northern Territory	219 900	54
Victoria	5 297 600	71
Queensland	4 279 400	46
South Australia	1 601 800	73.5
Tasmania	498 200	41
Western Australia	2 163 200	73.4

exercise 11.4 challenge

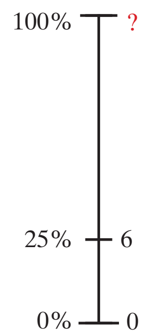
The interest rate on Jack's overdue credit card payment was 0.06328% per day that the payment was overdue. If Jack owed $864 and the payment was overdue by 33 days, how much interest did Jack have to pay?

11.5 Finding the whole

In this section we look at the third type of problem, where we know the amount that represents a certain percentage of the whole, but we do not know the whole amount.

In a class, 25% of students play a musical instrument. If 6 students play a musical instrument, how many students are there in the class?

What is the whole if 6 is 25%?



The easiest way to solve problems of this type is to find the amount that represents 1% of the whole, then multiply by 100 to find 100%.

$$\begin{aligned} 25\% \text{ of students} &= 6 \\ 1\% \text{ of students} &= \frac{6}{25} \\ 100\% \text{ of students} &= \frac{6}{25} \times \frac{100}{1} \\ &= 24 \end{aligned}$$

So there are 24 students in the class.

Example 19

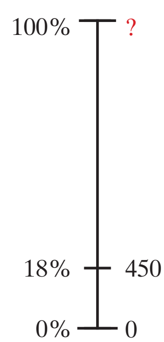
Eighteen percent of a number is 450. What is the number?

Working

$$\begin{aligned} 18\% \text{ of the number} &= 450 \\ 1\% \text{ of the number} &= \frac{450}{18} \\ &= 25 \\ 100\% \text{ of the number} &= 25 \times 100 \\ &= 2500 \end{aligned}$$

The number is 2500.

Reasoning



Find 1% by dividing by 18.

Multiply by 100 to find 100%.

Example 20

Of all the Year 7 students at Alpha Secondary College, 48 play soccer. This represents 40% of the Year 7 students. How many Year 7 students are there altogether at Alpha Secondary College?

Working

40% of Year 7 students = 48

1% of Year 7 students $= \frac{48}{40}$

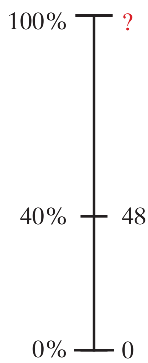
100% of Year 7 students

$= \frac{48}{40} \times \frac{100}{1}$

100% of Year 7 students = 120

There are 120 Year 7 students at Alpha Secondary College.

Reasoning



Find 1% by dividing by 40.

Multiply by 100 to find 100%.

Finding the whole when we know the value of a certain percentage of the whole can be applied to finding the original price when shops advertise percentage discounts. If we know the reduced price and the percentage discount, we can calculate the original price.

Example 21

A shop has a sale where all goods are reduced by 25%. This means that the sale price of goods is 75% of the original price. If a pair of shoes costs \$45 in the sale, what was the original price?

Working

75% of original price = \$45

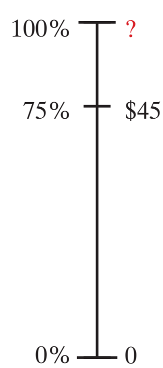
1% of original price $= \frac{45}{75}$

100% of original price $= \frac{45}{75} \times \frac{100}{1}$

$= 60$

The original price of the shoes was \$60.

Reasoning



\$45 is 75% of the original price so divide by 75 to find 1% of the original price.

Multiply by 100 to find 100% of the original price.

Tech tip

The TI-30XB MultiView calculator can find the whole amount when a percentage of a quantity is known.

For example, to find the whole if 70% of the whole is 28, type:

2 8 ÷ 7 0 × 1 0 0 enter.

exercise 11.5

LINKS TO Example 19

1. Find the whole quantity represented by each of these by first finding 1% then finding 100%.
   - **a** 30% of a quantity is 330
   - **b** 80% of a quantity is 960
   - **c** 45% of a quantity is 810
   - **d** 16% of a quantity is 720
   - **e** 32% of a quantity is 1792
   - **f** 9% of a quantity is 747
   - **g** 7% of a quantity is 875
   - **h** 14% of a quantity is 8120
   - **i** 24% of a quantity is 3840
   - **j** 35% of a quantity is 1960
   - **k** 18% of a quantity is 5760
   - **l** 75% of a quantity is 25 875

LINKS TO Example 20

2. In a class of Year 7 students, 15 came to school by bus. This represented 75% of the students in the class. How many students were in the class?

LINKS TO Example 20

3. In the school 8% of students played in the school band. If there were 52 students in the band, what was the total number of students at the school?

4. Of all the Year 7 students, 25% were on an excursion. If there were 48 Year 7 students on the excursion, what was the total number of students in Year 7?

LINKS TO Example 21

5. Students are offered a 12% discount on concert tickets. If the student price is $17.60, what is the full price of tickets?

LINKS TO Example 21

6. A shop has a sale where all goods are reduced by 15%. This means that the sale price of goods is 85% of the original price. If jeans cost $51 in the sale, what was the original price?

LINKS TO Example 21

7. A computer is reduced by 20% to $840.
   - **a** What percentage of the original price is the reduced price?
   - **b** What was the original price?

exercise 11.5 challenge

1. A water reservoir contained 111 930 megalitres of water. This was 39% of its maximum capacity. How many megalitres would it contain if it was 45% full?

11.6 Ratio

A ratio compares two different quantities of the same kind that have the same units. Because the units are the same, ratios do not have units.

Part:part and part:whole ratios

Suppose a class has 12 girls and 13 boys. We can compare the number of girls to the number of boys—we call this a part:part ratio. We can also compare the number of girls to the total number of students in the class. We call this a part:whole ratio.

Part:whole ratio

Number of girls to the total number of students = 12:25.

Part:part ratio

Number of girls to the number of boys = 12:13.

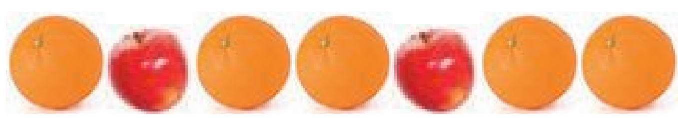
It is important that we write the numbers in the ratio in the same order as the quantities have been stated.

Example 22

Write each of these as a ratio and state whether it is a part:part ratio or a part:whole ratio.

a oranges to the total number of pieces of fruit

b apples to oranges



Working	Reasoning
a oranges to total number of pieces of fruit = 5:7 part:whole	The 5 oranges are being compared with the whole row of fruit.
b oranges to apples = 5:2 part:part	The numbers of oranges and apples are being compared. Oranges and apples represent two parts of the whole row of pieces of fruit.

Example 23

Of the passengers on a bus, 18 were children and 13 were adults.
Write each of these as a ratio and state whether it is a part:part ratio or a part:whole ratio.

a the ratio of children to adults

b the ratio of adults to children

c the ratio of children to the total number of passengers

Working

a children:adults = 18:13 (part:part)

b adults:children = 13:18 (part:part)

c children to total number of passengers = 18:31 (part:whole)

Reasoning

The ratio must be written in the same order that the quantities are stated.

The total number of passengers = 18 + 13 = 31

We can express part:whole ratios as fractions and percentages.

Example 24

a What is the ratio of cordial to water in this drink?

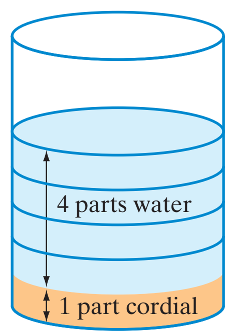
b What is the ratio of water to cordial?

c What is the ratio of cordial to the total number of parts of the drink?

d What fraction of the drink is cordial?

e What fraction of the drink is water?

f What percentage of the drink is water?



Working

a cordial:water = 1:4

b water:cordial = 4:1

c cordial to total drink = 1:5

d fraction of cordial in the drink $= \frac{1}{5}$

e fraction of water in the drink $= \frac{4}{5}$

f $\frac{4}{5} = 80\%$
80% of the drink is water.

Reasoning

a, b There is 1 part of cordial and 4 parts of water.

c Altogether there are 5 parts.

d 1 part out of 5 is cordial.

e 4 parts out of 5 are water.

f $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1}\%$

Before quantities can be written as a ratio, the quantities must be expressed in the same measurement units.

Example 25

Rewrite each of these as a ratio.

a 7 months to 1 year

b 47 cm to 1 m

Working	Reasoning
a 7 months to 1 year = 7 months to 12 months. The ratio is 7:12	Convert 1 year to 12 months. A ratio does not have units.
b 47 cm to 1 m = 47 cm to 100 cm The ratio is 47:100	Convert 1 m to 100 cm. A ratio does not have units.

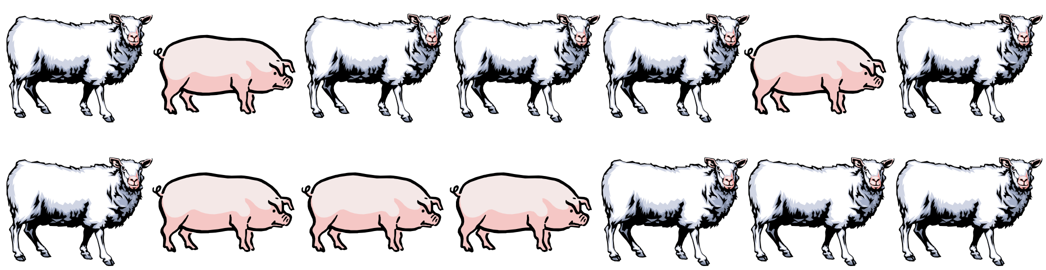
exercise 11.6

LINKS TO Example 22

1. Use this picture of sheep and pigs to answer the questions.

a What is the ratio of pigs to sheep?

b What is the ratio of sheep to pigs?



c What fraction of the animals are pigs?

d What fraction of the animals are sheep?

LINKS TO Example 22

2. 

The ratio of apples to the total number of pieces of fruit is

A 5:4 **B** 9:5 **C** 4:9 **D** 5:9 **E** 4:5

3. Write each of the following statements as a ratio. Remember that ratios do not have units.

a On a school excursion there was one teacher for every ten students.

b There were 12 girls and 11 boys in the class.

c Jack and Ted shared the profits equally.

d The instructions on the packet of rice said to use 2 cups of water for 1 cup of rice.

LINKS TO Example 23

To mix a certain color of paint, Will mixes 5 litres of red paint and 2 litres of blue paint.

a What is the ratio of red paint to the total amount of paint?

b What is the ratio of red paint to blue paint.

c What fraction of the paint mixture is red paint?

d What fraction of the paint mixture is blue paint?

LINKS TO Example 24

a What is the ratio of chocolate muffins to blueberry muffins?

b What is the ratio of chocolate muffins to the total number of muffins?

c What fraction of the muffins are chocolate?

d What fraction of the muffins are blueberry?

LINKS TO Example 24

a What is the ratio of children to adults?

b What is the ratio of children to the total number of people?

c What fraction of the people are children?

d What percentage of the people are children?

A concrete mix contains 3 parts of sand, 2 parts of gravel and 1 part of cement.

a Write this as a ratio.

b What is the ratio of cement to sand?

c What is the ratio of gravel to sand?

d What is the ratio of cement to concrete?

e What fraction of the concrete is cement?

LINKS TO Example 25

The length of a rectangle is 2 m and the width is 77 cm. What is the ratio of length to width of the rectangle?

LINKS TO Example 25

In an Australian rules match, the Magpies scored 5 goals and the Roosters scored 3 goals and 5 behinds. Express the scores for the Magpies and Roosters as a ratio.

LINKS TO Example 25

Emma took 5 minutes to walk home. Hannah took 3 minutes 17 seconds. Express their times as a ratio.

exercise 11.6 challenge

Great Aunt Ermyntrude is giving $1700 to her great niece and nephew in proportion to their ages. Nancy is 5, Jack is 9 and Emily is 11. How much will they each receive?

11.7 Simplifying ratios

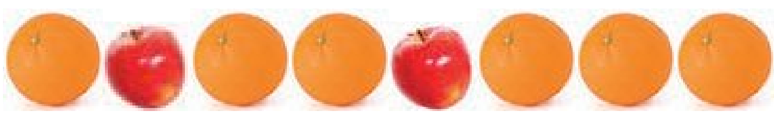
Ratios are usually written in their simplest form. We can divide both sides of a ratio by a common factor to produce an equivalent ratio. We can compare this with simplifying a fraction by dividing the numerator and denominator by a common factor to produce an equivalent fraction.

Example 26

Express these ratios in their simplest form.

a apples to oranges

b oranges to apples



Working

a apples to oranges
2:6
= 1:3

Reasoning

The numbers 2 and 6 have a common factor of 2.
Divide both sides of the ratio by 2.
There is 1 apple for every 3 oranges.

Working

b oranges to apples
6:2
3:1

Reasoning

Divide both sides of the ratio by 2.
There are 3 times as many oranges as apples.

Example 27

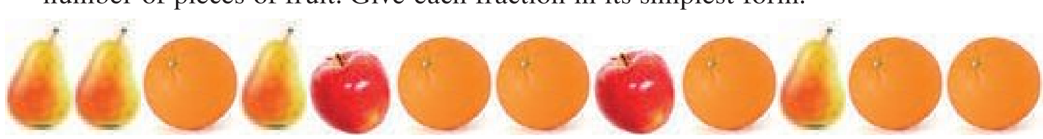
Express these ratios in their simplest form.

a pears to oranges

b pears to apples

c pears to the total number of fruit

d For each of the three different types of fruit, calculate the fraction of the total number of pieces of fruit. Give each fraction in its simplest form.



Working

a Pears to oranges
4:6
= 2:3

Reasoning

The numbers 4 and 6 have a common factor of 2.
Divide both sides of the ratio by 2.
There are 2 pears for every 3 oranges.

continued

Example 27 continued

Working	Reasoning
b Pears to apples 4:2 = 2:1	Divide both sides of the ratio by 2. There are 2 pears for every apple.
c Pears to total pieces of fruit 4:12 = 1:3	One-third of the pieces of fruit are pears.
d Apples: $\frac{2}{12} = \frac{1}{6}$	There are 12 pieces of fruit, 2 of which are apples.
Oranges: $\frac{6}{12} = \frac{1}{2}$	There are 12 pieces of fruit, 6 of which are oranges.
Pears: $\frac{4}{12} = \frac{1}{3}$	There are 12 pieces of fruit, 4 of which are pears.

Example 28

In a Year 7 class there are 12 boys and 10 girls.

a What is the ratio of boys to girls?

b What is the ratio of boys to the total number of students in the class?

Working	Reasoning
a Ratio of boys:girls = 12:10	Because we are looking at the ratio of boys to girls, rather than girls to boys, we must write 12:10, not 10:12.
= 6:5	Divide by common factor of 2.
b Boys to total number of students 12:22 = 6:11	There is a total of 22 students in the class. 6 out of each 11 students are boys.

Ratios are equivalent if they can be simplified to the same ratio.

Example 29

Which of these ratios are equivalent?

3:4 18:24 16:20 30:45

continued

Example 29 continued

Working

18:24 = 3:4
16:20 = 4:5
30:45 = 2:3
The equivalent ratios are 3:4 and 18:24.

Reasoning

Simplify all the ratios using the TI-30XB MultiView calculator.

[1] [8] [n/d] [2] [4] [enter]

[1] [6] [n/d] [2] [0] [enter]

[3] [0] [n/d] [4] [5] [enter]

Example 30

Gerard mixed 250 mL of cordial with 1 L of water. What was the ratio of cordial to water?

Working

Ratio of cordial to water = 250:1000
= 25:100
= 1:4

Reasoning

1 litre of water is 1000 millilitres.
Divide by the common factor of 10.
Divide by the common factor of 25.

Ratios can be used to find what percentage a part is of the whole.

Example 31

In a group of Year 7 students, 72 students learn Indonesian and 84 learn Italian. What percentage of students learn Indonesian?

Working

Ratio of students learning Indonesian to total number of students
= 72:156

Reasoning

Total number of students = 72 + 84 = 156

Working

Fraction of students learning Indonesian

$$= \frac{72}{156}$$

Reasoning

72 out of 156 students learn Indonesian.

Working

Percentage of students learning Indonesian

$$= \frac{72}{156} \times \frac{100}{1}\%$$

Approximately 46% of students learn Indonesian.

Reasoning

Using the TI-30XB MultiView calculator, type:

[7] [2] [÷] [1] [5] [6] [2nd] [▸%] [enter]

or

[7] [2] [÷] [1] [5] [6] [×] [1] [0] [0] [enter].

exercise 11.7

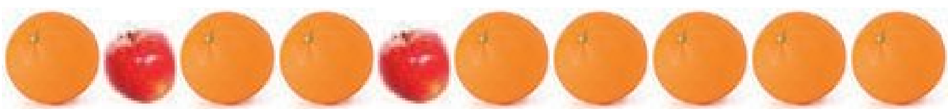
LINKS TO Example 26

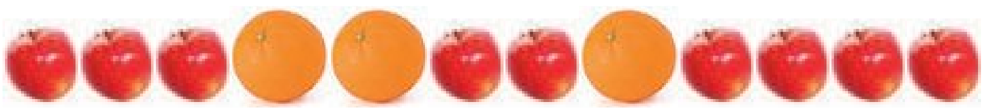
1 Write each of these ratios in simplest form.

i apples to oranges

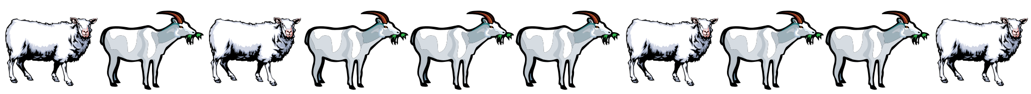
ii oranges to apples

iii oranges to the total number of pieces of fruit

a 

b 

2 a What is the ratio, in its simplest form, of sheep to goats?



b What fraction of the animals are sheep?

c What fraction of the animals are goats?

3 Write each of these ratios in their simplest form.

a 3:9	b 6:24	c 10:25	d 35:14
e 70:30	f 56:32	g 17:34	h 15:18
i 81:36	j 66:48	k 14:42	l 36:45
m 54:72	n 140:35	o 91:26	p 132:48

LINKS TO Example 28

4 

a Express the ratio of red balloons to blue balloons to white balloons in simplest form.

b What is the ratio of red balloons to the total number of balloons?

c What fraction of the balloons are red?

d What is the ratio of red balloons to white balloons?

e What is the ratio of red balloons to blue balloons?

11.7

LINKS TO Example 29

Complete these equivalent ratios.

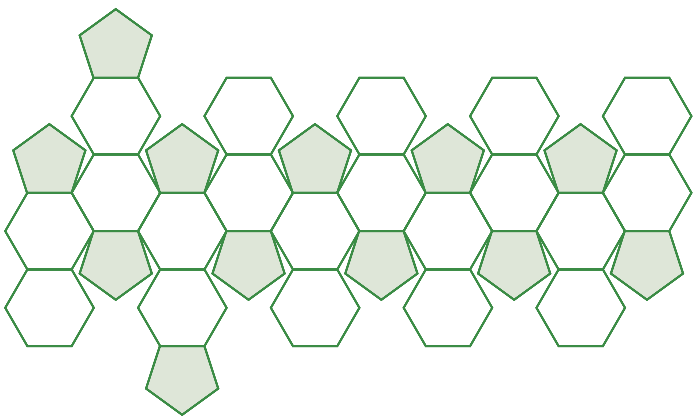
a 2:3 = 4:__ = __:12
b 24:32 = 3:__ = 9:__
c 5:6 = 15:__ = __:30
d 12:28 = 3:__ = __:21
e 9:5 = 36:__ = __:40
f 21:35 = __:20 = 6:__
g 3:7 = 15:__ = __:21
h 16:40 = __:30 = 10:__
i 45:27 = 25:__ = __:30
j 72:108 = 14:__ = __:3

The ratio 24:40 is equivalent to

A 1:64 **B** 3:5 **C** 2:3 **D** 3:8 **E** 40:24

This is a net for the truncated icosahedron on which the soccer ball is based.

a What is the ratio of pentagons (5 sides) to hexagons (6 sides)?
b What fraction of the shapes are pentagons?
c What fraction of the shapes are hexagons?



LINKS TO Example 30

Find the ratio in each of the following. Remember: (i) convert quantities to the same unit, (ii) write as a ratio without units, (iii) write the ratio in its simplest form.

a 400 g to 1 kg
b 600 mL to 1 L
c 750 mm to 1 m
d 45c to $1
e 250 kg to 1 tonne
f 60 cm to 2.4 m
g 24c to $2
h 40 seconds to 2 minutes

LINKS TO Example 30

A juice carton contains 225 mL of apple juice. A bottle of apple juice contains 2 L. What is the ratio of juice in the carton to juice in the bottle?

A 2:225 **B** 9:80 **C** 9:40 **D** 8:25 **E** 8:9

What can the ratio 156:132 be simplified to?

A 132:24 **B** 11:13 **C** 13:11 **D** 24:11 **E** 24:13

LINKS TO Example 31

a A concrete mixture contains 10 kg of cement, 20 kg of sand and 25 kg of gravel. Write this as a ratio in its simplest form.
b What fraction of the mixture is cement?
c What is the percentage of cement in the concrete mixture? Give to the nearest whole number percentage.

exercise 11.7 challenge

1. For a particular concert, an orchestra had the following instruments.

Strings		Woodwind		Brass		Percussion	
Violin	32	Piccolo	1	Trumpet	4	Drums, etc.	3
Viola	10	Flute	3	Trombone	4	Timpani	2
Cello	10	Clarinet	3	French horn	6		
Double bass	8	Oboe	3	Tuba	1		

Find the following ratios (express in their simplest form).

a strings to woodwind instruments
b strings to brass instruments
c strings to woodwind to brass
d violins to other stringed instruments
e French horns to other brass instruments
f stringed instruments to the rest of the orchestra
g strings to woodwind to brass to percussion

2. **a** Australian 5, 10 and 20 cent coins are composed of 75 parts nickel to 25 parts copper. Write the ratio of nickel to copper in its simplest form.
b What is the percentage of copper in 5, 10 and 20 cent coins?
c Australian $1 and $2 coins are composed of 2 parts nickel, 6 parts aluminium and 92 parts copper. Write the ratio in its simplest form.
d What is the percentage of copper in $1 and $2 coins?

e Brass is a mixture of copper and zinc in the ratio 7:3. What is the percentage of copper in brass?
f Bronze is a mixture of copper and tin in the ratio 9:1. What is the percentage of copper in bronze?

Analysis task

Pizza percentages

Look at the cartoon and answer the questions.

a Which fraction can be simplified? Express this fraction in its simplest form.

b What fraction of the pizza had extra cheese?

c Record the following conversions in a table as shown.

i Convert each of the fractions to equivalent fractions with the lowest common denominator.

ii Use your calculator to convert each fraction to a decimal.

iii Convert the decimals to percentages.

"I'll take a large pizza with half-onion, two-thirds olives, nine-fifteenths mushrooms, five-eights pepperoni, one-eighth anchovies, and extra cheese on five-ninths of the onion half."

Topping	Fractions expressed with lowest common denominator	Decimal	Percentage
Onion			
Olives			
Mushrooms			
Pepperoni			
Anchovies			
Extra cheese			

d Which topping covers the greatest part of the pizza?

For each of the following questions, include a diagram and percentages to help explain your answer.

e Did any of the pizza have both onion and olives?

f Could any of the pizza have had both anchovies and onions?

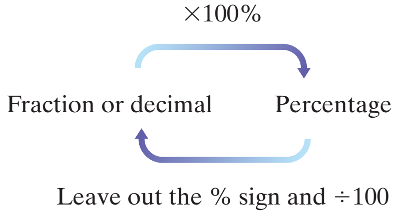
g Could any of the pizza have had onion, pepperoni and mushrooms?

h What is the largest percentage of the pizza that could have had 'the lot'?

Review *Percentages and ratios*

Summary

- Percentage means ‘out of 100’
- Converting between fractions, decimals and percentages
- Three different types of calculations involving percentages



1 Finding a percentage In a class of 24, 6 students play a musical instrument. What percentage of the students play a musical instrument?	What percentage is 6 of 24?
2 Finding a part In a class of 24, 25% of students play a musical instrument. How many students play a musical instrument?	What number is 25% of 24?
3 Finding the whole In a class, 25% of students play a musical instrument. If 6 students play a musical instrument, how many students are there in the class?	What is the whole if 6 is 25% of the whole?

Ratio

- A ratio is a comparison of two or more quantities with the same unit. A ratio does not have units.

Equivalent ratios

- Ratios are equivalent if they can be simplified to the same ratio, e.g. 4:7 and 8:14 are equivalent ratios.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key concepts covered in this chapter.

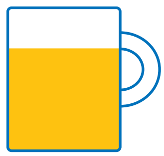
decimal	improper fraction	part:whole ratio
denominator	mixed number	percentage
discount	numerator	ratio
equivalent ratio	part	whole
fraction	part:part ratio	

Revision

Multiple-choice questions

What approximate percentage of orange juice is in the mug?

A 0 – 20%
B 21 – 40%
C 41 – 60%
D 61 – 80%
E 81 – 100%



What is $17\frac{1}{2}\%$ when expressed as a fraction in its simplest form?

A $\frac{7}{40}$ **B** $\frac{17}{100}$ **C** $\frac{17\frac{1}{2}}{100}$ **D** $\frac{17.5}{100}$ **E** $\frac{35}{200}$

6% can be written as

A 0.6 **B** $\frac{6}{10}$ **C** 6.0 **D** $\frac{1}{600}$ **E** 0.06

15% of \$400 =

A \$15 **B** \$40 **C** \$60 **D** \$150 **E** \$600

The measurement 24 metres is what percentage of 40 metres?

A 24% **B** 40% **C** 48% **D** 60% **E** 80%

Short-answer questions

Complete this table of percentage equivalents.

Percentage	Fraction	Decimal
50%		
	$\frac{3}{4}$	
		0.4
17%		
	$\frac{2}{3}$	
		0.06

A commission of 4% was paid to the salesperson when a \$20 000 car was sold. How much was this commission?

The price of a wide screen television was reduced by 15% in the June stocktaking sale. If the television was originally \$2500, what was its sale price?

30% of a bag of nuts weighs 66 grams. What was the weight of the nuts when the bag was full?

Hannah scored 17 out of 20 for her first assignment and 26 out of 30 for her second assignment.

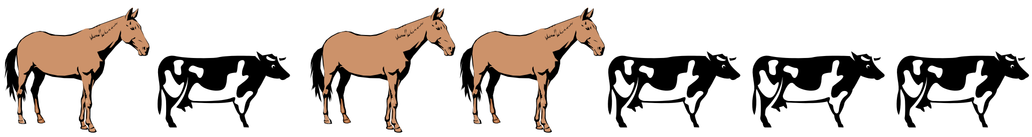
a What was her first score as a percentage?

b What was her second score as a percentage?

c Which of her scores was better?

Extended-response questions

Use this drawing of horses and cows to answer the questions.



a What is the ratio of cows to horses?

b What is the ratio of horses to cows?

c What is the ratio of cows to the total number of animals?

d What fraction of the animals are cows?

e What fraction of the animals are horses?

Use this collection of red and blue pens to answer the questions.

a What is the ratio in simplest form of blue pens to red pens?

b What is the ratio in simplest form of red pens to the total number of pens?

c What fraction of the pens are blue?

d What fraction of the pens are red?

e What percentage of the pens are blue?

f What percentage of the pens are red?

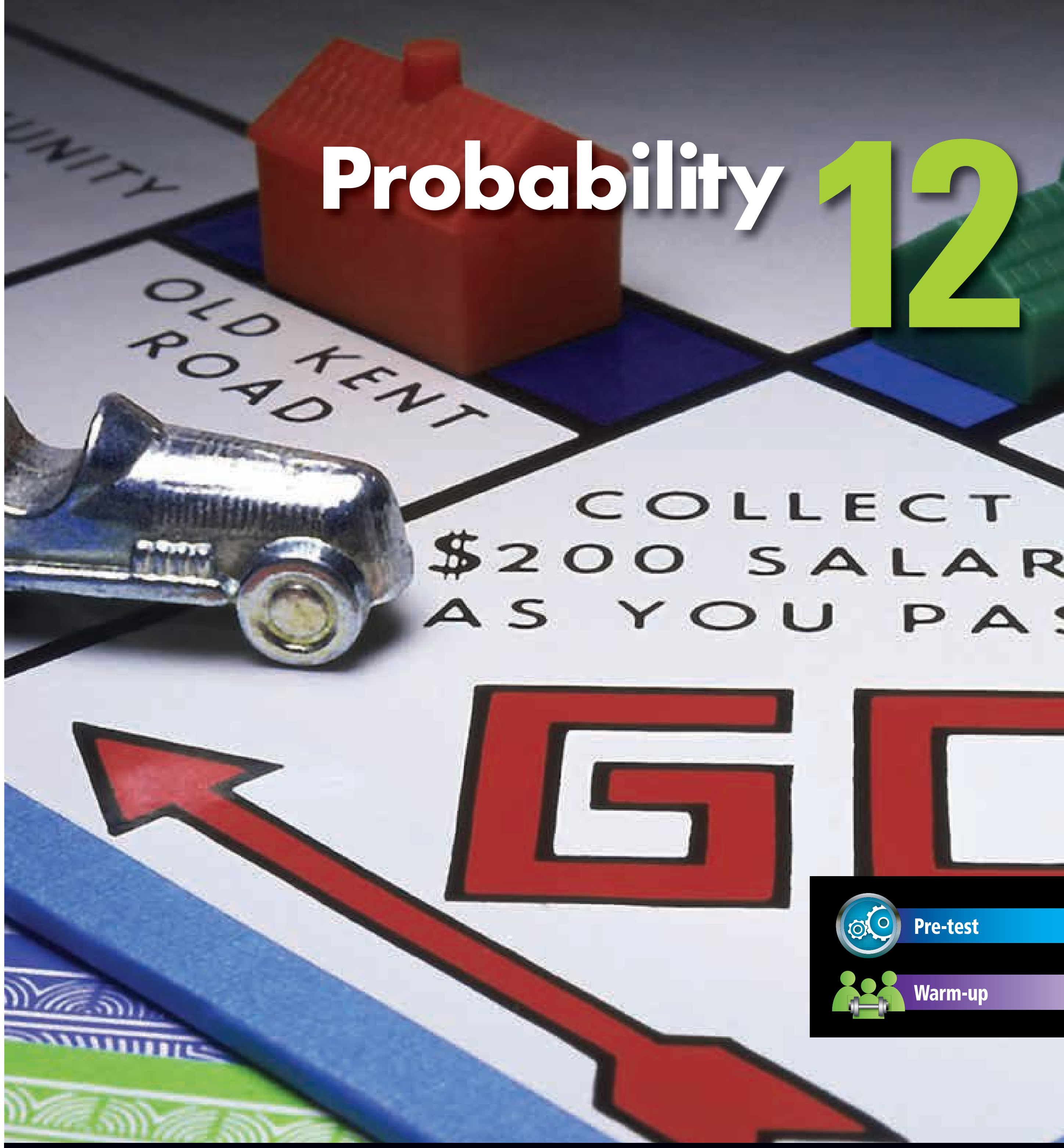
Write the ratio 72:45 in its simplest form.

Write these quantities as ratios in simplest form.

a 64 mm to 8 cm

b 2.7 L to 150 mL

Probability 12



Pre-test

Warm-up

Dice, coins, spinners and cards are used to introduce chance and risk into games. In many games where probability or chance is involved, no individual result or outcome is more likely than another. For example, when a die is rolled, each of the six numbers is an equally likely outcome. However, the outcomes are uncertain and the games are therefore somewhat unpredictable and fun! The term ‘random’ is often used to describe such situations where the outcomes are uncertain.

12.1 The language of chance

Probability gives a measure of how likely something is to happen. In everyday life there are many things where the chance of certain things happening can range from impossible to certain, with various levels of possibility in between. The mathematics of probability can often help us make predictions about how likely a particular outcome may be.

Example 1

For each of the following events, assign a 'chance' word to indicate the likelihood of it occurring. Use the words *impossible, unlikely, possible, likely* or *certain*.

a A 7 is thrown on a standard six-sided die.

b Ten coins are flipped and they all come up heads.

c There will be a story on the TV news tonight involving crime.

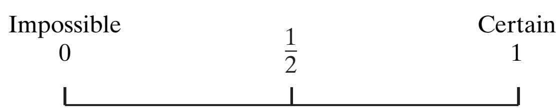
d In a family of 3 children, 2 will be boys.

e A head or a tail occurs when a coin is flipped.

Working	Reasoning
a impossible	Only the numbers 1 to 6 are possible.
b unlikely	It can happen, but it would not happen very often.
c likely	There is usually at least one TV story each night involving a crime.
d possible	Boys and girls are born in roughly equal numbers, so in a family of 3 children, 2 could be boys.
e certain	Assuming that the tossed coin will not land on its edge, it is certain to be either a head or a tail.

Probabilities are expressed as numbers between 0 and 1.

- A probability of 0 means that the event is **impossible**.
- A probability of 1 means that the event is **certain** to happen.
- A probability of $\frac{1}{2}$ means that there is an **equal chance** of the event happening or not happening.



Probabilities can be expressed as fractions or decimals between 0 and 1 inclusive. Probabilities can also be expressed as percentages. A probability of 100% means that the event is certain to happen.

Example 2

Express each of these probabilities as a decimal and as a percentage.

a $\frac{1}{2}$

b $\frac{1}{3}$

Working

a $\frac{1}{2} = 0.5 = 50\%$

b $\frac{1}{3} = 0.\overline{3} = 33\frac{1}{3}\%$

Reasoning

50% means 50 out of 100, which simplifies to $\frac{1}{2}$.

$\frac{1}{3}$ is equal to the recurring decimal 0.33…

In everyday language there are many expressions relating to how likely things are to happen.

Example 3

Match each of these expressions with the probabilities 0, $\frac{1}{2}$ or 1.

a It's a dead cert.

b You've got Buckley's chance.

c It's 50-50 whether I go or not.

d I'm 100% sure I saw it.

Working

a 1

b 0

c $\frac{1}{2}$

d 1

Reasoning

Dead cert means that it is certain to happen.

Sometimes this could mean there is only a very small chance, but normally it means there is no chance.

There is an equal chance of something happening or not happening.

100% is a whole, so this corresponds to certainty, that is, a probability of 1.

Example 4

Place these statements along the number line according to how likely something is to happen.

a I'm 90% sure to go.

b There's an even chance I'll go.

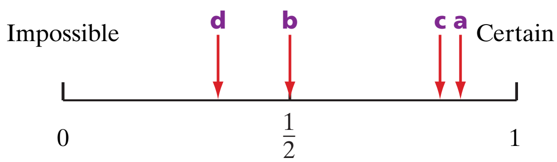
c Odds on I'll go.

d There's a 1 in 3 chance I'll go.

continued

Example 4 continued

Working



Reasoning

Certain means 100% sure, so 90% sure is 9 tenths along the line from 0 to 1.
Even chance means equal chance of going or not going.
Odds on means it is very likely.
A 1 in 3 chance is one-third of the way along the line from 0 to 1.

exercise 12.1

LINKS TO Example 1

For each of the following statements, assign one of these words: impossible (I), unlikely (U), possible (P), likely (L), certain (C).

- **a** You will shrink by 30 cm in the next year.
- **b** You will grow by 10 cm in the next year.
- **c** Sydney Harbour will freeze next summer.
- **d** Birdsville will be the largest city in Australia by 2015.
- **e** The day after Wednesday will be Thursday.
- **f** Melbourne will have a larger population than Sydney by 2020.
- **g** If you throw a standard die (with numbers from 1 to 6) you will get a number greater than 2.
- **h** If you throw a standard die, you will get a number less than 7.

One die, two dice —'die' is the singular of 'dice'

The probability of a six-sided die coming to rest on one of its corners is best described as

A impossible. **B** unlikely. **C** possible. **D** likely. **E** certain.

LINKS TO Example 2

Complete this table of probabilities.

Fraction	Decimal	Percentage
$\frac{1}{10}$		
		20%
	0.25	
		40%
$\frac{1}{2}$		
		99%

LINKS TO Example 3

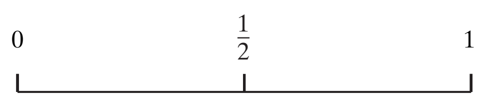
Find out the meanings of each of these expressions then match each expression with the probabilities 0, $\frac{1}{2}$ or 1.

a There's not a snowflake's hope in hell of that.
b All over bar the shouting.
c As good as done.
d She's 50–50 to win that race.
e Nothing surer.
f Lightning never strikes in the same place twice.

LINKS TO Example 4

Below are some more common expressions that relate to chance. Rule a 10 cm line and label 0, $\frac{1}{2}$ and 1 as shown. Place statements a to f along the line from 0 to 1 according to how likely you think they mean something will happen.

a I go to a movie once in a blue moon.
b I might tidy my room today but don't hold your breath.
c I've got an even chance of getting into the team.
d It's looking good for the Hawks winning on Saturday.
e Yeah, and pigs might fly.
f There's an 80% chance I'll come.



Write five statements of your own, similar to those in question 5 and place them along a line from 0 to 1 to show how likely something is to happen.

exercise 12.1 challenge

Place these words along a number line from 0 to 1 according to the probability you think they represent. Compare your number line with those of other students in your class. Explain why these words can be confusing.

always
occasionally
frequently
sometimes
rarely
often
never
usually

12.2 Predicting probability

Sample space

When we consider the probability of a certain outcome, we refer to the set of all possible outcomes or the set of things that can happen as the **sample space**.

Example 5

List the sample space for

a one roll of a standard six-sided die.

b selecting one card from a standard deck of playing cards and noting its colour.

Working	Reasoning
a $S = \{1, 2, 3, 4, 5, 6\}$	The sides of the die are numbered 1 to 6. So with any one roll you get one of the numbers 1, 2, 3, 4, 5 or 6.
b $S = \{\text{red, black}\}$	The colour of the card is being noted in this experiment. In a deck of 52 playing cards, the cards are either red (hearts and diamonds) or black (clubs and spades).

Theoretical probability

When we toss a coin, there are two possible outcomes: 'Heads' or 'Tails'. We have a chance of 1 in 2 of getting a head and a chance of 1 in 2 of getting a tail.

We write this as Probability (Head) $= \frac{1}{2}$. This is usually shortened to Pr(Head) $= \frac{1}{2}$.

Similarly, we can write Pr(Tail) $= \frac{1}{2}$.

Pr(Tail) $= \frac{1}{2}$ means the probability of the coin landing tails up is 1 out of 2.

In probability language, we use the word **event** to describe a particular set of outcomes that we are interested in. When we roll a die, there are many different events we could be interested in, such as

- getting a 6.
- getting an even number.
- getting a prime number.
- getting a number divisible by 3.

For some of these events, there is more than one outcome that would result in the event happening. If the event is 'getting an even number', there are three different outcomes that would meet this requirement: 2, 4 or 6. For the event 'getting a prime number' there are also three outcomes: 2, 3, or 5.

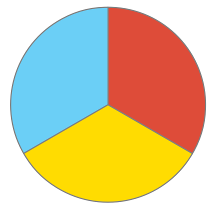
The outcomes that satisfy the requirements of a particular event are referred to as **favourable outcomes**. If the event is 'getting an even number', the favourable outcomes are 2, 4 or 6.

The probability that an event occurs is given by

$$\text{Pr(Event)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Example 6

A spinner is divided into three equal sectors as shown.



a What is the probability that it lands on the red sector?

b If the spinner was spun six times, how many times would you *expect* it to land on the red sector?

c If the spinner was spun six times, what are the possible number of times it *could* land on the red sector?

Working	Reasoning
a The spinner has a chance of 1 out of 3 of landing on red.	One-third of the spinner is coloured red.
Pr(Landing on red) $= \frac{1}{3}$	The sample space, S = {red, blue, yellow} and each of these three possible outcomes is equally likely.
b We expect that it will land on red on $\frac{1}{3}$ of all spins. Therefore, in six spins, we would expect it to land on red $\frac{1}{3}$ 6 = 2 times.	We expect it would land on the red sector in 2 out of 6 rolls.
c It could land on red 0, 1, 2, 3, 4, 5, or 6 times.	All of these answers are possible.

Example 7

A standard die is rolled once. Determine the probability of

a rolling a 3.

b rolling a number greater than 2.

c not rolling a 5.

continued

Example 7 continued

Working

a There is one favourable outcome: 3.
There are six possible outcomes: 1, 2, 3, 4, 5, 6.

$$\Pr(3) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$$= \frac{1}{6}$$

Reasoning

The sample space of all possible outcomes, $S = \{1, 2, 3, 4, 5, 6\}$.

b There are 4 favourable outcomes: 3, 4, 5, 6.
Pr(Number > 2)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

Numbers on the die that are greater than 2 are 3, 4, 5 and 6.
The sample space of all possible outcomes, $S = \{1, 2, 3, 4, 5, 6\}$.

Simplify the fraction.

c There are five favourable outcomes: 1, 2, 3, 4, 6.
There are six possible outcomes.
Pr(not 5)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$$= \frac{5}{6}$$

Numbers on the die that are not 5 are 1, 2, 3, 4 and 6.
The sample space of all possible outcomes, $S = \{1, 2, 3, 4, 5, 6\}$.

In some probability experiments, the outcomes are not all equal. For example, in a bag of red and blue marbles, there may be more red marbles than blue. The probability of taking a red marble out of the bag will then be greater than taking out a blue marble.

Example 8

Consider a game in which a student is blindfolded, and then asked to select one of the cards on the board shown below.

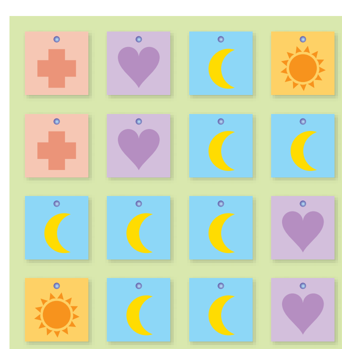
a What is the total number of possible outcomes?

b How many different types of cards are there?

c Does the student have an equal chance of selecting each type of card? Explain.

d How many moon cards are there?

e Use your answers to parts a and d to calculate the probability of selecting a moon card.



continued

Example 8 continued

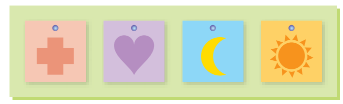
Working

a There are 16 cards so there are 16 possible outcomes.

Reasoning

The total number of cards is 16. Even though some of the cards are the same, each of the 16 cards has an equal chance of being selected.

b There are four different types of card.



c Each of the 16 cards is equally likely to be selected, but the chances of selecting particular types of cards (e.g. heart) are not equally likely, as there are different numbers of cards of each type (e.g. 4 hearts but 8 moons).

The chance of selecting a particular type of card depends on how many of that type of card there are compared with the total number of cards.

d Number of moon cards = 8

e Pr(Selecting a moon card)

$$= \frac{\text{Number of moon cards}}{\text{Total number of cards}}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

Because there are 8 moon cards and 16 cards altogether, theoretically we would expect the student to select a moon card on about half the occasions.

We write fractions in their simplest form.

$$\frac{8}{16} = \frac{1}{2}$$

exercise 12.2

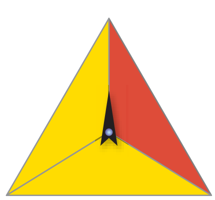
LINKS TO Example 5

1 For each of these spinners

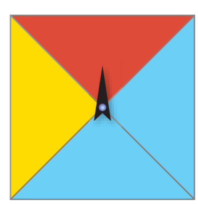
i list the sample space of all the possible outcomes.

ii state the total number of possible outcomes.

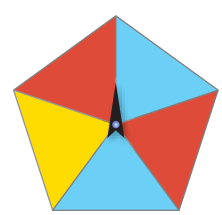
a



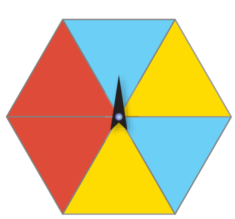
b



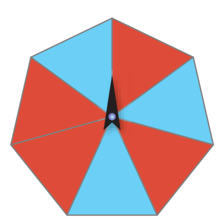
c



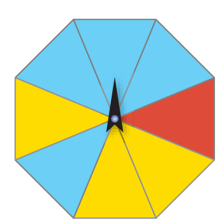
d



e



f



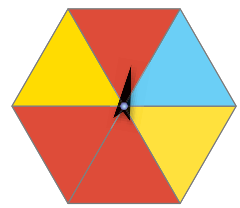
LINKS TO Example 6

For each of the spinners in question 1, calculate the probability of the spinner landing on

i red. **ii** yellow. **iii** blue.

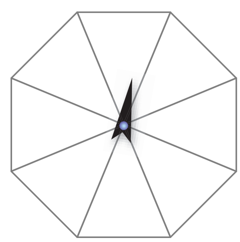
For the spinner below, complete the following table.

Colour	Number of sides	Probability
Red	3	
Yellow		$\frac{1}{3}$
Blue		$\frac{1}{6}$
Total		



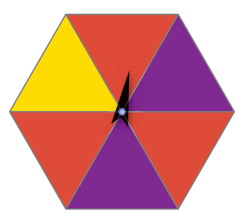
Copy the spinner in your book and colour the spinner so that $Pr(red) = \frac{1}{2}$, $Pr(blue) = \frac{1}{4}$

$Pr(yellow) = \frac{1}{8}$ and $Pr(white) = \frac{1}{8}$



LINKS TO Example 7

The probability that this spinner shown does not land on a purple segment is



A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** $\frac{5}{6}$

LINKS TO Example 7

A standard die is rolled.

a List all the possible outcomes.

b How many outcomes are odd numbers?

c What is the probability of rolling an even number?

Daniel rolls an eight-sided die once.

a What is the total number of possible outcomes?

b Determine the probability of rolling the following.

i a 7 **ii** an even number **iii** a prime number

iv a number greater than 3 **v** a number less than 4

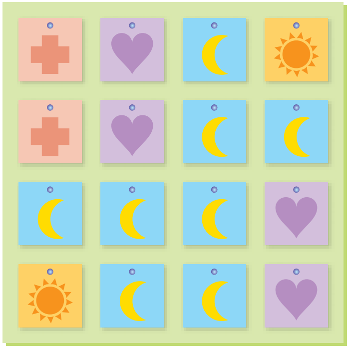
A 12-sided die is rolled. What is the probability of obtaining

a a number less than 8? **b** a multiple of 3?

c an even number? **d** an even number divisible by 3?

LINKS TO Example 8

Emma is blindfolded then asked to select one of the cards from the board shown.



a Is the probability of her selecting a heart card the same as the probability of her selecting a sun card? Give a reason for your answer.

b Are there two events which have the same probability? Give a reason for your answer.

c What is the probability of her selecting a heart card?

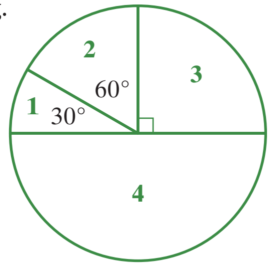
d What is the probability of her not selecting a sun card?

For the spinner shown, calculate the probability of the following.

a scoring a 2

b scoring a 1 or a 3

c not scoring a 3



A student selects a card from a standard deck of 52 cards.

What is the probability that it is

a a heart?

b a jack?

c a red card?

d a red card or a jack?

e a red card and a jack?

Clubs	Spades	Diamonds	Hearts
♣K	♠K	♦K	♥K
♣Q	♠Q	♦Q	♥Q
♣J	♠J	♦J	♥J
♣10	♠10	♦10	♥10
♣9	♠9	♦9	♥9
♣8	♠8	♦8	♥8
♣7	♠7	♦7	♥7
♣6	♠6	♦6	♥6
♣5	♠5	♦5	♥5
♣4	♠4	♦4	♥4
♣3	♠3	♦3	♥3
♣2	♠2	♦2	♥2
♣A	♠A	♦A	♥A

Look at the following photograph of a hand of cards.

If you were to pick a card from this hand without looking at the cards, what is the probability that you would pick a card that is

a a heart?

b a court card (a king, queen or jack)?

c from a black suit?

d either a black card or a diamond less than a 5?

exercise 12.2 challenge

A class of 24 students was asked how many children there were in their family. The following results were obtained.

2, 3, 3, 2, 2, 1, 3, 3, 2, 2, 2, 2, 2, 3, 3, 4, 2, 2, 2, 1, 4, 5, 1, 2.

What is the probability that a student selected at random would have

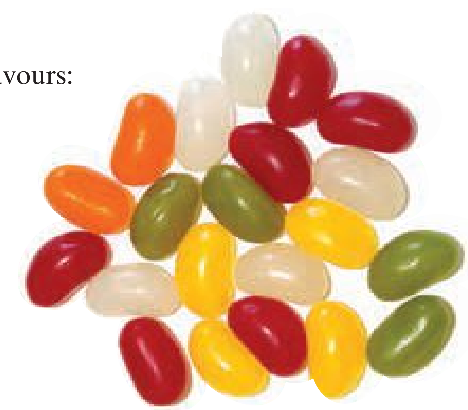
a 4 children in their family?

b an even number of children in their family?

c more than 3 children in their family?

d at most 2 children in their family?

A pack of 21 jelly beans contains the following flavours: vanilla, lemon, orange, lime, and cherry. The beans were placed into a paper bag, and a single bean was selected from the bag.



a Copy the following table and calculate the probabilities of selecting each flavour.

Event	Vanilla	Lemon	Orange	Lime	Cherry
Probability					

b What is the probability of selecting either an orange or a lemon jelly bean?

c Which flavour jelly bean are you least likely to select?

d Is it possible to have a group of 21 jelly beans where all 5 flavours are equally likely?

12.3 Probability experiments

Theoretical and actual frequencies

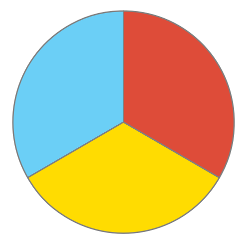
In probability experiments such as tossing a coin, we refer to each toss of the coin as a **trial**. If we toss a coin 100 times, the number of trials is 100. The number of times that a particular outcome occurs is called the **frequency** of the event.

Frequency means how often something happens.

The **expected frequency** and the actual frequency may not necessarily be the same. For example, if we toss a coin 100 times we expect to obtain heads 50 times. However, we may get heads only 47 times, so the actual frequency of heads is 47.

Example 9

A spinner is divided into three equal sectors, and spun 24 times. The colour that the spinner lands on is recorded.



Blue, blue, yellow, blue, yellow, yellow, red, blue, red, blue, yellow, red, red, blue, red, red, blue, red, yellow, red, blue, blue, red, blue

a What is the actual frequency of the spinner landing on blue?

b What is the expected frequency of the spinner landing on blue?

Working

a Actual frequency of blue = 10

Reasoning

According to the results, the spinner landed on blue 10 times out of 24 spins.

Working

b Pr(Spinner lands on the blue sector)

$$= \frac{\text{Number of 'Blue' outcomes}}{\text{Number of possible outcomes}}$$

$$= \frac{1}{3}$$

Therefore in 24 spins, we would expect that it lands on blue

$\frac{1}{3}$ 24 = 8 times.

Expected frequency = 8

Reasoning

We would expect one-third of all spins to land on blue.

Relative frequency

In probability experiments such as tossing a coin, we refer to each toss of the coin as a trial. The number of times that a particular outcome occurs in an experiment is called the frequency of the event. For example, if we toss a coin 100 times and we get 'Heads' 47 times, the frequency of heads is 47.

We can calculate the frequency of a particular event as a fraction (or decimal or percentage) of the total number of trials in the experiment. This is called the **relative frequency** of the event. If we got 47 heads from 100 trials, then the relative frequency of heads would be $\frac{47}{100}$. We could also express this as 0.47 or 47%.

$$\text{Relative frequency of an outcome} = \frac{\text{frequency of the outcome}}{\text{number of trials}}$$

We can compare the relative frequency with the theoretical probability. For example, when we toss a coin, the **theoretical probability** of getting heads is $\frac{1}{2}$ (or 0.5 or 50%).

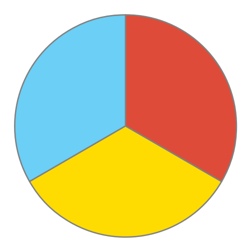
If we repeat an experiment a large number of times, the relative frequency will come closer to the theoretical probability. This means that the more times we toss the coin, the more likely it is that the experimental probability of heads will be closer to a half. For this reason, the theoretical probability is sometimes referred to as the **long-run probability**.

The more times an experiment is repeated, the closer the relative frequency will come to the predicted (long-run) probability. However, because the outcome of each individual trial is random, it is impossible to know the outcome of the next trial.

Example 10

The spinner landed on blue 12 times out of 30 trials.

Calculate as a decimal the relative frequency of the spinner landing on blue. Compare the relative frequency with the theoretical probability of landing on blue.



Working

Relative frequency

$= \frac{\text{frequency of blue}}{\text{total number of trials}}$

$= \frac{12}{30}$

$= 0.4$

The relative frequency of blue was 0.4 compared with the theoretical probability of $0.\overline{3}$.

Reasoning

Total number of trials = 30. In 12 of these the spinner landed on blue.

Theoretical probability of the spinner landing on blue is $\frac{1}{3}$, which is equal to $0.\overline{3}$.

Simulation

Class activity
Corny cards

A simulation is a situation that mimics a real situation. Simulations are useful because we can draw conclusions about real situations from them. Some examples of commonly-used simulations are flight simulators for training airline pilots and disaster simulations to train and prepare emergency services for real events. Simulations are used because they are cheaper and less time-consuming than trying to investigate a real situation. Often simulations are also safer than using a real situation.

We can sometimes use probability experiments to mimic or simulate real situations. For example, if we wanted to do a very simple simulation of a traffic light being red or green, we could toss a coin, with heads representing a red light and tails a green light. We can repeat the experiment many times and compare the experimental outcomes with the expected outcomes.

Example 11

Use a coin to simulate the numbers of boys and girls born in the next 20 births in Australia.

Calculate the relative frequency of boys and of girls and compare these with the theoretical probabilities.

Working

Let heads (H) be boys and let tails (T) be girls.
T T H H H T H T H H T H H H T T T H H H

Frequencies:
12 H
8 T

Relative frequency of boys

$= \frac{\text{frequency of H}}{\text{total number of trials}}$

$= \frac{12}{20}$

$= 0.6$

Relative frequency of girls

$= \frac{\text{frequency of T}}{\text{total number of trials}}$

$= \frac{8}{20}$

$= 0.4$

The theoretical probabilities are 0.5 for boys and for girls.

Reasoning

Define which outcome represents boys and which represents girls. Toss the coin 20 times and record the numbers of heads and tails. Count the frequencies of each outcome.

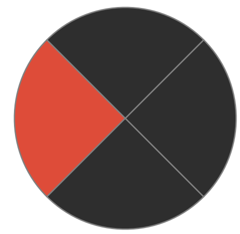
Calculate each frequency as a fraction of the total number of trials.

We would expect half the births to be boys and half to be girls.

exercise 12.3

LINKS TO Example 8

1 Construct a spinner similar to the one shown.



a Predict the probability that the spinner will land on black.

b If you conducted 40 trials (spins), roughly how many times would you expect it to land on black?

c Spin the spinner 40 times and record the number of times each colour occurs (using a tally sheet).

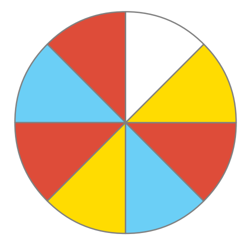
i Copy and complete the following table.

Results from 40 trials	Black	Red
Actual frequency		
Expected frequency		

ii Comment on the closeness or otherwise of the actual frequencies and the expected frequencies.

LINKS TO Example 9

2 Construct a spinner similar to the one shown.



a Spin the spinner 40 times and record the number of times each colour occurs (using a tally sheet).

b Construct a bar chart to illustrate the frequency of each colour across the 40 trials. (Don't forget to label the axes and give the graph an appropriate title.)

c Calculate what frequencies you would have expected for each colour over the 40 trials.

d Copy and complete the following frequency table of the results for your experiment.

Results from 40 trials	Blue	Red	Yellow	White
Actual frequency				
Expected frequency				

e Copy the following table. Calculate the theoretical probability and relative frequency by calculating each frequency as a fraction of 40. Give your answers as decimals. Record them in the table.

Results from 40 trials	Blue	Red	Yellow	White
Relative frequency				
Theoretical probability				

f Comment on the closeness or otherwise of the theoretical probabilities and relative frequencies.

Find a paper, plastic or polystyrene cup, and a wide flat space on which to conduct a 'landing' experiment.

Upright Inverted On side

a Estimate the probability that the cup will rest on its side when dropped from the height of your shoulders.

b Use this estimate to predict how many times in 20 trials that the cup will rest on its side.

c Conduct 20 trials to test the closeness of your prediction.

d Having observed 20 trials, refine your probability estimate and your prediction for the number of times the cup will rest on its side in 20 trials. Then conduct a further 20 trials and check whether your prediction is closer this time.

Erica wants to simulate rolling a die 60 times using random numbers.

a In the 60 trials

i how many 6s would you expect to get?

ii how many even numbers would you expect to get?

Erica's 60 random numbers are shown.

6	6	1	4	3	5	1	3	6	2
5	6	2	3	1	6	1	1	4	6
6	2	2	1	1	5	1	3	2	6
1	6	4	2	6	5	2	2	6	6
1	1	5	6	4	2	1	6	2	3
1	2	6	3	4	4	1	1	1	5

b In Erica's experiment, how many times were

i 6s rolled? **ii** even numbers rolled?

c What is the relevant frequency of rolling

i a 6? **ii** an even number?

Doyle draws a card from a pack, notes the suit and then replaces the card in the pack. He repeats the experiment until he has 60 results, which are summarised below.

♥	♣	♦	♠
18	13	12	17

a Write the theoretical probability of obtaining each suit as a decimal in the table below.

Suit	♥	♣	♦	♠
Theoretical probability				

b Calculate the relative frequency of obtaining each suit.

Suit	♥	♣	♦	♠
Relative frequency				

LINKS TO Example 10

Consider the experiment of tossing a coin.

a What is the probability that the coin will turn up heads?

b If the coin was tossed 10 times, list some possibilities for the numbers of heads and tails that you might obtain.

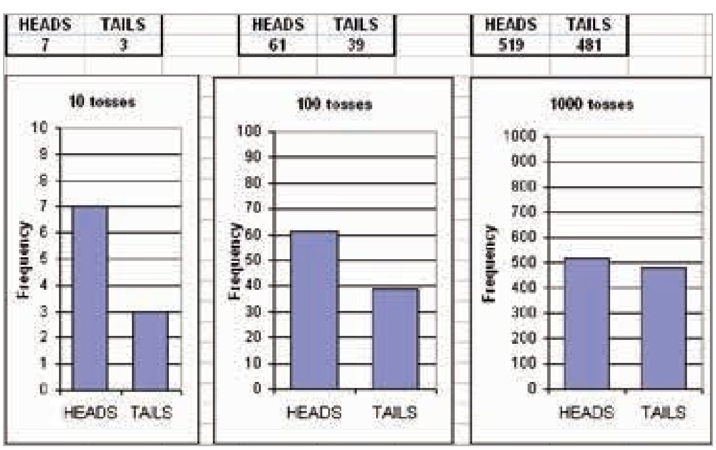
Tossing a coin

c Open the Excel file *Tossing a coin*, which simulates the tossing of a coin by using the number 1 for heads and 0 for tails. If you put the cursor in cell A1 you will notice that the formula is =RANDBETWEEN(0,1). Explain what this formula is doing.

d The formula in cell B1 is =IF(A1=1,"HEADS","TAILS"). Explain what you think this formula is doing.

e In this spreadsheet, the formula has been copied down to row 1000 to simulate 1000 tosses of a coin. Beside column B there are three tables and three graphs. The first of these counts and displays the numbers of heads and tails for the first 10 tosses (that is, the first 10 rows of column A), the second represents 100 tosses and the third represents 1000 tosses. Put the cursor in cell D2 and you will see the formula =COUNTIF(A1:A10,1). In cell E2 the formula is =COUNTIF(A1:A10,0). What do you think each of these formulas is doing?

f Press the F9 key and all the random numbers will automatically change, which makes the data in the tables and the graphs change too. Press the F9 key several times and watch the table and graph for 10 tosses. Do you ever get 10 heads and 0 tails, or 0 heads and 10 tails? Do you sometimes get 9 heads and 1 tail, or 1 head and 9 tails? Do you sometimes get 5 of each? Compare your observations with those of other students in the class. Sample results have been provided.



g Now look at the tables and graphs for 100 tosses and 1000 tosses. What numbers of heads and tails would you expect for

i 100 tosses? **ii** 1000 tosses?

h Compare the three graphs. Apart from the different numbers of tosses being simulated in each graph, how do the three graphs differ from each other? In the sets of data shown in the graphs, the frequencies of heads and tails for 1000 tosses are closest to the expected frequencies. Do you think this will always be the case?

i Using one of your sets of data for 1000 tosses, calculate the fraction of heads and of tails (i.e. the relative frequency of obtaining heads or tails). Give your answer as a proper fraction and as a decimal. How close is your answer to the theoretical probability for heads and tails?

Rolling a die

Consider the experiment of rolling a die.

a What is the theoretical probability that the die will show a 6? Give your answer as a fraction and as a decimal. Is this the same as for each of the other numbers on the die?

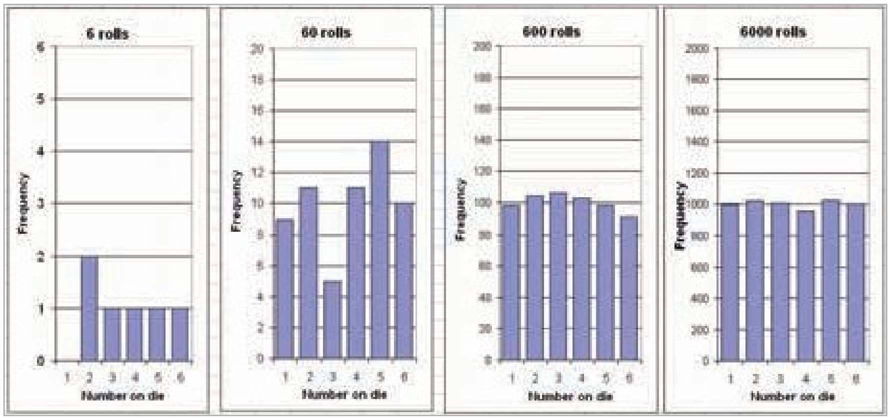
b If the die was rolled six times, list some possibilities for the numbers that you might obtain.

c Open the Excel file *Rolling a die*, which simulates the rolling of a die. Put the cursor in cell A1. What formula is in this cell? Explain what the formula is doing.

d The formula in cell A1 has been copied down 6000 rows to simulate the rolling of a die 6000 times. Beside column A there are four tables and four graphs. The first of these counts and displays the frequency of each of the six numbers on the die for the first 6 rolls (i.e. how often each number occurs in the first 6 rows of column A). The second table and graph represents the frequencies of the 6 numbers for 60 rolls, the third represents 600 rolls and the fourth represents 6000 rolls. Put the cursor in cell C5 and you will see the formula =COUNTIF(A1:A6,1). In cell D5 the formula is =COUNTIF(A1:A6,2). What do you think each formula is doing?

e Press the F9 key to change the random numbers and observe the tables and graphs. Copy and complete the following table and record the first three sets of numbers for 6 rolls, 60 rolls, 600 rolls and 6000 rolls. Compare your observations with those of other students in the class. Sample sets have been provided.

Number of rolls	Number of					
	Ones	Twos	Threes	Fours	Fives	Sixes
6						
60						
600						
6000						



f For the simulation of six rolls, did you ever get exactly one of each number on the die? Did you ever get the same number six times, for example, 6 fours?

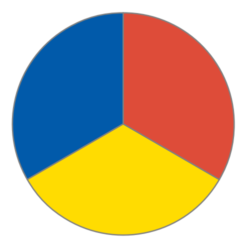
g Compare the four graphs. Apart from the different numbers of rolls being simulated in each graph, how do the four graphs differ from each other?

h For which of the four graphs are the results closest to what you would expect on the basis of the theoretical probability?

i Using one of the sets of data for 6000 rolls, calculate the fraction of sixes, that is, the experimental long-term probability of obtaining a 6. Give your answer as a proper fraction and as a decimal correct to three decimal places. How close is your answer to the theoretical probability of obtaining a 6?

exercise 12.3 challenge

● Using the same method that is used in the *Tossing a coin* and *Rolling a die* Excel files, construct your own simulation of 3, 30 and 300 spins of a spinner which is divided equally into three different colours.



a Make a table to show which number (1, 2 or 3) you are assigning to each colour.

b What formula will you use to generate random numbers? For how many rows will you need to drag your formula down? Type your formula in cell A1 of a spreadsheet and copy it down for as many rows as you need.

c Set up tables in the spreadsheet (as in questions 6 and 7) to show the colours obtained for 3 spins, 30 spins and 300 spins. What formulas will you use to count the numbers of times the spinner lands on each colour?

d Draw column graphs for the data for 3 spins, 30 spins and 300 spins.

e What is the theoretical probability of the spinner landing on red?

f For which one of your three graphs is the relative frequency closest to the theoretical probability?

Analysis task

BLM Dice cricket scoresheet

Dice cricket

In the game of One Day Dice Cricket, two teams play and runs are scored in the following manner.

Roll	1	2	3	4	5	6
Result	1 run	2 runs	3 runs	4 runs	Wicket	6 runs

Other rules are as follows.

- Both sides have one innings each.
- The innings for a side is complete when 10 wickets have fallen.
- The total number of runs scored is the sum of all runs scored in an innings.

a If we rolled the die six times

i how many runs would you expect to score?

ii how many wickets would you expect to fall?

b Use your answer to part a to predict the expected total innings score.

c With a partner, conduct a game of One Day Dice Cricket, with one innings each. Keep a tally of the runs and wickets using the score card provided by your teacher.

Wickets	Runs																			
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
3	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
4	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
5	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
6	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
7	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
8	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
9	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
10	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300

Challenge

d Scoring one 6 per over seems unlikely. The table displayed shows a more typical set of results from a one-day match. In it an 'X' means a wicket and a 'dot' means no run has been scored from that ball. Design a spinner that you believe better simulates a one day cricket match.

Over	Results
1	••••••
2	•X•4•4
3	••••1•
4	••••4•
5	••••••

Over	Results
6	24••••
7	••••42
8	•1•11•
9	4••1X•
10	•4•1••

Over	Results
11	•••4••
12	1••41•
13	•444•1
14	•44•41
15	2•2••1

Review Probability

Summary

- Probability is the mathematical study of chance.
- If all outcomes are known and equally likely, the theoretical probability of an event occurring can be calculated as follows:

$$\text{Pr(Event)} = \frac{\text{Number of ways the favourable outcome can occur}}{\text{Total number of possible outcomes}}$$

  - Pr(Event) is always a number between 0 and 1 (inclusive).
  - If Pr(Event) $= 0$, this means that it is impossible.
  - If Pr(Event) $= 1$, this means that it is certain.
  - The closer the probability is to 1, the more likely it is that the event will occur.
- Theoretical probability helps predict what is likely to happen over a large number of trials (long-run probability). It is not helpful in predicting what will happen in the next trial.
- The number of times that a particular outcome occurs in an experiment is called the **frequency** of the event.
- The frequency of a particular event as a fraction of the total number of trials in an experiment is called its **relative frequency**.
- $\text{Relative frequency} = \dfrac{\text{frequency of outcome}}{\text{total number of trials}}$
- The greater the number of trials, the closer the relative frequencies of the outcomes will be to the theoretical probabilities.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

actual frequency	experiment	random
certain	favourable outcome	relative frequency
chance	fraction	sample space
data	frequency	simulation
equally likely	long-run probability	theoretical probability
event	outcomes	trial
expected frequency	probability	

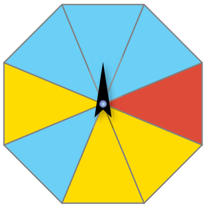
Revision

Multiple-choice questions

1. Which one of the following could not be a probability?

A 40% **B** 0.2 **C** $\frac{3}{8}$ **D** 0.3 **E** $\frac{5}{4}$

2. The probability that this spinner lands on a yellow segment is



A $\frac{1}{8}$ **B** 0.3 **C** $\frac{1}{3}$

D $\frac{3}{8}$ **E** 3

3. A die is tossed. The probability that it shows a multiple of 3 is

A $\frac{1}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{6}$ **D** $\frac{2}{3}$ **E** $\frac{1}{2}$

4. With one roll of a fair die, the probability of getting a number greater than 2 showing uppermost is

A $\frac{1}{6}$ **B** $\frac{5}{6}$ **C** $\frac{2}{3}$ **D** $\frac{1}{3}$ **E** $\frac{1}{2}$

5. A nursery has a lucky dip, which offers visitors one chance of getting a bag of spring bulbs for 50 cents. There are 50 bags of mixed bulbs, 20 of daffodils, 20 of jonquils and 10 of tulips, which are my favourites. What is the probability that I will get my favourites if I can only dip once?

A 0.5 **B** 0.2 **C** 0.3 **D** 0.1 **E** 0.4

Short-answer questions

1. The four cards shown are shuffled and placed face down in a row.

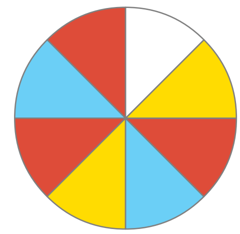
If Paul randomly selects one of the cards, what is the probability that it is

a black?

b a diamond?

c the king of hearts?

- Look at the spinner on the right and complete the following.



  a List all possible outcomes for one spin of this spinner.
  b Are all outcomes equally likely?
  c What is the probability of spinning
    i yellow? **ii** red?
    iii white? **iv** not blue?
  d In 600 spins, how many times would you expect to spin blue?

- A 12-sided die is rolled. What is the probability of obtaining
  a a 6? **b** a perfect square?
  c a number less than 5? **d** a prime number?

Extended-response questions

- At a fair, for \$1 you can throw three darts at a target made up of 100 squares, numbered 1 to 100. If you throw and get 55 or 77 you win \$1, if you throw a number that is a perfect square you win 50 cents.
  a Ron throws his first dart. What is the probability that he gets a perfect square?
  b For his second dart, what is the probability that he gets \$1?
  c For his third dart, what is the probability that he wins nothing?
  d If Ron won \$2, list three possible combinations of numbers he may have thrown. (It is possible to get the same number more than once.)

- In one city it rains an average of 10 days in November. This is a $\frac{1}{3}$ chance of rain. A die may be used to simulate this if we let a 1 or a 2 represent rainy days.
  a Grant rolls a die 7 times to represent a week in November and his results are: 1, 3, 6, 2, 3, 3, 5. How many rainy days are predicted for this week?
  b Roll a die 7 times and find the number of rainy days predicted for the week.
  c Roll a die 30 times to represent the month of November. How many rainy days did you predict?
  d Find your relative frequency of a rainy day.
  e Did the relative frequency equal $\frac{1}{3}$?
  f Did you expect it to?

Practice quiz
Chapter 12

13 Statistics

Pre-test

Warm-up

Customers at this store are given a plastic token when they make a purchase. They are invited to drop their token into one of three charity token boxes. At the end of each three-month period, the donation that the store makes to each nominated charity depends on the number of tokens each charity has received. The three charity boxes resemble a column graph of the customers' choice of charity. In this chapter we will look at different ways of displaying and analysing data.

13.1 Collecting data

Statistics is a branch of Mathematics concerned with collecting and interpreting **data**. Data are collected observations or facts. By organising and displaying data in more useful ways, statistics helps us change data into useful information.

Data is the plural of datum and hence we use the word datum if we are referring to a single piece or value of the data.

Primary data or raw data is collected directly from the source. Secondary data refers to data collected indirectly or when some of the data collection tasks have already been done.

Some examples of data sources are given in the table below.

Primary data sources	**Secondary data sources**
surveys	newspaper article
direct observation	edited audio/video footage
raw audio/video footage	hearsay, gossip
interviewing a person	books
using a data collection device (e.g. thermometer, speed camera)	databases (e.g. internet search engines)
	tables and graphs prepared from primary data

Any set of data contains information about some group of persons or other objects. The characteristic being measured, counted or observed is often referred to as a **variable**, since it is a characteristic that often varies (takes different values). We can also use the word **attribute** to describe a variable in statistics.

Data can be classified as **numerical data** or **categorical data**.

Categorical data is a reference to groups or categories. As an example, consider data collected by asking the question 'Which country were you born in?' Data collected from this question could include 'Australia', 'Vietnam', 'Lebanon' and 'Ireland'.

Numerical data can be counted or measured. Data collected from asking the question 'How tall are you?' would be numerical data. This type of data can be averaged or added to provide more information.

Example 1

Ten students were asked the following three questions and their answers were recorded in the table below. Classify each of the data as either categorical data or numerical data.

Sample survey question	Response data collected
Question 1 What is your favourite season?	winter, summer, summer, spring, autumn, summer, autumn, spring, summer, summer
Question 2 What year are you in at school?	Year 8, Year 7, Year 8, Year 9, Year 7, Year 8, Year 7, Year 7, Year 10, Year 7
Question 3 How many people live in your home?	4, 3, 2, 3, 5, 7, 2, 7, 6, 3

Working

a Categorical data

b Categorical data

c Numerical data

Reasoning

The responses are categories of season.

The responses include numbers but it would not make sense to add or average this data. Each year level is a category.

These responses could be added (e.g. To find the total number of people in all 10 homes) and averaged to find the mean number of people per home.

Questionnaires

Questionnaires are often used to obtain data. A questionnaire consists of a set of questions, usually focusing on a particular topic. Questionnaires can be administered by an interviewer, or respondents can fill in the questionnaire themselves.

Guidelines for effective questionnaire design

Questions should be unbiased.

In designing a questionnaire, it is important that the questions should not be biased. Here is an example of a biased question.

Smart people pay their bills using the internet.
Do you pay your bills using the internet?

