Back to contents

Edrolo

MATHEMATICS

Year 8

2ND EDITION

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YEAR 8 MATHEMATICS

2ND EDITION

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CONTENTS

GUIDE TO EDROLO YEAR 8 MATHEMATICS VI
BIG IDEAS VIII

01 Integers 1

Research summary ... 2
1A Adding and subtracting whole numbers *(Revision)* ... 6
1B Multiplying and dividing whole numbers *(Revision)* ... 13
1C Adding and subtracting integers ... 21
1D Multiplying and dividing integers ... 27
1E Order of operations ... 33
Chapter 1 extended application ... 41
Chapter 1 review ... 42

02 Fractions and decimals 47

Research summary ... 48
2A Equivalent fractions *(Revision)* ... 53
2B Adding and subtracting fractions *(Revision)* ... 61
2C Multiplying and dividing fractions *(Revision)* ... 70
2D Working with negative fractions ... 78
2E Four operations with decimals *(Revision)* ... 86
2F Terminating and recurring decimals ... 94
2G Rounding and estimating with decimals ... 100
Chapter 2 extended application ... 107
Chapter 2 review ... 109

03 Percentages and financial mathematics 113

Research summary ... 114
3A Percentages, decimals, and fractions *(Revision)* ... 118
3B Expressing as a percentage and percentages of quantities ... 125
3C Increasing and decreasing by a percentage ... 132
3D Percentage change ... 142
3E The unitary method *(Extension)* ... 151
Chapter 3 extended application ... 159
Chapter 3 review ... 161

04 Indices 165

Research summary ... 166
4A Index notation ... 169
4B Multiplying indices ... 176
4C Dividing indices ... 182
4D Raising powers ... 188
Chapter 4 extended application ... 195
Chapter 4 review ... 198

05 Algebra 201

Research summary ... 202
5A Introduction to algebra *(Revision)* ... 208
5B Substitution *(Revision)* ... 216

5C Adding and subtracting terms 223
5D Multiplying and dividing terms 231
5E Algebraic fractions *(Extension)* 239
5F Expanding brackets 246
5G Factorising expressions 253
Chapter 5 extended application 263
Chapter 5 review 265

06 Equations 269

Research summary 270
6A Introduction to equations *(Revision)* 274
6B Solving one-step and multi-step equations 281
6C Solving equations with unknowns on both sides 290
6D Formulas and applications of equations 297
6E Inequalities *(Extension)* 306
Chapter 6 extended application 314
Chapter 6 review 316

07 Congruence and transformations 319

Research summary 320
7A Angles and parallel lines *(Revision)* 324
7B Triangles and quadrilaterals 335
7C Transformations on the Cartesian plane 347
7D Congruent 2D shapes 363
7E Rules for congruent triangles 378
7F Congruence and quadrilaterals 388
Chapter 7 extended application 401
Chapter 7 review 402

08 Statistics 409

Research summary 410
8A Classifying data *(Revision)* 415
8B Collecting data 424
8C Interpreting data displays *(Revision)* 432
8D Frequency tables and histograms 443
8E Measures of centre and spread *(Extension)* 454
Chapter 8 extended application 464
Chapter 8 review 466

09 Linear graphing 471

Research summary 472
9A The Cartesian plane *(Revision)* 477
9B Tables of values 488
9C Linear rules 499
9D Gradient *(Extension)* 511
9E Gradient-intercept form *(Extension)* 524
9F Solving linear equations graphically 533
9G Non-linear graphs *(Extension)* 545
Chapter 9 extended application 555
Chapter 9 review 557

10 Probability 563

Research summary 564
10A Introduction to probability 570
10B Complementary events 578
10C Venn diagrams and two-way tables 586
10D Treediagramsandmulti-stepexperiments *(Extension)* 598
10E Experimental probability 610
Chapter 10 extended application 619
Chapter 10 review 621

11 Rates and ratios 627

Research summary 628
11A Introduction to ratios 633
11B Equivalent ratios 643
11C Dividing a quantity in a given ratio 651
11D Introduction to rates 660
11E Solving rate problems 667
11F Speed 676
Chapter 11 extended application 685
Chapter 11 review 687

12 Measurement 691

Research summary 692
12A Perimeter and units of length *(Revision)* 698
12B Circumference of a circle 706
12C Area of squares, rectangles and triangles 717
12D Area of special quadrilaterals 726
12E Area of a circle 738
12F Composite shapes with sectors *(Extension)* 750
12G Volume of prisms and cylinders 761
12H Time 772
Chapter 12 extended application 781
Chapter 12 review 785

13 Pythagoras' theorem 791

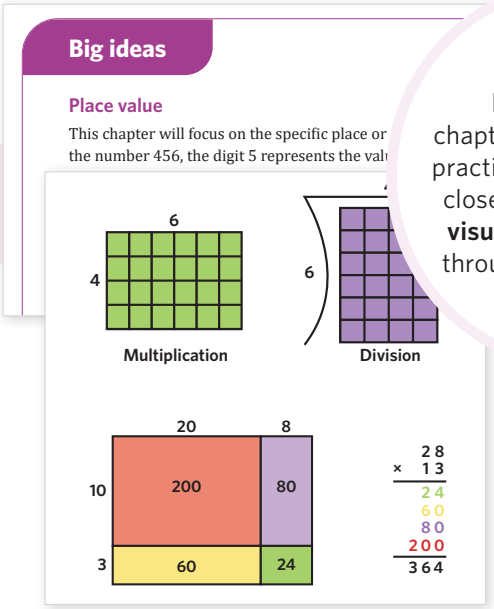
Research summary 792
13A Introduction to Pythagoras *(Extension)* 795
13B Calculating the hypotenuse *(Extension)* 807
13C Calculating the shorter side *(Extension)* 820
Chapter 13 extended application 836
Chapter 13 review 838

Answers 843

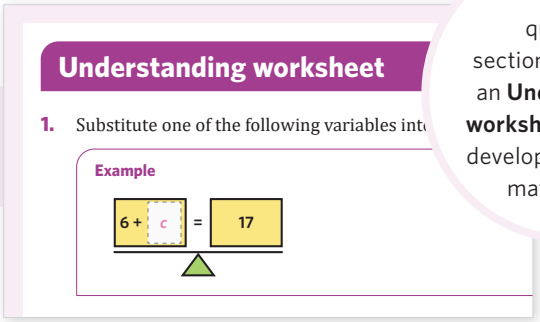
GLOSSARY 1031

Guide to Edrolo Year 8 Mathematics

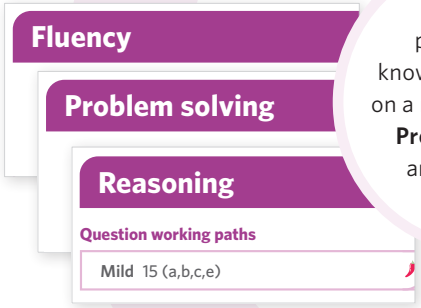
This resource deepens students' conceptual understanding to develop a strong foundation of knowledge through print and online video lessons.



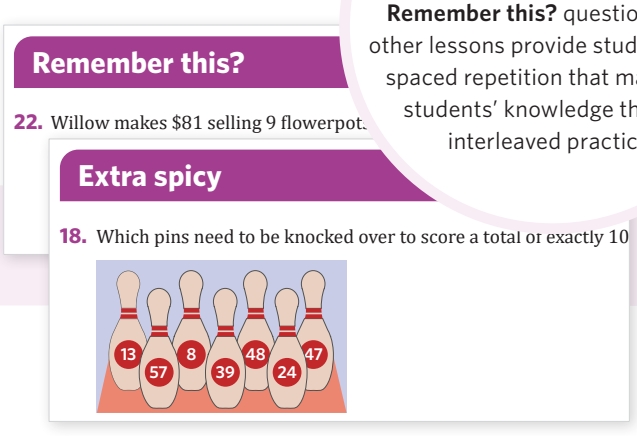
Big ideas are highlighted per chapter to guide teaching practice. Abstract math is closely intertwined with **visual representations** throughout each lesson.



Each question section begins with an **Understanding worksheet** to aid the development of core maths skills.



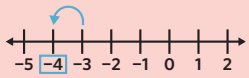
Students practice their knowledge and skills on a range of **Fluency**, **Problem solving**, and **Reasoning** questions.



Extra spicy questions challenge students wanting to stretch themselves, and **Remember this?** questions from other lessons provide students with spaced repetition that maintains students' knowledge through interleaved practice.

Visit Edrolo.com.au to access all the digital materials.

Misconceptions

Misconception	Incorrect
dding or subtract ng gers, students ong direction ne.	$3 + 1 = -4$ 

Misconceptions are actively highlighted, uncovered and addressed.

Worked example 1

Expanding brackets

Expand the brackets in each expression.

a. $4(3x - 5)$

Working

$4(3x - 5) = 4 \times 3x + 4 \times (-5)$

$= 12x - 20$

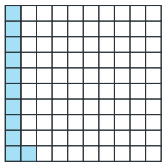
KEY TERMS AND DEFINITION

Converting is the process of rev
change to the value.

New ideas and vocabulary used are highlighted at the beginning of each lesson.

Key ideas

1. Percentages represent a value out of 100 parts.



x out of 100 $= \frac{x}{100} = x\%$

11 out of 100 $= \frac{11}{100} = 11\%$

Problem solving

11. Ingrid buys a new set of airpods for less than $350. Write this a
an inequality where the price of the airpods is represented as p

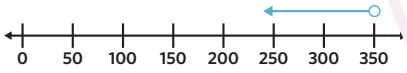
Key points

- Ingrid buys a new set of airpods for less than $350.
- Write this as an inequality where the price of the ai
represented as p.

Explanation

A closed circle is used to indicate that $350 is not in

The arrow points to the left as the price of the airpo
than $350.



Answer

$p < 350$

Understanding worksheet

1. **a.** $4.58 \times 100 = 458$
$458 \div 100 = 4.58$

c. $3.25 \times 100 = 325$
$325 \div 100 = 3.25$

2. **a.** $4.81 \times 4 = 19.24$

c. $2.34 \times 3 = 7.02$

3. subtract; dividing; multiplying; whole

Answers are given in the back of the book. Full worked problem solving solutions are stepped out in detail to understand the why behind the answer. Detailed solutions and videos are available online.

Back to contents

Big ideas

Place value

Chapters: 1, 2

'Capacity to recognise and work with place-value units and view larger numbers as counts of these units rather than a collection of ones. Appreciates the structure in terms of 10 of these is 1 of those' (Siemon, 2022, p. 9).

Multiplicative thinking

Chapters: 1, 2, 3, 4

'Works with multiple representations of multiplication and division (e.g., the 'for each', 'times as many' and 'area' ideas). Moving to factor-factor-product idea, efficient strategies for multiplication facts' (Siemon, 2022, p. 9).

Equi-partitioning

Chapters: 1, 2, 3

'Uses partitioning strategies to construct line and areas models for fractions and decimals, uses representations to compare, order and locate fractions and decimals on number lines, recognise that numbers can be divided to create new numbers, solves simple problems involving fractions' (Siemon, 2022, p. 9).

Back to contents

Proportional reasoning

Chapters: 3, 4, 5, 6

'Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, solves problems involving intensive quantities and proportional relationships' (Siemon, 2022, p. 9).

Generalising

Chapters: 4, 5, 6

'Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules and properties' (Siemon, 2022, p. 9).

Proportional reasoning

Chapters: 7, 10, 11, 12

'Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, solves problems involving intensive quantities and proportional relationships.' (Siemon, 2022, p. 9).

Generalising

Chapters: 9, 10, 11, 12, 13

'Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules and properties.' (Siemon, 2022, p. 9).

Variation with expectation and randomness

Chapters: 8, 10

Rolling a dice, spinning a spinner, drawing names or blocks from a bag are random events and we expect the outcomes to be different every time. An average, whether mean, median, or mode, expresses an expectation about a data set. The average is a description that varies as new data are added or data points are removed.

Back to contents

Variation with distribution and expectation

Chapters: 8, 10

Distribution describes the ways in which the data are spread out or distributed. A graph is a picture of a distribution and the variation it represents. In school it is useful to develop the idea that tables, graphs or other representations are ways of visualising variation through the distribution of the data.

Variation with informal inference

Chapters: 8, 10

Collecting data is a purposeful activity that describes a situation and can help people make decisions. When considering any data set we need to consider aspects such as the sample, how the data were collected, and the questions we are asking about the data.

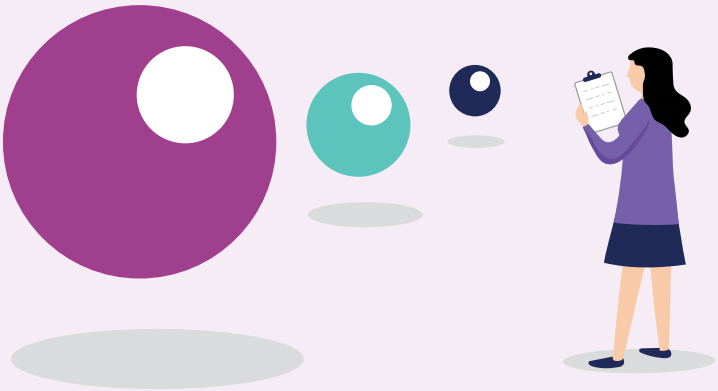
References and additional reading

Siemon, D. (2022). *ISSUES IN THE TEACHING OF MATHEMATICS Teaching with the Big Ideas in Mathematics*. https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/teaching-with-the-big-ideas-in-mathematics.pdf

Siemon, D. (2022). *ISSUES IN THE TEACHING OF MATHEMATICS Teaching with the Big Ideas in Mathematics*. https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/teaching-with-the-big-ideas-in-mathematics.pdf

http://www.mathseducation.org.au/wp-content/uploads/2020/11/Geometrical-Reasoning-Learning-Progression.pdf

http://www.mathseducation.org.au/wp-content/uploads/2020/11/Statistical-Reasoning-Learning-Progression.pdf



01

Integers

Number and Algebra

Research summary

1A Adding and subtracting whole numbers *(Revision)*

1B Multiplying and dividing whole numbers *(Revision)*

1C Adding and subtracting integers

1D Multiplying and dividing integers

1E Order of operations

Chapter 1 extended application

Chapter 1 review

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Back to contents

Chapter 1 research summary

Integers

Big ideas

Place value

This chapter will focus on the specific place or position in a number that defines the value of a digit. For example, the number 456, the digit 5 represents the value of 50 because the five is in the position of the tens place.

Additionally, this chapter will focus on the fact that each place value to the left is ten times greater than the digit on the right, and each place value to the right is ten times less than the digit to the left. For example, 20 is ten times greater than 2 ($2 \times 10 = 20$) and 2 is ten times less than 20 ($20 \div 10 = 2$).

Understanding this concept is built upon trusting the count and is essential for grasping future big idea topics such as multiplicative thinking, equi-partitioning, and for algebraic, statistical and geometric reasoning and micro topics such as fractions, decimals, and percentages.

Multiplicative thinking

This chapter will focus on efficient strategies to multiply and divide based on groups rather than the number in each group. For example, three groups of something is the same as doubling it and an additional group.

Multiplicative thinking is built upon an understanding of place value and will underpin future topics such as equi-partitioning, algebraic, statistical and geometric reasoning, fractions, decimals, percentages, rates and ratios.

Equi-partitioning

This chapter will focus on partitioning to deconstruct larger numbers into smaller numbers when adding, subtracting, multiplying and dividing. For example, $84 \div 6$ can be partitioned as $(60 + 24) \div 6$ to make the calculation easier to divide.

Equi-partitioning is built upon an understanding of place value and multiplicative thinking and is essential for grasping future big idea topics such as algebraic, statistical, and geometric reasoning and micro-topics such as fractions, decimals, rates, and ratios.

Visual representations

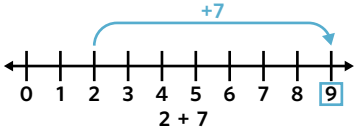
Number line

A number line is traditionally shown as a horizontal line or axis, but can be a vertical line. A number line shows the order and size of numbers. It helps deepen the understanding of mathematical concepts such as fractions, decimals, percentages, negative numbers, and mathematical operations.

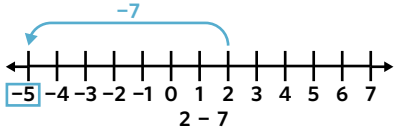
In this chapter a number line will be used to aid in:

- adding and subtracting with whole numbers and integers
- support understanding when multiplying and dividing by integers.

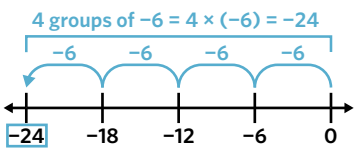
Addition:



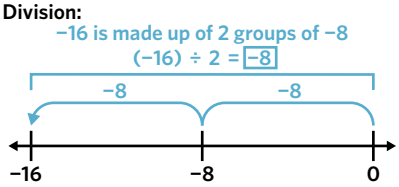
Subtraction:



Multiplication:

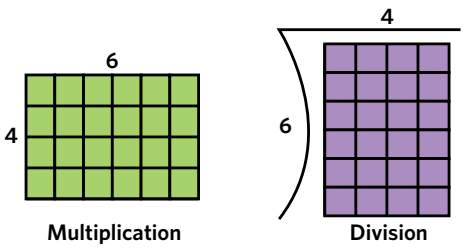


Division:



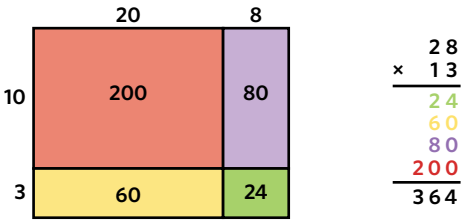
Arrays

An essential model for developing multiplicative thinking is arrays. When used frequently, they can provide prolonged support in linking skills in multiplication, division, decimals, fractions and algebra (Jacob & Mulligan, 2014). An array is formed by arranging objects into rows and columns to form a rectangle. The two kinds of arrays used in this chapter are grid arrays and open arrays.



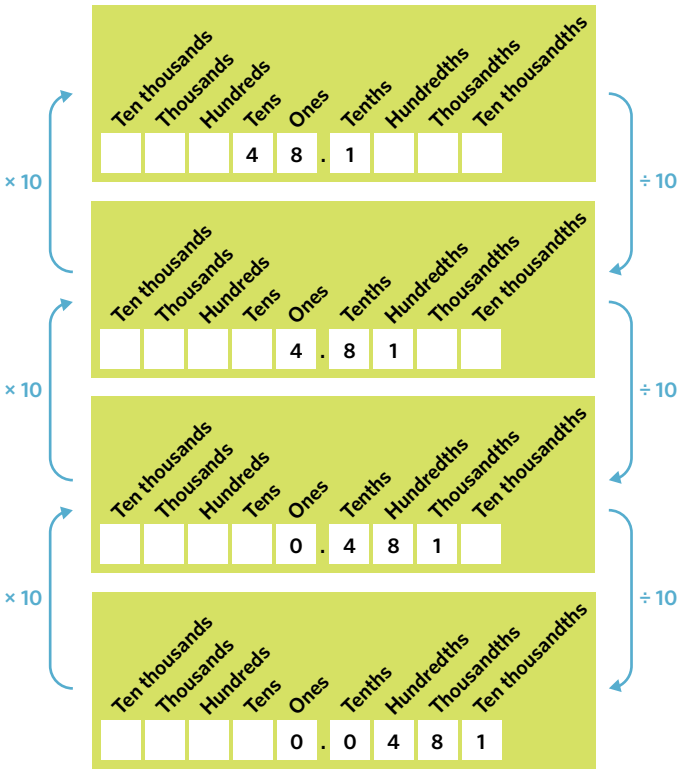
Grid arrays show units within the array and are used to give meaning to multiplication and division, because they are visual representations of these abstract concepts.

Open arrays are helpful for the multiplication of larger numbers and do not show units within the array. They promote the use of multiplication strategies rather than counting. Additionally, they are practical when using the distribution law for two and three-digit multiplications and provide a much deeper understanding of the multiplication algorithm.

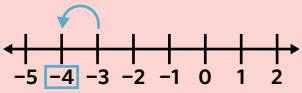
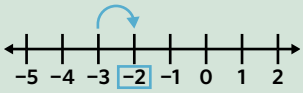
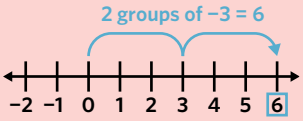
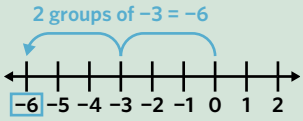


Number slide

Number slides can be used when multiplying and dividing decimals by powers of 10. They are useful when it comes to addressing misconceptions around the decimal point and building knowledge around decimal place value.



Misconceptions

Misconception	Incorrect	Correct	Lesson
Students add or subtract whole numbers without considering place value.	$2216 + 35 =$ $\begin{array}{r} 2\ 2\ 1\ 6 \\ +\ 3\ 5 \\ \hline 5\ 7\ 1\ 6 \end{array}$	$2216 + 35 =$ $\begin{array}{r} {\scriptstyle +1} \\ 2\ 2\ 1\ 6 \\ +\quad 3\ 5 \\ \hline 2\ 2\ 5\ 1 \end{array}$	1A
When adding whole numbers, students use the process of algorithms without an understanding of regrouping.	$\begin{array}{r} 3\ 8\ 2 \\ +\quad 5\ 6 \\ \hline 3\ 1\ 3\ 8 \end{array}$	$\begin{array}{r} {\scriptstyle +1} \\ 3\ 8\ 2 \\ +\quad 5\ 6 \\ \hline 4\ 3\ 8 \end{array}$	1A
When subtracting a whole number that has zero as a place holder, students do not consider place value.	$\begin{array}{r} 2\ 0\ 4 \\ -\quad 5\ 9 \\ \hline 2\ 5\ 5 \end{array}$	$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 1\ 10\ 14} \\ 2\ \not{0}\ \not{4} \\ -\quad 5\ 9 \\ \hline 1\ 4\ 5 \end{array}$	1A
Students think that subtraction and division are commutative.	$20 - 5 = 5 - 20$ $15 \div 3 = 3 \div 15$	$20 - 5 = 15$ $5 - 20 = -15$ $15 \div 3 = 5$ $3 \div 15 = \frac{1}{5}$	1A 1B
When multiplying, students use the process of algorithms without an understanding of place value.	$\begin{array}{r} 3\ 5 \\ \times\quad 9 \\ \hline 2\ 7\ 5 \end{array}$	$\begin{array}{r} {\scriptstyle +3\ +4} \\ 3\ 5 \\ \times\quad 9 \\ \hline 3\ 1\ 5 \end{array}$	1B
When multiplying and dividing by powers of 10, students do not consider place value.	$33 \times 100 = 330$ $5\,800 \div 100 = 580$	$33 \times 100 = 3\,300$ $5\,800 \div 100 = 58$	1B
In division, students think that remainders represent a whole number.	$100 \div 7 = 14$ r 2 or 14.2	$100 \div 7 = 14$ r 2 or $14\frac{2}{7}$	1B
When adding or subtracting positive integers, students move in the wrong direction on the number line.	$-3 + 1 = -4$ 	$-3 + 1 = -2$ 	1C
When adding a negative integer, students increase the value.	$-2 + 3 = -5$	$-2 + 3 = 1$	1C
When subtracting a negative integer, students decrease the value.	$2 - (-2) = 0$	$2 - (-2) = 4$	1C
When adding two negative integers, students get a positive sum.	$-10 + (-5) = 15$	$-10 + (-5) = -15$ $-10 - 5 = -15$	1C
When multiplying or dividing integers, students move in the wrong direction.	$2 \times (-3) = 6$ 	$2 \times (-3) = -6$ 	1D
Students assume that dividing or multiplying by a negative number always produces a negative result.	$\frac{-15}{-3} = -5$	$\frac{-15}{-3} = 5$	1D

Misconception	Incorrect	Correct	Lesson
When multiplying integers, students assume that the signs of the integers indicate addition or subtraction.	$2 \times -2 = 0$	$2 \times -2 = -4$	1D
Students work from left or right regardless of the operation sign.	Calculations are performed from left to right. $2 + 7 \times 5 - 3$ $= 9 \times 5 - 3$ $= 45 - 3$ $= 42$	Calculations are performed using order of operations. $2 + 7 \times 5 - 3$ $= 2 + 35 - 3$ $= 37 - 3$ $= 34$	1E
When working with inverse operations, students do not work from left to right.	Working right to left. $16 \div 4 \times 2$ $= 16 \div 8$ $= 2$	Work from left to right. $16 \div 4 \times 2$ $= 4 \times 2$ $= 8$	1E
Students work from left to right inside the brackets regardless of the operations.	$2 + (3 - 10 \times 5)$ $= 2 + (-7 \times 5)$ $= 2 + (-30)$ $= -28$	$2 + (3 - 10 \times 5)$ $= 2 + (3 - 50)$ $= 2 + (-47)$ $= -45$	1E

References and additional readings

Jacob, L., & Mulligan, J. (2014). Using Arrays. *Australian Primary Mathematics Classroom*, *19*(1), 2014. https://files.eric.ed.gov/fulltext/EJ1093251.pdf

Khalid, M., & Embong, Z. (2019). Sources and Possible Causes of Errors and Misconceptions in Operations of Integers. *International Electronic Journal of Mathematics Education*, *15*(2). https://doi.org/10.29333/iejme/6265

Mathematics Mastery. (2014). *Common errors and misconceptions in column addition and subtraction*. http://toolkit.mathematicsmastery.org/app/webroot/js/tiny_mce/plugins/moxiemanager/data/files/Common%20errors%20and%20misconceptions%20in%20column%20addition%20and%20subtraction.pdf

Department of Education and Training. (2018, June 24). *Assessment for common misunderstandings*. www.education.vic.gov.au; State Government of Victoria. https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx

Back to contents

1A Adding and subtracting whole numbers

When adding and subtracting whole numbers it is important that the correct place value of the digits are being added or subtracted. There are a variety of different mental or written strategies that can be used. Changing the order of numbers that are added together does not not affect the result. However, changing the order of numbers that are being subtracted does affect the result.

LEARNING INTENTIONS

Students will be able to:

- understand the commutative and associative laws of addition and subtraction
- add and subtract whole numbers using different mental strategies
- add and subtract whole numbers using the vertical algorithm.

KEY TERMS AND DEFINITIONS

The **compensation strategy** uses rounded numbers and then adjustments to make calculations simpler to solve.

The **partitioning strategy** deconstructs larger numbers into smaller numbers to make calculations simpler to solve.

The **vertical algorithm** is a written method used to add and subtract numbers.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

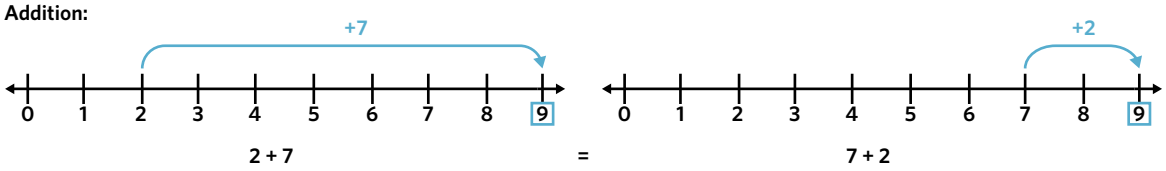
Image: Mitch Hutchinson/Shutterstock.com

Whole number addition and subtraction is used in many everyday activities. For example, adding up points when playing different board games.

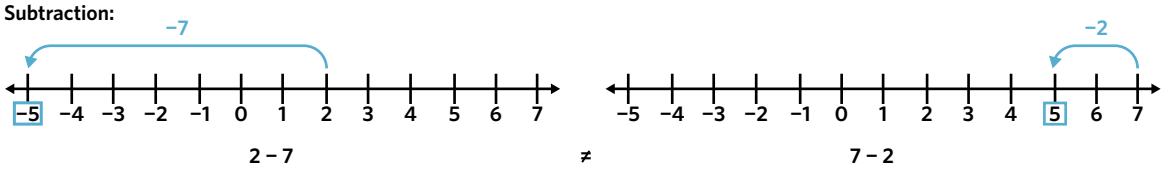
Key ideas

1. The commutative law means that when the order of the numbers in an addition is changed the result is not affected. The commutative law does not apply to subtraction.

Addition:



Subtraction:



2. The associative law means that when the grouping of the numbers in an addition is changed the result is not affected. The associative law does not apply to subtraction.

Addition:

$(6 + 3) + 2 = 6 + (3 + 2)$
$9 + 2 = 6 + 5$
$11 = 11$

Subtraction:

$(6 - 3) - 2 \neq 6 - (3 - 2)$
$3 - 2 \neq 6 - 1$
$2 \neq 5$

Worked example 1

Using mental strategies

Calculate using a mental strategy.

a. $57 - 29$ WE1a

Working

$57 - 29 = 57 - 30 + 1$

$= 27 + 1$

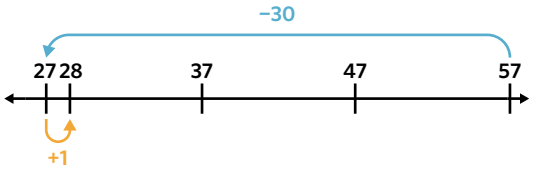
$= 28$

Thinking

Step 1: Round one of the numbers to the nearest ten and adjust the calculation so that both sides are equal. This is the compensation strategy.

Step 2: Simplify.

Visual support



b. $782 + 271$ WE1b

Working

$782 + 271 = 700 + 80 + 2 + 200 + 70 + 1$

$= 700 + 200 + 80 + 70 + 2 + 1$

$= 900 + 150 + 3$

$= 1053$

Thinking

Step 1: Partition each number into its place value parts.

Step 2: Regroup and simplify.

Student practice

Calculate using a mental strategy.

a. $84 - 39$ **b.** $697 - 486$

Back to contents

Worked example 2

Using the vertical algorithm

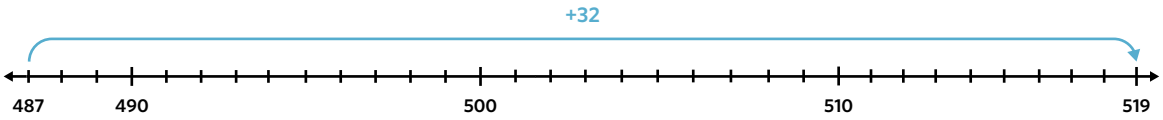
Calculate the following.

a. 487 + 32 WE2a

Working

$$\begin{array}{r} {\scriptstyle +1} \\ 4\ 8\ 7 \\ +\quad 3\ 2 \\ \hline 5\ 1\ 9 \end{array}$$

Visual support



Thinking

Step 1: Write the numbers vertically, aligning the digits in each place value.

Step 2: From right to left, add the digits in each place value column. If the digits sum to 10 or more, regroup to the place value column to the left.

b. 2004 − 632 WE2b

Working

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 1\ \cancel{10}\ 10} \\ \cancel{2}\ \cancel{0}\ \cancel{0}\ 4 \\ -\quad 6\ 3\ 2 \\ \hline 1\ 3\ 7\ 2 \end{array}$$

Thinking

Step 1: Write the numbers vertically ensuring the digits in each place value line up.

Step 2: From right to left, subtract the digit in the bottom row from the digit in the top row.

Step 3: If the value of the digit in the top row is less than the bottom digit, regroup using the next place value column to the left.

Student practice

Calculate using a mental strategy.

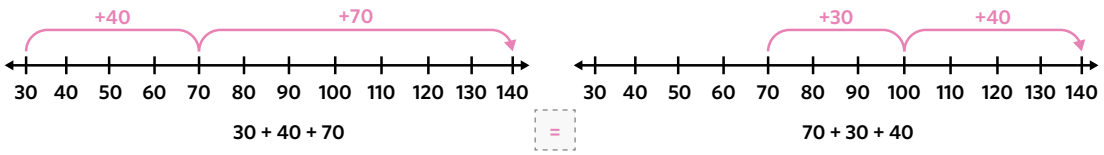
a. 576 + 82 **b.** 2008 − 255

1A Questions

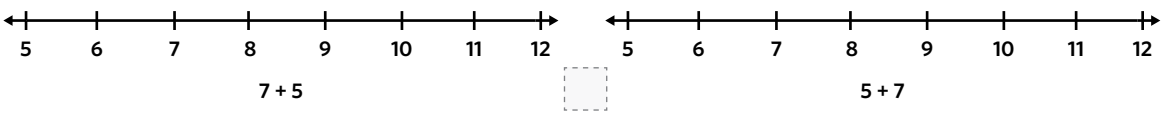
Understanding worksheet

1. Draw the 'jumps' on the number line and fill in the blank box with = or ≠.

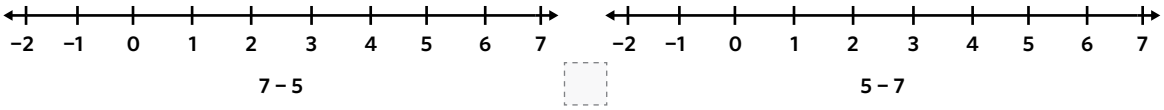
Example



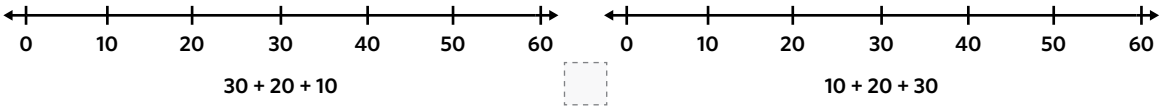
a.



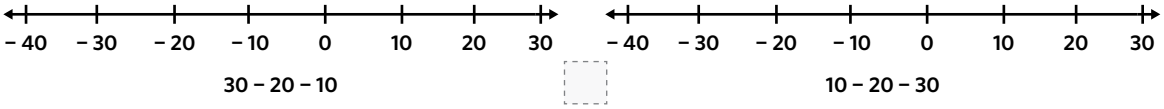
b.



c.



d.



2. Fill in the blank box with = or ≠ to make each number sentence true.

a. $(7 + 2) + 3 \square 7 + (2 + 3)$

b. $(15 - 10) - 5 \square 15 - (10 - 5)$

c. $100 - (50 - 20) \square (100 - 50) - 20$

d. $40 + (15 + 5) \square (40 + 15) + 5$

3. Fill in the blanks by using the words provided.

subtraction | associative | commutative | compensation | partitioning

The ____ law means that when the order of the numbers in an addition is changed the result is not affected. The ____ law means that when the grouping of the numbers in an addition is changed the result is not affected. Both laws do not apply to ____. The ____ strategy partitions numbers into place value parts, whereas the ____ strategy uses rounded numbers to make calculations simpler to solve.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Calculate using a mental strategy. WE1a

a. 56 − 39 **b.** 87 − 68 **c.** 36 + 38 **d.** 132 + 61

e. 284 − 73 **f.** 451 − 62 **g.** 126 + 147 **h.** 492 + 169

5. Calculate using a mental strategy. WE1b

a. 23 + 45 **b.** 362 + 127 **c.** 56 − 47 **d.** 875 − 723

e. 3254 + 7832 **f.** 480 + 294 + 113 **g.** 981 − 597 **h.** 54 540 − 1247

6. Calculate using the vertical algorithm.

a.
$$\begin{array}{r} 451 \\ +\ 325 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 5622 \\ +\ 364 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 8942 \\ -\ 4521 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 645 \\ -\ 475 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 252 \\ 478 \\ +\ 139 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 5452 \\ 573 \\ +\ 3041 \\ \hline \end{array}$$

g.
$$\begin{array}{r} 9500 \\ -\ 571 \\ \hline \end{array}$$

h.
$$\begin{array}{r} 8000 \\ -\ 1765 \\ \hline \end{array}$$

7. Calculate. WE2

a. 134 + 265 **b.** 820 + 39 **c.** 864 − 543 **d.** 738 − 83

e. 5736 + 457 **f.** 768 + 581 + 37 **g.** 7082 − 1491 **h.** 10 001 − 5642

8. Evaluate.

a. 88 + 11 − 58 **b.** 987 − 654 + 321

c. 567 − 214 − 145 **d.** 4382 + 567 − 258

e. 4627 − 1298 + 3384 **f.** 18 550 + 1250 − 16 304

g. 520 − 461 + 8969 **h.** 5003 − 2698 + 2985

9. Calculate.

2000 − 217 + 104

A. 274 **B.** 1887 **C.** 1679 **D.** 1870 **E.** 1783

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. 1469 + 2334 + 3264

Student A

$$\begin{array}{r} 1469 \\ 2334 \\ +\ 3264 \\ \hline 6957 \end{array}$$

Student B

$$\begin{array}{r} {\scriptstyle +1\ +1\ +1} \\ 1469 \\ 2334 \\ +\ 3264 \\ \hline 7067 \end{array}$$

b. 6000 − 364

Student A

$$\begin{array}{r} 6000 \\ -\ 364 \\ \hline 6364 \end{array}$$

Student B

$$\begin{array}{r} {\scriptstyle 9\ \ 9} \\ {\scriptstyle 5\ \not{10}\ \not{10}\ 10} \\ \not{6}\ \not{0}\ \not{0}\ \not{0} \\ -\ 364 \\ \hline 5636 \end{array}$$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Tao is an Uber driver. Over two weeks he earned $1045 and $632. How much did Tao earn in total?

12. There are 1213 students that attend Plight Street secondary school. 801 of the students travel to school by public transport. How many students do not travel by public transport to get to Plight Street secondary school?

13. Steven got a new job and needs to buy a whole new outfit. He has a budget of $900. He buys a $557 suit, a $125 tie, and matching $212 shoes. How much money does Steven have remaining after all his purchases?

14. When Charlotte started her bike ride her bike monitor already displayed 147 km. Halfway through her ride the monitor showed 169 km. By the end of her ride, how far had Charlotte travelled?

15. Winston spent 450 minutes in total working in his garden over four different Sundays. In the first week he spent two hours gardening, in the second week 118 minutes and in the third week an hour and a half. How much time did Winston spend gardening in the fourth week?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Tori and her brother Matthew live together. They both contribute money to a shared bank account. They use the bank account to pay for groceries and household bills. At the start of each month Tori contributes $870 and Matthew contributes $725.

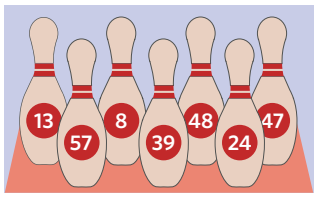
- **a.** Assuming that they have a balance of $0 at the end of July, what is the balance of the account after Tori and Matthew transfer on the 1st of August?
- **b.** They first use the money to pay $790 of rent on the 2nd of August. After paying rent, how much money do they have remaining in their account?
- **c.** The other monthly bills are $121 on gas and electricity, $69 for the internet and $25 on a Netflix account. Monthly, they normally spend $500 on groceries too. How much money can they expect to have in the account at the end of August?
- **d.** If Matthew contributed that same amount of money at the start of August, what would the balance have been at the end of August?
- **e.** What strategies could Tori and Matthew use to ensure that they never exceed their monthly budget?

17. Assuming that Sarah and Patrick both earn more than $100 and that their earnings are different, use the mathematical symbols = or ≠ to make each statement true.

- **a.** Sarah's earnings + Patrick's earnings ☐ Patrick's earnings + Sarah's earnings
- **b.** Sarah's earnings − Patrick's earnings ☐ Patrick's earnings − Sarah's earnings
- **c.** Compare your answers in part **a** and **b** and explain why they might be different.

Extra spicy

18. Which pins need to be knocked over to score a total of exactly 100 points?



19. Assuming all the symbols represent different whole numbers, select the option that is not true.

A. ★ + ● + △ + ■ = ■+ ★ + ● + △
B. ● + △ + (★ + ■) = (● + ■) + ★ + △
C. ● − △ − ★ − ■ ≠ ★ − △− ● − ■
D. (△ − ■) − ● − ★ − ■ = (★ − △) − ■ − ●
E. ★ + (△ + ● + ■) = ● + (■ + ★ + △)

20. How many times does the digit 9 appear in the solution to 30 010 000 − 4 010 101?

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

21. Which two numbers sum to 216 and have a difference of 82?

Remember this?

22. Willow makes $81 selling 9 flowerpots at the local market. All her flower pots are the same price.

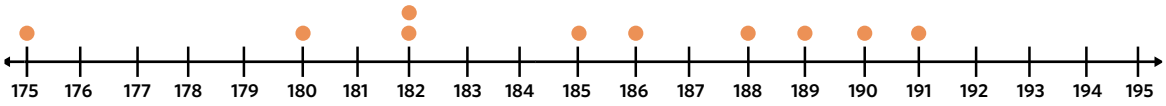
How much money will she make selling 13 flower pots?

A. $90 **B.** $100 **C.** $117 **D.** $130 **E.** $108

23. Lucille has a fruit bowl with 10 mangoes and 9 plums. What percentage of the fruit in the bowl is plums?

A. 53% **B.** 47% **C.** 9% **D.** 19% **E.** 90%

24. The heights, in centimetres, of ten AFL players are represented in the following dot plot.



Select the set of data values that corresponds to the dot plot.

A. 182, 182, 175, 190, 180, 185, 186, 191, 188, 189
B. 182, 175, 190, 180, 185, 186, 191, 188, 189
C. 182, 175, 190, 180, 185, 186, 191, 188
D. 182, 182, 175, 190, 180, 185, 186, 190, 188, 189
E. 181, 182, 175, 190, 180, 185, 186, 191, 188, 189

Back to contents

1B Multiplying and dividing whole numbers

Multiplication and division are basic mathematical skills. They build on skills of addition and subtraction, and are required for calculations involving fractions and powers.

LEARNING INTENTIONS

Students will be able to:

- understand the associative and commutative laws for multiplication
- multiply and divide using mental strategies
- multiply and divide using written algorithms.

KEY TERMS AND DEFINITIONS

A **dividend** is the first number in a division calculation. It is the number that is being divided by the second number.

A **divisor** is the second number in a division calculation. It is the number by which a given value is divided.

The **product** is the result when two or more values are multiplied together.

The **quotient** is the result of a division calculation.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Colleen Michaels/Shutterstock.com

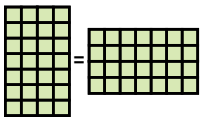
Multiplication and division are used as faster ways of counting and to calculate ways of sharing quantities equally. For example, calculating the total cost of buying five or more gift cards for a gift.

Key ideas

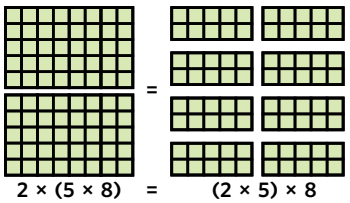
1. The commutative and associative laws apply to multiplication but do not apply to division.

Multiplication

The commutative law
$a \times b = b \times a$



The associative law
$a \times (b \times c) = (a \times b) \times c$



Division

The commutative law

$a \div b \neq b \div a$

$12 \div 3 \neq 3 \div 12$

$4 \neq 0.25$

The associative law

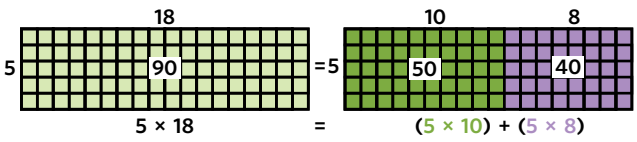
$a \div (b \div c) \neq (a \div b) \div c$

$24 \div (3 \div 2) \neq (24 \div 3) \div 2$

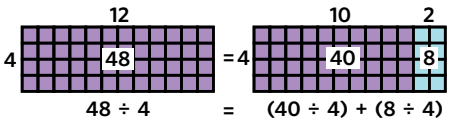
$16 \neq 4$

2. The distributive law can be used as a mental strategy for both multiplication and division.

Multiplication



Division



Back to contents

Worked example 1

Multiplying and dividing by powers of 10

Evaluate the following using a mental strategy.

a. 42×100 WE1a

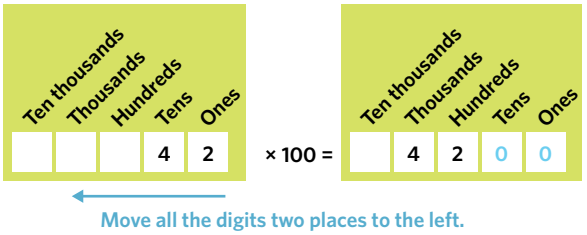
Working

$42 \times 100 = 4200$

Thinking

When multiplying by 100, move the numbers two places to the left. Use zeros as placeholders in the tens and ones place.

Visual support



b. $330 \div 10$ WE1b

Working

$330 \div 10 = 33$

Thinking

When dividing by 10, move the numbers one place to the right.

Student practice

Evaluate the following using a mental strategy.

a. 870×100 **b.** $2700 \div 10$

Worked example 2

Multiplying and dividing using mental strategies

Evaluate the following using a mental strategy.

a. 7×24 WE2a

Working

$7 \times 24 = 7 \times (20 + 4)$

$= (7 \times 20) + (7 \times 4)$

$= 140 + 28$

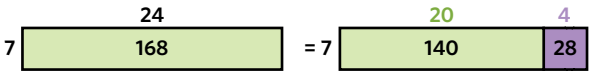
$= 168$

Thinking

Step 1: Identify the larger number and partition it into its place value parts.

Step 2: Use the distributive law. Multiply the two parts separately. Add the results together.

Visual support



Continues →

b. $84 \div 6$ WE2b

Working

$84 \div 6 = (60 + 24) \div 6$

$= (60 \div 6) + (24 \div 6)$

$= 10 + 4$

$= 14$

Thinking

Step 1: Identify the larger number and partition it into numbers that are easier to divide by.

Step 2: Use the distributive law. Complete the two division operations separately. Add the results together.

Student practice

Evaluate the following using a mental strategy.

a. 3×26 **b.** $65 \div 5$

Worked example 3

Multiplying and dividing using a vertical algorithm

Evaluate the following using a vertical algorithm.

a. 127×19 WE3a

Working

$$\begin{array}{rcccc} & & {\scriptstyle +2} & {\scriptstyle +6} & \\ & & 1 & 2 & 7 \\ \times & & & 1 & 9 \\ \hline & {\scriptstyle +1} & & & \\ & 1 & 1 & 4 & 3 \\ + & 1 & 2 & 7 & 0 \\ \hline & 2 & 4 & 1 & 3 \end{array}$$

Thinking

Step 1: Rewrite the numbers aligned vertically with the larger number on top.

Step 2: Multiply the top number by the digit in the ones place of the bottom number.

Step 3: Place a zero in the ones place under your product as a placeholder.

Step 4: Multiply the top number by the digit in the tens place of the bottom number.

Step 5: Add the two results together.

b. $1246 \div 12$ WE3b

Working

$$\begin{array}{r} 0\ \ 1\ \ 0\ \ 3\ \ r\ \ 10 \\ \hline 12\overline{)1\ {}^{1}2\ \ 4\ {}^{4}6} \end{array}$$

$1246 \div 12 = 103$ r 10

or

$103\frac{10}{12} = 103\frac{5}{6}$

Thinking

Step 1: Divide each digit of the dividend by the divisor from left to right. If the dividend is less than the divisor, use 0 as a placeholder and write the remainder alongside the next digit.

Step 2: Continue to perform the division and write the remainder alongside the next digit until the remainder is less than the divisor or 0.

Step 3: Write the final answer. Simplify the fraction.

Student practice

Evaluate the following using a vertical algorithm.

a. 245×21 **b.** $2681 \div 12$

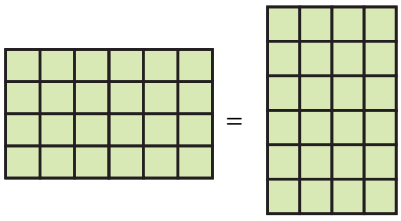
Back to contents

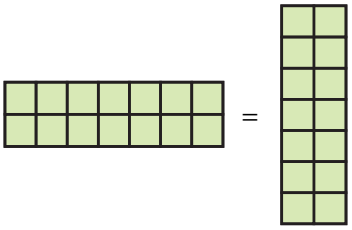
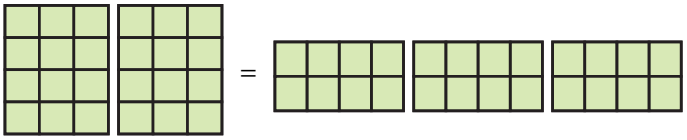
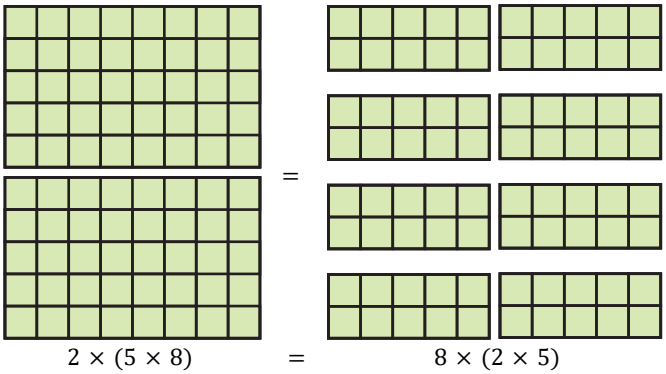
1B Questions

Understanding worksheet

1. Select whether the following show the commutative or associative laws by ticking the box.

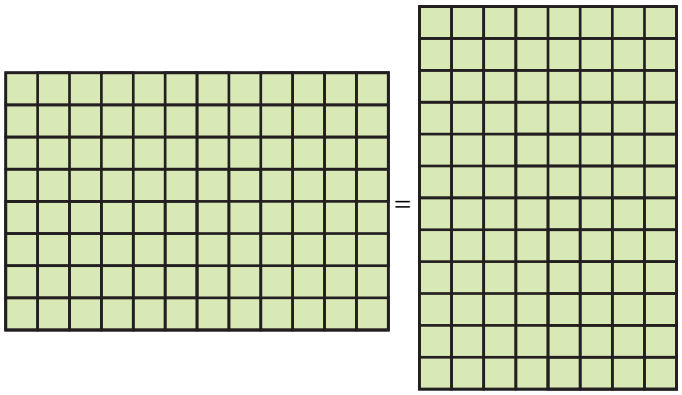
Example

	Commutative	Associative
 $4 \times 6 = 6 \times 4$	☑	☐

	Commutative	Associative
a.  $2 \times 7 = 7 \times 2$	☐	☐
b.  $2 \times (3 \times 4) = 3 \times (2 \times 4)$	☐	☐
c.  $2 \times (5 \times 8) = 8 \times (2 \times 5)$	☐	☐

Commutative **Associative**

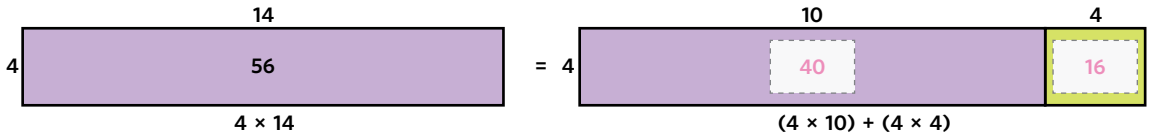
d.



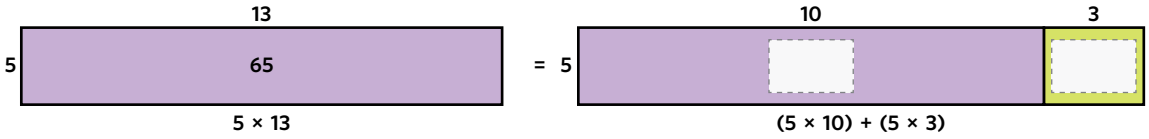
$8 \times 11 = 11 \times 8 = 88$ ☐ ☐

2. Fill in the blanks to apply the distributive law.

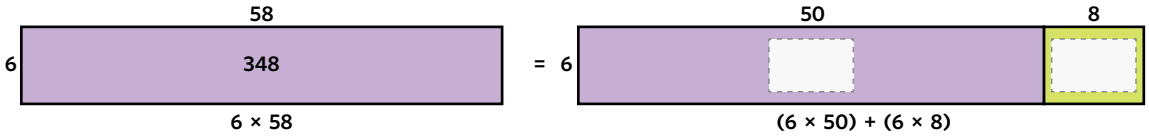
Example



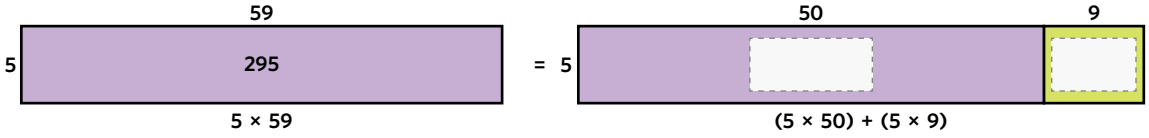
a.



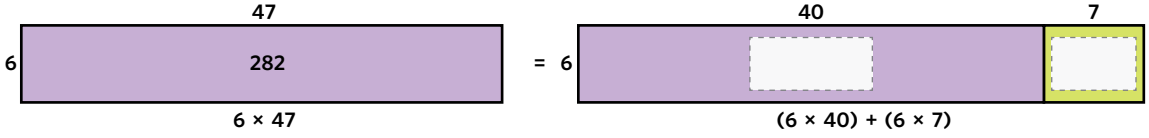
b.



c.



d.



3. Fill in the blanks by using the words provided.

product | quotient | divisor | dividend

In the equation $10 \div 5 = 2$, the number '10' is the _____ and the number '5' is the _____. The result of '2' is known as the _____ and applies specifically to division.

In the equation $3 \times 4 = 12$, the result of '12' is known as the _____ and applies only to multiplication.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Evaluate using a mental strategy. WE1a

a. 6×10 **b.** 41×100 **c.** 123×100 **d.** 257×1000

e. 650×1000 **f.** 8109×1000 **g.** $24 \times 100\,000$ **h.** $755 \times 100 \times 100$

5. Evaluate using a mental strategy. WE1b

a. $100 \div 10$ **b.** $320 \div 10$

c. $800 \div 100$ **d.** $4900 \div 100$

e. $53\,000 \div 1000$ **f.** $707\,000 \div 1000$

g. $2\,000\,000 \div 10\,000$ **h.** $81\,000\,000 \div 10 \div 100$

6. Evaluate using a mental strategy. WE2a

a. 3×16 **b.** 9×15 **c.** 16×7 **d.** 54×8

e. 12×16 **f.** 33×11 **g.** 9×112 **h.** 86×25

7. Evaluate using a mental strategy. WE2b

a. $48 \div 3$ **b.** $76 \div 4$ **c.** $98 \div 7$ **d.** $117 \div 9$

e. $168 \div 12$ **f.** $1133 \div 11$ **g.** $2550 \div 25$ **h.** $16\,032 \div 16$

8. Evaluate using a vertical algorithm. WE3a

a. $\begin{array}{r} 29 \\ \times\ 7 \\ \hline \end{array}$ **b.** $\begin{array}{r} 131 \\ \times\ 4 \\ \hline \end{array}$ **c.** $\begin{array}{r} 29 \\ \times\ 11 \\ \hline \end{array}$ **d.** 54×32

e. 176×19 **f.** 22×430 **g.** 2584×53 **h.** $767 \times 97\,531$

9. Evaluate using a vertical algorithm. WE3b

a. $3\overline{)54}$ **b.** $8\overline{)97}$ **c.** $5\overline{)132}$ **d.** $145 \div 11$

e. $266 \div 12$ **f.** $3383 \div 16$ **g.** $4144 \div 19$ **h.** $30\,111 \div 27$

10. Consider the division: $321 \div 5$.

Select the mental strategy that is correct.

A. $(321 \div 10) \div 2$

B. $(300 \div 5) + (21 \div 5)$

C. $5 \div 321$

D. $(300 \div 2) + (21 \div 3)$

E. $(3 \div 5) + (2 \div 5) + (1 \div 5)$

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. 89×13

Student A

$$\begin{array}{r} {\scriptstyle +2} \\ 8\ 9 \\ \times\quad 1\ 3 \\ \hline {\scriptstyle +1} \\ 2\ 6\ 7 \\ +\ 8\ 9\ 0 \\ \hline 1\ 1\ 5\ 7 \end{array}$$

Student B

$$\begin{array}{r} 89 \\ \times\ 13 \\ \hline 247 \\ +\ 890 \\ \hline 1037 \end{array}$$

b. $2346 \div 7$

Student A

$$\begin{array}{r} 0\ 3\ 3\ 5\ \text{r}\ 1 \\ \hline 7\overline{)2\ 3^{2}4^{3}6} \end{array}$$

Student B

$$\begin{array}{r} 3\ 3\ 3\ 5\ \text{r}\ 1 \\ \hline 7\overline{)2\ 3^{2}4^{3}6} \end{array}$$

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

12. Holly counts lollies to place into party bags for her birthday. If there are 120 lollies and a total of 10 party bags, how many lollies should she put into each bag?

13. Alex receives $5 pocket money each week and saves it all up for a $448 Xbox Series S Console. How many weeks would Alex need to save for?

14. The current time is 14:00. How many minutes have passed since midnight?

15. Cassie finds a cookie recipe that makes 10 cookies. The recipe requires 75 g flour, 50 g butter, 50 g condensed milk and 20 g caster sugar. How many grams of each ingredient does Cassie need to make 40 cookies?

16. Exercise books are on sale for 35c each. Karylle requires 2 books per subject and studies 7 subjects at school. How much will it cost Karylle to buy exercise books for every subject? Give your answer in dollars and cents.

Reasoning

Question working paths

Mild 17 (a,b,d)	Medium 17 (a,b,d), 18 (a,b)	Spicy All

17. Charlotte works at McDonald's and earns $12 an hour.

a. If she works 10 hours a week, how much would Charlotte earn?

b. Charlotte wants to buy a secondhand car worth $2000. How many hours does Charlotte need to work to save up for the car?

c. Charlotte turns sixteen at the beginning of her eleventh week working at McDonald's and her pay increases from $12 to $14 an hour. If she works 10 hours each week, how many weeks would it take Charlotte to save up for her car?

d. What is one way for Charlotte to earn more money in order to buy the car sooner?

Image: Sorbis/Shutterstock.com

18. Consider the two expressions.

$a \times 10$

$a \times 100$

Use the numbers 1–5 to substitute into a.

a. What is the smallest product for $a \times 10$?

b. What is the smallest product for $a \times 100$?

c. Comparing your results for **a** and **b**, what pattern do you notice when multiplying by 10 versus multiplying by 100?

Extra spicy

19. What time is exactly 2000 hours after 9:00 am?

20. Rita draws an owl on a piece of paper, which is approximately 210 mm wide by 30 cm long. She would like to photocopy it such that the drawing is 300% of its original size. What is the area of the photocopied page, in mm^2?

21. It cost Sorcha about 400 000 British pounds (£) to rebuild her home in the United Kingdom after the floods in 2021. This is roughly 500 times as much as the cost of the original build in 1923. Approximately how much did the home cost to build in 1923?

A. £80 **B.** £200 **C.** £800 **D.** £2000 **E.** £8000

22. The word 'thirty' contains six letters and $6 = 30 \div 5$. Similarly, the word 'forty' contains five letters and $5 = 40 \div 8$. Which of the following numbers is not a multiple of the number of letters it contains? Select the correct response.

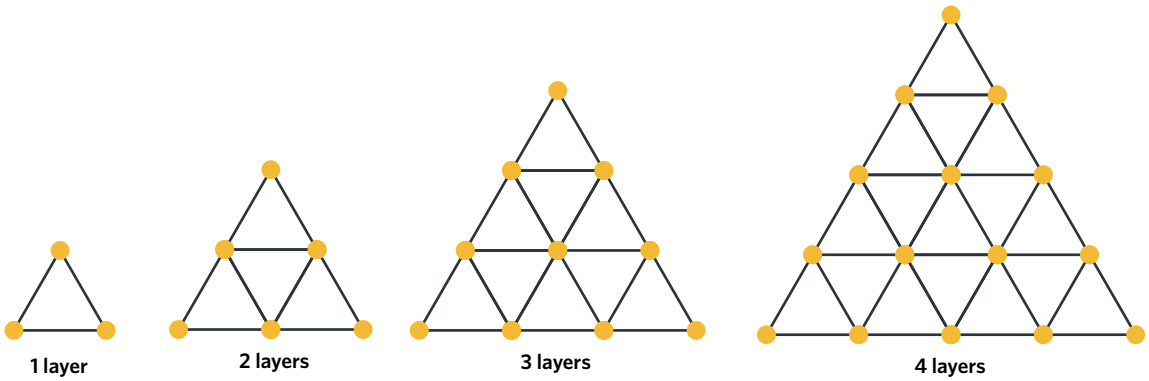
A. Six **B.** Twelve **C.** Eighteen **D.** Seventy **E.** Ninety

Remember this?

23. Louise has 1 cat, 3 dogs and 7 rabbits. What fraction of her pets are dogs?

A. $\frac{1}{11}$ **B.** $\frac{3}{10}$ **C.** $\frac{3}{11}$ **D.** 3 **E.** 11

24. Thanh drew a pyramid dot pattern.



If he continues the pattern, how many dots will 5 layers have? Select the correct response.

A. 5 **B.** 6 **C.** 18 **D.** 20 **E.** 21

25. Rachel is exactly 12 years old. Her sister Jill is 20 months old. Approximately how many months is Rachel older than Jill? Select the correct response.

A. −8 **B.** 8 **C.** 10 **D.** 11 **E.** 124

Back to contents

1C Adding and subtracting integers

When we add or subtract integers, the + and − signs of the numbers affect the overall outcome of the calculations. Although the same addition and subtraction strategies we use with whole numbers can be used with integers, there are further rules we must apply regarding the signs of the values.

LEARNING INTENTIONS

Students will be able to:

- use a number line to add and subtract positive and negative integers
- add and subtract positive and negative integers.

KEY TERMS AND DEFINITIONS

An **integer** is a whole number that is either positive, negative or zero.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

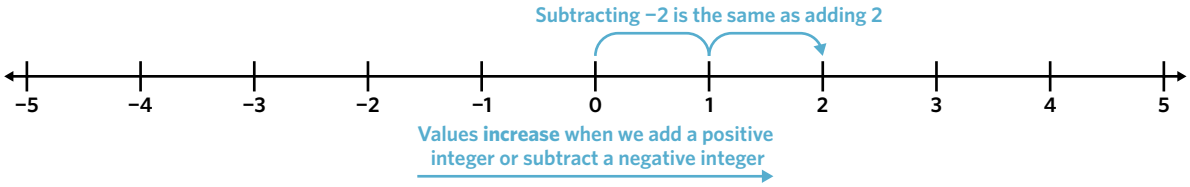
Hole in one		Eagle	−2
Par	0	Birdy	−1
Ablatross	−3	Bogey	+1

Hole	1	2	3
Par	4	5	3
Handicap	6	7	5

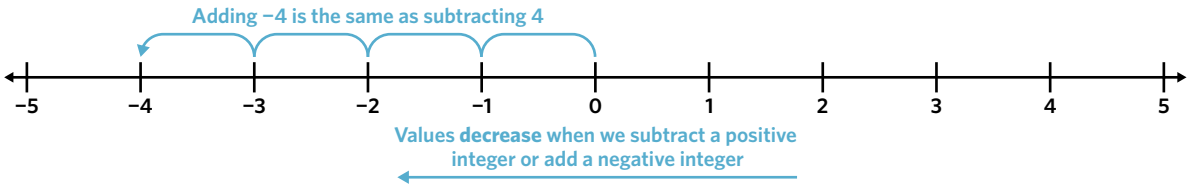
Adding and subtracting integers is an essential skill used in many situations. For example, keeping score of a sports game often involves the combination of both positive and negative values to calculate the total number of points given.

Key ideas

1. $0 - (-2) = 0 + 2$



2. $0 + (-4) = 0 - 4$



3. Zero sum pairs (opposite numbers) have the same value but different signs. They always sum to zero.

$$x + (-x) = x - x = 0$$

$$2 + (-2) = 2 - 2 = 0$$

Worked example 1

Adding integers

Calculate the following.

a. $-5 + 8$ WE1a

Working

$-5 + 8 = -5 + 5 + 3$

$= 0 + 3$

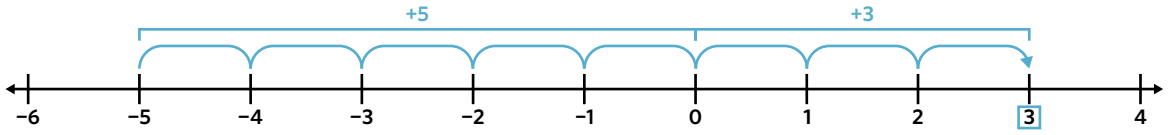
$= 3$

Thinking

Step 1: Partition 8 into 5, the opposite number to −5, and 3 in order to use 0 as the benchmark number.

Step 2: Evaluate the expression.

Visual support



b. $-2 + (-5)$ WE1b

Working

$-2 + (-5) = -2 - 5$

$= -7$

Thinking

Step 1: Adding a negative integer is the same as subtracting a positive integer.

Step 2: Evaluate the expression.

Student practice

Calculate the following.

a. $-4 + 6$ **b.** $-1 + (-2)$

Worked example 2

Subtracting integers

Calculate the following.

a. $-1 - 8$ WE2a

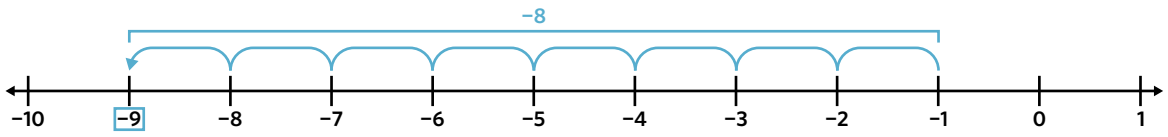
Working

$-1 - 8 = -9$

Thinking

A positive integer is being subtracted from a negative integer so move 8 units in the negative direction, away from 0.

Visual support



Continues →

b. $-4 - (-9)$ WE2b

Working

$-4 - (-9) = -4 + 9$

$= 5$

Thinking

Step 1: Subtracting a negative integer is the same as adding a positive integer.

Step 2: Evaluate the expression.

Student practice

Calculate the following.

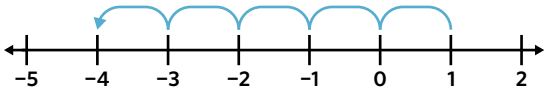
a. $-2 - 4$ **b.** $-3 - (-5)$

1C Questions

Understanding worksheet

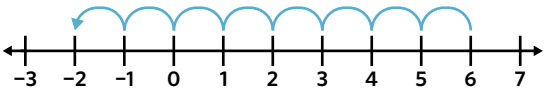
1. Match the following calculations with their number line representations.

Example



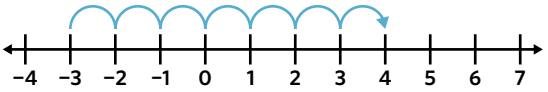
● ——— ● $1 + (-5)$

a.



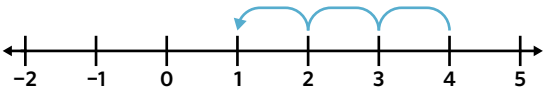
● ● $6 - 8$

b.



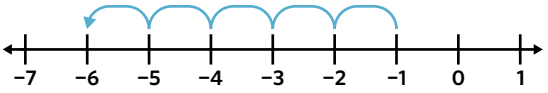
● ● $4 + (-3)$

c.



● ● $-3 + 7$

d.



● ● $-1 - 5$

2. Choose whether addition or subtraction will make each of the following calculations true.

Example

$3 \boxed{-} (-2) = 5$

a. $-7 \boxed{} 7 = 0$

b. $-9 \boxed{} 2 = -11$

c. $8 \boxed{} (-1) = 9$

d. $-12 \boxed{} (-5) = -17$

3. Fill in the blanks by using the words provided.

negative | positive | whole | increase | zero | decrease | subtracting

Integers are _____ numbers that can be positive, negative or _____.

Adding a _____ integer is the same as subtracting a positive integer. _____

a negative integer is the same as adding a _____ integer. Integers on the number line

_____ in value to the right and _____ in value to the left.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Calculate. WE1a

a. $-2 + 4$ **b.** $-11 + 3$ **c.** $-17 + 11$ **d.** $-21 + 30$

e. $-53 + 40$ **f.** $-47 + 100$ **g.** $-160 + 54$ **h.** $-124 + 253$

5. Calculate. WE1b

a. $6 + (-3)$ **b.** $-3 + (-1)$ **c.** $20 + (-13)$ **d.** $-5 + (-11)$

e. $23 + (-19)$ **f.** $-119 + (-24)$ **g.** $74 + (-105)$ **h.** $-107 + (-198)$

6. Calculate. WE2a

a. $5 - 7$ **b.** $-9 - 3$ **c.** $4 - 16$ **d.** $-15 - 10$

e. $48 - 53$ **f.** $-79 - 21$ **g.** $54 - 120$ **h.** $-209 - 192$

7. Calculate. WE2b

a. $2 - (-6)$ **b.** $-7 - (-1)$ **c.** $12 - (-8)$ **d.** $-22 - (-11)$

e. $-23 - (-23)$ **f.** $68 - (-37)$ **g.** $-124 - (-39)$ **h.** $-181 - (-195)$

8. Calculate.

a. $2 + (-6)$ **b.** $3 - (-7)$

c. $-2 + (-10)$ **d.** $-20 - (-6)$

e. $-11 - (-20)$ **f.** $19 + (-2) + (-7)$

g. $15 - (-7) + 3$ **h.** $-53 - (-21) - (-15)$

9. Consider the following incomplete calculation.

$(-17) - \square = -34$

Select the option to fill in the box to make the calculation true.

A. -34 **B.** -17 **C.** 17 **D.** 34 **E.** 51

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. $-14 + (-3)$

Student A

$= -14 + -3$

$= -14 + 3$

$= -11$

Student B

$= -14 + -3$

$= -14 - 3$

$= -17$

b. $-20 - (-6)$

Student A

$= -20 - -6$

$= -20 + 6$

$= -14$

Student B

$= -20 - -6$

$= -20 - 6$

$= -26$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. An iceberg floating in the Arctic Ocean is submerged to -18 metres. If 2 metres of the iceberg is sticking out of the water, how tall is the iceberg altogether?

12. Mikko's bank account has a balance of $-\$50$. \$67 is withdrawn from his account. What is the balance of Mikko's account after the withdrawal?

13. A scuba diver dives to 21 metres below sea level. She must make a safety stop at 5 metres below sea level on her way back up to the surface to decompress her body. How far towards the surface did the diver have to swim before making her safety stop?

14. In Australia, the highest temperature ever recorded was 51°C in Western Australia in 2022. The lowest temperature ever recorded was -23°C in New South Wales in 1994. What is the difference between the highest and lowest temperatures ever recorded in Australia?

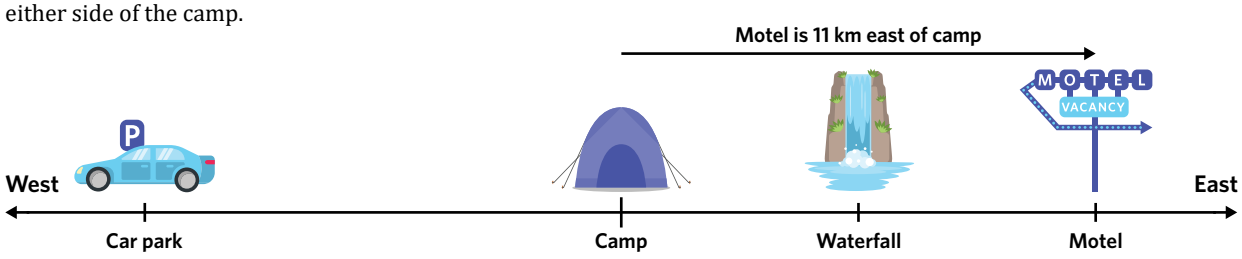
15. An ion is a charged atom because there are different numbers of positive protons and negative electrons inside of it. If an ion has a charge of -2 and contains 8 protons, how many electrons does it contain?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Paulie and Christopher are camping during a long weekend. They set up their camp to the west of the waterfall they would like to visit. The motel and car park are both an equal distance away on either side of the camp.



a. If the camp represents zero on an imaginary number line, express the distance to the motel from the camp in km, as an integer.

b. Express the distance to the car park from the camp as an integer.

c. The waterfall is 6 km west of the motel. Express the distance to the waterfall from the camp as an integer.

d. On the second morning of their camping trip, the two friends walk to the waterfall from camp and spend the day there before walking back to camp and packing up. After, they walk to the car park, pick up the car and drive to the motel. Assuming that they walk and drive on the same path, how much further have they driven that day compared to how far they walked?

e. Paulie and Christopher were very tired from all the walking they did when they got back to the motel. Suggest some ways that they could have organised their trip and set up camp so as to reduce the amount of travel by foot.

17. Consider the following expression, where x and y can be any integer.

$x - y$

a. Using only single digit integers -9 to 9, inclusive of zero, substitute values for x and y to make the largest result possible. State the values you chose for x and y as well as the largest result.

b. Using only single digit integers -9 to 9, inclusive of zero, substitute values for x and y to make the smallest result possible. State the values you chose for x and y as well as the smallest result.

c. Compare your answers from parts **a** and **b** and explain why subtraction does not always decrease the value in a calculation.

Extra spicy

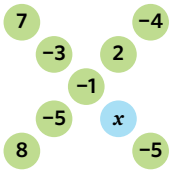
18. $-a + b + b - a$

Substitute $a = 5$ and $b = -3$ into the above expression and evaluate. Pick the correct answer option from the ones given below.

A. -16 **B.** -6 **C.** -4 **D.** 6 **E.** 16

19. Pick an integer to substitute for x, so that the five numbers along both diagonals in the following image sum to the same value.

A. -8
B. -6
C. -2
D. 0
E. 2



20. Marty and Emmett have invented a time machine. First they travelled back in time to when Marty was born 18 years ago. Then they travelled forward in time by 25 years and ended up in the year 2000. In which year did Marty and Emmett initially start their time travels?

21. All four digits of two 2-digit negative integers are different. What is the largest possible sum of the two negative integers?

Remember this?

22. Nancy made $400 from selling four paintings. After she sold the first painting, she earned $30 more for each following painting compared to the one sold directly before it. How much did Nancy sell her first painting for?

A. $30 **B.** $40 **C.** $55 **D.** $100 **E.** $180

23. When Pascal stands on Mickey's shoulders during an acrobatic performance, their total height is 3.5 metres. If there is a difference of 20 centimetres in Mickey's and Pascal's heights, what is the height of the taller acrobat?

A. 165 cm **B.** 170 cm **C.** 175 cm **D.** 185 cm **E.** 195 cm

24. At the International Mathematical Olympiad, each contestant is required to solve six problems in total. They can get a maximum of seven points for each correctly solved problem. If Ibrahim got 31 points, what is the greatest number of problems for which he could have received full points?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

Back to contents

1D Multiplying and dividing integers

When we multiply or divide integers, the signs of the numbers affect the outcome of the calculations. Although the same multiplication and division strategies we use with whole numbers can be used with integers, there are further rules we must apply regarding the signs of the values.

LEARNING INTENTIONS

Students will be able to:

- understand that the product or quotient of a positive and negative integer is a negative integer
- understand that the product or quotient of two negative integers is a positive integer
- calculate the product of more than two integers.

KEY TERMS AND DEFINITIONS

Directed numbers have a direction and value, one direction is positive (+) and the other direction is negative (−).

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: vinnstock/Shutterstock.com

When it comes to budgeting, we often have to deal with both positive and negative values. Tasks such as working out savings plans or ensuring that regular direct debits do not overdraw an account require knowledge of multiplication and division.

Key ideas

1. The product or quotient of two integers with the same directions or signs will always be positive.

Positive integer	×	Positive integer	=	Positive integer
Positive integer	÷	Positive integer	=	Positive integer
Negative integer	×	Negative integer	=	Positive integer
Negative integer	÷	Negative integer	=	Positive integer

2. The product or quotient of two integers with different directions or signs will always be negative.

Positive integer	×	Negative integer	=	Negative integer
Positive integer	÷	Negative integer	=	Negative integer
Negative integer	×	Positive integer	=	Negative integer
Negative integer	÷	Positive integer	=	Negative integer

3. The product of zero and any integer is zero.

$$x \times 0 = 0 \times x = 0$$
$$3 \times 0 = 0 \times 3 = 0$$

Back to contents

Worked example 1

Multiplying integers

Calculate the following.

a. $4 \times (-6)$ WE1a

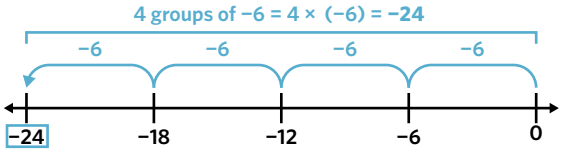
Working

$4 \times (-6) = -24$

Thinking

The product of a positive integer and a negative integer will be negative. Evaluate the product.

Visual support



b. $-3 \times (-5)$ WE1b

Working

$-3 \times (-5) = 15$

Thinking

The product of two negative integers will be positive. Evaluate the product.

c. $-2 \times 6 \times (-3)$ WE1c

Working

$-2 \times 6 \times (-3) = -12 \times (-3)$

$= 36$

Thinking

Step 1: Going from left to right, multiply the first two integers together.

Step 2: The product of two negative integers will be positive. Evaluate the product.

Student practice

Calculate the following.

a. $5 \times (-6)$ **b.** $-4 \times (-7)$ **c.** $-3 \times 5 \times (-4)$

Worked example 2

Dividing integers

Calculate the following.

a. $-16 \div 2$ WE2a

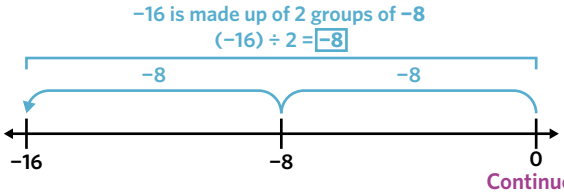
Working

$-16 \div 2 = -8$

Thinking

The quotient of a negative integer and a positive integer will be negative. Evaluate the quotient.

Visual support



Continues →

b. $\frac{-18}{-3}$ WE2b

Working

$\frac{-18}{-3} = -18 \div (-3)$

$= -18 \div (-3) = 6$

Thinking

Step 1: Rewrite the fraction as a division.

Step 2: The quotient of two negative integers will be positive. Evaluate the quotient.

Student practice

Calculate the following.

a. $-21 \div 7$ **b.** $\frac{-32}{-4}$

1D Questions

Understanding worksheet

1. Circle the direction of the outcomes for each of the following calculations.

Example

Negative integer × Positive integer = Positive integer or (Negative integer)

a. Negative integer × Negative integer = Positive integer or Negative integer

b. Positive integer × Negative integer = Positive integer or Negative integer

c. Negative integer ÷ Positive integer = Positive integer or Negative integer

d. Negative integer ÷ Negative integer = Positive integer or Negative integer

2. Show the direction of each outcome below by placing a negative or positive sign in each of the empty boxes.

Example

$-4 \times 3 = \boxed{-}\ 12$

a. $2 \times (-3) = \square\ 6$

b. $-8 \div 2 = \square\ 4$

c. $-5 \times (-3) = \square\ 15$

d. $-12 \div (-6) = \square\ 2$

3. Fill in the blanks by using the words provided.

left | positive | opposite | right | direction

When an integer has value and _____, it is called a directed number. Negatively directed numbers lie to the _____ of zero, while positively directed numbers lie to the _____ of zero. The product or quotient of two numbers with the same directions will always be _____. The product or quotient of two numbers with _____ directions will always be negative.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Calculate. WE1a,b

a. $4 \times (-2)$ **b.** -7×2 **c.** $0 \times (-11)$ **d.** $-6 \times (-1)$

e. $-9 \times (-3)$ **f.** $-10 \times (-6)$ **g.** $12 \times (-7)$ **h.** $-24 \times (-4)$

5. Calculate. WE1c

a. $-3 \times 1 \times 2$ **b.** $7 \times 2 \times (-1)$ **c.** $4 \times (-2) \times 2$ **d.** $3 \times (-8) \times 0$

e. $2 \times (-5) \times (-4)$ **f.** $-5 \times (-6) \times 2$ **g.** $-8 \times 3 \times (-2)$ **h.** $-7 \times (-3) \times (-4)$

6. Calculate. WE2

a. $-9 \div 3$ **b.** $-12 \div (-6)$ **c.** $-20 \div 5$ **d.** $\frac{35}{-7}$

e. $-42 \div (-6)$ **f.** $\frac{-14}{-7}$ **g.** $96 \div (-6)$ **h.** $-152 \div (-8)$

7. Calculate. WE1,2

a. $13 \times (-4)$ **b.** $-45 \div 15$ **c.** 554×0 **d.** $0 \times (-791)$

e. $1600 \div (-40)$ **f.** $-207 \times (-10)$ **g.** $\frac{-486}{-9}$ **h.** $-3133 \div (-13)$

8. Determine which of the following options is not equivalent to the below calculation.

$-4 \times 2 \times (-6)$

A. -24×2 **B.** $-8 \times (-6)$ **C.** $-4 \times (-12)$ **D.** 24×2 **E.** 48

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. 4×-8

Student A

$= 4 - 8$

$= -4$

Student B

$= 4 \times (-8)$

$= -32$

b. $\frac{-56}{-8}$

Student A

$= -56 \div (-8)$

$= 7$

Student B

$= -56 \div 8$

$= -7$

Problem solving

Question working paths

Mild	Medium	Spicy
10, 11, 12	11, 12, 13	12, 13, 14

10. Tyson pays $180 every week for his room in a sharehouse. This includes his rent and $30 for electricity, gas and internet access. Over 20 weeks, how much did Tyson pay for only his rent?

11. Jan is competing on a trivia game show. $250 is deducted every time a contestant gives an incorrect answer. If Jan has earned $2500 and then incorrectly answers three questions, how much money will she be taking home?

12. On a school camp, a box of apples is put out for the students. Throughout the day, a total of 266 students came and took apples out of the box until there were only 2 left. If there were 800 apples to begin with, then how many apples on average did each student take?

13. Jez is playing a computer game where he can respawn after failing a mission. Every time he respawns, it costs him 1200 coins in the game. Jez has already collected 34 000 coins. How many times can he respawn before he runs out of coins and has to restart the mission?

14. Gary is paying for a new gaming console in weekly instalments of $140. If Gary had −$40 in his account after 4 weeks of regular payments, then what was the balance of Gary's account to begin with?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. The Mariana Trench is the deepest part of any ocean on our planet. Using the diagram below:

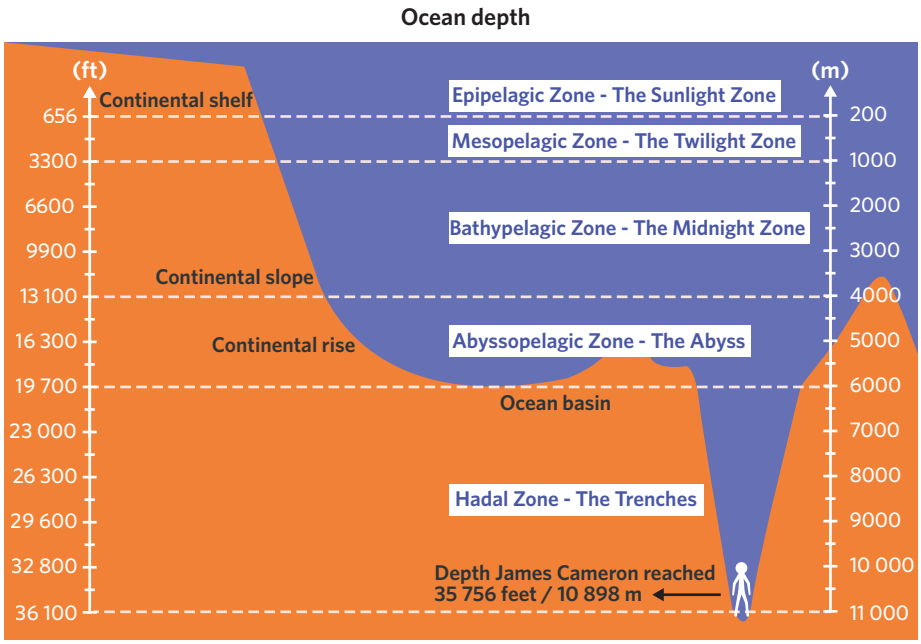


Image: VectorMine/Shutterstock.com

a. Express the depth of the Mariana Trench as an integer, in metres.

b. Light cannot travel beyond The Twilight Zone, but photosynthesis can only occur in the Sunlight Zone. How much further past The Sunlight Zone can light travel, in metres?

c. The deepest part of the ocean lies in the Hadal Zone, or The Trenches, which is very dark and cold. How many Sunlight Zones would fit inside the Hadal Zone?

d. An atmosphere (atm) is a unit of pressure. The total atm experienced by a diver increases by one unit every 10 metres below the surface. Including the 1 atm of pressure on the surface, how many atm would someone experience at the deepest part of the ocean?

e. The Mariana Trench is located in the Pacific Ocean, east of the Mariana Islands. Since its discovery in 1875, only seven descents, four manned and three unmanned, have ever been achieved. What are the challenges of organising an expedition such as this one, and do they outweigh the benefits of the data we might gather as a result? Explain your thinking.

16. Use the tables below to answer the following questions.

Table 1
$x =$
$x \times (-1) =$
$x \times (-1) \times (-1) =$
$x \times (-1) \times (-1) \times (-1) =$

Table 2
$y =$
$y \times (-1) =$
$y \times (-1) \times (-1) =$
$y \times (-1) \times (-1) \times (-1) =$

a. Pick any single digit positive integer to represent x and substitute it into the expressions in Table 1. Record your results for each of the substitutions.

b. Pick any single digit negative integer to represent y and substitute it into the expressions in Table 2. Record your results for each of the substitutions.

c. What kind of pattern do the results in each of the tables for x and y form and what does this show about the effects of multiplying by a negative?

Extra spicy

17. If a and b are both negative integers, which one of the following statements is false?

A. $a \div b > 0$
B. $a \times b > 0$
C. $-a \times b < 0$
D. $a \div (-b) < 0$
E. $-a \times (-b) < 0$

18. The mean of the average surface temperatures on Mars, the Moon, and Jupiter is -84°C. When the average surface temperature of the Earth is included in this mean, it becomes -59°C. What is the average surface temperature of the Earth?

A. -59°C **B.** -25°C **C.** -16°C **D.** 16°C **E.** 25°C

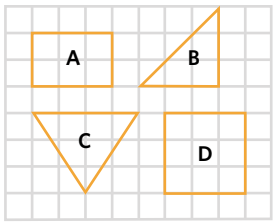
19. What is the largest even two-digit integer that is a product of two prime numbers?

20. A triangle's perimeter is 18 cm and all of the side lengths are positive integers. Two of its sides are 3 and 5 times as long as the third side. What are the side lengths of this triangle?

Remember this?

21. Which pair of shapes have the same area?

A. A and B
B. A and C
C. B and C
D. B and D
E. C and D



22. When Amber goes hiking, she can cover 14 km in 3.5 hours. How many kilometres can Amber cover in 2 hours?

A. 1.5 km **B.** 2 km **C.** 4 km **D.** 8 km **E.** 12 km

23. What is the whole number closest in value to 4.7^2?

A. 9 **B.** 10 **C.** 20 **D.** 23 **E.** 25

Back to contents

1E Order of operations

When calculating any mathematical expression with multiple operations, the order of operations is important. The order in which mathematical operations should be performed is as follows: brackets, exponents, multiplication or division, addition or subtraction. Sometimes not all the operations appear in a problem, but it is essential to check and continue with the order of operations.

LEARNING INTENTIONS

Students will be able to:

- understand the order of operations in expressions
- apply order of operations when evaluating expressions
- create and solve equations to represent problems.

KEY TERMS AND DEFINITIONS

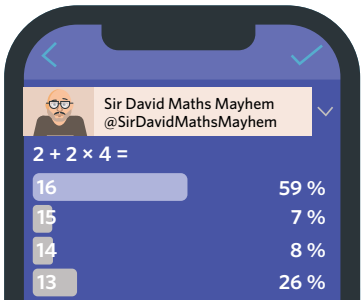
Inverse operations are mathematical operations that undo each other. The inverse of multiplication is division and the inverse of division is multiplication; the inverse of addition is subtraction and the inverse of subtraction is addition.

An **operation** in maths is a mathematical process used to solve problems. The four basic operations in maths are addition, multiplication, subtraction and division.

A **variable** is a letter used to represent a value that is unknown or may vary. This is also known as a pronumeral or an unknown.

Substitution is the process of replacing a variable or an unknown with a given value.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



The order in which we solve maths problems is universally the same.

If we did not have a universally known procedure to solve maths problems, everyone would get different answers (much like the poll above), causing global mathematics mayhem.

Note: The poll above does not show the correct answer.

Key idea

1. Expressions with multiple operations, follow a specific order known as the order of operations.

$[3 + 8 \div (3^2 - 10) \times 2]$	Brackets/Exponent	**Correct order of operations:**
$= [3 + 8 \div (9 - 10) \times 2]$	Brackets/Subtraction	**1st: Brackets**
$= [3 - 8 \div (-1) \times 2]$	Division	**2nd: Exponent**
$= [3 + 8 \times 2]$	Multiplication	**3rd: Multiplication or Division - working from left to right**
$= [3 + 16]$	Addition	**4th: Addition or Subtraction - working from left to right**
$= 19$		

Back to contents

Worked example 1

Completing order of operations calculations

Using the order of operations, complete the calculations.

a. $(-2 \times 3^2) \div 6$ WE1a

Working

$(-2 \times 3^2) \div 6 = (-2 \times 9) \div 6$

$= -18 \div 6$

$= -3$

Thinking

Step 1: Using order of operations, evaluate the exponent inside the bracket.

Step 2: Evaluate the multiplication.

Step 3: Evaluate the division.

Visual support

$(-2 \times 3^2) \div 6$ Brackets/Index
$= (-2 \times 9) \div 6$ Multiplication
$= -18 \div 6$ Division
$= -3$

b. $[\sqrt{49} - 12 + (7 - 3)]$ WE1b

Working

$[\sqrt{49} - 12 + (7 - 3)] = [\sqrt{49} - 12 + 4]$

$= 7 - 12 + 4$

$= -5 + 4$

$= -1$

Thinking

Step 1: Using order of operations, simplify the inner bracket by subtracting.

Step 2: Evaluate the square root.

Step 3: Addition and subtraction will be calculated from left to right.

Student practice

Using the order of operations, complete the calculations.

a. $(-12 \times 3^2) \div 2$

b. $[\sqrt{1} - 5 + (9 - 2)]$

Worked example 2

Evaluating expressions using substitution

Evaluate these expressions using $a = -1$ and $b = 2$.

a. $a - b$ WE2a

Working

$a - b = -1 - 2$

$= -3$

Thinking

Step 1: Substitute the variables for numbers.

Step 2: Evaluate the subtraction.

Visual support

$a = -1$ $b = 2$
$a - b$
$= -1 - 2$
$= -3$

Continues →

b. $2ab - b$ WE2b

Working

$2ab - b = 2(-1)(2) - 2$

$= 2 \times (-1) \times 2 - 2$

$= -2 \times 2 - 2$

$= -4 - 2$

$= -6$

Thinking

Step 1: Substitute the variables for numbers.

Step 2: Using order of operations, multiply the integers from left to right.

Step 3: Calculate the subtraction.

Student practice

Evaluate these expressions using $a = -1$ and $b = 2$.

a. $a + b$ **b.** $3ab - b$

Worked example 3

Solving worded order of operation questions

Represent the following sentences as expressions and solve.

a. Three minus the quotient of ten and two. WE3a

Working

$= 3 - 10 \div 2$

$= 3 - 5$

$= -2$

Note: This can also be expressed as: $3 - \left(\frac{10}{2}\right)$

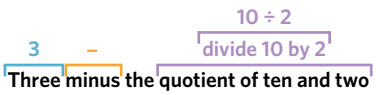
Thinking

Step 1: Represent the worded problem as an expression.

Step 2: Using order of operations, evaluate the division.

Step 3: Evaluate the subtraction.

Visual support



b. Four squared divided by the product of negative eight and two. WE3b

Working

$\frac{4^2}{(-8 \times 2)}$

$= \frac{4^2}{(-16)}$

$= \frac{16}{-16}$

$= -1$

Note: This can also be expressed as: $4^2 \div (-8 \times 2)$

Thinking

Step 1: Represent the worded problem as an expression.

Step 2: Using order of operations, evaluate the brackets.

Step 3: Evaluate the exponent.

Step 4: Evaluate the division.

Student practice

Represent the following sentences as expressions and solve.

a. Seven minus the quotient of twenty four and two.

b. Twelve squared divided by the product of negative three and six.

Back to contents

1E Questions

Understanding worksheet

1. Place brackets around the part that should be calculated first.

Example

$4 + (7 \times 4) - 6$

a. $13 + 12 \div 6 - 4$ **b.** $12 \times 3^3 \div 10$ **c.** $18 \div 3 \times 13 \div 13$ **d.** $-8 \times [19 - 3 \times 42]$

2. Replace the word with the matching mathematical operation.

Example

The product of negative four and two is added to the quotient of sixteen and two.

$(-4 \boxed{\times} 2) + (16 \boxed{\div} 2)$

a. Nine less than the sum of 3 and negative 6.

$(3 - 6) \boxed{} 9$

b. Half of the product of three and thirty.

$\frac{3 \boxed{} 30}{2}$

c. Three more than the difference of negative nine and negative four.

$3 \boxed{} (-9 \boxed{} - 4)$

d. Eight less than the squared quotient of negative twelve and two.

$(-12 \boxed{} 2)^2 \boxed{} 8$

3. Fill in the blanks by using the words provided.

brackets | order | operations | inverse

The order in which mathematical [ ] should be performed is as follows: brackets, exponents, multiplication or division, addition or subtraction. Even if some operations are not in the problem, it is important to follow the [ ] of operations. When working with [ ] operations we work from left to right. When solving an equation with multiple [ ] work from inner brackets to outer brackets.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Using the order of operations, complete the calculations. WE1a

a. $3 + 5 \times 6$ **b.** $6 \times 7 - 50$

c. $-8 \div 2 \times 4$ **d.** $-3 \times (0 - 4)$

e. $-9 - 10 \times 2$ **f.** $-3 - 5 + 7 \times (-9)$

g. $-14 - (25 - 60 + 5)$ **h.** $4^2 - \sqrt{16}$

5. Using the order of operations, complete the calculations. WE1a

a. $2^2 \times (3 - 5)$

b. $-5 \times 6 \div 5$

c. $2 \times (27 - 40) \div 2$

d. $(7 - \sqrt{49}) - (-6)^2$

e. $(10 - 21 + 80 \div 10)^2$

f. $-40 \times 30 - 9^2 + \frac{25}{5}$

g. $\frac{\sqrt{9}}{3} + \frac{36}{6}$

h. $\frac{-100}{(80 \times 2) \div (4 \times 4)}$

---

6. Complete the calculations. WE1b

a. $[(-3 \times 4) \div 2]$

b. $0 \times [(-100 - 20) \div 40]$

c. $50 \div ((3 - 5) \times 5^2)$

d. $[(8 - 3)^2 - (20 + 30)]$

e. $(3 - (-4)^2) \times (-2)$

f. $\frac{21}{7} + [3^2 - (19 - 26)]$

g. $[\sqrt{144} \div (2 - 3)^3 \div (-1)]$

h. $\sqrt{(10^2 - 8^2)}$

---

7. Evaluate these expressions using $a = 2$ and $b = -3$. WE2

a. $a + b$

b. ab

c. $2a - b$

d. $b^2 - 3ab$

e. $(-2a + 2b) \div a$

f. $(b - a)^2 - b$

g. $\frac{3a}{a} + \sqrt{b^2}$

h. $\sqrt{(2a)^2 + b^2}$

---

8. Rewrite the following equations and insert brackets to make them true.

a. $2 + 2 \times 4 = 16$

b. $-15 - 10 \times -8 = 65$

c. $-4 - 13 \times -2 = 34$

d. $6 - 4^2 \times -2 = 20$

e. $5 - -3^2 \times -7 = 28$

f. $-36 + \frac{\sqrt{144}}{2} = -30$

g. $-3 \times 2 \times 5 - 3 = -12$

h. $\frac{-30}{60 \times 2 \div 3 \times 4} = -3$

---

9. Represent the following sentences as expressions and solve. WE3

a. Five minus half of twenty.

b. Nine less than negative twenty added to thirteen.

c. The product of negative five and four plus thirty-three.

d. Three times the sum of negative eight and negative two.

e. Three groups of fifteen divided by negative five.

f. The product of negative two and three multiplied by the quotient of negative sixty-six and eleven.

g. The product of negative eight and two divided by four squared.

h. Six halves squared.

---

10. What is the answer to the following expression:

$[3 - 24 \div (3 - 6)] \times 3^2$

A. -45 **B.** 6 **C.** 9 **D.** 66 **E.** 99

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. $-6 - 36 \div (-9)$

Student A

$-6 - 36 \div (-9)$

$= -6 - 4$

$= -10$

Student B

$-6 - 36 \div (-9)$

$= -6 + 4$

$= -2$

b. $12 - 60 \div 12 \times (-4)$

Student A

$12 - 60 \div 12 \times (-4)$

$= -48 \div 12 \times (-4)$

$= -4 \times (-4)$

$= 16$

Student B

$12 - 60 \div 12 \times (-4)$

$= 12 - 5 \times (-4)$

$= 12 + 20$

$= 22$

Back to contents

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

12. In a game of AFL, each goal is worth six points and each behind is worth one point. Hawthorn scored 79 points in total. If they scored 11 goals, how many behinds did they kick?

13. Collyn is saving money for a new VR headset which costs $529.00. She has saved $33 a week over twelve weeks. How much more money does she need to save in order to purchase the VR headset?

14. Niamh has collected 20 Pop vinyl bobble head figures, half of them have been kept in their original packaging. Her dad gave her 152 more figures. Niamh noticed that only an eighth of the figures from her dad were in their original packaging. How many figures in original packaging does Niamh have?

15. Mateo wants to knit five jumpers and needs 1496 metres of yarn for each jumper. A single ball of yarn is 170 metres long and costs $5. A four pack of yarn is on sale for $16. How much will Mateo save by purchasing all the yarn for the jumpers when the price is discounted?

16. Zom-beads make necklaces to sell. A 40 cm necklace has fifty beads, and a 70 cm necklace has twice the amount of beads as the 40 cm. A 20 cm necklace is made with a quarter of the beads of a 70 cm necklace. How many beads are needed to make three 40 cm, five 70 cm and two 20 cm necklaces?

Reasoning

Question working paths

Mild 17 (a,b,c,e)	Medium 17 (a,b,c,e), 18 (a,b)	Spicy All

17. The crown-of-thorns sea star lives in the ocean near Cairns. One sea star has the capacity to destroy 10 square kilometres of coral every year. Trained scuba divers try to stop the spread of the crown-of-thorns sea star by injecting them with low doses of acid. A professional scuba diver can inject 1000 sea stars during a 50 minute dive.

Image: Rich Carey/Shutterstock

a. Over a span of 25 years, how many square kilometres of coral can one single crown-of-thorns sea star destroy?

b. The rate at which a professional scuba diver can inject the crown-of-thorns sea star in 1 minute is the quotient of the number of sea stars and the time it takes. Write an equation and determine how many sea stars the scuba divers can inject in 1 minute.

c. A diver can safely descend at a rate of 900 centimetres per minute. If a diver descends to 27 m below sea level, spends 40 minutes injecting crown-of-thorns sea stars, and makes a safety stop for five minutes before surfacing, then how long was their dive altogether?

d. One diver injected 1108 crown-of-thorns sea stars in a 60 minute dive. During the first half of the dive the diver administered half of the injections. In the next 10 minutes they injected a fourth of all the sea stars. How many sea stars did the diver inject in the last 20 minutes?

e. Crown-of-thorns sea stars often experience rapid rises in population. One of the reasons for this is the overfishing of their natural predators. Suggest some ways that we can control the population of crown-of-thorns sea stars.

18. Fill in the boxes using the numbers 0, 1, 2, 3, 4 only once to make the:

$\square \times (\square + \square - \square \div \square)$

a. smallest possible integer answer.

b. largest possible integer answer.

c. Compare your answers from **a** and **b** and explain why these results are possible using order of operations.

Extra spicy

19. Fill in the boxes to make the equation true.

$6 \times (4 - 9)^3 = (\square\ 9\ \square\ 4)^3 \times 6$

20. Casey operates an e-commerce business repairing phones. The following table shows the cost price of each repair and the price charged to customers.

	Cost price ($)	**Price for customers($)**
Screen repair	12	120
Back panel repair	10	60
Battery replacement	25	50

Last year Casey paid $67 plus a $7 shipping fee for the tools for the business and fixed 12 screens, half of which needed a back panel repair and a third of which needed battery replacements. This year they fixed 56 screens, 32 back panels, and replaced 22 batteries. How much more profit did the business make this year compared to last year?

21. If $a = -3$ and $b = 2$, which of the following equations is false?

A. $a + b = -1$ **B.** $-a - 1 = b$ **C.** $b + a = -1$ **D.** $-1 - b = a$ **E.** $b - a = -1$

22. For what value of the ♥ is the following expression true?

$$\frac{3^1 \times 2^2 \times 12^1 + 2^{\heartsuit}}{(-2)^2 \times (-5)} = (-2)^3$$

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

Remember this?

23. These are three different pictures of the same sculpture. The sculpture is made up of:

A. a rectangle and circle.

B. a rectangle and a sphere.

C. a cylinder and a circle.

D. a cylinder and a sphere.

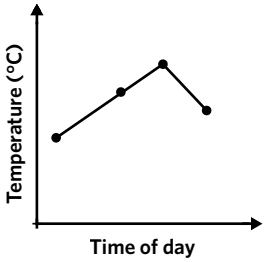
E. a cylinder and a balloon.

24. On Saturday, Karthik measured the temperature every 2 hours from 8 am to 3 pm.

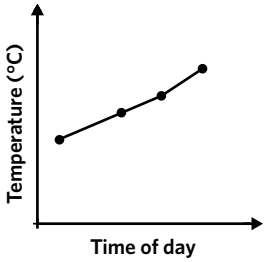
Time of day	8:00 am	11:00 am	1:00 pm	3:00 pm
Temperature (°C)	12	19	23	16

Which graph shows Karthik's results?

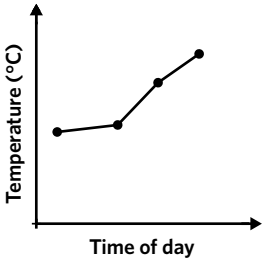
A.



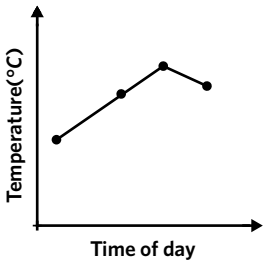
B.



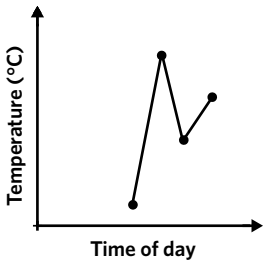
C.



D.



E.



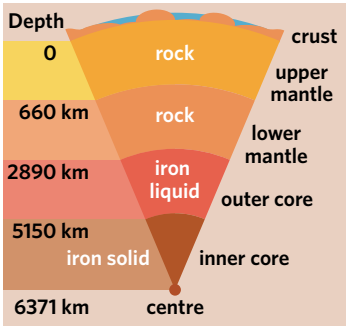
25. Clinton made 48 cookies. There was a mix of raisin cookies and chocolate chip cookies. Both types of cookies look the same. Satoshi took a cookie. He had a 1 in 8 chance of taking a raisin cookie.

How many chocolate chip cookies did Clinton make?

A. 6
B. 12
C. 36
D. 40
E. 42

Chapter 1 extended application

1. The Earth is made up of layers, with the crust being at the very surface. It is also where we can find our deepest oceans and highest mountain ranges. However, by comparison to the deeper layers inside our planet, the crust is by far the most shallow. Below is a diagram showing the Earth's layers as well as their depths.



a. Express the depth of the very centre of the Earth as a negative integer relative to the surface.

b. The top layers of the Earth are made out of rock whereas the bottom layers become iron. Express the depth at which rock ends and iron begins as a negative integer.

c. Which of the layers of Earth is the thickest? Name it and give the value of its thickness as a positive integer.

d. The inner core is a solid ball of iron at the centre of the Earth. According to the above diagram, what is its radius? Express the value as a positive integer.

e. Diamonds and other precious gems form in the mantle of our planet and often require special equipment to extract. For some time now, we have been able to grow precious stones artificially in a lab without having to disturb the natural environment or put miners' lives at risk. Give possible reasons why natural precious stones are still very popular among consumers, despite the devastating effects their extraction can have on the environment and people.

2. In an esports tournament, each team played each other once and the results were recorded in a table.

	Win	Draw	Loss	Points
Echidnas	2	1	0	7
Roos	1	1	1	3
Wombats	2	0	1	5
Koalas	1	0	2	1

a. Determine how many games were played by each team.

b. Determine the number of points given for a Win, Draw, and Loss.

c. What is the maximum number of points a single team can possibly get at the end of the tournament?

d. The Koalas' only win was against the Roos. Who did the Echidnas win against?

e. Competitions are a great way to test your skills and engage with other people who share your interests. Suggest some ways in which a participant can ensure that they are a good competitor who is fair and respectful.

3. Mateo and Silas use TikTok to promote their knitting business. Mateo and Silas take 24 hours each to handknit solid coloured jumpers and 32 hours to knit a striped jumper using various styles of stitches. Each jumper consists of $36 dollars worth of yarn and their hourly rate to knit is $21.

a. Mateo wants to knit four solid coloured jumpers and Silas wants to knit two striped jumpers. How many combined hours will Mateo and Silas spend knitting to complete these jumpers?

b. If Mateo knits for 6 hours a day and Silas knits for 4 hours a day and they both knit four days out of the week, how many weeks will it take for the pair to complete the jumpers from part **a**?

c. Mateo and Silas went viral on TikTok and now there is a waiting list to purchase one of their handmade jumpers. There are currently 1384 people who have signed up to purchase a striped jumper and 1830 people to purchase a solid colour jumper. How many combined weeks will Mateo and Silas spend knitting to complete these orders?

d. The cost of a jumper is the product of their hourly rate and the number of hours plus the cost of the yarn. Write an equation and determine how much each type of jumper costs.

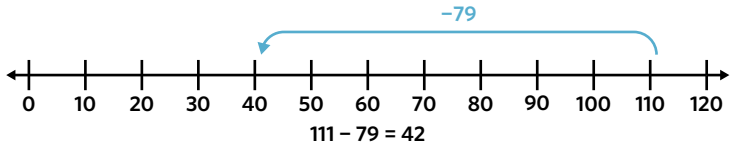
e. Is it sustainable for Mateo and Silas to handmake jumpers for this amount of people in a year? Suggest ways that they can accommodate more customers and expand their business efficiently.

Chapter 1 review

Multiple choice

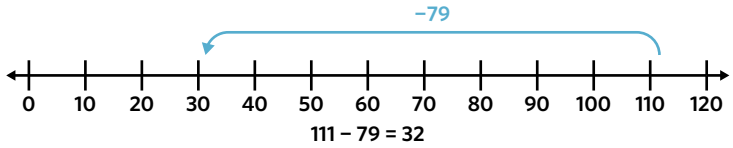
1. Which of the following number lines correctly represents its corresponding equation? 1A

A.



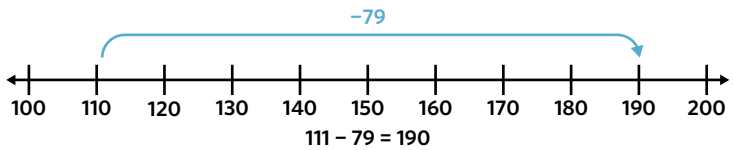
111 − 79 = 42

B.



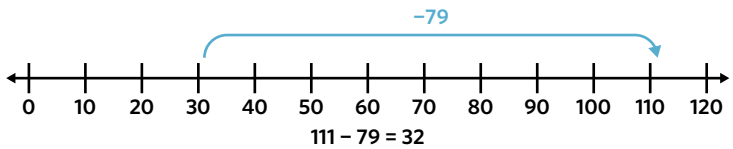
111 − 79 = 32

C.



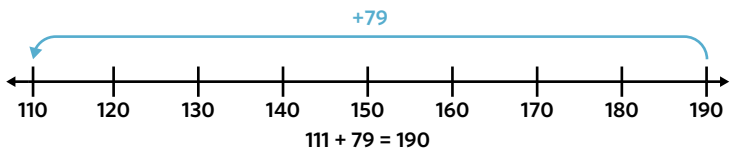
111 − 79 = 190

D.



111 − 79 = 32

E.



111 + 79 = 190

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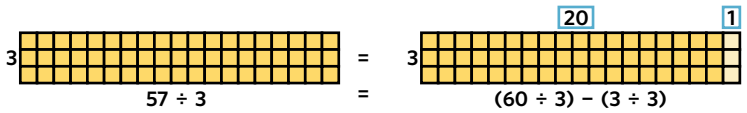
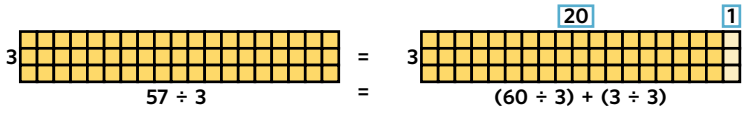
2. Which of the following visual representations correctly represents its corresponding equation? 1B

A.



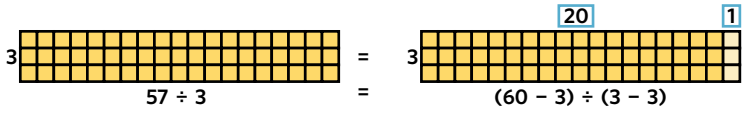
57 ÷ 3 = (60 ÷ 3) − (3 ÷ 3)

B.



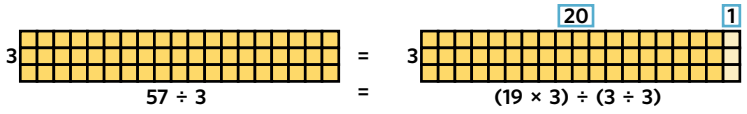
57 ÷ 3 = (60 ÷ 3) + (3 ÷ 3)

C.



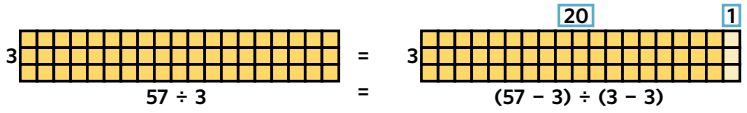
57 ÷ 3 = (60 − 3) ÷ (3 − 3)

D.



57 ÷ 3 = (19 × 3) ÷ (3 ÷ 3)

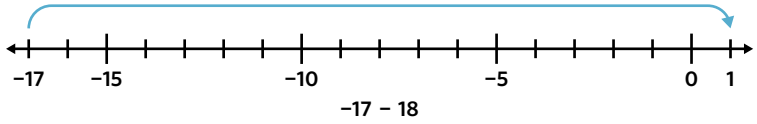
E.



57 ÷ 3 = (57 − 3) ÷ (3 − 3)

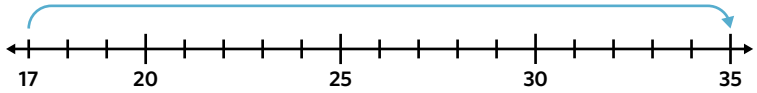
3. Which of the following number lines correctly represents its corresponding expression? 1C

A.



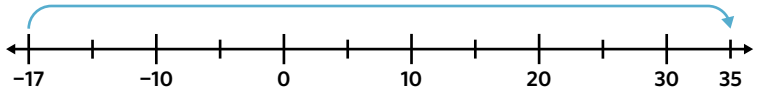
−17 − 18

B.



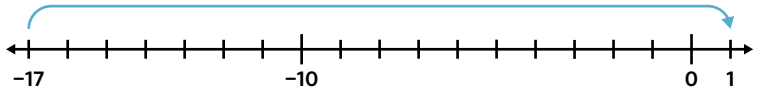
−17 − (−18)

C.



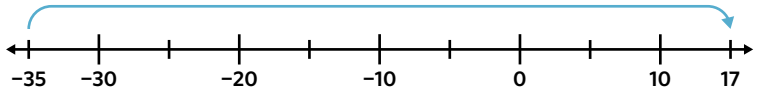
−17 + 18

D.



−17 − (−18)

E.



−17 − (−18)

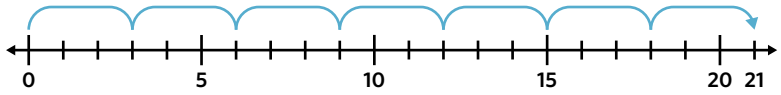
---

4. Which of the following visual representations has the correct corresponding equation? 1D

A.

21 is made up of 3 groups of 7

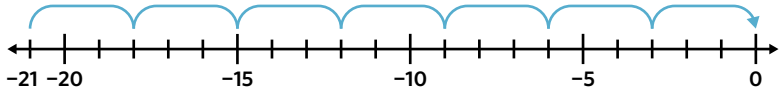
7 × 3 = 21



B.

−21 is made up of 7 groups of 3

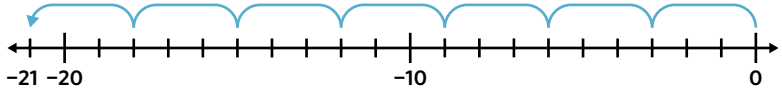
7 × 3 = −21



C.

−21 is made up of 7 groups of −3

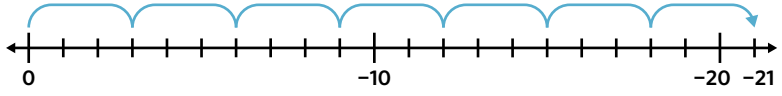
7 × (−3) = −21



D.

−21 is made up of −7 groups of −3

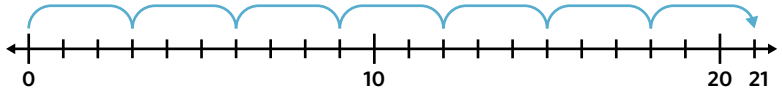
(−7) × (−3) = −21



E.

21 is made up of 7 groups of −3

7 × (−3) = 21



5. Which of the following sentences represents the correct expression? 1E

A. $(-9) + 3^2 \times \sqrt{2}$
Nine less than the product of square root two and three to the power of two

B. $9 - 3^2 \times \sqrt{2}$
Nine less than the product of square root two and three to the power of two

C. $(-9) - 3^2 \div \sqrt{2}$
Nine less than the product of square root two and three to the power of two

D. $9 - 3^2 \div \sqrt{2}$
Nine less than the product of square root two and three to the power of two

E. $9 + 3^2 \times \sqrt{2}$
Nine less than the product of square root two and three to the power of two

Fluency

6. Calculate using a mental strategy. 1A

a. $21 + 15$ **b.** $98 - 31$ **c.** $96 + 318$ **d.** $1208 - 879$

7. Calculate using a vertical algorithm. 1A

a. $45 + 54$ **b.** $32 + 78 + 61$ **c.** $912 - 789$ **d.** $100\,001 - 779$

8. Calculate using a mental strategy. 1B

a. 76×4 **b.** $2139 \div 3$ **c.** $4 \times 100 \times 100$ **d.** $480\,036 \div 12$

9. Calculate using a vertical algorithm. 1B

a. $343 \div 7$ **b.** 101×101 **c.** 724×23 **d.** $8190 \div 546$

10. Calculate. 1C

a. $3 - (+91)$ **b.** $12 + (-198)$
c. $51 - (-325) - (+523)$ **d.** $-291 - (-8) - (-23) + (-91)$

11. Calculate. 1D

a. $-900 \div (-5)$ **b.** $335 \div (-67)$
c. $207 \times (-15) \times (-11)$ **d.** $(-3822) \div [-(-42)]$

12. Calculate using the order of operations. 1E

a. $5 + 7 \times 18$ **b.** $-9 - (3^2 + 7^3)$
c. $\sqrt{[-7 - 4 \times (-1)]^2}$ **d.** $42 \div (6 + 1) - 56 \times (-2^4) \div 28$

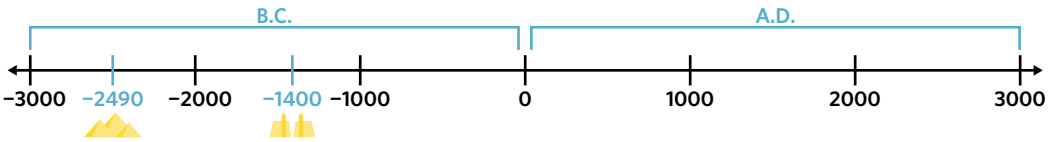
13. Represent the following sentences as expressions and solve. 1E

a. Seventeen multiplied by seventy.
b. Three less than the quotient of one hundred thirty-two and negative twelve.
c. The sum of negative ninety-two halved and negative twenty-two.
d. Four fifths of the square root of four less than the product of five and seventeen.

Problem solving

14. Vutha has received his maths test paper. Vutha sees that he has got 18 marks out of 20 for multiple choice questions, 31 marks out of 40 for fluency questions, 17 marks out of 20 for problem solving questions and 6 marks out of 10 for reasoning questions. What is Vutha's overall mark out of 90, for this test? 1A

15. The pyramids were built in 2490 B.C., and the Luxor Temple was built in 1400 B.C.. Given the following timeline, where B.C. years are represented as negative integers, write an equation and calculate the number of years there are between the building of the pyramids and the Luxor Temple. 1C



16. On Monday, Becky checked the weather and it forecasted 30°C. To calculate the degrees in Fahrenheit (°F), the degrees in Celsius are multiplied by 1.8 then 32 is added to the result. What is the temperature on Monday in °F, if Becky rounds 1.8 to the nearest whole number to complete the conversion? 1B

17. During mental health awareness month, St. Rose College decides to start a fundraising event to raise money for mental health research by selling chocolate. One chocolate company will be providing twenty boxes of chocolate, each containing fifty bars. If each bar of chocolate costs $2, how much money will St. Rose College raise if the students sell all the chocolates? 1D

18. Kirra wants to buy a new phone plan. Telstar offers her the following monthly deal: 1E

- $20 for the phone

For the first 2000 MB:

- $5 for every 500MB of data used

After the first 2000 MB:

- $14 for every 1000 MB of data used

In March Kirra used a total of 3500 MB of data. How much can Kirra expect to pay in March?

Reasoning

19. Jedda is finding it difficult to see the whiteboard in her maths class. Her parents decided to take her to see an optometrist. The optometrist asked Jedda to read off a letter chart. For each letter that Jedda reads correctly, the optometrist records three points. For each letter that Jedda reads incorrectly, the optometrist records one point.

a. Jedda reads eight letters in total, five were answered correctly and three were answered incorrectly. How many points would the optometrist have recorded?

b. The optometrist performed an eye test and wrote a prescription that Jedda's right eye has a power of -2.75. Given that Jedda knows that positive powers (such as $+2.75$) represent far-sightedness, how should Jedda interpret negative powers?

c. Jedda is trying to decide whether she should get glasses or contact lenses. The optometrist showed her the following price list. Over the course of a year, which of the two options is cheaper and how much will Jedda save by buying the cheaper option?

Glasses	Frame costs $219 Two lens costs $398 Expires after a year
Contact lenses	Each box costs $49 Each box provides a month's supply

d. Jedda eventually decides to buy contact lenses. She decided to invest in an insurance plan to cover some of the costs. She is introduced to two plans and the information is given below.

Plan A	Yearly cost $299 Covers 60% of the cost of contact lenses
Plan B	Yearly cost $159 Covers 40% of the cost of contact lenses

Based on this new information, and using information from part **c**, determine which plan is cheaper for Jedda.

e. Do you think it's worth having insurance? Why or why not? Justify your reasons.

20. Using only single digit integers from -9 to 9, inclusive of zero, substitute two different values for x and y in order to:

Expression 1	$x - y$
Expression 2	$x \times y$

a. find the largest possible answer for expression 1. State all possible values of x and y.

b. find the largest possible answer for expression 2. State all possible values of x and y.

c. Compare your answers from part **a** and part **b** and outline the pattern that you observed regarding the number of possible values of x and y.

Back to contents

02

Fractions and decimals

Number and Algebra

Research summary

2A Equivalent fractions *(Revision)*

2B Adding and subtracting fractions *(Revision)*

2C Multiplying and dividing fractions *(Revision)*

2D Working with negative fractions

2E Four operations with decimals *(Revision)*

2F Terminating and recurring decimals

2G Rounding and estimating with decimals

Chapter 2 extended application

Chapter 2 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 2 research summary

Fractions and decimals

Big ideas

Place value

This chapter will extend knowledge of the place value of whole numbers to decimals. Just as the base ten structure of '10 makes 1' extends to digits in larger and larger place values, it also extends to smaller and smaller place values.

Working with and associating decimal digits with whole numbers can cause many misconceptions and are difficult for students to overcome (Steinle, 2004). Instead, always view decimals as what they are; base ten fractions or, decimal fractions. These are fractions with denominators of 10, 100, 1000, and so forth. For example, the number 34.67 is also $34 + \frac{67}{100}$ and $34 + \frac{6}{10} + \frac{7}{100}$.

Understanding this concept is built upon trusting the count and place value of whole numbers. Therefore, grasping the topics of operating with decimals and fractions, performing computations with decimals and fractions, ordering and rounding decimals, and equi-partitioning, are essential for future topics such as percentages.

Multiplicative thinking

This chapter will extend multiplicative thinking about place value and powers of 10 to learn how to appropriately add, subtract, multiply and divide decimals. For example, 1 is ten times greater than 0.1 ($0.1 \times 10 = 1$) and 0.1 is ten times less than 1 ($1 \div 10 = 0.1$).

Additionally, this chapter will develop an understanding of integers, terminating and recurring decimals and the complexities involved with what a number represents, how a number is represented, and what we can use to represent a number. For example, $2\frac{1}{3} = \frac{7}{3} = 2.3333... = 2.\overline{3}$.

Understanding this big idea is built upon place value. It is essential for grasping the next chapter of how decimals, fractions, and percentages are related, equi-partitioning, and for future big idea topics such as algebraic, statistical and geometric reasoning.

Equi-partitioning

This chapter will extend on partitioning whole numbers to develop an understanding of part-whole comparisons. The **part** represents the numerator and the **whole** represents the fraction's denominator. Using various models besides area models for part-whole comparisons of fractions will add rigour and avoid potential misconceptions (Lamon, 2020).

Additionally, this chapter will focus on establishing the meaning of equivalent fractions. For example, eating 4 slices of a pizza that was cut into 8 slices $\left(\frac{4}{8}\right)$ is the same as eating one-half of the pizza $\left(\frac{1}{2}\right)$.

Partitioning fractions and the understanding of equivalent fractions will underpin the efficiency of how students will add, subtract, multiply and divide fractions within this chapter.

Furthermore, understanding this big idea is built upon place value and multiplicative thinking. Therefore, it is essential for the next chapter on how fractions relate to decimals, rates and ratios and future big idea topics such as algebraic, statistical, and geometric reasoning.

Visual representations

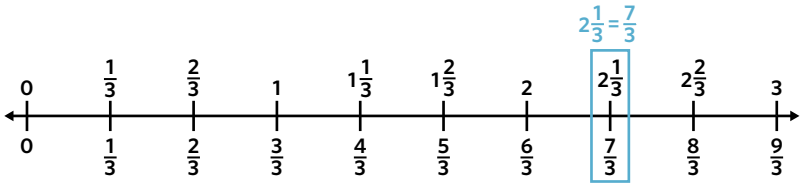
Number line

A number line is traditionally shown as a horizontal line or axis, but can also be a vertical line. A number line shows the order and size of numbers. It helps to deepen our understanding of mathematical concepts such as fractions, decimals, percentages, negative numbers and mathematical operations.

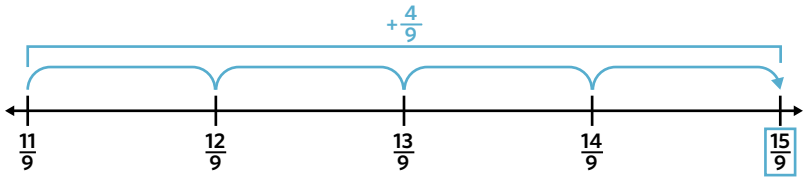
In this chapter a number line will be used to aid in:

- recognising fractions as rational numbers that fall between integers
- linking equivalent fractions, mixed numbers, and improper fractions
- support understanding when performing addition and subtraction with fractions
- ordering and investigating the magnitude of decimals
- connecting the addition, subtraction, multiplication, and division of decimals to the abstract process
- rounding numbers to any decimal place to make estimations.

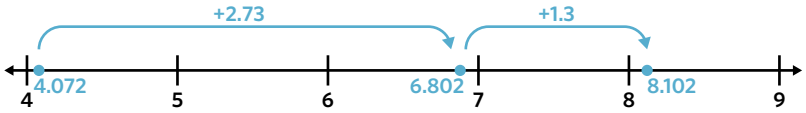
Linking fractions



Operations with fractions

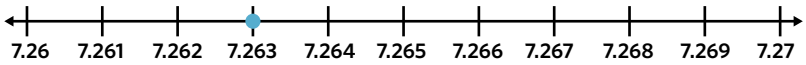


Operations with decimals



Rounding decimals

7.263 rounded to 2 decimal places is 7.26

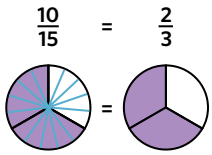


Area model

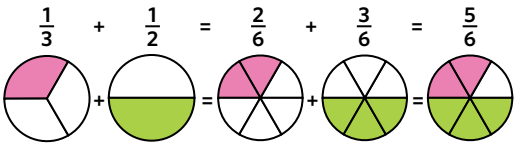
Area models help us understand the partitioning of a unit based on its area. In this chapter, area models will represent a fraction as a two-dimensional shape divided into equal parts, connect equivalent fractions, and aid in arithmetical operations with fractions.

However, relying solely on area models can limit our understanding of fractions as rational numbers and can cause common misconceptions within the topic (Lamon, 2020; Sidney et al., 2019; Ervin, 2017). Therefore, using the area model alongside other models and non-visual approaches is vital when teaching and learning fractions (E.g. Linear arithmetic blocks, cuisenaire rods, discrete models, number lines, dominos, geoboards, and volume models).

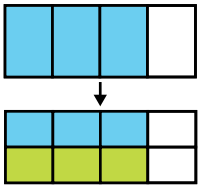
Equivalent fractions



Adding fractions



Multiplying fractions



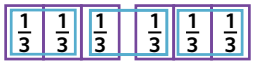
$\frac{1}{2}$ of $\frac{3}{4} = \frac{1}{2} \times \frac{3}{4}$

$= \frac{3}{8}$

Dividing fractions

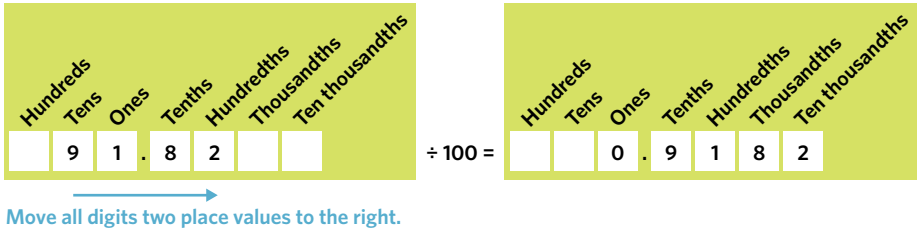
2 is made up of 3 groups of $\frac{2}{3}$ → $2 \div \frac{2}{3}$

$= 3$



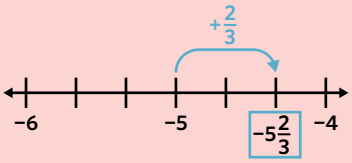
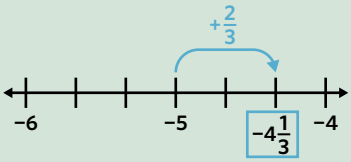
Number slides

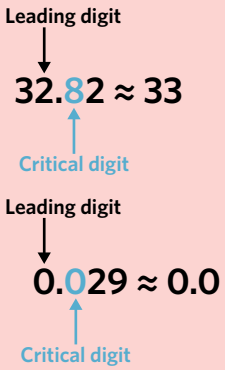
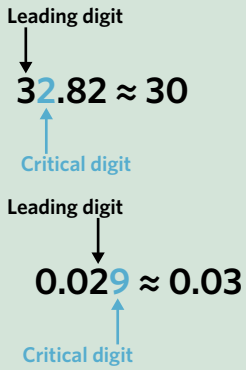
A number slide can be used when multiplying and dividing decimals by powers of 10. In addition they are helpful when addressing misconceptions around the decimal point and building knowledge around decimal place value.



Misconceptions

Misconception	Incorrect	Correct	Lesson
When finding equivalent fractions, students only multiply the numerators not the denominators.	$\frac{3}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$	$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$	2A
Students divide the whole number when simplifying a mixed number.	$2\frac{6}{12} \overset{\div 2}{\underset{\div 2}{}}$ $= 1\frac{1}{2}$	$2\frac{6}{12} \overset{\div 2}{\underset{\div 2}{}}$ $= 1\frac{1}{2}$	2A
Students think any equivalent fraction with a smaller denominator is the simplest form.	$\frac{4}{8} = \frac{2}{4}$ 4 has a smaller denominator than 8, therefore the fraction $\frac{2}{4}$ is in simplest form.	$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$ 1 and 2 do not have any common factors besides 1, therefore the fraction $\frac{1}{2}$ is in simplest form.	2A
Students add or subtract fractions by combining the numerator and denominators.	$\frac{3}{5} - \frac{1}{2} = \frac{2}{3}$ $\frac{3}{5} + \frac{1}{2} = \frac{4}{7}$	$\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$ $\frac{3}{5} + \frac{1}{2} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$	2B
When adding fractions with the same denominator, students add the numerators and denominators.	$\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$	$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	2B
When multiplying fractions, students only multiply the numerators or denominators.	$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5} = \frac{6}{5}$ $\frac{3}{5} \times \frac{2}{7} = \frac{3}{5 \times 7} = \frac{3}{35}$	$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$	2B

Misconception	Incorrect	Correct	Lesson
When multiplying a fraction by a whole number, students multiply both the numerator and the denominator by the same number.	$2 \times \frac{3}{4} = \frac{2 \times 3}{2 \times 4} = \frac{6}{8}$	$2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4}$ $= \frac{2 \times 3}{1 \times 4}$ $= \frac{6}{4}$	2C
When cross simplifying, students cross multiply.	$\frac{2}{3} \times \frac{9}{10} = \frac{2 \times 10}{3 \times 9} = \frac{20}{27}$	$\frac{1}{1}\frac{2}{3} \times \frac{9}{10}\frac{3}{5} = \frac{1 \times 3}{1 \times 5} = \frac{3}{5}$	2C
When dividing fractions, students multiply before finding the reciprocal of the divisor.	$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{2}{3}$ $= \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$	$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2}$ $= \frac{2 \times 3}{5 \times 2} = \frac{6}{10}$	2C
Students think multiplication always results in larger values and division always results in smaller values.	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\frac{1}{8}$ is bigger because multiplication makes numbers bigger.	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ Multiplying or dividing fractions increases or reduces the size by increasing or decreasing the number of parts.	2C
When using a number line to subtract fractions, students count in the wrong direction.	$-5 + \frac{2}{3} = -5\frac{2}{3}$ 	$-5 + \frac{2}{3} = -4\frac{1}{3}$ 	2D
When subtracting a negative fraction, students decrease the value.	$\frac{3}{4} - \left(-\frac{1}{4}\right) = \frac{2}{4}$	$\frac{3}{4} - \left(-\frac{1}{4}\right) = \frac{4}{4}$	2D
When dividing by negative fractions, students change the sign of the negative fraction when writing the reciprocal.	$\frac{2}{5} \div \left(-\frac{2}{3}\right) = \frac{2}{5} \times \frac{3}{2}$ $= \frac{6}{10}$	$\frac{2}{5} \div \left(-\frac{2}{3}\right) = \frac{2}{5} \times \left(-\frac{3}{2}\right)$ $= -\frac{6}{10}$	2D
Students add and subtract the whole numbers and decimal places separately.	$22.16 + 15.89 = 37.105$	$22.16 + 15.89 = 38.05$	2E
Students do not consider place value when multiplying decimals using the vertical algorithm.	$2.13 \times 1.2 =$ $\begin{array}{r} 213 \\ \times\ 12 \\ \hline 426 \\ +\ 213 \\ \hline 639 \end{array}$ $2.13 \times 1.2 = 0.639$	$2.13 \times 1.2 =$ $\begin{array}{r} 213 \\ \times\ 12 \\ \hline 426 \\ +\ 2130 \\ \hline 2556 \end{array}$ $2.13 \times 1.2 = 2.556$	2E
Students use mental strategies to multiply and divide decimals and do not consider place value.	$0.5 \times 0.5 = ?$ $5 \times 5 = 25$ So $0.5 \times 0.5 = 2.5$	$0.5 \times 0.5 = ?$ $5 \times 5 = 25$ So $0.5 \times 0.5 = 0.25$ (with two decimal places)	2E
Students think that recurring decimals end.	$1.\dot{6} = 1.66$	$1.\dot{6} = 1.666...$	2F
Students think only one or two digits repeat in recurring decimals.	$4.12123123...$ $= 4.\overline{12}$	$4.12123123...$ $= 4.12\overline{123}$	2F
Students use the digit to be rounded instead of the critical digit to round decimals.	2.37 to the nearest tenth is 2.3 because $3 < 5$.	2.37 to the nearest tenth is 2.4 because $7 > 5$.	2G

Misconception	Incorrect	Correct	Lesson
When the rounding digit is 9, students round to 10 instead.	Round 0.95 to the nearest whole number. $0.95 \approx 0.10$	Round 0.95 to the nearest whole number. $0.95 \approx 1.0$	2G
When lead digit rounding, students think the critical digit is the first decimal place.			2G

References and additional readings

Ervin, H. (2017). Fraction Multiplication and Division Models: A Practitioner Reference Paper. *International Journal of Research in Education and Science (IJRES)*, 3(1), 258–279. https://files.eric.ed.gov/fulltext/EJ1126700.pdf

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Solihull LA Primary Mathematics Forum. (2019). *Using resources effectively to teach fractions.* https://www.solgrid.org.uk/education/wp-content/uploads/sites/43/2019/09/Using-resources-effectively-to-teach-fractions-1.pdf

Sidney, P. G., Thompson, C. A., & Rivera, F. D. (2019). Number lines, but not area models, support children's accuracy and conceptual models of fraction division. *Contemporary Educational Psychology,* 58(2019), 288–298. https://doi.org/10.1016/j.cedpsych.2019.03.011

Steinle, V. (2004). Detection and remediation of decimal misconceptions. In B. Tadich, S. Tobias, C. Brew, B. Beatty, & P. Sullivan (Eds.), *Towards excellence in mathematics,* 460-478. Brunswick: The Mathematical Association of Victoria.

Back to contents

2A Equivalent fractions

There are three types of fractions: proper fractions, improper fractions, and mixed numbers. Equivalent fractions represent the same quantity but with a different number of parts. For example, $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent because they both represent the same quantity but with a different number of parts.

LEARNING INTENTIONS

Students will be able to:

- convert between mixed numbers and improper fractions
- identify equivalent fractions
- write fractions in the simplest form.

KEY TERMS AND DEFINITIONS

The **numerator** is the value on the top of a fraction and represents the number of parts of a unit we are considering.

The **denominator** is the value on the bottom of a fraction and represents the number of parts each unit is split into.

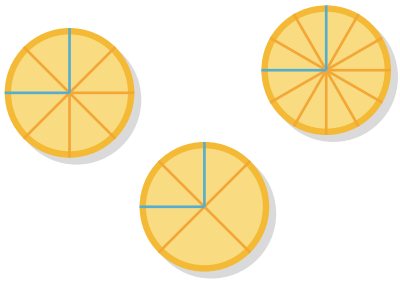
The **highest common factor (HCF)** is the largest number that is a factor of two or more numbers.

The **lowest common multiple (LCM)** is the smallest number that is a multiple of two or more numbers.

An **improper fraction** has a numerator greater than or equal to its denominator.

A **mixed number** is a combination of a whole number and a proper fraction.

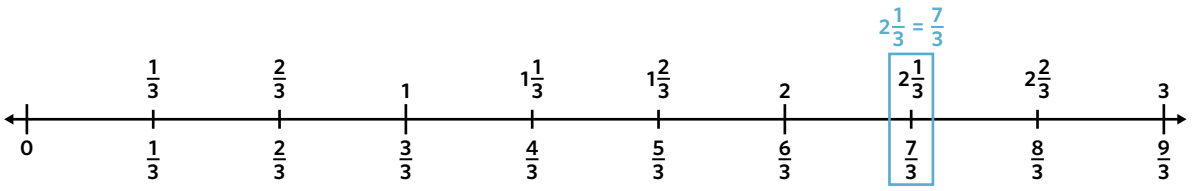
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



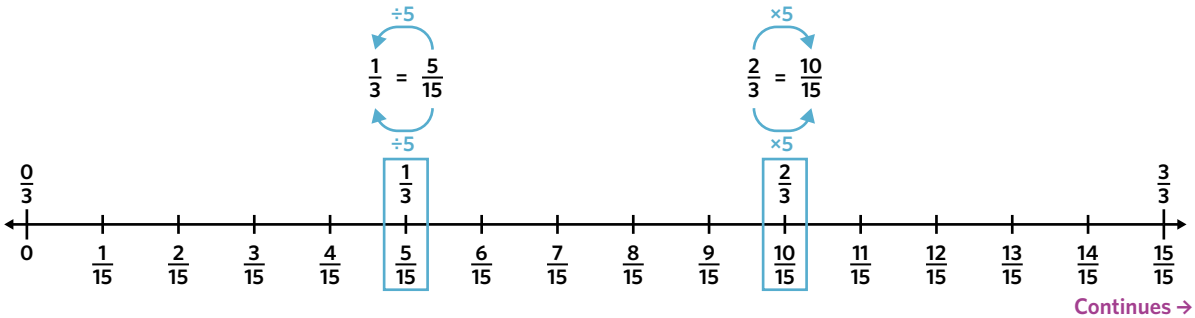
Equivalent fractions are commonly used when comparing quantities and amounts. For example, even though cakes of the same size can be cut into different sized slices, we can still compare the quantities using equivalent fractions.

Key ideas

1. A **mixed number** can be converted to an **improper fraction** by recognising how many unit fractions are in a whole.



2. Fractions remain **equivalent** if the numerator and denominator are both multiplied or divided by the same value.



Continues →

3. The **simplest form** of a fraction is when the numerator and denominator do not share any common factors.

Highest common factor of 10 and 15 is 5.

$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

Worked example 1

Converting between mixed numbers and improper fractions

Convert each of the following to the described form.

a. $2\frac{3}{5}$ (improper fraction) WE1a

Working

$2\frac{3}{5} = \frac{2 \times 5}{5} + \frac{3}{5}$

$= \frac{10}{5} + \frac{3}{5}$

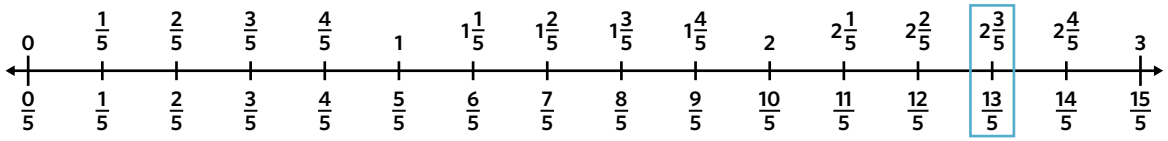
$= \frac{13}{5}$

Thinking

Step 1: Multiply the whole number by the denominator of the fraction and add the numerator of the fraction.

Step 2: Simplify.

Visual support



b. $\frac{22}{7}$ (mixed number) WE1b

Working

$22 \div 7 = 3$ remainder 1

$\frac{22}{7} = 3\frac{1}{7}$

Thinking

Step 1: Divide the numerator by the denominator.

Step 2: Write the remainder as a fraction with the same denominator as the improper fraction.

Student practice

Convert each of the following to the described form.

a. $2\frac{5}{6}$ (improper fraction)

b. $\frac{12}{5}$ (mixed number)

Worked example 2

Finding an equivalent fraction

Calculate the equivalent fractions.

a. $\frac{3}{5} = \frac{6}{\square} = \frac{\square}{15}$

WE2a

Working

$\frac{3}{5} \overset{\times 2}{=} \frac{6}{10} \overset{\times 2}{=} \frac{}{15}$

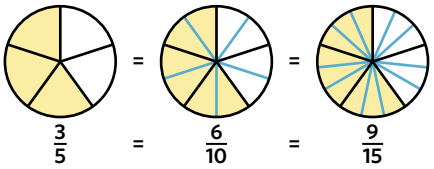
$\frac{3}{5} \overset{\times 3}{=} \frac{6}{10} \overset{\times 3}{=} \frac{9}{15}$

Thinking

Step 1: Find the relationship between the two known numerators. Apply this change to find the unknown denominator.

Step 2: Find the relationship between the two known denominators. Apply this change to find the unknown numerator.

Visual support



b. $\frac{16}{24} = \frac{\square}{12} = \frac{2}{\square}$

WE2b

Working

$\frac{16}{24} \overset{\div 8}{=} \frac{}{6} \overset{\div 8}{=} \frac{2}{3}$

$\frac{16}{24} \overset{\div 4}{=} \frac{4}{6} \overset{\div 4}{=} \frac{2}{3}$

Thinking

Step 1: Find the relationship between the two known numerators. Apply this change to find the unknown denominator.

Step 2: Find the relationship between the two known denominators. Apply this change to find the unknown numerator.

Student practice

Calculate the equivalent fractions.

a. $\frac{6}{8} = \frac{12}{\square} = \frac{\square}{40}$

b. $\frac{12}{48} = \frac{\square}{24} = \frac{1}{\square}$

Worked example 3

Converting fractions to the simplest form

Write the following fractions in the simplest form.

a. $\frac{9}{12}$ WE3a

Working

HCF of 9 and 12 is 3.

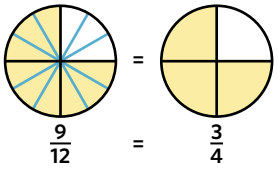
$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

Thinking

Step 1: Find the HCF of the numerator and denominator.

Step 2: Divide numerator and denominator by the HCF to simplify.

Visual support



b. $\frac{27}{18}$ WE3b

Working

HCF of 27 and 18 is 9.

$\frac{27 \div 9}{18 \div 9} = \frac{3}{2}$

Thinking

Step 1: Find the HCF of the numerator and denominator.

Step 2: Divide numerator and denominator by their HCF to find the simplest form of the fraction.

Student practice

Write the following fractions in the simplest form.

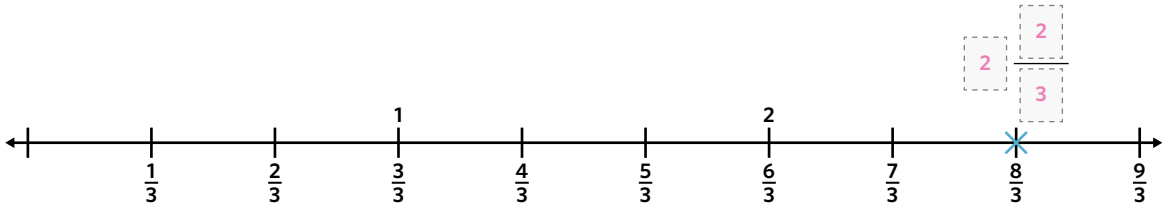
a. $\frac{10}{15}$ **b.** $\frac{24}{9}$

2A Questions

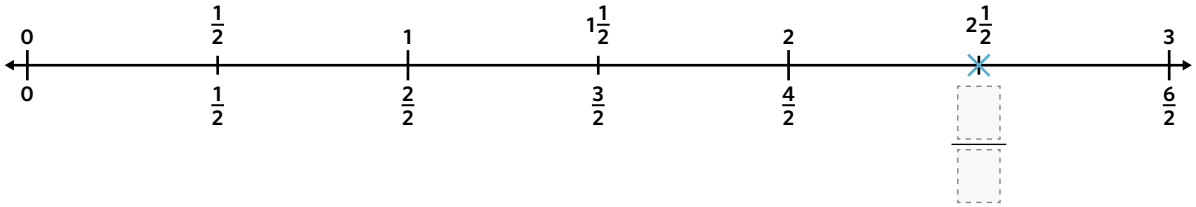
Understanding worksheet

1. Write the mixed number or improper fraction from the given number line.

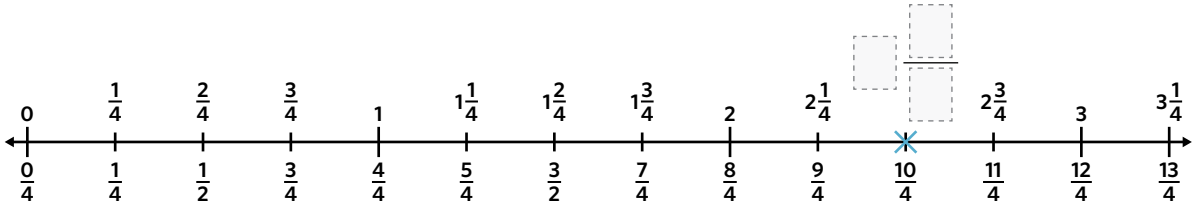
Example



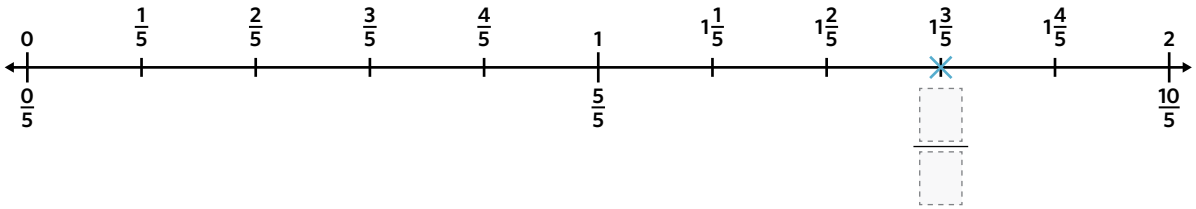
a.



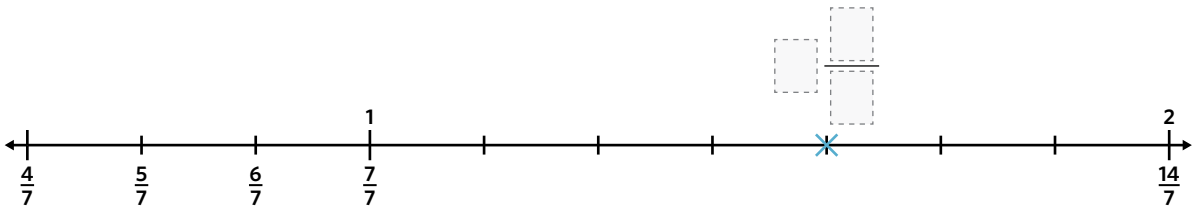
b.



c.

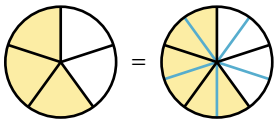


d.



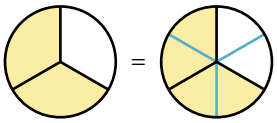
2. Use the area model to write the equivalent fractions.

Example



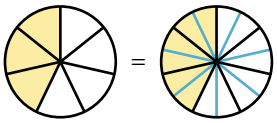
$\frac{3}{5} = \frac{6}{10}$

a.



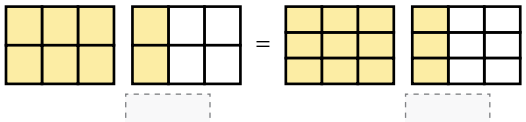
$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

b.



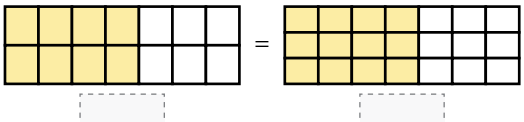
$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

c.



$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

d.



$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

3. Fill in the blanks by using the words provided.

improper | equivalent | simplest | mixed number

Fractions with a numerator that is greater than the denominator are _____ fractions. When an improper fraction is expressed as a whole number and a fraction, it is a _____. If we multiply or divide the numerator and denominator of a fraction by the same number, we form an _____ fraction. To find the _____ form of a fraction we divide the numerator and denominator by their highest common factor.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Convert the following mixed numbers to improper fractions. WE1a

a. $1\frac{3}{4}$ **b.** $2\frac{2}{5}$ **c.** $5\frac{3}{4}$ **d.** $3\frac{2}{7}$

e. $8\frac{1}{4}$ **f.** $13\frac{3}{5}$ **g.** $15\frac{17}{7}$ **h.** $61\frac{6}{3}$

5. Convert the following improper fractions to mixed numbers. WE1b

a. $\frac{5}{2}$ **b.** $\frac{5}{4}$ **c.** $\frac{8}{3}$ **d.** $\frac{17}{5}$

e. $\frac{17}{4}$ **f.** $\frac{41}{6}$ **g.** $\frac{124}{11}$ **h.** $\frac{73}{17}$

6. Calculate the equivalent fractions. WE2

a. $\frac{7}{10} = \frac{\square}{20} = \frac{21}{\square}$ **b.** $\frac{2}{5} = \frac{\square}{15} = \frac{\square}{30}$ **c.** $\frac{100}{80} = \frac{25}{\square} = \frac{\square}{8}$ **d.** $\frac{84}{48} = \frac{42}{\square} = \frac{\square}{12}$

e. $\frac{7}{4} = \frac{\square}{12} = \frac{28}{\square}$ **f.** $\frac{13}{6} = \frac{\square}{18} = \frac{78}{\square}$ **g.** $\frac{48}{32} = \frac{\square}{16} = \frac{6}{\square}$ **h.** $\frac{15}{36} = \frac{\square}{24} = \frac{5}{\square}$

7. Convert the following fractions to an equivalent fraction with a denominator of 48.

a. $\frac{1}{4}$ **b.** $\frac{3}{2}$ **c.** $\frac{11}{12}$ **d.** $\frac{8}{6}$

e. $3\frac{3}{8}$ **f.** $5\frac{7}{4}$ **g.** $\frac{5}{3}$ **h.** $\frac{52}{96}$

8. Write the following fractions in the simplest form. WE3

a. $\frac{6}{9}$ **b.** $\frac{3}{6}$ **c.** $\frac{18}{15}$ **d.** $\frac{18}{12}$

e. $\frac{35}{40}$ **f.** $5\frac{28}{42}$ **g.** $\frac{44}{72}$ **h.** $\frac{108}{84}$

9. Express $6\frac{18}{24}$ in its simplest form.

A. $1\frac{3}{4}$ **B.** $3\frac{9}{12}$ **C.** $6\frac{9}{12}$ **D.** $6\frac{6}{8}$ **E.** $6\frac{3}{4}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Find an equivalent fraction to $\frac{2}{3}$.

Student A

$\frac{2}{3} = \frac{2 \times 2}{3} = \frac{4}{3}$

Student B

$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

b. Express $\frac{24}{36}$ in the simplest form.

Student A

$\frac{24}{36} = \frac{24 \div 4}{36 \div 4} = \frac{6}{9}$

Student B

$\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. If six friends share four cantaloupes, what fraction of a cantaloupe does each person receive? Give your answer in its simplest form.

12. How long are each of Carlos' steps if he took 12 equal steps and crossed his 15 m front garden? Give your answer in its simplest form.

13. Papa Joe's Pizza cuts large pizzas into eighths. Mrs. Gooley wants to buy pizza for her 24 students. What is the minimum number of pizzas she needs to order so that each student receives three slices?

14. Yoanna's car drives 600 km on a tank of fuel. How many tanks of fuel will she need to drive 2600 km from Bendigo to Alice Springs?

15. 300 students were surveyed about school camps and 80 chose to camp at Gariwerd (Grampians National Park). If half of the remaining students chose to travel to the Gold Coast, express the fraction of students who wanted to travel to the Gold Coast in the simplest form.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

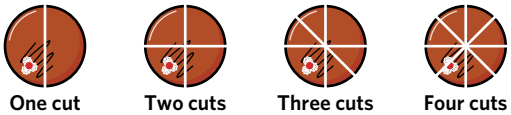
16. Max, Bob, Jane, and Mary organised a bake-a-thon fundraising event to raise awareness for climate change at their local scouts group. At the end of the bake-a-thon there were seven cakes left which they had baked together.

a. How many cakes does each of the organisers take home if they share the remaining cakes evenly between them?

b. Each of the remaining cakes were cut into eight slices. How many slices did each person receive?

c. It took Max, Bob, Jane, and Mary $3\frac{7}{12}$ hours to set up the scout hall for the bake-a-thon. How many minutes did it take them to set up the hall?

d. The following diagram shows the number of cuts required to divide the cake into a specific number of slices. What is the minimum number of cuts required so that everyone can take home their equal share?



One cut | Two cuts | Three cuts | Four cuts

e. Other than a bake-a-thon, suggest other ways local community groups can raise awareness for climate change.

17. Use a calculator to fill in the missing information to write equivalent fractions and determine whether they are in their simplest form.

a. $\frac{165}{200} = \frac{\square}{40}$ **b.** $\frac{504}{567} = \frac{\square}{18}$

c. Compare and contrast your answers from part **a** and **b** and explain how to determine if a fraction is in its simplest form.

Extra spicy

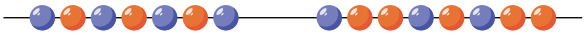
18. Select a number to fill in the missing numerator in the fraction $\frac{\square}{9}$ to create a value between 7 and 9.

A. 8 **B.** 63 **C.** 72 **D.** 81 **E.** 90

19. If Wendy runs 21 km in 2 hrs and 6 min, how far will Mark need to run in his first hour to be running at the same speed as Wendy?

20. Milgroode High School is going to award jellybeans to first, second and thirds in the times tables competition. If first gets three times as many as thirds and second gets two times as many as third, what fraction of the jellybeans does second place get?

21. If the beads on the following necklace are kept in two separate groups, how many need to slide from the left group to the right group so that the fraction of those on the left that are orange is equivalent to the fraction on the right that are blue. Beads can't be removed, they can only slide across

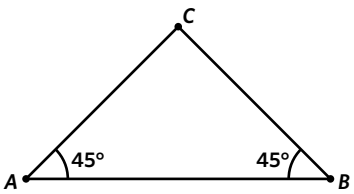


A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

Remember this?

22. Which statement defines the triangle *ABC*.

A. The triangle *ABC* is an equilateral triangle
B. The triangle *ABC* is a scalene triangle
C. The triangle *ABC* is an isosceles triangle
D. The triangle *ABC* is a right-angled triangle
E. Both C and D



23. Which number shows 7 thousands, 4 tens and 3 tenths?

A. 7342.3 **B.** 23.427 **C.** 7324.3 **D.** 73.423 **E.** 70.413

24. Ying has 7 blue balls and 5 red balls in a bag. She selects balls at random from the bag and records the colour of the ball, then puts it back in the bag. If she repeats this experiment 60 times how many times should Ying expect to select a blue ball?

A. 12 **B.** 24 **C.** 35 **D.** 42 **E.** 58

Back to contents

2B Adding and subtracting fractions

Like whole numbers we can add and subtract fractions. A fraction is a part of a whole, so when adding and subtracting fractions we are adding or subtracting parts of a whole. When adding and subtracting fractions the denominators must be the same. This means that when adding and subtracting fractions with different denominators we use equivalent fractions.

LEARNING INTENTIONS

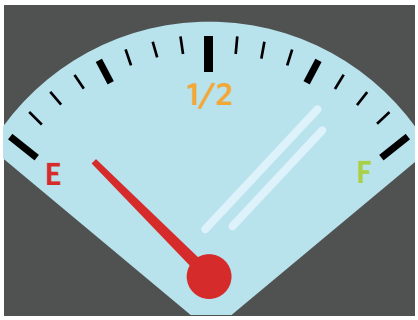
Students will be able to:

- use a number line to add and subtract fractions
- add and subtract fractions with the same denominator
- add and subtract fractions with different denominators.

KEY TERMS AND DEFINITIONS

Common denominators are when fractions have denominators that are the same.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



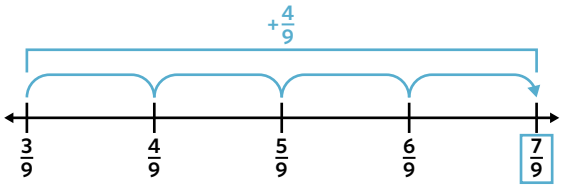
Adding and subtracting fractions are commonly used in many everyday activities. For example, fractions are used to add or subtract quantities and units of measurements. Fractions can be used to calculate the amount of petrol required to fill up a tank.

Key ideas

1. Fractions can only be added and subtracted when they have the same denominator.

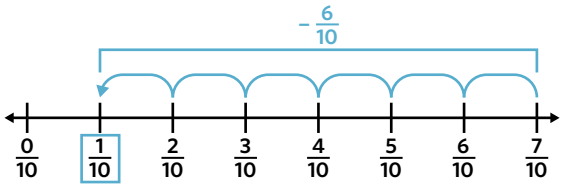
Addition

$\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

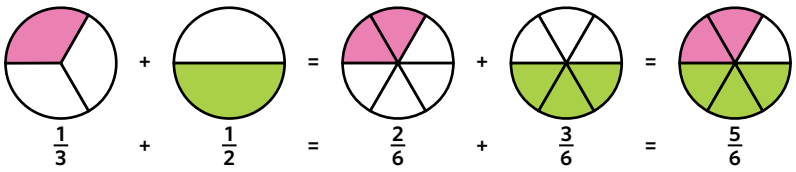


Subtraction

$\frac{7}{10} - \frac{6}{10} = \frac{1}{10}$



2. When fractions with different denominators are added or subtracted, the lowest common denominator (LCD) must be determined.



Worked example 1

Adding and subtracting fractions and mixed numbers with the same denominator

Simplify.

a. $\frac{11}{9} + \frac{4}{9}$ WE1a

Working

$\frac{11}{9} + \frac{4}{9} = \frac{11 + 4}{9}$

$= \frac{15}{9}$

$= \frac{5}{3}$

$= 1\frac{2}{3}$

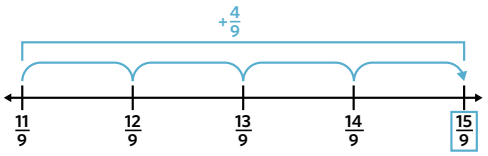
Thinking

Step 1: Add the numerators, the denominator values do not change as they are the same.

Step 2: Simplify.

Note: The final answer can be written in its simplest form as a mixed number or an improper fraction.

Visual support



b. $\frac{12}{10} - \frac{6}{10}$ WE1b

Working

$\frac{12}{10} - \frac{6}{10} = \frac{12 - 6}{10}$

$= \frac{6}{10}$

$= \frac{3}{5}$

Thinking

Step 1: Subtract the numerators, the denominator values do not change as they are the same.