The question implies that if you do not pay your bills using the internet, you are not smart. Therefore, people are more likely to say yes, even if they really don't pay their bills using the internet. To make this an unbiased question, the first sentence needs to be removed.

Questions should be easy to understand.

Questions should be as short as possible and presented in a sensible order. Related questions should be grouped together.

Questions should not be upsetting or embarrassing.

Many people feel uncomfortable when they are asked questions such as 'How much do you weigh?' or 'What is your income?'

Questions should be easy to analyse.

To make analysis of the data easier, questions usually include a choice of possible answers. Some examples are:

- What is your age?

  ☐ 20–29 ☐ 30–39 ☐ 40–49 ☐ 50–59 ☐ 60+
- Will you go away on a holiday this year?

  ☐ Yes ☐ No ☐ Don't know
- Politicians are underpaid.

  ☐ Strongly disagree ☐ Disagree ☐ Neither agree nor disagree

  ☐ Agree ☐ Strongly agree

Questions should be tested.

A questionnaire should be tested on a small group to check that the questions are clear and that they do bring out the information you require. The questionnaire can then be modified if necessary before it is used in the actual survey.

Example 2

Reword the survey question 'How many pets do you have at home?' so that the responses would contain categorical data rather than numerical data.

Working

A possible answer is,
'What types of pets do you have at home?'

Reasoning

It is necessary for the responses to fall into categories.
Dogs, cats, birds, fish, snakes etc. are examples of categorical data.

exercise 13.1

LINKS TO Example 1

1. Make a table with two columns headed 'Numerical data' and 'Categorical data'. Write each of the following in the appropriate column.

   a age in years
   b year level at school
   c students' heights
   d your favourite subject
   e time spent on homework last night
   f number of children in your family
   g your favourite TV show
   h the number showing on a die
   i your postcode
   j quantity of leaves on a tree
   k your arm span in centimetres
   l style of music

LINKS TO Example 2

- For the following survey questions, find a way of rewording the question so that responses would contain numerical data rather than categorical data.
  a Do you watch television every night?
  b What is your favourite fruit?
  c Which of the following phrases best describes your height relative to your classmates'?
  i taller than most **ii** similar to most **iii** shorter than most

- Which of these questions are biased? Explain why.
  a How many people live in your home?
  b Normal people enjoy eating a roast dinner. Do you enjoy eating a roast dinner?
  c It is important to drink water every day. How many glasses of water did you drink yesterday?
  d How do you travel to school each day?
  e What is your favourite colour?
  f Do you like rich, warm colours like red, orange and brown?
  g It is important to go to the dentist every six months. How often do you go to the dentist?
  h Do you agree that the new freeway is a great success?

- Match each of the questions or statements below (a to c) with the set of responses (A, B or C) you think is most appropriate.
  a How many pets do you have?
  b Swimming is fun.
  c Is Adelaide further north than Melbourne?
  A ☐ Yes ☐ No ☐ Don't know
  B ☐ 0 ☐ 1 ☐ 2 ☐ 3 or more
  C ☐ Strongly disagree ☐ Disagree ☐ Neither agree nor disagree ☐ Agree ☐ Strongly agree

- Hilton wants to find out how much time students spend on homework each night. Write a question for him to ask that will give the most specific answers possible.

exercise 13.1 challenge

- The following question was used in a questionnaire.

  Are you short? ☐ Yes ☐ No

  Out of 200 people who filled in the questionnaire, 40 people ticked yes, 60 ticked no and 100 did not give a response to this question.
  a What proportion of people who gave a response to this question ticked no?
  b Give two reasons why the non-response category cannot be ignored.
  c Reword the question so that it is a more suitable question for a survey.

13.2 Recording data

Frequency tables

When data is collected some data values may occur more than once. The number of times a particular data value occurs is called the **frequency** of that data value. A **frequency table** is a useful way of organising data. When recording data we can put a **tally mark** (/) beside the appropriate data value. Tally marks are bundled into groups of 5 using 𝍸 . This makes it easier to work out the frequency of each data value.

Example 3

A die was tossed 30 times and the following results were obtained:

2	3	4	4	2	5	3	4	1	1
5	5	1	5	4	2	4	5	6	2
5	4	5	1	4	5	4	6	4	6

a Display the data using a table with a tally column and a column for the total frequency of each die number.

b Using your tally and the frequencies obtained, comment on the distribution of this set of data.

Working

a

Die		
Number	**Tally**	**Frequency**
1	||||	4
2	||||	4
3	||	2
4	𝍸 ||||	9
5	𝍸 |||	8
6	|||	3
	Total	**30**

Reasoning

The categories for the die number are the numbers 1 to 6.

Note the use of '𝍸' to indicate a tally of five scores.

b The distribution of results shows that while each number occurred in the 30 trials, numbers such as 4 and 5 turned up most frequently, and 3 only turned up twice.

The distribution of results can be described by observing the frequency of occurrence of each of the possible values, and seeing which had higher and lower frequencies.

It is often convenient to group numerical data using intervals of values. The following example illustrates this process.

Example 4

The data below give the weights (in kg) of 30 eleven-year-old boys.
26, 33, 55, 50, 32, 25, 44, 31, 36, 35, 28, 28, 36, 48, 36,
31, 34, 32, 47, 37, 46, 36, 47, 33, 42, 32, 32, 29, 34, 30.

a Tally the measurements into suitable 5 kg intervals (e.g. 40–44, 45–49).

b Record the frequency of each weight interval.

c Which weight interval has the greatest frequency?

Working

a and **b**

Weight		
Interval	**Tally**	**Frequency**
25–29	𝍸	5
30–34	𝍸 𝍸 |	11
35–39	𝍸 |	6
40–44	||	2
45–49	||||	4
50–54	|	1
55–59	|	1
	Total	**30**

c The most frequent weight interval for this data is 30–34 kg.

Reasoning

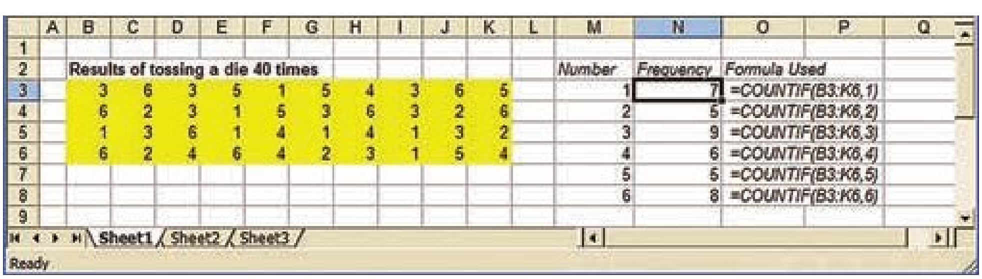
The intervals for the weight ranges are based on observing the minimum (25 kg) and maximum (55 kg) values. It is common to start intervals of five on numbers which are multiples of five (e.g. 25–29 kg, 30–34 kg).

Weights in this interval occur most frequently in this sample.

Spreadsheets

Tech tip

Spreadsheets like Microsoft Excel are useful for grouping data, particularly if the dataset is large. The =COUNTIF function counts the frequency that a given data value occurs in a specified dataset. For example, in the screen below the results from tossing a die 40 times are recorded in cells B3 to K6. To calculate how many times the number '1' showed up on the die, the function =COUNTIF(B3:K6,1) will count the number of 1s in the cells B3 to K6.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2		Results of tossing a die 40 times											Number	Frequency	Formula Used
3		3	6	3	5	1	5	4	3	6	5		1	7	=COUNTIF(B3:K6,1)
4		6	2	3	1	5	3	6	3	2	6		2	5	=COUNTIF(B3:K6,2)
5		1	3	6	1	4	1	4	1	3	2		3	9	=COUNTIF(B3:K6,3)
6		6	2	4	6	4	2	3	1	5	4		4	6	=COUNTIF(B3:K6,4)
7													5	5	=COUNTIF(B3:K6,5)
8													6	8	=COUNTIF(B3:K6,6)

exercise 13.2

The data below shows the responses of a group of Year 7 students to the Australian Census at School question: What is your favourite type of take-away food? Students responded by clicking on only one selection in the list provided.

Favourite take-away food	Tally	Frequency
Chicken		
Chips/fries	卌 卌 II	
Fish (e.g. fish and chips)	IIII	
Fruit/fruit salad	II	
Hamburgers	IIII	
Kebabs/wraps	卌 I	
Noodle dishes	II	
Pies/pasties	I	
Pizza/pasta	卌 卌 II	
Rice dishes (e.g. sushi)	II	
Rolls/sandwiches	II	
Salads		
Other	II	
None		
	Totals	

a Complete the table by filling in the frequency column.
b What were the two most popular take-away foods?
c How many students were in the group represented by the data?
d What percentage of students said pizza/pasta was their favourite take-away food?

LINKS TO Example 3

A die was tossed 40 times and the following results were obtained.

3, 6, 3, 5, 1, 5, 4, 3, 6, 5,
6, 2, 3, 1, 5, 3, 6, 3, 2, 6,
1, 3, 6, 1, 4, 1, 4, 1, 3, 2,
6, 2, 4, 6, 4, 2, 3, 1, 5, 4

a Tally the results and find the frequency for each number.
b Comment on the distribution of results.

LINKS TO Example 3

● Jane recorded the hair colour of each member of her class. Her raw data follow.

blonde, red, brown, brown, black, blonde, black, black, brown, blonde, red, blonde, brown, brown, black, red, brown, black, blonde, blonde, black, brown, black, brown, blonde, black, brown, brown

a Organise this data using a frequency table.

b Which hair colour had the highest frequency?

● The results of a survey to find out how students travel to school are shown below for a sample of 40 students.

bus, bicycle, skateboard, car, walk, bus, other, car, bicycle, walk,
car, car, car, car, bus, walk, bicycle, car, walk, walk,
car, car, car, bus, car, car, bus, bus, car, bus,
bus, bicycle, car, car, bus, car, car, walk, car, car

a Organise the data into a frequency table.

b Which was the most common way of getting to school?

c What percentage of students came to school by car?

d Suggest how this data might be useful for a school or local council.

● A survey was conducted where a group of kindergarten children were asked their favourite colour. The results are shown in the frequency table below.

Colour	Tally	Frequency
Red	𝍸	
Blue	𝍸 III	
Pink	𝍸 I	
Yellow	II	
Orange	III	
Purple	𝍸 I	
	Totals	

a Complete the table by filling in the frequency column.

b What was the most popular colour?

c How many students were questioned?

d What fraction of the children questioned chose red as their favourite colour?

LINKS TO Example 4

● Katina conducts a survey where she asks Year 7 students what their favourite sport is. The results are shown below.

tennis, football, cricket, football, hockey, cricket, swimming, netball, netball, football, soccer, tennis, cricket, swimming, football, soccer, netball, tennis, soccer, football, netball, basketball, netball, football, swimming, hockey, netball, cricket

She organises her results using a frequency table. The tally for netball is

A IIII **B** 𝍸 **C** 𝍸 I **D** 𝍸 II **E** 𝍸 III

LINKS TO Example 4

The data below give the heights (in cm) of 30 eleven-year-old girls.

135, 146, 153, 154, 139, 131, 149, 137, 143, 146, 141, 136, 154, 151, 155,
133, 149, 141, 164, 146, 149, 147, 152, 140, 143, 148, 149, 141, 137, 135

- **a** Tally the measurements using 5 cm intervals of 130–134, 135–139, 140–144, etc. Present your answer in a frequency table.
- **b** Record the frequency of each interval.
- **c** Which interval has the greatest frequency?

LINKS TO Example 4

The heights of tennis players at a tennis tournament, correct to the nearest centimetre, are shown in the frequency table.

- **a** Which height interval has the greatest frequency?
- **b** How many players have heights in the interval 145–149 cm?
- **c** Which height interval has a frequency of 10?

Height (cm)	Frequency
135–139	1
140–144	2
145–149	4
150–154	5
155–159	7
160–164	9
165–169	10
170–174	9
175–179	11
180–184	2

Angelo has planted some tomato seedlings. He measures their heights, correct to the nearest centimetre. His data are listed below.

8, 15, 14, 12, 20, 23, 14, 6, 9, 12, 15, 18, 20, 22, 28, 31, 16, 22, 18, 9, 6, 18, 13, 12, 6, 12, 15, 26

- **a** Tally the measurements in class intervals of five.
- **b** Record the frequency of each interval.
- **c** Which interval has the lowest frequency?

exercise 13.2 challenge

The pulse rates (beats/minute) of 48 hospital patients were recorded as follows:

68, 80, 84, 80, 80, 80, 92, 92, 80, 80, 80, 80, 80, 78, 90, 80,
72, 80, 82, 76, 84, 70, 80, 82, 84, 116, 80, 95, 80, 76, 100, 88,
90, 90, 90, 80, 76, 80, 84, 80, 80, 80, 80, 104, 80, 68, 84, 64

- **a** What were the minimum and maximum pulse rates?
- **b** Tally the data into:
  - **i** intervals of 5 beats per minute, starting at 60–64.
  - **ii** intervals of 10 beats per minute.
- **c** Record the frequencies in each case.
- **d** Comment on the distribution of results. What pulse rate seems to be the most common?

13.3 Summarising data: measures of centre and spread

There are a number of ways of summarising the 'shape' or distribution of a set of numerical data. In this section, we look at statistics that measure the centre and spread of a set of data. These statistics provide answers to the following questions.

- Where is the middle (or centre) of the data?
- How varied (or spread out) is the data?

Measures of centre

There are three ways of measuring the centre or middle of a set of data. These are the **mean**, the **median** and the **mode**.

The mean

The mean is calculated by adding all data values and dividing by the number of data values.

$$\text{mean} = \frac{\text{sum of all the data values}}{\text{number of data values}}$$

Example 5

The hand spans (to the nearest cm) of a group of 10 students were found to be: 15, 14, 17, 19, 14, 14, 13, 15, 21, 16. Calculate the mean hand span of students in the group.

Working

Mean hand span:

$$= \frac{15 + 14 + 17 + 19 + 14 + 14 + 13 + 15 + 21 + 16}{10}$$

$$= \frac{158}{10}$$

$$= 15.8\,\text{cm}$$

Reasoning

The mean is the sum of all values divided by the number of values.

The median

The median value of a data set is the value in the middle when the data is listed in either ascending order (smallest to largest) or descending order (largest to smallest).

If there is an odd number of data values, there will be a middle value. If there is an even number of data values, there will be two middle values, so we take the mean of those two middle values.

Example 6

The heights (in cm) of a group of 11 students were recorded as follows:

151, 139, 153, 154, 162, 194, 144, 119, 127, 110, 120.

Calculate the median height for the 11 students.

Working	Reasoning
110, 119, 120, 127, 139, 144, 151, 153, 154, 162, 194 110, 119, 120, 127, 139, (144), 151, 153, 154, 162, 194	First sort the data values in ascending order (from smallest to largest). This is necessary for locating the middle value.
The median value is 144 cm.	There is an odd number of data values so there is a single middle value.

Example 7

The maximum temperatures in Sunnyville over a 12-day period were as follows:

21, 41, 36, 28, 31, 28, 20, 23, 23, 24, 24, 31.

Calculate the median maximum temperature for the 12-day period.

Working	Reasoning
20, 21, 23, 23, 24, 24, 28, 28, 31, 31, 36, 41	List data in ascending order. This is necessary for locating the middle value.
20, 21, 23, 23, 24, (24, 28), 28, 31, 31, 36, 41	Since there is no single middle value, locate the middle two data values and circle them.
Median temperature $= \dfrac{24 + 28}{2}$ $= 26$	The median value is midway between the two middle values.

The mode

The mode is the most common or most frequently observed value of the data.

Example 8

Julian measures the weights of 18 dogs entered in a show. Their weights, correct to the nearest kilogram are given below.

16, 43, 39, 27, 45, 37, 32, 30, 34, 27, 37, 41, 37, 43, 34, 26, 28, 35

Find the modal weight.

Working	Reasoning
The mode is 37.	The number 37 occurs three times but the other data values only occur once or twice. This is the most commonly occurring value.

The mode is not always a useful measure of the centre of the data. In some data sets there may be more than one mode, that is, there may be two different data values with the same frequency.

Tech tip

The TI-30XB MultiView calculator can be used to find the mean and median of a data set.

Press [data] then enter the data in L1. Type one data value then press [enter] and continue this process until all data values are entered in the list. Press [2nd] [data] for [stat] and select 1-Var Stats by pressing [1]. The data is in L1 so make sure that L1 is highlighted by pressing ▶ if it is not already highlighted then press [enter]. For FRQ make sure that ONE is highlighted then press [enter]. CALC will now be highlighted in the bottom right of the screen. Press [enter]. The mean is $\bar{x}$. Use ▼ to find the median which is denoted by Med.

Measuring the spread of the data: the range

When describing a set of data, we are often interested in how varied the data is. One way of comparing the spread of two data sets is to calculate the difference between the lowest value and the highest value. This is called the **range** of the data.

Example 9

The following data are the heights in centimetres of a random sample of 20 Year 7 girls and 20 Year 7 boys from the 2010 Census at School data.

Girls	147	149	150	151	152	153	154	154	155	156
	159	161	163	164	164	165	165	169	170	173

Boys	144	149	149	150	151	152	153	154	155	156
	157	159	159	164	166	168	168	169	170	178

Source: Australian Bureau of Statistics

a What is the range for the girls' heights?
b What is the range for the boys' heights?
c Compare the ranges for girls and boys.

Working

a Girls
Range = 173 − 147
= 26

Reasoning: Range = largest data value − smallest data value

b Boys
Range = 178 − 144
= 32

c The boys' heights are spread over a slightly greater range than the girls' heights.

Reasoning: The range is a measure of how spread out the data values are.

Tech tip

Summarising data with spreadsheets

Most spreadsheet software includes features for summarising statistical data. For example, Microsoft Excel has a range of inbuilt functions for calculating **summary statistics** (numbers that summarise a set of data).

Suppose the data were entered into cells A6 to A17. The following formulae can be entered to calculate summary statistics for this data.

Summary statistic	Spreadsheet formula used
Number of data values (n)	=COUNT(A6:A17)
Total	=SUM(A6:A17)
Mean	=AVERAGE(A6:A17)
Median	=MEDIAN(A6:A17)
Mode	=MODE(A6:A17)
Minimum (smallest value)	=MIN(A6:A17)
Maximum (largest value)	=MAX(A6:A17)

exercise 13.3

LINKS TO Example 5

1. Find the mean for each of the following sets of numbers. Round answers to 1 decimal place.
   - **a** 8, 2, 3, 5, 8
   - **b** 3, 2, 7, 4, 10
   - **c** 36, 83, 74, 51, 26, 57
   - **d** 11, 11, 11, 12, 13, 14, 15, 16
   - **e** 5, 5, 5, 5, 4, 4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3
   - **f** 32, 37, 41, 43, 45, 45, 46, 48, 33, 42, 51
   - **g** 34, 27, 22, 21, 18, 19, 25, 38. 51, 110
   - **h** 1.5, 1.8, 2.2, 2.5, 2.6, 2.8, 3.1, 4.7

2. Which one of the following data sets has the greatest mean?
   A 2, 4, 6, 8, 10 **B** 2, 4, 2, 8, 4 **C** 4, 6, 10, 10, 2 **D** 4, 4, 4, 2, 6 **E** 2, 4, 2, 6, 6

LINKS TO Example 6

3. Find the median for each of the following sets of numbers.
   - **a** 1, 2, 3, 4, 5, 6, 7
   - **b** 5, 14, 22, 35, 70, 82, 93
   - **c** 1, 4, 3, 1, 3
   - **d** 2, 1, 5, 7, 8, 3, 2
   - **e** 13, 12, 15, 18, 17, 14, 11
   - **f** 4, 0, 7, 2, 5, 5, 3, 2, 5, 4, 1
   - **g** 6, 6, 7, 3, 9, 4, 2, 1, 7, 9, 3
   - **h** 10, 15, 18, 12, 27, 35, 41, 29, 25, 32, 41, 18, 13
   - **i** 1.5, 0.2, 4.3, 5.2, 1.7, 0.9, 2.2, 2.7, 3.3
   - **j** 0.43, 0.02, 0.50, 0.48, 0.76, 0.51, 0.27, 0.61, 0.48

LINKS TO Example 7

4. Find the median for each of the following sets of numbers.
   - **a** 2, 4, 8, 9
   - **b** 4, 6, 9, 12
   - **c** 1, 1, 2, 3, 4, 4, 4, 5, 5, 6
   - **d** 2, 4, 6, 8, 10, 12, 14, 16
   - **e** 1, 2, 3, 4, 5, 6, 7, 8
   - **f** 8, 0, 0, 5, 2, 7, 8, 9
   - **g** 10, 2, 6, 8, 2, 6, 10, 8, 2, 6, 8, 6, 10, 8
   - **h** 12, 4, 6, 7, 8, 3, 11, 10, 5, 3, 2, 16
   - **i** 15, 7, 19, 23, 45, 62, 37, 41, 29, 3
   - **j** 2.5, 3.2, 4.7, 5.4, 6.6, 7.2, 8.3, 9.7

5. 15, 16, 18, 20, 22, 15, 18, 14
   The median for the data is
   A 21 **B** 20 **C** 17 **D** 18 **E** 17.5

LINKS TO Example 8

6. Find the mode for each of the following sets of numbers.
   - **a** 1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 5
   - **b** 3, 5, 6, 8, 9, 9, 9, 10
   - **c** 5, 7, 5, 3, 8, 4
   - **d** 2, 3, 0, 2, 0, 0, 3, 0
   - **e** 2, 4, 7, 9, 3, 1, 7, 9, 1, 4, 8, 1, 3
   - **f** 5, 1, 3, 4, 1, 3, 5, 4, 1, 3, 4, 3, 5
   - **g** 9, 11, 13, 8, 6, 14, 6, 7, 9, 14, 11, 12, 6
   - **h** 0.1, 0.3, 0.5, 0.2, 0.4, 0.5, 0.7
   - **i** 3, 5, 3, 3, 2, 6, 2, 2, 7, 1, 5, 6
   - **j** 16, 18, 12, 14, 8, 3, 9, 21, 24, 13

7. Calculate the mean, median and mode for the following sets of numbers. Round answers for the mean to 1 decimal place.
   - **a** 1, 2, 2, 3, 3, 3, 3, 4, 5

b 0, 2, 3, 3, 3, 3, 3, 3, 25
c 0, 2, 3, 3, 4, 5, 6, 6, 7
d 3, 3, 3, 3, 12, 13, 13, 13, 13
e 7, 4, 8, 6, 5, 9, 7, 5, 7, 3, 8, 9
f 9, 7, 4, 4, 5, 3, 2, 8, 9, 7, 3, 2, 3, 5, 8
g 5, 6, 4, 2, 4, 1, 2, 5, 8, 2, 1, 6, 9, 3, 4, 5, 2, 7
h 2, 3.3, 6, 2.4, 2, 3.1, 8

Consider the two data sets below.

i 1, 2, 3, 4, 5 **ii** 1, 2, 3, 4, 10

a For each set of data calculate the mean and the median.
b Comment on the effect of changing the 5 in the first data set to a 10 in the second data set.

LINKS TO Example 9

Calculate the median and range for the following sets of numbers.

a 28, 28, 23, 33, 32, 40, 39, 28, 21, 34
b 97, 79, 96, 83, 55, 85, 11, 46, 10, 5
c 118, 182, 105, 166, 130, 135, 149, 157, 186, 133
d 179, 166, 32, 35, 18, 130, 180, 118, 22, 8
e 67, 55, 60, 56, 67, 64, 58, 62, 59
f 126, 134, 128, 197, 153, 167, 134, 122, 136, 185
g 0.2, 0.8, 0.3, 0.4, 0.9, 0.4, 0.5, 0.7, 0.6, 1.1, 0.9
h 96.7, 83.2, 85.8, 96.4, 99.1, 74.3, 85.6, 91.8, 68.5, 79.2

Rashid wanted to know his mean test score, as this was going to be used to calculate his end-of-year test score in his science class. His individual test grades were as follows (out of 100): 78, 65, 30, 59, 77, 89, 99, and 70. What was his mean score?

Newspapers regularly publish mean and median house prices for different suburbs. On a particular weekend five houses sold in one suburb. They sold for $300 000, $350 000, $400 000, $450 000 and $2 500 000.

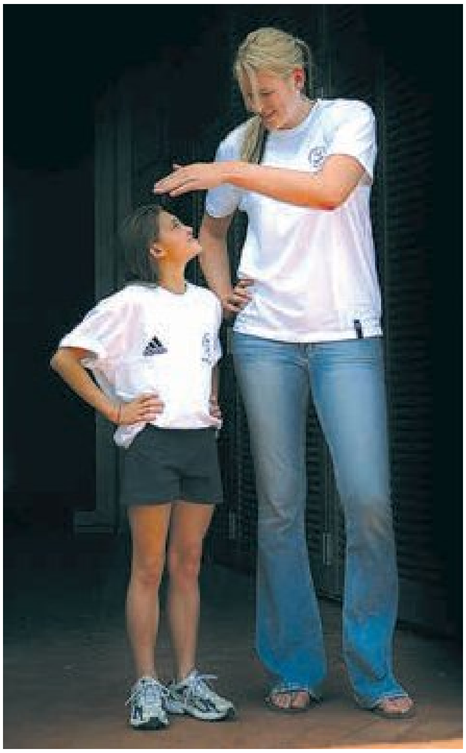
a Find the mean and the median price of the sales.
b If you were a real-estate agent who earns a percentage of the sales, would you be most interested in the mean or the median price?
c If you were a potential buyer deciding whether you could afford to buy in this suburb, would you be most interested in the mean or the median price?

A group of students' results for a maths test are shown below.

16, 14, 10, 15, 13, 12, 20, 3, 17, 15, 13, 15, 10, 12.

a Sara felt sick and had to leave after 5 minutes. Which mark do you think is hers?
b What is the most likely total mark for the test?
c If Sara's mark is included, find the mode, median and mean marks for the test.
d If Sara's mark is not included, find the mode, median and mean marks for the test.

- Imagine that all the members of the Australian Olympic team were nearly the same height and weight. The spread of their measurements would be very small.
  - **a** Is it desirable to have a small spread of measurements for members of an Olympic team? Why or why not?
  - **b** In which sports do people have an advantage if they are very tall, very heavy, very short or very light?
  - **c** For what team sports might it be an advantage if the team members' measurements had a very small spread?



Members of Australia's 2006 Commonwealth Games team, diver Melissa Wu and basketballer Lauren Jackson

- Five students gather evidence to convince their parents that they are expected to do too much work around the house. The students survey the number of hours they each spend on chores during the week. The results are: 1 hour, 4 hours, 5 hours, 5 hours, 6 hours.
  - **a** What is the mean amount of time spent on housework by each student in hours and minutes?
  - **b** What is the mean amount of time spent on housework by each student in hours and minutes?
  - **c** If the students tell their parents the mean, how many students can argue that they are over worked?
  - **d** Jessica, who does one hour of housework per week, realises that she can't gain from the plan, so she drops out of the group. Find the mean and median without Jessica.
  - **e** How many students are above the mean now?

exercise 13.3 challenge

- A student has an average test score of 67% based on six tests. If she scores 80% on her seventh test, what is her new average test score (to the nearest whole number)?
- Five students sat for a test and four of those students received marks of 5, 6, 8 and 9 respectively. Try to work out the score of the fifth student if the mean mark is equal to the median mark.

13.4 Displaying and interpreting data: column and pie graphs

Graphs of data usually include four key elements—scale, axis, legend and title (SALT). These are outlined below.

- Scale: the units of measurement should be clearly indicated—for example, percentage.
- Axis: each axis must be clearly labelled, indicating what is being shown (numerical amounts or categories can form the labels, depending on the type of data and variables being graphed).
- Legend: a legend (key) explains the colours, shading, lines or symbols used on the graph. Not all graphs will require a legend.
- Title: this is what the data being shown is about. Where possible, a graph title should include information about when the data was collected.

Example 10

Ruth collected some data about car colour from a nearby car park, and then displayed it in a bar chart.

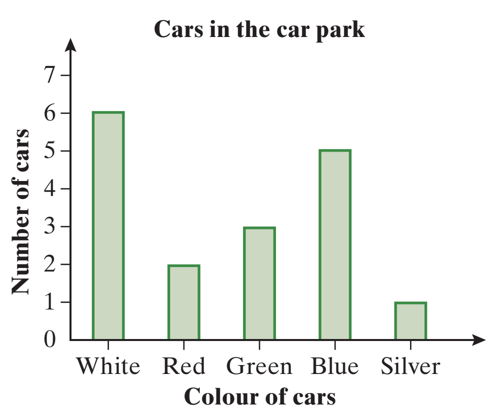
a What is the title for the graph?

b What is the label for the vertical axis?

c What is the label for the horizontal axis?

d Which axis shows a scale?

e Does this graph need a legend?



continued

Example 10 continued

Working	Reasoning
a The title of the graph is 'Cars in the car park'.	The title shows what the graph is about.
b The label for the vertical axis is 'Number of cars'.	The vertical axis shows how many cars of each colour there are.
c The label for the horizontal axis is 'Colour of cars'.	The horizontal axis shows the categories, in this case, the different colours of the cars in the car park.
d The vertical axis has a scale.	The numbers on the vertical axis show the number of cars of each colour.
e The graph does not need a legend.	There is no colour, shading or symbols that need explaining.

Column graphs

A column graph can be constructed from categorical data organised in a frequency table. In a column graph, each category has its own column, with the horizontal axis used as the **category axis** and the vertical axis used as the **frequency axis.**

Example 11

Jacquie, a video shop employee, noted the type of DVDs being borrowed on a Tuesday night.

DVD type	Frequency
Drama	13
Comedy	8
Horror	3
Kids	6
Music	2
Other	8

continued

Example 11 continued

a Construct a column graph for this data.

b What was the total number of DVDs borrowed on that night?

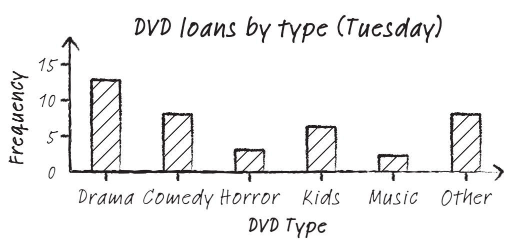
c Compare the column graph with the frequency table. Why might a column graph be more useful for representing the distribution of the data set?

d On the basis of this data, Jacquie suggests that they should not bother stocking horror or music DVDs. What do you think?

Working

a Construct the category axis to allow for six categories with a small gap between each column.

The frequency values vary from 2 to 13, so construct the frequency axis from 0 to 15.



Reasoning: Think about the scale on the category axis. How wide should it be so that all the categories will fit in.

Think about the scale on the frequency axis. How big should it be so that the different frequencies can be shown clearly?

Label each of the axes, so that it is clear what the chart represents. Also include a title.

b Total videos = 13 + 8 + 3 + 6 + 2 + 8
= 40 videos

Reasoning: Either sum the values from the table, or read and sum the heights (frequencies) of the columns from the chart.

c The column graph makes it easier to compare the frequencies of the different categories. It is easier to see which categories are the most popular and which are the least popular.

Reasoning: Looking at the heights of the columns gives a more immediate view of their relative frequencies than the numbers in the table.

d Horror and music videos were least popular on that Tuesday night, but this pattern may not be typical of other nights of the week. Jacquie would need to collect data on other days of the week and over a longer period of time, such as a year.

A bar graph or bar chart, as it is often called, is similar to a column graph but the vertical axis is used as the category axis and the horizontal axis is used as the frequency axis.

Example 12

The following data shows the hair colours of a group of Year 7 students.
Black, brown, brown, fair, black, fair, fair, red, brown, brown, red, black, fair, brown, brown, brown.

a Complete the frequency table.

b Construct a bar graph for this data.

Colour	Tally	Frequency
Black		
Brown		
Fair		
Red		
	TOTAL	

Working

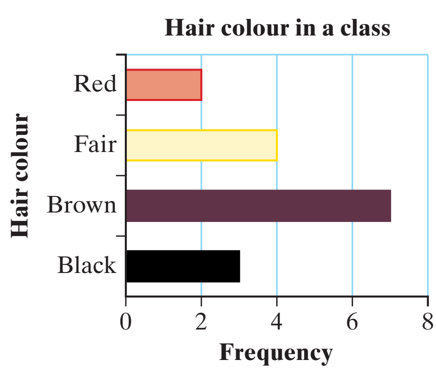
a

Colour	Tally	Frequency
Black	|||	3
Brown	~~||||~~ ||	7
Fair	||||	4
Red	||	2
	TOTAL	**16**

Reasoning

Use | for each occurrence of a hair colour. Count each of the tallies to get the frequency of each hair colour.
Add all the frequencies to get the total.

b

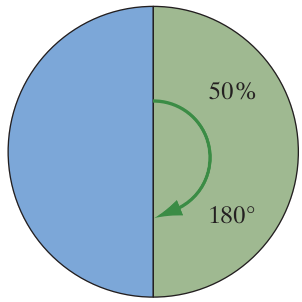


The category axis is the vertical axis.
The frequency axis is the horizontal axis.
The length of each bar gives an indication of the frequency of that particular hair colour.

Pie graphs

Pie graphs are useful for displaying categorical data. Before drawing a pie graph, it is necessary to find the percentage or fraction of the whole that each category represents.

When drawing a pie graph, the sector angle for each category is that *same* percentage or fraction of the full turn (360°) around the centre of the circle. For example, if a category percentage is 50%, its sector angle will be 50% of a full turn or 180°.



Example 13

Calculate each sector angles for a pie graph showing the following percentages. Round each angle to the nearest degree.

a 50% **b** 16% **c** 24% **d** 10%

Working

a $50\% \text{ of } 360° = \frac{50}{100} \times \frac{360°}{1} = 180°$

b $16\% \text{ of } 360° = \frac{16}{100} \times \frac{360°}{1} = 57.6°$

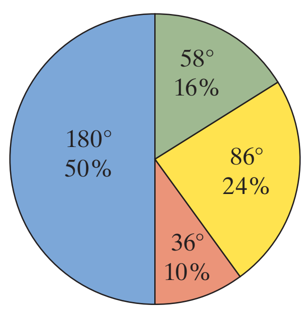
c $24\% \text{ of } 360° = \frac{24}{100} \times \frac{360°}{1} = 86.4°$

d $10\% \text{ of } 360° = \frac{10}{100} \times \frac{360°}{1} = 36°$

The four sector angles are 180°, 58°, 86° and 36°.

Reasoning

Use the percentage for each category to find each sector angle.



Check that the four angles add to 360°.
180° + 58° + 86° + 36° = 360°.

Example 14

Twenty-four students in a class were asked how many children are in their family. The following results were obtained.

2, 3, 3, 2, 2, 1, 3, 3, 2, 2, 2, 2, 2, 3, 3, 4, 2, 2, 2, 1, 4, 5, 1, 2

a What is the variable here, and what values can it take?

b Construct a frequency table for this data set.

c Construct a pie graph to represent this data.

Working

a The variable here is the 'number of children in the family', and it can take whole number values 0, 1, 2, 3, … and so on.

b

Number of children	Frequency
One	3
Two	12
Three	6
Three	3

c Calculating sector angles:

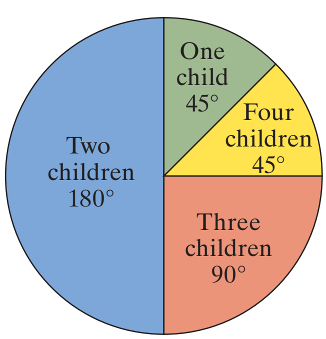
One child: $\frac{3}{24}$ 360° = 45°

Two children: $\frac{12}{24}$ 360° = 180°

Three children: $\frac{6}{24}$ 360° = 90°

Three children: $\frac{3}{24}$ 360° = 45°

Number of children in family



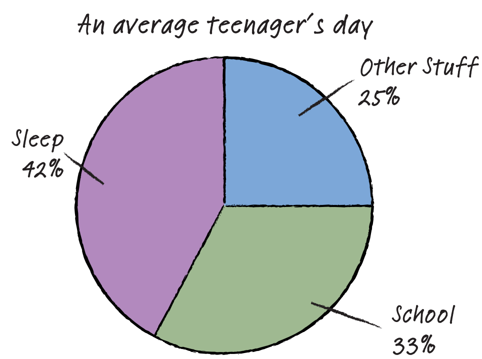
Reasoning

Record the frequencies.

Calculate the fraction of the pie that each category contributes. There are 24 children altogether so the fractions $\frac{3}{24}, \frac{12}{64}$, and so on, give the fraction of the total circle area that should be used for each category. There are 360° in a full circle so we need $\frac{3}{24}$ of 360°, etc.

Use a protractor to mark the angles needed for each part of the pie. (Refer to section 6.2 in chapter 6.)

Example 15



The following graph claims to represent the proportion of time an 'average teenager' spends on sleep, school and other stuff.

a According to the graph, how many hours would an 'average teenager' spend at school each day?

b If the claim is correct, how many years and months does this graph suggest teenagers sleep?

c What information about teenagers' sleeping habits is not clear from this graph?

Working

a $\frac{33}{100} \times 24 \approx 8$

A teenager spends about 8 hours per day at school.

Reasoning: 33% of the day is spent at school so calculate 33% of 24 hours.

b Number of years as a teenager = 7 years

Time spent sleeping

= 42% of 7 years

= 0.42 7

= 2.94 years

= 2 years + 0.94 12 months

= 2 years and 11 months

(to nearest month)

So this graph claims that nearly 3 years of teenage life is spent sleeping.

Reasoning: 42% of any day is spent sleeping so 42% of 7 years would be spent sleeping.

Since 42% is just less than half, we would expect that just under half of the 7 years would be spent sleeping.

c The graph does not explain how varied teenage sleeping patterns are. For instance, further information about when they sleep, and what proportion of teenagers sleep more or less than the 'average' may help here. Also, wouldn't we expect the sleeping patterns of most 13- and 19-year-olds to be different?

Reasoning: A single graph or statistic may be helpful for an overall picture, but it may well obscure how much variation exists in the data.

exercise 13.4

LINKS TO Example 10

1. Twenty students were asked to nominate their favourite season of the year. The results are shown below.

Season	Winter	Spring	Summer	Autumn
Frequency	4	6	8	2

a Constuct a column graph for this data. Make sure that you use the SALT conventions.

b How might the time of year influence the results obtained from such a question?

LINKS TO Example 12

2. Scores on a quick quiz are shown below:

3, 9, 8, 10, 7, 8, 6, 5, 7, 8, 6, 8, 4, 7, 7, 6, 7, 8, 6, 7, 7

a How many students took the quiz?

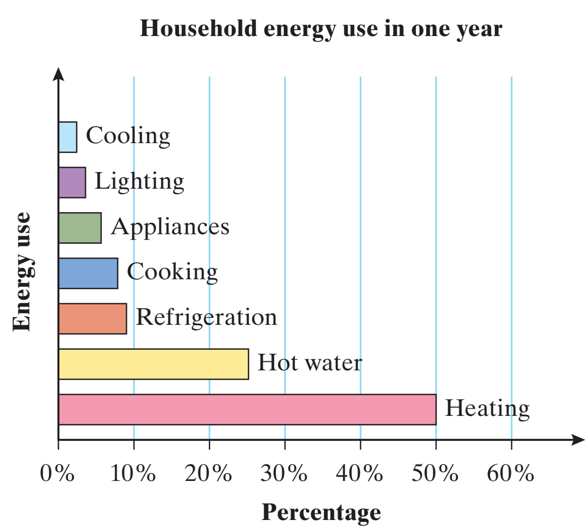
b Organise the data into a frequency table.

c Construct a column graph of the data.

d Was it a difficult quiz? Give reasons.

LINKS TO Example 11

3. The bar graph shown gives an indication of how energy is used in a home over a year.



a What percentage of household energy is used on heating?

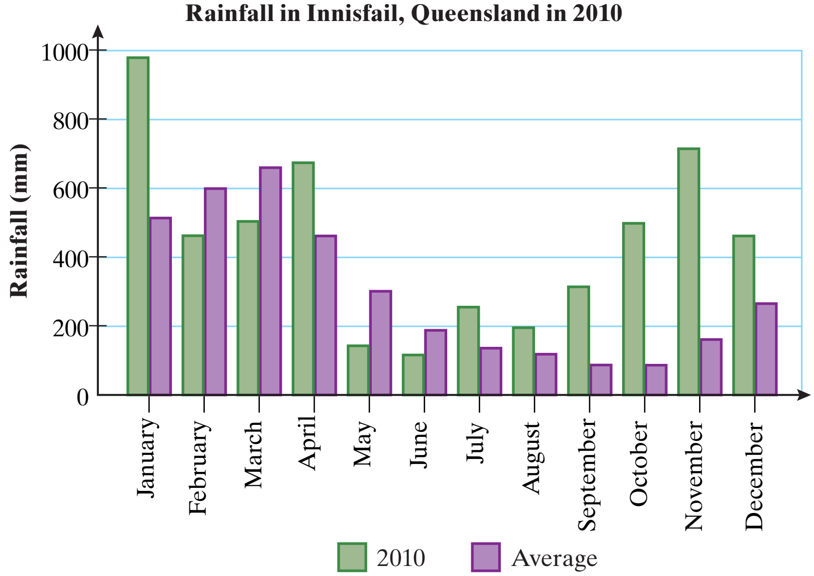
b Which energy source has the next highest percentage?

c Is more energy used on refrigeration or cooking?

d How might this graph change if it only gave an indication of how energy is used over the summer months?

The following graph shows the monthly rainfall for Innisfail in Queensland for 2010.

Rainfall in Innisfail, Queensland in 2010



a In which month of 2010 was there almost 1 metre of rain?

b In which month of 2010 was the rainfall over 5 times the average for that month?

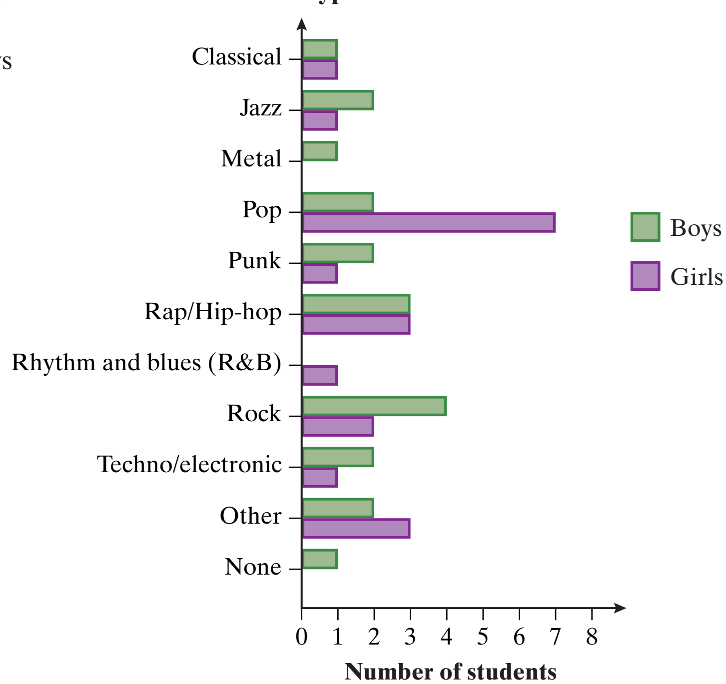
c In which months of 2010 was the rainfall below the average for those months?

d Use the graph to estimate the total rainfall for Innisfail for 2010.

e Use the graph to estimate the average total annual rainfall for Innisfail.

The following graph shows the favourite music of 40 Year 7 students (20 boys and 20 girls).

Favourite type of music of 40 Year 7 students



a What was the most popular music type for girls?
b What percentage of girls selected this type as their favourite music?
c What was the most popular music type for boys?
d What percentage of boys selected this type as their favourite?
e Why is a bar graph more useful than a column graph for displaying this data?

A class of 20 students keeps a record of all the TV they watch in a week. The students combine their results and divide them into the categories shown at right.

a Use a spreadsheet to draw a column graph of the data.
b If there are a total of 200 hours of TV watched, how many hours do the students spend watching music shows?

Category of show	Percentage of time
Movies	30
Drama series	20
Sitcoms	15
Music shows	10
Documentaries	10
Current affairs	5
News	5

LINKS TO Example 13

Students were surveyed on what they were doing at 7 pm last night. The results are summarised in the table below.

Activity	Frequency	Angle
Watching TV	4	
Eating dinner	2	
On the phone	5	
At the computer	5	
Reading	4	
Playing music	1	
Playing sport	3	
Listening to music	2	
Doing homework	10	
Total	**36**	

a If this information was presented in a pie chart, how many degrees would represent one student?
b In the right-hand column in the table above, fill in the angle each activity would make in a pie chart.
c Draw a pie chart representing the information in the table above.

LINKS TO Example 13

Zeev kept a record of how he spent his time in a day. He wishes to show this data in a pie graph. Complete the table below showing the sector angle corresponding to each activity.

Activity	Hours	Angle
Sleeping	9	
Eating	3	
In class	5	
Sport	2	
TV	2	
Homework	2	
Other	1	

LINKS TO Example 14

A group of 36 Year 7 students were surveyed about the number of pets they have at home. The results of the survey are given below.

1, 1, 2, 2, 0, 4, 3, 2, 2, 0, 1, 1, 2, 3, 4, 5, 0, 2, 1, 3, 1, 2, 1, 5, 4, 2, 2, 2, 0, 0, 1, 1, 2, 1, 2, 4

a What is the variable here and what values can it take?

b Construct a frequency table for the data set.

c Construct a pie chart to represent the data.

The following pie chart shows how a group of Year 7 students travel to school.

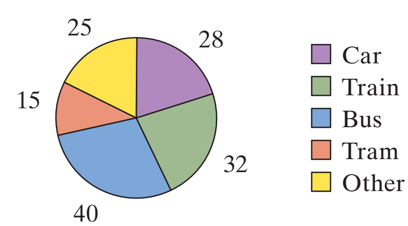
a Do the numbers on each sector represent percentages or numbers of students? How do you know?

b How many students travel to school by car?

c What is the most common method of transport?

d What percentage of students take the train?

How students travel to school



LINKS TO Example 15

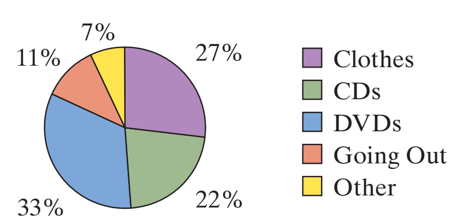
The pie chart to the right shows how Daniel spent the money he received for his birthday. If Daniel received $450 for his birthday

a how much did he spend on clothes?

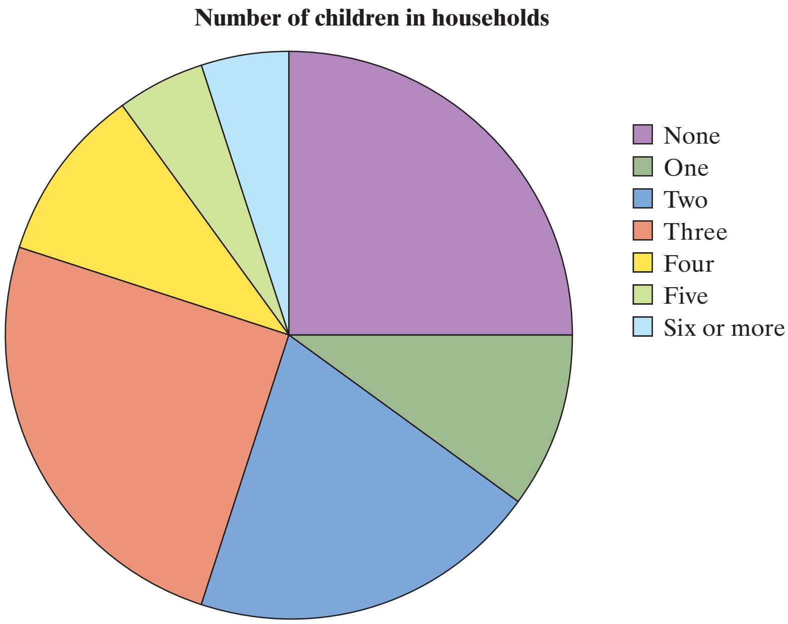
b how much did he spend on DVDs?

c What information about the DVDs he bought is not clear from this graph?

How Daniel spent his birthday money



● Town planners surveyed a community to find out how many children lived in each household. The results are shown below.



a Measure the angle for each sector and complete the table.

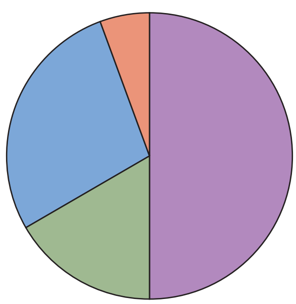
b What percentage of households has no children?

c What percentage of households has more than five children?

Children	Angle	Percentage
0		
1		
2		
3		
4		
5		
>5		

exercise 13.4 challenge

● This pie graph represents some data collected by your class. Design a question that the data on the pie graph might represent, and provide a frequency table of responses. Also add labels and a legend so that the graph makes sense.



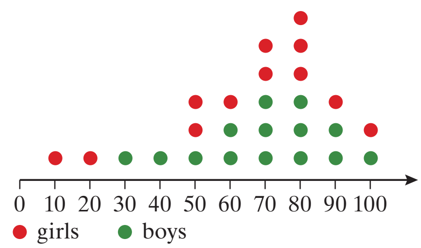
13.5 Displaying and interpreting data: dot plots and stem-and-leaf plots

Visual plots provide a quick picture of

- what values occur more frequently than others.
- how much **variation** there is among the data values.
- the **range** of the values (the difference between the smallest and largest values).

Dot plots

Dot plots are a useful way to look at the entire data set, and to see at a glance which values occur most frequently. They also illustrate how much variation exists in the data set.



A dot plot of algebra test scores for Year 7 White

To construct a dot plot

1. draw and label a horizontal axis long enough to fit all the numbers in the sample.
2. draw a dot over the position corresponding to each number.
3. if more than one dot falls in the same position, stack the dots above each other.

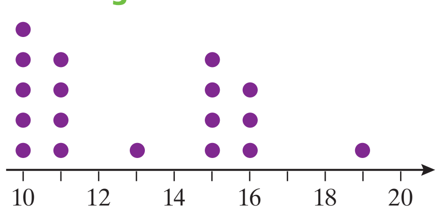
Example 16

Eighteen students are members of a youth group. Their current ages are as follows.

10, 13, 10, 15, 16, 11, 11, 16, 15, 10, 10, 11, 19, 16, 15, 11, 10, 15

Use a dot plot to represent the current ages of the members.

Working



Ages of youth group members

Reasoning

Use one dot to represent one person. It is not necessary to start the number line at zero, as the lowest data value is 10.

Example 17

The ages of 30 students in a school chess club are shown below.

15, 14, 13, 18, 16, 14, 13, 12, 14, 16, 16, 15, 16, 11, 17,

14, 15, 12, 17, 16, 14, 16, 13, 15, 16, 17, 16, 13, 16, 15

a Represent the ages of the students in the chess club in a dot plot.

b What is the range of ages?

c What is the modal age?

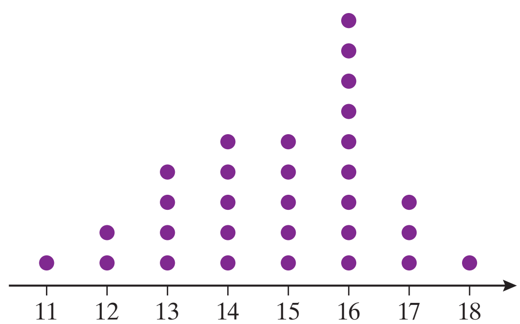
d What is the median age?

e What is the mean age?

f Suggest why the mean age is lower than the median age.

Working

a



Ages of members of the chess club

Reasoning

One dot represents one student.
It is not necessary to start the number line at 0 as the lowest age is 11.

b The range of ages is 7 years, from 11 to 18.

The range is the difference between the highest and lowest data values.
$18 - 11 = 7$

c The modal age is 16.

There are more 16-year-olds than any other age.

d The median age is 15.

There are 30 students so the median is halfway between the 15th and 16th data values—there must be 15 data values each side of the median.

$$\text{mean} = \frac{445}{30}$$

$$= 14.83$$

$$\text{mean} = \frac{\text{sum of all the ages of members}}{\text{total number of members}}$$

e The mean age is 14.8.

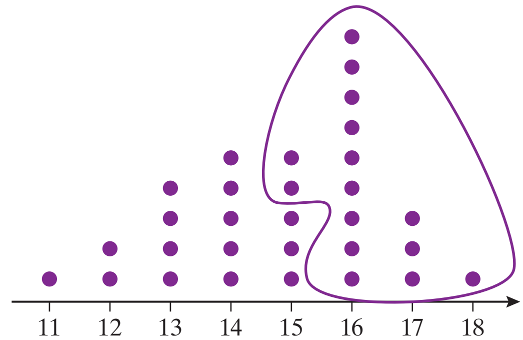
continued

Example 17 continued

Working

f Younger students in the club are spread over a wider range of ages than the older students. This brings the mean age down. So, the mean age is lower than the median age.

Reasoning



Ages of members of the chess club

Outliers

An **outlier** is an extreme value of the data. It is a value that is quite different from the rest of the data. There may be more than one outlier in a data set.

Sometimes outliers occur as a result of an error and should be ignored. Perhaps an incorrect measurement was made, a data value was recorded incorrectly or a person misinterpreted a survey question. Sometimes, though, outliers are important and should not be ignored. Investigating outliers may lead to interesting new information.

Outliers affect the mean more than they affect the mode and median.

Example 18

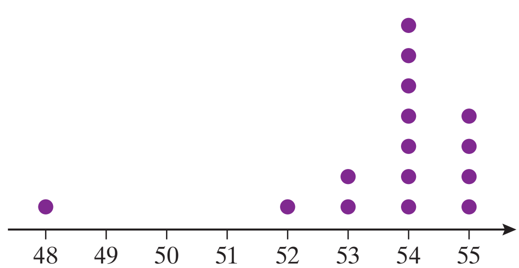
This dot plot shows 15 data values.

a Which data value is an outlier in the data shown in this dot plot?

b What is the median value?

c If the value 48 was found to be a mistake and it should have been 52, what would the median value be now?

d Do you think the mean will change when 48 is changed to 52? Explain.



continued

Example 18 continued

Working	Reasoning
a The value 48 is an outlier.	48 does not fit the pattern of the other data values, which range from 52–55.
b The median is 54.	There are 15 data values, so the 8th data value represents the median, so that there are 7 values either side of the median.
c The median is still 54.	Changing 48 to 52 does not change the median of this data set.
d Changing 48 to 52 does not affect the mode of this data set.	The number with the greatest frequency will still be 54.
e Yes, the mean will increase slightly because the sum of all the data values will be 4 greater than it was before.	$\text{Mean} = \dfrac{\text{sum of all the data values}}{\text{total number of data values}}$

The mean is affected by outliers (extremely high or extremely low data values).

The median is not influenced by outliers.

Stem-and-leaf plots

In a **stem-and-leaf plot** each piece of data can be split into a stem and a leaf. It is usual for the leaf to be the end digit of the number and for any numbers to the left of this digit to be called the stem. For example, 67 is usually split to have stem 6 and leaf 7 or 154 can have stem 15 and leaf 4. A number such as 6 has a stem of 0 and leaf of 6.

When constructing a stem-and-leaf plot, it is usual to arrange the leaves in ascending order, i.e. smallest to largest. This is called an **ordered stem-and-leaf plot**.

Example 19

The test results, out of 60, for a class of 30 students are shown below.

32, 60, 57, 42, 45, 52, 57, 60, 48, 45, 43, 38, 47, 56, 55, 39, 43, 56, 54, 38, 59, 46, 44, 45, 40, 59, 50, 40, 55, 35

Create an ordered stem-and-leaf plot for the data using

a one stem for each multiple of 10.

b two stems for each multiple of 10 (referred to as split stems).

Working	Reasoning
a 32, 35, 38, 38, 39, 40, 40, 42, 43, 43, 44, 45, 45, 45, 46, 47, 48, 50, 52, 54, 55, 55, 56, 56, 57, 57, 59, 59, 60, 60	Arranging the data in ascending order means that an ordered stem plot can be obtained directly.

continued

Example 19 continued

Working

Stem	Leaf
3	2 5 8 8 9
4	0 0 2 3 3 4 5 5 5 6 7 8
5	0 2 4 5 5 6 6 7 7 9 9
6	0 0

(Key: 3|2 means 32)

Reasoning

Split each piece of data into a stem and a leaf, starting with 32 having stem 3 and leaf 2.

Test scores out of 60

Stem	Leaf
3	2 5 8 8 9
4	0 0 2 3 3 4 5 5 5 6 7 8
5	0 2 4 5 5 6 6 7 7 9 9
6	0 0

(Key: 3|2 means 32)

Give the stem-and-leaf plot a title and a key.

b **Test scores out of 60**

Stem	Leaf
3	2
3	5 8 8 9
4	0 0 2 3 3 4
4	5 5 5 6 7 8
5	0 2 4
5	5 5 6 6 7 7 9 9
6	0 0

(Key: 3|2 means 32)

Where two stems are used for each multiple of 10 then the digits 0, 1, 2, 3 and 4 are placed with the first stem and the digits 5, 6, 7, 8 and 9 are placed with the second stem.

Note that only one stem is required for the 60s as there are no leaves above the value of 4.

If we wish to compare the distribution of data values in two different data sets, we can use back-to-back stem plots. The stem is shared and one set of leaves is shown to the left and the other set to the right of the stem.

Example 20

A PE teacher conducts a fitness test on two of her classes. She records the number of push ups completed in one minute by students in each class and the data is shown below.

Class 1: 38, 43, 47, 52, 35, 34, 36, 47, 64, 60, 33, 42, 58, 52, 40, 33, 43, 36, 52, 55

Class 2: 46, 54, 38, 39, 60, 62, 65, 58, 46, 53, 57, 42, 45, 49, 53, 58, 62, 54, 43, 47

a Construct an ordered back-to-back stem plot using two stems for each multiple of 10.

b What is the median number of push ups for Class 1?

c What is the median number of push ups for Class 2?

d Which class seems to be the fittest?

Working

a **Number of push ups in one minute for two classes**

Class 1 Leaf	Stem	Class 2 Leaf
4 3 3	3	
8 6 6 5	3	8 9
3 3 2 0	4	2 3
7 7	4	5 6 6 7 9
2 2 2	5	3 3 4 4
8 5	5	7 8 8
4 0	6	0 2 2
	6	5

(Key: 5|3| means 35
|3|8 means 38)

Reasoning

Decide which set of data is going to be displayed on each side and use headings to show this.

Decide on the stems you need to use. To do this, look at the two sets of data and find the smallest and largest values. These are 33 and 65. So the stems need to start at 3 and end at 6.

Split each piece of data for Class 1 and Class 2 into a stem and a leaf.
Add a title and legend.

b The median number of push ups for Class 1 is 43.

There are 20 data values so there must be 10 data values each side of the median. The median is halfway between 43 and 43.

c The median number of push ups for Class 2 is 53.

There are 20 data values so there must be 10 data values each side of the median.

d Class 2 seems to be the fittest. This class has a higher median number of push ups and there are more students with higher numbers, and fewer with lower numbers.

The back-to-back stem plot allows us to compare the two classes.

exercise 13.5

LINKS TO Example 16

1 Use a dot plot to represent each of the following data sets, starting each scale at a suitable value.

a 0, 0, 0, 1, 1, 2, 3, 3, 3, 3, 4, 5, 7

b 2, 3, 4, 4, 3, 5, 6, 3, 7, 2

c 1, 2, 3, 2, 3, 2, 4, 1, 2, 2, 6, 2, 3, 1, 4, 2

d 19, 15, 18, 15, 18, 19, 20, 20, 16, 20, 17, 15, 17, 18, 19, 17, 16, 19, 18, 20

e 31, 36, 36, 30, 30, 32, 36, 30, 37, 31, 40, 31, 39, 32, 38, 34, 37, 32, 34, 38

f 102, 97, 95, 99, 96, 102, 102, 100, 97, 102, 99, 99, 95, 97, 102, 100, 95, 99, 101, 100

LINKS TO Example 16

2 Zhen collects data from a group of students on the number of siblings (brothers and sisters) each student has. Her results are: 0, 5, 3, 3, 2, 1, 0, 3, 2, 2, 0, 5, 1, 4, 2, 2, 1, 2, 3, 1, 2

a Construct a dot plot to represent this data.

b What is the median number of siblings?

c What is the range?

d How many students have more than one sibling?

LINKS TO Example 17

3 One of the questions in the CensusAtSchool asks students to state how many people normally live at their house. The following data were obtained from a group of 40 students.

4, 4, 4, 7, 3, 4, 4, 4, 4, 4, 8, 4, 5, 4, 3, 4, 4, 5, 4, 4

5, 2, 4, 4, 4, 4, 4, 4, 5, 4, 4, 4, 3, 6, 4, 4, 5, 4, 4, 4

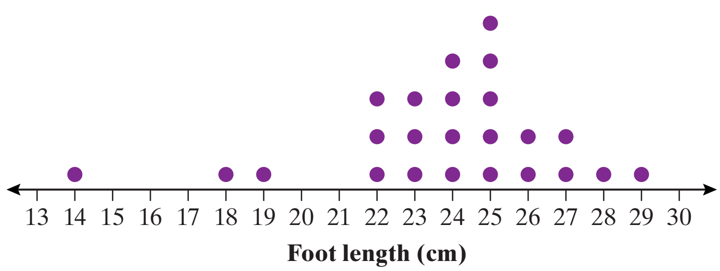
a Represent the data as a dot plot.

b What is the modal number of people per house?

c What is the median number of people per house?

LINKS TO Example 18

4 The CensusAtSchool project asked students to record their foot length. This dot plot shows foot lengths for a sample of 24 students.



a What is the modal foot length?

b What is the median foot length?

c What is the range?

d Which value can be regarded as an outlier?

e What is the range if the outlier is removed?

f Determine the effect of removing the outlier on

i the mode. **ii** the median.

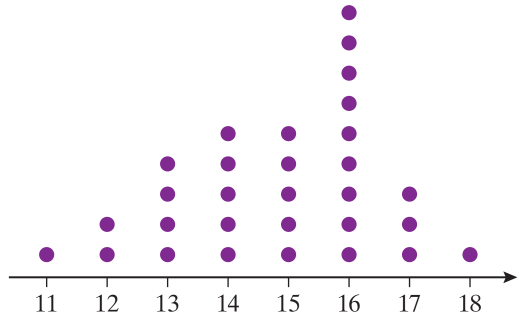
The dot plot below shows the ages of the players in a junior tennis squad.

a What is the most common age?

b How many players are 15 years or older?

c What is the range?

d What is the median age?



Ages of members of the junior tennis squad

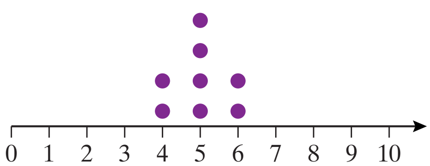
Two groups of eight students underwent a test of manual dexterity. Their scores out of 10 are shown below.

'Manual dexterity' means ability or skill in using the hands.

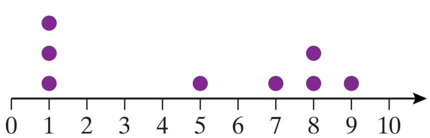
Group A	4	4	5	5	5	5	6	6
Group B	1	1	1	5	7	8	8	9

a Calculate the mean test result for each group. What do you notice?

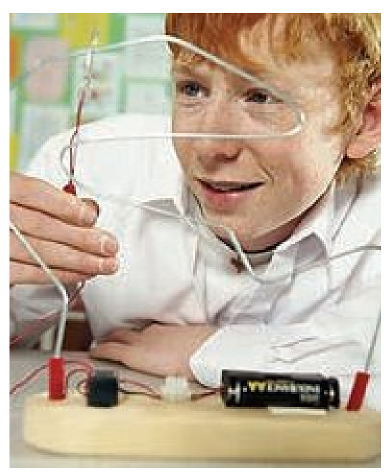
The following dot plots show the distribution of each set of data.



Group A



Group B



b Compare the variability in the distribution of each data set.

c What information would have been 'hidden' if only the means of the two variables had been given?

The following data show the length of the right foot in centimetres for 20 Year 7 boys and 20 Year 7 girls.

Boys: 26, 27, 26, 27, 25, 23, 24, 25, 24, 20, 24, 25, 28, 21, 27, 26, 23, 23, 22, 27

Girls: 25, 23, 22, 24, 23, 26, 23, 29, 23, 26, 26, 23, 21, 24, 27, 21, 20, 21, 23, 26

a Construct two dot plots for the data.
b Compare the median foot length for boys and for girls.
c Compare the mean foot length for boys and for girls.
d Do you think similar results would be obtained from a survey of Year 11 students?

LINKS TO Example 19a

Construct an ordered stem-and-leaf plot for each of the following sets of data using one stem for each multiple of 10.

a 57, 56, 33, 44, 45, 51, 38, 32, 45
b 71, 82, 65, 68, 82, 76, 61, 74, 82, 75
c 32, 25, 34, 36, 40, 36, 27, 31, 31, 42, 31, 24
d 98, 64, 70, 69, 68, 74, 84, 95, 61, 84, 59, 86, 96, 63
e 8, 11, 14, 21, 7, 17, 13, 12, 25, 7, 3, 18, 24, 16, 5, 22
f 7, 12, 26, 34, 18, 9, 52, 38, 46, 42, 37, 21, 26, 18, 5, 26, 33, 51, 30, 27
g 69, 65, 56, 56, 62, 49, 76, 48, 45, 34, 27, 55, 42, 36, 38, 40, 36, 29, 32, 32, 44, 32, 21
h 62, 21, 56, 62, 35, 58, 57, 49, 26, 65, 22, 25, 36, 53, 48, 35, 56, 67, 71, 54

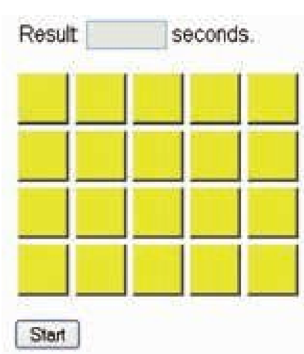
LINKS TO Example 19b

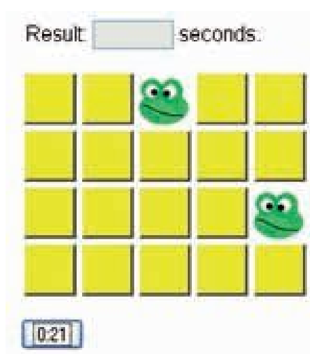
Construct an ordered stem-and-leaf plot for each of the following sets of data using two stems for each multiple of 10.

a 125, 132, 128, 119, 127, 130, 115, 136, 141, 110, 116, 129, 134, 135, 126, 128
b 84, 82, 76, 91, 103, 88, 76, 84, 82, 78, 100, 95, 93, 86, 77, 82, 93, 81, 104, 77
c 62, 20, 51, 61, 31, 58, 58, 45, 24, 67, 21, 22, 36, 53, 49, 32, 55, 66, 70, 54

LINKS TO Example 20

One of the items in the Australian CensusAtSchool online questionnaire measures students' concentration when they match pairs of pictures. There are 20 pictures, each of which appears when the student clicks on a yellow square. If the student clicks on another square that has the same picture, the two pictures remain visible, but if the pictures are different they are hidden again. Ten pairs of pictures must be matched.





Source: Australian Bureau of Statistics

The times for 15 boys and 15 girls are shown below.

Boys: 62, 36, 76, 47, 38, 38, 49, 61, 64, 74, 57, 36, 87, 61, 30

Girls: 42, 42, 48, 44, 43, 54, 56, 31, 35, 31, 48, 45, 72, 83, 146

a Construct a back-to-back stem-and-leaf plot to show the two data sets. Use one stem for each multiple of 10.

b Compare the range for the boys and the girls.

c Compare the median times for the boys and the girls.

LINKS TO Example 20

The following data show the arm spans in centimetres of 20 Year 7 boys and 20 Year 7 girls who took part in the 2010 Australian Census at School.

Boys: 146, 178, 156, 151, 168, 154, 163, 168, 156, 154,
155, 148, 149, 165, 154, 165, 152, 162, 178, 150

Girls: 150, 154, 167, 176, 144, 154, 180, 160, 160, 137,
160, 158, 144, 155, 155, 142, 133, 154, 157, 150

Source: Australian Bureau of Statistics

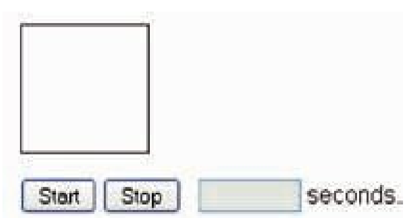
a Construct a back-to-back stem-and-leaf plot to show the two data sets. Use two stem for each multiple of 10.

b Compare the range for the boys and the girls.

c Compare the median arm span for the boys and the girls.

exercise 13.5 challenge

The 2010 Australian CensusAtSchool included a test of reaction time where students used their dominant hand to click the mouse when an image appeared in a box on the computer screen. In a second test students used their non-dominant hand.



Source: Australian Bureau of Statistics

The following data show the reaction time in seconds (that is, the delay between seeing the image and clicking on Stop) for 20 Year 7 students using their dominant hand.

0.28, 0.37, 0.35, 0.46, 0.46, 0.40, 0.36, 0.40, 0.34, 0.34,
0.29, 0.53, 0.34, 0.43, 0.43, 0.42, 0.30, 0.28, 0.39, 0.32

a Construct a stem-and-leaf plot for the data

b What is the median reaction time?

c What is the range?

d In what way might the median and the range differ for reaction times using the non-dominant hand?

Analysis task

Concentration time

In question 10 of exercise 13.5, a concentration exercise used in the CensusAtSchool test was described. This analysis task involves comparing the times taken to complete the task by a sample of girls and a sample of boys in Year 7. Random samples of data are obtained from the CensusAtSchool website:
www.cas.abs.gov.au/cgi-local/cassampler.pl

a Selecting the data for a sample of 30 girls

Once you have loaded the website you are able to choose the type of sample you want. Choose the following to obtain your data for a sample of 30 girls:

Reference year	**2010**
Questions	All
Sample size	30
Post code range	0000 - 9999
Year level	7
Sex	Female

Now click on **Select sample data**. Click on the Excel spreadsheet icon beside **Download data xls sample file**. Open the spreadsheet and scroll across to **column AO**.

Copy and paste the column of data into a new spreadsheet and label the column Female.

b Selecting the data for a sample of 30 boys

Repeat all the above steps, choosing Male so that you have data for 30 boys to paste into a column labelled Male.

c Unordered stem-and-leaf plot

Make sure that you have the same numbers of females and males, as some students may not have answered the question. Try another random sample if you have missing data. Using a table as shown, enter all your data into the tables so that you have an unsorted back-to-back stem-and-leaf plot.

Unordered concentration times

Males		**Females**
Leaf	Stem	Leaf

d Ordered stem-and-leaf plot

Using your unordered data, sort each leaf into ascending order, with the smallest values closest to the stem on each side.

Unordered concentration times

Males		**Females**
Leaf	Stem	Leaf

Give your stem-and-leaf plot a title and a key.

e Comparing male and female students

Use the shape, centre and spread of the stem and leaf plot to compare the distribution of times for males and females.

- How are the distributions similar and how are they different?
- Do you think different results would be obtained with a different year level?

f Collecting class data

Go to the CensusAtSchool website (www.abs.gov.au/censusatschool), select *Past Questionnaires* and choose the online sample version of the 2010 questionnaire. Go to question 25 and complete the task. Record the time you took as indicated on the screen. Collect together the times separately for the boys and girls in your class. Construct a back-to-back stem-and-leaf plot of your class data. Comment on the similarities or differences compared with the random sample you used in the first part of this analysis task.

Review Statistics

Summary

What are data?

- Data are collected observations or facts which can be numerical or non-numerical.
- The characteristic being measured, counted or observed is often referred to as a *variable*, since it is something that often varies (takes different values).

Data types

- Categorical data provide information about categories or groups.
- Numerical data provide information about characteristics that can be measured.

Summarising data: measuring centre and spread

- $\text{Mean} = \dfrac{\text{Sum of all the data values}}{\text{Number of data values}}$
- The median is the middle value when the data set is placed in sorted order.
- The mode is the most frequent data value.
- The mean is affected by outliers (extremely high or extremely low data values).
- The median is not influenced by outliers.
- The range is the difference between the highest and lowest data values.

Representing and interpreting data in graphs

- Graphs of data usually include four key elements—scale, axis, legend and title (SALT).
- Pie graphs and column graphs are most useful for displaying categorical data.
- Line graphs are useful for showing changes in variables over time.
- Scatter plots show the relationship between two variables.

Summarising data: visually

- Dot plots and stem-and-leaf plots are useful tools for creating a visual summary of the distribution of small sets of numerical data where the values are reasonably close in size.
- Dot plots and stem-and-leaf plots show every data value.
- The median can easily be found from dot plots and stem-and-leaf plots because the data is in order.
- Back-to-back stem-and-leaf plots are useful for comparing sets of data for two groups, e.g. male and female.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

ascending	maximum	primary source
average	mean	range
axis	median	scale
categorical	minimum	statistics
column graph	mode	stem plot
descending	numerical	secondary source
distribution	ordered	variable
dot plot	outlier	variation
frequency	pie graph	

Revision

Multiple-choice questions

Which of the following is not categorical data?

A red, green, black, green, red, blue, orange, orange
B 1st, 3rd, 1st, 2nd, 2nd, 3rd, 5th, 4th
C 3, 12, 13, 12, 7, 18, 8, 19
D cat, dog, dog, mouse, dog, cat, cat
E high, medium, low, high, high, low

Use the following data for questions 2 and 3.

The mass in grams of 10 eggs was 58, 64, 60, 58, 59, 66, 57, 58, 62, 61

The range is

A 3 g **B** 9 g **C** 58 g **D** 60 g **E** 66 g

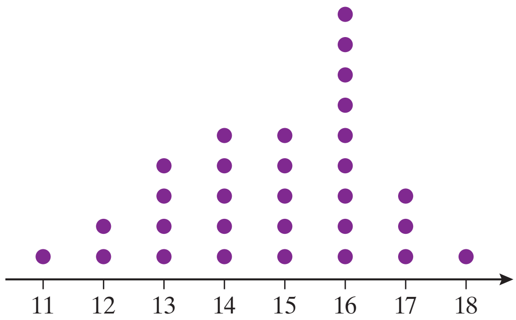
The mean mass is

A 57 g **B** 58 g **C** 59.5 g **D** 60.3 g **E** 61.5 g

- The dot plot shows the ages of students in a school chess club.

  Which of the following is NOT correct?

  A The range is seven years
  B Twelve students were younger than 15
  C The median age is 14.5
  D The modal age is 16
  E Sixty per cent of the students were 15 or over



The ages of members of a school chess club

- The following stem-and-leaf plot represents the ages of people on a bus.
  The median age is

  A 25.5
  B 26
  C 26.5
  D 30
  E 62

Stem	Leaf
0	5 7 9
1	2 8 8
2	1 6 6 6 7
3	0 3 4 7 8
4	2 6
5	1 2

(Key: 1|2 means 12)

Short-answer questions

- For each of the following survey items, decide whether the response data would be categorical (C) or numerical (N).

  a students' birthday months
  b baby birth weight
  c total weight of fat in burger
  d final placing of eight students in the 25 m novelty race

- The data below give the lengths of the hand spans of 18 girls in millimetres.

  116, 148, 148, 145, 154, 138, 115, 135, 122, 153, 120, 130, 121, 140, 123, 132, 121, 111

  a Tally the measurements into 10 mm intervals.
  b Calculate the frequency of each interval.
  c Which interval had the highest frequency?

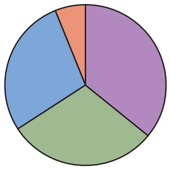
- A school librarian records the number of books borrowed by each of 25 students over lunchtime one day. This frequency table summarises the data.

  a Construct a column graph representing this data.
  b State the mode.

Number of books	Frequency
1	7
2	9
3	5
4	2
5	2

- The world loses about seven million hectares of fertile land each year due to soil degradation. Use the information in the graph to calculate how many hectares of land are lost due to deforestation.

Worldwide causes of soil degradation



- The following scores out of 10 were received by students for their science practical report. Find the median score.

  4, 7, 3, 6, 8, 2, 6, 8, 5, 8, 9, 5, 7, 5, 6, 10, 4, 9, 6, 7

Extended-response questions

- Twenty Year 7 students each measured and recorded the height of their belly button above the floor. The following data gives the heights in centimetres.

  99, 100, 74, 87, 90, 94, 97, 100, 94, 87, 94, 94, 93, 101, 97, 101, 90, 100, 98, 96

  a Construct a dot plot of the data
  b What is the median height?
  c What is the mean height?
  d Which height is an outlier?
  e What effect does removal of this outlier have on the median?
  f What effect does removal of this outlier have on the mean?

- A road intersection is to be modified based on data collected in a traffic survey. The number of cars travelling through the intersection each minute is measured with the results shown in the stem-and-leaf plot below.

  a Each piece of data represents one minute of observations. For how many minutes was the intersection observed?
  b What is the median number of cars passing through the intersection in one minute?

Number of cars passing through the intersection in one minute

Stem	Leaf
0	3 3 6 7 7 8 8 9
1	0 0 1 3 4 5
2	1 5 6 6 8 8 9
3	0 1 1 4

(Key: 2|1 means 21)

- The following data represents the number of hours that 50 Year 7 students watched television in a particular week.

  3, 8, 19, 8, 0, 1, 7, 10, 4, 20, 7, 9, 1, 11, 8, 13, 2, 1, 30, 7, 40, 18, 40, 4, 10,
  15, 10, 2, 4, 40, 0, 5, 10, 2, 28, 2, 9, 5, 5, 17, 25, 12, 3, 6, 1, 4, 3, 17, 11, 4

  a Construct an ordered stem-and-leaf plot of the data
  b What is the median number of hours that students watched television?
  c What percentage of students watched television for more than five hours in the week?

Eliza and Henry compete in a maths problem-solving competition to see who can solve problems the fastest. Their times, measured in seconds, to complete the problems are as shown.

Eliza | **Henry**

Leaf	Stem	Leaf
	3	0 1 2
3 1	4	1 1 1 4 5
8 4 1 0 0	5	2
8 5 3	6	9

Key Left side: 1|4 means 41 Right side 4|1 means 41

a Complete this table to compare Eliza's and Henry's times.

b Explain why you think Henry was chosen as the winner.

	Eliza	Henry
Shortest time		
Longest time		
Median time		

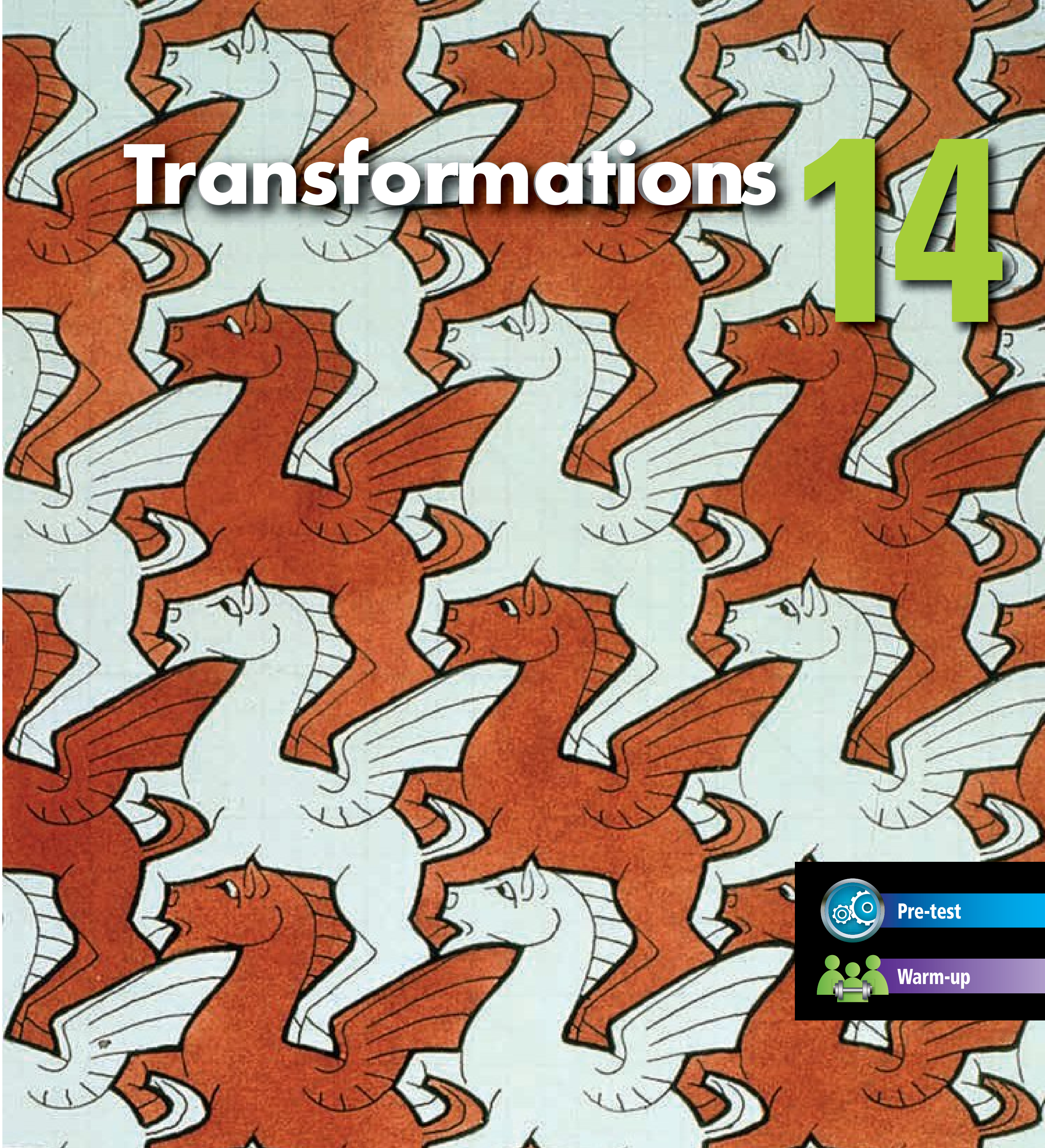
The following list shows six different ways of representing data.

pie graph
bar graph
back-to-back stem-and-leaf plot
column graph
stem-and-leaf plot
dot plot

Match each of the following data descriptions with the most suitable way of representing the data from the list above.

Data	Representation
Scores out of 40 on a test for 50 boys and 50 girls	
Number of TVs in the home of each student in a Year 7 class.	
Favourite music styles of students in Year 9.	
Percentage of Australians living in each of the Australian states and territories	
Method of travel to school of students at a particular school.	
Concentration in minutes on a task for 50 students	

Transformations 14



Pre-test

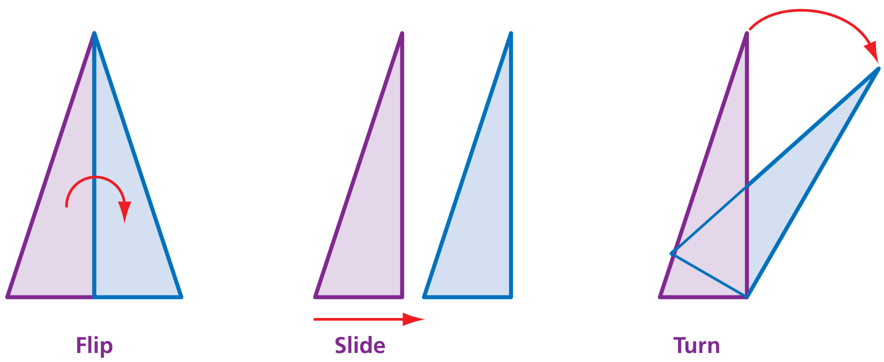
Warm-up

Changing the position or size of a shape is called a *transformation*. The Dutch artist Maurits Cornelis Escher created many beautiful tessellating designs based on geometric transformations—translation, reflection and rotation—of animal shapes. Which transformation has Escher used in this design of winged horses?

14.1 Flips, slides and turns

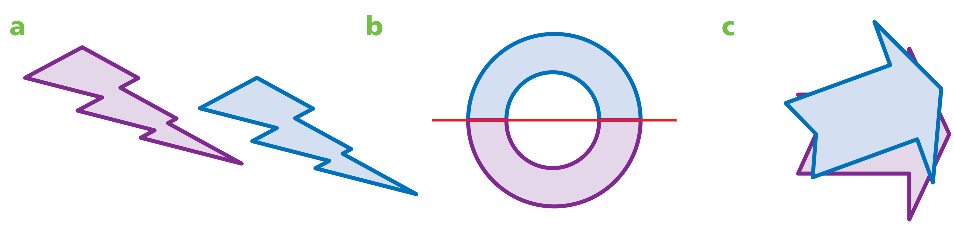
Transformations

The change in the position of a shape is called a **transformation**. You have probably used the three terms **flip**, **slide** and **turn** to describe transformations.



Example 1

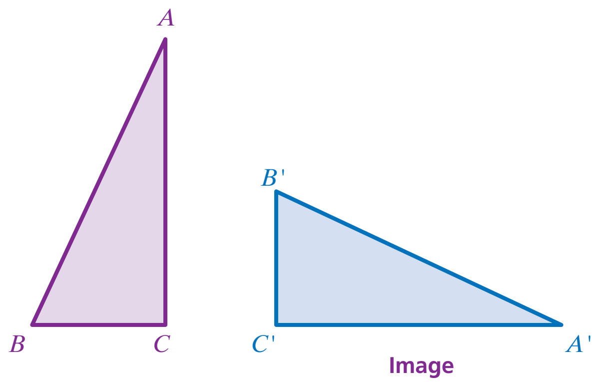
State the transformation represented by each of these diagrams.



Working	Reasoning
a Slide	The position of the shape has changed but it has not been flipped or turned.
b Flip	The top shape is a mirror image of the bottom shape.
c Turn	The arrow has been turned in an anticlockwise direction.

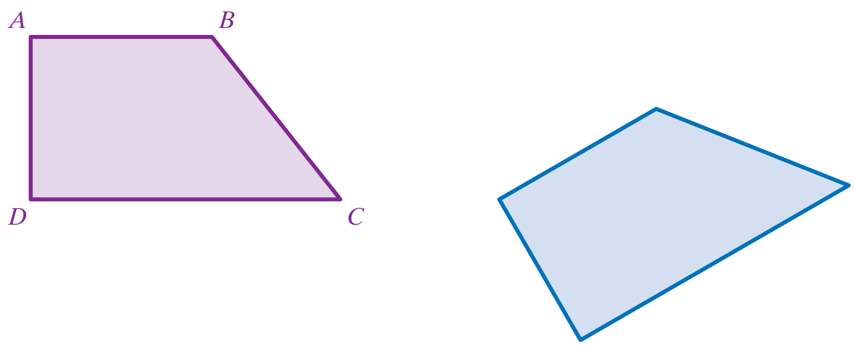
The transformed shape is called the **image** of the original shape. If we use the letters A, B, C, etcetera to label the vertices of the shape, we normally use A', B', C', etc. to label the vertices of its image. For example, if a triangle is labelled ABC, its image would be labelled $A'B'C'$. We write 'A''. We say 'A dash'.

When showing the transformations of shapes in this chapter, we will use different colours to fill the shape and its image, as shown below. In the following diagram, triangle $A'B'C'$ is the image of triangle ABC after triangle ABC has been transformed.

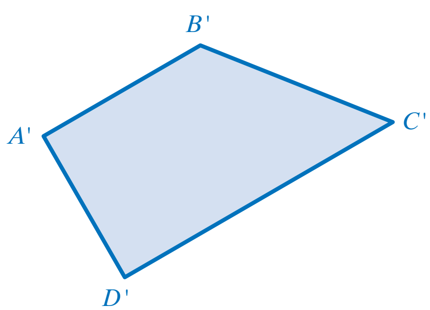


Example 2

Label the vertices of the image of $ABCD$.



Working



Reasoning

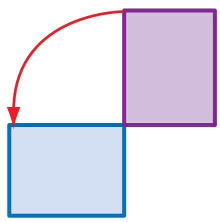
Each matching vertex of $ABCD$ is labelled with the same letter followed by a dash$'$.

exercise 14.1

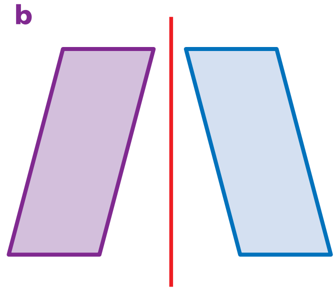
LINKS TO Example 1

State whether each of these transformations is a flip, a slide or a turn.

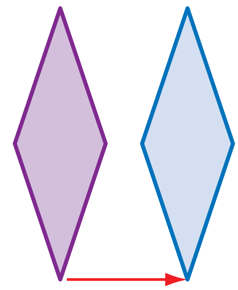
a



b



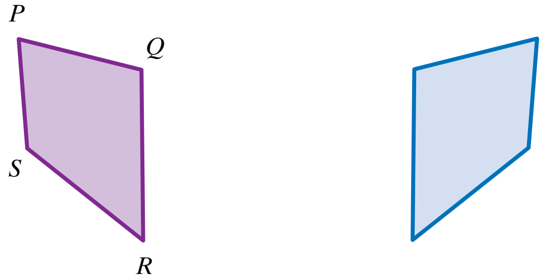
c



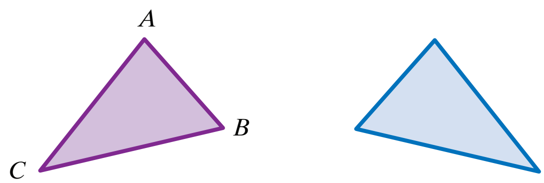
LINKS TO Example 2

Label the vertices of each image.

a



b



exercise 14.1 challenge

To produce the blue image, the shape *ABCDEF* has been transformed by

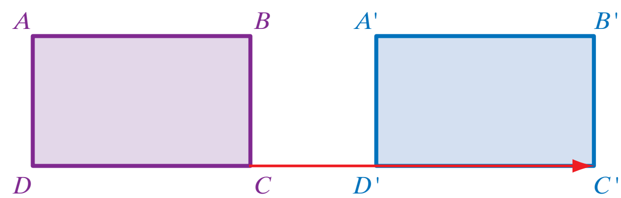
A flipping and sliding but not turning.
B turning and sliding but not flipping.
C turning, flipping and sliding.
D sliding only.
E turning only.

14.2 Translation

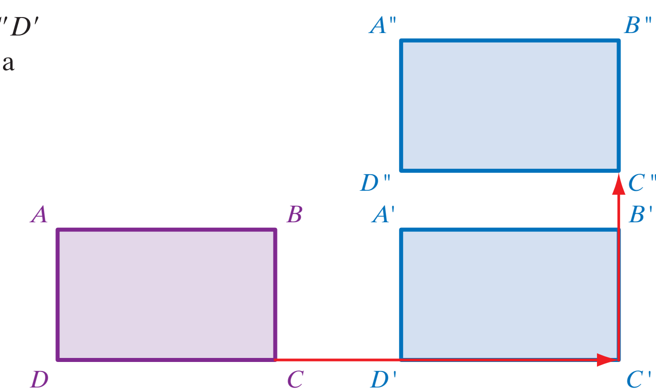
Translation

The transformation of a shape by sliding into a new position is called **translation**. Translating a shape is simply what you have previously called sliding. In the design on the opening page of this chapter, Escher has used translation to repeat the pattern of the winged horse. Translating a shape involves the **direction** and the **distance** that it slides.

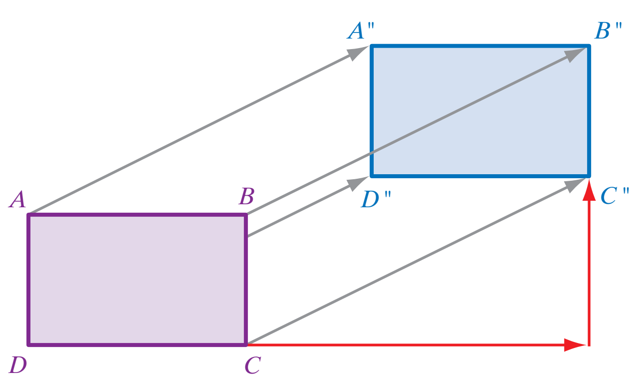
In the diagram below, the red arrow shows the distance and direction of the translation. Notice that each vertex of rectangle $ABCD$ has been translated by the same amount in the same direction. The image is called $A'B'C'D'$.



In the next diagram, the image $A'B'C'D'$ has been translated vertically to form a new image $A\ B\ C\ D$.

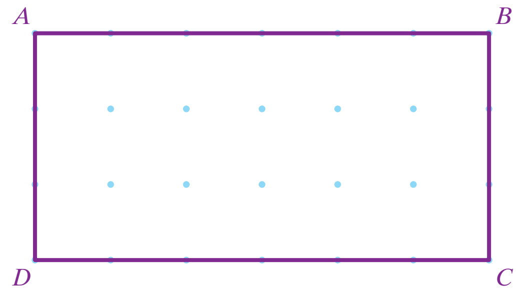


The two separate translations (shown by the grey arrows) can be represented by a single translation (shown by the red arrows). Notice how each vertex of the rectangle has been translated in the same direction and by the same distance.

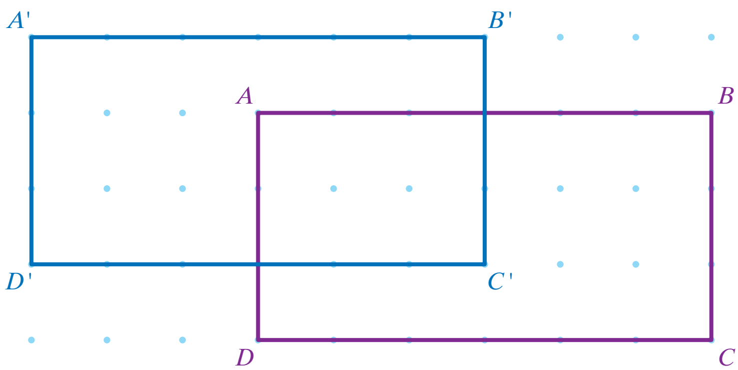


Example 3

Translate rectangle $ABCD$ 3 units to the left and 1 unit up. Label the image rectangle $A'B'C'D'$.



Working



Reasoning

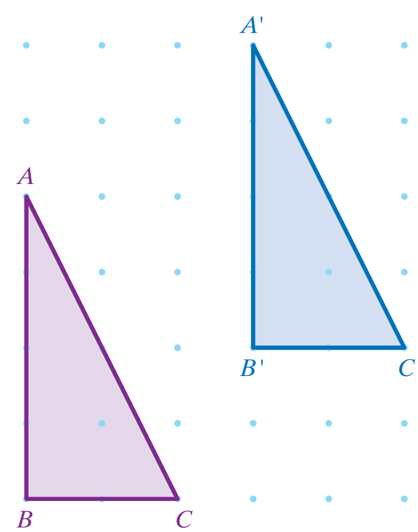
Each vertex of the rectangle has moved 2 units to the left and 1 unit up.

Example 4

Consider triangle ABC and its image.

a Describe in two steps how the triangle has been translated.

b Draw arrows AA', BB' and CC' to show the translation as a single translation.



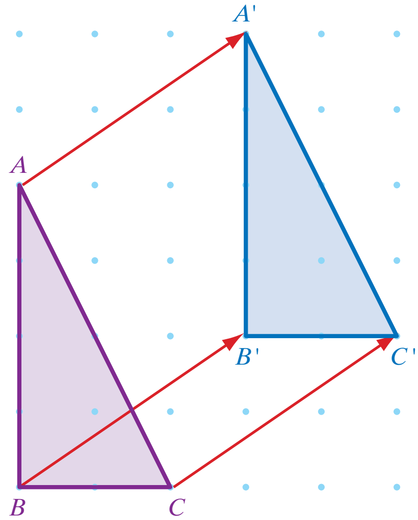
continued

Example 4 continued

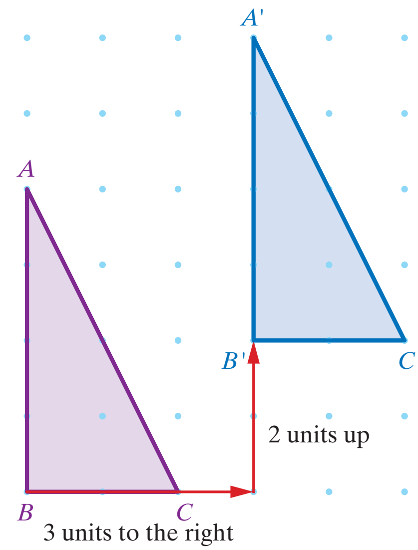
Working

a The triangle has been translated 2 units up and 3 units to the right (or 3 units to the right and 2 units up).

b



Reasoning



Translation on the Cartesian plane

Translation on the Cartesian plane

When we translate a point in a horizontal direction on the Cartesian plane, the x-coordinate changes but the y-coordinate stays the same.

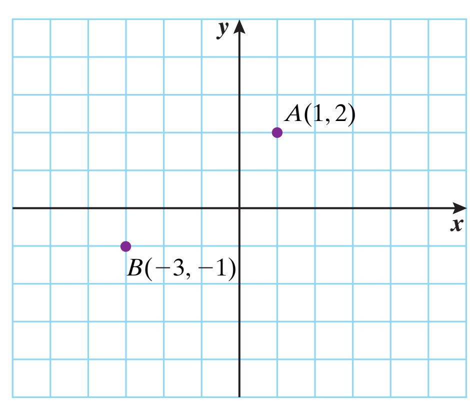
When we translate a point in a vertical direction on the Cartesian plane, the y-coordinate changes but the x-coordinate stays the same.

Example 5

Translate the points A and B in the following directions, in each case labelling points A' and B' and giving their coordinates.

a 4 units to the right

b 3 units down

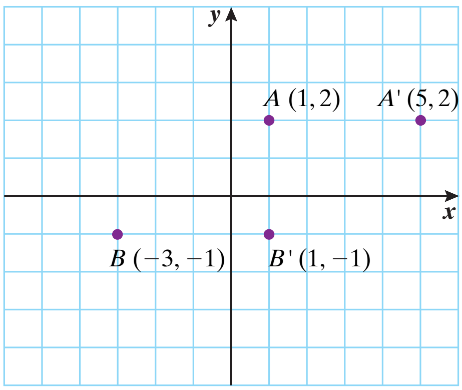


continued

Example 5 continued

Working

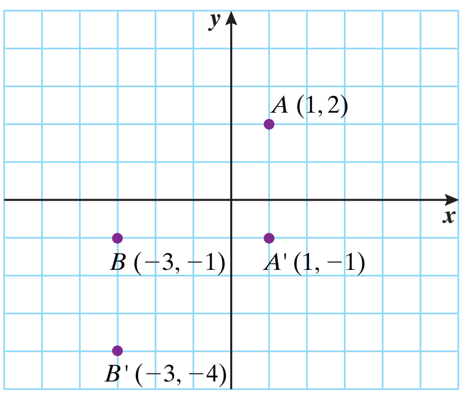
a



Reasoning

Each x-coordinate increases by 4.
$1 + 4 = 5$
$-3 + 4 = 1$
The y-coordinates stay the same.

b



The x-coordinates stay the same. Each y-coordinate decreases by 3.
$2 - 3 = -1$
$-1 - 3 = -4$

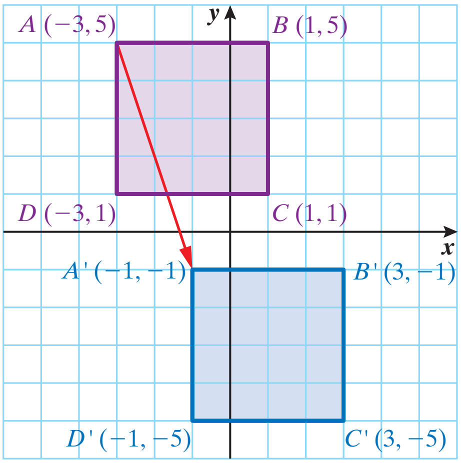
Example 6

The vertices of square $ABCD$ are $A(-3, 5)$, $B(1, 5)$, $C(1, 1)$ and $D(-3, 1)$. Translate the square 2 units to the right and 6 units down. Label the vertices of the image and give their coordinates. Draw an arrow from A to A' to show the combined effect of the translations in the two directions.

continued

Example 6 continued

Working



Reasoning

Each x-coordinate increases by 2.
Each y-coordinate decreases by 6.

exercise 14.2

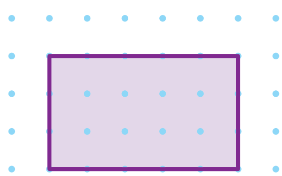
LINKS TO Example 3

BLM Dotted paper

BLM 1 cm grid

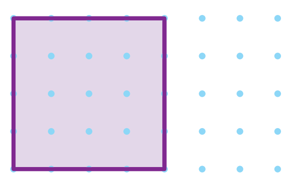
Copy each of these shapes onto squared or dotted paper. Translate each of these shapes in the direction indicated.

a



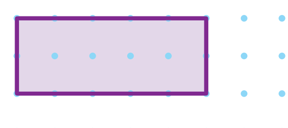
3 units down

b



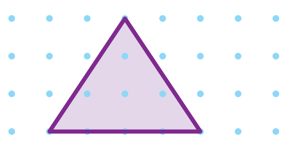
6 units to the right

c



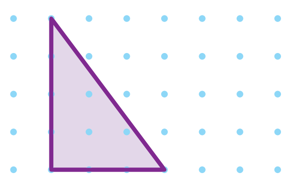
2 units up and 3 units to the left

d



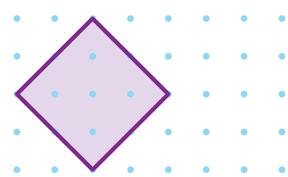
3 units up and 6 units to the right

e



1 unit up and 4 units to the left

f

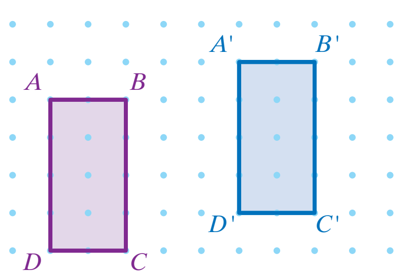


3 units up and 2 units to the right

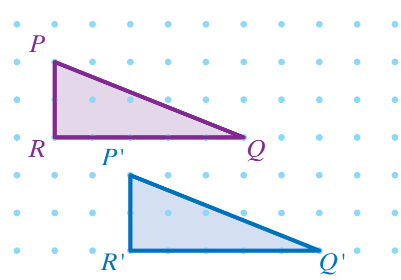
LINKS TO Example 3

Describe each of these translations.

a



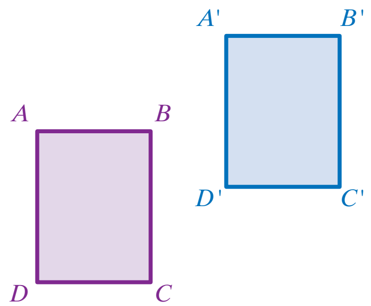
b



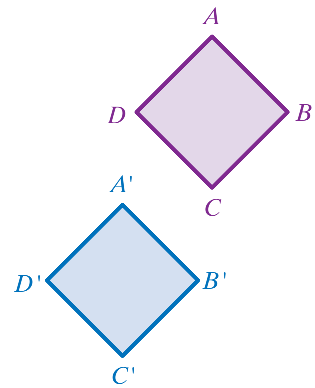
LINKS TO Example 4

For each of the following draw an arrow from A to show the distance of the translation.

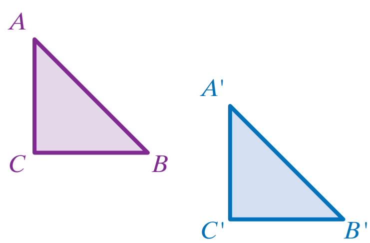
a



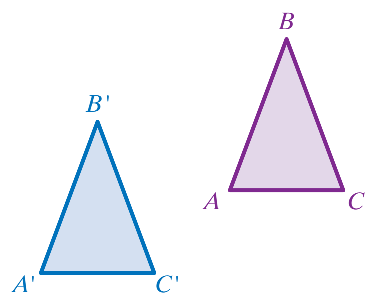
b



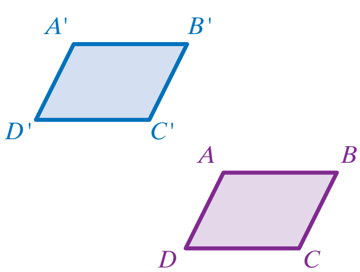
c



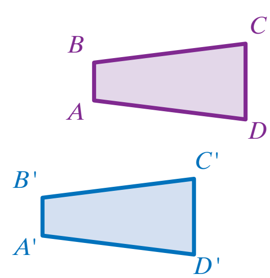
d



e



f



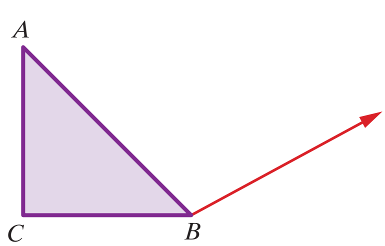
For each of these shapes,

i copy the shape and the arrow.

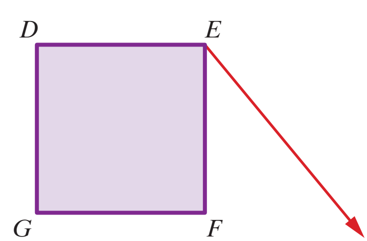
ii use the arrow to translate the shape.

iii label the image.

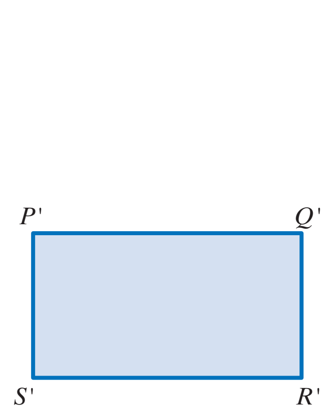
a



b



Carefully copy rectangle $PQRS$ and its image $P'Q'R'S'$. Draw an arrow at P to show the distance and direction of the translation.



The arrow $ABCDEFG$ has been translated

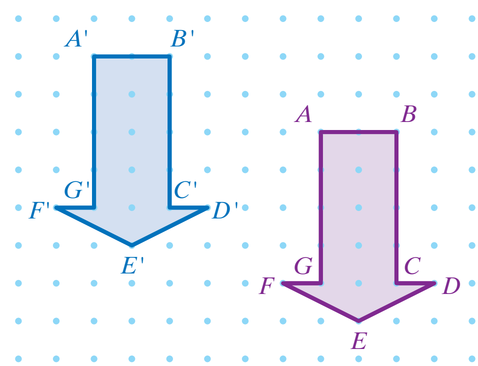
A 4 units to the right and 2 units down.

B 6 units to the right and 2 units down.

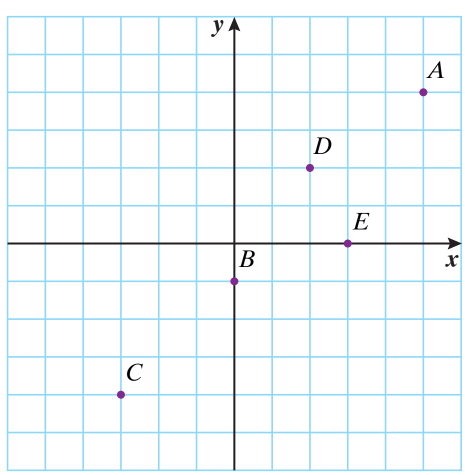
C 4 units to the left and 2 units up.

D 6 units to the left and 2 units up.

E 2 units to the left 2 and units up.



Use the five points A (5, 4), B (0, −1), C (−3, −4), D (2, 2) and E (3, 0) for questions 7 and 8.



LINKS TO Example 5

Translate each of the points 3 units to the left. Copy and complete the table to show the coordinates of each point and its translated image.

Coordinates	
Point	**Image**
A (5, 4)	A'
B	B'
C	C'
D	D'
E	E'

LINKS TO Example 5

Translate each of the points 2 units down. Copy and complete the table to show the coordinates of each point and its translated image.

Coordinates	
Point	**Image**
A (5, 4)	A'
B	B'
C	C'
D	D'
E	E'

Copy and complete.

When points are translated in a horizontal direction on the Cartesian plane, each ___ coordinate changes by the same amount. The ___ coordinates stay the same.

When points are translated in a vertical direction on the Cartesian plane, each ___ coordinate changes by the same amount. The ___ coordinates stay the same.

Mark and label the image of each of these points when they are translated in the given directions on the coordinate plane.

a A (1, 3) is translated 3 units to the right and 5 units up.

b B (4, 6) is translated 4 units to the left and 5 units down.

c C (8, 2) is translated 2 units to the left and 4 units up.

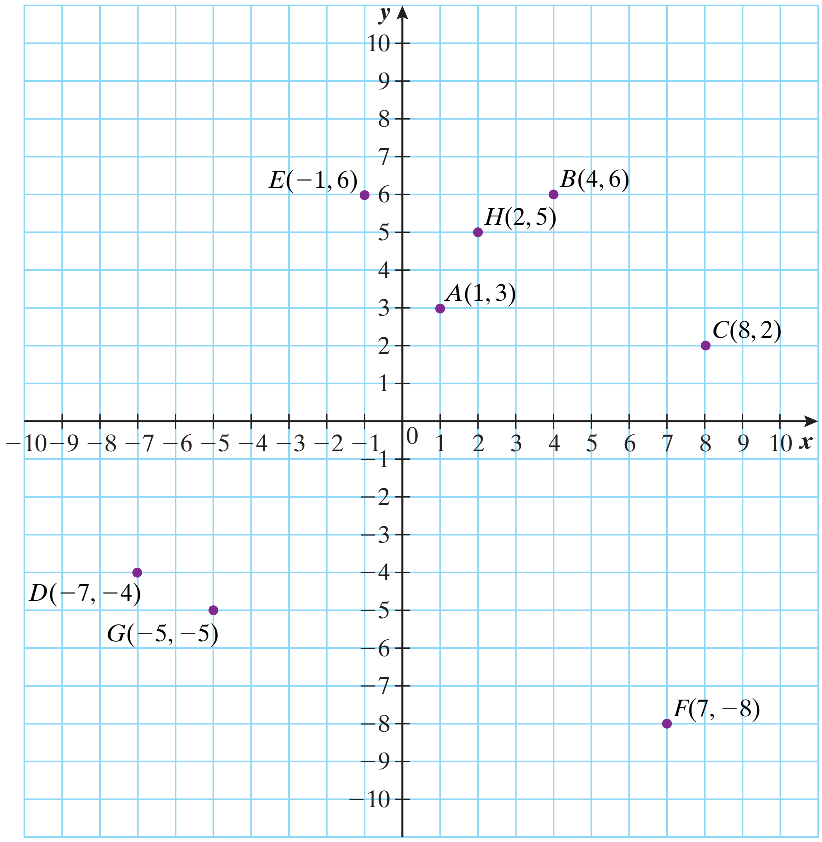
d D (−7 , −4) is translated 4 units to the right and 6 units up.

e E (−1, 6) is translated 4 units to the left and 5 units down.

f F (7, −8) is translated 5 units to the left and 1 unit down.

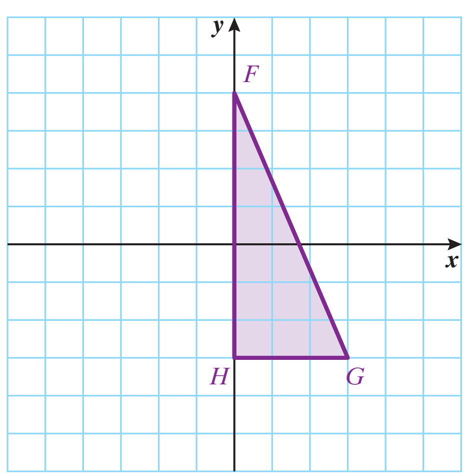
g G (−5, −5) is translated 3 units to the left and 3 units up.

h H (2, 5) is translated 6 units to the left and 8 units down.



LINKS TO Example 6

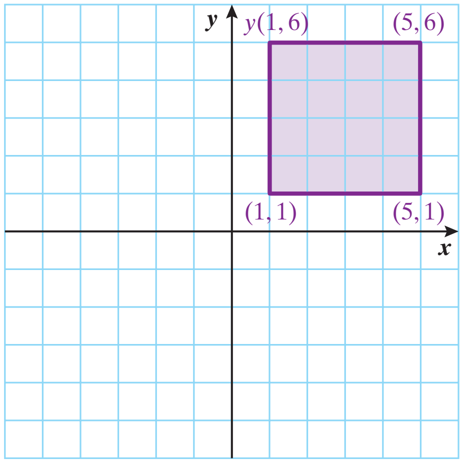
Translate the triangle 6 units to the left. Make a table to show the coordinates of each point and its image. (Each grid square represents one unit.)



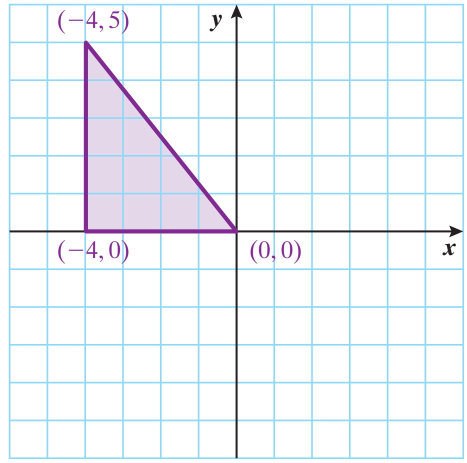
LINKS TO Example 6

Translate each of the following figures in the given direction.

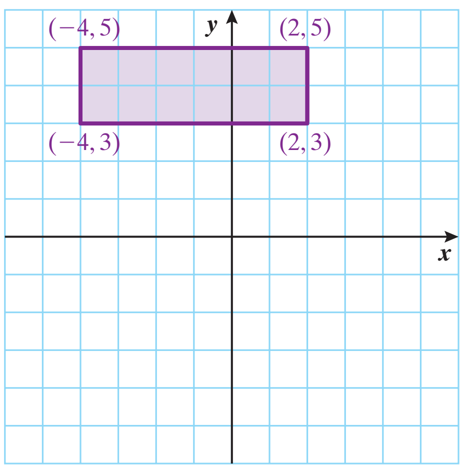
a 6 units to the left and 5 units down



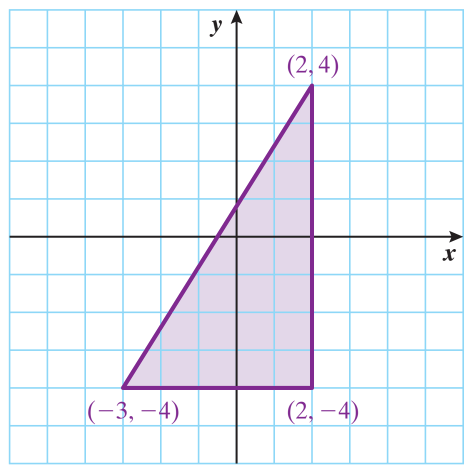
b 5 units to the right and 1 unit up



c 3 units to the right and 7 units down



d 2 units to the left and 1 unit up



exercise 14.2 challenge

Consider the following translations.

a The triangle with vertices A (2, 0), B (−3, 0) and C (0, 4) is translated 2 units down. State the coordinates of the vertices of the image, A', B' and C'.

b The rectangle with vertices A (4, 2), B (4, −1), C (−1, −1) and D (−1, 2) is translated 3 units left and 2 units up. State the coordinates of the vertices of the image, A', B', C' and D'.

c The triangle with vertices A (2, 0), B (−3, −3) and C (0, 4) is translated 5 units to the right and 2 units down. State the coordinates of the vertices of the image, A', B' and C'.

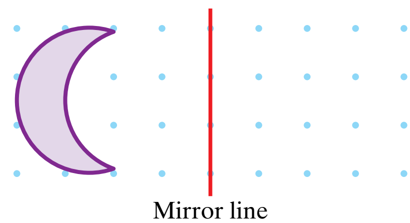
14.3 Reflection

Reflection

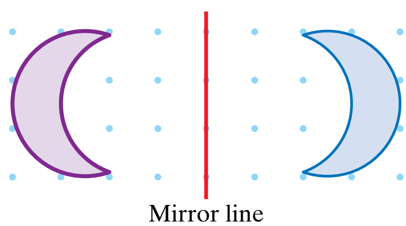
A **reflection** is a flip. When a shape is reflected we sometimes say that a **mirror image** is formed. When you look in a mirror and see your image, you will have noticed that you seem to be behind the mirror by the same distance that you are in front of it. When a shape is reflected in a line, we call that line the **mirror line**. The mirror line corresponds to the position of the mirror.

Example 7

Reflect the shape in the mirror line.



Working

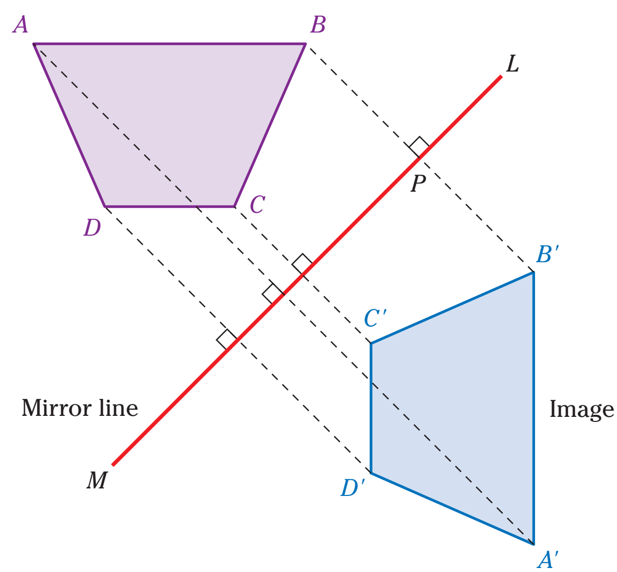


Reasoning

The reflected image is the same distance from the mirror line as the original shape, but on the opposite side.

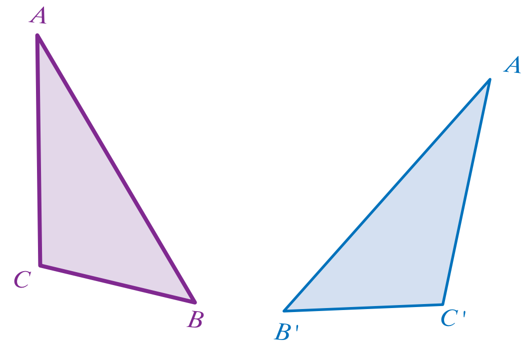
In this diagram, trapezium $ABCD$ is reflected in the mirror line ML. Notice how each vertex of the image $A'B'C'D'$ is the same distance from the mirror line as the corresponding vertex of $ABCD$. To find the position of the image, we draw lines from each vertex of $ABCD$ perpendicular to ML. The vertices of the image must be the same distance from ML as the vertices of $ABCD$. For example, $B'P$ must be the same distance as BP.

Notice that if the diagram were folded along the mirror line, the image $A'B'C'D'$ would coincide with $ABCD$. (See the GeoGebra or HTML files *Reflection* in the ebook.)

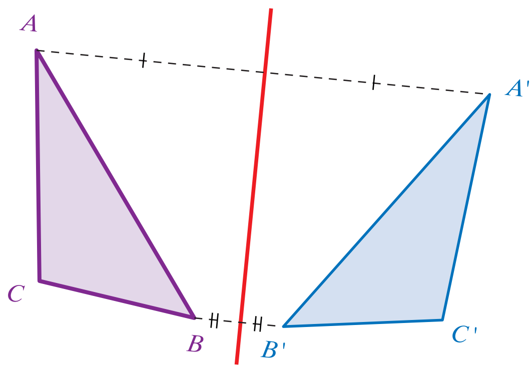


Example 8

Find the position of the mirror line.



Working

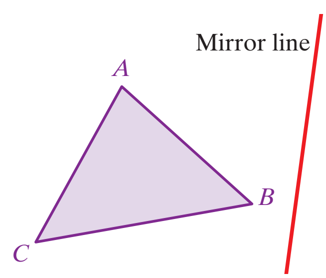


Reasoning

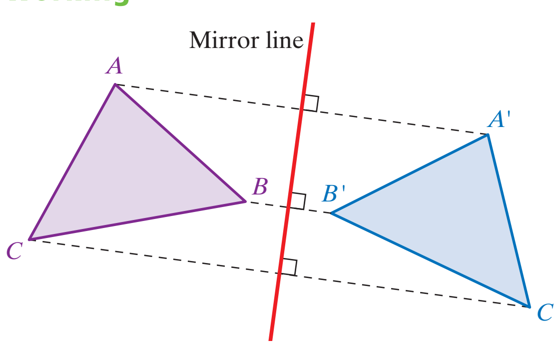
Join A and A'.
Join B and B'.
Find the midpoint of AA'.
Find the midpoint of BB'.
Join the midpoints. This is the mirror line.
The mirror line is perpendicular (at right angles) to AA' and to BB'.

Example 9

Reflect triangle ABC in the mirror line.



Working



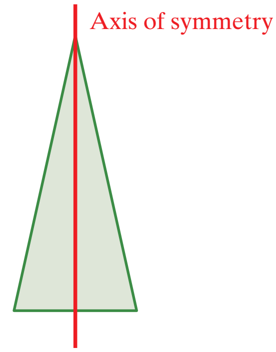
Reasoning

Draw lines from each vertex of the triangle perpendicular to the mirror line.
Measure the distance from each vertex to the mirror line and mark off points an equal distance on the other side of the mirror line.

Label the reflected points A', B' and C'. Join the three points.

Line symmetry

If a figure can be folded so that the two halves are exact reflections of each other, then we say that the figure has **line symmetry**. This isosceles triangle has line symmetry and the dotted line is called an **axis of symmetry** because it is the line about which the figure is symmetrical. An axis of symmetry is like a mirror line.



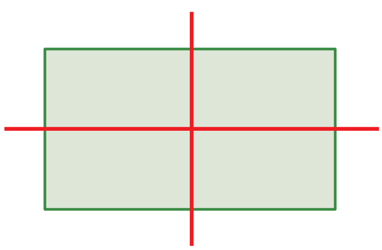
This photograph of the Taj Mahal temple in India has a vertical axis of symmetry. The photograph shows the symmetry of the temple as well as the symmetry of the rows of trees, the pool and the archway.

Some figures have more than one axis of symmetry.

Example 10

Draw a rectangle and then draw the axes of line symmetry.

Working



Reasoning

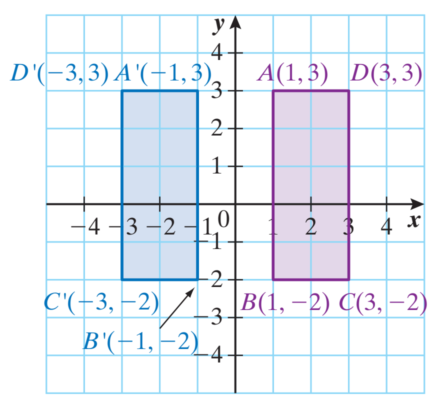
A rectangle has two axes of line symmetry because there are two lines along which the rectangle can be folded exactly in half.

Reflections on the Cartesian plane

Reflection on the Cartesian plane

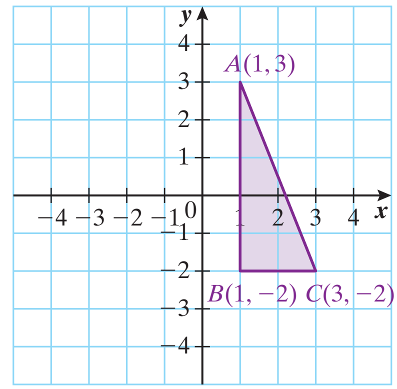
In this diagram $ABCD$ has been reflected in the y-axis to produce the image $A'B'C'D'$.

For reflection in the y-axis, notice that the y-coordinates remain the same, but the signs of the x-coordinates change.

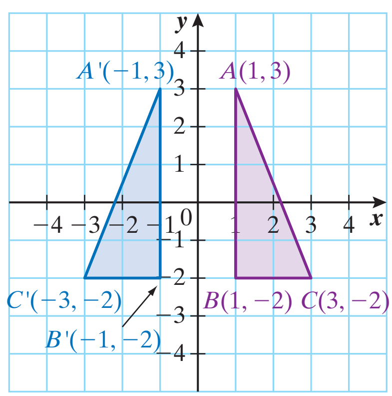


Example 11

Reflect $\triangle ABC$ in the y-axis to produce the image $\triangle A'B'C'$.



Working



Reasoning

When reflected in the y-axis, each y-coordinate stays the same, but the sign of each x-coordinate has the corresponding negative value.

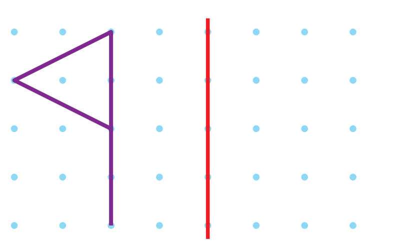
exercise 14.3

LINKS TO Example 7

1 Copy the flag and mirror line onto dotted paper or 1 cm graph paper. Draw the reflection of the flag in the mirror line.

BLM Dotted paper

BLM 1 cm grid



Mirror line

Which of the following capital letters will look the same when reflected

a in a vertical mirror?

A B C D E M P S T W

Mirror

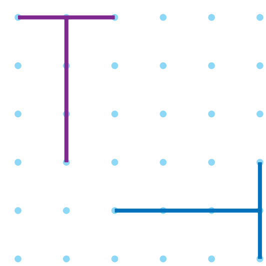
b in a horizontal mirror?

A B C D E M P S T W

Mirror

LINKS TO Example 8

Copy each T shape onto dotted paper or 1 cm graph paper. Locate and draw the mirror line accurately.

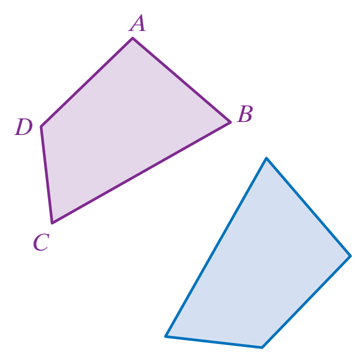


LINKS TO Example 8

Quadrilateral $ABCD$ and its image are shown on the right.

a Trace the figures and label the vertices of the image, A', B', C' and D'.

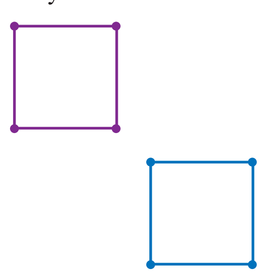
b Locate and draw the mirror line accurately.



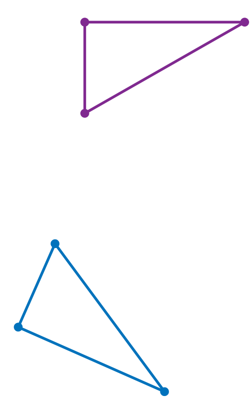
LINKS TO Example 8

Accurately locate and draw the mirror line.

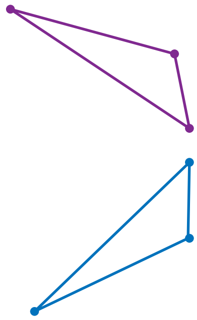
a



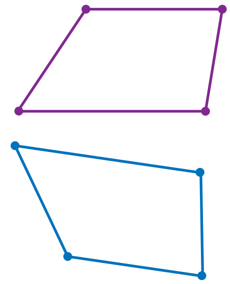
b



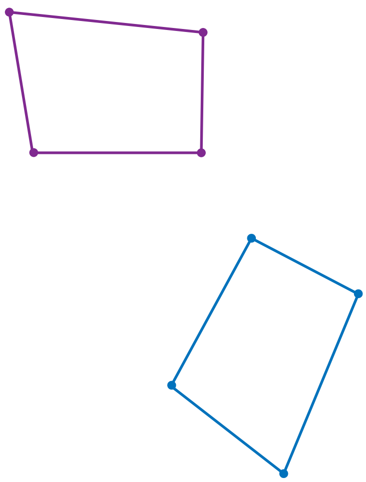
c



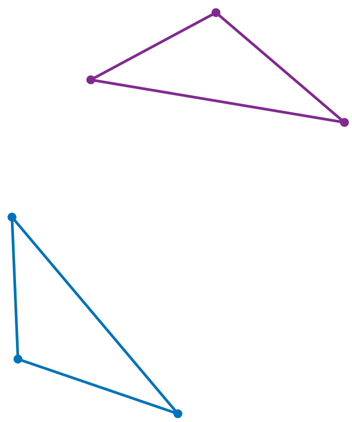
d



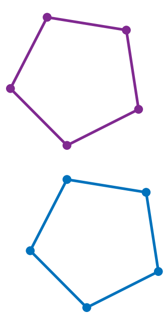
e



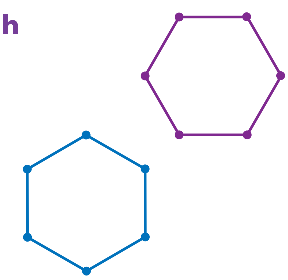
f



g



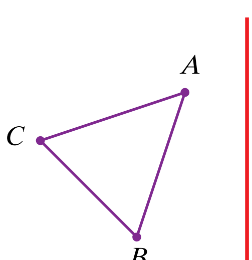
h



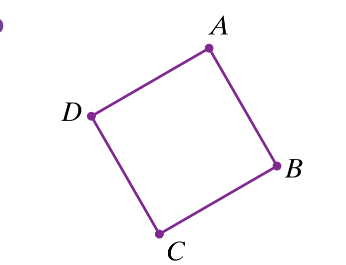
LINKS TO Example 9

Reflect each of these figures in the mirror line.

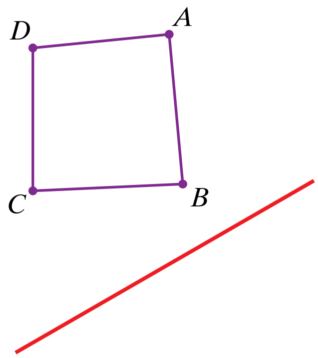
a



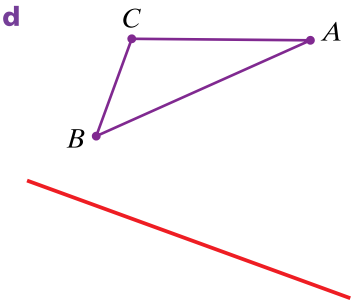
b



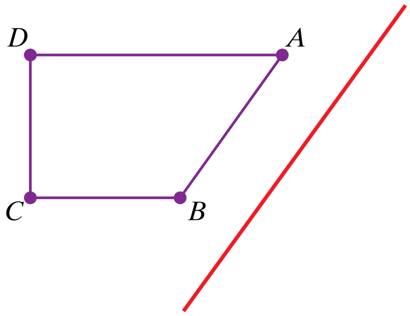
c



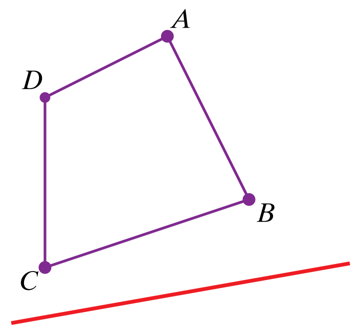
d



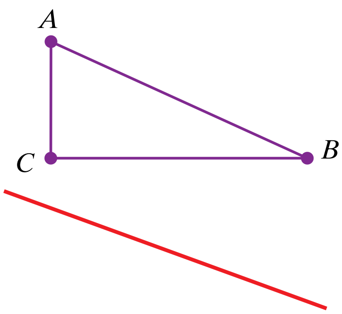
e



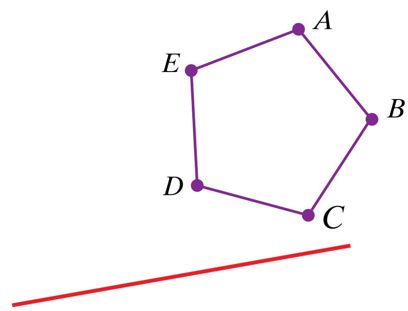
f



g



h

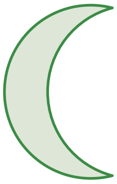


LINKS TO Example 10

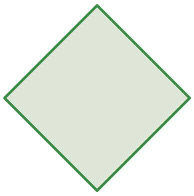
Mark in all the axes of line symmetry for each of the following figures.

a

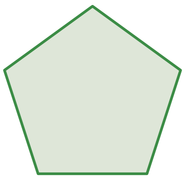
b



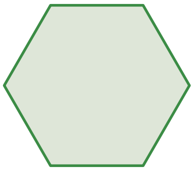
c



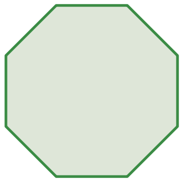
d



e

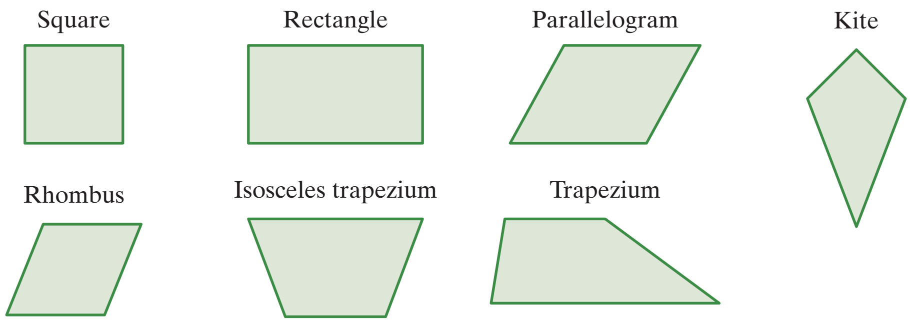


f



LINKS TO Example 10

Mark in all the axes of line symmetry for each of the following quadrilaterals.



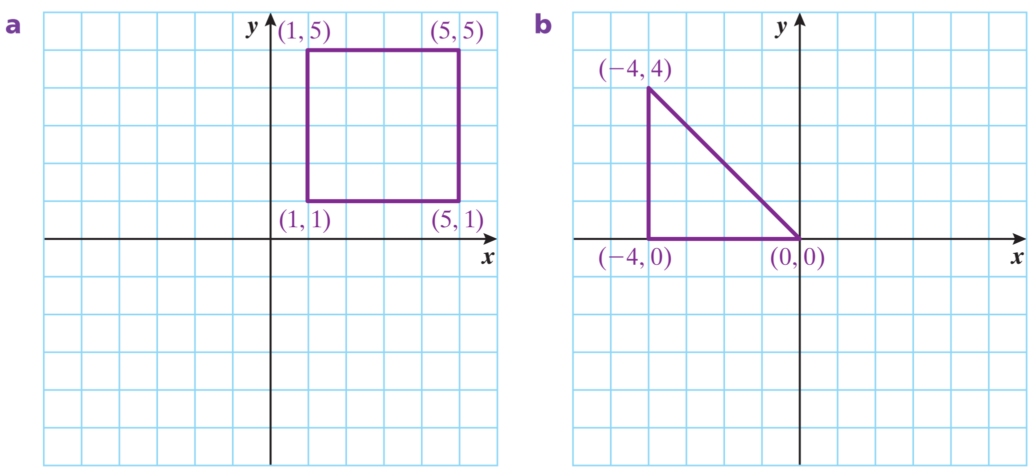
LINKS TO Example 11

Reflect each of these shapes

i in the x-axis.

ii in the y-axis.

Write the coordinates of each vertex of the image.

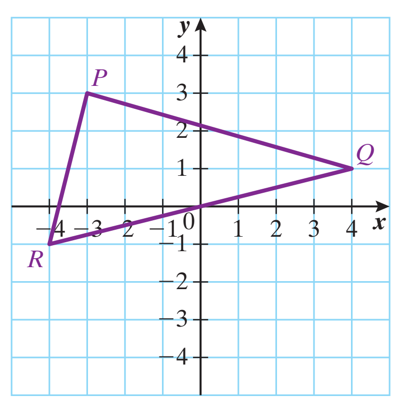


exercise 14.3 challenge

Reflect triangle PQR in the x-axis.

a Give the coordinates of the vertices of the image.

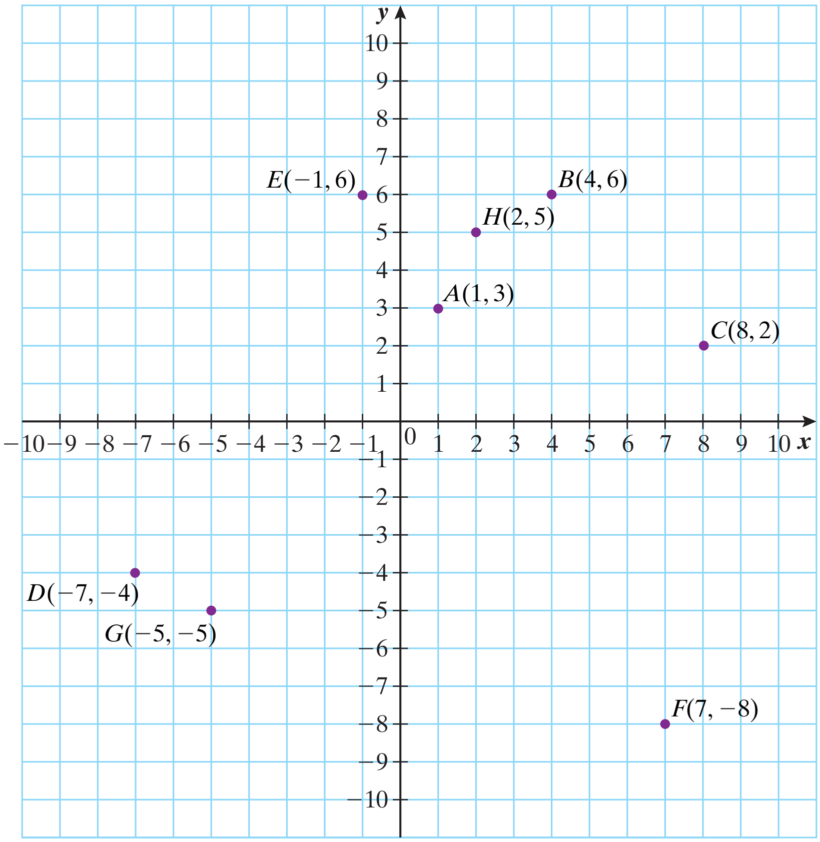
b What do you notice about the coordinates of the image compared with the original?



Consider the points A, B, C, D, E, F, G and H on the Cartesian plane below.

a Mark and label the image of each of these points when they are reflected in the y-axis.
A (1, 3) B (4, 6) C (8, 2) D (−7 , −4)

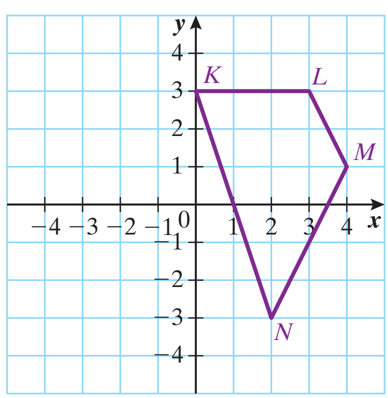
b Mark and label the image of each of these points when they are reflected in the x-axis.
E (−1, 6) F (7, −8) G (−5, −5) H (2, 5)



Reflect quadrilateral $KLMN$ in the y-axis.

a Give the coordinates of the vertices of the image.

b What do you notice about the coordinates of the image compared with the original?



14.4 Rotation

Rotational symmetry

The transformation that we call **rotation** is a turning.

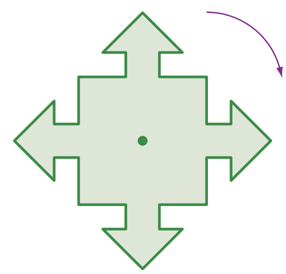
Rotation occurs when a figure is turned about a point. In describing a rotation we must state

- the angle of rotation,
- the direction of rotation (clockwise or anticlockwise) and
- the point about which the figure is to be rotated.

If a figure is rotated through 360° it will end up back in its original position.

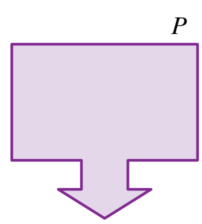
Rotational symmetry

Some shapes appear the same when they are rotated about a point at their centre. These shapes are said to have **rotational symmetry**. If the figure on the right is rotated about its centre through 90°, 180°, 270° or 360°, it will look just the same. We say that it has *four-fold rotational symmetry* (or that the *order of the rotational symmetry is four*) since there are four positions where it will look the same as it turns around.

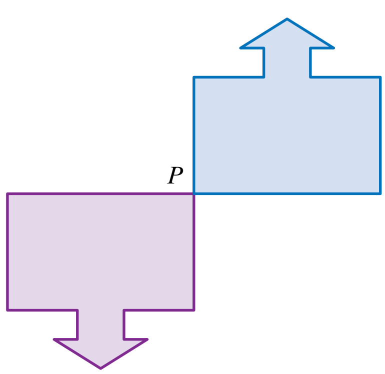


Example 12

Rotate this shape through 180° in a clockwise direction about point P.



Working



Reasoning

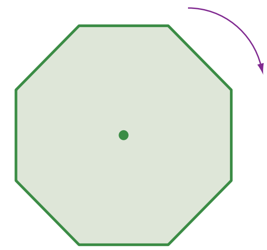
Point P stays in the same position. Every side of the shape is rotated through 180° in a clockwise direction.

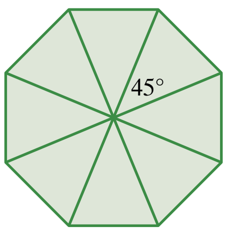
Example 13

The regular octagon at right is rotated.

a List the angles for which this regular octagon will look the same when it is rotated about its centre.

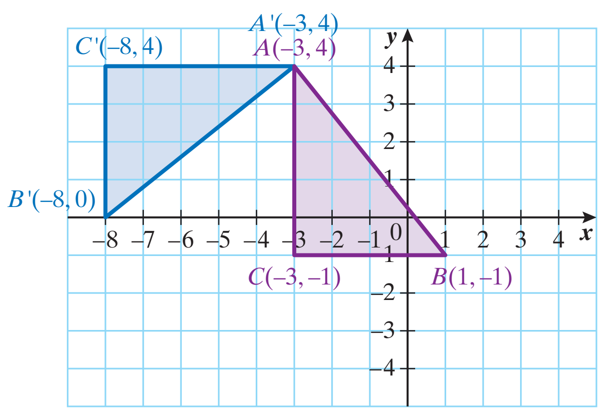
b What is the order of rotational symmetry?



Working	**Reasoning**
a  45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°	If the regular octagon is rotated through 360° ÷ 8, that is, 45° (or any multiple of 45°) it will look just the same.
b The regular octagon has eight-fold rotational symmetry because there are eight different positions where it will look the same as it turns around	The order of the rotational symmetry is 8.

Rotation on the Cartesian plane

Triangle ABC has been rotated through 90° about vertex A in a clockwise direction. Notice that each of the sides of the image triangle $A'B'C'$ are at right angles to the position they were in before.



Rotation about *A*

Example 14

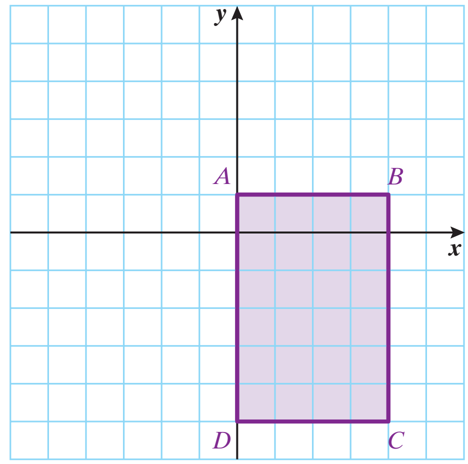
Example 14

Rotate rectangle $ABCD$

a through 90° in a clockwise direction about point A.

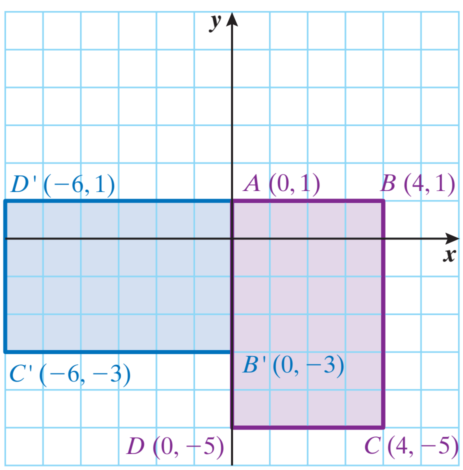
b through 180° in an anticlockwise direction about point A.

In each case label the coordinates of A, B, C, D and their images (each grid square is 1 unit).



Working

a

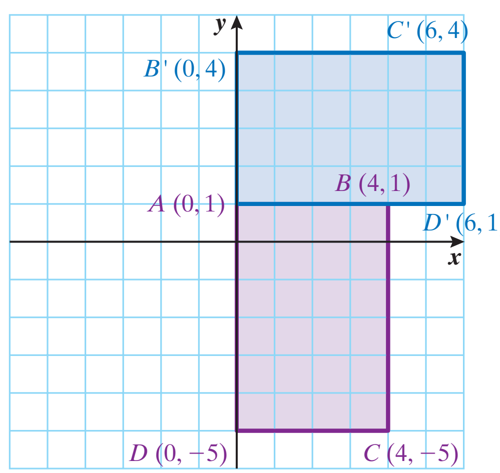


Reasoning

Each side of the rectangle is rotated through 90° in a clockwise direction.

A' is at the same position as A because this is the point about which the rectangle is rotated.

Rotation through 90° about A in an anticlockwise direction would produce a different image.

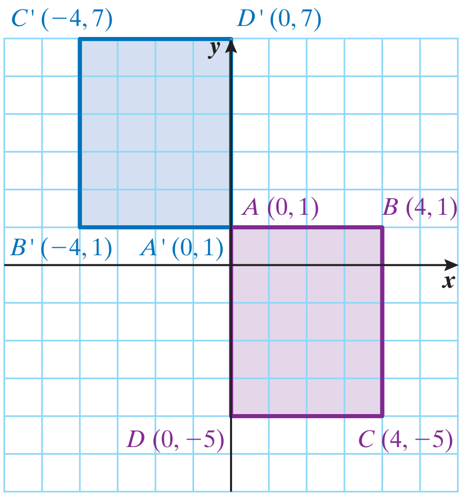


continued

Example 14 continued

Working

b



Reasoning

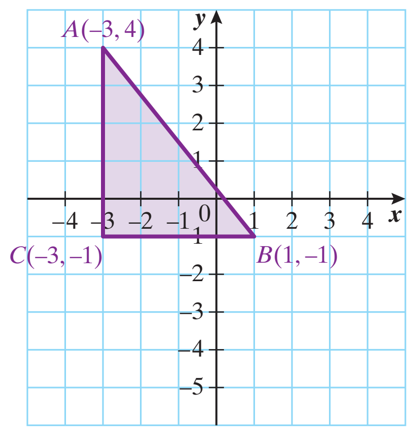
Each side of the rectangle is rotated through 180° in an anticlockwise direction.

A' is at the same position as A because this is the point about which the rectangle is rotated.

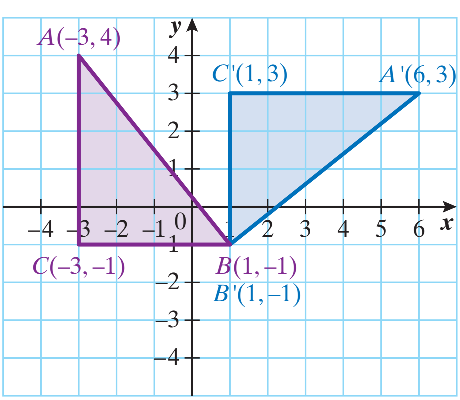
Rotation through 180° in a clockwise direction would produce the same image.