Step 2: Simplify.

c. $2\frac{5}{8} + 3\frac{2}{8}$ WE1c

Working

$2\frac{5}{8} + 3\frac{2}{8} = 2 + 3 + \frac{5}{8} + \frac{2}{8}$

$= 5 + \frac{5}{8} + \frac{2}{8}$

$= 5\frac{7}{8}$

Thinking

Step 1: Rewrite the addition by partitioning each mixed number into a whole number and fraction.

Step 2: Add the whole numbers.

Step 3: Add the fraction parts.

Continues →

d. $3\frac{3}{6} - 2\frac{1}{6}$

WE1d

Working

$$3\frac{3}{6} - 2\frac{1}{6} = \frac{21}{6} - \frac{13}{6}$$
$$= \frac{21 - 13}{6}$$
$$= \frac{8}{6}$$
$$= \frac{4}{3}$$
$$= 1\frac{1}{3}$$

Thinking

Step 1: Convert the mixed numbers to improper fractions.

Step 2: Subtract the numerators, the denominator values do not change as they are the same.

Step 3: Simplify.

Student practice

Simplify.

a. $\frac{12}{6} + \frac{4}{6}$ **b.** $\frac{8}{5} - \frac{4}{5}$ **c.** $1\frac{3}{6} + 2\frac{2}{6}$ **d.** $4\frac{4}{9} - 1\frac{1}{9}$

Worked example 2

Adding and subtracting fractions and mixed numbers with different denominator

Simplify.

a. $\frac{1}{3} + \frac{2}{5}$

WE2a

Working

LCD of 3 and 5 is 15.

$$\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$$
$$= \frac{5 + 6}{15}$$
$$= \frac{11}{15}$$

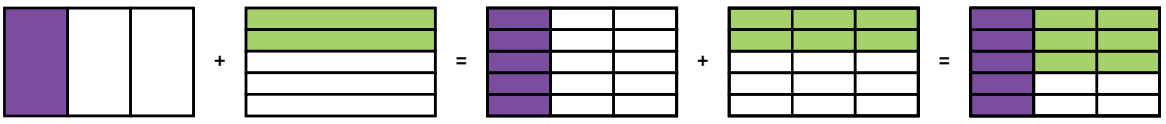
Thinking

Step 1: Identify the LCD for both fractions as the denominators are different.

Step 2: Write equivalent fractions with equal denominators.

Step 3: Add the numerators, the denominator values do not change as they are the same.

Visual support



$\frac{1}{3}$ + $\frac{2}{5}$ = $\frac{5}{15}$ + $\frac{6}{15}$ = $\frac{11}{15}$

Continues →

b. $\frac{12}{10} - \frac{3}{6}$ WE2b

Working

LCD of 10 and 6 is 30.

$$\frac{12}{10} - \frac{3}{6} = \frac{36}{30} - \frac{15}{30}$$
$$= \frac{36 - 15}{30}$$
$$= \frac{21}{30}$$
$$= \frac{7}{10}$$

Thinking

Step 1: Identify the LCD for both fractions as the denominators are different.

Step 2: Write equivalent fractions with equal denominators.

Step 3: Subtract the numerators, the denominator values do not change as they are the same.

Step 4: Simplify.

c. $2\frac{2}{4} + 3\frac{1}{8}$ WE2c

Working

$$2\frac{2}{4} + 3\frac{1}{8} = \frac{10}{4} + \frac{25}{8}$$

LCD of 4 and 8 is 8.

$$\frac{10}{4} + \frac{25}{8} = \frac{20}{8} + \frac{25}{8}$$
$$= \frac{20+25}{8}$$
$$= \frac{45}{8}$$
$$= 5\frac{5}{8}$$

Thinking

Step 1: Convert the mixed numbers to improper fractions.

Step 2: Identify the LCD for both fractions as the denominators are different.

Step 3: Write equivalent fractions with equal denominators.

Step 4: Add the numerators, the denominator values do not change as they are the same.

Step 5: Simplify.

d. $3\frac{4}{5} - 2\frac{3}{6}$ WE2d

Working

$$3\frac{4}{5} - 2\frac{3}{6} = \frac{19}{5} - \frac{15}{6}$$

LCD of 5 and 6 is 30.

$$\frac{19}{5} - \frac{15}{6} = \frac{114}{30} - \frac{75}{30}$$
$$= \frac{114 - 75}{30}$$
$$= \frac{39}{30}$$
$$= \frac{13}{10}$$
$$= 1\frac{3}{10}$$

Thinking

Step 1: Convert the mixed numbers to improper fractions.

Step 2: Identify the LCD for both fractions as the denominators are different.

Step 3: Write equivalent fractions with equal denominators.

Step 4: Subtract the numerators, the denominator values do not change as they are the same.

Step 5: Simplify.

Student practice

Calculate the following.

a. $\frac{5}{10} + \frac{3}{4}$ **b.** $\frac{9}{7} - \frac{2}{5}$ **c.** $1\frac{3}{5} + 2\frac{2}{3}$ **d.** $3\frac{2}{4} - 3\frac{1}{6}$

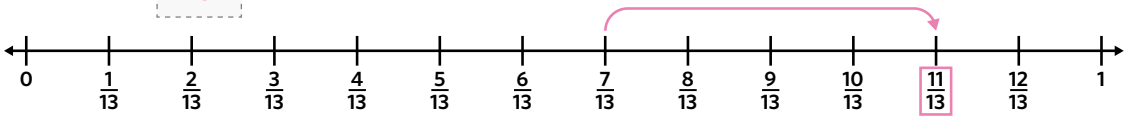
2B Questions

Understanding worksheet

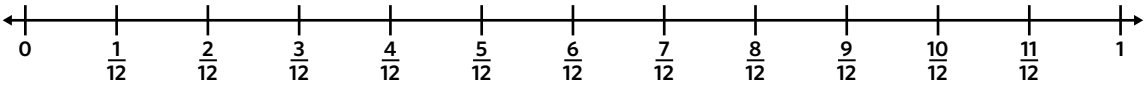
1. Complete the jumps on the number line to complete each calculation.

Example

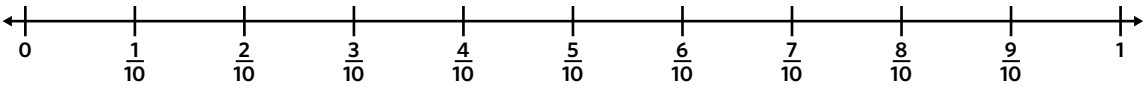
$\frac{7}{13} + \frac{4}{13} = \frac{11}{13}$



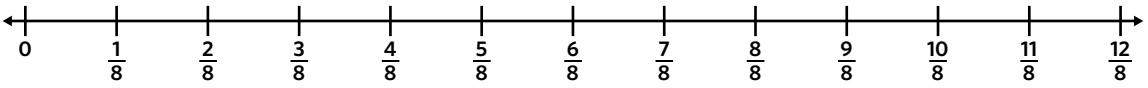
a. $\frac{3}{12} + \frac{6}{12} = \frac{\square}{\square}$



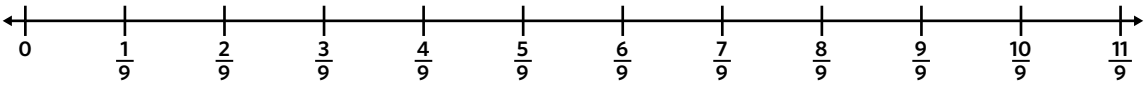
b. $\frac{9}{10} - \frac{7}{10} = \frac{\square}{\square}$



c. $\frac{5}{8} + \frac{7}{8} = \frac{\square}{\square}$

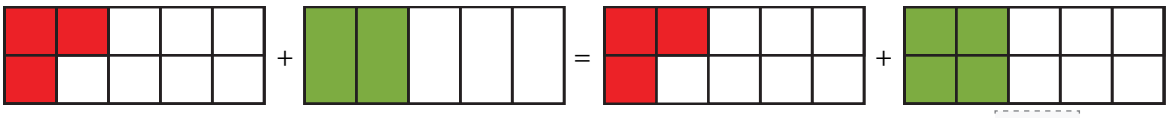


d. $1\frac{2}{9} - \frac{4}{9} = \frac{11}{9} - \frac{4}{9} = \frac{\square}{\square}$



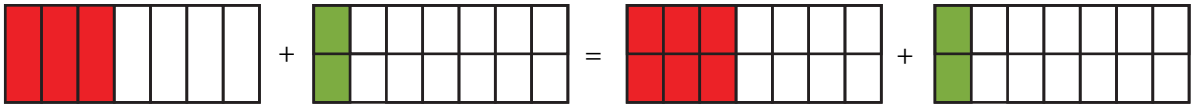
2. Fill in the missing boxes to write the value of each fraction and find the result.

Example



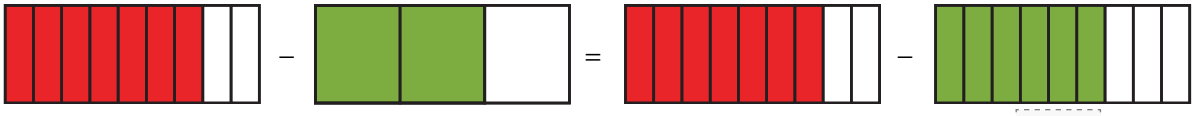
$\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10}$

a.



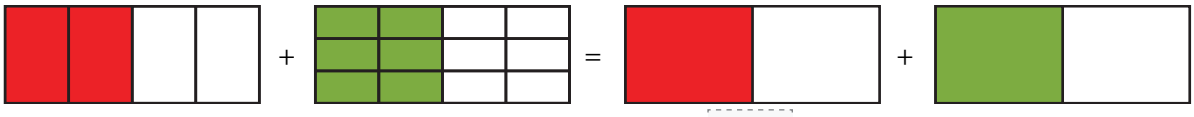
$\frac{3}{7} + \frac{2}{14} = \frac{6}{14} + \frac{\square}{\square}$

b.



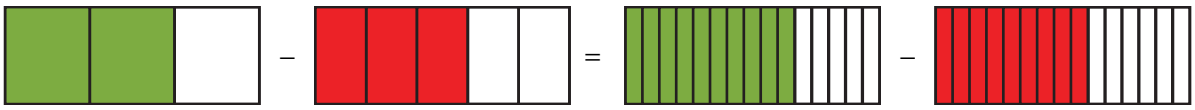
$\frac{7}{9} - \frac{2}{3} = \frac{7}{9} - \frac{\square}{\square}$

c.



$\frac{2}{4} + \frac{6}{12} = \frac{\square}{\square} + \frac{1}{2}$

d.



$\frac{2}{3} - \frac{3}{5} = \frac{\square}{\square} - \frac{9}{15}$

3. Fill in the blanks by using the words provided.

simplest | lowest | common | denominator | subtract

To add and [ ] fractions, the fractions must have a common [ ].

If the fractions have denominators that are not equal, we are required to find the [ ]

[ ] denominator for the fractions. The final answer must be in its [ ] form.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Simplify. WE1

a. $\frac{3}{7} - \frac{2}{7}$ b. $\frac{5}{8} - \frac{1}{8}$ c. $\frac{1}{2} + \frac{3}{2}$ d. $\frac{13}{14} + \frac{15}{14}$

e. $\frac{12}{8} + \frac{13}{8}$ f. $\frac{5}{12} + \frac{15}{12}$ g. $\frac{16}{15} - \frac{13}{15}$ h. $\frac{16}{8} - \frac{4}{8}$

5. Simplify. WE1

a. $1\frac{5}{8} - \frac{7}{8}$ b. $3\frac{1}{7} + \frac{3}{7}$ c. $2\frac{5}{11} - \frac{2}{11}$ d. $2\frac{7}{13} + \frac{2}{13}$

e. $2\frac{2}{4} + 4\frac{3}{4}$ f. $3\frac{2}{11} - 1\frac{9}{11}$ g. $3\frac{2}{6} + 2\frac{3}{6}$ h. $2\frac{4}{6} - 1\frac{2}{6}$

6. State the LCD for the following pairs of fractions.

a. $\frac{10}{12} + \frac{4}{24}$ b. $\frac{3}{5} + \frac{3}{15}$ c. $\frac{3}{9} - \frac{7}{27}$ d. $\frac{2}{6} - \frac{3}{9}$

e. $2\frac{8}{16} - 1\frac{4}{8}$ f. $1\frac{4}{5} - 2\frac{8}{15}$ g. $1\frac{4}{6} + 3\frac{2}{8}$ h. $2\frac{2}{3} + 2\frac{4}{5}$

7. Simplify. WE2

a. $\frac{3}{2} - \frac{8}{6}$ b. $\frac{5}{12} + \frac{7}{6}$ c. $\frac{11}{3} - \frac{6}{9}$ d. $\frac{4}{3} + \frac{1}{4}$

e. $\frac{9}{8} - \frac{2}{3}$ f. $\frac{6}{8} + \frac{7}{6}$ g. $\frac{5}{3} - \frac{4}{5}$ h. $\frac{9}{4} + \frac{4}{8}$

8. Simplify. WE2

a. $4\frac{10}{12} - 2\frac{2}{12}$ b. $1\frac{12}{15} + 2\frac{3}{15}$ c. $2\frac{10}{18} - 2\frac{3}{9}$ d. $3\frac{8}{12} + 2\frac{12}{24}$

e. $3\frac{5}{7} - 3\frac{1}{3}$ f. $1\frac{11}{3} + 4\frac{1}{6}$ g. $4\frac{2}{9} - 3\frac{1}{5}$ h. $2\frac{10}{4} + 4\frac{1}{9}$

9. Calculate.

$\frac{5}{8} + \frac{12}{16} - \frac{4}{8}$

A. $\frac{13}{1024}$ B. $\frac{13}{16}$ C. $\frac{14}{16}$ D. $\frac{7}{8}$ E. $\frac{8}{7}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. $\frac{2}{9} + \frac{5}{9}$

Student A

$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9}$

$= \frac{7}{9}$

Student B

$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9+9}$

$= \frac{7}{18}$

b. $\frac{3}{4} - \frac{3}{8}$

Student A

$\frac{3}{4} - \frac{3}{8} = \frac{3-3}{4 \times 8}$

$= \frac{0}{32}$

$= 0$

Student B

LCD of 4 and 8 is 8

$\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8}$

$= \frac{6-3}{8}$

$= \frac{3}{8}$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. The junior boys' basketball team won six games, lost three games, and drew two games. What fraction of all the games did the basketball team win or draw?

12. Dusty made breakfast by combining $\frac{2}{5}$ of a bowl of Weet-Bix with $\frac{1}{3}$ of a bowl of chopped bananas and $\frac{3}{15}$ of a bowl of milk. What fraction of the bowl is filled with Weet-Bix and bananas?

13. During a training session, Marta spent $\frac{2}{8}$ practising free kicks, $\frac{3}{8}$ doing fitness drills and $\frac{2}{16}$ kicking penalties. If Marta was in the gym for the rest of her training session, what fraction of the entire training session did she spend in the gym?

14. Students in the year 8 cohort either play the guitar, the piano, or do not play an instrument. What fraction of the students do not play any instrument if $\frac{3}{7}$ of the students play guitar and $\frac{6}{14}$ play the piano?

15. Cheryl baked a small cake for the Edrolo Christmas party. Meeky ate $\frac{3}{20}$ of the cake, Bretty ate $\frac{2}{5}$ and also took some home for his children. How much cake did Bretty take home for his children, if there was $\frac{1}{10}$ of the cake remaining?

Reasoning

Question working paths

Mild 16 (a,b,d)	Medium 16 (a,b,d), 17 (a,b)	Spicy All

16. Julian is training for the interschool running competition. Julian is a sprinter and does not like running long distances. During his training on Monday, he ran $\frac{4}{5}$ km and on Tuesday he ran $\frac{7}{10}$ km.

- **a.** Over the two days, what is the total distance Julian ran? Provide your answer in its simplest form.
- **b.** The furthest that Julian has ever run over two days is $1\frac{7}{12}$ km. How much further would Julian have had to run to match his record.
- **c.** Julian's sister, Ambrosia, also trained on Monday and Tuesday and ran a total of $\frac{24}{15}$ km. Who ran more and by how far?
- **d.** What strategies could Julian and Ambrosia implement to increase the distance of their training runs?

17. For parts **a** and **b**, use the mathematical symbols to fill in the blank box with either $=$ or $\neq$ to make each mathematical statement true.

- **a.** $\frac{a}{b} + \frac{c}{b} \square \frac{a+c}{b}$
- **b.** $\frac{a}{b} + \frac{c}{d} \square \frac{ad}{bd} + \frac{cb}{db}$
- **c.** Compare part **a** and **b** and explain why fractions with different denominators may need to be changed before being added or subtracted.

Extra spicy

18. What value is added to $\frac{12}{8}$ and results in $4\frac{6}{16}$.

A. $1\frac{4}{8}$ **B.** $2\frac{7}{8}$ **C.** $3\frac{2}{8}$ **D.** $4\frac{6}{16}$ **E.** $4\frac{1.5}{2}$

19. In the morning Millhouse drinks $1\frac{6}{14}$ cups of water. Every hour from midday until 6:00 pm he drinks $\frac{3}{7}$ cups of water. To drink $5\frac{1}{4}$ cups in total, how much more water does he need to drink after 6:00 pm?

20. Calculate the length of the unknown side of the rectangle if the perimeter is $3\frac{12}{20}$ metres and the width is $1\frac{2}{3}$ metres.

21. What fraction of a 24-hour day does school take up, if school starts at 8:45 am and finishes at 3:15 pm?

A. $\frac{6.5}{24}$ **B.** $\frac{13}{48}$ **C.** $\frac{7}{24}$ **D.** $\frac{6\frac{1}{2}}{24}$ **E.** 6.5 hours

Remember this?

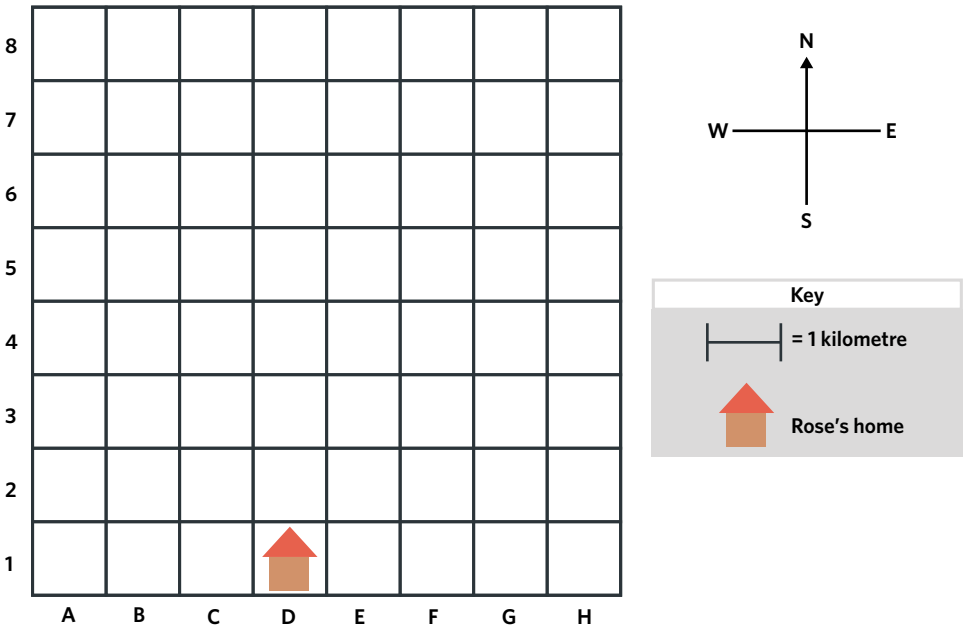
22. The AFL grand final may be played in Brisbane at the Gabba every two years commencing in 2020. Harry wants to attend the grand final whenever it is held at the Gabba. After the 2020 grand final, in which year may Harry next expect to attend?

A. 2021 **B.** 2022 **C.** 2023 **D.** 2024 **E.** 2025

23. Lucille loves cartons of vanilla milk. Each carton contains 250 millilitres of milk. Lucille buys a pack of six. How many litres of the milk are in the pack?

A. 0.25 litres **B.** 1.25 litres **C.** 1.5 litres
D. 1.75 litres **E.** 1075 millilitres

24. Rose's home is shown on the following map.



Rose's daughter lives 6 kilometres north and then 3 kilometres east from her home.
In which cell on the map is Rose's daughter's home?

A. A6 **B.** A7 **C.** F6 **D.** G6 **E.** G7

Back to contents

2C Multiplying and dividing fractions

Multiplying and dividing fractions by whole numbers is similar to working with whole numbers. Division with fractions can be thought of as 'how many groups of some amount'. Multiplication with only fractions is different. It can be thought of as finding a 'fraction of a fraction'.

LEARNING INTENTIONS

Students will be able to:

- multiply with whole numbers and fractions
- divide with whole numbers and fractions
- simplify fractions to make it easier to multiply and divide fractions.

KEY TERMS AND DEFINITIONS

The **reciprocal** of a number is 1 divided by that number.

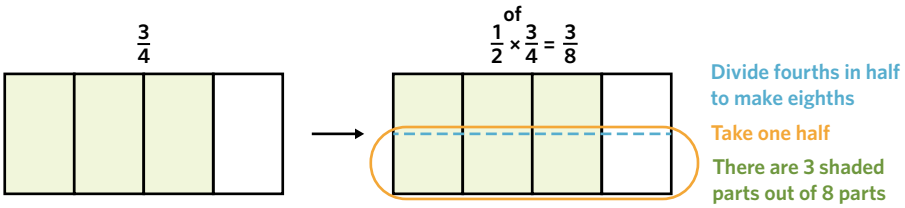
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Dusan Petkovic/Shutterstock.com

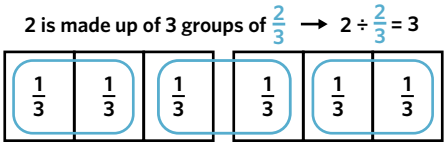
Multiplication and division with fractions can be used in many situations. For example carpenters often need to cut pieces of wood with varying lengths into equally sized pieces.

Key ideas

1. Multiplication is the same as 'of'.



2. Division is the same as 'made up of how many'.



3. Dividing a fraction by a value is the same as multiplying the fraction by the reciprocal of that value.

$$4 \div \frac{3}{2} = 4 \times \frac{2}{3} \qquad a \div \frac{b}{c} = a \times \frac{c}{b}$$

(Reciprocal)

Worked example 1

Multiplying with fractions

Evaluate each expression.

a. $\frac{2}{3} \times \frac{1}{2}$ WE1a

Working

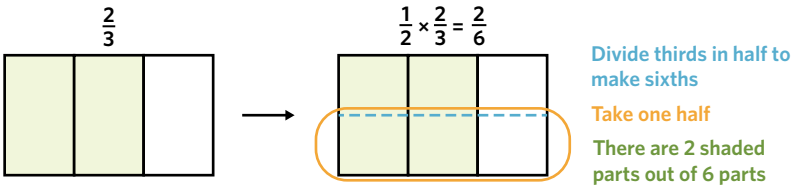
$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

Thinking

Step 1: Multiply the numerators and multiply the denominators.

Step 2: Simplify.

Visual support



b. $\frac{3}{5} \times 3$ WE1b

Working

$$\frac{3}{5} \times 3 = \frac{3}{5} \times \frac{3}{1}$$
$$= \frac{3 \times 3}{5 \times 1}$$
$$= \frac{9}{5} = 1\frac{4}{5}$$

Thinking

Step 1: Rewrite the whole number as a fraction with a denominator of 1.

Step 2: Multiply the numerators and multiply the denominators.

Step 3: Simplify.

c. $2\frac{1}{5} \times \frac{7}{2}$ WE1c

Working

$$2\frac{1}{5} \times \frac{7}{2} = \frac{2 \times 5 + 1}{5} \times \frac{7}{2}$$
$$= \frac{11}{5} \times \frac{7}{2}$$
$$= \frac{11 \times 7}{5 \times 2}$$
$$= \frac{77}{10} = 7\frac{7}{10}$$
$$= 7\frac{7}{10}$$

Thinking

Step 1: Convert the mixed number into an improper fraction.

Step 2: Multiply the numerators and multiply the denominators.

Step 3: Simplify.

Note: The final answer can be written as a mixed number or an improper fraction.

Student practice

Evaluate each expression.

a. $\frac{5}{7} \times \frac{1}{3}$ **b.** $8 \times \frac{3}{4}$ **c.** $1\frac{3}{5} \times \frac{5}{3}$

Back to contents

Worked example 2

Dividing with fractions

Evaluate each expression.

a. $\frac{2}{3} \div \frac{4}{6}$ WE2a

Working

$\frac{4}{6} \rightarrow \frac{6}{4}$

$\frac{2}{3} \div \frac{4}{6} = \frac{2}{3} \times \frac{6}{4}$

$= \frac{2 \times 6}{3 \times 4}$

$= \frac{12}{12}$

$= 1$

Thinking

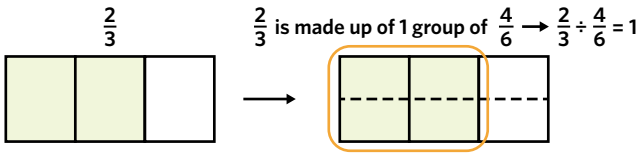
Step 1: Find the reciprocal of the divisor.

Step 2: Rewrite the division as multiplication.

Step 3: Multiply the numerators and multiply the denominators.

Step 4: Simplify.

Visual support



b. $\frac{2}{9} \div 6$ WE2b

Working

$6 = \frac{6}{1} \rightarrow \frac{1}{6}$

$\frac{2}{9} \div 6 = \frac{2}{9} \times \frac{1}{6}$

$= \frac{2 \times 1}{9 \times 6}$

$= \frac{2}{54}$

$= \frac{1}{27}$

Thinking

Step 1: Find the reciprocal of the divisor.

Step 2: Rewrite the division as multiplication.

Step 3: Multiply the numerators and multiply the denominators.

Step 4: Simplify.

c. $1\frac{1}{2} \div \frac{3}{7}$ WE2c

Working

$1\frac{1}{2} \div \frac{3}{7} = \frac{1 \times 2 + 1}{2} \div \frac{3}{7}$

$= \frac{3}{2} \div \frac{3}{7}$

$\frac{3}{7} \rightarrow \frac{7}{3}$

$= \frac{3}{2} \times \frac{7}{3}$

$= \frac{3 \times 7}{2 \times 3}$

$= \frac{21}{6}$

$= \frac{7}{2} = 3\frac{1}{2}$

Thinking

Step 1: Convert the mixed number into an improper fraction.

Step 2: Find the reciprocal of the divisor.

Step 3: Rewrite the division as multiplication.

Step 4: Multiply the numerators and multiply the denominators.

Step 5: Simplify.

Continues →

Student practice

Evaluate each expression.

a. $\frac{4}{5} \div \frac{2}{25}$ **b.** $\frac{5}{6} \div 8$ **c.** $2\frac{1}{4} \div \frac{5}{3}$

Worked example 3

Simplifying fractions before multiplying and dividing

Evaluate each expression by first simplifying.

a. $\frac{1}{5} \times \frac{4}{6}$ WE3a

Working

$\frac{1}{5} \times \frac{4}{6} = \frac{1}{5} \times \frac{2}{3}$

$= \frac{1 \times 2}{5 \times 3}$

$= \frac{2}{15}$

Thinking

Step 1: Rewrite the calculation and simplify where possible.

Step 2: Multiply the numerators and multiply the denominators.

b. $\frac{7}{4} \div \frac{14}{9}$ WE3b

Working

$\frac{14}{9} \rightarrow \frac{9}{14}$

$\frac{7}{4} \div \frac{14}{9} = \frac{7}{4} \times \frac{9}{14}$

$\frac{\cancel{7}^{1}}{4} \times \frac{9}{\cancel{14}_{2}} = \frac{1}{4} \times \frac{9}{2}$

$= \frac{1 \times 9}{4 \times 2}$

$= \frac{9}{8} = 1\frac{1}{8}$

Thinking

Step 1: Find the reciprocal of the divisor.

Step 2: Rewrite the division as multiplication.

Step 3: Cross-simplify if the numerator of one fraction and the denominator of the other share a common factor.

Step 4: Multiply the numerators and multiply the denominators.

Student practice

Evaluate each expression by first simplifying.

a. $\frac{5}{7} \times \frac{6}{24}$ **b.** $\frac{10}{11} \div \frac{3}{22}$

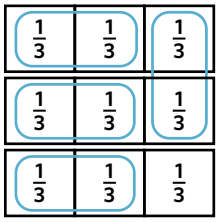
2C Questions

Understanding worksheet

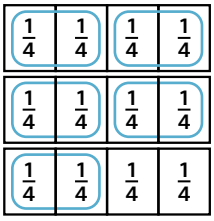
1. Use each diagram to fill in the boxes.

Example

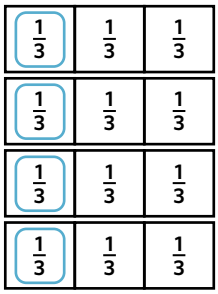
4 groups of $\frac{2}{3}$ is $\frac{8}{3}$



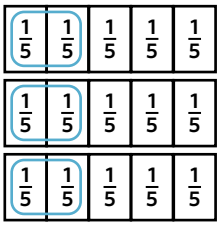
a. 5 groups of $\frac{2}{4}$ is $\frac{\square}{\square}$



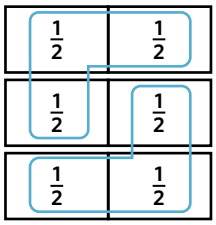
b. $\frac{1}{3}$ of 4 is $\frac{\square}{\square}$



c. $\frac{2}{5}$ of 3 is $\frac{\square}{\square}$



d. 2 groups of $\frac{3}{2}$ is $\frac{\square}{\square}$



2. Fill in the boxes to turn the divisions into multiplications.

Example

$\frac{1}{5} \div \frac{3}{7} = \frac{1}{5} \times \frac{7}{3}$

a. $\frac{6}{7} \div \frac{7}{10} = \frac{6}{7} \times \frac{\square}{\square}$

b. $\frac{8}{9} \div \frac{5}{2} = \frac{8}{9} \times \frac{\square}{\square}$

c. $\frac{3}{5} \div \frac{1}{4} = \frac{3}{5} \times \frac{\square}{\square}$

d. $\frac{9}{2} \div 3 = \frac{9}{2} \times \frac{\square}{\square}$

3. Fill in the blanks by using the words provided.

reciprocal | denominators | cross-simplifying | improper

Multiplication involving fractions involves multiplying the numerators and _____.

Division involving fractions can be turned into multiplication by using a number's _____.

_____ fractions first can make multiplication and division easier. Mixed numbers must first be converted into _____ fractions.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Evaluate each expression. WE1

a. $\frac{3}{6} \times \frac{3}{4}$ **b.** $\frac{2}{3} \times \frac{7}{2}$ **c.** $5 \times \frac{3}{5}$ **d.** $1\frac{2}{3} \times \frac{2}{7}$

e. $2\frac{2}{5} \times \frac{3}{2}$ **f.** $2\frac{6}{12} \times 2$ **g.** $2\frac{6}{11} \times 2\frac{3}{4}$ **h.** $1\frac{2}{3} \times 3\frac{1}{5} \times 3\frac{1}{3}$

5. Evaluate each expression. WE2

a. $\frac{1}{3} \div \frac{1}{12}$ **b.** $\frac{4}{5} \div \frac{5}{10}$ **c.** $\frac{4}{7} \div 3$ **d.** $1\frac{1}{10} \div \frac{3}{6}$

e. $4\frac{2}{7} \div 3$ **f.** $2\frac{4}{9} \div \frac{9}{2}$ **g.** $2\frac{6}{11} \div 2\frac{3}{4}$ **h.** $2\frac{5}{6} \div 3\frac{1}{3} \div 6\frac{1}{2}$

6. Evaluate each expression by first simplifying. WE3

a. $\frac{5}{7} \times \frac{2}{6}$ **b.** $\frac{4}{3} \times \frac{12}{7}$ **c.** $\frac{8}{3} \div \frac{4}{5}$ **d.** $\frac{8}{10} \times \frac{5}{16}$

e. $2\frac{3}{8} \times \frac{10}{17}$ **f.** $3\frac{3}{6} \div 1\frac{10}{14}$ **g.** $\frac{30}{7} \div \frac{5}{21} \times \frac{15}{6}$ **h.** $\frac{22}{21} \times \frac{4}{2} \div \frac{2}{5}$

7. Evaluate each expression using order of operations.

a. $\frac{1}{2} \times \frac{4}{9} + \frac{2}{5}$ **b.** $\frac{2}{3} - \frac{1}{8} \times \frac{1}{9}$

c. $\left(\frac{5}{6} + \frac{2}{5}\right) \div \frac{3}{4}$ **d.** $\frac{4}{7} + \frac{3}{14} \div \frac{5}{7} - \frac{3}{5}$

e. $\left(\frac{3}{5} - \frac{2}{5} + \frac{1}{4}\right) \div \frac{1}{6}$ **f.** $\left(\frac{4}{5} + \frac{24}{3}\right) \times \left(\frac{6}{9} - \frac{2}{11}\right)$

g. $\left(\frac{1}{8} \times \frac{2}{3}\right) \div \frac{1}{3} + \frac{4}{5}$ **h.** $\frac{3}{5} \times \left(\frac{8}{9} - \frac{2}{3} + \frac{5}{8}\right) \div \frac{1}{3}$

8. Which of the following is equivalent to $3\frac{3}{5} \div 6\frac{2}{3}$?

A. $\frac{18}{5} \times \frac{20}{3}$

B. $(3 \div 6) + \left(\frac{3}{5} \div \frac{2}{3}\right)$

C. $\frac{5}{18} \times \frac{20}{3}$

D. $\frac{18 \times 3}{5 \times 20}$

E. $\frac{18 \div 3}{5 \div 20}$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. $\frac{7}{6} \times 2$

Student A

$$\frac{7}{6} \times 2 = \frac{7}{6} \times \frac{2}{2}$$
$$= \frac{7 \times 2}{6 \times 2}$$
$$= \frac{14}{12}$$
$$= \frac{7}{6}$$

Student B

$$\frac{7}{6} \times 2 = \frac{7}{6} \times \frac{2}{1}$$
$$= \frac{7 \times 2}{6 \times 1}$$
$$= \frac{14}{6}$$
$$= \frac{7}{3}$$

b. $\frac{13}{20} \div \frac{3}{2}$

Student A

$$\frac{13}{20} \div \frac{3}{2} = \frac{13}{20} \times \frac{2}{3}$$
$$= \frac{13 \times 2}{20 \times 3}$$
$$= \frac{26}{60}$$
$$= \frac{13}{30}$$

Student B

$$\frac{13}{20} \div \frac{3}{2} = \frac{13}{20} \times \frac{3}{2}$$
$$= \frac{13 \times 3}{20 \times 2}$$
$$= \frac{39}{40}$$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Hawi initially charged the battery of her electric car to $\frac{4}{5}$ of its full capacity. After driving for a few days, she saw that she used $\frac{3}{7}$ of the initial charge. What fraction of a full charge did she use?

11. A group of students are planning a 24-hour treadmill challenge to raise money for charity. If each person runs for two-fifths of an hour, how many students are needed for the full 24 hours?

12. Georgia buys cans of dog food in boxes of 10 cans. She feeds her dog three-quarters of a can every day. If she has three-fifths of a full box left, how many days' worth of food does she have for her dog?

13. The floors in Harish's rectangular dining room need to be covered in varnish. The room has a length of $6\frac{2}{3}$ m and a width of $3\frac{2}{7}$ m. If $4\frac{8}{21}$ tins were needed, how much area did each tin cover?

14. Maddie and Hugh are playing the board game Blokus that involves placing differently shaped tiles on the board. $\frac{3}{8}$ of the board is covered in tiles, and $\frac{2}{5}$ of those are Maddie's red tiles. What fraction of the board does Maddie still need to cover so that her tiles take up at least half of the board?

Reasoning

Question working paths

Mild 15 (a,b,c,e), 16 (a,b)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. A cinema has one theatre with 300 seats. For one night only, it is showing the blockbuster movie Shrek. Each session runs for $1\frac{1}{2}$ hours and tickets cost $24 per person. Those who buy popcorn and drinks usually spend an additional $17. Multiple sessions will be run throughout the night.

a. How many people bought tickets for the first session if $\frac{3}{5}$ of the available seats were occupied?

b. $\frac{1}{3}$ of the people who bought a ticket also bought popcorn and drinks. What fraction of the total available seats were occupied by people who bought snacks?

c. If a quarter of an hour is needed to clean the cinema between sessions, how many sessions will run between 5:30 pm and 11 pm.

d. Assuming the same fraction of the theatre was filled for each session and the same fraction of each audience bought popcorn and drinks, how much revenue will the cinema earn from all sessions on the night?

e. Propose two different ways that the cinema could increase the amount of money they make each night.

16. Fill in the boxes using any of the the digits from 1 to 9 one time each to make the:

$$\frac{\square}{\square} \div \frac{\square}{\square}$$

a. smallest possible answer.

b. largest possible answer.

c. Compare your answers to part **a** and **b** to explain whether a fraction divided by a smaller valued fraction will result in a larger or smaller fraction.

Extra spicy

17. Which option represents the smallest value?

A. $\frac{5+4}{6+5}$ **B.** $\frac{5}{6} \div \frac{4}{5}$ **C.** $\frac{54}{65}$ **D.** $\frac{5-4}{6-5}$ **E.** $\frac{5}{6} \times \frac{4}{5}$

18. A ball is dropped from a height of 110 cm. Each time it hits the ground, it bounces back up to $\frac{4}{5}$ of the height it fell from. What approximate height does the ball bounce to after hitting the ground three times?

A. 1 cm **B.** 22 cm **C.** 29 cm **D.** 56 cm **E.** 88 cm

19. For what value of ♥ is the expression $\frac{♥ - 3}{2} \div \frac{1}{4}$ equal to 3.

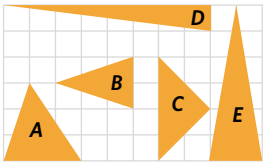
20. Kathy, Patrick and Warren are eating a cake.

Kathy ate $\frac{1}{5}$ of the cake. Then, Patrick ate $\frac{2}{5}$ of the remaining cake. Finally, Warren ate $\frac{3}{5}$ of the remaining cake.

What fraction of the cake is left?

Remember this?

21. Which triangle has the largest area?



22. Which of the following shows 468 expressed as a product of its prime factors?

A. 36×13 **B.** $4 \times 9 \times 13$ **C.** $3 \times 2 \times 78$ **D.** $2^2 \times 3^2 \times 13$ **E.** $6^2 \times 13$

23. Yindi is constructing a house out of blocks. On one day, she built $\frac{7}{30}$ of the house. On the following day, she built $\frac{3}{10}$ of the house.

What fraction of the house does Yindi have left to build?

A. $\frac{3}{10}$ **B.** $\frac{1}{3}$ **C.** $\frac{7}{15}$ **D.** $\frac{8}{15}$ **E.** $\frac{2}{3}$

Back to contents

2D Working with negative fractions

When applying the four operations to negative fractions we can use the same rules and processes that are used for negative integers. When we multiply or divide with fractions, the + and − signs of the fractions affect the outcome of the calculations.

LEARNING INTENTIONS

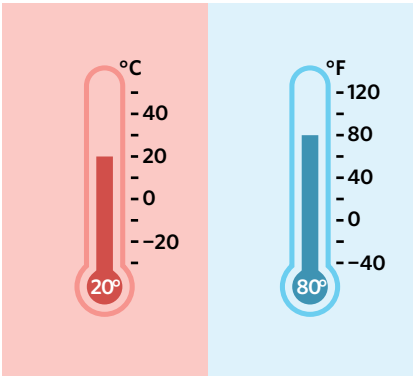
Students will be able to:

- add and subtract negative fractions and mixed numbers with common denominators on a number line
- add and subtract negative fractions and mixed numbers
- multiply and divide negative fractions and mixed numbers.

KEY TERMS AND DEFINITIONS

Directed numbers have a direction and value, one direction is positive (+) and the other direction is negative (−).

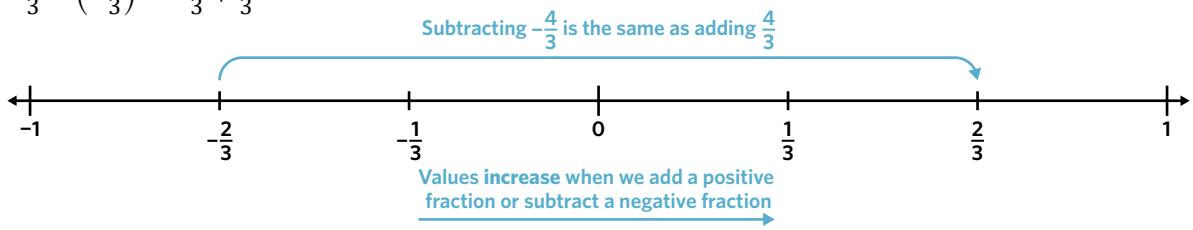
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



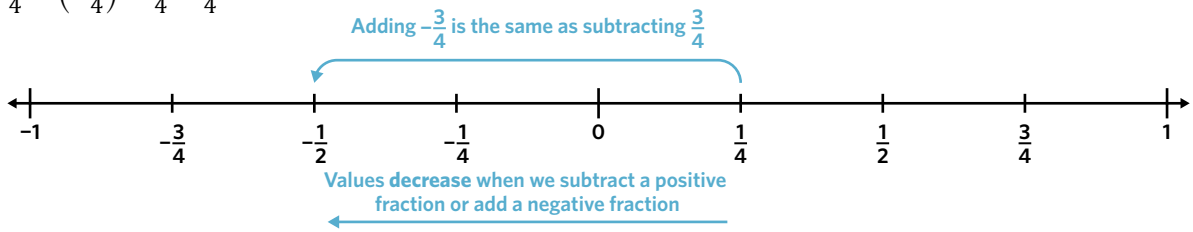
One of the most common uses for negative numbers is measuring temperatures. Calculating the average temperature with positive and negative degrees involves adding, subtracting, multiplying, and dividing fractions of positive and negative numbers.

Key ideas

1. $-\frac{2}{3} - \left(-\frac{4}{3}\right) = -\frac{2}{3} + \frac{4}{3}$



2. $\frac{1}{4} + \left(-\frac{3}{4}\right) = \frac{1}{4} - \frac{3}{4}$



3. The product or quotient of two fractions with the same directions or signs will always be positive.

Positive fraction	×	Positive fraction	=	Positive fraction
Positive fraction	÷	Positive fraction	=	Positive fraction
Negative fraction	×	Negative fraction	=	Positive fraction
Negative fraction	÷	Negative fraction	=	Positive fraction

Continues →

4. The product or quotient of two fractions with different directions or signs will always be negative.

Positive fraction	×	Negative fraction	=	Negative fraction
Positive fraction	÷	Negative fraction	=	Negative fraction
Negative fraction	×	Positive fraction	=	Negative fraction
Negative fraction	÷	Positive fraction	=	Negative fraction

Worked example 1

Adding and subtracting negative fractions and mixed numbers

Evaluate and simplify.

a. $\frac{2}{7} + \left(-\frac{5}{7}\right)$ WE1a

Working

$\frac{2}{7} + \left(-\frac{5}{7}\right) = \frac{2}{7} - \frac{5}{7}$

$= -\frac{3}{7}$

Thinking

Step 1: Adding a negative fraction is the same as subtracting a positive fraction.

Step 2: Evaluate the expression.

b. $-\frac{1}{5} - \left(-\frac{3}{2}\right)$ WE1b

Working

$-\frac{1}{5} - \left(-\frac{3}{2}\right) = -\frac{1}{5} + \frac{3}{2}$

$= -\frac{2}{10} + \frac{15}{10}$

$= \frac{13}{10}$

$= 1\frac{3}{10}$

Thinking

Step 1: Subtracting a negative fraction is the same as adding a positive fraction.

Step 2: Write equivalent fractions with equal denominators then evaluate the expression.

c. $1\frac{4}{9} + \left(-2\frac{1}{3}\right)$ WE1c

Working

$1\frac{4}{9} + \left(-2\frac{1}{3}\right) = 1\frac{4}{9} - 2\frac{1}{3}$

$= \frac{13}{9} - \frac{7}{3}$

$= \frac{13}{9} - \frac{21}{9}$

$= -\frac{8}{9}$

Thinking

Step 1: Adding a negative fraction is the same as subtracting a positive fraction.

Step 2: Convert the mixed numbers to improper fractions.

Step 3: Write equivalent fractions with equal denominators then evaluate the expression.

Student practice

Evaluate and simplify.

a. $\frac{3}{8} + \left(-\frac{5}{8}\right)$ **b.** $\frac{2}{3} - \left(-\frac{1}{2}\right)$ **c.** $1\frac{2}{5} + \left(-2\frac{3}{4}\right)$

Worked example 2

Multiplying and dividing negative fractions and mixed numbers

Evaluate and simplify.

a. $-\frac{3}{4} \times \left(-\frac{5}{7}\right)$ WE2a

Working

$-\frac{3}{4} \times \left(-\frac{5}{7}\right) = \frac{3 \times 5}{4 \times 7}$

$= \frac{15}{28}$

Thinking

Step 1: The product of a negative fraction and a negative fraction will be positive.

Step 2: Multiply the numerators and multiply the denominators.

b. $\frac{3}{5} \div \left(-\frac{6}{7}\right)$ WE2b

Working

$\frac{3}{5} \div \left(-\frac{6}{7}\right) = -\left(\frac{3}{5} \div \frac{6}{7}\right)$

$= -\left(\frac{\cancel{3}^{1}}{5} \times \frac{7}{\cancel{6}_{2}}\right)$

$= -\left(\frac{1}{5} \times \frac{7}{2}\right)$

$= -\left(\frac{1 \times 7}{5 \times 2}\right)$

$= -\frac{7}{10}$

Thinking

Step 1: The quotient of a positive fraction and a negative fraction will be negative.

Step 2: Find the reciprocal of the second number and rewrite the division as multiplication.

Step 3: Cross-simplify if the numerator of one fraction and the denominator of the other share a common factor.

Step 4: Multiply the numerators and multiply the denominators.

c. $\left(-1\frac{1}{4}\right) \div \frac{2}{3}$ WE2c

Working

$\left(-1\frac{1}{4}\right) \div \frac{2}{3} = -\left(1\frac{1}{4} \div \frac{2}{3}\right)$

$= -\left(\frac{5}{4} \div \frac{2}{3}\right)$

$= -\left(\frac{5}{4} \times \frac{3}{2}\right)$

$= -\left(\frac{5 \times 3}{4 \times 2}\right)$

$= -\frac{15}{8}$

$= -1\frac{7}{8}$

Thinking

Step 1: The quotient of a negative fraction and a positive fraction will be negative.

Step 2: Convert the mixed number to an improper fraction.

Step 3: Find the reciprocal of the second number and rewrite the division as multiplication.

Step 4: Multiply the numerators and multiply the denominators.

Student practice

Evaluate and simplify.

a. $-\frac{2}{3} \times \left(-\frac{5}{11}\right)$ **b.** $\frac{2}{5} \div \left(-\frac{5}{9}\right)$ **c.** $\left(-1\frac{1}{6}\right) \div \frac{3}{4}$

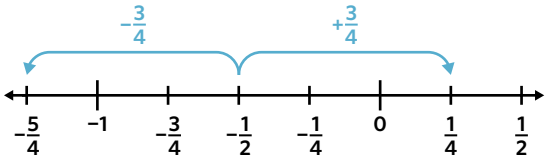
2D Questions

Understanding worksheet

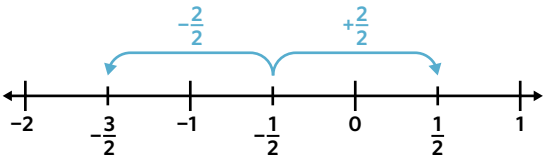
1. Use the number line to choose the correct direction to complete the following calculations.

Example

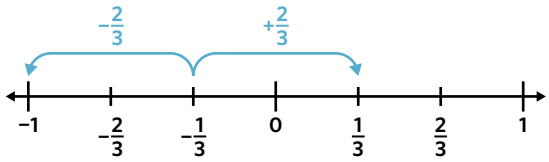
$-\frac{1}{2} - \left(-\frac{3}{4}\right) =$ $\frac{1}{4}$



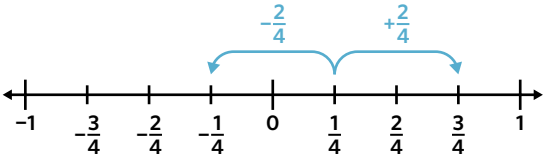
a. $-\frac{1}{2} - \frac{2}{2} =$ ☐



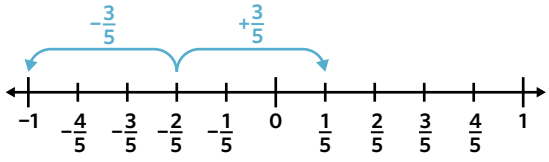
b. $-\frac{1}{3} + \frac{2}{3} =$ ☐



c. $\frac{1}{4} - \left(-\frac{2}{4}\right) =$ ☐



d. $-\frac{2}{5} - \left(-\frac{3}{5}\right) =$ ☐



2. Fill in the blank with the positive or negative sign.

Example

$\frac{5}{7} \div -\frac{4}{9} =$ $-$ $\frac{45}{28}$

a. $-\frac{10}{11} \times \frac{8}{9} =$ ☐ $\frac{80}{99}$

b. $-\frac{15}{7} \times -\frac{3}{10} =$ ☐ $\frac{9}{14}$

c. $-\frac{13}{20} \div -\frac{4}{5} =$ ☐ $\frac{13}{16}$

d. $-\frac{2}{11} \div \frac{4}{5} =$ ☐ $\frac{5}{22}$

3. Fill in the blanks by using the words provided.

positive | negative | direction | reciprocal

When adding a ☐ fraction on the number line we move to the right. When subtracting we move to the left. The ☐ of addition and subtraction reverses on the number line when working with ☐ fractions. Dividing fractions is the same as multiplying by the ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Evaluate and simplify. WE1a,b

a. $-\frac{4}{8}+\frac{1}{8}$ b. $\frac{2}{11}-\left(-\frac{5}{11}\right)$ c. $-\frac{4}{5}-\left(-\frac{3}{5}\right)$ d. $\frac{2}{3}+\left(-\frac{7}{9}\right)$

e. $-\frac{1}{4}+\frac{1}{8}$ f. $\frac{2}{3}+\left(-\frac{3}{5}\right)$ g. $-\frac{5}{7}-\left(-\frac{7}{5}\right)$ h. $-\frac{5}{8}-\left(-\frac{5}{6}\right)$

5. Evaluate and simplify. WE1c

a. $3\frac{3}{4}-\frac{1}{4}$ b. $-1\frac{8}{11}+\frac{5}{11}$ c. $2\frac{1}{3}-3\frac{2}{3}$ d. $-1\frac{4}{9}-\left(-2\frac{2}{9}\right)$

e. $-1\frac{1}{2}+\frac{5}{8}$ f. $-2\frac{2}{5}-\left(-3\frac{2}{3}\right)$ g. $\frac{5}{6}+\left(-3\frac{8}{9}\right)$ h. $-\frac{5}{7}+\left(-3\frac{3}{4}\right)$

6. Evaluate and simplify. WE2a,b

a. $\frac{1}{2}\times\left(-\frac{3}{5}\right)$ b. $\left(-\frac{3}{8}\right)\times\left(-\frac{5}{9}\right)$

c. $\left(-\frac{2}{5}\right)\div\frac{5}{9}$ d. $\left(-\frac{12}{21}\right)\div\frac{6}{7}$

e. $\frac{2}{15}\div\left(-\frac{5}{9}\right)$ f. $\left(-\frac{8}{15}\right)\times\left(-\frac{2}{3}\right)$

g. $\left(-\frac{9}{10}\right)\div\left(-\frac{6}{25}\right)\times\left(-\frac{1}{4}\right)$ h. $\left(-\frac{7}{12}\right)\times\left(-\frac{3}{10}\right)\div\frac{14}{15}$

7. Evaluate and simplify. WE2c

a. $\left(-1\frac{1}{4}\right)\times\frac{5}{6}$ b. $2\frac{1}{4}\times\left(-1\frac{2}{3}\right)$ c. $-1\frac{3}{8}\div3\frac{1}{3}$ d. $-24\times2\frac{5}{12}$

e. $36\div\left(-1\frac{1}{2}\right)$ f. $\left(-2\frac{8}{15}\right)\div\left(-1\frac{7}{14}\right)$ g. $\left(-1\frac{3}{7}\right)\div1\frac{8}{11}$ h. $1\frac{2}{25}\div\left(-4\frac{3}{8}\right)$

8. Evaluate and simplify.

a. $\frac{1}{3}\times\left(\frac{4}{8}+\frac{1}{8}\right)$ b. $\frac{2}{5}\times\left(2-\frac{4}{9}\right)$

c. $\frac{1}{3}\times\frac{4}{8}+\frac{1}{8}$ d. $\frac{4}{7}-\frac{5}{14}\div\frac{5}{12}$

e. $\left(1\frac{11}{12}-\frac{3}{8}\right)\div\left(\frac{3}{5}-2\frac{1}{10}\right)$ f. $\frac{4}{7}-\frac{3}{14}\div\left(-\frac{5}{7}\right)-\frac{3}{5}$

g. $\frac{11}{10}\times\left(-\frac{1}{6}\right)\div\left(-\frac{22}{15}\right)$ h. $\frac{13}{24}-\left(\frac{2}{5}-\left(-\frac{5}{9}\right)\right)\div\frac{2}{5}$

9. Evaluate and simplify.

$\left(-\frac{12}{25}\right)\div\left(-\frac{9}{10}\right)$

A. $-\frac{8}{15}$ B. $\frac{8}{15}$ C. $\frac{54}{125}$ D. $\frac{120}{225}$ E. $\frac{15}{8}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Evaluate and simplify:

$-\frac{5}{6} - \left(-\frac{3}{8}\right)$

Student A

$= +\frac{5}{6} + \frac{3}{8}$

$= +\frac{20}{24} + \frac{9}{24}$

$= \frac{+20 + 9}{24}$

$= \frac{29}{24}$

Student B

$= -\frac{5}{6} + \frac{3}{8}$

$= -\frac{20}{24} + \frac{9}{24}$

$= \frac{-20 + 9}{24}$

$= -\frac{11}{24}$

b. Evaluate and simplify:

$-\frac{17}{24} \div \left(-\frac{9}{12}\right)$

Student A

$= \frac{17}{24} \div \frac{9}{12}$

$= \frac{17}{24} \times \frac{12}{9}$

$= \frac{17}{2} \times \frac{1}{9}$

$= \frac{17 \times 1}{2 \times 9}$

$= \frac{17}{18}$

Student B

$= -\frac{17}{24} \times \frac{12}{9}$

$= -\frac{17}{2} \times \frac{1}{9}$

$= -\frac{17 \times 1}{2 \times 9}$

$= -\frac{17}{18}$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Ciara needs $3\frac{1}{5}$ rolls of fabric to create her lounge room curtains and half as much for her bedroom curtains. If she already has 4 rolls of fabric, what fraction of another roll does she need to complete the curtains for both rooms?

12. Sarah brought $\frac{3}{4}$ of a large block of chocolate to class, and Nadia brought $\frac{3}{5}$ of the same size block. They combined their chocolate and gave their teacher Mr Ellis the equivalent of half of a large block of chocolate. What fraction of a large block do Sarah and Nadia have left?

13. Mackenzie spends three eighths of her monthly salary on bills. If she buys Christmas gifts with a third of the remaining amount, what fraction of her salary does she have left to save?

14. Patrick is learning the waltz. After every two steps forwards he takes one step backwards. Each of his forward dancing steps are four fifths of a metre long. If the size of his backwards steps are three quarters of the length of a forwards step, how far forward has he moved after 12 steps in total?

15. Liza fills an empty bucket for $2\frac{3}{5}$ minutes at a rate of $12\frac{1}{2}$ litres per minute. Henry then puts a hole in the bucket. How much water is left in the bucket if it was leaking for $4\frac{1}{3}$ minutes at a rate of $7\frac{1}{6}$ litres per minute?

Reasoning

Question working paths

Mild	Medium	Spicy
16 (a,b,c,e)	16 (a,b,c,e), 17 (a,b)	All

16. Vicky brings $3\frac{3}{2}$ kg of flour to her cooking class of nine students. Today they will be cooking pizza. Use fractions or mixed numbers to answer the following questions.

- **a.** Each student will require $\frac{1}{8}$ kg of flour to make a pizza. If two of Vicky's students are absent, how many kg of flour are needed so that each present student can make a pizza?
- **b.** At the beginning of class, Vicky made two demonstration pizzas. How many kg of flour is left after the demonstration pizzas and the students' pizzas have been made?
- **c.** The two absent students arrive after everyone has completed their first pizza. How many more pizzas can the students make with the remaining flour?
- **d.** Five anchovies weigh $\frac{1}{50}$ kg and a 20 slice packet of salami weighs 100 g. Vicky bought 1 kg of salami and 250 g of anchovies. How many slices of salami and how many anchovies were available for the class to use?
- **e.** If you were judging for the world pizza championship, what ingredients and cooking methods would win your vote?

17. Observe the function machines in the following diagram.

- **a.** What is the missing value in the blue square?
- **b.** What is the missing value in the red square?
- **c.** Consider your answers to parts **a** and **b** to explain why the values in the blue and red squares are equal.

Extra spicy

18. Daniel's coffee pot is $\frac{4}{5}$ full. After pouring out another $\frac{1}{3}$ of the pot, Daniel has exactly $2\frac{1}{3}$ cups of coffee left in the pot. How many cups in total does his pot hold?

A. 5 **B.** 6 **C.** 7 **D.** 8 **E.** 9

19. Two months ago, Vutha moved into a house with two friends. They received an electricity bill for the past three months. What fraction of the bill is Vutha required to pay if they split the entire bill equally for the time he has been living there?

20. Ronald harvests $16\frac{2}{3}$ pounds (lbs) of soybeans. Henry has 14 bags and each bag can carry $\frac{3}{4}$ of a lb. What fraction of Ronald's soybeans can Henry put into his bags?

21. The temperature for a third of the day was $-1\frac{1}{5}$°C, and the temperature for $\frac{1}{5}$ of the day was $-2\frac{1}{3}$°C. The temperature for the rest of the day was constant. What was the temperature for the rest of the day if the average for the whole day was 1°C?

A. $\frac{7}{15}$°C **B.** $\frac{8}{15}$°C **C.** 1°C **D.** $1\frac{7}{15}$°C **E.** 4°C

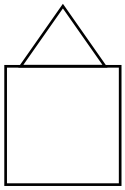
Remember this?

22. Colleen is planning the daily removal of litter from the beach at her surf club. She wants to estimate the number of pieces of litter that are left on the beach every day. Which data sampling method should she use to get the most accurate estimate?

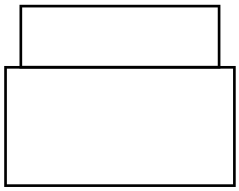
A. Count the daily number of people that catch the bus that passes the beach.
B. Post a survey online asking people to state whether they litter on the beach.
C. Count the amount of pieces of litter left on the beach at the end of a few different days.
D. Count the number of people who buy ice creams at the beach.
E. Use the government figures for the daily average number of people who visit any beach in Australia.

23. Victoria and Stephen are tiling along their wall. They use a pattern of 6 green tiles to 5 purple tiles. How many green tiles do they use if they use a total of 77 tiles?

24. Vida put one block on top of another block and drew the following drawings of the front and side views.

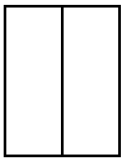


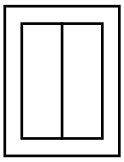
Front view

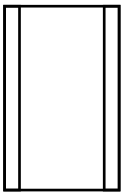


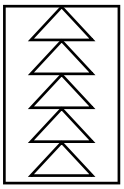
Side view

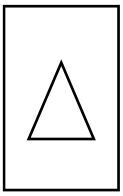
Which of the following is the top view?

A. 

B. 

C. 

D. 

E. 

Back to contents

2E Four operations with decimals

Written strategies can be used to add, subtract, multiply, and divide decimals. The vertical algorithm is applied differently with each operation. When multiplying and dividing decimals by powers of 10 the place value of each digit will change.

LEARNING INTENTIONS

Students will be able to:

- use the four operations with decimals
- multiply and divide decimals by powers of 10.

KEY TERMS AND DEFINITIONS

Powers of 10 are the result of multiplying ten by itself a certain number of times.

Multiplier is the number by which a given value is multiplied.

Product is the result when two or more values are multiplied together.

Divisor is the second number in a division calculation. It is the number by which a given value is divided.

Dividend is the first number in a division calculation. It is the number that is being divided by the second number.

Quotient is the result of a division calculation.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Operations with decimals are commonly used with the weight of food. For example, customers at Yo-get-it make their own cup with toppings. The price is calculated based on the weight of a serving. For every 100 grams it costs $2.80.

Key ideas

1. When adding and subtracting using the vertical algorithm, the decimal points and place values must line up.

1.52 + 4.27 + 3.34

	Ones	Decimal point	Tenths	Hundredths
	1	.	5	2
	4	.	2	7
+	3	.	3	4

8.73 − 2.31

	Ones	Decimal point	Tenths	Hundredths
	8	.	7	3
−	2	.	3	1

Continues →

2. When multiplying decimals using the vertical algorithm the decimal point is initially ignored. The number of decimal places in the answer is the same as the total number of decimal places from the numbers in the question.

$$\begin{array}{r} {\scriptstyle +1\ +1} \\ {\scriptstyle +2\ +1} \\ 6\,5\,4 \\ \times\quad 3\,4 \\ \hline {\scriptstyle +1} \\ 2\,6\,1\,6 \\ {\scriptstyle +1} \\ +\,1\,9\,6\,2\,0 \\ \hline 2\,2\,2\,3\,6 \end{array}$$

6.54×3.4

3 numbers after decimal point in question

$6.54 \times 3.4 = 22.236$

3 numbers after decimal point in answer

3. When dividing with decimals, the divisor is converted into a whole number by multiplying by a power of 10. The dividend is multiplied by the same power of 10. The decimal point in the dividend is aligned with the decimal point in the quotient.

$5.25 \times 10 \div 0.4 \times 10 = 52.5 \div 4$

Quotient (answer)

$$\begin{array}{r} 1\,3.1\,2\,5 \\ 4\overline{)\,5\,{}^{1}2.5\,{}^{1}0\,{}^{2}0} \end{array}$$

Divisor → 4) 5 12 . 5 10 20 ← Dividend

Worked example 1

Multiplying and dividing by powers of 10

Calculate.

a. $91.82 \div 100$ WE1a

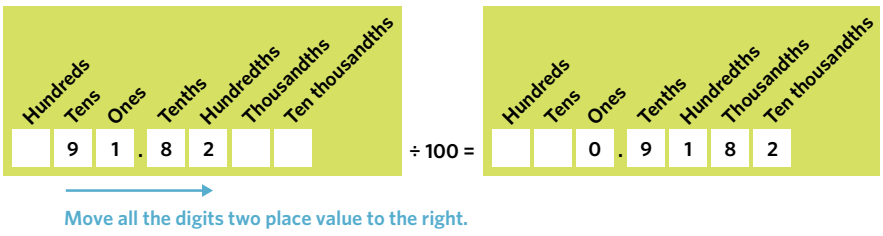
Working

$91.82 \div 100 = 0.9182$

Thinking

When dividing by 100, move the digits two places to the left. Use zeros as placeholders in the ones place.

Visual support



b. $6.327 \times 10\,000$ WE1b

Working

$6.327 \times 10\,000 = 63\,270$

Thinking

When multiplying by 10 000, move the digits four places to the left. Use zeros as placeholders in the ones place.

Continues →

Student practice

Calculate.

a. $3.74 \div 100$ **b.** $25.39 \times 10\,000$

Worked example 2

Adding and subtracting decimals

Calculate.

a. $4.072 + 2.73 + 1.3$ WE2a

Working

$$\begin{array}{r} {\scriptstyle +1\ +1\ \ \ \ } \\ 4.0\ 7\ 2 \\ 2.7\ 3\ 0 \\ +\ 1.3\ 0\ 0 \\ \hline 8.1\ 0\ 2 \end{array}$$

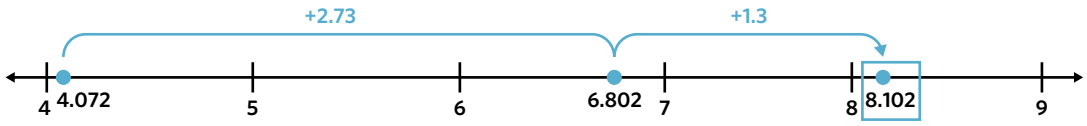
Thinking

Step 1: Write the decimals vertically, ensuring the decimal points and the digits in each place value line up.

Step 2: Use zeros as a placeholder if there is a place value digit missing.

Step 3: From right to left, add the digits in each place value column. If the digits sum to 10 or more, regroup to the place value column to the left.

Visual support



b. $8.477 - 3.38$ WE2b

Working

$$\begin{array}{r} {\scriptstyle \ \ 3\ 17\ \ } \\ 8.\cancel{4}\ \cancel{7}\ 7 \\ -\ 3.3\ 8\ 0 \\ \hline 5.0\ 9\ 7 \end{array}$$

Thinking

Step 1: Write the decimals vertically, ensuring that the decimal points and the digits in each place value line up.

Step 2: Use zeros as a placeholder if there is a place value digit missing.

Step 3: From right to left, subtract the digits in each place value column, regrouping where necessary.

Student practice

Calculate.

a. $5.381 + 1.53 + 2.4$ **b.** $9.856 - 2.56$

Worked example 3

Multiplying and dividing decimals

Calculate.

a. 1.52×6 WE3a

Working

$$\begin{array}{r} {\scriptstyle 3\ 1\ \ } \\ 1\ 5\ 2 \\ \times \qquad 6 \\ \hline 9\ 1\ 2 \end{array}$$

$1.52 \times 6 = 9.12$

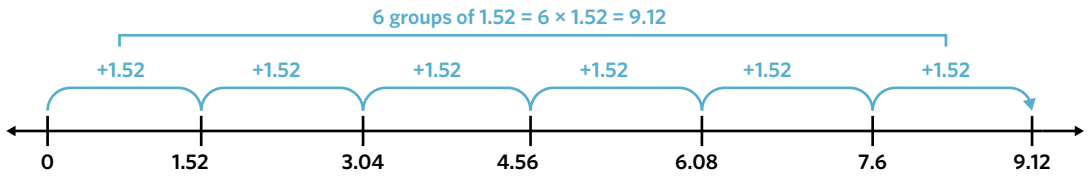
Thinking

Step 1: Set up the vertical algorithm as if multiplying whole numbers and removing decimal points.

Step 2: Perform the multiplication by multiplying each place value column, from right to left, regrouping where necessary.

Step 3: Count the number of decimals in the question, this is the same for the answer.

Visual support



b. $68.94 \div 0.4$ WE3b

Working

$68.94 \times 10 = 689.4$

$0.4 \times 10 = 4$

$689.4 \div 4$

$$\begin{array}{r} 1\ 7\ 2\ .\ 3\ 5 \\ 4\overline{)6\ {}^{2}8\ \ 9\ .\ {}^{1}4\ {}^{2}0} \end{array}$$

$68.94 \div 0.4 = 172.35$

Thinking

Step 1: Multiply the dividend and the divisor by the same power of 10 so the divisor is a whole number.

Step 2: Set up the short division. Align the decimal point in the quotient with the decimal point in the dividend.

Step 3: Perform short division the same way as with whole numbers.

Student practice

Calculate.

a. 2.34×4

b. $58.95 \div 0.6$

2E Questions

Understanding worksheet

1. Fill in the missing boxes to make the equations true.

Example

$4.9 \times$ [1000] $= 4900$

$4900 \div$ [1000] $= 4.9$

a. $4.58 \times 100 =$ [ ]

$458 \div 100 =$ [ ]

b. $5.87 \times$ [ ] $= 58.7$

$58.7 \div$ [ ] $= 5.87$

c. $3.25 \times$ [ ] $= 325$

$325 \div$ [ ] $= 3.25$

d. [ ] $\times 1000 = 62\,500$

[ ] $\div 1000 = 62.5$

2. Use the vertical algorithm to write the answer with the decimal place in the correct position.

Example

$$\begin{array}{r} {\scriptstyle +3\ +3} \\ 3\ 7\ 8 \\ \times\quad\quad 4 \\ \hline 1\ 5\ 1\ 2 \end{array}$$

$3.78 \times 4 =$ [15.12]

a.

$$\begin{array}{r} {\scriptstyle +3} \\ 4\ 8\ 1 \\ \times\quad\quad 4 \\ \hline 1\ 9\ 2\ 4 \end{array}$$

$4.81 \times 4 =$ [ ]

b.

$$\begin{array}{r} {\scriptstyle +1} \\ 1\ 4\ 6 \\ \times\quad\quad 2 \\ \hline 2\ 9\ 2 \end{array}$$

$1.46 \times 2 =$ [ ]

c.

$$\begin{array}{r} {\scriptstyle +1\ +1} \\ 2\ 3\ 4 \\ \times\quad\quad 3 \\ \hline 7\ 0\ 2 \end{array}$$

$2.34 \times 3 =$ [ ]

d.

$$\begin{array}{r} {\scriptstyle +5\ +5} \\ 2\ 8\ 9 \\ \times\quad\quad 6 \\ \hline 1\ 7\ 3\ 4 \end{array}$$

$2.89 \times 6 =$ [ ]

3. Fill in the blanks by using the words provided.

subtract | multiplying | dividing | whole

To add and [ ] decimals using the vertical algorithm the decimal points must line up.

When [ ] decimals by 1000, each digit moves three place values to the right, however when [ ] decimals by 1000, each digit moves three places to the left. When dividing decimals by decimals, the first step is to multiply the divisor by a power of 10 to make it a [ ] number.

Back to contents

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Calculate. WE1

a. $905.21 \div 10$ **b.** 65.477×100 **c.** $53.434 \div 100$ **d.** 73.434×1000
e. $35.719 \div 1000$ **f.** $425.26 \times 10\,000$ **g.** $7.583 \div 1000$ **h.** 788.73×1000

5. Calculate. WE2

a. $113.7 + 124.1$ **b.** $948.7 - 528.1$ **c.** $37.81 + 2.116$ **d.** $73.5 - 63.22$
e. $572.9 + 4.385$ **f.** $26.42 - 0.2082$ **g.** $0.5776 + 345.4$ **h.** $935.9 - 5.886$

6. Calculate. WE3a

a. 97.1×5 **b.** 99.8×7 **c.** 8.59×4 **d.** 91.4×13
e. 54.4×15 **f.** 2.48×2.7 **g.** 88.7×7.2 **h.** 8.62×65.8

7. Calculate. WE3b

a. $4.53 \div 3$ **b.** $8.96 \div 8$ **c.** $140.4 \div 4$ **d.** $35.64 \div 0.9$
e. $177.9 \div 0.3$ **f.** $1.782 \div 0.8$ **g.** $3.48 \div 0.04$ **h.** $2.856 \div 1.4$

8. Calculate.

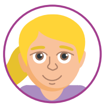
$4.75 + 3.9 + 2.65$

A. 9.230 **B.** 9.150 **C.** 11.3 **D.** 9.2 **E.** 12

Spot the mistake

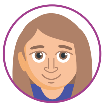
9. Select whether Student A or Student B is incorrect.

a. 2.15×1.8



Student A

```
   +1 +4
      2  1  5
×        1  8
   1  7  2  0
+  2  1  5  0
   3 . 8  7  0
```



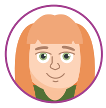
Student B

```
   +1 +4
      2  1  5
×        1  8
   1  7  2  0
+     2  1  5
   1 . 9  3  0
```

b. $1.52 + 3.87$

Student A

```
   1 . 5  2
+  3 . 8  7
   4 . 1  3  9
```



Student B

```
  +1
   1 . 5  2
+  3 . 8  7
   5 . 3  9
```

Problem solving

Question working paths

Mild	Medium	Spicy
10, 11, 12	11, 12, 13	12, 13, 14

10. Carlos is at JB-HiFi with \$120 to spend. He wants to buy a wireless headset for \$32.87, a power bank for \$48.19 and a mouse for \$37.51. How much change should Carlos expect?

11. Macca had a drum of cable that was 100.50 metres, which he cut into three pieces. One of the pieces was 22.34 metres and another was 58.89 metres. What was the length of the final cut?

12. Annora jogs to work and Surendra drives to work. Surendra's average driving speed is 10 times faster than Annora's average jogging pace. Annora's average pace is 10.24 km/h. What is Surendra's average driving speed in km/h?

13. André's car has a total fuel capacity of 45.5 litres. When Andre goes to fill up his tank the price of each litre of petrol is $2.05. Assuming that his tank is completely empty, how much will it cost André to fill the tank? Provide your final answer to the nearest cent.

14. The Melbourne Cricket Ground (MCG) is 172.8 metres long. Max calculates that he can kick the ball six times in order for the ball to traverse the total length of the MCG. If all of his kicks have the same length, what is the length of one of Max's kicks?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Nazar, Ari, and Jola went to primary school together and now all study at the same secondary school. Nazar lives the closest to the secondary school so he walks. He often changes the routes he walks along. Ari and Jola live further away so they travel by bus and bike respectively. Nazar records the distance walked and presents it in the table below.

	Distance walked to school (km)	**Distance walked from school (km)**
Monday	2.79	3.135
Tuesday	3.54	1.5
Wednesday	1.23	1.23
Thursday	3.65	1.7
Friday	2.29	2.87

a. On Monday, how far did Nazar walk in total?

b. How long is Ari's bus journey to school if it is 10 times longer than the total distance Nazar walked on Thursday?

c. On Friday, Jola's bike journey to school was double the total distance walked by Nazar on Tuesday. She took a shortcut home which made the return journey 1.5 km less than her ride to school. How far did Jola ride on Friday?

d. The average distance walked is calculated by using the formula $\frac{\text{total distance walked}}{\text{number of walks}}$.
What was the average distance of Nazar's walk home?

e. Nazar, Ari and Jola are frustrated by the number of their friends that travel in a car as it has a negative impact on traffic and the environment. Propose a strategy to encourage more of their school friends to not travel by car to school.

16. Using the calculation $4.67 \times 12.23 = 57.1141$:

a. record your answer to 46.7×12.23.

b. record your answer to 4.67×1223.

c. Compare and contrast your answer from part **a** and **b** to explain the effect of multiplying one of the numbers in a decimal multiplication by a power of 10.

Extra spicy

17. Sid's square garden box has a total volume of 1.125 metres cubed, where volume = length × width × height. The length is 1.5 metres. Calculate the width and the height of Sid's garden box.

18. 1425.36×10^3

A. $-1\,425\,360.0$ **B.** -0.01425360 **C.** 1.42536 **D.** $142\,536$ **E.** $1\,425\,360$

19. $78.9 \div 0.5 + 56.55 \div 0.04$

A. -1255.95 **B.** 16.425 **C.** 299.175 **D.** 5812.8 **E.** 1571.55

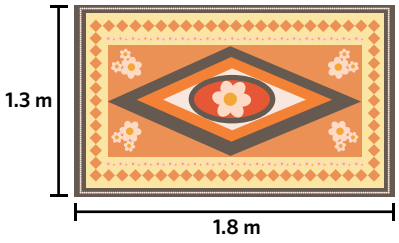
20. Oliver and Jay each ride a total of 15.4 km. Oliver's average pace is 18.9 km/h and Jay's average pace is 21.3 km/h. How much longer does it take Oliver to complete the ride? Express your answer to the nearest minute.

Remember this?

21. The diagram shows the length and width of a rug.

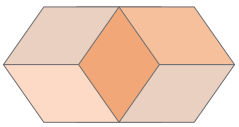
What is the area of the rug?

A. 2.34 square centimetres
B. 2.34 square metres
C. 3.1 square metres
D. 6.2 square metres
E. Cannot be calculated



22. Fred has 38 of the identical shape tiles and is using them to make steps on a pathway. Each step needs 5 tiles. How many **complete** steps can Fred make?

A. 6
B. 7
C. 7.6
D. 8
E. 40



23. Joshua places one tissue box on top of another tissue box. The total height of the two tissue boxes is 21 centimetres. One box is two times taller than the other.

What is the height of the taller tissue box?

A. 7 cm **B.** 21 cm **C.** 14 cm **D.** 2 cm **E.** 42 cm

Back to contents

2F Terminating and recurring decimals

Terminating and non-terminating decimals are two ways to classify numbers. Decimals that continue in a pattern forever are known as recurring decimals. Decimals that stop, or terminate, after a certain number of digits are known as terminating decimals.

LEARNING INTENTIONS

Students will be able to:

- identify terminating, non-terminating decimals, rational and irrational numbers
- write recurring decimals using mathematical notation (dots and dashes)
- convert a fraction to a terminating or recurring decimal with and without digital technology.

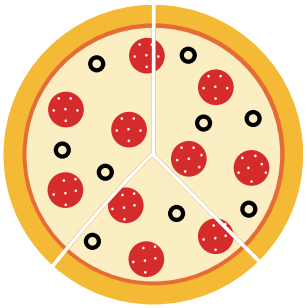
KEY TERMS AND DEFINITIONS

A **finite** amount can be counted, measured or given a value.

An **infinite** amount cannot be counted, measured or given a value.

A **ratio** is a set of numbers in the form $a : b$ or $\frac{a}{b}$ that expresses the relationship between two quantities or sizes.

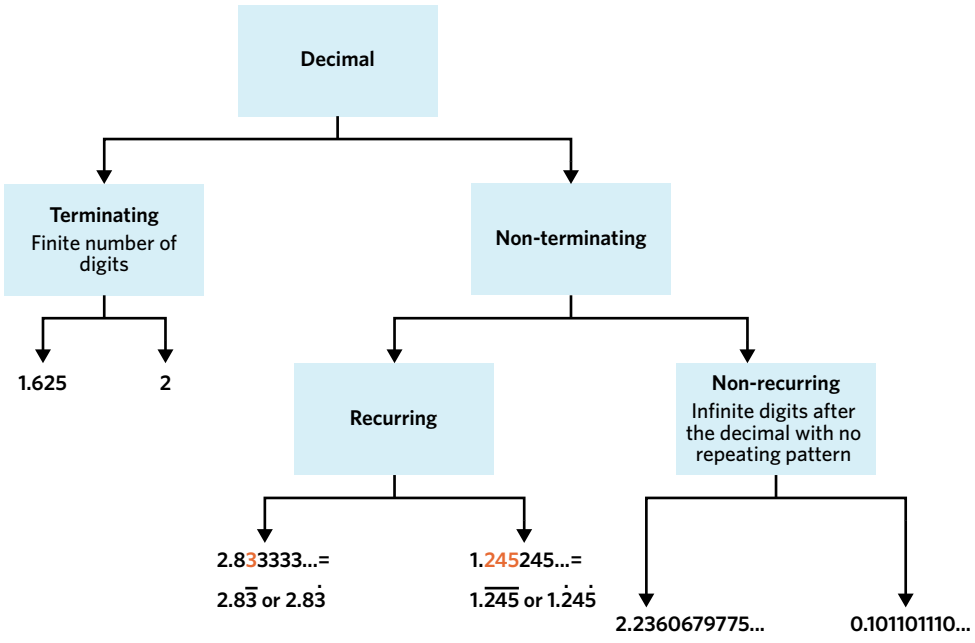
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Knowing whether a number is terminating or non-terminating can help determine if something can be divided into equal groups. For example, when dividing a regular circular pizza into thirds, since $\frac{1}{3}$ is a recurring decimal, each person will not get the exact same amount.

Key ideas

1. Numbers can be classified as terminating, non-terminating and recurring decimals. Repeating patterns of digits are indicated using dots and dashes



Continues →

2. A rational number is a number that can be expressed as a ratio of two integers, such as a fraction, whereas an irrational number cannot.

Rational – terminating and recurring decimals	Irrational – non-recurring decimals
$\frac{3}{4} = 0.75$ $\frac{1}{3} = 0.3333333... = 0.\dot{3}$ $\sqrt{4} = \frac{2}{1} = 2$	$\sqrt{5} = 2.2360679775...$ $\pi = 3.14159265358979...$

Worked example 1

Writing fractions as terminating and recurring decimals

Convert each fraction to a decimal without a calculator.

a. $\frac{1}{8}$ WE1a

Working

$\frac{1}{8} = 1 \div 8$

$$8\overline{)1^{1}.0^{2}2^{4}0^{4}5\,0}$$

$\frac{1}{8} = 0.125$

Thinking

Step 1: Write the fraction as division.

Step 2: Complete the division using the short division algorithm.

Step 3: Write the answer.

b. $\frac{2}{11}$ WE1b

Working

$\frac{2}{11} = 2 \div 11$

$$11\overline{)2^{2}.0^{9}0^{2}2^{9}0^{2}2^{9}0^{2}2\,0}$$

$\frac{2}{11} = 0.\dot{1}\dot{8}$ or $0.\overline{18}$

Thinking

Step 1: Write the fraction as division.

Step 2: Complete the division using the short division algorithm.

Step 3: Write the answer using dot or dash notation.

Student practice

Convert each fraction to a decimal.

a. $\frac{5}{16}$ **b.** $\frac{7}{12}$

Worked example 2

Identifying rational and irrational numbers

Use a calculator or otherwise to determine whether each value is rational or irrational.

a. $\frac{412}{999}$ WE2a

Working

$\frac{412}{999}$ is a rational number.

Thinking

State whether the number is rational or irrational. Fractions with integer denominators and numerators are rational numbers.

Continues →

b. $\sqrt{26}$ WE2b

Working

$\sqrt{26} = 5.0990195...$

$\sqrt{26}$ is an irrational number.

Thinking

Step 1: Use a calculator to write down the decimal to 7 decimal places.

Step 2: State whether the number is rational or irrational. Non-recurring decimals are irrational numbers.

Student practice

Use a calculator or otherwise to determine whether each value is rational or irrational.

a. $\frac{206}{495}$ **b.** $\sqrt{49}$

2F Questions

Understanding worksheet

1. Tick the box to state whether each number is a terminating, recurring or non-recurring decimal.

Example

	Terminating	Recurring	Non-recurring
0.142142142...	☐	☑	☐

		Terminating	Recurring	Non-recurring
a.	0.277777...	☐	☐	☐
b.	6.40312436...	☐	☐	☐
c.	0.234234234...	☐	☐	☐
d.	1.625	☐	☐	☐

2. Identify the repeating pattern of each number and write the number using dot and dash notation.

Example

	Dot notation	Dash notation
4.561561561...	$4.\dot{5}6\dot{1}$	$4.\overline{561}$

		Dot notation	Dash notation
a.	2.666666...		
b.	78.41254125...		
c.	0.13257132571...		
d.	0.2307923079...		

3. Fill in the blanks by using the words provided.

fini e | irrational | non-recurring | infini e

Decimals can be classified as terminating or non-terminating. Terminating decimals have a _____ number of digits, whereas non-terminating numbers have an _____ number of digits. Recurring and non-recurring decimals are types of non-terminating decimals. _____ decimals are also known as _____ numbers.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Use a calculator to write the following decimals using dot or dash notation.

a. $\frac{4}{3}$ **b.** $\frac{5}{9}$ **c.** $\frac{8}{11}$ **d.** $\frac{13}{33}$

e. $\frac{11}{36}$ **f.** $\frac{52}{99}$ **g.** $\frac{5}{7}$ **h.** $\frac{21}{13}$

---

5. Convert each fraction to a decimal without a calculator. WE1

a. $\frac{7}{10}$ **b.** $\frac{1}{9}$ **c.** $\frac{3}{8}$ **d.** $\frac{11}{6}$

e. $\frac{11}{25}$ **f.** $\frac{17}{12}$ **g.** $\frac{13}{18}$ **h.** $\frac{8}{7}$

---

6. Use a calculator or otherwise to determine whether each value is rational or irrational. WE2

a. $\frac{4}{9}$ **b.** $\sqrt{2}$ **c.** $\frac{25}{3}$ **d.** $\sqrt{64}$

e. $\frac{199}{333}$ **f.** $\sqrt{7} \div 4$ **g.** $\frac{154}{125}$ **h.** $\sqrt{7.41}$

---

7. Select the option that shows 4.2076420764207... using dash notation.

A. $0.\overline{20764}$ **B.** $4.\overline{2}$ **C.** $4.\overline{207}$ **D.** $4.\overline{20764}$ **E.** $4.\overline{76420}$

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Convert $\frac{169}{333}$ to a decimal.

Student A

$\frac{169}{333} = 0.50750750...$

$= 0.\overline{50}$

Student B

$\frac{169}{333} = 0.50750750...$

$= 0.\overline{507}$

b. Determine whether $\sqrt{50}$ is rational or irrational.

Student A

$\sqrt{50} = 7.07106781187$

$\sqrt{50}$ is a terminating decimal, so it is rational.

Student B

50 is not a perfect square, so $\sqrt{50}$ is a non-recurring decimal. This means it is irrational.

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. Rob is making jelly cups for his daughter's birthday. He has 4 litres of jelly mix that he will divide equally into 25 cups. In litres, how much jelly will each cup contain?

10. Sahir is running a half-marathon, which is 21 km long. He ran $\frac{4}{9}$ of the marathon along a beach. How many kilometres, written as a decimal, did Sahir run along the beach?

11. There are 99 students in the Year 8 class at McCrawley Grammar. 47 students voted in the school election. What portion of the Year 8 students did not vote in the election? Express your answer as a decimal.

12. Rowena is packaging 200 colouring pencils into bundles to sell on her online store. Explain whether Rowena should divide the pencils into groups of four or six so that every bundle has the exact same number of pencils.

13. Warren just started a new book. He read $\frac{13}{20}$ of it in one night and then read half of the rest of the book the next night. A week later, he read a third of the pages he had not yet read. Written as a decimal, how much of the book does he have left to read?

Reasoning

Question working paths

Mild 14 (a,b,d)	Medium 14 (a,b,d), 16 (a,b)	Spicy All

14. Niamh is landscaping her backyard and wants to include a large square veggie patch to grow vegetables. She is planning for the veggie patch to have an area of 74 m^2.

a. What is the side length of the planned veggie patch? Write to 2 decimal places.

b. Niamh wants to plant 24 vegetables. How much area will each vegetable plant have, correct to two decimal places?

c. Explain why it is not possible for Niamh to have a square veggie patch with an exact area of 74 m^2.

d. Suggest a possible side length for Niamh's veggie patch and state the number of vegetable plants that she will be able to plant. Your suggestion must meet the following criteria:

- Each vegetable plant must have at least an area of 3 m^2 to grow in.
- The side length of the veggie patch is physically possible.
- At least 24 vegetable plants need to be planted.
- The veggie patch is a square.

15. The following set of fractions contains both terminating and recurring decimals.

$\frac{4}{5}, \frac{3}{10}, \frac{7}{8}, \frac{1}{6}, \frac{5}{9}, \frac{11}{12}, \frac{7}{15}$

a. Which of the fractions represent terminating decimals?

b. Which of the fractions represent recurring decimals?

c. Write the prime factorisation of the denominators of the fractions in parts **a** and **b**. Compare them and explain how you can determine whether a fraction represents a recurring or terminating decimal.

Extra spicy

16. The time 15 hours, 3 minutes and 20 seconds can be written in hours as $15 + \frac{3}{60} + \frac{20}{3600}$.

Write the time 7 hours, 23 minutes and 41 seconds as a decimal in hours.

17. Without using a calculator, determine which of the following lists is in ascending order.

A. $\left(\sqrt{0.4}\right)^2, \frac{7}{11}, 0.36, \sqrt{4}$

B. $0.36, \frac{7}{11}, (\sqrt{0.4})^2, \sqrt{4}$

C. $0.36, (\sqrt{0.4})^2, \frac{7}{11}, \sqrt{4}$

D. $\sqrt{4}, \frac{7}{11}, (\sqrt{0.4})^2, 0.36$

E. $\frac{7}{11}, \sqrt{4}, 0.36, (\sqrt{0.4})^2$

18. Which of the following statements is always true?

A. recurring + recurring = recurring

B. recurring + terminating = recurring

C. $(\text{recurring})^2 = \text{terminating}$

D. $\sqrt{\text{terminating}} = \text{terminating}$

E. recurring − recurring = recurring

19. The width of A0 paper is 0.841 m. Two possible lengths of A0 paper can be found using either of the following facts:

- The paper size A0 has an area 1 m^2.
- The length is $\sqrt{2}$ times the width.

Use both facts to calculate the two possible lengths of the A0 paper. What is the difference between the two lengths in millimetres, correct to 3 decimal places?

Remember this?

20. Consider the number sentence $(3 + 5) \times (6 - 1) + 2 = \star$.

What number does $\star$ represent?

A. 34 **B.** 36 **C.** 42 **D.** 49 **E.** 58

21. Qantas paid \$250.25 million for four aeroplanes. The price did not include the delivery cost of \$0.35 million per plane or the discount of \$2.5 million per plane.

What was the total cost of purchasing one aeroplane?

A. \$60.41 million

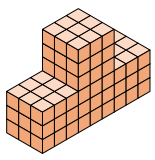
B. \$62.03 million

C. \$63.10 million

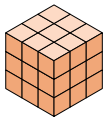
D. \$248.10 million

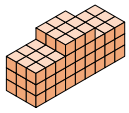
E. \$252.40 million

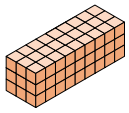
22. Yindi made the following 3D object using 108 wooden blocks.

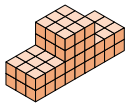


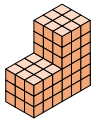
Which of these has volume that is one third less than Yindi's object?

A. 

B. 

C. 

D. 

E. 

Back to contents

2G Rounding and estimating with decimals

Rounding is commonly used because some decimal numbers have lots of decimal places and it is not practical to work with these numbers. Estimates are used as a faster way to calculate a 'rough idea' of a quantity rather than calculate an exact quantity.

LEARNING INTENTIONS

Students will be able to:

- round numbers to any decimal place
- use rounded numbers to make estimations
- use rounding and estimation in real-world applications.

KEY TERMS AND DEFINITIONS

Approximately equal to is represented in calculations using the ≈ sign.

Estimates are approximate values that are close to the actual value.

Lead digit rounding rounds the number based on the first digit and replaces all other digits with zeros.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: BMJ/Shutterstock.com

Rounding numbers and other forms of estimation are used to measure animal population sizes and other key indicators. For example, a biologist may need to estimate the number of penguins in a colony.

Key ideas

1. When rounding to a specific place value, consider the first digit to the right of the digit being rounded. This is the critical digit.

Round to 4 decimal places

Critical digit

$78.39845 \approx 78.3985$

The digit is increased as the critical digit is 5 or more

Round to 3 decimal places

Critical digit

$78.39846 \approx 78.398$

The digit is unchanged as the critical digit is 4 or less

2. Lead digit rounding is used to make estimates.

Rounded numbers

$18\,564.45 \div 9.765 \approx 20\,000 \div 10 = 2000$

Critical digits

Worked example 1

Rounding decimals

Round the following numbers to the specified decimal places.

a. 4.364 (2) WE1a

Working

4.36|4

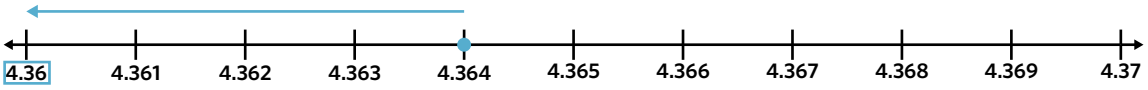
4.364 ≈ 4.36

Thinking

Step 1: Draw a line between the digit being rounded and the critical digit.

Step 2: The critical digit is less than 5, so the digit being rounded stays the same. Write the number with two decimal places.

Visual support



b. 11.3799 (3) WE1b

Working

11.379|9

11.3799 ≈ 11.380

Thinking

Step 1: Draw a line between the digit being rounded and the critical digit.

Step 2: The critical digit is more than 5, so the digit being rounded increases. As the digit being rounded is 9, increase the next digit to the left. Write the number with three decimal places, using zero as a place holder.

Student practice

Round the following numbers to the specified decimal places.

a. 64.34 (1) **b.** 7.896 (2)

Worked example 2

Estimating with lead digit rounding

Estimate the following calculations by rounding each number to the lead digit.

a. 581 × 1.763 WE2a

Working

581 × 1.763 ≈ 600 × 2

= 1200

Thinking

Step 1: Round all numbers in calculation to the lead digit.

Step 2: Complete the calculation.

Continues →

b. $313 \div 21$ WE2b

Working

$313 \div 21 \approx 300 \div 20$

$= 15$

Thinking

Step 1: Round all numbers in calculation to the lead digit.

Step 2: Complete the calculation.

Student practice

Estimate the following calculations by rounding each number to the lead digit.

a. 722×6.39 **b.** $566 \div 57$

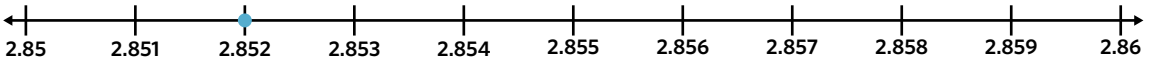
2G Questions

Understanding worksheet

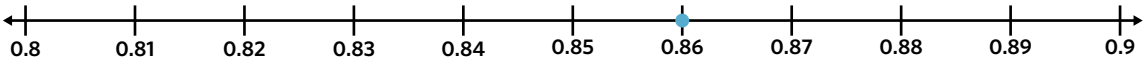
1. Use the number lines to round the following numbers.

Example

2.852 rounded to 2 decimal places is 2.85

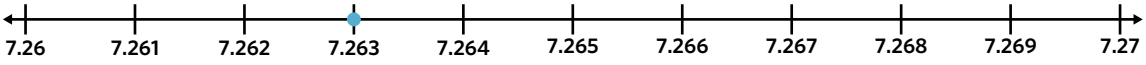


a. 0.86 rounded to the nearest 1 decimal place is

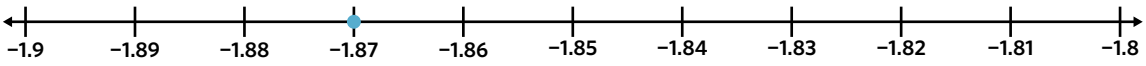


b. 2.5 rounded to a whole number is

c. 7.263 rounded to 2 decimal places is



d. −1.87 rounded to 1 decimal place is



2. Round both numbers to the leading digit to show the approximation.

Example

$314.7 \times 18.38 \approx$ 300 $\times$ 20 $= 6000$

a. $53 - 29 \approx$ ☐ $-$ ☐ $= 20$

b. $762 \times 23 \approx$ ☐ $\times$ ☐ $= 16\,000$

c. $11.63 \times 67.4 \approx$ ☐ $\times$ ☐ $= 700$

d. $\frac{2823}{613} \approx \frac{\square}{\square} = 5$

3. Fill in the blanks by using the words provided.

critical | lead | approximately | equal | estimate

If the ☐ digit is five or greater we increase the digit being rounded. If it is four or less the digit being rounded is unchanged. When trying to simplify a difficult calculation we can use ☐ digit rounding. This approximate calculation will give an ☐ of the answer. When simplifying a calculation it is important to make sure the solution is shown to not be exact by using the ☐ sign.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Round the following numbers to the specified decimal places. WE1

a. 0.231 (2) **b.** 3.8768 (2) **c.** 6.30045 (3) **d.** 1.72688712 (4)

e. 4.51098 (3) **f.** 14.51098 (4) **g.** 23.4999 (3) **h.** 19.9999 (2)

5. Estimate the following calculations by rounding each number to the lead digit. WE2

a. $208 - 15$ **b.** $918 + 78 - 31$ **c.** $78 \div 23$ **d.** 612×43

e. $67 \times (31 - 15)$ **f.** $584 \div 76$ **g.** $\frac{23}{77}$ **h.** 38×0.042

6. Estimate the following calculations by rounding each number to the digit after the lead digit.

a. $1884 + 513$ **b.** $208 - 15$ **c.** $184 - 615$ **d.** $235 \div 30$

e. $(164 + 22) \div 91$ **f.** $178\,057 \times 19.601$ **g.** $39\,458 - 15 \times 495$ **h.** $178\,057 \div 8.9701$

7. State whether the following estimates will be less than or greater than the exact calculation.

a. $156 + 85 \approx 200 + 100$

b. $1518 - 1089 \approx 2000 - 1000$

c. $65 \div 13 \approx 70 \div 10$

d. $518 \times 411 \approx 500 \times 400$

e. $16.7 \times 0.028 \approx 17 \times 0.03$

f. $629 + 118 - 72 \approx 600 + 100 - 70$

g. $523 \div 45 \approx 500 \div 50$

h. $(214 + 11 \times 9) \div 131 \approx (200 + 10 \times 10) \div 100$

---

8. Convert the following fractions to decimals and then round them to the specified decimal places.

a. $\frac{1}{8}$ (2) **b.** $\frac{1}{3}$ (3) **c.** $\frac{2}{6}$ (4) **d.** $\frac{2}{7}$ (3)

e. $\frac{5}{12}$ (3) **f.** $\frac{8}{7}$ (4) **g.** $\frac{13}{9}$ (4) **h.** $\frac{9}{13}$ (4)

---

9. Round 72.29993 to 3 decimal places.

A. 72.2 **B.** 72.2910 **C.** 72.299 **D.** 72.2999 **E.** 72.300

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Round 0.5496 to 3 decimal places.

Student A

$0.549|6 \approx 0.550$

Student B

$0.549|6 \approx 0.5410 = 0.541$

b. Use lead digit rounding to estimate $237 \div 41$.

Student A

$237 \div 41 \approx 240 \div 40 = 6$

Student B

$237 \div 41 \approx 200 \div 40 = 5$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. According to her GPS, Tracey needs to drive west for 28.74 km to arrive at her desired destination. If her speedometer says she has already driven 17.6 km, how much further does she need to drive, if she rounded the remaining distance to the nearest km?

12. Aaron is standing next to a stack of 1.9 cm thick books and is as tall as 87 of them. How tall is Aaron, if he rounds his height to the nearest cm?

13. 29 of Keenan's students have booked a parent teacher interview with him. Keenan has 5 hours to complete the interviews. If Keenan uses lead digit rounding, how much time does he approximately have for each student interview?

14. Radio signals and other electromagnetic waves travel at 299 792.458 km per second. The distance from Earth to the Moon is 384 399 km. Use lead digit rounding to estimate the time it takes for electromagnetic waves to travel from the Earth to the Moon. Round your answer to two decimal places.

15. Esther and Katie had a very close finish in the 70 m sprint. Their times were both rounded to the nearest hundredth of a second. They both finished in 9.18 seconds, but the photo finish showed that Esther won. What is the largest possible time difference between Esther and Katie's race times, in thousandths of a second?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Alistar tried to calculate the attendance for the Anzac day game at the Melbourne Cricket Ground (MCG) which has a capacity of 100 024 spectators. Alistar rounded the capacity of the MCG using lead digit rounding to complete all estimates.

Image: Neale Cousland/Shutterstock.com

a. The Shane Warne stand holds half of the stadium's seats and appears to have $\frac{9}{10}$ of its seats occupied. Estimate how many people are in the Shane Warne stand, to the nearest thousand.

b. The Members Reserve stand has $\frac{1}{5}$ of the stadium's seats and appears to have $\frac{3}{4}$ of its seats occupied. Estimate how many people are in the Members Reserve stand, to the nearest thousand.

c. The Ponsford stand holds $\frac{1}{5}$ of the stadium's seats and appears to have $\frac{8}{10}$ of its seats occupied. Estimate how many people are in the Ponsford stand, to the nearest thousand.

d. The remaining seats are in the Northern stand which appears to have $\frac{95}{100}$ of its seats occupied. Estimate how many people are attending the game in total and round your answer to the nearest thousand.

e. Other than looking at how full each stand appears, propose another strategy to estimate how many people attended the game.

17. The first thirteen decimal places of π are 3.141592653589.

According to a scientific calculator the circumference of the Earth is:

$\pi \times 12\,742 \text{ km} = 40\,030.17359 \text{ km}$

a. Round π to 4 decimal places to estimate the circumference of the Earth. Round your answer to the nearest km.

b. Round π to 3 decimal places to estimate the circumference of the Earth. Round your answer to the nearest km.

c. Compare the estimates in parts **a** and **b** to the exact result and explain how rounding to different decimal places can affect the accuracy of an estimation.

Image: gladder/Shutterstock.com

Extra spicy

18. Which of the following calculations can be estimated by 30×60?

A. 24×56 **B.** 31×53 **C.** 26×51 **D.** 34×56 **E.** 35×55

19. Reece selected two numbers from the following list:

44.37 44.44 44.48 44.48 44.53
44.55 44.63 44.67 44.71

When he rounded the numbers to 1 decimal place they were the same. When he rounded them to the digit following the lead digit they were not equal.

Which two numbers did he choose?

20. Albert was asked to list the value of every number from 1 to 100 divided by 7 rounded to 3 decimal places. How many of the numbers will have the decimals □.143?

A. 6 **B.** 13 **C.** 15 **D.** 85 **E.** 94

21. The exchange rate between the Australian dollar and the Euro is 1 to 0.67 respectively. Estimate how many Australian dollars can be exchanged for 50 Euros using lead digit rounding, to the nearest dollar.

Remember this?

22. Which of these is not equal to 48×30

A. $48 \times 10 \times 3$
B. 480×3
C. $40 \times 30 + 8 \times 30$
D. $40 + 8 \times 30$
E. $(4 \times 10 + 8) \times 30$

23. Drago uses the same rule to change each ● into a ▲.

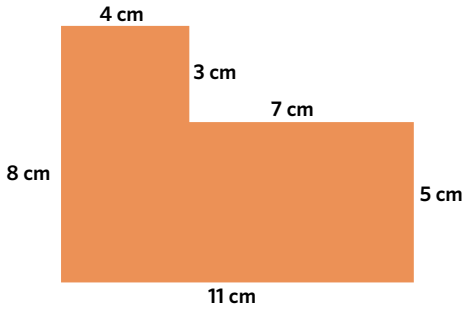
●	▲
2	4
3	7
7	19

Select the option that represents the rule.

A. ▲ = ● $\times 1 + 2$
B. ▲ = ● $\times 2 + 5$
C. ▲ = ● $\times 3 - 2$
D. ▲ = ● $\times 4 - 5$
E. ▲ = ● $\times 2 + 1$

24. Which of the following expressions could be used to calculate the area of the shape in cm squared.

A. $(8 \times 4) + (11 \times 5)$
B. $(11 \times 8) - (7 \times 5)$
C. $(8 \times 3) + (7 \times 3)$
D. $(11 \times 8) - (7 \times 3)$
E. $(5 \times 11) + (3 \times 7)$



Chapter 2 extended application

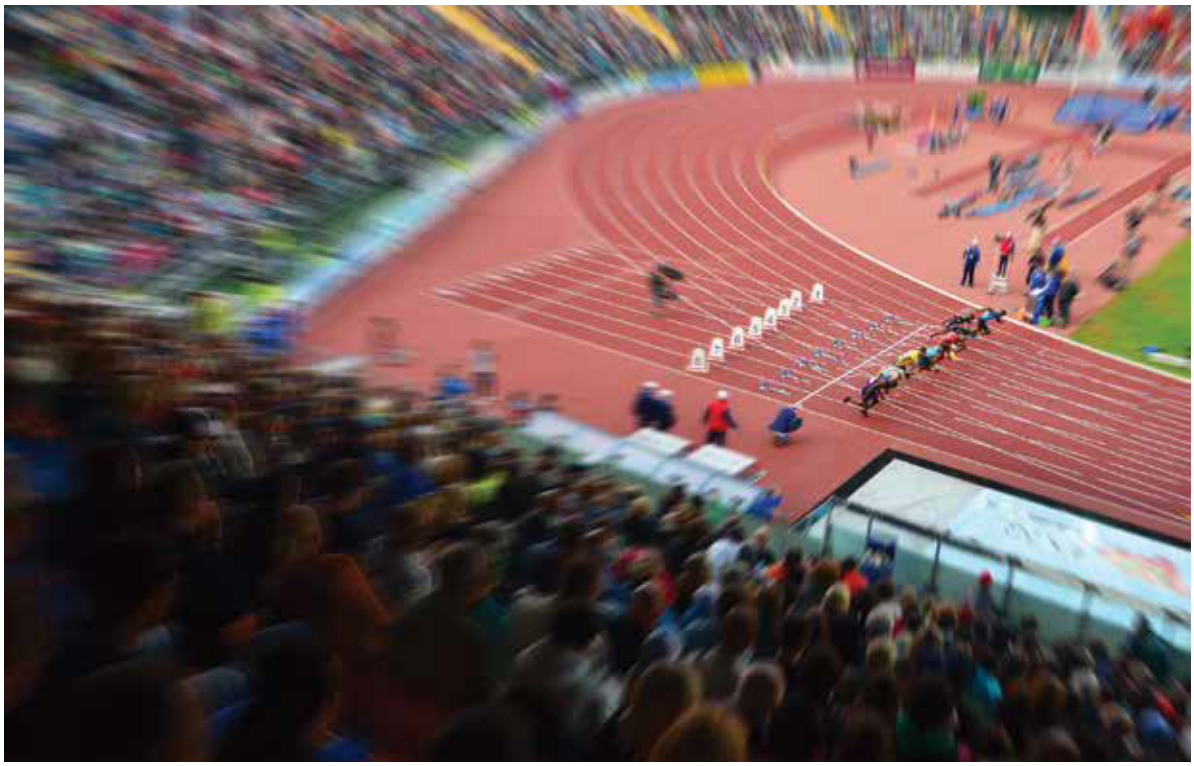


Image: kovop58/Shutterstock.com

1. The following table shows the five fastest recorded 100 m times. Usain Bolt, who is considered one of the greatest sprinters of all time, holds the world record for the fastest 100 m time.

Average speed (m/sec)	Time	Athlete	Country	Rank	Venue	Date
10.43841336	9.58	Usain BOLT	JAM	1	Berlin (GER)	16-Aug-09
10.31991744	9.69	Tyson GAY	USA	4	Shanghai (CHN)	20-Sep-09
10.31991744	9.69	Yohan BLAKE	JAM	5	Lausanne (SUI)	23-Aug-12
10.28806584	9.72	Asafa POWELL	JAM	8	Lausanne (SUI)	2-Sep-08
10.26694045	9.74	Justin GATLIN	USA	10	Doha (QAT)	15-May-15

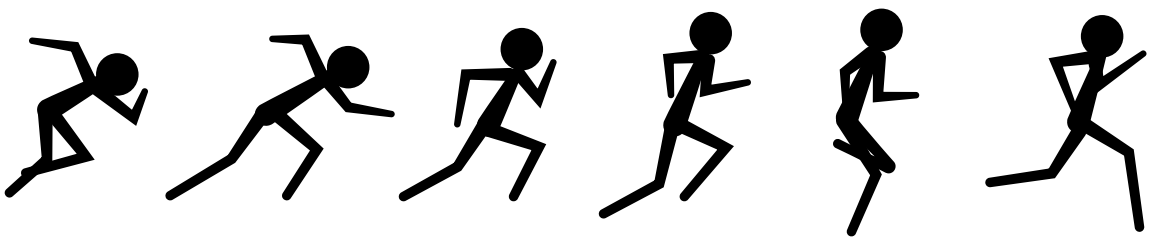
a. State how far each athlete runs in a tenth of a second. Round your answers to two decimal places.

b. Average speed × Usain Bolt's record time = distance traveled compared to Bolt. Using a calculator and the formula, determine the distance, in m, traveled by each athlete in the time that it took Usain Bolt to complete his world record. Round the answer to 2 decimal places.

c. Convert the distances from part **b** to cm (hundredth of a m). Calculate the distances, in cm, by which Usain Bolt would have beaten each athlete if they were all running in the same 100 m race.

d. Super slow motion cameras take 1000 pictures per second. Rounded to the nearest mm, how far would Usain Bolt have traveled in 0.001 seconds?

e. In 1922 the world record for the 100 m sprint was 12.8 seconds. Explain the possible reasons why a human today is able to run so much faster than a human 100 years ago.



2. Gold is an important part of computer circuits and chip boards due to its high electrical conductivity and resistance to corrosion (rusting). The gold from old discarded computers can be recycled. Some computer shops offer trade-ins for old computers as they can recycle a computer's old parts.

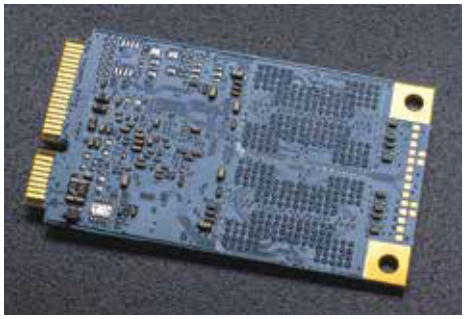


Image: Troggt/Shutterstock.com

Image: Krzysztof Bubel/Shutterstock.com

a. Half of a standard circuit board has a third of its surface covered by metals. The other half has a quarter of its surface covered by metal. What fraction of a standard circuit board is covered by metals?

b. If a circuit board was covered completely with metal, all the metal alone would weigh $2\frac{2}{11}$ grams. How much does all the metal on a standard circuit board, like the one in part **a**, weigh?

c. Calculate the weight of copper, silver and gold on a standard circuit board, in fractions of a gram, if $\frac{1}{6}$ of the metal is gold, $\frac{1}{3}$ is silver and $\frac{1}{2}$ is copper.

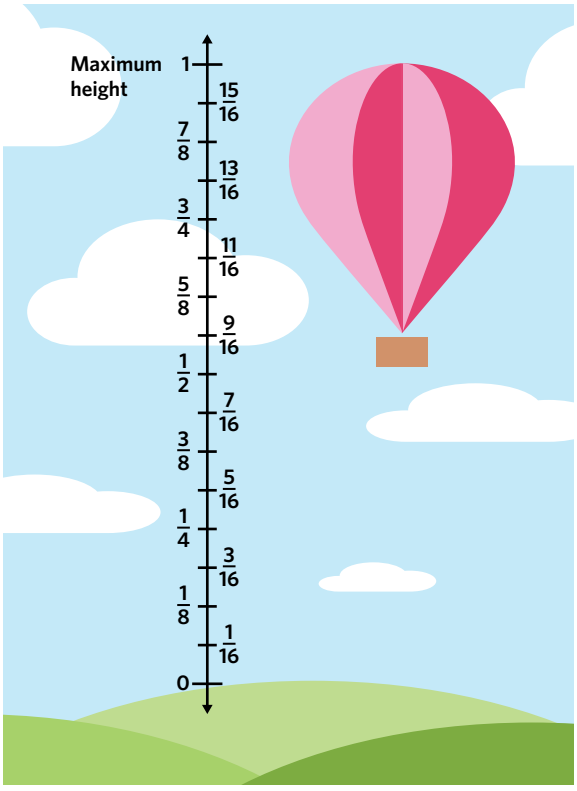
d. How many standard circuit boards would need to be collected to accumulate five grams of recycled gold?

e. Gold sells for \$59.72 per gram, silver sells for \$0.71 per gram and copper sells for \$0.01 per gram. Using a calculator, determine the total value of all the metals gathered from enough circuit boards for 5 g of gold.

f. Many computer shops offer a trade in service for customers to recycle their old computers. What are other strategies to encourage people to recycle old computer products?

3. Marcel loves to go hot air ballooning every Saturday morning. Each Saturday, the maximum altitude that his balloon can ascend to changes based on the atmospheric conditions and how many sandbags he flies with.

The sandbags on his balloon are used to control the altitude he can float to. If he flies without any sandbags he is able to reach the balloon's maximum altitude. The weight of each sandbag decreases his altitude by $\frac{3}{16}$ of the balloon's maximum.



a. If Marcel was carrying one sandbag, what fraction of the maximum altitude would he be able to float to?

b. How many sandbags does Marcel need to have in his balloon to be sitting on the ground?

c. How many sandbags would he need to be floating at a quarter of the maximum altitude?

d. How many sandbags would he have if he removed another $2\frac{1}{4}$ bags from when he was floating at a quarter of the maximum altitude?

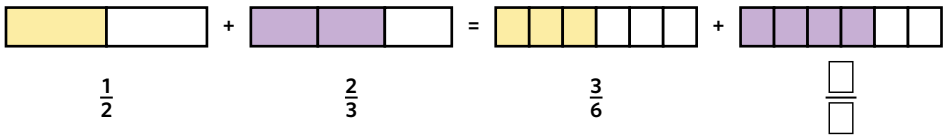
e. Use the number of bags you calculated that he had left in part **d** to determine the altitude he would be floating at.

f. Where in the world would you like to go hot air ballooning and spend a few hours floating above with a bird's eye view?

Chapter 2 review

Multiple choice

1. Fill in the missing boxes. 2A, 2B

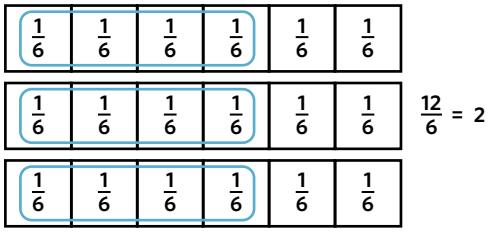


A. $\frac{2}{6}$ **B.** $\frac{4}{6}$ **C.** $\frac{1}{3}$ **D.** $\frac{2}{3}$ **E.** $\frac{5}{6}$

---

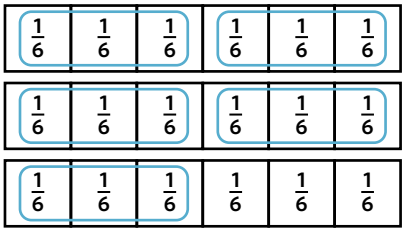
2. Three groups of $\frac{5}{6}$ is: 2C

A.



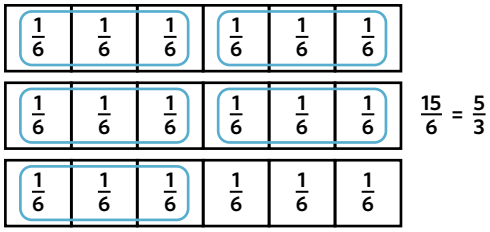
$\frac{12}{6} = 2$

B.



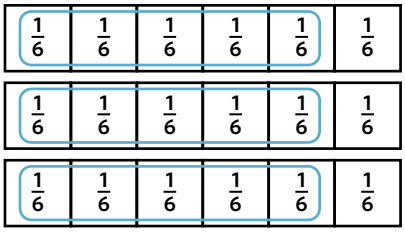
$\frac{15}{6} = \frac{5}{2}$

C.



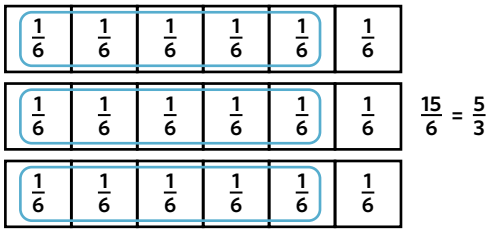
$\frac{15}{6} = \frac{5}{3}$

D.



$\frac{15}{6} = \frac{5}{2}$

E.

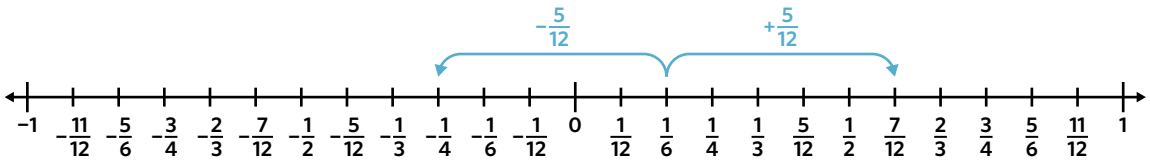


$\frac{15}{6} = \frac{5}{3}$

---

3. Use the number line to choose the correct direction of the arrow to complete the following calculation: 2D

$\frac{1}{6} - \left(-\frac{5}{12}\right)$



A. Left, because subtracting a negative fraction is the same as subtracting a positive fraction.

B. Left, because subtracting a negative fraction is the same as adding a positive fraction.

C. Right, because subtracting a negative fraction is the same as subtracting a positive fraction.

D. Right, because subtracting a negative fraction is the same as adding a positive fraction.

E. Both arrows produce the same answer.

4. A student is using the vertical algorithm to solve 76.2×0.41. Has the student done this correctly and, if not, what mistake have they made? 2E

$76.2 \times 0.41 = 3.712$

A. The student has done this correctly.

B. The student has done this incorrectly, since the student ignored the decimal point during multiplication.

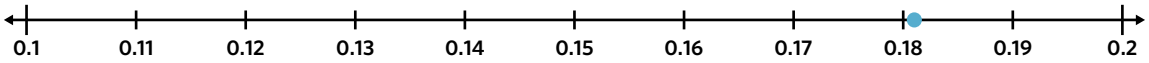
C. The student has done this incorrectly, since they have not used 0 as a placeholder.

D. The student has done this incorrectly, since the number of decimal places in the final answer is incorrect.

E. The student has done this incorrectly, since whole numbers and decimals were dealt with separately.

$$\begin{array}{r} {\scriptstyle +2} \\ 762 \\ 41 \\ \hline {\scriptstyle +1} \\ 762 \\ \times\ 3048 \\ \hline 3712 \end{array}$$

5. Use the following number line to round $0.\overline{18}$ to one decimal place. 2F, 2G



A. 0.1 **B.** 0.18 **C.** 0.19 **D.** 0.2 **E.** 1

Fluency

6. Convert each of the following to the described form. 2A

a. $2\frac{1}{10}$ (improper fraction)

b. $5\frac{2}{3}$ (improper fraction)

c. $\frac{15}{2}$ (mixed number)

d. $\frac{95}{12}$ (mixed number)

7. Calculate the equivalent fractions by finding the missing values. 2A

a. $\frac{1}{5} = \frac{\square}{10} = \frac{\square}{50}$

b. $\frac{6}{18} = \frac{\square}{36} = \frac{2}{\square}$

c. $2\frac{3}{4} = 2\frac{36}{\square} = \frac{\square}{4}$

d. $\frac{54}{\square} = \frac{45}{25} = 1\frac{\square}{5}$

8. Simplify. 2B

a. $\frac{12}{13} - \frac{11}{13}$

b. $\frac{8}{4} + \frac{1}{4}$

c. $\frac{15}{21} - \frac{1}{7}$

d. $\frac{31}{4} + \frac{1}{12}$

9. Evaluate each expression. 2C

a. $\frac{3}{2} \times \frac{4}{9}$

b. $\frac{1}{3} \div \frac{13}{9}$

c. $\frac{18}{11} \times \frac{11}{3} \div 4$

d. $\frac{16}{5} \div \frac{24}{7} \div \frac{3}{5}$

10. Evaluate and simplify. 2C

a. $\left(\frac{1}{5} \div \frac{15}{60}\right) - \frac{2}{12}$

b. $\frac{1}{8} \div \left(\frac{1}{48} + \frac{11}{12} - \frac{1}{10}\right)$

c. $\frac{1}{2} \times \left(\frac{3}{4} + \frac{4}{5} - \frac{6}{7}\right)$

d. $4\frac{2}{5} \div 2\frac{1}{5} \times 3\frac{1}{10}$

11. Evaluate and simplify. 2D

a. $-\frac{1}{5} - \frac{1}{6}$

b. $\frac{8}{3} \times \left(-\frac{1}{6}\right)$

c. $-\frac{7}{8} - \left(-\frac{4}{3} + 1\right)$

d. $\frac{9}{55} - \left(\frac{4}{5} - \left(-\frac{1}{11}\right)\right)$

12. Calculate. 2E

a. $14.1 + 3.8$

b. $23.1 + 1.9$

c. $8.32 - 2.44$

d. $13.1 - 7.3292$

13. Calculate. 2E

a. 10.55×4

b. $410.05 \div 2$

c. 99.1×1.16

d. $82.08 \div 3.42$

14. Convert each fraction to a decimal without a calculator. 2F

a. $\frac{3}{8}$ **b.** $\frac{7}{250}$ **c.** $\frac{5}{3}$ **d.** $\frac{44}{13}$

15. Round the following numbers as specified. 2G

a. 639.17 (1) **b.** 4.99999 (2)

c. 451.21 (round to the lead digit) **d.** 81.11 (round to digit after the lead digit)

16. Estimate the following calculations by rounding each number as specified. 2G

a. $304.2 \div 18.97$ (round to lead digit)

b. $(212.4482 + 104.5119) \times 2.01888$ (round to lead digit)

c. $1.1234 + [(7.340912 \times 7.0001) \div (8.7005 - 6.70102)]$ (round to lead digit)

d. $(12.3112 \times 0.9348) \div (2.555 - 1.01)$ (round to digit after lead digit)

Problem solving

17. Sienna spends \$4 per day on chai lattes. How many chai lattes will she drink to have spent \$24 on chai lattes? 2A

18. Clelia has written a short story for a competition, and is checking the word count at the beginning, middle and end. The word count at the beginning and middle sum to $\frac{4}{5}$ of the whole story. The middle and ending sum to $\frac{2}{3}$ of the whole story. What fraction of the whole story was the middle? 2B

19. A group of friends are comparing their hourly rates at their part-time jobs. John makes $\frac{2}{5}$ of what Sam makes. Sam makes $\frac{7}{6}$ of what Tracey makes. Ishir makes $\frac{4}{5}$ of what Tracey makes. If Ishir makes \$24 per hour, how much does John make per hour? 2C

20. On a particular day, the temperature in the South Pole in Antarctica was $-62\frac{4}{10}$°C and in Mount Buller it was $-1\frac{4}{5}$°C. Find how much warmer Mount Buller was than the South Pole, as a mixed number. 2D

21. At the beginning of last year, one ounce of silver was \$15.65. At the end of last year, one ounce of silver was \$17.90. If Nadia bought 0.4 ounces of silver at the beginning of last year and sold it at the end of last year, what profit did she make? 2E

22. In a laboratory, Ellett is measuring the speed of a snail using very precise equipment. If the distance a snail travels in two minutes is $1.\overline{3}$ m, calculate its speed in metres per minute. Write your answer as a decimal using appropriate notation. 2F

23. A doctor is prescribing some medication for a patient. The dosage depends on the weight of the patient, and is described by the following rule: 2G

$\text{dosage} = 1.105 \times 80.54 - 1.0418 \text{ mg}$

It is important to ensure that a patient has the correct dosage, so after performing the calculation, she performs lead digit rounding to check the reasonableness of her answer. What answer does lead digit rounding obtain?

Reasoning

24. A soft drink company has just released a new soft drink called Berry Blitz.

a. Some of Berry Blitz's nutritional information is provided below. By law, the nutritional information needs to be listed on the packet to one decimal place. Round each value to one decimal place.

	Per 100 mL (to 7 decimal places)	**Per 100 mL (to 1 decimal place)**
Fat (total)	0.1705473 g	
Saturated fat	0.0058101 g	
Carbohydrate (total)	14.2188915 g	
Sugar	14.0611112 g	
Protein	0.0000101 g	
Salt	0.0000001 g	

b. Use these rounded values to estimate the amount of each component in 250 mL.

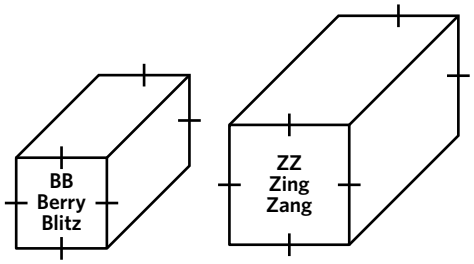
c. Claire introduces her friend to Berry Blitz. She pours out one third of her drink for him. If she also drinks one third of her drink, how much of the drink is left? Provide your final answer as a fraction.

d. Write your answer from part **c** as a recurring decimal.

The company also sells another soft drink called Zing Zang. Customers can purchase multipacks of each drink. These are sold in rectangular prism cartons as shown below.

The area of the square face of Berry Blitz is $\frac{2}{3}$ of the area of the square face of Zing Zang.

The length of Berry Blitz is $\frac{2}{3}$ of the length of Zing Zang.



e. The volume of rectangular prism = area of square face × height. If the volume of the Zing Zang carton is 450 cm^3, calculate the volume of Berry Blitz.

f. Claire's school is looking into having more healthy drink options in the canteen. What strategies could Claire's school implement to improve their students' health?

25. Yvonne is exploring different ways to multiply lots of fractions together.

a. Evaluate $\frac{1}{2} \times \frac{2}{3}$ and $\frac{998}{999} \times \frac{999}{1000}$.

b. Evaluate $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ and $\frac{997}{998} \times \frac{998}{999} \times \frac{999}{1000}$

c. Compare the strategies you used to solve parts **a** and **b** to efficiently evaluate $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{999}{1000}$. Explain the technique you used.

Back to contents

03

Percentages and financial mathematics

Number and Algebra

Research summary

3A Percentages, decimals, and fractions *(Revision)*

3B Expressing as a percentage and percentages of quantities

3C Increasing and decreasing by a percentage

3D Percentage change

3E The unitary method *(Extension)*

Chapter 3 extended application

Chapter 3 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 3 research summary

Percentages and financial mathematic

Big ideas

Multiplicative thinking

This chapter will extend multiplicative thinking about place value, powers of 10, decimals, and fractions to build knowledge of how to work with the language of percentages appropriately.

Scattered throughout the chapter is the use of multiplicative and additive language structures. This chapter will address how to decode some of the confusing multiplicative and additive words when working with percentages. For example increasing by 10% sounds additive, but it has multiplication and addition (first calculate 10% and then add it to the original value), or only multiplication (multiply the original value by 110%).

The language structures of percentages can be ambiguous and suppress an intuitive understanding of percentages (Parker & Leinhardt, 1995); thus, the most effective way to start percentage topics is to explore students' existing knowledge of what percentages mean to them (Reys et al., 2004). For example, students can often be confused when percentages are over 100%. It is impossible to give more than 100% of what you have, but going over 100% of the screen time limit your parents set is possible.

Understanding this big idea is built upon the knowledge of decimals, fractions, and percentages. It is crucial for grasping topics within this chapter, for future lessons such as rates and ratios, and for future big idea topics such as algebraic, statistical and geometric reasoning.

Equi-partioning

This chapter will extend on partitioning fractions and decimals to understand how to find percentages of quantities efficiently.

While using an algorithm and pictures are methods to solve percentage problems, benchmarking or partitioning is one sustainable method to solve percentages (Lembke & Reys, 1994). Starting with familiar benchmarks allows students to develop competence in 10%, and then the unitary method (Burfitt, 2014). For example, students who know half is 12 and 50% can use this knowledge to find 25%.

Understanding this big idea is built upon place value and multiplicative thinking. Therefore, equi-partioning percentages is essential for this chapter and future lessons, such as rates and ratios, and future big idea topics, such as algebraic, statistical, and geometric reasoning.

Proportional reasoning

This chapter will focus on students' ability to identify and apply their extensive multiplication, division, and number sense knowledge to solve problems with varying levels of proportional relationships.

In this chapter, students will connect the relationship of percentages with fractions, decimals, multiples, portions, and proportional differences. Having a solid understanding and connection to decimals and fractions, connecting the part-whole relationships of an object or set of objects with numbers, connecting language they are already familiar with, and connecting real-world experiences can remedy common misconceptions (Parker & Leinhardt, 1995), thus increasing proportional reasoning skills within this chapter.

For example, decimal fractions aid in recognising what the term per cent and the percentage symbol (%) means. Students should understand that percentages and the symbol (%) are a standard ratio where the denominator is 100 and can be substituted for the term hundredths. For example, 37% is 37 per 100 and is the same as $\frac{37}{100}$ or 0.37.

Within this chapter, students will be exposed to readily accessible workable procedures (benchmarks, proportional thinking strategies, formal proportions, visual representations). While some researchers agree that percentages are an extension of identifying fractions, decimals, ratios and proportions and require no new concepts and skills beyond this (Drews et. al., 2005; Killen & Hindhaugh, 2018; Reys et al., 2004) percentages are multifaceted, complex, and lush. Major misconceptions can occur if instruction is oversimplified to 'no new skills required' or to package percentages into an algorithm.

Understanding this big idea is built upon place value and multiplicative thinking. Therefore, it is not only essential for this chapter and future lessons such as rates and ratios but heavily impacts future big ideas in mathematics such as generalising and influencing students' learning journey, beliefs and attitudes toward mathematics.

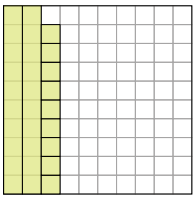
Visual representations

Grids

Grids are visual representations that aid in performing operations with decimals and fractions and promote the language to describe fractions and decimals. Additionally, they can be used to provide a conceptual link when it comes to computation with percentages and can make working with these ratios simpler. Grids can be broken into fractions, such as tenths, hundredths, and thousandths. These grids can be used to connect the relationship between decimals, fractions and percentages.

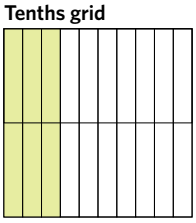
This chapter will use various numbered grids to connect percentages with decimals and fractions. Students with a solid pictorial representation that 'per cent' is 'per hundred' are more likely to solve percentage problems (Reyes, 2004). Students who use various tools and visual representations, such as discrete objects and models and other methods of calculating percentages, can solve percentage problems effectively and quickly.

Hundredths grid



Fraction		Decimal		Percentage
$\frac{29}{100}$	=	0.29	=	29%

Tenths grid

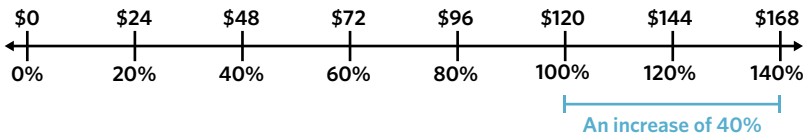


Fraction		Fraction		Decimal		Percentage
$\frac{6}{20}$	=	$\frac{30}{100}$	=	0.3	=	30%

Number line

A number line is traditionally shown as a horizontal line or axis but can be a vertical line. A number line shows the order and size of numbers and helps deepen our understanding of mathematical concepts such as fractions, decimals, percentages, negative numbers and mathematical operations.

This chapter uses a double number line to aid in learning about percentages. Number lines have been shown to support student learning, organise thinking of what is known and unknown, and aid in the teaching of the three common percentage problems (State Government Victoria, 2008). In addition to number lines, discrete models, pie charts, percentage tables, and area models can be used to aid in student's conceptual understanding of percentages (Dole, 1999).



Misconceptions

Misconception	Incorrect	Correct	Lesson
Students use the numbers in a fraction to convert them to decimals and percentages.	$\frac{2}{5} = 0.25 = 25\%$	$\frac{2}{5} = 0.4 = 40\%$	3A
Students believe that percentages are the same as whole numbers.	$23\% = 23$	$23\% = \frac{23}{100} = 0.23$	3A 3B 3C 3D
When converting fractions to decimals, students divide the denominator by the numerator.	$\frac{5}{8}$ = 8 divided by 5 $= 5\overline{)8.^{3}0}^{1.6}$	$\frac{5}{8}$ = 5 divided by 8 $= 8\overline{)5.0^{4}0^{4}0}^{0.625}$	3A
Students believe percentages are between 1 and 100%.	12 out of 10 as a percentage is $\frac{10}{12} \times 100\% = \frac{10}{12} \times \frac{100}{1}$ $= 83\frac{1}{3}\%$	12 out of 10 as a percentage is $\frac{12}{10} \times 100\% = \frac{12}{10} \times \frac{100}{1}$ $= 120\%$	3B
When calculating percentage amounts, students decode the word 'of' to represent 'out of' and divide values.	75% of 30 $\frac{75}{30} = \frac{5}{2} \times \frac{100}{1}$ $= \frac{5}{1} \times \frac{50}{1}$ $= 250$	75% of 30 $\frac{75}{100} \times 30 = \frac{3}{4} \times \frac{30}{1}$ $= \frac{3}{2} \times \frac{15}{1}$ $= \frac{45}{2}$ $= 22\frac{1}{2}\%$	3B
When calculating the final price, students calculate the increase/decrease amount.	Increase 60 by 20%: $60 \times 0.20 = 12$	Increase 60 by 20%: $60 \times 0.20 = 12$ $60 + 12 = 72$	3C
When calculating the original price, students multiply the percentage change by the final price.	Original price: ? Percentage increase: 50% Final price: \$3.00 Original price $= 3 \times \frac{50\%}{100\%} = \$1.50$	Original price: ? Percentage increase: 50% Final price: \$3.00 Final value percentage $= 150\%$ 1% of original price $= \frac{3}{150\%} = \$0.20$ 100% of original price $= \$0.20 \times 100\% = \$2.00$	3C
Students believe that increasing and decreasing by percentages are additive.	Decrease 80 by 20% $80 - 0.20 = 79.80$ or $80 - 20 = 60$	Decrease 80 by 20%: $80 \times 0.20 = 16$ $80 - 16 = 64$	3C
When calculating percentage change or error, students believe it is additive.	What is the percentage decrease from 60 to 24? $60 - 24 = 34\%$	What is the percentage decrease from 60 to 24? $\frac{60 - 24}{60} \times 100\% = 60\%$	3D
Students do not understand the importance of number order when calculating the difference.	What is the percentage change from 3 to 5? $\frac{3 - 5}{3} \times 100\%$ = decrease of $66\frac{2}{3}\%$	What is the percentage change from 3 to 5? $\frac{5 - 3}{3} \times 100\%$ = increase of $66\frac{2}{3}\%$	3D

Misconception	Incorrect	Correct	Lesson
When calculating percentage change or error using a formula, students use the wrong value in the denominator.	What is the percentage increase from 120 to 150? $\frac{150 - 120}{150} \times 100\% = 20\%$	What is the percentage increase from 120 to 150? $\frac{150 - 120}{120} \times 100\% = 25\%$	3D
Students believe that the 'best buy' is the one that has a lower total cost.	What is the best buy: 220 mL bottle that costs $1.70 or 1 L bottle that costs $6. $1.70 is cheaper than $6.00. The 220 mL bottle is the best buy.	What is the best buy: 220 mL bottle that costs $1.70 or 1 L bottle that costs $6. 220 mL costs $1.70 100 mL costs $1.70 ÷ 2.20 = $0.77 and 1 L costs $6.00 100 mL costs $6.00 ÷ 100 = $0.60 The 1 L bottle is the best buy	3E
Students do not understand how to use division to calculate 1 unit in cost or quantity.	What is the cost of 1 unit if 250 mL costs $2? 250 ÷ 2.5 = 100 or $1.00 is the unit price.	What is the cost of 1 unit if 250 mL costs $2? 250 ÷ 2.5 =100 mL $2 ÷ 2.5 = $0.8 per 100 mL	3E
When calculating the original value from a final value, students do not understand how to use the percentage.	Original price: ? Percentage decrease: 20% Final price: $5.20 1% = $5.20 ÷ 20 = $0.26 100% = $0.26 × 100 = $26 The retail price is $26.	Original price: ? Percentage decrease: 20% Final price: $5.20 Final value percentage = 80% 1% = $5.20 ÷ 80 = $0.065 100% = $0.065 × 100 = $6.50 The retail price is $6.50.	3E

References and additional readings

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Back to contents

3A Percentages, decimals, and fractions

Percentages, decimals, and fractions are three different ways of expressing the same value. The ability to convert between the three representations is useful in situations when whole numbers are not appropriate, or are not accurate enough.

LEARNING INTENTIONS

Students will be able to:

- convert between percentages, decimals, and fractions.

KEY TERMS AND DEFINITIONS

Converting is the process of rewriting a value in a different form, with no change to the value.

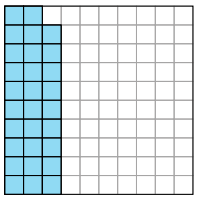
A **percentage** represents a value out of 100 parts, where 100 parts represents one whole.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Percentages, decimals, and fractions are used together in the real world. For example, to calculate the price of runners in a storewide sale, it is helpful to first convert the percentage to a fraction or a decimal.

Key idea

1. Fractions, decimals, and percentages are three equivalent ways of expressing any given value.



Fraction		Decimal		Percentage
$\frac{29}{100}$	=	0.29	=	29%

The following are some commonly used fractions, decimals and percentages.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{4}$
Decimal	0.5	$0.\overline{3}$	0.25	0.2	0.1	0.75
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	10%	75%

Worked example 1

Converting percentages to decimals and fractions

For the following percentages.

a. Convert 45% to a decimal. WE1a

Working

$45\% = \frac{45}{100}$

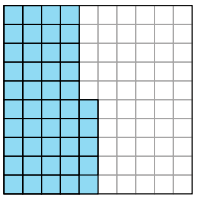
$45 \div 100 = 0.45$

Thinking

Step 1: Write the percentage as a fraction out of 100.

Step 2: Write the fraction as a division and calculate.

Visual support



Percentage		Decimal
45%	=	0.45

b. Convert 120% to a mixed number. WE1b

Working

$120\% = \frac{120}{100}$

$\frac{120}{100} = \frac{6}{5}$

$= 1\frac{1}{5}$

Thinking

Step 1: Write the percentage as a fraction out of 100.

Step 2: Simplify the fraction and convert to a mixed number.

Student practice

For the following percentages.

a. Convert 27% to a decimal.

b. Convert 250% to a mixed number.

Worked example 2

Converting decimals to percentages and fractions

For the following decimals.

a. Convert 0.06 to a percentage. WE2a

Working

$0.06 \times 100 = 6$

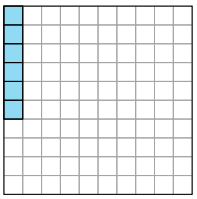
$0.06 = 6\%$

Thinking

Step 1: Multiply by 100.

Step 2: Insert a percentage sign (%).

Visual support



Decimal		Percentage
0.06	=	6%

Continues →

b. Convert 0.125 to a fraction. WE2b

Working

$0.125 = \frac{125}{1000}$

$\frac{125}{1000} = \frac{1}{8}$

Thinking

Step 1: Write the decimal as a fraction.

Step 2: Simplify the fraction.

Student practice

For the following decimals.

a. Convert 0.01 to a percentage.

b. Convert 0.75 to a fraction.

Worked example 3

Converting fractions to decimals and percentages

For the following fractions.

a. Convert $\frac{3}{5}$ to a decimal. WE3a

Working

$\frac{3}{5} = 3 \div 5$

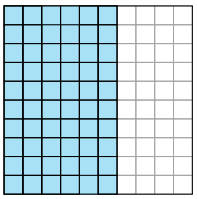
$5\overline{)3.0}$ 0.6

Thinking

Step 1: Write the fraction as division.

Step 2: Evaluate using short division.

Visual support



Fraction		Decimal
$\frac{3}{5}$	=	0.6

b. Convert $\frac{2}{9}$ to a percentage. WE3b

Working

$\frac{2}{9} \times 100 = \frac{200}{9}$

$9\overline{)2\ 0\ 2\ 0\ 2\ 0}\ r\ 2$

$= 22\frac{2}{9}$

$\frac{2}{9} = 22\frac{2}{9}\%$

Thinking

Step 1: Multiply by 100.

Step 2: Set out the fraction as short division.

Step 3: Add a percentage sign (%).

Student practice

For the following fractions.

a. Convert $\frac{4}{5}$ to a decimal.

b. Convert $\frac{2}{3}$ to a percentage.

3A Questions

Understanding worksheet

1. Select if each of the following is a fraction, decimal or percentage.

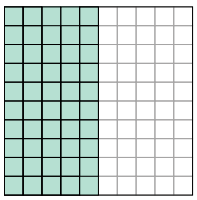
Example

	Fraction	Decimal	Percentage
0.87	☐	☑	☐

		Fraction	Decimal	Percentage
a.	$\frac{1}{5}$	☐	☐	☐
b.	1.36	☐	☐	☐
c.	8%	☐	☐	☐
d.	0.1%	☐	☐	☐

2. Complete the missing values.

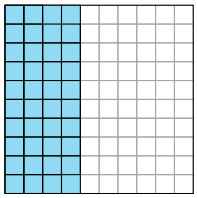
Example



Fraction Decimal Percentage

$\frac{1}{2}$ = 0.5 = 50%

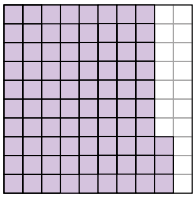
a.



Fraction Decimal Percentage

$\frac{2}{5}$ = 0.4 = ☐

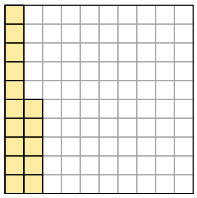
b.



Fraction Decimal Percentage

☐ = 0.83 = 83%

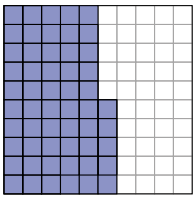
c.



Fraction Decimal Percentage

$\frac{3}{20}$ = ☐ = 15%

d.



Fraction Decimal Percentage

☐ = 0.55 = 55%

3. Fill in the blanks by using the words provided.

percentage | decimal | fraction | convert

$\frac{1}{10}$ is an example of a ☐. Fractions, decimals, and percentages are three different ways of representing equivalent values. We can ☐ $\frac{1}{10}$ to the ☐ 0.1, or write it as a ☐, which would be 10%.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Convert the following percentages to decimals. WE1a

a. 25% **b.** 78% **c.** 123% **d.** 4%
e. 39.1% **f.** 602.7% **g.** 5.3% **h.** 0.02%

5. Convert the following percentages to fractions. Give your answer as a proper fraction or mixed number. WE1b

a. 13% **b.** 7% **c.** 40% **d.** 130%
e. 8% **f.** 225% **g.** 0.7% **h.** 2.5%

6. Convert the following decimals to percentages. WE2a

a. 0.25 **b.** 1.6 **c.** 0.473 **d.** 0.08
e. 56.02 **f.** 110.9 **g.** 0.005 **h.** 0.0104

7. Convert the following decimals to proper fractions or mixed numbers. WE2b

a. 0.9 **b.** 0.8 **c.** 0.58 **d.** 1.1
e. 3.09 **f.** 0.05 **g.** 18.26 **h.** 0.072

8. Convert the following fractions and mixed numbers to decimals. WE3a

a. $\frac{1}{2}$ **b.** $\frac{7}{10}$ **c.** $\frac{1}{3}$ **d.** $\frac{5}{8}$
e. $1\frac{1}{4}$ **f.** $2\frac{9}{20}$ **g.** $\frac{4}{9}$ **h.** $\frac{32}{3}$

9. Convert the following fractions and mixed numbers to percentages. WE3b

a. $\frac{1}{4}$ **b.** $\frac{2}{100}$ **c.** $\frac{99}{100}$ **d.** $\frac{3}{8}$
e. $1\frac{4}{5}$ **f.** $3\frac{2}{3}$ **g.** $\frac{36}{5}$ **h.** $\frac{7}{9}$

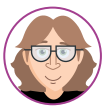
10. Select the correct response. $\frac{5}{9}$ is equal to:

A. $0.\overline{5}$ **B.** 0.59 **C.** 1.8% **D.** 5.9 **E.** 59%

Spot the mistake

11. Select whether Student A or Student B is incorrect.

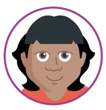
a. Convert 127% to a decimal.



Student A

$127\% = \frac{127}{100}$

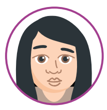
$127 \div 100 = 1.27$



Student B

$127\% = 127.0$

b. Convert $\frac{7}{8}$ to a percentage.



Student A

$\frac{7}{8} = 7.8\%$

Student B

$\frac{7}{8} \times 100 = \frac{700}{8}$

$\begin{array}{r} 0\ \ 8\ \ 7\ .\ 5 \\ 8\overline{)7\ \ 0\ \ {}^{6}0\ .\ {}^{4}0} \end{array}$

$\frac{7}{8} = 87.5\%$

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

12. Bela estimates that three-quarters of the Australian population own a mobile phone. What percentage does this represent?

13. It takes $5\frac{1}{2}$ minutes for Matchoo to heat up his frozen lasagne for lunch. Express the time as a decimal.

14. In its simplest form, what fraction of Tori's hair is straight if 55% of it is curly?

15. Kathy cuts her pizza into six equal pieces. Write down the value that one piece represents, as a fraction, decimal, and percentage.

16. Patrick has 0.8 hours left to finish a science experiment. Rewrite the amount of time as a fraction out of 60, so that Patrick knows how many minutes he has left.

Reasoning

Question working paths

Mild 17 (a,b,c,e), 18 (a,b)	Medium 17 (a,b,c,e), 18 (a,b)	Spicy All

17. Vutha is baking ANZAC biscuits for his friends. He finds the following recipe for 12 biscuits and goes to the supermarket to purchase the ingredients.

ANZAC Biscuits Recipe

100 g	*plain flour*
90 g	*rolled oats*
80 g	*dessicated coconut*
150 g	*unsalted butter*
80 g	*golden syrup (approx 4 tablespoons)*

a. What percentage of the recipe is plain flour?

b. In its simplest form, what fraction of the recipe are rolled oats?

c. The ingredients normally cost $13.30 to purchase. How much would it cost Vutha if he was able to use his cousin's 5% staff discount? Round your answer to the nearest cent.

d. List the amount of each ingredient Vutha will need if he wants to make 15 cookies.

e. Suggest a way Vutha can save money but still do something nice for his friends.

18. Consider the following values: $\frac{1}{5}$, 8%, 0.12, 35%, and $\frac{2}{3}$.

a. Which is the smallest value?

b. Which is the largest value?

c. Compare your answers from parts **a** and **b** and propose an efficient method to compare the values in the list.

Extra spicy

19. Lena leaves home and walks one-fifth of the way to school when she realises that she forgot her homework. She returns home to get it, then walks to school. As a percentage of the distance from home to school, how much does she walk in total? Select the correct response.

A. 1.4% **B.** $1\frac{2}{5}$ **C.** 120 **D.** 125% **E.** 140

20. Warren wants to score at least 90% on his Maths test and receives the mark 51 out of 55.
By what whole percentage was Warren above or below his goal? Select the correct response.

A. 2% below **B.** 2% above **C.** 3% below **D.** 3% above **E.** 4 marks below

21. Human infants are born with approximately 270 bones. By adulthood, humans have 206 bones.
What percentage of bones fuse with others in the growth process? Round your answer to the nearest percent.

22. What fraction is halfway between 25% and 0.0625?

Remember this?

23. Which of the following has the same value as 15×3?

A. $10 \times 3 + 5 \times 1$
B. $10 + 5 \times 3$
C. $15 \times 2 + 1$
D. $10 \times 5 \times 3$
E. $(10 + 5) \times 3$

24. A number is doubled and then 3 is added. The answer is 11. What is the number?

A. 2 **B.** 4 **C.** 8 **D.** 14 **E.** 28

25. Another way of writing 9^3 is

A. 9×3 **B.** $9 \times 9 \times 9$ **C.** 27 **D.** 81 **E.** 93

Back to contents

3B Expressing as a percentage and percentages of quantities

The words 'per cent' come from the Latin *per centum*, which means 'out of one hundred'. This gives rise to words such as century (100 years), cents (where 100 cents = \$1) and centimetres (where 100 cm = 1 m). Much as a century is always 100 years, percentages always represent values out of 100.

LEARNING INTENTIONS

Students will be able to:

- express one quantity as a percentage of another
- calculate the percentage of a given quantity.

KEY TERMS AND DEFINITIONS

A **percentage** represents a value out of 100 parts, where 100 parts represents one whole.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

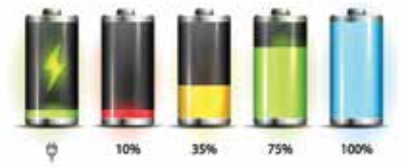
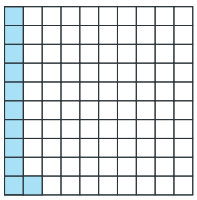


Image: MarySan/Shutterstock.com

Percentages are used in many everyday situations, such as topic test results, battery level and the amount of a file that has been downloaded.

Key ideas

1. Percentages represent a value out of 100 parts.



x out of 100 = $\frac{x}{100}$ = x%

11 out of 100 = $\frac{11}{100}$ = 11%

2. Expressing one quantity as a percentage of another is the same as finding the fraction of a whole.

11 out of 20 can be rewritten as the fraction

$\frac{11}{20}$ — The first number is the numerator or 'part amount'.
— The second number is the denominator or 'whole amount'.

Worked example 1

Expressing one number as a percentage of another

Express the following as a percentage.

a. 12 out of 25 WE1a

Working

12 out of 25 = $\frac{12}{25}$

$\frac{12}{25} \times \frac{100}{1}\% = \frac{12}{1} \times \frac{4}{1}\%$

Thinking

Step 1: Rewrite as a fraction.

Step 2: Multiply the fraction by 100%.

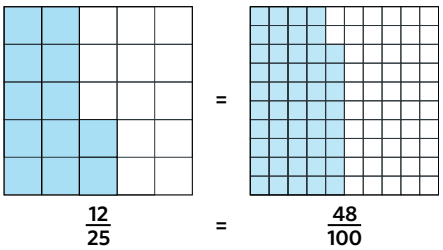
Continues →

$= \frac{48}{1}\%$

$= 48\%$

Step 3: Simplify.

Visual support



b. 15 out of 24, as a mixed number percentage

WE1b

Working

$\frac{15}{24} \times \frac{100}{1}\% = \frac{5}{8} \times \frac{100}{1}\%$

$= \frac{500}{8}\%$

$= 62\frac{1}{2}\%$

Thinking

Step 1: Rewrite as a fraction.

Step 2: Multiply the fraction by 100%.

Step 3: Simplify.

Student practice

Express the following as a percentage.

a. 13 out of 20

b. 18 out of 27, as a mixed number percentage

Worked example 2

Expressing quantities with units as percentages of each other

Express the following as a percentage.

a. 6 cm out of 20 cm

WE2a

Working

$\frac{6}{20} \times \frac{100}{1}\% = \frac{6}{1} \times \frac{5}{1}\%$

$= \frac{30}{1}\%$

$= 30\%$

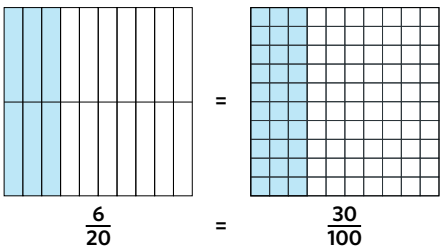
Thinking

Step 1: Rewrite as a fraction.

Step 2: Multiply the fraction by 100%.

Step 3: Simplify.

Visual support



Continues →

b. 80 cm as a percentage of 3 m, rounded to the nearest percentage **WE2b**

Working

$3 \text{ m} = 300 \text{ cm}$

$\frac{80}{300} \times \frac{100}{1}\% = \frac{80}{3} \times \frac{1}{1}\%$

$= \frac{80}{3}\%$

$= 26.\dot{6}\%$

$= 27\%$

Thinking

Step 1: Convert both quantities to the same unit.

Step 2: Rewrite as a fraction and multiply by 100%.

Step 3: Write the answer as a decimal.

Step 4: Round to the nearest percentage.

Student practice

Express the following as a percentage.

a. 80 mL as a percentage of 400 mL

b. 35 cm as a percentage of 2 m, rounded to the nearest percentage.

Worked example 3

Calculating the percentage of a given quantity

Evaluate the following.

a. 30% of 50 **WE3a**

Working

$\frac{30}{100} \times \frac{50}{1} = \frac{3}{1} \times \frac{5}{1}$

$= \frac{15}{1}$

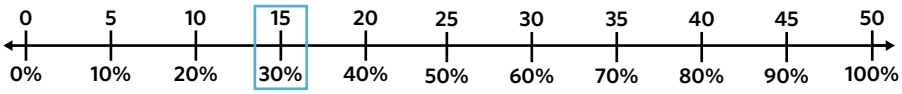
$= 15$

Thinking

Step 1: Write the percentage and whole number as fractions. Replace 'of' with a multiplication sign.

Step 2: Multiply the fraction by 100%.

Visual support



b. 11% of 167, to 1 decimal place. **WE3b**

Working

$\frac{11}{100} \times \frac{167}{1} = \frac{1837}{100}$

$= 18.37$

≈ 18.4

Thinking

Step 1: Write the percentage and whole number as fractions. Replace 'of' with a multiplication sign.

Step 2: Convert the answer to a decimal, rounded to 1 decimal place.

Student practice

Evaluate the following.

a. 40% of 75

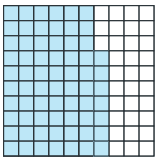
b. 17% of 230, to 1 decimal place.

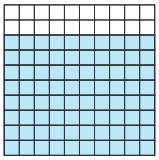
3B Questions

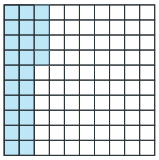
Understanding worksheet

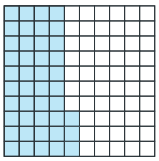
1. State the percentage described in the following visuals.

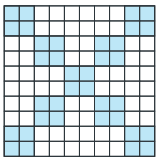
Example

 = 67 %

a.  = ____ %

b.  = ____ %

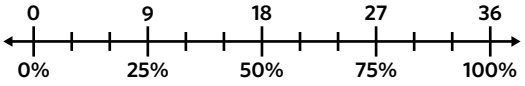
c.  = ____ %

d.  = ____ %

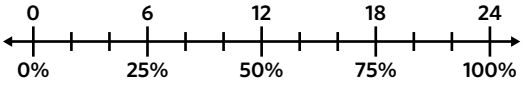
2. Use the number lines to calculate the percentage of each quantity.

Example

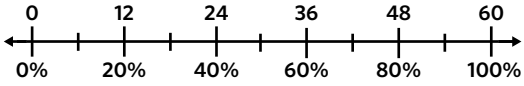
25% of 36 is: 9



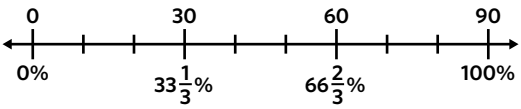
a. 50% of 24 is: ____



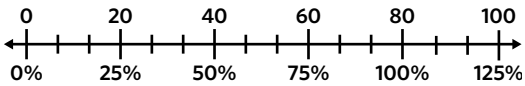
b. 40% of 60 is: ____



c. $33\frac{1}{3}$% of 90 is: ____



d. 125% of 80 is: ____



3. Fill in the blanks by using the words provided.

out of | fractions | percentages | multiplication

____ are always ____ one hundred. When expressing one quantity as a percentage of another, or calculating the percentage of a quantity, we often convert the numbers to ____ first. Calculating the percentage of a quantity replaces the word 'of' with the ____ symbol.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Express the following as a percentage. WE1a

a. 28 out of 100 **b.** 15 out of 30 **c.** 42 out of 200 **d.** 15 out of 10
e. 57 out of 76 **f.** 39 out of 65 **g.** 72 out of 180 **h.** 117 out of 450

5. Express the following as mixed number percentages. WE1b

a. 3 out of 8 **b.** 25 out of 200 **c.** 7 out of 12 **d.** 11 out of 8
e. 39 out of 45 **f.** 48 out of 112 **g.** 91 out of 84 **h.** 117 out of 143

6. Express the following as a percentage. WE2

a. 50 g out of 200 g
b. Level 12 of 120 levels in a video game
c. 80 cents out of $2.50
d. 0.25 L out of 200 mL
e. 1750 g out of 5 kg
f. 185 cents out of $0.74
g. 81 mm out of 13.5 cm
h. 21 minutes out of 2.5 hours

7. Evaluate the following. WE3a

a. 50% of 12 **b.** 25% of 60 **c.** 70% of 40 **d.** 130% of 50
e. 15% of 180 **f.** 42% of 250 **g.** 98% of 2450 **h.** 264% of 525

8. Evaluate the following to 1 decimal place. WE3b

a. 10% of 25 **b.** 35% of 63 **c.** 47% of 106 **d.** 54% of 172
e. 108% of 99 **f.** 262% of 162 **g.** $33\frac{1}{3}$% of 50 **h.** $56\frac{1}{5}$% of 778

9. Which of the following represents the largest value? Select the correct response.

A. 0.17% of 39 250 **B.** 33% of 2022 **C.** 51% of 1300 **D.** 98% of 681 **E.** 124% of 539

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Express 42 out of 35 as a percentage.

Student A

$$\frac{35}{42} \times 100\% = \frac{35}{42} \times \frac{100}{1}$$
$$= \frac{5}{3} \times \frac{50}{1}$$
$$= \frac{250}{3}$$
$$= 83\frac{1}{3}\%$$

Student B

$$\frac{42}{35} \times 100\% = \frac{42}{35} \times \frac{100}{1}$$
$$= \frac{6}{1} \times \frac{20}{1}$$
$$= \frac{120}{1}$$
$$= 120\%$$

b. Calculate 70% of 20.

Student A

$$\frac{70}{100} \times 20 = \frac{7}{1} \times \frac{2}{1}$$
$$= \frac{14}{1}$$
$$= 14$$

Student B

$$\frac{70}{20} = \frac{7}{2} \times \frac{100}{1}$$
$$= \frac{7}{1} \times \frac{50}{1}$$
$$= 350$$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Hunter has 1 yellow, 2 green and 3 pink chocolate clinkers. What percentage of clinkers are pink?

12. Casio piano mini-keyboards have a marked price of $110 but are discounted by 20% as part of an end of financial year (EOFY) sale. What dollar amount is the discount?

13. A standard running track is 400 m. What percentage of the track does Boyd run, if the race is 5 km?

14. 23 minutes have elapsed in a 90-minute multiple choice exam, where each question is worth one mark. If Cas has completed 8 out of 40 questions, would they want to go faster or slower in order to finish the exam in time?

15. Sally-Anne owns a veterinary business and pays 25% company tax, down from the old tax rate of 27.5%. If the business had taxable income of 1.78 million dollars, how much tax is saved at the new tax rate, compared to the old one? Give your answer in dollars.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. The minimum wage in Australia for employees aged 21 and over is $20.33. The following table shows the minimum wage of workers aged under 21. Casual workers receive 25% more pay compared to the minimum wage of part-time and full-time staff.

Age of employee	Wage as a percentage of $20.33	Part-time and full-time wage (per hour)	Casual wage (per hour)
Under 16	36.8%	$7.48	**a**
16	47.3%	$9.62	$12.03
17	57.8%	$11.75	$14.69
18	68.3%	$13.89	$17.36
19	82.5%	$16.77	$20.96
20	97.7%	$19.86	$24.83

a. Calculate the minimum wage of a 15-year-old casual employee.

b. Express the minimum wage of a 17-year-old casual employee as a percentage of the wage of an 18-year-old casual employee. Round your answer to 2 decimal places.

c. Tajah-Rose is a 17-year-old casual employee. She earned $293.80 last week and wants to save $200 towards a hairdressing course. What percentage of pay would Tajah-Rose need to save? Round your answer to the nearest percentage.

d. Jordan is an 18-year-old part-time employee and estimates that he is spending 45% of his pay on food. If he works 25 hours a week on average, how much money is Jordan spending on food? Round your answer to 2 decimal places.

e. Many people wish to save for big items, such as a new phone, car or house. What is one way to better manage money in order to save for these items?

17. Complete the following.

a. Calculate 40% of 120.

b. Calculate 20% of 120.

c. Compare your answers for **a** and **b**, explain how changes to the percentage might affect the answer.

Extra spicy

18. Donald's height is 87% of Eddie's height. Donald is 1.32 m tall. What is Eddie's height in metres? Round your answer to 2 decimal places.