Example 15

Rotate triangle ABC 90° in a clockwise direction about Vertex B.



Working



Reasoning

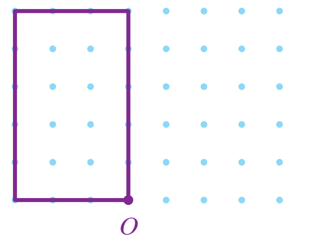
When triangle ABC is rotated through 90° about point B, each side of the triangle is rotated through 90°. The image of B stays in the same position as B.

exercise 14.4

LINKS TO Example 12

BLM Dotted paper

Rotate this rectangle through 90° in a clockwise direction about point O.



LINKS TO Example 13

For each of the following

i state the order of rotational symmetry.

ii what is the smallest angle through which the object must turn so that it looks the same?

a

b

c

d

LINKS TO Example 13

Three different wheels are illustrated below.

a

b

c

For each of the wheels

- **i** state the order of the rotational symmetry. (Ignore the bolts and logo at the centre of the wheels.)
- **ii** calculate the angle through which the wheel would have to turn for it to look the same.

Which of these animals has

a rotational symmetry?

b line symmetry?

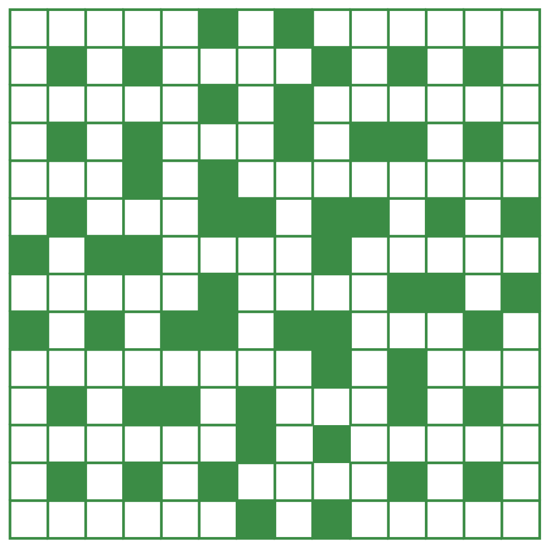
Seastar

Butterfly

Crossword designs often use symmetry.

a State the order of rotational symmetry for the design shown.

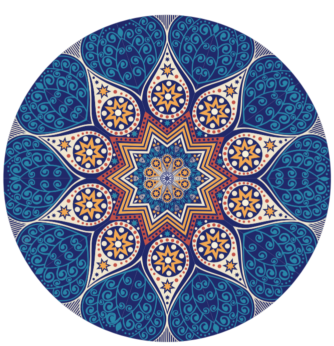
b Does the crossword design also have line symmetry?



For each of these designs state

- **i** the order of rotational symmetry.
- **ii** the number of axes of line symmetry.

a



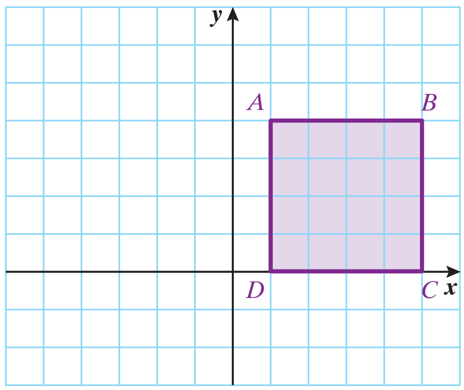
b

LINKS TO Examples 14, 15

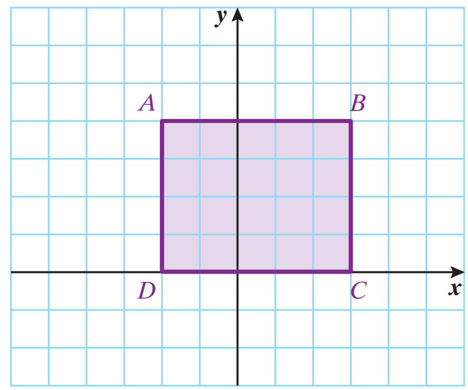
BLM 1 cm grid

Rotate the following shapes through the given angle in the stated direction. In each case, label the coordinates of A, B, C, D and their images (each grid square is 1 unit).

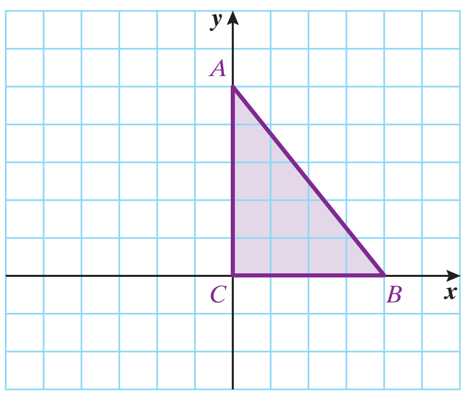
a through 90° in an anticlockwise direction about point A.



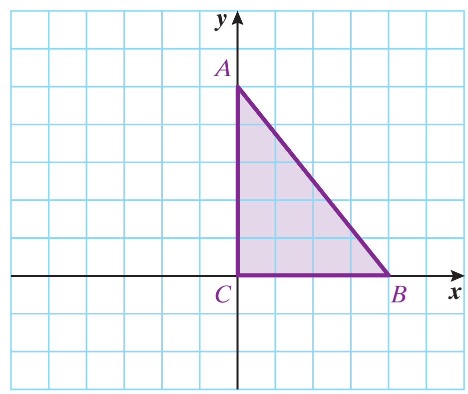
b through 90° in a clockwise direction about point A.



c through 180° in a clockwise direction about point C.



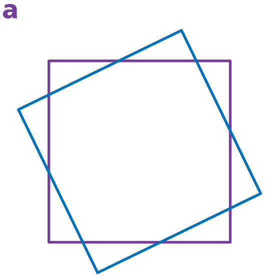
d through 270° in an anticlockwise direction about point C.



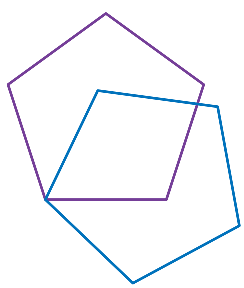
exercise 14.4 challenge

Copy each figure and its rotated image, then locate the point about which the figure has been rotated.

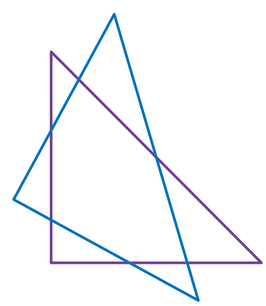
a



b

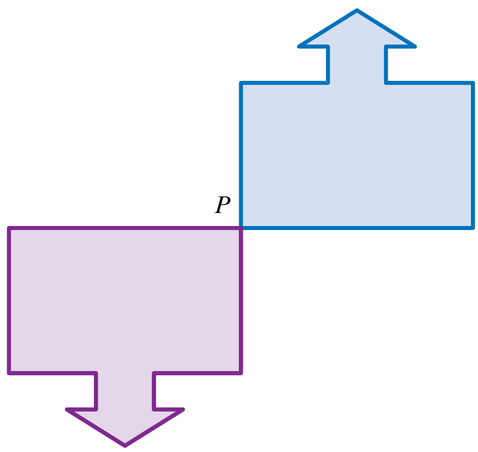


c

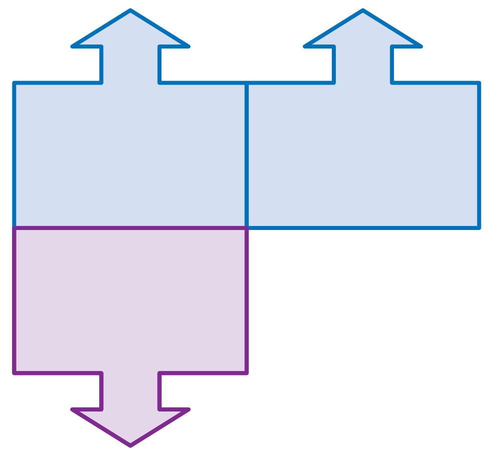


14.5 Combining transformations

In example 12 in section 14.4 a shape was rotated about a point *P*. The same result could have been obtained by reflecting the shape along its horizontal upper edge then translating the image or reflecting it again along its right hand edge.



Rotation around *P* through 180° in a clockwise direction



Reflection followed by translation or two reflections

Combinations of transformations are often used in art and design such as wallpapers and decorative borders.

Example 16

Create three different border patterns by transforming a capital F using these combinations of transformations.

i translation

ii reflection in a horizontal mirror line and translation through the middle of the F

iii rotation and translation

Working

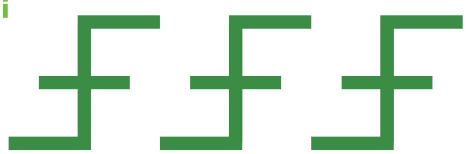
i

Reasoning

The F is translated without being reflected or rotated.

continued

Example 16 continued

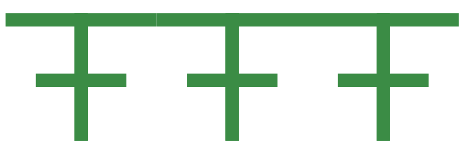
Working	Reasoning
ii 	The F is reflected in a horizontal mirror line through the middle of the F then translated.
iii 	The F is translated then rotated through 180°.

exercise 14.5

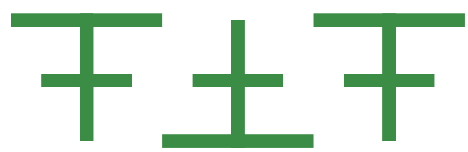
LINKS TO Example 16

Describe the transformations of the letter *F* in each of these patterns.

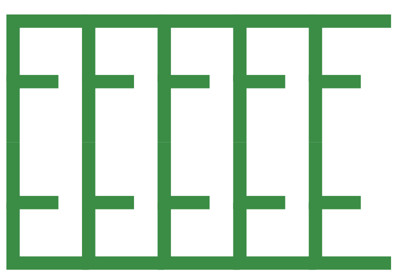
a



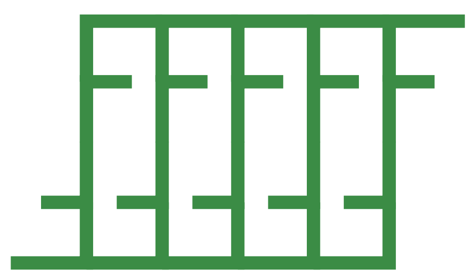
b



c



d

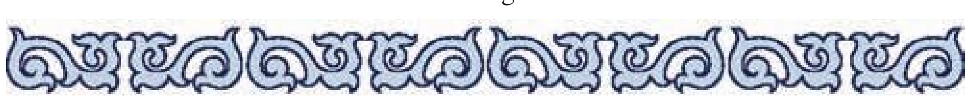


Design your own border pattern by transforming the letter *K*.

Design your own border pattern by transforming the letter *P*.

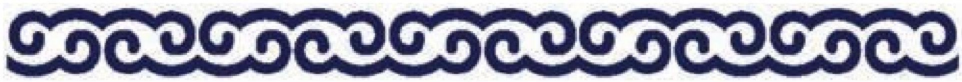
Describe the transformations used in the design of these decorative borders.

a



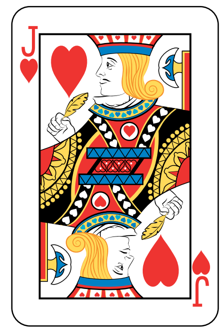
b

c

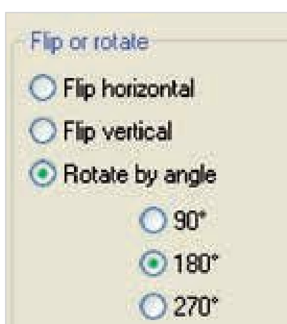


d

Describe two different ways in which this playing card could be designed using transformations.



Jake was trying to turn a photo up the right way on his computer. He wasn't sure whether to flip the photo vertically or whether to rotate it through 180°.



Original photo

Image 1

Image 2

a Which option did Jake choose to get Image 1?
b Which option did Jake choose to get Image 2?
c Which photo is correct—Image 1 or Image 2?
d Which transformation could Jake use to correct the incorrect image?

● What transformations have been used to create these designs?

a

b

exercise 14.5 challenge

● Look at Escher's *Winged horse* design at the beginning of this chapter. What transformation has Escher used in this design?

Now go to the website www.mcescher.com/, select Picture gallery then Symmetry. Look at the transformations that Escher has used to create his designs.

At the following website you will find animated illustrations of transformations in Escher's designs.

www.math.nus.edu.sg/aslaksen/gem-projects/maa/0203-2-03-Escher/main3.html

Scrolling down the screen, the animations show:

- adding and subtracting to produce tessellating shapes
- translation
- rotation
- glide reflection
- combination.

● Create your own design using a combination of transformations.

14.6 Enlargement and reduction

Enlargement and reduction

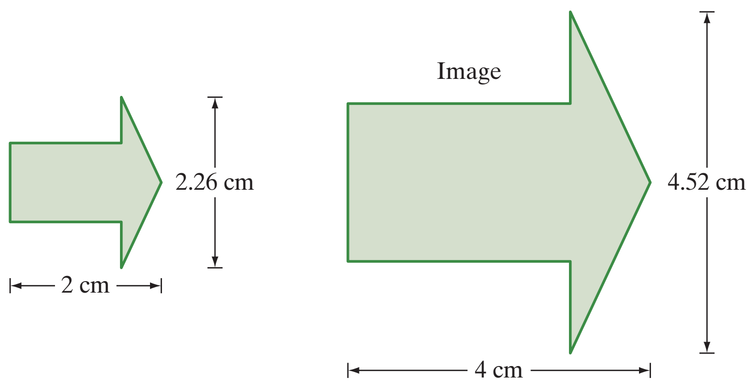
In all the transformations we have looked at so far, the image is exactly the same size and shape as the original shape. Translation, reflection and rotation are called **isometric transformations**.

'iso' meaning 'the same' and 'metric' meaning 'measure'

We can also transform a shape by **enlarging** it or **reducing** it so that its shape stays the same. Enlarging and reducing are not isometric transformations.

In the diagram below, the arrow on the left has been enlarged to form an image that has identical shape, but the length and the height are each twice the length and height of the original arrow.

We say that there is a **scale factor** of 2. This can also be expressed as a percentage as 200%.



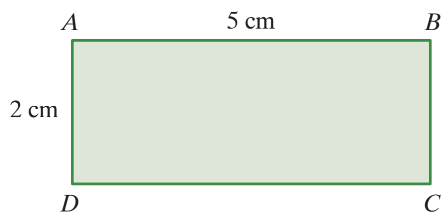
$$\text{Scale factor} = \frac{\text{length of image}}{\text{length of original shape}}$$

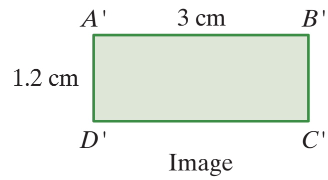
If the scale factor is greater than 1 (for example, 2, 150%), the transformation is an enlargement.

If the scale factor is less than 1 (for example, $\frac{1}{2}$, 0.4 , 60%) the transformation is a reduction.

Example 17

Find the scale factor for the reduction of rectangle $ABCD$.





Working

Scale factor
= length of image   length of figure
= 3 cm   5 cm
= 0.6

Alternatively:
Scale factor
= height of image   height of figure
= 1.2 cm   2 cm
= 0.6

Reasoning

The scale factor compares the size of the image with the corresponding size of the original figure.
If the image is smaller than the original figure, the scale factor is less than one.

Corresponding dimensions must be compared, e.g. length to length or height to height.

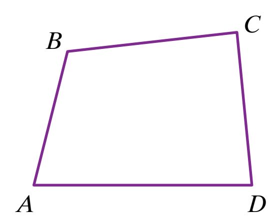
Example 17 shows a method for enlarging a shape.

Example 18

Transform quadrilateral $ABCD$ by the following scale factors.

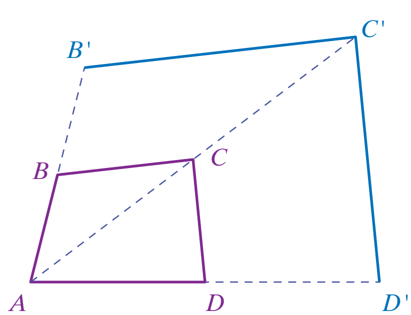
a 2

b 50%



Working

a



Reasoning

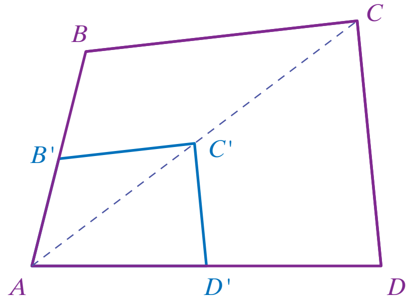
Enlargement by 2 means that each side of $AB'C'D'$ will be twice the length of the matching side of $ABCD$.
Extend sides AB and AD to B' and D' so that $AB' = 2 \times AB$ and $AD' = 2 \times AD$
Draw the diagonal AC and extend it to C' so that $AC' = 2 \times AC$
Draw the sides $B'C'$ and $C'D'$.
Quadrilateral $AB'C'D'$ is an enlargement of quadrilateral $ABCD$ with a scale factor of 2.

continued

Example 18 continued

Working

b



Reasoning

Reduction by 50% means that each side of $AB'C'D'$ will be half the length of the matching side of $ABCD$.
Measure the midpoint of AB and label it B'. Find the midpoint of AD and label it D'. Draw the diagonal AC and label the midpoint C'.
Quadrilateral $AB'C'D'$ is a reduction of quadrilateral $ABCD$ with a scale factor of 50%.

exercise 14.6

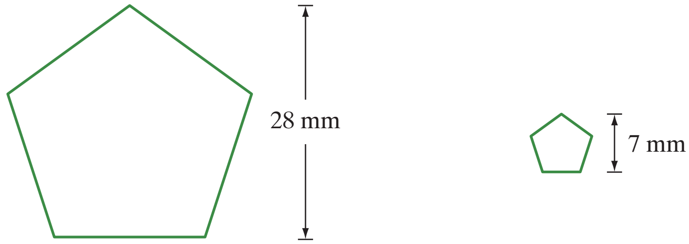
LINKS TO Example 17

1 State whether each of the following represents an enlargement or a reduction, then calculate the scale factor. Give your answer

- **i** as a fraction or decimal.
- **ii** as a percentage.

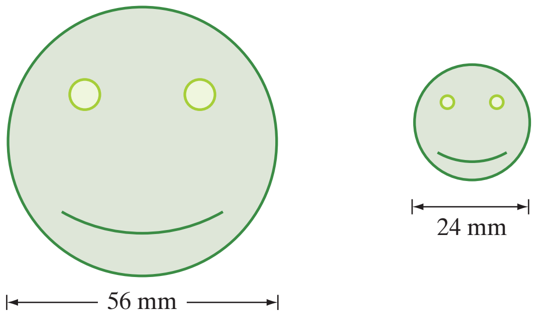
a

b

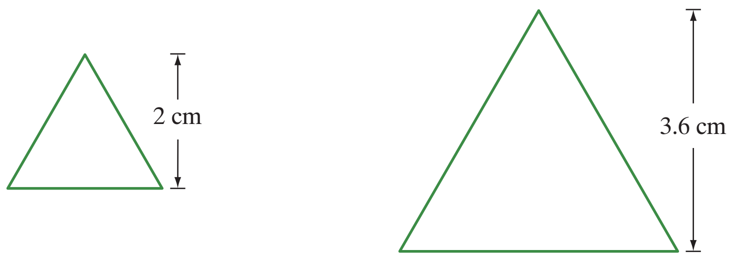


c

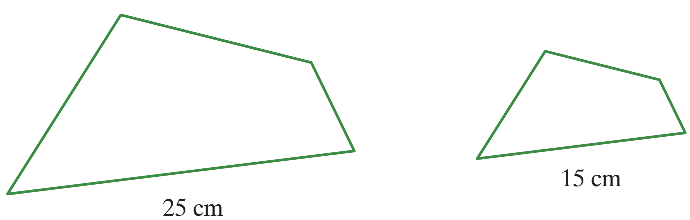
d



e



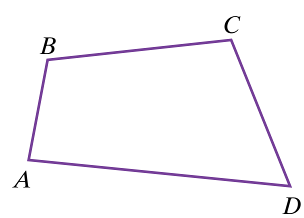
f



BLM Dotted paper

Copy quadrilateral *ABCD* and transform it by the following scale factors.

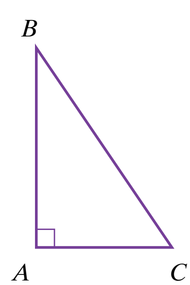
a 3
b 1.5
c 250%
d 50%



LINKS TO Example 18

Copy triangle *ABC* and transform it by the following scale factors.

a 2
b 4
c 250%
d 50%



exercise 14.6 challenge

For each of the following enlargements, how will the area change?

a scale factor 2
b scale factor 3

Analysis task

BLM Rep-tiles

Rep-tiles

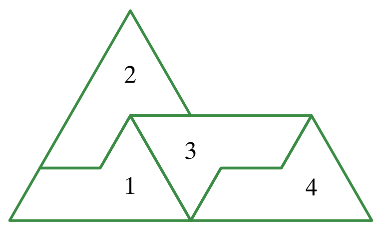
Tiles that can be put together to form a larger version of the same shape have been called rep-tiles. This can then be repeated, with the larger tiles then making even larger tiles with the same shape.

In the tessellation below, four tiles make an identical, but larger, shape. Starting with tile 1, we can see that the other three tiles have been transformed in one or more ways.

a Which tile is a translation of tile 1?

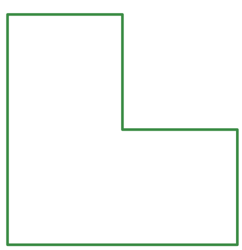
b Which tiles have been rotated?

c Have any of the tiles been reflected?



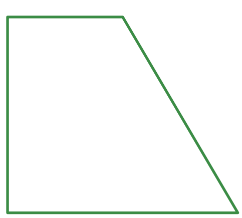
d Each of the four tiles shown below are rep-tiles. Using cut-out copies of the rep-tiles, work out the transformations needed to make a larger version of each tile. In each case, paste the larger rep-tile into your workbook, number the tiles as in the diagram above and describe the transformations for each smaller tile.

i

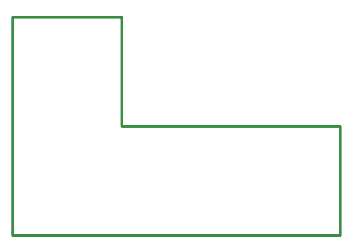


ii

iii



iv



BLM Federation Square triangles

e The template file *Federation Square triangles* in the ebook, shows how five of the triangles can be arranged to form a larger triangle with exactly the same shape, and how five of these larger triangles can be arranged to form an even larger triangle with the same shape. This is the pattern on which the Federation Square design is based.

Review Transformations

Summary

Translation (slide)

- A translation vector shows the direction and distance an object is translated.

Reflection (flip)

- Each point on the object and its matching point on the image are an equal distance from the mirror line.
- The line joining each pair of matching points is perpendicular to the mirror line.

Line symmetry

- Shapes have line symmetry if mirror lines can be drawn on them.

Rotation (turn)

- Shapes can be rotated through an angle about a particular point.

Rotational symmetry

- Shapes have rotational symmetry if they can be rotated about the centre through a particular angle and look the same. e.g. if an object looks the same when rotated through 90° it has order 4 rotational symmetry because $360° \div 90° = 4$.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

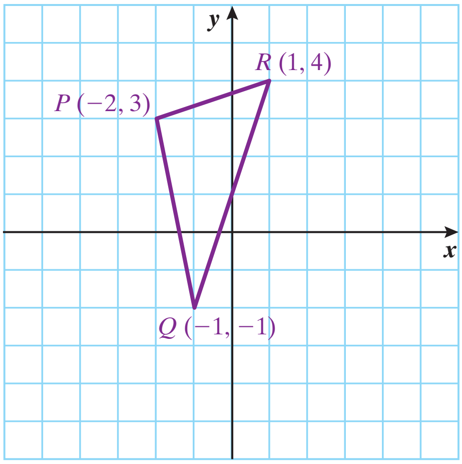
anticlockwise	line symmetry	regular polygon
axis	mirror line	rotation
axis of symmetry	order of rotational symmetry	rotational symmetry
Cartesian plane	parallel	tessellation
clockwise	perpendicular	transformation
coordinates	polygon	translation
enlargement	reduction	vertex
isometric transformation	reflection	

Revision

Multiple-choice questions

If triangle PQR is translated 4 units to the right and 3 units down, the coordinates of the image $P'Q'R'$ will be

A $P'(-6, 0)$, $Q'(-5, -5)$, $R'(-3, 1)$.
B $P'(-5, 7)$, $Q'(-4, 2)$, $R'(-2, 8)$.
C $P'(2, 0)$, $Q'(3, -5)$, $R'(5, 1)$.
D $P'(2, 6)$, $Q'(3, 1)$, $R'(5, 7)$.
E $P'(-3, 2)$, $Q'(-2, 2)$, $R'(0, 3)$.

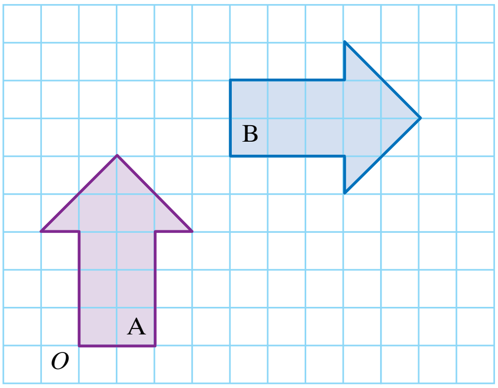


This figure has

A one axis of line symmetry and order. two rotational symmetry.
B two axes of line symmetry and order four rotational symmetry.
C one axis of line symmetry and no rotational symmetry.
D two axes of line symmetry and no rotational symmetry.
E two axes of line symmetry and order two rotational symmetry.

In the diagram at right, the blue arrow, B, is the image of the purple arrow, A. The purple arrow has been

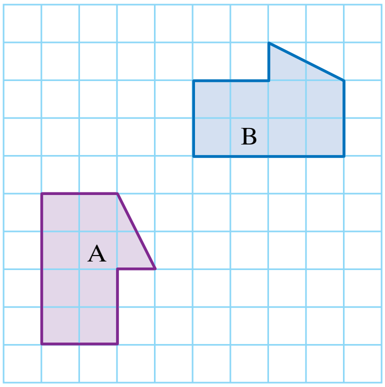
A translated 7 units up, 4 units to the right and rotated through 90° in a clockwise direction about O.
B translated 7 units up, 4 units to the right and rotated through 90° in an anticlockwise direction about O.
C translated 5 units up, 2 units to the right and rotated through 90° in a clockwise direction about O.
D translated 5 units up, 4 units to the right and rotated through 90° in a clockwise direction about O.
E translated 5 units up, 4 units to the right and rotated through 90° in an anticlockwise direction about O.



In the diagram to the right, the blue shape, B, is the image of the purple shape, A.

The purple shape has been

A translated, rotated and reflected.
B translated and rotated but not reflected.
C translated and reflected but not rotated.
D reflected and rotated but not translated.
E rotated but not reflected or translated.



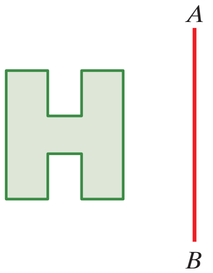
If the point $(-5, 4)$ is reflected in the x-axis, the image is the point

A $(5, 4)$. **B** $(4, 5)$. **C** $(-5, 4)$. **D** $(-5, -4)$. **E** $(5, -4)$.

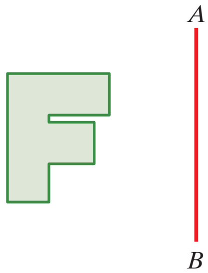
Short-answer questions

Draw the reflection of each letter in the mirror line AB.

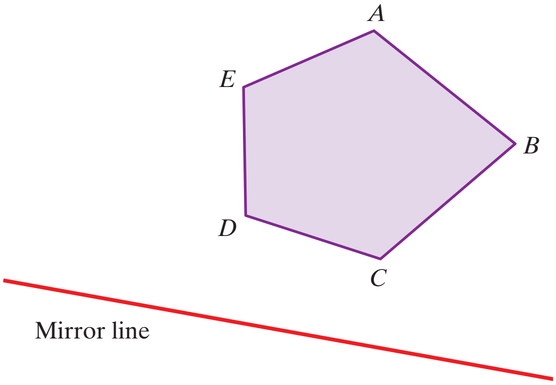
a



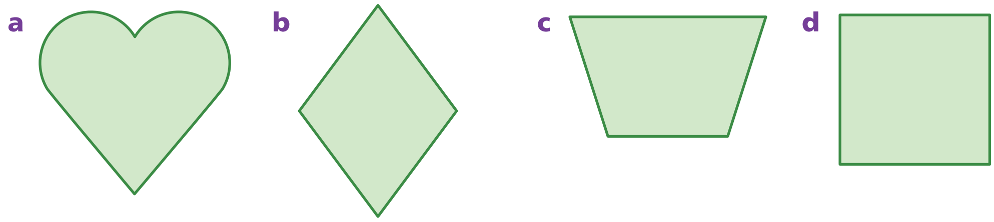
b



Reflect $ABCDE$ in the mirror line. Label the vertices of the image $A'B'C'D'E'$.



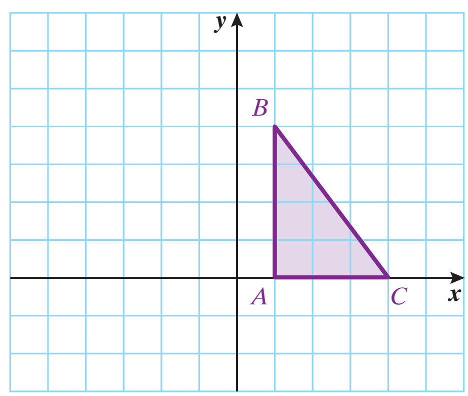
- Draw all axes of line symmetry on each of these shapes.



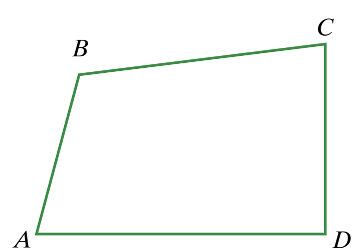
- Refer to the diagrams of tiles A and B.
  - **a** How many axes of line symmetry does tile A have?
  - **b** How many axes of rotational symmetry does tile A have?
  - **c** How many axes of line symmetry does tile B have?
  - **d** How many axes of rotational symmetry does tile B have?

Tile A **Tile B**

- Transform triangle ABC in each of the following ways.
  - **a** Rotate ABC through 270° in an anticlockwise direction about point A.
  - **b** Rotate ABC through 90° in a clockwise direction about point B.
  - **c** Reflect ABC in the x-axis.
  - **d** Reflect ABC in the y-axis.
  - **e** translate ABC 4 units to the left and 3 units down.

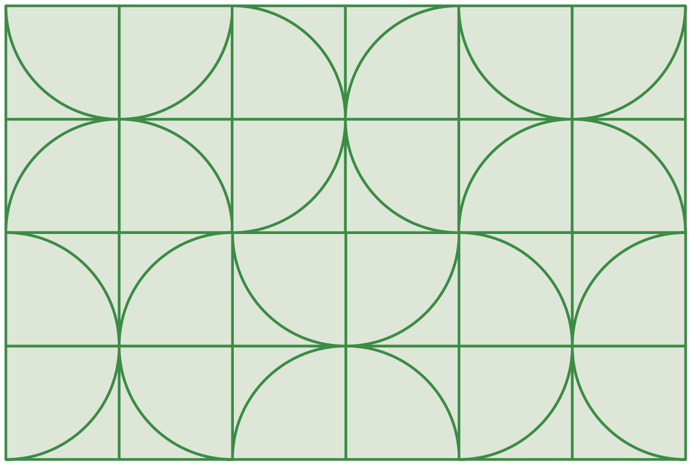


- Copy the quadrilateral $ABCD$ and transform it by the following scale factors. State whether each transformation is an enlargement or a reduction.
  - **a** 2
  - **b** 50%



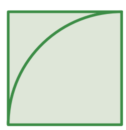
Extended-response question

This design is made up of squares and quarter circles.

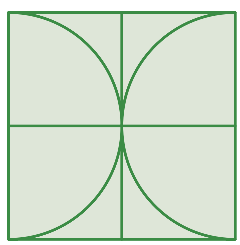


On a copy of the design shade or outline parts of the design to show the following transformations.

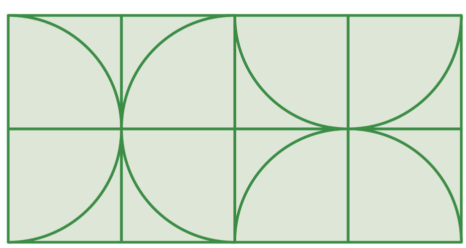
a reflection of this shape



b rotation of this shape



c translation of this shape



Practice quiz
Chapter 14

Extending and investigating 15

The pencils in this triangular stack form a pattern. The number of pencils in each row depends on how many pencils in the row below. Algebra can help us explore the pattern and generalise so we can tell how many pencils there will be in any particular row. This chapter provides an opportunity for you to use mathematics to explore and investigate interesting problems.

15.1 Magic squares

A magic square is an array of numbers where each row, column and diagonal have the same sum. The first known magic square originated in China about 4000 years ago. This was a 3 × 3 square with the digits 1 to 9 in the 9 cells. The magic number was 15; that is, all the rows, columns and diagonals added to 15. This 3 × 3 magic square spread via India to Arab mathematicians about 2000 years ago.

	5	

a Place the digits 1, 2, 3, 4, 6, 7, 8 and 9 in the eight empty cells so that each row, column and diagonal add to 15.

b Compare your magic square with those of other students in your class. Can you find more than one way of arranging the digits?

The first records of 4 × 4 magic squares are from India about 1000 years ago. In 1514, the German artist Albrecht Dürer included this 4 × 4 magic square in one of his etchings.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

c What is the magic number for this magic square?

d What do you notice about the bottom row of the magic square?

e There are many ways of arranging the numbers 1 to 16 in a 4 × 4 magic square. Can you find another way?

f Show why all 4 × 4 magic squares with the numbers 1 to 16 must have the same magic number.

The next magic square also contains the numbers 1 to 16, but it is called a **diabolical magic square**. As well as the rows, columns and diagonals, it has many other combinations of numbers that will add to the magic number; for example, the four numbers 10, 5, 8 and 11 in the bottom left corner add to the magic square number.

1	14	7	12
15	4	9	6
10	5	16	3
8	11	2	13

BLM
Magic squares

g Using the sheet of copies of this magic square, on each copy of the magic square, colour in sets of four numbers which add to the magic number.

This 5 × 5 magic square contains all the numbers 1 to 25.

h Find the magic number for this magic square.

i Show that all 5 × 5 magic squares containing the numbers 1 to 25 must have this same magic number.

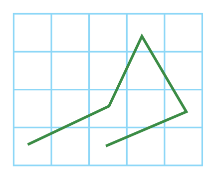
10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

The Swiss mathematician, Leonhard Euler, who lived in the 18th century, created an 8 × 8 square with the numbers 1 to 64.

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

BLM Magic squares

Although Euler's square was not a perfect magic square, it has another very unusual feature. Starting at 1, you can make a 'knight's move' to 2, then to 3 and so on, finishing at 64. On a chessboard, a knight's move is a move two squares across and one square up or down, or one square across and two squares up or down, as shown in this diagram.



j What is the magic number for an 8 × 8 square with the numbers 1 to 64?

k Show why Euler's square is not a perfect magic square.

l Starting at 1, use a ruler and pencil to draw a knight's move path in order from 1 to 64. Draw your lines from the centres of each square. What do you notice about your pattern of lines?

m Divide the 8 × 8 square into four 4 × 4 squares. What do you notice about each of the four 4 × 4 squares?

n Look at the two 4 × 4 squares at the top of Euler's square. Can you see a pattern in the sets of numbers in the right-hand 4 × 4 square compared with the left-hand 4 × 4 square?

o Is there a similar pattern between the numbers of the right-hand and left-hand 4 × 4 squares in the bottom half of Euler's square?

p If the 64 squares were coloured either black or white as in a chessboard, what pattern of would occur with the 64 numbers in Euler's square?

q Can you find any more patterns in the numbers of Euler's square?

15.2 The four fours

Four fours is an old mathematical puzzle. The aim is to make as many whole numbers as possible using only four fours and the common operations of adding, subtracting, multiplying and dividing. Brackets can also be used. Two fours can also be used to write 44. There may be more than one way you can obtain a number.

Zero, for example, could be obtained in the following ways:

$4 + 4 - 4 - 4 = 0$

$44 - 44 = 0$

a Find another way of obtaining 0.

b Show how you could obtain 1 using only four fours and any of the symbols $+$, $-$, $\times$, $\div$ and ( ).

c Show how you could obtain 2, 3, 4 and 5 using only four fours and any of the symbols $+$, $-$, $\times$, $\div$ and ( ).

d Show how you could obtain 6, 7, 8, 9 and 10 using four fours and any of the symbols $+$, $-$, $\times$, $\div$ and ( ).

By allowing decimal numbers, for example, 0.4, and square root, $\sqrt{4}$, it is possible to make many other whole numbers beyond 10. Sometimes 4 factorial, written as 4!, is also included. The factorial of a number is the product of all the whole numbers from the number itself down as far as 1. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

e Using 0.4, $\sqrt{4}$ and 4! as well as $+$, $-$, $\times$, $\div$ and ( ) try to make all the whole numbers up to 50.

f Using five 6s try to make as many numbers from 0 to 25 as possible, using any of the symbols $+$, $-$, $\times$, $\div$ and ( ) as well as 6 factorial.

15.3 How many candles?

The candles in the photograph on the next page are arranged in rows. Starting from the top, there is one candle in the top row, two in the second row, three in the third row, and so on down to the 17th row, where there are 17 candles. To find the total number of candles we could count each candle. But is there an easier way?

What we need is the sum of the following consecutive integers:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17$

One way of finding the sum of these numbers is to start from each end and group the numbers in pairs until you come to the middle; for example, $1 + 17 = 18$.

a Use this method to find the total number of candles in the stack.

According to a story, a German schoolboy called Carl Gauss (1777–1855) surprised his teacher by how quickly he found the sum of the first 100 integers. By using this method of pairing, we are told that Gauss was able to find the sum very quickly. Gauss went on to become a famous mathematician and scientist. He is shown here on a German banknote.

b By grouping the numbers 1 to 100 in pairs, find the sum of all the integers from 1 to 100.

When we add integers we produce a sequence of numbers as shown below:

$1=1$

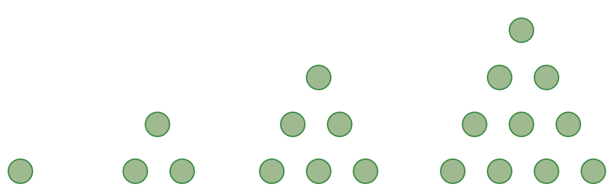
$1+2=3$

$1+2+3=6$

$1+2+3+4=10$

$1+2+3+4+5=15$

The numbers in this sequence: 1, 3, 6, 10, 15 … are called **triangle numbers**. As you can see in the stack of candles and in the diagrams below, each of these numbers can be represented as a triangle.



1 row	2 rows	3 rows	4 rows
1	3	6	10
1st term	2nd term	3rd term	4th term

c Copy the diagrams for the first four terms of the triangle number sequence then continue the diagrams for the fifth and sixth terms.

d Double each number in the sequence so that you have 2, 6, 12, 20… Look for a pattern in the factors of each of these numbers. Hint: $2 = 2 \times 1, 6 = 3 \times 2$.

Record your pattern in a table as shown below. Continue down to the 10th row of candles.

Number of rows of candles	Total number of candles	Number of candles × 2	Product of factors
1	1	2	2×1
2	3	6	3×2
3	6	12	

e Using the pattern of factors you have worked out, what is the total number of candles in the first 10 rows? (Don't forget to divide the product of factors by 2 to allow for the multiplying by 2 in column 3.)

f Use the pattern of factors to find the total number of candles in the 17 rows.

g Use the triangle number factor pattern to find the sum of the first 100 integers.

$1 + 2 + 3 + 4 + 5 + \ldots 47 + 48 + 49 + 50 + 51 + 52 + \ldots 96 + 97 + 98 + 99 + 100$

(We use dots to represent numbers in between to save having to write every number up to 100.)

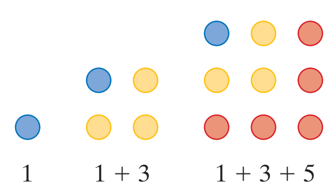
h Work out why the pairing method used by Gauss gives the same answer as the triangle numbers factor pattern.

i Set up a spreadsheet to calculate the sum of the integers. To produce the list of consecutive integers in column A, type 1 in cell A1, then in cell A2 enter the formula =A1+1. Copy the formula down to about row 120. Enter an appropriate formula in cell B1 to calculate the sum. (Base your formula on your findings from part e.)

15.4 Adding odd integers

There are many sequences of numbers that follow some sort of pattern.

a The diagrams on the right show the pattern formed by adding consecutive odd integers. Copy and complete the diagrams for $1 + 3 + 5 + 7$ and $1 + 3 + 5 + 7 + 9$.



'Consecutive' means 'coming after each other in order', e.g, 1, 3, 5.

b The table below shows the sums of odd integers as far as the sum of the first 10 odd integers. Copy and complete column 2 of the table.

1	1
1 + 3	4
1 + 3 + 5	
1 + 3 + 5 + 7	
1 + 3 + 5 + 7 + 9	
1 + 3 + 5 + 7 + 9 + 11	
1 + 3 + 5 + 7 + 9 + 11 + 13	
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15	
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17	
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19	

c Look at the numbers in the second column. What do you notice about them all? Use this observation to explain the pattern of dots in part a.

d Use index notation to write the numbers in column 2 in another way in column 3.

e Use the pattern to find the sum of the first 20 odd integers.

f The sum of the first N odd integers was found to be 961.

$1 + 3 + 5 + 7 + \ldots. + N = 961$

How many odd integers were there? In other words, what was the value of N?

15.5 Egyptian fractions

About 5000 years ago, the ancient Egyptians had a method for representing fractions such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. These fractions are called **unit fractions** because in each case the numerator is 1. However the Egyptians did not have a method for writing fractions such as $\frac{3}{4}$ or $\frac{5}{6}$. Instead they wrote fractions as a sum of different unit fractions.

For example, $\frac{3}{4}$ can be written as $\frac{1}{2} + \frac{1}{4}$ and $\frac{4}{5}$ can be written as $\frac{1}{2} + \frac{1}{4} + \frac{1}{20}$ or as $\frac{1}{2} + \frac{1}{5} + \frac{1}{10}$.

A fraction written as the sum of unit fractions in this way is called an **Egyptian fraction**. The unit fractions all had to be different, so $\frac{3}{4}$ could not be written as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

a Write each of these Egyptian fractions as a single fraction.

i $\frac{1}{2} + \frac{1}{5}$ **ii** $\frac{1}{3} + \frac{1}{4}$

iii $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$ **iv** $\frac{1}{3} + \frac{1}{4} + \frac{1}{7}$

If a fraction is greater than $\frac{1}{2}$, we can start the Egyptian fraction with $\frac{1}{2}$ and then work out which other fractions need to be added. For example, $\frac{5}{9}$ is greater than $\frac{1}{2}$, so we start by writing $\frac{5}{9} = \frac{1}{2} + \ldots$ If we then write $\frac{5}{9}$ and $\frac{1}{2}$ with a common denominator of 18, we see that $\frac{5}{9} = \frac{10}{18}$ and $\frac{1}{2} = \frac{9}{18}$ so we must add $\frac{1}{18}$. So $\frac{5}{9} = \frac{1}{2} + \frac{1}{18}$.

If the fraction is less than $\frac{1}{2}$, we might start with $\frac{1}{3}$ or $\frac{1}{4}$ then use common denominators to find what needs to be added on. This method of starting with the largest possible unit fraction is called the **greedy method** for turning fractions into Egyptian fractions.

b Find the missing denominators in these Egyptian fractions.

i $\frac{3}{5} = \frac{1}{2} + \frac{1}{\_}$ **ii** $\frac{11}{20} = \frac{1}{2} + \frac{1}{\_}$

iii $\frac{5}{12} = \frac{1}{3} + \frac{1}{\_}$ **iv** $\frac{7}{12} = \frac{1}{2} + \frac{1}{\_}$

c Use the greedy method to complete each of these Egyptian fractions.

i $\frac{6}{7} = \frac{1}{2} + \frac{1}{\_} + \frac{1}{\_}$ **ii** $\frac{8}{9} = \frac{1}{2} + \frac{1}{\_} + \frac{1}{\_}$

iii $\frac{9}{11} = \frac{1}{\_} + \frac{1}{\_} + \frac{1}{\_}$ **iv** $\frac{8}{13} = \frac{1}{\_} + \frac{1}{\_} + \frac{1}{\_}$

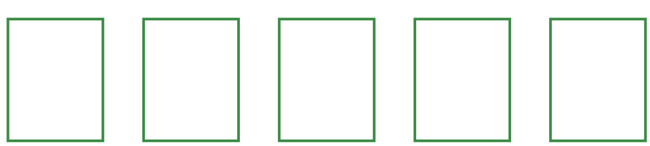
v $\frac{4}{11} = \frac{1}{\_} + \frac{1}{\_}$ **vi** $\frac{2}{7} = \frac{1}{\_} + \frac{1}{\_}$

d By writing each of the fractions as an Egyptian fraction, show that $\frac{4}{7}$ is smaller than $\frac{5}{8}$.

e Ahmed had 5 sacks of wheat that he wanted to divide equally between his 8 farm workers.

i Write $\frac{5}{8}$ as an Egyptian fraction.

ii Use this diagram of the five sacks to represent how Ahmed would use the Egyptian fraction to divide the 5 sacks equally between the 8 workers.

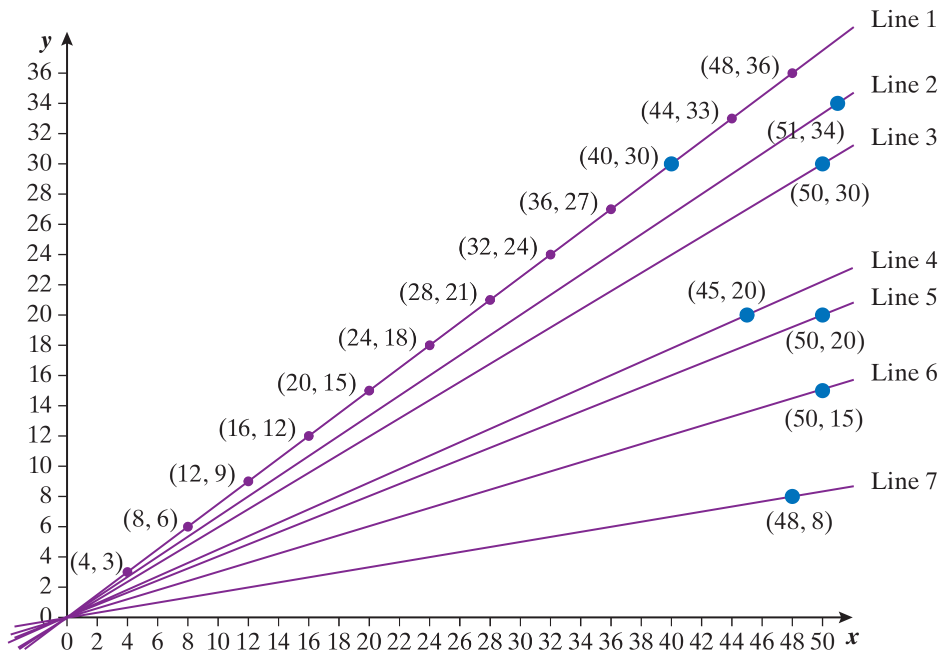


f Find three different ways of writing $\frac{5}{7}$ as an Egyptian fraction.

15.6 Equivalent fractions from graphs

Equivalent fractions from graphs

For this investigation you will need the GeoGebra file *Equivalent fractions from graphs* in the ebook.



Line 1 passes through the points (0, 0) and (40, 30).

We can make a fraction by writing the *y*-coordinate over the *x*-coordinate, for example, $\frac{30}{40}$, which of course is equivalent to $\frac{3}{4}$.

All the other blue points on Line 1 are at points where grid lines intersect. This means that the *x*-coordinates and *y*-coordinates are whole numbers. The *y*-coordinate over the *x*-coordinate for each of these points will tell us an equivalent fraction to $\frac{3}{4}$.

a Look at all the points along Line 1 that have whole number coordinates and list all the equivalent fractions to $\frac{3}{4}$.

b Now look at Line 2. What fraction can we make from the coordinates of the red point?

c Write this fraction in its simplest form.

d Using the GeoGebra file, drag the red point slowly along Line 2 and find the coordinates of all the other points where grid lines intersect. Then list all of the fractions equivalent to the fraction you wrote in part c.

e Repeat for each of the lines 3–7. Record all your equivalent fractions in a table as shown:

Line	Fraction in simplest form	Equivalent fractions	Rule (part f)
Line 1	$\frac{3}{4}$		$y = \frac{3x}{4}$
Line 2			
Line 3			
Line 4			
Line 5			
Line 6			
Line 7			

f The first graph was made by joining the origin and the point (40, 30). For every point on this graph the y-coordinate was $\frac{3}{4}$ of the x-coordinate. Because of this, the rule for the line is $y = \frac{3x}{4}$. Write the rule for each of the other lines.

15.7 Tom and Tori's towers

Tom and Tori decided to make a tower surrounded by a path. They intended to use ceramic tiles for each surface of the tower and for the path. They each drew a plan.

For Tori's tower,

a how many ceramic tiles would be needed for the tower and path shown at left?

b how many ceramic tiles would be needed for a tower one tile high? Two tiles high?

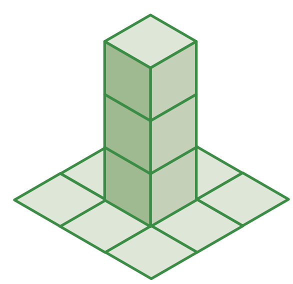
c choose and explain symbols for the height of the tower and for the total number of ceramic tiles needed.

d summarise your information in a table of values.

e find the rule connecting the height of Tori's tower and the total number of tiles needed.

f explain why the rule works.

g use the rule to predict the number of tiles needed for a tower 20 tiles high.



Tori's plan

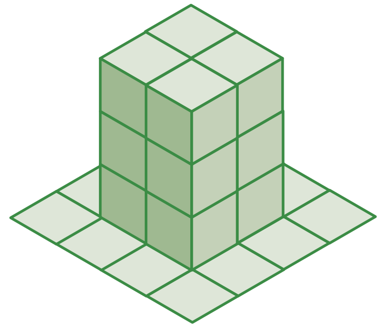
For Tom's tower,

h how many ceramic tiles would be needed if the tower was one, two or three tile high?

i summarise our information in a table of values.

j find the rule connecting the height of Tom's tower and the total number of tiles needed.

k use this rule to predict the number of tiles needed with a tower 20 tiles high.



Tom's plan

15.8 Coordinate tracks

BLM Coordinate tracks

This investigation explores the tracks created when sets of coordinates follow a particular rule. You may use 1 cm graph paper or you may prefer to use computer software such as a spreadsheet, or geometry software which allows you to place points on a coordinate grid.

a **i** Carefully mark a set of 10 points for which the *y*-coordinate has the same value as the *x*-coordinate; for example, (0, 0), (1, 1), (2, 2), (2.1, 2.1), (3.5, 3.5), and so on.

ii What do you notice about your 10 points?

iii Place your ruler carefully along the points and rule a line connecting the points.

iv The 10 points can all be described by the rule: *y*-coordinate = *x*-coordinate. We can abbreviate this to $y = x$. Write this rule along the line connecting the points.

b **i** Using the same coordinate grid, mark a second set of 10 points for which each *y*-coordinate is twice the corresponding *x*-coordinate; for example, (2, 4).

ii Does the point (0, 0) satisfy this rule? Explain.

iii Write a rule in the form $y =$ _____ for the second set of points.

iv Rule a line through the second set of points and write the rule along the line.

v Write a sentence comparing the lines joining the first and second sets of points.

c **i** Mark a third set of 10 points for which each *y*-coordinate is three times the *x*-coordinate; for example, (1.5, 4.5).

ii Does the point (0, 0) satisfy this rule? Explain.

iii Write a rule in the form $y =$ _____ for the third set of points.

iv Rule a line passing through the third set of points and write the rule along the line.

v Write a sentence comparing the line joining the third set of points with the lines through the other two sets of points.

d **i** Mark a fourth set of 10 points for which each *y*-coordinate is three more than the *x*-coordinate; for example, (2, 5).

ii Does the point (0, 0) satisfy this rule? Explain.

iii Write a rule in the form $y =$ _____ for the fourth set of points.

iv Rule a line passing through the fourth set of points and write the rule along the line.

v Write a sentence comparing the line joining the fourth set of points with the line joining the first set of points.

e **i** Mark the points (0, 0), (1, 1), (2, 4), (3, 9).

ii Write a rule in the form $y =$ _____

iii How does this set of points differ from the other four sets of points?

iv Give the y-coordinate for the point which has x-coordinate 5.

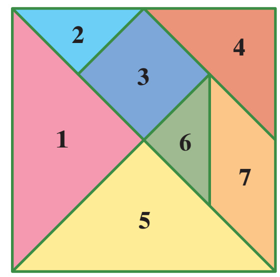
15.9 Tangram pieces


Class activity
Tangram pieces


Tangram pieces

The tangram shown below is an ancient Chinese puzzle where the seven pieces fit together to make a square. The pieces can be rearranged to make various figures such as animals. The file named *Tangram pieces* in the ebook is an interactive GeoGebra file or HTML file in which the seven shapes can be moved and rotated to make the required shapes.

This analysis task, however, is looking at the different triangles and quadrilaterals that either occur in the tangram shapes or can be constructed from them. You can either copy the shapes and cut them out or use the shapes in the *Tangram pieces* file to help you to answer the following questions.



a What is the special quadrilateral shape of piece 3?

b What is the special quadrilateral shape of piece 7?

c Which special triangle shape are pieces 1, 2, 4, 5 and 6?

d Congruent figures have exactly the same shape and size. Find two pieces that are congruent shapes. Is there more than one pair of congruent shapes?

e Pieces 2 and 6 can be rearranged in different ways to make three of the other tangram pieces. Draw three diagrams to show how you would arrange the two pieces to make the other three shapes.

f Draw diagrams to show all the different combinations of pieces that could be arranged to form different sized squares.

g Which three of the tangram pieces could be arranged to form a rectangle that is not a square?

h Find a different combination of three pieces that could be arranged to form a rectangle that is not a square.

i Copy piece 7 and label the four angle sizes.

j Draw a diagram to show how two tangram pieces could be arranged to form a trapezium. Which two pieces have you used? Make a different trapezium using a different pair of pieces. Which two pieces have you used?

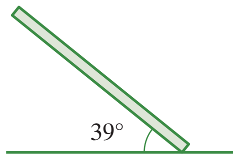
k Draw a diagram to show how three tangram pieces could be arranged to form a trapezium. Which three pieces have you used? Draw another diagram to show how three tangram pieces could be arranged to form a different trapezium. Which three pieces have you used this time?

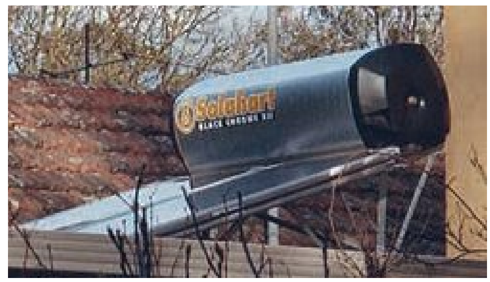
l Figures that have exactly the same shape but not necessarily the same size are called *similar* figures. Which of these sets of three tangram pieces are similar?

1, 3 and 5   1, 2 and 4   4, 5 and 6   2, 6 and 7

15.10 Catching the Sun's heat

Solar hot water panels need to be tilted towards the sun so they receive as much of the sun's heat as possible. Ideally the panels should be tilted at different angles through the year, but for the best all-year-round angle, it is recommended that the panels should be tilted at an angle to the horizontal equal to the latitude of the location plus 12°. For example, Brisbane has a latitude of 27°S, so solar panels should be tilted at an angle of 27° + 12°, that is 39°, to the horizontal, as shown in the diagram.



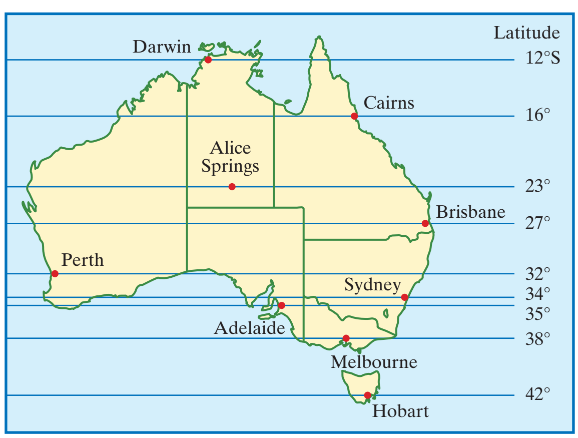


a Use the information in the map to calculate the best tilt angles for solar panels in

i Melbourne. **ii** Darwin. **iii** Hobart.

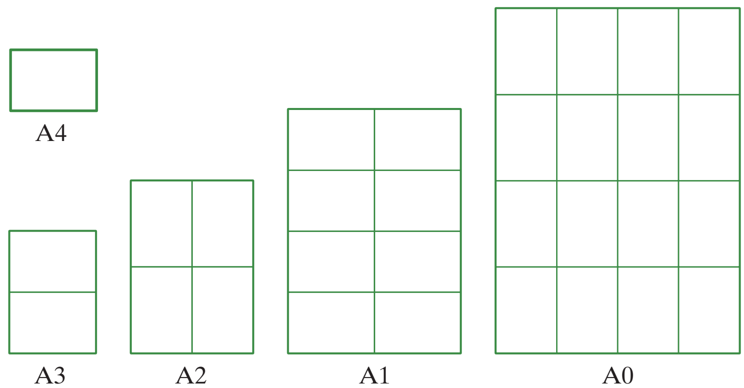
b Using your ruler and protractor, make careful diagrams to show the tilt angles you have calculated for Melbourne, Darwin and Hobart.

c The angles you have calculated represent the best angles for the whole year. Ideally, it would be better if the solar panels were able to be tilted at different angles through the year. In summer, when the sun is at a higher angle for a greater part of the day compared with in winter, would the solar panels need to be set at a steeper or less steep angle? Use diagrams to explain your answer.



15.11 Paper sizes

The most common paper size in Australia is A4. As shown in the diagram, the area of A3 paper is twice that of A4 paper, the area of A2 paper is four times the area of A4 paper, and so on, down to A0 paper, which is the largest size in this series of paper sizes.

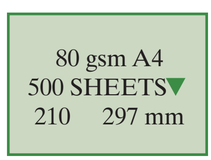


a A sheet of A0 paper is 0.841 m by 1.189 m. What is the area of a sheet of A0 paper? Give your answer correct to one decimal place.

b What is the area of a sheet of A1 paper? Give your answer as a fraction of a square metre.

c Now give the areas of sheets of A2, A3, and A4 paper as fractions of a square metre.

d Standard office paper (A4) has a mass of 80 grams per square metre (80 gsm). What is the mass of a piece of A0 paper?



Make a table to show the masses of sheets of A0, A1, A2, A3, and A4 paper.

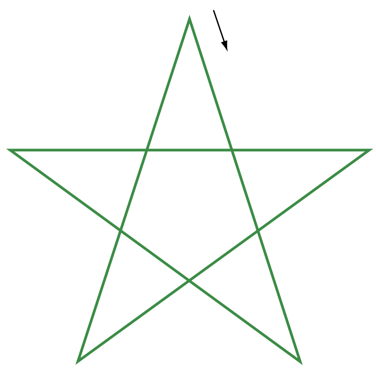
e What is the mass of 500 sheets of A4 paper?

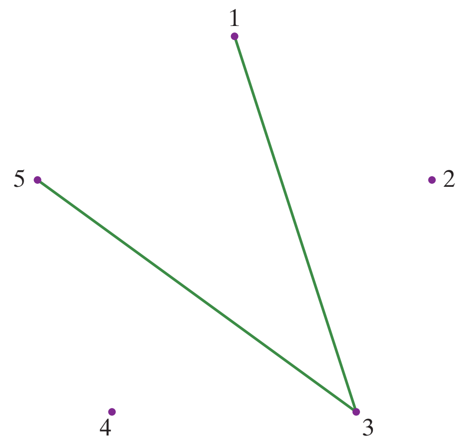
15.12 Star polygons

Class activity
Star polygons dot templates

The five-pointed star is an example of a **star polygon**.

Starting at the top, trace over the star shape with your finger. Notice that you can return to your starting point without lifting your finger. This is a property of all star polygons.





Notice how the star has five vertices that are joined by skipping over one vertex each time. This star polygon is called a 5/2 star polygon, where 5 is the number of vertices and 2 tells us that we join every second vertex, that is, from 1 to 3 to 5, and so on.

a Using the 5 dots template, draw
- **i** a 5/2 star polygon.
- **ii** a 5/3 star polygon.

b Use the 7 dots template to draw
- **i** a 7/3 star polygon.
- **ii** a 7/4 star polygon.

c Use new sets of 7 dots to draw 7/2 and 7/5 star polygons. What do you notice?

d Which star polygon do you think would be the same as a 9/2 star polygon?

The six-pointed Star of David is not a star polygon because it cannot be drawn without lifting the pencil. Instead it is called a star figure.

e Use a set of 6 dots to see what happens when you try to draw a 6/2 star polygon. Lift your pencil and start again to complete the Star of David.

f Use a new set of 6 dots to try to draw a 6/3 star polygon. What do you notice?

g Use a set of 8 dots to draw an 8/3 star polygon.

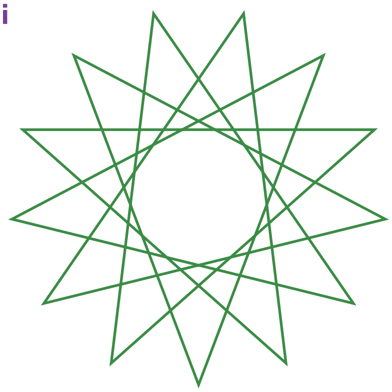
h Which star polygon will be the same shape as the 8/3 star polygon?

i How many star polygons are possible with 8 vertices?

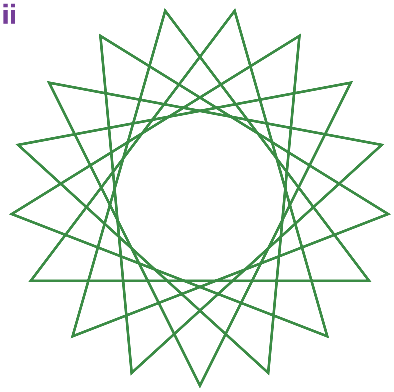
j Is there a star figure with 8 vertices?

k Name these star polygons.

i



ii



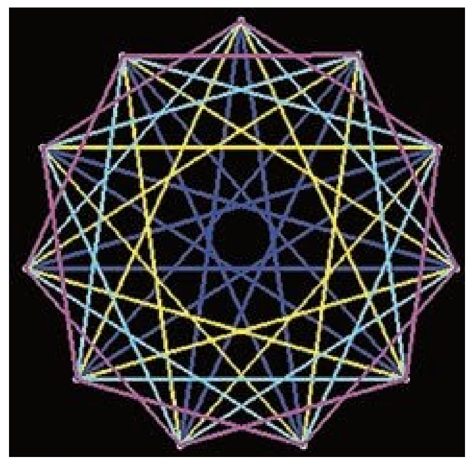
Patterns can be made by constructing a series of overlapping star polygons in different colours. In this picture, there are four overlapping star polygons.

Pink $^{11}/_{2}$

Green $^{11}/_{3}$

Yellow $^{11}/_{4}$

Blue $^{11}/_{5}$



l Explain why there are no other possible star polygons with 11 vertices.

m On which star polygon or star figure is this wheel cover based?

n How many different star polygons are possible with 13 vertices?

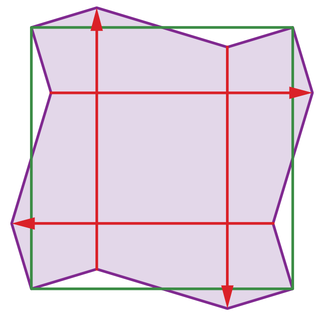
o We can represent a star polygon as a fraction $\frac{n}{d}$ where $n > d$.

Find an expression in terms of n and d for how many different star polygons are possible if n is a prime number.

Class activity
Using GeoGebra to create a tessellation

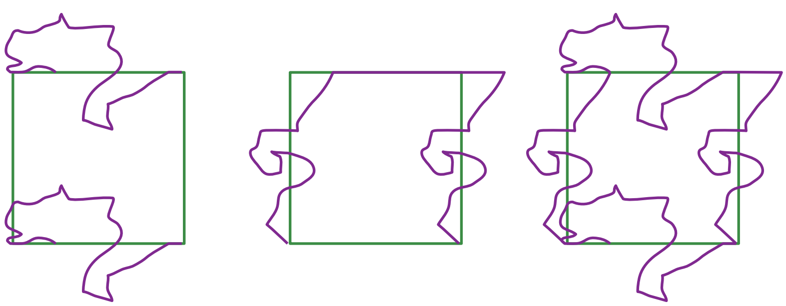
15.13 Designing a tessellation

Tessellating designs can be made by starting with any shape that will tessellate, for example, a square or parallelogram, and then cutting a shape from one edge and moving (translating) it to the opposite edge of the shape. The paving bricks below are based on a square, as shown. Triangles are cut from one edge and translated to the opposite edge. Two of the resulting shapes are then combined to form each paving stone.



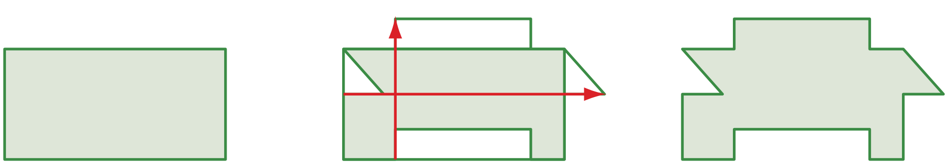
The red arrows are translation vectors.

The title page of Chapter 14 shows a tessellating design of winged horses by Escher. In the diagrams below you can see the design starts with a square. Each shape that is cut out of one side is translated to the opposite side. The pieces added on to one side then fits into the gap left behind on the opposite side in much the same way as the pieces of a jigsaw puzzle fit together.

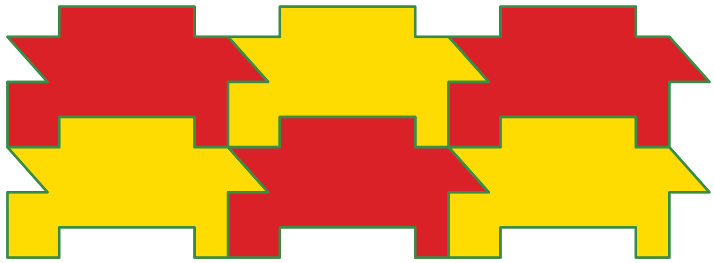


The following diagrams show how you can construct a simple tessellating design.

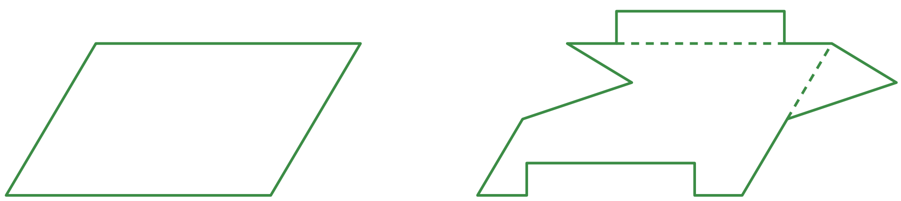
A triangle has been cut from the short side of the rectangle and translated to the opposite side. Similarly, a rectangle has been cut from the long side and translated to the opposite side. The red arrows show the two translation vectors.



The tessellation is then formed by translating the shape.

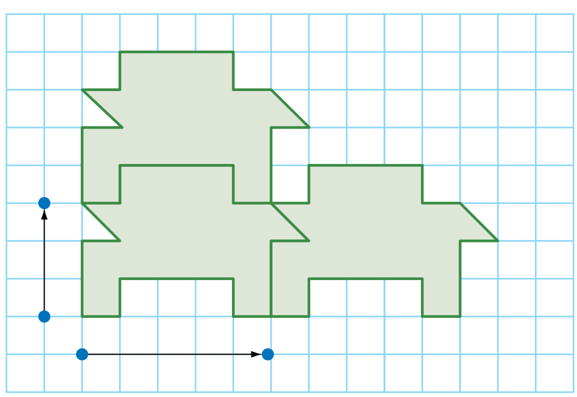


The following diagrams show how the same process can be used starting with a parallelogram instead of a rectangle.



Starting with either a rectangle or a parallelogram, design your own tessellation.

The file *Using GeoGebra to create a tessellation* in the ebook shows how you can copy your pencil and paper shape into GeoGebra then use the *Translation vector* tool and the transformation tool, *Translation* to create a tessellation.



Answers

Chapter 1

exercise 1.1

1

		Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
a	4086				4	0	8	6
b	12 792			1	2	7	9	2
c	1 040 806	1	0	4	0	8	0	6
d	390 600		3	9	0	6	0	0
e	2 340 000	2	3	4	0	0	0	0
f	964 005		9	6	4	0	0	5

2 **a** 40 **b** 600 **c** 5000 **d** 700
e 80 **f** 20 000 **g** 800 000 **h** 2 000 000

3 **a** 50 **b** 5000 **c** 500 **d** 50 000
e 5 **f** 500 **g** 5000 **h** 500 000

4 **a** five hundred and eighty-seven
b three thousand four hundred and seventy-six
c seventy-five thousand, eight hundred and ninety-six
d one hundred and thirty-four thousand, eight hundred and seventy-six
e two hundred thousand, four hundred and thirteen
f seven million, eight hundred and ninety-seven thousand, seven hundred and eighty-four
g six million, nine hundred thousand five hundred
h two million, three hundred and thirty-three thousand, three hundred and thirty-three

5 **a** 257 **b** 1143 **c** 11 629 **d** 145 057

6 Thirteen thousand and seventeen dollars

7 **a** 1345 **b** 2080 **c** 12 382
d 7 357 000 **e** 65 021 **f** 739 000
g three thousand, four hundred and seventy-two
h nineteen thousand and fifty-six
i three hundred and seventy-five thousand, one hundred and twenty-four
j forty-one thousand and one
k three hundred and five thousand and eighty
l twenty-one million five-hundred and sixty-seven thousand and fifty-three

8 **a** $>$ **b** $<$ **c** $>$
d $>$ **e** $>$ **f** $<$
g $<$ **h** $>$ **i** $>$

9 **a** 115, 511, 1998, 2009, 2089, 2890
b 287, 516, 728, 1506, 1560, 1800
c 998, 1098, 1800, 8900, 10 800, 80 100
d 5003, 5010, 5698, 5698, 8965, 53 000

10 **a** 10 700, 10 070, 7100, 1700, 1070, 710
b 20 030, 3020, 2300, 2030, 320, 302
c 10 090, 10 009, 9001, 1090, 1009, 901
d 41 000, 8004, 4800, 4080, 1400, 804

11 **a** $3 \times 100 + 4 \times 10 + 6$
b $6 \times 1000 + 4 \times 100 + 7 \times 10 + 8$
c $8 \times 1000 + 9 \times 100 + 6 \times 10 + 5$
d $1 \times 10\,000 + 2 \times 1000 + 7 \times 100 + 8 \times 10 + 6$
e $5 \times 10\,000 + 4 \times 1000 + 6 \times 100 + 5 \times 10 + 8$
f $1 \times 100\,000 + 2 \times 10\,000 + 3 \times 1000 + 7 \times 100 + 6 \times 10 + 5$
g $7 \times 100\,000 + 8 \times 10\,000 + 6 \times 1000 + 5 \times 100 + 4 \times 10 + 6$
h $1 \times 1\,000\,000 + 4 \times 100\,000 + 3 \times 10\,000 + 2 \times 1000 + 5 \times 100 + 6 \times 10 + 7$

12 **a** $6 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 9$
b $3 \times 10^2 + 9 \times 10^1 + 6$
c $2 \times 10^3 + 5 \times 10^2 + 8 \times 10^1 + 0$
d $5 \times 10^4 + 6 \times 10^3 + 9 \times 10^2 + 1 \times 10^1 + 4$
e $1 \times 10^4 + 7 \times 10^3 + 0 \times 10^2 + 8 \times 10^1 + 5$
f $5 \times 10^3 + 0 \times 10^2 + 9 \times 10^1 + 2$
g $2 \times 10^4 + 4 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 5$
h $6 \times 10^5 + 7 \times 10^4 + 5 \times 10^3 + 8 \times 10^2 + 5 \times 10^1 + 4$
i $2 \times 10^4 + 0 \times 10^3 + 8 \times 10^2 + 0 \times 10^1 + 0$
j $1 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 5$
k $7 \times 10^6 + 6 \times 10^5 + 5 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 0 \times 10^1 + 5$
l $6 \times 10^6 + 0 \times 10^5 + 3 \times 10^4 + 5 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 2$

13 **a** 5714 **b** 63 185 **c** 807 356
d 900 115 **e** 672 012 **f** 930 709
g 5 800 000 **h** 809 735 **i** 703 902
j 60 938 **k** 706 309 404 **l** 2 500 896

14 **a** 7×10^3 **b** 9×10^2 **c** 4×10^4
d 6×10^5 **e** 3×10^3 **f** 8×10^5
g 2×10^4 **h** 7×10^6 **i** 33×10^5
j 95×10^4 **k** 27×10^8 **l** 84×10^{15}

15 **a** 5000 **b** 70 000
c 20 **d** 5 000 000
e 300 000 **f** 900
g 600 000 000 **h** 12 000
i 45 000 000 000 **j** 830 000 000
k 75 000 000 **l** 36 000 000 000 000

16 **a** 5×10^3 **b** 6×10^6
c 2×10^5 **d** 3×10^4
e 7×10^3 **f** 2×10^6
g 64×10^3 **h** 52×10^4

17 **a** 88 753 **b** 8 765 200

18 **a** 2035 **b** 1 002 389

19 **a** 9 999 999; nine million, nine hundred and ninety nine thousand, nine hundred and ninety nine
b 1 000 000; one million

20 **a** 4×10^4 **b** 3×10^8 **c** 15×10^7

exercise 1.2

1 **a** 20 **b** 30 **c** 20 **d** 30
e 31 **f** 30 **g** 31 **h** 30
i 31 **j** 30 **k** 33 **l** 52

2 **a** 100 **b** 57 **c** 75 **d** 200
e 51 **f** 190 **g** 211 **h** 150
i 151 **j** 159 **k** 141 **l** 113

3 **a** 100 **b** 150 **c** 140 **d** 110
e 160 **f** 170 **g** 140 **h** 170
i 200 **j** 101 **k** 141 **l** 172

4 **a** 177 **b** 155 **c** 143 **d** 127
e 631 **f** 931 **g** 1804 **h** 1885
i 4080 **j** 17 674 **k** 4 010 000 **l** 127 070

5 **a** 47 **b** 27 **c** 8 **d** 18
e 18 **f** 43 **g** 29 **h** 12
i 28 **j** 15 **k** 23 **l** 54

6 **a** 14 **b** 38 **c** 9 **d** 18
e 124 **f** 86 **g** 77 **h** 1087
i 819 **j** 2212 **k** 2920 **l** 12 867

7 **a** 39 **b** 59 **c** 57
d 79 **e** 578 **f** 1668

8 **a** 22 339 900 **b** 7 682 300

9 **a** 19 **b** \$68 **c** 683
d 68 **e** 5557 **f** 350 kg

10 **a**
$$\begin{array}{r} 1\,2\,8 \\ +\ 2\,4\,3 \\ \hline 3\,7\,1 \end{array}$$
b
$$\begin{array}{r} 4\,9\,8 \\ +\ 1\,3\,7 \\ \hline 6\,3\,5 \end{array}$$
c
$$\begin{array}{r} 5\,4\,0 \\ +\ 6\,7\,3 \\ \hline 1\,2\,1\,3 \end{array}$$
d
$$\begin{array}{r} 2\,7\,8 \\ +\ 9\,9\,7 \\ \hline 1\,2\,7\,5 \end{array}$$

11 **a** 0 **b** 13 **c** 22 **d** 9
e 26 **f** 0 **g** 21 **h** 38
i 6 **j** 20 **k** 15 **l** 45

12 **a**

8	9	4
3	7	11
10	5	6

b

20	17	8
3	15	27
22	13	10

c

40	37	16
7	31	55
46	25	22

d

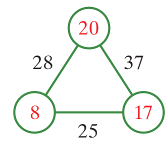
21	23	19
19	21	23
23	19	21

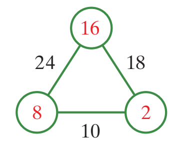
13 **a** 276 kg **b** 64 kg

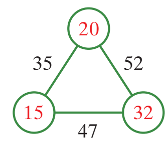
14 **a** 73 and 67 **b** 92 and 76

15 **a** **i** 111 **ii** 108
b Mudlarks by 3 points

16 \$41 750

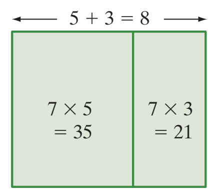
17 **a**


b


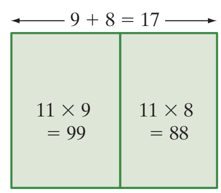
c


18 **a** 982 km **b** 755 km

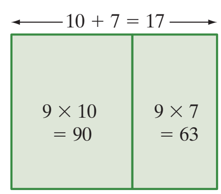
exercise 1.3

1 **a**


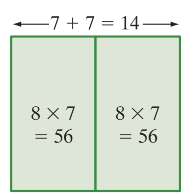
$7 \times 8 = 35 + 21 = 56$

b


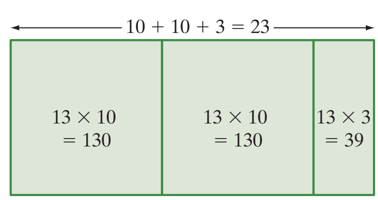
$11 \times 17 = 99 + 88 = 187$

c


$9 \times 17 = 90 + 63 = 153$

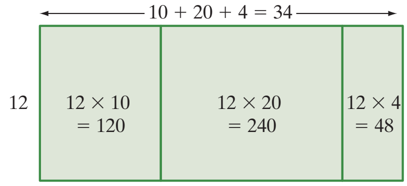
d


$8 \times 14 = 56 + 56 = 112$

e


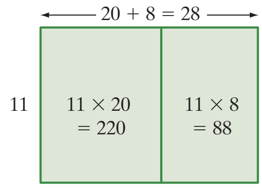
$13 \times 23 = 130 + 130 + 39 = 299$

f



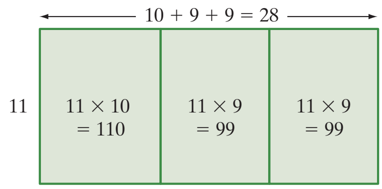
12 × 34 = 120 + 240 + 48 = 408

g



11 × 28 = 220 + 88 = 308

h



11 × 28 = 110 + 99 + 99 = 308

2 **a** 240
b 290

3 **a** 6 × 43 = 6 × 40 + 6 × 3
= 240 + 18
= 258
b 8 × 37 = 8 × 30 + 8 × 7
= 240 + 56
= 296
c 9 × 62 = 9 × 60 + 9 × 2
= 540 + 18
= 558
d 4 × 93 = 4 × 90 + 4 × 3
= 360 + 12
= 372
e 7 × 79 = 7 × 70 + 7 × 9
= 490 + 63
= 553
f 8 × 46 = 8 × 40 + 8 × 6
= 320 + 48
= 368
g 5 × 93 = 5 × 90 + 5 × 3
= 450 + 15
= 465
h 7 × 85 = 7 × 80 + 7 × 5
= 560 + 35
= 595
i 8 × 69 = 8 × 60 + 8 × 9
= 480 + 72
= 552
j 4 × 79 = 4 × 70 + 4 × 9
= 280 + 36
= 316
k 9 × 87 = 9 × 80 + 9 × 7
= 720 + 63
= 783
l 9 × 58 = 9 × 50 + 9 × 8
= 450 + 72
= 522

4 **a** 6 × 49 = 6 × 50 − 6 × 1
= 300 − 6
= 294
b 8 × 37 = 8 × 40 − 8 × 3
= 320 − 24
= 296
c 9 × 58 = 9 × 60 − 9 × 2
= 540 − 18
= 522
d 4 × 79 = 4 × 80 − 4 × 1
= 320 − 4
= 316
e 4 × 58 = 4 × 60 − 4 × 2
= 240 − 8
= 232
f 8 × 38 = 8 × 40 − 8 × 2
= 320 − 16
= 304
g 7 × 49 = 7 × 50 − 7 × 1
= 350 − 7
= 343
h 5 × 78 = 5 × 80 − 5 × 2
= 400 − 10
= 390
i 8 × 68 = 8 × 70 − 8 × 2
= 560 − 16
= 544
j 8 × 89 = 8 × 90 − 8 × 1
= 720 − 8
= 712
k 9 × 99 = 9 × 100 − 9 × 1
= 900 − 9
= 891
l 7 × 79 = 7 × 80 − 7 × 1
= 560 − 7
= 553

5 **a** 476 **b** 378 **c** 342 **d** 384
e 744 **f** 329 **g** 516 **h** 238

6 **a** 133 **b** 738 **c** 392 **d** 552
e 445 **f** 819 **g** 174 **h** 336
i 792 **j** 648 **k** 354 **l** 234

7 **a** **i** 190 **ii** 114 **iii** 304
b Multiply the number by 10 and by 6, then add these two results
c Multiply 16 by 20 and subtract 1, then times 16

8 **a** 100 **b** 5000 **c** 660
d 98 000 **e** 800 000 **f** 345 000
g 7 860 000 **h** 4 567 500 **i** 34 760 000
j 2 673 480 000 **k** 12 399 900 000 **l** 489 821 000 000

9 **a** 60 **b** 280 **c** 120 **d** 180
e 140 **f** 270 **g** 280 **h** 720
i 840 **j** 810 **k** 280 **l** 1080

10 **a** 135 **b** 644 **c** 6083
d 6237 **e** 51 403 **f** 215 112

11 **a** 192 **b** 432 **c** 306 **d** 560
e 848 **f** 532 **g** 540 **h** 666
i 638 **j** 1225 **k** 2790 **l** 1722

12 **a** 720 **b** 1800 **c** 3500 **d** 18 000
e 72 000 **f** 21 000 **g** 540 000 **h** 400 000
i 2 100 000 **j** 2 800 000 **k** 750 000 **l** 84 000 000

13 **a**
326
3
978

b
873
8
6984

c
546
7
3822

d
853
9
7677

14 **a** 1728 **b** 1235 **c** 2324 **d** 2772
e 33 705 **f** 46 880 **g** 13 041 **h** 16 558
i 42 240 **j** 209 684 **k** 563 136 **l** 1 248 972
m 29 008 **n** 29 568 **o** 42 476

15 **a** 782 **b** 2530 **c** 5145
d 29 193 **e** 665 898 **f** 518 964
g 358 425 **h** 690 108 **i** 809 932
j 10 823 967 **k** 284 361 125 **l** 118 155 268

16 **a** $158 **b** $7

17 **a** 180 **b** 12

18 **a** 839 minutes **b** $525

19 **a** 296 cm **b** 592 cm **c** 2368 cm

20 **a** 6
b $20 \times 1 \times 1$, $1 \times 20 \times 1$, $1 \times 1 \times 20$, $10 \times 2 \times 1$, $1 \times 10 \times 2$, $2 \times 1 \times 10$, $5 \times 4 \times 1$, $1 \times 5 \times 4$, $4 \times 1 \times 5$, $5 \times 2 \times 2$, $2 \times 5 \times 2$, $2 \times 2 \times 5$

21 36

22 **a** 57 and 13 **b** 27 and 15

23 **a** 25
b 176
c 17 ants, 8 spiders

24 **a** 18 **b** 54 **c** 10 chickens, 8 cows

exercise 1.4

1 **a** $88 = 8 \times 11$ **b** $144 = 18 \times 8$
c $91 = 13 \times 7$ **d** $57 = 3 \times 19$
e $64 = 16 \times 4$ **f** $156 = 12 \times 13$
g $84 = 12 \times 7$ **h** $256 = 32 \times 8$
i $85 = 17 \times 5$

2 **a** **i** $37 \div 6 = 6$ remainder 1
ii $37 = 6 \times 6 + 1$
b **i** $43 \div 8 = 5$ remainder 3
ii $43 = 5 \times 8 + 3$
c **i** $59 \div 11 = 5$ remainder 4
ii $59 = 5 \times 11 + 4$
d **i** $72 \div 7 = 10$ remainder 2
ii $72 = 7 \times 10 + 2$
e **i** $29 \div 8 = 3$ remainder 5
ii $29 = 3 \times 8 + 5$
f **i** $123 \div 12 = 10$ remainder 3
ii $123 = 10 \times 12 + 3$
g **i** $78 \div 9 = 8$ remainder 6
ii $78 = 8 \times 9 + 6$
h **i** $67 \div 7 = 9$ remainder 4
ii $67 = 9 \times 7 + 4$

3 **a**
$$\begin{aligned} 54 \div 6 &= (48 + 6) \div 6 \\ &= 48 \div 6 + 6 \div 6 \\ &= 8 + 1 \\ &= 9 \end{aligned}$$
b
$$\begin{aligned} 84 \div 12 &= (96 - 12) \div 12 \\ &= 96 \div 12 - 12 \div 12 \\ &= 8 - 1 \\ &= 7 \end{aligned}$$
c
$$\begin{aligned} 63 \div 9 &= (45 + 18) \div 9 \\ &= 45 \div 9 + 18 \div 9 \\ &= 5 + 2 \\ &= 7 \end{aligned}$$
d
$$\begin{aligned} 55 \div 11 &= (88 - 33) \div 11 \\ &= 88 \div 11 - 33 \div 11 \\ &= 8 - 3 \\ &= 5 \end{aligned}$$
e
$$\begin{aligned} 48 \div 6 &= (12 + 36) \div 6 \\ &= 12 \div 6 + 36 \div 6 \\ &= 2 + 6 \\ &= 8 \end{aligned}$$
f
$$\begin{aligned} 100 \div 5 &= (150 - 50) \div 5 \\ &= 150 \div 5 - 50 \div 5 \\ &= 30 - 10 \\ &= 20 \end{aligned}$$
g
$$\begin{aligned} 56 \div 7 &= (70 - 14) \div 7 \\ &= 70 \div 7 - 14 \div 7 \\ &= 10 - 2 \\ &= 8 \end{aligned}$$
h
$$\begin{aligned} 98 \div 7 &= (63 + 35) \div 7 \\ &= 63 \div 7 + 35 \div 7 \\ &= 9 + 5 \\ &= 14 \end{aligned}$$

4 **a** 5 **b** 9 **c** 8 **d** 5
e 11 **f** 9 **g** 11 **h** 9

5 **a** 87 **b** 82 **c** 941 **d** 285
e 5462 **f** 29 218 **g** 283 893 **h** 164 417

6 **a** 229 **b** 97 **c** 1099 **d** 1039
e 1132 **f** 2898 **g** 18 196 **h** 641 057

7 **a** 1 **b** 10 **c** 2 **d** 1
e 3 **f** 5 **g** 4 **h** 8

8 **a** 4 **b** 3 **c** 9 **d** 50
e 90 **f** 1200 **g** 1100 **h** 4000

9 **a** 5 **b** 11 **c** 9 **d** 15
e 10 **f** 17 **g** 23 **h** 37

10 **a** 29 **b** 56 **c** 29 **d** 43
e 78 **f** 81 **g** 208 **h** 175
i 309 **j** 532 **k** 961 **l** 648

11 **a** 36 remainder 7 **b** 33 remainder 11
c 54 remainder 9 **d** 68 remainder 1

e 96 remainder 5 **f** 87 remainder 3
g 102 remainder 1 **h** 207 remainder 15
i 325 remainder 4 **j** 512 remainder 6
k 1250 remainder 6 **l** 512 remainder 9

12 **a** 1 096 216 turtles **b** 145 cm

13 **a** \$3510 **b** \$810

14 **a** 239 **b** 1

15 30

16 **a**

```
       265
  13)3456
     2600
     ----
      856
      780
     ----
       76
       65
     ----
       11 remainder
```

b

```
       390
  14)5463
     42
     ---
     126
     126
     ---
     03 remainder
```

17 **a** 1 kg/day
b 5 tonnes
c 4868 days
d about 13 years

18 11 and 121

19 **a** 10 370 **b** 612 974

exercise 1.5

1 **a** 32 **b** 27 **c** 29
d 10 **e** 6 **f** 46
g 6 **h** 43 **i** 480

2 C

3 E

4 D

5 **a** 13 **b** 108 **c** 24 **d** 1
e 37 **f** 79 **g** 8 **h** 4
i 100 **j** 37 **k** 36 **l** 9
m 4 **n** 60 **o** 55 **p** 271
q 100 **r** 42

6 **a** 5 **b** 9 **c** 11 **d** 3
e 0 **f** 13 **g** 46 **h** 3

7 **a** false **b** false **c** true **d** true
e true **f** false **g** true **h** false

8 **a** $(5 + 4) \times 9 = 81$
b $(9 + 6) \div 3 = 5$
c $11 \times (8 + 3) = 121$
d $12 + 8 \times (3 + 2) = 52$
e $(24 + 16) \div 8 + 16 = 21$
f $32 + 64 \div (4 + 4) = 40$

9 **a** no, she needed to use brackets around the $6 - 3$
b yes, $\dfrac{4 \times 15}{6 - 3} = \dfrac{60}{3} = 20$
c

4	×	1	5	÷	(	6	−	3	)	=

10

Team	Goals	Behinds	Score	Working
Brisbane Lions	18	5	113	$18 \times 6 + 5$
Geelong Cats	14	12	96	$\dfrac{96 - 12}{6}$
Sydney Swans	15	18	108	$108 - 15 \times 6$
West Coast Eagles	12	16	88	$\dfrac{88 - 16}{6}$

11 **a** $50 \times 110 \times 15 + 66 \times 130 \times 15 = 211\,200$
b $(50 \times 110 + 66 \times 130) \times 15$

exercise 1.6

1 750 approximately

2 **a** 60 **b** 100 **c** 130 **d** 2350
e 7860 **f** 10 790 **g** 12 650 **h** 109 780

3 **a** 100 **b** 400 **c** 400 **d** 1000
e 1700 **f** 2400 **g** 34 700 **h** 897 800

4 **a** 3000 **b** 9000 **c** 12 000 **d** 99 000
e 678 000 **f** 879 000 **g** 1 234 000 **h** 5 556 000

5 **a** 2000 **b** 60 000 **c** 700 000
d 3 000 000 **e** 50 000 000 **f** 80 000 000

6 **a** 190 **b** 300 **c** 1600
d 800 **e** 15 000 **f** 6000

7 **a** 150 000 **b** 280 000 **c** 1 600 000
d 50 **e** 60 **f** 20

8 **a** \$440 000
b \$540 474
c The estimate was over \$100 000 less
d \$550 000
e The two leading digits was a much better estimate

9 **a** 1200 people **b** 7200 people

10 **a** 300 cows **b** 600 L

11 **a** 400 **b** 300
c The two leading digits was much closer to the exact answer of 292

12 **a** \$8800
b Not enough by \$520
c \$8776

13 **a** 10 000 m^2 **b** 10 weeds
c 2500 squares **d** 25 000 weeds

Revision

1 E **2** D **3** C **4** D **5** E

6 **a** 8000 **b** 80 000

7 a $2 \times 10^4 + 4 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 2$
b $1 \times 10^5 + 8 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5$
c $8 \times 10^4 + 4 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 6$
d $7 \times 10^3 + 0 \times 10^2 + 8 \times 10^1 + 5$

8 a 720 519 b 800 785

9 a 8×10^4 b 3×10^5
c 7×10^3 d 18×10^6

10 a 140 b 170 c 160 d 140

11 a 19 b 33

12 a 1043 b 2975

13 a 333 b 552

14 a 468 b 480

15 a 6408 b 78 750

16 a 38 b 157
c 365 d 450 remainder 8

17 a 2000 b 37 000

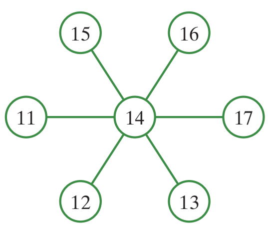
18 a 270 000 b 2

19 300 approximately

20 $\frac{1}{2}$ of $1500 - 110 \times 5 = 200$

21 a 10 950 b 3 years

22



Chapter 2

exercise 2.1

1 a 8, 10, 12 b 20, 25, 30
c 28, 35, 42 d 48, 60, 72
e 200, 250, 300 f 240, 300, 360
g 4000, 5000, 6000 h 60, 75, 90
i 68, 85, 102

2 a 3, 6, 9, 12 b 4, 8, 12, 16
c 8, 16, 24, 32 d 11, 22, 33, 44
e 9, 18, 27, 36 f 13, 26, 39, 52
g 17, 34, 51, 68 h 21, 42, 63, 84
i 25, 50, 75, 100 j 32, 64, 96, 128
k 101, 202, 303, 404 l 687, 1374, 2061,2748

3 a 60 b 49 c 44 d 72
e 91 f 175 g 258 h 5868

4 a no b yes c no d yes
e yes f no

5 a 8 b 10 c 35 d 24
e 30 f 33 g 12 h 24

6 a 6, 12, 18, 24, 30, 36 b 9, 18, 27, 36
c 18, 36 d 18 e 54

7 a 84, equal b 33, equal
c 28, less than d 56, less than
e 30, less than f 36, less than
g 65, equal h 57, equal

8 a If the two numbers have no common factor, the lowest common multiple is the product of the two numbers.
b If the two numbers have a common factor, the lowest common multiple is less than the product of the two numbers.

9 a 15, 30, 45 b 12, 24, 36
c 21, 42, 63 d 35, 70, 105
e 84, 168, 252 f 60, 120, 180
g 420, 840, 1260 h 48, 96, 144

10 a 70 b 210 c 24 d 36
e 60 f 36 g 2145 h 120

11 a 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
b 14, 28, 42
c 6 weeks

12 a 180
b Trang three times, Grace two times

13 a 12
b once
c 9.12 am, 9.24 am, 9.36 am
d 11.29 am

14 a 3457, 5679, 5689, 5789
b 9669

exercise 2.2

1 a 1, 2, 3, 4, 6, 12 b 1, 2, 4, 5, 10, 20
c 1, 5, 25 d 1, 2, 3, 5, 6, 8, 15, 30
e 1, 3, 7, 21 f 1, 2, 4, 8, 16
g 1, 2, 3, 6, 7, 14, 21, 42 h 1, 2, 4, 8, 16, 32, 64

2 a 100, 50, 25, 20, 10, 5, 4, 2, 1
b 133, 66, 44, 33, 22, 12, 11, 6, 4, 3, 2, 1
c 121, 11, 1
d 144, 72, 48, 36, 24, 18, 16, 12, 9, 8, 6, 4, 3, 2, 1
e 108, 54, 36, 27, 18, 12, 9, 6, 4, 3, 2, 1
f 169, 13, 1
g 196, 98, 49, 28, 14, 7, 4, 2, 1
h 245, 49, 35, 7, 5, 1

3 a 3 b 12 c 5 d 6 e 33 f 16

4 a 33: 1, 3, 11, 33
99: 1, 3, 9, 11, 33, 99
highest common factor is 33
b 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
125: 1, 5, 25, 125
highest common factor is 5
c 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
98: 1, 2, 7, 14, 49, 98
highest common factor is 2
d 144: 1, 2, 4, 6, 8, 12, 18, 24, 36, 72, 144
152: 1, 2, 4, 8, 19, 38, 76, 152
highest common factor is 8

e 150: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
225: 1, 3, 5, 15, 45, 75, 225
highest common factor is 75

f 230: 1, 2, 5, 10, 23, 46, 115, 230
300: 1, 2, 3, 4, 5, 6, 10, 15, 20, 30, 50, 60, 75, 100, 150, 300
highest common factor is 10

5 **a** 121: 1, 11, 121
226: 1, 2, 113, 226
990: 1, 2, 3, 5, 6, 9, 11, 15, 66, 90, 110, 165, 198, 330, 495, 990
highest common factor is 1

b 27: 1, 3, 9, 27
81: 1, 3, 9, 27, 81
135: 1, 3, 5, 9, 15, 27, 45, 135
highest common factor is 27

c 132: 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132
164: 1, 2, 4, 41, 82, 168
168: 1, 2, 4, 6, 8, 12, 14, 21, 28, 42, 84, 168
highest common factor is 4

d 21: 1, 3, 7, 21
63: 1, 3, 7, 9, 21, 63
126: 1, 2, 3, 6, 9, 14, 21, 42, 61, 121
highest common factor is 21

e 35: 1, 5, 7, 35
105: 1, 5, 7, 15, 21, 105
210: 1, 2, 3, 7, 15, 70, 105, 210
highest common factor is 7

f 44: 1, 2, 4, 11, 22, 44
64: 1, 2, 4, 8, 16, 32, 64
192: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192
highest common factor is 4

6 **a** 18 teams **b** 14 teams

7 **a** 24 sets
b green cardboard 3, yellow dots 16, black beads 4

8 **a** 6 clauses **b** 22 students

9 **a** 39 bags
b 3 jelly snakes, 2 chocolate frogs, 5 mini-Pluto bars

10 28: $1 + 2 + 4 + 7 + 14 = 28$
496: $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$

11 220: $1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$
284: $1 + 2 + 4 + 71 + 142 = 220$

exercise 2.3

1 **a** 32, 66, 260, 232, 2768, 12 346, 108 556
b 32, 260, 232, 2768, 108 556
c 32, 232, 2768

2 **a** 24, 99, 123, 7677, 54 879
b 24
c 99, 7677

3 **a** 25, 30, 765, 870, 1345, 76 540, 109 900
b 30, 870, 76 540, 109 900

4 a, b, c, e, h

5 a, c, d, g

6 b, e

7 a, c, d, e, g

8 a, b, d, f, g

9 a, c, e, g, h

10 b, c, e, g

11 a, d

12 c

13 **a** 0, 2, 4, 6, 8 **b** 0, 6
c 6 **d** 0, 2, 4, 6, 8

14 **a** true **b** false **c** true **d** true

15 **a** yes **b** 307 Smarties

16 **a** no
b no, the last three digits of the number would still be 452 and this is not divisible by 8

17 **a** 2004 **b** 2092

18 999 996

19 45 (a 90-year-old would be retired from teaching!)

20 **a** 0, 9 **b** 2, 8 **c** 1, 4, 7

21 **a** 9 999 990 **b** 1 000 080 **c** 270

exercise 2.4

1

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
1	4	9	16	25	36	49	64	81	100
11^2	12^2	13^2	14^2	15^2	16^2	17^2	18^2	19^2	20^2
121	144	169	196	225	256	289	324	361	400

2 **a** 9 **b** 1156
c 9801 **d** 186 624
e 976 144 **f** 767 376
g 46 090 521 **h** 76 650 025
i 152 399 025 **j** 1 886 338 624
k 152 399 025 **l** 1 886 338 624

3 441, 484

4 **a** 28 **b** 29 **c** 35 **d** 32
e 36 **f** 31 **g** 45 **h** 64

5 **a** 400 **b** 900 **c** 2500 **d** 10 000
e 14 400 **f** 40 000 **g** 490 000 **h** 4 000 000
The number of zeros in each answer is double that of the starting number

6 **a** 25 **b** 31 **c** 33 **d** 21
e 37 **f** 1 **g** 0 **h** 17
i 10 **j** 100 **k** 80 **l** 900

7 **a** $6 \times 6 \times 6 = 216$
b $10 \times 10 \times 10 = 1000$
c $1 \times 1 \times 1 = 1$
d $8 \times 8 \times 8 = 512$
e $12 \times 12 \times 12 = 1728$
f $20 \times 20 \times 20 = 8000$

g $100 \times 100 \times 100 = 1\,000\,000$
h $1000 \times 1000 \times 1000 = 1\,000\,000\,000$

8 6343

9 a i 121 ii 12 321
iii 1 234 321 iv 123 454 321
b i 12 345 654 321 ii 1 234 567 654 321
iii 123 456 787 654 321 iv 12 345 678 987 654 321
c check they match your answers in part b
d 12 345 678 900 987 654 321

exercise 2.5

1

Words	Index notation	Expanded notation	Value
Three to the power of six	3^6	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	729
Seven cubed	7^3	$7 \times 7 \times 7$	343
Five to the power of four	5^4	$5 \times 5 \times 5 \times 5$	625
Six to the power of five	6^5	$6 \times 6 \times 6 \times 6 \times 6$	7776
Thirteen squared	13^2	13×13	169
Three to the power of eight	3^8	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	6561
Eleven to the power of four	11^4	$11 \times 11 \times 11 \times 11$	14 641
Twenty-nine cubed	29^3	$29 \times 29 \times 29$	24 389
Two to the power of five	2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
Ten to the power of seven	10^7	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10 000 000

2

		Base	Index
a	2^8	2	8
b	8^2	8	2
c	17^3	17	3
d	10^5	10	5
e	1^6	1	6
f	4^7	4	7
g	3^{11}	3	11
h	11^3	11	3
i	15^2	15	2
j	25^6	25	6

3 a 3^4 b 5^6 c 6^3 d 7^8
e 9^5 f 12^6 g 31^7 h 10^5
i 57^3 j 100^4 k 4^8 l 8^4

4 a $2^3 \times 3$ b $7^2 \times 9^4$
c $4^2 \times 6^3$ d $2^4 \times 5^4$
e $2 \times 3^3 \times 5^2$ f $5^3 \times 12^3$
g $4 \times 6^2 \times 7^2$ h $7^3 \times 9 \times 13 \times 17^3$
i $2^2 \times 5^2 \times 19^3$ j $11 \times 13^3 \times 15^3$
k $3^2 \times 8^3 \times 11^2$ l $2^3 \times 5^3 \times 13^2$

5 a $2 \times 2 \times 2 \times 2 \times 2$
b $7 \times 7 \times 7$
c $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
d $5 \times 5 \times 5 \times 5$
e $1 \times 1 \times 1$
f $3 \times 3 \times 3 \times 3 \times 3$
g $9 \times 9 \times 9 \times 9 \times 9 \times 9$
h $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
i $4 \times 4 \times 4$
j $3 \times 3 \times 3 \times 3$
k $15 \times 15 \times 15 \times 15 \times 15$
l $36 \times 36 \times 36 \times 36$

6 a $2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
b $3 \times 3 \times 6 \times 6 \times 6 \times 6 \times 6$
c $7 \times 7 \times 7 \times 8 \times 8 \times 8 \times 8$
d $4 \times 4 \times 4 \times 4 \times 4 \times 9 \times 9 \times 9$
e $3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4$
f $11 \times 11 \times 11 \times 11 \times 11 \times 7 \times 7 \times 7 \times 7$
g $13 \times 13 \times 13 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
h $29 \times 29 \times 31 \times 31 \times 31 \times 31$
i $43 \times 43 \times 43 \times 43 \times 43 \times 43 \times 57 \times 57$
j $134 \times 134 \times 134 \times 134 \times 234 \times 234 \times 234$
k $1243 \times 1243 \times 9899 \times 9899 \times 9899 \times 9899 \times 9899$
l $8788 \times 8788 \times 8788 \times 333 \times 333 \times 333 \times 333 \times 333$

7 B

8 a 390 625 b 512 c 1024
d 128 e 2401 f 6561
g 59 049 h 1 i 2 985 984
j 91 125 k 916 132 832 l 108 243 216

9 a 1944 b 400 c 6125
d 5488 e 20 000 f 1296
g 72 h 11 664

10 399

11 $2^6, 4^3, 8^2$

12 $12^2, 2^4 \times 3^2, 4^2 \times 3^2$

exercise 2.6

1 **a** 91 **b** 1, 7, 13, 91

c

Category	Prime numbers	Number of primes
Between 0 and 10	2, 3, 5, 7	4
Between 10 and 20	11, 13, 17, 19	4
Between 20 and 30	23, 29	2
Between 30 and 40	31, 37	2
Between 40 and 50	41, 43, 47	3
Between 50 and 60	53, 59	2
Between 60 and 70	61, 67	2
Between 70 and 80	71, 73, 79	3
Between 80 and 90	83, 89	2
Between 90 and 100	97	1

d 15 **e** 24

2 A

3 **a** prime **b** composite
c composite **d** prime
e composite **f** prime
g composite **h** prime

4 101 and 103

5 **a** prime
b prime
c not prime, factors are 1, 3, 43, 129
d not prime, factors are 1, 3, 7, 9, 17, 21, 51, 63, 119, 153, 357, 1071

6 **a** true **b** true **c** true **d** false
e false **f** true **g** true **h** true

7 17 and 71, 37 and 73, 79 and 97

8 **a** $2 + 3 + 5 + 7 + 11 = 28$
b $47 - 23 = 24$
c $2 \times 83 = 166$
d $41 + 43 = 84$
e $11 + 13 + 17 + 19 = 60$

9 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

10 **a** 5, 10, 15, 20
b 3, 6, 9, 12, 18
c 2, 4, 8, 14, 16
d 1, 7, 11, 13, 17, 19
e Olivia: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
Michael: 3, 9, 15
Mia: 5

f Same as before, because the cards left are the ones which are not multiples of 2, 3, or 5.

11 All square numbers have the number they are squaring as a factor, so they never have just one and itself as factors and can never be prime.

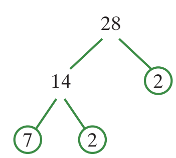
12 **a** 101, 131, 151 are three examples
b 73
c 257
d When looking at three numbers which are two apart, one will always be divisible be three. For 3, 5 and 7 it is the 3 and the numbers can all only be divided by one and itself. But for any other numbers one of the numbers would also be divisible by three and so 3, 5 and 7 are the only triple primes.
e 2, 5, 17, 37, 101, 197
f 3

13 **a** $2 + 2$
b $23 + 3, 19 + 7, 13 + 13$
c $31 + 7, 19 + 19$
d $43 + 3, 41 + 5, 29 + 17, 23 + 23$
e $91 + 5, 89 + 7, 83 + 13, 79 + 17, 73 + 23, 67 + 29, 59 + 37, 53 + 43, 47 + 47$
f $91 + 7, 79 + 19, 67 + 31, 61 + 37$

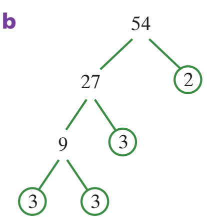
14 The following are a sample of correct answers, but there are more as indicated.
a $2 + 2 + 3$
b $2 + 2 + 7, 3 + 3 + 5$
c $23 + 2 + 2, 19 + 7 + 3, 13 + 7 + 7, \ldots$
d $23 + 5 + 3, 19 + 7 + 5, \ldots$
e $173 + 61 + 23, 131 + 67 + 59, \ldots$
f $173 + 173 + 67, 307 + 89 + 17, \ldots$

exercise 2.7

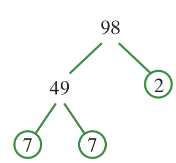
1 **a**



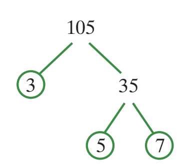
b



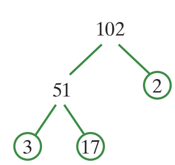
c



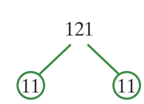
d

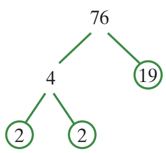


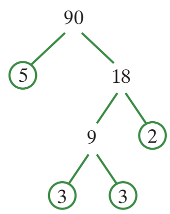
e

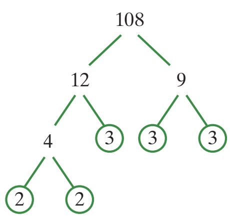


f



g


h


i


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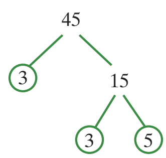
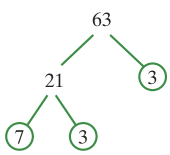
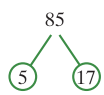
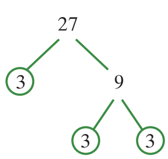
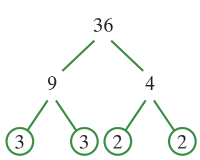


k


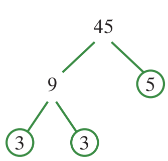
l


2 a i


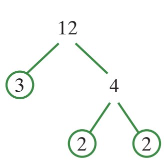
ii 3
iii 3^3

b i


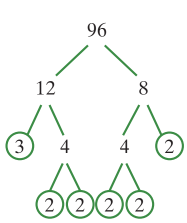
ii 2, 3
iii $2^2 \times 3^2$

c i


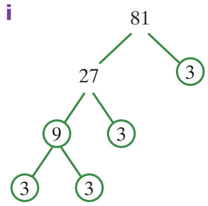
ii 3, 5
iii $3^2 \times 5$

d i


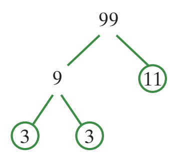
ii 2, 3
iii $2^2 \times 3$

e i


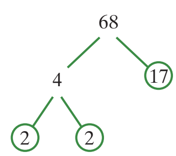
ii 2, 3
iii $2^5 \times 3$

f i


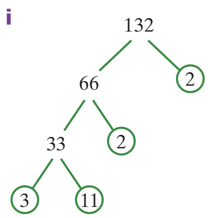
ii 3
iii 3^4

g i


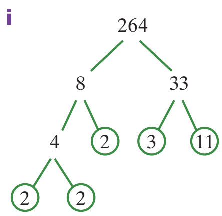
ii 3, 11
iii $3^2 \times 11$

h i


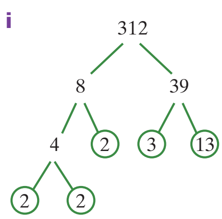
ii 2, 17
iii $2^2 \times 17$

i i


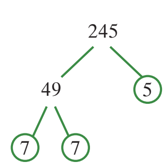
ii 2, 3, 11
iii $2^2 \times 3 \times 11$

j i


ii 2, 3, 11
iii $2^3 \times 3 \times 11$

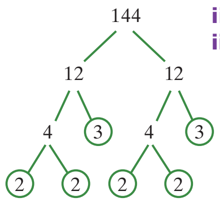
k i


ii 2, 3, 13
iii $2^3 \times 3 \times 13$

l i


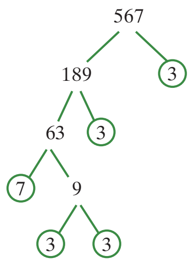
ii 5, 7
iii 5×7^2

m i



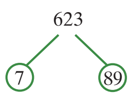
ii 2, 3
iii $2^4 \times 3^2$

n i



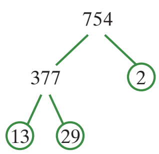
ii 3, 7
iii $3^4 \times 7$

o i



ii 7, 89
iii 7×89

p i



ii 2, 13, 29
iii $2 \times 13 \times 29$

3 a

2	18
3	9
3	3
	1

$18 = 2 \times 3^2$

b

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2^4 \times 3$

c

2	84
2	42
3	21
7	7
	1

$84 = 2^2 \times 3 \times 7$

d

2	42
3	21
7	7
	1

$42 = 2 \times 3 \times 7$

e

2	1430
5	715
11	143
13	13
	1

$1430 = 2 \times 5 \times 11 \times 13$

f

2	7350
3	3675
5	1225
5	245
7	49
7	7
	1

$7350 = 2 \times 3 \times 5^2 \times 7^2$

g

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$1800 = 2^3 \times 3^2 \times 5^2$

h

3	2475
3	825
5	275
5	55
11	11
	1

$2475 = 3^2 \times 5^2 \times 11$

i

5	1225
5	245
7	49
7	7
	1

$1225 = 5^2 \times 7^2$

j

2	3024
2	1512
2	756
2	378
3	189
3	63
3	21
7	7
	1

$3024 = 2^4 \times 3^3 \times 7$

k

2	79 380
2	39 690
3	19 845
3	6615
3	2205
3	735
5	245
7	49
7	7
	1

l

2	89 100
2	44 550
3	22 275
3	7425
3	2475
3	825
5	275
5	55
11	11
	1

$79\,380 = 2^2 \times 3^4 \times 5 \times 7^2$

$89\,100 = 2^2 \times 3^4 \times 5^2 \times 11$

4 a i

2	140
2	70
5	35
7	7
	1

ii 2, 5, 7
iii $2^2 \times 5 \times 7$

b i

2	3060
2	1530
3	765
3	255
5	85
17	17
	1

ii 2, 3, 5, 17
iii $2^2 \times 3^2 \times 5 \times 17$

c **i**

2	10 584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	7
	1

ii 2, 3, 7
iii $2^3 \times 3^3 \times 7^2$

d **i**

2	1872
2	936
2	468
2	234
3	117
3	39
13	13
	1

ii 2, 3, 13
iii $2^4 \times 3^2 \times 13$

e **i**

2	7200
2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

ii 2, 3, 5
iii $2^5 \times 3^2 \times 5^2$

f **i**

2	39 600
2	19 800
2	9900
2	4950
3	2475
3	825
5	275
5	55
11	11
	1

ii 2, 3, 5, 11
iii $2^4 \times 3^2 \times 5^2 \times 11$

g **i**

3	72 765
3	24 255
3	8085
5	2695
7	539
7	77
11	11
	1

ii 3, 5, 7, 11
iii $3^3 \times 5 \times 7^2 \times 11$

h **i**

2	4480
2	2240
2	1120
2	560
2	280
2	140
2	70
5	35
7	7
	1

ii 2, 5, 7
iii $2^7 \times 5 \times 7$

5 E

6 D

7 **a** **i** $16 = 2^4; 24 = 2^3 \times 3$
ii $\text{HCF} = 2^3 = 8$
b **i** $60 = 2^2 \times 3 \times 5; 126 = 2 \times 3^2 \times 7$
ii $\text{HCF} = 2 \times 3 = 6$
c **i** $70 = 2 \times 5 \times 7; 98 = 2 \times 7^2$
ii $\text{HCF} = 2 \times 7 = 14$
d **i** $28 = 2^2 \times 7; 182 = 2 \times 3^4$
ii $\text{HCF} = 2$
e **i** $72 = 2^3 \times 3^2; 108 = 2^2 \times 3^3$
ii $\text{HCF} = 2^2 \times 3^2 = 36$
f **i** $44 = 2^2 \times 11; 242 = 2 \times 11^2$
ii $\text{HCF} = 2 \times 11 = 22$
g **i** $16 = 2^4; 96 = 2^5 \times 3$
ii $\text{HCF} = 2^4 = 16$
h **i** $27 = 3^3; 162 = 2 \times 3^4$
ii $\text{HCF} = 3^3 = 27$
i **i** $45 = 3^2 \times 5; 204 = 2^2 \times 3 \times 17$
ii $\text{HCF} = 3$
j **i** $98 = 2 \times 7^2; 126 = 2 \times 3^2 \times 7$
ii $\text{HCF} = 2 \times 7 = 14$
k **i** $144 = 2^4 \times 3^2; 168 = 2^3 \times 3 \times 7$
ii $\text{HCF} = 2^3 \times 3 = 24$
l **i** $112 = 2^4 \times 7; 196 = 2^2 \times 7^2$
ii $\text{HCF} = 2^2 \times 7 = 28$

8 **a** **i** $16 = 2^4; 24 = 2^3 \times 3$
ii $\text{LCM} = 2^4 \times 3 = 48$
b **i** $18 = 2 \times 3^2; 32 = 2^5$
ii $\text{LCM} = 2^5 \times 3^2 = 288$
c **i** $15 = 3 \times 5; 54 = 2 \times 3^3$
ii $\text{LCM} = 2 \times 3^3 \times 5 = 2700$
d **i** $64 = 2^6; 108 = 2^2 \times 3^3$
ii $\text{LCM} = 2^6 \times 3^3 = 1728$
e **i** $72 = 2^3 \times 3^2; 108 = 2^2 \times 3^3$
ii $\text{LCM} = 2^3 \times 3^3 = 216$
f **i** $44 = 2^2 \times 11; 64 = 2^6$
ii $\text{LCM} = 2^6 \times 11 = 704$
g **i** $16 = 2^4; 88 = 2^3 \times 11$
ii $\text{LCM} = 2^4 \times 11 = 176$
h **i** $27 = 3^3; 486 = 2 \times 3^5$
ii $\text{LCM} = 2 \times 3^5 = 486$
i **i** $45 = 3^2 \times 5; 100 = 2^2 \times 5^2$
ii $\text{LCM} = 2^2 \times 3^2 \times 5^2 = 900$

j **i** $36 = 2^2 \times 3^2$; $98 = 2 \times 7^2$
ii $\text{LCM} = 2^2 \times 3^2 \times 7^2 = 1764$
k **i** $24 = 2^3 \times 3$; $144 = 2^4 \times 3^2$
ii $\text{LCM} = 2^4 \times 3^2 = 144$
l **i** $32 = 2^5$; $120 = 2^3 \times 3 \times 5$
ii $\text{LCM} = 2^5 \times 3 \times 5 = 480$

9 **a** **i** 3^4
ii 5 factors
iii 1, 3, 9, 27, 81
b **i** $2^2 \times 5^2$
ii 9 factors
iii 1, 2, 4, 5, 10, 20, 25, 50, 100
c **i** 2^5
ii 6 factors
iii 1, 2, 4, 8, 16, 32
d **i** $3^3 \times 5 \times 19$
ii 16 factors
iii 1, 3, 5, 9, 15, 19, 27, 45, 57, 95, 135, 171, 285, 513, 855, 2565

10 **a** True **b** False **c** True
d True **e** False **f** True

11 3675

12 72, 96, 108, 486

Revision

1 E **2** D **3** B **4** C **5** C

6 **a** 12, 24, 36 **b** 14, 28, 42 **c** 35, 70, 105

7 **a** 1, 3, 7, 21
b 1, 2, 4, 5, 10, 20, 25, 50, 100
c 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132

8 **a** 85 **b** 315 **c** 84

9 **a** 48 **b** 3 **c** 9

10 **a** 4 **b** 4 and 8 **c** no **d** 4

11 **a** 3 **b** 3 **c** no **d** 3 and 9

12 **a** 16 **b** 32 **c** 72 **d** 7

13 **a** 1024 **b** 625
c 110 270 727 **d** 5184

14 **a** 2^9 **b** 5^6 **c** 3^8 **d** 4^{10}

15 144 factors

16 **a**

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

b $576 = 2^6 \times 3^2$
c 21 factors

17 **a**

2	27720
2	13860
2	6930
3	3465
3	1155
5	385
7	77
11	11
	1

b $27720 = 2^3 \times 3^2 \times 5 \times 7 \times 11$
c 96 factors

18 **a** **i**

2	32
2	16
2	8
2	4
2	2
	1

2	108
2	54
3	27
3	9
3	3
	1

ii $32 = 2^5$; $108 = 2^2 \times 3^3$
iii $\text{HCF} = 2^2 = 4$

b **i**

2	70
5	35
7	7
	1

2	392
2	196
2	98
7	49
7	7
	1

ii $70 = 2 \times 5 \times 7$; $392 = 2^3 \times 7^2$
iii $\text{HCF} = 2 \times 7 = 14$

19 **a** **i**

2	28
2	14
7	7
	1

2	36
2	18
3	9
3	3
	1

ii $28 = 2^2 \times 7$; $36 = 2^2 \times 3^2$
iii $\text{LCM} = 2^2 \times 3^2 \times 7 = 252$

b **i**

2	16
2	8
2	4
2	2
	1

2	18
3	9
3	3
	1

ii $16 = 2^4$; $18 = 2 \times 3^2$
iii $\text{LCM} = 2^4 \times 3^2 = 144$

20 11:30 am, 2 pm

21 **a** 13
b 7 packets of pasta, 12 tins of soup, 13 packets of noodles and 6 tins of baked beans

22 **a** 4, 9, 25, 49 **b** 10 020 **c** 9990
d 420 **e** 49

Chapter 3

exercise 3.1

1 **a** **i** $\frac{3}{4}$ **ii** $\frac{1}{4}$ **b** **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$ **c** **i** $\frac{5}{7}$ **ii** $\frac{3}{7}$

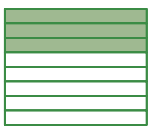
2 **a** **i** $\frac{3}{5}$ **ii** $\frac{2}{5}$ **b** **i** $\frac{5}{6}$ **ii** $\frac{1}{6}$ **c** **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$

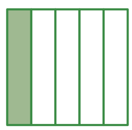
d **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$ **e** **i** $\frac{3}{4}$ **ii** $\frac{1}{4}$ **f** **i** $\frac{5}{9}$ **ii** $\frac{4}{9}$

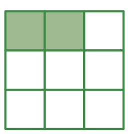
g **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **h** **i** $\frac{3}{5}$ **ii** $\frac{2}{5}$ **i** **i** $\frac{1}{6}$ **ii** $\frac{5}{6}$

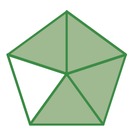
j **i** $\frac{3}{8}$ **ii** $\frac{5}{8}$ **k** **i** $\frac{5}{7}$ **ii** $\frac{2}{7}$ **l** **i** $\frac{4}{9}$ **ii** $\frac{5}{9}$

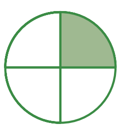
3 The following diagrams show how many sections should be shaded. There are many possible arrangements of those sections.

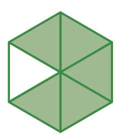
a


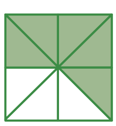
b


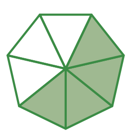
c


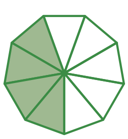
d


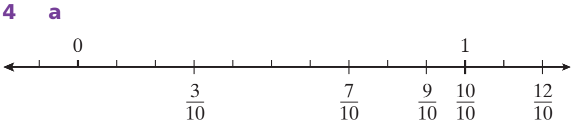
e


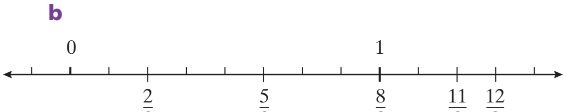
f


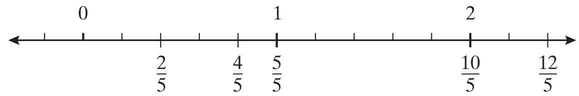
g


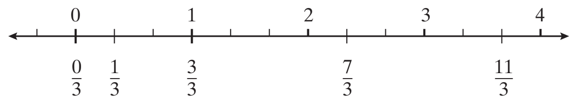
h


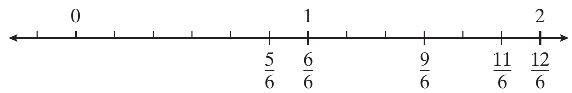
i


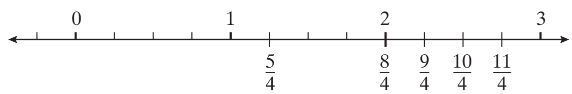
4 **a**


b


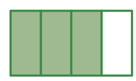
c


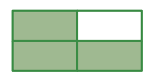
d


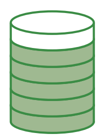
e


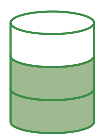
f


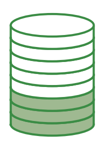
5 **a** $\frac{3}{7}$ **b** $\frac{2}{7}$ **c** $\frac{4}{7}$

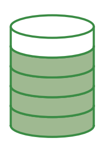
6 **a**


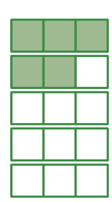
b


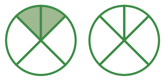
7


8


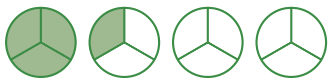
9


10


11 **a** **i** $\frac{5}{3}$ **ii**


b **i** $\frac{1}{5}$ **ii** 

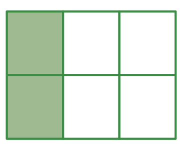
c **i** $\frac{3}{8}$ **ii** 

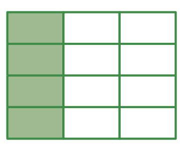
d **i** $\frac{4}{3}$ **ii** 

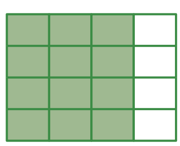
e **i** $\frac{5}{3}$
ii 

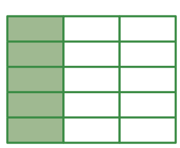
12 12

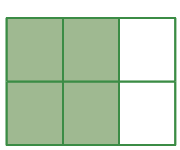
exercise 3.2

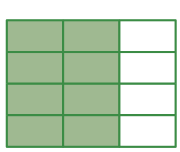
1 **a** **i** $\frac{1}{3}$ **ii** 
iii $\frac{2}{6}$

b **i** $\frac{1}{3}$ **ii** 
iii $\frac{4}{12}$

c **i** $\frac{3}{4}$ **ii** 
iii $\frac{12}{16}$

d **i** $\frac{1}{3}$ **ii** 
iii $\frac{5}{15}$

e **i** $\frac{2}{3}$ **ii** 
iii $\frac{4}{6}$

f **i** $\frac{2}{3}$ **ii** 
iii $\frac{8}{12}$

2 **a** $\frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{22}{55}$ **b** $\frac{4}{7}=\frac{8}{14}=\frac{20}{35}=\frac{28}{49}$
c $\frac{1}{2}=\frac{2}{4}=\frac{5}{10}=\frac{10}{20}$ **d** $\frac{2}{3}=\frac{4}{6}=\frac{8}{12}=\frac{10}{15}$
e $\frac{3}{4}=\frac{9}{12}=\frac{12}{16}=\frac{18}{24}$ **f** $\frac{1}{8}=\frac{3}{24}=\frac{2}{16}=\frac{5}{40}$
g $\frac{4}{5}=\frac{12}{15}=\frac{16}{20}=\frac{24}{30}$ **h** $\frac{5}{6}=\frac{10}{12}=\frac{25}{30}=\frac{55}{66}$
i $\frac{2}{7}=\frac{4}{14}=\frac{8}{28}=\frac{16}{56}$ **j** $\frac{3}{5}=\frac{12}{20}=\frac{18}{30}=\frac{36}{60}$
k $\frac{9}{10}=\frac{18}{20}=\frac{45}{50}=\frac{72}{80}$ **l** $\frac{5}{11}=\frac{15}{33}=\frac{20}{44}=\frac{40}{88}$

3 **a** $\frac{8}{12}$ **b** $\frac{3}{12}$ **c** $\frac{10}{12}$ **d** $\frac{6}{12}$
4 **a** $\frac{10}{20}$ **b** $\frac{6}{20}$ **c** $\frac{8}{20}$ **d** $\frac{15}{20}$
5 **a** $\frac{6}{18}$ **b** $\frac{9}{18}$ **c** $\frac{15}{18}$ **d** $\frac{8}{18}$
6 **a** $\frac{12}{24}$ **b** $\frac{16}{24}$ **c** $\frac{18}{24}$ **d** $\frac{15}{24}$
7 **a** $\frac{7}{28}$ **b** $\frac{14}{28}$ **c** $\frac{20}{28}$ **d** $\frac{6}{28}$
8 **a** $\frac{20}{30}$ **b** $\frac{18}{30}$ **c** $\frac{25}{30}$ **d** $\frac{21}{30}$

9 **a** **i** $\frac{5}{10}$ **ii** $\frac{1}{2}$
b **i** $\frac{6}{9}$ **ii** $\frac{2}{3}$
c **i** $\frac{6}{8}$ **ii** $\frac{3}{4}$
d **i** $\frac{9}{15}$ **ii** $\frac{3}{5}$
e **i** $\frac{12}{20}$ **ii** $\frac{3}{5}$
f **i** $\frac{8}{12}$ **ii** $\frac{2}{3}$
g **i** $\frac{12}{30}$ **ii** $\frac{2}{5}$
h **i** $\frac{25}{30}$ **ii** $\frac{5}{6}$
i **i** $\frac{5}{20}$ **ii** $\frac{1}{4}$

10 **a** $\frac{3}{4}$ **b** $\frac{1}{3}$ **c** $\frac{6}{7}$ **d** $\frac{2}{3}$
e $\frac{1}{4}$ **f** $\frac{4}{5}$ **g** $\frac{1}{4}$ **h** $\frac{7}{9}$
i $\frac{3}{4}$ **j** $\frac{2}{5}$ **k** $\frac{4}{7}$ **l** $\frac{5}{8}$
m $\frac{9}{10}$ **n** $\frac{2}{3}$ **o** $\frac{2}{5}$ **p** $\frac{3}{4}$

11 **a** iii **b** iv **c** ii **d** i
12 **a** iv **b** ii **c** i **d** iii
13 **a** iv **b** ii **c** i **d** iii

14 **a** iii **b** ii **c** iv **d** i

15 **a** ii **b** i **c** iv **d** iii

16 **a** $\frac{1}{4}$ **b** $\frac{5}{6}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$

17 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{11}{15}$ **d** $\frac{5}{6}$

18 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{3}{10}$ **d** $\frac{11}{25}$

19 **a** $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{7}{10}$ **d** $\frac{18}{25}$

20 **a** 4 **b** 4 **c** 4 **d** 16

e **i** ♩♪♪♩ **ii** 𝅗𝅥♩

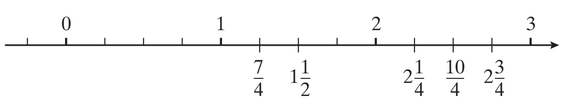
iii ♩♪♪ **iv** ♪♪♪♪♪♪

exercise 3.3

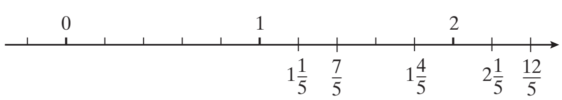
1

Proper fractions	Improper fractions	Mixed numbers
$\frac{5}{8}$ $\frac{1}{7}$ $\frac{3}{11}$	$\frac{9}{4}$ $\frac{12}{5}$ $\frac{4}{3}$	$4\frac{2}{5}$ $3\frac{3}{4}$ $1\frac{1}{2}$
$\frac{4}{5}$ $\frac{3}{10}$ $\frac{5}{12}$	$\frac{15}{8}$ $\frac{10}{9}$ $\frac{8}{3}$	$2\frac{4}{7}$ $1\frac{5}{6}$ $4\frac{1}{3}$

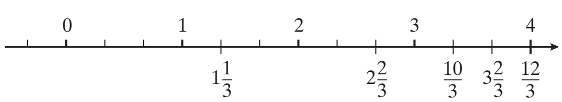
2 **a**



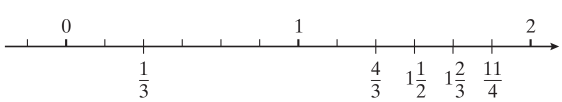
b



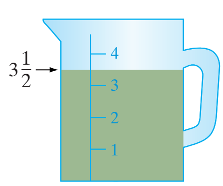
c



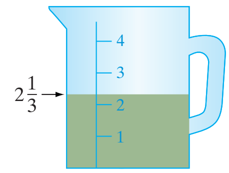
d



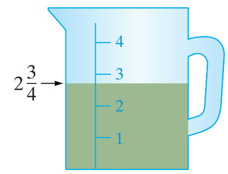
3 **a** $\frac{7}{2}$



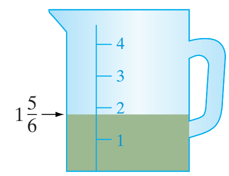
b $\frac{7}{3}$



c $\frac{11}{4}$



d $\frac{11}{6}$

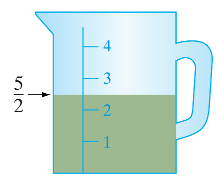


4 **a** $\frac{5}{3}$ **b** $\frac{9}{4}$ **c** $\frac{23}{6}$ **d** $\frac{3}{2}$

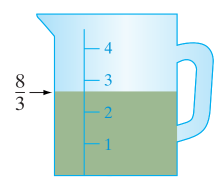
e $\frac{17}{7}$ **f** $\frac{10}{3}$ **g** $\frac{25}{6}$ **h** $\frac{23}{4}$

i $\frac{19}{8}$ **j** $\frac{13}{2}$ **k** $\frac{14}{5}$ **l** $\frac{29}{9}$

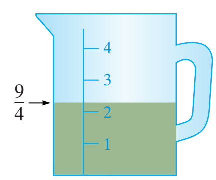
5 **a** $2\frac{1}{2}$



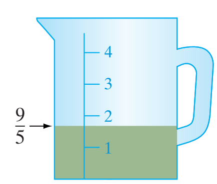
b $2\frac{2}{3}$



c $2\frac{1}{4}$



d $1\frac{4}{5}$



6 **a** $2\frac{2}{3}$ **b** $1\frac{4}{7}$ **c** $2\frac{1}{4}$ **d** $1\frac{7}{8}$

e $2\frac{2}{5}$ **f** $3\frac{1}{2}$ **g** $2\frac{5}{6}$ **h** $4\frac{1}{4}$

i $1\frac{7}{9}$ **j** $1\frac{1}{12}$ **k** $2\frac{2}{7}$ **l** $3\frac{3}{5}$

7 **a**

Fraction	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$
Simplest form of fraction	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1

b

Fraction	$1\frac{4}{10}$	$1\frac{2}{10}$	1	$\frac{8}{10}$	$\frac{6}{10}$	$\frac{4}{10}$
Simplest form of fraction	$1\frac{2}{5}$	$1\frac{1}{5}$	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$

c

Fraction	$1\frac{1}{8}$	$1\frac{6}{8}$	$2\frac{3}{8}$	3	$3\frac{5}{8}$	$4\frac{2}{8}$
Simplest form of fraction	$1\frac{1}{8}$	$1\frac{3}{4}$	$2\frac{3}{8}$	3	$3\frac{5}{8}$	$4\frac{1}{4}$

d

Fraction	$2\frac{1}{4}$	3	$3\frac{3}{4}$	$4\frac{2}{4}$	$5\frac{1}{4}$	6
Simplest form of fraction	$2\frac{1}{4}$	3	$3\frac{3}{4}$	$4\frac{1}{2}$	$5\frac{1}{4}$	6

e

Fraction	$1\frac{5}{11}$	$1\frac{8}{11}$	2	$2\frac{3}{11}$	$2\frac{6}{11}$	$2\frac{9}{11}$
Improper fraction	$\frac{16}{11}$	$\frac{19}{11}$	$\frac{22}{11}=\frac{2}{1}$	$\frac{25}{11}$	$\frac{28}{11}$	$\frac{31}{11}$

f

Fraction	$1\frac{3}{6}$	$1\frac{4}{6}$	$1\frac{5}{6}$	2	$2\frac{1}{6}$	$2\frac{2}{6}$
Improper fraction	$\frac{9}{6}=\frac{3}{2}$	$\frac{10}{6}=\frac{5}{3}$	$\frac{11}{6}$	$\frac{12}{6}=\frac{2}{1}$	$\frac{13}{6}$	$\frac{14}{6}=\frac{7}{3}$

g

Fraction	$3\frac{1}{7}$	$3\frac{5}{7}$	$4\frac{2}{7}$	$4\frac{6}{7}$	$5\frac{3}{7}$	6
Improper fraction in sevenths	$\frac{22}{7}$	$\frac{26}{7}$	$\frac{30}{7}$	$\frac{34}{7}$	$\frac{38}{7}$	$\frac{42}{7}$

h

Fraction	$2\frac{1}{5}$	$2\frac{3}{5}$	3	$3\frac{2}{5}$	$3\frac{4}{5}$	$4\frac{1}{5}$
Improper fraction in fifths	$\frac{11}{5}$	$\frac{13}{5}$	3	$\frac{17}{5}$	$\frac{19}{5}$	$\frac{21}{5}$

8 **a** $1\frac{3}{4}$ **b** $1\frac{5}{6}$ **c** $1\frac{1}{4}$ hrs

9 **a** $\frac{4}{3}$ **b** $1\frac{1}{3}$

10 **a** $\frac{1}{10}$ **b** $\frac{17}{10}$ **c** $1\frac{7}{10}$

11 **a** $\frac{39}{4}$

b You would expect a platform number to be a whole number.

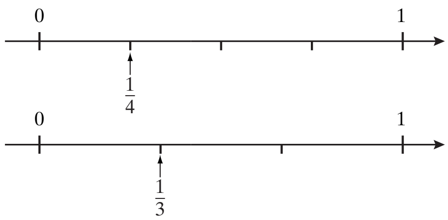
c 10

12 **a** **i** $2\frac{2}{3}$ **ii** $\frac{2}{3}$

b **i** $2\frac{1}{2}$ **ii** $\frac{3}{4}$

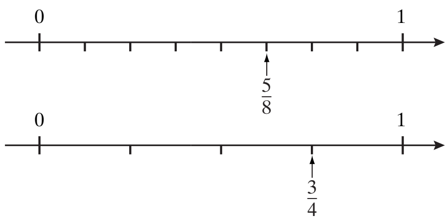
exercise 3.4

1 **a**



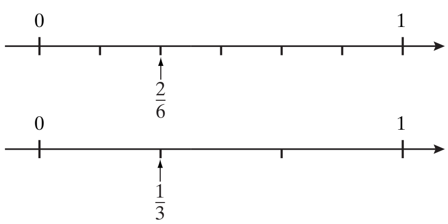
So, $\frac{1}{3} > \frac{1}{4}$

b



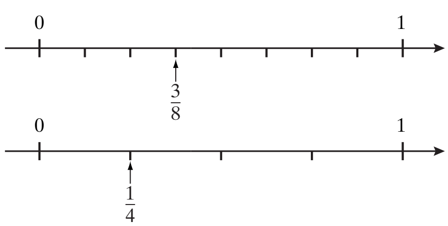
So, $\frac{3}{4} > \frac{5}{8}$

c



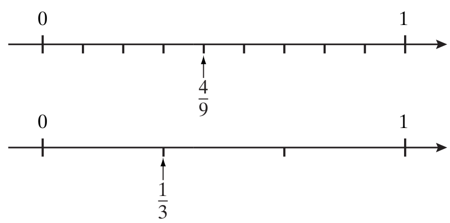
So, $\frac{1}{3} = \frac{2}{6}$

d



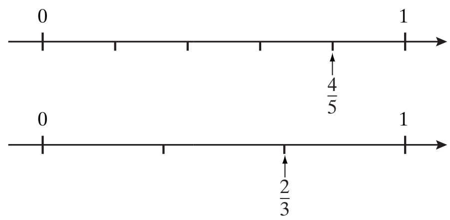
So, $\frac{1}{4} < \frac{3}{8}$

e



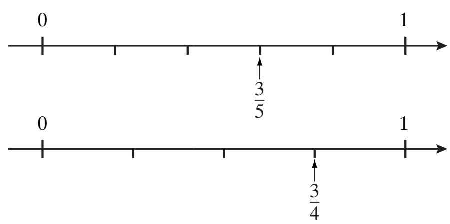
So, $\frac{1}{3} < \frac{4}{9}$

f



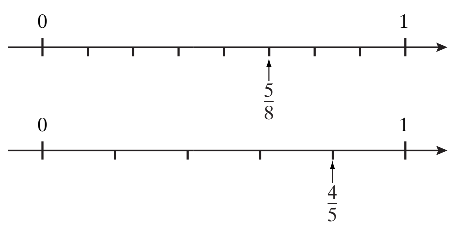
So, $\frac{2}{3} < \frac{4}{5}$

g



So, $\frac{3}{4} > \frac{3}{5}$

h



So, $\frac{4}{5} > \frac{5}{8}$

2 **a** 20 **b** 12 **c** 35 **d** 24
e 12 **f** 9 **g** 30 **h** 21

3 **a** 30 **b** 42 **c** 36 **d** 24

4 **a** $\frac{4}{6} > \frac{3}{6}$ **b** $\frac{2}{8} < \frac{3}{8}$ **c** $\frac{6}{9} = \frac{6}{9}$ **d** $\frac{7}{15} < \frac{9}{15}$
e $\frac{2}{9} < \frac{3}{9}$ **f** $\frac{4}{10} > \frac{3}{10}$ **g** $\frac{24}{40} < \frac{25}{40}$ **h** $\frac{20}{30} < \frac{21}{30}$
i $\frac{14}{21} < \frac{15}{21}$ **j** $\frac{15}{18} > \frac{14}{18}$ **k** $\frac{8}{30} < \frac{9}{30}$ **l** $\frac{21}{24} < \frac{22}{24}$

5 **a** $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}$ **b** $\frac{3}{10}, \frac{2}{5}, \frac{13}{30}, \frac{7}{15}$ **c** $\frac{1}{2}, \frac{11}{20}, \frac{3}{5}, \frac{7}{10}$
d $\frac{4}{9}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ **e** $\frac{2}{3}, \frac{7}{9}, \frac{5}{6}, \frac{11}{12}$ **f** $\frac{7}{16}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}$
g $\frac{1}{4}, \frac{3}{11}, \frac{7}{22}, \frac{1}{2}$ **h** $\frac{1}{4}, \frac{9}{28}, \frac{5}{14}, \frac{3}{7}$ **i** $\frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{5}{12}$

6 **a** $\frac{5}{2}, 2\frac{1}{4}, 1\frac{5}{6}, \frac{5}{3}$ **b** $\frac{3}{2}, 1\frac{3}{8}, 1\frac{1}{4}, \frac{9}{8}$
c $3\frac{4}{5}, \frac{7}{2}, 2\frac{7}{10}, \frac{11}{5}$ **d** $\frac{10}{3}, 2\frac{3}{4}, \frac{5}{2}, 1\frac{7}{12}$
e $\frac{8}{3}, 2\frac{1}{9}, \frac{11}{6}, 1\frac{13}{18}$ **f** $2\frac{3}{5}, \frac{7}{3}, \frac{31}{15}, 1\frac{14}{15}$
g $2\frac{7}{10}, \frac{37}{20}, 1\frac{3}{4}, 1\frac{5}{8}$ **h** $\frac{7}{2}, 2\frac{7}{16}, \frac{15}{8}, 1\frac{1}{4}$
i $\frac{7}{4}, \frac{13}{6}, 2\frac{1}{12}, \frac{7}{4}$

7 $\frac{3}{8}$ because $\frac{9}{24} > \frac{8}{24}$

8 **a** $\frac{3}{4}, \frac{4}{5}$ **b** 20 **c** $\frac{15}{20}, \frac{16}{20}$ **d** test 2

9 **a** 15 **b** $\frac{9}{15}, \frac{10}{15}$ **c** the one with $\frac{3}{5}$

10 the new mix

11 three examples are $\frac{13}{20}, \frac{7}{10}, \frac{3}{4}$ but there are others

12 three examples are $\frac{5}{12}, \frac{1}{2}, \frac{7}{12}$ but there are others

13 **a** the fraction gets smaller $\frac{5}{10}, \frac{4}{9}, \frac{3}{8}, \ldots$
b the fraction gets closer to 1 $\frac{7}{12}, \frac{8}{13}, \frac{9}{14}, \ldots$

exercise 3.5

1 **a** 3 fifths **b** 3 sevenths **c** 7 ninths
d 8 tenths **e** 5 eighths **f** 2 fifths

2 **a** $\frac{5}{7}$ **b** $\frac{3}{5}$ **c** $\frac{6}{11}$ **d** $\frac{7}{9}$
e $\frac{1}{2}$ **f** $\frac{5}{9}$ **g** $\frac{7}{9}$ **h** $\frac{9}{10}$
i $\frac{7}{12}$ **j** $\frac{11}{15}$ **k** $\frac{11}{12}$ **l** $\frac{5}{6}$
m $\frac{13}{15}$ **n** $\frac{13}{18}$ **o** $\frac{13}{18}$ **p** $\frac{19}{24}$

3 **a** $\frac{3}{5}$ **b** $\frac{1}{9}$ **c** $\frac{2}{7}$ **d** $\frac{5}{11}$
e $\frac{1}{3}$ **f** $\frac{1}{8}$ **g** $\frac{3}{10}$ **h** $\frac{2}{9}$
i $\frac{2}{9}$ **j** $\frac{1}{12}$ **k** $\frac{2}{15}$ **l** $\frac{4}{21}$
m $\frac{11}{18}$ **n** $\frac{3}{20}$ **o** $\frac{11}{24}$ **p** $\frac{13}{28}$

4 **a** $1\frac{2}{5}$ **b** $\frac{1}{3}$ **c** $1\frac{3}{7}$ **d** $\frac{1}{5}$
e $\frac{1}{2}$ **f** $1\frac{5}{8}$ **g** $\frac{1}{2}$ **h** $1\frac{5}{9}$
i $1\frac{7}{12}$ **j** $\frac{1}{4}$ **k** $1\frac{4}{15}$ **l** $\frac{1}{3}$
m $\frac{1}{5}$ **n** $1\frac{1}{24}$ **o** $\frac{1}{10}$ **p** $1\frac{7}{30}$

5 **a** $\frac{5}{12}$ **b** $\frac{7}{12}$

6 **a** $\frac{17}{24}$ **b** $\frac{7}{24}$

7 **a** $1\frac{1}{4}$ hours **b** 75 minutes

8 **a** $\frac{17}{20}$ **b** $\frac{17}{20}$ of a tank

9 **a** $\frac{1}{2}$ hr = 30 mins
b $\frac{7}{12}$ hr = 35 mins
c $\frac{1}{12}$ hr = 5 mins

10 24 litres

exercise 3.6

1 **a** $4\frac{1}{3}$ **b** $3\frac{5}{7}$ **c** $2\frac{10}{11}$ **d** $3\frac{5}{9}$
e $4\frac{1}{2}$ **f** $4\frac{2}{7}$ **g** $4\frac{1}{6}$ **h** $2\frac{5}{6}$
i $4\frac{1}{4}$ **j** $3\frac{7}{9}$ **k** $4\frac{3}{10}$ **l** $2\frac{17}{18}$

2 **a** $2\frac{1}{5}$ **b** $\frac{2}{3}$ **c** $1\frac{2}{3}$ **d** $3\frac{2}{5}$
e $2\frac{1}{2}$ **f** $2\frac{3}{5}$ **g** $2\frac{1}{12}$ **h** $\frac{9}{20}$
i $1\frac{8}{15}$ **j** $1\frac{3}{20}$ **k** $1\frac{5}{8}$ **l** $3\frac{1}{6}$

3 **a** $1\frac{2}{5}$ **b** $2\frac{3}{7}$ **c** $\frac{1}{3}$ **d** 3
e $4\frac{1}{8}$ **f** $1\frac{4}{9}$ **g** $3\frac{1}{6}$ **h** $1\frac{1}{2}$
i $\frac{8}{15}$ **j** $\frac{11}{12}$ **k** $2\frac{4}{15}$ **l** $2\frac{1}{6}$

4 $3\frac{3}{4}$ hours

5 $\frac{13}{20}$

6 $1\frac{3}{4}$ hours

7 $4\frac{1}{4}$ hours

8 $8\frac{5}{8}$ pizzas

9 $3\frac{1}{12}$ years

10 $\frac{7}{12}$ km

11 $2\frac{5}{6}$ kg

12 **a** $5\frac{11}{12}$ bags
b $2\frac{1}{12}$ bags

13 **a** $1\frac{11}{24}$ kg
b $7\frac{19}{24}$ kg

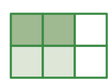
14 **a** **i** 1 **ii** $1\frac{1}{2}$ **iii** 2 **iv** $2\frac{1}{2}$
b Each sum in parts a i to iv is larger by a $\frac{1}{2}$, so we would expect this to add to 3.
c The sum is 3, so the prediction is correct.