19. For the numbers 1 to 24 (inclusive), what percentage of whole numbers are exact multiples of 5? Give your answer as a mixed number.

20. A town has 7500 residents, of whom 60% voted in the latest local council elections. Of those who voted, 29% voted for candidate A, 38% for candidate B and 33% for candidate C. The number of residents who voted for the winning candidate was:

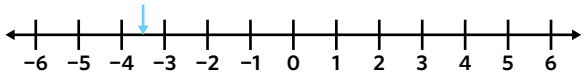
A. 1305 **B.** 1485 **C.** 1710 **D.** 2850 **E.** 4500

21. Approximately 100 cm of organic matter (peat) forms in a bog every thousand years. Roughly what percentage forms each day? Select the correct response.

A. 0.0003% **B.** 0.003% **C.** 0.03% **D.** 0.3% **E.** 3%

Remember this?

22. What value is the blue arrow pointing to on the number line?



What number does the arrow point to?

A. -3 **B.** $-3\frac{1}{2}$ **C.** -4 **D.** $-4\frac{1}{2}$ **E.** -4.5

23. The following table shows the times for 3 of the first 4 runners in a sprint.

Place	Time (seconds)
1st	12.39
2nd	12.93
3rd	?
4th	13.01

The time of the runner in 3rd place could be:

A. 12.01 seconds **B.** 12.26 seconds **C.** 12.31 seconds **D.** 12.98 seconds **E.** 13.95 seconds

24. Two friends' houses are 3.2 cm apart on a map. If 1 cm represents 5 km on the map, what is the actual distance between the two places? Select the correct response.

A. 3.2 km **B.** 8.2 cm **C.** 15.10 km **D.** 16 km **E.** 32 km

Back to contents

3C Increasing and decreasing by a percentage

It is very common to increase or decrease values by percentages, rather than by a whole number amount. There are two methods for increasing and decreasing by a percentage, both methods involve using multiplication and either addition if increasing, or subtraction if decreasing.

LEARNING INTENTIONS

Students will be able to:

- increase and decrease a number or quantity by a given percentage
- calculate discounts and mark-ups
- calculate prices before and after GST is applied.

KEY TERMS AND DEFINITIONS

The **cost price** or **wholesale price** is the amount spent by businesses to make or buy an item for sale.

A **mark-up** describes the percentage or amount that the cost price is increased by.

The **marked price**, also known as the **selling price**, **retail price**, or **recommended retail price (RRP)** is the price an item is sold for after mark-up.

A **discount** describes the percentage or amount that the marked price is decreased by.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Ian_Stewart/Shutterstock.com

Decreasing by a percentage is commonly seen in sales events, such as a 20% storewide sale. Inflation is an example of general prices rising by a percentage.

Key ideas

1. Different types of prices are created by increasing or decreasing by a given value or percentage.

The retail price is found by applying a mark-up to the cost price.

Retail price = cost price + mark-up

The retail price can be lowered by applying a discount.

Discounted price = retail price − discount

Continues →

2. **Goods and Services Tax (GST)** in Australia is 10% of the cost of goods, and charged on all non-essential goods.

In Australia, it is a legal requirement that businesses display the price including GST. Consumers (people who buy goods and services) pay the GST to businesses as part of payment. Businesses then pass on the GST to the government.

Example of **essential** goods and services that **do not attract GST**

Fruit and vegetables

Flour

Rice

Healthcare and basic medicine

Housing

Water, gas and electricity (utilities)

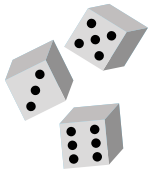
Example of **non-essential** goods and services that **do attract GST**

Clothes

Restaurant meals

Musical instruments

Airfares



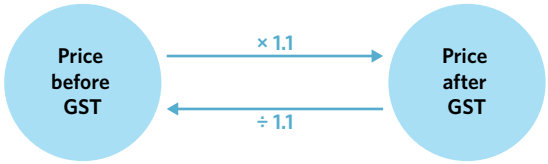
Toys and games

Laptops

3. Formulas for calculating GST:

Price after GST = Price before GST × 1.1

Price before GST = Price after GST ÷ 1.1



Worked example 1

Increasing and decreasing by a percentage

Determine the new price of the following.

a. $120 increased by 40% WE1a

Working

Method 1

$$40\% \text{ of } \$120 = \frac{40}{100} \times \frac{120}{1}$$
$$= \frac{4}{1} \times \frac{12}{1}$$
$$= \$48$$

$$\text{New price} = \$120 + \$48$$
$$= \$168$$

Thinking

Method 1

Step 1: Calculate the dollar amount of the increase.

Step 2: Add the dollar amount to the original amount.

Continues →

Method 2

$100\% + 40\% = 140\%$

New price $= 140\%$ of $\$120$

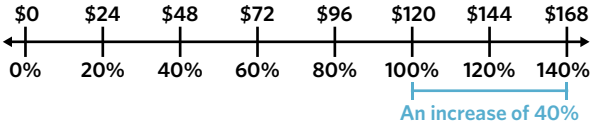
$$= \frac{140}{100} \times \frac{120}{1}$$
$$= \frac{14}{1} \times \frac{12}{1}$$
$$= \$168$$

Method 2

Step 1: Calculate the percentage after the increase.

Step 2: Calculate the dollar amount of the percentage after the increase.

Visual support



b. $60 decreased by 30%

WE1b

Working

Method 1

$$30\% \text{ of } \$60 = \frac{30}{100} \times \frac{60}{1}$$
$$= \frac{3}{1} \times \frac{6}{1}$$
$$= \$18$$

New price $= \$60 - \$18$

$$= \$42$$

Method 2

$100\% - 30\% = 70\%$

New price $= 70\%$ of $\$60$

$$= \frac{70}{100} \times \frac{60}{1}$$
$$= \frac{7}{1} \times \frac{6}{1}$$
$$= \$42$$

Thinking

Method 1

Step 1: Calculate the dollar amount of the decrease.

Step 2: Subtract the dollar amount from the original amount.

Method 2

Step 1: Calculate the percentage after the decrease.

Step 2: Calculate the dollar amount of the percentage after the decrease.

Student practice

Determine the new price of the following.

a. $80 increased by 25%

b. $50 decreased by 20%

Worked example 2

Calculating the new price after a discount or mark-up

Determine the new price of the following, rounding to the nearest cent where relevant.

a. A \$2 yoga mat that is marked up by 150%. WE2a

Working

Method 1

$$150\% \text{ of } \$2 = \frac{150}{100} \times \frac{2}{1}$$
$$= \frac{3}{1} \times \frac{1}{1}$$
$$= \$3$$

$$\text{New price} = \$2 + \$3$$
$$= \$5$$

Method 2

$$100\% + 150\% = 250\%$$

$$\text{New price} = 250\% \text{ of } \$2$$
$$= \frac{250}{100} \times \frac{2}{1}$$
$$= \frac{5}{1} \times \frac{1}{1}$$
$$= \$5$$

Thinking

Method 1

Step 1: Calculate the dollar amount of the mark-up.

Step 2: Add the dollar amount to the original amount.

Method 2

Step 1: Calculate the percentage after the mark-up.

Step 2: Calculate the dollar amount of the percentage after the mark-up.

Visual support

b. A \$399 mobile phone that is discounted by 10%. WE2b

Working

Method 1

$$10\% \text{ of } \$399 = \frac{10}{100} \times \frac{399}{1}$$
$$= \frac{1}{10} \times \frac{399}{1}$$
$$= \$39.90$$

$$\text{New price} = \$399 - \$39.90$$
$$= \$359.10$$

Method 2

$$100\% - 10\% = 90\%$$

$$\text{New price} = 90\% \text{ of } \$399$$
$$= \frac{90}{100} \times \frac{399}{1}$$
$$= \frac{9}{10} \times \frac{399}{1}$$
$$= \$359.10$$

Thinking

Method 1

Step 1: Calculate the dollar amount of the discount.

Step 2: Subtract the dollar amount from the original amount.

Method 2

Step 1: Calculate the percentage after the discount.

Step 2: Calculate the dollar amount of the percentage after the discount.

Student practice

Determine the new price of the following, rounding to the nearest cent where relevant.

a. A \$5 wireless mouse is marked up by 175%

b. A \$7 pair of socks is discounted by 20%.

Worked example 3

Calculating prices before and after GST is applied

Determine the following, rounding to the nearest cent where relevant.

a. The price after GST of a glass jar whose price excluding GST is $4.54. WE3a

Working

Price after GST = Price before GST × 1.1

= 4.54 × 1.1

= 4.994

≈ $4.99

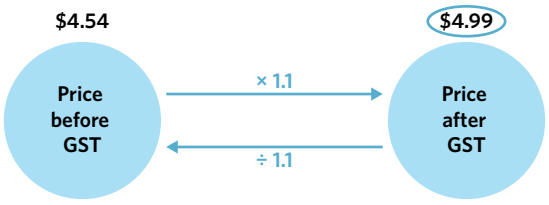
Thinking

Step 1: Write down the formula for price after GST.

Step 2: Substitute the price before GST into the formula and calculate.

Step 3: Round to the nearest cent, which is 2 decimal places.

Visual support



b. The price before GST of a set of headphones whose selling price is $65. WE3b

Working

Price before GST = Price after GST ÷ 1.1

= 65 ÷ 1.1

= 59.0909...

≈ $59.09

Thinking

Step 1: Write down the formula for price before GST.

Step 2: Substitute the price after GST into the formula and calculate.

Step 3: Round to 2 decimal places where necessary.

Student practice

Determine the following, rounding to the nearest cent where relevant.

a. The price after GST of a video game whose price excluding GST is $63.64.

b. The price before GST of nail polish whose selling price is $10.

3C Questions

Understanding worksheet

1. Fill in the blanks.

Example

Cost price = $21

Mark-up = $27

Retail price = $ 48

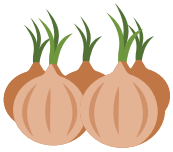
a.

Cost price = $12

Mark-up = $33

Retail price = $ ____

b.



Cost price = $7 per kg

Mark-up = $2 per kg

Retail price = $ ____ per kg

c.

Cost price = $15

Mark-up = $ ____

Retail price = $50

d.

Cost price = $ ____

Mark-up = $22

Retail price = $34

2. Fill in the blanks.

Example

Retail price = $4

Discount = $1.50

Discounted price = $ 2.50

a.

Retail price = $50

Discount = $10

Discounted price = $ ____

b.

Retail price = $20

Discount = $4

Discounted price = $ ____

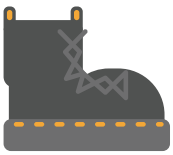
c.

Retail price = $15

Discount = $ ____

Discounted price = $12.50

d.



Retail price = $ ____

Discount = $75

Discounted price = $225

3. Fill in the blanks by using the words provided.

discount | GST | mark-up | cost price

When selling a product, '20% off' is an example of a _____. The amount added to the _____ is called the _____, and together, these form the selling price.

For non-essential goods and services, the government imposes a spending tax, called _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Determine the new price of the following, rounding to the nearest cent where relevant. WE1a

a. $100 is increased by 20%
b. $20 is increased by 50%
c. $75 is increased by 10%
d. $50 is increased by 100%
e. $380 is increased by 25%
f. $1.30 is increased by 1%
g. $7.20 is increased by 4%
h. $432 is increased by 260%

5. Determine the new price of the following, rounding to the nearest cent where relevant. WE1b

a. $40 is decreased by 10%
b. $60 is decreased by 25%
c. $80 is decreased by 40%
d. $110 is decreased by 100%
e. $240 is decreased by 30%
f. $4.80 is decreased by 60%
g. $2.45 is decreased by 5%
h. $3003 is decreased by 99%

6. Determine the new price of the following, rounding to the nearest cent where relevant. WE2a

a. A $2 stapler is marked up by 50%
b. Broccoli sold at $5 per kg is marked up by 20%
c. A $12 backpack is marked up by 150%
d. A $2.50 lunchbox is marked up by 160%
e. A $1.70 can of cat food is marked up by 40%
f. A $20.56 scientific calculator is marked up by 60%
g. A $2.07 cup of hot chocolate is marked up by 232%
h. A $0.78 box of tissues is marked up by 321%

7. Determine the new price of the following, rounding to the nearest cent where relevant. WE2b

a. A $10 pack of highlighters is discounted by 20%.
b. A $20 pair of headphones is discounted by 10%.
c. A $80 space heater is discounted by 25%.
d. An avocado priced at $2 that is discounted by 5%.
e. A $25 bag of cat litter is discounted by 32%.
f. A giant soft toy priced at $50 that is discounted by 7%.
g. A glitter pen pack priced at $5.99 that is discounted by 15%.
h. A $39.99 pair of jeans on clearance with a discount of 55%.

8. Determine the price after GST for the following, rounding to the nearest cent where relevant. WE3a

- **a.** A plastic ruler whose price excluding GST is $1.
- **b.** A dog toy whose price excluding GST is $6.82.
- **c.** A USB cable whose price excluding GST is $8.18.
- **d.** A lasagna whose price excluding GST is $9.05.
- **e.** A pair of sunglasses whose price without GST is $19.09.
- **f.** A lawn mowing service whose price excluding GST is $54.55.
- **g.** A relaxation massage whose price excluding GST is $81.82.
- **h.** A vacuum cleaner whose price excluding GST is $272.72.

---

9. Determine the price before GST for the following, rounding to the nearest cent where relevant. WE3b

- **a.** A game of Uno whose selling price is $8.
- **b.** A t-shirt whose selling price is $15.
- **c.** A haircut where the customer pays $30.
- **d.** A box of green tea whose RRP is $3.50.
- **e.** A Spotify subscription whose selling price is $11.99 per month.
- **f.** A Billie Eilish concert ticket priced at $139.90.
- **g.** A smart watch whose retail price is $249.95.
- **h.** A 55 inch television whose RRP is $1199.99.

---

10. Calculate the price before GST of a $35 newspaper subscription.

A. $3.18 **B.** $3.50 **C.** $31.82 **D.** $31.50 **E.** $38.50

Spot the mistake

11. Select whether Student A or Student B is incorrect.

- **a.** Calculate the new price when $50 is increased by 12%.

Student A

$$\text{New price} = 12\% \text{ of } \$50$$
$$= \frac{12}{100} \times \frac{50}{1}$$
$$= \frac{6}{1} \times \frac{1}{1}$$
$$= \$6$$

Student B

$$12\% \text{ of } \$50 = \frac{12}{100} \times \frac{50}{1}$$
$$= \frac{6}{1} \times \frac{1}{1}$$
$$= \$6$$
$$\text{New price} = \$50 + \$6$$
$$= \$56$$

- **b.** Calculate the discounted price when a $15 drink bottle is discounted by 20%.

Student A

$$100\% - 20\% = 80\%$$
$$\text{New price} = 80\% \text{ of } \$115$$
$$= \frac{80}{100} \times \frac{15}{1}$$
$$= \frac{4}{1} \times \frac{3}{1}$$
$$= \$12$$

Student B

$$\text{New price} = \$15 - \$0.20$$
$$= \$14.80$$

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

12. Lorelai buys a monkey lamp online that is usually $30. If there is a 10% storewide sale and free delivery, how much does she pay for the lamp?

13. Luke owns a cafe and sells sprinkled and glazed donuts at a 100% mark-up. If the retail price per donut is $4, what is the cost price?

14. Rory wants to earn $50 per hour for writing magazine articles. Including GST, how much would clients need to pay per hour?

15. Ceol buys gravy cheese chips at a chip shop in St Kilda for $12. What is the price before GST?

16. Dashy sells personalised mugs online. The wholesale price of each mug is $5. If she puts a mark-up of 120% and then GST of 10% onto the marked-up price, what is the price that customers are paying for the mug?

Reasoning

Question working paths

Mild 17 (a,b,c,e)	Medium 17 (a,b,c,e), 18 (a,b)	Spicy All

17. Beatrix owns a small cake shop on the Gold Coast.

- **a.** At the shop, Beatrix sells packs of candles at $5 a pack. She decides to discount the price by 20%. What is the discounted price of a pack of candles?
- **b.** A blackforest cake costs approximately $12 in ingredients and takes roughly 2 hours to make. Beatrix marks up the cost by 450% to cover her rent, electricity, wages and other bills. How much does Beatrix sell each blackforest cake for?
- **c.** If a piece of vanilla slice costs $5.91 before GST, what is the price after GST? Give your answer to the nearest cent.
- **d.** Cakes attract GST because they are considered a non-essential product. What is the price before GST of a $50 cake? Give your answer to the nearest cent.
- **e.** Why might a business sell a product if it results in a loss?

18. Start with the number 199.

- **a.** Decrease the value by 20%.
- **b.** Increase the lower value by 20%.
- **c.** Compare your answer for **b** with the original number. Comment on why the values might be the same or different.

Extra spicy

19. Boris' wage increases by 2.4% to $86 243. How much did Boris earn before his wage increase? Round your answer to the nearest dollar.

20. Rawnak collects 3 coupons to use on eBay. The coupons are for 20% off, 10% and 5%. Due to a system issue, Rawnak manages to use all 3 coupons in a single purchase when she manually enters one at a time, which results in the second and third coupons providing further discounts on the already discounted price. What is the best percentage discount that Rawnak can receive on her order, assuming she can use them in any order?

21. If each side length of a square is increased by 50%, what percentage does the area of the square increase by? Select the correct response.

A. 50 **B.** 100 **C.** 125 **D.** 150 **E.** 225

22. Yan buys wireless headphones just before receiving a set for Christmas. At first, she decides to sell her headphones for 20% less than the price she paid. However, Yan eventually decides to increase the price by 10% of the lower price. What is the net percentage change? Select the correct response.

A. 10% increase **B.** 10% decrease **C.** 12% increase **D.** 12% decrease **E.** 30% decrease

Remember this?

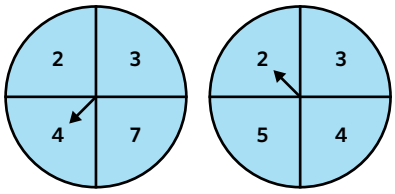
23. In Victoria, women were first granted the right to vote in 1908.
By 2024, women in Victoria will have been able to vote for:

A. 76 years **B.** 98 years **C.** 104 years **D.** 116 years **E.** 124 years

24. A standard bathtub has a capacity of 0.18 kilolitres. How many litres of water does the bathtub hold when it is filled to capacity?

A. 18 **B.** 108 **C.** 180 **D.** 1800 **E.** 18 000

25. Lena spins the arrows on the following circular boards. She adds up the numbers where the arrows stop and gets a total of 6.



Lena spins both arrows again. How many different ways can she get a total of 7?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

Back to contents

3D Percentage change

Percentage change captures the amount that a value has increased or decreased by, compared to the original value. Different versions of the same formula are used for percentage change, percentage profit, percentage loss and percentage error.

LEARNING INTENTIONS

Students will be able to:

- calculate the percentage change when values are increased or decreased
- calculate percentage error
- calculate profit and loss.

KEY TERMS AND DEFINITIONS

Revenue is the income that a business receives from selling goods and services.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: photocritical/Shutterstock.com

Percentage change calculations are often used in business. Whether a plumber, cafe owner or a large store like Woolworths, percentage profit and loss calculations are carried out to make sure that there is enough money for owners to pay for business expenses, and their own living expenses.

Key ideas

1. Percentage change calculates the increase or decrease in value, as a portion of the original value.

$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100\%$$

2. Profit and loss describe whether a business makes money.

Profit	Revenue is greater than costs.
Loss	Revenue is less than costs.
Break-even	Revenue is equal to costs.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost}} \times 100\%$$

3. Percentage error describes the difference between a predicted and the actual value, relative to the actual value. If the predicted value is the same as the actual value, percentage error $= 0$.

$$\text{Percentage error} = \frac{\text{difference}}{\text{actual}} \times 100\%$$

Worked example 1

Calculating percentage increase and decrease

Calculate the percentage increase or decrease for the following.

a. 20 becomes 45 WE1a

Working

$$\text{Increase} = 45 - 20$$
$$= 25$$

$$\text{Percentage change} = \frac{25}{20} \times 100\%$$
$$= 125\% \text{ increase}$$

Thinking

Step 1: Write down if it is an increase or decrease and calculate the difference.

Step 2: Substitute values into the percentage change formula and evaluate.

Visual support

$$\text{Percentage change} = \frac{\overset{45 - 20 = 25}{\overset{\downarrow}{\text{change}}}}{\underset{\uparrow}{\text{original value}}} \times 100\%$$
$$\underset{20}{\uparrow}$$

b. The population changes from 6.89 million to 6.78 million, rounded to one decimal place. WE1b

Working

$$\text{Decrease} = 6.89 - 6.78$$
$$= 0.11$$

$$\text{Percentage change} = \frac{0.11}{6.89} \times 100\%$$
$$= 1.597\%$$
$$\approx 1.6\% \text{ decrease}$$

Thinking

Step 1: Write down if it is an increase or decrease and calculate the difference.

Step 2: Substitute values into the percentage change formula and evaluate.

Step 3: Round the answer as specified.

Student practice

Calculate the percentage increase or decrease for the following, rounded to one decimal place where necessary.

a. 30 becomes 36

b. The population changes from 1540 to 1370.

Worked example 2

Calculating percentage profit and lo s

Determine whether the following describe a profit or loss and calculate the percentage. Round your answer to the nearest percentage where necessary.

a. Cost price = \$12.50, retail price = \$18 WE2a

Working

$$\text{Profit} = 18 - 12.50$$
$$= \$5.50$$

$$\text{Percentage profit} = \frac{5.50}{12.50} \times 100\%$$
$$= 44\%$$

A 44% profit was made.

Thinking

Step 1: Write down if it is a profit or loss and calculate the difference.

Step 2: Substitute values into the percentage profit formula and evaluate.

Step 3: Write the answer in words.

Visual support

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

18 − 12.5 = \$5.50 (profit)

\$12.50 (cost)

b. Michelle buys a theatre ticket for \$120, but falls sick. She sells it for \$75. WE2b

Working

$$\text{Loss} = 120 - 75$$
$$= \$45$$

$$\text{Percentage loss} = \frac{45}{120} \times 100\%$$
$$= 37.5\%$$
$$\approx 38\%$$

Michelle made a 38% loss.

Thinking

Step 1: Write down if it is a profit or loss and calculate the difference.

Step 2: Substitute values into the percentage loss formula and evaluate.

Step 3: Round the answer as specified.

Step 4: Write the answer in words.

Student practice

Determine whether the following describe a profit or loss and calculate the percentage. Round your answer to the nearest percentage where necessary.

a. Cost price = \$4, retail price = \$7

b. John buys a clock for \$25 and later sells it for \$10.

Worked example 3

Calculating percentage error

Calculate the percentage error of the following, to two decimal places.

a. 4.2 mm of rainfall is predicted, while actual rainfall is 4.5 mm. WE3a

Working

$$\text{Error} = 4.5 - 4.2$$
$$= 0.3$$

$$\text{Percentage error} = \frac{0.3}{4.5} \times 100\%$$
$$= 6.\overline{6}\%$$
$$\approx 6.67\%$$

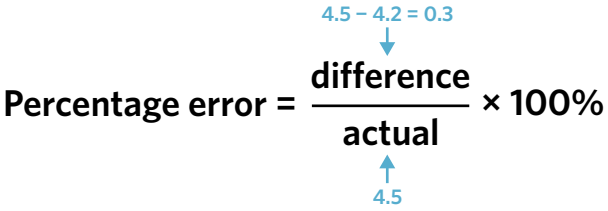
Thinking

Step 1: Calculate the error by taking the difference between predicted and actual values.

Step 2: Substitute values into the percentage error formula and evaluate.

Step 3: Round the answer as specified.

Visual support



b. Dante measures the length of a table as 80 cm but the actual length is 82 cm. WE3b

Working

$$\text{Error} = 82 - 80$$
$$= 2$$

$$\text{Percentage error} = \frac{2}{82} \times 100\%$$
$$= 2.\overline{43902}\%$$
$$\approx 2.44\%$$

Thinking

Step 1: Calculate the error by taking the difference between assumed and actual values.

Step 2: Substitute values into the percentage error formula and evaluate.

Step 3: Round the answer as specified.

Student practice

Calculate the percentage error of the following, to one decimal place.

a. Jayde predicts the temperature is 10°C when the actual temperature is 12°C.

b. Rhys weighs his dog and reads 7.80 kg on the scales but the actual reading is 7.00 kg.

3D Questions

Understanding worksheet

1. Circle the correct response.

Example

Cost/wholesale price = $1.50
Retail price = $1
This results in a
profit (loss) break-even

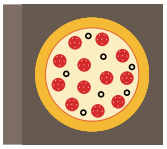
a.

Wholesale price = $12
Retail price = $39
This results in a
profit loss break-even

b.

Cost price = $19
Retail price = $15
This results in a
profit loss break-even

c.



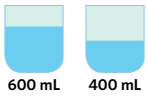
Wholesale price = $12.63
Retail price = $14.95
This results in a
profit loss break-even

d.

Cost price = $1.75
Retail price = $1.75
This results in a
profit loss break-even

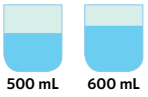
2. Fill in the blanks to calculate the percentage change for the following.

Example



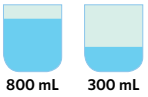
This is a $\frac{200}{600}$ × 100% decrease.

a.



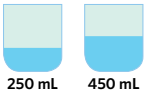
This is a $\frac{100}{}$ × 100% increase.

b.



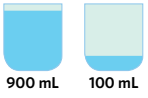
This is a $\frac{500}{}$ × 100% decrease.

c.



This is a $\frac{200}{}$ × 100% increase.

d.



This is a $\frac{800}{}$ × 100% decrease.

3. Fill in the blanks by using the words provided.

break-even | profi | loss | retail price

A _____ occurs when the cost price is lower than the _____, allowing the business to make money. _____ is when the cost price is equal to the retail price, whereas a _____ is when the cost price is higher than the retail price.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Calculate the percentage change for the following. WE1a

a. 10 becomes 15 **b.** 24 becomes 30 **c.** 35 becomes 28 **d.** 20 becomes 100
e. 48 becomes 72 **f.** 80 becomes 76 **g.** 60 becomes 135 **h.** 110 becomes 44

5. Calculate the percentage population increase or decrease for the following, rounded to one decimal place where necessary. WE1b

a. The population changes from 200 to 250.
b. The population changes from 1100 to 1000.
c. The population changes from 2350 to 2200.
d. The population changes from 10 500 to 15 000.
e. The population changes from 523 800 to 522 700.
f. The population changes from 4.99 million to 5.05 million.
g. The population changes from 3.08 million to 3.06 million.
h. The population changes from 123.6 million to 123.9 million.

6. Determine whether the following describe a profit or loss and calculate the percentage. WE2a

a. Cost price = $5, retail price = $6 **b.** Cost price = $10, retail price = $11
c. Cost price = $2, retail price = $1 **d.** Cost price = $1.20, retail price = $0.90
e. Cost price = $2.50, retail price = $2.75 **f.** Cost price = $2.20, retail price = $2.09
g. Cost price = $1.80, retail price = $4.95 **h.** Cost price = $9.20, retail price = $7.82

7. Determine whether the following describe a profit or loss and calculate the percentage. Round your answer to the nearest percentage where necessary. WE2b

a. Yas buys highlighter packs for $3 and sells them for $5.
b. Hussein sells drawings for $6 each where materials cost him $3 per drawing.
c. Suzi spends $10 on a lamp that she later sells for $8.
d. Liv buys a Science textbook for $60 and sells it for $35.
e. It costs Richmond $100 to buy paint and he charges $400 to complete a paint job.
f. Shivantha buys a printer for $120 and later sells it for $75.
g. Jac spends $450 on a smart TV that they sell for $300 a few years later.
h. Omid buys a car for $27 000 and sells it for $15 000 after three years.

8. Calculate the percentage error for the following, to two decimal places. WE3

- **a.** Paige predicts that their AFL team will win 15 games and there are 17 actual wins.
- **b.** Yifei estimates that she will give birth after 40 weeks, but her baby is born after 38 weeks.
- **c.** Jared guesses that there are 60 skittles in his bag, but counted 52 when he opened it.
- **d.** June believes the Eureka Tower has 89 floors but it actually has 91 floors.
- **e.** Starr estimates that he will get 95 questions correct, but he gets 99 questions correct.
- **f.** Noah won a jelly bean guessing competition, guessing 205 when the answer was 202.
- **g.** Leif measures his height as 1.60 m but his actual height is 1.62 m.
- **h.** Nandini expects 0.5 mm of rain but the actual rainfall is 10.2 mm.

9. Which of the following is the correct formula for calculating percentage loss?

- **A.** $\text{Percentage loss} = \frac{\text{profit}}{\text{cost}} \times 100\%$
- **B.** $\text{Percentage loss} = \frac{\text{loss}}{\text{retail price}} \times 100\%$
- **C.** $\text{Percentage loss} = \frac{\text{loss}}{\text{profit}} \times 100\%$
- **D.** $\text{Percentage loss} = \frac{\text{loss}}{\text{cost}} \times 100\%$
- **E.** $\text{Percentage loss} = \frac{\text{loss}}{\text{cost}}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the percentage increase or decrease when 30 becomes 24.

Student A

$\text{Increase} = 30 - 24$
$= 6$
Percentage change
$= \frac{6}{30} \times 100\%$
$= 20\%$ increase

Student B

$\text{Decrease} = 30 - 24$
$= 6$
Percentage change
$= \frac{6}{30} \times 100\%$
$= 20\%$ decrease

b. Calculate the percentage error where the expected result is 6 and the actual result is 5.

Student A

$\text{Error} = 6 - 5$
$= 1$
Percentage error
$= \frac{1}{5} \times 100\%$
$= 20\%$

Student B

$\text{Error} = 6 - 5$
$= 1$
Percentage error
$= \frac{1}{6} \times 100\%$
$= 16\frac{2}{3}\%$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. The height of a tree is measured each year. If it measured 2.5 m in 2022 and 3 m in 2023, by what percentage did the tree's height increase?

12. Raoul owns a furniture store and sells a particular type of rug for $150. If the cost price is $60, what is Raoul's percentage profit?

13. At the local supermarket, pastries that are close to their use by date are heavily reduced. If the cost price of a croissant is $1.83 and the discounted price is $0.95, what is the percentage loss? Round your answer to the nearest percentage.

14. Alanna thinks that she will get $\frac{39}{50}$ on her Physical Education test, but her actual mark is $\frac{43}{50}$. What was the percentage error in Alanna's prediction? Give your answer to one decimal place.

15. The maximum temperature in Melbourne on a winter day was 13°C, but decreased by 115% overnight. What was the overnight temperature, rounded to the nearest degree Celsius?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. The population in Australia over time is shown in the following table.

Year	Population
1970	12 595 034
1980	14 706 322
1990	17 048 003
2000	19 017 963
2010	22 019 168
2020	25 670 051

a. Round the population figures for 2010 and 2020 to the nearest million.

b. Using your answers from part **a**, what was the percentage increase in population between 2010 and 2020? Give your answer to one decimal place.

c. By what percentage did the population increase or decrease between 1970 and 2020? Round your answer to the nearest percentage.

d. Bingley believed that the population was at 26 million in 2020. Calculate the percentage error in Bingley's prediction, rounded to two decimal places.

e. Explain whether there might be more harm in human overpopulation or underpopulation for future generations.

17. Consider the numbers 3 and 4.

a. Calculate the percentage increase when 3 becomes 4.

b. Calculate the percentage decrease when 4 becomes 3.

c. Compare your answers for parts **a** and **b**. Explain why the order might or might not matter, in terms of which number is first and which number is second when calculating percentage change.

Extra spicy

18. When $\frac{13}{7}$ is written as a recurring decimal complete with the recurrence symbol, what percentage of digits are less than 5? Round your answer to the nearest percentage.

19. A clock is set correctly at 9 am. At 7 am the next day, the clock reads 5:54 am. What percentage of time has been lost?

20. In a square, the lengths of two sides are reduced by 10% whilst the lengths of the other two sides are increased by 10%, creating a rectangle. Compared to the area of the square, the area of the rectangle is:

A. The same **B.** 1% greater **C.** 1% less **D.** 5% greater **E.** 5% less

21. Profit of $800 is divided between Ben, Duncan and Jeremy. Ben receives $120 and Duncan receives two and a half times as much as Ben. Each person receives a set percentage of the total profit. If Jeremy wishes to receive exactly $570, what percentage would total profit need to increase or decrease by?

A. 16.25% decrease
B. 50% increase
C. 71.25% increase
D. 100% increase
E. 190% increase

Remember this?

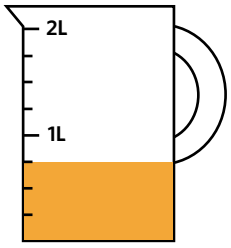
22. Determine which of the following options is equivalent to the below calculation.

2.305×100

A. 0.2305 **B.** 2.305 **C.** 23.05 **D.** 230.5 **E.** 2305

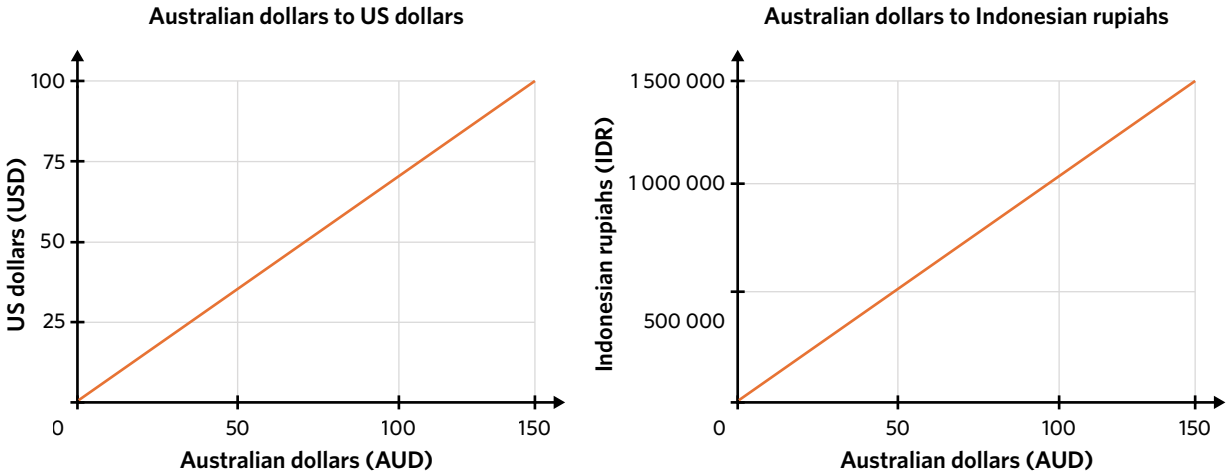
23. The following jug is partially filled with orange juice.

If Greg adds an extra 300 mL of orange juice to the jug, how many millilitres (mL) of orange juice will then be in the jug?



A. 1.2 L
B. 300 mL
C. 1050 mL
D. 1100 mL
E. 1200 mL

24. Julie uses the following currency conversion graphs.



Approximately how many Indonesian rupiahs are equal in value to 100 US dollars?
Select the best response.

A. 100 Indonesian rupiahs
B. 6667 Indonesian rupiahs
C. 15 000 Indonesian rupiahs
D. 1 000 000 Indonesian rupiahs
E. 1 500 000 Indonesian rupiahs

Back to contents

3E The unitary method

The unitary method is based off calculating the value when the quantity is one, and using that to either calculate original values, whole amounts or a particular percentage. It is also useful for determining what purchases are better value when decisions are based purely on price and quantity.

LEARNING INTENTIONS

Students will be able to:

- calculate best buys using the unitary method
- apply the unitary method with percentages
- determine original values after a percentage increase or decrease has been applied.

KEY TERMS AND DEFINITIONS

Best buy refers to comparing the price of two or more goods for the same quantity.

The **unitary method** is a process of calculation where the first step is to determine the value of one unit.

The **unit price** refers to the price for when the quantity is one.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

When looking at prices alone, it can be difficult to compare different quantities of the same product. Using the unitary method, it is possible to see what the cost would be if the quantity was the same. This is a possible strategy for saving money.

Key ideas

1. In the context of best buys, the unitary method calculates the price when the quantity is one unit. This is called the unit price. This allows prices to be compared when the quantities are different. The unit price may refer to the price per 100 grams, per 100 mL or for a single item.

2. Percentage mark-ups and percentage discounts are treated the same way as percentage increases and decreases.

For mark-ups:

Percentage of cost price = 100% + percentage of mark-up

For discounts:

Percentage of retail price = 100% − percentage of discount

Worked example 1

Determining the best buy

Determine the best buy out of the following.

a. Two mangoes for $3 or three mangoes for $5. WE1a

Working

Two mangos cost $3

One mango costs $3 ÷ 2 = $1.50

or

Three mangos cost $5

One mango costs $5 ÷ 3 = $1.67

Two mangos for $3 is the best buy.

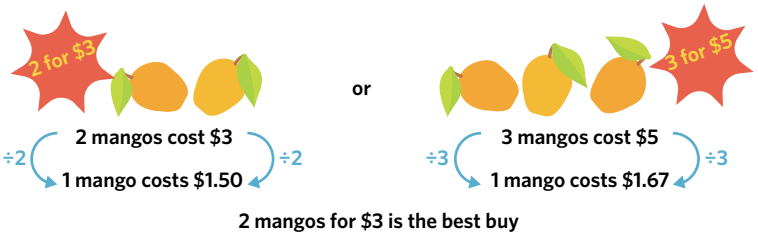
Thinking

Step 1: Calculate the cost for one unit.

Step 2: Compare and determine which option is less expensive per unit.

Step 3: State which item is the best buy.

Visual support



b. 500 mL bottle of body wash for $6.50 or 1 L bottle of body wash for $10. WE1b

Working

One unit = 100 mL

÷5 (500 mL costs $6.50 / 100 mL costs $1.30) ÷5

÷10 (1 L costs $10 / 100 mL costs $1) ÷10

The 1 L bottle is the best buy.

Thinking

Step 1: Determine the quantity of one unit.

Step 2: Calculate the cost for one unit.

Step 3: Compare and determine which option is less expensive per unit. State which item is the best buy.

Student practice

Determine the best buy out of the following.

a. Two jam donuts for $3 or five for $10.

b. 300 g of coconut sugar for $6.90 or 500 g for $17.

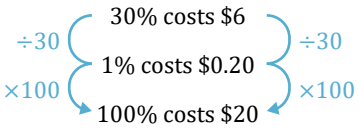
Worked example 2

Calculating the new percentage using the unitary method

For the following:

a. Calculate 100% of the cost if 30% is $6. WE2a

Working



100% of the cost is $20.

Thinking

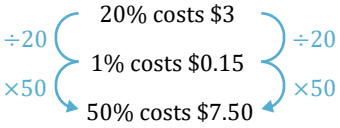
Step 1: Calculate 1% of the price using division.

Step 2: Calculate 100% of the price by multiplying by 100.

Step 3: Write the answer in a sentence.

b. If 20% of the price is $3, what is the cost of 50%? WE2b

Working



The cost of 50% is $7.50.

Thinking

Step 1: Calculate 1% of the price using division.

Step 2: Calculate 50% of the price by multiplying by 50.

Step 3: Write the answer in a sentence.

Student practice

For the following:

a. Calculate 100% of the cost if 20% is $3.

b. If 10% of the price is $2, what is the cost of 40%?

Worked example 3

Determining the original value after a percentage change has been applied

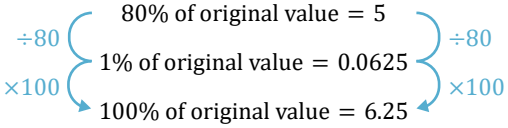
Calculate the following.

a. Original value if final value $= 5$, percentage decrease $= 20\%$ WE3a

Working

Final value percentage $= 100\% - 20\%$

$= 80\%$



The original value is 6.25.

Thinking

Step 1: Express the final value as a percentage given the decrease.

Step 2: Calculate 1% of the value using division.

Step 3: Calculate 100% of the value by multiplying by 100.

Step 4: Write the answer in a sentence.

Continues →

b. The cost price of a picture frame if the retail price is $9 and it is marked up by 80%. WE3b

Working

Percentage after mark-up = 100% + 80%
= 180%

÷180 ×100 180% costs $9 → 1% costs $0.05 → 100% costs $5 ÷180 ×100

The cost price is $5.

Thinking

Step 1: Express the final value as a percentage after the mark-up.

Step 2: Calculate 1% of the value using division.

Step 3: Calculate 100% of the value by multiplying by 100.

Step 4: Write the answer in a sentence.

Student practice

Calculate the following.

a. Original value if final value = 3, percentage decrease = 40%

b. The cost price of a folder if the retail price is $3 and it is marked up by 50%.

3E Questions

Understanding worksheet

1. Circle the best buy.

Example

The best buy is:

300g for $2.00

Price per 100g = $0.67

555g for $3.20

Price per 100g = $0.58

a. The best buy is:

4 for $6.70

$1.68 each

6 for $8.80

$1.47 each

b. The best buy is:

120 g for $4

$3.33 per 100 g

174 g $6.50

$3.47 per 100 g

c. The best buy is:

200 g for $2.75

$1.38 per 100 g

1 kg for $14.50

$1.45 per 100 g

d. The best buy is:

450 g for $6

$1.33 per 100 g

1.4 kg for $17.50

$1.25 per 100 g

2. Fill in the blanks.

Example

A mobile phone that has been marked up by 60%. The retail price is [160] % of the cost price.

a.

A book that has been marked up by 20%

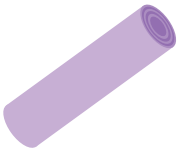
The retail price is [ ] % of the cost price.

b.

A bag of potatoes that has been marked up by 30%.

The retail price is [ ] % of the cost price.

c.



A yoga mat is discounted by 40%.

The discounted price is [ ] % of the original price.

d.

A hairdresser that has a 10% discount on all services.

The discounted price is [ ] % of the original price.

3. Fill in the blanks by using the words provided.

best buy	unitary method	unit price	one

The [ ] is used for a variety of calculations and the method involves finding the [ ], or the value when the quantity is [ ]. The [ ] describes the item that is better value when comparing identical quantities of the same item.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (b,c), 5 (b,c), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (c,d), 5 (c,d), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Determine the best buy out of the following. WE1a

a. One pack of Shapes biscuits for $2.75 or two packs for $5.

b. Two avocados for $5 or three avocados for $7.

c. A four-pack of highlighters for $7.98 or a six-pack for $9.99.

d. A 20-pack of toilet paper for $11 or a 24-pack for $13.60.

5. Determine the best buy out of the following. WE1b

- **a.** A 500 g bag of frozen peas for $2.30 or a 1 kg bag for $4.70.
- **b.** A 1.25 L bottle of spring water for $1.65 or a 1.5 L bottle for $2.
- **c.** A 140 g tube of toothpaste for $1.02 or a 180 g tube for $3.50.
- **d.** A 330 mL bottle of kombucha for $3.20 or a 700 mL bottle for $7.

6. Calculate 100% of the cost given the information provided. WE2a

- **a.** 50% of the cost is $2
- **b.** 20% of the cost is $5
- **c.** 25% of the cost is $8
- **d.** 200% of the cost is $30
- **e.** 40% of the cost is $24
- **f.** 75% of the cost is $81
- **g.** 92% of the cost is $67.16
- **h.** 135% of the cost is $166.05

7. Calculate the price for the percentage specified. WE2b

- **a.** If 50% of the price is $4, what is the cost of 25%?
- **b.** If 10% of the price is $3, what is the cost of 30%?
- **c.** If 20% of the price is $5, what is the cost of 75%?
- **d.** If 40% of the price is $6, what is the cost of 60%?
- **e.** If 25% of the price is $8, what is the cost of 80%?
- **f.** If 30% of the price is $12, what is the cost of 55%?
- **g.** If 15% of the price is $2.55, what is the cost of 40%?
- **h.** If 110% of the price is $9.90, what is the cost of 35%?

8. Calculate the original value of the following. WE3a

- **a.** Final value = 3, percentage decrease = 50%
- **b.** Final value = 4, percentage decrease = 20%
- **c.** Final value = 5, percentage increase = 25%
- **d.** Final value = 11, percentage increase = 10%
- **e.** Final value = 14, percentage decrease = 30%
- **f.** Final value = 24, percentage increase = 60%
- **g.** Final value = 32, percentage decrease = 80%
- **h.** Final value = 51, percentage increase = 2%

9. Calculate the specified price, rounding to the nearest cent where relevant. WE3b

- **a.** The retail price of a pack of batteries which is discounted by 50%, if the discounted price is $9.
- **b.** The retail price of a herbal tea pack which is discounted by 20%, if the discounted price is $2.40.
- **c.** The cost price of a shampoo bottle which is marked up by 40%, if the retail price is $5.
- **d.** The cost price of a 7.5 kg bag of dog food which is marked up by 30%, if the retail price is $19.23.
- **e.** The retail price of a pair of jeans which is discounted by 20%, if the discounted price is $31.96.
- **f.** The cost price of a roll of masking tape which is marked up by 60%, if the retail price is $2.80.
- **g.** The retail price of an oil paint set which is discounted by 20%, if the discounted price is $71.17.
- **h.** The cost price of a large woollen quilt which is marked up by 55%, if the retail price is $129.95.

10. If 15% of the cost is $3, what is the full cost?

A. $0.45 **B.** $2.55 **C.** $3.15 **D.** $6.67 **E.** $20

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Calculate the best buy if a 220 mL bottle of tomato sauce costs $1.70 and a 1 L bottle costs $6.

Student A

220 mL costs $1.70

100 mL costs $1.70 ÷ 2.20 = $0.77

or

1 L costs $6.00

100 mL costs $6.00 ÷ 100 = $0.60

The 1 L bottle is the best buy.

Student B

220 mL costs $1.70

or

1 L costs $6.00

$1.70 is cheaper than $6.00.

The 220 mL bottle is the best buy.

b. Dog treats are discounted by 20% and the discounted price is $5.20. Calculate the retail price.

Student A

Final value percentage = 20%

20% costs $5.20

1% = $5.20 ÷ 20

= $0.26

100% = $0.26 × 100

= $26

The retail price is $26.

Student B

Final value percentage = 100% − 20%

= 80%

80% costs $5.20

1% = $5.20 ÷ 80

= $0.065

100% = $0.065 × 100

= $6.50

The retail price is $6.50.

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

12. Prasad is looking to buy chew toys for his guinea pigs. He can buy a twin pack for $4 or a four-pack for $7. Which is less expensive per chew toy?

13. Emile creates handmade face cream at home and sells it online. If the mark-up is 150% and the retail price is $30, what is Emile's cost price?

14. Brian drinks 40% of the water from his drink bottle and there is 300 mL left. How much water was in the drink bottle to begin with?

15. Dexter buys ingredients to bake cookies for friends. He is choosing between a 200 g bag of chocolate chips for $3 and a 375 g bag for $5.50. Which option is the best buy?

16. Diep buys a portable speaker during a 10% storewide sale. She then decides to buy one for her sister the following year for $103.95, even though the sale has ended and the retail price has increased by 5%. What price did Diep pay for her own speaker?

Reasoning

Question working paths

Mild	Medium	Spicy
17 (a,b,d)	17 (a,b,d), 18 (a,b)	All

17. Reina is buying new school shoes for her twin sons and has the following options.

Spendlittle Shoes: $59 each

Larks Shoes: three pairs for $100

Billy's Shoes: $80 for the first pair and half-price on the second pair

a. If choosing between Spendlittle and Larks, which is the best buy?

b. If choosing between all three shoe shops, which is the best buy?

c. Splendlittle Shoes have a 40% mark-up on their shoes. What is their cost price?

d. Apart from price per unit, what other factors might be important when deciding which shoes to buy?

18. For the following:

a. solve for x, if $1.25x = 4.8$

b. solve for x, if $x + 0.25x = 4.8$

c. Compare your solutions for parts **a** and **b**. Why might someone prefer one method over the other?

Extra spicy

19. Ms Withers buys stationery as prizes for her students and needs to decide which prize is the least expensive per unit. Out of the following options, which is the best buy:

A. One plastic ruler for $0.50

B. Two-pack of correction tape for $1

C. Six-pack retractable gel pens for $2.25

D. Six-pack of highlighters for $3

E. Ten-pack gel ink pens for $4.50

20. Rupi is a textbook writer and it takes her longer to write her first lesson than subsequent lessons. It takes 49.1 hours to complete the second lesson, which is 18% less time than was needed for the first lesson. How long did Rupi take to write her first lesson? Round your answer to one decimal place.

21. During a sale, Hawra buys a pair of earrings at a 20% discount, but noted that the original price had been raised by 5% from when she first saw them. If the final price was $2.31, what was the original price of the earrings?

22. Felix buys a box of 50 chocolate frogs for $30 to fundraise money for his school musical. If he expects to sell 40 chocolate frogs and needs to make a profit of $20, what percentage mark-up should Felix use?

A. $8\frac{1}{3}\%$ **B.** 20% **C.** 50% **D.** $66\frac{2}{3}\%$ **E.** $108\frac{1}{3}\%$

Remember this?

23. Rune has $5.35 in coins. What is the least number of coins he can have?

A. 3 coins **B.** 4 coins **C.** 6 coins **D.** 8 coins **E.** 107 coins

24. Last week 1748 people attended a local footy match. The number of people attending this week is $\frac{3}{4}$ of last week's figure. How many people attended this week's match?

A. 437 **B.** 1311 **C.** 1714 **D.** 2185 **E.** 2331

25. Duyen collects Pokemon cards and decides to sell some of the duplicates at the following prices:

$1, $1, $2, $2, $5, $5, $7, $7, $10, $50. What is the median price?

A. $1 **B.** $5 **C.** $6 **D.** $9 **E.** $49

Chapter 3 extended application

1. Reina is deciding on a new mobile phone plan and chooses between the following.

Plan 1: $49 per month with 30 gigabytes (GB) of data and unlimited calls.

Plan 2: $58 per month with 40 GB of data and unlimited calls.

Plan 3: $68 per month with 180 GB of data and unlimited calls.

a. Express the monthly cost of Plan 1 as a fraction of Plan 2.

b. Rewrite your answer from part **a** as a percentage and a decimal, rounded to two decimal places.

c. The phone company for Plan 1 offers a 15% discount for a limited time. Calculate the discounted monthly price of Plan 1.

d. If choosing based on price per gigabyte of data, which plan is the best buy?

e. Price and data allowance are factors to consider when choosing a mobile plan. Name at least two other factors that should be considered.

f. Use the internet to research mobile plans to find the one that you think is best for you. Write down the phone company, monthly fee, data inclusion and an explanation of why the plan is right for you.

2. Ms Muscat wants to buy a new car and considers the following options.

	Razor RX-3	**Vroom Vell**	**Motor Moon Cross (hybrid)**	**Haiku HR-V**
Cost	$29 500	$31 300	$33 100	$36 700
Fuel economy	6.3 L/100 km	6.8 L/100km	5.4 L/100km	5.8 L/100km

a. Prices are currently increasing approximately 5% per year for vehicles. If the listed price is for this year, how much would the Razor RX-3 cost next year?

b. If it costs the dealership $30 615 to buy the Motor Moon Cross, what is their percentage profit? Round your answer to the nearest percentage.

c. All prices quoted include Goods and Services Tax (GST) of 10%. State the price of the Vroom Vell before GST, rounded to the nearest dollar.

d. Determine the cost price of the Haiku HR-V if a mark-up of 12% was applied.

e. Is a lower or higher value better for fuel economy? Based on your answer, state which car has the best fuel economy.

f. Based on the information provided, what car should Ms Muscat choose? Justify your choice.

3. Bhutan was the first country to officially consider the concept of happiness as a goal.

In 1972, the fourth King of Bhutan declared 'Gross National Happiness is more important than Gross Domestic Product'. In 2019, New Zealand was one of the first countries to base their government budget on the idea of well–being ahead of indicators for income and wealth. In 2022, the Australian government announced plans to follow in New Zealand's footsteps, which may lead to changes in Australia's happiness index ratings.

The following table shows Australia's happiness index ratings from 2017–2022.

Year	Happiness Index Rating (out of 10)
2017	7.284
2018	7.272
2019	7.228
2020	7.223
2021	7.183
2022	7.162

a. What was the percentage increase or decrease in Australia's happiness index rating between 2017 and 2022? Round your answer to one decimal place.

b. Mr Robertson predicts that the happiness index rating for 2023 will increase by 0.5% compared to 2022. Calculate Mr Robertson's index rating, rounded to three decimal places.

c. Miss Qin predicted that the rating for 2022 would be the same as the figure for 2019. What was the percentage error in her prediction? Round your answer to two decimal places.

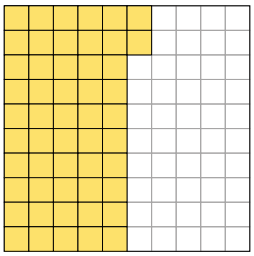
d. If the 2017 happiness index rating in Australia is 0.4% higher than the figure for 2016, what was the rating for 2016? Give your answer to three decimal places.

e. Dr Victor Frankl wrote that, 'when we are no longer able to change a situation, we are challenged to change ourselves.' In the context of happiness and the events of the Coronavirus pandemic, do you believe that it is possible to adapt to circumstances and find happiness, despite hardship? Discuss.

Chapter 3 review

Multiple choice

1. What percentage, fraction and decimal is coloured in the following visual? 3A



A. 0.42, 42%, $\frac{42}{100}$

B. 0.52, 52%, $\frac{52}{100}$

C. 0.62, 62%, $\frac{62}{100}$

D. 0.72, 72%, $\frac{72}{100}$

E. 0.82, 82%, $\frac{82}{100}$

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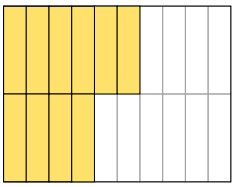
2. Which of the following is the correct percentage described in the visuals? 3B



A. 10%

B. 30%

C. 50%

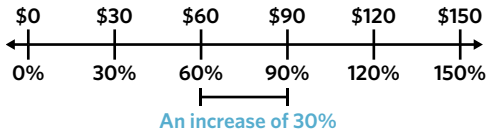
D. 70%

E. 100%

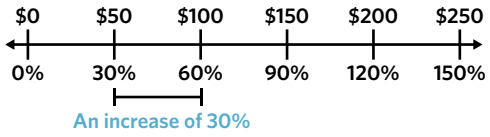
---

3. Which of the following represents \$50 increased by 30%? 3C

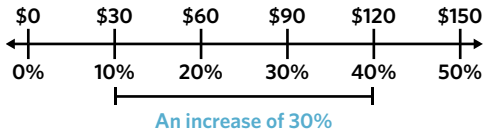
A.



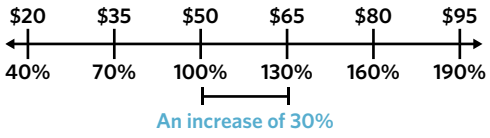
B.



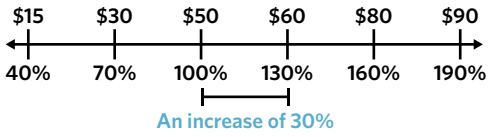
C.



D.



E.



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4. Tia bought a \$36 jacket and resold it for \$45. Which of the following equations represent her percentage profit? 3D

A. $\frac{9}{45} \times 100\% = 20\%$

B. $\frac{9}{36} \times 100\% = 25\%$

C. $\frac{45}{100} \times 100\% = 45\%$

D. $\frac{75}{100} \times 100\% = 75\%$

E. $\frac{36}{45} \times 100\% = 80\%$

5. Determine the best buy out of the following. 3E

A.  5 for $20

B.  2 for $10.50

C.  1 for $3.50

D.  6 for $19

E.  4 for $12

Fluency

6. Convert the following decimals to percentages. 3A

a. 0.75 **b.** 0.95 **c.** 1.97 **d.** 0.035

7. Convert the following decimals to a proper fraction or mixed numbers. 3A

a. 0.45 **b.** 2.5 **c.** 380.3 **d.** 0.672

8. Express the following as a percentage. Give your answer to 1 decimal place. 3B

a. 34 out of 95 **b.** 7 out of 24 **c.** 6.5 out of 3 **d.** 85 out of 135

9. Evaluate the following. Give your answer to 1 decimal place. 3B

a. 25% of 65 **b.** 87% of 124 **c.** 120% of 73 **d.** $35\frac{1}{4}$% of 180

10. Determine the new price of the following. 3C

a. $60 is increased by 15%

b. $50 is increased by 60%

c. $5.20 is decreased by 10%

d. $414 is decreased by 70%

11. Determine the price before GST for the following. 3C

a. A ring whose selling price is $95.

b. A massage where the customer pays $37.50.

c. A laptop whose RRP is $1280.99.

d. A scientific calculator whose selling price is $42.85.

12. Calculate the percentage change for the following. 3D

a. 10 becomes 11 **b.** 15 becomes 12 **c.** 30 becomes 150 **d.** 125 becomes 85

13. Determine whether the following describe a profit or loss and calculate the percentage. 3D

a. Cost price = $8, retail price = $10

b. Cost price = $12, retail price = $18

c. Cost price = $50, retail price = $28

d. Cost price = $2.50, retail price = $1.50

14. Determine the best buy out of the following. 3E

a. One can of corn for $0.90 or five cans for $3.90.

b. A five-pack of pens for $3.20 or a twenty-pack for $22.50.

c. 1 L of milk for $2.16 or 3 L of milk for $4.35.

d. A 250 mL bottle of Fanta for $2.24 or a 750 mL bottle for $7.45.

---

15. Calculate the original value of the following. 3E

a. Final value = 15, percentage decrease = 50%

b. Final value = 30, percentage increase = 20%

c. Final value = 45, percentage decrease = 25%

d. Final value = 204, percentage increase = 36%

Problem solving

16. Jimmy ate three-eighths of a pizza and saved the rest for the next day. What percentage of pizza did Jimmy save for the next day? 3A

---

17. Daisy was aiming to do 15 laps at the local swimming pool. She was only able to complete 60% of her goal. How many laps did she swim in total? 3B

---

18. Berty has a budget of $500 and wants to buy a new phone. The one he wants costs $756. After a couple of months, the phone has been discounted by 25%. Calculate and determine whether or not Berty has enough money? 3C

---

19. Lilly bought all of the art supplies to create a painting. The resources were $95 in total. Lilly then had to pay $12 to advertise her artwork online. She sold the painting for $115. Calculate the percentage profit or loss. Give your answer to one decimal place. 3D

---

20. Morgan needs fresh fruit for a grazing box. He looks at two different stalls at a market. Stall A has a 1 kg bag of grapes for $15.20 and 250 g of strawberries for $5.25. Stall B has a 500 g bag of grapes for $11 and 500 g of strawberries for $8.50. Which stall has the best buy for grapes and which stall has the best buy for strawberries? 3E

Reasoning

21. Ava has a small shop, where she sells an array of items. The items are listed below with the prices.

Item	Button-up shirts: 	Fun socks: 	Bucket hats: 	Hoodies: 
Sale price per unit	$15	$7.50	$12.50	$45.00

a. On the same day, Ava had 75 pairs of socks in stock. Ava was able to sell 20% of the stock by the end of the day. How many pairs of socks did Ava sell?

b. On a particular day Ava had 40 button up shirts in stock. She was able to sell $\frac{1}{4}$ of her stock that day. Write the percentage that represents portion shirts she sold on that particular day.

c. Ava decided to mark-up the price of the bucket hats by 46%. What is the new price of the bucket hats?

d. Ava bought a bundle of 35 shirts from her supplier for $420. What is the profit percentage when she sells the shirts in her shop?

e. What are some other items of clothing that would be appropriate for Ava to sell at her stall?

22. Consider the number 120.

a. Increase the value by 30%.

b. Using your answer from part **a**, decrease the value by 30%.

c. Compare your answer from **b** with 120. Comment on why the values may be the same or may vary.

Back to contents

04

Indices

Number and Algebra

Research summary

4A Index notation

4B Multiplying indices

4C Dividing indices

4D Raising powers

Chapter 4 extended application

Chapter 4 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 4 research summary

Indices

Big ideas

Multiplicative thinking

This chapter will extend multiplicative thinking regarding the negative sign, additive and multiplicative inverses, and additive and multiplicative identities to build structural and conceptual knowledge of indices.

This chapter will combine students' understanding of the negative sign as a subtraction (a binary operation) and additive inverse (a unary operation). For example, the expression $(-4)^2 - 9^2$ has negative signs representing a binary operation and a unary operation.

Using colloquial language, such as 'opposite', is a great way to engage students in the language of mathematics, however, using more formal language such as 'additive inverse' can strengthen students' understanding of exponents when bases are negative (Cangelosi et al., 2013).

For example, -4^2 and $(-4)^2$ is often described as 'negative four to two', but this phrase makes no distinction between the two numbers and can lead to misconceptions about exponents into college years (Cangelosi et al., 2013) and impact algebraic reasoning (Booth et al., 2016; Booth, 2014; Bush 2011). Cangelosi et al., (2013) writes one way to avoid ambiguity is by reading -4^2 as 'the additive inverse of [pause] four to two' and by reading $(-4)^2$ as 'the quantity negative four [pause] to two'.

This big idea is built upon a strong understanding of the negative sign, additive and multiplicative inverses and identities. Therefore, understanding the relationship of the negative and positive bases of indices is essential for this chapter and future lessons such as algebra, equations, and geometry and impacts the future big ideas of generalising.

Proportional reasoning

This chapter will focus on students' ability to identify and apply their extensive multiplication, division, fractions, and number sense knowledge to build structural and conceptual knowledge of indices.

Within this chapter, students' knowledge of multiples, factors, and fractions will provide additional information about relationships of those numbers in the form of exponents. For example, $4 \times 4 = 4^2$ and is less than $\frac{4^3}{4^3}$.

Additionally, students need a strong foundation of multiples, factors and repeated addition to work with indices (Ulusoy, 2019). For example $y + y + y + y = 4y$, and $y^4 = y \times y \times y \times y$.

Strong knowledge of multiples, factors and fractions is needed to connect exponential proportions and their laws to increase conceptual understanding (Bush, 2011), and reduce the misconception of numbers raised to the 1 and 0 power. For example, $\frac{3 \times 3}{3 \times 3} = \frac{9}{9} = 1$ and $\frac{3^2}{3^2} = 3^{2-2} = 3^0 = 1$.

Understanding this big idea is built upon place value, multiplicative thinking, and fractions. Therefore, it is not only essential for this chapter and future lessons but heavily impacts future big ideas in mathematics, such as generalising and algebraic reasoning.

Generalising

This chapter will focus on combining multiplicative thinking and proportional reasoning skills to recognise and represent patterns and relationships of indices. Additionally, this chapter will build structural and conceptual knowledge of the properties of indices and their symbolic relationship.

Within this chapter, students will be exposed to workable procedures of index laws. However, misconceptions can occur when index rules are over generalised (Cangelosi et al., 2013). Relating indices to functions and geometry can lead to an increased conceptual understanding of index laws. For example, x is linear and represents length, x^2 is quadratic and represents area, and x^3 is cubic and represents volume; therefore, is it possible to add the area of an object to its volume $(x^2 + x^3)$?

By using patterns to explain indices, students can expand their knowledge about patterns of numbers which could lead to students relying more on their sense-making skills and lessen their reliance on computation (Slavit, 2006). For example, if you were to evaluate 5^{16}, the digit in the ones' place will be 5 and $(-4)^{13}$ will result in a negative number.

Additionally, a structural understanding of indices is needed within this chapter. Good mathematical conventions underpin conceptual understanding of indices, and these conventions are essential for the communication of mathematics. Unfortunately, students often come to the correct answer without the correct mathematical conventions (Cangelosi et al., 2013) and often see that the parentheses are optional or have personal shorthand notation. For example, the problem might ask students to evaluate $(-3)^2$, and students write, $-3^2 = 9$ ignoring the brackets.

Understanding the idea of exponents, negative exponents, and how to generalise properties of indices and their symbolic relationship is needed for factoring, solving equations, identifying shapes, and shapes of functions. Furthermore, understanding the conceptual and procedural knowledge of indices underpins students' algebraic reasoning skills throughout the rest of the book.

Misconceptions

Misconception	Incorrect	Correct	Lesson
Students multiply the base number by the index.	$2^3 = 2 \times 3$	$2^3 = 2 \times 2 \times 2$	4A
Students do not include the base number when writing numbers in index form.	$6 \times 6 \times 6 \times 6 = 6^3$	$6 \times 6 \times 6 \times 6 = 6^4$	4A
Students use repeated addition to express numbers in expanded form.	$3^5 = 3 + 3 + 3 + 3 + 3$	$3^5 = 3 \times 3 \times 3 \times 3 \times 3$	4A
Students misunderstand the meaning of the negative sign when calculating indices.	$-4^2 = 16$ $(-4^2) = -16$	$-4^2 = -16$ $(-4^2) = 16$	4A 4B
Students multiply/divide different bases and add/subtract the indices when simplifying numbers with indices.	$5^3 \times 4^3 = (5 \times 4)^{3+3} = 20^6$ $\frac{12^4}{4^2} = (12 \div 3)^{4-2} = 3^2$	$5 \times 5 \times 5 \times 4 \times 4 \times 4 = 5^3 \times 4^3$ and cannot be simplified further. $\frac{12 \times 12 \times 12 \times 12}{4 \times 4} = \frac{12^4}{4^2}$ and cannot be simplified further.	4B 4C
Students multiply/divide the bases when simplifying indices.	$7^4 \times 7^3 = (7 \times 7)^{4+3} = 49^7$ $\frac{9^6}{9^3} = \left(\frac{9}{9}\right)^{6-3} = 1^3$	$7^4 \times 7^3 = 7^{4+3} = 7^7$ $\frac{9^6}{9^3} = 9^{6-3} = 9^3$	4B 4C
Students use repeated addition, or multiply the indices or a combination of both when simplifying numbers with indices.	$5^3 \times 5^2 = (5 + 5)^{3+2} = 10^5$ $8^4 \times 8^5 = 8^{4\times5} = 8^{20}$	$5^3 \times 5^2 = 5^5$ $8^4 \times 8^5 = 8^9$	4B
Students divide indices similar to how they divide integers when simplifying numbers with indices.	$\frac{11^8}{11^2} = 11^{8\div2} = 11^4$	$\frac{11^8}{11^2} = 11^{8-2} = 11^6$	4C
When raising powers, students add the powers together.	$(3^4)^2 = 3^{4+2} = 3^6$	$(3^4)^2 = 3^{4\times2} = 3^8$	4D
Students do not raise the coefficients inside the brackets to the power outside the bracket.	$(5r^5)^3 = 5r^{5\times3} = 5r^{15}$	$(5r^5)^3 = 5^3r^{5\times3} = 5^3r^{15}$	4D

References and additional readings

Booth, J. L., Barbieri, C., Eyer, F., & Pare-Blagoev, E. J. (2014). Persistent and Pernicious Errors in Algebraic Problem Solving. *The Journal of Problem Solving*, 7(1). https://doi.org/10.7771/1932-6246.1161

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Back to contents

4A Index notation

Indices are used to represent repeated factors of the same number. They can be used to investigate things that grow by repeated multiplication. There is a particular way that we write numbers with indices so that they can be understood by everyone who uses them.

LEARNING INTENTIONS

Students will be able to:

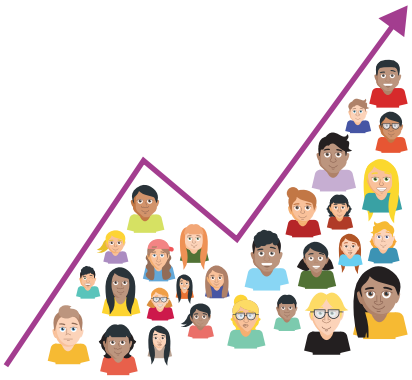
- know the meaning of the terms base, index, expanded form, and index form
- convert between expanded form and index form
- evaluate indices as whole numbers in expressions.

KEY TERMS AND DEFINITIONS

Index notation is a way of representing repeated factors of the same number.

Expanded form is a way of representing repeated multiplications (factors) of all numbers in an expression.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Numbers with indices are used to predict the growth of the human population. This is important to help plan food production, how many houses and schools to build.

Key ideas

1. A number written in index notation consists of a base and an index (also called power).

Base — 8^3 — Index

2. When expressed in expanded form, the index (or power) indicates the number of repeated factors of the base.

$$8^3 = \overbrace{8 \times 8 \times 8}^{\text{3 factors of 8}}$$

Worked example 1

Identifying the base and index of a number

Identify the base and index of the following expressions.

a. 5^3 WE1a

Working

The base is 5.

The index is 3.

Thinking

Step 1: Identify the base (first number).

Step 2: Identify the index (top right of the base).

Visual support

Base — 5^3 — Index

Continues →

b. x^5 WE1b

Working

The base is x.

The index is 5.

Thinking

Step 1: Identify the base (first number).

Step 2: Identify the index (top right of the base).

Student practice

Identify the base and index of the following expressions.

a. 2^7 **b.** z^4

Worked example 2

Converting between expanded form and index form

Write the following expressions in the described form.

a. $7 \times 7 \times y \times y \times y$ (index form) WE2a

Working

$7 \times 7 = 7^2$

$y \times y \times y = y^3$

$7 \times 7 \times y \times y \times y = 7^2 \times y^3$

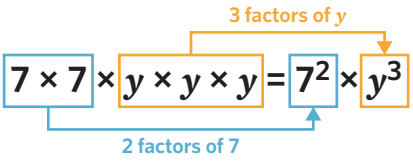
Thinking

Step 1: Write the first base in index form by counting the number of repeated factors.

Step 2: Repeat for the other base.

Step 3: Write the expression in index notation.

Visual support



b. $a^4 \times 4^2$ (expanded form) WE2b

Working

$a^4 = a \times a \times a \times a$

$4^2 = 4 \times 4$

$a^4 \times 4^2 = a \times a \times a \times a \times 4 \times 4$

Thinking

Step 1: Write the first base as repeated factors. The index indicates the number of repeated factors.

Step 2: Repeat this for the other bases.

Step 3: Write the expression in expanded form.

Student practice

Write the following expressions in the described form.

a. $8 \times 8 \times 8 \times 8 \times m \times m$ (index form)

b. $2^5 \times y^2$ (expanded form)

Worked example 3

Evaluating expressions with indices

Evaluate each expression using order of operations.

a. $3^4 - 4 \times 2$ WE3a

Working

$3^4 - 4 \times 2 = 81 - 4 \times 2$

$= 81 - 8$

$= 73$

Thinking

Step 1: Evaluate the expression according to the order of operations. Indices are evaluated first.

Step 2: Evaluate the multiplication.

Step 3: Evaluate the subtraction.

b. $(8 \div 4) + 2^5$ WE3b

Working

$(8 \div 4) + 2^5 = 2 + 2^5$

$= 2 + 32$

$= 34$

Thinking

Step 1: Evaluate the expression according to the order of operations. The expression in the brackets is evaluated first.

Step 2: Evaluate the index.

Step 3: Evaluate the addition.

Student practice

Evaluate each expression using order of operations.

a. $16 \div 4 + 2^4$ **b.** $8 \times 3^3 - (12 - 9)$

4A Questions

Understanding worksheet

1. Fill in the missing boxes with the bases and indices.

Example

Base	Index	Index form
8	6	8^6

	Base	Index	Index form
a.	4	6	
b.	7	2	
c.	x	3	
d.	12	b	

2. Fill in the boxes to convert from expanded form to index form.

Example

$4 \times 4 \times 4 \times 4 \times 4 = 4^{5}$

a. $2 \times 2 \times 2 \times 2 = 2^{\square}$

b. $9 \times 9 \times 9 = 9^{\square}$

c. $y \times y \times y \times y \times y \times y = y^{\square}$

d. $3 \times 3 \times 5 \times 5 \times 5 = 3^{\square} \times 5^{\square}$

3. Fill in the blanks by using the words provided.

base | power | factors | expanded form

An expression written in index form consists of a _____ and an index. The index indicates how many _____ of the base are in written in the _____. The index is also referred to as a _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Identify the bases and indices in the following expressions. WE1

a. 7^5 **b.** 2^3 **c.** 1^8 **d.** 12^3

e. 9^{11} **f.** p^2 **g.** y^{15} **h.** a^c

5. Write the following expressions in index form. WE2a

a. $8 \times 8 \times 8 \times 8$

b. $3 \times 3 \times 3$

c. $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

d. $5 \times 6 \times 5$

e. $0.2 \times 4 \times 4 \times 0.2 \times 4 \times 7$

f. $k \times 9 \times k \times k \times k \times 9$

g. $3 \times x \times y \times x \times z \times 3 \times 4$

h. $m \times 5.3 \times 5.3 \times 1 \times m \times n \times n \times n \times 5.3$

6. Write the following expressions in expanded form. WE2b

a. 6^3

b. 4^6

c. $2^2 \times 9^4$

d. $a^3 \times 8^6$

e. $1.7^4 \times 10^1 \times 5^4$

f. $a^3 \times 8^2 \times 13.2^1 \times x^2 \times y^5$

g. $q^3 \times r^4 \times s^1$

h. $n^3 \times m^2 \times 8 \times 3^4 \times k^4$

7. Evaluate the following expressions using order of operations. WE3a

a. $8^2 + 9 - 6$

b. $3 \times 4 - 1^3$

c. $8 \div 2^2 - 4$

d. $3^2 + (12 - 1)$

e. $(19 + 2^2) \times 3 - 8^2$

f. $(6 + 7) \times (9 - 4) - 9^2$

g. $(3^4 - (2 + 3^2)) \div 5$

h. $(9 - 2)^2 + 20 \div (26 - 2^4)$

8. Which option shows the expression $3^2 \times 7^3$ in expanded form?

A. $3 + 3 + 7 + 7 + 7$

B. $3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 7$

C. $3 \times 2 \times 7 \times 3$

D. $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

E. $3 \times 3 \times 7 \times 7 \times 7$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Write $3^2 \times b^3 \times j$ in expanded form.

Student A

$3^2 \times b^3 \times j = 3 \times 2 \times 3 \times b \times j$

$= 6 \times 3b \times j$

Student B

$3^2 \times b^3 \times j = 3 \times 3 \times b \times b \times b \times j$

b. Write $4 \times 3 \times 4 \times a \times 3 \times a \times 4$ in index form.

Student A

$4 \times 3 \times 4 \times a \times 3 \times a \times 4 = 4^3 \times 3^2 \times a^2$

Student B

$4 \times 3 \times 4 \times a \times 3 \times a \times 4 = 4^2 \times 3^1 \times a^1$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Julia received 5 Amazon gift cards for Christmas. If she can buy 5 movies with each card, how many movies can she buy in total?

11. Fatima is stacking LEGO blocks. Every day, the number of blocks she stacks is equal to double the number of blocks she had the previous day. On day 1, she stacks 2 blocks. Write an expression in expanded form to represent the number of blocks she stacks on day 4.

12. When bread is left out at room temperature, it begins to spoil and mould begins to grow. The initial area of the mould is 2 cm^2 and multiplies by 1.5 every day. How big is the area of mould on a piece of bread after 3 days?

13. John receives $10 of pocket money every week. In the first week that Tanaka was given pocket money she received $2. Afterwards, her parents gave double the amount from the previous week. Assuming that they both started receiving pocket money in the same week, after seven weeks, who has more money and by how much?

14. At 1 pm, Kareem, Geoff and Bela, were told a secret. One hour later, they each told the secret to three of their own friends. Every hour each person who hears the secret tells three other people. At what time were 243 people told the secret, excluding Kareem, Geoff, and Bela?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Megan is studying two species of bacteria (singular bacterium) called *E. coli* and *D. coli*. She observed two samples of species bacteria under a microscope. She counted 40 individuals each. She knows that *E. coli* reproduces two identical copies every hour whereas *D. coli* reproduces three identical copies.

Image: fusebulb/Shutterstock.com

For *E. coli*

- 1 hour after counting the bacteria, Megan expects there to be $40 \times 2 = 80$ bacteria.
- 2 hours after counting the bacteria, Megan expects there to be $80 \times 2 = 40 \times 2 \times 2 = 160$ bacteria.

a. Write the expression $40 \times 2 \times 2 \times 2$ in index form.

b. How many *E. coli* bacteria does Megan expect there to be 4 hours after counting them?

c. What does the index of the base 2 represent?

d. How many more *D. coli* compared to *E. coli* does Megan expect after 4 hours?

e. After many days, the number of bacteria will be very large. Devise a way that Megan could approximate the number of bacteria, instead of counting, to verify her predictions.

16. Fill in the box with 1, 2, 3 or 4 to find the number with the:

$(-3)^{\square}$

a. Greatest number

b. Smallest number

c. Compare the values of your answers from parts **a** and **b** and explain the difference between negative bases with odd powers and negative bases with even powers.

Extra spicy

17. Jared deposits $500 in a bank account. Every year, the amount in the bank account increases by 3%.

How much money, to the nearest dollar, will be in the bank account after 4 years?

A. $560 **B.** $563 **C.** $672 **D.** $1428 **E.** $2060

18. A 10 g sample of radioactive material halves in mass every 45 minutes.

Which of the following expressions evaluates the mass of the sample after 24 hours?

A. 10×0.5^{32} **B.** 10×2^{32} **C.** 0.5^{32} **D.** 0.5^{45} **E.** 10×2^{24}

19. 78 written as a sum of powers of 2 is $2^6 + 2^3 + 2^2 + 2^1$.

Write the number 90 as a sum of powers of 2.

20. A particular combination lock has five spinners that can be combined in different ways to create a passcode. The first three of the spinners have the digits from 0 to 9. The final two spinners have the letters a, b and c.

How many possible passcode combinations are there?

Remember this?

21. Luk painted $\frac{5}{12}$ of his bedroom wall.

What fraction of their bedroom room did Amiel paint if they painted the same amount as Luk?

A. $\frac{5}{15}$ **B.** $\frac{9}{12}$ **C.** $\frac{5}{6}$ **D.** $\frac{10}{24}$ **E.** $\frac{7}{12}$

22. In which of the following options are all numbers equivalent?

A. 65%, 0.65, $\frac{5}{6}$

B. 650%, 65, $\frac{650}{100}$

C. 0.65%, 0.0065, $\frac{65}{10000}$

D. 0.65%, 0.65, $\frac{65}{100}$

E. 65%, 65, $\frac{65}{100}$

23. Which one of these has the same value as 16^2?

A. 16×2 **B.** $4 \times 4 \times 4$ **C.** 2×8^2 **D.** $8 \times 4^2 \times 2$ **E.** $32^2 \div 2$

Back to contents

4B Multiplying indices

Operations on numbers written in index form are governed by the Index Laws. The first of these laws outlines the rule for multiplying indices. The base and index of any number written in index form must be considered before multiplication can be performed.

LEARNING INTENTIONS

Students will be able to:

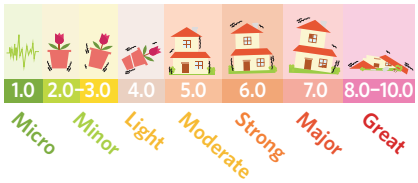
- multiply terms with the same base in index form
- simplify expressions containing multiplication of terms in index form.

KEY TERMS AND DEFINITIONS

The **First Index Law** states to add the indices (or powers) when multiplying terms with the same base in index form.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Earthquake magnitude scale



The multiplication of numbers in index form is the basis for many complex systems. For example, a logarithmic scale is used to measure the strength of earthquakes and involves the multiplication of index terms with base 10.

Key ideas

1. The First Index Law only applies to terms with the same base.

$8^2 \times 8^3$ — Same base – law applies

$4^5 \times 3^2$ — Different bases – law **does not** apply

2. When multiplying terms with the same base, add the indices.

$$x^p \times x^q = x^{p+q}$$

$$8^2 \times 8^3 = 8^{2+3} = 8^5$$

Worked example 1

Using the First Index Law

Multiply the following.

a. $2^4 \times 2^6$ WE1a

Working

$= 2^{(4+6)} = 2^{10}$

Thinking

Identify the common base of the terms being multiplied. Add the indices of the terms to get the index of the product.

Visual support

Add the indices 4 and 6

$2^4 \times 2^6 = 2^{4+6} = 2^{10}$

Continues →

b. $x^7 \times x^2$ WE1b

Working

$= x^{(7+2)} = x^9$

Thinking

Identify the common base of the terms being multiplied. Add the indices of the terms to get the index of the product.

Student practice

Multiply the following.

a. $3^5 \times 3^4$ **b.** $y^2 \times y^3$

Worked example 2

Simplifying expressions by using the First Index Law

Simplify the following expressions.

a. $3 \times 7^5 \times 3^4 \times 7^6$ WE2a

Working

$= 3^{(1+4)} \times 7^{(5+6)}$

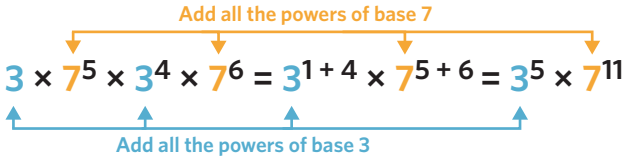
$= 3^5 \times 7^{11}$

Thinking

Step 1: Identify the terms with common bases and add their indices.

Step 2: Simplify the expression.

Visual support



b. $a^4 \times b^4 \times a^3 \times b^5$ WE2b

Working

$= a^{(4+3)} \times b^{(4+5)}$

$= a^7 \times b^9 = a^7b^9$

Thinking

Step 1: Identify the terms with common bases and add their indices.

Step 2: Simplify the expression.

c. $2x^3 \times 3y^6 \times x^4 \times 5y^2$ WE2c

Working

$= 2 \times 3 \times 5 \times x^3 \times x^4 \times y^6 \times y^2$

$= 30 \times x^{(3+4)} \times y^{(6+2)}$

$= 30x^7y^8$

Thinking

Step 1: Identify the terms with common bases and group them together.

Step 2: Multiply the numbers together. Add the indices of terms with common bases.

Step 3: Simplify the expression.

Student practice

Simplify the following expressions.

a. $2^2 \times 5^3 \times 2 \times 5^5$ **b.** $u^6 \times v^7 \times u^4 \times v^3$ **c.** $4t^3 \times 5r^2 \times t^5 \times 2r^6$

4B Questions

Understanding worksheet

1. Determine whether the First Index Law applies to each of the following expressions.

Example

Expression	Law applies	Law does not apply
$3^7 \times 3^9$	☑	☐

	Expression	Law applies	Law does not apply
a.	$5^6 \times 5^7$	☐	☐
b.	$2^{10} \times 4^6$	☐	☐
c.	$b^4 \times c^5$	☐	☐
d.	$2x \times x^4$	☐	☐

2. Fill in the missing indices so that each of the following expressions is true.

Example

$5^3 \times 5^{\boxed{7}} = 5^{10}$

a. $4^2 \times 4^{\square} = 4^7$

b. $7^{\square} \times 7 = 7^5$

c. $(-3)^4 \times (-3)^{\square} = (-3)^6$

d. $5y^{\square} \times 2y^9 = 10y^{12}$

3. Fill in the blanks by using the words provided.

added | index | bases | First | simplify

When we _____ expressions, we must always check if the _____ Index Law can be applied to any terms written in _____ form. It states that only terms with common _____ can be multiplied and their indices must be _____ together.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Multiply. WE1

a. $6^2 \times 6^4$
b. $1^3 \times 1^9$
c. $10^7 \times 10^4$
d. $t^3 \times t^{10}$
e. $(-4)^5 \times (-4)^3$
f. $5^9 \times 5^2 \times 5^{10}$
g. $y \times y^{11} \times y^3$
h. $(-z)^7 \times (-z)^5 \times (-z)$

5. Simplify. WE2a

a. $2^5 \times 2^2 \times 4^6 \times 4^3$
b. $3^{10} \times 3^4 \times 8 \times 8^7$
c. $5^2 \times 7^3 \times 5^2 \times 7^3$
d. $10^3 \times 4^7 \times 4^{10} \times 10^5$
e. $12^4 \times 8^3 \times 12^{10} \times 8^4$
f. $(-11)^3 \times (-11)^9 \times 6^{11} \times 6^9$
g. $(-7)^2 \times (-8)^4 \times (-7)^8 \times (-8)^{12}$
h. $4 \times 5^7 \times (-3)^{13} \times 5^8 \times (-3)^{11}$

6. Simplify. WE2b,c

a. $x^3 \times x^2 \times y^6 \times y^2$
b. $t^4 \times r^2 \times t^7 \times r^{10}$
c. $u^{10} \times v^4 \times u \times v^5$
d. $a \times c \times a \times c$
e. $2y^3 \times y^2 \times x^5 \times x^3$
f. $4x^2 \times 3y^4 \times x^{10} \times y^8$
g. $7g^4 \times 3h \times 3g^9 \times 10h^8$
h. $-5d^6 \times 2f^3 \times 3d^5 \times 2f^9$

7. Simplify.

a. $4x^2 \times x \times 7x^5$
b. $4 \times a^3 \times b^4 \times a^2$
c. $2^5 \times x^2 \times 2^2 \times x^4$
d. $(-j)^6 \times 5^2 \times (-j)^7 \times 5$
e. $3y^7 \times 3^2 \times 5y \times 3^4$
f. $-6t^2 \times 5^2 \times 3t^{13} \times 2^7$
g. $k^4 \times (-k)^2 \times 5k^8 \times 2(-k)^7$
h. $3m^4 \times (-4)n^5 \times (-m)^3 \times 2n^5$

8. Which of the following is not equivalent to the below expression?

$2^5 \times 3^4$

A. $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
B. $2^3 \times 2^2 \times 3^3 \times 3$
C. $2^2 \times 3 \times 2^3 \times 3^3$
D. $(2 \times 3)^{5+4}$
E. $3^4 \times 2^5$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. $4^8 \times 4^5$

Student A
$= (4 \times 4)^{8+5}$
$= 16^{13}$

Student B
$= 4^{8+5}$
$= 4^{13}$

b. $7^2 \times 7^{13}$

Student A
$= 7^{2+13}$
$= 7^{15}$

Student B
$= (7 \times 7)^{2+13}$
$= 14^{15}$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. A sunflower's height increases by a factor of six every month. If the height of a fully grown sunflower is 216 cm, then how many months have passed since the sunflower was 1 cm tall?

11. At Papa's Gelateria, there are five flavours of gelato available. A customer can also choose to add one of three types of sauces and one of three types of sprinkles. Express the number of combinations of ice cream, sauce, and sprinkles in index notation.

12. A single dandelion can spread up to 200 seeds. If each one of these seeds grows and produces its own seeds, then what is the expected number of seeds produced by the second generation of one plant? Express your answer in index notation.

13. Simon posts a funny picture on his personal social media page. Forty of his friends proceed to share the picture with four of their own friends each. Then all of these people share the post with another four friends each. Express the number of times Simon's post has been shared by friends of friends, in index notation.

14. All of Mrs Doubtfire's offspring, including children and grandchildren have had two kids of their own each. If Mrs Doubtfire has 12 great-grandchildren altogether, then how many of her own children did she have?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. The following table shows a list of commonly used units for storing digital information.

Unit	Value (bytes)	Abbreviation
Byte	1	B
Kilobyte	10^3	kB
Megabyte	10^6	MB
Gigabyte	10^9	GB
Terabyte	10^{12}	TB

a. How many bytes are there in one kB?

b. How many kB can be multiplied together to make up one GB?

c. Express the size of a TB in terms of kB.

d. Amber loves movies and prefers to watch them in 4K resolution. If each film of this quality is around 20 GB in size, then how many of them can Amber store on a 1 TB hard drive?

e. With the kind of technology available in the world today, we store most of our important information digitally. List some ways in which you use digital data storage every day of your life.

16. Substitute a single-digit positive integer x into the following expressions and calculate the results for parts **a** and **b**.

a. $(-x)^4 \times (-x)^2$

b. $(-x)^4 \times (-x)^3$

c. Compare your answers from parts **a** and **b** and note their similarities and differences. How does the index affect the directionality of the number?

Extra spicy

17. Consider the following equation

$u \times u^4 = 1024$

Pick a value for u from the options given below, so that the above equation is true.

A. $u = 2$ **B.** $u = 3$ **C.** $u = 4$ **D.** $u = 5$ **E.** $u = 6$

18. A colony of rabbits initially consists of six members. If rabbit populations can grow at a rate of 20% per month, then which of the following calculations represents the expected number of rabbits in the colony after a year?

A. 1.2^{12} **B.** 6×1.2^{12} **C.** 6×12 **D.** 6×20 **E.** 20^{12}

19. Find two single-digit non-zero integers a and b so that $a^b = b^a$.

20. Simplify the following expression using the First Index Law.

$x^y \times y^x \times 3x^{2y} \times (-4)y^{6x}$

Remember this?

21. Five postal workers have to deliver 120 parcels each. This table shows how many hours it took each of the workers to deliver all the parcels.

Postal worker	Number of hours
Sam	3.5
John	4
Eileen	5
Michael	3
Christian	7

Which worker delivered approximately 35 parcels per hour?

A. Christian **B.** Eileen **C.** John **D.** Michael **E.** Sam

22. Eduardo gets paid the same amount every month. In a year he earns $54 000. How much does he earn in 7 months?

A. $4500 **B.** $7714 **C.** $22 500 **D.** $31 500 **E.** $37 800

23. Which of the following calculations is equivalent to 4^3?

A. 4×3 **B.** $4 + 4 + 4$ **C.** $4 \times (4 + 4)$ **D.** 16×4 **E.** $(4 + 4 + 4)^3$

Back to contents

4C Dividing indices

Operations on numbers written in index form are governed by the Index Laws. The second of these laws outlines the rule for dividing indices. The base and index of any number written in index form must be considered before division can be performed.

LEARNING INTENTIONS

Students will be able to:

- divide terms with the same base in index form
- simplify expressions containing division of terms in index form.

KEY TERMS AND DEFINITIONS

The **Second Index Law** states to subtract the index of the divisor (or denominator) from the index of the dividend (or numerator) when dividing terms with the same base in index form.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

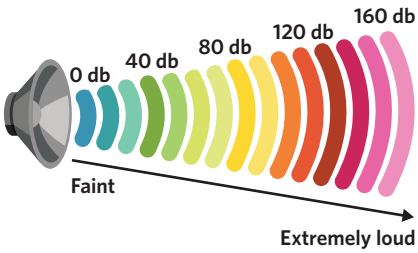


Image: BlueRingMedia/Shutterstock.com

The scale on which we measure sound intensity increases and decreases by factors in index form. This means that when you change the volume of your music, it becomes loud and intense very rapidly, but needs to be muted to disappear completely.

Key ideas

1. The Second Index Law only applies to terms with the same base.

Same base – law applies: $\frac{5^9}{5^3}$ $\quad$ Different bases – law **does not** apply: $\frac{7^6}{4^2}$

2. When dividing terms with the same base, subtract the index of the divisor from the index of the dividend.

$$\frac{x^p}{x^q} = x^{p-q}$$

$$\frac{5^9}{5^3} = 5^{9-3} = 5^6$$

3. Any non-zero number, raised to the power of zero has the value 1.

$$\frac{x^r}{x^r} = x^{r-r} = x^0 = 1$$

$$\frac{2^8}{2^8} = 2^{8-8} = 2^0 = 1$$

Worked example 1

Using the Second Index Law

Divide the following.

a. $\frac{8^{11}}{8^5}$

WE1a

Working

$8^{(11-5)} = 8^6$

Thinking

Identify the common base of the terms being divided. Subtract the indices of the terms to get the index of the quotient.

Visual support

Subtract the index of the denominator (5) from the index of the numerator (11)

$$\frac{8^{11}}{8^5} = 8^{11-5} = 8^6$$

b. $\frac{x^{13}}{x^{10}}$

WE1b

Working

$x^{(13-10)} = x^3$

Thinking

Identify the common base of the terms being divided. Subtract the indices of the terms to get the index of the quotient.

Student practice

Divide the following.

a. $\frac{3^{14}}{3^8}$ **b.** $\frac{z^{17}}{z^9}$

Worked example 2

Simplifying expressions by using the Second Index Law

Simplify the following expressions.

a. $\frac{12u^8}{4u^6}$

WE2a

Working

$= (12 \div 4)u^{(8-6)}$

$= 3u^2$

Thinking

Step 1: Simplify the coefficients and subtract the indices of the terms.

Step 2: Simplify the expression.

Visual support

Simplify the coefficients by dividing both by 4

Subtract the indices

$$\frac{\cancel{12}u^8}{\cancel{4}u^6} = 3u^{8-6} = 3u^2$$

Continues →

b. $\frac{11^6 \times 9^4}{9^2 \times 11^3}$ WE2b

Working

$= 11^{(6-3)} \times 9^{(4-2)}$

$= 11^3 \times 9^2$

Thinking

Step 1: Identify the terms with common bases and subtract their indices.

Step 2: Simplify the expression.

c. $\frac{6a^{15}b^{10}}{3a^{11}b^7}$ WE2c

Working

$= (6 \div 3)a^{(15-11)}b^{(10-7)}$

$= 2a^4b^3$

Thinking

Step 1: Simplify the coefficients. Identify the terms with common bases and subtract their indices.

Step 2: Simplify the expression.

Student practice

Simplify the following expressions.

a. $\frac{15t^9}{3t^4}$ **b.** $\frac{2^{15} \times 5^{10}}{5^3 \times 2^7}$ **c.** $\frac{8u^6v^{14}}{2u^3v^6}$

4C Questions

Understanding worksheet

1. Determine whether the Second Index Law applies to each of the following expressions.

Example

Expression	Law applies	Law does not apply
$\frac{12^7}{4^5}$	☐	☑

	Expression	Law applies	Law does not apply
a.	$\frac{6^8}{3^4}$	☐	☐
b.	$\frac{4^5}{4^2}$	☐	☐
c.	$\frac{x^6}{y^3}$	☐	☐
d.	$\frac{5t^7}{t^2}$	☐	☐

2. Fill in the missing indices so that each of the following expressions is true.

Example

$\frac{11^{10}}{11^7} = 11^3$

a. $\frac{3^5}{3^{\square}} = 3^3$ **b.** $\frac{7^{\square}}{7^5} = 7^4$ **c.** $\frac{8y^{\square}}{4y^3} = 2y^4$ **d.** $\frac{(-2)^{10}}{(-2)^{\square}} = (-2)^5$

3. Fill in the blanks by using the words provided.

common | dividend | zero | subtract | Second

The _____ Index Law is used to divide terms in index form. It only applies to indices with a _____ base. To divide the terms, _____ the index of the divisor from the index of the _____. The value of any non-zero number to the power of _____ is always equal to one.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Divide. WE1

a. $\frac{3^{10}}{3^6}$ **b.** $\frac{9^{12}}{9^7}$ **c.** $\frac{y^{13}}{y^{13}}$ **d.** $\frac{(-2)^6}{(-2)^4}$

e. $\frac{(-x)^{20}}{(-x)^{12}}$ **f.** $\frac{7^4 \times 7^9}{7^8}$ **g.** $\frac{8^{16}}{8^4 \times 8^5}$ **h.** $\frac{y^{11} \times y^3}{y^4 \times y^4}$

5. Simplify. WE2a

a. $\frac{10x^8}{5x^2}$ **b.** $\frac{24v^{10}}{3v^3}$ **c.** $\frac{y^6}{3y^2}$ **d.** $\frac{4t^5}{16t^2}$

e. $\frac{12r^7}{8r^3}$ **f.** $\frac{-2w^{17}}{w^{14}}$ **g.** $\frac{4b^{12} \times 2b^5}{6b^{17}}$ **h.** $\frac{3d^{20} \times 5d^{22}}{35d^{29}}$

6. Simplify. WE2b

a. $\frac{2^5 \times 3^9}{2^2 \times 3^4}$ **b.** $\frac{9^{10} \times 7^{11}}{7^6 \times 9^4}$

c. $\frac{5^2 \times 8^4}{8^2 \times 5}$ **d.** $\frac{(-4)^9 \times 10^{14}}{10^8 \times (-4)^3}$

e. $\frac{6^5 \times t^7}{6^3 \times t^4}$ **f.** $\frac{f^{18} \times 15^{19}}{15^{13} \times f^{17}}$

g. $\frac{x^5 \times 9^3 \times x^{15}}{9^2 \times x^{13}}$ **h.** $\frac{(-z)^7 \times 12^{30} \times (-z)^8 \times 12^{10}}{12^{40} \times (-z)^{15}}$

7. Simplify. WE2c

a. $\frac{b^{10}c^{10}}{c^3b^7}$ **b.** $\frac{x^4y^9}{xy^6}$ **c.** $\frac{2t^7s^4}{t^5s^2}$ **d.** $\frac{4u^{12}v^{16}}{2v^9u}$

e. $\frac{18x^{11}y^3}{9xy}$ **f.** $\frac{n^{18}m^{24}}{13n^6m^8}$ **g.** $\frac{10k^{17}p^{34}}{70p^{19}k^5}$ **h.** $\frac{-5v^{21}w^{29}}{25w^{18}v^7}$

8. $\frac{10^4}{5^2}$

Which of the following is not equivalent to the above expression?

A. $\frac{10 \times 10 \times 10 \times 10}{5 \times 5}$ **B.** $\frac{10^2 \times 10^2}{5^2}$ **C.** $\frac{10^4}{5 \times 5}$ **D.** $10^4 \times \frac{1}{5^2}$ **E.** 2^2

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. $\frac{6^{14}}{6^7}$

Student A

$= 6^{14-7}$

$= 6^7$

Student B

$= 6^{14 \div 7}$

$= 6^2$

b. $\frac{3 \times 12^{10}}{12^6}$

Student A

$= 3 \times \left(\frac{12}{12}\right)^{10-6}$

$= 3 \times 1^4$

Student B

$= 3 \times 12^{10-6}$

$= 3 \times 12^4$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. A competition starts with a group of 64 players. The final round occurs when only two players are left. If in each round, half of the players are eliminated, then how many rounds does it take to get to the final two players?

11. The number of people employed at a company has grown from just four to well over 200 staff. If staff numbers quadrupled every year, then what is the minimum number of years that the company has been around for?

12. A small piece of algae with an area of 1 cm^2 doubles in size every day for 28 days until it completely covers the whole lake. When copper is added to the water to remove the algae, it halves in size every day until it disappears completely. What was the area of the algae patch on the 15th day of treatment? Express your answer in index notation.

13. When Tori opened up an online tutoring service, she had only two students. This number has been increasing steadily by a factor of five every six months. If at the beginning of January 2022, the number of students subscribing to Tori's service has grown to 250, then when did she first open her business?

14. When Gavin was born, his parents invested $1000 for him. The investment tripled every four years and reduced by a factor of three every five years. Including the initial investment amount, express how much money Gavin has accumulated after 60 years, in index form.

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Sione bakes a brownie for his family. Using the information given in the image of the brownie below, answer the following questions.

a. Find the area of the brownie using the dimensions given, in cm^2.

b. If the brownie is cut in equal pieces, as pictured above, then what are the dimensions of one square piece, in cm?

Image: Atsushi Hirao/Shutterstock.com

c. Using your answers from parts **a** and **b**, express the area of the whole brownie as powers of four using index notation.

d. Each time someone comes into the kitchen, the brownie halves in size. How many people have helped themselves to the brownie by the time only one 1 cm^2 piece remains?

e. Sione thinks it is unfair that whoever got to the brownie first ate most of it. Suggest some ways in which he can ensure that next time, everyone gets an equal share.

16. Consider the following expression.

$$\frac{3^p}{3^q} = 3^{(p-q)}$$

a. Substitute p for 4 and q for 2. Evaluate the expression, with or without a calculator.

b. Swap the values of p and q so that the index of the quotient is negative. Evaluate the expression, with or without a calculator, leaving the answer as a fraction.

c. Compare and contrast your answers for parts **a** and **b**. How does a negative index affect the outcome?

Extra spicy

17. If a, and b, and x are all integers greater than one where $a > b$, then which of the following expressions is **not** true?

A. $\frac{x^a}{x^b} = x^{(a-b)}$

B. $x^a > x^b$

C. $x^a \times x^b = x^b \times x^a$

D. $\frac{x^a}{x^b} \times \frac{x^b}{x^a} = 1$

E. $\frac{x^a}{x^b} = \frac{x^b}{x^a}$

18. Luke uses a hard drive to store all his computer games. He has found that on average, he manages to fill up a third of the empty space with content every week. Which of the following expressions shows how much data is left on a two TB hard drive after n weeks of usage?

A. $\frac{1}{3^n}$ **B.** 2×3^n **C.** $\frac{2}{3^n}$ **D.** $2 - \frac{1}{3^n}$ **E.** $2 - \frac{2}{3^n}$

19. Find the value of p.

$$3^{4p} = 3^p \times \frac{3^8}{3^p}$$

20. Without using a calculator, find the fraction that has the same value as the following.

$$\sqrt{\frac{6^2}{2^6}}$$

Remember this?

21. Sally works from 9 am until 5 pm every day Tuesday to Thursday. She stays for a longer shift on Fridays from 9 am until 8 pm. How many hours a week does Sally work altogether?

A. 19 **B.** 32 **C.** 35 **D.** 40 **E.** 43

22. The Amazon River is 6 992 000 metres long. The Murray River is 2 508 000 metres long. How many kilometres longer is the Amazon River than the Murray River?

A. 4.848 m **B.** 4848 m **C.** 4.484 km **D.** 4484 km **E.** 4 484 000 m

23. Which of these has the same value as $2^2 \times 3^3 \times 5^3$?

A. $4 \times 9 \times 5$ **B.** $4 \times 9 \times 15$ **C.** $4 \times 27 \times 5$ **D.** $4 \times 27 \times 15$ **E.** $4 \times 27 \times 125$

Back to contents

4D Raising powers

Operations on numbers written in index form are governed by the Index Laws. The third of these laws outlines the rule for raising indices to a power. The indices of all components of a term are affected by the power to which it is being raised.

LEARNING INTENTIONS

Students will be able to:

- raise a power to another power
- simplify expressions using index laws.

KEY TERMS AND DEFINITIONS

The **Third Index Law** states to multiply the indices when raising a term in index form to another power.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Martin Bergsma/Shutterstock.com

Games like pinball and Scrabble allow the player to multiply their score repeatedly if certain conditions are met. This means that a strategic and skilled player can increase their score rapidly with the right succession of moves and the application of Index Laws.

Key ideas

1. The Third Index Law applies to terms in index form raised to a power.

$$(5^2)^4 = \overbrace{5^2 \times 5^2 \times 5^2 \times 5^2}^{\text{Multiplied 4 times}}$$

2. When raising a power to another power, multiply the indices.

$$(x^p)^q = x^{p \times q} = x^{pq}$$

$$(5^2)^4 = 5^{2 \times 4} = 5^8$$

Worked example 1

Using the Third Index Law

Simplify the following expressions.

a. $(4^2)^5$ WE1a

Working

$4^{2\times5} = 4^{10}$

Thinking

Multiply the index of the number inside the brackets by the index on the outside of the brackets.

Visual support

$$(4^2)^5 = \overbrace{4^2 \times 4^2 \times 4^2 \times 4^2 \times 4^2}^{\text{Multiplied 5 times}}$$
$$= 4^{2+2+2+2+2}$$
$$= 4^{2\times5} = \boxed{4^{10}}$$

b. $(7y^3)^4$ WE1b

Working

$7^{1\times4}y^{3\times4} = 7^4y^{12}$

Thinking

Multiply the indices of the coefficient and pronumeral inside the brackets by the index on the outside of the brackets.

Student practice

Simplify the following expressions.

a. $(5^3)^8$ **b.** $(4p^7)^5$

Worked example 2

Simplifying expressions by using index laws

Simplify the following expressions.

a. $(3b^5)^2 \times (b^6)^3$ WE2a

Working

$3^2b^{5\times2} \times b^{6\times3} = 3^2b^{10} \times b^{18}$

$3^2b^{(10+18)} = 3^2b^{28}$

Thinking

Step 1: Use the Third Index Law to simplify the indices of terms raised to a power.

Step 2: Simplify the expression by using the First Index Law on indices with common bases.

Visual support

$$(3b^5)^2 \times (b^6)^3 = \overbrace{3b^5 \times 3b^5}^{\text{Multiplied 2 times}} \times \overbrace{b^6 \times b^6 \times b^6}^{\text{Multiplied 3 times}}$$
$$= 3^2b^{5\times2} \times b^{6\times3}$$
$$= 3^2b^{10} \times b^{18}$$
$$= 3^2b^{10+18} = \boxed{3^2b^{28}}$$

Continues →

b. $\frac{(8a^9)^4}{(a^3)^2}$ WE2b

Working

$$\frac{8^4a^{9\times4}}{a^{3\times2}} = \frac{8^4a^{36}}{a^6}$$

$$8^4a^{(36-6)} = 8^4a^{30}$$

Thinking

Step 1: Use the Third Index Law to simplify the indices of terms raised to a power.

Step 2: Simplify the expression by using the First Index Law on indices with common bases.

Student practice

Simplify the following expressions.

a. $(2x^8)^2 \times (x^4)^3$ **b.** $\frac{(10b^5)^4}{(b^6)^3}$

4D Questions

Understanding worksheet

1. Write the expanded form of each of the following expressions.

Example

$(7^2)^6 =$ 7^2 × 7^2 × 7^2 × 7^2 × 7^2 × 7^2

a. $(3^8)^3 =$ ☐ × ☐ × ☐

b. $(x^4)^5 =$ ☐ × ☐ × ☐ × ☐ × ☐

c. $(5t^5)^2 =$ ☐ × ☐

d. $(4^3z^6)^4 =$ ☐ × ☐ × ☐ × ☐

2. Fill in the missing indices so that each of the following expressions is true.

Example

$5^4 \times 5^4 \times 5^4 \times 5^4 \times 5^4 \times 5^4 = (5^4)^{6}$

a. $8^2 \times 8^2 \times 8^2 = (8^2)^{\square}$

b. $y^5 \times y^5 \times y^5 \times y^5 \times y^5 \times y^5 \times y^5 = (y^5)^{\square}$

c. $3d^6 \times 3d^6 \times 3d^6 \times 3d^6 = (3d^6)^{\square}$

d. $9^4f^3 \times 9^4f^3 \times 9^4f^3 \times 9^4f^3 \times 9^4f^3 = (9^{\square}f^3)^{\square}$

3. Fill in the blanks by using the words provided.

simplify | coefficien | indices | Third | multiplied

The _____ Index Law states that in order to raise a term to a power, the indices must be _____ together. It applies to all parts of a term shown to be in brackets, including any _____. A combination of index laws can be used to _____ expressions involving _____ with common bases.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Simplify. WE1a

a. $(3^4)^2$ **b.** $(9^{10})^5$ **c.** $(13^6)^4$ **d.** $(a^9)^2$

e. $(x^{12})^3$ **f.** $(p^{25})^4$ **g.** $(t^{16})^0$ **h.** $(y^0)^{154}$

5. Simplify. WE1b

a. $(2b^4)^3$ **b.** $(7x^5)^4$ **c.** $(10p^2)^8$ **d.** $(9u^8)^9$

e. $(6s^7)^{10}$ **f.** $(11x^8)^0$ **g.** $(5^2t)^7$ **h.** $(3^3r^{11})^3$

6. Simplify. WE2a

a. $(5^4)^2 \times 5^6$ **b.** $4^8 \times (4^3)^4$ **c.** $(y^3)^2 \times y^5$ **d.** $x^{10} \times (x^4)^8$

e. $(2^7)^3 \times (2^2)^3$ **f.** $(v^3)^3 \times (v^6)^2$ **g.** $(a^4)^3 \times (4a^3)^5$ **h.** $(2w^5)^4 \times (w^4)^2$

7. Simplify. WE2b

a. $\frac{(7^2)^4}{7^3}$ **b.** $\frac{6^{15}}{(6^3)^2}$ **c.** $\frac{(x^5)^3}{x^6}$ **d.** $\frac{y^{20}}{(y^4)^2}$

e. $\frac{(2p^{10})^4}{p^{15}}$ **f.** $\frac{u^{17}}{(4u^8)^2}$ **g.** $\frac{12k^{25}}{(k^6)^3}$ **h.** $\frac{(8r^{11})^8}{(r^8)^{11}}$

8. Simplify.

a. $\frac{(3^4)^2}{(3^2)^3}$ **b.** $\frac{(x^9)^2}{(x^5)^2}$ **c.** $(5q^3)^4 \times (5q^5)^3$ **d.** $(6c^5)^2 \times (2c^9)^4$

e. $\frac{7^5(v^{10})^5}{7^2(v^6)^4}$ **f.** $2(q^3)^5 \times (2q^5)^4$ **g.** $\frac{(3^4n^7)^6}{3^5n^{22}}$ **h.** $(8^2x^{10})^5 \times (8^3x^2)^7$

9. $(3x^7)^3$

Which of the following is not equivalent to the above expression?

A. $3x^7 \times 3x^7 \times 3x^7$

B. 3^3x^{21}

C. $3 \times 3 \times 3 \times x^{21}$

D. $3x^{21}$

E. $3^3 \times x^7 \times x^7 \times x^7$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. $(9^6)^3$

Student A

$= 9^{6+3}$

$= 9^9$

Student B

$= 9^{6\times3}$

$= 9^{18}$

b. $(6r^7)^5$

Student A

$= 6r^{7\times5}$

$= 6r^{35}$

Student B

$= 6^5r^{7\times5}$

$= 6^5r^{35}$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Genna shares a link to her online store with 10^2 people. In a few days, the number of people who have the link to Genna's store has increased by a power of three. Express the number of people who received the link in index form.

12. Alex's current phone contains 2^4 gigabytes (GB) of storage space. The store manager says she can offer her more than double her storage if she upgrades to the new model which has 2^8 GB of storage space. Calculate and state whether the manager is correct or incorrect.

13. A meal deal at the Double R Diner consists of a hot dog with a piece of cake. A customer can have one of five different types of toppings on their hot dog. There are five types of cake available, which comes with one of five types of ice cream. In index form, how many possible combinations of hot dogs with toppings and cakes with ice cream are possible?

14. The basic version of a computer game has four different types of helmets, armour and boots that the player can mix and match to dress their character. Downloading extra content will raise the number of available combinations of outfits by a power of two. Express this number in index form.

15. Shelby breeds tropical fish. Over three months, the number of fish Shelby has increased by a power of three. When Shelby bought another fish tank and more fish, the total number of fish increased by a power of two. If Shelby initially had three fish, then how many fish does she have now?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. In a game of Meltdown, each contestant starts with one point. This score is multiplied by three every time they clear a platform. This total is then raised to the power of two for every time they avoid touching the rotating barrier as well. The following table shows the results after one round of Meltdown.

Image: Atfie SahidMY/Shutterstock.com

Contestant	Number of platforms cleared	Number of barriers cleared	Final score
Ali	7	1	$(1 \times 3^7)^2 = (3^7)^2$
Anna	**a**	2	$(3^9)^4$
Oleg	11	**b**	$(3^{11})^2$
Patrick	8	1	$(3^8)^2$
Rebecca	6	2	**c**

a. Find the number of platforms Anna cleared.

b. How many times did Oleg clear the barriers?

c. Express Rebecca's final score in index form.

d. Who got the highest score in the round? State their name and score.

e. Group activities, like sports, are a great way to spend more time with the people in your life and make new friends. List some other ways in which you can meet people in a safe and fun environment.

17. Substitute x for 3 into the below expressions and evaluate the results for parts **a** and **b**, with or without a calculator.

a. $6(x^2)^3 = 6 \times x^2 \times x^2 \times x^2$

b. $(6x^2)^3 = 6x^2 \times 6x^2 \times 6x^2$

c. Compare and contrast your answers for parts **a** and **b**. How do brackets affect the outcome of raising a term in index form to another power?

Extra spicy

18. An investment of \$1500 is expected to grow according to the following formula, where t represents the number of years the money has been in the account.

$1500 \times (1.3^2)^t$

How much money will there be in the account after four years, to the nearest thousand?

A. \$7000 **B.** \$8000 **C.** \$12 000 **D.** \$13 000 **E.** \$14 000

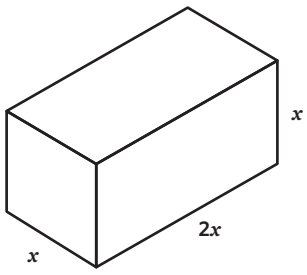
19. Helen is a seasonal worker at an apple orchard. Over six weeks, she works for six hours per day and six days per week. She gets paid \$0.50 for every tree that she plants. How many trees should Helen aim to plant in an hour of work if she wants to make at least \$7000 over the whole season?

A. 32 **B.** 64 **C.** 65 **D.** 195 **E.** 216

20. Solve for x.

$4^x = (2^4)^6$

21. Given that the volume of the cuboid is 54 cm^3, what is the area of one square face, in index form?



Remember this?

22. Harvey attends hockey training three times a week during winter sports season. If term two was 11 weeks long, then how many times has Harvey attended hockey training in all of term two?

A. 3 **B.** 11 **C.** 14 **D.** 33 **E.** 77

23. Chloe is saving up for a new skateboard. If the skateboard costs $220 and Chloe has already saved 55% of it, then how much more money does she need to save?

A. $45 **B.** $99 **C.** $121 **D.** $165 **E.** $219.45

24. In a square pattern, half of the tiles are black. There are half as many red tiles as there are black and the rest are white. If the area of the whole pattern is 64 tiles, then how many tiles are white?

A. 4 **B.** 8 **C.** 16 **D.** 32 **E.** 48

Chapter 4 extended application

1. In the study of Astronomy, numbers become relatively large. For example, the distance between Neptune and the Sun is approximately 4 474 000 000 km in decimal notation. Given the length of the number, scientific notation is a common way to write the distance.

For example, the distance 4 474 000 000 km between Neptune and the Sun can be rewritten as 4.474×10^9 in scientific notation. The index of '9' indicates that there are nine zeros after the first digit.

Base Index

$4\,474\,000\,000 = 4.474 \times 10^9$

Decimal notation — Index notation

Another way of describing distances in space is to use astronomical units (AU). Astronomical units are based on the distance between Earth and the Sun, which is defined as 1 AU. All other distances in the solar system are therefore relative to this distance. For example, the distance between Earth and Jupiter is 4.2 AU, which means that it is a little over four times the distance compared to how far Earth is from the Sun.

Image: ESA/Hubble

The distance between planets change, due to planets having elliptical orbits which are not perfect circles. The distance between Mercury and Earth can range from 77 million kilometres at the closest point (perihelion) and 222 million kilometres at the furthest point (aphelion). The following table shows the average distance from Earth to the destination listed, rounded to the nearest million kilometres.

Destination	Distance (km)	Distance (AU)
The Sun	150 000 000	1
Mercury	92 000 000	0.61
Venus	41 000 000	0.28
Mars	78 000 000	0.52
Jupiter	629 000 000	4.2
Saturn	1 275 000 000	8.52
Uranus	2 724 000 000	18.21
Neptune	4 351 000 000	29.09

a. The distance between Earth and the Sun can be written as 1.5×10^8 km in scientific notation. State the base and the index for 10^8.

b. What is the kilometre distance between Earth and Venus, in scientific notation?

c. The distance between Earth and dwarf planet Pluto is 5.0488×10^9 km. Convert this to decimal notation.

d. In the Pleiades open star cluster, there is a star called Atlas, which is 431 light years away from Earth. If one light year is approximately 63 000 AU, calculate the distance in kilometres between Earth and Atlas, giving your answer in scientific notation to 1 decimal place.

e. The concept of using the human foot as a unit of measurement was introduced at least 4000 years ago. The idea of measuring the distance between the Earth and the Sun, which became known as the astronomical unit, is also suspected to be thousands of years old. What other unit of measurement might humans have invented, or invent in the future, that is based on ourselves and our planet?

2. The seahorse is one of the fastest reproducing creatures in the animal kingdom. There are several oddities in the way that seahorses reproduce. For example, couples dance together for hours or days, change colour and entwine their tails as part of the mating ritual. Couples also tend to remain monogamous for an entire breeding season.

Unlike most other animals, male seahorses become pregnant and carry the eggs to term. The female deposits between 5 and 1500 unfertilised eggs into the male's brood pouch, which he fertilises and carries for two to three weeks. After giving birth, male seahorses can become pregnant again within hours. Sadly, only 0.5% of babies survive to adulthood.

Image: Bernard S Tjandra/Shutterstock.com

a. A particular male seahorse births an average of $10 \times 10 \times 10$ babies, called fry, every fortnight. Determine the number of fry that would be birthed in an 8-week period.

b. The large Australian pot-bellied female seahorse deposits $4^2 \times 4^3$eggs into the brood pouch of her mate. Assuming all eggs survive and are birthed, how many babies will this result in? Give your answer in index form.

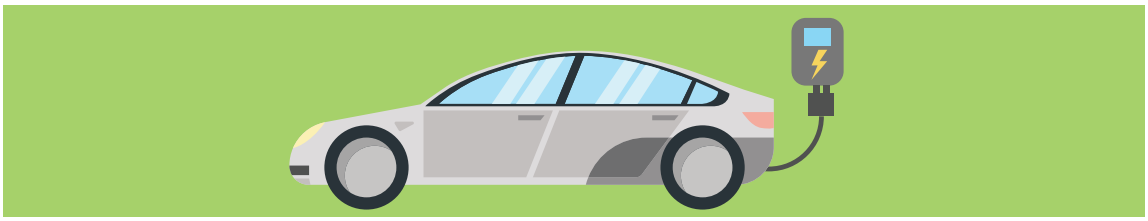
c. If the number of fry that survive to adulthood from each clutch (group of eggs) is 5^0, how many babies would survive from 3^4 clutches?

d. Assuming there are 2^4 clutches a year, 2^9 eggs per clutch and 2^2 fertile years, how many fry would be birthed over the lifespan of one male seahorse? Give your answer in index form.

e. Many seahorse species are either endangered or vulnerable. This is due to a combination of pollution, habitat loss, fishing nets and the capture of seahorses for traditional medicine. Suggest one way that we can protect seahorses from being further endangered.

3. Cars are known to depreciate (fall in value) year on year. The average depreciation of a car's values is 10% every year. For example, the price of a brand new Electro car sedan is $54 000. After one year, it is worth 90% of the original price. Every year the car depreciates, it is only worth 90% of the price of the previous year. In other words, the value of the car over time = original price $\times$ 0.9 $\times$ 0.9 $\times$ 0.9 ... for the amount of time the car is owned.

$$\text{Value of car over time} = \text{original price} \times \underbrace{0.9 \times 0.9 \times 0.9 \times \ldots \times 0.9}_{\text{Number of 0.9 = number of years after purchase}}$$



a. Determine the value of a three year old Electro car sedan with the equation $\$54\,000 \times (0.9)^3$. Rounded to the nearest dollar, how much has it reduced in price?

b. Rounded to the nearest dollar, determine the value of the car when it is 10 years old.

Andrew turns 14 years old in 2030 and wants to save up to purchase a car for his 18th birthday. He already has $3000 in his saving account and has figured out he can save $2500 per year from his paper round.

c. Calculate how much Andrew will have saved in 4 years time.

d. Andrew does not have enough money to buy a brand new car and will have to buy an older pre-owned car. Determine the year of the Electro car sedan Andrew can afford by using the equation $54\,000 \times (0.9)^{years}$ and substituting different numbers of years.

e. Andrew is trying to decide if he should buy a car at 18 or wait a few more years to buy a better car. He creates the table below to help him calculate his options. Complete the table and show all working below to help Andrew decide if he should buy a car at 18 or wait.

Year	Savings	Age of car (years)	Year car made
2031	$5500	22 years	2009
2032	$8000	19 years	2013
2033	$10 500	16 years	2017
2034	$13 000		
2035	$15 500		
2036	$18 000		

f. Based on the information calculated in the table should Andrew buy a car at 18 or wait a few more years to buy a car? Explain your reasoning using the calculations from the table.

Chapter 4 review

Multiple choice

1. Which of the following has been written incorrectly in index form? 4A

A. $12 \times a \times b \times a \times 12 \times 12 = 12^3 \times a^2 \times b$

B. $9 \times x \times x \times y \times y \times x = 9 \times x^3 \times y^2$

C. $4.2 \times m \times 4.2 \times n \times m \times n \times 1 \times x = 4.2^2 \times m^3 \times n^2 \times x$

D. $3.6 \times 7 \times 7 \times 3.6 \times x \times x \times z = 3.6^2 \times 7^2 \times x^2 \times z$

E. $8 \times q \times r \times 8 \times r \times q \times p \times r = 8^2 \times q^2 \times r^3 \times p$

---

2. Which of the following indices would make this expression true? 4B

$(-8)^{\square} \times (-8)^2 = (-8)^8$

A. 4 **B.** 6 **C.** 8 **D.** 10 **E.** 16

---

3. Which of the following cannot be simplified using the First Index Law? 4B

A. $4^8 \times 4^3 \times x \times y$

B. $2^4 \times 4^6 \times b^5 \times c$

C. $3x \times x^3 \times y^4 \times y^2$

D. $6^2 \times m^2 \times n^3 \times 6^3 \times n \times p$

E. $a^5 \times 5^2 \times b^4 \times b \times 5$

---

4. Which of the following has been incorrectly simplified? 4C

A. $\frac{12x^7}{6x^5} = 2x^2$

B. $\frac{10a^{11}b^3}{5a^4b} = 2a^7b^2$

C. $\frac{14^3}{14^3} = 14^0 = 1$

D. $\frac{(-3)^8}{(-3)^2} = 3^4$

E. $\frac{30t^9}{2t^8} = 15t$

---

5. Which of the following is not equivalent to the below expression? 4C, 4D

$\frac{8^8}{2^4}$

A. $\frac{8^4 \times 8^4}{2^2 \times 2^2}$

B. $\frac{8^6 \times 8^2}{2 \times 2 \times 2 \times 2}$

C. $\frac{(8^2)^4}{(2^2)^2}$

D. $8^8 \times \frac{1}{2^2 \times 2^2}$

E. 4^4

Fluency

6. Write the following expressions in index form. 4A

a. $4 \times 4 \times 4 \times 4$

b. $8 \times 7 \times 8 \times 7 \times 8$

c. $5 \times 2 \times a \times b \times a \times b \times c \times 5$

d. $-2.3 \times x \times y \times y \times y \times x \times 2.3 \times z$

---

7. Evaluate the following expressions using order of operations. 4A

a. $5^2 + 2^3 - 6 + 3$

b. $20 \div 2^2 + 3^2 - 10$

c. $(8 + 2) \times (10 + 6) - 4^2 \times 2$

d. $(15 - 11)^2 + 3 \times (40 - 2^5)$

---

8. Simplify. 4B

a. $3^2 \times 3^4 \times 5^7 \times 5^3$

b. $12^2 \times 8^4 \times 8^6 \times 12$

c. $(-9)^2 \times 6^9 \times (-9)^{10} \times 6^8$

d. $15^2 \times (-2)^{10} \times 4^5 \times (-2)^3 \times 15^5$

---

9. Simplify. 4B

a. $5^2 \times a^4 \times b^6 \times a^2$

b. $3^3 \times (-p)^5 \times 3 \times (-p)^8$

c. $-7a^3 \times 4^2 \times 2^5 \times 3a^{15}$

d. $-5x^2 \times 8y^5 \times (-x)^{10} \times 3y^8$

---

10. Simplify. 4C

a. $\dfrac{20x^5}{4x^2}$ **b.** $\dfrac{4p^6}{32p^3}$ **c.** $\dfrac{-9w^{20}}{36w^{12}}$ **d.** $\dfrac{5c^{18} \times 6c^{20}}{48c^{17}}$

---

11. Simplify. 4C

a. $\dfrac{a^{10}b^9}{b^5a^3}$ **b.** $\dfrac{16p^{17}q^6}{12p^8q^6}$ **c.** $\dfrac{-7k^{39}r^{13}}{42k^{22}r^5}$ **d.** $\dfrac{48v^{14}w^{28}}{70w^{15}v^{12}}$

---

12. Simplify. 4D

a. $(8^2)^3 \times 8^4$ **b.** $(3a^5)^5 \times (2a^3)^7$ **c.** $\dfrac{(4v^{13})^3}{(v^2)^{17}}$ **d.** $\dfrac{(5p^4)^4}{(p^8)^2}$

---

13. Simplify. 4D

a. $6(a^4)^6 \times (6a^3)^8$ **b.** $\dfrac{2^{10}(x^8)^4}{2^5(x)^8}$

c. $(12^3v^{16})^4 \times (12v^{10})^7$ **d.** $\dfrac{(4^5n^7)^6}{4^{28}n^{14}}$

Problem solving

14. The force of gravity on the Sun's surface is 3^3 times stronger than it is on Earth. If the force of gravity on Earth is 9.8 Newtons (N), then what is the force of gravity on the Sun's surface, written in index and expanded form? 4A

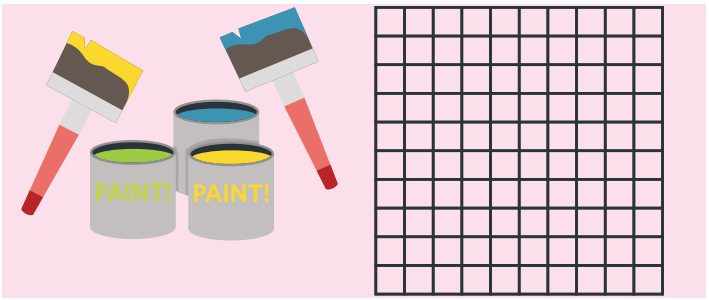
15. Maria is visiting her local Mexican restaurant Flexica Mexicana. There are four different types of meals available (tacos, quesadilla, burrito, nachos). She can choose to add onto those meals one of four different types of meat, one of three different types of filling and one of three different types of sauces. What are the combinations of meals, meat, filling and sauce that she can have? Express in index notation. 4B

16. Tom's parents invested $10 000 into a savings fund the day Tom was born. Every five years, the investment would quadruple, but then would be reduced by a factor of four every six years. How much money would Tom have on his 60th birthday, including the initial investment amount? Express in index form. 4C

17. Alannah is trying to grow her Tiktok account by posting more videos. She initially had three followers. After one week, her number of followers increased by a power of two. Two days later, Alannah posted a video of herself singing, and the total number of followers increased by a power of six. Express how many followers Alannah has now in index form. 4D

Reasoning

18. Brad is painting floor tiles. One section contains 10 rows of tiles, with each row containing 10 tiles. He is assigned to paint 3 sections.



a. How many tiles are in one section? Express in index form.

b. Express the entire paint job (including all three sections) in index notation, and find the number of tiles Brad has to paint in total.

c. Brad estimates it will take 5 minutes to paint one tile. How long will the entire paint job take? First express in index form and then evaluate.

d. Is it possible for Brad to finish the entire paint job within 20 hours? If not, how many more hours does he need to complete it?

e. Paints (especially acrylic paint) are known to be harmful for the environment in many ways. Suggest some ways that Brad can ensure his painting is eco-friendly.

19. Evaluate the following expressions.

a. $(2^3)^4$

b. $(2^4)^3$

c. Compare your answers from parts **a** and **b** and determine if they are equal. Explain the reasoning behind your findings.

Back to contents

05

Algebra

Number and Algebra

Research summary

5A Introduction to algebra *(Revision)*

5B Substitution *(Revision)*

5C Adding and subtracting terms

5D Multiplying and dividing terms

5E Algebraic fractions *(Extension)*

5F Expanding brackets

5G Factorising expressions

Chapter 5 extended application

Chapter 5 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 5 research summary

Algebra

Big ideas

Proportional reasoning

This chapter will focus on students' ability to identify and apply their extensive conceptual and structural understanding of numbers and their properties, order of operations, ratios and proportions, equivalence, and number sense to build structural and conceptual knowledge of algebra.

Students will use their understanding of number (especially fractions and integers) and number properties (addition, subtraction, multiplication, division) to develop a conceptual understanding of variables that underpins the conceptual understanding of algebra (Bush, 2011; Darley, 2009). For example, this chapter will get students to investigate why $a \times b$ is the same as $b \times a$, but $a \div b$ is not always the same as $b \div a$.

Reviewing number properties and using a calculator can strengthen students' understanding of the connection between number and algebra (Bush, 2011; Darley, 2009). For example, this chapter will get students to justify the reasoning why $\frac{y}{y} = 1$ and $\frac{a}{b} \times \frac{b}{a} = 1$.

Using the phrase 'cancelling' or 'goes away' may cause misconceptions in algebra. Instead, students and teachers are encouraged to use phrases such as 'sums to zero' and 'simplifies to 1'. This will help strengthen students' connection of number properties in algebra. For example, $3x - 3x$ sums to zero and $\frac{4y}{4y}$ simplifies to 1.

Additionally, this chapter will draw upon students' interpretation of the equal sign and equivalence. Students will need a relational understanding of the equal sign to strengthen algebraic reasoning (Carpenter et. al. 2000) and to avoid misconceptions in algebra (Welder, 2012). For example, $30 - 3x = y$ is the same as $y = 30 - 3x$.

Supporting the idea of equivalence in arithmetic is critical before students apply it to algebra so they do not see this idea as an outlier. Balance scales, physical and pictorial models, reversing the amount of quantities on either side of the equal sign, and using number sentences with operations on either side can be applied to strengthen the idea of equivalence in both arithmetic and algebraic reasoning and thinking (Welder, 2012; Kieran 1981).

This big idea is built upon a strong understanding of number, number properties, ratios and proportions, and equality. Therefore, understanding the connection between number, number properties, equality, and algebra is pivotal for future success in mathematics. Progress in this chapter and future lessons such as equations and geometry impacts the future big ideas of generalising.

Generalising

This chapter will focus on combining multiplicative thinking and proportional reasoning skills to recognise and represent patterns and relationships of algebra. Additionally, this chapter will build structural and conceptual knowledge of the properties of algebraic notation, letter usage, and their symbolic relationship within expressions and the language of mathematics.

It is essential to realise that it takes time for students to make algebraic generalisations (Bush, 2011). Algebra is plagued with potential pitfalls for students. Algebra requires students to navigate and combine their skills of analysing mathematical words and text, graphs, and tables. They must manoeuvre familiar and foreign mathematical symbols to work with various numbers and properties to conclude evidence or assumptions.

In this chapter, algebra will be introduced using letters and symbols to represent unknown numbers and relationships between quantities in the form of expressions and equations. The success of algebra and general problem solving is underpinned by the conceptual understanding and mastery of variables (Tooher & Johnson, 2020). For example, there are six times as many students as teachers at this school can be represented by $s = 6t$, where $s =$ the number of students and $t =$ the number of teachers.

Additionally, this chapter will assess students' ability to translate words into equations and expressions. Word to equation/expression translation plays a vital role in the developmental success of algebra. Practicing problems that are relatable, substituting numbers to make sure their equation makes mathematical sense, and using cognitive dissonance can promote algebraic generalisation (Tooher & Johnson, 2020). For example, $4s + 7r = 42$, where scones cost s and rolls cost r, makes students stop and consider what the variables represent since 4 scones and 7 rolls does not equal 42.

Furthermore, to promote algebraic thinking and generalisations, representing unknown quantities with completely different abbreviations than the item's name can reduce the misconception that letters are objects or labels instead of numbers (McNeil et al., 2010). For example, $u =$ number of apples.

Lastly, this chapter will use tables and graphs to promote algebraic generalisations. Connecting patterns with equations, tables, graphs, and words promotes algebraic thinking and understanding of functions (Kieran & Sfard, 1999). Additionally, these connections are fundamental for generalising mathematical proofs. For example, creating a rule that represents the n^{th} term in the sequence of numbers 3, 6, 9, 12, n^{th}.

This big idea is built upon a strong understanding of proportional understanding, number and number properties, patterns, and equality. Therefore, a conceptual understanding of variables and the relationship and connection of words, equations, tables, graphs, and patterns is essential for success in future mathematics topics.

Visual representations

Algebra tiles

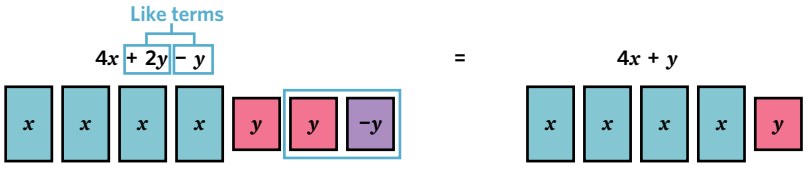
Algebra tiles are an extension of the area model. Algebra tiles represent constants and variables and are used to support students in conceptualising and visualising the abstract ideas of algebra. They usually represent a small square with dimensions 1 by 1 square unit and area of 1 square unit; an oblong rectangular shape with dimensions 1 by x and an area of x square units; and a large square with dimensions x by x and an area of x^2 square units. The side length of the larger square is designed so that it cannot be made using a certain number of smaller squares.

In this chapter, algebra tiles will be used to aid with:

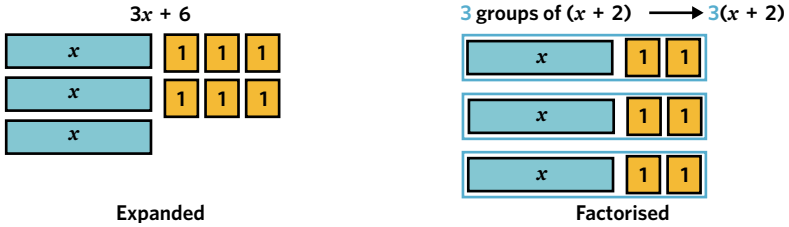
- adding and subtracting like terms and integers
- multiplying and dividing integers
- the distributive property
- factorising and grouping.

While concrete manipulatives increase understanding of abstract algebraic ideas, if they are not applied correctly, they could reinforce misconceptions regarding equality and variables (MacGregor & Stacey, 1997). Therefore, it is vital to use a range of manipulatives and visual models, such as number lines and balance scales, to support students understanding of algebraic concepts.

Adding and subtracting

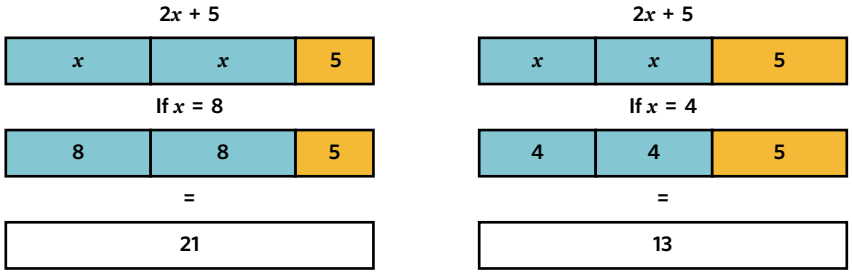


Distributive property and factorising



Bar model

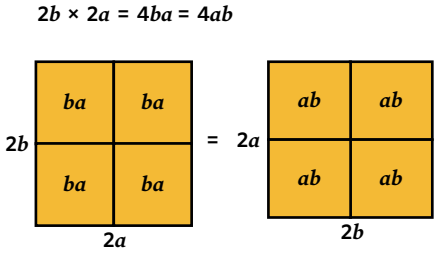
Bar models, also known as tape or strip diagrams, help students construct a pictorial equation from a word problem as a whole rather than as distinct parts. The bar model is usually represented by a rectangle representing the relationship between the known and unknown quantities and can be divided to represent proportional relationships. In this chapter, bar models will aid in relational thinking, algebraic thinking, and substitution.



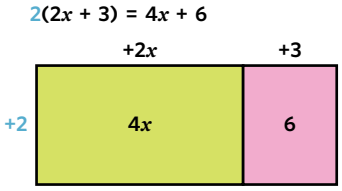
Open array

Open arrays are helpful for the multiplication of larger numbers and do not show units within the array. They promote the use of multiplication strategies rather than counting. While they are practical for the distribution law for digits, they are also practical for conceptualising algebraic expressions and equations. This chapter will use open arrays to aid in understanding multiplying and dividing terms, factorising and expanding algebraic expressions.

Multiplying and dividing

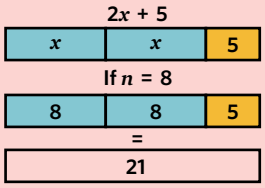
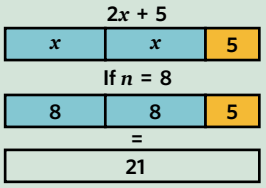


Expanding and factorising



Misconceptions

Misconception	Incorrect	Correct	Lesson
Students include some or none of the fraction when describing terms.	$\frac{m}{3} + 3$ Terms: $\frac{1}{3}, m, 3$	$\frac{m}{3} + 3$ Terms: $\frac{m}{3}, 3$	5A
Students do not include the operation sign when describing coefficients.	$4 - 3i$ Coefficient of i is 3	$4 - 3i$ Coefficient of i is -3	5A
Students think the coefficient of a variable with no number in front is 0.	$s + 81$ Coefficient of s is 0	$s + 81$ Coefficient of s is 1	5A
Students reverse the order when translating worded problems to algebraic equations or expressions.	There are six times as many capybaras (c) as numbats (n). $6c = n$	There are six times as many capybaras (c) as numbats (n). $6n = c$	5A
Students match the order of words when translating worded problems to algebraic equations or expressions.	Three less than n. $3 - n$	Three less than n. $n - 3$	5A

Misconception	Incorrect	Correct	Lesson
Students place the larger number with the larger group when translating worded problems to algebraic equations or expressions.	There are seven times as many crocodiles (c) as echidnas (e). $7c = e$	There are seven times as many crocodiles (c) as echidnas (e). $7e = c$	5A
Students think that letters represent a word rather than a value.	$5p + 6t$ 5 pizzas + 6 tacos	$5p + 6t$ $5p + 6t$, where p = cost of pizza and t = cost of taco	5B
Students do not apply order of operations when solving algebraic equations.	$4 + 6i$, where $i = 3$ $4 + 6i = 4 + 6 \times 3$ $= 10 \times 3$ $= 13$	$4 + 6i$, where $i = 3$ $4 + 6i = 4 + 6 \times 3$ $= 4 + 18$ $= 22$	5B
Students think that a variable only represents one specific number.	 n is always equal to 8.	 n is only equal to 8 for this question.	5B
Students think that variables cannot represent the same number.	$s = 6$ and $p = 6$ is not true because variables must always represent a different number of quantities.	$s = 6$ and $p = 6$ is true because variables can represent the same number of quantities.	5B
Students will combine unlike terms together to form a single term when simplifying expressions.	$2l + 3a = 5al$	$2l + 3a$ This expression cannot be simplified further because $2l$ and $3a$ are unlike terms.	5C
Students ignore the sign of the coefficient of the term when combining like terms.	$10g - 6g = 16g$	$10g - 6g = 4g$	5C
Students will combine unlike terms together when simplifying terms with powers.	$5u^2 + u + 3eu^2 + e^2u$ $= 4e^2u^3 + 6u^3$	$5u^2 + u + 3eu^2 + e^2u$ This expression cannot be simplified further because $5u^2$, u, $3eu^2$, and e^2u are unlike terms.	5C
Students believe that only like terms can be multiplied and divided.	$3a \times 2L \times 4a = 12a \times 2L$	$3a \times 2L \times 4a = 24a^2L$	5D
When multiplying two terms students multiply each factor from one term by the coefficient of the other term.	$3 \times 6g = 3 \times 6 \times 3 \times g = 54g$	$3 \times 6g = 18g$	5D
When simplifying expressions with multiplication and powers, students use repeated addition or combine terms incorrectly.	$4b^2 \times 2b^3 = 8b^2b^3$ or $3b^2 \times 4r = 12(br)^2$	$4b^2 \times 2b^3 = 8b^5$ or $3b^2 \times 4r = 12b^2r$	5D
When dividing terms, students believe that terms with the same variable 'cancel' and 'go away' instead of simplifying.	$8a^2b \div 2a = \frac{8a^2b}{2a}$ $= 4b$	$8a^2b \div 2a = \frac{8a^2b}{2a}$ $= \frac{8 \times a \times a \times b}{2 \times a}$ $= 4ab$	5D

Misconception	Incorrect	Correct	Lesson
When adding algebraic fractions, students multiply the numerators and/or denominators.	$\frac{2e}{7} + \frac{4e}{7} = \frac{4 \times 2 \times e \times e}{7 \times 7} = \frac{8e^2}{49}$	$\frac{2e}{7} + \frac{4e}{7} = \frac{6e}{7}$	5E
When converting a whole term to a fraction, students use the entire term as both the numerator and the denominator.	$2w = \frac{2w}{2w}$	$2w = \frac{2w}{1}$	5E
Students don't multiply the reciprocal of the divisor.	$\frac{7}{a} \div \frac{a}{3} = \frac{7}{a} \times \frac{a}{3} = \frac{7a}{3a}$	$\frac{7}{a} \div \frac{a}{3} = \frac{7}{a} \times \frac{3}{a} = \frac{21}{a^2}$	5E
When expanding, students multiply the first term inside the bracket only.	$6(r + 8) = 6r + 8$	$6(r + 8) = 6r + 48$	5F
Students add or subtract when expanding terms.	$10(4e - 8) = 14e - 2$	$10(4e - 8) = 40e - 80$	5F
Students ignore negative signs when expanding.	$-2(3k - 7) = 6s - 14$	$-2(3k - 7) = -6s + 14$	5F
Students find a common factor and not the highest common factor when factorising.	$27 + 18i = 3(9 + 6i)$ 3 is the highest factor	$27 + 18i = 9(3 + 2i)$ 9 is the highest factor	5G
Students factor some of the terms.	$15d - 10 = 5(3d - 10)$	$15d - 10 = 5(3d - 2)$	5G
Students factor some of the operational signs.	$-24 - 6s = -6(4 - s)$	$-24 - 6s = -6(4 + s)$	5G

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5A Introduction to algebra

This lesson covers the foundational language of algebra. It is essential to understand this language so that equations and expressions can properly be represented and manipulated in the future. Equations and expressions can model real life situations.

LEARNING INTENTIONS

Students will be able to:

- define and identify the pronumerals, coefficients, terms and constants
- understand the difference between an expression and an equation
- write algebraic expressions and equations from worded situations
- write worded statements that represent algebraic expressions and equations.

KEY TERMS AND DEFINITIONS

A **coefficient** is the number that a variable is being multiplied by.

A **constant** is a number that cannot change its value in an expression or equation.

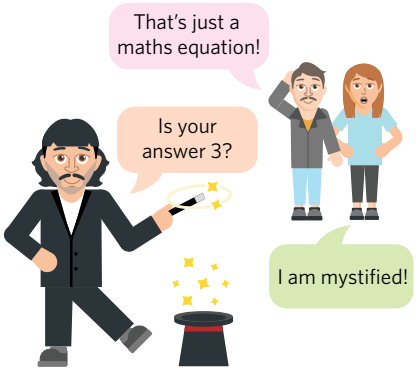
An **equation** is another name for a number sentence. An equation can contain letters, numbers and operations, as well as an equals sign. Each side of an equation is equal.

An **expression** is a number of terms grouped together by operations.

A **term** is a part of an expression or equation made up of a number and/or pronumerals. Terms are separated by addition or subtraction.

A **variable** is a letter used to represent a value that is unknown or may vary. This is also known as a pronumeral or an unknown.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Magic and algebra are closely related. Magic and algebra both rely on producing illusions to complicate and distract the audience. However, when we unveil the charms, spells and trickery we can see that the unknown was known all along.

A classic maths trick goes like this:

Pick any number from 1–10. Multiply that number by 2. Multiply the new number by 5. Divide that number by your original number. Then subtract 7.

Key ideas

1. Variables represent unknown quantities.

Words	Pictorial	Algebraic
Three rectangles that each have an area of y^2 cm^2, $1y$ cm^2, and ♥ cm^2.	y^2 y ♥	Area $= y^2 + y +$ ♥

2. Equations and expressions can both contain variables, coefficients, and constants with operations. Equations also contain an equal sign.

Coefficient Variable Constant

$-2a + 5 = 91$

Terms: $-2a$, 5, 91

Continues →

3. When writing expressions and equations, the coefficient is placed before the variable.

Correct maths conventions:	Incorrect maths conventions:
$-4a$	$-a4$

Worked example 1

Identifying parts of algebraic expressions and equations

For each expression or equation state:

i. if it is an expression or equation.
ii. the terms.
iii. the coefficient of b.
iv. the constants.

a. $3a - b - 9$ WE1a

Working

i. Expression

ii. The terms are $3a$, $-b$, -9.

iii. The coefficient of b is -1.

iv. The constant is -9.

Thinking

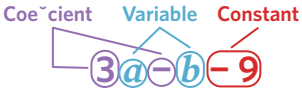
Step 1: If the (LHS) of the equal sign equals the (RHS) of the equals sign, then it will be an equation. Otherwise it is an expression.

Step 2: Determine the terms. Terms are separated by addition or subtraction.

Step 3: Identify the coefficients. Coefficients are the numbers multiplied by the variables.

Step 4: Identify the constant. The constant is a number without a variable.

Visual support



Terms: $3a$, $-b$, -9

b. $7a - 9b - 81 = 71 - c$ WE1b

Working

i. Equation

ii. The terms are $7a$, $-9b$, -81, 71, $-c$

iii. The coefficient of b is -9.

iv. The constants are -81 and 71.

Thinking

Step 1: An equation has an equal sign, whereas expressions do not.

Step 2: Determine the terms. Terms are separated by addition or subtraction.

Step 3: Identify the coefficients. Coefficients are the numbers multiplied to the variables.

Step 4: Identify the constant. The constant is a number without a variable.

Continues →

Student practice

For each expression and equation state:

i. if it is an expression or equation.
ii. the terms.
iii. the coefficient of b.
iv. any constants.

a. $7a - 2b + 9$

b. $4a - b - 161 = 43 - c$

Worked example 2

Writing algebraic expressions and equations

Write an expression or equation to represent each of the statements.

a. The product of b and negative two is added to five. WE2a

Working

$-2 \times b$ or $-2b$

$-2b + 5$

Thinking

Step 1: 'product of b and negative two' means we need to multiply b and -2.

Step 2: 'is added to five' means we add 5.

Visual support

The product of b and negative two is added to five.
$-2 \times b$ $+5$

b. One third of negative e is added to eight and is equal to thirty-two. WE2b

Working

$-\frac{e}{3}$ or $-\frac{1}{3}e$

$-\frac{e}{3} + 8$

$-\frac{e}{3} + 8 = 32$

Thinking

Step 1: 'one third of negative e' means we need to divide $-e$ by 3 or multiply $\frac{1}{3}$ and $-e$.

Step 2: 'is added to eight' means we add 8.

Step 3: 'and is equal to thirty-two' means we equal the two terms to 32.

Student practice

Write an expression or equation to represent each of the statements.

a. The product of e and negative five is added to twelve.

b. One half of negative m is added to four and is equal to thirty-five.

Worked example 3

Writing algebraic equations

Write a statement that describes the following expression or equation.

a. $15 - a = 9$ WE3a

Working

a is subtracted from fifteen.

a is subtracted from fifteen and equals nine.

Thinking

Step 1: The first term is 15 and is being subtracted by a.

Step 2: The two terms 15 and $-a$ equals 9.

Note: There are multiple answers.

Visual support

Subtract/Subtracted
Minus
Di erence

$15 - a = 9$

Equal to
Equivalent
Total

b. $\frac{n}{2} + 3y$ WE3b

Working

Half of n.

Half of n is added to three groups of y.

Thinking

Step 1: The first term is n divided by 2.

Step 2: The second term is added to three groups of y.

Student practice

Write a statement that describes the following expression or equation.

a. $18 - b = 9$ **b.** $\frac{e}{4} + 2t$

5A Questions

Understanding worksheet

1. Circle the coefficients of the expression or equation.

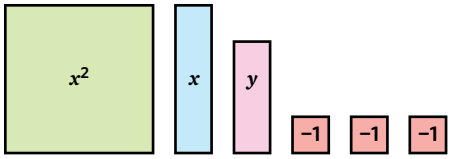
Example

$8b = 19 - 4a$

a. $8y + 7 = 3$ **b.** $3 - 9a$ **c.** $6y - 0.5m$ **d.** $\frac{1}{3}at = 3hs$

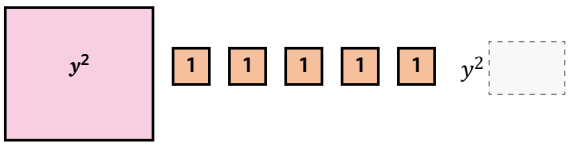
2. Fill in the blanks so that the expression shows the sum of the visual.

Example

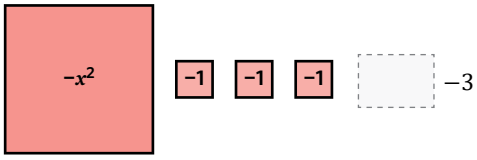


x^2 [$+ x$] $+ y$ [$- 3$]

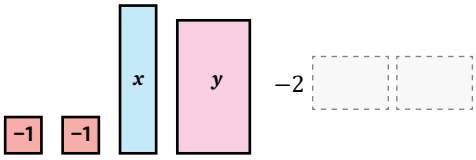
a.



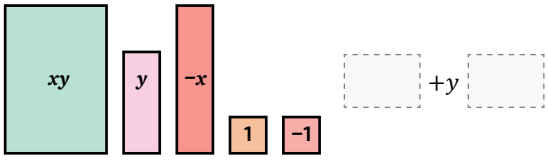
b.



c.



d.



3. Fill in the blanks by using the words provided.

constant | 1 | variable | term | equation

In the expression $2 + x$, the coefficient in front of the variable is _____. The number that cannot change its value in the expression or equations is called a _____. A constant and a _____ can also be called a _____ if they are separated by addition or subtraction. If $2 + x$ should equal 14, then this expression will change to an _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Use the words coefficient, term, variable, constant, expression, or equation to describe what is circled.

a. $8t - 6r + 3e + x$ (expression)

b. $3m + 2a - 4t - s = 54$ (equation)

c. $5w - 3.2a + x$ (w: variable)

d. $\frac{1}{4}y - \frac{3}{5}a = \frac{1}{2}$ ($\frac{1}{2}$: constant)

e. $42g + \frac{1}{2}a - m = e$ (m: variable)

f. $0.5ja - 87ws$ ($-87ws$: term)

g. $27b - a + m = 54$ ($-$: operation)

h. $\frac{a}{54} - 8.7 = 10.4$ ($\frac{a}{54}$: term)

5. For each expression and equation state: WE1

i. if it is an expression or equation.
ii. the terms.
iii. the coefficient of b.
iv. any constants.

a. $2b + 3u - g$

b. $\frac{1}{2}a - 7b - 2.5 = 5$

c. $c^2 - a^2 = 3^2$

d. $-\frac{11}{6}b - at$

e. $ca - 0.5b = s$

f. $\frac{b}{4} - ak + 9e - 8r$

g. $\frac{d}{5} = 25a^2 + 6 - b$

h. $\frac{3b}{4} + \frac{1a}{4}$

6. State all the terms in the following expressions.

a. $y + 8 + 4e$

b. $8h - a - 56w$

c. $\frac{1}{2} - 2sp - 32a$

d. $0.5s - 2k + 78 + \frac{1}{2}y$

e. $e^2 + 2el + s^2 + 1$

f. $\frac{a}{4} - 4rc - 100$

g. $8.9w^2 - \frac{1}{12}r + \sqrt{y}$

h. $\frac{a}{100} - \frac{a}{b} - \frac{b}{100}$

7. Write an expression or equation to represent each of the statements. WE2

a. Nine less than a.

b. The difference between thirteen and the sum of a and b.

c. The product of b and c is equal to negative ten.

d. The quotient of negative d and eight is equal to thirty-two.

e. One third of the product of f, g, and h is equal to ninety.

f. Half of three groups of d is added to four groups of e.

g. A quarter of e is added to f squared.

h. g is multiplied by itself and equals the square root of h.

---

8. Write a statement that describes the equation or expression. WE3

a. $a + 7$

b. $a - b$

c. $10 = 7 - b$

d. $ab = -15$

e. $2a - 3b$

f. $\frac{a}{2} + 3y$

g. $81 \div (c - 2)$

h. $g^2 + r + r + r = \sqrt{25}$

---

9. Write an equation or expression to represent each description

a. Each basketball team has t players. There are 10 teams with a total of 110 players.

b. A hotel had 234 rooms and had w rooms on each floor. There were a total of 18 floors.

c. Uzair has a total of $\$B$ in his checking account. He started with \$8200 in his checking account and paid \$62 each month for his internet over the next p months.

d. The total distance (d) in kilometres a skier can travel is equal to the product of speed (s) in kilometres per hour and time (t) in hours.

e. The perimeter (P) of a rectangle is equal the sum of two times the length (l) and two times the width (w).

f. The area of a square that has a side length of x metres is added the area of a rectangle with a side length of x metres and a width of 1 metre.

g. Three classes from Edrolo High School went to the airport with r students from each class. They met another school at the airport that had exactly half as many students. There were a total of z students at the airport.

h. The volume of a cube is calculated by cubing the side length y cm and is added to the volume of a rectangular prism with a side length y cm width of e cm and depth of p cm.

---

10. Which option shows the coefficients of m and t for the expression $7m - \frac{t}{3}$.

A. m: 7 and t: -1 **B.** m: -7 and t: $\frac{1}{3}$ **C.** m: 7 and t: $-\frac{t}{3}$ **D.** m: 7 and t: $-\frac{1}{3}$ **E.** m: $7m$ and t: $-\frac{t}{3}$

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. State the terms in the expression.

$\frac{a}{3} - 9b + c$

Student A

Terms: $\frac{a}{3}, -9b, c$

Student B

Terms: $\frac{1}{3}, a, 9b, c$

b. State the coefficients of a, b and c in the equation

$\frac{3a}{5} - 6b = 5$

Student A

Coefficient of a is 3

Coefficient of b is 6

Coefficient of c is 0

Student B

Coefficient of a is $\frac{3}{5}$

Coefficient of b is -6

Coefficient of c is 0

Problem solving

Question working paths

Mild 12, 13, 14	Medium 13, 14, 15	Spicy 14, 15, 16

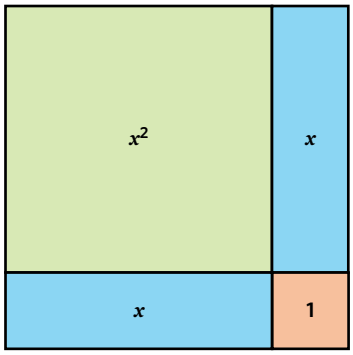
12. Kavitha's sister is s years old and Kavitha is currently half of her sister's age. Write an expression to represent how old Kavitha is.

13. A person's maximum heart rate can be calculated by subtracting seven tenths of their age from 207. Write an equation that describes the maximum heart rate (R) for somebody that is y years old.

14. Atsuko and Lena bought art supplies. Atsuko bought seven tubes of paint that cost \$$k$ each and \$72.20 worth of paintbrushes. Lena purchased three of the same tubes of paint and three canvases that cost \$$r$ each. Write an expression that represents the difference that Atsuko and Lena spent.

15. Alex wants to improve his magic show by using maths. He tells the audience to select any number (x) and multiply it by three and then by two. Afterward he instructs them to divide the result by the number they started with and then subtract one. Without simplifying, write an expression that describes the scenario of Alex's mathematical magic act in terms of x.

16. A large square is made up of two rectangles each with an area of x cm^2, a small square with an area of x^2 cm^2 and a smaller square with an area equal to 1 cm^2. Express the area of the large square.



Reasoning

Question working paths

Mild 17 (a,b,c,e)	Medium 17 (a,b,c,e), 18 (a,b)	Spicy All

17. Amir and his friends got a new job at Coolworths. Employees receive a percentage of the base rate of pay depending on their age. The current base pay rate is \$$r$ per hour.

Age	% of base rate of pay
16 years and under	50%
17 years	60%
18 years	70%
19 years	80%
20 years	90%

a. Amir's youngest friend, Kofi is 16. Write an equation that represents how much money (\$$K$) Kofi would receive per hour.

b. Using part **a**, write an expression that represents how much Kofi would receive in a week if he works four hours twice a week, and five hours once a week.

c. Amir turns 18 in two days. Write an expression that represents the difference between his pay rate as a 17 and 18 year old.

d. Hanna is 18 years old and wants to work over various public holidays. The holiday rate is calculated by adding 125% of the base rate to the employee's regular pay. Write an equation that represents how much total money ($\$H$) Hanna would receive if she worked on two public holidays for 6 hours each day.

e. If the base pay rate is \$22 per hour, do you think this base pay rate is a livable wage for a person who is aged 16–20 years old? Why or why not?

18. Apply the number 0 as the coefficient of a or b so that the expression $a - b$ is transformed into an;

a. expression containing one term with a positive coefficient.

b. expression containing one term with a negative coefficient.

c. Why is it possible to have a positive or a negative coefficient when there is no number written in front of the variable?

Extra spicy

19. Which of the following options is not equivalent to $3r + 1.5y$?

A. $3r + \frac{3}{2}y$ **B.** $3r + \frac{3y}{2}$ **C.** $3r + \frac{3}{2y}$ **D.** $3r + (3y \div 2)$ **E.** $3r + \frac{3}{2} \times y$

20. The magician Alex wants to switch up his mathematics magic act. He is currently using the equation $\frac{4(2p)}{p} - 1 = 7$, where p is any number the audience chooses and through various operations the audience gets an answer of 7. He wants the audience to get an answer of 15. What should the coefficient of p change to?

21. Which of the following statements always produces an even number if x and y are integers?

A. x^2 **B.** $3x + 2$ **C.** $2(x + y)$ **D.** $3(x + 2y)$ **E.** $x + y + 1$

22. A square with side length s, has a quarter of its area removed from one corner. What is the perimeter of the new square in terms of s?

Remember this?

23. Clinton wants to give $\frac{1}{5}$ of his 80 stickers to his brother.

How many stickers will he give to his brother?

A. 10 **B.** 16 **C.** 20 **D.** 40 **E.** 64

24. Edrolo bought brand new monitors and laptops for their employees worth \$ 1 750 000. They were given a 10% discount. What was the price Edrolo paid after the discount?

A. \$15 750 **B.** \$17 500 **C.** \$157 500 **D.** \$175 000 **E.** \$1 575 000

25. At the food stall, a can of soda and a burger each cost \$4. The food stall owner uses a formula to calculate the amount of money collected from the sale of w cans of soda and z burgers daily. Select all the formulas the owner could use to determine the correct amount of money collected daily in dollars.

A. $4wz$ **B.** $8wz$ **C.** $4w + 4z$ **D.** $4(wz)$ **E.** $8(w + z)$

Back to contents

5B Substitution

Substitution is an essential building block for algebra. If a value of a variable is known, we substitute that value into the equation and evaluate. Substitution can also be used for a formula or a rule.

LEARNING INTENTIONS

Students will be able to:

- substitute positive integers into expressions
- substitute negative integers into expressions
- use substitution and formulas to solve real world problems.

KEY TERMS AND DEFINITIONS

Substitution is the process of replacing a variable with a given value.

Evaluate means to carry out mathematical operations in the correct order to find a value of an expression or equation.

Formula is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.

Order of operations is a set of rules that tells us what calculations are performed first in a number sentence or equation. The order follows brackets, indices, division or multiplication and then addition or subtraction.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Noles

Register: 126 Receipt: 1179
Date: 01/08/2022 Time: 18:12

Snickers	6. 00
6 @ $1. 00 EACH	
Chupa Chup	0. 60
3 @ $0. 20 EACH	
Total of 9 items	$6. 60
Cash	$6. 60
GST INCLUDED IN TOTAL	$0. 66

Algebraic substitution is used in many real world situations. For example, we purchase three Chupa Chups and six Snickers. For each Chupa Chup it costs $0.20 and for each Snickers bar it costs $1.00. Our receipt will show that 0.20 x 3 and 1.00 x 6.