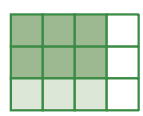
15 $\frac{4}{7}$

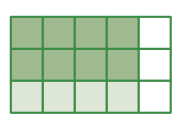
exercise 3.7

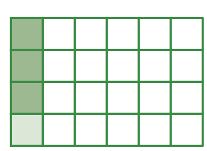
1 **a** $\frac{6}{7}$ **b** $2\frac{2}{3}$ **c** $1\frac{1}{4}$ **d** $1\frac{1}{5}$
e $3\frac{1}{3}$ **f** $1\frac{1}{2}$ **g** 10 **h** 9
i $1\frac{3}{5}$ **j** $1\frac{1}{6}$ **k** $1\frac{1}{9}$ **l** $2\frac{2}{5}$
m 4 **n** $1\frac{1}{2}$ **o** $2\frac{1}{2}$ **p** 7

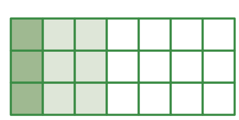
2 **a** $5\frac{1}{3}$ **b** $1\frac{1}{5}$ **c** $3\frac{3}{4}$ **d** 6
e $2\frac{2}{5}$ **f** $1\frac{1}{2}$ **g** $2\frac{2}{3}$ **h** 4
i $1\frac{3}{4}$ **j** $2\frac{2}{3}$ **k** $1\frac{1}{5}$ **l** $6\frac{2}{3}$
m 4 **n** $4\frac{1}{2}$ **o** $1\frac{2}{3}$ **p** 8

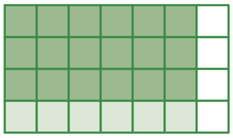
3 **a**

$\frac{2}{6} = \frac{1}{3}$

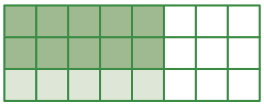
b

$\frac{6}{12} = \frac{1}{2}$

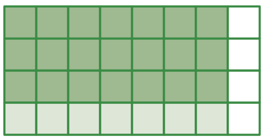
c

$\frac{8}{15}$

d

$\frac{3}{24} = \frac{1}{8}$

e

$\frac{3}{21} = \frac{1}{7}$

f  $\frac{18}{28} = \frac{9}{14}$

g  $\frac{10}{24} = \frac{5}{12}$

h  $\frac{21}{32}$

4 **a** $\frac{3}{10}$ **b** $\frac{3}{10}$ **c** $\frac{4}{21}$ **d** $\frac{3}{40}$
e $\frac{9}{14}$ **f** $\frac{1}{3}$ **g** $\frac{7}{10}$ **h** $\frac{1}{6}$
i $\frac{1}{6}$ **j** $\frac{2}{7}$ **k** $\frac{1}{2}$ **l** $\frac{1}{4}$

5 **a** Incorrect, can't cancel from both numerators.
$\frac{4}{9} \times \frac{2}{5} = \frac{8}{45}$
b Incorrect, the denominators were not multiplied.
$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{1}{3}$

6 **a** $1\frac{3}{4}$ **b** 4 **c** $1\frac{5}{7}$ **d** $4\frac{2}{5}$
e 7 **f** $3\frac{1}{5}$ **g** $3\frac{1}{2}$ **h** 2
i $1\frac{1}{2}$ **j** 2 **k** $4\frac{2}{5}$ **l** 4
m 3 **n** $3\frac{3}{5}$ **o** 5 **p** $4\frac{1}{8}$

7 **a** $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{21}$ **d** $\frac{1}{4}$
e 1 **f** $1\frac{1}{4}$ **g** 9 **h** 7
i 7 **j** $1\frac{1}{2}$ **k** $3\frac{1}{2}$ **l** $4\frac{1}{2}$

8 8

9 **a** \$4000 **b** \$2000 **c** \$18 000

10 **a** 2 **b** 3
c **i** $6 \times \frac{1}{8}$ **ii** $\frac{3}{4}$ kg
d **i** $6 \times \frac{3}{4}$ **ii** $4\frac{1}{2}$ cups
e **i** $6 \times 1\frac{1}{4}$ **ii** $7\frac{1}{2}$ cups

11 **a** 6 km
b 12 km
c distance = speed × time taken
d 14 km

12 $\frac{1}{8}$

13 $\frac{1}{64}$

exercise 3.8

1 **a** $\frac{3}{2}$ **b** $\frac{5}{1}$ **c** $\frac{8}{3}$ **d** $\frac{7}{3}$
e $\frac{2}{5}$ **f** $\frac{3}{7}$ **g** $\frac{5}{6}$ **h** $\frac{9}{11}$

2 **a** 6 **b** 12 **c** 10 **d** 18
e $7\frac{1}{2}$ **f** $4\frac{2}{3}$ **g** $4\frac{4}{5}$ **h** $7\frac{1}{2}$

3 **a** $\frac{4}{5}$ **b** $\frac{5}{18}$ **c** $\frac{9}{16}$ **d** $4\frac{2}{3}$
e $2\frac{2}{5}$ **f** $1\frac{1}{6}$ **g** $2\frac{4}{9}$ **h** $1\frac{1}{3}$
i $2\frac{1}{12}$ **j** $2\frac{2}{3}$ **k** $5\frac{1}{3}$ **l** $5\frac{1}{4}$

4 **a** $2\frac{1}{4}$ **b** $2\frac{2}{9}$ **c** $1\frac{1}{10}$ **d** $2\frac{2}{7}$
e $\frac{3}{4}$ **f** $1\frac{1}{2}$ **g** $1\frac{1}{3}$ **h** $2\frac{2}{3}$
i $1\frac{2}{3}$ **j** $1\frac{1}{3}$ **k** $2\frac{2}{3}$ **l** $1\frac{1}{2}$

5 16

6 $4\frac{1}{2}$

7 $5\frac{3}{5}$

8 15

9 $51\frac{1}{5}$ km

exercise 3.9

1 **a** $\frac{1}{4}$ **b** $1\frac{1}{2}$ **c** 24 **d** $\frac{1}{24}$ **e** $\frac{1}{24}$
f $\frac{1}{3}$ **g** $\frac{1}{2}$ **h** $\frac{1}{2}$ **i** $\frac{3}{10}$

2 **a** $2\frac{5}{6}$ **b** $2\frac{1}{4}$ **c** 2 **d** $\frac{1}{2}$ **e** $\frac{11}{12}$
f $1\frac{11}{14}$ **g** $5\frac{3}{4}$ **h** 0 **i** $2\frac{1}{22}$

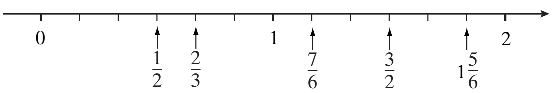
3 **a** $\frac{4}{5}$ **b** 0.8 **c** $3\frac{1}{5}$ **d** 3.2 **e** yes
f without the brackets the division is calculated first, but with the brackets the addition is calculated first

4 5

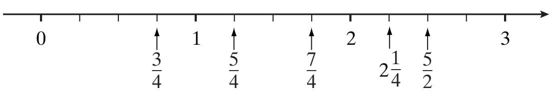
Revision

1 B **2** C **3** D **4** A **5** D

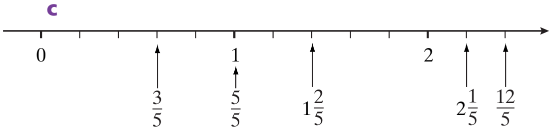
6 a



b



c



7 a > **b** <

8 a $\frac{7}{9}$ **b** $\frac{3}{5}$ **c** $1\frac{3}{10}$ **d** $4\frac{1}{8}$

9 a $\frac{4}{7}$ **b** $\frac{7}{12}$ **c** $1\frac{4}{11}$ **d** $\frac{3}{10}$ **e** $1\frac{5}{12}$

10 a $\frac{\overset{1}{\cancel{7}}}{\underset{6}{\cancel{18}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{14}}}$

b $\frac{\overset{1}{\cancel{\overset{7}{\cancel{14}}}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{1}{\cancel{11}}}{\underset{3}{\cancel{21}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{\underset{11}{\cancel{22}}}}}$

11 a 4 **b** 9 **c** $\frac{2}{21}$

d $\frac{1}{6}$ **e** $\frac{3}{14}$ **f** $4\frac{1}{5}$

12 a $\frac{6}{7}$ **b** $\frac{16}{21}$ **c** $\frac{4}{5}$ **d** $\frac{3}{4}$

13 $\frac{7}{40}$

14 $1\frac{1}{4}$ hours

15 a 9 **b** 8 **c** 19

d $\frac{19}{36}$ **e** $\frac{1}{4} + \frac{2}{9} + \frac{19}{36}$ does equal 1

16 a $1\frac{3}{4}$ hours **b** 17 cars **c** $4\frac{1}{4}$ hours

17 a $1\frac{7}{12}$ cans **b** 3

c $2\frac{1}{4}$ cans of yellow, $\frac{1}{2}$ cans of blue and 2 cans of white

Chapter 4

exercise 4.1

1 a 3 **b** 1 **c** 4 **d** 6
e 5 **f** 0 **g** 1 **h** 2

2 a i 4 tens **ii** 2
b i 4 tenths **ii** 1
c i 4 ones **ii** 1
d i 4 hundredths **ii** 2
e i 4 hundreds **ii** 1
f i 4 thousandths **ii** 3
g i 4 hundredths **ii** 3
h i 4 tenths **ii** 3
i i 4 ones **ii** 2
j i 4 hundredths **ii** 3
k i 4 ten thousandths **ii** 4
l i 4 thousandths **ii** 4

3 a i twelve point six
ii 10, 2, $\frac{6}{10}$
iii 1 ten, 2 ones, 6 tenths

b i one point seven four
ii 1, $\frac{7}{10}$, $\frac{4}{100}$
iii 1 one, 7 tenths, 4 hundredths

c i twenty four point eight
ii 20, 4, $\frac{8}{10}$
iii 2 tens, 4 ones, 8 tenths

d i zero point seven three six
ii $\frac{7}{10}$, $\frac{3}{100}$, $\frac{6}{1000}$
iii 7 tenths, 3 hundredths, 6 thousandths

e i three hundred and eight point two
ii 300, 8, $\frac{2}{10}$
iii 3 hundreds, 8 ones, 2 tenths

f i seven point zero nine three
ii 7, $\frac{9}{100}$, $\frac{3}{1000}$
iii 7 ones, 9 hundredths, 3 thousandths

g i zero point two nine four
ii $\frac{2}{10}$, $\frac{9}{100}$, $\frac{4}{1000}$
iii 2 tenths, 9 hundredths, 4 thousandths

h i thirty point four zero eight
ii 30, $\frac{4}{10}$, $\frac{8}{1000}$
iii 3 tens, 4 tenths, 8 thousandths

i **i** seven point one zero six

ii $7, \frac{1}{10}, \frac{6}{1000}$

iii 7 ones, 1 tenths, 6 thousandths

j **i** twenty point zero eight nine

ii $20, \frac{8}{100}, \frac{9}{1000}$

iii 2 tens, 8 hundredths, 9 thousandths

k **i** three hundred and one point zero six five

ii $300, 1, \frac{6}{100}, \frac{5}{1000}$

iii 3 hundreds, 1 one, 6 hundredths, 5 thousandths

l **i** seven thousand and ninety point zero zero four one

ii $7000, 90, \frac{4}{1000}, \frac{1}{10000}$

iii 7 thousands, 9 tens, 4 thousandths, 1 ten thousandth

4 **a** **i** $3 + \frac{4}{10}$ **ii** 3.4

b **i** $6 + \frac{9}{100}$ **ii** 6.09

c **i** $5 + \frac{2}{10} + \frac{6}{100}$ **ii** 5.26

d **i** $7 + \frac{4}{10} + \frac{8}{100}$ **ii** 7.48

e **i** $\frac{2}{100} + \frac{9}{1000}$ **ii** 0.029

f **i** $\frac{3}{10} + \frac{1}{10\,000}$ **ii** 0.3001

g **i** $80 + \frac{3}{10} + \frac{4}{1000}$ **ii** 80.304

h **i** $5 + \frac{2}{100} + \frac{7}{1000}$ **ii** 5.027

i **i** $\frac{6}{10} + \frac{3}{100} + \frac{9}{1000}$ **ii** 0.639

j **i** $200 + \frac{1}{100} + \frac{3}{1000}$ **ii** 200.013

k **i** $4 + \frac{8}{10} + \frac{2}{10\,000}$ **ii** 40.8002

l **i** $7000 + 6 + \frac{4}{1000}$ **ii** 7006.004

5 **a** 3.16 **b** 12.07 **c** 47 **d** 47.2
e 47.29 **f** 47.291 **g** 56.08 **h** 74.603
i 409.018 **j** 25.0403 **k** 738.209 **l** 9002.6008

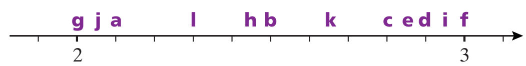
6 **a** 8.36
b 4 tenths, 8 hundredths
c 69 hundredths
d 4 units, 0 tenths
e 0 tenths, 3 hundredths
f 5 units, 5.21
g 2 tens, 1 tenth, 23.14
h 8 units, 0 hundredths, 8.106

7

Fraction (words)	Fraction	Decimal
7 tenths	$\frac{7}{10}$	0.7
2 hundredths	$\frac{2}{100}$	0.02
43 hundredths	$\frac{43}{100}$	0.43
3 thousandths	$\frac{3}{1000}$	0.003

8 **a** 0.07 **b** 0.04 **c** 0.11 **d** 0.27
e 3.06 **f** 7.01 **g** 2.39 **h** 5.44
i 7.3 **j** 8 **k** 1.64 **l** 5.3
m 2.03 **n** 4.57 **o** 6.70 **p** 8.10
q 3.4 **r** 5.8

9



10 **a** 4.40 **b** 5.10 **c** 4.10 **d** 4.70
e 4.05 **f** 4.25 **g** 4.350 **h** 4.310
i 4.370 **j** 4.315 **k** 4.365 **l** 4.305

11 **a** 0.9 **b** 0.86 **c** 1.43
d 1.3 **e** 1.8 **f** 0.37

12 **a** 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, …
b 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, …
c 2.1, 2.3, 2.5, 2.7, 2.9, 3.1, …
d 4.1, 4.4, 4.7, 5.0, 5.3, 5.6, …
e 0.56, 0.57, 0.58, 0.59, 0.60, 0.61, …
f 1.37, 1.38, 1.39, 1.40, 1.41, 1.42, …

13 **a** any five numbers in the sequence 6.11, 6.12, … , 6.19
b 6.11, 6.12, 6.13, 6.14
c 6.15, 6.16, 6.17, 6.18, 6.19
d it is in the middle, usually it is included with those closer to 6.2

14 41 and 8

exercise 4.2

1 **a** 1.6 > 1.4 **b** 4.08 < 4.13
c 2.06 < 2.6 **d** 7.24 > 7.2
e 0.354 < 0.435 **f** 2.45 < 4.553
g 31.54 > 31.276 **h** 7.2 < 7.319
i 4.13 < 4.138 **j** 9.3333 > 9.3
k 0.299 < 0.30 **l** 5.17 > 5.017

2 **a** 0.12 **b** 3.7 **c** 4.665 **d** 0.37
e 2.508 **f** 0.27 **g** 4.771 **h** 6.24
i 0.47 **j** 1.9213 **k** 3.8 **l** 7.8

3 **a** 6.01, 6.02, 6.12, 6.21 **b** 4.67, 4.76, 7.46, 7.64
c 3.003, 3.03, 3.3, 3.33 **d** 0.058, 0.508, 0.58, 0.85
e 2.115, 2.15, 2.151, 2.51 **f** 9.026, 9.206, 9.26, 9.6

4 **a** 1.54, 1.45, 1.05, 1.04
b 8.921, 8.219, 8.192, 8.129
c 4.73, 4.37, 3.74, 3.47

d 5.22, 5.202, 5.2, 5.022
e 7.646, 7.64, 7.464, 7.46
f 3.99, 3.909, 3.9, 3.099

5 303.4825, 330.9945, 780.9, 994.04

6 wandering albatross, marabou stork, white pelican, lammergeier, Andean condor

7 56.73, 57.10, 57.25, 57.84, 57.99, 58.06, 58.10, 58.54

8 Australia, Greenland, New Guinea, Borneo, Madagascar

9 1st Steve Hooker 5.96 m, 2nd Evgeny Lukyanenko 5.85 m, 3rd Danny Ecker, Dimitry Starodubtsev, Denys Yurchenko and Derek Miles 5.70 m

10 a 0.24 is 24 hundredths but 0.3 is 30 hundredths, so 0.3 is larger
b 0.2 is just two tenths, but 0.24 is two tenths plus four hundredths, so 0.24 is larger

11 a 0.90, 0.976, 0.985, 1.014, 1.064, 1.21
b 1.014 is only 0.014 away from 1, whilst 0.985 is 0.015 away
c 0.976 is only 0.026 away from 0.95, whilst 0.9 is 0.05 away

exercise 4.3

1 a 17 **b** 65 **c** 23 **d** 0
e 568 **f** 1 **g** 0 **h** 42
i 7 **j** 83 **k** 9 **l** 25
m 176 **n** 20 **o** 1 **p** 64

2 a 6.49 **b** 28.52 **c** 8.05 **d** 42.18
e 281.40 **f** 7.70 **g** 0.31 **h** 36.29
i 8.00 **j** 93.63 **k** 4.10 **l** 71.82
m 12.50 **n** 33.56 **o** 8.15 **p** 53.59

3 a 7.729 **b** 0.479 **c** 4.753 **d** 8.003
e 19.148 **f** 24.698 **g** 63.592 **h** 87.564
i 0.175 **j** 8.331 **k** 4.830 **l** 3.000
m 21.500 **n** 30.250 **o** 26.310 **p** 31.773

4 a 3.1 **b** 98.9 **c** 7.4 **d** 131.6
e 0.0 **f** 3.8 **g** 225.0 **h** 53.0
i 9.8 **j** 9.0 **k** 3.0 **l** 56.9
m 34.5 **n** 2.0 **o** 8.3 **p** 7.2

5

Number	Nearest whole number	Nearest tenth (one decimal place)	Nearest hundredth (two decimal places)	Nearest thousandth (three decimal places)
23.5284	24	23.5	23.53	23.528
0.72053	1	0.7	0.72	0.721
3.6193	4	3.6	3.62	3.619
28.4075	28	28.4	28.41	28.408
7.158937	7	7.2	7.16	7.159
0.63927	1	0.6	0.64	0.639
7.1839	7	7.2	7.18	7.184
0.99872	1	1.0	1.00	0.999
36.2949	36	36.3	36.29	36.295
8.9997	9	9.0	9.00	9.000

6 a 4.23, 4.24 **b** 8.74, 8.75
c 3.14, 3.15 **d** 5.63, 5.64
e 12.54, 12.55 **f** 0.00, 0.01
g 50.29, 50.30 **h** 26.19, 26.20
i 5.39, 5.40 **j** 8.99, 9.00

7 42.6 square metres

8 3.5 km

9 \$21.46

10 \$672.25

11 162.3 cm

12 17.484 km/hr

13 a 8 million square kilometres
b 7.7 million square kilometres
c 7.69 million square kilometres

14 Answers will vary, one set of solutions are shown below.
a 2.17, 2.23, 2.24
b 2.162, 2.195, 2.224
c 2.1536, 2.204, 2.216

15 a 13.5755017 **b** 13.58 **c** 3.3
d 4.2 **e** 13.86
f yes, it does make a difference—the answer is more accurate if you round afterwards

16 a \$37.20, \$46.45, \$28.95
b If amount ends in 1 or 2 cents, round down to 0. If amount ends in 3 or 4 round up to 5. If amount ends in 6 or 7 round down to 5. If amount ends in 8 or 9 round up to next 0.

17 a 0.538 hours
b 55 km/hr
c Yes. In the city the maximum speed is usually 60 km/hr. The average speed will be less than this because for some of the time the car is going slower than 60 km/hr due to traffic lights and other junctions.
d It depends how long the freeway section is compared with the rest of the trip and will also depend on other road conditions as mentioned in the answer to part c.

18 a none **b** 1 **c** 2

exercise 4.4

1 a $\frac{3}{10}$ **b** $\frac{7}{10}$ **c** $\frac{1}{10}$ **d** $\frac{9}{10}$
e $\frac{1}{100}$ **f** $\frac{43}{100}$ **g** $\frac{29}{100}$ **h** $\frac{87}{100}$
i $\frac{619}{1000}$ **j** $\frac{433}{1000}$ **k** $\frac{367}{10000}$ **l** $\frac{41}{10000}$
m $\frac{699}{1000}$ **n** $\frac{917}{10000}$ **o** $\frac{29}{10000}$ **p** $\frac{561}{10000}$

2 a $3\frac{7}{10}$ **b** $6\frac{1}{10}$ **c** $4\frac{9}{10}$ **d** $8\frac{3}{10}$
e $7\frac{3}{100}$ **f** $8\frac{21}{100}$ **g** $5\frac{67}{100}$ **h** $4\frac{9}{100}$

i $2\frac{589}{1000}$ j $3\frac{211}{1000}$ k $19\frac{613}{1000}$ l $24\frac{803}{1000}$
m $7\frac{391}{10\,000}$ n $82\frac{69}{10\,000}$ o $9\frac{607}{10\,000}$ p $63\frac{57}{10\,000}$

3 a $\frac{7}{10}$ b $3\frac{2}{5}$ c $6\frac{1}{2}$ d $4\frac{4}{5}$
e $7\frac{1}{50}$ f $\frac{9}{20}$ g $1\frac{1}{4}$ h $\frac{2}{25}$
i $8\frac{11}{50}$ j $7\frac{7}{20}$ k $\frac{4}{25}$ l $\frac{3}{1000}$
m $\frac{61}{250}$ n $\frac{69}{1000}$ o $1\frac{61}{200}$ p $\frac{301}{500}$

4 C

5 a $\frac{2}{3}$ b $\frac{2}{9}$ c $\frac{5}{9}$
d $\frac{4}{11}$ e $\frac{82}{99}$ f $\frac{71}{333}$

exercise 4.5

1 a 53.8 b 8.66 c 409.71 d 168.391
e 43.536 f 1.248 g 28.28 h 568.9

2 a 9.94 b 38.8 c 356.2 d 0.916
e 10.75 f 28.11 g 51.2 h 20.523

3 0.425 kg

4 19.8 km

5 3.35 litres

6 9.29 minutes

7 E

8 4.6 km

9 a 0.54 b 32.0 c 13.19 d 0.505
e 0.454 f 0.445 g 0.407 h 0.1688

10 a 31.6 b 0.502 c 0.311.19 d 21.16
e 0.152 f 1.895 g 2.45 h 8.405
i 0.664 j 3.313 k 0.132 l 12.73
m 5.048 n 42.277 o 0.541 p 19.097

11 6.1 km

12 $181.85

13 $63.55

14 a 3.08 seconds b 332.69
c 3 minutes 52.69 seconds

15 a
$$\begin{array}{r} 527.13 \\ -\ 164.74 \\ \hline 362.39 \\ \hline \end{array}$$
b
$$\begin{array}{r} 2.431 \\ -\ \ 7.59 \\ 2.104 \\ \hline 12.125 \\ \hline \end{array}$$

exercise 4.6

1 a 2 b 4 c 6
d 1 e 3 f 5

2 a 40 b 80 c 38 d 2
e 125.31 f 67.2 g 9100 h 2 433 000
i 607 400 j 9824.1 k 523.83 l 24 102
m 4735.2 n 236 o 21 330 p 6
q 14.3 r 70 s 0.018 t 50

3 a 3 b 2 c 1
d 5 e 6 f 4

4 a 3 b 0.3 c 4.16 d 2.04
e 4.6169 f 56.037 g 0.34213 h 8.129
i 0.3 j 0.7 k 0.41 l 0.3402
m 0.00473 n 2.36 o 0.0213 p 0.08
q 0.0025 r 0.023 s 0.00018 t 0.0007

5 a 9 mm b 0.2 cm c 30 cm

6 a 35 mm b 0.35 mm

7 384 000 km

exercise 4.7

1 a 0.8 b 0.21 c 0.018 d 0.08
e 1.6 f 0.015 g 0.42 h 4.5

2 a 18.81 b 34.12 c 2058.5 d 48.552
e 205.17 f 1455.6 g 49.544 h 4.312
i 0.872 j 4.38 k 0.992 l 6.327
m 16.156 n 421.8 o 24.56 p 300.35

3 a 37.1×4 b 23.5×6
c 6320×3 d 90.25×8
e 8.25×7 f 3618×2
g 5732×6 h $414\,020 \times 7$
i 426×7 j 76×3
k 7.93×5 l 8.2×6
m 30.5×4 n 4060×9
o 34.1×8 p 20090×2

4 a $61.3 \times 4 = 245.2$ b $520 \times 3 = 1560$
c $452.6 \times 7 = 3168.2$ d $212.4 \times 5 = 1062$
e $4.83 \times 2 = 9.66$ f $5233 \times 6 = 31\,398$
g $58.3 \times 8 = 466.4$ h $522.3 \times 4 = 2089.2$
i $123 \times 4 = 492$ j $22 \times 6 = 132$
k $1.62 \times 3 = 186$ l $5.2 \times 4 = 20.8$
m $40.5 \times 5 = 202.5$ n $7080 \times 2 = 14160$
o $41.2 \times 6 = 247.2$ p $30070 \times 7 = 210\,490$

5 a 0.09 b 0.024 c 0.00016
d 0.021 e 0.25 f 0.032
g 0.0006 h 0.0015 i 0.014
j 0.024 k 0.0036 l 0.0018
m 0.55 n 0.072 o 0.0044

6 a 1.55 b 0.192 c 0.074
d 6.44 e 0.164 f 10.28
g 0.0813 h 9.536 i 0.318
j 0.3472 k 0.2439 l 0.007905
m 10.44 n 1.88 o 6.342

7 a 12.92 b 2.624 c 10.836
d 20.296 e 104.16 f 2.1736
g 8.526 h 4.5549 i 0.928
j 37.842 k 1.7986 l 1.33484
m 240.34 n 91.756 o 35.583

8 $5.25

9 2.48 km

10 4612 cents = $46.12

11 $61.50

12 78 kg

13 1.2 g

14 $43.55

15 **a** 19.44 km/hr
b cheetah, domestic cat, Simon

16 8.228 pounds

17 **a** 0.08, smaller **b** 0.015, smaller
c 0.88, between **d** 1.47, between
e 2.53, larger **f** 6.72, larger
g **i** when both numbers are greater than one
ii when one is greater than one and one is between zero and one
iii when both are between zero and one

18 **a** **i** 48.99 **ii** 51.7 **iii** 51.48
iv 56.56 **v** 50.16
b 258.89, no, Olivia scored less

exercise 4.8

1 **a** 0.7 **b** 0.8 **c** 0.5
d 0.9 **e** 0.4 **f** 0.41
g 0.03 **h** 0.14 **i** 0.55
j 1.18 **k** 4.3 **l** 2.24
m 6.35 **n** 3.57 **o** 5.4

2 **a** 0.375 **b** 0.625 **c** 1.75 **d** 1.875 **e** 4.875
f 2.25 **g** 2.375 **h** 4.75 **i** 2.125 **j** 2.75

3 **a** $0.\overline{3}$ **b** $0.\overline{6}$ **c** $4.\overline{18}$ **d** $7.\overline{45}$
e $1.217\overline{3}$ **f** $8.\overline{4227}$ **g** $3.\overline{8}$ **h** $7.\overline{36}$
i $2.1\overline{6}$ **j** $1.\overline{72}$ **k** $5.41\overline{6}$ **l** $0.\overline{307692}$

4 **a** $0.\overline{428571}$ **b** $0.\dot{4}$ **c** $0.\dot{3}$ **d** $0.\dot{6}$
e $0.\overline{54}$ **f** $0.8\dot{3}$ **g** $0.\dot{7}$ **h** $3.\dot{6}$
i $3.8\dot{3}$ **j** $1.\overline{09}$ **k** $5.\dot{6}$ **l** $1.\dot{6}$
m $7.1\dot{6}$ **n** $2.\overline{36}$ **o** $5.58\dot{3}$

5 **a** Recur: $\frac{2}{3}, \frac{1}{6}, \frac{4}{9}, \frac{2}{11}, \frac{7}{12}$
Terminate: $\frac{1}{2}, \frac{4}{5}, \frac{9}{10}$
b $2; 5; 10 = 2 \times 5$
c $3; 6 = 2 \times 3; 9 = 3^2; 11; 12 = 2^2 \times 3$
d 2, 3, 5, 11

exercise 4.9

1 **a** 7.6 **b** 2.7 **c** 0.24 **d** 57.8
e 79.23 **f** 0.057 **g** 26.68 **h** 1.095
i 14.6 **j** 43.6 **k** 0.381 **l** 0.0538
m 3.06 **n** 2.07 **o** 4.206 **p** 0.0803

2 **a** 11.55 **b** 8.26 **c** 0.215 **d** 4.25
e 7.835 **f** 0.3345 **g** 61.28 **h** 0.020125
i 3.426 **j** 1.7835 **k** 70.7 **l** 0.1815
m 0.9485 **n** 0.00155 **o** 2.2515 **p** 0.01052

3 **a** 15.57 **b** 4.40 **c** 0.89 **d** 2.39
e 3.92 **f** 0.34 **g** 35.74 **h** 0.03
i 0.50 **j** 0.08 **k** 3.04 **l** 0.16
m 0.71 **n** 70.21 **o** 11.40 **p** 0.01

4 **a** 3.95 **b** 13.7 **c** 13.51 **d** 35
e 270.5 **f** 7.125 **g** 57.75 **h** 10.46
i 4.32 **j** 13.68 **k** 450.3 **l** 7.8
m 14.4 **n** 27 **o** 0.42 **p** 7.2

5 **a** 271.33 **b** 1.569 **c** 250 **d** 7.05
e 32.7 **f** 17.55 **g** 4.02 **h** 48.62
i 6.12 **j** $45.\overline{63}$ **k** $101.7\dot{6}$ **l** 77.8875
m $2043.\dot{3}$ **n** 205.05 **o** 12.26 **p** $24.4\dot{6}$

6 B

7 $1.65

8 $14.40

9 1.53 m

10 $1.49

11 $23.75 for 5 kg

12 Ice-cream $7.62 for 2 L
Pineapple juice $3.80 for 2 L
Cornflakes $4.57 for 800 g
Pasta sauce $4.46 for 785 g

13 **a** $2.10 cents/L
b $2.50 cents/L
c $4.87 cents/L

14 5 for $12

15 5.25 g

16 8

17 45

18 28

19 **a** 22.39 litres **b** 224 km
c no **d** 0.94 litres

Revision

1 C **2** A **3** B **4** C **5** B

6 178.82

7 **a** 482.08 **b** 18.353

8 250.12

9 **a** 5132.4 **b** 86 600 **c** 27.431 **d** 0.007846

10 2526

11 0.00077

12 715.392

13 6.8

14 14.626

15 **a** $\frac{4}{5}$ **b** $\frac{31}{100}$ **c** $2\frac{1}{20}$ **d** $7\frac{347}{1000}$

16 **a** $0.\dot{7}$ **b** $0.5\dot{3}$ **c** $0.\overline{18}$ **d** $0.3\overline{214}$

17 **a** 0.625 **b** $0.\overline{54}$ **c** 1.65 **d** 3.25

18 **a** 27.8 megabytes
b 7.2 megabytes
c Dragon Flyers with Wizard Wonders, Magic Arts with Wizard Wonders

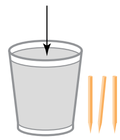
19 **a** \$1.49/100 g
b \$1.63/100 g
c \$1.83/100 g
d \$2.17/100 g

20 **a** 0.6 m
b 511.2 m
c 400 m, approximately 667 paces

Chapter 5

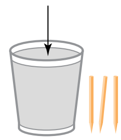
exercise 5.1

1 **a** $m = 5$



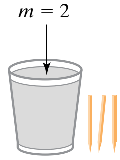
, 8

b $m = 9$



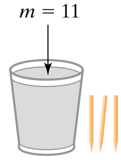
, 12

c $m = 2$



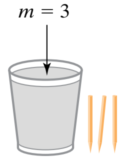
, 5

d $m = 11$



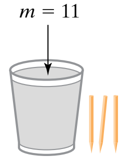
, 14

e $m = 3$



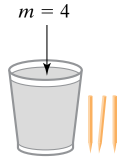
, 6

f $m = 11$



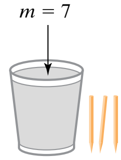
, 14

g $m = 4$



, 7

h $m = 7$



, 10

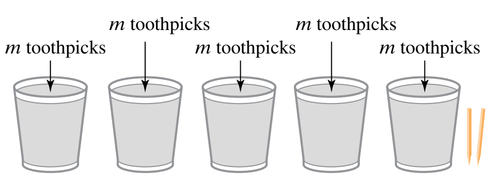
2 **a** $m + 5$ **b** $3m + 2$ **c** $2m + 2$ **d** $4m + 6$
e $3m + 4$ **f** $2m + 4$ **g** $3m + 1$ **h** $4m + 3$

3 **a** 14 **b** 22 **c** 8 **d** 26
e 10 **f** 26 **g** 12 **h** 18

4 **a** 25 **b** 10 **c** 22 **d** 13
e 28 **f** 7 **g** 19 **h** 16

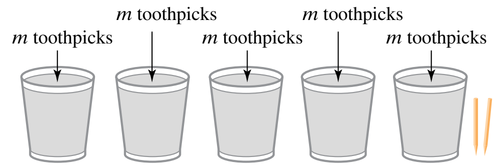
5 **a** 8 **b** 17 **c** 35 **d** 23
e 62 **f** 47 **g** 98 **h** 77

6 **a**

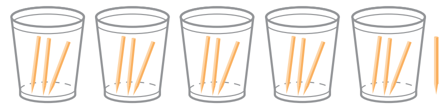


b **i** 7 **ii** 17 **iii** 22

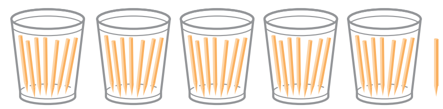
7 **a**



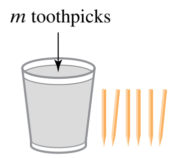
b



c

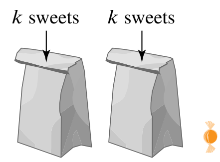


8 **a**



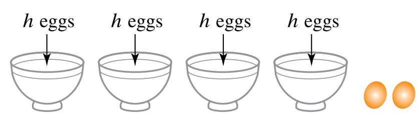
b **i** 8 **ii** 9 **iii** 10

9 **a**



b **i** 5 **ii** 7 **iii** 9

10 **a**



b **i** 50 **ii** 38 **iii** 14

11 **a** $n + 5$ **b** $n - 16$ **c** $5n$
d $\frac{n}{9}$ **e** $14 - 3n$ **f** $4n + 2$

12 **a** three times a certain number
b seven more than a certain number
c five minus a certain number
d eight less than a certain number
e six less than twice a certain number
f six plus half of a certain number

13 **a** 20 **b** 22 **c** 9 **d** 16 **e** 16 **f** 48

14 **a** 11 **b** 2 **c** 23 **d** 11 **e** 8 **f** 15

15 **a** **i** m **ii** 3: $3m$, $4m$, 11 **iii** 4
b **i** x **ii** 4: 5, x, 11, $3x$ **iii** 2
c **i** k **ii** 5: $7k$, $2k$, 5, $4k$, 8 **iii** 2
d **i** b **ii** 3: 6, b, 9 **iii** 1
e **i** y **ii** 3: 1, $3y$, $4y$ **iii** 3
f **i** n **ii** 7: n, n, $4n$, 7, $6n$, 11, 3 **iii** 1
g **i** h **ii** 6: 5, $2h$, $4h$, 7, $6h$, h **iii** 2
h **i** a **ii** 7: a, $5a$, 2, $3a$, 6, $5a$, 3 **iii** 5

16 **a** 6 **b** 8 **c** 15

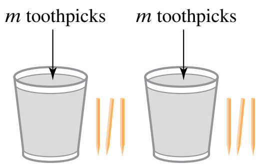
17 **a** 14 **b** 8 **c** 22 **d** 38

18 **a** $10t + 4$
b **i** 54 **ii** 64 **iii** 84

19 **a** 3 **b** 6 **c** 11

20 **a** $6s - 4$
b **i** 56 **ii** 86 **iii** 116

21 **a**

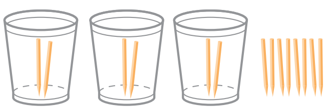


b **i** 14 **ii** 18 **iii** 32

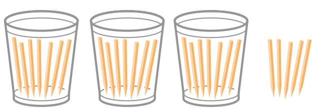
22 **a** **i** $b - 5$ **ii** $3b$
b Julia 2, Marco 15, Aunt Sofie 21
c Julia 8, Marco 21, Aunt Sofie 39

exercise 5.2

1 **a** $3m + 8$
b

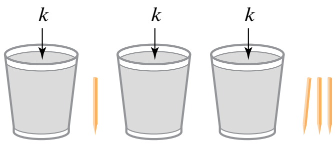


2 **a** $3m + 5$
b



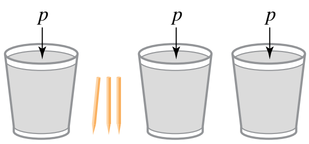
3 $3k + 6$

4 **a**



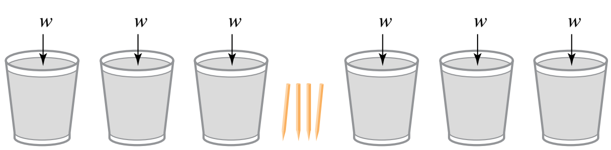
b $3k + 4$

5 **a**



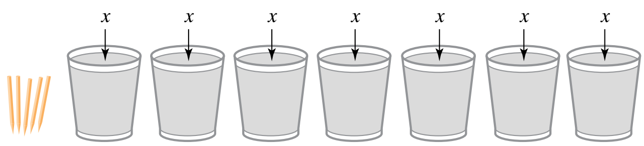
b $3p + 3$

6 **a**



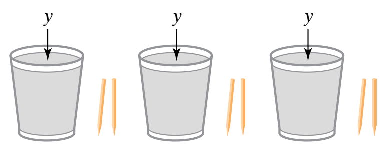
b $6w + 4$

7 **a**



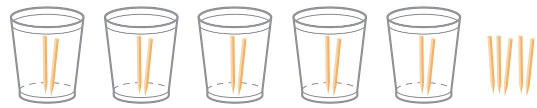
b $7x + 5$ **c** 40

8 **a**

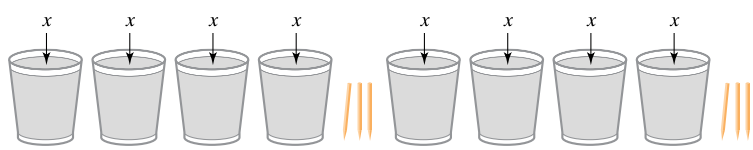


b $3y + 6$ **c** 18

9 **a** $5m + 5$
b

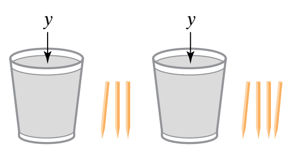


10 **a**



b 46 **c** 6

11 **a**



b $2y + 7$ **c** 15

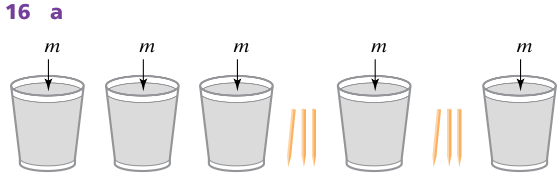
12 $3k + 22$

13 **a** $3m + 7$ **b** $3z + 11$ **c** $5p + 14$
d $2d + 10$ **e** $13y + 4$

14 **a** 9 **b** 4 **c** 29
d 8 **e** 22 **f** 0
g 20 **h** 60 **i** 42
j 7 **k** 8 **l** 11

15 **a** many answers are possible, e.g. $2m + 3 + 4m + 5$, check with your teacher
b when $m = 3$, all expressions should have the value 26

16 **a**



b $5m + 6$ **c** 16

exercise 5.3

1 **a** number of toothpicks in the cup
b total number of toothpicks
c

e	1	3	4	7	9	17
d	4	6	7	10	12	20

2 **a** number of toothpicks in the cup
b total number of toothpicks
c

j	1	3	4	7	5	11
k	4	12	16	28	20	44

3

w	2	5	8	11	12	20
g	9	15	21	27	29	45

4 **a** number of toothpicks in the cup
b total number of toothpicks
c

z	2	4	7	9	10	19
w	9	15	24	30	33	60

5 **a** number of toothpicks in the cup
b total number of toothpicks
c

m	1	2	4	6	8	16
p	10	12	16	20	24	40

6 **a**

t	0	2	5	9
u	6	8	11	15

b

b	0	2	5	7
c	3	9	18	30

c

j	1	3	6	8
k	7	5	2	0

d

t	0	2	3	12
e	7	15	19	55

e

g	0	5	15	30
h	3	4	6	9

f

q	2	3	4	5
r	9	12	15	18

g

y	0	6	9	21
z	0	4	6	14

h

w	3	11	15	27
x	1	3	4	7

i

p	2	5	7	12
q	0	12	20	40

j

r	7	11	17	21
s	2	4	7	9

7 **a**

a	1	3	4	6	9	15
y	6	8	9	11	14	20

b

w	0	2	4	5	7	10
p	1	5	9	11	15	21

c

k	0	4	12	16	20	28
m	0	1	3	4	5	7

d

x	1	2	4	5	7	8
t	2	5	11	14	20	23

e

b	4	12	16	24	40
c	1	3	4	6	10

f

x	1	2	3	7	10
y	6	8	10	18	24

8 **a**

e	*d*
6	1
10	5
14	9
17	12
35	30

b

a	*b*
0	0
3	18
5	30
8	48
11	66

c

u	*w*
0	7
2	11
3	13
10	27
13	33

d

m	*n*
0	15
2	11
4	7
3	9
7	1

e

x	y
0	12
1	10
3	24
5	32
8	44

f

p	q
1	2
7	5
9	6
11	7
21	10

9 **a** $n = 3x + 2$

b

x	7	8	10	12	15
n	23	26	32	38	47

10 **a** $m = \frac{t}{4}$

b

t	20	28	36	400
m	5	7	9	100

11 **a** $k = \frac{h}{2} + 2$

b

h	12	16	22	30
k	8	10	13	17

exercise 5.4

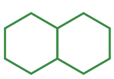
1 **a**

n	1	2	3
p	3	6	9

b $p = 3n$
c it works because three people can sit at each table
d 30 people

2 **a** 16
b


, 6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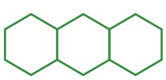
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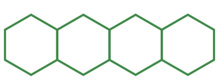
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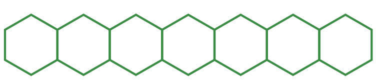

, 36

c

H	1	2	3	4	7
P	6	11	16	21	36

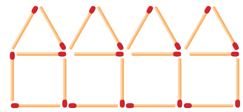
d yes
e $P = 5H + 1$
f 51
g 11

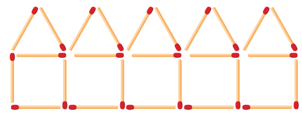
3 **a**

n	1	2	3	5	8
p	5	8	11	17	26

b three times the number of rungs plus 2
c $p = 3n + 2$
d 41
e 14

4 **a**


, 6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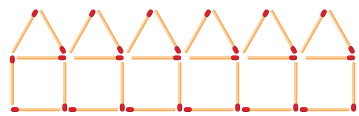

, 31

b

n	1	2	3	4	5	6
m	6	11	16	21	26	31

c five times the number of houses plus one
d $m = 5n + 1$
e 151
f 12

5 **a** 32
b

L	10	11	12	13	14
n	28	30	32	34	36

c twice the length of the pool plus eight
d $n = 2L + 8$
e 48
f 27

6 **a**

n	1	2	3	4	5	9	12
t	1	4	9	16	25	81	144

b the pattern number squared
c $t = n^2$
d 81
e 12

7 **a** $58 **b** $70 **c** $50
d

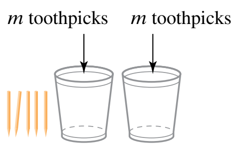
k	0	1	2	3	4
d	50	54	58	62	66

e four times the number of kilometres plus fifty
f $d = 4k + 50$
g $82
h 6 km

Revision

1 C **2** D **3** B **4** C **5** D

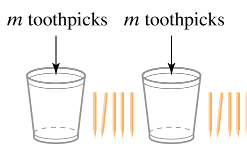
6 **a**



b **i** 7 **ii** 11 **iii** 13

7 **a** **i** 3 **ii** 7
b **i** 13 **ii** 19
c **i** 4 **ii** 5
d **i** 14 **ii** 18

8 **a**



b $2m + 10$
c 16

9 **a** $b = \frac{T}{5}$

b

T	20	30	35	60
b	**4**	**6**	**7**	**12**

10 **b**

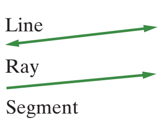
p	1	2	3	4	5	6
b	7	13	19	25	31	37

c $b = 6p + 1$ **d** 61 **e** 16

Chapter 6

exercise 6.1

1

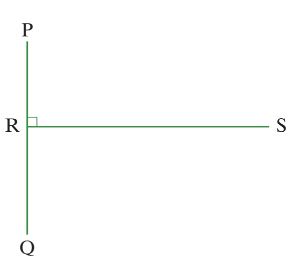


2 There are parallel segments in the rows of bricks. The legs of the parallel bars are parallel to each other.

3 **a** *CD*, *EF* **b** *AB*, *CD*, *EF*

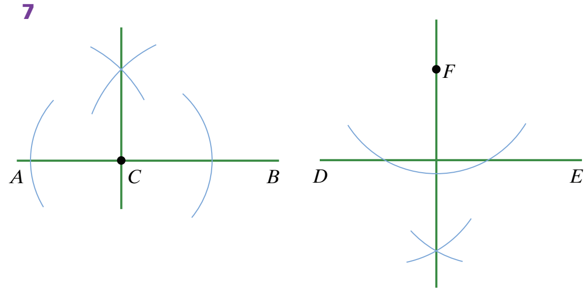
4

5



6 In striped material, the stripes are parallel to each other. In check material the horizontal stripes are perpendicular to the vertical stripes.

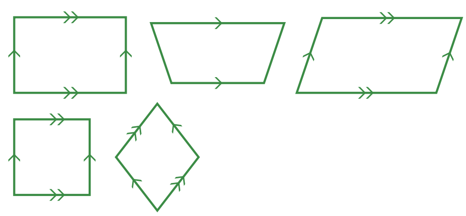
7



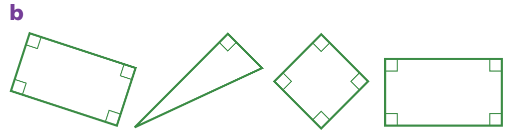
8 Chris is building a house with rectangular rooms. He has checked that the floor is horizontal. The walls must be vertical and perpendicular to the floor. The walls on opposite sides of the room must be parallel to each other. Two walls meeting at a corner of the room must be perpendicular to each other.

9 **a** perpendicular **b** vertical
c parallel **d** horizontal

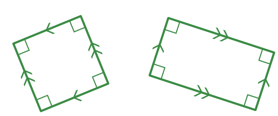
10 **a**



b



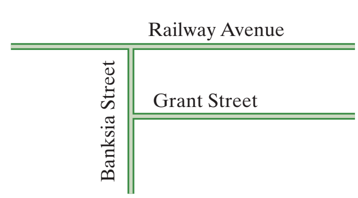
c



11 **a** meet
b don't meet, but not parallel
c don't meet, but not parallel
d don't meet and parallel

12 **a** Bay Street, Bank Street or Junction Street
b Main Street, Tower Street or River Street

13



exercise 6.2

1 **a** $\angle TOP$ (or $\angle POT$), acute, 50°
b $\angle JCP$ (or $\angle PCJ$), obtuse, 130°

c ∠BUS (or ∠SUB), reflex, 296°
d ∠PAT (or ∠TAP), reflex, 270°
e ∠HDG (or ∠GDH), obtuse, 163°
f ∠XYZ (or ∠ZYX), acute, 25°
g ∠MOP (or ∠POM), right, 90°
h ∠KTP (or ∠PTK), obtuse, 130°
i ∠HFK (or ∠KFH), straight, 180°

2 **a** obtuse angle **b** acute angle
c reflex angle **d** right angle
e obtuse angle **f** reflex angle
g revolution **h** straight

3 **a** line **b** perpendicular
c obtuse angle **d** line segment
e reflex angle **f** ray
g acute angle **h** parallel

4 **a** 66° **b** 301° **c** 125° **d** 240°
e 75° **f** 153° **g** 290° **h** 105°

5 **a** 70° **b** 260° **c** 150° **d** 45° **e** 132°
f 95° **g** 205° **h** 63° **i** 285° **j** 345°

6 **a** 33° **b** 40°

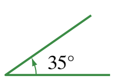
7 **a** The photograph at the left, because it has been taken from a position more directly in front of the beam.
b approximately 30°
c 60°

8 **a** A wheelchair ramp cannot be too steep otherwise the person will go down the slope too quickly and will not be able to stop. The ski slope needs to be steep enough for the skier to build up enough speed to keep going. The escalator needs to be steep enough so that it does not take up too much horizontal space, but not so steep that people could fall over on it.
b **i** 3° **ii** 30° **iii** 30°
c The photograph has been taken from a position side on to the escalator. If the angle is to be measured accurately, the photograph must be taken from a position directly in front of the escalator, at right angles to the tiled wall.

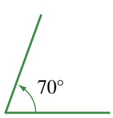
9 **a** 40°
b 53°; yes, the angle is greater than 50°

10 **a** $a = 49, b = 50, c = 51, d = 48, e = 41, f = 39, g = 36, h = 39$
b answers will vary

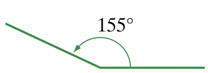
11 **a** obtuse



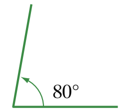
b acute



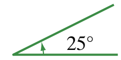
c reflex



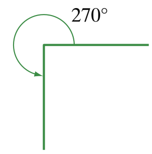
d reflex



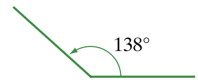
e acute



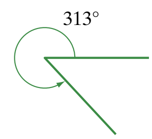
f acute



g obtuse

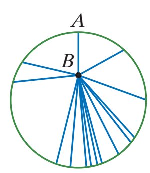


h acute



12 answers will match question 11

13 **a**



b mid-on

exercise 6.3

1 **a** 145° **b** 23° **c** 90° **d** 58° **e** 171° **f** 42°

2 **a** 21° **b** 109° **c** 72° **d** 160° **e** 115° **f** 125°

3 **a** 76° **b** 65° **c** 94° **d** 100° **e** 58° **f** 40°

4 **a** $a = 38, b = 38, c = 114$
b $d = 26, e = 116, f = 64$
c $a = 61, b = 110, c = 110$
d $a = 48, b = 132, c = 42$
e $a = 82, b = 59, c = 39, d = 59$
f $a = 110, b = 70, c = 70$
g $a = 70, b = 39, c = 71, d = 70$
h $a = 36$
i $k = 72$

5

Angle	Complement
14°	76°
88°	2°
45°	45°
90°	0°

Angle	Complement
32°	58°
75°	15°
15°	75°
1°	89°

6 **a** $a = 50$ **b** $b = 10$ **c** $c = 32$
d $d = 45$ **e** $e = 64$ **f** $f = 24$

7

Angle	Supplement
160°	20°
23°	157°
91°	89°
100°	80°

Angle	Supplement
90°	90°
175°	5°
78°	102°
1°	179°

8 **a** $a = 32$ **b** $b = 86$ **c** $c = 138$
d $d = 45$ **e** $e = 65$ **f** $f = 90$

9 **a** 107°
b ADB is a straight line, so the angles must add to 180°

10 **a** $e = 46$ **b** $d = 70$ **c** $y = 107$
d $m = 23$ **e** $h = 68$ **f** $w = 33$
g $a = 45, b = 45, c = 85$
h $x = 66$ **i** $x = 40$ **j** $x = 285$
k $x = 97$ **l** $x = 90$

11 **a** $a = 60; b = 120$
b $c = 60; d = 120; e = 60; f = 120$

12 **a** 30°
b 30° ; same size as angle at A
c 60°
d 120°; supplementary angles
e Queensberry Street
f Elizabeth Street, Swanston Street

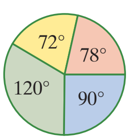
13 $a = 24$

14 **a** $\angle AOB$ or $\angle AOC$ or straight angle $\angle AOD$
b $\angle AOC$

15 80°

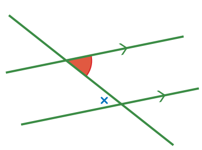
16 **a** 36°
b Bec and Sarah's total angle = 108°, Emma's total angle = 144°

17 **a** Blue 90°; Green 120°; Yellow 72°
b Red 78°
c

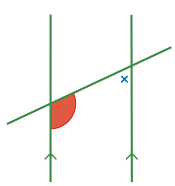


exercise 6.4

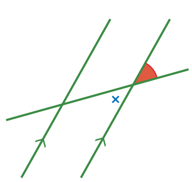
1 **a**



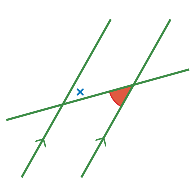
b



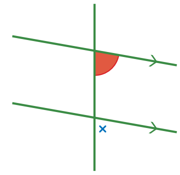
c



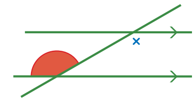
d



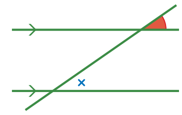
e



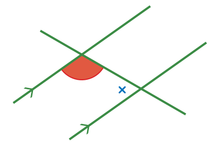
f



g



h



2 **a** 47 **b** 94
c $m = 152, n = 152$ **d** $d = 61, k = 119$
e 48 **f** 88
g $x = 101, y = 101$ **h** $a = 36, b = 144$
i $a = 31, b = 149, c = 149$

3 **a** 56 **b** 124
c 143 **d** 115
e $e = 59, f = 59$ **f** $m = 42, n = 42$
g 134
h $a = 57, b = 123, c = 57$
i $w = 112, x = 68, y = 68, z = 112$

4 **a** AB is parallel to CD because allied angles add to 180°
b AB is not parallel to CD because allied angles add to 182°
c AB is parallel to CD because allied angles add to 184°
d AB is parallel to CD because allied angles add to 180°

5 **a** $\angle ACH$ **b** $\angle BCG$ **c** $\angle BCG$ **d** $\angle BCH$
e $\angle MBA$ **f** $\angle CBM$ **g** $\angle BCH$
h $\angle ABN, \angle MBC, \angle BCH, \angle GCD$

6 35° Harry's line of sight and the sea are parallel, so $d°$ and 35° are alternate angles which are equal.

exercise 6.5

1 **a** $\triangle ABC$ (or $\triangle CAB$ or $\triangle BCA$)
b $\triangle BOX$ (or $\triangle OXB$ or $\triangle XBO$)
c $\triangle THA$ (or $\triangle HAT$ or $\triangle ATH$)
d $\triangle ENP$ (or $\triangle PEN$ or $\triangle NPE$)

2 Equilateral triangle: angles are 60°, sides are equal.

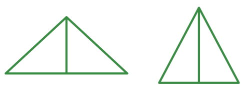
3 **a** right-angled scalene triangle
b acute-angled equilateral triangle
c right-angled isosceles triangle
d acute-angled isosceles triangle
e obtuse-angled isosceles triangle
f equilateral triangle (acute-angled)
g obtuse-angled scalene triangle
h acute-angled scalene triangle
i acute-angled isosceles triangle

4 **a** $a = 63$ **b** $b = 49$ **c** $c = 76$ **d** $d = 36$
e $e = 86$ **f** $f = 53$ **g** $g = 50$ **h** $h = 30$
i $i = 61$ **j** $j = 90$ **k** $k = 30$ **l** $l = 108$

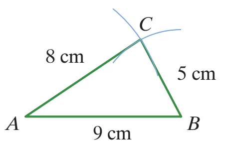
5 **a** $a = 32$ **b** $b = 64$
c $c = 70, d = 40$ **d** $x = 60, y = 60, z = 60$
e $e = 74$ **f** $y = 45, z = 75$
g $g = 60$ **h** $h = 106$
i $t = 30, u = 120$ **j** $d = 50, e = 80$
k $m = 45$ **l** $x = 60, y = 60$

6 **a** $a = 65$ **b** $b = 77$ **c** $c = 90$ **d** $d = 108$
e $e = 125$ **f** $f = 52$ **g** $g = 70$ **h** $h = 110$
i $m = 44$ **j** $j = 39$ **k** $a = 75$ **l** $x = 70$

7 **a** right-angled scalene triangle
b

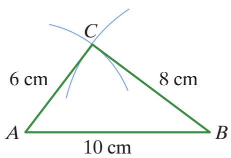


8 **a** **i**



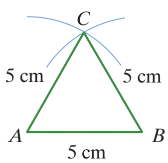
ii acute-angled triangle
iii scalene triangle

b **i**



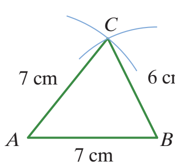
ii right-angled triangle
iii scalene triangle

c **i**



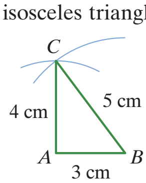
ii acute-angled triangle
iii equilateral triangle

d **i**



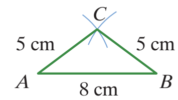
ii acute-angled triangle
iii isosceles triangle

e **i**



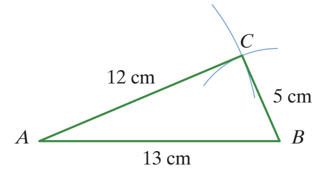
ii right-angled triangle
iii scalene triangle

f **i**



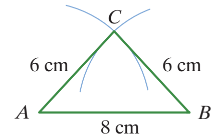
ii obtuse-angled triangle
iii isosceles triangle

g **i**



ii right-angled triangle
iii scalene triangle

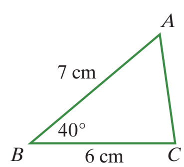
h **i**



ii acute-angled triangle
iii isosceles triangle

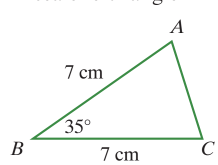
9 **a** no; 5 cm + 3 cm < 9 cm
b yes; 5 cm + 6 cm > 7.5 cm
c no; 2 cm + 2 cm = 4 cm
d yes; 3 cm + 3 cm > 4 cm
e no; 5 cm + 4 cm = 9 cm
f no; 6 cm + 8 cm < 16 cm
g yes; 10 cm + 3 cm > 11 cm
h no; 7 cm + 9 cm < 20 cm

10 **a**



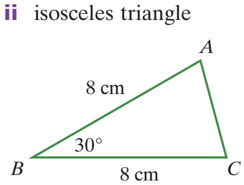
i acute-angled triangle
ii scalene triangle

b



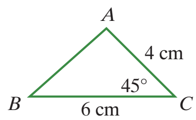
i acute-angled triangle
ii isosceles triangle

c



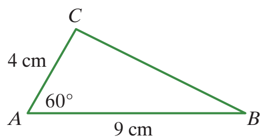
i acute-angled triangle
ii isosceles triangle

d



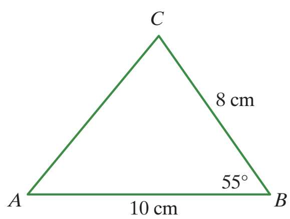
i obtuse-angled triangle
ii scalene triangle

e



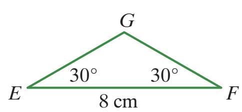
i obtuse-angled triangle
ii scalene triangle

f



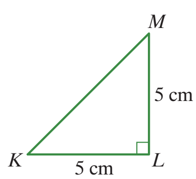
i acute-angled triangle
ii scalene triangle

g



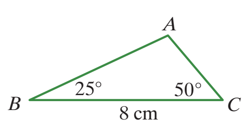
i obtuse-angled triangle
ii isosceles triangle

h



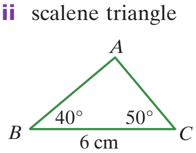
i right-angled triangle
ii isosceles triangle

11 a



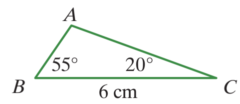
i obtuse-angled triangle
ii scalene triangle

b



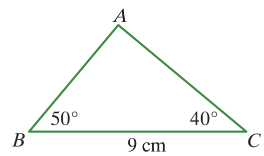
i right-angled triangle
ii scalene triangle

c



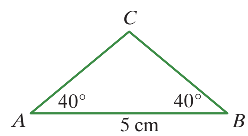
i obtuse-angled triangle
ii scalene triangle

d



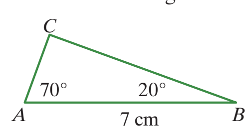
i right-angled triangle
ii scalene triangle

e



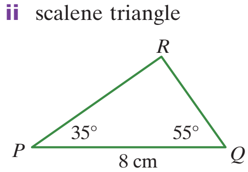
i obtuse-angled triangle
ii isosceles triangle

f



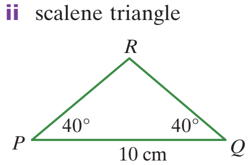
i right-angled triangle
ii scalene triangle

g



i right-angled triangle
ii scalene triangle

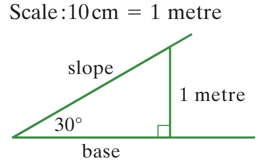
h



i obtuse-angled triangle
ii isosceles triangle

12 base 1.73 m, slope 2 m

Scale : 10 cm = 1 metre



slope measures 20 cm = 2 metres
base measures 17.3 cm = 1.73 metres

exercise 6.6

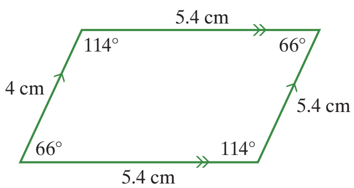
1 **a** $a = 55$ **b** $b = 49$ **c** $c = 124$ **d** $d = 113$
e $e = 103$ **f** $f = 56$ **g** $g = 108$ **h** $h = 114$
i $i = 110$ **j** $j = 132$ **k** $k = 58$ **l** $l = 82$

2 **a** square **b** rhombus
c trapezium **d** rectangle
e parallelogram **f** kite
g parallelogram **h** square
i rhombus

3 rectangle, trapezium, square

4 **a** $k = 93$ **b** $p = 90, q = 90, r = 90$
c $t = 45$ **d** $d = 69$
e $e = 101, f = 50$ **f** $x = 62, y = 118$
g $m = 110$ **h** $u = 40, v = 140$
i $m = 72, n = 108$
j $x = 118, y = 62, z = 118$
k $p = 40, q = 140, r = 40$
l $x = 95, y = 97$ **m** $a = 121, b = 121$
n $d = 70$ **o** $a = 125, b = 60$

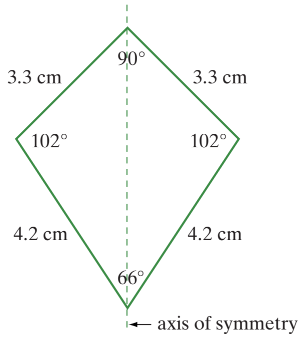
5 **a**



b Both pairs of opposite sides of a parallelogram are parallel and equal.
c The opposite angles of a parallelogram are equal. The adjacent angles of a parallelogram add to 180°.

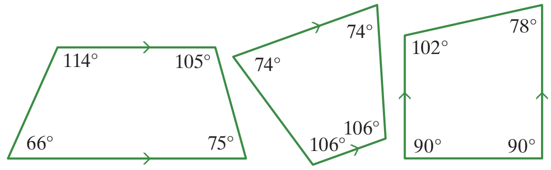
6 **a** Yes, they all have both pairs of opposite sides parallel and equal.
b A rhombus is a parallelogram with four equal sides. A rectangle is a parallelogram with four right angles. A square is a parallelogram, with four equal sides and four right angles.

7 **a, c**



b The kite has two pairs of equal sides and one pair of equal angles.

8 **a**



b two

9 **a** rhombus
b The bars stay parallel to each other. Both pairs of opposite sides of the rhombus are parallel and equal, so the bars close up neatly and compactly. The diagonals of each rhombus are at right angles to each other so the rhombuses move up and down (or in and out) at right angles to the base. This is particularly important in the case of the scissor lift to ensure that the work platform remains parallel to the ground.

10 **a** kite
b $\angle DCB = 90°$

$\angle CBA = 110°$

$\angle BAD = 50°$

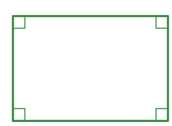
11 **a** trapezium
b It is the best shape to fit a bicycle with the least amount of wasted space. The two trapeziums fit beside each other to make a rectangle.

12 60° and 120°

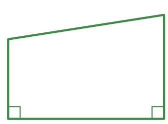
13 $h = 72, s = 74$

14 **a** examples are shown below:

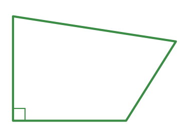
i



ii



iii



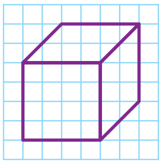
b No, the angles in a quadrilateral always add up to 360°, so if you had three right-angles then the fourth angle would also have to be a right-angle.

15 **a** $a = 29$ **b** $b = 120$
c $c = 123, d = 57$ **d** $x = 106$

16 James is correct. Both a square and a rhombus have both pairs of opposite sides parallel and all sides equal. They also both have opposite angles equal. The difference is that for a square all the angles must be 90°, but for the rhombus there is no such limitation.

exercise 6.7

1



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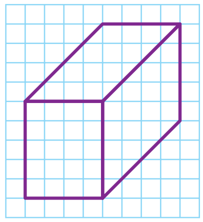

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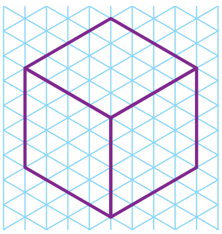

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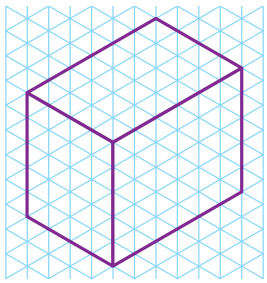
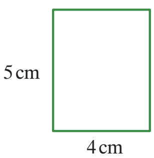


5 a i



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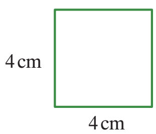

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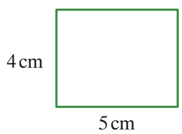
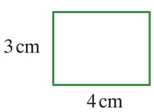


b i



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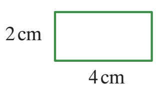

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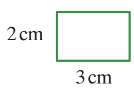
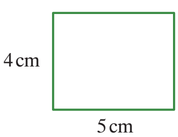


c i



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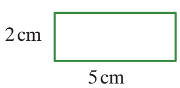

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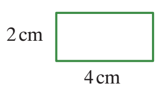
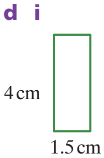


d i



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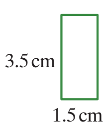

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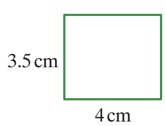
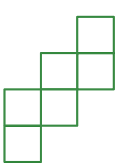


6 a i



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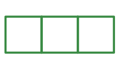

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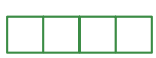

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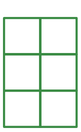

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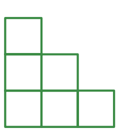

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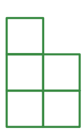

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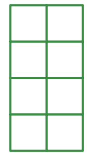

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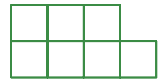

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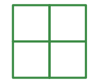

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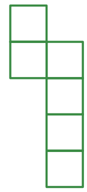

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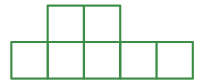

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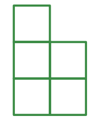
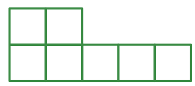


e i



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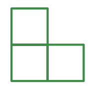

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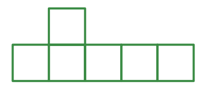
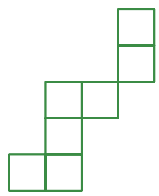


f i



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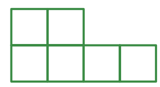

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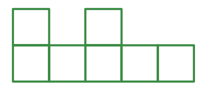
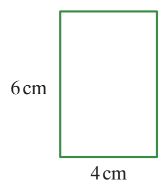


7 a i



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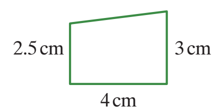

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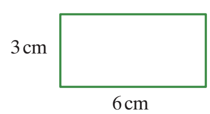
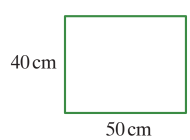


b i



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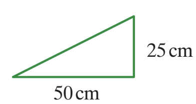

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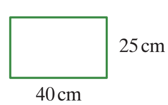

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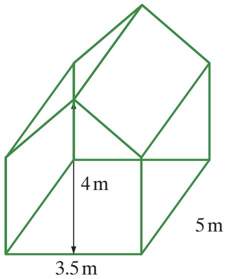
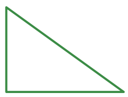
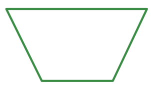


9 **a**

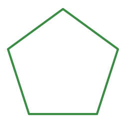


b

c



d



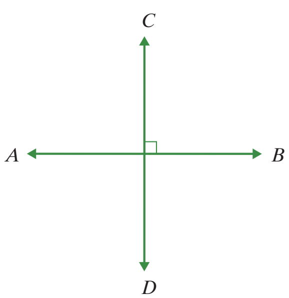
10 Answers will vary according to prisms used. Check with your teacher.

11 **a** plan **b** side elevation
c isometric drawing **d** oblique drawing

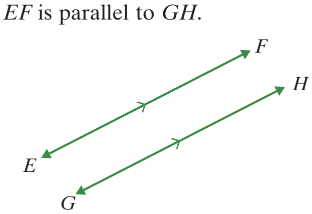
Revision

1 C **2** E **3** C **4** B **5** E

6 **a** The line *PQ* is parallel to the line *RS*.
b **i** *AB* is perpendicular to *CD*.



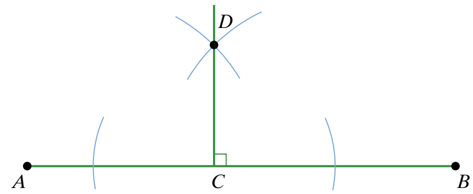
ii *EF* is parallel to *GH*.



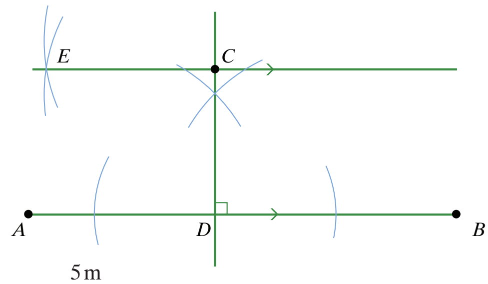
7 **a** 144° **b** 215°

8 **a** **i** $\angle MLG$ or $\angle GLM$
ii acute
iii 65°
b **i** $\angle PHC$ or $\angle CHP$
ii reflex
iii 280°

9

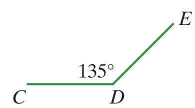


10



11 **a** 53° and 37° **b** 139° and 41°

12 **a**



b 45°

13 **a** $k = 28$ **b** $\angle EXY = 73°$
c $a = 80$, $b = 100$, $c = 43$

14 **a** $a = 50$ (cointerior angles in parallel lines add to 180°)
b $b = 146$ (corresponding angles in parallel lines are equal)

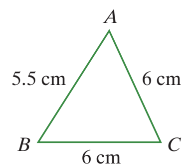
15 **a** $a = 124$ **b** $b = 22$

16 **a** rectangle; $a = 30$ **b** parallelogram; $b = 98$
c kite; $c = 115$ **d** square; $d = 90$
e rhombus; $e = 105$
f trapezium; $f = 90$, $g = 130$

17 **a** Yes, the sum of the two shortest sides is more than the third side, so the sides will meet.
b Yes, the sum of the two shortest sides is more than the third side, so the sides will meet.
c No, the sum of the two shortest sides is equal to the third side, so the sides will not meet.