Key idea

1. Variables are letters or symbols that represent an unknown value. Variables are replaced with values to then evaluate the expression.

$2x + 5$

x	x	5

If $x = 8$

8	8	5

=

21

$2x + 5$

x	x	5

If $x = 4$

4	4	5

=

13

Worked example 1

Substitution

Substitute $p = 4$ and $q = -5$.

a. $8p + 3$ WE1a

Working

$8 \times 4 + 3$

$= 32 + 3$

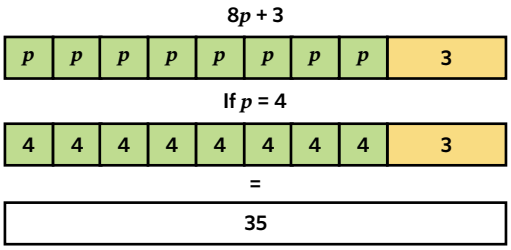
$= 35$

Thinking

Step 1: Substitute the value for the variable.

Step 2: Simplify.

Visual support



b. $-6p - \frac{4q}{2}$ WE1b

Working

$-6 \times (4) - \frac{4 \times (-5)}{2}$

$= -24 - \frac{-20}{2}$

$= -24 - (-10)$

$= -24 + 10$

$= -14$

Thinking

Step 1: Substitute the values for the variables.

Step 2: Simplify.

Student practice

Substitute $a = 9$ and $b = -8$

a. $7a + 5$ **b.** $-3a - \frac{6b}{3}$

Worked example 2

Using a rule

A bathtub is being filled up at a constant rate. The rule $W = 100t + 20$ is used to calculate how many litres of water is in the bathtub at any given time. W represents water in litres, and t represents time in hours.

Use the rule to construct and complete a table of values when $t = 1$, 3, and 5. WE2

Working

$t = 1, W = 100t + 20$

$= 100(1) + 20$

$= 100 + 20$

$= 120$

$t = 3, W = 100t + 20$

$= 100(3) + 20$

$= 300 + 20$

$= 320$

$t = 5, W = 100t + 20$

$= 100(5) + 20$

$= 500 + 20$

$= 520$

Time(t), in hours	1	3	5
Water(w), in litres	120	320	520

Thinking

Step 1: Substitute the t values into the rule to find the W values.

Step 2: Complete the table using the W values calculated.

Student practice

A bathtub is being filled up at a constant rate. The rule $W = 100t + 20$ is used to calculate how much litres of water is in the bathtub at any given time. W represents water in litres, and t represents time in hours.

Use the rule to construct and complete a table of values when $t = 2$, 4, and 6.

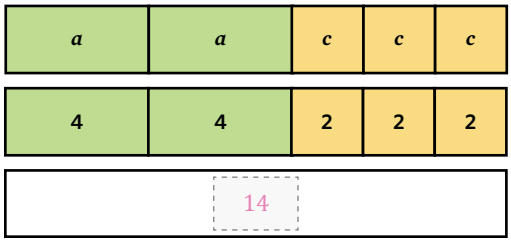
5B Questions

Understanding worksheet

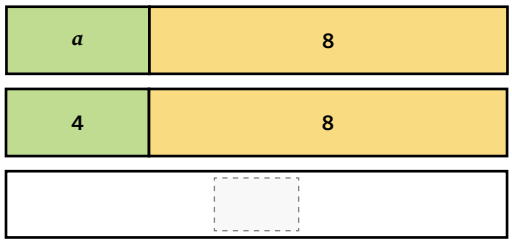
1. Substitute $a = 4$ and $c = 2$ to solve each bar model.

Example

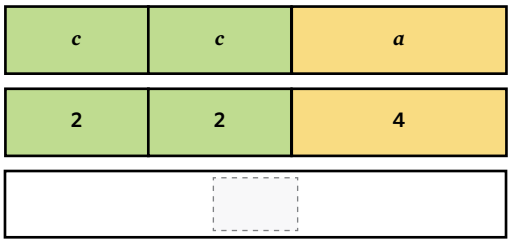
$2a + 3c$



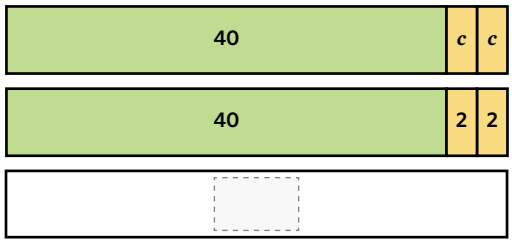
a. $a + 8$



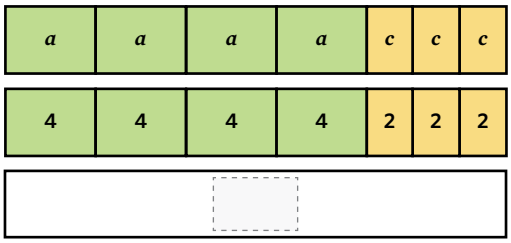
b. $2c + a$



c. $40 + 2c$



d. $4a + 3c$



2. Substitute $a = 2$, $b = 5$ and $c = 3$ for each expression.

Example

$3c + a$

$3 \times \boxed{3} + \boxed{2} = 11$

a. $4a$

$4 \times \boxed{} = 8$

b. $a + 3c$

$\boxed{} + 3 \times \boxed{} = 11$

c. $a + b - c$

$\boxed{} + \boxed{} - \boxed{} = 4$

d. $-b - 3b = -20$

$-\boxed{} - 3 \times \boxed{} = -20$

3. Fill in the blanks by using the words provided.

value | variable | substitute | operations

If a [ ] has a known value, it is possible to evaluate the expression. To evaluate an expression, [ ] the variable for the [ ] and simplify. The order in which the expression is simplified is referred to as the order of [ ].

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Substitute $a = 3$, $b = 2$ and $c = 4$ to evaluate the expression. WE1a

a. $3b + 4$ **b.** $9a + 2c$ **c.** $-8b + a$ **d.** $a(4 - 7c)$

e. $4b + \frac{3c}{a}$ **f.** $1.5(b + 4a)$ **g.** $-6(a - 4c)$ **h.** $-1.2(a^2 - 2c)$

---

5. Substitute $l = -2$, $m = -4$ and $n = -3$ to evaluate the expression. WE1b

a. $2m - 3$ **b.** $3n + 4$ **c.** $-4l - 6m$ **d.** $8l + 2n$

e. $5n + \frac{3m}{n}$ **f.** $2.5(n - 3l)$ **g.** $-4(m - 2n)$ **h.** $-1.5(m^2 + 3n)$

---

6. The cost of hiring a set of skis is given by the rule $\$C = 20 + 60d$. Where C is the total cost and d is the number of days. Substitute and determine the specified unknown value. WE2

a. One day (C) **b.** Three days (C)

c. Three and a half days (C) **d.** One week and a half (C)

e. The month of June (C) **f.** \$140 (d)

g. \$320 (d) **h.** \$500 (d)

---

7. Using the following expression $-3(-6 + ab)$ substitute and solve when $a = 7$ and $b = -9$.

A. -1134 **B.** 27 **C.** 207 **D.** 351 **E.** 1134

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Using the following expression $3(c + 4a)$, substitute and evaluate when $a = 3$ and $c = 8$.

Student A

$$\begin{aligned} 3(c + 4a) &= 3(8 + 4 \times 3) \\ &= 3(12 \times 3) \\ &= 3(36) \\ &= 108 \end{aligned}$$

Student B

$$\begin{aligned} 3(c + 4a) &= 3(8 + 4 \times 3) \\ &= 3(8 + 12) \\ &= 3(20) \\ &= 60 \end{aligned}$$

b. Substitute $p = -4$ and $r = -6$ to evaluate the expression $-5(3p - r)$.

Student A

$$\begin{aligned} -5(3p - r) &= -5(3 \times (-4) - (-6)) \\ &= -5(-12 + 6) \\ &= -5(-6) \\ &= 30 \end{aligned}$$

Student B

$$\begin{aligned} -5(3p - r) &= -5(3 \times (-4) - (-6)) \\ &= -5(12 - 6) \\ &= -5(6) \\ &= -30 \end{aligned}$$

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. Zora wants to print out the photos from her thirteenth birthday. The cost of printing out her photos from Officeworks costs $\$C = 1.5 + 0.25p$, where p is the number of photos and C is the cost. How much should Zora expect to pay for 50 photos?

10. Arnie is focusing on her fitness skills and wants to make sure that every time she attends the gym she reaches her maximum heart rate. This is calculated using $R = 207 - 0.7a$, where R is heart rate and a is age. Arnie is 29 years old, what is her maximum heart rate?

11. Kathy takes an Uber home from work everyday. The total cost is calculated by adding \$2.50 per kilometre travelled plus an additional \$0.75 per minute of idle time in traffic. How much will Kathy's total ride cost if she travelled 7.25 km and spent 12.5 minutes sitting idly in traffic?

12. Claire went skiing at Mt. Hotham. She went down the Big D slope at a constant speed of 18 km/h. The rule used is $k = 18x$, where k is total kilometres and x is time, in hours. How many kilometres can Claire expect to have skied after 45 minutes?

13. Cindy is baking some cupcakes in an oven.

$t =$ time in minutes

$c =$ number of cupcakes

She has two ovens:

Convection oven: $t = 15 + 7.5c$

Fan forced oven: $t = 25 + 5c$

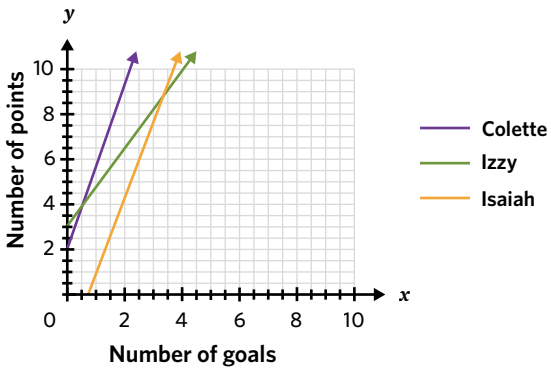
How much quicker is it to cook 10 cupcakes in a fan forced oven?

Reasoning

Question working paths

Mild 14 (a,b,d)	Medium 14 (a,b,d), 15 (a,b)	Spicy All

14. Izzy, Colette and Isaiah all play football. Izzy has scored $2j + 3$ points. Colette has scored $3j + 2$ points. Isaiah has scored $4j - 3$ points. Where j represents the number of goals. At the end of the season their coach has displayed their performance on the following graph.



a. How many goals will it take for Izzy and Collete to have an equal number of points?

b. After three goals, Izzy and Isaiah have the same amount of points. How many points will each have?

c. What is the difference in total points between Colette and Isaiah after 20 goals?

d. Izzy, Colette and Isaiah all want to increase their accuracy of goals for next season. What are some strategies they could implement?

15. Both a and b are different unknown numbers.

a. Substitute two values for the variables in $(a + b)^2$ and solve.

b. Substitute two values for the variables in $a^2 + b^2$ and solve.

c. Compare and contrast your answer from part **a** and part **b** and explain the effects to expressions with and without brackets.

Extra spicy

16. Ricardo multiplies an unknown number (r) by 2, then adds 5, then divides all this by 4.

What number needs to be substituted for r to get 17 as a result?

17. If $n = 2$, solve $n^2 - 4$ and calculate which expression will also give the same answer.

A. $3(2n - 3)$ **B.** $6(n - 2)$ **C.** $2(n - 1)$ **D.** $5(n - 3)$ **E.** $4(n - 1)$

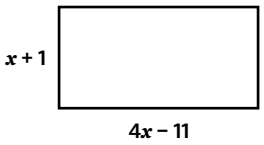
18. For the following equation, n is equal to 9.

$$\frac{6\left(\frac{n}{3} + 2\right)}{5} - 4 = 2$$

If the value of n is doubled, what will the equation now be equal to?

19. The value of $x = 7$. Substitute and calculate the perimeter, in cm, of the rectangle.

A. 28 cm
B. 50 cm
C. 70 cm
D. 98 cm
E. 1064 cm

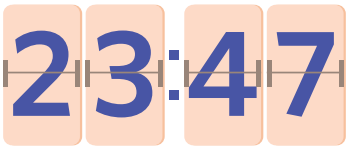


Remember this?

20. The clock below shows the time in the 24-hour system.

Using the 12-hour system, what is the equivalent time?

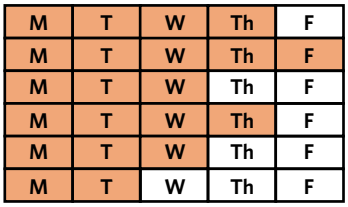
A. 2:47 am
B. 11:47 am
C. 2:47 pm
D. 3:47 pm
E. 11:47 pm



21. The shaded area on this graph shows the days that Gemma ran over a 30 day period.

What percentage of days has Gemma run?

A. 30%
B. 40%
C. 50%
D. 70%
E. Not enough information



22. Beacon Lighting is wanting to buy a new shop for a new location. The shop is worth $2 650 000. The agent is currently offering a 5% discount. What is the price Beacon Lighting will pay after the discount?

A. $1325 **B.** $2650 **C.** $132 500 **D.** $2 517 500 **E.** $2 782 500

Back to contents

5C Adding and subtracting terms

Expressions can be simplified if they contain like terms. Like terms contain the same variables, and can be combined by adding and subtracting the coefficients of the like terms. All constants are like terms and can just be added or subtracted. A fully simplified expression contains no like terms.

LEARNING INTENTIONS

Students will be able to:

- identify like terms
- simplify expressions by combining like terms.

KEY TERMS AND DEFINITIONS

A **constant** is a number that cannot change its value in an expression or equation.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: photocritical/Shutterstock.com

We create expressions when we don't know the values of some variables. For example if we want to calculate the costs of different sized perimeters of a paddock on a farm, we can use the expression $P = L + L + W + W$ and simplify it to $P = 2L + 2W$.

Key ideas

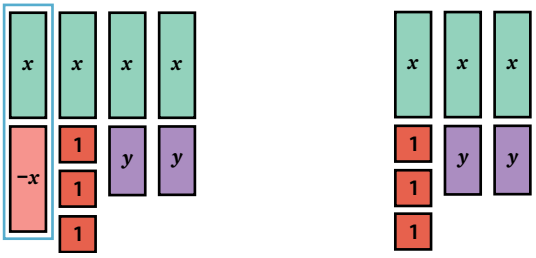
1. Terms with the same variables (and their powers) are like terms. This includes constants.

Like terms	Unlike terms
$7y$ and $-2y$	$7y$ and 7
$3n^2$ and n^2	$3n^2$ and $3n$
$2qr$ and $5rq$	$2q^2r$ and $5r^2q$
3, 4 and -5	3, $4x$ and $5y$

2. Expressions with like terms can be simplified by collecting the like terms.

Like terms

$4x + 3 + -x + 2y = 3x + 3 + 2y$



Worked example 1

Identifying like terms

Identify groups of like terms in each expression.

a. $3x - 2x + 9y$ WE1a

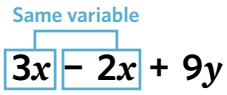
Working

$3x$ and $-2x$ are like terms.

Thinking

Identify terms that have the same variables or are constants.

Visual support



b. $8ab - 3 + 9ba + 4$ WE1b

Working

$8ab$ and $9ba$ are like terms.

-3 and 4 are like terms.

Thinking

Identify terms that have the same variables or are constants.

Student practice

Determine the like terms in each expression.

a. $7f - 2g + 3f$

b. $-2qw + 5qw - 2 + 8wq$

Worked example 2

Simplifying expressions with like terms

Simplify each expression by collecting like terms.

a. $4x + 2y - y$ WE2a

Working

y and $2y$ are like terms.

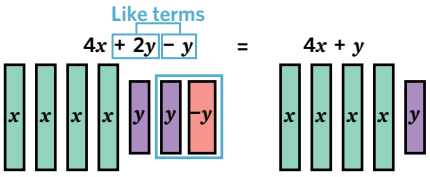
$4x + 2y - y = 4x + y$

Thinking

Step 1: Identify terms that have the same variables or are constants.

Step 2: Simplify the expression by collecting like terms.

Visual support



Continues →

b. $-2nm + 5mn + 3m + 8nm$ WE2b

Working

$-2nm$, $5mn$ and $8nm$ are like terms.

$-2nm + 5mn + 3m + 8nm$

$= -2nm + 5mn + 8nm + 3m$

$= 11nm + 3m$

Thinking

Step 1: Identify terms that have the same variables or are constants.

Step 2: Group together the like terms.

Step 3: Simplify the expression.

Student practice

Simplify each expression by collecting like terms.

a. $4a - b + 6a$

b. $2p + 9pq + 8qp - 2q$

Worked example 3

Substituting by fi st simplifying expressions

Simplify each expression and substitute the values for the variables.

a. $3i + 4j - 2i \ (i = 2, j = 3)$ WE3a

Working

$3i + 4j - 2i = 3i - 2i + 4j$

$= i + 4j$

$= 2 + 4 \times 3$

$= 2 + 12$

$= 14$

Thinking

Step 1: Simplify the expression by collecting like terms.

Step 2: Substitute the values for the variables.

b. $-5xy + 9xy + 3z + xy \ (x = 1, y = 2, z = 4)$ WE3b

Working

$-5xy + 9xy + 3z + xy = -5xy + 9xy + xy + 3z$

$= 5xy + 3z$

$= 5 \times 1 \times 2 + 3 \times 4$

$= 10 + 12$

$= 22$

Thinking

Step 1: Simplify the expression by collecting like terms.

Step 2: Substitute the values for the variables.

Student practice

Simplify each expression and substitute the values for the variables.

a. $10b - 5a - 4b \ (a = 5, b = 3)$

b. $fg + 3fg - 3h - 7h \ (f = 2, g = 3, h = 4)$

5C Questions

Understanding worksheet

1. In each expression, circle the like terms.

Example

$(6d) + 6e + (-2d) + 4e^2$

a. $2a + 4b + a$

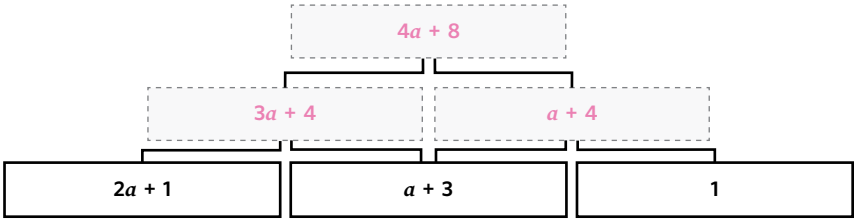
b. $10p - 2p + 3n - 7p$

c. $4km + 6k + 2m + 8mk$

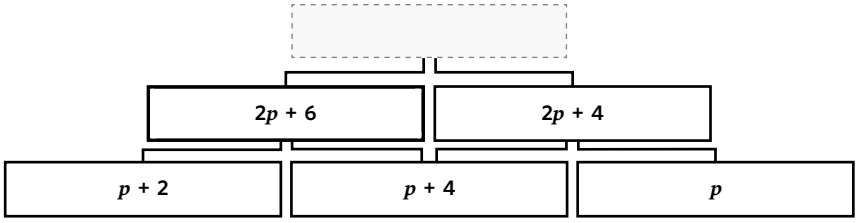
d. $x^2 + 6x - 9x^2 + 2$

2. Complete each pyramid. Each expression is the sum of the two expressions below it.

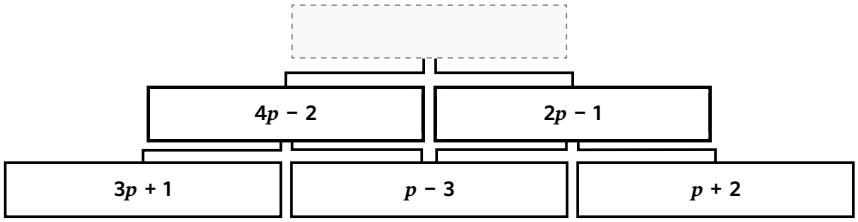
Example



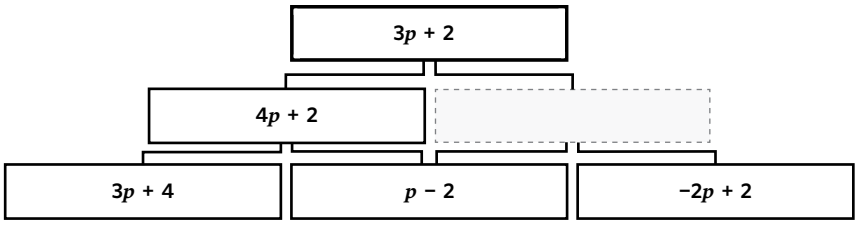
a.



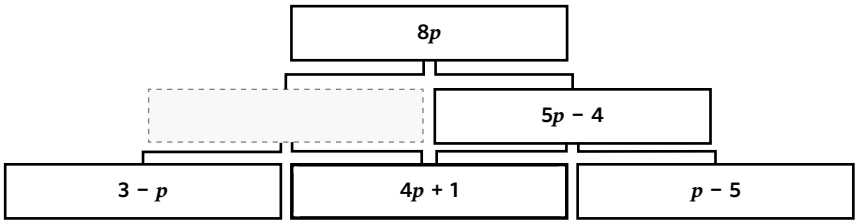
b.



c.



d.



3. Fill in the blanks by using the words provided.

simplify | constants | powers | commutative

Like terms have the same variables, which means they can be added or subtracted to _____ expressions. Since multiplication is _____, terms with the same variables but in different orders are still like terms. Terms that are just numbers are called _____ and are also like terms. If two terms have the same variables, but different _____ of those variables, they are not like terms.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Identify groups of like terms in each expression. WE1

a. $3x + 2x$
b. $1 + 2k + 4$
c. $7m + 2l - 3m$
d. $4 - 6p + 2 + p$
e. $4xy + 8x + 4y + 3x$
f. $10xy - 7xy - 10z - 8yx$
g. $10x^2 - 5y + 4x^2$
h. $6ab + 9ab + 10c^2 - b + 7c^3$

5. Simplify each expression by collecting like terms. WE2a

a. $8x + 7x$
b. $2p + 3 + 2$
c. $9u - 4u + 2j$
d. $3q + 2 - 2q + 8$
e. $0.5a + 4.2 + 3.5a$
f. $\frac{4}{5}b + \frac{2}{5} - \frac{1}{5}b + \frac{1}{5}$
g. $3x - 2y + 4x + 7y$
h. $-\frac{2}{3}m + \frac{5}{6}n - \frac{5}{3}m - \frac{2}{3}n$

6. Simplify each expression by collecting like terms. WE2b

a. $3jk + 2jk$
b. $4pr - 2p + 4p$
c. $2a^2 + 3a - 2a$
d. $4k^2 + 6 - 2k^2 + 2$
e. $-7h^2 + 4h^3 + 3h^2 - h$
f. $6m + n + 7mn - 5m$
g. $13ab - 3c + 4ba - 4$
h. $3mn + 2l^2m - 2nm + 5m^2l$

7. Simplify each expression and substitute the values for the variables. WE3

a. $15f - 6f\ (f = 5)$
b. $13t + 4t - 5t\ (t = 3)$
c. $12y + 3x - 2y\ (x = 2, y = 5)$
d. $-3r + 6u + 10r - 9u\ (r = 3, u = 1)$
e. $5a - 7ab + 4a + 7ba\ (a = 6, b = 7)$
f. $14tu - 3tu - ut - 5ut\ (t = 3, u = 6)$
g. $3a^2b - ba^2 + 2\ (a = 2, b = 3)$
h. $7x^2 - z - 12x^2 - 5z + xy\ (x = 2, y = -1, z = 3)$

8. Which option is the fully simplified expression of $6a^2 + 2a - 4a + 2$?

A. $6a^2$ **B.** $6a^2 - 2a + 2$ **C.** $6a^2 + 6a + 2$ **D.** $10a + 2$ **E.** $4a^2 + 2$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Simplify $4x + 8y + 2y$

Student A

$4x + 8y + 2y = 14xy$

Student B

$4x + 8y + 2y = 4x + 10y$

b. Simplify $3ab + 5ab^2 - 2a^2b + 4ab$

Student A

$3ab + 5ab^2 - 2a^2b + 4ab$
$= 7ab + 3ab^2$

Student B

$3ab + 5ab^2 - 2a^2b + 4ab$
$= 7ab + 5ab^2 - 2a^2b$

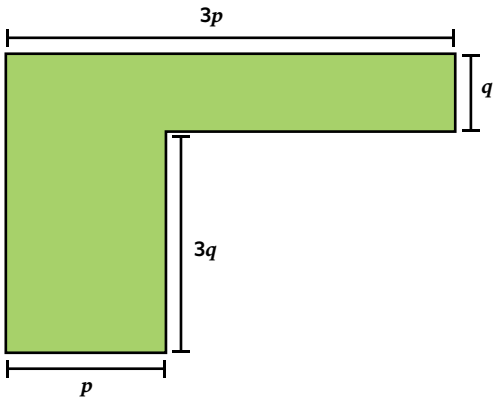
Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Vasili and Ivy went shopping at Officeworks. Vasili bought 4 pencils that cost $\$n$ each and 5 exercise books that cost $\$m$ each. Ivy bought 2 of the same pencils and 1 of the same exercise book. Write an expression to represent the total amount of money they both spent together.

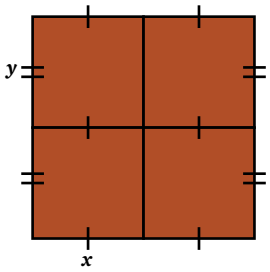
11. What expression can be written for the perimeter of the shape below?



12. The Miranda family has a shared data pool for their mobile internet plans. Fahrad used p gigabytes of data. Jina used 5 gigabytes less than Fahrad. Garath used 2 times as many gigabytes as Fahrad. What is the simplified expression for the total amount of data the family used?

13. Naheed took an Uber to get home from a concert. The Uber costs \$1.45 per kilometre (d), plus a \$0.55 booking fee. If the Uber distance is 4.5 km, how much will it cost Naheed to get home?

14. Rebecca has drawn a design for a wooden roof of an outdoor area in her backyard. It consists of four brown rectangles.



Write a simplified expression that represents the total area of the roof.

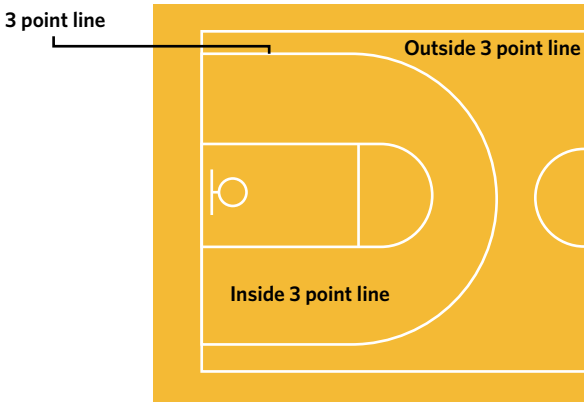
Reasoning

Question working paths

Mild 15 (a,b,d)	Medium 15 (a,b,d), 16 (a,b)	Spicy All

15. In a modified game of basketball, a free throw is worth a points, a goal made within the 3 point line is worth b points, and a goal made outside the 3 point line is worth c points.

A goal is considered a field goal if it's made within or outside the 3 point line, and is not a free throw.



a. In a game, Kai scored 5 goals inside the 3 point line, 4 goals outside the 3 point line, and made 3 free throws. Write an expression to find the total number of points Kai scored in the game.

b. If free throws and goals made outside the 3 point line are worth the same number of points, write a new expression for the total number of points Kai scored in the game.

c. If:

- goals inside the 3 point line are worth 4 points
- goals outside the 3 point line are worth 6 points
- free throws are worth 3 points

Use the expression from part **a** to calculate how many points Kai scored in the game.

d. List two things that a game designer should consider when determining how many points each type of goal is worth.

16. Find 2 equivalent expressions that simplify to $3a + 4b - 2$ and have:

a. 5 terms where only one term is a constant.

b. 6 terms where one term has a negative coefficient of a.

c. Substitute $a = 3$ and $b = -2$ into each expression and evaluate. What can you say about the values of equivalent expressions?

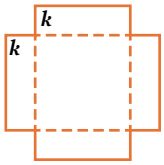
Extra spicy

17. Write an expression that represents the sum of five consecutive even integers, where x is the first integer.

18. Expressions inside brackets can be considered as single terms which can be collected as like terms. Which of the following options is a simplified expression of $(2x + 1) + 2(2x + 1) - 3(x + 1) + (1 + 2x)$?

A. $(1 + 2x)$
B. $(7x + 4)$
C. $4(6x + 3) - 3(x + 1)$
D. $4(2x + 1) - 3(x + 1)$
E. $3(2x + 1) - 3(x + 1) + (1 + 2x)$

19. A large square with side length j cm has 4 smaller squares cut from its corners.



The side lengths of the smaller squares are all k cm.

Write an expression for the area of the shape?

20. Two groups of $\sqrt{2}$ can be written as $2\sqrt{2}$. Using this, determine which of the following options is the simplified expression of $2x + x\sqrt{2} + 3\sqrt{2} - 4x - 5\sqrt{2}$.

A. $-6x\sqrt{2}$
B. $6x - 2x\sqrt{2}$
C. $-2x + x\sqrt{2} - 2\sqrt{2}$
D. $-2x - 2x\sqrt{2}$
E. $\frac{5x}{2} - 2\sqrt{2}$

Remember this?

21. Which number will make the number sentence correct?

$2.\overline{3} \times ? = 1.\overline{6}$?

A. $\frac{2}{3}$ **B.** $\frac{5}{7}$ **C.** $1\frac{2}{5}$ **D.** $3\frac{8}{9}$ **E.** 4

22. Jareem spends \$20, which is 40% of his pocket money. How much pocket money did he have before spending the \$20?

A. \$28 **B.** \$40 **C.** \$50 **D.** \$60 **E.** \$100

23. Matthew and Tori saved \$200. Tori saved three times as much money as Matthew.

Which expression represents the amount of money Matthew saved, if Tori saved \$$x$?

A. $3x - 200$ **B.** $x + 200$ **C.** $200 - 3x$ **D.** $200 - x$ **E.** $197 + 3x$

Back to contents

5D Multiplying and dividing terms

An algebra term is a variable, multiplied by a coefficient. When multiplying algebra terms together all of the factors are multiplied together to create a new term. Dividing two terms can be written as a fraction. A new term is created by simplifying all common factors.

LEARNING INTENTIONS

Students will be able to:

- multiply positive and negative algebra terms
- divide positive and negative algebra terms
- express algebra terms in simplest form.

KEY TERMS AND DEFINITIONS

Common factors are the same factors that occur between two or more numbers.

The **commutative law** means that the order of the numbers in an addition and multiplication calculation can change without affecting the result.

A **divisor** is the second number in a division calculation. It is the number by which a given value is divided.

The **product** is the result when two or more values are multiplied together.

A fraction is in its **simplest form** when the numerator and denominator have no common factors other than 1.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

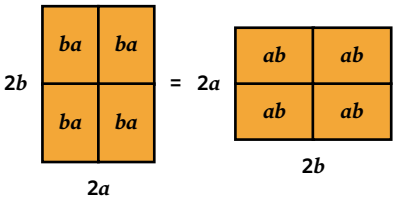
Image: Jasen Wright/Shutterstock.com

Multiplying and dividing algebra terms can be used by businesses. For example, factories may use the productivity formula $\frac{x}{y}$, where x represents the total output of goods, and y represents the total input of labour, materials, and other production expenses.

Key ideas

1. Multiplication is commutative, therefore order does not matter.

$2b \times 2a = 4ba = 4ab$



2. Exponent notation is used to indicate repeated variable factors.

$$3a \times ac = 3 \times a \times a \times c = 3a^2c$$

Exponent form

Repeated factor

3. Express the division of algebra terms as a fraction and simplify all common factors.

$$6a^2 \div 3ac = \frac{6a^2}{3ac} = \frac{\overset{1}{\cancel{3}} \times 2 \times \overset{1}{\cancel{a}} \times a}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{a}} \times c} = \frac{2a}{c}$$

3 and a are common factors

Worked example 1

Multiplying terms

Simplify.

a. $3b \times 6c$ WE1a

Working

$3b \times 6c = 3 \times b \times 6 \times c$

$= 3 \times 6 \times b \times c$

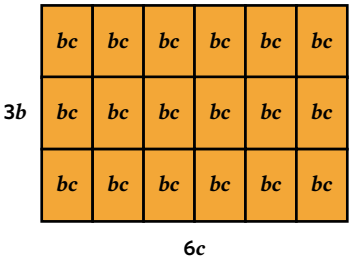
$= 18bc$

Thinking

Step 1: Write the expression with multiplication signs.

Step 2: Rearrange the expression using the commutative law, combining the common factors and simplify the product.

Visual support



b. $-5cf \times -2y$ WE1b

Working

$-5cf \times -2y = 5cf \times 2y$

$= 5 \times c \times f \times 2 \times y$

$= 5 \times 2 \times c \times f \times y$

$= 10cfy$

Thinking

Step 1: Determine if the product is positive or negative.

Step 2: Write the expression with multiplication signs.

Step 3: Rearrange the expression using the commutative law, combining the common factors and simplify the product.

Student practice

Simplify.

a. $8a \times 5b$ **b.** $-4de \times -3f$

Worked example 2

Dividing terms

Simplify.

a. $2xy \div 5x$ WE2a

Working

$2xy \div 5x = \frac{2xy}{5x}$

$= \frac{2 \times y}{5}$

$= \frac{2y}{5}$

Thinking

Step 1: Represent the expression as a fraction.

Step 2: Simplify all common factors.

Continues →

b. $8qrt \div (-4qtu)$ WE2b

Working

$$8qrt \div (-4qtu) = -(8qrt \div 4qtu)$$

$$= -\frac{8qrt}{4qtu}$$

$$= -\frac{\overset{2}{\cancel{8}}\ \overset{1}{\cancel{q}}\ r\ \overset{1}{\cancel{t}}}{\underset{1}{\cancel{4}}\ \underset{1}{\cancel{q}}\ \underset{1}{\cancel{t}}\ u}$$

$$= -\frac{2r}{u}$$

Thinking

Step 1: Determine if the result of the division is positive or negative.

Step 2: Represent the expression as a fraction.

Step 3: Simplify all common factors.

Student practice

Simplify.

a. $4ts \div 3s$ **b.** $12abc \div (-4acd)$

Worked example 3

Multiplying and dividing terms with exponents

Simplify.

a. $5by \times bz$ WE3a

Working

$$5by \times bz = 5 \times b \times y \times b \times z$$

$$= 5 \times b \times b \times y \times z$$

$$= 5b^2yz$$

Thinking

Step 1: Write the expression with multiplication signs.

Step 2: Rearrange the expression using the commutative law, combining the common factors and simplify the product.

b. $5p^3y \div pyz$ WE3b

Working

$$5p^3y \div pyz = \frac{5p^3y}{pyz}$$

$$= \frac{5 \times p \times p \times p \times y}{p \times y \times z}$$

$$= \frac{5 \times p \times p \times \overset{1}{\cancel{p}} \times \overset{1}{\cancel{y}}}{\underset{1}{\cancel{p}} \times \underset{1}{\cancel{y}} \times z}$$

$$= \frac{5p^2}{z}$$

Thinking

Step 1: Represent the expression as a fraction.

Step 2: Write the expression with multiplication signs.

Step 3: Simplify all common factors. Repeated multiplication is simplified using exponent notations.

Student practice

Simplify.

a. $2ab \times ac$ **b.** $7b^3a \div abc$

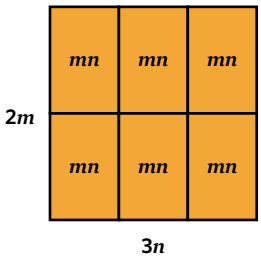
5D Questions

Understanding worksheet

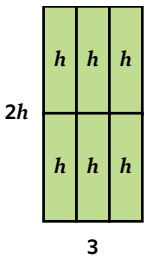
1. Use the area models to complete the multiplication.

Example

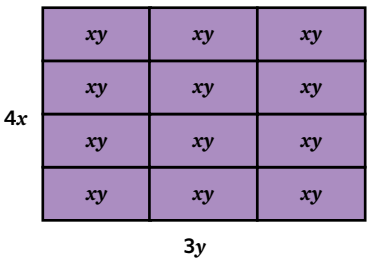
$2m \times 3n =$ $6mn$



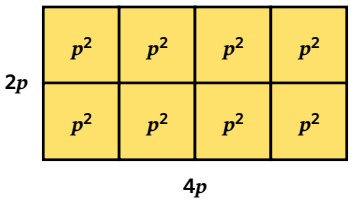
a. $2h \times 3 =$



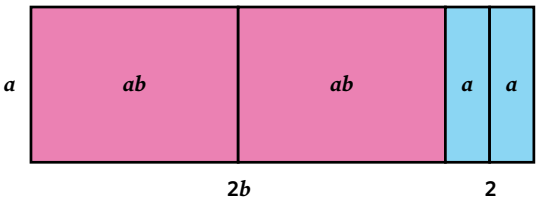
b. $4x \times 3y =$



c. $2p \times 4p =$

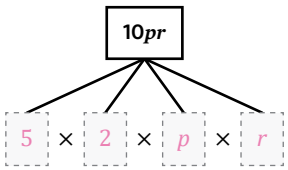


d. $a \times (2b + 2) =$

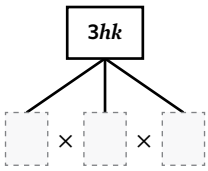


2. Find all the factors of the following terms.

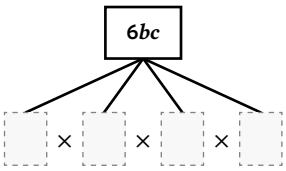
Example



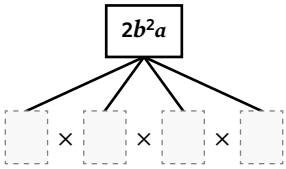
a.



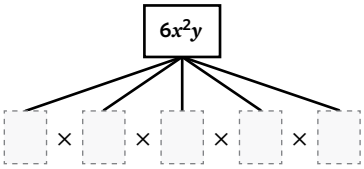
b.



c.



d.



3. Fill in the blanks by using the words provided.

coefficie | variables | factors | exponent

An algebra term is defined as one or multiple _____ and a _____.

Because they are all multiplied together to create a term, they are all _____ of the term.

The _____ notation is used when a variable is multiplied by itself more than one time.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Simplify. WE1a

a. $2g \times 5$ **b.** $7 \times 3r$ **c.** $j \times 8k$ **d.** $6x \times y$

e. $3a \times 5x$ **f.** $4xy \times 9z$ **g.** $1.5s \times 6ty$ **h.** $\frac{2}{3}d \times 12ef$

5. Simplify. WE2a

a. $3x \div 5$ **b.** $2 \div 5p$ **c.** $2g \div 3gh$ **d.** $10x \div 5yz$

e. $2qr \div 8q$ **f.** $12wxy \div 18xyz$ **g.** $51mg \div 27bmx$ **h.** $3.5p \div 7tp$

6. Simplify. WE1b,2b

a. $5e \times -3f$ **b.** $-8jk \div 2k$ **c.** $-x \times -6ap$ **d.** $12qp \div -2k$

e. $-24x \div -18xk$ **f.** $-7t \times 2.3r$ **g.** $-\frac{4}{3}x \times 9z$ **h.** $-12b \div -0.6q$

7. Simplify. WE3

a. $x \times 9x$ **b.** $9bz \times b$ **c.** $2q^2 \div q$ **d.** $5y^2x \times 4zy$

e. $d^4 \div cd^2$ **f.** $k^2p \times 8p^2k^3$ **g.** $30u^4w \div 18u^3w$ **h.** $26r^2s \div 18rs^2$

8. Simplify.

a. $c \times 9x \times 2xc$ **b.** $(a^3 \div 9b) \times 3ab^2$

c. $(3a^3b)^2$ **d.** $(2xy^2)^3 \div 16y^4$

e. $-\frac{3}{8} \times (2a^2 \div ab)$ **f.** $(25d \div 6b^4) \times (8ab^2 \div 15cd^2)$

g. $7wq \times (11q^2 + 15hw)$ **h.** $(9x + 15f) \div 2y$

9. Simplify.

$(4a^2b \times 3ab) \div 6ab$

A. $4a^2b \times \frac{3ab}{6ab}$

B. $\frac{12a^2b}{6ab}$

C. $\frac{12a^3b^2}{6ab}$

D. $2a^2b$

E. $\frac{4a^2b \times 3 \times 4a^2b \times a \times 4a^2b \times b}{6ab}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Simplify $10a \times 5ab$

Student A

$$\begin{aligned} 10a \times 5ab &= 10 \times 5 \times a \times a \times b \\ &= 50 \times a^2 \times b \\ &= 50a^2b \end{aligned}$$

Student B

$$\begin{aligned} 10a \times 5ab &= (10a \times 5a) \times b \\ &= 50a \times b \\ &= 50ab \end{aligned}$$

b. Simplify $15e^2f \div 12ef^2$

Student A

$$\begin{aligned} 15e^2f \div 12ef^2 &= 15 \times e^2f \div 12 \times ef^2 \\ &= 15 \times 12 \div e^2f \times ef^2 \\ &= 180 \div e^3f^3 \\ &= \frac{180}{e^3f^3} \end{aligned}$$

Student B

$$\begin{aligned} 15e^2f \div 12ef^2 &= \frac{15e^2f}{12ef^2} \\ &= \frac{\overset{1}{\cancel{3}} \times 5 \overset{1}{\cancel{e}} e \overset{1}{\cancel{f}}}{\underset{1}{\cancel{3}} \times 4 \underset{1}{\cancel{e}} \underset{1}{\cancel{f}} \underset{1}{\cancel{f}}} \\ &= \frac{5e}{4f} \end{aligned}$$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

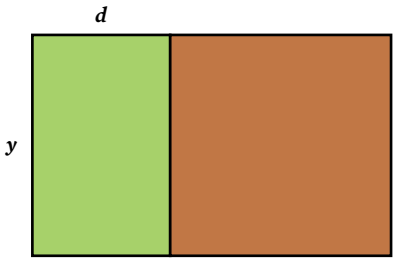
11. Hera wants to calculate the area of her rectangular cargo bay. She uses the arm spans of Ezra and Kanan. Write an equation for the area if the width is three times Kanan's arm span (k) and the length is four times Ezra's arm span (e).

12. Maxine's Hardware shop wanted to keep track of the number of nails that arrive in a particular order. Write an expression to determine how many nails are in an order that arrives with (c) amount of crates. The crates contain (b) amount of boxes. The boxes contain (n) amount of nails.

13. Freeda was organising fruit snacks for her childcare centre. Each day a number of parent volunteers (v) would cut the same amount of pieces (p) each and Freeda would share them equally among the children (c). Write an expression that would allow Freeda to determine how many pieces each child would receive once she knew how many parents and how many children attended that day.

14. How many tiles would Dimitry need for the great dining hall in his palace if it is x metres wide and y metres long and the tiles are a millimetres wide and b millimetres long?

15. Dylan has enough lawn seed for 16 m^2. His back fence is y metres long. Write an algebraic expression to describe the distance (d) away from the fence he can plant the seed for a rectangular lawn.



Reasoning

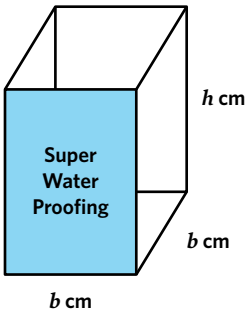
Question working paths

Mild 16 (a,b,d)	Medium 16 (a,b,d), 17 (a,b)	Spicy All

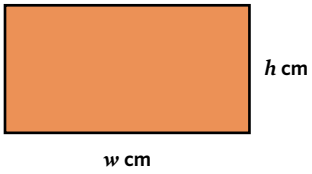
16. Pinewall College has constructed a large new building and have contacted Mario to give them a quote to waterproof the roof. Mario has decided to arrive with prepared expressions to use once they give him the measurements so he can quickly calculate the pricing for them.

Image: Aisyaqilumaranas/Shutterstock.com

a. Use the following diagram to calculate the volume of one container of waterproofing membrane.



b. The membrane needs to be applied 0.4 cm thick. Use the diagram of the roof to calculate the volume of membrane needed.



c. Write an expression that will calculate the amount of containers required.

d. Water damage is detrimental to concrete, would you focus on spending more on the initial waterproofing, or would you keep costs down initially and plan to regularly maintain the water proofing?

17. Algebra allows mathematicians to test general cases and define rules and conventions.

a. Express $(3x \div 4y) \div 5z$ as a fraction.

b. Express $3x \div (4y \div 5z)$ as a fraction.

c. Compare and contrast your answers in part **a** and **b** to then explain why brackets are important when writing expressions.

Extra spicy

18. If

$a + a = b$

$a + b = c$

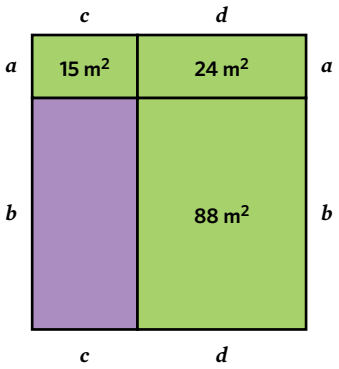
$a + b + c = d$

$xa = d$

Then x is equal to:

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

19. Use the following rectangle to determine the value of bc.



20. Last year at Matthew's Mathematics college there were 315 girls out of 600 students. This year the number of students has increased to 640. If the proportion of girls is the same as last year, how many girls are there?

21. Let n and m be any positive single digit whole number. How many possible solutions are there to nm?

A. 9 **B.** 36 **C.** 45 **D.** 81 **E.** 99

Remember this?

22. Rebecca's flight was scheduled to depart at 10:30 pm. If the flight took $1\frac{1}{2}$ hours but was delayed by $1\frac{1}{4}$ hours, what time did she arrive?

A. 12:45 am **B.** 1:00 pm **C.** 1:00 am **D.** 1:15 am **E.** 1:15 pm

23. Cameron and Michael rounded the number 129 988 in different ways but got the same answer. Which two ways of rounding did they use?

A. Rounding to the nearest unit and nearest ten

B. Rounding to the nearest ten and nearest hundred

C. Rounding to the nearest ten and nearest thousand

D. Rounding to the nearest hundred and nearest thousand

E. Rounding to the nearest thousand and nearest hundred thousand

24. Two 6 sided dice are rolled and the outcomes are added. Which of the following totals is most likely to occur?

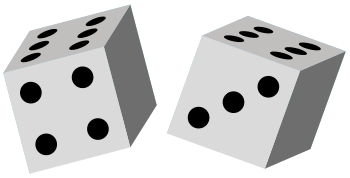
A. 2

B. 4

C. 7

D. 10

E. 12



Back to contents

5E Algebraic fractions

The concepts with algebraic fractions are the same as those for other fractions. To add and subtract a pair of fractions they need to have the same denominator. To multiply fractions, multiply the numerators and the denominators. To divide fractions, multiply by the reciprocal of the divisor.

LEARNING INTENTIONS

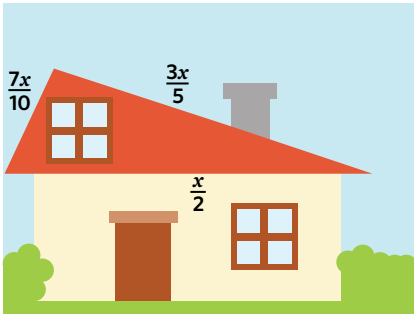
Students will be able to:

- add and subtract algebraic fractions
- multiply algebraic fractions
- divide algebraic fractions.

KEY TERMS AND DEFINITIONS

Algebraic fraction is a fraction using a variable in the numerator, or the denominator, or in both the numerator and the denominator.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Algebraic fractions are used in many real world situations. For example we can use algebraic fractions to scale the dimensions of a triangular roof on blueprints.

Key ideas

1. The arithmetic for an algebraic fraction is the same as a fraction.

Fraction

$$\frac{3}{8} + \frac{2}{8} = \frac{2+3}{8} = \frac{5}{8}$$

Same denominator = no change

Algebraic fraction

$$\frac{3p}{8} + \frac{2p}{8} = \frac{2p+3p}{8} = \frac{5p}{8}$$

Same denominator = no change

2. Multiplying and dividing algebraic fractions is calculated the same way as multiplying and dividing fractions.

Fraction

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15}$$

Algebraic fraction

$$\frac{2r}{3} + \frac{3r}{5} = \frac{2r \times 3r}{3 \times 5} = \frac{6r^2}{15}$$

Fraction

$$\frac{4}{7} \div \frac{3}{5} = \frac{4}{7} \times \frac{5}{3} = \frac{20}{21}$$

Algebraic fraction

$$\frac{4a}{7} \div \frac{3b}{5} = \frac{4a}{7} \times \frac{5}{3b} = \frac{20a}{21b}$$

Worked example 1

Adding and subtracting algebraic fractions

Simplify.

a. $\frac{2a}{9} + \frac{4a}{9}$ WE1a

Working

$2a$ and $4a$ are like terms.

$$\frac{2a}{9} + \frac{4a}{9} = \frac{2a + 4a}{9}$$

$$= \frac{6a}{9}$$

$$= \frac{2a}{3}$$

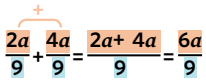
Thinking

Step 1: Identify terms with the same variables.

Step 2: Add the numerators. The denominator values do not change as they are the same.

Step 3: Simplify.

Visual support



Same denominator = no change

b. $\frac{9x}{8} - \frac{3x}{8}$ WE1b

Working

$9x$ and $3x$ are like terms.

$$\frac{9x}{8} - \frac{3x}{8} = \frac{9x - 3x}{8}$$

$$= \frac{6x}{8}$$

$$= \frac{3x}{4}$$

Thinking

Step 1: Identify terms with the same variables.

Step 2: Subtract the numerators. The denominator values do not change as they are the same.

Step 3: Simplify.

Student practice

Simplify.

a. $\frac{2s}{7} + \frac{3s}{7}$ **b.** $\frac{7d}{12} - \frac{2d}{12}$

Worked example 2

Multiplying algebraic fractions

Simplify.

a. $\frac{3}{a} \times \frac{5}{b}$ WE2a

Working

$$\frac{3}{a} \times \frac{5}{b} = \frac{3 \times 5}{a \times b}$$

$$= \frac{15}{ab}$$

Thinking

Multiply the numerators and multiply the denominators.

Continues →

Visual support

$$\frac{3}{a} \times \frac{5}{b} = \frac{3 \times 5}{a \times b} = \frac{15}{ab}$$

b. $7 \times \frac{c}{6}$ WE2b

Working

$$7 \times \frac{c}{6} = \frac{7}{1} \times \frac{c}{6}$$

$$= \frac{7 \times c}{1 \times 6}$$

$$= \frac{7c}{6}$$

Thinking

Step 1: Rewrite the whole number as a fraction with a denominator of 1.

Step 2: Multiply the numerators and multiply the denominators.

Student practice

Simplify.

a. $\frac{2}{g} \times \frac{6}{h}$ **b.** $4 \times \frac{q}{3}$

Worked example 3

Dividing algebraic fractions

Simplify.

a. $\frac{y}{6} \div \frac{z}{4}$ WE3a

Working

$$\frac{z}{4} \rightarrow \frac{4}{z}$$

$$\frac{y}{6} \div \frac{z}{4} = \frac{y}{6} \times \frac{4}{z}$$

$$= \frac{y \times 4}{6 \times z}$$

$$= \frac{4y}{6z}$$

$$= \frac{2y}{3z}$$

Thinking

Step 1: Find the reciprocal of the divisor.

Step 2: Rewrite the division as multiplication.

Step 3: Multiply the numerators and multiply the denominators.

Step 4: Simplify.

Visual support

$$\frac{y}{6} \div \frac{z}{4} = \frac{y}{6} \times \frac{4}{z} = \frac{4y}{6z}$$

b. $4 \div \frac{x}{2y}$ WE3b

Working

$$4 = \frac{4}{1}$$

$$\frac{x}{2y} \rightarrow \frac{2y}{x}$$

Thinking

Step 1: Rewrite the whole number as a fraction with a denominator of 1.

Step 2: Find the reciprocal of the divisor.

Continues →

$$4 \div \frac{x}{2y} = \frac{4}{1} \times \frac{2y}{x}$$
$$= \frac{4 \times 2y}{1x}$$
$$= \frac{8y}{x}$$

Step 3: Rewrite the division as multiplication.

Step 4: Multiply the numerators and multiply the denominators.

Step 5: Simplify.

Student practice

Simplify.

a. $\frac{c}{4} \div \frac{d}{8}$ **b.** $6 \div \frac{6j}{k}$

5E Questions

Understanding worksheet

1. Determine the missing value to make the following equations true.

Example

$\frac{2x}{4} + \frac{x}{4} = \frac{3x}{4}$

a. $\frac{4a}{7} + \frac{2a}{7} = \frac{\square}{7}$ **b.** $\frac{8t}{5} - \frac{2t}{5} = \frac{\square}{5}$

c. $\frac{3a}{5} = \frac{\square}{10}$ **d.** $\frac{2q}{13} = \frac{6q}{\square}$

2. Determine the missing value to make the following equations true.

Example

$\frac{a}{7} \times \frac{c}{6} = \frac{ac}{42}$

a. $\frac{3rs}{4} \times \frac{8}{2} = \frac{24rs}{\square}$ **b.** $\frac{5p}{2} \times \frac{2}{7} = \frac{\square}{14}$

c. $\frac{2a}{4} \div \frac{2}{8} = \frac{2a}{4} \times \frac{8}{\square}$ **d.** $\frac{3m}{4} \div \frac{2n}{9} = \frac{3m}{4} \times \frac{9}{\square}$

3. Fill in the blanks by using the words provided.

algebraic | denominator | reciprocal | product

The sum or difference of $\square$ fractions must have a common $\square$.

To divide algebraic fractions, the dividend must be multiplied by the $\square$ of the divisor.

To multiply algebraic fractions, find the $\square$ of the numerators and the denominators.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Simplify. WE1

a. $\frac{l}{5} + \frac{3l}{5}$ **b.** $\frac{12a}{7} - \frac{9a}{7}$ **c.** $\frac{2t}{15} + \frac{3t}{15}$ **d.** $\frac{5y}{16} - \frac{y}{8}$

e. $\frac{5b}{4} + \frac{3b}{8}$ **f.** $\frac{5g}{7} - \frac{3g}{14}$ **g.** $\frac{2e}{4} + \frac{8e}{16}$ **h.** $\frac{t}{4} - \frac{t}{5}$

5. Simplify. WE2

a. $\frac{c}{7} \times \frac{d}{5}$ **b.** $\frac{r}{5} \times \frac{s}{4}$ **c.** $\frac{1}{2} \times \frac{m}{n}$ **d.** $3 \times \frac{5}{c}$

e. $4 \times \frac{4}{2d}$ **f.** $\frac{a}{3} \times \frac{5a}{5}$ **g.** $\frac{2}{m} \times n$ **h.** $\frac{x^2}{2} \times \frac{2x}{4}$

6. Simplify. WE3

a. $\frac{a}{3} \div \frac{2}{x}$ **b.** $\frac{5}{a} \div \frac{b}{a}$ **c.** $\frac{1}{5} \div \frac{m}{n}$ **d.** $\frac{7}{t} \div \frac{t}{3}$

e. $12 \div \frac{c}{d}$ **f.** $\frac{4}{9} \div z$ **g.** $\frac{2a}{5} \div \frac{3c}{7}$ **h.** $\frac{5}{12b} \div \frac{2b}{3}$

7. Simplify.

$\frac{7}{h} \div \frac{h}{3}$

A. $\frac{7h}{3h}$ **B.** $\frac{7}{3}$ **C.** $\frac{21}{h^2}$ **D.** $\frac{21}{h}$ **E.** $\frac{h^2}{21}$

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. $\frac{2z}{7} + \frac{4z}{7}$

Student A

$\frac{2z}{7} + \frac{4z}{7} = \frac{2z + 4z}{7}$

$= \frac{6z}{7}$

Student B

$\frac{2z}{7} + \frac{4z}{7} = \frac{z \times z \times 4 \times 2}{7}$

$= \frac{8z^2}{7}$

b. $\frac{2v}{3} \times 3v$

Student A

$\frac{2v}{3} \times 3v = \frac{2v}{3} \times \frac{3v}{3v}$

$= \frac{6v^2}{9v}$

Student B

$\frac{2v}{3} \times 3v = \frac{2v}{3} \times \frac{3v}{1}$

$= \frac{2v \times 3v}{3 \times 1}$

$= \frac{6v^2}{3}$

$= \frac{2v^2}{1}$

$= 2v^2$

Problem solving

Question working paths

Mild	Medium	Spicy
9, 10, 11	10, 11, 12	11, 12, 13

9. The Muscat and Axiak family are having pizza for dinner. The bill for the pizza comes to a total of $\$p$ and each family pays half. Write an algebraic fraction for the amount the Muscat family needs to pay.

10. Lewis has gone to the shops. He spent half of his money on Christmas gifts and a third of his money on a pair of shoes. Let m be the total amount of Lewis' money. Write an expression to show how much money Lewis has left.

11. Blossom Collective buys flowers from the local flower market. Let d represent the total money Marielle spent at the market. She spent one eighth on tulips, one quarter on dahlias and one half on ranunculus. Write an expression to show how much money Marielle has spent on flowers.

12. Jaxson has a number, represented as h, written on a piece of paper. On a different piece of paper he has a number written that is three times h. The sum of their reciprocals is unknown. Construct an algebraic expression to represent this scenario and simplify it.

13. In a particular odd taxi, Odokawa the driver charges $\$\frac{7d}{2}$ per kilometre. There is also a base fee of $\frac{c}{3}$ regardless of the distance travelled. If Shirakawa takes this taxi and travels $\frac{n}{6}$ kilometres, how much does Shirakawa have to pay Odokawa, in terms of c, d and n?

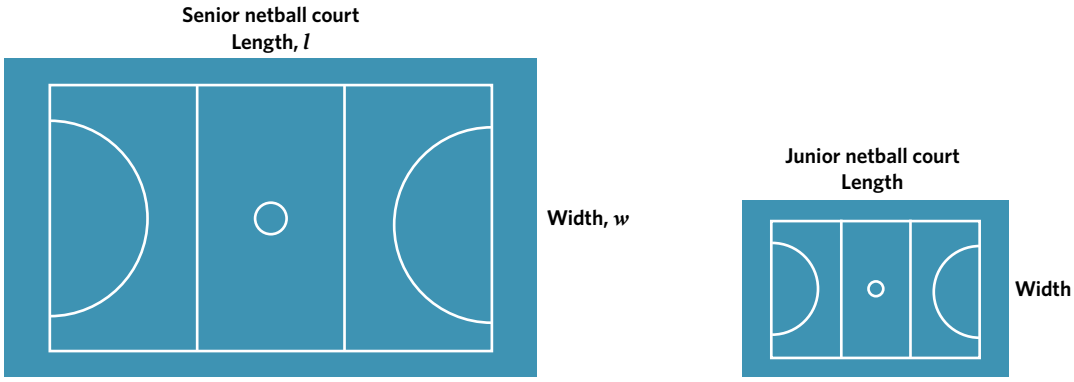
Reasoning

Question working paths

Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. Mary has started playing senior netball. The senior and junior netball courts for her team are shown below. The junior netball court has a length that is a half of the senior netball court and a width that is two-thirds of the senior netball court.

Note: Images are not to scale.



a. Write an expression for the area of the senior netball court.

b. Write an expression for the length and an expression for the width of the junior netball court.

c. Write an expression for the area of the junior netball court.

d. Write an expression to find the difference between the senior netball court and the junior netball court.

e. Mary is coaching the junior netball team and needs to provide them some suggestions on how they can best prepare for the transition to the senior team. Outline two suggestions on how they can go about improving their skills.

15. Calculate the following.

a. $1 \times \frac{x}{y}$

b. $1 \div \frac{x}{y}$

c. Compare and contrast your answer from part **a** and **b**. Explain the difference between multiplying and dividing algebraic fractions.

Extra spicy

16. For the following expression, find an equivalent algebraic fraction.

$\frac{2e}{5} + \frac{e}{2} - \frac{e}{5}$

17. $3x^{-2}$ is equal to:

A. $\frac{3}{x}$ **B.** $\frac{9}{x^2}$ **C.** $\frac{3}{x^2}$ **D.** $\frac{1}{3x^2}$ **E.** $\frac{1}{9x^2}$

18. Simplify.

$\frac{x-1}{3z} \times \frac{x+2}{6z}$

19. $\frac{1}{4}$ is tripled by adding the same number to both the numerator and denominator. That number is:

A. 2 **B.** 3 **C.** 5 **D.** 8 **E.** 9

Remember this?

20. Jason plans to leave Melbourne at 10:30 am and drive through Bright to Falls Creek.

He generates some information regarding travel times.

Melbourne to Bright	3 hours 45 minutes
Bright to Falls Creek	1 hour 30 minutes

Jason plans to have a 45 minute rest break stop at Bright.

With reference to these driving times and rest breaks, when should Jason arrive at Falls Creek?

A. 2:15 pm **B.** 3:30 pm **C.** 4:00 pm **D.** 4:15 pm **E.** 4:30 pm

21. Boyd needs to measure the distance from his friend's house back to his house. Which of these units of measurement would be the most appropriate?

A. Hectares **B.** Metres **C.** Kilometres **D.** Cubic metres **E.** Metres squared

22. This table shows the length and width of five different blocks of land in a new estate.

Block	Length (m)	Width (m)
Block A	20	15
Block B	11	22
Block C	19	13
Block D	17	19
Block E	22	30

Which block has a perimeter of 66 metres?

A. Block A **B.** Block B **C.** Block C **D.** Block D **E.** Block E

Back to contents

5F Expanding brackets

Brackets can be used to group expressions together that are being multiplied or divided by some number. When simplifying expressions that contain multiple terms and expressions inside brackets, it is usually helpful to expand, or remove, the brackets first. This involves using the distributive law.

LEARNING INTENTIONS

Students will be able to:

- expand brackets using the distributive law
- simplify expressions by expanding brackets and collecting like terms.

KEY TERMS AND DEFINITIONS

The **distributive law** for multiplication means that multiplying a number by a group of numbers is the same as multiplying the number by the sum of the other numbers.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

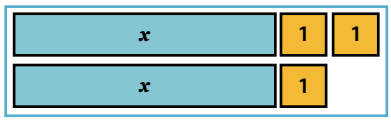
Image: fototip/Shutterstock.com

Brackets help group together expressions that involve variables. The expression 2w + *l* can represent the volumes of water and lemon juice in a bottle of lemonade. The volume in 20 bottles can be represented as 20(2*w* + *l*) to help figure out how many litres will be produced.

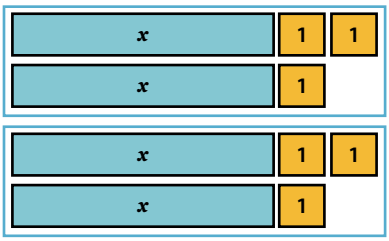
Key ideas

1. Brackets are used to represent multiplying expressions.

$2x + 3$

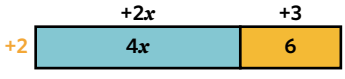


$2(2x + 3)$



2. Expanding brackets involves applying the distributive law to find an equivalent expression without brackets.

$2(2x + 3)$



$2(2x + 3) = 2 \times 2x + 2 \times 3$

$= 4x + 6$

Worked example 1

Expanding brackets

Expand the brackets in each expression.

a. $4(3x - 5)$ WE1a

Working

$4(3x - 5) = 4 \times 3x + 4 \times (-5)$

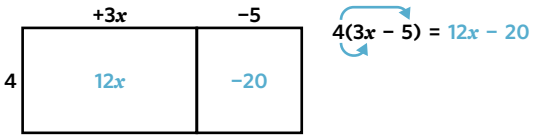
$= 12x - 20$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify.

Visual support



b. $3a(2a + 1)$ WE1b

Working

$3a(2a + 1) = 3a \times 2a + 3a \times 1$

$= 6a^2 + 3a$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify.

Student practice

Expand the brackets in each expression.

a. $3(4m - 6)$ **b.** $8b(1 + 2b)$

Worked example 2

Simplifying expressions by expanding brackets

Expand the brackets in each expression and simplify.

a. $2(t + 3) + 4t$ WE2a

Working

$2(t + 3) + 4t = 2 \times t + 2 \times 3 + 4t$

$= 2t + 6 + 4t$

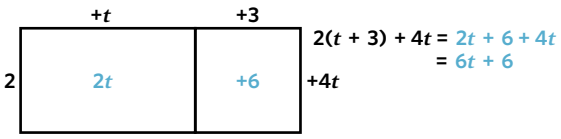
$= 6t + 6$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify by collecting like terms.

Visual support



Continues →

b. $5(q + 2) + 3(q + 6)$ WE2b

Working

$5(q + 2) + 3(q + 6)$

$= 5 \times q + 5 \times 2 + 3 \times q + 3 \times 6$

$= 5q + 10 + 3q + 18$

$= 8q + 28$

Thinking

Step 1: Multiply each term inside the brackets by the coefficient of the bracket.

Step 2: Simplify by collecting like terms.

Student practice

Expand the brackets in each expression and simplify.

a. $4 + 7(v - 2)$ **b.** $3(j - 2) + 4(j + 5)$

5F Questions

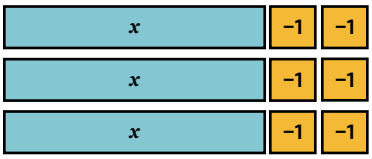
Understanding worksheet

1. Fill in the blanks to expand the brackets.

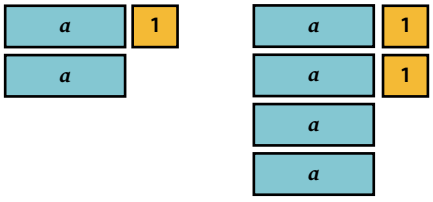
Example

$x - 2$

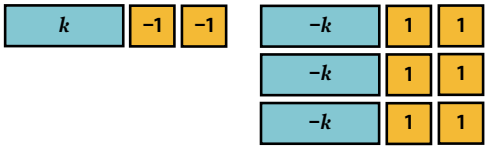
3 $(x - 2) =$ $3x - 6$



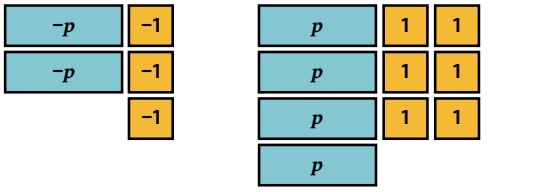
a. $2a + 1$ $2(2a + 1) =$ ____



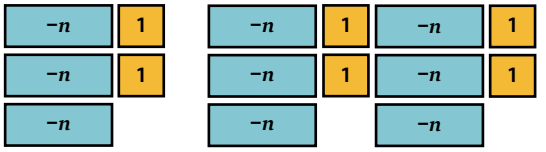
b. $k - 2$ $-3(k - 2) =$ ____



c. $-2p - 3$ $-2(-2p - 3) =$ ____



d. $-3n + 2$ ____ $(-3n + 2) =$ ____



2. Expand the expression using the distributive law.

Example

$3(x + 4) =$ $3x + 12$

a.

$4(h + 5) =$

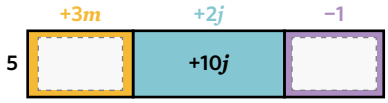
b.

$3(-4 + 3l) =$

c.

$f(b + 7) =$

d.



$5(3m + 2j - 1) =$

3. Fill in the blanks by using the words provided.

distributive law | simplifying | expanding | coefficie

_____ brackets means to find an equivalent expression without brackets.

The _____ states that each term inside the brackets is multiplied by the _____ of the brackets. This is a useful first step when _____ expressions.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Expand the brackets in each expression. WE1a

a. $6(f + 1)$ **b.** $3(c - 4)$ **c.** $8(-2 + x)$ **d.** $-10(p - 8)$

e. $-5(-s - 4)$ **f.** $k(k + 3)$ **g.** $3(z + g - 5)$ **h.** $2(10 + h + 3w)$

5. Expand the brackets in each expression. WE1b

a. $x(x + 5)$ **b.** $4u(2u + 5)$ **c.** $\frac{1}{2}(8k + 4)$ **d.** $-2g(6 - g)$

e. $0.3v(0.6h + 1.2f)$ **f.** $9h(3r - 2s)$ **g.** $\frac{3}{4}p(-2d - 4)$ **h.** $5w(3wc - 6w + 4f)$

6. Expand the brackets in each expression and simplify. WE2

a. $2(v + 7) + 8v$ **b.** $9(w - 2) + 1$

c. $10f + 3(6 - f) + 2$ **d.** $8(b + 3) - 10(b + 1)$

e. $24 + 6y(y - 4) + 6y$ **f.** $4(n + h) + 8h + 5n(n - 2)$

g. $3r(2g + 1) - 8g(r + 4)$ **h.** $3q(2q - 3m) + 4q(7m - 2q) + 2$

7. Write expressions with brackets for each of the following statements.

a. A number a is added to 4 and the result is multiplied by 2.

b. 1 is subtracted from a number p and the result is multiplied by 6.

c. 9 has the number r subtracted from it, which is then multiplied by 3.

d. The result of 8 subtracted from a number m is tripled.

e. The result of 3 added to half a number f is multiplied by 8.

f. Four times a number g is subtracted from 10. The result is multiplied by a quarter.

g. Negative 4 is added to a number y and the result is multiplied by w.

h. A number k is doubled and subtracted from five-halves. The result is halved.

---

8. Expand and simplify the expression $3(u - 6) + 7u$.

A. $4u$ **B.** $8u - 3$ **C.** $10u - 6$ **D.** $10u - 18$ **E.** $10u + 18$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Expand $6(h + 4)$

Student A

$6(h + 4) = 6 \times h + 6 \times 4$

$= 6h + 24$

Student B

$6(h + 4) = 6 \times h + 4$

$= 6h + 4$

b. Expand $-10(y - 2)$

Student A

$-10(y - 2) = -10 \times y - 10 \times 2$

$= -10y - 20$

Student B

$-10(y - 2) = -10 \times y - 10 \times (-2)$

$= -10y + 20$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Stacey spends \$15 flat rate for parking at uni from 9 am to 3 pm. The fee for parking is an additional \$5 an hour for every hour h after 3 pm. Write an expression with brackets to represent how much Stacey pays for parking for 5 days, if she parks for the same time every day.

11. Cindy is building a table that has a width of 0.5 m. She doesn't know how long it will be, but it will have a length of at least 1.5 m. What expanded expression represents the area of this table?

12. Bihai is creating a footpath with rectangular tiles. Each tile has a length of $a + 3$ m and a width of $a - 2$ m. He puts three together, as shown below.

Write an expanded and fully simplified expression for the total perimeter of this shape.

13. Tickets for a theme park cost $\$x$ per adult ticket and $\$y$ per child ticket. A family package consists of two adult tickets and two children's tickets and is \$5 less than buying the tickets individually. A group of adults and kids buy 3 family packages, and two extra adult tickets. Find an expression that represents the total cost of all the tickets.

14. John, Jeremy, Melissa and Fatima are running a relay marathon. At different intervals, a new runner takes over. John is the first runner. Jeremy is the second runner, and thinks he can run 5 km more than John. Melissa is next, and thinks she can run twice as far as Jeremy. Fatima is the last runner, who can run 3 times as far as John. Construct a fully simplified expression to represent the total distance run by all runners, where x is the distance run by John.

Reasoning

Question working paths

Mild 15 (a,b,d)	Medium 15 (a,b,d), 16 (a,b)	Spicy All

15. Felix runs an online store where he sells pottery items he makes, such as plates, vases and plant pots. If a customer orders less than five items, Felix pays for the materials only at \$3 per item, with the shipping paid for by the customer. If a customer orders five or more items, Felix also offers free shipping, which costs him an additional \$8. Use s to represent the selling price of each item in Felix's store when answering the questions below.

a. Construct an expression that represents the profit he earns per item, for an order with less than 5 items.

b. A customer orders 5 items. Write an expanded and simplified expression to represent the profit he earns from this order.

c. What price should each item be if Felix wants to earn \$52 profit from the customer's order in part **b**?

d. List two other things that Felix needs to consider when figuring out how much he should sell his pottery items for.

16. Expand and simplify the expressions:

a. $12(n + 2) - 2(n + 2)$ and $10(n + 2)$.

b. $-3(2 - p) + 6(-p + 2)$ and $3(2 - p)$.

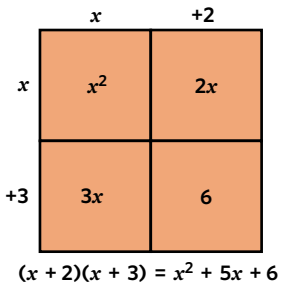
c. What similarities are in the expressions in part **a**, and the expressions in part **b**? If you have multiple of the same sets of brackets, how can you simplify equations without expanding?

Extra spicy

17. Simplify the expression $\frac{y-2}{2} + \frac{y}{3}$

A. $\frac{2y-2}{5}$ **B.** $\frac{2y-2}{6}$ **C.** $\frac{4y-2}{6}$ **D.** $\frac{4y-6}{6}$ **E.** $\frac{5y-6}{6}$

18. The expression $(x + 2)(x + 3)$ can be expanded as shown below.



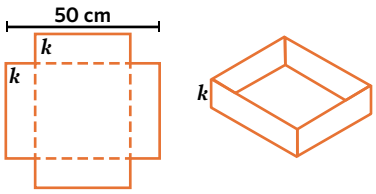
Using the above example, expand the expression $(x - 3)(x + 4)$.

A. $3x$ **B.** $x^2 + 7x + 12$ **C.** $x^2 + x + 1$ **D.** $x^2 + x - 12$ **E.** $-3x^2 + 4x - 12$

19. Prove that $3(5n - 2) - 3(3n + 4)$ is always a multiple of 6, where n is an integer.

20. A large square with side length 50 cm has 4 smaller squares cut from its corners.

The side lengths of the smaller squares are all k cm. The sides fold up to form a box.



Write an expression involving brackets for the volume of the box.

Remember this?

21. Reid recorded the total distance they ran each week over a month.

Week	Distance
1	24.92 km
2	12.23 km
3	33.11 km
4	29.87 km

To estimate how far they ran for the month, Reid rounded the weekly distances to the nearest 10 km and added them.

How far did Reid run?

A. 80 km **B.** 90 km **C.** 98 km **D.** 100 km **E.** 120 km

22. What is an equivalent expression for $x \times 2^5 \times x^3$?

A. $2x^9$ **B.** $10x^4$ **C.** $10x^5$ **D.** $32x^4$ **E.** $32x^5$

23. Ruergan is 18 years old.

Jack is 3 years more than twice Ruergan's age.

How old is Jack?

A. 21 **B.** 23 **C.** 36 **D.** 39 **E.** 56

Back to contents

5G Factorising expressions

The opposite of expanding algebraic expressions is factorising algebraic expressions. Much like the opposite of multiplication is division. While expanding focuses on multiplying algebraic terms, factorising is similar to division by creating groups of terms from expressions.

LEARNING INTENTIONS

Students will be able to:

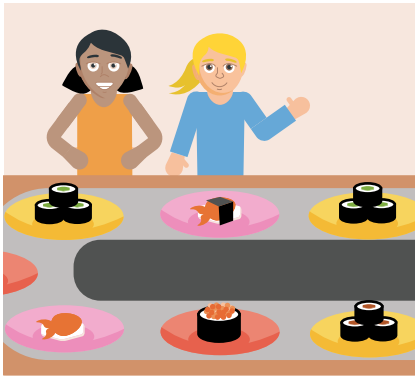
- understand the relationship between expanding and factorising
- determine the highest common factor(s) of algebraic terms
- factorise algebraic expressions.

KEY TERMS AND DEFINITIONS

Factorising in algebra is the opposite of expanding brackets.

The **highest common factor (HCF)** is the largest number that is a factor of two or more numbers.

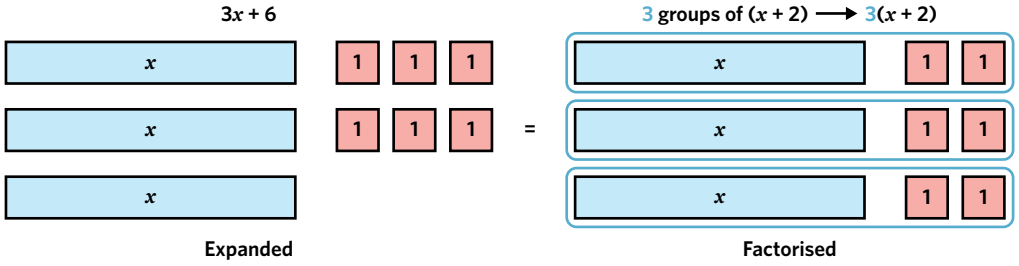
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Factorising is used to create common groups of the unknown. When ordering from a sushi train, the colours of the plates indicate the different prices of the dishes. We can group the coloured plates to quickly determine the cost and split the bills amongst friends.

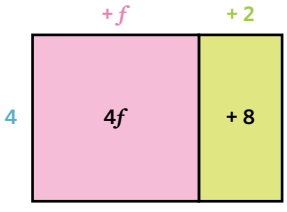
Key ideas

1. The purpose of factorising is to group expressions, which is the opposite of expanding expressions.



2. To factorise expressions, group terms by their highest common factor (HCF).

$4f + 8$



HCF is 4

$4f + 8 = 4 \times f + 4 \times 2$

$= 4(f + 2)$

3. When factorising expressions, the leading term inside the brackets should be positive.

Correct maths conventions:	Incorrect maths conventions:
$-4(2x + 1)$	$4(-2x + 1)$

Worked example 1

Finding the HCF of algebraic terms

Find the highest common factor of the following pairs of terms.

a. $6n$ and 36 WE1a

Working

$6n = (1 \times 6)n$

$6n = (2 \times 3)n$

$36 = 1 \times 36$

$36 = 2 \times 18$

$36 = 3 \times 12$

$36 = 4 \times 9$

$36 = 6 \times 6$

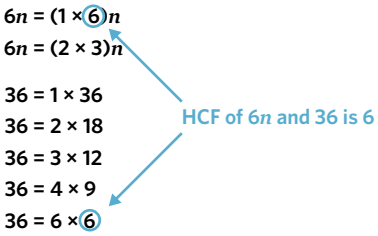
The highest common factor of $6n$ and 36 is 6.

Thinking

Step 1: Calculate the factors of each term.

Step 2: Determine the HCF.

Visual support



b. $20s$ and $24s$ WE1b

Working

$20s = (1 \times 20)s$

$20s = (2 \times 10)s$

$20s = (4 \times 5)s$

$24s = (1 \times 24)s$

$24s = (2 \times 12)s$

$24s = (3 \times 8)s$

$24s = (4 \times 6)s$

The highest common factor of $20s$ and $24s$ is $4s$.

Thinking

Step 1: Calculate the factors of each term.

Step 2: Determine the HCF.

Student practice

Find the highest common factor of the following pairs of terms.

a. $8k$ and 30

b. $27w$ and $63w$

Worked example 2

Factorising algebraic expressions

Factorise the expressions.

a. $7t - 14$ WE2a

Working

$7t - 14 = 7(t - 2)$

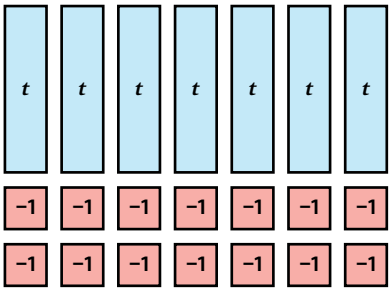
$7(t - 2) = 7t - 14$

Thinking

Step 1: Determine the HCF of all the terms in order to factorise the expression.

Step 2: Check the answer by expanding.

Visual support



7 groups of $(t - 2) = 7(t - 2)$

b. $-121h + 22$ WE2b

Working

$-121h + 22 = 11(-11h + 2)$

$= -11(11h - 2)$

$-11(11h - 2) = -121h + 22$

Thinking

Step 1: Determine the HCF of all the terms in order to factorise the expression.

Step 2: Make the leading term positive by factorising -1 from the terms inside the brackets.

Step 3: Check the answer by expanding.

Student practice

Factorise the expressions.

a. $8e - 16$ **b.** $-81n + 18$

Worked example 3

Factorising and simplifying expressions

Factorise the expressions.

a. $-3ke - ep$ WE3a

Working

$-3ke - ep = e(-3k - p)$

$= -e(3k + p)$

$-e(3k + p) = -3ke - p$

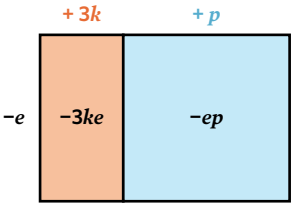
Thinking

Step 1: Determine the HCF of all the terms in order to factorise the expression.

Step 2: Make the leading term positive by factorising -1 from the terms inside the brackets.

Step 3: Check the answer by expanding.

Visual support



$-3ke - ep = -e(3k + p)$

b. $\frac{3a + 9}{3}$ WE3b

Working

$\frac{3a + 9}{3} = \frac{3(a + 3)}{3}$

$= \frac{{}^{1}\cancel{3}(a + 3)}{\cancel{3}_{1}}$

$= a + 3$

Thinking

Step 1: Factorise the numerator.

Step 2: Simplify the terms of the numerator and denominator.

Student practice

Factorise the expressions

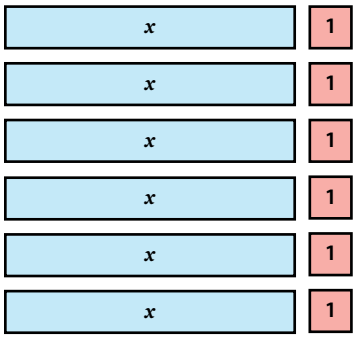
a. $-9ne - ed$ **b.** $\frac{12a + 24}{12}$

5G Questions

Understanding worksheet

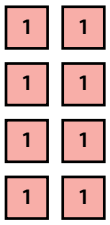
1. Factorise the expressions by filling in the blanks.

Example



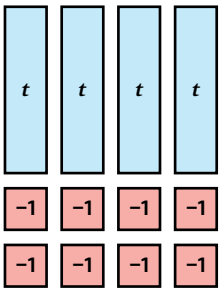
6 groups of $x + 1$ $= 6(x + 1)$

a.



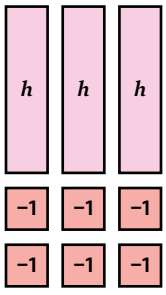
4 groups of $\square$ $= 4 \times 2$

b.



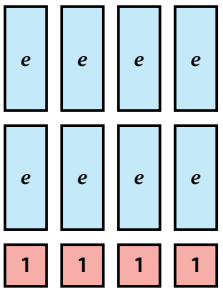
4 groups of $\square$ $= 4(t - 2)$

c.



$\square$ groups of $h - 2 = \square$ $(h - 2)$

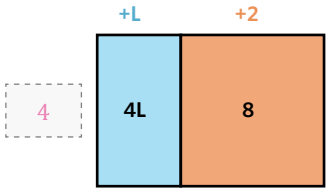
d.



4 groups of $\square$ $= 4($ $\square$ $)$

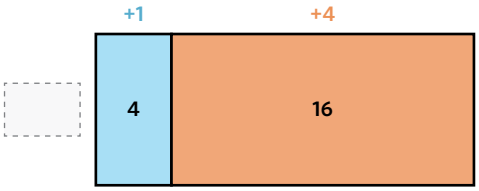
2. Factorise the expressions by filling in the blanks.

Example



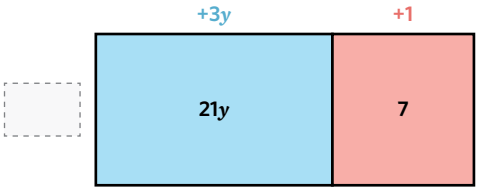
$4L + 8 = \square\ (L + 2)$

a.



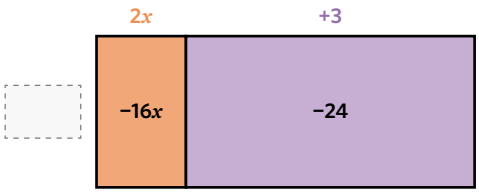
$4 + 16 = \square\ (1 + 4)$

b.



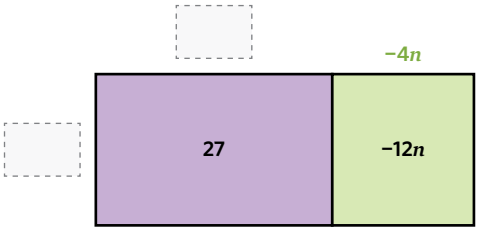
$21y + 7 = \square\ (3y + 1)$

c.



$-16x - 24 = \square\ (2x + 3)$

d.



$27 - 12n = \square\ (\square - 4n)$

3. Fill in the blanks by using the words provided.

expanded	multiplied	8	−4

In the factorised expression $-4(x - 2)$, the number ☐ is the highest common factor.

The number -4 is ☐ by the sum of x and -2. When ☐, this expression is equivalent to the sum of $-4x$ and ☐.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9 (a,b,c,d), 10	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9 (c,d,e,f), 10	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9 (e,f,g,h), 10

4. Find the highest common factor. WE1

a. 12 and 8 **b.** 81 and 63 **c.** $5j$ and 25 **d.** 14 and $49a$

e. $144b$ and $36b$ **f.** $8er$ and $2ers$ **g.** $80a^2$ and $94a$ **h.** $28rg$ and $4h$

5. Factorise. WE2a

a. $2u + 4$ **b.** $49n - 14$ **c.** $60 - 120p$ **d.** $9u - 9$

e. $32 - 8g$ **f.** $72L + 140$ **g.** $63 + 54g$ **h.** $81e - 99d + 45$

6. Factorise. WE2b

a. $-6t - 6$ **b.** $-4 - 4e$ **c.** $-n - 1$ **d.** $-25c + 75$

e. $-28 + 46h$ **f.** $-44 - 46i$ **g.** $-115 + 180q$ **h.** $-36u + 34e - 22$

---

7. Factorise. W3a

a. $5m + 6um$ **b.** $4d + 12ad$ **c.** $63si - 27s$ **d.** $-18to + 3t$

e. $-10ke - 2ep$ **f.** $-e - els$ **g.** $-6as + 8si + 2st$ **h.** $4da + 16ta^2$

---

8. Correct the factorised expression so that it is equivalent to the expanded expression on the left hand side.

a. $10a + 14 = 10(a + 4)$ **b.** $9n - 18 = 9(n - 18)$

c. $-3s + 3 = -3(s + 1)$ **d.** $-14w - 14 = -14(w - 14)$

e. $10er + 5r = 5r(2es + 0)$ **f.** $12e + 4er + 2ie = 2e(10 + 2r + 0i)$

g. $t^2u + 14tu = tu(t + 14u)$ **h.** $6e^2 + 10pe + 2e = 2e(4e^2 + 8p + e)$

---

9. Factorise the algebraic expressions and simplify. WE3b

a. $\frac{4x + 4}{4}$ **b.** $\frac{16x - 32}{8}$ **c.** $\frac{6}{3x + 12}$ **d.** $\frac{3x - 3}{x - 1}$

e. $\frac{2x - 4}{-4x + 8}$ **f.** $\frac{-6r + 12b}{2r - 4b}$ **g.** $\frac{-144r - 12br}{-8br - 72r}$ **h.** $\frac{100r^2 + 20r}{2r^2 - 4r}$

---

10. Which statement is equivalent to the expression: $-16s + 16$

Statement 1: $16(1 - s)$

Statement 2: $-16(s + 1)$

Statement 3: $-16(s - 1)$

A. Statement 1 only

B. Statement 2 only

C. Statement 3 only

D. Statement 1 and 2

E. Statement 1 and 3

Spot the mistake

11. Select whether Student A or Student B is incorrect.

a. Factorise the expression.

$-4 - 44w$

Student A

$-4 - 44w = -4(1 + 11w)$

Student B

$-4w - 44 = -4(0 - 11w)$

b. Factorise the expression.

$-54er + 36e$

Student A

$-54er + 36e = -18e(3r - 2)$

Student B

$-56er + 36e = -18e(3r - 2e)$

Problem solving

Question working paths

Mild	Medium	Spicy
12, 13, 14	13, 14, 15	14, 15, 16

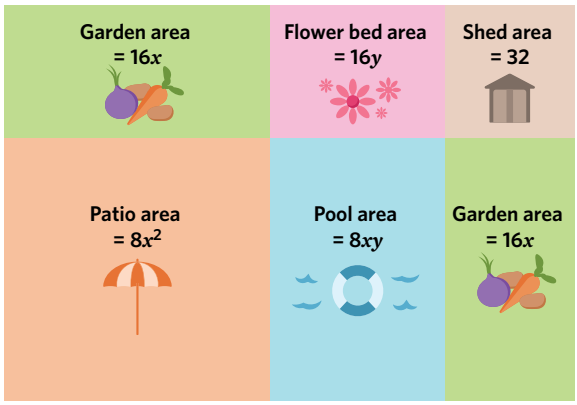
12. A Year 8 class is planning where to sit for a play. Currently there are 8 rows with s students in each row. An additional 16 students want to watch the play and sit with their classmates. If the Year 8 class wants to sit together and divide themselves equally, write an expression that represents the total number of rows and students that are now in each row.

13. The number of family membership plans sold by the Sydney Living Museum can be represented by the expression $x + 4y$, where $\$x$ is the cost per adult ticket and $\$y$ is the cost per child ticket. How many family membership plans did the museum sell if they sold 4 adult tickets and 16 children tickets?

14. Scientists are studying ancient life in Australia. They dig in two different sites and their total findings can be represented by the expression $6p + 4a$, where p is the number of fossilised plants, and a is the number of fossilised animals. Write an expression to represent the number of fossilised plants and animals if two of the scientists split the findings equally between them.

15. Riley forgot their measuring tape and used the length of their shoe (s) in cm and the width of their phone (p) in cm to calculate the length and width of a rectangular room. The area equaled $144s^2 + 12sp$. Factorise the expression so that area represents the multiplication of the length and width of the room in terms of s and p.

16. Emmanuelle is re-designing her rectangular backyard and only knows the area of the shed which is 32 m^2. She mocks up a design that includes 2 rectangular garden beds with the same area, a rectangular flower bed, a rectangular pool, and a square shaped patio.



What is the length and width of her backyard in terms of x and y?

Reasoning

Question working paths

Mild	Medium	Spicy
17 (a,b,c,e)	17 (a,b,c,e), 18 (a,b)	All

17. Jeremy and Lena go to a sushi train restaurant. Various types of nigiri, sashimi, maki, and hot dishes are placed on coloured plates that indicate the prices of each dish.

	Colour	Cost per plate in dollars
	Red	$\$r$
	Pink	$\$p$
	Gold	$\$g$

a. Jeremy eats from six red plates and four gold plates. Lena eats from two red plates and six gold plates. Write an expression that represents the total cost of their combined bills.

b. They want to split the bill equally and will need to create two equal groups of red and gold plates. Write an expression that shows how much they will each pay in terms of red and gold plates.

c. On another occasion, Jeremy and Lena go out for dinner with two other friends to eat sushi. They group the plates by colour and count that they have eaten a combined amount of 16 red plates, 24 pink plates and eight gold plates. Write an expression that shows how much each friend will pay if they split the total cost into four equal groups.

d. The waiter groups the total amount of the four friends' plates into two equal groups. Write an expression that shows how much the bill will be if the total cost is split into two equal groups.

e. Is it fair to split bills equally? Why or why not?

18. Fill in the missing boxes to make the expressions equivalent.

a. Fill in the missing boxes to make the expression equivalent.

$-33 + 11x = \square(3 - x)$

b. Fill in the missing boxes to make the expression equivalent.

$-33 + 11x = 11(x \ \square)$

c. Why are the factorised expressions from part **a** and **b** equivalent?

Extra spicy

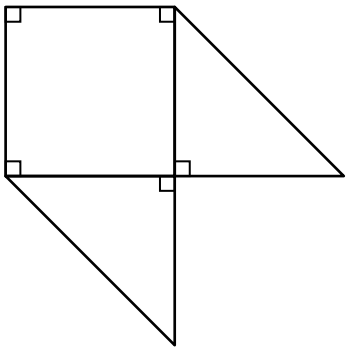
19. Using the expression product puzzle as an example, fill in the missing boxes to make the second puzzle true.

Expression Product Puzzle Example

$2n + 1$	5	$10n + 5$
2	$2n - 1$	$4n - 2$
$4n + 2$	$10n - 5$	

	2	$10e - 4$
		$12e + 9$
$15e - 6$	$8e + 6$	

20. The area of the shape is equal to $x^2(1 + 1)$. What assumptions do you have to make about the shape in order for the area to be true?



A. The shape has a square with dimensions x by x, and two triangles are the same and have a base of 1 and height of x.

B. The shape has a square with dimensions x by x, and the two triangles are the same and have a base of 1 and height of 1.

C. The shape has a square with dimensions of x by x, and the two triangles are the same and have a base of x and height of x.

D. The shape has a rectangle with dimensions of x by 1, and the two triangles are the same and have a base of x and height of 1.

E. The shape has a rectangle with dimensions of x by 1, and the two triangles are the same and have a base of x and height of x.

21. The expression $-x^3 - 3x^2 + 6x - 18$ is not fully factorised below.

$-x^3 - 3x^2 + 6x - 18$

$-x^2(x + 3) + 6(x + 3)$

Which expression shows the full factorisation of the expression?

A. $-x^2(x + 3) + 6$

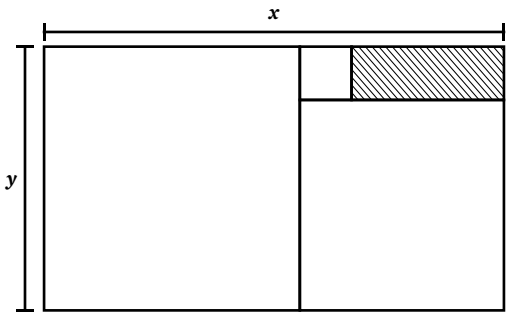
B. $(x + 3) - x^2 - 6$

C. $(x + 3) - (x^2 + 6)$

D. $(x + 3)(x^2 + 6)$

E. $(x + 3)(6 - x^2)$

22. A large rectangle has dimensions x by y. What is the area of the shaded rectangle in terms of x and y, if all the other shapes inside the larger rectangle are squares?



Remember this?

23. Which of the expressions is equal to p^5?

A. $\sqrt[5]{p}$

B. $p^3 + p^2$

C. $p \times 5$

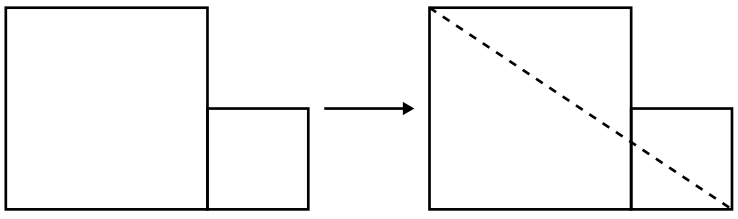
D. $p + p + p + p + p$

E. $p \times p \times p \times p \times p$

24. The fraction $\frac{1}{8}$ is equal to 0.125 as a decimal. What does $\frac{3}{16}$ equal as a decimal?

A. 0.042 **B.** 0.0625 **C.** 0.1875 **D.** 0.25 **E.** 0.375

25. Sourab makes a large cake by placing two square cakes of different sizes next to each other. The cakes have areas of 4 cm^2 and 16 cm^2. Sourab uses icing to draw a line from the bottom right to the top left, and fills in the icing above the dotted line.



How much icing covers the cake?

A. 8 cm^2 **B.** 10 cm^2 **C.** 18 cm^2 **D.** 20 cm^2 **E.** 64 cm^2

Chapter 5 extended application

1. Dion and Rebecca are going to Hobart for 6 days and each want to rent e-scooters to scoot around the city. They plan out how much they will need to use the scooter over the next 6 days.

	From hotel to city	Explore the city	From city to hotel	Travel to and from city in the morning	Travel to and from city in the afternoon	Travel to and from city in the evening
Day 1–4	10 minutes	No scooter needed	15 minutes	✓	✓	✓
Day 5	10 minutes	No scooter needed	15 minutes	×	×	✓
Day 6	Rest					

Proton Scooter has a few options for how they price their rides. There is a normal fee of $1 dollar to activate the scooter and 45 cents per ride minute. They also offer a 7-day pass that is paid in full and costs $4.71 per day with a 90 minute per day ride limit and no activation fee. The third option is a 3-day pass that must be paid in full and is represented by the equation $C = 8.33d$, where C represents total cost per (d) day.

a. Write an equation that represents the total (C) cost in dollars of how much they will each pay when they purchase a 7-day pass from Proton Scooter every (d) day.

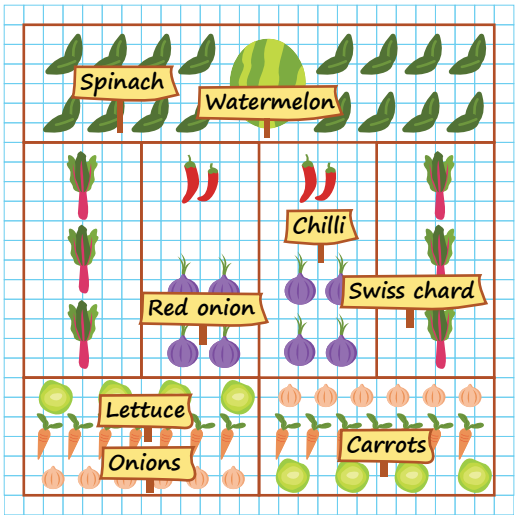
b. The activation fee will be applied each time they use the scooter to travel to and from the city. If they don't purchase a pass, calculate the total cost of how much they will each pay on the first day.

c. Using part **b** if they don't purchase a pass, calculate the total cost of how much they each will pay over the 6 days.

d. Dion proposes that they purchase a 3-day pass and then pay the normal rate for their last 3 days. Rebecca says they should pay for a 7-day pass. Calculate the cost difference and determine which plan saves them more money.

e. When travelling you often don't have a car and are limited to the available public transport. Determine at least two cost effective modes of transportation that Rebecca and Dion should consider when travelling.

2. Bernadine is designing her square garden to maximise the amount of produce she can grow. She draws her design on grid paper where each grid square represents an area of s^2 units^2.

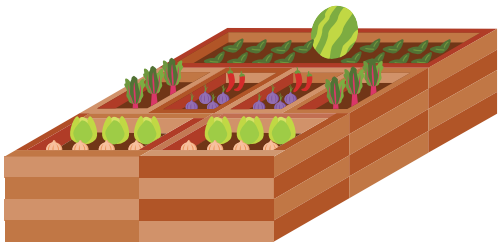


a. What is the perimeter and area of her garden in s units?

b. Currently the length and width of the garden box for the lettuce, onions and carrots is $12s$ by $6s$. She wants to divide the lettuce, onions, and carrots into three equal rows. What will be the width of each of these rows in s units?

c. Bernadine researched that plants thrive better in raised garden beds and created a sketch of her garden beds side by side. She wants each of her 7 rectangular garden beds to be enclosed by a wooden frame. What will be the total perimeter of the 7 rectangular beds in s units?

d. Using part **a** Bernadine redesigns her garden bed so that each crop has space to grow in an area of $4s$ by $4s$. Determine the number of crops she can grow with these requirements.

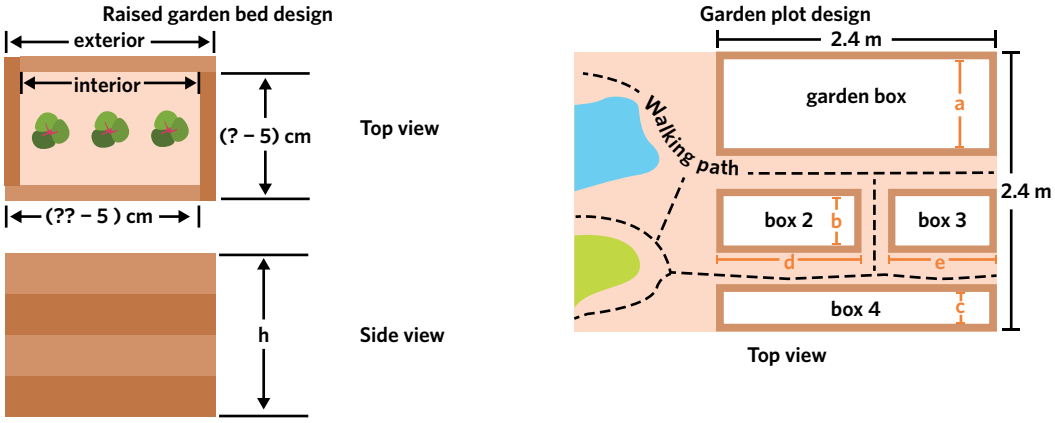


e. Bernadine wants the swiss chard spaced equally. Each swiss chard plant needs approximately 20 cm of room on all four sides to grow and when fully grown will take up 40 cm^2 of area. If $s = 10$ cm, determine how many swiss chards plants are in each of the two rectangular plots.

f. A good garden bed design should consider being able to reach past the middle of the garden bed. If $s = 10$ cm, and the garden beds are placed next to each other with no gaps, will Bernadine be able to reach her arm across all the garden beds to tend to her produce? If not, propose a solution so she is able to tend to all of her crops.

3. Bernadine re-draws her garden to include walking paths and four garden beds and sketches the design of her new garden plot and the raised garden beds. She isn't sure what the new length and width of each box is and denotes this with ? and ?? in her design. She does know the following information:

- The thickness of the boxes are all the same.
- The garden plot is is 2.4 m by 2.4 m.
- Garden box 1 and 4 will have the same exterior length as the garden plot.
- Garden box 2 and 3 will have the same interior widths but have different interior lengths.



a. Describe what ?, ??, and h represent in the raised garden bed design.

b. Describe what '5 cm' represents in the raised garden bed design.

c. The interior dimensions of the garden boxes are measured in cm and represented by variables a, b, c, d, and e. Assume the width of all walking paths are exactly 50 cm, determine the length of the entire garden plot using the variables d and e, and the width of the entire garden bed using the variables a, b, and c.

d. If the walking path is at least 50 cm wide between the garden boxes, determine if the following dimensions will meet the criteria of the garden design:

- $a = 45$ cm and b is one half of a.
- c is a two thirds of b.
- Garden box 2 is half the exterior length of garden box 1.
- The exterior length of garden box 3 is a fourth of the exterior length of garden box 4.

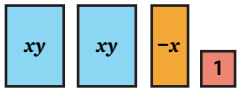
e. Bernadine uses different dimensions to maximise her garden bed dimensions to replace the diagram given in the question stem. Determine the exterior dimensions of garden boxes 1, 2 and 3, and 4, if garden box 1 and 4 have the same width, and garden box 3 has an exterior length of 52.10 cm and area of 1953.75 cm^2 and the walking path is 50 cm between all garden boxes.

f. Using the whole length of railway sleepers without cutting them avoids waste. Propose two other options why someone would want to use the entire length of the railway sleepers.

Chapter 5 review

Multiple choice

1. Which of the following expressions shows the fully simplified sum of the visual? 5A, 5C



A. $2xy - x + 1$
B. $xy + xy - x + 1$
C. $2xy - xy + 1$
D. $2xy + x + 1$
E. $x - 2xy + 1$

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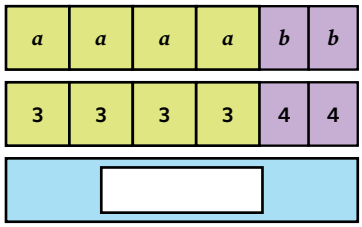
2. Substitute $a = 3$ and $b = 4$ to solve the bar model. 5B



A. 16 **B.** 20 **C.** 24 **D.** 28 **E.** 32

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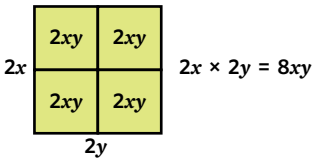
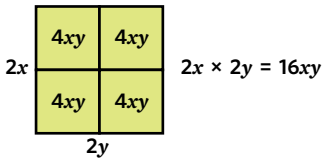
3. Which area model correctly evaluates $2x \times 2y$? 5D

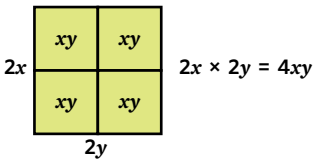
A.



B.



C.



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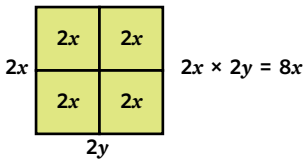

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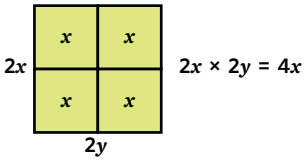


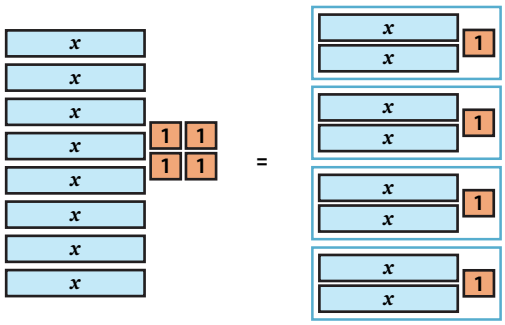
---

4. Determine the missing value to make the equation true. 5E

$\frac{m}{6} \div \frac{8}{n} = \frac{\square}{48}$

A. 1 **B.** $8m$ **C.** $6n$ **D.** $48mn$ **E.** mn

5. Using the following visual, determine which option correctly shows an expanded expression and its fully factorised equivalent. 5F, 5G



A. $8x + 4 = 4x - 8$ **B.** $8x + 4 = 4(2x + 1)$ **C.** $8x + 4 = 2(4x + 2)$
D. $8x + 4 = 4(8x + 1)$ **E.** $8x + 4 = 4(2x + 4)$

Fluency

6. Write an equation or expression to represent each statement. 5A

a. 12 more than x.
b. Four groups of u are added to v.
c. Half of five groups of a is equal to the product of b, c and d.
d. m is squared. This equals one third of the square root of n.

---

7. Substitute $a = -1$, $b = 2$ and $c = -3$ to evaluate each expression. 5B

a. $3b + 10$
b. $2(b - a)$
c. $4c + \frac{2a}{b}$
d. $-0.1(c^2 - 2b)$

---

8. Simplify each expression by collecting like terms. 5C

a. $3 + 4y + 5y + 6$
b. $m^3 + 12 - 8m + 3m^2$
c. $7cd - c^2d - 3cd + 5c^2d$
d. $\frac{1}{2}a - \frac{1}{4}b - \frac{3}{4}a + \frac{3}{2}b$

---

9. Simplify. 5D

a. $3a \times 5b$ **b.** $10x \div 2xy$ **c.** $mn \times 9m$ **d.** $27p^2q \div 10pq^2$

---

10. Simplify. 5D

a. $3a \times 4$ **b.** $6p \div 12$ **c.** $8ab \div -2b$ **d.** $\frac{2}{3}f \times 6g$

---

11. Simplify. 5E

a. $\frac{x}{7} + \frac{2x}{7}$ **b.** $\frac{m}{8} + \frac{5m}{24}$ **c.** $\frac{8a}{3} - \frac{3a}{8}$ **d.** $\frac{q}{3} - \frac{q}{5}$

---

12. Simplify. 5E

a. $\frac{a}{4} \times \frac{b}{3}$ **b.** $\frac{6}{c} \div \frac{d}{c}$ **c.** $y \times \frac{5}{x}$ **d.** $\frac{9}{16u} \div \frac{2u}{3}$

13. Expand the brackets in each expression. 5F

a. $5(x + 2)$ **b.** $-10(9 - y)$ **c.** $d(d - 3)$ **d.** $3(5 + f + 2g)$

14. Expand the brackets in each expression. 5F

a. $7w(7w + 2)$ **b.** $-4x(5 - x)$ **c.** $3x(4y - 5z)$ **d.** $2f(5fg - 8f + g)$

15. Factorise. 5G

a. $10 + 20x$ **b.** $-f - 4$

c. $14x + 49y - 21z$ **d.** $2m - 2mno - 2mo$

16. Factorise each algebraic expression and simplify. 5G

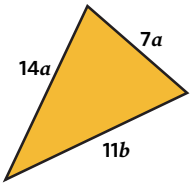
a. $\frac{11a + 22}{11}$ **b.** $\frac{-2}{2x + 40}$ **c.** $\frac{-r + 2b}{5r - 10b}$ **d.** $\frac{r^2 - 4r}{2r^2 - 8r}$

Problem solving

17. Elsa bought x cupcakes for \$20 from the supermarket to decorate and sell at her school fete. She aims to make a profit of \$$y$ per cupcake. Find an expression for the selling price of each cupcake. 5A

18. At an animal shelter, there are birds, cats and dogs. Bill collects food for all the animals each week. The kilograms of food every week is $0.01b + 1.4c + 2d$, where b is the number of birds, c is the number of cats and d is the number of dogs. If there are 10 birds, 20 cats and 30 dogs, what is the weight of food that Bill is collecting? 5B

19. What expression can be written for the perimeter of the shape below? 5C



20. A recipe CJ wants to make requires x grams of sugar and twice as much flour. The sugar costs \$$y$ per gram. The flour is half this price per gram. How much will CJ pay for sugar and flour in total, in terms of x and y? 5D

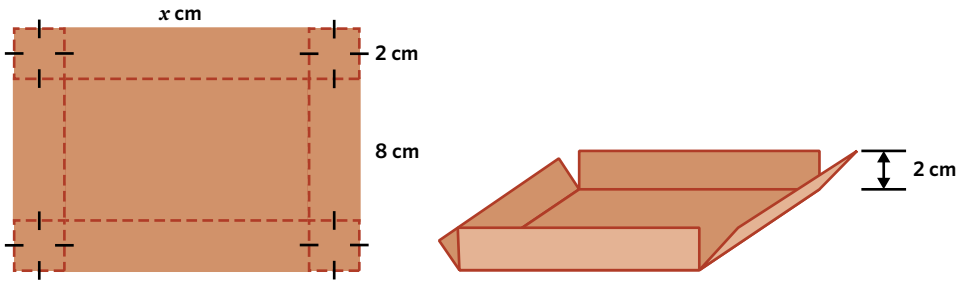
21. Amaja queues up with an empty plate at an all-you-can-eat buffet restaurant. Let p represent the area of the plate. She fills one third of her plate with a steak and one quarter with a salad. She then fills half of the remaining space with garlic bread. Write an expression to show how much of Amaja's plate is left to fill up with dessert. 5E

22. Dr W's Year 8 mathematics class sits an algebra test. The average score per student can be represented by the expression $5x + 20$. Find the sum of all marks in Dr W's class of 25 students, in terms of x. 5F

23. The area of Vicki's rectangular canvas is $13x + 26xy + 39xz$ m^2. The canvas does not have a length or width of 1 m. Vicki's assistant attempts to express this area as the product of its dimensions and writes down three potential solutions. Use factorisation to determine these three potential solutions. 5G

Reasoning

24. Hankan is making an open tray from a piece of thick recycled cardboard x cm long and 8 cm wide. Equal squares of side length 2 cm are cut from each corner. The tray is then constructed by folding up each of the four sides.



a. Write an expression for the length of the base of the tray, and use it to calculate the length of the base of the tray if $x = 12$ cm.

b. In terms of x, write a factorised and expanded expression for the area of the base of the tray.

Hankan decides that the tray is too small to be useful. He creates two new designs. In design 1, he multiplies the area of the base by z. In design 2, he multiplies the width of the base and the length of the base by z.

c. Express the areas of the base for each design in terms of x and z, giving your answer as expanded expressions.

d. Hankan decides to sell these recycled trays, and makes $\$c$ profit. He spends one half of this amount on movie tickets, and one third of the remaining amount on paint for future trays. If he already has $\$10c$ in savings, how much money does he now have in his savings account?

e. Hankan made the open tray from recycled materials. Give another example of something you could make at home from recycled materials.

25. Roslyn is trying to write the following algebraic expression as a single fraction.

$$\frac{1}{a} - \frac{1}{a+h}$$

a. Substitute $a = 2$ and $h = 3$ into $\frac{1}{a} - \frac{1}{a+h}$ and simplify. Write your answer as a single fraction.

b. Substitute $a = 5$ and $h = 4$ into $\frac{1}{a} - \frac{1}{a+h}$ and simplify. Write your answer as a single fraction.

c. Compare your answers from parts **a** and **b**. How can we write expressions of the form $\frac{1}{a} - \frac{1}{a+h}$ as a single fraction?

Back to contents

06

Equations

Number and Algebra

Research summary

6A Introduction to equations *(Revision)*

6B Solving one-step and multi-step equations

6C Solving equations with unknowns on both sides

6D Formulas and applications of equations

6E Inequalities *(Extension)*

Chapter 6 extended application

Chapter 6 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 6 research summary

Equations

Big ideas

Proportional reasoning

This chapter will focus on students' ability to identify and apply their extensive conceptual and structural connection of algebra and number and their properties, order of operations, and equivalence, to build structural and conceptual knowledge of equations and inequalities.

Using the phrase 'cancelling' or 'goes away' may cause misconceptions in algebra and when solving equations. Instead, students and teachers are encouraged to use phrases such as 'sums to zero' and 'simplifies to 1'. This will help strengthen students' connection of number properties in algebra. For example, $3x - 3x$ sums to zero and $\frac{4y}{4y}$ simplifies to 1.

This chapter will ask students to solve equations with variables on both sides of the equal signs. This topic can be challenging for students because they cannot rely solely on arithmetic. Students need a solid conceptual understanding of equality, inequality, brackets, negative signs, subtraction signs, and inverse operations to solve algebraic equations (Booth; 2017). For example, $3x - 7 = 5x + 9$.

Lastly, another helpful activity students can do is check their work by substituting their answers into the original equation. This process helps students understand the idea of equivalence. By having students use order of operations to check their work, they realise the importance of how inverse operations are needed to solve equations.

Supporting the idea of equivalence in arithmetic is critical before students apply it to algebra so they do not see this idea as an outlier. This chapter will use balance scales, bar models, reverse which side the variable is placed, inverse operations, and use number sentences with operations on either side to strengthen arithmetic and algebraic thinking.

Generalising

This chapter will focus on combining multiplicative thinking and proportional reasoning skills to build conceptual and procedural knowledge of equations and inequalities. Additionally, this chapter will use the general properties of algebraic notation, letter usage, and their symbolic relationship to formulate and solve equations and inequalities.

As a reminder, it is essential to realise that it takes time for students to make algebraic generalisations (Bush, 2011). They must manoeuvre familiar and foreign mathematical symbols to work with various numbers and properties to conclude evidence or assumptions.

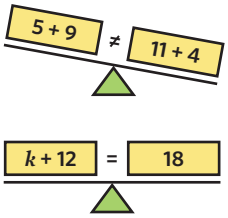
Students are often better at verbally explaining what they need to do than writing symbolically when solving equations (Swafford & Langrall, 2000). Having students connect their verbal and written skills through balance models, number lines, tables, and graphs can help them bridge their symbolic, conceptual and procedural knowledge. Additionally the use of these skills will help students understand the connection between inverse operations and solving equations.

This big idea is built upon a strong understanding of proportional understanding, number and number properties, patterns, and equality. Therefore, a conceptual understanding of how to manipulate equations and inequalities is essential for success in future mathematics topics.

Visual representations

Balance scale

Like the traditional pan balances, balance scales help develop relational thinking by drawing attention to the idea that for the scales to be balanced, both sides have to be the same or equal. For this to be true, whatever operation is applied to one side must also be applied to the other side. In this chapter, balance scales will aid in relationship thinking, algebraic thinking, substitution, and provide students with a conceptual and procedural understanding of how to solve equations.

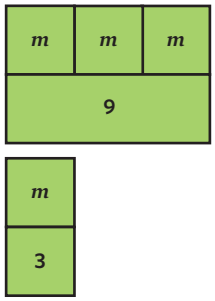


Bar model

Bar models, also known as tape or strip diagrams, help students construct a pictorial equation from a word problem as a whole rather than as distinct parts. The bar model is usually represented by a rectangle representing the relationship between the known and unknown quantities and can be divided to represent proportional relationships. In this chapter, bar models will aid in relational thinking, algebraic thinking, substitution, and provide students a conceptual and procedural understanding of solving equations.

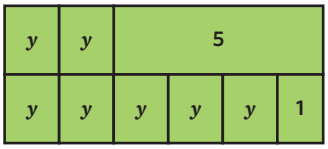
Solving one and multi-step equations

$3m = 9$



Solving equations with unknowns on both sides

$2y + 5 = 5y + 1$

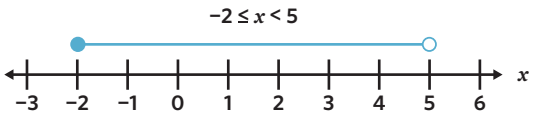


Number line

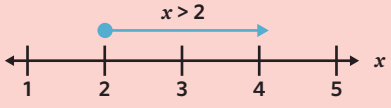
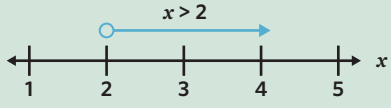
A number line is traditionally shown as a horizontal line or axis but can also be a vertical line. A number line shows the order and size of numbers. It helps deepen our understanding of mathematical concepts such as fractions, decimals, percentages, negative numbers and mathematical operations.

In this chapter, a number line will be used to aid in:

- representing inequalities
- formulating inequalities
- solving inequalities.



Misconceptions

Misconception	Incorrect	Correct	Lesson
When the same variable is on either side of the equal sign, students don't realise that it is part of the sum.	$3e + 2 = e + 3$ where $e = 3$ $3(3) + 2 = e + 3$ $11 = e + 3$ $8 = e$	$3e + 2 = e + 3$ where $e = 3$ $3(3) + 2 = 3 + 3$ $11 \neq 6$	6A
Students see the equal sign as 'the answer is' rather than a statement of equal value.	$10 + 5 = q + 7$ $q = 15$	$10 + 5 = q + 7$ $10 + 5 = 8 + 7$ $q = 8$	6A
Students multiply some of the terms on either side of the equation.	$\times 2 \left(\frac{u}{2} = 5 - 2 \right) \times 2$ $u = 10 - 2$	$\times 2 \left(\frac{u}{2} = 5 - 2 \right) \times 2$ $u = 10 - 4$	
Students don't understand the order of operations when solving for an unknown variable.	$\frac{a + 2}{5} = 5$ $\frac{a + 2}{5} - 2 = 5 - 2$	$\frac{a + 2}{5} = 5$ $\frac{a + 2}{5} \times 5 = 5 \times 5$	6B
Students only apply the inverse operation to one side of the equation.	$9t + 2 = t + 4$ $9t + t = 4 - 2$	$9t + 2 = t + 4$ $9t - t = 4 - 2$	6B, 6C
Students collect like terms by adding all the algebraic and numerical terms together on either side of the equals sign.	$3i + 4 = i + 8$ $3i + i = 8 + 4$	$3i + 4 = i + 8$ $3i - i = 8 - 4$	6B 6C
Students don't apply inverse operations when isolating variables on one side of the equals sign.	$2o + 5 = -3o - 11$ $2o - 3o = -11 + 5$	$2o + 5 = -3o - 11$ $2o + 3o = -11 - 5$	6B 6C
Students try to solve a formula for a value when it contains more than 1 variable.	$3n + s = 6$ $s = 6 - 3$ $s = 1$	$3n + s = 6$	6D
Students solve equations as separate steps instead of line-by-line.	$2a + 1 = 3$ $3 - 1 = 2$ $2a = 2$ $2 \div 2 = 1$ $a = 1$	$2a + 1 = 3$ $2a + 1 - 1 = 3 - 1$ $2a = 2$ $2a \div 2 = 2 \div 2$ $a = 1$	6D
Students reverse the order when translating worded problems to algebraic equations or expressions.	There are six times as many capybaras (c) as numbats (n). $6c = n$	There are six times as many capybaras (c) as numbats (n). $6n = c$	6D
Students match the order of words when translating worded problems to algebraic equations or expressions.	Three less than n. $3 - n$	Three less than n. $n - 3$	6D
Students place the larger number with the larger group when translating worded problems to algebraic equations or expressions.	There are seven times as many crocodiles (c) as echidnas (e). $7c = e$	There are seven times as many crocodiles (c) as echidnas (e). $7e = c$	6D
Students misinterpret the empty and full circle notation on a number line.			6E

Misconception	Incorrect	Correct	Lesson
Students misinterpret the meaning of the inequality symbols.	x is greater than or equal to -8 $-8 < x$	x is greater than or equal to -8 $-8 \leq x$	6E
When solving inequalities, students only apply the inverse operation to one side of the statement.	$2r + 5 > -3r - 11$ $2r - 3r > -11 - 5$	$2r + 5 > -3r - 11$ $2r + 3r > -11 - 5$	6E

References and additional readings

Bush, S. B. (2011). *Bush, S. B. (2011). Analyzing common algebra-related misconceptions and errors of middle school students.* [Doctoral Thesis, University of Louisville]. https://doi.org/10.18297/etd/187

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Back to contents

6A Introduction to equations

An equation is two expressions that are set apart using an equals sign (=). The equals sign shows that the left-hand side (LHS) of the equation is equal to the right-hand side (RHS). If an equation has a variable, it requires you to substitute a value for the pronumeral, that will make the equation true and balanced.

LEARNING INTENTIONS

Students will be able to:

- generate equivalent equations
- use substitution and test if an equation is true.

KEY TERMS AND DEFINITIONS

The **equals sign** (=) in an equation or number sentence indicates that the value on the left of the sign is the same value as on the right.

An **equation** is another name for a number sentence. An equation can contain letters, numbers and operations, as well as an equals sign. Each side of an equation is equal.

A **false equation** is when the left-hand side and right-hand side of an equation are not equal.

A **number sentence** is an equation expressed using numbers and operations.

Substitution is the process of replacing a variable with a given value.

A **true equation** is when the left-hand side and right-hand side of an equation are equal.

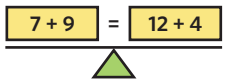
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Equations can be used in a variety of different transportation companies. For example, John's taxi ride from home to work was calculated using $\$c = 10 + 1.5d$, where $\$c$ is cost in dollars and d is distance in km. John was charged \$85, which then requires \$85 to be substituted for $\$c$.

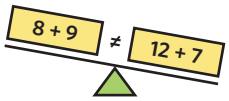
Key ideas

1. Balance scales can be used to show if equations are true or false.

Balanced or true equation



Unbalanced or false equation



2. Different keywords are used to generate equations.

Operation	Key words
Addition (+)	sum, added to, plus, increase, more, combined, together, more than
Subtraction (−)	subtracted from, less than, difference, decreased, reduced, minus
Multiplication (×)	per, times, of, factor, multiplied by, product, each
Division (÷)	out of, divided, quotient of
Equal (=)	result, equals, equal to, total

Worked example 1

Classifying equations

State whether the equations are true or false.

a. $5 + 9 = 11 + 4$ WE1a

Working

$\text{LHS} = 5 + 9$

$= 14$

$\text{RHS} = 11 + 4$

$= 15$

$5 + 9 = 11 + 4$ ✗

False

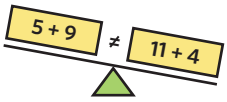
Thinking

Step 1: Calculate the left-hand side (LHS).

Step 2: Calculate the right-hand side (RHS).

Step 3: Check to see if the LHS equals the RHS.

Visual support



b. $22 - 4 = 9 \times 2$ WE1b

Working

$\text{LHS} = 22 - 4$

$= 18$

$\text{RHS} = 9 \times 2$

$= 18$

$22 - 4 = 9 \times 2$ ✓

True

Thinking

Step 1: Calculate the LHS.

Step 2: Calculate the RHS.

Step 3: Check to see if the LHS equals the RHS.

Student practice

State whether the equation is true or false.

a. $6 + 15 = 7 + 14$ **b.** $9 - 4 = 1 \times 5$

Worked example 2

Find the solution to an equation

Determine which value of m is required to make each equation balanced. $m = 4, m = 7, m = 12$

a. $2m + 12 = 26$ WE2a

Working

$2 \times 4 + 12 \neq 26$ ✗

$2 \times 7 + 12 = 26$ ✓

$2 \times 12 + 12 \neq 26$ ✗

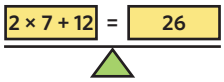
$m = 7$

Thinking

Step 1: Substitute each value of m into the equation.

Step 2: Determine which value of m makes the equation true.

Visual support



b. $m + 15 = 4m + 3$ WE2b

Working

$4 + 15 = 4 \times 4 + 3$ ✓

$7 + 15 \neq 4 \times 7 + 1$ ✗

$12 + 15 \neq 4 \times 12 + 1$ ✗

$m = 4$

Thinking

Step 1: Substitute each value of m into the equation.

Step 2: Determine which value of m makes the equation true.

Student practice

Determine which value of p is required to make each equation balanced. $p = 4, p = 5, p = 12$

a. $3p + 10 = 25$ **b.** $p + 16 = 4p + 1$

Worked example 3

Writing balanced equations

Write an equation for each of the given descriptions.

a. Twelve more than k and the result is 18. WE3a

Working

$k + 12$

$= 18$

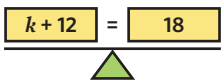
$k + 12 = 18$

Thinking

Step 1: Twelve more than k.

Step 2: Result is 18.

Visual support



Continues →

b. Eight plus double h and the result is 16. WE3b

Working

$8 +$

$2h$

$= 16$

$8 + 2h = 16$

Thinking

Step 1: Eight plus

Step 2: Double h

Step 3: Result is 16.

Step 4: Construct a formula to represent the equation.

Student practice

Write an equation for the given scenarios.

a. Nine more than p and the result is 15.

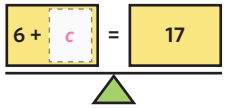
b. Four plus double d and the result is 18.

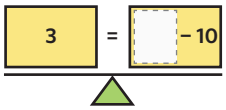
6A Questions

Understanding worksheet

1. Substitute one of the following variables into the equation to make it true. $a = 6$, $b = 13$, $c = 11$

Example



a.


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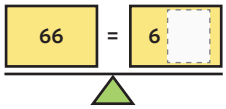

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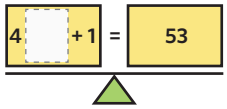
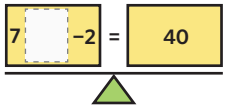


d.


2. Classify each of the following equations as true or false.

Example

	True	False
$4 + 8 = 16 - 4$	☑	☐

	True	False
a. $3 + 9 = 5 + 8$	☐	☐
b. $14 - 10 = 20 - 16$	☐	☐
c. $3 + 14 = 6 + (10 + 1)$	☐	☐
d. $14 - 10 = 8 + (14 - 11)$	☐	☐

3. Fill in the blanks by using the words provided.

false | substituted | balanced | equations

Mathematical statements can be represented as an equation. _____ can be true or false.

$4 + 6 = 20 - 9$ is an equation that is _____, whereas $20 - 12 = 19 - 11$ is an equation that is true.

The left-hand side (LHS) of an equation is _____ when it is equal to the right-hand side (RHS). If a is _____ for 5, then $8 + 7a = 42$ is a true and balanced equation.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. State whether the equations are true or false. WE1

a. $10 - 4 = -5$ **b.** $24 + 36 = 60$ **c.** $8 + 5 = 12 + 1$ **d.** $5 - 24 = 3 \times 8$

e. $3 + 3 - 4 = 5 \times 0$ **f.** $5 + 20 = 25 \div 5$ **g.** $2.5 - 0.1 = 6 \times 0.4$ **h.** $5 - 2.4 = 0.3 \times 8$

5. Determine which value of a is required to make each equation balanced. WE2a

$a = 5$, $a = 8$, $a = 12$

a. $3a = 15$ **b.** $a + 3 = 15$ **c.** $8 - a = 0$ **d.** $6a - 4 = 44$

e. $3a + 12 = 48$ **f.** $3 - 9a = -42$ **g.** $\frac{5a}{4} - 3 = 12$ **h.** $24 - \frac{6a}{6} = 19$

6. Determine which value of x is required to make each equation balanced. WE2b

$x = 3$, $x = 5$, $x = 8$

a. $2x + 6 = 4x$ **b.** $6x - 6 = 4x$ **c.** $x + 9 = 3x + 3$ **d.** $x + 18 = 4x + 3$

e. $-4x - 30 = 2 - 8x$ **f.** $3x + 6 = 4x - 2$ **g.** $-5x + 2 = -4x - 3$ **h.** $-3x + 45 = 3x - 3$

7. Write an equation for each of the given descriptions. WE3

a. A number m is multiplied by three and this is equal to twelve.

b. Two is added to a number v and the result is equal to nine.

c. Four is subtracted from a number q to give a result of sixteen.

d. Seven is equal to the result of a number k multiplied by three and added to one.

e. A number a is added to four and the result is multiplied by two. This is equal to eight.

f. Nine is equal to the result of three multiplied by the expression six subtracted from a number j.

g. Three is added to twice a number v. This is equal to the same unknown number v.

h. Nine is added to a number p, then the result is divided by seven to give six.

8. Determine a value for a to make the equation true.

$3 + a = 2a$

A. -1 **B.** 3 **C.** -6 **D.** 6 **E.** Not possible

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine a value for r to make the equation true.

$21 + 10 = r - 10$

Student A

$\text{LHS} = 21 + 10$

$= 31$

$\text{RHS} = 41 - 10$

$= 31$

$\therefore r = 41$

Student B

$\text{LHS} = 21 + 10$

$= 31$

$\text{RHS} = 31$

$\therefore r = 31$

b. Determine a value for a to make the equation true.

$a + a + a + 4 - 3 = 16$

Student A

$\text{LHS} = a + a + a + 1$

$= 5 + 5 + 5 + 1$

$= 16$

$\text{RHS} = 16$

$\therefore a = 5$

Student B

$\text{LHS} = a + a + a + 1$

$= 6 + 5 + 4 + 1$

$= 16$

$\text{RHS} = 16$

$\therefore a = 6, 5, 4$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Mary's height is h cm and her sister Katie is 23 cm taller. Katie's height is 161 cm. Write an equation to show Katie's height.

11. Robert buys k kg of dried apricots at $2.40 per kg. He spends a total of $4.80. Write an equation to describe this situation.

12. Rebecca is working on a group project and buys everyone in her group a hot chocolate. Construct an equation for an order of six large hot chocolates and five medium hot chocolates which costs a total of $60, if a large hot chocolate costs $\$l$ per cup and a medium hot chocolate costs $\$m$.

13. Joanne wants to hire a bus to go to Phillip Island. There is a fixed rate of $10 and then it costs $1.20 per km, ($k$). At the end of the trip she is charged $370. Write an equation that displays Joanne's bus fees.

14. Construct an equation for the area of a trapezium (A), where the sum of the parallel sides(a and b) is halved and then multiplied by the height (h).

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Danielle is going to Six the Musical. Tickets go on sale in six weeks and each ticket costs $100. Over the last few weeks, she has stopped buying coffees before work and has saved a total of $25, which will go towards her ticket.

a. Write an equation to show how much money ($\$m$) she will have in six weeks if she continues to not buy coffees and save $\$c$ per week.

b. If Danielle saves $15 per week on coffees, use the equation generated in part **a** to show that she will have enough money for a ticket to the musical.

c. Danielle's mother, Rose, also wants to go to the musical. Rose saves $\$l$ per week on coffees but owes her daughter Tori $10. Write an equation to show how much money ($\$m$) Rose will have in six weeks' time if she repays Tori.

d. If Rose saves $17 per week on coffees, use the equation generated in part **c** to determine whether or not she will have enough money for a ticket to the musical.

e. After the show, Danielle and Rose start to buy coffees from their favourite cafe again. List two suggestions for how they can continue to buy coffees but help the environment.

16. State whether the equation in part **a** and **b** are true or false when $p = 16$.

a. $\frac{p}{4} + 2 = 6$

b. $\frac{p + 2}{4} = 6$

c. Compare and contrast your answers from part **a** and **b** Note the similarities and differences. Are both equations true if p is substituted for 16? Why/Why not?

Extra spicy

17. Determine the value for n in the equation $n^2 = 36$.

18. Determine what values need to be substituted for c and d to make the equations true.

$d + c = 20$

$c - d = 10$

A. $c = 5, d = 15$

B. $c = 15, d = 5$

C. $c = 10, d = 10$

D. $c = -10, d = -10$

E. Not possible

19. State a solution for f to the following equation $-(2d)^2 + e^3 + f = 88$, if $d = 3$, $e = 5$ and $f = \square$.

A. -1 **B.** -73 **C.** 0 **D.** 73 **E.** 89

20. In order for the equation below to be true, the value of y must have a value of what?

$x + y = 2 + x$

Remember this?

21. Michelle drives her electric scooter a distance of 30 km at a constant speed of 40 km/h. How long will it take Michelle to travel 30 km?

A. 25 minutes

B. 30 minutes

C. 45 minutes

D. 1 hour 30 minutes

E. 1 hour 45 minutes

22. Calculate $\frac{2.84 + 7.1}{2.84} = \square$

A. 2.84 **B.** 3.5 **C.** 7.1 **D.** 8.1 **E.** 9.94

23. The fraction $\frac{1}{5}$ is equal to 0.2 as a decimal. What does $\frac{1}{10}$ equal to as a decimal?

A. 0.01 **B.** 0.1 **C.** 0.2 **D.** 1.0 **E.** 10%

Back to contents

6B Solving one-step and multi-step equations

Solving one, two and multi-step equations are all similar. Solving equations requires the inverse operation(s) to isolate the variable. Some equations include fractions and brackets. Fractions are used to represent division and brackets are used to clarify the order of operations.

LEARNING INTENTIONS

Students will be able to:

- solve one-step equations algebraically
- solve multi-step equations algebraically
- verify the value of unknowns using substitution.

KEY TERMS AND DEFINITIONS

Solving equations is a process to find the value of the unknown by performing a series of inverse operations.

A **one-step equation** is an equation that can be solved by applying a single inverse operation.

A **two-step equation** is an equation that can be solved by applying two inverse operations.

A **multi-step equation** is an equation that can be solved by applying more than two inverse operations.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Solving equations can be used to help make decisions.

Steve, an electrician with his own business, can build and solve equations to determine the number of customers he has to work for to make a weekly profit of $1150.

Key ideas

1. To isolate and find the value of the unknown in an equation, use the inverse operation.

Addition +	⟷	Subtraction −
Multiplication ×	⟷	Division ÷
Square x^2	⟷	Square root $\sqrt{x}$

2. When solving equations, it is important to understand the order of operations involved in constructing the equation.

1	Brackets	()
2	Exponents	a^n
3	Multiplication and Division (left to right)	× ÷
4	Addition and Subtraction (left to right)	+ −

Worked example 1

Solving one and two-step equations

Solve.

a. $x + 6 = 17$ WE1a

Working

$+6$

$x + 6 - 6 = 17 - 6$

$x = 11$

Check: $11 + 6 = 17$ ✓

Thinking

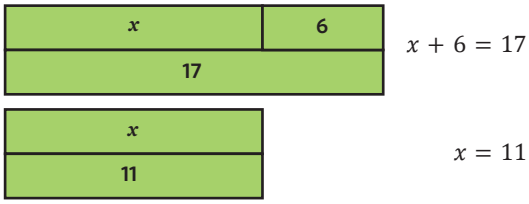
Step 1: Identify the operation being applied to the pronumeral x.

Step 2: Isolate x by applying the inverse operation (subtraction). To keep the equation true, apply the same operation on both sides of the equal sign.

Step 3: Simplify both sides of the equation to find the solution for x.

Step 4: Check to see if the LHS equals the RHS.

Visual support



$x + 6 = 17$

$x = 11$

b. $7p + 7 = 140$ WE1b

Working

$+7$ and $\times 7$

$7p + 7 - 7 = 140 - 7$

$7p = 133$

$\frac{7p}{7} = \frac{133}{7}$

$p = 19$

Check: $7 \times 19 + 7 = 140$ ✓

Thinking

Step 1: Identify the operations being applied to the pronumeral p. The first inverse operation will be the opposite of the last operation used to construct the equation.

Step 2: Isolate p by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 3: Simplify both sides of the equation to find the solution for p.

Step 4: Check to see if the LHS equals the RHS.

Student practice

Solve.

a. $a + 7 = 19$

b. $10b + 15 = 65$

Worked example 2

Solving equations with fractions

Solve.

a. $\frac{8d + 15}{2} = 8$ WE2a

Working

÷2, +15 and ×8

$$\frac{8d + 15}{2} \times 2 = 8 \times 2$$
$$8d + 15 = 16$$
$$8d + 15 - 15 = 16 - 15$$
$$8d = 1$$
$$\frac{8d}{8} = \frac{1}{8}$$
$$d = \frac{1}{8}$$

Check: $\frac{8 \times \frac{1}{8} + 15}{2} = 8$ ✓

Thinking

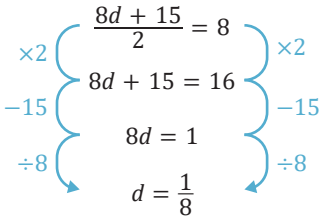
Step 1: Identify the operations being applied to the pronumeral *d*. The first inverse operation will be the opposite of the last operation used to construct the equation.

Step 2: Isolate *d* by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 3: Simplify both sides of the equation to find the solution for *d*.

Step 4: Check to see if the LHS equals the RHS.

Visual support



b. $\frac{5a}{2} - 8 = 12$ WE2b

Working

−8, ÷2 and ×5

$$\frac{5a}{2} - 8 + 8 = 12 + 8$$
$$\frac{5a}{2} = 20$$
$$\frac{5a}{2} \times 2 = 20 \times 2$$
$$5a = 40$$
$$\frac{5a}{5} = \frac{40}{5}$$
$$a = 8$$

Check: $\frac{5 \times 8}{2} - 8 = 12$ ✓

Thinking

Step 1: Identify the operations being applied to the pronumeral *a*. The first inverse operation will be the opposite of the last operation used to construct the equation.

Step 2: Isolate *a* by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 3: Simplify both sides of the equation to find the solution for *a*.

Step 4: Check to see if the LHS equals the RHS.

Student practice

Solve.

a. $\frac{4c + 15}{9} = 3$ **b.** $\frac{4f}{10} + 11 = 13$

Worked example 3

Solving equations with brackets

Solve.

a. $6(4l - 2) = 204$ WE3a

Working

$\times 6$, -2 and $\times 4$

$$\frac{6(4l - 2)}{6} = \frac{204}{6}$$
$$4l - 2 = 34$$
$$4l - 2 + 2 = 34 + 2$$
$$4l = 36$$
$$\frac{4l}{4} = \frac{36}{4}$$
$$l = 9$$

Check: $6(4 \times 9 - 2) = 204$ ✓

Thinking

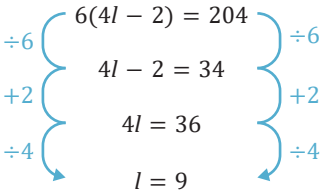
Step 1: Identify the operations being applied to the pronumeral l. The first inverse operation will be the opposite of the last operation used to construct the equation.

Step 2: Isolate l by applying the inverse operations. To keep the equation true apply the same operations on both sides of the equal sign.

Step 3: Simplify both sides of the equation to find the solution for l.

Step 4: Check to see if the LHS equals the RHS.

Visual support



b. $12 = \frac{2(k - 10)}{5}$ WE3b

Working

$\div 5$, $\times 2$ and -10

$$12 \times 5 = \frac{2(k - 10)}{5} \times 5$$
$$60 = 2(k - 10)$$
$$\frac{60}{2} = \frac{2(k - 10)}{2}$$
$$30 = k - 10$$
$$30 + 10 = k - 10 + 10$$
$$40 = k$$

Check: $12 = \frac{2(40 - 10)}{5}$ ✓

Thinking

Step 1: Identify the operations being applied to the pronumeral k.

Step 2: The first inverse operation will be the opposite of the last operation used to construct the equation.

Step 3: Isolate k by applying the inverse operations. To keep the equation true apply the same operations on both sides of the equal sign.

Step 4: Simplify both sides of the equation to find the solution for k.

Step 5: Check to see if the LHS equals the RHS.

Student practice

Solve.

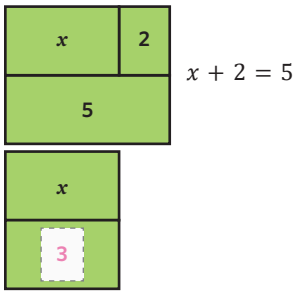
a. $2(9a - 5) = 80$ **b.** $15 = \frac{2(y - 20)}{10}$

6B Questions

Understanding worksheet

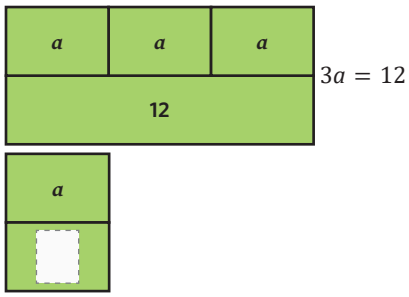
1. Complete the missing boxes to show how each equation is solved.

Example



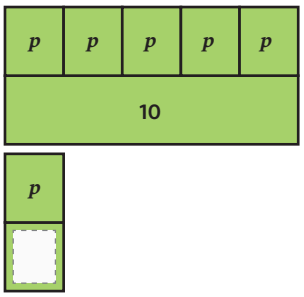
$x + 2 = 5$

a. $3a = 12$



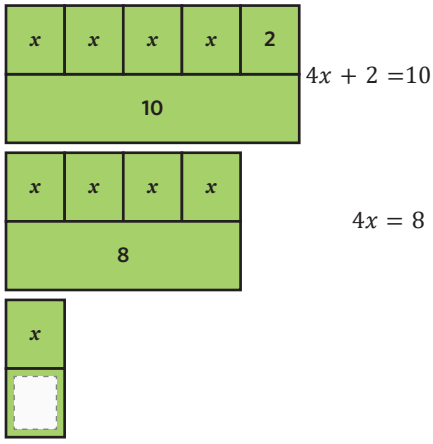
$3a = 12$

b. $5p = 10$



$5p = 10$

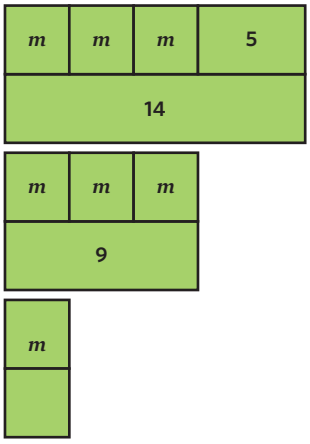
c. $4x + 2 = 10$



$4x + 2 = 10$

$4x = 8$

d. $3m + 5 = 14$

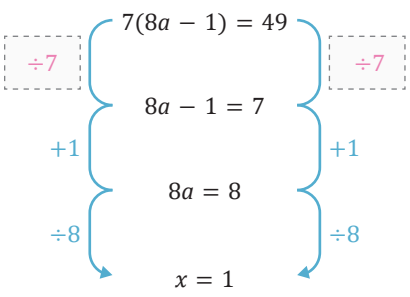


$3m + 5 = 14$

$3m = 9$

2. Complete the missing boxes to show how each equation is solved.

Example



a.

$3(3p + 8) = 159$

$3p + 8 = 53$ (−8, −8)

$3p = 45$ (÷3, ÷3)

$p = 15$

b.

$8(7j - 1) = 216$

$7j - 1 = 27$ (÷8, ÷8)

$7j = 28$ (÷7, ÷7)

$j = 4$

c.

$\frac{8l - 10}{9} = 6$

$(8l - 10) = 54$ (+10, +10)

$8l = 64$ (÷8, ÷8)

$l = 8$

d.

$\frac{5(x + 2)}{15} = 2$

$5(x + 2) = 30$ (×15, ×15)

$x + 2 = 6$ (−2, −2)

$x = 4$

3. Fill in the blanks by using the words provided.

undo | subtracting | inverse | multiplying

$\frac{4x + 6}{5} = 10$ is an example of an algebraic fraction. To solve this equation, the _____ operation is applied to both sides of the equal sign. To isolate x, start by _____ by five on both sides, followed by _____ six, then divide by four on both sides. Inverse operations are mathematical operations that _____ each other.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Solve. WE1a

a. $z + 10 = 25$ **b.** $m - 8 = 18$ **c.** $5 = x + 3$ **d.** $\frac{x}{4} = 4$

e. $-36 = -4x$ **f.** $h \div 4 = -4$ **g.** $-4c = 8$ **h.** $-8b = 24$

5. Solve. WE1b

a. $2x + 2 = 22$ **b.** $10x - 10 = 20$ **c.** $5e - 12 = -7$ **d.** $5 = \frac{x}{12} + 3$

e. $\frac{x}{2} - 10 = -13$ **f.** $14 + -8x = 6$ **g.** $-12 = 4x + 8$ **h.** $x^2 - 15 = 21$

6. Solve. WE2

a. $\frac{c + 3}{4} = 2$ **b.** $\frac{c - 7}{2} = 5$ **c.** $\frac{10a}{10} = 4$ **d.** $-1 = \frac{s - 2}{8}$

e. $\frac{4v + 15}{9} = 3$ **f.** $\frac{4b}{3} - 8 = -8$ **g.** $2 + \frac{5k}{3} = 12$ **h.** $10 - \frac{2m}{3} = 4$

7. Solve. WE3a

a. $2(k + 3) = 20$
b. $-4(x - 2) = 4$
c. $15 = 5(-4 + 7x)$
d. $4(5h - 1) = 56$
e. $5(2f + 5) = 25$
f. $9(-4 - 3b) = -45$
g. $-8(-5 + 3v) = 54.4$
h. $-2(-9e + 7) = -12.2$

8. Solve. WE3b

a. $\frac{2(l + 1)}{4} = 5$
b. $\frac{3(x - 10)}{3} = 10$
c. $4 = \frac{6(2 - j)}{9}$
d. $-5 = \frac{15(a - 4)}{3}$
e. $\frac{2(10 - h)}{6} = \frac{2}{3}$
f. $\frac{3(-h - 40)}{12} = 0.5$
g. $\frac{1.5(a + 2)}{3} = 1.5$
h. $-\frac{14(-x - 10)}{7} = 7$

9. Solve.

$$\frac{2p - 4}{5} = 1$$

A. 1 **B.** 4.5 **C.** 6.5 **D.** 22.5 **E.** $2\left(\frac{1}{5} - 4\right)$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. $\frac{a + 4}{5} = 2$

Student A

$$\frac{a + 4}{5} = 2$$
$$\frac{a + 4}{5} \times 5 = 2 \times 5$$
$$a + 4 = 10$$
$$a + 4 - 4 = 10 - 4$$
$$a = 6$$

Student B

$$\frac{a + 4}{5} = 2$$
$$\frac{a + 4}{5} - 4 = 2 - 4$$
$$\frac{a}{5} = -2$$
$$\frac{a}{5} \times 5 = 2 \times 5$$
$$a = 10$$

b. $2a + 4 = 20$

Student A

$$2a + 4 = 20$$
$$2a + 4 - 4 = 20 - 4$$
$$2a = 16$$
$$\frac{2a}{2} = \frac{16}{2}$$
$$a = 8$$

Student B

$$2a + 4 = 20$$
$$2a + 4 - 4 = 20$$
$$\frac{2a}{2} = 20$$
$$a = 20$$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Sally wants to buy doughnuts for her colleagues. She has \$15. The price (p) of doughnuts is given by the equation $p = 1.2b + 3$. Where b is the number of doughnuts being bought. How many doughnuts can Sally expect to purchase?

12. Joanne had l number of lollies. She divided these lollies amongst eight people, including herself. Her nanna gave her an additional 10 lollies. She was left with 14 lollies. How many lollies (l) did Joanne have to begin with?

13. 32 is two times the difference between Jonah's age (j) and 36. Assume Jonah is older than 36. Write an equation and solve for Jonah's age, j.

14. Amber is participating in a star jump challenge and does a total of 16 star jumps at the start of each day. She wants to increase the number of star jumps she does each day by four until she reaches 40 star jumps each day. Using d to represent the number of days since Amber started her star jump challenge, what day can Amber expect to reach her goal?

15. Moana and Billy both ran 12 kilometres together. Moana then ran twice as far as Billy. Altogether she ran 35 km. Let Billy's individual running distance be represented as b. Create and solve an equation for the number of kilometres Billy ran on his own.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. The height of a sunflower can be found using the equation $h = \frac{3(2d + 15)}{4}$, where h is the height in cm and d is the number of days.

- **a.** Solve for the number of days (d), it will take for the sunflower to have a height of 15 cm.
- **b.** Solve for the number of days (d), it will take for the sunflower to double the height given in part **a**.
- **c.** Rearrange the equation to solve for d instead of h?
- **d.** Using part **c** solve for the height (h), in cm, of the sunflower, on day 0.
- **e.** Suggest some ways the growth of the sunflower can be enhanced.

17. Solve for a.

- **a.** $-\frac{2(a - 5)}{3} = 2$
- **b.** $\frac{2(a - 5)}{3} = 2$
- **c.** Compare the value of a in both parts **a** and **b.** Note their similarities and differences. Is the final answer the same value for a? Why/why not?

Extra spicy

18. The area of a triangle is $A = \frac{bh}{2}$. Solve for h if $b = \sqrt{64}$ cm and $A = 20$ cm^2

19. Solve for a.

$a + b = 1000$

$a - b = 700$

- **A.** -150
- **B.** -850
- **C.** 150
- **D.** 850
- **E.** no solution

20. The solution to $6(a^2 + 2) = 18$ is:

- **A.** $a = \pm 2.667$
- **B.** $a = \pm 1.663$
- **C.** $a = \pm 1$
- **D.** $a = 0$
- **E.** $a = \sqrt{a}$

21. Lou hires a scooter and travels k kilometres, then walks half as far as she scoots. If the total distance traveled is 38 km, solve to find how many kilometres Lou is on the scooter. Provide your final answer to two decimal places.

Remember this?

22. Remi measures a distance of 4 kilometres and 20 metres.

How far did Remi measure in kilometres?

A. 4.020 km **B.** 4.20 km **C.** 5.20 km **D.** 5.020 km **E.** 420 km

23. $185.9 + 56.7 = \square$

A. 131.6 **B.** 242.6 **C.** 752.9 **D.** 1426 **E.** 7529

24. Augustus is facing south-west. He makes a three quarter turn to his right.

Which direction is he facing after the turn?

A. North **B.** West **C.** East **D.** North-west **E.** South-east

Back to contents

6C Solving equations with unknowns on both sides

When two expressions are equal to each other, this will give an equation with variables on both sides. These equations can be solved for the variable in the same way as having a variable only on one side. The same rules for using inverse operations with numbers can be applied to variables.

LEARNING INTENTIONS

Students will be able to:

- solve equations with unknowns on both sides.

KEY TERMS AND DEFINITIONS

An **expression** is a number of terms grouped together by operations.

Solving equations is a process to find the value of the unknown by performing a series of inverse operations.

When you **equate** expressions, you make them equal to each other.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: TP71/Shutterstock.com

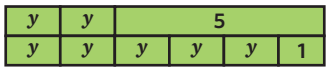
Equating expressions is useful to compare situations involving the same quantity, such as money. For example, you can compare the cost of buying a car to using fuel with using the train every day and figure out how many days it will take for buying and driving the car to be cheaper than taking the train.

Key idea

1. Equating two expressions results in equations with variables on both sides of the equals sign.

$2y + 5$

$5y + 1$



$2y + 5 = 5y + 1$

Worked example 1

Solving equations with variables on both sides

Solve each equation for the given variable.

a. $3x - 3 = 2x + 4$ WE1a

Working

$$3x - 3 = 2x + 4$$
$$3x - 3 - 2x = 2x + 4 - 2x$$
$$x - 3 = 4$$

Thinking

Step 1: Simplify the expression by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Continues →

$x - 3 + 3 = 4 + 3$

$x = 7$

Check:

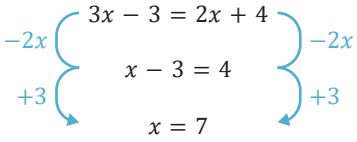
$3 \times 7 - 3 = 2 \times 7 + 4$

$18 = 18$ ✓

Step 2: Solve for x.

Step 3: Check to see if the LHS equals the RHS.

Visual support



b. $3k = 2k - 2$ WE1b

Working

$3k = 2k - 2$

$3k - 2k = 2k - 2k - 2$

$k = -2$

Check:

$3 \times (-2) = 2(-2) - 2(-2) - 2$

$-6 = -6$ ✓

Thinking

Step 1: Simplify the expression by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 2: Solve for k.

Step 3: Check to see if the LHS equals the RHS.

c. $3(k - 5) = 2(k - 1)$ WE1c

Working

$3(k - 5) = 2(k - 1)$

$3k - 15 = 2k - 2$

$3k - 15 - 2k = 2k - 2k - 2$

$k - 15 + 15 = -2 + 15$

$k = 13$

Check:

$3 \times (13 - 5) = 2 \times (13 - 1)$

$24 = 24$ ✓

Thinking

Step 1: Expand the brackets.

Step 2: Simplify the expression by collecting like terms. When collecting like terms, apply the inverse operation where necessary.

Step 3: Solve for k.

Step 4: Check to see if the LHS equals the RHS.

Student practice

Solve each equation for the given variable.

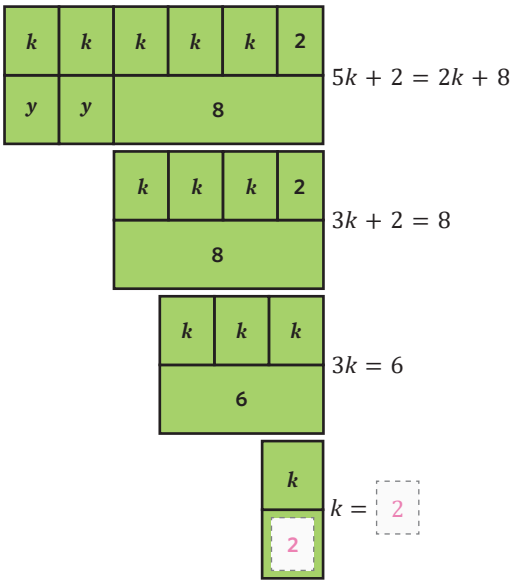
a. $4x - 1 = 3x + 2$ **b.** $4k = 5k + 4$ **c.** $2(k + 5) = 3(k - 1)$

6C Questions

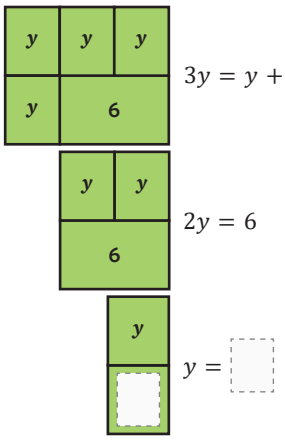
Understanding worksheet

1. Complete the missing boxes to show how each equation is solved.

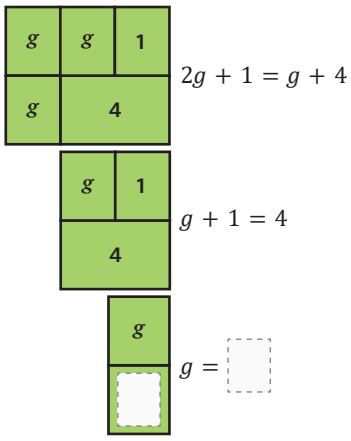
Example



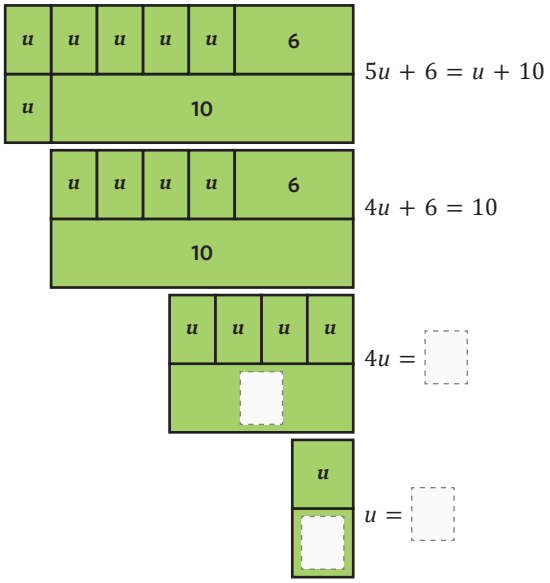
a.



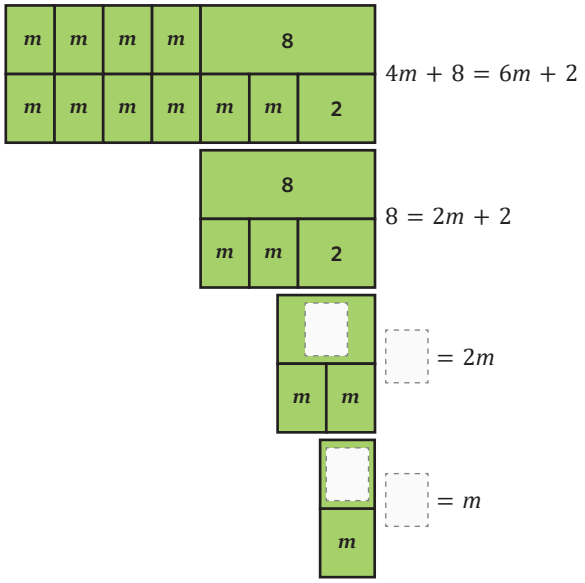
b.



c.



d.



2. Complete the missing boxes to show how each equation is solved.

Example

$3a + 3 = 2a + 5$

$-2a$ $-2a$

$a + 3 = 5$

-3 -3

$a = 2$

a.

$4g - 2 = 3g + 3$

$-3g$ $-3g$

$g - 2 = 3$

$\square$ $\square$

$g = 5$

b.

$2 + 4k = 5k + 1$

$-4k$ $-4k$

$2 = k + 1$

$\square$ $\square$

$1 = k$

c.

$6a - 5 = 4a + 5$

$\square$ $\square$

$2a - 5 = 5$

$+5$ $+5$

$2a = 10$

$\div 2$ $\div 2$

$a = 5$

d.

$5t + 8 = 8t - 5$

$\square$ $\square$

$8 = 3t - 4$

$+4$ $+4$

$12 = 3t$

$\div 3$ $\div 3$

$4 = t$

3. Fill in the blanks by using the words provided.

expanded | inverse | constants | variables

When two expressions are made equal to each other, this will result in an equation where there are $\square$ on both sides of the equation. $\square$ operations can be applied to variables as well as $\square$ to solve these equations. If there are brackets involved, they should be $\square$ first.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Solve each equation for the given variable. WE1a,b

a. $4m + 3 = 3m + 8$ **b.** $5e - 3 = 4e + 1$ **c.** $5g + 6 = 6g - 5$ **d.** $4r - 6 = 2r$

e. $y + 3 = 5 + \frac{y}{2}$ **f.** $-2 - 3w = 5w - 34$ **g.** $-6j - 6 = 2j + 10$ **h.** $-9 + \frac{3t}{2} = \frac{5t}{6} + 1$

5. Solve each equation for the given variable. WE1c

a. $4(w + 3) = 7w + 15$ **b.** $2(2v + 3) = 27 - 3v$

c. $2(4j + 5) = 9j$ **d.** $7d = 3(2d - 3)$

e. $3(5z + 2) = 2(5z + 8)$ **f.** $2(3x - 15) = 3(6x - 16)$

g. $3(7e - 23) = 9(-13 + 3e)$ **h.** $3(4 + 2y) = 2(-4 - 2y)$

6. Solve each equation, when necessary write fractions as improper fractions.

a. $4a + 3 = 2a + 8$

b. $5j - 1 = j + 1$

c. $5c - 6 = 2c + 2$

d. $2n - 1 = 7n - 2$

e. $8z + 5 = 1 + 3z$

f. $-6r + 5 = -2r + 8$

g. $-2 + 9b = 4b - 7$

h. $-8s - 1 = -7 - 2s$

---

7. The solution to the equation $7x - 3 = 5 + 3x$ is

A. $x = \frac{1}{5}$ **B.** $x = \frac{1}{2}$ **C.** $x = \frac{4}{5}$ **D.** $x = 1$ **E.** $x = 2$

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Solve the equation $8f + 3 = -2f - 9$

Student A

$$\begin{aligned} 8f + 3 &= -2f - 9 \\ 8f - 2f + 3 &= -9 \\ 6f + 3 &= -9 \\ 6f &= -12 \\ f &= -2 \end{aligned}$$

Student B

$$\begin{aligned} 8f + 3 &= -2f - 9 \\ 8f + 2f + 3 &= -9 \\ 10f + 3 &= -9 \\ 10f &= -12 \\ f &= \frac{-6}{5} \end{aligned}$$

b. Solve the equation $8q + 3 = 10q + 6$

Student A

$$\begin{aligned} 8q + 3 &= 10q + 6 \\ 8q + 10q &= 6 + 3 \\ 18q &= 9 \\ q &= \frac{9}{18} \\ &= \frac{1}{2} \end{aligned}$$

Student B

$$\begin{aligned} 8q + 3 &= 10q + 6 \\ 3 &= 2q + 6 \\ -3 &= 2q \\ \frac{-3}{2} &= q \end{aligned}$$

Problem solving

Question working paths

Mild 9, 10, 11 '	Medium 10, 11, 12 ''	Spicy 11, 12, 13 '''

9. An equilateral triangle has side lengths $x + 1$, and a rectangle has width x and length $x + 1$.

For what value of x are the perimeters of both shapes the same?

10. Get Fit! Gym charges $20 per month plus a $80 sign-up fee. Just Do It! Gym charges $40 per month but charges no sign-up fee. After how many months will the cost of gym membership be equal at both locations?

11. Two 2000 L water tanks are being filled. Tank 1 was initially one-quarter full and is filled at a rate of 200 L per minute. Tank 2 was initially half-full and is filled at a rate of 100 L per minute. After how many minutes will the water tanks have the same amount of water?

12. In 3 years' time, Joe's current age will double. What is Joe's current age?

13. In 2019, a school had the same number of Year 7 boys and girls. The following year, 50 female students joined in Year 8 which meant there were now twice as many girls compared to boys. How many students were there in Year 7?

Reasoning

Question working paths

Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. The table below compares the taxi fares in Melbourne and Sydney. The flat fee is the cost of ordering the taxi and is the same regardless of distance travelled.

	Flat fee ($)	Cost per kilometre
Sydney	5.20	1.20
Melbourne	2.50	1.43

a. Write an expression that calculates the cost of a taxi in Sydney using the distance (d) travelled.

b. Write an expression that calculates the cost of a taxi in Melbourne using the distance (d) travelled.

c. Equate the expressions in part **a** and **b** to each other. Solve this equation to find the travel distance where the cost of taking a taxi is the same in both cities. Round to two decimal places.

d. How much does it cost to travel the distance found in part **c** in Sydney and Melbourne?

e. Some taxi services also charge per minute of driving time. Discuss which should have the highest dollar value: cost per minute or cost per kilometre.

15. Using the digits 1 to 9 at most two times each, make equations where:

$\square x + \square = \square x + \square$

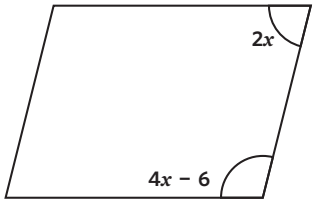
a. there is a whole number solution for x.

b. it is not possible to solve for x.

c. Comparing your answers from parts **a** and **b**, when is it the case that there is no possible solution for any equation with unknowns on both sides of the equals sign?

Extra spicy

16. The shape below is a parallelogram.



What is the value of x?

17. For the three angles in a triangle, one is double the smallest and the other is 40° more than the smallest. What are the three angles?

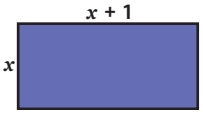
A. 46°, 92°, 42°
B. 40°, 80°, 60°
C. 35°, 70°, 75°
D. 15°, 55°, 110°
E. 30°, 60°, 90°

18. What is the solution to $\frac{y+1}{2} = \frac{y+3}{5}$?

19. The shape below is a rectangle with width x and length $x + 1$:

Which equation can be solved to find the value of x for which the perimeter is equal to the area?

A. $x = x + 1$
B. $2x = 2x + 1$
C. $4x + 2 = x + 1$
D. $4x + 2 = x^2 + x$
E. $2x + 1 = x^2 + x$



Remember this?

20. Emma is looking through a catalogue and sees a number of items that have been discounted from their original prices.

Which of the following represents the largest percentage discount?

	Original price	New price
A.	$25	$12
B.	$15	$10
C.	$32	$22
D.	$50	$38
E.	$100	$90

21. The table shows women's shoe sizes in Australia and Japan.

Australian size	4	5	6	7	8	9
Japanese size	21	22	23	24	25	26

Which rule describes the relationship between Japanese and Australian women's shoe sizes?

A. Japan $=$ Australia $+ 17$

B. Japan $= (5 \times \text{Australia}) + 1$

C. Japan $=$ Australia $- 17$

D. Japan $= \frac{\text{Australia}}{2} - 6$

E. Japan $= (2 \times \text{Australia}) + 8$

22. The table shows the amount of revenue that a paint store earned from sales over four days.

Day	**Revenue from sales**
Thursday	$12 321
Friday	$15 642
Saturday	$21 546
Sunday	$22 312

Each litre of paint costs $30.

What was the mean volume of paint, to the nearest litre, sold per day?

A. 333 L **B.** 411 L **C.** 599 L **D.** 17 955 L **E.** 538 650 L

Back to contents

6D Formulas and applications of equations

Solving equations is a powerful tool in problem solving. However, a mathematician first needs to read a problem and then build an equation that can be used to solve the problem. Formulas are also used a lot in problem solving, particularly in science when calculations are needed. This can include converting temperatures or distances or determining the speed of an object.

LEARNING INTENTIONS

Students will be able to:

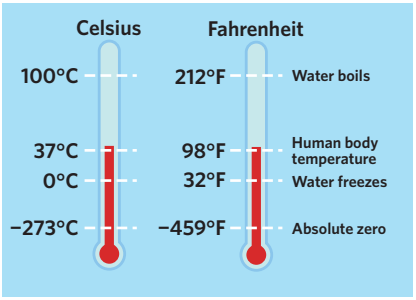
- understand what a formula is
- use substitution with formulas
- apply formulas to real world problems
- build and solve equations from worded situations.

KEY TERMS AND DEFINITIONS

A **rule** is another term to describe an equation or formula. It usually refers to equations that produce sequences of numbers.

The **subject** of a formula is a variable that occurs by itself, usually on the left-hand-side of an equation. It indicates what the formula calculates.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



In the United States and other countries and territories, the Fahrenheit scale is used to measure temperature. However, in most parts of the world the Celsius scale is used. The formula $F = \frac{9}{5}C + 32$ is used to convert Celcius to Fahrenheit.

Key idea

1. A **formula** is an equation that describes the relationship between two or more variables.

Perimeter of a rectangle

$$P = 2l + 2w$$

Perimeter · Length · Width

Converting between Celsius and Fahrenheit

$$F = \frac{9}{5}C + 32$$

Temperature in Fahrenheit · Temperature in Celsius

Worked example 1

Applying formulas

Substitute the given values for the variables and solve for the unknown variable.

a. Perimeter of a rectangle: $P = 2l + 2w$ ($l = 3$ m and $w = 5$ m) WE1a

Working	Thinking
$P = 2 \times 3 + 2 \times 5$	**Step 1:** Substitute the values for the variables.
$= 6 + 10$	**Step 2:** Simplify.
$= 16$	
$P = 16$ m	**Step 3:** State the units.

Continues →

Visual support

l = 3 m w = 5 m

$P = 2l + 2w$

$P = 2 \times 3 + 2 \times 5$

$P = 16$ m

b. Volume of a cuboid: $V = lwh$ ($V = 40$ cm^3, $l = 8$ cm, $h = 10$ cm) WE1b

Working	Thinking
$40 = 8 \times w \times 10$	**Step 1:** Substitute the values for the variables.
$40 = 80 \times w$	**Step 2:** Simplify.
$0.5 = w$	**Step 3:** Solve for the unknown variable.
$w = 0.5$ cm	**Step 4:** State the units.

Student practice

Substitute the given values for the variables and solve for the unknown variable.

a. Area of a triangle: $A = \frac{1}{2}bh$ ($b = 3$ m, $h = 2$ m)

b. Newton's second law: $F = ma$ ($F = 20$ N, $m = 4$ kg and the unit of a is m/s^2)

Worked example 2

Building formulas

For each scenario, build a simplified formula that represents the relationship between the given variables.

a. A baker used 192 sticks of butter to make 32 pie crusts. WE2a
(s = number of sticks of butter, c = total number of pie crusts)

Working	Thinking
$192 \div 32 = 6$ sticks per pie crust	**Step 1:** Determine how many sticks of butter are needed to make 1 pie crust.
$s = 6c$	**Step 2:** Write a formula with s as the subject.

b. A chicken cooks for 45 minutes per kilogram plus an extra 20 minutes. WE2b
(T = total cooking time, w = weight of chicken)

Working	Thinking
$45w$	**Step 1:** Determine the cooking time per kilogram. 'Per' refers to multiplication.
$45w + 20$	**Step 2:** Add the extra time.
$T = 45w + 20$	**Step 3:** Write the formula with T as the subject.

Student practice

For each scenario, build a simplified formula that represents the relationship between the given variables.

a. Beatrice bought 12 kg of honey for $180.
(w = weight of honey, p = total price)

b. People can claim a flat fee of $50 in addition to $5 per hour in tax back from the government for working from home.
(t = tax return, c = amount of hours)

Worked example 3

Building and solving equations

For each scenario:

i. Choose a variable to represent the unknown quantity.
ii. Construct an equation to solve.
iii. Solve the equation for the unknown.
iv. State the answer.

a. A business makes and sells doughnuts for $3 per doughnut. The total cost of electricity, wages and salaries is $6000 per month. How many doughnuts must be sold to cover the business' costs? WE3a

Working	Thinking
Let n be the number of doughnuts sold.	i. Choose a variable to represent the unknown quantity.
$3n = 6000$	ii. Construct an equation to solve. 'Per' refers to multiplication.
$3n \div 3 = 6000 \div 3$ $n = 2000$	iii. Solve the equation for the unknown.
2000 doughnuts need to be sold.	iv. State the answer.

b. Elanor bought 5 lobsters and a jumbo tray of oysters from the fishmonger for a total of $140. If the tray of oysters cost $40, what is the price per lobster, in dollars? WE3b

Working	Thinking
Let c be the price, in dollars, per lobster.	i. Choose a variable to represent the unknown quantity.
$5c + 40 = 140$	ii. Construct an equation to solve. 'Total' refers to an equal sign.
$5c + 40 - 40 = 140 - 40$ $5c \div 5 = 100 \div 5$ $c = 20$	iii. Solve the equation for the unknown.
The price per lobster is $20.	iv. State the answer.

Student practice

For each scenario:

i. Choose a variable to represent the unknown quantity.
ii. Construct an equation to solve.
iii. Solve the equation for the unknown.
iv. State the answer.

a. Dion has $360 and wants to buy video games that each cost $60. How many video games can he buy?

b. Rob bought a meat pie for $7 and 6 bags of chips for his family. He spent a total of $37. How much does each bag of chips cost?

6D Questions

Understanding worksheet

1. Match the descriptions to the equations from the list below.

Example

Description	Equation
Two more than the product of nine and a number q is equal to seven.	$2 + 9q = 7$

- $9k - 2 = 4$
- $\frac{g}{2} + 4 = 10$
- $6r = 36$
- $8 = h - 4$

	Description	Equation
a.	The product of six and a number r is equal to thirty-six.	
b.	Eight is the result of four less than a number h.	
c.	A number k is multiplied by nine minus two equals four.	
d.	The result of half a number g added to four is ten.	

2. For each description, tick which is the unknown variable.

Example

Description	Unknown variable
Elise has a collection of comic books. One day she bought 4 new comic books so she now has a total of 10 comic books.	☑ Original number of comic books ☐ Number of comic books bought

	Description	Unknown variable
a.	Duyen has \$45 to spend on books that cost \$9 each.	☐ Price per book ☐ Number of books
b.	After buying 4 house plants, Ying had a total of 23 house plants.	☐ Total number of house plants ☐ Original number of house plants
c.	A water tank fills up at a rate of 200 L per minute which has a total volume of 2000 .	☐ Current volume of the tank ☐ Total time taken to fill tank
d.	Chris used half of his weekly salary on rent, and spent \$200 on groceries.	☐ Cost of weekly rent ☐ Weekly salary

3. Fill in the blanks by using the words provided.

substitute | variables | rule | subject

An equation with two or more _____ is called a formula. The _____ indicates what it calculates. When all but one of the variables have a value, you can _____ to find the value. Another term that describes an equation is a _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c,d), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (c,d,e), 7

4. Substitute the given values for the variables, and solve for the unknown variable. Round any decimals to two decimal places. WE1

a. Area of a rectangle: $A = lw$ ($l = 6$ m and $w = 10$ m)

b. Average speed of an object: $v = \frac{x}{t}$ ($x = 10$ km, $t = 2$ hours and the unit of speed is km/h)

c. Volume of a cone: $V = \frac{1}{3} \times b \times h$ ($b = 4$ mm^2, $h = 2.5$ mm)

d. Area of a trapezium: $A = \frac{1}{2}h(a + b)$ ($h = 2$ cm, $a = 3$ cm and $b = 7$ cm)

e. $I = PRt$ ($I = \$10$, $R = 0.03$, $t = 3$ months and the unit of P is dollars)

f. Distance travelled by an object: $x = vt + \frac{1}{2}at^2$ ($v = 3$ m/s, $t = 5$ seconds and $a = 2$ m/s^2)

g. Converting from Fahrenheit to Celsius: $F = \frac{9}{5}C + 32$ ($F = -5$°F)

h. Volume of a sphere: $V = \frac{4}{3}\pi r^3$ ($V = 56$ m^3)

---

5. For each scenario, build a simplified formula that represents the relationship between the given variables. WE2

Use the first variable given as the subject of the formula. Round any decimals to two decimal places.

a. A chef bought 4 bags of potatoes for her restaurant and it cost $12.
(c = total cost of buying potatoes, b = number of bags of potatoes)

b. Pam travelled 50 kilometres in 2 hours.
(d = distance travelled, t = time taken)

c. A painter was able to paint 80 vases, using 400 millilitres of paint.
(v = number of litres of paint, n = number of painted vases)

d. Adult tickets to a movie cost $24, plus an online booking fee of $2 for the whole order.
(c = total cost of buying adult tickets online, a = number of adult tickets)

e. A company is buying loaves of bread in bulk. Each loaf costs $1.50 and they get a discount of $20 on the whole order.
(c = total cost of buying the bread, l = number of loaves of bread).

f. Fahrad wants to extend the length of his veggie patch that is already 3 metres long and 2 metres wide.
(A = area of the veggie patch, x = extra length of veggie patch)

g. A farm has a number of chickens (that have two legs) and sheep (that have four legs).
(l = total number of legs, c = number of chickens, s = number of sheep)

h. Kirsty makes pies, which cost $3.50 each, and sausage rolls, which cost $2.60 each, for a football canteen. It costs $0.60 cents to make each item.
(t = total profit made from selling pies and sausage rolls, p = number of pies sold, s = number of sausage rolls sold).

6. For each scenario: WE3

i. Choose a variable to represent the unknown quantity.
ii. Construct an equation to solve.
iii. Solve the equation for the unknown.
iv. State the answer.

a. Feliks is raising money for a charity by running 80 km. He thinks he can run 4 km per day. How many days will it take to reach his goal?

b. The cost of an ad on YouTube is approximately \$0.20 per view. How many views can a video game studio get with a budget of \$3000?

c. Teagan is told that two-thirds of all the customers on one day at her coffee shop ordered food. If 50 customers ordered food, how many customers came to the coffee shop?

d. Two classes, 7A and 7B, have a total of 43 students. One class has 5 more students than the other. How many students are in each class?

e. Surae bought some PS5 controllers that each cost \$75 and two games that cost \$79 each. The total cost of her purchase was \$383. How many PS5 controllers did she buy?

---

7. Kai's Kayaks rents out their kayaks with a base rate of \$15 plus \$10 per hour. Kathy hired a kayak and prepaid \$60, but can only rent the kayak in whole number of hours. How long can she rent the kayak for?

A. 2 hours **B.** 3 hours **C.** 4 hours **D.** 5 hours **E.** 6 hours

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. 96 students went on an excursion to the museum. Two buses were completely filled and 4 students travelled in a car. How many students fit in each bus?

Student A

Let s = number of students per bus

$2s + 4 - 4 = 96 - 4$

$2s \div 2 = 92 \div 2$

$s = 46$

There are 46 students per bus.

Student B

Let s = number of students per bus

$\frac{s}{2} + 4 - 4 = 96 - 4$

$\frac{s}{2} \times 2 = 92 \times 2$

$s = 184$

There are 184 students per bus.

b. The sum of three consecutive numbers is 84. What is the value of the smallest number?

Student A

Let x be the smallest number.

$x + (x + 1) + (x + 2) = 84$

$3x + 3 - 3 = 84 - 3$

$3x \div 3 = 81 \div 3$

$x = 27$

The smallest number is 27.

Student B

Let x be the smallest number.

$x + 1 + 2 = 84$

$x + 3 - 3 = 84 - 3$

$x = 81$

The smallest number is 81.

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. A bag of 12 hockey balls cost $120. Individually each ball costs $12. How much cheaper is it to buy the bag of hockey balls compared to buying 12 individual balls?

10. A basketball team played 21 games. They lost twice as many games as they won. How many games did the team lose?

11. Lena is paving a rectangular area of her backyard with stone slabs. The width of the area is 3.5 metres and she has budgeted $2000 for the slabs. If the stone slabs cost $90 per square metre, what is the maximum length of the area she can pave? Round your answer to two decimal places.

12. Janine is a sheep farmer, and she knows she can shear 6 kg of wool per sheep. Usually, 5 kg of wool is wasted from a flock of sheep (no matter how many sheep) because it is low quality and cannot be sold. How many sheep does Janine need if she wants to sell 350 kg of wool?

13. Daniel owns a car washing business. The cost of detergent and labour is $3 per car. The buckets, hoses, sponges and brushes cost $24 for a day's work. If Daniel charges $6 per car, what is the minimum amount of cars they need to clean to make a profit?

Reasoning

Question working paths

Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. A family decides to take two cars to travel from Melbourne for a holiday. Car B leaves 1 hour after Car A.

Car A travels at a speed of 60 km/h and Car B travels at a speed of 80 km/h. The distance travelled can be calculated by multiplying the speed by the time. Use t as the time travelled by Car B.

a. Write an expression for the time travelled by Car A in terms of t.

b. Write an expression for the distance travelled by Car A and an expression for the distance travelled by Car B.

c. Write an equation that can be solved to find when Car B catches up to Car A (this is when they have travelled the same distance).

d. How long does it take for Car B to catch up to Car A?

e. List a reason for, and a reason against, taking two cars instead of one for a holiday.

15. The formula $v = \frac{x}{t}$ is used to calculate the average speed of an object that has travelled a distance x in a time period t, in hours. The average speed of a car that travelled 120 km in 2 hours is 60 km/h.

a. Calculate the average speed of a car that travelled 240 km in 2 hours.

b. Calculate the average speed of a car that travelled 120 km in 4 hours.

c. Compare what happens to v when the numerator is doubled to what happens when the denominator is doubled.

Extra spicy

16. The average of a number g and 15 is 6.5. Calculate the value of g.

17. The equation to convert Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Rearrange the equation to make Celsius (C) the subject.

A. $C = \frac{9}{5}F + 32$

B. $C = \frac{5}{9}F - 32$

C. $C = \frac{5}{9}(F - 32)$

D. $C = \frac{9}{5}(F - 32)$

E. $C = \frac{9}{5}(F + 32)$

18. Heron's formula can be used to find the area of any triangle if all the side lengths are known:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$ (this is known as the semi-perimeter or half-perimeter) and a, b and c are the side lengths of the triangle.

Calculate the area of a triangle with side lengths 4 cm, 5 cm and 6 cm to two decimal places.

A. 3.62 cm^2 **B.** 9.92 cm^2 **C.** 30.00 cm^2 **D.** 44.05 cm^2 **E.** 120.64 cm^2

19. The formula below was created to predict the prices of houses (in $USD) in Boston:

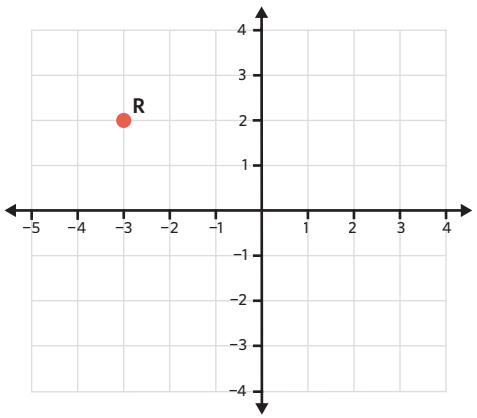
House price $= 1000(14.86 + 0.22 \times a - 0.036 \times y + 0.033 \times f - 0.081 \times t)$

Calculate the difference in house price for the following two properties:

Feature	House 1	House 2
Area (a)	96.5 m^2	120 m^2
Age (y)	122	105
Number of floors (f)	2	3
Time to treavel to CBD (t)	22	12

Remember this?

20. Point R is translated 4 units right and 5 units down.

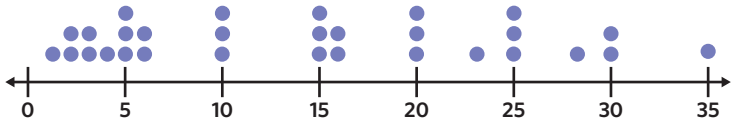


What are the coordinates of the new position of point R?

A. (−3,2) **B.** (−3,1) **C.** (1,−3) **D.** (2,−2) **E.** (2,−3)

21. Ms González's class of 30 Year 8 students recorded the time it took to travel to school one day.

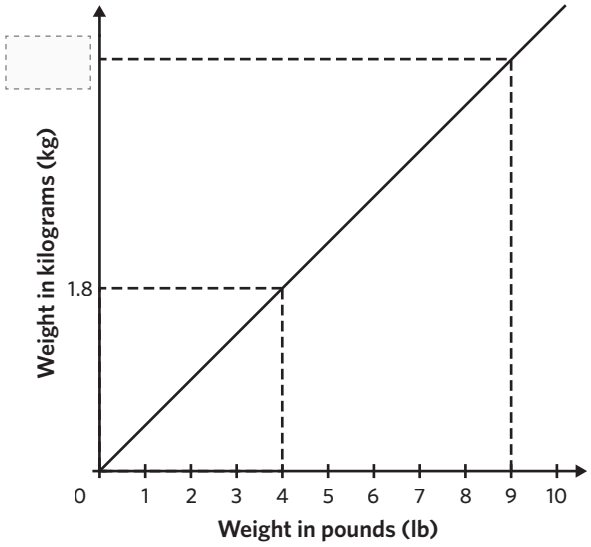
Ms González made a dot plot to show the travel times.



What fraction of the Year 8 students' travel times is greater than 10 minutes but less than 25 minutes?

A. $\frac{3}{10}$ **B.** $\frac{4}{10}$ **C.** $\frac{1}{2}$ **D.** $\frac{8}{15}$ **E.** $\frac{21}{30}$

22. The graph below can be used to approximately convert pounds to kilograms.



Using the graph, a weight of 9 pounds is closest to

A. 0.45 kg **B.** 3.6 kg **C.** 4.1 kg **D.** 5.4 kg **E.** 6.8 kg

Back to contents

6E Inequalities

An inequality is like an equation, however equations are used to represent when two expressions are equal. Inequalities are used to indicate when two expressions are not equal and are greater than or less than each other.

LEARNING INTENTIONS

Students will be able to:

- represent inequalities on a number line
- solve inequalities
- describe real-life situations using inequalities.

KEY TERMS AND DEFINITIONS

Inequality is a statement when one value or algebraic expression is less than or greater than another.

Inclusive is including the value(s).

Exclusive is excluding the value(s).

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Giorgio Souza/Shutterstock.com

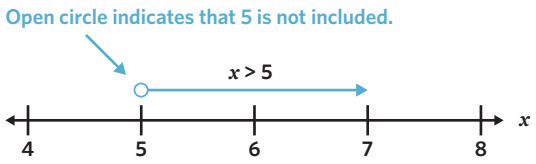
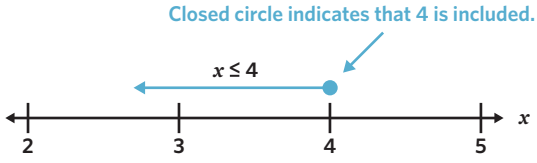
Speed limits are an example of inequalities. Inequalities don't represent an exact amount, however represent a limit of what is allowed. For example, the limit on a particular street is 70 km/h. As an inequality this would be represented as $s \leq 70$ where s represents speed.

Key ideas

1. Inequalities can be represented as different symbols that mean different things.

Symbol	Symbol to words	Inequation example	Example in words
$>$	Greater than	$x > 4$	x is greater than four
$\geq$	Greater than or equal to	$x \geq 12$	x is greater than or equal to twelve
$<$	Less than	$x < 5$	x is less than five
$\leq$	Less than or equal to	$x \leq 11$	x is less than or equal to eleven

2. To display inequalities on a number line, there are different symbols that represent the inequality symbols.

Symbol	Symbol to word	Number line diagram
○	Greater than, $>$ Less than, $<$	
●	Greater than or equal to, $\geq$ Less than or equal to, $\leq$	

Continues →

3. To solve an inequality, isolate the unknown variable by using the inverse operation.

Addition +	⟷	Subtraction −
Multiplication +	⟷	Division ÷

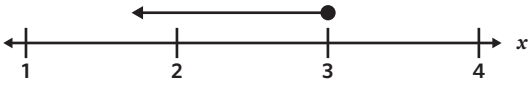
Worked example 1

Representing inequalities on a number line

Represent the inequalities on a number line.

a. $x \leq 3$ WE1a

Working



Thinking

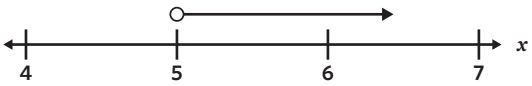
Step 1: To show the solution on a number line, use a closed circle to indicate the number given is also included in the set.

Step 2: Generate a number line that includes the given number.

Step 3: Draw an arrow pointing to the left to all numbers less than or equal to the given number.

b. $x > 5$ WE1b

Working



Thinking

Step 1: To show the solution on a number line, use an open circle to indicate the number given is not included in the set.

Step 2: Generate a number line that includes the given number.

Step 3: Draw an arrow pointing to the right to all numbers greater than the given number.

c. $-2 \leq x < 5$ WE1c

Working

Thinking

Step 1: To show the solution on a number line, use an open circle to indicate the number(s) given is not included in the set and use a closed circle to indicate the number(s) given is also included in the set.

Step 2: Generate a number line that includes the given numbers.

Step 3: Draw a line between the two circles.

Student practice

Represent the inequalities on a number line.

a. $x \leq 4$ **b.** $x > 3$ **c.** $-4 \leq x < 2$

Back to contents

Worked example 2

Using inequalities in real-life situations

Represent the following scenarios as an inequality, using x as the unknown quantity.

a. The product of two and an unknown number is less than twelve. WE2a

Working	Thinking
	Step 1: Use x to represent the unknown number.
$2x < 12$	**Step 2:** x is less than, $\therefore x$ is $<$.

b. Harry is at least as tall as Kerry, Kerry is 164 cm. WE2b

Working	Thinking
	Step 1: Use x to represent the unknown number.
$x \geq 164$	**Step 2:** x is at least, which indicates that it is greater than or equal to, $\therefore x$ is $\geq$.

c. Johnny ran between 25 and 40 minutes, inclusive. WE2c

Working	Thinking
	Step 1: Use x to represent the unknown number.
$25 \square x \square 40$	**Step 2:** x is between 25 and 40.
$25 \leq x \leq 40$	**Step 3:** Inclusive indicates that 25 and 40 are both included, $\therefore \leq x \leq$.

Student practice

Represent the following scenarios as an inequality, using x as the unknown quantity.

a. The product of three and an unknown number is less than ten.

b. Bobby is at least as tall as Lewis, Lewis is 155 cm.

c. Sammy ran between 40 and 55 minutes, inclusive.

Worked example 3

Solving inequalities

Solve for x.

a. $3x + 2 > 20$ WE3a

Working

$+ 2$ and $\times 3$

$3x + 2 - 2 > 20 - 2$

$3x > 18$

$\frac{3x}{3} > \frac{18}{3}$

$x > 6$

Thinking

Step 1: Identify the operations being applied to the pronumeral x. The first inverse operation will be the opposite of the last operation used to construct the inequation.

Step 2: Isolate x by applying the inverse operations.

Step 3: Simplify both sides of the inequation to solve for x.

b. $\frac{-6 + 4x}{12} \leq 8$ WE3b

Working

$\div 12$, -6 and $\times 4$

$\frac{-6 + 4x}{12} \times 12 \leq 8 \times 12$

$-6 + 4x \leq 96$

$-6 + 6 + 4x \leq 96 + 6$

$4x \leq 102$

$\frac{4x}{4} \leq \frac{102}{4}$

$x \leq 25.5$

Thinking

Step 1: Identify the operations being applied to the pronumeral x. The first inverse operation will be the opposite of the last operation used to construct the inequation.

Step 2: Isolate x by applying the inverse operations.

Step 3: Simplify both sides of the inequation to solve for x.

c. $5(2x - 4) \geq 40$ WE3c

Working

$\times 5$, -4 and $\times 2$

$\frac{5(2x - 4)}{5} \geq \frac{40}{5}$

$2x - 4 \geq 8$

$2x - 4 + 4 \geq 8 + 4$

$2x \geq 12$

$\frac{2x}{2} \geq \frac{12}{2}$

$x \geq 6$

Thinking

Step 1: Identify the operations being applied to the pronumeral x. The first inverse operation will be the opposite of the last operation used to construct the inequation.

Step 2: Isolate x by applying the inverse operations.

Step 3: Simplify both sides of the inequation to solve for x.

Student practice

Solve for x.

a. $4x + 8 > 32$ **b.** $\frac{-5 + 15x}{10} \leq 10$ **c.** $6(5x - 9) < 36$

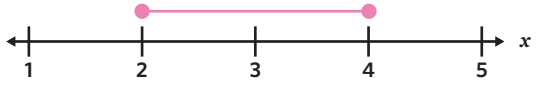
6E Questions

Understanding worksheet

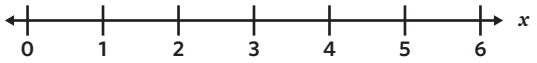
1. Represent the inequality statement on the number line given.

Example

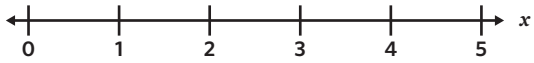
$2 \leq x \leq 4$



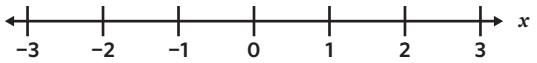
a. $0 < x < 5$



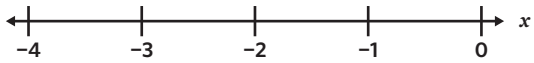
b. $1 \leq x \leq 3$



c. $-2 \leq x < 3$



d. $-3 \leq x < -1$



2. Determine what inequality symbol is listed in each statement.

Example

	Greater than	Greater than or equal to	Less than	Less than or equal to
$x > 12$	☑	☐	☐	☐

	Greater than	Greater than or equal to	Less than	Less than or equal to
a. $x \geq 6$	☐	☐	☐	☐
b. $x < 8$	☐	☐	☐	☐
c. $x \leq 24$	☐	☐	☐	☐
d. $x > 13$	☐	☐	☐	☐

3. Fill in the blanks by using the words provided.

included | number line | inequality | symbols

Inequalities can be represented using different _____. An inequation such as $4x \leq 2$ is an _____ statement. When displaying an inequality using a _____, a closed circle is used to represent when a value is _____, whereas an open circle is used to represent when a value is not included.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Represent the inequalities on a number line. WE1a,b

a. $x > 2$ **b.** $x \leq 5$ **c.** $x < 3$ **d.** $x \geq 4$

e. $6 > x$ **f.** $4 \leq x$ **g.** $x < -5$ **h.** $x \geq -3$

5. Represent the inequalities on a number line. WE1c

a. $4 < x < 6$ **b.** $2 \leq x \leq 7$ **c.** $9 < x < 12$ **d.** $5 \leq x < 10$

e. $1 < x \leq 5$ **f.** $-4 \leq x \leq 2$ **g.** $-7 \leq x \leq -3$ **h.** $-3 \leq x < 3$

6. Represent the following scenarios as an inequality, using x as the unknown quantity. WE2a,b

a. Five is added to an unknown and the result is greater than ten.

b. Three is subtracted from an unknown number and the result is less than five.

c. The sum of an unknown number and six is less than or equal to eleven.

d. The temperature on a particular Tuesday is less than 14°C.

e. Claire is more than twenty seven years old.

f. On Wednesday night, Harry spent less than or equal to forty five minutes on TikTok.

g. The temperature on Wednesday is between −10°C and 4°C inclusive.

h. John's age is between 28 and 34 exclusive.

7. Solve for x. WE3a,b

a. $x + 3 < 9$ **b.** $x - 3 > 7$ **c.** $5x \geq 15$ **d.** $\frac{x}{3} \geq 4$

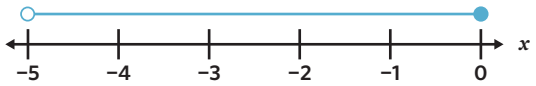
e. $3x - 5 > 16$ **f.** $4x + 8 < 32$ **g.** $9 \leq 2x + 1$ **h.** $68 \geq 5x - 2$

8. Solve for x. WE3c

a. $\frac{x}{2} + 1 \leq 5$ **b.** $\frac{x}{9} - 6 > 4$ **c.** $\frac{x+3}{2} > 6$ **d.** $\frac{x-5}{4} \geq 2$

e. $12(x + 4) \leq 36$ **f.** $6(x + 8) < 42$ **g.** $2(5x + 1) \leq 5$ **h.** $3(2x + 7) \leq 9$

9. State the inequality displayed on the number line.



A. $0 < x < 5$ **B.** $0 < x \leq -5$ **C.** $-5 < x \leq 0$ **D.** $-5 \leq x \leq 0$ **E.** $x < -5$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Represent the following scenarios as an inequality, using x as the unknown quantity.

x is greater than or equal to -8, and less than 3.

Student A

$x =$ unknown number

$-8 \square x \square 3$

$-8 \leq x < 3$

Student B

$x =$ unknown number

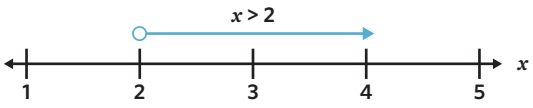
$-8 \square x \square 3$

$-8 < x < 3$

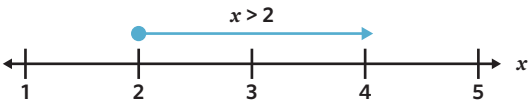
b. Show these inequalities on a number line.

$x > 2$

Student A



Student B



Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Ingrid buys a new set of airpods for less than \$350. Write this as an inequality where the price of the airpods is represented as p.

12. The price of a new electric scooter (s) is \$2500 or more. Write this as an inequality.

13. Form an inequality in terms of s. Julia has saved \$150. She went online and purchased four pairs of shoes and a dress. The dress cost \$45. Each pair of shoes cost \$$s$.

14. Mike has \$600 in a savings account at the beginning of winter. He wants to have at least \$900 in the account by the end of winter. He adds \$30 each week. Solve and calculate how many weeks Mike is expected to add \$30 into his account in order to reach his goal.

15. Dylan and Brittany are twins. It is known that Dylan is between 17 and 22, inclusive and Brittany is between 19 and 24, exclusive. Generate two inequalities on the same number line, for both Dylan and Brittany to evaluate what the range of their age is.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Jessica is going on a holiday and is calculating whether it would be cheaper to use Uber or Yellow Taxi. Uber charges \$2.50 flat rate and an additional \$1.50 per kilometre (k). Yellow Taxi charges \$3.00 flat rate and an additional \$1.00 per kilometre (k). Jessica has no more than \$500 to spend.

a. Write an inequality to represent Uber as an option.

b. Write an inequality to represent Yellow Taxi as an option.

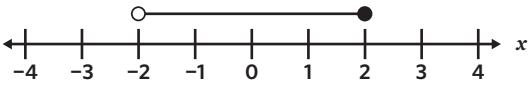
c. How many kilometres can Jessica travel with Uber without exceeding her budget?

d. How many more kilometres can Jessica expect to travel using Yellow Taxi compared to Uber?

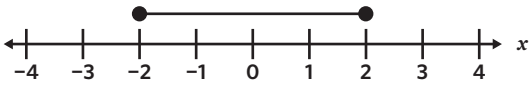
e. List two physical activity options Jessica could use during her holiday.

17. In part **a** and **b** state the inequalities from the given number lines.

a.



b.



c. Compare the inequalities in both parts **a** and **b**. Note their similarities and differences.

Extra spicy

18. Solve the following inequality.

$4x + 3 > 2x + 11$

19. Which two whole numbers can be substituted into the inequality statement.

$-2 < x \leq 3$

A. $-2.5, -2$ **B.** $-1, -3$ **C.** $-2, -1$ **D.** $-1, 3$ **E.** $0, 4$

20. x is a prime number and also satisfies $7 < 2x - 3 \leq 25$. List the three possible values of x.

21. The sides of a triangle have lengths of 7.5 cm and 11 cm and x, where x is a whole number. What is the smallest possible value of x?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

Remember this?

22. How many degrees does the minute hand turn in 45 minutes?

A. $\frac{3}{4}$ **B.** 45° **C.** 90° **D.** 180° **E.** 270°

23. Chirag has a bag of Skittles. 20% of the Skittles are yellow. He takes a purple Skittle from the bag and eats it. Without looking, he takes another Skittle from the bag.

What is the chance the skittle is yellow?

A. Equal to 20%

B. Less than 20%

C. Greater than 20%

D. Greater than 30%

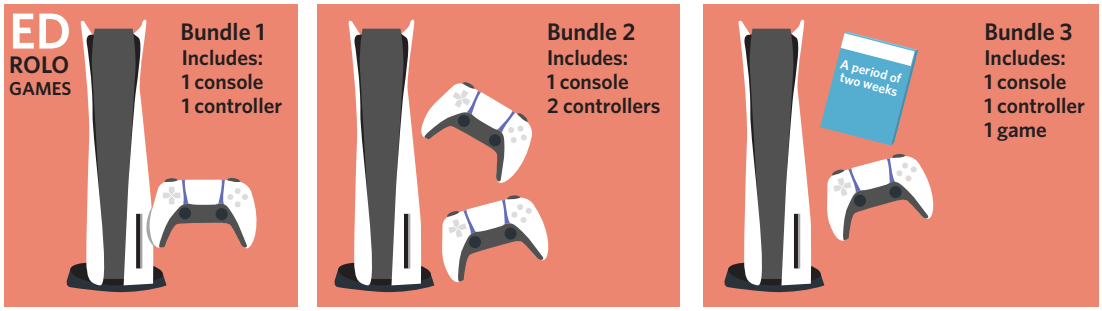
E. No chance

24. Simplify $\frac{4}{7} + \frac{2}{3}$.

A. $\frac{3}{5}$ **B.** $\frac{2}{7}$ **C.** $\frac{6}{10}$ **D.** $\frac{6}{21}$ **E.** $1\frac{5}{21}$

Chapter 6 extended application

1. An electronics store has a number of bundles available for purchase. The contents of each bundle are outlined below.



a. Bundle 2 has a value of $850. Write an equation for the value of the bundle, using x to represent the value of the console and y to represent the value of one controller.

b. The retail value of one controller is $79. Determine the value of one console.

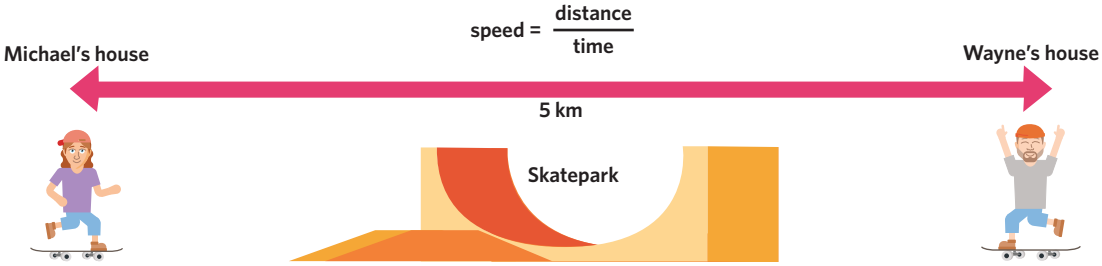
c. Determine the value of Bundle 1.

d. Determine the value of the game in Bundle 3 if its total price is $849.95.

e. Bundle 4 has been added and it includes a console, one game, and two controllers. This new bundle has a special price of $859.90. Calculate the amount of money saved by buying the bundle instead of each included item individually.

f. Shops often advertise items at discounted prices. Considering that all successful businesses must make a profit, suggest a way that they can afford to lower prices and continue to operate.

2. Michael and Wayne decide to meet at the skatepark. They live 5 km away from each other and the skatepark is located between their houses. Use the information provided in the diagram to answer the questions below.



a. It takes Michael 30 minutes to skate from his house to Wayne's without making any stops. What is Michael's skating speed, in km/h?

b. Michael leaves his house and gets to the skatepark in 12 minutes. How far away does Wayne live from the skatepark, in km?

c. Wayne's skating speed is 1 km slower per hour than Michael's. How long will it take Wayne to get to the skatepark from his house, in minutes?

d. How long does it take Wayne to skate to Michael's house without making any stops, to the nearest minute?

e. Michael and Wayne both leave their houses and skate towards each other at their normal speeds. They meet exactly in the middle between their houses. How much earlier did Wayne have to leave in order to meet Michael on time? Round your answer to the nearest minute.

f. Skateparks are generally communal places open to the public at no cost. How can the local community ensure that these kinds of spaces are looked after and able to be enjoyed by everyone?

3. A group of 20, consisting of adults, elderly people, and children go to a school fair. They all pay a total of $20 to attend and there are five elderly people in the group. Use the information provided in the diagram to answer the following questions.

a. Write an equation to represent the total number of people in the group, using x for the number of adults and y for the number of children.

b. Write an equation to represent the total amount of money paid by the group to attend the fair.

c. How much of the total amount paid can be attributed to the adults and children?

d. Write the number of adults in the group as an inequality.

e. Determine the number of adults and children in the group.

f. Tickets to events are often priced differently based on the age of the customer. Suggest a possible reason why this might be the case.

Chapter 6 review

Multiple choice

1. Which of the following equations is correctly balanced? 6A

A. $3 + 9 = 4 + 10$

B. $5 = 2 \times 3 - 1$

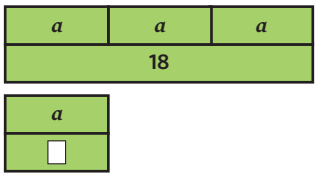
C. $77 = 7 \times 7$

D. $6 - (-4) = 2$

E. $4 - 3 = 3 - 4$

---

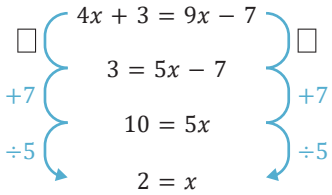
2. Using the bar model, solve for a. 6B



A. $a = 6$ **B.** $a = 7$ **C.** $a = 8$ **D.** $a = 9$ **E.** $a = 10$

---

3. Which of the following complete the missing boxes to show how this equation is solved? 6C

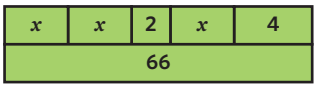


A. $+4x, +4x$ **B.** $-4x, -4x$ **C.** $\div 4x, \div 4x$ **D.** $\times 4x, \times 4x$ **E.** $+3, +3$

---

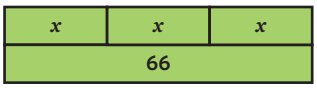
4. Which bar model matches the following scenario: 6D
Three friends are comparing their ages. The oldest of them is two years older than the middle one. The middle one is two years older than the youngest one, who is x years old. Their age sums to 66.

A.

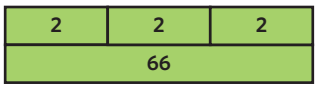


B.

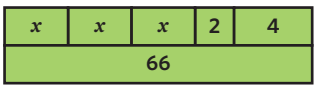
C.



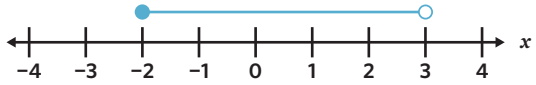
D.

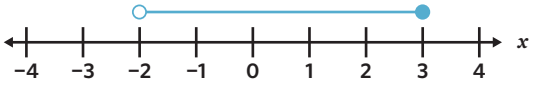


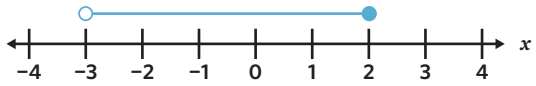
E.

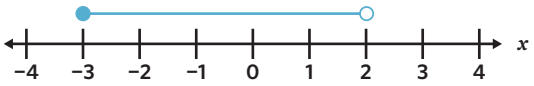


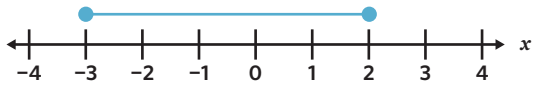
5. Which of the following correctly represents the inequality $-3 \leq x < 2$ statement on the number line? 6E

A.


B.


C.


D.


E.


Fluency

6. Determine which value of x is required to make each equation balanced. 6A

$x = 1, x = 8, x = 10$

a. $x - 7 = 1$ **b.** $x + 2 = 3x$ **c.** $3x + 8 = 48 - x$ **d.** $x - 10 = 20 - 2x$

---

7. Solve. 6B

a. $11 = x + 6$ **b.** $2y - 1 = 3$ **c.** $4 + \frac{2k}{5} = 14$ **d.** $24 = 6(8x - 4)$

---

8. Solve. 6B

a. $\frac{1 + u}{6} = 3$ **b.** $\frac{3(x + 10)}{8} = 6$ **c.** $-4 = \frac{16(a - 1)}{4}$ **d.** $\frac{2(3b - 1)}{8} = \frac{2}{4}$

---

9. Solve each equation for the given variable. 6C

a. $5k + 1 = k + 13$ **b.** $5j - 5 = 2j + 7$ **c.** $-x - 4 = 2 - 2x$ **d.** $z = 3 - \frac{z}{2}$

---

10. Solve each equation for the given variable. 6C

a. $3a = 7(a - 4)$ **b.** $6b = 3(b + 3)$

c. $5(2c + 1) = 35 - 5c$ **d.** $2(8d - 10) = -4(4d - 3)$

---

11. For each scenario: 6D

i. Choose a variable to represent the unknown quantity.
ii. Construct an equation to solve.
iii. Solve the equation for the unknown.
iv. State the answer.

a. Vicky buys four pencils for $12. How much was each pencil?

b. CJ spent a third of his weekly pay on a $120 coat. How much does CJ get paid per week?

c. Madison spent eight hours in total watching anime and drawing. If she spent three more hours watching anime than drawing, how long did she draw for?

d. Clement is adding up all the marks he received over the semester in maths. He got 40 marks per test in the first few topics, and in the last two topics got 45 marks. If he got 210 marks overall, how many tests did he score 40 marks in?

---

12. Represent the inequalities on a number line. 6E

a. $x < 5$ **b.** $4 \geq x$ **c.** $-1 < x < 4$ **d.** $-8 \leq x \leq -1$

13. Solve for x. 6E

a. $x - 1 > 8$ **b.** $3x \geq 18$ **c.** $\frac{x}{5} - 4 > 6$ **d.** $4(x + 9) \geq 48$

Problem solving

14. Mandy spends \$200 on x books. Each book costs \$16. Write an equation to describe this situation. 6A

15. Damien bought some cupcakes from a local bakery to share with his x students. Every student receives three cupcakes. Damien found that there were six leftover cupcakes. If there are 30 cupcakes in total, how many students does Damien have? 6B

16. Daniel did $2t$ push-ups on one day, and on the next day Daniel did four push-ups. Tasha did t push-ups each day for 4 days. Overall, Daniel and Tasha did the same number of push-ups. How many push-ups does t represent? 6C

17. David spent \$33 dollars in total to prepare for his party. He purchases a \$25 jumbo pack of lollies, then purchases mesh bags to divide up the lollies so that each of his guests gets one mesh bag. The cost for each mesh bag is 50 cents. How many guests are there at the party? 6D

18. Vutha and Jayden are stacking lego blocks. It is known that Jayden has between 90 and 95 lego blocks, inclusive, and Vutha has between 87 and 93 lego blocks, exclusive. Generate two inequalities on the same number line, for both Vutha and Jayden, to evaluate what the range of their number of lego blocks are. 6E

Reasoning

19. James and Bethany are planning a fundraising raffle for a school fete. During the school fete, students will enter competitions to win either first or second prize. James and Bethany have a budget of \$300, but they have different proposals in terms of how to spend the money.

James' proposal	Bethany's proposal
Spend the entire budget on prize money. First prize is \$3j. Second prize is \$2j.	Spend \$$b$ on advertising. First prize is \$2b. Second prize is \$$b$.

a. Write an equation to represent James' proposal.

b. Solve the equation from part **a** to determine how much money the first prize winner receives under James' proposal.

c. For Bethany's proposal, write an equation to show that:

Prize money = budget money − advertising money

d. Solve the equation from part **c** to determine how much money Bethany proposes to spend on advertising.

e. Ms Muscat thinks the budget is not enough. She proposes a budget that is greater than \$300, exclusive, and smaller than \$450, inclusive. Represent this inequality on a number line.

f. Do you think James or Bethany's proposal is better? Explain your reasoning.

20. Renae is solving the following inequalities in class. For parts **a** and **b**:

a. solve $7y < 21$. Represent your solution on a number line.

b. solve $21 < y + 2$. Represent your solution on a number line.

c. Compare your number lines in parts **a** and **b**. Explain why both inequalities cannot be true at the same time.

Back to contents

07

Congruence and transformations

Measurement and Geometry

Research summary

7A Angles and parallel lines *(Revision)*

7B Triangles and quadrilaterals

7C Transformations on the Cartesian plane

7D Congruent 2D shapes

7E Rules for congruent triangles

7F Congruence and quadrilaterals

Chapter 7 extended application

Chapter 7 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 7 research summary

Congruence and transformations

Big ideas

Hierarchy and properties

This chapter will focus on the properties and classifications of angles, parallel lines, and the properties of quadrilaterals. Additionally this chapter will focus on the properties of congruent 2-D shapes, quadrilaterals, and triangles and how to prove congruence for quadrilaterals and triangles.

It is important to understand that angles can have multiple definitions and the views of angles can hinder students' conceptual understanding of angles and future topics of geometry such as right angles and trigonometry (Mullins, 2020; Biber et al., 2013). An angle can be defined as a relation, a quantity, or a quality and angles can be dynamic and static (Mullins, 2020: Tallman & Frank, 2018). One way to help students develop their understanding of angles, is to show angles using multiple representations (Keiser, 2004).

To fully understand how to find missing angles, students must combine their skills for memorization and understanding of arbitrary and necessary things (Salim, 2019). Arbitrary things are concepts in the realm of students' memory, while necessary things are in the students' realm of awareness (Salim, 2019).

Students can have difficulty memorising, creating, and implementing the hierarchical properties of shapes when they do not look like the 'typical' example (Sinclair et al., 2016). For example, parallelograms may be overlooked because the shape is not slanted.

One way to improve student's hierarchical definitions is to give students the opportunity to create definitions and properties of triangles and quadrilaterals and discuss what are not properties of the shapes (Sinclair et al., 2016; Fujita et al., 2019). It is important that students are exposed to less familiar quadrilaterals so they can understand the need for explicit and sufficient definitions (Sinclair et al., 2016). For example, have students define an isosceles trapezium.

Congruence is a major learning curve for students in this grade (Sinclair et al., 2016; Winer & Battista, 2022). Students can have difficulty in recognising what appears to be true, what justifications they can use based on reason and the various forms of mathematical reasoning such as explanation, argument, verification, and proof, and devising a plan to the proof process (Herbst, 2002; Jones 2000; Wang et al., 2018).

However, students can form logical arguments (Winer & Battista, 2022) and when given the opportunities and appropriate strategies students can develop their skill in constructing and formalising proofs (Ding et al., 2015).

One strategy to improve students' understanding of congruence is to recognise the importance of language in geometry (Ding et al., 2015; Uyen et al., 2021). For example, have students practise how to correctly translate words of a theorem to the symbols and diagrams. It is crucial that students are provided time to analyse the structure of proof to develop their analytic thinking and to transform their logical arguments to formulising their own proofs (Ding et al., 2015; Uyen et al., 2021; Winer & Battista, 2022). For example, have students start to think about 'what are we proving' and 'what is given' to reason about what is given, and make connections between theorems and definitions and the structure of writing proofs (Ding et al., 2015).

The hierarchy and properties of angles, lines, congruence, and 2D shapes is essential for further topics in mathematics such as trigonometry and hierarchy, properties and measurement of 3D figures.

Transformations of relationships

This chapter will include transformations of shapes on the Cartesian plane to support the ideas congruence. Students will view objects from different perspectives and will transform objects on a Cartesian plane. Transformations of shapes are necessary for spatial thinking in geometry.

Spatial thinking is essential for thinking about how 2-D and 3-D shapes and objects move, change, interact, and is a core concept in geometry (Bruce & Hawes, 2014). For example, interlocking cubes, tangrams, and pattern blocks can be used to create and physically rotate shapes.

Transformation of relationships and spatial thinking is necessary for geometry and a predictor of mathematics achievement at all grade levels (Bruce & Hawes, 2014). Spatial thinking underpins further ideas in mathematics such as scaling objects, transformations of functions, and area and volume of figures.

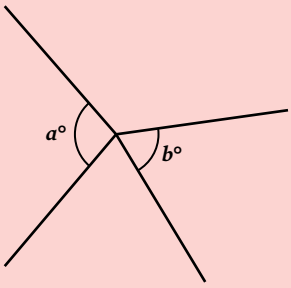
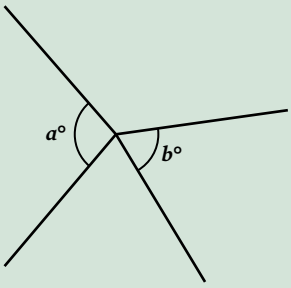
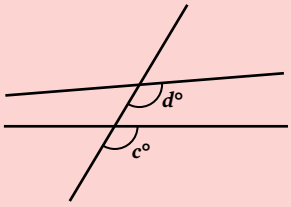
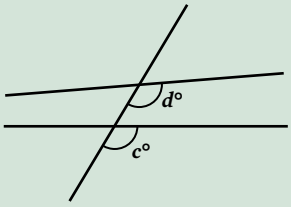
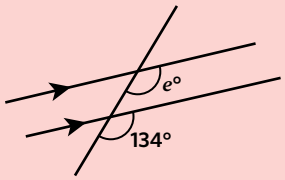
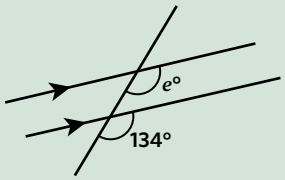
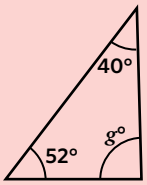
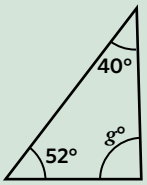
Geometric measurement

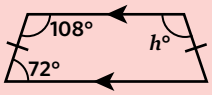
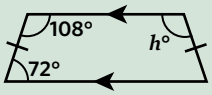
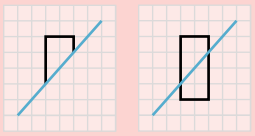
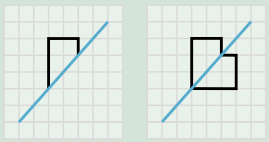
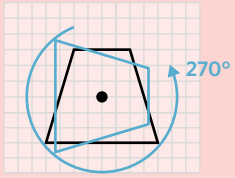
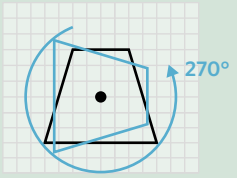
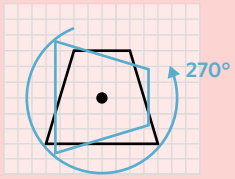
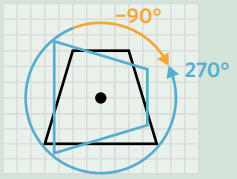
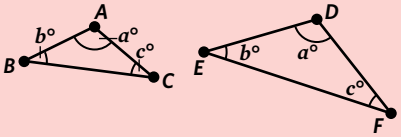
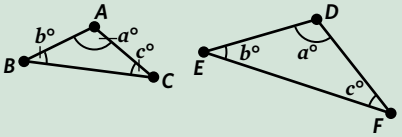
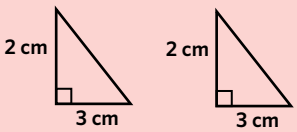
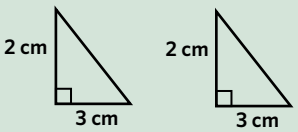
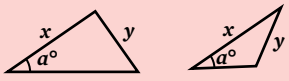
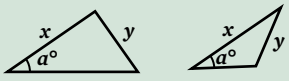
This chapter will include key ideas around measurement of length to determine congruence of shapes, and transformation of shapes on a Cartesian plane.

It is crucial that there is consistency in explaining angles depending on the representation of angles. When students are working with angles particularly with circles, it is essential that quantitative reasoning is applied rather than portraying the angle as an arithmetic measurement or non-quantitative measurement.

Students who have a strong understanding of the multiple representations of geometric measurement, spatial thinking, and mental rotation of shapes will find it beneficial for proving symmetry, understanding area measurement tasks, creating and decomposing 2D and 3D figures (Bruce & Hawes, 2014).

Misconceptions

Misconception	Incorrect	Correct	Lesson
Students identify vertically opposite angles when four different lines meet at a point.	 $\angle a = \angle b$	 $\angle a \neq \angle b$	7A
Students apply the angle properties of parallel lines when the lines are not parallel.	 $\angle c$ and $\angle d$ are equal corresponding angles.	 $\angle c$ and $\angle d$ are not equal corresponding angles.	7A
Students think that alternate or corresponding angles in parallel lines sum to 180°.	 $\angle e + 134° = 180°$ $\angle e = 46°$	 $\angle e = 134°$	7A
Students identify the sizes of the angles based on their appearance.	 $\angle g = 90°$ because the angle looks like a right angle.	 $\angle g + 52° + 40° = 180°$ $\angle g = 88°$	7B

Misconception	Incorrect	Correct	Lesson
Students think that the sum of interior angles of all shapes does not change with the number of sides.	The sum of interior angles of both triangles and quadrilaterals is 180°.	The sum of interior angles of triangles is 180° and quadrilaterals is 360°.	7B
Students believe that opposite angles in quadrilaterals are always equal because opposite angles in parallelograms are equal.	 $\angle h = 72°$, because the angles are opposite to each other.	 $\angle h = 108°$, because they are adjacent angles.	7B
Students incorrectly draw a reflection when the mirror line is not horizontal or vertical but the object is horizontal or vertical.			7C
Students misinterpret Cartesian plane coordinates and confuse horizontal and vertical movements of a translation.	[8,7] is 8 units up and 7 units right.	[8,7] is 8 units right and 7 units up.	7C
Students reverse clockwise and anticlockwise direction.	 Clockwise	 Anticlockwise	7C
Students think there is only one rotation size and angle for a given rotation.	 270° clockwise is the only way to achieve this rotation.	 There are multiple ways to achieve this rotation. It could be achieved with either a 90° clockwise rotation or a 270° anticlockwise roation.	7C
Students think that similar figures are congruent.	 $\triangle ABC$ is congruent to $\triangle DEF$ because they have the same angle measurements.	 $\triangle ABC$ is not congruent to $\triangle DEF$ because they have different side lengths.	7D 7E
Students use RHS for all right angled triangles when SAS is appropriate.	 Triangles are congruent by RHS.	 Triangles are congruent by SAS.	7F
When using SAS, students use any angle to prove the triangles are congruent.	 Triangles are congruent by SAS, because they both have the same side length and angle measurement.	 Triangles are not congruent by SAS, because not all angles and sides are the same.	7F

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Back to contents

7A Angles and parallel lines

Geometry is the basis of humans' pursuit to understand the natural world. The foundation of geometry are straight lines and the angles they create at their intersections. Thousands of years ago ancient Greeks began to establish and write down the laws of geometry. The most important geometry book was written by Euclid of Alexandria.

LEARNING INTENTIONS

Students will be able to:

- identify complementary, supplementary and vertically opposite angles
- identify corresponding, alternate and co-interior angles
- calculate missing angles around a point or in parallel lines using angle properties.

KEY TERMS AND DEFINITIONS

Adjacent angles are angles that share a vertex and a common side.

Parallel lines never touch and are always the same distance apart.

Perpendicular lines meet at a right angle (90°).

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

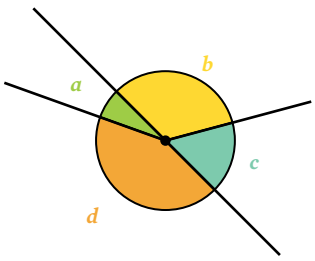
Image: metamorworks/Shutterstock.com

A number of car companies are attempting to create autonomous cars. They use angles and straight lines to determine the position and direction of other cars and objects around them.

Key ideas

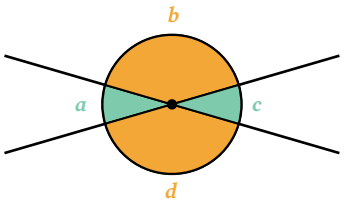
1. Angles meeting at a vertex can share special relationships.

A full **revolution** is when all angles at a point sum to 360°.



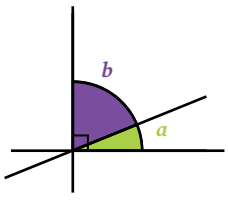
$a + b + c + d = 360°$

Vertically opposite angles are equal to each other.



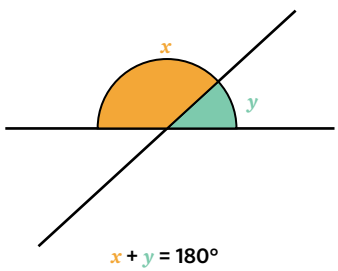
$b = d$ and $a = c$

Complementary angles sum to 90°.



$a + b = 90°$

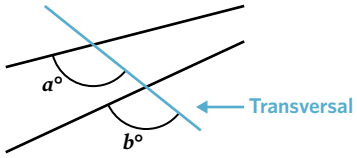
Supplementary angles sum to 180°.



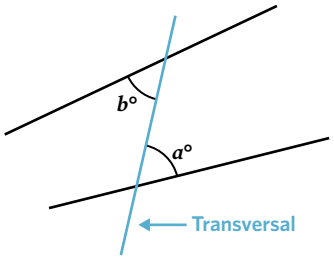
$x + y = 180°$

Continues →

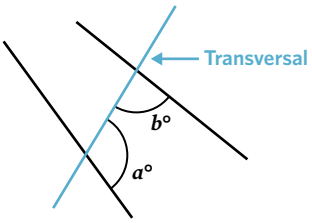
2. When a transversal line crosses two or more lines it forms pairs of angles:



Corresponding angles are in matching positions. They are on the same side of the transversal.

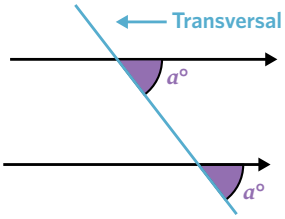


Alternate angles are on either side of the transversal. They are either both in between two straight lines, or both outside of two straight lines.

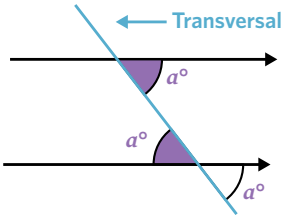


Co-interior angles are in between two straight lines and on the same side of the transversal.

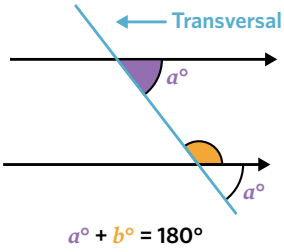
3. When a transversal crosses two **parallel lines**:



Corresponding angles must be equal.



Alternate angles are equal.



Co-interior angles sum to 180° (supplementary).

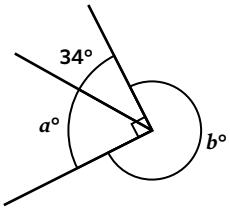
Worked example 1

Angles at a point

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a.

WE1a



Working

$a°$ and 34° are complementary angles.

$a° + 34° = 90°$

$a° = 56°$

$a°$, 34° and $b°$ form a full revolution.

$a° + 34° + b° = 360°$

$90° + b° = 360°$

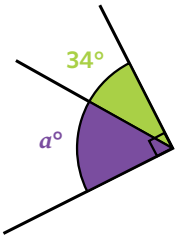
$b° = 270°$

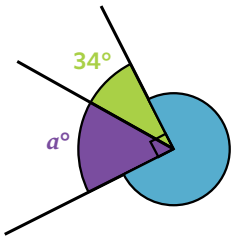
Thinking

Establish the relationships between the unknown angles and known angles, then solve.

Continues →

Visual support



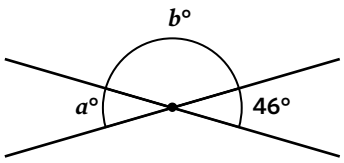


$a° + 34° = 90°$

$a° + 34° + b° = 360°$

b.

WE1b



Working

$a°$ and $46°$ are vertically opposite angles.

$a° = 46°$

$a°$ and $b°$ are supplementary angles.

$a° + b° = 180°$

$46° + b° = 180°$

$b° = 134°$

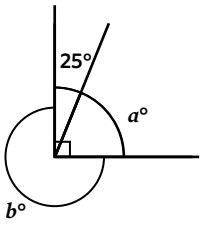
Thinking

Establish the relationships between the unknown angles and known angles, then solve.

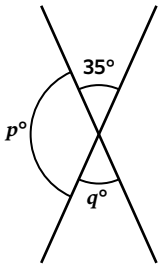
Student practice

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a.



b.

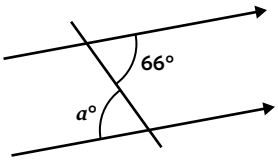


Worked example 2

Angles with transversals of parallel lines

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a. WE2a



Working

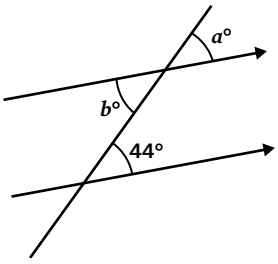
$a°$ and $66°$ are alternate angles.

$a° = 66°$

Thinking

Establish the relationships between the unknown angles and known angles, then solve.

b. WE2b



Working

$a°$ and $44°$ are corresponding angles.

$a° = 44°$

$a°$ and $b°$ are vertically opposite angles.

$b° = a°$

$b° = 44°$

or

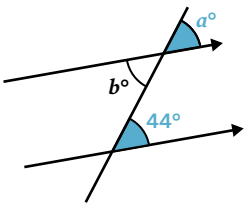
$b°$ and $44°$ are alternate angles.

$b° = 44°$

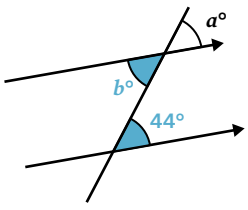
Thinking

Establish the relationships between the unknown angles and known angles, then solve.

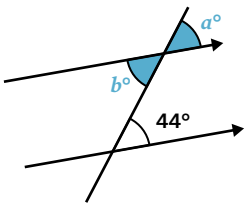
Visual Support



Corresponding



Alternate

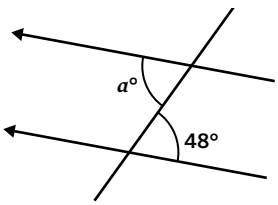


Vertically opposite

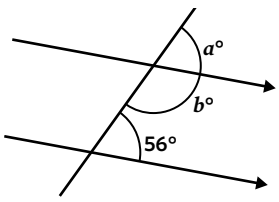
Student practice

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a.



b.

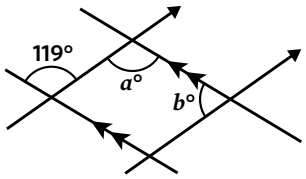


Worked example 3

Parallel lines with multiple transversals

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a. WE3a



Working

$a°$ and 119° are alternate angles.

$a° = 119°$

$a°$ and $b°$ are co-interior angles.

$a° + b° = 180°$

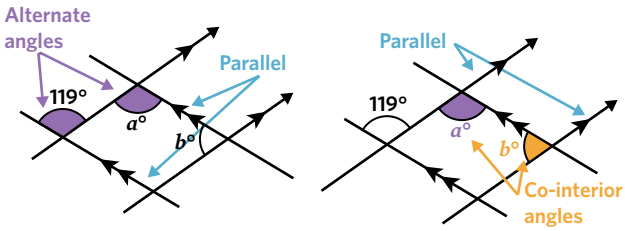
$119° + b° = 180°$

$b° = 61°$

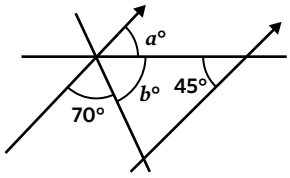
Thinking

Establish the relationships between the unknown angles and known angles, then solve.

Visual support



b. WE3b



Working

$a°$ and 45° are alternate angles.

$a° = 45°$

$a°$, $b°$ and 70° are supplementary angles.

$a° + b° + 70° = 180°$

$45° + b° + 70° = 180°$

$b° + 115° = 180°$

$b° = 65°$

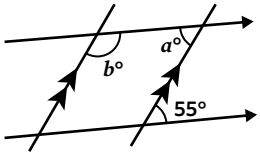
Thinking

Establish the relationships between the unknown angles and known angles, then solve.

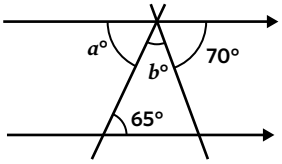
Student practice

Determine the value(s) of the pronumeral(s), with reasons, for each diagram.

a.



b.

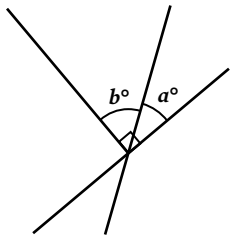


7A Questions

Understanding worksheet

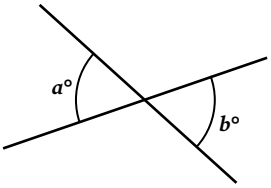
1. Circle the relationship between the angles in these diagrams.

Example



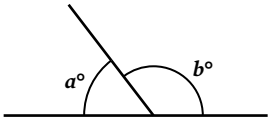
(Complementary) / Supplementary / Vertically opposite

a.



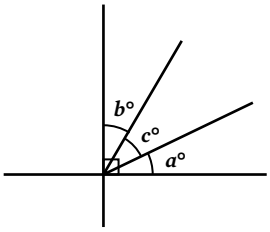
Complementary / Supplementary / Vertically opposite

b.



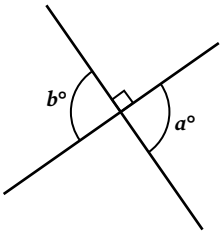
Complementary / Supplementary / Vertically opposite

c.



Complementary / Supplementary / Vertically opposite

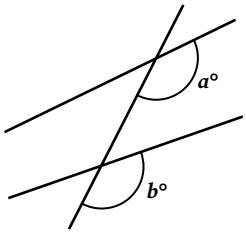
d.



Complementary / Supplementary / Vertically opposite

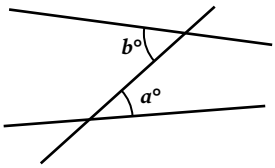
2. Circle the relationship(s) between the angles in these diagrams.

Example



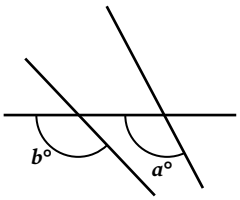
(Corresponding) / Alternate / Co-interior

a.



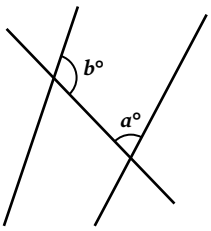
Corresponding / Alternate / Co-interior

b.



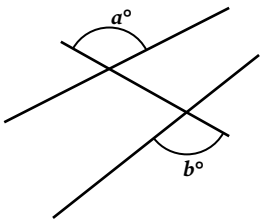
Corresponding / Alternate / Co-interior

c.



Corresponding / Alternate / Co-interior

d.



Corresponding / Alternate / Co-interior

3. Fill in the blanks by using the words provided.

supplementary | complementary | alternate | co-interior

If angles at a point sum to 90° they are _____ angles and if the angles sum to 180° they are _____ angles. If a transversal crosses two parallel lines then the corresponding angles must be the same. By combining this fact with the rules about angles at a point we can reason that _____ angles are equal and _____ angles are supplementary.

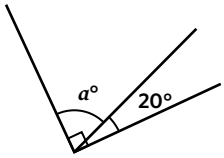
Fluency

Question working paths

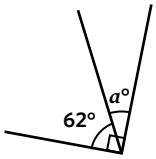
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Determine the value(s) of the pronumeral(s), with reasons, for each diagram. WE1a

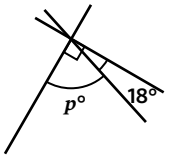
a.



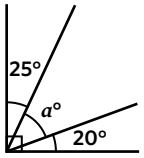
b.



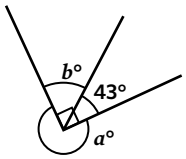
c.



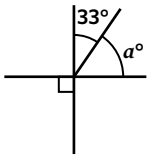
d.



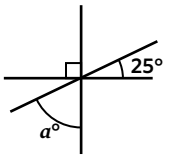
e.



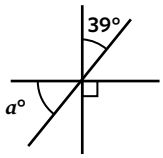
f.



g.

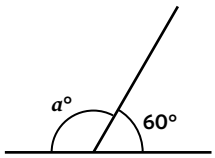


h.

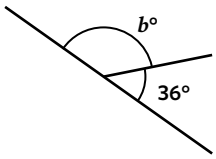


5. Determine the value(s) of the pronumeral(s), with reasons, for each diagram. WE1b

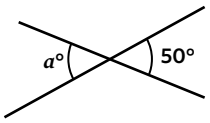
a.



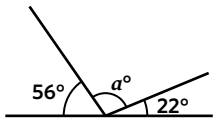
b.



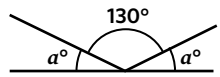
c.



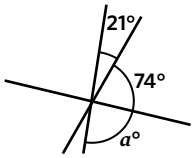
d.



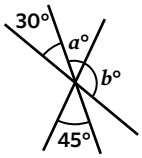
e.



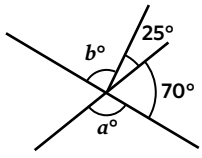
f.



g.

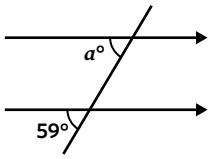


h.

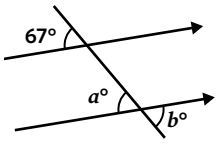


6. Determine the value(s) of the pronumeral(s), with reasons, for each diagram. Name the relationship between each angle. WE2

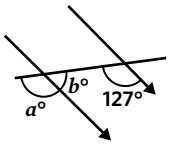
a.



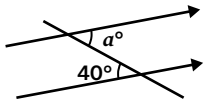
b.



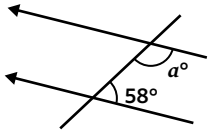
c.



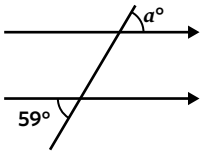
d.



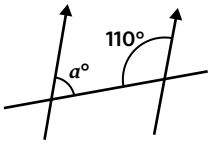
e.



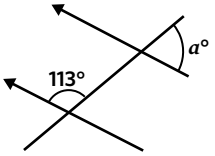
f.



g.

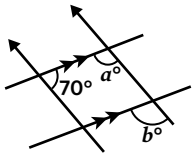


h.

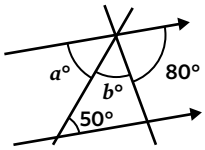


7. Determine the value(s) of the pronumeral(s), with reasons, for each diagram. WE3

a.



b.



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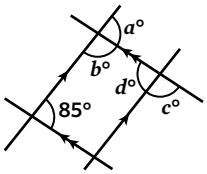

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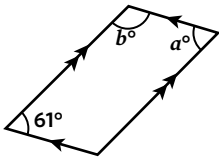

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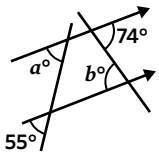
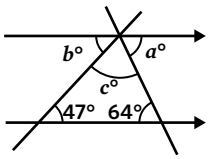
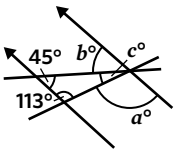


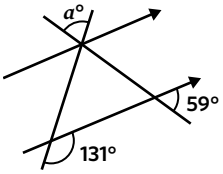
f.



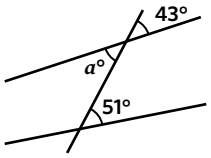
g.



h.



8. Determine the value of the pronumeral, with reasons.

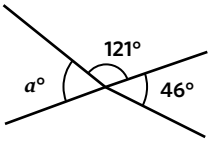


A. 43°, vertically opposite angles
B. 137°, supplementary angles
C. 51°, alternate angles
D. 129°, co-interior angles
E. 47°, complementary angles.

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the value of the pronumeral, with reasons.



Student A

$a°$ and 121° are supplementary angles:

$a° + 121° = 180°$

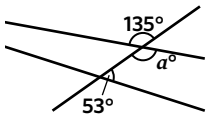
$a° = 59°$

Student B

$a°$ and 46° are vertically opposite angles:

$a° = 46°$

b. Determine the value of the pronumeral, with reasons.



Student A

$a°$ and 53° are co-interior angles:

$a° + 53° = 180°$

$a° = 127°$

Student B

$a°$ and 135° are vertically opposite angles:

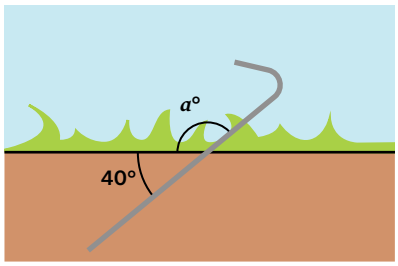
$a° = 135°$

Problem solving

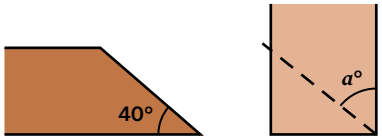
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

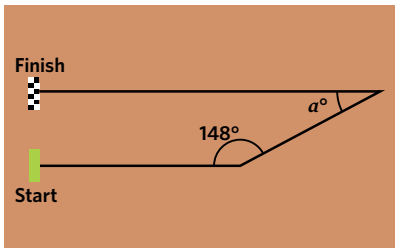
10. Nadia and Lena wanted to insert their tent pegs at an angle of 40° to the ground. What angle could they measure above the ground to ensure they had placed them correctly?



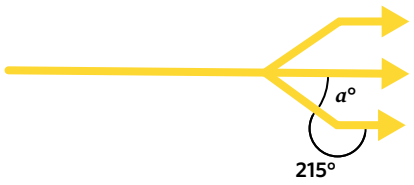
11. Kathy is renovating her house and needs to place a wedge in the door to keep it in place. What angle (a) should she cut the bottom of her door so the wedge fits perfectly.



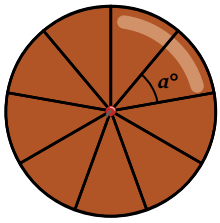
12. Mr Morrow built a two-turn track for BMX sprint racing as a lunch time activity. He wanted one easy turn and one sharp turn. If the start and finish straights were parallel to each other, and the easy turn was 148°, what was the angle of the sharp turn?



13. An ancient trident of Neptune had parallel prongs. If the outside angle was measured at 215°, what is the measurement of angle a?



14. Tony had eight friends visit for his birthday. He decided to cut the cake into nine equal slices. What angle is each slice?

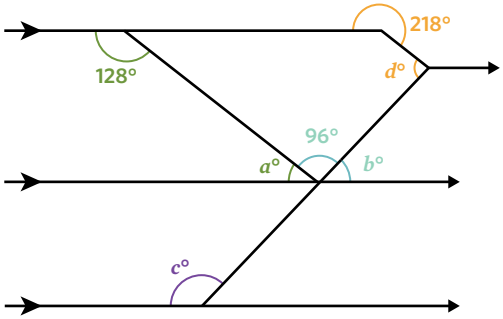


Reasoning

Question working paths

Mild 15 (a,b,c,e), 16 (a,b)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Susan is the engineering manager of a large network of gas pipes. She wants to determine the angles where all the points join to ensure that the network is compliant with safety regulations. Gas flows from left to right, parallel pipes are marked with arrows.



a. A join is measured as 128°, determine the value of the angle $a°$, explain your reasoning.

b. A join is measured as 96°, determine the value of the angle $b°$, explain your reasoning.

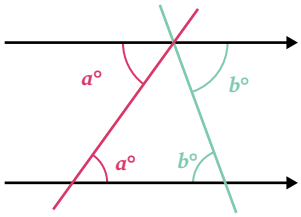
c. Determine the value of the angle $c°$, explain your reasoning.

d. A join is measured as 218°, determine the value of the angle $d°$.

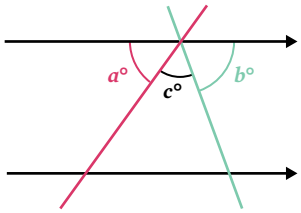
e. Human civilisation has many complex networks and systems that can be mapped schematically like this. Can you think of any networks or systems that you interact with?

16. Consider the following angles below.

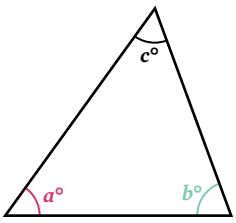
a. What geometric rule makes the two angles marked $a°$ equal, and the two angles marked $b°$ equal?



b. What is $a° + b° + c°$ equal to and what geometric rule allows us to know this?



c. Using the answers from part **a** and **b**, what can we prove about the internal angle sum of all triangles?

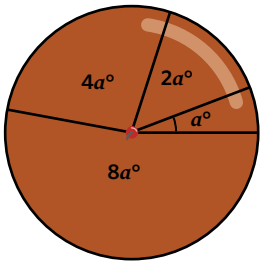


Extra spicy

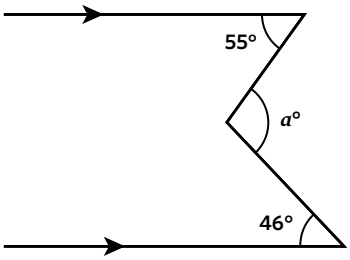
17. Two parallel lines are transversed by another line. This creates eight equal angles. What is the size of each angle?

A. 45° **B.** 90° **C.** 270° **D.** 360° **E.** 720°

18. A circular cake is cut into slices that have the angles $a°$, $2a°$, $4a°$, and $8a°$. Determine the value of $a°$.



19. Determine the value of $a°$.



20. The larger angle between the hands of a clock at 1:24 is:

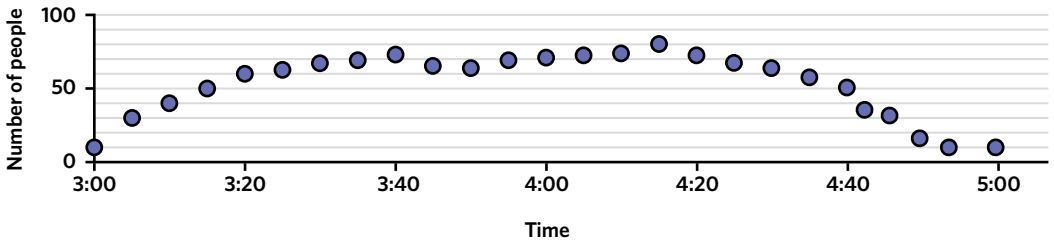
A. 102° **B.** 210° **C.** 240° **D.** 258° **E.** 317.5°

Remember this?

21. Which of these has the same value as $47 \times 3 + 5$.

A. $100 - 6$
B. $47 + 50$
C. $70 \times 2 - 3 \times 2$
D. $70 \times 2 + 6$
E. 47×5

22. The number of people at Central Station was recorded every 5 minutes. Marita graphed the data:



At what time was the greatest number of people at the station?

A. 3:40 **B.** 4:00 **C.** 4:05 **D.** 4:15 **E.** 4:20

23. Ajay paid $24.80 for eight Slinkys. What is the price of one Slinky?

A. $3.00 **B.** $3.10 **C.** $3.80 **D.** $4.10 **E.** $4.80

Back to contents

7B Triangles and quadrilaterals

Triangles are three-sided polygons that have internal angles that sum to 180°. Quadrilaterals are four-sided polygons that have internal angles that sum to 360°. We classify triangles and quadrilaterals by their angles and side lengths.

LEARNING INTENTIONS

Students will be able to:

- classify triangles using either angles or side lengths
- classify quadrilaterals using angles and side lengths
- calculate missing side lengths and angles in triangles and quadrilaterals.

KEY TERMS AND DEFINITIONS

An **acute angle** is any angle that is greater than 0° but less than 90°.

An **obtuse angle** is any angle greater than 90° but less than 180°.

To **bisect** a line or an angle is to divide it into two equal parts.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

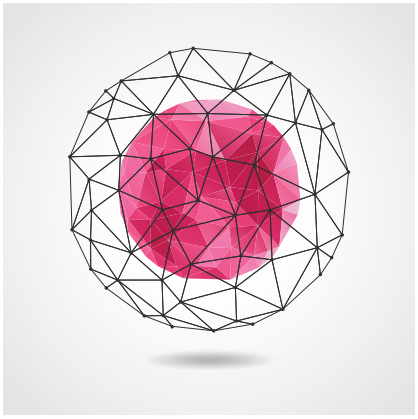


Image: Happy_Soul/Shutterstock.com

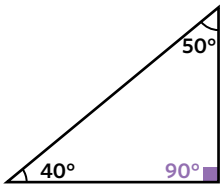
Triangles are used to build all 3D computer graphics. Any surface can be approximated with triangles for the required level of clarity.

Key ideas

1. Triangles can be classified by either angles or side lengths.

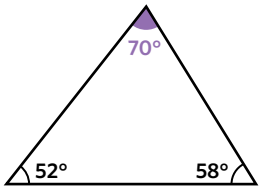
Angles:

Right-angled triangle



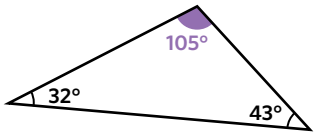
One interior angle is 90°.

Acute-angled triangle



All three interior angles are acute.

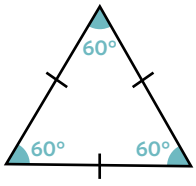
Obtuse-angled triangle



One interior angle is obtuse.

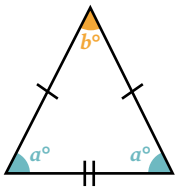
Sides:

Equilateral triangle



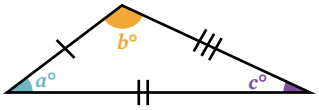
All three side lengths are equal.
All three angles are equal (60°).

Isosceles triangle



Two side lengths are equal.
Two angles are equal.

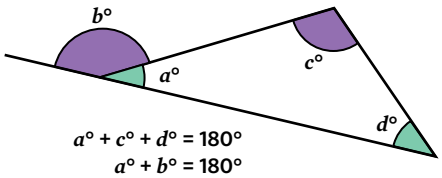
Scalene triangle



All three side lengths are different.
No angles are equal.

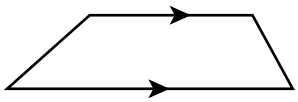
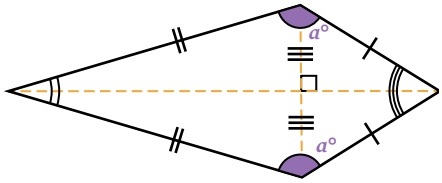
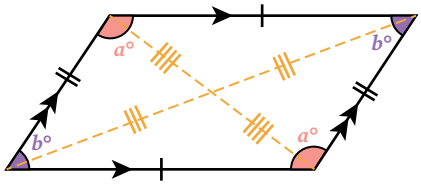
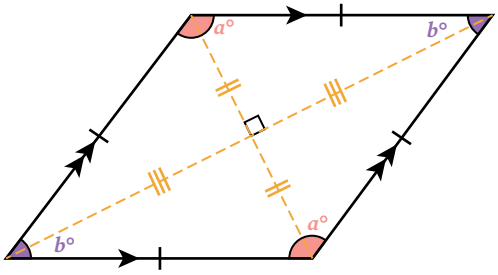
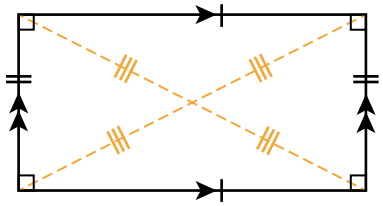
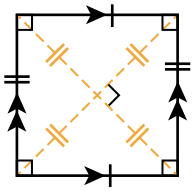
Continues →

2. The exterior angle of a triangle is equal to the two opposite interior angles.



$a° + c° + d° = 180°$
$a° + b° = 180°$
$b° = c° + d°$

3. A quadrilateral is a four-sided polygon where all the angles sum to 360°. Quadrilaterals are defined by the following conditions:

	**Trapezium:** One pair of opposite sides are parallel. Two separate pairs of internal angles add up to 180°.
	**Kite:** Two pairs of adjacent sides have lengths that are equal. One pair of opposite interior angles are equal. Diagonals are perpendicular to each other.
	**Parallelogram**: Both pairs of opposite sides are parallel. Both pairs of opposite side lengths are equal. Both pairs of opposite interior angles are equal. Adjacent interior angles are supplementary (sum to 180°). Diagonals bisect each other.
	**Rhombus:** All side lengths are equal. Both pairs of opposite interior angles are equal. Diagonals bisect each other. Diagonals are perpendicular to each other.
	**Rectangle:** Opposite side lengths are equal. All interior angles are 90°. Diagonals bisect each other. Diagonals have an equal length.
	**Square:** All side lengths are equal All interior angles are 90° Diagonals bisect each other. Diagonals are perpendicular to each other. Diagonals are equal length.

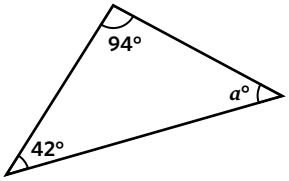
Worked example 1

Finding angles in a triangle

Find the value(s) of the pronumeral(s) in the diagrams.

a.

WE1a



Working

$a° + 42° + 94° = 180°$

$a° + 136° = 180°$

$a° = 44°$

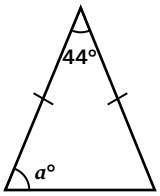
Thinking

Step 1: Determine the unknown value by making the sum of all three angles equal to 180°.

Step 2: Solve.

b.

WE1b



Working

$a° + a° + 44° = 180°$

$2a° + 44° = 180°$

$2a° - 44° = 180° - 44°$

$\frac{2a°}{2} = \frac{136°}{2}$

$a° = 68°$

Thinking

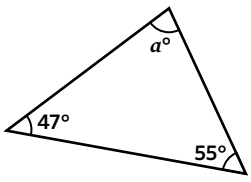
Step 1: Determine the unknown values by making the sum of all three angles equal to180°, with reference to an isosceles triangle.

Step 2: Solve.

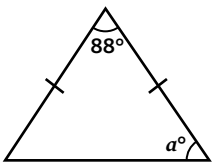
Student practice

Find the value(s) of the pronumeral(s) in the diagrams.

a.



b.

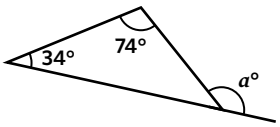


Worked example 2

Using the exterior angle theorem

Find the value(s) of the pronumeral(s) in the diagrams.

a.



WE2a

Working

$a° = 34° + 74°$

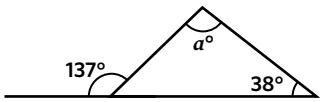
$a° = 108°$

Thinking

Step 1: Add the two interior angles to give the value of the opposite exterior angle.

Step 2: Solve.

b.



WE2b

Working

$a° + 38° = 137°$

$a° = 99°$

Thinking

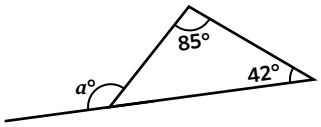
Step 1: Find the unknown interior angle by finding the difference of the opposite exterior angle and the known interior angle.

Step 2: Solve.

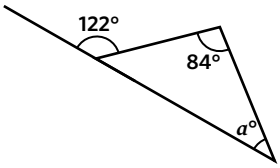
Student practice

Find the value(s) of the pronumeral(s) in the diagrams.

a.



b.

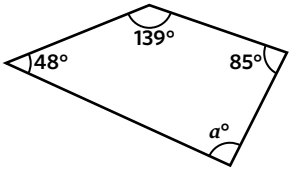


Worked example 3

Finding angles in a quadrilateral

Find the value(s) of the pronumeral(s) in the diagrams.

a.



WE3a

Working

$a° + 48° + 139° + 85° = 360°$

$a° + 272° = 360°$

$a° = 88°$

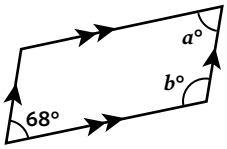
Thinking

Step 1: Determine the unknown value by making the sum of all four interior angles equal to 360°.

Step 2: Solve.

Continues →

b.



WE3b

Working

$a° = 68°$

$b° + 68° = 180°$

$b° = 112°$

Thinking

Step 1: Opposite angles in a parallelogram are equal.

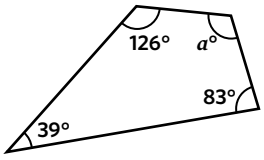
Step 2: Adjacent angles in a parallelogram sum to 180°.

Step 3: Solve.

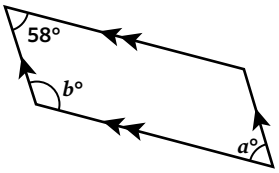
Student practice

Find the value(s) of the pronumeral(s) in the diagrams.

a.



b.

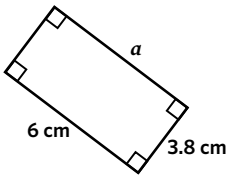


Worked example 4

Finding unknown lengths in special quadrilaterals

Find the value(s) of the pronumeral(s) in the diagrams.

a.



WE4a

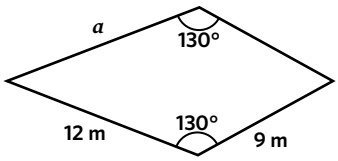
Working

$a = 6$ cm

Thinking

Opposite sides of a rectangle are parallel and have lengths that are equal.

b.



WE4b

Working

$a = 12$ cm

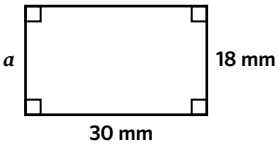
Thinking

Adjacent side lengths of a kite are equal.

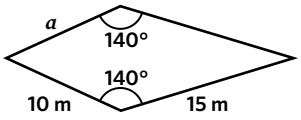
Student practice

Find the value(s) of the pronumeral(s) in the diagrams.

a.



b.

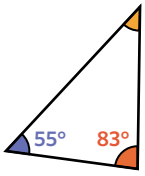


7B Questions

Understanding worksheet

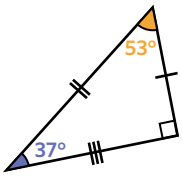
1. Circle all of the types of triangles the following could be classified as.

Example



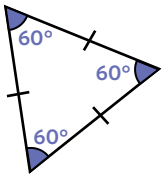
Equilateral Acute
Isosceles Right
Scalene Obtuse

a.



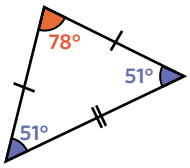
Equilateral Acute
Isosceles Right
Scalene Obtuse

b.



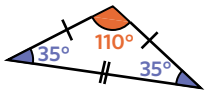
Equilateral Acute
Isosceles Right
Scalene Obtuse

c.



Equilateral Acute
Isosceles Right
Scalene Obtuse

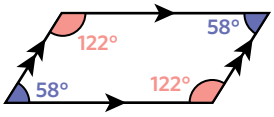
d.



Equilateral Acute
Isosceles Right
Scalene Obtuse

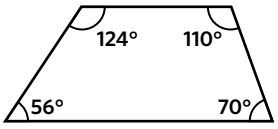
2. Circle all of the shapes the following diagrams could be classified as.

Example



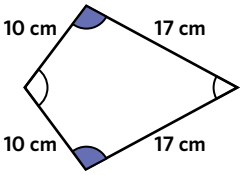
Quadrilateral Kite
Trapezium Rhombus
Parallelogram Square
Rectangle

a.



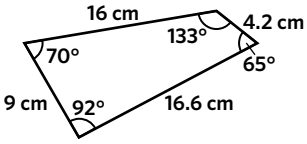
Quadrilateral Kite
Trapezium Rhombus
Parallelogram Square
Rectangle

b.



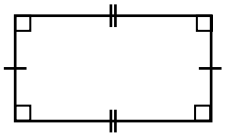
Quadrilateral Kite
Trapezium Rhombus
Parallelogram Square
Rectangle

c.



Quadrilateral Kite
Trapezium Rhombus
Parallelogram Square
Rectangle

d.



Quadrilateral Kite
Trapezium Rhombus
Parallelogram Square
Rectangle

3. Fill in the blanks by using the words provided.

parallelogram | rhombuses | internal angles | side lengths

When triangles are classified by their _____ they can be called equilateral, isosceles, or scalene.

If they are classified by their _____ they are called right-angled, acute, or obtuse.

Quadrilateral classifications share many characteristics. For example rectangles, _____, or squares could also be classified as a _____.

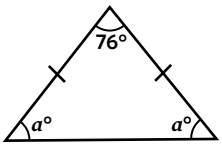
Fluency

Question working paths

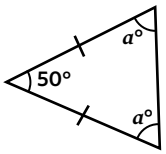
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Find the value(s) of the pronumeral(s) in the diagrams. WE1

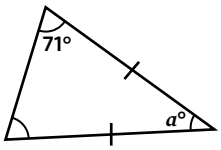
a.



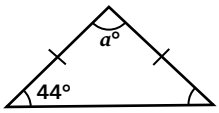
b.



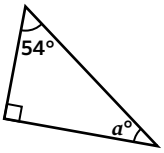
c.



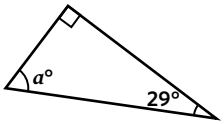
d.



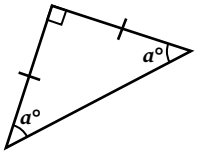
e.



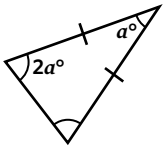
f.



g.

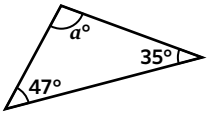


h.

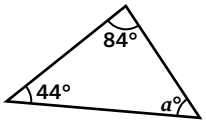


5. Find the value(s) of the pronumeral(s) in the diagrams. WE1a,3a

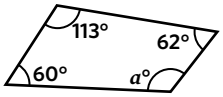
a.



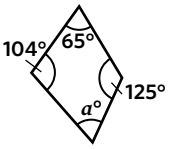
b.



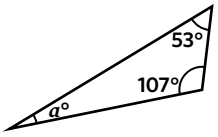
c.



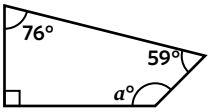
d.



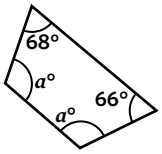
e.



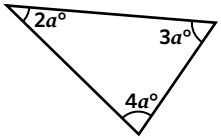
f.



g.

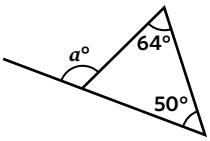


h.

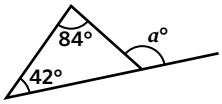


6. Find the value(s) of the pronumeral(s) in the diagrams. WE2

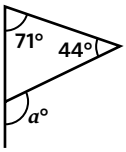
a.



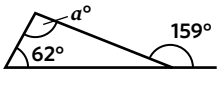
b.



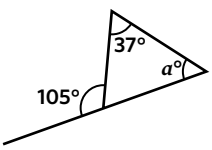
c.



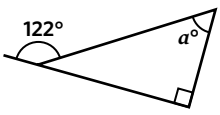
d.



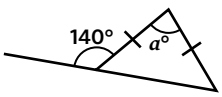
e.



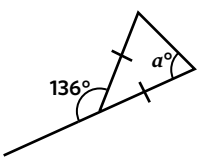
f.



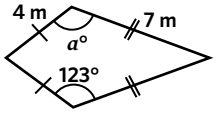
g.

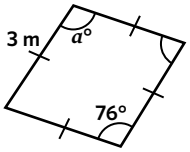


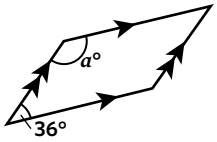
h.

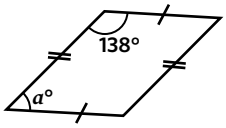


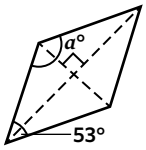
7. Find the value(s) of the pronumeral(s) in the diagrams. WE3b

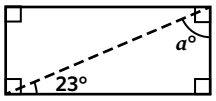
a. 

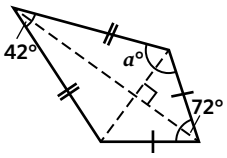
b. 

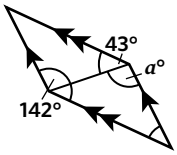
c. 

d. 

e. 

f. 

g. 

h. 

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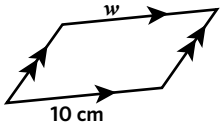
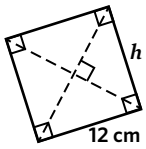
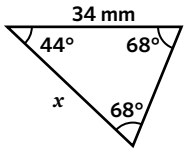
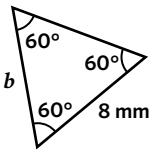
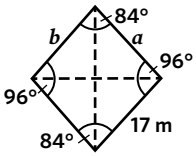
8. Find the value(s) of the pronumeral(s) in the diagrams. WE4

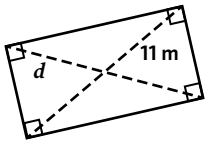
a. 

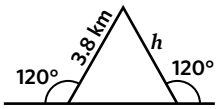
b. 

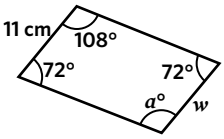
c. 

d. 

e. 

f. 

g. 

h. 

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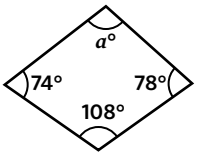
9. Find the value of the pronumeral in the diagram.

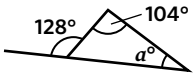


A. 98° **B.** 100° **C.** 102° **D.** 106° **E.** 108°

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Find the value of the pronumeral in the diagram.



Student A

$a° + 128° = 180°$

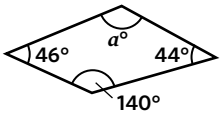
$a° = 52°$

Student B

$a° + 104° = 128°$

$a° = 24°$

b. Find the value of the pronumeral in the diagram.



Student A

$a°$ and $140°$ are opposite interior angles.

$a° = 140°$

Student B

$a° + 46° + 140° + 44° = 360°$

$a° + 230° = 360°$

$a° = 130°$

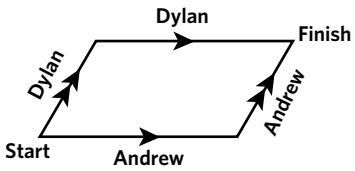
Problem solving

Question working paths

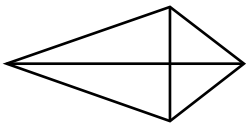
Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Nillah leaned a ladder up against a wall. If the ladder was leaning at an angle of 35° to the ground, what angle would it make at the wall?

12. Dylan and Andrew are racing on the parallelogram track. Dylan starts from the starting point, travels up and then across. Andrew starts from the starting point, travels across and then up. Who travels the furthest?



13. Phillipa is making kites for her friends. She is using fibreglass sticks as structural support. What angle does she need to ensure they join at?



14. Dorian and Raja are deciding what shape to frame their business logo. Dorian wants the frame to have four equal side lengths. Raja wants the pairs of opposite interior angles of the frame to be equal. What shape can satisfy both of them?

15. Sameera is building a triangular frame for ivy to grow along her wall. She has two beams that are three metres long. She has placed one at an angle of 50° to the ground. What angle to the ground should Sameera set the other beam at to meet the current beam at the tip?

Reasoning

Question working paths

Mild 16 (a,b,e), 17 (a,b)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Cindy has been asked to design walking tracks in a new park. She wants to recreate the geometric designs of French Gardens and settles on this design:

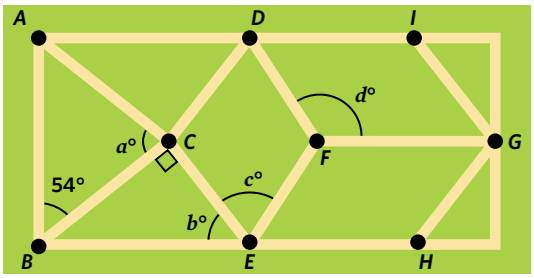


Image: Tomas Hasl/Shutterstock.com

Her understanding of triangles and quadrilaterals is going to be required. She will only measure one angle, the rest will be deduced.

a. Cindy begins by making an isosceles triangle with the points A, B and C, where $\overline{AC} = \overline{BC}$ She measures the angle at B as 54°. What is the angle $a°$ at point C.

b. Cindy then marks the points D and E by creating two right-angled triangles ACD and BCE with the right angle for both at C. What is the angle $b°$ at point E?

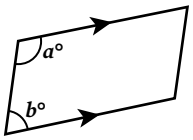
c. Cindy then creates a parallelogram with point F. What is the angle $c°$ at the point E?

d. Cindy creates two identical parallelograms with points H, I, and G. Determine the size of angle $d°$ at point F.

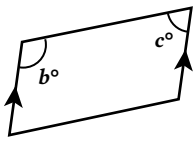
e. Maintaining a manicured garden can use a lot of water. Suggest some methods of collecting water that is not from the main drinking supply.

17. Use the rules of angles created by transversals of parallel lines for parts **a** and **b**.

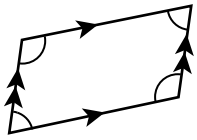
a. What does $a° + b°$ equal?



b. What does $b° + c°$ equal?

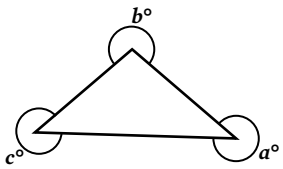


c. Demonstrate how this implies that $a° = c°$ and that the opposite angles of a parallelogram must be equal.



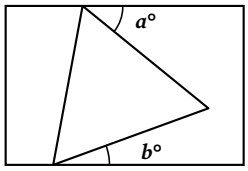
Extra spicy

18. What is the value of $a° + b° + c°$?



A. 180° **B.** 360° **C.** 720° **D.** 900° **E.** 1080°

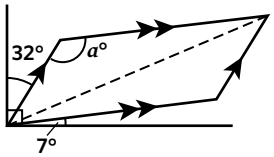
19. The diagram has an equilateral triangle inside a rectangle.



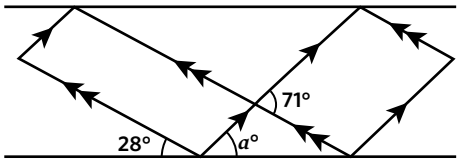
What does $a° + b°$ equal?

A. 30° **B.** 45° **C.** 60° **D.** 75° **E.** 90°

20. Determine the value of $a°$ in the following diagram.



21. Determine the value of $a°$ in the following diagram.



Remember this?

22. This table shows some information about cities in Queensland and New South Wales

Name of city	Amount of people	State
Albury	146 748	New South Wales
Central Coast	352 249	New South Wales
Greater Brisbane	5 338 993	Queensland
Greater Sydney	5 445 244	New South Wales
Sunshine Coast	456 345	Queensland
Toowoomba	187 434	Queensland
Wollongong	248 726	New South Wales

How much busier is New South Wales' busiest city than Queensland's busiest city?

A. 97 920 people
B. 98 145 people
C. 106 161 people
D. 106 251 people
E. 106 730 people

23. This list shows the ages of 15 workers.

32, 45, 19, 21, 48, 35, 61, 53, 19, 22, 36, 21, 42, 31, 39

What is the range of ages?

A. 12 **B.** 18 **C.** 22 **D.** 36 **E.** 42

24. To make jelly, 4 cups of boiling water need to be used for every 6 cups of gelatine.

How many cups of boiling water are needed for 9 cups of gelatine?

A. 2 cups **B.** 4 cups **C.** 6 cups **D.** 8 cups **E.** 10 cups

Back to contents

7C Transformations on the Cartesian plane

Translation, rotation and reflection are different ways of transforming the location of an object in 2D space. It is important to consider carefully how each describes a very different type of movement.

LEARNING INTENTIONS

Students will be able to:

- translate points and shapes on the Cartesian plane using worded descriptions or vectors
- reflect points and shapes over a mirror line on the Cartesian plane
- draw the rotation of a point or shape around a central point in the Cartesian plane.

KEY TERMS AND DEFINITIONS

A **Cartesian plane** is a set of two perpendicular number lines that intersect at the origin.

Cartesian coordinates are the numbers that indicate the location of a point on a Cartesian plane. The x–coordinate is written before the y-coordinate.

The ***x*-axis** is the horizontal axis that runs left to right through zero.

The ***y*-axis** is the vertical axis that runs up and down through zero.

Clockwise is the same direction that the hands of a clock move around.

Anticlockwise is the opposite direction that the hands of a clock move around.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

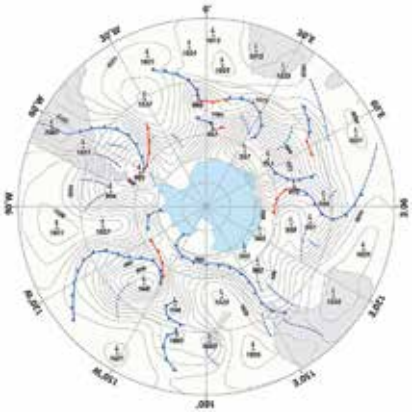
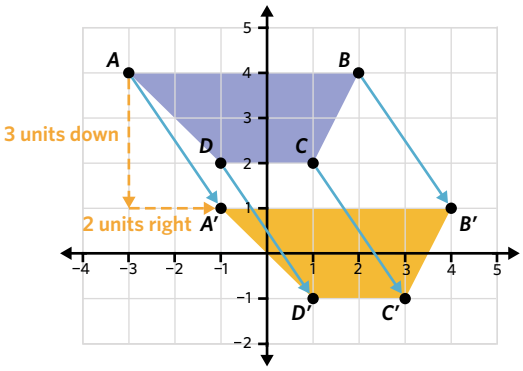


Image: Rainer Lesniewski/Shutterstock.com

Transformations mapped with a coordinate system are used every day. One of the most important applications is weather predictions. A coordinate system is used to track the movement of weather events and predict patterns in those movements.

Key ideas

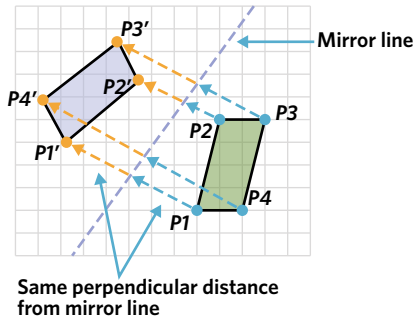
1. A point or shape can be translated in the Cartesian plane using the words up, down, left,right or by a vector.



The trapezium is translated by the vector $(2,-3)$ which is two units to the right and three units down (plus 2 in the x direction, minus 3 in the y direction).

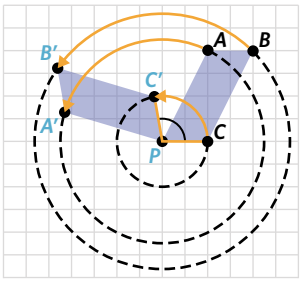
Continues →

2. A point or shape can be reflected in any line.

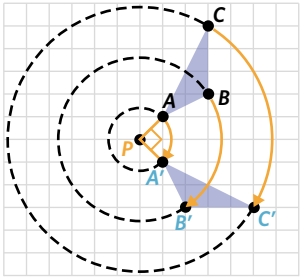


3. A point or shape can be rotated about a point.

Anticlockwise is a positive rotation.
E.g. The parallelogram below has been rotated by 100°.



Clockwise is a negative rotation.
E.g. The parallelogram below has been rotated by −90°.

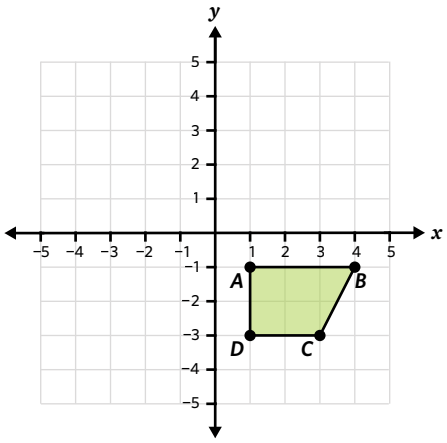


Worked example 1

Translating points and shapes on the Cartesian plane

Translate.

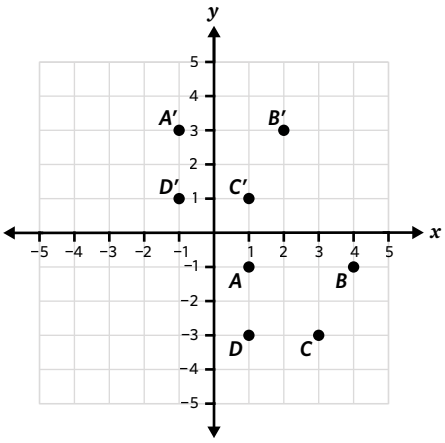
a. The trapezium *ABCD* by the vector $(-2,4)$. WE1a

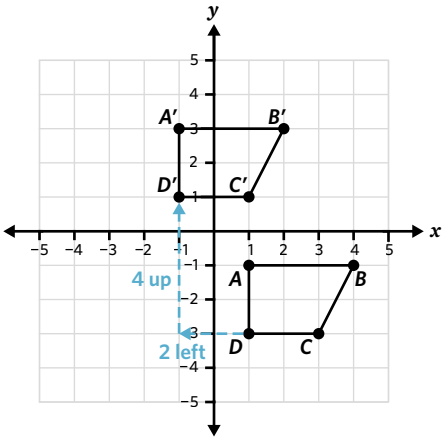


Continues →

Working

$(-2,4)$ is moving two units to the left and four units up.





Thinking

Step 1: Translate all of the vertices, by the given vector.

Step 2: Join the translated vertices.

b. $A = (3,-5)$ by the vector $(-1,2)$. WE1b

Working

x–value $= 3 + (-1)$

y–value $= -5 + 2$

$A' = (2,-3)$

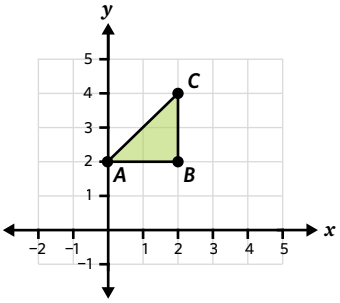
Thinking

Step 1: Add the x-value of the vector to the x-value of the original point and then add the y-value of the vector to the y-value of the original point.

Step 2: Complete the sums to determine the new point.

Student practice

Translate.



a. The triangle ABC by $(2,-1)$.

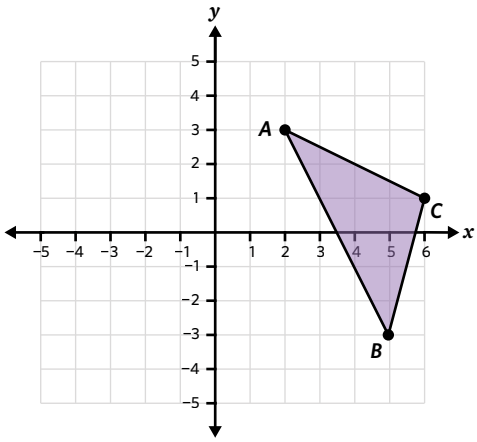
b. $B = (4,-3)$ by the vector $(-2,1)$.

Worked example 2

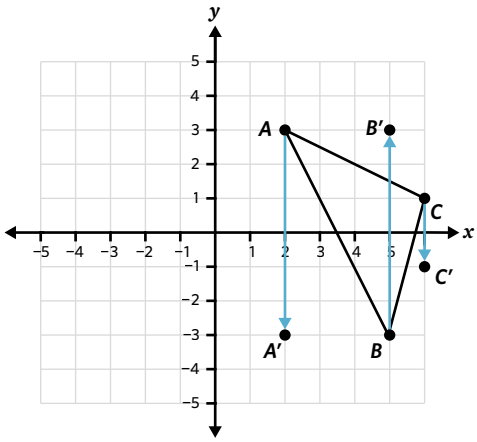
Reflecting points and shapes in mir or lines

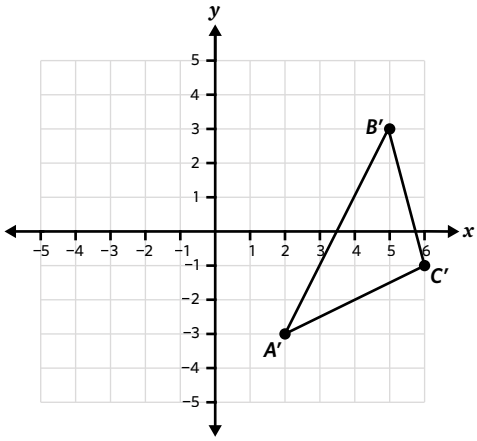
Reflect.

a. The triangle ABC in the x-axis. WE2a



Working





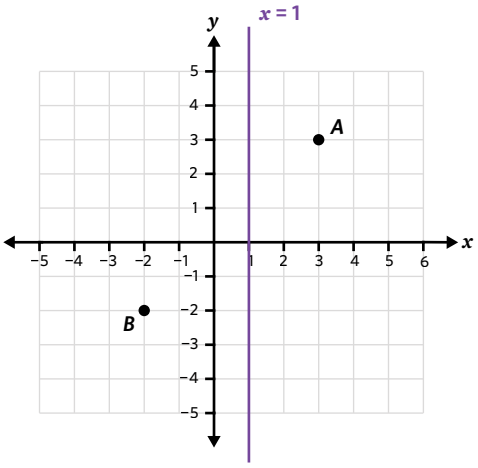
Thinking

Step 1: Reflect each vertex in the x–axis.

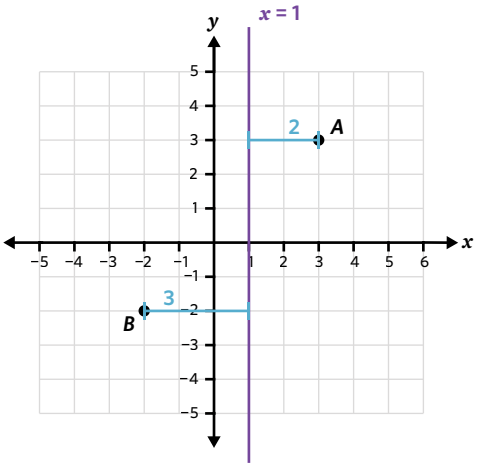
Step 2: Connect the image of the reflected vertices to create the reflected triangle.

Continues →

b. $A = (3,3)$ and $B = (-2,-2)$ in the line $x = 1$. WE2b

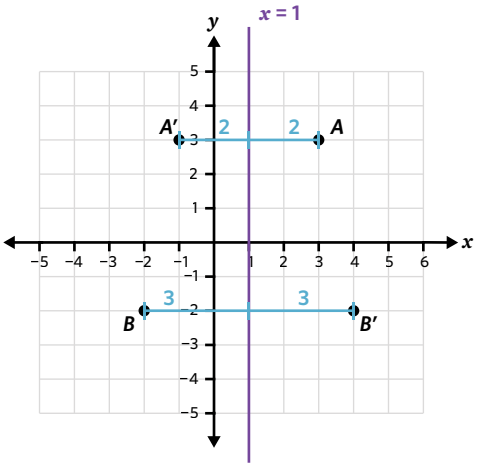


Working



A is 2 units from $x = 1$

B is 3 units from $x = 1$



Thinking

Step 1: Find the perpendicular distance from the point to the line.

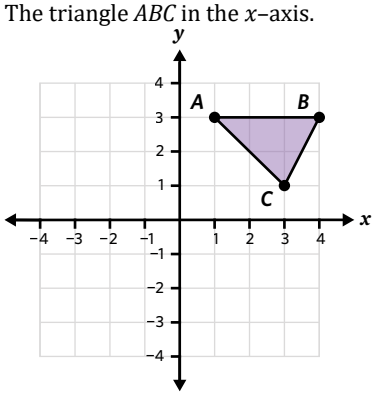
Step 2: Find the point that is the same distance, but opposite direction from the line.

Continues →

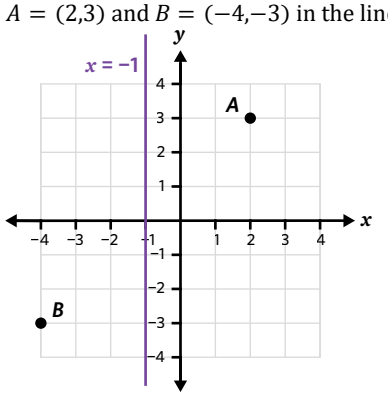
Student practice

Reflect.

a. The triangle ABC in the x–axis.



b. $A = (2,3)$ and $B = (-4,-3)$ in the line $x = -1$.

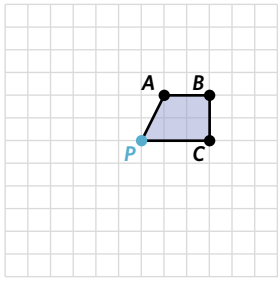


Worked example 3

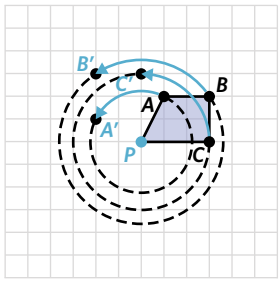
Rotating points and shapes on the Cartesian plane

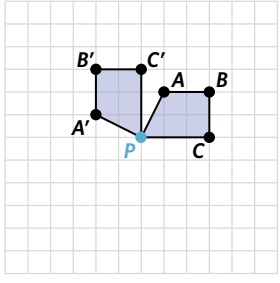
Rotate.

a. The trapezium $ABCP$ 90° anticlockwise about P. WE3a



Working





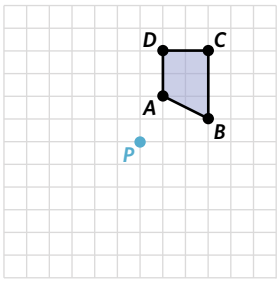
Thinking

Step 1: Rotate each vertex 90° anticlockwise around the point of rotation.

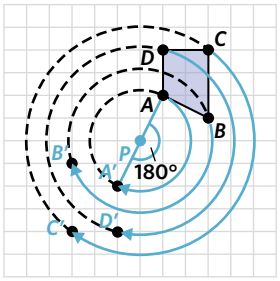
Step 2: Connect the rotated vertices.

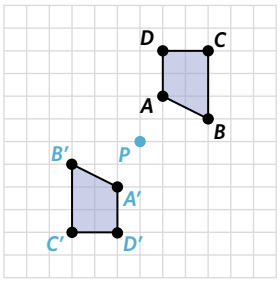
Continues →

b. The trapezium *ABCD* 180° clockwise about *P*. WE3b



Working





Thinking

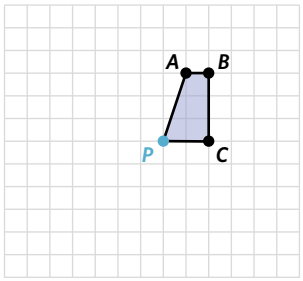
Step 1: Rotate each vertex 180° clockwise around the point of rotation.

Step 2: Connect the rotated vertices.

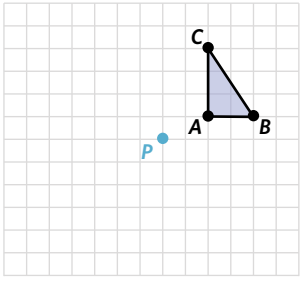
Student practice

Rotate.

a. The trapezium *ABCP* 90° anticlockwise about *P*.



b. The triangle *ABC* 180° clockwise about *P*.



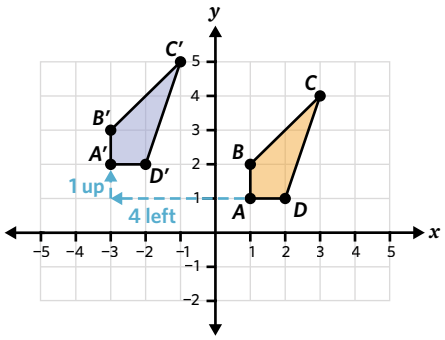
7C Questions

Understanding worksheet

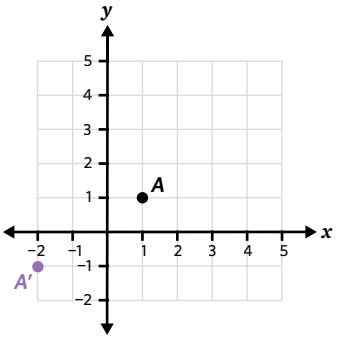
1. Complete the descriptions of the translations below.

Example

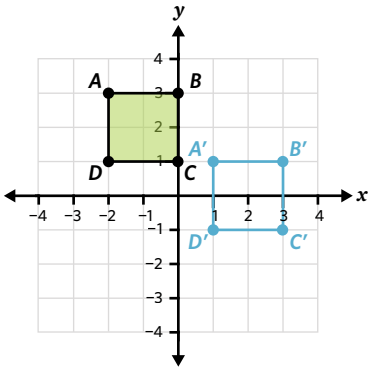
The kite *ABCD* by the vector (−4 , 1)



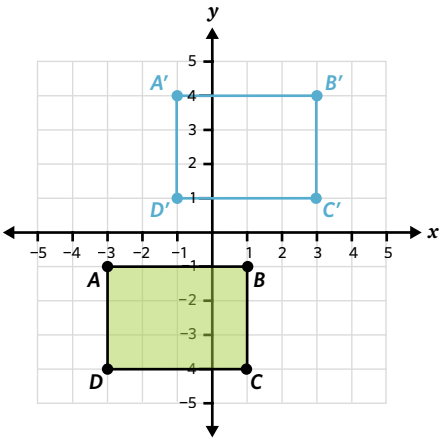
a. $A = (1,1)$ was translated by the vector (,)



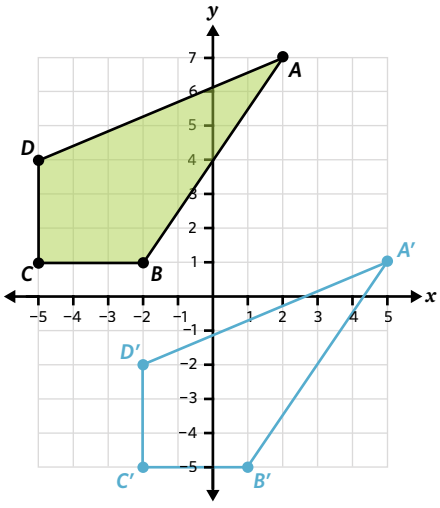
b. The square *ABCD* was translated by the vector (,)



c. The rectangle *ABCD* was translated by the vector (,)

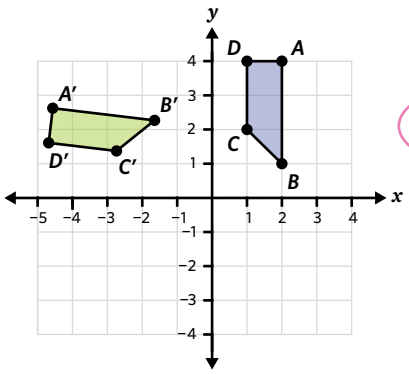


d. The kite *ABCD* was translated by the vector (,)



2. Determine what type of transformation was made.

Example



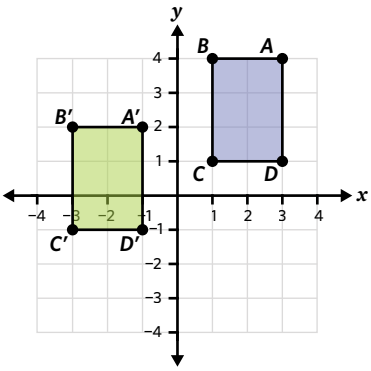
Translation

Rotation about origin

Reflection in y–axis

Reflection in x–axis

a.



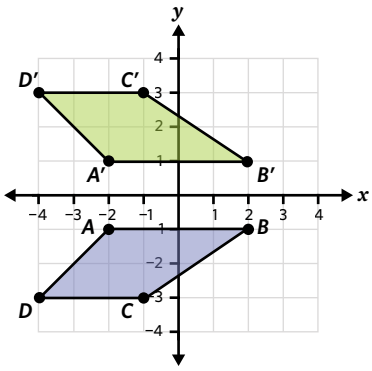
Translation

Rotation about origin

Reflection in y–axis

Reflection in x–axis

b.



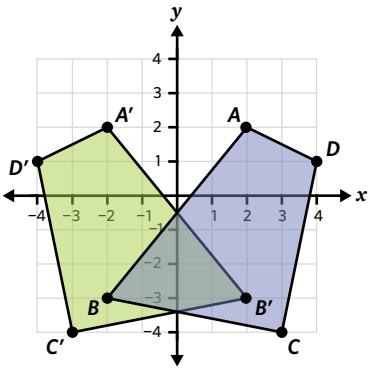
Translation

Rotation about origin

Reflection in y–axis

Reflection in x–axis

c.



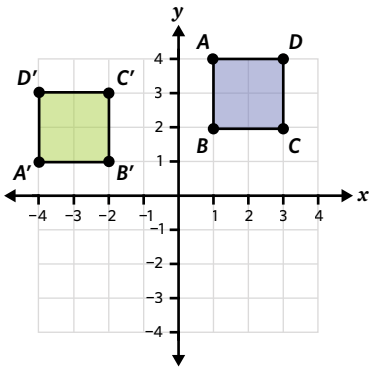
Translation

Rotation about origin

Reflection in y–axis

Reflection in x–axis

d.



Translation

Rotation about origin

Reflection in y–axis

Reflection in x–axis

3. Fill in the blanks by using the words provided.

vector | reflectio | rotation | translation

There are three ways to transform points and shapes on the Cartesian plane. A _____ moves all points either left or right and up or down, a _____ creates a mirror image and a _____ moves in a circular motion around a point. A _____ can be used to replace the words 'left' or 'right', which are represented by its x–value and 'up' or 'down', which are represented by its y–value.

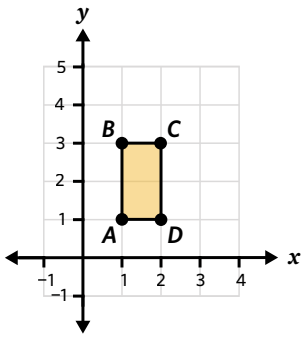
Fluency

Question working paths

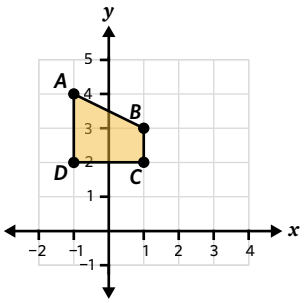
Mild	Medium	Spicy
4 (a,b), 5 (a,b,c,d), 6 (a,b), 7 (a,b), 8	4 (b,c), 5 (c,d,e,f), 6 (b,c), 7 (b,c), 8	4 (c,d), 5 (e,f,g,h), 6 (c,d), 7 (c,d), 8

4. Translate. WE1a

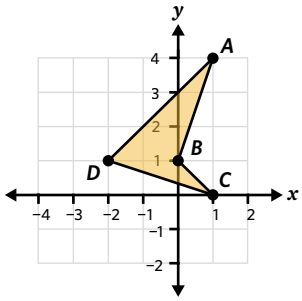
a. The rectangle $ABCD$ by the vector $(2,2)$.



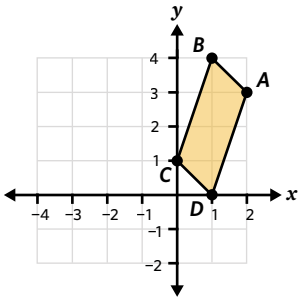
b. The trapezium $ABCD$ by the vector $(3,-1)$.



c. The quadrilateral $ABCD$ by the vector $(-1,-2)$.



d. The parallelogram $ABCD$ by the vector $(-4,-2)$.



5. Translate. WE1b

a. $A = (2,4)$ by the vector $(1,1)$

b. $B = (3,1)$ by the vector $(3,4)$

c. $C = (3,4)$ by the vector $(3,1)$

d. $D = (-5,1)$ by the vector $(-1,3)$

e. $E = (11,-4)$ by the vector $(-15,13)$

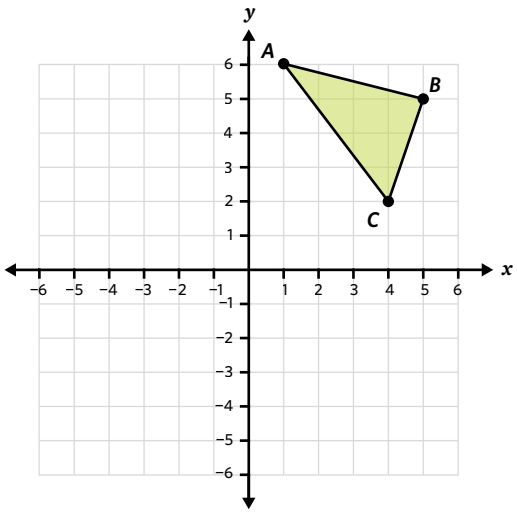
f. $F = (-27,-15)$ by the vector $(-11,12)$

g. $G = (29,-51)$ by the vector $(-14,14)$

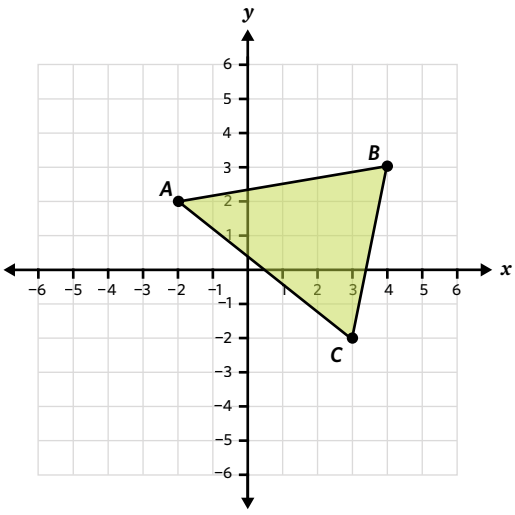
h. $H = (33,-41)$ by the vector $-(12,-23)$

6. Reflect. WE2

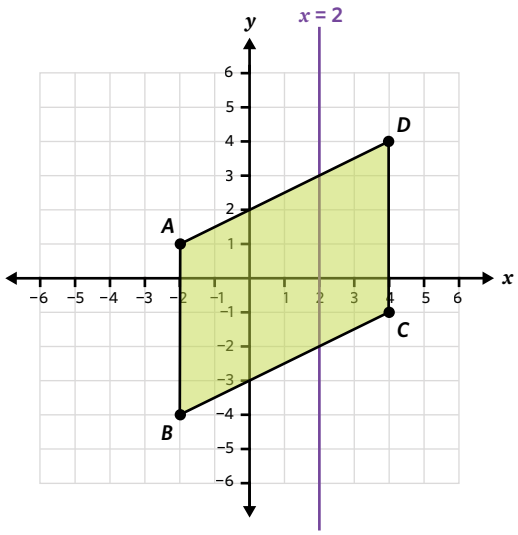
a. The triangle ABC in the y–axis.



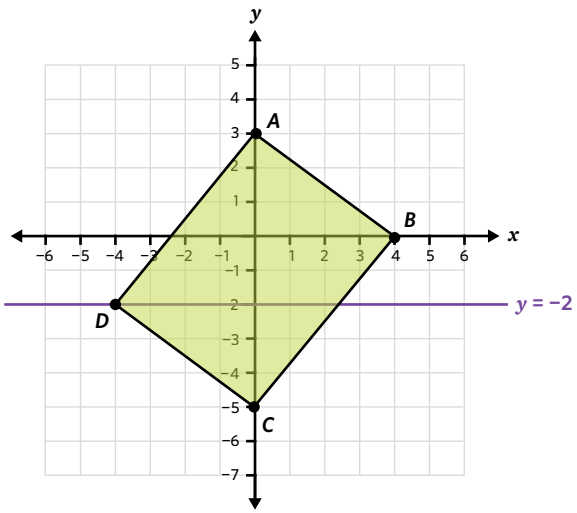
b. The triangle ABC in the x–axis.



c. The parallelogram $ABCD$ in the line $x = 2$.

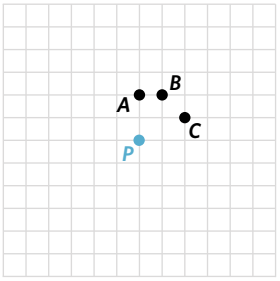


d. The rectangle $ABCD$ in the line $y = -2$.

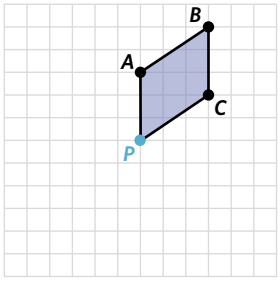


7. Rotate. WE3

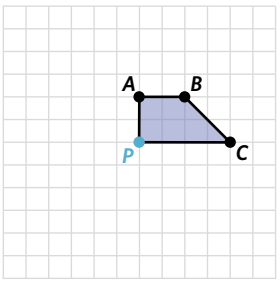
a. The points A, B, C and D 90° anticlockwise about P.



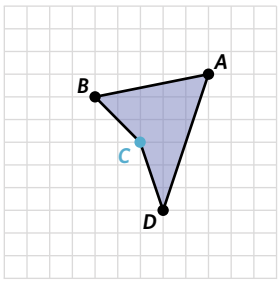
b. The parallelogram $ABCP$ 90° anticlockwise about P.



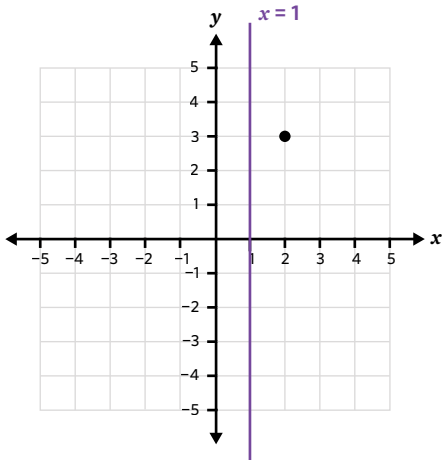
c. The trapezium $ABCP$ 90° clockwise about P.



d. The quadrilateral $ABCD$ 180° clockwise about C.



8. Reflect the point (2,3) in the line $x = 1$.

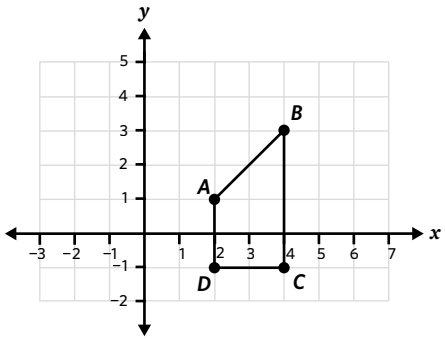


A. (−3,2) **B.** (2,−3) **C.** (−2,3) **D.** (2,−1) **E.** (0,3)

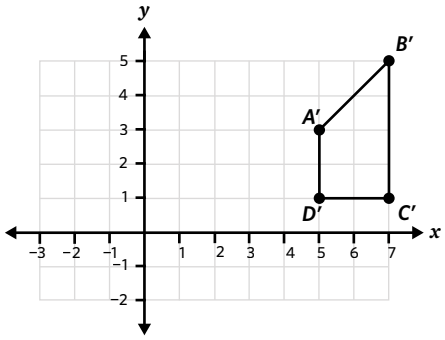
Spot the mistake

9. Select whether Student A or Student B is incorrect.

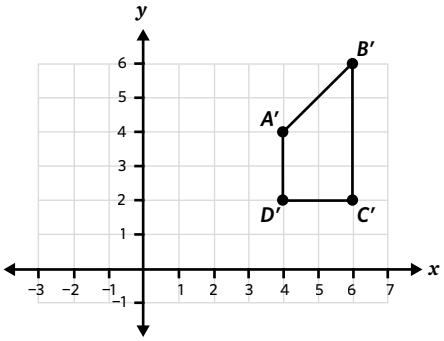
a. Translate the trapezium *ABCD* by the vector (3,2).



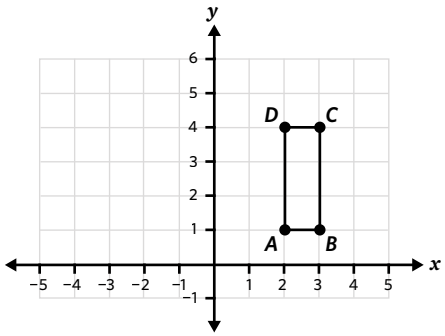
Student A



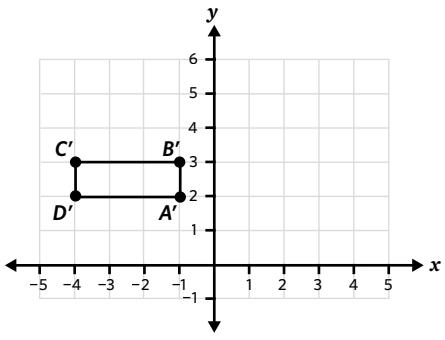
Student B



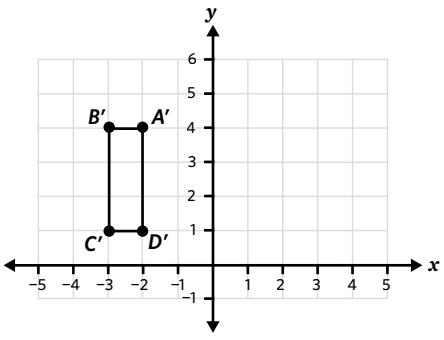
b. Rotate the rectangle *ABCD* 90° anticlockwise about the origin.



Student A



Student B

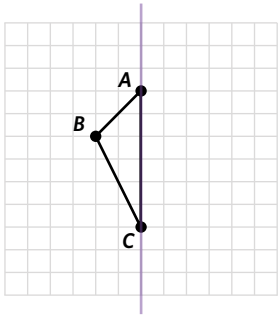


Problem solving

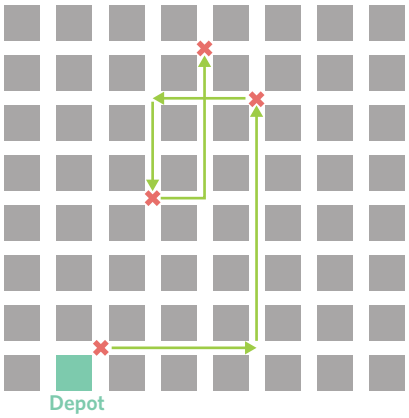
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

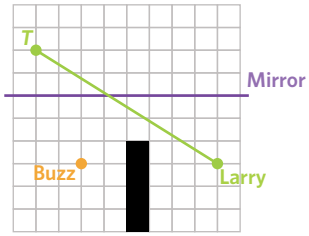
10. Matthew is studying graphic design. He makes a design by reflecting triangle *ABC* over the mirror line. What shape is the design that Matthew has made?



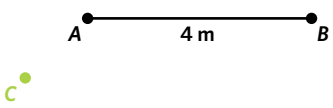
11. Grid town's major delivery company is called Vectors and Rogers. Drivers are given vectors to describe the journey from one stop to the next. A driver leaves the depot with the instructions: (3,5), (−2,−2) and (1,3). What is the delivery vector that goes directly from the depot to the final delivery?



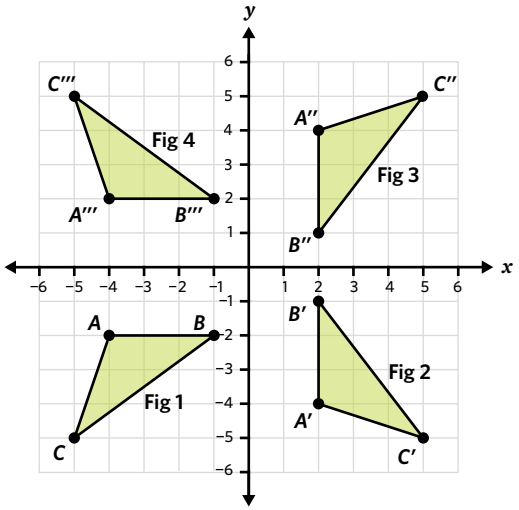
12. Larry and his friend Buzz are playing Laser Tag. Buzz is hiding behind a wall. Larry thinks he can tag his friend buzz by reflecting a laser off the mirror labeled in the diagram below. If *T* represents Larry's aim through the mirror, will Larry be able to tag Buzz?



13. The points *A* and *B* are 4 m apart. If both points are rotated 90° around point *C*, how far apart are their images *A'* and *B'*?



14. Name the transformations from Fig 1 to Fig 2 to Fig 3 to Fig 4 that Patrick used to create the following pattern so that Kysaiah can replicate it.

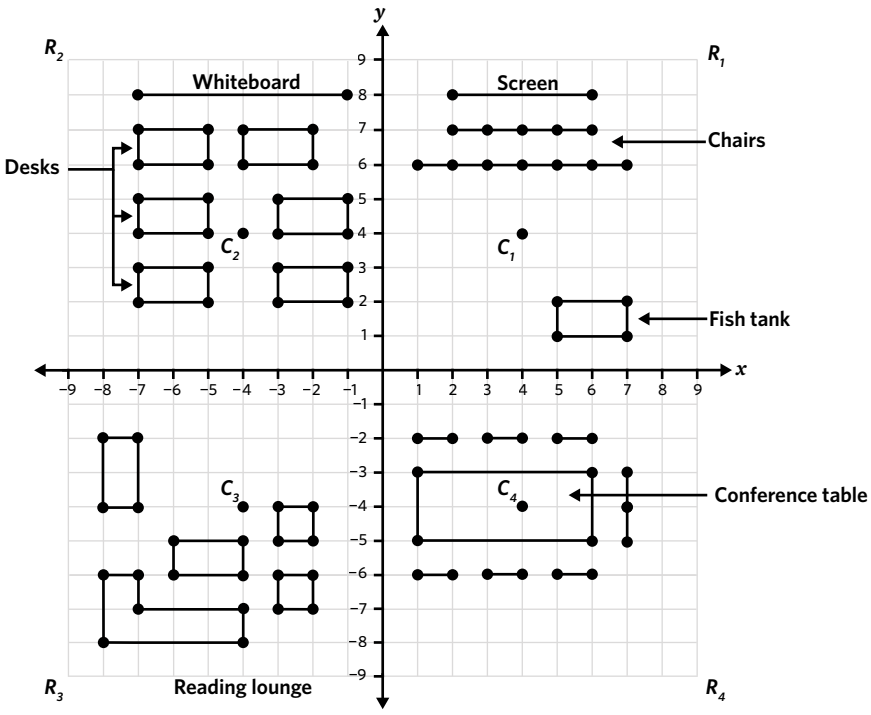


Reasoning

Question working paths

Mild 15 (a,b,e)	Medium 15 (a,b,e), 16 (a,b)	Spicy All

15. Mr Nicholas has divided his learning area into four rooms. He wants to transform the rooms around for term two. Help Mr Nicholas decide which layout to use by drawing the new lay out for each part below.



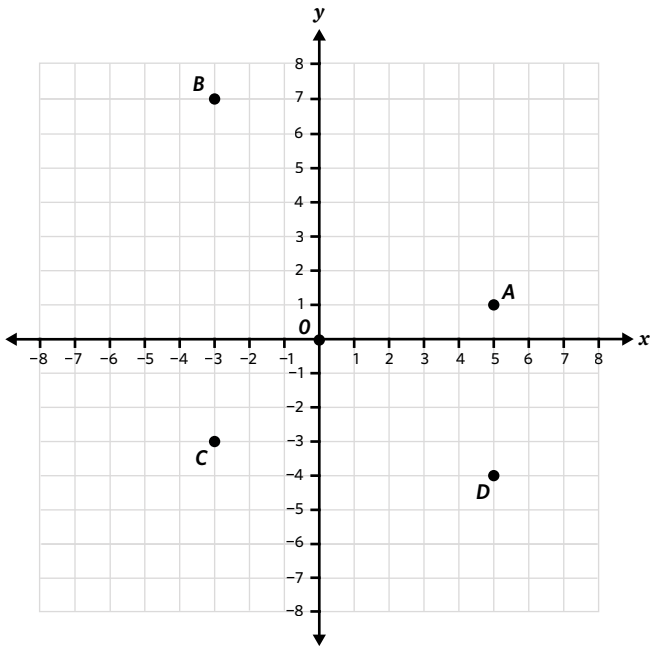
a. He begins by translating all of the desks in room 2 $(-1,-1)$ to create more space at the front of the classroom.

b. Next he rotates room 1 180° clockwise around C_1 so the screen is not distracting the students in room 2.

c. He swaps room 3 and room 4 by reflecting both of them in the y–axis. Draw the final layout of Mr Nicholas' learning space following all transformations.

d. Does altering the layout of a learning space help students be more engaged with their lessons?

16. Complete the following transformations.



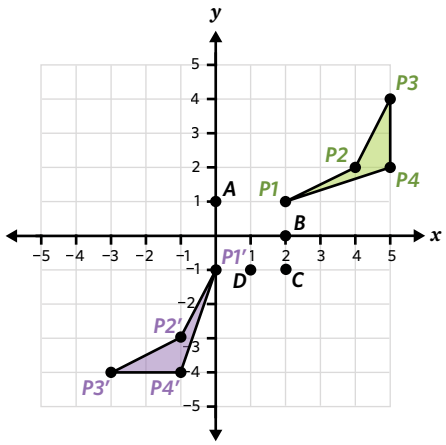
a. Rotate the points A, B, C and D 180° anticlockwise around O.

b. Reflect the points A, B, C and D in the x–axis and then the y–axis

c. Compare your answers in parts **a** and **b** to determine what happens to any coordinates (x,y) when rotated anticlockwise 180°.

Extra spicy

17. Which two points does the line of reflection for the two shapes in this image pass through?

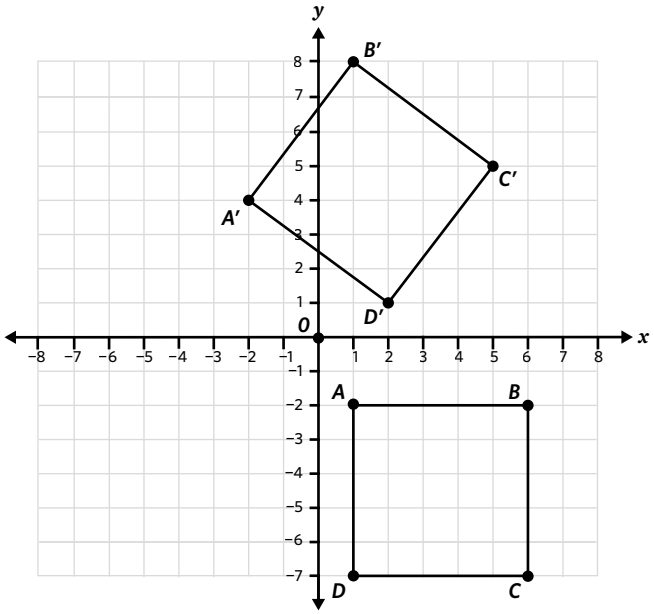


A. A,B **B.** A,C **C.** A,D **D.** B,C **E.** C,D

18. What is the minimum amount of times a point need to be rotated by 50° to be rotated the equivalent of 190°?

19. A point was rotated anticlockwise 90° then translated $(-1,-1)$ and then reflected in the x–axis. If the image after transformation is $(3,3)$, what was the original point?

20. What is the minimum amount of transformation(s) required for the image of the square *ABCD*.



A. Reflection
B. Rotation
C. Translation
D. Rotation and translation
E. Translation and reflection

Remember this?

21. This stem–and–leaf plot shows the number of children that went to the local pool each day for the last three weeks.

Stem	Leaf
0	2 3
1	1 5 7 8
2	3 5 6 8 9
3	0 3 6 8
4	0 7 9
5	2 4 7

Key

3 | 6 = 36

How many days were there less than 49 children?

A. 9 **B.** 11 **C.** 17 **D.** 28 **E.** 36

22. Which of these is 53.29892 rounded to 3 decimal places?

A. 53.298 **B.** 53.2989 **C.** 53.299 **D.** 53.2999 **E.** 5329

23. James decides to join in on a running marathon to raise money.

He has 4 sponsors.

His mother donates $6.50 for every kilometre he runs.

His grandfather donates $4 for every kilometre.

His sister donates $15 in total.

His friend donates $26 in total.

How many kilometres does James need to run to raise at least $210?

A. 15 kilometres **B.** 30 kilometres **C.** 45 kilometres **D.** 60 kilometres **E.** 90 kilometres

7D Congruent 2D shapes

Two shapes are congruent if they are essentially the same shape but in different positions or orientations. They share exactly the same features such as side lengths and internal angles. Translations, reflections and rotations are called isometric transformations because the image following transformation is congruent with the original shape.

LEARNING INTENTIONS

Students will be able to:

- define and identify congruent shapes
- identify corresponding geometric features of congruent shapes
- recognise that reflections, translations and rotations of a shape are congruent to the original shape.

KEY TERMS AND DEFINITIONS

A **line segment** between vertices A and B is denoted $\overline{AB}$.

The **angle** between line segments AB and BC is denoted $\angle ABC$.

An **isometric transformation** does not change the size or shape of the original image after the transformation.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

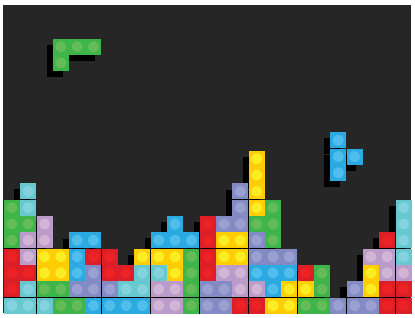
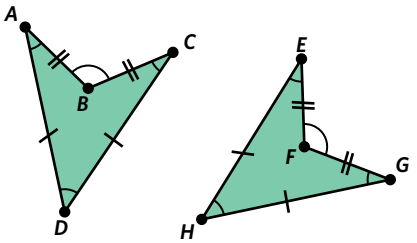


Image: ace03/Shutterstock.com

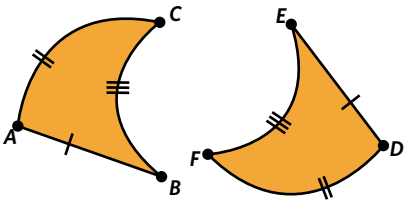
Congruent figures are very important for 2D video games. The greatest video game ever made, Tetris, challenges players to stack congruent shapes together as tightly as possible.

Key ideas

1. A pair of shapes are congruent if they are exactly the same shape and size. The symbol ≡ or ≅ indicates shapes are congruent.

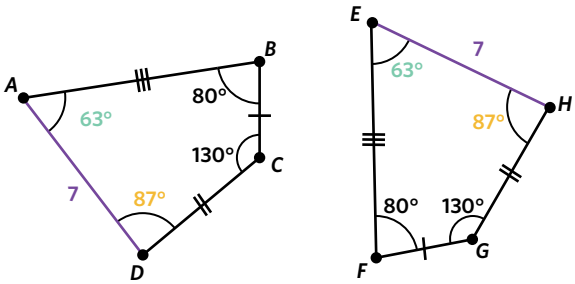


ABCD ≡ *EFGH*



ABC ≡ *DEF*

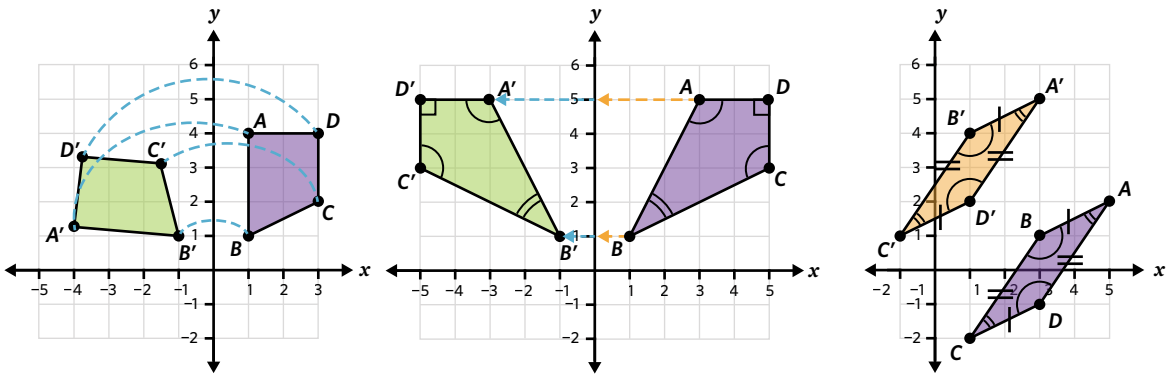
2. Two congruent figures have corresponding geometric properties $ABCD \equiv EFGH$.