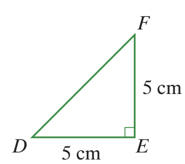
18 **a** $p = 30$ **b** 150° **c** 4 pieces **d** 90°

19 **a**



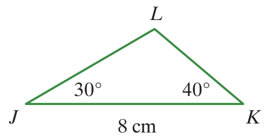
b isosceles triangle
c acute-angled triangle

20 **a**



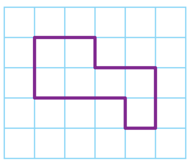
b isosceles triangle
c right-angled triangle

21 a

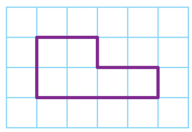


b scalene triangle
c obtuse angled triangle

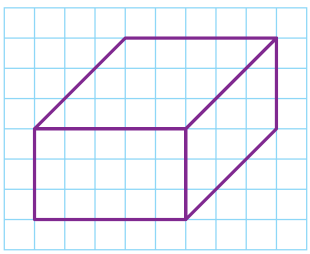
22 i



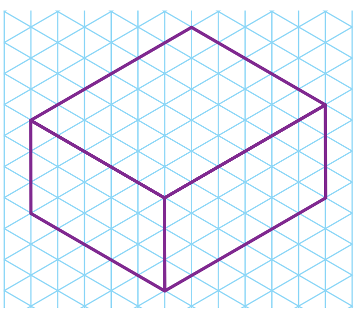
ii



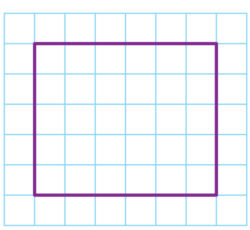
23 i



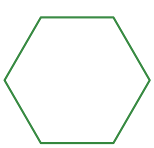
ii



iii



24



Chapter 7

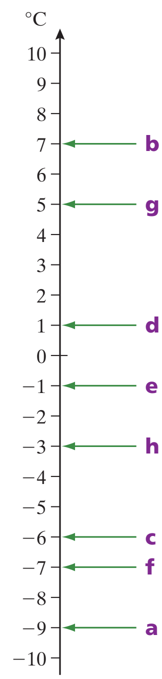
exercise 7.1

1 **a** ten degrees above zero
b having eighty dollars
c 2.6 m below sea level
d 3 cm to the right of zero
e the fourth basement level
f 30 km south of a town
g a turn of 45° clockwise
h 120 km east of a city

2 **a** 12 **b** -15 **c** -3 **d** 4
e -2 **f** -6 **g** -5 **h** -15

3 **a** 6°C **b** -1°C **c** -8°C **d** -6°C
e 3°C **f** -4°C **g** 0°C **h** -9°C

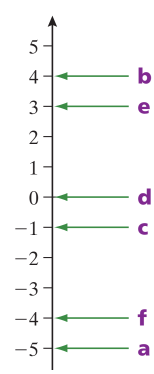
4

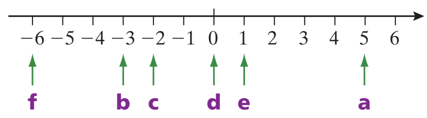


5 **a** 6 **b** 8 **c** 1 **d** -3

6 **a** 2 **b** -1 **c** 3 **d** -3

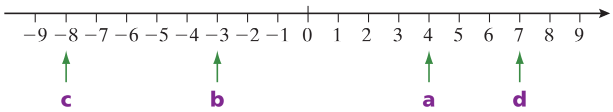
7



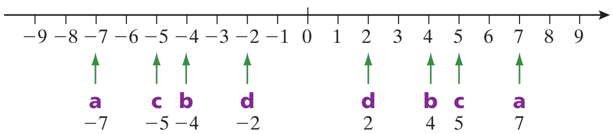
8 

9 **a** **i** 4 **b** −3 **c** −8 **d** 7

ii



10



11 **a** 8 **b** −5 **c** 2 **d** −9

12 **a** −50 **b** −10 **c** 2 **d** 50 **e** −20

13 **a** **i** 904 **ii** 1177
b **i** 273 **ii** −373 **iii** −621 **iv** −904

exercise 7.2

1 **a** −7, −4, −2, 1, 6 **b** −8, −7, −3, 0, 7
c −5, −4, −1, 3, 4 **d** −6, −4, −3, 3, 9
e −6, −2, 0, 1, 5 **f** −8, −5, −1, 0, 7

2 **a** 7, 0, −3, −6, −7 **b** 7, 0, −1, −5, −8
c 4, 3, −1, −4, −5 **d** 1, 0, −2, −6, −8
e 6, 1, −2, −4, −7 **f** 9, 5, 3, −4, −5

3 Moscow, Beijing, Tokyo, Paris, Rome

4 **a** −2, −1, 0, 1 **b** −7, −6, −5
c 3, 2, 1, 0, −1 **d** 6, 7, 8, 9
e −2, −3, −4, −5 **f** 2, 1, 0, −1, −2
g −5, −4, −3, −2, −1 **h** −3, −2, −1, 0, 1, 2, 3

5 **a** 6 **b** −2 **c** 7 **d** 10
e −1 **f** 5 **g** 0 **h** −6

6 **a** $-2 < 1$ **b** $-7 < -4$
c $8 > -1$ **d** $3 < 9$
e $-3 > -5$ **f** $4 > -8$
g $-6 < 0$ **h** $5 > -5$

7 **a** 3, 2 **b** −2, −3 **c** −4, −3
d 1, 2 **e** 0, −2 **f** 4, 7
g −1, 1 **h** 1, −2 **i** −3, 2

8 **a** 4 **b** 7 **c** 12 **d** 15

9 **a** \$3 **b** −8°C **c** −17 m

10 **a** \$2 **b** \$0 **c** \$1 **d** \$0 **e** \$5

11 **a** **i** 0 **ii** 3 **iii** −2
b 5 **c** 3 **d** 9 **e** 2

exercise 7.3

1 **a** 5°C **b** 7°C **c** −2°C **d** 18°C

2 **a** −4°C **b** −3°C **c** −6°C **d** −7°C

3 **a** 3rd floor **b** 1 floor below ground level
c 5th floor **d** 12th floor

4 **a** 2 floors below ground level
b 3 floors below ground level
c 3 floors below ground level
d 1 floor below ground level

5 **a** 9 m below sea level **b** 2 m below sea level
c 7 m below sea level **d** 9 m below sea level

6 **a** 4 **b** −3 **c** 0 **d** −3
e −5 **f** 2 **g** 4 **h** 0
i 5 **j** −4 **k** −1 **l** −6
m 5 **n** −2 **o** 3 **p** −7
q −2 **r** 5 **s** −8 **t** 0

7 **a** −5 **b** 5 **c** −5 **d** −7
e 0 **f** −6 **g** −9 **h** 2
i −7 **j** −3 **k** −7 **l** 2
m −6 **n** −4 **o** −2 **p** −8
q 0 **r** −1 **s** −7 **t** −6

8 **a** 1 **b** −5 **c** 0 **d** 3
e −7 **f** 5 **g** −2 **h** 8
i −9 **j** 10 **k** −12 **l** 19
m −13 **n** 14 **o** −4 **p** 4
q −9 **r** −4 **s** 0 **t** 4

9 **a** −5 **b** 0 **c** −3 **d** −8
e −1 **f** −5 **g** −7 **h** 4
i −11 **j** −9 **k** −13 **l** −12
m −15 **n** −0 **o** −14 **p** 6
q −14 **r** −6 **s** 17 **t** −11

10 **a** −2 **b** −3 **c** 7 **d** −8
e −1 **f** 2 **g** −7 **h** −3
i −10 **j** −4 **k** −5 **l** 7
m −2 **n** 0 **o** −16 **p** 4
q −7 **r** 6 **s** −6 **t** −7

11 **a** −3 **b** 1 **c** −5 **d** −7
e −1 **f** −9 **g** −3 **h** −5
i −7 **j** 5 **k** 2 **l** −4

12 B

13 **a** $4 - 7 = -3$; −3°C
b $-4 + 6 = 2$; 2°C
c $5 - 10 + 3 = -2$; −2°C
d $-2 + 4 - 7 = -5$; −5°C

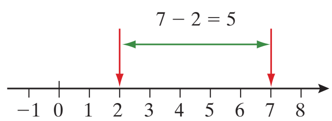
14 **a** $8 - 6 - 5 = -3$; 3 floors below ground level
b $4 - 5 + 2 = 1$; 1st floor
c $-1 + 6 - 2 = 3$; 3rd floor
d $7 - 1 - 8 = -2$; 2 floors below ground level

15 **a** $-480 + 20 + 100 + 240 = -120$
b $-260 + 140 + 100 + 60 = 40$
c $-480 + 140 + 10 + 60 = 270$
d $-260 + 40 + 30 + 240 = 50$
e $-320 + 10 + 10 + 200 = -100$
f Jane, Tai, Troy, Chris, Kate

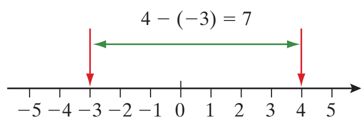
16 **a** $0 + 5 = 5$ **b** $5 - 10 = -5$
c $-5 + 12 = 7$ **d** $7 - 25 + 6 = -12$

exercise 7.4

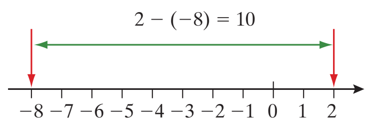
1 **a**



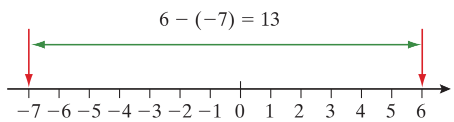
b



c



d



2 **a** 2 **b** 0 **c** 0 **d** 6
e 5 **f** −10 **g** −4 **h** 8
i −4 **j** −14 **k** 6 **l** −14
m −6 **n** −9 **o** 0 **p** −12

3 **a** 8 **b** 8 **c** 0 **d** 7
e −3 **f** 10 **g** 5 **h** 7
i 1 **j** −3 **k** 5 **l** 6
m 1 **n** 3 **o** 0 **p** −6

4 **a** 4 **b** 5 **c** 0 **d** 2
e 3 **f** −1 **g** −18 **h** 9
i −6 **j** −9 **k** −5 **l** −19
m 3 **n** −11 **o** −18 **p** 0

5

+	−2	4	−7	2
−3	−5	1	−10	−1
7	5	11	0	9
−6	−8	−2	−13	−4
−1	−3	3	−8	1

6

−	−1	5	−3	4
−6	−5	−11	−3	−10
2	3	−3	5	−2
−7	−6	−12	−4	−11
3	4	−2	6	−1

7 D

8 E

9 **a** 5°C **b** 8°C **c** 10°C **d** 15°C

10 **a** −10°C on December 22
b −6°C on December 22
c 6°C higher
d 7°C higher
e 3°C **f** 5°C **g** 16°C **h** 14°C

11 **a** 0

b

−3	2	1
4	0	−4
−1	−2	3

c

−5	0	−1
2	−2	−6
−3	−4	1

d −6
e you took 2 off each number, so in a row of 3 that is $3 \times (-2) = -6$

exercise 7.5

1 **a** 12 **b** −12 **c** −12 **d** 12
e 22 **f** −22 **g** −22 **h** 22
i 30 **j** −30 **k** −30 **l** 30

2 **a**

$5 \times 3 = 15$ $3 \times 3 = 9$
$5 \times 2 = 10$ $3 \times 2 = 6$
$5 \times 1 = 5$ $3 \times 1 = 3$
$5 \times 0 = 0$ $3 \times 0 = 0$
$5 \times (-1) = -5$ $3 \times (-1) = -3$
$5 \times (-2) = -10$ $3 \times (-2) = -6$
$5 \times (-3) = -15$ $3 \times (-3) = -9$

$-4 \times 3 = -12$ $10 \times 3 = 30$
$-4 \times 2 = -8$ $10 \times 2 = 20$
$-4 \times 1 = -4$ $10 \times 1 = 10$
$-4 \times 0 = 0$ $10 \times 0 = 0$
$-4 \times (-1) = 4$ $10 \times (-1) = -10$
$-4 \times (-2) = 8$ $10 \times (-2) = -20$
$-4 \times (-3) = 12$ $10 \times (-3) = -30$

b

×	−	+
+	−	+
−	+	−

3 **a** 18 **b** −40 **c** 21 **d** −24
e −30 **f** 7 **g** 99 **h** −48
i 20 **j** −18 **k** −14 **l** 50
m −35 **n** 36 **o** −80 **p** 33

4 a −30 b 12 c −18
d 30 e −12 f 30
g −21 h −24 i 14
j 36 k −40 l −18
m 60 n −18 o 16

5 E

6 B

7 a 49 b 49 c 1 d 1
e −4 f 4 g −25 h 25

8 a

x	−2	−1	0	1	2
y	−6	−3	0	3	6

b

x	−2	−1	0	1	2
y	8	4	0	−4	−8

c

x	−2	−1	0	1	2
y	−14	−7	0	7	14

d

x	−2	−1	0	1	2
y	10	5	0	−5	−10

9 a

×	−5	3	−4
−7	−35	−21	28
6	30	18	−24
−2	−10	−6	8

b

×	3	−6	5
11	33	−66	55
−4	−12	24	−20
−7	−21	42	−35

exercise 7.6

1 a −10 b 1 c 11 d −4
e 2 f −5 g 3 h −6
i −7 j 4 k −8 l 11
m 5 n −3 o −2 p 10
q 4 r −3 s −10 t 11
u −3 v 4 w −6 x 0

2 a 6 b −2 c −4 d 8
e 12 f 5 g −3 h −2
i −4 j 9 k 5 l −7
m −11 n 4 o −5 p −6
q −5 r 8 s −3 t 5
u −4 v −7 w 8 x 0

3 a $5 \times (-3) = 15$
$-15 \div 5 = -3$
$-15 \div (-3) = 5$
b $-4 \times 6 = -24$
$-24 \div (-4) = 6$
$-24 \div 6 = -4$
c $-2 \times (-7) = 14$
$14 \div (-2) = -7$
$14 \div (-7) = -2$
d $4 \times +11 = 44$
$44 \div 11 = 4$
$44 \div 4 = 11$
e $-6 \times 3 = -18$
$-18 \div (-6) = 3$
$-18 \div 3 = -6$
f $-11 \times (-3) = 33$
$33 \div (-3) = -11$
$33 \div (-11) = -3$
g $6 \times (-8) = -48$
$-48 \div 6 = -8$
$-48 \div (-8) = 6$
h $5 \times (-5) = -25$
$-25 \div 5 = -5$
$-25 \div (-5) = 5$

4 E

5 C

6 a −13 b 5 c −30

7 a 4 b −4 c 2 d −1
e −3 f 5 g −10 h 8
i −5 j −4 k 4 l −2
m 6 n −2 o −3 p 10

8 a 5 b 4 c −1 d 12
e 20 f 12 g 10 h 10

9 a 4 b −4 c 7 d −1
e −3 f 8 g −4 h 0

10 A

11 a −3 b −1 c −6 d 21

12 a

x	−12	−6	0	6	12
y	−4	−2	0	2	4

b

x	−12	−6	0	6	12
y	6	3	0	−3	−6

c

x	−12	−6	0	6	12
y	−2	−1	0	1	2

d

x	−12	−6	0	6	12
y	12	6	0	−6	−12

13 a −1°C b −2°C

Revision

1 C **2** D **3** C **4** A **5** E

6 **a** 6 **b** −4 **c** −8 **d** 0

7 **a** −10 **b** −11 **c** 5 **d** −3

8 **a** 18 **b** −28 **c** 40 **d** −20

9 **a** −5 **b** −9 **c** −6 **d** 11

10 **a** 1 **b** −3 **c** −12 **d** 3

11 **a** −9°C
b **i** −9 + 8 **ii** −1°C
c **i** −1 – 5 **ii** −6°C
d −2°C

12 **a** 1 **b** 0
c **i** −2 **ii** 3 m
d **i** −2 – 4 **ii** 6 m below sea level
e **i** − 3 × 4
ii −12, they have descended 12 m
iii 18 m below sea level
f sea slug

Chapter 8

exercise 8.1

1 **a** 5 **b** 6 **c** 16 **d** 5
e 2 **f** 3 **g** 2 **h** 4
i 2 **j** 3

2 **a** true **b** false **c** false **d** true
e true **f** false **g** false **h** true
i true **j** true **k** true **l** false

3 E

4 B

5 Yes, she is right. If one expression is greater than another, it is also 'not equal' to the other. The '>' sign just provides more detail about the comparison of their values.

6 Answers may vary — sample answers given
a $2 \times 6 = 2 \times 5 + 2$
b $19 - 4 + 3 = 3 \times 5 + 3$
c $7 \times 3 - 5 \times 2 \neq 3 + 8$
d $18 - 6 \div 2 = 5 \times 3$
e $(18 - 9) = 26 \div 2 - 4$
f $3 + 3 = 10 - 5 + 1$
g $30 - 5 - 10 \div 2 = 24 - 4$
h $50 \div 10 - 4 = 20 \div 5 - 3$
i $21 - 3 \times 4 = 32 \div 8 + 5$
j $(12 - 8) \div 4 = 16 \div 8 - 1$
k $(18 - 6) \div 3 \times 5 = (5 + 1) \times 5$
l $16 \div 2 \div 8 = 10 \div 5 \div 2$

exercise 8.2

1 **a** $4n = 20$ **b** $n + 7 = 35$
c $n - 5 = 18$ **d** $\frac{n}{3} = 4$
e $10 - n = 6$ **f** $2n + 2 = 18$
g $3n - 1 = 11$ **h** $\frac{n}{2} - 7 = 4$
i $\frac{n}{3} + 2 = 11$ **j** $\frac{4n}{5} = 8$
k $3(n + 5) = 33$ **l** $4(n - 2) = 20$

2 E

3 C

4 **a** $n + 7 = 26$ **b** $n - 4 = 35$
c $8n = 64$ **d** $\frac{n}{3} = 7$
e $5 - n = 2$ **f** $4n + 5 = 29$
g $3n - 8 = 19$ **h** $\frac{4n}{5} = 8$
i $7n + 3 = 17$ **j** $\frac{n}{3} + 6 = 11$
k $2n - 5 = 7$ **l** $3n + 7 = 13$

5 D

6 **a** 13 **b** 11 **c** $3x - 2$

exercise 8.3

1 **a** yes **b** no **c** no **d** yes
e yes **f** no **g** yes **h** no
i no **j** no **k** yes **l** no

2 **a** true **b** false **c** true
d false **e** true **f** false

3 **a** $x = 4$ **b** $y = 8$ **c** $x = 5$

4 **a** $x = 5$

x	$3x - 1$
1	2
2	5
3	8
4	11
5	14

b $x = 4$

x	$5x + 7$
0	7
1	12
2	17
3	22
4	27

c $x = 6$

x	$2x - 3$
4	5
5	7
6	9
7	11
8	13

d $p = 3$

p	$4p + 1$
0	1
1	5
2	9
3	13
4	17

e $x = 9$

x	$5x - 1$
6	29
7	34
8	39
9	44
10	49

f $m = 12$

m	$\frac{m}{2} - 5$
10	0
12	1
14	2
16	3
18	4

5 **a** $a = 4$ **b** $b = 3$ **c** $c = 13$
d $d = 19$ **e** $e = 15$ **f** $f = 27$
g $g = 6$ **h** $h = 8$ **i** $i = 11$
j $j = 36$ **k** $k = 20$ **l** $l = 42$

6 **a** $x = 16$ **b** $x = 9$ **c** $x = 9$
d $x = 7$

7 **a** $x = 22$ **b** $x = 19$ **c** $x = 8$ **d** $x = 11$

exercise 8.4

1 **a** output number is 17

5 → ($\times 3$) → 15 → ($+ 2$) → 17

b output number is 3

7 → ($+ 5$) → 12 → ($\div 4$) → 3

c output number is 1

20 → ($\div 10$) → 2 → ($\div 2$) → 1

d output number is 55

18 → ($- 7$) → 11 → ($\times 5$) → 55

e output number is 2.25

12 → ($\div 8$) → 1.5 → ($\times 1.5$) → 2.25

f output number is 10

14 → ($\times \frac{1}{2}$) → 7 → ($+ 3$) → 10

g output number is 21

1 → ($+ 3.2$) → 4.2 → ($\times 5$) → 21

h output number is 5.2

18 → ($\times 0.1$) → 1.8 → ($+ 3.4$) → 5.2

i output number is $\frac{1}{4}$

$\frac{1}{4}$ → ($+\frac{1}{2}$) → $\frac{3}{4}$ → ($\div 3$) → $\frac{1}{4}$

j output number is 8

9 → ($+ 3$) → 12 → ($\times 2$) → 24 → ($\div 3$) → 8

k output number is 22

3 → ($+ 5$) → 8 → ($\times 2$) → 16 → ($+ 6$) → 22

l output number is 28

18 → ($\div 3$) → 6 → ($+ 1$) → 7 → ($\times 4$) → 28

m output number is 4

30 → ($\div 5$) → 6 → ($\div 3$) → 2 → ($\times 2$) → 4

n output number is 10

8 → ($\times 9$) → 72 → ($\div 12$) → 6 → ($+ 4$) → 10

2 **a** output number is 13

12 → ($\div 3$) → 4 → ($+ 9$) → 13

b output number is 7

12 → ($+ 9$) → 21 → ($\div 3$) → 7

c No, when Kristopher completes his flowchart the larger number 9 is not divided by 3, but when Toby completes his the 9 is divided by 3 and so his answer is less.

3 **a** input number is 8

$8 \xrightarrow{\times 2} 16 \xrightarrow{-1} 15$

b input number is 9

$9 \xrightarrow{\times 2} 18 \xrightarrow{+7} 25$

c input number is 24

$24 \xrightarrow{\div 3} 8 \xrightarrow{\times 2} 16$

d input number is 10

$10 \xrightarrow{-5} 5 \xrightarrow{\times 4} 20$

e input number is 16

$16 \xrightarrow{\div 4} 4 \xrightarrow{-3} 1$

f input number is 8

$8 \xrightarrow{\times 7} 56 \xrightarrow{-5} 51$

g input number is 3

$3 \xrightarrow{\times 2} 6 \xrightarrow{+3} 9$

h input number is 16

$16 \xrightarrow{+4} 20 \xrightarrow{\times 3} 60$

i input number is 8

$8 \xrightarrow{+4} 12 \xrightarrow{-2} 10 \xrightarrow{\div 5} 2$

j input number is 18

$18 \xrightarrow{-3} 15 \xrightarrow{\div 5} 3 \xrightarrow{+4} 7$

k input number is 66

$66 \xrightarrow{\div 11} 6 \xrightarrow{+2} 8 \xrightarrow{\times 5} 40$

l input number is 2

$2 \xrightarrow{\times 50} 100 \xrightarrow{+8} 108 \xrightarrow{\div 12} 9$

4 **a** $x \xrightarrow{\times 2} 2x \xrightarrow{+4} 2x + 4$

b $y \xrightarrow{\div 3} \frac{y}{3} \xrightarrow{+7} \frac{y}{3} + 7$

c $p \xrightarrow{+5} p + 5 \xrightarrow{\div 4} \frac{p+5}{4}$

d $y \xrightarrow{+2} y + 2 \xrightarrow{\times 9} 9(y + 2)$

e $k \xrightarrow{-7} k - 7 \xrightarrow{\div 2} \frac{k-7}{2}$

f $z \xrightarrow{\times 5} 5z \xrightarrow{-1} 5z - 1$

g $m \xrightarrow{\times 4} 4m \xrightarrow{+1} 4m + 1 \xrightarrow{\div 3} \frac{4m+1}{3}$

h $c \xrightarrow{-7} c - 7 \xrightarrow{\div 2} \frac{c-7}{2} \xrightarrow{+5} \frac{c-7}{2} + 5$

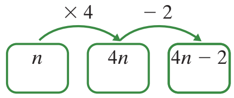
i $a \xrightarrow{\div 4} \frac{a}{4} \xrightarrow{+12} \frac{a}{4} + 12 \xrightarrow{\times 3} 3\left(\frac{a}{4} + 12\right)$

j $h \xrightarrow{\times 3} 3h \xrightarrow{\div 5} \frac{3h}{5} \xrightarrow{+4} \frac{3h}{5} + 4$

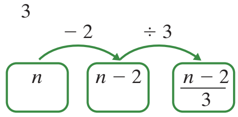
5 **a** **i** $\times 2$ **ii** $2n + 1$

$n \xrightarrow{\times 2} 2n \xrightarrow{+1} 2n + 1$

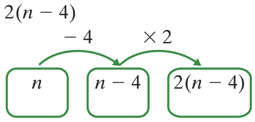
b **i** × 4 **ii** $4n - 2$



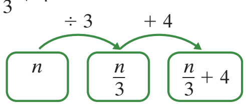
c **i** − 2 **ii** $\frac{n-2}{3}$



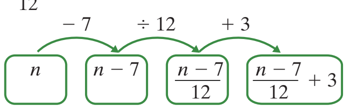
d **i** − 4 **ii** $2(n - 4)$



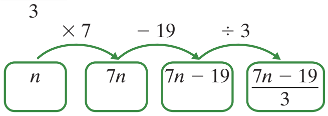
e **i** ÷ 3 **ii** $\frac{n}{3} + 4$



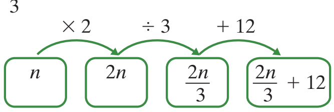
f **i** − 7
ii $\frac{n-7}{12} + 3$



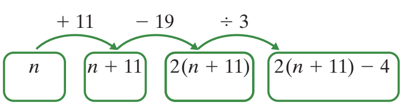
g **i** × 7
ii $\frac{7n-19}{3}$



h **i** × 2 (÷ 3 will also work)
ii $\frac{2n}{3} + 12$

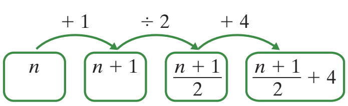


6 **a**



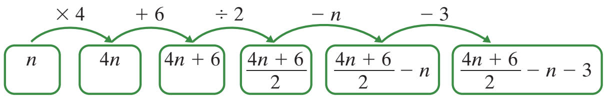
b $2(n + 11) - 4$ **c** 32 **d** 2

7 **a**



b $\frac{n+1}{2} + 4$ **c** 7 **d** 5

8 **a**



b $\frac{4n+6}{2} - n - 3$

c **i** 1 **ii** 2 **iii** 3

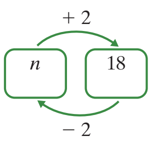
d The finishing numbers are always the same as the starting numbers. This has occurred because $\frac{4n+6}{2} - n - 3$ can be simplified to n.

$$\frac{4n+6}{2} - n - 3 = \frac{2(2n+3)}{2} - n - 3$$
$$= 2n + 3 - n - 3$$
$$= n$$

exercise 8.5

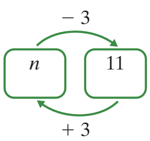
Answers to questions 1, 2, 3 and 6 are shown as single flow charts with the values at each step shown in red, to save space. Double flow chart versions are available in the student ebook and teacher book.

1 **a** **i**



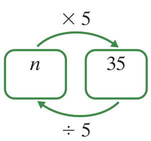
ii 16

b **i**



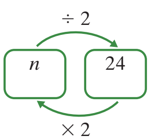
ii 14

c **i**



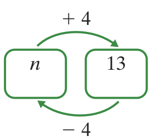
ii 7

d **i**



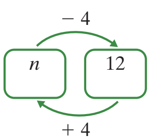
ii 48

e **i**



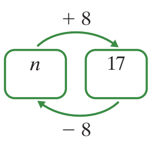
ii 9

f **i**



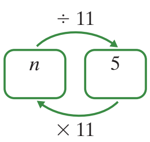
ii 16

g **i**



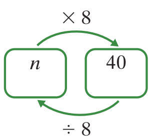
ii 9

h **i**



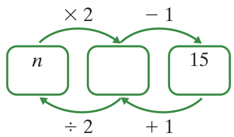
ii 55

i **i**



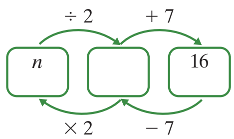
ii 5

2 **a** **i**



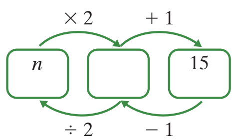
ii 8

b **i**



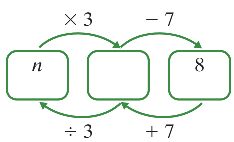
ii 18

c **i**



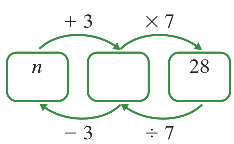
ii 7

d **i**



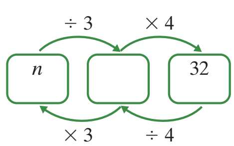
ii 5

e **i**



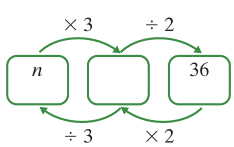
ii 1

f **i**



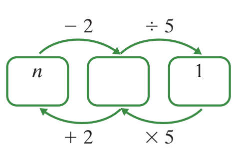
ii 24

g **i**



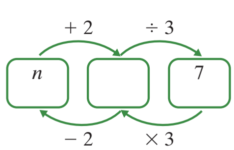
ii 24

h **i**



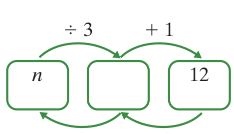
ii 7

i **i**



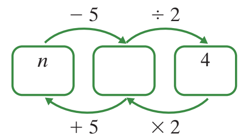
ii 19

j **i**



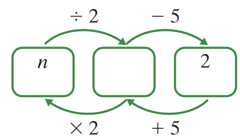
ii 33

k **i**



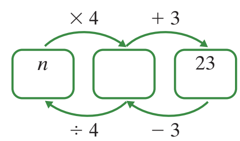
ii 13

l **i**



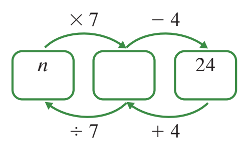
ii 14

m **i**



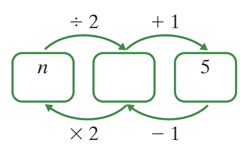
ii 5

n **i**



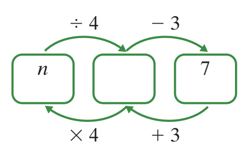
ii 4

o **i**



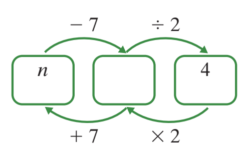
ii 8

p **i**



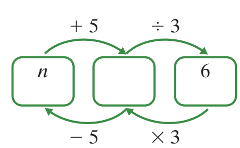
ii 40

q **i**



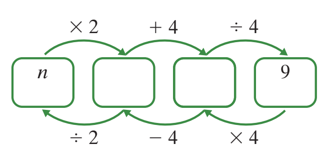
ii 15

r **i**



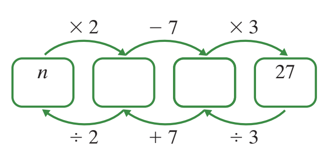
ii 13

3 **a** **i**



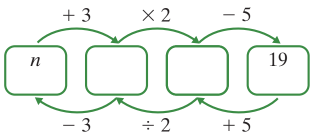
ii 16

b **i**



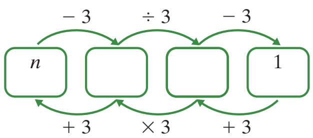
ii 8

c **i**



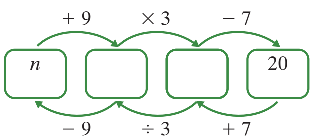
ii 9

d **i**



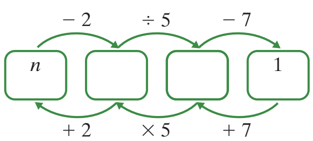
i 15

e **i**



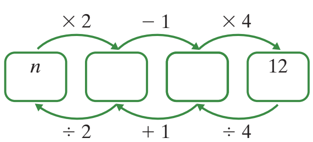
ii 0

f **i**



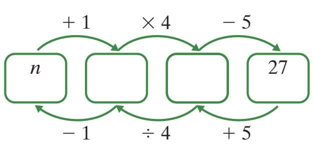
ii 42

g **i**



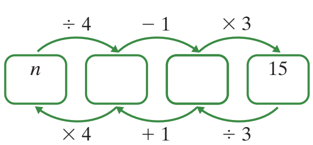
ii 2

h **i**



ii 7

i **i**

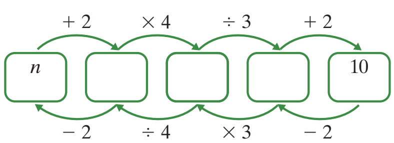


ii 24

4 **a** number = 50
b number = 6
c number = 17

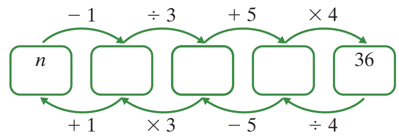
5 **a** $a = 3$ **b** $b = 15$ **c** $a = 6$
d $s = 16$ **e** $a = 17$ **f** $m = 2$
g $k = 3$ **h** $n = 5$ **i** $n = 10$

6 **a** **i**



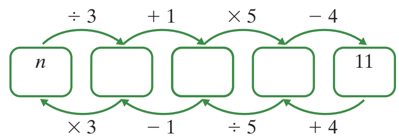
ii 4

b **i**



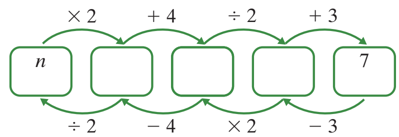
ii 13

c **i**



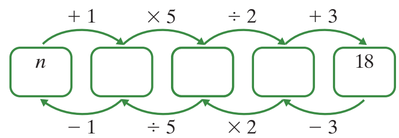
ii 6

d **i**



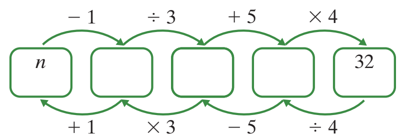
ii 2

e **i**



ii 5

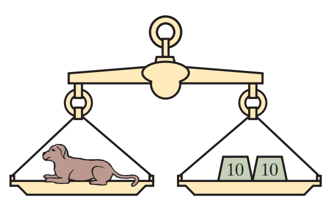
f **i**



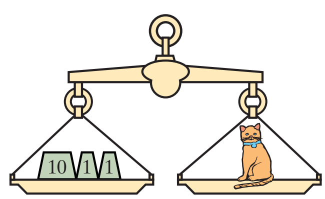
ii 10

exercise 8.6

1



2



3 **a** **i** $2x + 1 = 21$ **ii** $x = 10$
b **i** $3x + 10 = 25$ **ii** $x = 5$
c **i** $2x + 1 = 11$ **ii** $x = 5$
d **i** $4x = 20$ **ii** $x = 5$

4 **a** **i** $x + 9 = 20$
ii take away 9 from both sides
iii $x = 11$
b **i** $x - 7 = 13$
ii add 7 to both sides
iii $x = 20$
c **i** $2x = 14$
ii divide both sides by 2
iii $x = 7$
d **i** $2x - 7 = 9$
ii add 7 to both sides and then divide by 2
iii $x = 8$
e **i** $2x - 5 = 12$
ii add 5 to both sides and then divide by 2
iii $x = 8\frac{1}{2}$
f **i** $3x + 3 = 18$
ii take away 3 from both sides and then divide by 3
iii $x = 5$
g **i** $5x - 4 = 11$
ii add 4 to both sides and then divide by 5
iii $x = 3$
h **i** $3x - 2 = 28$
ii add 2 to both sides and then divide by 3
iii $x = 10$

5 **a** **i** $2(x + 5) = 22$
ii divide both sides by 2 and take away 5 from both sides
iii $x = 6$
b **i** $\frac{3x}{4} = 9$
ii multiply both sides by 4 and then divide both sides by 3
iii $x = 12$

exercise 8.7

1 **a** $x = 4$ **b** $x = 18$ **c** $a = 3$
d $a = 3$ **e** $b = 7$ **f** $b = 33$
g $a = 21$ **h** $d = 9$ **i** $d = 40$
j $m = 23$ **k** $m = 5.8$ **l** $p = 34.7$
m $x = 8.3$ **n** $x = 2.5$ **o** $t = 28$

2 **a** $m = 7$ **b** $b = 5$ **c** $m = 7$
d $x = 13$ **e** $m = 7$ **f** $h = 10$
g $k = 4$ **h** $a = 5$ **i** $x = 3.2$
j $a = 2.4$ **k** $b = 3.5$ **l** $h = 10.5$

3 **a** $x = 14$ **b** $b = 63$ **c** $k = 100$
d $n = 50$ **e** $a = 96$ **f** $d = 10$
g $x = 135$ **h** $m = 66$ **i** $m = 4.8$
j $y = 7.5$ **k** $b = 4.5$ **l** $a = 15.4$

4 D

5 **a** $a = 10$ **b** $a = 8$ **c** $a = 10$
d $a = 8$ **e** $a = 4$ **f** $a = 7$
g $a = 1.5$ **h** $a = 6$ **i** $a = 15$
j $a = 8$ **k** $a = 8$ **l** $a = 7$
m $a = 2$ **n** $a = 2$ **o** $a = 3.4$
p $a = 3$ **q** $a = 2.2$ **r** $a = 0.9$

6 D

7 **a** $n = 8$ **b** $n = 48$ **c** $n = 36$
d $n = 12$ **e** $n = 42$ **f** $n = 48$
g $n = 64$ **h** $n = 72$ **i** $n = 120$
j $n = 56$ **k** $n = 240$ **l** $n = 98$
m $n = 99$ **n** $n = 2.4$ **o** $n = 3.5$
p $n = 4.5$ **q** $n = 3.2$ **r** $n = 161$

8 **a** $d = 54$ **b** $x = 69$ **c** $y = 3.6$
d $x = 108$ **e** $b = 77$ **f** $a = 5$
g $m = 49.6$ **h** $k = 72$ **i** $x = 2.1$
j $n = 8.6$ **k** $b = 20$ **l** $a = 2$
m $b = 17.6$ **n** $a = 30.5$ **o** $a = 19$
p $a = 2.5$ **q** $h = 6.3$ **r** $n = 11$

9 $3x - 4 = 5$, number $= 3$

10 **a** $p = 15.67$ **b** $p = 1.39$ **c** $p = 2.58$
d $p = 4.41$ **e** $p = 5.20$ **f** $p = 6.67$

11 **a** $v = 2$ **b** $a = 5$ **c** $q = 1$
d $a = \frac{1}{5}$ **e** $a = 3\frac{1}{2}$ **f** $k = 2\frac{10}{11}$

exercise 8.8

1 **a** $p = 2$ **b** $c = 7$ **c** $m = 10$
d $x = 15$ **e** $m = 0$ **f** $k = 8$
g $z = 6$ **h** $x = 7$ **i** $x = 1$

2 **a** $m = 1\frac{1}{2}$ **b** $m = 3\frac{1}{2}$ **c** $m = 11\frac{1}{2}$
d $m = 7\frac{2}{5}$ **e** $m = 2\frac{1}{3}$ **f** $m = 3\frac{1}{5}$
g $m = 15\frac{1}{2}$ **h** $m = 3\frac{3}{4}$ **i** $m = \frac{2}{3}$

3 **a** $a = 6$ **b** $a = 8$ **c** $a = 3$
d $a = 1.5$ **e** $a = 2.4$ **f** $a = 12$
g $a = 0$ **h** $a = 0$ **i** $a = 2.8$

4 **a** $x = 2$ **b** $x = 24$ **c** $x = 33$
d $x = 32$ **e** $x = 15$ **f** $x = 0$
g $x = 35$ **h** $x = 17$ **i** $x = 12.8$

5 **a** $b = 2$ **b** $b = 5$ **c** $b = 5$
d $b = 8$ **e** $b = 4$ **f** $b = 5$
g $b = 3$ **h** $b = 4$ **i** $b = 2.2$

6 **a** $x = -5$ **b** $x = -11$ **c** $x = -12$
d $x = -2$ **e** $x = -2$ **f** $x = -13$
g $x = 0$ **h** $x = -20$ **i** $x = -4$

7 **a** $m = -6$ **b** $m = -11$ **c** $m = -7$
d $m = -4$ **e** $m = -4$ **f** $m = 7$
g $m = -18$ **h** $m = -66$ **i** $m = -28$
j $m = -35$ **k** $m = 12$ **l** $m = 15$

8 **a** $n = -9$ **b** $n = 5$ **c** $n = -3$
d $n = -4$ **e** $n = 8$ **f** $n = -2$
g $n = -8$ **h** $n = -4$ **i** $n = -12$

9 **a** **i** $2x = x + 30$
ii $x = 30$
iii LS = 60, RS = 60, LS = RS
b **i** $2x + 22 = 4x + 2$
ii $x = 10$
iii LS = 42, RS = 42, LS = RS

c i $3x = x + 10$
 ii $x = 5$
 iii LS = 15, RS = 15, LS = RS
d i $2x + 12 = 4x + 2$
 ii $x = 5$
 iii LS = 22, RS = 22, LS = RS
e i $2x + 2 = x + 5$
 ii $x = 3$
 iii LS = 8, RS = 8, LS = RS
f i $x + 5 = 4x$
 ii $x = 1\frac{2}{3}$
 iii LS = $6\frac{2}{3}$, RS = $6\frac{2}{3}$, LS = RS

10 a $x = 1$ b $x = 4$ c $x = 14$ d $x = 2$
 e $x = 2$ f $x = 1$ g $x = -3$ h $x = -1.5$

11 a $2\left(\frac{x}{3} - 2\right) = 8$ b 18

12 a $\frac{x - 3}{5} + 9 = 11$ b 13

13 7

14 a $p = 11$ b $m = 20$ c $m = 10$
 d $k = 18$ e $a = 6$ f $a = 6.5$
 g $x = 5$ h $m = 16$ i $x = 19$

exercise 8.9

1 a $b + 7$ b $b + 7 = 23$
 c $b = 16$ d 16 jellybeans

2 a $c + 13$
 b $c + (c + 13) = 71$ or $2c + 13 = 71$
 c $c = 29$
 d 29 and 42 chocolate buttons

3 a $4k + 3$ b $4k + 3 = 23$
 c $k = 5$ d 5 lollies

4 a $6x - 5$ b $6x - 5 = 175$
 c $x = 30$ d 30 sheets

5 a $x + (x + 10) = 154$ or $2x + 10 = 154$
 b Sarah is 72 and William is 82

6 Michael is correct. Denise has only divided the 6 by 3 not the n and the 6.

7 a $n + n + 1 = 29$ or $2n + 1 = 29$
 b $n = 14$
 c The two numbers are 14 and 15

8 a $n + n + 1 + n + 2 = 27$
 b $n = 8$
 c The three numbers are 8, 9, 10

9 a $4x$ b $4x = 76$ c $x = 19$

10 a i $4y = 64$
 ii $y = 16$
 iii all sides 16 cm
 b i $4x = 24.8$
 ii $x = 6.2$
 iii all sides 6.2 cm

c i $2x + 10 = 26$
 ii $x = 13$
 iii 13 cm and 5 cm, 13 cm, 5 cm

11 a $6n - 4 = 4.1$ b $n = 1.35$

12 a $12p = 132$ b 11 petunias

13 a $7.5n$
 b $7.5n = 37.5$
 c 5 hours

14 a $16.5n + 34 = 265$
 b $n = 14$
 c 14 hours

15 a $4n + 12 = 48$
 b $n = 9$
 c 9 hours

16 a Lucy: $d + 6$
 b Peter: $d + 3$
 c $d + d + 3 + d + 6 = 30$ or $3d + 9 = 30$
 d $d = 7$
 e Dominic is 7 years old, Peter is 10 years old and Lucy is 13 years old

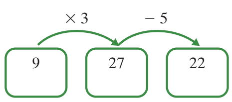
Revision

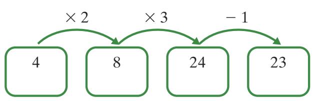
1 D **2** A **3** C **4** C **5** A

6 a > b =
 c = d <

7 a $2(n + 3) = 18$ b $5n - 4 = 11$

8 a $h = 3$ b $k = 4$ c $x = 27$

9 a


b


10 a $n = -6$ b $n = 9$

11 a $m = 17$ b $b = 11$ c $x = 12$
 d $y = 12$ e $d = 9$ f $n = 33$
 g $k = 25$ h $n = 7.5$ i $y = 8$
 j $a = 3$ k $a = 3$

12 a n b $n + 11$
 c $n + n + 11 = 103$ d $n = 46$
 e 46 and 57

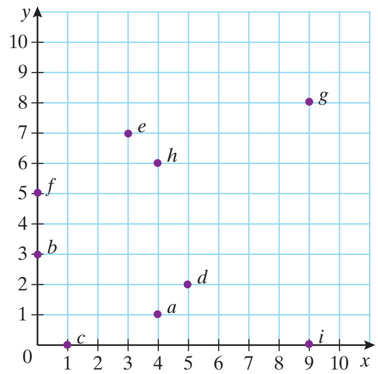
13 a $n + n + 1 = 121$ b $n = 60$
 c the two numbers are 60 and 61

Chapter 9

exercise 9.1

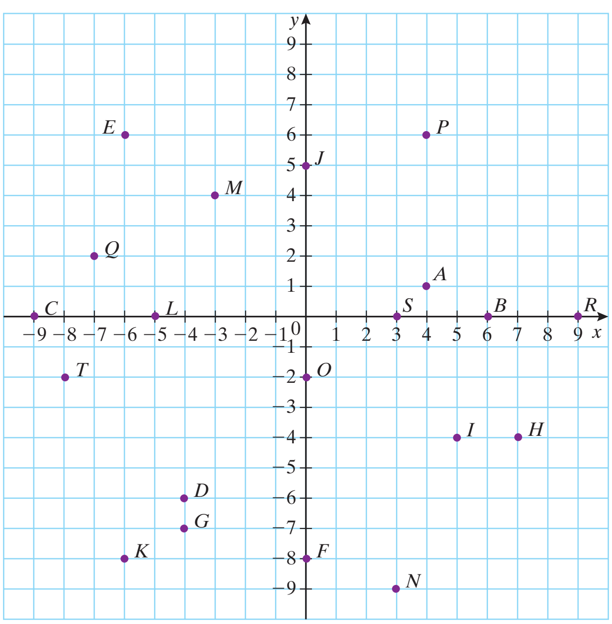
1 *A* (1, 7); *B* (0, 2); *C* (4, 1); *D* (8, 9); *E* (6, 5); *F* (9, 8)

2



3 *A* (−3, 2); *B* (7, 0); *C* (−3, −4); *D* (4, −6); *E* (−2, 4); *F* (9, 7); *G* (−1, 6); *H* (5, 4); *I* (3, 6); *J* (6, −3); *K* (−6, −1); *L* (−8, 7), *M* (−9, −8); *N* (−5, 9); *P* (6, 9); *Q* (2, −8); *R* (8, −1), *S* (0, 8); *T* (9, −5); *U* (−5, −6)

4

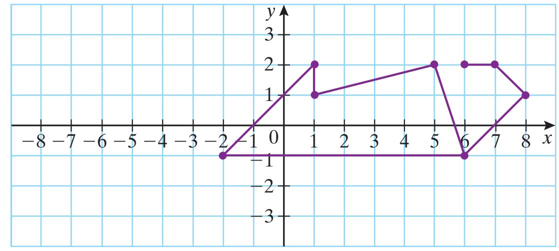


5 B

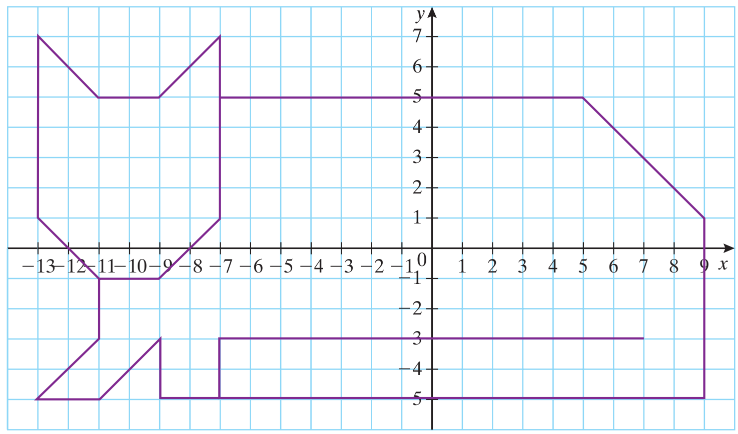
6 D

7 **a** *D*, *H* **b** *B*, *E*
c *G* **d** *J*

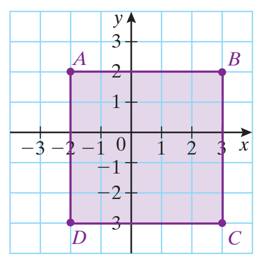
8



9

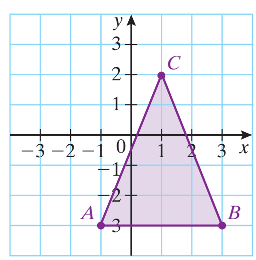


10 **a**



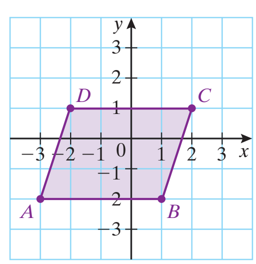
b *C* = (3, −3); *D* = (−2, −3)

11 **a**



b *C* = (1, 2)

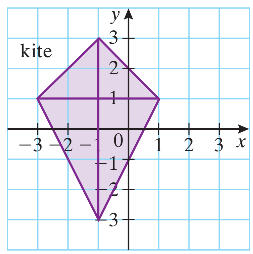
12 **a**



b *D* = (−2, 1)

13 **a** $A = (-2, 3)$; $B = (2, 2)$; $C = (3, -1)$; $D = (-3, -2)$
b $(0, 0)$; $(2, 1)$; $(-2, -1)$

14 **a** x-axis **b** y-axis **c** $(1, 2)$

15 **a**


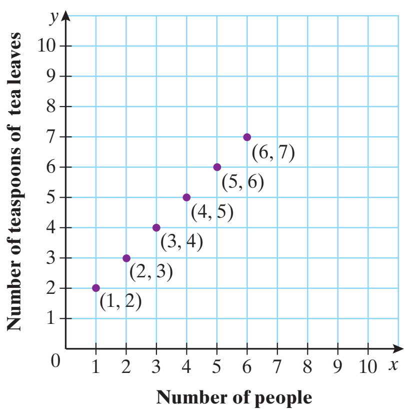
b $(-1, 1)$

16 answers will vary

exercise 9.2

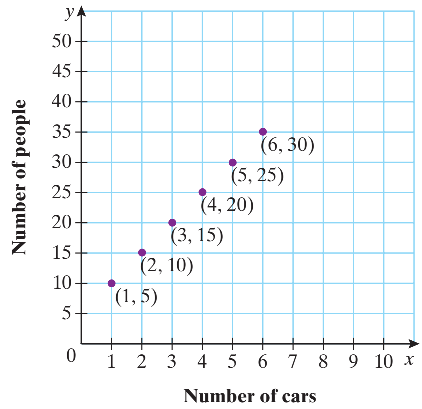
1 **a**

Number of people (x)	1	2	3	4	5	6
Number of teaspoons of tea leaves (y)	2	3	4	5	6	7

b (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)
c $y = x + 1$
d


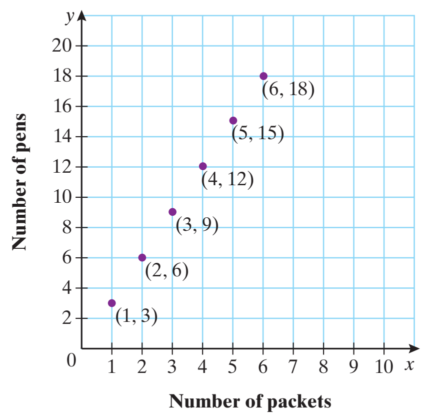
2 **a**

Number of cars (x)	1	2	3	4	5	6
Number of people (y)	5	10	15	20	25	30

b $y = 5x$
c (1, 5), (2, 10), (3, 15), (4, 20), (5, 25), (6, 30)
d


3 **a**

x	1	2	3	4	5	6
y	3	6	9	12	15	18

b (1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18)
c


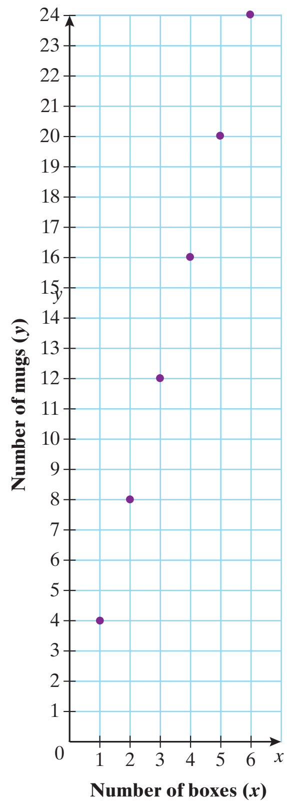
d number of pens equals three times the number of packets
e $y = 3x$

4 **a**

x	1	2	3	4	5	6
y	4	8	12	16	20	24

b (1, 4), (2, 8), (3, 12), (4, 16), (5, 20), (6, 24)
c number of mugs equals four times the number of boxes
d $y = 4x$

e



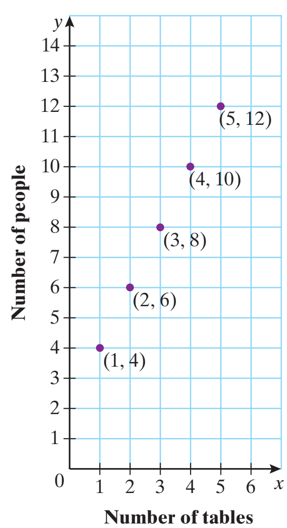
5 **a** 6

b

x	1	2	3	4	5
y	4	6	8	10	12

c (1, 4), (2, 6), (3, 8), (4, 10), (5, 12)

d

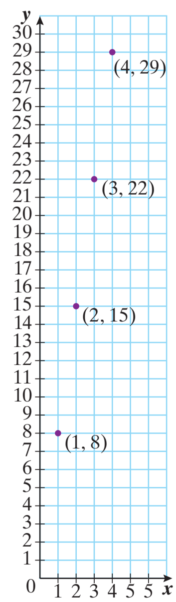


6 **a**

Number of flower beds (x)	1	2	3	4
Number of edge strips (y)	8	15	22	29

b (1, 8); (2, 15); (3, 22); (4, 29)

c



d 36 edge strips

e $y = 7x + 1$

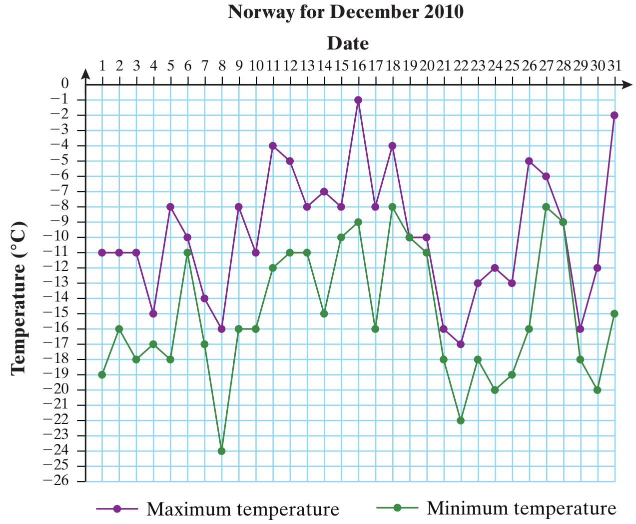
exercise 9.3

1 D

2 **a** 4 days **b** 19°C on 2 January
c 15 degrees **d** answers will vary

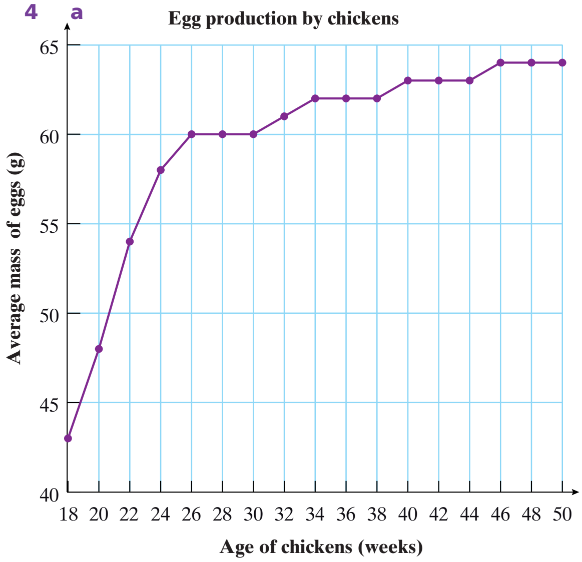
3 **a, b**

Maximum and minimum daily temperatures for Oslo, Norway for December 2010



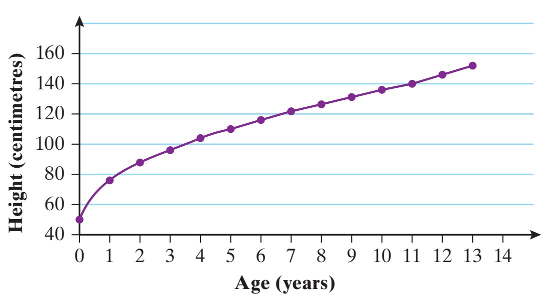
- **c** −24 °C
- **d** −17 °C
- **e** −8 °C
- **f** a difference of 13 degrees on 31 December
- **g** 14 days
- **h** 3 December
- **i** 11 degrees

4 **a** Egg production by chickens



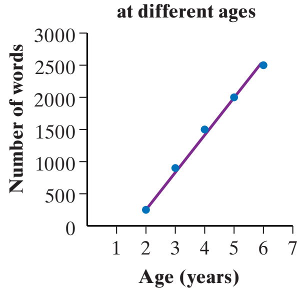
- **b** The chickens did not lay eggs before 18 weeks.
- **c** The mass of the eggs increased steadily from 18 weeks to 24 weeks.
- **d** After 24 weeks the increase in the mass of eggs was slower. The eggs would eventually reach a maximum mass and would not be any larger.

5 **a** Grace's height since birth

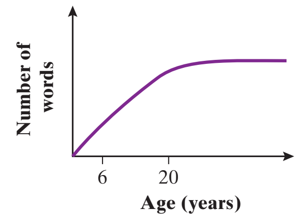


- **b** about 70 cm

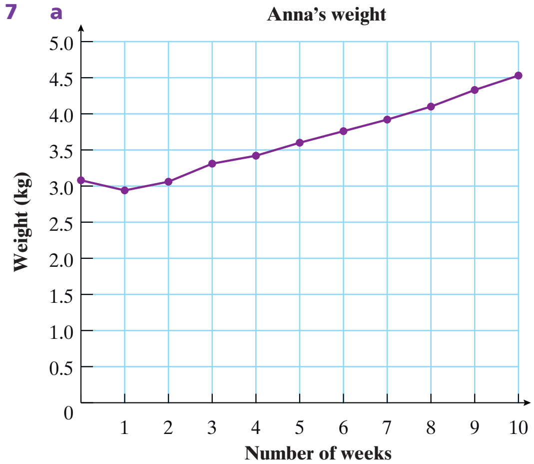
6 **a** Average number of words at different ages



- **b** approximately 3000
- **c** no, because the graph may not continue in a straight line—growth in vocabulary may increase faster during secondary school years then slow down
- **d**



7 **a** Anna's weight



- **b** there was a small decrease in Anna's weight after one week, then her weight increased fairly steadily

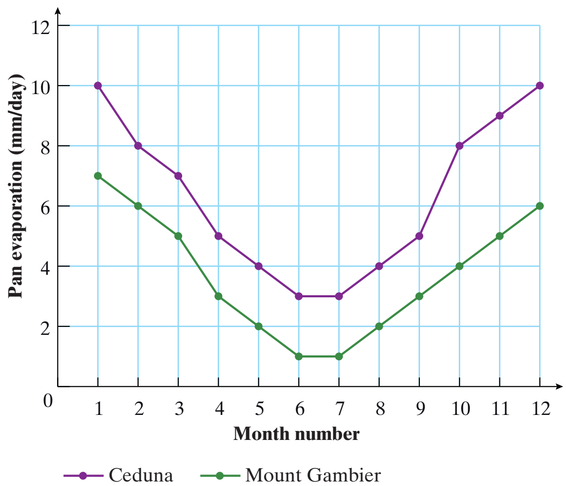
8
- **a** 1 h (or 60 min)
- **b** 125 km
- **c** 1.5 h (or 90 min)
- **d** 25 km
- **e** 30 min
- **f** 50 km
- **g** 30 min
- **h** 1 h (or 60 min)
- **i** 6 h

9
- **a** 60 min
- **b** 5 km
- **c** 60 min
- **d** They may have forgotten to post the letter so they return to Seaville.
- **e** They walk fastest when they walk from the campsite to Seaville because they walk 5 km in 1 hour. When they walk back from Seaville and return again to post the letter, they walk 2 km in half and hour, which is only 4 km per hour. When they return home they walk 3 km in 1 hour.
- **f** 3.5 h

10
- **a** month 4 (April)
- **b** 5 months
- **c** month 5 (May
- **d** 4 months
- **e** month 10 (October)
- **f** pan evaporation was not over 15 mm per day for any months

g months 2 and 3 (February and March)
h 11 months
i Alice Springs is much drier than Darwin. The hottest months, over the summer, are when evaporation is greatest. In Darwin there is high humidity, particularly in the monsoon season, so the evaporation rate is lower.

11 a **Evaporation rates in Cedune and Mount Gambier**

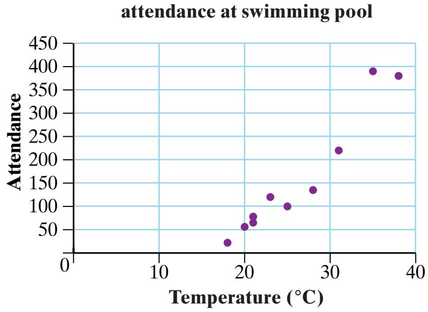


b in the summer months (October – February)
c in the winter months (May to August)
d in the hot months when they need more water for irrigation, they are losing water by evaporation at a greater rate
e Ceduna is a hotter place than Mount Gambier

exercise 9.4

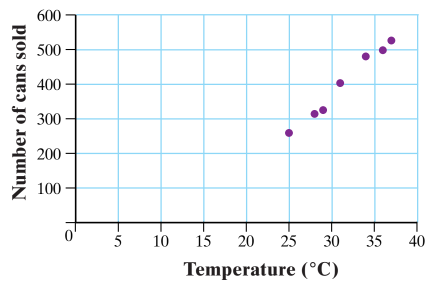
1 a There is a slight tendency for taller people to have longer feet, but it is not a strong relationship.
b no, because the relationship is not strong

2 a **Maxmimum daily temperature vs. attendance at swimming pool**



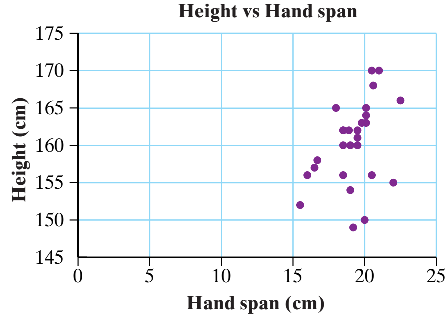
b There appears to be a strong relationship: the hotter the day, the higher the attendance at the swimming pool.

3 a **Temperature vs Number of cans sold**



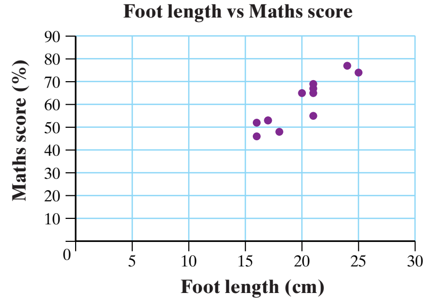
b There is a strong relationship: the higher the temperature, the higher the number of cans sold.

4 a **Height vs Hand span**



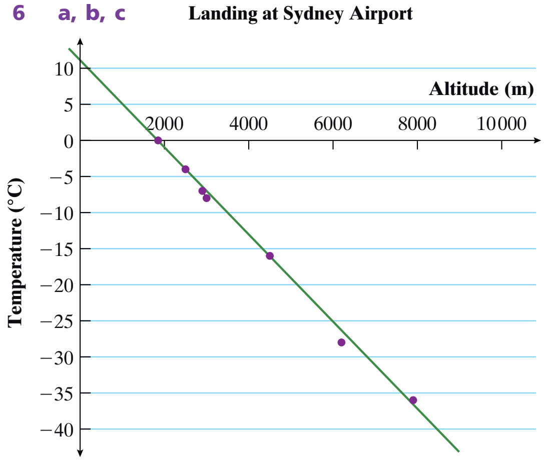
b There appears to be a weak relationship.
c There is only a weak relationship so handspan would not be a very good predictor of height.

5 a **Foot length vs Maths score**



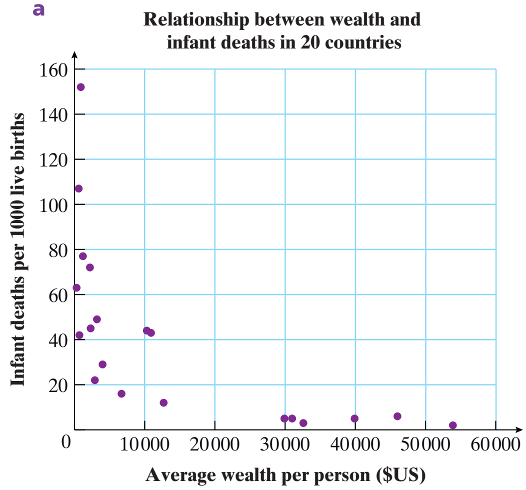
b There is a strong association: the longer the foot the higher the Maths score.
c Answers will vary, but a possible reason is that those with longer feet are older and so their higher Maths score could be related to that not their foot length.
d Even though a graph may show a strong relationship between two variables, we cannot assume that one is caused by the other. Both might be related to some other variable.

6 **a, b, c**



d 12°C
e approximately 6°C

7 **a**



b in countries with greater wealth there are fewer infant deaths compared with poor countries where there are high rates of infant deaths
c Iran and South Africa had higher infant death rates than we might expect for the average wealth
d e.g. food shortages, poor medical facilities

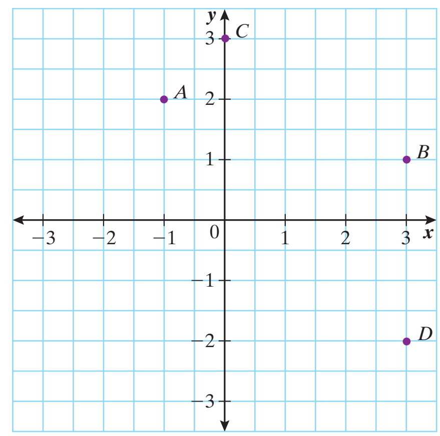
Revision

1 C **2** C **3** D **4** A **5** D

6 x-coordinate $= 3$, y coordinate $= 7$

7 $A = (2, 3)$, $B = (0, 2)$, $C = (1, -2)$, $D = (-3, 0)$

8



9 **a** x-axis **b** y-axis **c** (2, 4)

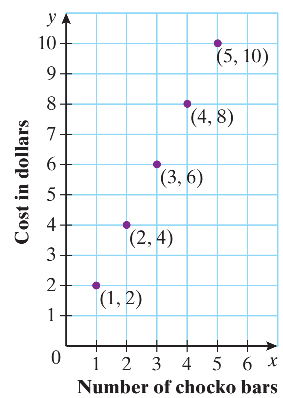
10 **a** A (−3, 2) **b** B (5, 10)

11 **a**

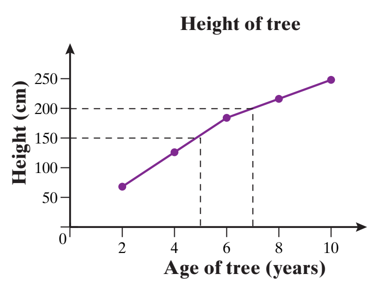
x	1	2	3	4	5
y	2	4	6	8	10

b (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)

c



12 **a**



b 150 cm
c 7 years
d The measurements weren't taken and so we are using the other measurements to predict what happened in between.
e approximately 270 cm

13 There is a strong relationship. It appears that the taller the player is, the heavier they tend to be.

Chapter 10

exercise 10.1

1

Millimetre	the length of a small ant
Centimetre	the height of your Mathematics textbook
Metre	the length of a netball court
Kilometre	the distance from Adelaide to Perth

2 **a** metre **b** kilometre **c** millimetre
d centimetre **e** metre **f** metre
g millimetre **h** centimetre **i** centimetre
j centimetre **k** kilometre
l centimetre (or metre)

3 answers will vary

4 **a** centimetre **b** decimetre **c** millimetre
d nanometre **e** millimetre

5 **a** five thousand metres
b two hundredths of a metre
c six thousandths of a metre
d eight thousand metres
e three tenths of a metre
f eleven thousandths of a metre
g twelve hundredths of a metre
h nine tenths of a metre
i two hundredths of a metre
j eleven thousand metres
k eight tenths of a metre
l five thousandths of a metre

6 **a** five metres **b** two metres
c three metres **d** eight kilometres
e seven metres **f** four kilometres
g three metres **h** thousand kilometres
i five metres **j** eight metres
k seventy metres **l** twelve metres

7 answers may vary, but three examples are century = 100 years, 100 cents = $1, centurion = commander of 100 soldiers in the roman army

8 **a** 5 m **b** 20 cm **c** 18 mm
d 3 km **e** 20 m **f** 3.6 mm
g 4.5 cm **h** 6.3 m **i** 25.6 km
j 11 cm **k** 7 mm **l** 2.5 km

9 **a** six centimetres **b** nine metres
c one millimetre **d** eight kilometres
e three millimetres **f** seven metres
g two kilometres **h** ten centimetres

10 **a** 11 000 m **b** 4300 cm **c** 180 mm
d 340 m **e** 3500 mm **f** 2400 mm
g 238 mm **h** 1400 m **i** 18 400 mm
j 14 000 cm **k** 80 cm **l** 20 000 cm
m 48.6 mm **n** 450 mm **o** 45 m

11 **a** 72.4 cm **b** 4.5 km **c** 0.15 m
d 3.2 km **e** 2.5 cm **f** 0.58 m
g 1.68 cm **h** 0.14 cm **i** 2.4 km
j 1.84 cm **k** 650 cm **l** 0.18 km
m 1.3 cm **n** 0.006 m **o** 23.5 m

12 **a** 189 mm **b** 37.75 m **c** 0.065 km
d 4 mm **e** 0.4 mm **f** 1.87 m
g 75 cm **h** 25 m **i** 95.8 cm
j 1.638 m **k** 36.84 m **l** 38 650 mm
m 1.8 km **n** 60 m **o** 19.485 m

13 **a** 58 mm **b** 14.3 cm **c** 129 mm
d 2.46 m **e** 305 cm **f** 1320 mm
g 0.679 m **h** 2570 m **i** 3.275 m
j 1.024 km

14 D

15 A

16 **a** **i** 3.7 cm **ii** 37 mm
b **i** 2.8 cm **ii** 28 mm
c **i** 7.4 cm **ii** 74 mm
d **i** 9.2 cm **ii** 92 mm

17 **a** 55 mm **b** 5.5 cm

18 **a** **i** 500 000 cm **ii** 5 km
b 75 pairs of jeans

19 **a** 55 cm, 23.5 cm, 24.5 cm
b 2.04 m by 0.82 m

20 **a** 1150 m = 1.15 km **b** 50 laps

21 **a** 53 mm **b** 600 mm **c** 2200 mm

22 **a** = **b** > **c** = **d** <
e = **f** > **g** < **h** <
i > **j** = **k** = **l** <

23 **a** 7.6 km **b** 7600 m

24 **a** 2.228 km **b** 4 times **c** 11.032 km

25 **a** 20 000 millipedes
b The name suggests that they have 1000 legs. They do have a lot of legs, but not 1000!

26 no, Will would only get 56 slices

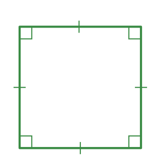
exercise 10.2

1 **a** 458 mm **b** 222.4 m

2 **a** 72 m **b** 78 cm
c 18.0 m **d** 142 m
e 114.4 m **f** 3.78 m or 378 cm
g 16.2 m or 1620 cm **h** 125.0 m
i 74.0 cm **j** 117.6 cm
k 93.6 cm **l** 1190 mm

3 **a** 3.18 m **b** 9.2 m **c** 5.96 m

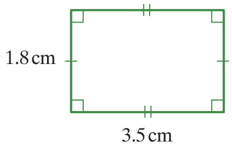
4 **a**



4.2 m

b 16.8 m

5 **a**



b 10.6 m

6 **a** 10 m **b** $a = 4, b = 3$ **c** 14 m

7 61 m

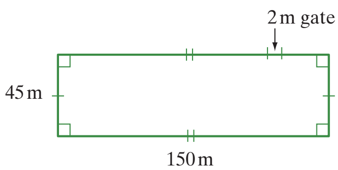
8 **a** 165 mm **b** 128 mm **c** 77 mm **d** 878 cm
e 11 m **f** 5.4 m **g** 18.32 m **h** 4 m
i 18.4 m **j** 5.66 m **k** 9.5 cm **l** 35 cm

9 **a** 202 cm **b** 256 m **c** 192.4 m
d 700 cm **e** 280.8 m **f** 459.2 m
g 109.0 m **h** 15.46 m **i** 157.2 m

10 124 cm

11 **a** 1120 cm **b** 11.2 m

12 **a** 309 m
b



c 388 m
d 1394 m

13 **a** 7 cm **b** 35 cm **c** 9.5 m
d 21.2 mm **e** 21.4 m **f** 26.5 mm

14 **a** 29 m **b** 189 cm **c** 8.1 m
d 7.4 m **e** 48 cm **f** 11 mm

15 27.5 m

16 **a** 14.4 m **b** 6.6 m, 4.6 m **c** 21.2 m

17 **a** 14.80 m **b** 13.96 m

18 The distance is the same. The combined distance Rani walks along Market Street and South Street is equal to the distance Suri walks along Station Road to the New Street corner. The combined distance Rani walks along Bank Street and New Street is the same as the distance Suri walks along Wood Street.