$\overline{AD}$ is adjacent to $\angle BAD = 63°$ and $\angle ADC = 87°$.
$\overline{EH}$ is adjacent to $\angle FEH = 63°$ and $\angle EHG = 87°$.
$\therefore \overline{AD}$ and $\overline{EH}$ are corresponding sides and $\overline{AD} = \overline{EH}$.

Continues →

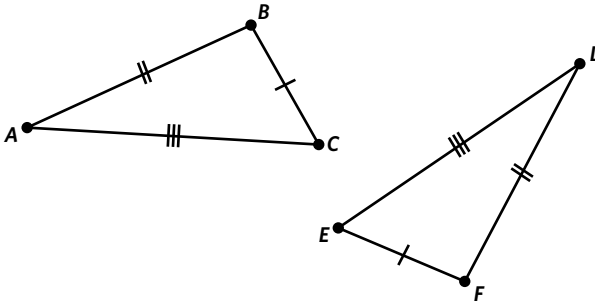
3. Translations, rotations and reflections are isometric transformations therefore, they are congruent.



Worked example 1

Corresponding features in congruent shapes

Name the corresponding features in these congruent shapes.



a. The side in $\triangle DEF$ that corresponds to $\overline{AC}$. WE1a

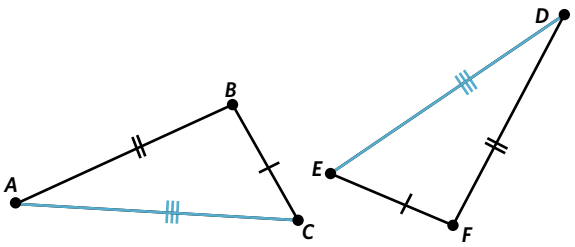
Working

$\overline{DE}$

Thinking

$\overline{AC}$ is the longest side in $\triangle ABC$ and $\overline{DE}$ is the longest side in $\triangle DEF$.

Visual support



Continues →

b. The vertex in $\triangle ABC$ that corresponds to vertex F. WE1b

Working

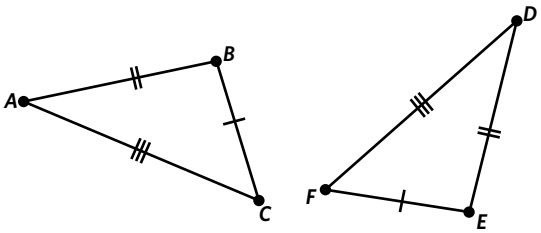
Vertex B

Thinking

Vertex F is connected to the two shortest sides. Vertex B is also connected to the two shortest sides.

Student practice

Name the corresponding features in these congruent shapes



a. The side in $\triangle ABC$ that corresponds to $\overline{FE}$

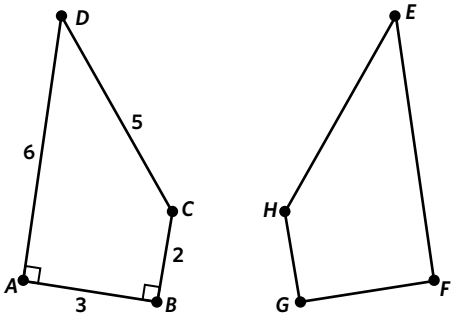
b. The vertex in $\triangle DEF$ that corresponds to vertex A

Worked example 2

Missing values in congruent shapes

Find the values of the missing side lengths or angles for the congruent shapes.

a. $\overline{HG}$ WE2a



Working

Side $\overline{HG}$ is the shortest side, it is also connected to an obtuse angle and a right angle.

Side $\overline{CB}$ is also the shortest side and connected to an obtuse angle and a right angle.

$\overline{HG} = \overline{CB}$

$\overline{HG} = 2$

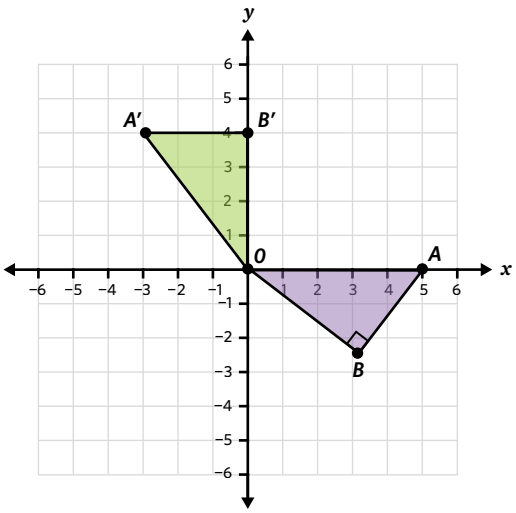
Thinking

Step 1: Compare the lengths and the angles at the vertices.

Step 2: Solve

Continues →

b. $\overline{OA'}$ WE2b



Working

$\overline{OA'}$ and $\overline{OA}$ are the sides that are opposite to the right angle.

$\overline{OA'} = \overline{OA}$

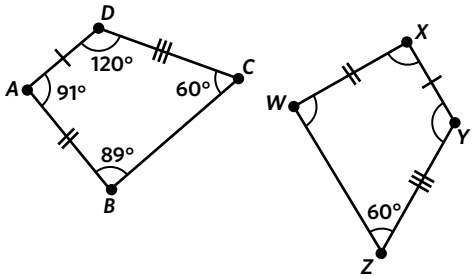
$\overline{OA'} = 5$

Thinking

Step 1: Compare the lengths and the angles at the vertices.

Step 2: Solve

c. $\angle XYZ$



Working

$\overline{DC}$ and $\overline{YZ}$ are corresponding sides connected to the 60° angle.

$\angle XYZ$ and $\angle ADC$ are both adjacent to the 60° angle.

$\angle XYZ = \angle ADC$

$\angle XYZ = 120°$

Thinking

Step 1: Compare the lengths and the angles at the vertices.

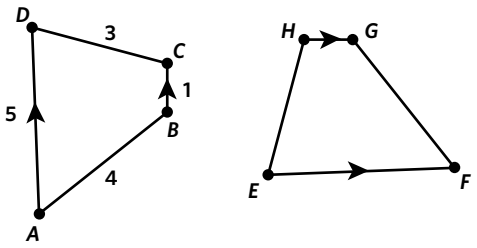
Step 2: Solve

Continues →

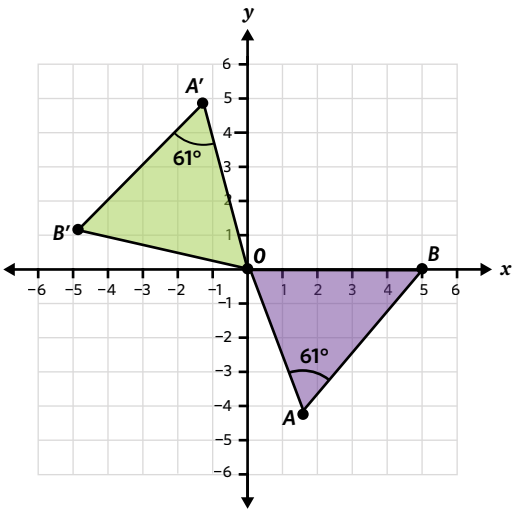
Student practice

Find the values of the missing side lengths or angles for the congruent shapes.

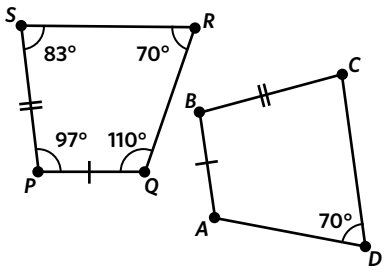
a. $\overline{HG}$



b. $\overline{OB'}$



c. $\angle ABC$

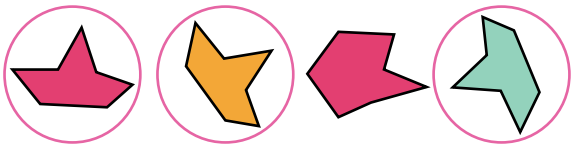


7D Questions

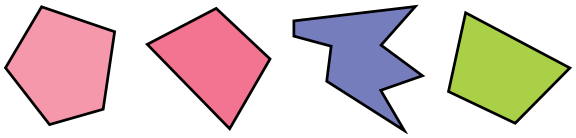
Understanding worksheet

1. Circle the congruent shapes.

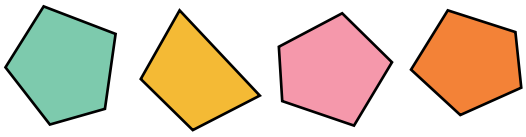
Example



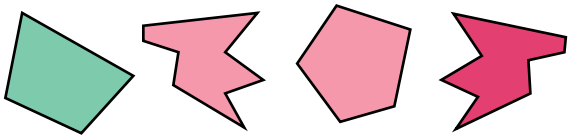
a.



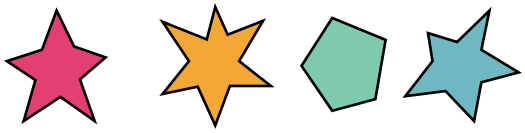
b.



c.



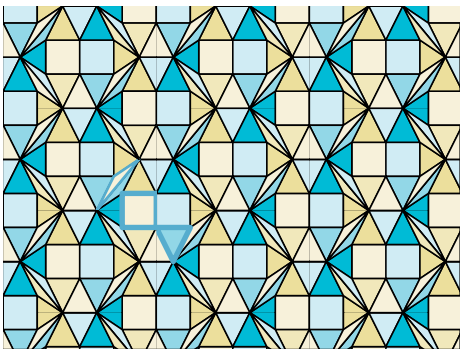
d.



2. How many types of congruent shapes are there in the following tessellations.

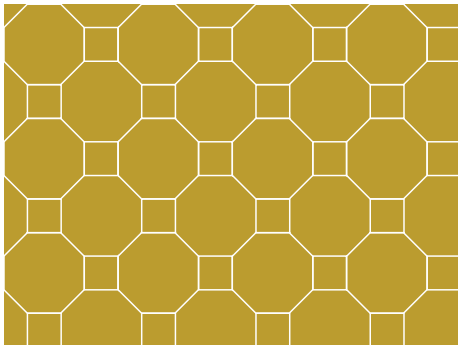
Example

3 different shapes create this tessellation.



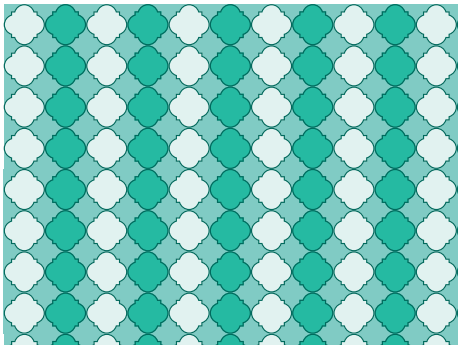
Hi-Point/Shutterstock.com

a. ____ different shapes create this tessellation.



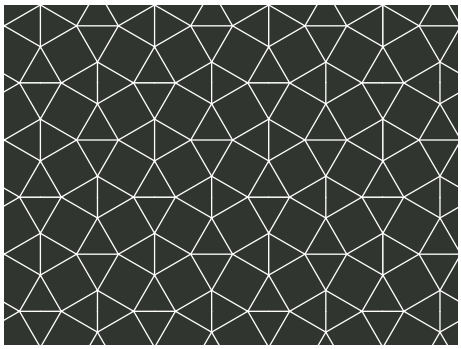
Hi-Point/Shutterstock.com

b. ____ different shapes create this tessellation.



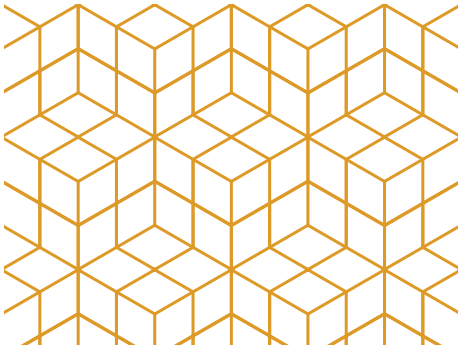
Yeshe-la/Shutterstock.com

c. [ ] different shapes create this tessellation.



Hi-Point/Shutterstock.com

d. [ ] different shapes create this tessellation.



Hi-Point/Shutterstock.com

3. Fill in the blanks by using the words provided.

isometric | congruent | equal | reflec ed

Two [ ] shapes are essentially the same shape. They may be in a different position or oriented differently, but their corresponding side lengths and interior angles are [ ]. Because the image of a shape that has been translated, rotated or [ ] is congruent with the original shape, they are called [ ] transformations.

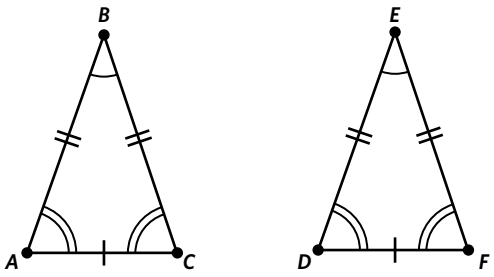
Fluency

Question working paths

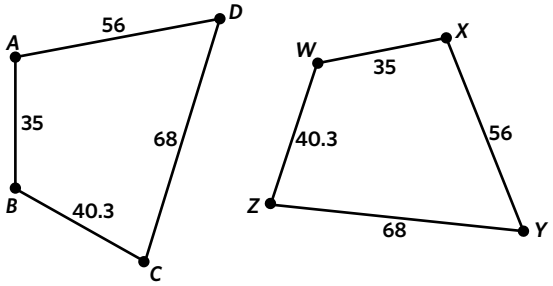
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b), 7 (a,b,c), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (b,c), 7 (b,c,d), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (c,d), 7 (c,d,e,f), 8

4. Name the corresponding features in these congruent shapes. WE1

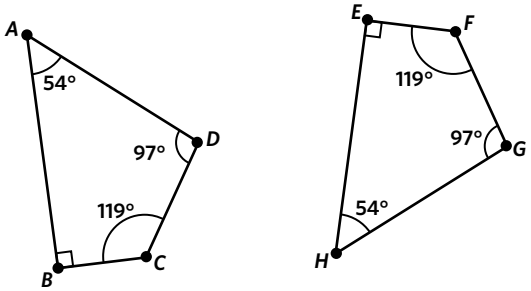
a. The side in $\triangle ABC$ that corresponds to side $\overline{DF}$.



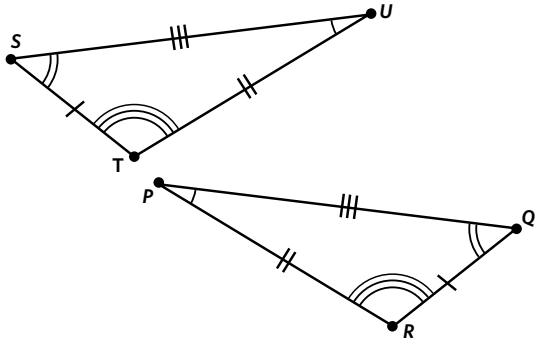
b. The vertex in $WXYZ$ that corresponds to vertex B.



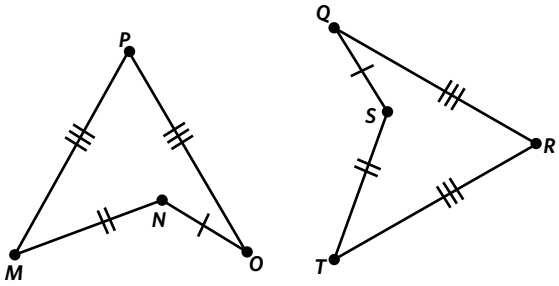
c. The side in $ABCD$ that corresponds to side $\overline{HG}$.



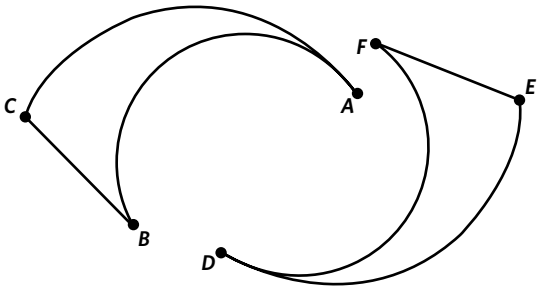
d. The vertex in $\triangle PQR$ that corresponds to vertex S.



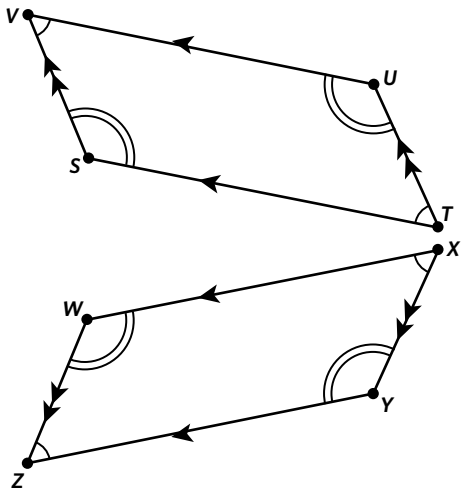
e. The vertex in $QRTS$ that corresponds to vertex O.



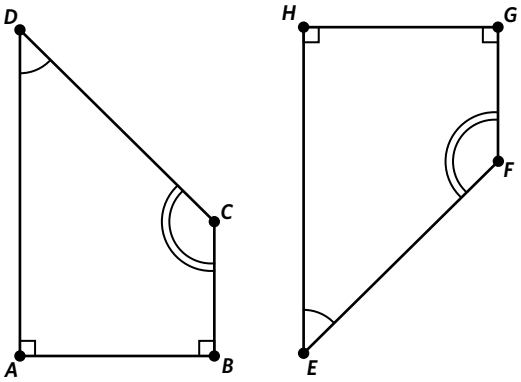
f. The vertex in ABC that corresponds to vertex F.



g. All sides in $STUV$ that could correspond with side $\overline{ZY}$.



h. The vertex in $EFGH$ that corresponds to vertex B.



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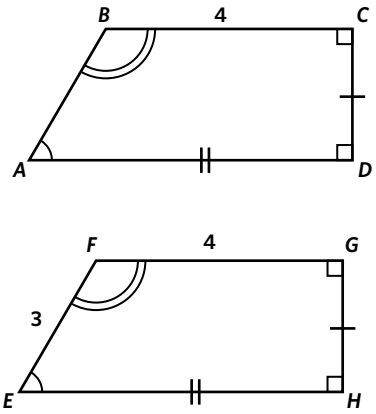
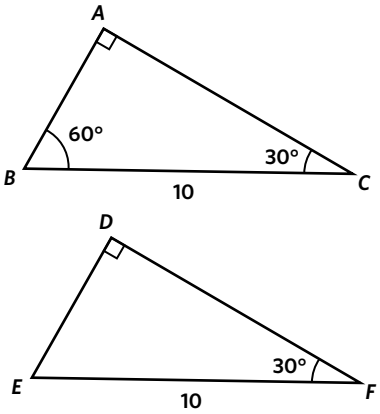
5. Find the values of the missing side lengths or angles for the congruent shapes. WE2a,c

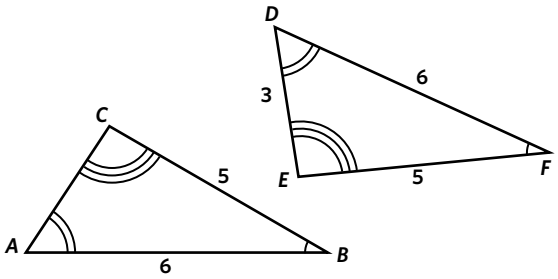
a. $\overline{AB}$



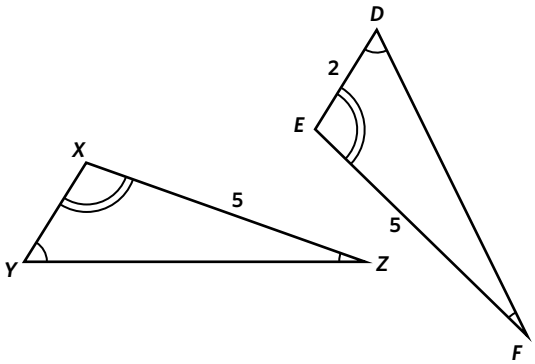
b. $\angle DEF$



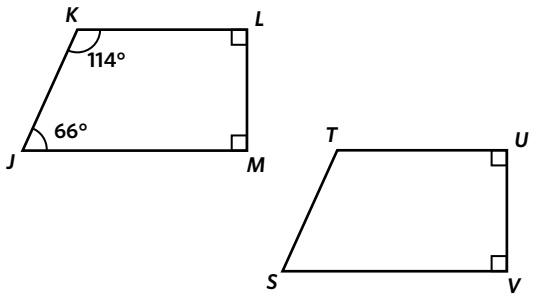
c. $\overline{AC}$



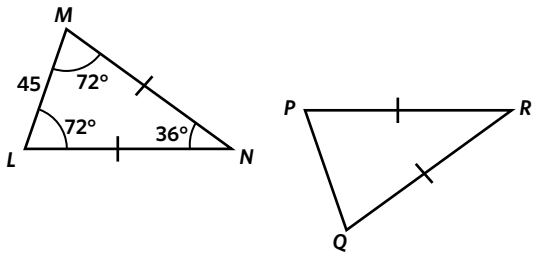
d. $\overline{XY}$



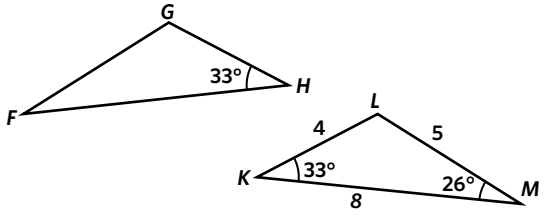
e. $\angle TSV$



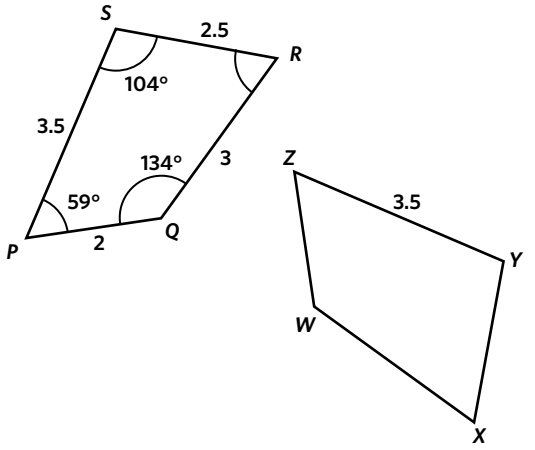
f. $\angle PRQ$



g. $\overline{FG}$

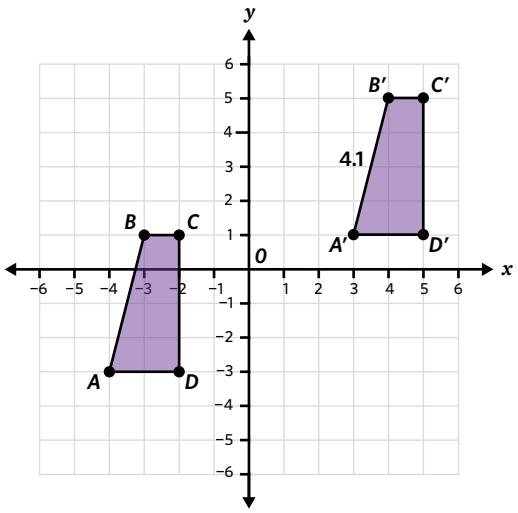


h. $\angle WXY$

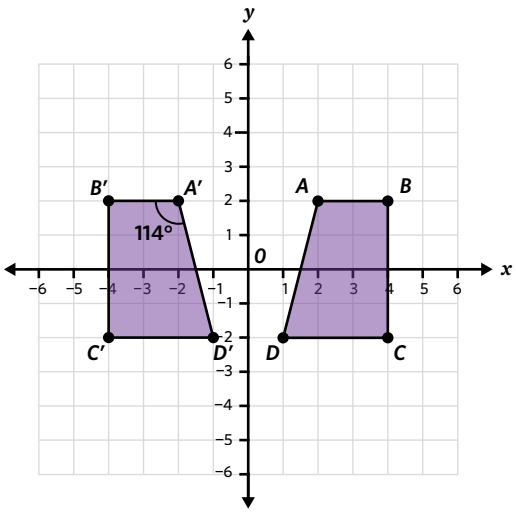


6. Find the values of the missing side lengths or angles for the congruent shapes. WE2b,c

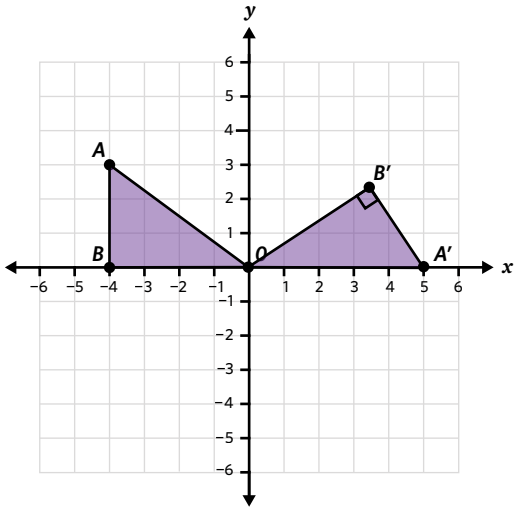
a. $\overline{AB}$



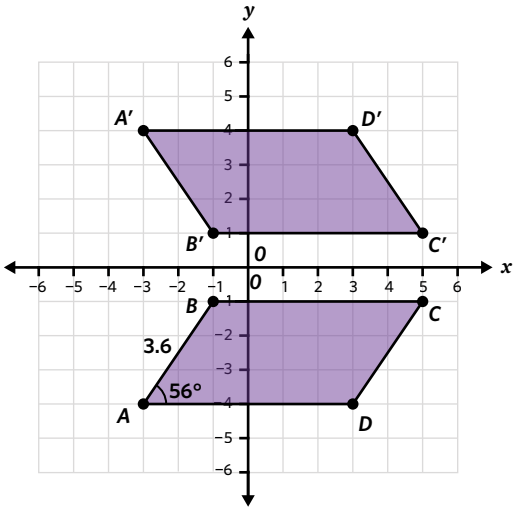
b. $\angle DAB$



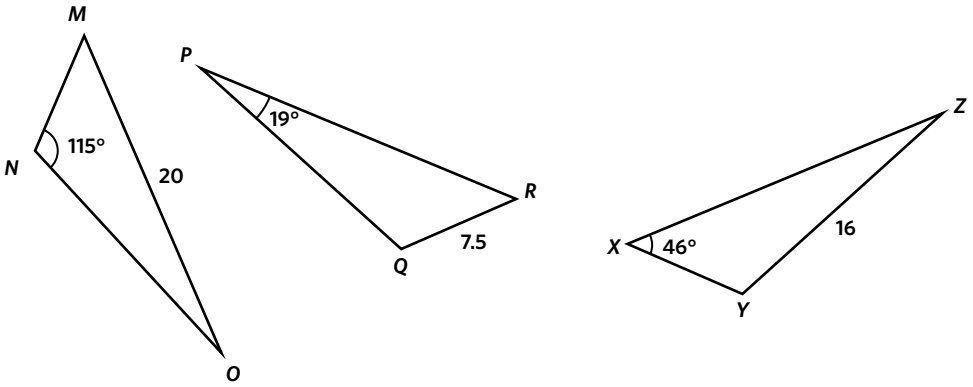
c. $\overline{OA}$



d. $\angle B'C'D'$



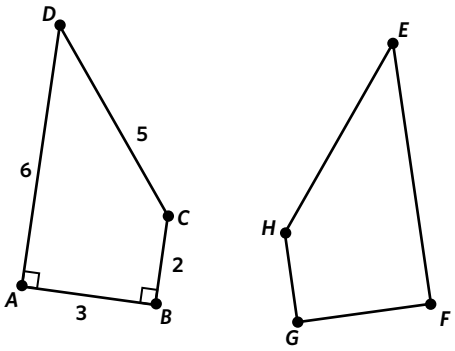
7. Find the values of the missing side lengths or angles for the congruent shapes.



a. $\overline{PR}$ **b.** $\overline{XY}$ **c.** $\overline{NO}$ **d.** $\angle RQP$

e. $\angle XYZ$ **f.** $\angle NMO$

8. $ABCD \equiv EFGH$. Determine the length of side $\overline{FG}$.

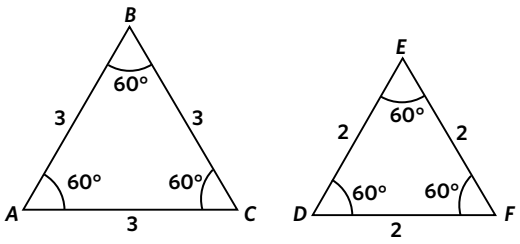


A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 90

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine if $\triangle ABC$ is congruent to $\triangle DEF$.



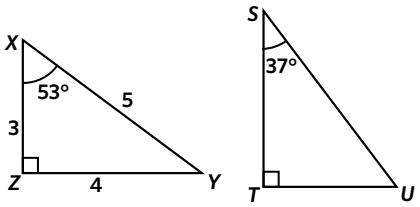
Student A

Yes they are congruent because they are both equilateral triangles.

Student B

No they are not congruent because they have different side lengths.

b. $\triangle XYZ \equiv STU$. Determine the value of $\overline{TU}$.



Student A

Sides $\overline{TU}$ and $\overline{ZY}$ are both on the bottom of congruent right triangles.

$\therefore \overline{TU} = \overline{ZY}$

$\overline{TU} = 4$

Student B

$\angle TSU = 37°$

$\therefore \angle SUT = 53°$

Side $\overline{TU}$ is adjacent to $\angle SUT$ and the right angle.

Side $\overline{XZ}$ is adjacent to angle of 53° and the right angle.

$\therefore \overline{TU} = \overline{XZ}$

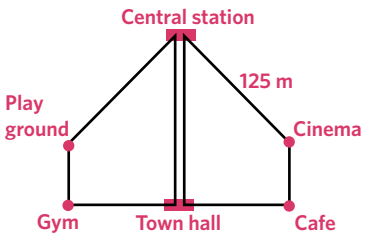
$\overline{TU} = 3$

Problem solving

Question working paths

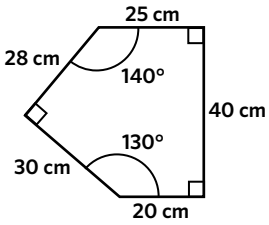
Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Euclid town was planned with streets creating congruent shapes alongside each other. We can see a pair of congruent shapes in the diagram.

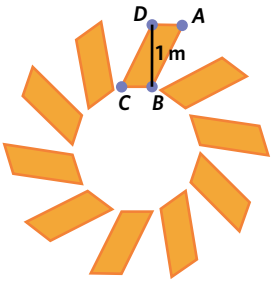


What is the distance from the playground to Central station if the distance from the cinema to Central station is 125 m?

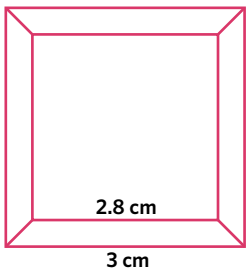
11. Dakota is welding metal pieces together following the design in the diagram. She has cut a 20 cm piece and a 30 cm piece as shown in the diagram. What angle does she need to join them at?



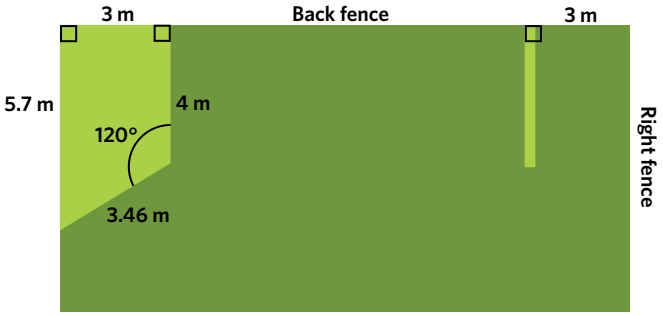
12. Gabriel decided to paint a mural on the wall of his solar panel shop. The design was a parallelogram that has a height of 1 m and width of 0.5 m rotated repeatedly 36°. Use the rule for the area of a parallelogram, base × height, to determine the total area Gabriel will need to paint.



13. Noah built a square box frame with 4 congruent trapeziums. If the area of the box including the frame was 3^2 cm^2 and the area inside the frame was 2.8^2 cm^2, what was the area of one trapezium?



14. Tran wants to mow a shape in the back right corner of the lawn to mirror the shape Trevor cut in the back left corner. Tran measured 3 m along the back fence and began mowing parallel to the right fence. Tran mows 4 m and then needs to turn to complete the shape. What angle does Tran need to turn to ensure they mow the same shape as Trevor?

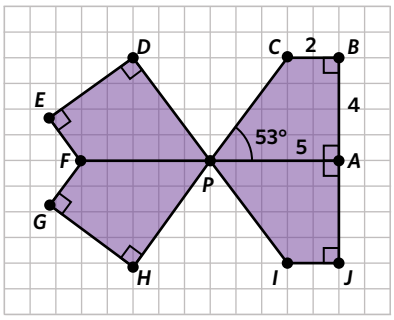


Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Toby is using trapeziums to create a geometric logo for his gym shirt brand called Traps. He draws the line segment $\overline{PA}$ with length 5, $\overline{AB}$ with length 4, and $\overline{BC}$ with length 2. To complete the trapezium $PABC$ he draws the line segment $\overline{PC}$ with $\angle CPA = 53°$. He then rotates it around P to create $PDEF$ and finally reflects them both in the line $\overline{FA}$.



a. List all of the line segments with length 2.

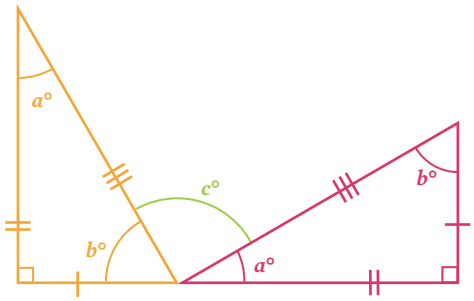
b. List all of the line segments with length 4.

c. List all of the line segments with length 5.

d. Determine the value of all of the internal angles that are not right angles.

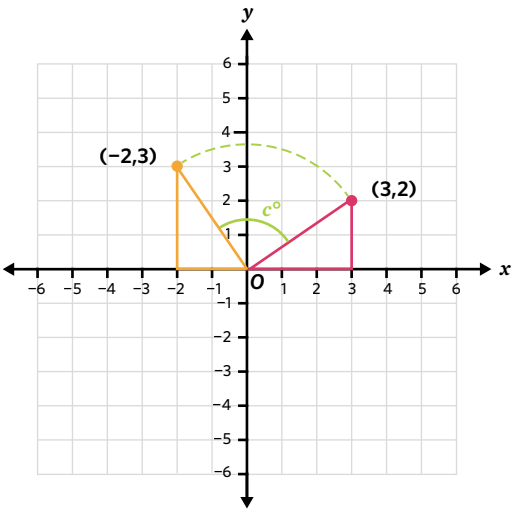
e. Toby wants his company to make a positive impact. Should he have the t-shirts made overseas so that they are cheaper and more people can afford them or should he have them made in Australia to create more jobs here?

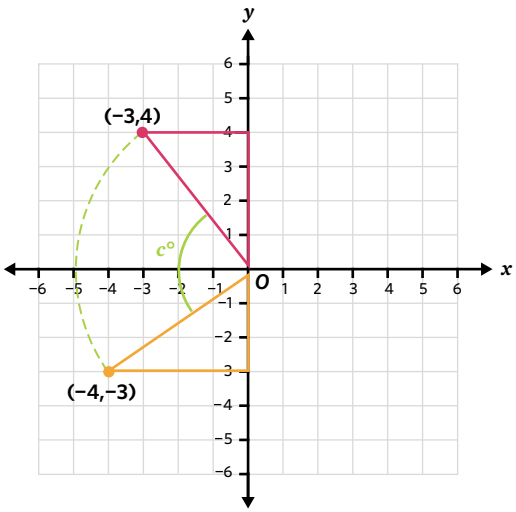
16. Two congruent right–angled triangles are placed next to each other as is displayed below.



a. What must the value of $c°$ equal?

b. What is the angle of rotation around O from $(3,2)$ to $(-2,3)$ and from $(-3,4)$ to $(-4,-3)$?

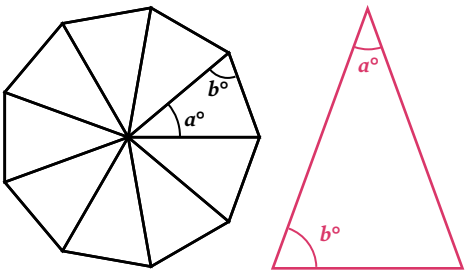




c. Use your answers to part **b** to prove that any point (x,y) rotated 90° anticlockwise around O is $(-y,x)$.

Extra spicy

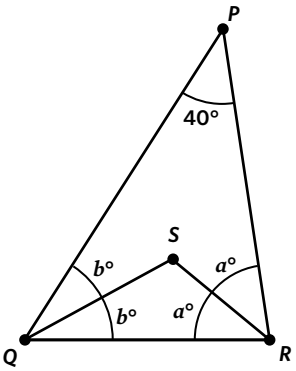
17. Nine isosceles triangles form a regular nonagon. What are the values of $a°$ and $b°$.



18. Each side of an isosceles triangle is a whole number in centimetres. Its perimeter has a length of 20 cm. How many possibilities are there for the lengths of its sides?

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

19. $\triangle PQR$ has an internal angle $\angle QPR = 40°$. If $\triangle QRS$ is formed with $\overline{QR}$ and line segments $\overline{QS}$ and $\overline{RS}$ that bisect $\angle PQR$ and $\angle PRQ$ respectively, what is the size of $\angle QSR$?



A. 110° **B.** 120° **C.** 130° **D.** 135° **E.** 140°

20. Eirron takes a sheet of paper and folds it in half 4 times and cuts a hole all the way through the sheet, as shown in the diagram.

How many holes are showing after they unfold the sheet of paper?

Remember this?

21. Simplify: $\dfrac{3^2 \times 4^2 \times 5^4 \times 8}{3 \times 3 \times (5 \times 2)^5}$

A. 0.8 **B.** 1.25 **C.** $\frac{64}{45}$ **D.** 64 **E.** 6445

22. What is the rule connecting shoe sizes in Australia and Europe?

Women's Shoe Sizes in Australia and Europe							
Australian size	5	6	7	8	9	10	11
European size	36	37	38	39	40	41	42

A. European Size $=$ (Australian size $\times$ 2) $+$ 20

B. European Size $=$ (Australian size $\times$ 5) $+$ 11

C. European Size $=$ Australian size $+$ 31

D. European Size $=$ Australian size $-$ 31

E. None of the above

23. Jessica has 2 kilograms of prosciutto.

She uses 1.25 kilograms for charcuterie boards.

How much prosciutto does she have left?

A. $\frac{1}{4}$ kg **B.** $\frac{1}{2}$ kg **C.** $\frac{3}{4}$ kg **D.** $1\frac{1}{4}$ kg **E.** $1\frac{3}{4}$ kg

Back to contents

7E Rules for congruent triangles

A triangle has six geometric features: three side lengths and three angles. It is possible to determine if two triangles are congruent with just the knowledge of the same three geometric features in each triangle, as long as one of those features is a side length.

LEARNING INTENTIONS

Students will be able to:

- understand that SSS, SAS, ASA and RHS rules are sufficient and necessary conditions for congruent triangles
- use the SSS, SAS, ASA and RHS rules to show that two triangles are congruent
- solve problems with congruent triangles.

KEY TERMS AND DEFINITIONS

The **hypotenuse** is the longest side of a right-angled triangle; it is always opposite to the right angle.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

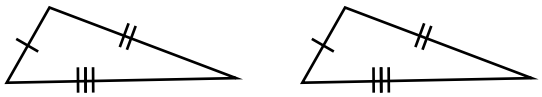
Image: satit_srihin/Shutterstock.com

A geodisic dome is constructed with multiple congruent triangles. This is an example of how a 3D surface can be approximated with triangles.

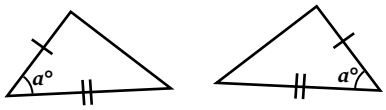
Key idea

1. Two triangles are congruent if they satisfy the conditions of these tests:

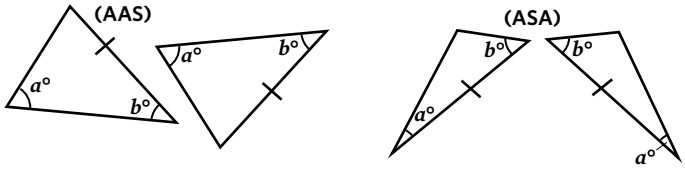
All three sides in each triangle have a corresponding side length **(SSS)**.



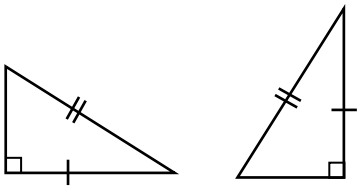
Two pairs of corresponding sides and a corresponding angle (the angle between them) **(SAS)**.



Two corresponding pairs of angles and one corresponding pair of side lengths.



Two right-angled triangles have a corresponding hypotenuse and another pair of corresponding side lengths **(RHS)**.

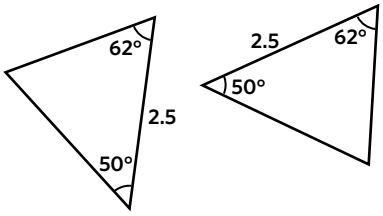


Worked example 1

Testing triangles for congruency

Determine if the following triangles are congruent. Explain using the congruence tests.

a. WE1a



Working

Two corresponding internal angles: 50° and 62°.

One corresponding side length: 2.5 units.

The corresponding side lengths are in the same position with respect to the corresponding angles.

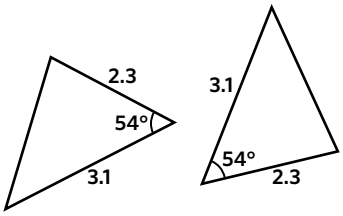
∴ These triangles are congruent by ASA.

Thinking

Step 1: Find corresponding features.

Step 2: Decide if corresponding features satisfy a congruence test.

b. WE1b



Working

Two corresponding side lengths of 2.3 and 3.1.

One corresponding angle of 54°.

The corresponding angle is adjacent to both corresponding side lengths.

∴ These triangles are congruent by SAS

Thinking

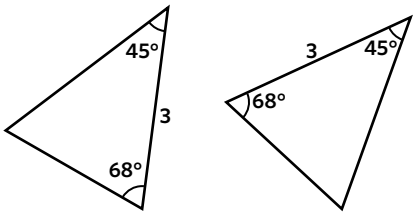
Step 1: Find corresponding features.

Step 2: Decide if corresponding features satisfy a congruence test.

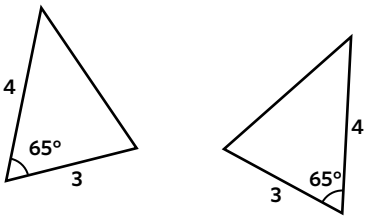
Student practice

Determine if the following triangles are congruent. Explain using the congruence tests.

a.



b.



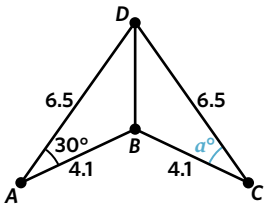
Back to contents

Worked example 2

Congruent triangles in geometry

Determine the values of the pronumerals. Explain using the congruence tests.

a. WE2a



Working

$\triangle ABD$ has sides $\overline{AB} = 4.1$, $\overline{AD} = 6.5$ and $\overline{BD}$.

$\triangle BCD$ has sides $\overline{BC} = 4.1$, $\overline{CD} = 6.5$ and $\overline{BD}$.

$\therefore \triangle ABD \equiv \triangle BCD$ by SSS.

$\angle DAB$ is corresponding with $\angle DCB$.

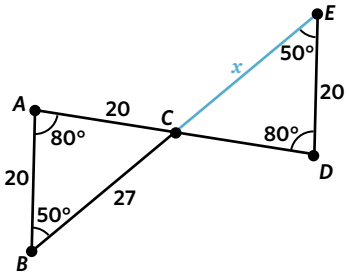
$a° = 30°$

Thinking

Step 1: Outline key features from the two triangles to identify if they are congruent and if they satisfy the conditions.

Step 2: Find the side or angle that corresponds with the pronumeral.

b. WE2b



Working

$\triangle ABC$ has angles $\angle CAB = 80°$ and $\angle ABC = 50°$ with side length $\overline{AB} = 20$ between them.

$\triangle CDE$ has angles $\angle CDE = 80°$ and $\angle DEC = 50°$ with side length $\overline{ED} = 20$ between them.

$\therefore \triangle ABC \equiv \triangle$

$\therefore \triangle ABC \equiv \triangle CDE$ by ASA or AAS.

$\overline{BC}$ is corresponding with $\overline{CE}$.

$x = 27$

Thinking

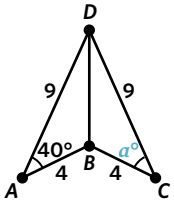
Step 1: Outline key features from the two triangles to identify if they are congruent and if they satisfy the conditions.

Step 2: Find the side or angle that corresponds with the pronumeral.

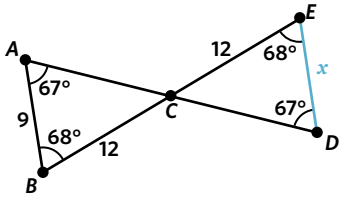
Student practice

Determine the values of the pronumerals. Explain using congruence tests.

a.



b.

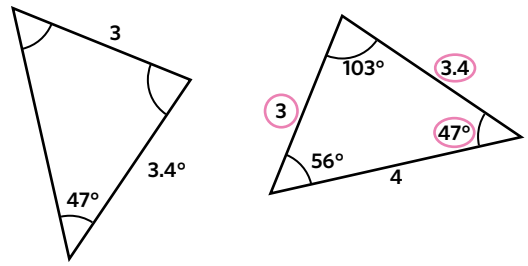


7E Questions

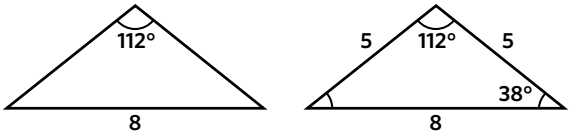
Understanding worksheet

1. Circle the corresponding sides and angles.

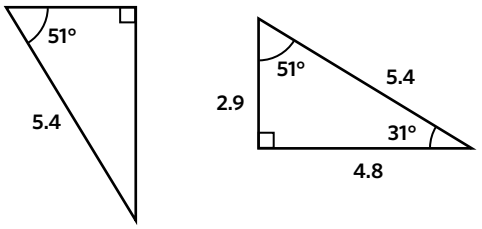
Example



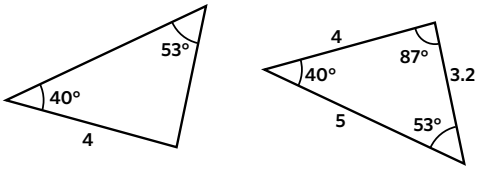
a.



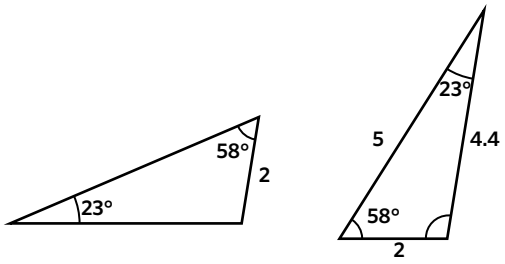
b.



c.

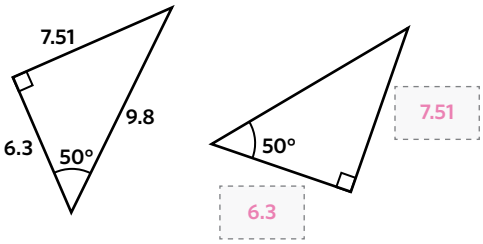


d.

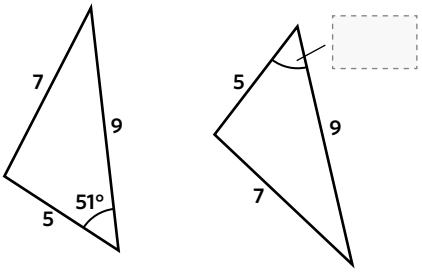


2. Fill in the blanks for the following congruent triangles.

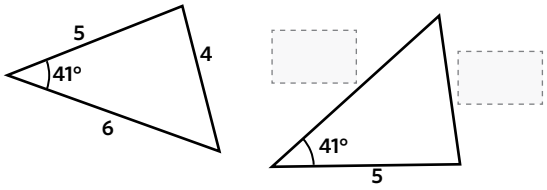
Example



a.



b.



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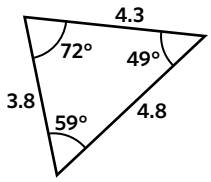
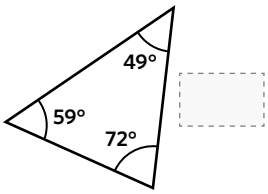
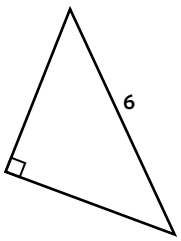
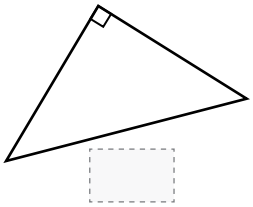




d.





3. Fill in the blanks by using the words provided.

side lengths | equal | hypotenuse | angles | adjacent

It is necessary and sufficient to know if 3 particular features of two triangles are equal to know that they are congruent and all 6 features are _____. The SSS test is satisfied if all three _____ are equal. The SAS test is satisfied if the lengths of two sides and an _____ angle are equal. The AAS test is satisfied if two _____ and a corresponding side length are equal. The RHS test is satisfied if two right-angled triangles have an equal _____ and one other corresponding side length.

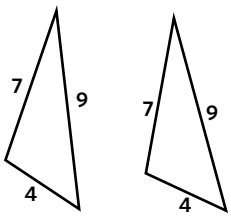
Fluency

Question working paths

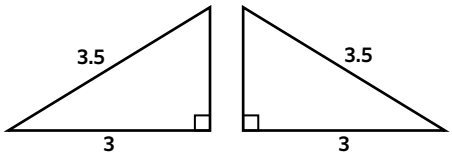
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Determine if the following triangles are congruent with reasons. WE1

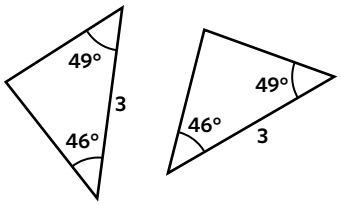
a.



b.



c.



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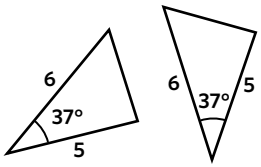

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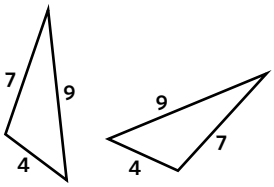

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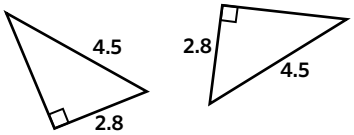
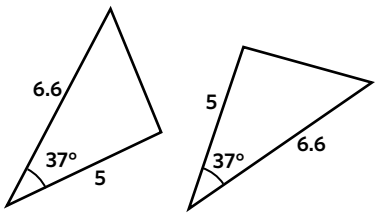
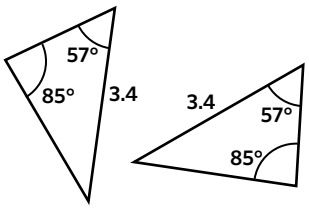


g.

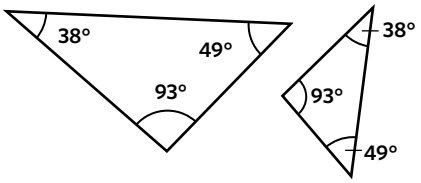


h.

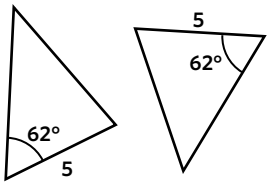


5. Determine if the following triangles satisfy a test for congruence.

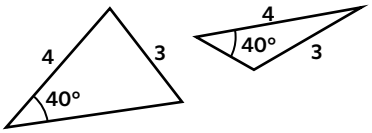
a.



b.



c.



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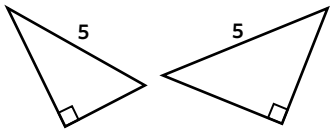

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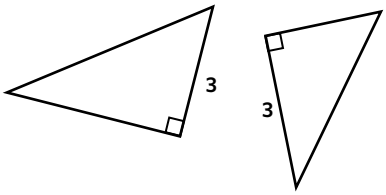

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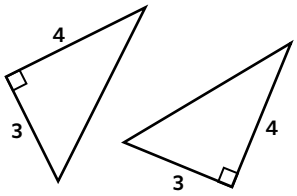

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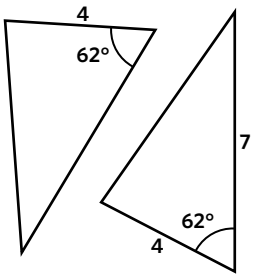

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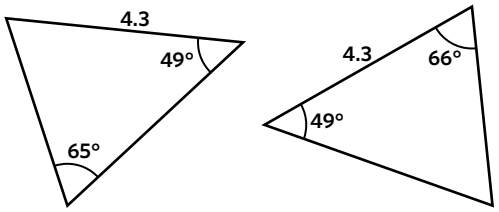

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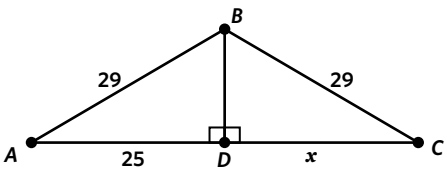
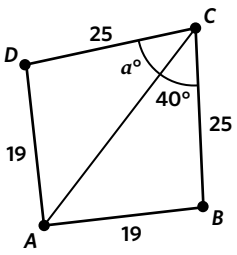
6. Determine the values of the pronumerals. Explain using the congruence tests. WE2

a.



b.



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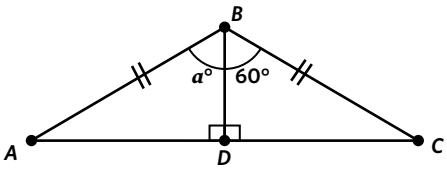

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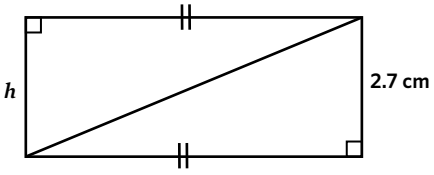

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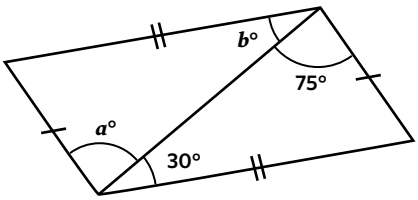

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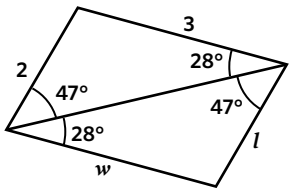
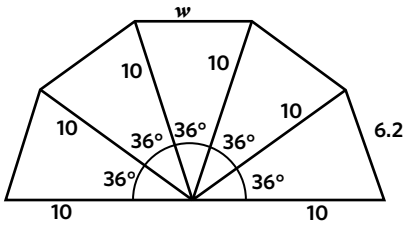
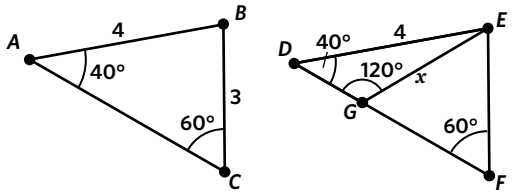


g.



h.



7. How do we know these triangles are congruent?

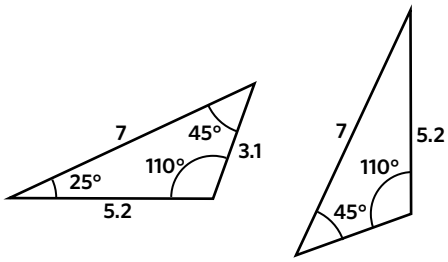
A. All three angles are equal.

B. Two sides are equal.

C. One angle and a side are equal.

D. Two angles are equal.

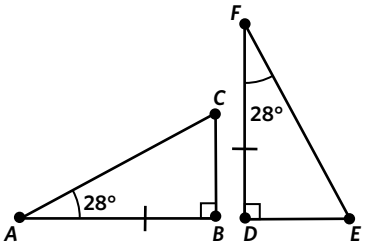
E. Two angles and a side are equal.



Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Determine if $\triangle ABC \equiv \triangle DEF$ with reasons.



Student A

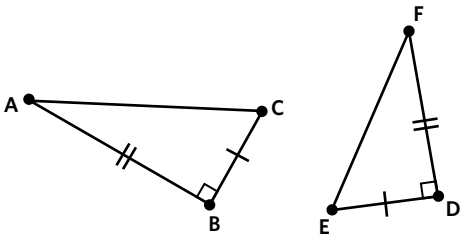
$\angle ABC = \angle FDE$ and $\angle BAC = \angle DFE$ and the side between them is the same length.

$\therefore \triangle ABC \equiv \triangle DEF$ by (ASA).

Student B

$\triangle ABC$ and $\triangle DEF$ are not congruent because (RHS) requires the hypotenuse to be equal.

b. Determine if $\triangle ABC \equiv \triangle DEF$ with reasons.



Student A

$\triangle ABC$ and $\triangle DEF$ are not congruent because (RHS) requires the hypotenuse to be equal.

Student B

$\angle ABC = \angle FDE$ and both adjacent sides to the angles are equal.

$\therefore \triangle ABC \equiv \triangle DEF$ by (SAS).

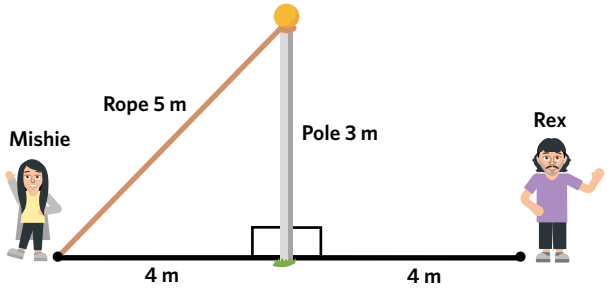
Problem solving

Question working paths

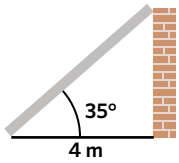
Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. I have three logs and make a triangle. I don't like the triangle that I have made, can I make a different triangle with the same three logs? Why or why not?

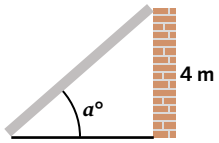
10. Mishie and Rex are securing a pole in the ground and using ropes and pegs to secure the pole. The pole is 3 m high. Mishie places a peg at 4 m along the ground from the pole. Mishie measures the rope from the top of the pole to the peg. This has a measurement of 5 m. Rex places his peg directly opposite to Mishie's peg, also 4 m away from the pole. Will Rex also require 5 m of rope? Explain.



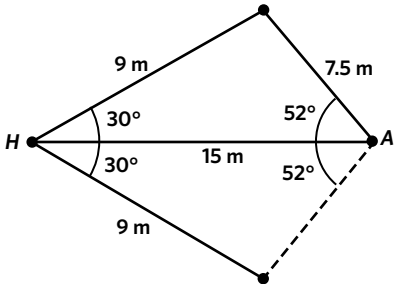
11. Petro's ladder is touching the ground 4 m away from the wall. It has an angle of 35° with the ground.



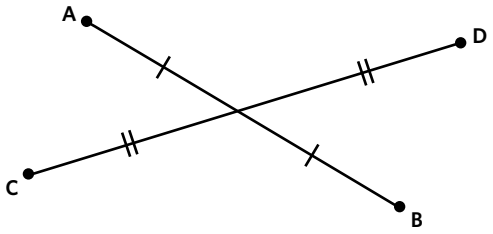
At what angle with the ground does Petro's ladder need to be to reach 4 m up the wall?



12. Héritier and Alywn are playing rugby. They face off 15 m apart. Round one Héritier turned left 30° and Alywn turned right 52°. Alwyn ran 7.5 m to intercept Héritier where he would be after running 9 m. In the next round, Héritier turns right at 30° and Alwyn turns left at 52°. How far does Alwyn need to run to intercept Héritier when he runs 9 m?



13. Alice and Bob hold a rope at either end, while Chris and Dianne hold another rope at either end. If their ropes cross exactly in the middle of each other, how do we know that there is the same distance between Alice and Charlie as there is between Bob and Dianne?

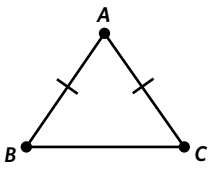


Reasoning

Question working paths

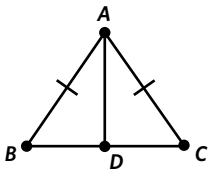
Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. Maggie and Michael are decorating their guest room with triangle nanoleaf LED lights. They begin with an isosceles triangle $\triangle ABC$.



a. What is the relationship between side lengths $\overline{AB}$ and $\overline{AC}$ and between the angles $\angle ABC$ and $\angle ACB$?

Maggie and Michael decide to cut the triangle in half, so they can work on a different design each.



b. If side lengths $\overline{BD}$ and $\overline{DC}$ are the same length, how do Maggie and Michael know that $\triangle ABD \equiv \triangle ADC$?

c. What is the relationship between $\angle BAD$ and $\angle DAC$?

d. Explain if Maggie and Michael have a right-angled triangle each.

e. Create your own nanoleaf light design.

15. Draw a 5 cm line marking the vertices A and B at either end.

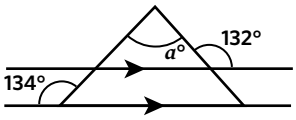
a. Draw a line that is at 35° to the line $\overline{AB}$ starting at A. Draw another line that is at 65° to the line $\overline{AB}$ starting at B. Mark the intersections of the lines with the vertex C. How many possible points could these lines meet at?

b. Draw a line that is at 35° to the line $\overline{AB}$ starting at B and another a line that is at 65° to the line $\overline{AB}$ starting at A. Mark the intersections of the lines with the vertex C. How many possible points could these lines meet at?

c. Using your answers in parts **a** and **b**, demonstrate why AAS as a congruence test must be true.

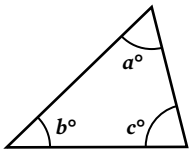
Extra spicy

16. What is the value of $a°$ in the diagram?

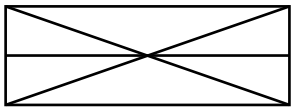


A. 46° **B.** 48° **C.** 86° **D.** 92° **E.** 94°

17. The value of $a°$ in the diagram is equal to the mean of $b°$ and $c°$. What is the value of $a°$?



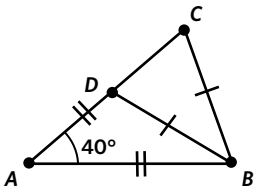
18. How many triangles are there of any size in this diagram?



19. In the diagram $\overline{AB} = \overline{AC}$ and $\overline{BD} = \overline{BC}$

What is the value of $\angle ABD$?

A. 15° **B.** 20° **C.** 25° **D.** 30° **E.** 35°



Remember this?

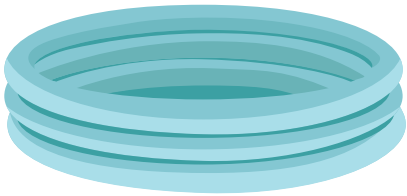
20. Stefanie is a manager at a bakery.

She notices that 64 of their 1000 orders are for chocolate croissants.

Which decimal represents this?

A. 0.0064 **B.** 0.064 **C.** 0.64 **D.** 6.4 **E.** 64

21. What is the best estimate for the total capacity of this blowup pool?



A. 270 millilitres **B.** 2700 millilitres **C.** 2.7 litres **D.** 27 litres **E.** 270 litres

22. How many degrees does the **minute hand** turn in 45 minutes?

A. 0° **B.** 90° **C.** 180° **D.** 270° **E.** 360°

Back to contents

7F Congruence and quadrilaterals

All convex quadrilaterals can be represented as triangles by drawing diagonals between opposite vertices. The properties of special quadrilaterals and their relationship to each other can be deduced by investigating the relationship between the triangles using tests for congruence and angles created at a point.

LEARNING INTENTIONS

Students will be able to:

- understand the relationship between special quadrilaterals (parallelograms and rectangles, squares and rhombuses)
- establish properties of quadrilaterals using congruent triangles.

KEY TERMS AND DEFINITIONS

A **diagonal** line joins one corner to the opposite corner of a four-sided shape and is not an edge.

All internal angles of a **convex polygon** are less than 180°.

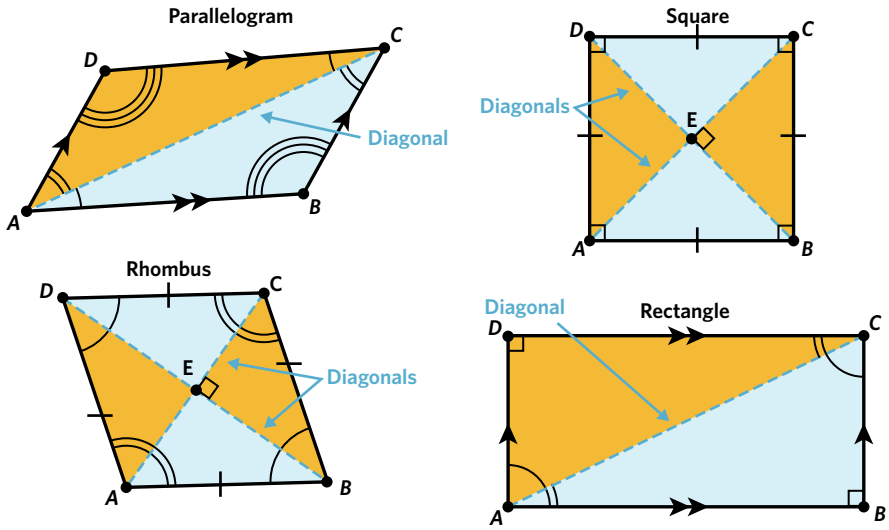
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: VTT Studio/Shutterstock.com

Almost every wall in a building is a quadrilateral. Triangles create the strongest possible structure. Engineers and architects usually rely upon triangles to increase the structural integrity of large buildings.

Key ideas

1. Properties of quadrilaterals can be investigated by breaking them into triangles with diagonals.



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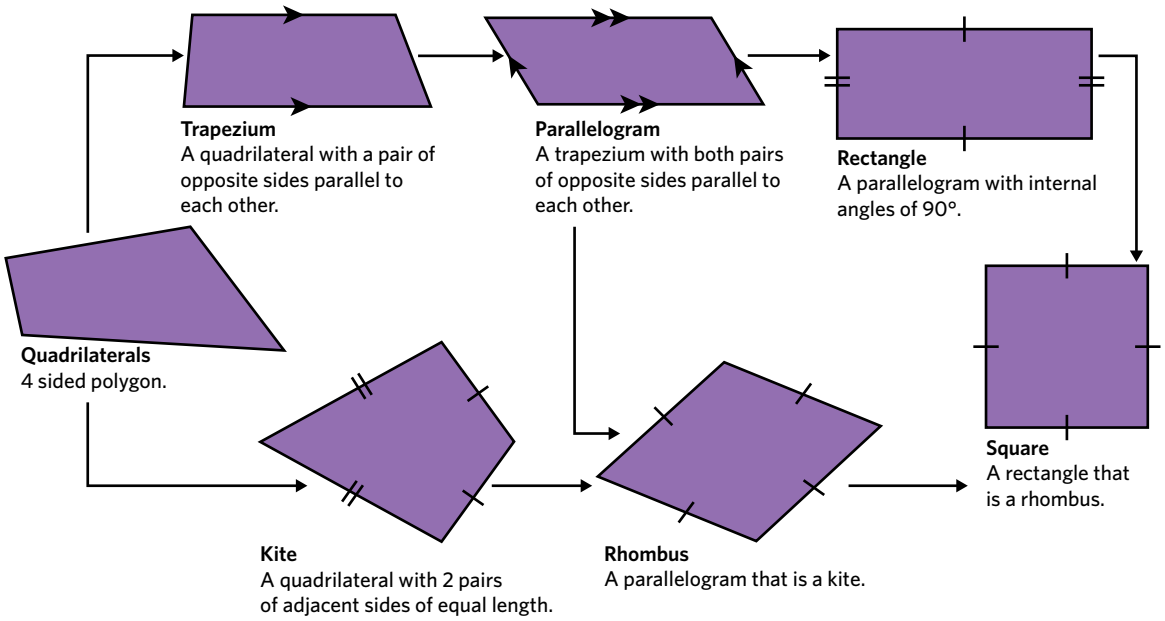
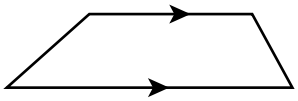
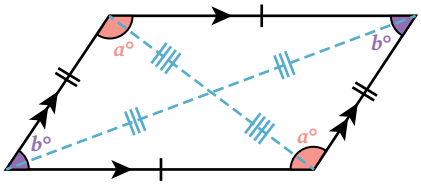
2. A flow chart can represent the relationships between special quadrilaterals.



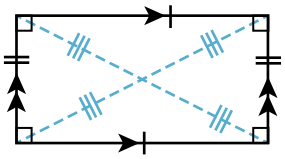
3. Some quadrilaterals are special cases of other quadrilaterals.



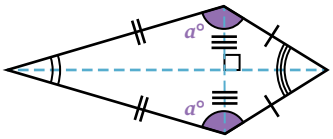
Trapezium
All trapeziums are quadrilaterals.



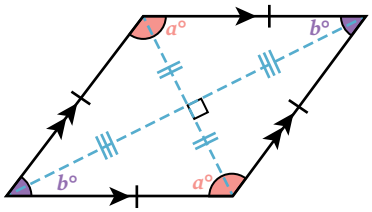
Parallelogram
All parallelograms are quadrilaterals.



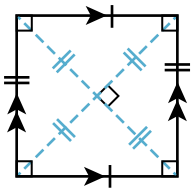
Rectangle
All rectangles are parallelograms.



Kite
All kites are quadrilaterals.



Rhombus
All rhombuses are kites and parallelograms.



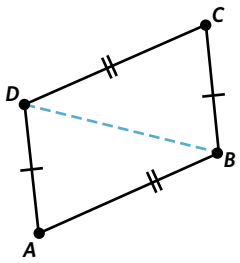
Square
All squares are rhombuses and rectangles.

Worked example 1

Finding congruent triangles in quadrilaterals

Determine which test establishes the triangles are congruent.

a. WE1a



$\triangle ADB$ and $\triangle DBC$

Working

$\overline{AD} = \overline{BC}, \overline{AB} = \overline{DC}$ and $\overline{BD}$ is shared.

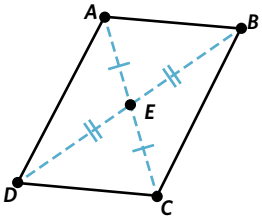
$\therefore \triangle ABD \equiv \triangle DBC$ by SSS

Thinking

Step 1: Establish corresponding sides or angles.

Step 2: Determine if they satisfy a congruence test.

b. $\triangle AEB$ and $\triangle DEC$ WE1b



Working

$\overline{AE} = \overline{EC}, \overline{BE} = \overline{ED}$ and $\angle AEB = \angle DEC$ (vertically opposite angles)

The corresponding angle is adjacent to two corresponding sides.

$\therefore \triangle AEB \equiv \triangle DEC$ by SAS

Thinking

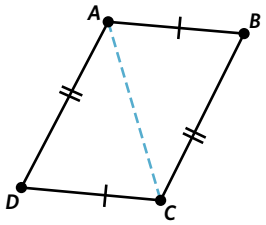
Step 1: Establish corresponding sides or angles.

Step 2: Determine if they satisfy a congruence test.

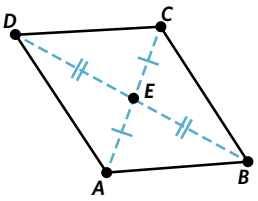
Student practice

Determine which test establishes the triangles are congruent.

a. $\triangle ABC$ and $\triangle DAC$



b. $\triangle AED$ and $\triangle CEB$

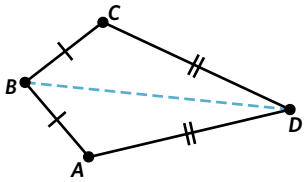


Worked example 2

Proving properties of special quadrilaterals

Prove the following properties of the quadrilaterals.

a. $\angle DAB = \angle DCB$ WE2a



Working

The diagonal $\overline{BD}$ creates triangles $\triangle BAD$ and $\triangle BCD$.

$\overline{BA} = \overline{BC}, \overline{AD} = \overline{CD}$ and $\overline{BD}$ is shared.

$\therefore \triangle BAD \equiv \triangle BCD$ by SSS.

$\angle DAB$ is adjacent to $\overline{BA}$ and $\overline{AD}$.

$\angle DCB$ is adjacent to $\overline{BC}$ and $\overline{CD}$.

$\therefore \angle DAB = \angle DCB$

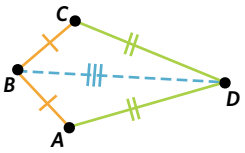
Thinking

Step 1: Determine the relevant triangles.

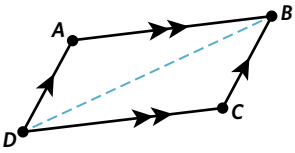
Step 2: Determine if they are congruent.

Step 3: Explain how that proves the property.

Visual support



b. $\overline{AB} = \overline{DC}$ WE2b



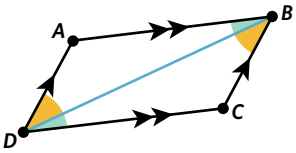
Working

The diagonal $\overline{BD}$ creates triangles $\triangle BAD$ and $\triangle BCD$.

$\angle BDC = \angle DBA$ (alternate angles)

$\angle ADB = \angle DBA$ (alternate angles)

$\overline{BD}$ is shared and corresponding to both equal angles.



$\therefore \triangle BAD \equiv \triangle BCD$ by ASA

$\overline{AB}$ and $\overline{DC}$ are corresponding because they are adjacent to corresponding angles.

$\therefore \overline{AB} = \overline{DC}$

Thinking

Step 1: Determine the relevant triangles.

Step 2: Determine of they are congruent.

Step 3: Explain how that proves the property.

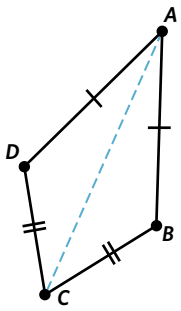
Continues →

Back to contents

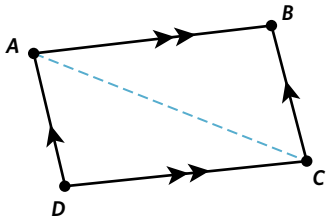
Student practice

Prove the following properties of the quadrilaterals below.

a. $\angle CDA = \angle CBA$



b. $\overline{AD} = \overline{BC}$

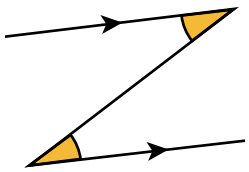


7F Questions

Understanding worksheet

1. Circle the relationship between the following pairs of angles.

Example



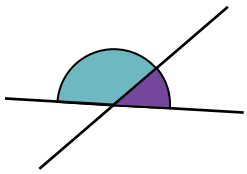
Alternate angles

Vertically opposite angles

Supplementary angles

Complementary angles

a.



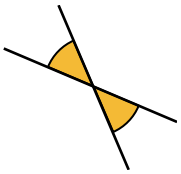
Alternate angles

Vertically opposite angles

Supplementary angles

Complementary angles

b.



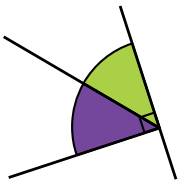
Alternate angles

Vertically opposite angles

Supplementary angles

Complementary angles

c.



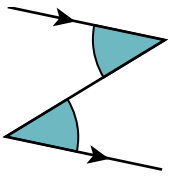
Alternate angles

Vertically opposite angles

Supplementary angles

Complementary angles

d.



Alternate angles

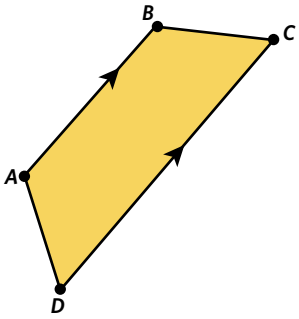
Vertically opposite angles

Supplementary angles

Complementary angles

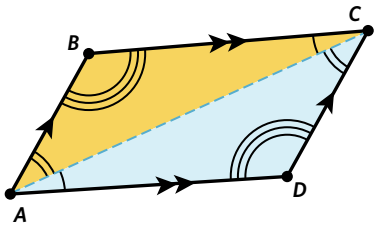
2. Circle the names of the following shapes.

Example



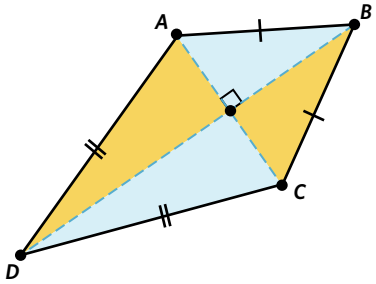
Quadrilateral
Trapezium
Kite
Parallelogram
Rectangle
Rhombus
Square

a.



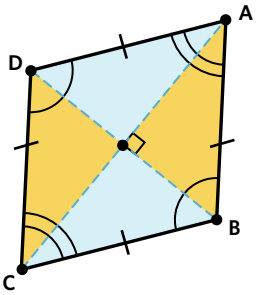
Quadrilateral
Trapezium
Kite
Parallelogram
Rectangle
Rhombus
Square

b.



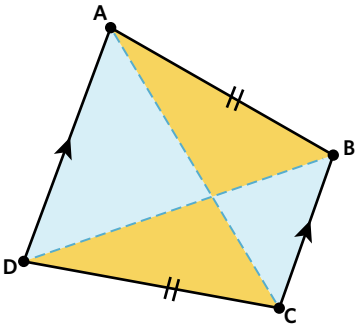
Quadrilateral
Trapezium
Kite
Parallelogram
Rectangle
Rhombus
Square

c.



Quadrilateral
Trapezium
Kite
Parallelogram
Rectangle
Rhombus
Square

d.



Quadrilateral
Trapezium
Kite
Parallelogram
Rectangle
Rhombus
Square

3. Fill in the blanks by using the words provided.

intersections	quadrilateral	transversals	triangles

A diagonal of a _____ is a line between two opposite vertices. The _____ that are created with diagonals make it possible to establish all of the known properties of quadrilaterals. We can use the diagonal lines as _____ when opposite sides are parallel. The _____ made by the diagonals form vertically opposite and supplementary angles.

Fluency

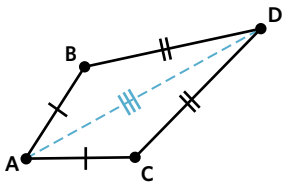
Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b,c), 6 (a,b,c,d), 7 (a,b,c), 8 (a,b,c), 9	4 (b,c), 5 (b,c,e), 6 (c,d,e,f), 7 (c,d,e,f), 8 (b,c,e), 9	4 (c,d), 5 (c,d,e), 6 (e,f,g,h), 7 (e,f,g,h), 8 (c,d,e), 9

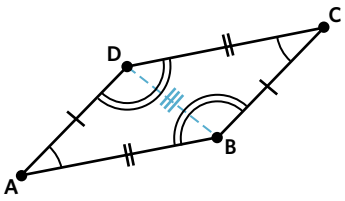
4. State whether the triangles formed by the following quadrilaterals and diagonals are:

- scalene
- isosceles
- right-angled.

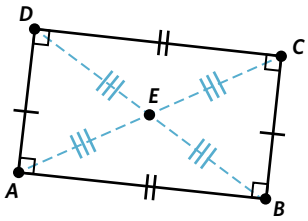
a.



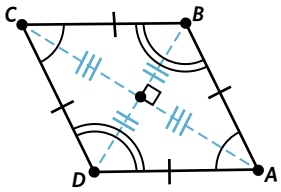
b.



c.



d.



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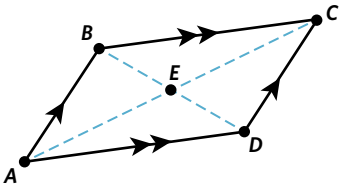
5. State whether the following groups of angles are: WE1a,b

- supplementary
- alternate
- vertically opposite.



a. $\angle AED$ and $\angle BEC$

b. $\angle BEC$ and $\angle DEC$

c. $\angle ECB$ and $\angle EAD$

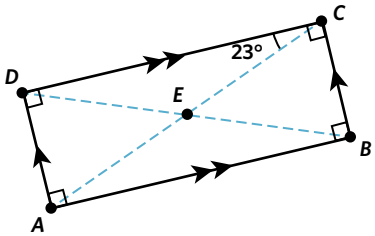
d. $\angle ACD$ and $\angle CAB$

e. $\angle DAB$ and $\angle ABC$

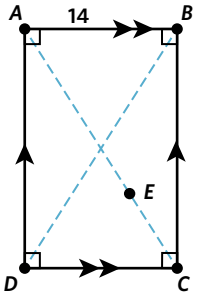
---

6. Determine the values of the following angles and side lengths. WE2

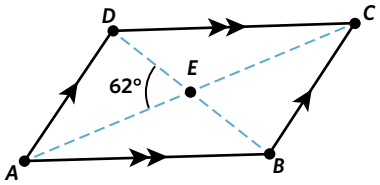
a. $\angle ECB$



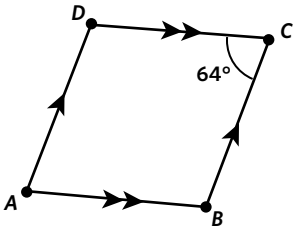
b. $\overline{DC}$.



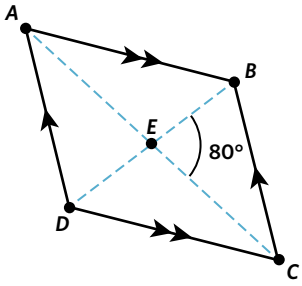
c. $\angle CEB$



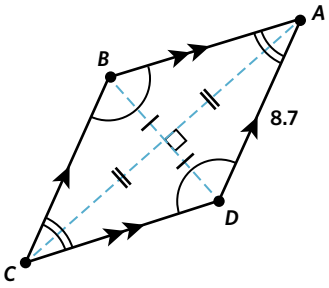
d. $\angle ABC$



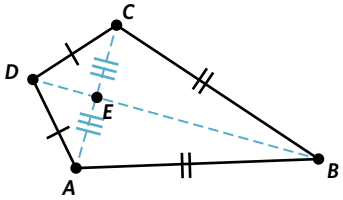
e. $\angle CED$



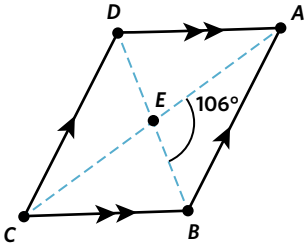
f. $\overline{CD}$



g. $\angle DEA$



h. $\angle ECD + \angle EDC$



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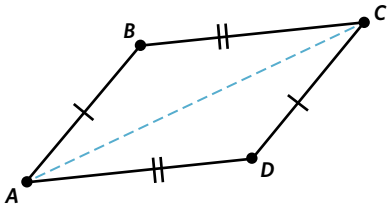
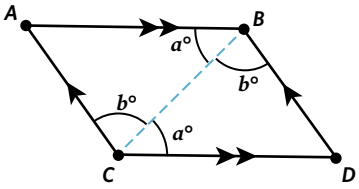
7. Determine which test establishes the triangles are congruent. WE1

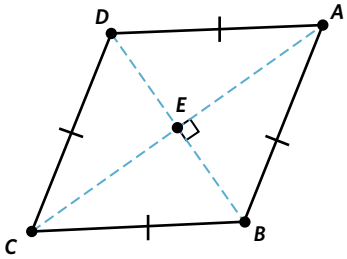
a. $\triangle ADC$ and $\triangle ABC$



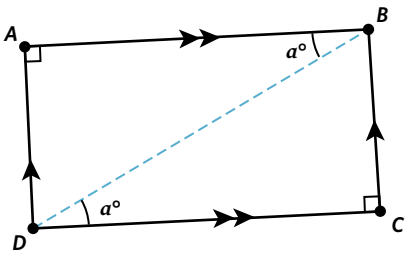
b. $\triangle ABC$ and $\triangle DBC$



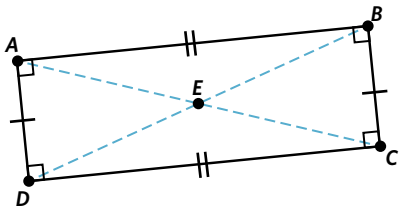
c. $\triangle BEA$ and $\triangle DEA$



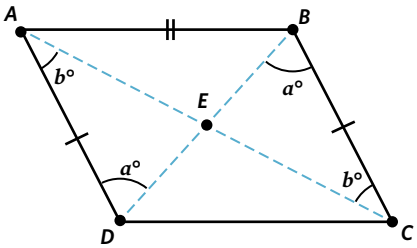
d. $\triangle ABD$ and $\triangle DBC$



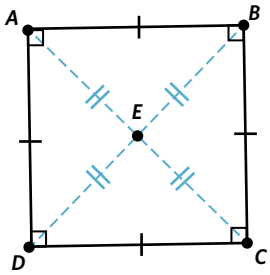
e. $\triangle ABC$ and $\triangle DBC$



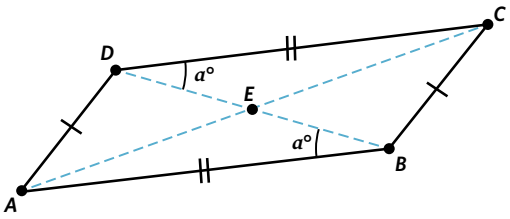
f. $\triangle AED$ and $\triangle BEC$



g. $\triangle AEB$, $\triangle AED$, $\triangle DEC$, and $\triangle CEB$

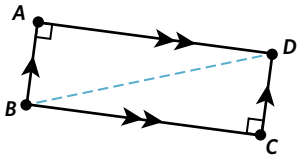


h. $\triangle AEB$ and $\triangle DEC$

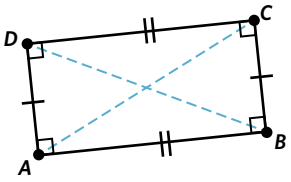


8. Prove the following properties for the quadrilaterals below. WE2

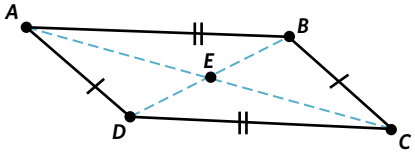
a. $\overline{AD} = \overline{BC}$



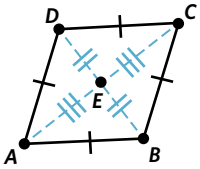
b. $\overline{AC} = \overline{BD}$



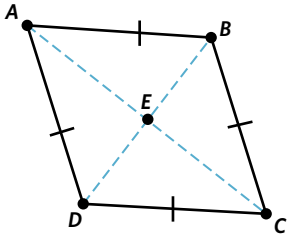
c. $\overline{AE} = \overline{EC}$



d. $\angle DCE = \angle DAE$



e. $\angle DEC = 90°$



9. Which quadrilateral has diagonals that bisect each other?

A. Square
B. Rectangle
C. Parallelogram
D. Rhombus
E. All of the above

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. List the quadrilaterals that have diagonals that are perpendicular to each other.

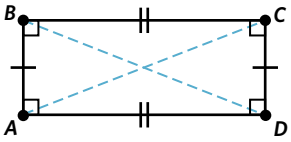
Student A

Rhombus, kite

Student B

Rhombus, kite, square

b. Prove $\triangle ADC \equiv \triangle BCD$



Student A

$\overline{CD} = \overline{BA}, \overline{AD} = \overline{BC}$ and both diagonals are the same length.

$\therefore \triangle ADC \equiv \triangle BCD$ by SSS

Student B

$\overline{CD} = \overline{BA}, \overline{AD} = \overline{BC}$ and $\angle ADC = \angle BCD$

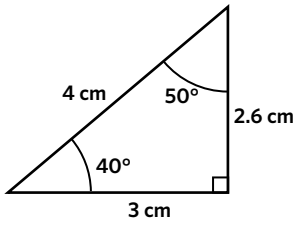
$\therefore \triangle ADC \equiv \triangle BCD$ by SAS

Problem solving

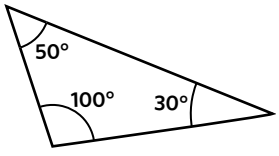
Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

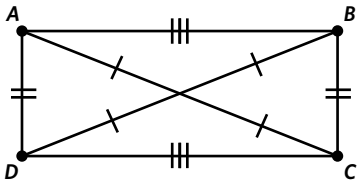
11. Makena has four right angled triangular shaped blocks, as shown in the diagram. He puts them together to make a quadrilateral. What are the names of the shapes Makena can make with the triangular blocks?



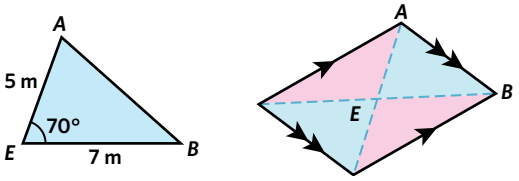
12. Rebecca has two triangular stickers that are scalene and congruent. Draw a diagram of the three parallelograms Rebecca can make with the stickers, including any measurements.



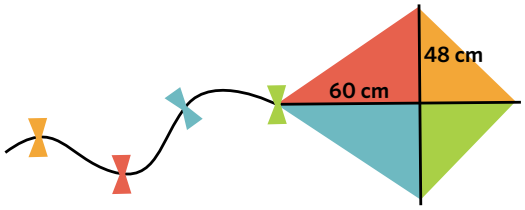
13. When Angelo is creating house frames he uses a tape measure to check for right angles in the quadrilateral shapes he creates. He measures all four sides and the diagonals. Show how Angelo can prove that all interior angles in a quadrilateral are equal to exactly 90° by using the diagram below.



14. Donato has a pair of blue carpet triangles, pictured below, that he knows are congruent because they both have an angle of 70° between side lengths of 7 m and 5 m. Use the given information to determine as many of the length and angle measurements as possible in the diagram of the pattern below.



15. Toula has a kite that has diagonals with lengths 48 cm and 60 cm, pictured below. Calculate the area of Toula's kite using the formula for the area of a triangle, $\frac{\text{base} \times \text{height}}{2}$.

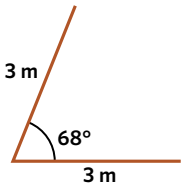


Reasoning

Question working paths

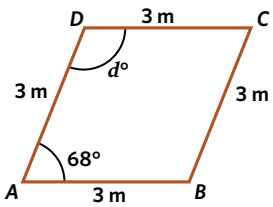
Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Salvadore was planning to build a slanted garden feature for hanging plants with four 3 m beams of wood. He began by laying one along the ground and another that was raised at an angle of 68° to the beam on the ground, pictured below.



a. What shape could Salvadore make with his four identical beams, using the 68° join?

Salvadore marks the corners of the shape from part **a** with letters *A*, *B*, *C*, and *D* to help explain his plans to his friends.



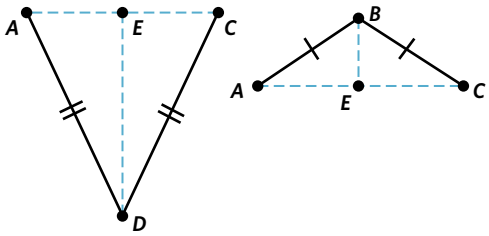
b. What is the size of the angle joining the beams at point *D*?

c. To help stabilise his feature with structural supports, Salvadore is going to install an extra beam between points A and C, and another one between points B and D. At what angle will these two beams intersect?

d. What is the angle the structural supports from part **c** will make with the horizontal ground?

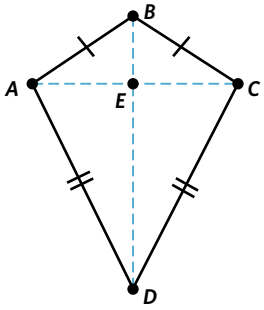
e. What sorts of plants can be displayed in a hanging garden?

17. Consider the isosceles triangles below.



a. If side $\overline{AE} = \overline{EC}$ how do we know that $\triangle ABE \equiv \triangle BEC$ and $\triangle ADE \equiv \triangle DEC$?

b. How does your answer in part **a** demonstrate that the diagonals are perpendicular to each other in a kite and the diagonal $\overline{BD}$ bisects the angles $\angle ABC$ and $\angle ADC$?



c. How does your answer to part **b** demonstrate that the diagonals of a rhombus and square are perpendicular to each other and bisect the internal angles?

Extra spicy

18. A square is folded exactly in half and folded in half again. Which of the following could not be the resulting shape?

A. 

B. 

C. 

D. 

E. 

19. A square is divided into three congruent rectangles.

The middle rectangle is removed from the square and placed on the side to create an octagon, see diagram.

What is the ratio of the length of the perimeter of the square to the length of the perimeter of the octagon?

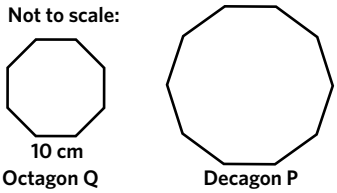
A. 3 : 5 **B.** 2 : 3 **C.** 5 : 8 **D.** 1 : 2 **E.** 1 : 1

20. The perimeter of a regular decagon P is 8 times the perimeter of a regular octagon Q.

Each edge of the regular octagon Q is 10 cm long.

How long is each edge of the regular decagon P?

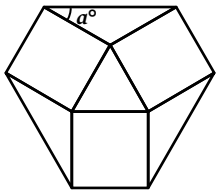
Not to scale:



Back to contents

21. An equilateral triangle is surrounded by three squares, as shown.

What is the value of $a°$?



Remember this?

22. There are 15 family members at an all you can eat restaurant for dinner.

The cost was \$22.50 per person, plus \$45 for the drinks.

Which of these shows how to calculate the total cost of the dinner in dollars?

A. $22.50 + 45 + 15$

B. 45×22.50

C. $(22.50 \times 15) + 45$

D. $(45 \times 15) + 22.50$

E. $(22.50 \times 45) + 15$

23. Jane created an equation $4x + 2 = 22$.

What does x equal?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

24. This table shows the attendance of people at a festival.

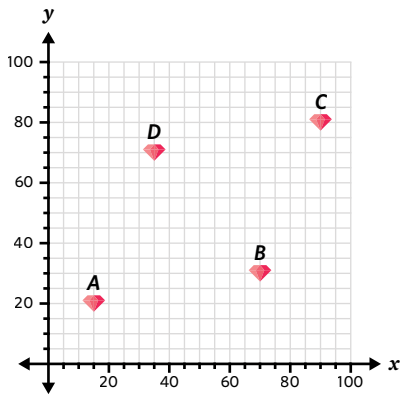
	Male	**Female**
18-25 years old	554	624
25+ years old	432	390

What fraction of the females at the festival were 18–25 years old?

A. $\frac{390}{2000}$ **B.** $\frac{625}{2000}$ **C.** $\frac{624}{1014}$ **D.** $\frac{390}{554}$ **E.** $\frac{625}{432}$

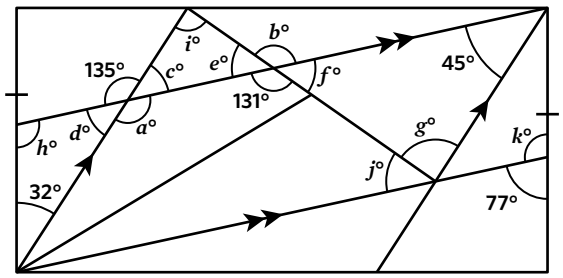
Chapter 7 extended application

1. A player is moving around the board shown below. Starting at any gem, the aim of this game is to collect all gems using the shortest path possible.



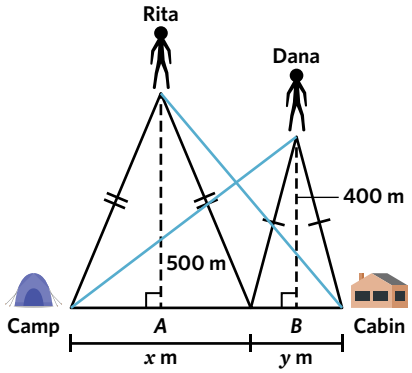
- **a.** Describe the translation from gem A (15,20) to gem B (70,30).
- **b.** Describe the translation from gem B to gem C (90,80).
- **c.** Determine the type and properties of the quadrilateral formed by joining the points A, B, C and D with straight lines.
- **d.** Using congruent triangles, prove that the direct distance from A to B to C is equal to the distance from A to D to C.
- **e.** The shortest direct distance between any two gems is D to B. Starting at any gem on the board and moving in any direction, including diagonally, show the shortest path possible whilst collecting all gems. List the vectors describing this translation.
- **f.** Board games often require us to use arithmetic and spatial reasoning. Name at least one game that can be won by applying maths.

2. Ben is making a wooden puzzle for his daughter. He sketched the design below on a piece of A4 paper using a ruler, and he measured three of the angles using a protractor. The puzzle must be cut accurately, so that it fits perfectly when assembled the correct way.



- **a.** Count the number of pieces in the puzzle.
- **b.** Determine the sizes of angles A to F.
- **c.** Determine the sizes of angles G to K.
- **d.** Use different colours to identify the pairs of congruent shapes made by the lines in Ben's design.
- **e.** Determine the sizes of the interior angles of the shapes from part **d**.
- **f.** Do you like to receive homemade gifts? Give a reason for your preference.

3. Ria and Dana have reached the tops of two mountains, 500 and 400 metres tall. They will zipline along the blue lines down to camp and the cabin. The zip lines are equal in length in the view below.

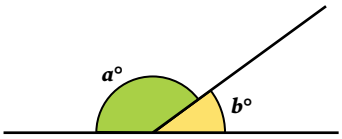


- **a.** Determine the distance between point A and the cabin.
- **b.** Determine the distance between point B and camp.
- **c.** The horizontal distance between the camp and the cabin is 600 m. Write an expression for the length of x in terms of y.
- **d.** Determine the lengths of x and y, using the information provided.
- **e.** Determine the distance between points A and B.
- **f.** Name three things you should always take with you when going camping.

Chapter 7 review

Multiple choice

1. Which option describes the relationship between the two angles in the following diagram? 7A



A. Complementary
B. Supplementary
C. Vertically opposite
D. Alternate
E. Co-interior

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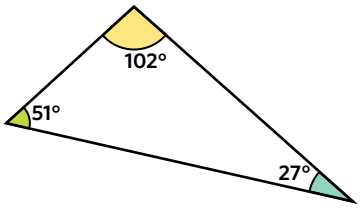
2. What type(s) of triangle could the following triangle be classified as? 7B



A. Scalene triangle
B. Acute angled triangle
C. Obtuse angled triangle
D. Options A and B
E. Options A and C

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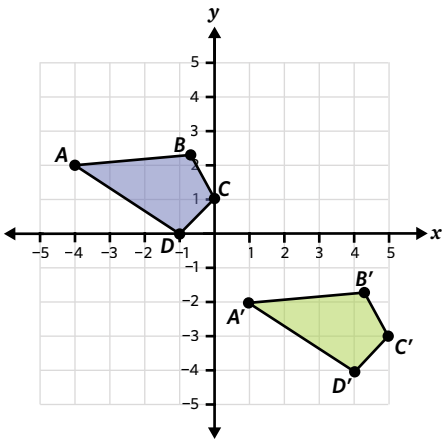
3. Which vector translated *ABCD* to its image? 7C



A. $(-1,-1)$ **B.** $(1,4)$ **C.** $(5,-4)$ **D.** $(-5,4)$ **E.** $(5,1)$

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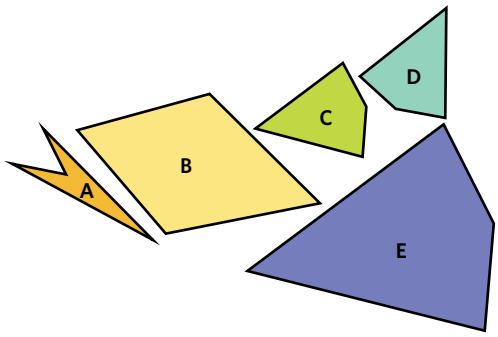
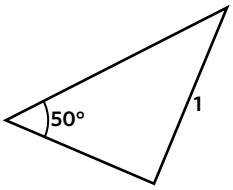
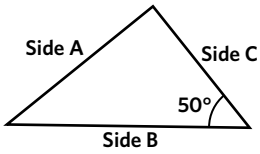
4. Identify which shapes are congruent in the scaled diagram below. 7D,7F



A. B and E **B.** C and D **C.** C, D and E **D.** A, B and C **E.** C, D and E

5. The following two scalene triangles are congruent. Which side is equal to 1? 7E



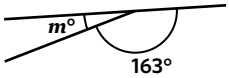


A. Side A
B. Side B
C. Side C
D. Sides B and C
E. Impossible to determine from the given information.

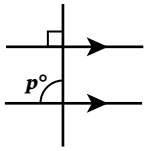
Fluency

6. Determine the value(s) of the pronumeral(s), with reasons, for each diagram. 7A

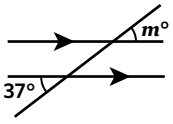
a.



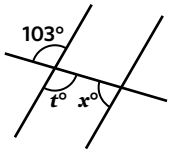
b.



c.

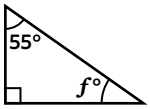


d.

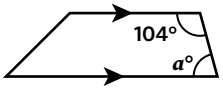


7. Determine the value(s) of the pronumeral(s), for each diagram. 7A

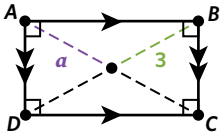
a.



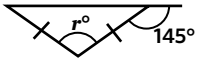
b.



c.

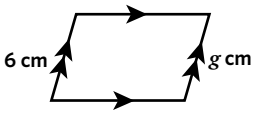


d.

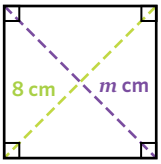


8. Determine the value(s) of the pronumeral(s), for each diagram. 7B

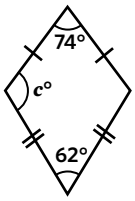
a.



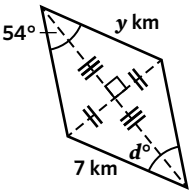
b.



c.

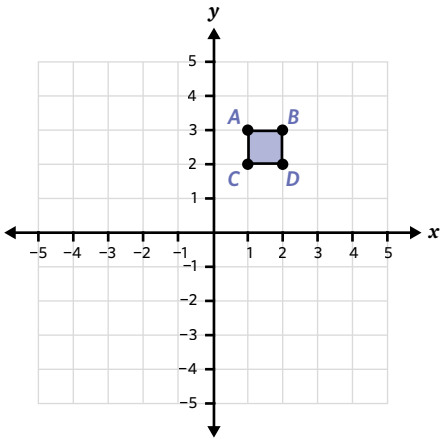


d.

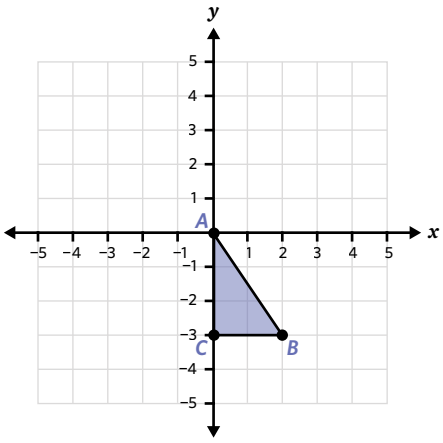


9. Translate. 7C

a. The square $ABCD$ by the vector $(2,1)$.



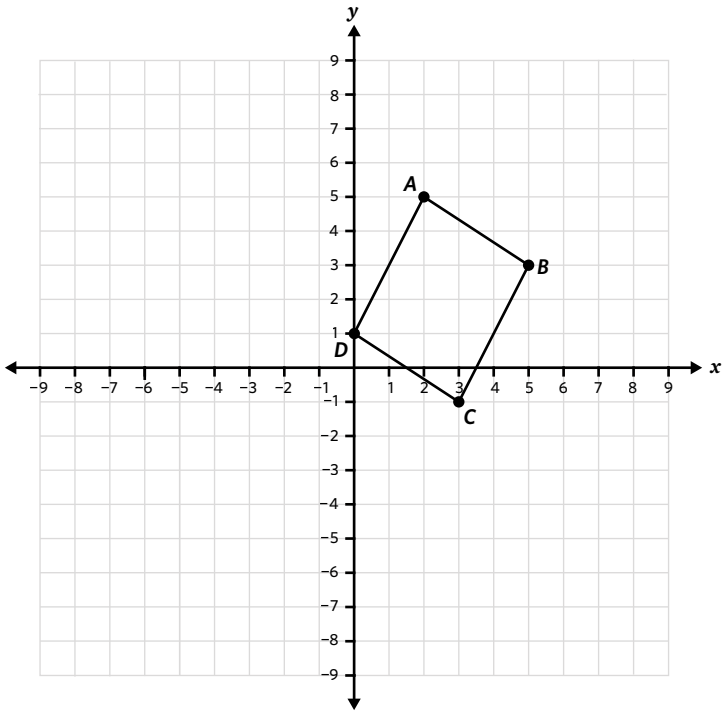
b. The triangle ABC by the vector $(-3,5)$.



c. The point $B = (8,1)$ by the vector $(1,10)$.

d. The point $D = (-7,-25)$ by the vector $(10,11)$.

10. Perform the following transformations on the rectangle $ABCD$. 7C



a. Reflect in the y-axis.

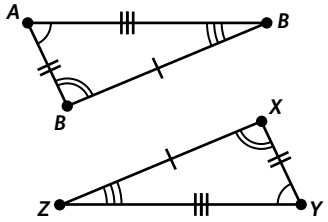
b. Reflect in the line $y = -2$

c. Rotate clockwise by 270° about the origin.

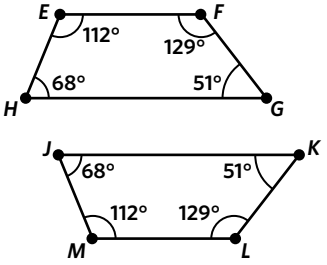
d. Rotate clockwise by 270° about $(0,1)$.

11. Name the corresponding features in these congruent shapes. 7D

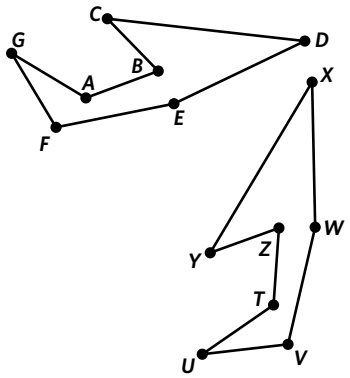
a. The side in $\triangle ABC$ that corresponds to $\overline{XY}$.



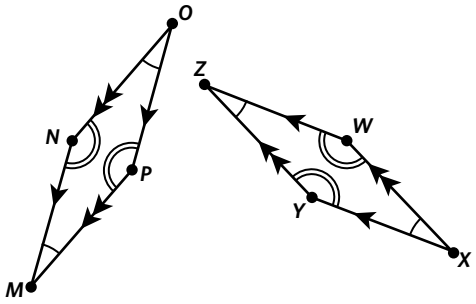
b. The side in $JKLM$ that corresponds to side $\overline{EH}$.



c. The vertex in TUVWXYZ that corresponds to vertex A.

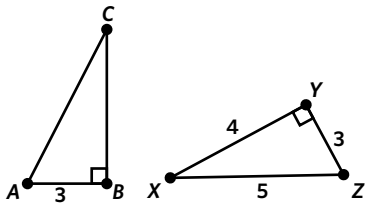


d. All sides in $WXYZ$ that could correspond with side $\overline{OP}$.

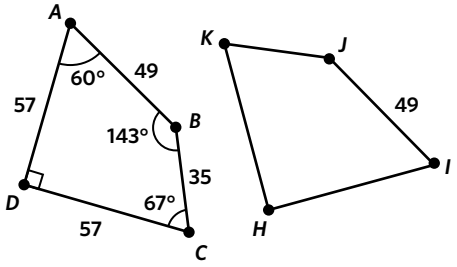


12. Find the values of the missing side lengths or angles for the pairs of congruent shapes. 7D

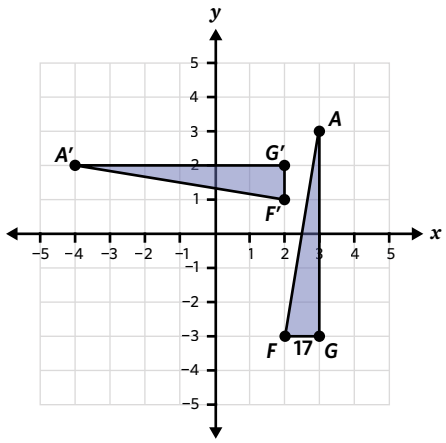
a. Side $\overline{BC}$



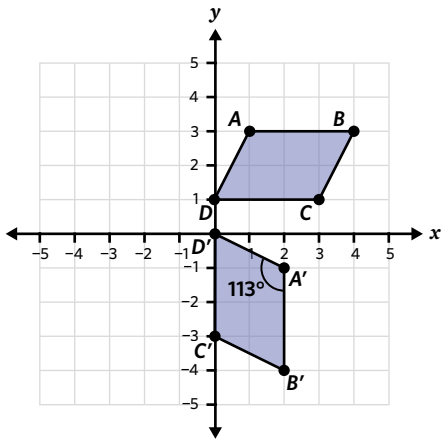
b. $\angle HKJ$



c. Side $\overline{F'G'}$

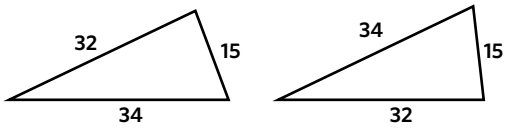


d. $\angle DAB$.

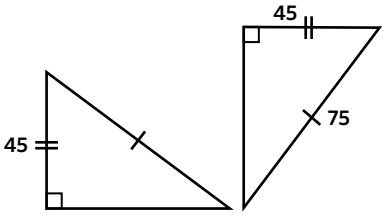


13. Determine if the following triangles are congruent, with reasons. 7E

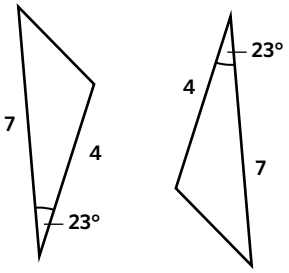
a.



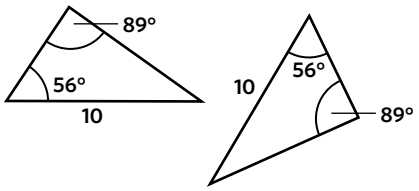
b.



c.

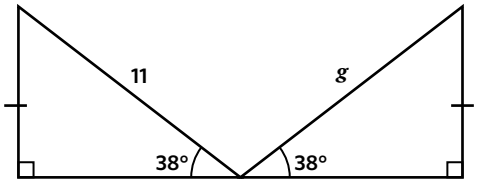


d.

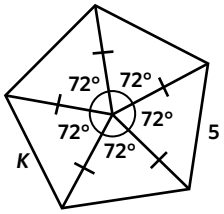


14. Determine the value(s) of the pronumeral(s). Explain using the congruence tests 7E

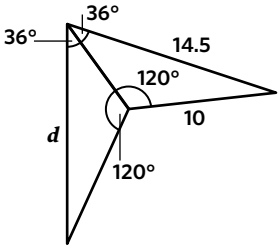
a.



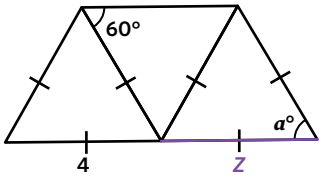
b.



c.

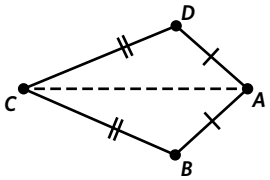


d.

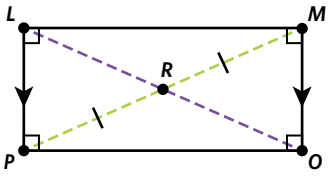


15. Prove the following properties of the quadrilaterals below. 7F

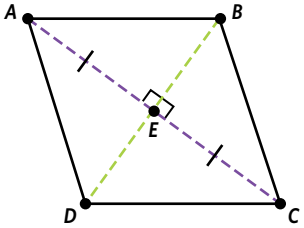
a. $\angle ABC = \angle ADC$



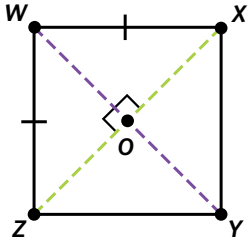
b. $\overline{LP} = \overline{MO}$



c. $AB = BC$.

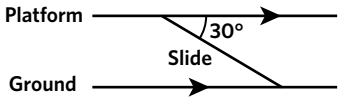


d. $\angle ZWO = \angle XWO$

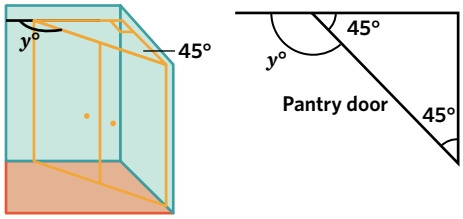


Problem solving

16. At the local playground, the slide is at an incline of 30°. If the platform at the top of the slide is parallel to the ground, what is the angle between the slide and the ground? 7A

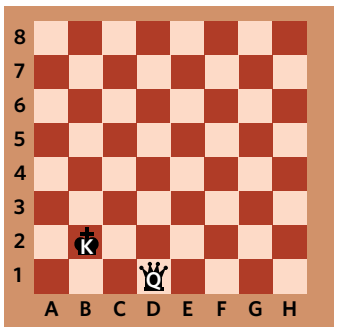


17. A pantry is being built into the corner of a kitchen as shown in the diagram. The second diagram shows the top of the pantry. 7B



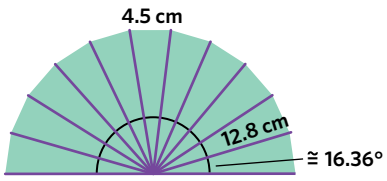
The walls of the kitchen are perpendicular to one another. Inside the pantry, the pantry doors form a 45° angle with the right-hand wall. Calculate the exterior angle that the pantry doors make along the left-hand wall ($y°$).

18. Alistair and Zoe are playing a game of chess, but they don't know the rules. The starting position of Alistair's knight (K) at B2 and Zoe's queen (Q) at D1 is shown below. Alistair makes two moves with his knight, which can be described by the vectors: (1,2), (−1,2). Then Zoe makes one move with her queen, which can be described by the following vectors (2,1), (−3,4). State the final positions of the knight and queen and determine whether they are next to each other. 7C



19. A decorative tiling pattern is made from congruent trapeziums. So far, the tiler has laid down sixteen tiles, as shown in the diagram below. If the tiler needs to create a row of tiles with a length of 200 cm, how many tiles do they still need to lay? 7D

20. Mahima wishes to make the following paper fan out of blue and yellow sticks which she will need to purchase. As the blue sticks are all identical, Mahima thinks that she will need nearly 50 cm of yellow sticks in total because $4.5 \times 11 = 49.5$. Is Mahima correct? How can we verify her measurements? 7E

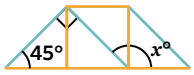


21. Pam has drawn a quadrilateral. Both diagonals bisect internal angles. If the diagonals are of equal length, prove that Pam's quadrilateral must be a square. 7F

Reasoning

22. A group of engineers are planning a bridge, as shown in the diagram below. The sides of the bridge are made from five congruent squares (orange), as well as eight identical supports (blue).

a. What is the value of angle x?



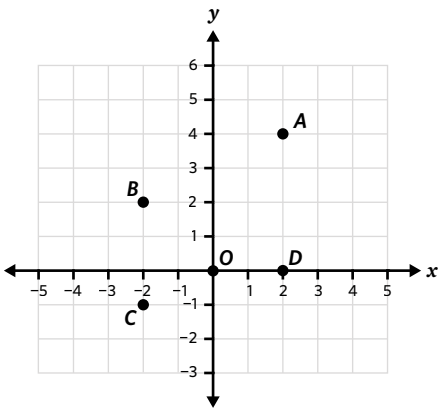
b. Given that the five squares are congruent, can we conclude that all five squares have the same side length? Explain why or why not.

c. Consider the leftmost and rightmost triangles formed by the supports. Are they congruent? Justify your answer.

d. Consider the middle square. Prove that the blue supports bisect each other.

e. What are some different materials that could be used for bridge construction?

23.



a. Rotate the points A, B, C and D clockwise 90° around O.

b. Reflect the points A, B, C and D over the line $y = x$, and then over the x-axis.

c. Compare your answers in parts **a** and **b** to determine the new coordinates when (x, y) is rotated 90° clockwise about the origin for any x and y.

Back to contents

08

Statistics

Statistics and Probability

Research summary

8A Classifying data *(Revision)*

8B Collecting data

8C Interpreting data displays *(Revision)*

8D Frequency tables and histograms

8E Measures of centre and spread *(Extension)*

Chapter 8 extended application

Chapter 8 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 8 research summary

Statistics

Big ideas

Variation with expectation and randomness

This chapter will focus on students' ability to make reasonable expectations about data sets using the mean, median, mode, range, and various types of visual representations such as histograms and stem and leaf plots. Statistical literacy is an essential skill for students inside and outside the classroom.

The language of statistics can cause many misconceptions for students. This chapter will use words such as mean, median, mode, range, and the names for various data types (numerical, categorical, discrete, continuous, nominal, and ordinal) which all take on new altered meanings. It is important that students understand not only the noun of statistics but also have clear definitions of the verbs. For example, the words 'formulate' questions, and 'interpret', 'analyse', and 'collect' data are all verbs that have distinct meanings but vast agreements of what they mean in statistics.

It is up to the teacher on how students should learn the definitions for these words. Some students see success when the words are defined using student-friendly language before they work on statistical problems and then re-define the definitions after (Sutherland et al., 2022). While other students see success when they work on problems without any definitions beforehand and then learn the definitions later (Konold & Harradine, 2014; Watson, 2018).

This big idea is built upon a strong understanding of number, number properties, ratios and proportions. Therefore, understanding the connection between number, number properties, equality, and algebra is pivotal for future success in mathematics. Progress in this chapter and lessons such as probability impacts connection between probabilistic and statistical thinking and reasoning.

Variation with informal inference

This chapter will explore the concepts of different types of data and how data is collected. Students will understand how sampling and population affects data and draw inferences and conclusions of data.

Distribution of data within various visual representations is a powerful cognitive tool that allows learners to interact with complex ideas. When given data (created or collected), students work with variation, think about the context and model to determine what graphical representations are appropriate, and draw informal inferences (Pfannkuch et al., 2018).

While creating visual representations are relevant when students are learning statistics, digital technology can aid in the understanding of distribution and expectation (Watson, 2018). Students can develop an origin story of how the results came to be and create a deeper meaning of the results, without the constraints of having to create graphs or perform the experiment themselves (Watson, 2018; Konald & Miller 2015). For example, the digital tool Tinkerplot and Wolfram Alpha can create graphical representations of many different various search results.

Learning about statistics through the PPDAC cycle (Pose a question, Plan the investigation to answer that question, collect the Data, Analyse the data, draw a Conclusion and write a report), can be a lengthy and time-consuming practice, however worthwhile (Watson & English, 2015). Teachers must be aware that students benefit from their feelings of uncertainty, backtracking, frustration, and re-thinking certain points of the investigation and outcome as this leads to a stronger understanding of statistical reasoning (Watson, 2018).

This big idea is built upon a strong understanding of number, number properties, and statistical reasoning. Therefore, understanding the connection between probabilistic and statistical reasoning is pivotal for future success in each of these topics and for real life situations. For example, drawing inferences from data is a key to understanding sports scores, rates of loans, and election results.

Variation with distribution and expectation

This chapter will focus on students' ability to interpret, create, and organise data to represent variation that occurs within statistics. Students will expand their existing knowledge of tables and tree diagrams to create and interpret frequency tables, pie charts, dot plots, and histograms in order to determine how data is spread out or distributed. Additionally, students will understand how to collect data, and the different types of data sources.

When students collect data, they often rely on their intuitions about samples and fairness rather than considering the likelihood of the sample being representative and unbiased (Watson, 2018). Rather than students collect data themselves, one way to promote the thinking and rich discussion of unbiased sampling is to give data to students to make informal inferences or from self-generated statistical inquiries (Watson, 2018; Meletiou-Mavrotheris & Paparistodemou, 2015).

Students can often have underdeveloped expectations and representatives of mean (Watson, 2018). Some students recognize the algorithm, but cannot apply this to open-ended questions, and visual representations such as box plots due to density of the data rather than the size of the box (Watson, 2018). Students can already intuitively look at data and understand how the data is 'clumped' and the shapes of clusters and distribution (Watson, 2018). Talking about 'clumps' of data rather than formal language and using inquiry based learning experiences can support the conceptions of central tendency (Konold & Harradine, 2014; Watson, 2018).

This big idea is built upon a strong understanding of number, number properties, and statistical reasoning. Therefore, understanding the connection between probabilistic and statistical reasoning is pivotal for future success in each of these topics. Progress in this chapter and lessons such as statistics impacts the big ideas of probability.

Visual representations

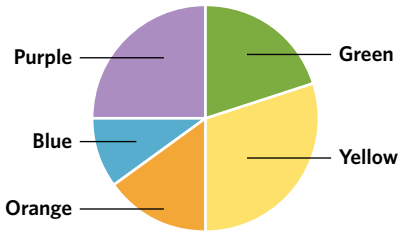
Frequency tables

A frequency table is used to summarise the tally of unique data in a data set. Some statisticians analyse very large sets of data. Frequency tables are a useful way to group these large data sets for easier analysis.

Category	Tally	Frequency
<Category 1>	𝍸 𝍸 III	13

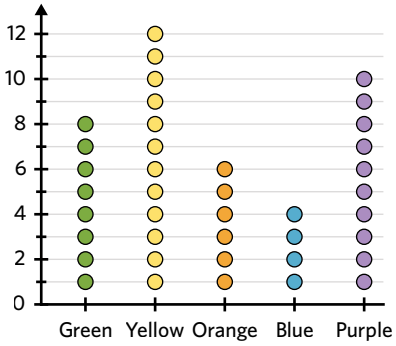
Pie charts

A pie chart is used to show the frequency of unique data in the form of a percentage and splits categories up into sectors that represent proportions of each category.



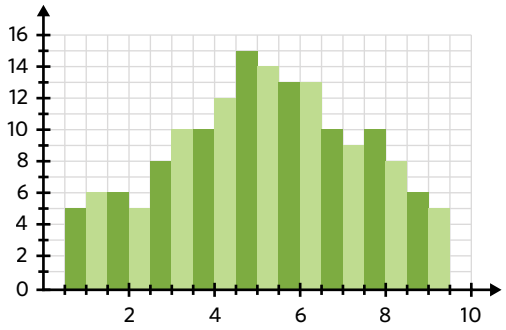
Dot plots

A dot plot is used to display a visual frequency by stacking equally sized and spaced dots to represent the frequency of each unique data in a data set. By keeping the dots equally sized and spaced, it is easy for us to determine the mode of the data by visual inspection of the tallest stack.



Histograms

Histograms contain data grouped in exclusive intervals, and have no gaps between the bars. Histograms can be symmetric, skewed, or bi-modal.



Stem-and-leaf plots

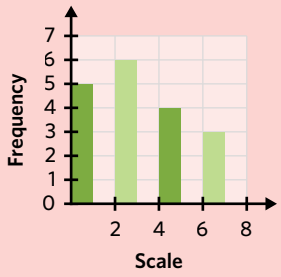
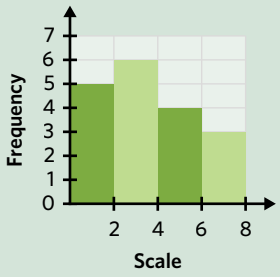
A stem-and-leaf plot is used in a similar way to dot plots and column graphs. A stem-and-leaf plot allows visualisation of data in horizontal stacks, but also retains the integrity of each individual data value for future analysis. A key is always included with a stem-and-leaf plot.

Leaf	Stem
0	9
1	1 2 2 3 4 7
2	0 4 5 9
3	4
4	1
5	7

Key
3 | 4 = 34

Misconceptions

Misconception	Incorrect	Correct	Lesson
Students think that number-based data is always numerical.	Rating out of 5 is numerical because it is number based.	Rating out of 5 is categorical ordinal because that data is ordered into categories.	8A
Students think that word based data is always categorical.	Excellent — Very good — Satisfactory — Ok — Poor The rating is categorical because it is word-based.	Excellent — Very good — Satisfactory — Ok — Poor The rating is categorical ordinal because the order matters.	8A
Students think that primary data does not have to be gathered first hand because it is raw and unfiltered.	Public archives that are accessed by citizens is primary data because it is raw data gathered by the government.	Public archives that are accessed by citizens is secondary data because it was not collected by the citizens.	8A
Students think all sample sizes should be 30.	Minimum sample size for any population is 30, therefore the minimum sample size for a population of 2000 is 30.	Minimum sample size of 2000 is $\approx \sqrt{2000}$ ≈ 45	8B
Students use sampling techniques that lead to biases.	Surveying a random group of people walking by about local playgrounds is not biased, because the sample is random.	Surveying a random group of passers by about local playgrounds is biased. The sample must consist of parents, childrens, or people who use the local playground for the survey to be meaningful.	8B

Misconception	Incorrect	Correct	Lesson
Students do consider place value when interpreting stem-and-leaf plots.	**Stem** | **Leaf** — **Key** 8 | 4 — 9 | 0 = 90 9 | 5 10 | 3 The number highlighted is 3.	**Stem** | **Leaf** — **Key** 8 | 4 — 9 | 0 = 90 9 | 5 10 | 3 The number highlighted is 103.	8C
When the data set contains 0, students do not write this in the stem and leaf plot.	{0, 1, 2, 3, 16, 23} **Stem** | **Leaf** — **Key** 0 | 1 2 3 — 9 | 0 = 90 1 | 6 2 | 3	{0, 1, 2, 3, 16, 23} **Stem** | **Leaf** — **Key** 0 | 0 1 2 3 — 9 | 0 = 90 1 | 6 2 | 3	8C
Students put gaps between the bars of a histogram.			8D
Students group data in overlapping or incomplete intervals.	Group the data in exclusive class intervals of size 2 {21, 25, 28, 29, 29, 30, 31, 31, 31, 33, 35} 21–23 23–25 25–27 27–29 29–31 31–33 33–35	Group the data in exclusive class intervals of size 2 {21, 25, 28, 29, 29, 30, 31, 31, 31, 33, 35} 21–<23 23–<25 25–<27 27–<29 29–<31 31–<33 33–<35	8D
Students do not consider the interval when grouping values.	15 cm is in the interval 10–<15	15 cm is in the interval 15–<20	8D
Students assume that the mean and median are always the same value.	{1, 3, 7, 8, 11, 11, 12, 15, 17} Median = 11 Median = mean Mean = 11	{1, 3, 7, 8, 11, 11, 12, 15, 17} Median = 11 Mean $= \frac{85}{9} = 9.44$	8E
Students assume that all the averages are affected by outliers.	The mean, median, and mode are all affected by outliers.	The mean is generally the only average that is significantly affected by outliers.	8E
Students find the median of unordered data.	{7, 5, 8, 1, 3, 4, 5, 7} Median $= \frac{1+3}{2} = \frac{4}{2} = 2$	{7, 5, 8, 1, 3, 4, 5, 7} {1, 3, 4, 5, 5, 7, 7, 8} Median $= \frac{5+5}{2} = \frac{10}{2} = 5$	8E

Misconception	Incorrect	Correct	Lesson
Students treat the stem and leaf as individual numbers when calculating mean, median, mode or range.	**Stem** | **Leaf** 5 | 9 6 | 3 5 7 | 0 2 **Key** 10 | 2 = 102 customers Mean: $\frac{5+9+6+3+5+7+0+2}{8}$ $= \frac{37}{8} = 4.625$ Range: $2 - 9 = -7$ Mode: 5 Median: **Stem** | **Leaf** 5 | 9 6 | 3 5 7 | 0 2 Median $\frac{3+5}{2} = 4$	**Stem** | **Leaf** 5 | 9 6 | 3 5 7 | 0 2 **Key** 10 | 2 = 102 customers Mean: $\frac{59+63+65+70+72}{5}$ $= \frac{329}{5} = 65.8$ Range: $72 - 59 = 13$ Mode: None Median: **Stem** | **Leaf** 5 | 9 6 | 3 5 7 | 0 2 Median = 65	8E

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Back to contents

8A Classifying data

Classifying data is the process by which we organise data so that it can be used efficiently. Data can be numerical or categorical, drawn from a sample or population, and come from a primary or secondary source.

LEARNING INTENTIONS

Students will be able to:

- define the terms population, sample, survey, and census
- understand the difference between primary and secondary data sources
- determine whether a variable is numerical discrete, numerical continuous, categorical nominal or categorical ordinal.

KEY TERMS AND DEFINITIONS

Numerical data is quantitative data that can either be discrete or continuous.

Categorical data is descriptive information and separated into groups.

A **variable** is a quantity that changes and can include qualities that are given numerical values.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

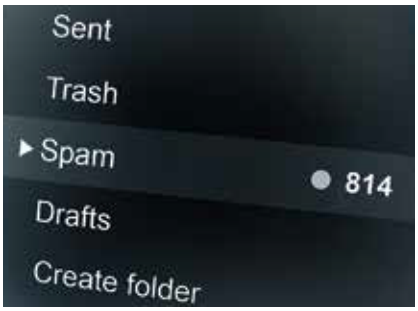
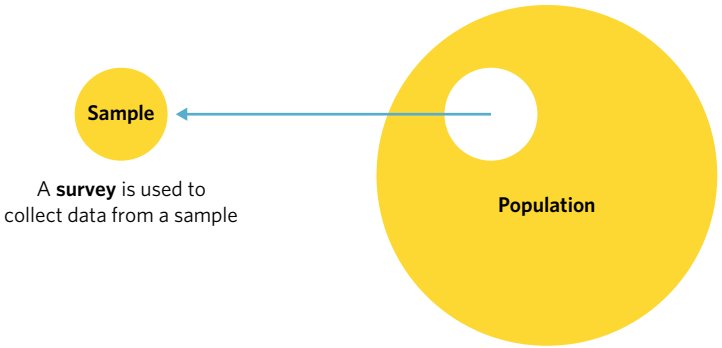


Image: GRADIENT BACKGROUND/Shutterstock.com

Data classification plays an important role in our everyday lives. From customer satisfaction surveys to more complex processes, such as email filtering systems, data is constantly being gathered and analysed around us.

Key ideas

1. Sampling can be used to statistically analyse a population. A sample is a selected group from an entire population. Data can be collected using a census or a survey.

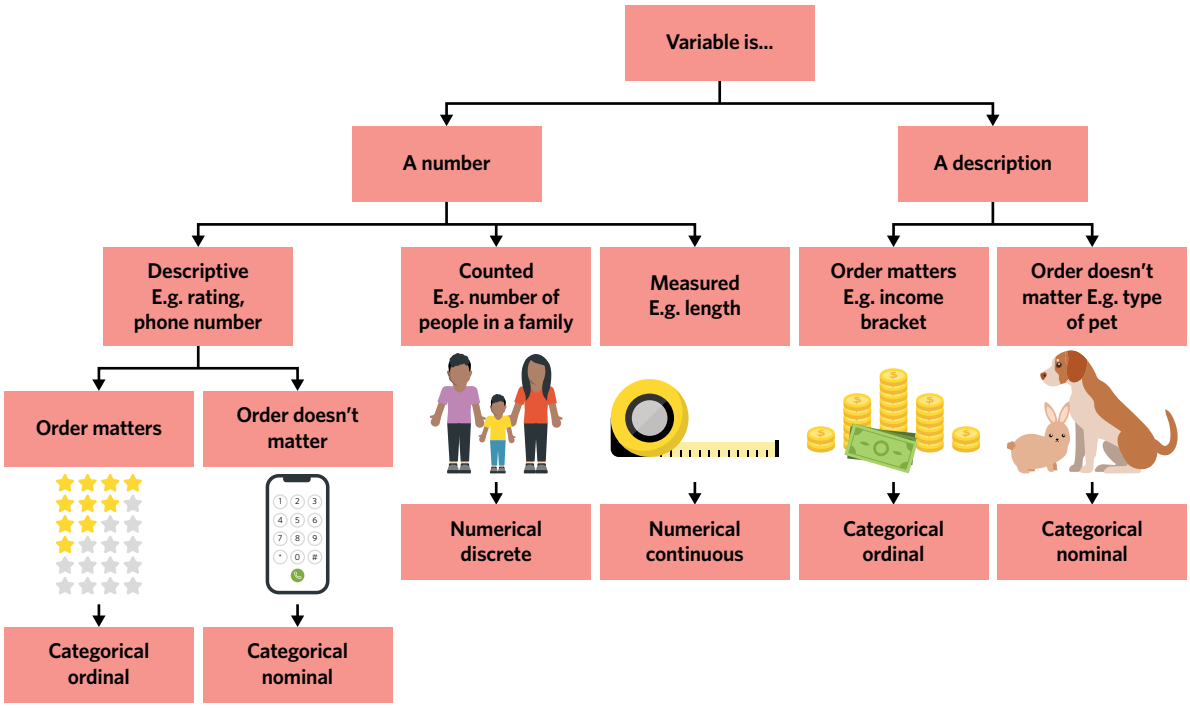


A **survey** is used to collect data from a sample

A **census** is used to collect data from the population

Continues →

2. Variables can be descriptive or quantitative.



Worked example 1

Defining the sample and populatio

For each of the following

i. State whether a sample or population is being used.

ii. State the name of the method used for data collection.

a. 100 randomly selected 13-year-olds from around Australia were asked to complete an online task as part of a study on reaction times. WE1a

Working

A sample of 100 13-year-olds in Australia is being used.

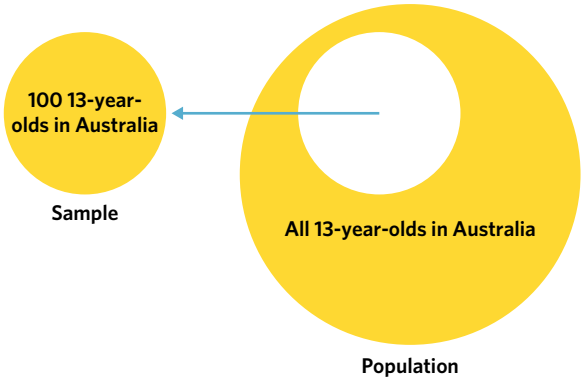
The reaction times were collected using a survey.

Thinking

i. Identify whether the whole group (population) or part of the group (sample) is included.

ii. State whether a survey or census is used to obtain data. A survey is used to collect data from samples.

Visual support



Continues →

b. A school sends out a questionnaire about the uniform to every student in the school. WE1b

Working	Thinking
The data comes from the population of students.	**i.** Identify whether the whole group (population) or part of the group (sample) is included.
The school is using a census.	**ii.** State whether a survey or census is used to obtain data. A census is used to collect data from populations.

Student practice

For each of the following

i. State whether a sample or population is being used.

ii. State the name of the method used for data collection.

a. A journalist asks 30 random people on the street for their opinions of local politicians.

b. All the residents in a rest home are asked to rate the new dinner menu.

Worked example 2

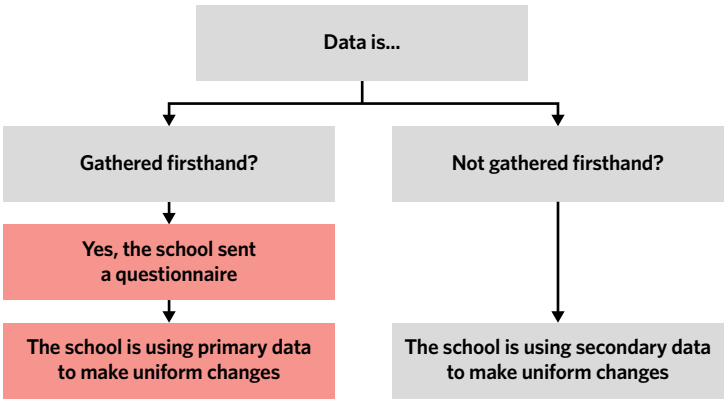
Defining ypes of data

For each of the following, state whether primary or secondary data is being used.

a. A school using a questionnaire they sent to every student to make uniform changes. WE2a

Working	Thinking
The school is using primary data.	Identify whether the data is collected firsthand or by an external source. Firsthand data is a primary source.

Visual support



b. A high school student using public records for a maths assignment. WE2b

Working	Thinking
The student is using secondary data.	Identify whether the data is collected firsthand or by an external source. Data not collected firsthand is a secondary source.

Continues →

Student practice

For each of the following, state whether primary or secondary data is being used.

a. Jack measures the pH levels in his tropical fish tank every day and keeps records of the data.

b. Miriam uses the data from a DNA ancestry website to complete her family tree.

Worked example 3

Organising data

For each of the following

i. State the variable.

ii. Determine whether the variable is numerical discrete or continuous, categorical nominal or ordinal.

a. A computer game company obtains data on reaction times, in milliseconds. WE3a

Working

The variable is reaction times, in milliseconds.

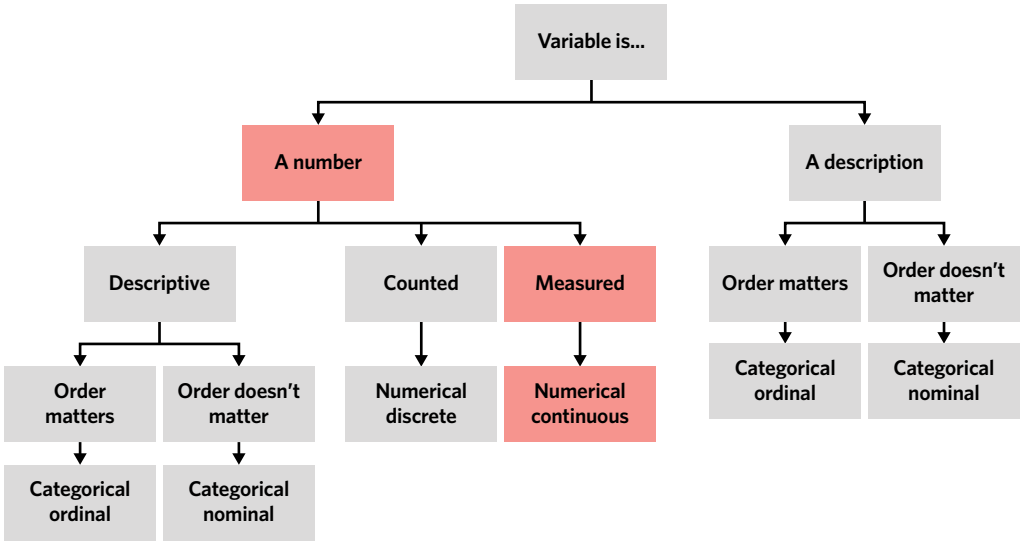
Reaction time is a numerical continuous variable.

Thinking

i. Determine which quantity or quality is being measured, counted, or observed.

ii. Determine whether the variable is numerical or categorical first, then whether it is counted or measured, if it is numerical.

Visual support



b. The CEO of a company asks employees to rate their work happiness on a whole number scale from 1 to 10. WE3b

Working

The variable is happiness rating as a whole number out of 10.

Work happiness is an categorical ordinal variable.

Thinking

i. Determine which quantity or quality is being measured, counted, or observed.

ii. Determine whether the variable is numerical or categorical first, then whether it is counted or measured, if it is numerical.

Continues →

Student practice

For each of the following

i. State the variable.

ii. Determine whether the variable is numerical discrete or continuous, categorical nominal or ordinal.

a. A teacher gets the students in their maths class to measure their arm spans, in centimetres.

b. A teacher asks their students to rate an activity out of 5 stars through an anonymous survey.

8A Questions

Understanding worksheet

1. For each of the following, determine whether a sample or the population is being used.
Circle the correct option.

Example

Image: angellodeco/Shuttertock.com

Sample

(Population)

a.

Image: Happy_Nati/Shuttertock.com

Sample

Population

b.

Image: mundissima/Shuttertock.com

Sample

Population

c.

Image: Beekeepx/Shuttertock.com

Sample

Population

d.

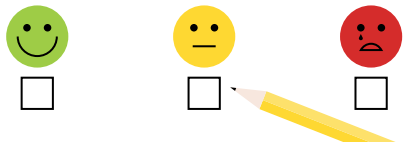
Image: Rich Carey/Shuttertock.com

Sample

Population

2. Determine whether the following will result in numerical or categorical data.
Circle the correct option.

Example



Numerical

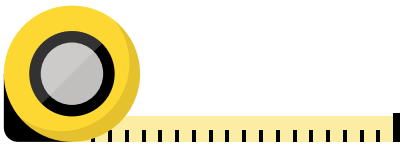
(Categorical)

a.

Numerical

Categorical

b.



Numerical

Categorical

c.

Numerical

Categorical

d.

Numerical

Categorical

3. Fill in the blanks by using the words provided.

numerical | categorical | continuous | discrete | sample | population

We can use a _____ to statistically analyse a _____. Data we obtain can be _____ or categorical. Numerical data is either counted, in which case it is _____, or measured, in which case it is _____. _____ data can be word or number-based and is descriptive in nature.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. For each of the following WE1

i. State whether a sample or population is being used.
ii. State the name of the method used for data collection.

a. Everyone in the house getting to vote on which board game to play next.

b. Trying flavours of ice cream before you decide which one you would like to buy.

c. Watching trailers of movies to decide which one you and your friends would like to go to.

d. A scientist gathering soil to analyse the types of minerals present in the area.

e. Every second shopper at a supermarket is asked if they would like to try out a new product for free.

f. A referendum where everyone in a country gets to vote on a new law change.

g. Reading online reviews on a specific website to make a decision about your next holiday destination.

h. A shop owner using an automatic counter above the entrance to keep track of how many customers come in every day.

5. For each of the following, state whether primary or secondary data is being used. WE2

- **a.** Using Wikipedia to look up information for an assignment.
- **b.** Keeping a diary to write down your thoughts every day.
- **c.** A sports team changing their name based on poll results gathered on their official site.
- **d.** A journalist publishing an interview they conducted with a movie star.
- **e.** Making a post online about something that happened to you.
- **f.** A doctor prescribing medicine using laboratory test results for reference.
- **g.** Using the photographs you took in Europe to tell people about your holiday.
- **h.** A police officer using a portable speed camera to monitor the speeds of cars driving past.

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6. For each of the following WE3

- **i.** State the variable.
- **ii.** Determine if the variable is numerical discrete or continuous, categorical nominal or ordinal.

- **a.** Asking each member of your household what their favourite food is.
- **b.** Suparna keeping a record of how fast she can solve a Rubik's Cube every day.
- **c.** Song Hee recording her daughter's height on the wall every year.
- **d.** A birdwatcher keeping a log of the types of birds they see in the park.
- **e.** A personal trainer keeping a log of how many crunches their client can do every session.
- **f.** Making a class birthday calendar to put on the wall in your homeroom.
- **g.** A marine biologist monitoring the temperature of a salt water lake every day for a month.
- **h.** A car park monitoring the registration numbers of all the cars that come in and out of the building.

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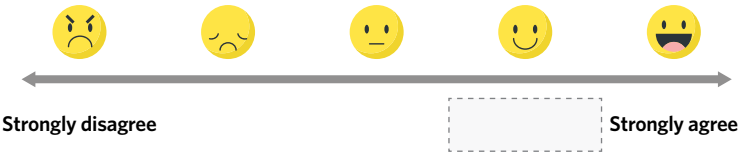
7. A Likert scale is often used to measure attitudes and opinions. A version of it is shown below visually, as well as on a spectrum.



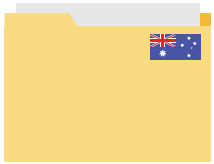
Which of the following words is the most appropriate to fill in the blank space on the scale above?

A. Disagree **B.** Neutral **C.** Agree **D.** Likely **E.** Dissatisfied

Spot the mistake

8. Select whether Student A or Student B is incorrect.

- **a.** Determine if the National Archives of Australia website is a primary or secondary data source for a school assignment.



Student A

It is a primary data source because it is raw and has not been analysed by anyone else before the student.

Student B

It is a secondary data source because although it is raw, it was not gathered firsthand by the student.

b. Determine whether car number plates represent categorical ordinal or categorical nominal data.

Student A

They represent categorical nominal data because they are randomly generated and used for identification only.

Student B

They represent categorical ordinal data because the numbers and letters have an order.

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. Mr Aykroyd is a Year 8 coordinator and wants to get feedback about recent uniform changes from the students. Should he use a sample or the population to conduct his investigation?

10. Eddie wants to investigate how long his classmates spend on homework every night.
State the variable of this investigation, with appropriate units, and determine its type.

11. Ying is trying to decide what kind of outfits she needs to pack for her trip to Bali. She does some research online on the kind of weather to expect during the time of the year she will be visiting.
Is Ying using primary or secondary data sources to make her decision?

12. A confectionery company weighs every second bag of gummy worms on the production line to make sure it matches with what it says on the packet. What is the sample and population of this process?

13. Donnie has to complete an assignment on the most popular music in the year he was born. To obtain the appropriate data, should Donnie just talk to his parents and relatives or go online to look up top ten hits from the year he was born?

Reasoning

Question working paths

Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. As part of a study on the extracurricular activities of Australian teenagers, a group of 50 people aged 13 to 16 were given the questionnaire shown in the table.

a. What is the sample and population of this study?

b. State the variable resulting from question 3 and suggest appropriate units for it.

c. Which of the questions will result in categorical nominal data?

d. Once the study is complete, the results are published on a website. Mr Cronenberg is a maths teacher who would like to use some of this data in his lessons.
Is Mr Cronenberg using primary or secondary data?

e. List two extra-curricular activities 13 to 16 year olds may be involved in.

Question #	Question
1	How old are you, in years and months?
2	Are you involved in extracurricular activities this year?
3	How much time per week do you spend on extracurricular activities?
4	What is your favourite activity?
5	Rate the importance you place on extracurricular activities as a whole number out of 10.
6	How likely are you to continue pursuing your favourite activity again next year?

15. 1, 1, 5, 3, 2, 7, 7, 6, 3, 5, 4, 1, 2, 8, 7, 9, 2, 4, 5

- **a.** Put the above data in any order and classify it as numerical continuous, numerical discrete, categorical nominal or categorical ordinal.
- **b.** Group the data by breaking it up into categories of your choice and classify it as numerical continuous, numerical discrete, categorical nominal or categorical ordinal.
- **c.** Compare your answers for parts **a** and **b** and determine which aspects of data sets help with classification.

Extra spicy

16. An online content provider is trying to come up with some ideas for a new teenage drama series. Which of the following sample groups is the most appropriate for their focus group?

- **A.** 50 primary school students
- **B.** 2 secondary school students
- **C.** 25 adults and 25 secondary school students
- **D.** 30 secondary school students
- **E.** 20 adults and 30 three-year-olds

17. From the questions listed below, choose the most appropriate for obtaining census data on education levels around the country.

- **A.** Did you graduate from a university?
- **B.** List all of your qualifications in the space provided.
- **C.** What is the highest qualification you currently hold?
- **D.** What is your current occupation?
- **E.** What is your annual income?

18. The variable describing the area of a house is metres squared, what type of variable is this?

19. If possible, it is always much better to use the target population as opposed to a sample for statistical analysis. What is the main reason for this?

Remember this?

20. There are 27 853 people at a BTS concert. One-quarter of them are over the age of 25. Which of these is the closest to the number of people at the concert who are over 25?

A. 70 **B.** 700 **C.** 800 **D.** 7000 **E.** 8000

21. Junji spent 95% of his money on comic books. The comic books cost $250 in total.

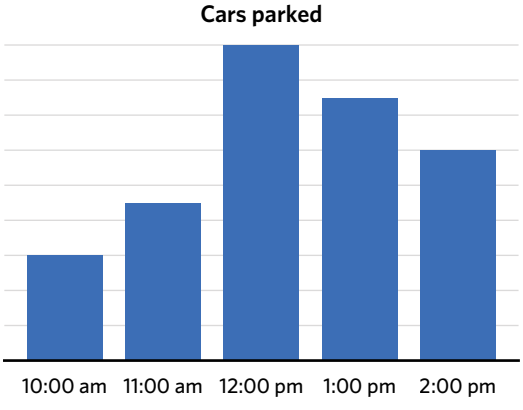
How much money does Junji have left after he bought the books?

A. $13.16 **B.** $62.50 **C.** $187.50 **D.** $250 **E.** $312.50

22. Matthew recorded how many cars were parked in the car park next to his fish and chip shop. He created a column chart of the results below.

In total, he recorded 100 cars that day. How many cars were parked next to the fish and chip shop at 2 pm?

- **A.** 6
- **B.** 20
- **C.** 40
- **D.** 60
- **E.** 80



Back to contents

8B Collecting data

Collecting data is an essential part of statistical research. Carefully chosen sampling techniques can help us gather data on a population, thus avoiding the time and expense of conducting a census. In order to draw meaningful conclusions from a sample, it must be appropriately sized and selected.

LEARNING INTENTIONS

Students will be able to:

- understand that a sample has to be representative of the population in order for the conclusions to be meaningful
- determine whether a sample contains bias
- analyse the strengths and weaknesses of sampling as a data collection method.

KEY TERMS AND DEFINITIONS

A **population** refers to the entire group that the conclusions of the study are applied to as opposed to part of the group.

A **sample**, or **sampling**, refers to the collection of data from a small group that is representative of the population.

Statistical **bias** occurs when a sample is not representative of the target population.

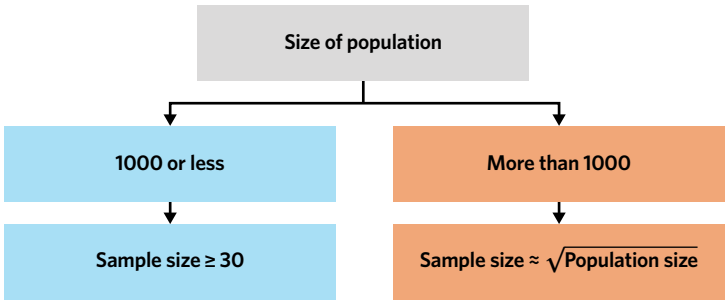
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: SmartPhotoLab/Shutterstock.com

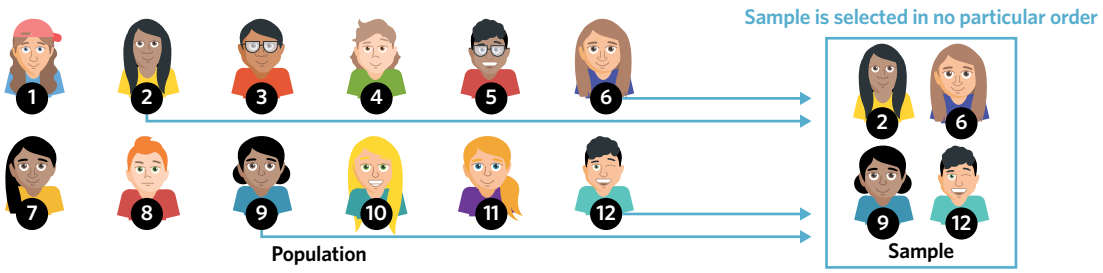
Sampling at the very basic level helps us make financial decisions, such as buying furniture or painting the house. Retailers often rely on samples to showcase and sell products.

Key ideas

1. Sample size must be representative of the population.



2. When an unbiased random sample is selected, each member of the population has an equal chance of being picked.



Worked example 1

Sampling techniques

Determine which of the given methods will produce a random sample that is representative of the population.

a. Conducting a survey about the well-being of staff in a company with just over 100 workers. WE1a

Method 1: Sending an anonymous survey link to ten random employees.

Method 2: Sending an anonymous survey link to thirty random employees.

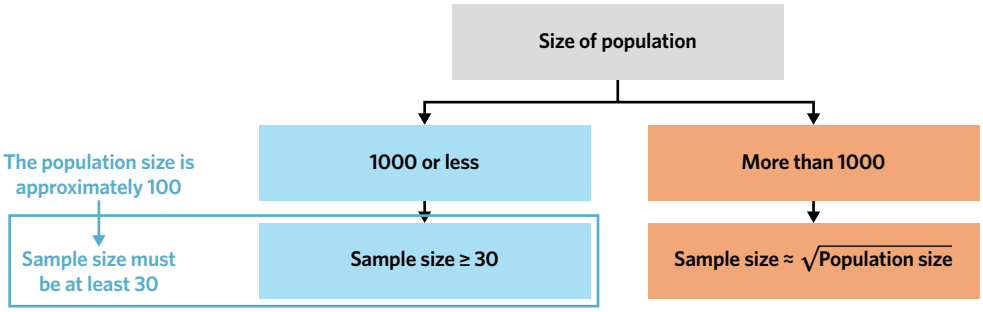
Working

Method 2

Thinking

Determine which of the two given methods uses a sample size appropriate for a population of just over 100.

Visual support



b. Picking participants for a survey on employment satisfaction. WE1b

Method 1: Interviewing people in the central city at lunchtime on a Wednesday.

Method 2: Interviewing people walking out of a school on a Friday afternoon.

Working

Method 1

Thinking

Determine which of the two given methods ensures that a sample is taken from the target population and will not introduce bias based on location.

c. Selecting students from a school of 400 for a survey on school facilities. WE1c

Method 1: Picking every second entry on an alphabetical roll of students' names until 50 students are selected in total.

Method 2: Putting all of the students' names in a box and picking 50 names out one by one without looking.

Working

Method 2

Thinking

Determine which of the two given methods ensures that each member of the population has an equal chance of being chosen for the sample.

Student practice

Determine which of the given methods will produce a random sample that is representative of the population.

Selecting a sample for a study on average household income.

Method 1: Putting an ad on the internet asking for participants in exchange for a $50 supermarket voucher.

Method 2: Using the household income data gathered by the Australian Bureau of Statistics to randomly select a sample.

Worked example 2

Biased sampling

Determine the source of bias in the described sampling methods.

a. Conducting a survey about the well-being of staff in a company with just over 100 workers. WE2a

Data was collected by sending an anonymous survey link to 10 random employees.

Working

The method will produce a sample that is too small to be representative of the population.

Thinking

Check if the described data collection method will produce an appropriately sized sample representative of the whole target population.

Visual support

b. Picking participants for a survey on career satisfaction. WE2b

Data was collected by interviewing people walking out of a school on a Friday afternoon.

Working

The method will produce a sample which is not representative of the target population of employed adults due to the location of the survey.

Thinking

Check if the described data collection method will produce an appropriately sized sample representative of the whole target population.

c. Selecting students from a school of 400 for a survey on school facilities. WE2c

Data was collected by picking every second entry on an alphabetical roll of students' names until 50 students were selected in total.

Working

The method will produce a sample where only the first 100 people on the school roll have a chance of being selected.

Thinking

Check if the described data collection method will produce an appropriately sized sample representative of the whole target population.

Student practice

Determine the source of bias in the described sampling method.

Selecting a sample for a study on average household income.

Data was collected from participants in exchange for a $50 supermarket voucher.

8B Questions

Understanding worksheet

1. Pick the correct sample size option for each population.

Example

Population description	Population size	Sample size ≥ 30	Sample size ≈ $\sqrt{\text{(population size)}}$
All the people attending a footy game	35 000	☐	☑

	Population description	Population size	Sample size ≥ 30	Sample size ≈ $\sqrt{\text{(population size)}}$
a.	Every person who lives in Melbourne.	5.1 million	☐	☐
b.	All the people who have travelled to space.	566	☐	☐
c.	All the billionaires in the world.	More than 2700	☐	☐
d.	A current Year 8 cohort in an Australian secondary school.	50–400	☐	☐

2. Determine if the samples will be biased using the given descriptions.

Example

Sample description	Biased	Unbiased
The supporters at a children's soccer game are asked to complete a survey on the average number of children.	☑	☐

	Sample description	Biased	Unbiased
a.	3 random people in Australia are asked to complete a government survey about the Prime Minister.	☐	☐
b.	A random group of people at a shopping centre are surveyed about the shops they visit most often.	☐	☐
c.	500 Northern Territory residents are asked to complete a survey about public transport in Victoria.	☐	☐
d.	2 people from each year level at a secondary school are chosen randomly to complete an anonymous survey about the principal.	☐	☐

3. Fill in the blanks by using the words provided.

thirty | equal | randomly | representative

With the right sampling techniques, a _____ sample can be drawn from the population.

The sample must have a size of at least _____ for a population under 1000. It must be chosen _____ and fairly, so that every member of the population has an _____ chance of being selected.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b), 6 (a,b), 7 (a,b,c,d), 8	4 (b,c), 5 (b,c), 6 (b,c), 7 (c,d,e,f), 8	4 (c,d), 5 (c,d), 6 (c,d), 7 (e,f,g,h), 8

4. Determine which of the given methods will produce a random sample that is representative of the population. WE1a

a. Conducting a study on the kind of chores your peers do at home by

Method 1: Asking your closest four friends at school to complete a questionnaire.

Method 2: Asking everyone in your homeroom at school to complete a questionnaire.

b. Conducting a study on your workmates' favourite sport by

Method 1: Sending a link to an anonymous survey about sport to everyone you work with.

Method 2: Asking the same small group of six people you sit with at lunch what their favourite sport is.

c. Surveying university students about their employment status by

Method 1: Posting an ad on the university notice board asking for 100 participants for your study.

Method 2: Spending an hour walking around university on a weekday and interviewing 20 random students.

d. Working out the average size of fish in a lake you visit by

Method 1: Weighing three fish you have caught in one trip and using this data to work out their average mass.

Method 2: Weighing every fish you have caught in the lake over at least five fishing trips and working out their average mass.

---

5. Determine which of the given methods will produce a random sample that is representative of the population. WE1b

a. Researching the popularity of the school library by

Method 1: Asking people in the library at lunchtime how often they come to the library per week.

Method 2: Asking a random group of students across the school how often they visit the school library.

b. Researching the average number of people in an Australian family by

Method 1: Using census results on household numbers from the Australian Bureau of Statistics.

Method 2: Putting a questionnaire about family in every household's letterbox on your street.

c. Researching the number of hours of exercise that 14-year-olds complete each week by

Method 1: Surveying students as they enter the school in the morning.

Method 2: Surveying all the players at footy training.

d. Investigating the age at which the children in your suburb first started using digital devices by

Method 1: Visiting a grade one class at the local primary school to ask the children.

Method 2: Asking parents to complete a survey in a local Facebook group.

---

6. Determine which of the given methods will produce a random sample that is representative of the population. WE1c

a. Investigating the most popular music genre in Australia by

Method 1: Searching in the Spotify music app for top played songs in Australia.

Method 2: Asking a large number of students at school.

b. Researching the number of hours of sleep high school students get by

Method 1: Going to the city and surveying the sleeping habits of anyone who looks under 18.

Method 2: Sending a link to an anonymous survey about sleep through to the school email system to everyone at your school.

c. Studying the average age that people get married in Victoria by

Method 1: Asking people at the local church about when they got married.

Method 2: Checking the Australian Bureau of Statistics website for data on the ages of Victorians at the time of their weddings.

d. Studying the wages of workers across different countries by

Method 1: Surveying tourists on the street and asking them about average wage in their home country.

Method 2: Surveying friends whose parents are immigrants for information on wages in their home country.

---

7. Determine the source of bias in the described sampling methods. WE2

a. Researching the most popular mobile games among teenagers by asking your teenage male relatives.

b. Researching the number of siblings people at your school have by writing down what you remember about your friends' families.

c. Finding the percentage of people in New South Wales who have been to university by asking people in Sydney central city about their education levels.

d. Finding the average lifespan of large dogs by interviewing people about their dogs at the greyhound rescue group.

e. Researching the average number of kittens a cat has by recording how many kittens were in your pet cat's last two litters.

f. Finding out how much water adults drink per day by asking five of your teachers about it at school.

g. Gathering one cup of water from the Pacific Ocean for a study on the presence of microbeads in the world's oceans.

h. Conducting research on the numbers of students involved in extracurricular activities by asking for volunteers for your survey.

---

8. A cafe changes their opening hours after surveying all of their customers who get coffees on the way to work in the morning for a week. Determine the source of bias of this sampling method.

A. The sample is too small.

B. The sample was selected in the wrong location.

C. The sampling method was not fair or random.

D. The sample was not chosen from the right population.

E. The sample is not biased.

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Choose an appropriate sample size for a population of 3000.

Student A

Sample size for any population ≥ 30

Therefore, an appropriate sample size is 30.

Student B

For a population of over 1000:

sample size $\approx \sqrt{(\text{population size})}$

So, for a population of 3000:

sample size $\approx \sqrt{3000} = 54.8 \approx 55$

Therefore, an appropriate sample size is approximately 55.

b. Suggest a location for gathering a sample of soil to study the effects of mining on the Australian environment.

Student A

The soil sample should be taken from a rural area in Australia close to a mining site and away from major cities.

Student B

The soil sample can be taken from anywhere in Australia.

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Miriam is a scientist who lives in New Zealand and likes to birdwatch every day. She keeps a record of all the native birds she sees when she travels around the country. What is the target population of Miriam's research?

11. The director of a large company is organising a staff lunch. She wants to know if the employees have any dietary requirements before booking a caterer. Explain why she must use the population as opposed to a sample of her employees in order to get this data.

12. Marco is planning on travelling to Thailand. He gathers data on yearly rainfall and finds that the probability of heavy rain on any day in Thailand is 34%. The majority of rainfall in Thailand occurs during the monsoon season late in the year, whereas Marco is travelling there in March. Did Marco's data provide him with a meaningful conclusion relevant to his situation? Explain your reasoning.

13. Dawn has found that out of the 100 cars that passed her house on a Friday evening between 8 pm and 10 pm, two were speeding. She presents her findings to the local council and asks for speed bumps to be installed due to the high likelihood of drivers speeding on her street. Does Dawn's survey support her claim? Explain your reasoning.

14. Callum is an engineer at a robotics company. His job is to test every third robot on the assembly line over the course of a month. If a total of 1500 robots were produced that month, find the number of robots in Callum's test sample and comment on whether it is representative of the population.

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Luke is studying film at university and would like to find the top ten films of all time based on a sample of 50 of his peers. He records the following responses to the question 'Which of the listed movies is your favourite of all time?'.

a. According to Luke's survey results, which of the listed films is the most popular?

b. What is the target population of Luke's survey?

c. Is the sample chosen by Luke likely to be representative of the target population? Explain your reasoning.

d. Identify at least one problem and/or source of bias in Luke's sampling and survey methods and suggest an improvement.

e. With the introduction of new technology, we have unlimited access to any media we desire, including movies, TV, and video games. List one advantage and disadvantage of having such a variety of content at your fingertips.

Movie	Frequency
2001: A Space Odyssey	18
Casablanca	0
Citizen Kane	2
The Godfather	7
Gone with the Wind	0
Parasite	7
Shawshank Redemption	5
Singin' in the Rain	1
Spirited Away	8
Vertigo	2

16. Determine whether the following samples are representative of the target population.

- **a.** A group of pensioners participate in a study on average reaction times.
- **b.** A group of pensioners participate in a study on average reaction times of people over 65.
- **c.** Compare your answers for parts **a** and **b** and give reasons for your decisions in each case.

Extra spicy

17. The current population of the world is 7.753 billion people. Which of the following random sample sizes is representative of the world's population?

- **A.** 30 people
- **B.** 3500 people
- **C.** 88 000 people
- **D.** 3.5 million people
- **E.** 2.5 billion people

18. If the population of the world was reduced to 100 people, 26 of them would be children. How many children would you expect to find in a random sample of 5000 people?

19. A sample was selected by approaching 15 random people in the city every day for 5 days. What is the most suitable population size for this sample?

A. 50 **B.** 75 **C.** 150 **D.** 5500 **E.** 15 000

20. Todd is an editor who is checking every third page of a book with 364 pages. He finds mistakes on 4 of the pages he checks. He has to return the book to the writer for revisions if the percentage of pages with mistakes in his sample exceeds 4%. Calculate the percentage of pages with mistakes in Todd's sample and state whether he should return the book for revisions.

Remember this?

21. Sarah is painting a fence with a total area of 92 m^2. Each square metre requires two litres of paint. If the hardware store only sells the paint Sarah needs in three litre tins, then how many tins will she need altogether to paint the whole fence?

A. 60 **B.** 61 **C.** 62 **D.** 63 **E.** 64

22. An adult's ticket to an amusement park costs 1.5 times as much as a child's. If a family of 2 adults and 2 children paid $120 for all their tickets, then what is the price of a ticket for 1 child?

A. $24 **B.** $30 **C.** $36 **D.** $40 **E.** $48

23. Matthew would like to open a fish and chip shop. He wants to estimate the number of cars that drive past the corner shop he wants to lease every day during lunchtime. Which method of data collection should he use?

- **A.** Count the number of cars in the car parks close to the shop during lunchtime.
- **B.** Ask his friends about how often they drive past the corner at lunchtime.
- **C.** Survey other businesses around the shop about how many customers they get in at lunchtime.
- **D.** Count the number of cars that drive past the shop on a random day in the week during lunchtime hours.
- **E.** Count the number of cars that drive past the corner shop every day between 12 pm and 2 pm for a week.

Back to contents

8C Interpreting data displays

Raw data alone does not provide us with useful information until we statistically analyse it. Data displays can reveal insights that we can use to draw meaningful conclusions. Different types of data are suited for a variety of charts and displays and it is important to know how to read and interpret them.

LEARNING INTENTIONS

Students will be able to:

- interpret frequency tables
- interpret dot plots, pie charts, and stem-and-leaf plots.

KEY TERMS AND DEFINITIONS

Frequency refers to how many times something occurs.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

The entertainment industry relies heavily on data analysis. Lists of top songs and movies provide consumers and companies with reliable information. Artists who top the charts are able to demand higher payments for performances and merchandise.

Key ideas

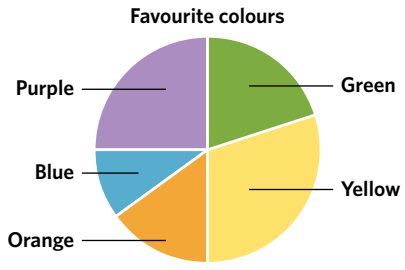
1. A frequency table can be used to record and organise data.

Category	Tally	Frequency
<Category 1>	𝍸 𝍸 III	13

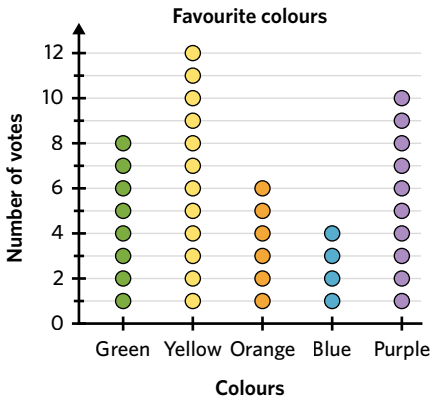
Tally is converted to a numerical value, i.e. frequency

2. Displays can be used to visualise data for different purposes.

A **pie chart** splits categories up into sectors that represent proportions of each category.



A **dot plot** shows the frequency of data points.



A **stem-and-leaf plot** shows the overall shape of data.

Leaf	Stem
0	9
1	1 2 2 3 4 7
2	0 4 5 9
3	4
4	1
5	7

Key: 3 | 4 = 34

Worked example 1

Interpreting frequency tables

The following table shows the tallied votes for favourite Pokémon characters.

Favourite Pokémon character	Tally
Bulbasaur	𝍸
Mewtwo	𝍸 IIII
Muk	I
Pikachu	𝍸 𝍸 𝍸 III
Squirtle	III

a. Convert the tallies to frequencies for each Pokémon character in the table. WE1a

Working

Bulbasaur = 5

Mewtwo = 5 + 4 = 9

Muk = 1

Pikachu = 5 + 5 + 5 + 3 = 18

Squirtle = 3

Thinking

Add the tally marks for each Pokémon character to find their frequencies.

b. What was the total number of votes? WE1b

Working

5 + 1 + 9 + 18 + 3 = 36

Total number of votes = 36

Thinking

Add all the frequencies together for the total number of votes.

Student practice

The following table shows the tallied votes for favourite computer game characters.

Favourite character	Tally
Donkey Kong	I
Kirby	III
Mario	𝍸 𝍸 𝍸 I
Pac-Man	𝍸 III
Sonic the Hedgehog	IIII

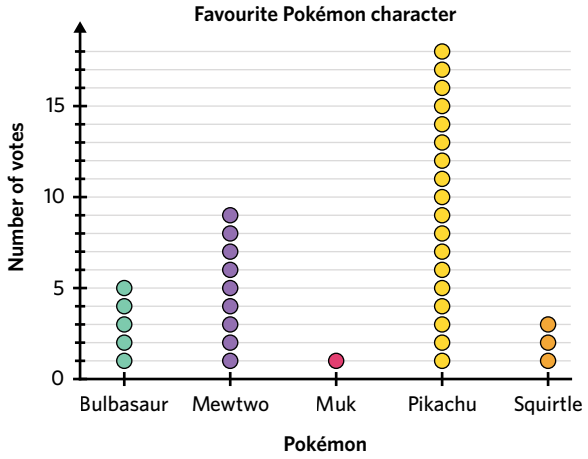
a. Convert the tallies to frequencies for each character in the table.

b. What was the total number of votes?

Worked example 2

Interpreting dot plots

The following dot plot shows the frequency of votes for favourite Pokémon characters.



a. Which Pokémon character is the least favourite? WE2a

Working

Muk is the least favourite Pokémon.

Thinking

Find the Pokémon character with the least amount of plotted dots.

b. How many more people voted for Pikachu compared to Squirtle? WE2b

Working

Pikachu $= 18$

Squirtle $= 3$

$18 - 3 = 15$

Pikachu had 15 more votes than Squirtle.

Thinking

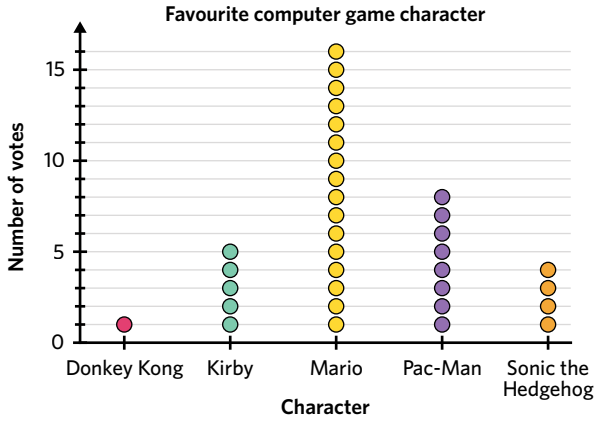
Step 1: Use the dot plot to determine the number of votes for Pikachu.

Step 2: Use the dot plot to determine the number of votes for Squirtle.

Step 3: Find the difference between the number of votes for Pikachu and Squirtle.

Student practice

The following dot plot shows the frequency of votes for favourite computer game characters.



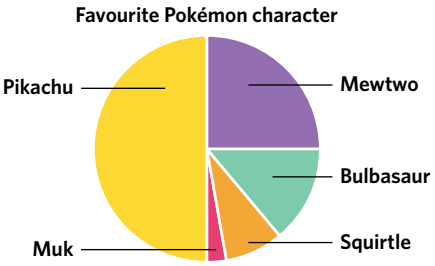
a. Which character is the least favourite?

b. How many more votes did Mario get compared to Kirby?

Worked example 3

Interpreting pie charts

The following pie chart shows the proportions of votes for favourite Pokémon characters.



a. Which Pokémon character has the largest proportion of the votes? WE3a

Working

Pikachu has the largest proportion of the votes.

Thinking

Find the sector with the largest area. It represents which Pokémon has the largest proportion of votes.

b. If nine of the votes were cast for Mewtwo, then how many votes were cast altogether? WE3b

Working

$\frac{1}{4}$ or 25% of the votes were cast for Mewtwo.

25% of the total number of votes $= 9$

25% of $x = 9$

$\frac{25}{100} \times x = 9$

$x = 9 \div \frac{25}{100} = \frac{9}{1} \times \frac{100}{25}$

$= \frac{900}{25} = 36$

There was a total of 36 votes.

Thinking

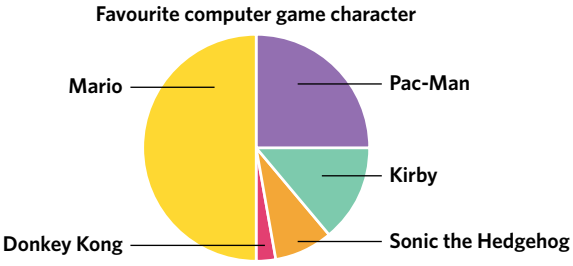
Step 1: Determine the percentage out of 100 represented by Mewtwo's sector in the pie chart.

Step 2: Write an equation showing 9 as a percentage of the total number of votes, using x to represent the total.

Step 3: Solve the equation for x.

Student practice

The following pie chart shows the proportions of votes for favourite computer game characters.



a. Which character has the largest proportion of the votes?

b. If 8 of the votes were cast for Pac-Man, then how many votes were cast altogether?

Worked example 4

Interpreting stem-and-leaf plots

The following stem-and-leaf plot shows the ages of 30 people at a Pokémon convention.

Stem	Leaf	Key
1	1 1 2 2 4 4 5 5 5 6 6 6 7 7 8 8 9 9	4 | 0 = 40 years
2	0 4 9	
3	3 4 5 6 7	
4	0 4 5 8	

a. Determine the ages of the youngest and oldest people in the group and find their difference. WE4a

Working

Youngest person's age = 11 years

Oldest person's age = 48 years

48 − 11 = 37 years

Thinking

Step 1: Determine the lowest value. This is represented by the first number, in the first row, listed in the 'leaf' column.

Step 2: Determine the highest value. This is represented by the last number, in the last row, listed in the 'leaf' column.

Step 3: Find the difference between the highest and lowest values.

Visual support

Stem	Leaf	Key
1	1 1 2 2 4 4 5 5 5 6 6 6 7 7 8 8 9 9	4 | 0 = 40 years
2	0 4 9	
3	3 4 5 6 7	
4	0 4 5 8	

Youngest

48 − 11 = 37 years

Oldest

b. According to the plot, the Pokémon convention was most popular with people of which age group? WE4b

Working

The Pokémon convention was most popular with people whose ages ranged from 11 to 19.

Thinking

Determine the age range with the most data points by looking at the longest 'leaf' on the plot.

Student practice

The ages of 30 people at a video game convention were recorded on a stem-and-leaf plot below.

Stem	Leaf	Key
1	4 5 6 7 8 8 8 9 9	2 | 6 = 26 years
2	0 0 1 1 1 1 2 2 3 3 4 5 6 7 7 9	
3	0 0 1 2 4	

a. Determine the ages of the youngest and oldest people in the group and find their difference.

b. According to the plot, the video game convention was most popular with people of which age group?

8C Questions

Understanding worksheet

1. Fill in the table by converting the frequencies to tallies.

Example

Favourite Pikachu	Tally	Frequency
Cosplay Pikachu	𝍸 III	8

	Favourite Pikachu	Tally	Frequency
a.	Summer Style Pikachu		4
b.	Detective Pikachu		10
c.	Flying Pikachu		6
d.	Pretend Boss Pikachu		12

2. Choose whether a dot plot, pie chart or a stem-and leaf plot is suitable for each of the following.

Example

Purpose	Pie chart	Dot plot	Stem-and-leaf plot
Recording and organising marathon finish times, in minutes.	☐	☐	☑

	Purpose	Pie chart	Dot plot	Stem-and-leaf plot
a.	Recording and organising heights of apple trees, in cm.	☐	☐	☐
b.	Comparing the frequency of 30 votes for favourite ice cream flavours.	☐	☐	☐
c.	Comparing percentages or proportions of people who voted for their favourite TV show.	☐	☐	☐
d.	Analysing the results of whole school votes for the next School Captain.	☐	☐	☐

3. Fill in the blanks by using the words provided.

proportions | dot | numerical | frequency

_____ tables are used to record and organise data resulting from a repeated number of votes or trials.

_____ plots can be used to show relative frequencies of categories, while pie charts show

_____ or percentages out of a whole. Stem-and-leaf plots are used for _____

data, which is either counted or measured.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b), 6 (a,b), 7 (a,b), 8	4 (b,c), 5 (b,c), 6 (b,c), 7 (b,c), 8	4 (c,d), 5 (c,d), 6 (c,d), 7 (c,d), 8

4. The following table shows the tallied votes for favourite family movies in 2022. WE1

Favourite family movie	Tally
The Bad Guys	𝍸 𝍸 |
Marmaduke	𝍸 𝍸 |||
The Bob's Burgers Movie	𝍸 𝍸 𝍸 𝍸 /
Turning Red	𝍸 𝍸 𝍸 𝍸 𝍸 ||
Home Team	𝍸 𝍸 𝍸 |
Cheaper by the Dozen	𝍸 𝍸 𝍸 ||

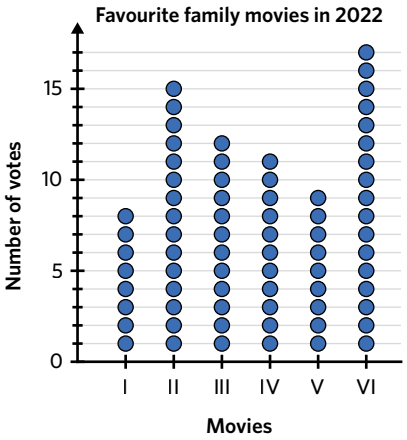
a. Which of the movies was the overall favourite?

b. Which of the movies was least favourite?

c. What was the total number of votes?

d. Write the number of votes for The Bob's Burgers Movie as a simplified fraction, out of the total number of votes.

5. The following dot plot shows the frequency of votes for favourite family movies in 2022. WE2



a. Label each of the categories on the horizontal axis using the frequency table from Q4.

b. Which of the movies had 9 votes?

c. How many more people voted for Cheaper by the Dozen compared to The Bad Guys?

d. How many people voted for Marmaduke or Turning Red?

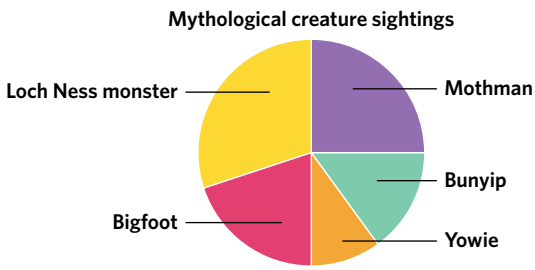
6. The following pie chart shows the proportions of reported mythological creature sightings. WE3

a. Which of the creatures had the most sightings reported?

b. Which of the creatures had the least sightings reported?

c. Name the creature whose sightings make up 25% of all the reports.

d. There were 18 Bunyip sightings reported. If this number makes up 15% of all the reports, then what was the total number of sightings of all the creatures in the pie chart?



7. The following stem-and-leaf plot shows the number of customers a movie theatre has each day in September.

WE4

Stem	Leaf
5	9
6	3 4 7 7 7
7	0 2 3 9 9
8	2 4 5 5 5 5 6 9
9	3 3 5 6 9
10	0 1 1 1 2 4

Key

10 | 2 = 102 customers

a. What was the highest number of customers the theatre had in one day?

b. For how many days did the theatre have 79 customers?

c. What is the difference between the most and least number of daily customers the movie theatre had in September?

d. If the movie theatre has less than 80 customers in a day, it will not make a profit. Write a fraction to represent the number of days out of the total number of days in September on which the movie theatre did not make a profit.

---

8. State the number represented by the highlighted value in the stem-and-leaf plot below.

A. 0.09
B. 0.9
C. 9
D. 90
E. 109

Stem	Leaf
0	9
1	5
2	1 2 9

Key

2 | 1 = 21

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Determine the value of the highlighted number.

Stem	Leaf
8	4
9	5
10	3

Key

9 | 5 = 9.5

Student A

10.3

Student B

3

b. Write 13 on a stem-and-leaf plot.

Student A

Stem	Leaf
0	1 3

Key

0 | 17 = 17

Student B

Stem	Leaf
1	3

Key

1 | 7 = 17

Problem solving

Question working paths

Mild 10, 11, 12 | Medium 11, 12, 13 | Spicy 12, 13, 14

10. Zeus asks 11 members of his council to vote on a new secretary. Each member will vote only once. Zeus wants to use a dot plot to record the data. How long should Zeus make the vertical scale of his dot plot to ensure that all the potential votes can be included?

11. Ms Muscat is a Year 8 coordinator who has 400 grades to analyse for an end-of-year mathematics exam. If each student received a grade from A to E for this assessment, suggest a suitable data display that Ms Muscat can use in order to compare the proportions of grades.

12. The winner of an election must have at least half of all the votes. What is the minimum size of the angle of the sector that represents the winner in a pie chart?

13. The following back-to-back stem-and-leaf plot shows the marks out of 100 for an English assignment for two classes, 8X and 8Y.

8X	Stem	8Y
9 5 2 0	3	1
8	4	0
9 9 8 8 5 4 3 3	5	4
9 3 2 2	6	1 4 5 8
6 5 5 3 3	7	3 7 8 8
0 0	8	2 3 4 5 5 6 7 8 9
2	9	0 1 4 5 9

Key

6 | 1 = 61 marks

Which class received higher marks for the English assignment overall? Explain your reasoning.

14. Sammy is a wedding planner. He asks a sample of 30 guests to try different entrees and vote for their favourite. He recorded the results in a frequency table.

Entree	Tally	Frequency
Fish cakes	𝍸	5
Spring rolls	𝍸 I	6
Arancini balls	𝍸 III	8
Samosas	𝍸 II	7
Bread and dips	IIII	4

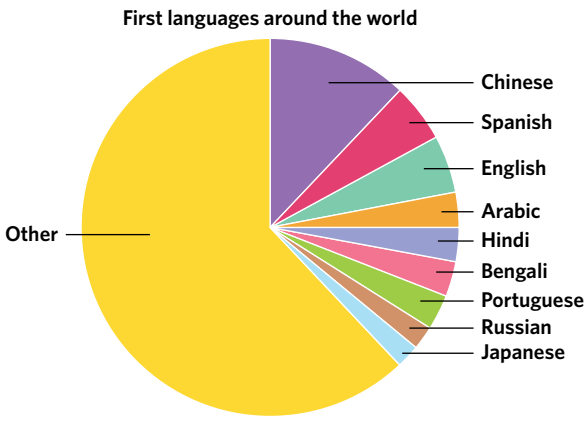
There are 240 guests coming to the wedding. According to the table, at least how many servings of arancini balls should be prepared for the event?

Reasoning

Question working paths

Mild 15 (a,b,c,e) | Medium 15 (a,b,c,e), 16 (a,b) | Spicy All

15. The following pie chart shows the proportions of first languages around the world.



a. Besides 'Other', which of the listed languages is spoken by the highest percentage of people in the world?

b. 62% of people in the world speak a language that falls in the 'Other' category. What percentage of the world speaks all the listed languages, not in the 'Other' category?

c. An equal number of people speak Arabic, Hindi, Bengali, and Portuguese. If they make up 12% of the world's population, then what percentage of the world speaks each of the aforementioned languages individually?

d. Spanish and English are first languages for 5% of the world's population each. Russian and Japanese speakers make up 4% of the world's population together. Determine the percentage of people who consider Chinese to be their first language.

e. The majority of the world's population speaks a language that falls in the 'Other' category. List two languages that could be included in this category.

16. State the type of variable (numerical or categorical) described in parts **a** and **b** and determine whether a dot plot, pie chart, or stem-and-leaf plot is suitable to display the data.

a. A survey where 30 participants have to rate an activity as a whole number out of five.

b. A study on the number of seconds it takes people to solve a Rubik's Cube.

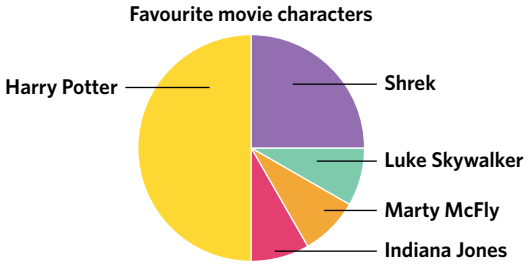
c. Compare your answers for parts **a** and **b** and comment on the restrictions of displaying certain types of variables.

Extra spicy

17. Matthew wants to sort all the numbers from 1 to 100 into prime, composite, and neither. What percentage of the numbers will be composite?

A. 1% **B.** 25% **C.** 74% **D.** 75% **E.** 76%

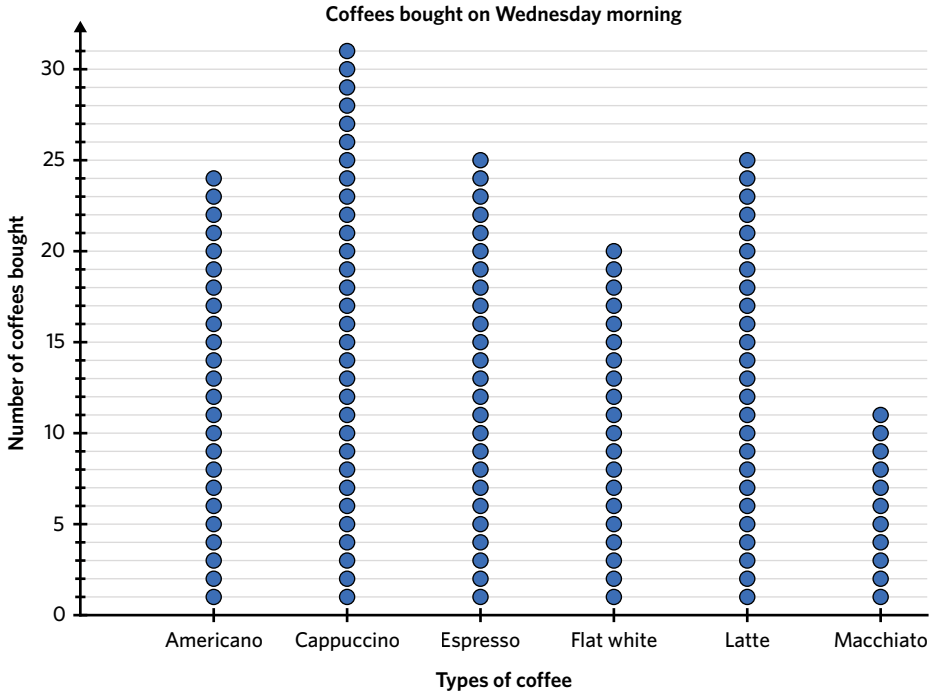
18. The following pie chart shows the proportions of votes for favourite movie characters of 120 people.



How many people chose Marty McFly as their favourite character?

A. 8 **B.** 10 **C.** 24 **D.** 30 **E.** 40

19. The following dot plot shows the number of coffees bought at a cafe between 7 am and 8 am on a Wednesday morning.



If each customer bought an average of two coffees, then how many customers did the cafe have during this time?

20. Artemis counted all the wild animals in the park. She counted 400 deer, 300 rabbits, 250 squirrels, 150 foxes, 75 wolves, and 25 bears. In a pie chart of this data, which type of animal would be represented by a 45° sector?

Remember this?

21. What is ★?

$8 - (2 + 4 \times 6 - 18) = ★$

A. -10 **B.** 0 **C.** 5 **D.** 16 **E.** 42

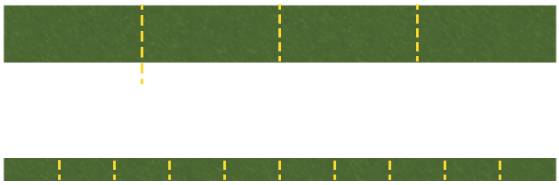
22. A mobile phone has a storage capacity of 256 GB. If 174 GB is available, what percentage of the storage has been used? Express your answer to the nearest percentage.

A. 19% **B.** 32% **C.** 54% **D.** 68% **E.** 73%

23. Masaki made two long pieces of sushi rolls.

She cuts the first piece in quarters and the second in tenths.

The longer pieces were 6 cm longer than the shorter pieces.



What is the length of the original piece before cutting?

A. 10 cm **B.** 18 cm **C.** 24 cm **D.** 32 cm **E.** 40 cm

Back to contents

8D Frequency tables and histograms

Frequency tables and histograms are used to organise and display large sets of data. The purpose of a histogram is to show the shape of a distribution. This is useful for identifying any special features or trends within a data set.

LEARNING INTENTIONS

Students will be able to:

- construct and interpret grouped frequency tables
- construct and interpret histograms
- define the terms symmetric, skewed, and bi-modal.

KEY TERMS AND DEFINITIONS

A **class interval** defines the width of the groups into which a set of data has been organised.

The **range** measures the spread of data. It is the difference between the maximum and minimum values.

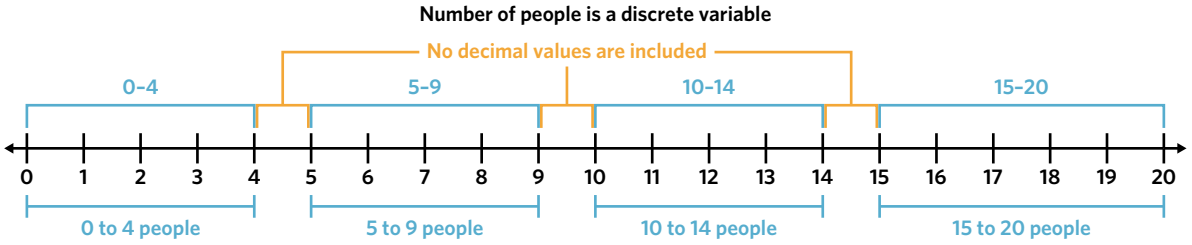
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Antonio Guillem/Shutterstock.com

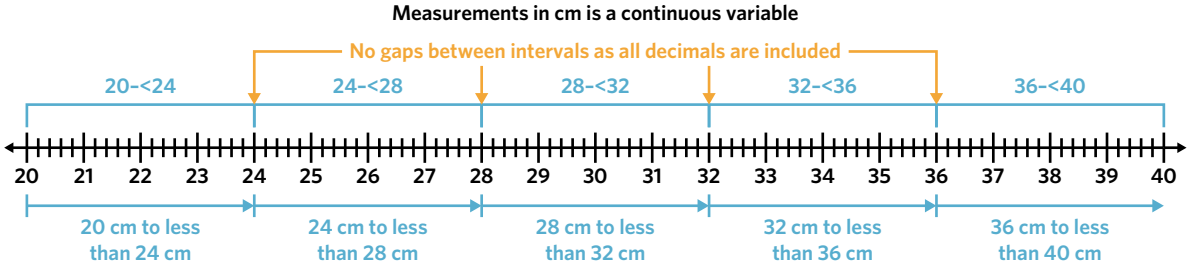
Histograms are often used by companies to analyse data, such as customer wait times. A company that cares about their customers will do this regularly. This allows for processes to be put in place to ensure all customers feel valued and supported.

Key ideas

1. **Discrete** data is grouped in **inclusive** class intervals, where the minimum and maximum values of each interval are included.



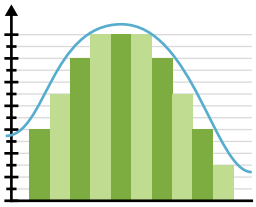
2. **Continuous** data is grouped in **exclusive** class intervals, where the maximum values of each interval are not included.



Continues →

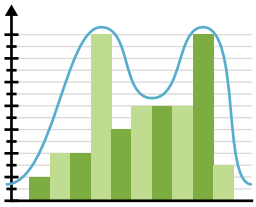
3. Histograms must contain data grouped in exclusive intervals only as there must not be any gaps between the bars. Histograms can be symmetric, skewed, or bi-modal.

Symmetric histogram



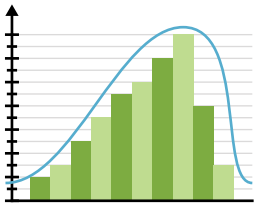
The overall shape is symmetrical.

Bi-modal histogram



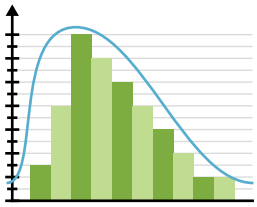
The overall shape shows two peaks.

Negatively skewed histogram



'Tail' on the left, closer to negative values indicates the direction of skew.

Positively skewed histogram



'Tail' on the right, closer to positive values indicates the direction of skew.

Worked example 1

Constructing grouped frequency tables of discrete data

The following list contains the letter counts of the 20 longest words in the English language.

25, 15, 36, 45, 28, 26, 27, 22, 25, 20, 21, 15, 30, 28, 15, 15, 29, 26, 15, 25

a. Organise the data into inclusive class intervals of size 5. WE1a

Working

Maximum value = 45

Minimum value = 15

Range = 45 − 15

= 30

Number of class intervals = 30 ÷ 5 = 6

The intervals will be 15–20, 21–26, 27–32, 33–38, 39–44, and 45–50 letters per word.

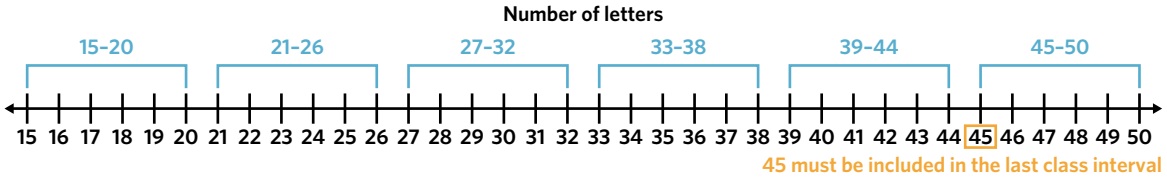
Thinking

Step 1: Calculate the range of the data by identifying the minimum and maximum values.

Step 2: Divide the range by 5 and round up to the nearest whole number. This will be the number of ordered groups, containing 5 values.

Step 3: Starting with the minimum value, group the data in inclusive class intervals, 5 letters long each. Make sure that the last interval contains the maximum value.

Visual support



Continues →

b. Record the grouped data in a frequency table. WE1b

Working

Letters per word	Tally	Frequency
15–20	𝍸 I	6
21–26	𝍸 I	7
27–32	𝍸	5
33–38	I	1
39–44		0
45–50	I	1

Thinking

Step 1: Draw a frequency table and list the class intervals in the first column.

Step 2: Go through the list of values and tally each class interval occurrence in the middle column.

Step 3: Convert the tallied results to frequencies.

Student practice

The following list contains the number of times 20 people have been to a hairdresser in one year.

16, 25, 4, 0, 1, 13, 15, 20, 9, 12, 12, 5, 2, 4, 10, 17, 8, 10, 12, 10

a. Organise the data into inclusive class intervals of size 5.

b. Record the grouped data in a frequency table.

Worked example 2

Constructing grouped frequency tables of continuous data

The following list contains the heights of the tallest 20 people in human history, in cm.

269, 258, 251.4, 250, 267, 264, 272, 262, 257, 258, 256, 249, 251, 268, 262, 249, 248, 255, 255, 254

a. Organise the data into exclusive class intervals of size 5. WE2a

Working

Maximum value = 272

Minimum value = 248

Range = 272 − 248

= 24

Number of class intervals = 24 ÷ 5 = 4.8 ≈ 5

The intervals will be 248–< 253 cm, 253–<258 cm, 258–<263 cm, 263–<268 cm, and 268–<273 cm.

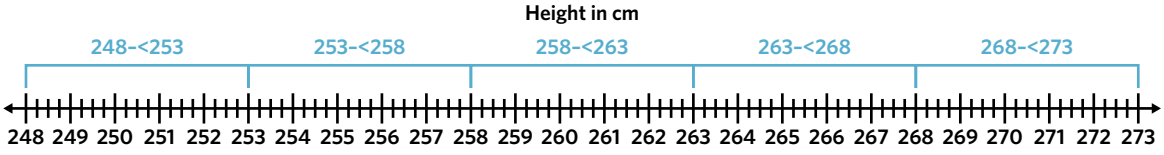
Thinking

Step 1: Calculate the range of the data by identifying the minimum and maximum values.

Step 2: Divide the range by 5 and round up to the nearest whole number. This will be the number of ordered groups, each containing 5 values.

Step 3: Starting with the minimum value, group the data in exclusive and equally sized class intervals. Make sure that the maximum value is included in the last class interval.

Visual support



Continues →

b. Record the grouped data in a frequency table. WE2b

Working

Height in cm	Tally	Frequency
248–<253	𝍸 I	6
253–<258	𝍸	5
258–<263	IIII	4
263–<268	II	2
268–<273	III	3

Thinking

Step 1: Draw a frequency table and list the class intervals in the first column.

Step 2: Go through the list of values and tally each class interval occurrence in the middle column.

Step 3: Convert the tallied results to frequencies.

Student practice

The following list contains the heights of 20 Year 8 netball players, in cm.
147, 160, 171, 170, 161, 165, 150, 157, 155.6, 168, 152, 169, 171, 175, 165, 166, 163, 168, 170, 171

a. Organise the data into exclusive class intervals of size 5.

b. Record the grouped data in a frequency table.

Worked example 3

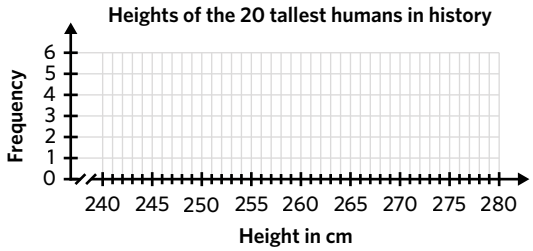
Constructing and interpreting histograms

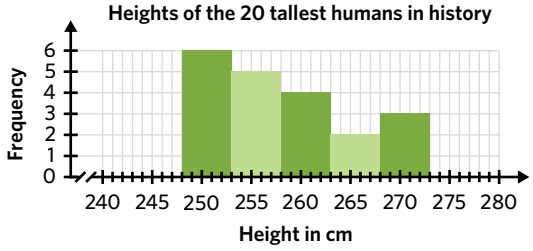
The following frequency table shows the heights of the 20 tallest people in human history.

Height in cm	Tally	Frequency
248–<253	𝍸 I	6
253–<258	𝍸	5
258–<263	IIII	4
263–<268	II	2
268–<273	III	3

a. Construct a histogram of the data. WE3a

Working





Thinking

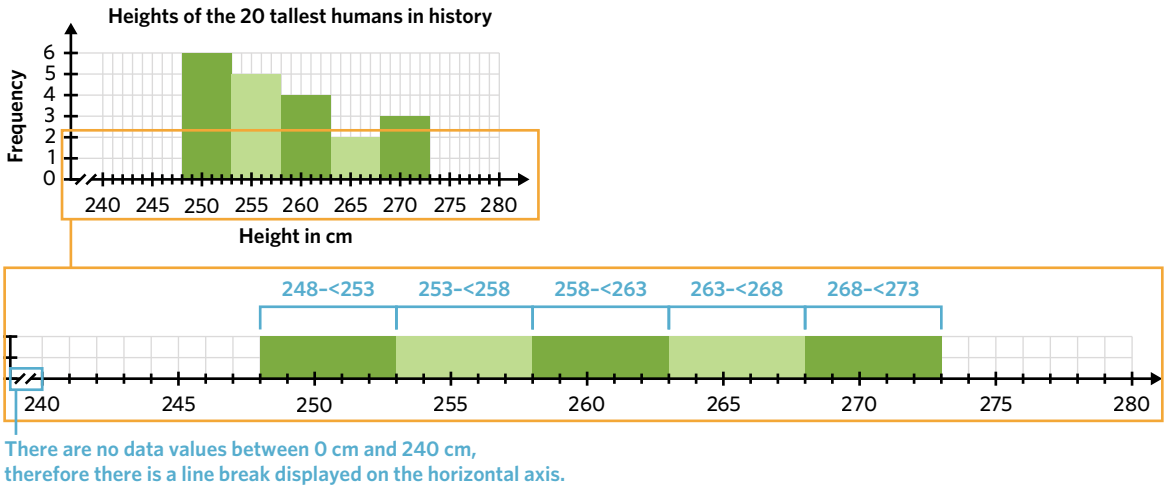
Step 1: Identify the smallest and largest value from the frequency table to determine how to scale the horizontal (x) axis.

Step 2: Identify the smallest and largest value from the frequency table to determine how to scale the vertical (y) axis.

Step 3: Draw the bars of the histogram using the frequencies and class intervals given in the table. Title the histogram.

Continues →

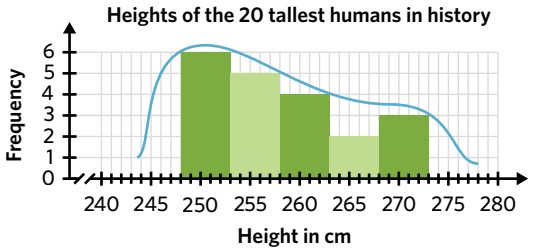
Visual support



There are no data values between 0 cm and 240 cm, therefore there is a line break displayed on the horizontal axis.

b. Determine the shape of the histogram. WE3b

Working



The histogram has a 'tail' on the right, so it is positively skewed.

Thinking

Step 1: Trace a rough outline of the shape of the histogram.

Step 2: Determine whether the histogram is symmetric, skewed, or bi-modal.

Student practice

The following frequency table shows the rent amounts, in Australian dollars, per week of 20 three-bedroom properties in Melbourne.

Rent per week (AUD)	Tally	Frequency
460–<560	𝍸	5
560–<660	𝍸	5
660–<760	IIII	4
760–<860	I	1
860–<960	III	3
960–<1060	I	1
1060–<1160	I	1

a. Construct a histogram of the data.

b. Determine the shape of the histogram.

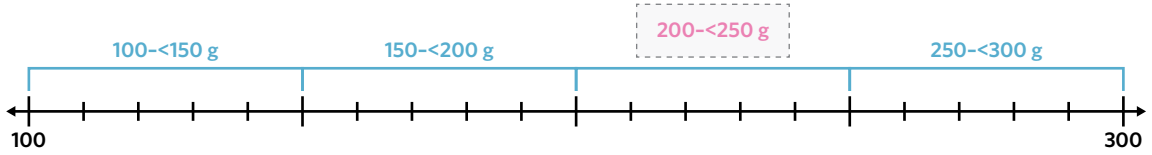
8D Questions

Understanding worksheet

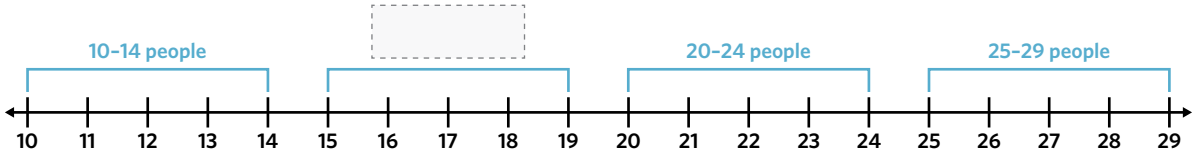
1. Fill in the missing class intervals.

Example

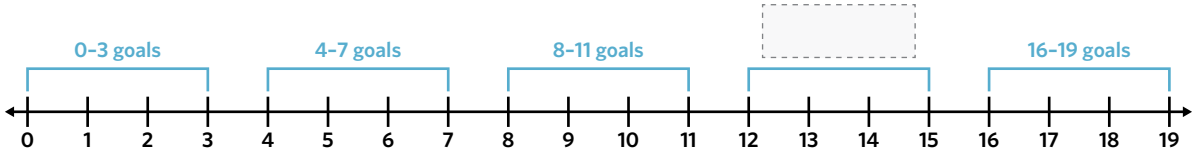
The number line represents weights, in grams.



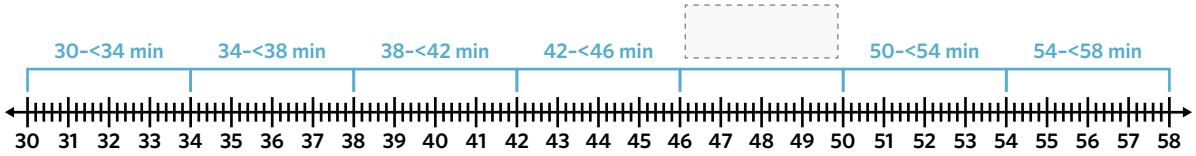
a. The number line represents the number of people.



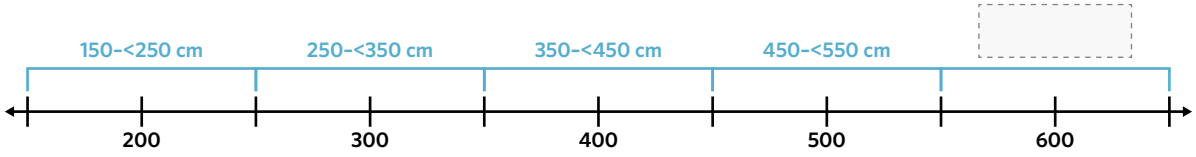
b. The number line represents the number of goals.



c. The number line represents times, in minutes.

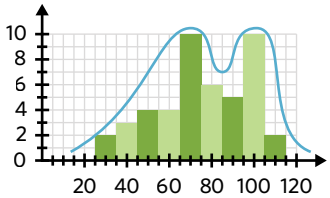


d. The number line represents lengths, in cm.



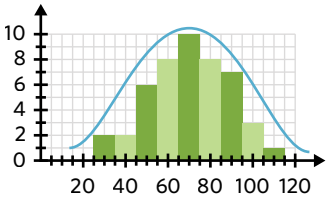
2. Circle the correct description for the overall shape of each histogram.

Example



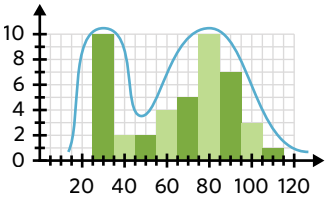
Symmetric
(Bi-modal)
Negatively skewed
Positively skewed

a.



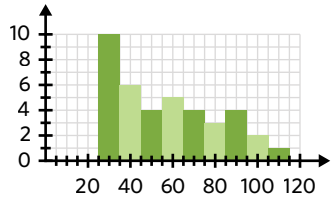
Symmetric
Bi-modal
Negatively skewed
Positively skewed

b.



Symmetric
Bi-modal
Negatively skewed
Positively skewed

c.



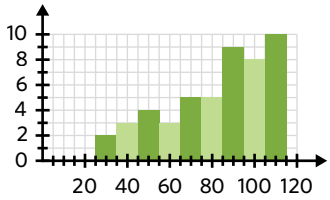
Symmetric

Bi-modal

Negatively skewed

Positively skewed

d.



Symmetric

Bi-modal

Negatively skewed

Positively skewed

3. Fill in the blanks by using the words provided.

continuous | shape | histogram | intervals | inclusive

Numerical data can be grouped in class _____. Discrete values are organised in _____ class intervals, whereas _____ data must be organised in exclusive class intervals. Grouped continuous data can be displayed on a _____. It is a chart used for analysing the overall _____ of a distribution.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b), 6 (a,b,c), 7 (a,b,c), 8	4 (b,c), 5 (b,c), 6 (b,c,d), 7 (c,d,e), 8	4 (c,d), 5 (c,d), 6 (c,d,e), 7 (d,e,f), 8

4. The following list contains the average class sizes in 20 countries around the world. WE1

24, 25, 36, 27, 21, 23, 17, 20, 22, 32, 33, 26, 25, 24, 18, 19, 32, 32, 23, 24

a. Find the range of the data.

b. Organise the data into inclusive class intervals of size 5.

c. Record the grouped data in a frequency table using the class intervals from part **b**.

d. According to the frequency table in part **c** what is the most common class size range in the world?

---

5. The following list contains the heights of 20 randomly chosen Year 8 students, in cm. WE2

136, 164, 152, 153, 155, 170, 150, 165, 151, 170, 162, 170, 154, 149, 152, 140, 156, 158, 164, 160

a. Find the range of the data.

b. Organise the data into exclusive class intervals of size 5.

c. Record the grouped data in a frequency table using the class intervals from part **b**.

d. If a random student was picked from the group, approximately how tall are they most likely to be?

---

6. The following list contains the weekly pay of 20 Victorians, in Australian dollars. WE3

658, 795, 1247, 982, 703, 594, 795, 1168, 912, 788, 655, 817, 659, 481, 701, 529, 1015, 1019, 1013, 1008

a. Using the data provided, fill in the tally and frequency columns of the following table.

b. Determine the size of the exclusive class intervals in the frequency table.

c. Construct a histogram of the data.

d. What is the most common salary range?

e. Determine the shape of the histogram.

Weekly salary (AUD)	Tally	Frequency
481–<581		
581–<681		
681–<781		
781–<881		
881–<981		
981–<1081		
1081–<1181		
1181–<1281		

7. The following list contains the rent amounts per week of 20 one-bedroom properties in Melbourne, in Australian dollars.

450, 350, 540, 600, 620, 440, 550, 600, 600, 700, 410, 450, 575, 430, 470, 730, 380, 540, 650, 575

a. Calculate the range of the data.

b. Group the data into exclusive class intervals of size 50.

c. Construct a frequency table of the data.

d. Construct a histogram of the data.

e. What is the most common price range for a one-bedroom property in Melbourne, according to the histogram?

f. Determine the shape of the histogram.

g. What does the shape of the histogram tell you about the distribution?

---

8. Rex wants to group values ranging from 45 minutes to 90 minutes in exactly 5 exclusive class intervals. Determine the size of each class interval.

A. 5 **B.** 8 **C.** 9 **D.** 10 **E.** 11

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Enter a measurement of 150 m in the given frequency table.

Student A

Length (meters)	Tally
0–<50	
50–<100	
100–<150	
150–<200	I
200–<250	

Student B

Length (meters)	Tally
0–<50	
50–<100	
100–<150	I
150–<200	
200–<250	

b. Group 0 to 60 minutes in exclusive class intervals of size 10.

Student A

Time (minutes)
0–<10
11–<20
21–<30
31–<40
41–<50
51–<60

Student B

Time (minutes)
0–<10
10–<20
20–<30
30–<40
40–<50
50–<60

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Giuseppe is analysing his profits for the month of April. He collects data in Australian dollars every day and records it in a frequency table. Should Giuseppe choose inclusive or exclusive class intervals to group the data for a histogram?

11. Patrick is a teacher who wants to record the assignment results for his class of 28 students in a frequency table. The marks of the students ranged from 25 to 100, including decimals. How many exclusive class intervals of size 10 will be in Patrick's table?

12. Stacey has collected data ranging from 50 cm to 350 cm. Determine the size of each exclusive class interval if Stacey organises the measurements in 6 equal ordered groups.

13. Lena is an accountant who is analysing her client's daily spendings during the previous financial year. She has to group amounts ranging from \$15 to \$500 dollars in exclusive class intervals of size 100. List the class intervals starting with the minimum of \$15 and including the maximum of \$500.

14. Sally asks 29 people how long it takes them to get to school in the morning. She records the results in the following table.

Time to get to school (minutes)	Tally	Frequency
1–11	𝍸 𝍸 II	12
11–21	𝍸 III	8
21–31	𝍸	5
31–41	II	2
41–51	I	1
51–61	I	1

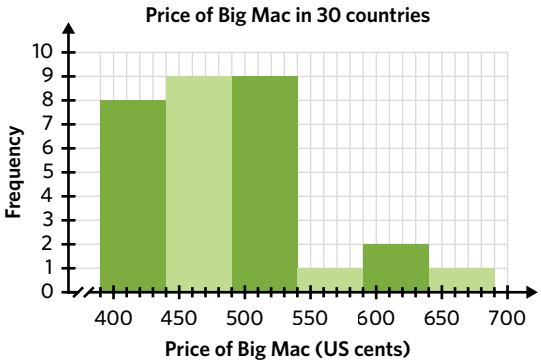
Predict the overall shape of the histogram Sally constructs from the data.

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. The following histogram shows the prices of Big Macs in 30 countries, in US cents.



a. The cheapest Big Mac on the list is in Estonia at \$3.90, while the most expensive is in Switzerland at \$6.80. What is the range of the data, in cents?

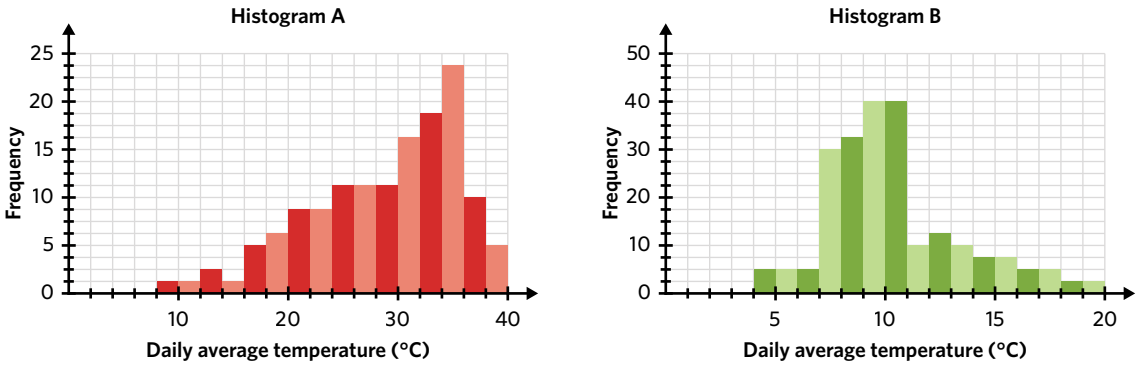
b. Determine the size of the exclusive class intervals in the histogram.

c. What is the most common price range for a Big Mac, according to the histogram, in dollars?

d. Transfer the data from the histogram to a frequency table.

e. Suggest a reason why a common item, such as a Big Mac may be priced differently, depending on the country.

16. Match the histograms with the descriptions in parts **a** and **b**.



a. The distribution of daily average temperatures in Melbourne, Victoria during summer.

b. The distribution of daily average temperatures in Melbourne, Victoria during winter.

c. Compare your answers for parts **a** and **b**. What does the shape of a distribution tell you about a data set?

Extra spicy

17. Sam collects and graphs the data for customer wait times on a histogram. The values range from 1.5 minutes to 25 minutes and the data has been grouped into exclusive class intervals of size 5. The distribution is symmetric with a distinct peak in the middle of the graph. What is the most common wait time for a customer?

A. 6–11 minutes
B. 6.5–11.5 minutes
C. 11–16 minutes
D. 11.5–16.5 minutes
E. 11.75–16.75 minutes

18. Warren collects data on the heights of 1000 children ranging in age from 10 to 16 years old.

When he constructs a diagram of the data, he finds that the distribution is bi-modal. Choose the most likely reason for the two modes at 140 cm and 165 cm.

A. Warren did not graph the data accurately.
B. The average heights of younger children and teenagers are significantly different.
C. Warren's data was not collected properly.
D. The sample is not large enough to be accurate.
E. It is a coincidence and does not mean anything.

19. An outlier in a set of data is a value that differs significantly from the others in the list. How does an outlier affect the shape of a histogram?

20. Anita is analysing her screen time for the week. She spent at least half of the total 40 hours playing World of Warcraft and no more than 5 hours doing her accounts. The rest of the time Anita spent watching movies and listening to music. Express the amount of time Anita spent watching movies and listening to music as an exclusive time interval.

Remember this?

21. Dom is trying to write down all the prime numbers between 20 and 40.

He writes: 21, 23, 29, 31, 37.

One of these numbers is not a prime number. Which is it?

A. 21 **B.** 23 **C.** 29 **D.** 31 **E.** 37

22. Uncle Bray bought three 4-litre cartons of apple juice for his party. There are 23 guests at his party.

If he is to serve his guests equal volumes of apple juice, how much apple juice is each guest receiving? Write your answer to 2 decimal places.

A. 0.13 L **B.** 0.17 L **C.** 0.34 L **D.** 0.52 L **E.** 0.60 L

23. Joey is trying to build a small ship. He uses 3 cylindrical blocks as smokestacks and places them on top of a large block.

Below are drawings of the front view and side view.

Front view **Side view**

Which of these is the correct top view?

A.

B.

C.

D.

E.

8E Measures of centre and spread

In order to draw meaning from gathered data, its central tendency and spread must be analysed. There are a number of factors that can affect the accuracy of data analysis, outliers being one of them. Outliers can be identified by graphing the distribution and assessing summary statistics of data sets both including and excluding outliers.

LEARNING INTENTIONS

Students will be able to:

- understand that the range is a measure of spread for numerical data
- compare the range of two or more sets of numerical data
- understand and compare the effect of outliers on the mean, median, and range.

KEY TERMS AND DEFINITIONS

A statistical **average** is a value which measures the central tendency of a set of numbers. It can be a mean, median, or mode.

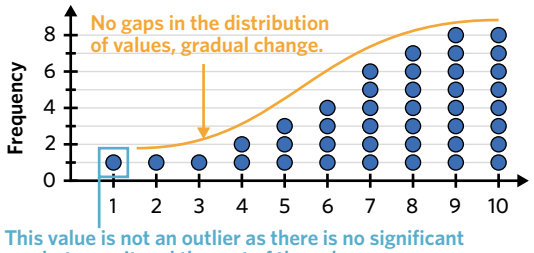
A statistical **distribution** describes how values are distributed on a scale. The shape of a distribution tells us which values are common and which are not.

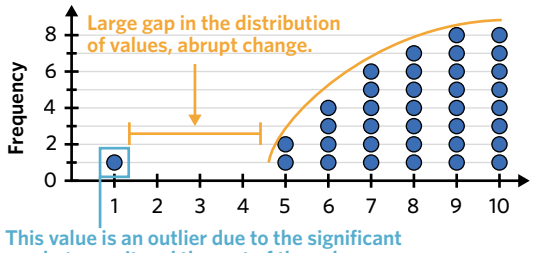
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Including outliers in summary statistics is one way that companies and institutions can present the public with inaccurate information. Universities often release data on the salaries of their past alumni, making sure to include any maximum outliers. This distorts the mean of the data and gives the illusion of above average employment opportunities to any prospective students.

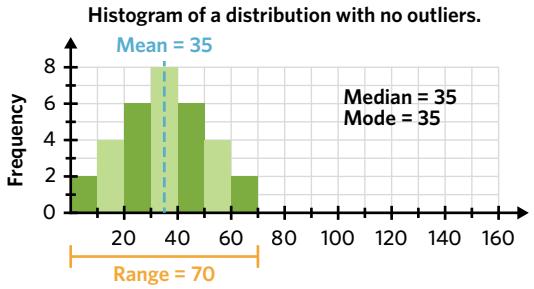
Key ideas

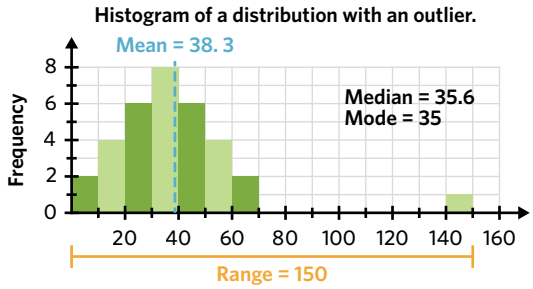
1. To be considered an **outlier**, a numerical value must differ significantly from others in the distribution.





2. Outliers affect the range and mean of numerical data, and have little to no effect on the median and mode.





Worked example 1

Calculating summary statistics

Determine the mean, median, mode, and range of the following data sets. Round your answers to 1 decimal place, where applicable.

a. 6, 1, 14, 1, 0, 8, 5, 3, 1, 6, 2, 11, 8, 9, 13, 10, 1, 2, 0, 6 WE1a

Working

0, 0, 1, 1, 1, 1, 2, 2, 3, 5, 6, 6, 6, 8, 8, 9, 10, 11, 13, 14

$$\text{Mean} = \frac{\text{sum of all values}}{\text{number of values}} = \frac{107}{20} = 5.35 \approx 5.4$$

$$\text{Median} = \frac{5 + 6}{2} = \frac{11}{2} = 5.5$$

$$\text{Mode} = 1$$

$$\text{Range} = 14 - 0 = 14$$

Thinking

Step 1: Organise the data in ascending order.

Step 2: Calculate the mean by adding all the values together and dividing by the total number of values.

Step 3: Identify the median by the 'middle' value in the ordered set. If there is no distinct middle number, then add the two middle numbers together and divide the result by two.

Step 4: Identify the mode by the value with the highest frequency in the data set.

Step 5: Calculate the range of the data by subtracting the minimum from the maximum value in the data set.

Visual support

Range = 14 − 0 = 14

0, 0, [1, 1, 1, 1,] 2, 2, 3, [5, 6,] 6, 6, 8, 8, 9, 10, 11, 13, 14

Mode = 1 Median = $\frac{5 + 6}{2} = \frac{11}{2} = 5.5$

b. Complete the third column ($n \times f$) of the table below and determine the mean, median, mode, and range of the data set. Round your answers to 1 decimal place, where applicable. WE1b

Number	Frequency
1	3
2	4
3	5
4	8
5	5
6	4
Total	29

Continues →

Working

Number (n)	Frequency (f)	$n \times f$
1	3	$1 \times 3 = 3$
2	4	$2 \times 4 = 8$
3	5	$3 \times 5 = 15$
4	8	$4 \times 8 = 32$
5	5	$5 \times 5 = 25$
6	4	$6 \times 4 = 24$
Total	29	107

$\text{Mean} = \frac{107}{29} = 3.7$

$\text{Location of median value} = \frac{\text{total frequency}}{2}$

$= \frac{29}{2} = 14.5 \approx 15$

When the total frequency is equal to 15, $n = 4$

$\therefore \text{Median} = 4$

$\text{Mode} = 4$

$\text{Range} = 6 - 1 = 5$

Thinking

Step 1: Multiply the numbers (n) by the corresponding frequencies (f). Enter these values in a third column in the table and find their total.

Step 2: Calculate the mean by dividing the total ($n \times f$) by the total frequency (f).

Step 3: Identify the location of the median value by dividing the total frequency by two and rounding the answer to the nearest whole number.

Step 4: Determine the median from the frequency table by locating the interval where the median value falls.

Step 5: Identify the mode by the number (n) with the highest corresponding frequency (f).

Step 6: Calculate the range of the data by subtracting the minimum from the maximum value in the set.

Visual support

Number (n)	Frequency (f)
1	3
2	4
3	5
4	8
5	5
6	4
Total	29

Sum of $f = 3 + 4 = 7$, so $f < 15$

Sum of $f = 7 + 5 = 12$, so $f < 15$

Sum of $f = 12 + 8 = 20$ $f = 15$ in this interval

The median is 4 because the sum of frequencies, $f = 15$ occurs when $n = 4$

Student practice

Determine the mean, median, mode, and range of the following data sets. Round your answers to 1 decimal place, where applicable.

a. 5, 1, 3, 5, 12, 3, 8, 2, 9, 6, 2, 12, 15, 12, 5, 15, 11, 14, 2, 12, 8

b.

Number (n)	Frequency (f)
1	2
2	5
3	6
4	5
5	2
6	3
Total	23

Worked example 2

Determining the effects of outliers

Using the following set of data, answer the questions. Round your answers to 1 decimal place, where applicable.

6, 1, 14, 1, 0, 8, 5, 3, 50, 1, 6, 2, 11, 8, 9, 13, 10, 1, 2, 0, 6

a. Identify the outlier in the data set. WE2a

Working	Thinking
0, 0, 1, 1, 1, 1, 2, 2, 3, 5, 6, 6, 6, 8, 8, 9, 10, 11, 13, 14, 50	**Step 1:** Organise the data in ascending order.
The maximum value (50) is significantly larger than the previous value, 14.	**Step 2:** Determine if the minimum or maximum value in the data set is an outlier.
$\therefore$ Outlier $= 50$	

b. Determine the mean, median, mode, and range of the data with and without the outlier included. WE2b

Working	Thinking
Without the outlier the summary statistics are:	**Step 1:** Calculate the mean, median, mode, and range of the data without the outlier included.
Mean $= 5.4$	
Median $= 5.5$	
Mode $= 1$	
Range $= 14$	
With the outlier the summary statistics are:	**Step 2:** Calculate the mean, median, mode, and range of the data with the outlier included.
Mean $= 7.5$	
Median $= 6$	
Mode $= 1$	
Range $= 50$	

c. Determine the effects of the outlier on the distribution. WE2c

Working	Thinking
The mean increased from 5.4 to 7.5.	Identify the statistics that changed when an outlier was included in the data, compared to when it was not included.
The median increased from 5.5 to 6.	
The range increased from 14 to 50.	
The mode did not change from 1.	

Student practice

Using the following set of data, answer the questions. Round your answers to 1 decimal place, where applicable.

5, 1, 3, 5, 12, 3, 8, 2, 9, 6, 2, 12, 15, 12, 5, 60, 15, 11, 14, 2, 12, 8

a. Identify the outlier in the data set.

b. Determine the mean, median, mode, and range of the data with and without the outlier included.

c. Determine the effects of the outlier on the distribution.

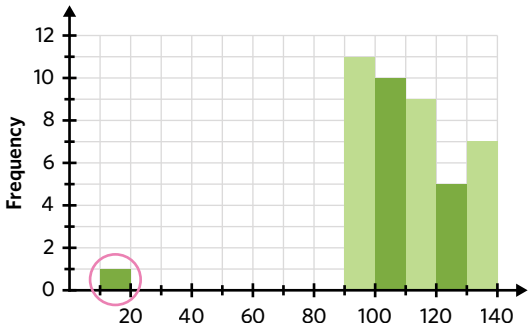
Back to contents

8E Questions

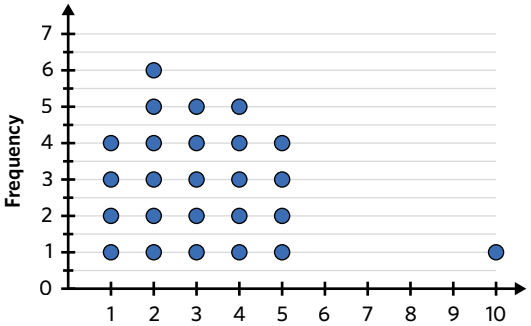
Understanding worksheet

1. Locate the outliers in the following distributions.

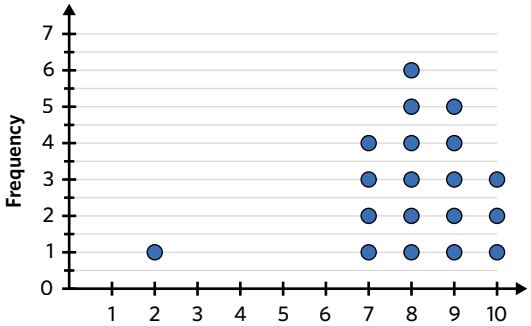
Example



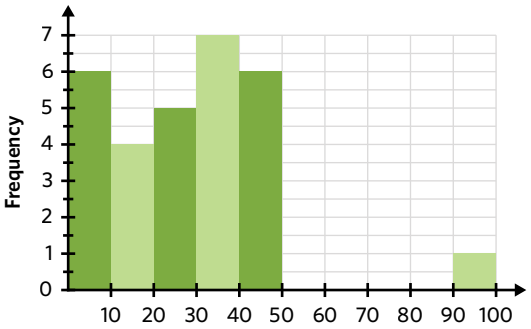
a.



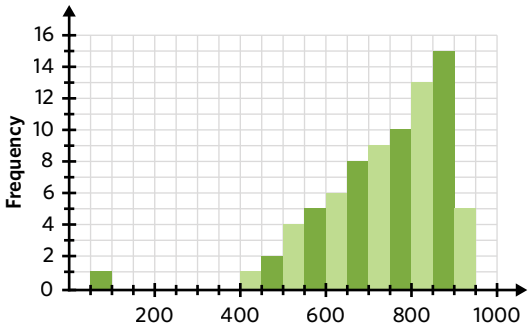
b.



c.



d.



2. Determine if the following statements are true or false.

Example

Statement	True	False
The median of a distribution is significantly affected by outliers.	☐	☑

	Statement	True	False
a.	The mean of a distribution will increase if a large maximum outlier is included.	☐	☐
b.	The mean of a distribution will decrease if a small minimum outlier is included.	☐	☐
c.	The range of a distribution is always affected by outliers.	☐	☐
d.	The mode of a distribution can often be affected by outliers.	☐	☐

3. Fill in the blanks by using the words provided.

maximum | mean | range | median

The average of a numerical data set can be shown by calculating the mean, _____, or mode.

The spread of a distribution is represented by the _____. An outlier is a minimum or _____ value that varies significantly from the others in the distribution. Outliers have an effect on the _____ and range of a numerical data set or distribution.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b), 6 (a,b,c), 7 (a,b,c), 8	4 (c,d,e), 5 (b,c), 6 (c,d,e), 7 (b,c,d), 8	4 (e,f,g), 5 (c,d), 6 (e,f,g), 7 (c,d,e), 8

4. Determine the mean, median, mode, and range of the following data sets. Round your answers to 1 decimal place, where applicable. WE1a

a. 3, 9, 8, 9, 7, 11, 15, 7, 13, 9, 11, 4, 5, 15, 7, 2, 11, 2, 4, 8

b. 8, 12, 12, 12, 19, 6, 20, 7, 13, 9, 7, 5, 6, 7, 5, 17, 8, 10, 10

c. 21, 12, 10, 15, 22, 22, 21, 29, 12, 18, 16, 10, 17, 28, 21, 25, 20, 17, 25, 22

d. 67, 42, 57, 14, 29, 48, 10, 69, 16, 13, 44, 59, 64, 30, 67, 15, 63, 18, 40

e. 9.5, 7.5, 1, 15.5, 5.5, 15.5, 5, 3, 12, 7, 9.5

f. 14, 10, 18.2, 10.4, 16.6, 16, 21.4, 16.8, 18, 10, 21.6, 11.3, 18, 19.5, 14, 28

g. 0.3, 14, 3, 1.5, 12.4, 5.4, 5.3, 10.9, 19.1, 11.6, 15.1, 18.8

---

5. Complete the third column ($n \times f$) of the tables below and determine the mean, median, mode, and range of each data set. Round your answers to 1 decimal place, where applicable. WE1b

a.

Number (n)	Frequency (f)	$n \times f$
0	3	
1	4	
2	6	
3	8	
Total	21	

b.

Number (n)	Frequency (f)	$n \times f$
1	2	
2	4	
3	6	
4	2	
5	4	
Total	18	

c.

Number (n)	Frequency (f)	$n \times f$
5	1	
6	4	
7	4	
8	5	
9	3	
10	2	
Total	19	

d.

Number (n)	Frequency (f)	$n \times f$
10	5	
20	3	
30	4	
40	6	
50	4	
Total	22	

6. Determine whether the following data sets contain outliers. State the value(s) of the outlier(s), where applicable. WE2a

- **a.** 5, 40, 8, 4, 10, 4, 13, 13, 16, 10, 5, 11, 10
- **b.** 16, 11, 14, 13, 14, 17, 19, 12, 15, 15, 15, 14 , 16, 10
- **c.** 11, 18, 25, 25, 15, 20, 14, 20, 23, 12, 22, 21, 21, 23, 18
- **d.** 21, 32, 30, 20, 36, 9, 35, 24, 24, 37, 38, 25, 37, 37, 34, 32, 35, 20, 28
- **e.** 7, 13, 5, 10, 9, 11, 8, 8, 0, 7, 5, 9, 9, 11,13
- **f.** 50, 27, 23, 29, 26, 33, 31, 23, 29, 26, 20, 24, 27, 23, 25, 24, 29, 32, 31
- **g.** 80, 107, 82, 115, 93, 94, 106, 98, 117, 99, 101, 102, 102, 115, 138, 103, 105, 107, 108, 115, 116

---

7. The following data set contains the times 30 random secondary school students spent standing on their left legs, in seconds. WE2

25, 6, 69, 88, 20, 50, 90, 46, 36, 15, 87, 48, 27, 30, 23, 44, 22, 42, 5, 49, 20, 7, 14, 30, 130, 60, 22, 36, 1, 23

- **a.** Identify the outlier in the data set.
- **b.** Determine the mean, median, mode, and range of the data with the outlier included. Round your answers to 1 decimal place, where applicable.
- **c.** Determine the mean, median, mode, and range of the data without the outlier included. Round your answers to 1 decimal place, where applicable.
- **d.** Which of the summary statistics were/were not affected by the outlier?
- **e.** Calculate the change in the affected statistics from part **d** and state whether each change was an increase or decrease.

---

8. Determine the value of the median in the following data set.

14, 20, 25, 14, 26, 2, 27, 28, 14, 21, 28, 9, 29

A. 14 **B.** 19.8 **C.** 20.5 **D.** 21 **E.** 27

Spot the mistake

9. Select whether Student A or Student B is incorrect.

- **a.** Determine how the mean, median and mode are affected by the minimum outlier in the following data set.

  7, 17, 18, 21, 24, 24, 24, 25, 25, 27, 30

Student A

The mean decreases, while the median and mode do not change.

Student B

The mean, median and mode all decrease.

- **b.** Determine the median value of the following data set.

  7, 8, 16, 9, 16, 10, 14, 17, 14, 2, 3, 14, 17, 19, 15

Student A

The median is 17.

Student B

The median is 14.

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Zoe measures the temperature inside a terrarium every day for a month. After gathering the data, she can see one outlier caused by the wrong setting on her thermometer. Should Zoe use the mean or median to estimate the average temperature inside the terrarium that month?

11. Cindy asks 10 of her classmates how many times they have been overseas. When she organised the responses, the minimum value was 0, and the range was 7. Cindy then records one more friend's response and the range of her data increases by 5. How many times has the last friend Cindy surveyed been overseas?

12. Jeffrey wants to become a court typist. He needs to be able to type an average of 200 spoken words per minute in order to qualify. It took Jeffrey 1.5 hours to type up a speech 19 000 words long. Does he qualify for the job?

13. Carol is studying for an exam every day of the school week Monday until Friday. She studied for 4 hours on Monday, 2 hours on Wednesday, 3 hours on Thursday, and 2 hours on Friday. How long did Carol study for on Tuesday if the mean number of hours she studied per day was 3 that school week?

14. 5 friends are comparing their scores for a test. The median and mode of their scores are both equal to 75 out of 100. If the range of their scores is 50, then what is the lowest possible mark in the data set?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. The following stem-and-leaf plot shows the total number of red cards of 20 professional footballers.

Stem	Leaf
1	4 4 4 5 6 7 7 7 8 8 8 9
2	0 0 1 1 2 7 7
3	
4	6

Key

2 | 0 = 20 red cards

a. Determine the value of the outlier in the data set.

b. What are the values of the three modes in the distribution?

c. Calculate the mean, median and range of the data, including the outlier. Round your answers to 1 decimal place, where applicable.

d. Calculate the mean, median and range of the data, not including the outlier. Round your answers to 1 decimal place, where applicable. Compare your answer to part **c** and comment on any changes in the statistics.

e. Organised sports and other group activities often have clear rules and guidelines that must be followed by all participants. List two rules you think is most important for an activity you enjoy watching or participating in.

16. Calculate the mean, median, mode and range for each of the following sets of data. Round any answers to 1 decimal place, where applicable.

a. 2, 2, 3, 4, 4, 5, 6, 8, 9, 10

b. 2, 5, 10,10, 22, 25, 28, 29, 30, 50

c. Compare the statistics you calculated in parts **a** and **b** and state which of the data sets had more consistent results, i.e. less variation between the averages. How does the range affect the reliability of small data sets?

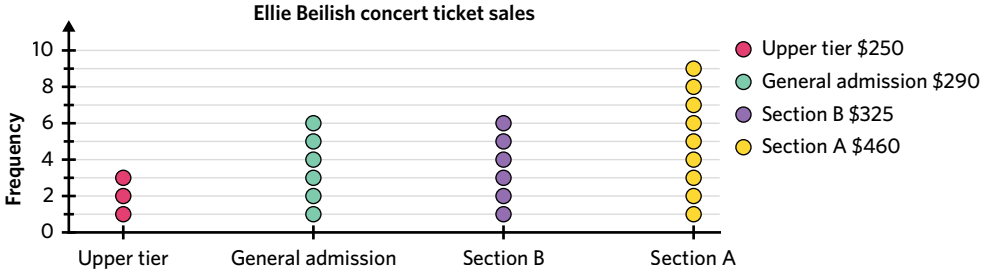
Extra spicy

17. On a street where the house numbers are odd on one side and even on the other, the mean house number on each of the sides is 13. Assuming that the house numbers begin at 1, find the total number of houses on the street.

18. A set of 21 values has a mean of 12 before an outlier with a value of 50 is included. Calculate the difference between the means of the set with and without the outlier. Round your answer to 1 decimal place.

A. 0.5 **B.** 1.7 **C.** 2.3 **D.** 2.4 **E.** 4.2

19. The following dot plot shows the number of sales of each ticket type to an Ellie Beilish concert. What is the mean price per ticket a concert goer paid to see the show?



A. $55.21 **B.** $325 **C.** $331.25 **D.** $357.50 **E.** $460

20. A set of 5 single digit numbers has the same mean, median, mode and range, which is equal to 4. What are the 5 numbers?

Remember this?

21. Steve pays $58 per month for his mobile data plan.

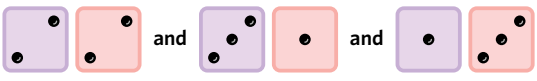
If he goes over his data allowance, he automatically is given a 1GB block of data for $10. Every time he uses an extra 1GB block before the end of the month, another 1GB is automatically added for another $10.

In May, Steve used his monthly allowance plus an additional y GB. How much was he charged in May?

A. $10y$ **B.** $58y$ **C.** $58y + 10$ **D.** $58 + 10y$ **E.** $68y$

22. Vutha and Zoe are playing snakes and ladders with 2 six-sided dice. 1 is purple and 1 is red. Zoe needs to roll a 4 to win.

There are 3 different ways Zoe can roll a 4:



Vutha needs to roll a 7 to win.

How many ways can Vutha roll a 7?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

23. Clo Yee is revising for her upcoming Statistics test by doing practice tests. All tests have the same number of marks available.

On her first practice test, she scored 25 marks.

On her second practice test, her score increased by 40%.

On her final practice test, her score increased by 20%.

How many marks did Clo Yee get on her final practice test?

A. 40 **B.** 41 **C.** 42 **D.** 43 **E.** 44

Chapter 8 extended application

1. Andy, Barbara, Conrad, Donna and Ernie are siblings who are standing in order from youngest to oldest, as shown in the diagram. Use the pronumerals a, b, c, d, and e to represent the ages of the siblings, respective to the order in which they are standing.

a. The mean age of the five siblings is 17 years. Write an equation for and calculate their combined age.

b. Assume that the mean age of the youngest three siblings is $15\frac{1}{3}$ years. Write an equation for and calculate the youngest three siblings' combined age.

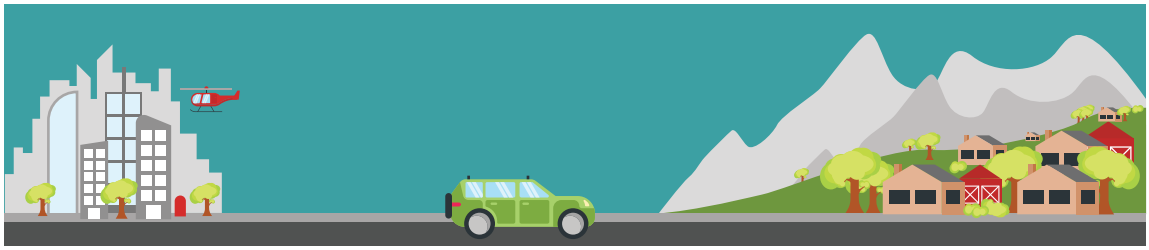
c. Assume that the mean age of the oldest three siblings is $18\frac{1}{3}$ years. Write an equation for and calculate the oldest three siblings' combined age.

d. Determine Conrad's age.

e. Andy and Barbara are twins, and the range of the five siblings' ages is 6 years. Determine the ages of all the siblings.

f. Living in a large family can teach you how to communicate and share with other people. What is the most important rule to you when it comes to sharing a house with others?

2. Milo is moving from Average Town to Sleepy Village. He works from home, and his annual income is $80 000. Use the given information to answer the questions.



Average Town:
Population = 2000
Mean annual income = $85 000

Sleepy Village:
Population = 250
Mean annual income = $40 000

a. Calculate the total annual income of Average Town.

b. Calculate the mean annual income of Average Town after Milo moves away, correct to the nearest dollar.

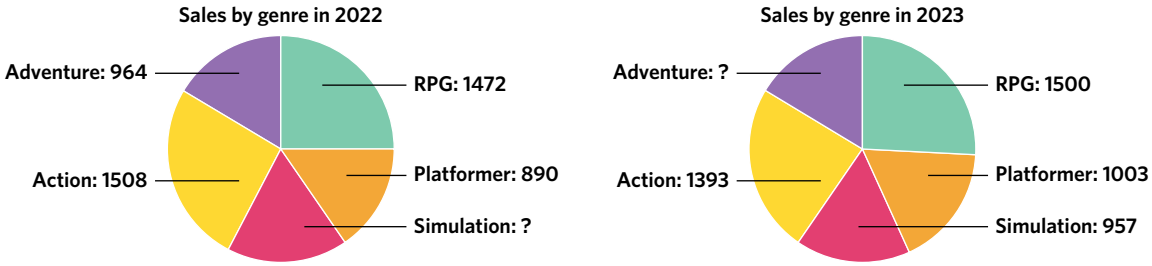
c. Calculate the mean annual income of Sleepy Village after Milo moves there, correct to the nearest dollar.

d. Before Milo moves there, if Sleepy Village was to try to match the mean annual income of Average Town, then by how much should their total annual income increase by?

e. A person from Average Town moves to Sleepy Village, thus doubling Sleepy Village's mean annual income. Calculate the mean annual income of Average Town after the move.

f. Give a reason for moving from the city to the countryside or vice versa.

3. At the GamerGo store, computer games can be categorised into five main genres, including action, adventure, platformer, role-playing games (RPG) and simulation. The following two pie charts show GamerGo sales by genre in 2022 and 2023.



a. Platformer games made up 15.203% of all sales in 2022. To the nearest whole number, how many computer games in total were sold in 2022?

b. Action games made up 24.005% of all sales in 2023. To the nearest whole number, how many adventure games were sold in 2023?

c. Calculate the percentage increase or decrease in game sales from 2022 to 2023. Round to 1 decimal place.

d. Complete the following table. Round all percentages to 3 decimal places.

Game genre	Number of sales in 2022	Number of sales in 2023	Sales increase/ decrease (%)
Action	1508	1393	−7.626%
Adventure	964		
Platformer	890	1003	12.697%
RPG	1472	1500	
Simulation		957	

e. GamerGo predicts that the genres will continue to grow and decrease in popularity at a similar rate in 2024. Construct a new pie chart showing the predicted sales proportions of each game genre based on these estimations. Round the number of sales for each genre to the nearest whole number.

f. Most types of entertainment, such as music, movies, books and video games can be bought and stored digitally. What is an advantage of acquiring these items in physical form?

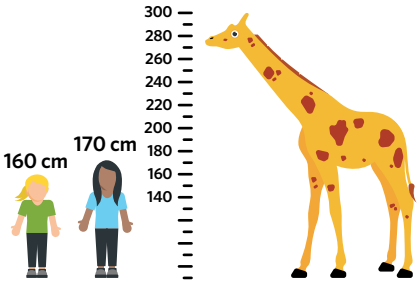
Chapter 8 review

Multiple choice

1. Which image represents numerical data? 8A

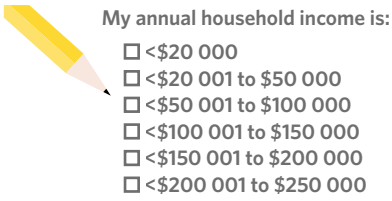
A.

B.



C.

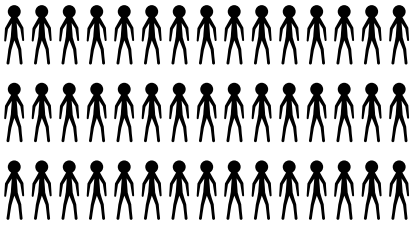
D.



E.

---

2. Below are the people in a given workplace. 8B



The manager wants to do an anonymous survey with a sample of individuals in the workplace. What would be the best option for the random sample to ensure the survey is not biased.

A. Sending an anonymous survey to all of the female employees.
B. Sending an anonymous survey to all of the male employees.
C. Sending an anonymous survey to fifteen random staff.
D. Sending an anonymous survey to thirty random staff.
E. Sending an anonymous survey to all of the staff.

---

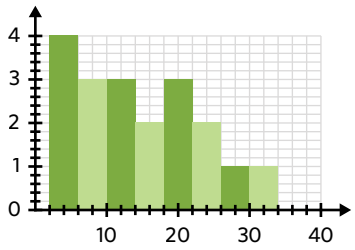
3. The following table shows the tallied results of a survey on the types of pets students prefer. Each student gave one response. 8C

Type of pet	Tally
Dog	𝍸 𝍸 I
Cat	𝍸 IIII
Guinea Pig	III
Rabbit	𝍸 II
No Animal	𝍸

What is the total number of students in this class?

A. 15
B. 25
C. 35
D. 45
E. 55

4. What is the correct description for the following histogram? 8D



A. Symmetric
B. Bi-modal
C. Negatively skewed
D. Positively skewed
E. None of the above

---

5. Which of the following sets of data has a median of 6? 8E

A. 2, 2, 2, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 9
B. 6, 6, 6, 6, 6, 6, 7, 7, 8, 8, 8, 8, 8, 9, 9
C. 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 6, 6, 7, 9, 9
D. 0, 0, 1, 1, 2, 3, 4, 5, 6, 6, 6, 6, 8, 9, 9
E. 1, 1, 1, 2, 3, 4, 6, 6, 7, 7, 8, 9, 9, 9, 9

Fluency

6. For each of the following 8A

i. State whether a sample or population is being used.
ii. State the name of the method used for data collection.

a. Asking everyone in a classroom what their favourite colour is.
b. Having grade 5 and 6 students decide on what the new uniform should look like.
c. Reading every second review on restaurants when deciding where to eat out.
d. Everyone votes on what song to play next when on a road trip.

---

7. For each of the following, state whether primary or secondary data is being used. 8A

a. Writing a review about a personal experience at a restaurant on their website.
b. Posting photos and videos on social media from a concert you went to.
c. Using the information in a textbook to complete a project.
d. An individual publishing a podcast interview they conducted with an artist.

---

8. Determine which of the given methods is more likely to produce a random sample that is representative of the population. 8B

a. Surveying an office of 50 people to find out everyone's favourite restaurant.

Method 1: Asking the 5 individuals that sit at your desk.

Method 2: Asking at least 50% of the office.

b. Voting on what movie to watch with your extended family.

Method 1: Getting half of your family to vote on what movie they want to watch.

Method 2: Getting 2 of your sisters to vote on what movie they want to watch.

c. Researching the average number of hours adults exercise each week.

Method 1: Surveying individuals that are walking into a gym.

Method 2: Surveying every third adult on the street.

d. Studying the wages of women in various industries.

Method 1: Putting out a survey in your work office for the women to respond to with their yearly salary.

Method 2: Putting out a survey to random women on the street to respond to with their yearly salary.

9. Determine the source of bias in the described sampling methods. 8B

a. Researching how many young people, aged from 10–17 years old, are on social media by asking a group of 16 year olds.

b. Researching what is the best-selling drink at a cafe by asking 10 customers on a particular day.

c. Researching how many people from the office are going away over the Christmas break by asking the individuals at your desk.

d. Conducting research on the most popular beach in Victoria by asking the people who live on your street.

---

10. The following table shows the tallied votes for favourite dessert in an office. 8C

a. What is the overall favourite dessert?

b. Which of the desserts was the least favourite?

c. How many people are there in the office in total, if each person voted once?

d. Write the number of votes for pastries as a simplified fraction, out of the total number of votes.

Favourite food	Tally
Cake	𝍸 𝍸 II
Ice-cream	𝍸 𝍸 𝍸 I
Cookies	𝍸 II
Pastries	𝍸 𝍸 II
Jelly	IIII
Chocolate	𝍸 III
Caramel	I

---

11. The following stem-and-leaf shows the number of pets that go to a vet each day in March. 8C

a. What was the lowest number of animals the vet had in one day?

b. For how many days did the vet have more than 53 animals?

c. What is the difference between the most and least number of daily animals the vet had in March?

d. If the vet has less than 35 animals in one day, it will not make a profit. Write a fraction to represent the number of days out of the total number of days in March on which the vet did not make a profit.

Stem	Leaf
2	5 8
3	0 0 1 7 8 9
4	0 3 6 7
5	0 2 3 4 6 8 8 8 9
6	1 3 5 7 8
7	0 2 3
8	0 2

Key

4 | 3 = 43 animals

---

12. The following list contains the number of people that still work from home, from 20 offices in Melbourne CBD. 8D

21, 30, 18, 20, 20, 29, 35, 22, 31, 24, 17, 17, 25, 25, 10, 19, 27, 21, 24, 20

a. What is the range of the data?

b. Organise the data into inclusive class intervals of size 5.

c. Record the grouped data into a frequency table, using the grouped data from part **b**.

d. According to the frequency table in part **c**, what is the most common number range of people per office still working from home?

---

13. The following list contains the ages of 20 randomly selected students at a school. 8D

12, 18, 14, 13, 11, 15, 11, 16, 17, 12, 14, 12, 13, 16, 16, 18, 14, 13, 15, 13

a. What is the range of the data?

b. Construct a frequency table of the data.

c. Construct a histogram of the data.

d. Determine the shape of the histogram.

14. Determine the mean, median, mode, and range of the following data sets. Round your answers to 1 decimal place, where applicable. 8E

a. 12, 16, 19, 11, 21, 13, 18, 19, 12, 10, 20, 22, 10, 14, 9

b. 25, 20, 31, 24, 19, 35, 45, 23, 27

c. 54, 65, 34, 57, 62, 59, 51, 49, 42, 58, 53, 50, 63, 67, 78, 50, 53, 42, 42, 54

d. 89, 99, 90, 83, 81, 86, 80, 80, 93, 93, 84, 89, 94, 85, 90, 100, 90, 78, 87, 98, 101

---

15. The following data set contains the times in minutes that it took 40 secondary school students to run 2 km. 8E

3, 14, 21, 14, 18, 11, 10, 9, 13, 12, 16, 18, 9, 10, 11, 15, 16, 18, 22, 11, 15, 12, 11, 18, 17, 14, 17, 19, 16, 15, 12, 13, 15, 17, 10, 12, 17, 18, 13, 17

a. Identify the outlier in the data set.

b. Determine the mean, median, mode and range of the data with the outlier included. Round your answers to 1 decimal place, where applicable.

c. Determine the mean, median, mode, and range of the data without the outlier included. Round your answers to 1 decimal place, where applicable.

d. Which of the summary statistics were/were not affected by the outlier?

Problem solving

16. Jessica wants to investigate how long her classmates listen to music everyday. State the variable of this investigation, with appropriate units, and determine its type. 8A

---

17. Lauren is a food critic and reviews food items at restaurants. She reviewed every second item on the menu at 12 restaurants. If there is an average of 36 food items on each menu, find the number of food items in Lauren's testing sample and comment on whether it is representative of the population. 8B

---

18. Ms Suzanna is a Year 8 English teacher for 4 classes, who has 180 grades to analyse for an essay assessment. If each student received a grade from A to E for the assessment, suggest a suitable data display that she can use in order to compare the proportions of grades. 8C

---

19. Ryan is analysing his business' profits for the month of March. He collects data in Australian dollars everyday and records it in a frequency table. Should Ryan choose inclusive or exclusive class intervals to group the data for a histogram? Explain why. 8D

---

20. Jimmy is studying for a test every day of the school week, Monday until Friday. He studied for 3 hours on Monday, 4 hours on Tuesday, 4 hours on Wednesday and 1 hour on Friday. How long did Jimmy study for on Thursday night if the mean number of hours he studied in the week was 3 hours? 8E

Reasoning

21. Tina is composing a playlist for her school formal. She wants to determine whether she should include songs by Taylor Swift in this playlist, so she asks twenty peers 'how many hours did you listen to music by Taylor Swift last month?' The following list contains the number of hours twenty students spent listening to Taylor Swift last month.

11, 33, 26, 3, 14, 12, 9, 11, 10, 19, 5, 10, 13, 25, 2, 0, 12, 34, 6, 0

a. What type of data has Tina recorded, primary or secondary?

b. Construct a frequency table of the data, grouping the times into exclusive class intervals of size 5.

c. Create a histogram for the data.

d. Calculate the mean, median, and range of the data. Round your answers to 1 decimal place, where applicable.

e. Tina had the option to hire a band but decided to make a playlist instead. List one disadvantage and advantage of having a pre-made playlist for the formal compared to a band.

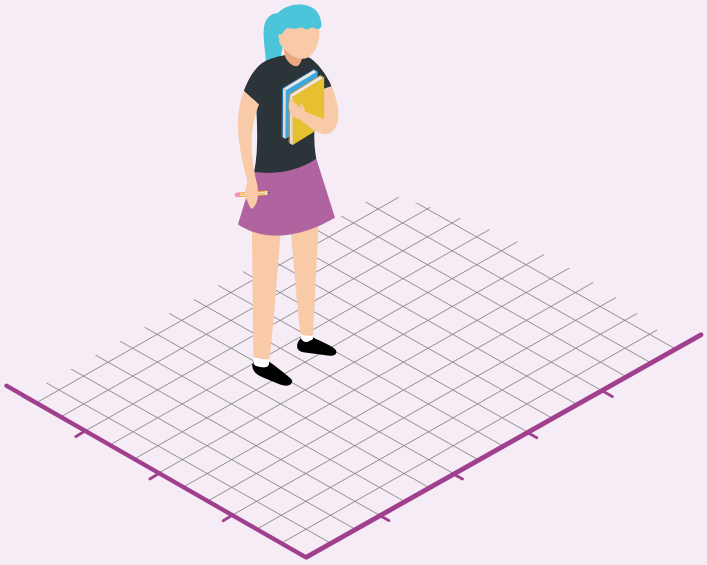
22. Select the type of variable (numerical or categorical) described in parts **a** and **b** and determine what is the best suited display for the data.

a. A survey where 25 individuals have to rate a menu item at a restaurant out of 5.

b. A study on the number of seconds it takes 20 people to run 6 km.

c. Compare your answers for parts **a** and **b** and comment on the restrictions of displaying certain types of variables.

Back to contents



09

Linear graphing

Number and Algebra

Research summary

9A The Cartesian plane *(Revision)*

9B Tables of values

9C Linear rules

9D Gradient *(Extension)*

9E Gradient-intercept form *(Extension)*

9F Solving linear equations graphically

9G Non-linear graphs *(Extension)*

Chapter 9 extended application

Chapter 9 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 9 research summary

Linear graphing

Big ideas

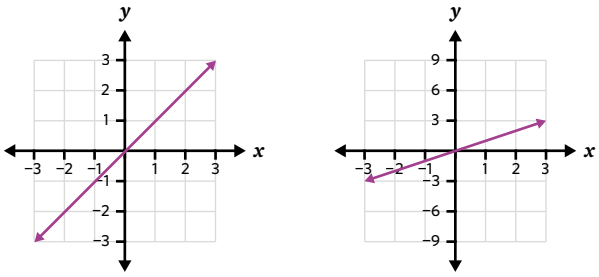
Generalisation

This chapter will build upon students' arithmetic and factual generalisation. Students will apply their ability to notice patterns and use words, gestures, drawings, and spoken words to deepen their algebraic thinking to contextually and symbolically generalise linear and non-linear functions.

It is essential that students have a solid understanding of the relationships of variables as quantities that have variability as it underpins the use of variables in multiple ways (Wilkie, 2016). For example, the x and y variables can be used to determine the domain, intercepts, and parameters. Furthermore, formulas that describe rules will lead to function notation (E.g. $y = 2x + 3 \rightarrow f(x) = 2x + 3$).

However, it is essential that algebraic thinking approaches indeterminacy analytically and designs its objects symbolically. (Radford, 2010). For example, students might say 'we found the rule on accident', or 'that's the procedure to find the equation' but algebra is not guessing or using signs, 'it is about using signs to think in a distinctive way' (Radford, 2010 p. 54). One way teachers can accomplish this is to have students notice local communities through arithmetic generalisations, and then to generalise the commonalities using factual generalisation and contextual generalisation, and finally make an symbolic generalisation.

This chapter will also focus on gradient. Students in high school and college often have limited conception of gradient (Stump, 2001) and the analytic and visual perspective of slope (Zaslavsky et al., 2002). To enhance the understanding of gradient and reduce misconceptions it is essential to distinguish between the analytic and visual concepts of gradient (Zaslavsky et al., 2002) and other attributions we have for gradient (Moore-Russo et al., 2011). For example, the following graphs represent the same function, but they visually appear to have different gradients.



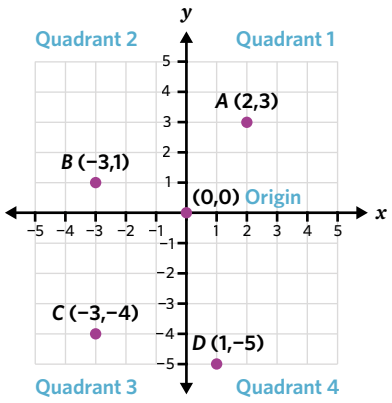
This big idea is built upon a strong understanding of proportional understanding, number and number properties, patterns, and equality. Therefore, a conceptual understanding of how to manipulate equations and inequalities is essential for success in future mathematics topics especially in statistics, calculus and physics.

Visual representations

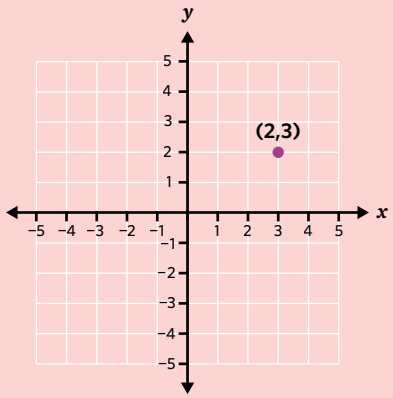
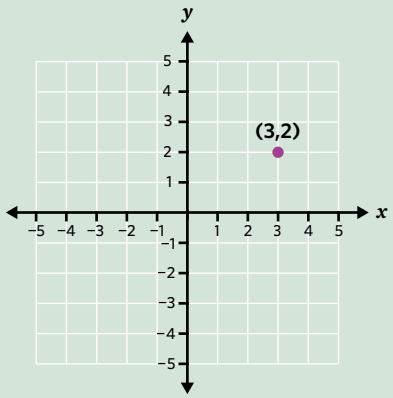
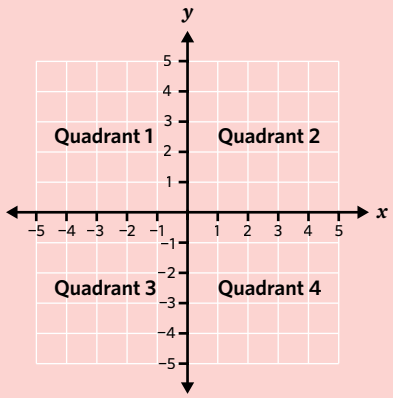
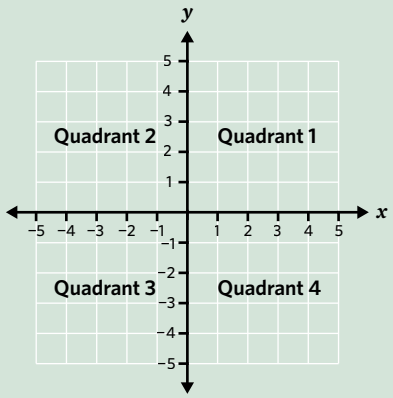
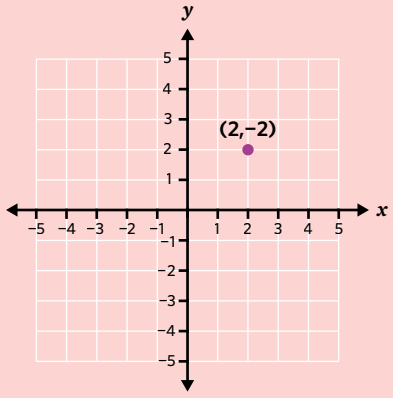
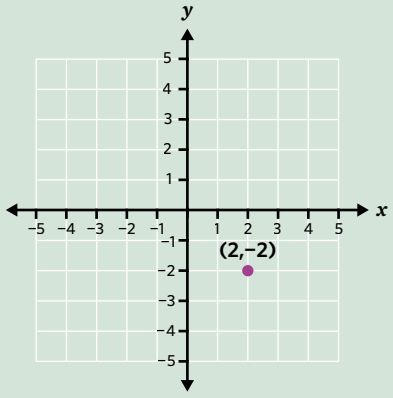
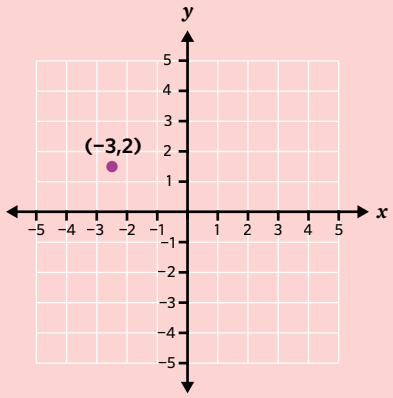
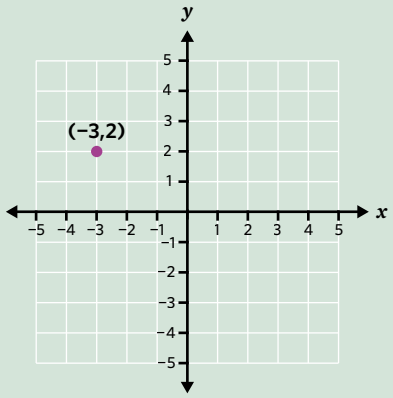
Cartesian plane

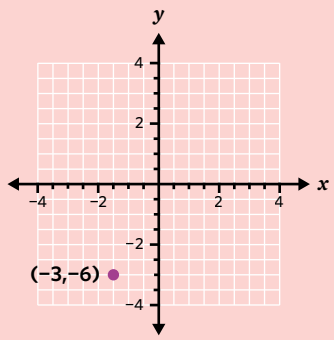
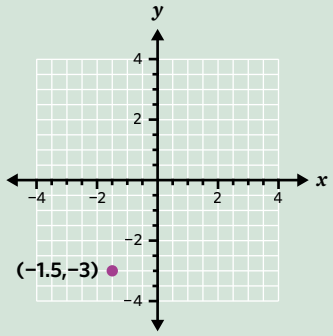
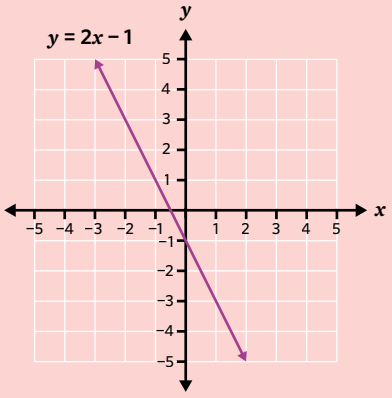
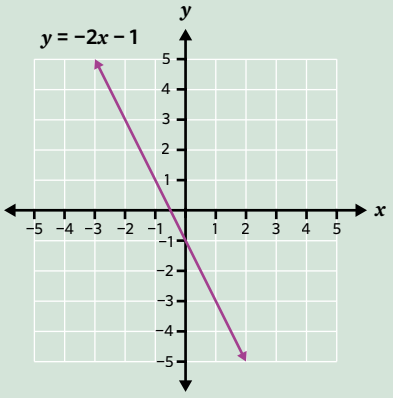
Cartesian coordinates describe a plane with two intersecting perpendicular lines, representing each dimension of the plane, that form four quadrants. Points are located on the plane by their coordinates, which provide their location within the plane. Points can be connected to create figures that lie on the plane. The visual representation of the Cartesian plane relies both as a process and an object for students. Students must be able to move fluidly from constructing and interpreting the Cartesian plane and algebraic functions as this is foundational to multiple representations within algebra (Moschkovich et al., 1993).

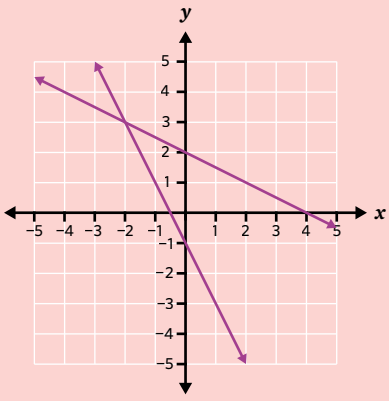
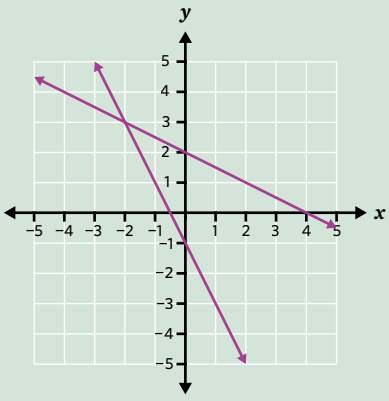
Additionally the seamless transition of using coordinate points, Cartesian planes, tables and functions and words can lead to a robust and flexible understanding of functions (Knuth 2000).

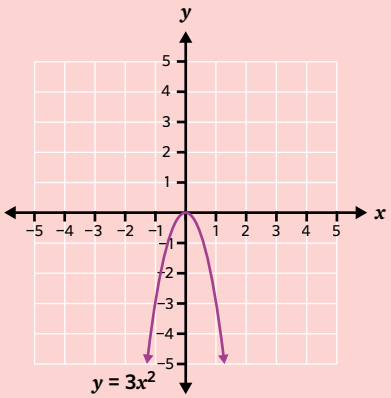
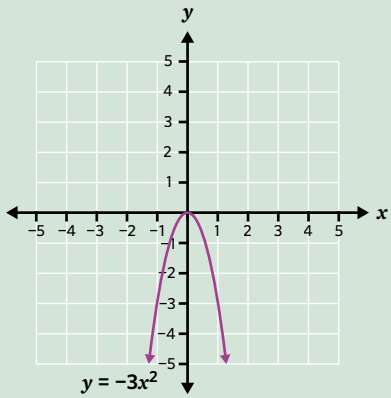


Misconceptions

Misconception	Incorrect	Correct	Lesson
Students think order does not matter when writing or locating coordinate points.			9A 9F
Students label the quadrants from left to right.			9A
Students think all coordinate points are positive.	Plot (2,−2) 	Plot (2,−2) 	9A
When a coordinate point is where two lines meet, students identify the location on the coordinate grid as a space.			9A

Misconception	Incorrect	Correct	Lesson
Students count the ticks to determine the coordinate point.			9A
Students think a table of values must have consecutive numbers.	$y = x + 4$ | x | −3 | −1 | 2 | | y | | | | $y = -3 + 4 = 1$ $y = -2 + 4 = 2$ $y = -1 + 4 = 3$	$y = x + 4$ | x | −3 | −1 | 2 | | y | | | | $y = -3 + 4 = 1$ $y = -1 + 4 = 2$ $y = 2 + 4 = 6$	9B
Students apply a different operator than multiplication to the coefficient of x.	$y = 2x$ | x | −1 | 0 | 1 | | y | −20 | 0 | 20 |	$y = 2x$ | x | −1 | 0 | 1 | | y | −2 | 0 | 2 |	9B
Students substitute the y-values into the x-value.	Fill in the table using the y-values $-1, 0, 1$ $y = 2x$ | x | −2 | 0 | 1 | | y | −1 | 0 | 1 |	Fill in the table using the y-values $-1, 0, 1$ $y = 2x$ | x | −1 | 0 | $\frac{1}{2}$ | | y | −2 | 0 | 1 |	9B
Students use non-consecutive coordinates to calculate the coefficient of x.	Determine the rule | x | −3 | 0 | 1 | | y | 1 | 4 | 5 | y-value changes by 3 for every one x-value. Constant: 4 Rule: $y = 3x + 4$	Determine the rule | x | −3 | 0 | 1 | | y | 1 | 4 | 5 | y-value changes 1 for every one x-value. Constant: 4 Rule: $y = x + 4$	9C
Students think all lines are positive.			9C
Students substitute the first coordinate as the coefficient and constant.	Determine the rule (2,4) (3,2) $y = 2x + 4$	Determine the rule (2,4) (3,2) $y = -2x + 9$	9C

Misconception	Incorrect	Correct	Lesson
When calculating the gradient, students determine the difference between the coordinates.	Determine the gradient (0,6) and (2,12) $m = 12 - 6 = 6$	Determine the gradient (0,6) and (2,12) $m = \frac{12 - 6}{2 - 0} = \frac{6}{2} = 3$	9D
When the gradient is a whole number, students assume the change in y values is 0.	$m = 15 = \frac{\text{rise}}{\text{run}} = \frac{15}{0}$	$m = 15 = \frac{\text{rise}}{\text{run}} = \frac{15}{1}$	9D
Students think equivalent fractions have different gradients.	$m = \frac{24}{6} \neq \frac{12}{3} \neq 4$ $\therefore \frac{12}{3}$ is a different gradient than 4.	$m = \frac{24}{6} = \frac{12}{3} = \frac{4}{1} = 4$ $\therefore \frac{24}{6}, \frac{12}{3}, \frac{4}{1}$, 4 are equivalent and show the same gradient.	9D
When calculating the gradient, students calculate the difference in x-values over the change in y-values.	Determine the gradient (0,6) and (2,12) $m = \frac{2 - 0}{12 - 6} = \frac{2}{6} = \frac{1}{3}$	Determine the gradient (0,6) and (2,12) $m = \frac{12 - 6}{2 - 0} = \frac{6}{2} = 3$	9D
When calculating the gradient, students assume the gradient is always positive.	Determine the gradient (1,2) and (0,5) $m = \frac{5 - 2}{0 - 1} = \frac{3}{-1} = 3$	Determine the gradient (1,2) and (0,5) $m = \frac{5 - 2}{0 - 1} = \frac{3}{-1} = -3$	9D
Students think that the gradient is always the first number in the equation and the y-intercept is the second number.	$y = 6 + 7x$ $m = 6$ $c = 7$	$y = 6 + 7x$ $m = 7$ $c = 6$	9E
Students assume they can determine the gradient and y-intercept from any equation.	$4x + 2y = 8$ $m = 4$ $c = 2$	$4x + 2y = 8$ $2y = 8 - 4x$ $y = 4 - 2x$ $m = -2$ $c = 4$	9E
Students think transposing equations requires additional information to solve.	Rearrange $4x + 2y = 8$ in gradient-intercept form. Not possible because there are two variables and one equation.	Rearrange $4x + 2y = 8$ in gradient-intercept form. $4x + 2y = 8$ $2y = 8 - 4x$ $y = 4 - 2x$	9E
Students list the y-intercepts as points of intersections of two lines.	 Point of intersection $(2,-1)$	 Point of intersection $(-2,3)$	9F
Students think $x^2 = 1 \times x$.	$y = x^2 - 1$ x: -1, 0, 1 y: -2, -1, 0	$y = x^2 - 1$ x: -1, 0, 1 y: 0, -1, 0	9F

Misconception	Incorrect	Correct	Lesson
Students think $-x^2 = (-x)^2$.	$-4^2 = 16$ $(-4)^2 = 16$ or $-4^2 = -16$ $(-4)^2 = -16$	$-4^2 = -16$ $(-4)^2 = 16$	9F
Students think the coefficients of non-linear lines are always positive.			9F

References and additional readings

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Zaslavsky, O., Sela, H., & Leron, U. (2002). *Being sloppy about slope: The effect of changing the scale.* Educational Studies in Mathematics, 49(1), 119–140. https://doi.org/10.1023/a:1016093305002

Back to contents

9A The Cartesian plane

The Cartesian plane (or number plane) is formed by two perpendicular lines (x-axis and y-axis). To locate any point on the Cartesian plane, a pair of numbers called the Cartesian coordinates (or points) are used and written in the form of (x,y). The value of x and y can be any number. The x value is the horizontal position and the y value is the vertical position.

LEARNING INTENTIONS

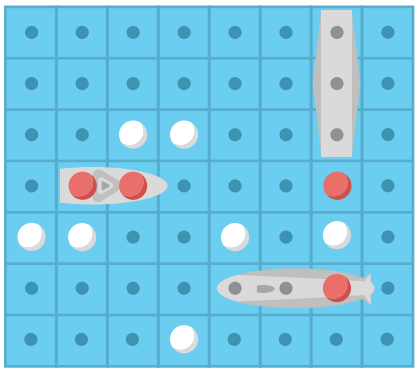
Students will be able to:

- label Cartesian coordinates on the Cartesian plane
- determine the quadrant or axis in which a Cartesian coordinate lies
- plot Cartesian coordinates on the Cartesian plane.

KEY TERMS AND DEFINITIONS

A **Cartesian plane** is a set of two perpendicular number lines that intersect at the origin.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

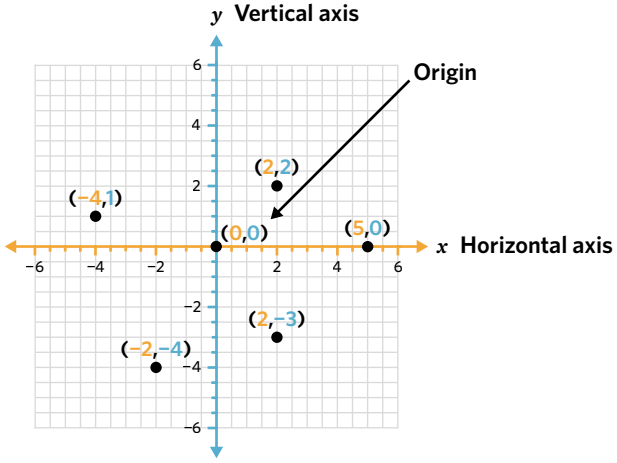


Battleship is a game where the main objective is to sink all of your opponents' ships. Each ship is located on different sections of the grid. Each player takes turns in attempting to plot the location of their opponents' ships with reference to Cartesian coordinates.

Key ideas

1. The x-axis is the horizontal line and the y-axis is the vertical line. The Cartesian coordinates of a point shows where a point is located on the Cartesian plane where the x-coordinate is listed first followed by the y-coordinate.

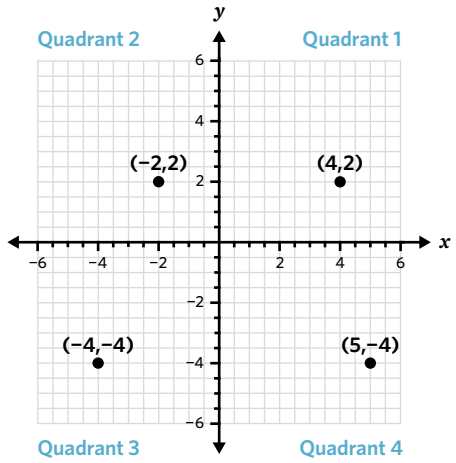
$(x,y) = ($horizontal units from the origin $/$ vertical units from the origin$)$



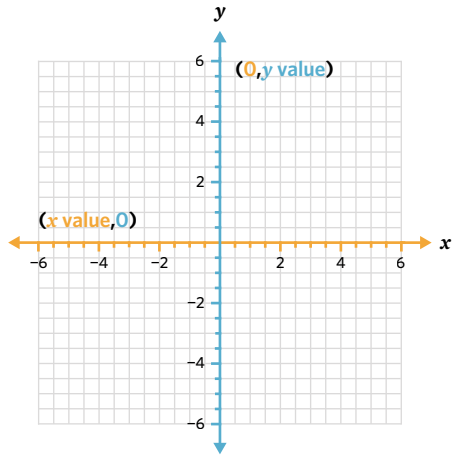
2. The positive and negative signs of the coordinates indicate which quadrant the point is located in.

Quadrant	x-coordinate	y-coordinate	Example
Quadrant 1	Positive	Positive	(4,2)
Quadrant 2	Negative	Positive	(−2,2)
Quadrant 3	Negative	Negative	(−4,−4)
Quadrant 4	Positive	Negative	(5,−4)

Continues →



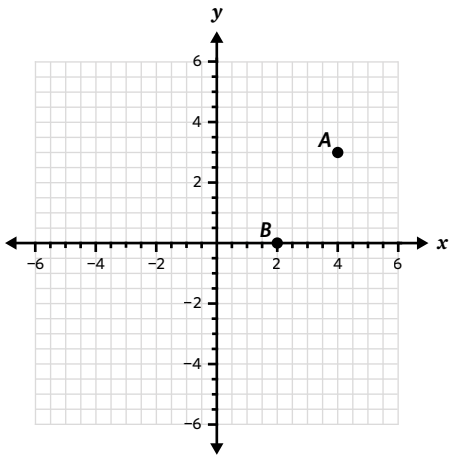
3. Any point on the x-axis will always have a y-coordinate that is zero. Any point on the y-axis will always have a x-coordinate that is zero.



Worked example 1

Stating the coordinates on a Cartesian plane

State the coordinates on the following Cartesian plane.



Continues →

a. Point A WE1a

Working

$A(4,3)$

Thinking

Point A is 4 units right and 3 units up from the origin.

b. Point B WE1b

Working

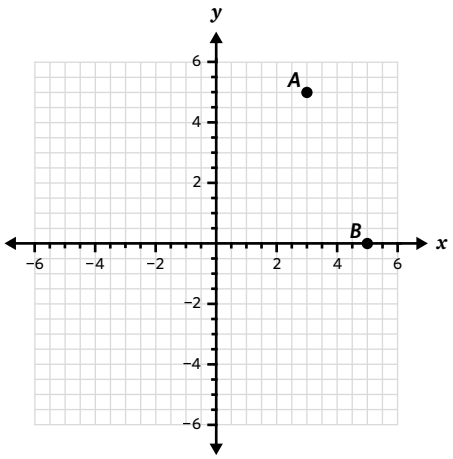
$B(2,0)$

Thinking

Point B is 2 units right from the origin.

Student practice

State the coordinates on the following Cartesian plane.



a. Point A **b.** Point B

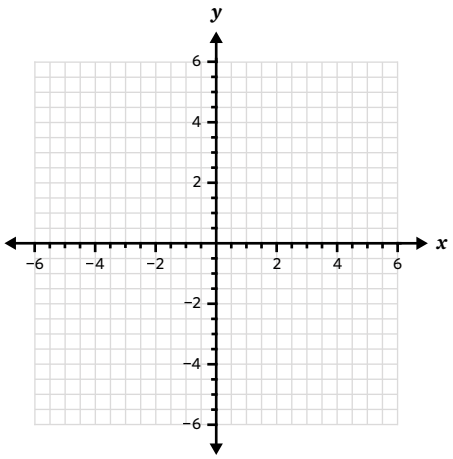
Worked example 2

Plotting points on a Cartesian plane

Plot the following coordinates on a Cartesian plane. WE2

$A(-3,5)$ and $B(-2,-3)$

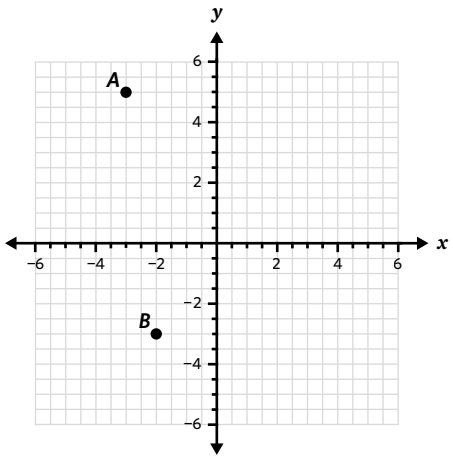
Working



Thinking

Step 1: Draw a set of axes, ensuring that they are long enough to fit all the coordinates.

Continues →



Step 2: Point A is 3 units left and 5 units up from the origin.

Step 3: Point B is 2 units left and 3 units down from the origin.

Student practice

Plot the following coordinates on a Cartesian plane.

$A(-2,3)$ and $B(-5,-4)$

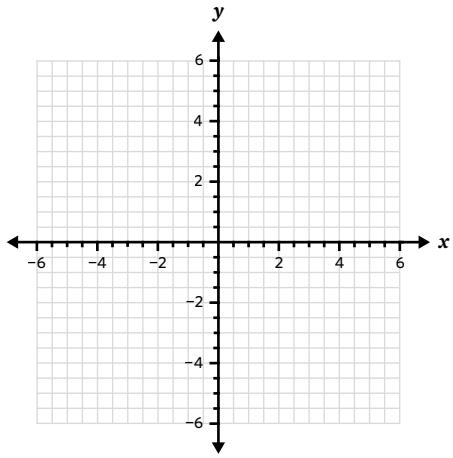
Worked example 3

Plot and state the location of each point on the Cartesian plane

Plot and state the quadrant or axis in which each point is located on the Cartesian plane. WE3

$A(2,-4)$ and $B(0,-2)$

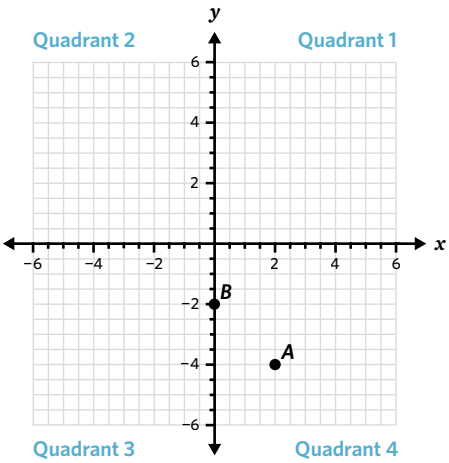
Working



Thinking

Step 1: Draw a set of axes, ensuring that they are long enough to fit all the coordinates. Then plot and label each point using the Cartesian coordinates.

Continues →



Step 2: Point A is 2 units right, 4 units down from the origin and is in the fourth quadrant.

Step 3: Point B is 2 units down from the origin and is on the y-axis.

Student practice

Plot and state the quadrant or axis in which each point is located on the Cartesian plane.

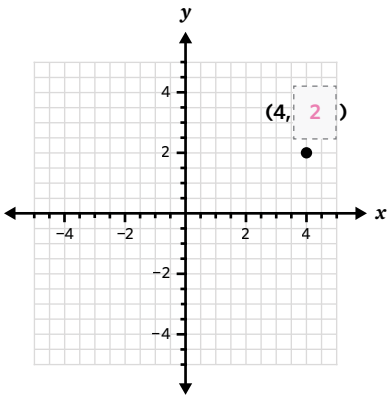
$A(3,-2)$ and $B(0,-5)$

9A Questions

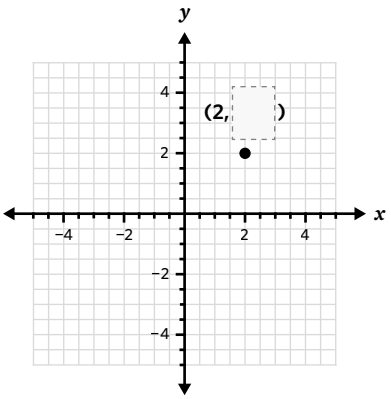
Understanding worksheet

1. State the missing coordinate plotted on the Cartesian plane.

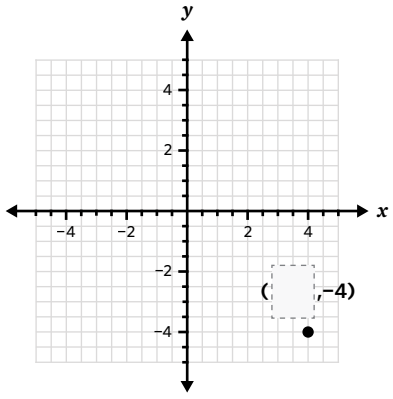
Example



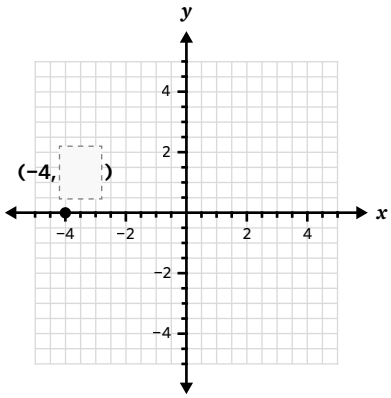
a.



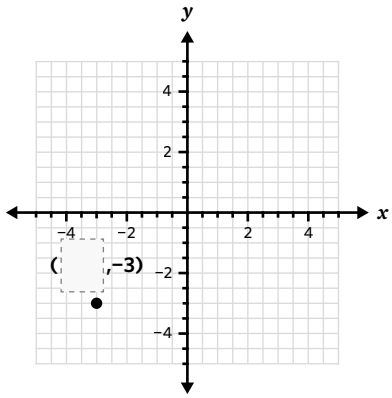
b.



c.

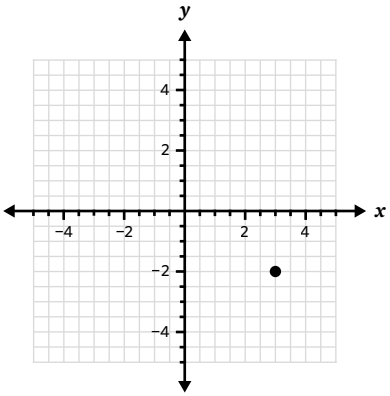


d.



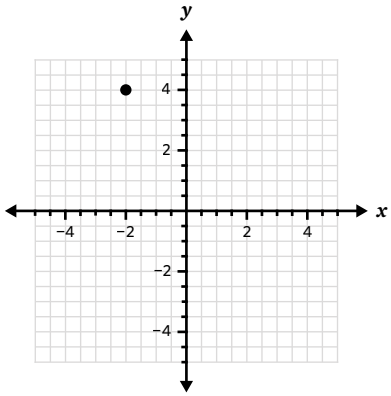
2. Circle the quadrant or axis for the given coordinate.

Example



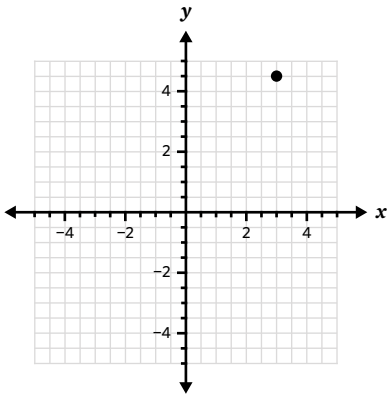
x-axis
y-axis
Quadrant 1
Quadrant 2
Quadrant 3
(Quadrant 4)

a.



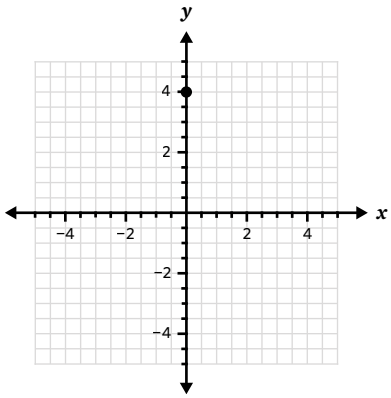
x-axis
y-axis
Quadrant 1
Quadrant 2
Quadrant 3
Quadrant 4

b.



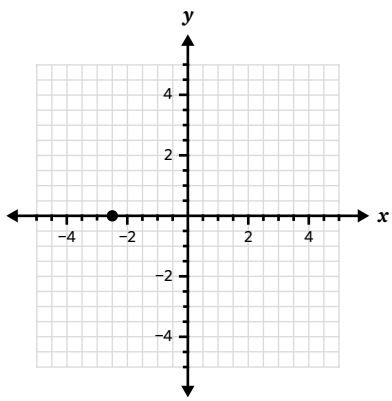
x-axis
y-axis
Quadrant 1
Quadrant 2
Quadrant 3
Quadrant 4

c.



x-axis
y-axis
Quadrant 1
Quadrant 2
Quadrant 3
Quadrant 4

d.



x-axis
y-axis
Quadrant 1
Quadrant 2
Quadrant 3
Quadrant 4

3. Fill in the blanks by using the words provided.

Cartesian | (x, y) | horizontal | four

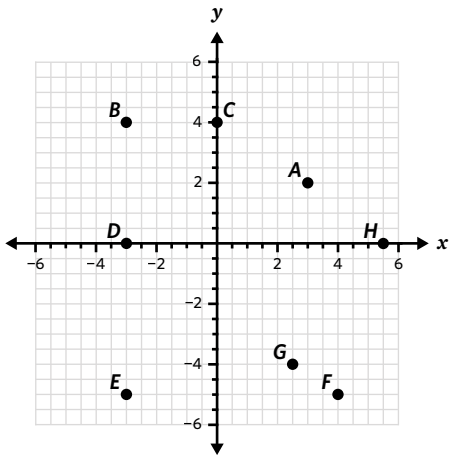
To locate any point on the _____ plane, a pair of numbers are used and are called the Cartesian coordinates. When the pair of numbers are written, they are written in the form _____. The x-axis refers to the _____ position, whereas the y-axis refers to the vertical position. The Cartesian plane is made up of _____ quadrants, where these are numbered in an anticlockwise direction, starting in the top right corner.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. State the coordinates on the following Cartesian plane. WE1



5. Plot the following coordinates on a Cartesian plane. WE2

a. $A(2,3)$ **b.** $B(-1,4)$ **c.** $C(-3,-5)$ **d.** $D(3,-5)$
e. $E(0,3)$ **f.** $F(4,0)$ **g.** $G(0,-4.5)$ **h.** $H(-2.5,0)$

6. Plot and state the quadrant or axis in which each point is located on the Cartesian plane. WE3

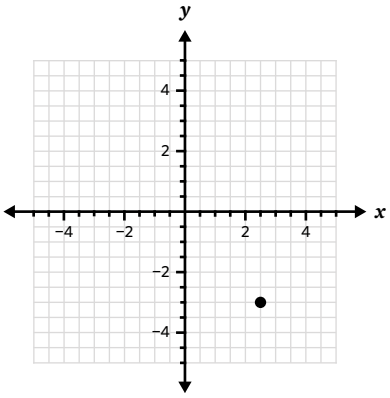
a. $A(2,2)$ **b.** $B(-4,4)$ **c.** $C(-2,-3)$ **d.** $D(5,-2)$
e. $E(0,-3)$ **f.** $F(4,0)$ **g.** $G(0,4.5)$ **h.** $H(-2.5,0)$

7. State the quadrant in which each point lies on the Cartesian plane.

a. $(2,5)$ **b.** $(6,1)$ **c.** $(-1,3)$ **d.** $(-3,1)$
e. $(-4,-1)$ **f.** $(3,-4)$ **g.** $(0,5.5)$ **h.** $(-3.5,0)$

8. State the coordinate on the following Cartesian plane.

A. (2.5,3)
B. (2.5,−3)
C. (3,2.5)
D. (−3,−2.5)
E. (−3,2.5)



Spot the mistake

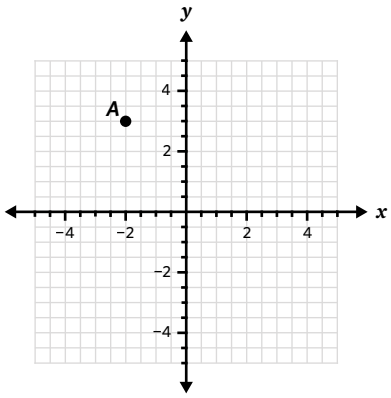
9. Select whether Student A or Student B is incorrect.

a. Plot the following coordinates on the given Cartesian plane.

$A(-2,3)$

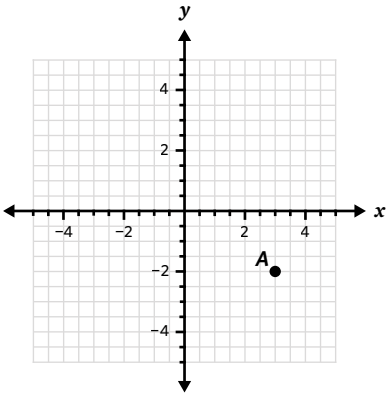
Student A

Point A is 2 units left and 3 units up.



Student B

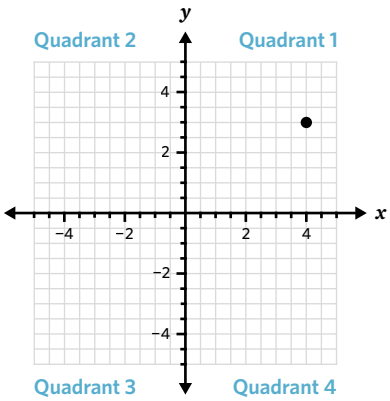
Point A is 2 units down and 3 units right.



b. Plot and state the quadrant or axis in which point B is located on the Cartesian plane.

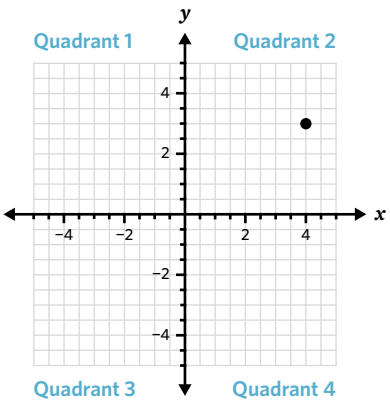
Student A

Point B is four units right and 3 units up and is in the first quadrant.



Student B

Point B is four units up and 3 units right and is in the second quadrant.



Problem solving

Question working paths

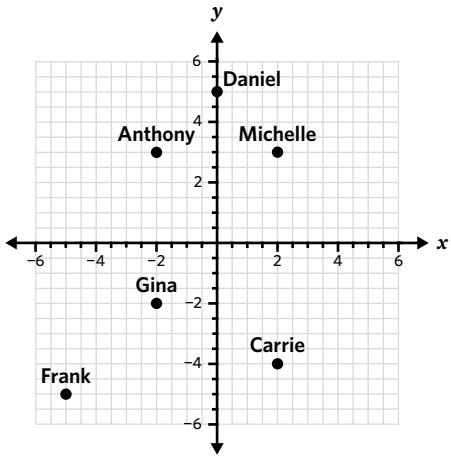
Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Nora lives on 3rd street and 5th road, represented as (3,5). Her grandmother lives at 5th street and 9th road, represented as (5,9). Plot the coordinates on a Cartesian plane.

11. Stanley is keeping track of the amount of push ups and sit ups he does per day by using a Cartesian plane. He uses the x-axis for the number of push ups and the y-axis for the number of sit ups. Plot the following: Monday (10,32), Tuesday (15,25), Wednesday (5,63) and Thursday (32,20) to then determine what day Stanley did the most push ups?

12. On a Cartesian plane, Jessie's work is located at $(-3,-2)$. Jessie lives four blocks north and five blocks east. State the ordered coordinates for Jessie's home.

13. The Cartesian plane below displays information about where six family members live.



Show that the distance between Michelle and Carrie is equal to seven units.

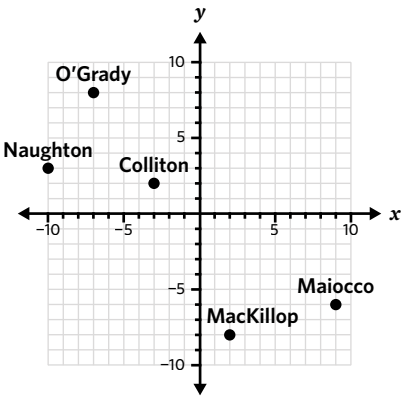
14. A particular shopping centre displays the following information on a Cartesian plane. Coles is located at the origin, Bank of Melbourne is located at $(-3,-4)$, the food court has the same x-coordinate as Bank of Melbourne and a y-coordinate of 1. The bathrooms are one unit left and three units up from the food court. The bathrooms are located in which quadrant?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Renae's high school has five different houses. Each student is allocated to a house. Renae has plotted these five buildings on a Cartesian plane. There is also a Student Hub that is exactly halfway between O'Grady and MacKillop houses.



a. Renae is in Colliton house, what are the coordinates for Colliton house?

b. Which quadrant on the Cartesian plane is Naughton, O'Grady and Colliton?

c. The y-coordinate for Naughton house is incorrectly plotted. The correct coordinate for Naughton house needs to be on the x-axis. What is the correct coordinate for Naughton house?

d. What is the coordinate for the Student Hub?

e. MacKillop and Maiocco students have the furthest to travel when going to classes, as most of their classes are in the second quadrant. Suggest some ways Renae's high school could make it fair for all students.

16. For parts **a** and **b**, state which quadrant the coordinate is found in.

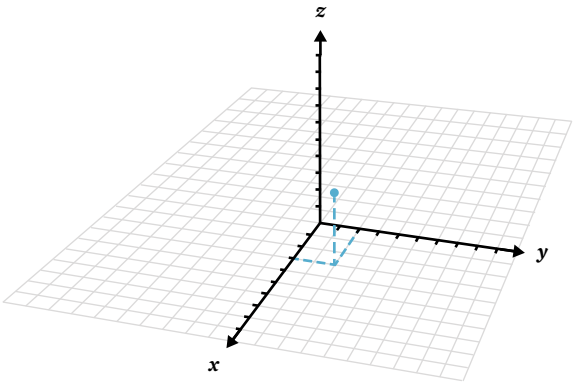
- **a.** $(2,-3)$
- **b.** $(-3,2)$
- **c.** Compare the answers from part **a** and **b** and explain why the coordinates are not found in the same quadrant?

Extra spicy

17. Using $(3,0)$ and $(-2,0)$, list the four coordinates (only integers) of x and y that lie on the line segment joining these two coordinates.

18. The midpoint of a line segment (or interval) is the Cartesian coordinate that cuts the segment in half. Find the midpoint for $(-4,-5)$ and $(6,-2)$.

19. State the coordinate for the point (x,y,z) displayed in the following Cartesian plane.



- **A.** $(2,3)$
- **B.** $(3,2)$
- **C.** $(3,2,4)$
- **D.** $(2,3,4)$
- **E.** Cannot be determined

20. A set of points have the following coordinates $(x,0)$, where x can be substituted for any number. What does this coordinate represent?

- **A.** y-axis
- **B.** x-axis
- **C.** Origin
- **D.** Second quadrant
- **E.** Fourth quadrant

Remember this?

21. Victoria has saved \$53 for her party. A pack of lollies costs \$7. What is the greatest number of packs of lollies Victoria can buy for her party?

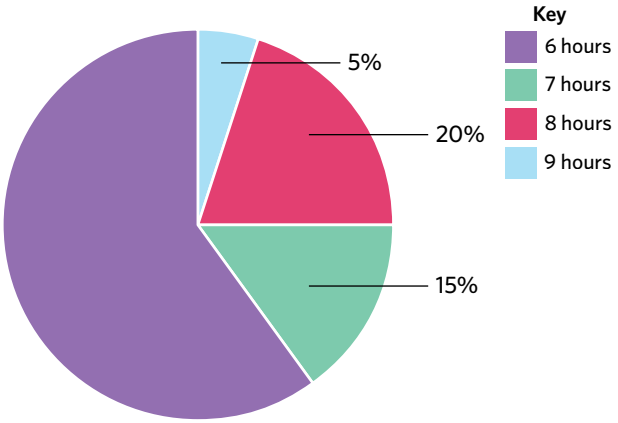
A. 5 **B.** 6 **C.** 7 **D.** 8 **E.** 9

22. Ying needs to buy a carpet that covers her whole bedroom. Ying's bedroom is a rectangle with dimensions 550 cm × 850 cm. One square metre of carpet costs \$25. What will the total cost of Ying's carpet be?

A. \$46.75 **B.** \$1168.75 **C.** \$116 875 **D.** \$467 500 **E.** \$11 687 500

23. Students at St. Rose College were surveyed to find how many hours they sleep each night.

The graph below shows the results.



Given that there were 100 students who slept 8 hours each night, how many students at St. Rose College sleep 6 hours each night?

A. 60% **B.** 80% **C.** 300 **D.** 400 **E.** 600

Back to contents

9B Tables of values

A rule with two variables can be used to show the relationship between the pair of coordinates (x, y). The coordinates for a rule can be displayed in a table of values and plotted on a Cartesian plane. A set of coordinates on a Cartesian plane can be transferred to a table of values.

LEARNING INTENTIONS

Students will be able to:

- complete a table of values using substitution
- write a table of values as (x,y) coordinates
- construct a linear graph using a table of values.

KEY TERMS AND DEFINITIONS

A **rule** is a formula written using mathematical symbols and pronumerals to describe a pattern.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Day (d)	1	2	3
Kilometres (k)	20	22	24
Coordinate	(1,20)	(2,22)	(3,24)

Harry is training for a marathon. Each day (d), he wants to run two more kilometres (k) than the previous day. Harry displays the first three runs in a table of values. Harry can plot the coordinates from the table of values to display his runs over the first three days.

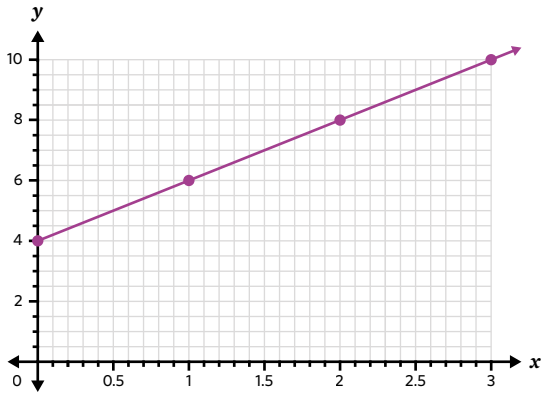
Key ideas

1. A table of values can be converted to coordinates on a Cartesian plane.

x	-2	-1	0	1	2
y	-4	-1	2	5	8
Coordinate	$(-2,-4)$	$(-1,-1)$	(0,2)	(1,5)	(2,8)

2. A table of values can be used to graph a straight line when given a rule.

x	0	1	2	3
y	4	6	8	10
Coordinate	(0,4)	(1,6)	(2,8)	(3,10)



Worked example 1

Constructing coordinates from a table of values

Use the table of values to generate coordinates.

a.

WE1a

x	−3	−2	−1	0	1
y	−3	−1	1	3	5
Coordinate					

Working

x	−3	−2	−1	0	1
y	−3	−1	1	3	5
Coordinate	(−3,−3)	(−2,−1)	(−1,1)	(0,3)	(1,5)

Thinking

Step 1: Identify the x-coordinate by referring to the first row of values. The corresponding y-coordinate is directly below the x-coordinate.

Step 2: State all the coordinates displayed in the table of values.

b.

WE1b

x	−2	−1	0	1	2
y	9	6	3	0	−3
Coordinate					

Working

x	−2	−1	0	1	2
y	9	6	3	0	−3
Coordinate	(−2,9)	(−1,6)	(0,3)	(1,0)	(2,−3)

Thinking

Step 1: Identify the x-coordinate by referring to the first row of values. The corresponding y-coordinate is directly below the x-coordinate.

Step 2: State all the coordinates displayed in the table of values.

Student practice

Use the table of values to generate coordinates.

a.

x	−3	−2	−1	0	1
y	−10	−6	−2	2	6
Coordinate					

b.

x	−3	−2	−1	0	1
y	8	6	4	2	0
Coordinate					

Worked example 2

Completing a table of values

Complete the table of values with the given rule.

a. $y = 2x + 4$ WE2a

x	0	1	2	3
y				

Working

$2(0) + 4 = 4$

$2(1) + 4 = 6$

$2(2) + 4 = 8$

$2(3) + 4 = 10$

x	0	1	2	3
y	4	6	8	10

Thinking

Step 1: Substitute each x-coordinate in the table into the rule to determine the y-coordinate.

Step 2: Complete the table of values.

b. $y = -3x + 2$ WE2b

x	-1	0	1	2
y				

Working

$-3(-1) + 2 = 5$

$-3(0) + 2 = 2$

$-3(1) + 2 = -1$

$-3(2) + 2 = -4$

x	-1	0	1	2
y	5	2	-1	-4

Thinking

Step 1: Substitute each x-coordinate in the table into the rule to determine the y-coordinate.

Step 2: Complete the table of values.

Student practice

Complete a table of values with a given rule.

a. $y = 3x + 2$

x	0	1	2	3
y				

b. $y = -2x + 3$

x	-1	0	1	2
y				

Worked example 3

Constructing a graph from a table of values

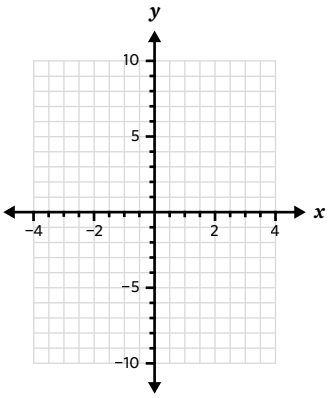
Construct a graph for each of the following.

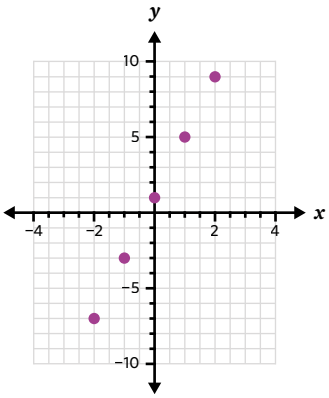
a. $y = 4x + 1$ WE3a

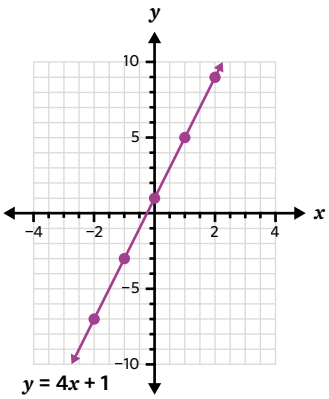
Working

x	−2	−1	0	1	2
y	−7	−3	1	5	9

x	−2	−1	0	1	2
y	−7	−3	1	5	9
Coordinate	(−2,−7)	(−1,−3)	(0,1)	(1,5)	(2,9)







Thinking

Step 1: Construct the table of values.

Step 2: Determine the coordinates given by the table of values.

Step 3: Draw and label the x and y axes. Identify the smallest and largest values in the table (for both x and y) to determine how to scale the graph.

Step 4: Plot each coordinate.

Step 5: Use a ruler to connect the coordinates and add arrows to each end.

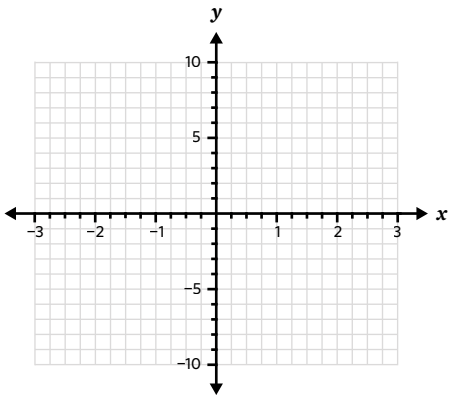
Step 6: Label the line with reference to the rule.

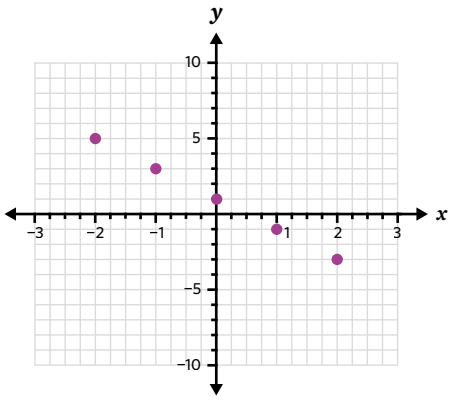
Continues →

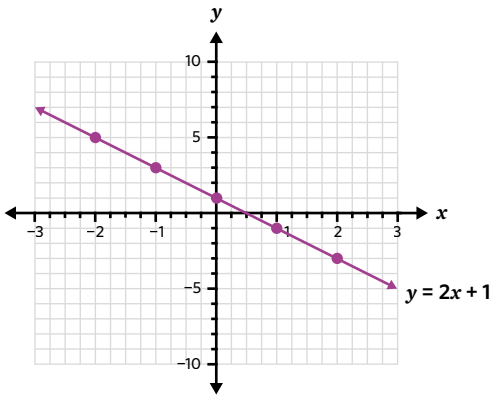
b. $y = -2x + 1$, where x is between -2 and 2. WE3b

Working

x	-2	-1	0	1	2
y	5	3	1	-1	-3
Coordinate	$(-2,5)$	$(-1,3)$	$(0,1)$	$(1,-1)$	$(2,-3)$







Thinking

Step 1: Determine the coordinates given by the table of values.

Step 2: Draw and label the x and y axes. Identify the smallest and largest values in the table (for both x and y) to determine how to scale the graph.

Step 3: Plot each coordinate.

Step 4: Use a ruler to connect the coordinates and add arrows to each end.

Step 5: Label the line with reference to the rule.

Student practice

Construct a graph for the rule using the table of values given.

a. $y = 2x + 3$

x	-2	-1	0	1	2
y	-1	1	3	5	7

b. $y = -3x + 4$, where x is between -2 and 2.

9B Questions

Understanding worksheet

1. Complete the table of values.

Example

x	-1	0	1	2
y	2	4	6	8
Coordinate	$(-1,2)$	(0,4)	(1,6)	(2,8)

a.

x	1	2	3	4
y	2		8	11
Coordinate	(1,2)	(2,5)	(3,8)	

b.

x	0	1	2	3
y		1.5	2	2.5
Coordinate	(0,1)	(1,1.5)		(3,2.5)

c.

x	-1	0	1	2
y	7		3	1
Coordinate	$(-1,7)$	(0,5)		(2,1)

d.

x	-2	-1	0	1
y		10	6	2
Coordinate	$(-2,14)$	$(-1,10)$	(0,6)	

2. Using the table of values, substitute the correct value into the equation.

Example

x	**-1**	**0**	**1**	**2**
$y = 2x + 1$	$2(-1) + 1 = -1$	$2(0) + 1 = 1$	$2(1) + 1 = 3$	$2(2) + 1 = 5$

a.

x	**1**	**2**	**3**	**4**
$y = 3x + 3$	$3(1) + 3 = 6$	$3(\quad) + 3 = 9$	$3(3) + 3 = 12$	$3(4) + 3 = 15$

b.

x	**0**	**1**	**2**	**3**
$y = 6x + 4$	$6(0) + 4 = 4$	$6(1) + 4 = 12$	$6(\quad) + 4 = 16$	$6(3) + 4 = 22$

c.

x	**-1**	**0**	**1**	**2**
$y = 3x - 5$	$3(\quad) - 5 = -8$	$3(0) - 5 = -5$	$3(1) - 5 = -2$	$3(2) - 5 = 1$

d.

x	**-2**	**-1**	**0**	**1**
$y = 7x - 1$	$7(-2) - 1 = -15$	$7(\quad) - 1 = -8$	$7(0) - 1 = -1$	$7(1) - 1 = 6$

3. Fill in the blanks by using the words provided.

Cartesian | *y*-coordinates | *x*-coordinates | smallest

A table of values displays a set of Cartesian coordinates. The coordinates are used to plot on a _____ plane. The values found in the top row of a table of values display the _____, whereas the values found in the second row display the _____. When plotting coordinates from a table of values to a Cartesian plane, it is important to identify the _____ and largest values in the table (for both x and y) to scale the graph appropriately.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (b,c), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (c,d), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Use the table of values to generate coordinates. WE1

a.

x	−3	−2	−1	0	1
y	−2	−1	0	1	2
Coordinate					

b.

x	−2	−1	0	1	2
y	−6	−4	−2	0	2
Coordinate					

c.

x	−1	0	2	4	6
y	1	0	−2	−4	−6
Coordinate					

d.

x	−2	0	2	4	6
y	4	−2	−8	−12	−16
Coordinate					

5. Complete the table of values with the given rule. WE2a

x	−2	−1	0	1	2
y					

a. $y = 3x$ **b.** $y = 5x$ **c.** $y = 2x + 2$ **d.** $y = 3x + 1$

e. $y = x + 3$ **f.** $y = 3 + 2x$ **g.** $y = \frac{1}{2}x + 1$ **h.** $y = 2 + \frac{3}{4}x$

6. Complete the table of values with the given rule. WE2b

x	−2	−1	0	1	2
y					

a. $y = -4x$ **b.** $y = -2x$ **c.** $y = -3x + 3$ **d.** $y = -2x - 4$

e. $y = -x$ **f.** $y = -2 - x$ **g.** $y = -\frac{1}{4}x + 2$ **h.** $y = 3 - \frac{1}{2}x$

7. Construct a graph for each of the following. WE3a

a. $y = 2x$

x	−2	−1	0	1	2
y	−4	−2	0	2	4

b. $y = x$

x	−3	−1	4	6	7
y	−3	−1	4	6	7

c. $y = x - 1$

x	-3	-2	-1	0	1
y	-4	-3	-2	-1	0

d. $y = -2x + 3$

x	0	1	2	3	4
y	3	1	-1	-3	-5

e. $y = 2x + 4$

x	-1	0	1	2	3
y	2	4	6	8	10

f. $y = -\frac{1}{3}x$

x	-3	0	3	6	9
y	1	0	-1	-2	-3

g. $y = \frac{1}{2}x + 6$

x	0	2	4	6	8
y	6	7	8	9	10

h. $y = -0.25x + 2$

x	-3	0	2	3	5
y	2.75	2	1.5	1.25	0.75

8. Construct a graph for the rule using a table of values when x is between -2 and 2. WE3b

a. $y = 4x$ **b.** $y = -2x$ **c.** $y = 3x + 2$ **d.** $y = -4x - 2$

e. $y = -\frac{2}{3}x$ **f.** $y = \frac{1}{2}x + 4$ **g.** $y = 0.75x - 3$ **h.** $y = x - 2x$

9. Identify which of the coordinates are for the given rule.

$y = 3x - 1$

A. $(-1,4), (2,6), (5,14), (8,23)$

B. $(2,1), (5,2), (8,3), (11,4)$

C. $(1,30), (3,32), (5,34), (10,309)$

D. $(-1,0), \left(1,-\frac{2}{3}\right), (2,-1), (5,-2)$

E. None of the above

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Complete the table of values for $y = 3x$.

x	0	1	2	3
y				

Student A

x	0	1	2	3
y	30	31	32	33

Student B

x	0	1	2	3
y	0	3	6	9

b. The calculations to complete the table of values for $y = 2x - 3$ are:

x	-3	-1	2	5
y				

Student A

$y = 2(-3) - 3 = -9$

$y = 2(-1) - 3 = -5$

$y = 2(2) - 3 = 1$

$y = 2(5) - 3 = 7$

Student B

$y = 2(-3) - 3 = -9$

$y = 2(-2) - 3 = -7$

$y = 2(-1) - 3 = -5$

$y = 2(0) - 3 = -3$

Problem solving

Question working paths

Mild 11, 12, 13 | Medium 12, 13, 14 | Spicy 13, 14, 15

11. Lester starts the Bolt for Gold running competition that his high school sports department is currently enrolled in. The distance that he has achieved over the first five days generates a linear pattern. State the coordinates of Lester's first five consecutive runs.

Day (d)	1	2	3	4	5
Kilometres (k)	3	3.25	4.5	5.75	7
Coordinate					

12. Clinton loves to bake cookies. The time (t) it takes him to bake (c) number of cookies is found by the rule $t = 2c + 15$. Generate a table of values that displays how long it will take Clinton to bake 1, 4, 6, 10 and 20 cookies.

13. For Christmas, Edrolo purchases travel vouchers for their employees. Each employee receives a \$150 voucher, plus an additional one off \$30 transaction fee. The rule that displays the cost is represented as $\$c = 150v + 30$, where v represents the number of vouchers and c represents the cost in dollars (\$). Generate a table of values that displays how much it will cost Edrolo to purchase 50, 70, 90, 120 and 200 travel vouchers.

14. Steve loves ordering pizza from Cagney's restaurant. All medium sized pizza costs \$14. Each additional topping costs \$2. Generate a table of values and a graph to display this scenario. Toppings on the pizza is represented as (x) and is between 0 and 6, and total cost is represented as (y).

15. Daniel has been saving to purchase his first car. After one year he has saved \$2000, after three years he has \$3500. Generate a table of values displaying Daniel's savings and construct a graph using this information.

Reasoning

Question working paths

Mild 16 (a,b,c,e) | Medium 16 (a,b,c,e), 17 (a,b) | Spicy All

16. It's Matt's birthday this weekend and he is cooking burgers for his family. His local butcher charges $1.95 for each burger and then a one off delivery fee of $10.

a. Copy and complete the table of values to show how much it would cost Matt to order burgers for his birthday.

Burgers (b)	15	18	20	25	40	50	60
Cost \$(c)							

b. Using your answers from part **a** state the coordinates for the table of values.

c. The rule for this table of values is $\$cost = 1.95b + 10$. Using the rule substitute and calculate how much it will cost for 100 burgers.

d. Construct a table of values where burgers (b) is between 80 and 85, inclusive and then plot on a Cartesian plane.

e. What are some other alternative barbeque options Matt could consider making for his birthday?

17. For part **a** and **b** complete a table of values with the given rule where x is between -3 and 3.

a. $y = 2x - 2$

b. $y = -2x - 2$

c. Compare your answers for parts **a** and **b** and comment on the differences between the coefficient values.

Extra spicy

18. Determine if the points $(-3,11)$ and $(2,-14)$ are coordinates for the rule $y = -5x - 4$.

19. Plot the points on a Cartesian plane for $y - \frac{1}{2} = \frac{3}{4}x$.

20. Determine the rule that will generate the following table of values.

x	1	2	3	4	5
y	1.5	4.5	7.5	10.5	13.5

A. $x = \frac{y}{3} + \frac{1}{2}$ **B.** $y = 3x + 1.5$ **C.** $x = 3y - 1.5$ **D.** $x = 3y + 1.5$ **E.** $y = 1.5x + 3$

21. The correct table of values for the rule $x = 0.5y - 1.375$ and coordinates $(-4,5.25)$ and $(4,10.75)$ is:

A.

x	-2	-1	0	1	2
y	-1.875	-1.375	-0.875	-0.375	0.125

B.

x	-2	-1	0	1	2
y	-1.25	0.75	2.75	4.75	6.75

C.

x	-2	-1	0	1	2
y	-2.375	-1.875	-1.375	-0.875	-0.375

D.

x	-2	-1	0	1	2
y	-3.25	-1.25	0.75	2.75	4.75

E. y is not the subject, so a table of values cannot be generated.

Remember this?