19 **a** 8 m **b** 50 plants

exercise 10.3

1 **a** 2 500 000 m^2 **b** 5000 cm^2
c 1800 mm^2 **d** 560 000 m^2
e 80 000 mm^2 **f** 24 000 mm^2
g 238 000 cm^2 **h** 1 400 000 m^2
i 1 300 000 mm^2 **j** 240 000 cm^2
k 8000 cm^2 **l** 200 000 000 cm^2

2 D

3 D

4 **a** 7.24 cm^2 **b** 4.5 km^2 **c** 1.5 m^2
d 32 cm^2 **e** 0.25 cm^2 **f** 0.58 m^2
g 160 cm^2 **h** 0.14 km^2 **i** 0.0024 km^2
j 0.184 cm^2 **k** 65 cm^2 **l** 180 m^2

5 A

6 C

7 **a** **i** 277 mm^2 **ii** 2.77 cm^2
b **i** 350 mm^2 **ii** 3.5 cm^2
c **i** 482 mm^2 **ii** 4.82 cm^2
d **i** 319 mm^2 **ii** 3.19 cm^2
e **i** 480 mm^2 **ii** 4.80 cm^2
f **i** 22 mm^2 **ii** 0.22 cm^2
g **i** 306 mm^2 **ii** 3.06 cm^2
h **i** 250 mm^2 **ii** 2.50 cm^2
i **i** 167 mm^2 **ii** 1.67 cm^2
j **i** 396 mm^2 **ii** 3.96 cm^2

8 **a** m^2 **b** cm^2 **c** mm^2
d km^2 **e** m^2 **f** cm^2

9 80 km^2

10 **a** 12 000 cm^2 **b** 1.2 m^2

11 **a** 0.35 km^2 **b** 350 000 m^2

exercise 10.4

1 **a** 25 cm^2 **b** 12 cm^2 **c** 45 cm^2
d 36 cm^2 **e** 36 cm^2 **f** 24 cm^2
g 28 cm^2 **h** 18 cm^2

2 **a** 45 cm^2 **b** 63 m^2 **c** 1500 cm^2
d 28 m^2 **e** 49 mm^2 **f** 168 m^2
g 18.02 cm^2 **h** 225 m^2 **i** 60 m^2
j 13.69 cm^2 **k** 6.82 m^2 **l** 3.78 m^2

3 **a** 62 370 mm^2 **b** 623.70 cm^2

4 Jay receives one mark for using the correct method but multiplying incorrectly. Kay receives zero for finding the perimeter not the area.

5 **a** 763 m^2 **b** 1651 m^2

6 **a** 1.0 m^2 **b** 120 m^2 **c** 120 strips

7 **a** 4 m **b** 12 cm **c** 8 cm **d** 30.5 cm
e 36 cm **f** 27 m **g** 45 mm **h** 23 km

8 **a** 9 m **b** 17 cm **c** 70 mm **d** 32 m
e 22 m **f** 37 mm **g** 2800 km **h** 175 mm

9 3.6 m

10 **a** 0.7 km **b** 700 m

11 3.5 m

12 **a** 2.7 m **b** 99 tiles

exercise 10.5

1 **a** 80 m^2 **b** 79 m^2 **c** 516 cm^2
d 19 m^2 **e** 3400 cm^2 **f** 78 cm^2
g 174 m^2 **h** 541.71 m^2

2 26.3 m^2

3 **a** 64 m^2 **b** 22 cm^2 **c** 17.5 m^2 **d** 16.6 m^2 **e** 1370 cm^2 **f** 1825 cm^2

4 18.4 m^2

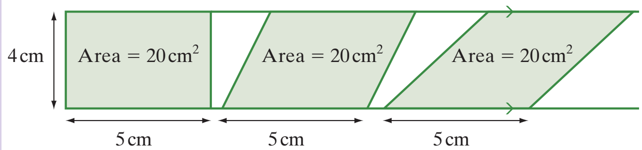
5 **a** 6.6 m^2 **b** 330 bricks

6 279 m^2

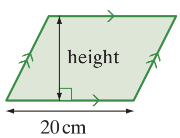
exercise 10.6

1 **a** 20 m^2 **b** 1100 cm^2 **c** 750 cm^2 **d** 192 m^2 **e** 576 000 mm^2 **f** 4968 cm^2 **g** 120.96 mm^2 **h** 643.8 cm^2 **i** 39.52 cm^2 **j** 3.96 m^2 **k** 104.2065 mm^2 **l** 0.9135 m^2

2

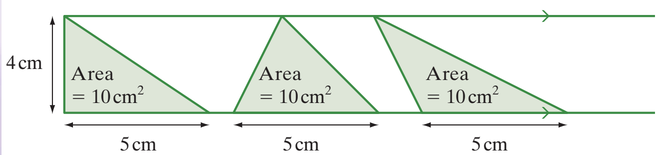


3 **a**



b 14 cm

4



a the areas are all the same, 10 cm^2

5 **a** 9 m^2 **b** 936 cm^2 **c** 522 m^2 **d** 805 cm^2 **e** 8.16 cm^2 **f** 1440 mm^2 **g** 17.25 m^2 **h** 1638 cm^2 **i** 1.5 m^2 **j** 85.68 m^2 **k** 7.74 m^2 **l** 43.4 cm^2 **m** 312.5 mm^2 **n** 27.95 cm^2 **o** 116.84 mm^2

6 173cm^2

7 **a** 3 000 000 m^2 **b** 3 km^2

8 **a** 7.14 m^2 **b** 6.70 m^2

9 6.2 m^2

10 1080 cm^2

11 No, the area is not the same because the perpendicular height of the parallelogram is less than the height of the rectangle. The rectangular frame encloses a larger area.

12 208 m^2

exercise 10.7

1 **a** 25 000 mm^3 **b** 0.0003 m^3 **c** 0.3 cm^3 **d** 450 cm^3 **e** 1 800 000 cm^3 **f** 70 000 mm^3 **g** 50 m^3 **h** 1 200 000 cm^3 **i** 8000 mm^3 **j** 0.0024 m^3 **k** 2 cm^3 **l** 350 cm^3 **m** 1 500 000 000 mm^3 **n** 4000 mm^3 **o** 5 600 000 000 m^3

2 C

3 **a** 58 000 mL **b** 12 kL **c** 2400 mL **d** 48 kL **e** 0.00375 L **f** 900 L **g** 0.720 kL **h** 237 L **i** 0.018 L **j** 0.015 kL **k** 670 000 kL **l** 1400 L **m** 0.00018 kL **n** 5.820 L **o** 17.5 kL **p** 4500 kL

4 **a** 25 mL **b** 1400 cm^3 **c** 40 kL **d** 2 L **e** 5 mL **f** 60 m^3 **g** 8000 cm^3 **h** 7500 mm^3 **i** 3.6 mL **j** 240 mL

5 **a** cm^3 **b** kL **c** m^3 **d** L **e** cm^3 **f** m^3 **g** mL **h** mm^3

6 9.5 L

7 **a** 500 mL **b** 0.5 L

8 3 L

exercise 10.8

1

	i	ii	iii
a	9	4	36 cm^3
b	9	3	27 cm^3
c	4	2	8 cm^3
d	16	2	32 cm^3
e	12	3	36 cm^3
f	25	5	125 cm^3
g	12	4	48 cm^3
h	6	6	36 cm^3
i	18	3	54 cm^3

2 **a** 64 cm^3 **b** 125 m^3 **c** 60 m^3 **d** 210 m^3 **e** 30 m^3 **f** 384 mm^3 **g** 315 m^3 **h** 35 cm^3 **i** 19.38 m^3 **j** 720 cm^3 **k** 0.036 m^3 **l** 0.2975 m^3

3

	i	ii
a	135 cm^3	135 mL
b	960 cm^3	960 mL
c	80 cm^3	80 mL
d	24 192 cm^3	24 192 mL
e	14 400 cm^3	14 400 mL
f	46 800 cm^3	46 800 mL

4

	i	ii
a	8 m^3	8 kL
b	1537.5 m^3	1537.5 kL
c	18 m^3	18 kL
d	2.964 m^3	2.964 kL
e	0.288 m^3	0.288 kL
f	0.01701 m^3	0.01701 kL

5 Check with your teacher. Four possible sets are as follows:
$120 \times 1 \times 1$, $60 \times 2 \times 1$, $30 \times 2 \times 2$, $10 \times 3 \times 4$

6 6 m^3

7 a i $2400\,cm^3$ ii $1920\,cm^3$
b tin i

8 $60\,000\,cm^3$

9 $99.84\,m^3$

10 $41.6\,m^3$

11 $38.064\,m^3$

12 a $250\,cm^3$ b 250 mL

13 a i $52\,500\,cm^3$ ii 52.5 L
b i $15\,000\,cm^3$ ii 15 L

14 a $1923\,cm^3$
b 2 L
c i Assuming 10 flushes per day, the saving per day would be 20 L
ii Assuming 10 flushes per day, the saving per year would be 7300 L

15 a $17.5\,m^3$ b 17 500 000 mL
c 17 500 L d 17.5 kL

16 check with your teacher

17 a $180\,km^3$ b $1802\,km^3$

18 24

Revision

1 D 2 A 3 E 4 A 5 C

6 a 8.8 m b 224 tiles

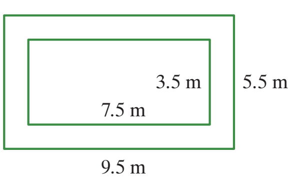
7 a 5.2 m b 24 cm

8 a $1750\,cm^2$ b $9\,m^2$

9 a $33\,m^2$ b $231 c 26 m

10 a $1614\,cm^2$ b $1765\,cm^2$

11 a $26.25\,m^2$
b



c $26\,m^2$

12 a $8640\,cm^3$ b 8.64 L

Chapter 11

exercise 11.1

1 a i 22% ii 78%
b i 52% ii 48%
c i 48% ii 52%
d i 10% ii 90%
e i 63% ii 37%
f i 25% ii 75%

2

Month	Small	Medium	Large
January	20%	50%	30%
February	10%	45%	45%
March	40%	35%	25%
April	18%	49%	33%
May	22%	31%	47%
June	21%	43%	36%

3 a

Model	First class seats	Business class seats	Economy class seats
A330-200	5%	14%	81%
A330-300	4%	14%	82%
A340-200	5%	10%	85%
A340-300	5%	14%	81%
A340-500	4%	13%	83%
A340-600	3%	14%	83%

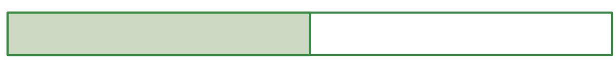
b economy c it is more affordable

4 B

5 a 25% b 100% c 50%
d 75% e 40% f 10%

6 a 36% b 50% c 70% d 80%

7 a



b

c

d

e

f

8 a you will be totally satisfied
b it is completely eco-friendly

exercise 11.2

1 **a** $\frac{17}{100}$ **b** $\frac{29}{100}$ **c** $\frac{3}{10}$ **d** $\frac{6}{25}$
e $\frac{4}{5}$ **f** $\frac{24}{25}$ **g** $1\frac{1}{2}$ **h** $\frac{1}{20}$
i $\frac{3}{20}$ **j** $\frac{9}{25}$ **k** $\frac{19}{50}$ **l** $2\frac{2}{5}$
m $\frac{13}{20}$ **n** $\frac{3}{4}$ **o** $1\frac{3}{4}$ **p** $\frac{1}{25}$

2 **a** $\frac{1}{40}$ **b** $\frac{1}{16}$ **c** $\frac{7}{200}$ **d** $\frac{3}{8}$
e $\frac{1}{3}$ **f** $\frac{2}{3}$ **g** $\frac{5}{8}$ **h** $\frac{1}{80}$

3 **a** 0.35 **b** 0.7 **c** 0.25 **d** 0.92
e 1.44 **f** 0.3 **g** 0.03 **h** 0.8
i 0.08 **j** 0.23 **k** 2.3 **l** 0.023
m 0.85 **n** 0.085 **o** 1.15 **p** 0.12

4 **a** **i** $\frac{1}{5}$ **ii** 0.2 **b** **i** $\frac{7}{10}$ **ii** 0.7
c **i** $\frac{3}{4}$ **ii** 0.75 **d** **i** $\frac{3}{20}$ **ii** 0.15
e **i** $\frac{9}{20}$ **ii** 0.45 **f** **i** $\frac{27}{50}$ **ii** 0.54
g **i** $\frac{16}{25}$ **ii** 0.64 **h** **i** $\frac{18}{25}$ **ii** 0.72

5 **a** 37% **b** 810% **c** 30% **d** 70%
e 90% **f** 79% **g** 12.7% **h** 37.5%

6 **a** 55% **b** 28% **c** 64% **d** 64%
e 75% **f** 95% **g** 92% **h** $62\frac{1}{2}\%$
i 38% **j** 84% **k** 54% **l** $87\frac{1}{2}\%$
m 16% **n** $37\frac{1}{2}\%$ **o** $31\frac{1}{4}\%$ **p** $93\frac{3}{4}\%$

7 **a** 140% **b** 250% **c** 125% **d** 130%
e 370% **f** 175% **g** 225% **h** 140%

8 **a** 85% **b** 12% **c** 48% **d** 40%
e 4% **f** 99% **g** 70% **h** 7%
i 0.7% **j** 65% **k** 6.5% **l** 140%
m 165% **n** 225% **o** 100% **p** 300%

9

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$

10

Fraction	Decimal	Percentage
$\frac{7}{10}$	0.7	70%
$\frac{2}{25}$	0.08	8%
$\frac{2}{5}$	0.4	40%
$\frac{17}{50}$	0.34	34%
$\frac{3}{20}$	0.15	15%
$\frac{6}{25}$	0.24	24%
$\frac{3}{50}$	0.06	6%
$\frac{3}{10}$	0.3	30%
$\frac{4}{5}$	0.8	80%
$\frac{3}{25}$	0.12	12%

11 D
12 E
13 E
14 **a** 30% **b** 42% **c** 67%
15 **a** $\frac{1}{20}$ **b** 5%
16 **a** **i** $0.\dot{8}$ **ii** $88.\dot{8}\%$
b $\frac{1}{9}$ **c** $11.\dot{1}\%$

exercise 11.3

1 **a** 45% **b** 62% **c** 40% **d** 60%
e 90% **f** 24% **g** 70% **h** 86%

2 **a** 30% **b** 36% **c** 40% **d** 45%
e 58% **f** 60% **g** 60% **h** 60%
i 40% **j** 35% **k** 48% **l** 40%

3 **a** 27% **b** 39% **c** 57% **d** 53%
e 35% **f** 9% **g** 29% **h** 28%
i 19% **j** 14% **k** 35% **l** 57%

4 C
5 C
6 **a** $\frac{23}{100}$ **b** 23%
7 **a** 36% **b** 64% **c** 32%
8 **a** 15% **b** 20% **c** 65%
9 **a** 25% **b** 20% **c** 20% **d** 18.75%
10 **a** 56%
b 44%
c bus = 52%, not bus = 48%

exercise 11.4

1 a 24 b 400 c 4 d 10
e 10 f 180 g 6000 h 13

2 a 21 b 50 c 200 d 300
e 80 f 120 g 3000 h 100

3 a 36 b 84 c 168 d 288
e 720 f 540 g 96 h 42
i 250 j 1200 k 448 l 1080

4 a 40 b 217 c 30 d 998
e 700 000 f 136 g 232 h 438
i 1018 j 546 k 15 l 46
m 4680 n 46 o 273 p 1042

5 a 4% b 10 g

6 a 369 girls b 53%

7 a 3.6 MB b 3.9 MB

8 $67.50

9 caramel = 6, lime = 24, strawberry = 30 and chocolate = 60

10 a $\frac{1}{10}$ b $\frac{9}{10}$ c $54

11 a $\frac{1}{5}$ b $\frac{4}{5}$ c $68

12 a $\frac{1}{3}$ b $\frac{2}{3}$ c $76

13 ACT 342 800, NSW 4 389 300, NT 118 700, VIC 3 761 300, QLD 1 968 500, SA 1 177 300, TAS 204 300, WA 1 587 800

14 $18.04

exercise 11.5

1 a 1100 b 1200 c 1800 d 4500
e 5600 f 8300 g 12 500 h 58 000
i 16 000 j 5600 k 32 000 l 34 500

2 20

3 650

4 192

5 $20

6 $60

7 a 80% b $1050

8 129 150 ML

exercise 11.6

1 a 5:9 b 9:5 c $\frac{5}{14}$ d $\frac{9}{14}$

2 D

3 a 1:10 b 12:11 c 1:1 d 2:1

4 a 5:7 b 5:2 c $\frac{5}{7}$ d $\frac{2}{7}$

5 a 9:7 b 9:16 c $\frac{9}{16}$ d $\frac{7}{16}$

6 a 9:11 b 9:20 c $\frac{9}{20}$ d 45%

7 a 3:2:1 b 1:3 c 2:3
d 1:6 e $\frac{1}{6}$

8 200:77

9 30:23

10 300:197

11 Nancy $340; Jack $612; Emily $748

exercise 11.7

1 a i 1:4 ii 4:1 iii 4:5
b i 3:1 ii 1:3 iii 1:4

2 a 2:3 b $\frac{2}{5}$ c $\frac{3}{5}$

3 a 1:3 b 1:4 c 2:5 d 5:2
e 7:3 f 7:4 g 1:2 h 5:6
i 9:4 j 11:8 k 1:3 l 4:5
m 3:4 n 4:1 o 7:2 p 11:4

4 a 4:6:3 b 4:13 c $\frac{4}{13}$ d 4:3 e 2:3

5 a 4:6; 8:12 b 3:4; 9:12
c 15:18; 25:30 d 3:7; 9:21
e 36:20; 72:40 f 12:20; 6:10
g 15:35; 9:21 h 12:30; 10:25
l 25:15; 50:30 j 14:21; 2:3

6 B

7 a 3:5 b $\frac{3}{8}$ c $\frac{5}{8}$

8 a i 400 g to 1000 g
ii 400:1000
iii 2:5
b i 600 mL to 1000 mL
ii 600:1000
iii 3:5
c i 750 mm to 1000 mm
ii 600:1000
iii 3:4
d i 45 c to 100 c
ii 45:100
iii 9:20
e i 250 g to 1000 g
ii 250:1000
iii 1:4
f i 60 cm to 240 cm
ii 60:240
iii 1:4
g i 24 c to 200 c
ii 24:200
iii 3:25
h i 40 seconds to 120 seconds
ii 40:120
iii 1:3

9 B

10 C

11 a 2:4:5 b $\frac{2}{11}$ c 18%

12 **a** 6:1 **b** 4:1 **c** 12:2:3
d 8:15 **e** 2:5 **f** 2:3
g 12:2:3:1

13 **a** 3:1 **b** 25% **c** 1:3:46
d 92% **e** 70% **f** 90%

Revision

1 D **2** A **3** E **4** C **5** D

6

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
75%	$\frac{3}{4}$	0.75
40%	$\frac{2}{5}$	0.4
17%	$\frac{17}{100}$	0.17
$66.\dot{6}\%$	$\frac{2}{3}$	$0.\dot{6}$
6%	$\frac{3}{50}$	0.06

7 $800

8 $2125

9 220 g

10 **a** 85%
b $86.\dot{6}\%$
c the second assignment

11 **a** 4:3 **b** 3:4 **c** 4:7
d $\frac{4}{7}$ **e** $\frac{3}{7}$

12 **a** 3:2 **b** 2:5 **c** $\frac{3}{5}$
d $\frac{2}{5}$ **e** 60% **f** 40%

13 8:5

14 **a** 4:5 **b** 18:1

Chapter 12

exercise 12.1

1 **a** *U* **b** *P* **c** *I* **d** *I*
e *C* **f** *P* **g** *I* **h** *C*

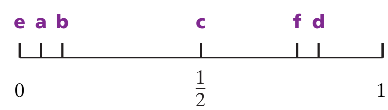
2 A

3

Fraction	Decimal	Percentage
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{2}{5}$	0.4	40%
$\frac{1}{2}$	0.5	50%
$\frac{99}{100}$	0.99	99%

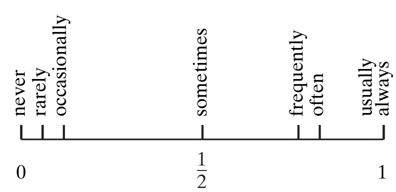
4 **a** 0 **b** 1 **c** 1
d $\frac{1}{2}$ **e** 1 **f** 0

5



6 answers will vary

7 The words can be confusing because different people interpret them differently.



exercise 12.2

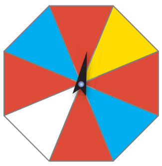
1 **a** **i** S = {red,yellow} **ii** 3
b **i** S = {red,blue,yellow} **ii** 4
c **i** S = {red,blue,yellow} **ii** 5
d **i** S = {red,blue,yellow} **ii** 6
e **i** S = {red,blue} **ii** 7
f **i** S = {red,blue,yellow} **ii** 8

2 **a** **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** 0
b **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$
c **i** $\frac{2}{5}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{5}$
d **i** $\frac{1}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{3}$
e **i** $\frac{4}{7}$ **ii** 0 **iii** $\frac{3}{7}$
f **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$

3

Colour	Number of sides	Probability
Red	3	$\frac{1}{2}$
Yellow	2	$\frac{1}{3}$
Blue	1	$\frac{1}{6}$
Total	6	1

4



5 D

6 **a** 1, 2, 3, 4, 5, 6 **b** 3 **c** $\frac{1}{2}$

7 **a** 8

b **i** $\frac{1}{8}$ **ii** $\frac{1}{2}$

iii $\frac{1}{2}$ **iv** $\frac{5}{8}$

v $\frac{3}{8}$

8 **a** $\frac{7}{12}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$

9 **a** no, there are 4 heart cards but only 2 suns

b the sun and the cross have the same probability are they have 2 cards each

c $\frac{1}{4}$

d $\frac{7}{8}$

10 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$

11 **a** $\frac{1}{4}$ **b** $\frac{1}{13}$ **c** $\frac{1}{2}$ **d** $\frac{7}{13}$ **e** $\frac{1}{26}$

12 **a** $\frac{3}{13}$ **b** $\frac{2}{13}$ **c** $\frac{6}{13}$ **d** $\frac{9}{13}$

13 **a** $\frac{1}{12}$ **b** $\frac{7}{12}$ **c** $\frac{1}{8}$ **d** $\frac{5}{8}$

14 **a**

Event	Vanilla	Lemon	Orange	Lime	Cherry
Probability	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{2}{21}$	$\frac{4}{21}$	$\frac{2}{7}$

b $\frac{2}{7}$ **c** orange **d** no

exercise 12.3

1 **a** $\frac{3}{4}$ **b** 30

c answers will vary

2 **a** answers will vary

b answers will vary

c red — 15, blue — 10, yellow — 10, green — 5

d

Results from 40 trials	Blue	Red	Yellow	Green
Actual frequency	Answers will vary			
Expected frequency	10	15	10	5

e

Results from 40 trials	Blue	Red	Yellow	Green
Relative frequency	Answers will vary			
Theoretical probability	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

f answers will vary

3 answers will vary

4 **a** **i** 10 **ii** 30

b **i** 15 **ii** 32

c **i** $\frac{1}{4}$ **ii** $\frac{8}{15}$

5 **a**

Suit	♥	♣	♦	♠
Theoretical probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b

Suit	♥	♣	♦	♠
Experimental probability	$\frac{3}{10}$	$\frac{13}{60}$	$\frac{1}{5}$	$\frac{17}{60}$

6 **a** $\frac{1}{2}$

b 10 heads, 0 tails
8 heads, 2 tails
3 heads, 7 tails, etc.

c it is randomly giving you a 0 or a 1

d If the random number comes up 1, then 'Heads' will be inserted in cell B1. Otherwise 'Tails' will be inserted.

e The first counts the number of 1's in A1 to A10 and puts it in D2. The second counts the number of 0's in A1 to A10 and puts it in E2.

f answers will vary

g **i** expect 50 heads and 50 tails

ii expect 500 heads and 500 tails

h answers will vary

i answers will vary

7 **a** $\frac{1}{6} = 0.1\dot{6}$ yes, all numbers have the same probability of being rolled
b answers will vary
c The formula in cell A1 is =RANDBETWEEN(1,6). This formula produces a random whole number from 1 to 6.
d Replace 'first' with 'formula in cell C5' and replace 'second' with 'formula in cell D5'
e–i answers will vary

8 **a** For example:

Blue	Red	Yellow
1	2	3

b =RANDBETWEEN(1,3)
c =COUNTIF(A1:A3, 1), =COUNTIF(A1:A30, 1), =COUNTIF(A1:A300, 1), then the same for 2 and 3
e $\frac{1}{3}$
f answers will vary

Revision

1 E **2** D **3** A **4** C **5** D

6 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$

7 **a** red, blue, yellow, green **b** no
c **i** $\frac{1}{4}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{3}{4}$
d 150

8 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{5}{12}$

9 **a** $\frac{1}{10}$ **b** $\frac{1}{50}$ **c** $\frac{22}{25}$
d e.g. 55, 1, 64, or 77, 81, 100

10 **a** 2 days **b–f** answers will vary

Chapter 13

exercise 13.1

1

Numerical data	Categorical data
age in years	year level at school
students' heights	your favourite subject
time spent on homework last night	your favourite TV show
number of children in your family	your postcode
the number showing on a die	style of music
quantity of leaves on a tree	
your arm span in centimetres	

2 **a** How many hours of television do you watch each night?
b How many pieces of fruit do you each day?
c What is your height in centimetres?

3 **a** Unbiased
b Biased, because it implies that if you consider yourself normal you should answer yes.
c Biased, because after being told it is important to drink water it is likely that you would over estimate how many glasses of water you drank yesterday
d Unbiased
e Unbiased
f Biased, the use of the words warm and rich implies that you should like them
g Biased, because after being told it is important to visit the dentist it is likely that you would over estimate how often you visited the dentist
h Biased, the wording implies that it is a great success and so you would be more likely to say yes

4 **a** B **b** C **c** A

5 How many minutes do you spend on homework each night?

6 **a** 30%
b Half of the total number who filled out the questionnaire did not respond and if they had it would change the results greatly. Also need to know why they didn't answer: were people embarrassed to answer or were they unsure how to define short.
c What is your height in centimetres?

exercise 13.2

1 **a**

Favourite take-away food	Tally	Frequency
Chicken		0
Chips/fries	卌 卌 ||	12
Fish (e.g. fish and chips)	||||	4
Fruit/fruit salad	||	2
Hamburgers	||||	4
Kebabs/wraps	卌 |	6
Noodle dishes	||	2
Pies/pasties	|	1
Pizza/pasta	卌 卌 ||	12
Rice dishes (e.g. sushi)	||	2
Rolls/sandwiches	||	2
Salads		0
Other	||	2
None		0

b Chips/fries and pizza/pasta
c 49 students
d 24.5%

2 a

Number	Tally	Frequency
1	𝍸 𝍷𝍷	7
2	𝍷𝍷𝍷𝍷	5
3	𝍸 𝍷𝍷𝍷𝍷	9
4	𝍸 𝍷	6
5	𝍸	5
6	𝍸 𝍷𝍷𝍷	8

b The results are quite evenly distributed with slightly more 3's and 6's and fewer 2's and 5's.

3 a

Hair colour	Tally	Frequency
Blonde	𝍸 𝍷𝍷	7
Red	𝍷𝍷𝍷	3
Brown	𝍸 𝍸	10
Black	𝍸 𝍷𝍷𝍷	8

b brown

4 a

Method of travel	Tally	Frequency
bicycle	𝍷𝍷𝍷𝍷	4
bus	𝍸 𝍷𝍷𝍷𝍷	9
car	𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	19
skateboard	𝍷	1
walk	𝍸 𝍷	6
other	𝍷	1

b car
c 47.5%
d Answers will vary, for example, to decide how many buses to provide or how much bicycle parking space will be needed.

5 a

Colour	Tally	Frequency
Red	𝍸	5
Blue	𝍸 𝍷𝍷𝍷	8
Pink	𝍸 𝍷	6
Yellow	𝍷𝍷	2
Orange	𝍷𝍷𝍷	3
Purple	𝍸 𝍷	6

b blue **c** 30 **d** $\frac{1}{6}$

6 C

7 a, b

Height (cm)	Tally	Frequency
130–134	𝍷𝍷	2
135–139	𝍸 𝍷	6
140–144	𝍸 𝍷	6
145–149	𝍸 𝍷𝍷𝍷𝍷	9
150–154	𝍸	5
155–159	𝍷	1
160–164	𝍷	1

c 145–149 cm

8 a 175–179 cm **b** 4 **c** 165–169 cm

9 a, b

Height (cm)	Tally	Frequency
5–9	𝍸 𝍷	6
10–14	𝍸 𝍷𝍷	7
15–19	𝍸 𝍷𝍷	7
20–24	𝍸	5
25–29	𝍷𝍷	2
30–34	𝍷	1

c 30–34 cm

10 a minimum = 64, maximum = 116
b, c
i

Pulse rate (beats/min)	Tally	Frequency
60–64	𝍷	1
65–69	𝍷𝍷	2
70–74	𝍷𝍷	2
75–79	𝍷𝍷𝍷𝍷	4
80–84	𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷	28
85–89	𝍷	1
90–94	𝍸 𝍷	6
95–99	𝍷	1
100–104	𝍷𝍷	2
105–109		0
110–114		0
115–119	𝍷	1

b, c
ii

Pulse rate (beats/min)	Tally	Frequency
60–69	|||	3
70–79	卌 |	6
80–89	卌 卌 卌 卌 卌 ||||	29
90–99	卌 ||	7
100–109	||	2
110–119	|	1

d The results are spread from 60 to 119, but there is a bunching around 75 to 94. The most common pulse rate is between 80 and 84.

exercise 13.3

1 **a** 5.2 **b** 5.2 **c** 54.5 **d** 12.9
e 3.9 **f** 42.1 **g** 36.5 **h** 2.7

2 C

3 **a** 4 **b** 35 **c** 3 **d** 3 **e** 14
f 4 **g** 6 **h** 25 **i** 2.2 **j** 0.48

4 **a** 6 **b** 7.5 **c** 4 **d** 9 **e** 4.5
f 6 **g** 7 **h** 6.5 **i** 26 **j** 6

5 C

6 **a** 2 **b** 9 **c** 5 **d** 0
e 1 **f** 3 **g** 6 **h** 0.5
i 2, 3 **j** There is no mode.

7 **a** mean = 2.9, median = 3, mode = 3
b mean = 5, median = 3, mode = 3
c mean = 4, median = 4, mode = 3, 6
d mean = 8.4, median = 12, mode = 3, 13
e mean = 6.5, median = 7, mode = 7
f mean = 5.3, median = 5, mode = 3
g mean = 4.2, median = 4, mode = 2
h mean = 3.8, median = 3.1, mode = 2

8 **a** **i** mean = 3, median = 3
ii mean = 4, median = 3
b The median stayed the same but the mean rose by 1

9 **a** range = 19, median = 30
b range = 92, median = 67
c range = 81, median = 142
d range = 172, median = 76.5
e range = 12, median = 60
f range = 75, median = 135
g range = 0.9, median = 0.6
h range = 30.6, median = 85.7

10 70.9

11 **a** mean = \$800 000, median = \$400 000
b mean
c median

12 **a** 3
b 20
c mode = 15, median = 13.5, mean = 13.2
d mode = 15, median = 14, mean = 14

13 **a** No, different sports require different physical characteristics and so range is required
b answers will vary, but some examples are — very tall: basketball, high-jump, volleyball; very heavy: shot put, discus, hammer; very short: gymnastics; very light: gymnastics
c answers will vary, but examples would be volleyball and rowing

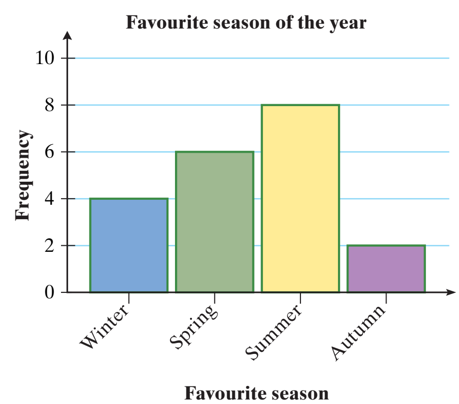
14 **a** 4 h 12 min
b 5 h
c three students (students who did 5 or 6 hours)
d mean = 5 h; median = 5 h
e one student (the student who did 6 hours)

15 69%

16 2 or 7 or 12

exercise 13.4

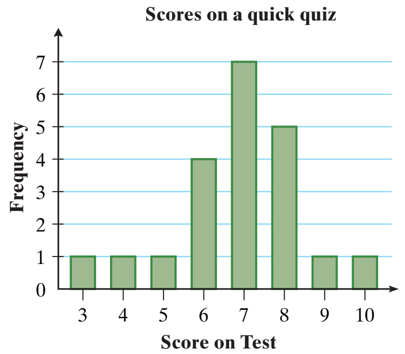
1 **a**



b It is quite likely that the time of year will influence the data. For example in the middle of July summer may be more highly favoured than in the middle of February.

2 **a** 21 students
b

Score	Tally	Frequency
3	|	1
4	|	1
5	|	1
6	||||	4
7	卌 ||	7
8	卌	5
9	|	1
10	|	1

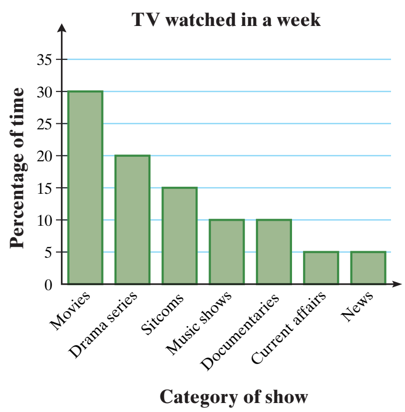
c 

d No, most students got six, seven or eight so it wasn't too difficult. There may have been one or two hard questions.

3 a 50%
b hot water
c refrigeration
d The heating would be less and the cooling would be higher, hot water would also probably be lower.

4 a January
b October
c February, March, May, June
d about 5.3 m
e about 3.6 m

5 a pop
b 35% of girls
c rock
d 20% of boys
e some of the music style names are long and with a bar graph the labels can be written horizontally

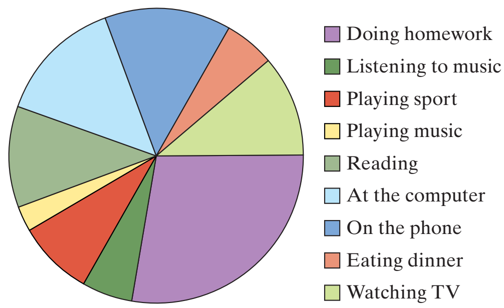
6 a 

b 20 hours

7 a 10°

b

Activity	Frequency	Angle
Watching TV	4	40°
Eating dinner	2	20°
On the phone	5	50°
At the computer	5	50°
Reading	4	40°
Playing music	1	10°
Playing sport	3	30°
Listening to music	2	20°
Doing homework	10	100°
Total	36	360°

c 

8

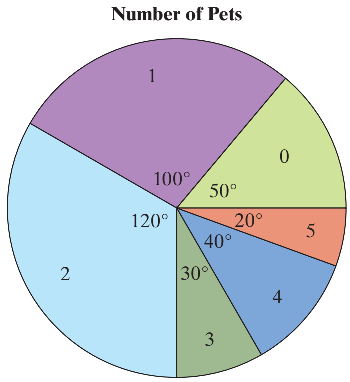
Activity	Hours	Angle
Sleeping	9	135°
Eating	3	45°
In class	5	75°
Sport	2	30°
TV	2	30°
Homework	2	30°
Other	1	15°

9 a number of pets, can take values from 0 to 5

b

Number of pets	Tally	Frequency
0	𝍸	5
1	𝍸 𝍸	10
2	𝍸 𝍸 II	12
3	III	3
4	IIII	4
5	II	2

c



10 **a** Numbers of students, because the total is 140 not 100 as it would be if they were percentages.
b 28 **c** bus **d** 20%

11 **a** \$121.50 **b** \$148.50
c Don't know how many DVDs he bought, just the amount of money he spent on them.

12 **a**

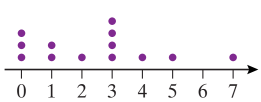
Children	Angle	Percentage
0	90°	25%
1	36°	10%
2	72°	20%
3	90°	25%
4	36°	10%
5	18°	5%
>5	18°	5%

b 25% **c** 5%

13 answers will vary — check with your teacher

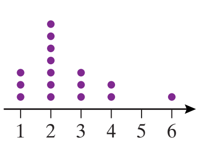
exercise 13.5

1 **a**

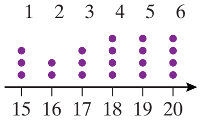


b

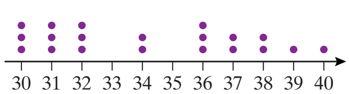
c



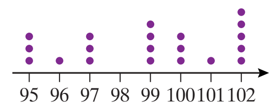
d



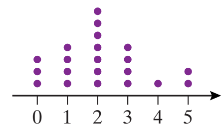
e



f

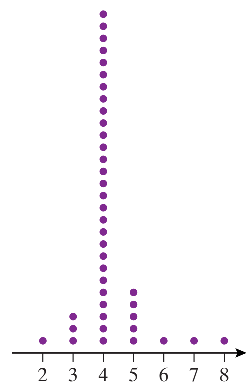


2 **a**



b median = 7 siblings
c range = 5
d 14 students had more than one sibling.

3 **a**



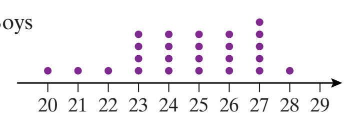
b 4
c 4.2

4 **a** 25 cm **b** 24 cm **c** 15 cm
d 14 cm **e** 11 cm
f **i** mode is still 25 cm
ii median is still 24 cm

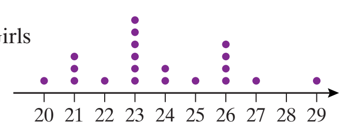
5 **a** 16 **b** 18 **c** 7 **d** 15

6 **a** They both have a mean of 5.
b Group A is clustered together from 4 to 6, but group B is very spread out from 1 to 9.
c The spread of the data.

7 **a** Boys



Girls



b median for boys = 25, median for girls = 23
c mean for boys = 24.6, mean for girls = 23.8
d data for Year 11 may show a greater difference between mean and median foot length of boys and girls

8 **a**

stem	leaf
3	2 3 8
4	4 5 5
5	1 6 7

3|2 means 32

b

stem	leaf
6	1 5 8
7	1 4 5 6
8	2 2 2

6|1 means 61

c

stem	leaf
2	4 5 7
3	1 1 1 2 4 6 6
4	0 2

2|4 means 24

d

stem	leaf
5	9
6	1 3 4 8 9
7	0 4
8	4 4 6
9	5 6 8

5|9 means 59

e

stem	leaf
0	3 5 7 7 8
1	1 2 3 4 6 7 8
2	1 2 4

0|3 means 3

f

stem	leaf
0	5 7 9
1	2 8 8
2	1 6 6 6 7
3	0 3 4 7 8
4	2 6
5	1 2

0|5 means 5

g

stem	leaf
2	1 7 9
3	2 2 2 4 6 6 8
4	0 2 4 5 8 9
5	5 6 6
6	2 5 9
7	6

2|1 means 21

h

stem	leaf
2	1 2 5 6
3	5 5 6
4	8 9
5	3 4 6 6 7 8
6	2 2 5 7
7	1

2|1 means 21

9 **a**

stem	leaf
11	0
11	5 6 9
12	
12	5 6 7 8 8 9
13	0 2 4
13	5 6
14	1

11|0 means 110

b

stem	leaf
7	6 6 7 7 8
8	1 2 2 2 4 4
8	6 8
9	1 3 3
9	5
10	0 3 4

7|6 means 76

c

stem	leaf
2	0 1 2 4
2	
3	1 2
3	6
4	
4	5 9
5	1 3 4
5	5 8 8
6	1 2
6	6 7
7	0

2|0 means 20

10 **a**

boys		**girls**
leaf	stem	leaf
8 8 6 6 0	3	1 1 5
9 7	4	2 2 3 4 5 8 8
7	5	4 6
4 2 1 1	6	
6 4	7	2
7	8	3
	9	
	10	
	11	
	12	
	13	
	14	6

0|3 means 30

3|1 means 31

b range for boys = 57 seconds, range for girls = 115 seconds

c median for boys = 57 seconds, median for girls = 45 seconds

11 a

boys leaf	stem	girls leaf
	13	3
	13	7
	14	2 4 4
9 8 6	14	
4 4 4 2 1 0	15	0 0 2 4 4
6 6 5	15	5 5 7 8
3 2	16	0 0 0
8 8 5 5	16	7
	17	
8 8	17	6
	18	0
	18	

6|14 means 146
13|3 means 133

b range for boys = 32 cm, range for girls = 47 cm
c median for boys = 156 cm, median for girls = 154.5 cm

12 a

stem	leaf
2	
2	8 8 9
3	0 2 4 4 4
3	5 6 7 9
4	0 0 2 3 3
4	6 6
5	3
5	

2|8 means 0.28

b median = 0.365 seconds
c range = 0.27 seconds
d Median would probably be higher. Range may be similar.

Revision

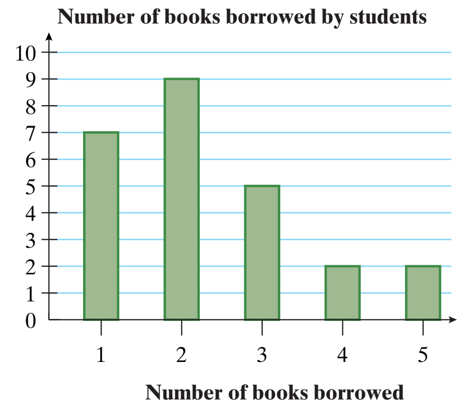
1 C **2** B **3** D **4** C **5** B
6 a C **b** N **c** N **d** C

7 a, b

Handspan	Tally	Frequency
110–119	|||	3
120–129	卌	5
130–139	||||	4
140–149	||||	4
150–159	||	2

c 120–129 mm

8 a

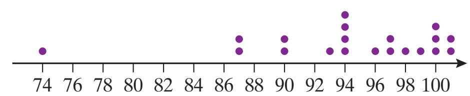


b 2

9 2.1 million hectares

10 6

11 a



b median = 95 cm
c mean = 94.3 cm
d 74 cm is an outlier
e median = 96 cm
f mean = 95.4 cm

12 a 25 minutes **b** 14 cars

13 a

stem	leaf
0	0 0 1 1 1 1 2 2 2 2 3 3 3 4 4 4 4 4
0	5 5 5 6 7 7 7 8 8 8 9 9
1	0 0 0 0 1 1 2 3
1	5 7 7 8 9
2	0
2	5 8
3	0
3	
4	0 0 0
4	

b 7.5 hours **c** 58%

14 a

	Eliza	Henry
shortest time	41	30
longest time	68	69
median	52.5	41

b Overall Henry was much faster, as shown by comparing the medians.

15

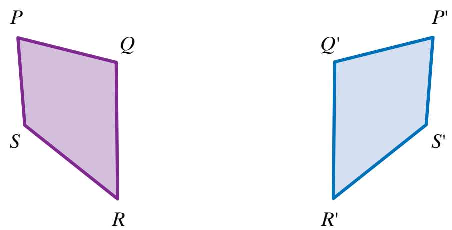
Data	Representation
Scores out of 40 on a test for 50 boys and 50 girls	Back-to-back stem-and-leaf plot
Number of TVs in the home of each student in a Year 7 class.	Column chart
Favourite music styles of students in Year 9.	Bar chart
Percentage of Australians living in each of the Australian states and territories	Pie graph
Method of travel to school of students at a particular school.	Bar chart
Concentration in minutes on a task for 50 students	Dot plot

Chapter 14

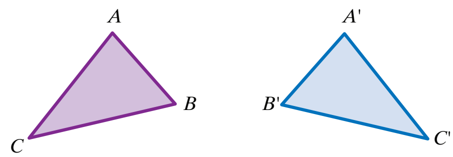
exercise 14.1

1 **a** turn **b** flip **c** slide

2 **a**



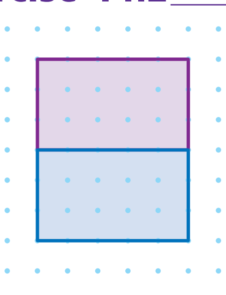
b



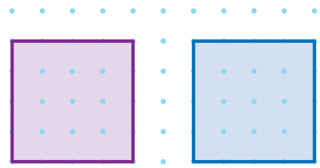
3 B

exercise 14.2

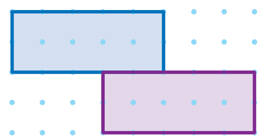
1 **a**



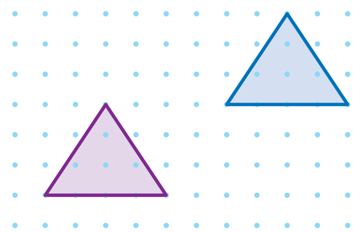
b



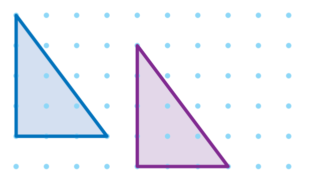
c



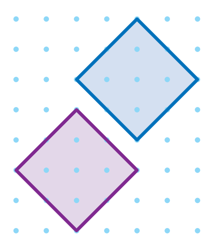
d



e

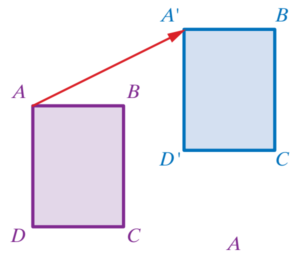


f

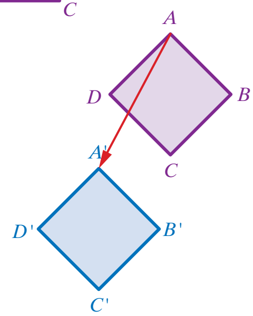


2 **a** 1 unit up and 5 units right
b 3 units down and 2 units right

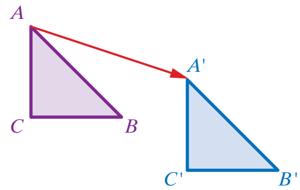
3 **a**



b



c



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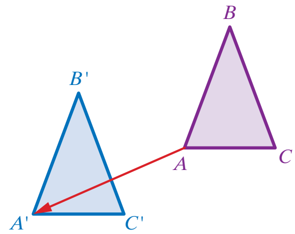

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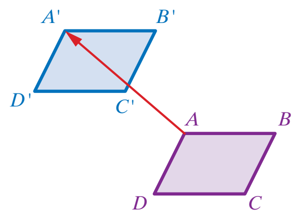

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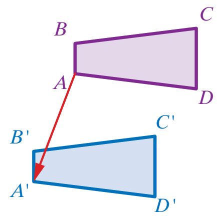
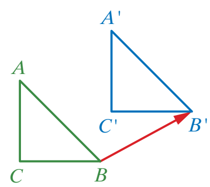
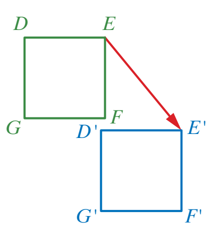


4 **a**



b



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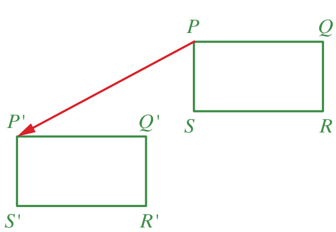


6 D

7

Coordinates	
Point	**Image**
$A(5, 4)$	$A'(2, 4)$
$B(0, -1)$	$B'(-2, -1)$
$C(-3, -4)$	$C'(-5, -4)$
$D(2, 2)$	$D'(0, 2)$
$E(3, 0)$	$E'(1, 0)$

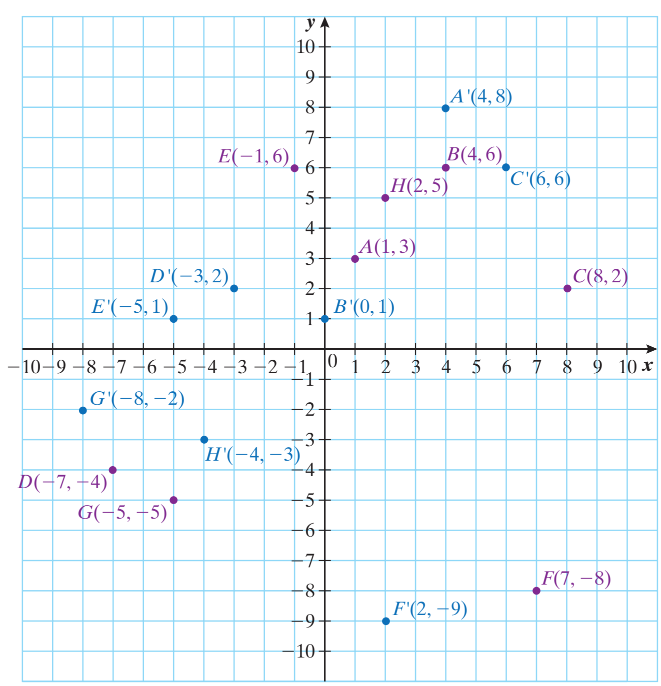
8

Coordinates	
Point	**Image**
$A(5, 4)$	$A'(5, 2)$
$B(0, -1)$	$B'(0, -3)$
$C(-3, -4)$	$C'(-3, -6)$
$D(2, 2)$	$D'(2, 0)$
$E(3, 0)$	$E'(3, -2)$

9 When points are translated in a horizontal direction on the Cartesian plane, each $\underline{x}$ coordinate changes by the same amount. The $\underline{y}$ coordinates stay the same.

When points are translated in a vertical direction on the Cartesian plane, each $\underline{y}$ coordinate changes by the same amount. The $\underline{x}$ coordinates stay the same.

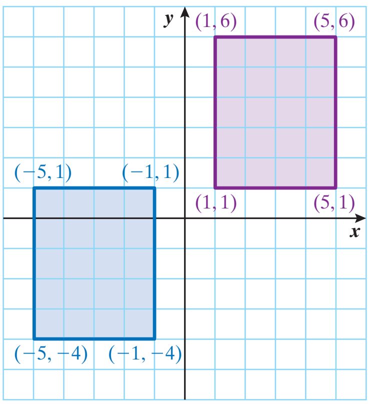
10



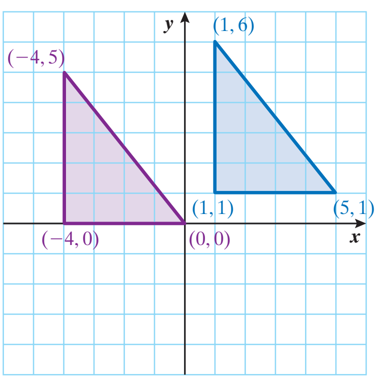
11

Coordinates	
Point	**Image**
$F(0, 4)$	$F'(-6, 4)$
$G(3, -3)$	$G'(-3, -3)$
$H(0, -3)$	$H'(-6, -3)$

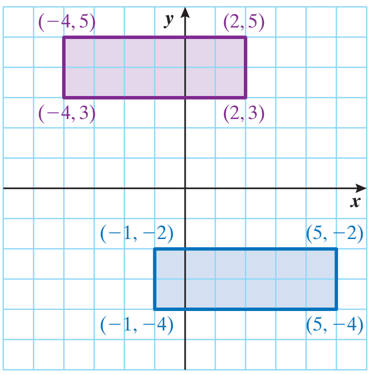
12 a



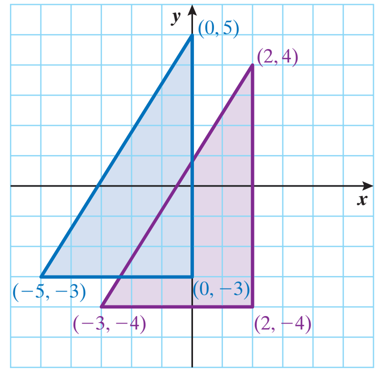
b



c



d



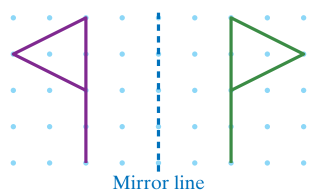
13 a $A' = (2, -2), B' = (-3, -2), C' = (0, 2)$

b $A' = (1, 4), B' = (1, 1), C' = (-4, 1), D' = (-4, 4)$

c $A' = (7, -2), B' = (2, -5), C' = (5, 2)$

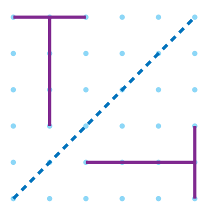
exercise 14.3

1

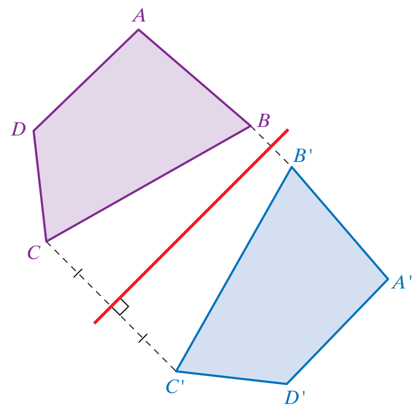


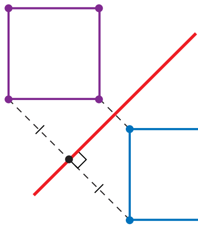
2 a A, M, T, W **b** B, C, D, E

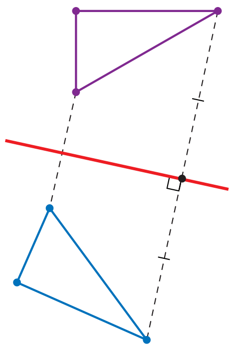
3



4 a, b



5 a


b


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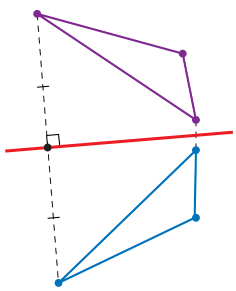

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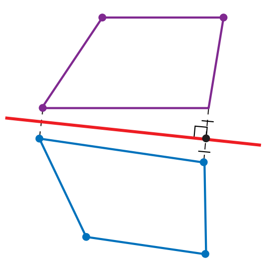
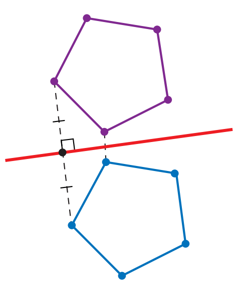


e


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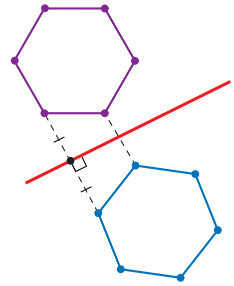
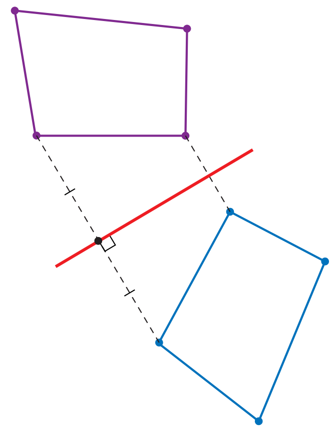
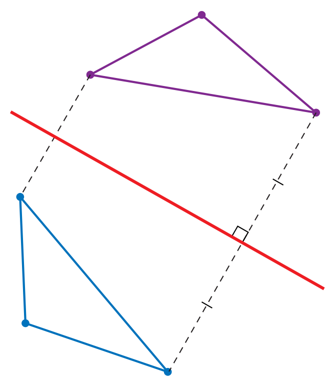
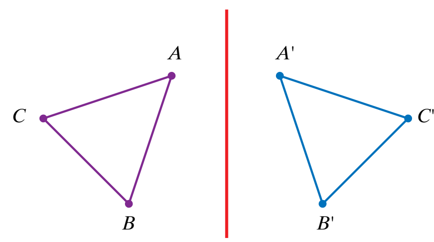
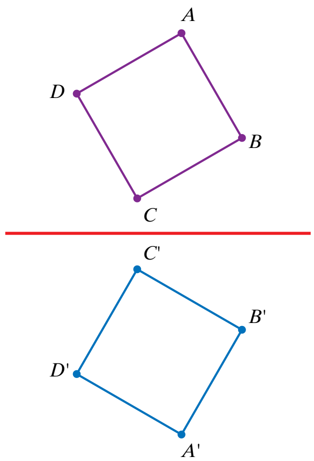


g


h


6 a


b



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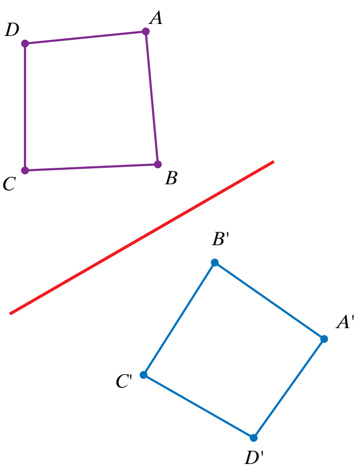

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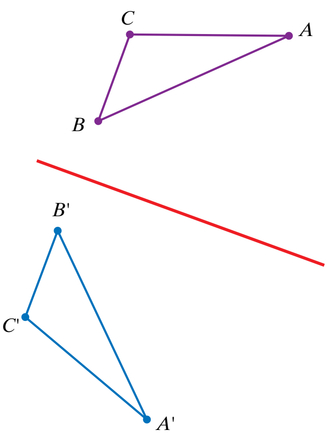

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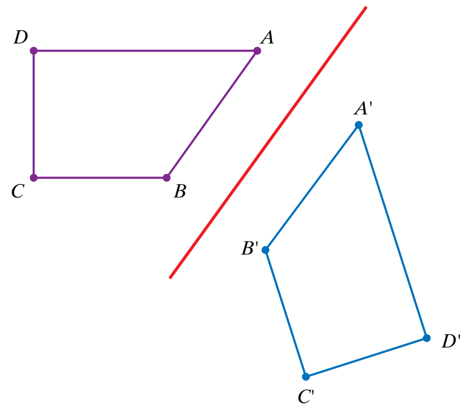

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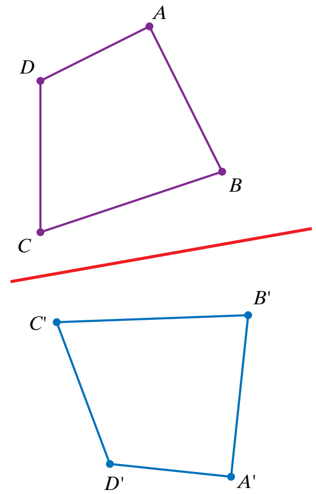

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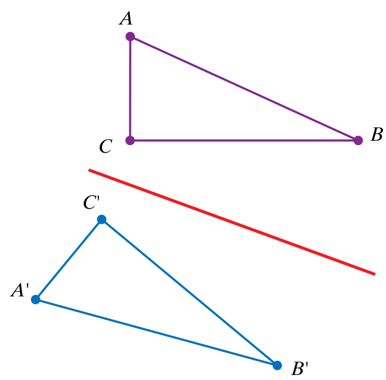

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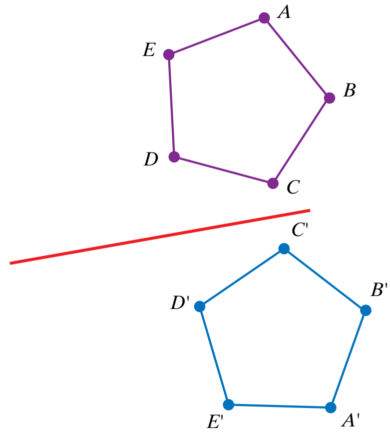
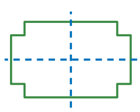
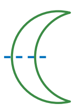


7 **a**



b



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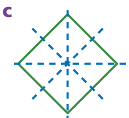

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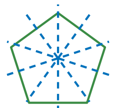

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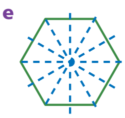

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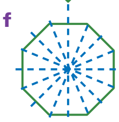
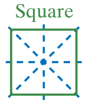
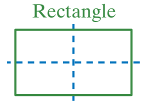


8 **a**

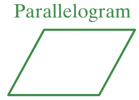
Square



Rectangle



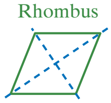
Parallelogram



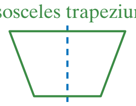
No axes of symmetry

b

Rhombus

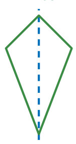


Isosceles trapezium

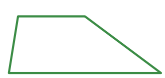


c

Kite

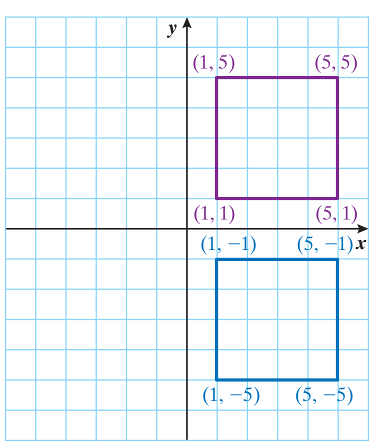


Trapezium

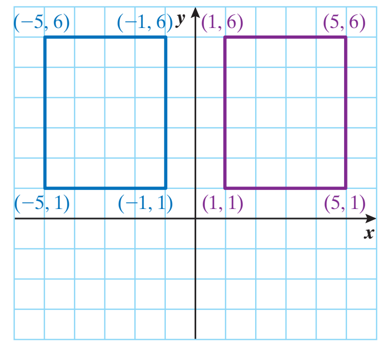


No axes of symmetry

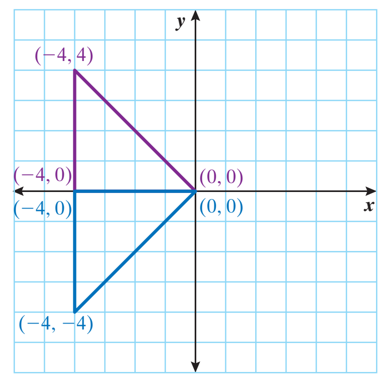
9 **a** **i**



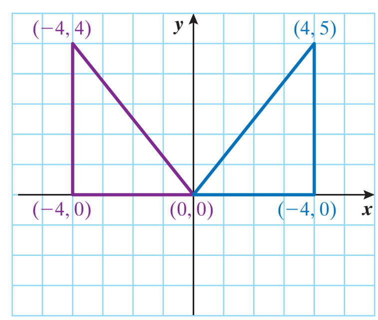
ii



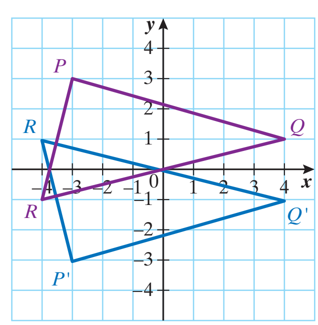
b **i**



ii



10

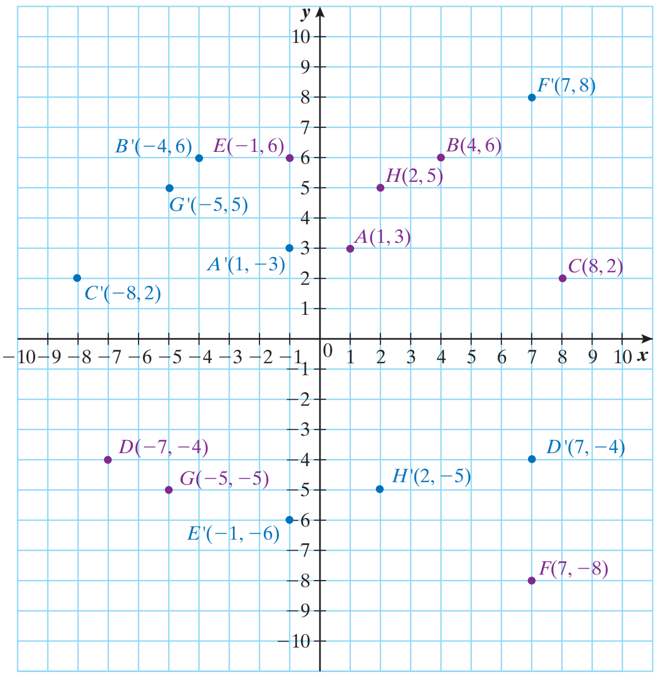


a

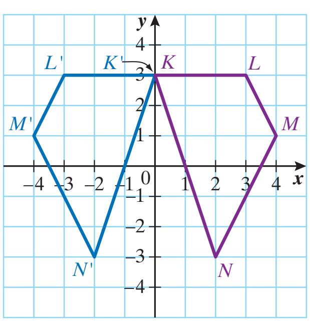
Coordinates	
Point	**Image**
$P(-3, 3)$	$P'(-3, -3)$
$Q(4, 1)$	$Q'(4, -1)$
$R(-4, -1)$	$R'(-4, 1)$

b the x-coordinates stay the same and the y-coordinates change in sign

11



12



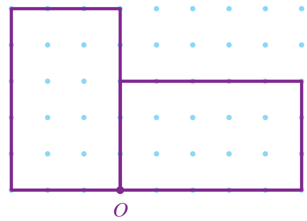
a

Coordinates	
Point	**Image**
$K(0, 3)$	$K'(0, 3)$
$L(3, 3)$	$L'(-3, 3)$
$M(4, 1)$	$M'(-4, 1)$
$N(2, -3)$	$N'(-2, -3)$

b the x-coordinates change in sign and the y-coordinates stay the same

exercise 14.4

1



2 **a** **i** 8 **ii** 45°
b **i** 12 **ii** 30°
c **i** 3 **ii** 120°
d **i** 6 **ii** 60°

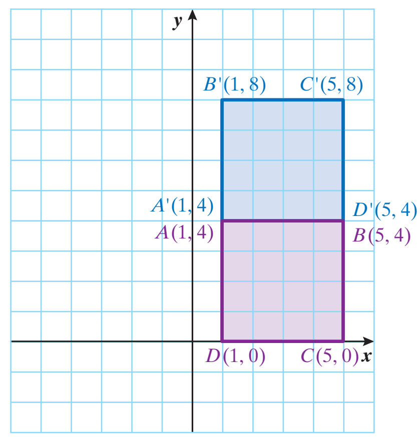
3 **a** **i** 5 **ii** 72°
b **i** 7 **ii** $51\frac{3}{7}°$
c **i** 11 **ii** $32\frac{8}{11}°$

4 **a** seastar **b** seastar, butterfly

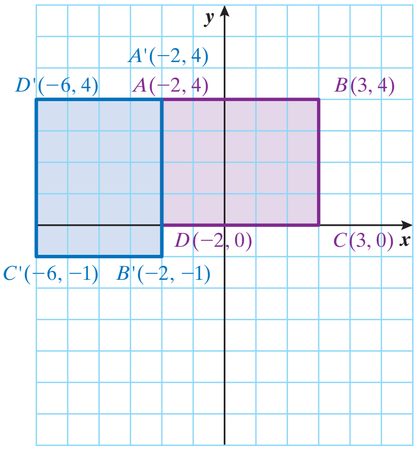
5 **a** 4 **b** no

6 **a** **i** 10 **ii** 10
b **i** 4 **ii** 4

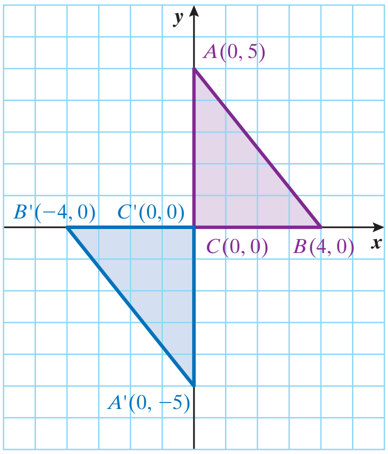
7 **a**



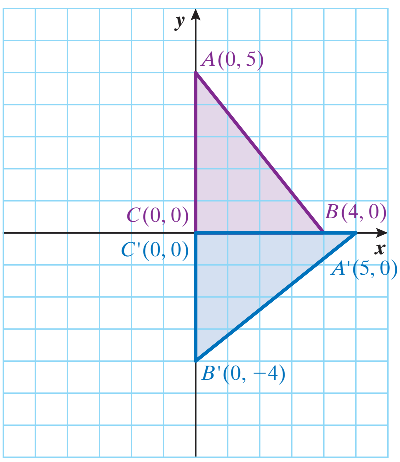
b



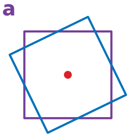
c



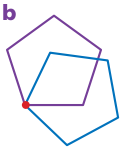
d



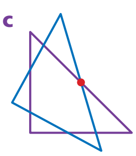
8 **a**



b



c



exercise 14.5

1 **a** reflection in vertical mirror and horizontal translation
b reflection in vertical mirror then reflection in horizontal mirror through the middle of the F then horizontal translation.
c reflection in a horizontal mirror line through the bottom of the F then horizontal translation of each F and its reflection
d rotation through 180° about the bottom of the F then horizontal translation of each F and its rotated image

2 answers will vary

3 answers will vary

4 **a** reflection in a vertical mirror line then horizontal translation of each shape and its reflection.
b horizontal translation
c reflection in horizontal mirror line then horizontal translation half the length of the pattern (glide reflection)
d reflection in a vertical mirror line then horizontal translation of each shape and its reflection

5 **i** rotation of the top half through 180° about the centre
ii reflection of the top half in a horizontal mirror line through the centre of the card, then reflection in a vertical mirror line through the centre of the card

6 **a** rotate by 180° **b** flip vertical
c image 1 **d** flip horizontal

7 **a** translation
b rotation and translation

8 translation

exercise 14.6

1 **a** Enlargement; (i) $\frac{3}{2}$ or 1.5 (ii) 150%

b Reduction; (i) $\frac{1}{4}$ or 0.25 (ii) 25%

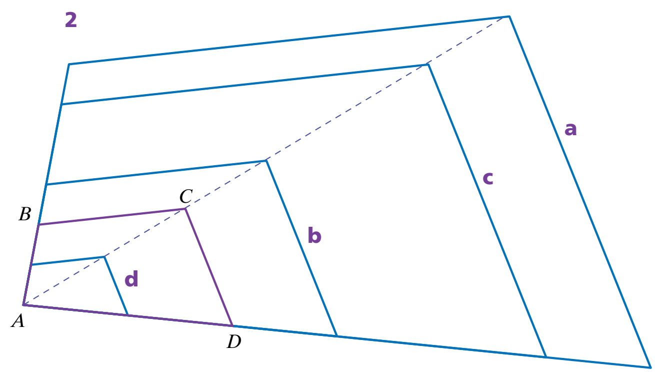
c Enlargement; (i) $\frac{4}{3}$ (ii) $133\frac{1}{3}$%

d Reduction; (i) $\frac{3}{7}$ (ii) $42\frac{6}{7}$%

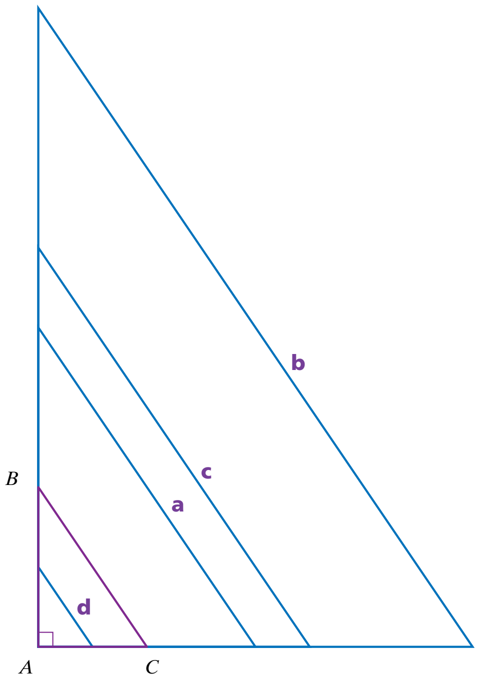
e Enlargement; (i) $\frac{9}{5}$ or 1.8 (ii) 180%

f Reduction; (i) $\frac{3}{5}$ or 0.6 (ii) 60%

2



3

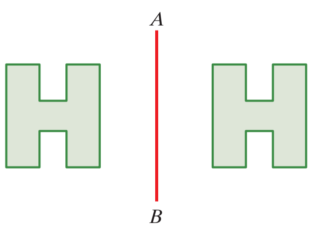


4 **a** four times **b** nine times

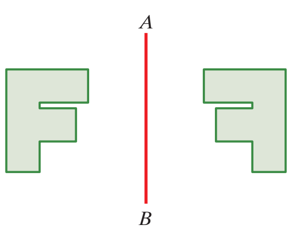
Revision

1 C **2** E **3** C **4** A **5** D

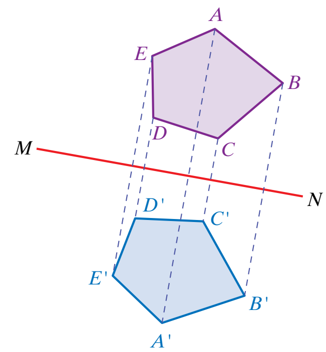
6 **a**



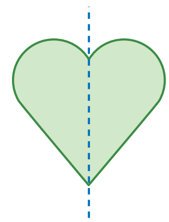
b



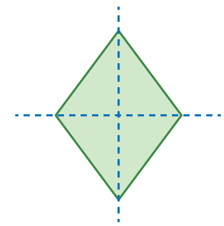
7



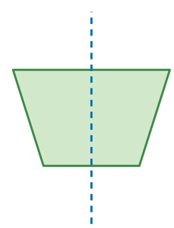
8 **a**



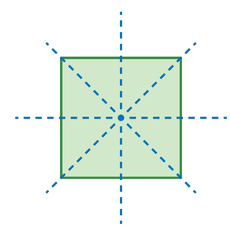
b



c

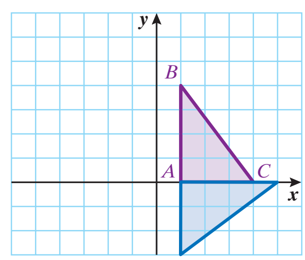


d

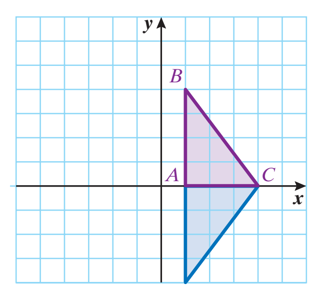


9 **a** 4 **b** 4 **c** 2 **d** 2

10 **a, b**



c



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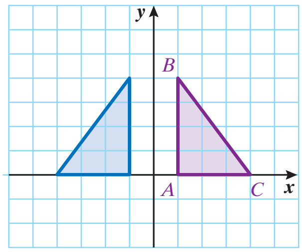
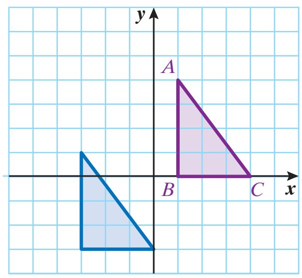


e



11 **a** enlargement
b reduction

Chapter 15

Answers to all the extending and investigating problems are contained in the Teacher book.