22. This table shows the number of questions Ms. Clifton's maths students answered correctly in a quiz.

Number of questions answered correctly	Number of students
3	4
4	5
5	11
6	3
7	2

What is the mode number of questions Ms. Clifton's students answered correctly for this quiz?

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

23. Two fair dice with six sides each are tossed at the same time. What is the probability that both dice will show the number six?

A. $\frac{1}{36}$ **B.** $\frac{1}{30}$ **C.** $\frac{1}{26}$ **D.** $\frac{1}{12}$ **E.** $\frac{1}{6}$

24. Patrick counted the number of gummy bears in a bowl. A sixth of the lollies are orange-flavoured, a third are pineapple-flavoured and the rest are banana-flavoured. There are 3 more pineapple-flavoured gummy bears than orange-flavoured gummy bears. How many banana-flavoured gummy bears are there in the bowl?

A. 3 **B.** 6 **C.** 9 **D.** 12 **E.** 18

Back to contents

9C Linear rules

A linear rule is an efficient way of describing the relationship between two variables. A linear rule can be established and calculated by referring to a set of coordinates, a table of values or a graph. Once a linear rule has been established, this allows us to calculate additional values that may not have been given originally.

LEARNING INTENTIONS

Students will be able to:

- understand and find the linear rule from a set of coordinates
- understand and find the linear rule from a table of values
- understand and find the linear rule from a graph.

KEY TERMS AND DEFINITIONS

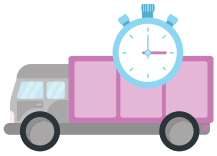
A **coefficient** is the number that a pronumeral is being multiplied by.

A **constant** is a number that cannot change its value in an expression or equation.

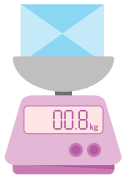
A **linear rule** is an equation used for a straight line.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

INTERNATIONAL SHIPMENT



EXPRESS DELIVERY SERVICE



POST OFFICE

There are different ways an item can be delivered. The cost of each method can be calculated using a different linear equation. These linear equations differ in coefficient and constant values and are dependent on some of the following factors: location, weight, company etc.

Key ideas

1. A linear rule is in the form of:

$$y = mx + c$$

or

$$y = c + mx$$

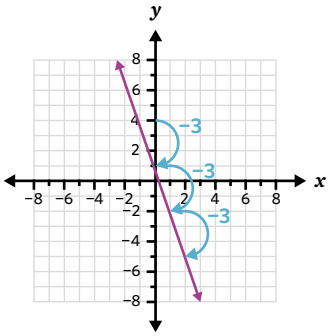
m = coefficient of x
c = constant
x and y are variables

2. Finding the coefficient value for a linear rule varies on what information has been given.

From a set of coordinates

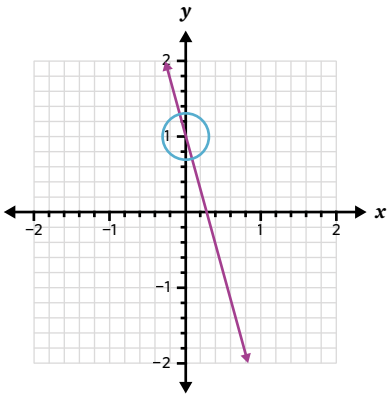
$(-1,7), (0,4), (1,1)$
For each one unit increase of x, y decreases by 3.
Coefficient of x is -3.
$\therefore y = -3x +$ constant

Continues →

From a table of values	From a graph
Ordered pairs of coordinates +1 +1 +1 x: 1, 2, 3, 4 y: 6, 8, 10, 12 +2 +2 +2 Coefficient of x is 2 For each one unit increase of x, the y-value increases by 2. Coefficient of x is 2. $\therefore y = 2x +$ constant	 For each one unit increase of x, the y-value decreases by 3. Coefficient of x is -3. $\therefore y = -3x +$ constant

3. Finding the constant value for a linear rule varies on what information has been given.

Finding the constant is when $x = 0$.

From a set of coordinates	From a table of values
$(-1,7), (0,4), (1,1)$ $\therefore (0,4)$ $y = mx + 4$	x: 0, 1, 2, 3, 4 y: 4, 6, 8, 10, 12 $\therefore (0,4)$ $y = mx + 4$
From a graph	**Substituting a set of given coordinates and solving for the constant**
 $\therefore (0,1)$ $y = mx + 1$	$(2,5.75), (3,7.75)$ $(4,9.75), (5,11.75)$ If the coordinate (0,constant) is not given, calculate the coefficient first. **Coefficient** y-value increases by 2. Coefficient of x is 2. $\therefore y = 2x +$ constant **Constant** Substitute any one coordinate into the rule and solve for the constant. $7.75 = 2 \times 3 +$ constant $7.75 = 6 +$ constant $1.75 =$ constant $\therefore y = 2x + 1.75$

Worked example 1

Finding a rule from a set of coordinates

Find the rule from the given set of coordinates.

a. $(-1,1), (0,3), (1,5), (2,7)$ WE1a

Working

$(1,5), (2,7)$

y-value increases by 2.
Coefficient of x is 2.

Coordinate where $x = 0$ is $(0,3)$.
Constant is 3.

$y = 2x + 3$

Thinking

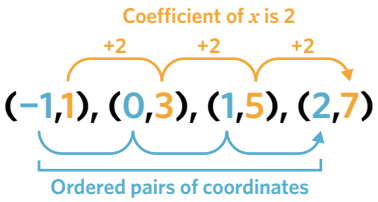
Step 1: Identify any two consecutive ordered pairs of coordinates.

Step 2: To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

Step 3: Identify the constant by referring to the coordinate where $x = 0$.

Step 4: Outline the rule in the form of $y = mx + c$.

Visual support



b. $(-1,7), (0,4), (1,1), (2,-2), (3,-5)$ WE1b

Working

$(2,-2), (3,-5)$

y-value decreases by 3.
Coefficient of x is -3.

Coordinate where $x = 0$ is $(0,4)$.
Constant is 4.

$y = -3x + 4$

Thinking

Step 1: Identify any two consecutive ordered pairs of coordinates.

Step 2: To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

Step 3: Identify the constant by referring to the coordinate where $x = 0$.

Step 4: Outline the rule in the form of $y = mx + c$.

Student practice

Find the rule from the given set of coordinates.

a. $(-1,-1), (0,2), (1,5), (2,8)$

b. $(-1,5), (0,3), (1,1), (2,-1), (3,-3)$

Worked example 2

Finding a rule from a table of values

Find the rule from the given table of values.

a.

WE2a

x	−1	0	1	2
y	−5	−2	1	4

Working

(1,1), (2,4)

y-value increases by 3.
Coefficient of x is 3.

Coordinate where $x = 0$ is (0,−2).
Constant is −2.

$y = 3x - 2$

Thinking

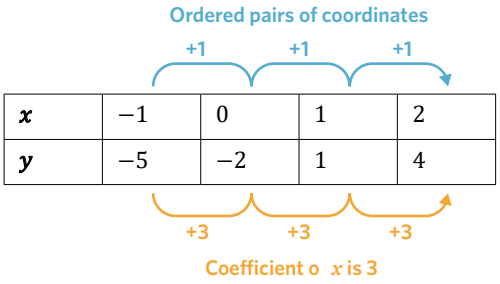
Step 1: Identify any two consecutive ordered pairs of coordinates.

Step 2: To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

Step 3: Identify the constant by referring to the coordinate where $x = 0$.

Step 4: Outline the rule in the form of $y = mx + c$.

Visual support



b.

WE2b

x	1	2	3	4
y	−1	−5	−9	−13

Working

(1,−1), (2,−5)

y-value decreases by 4.
Coefficient of x is −4.

(3,−9)

$-9 = -4(3) + \text{constant}$

$-9 = -12 + \text{constant}$

$3 = \text{constant}$

$y = -4x + 3$

Thinking

Step 1: Identify any two consecutive ordered pairs of coordinates.

Step 2: To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

Step 3: Substitute any one coordinate into the rule and solve for the constant.

Step 4: Outline the rule in the form of $y = mx + c$.

Student practice

Find the rule from the given table of values.

a.

x	−1	0	1	2
y	−4	−2	0	2

b.

x	1	2	3	4
y	1	−2	−5	−8

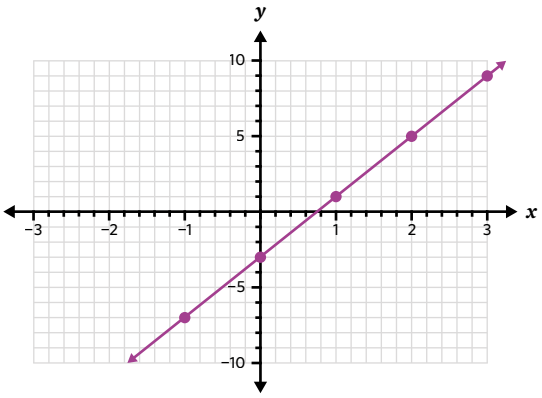
Worked example 3

Finding a rule from a graph

Find the rule from the graph.

a.

WE3a



Working

(1,1), (2,5)

y-value increases by 4.
Coefficient of x is 4.

Coordinate where $x = 0$ is $(0,-3)$.
Constant is -3.

$y = 4x - 3$

Thinking

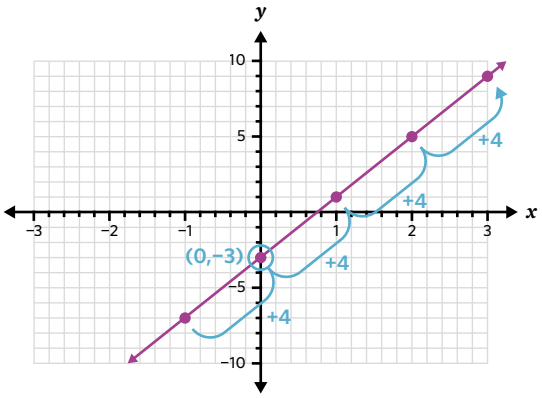
Step 1: Identify any two consecutive ordered pairs of coordinates.

Step 2: To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

Step 3: Refer to the y-axis to determine the constant, this is when $x = 0$.

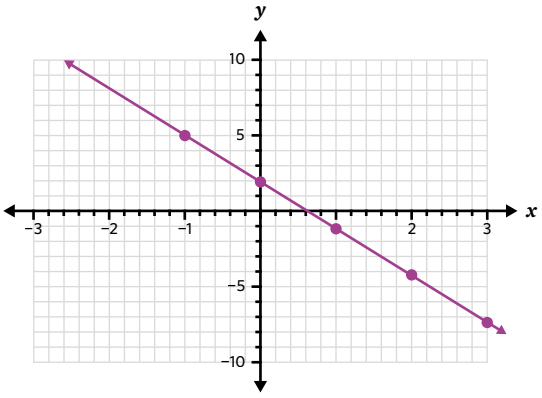
Step 4: Outline the rule in the form of $y = mx + c$.

Visual support



b.

WE3b



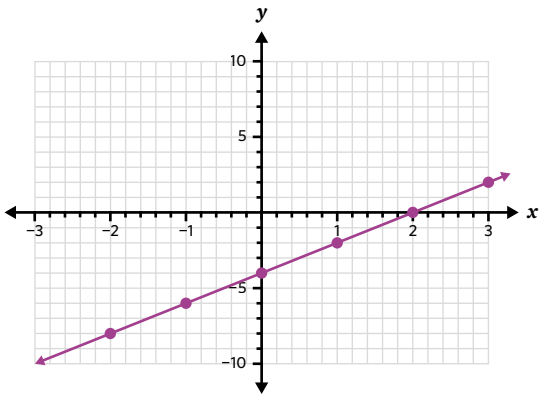
Continues →

Working	Thinking
$(1,-1.25), (2,-4.25)$	**Step 1:** Identify any two consecutive ordered pairs of coordinates.
y-value decreases by 3. Coefficient of x is -3.	**Step 2:** To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.
$(-1,4.75)$	**Step 3:** Substitute any one coordinate into the rule and solve for the constant.
$4.75 = -3(-1) + \text{constant}$	
$4.75 = 3 + \text{constant}$	
$1.75 = \text{constant}$	
$y = -3x + 1.75$	**Step 4:** Outline the rule in the form of $y = mx + c$.

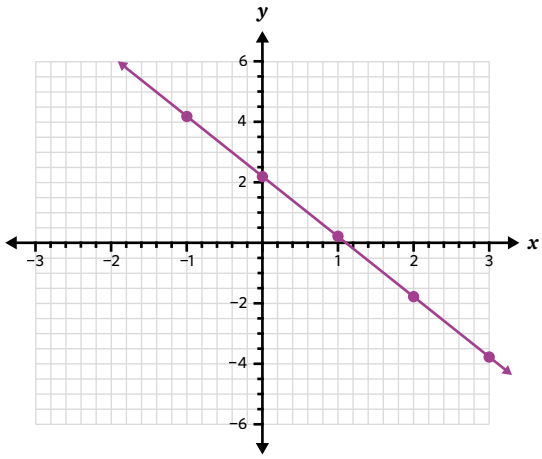
Student practice

Find the rule from the graph.

a.



b.



9C Questions

Understanding worksheet

1. Complete the rule with reference to the table of values.

Example

x	-1	0	1	2
y	-7	-4	-1	2

+3 +3 +3

$y =$ 3 $x - 4$

a.

x	-1	0	1	2
y	-6	-4	-2	0

$+2$ $+2$ $+2$

$y =$ ☐ $x - 4$

b.

x	-1	0	1	2
y	9	14	19	24

$+5$ $+5$ $+5$

$y =$ ☐ $x + 14$

c.

x	-1	0	1	2
y	3.5	3	2.5	2

-0.5 -0.5 -0.5

$y = 3 +$ ☐ x

d.

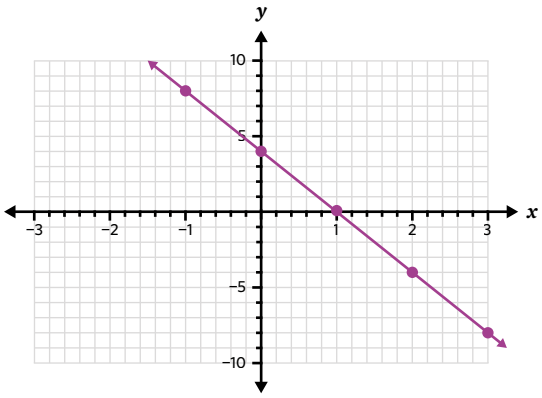
x	-1	0	1	2
y	-2.2	-3	-3.8	-4.6

-0.8 -0.8 -0.8

$y =$ ☐ $x - 3$

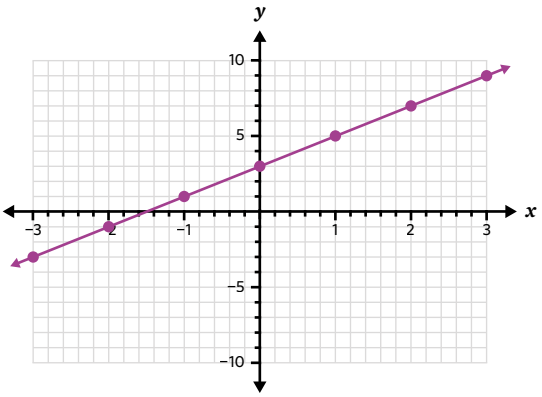
2. Complete the rule with reference to the graph.

Example



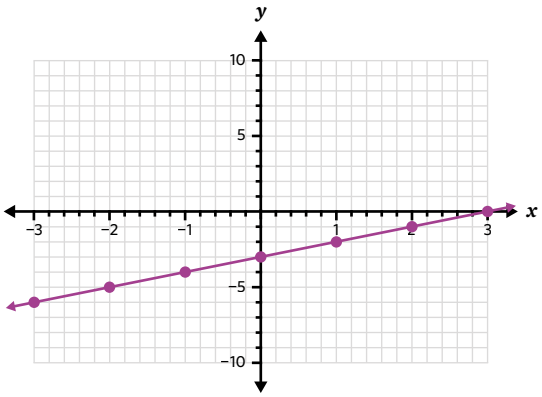
$y = -4x$ ☐ $+ 4$

a.



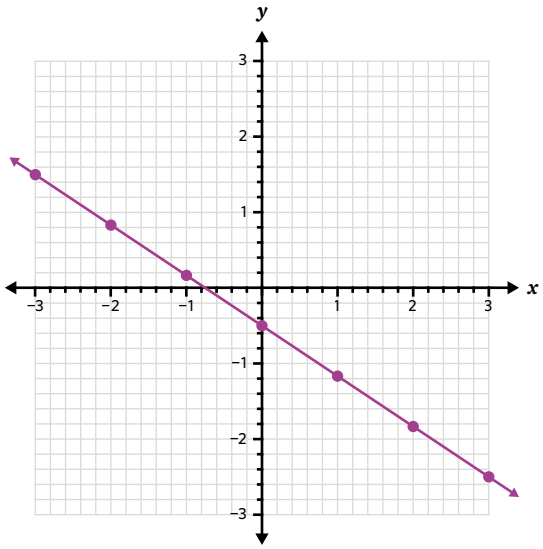
$y = 2x$ ☐

b.



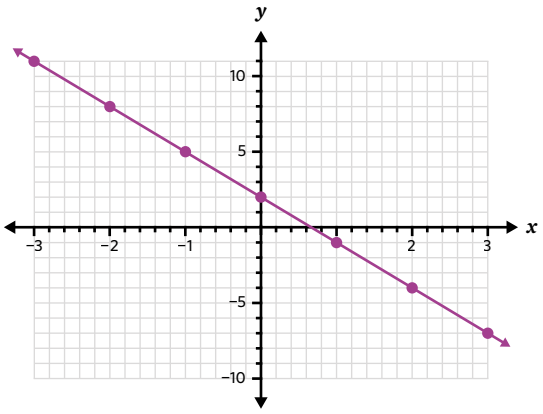
$y = x$ ☐

c.



$y = -\frac{2}{3}x$ ☐

d.



$y =$ ☐ $- 3x$

3. Fill in the blanks by using the words provided.

equation | coordinates | relationship | coefficie

A linear rule is an ☐ that describes the ☐ between two variables.

These values can be represented as ☐, in a table of values, or plotted on a graph.

A rule has a value which is known as the ☐ of x and a constant.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Find the rule from the given set of coordinates. WE1

a. (0,3), (1,7), (2,11), (3,15), (4,19)

b. (0,2), (1,5), (2,8), (3,11), (4,14)

c. (−1,−4), (0,0), (1,4), (2,8), (3,12)

d. (−2,8), (1,17), (2,20)

e. (−1,−10), (1,−8), (4,−5), (6,−3)

f. (−3,−12), (−1,−8), (1,−4), (5,4)

g. (−3,2.5), (−1,5.5), (2,10)

h. (−3,4.5), (3,31.5)

5. Find the rule from the given table of values. WE2

a.

x	0	1	2	3
y	5	7	9	11

b.

x	0	1	2	3
y	3	10	17	24

c.

x	0	1	2	3
y	2	2	2	2

d.

x	−2	2	4	5
y	−5	3	7	9

e.

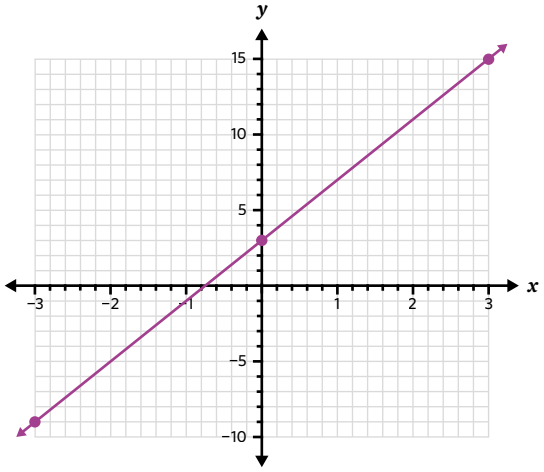
x	2	4	8
y	7	6	4

f.

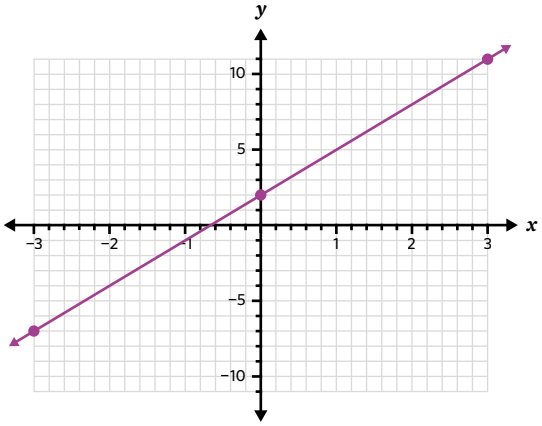
x	−4	−1	2
y	44	11	−22

6. Find the rule from the graph. WE3

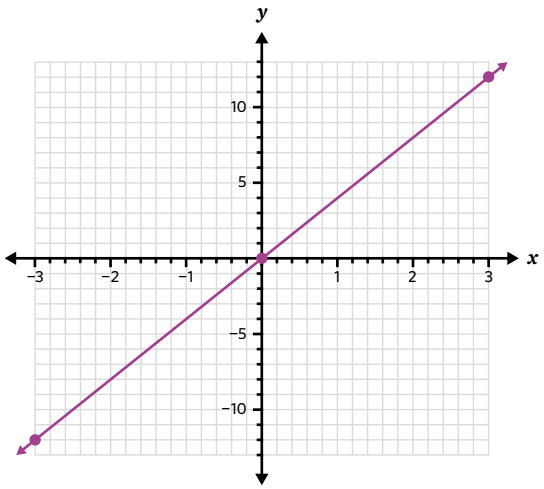
a.



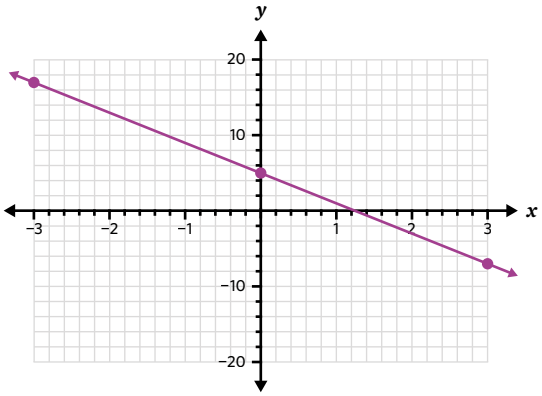
b.



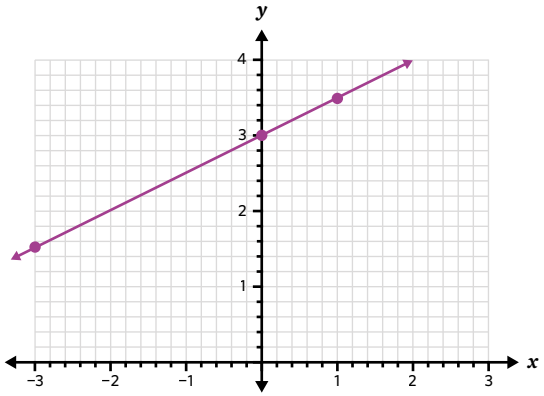
c.



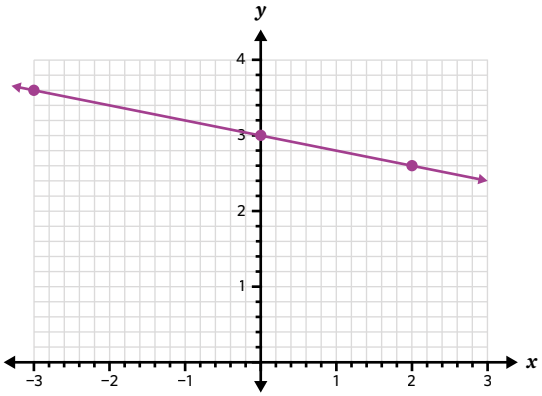
d.



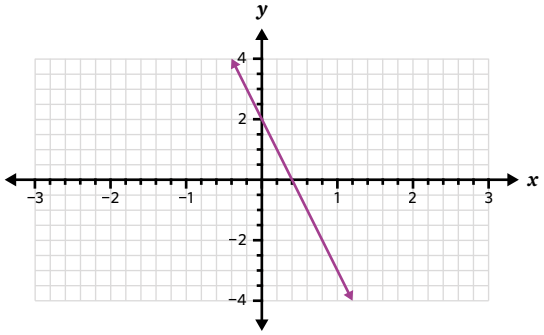
e.



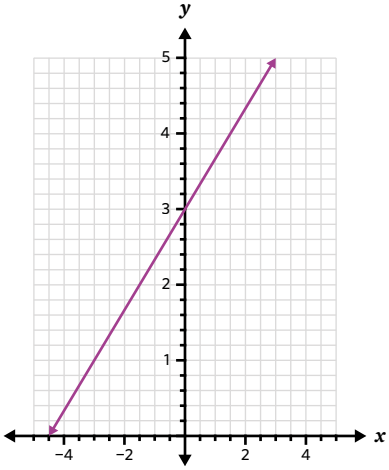
f.



g.



h.



7. State the rule using the provided information.

a. A rule that has a coefficient of x equal to 4 and a constant value of 6.

b. A rule that has a coefficient of x equal to 2 and a constant value of 2.

c. A rule that has a coefficient of x equal to 1 and a constant value of -3.

d. A rule that has a coefficient of x equal to -3 and a constant value of -3.

e. A rule that has a coefficient of x equal to $\frac{1}{2}$ and a constant value of 2.

f. A rule that has a coefficient of x equal to $\frac{2}{3}$ and coordinates (9,15).

g. A rule that has a coefficient of x equal to $\frac{1}{5}$ and coordinates (5,5)

h. A rule that has a coefficient of x equal to $\frac{1}{4}$ and coordinates (2,-3).

---

8. Find the rule from the set of coordinates.

(2,-5), (3,-8), (4,-11)

A. $y = 2x + 5$ **B.** $y = 3x + 1$ **C.** $y = -3x + 1$ **D.** $y = 3x$ **E.** $y = 3x$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Find the rule for the following table of values.

x	0	2	4	5
y	-3	1	5	7

Student A

y-value increases by 4.

Coefficient of $x = 4$.

$y = 4x + c$

Constant

$4 = 4 \times 5 + c$

$4 = 20 + c$

$-16 = c$

$y = 4x - 16$

Student B

y-value increases by 2.

Coefficient of $x = 2$.

$y = 2x + c$

Constant

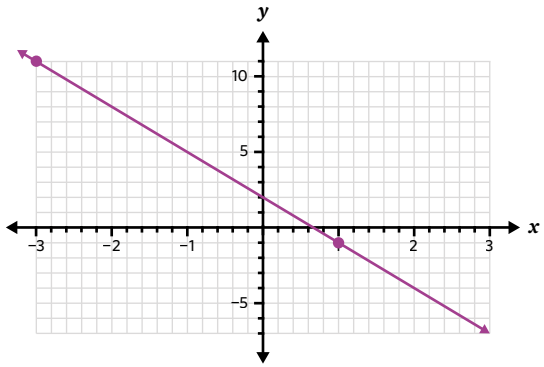
$5 = 2 \times 4 + c$

$5 = 8 + c$

$-3 = c$

$y = 2x - 3$

b. Find the coefficient of x from the graph.



Student A

y-value increases by 3.

Coefficient of $x = 3$.

Student B

y-value decreases by 3.

Coefficient of $x = -3$.

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

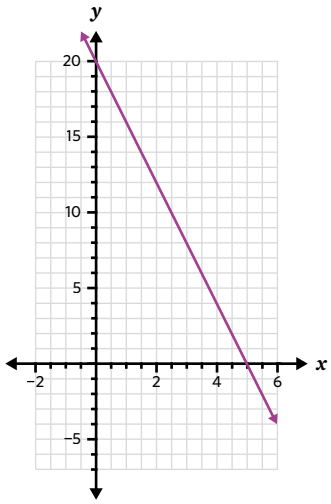
10. Georgie generates a linear equation $y = -2x + 8$. State the constant and coefficient for this linear equation.

11. Clinton is the Head of Marketing at Butter Chicken Co. He has calculated that for each new post he puts up on social media, the number of followers increases by 15. Write a rule to show the linear relationship between the number of posts (x) and number of followers (y).

Post, x	0	12	20	21	35
Number of followers, y	15	195	315	330	540

12. Tori is currently going through physio rehab and is having an ice salt bath each night to help with muscle recovery. For each bath she applies 125 grams of ice salt. The bag has a total of 1600 grams. On the tenth night she has 350 grams remaining. Generate two known coordinates and a rule that displays how much ice salt (s, grams) is left in the packet on any given day (d).

13. Lenny is working on her maths homework with her sister, Penny. Lenny explains to Penny that the following graph does not represent $y = -4x + 5$. Explain where Penny has gone wrong.



14. Xavier is in grade one. Each new day he learns to spell an additional three words. The following coordinates display how many words he has learnt on three particular days in grade one. Calculate and explain what the value of w represents when $d = 0$.

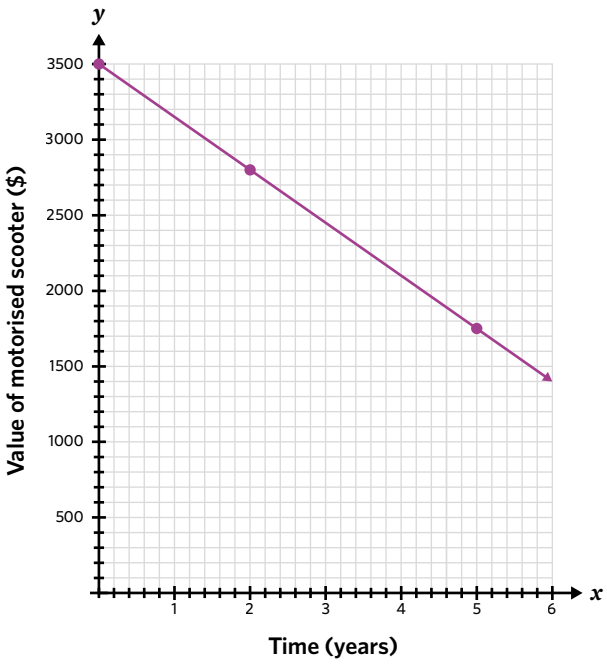
Day (d)	20	30	35
Number of words (w)	240	270	285
Coordinate	(20,240)	(30,270)	(35,285)

Reasoning

Question working paths

Mild 15 (a,b,c,e) | Medium 15 (a,b,c,e), 16 (a,b) | Spicy All

15. The graph displays the value of a motorised scooter over a six year period since the start of 2018. Consider the linear rule displayed in the graph.



a. What does the value y represent when $x = 0$?

b. What is the value of the motorised scooter in the third year?

c. Find the value of the coefficient of x.

d. What does the value of the coefficient represent in this scenario?

e. From this graph, we can see that the value of the motorised scooter is depreciating. What are some other items in our day-to-day life that may have a linear depreciation model?

16. Using the table of values, find the coefficient of x for both parts **a** and **b**.

a.

x	0	1	2	3
y	-1	2	5	8

b.

x	-2	0	4	7
y	-9	-1	15	27

c. Compare the different methods used to calculate the coefficient of x for parts **a** and **b**.

Extra spicy

17. A straight line has the equation $6x + 2y + 9 = 0$. The coefficient of x is:

A. -6 **B.** -3 **C.** 3 **D.** 6 **E.** 9

18. Show that the lines $y = 3x + 1$ and $4x - 2y + 3 = 0$ have different constant values.

19. Show that the coordinates (1,4), (4,10) and (9,20) generate a straight line by determining the rule.

20. (5,−2) are coordinates for which of the following linear equations?

A. $x = \frac{-y}{3} + \frac{13}{4}$ **B.** $x = y - 7$ **C.** $x = \frac{y}{2} + 6$ **D.** $x = \frac{y}{4} + \frac{9}{2}$ **E.** $x = \frac{-y}{2} - 4$

Remember this?

21. Ruby has a picture of her dog on her iPhone and she wants to make this picture her iPad wallpaper. The picture in her iPhone has dimensions 14 cm × 7 cm. She wants to enlarge it so that it has a width of 17.5 cm. What will be the length of this picture?

Image: Lelusy/Shutterstock.com

A. 14 cm **B.** 21 cm **C.** 31.5 cm **D.** 35 cm **E.** 98 cm

22. There are approximately 26 million people in Australia. Each day, an average Australian uses 18 KW electricity. Which of the following gives the best estimate of the total electricity in KW that all Australians use each day?

A. 468 KW
B. 46 800 KW
C. 468 000 KW
D. 46 800 000 KW
E. 468 000 000 KW

23. Which of the following statements is always true?

A. A parallelogram has four angles of 90°.
B. A triangle has three sides of equal lengths.
C. A square has four angles of the same size.
D. A rhombus has four right angles.
E. A trapezium has two pairs of sides of equal length.

9D Gradient

The gradient or slope gives a measure of the steepness of a line. Specifically it measures the change in vertical height in relation to the change in horizontal distance. When the gradient is calculated, it is given in the ratio $\frac{\text{rise}}{\text{run}}$. The gradient can be measured anywhere along the line, assuming it is a straight line.

LEARNING INTENTIONS

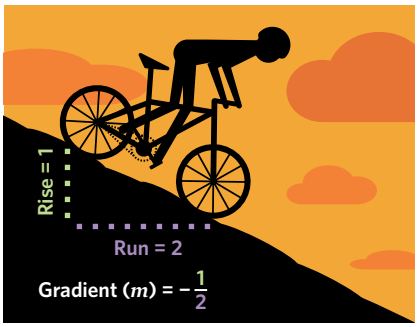
Students will be able to:

- identify different types of gradients
- determine the gradient of a line from a graph.

KEY TERMS AND DEFINITIONS

The **gradient** is the slope and direction of a line.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

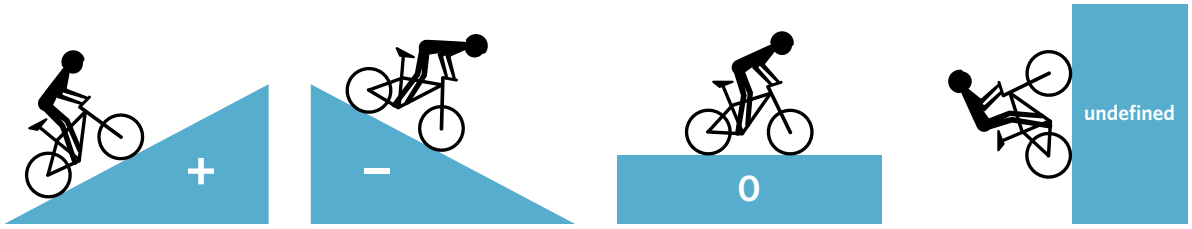


Quite often we may go on a bike ride and comment on the incline and decline of certain sections of the ride. The gradient is a measure of how far up or down compared to how far we have gone across on a bike ride. This is commonly referred to as the ratio $\frac{\text{rise}}{\text{run}}$.

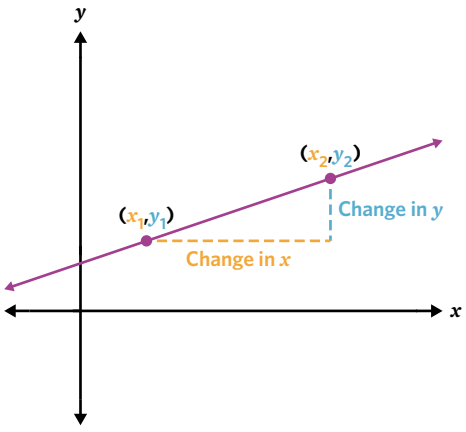
Key ideas

1. Different types of gradients can be categorised.

Positive	Negative	Zero	Undefined
Line has an upward trend	Line has a downward trend	Line is horizontal	Line is vertical



2. The gradient (m) is calculated using $\frac{\text{rise}}{\text{run}}$.



$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

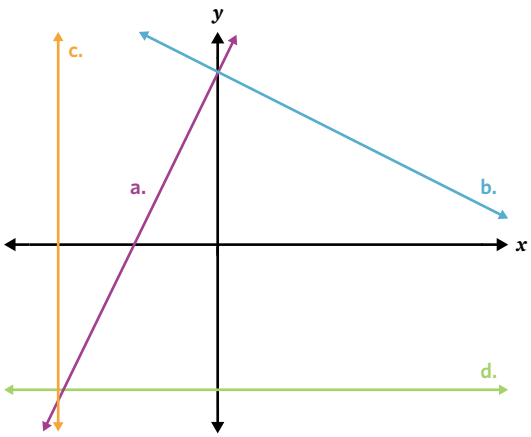
Rise	The vertical distance that the line goes up or down.
Run	The horizontal distance that the line goes across.

Back to contents

Worked example 1

Identifying different types of gradients

Using the graphs, state the type of gradient displayed. WE1



Working

a. Positive gradient

b. Negative gradient

c. Undefined gradient

d. Zero gradient

Thinking

a. As x increases, y increases.

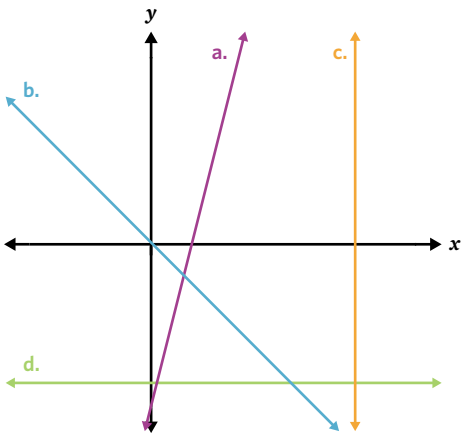
b. As x increases, y decreases.

c. The line is vertical.

d. The line is horizontal.

Student practice

Using the graphs, state the type of gradient displayed.



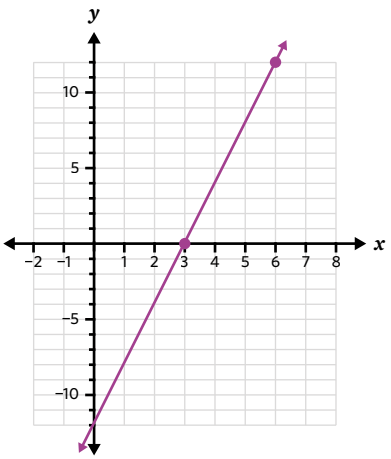
Worked example 2

Finding the gradient from a graph

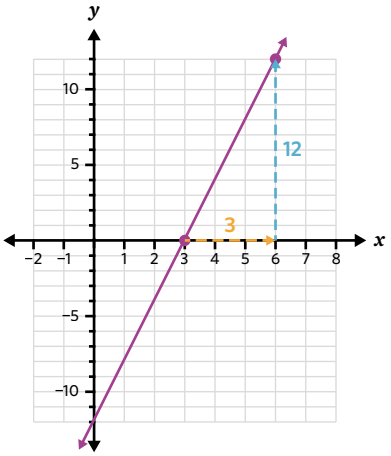
Determine the gradients using the graphs.

a.

WE2a



Working



$m = \frac{12}{3}$

$m = 4$

Thinking

Step 1: Use the two outlined coordinates in the graph.

Step 2: Determine the change in y between the two coordinates.

Step 3: Determine the change in x between the two coordinates.

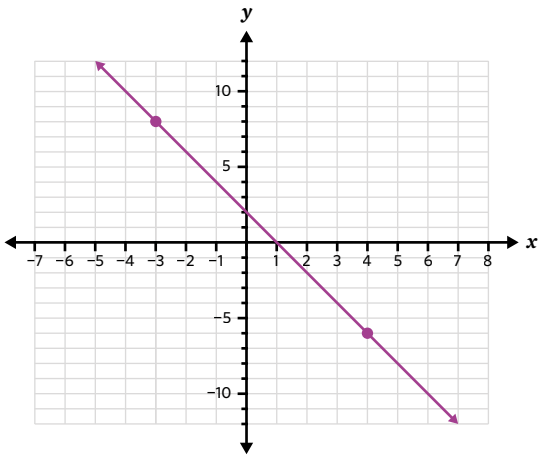
Step 4: Substitute the rise and run values into the rule to then calculate the gradient.

Step 5: As x values increase the y values increase, therefore this is a positive gradient.

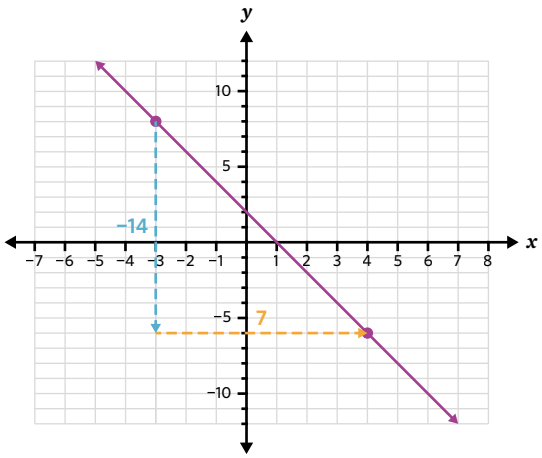
Continues →

b.

WE2b



Working



$m = \frac{-14}{7}$

$m = -2$

Thinking

Step 1: Use the two outlined coordinates in the graph.

Step 2: Determine the change in y between the two coordinates.

Step 3: Determine the change in x between the two coordinates.

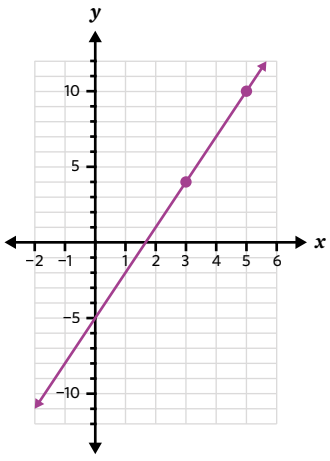
Step 4: Substitute the rise and run values into the rule to then calculate the gradient.

Step 5: As x values increase the y values decrease, therefore this is a negative gradient.

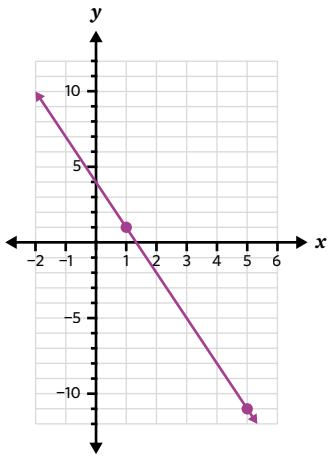
Student practice

Determine the gradients using the graphs.

a.



b.



Worked example 3

Calculating the gradient from a set of coordinates

Calculate the gradient using coordinates that are on the same line.

a. $A(3,0)$ and $B(6,12)$ WE3a

Working

Gradient $= \frac{\text{rise}}{\text{run}}$

$(3,0) \quad (6,12)$
$(x_1,y_1) \quad (x_2,y_2)$

$m = \frac{12 - 0}{6 - 3}$

$m = \frac{12}{3}$

$m = 4$

Thinking

Step 1: Use $\frac{y_2 - y_1}{x_2 - x_1}$.

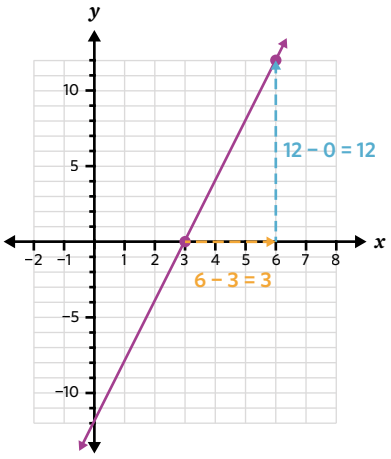
Step 2: Match coordinates A and B with (x_1,y_1) and (x_2,y_2).

Step 3: Determine the change in x and y between the two coordinates by substituting the correct values into the rule.

Step 4: Simplify.

Step 5: As x values increase the y values increase, therefore this is a positive gradient.

Visual support



b. $A(1,13)$ and $B(5,9)$ WE3b

Working

$m = \frac{\text{rise}}{\text{run}}$

$(1,13) \quad (5,9)$
$(x_1,y_1) \quad (x_2,y_2)$

$m = \frac{9 - 13}{5 - 1}$

$m = \frac{-4}{4}$

$m = -1$

Thinking

Step 1: Use $\frac{y_2 - y_1}{x_2 - x_1}$.

Step 2: Match coordinates A and B with (x_1,y_1) and (x_2,y_2).

Step 3: Determine the change in x and y between the two coordinates by substituting the correct values into the rule.

Step 4: Simplify.

Step 5: As x values increase the y values decrease, therefore this is a negative gradient.

Student practice

Calculate the gradient using coordinates that are on the same line.

a. $A(0,4)$ and $B(5,19)$

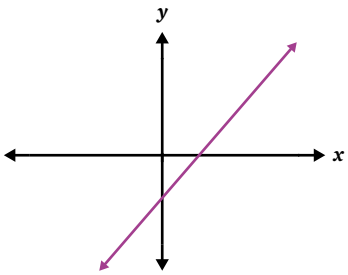
b. $A(3,2)$ and $B(5,-2)$

9D Questions

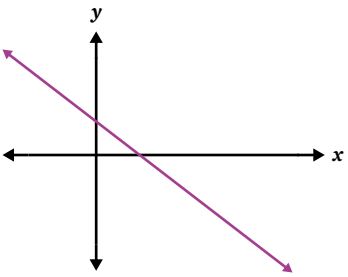
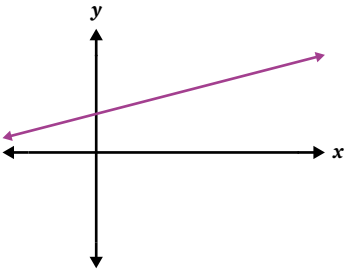
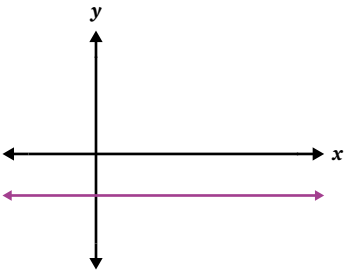
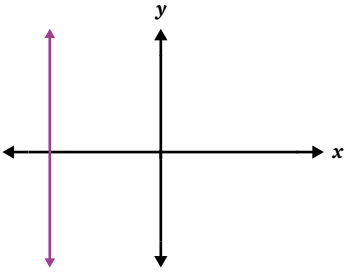
Understanding worksheet

1. State if the following line is positive, negative, undefined or zero.

Example

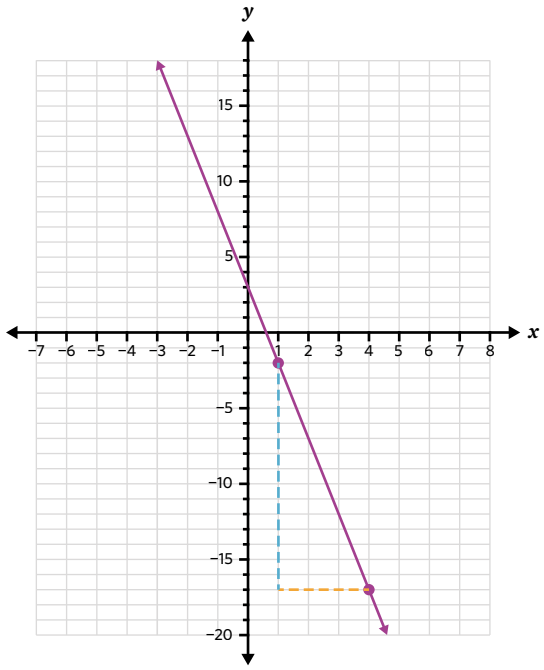


Positive	Negative	Zero	Undefined
☑	☐	☐	☐

	Positive	Negative	Zero	Undefined
a. 	☐	☐	☐	☐
b. 	☐	☐	☐	☐
c. 	☐	☐	☐	☐
d. 	☐	☐	☐	☐

2. Complete the calculations to determine the value of the gradient (m).

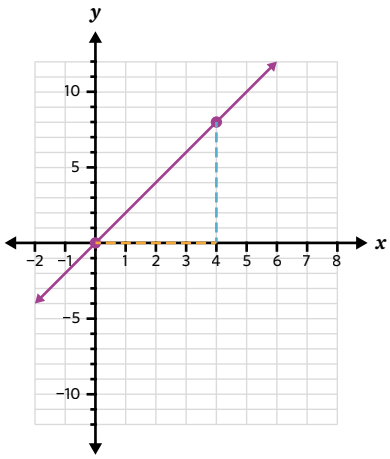
Example



$m = \frac{15}{3}$

$m = -5$

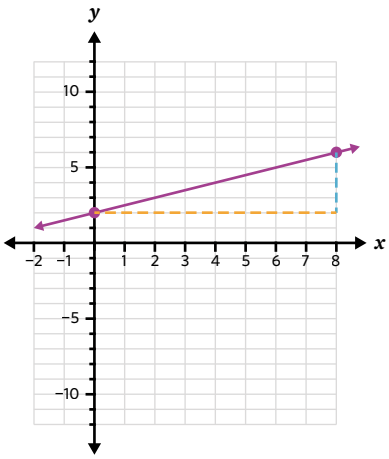
a.



$m = \frac{\quad}{\quad}$

$m = 2$

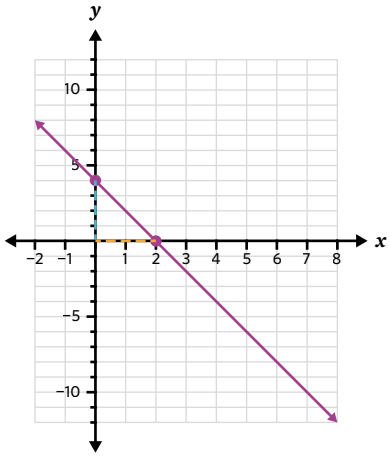
b.



$m = \frac{\quad}{\quad}$

$m = 0.5$

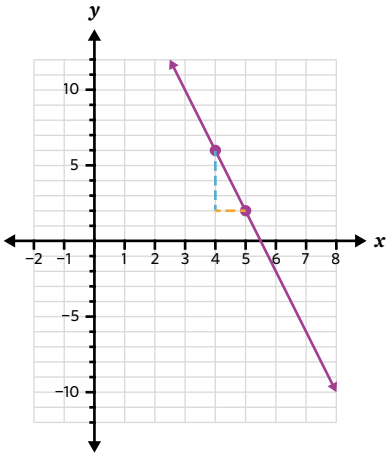
c.



$m = \frac{\quad}{\quad}$

$m = -2$

d.



$m = \frac{\quad}{\quad}$

$m = -4$

3. Fill in the blanks by using the words provided.

vertical | rise / run | steeper | zero

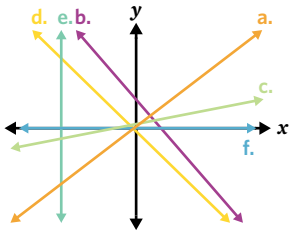
The gradient is another word for the slope, which is a measure of steepness. The gradient of a straight line is constant, this means that it will be the same when measured anywhere along the line. The gradient is calculated by using _____. The greater the value of the gradient the _____ the line. If the line is _____, this is an undefined gradient. If the line is horizontal, this line will have a gradient equal to _____.

Fluency

Question working paths

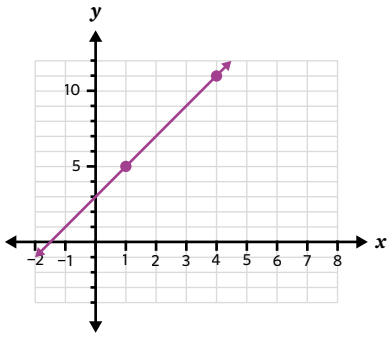
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (b,c,d,e), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (c,d,e,f), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. Using the graphs, state the type of gradient displayed. WE1

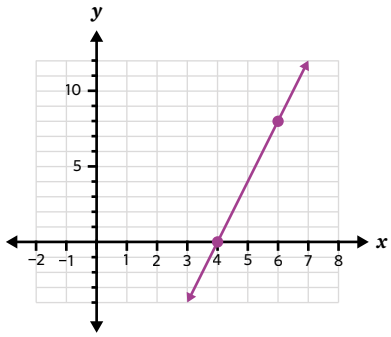


5. Determine the gradients using the graphs. WE2

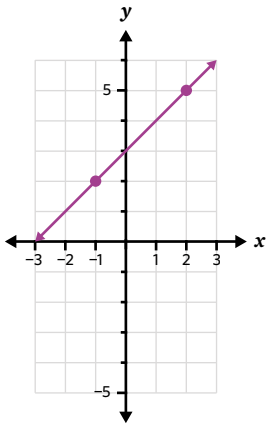
a.



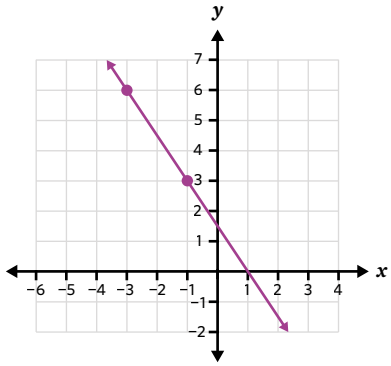
b.



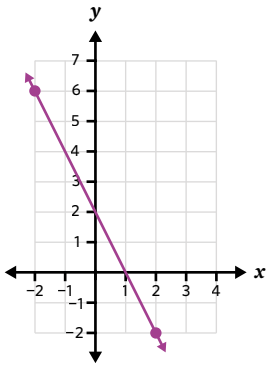
c.



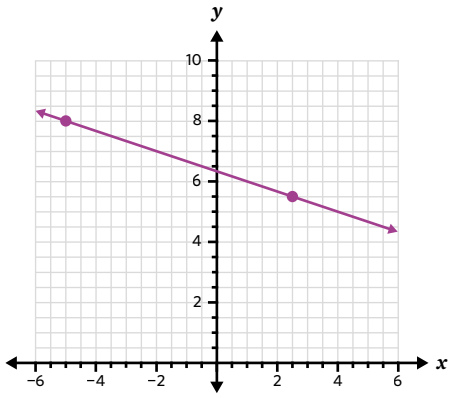
d.



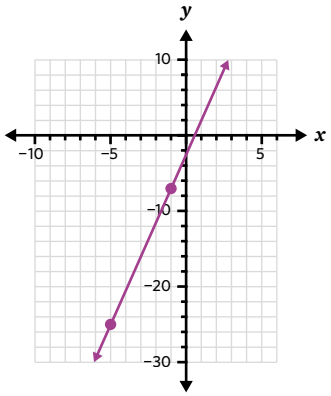
e.



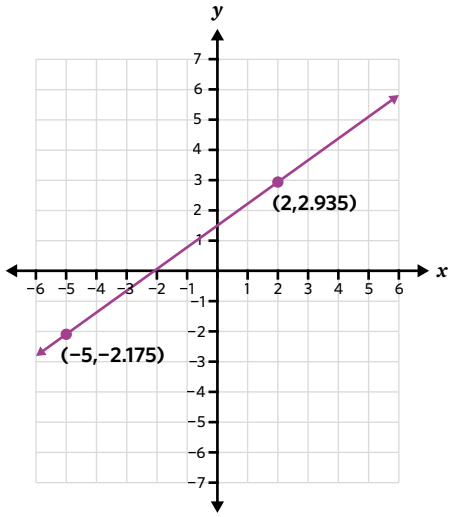
f.



g.



h.



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6. Calculate the gradient using coordinates that are on the same line. WE3

a. $A(1,4)$ and $B(4,13)$

b. $A(0,0)$ and $B(4,16)$

c. $A(4,-6)$ and $B(8,10)$

d. $A(-9,7)$ and $B(0,-2)$

e. $A(-3,-3)$ and $B(5,-5)$

f. $A(-2,1)$ and $B(3,-1)$

g. $A(-8,11)$ and $B(-2,9)$

h. $A(-5.5,4)$ and $B(2,-3.5)$

---

7. What are the rise and run values for this line?

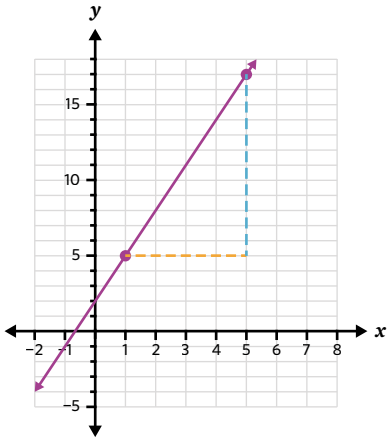
A. Rise = 12, run = 4

B. Rise = 4, run = 12

C. Rise = 17, run = 5

D. Rise = 5, run = 17

E. Positive gradient



Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. Calculate the gradient using coordinates that are on the same line.

$A(0,4)$ and $B(5,19)$

Student A

$A(0,4)$ and $B(5,19)$

Gradient $= 19 - 4$

Gradient $= 15$

Student B

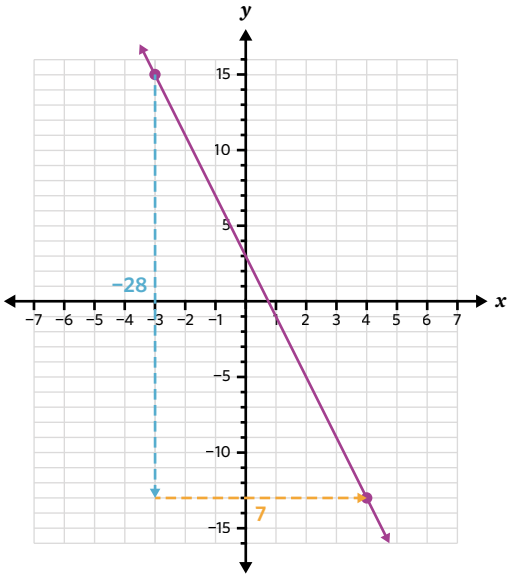
$A(0,4)$ and $B(5,19)$

Gradient $= \frac{19 - 4}{5 - 0}$

Gradient $= \frac{15}{5}$

Gradient $= 3$

b. Calculate the gradient using the coordinates on the graph.



Student A

Gradient $= \frac{-28}{7}$

Gradient $= -4$

Student B

Gradient $= \frac{7}{-28}$

Gradient $= -\frac{1}{4}$

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. Antonia plays Overcooked on her new Nintendo Switch. For every hour she plays she does 30 star jumps. After three hours, Antonia has done 90 star jumps. State the gradient for this scenario.

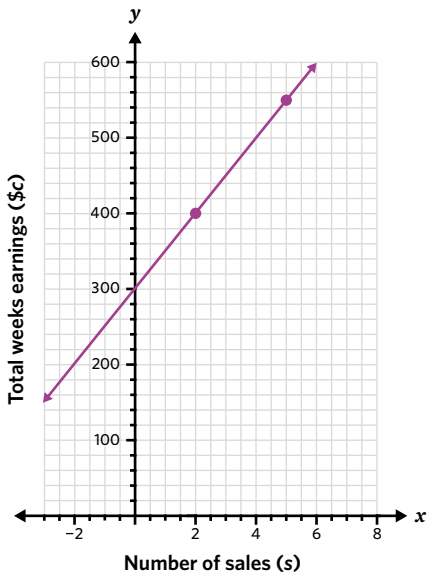
10. Duyen has a Frank Green reusable cup. Duyen fills up her coffee and leaves it open. The coffee has an initial temperature of 90°C. The temperature decreases at a constant rate. For every one minute, it decreases by 2°C. Using the table of values, calculate the gradient.

Time (minutes)	1	2	3	4	5
Temperature	88	86	84	82	80

11. Matthew rode his bike to work and each week he measured his average time. His initial average time was 30 minutes. After week four he has an average time of 18 minutes. Calculate the gradient for Matthew's average riding time.

12. Emilia needs to get her washing machine serviced. The technician has estimated that it will take 1–2 hours to service. For one hour they charge $75 and for 2 hours they charge $125. Find the gradient for this scenario and determine if it is positive or negative.

13. Henry is a car salesperson. The following graph displays Henry's sales data. Using the gradient, determine how much Henry earns per sale.

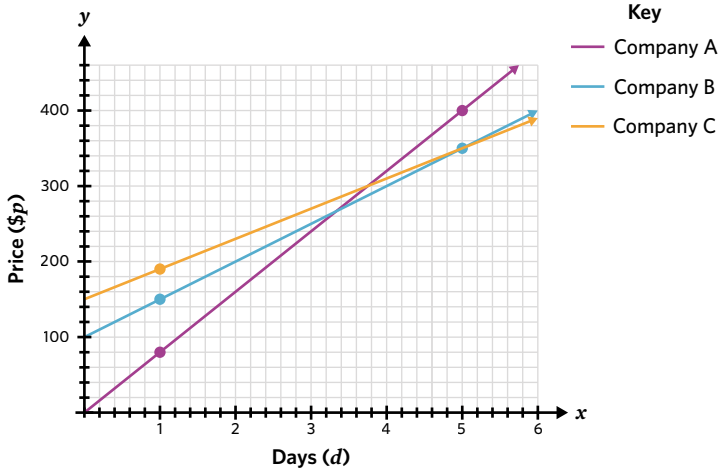


Reasoning

Question working paths

Mild 14 (a,b,d)	Medium 14 (a,b,d), 15 (a,b)	Spicy All

14. Bronwen is organising her trip to Queensland. She wants to hire a car and has called four different car rental companies. All four companies have a fee that they charge per day. They may also have a one off flat rate fee. The information for three companies are displayed in the graph below:



a. Calculate the gradient for Company A.

b. Company D is not displayed on the graph, however it has a gradient value that is exactly halfway between the amount of company A and C. What is the daily rate for Company D?

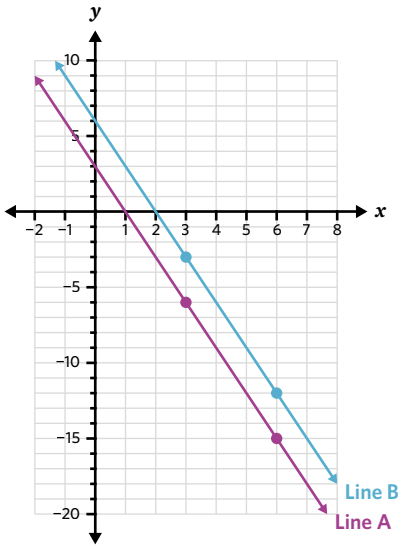
c. Using the graph, prove that the gradients for Company B and C are not the same for the first two days.

d. Outline some other alternative options Bronwen can consider for her trip in Queensland?

15. In part **a** and **b** calculate the gradient using the two coordinates given.

a. Line A

b. Line B



c. Compare the gradient values for lines A and B. Note the similarities and differences when calculating the gradient.

Extra spicy

16. A linear rule described as $y = 0$, passes through the origin, and has a gradient defined as:

A. No solution **B.** Origin **C.** Undefined **D.** y-intercept **E.** Zero

17. Find the value of a for the line passing through $(-2,5)$ and $(2,a)$ and has a gradient of -0.5.

18. A certain linear rule has coordinates in fraction form as (x,y).

$\left(\frac{1}{6},-\frac{1}{4}\right),\left(\frac{1}{3},-\frac{1}{6}\right)$

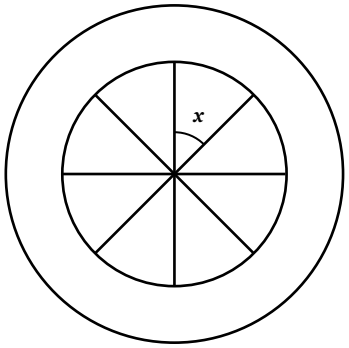
The gradient is:

A. 0 **B.** 0.5 **C.** 1.2 **D.** 2 **E.** Undefined

19. Find the gradient of the line passing through the points $(4x,x)$ and $(6x,5x)$.

Remember this?

20. Luke notices his dad's car tyre is in the following shape.



The tyre is divided into eight equal parts, what is the size of angle x?

A. 15° **B.** 20° **C.** 30° **D.** 45° **E.** 60°

21. Jimmy draws a random card from a deck of 52 cards. Which of the following is Jimmy most likely to draw out?

A. A queen
B. A diamond
C. A queen that is not a diamond
D. A diamond that is not a queen
E. Not a queen

22. Daniel is walking to his school campsite. He was heading north when he started. After 10 minutes of walking, he turned 90° to the left. After a following 20 minutes of walking, he turned 180°. Which direction is Daniel facing when he arrives at the campsite?

A. North **B.** South **C.** East **D.** West **E.** North-west

9E Gradient-intercept form

A straight line can be written as an equation, also known as a linear equation. This equation is often written in the gradient-intercept form where the equation contains information about both the gradient and the y-intercept.

LEARNING INTENTIONS

Students will be able to:

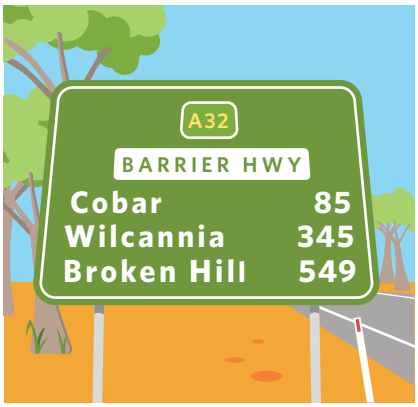
- state the gradient and y-intercept from a straight line in the form $y = mx + c$
- determine the rule/equation of a straight line from a graph.

KEY TERMS AND DEFINITIONS

The ***y*-intercept** of a straight line is where a line crosses the y-axis of a graph and when x is equal to 0.

The ***x*-intercept** of a straight line is where a line crosses the x-axis of a graph and when y is equal to 0.

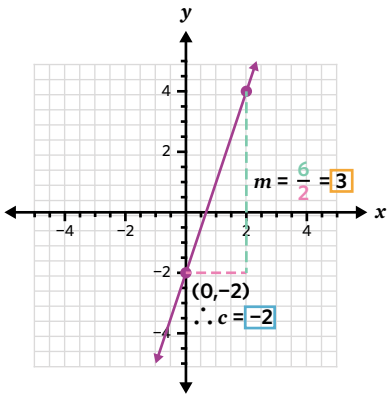
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Driving at a constant speed and determining how much petrol you've used after travelling a certain distance, in kilometres, can be described using a linear equation in the gradient-intercept form.

Key ideas

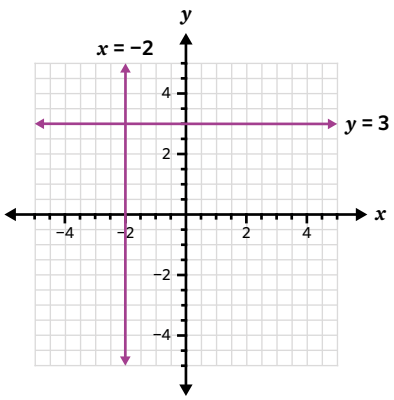
1. The gradient-intercept form is written as $y = mx + c$, where m is the gradient, c is the y-intercept and y is the subject.



$\therefore y = mx + c$ is:

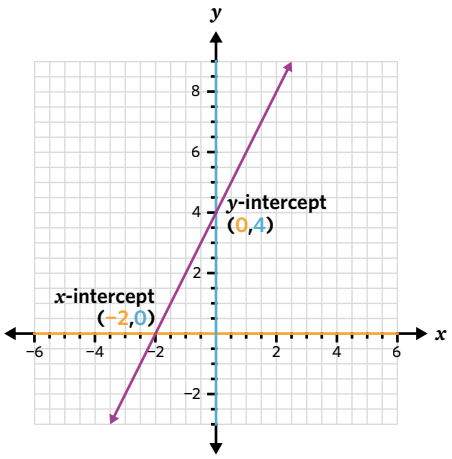
$y = 3x - 2$

2. A horizontal line has the rule $y = c$, as the gradient is equal to 0. A vertical line has the rule $x = a$.



Continues →

3. The x-intercept is where the line crosses the x-axis and will always have a y-coordinate that is 0. The y-intercept is where the line that crosses the y-axis will always have a x-coordinate that is 0.



Worked example 1

Identifying the gradient and y-intercept from a rule

State the gradient and y-intercept for the following equations.

a. $y = -2x + 4$ WE1a

Working

$y = -2x + 4$ ✓

-2 is the gradient (m).

4 is the y-intercept (c).

Thinking

Step 1: Confirm that the equation is in $y = mx + c$ form.

Step 2: Identify the coefficient of x, known as the gradient. This is the number that is being multiplied by x.

Step 3: Identify the constant (c), known as the y-intercept.

Visual support

$$y = \boxed{-2}x \boxed{+ 4}$$

b. $y = -3 + 4x$ WE1b

Working

$y = -3 + 4x$ is the same as $y = 4x - 3$. ✓

4 is the gradient (m).

-3 is the y-intercept (c).

Thinking

Step 1: Confirm that the equation is in $y = mx + c$ form.

Step 2: Identify the coefficient of x, known as the gradient (m). This is the number that is being multiplied by x.

Step 3: Identify the constant (c), known as the y-intercept.

Student practice

State the gradient and y-intercept for the following equations.

a. $y = -4x + 2$ **b.** $y = -3 + 3x$

Back to contents

Worked example 2

Generating lines using the equation in gradient-intercept form

Generate the lines using the equation in gradient-intercept form.

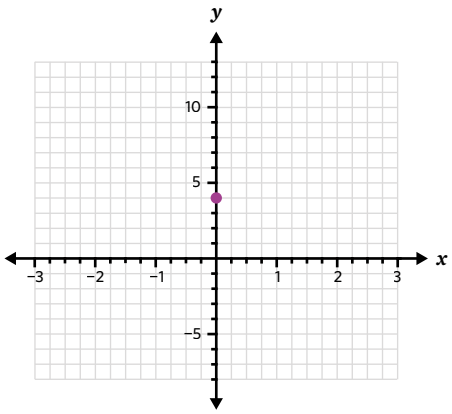
a. $y = -2x + 4$ WE2a

Working

$y = -2x + 4$ ✓

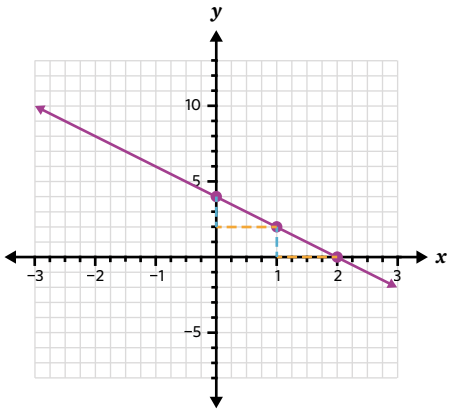
4 is the y-intercept (c).

$\therefore (0,4)$



Gradient is negative.

$\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$



Thinking

Step 1: Confirm that the equation is in $y = mx + c$ form.

Step 2: Identify the coordinates for the y-intercept by referring to the constant (c).

Step 3: Plot the coordinate for the y-intercept.

Step 4: Identify if the gradient is positive or negative. This is determined by the sign in front of the coefficient of x and/or determining if the y-value increases or decreases for each unit increase in x.

Step 5: Using the $\frac{\text{rise}}{\text{run}}$ and the gradient value, plot any two coordinates of the line.

Continues →

b. $2y = x + 3$ WE2b

Working

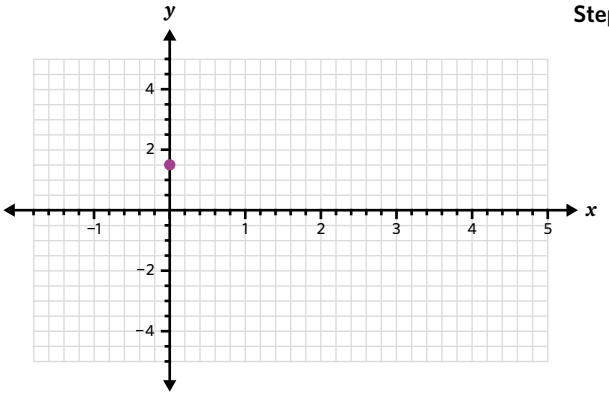
$2y = x + 3$ is not in gradient intercept form.

$\frac{2y}{2} = \frac{x}{2} + \frac{3}{2}$

$y = \frac{x}{2} + \frac{3}{2}$ ✓

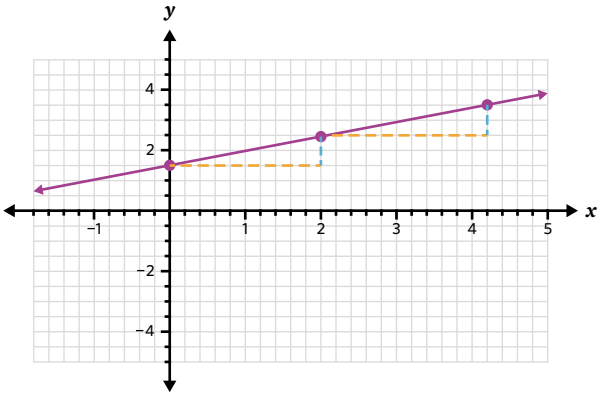
$\frac{3}{2}$ is the y-intercept (c).

$\therefore \left(0, \frac{3}{2}\right)$



Gradient is positive.

$\frac{\text{rise}}{\text{run}} = \frac{x}{2} = \frac{1}{2}$



Thinking

Step 1: Confirm that the equation is in $y = mx + c$ form.

Step 2: Rearrange equation in $y = mx + c$ form.

Step 3: Identify the coordinates for the y-intercept by referring to the constant (c).

Step 4: Plot the coordinate for the y-intercept.

Step 5: Identify if the gradient is positive or negative. This is determined by the sign in front of the coefficient of x and/or determining if the y-value increases or decreases for each unit increase in x.

Step 6: Using the $\frac{\text{rise}}{\text{run}}$ and the gradient value, plot any two coordinates of the line.

Student practice

Generate the lines using the equation in gradient-intercept form.

a. $y = -4x + 2$ **b.** $y = -3 + 3x$

Back to contents

Worked example 3

Identifying the x- and y-intercepts

Calculate the x- and y-intercepts using the equation in gradient-intercept form.

a. $y = 3x + 2$ WE3a

Working

$y = 3x + 2$ ✓

2 is the y-intercept (c).

$0 = 3x + 2$

$x = -\frac{2}{3}$

$\therefore \left(-\frac{2}{3}, 0\right)$

Thinking

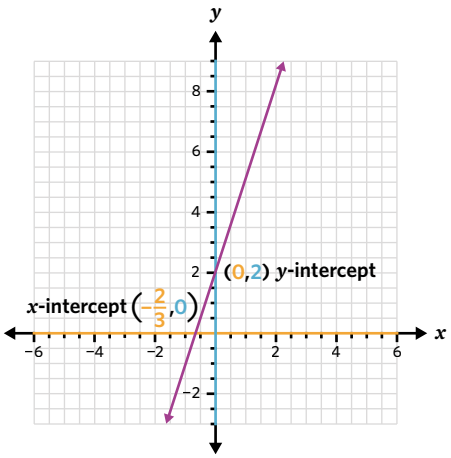
Step 1: To identify the y-intercept, confirm that the equation is in $y = mx + c$ form.

Step 2: Identify the constant (c), known as the y-intercept.

Step 3: To identify the x-intercept, make $y = 0$.

Step 4: Solve for x to determine the x-intercept.

Visual support



b. $y = -4 + 2x$ WE3b

Working

$y = -4 + 2x$ is the same as $y = 2x - 4$. ✓

-4 is the y-intercept (c).

$0 = -4 + 2x$

$x = 2$

$\therefore (2,0)$

Thinking

Step 1: To identify the y-intercept, confirm that the equation is in $y = mx + c$ form.

Step 2: Identify the constant (c), known as the y-intercept.

Step 3: To identify the x-intercept, make $y = 0$.

Step 4: Solve for x to determine the x-intercept.

Student practice

Calculate the x and y-intercepts using the equation in gradient-intercept form.

a. $y = 2x + 5$ **b.** $y = -3 + 4x$

9E Questions

Understanding worksheet

1. Place a circle around the gradient and a square around the y-intercept.

Example

$y = \textcircled{2}x \boxed{+ 1}$

a. $y = 3x + 1$ **b.** $y = -2x + 4$ **c.** $y = \frac{1}{2}x + 3$ **d.** $y = -\frac{1}{3}x - 4$

2. Match the gradient-intercept equation to its rearranged equation.

Example

Gradient-intercept form		Rearranged equations
$y = 3x + 2$	● —	● $y - 2 = 3x$

	Gradient-intercept form		Rearranged equations
a.	$y = 4x - 1$	●	● $x = \frac{-3y}{2} + \frac{3}{2}$
b.	$y = -2x + 4$	●	● $y + 3 = 3x$
c.	$y = 3x - 3$	●	● $x = -\frac{y}{2} + 2$
d.	$y = -\frac{2}{3}x + 1$	●	● $y - 4x = -1$

3. Fill in the blanks by using the words provided.

y-intercept | $y = mx + c$ | gradient | rearranged

The gradient-intercept rule is in the form of _____, where the c represents the _____. The coefficient of x, also known as the _____ is denoted as m.

The gradient-intercept form is in the form whereby y is the subject. In order for the gradient and y-intercept to be stated the equation may need to be _____ in order for these values to be stated.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (b,c,d,e), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (c,d,e,f), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. State the gradient and y-intercept for the following equations. WE1a

a. $y = 3x + 1$ **b.** $y = 5x - 1$ **c.** $y = -4x - 2$ **d.** $y = \frac{2}{3}x + 1$

e. $y = -\frac{1}{2}x + 3$ **f.** $y = \frac{1}{4}x$ **g.** $y = -x + 2$ **h.** $y = 2$

5. State the gradient and y-intercept for the following equations. WE1b

a. $y = 2 + 2x$ **b.** $y = -3 + 4x$ **c.** $y = 2 - 2x$ **d.** $y = 2 + \frac{1}{2}x$

e. $y = -3 - \frac{3}{4}x$ **f.** $y = -\frac{1}{2}x$ **g.** $y = -3 - x$ **h.** $y = -4$

---

6. Generate the lines using the equation in gradient-intercept form. WE2a

a. $y = 3x + 2$ **b.** $y = 4x - 3$ **c.** $y = -5x + 6$ **d.** $y = -2x - 9$

e. $y = \frac{1}{2}x - 5$ **f.** $y = 2x$ **g.** $y = \frac{3}{4}x$ **h.** $y = 4$

---

7. Generate the lines using the equation in gradient-intercept form. WE2b

a. $x + y = -15$ **b.** $2x + y = -11$ **c.** $3y = 4x - 27$ **d.** $x - 4y = 8$

e. $8x - 4y = 20$ **f.** $2x + 3y = 12$ **g.** $0.8x + 0.4y = 1.2$ **h.** $y - 8 = -\frac{1}{2}(x + 4)$

---

8. Calculate the x and y-intercepts using the equation in gradient-intercept form. WE3

a. $y = x - 1$ **b.** $y = 4x + 5$ **c.** $y = -5x + 2$ **d.** $y = -2x - 3$

e. $y = \frac{3}{2}x + 3$ **f.** $y = \frac{x}{9} + 5$ **g.** $y = \frac{-x}{2} + 3$ **h.** $y = \frac{-7x}{5} + \frac{42}{5}$

---

9. Rearrange the following equation to the gradient-intercept form.

$-3x - 4y = 2$

A. $y = \frac{1}{2} - \frac{3}{4}x$ **B.** $y = -\frac{1}{2} + \frac{3}{4}x$ **C.** $y = \frac{1}{2} + \frac{3}{4}x$ **D.** $y = -\frac{1}{2} - \frac{3}{4}x$ **E.** Not possible

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the x and y-intercepts using the equation in gradient-intercept form.

$x = -2y - 6$

Student A

To identify the *y*-intercept

-6 is the y-intercept (c)

$\therefore (-6,0)$

To identify the *x*-intercept

$0 = -2y - 6$

$0 + 6 = -2y - 6 + 6$

$6 = -2y$

$\frac{6}{-2} = \frac{-2y}{-2}$

$-3 = y$

$\therefore (0,-3)$

Student B

To identify the *y*-intercept

$x = -2y - 6 \Rightarrow y = \frac{1}{2}x - 3$

-3 is the y-intercept (c)

$\therefore (0,-3)$

To identify the *x*-intercept

$y = \frac{1}{2}x - 3$

$0 = \frac{1}{2}x - 3$

$x = -6$

$\therefore (-6,0)$

b. State the gradient and the y-intercept from the following equation.

$3y - 6x = 9$

Student A

$3y - 6x = 9$

$3y - 6x + 6x = 9 + 6x$

$3y = 9 + 6x$

$\frac{3y}{3} = \frac{9}{3} + \frac{6x}{3}$

$y = 3 + 2x$

Gradient $= 2$

y-intercept $= 3$

Student B

$3y - 6x = 9$

$3y - 6x + 6x = 9 + 6x$

$3y = 9 + 6x$

$\frac{3y}{3} = \frac{9}{3} + \frac{6x}{3}$

$y = 3 + 2x$

Gradient $= 3$

y-intercept $= 2$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Vaike went on a walk and the rule linking her distance (d) and time (t) in the gradient-intercept form is $d = 4t$. State the gradient and y-intercept of the rule.

12. Stefan is making a BBQ for the footy finals. His BBQ gas bottle starts with 9 kg of gas. Gas is used at a constant rate of 0.5 kg per hour. Generate an equation in the gradient intercept form to display Stefan's gas bottle in terms of mass of gas (G) and (t) time, per hour.

13. Bonnie is doing her maths homework and has been asked the following question: 'If the x-intercept of a line is positive and the y-intercept is negative, does the line have a positive or negative gradient?' What should Bonnie's answer to this be?

14. Tasha is eating an ice cream. The ice cream starts at a height of 15 cm. The ice cream is melting at a constant rate of 2.5 cm/minute. Generate an equation in the gradient-intercept form to represent this scenario.

15. A cyclist rides at a constant rate of 29 km per hour. The equation in gradient-intercept form can be written as $D = 29t$, where D represents total distance, in km and t represents time, in hours. Using the context of the question, explain why the y-intercept is equal to zero.

Reasoning

Question working paths

Mild 16 (a,b,d)	Medium 16 (a,b,d), 17 (a,b)	Spicy All

16. The value of new cars depreciates with time. The value (V), in dollars of a new car depreciates with time (t), in years. The car was initially purchased for $23 500 and the car depreciates by $1250 every year.

a. What was the value of the car when it was new? What is the mathematical name for this?

b. Construct an equation in the gradient-intercept form, in terms of V and t.

c. Interpret what the value of the gradient (m) means in terms of the variables.

d. State two other examples where the value of an item will depreciate over time.

17. Rearrange the following equations in the gradient-intercept form:

- **a.** $2y + 6x = 10$
- **b.** $3y + 9x = 15$
- **c.** Using the equations in part **a** and **b**, comment on the similarities and differences, before and after rearranging the equations.

Extra spicy

18. A line cuts through the x-axis when $x = -\frac{3}{5}$ and the slope of the line is -10. Generate the equation, in the gradient-intercept form.

19. Determine the equation, in the gradient-intercept form when a line has a gradient of -2 and an x-intercept of 2.

20. The rule $-3x + 2y = 20$, has a x-intercept of:

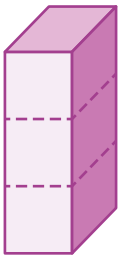
A. $-6\frac{2}{3}$ **B.** -3 **C.** $\frac{3}{2}$ **D.** 0 **E.** 3

21. $2(x + 1) + 3(y + 1) = 5$ transposed into the gradient-intercept form is equal to:

- **A.** $y = -\frac{2}{3}x$
- **B.** $y = 3 - 2x$
- **C.** $y = 21 + 6x$
- **D.** $y = 2x + 7$
- **E.** None of the above

Remember this?

22. A rectangular prism is cut into three identical cubes. Given that the surface area of each cube is 6 cm^2, what is the surface area of the rectangular prism?



- **A.** 18 cm^2
- **B.** 9 cm^2
- **C.** 8 cm^2
- **D.** 14 cm^2
- **E.** 36 cm^2

23. Three students, Kathy, Zoe and Bela, are running a 42 km marathon together. After an hour, Kathy has run 9.1 km, Zoe has run 7.9 km and Bela has run 10.2 km. Who is the closest towards finishing the marathon after an hour?

- **A.** Kathy
- **B.** Zoe
- **C.** Bela
- **D.** All of them
- **E.** Not enough information

24. The base of a triangle is 6 cm, and the height of the triangle is 5 cm. Two identical triangles are put together to make a parallelogram. What is the base : height ratio of the parallelogram?

A. 5 : 12 **B.** 12 : 10 **C.** 5 : 10 **D.** 6 : 5 **E.** 5 : 6

9F Solving linear equations graphically

A straight line shows the linear relationship between the x- and y-coordinate(s). A straight line on a Cartesian plane can be used to solve an equation graphically. Using the y-axis from a Cartesian plane and the straight line allows us to solve for x. The point where two straight lines cross over is referred to as the point of intersection. The point of intersection is a shared point between the two lines.

LEARNING INTENTIONS

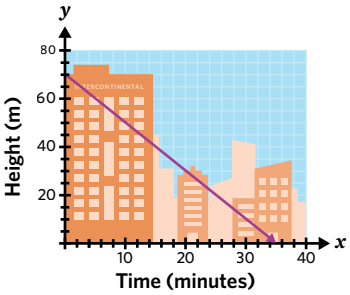
Students will be able to:

- determine the value of an unknown using graphical techniques
- apply graphical techniques to solve real world problems.

KEY TERMS AND DEFINITIONS

A **true equation** is when the left-hand side and right-hand side of an equation are equal.

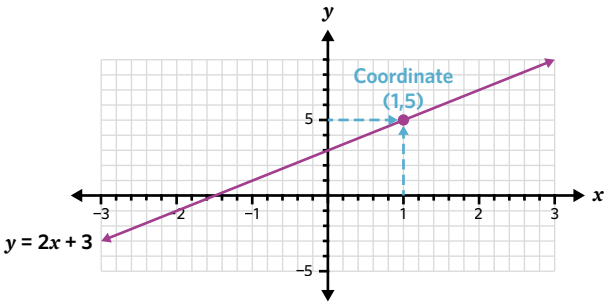
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Anna participated in the Adelaide City Plunge, a 70 metre abseil down the Adelaide Intercontinental Hotel. The graph below shows the height and time as she abseils down. The graph also shows that after 20 minutes Anna was 30 metres from the ground.

Key ideas

1. Solving equations graphically involves finding the x-coordinate on the graph of a straight line corresponding to a given y-coordinate. A point (x,y) on the line makes the equation true (LHS = RHS).



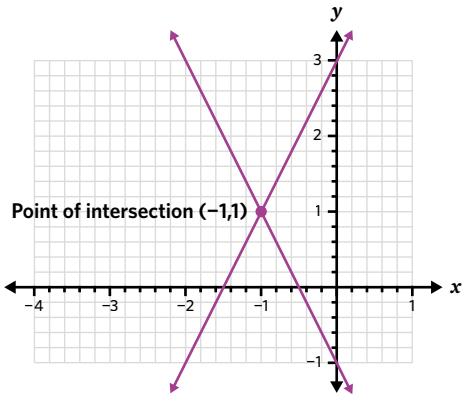
Equation
$2x + 3 = 5$

Solution
$x = 1$

LHS = RHS
$2 \times 1 + 3 = 5$

The point (1,5) on the line shows that when $2x + 3 = 5$, the solution is $x = 1$.

2. Point of intersection is when two lines intersect. Where the two lines intersect shows the only solution that satisfies both equations.



In this graph, $(-1,1)$ is the only point that makes both $y = 2x + 3$ and $y = -2x + 1$ true.

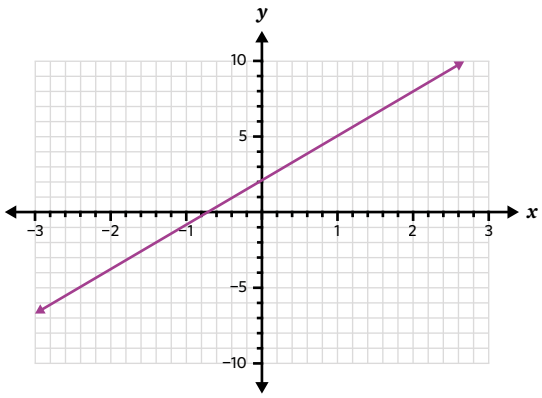
Back to contents

Worked example 1

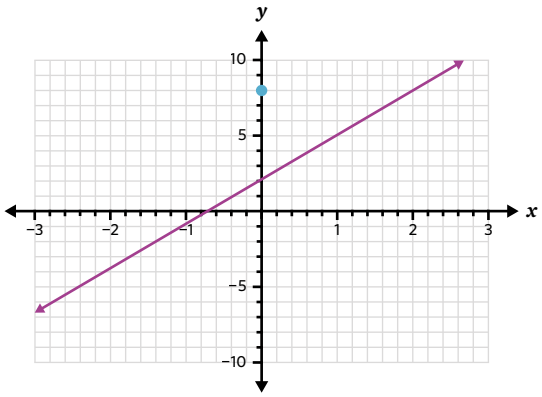
Solving equations using a linear graph

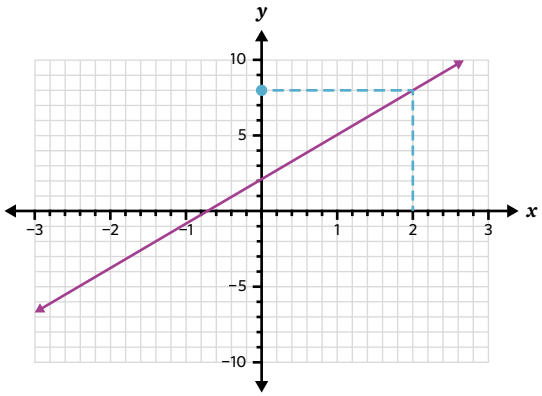
Use the given graph to solve the following equations.

a. $3x + 2 = 8$ WE1a



Working





$x = 2$

$3 \times 2 + 2 = 8$ ✓

Thinking

Step 1: Using the graph, locate the given y-coordinate point on the graph.

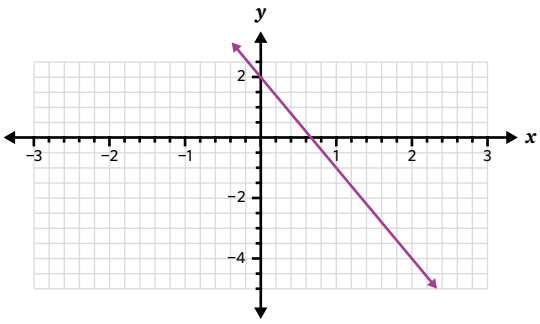
Step 2: Using the given y-coordinate, locate the x-coordinate for the given linear graph.

Step 3: State the solution by referring to the x-coordinate.

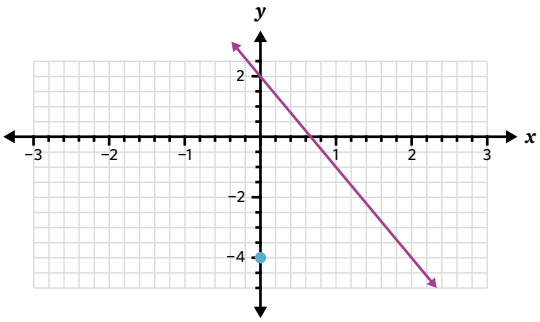
Step 4: Check to see if the LHS equals the RHS.

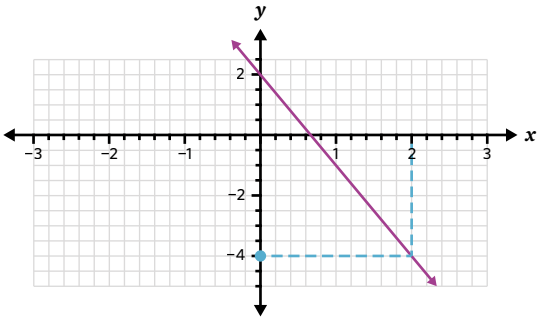
Continues →

b. $-3x + 2 = -4$ WE1b



Working





$x = 2$

$-3 \times 2 + 2 = -4$ ✓

Thinking

Step 1: Using the graph, locate the given y-coordinate point on the graph.

Step 2: Using the given y-coordinate, locate the x-coordinate for the given linear graph.

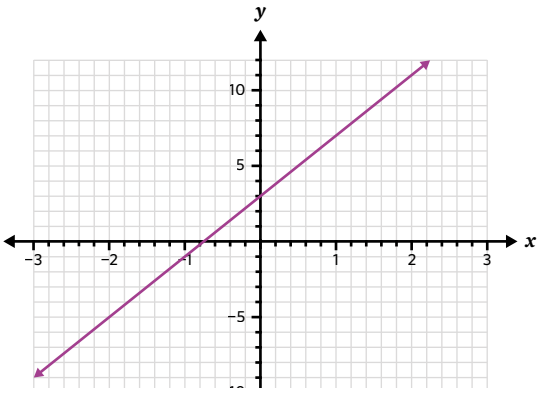
Step 3: State the solution by referring to the x-coordinate.

Step 4: Check to see if the LHS equals the RHS.

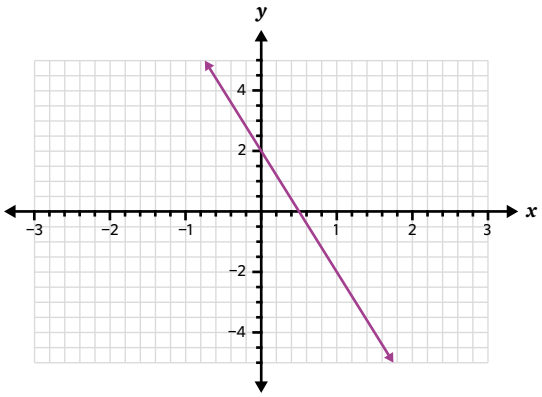
Student practice

Use the given graph to solve the following equations.

a. $4x + 3 = 11$



b. $-4x + 2 = -2$



Back to contents

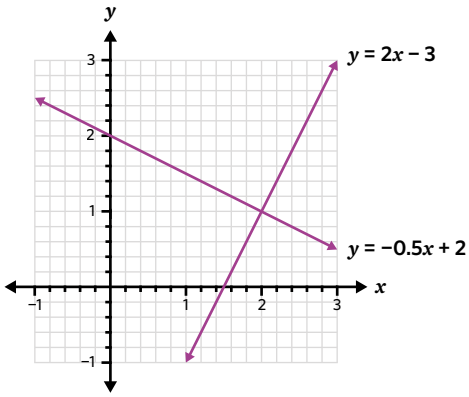
Worked example 2

Solving pairs of equations using a linear graph

Generate the lines using the gradient-intercept form or a digital technology of your choice to solve the following pairs of equations.

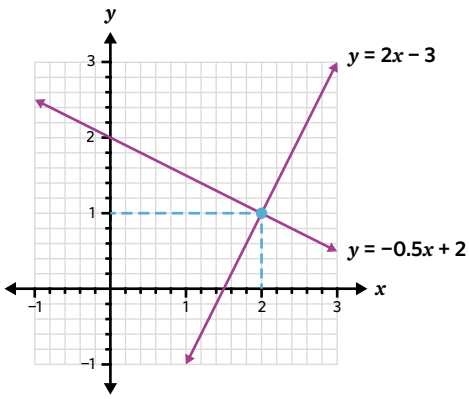
a. $y = 2x - 3$ and $y = -0.5x + 2$ WE2a

Working



Thinking

Step 1: Plot and label both of the graphs on the same set of axes.



Step 2: Locate the point of intersection. This is where the two lines cross over.

Point of intersection (2,1)

$x = 2$ and $y = 1$

Step 3: State the solution.

$1 = 2 \times 2 - 3$ ✓

$1 = -0.5 \times 2 + 2$ ✓

Step 4: Check to see if the LHS equals the RHS.

b. $y - 3 = 3x$ and $x = -\frac{2}{3}y$ WE2b

Working

$y - 3 = 3x$

$y = 3x + 3$ ✓

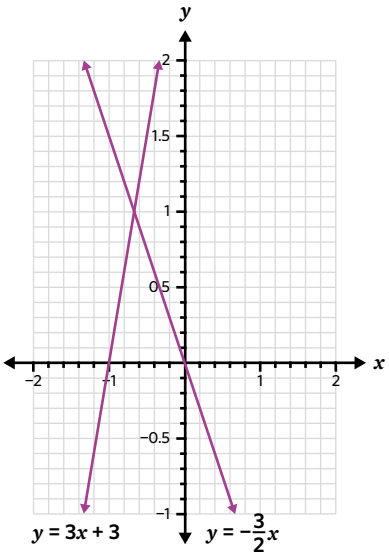
$x = -\frac{2}{3}y$

$y = -\frac{3}{2}x$ ✓

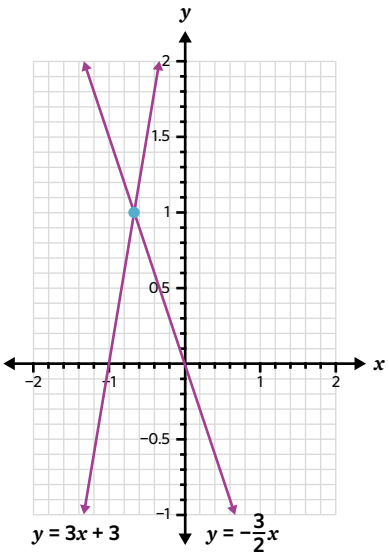
Thinking

Step 1: Rearrange the equations into $y = mx + c$ form.

Continues →



Step 2: Plot and label both of the graphs on the same set of axes.



Step 3: Locate the point of intersection. This is where the two lines cross over.

Point of intersection $(-0.667, 1)$

$x = -0.667$ and $y = 1$

Step 4: State the solution.

$1 = 3 \times (-0.667) + 3$ ✓

$1 = -\frac{3}{2} \times -0.667$ ✓

Step 5: Check to see if the LHS equals the RHS.

Student practice

Generate the lines using the gradient-intercept form or a digital technology of your choice to solve the following pairs of equations.

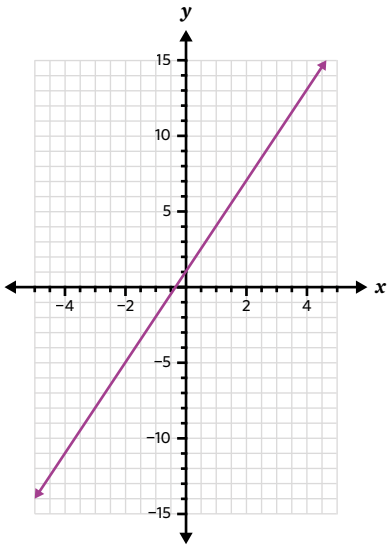
a. $y = 2x + 3$ and $y = -0.75x + 3$

b. $y - 5 = 2x$ and $x = -2y$

9F Questions

Understanding worksheet

1. Using the following graph, state the x-coordinate.



Example

$3x + 1 = 4$

$\therefore$ (1 ,4)

a. $3x + 1 = 7$

$\therefore$ (,7)

b. $3x + 1 = 10$

$\therefore$ (,10)

c. $3x + 1 = -5$

$\therefore$ (,−5)

d. $3x + 1 = -12.5$

$\therefore$ (,−12.5)

2. Match the equation to a possible solution.

Example

Equations		Possible solutions
$4 = 3x + 1$	● — ●	(1,4)

	Equations		Possible solutions
a.	$15 = 3x + 6$	●	● $x = 0.5$
b.	$5 = 2x - 1$	●	● $x = 3$
c.	$16 = -4x$	●	● (3,5)
d.	$\frac{11}{6} = -\frac{1}{3}x + 2$	●	● (−4,16)

3. Fill in the blanks by using the words provided.

intersection | RHS | straight | shared

To make an equation true the LHS needs to be equal to the _____. The equation for a straight line shows the connection between the x and y coordinates. Two _____ lines that are not parallel will have a point of _____. This is where the two lines have a _____ point.

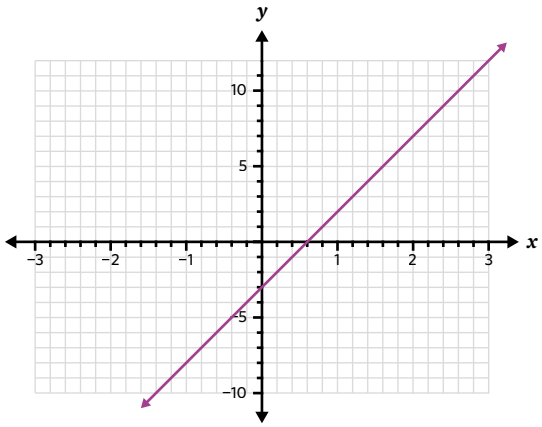
Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (b,c,d,e), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (c,d,e,f), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Use the given graph to solve the following equations. WE1a

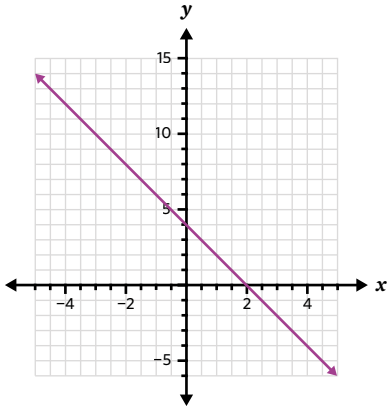
$y = 5x - 3$



a. $5x - 3 = 2$
b. $5x - 3 = 7$
c. $5x - 3 = 12$
d. $5x - 3 = -3$
e. $5x - 3 = 4.5$
f. $5x - 3 = -5.5$
g. $5x - 3 = -7.5$
h. $5x - 3 = 0$

5. Use the given graph to solve the following equations. WE1b

$y = -2x + 4$



a. $-2x + 4 = 2$
b. $-2x + 4 = 0$
c. $-2x + 4 = 3$
d. $-2x + 4 = -2$
e. $-2x + 4 = 1.5$
f. $-2x + 4 = -3$
g. $-2x + 4 = 4.5$
h. $-2x + 4 = -4.5$

6. Generate the lines using the gradient-intercept form or a digital technology of your choice to solve the following pairs of equations. WE2a

a. $y = 3x - 1$ and $y = x + 3$
b. $y = 3x + 1$ and $y = 2x + 3$
c. $y = 2x - 3$ and $y = x - 3$
d. $y = -2x - 1$ and $y = -3x + 3$
e. $y = x + 1$ and $y = -x - 3$
f. $y = x + 5$ and $y = -2x + 9$
g. $y = \frac{3}{2}x + 1$ and $y = \frac{2}{5}x + 2$
h. $y = -2x + 1$ and $y = -2x + 5$

7. Generate the lines using the gradient-intercept form or a digital technology of your choice to solve the following pairs of equations. WE2b

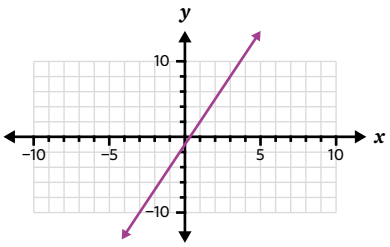
a. $x + y = 10$ and $y = x - 8$
b. $x + 3y = 15$ and $y = x + 1$
c. $x + 4y = 21$ and $y = 6 - x$
d. $2y - x = 12$ and $y = 3 - x$
e. $2y + x = -8$ and $3y - 4x = -1$
f. $x - 6y = 23$ and $8x - 9y = 28$
g. $2x - 5y = 11$ and $2x - 3y = 7$
h. $2x + 3y = 8$ and $3x + 3y = 5$

---

8. Use the given graph to solve the following equation.

$8 = 3x - 1$

A. (3,8)
B. (8,3)
C. (8,23)
D. (23,8)
E. (1.5,8)

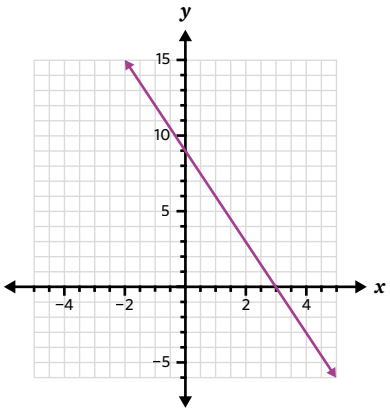


Spot the mistake

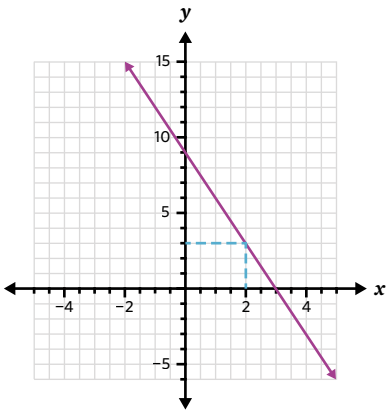
9. Select whether Student A or Student B is incorrect.

a. Use the given graph to solve the following equation.

$3 = -3x + 9$

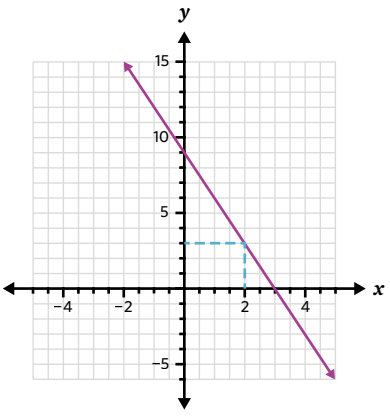


Student A



∴ (2,3)

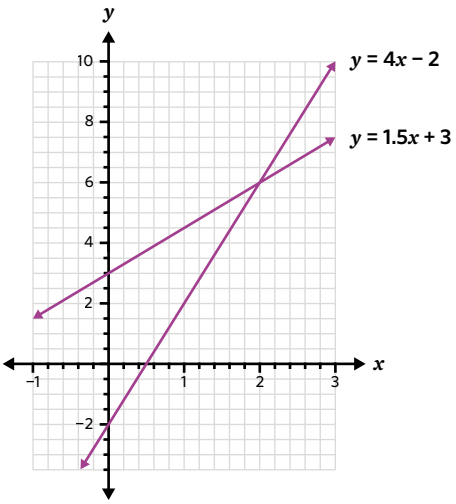
Student B



∴ (3,2)

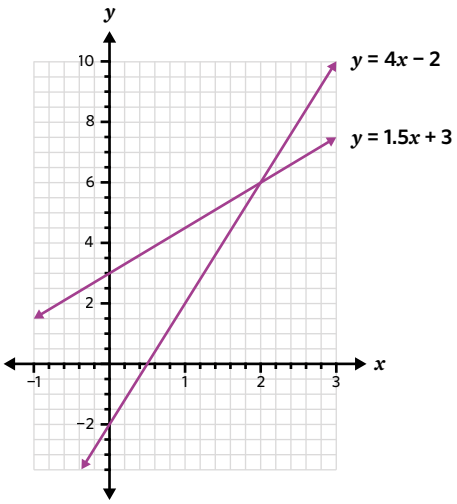
b. Generate the lines using the gradient-intercept form to solve the following pairs of equations.
$y = 4x - 2$ and $y = 1.5x + 3$

Student A



$\therefore (2,6)$

Student B



$\therefore (-2,3)$

Problem solving

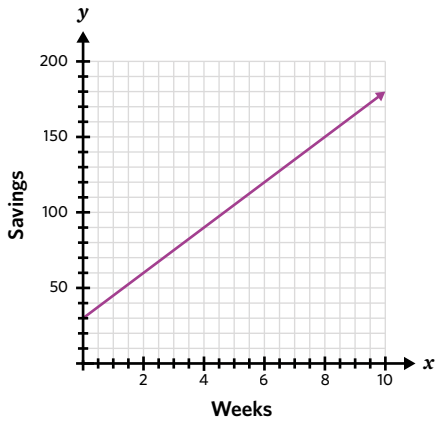
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

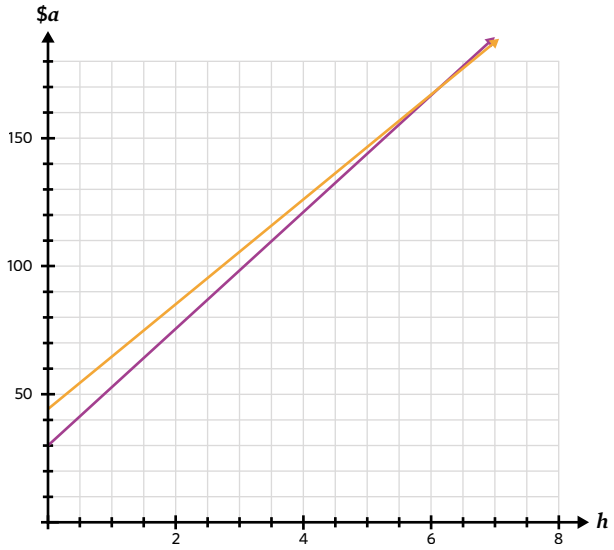
10. Two friends are heading out to dinner. They are both travelling from different locations. They are also travelling at different speeds (km/h). There is a point where their travelling paths cross over. The point at which they meet on the way to dinner is mathematically referred to as what? Explain.

11. Matheus has graphed the equation $y = 3x + 2$ to represent the rate it takes to build a snowman. His friend Olaf has graphed $x = -0.5y + 3.5$ to represent the rate it takes for the snowman to melt. Graphically show the point where the rate of building a snowman equals the rate of the snowman melting.

12. You are saving to go to Ed Sheeran's next music concert. Your savings are displayed in the following graph. You will need to have a total of $150 by a certain date. How many weeks will it take to save enough money to go to the concert?



13. The following graph displays the number of hours worked (h) and the total amount saved ($\$a$) over a month for Mimi and Yuri. Evaluate how many hours it will take both Mimi and Yuri in order for them to have saved the same amount.



14. Aram knows that two lines have a point of intersection at the coordinate $(2,-2)$. The rules for the lines are:

Rule 1: $-x + 2y = -6$

Rule 2: $3x + 2y = 2$

Using the gradient-intercept method or a digital technology of your choice, graph and prove that the solution $(2,-2)$ is true for both lines.

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

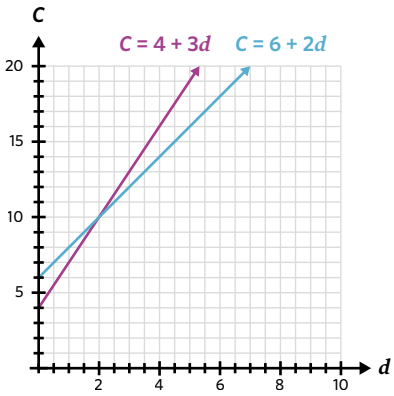
15. Anthony and Simon are going to the Monster Truck show. They both love gingerbread cookies and have decided that instead of eating them every night before bed they are going to put them aside for their upcoming show.

On Monday:

- Anthony starts with 6 and usually has 2 every day.
- Simon starts with 4 and usually has 3 every day.

The two rules to determine how many gingerbread cookies they have after d days is:

$C = 6 + 2d$ and $C = 4 + 3d$



a. Which rule applies to Simon and which to Anthony? Explain.

b. Using the given graph above, how many days does it take Anthony to have 12 cookies?

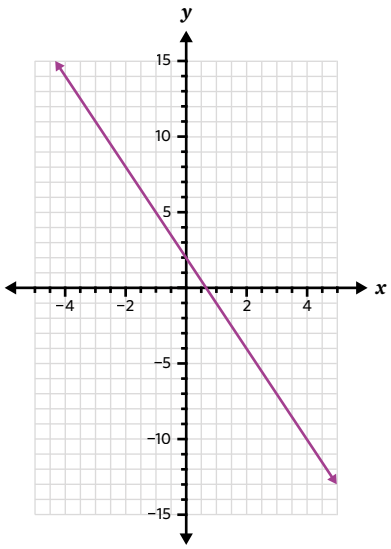
c. Using the given graph above, how many days does it take Simon to have 19 cookies?

d. Using the graph, how many days does it take for Simon and Anthony to have the same amount of cookies?

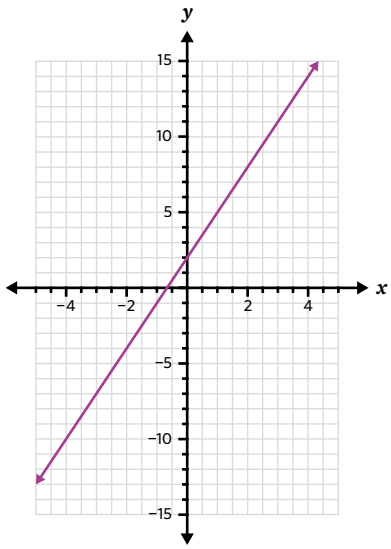
e. Some shows allow you to bring your own snacks, while others do not allow outside food. What are some benefits to being able to bring your own snacks to a show rather than buying only the food provided for the venue?

16. Using the given graphs for both equations in parts **a** and **b**, find the solution to:

a. $10 = -3x + 2$



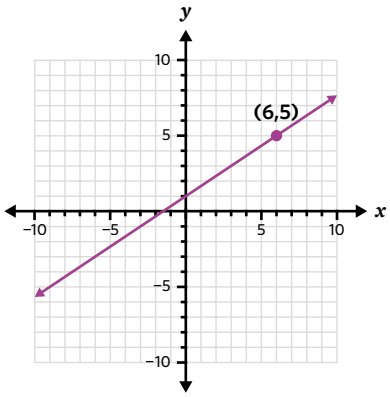
b. $10 = 3x + 2$



c. Compare and contrast the equations and solutions found in parts **a** and **b**.

Extra spicy

17. Use the following graph to find the equation of the line and verify your answer with the given point.



18. Line A passes through the points (1,5) and (3,11) and line B passes through the points (1,2) and (5,18). What is the coordinate for the point of intersection for line A and line B?

A. (5,14) **B.** (4,12) **C.** (3,11) **D.** (5,18) **E.** (4,14)

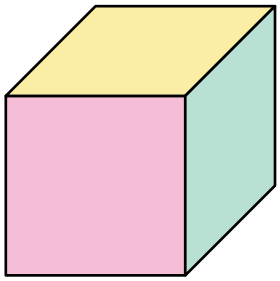
19. On Monday, Maria purchases 5 apples and 3 bananas for $3.90 from Coles. On Tuesday, she purchases 7 apples and 2 bananas which costs $4.25. What is the price for one apple and one banana?

A. Apple = $0.55, banana = $0.45
B. Apple = $0.45, banana = $0.55
C. Apple = $4.25, banana = $3.90
D. Apple = $3.90, banana = $4.25
E. None of the above

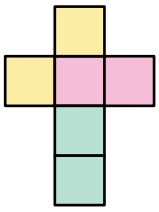
20. Rearrange in the form of $y = mx + c$ and find the solution for x, when $y = -3$ using $3(y - 3) - 2x = 4y + 3 - 5x$.

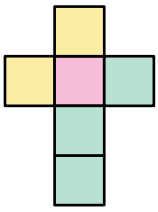
Remember this?

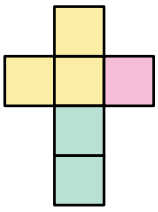
21. Seong-Min folds a net into a cube that is shown below.

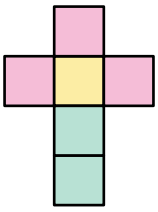


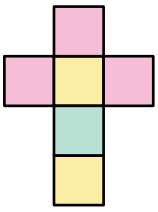
Which of the following are possible nets for the cube?

A. 

B. 

C. 

D. 

E. 

22. A point with coordinates $(7,9)$ is translated to $(-3,-1)$. Which one of the following is the correct description of the translation?

A. 4 units left and 8 units right
B. 4 units right and 8 units down
C. 10 units left and 10 units up
D. 10 units up and 10 units down
E. 10 units left and 10 units down

23. During a freestyle skiing event, Anong made a two and a quarter rotation in the air. By how many degrees did Anong rotate her body in the air?

A. 360° **B.** 405° **C.** 450° **D.** 720° **E.** 810°

9G Non-linear graphs

The relationship between two variables is not always linear, it may also be non-linear, which means that it forms a curve rather than a straight line. A parabola is a type of a non-linear curve that is symmetrical, meaning it is formed by mirroring the same amount of data on each side of the vertex.

LEARNING INTENTIONS

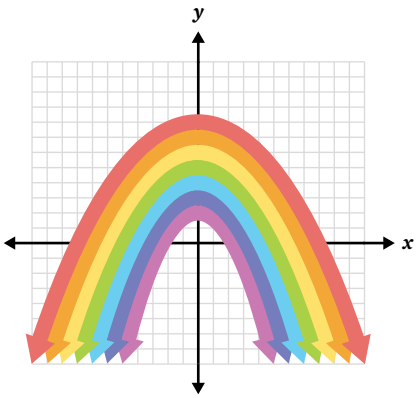
Students will be able to:

- recognise linear and non-linear relationships from a table of values
- identify linear and non-linear relationships from graphs on a Cartesian plane
- plot a non-linear relationship using Cartesian coordinates
- explore non-linear relationships for real life data.

KEY TERMS AND DEFINITIONS

A **parabola** forms an equation where x is raised to the power of 2.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



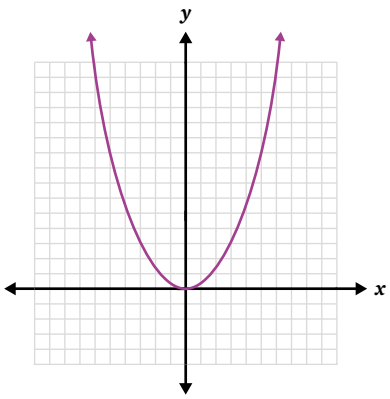
Quite often after it rains or showers, a rainbow is seen. The arc a rainbow makes in the sky creates a curved line. This curved line that a rainbow makes is referred to as a parabola.

Key ideas

1. A parabola generates a curved line and is not linear.

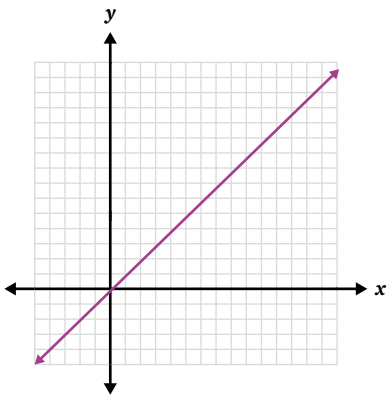
Non-linear (parabola)

$y = 3x^2$



Linear

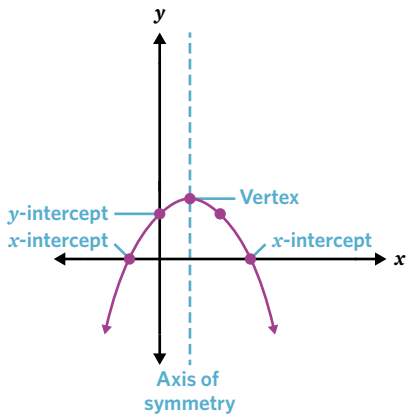
$y = 3x$



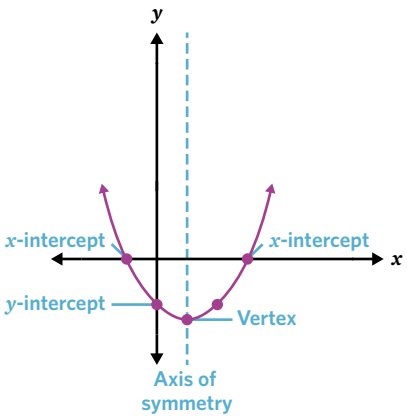
Continues →

2. A parabola has its axis of symmetry at the centre of the curve. The vertex of a parabola is the point on the axis of symmetry. There are two types of parabolas: upward-facing and downward-facing.

Downward-facing



Upward-facing



Worked example 1

Completing a table of values

Complete the table of values with the given rule.

a. $y = x^2 + 4$ WE1a

x	-3	-1	0	1	3
y					

Working

$(-3)^2 + 4 = 13$

$(-1)^2 + 4 = 5$

$0^2 + 4 = 4$

$1^2 + 4 = 5$

$3^2 + 4 = 13$

x	-3	-1	0	1	3
y	13	5	4	5	13

Thinking

Step 1: Substitute each x-coordinate in the table into the rule to determine the y-coordinate.

Step 2: Complete the table of values.

Continues →

b. $y = -x^2 + 3$ WE1b

x	−3	−1	0	1	3
y					

Working

$-(-3)^2 + 3 = -6$

$-(-1)^2 + 3 = 2$

$-(0)^2 + 3 = 3$

$-(1)^2 + 3 = 2$

$-(3)^2 + 3 = -6$

x	−3	−1	0	1	3
y	−6	2	3	2	−6

Thinking

Step 1: Substitute each x-coordinate in the table into the rule to determine the y-coordinate.

Step 2: Complete the table of values.

Student practice

Complete the table of values with the given rule.

a. $y = x^2 + 3$

x	−3	−1	0	1	3
y					

b. $y = -x^2 + 2$

x	−3	−1	0	1	3
y					

Worked example 2

Constructing a graph from a table of values

Using the table of values generated in **WE1**, plot the following graphs.

a. $y = x^2 + 4$ WE2a

x	−3	−1	0	1	3
y	13	5	4	5	13

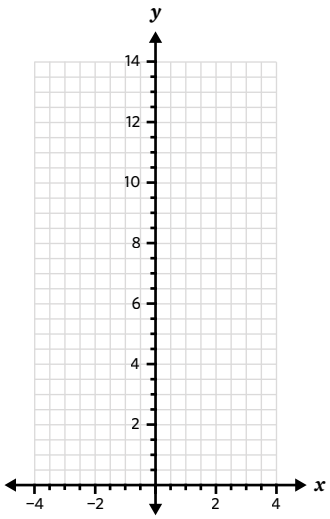
Working

x	−3	−1	0	1	3
y	13	5	4	5	13
Coordinate	(−3,13)	(−1,5)	(0,4)	(1,5)	(3,13)

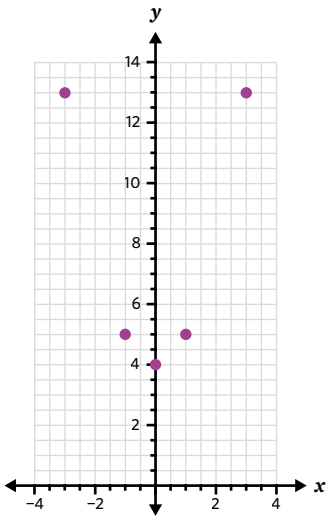
Thinking

Step 1: Determine the coordinates given by the table of values.

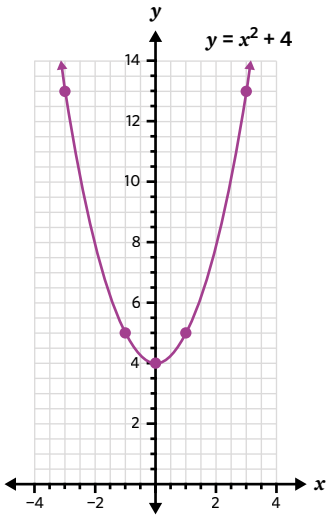
Continues →



Step 2: Draw and label the x and y axes. Identify the smallest and largest values in the table (for both x and y) to determine how to scale the graph.



Step 3: Plot each coordinate.



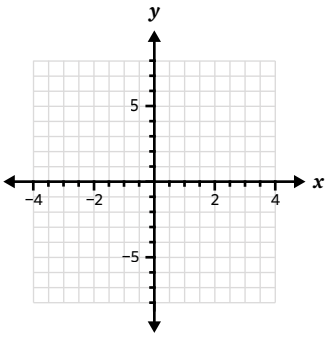
Step 4: Join the points with a smooth curve and label the line with reference to the rule.

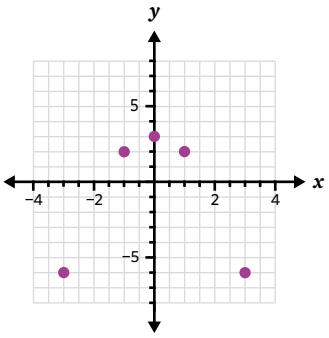
Continues →

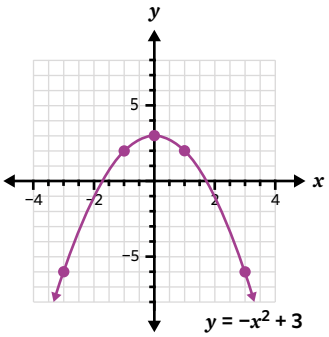
b. $y = -x^2 + 3$ WE2b

Working

x	−3	−1	0	1	3
y	−6	2	3	2	−6
Coordinate	(−3,−6)	(−1,2)	(0,3)	(1,2)	(3,−6)







Thinking

Step 1: Determine the coordinates given by the table of values.

Step 2: Draw and label the x and y axes. Identify the smallest and largest values in the table (for both x and y) to determine how to scale the graph.

Step 3: Plot each coordinate.

Step 4: Join the points with a smooth curve and label the line with reference to the rule.

Student practice

Using the table of values generated in **WE1**, plot the following graphs.

a. $y = x^2 + 3$ **b.** $y = -x^2 + 2$

9G Questions

Understanding worksheet

1. Using the following table, tick whether the equation is linear or non-linear.

Example

Equation	Non-linear	Linear
$y = x^2 + 4$	☑	☐

	Equation	Non-linear	Linear
a.	$y = x^2$	☐	☐
b.	$y = 2x + 2$	☐	☐
c.	$y = -4 - x^2$	☐	☐
d.	$y = x$	☐	☐

2. Using the following values of x, substitute and solve for y when $y = x^2 - 2$ to complete the table of values.

Example

$y = x^2 - 2$

x	3
y	7

x	1	2	1.5	-2
y	**a.**	**b.**	**c.**	**d.**

3. Fill in the blanks by using the words provided.

non-linear | symmetrical | parabola | power

The relationship between two variables is not always a linear association. Such relationships are referred to as _____ relationships. A _____ is a type of non-linear graph in which x is raised to the _____ of 2. A parabola is also _____ on either side of the vertex.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Complete the table of values with the given rule. WE1a

x	-3	-1	0	1	3
y					

a. $y = x^2$ **b.** $y = 3x^2$ **c.** $y = 4x^2$ **d.** $y = 0.25x^2$

e. $y = x^2 + 1$ **f.** $y = x^2 - 0.5$ **g.** $y = x^2 + 3x$ **h.** $y = x^2 - 2x$

---

5. Complete the table of values with the given rule. WE1b

x	-3	-1	0	1	3
y					

a. $y = -x^2$ **b.** $y = -2x^2$ **c.** $y = -5x^2$ **d.** $y = -0.5x^2$

e. $y = -x^2 + 2$ **f.** $y = -x^2 - 4$ **g.** $y = -x^2 + 2x$ **h.** $y = -x^2 - 3x$

---

6. Using the table of values generated in **Q4**, plot the following graphs. WE2a

a. $y = x^2$ **b.** $y = 3x^2$ **c.** $y = 4x^2$ **d.** $y = 0.25x^2$

e. $y = x^2 + 1$ **f.** $y = x^2 - 0.5$ **g.** $y = x^2 + 3x$ **h.** $y = x^2 - 2x$

---

7. Using the table of values generated in **Q5**, plot the following graphs. WE2b

a. $y = -x^2$ **b.** $y = -2x^2$ **c.** $y = -5x^2$ **d.** $y = -0.5x^2$

e. $y = -x^2 + 2$ **f.** $y = -x^2 - 4$ **g.** $y = -x^2 + 2x$ **h.** $y = -x^2 - 3x$

---

8. Using $y = -x^2 + 3$ when $x = 3$, the value for y is:

A. -6
B. 3
C. 6
D. 12
E. None of the above

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Complete the table of values with the given rule.

$y = x^2 - 4$

x	-3	-1	1	3
y				

Student A

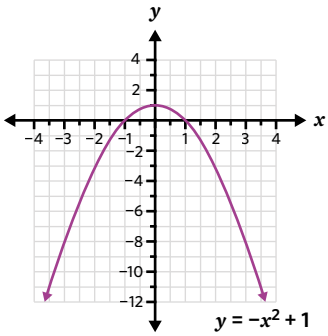
x	-3	-1	1	3
y	5	-3	-3	5

Student B

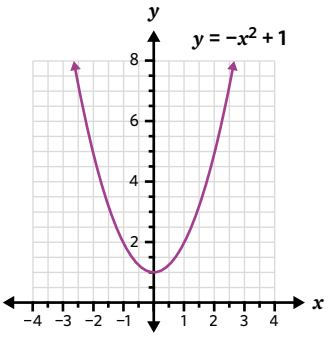
x	-3	-1	1	3
y	-7	-5	-3	-1

b. Construct a graph of $y = -x^2 + 1$ using a table of values.

Student A



Student B

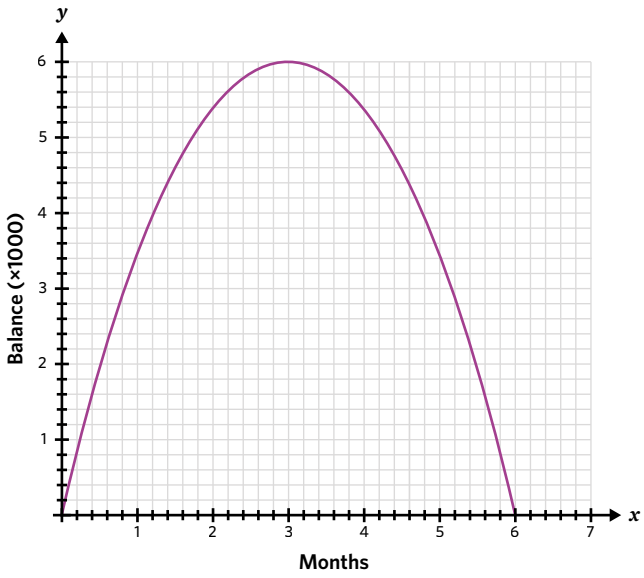


Problem solving

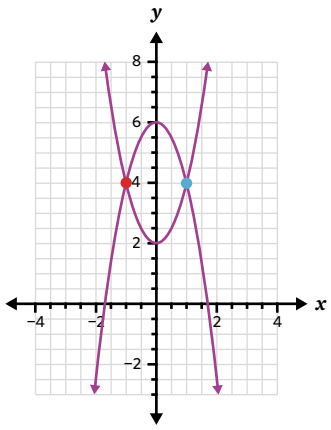
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. The following graph displays Kayla's savings over a certain period within a year. Kayla started saving at the beginning of January. Calculate Kayla's maximum savings amount.



11. Ron and Kyle are running around a local football oval. They are running in opposite directions but at different speeds. During their run, they cross paths twice. The first point of intersection is marked with a red dot. The second point of intersection is marked with a blue dot. State the coordinates for both points of intersection.



12. A tennis ball is dropped from a height of 15 metres. The equation $d = -\frac{5}{3}t^2 + 15$ provides the distance above ground, (d), of the tennis ball, after t seconds. After how many seconds does it take for the ball to hit the ground?

13. Jacob threw a rock into a lake and the equation $h = -5s^2 + 40s + 1.2$ can be used to find the height (h), in centimetres of the rock at any given point. Calculate the height of the rock after 6 seconds (s).

14. Jenny is playing Angry Birds. She has taken a shot that will move her distance away from the point of origin by 2.83 metres, rounded to two decimal places. The new position can be represented as (2.83,0). The non-linear equation that this shot creates is $y = -0.25x^2 + 2$, where x represents the distance in metres and y represents the height in metres. Calculate the distance the bird has travelled when the bird is at a height of 1.4 m, correct to one decimal place.

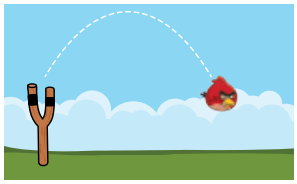


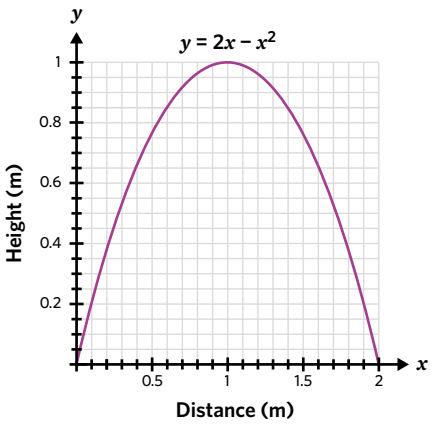
Image: Mahtab Shams/Shutterstock.com

Reasoning

Question working paths

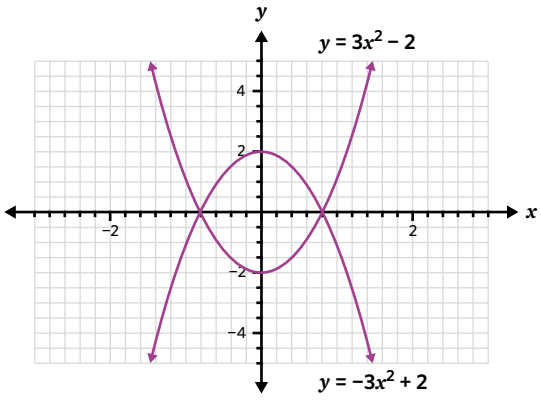
Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Rose's dog loves hunting for birds. Her dog, Arlo, is sitting on the grass and spots a bird. The bird is 2 metres away. Arlo jumps for the bird by taking a parabolic path.



- **a.** State the coordinates for Arlo's starting location.
- **b.** When $x = 2$, calculate the value of y.
- **c.** Determine the maximum height of Arlo's jump?
- **d.** When $x = 1.5$, calculate the value of y and explain what this value represents.
- **e.** From this graph, we can see that the line showing the relationship between distance and height is non-linear. What are some other examples of non-linear relationships?

16. For parts **a** and **b**, generate a table of values where x is between -2 to 2.



- **a.** $y = 3x^2 - 2$.
- **b.** $y = -3x^2 + 2$.
- **c.** With reference to the tables generated in parts **a** and **b**, comment on their similarities and differences.

Extra spicy

17. Solve the pair of equations for $y = 2x - 1$ and $y = x^2 - 4$ by generating a table of values and plotting the graphs.

18. State the x-intercepts of $y = x^2 - 4$.

19. Which point lies on the parabola $y = (x + 3)(x - 2)$?

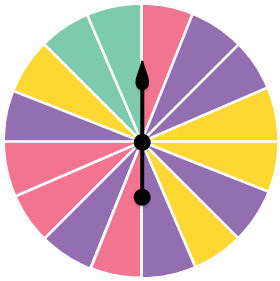
A. (2,4) **B.** (2,0) **C.** (2,5) **D.** (2,20) **E.** (3,−2)

20. From the following equations, which will not generate a parabola?

A. $y^2 + x^2 = 9$
B. $y = x(4 - x)$
C. $y + 9 = (x - 3)^2$
D. $y = (x - 3)(x + 3)$
E. None of the above

Remember this?

21. Niliam spins the spinner below. All sections of the spinner are divided evenly.



What colour is Niliam most likely to spin?

A. Yellow **B.** Purple **C.** Pink **D.** Green **E.** None of them

22. The number of Zhubin's classmates in her English class is 10% more than her Art class. Which of the following could be the number of Zhubin's classmates in her Art class?

A. 21 **B.** 22 **C.** 25 **D.** 27 **E.** 30

23. Aneta goes to school every morning. She takes 10 minutes to walk 900 m to the nearest bus station. She then rides 2.1 km on the bus that takes 14 minutes. She then spends 15 minutes taking a train for 4.095 km to arrive at her school. What is Aneta's average speed for her entire journey in metres per minute, to the nearest whole number?

A. 90 **B.** 150 **C.** 182 **D.** 273 **E.** 363

Chapter 9 extended application

1. Electricity rates are increasing and Dan Andrews has advised all citizens to compare what is out in the market. Rose has found Electabuzz to be the cheapest. The monthly figures for Electabuzz are displayed below.

Electabuzz	
Cost per kWh (k)	\$0.08
Supply charge	\$10.00

a. For the following coordinates, plot and state the quadrant or axis in which each point is located on the Cartesian plane.

(0,10), (20,11.6), (205,26.4), (350,38), (800,74)

b. What is the constant value? Explain what it represents in this scenario.

Another electricity company, Megawatt, charges \$0.12 per kWh, and has a supply charge of \$8.50.

c. With reference to appropriate values, explain whether Electabuzz or Megawatt will produce a steeper line.

d. Construct a table of values for Megawatt, where k is between 0 and 800, with intervals of 100.

e. After how many kWh is Megawatt the more expensive option?

f. With the rising cost of electricity rates, outline two ways you can reduce the electricity usage in your household?

2. Lena and her Mum have decided to travel up to the top of Mount Fuji. They will complete the trip over two days and sleep in a mountain hut after the first day. The following equations and table of values display the times and distances for their hike over the two days.

Day 1: $d = 1.4h$

Day 2: $d = 0.7h + 6.3$

Day 1:

Hours (h)	1	2	3	4	4.5
Distance (d), km	1.4	2.8	4.2	5.6	6.3

Day 2:

Hours (h)	1	2	3	4	5
Distance (d), km	7	7.7	8.4	9.1	9.8

Image: Nackoper/Shutterstock.com

a. Use the data from the table of values to represent the distance covered on day 1 of their trip graphically.

b. With reference to the gradient value in the equation for day 2, name and comment on the type of graph that will be formed.

c. Using the gradient rule, show that the average distance per hour of day 2 is half of day 1.

d. Prove that on the first day Lena and her Mum have travelled 3.85 km after 2 hours and 45 minutes.

e. Explain why the equation for day 2 has a y-intercept of 6.3, whereas the equation for day 1 has a y-intercept equal to zero.

f. When hiking up a mountain, what are some key items you should consider taking with you?

3. Your school has an agriculture program in which they use recycled water for their crops and animals. They have two tanks. Tank one is set to be used at a constant rate of 95 litres per minute. Tank two is set to be used at a constant rate of 75 litres per minute. Tank one has a total capacity of 100 000 litres and tank two has a total capacity of 95 000 litres.

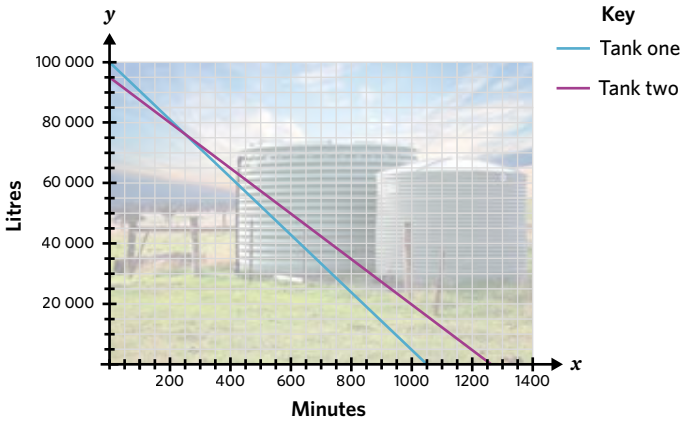


Image: kezza/Shutterstock.com

a. Using the provided information and the graphical display, generate an equation for both tank one and tank two. Represent minutes as x and litres as y.

b. The point of intersection has coordinates (250,76 250), explain what the point of intersection represents in this scenario with reference to the variables.

c. Complete the following table of values for tank one.

Minutes (x)	50	250	700		1050
Litres (y)		76 250		28 750	

d. Calculate how many minutes it will take for tank one to reach zero litres. Define this coordinate.

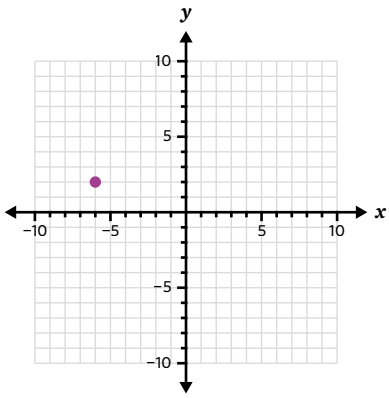
e. With reference to the graph, and starting with $y = -75x + c$, use a set of coordinates on the graph, where $x \neq 0$, to verify that the y-intercept of tank 2 is 95 000.

f. In order to encourage the growth of the crops, outline two changes the agriculture department could implement to help the growth rate.

Chapter 9 review

Multiple choice

1. State the marked coordinate on the following Cartesian plane. 9A

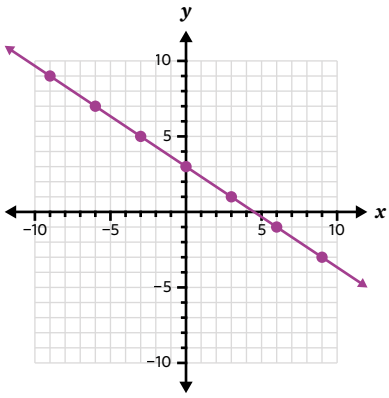


A. $(-6,-2)$ **B.** $(-6,2)$ **C.** $(6,-2)$ **D.** $(6,2)$ **E.** $(-5,2)$

---

2. Which of the following rules correctly describes the table of values and its corresponding graph? 9B, 9C

x	-9	-6	-3	0	3	6	9
y	9	7	5	3	1	-1	-3

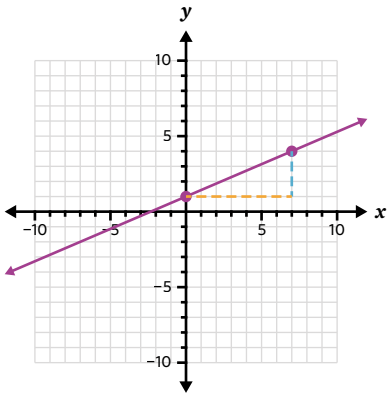


A. $y = -3x$ **B.** $y = -3x + 3$ **C.** $y = \frac{2}{3}x + 3$

D. $y = -\frac{2}{3}x + 3$ **E.** $y = -\frac{2}{3}x - 3$

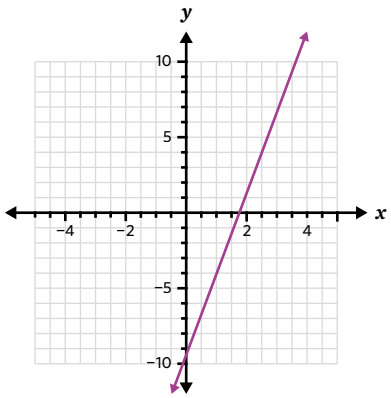
---

3. Determine the gradient using the following graph. 9D



A. $m = \frac{3}{7}$ **B.** $m = \frac{4}{7}$ **C.** $m = \frac{7}{3}$ **D.** $m = 3$ **E.** $m = 7$

4. Use the given graph to solve $5x - 9 = 3$. 9F



A. $x = -2.4$ **B.** $x = -2$ **C.** $x = 1$ **D.** $x = 2$ **E.** $x = 2.4$

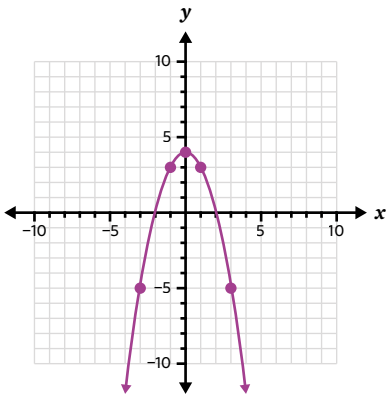
---

5. Which of the following graphs is the correct depiction of the equation and its corresponding table of values? 9G

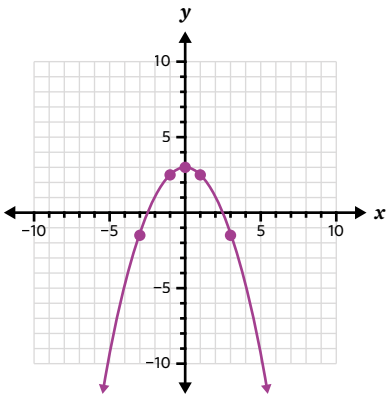
$y = -x^2 + 3$

x	−3	−1	0	1	3
y	−6	2	3	2	−6

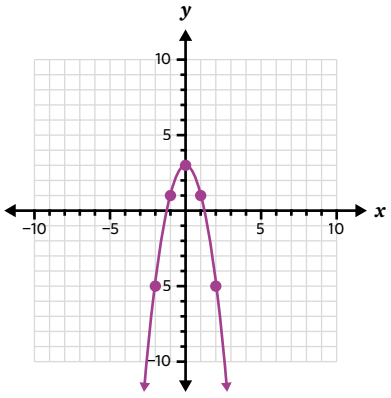
A.



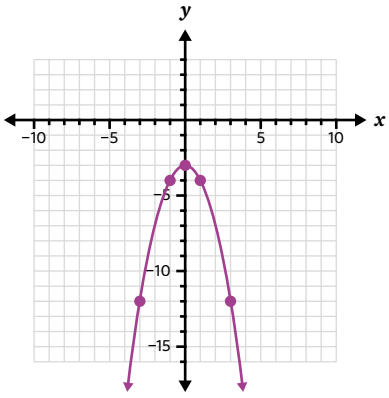
B.



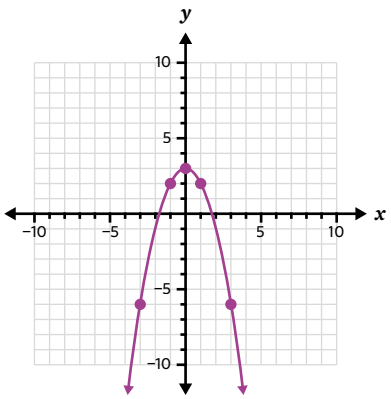
C.



D.



E.



Fluency

6. Plot and state the quadrant or axis in which each point is located on the Cartesian plane. 9A

a. (1,0) **b.** (2,4) **c.** (−1,3) **d.** (−4,−0.5)

7. Complete the table of values with the given rule. 9B

x	−2	−1	0	1	2
y					

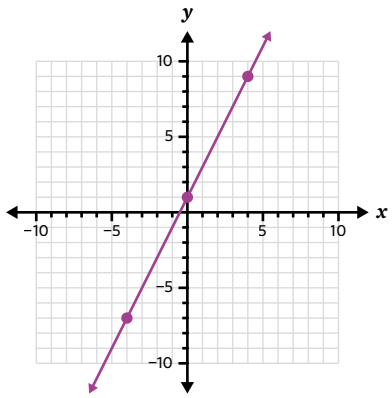
a. $y = 2x + 4$ **b.** $y = -x - 3$ **c.** $y = 3 + \frac{x}{2}$ **d.** $y = 4 - \frac{2}{3}x$

8. Construct a graph for the rule using a table of values where x is between −2 and 2. 9B

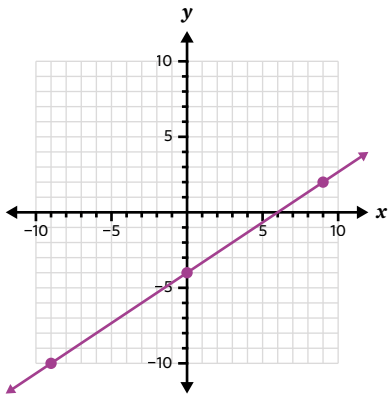
a. $y = 2x$ **b.** $y = 5x + 3$ **c.** $y = -\frac{1}{2} - x$ **d.** $y = -\frac{3}{4}x$

9. Find the rule from the graph. 9C

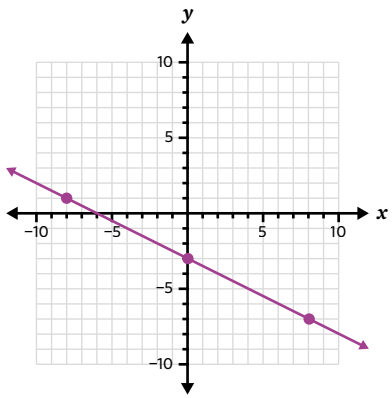
a.



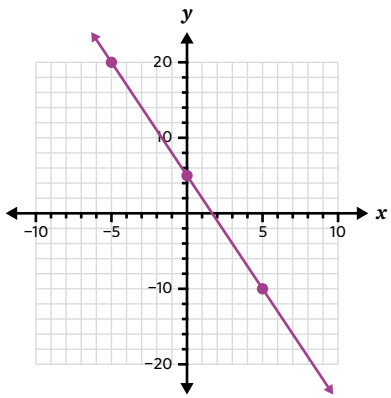
b.



c.

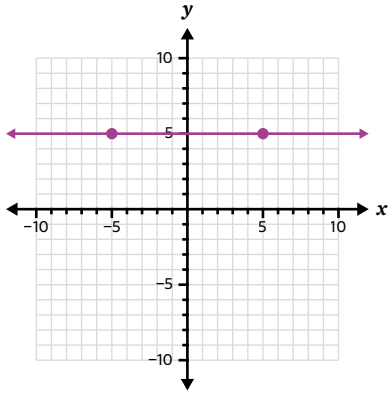


d.

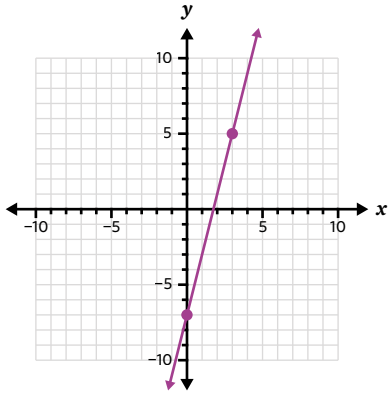


10. Determine the gradients using the graphs. 9D

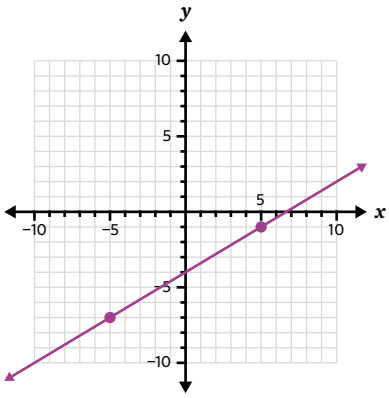
a.



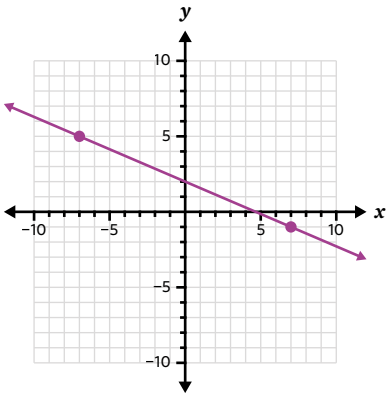
b.



c.



d.



11. Calculate the gradient using coordinates that are on the same line. 9D

a. $A(0,0)$ and $B(2,4)$ **b.** $A(3,5)$ and $B(9,7)$

c. $A(-7,3)$ and $B(0,-4)$ **d.** $A(-3,-4)$ and $B(6,-14)$

12. Generate the lines using the equation in gradient-intercept form. 9E

a. $y = 5x - 9$ **b.** $x + y = -3$

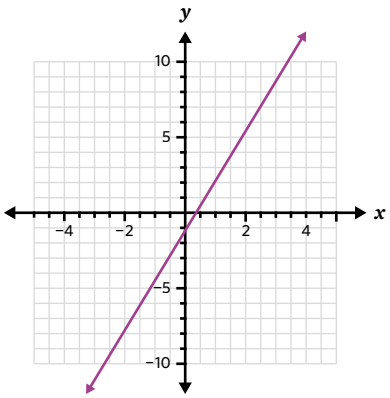
c. $3x + 4y = 5$ **d.** $-y - 9 = \frac{2}{9}(x - 3)$

13. Calculate the x and y intercepts using the equation in gradient-intercept form. 9E

a. $y = 7x - 2$ **b.** $y = -3x + 8$ **c.** $y = \frac{x}{7} - 13$ **d.** $y = 3x + \frac{17}{19}$

14. Use the given graph to solve the following equations. 9F

$y = 3x - 1$



a. $3x - 1 = 5$ **b.** $3x - 1 = -7$ **c.** $3x - 1 = 0.5$ **d.** $3x - 1 = -5.5$

15. Complete the table of values with the given rule. 9G

x	−3	−1	0	1	3
y					

a. $y = 2x^2$ **b.** $y = -3x^2$ **c.** $y = x^2 - 4x$ **d.** $y = -x^2 + 7x$

Problem solving

16. A battleship game is represented on a Cartesian plane. A submarine is located at (7,11). An enemy submarine is located seven units west and five units south. State the ordered coordinates for the enemy submarine. 9A

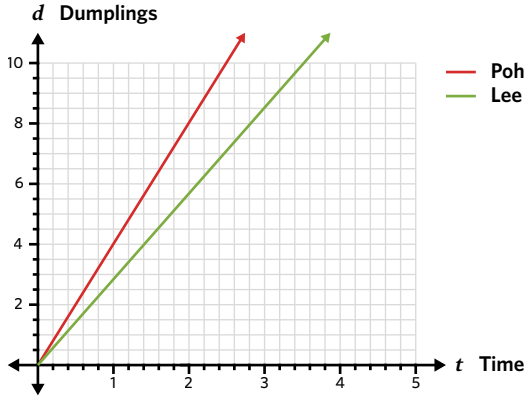
17. A large cup of boba from Ling Ling's Tea costs $6.80. A customer may choose to add up to six toppings, where each topping costs $0.50. Generate a table of values and a graph to display the cost of a cup of boba, C, with any given number of toppings, t, where $0 \leq t \leq 6$. 9B

18. Joey and Jack are keeping track of the velocity, in cm/s, of a projectile in a physics experiment. The following table displays their record of the projectile's instantaneous velocity, v, at a given time, t. 9C

Time (t)	3	6	9
Velocity (v)	42	54	66

Determine v when $t = 0$ and explain what the value represents.

19. Poh and Lee are competing in a dumpling eating competition. The number of dumplings eaten for each of them are displayed in the following graph. Determine the gradient of each line and explain what they represent. 9D



20. A tub of water initially has 5 L of water in it. Additional water is poured into it at a rate of $\frac{1}{5}$ L per second. Generate an equation in the gradient-intercept form to represent the volume of water, represented as V in the tub at a given time, represented as t. 9E

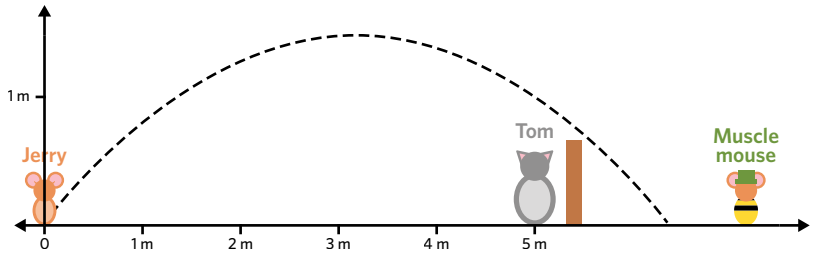
21. The neighbourhood around Khemarak Phoumin Street and Hun Sen Street is represented on a Cartesian plane. The two streets are represented by the following equations: 9F

Khemarak Phoumin Street: $-7x + y = 13$

Hun Sen Street: $-3x + 6y = 0$

Using the gradient-intercept method, graph the two lines to determine where the two boulevards intersect.

22. Jerry is trying to send his cousin, Muscle Mouse, who lives next door, a letter that he had stored in a glass bottle. To send the letter, Jerry has to slingshot the bottle so that it flies over the fence and lands on the other side. Tom, who is 0.7 m tall, is standing 5 m east of Jerry next to a fence. Using the diagram below, determine how high Tom needs to jump in order to catch the bottle. 9G



Reasoning

23. BestOven Inc. manufactures ovens for household use. They have designed a new model and are running tests on a prototype. They set the oven to 200°C and record the internal temperature of the oven after every minute. The initial internal temperature of the oven is the same as room temperature, which can be assumed as 25°C.

a. For the first ten minutes, the internal temperature of the oven rises by 10°C per minute. Generate a table of values that represents this situation.

b. Using the table of values generated in part **a**, plot every second point and determine the rule that describes the resulting line in gradient-intercept form.

c. The person running the test got distracted and came back five minutes later to record that the temperature was now 150°C. Plot this point on the same graph and determine the gradient between this point and the point preceding it. Explain what the gradients found in part **b** and part **c** represent and what the discrepancy in the gradients mean.

d. At this moment, the tester turns the oven off to observe the fall in internal temperature of the oven. The tester concludes that the internal temperature of the oven falls according to the rule $y = -\frac{x^2}{5} + 105$, where y is the internal temperature of the oven, in °C, and x is the time in minutes, after the oven has been turned off. How long does it take for the oven to get back to room temperature?

e. Why is it important to run tests on the internal temperature of an oven?

---

24. Consider the following equation:

$y = mx + 2$.

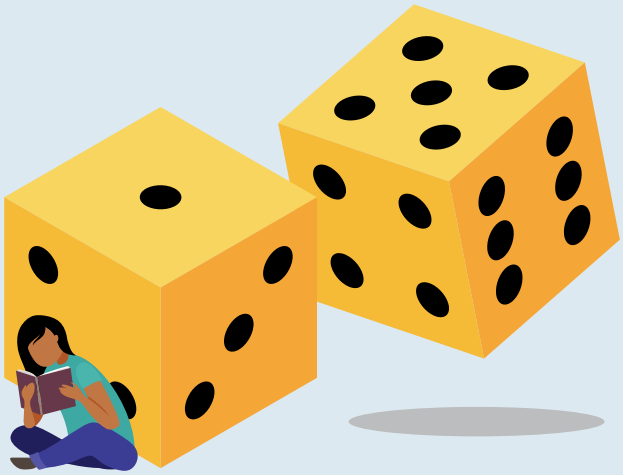
Generate a table of values for $-2 \leq x \leq 2$ and hence plot the graph for the following values of m.

a. $m = 2$

b. $m = 5$

c. Compare the graphs in parts **a** and **b**. How does the size of the gradient affect the slope of a linear line?

Back to contents



10

Probability

Statistics and Probability

Research summary

10A Introduction to probability

10B Complementary events

10C Venn diagrams and two-way tables

10D Tree diagrams and multi-step experiments *(Extension)*

10E Experimental probability

Chapter 10 extended application

Chapter 10 review

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Back to contents

Chapter 10 research summary

Probability

Big ideas

Variation with expectation and randomness

This chapter will focus on students' ability to make reasonable expectations given real-life situations, understand the randomness of events, help develop and apply the idea of the law of large numbers, and challenge their idea, experience and belief with fairness.

Students must have a solid understanding of variation and expectation before linking these ideas to randomness (Watson & Fitzallen, 2019) . Conducting trials such as rolling a 6-sided die is one way of reinforcing the concept of expectation and likelihood. However, variation can sometimes not be learned through this type of experiment and randomness is partially established (Chick, 2018). Classroom discussions are a great way to engage prior knowledge of expectation and variation and connect these topics with mathematics (Watson & Fitzallen, 2022).

Randomness is the central idea for probability and also one of the most complex ideas. The concept of randomness can be a difficult topic for students and teachers to understand and teach (Watson & Fitzallen, 2022; Bantanero 2015; Chick, 2018). For example students sometimes associate that rolling a die is random and equate randomness to fairness. To deepen student's learning about randomness, expectation and variance it is important to implement cognitive conflict for students (Watson & Fitzallen, 2022) For example, use a digital spinner which hides the proportions for students, and ask them to draw the spinner, estimate the percentages, and their confidence level after every 20, 50, 100, 500 spins.

The language of probability can cause many misconceptions for students. For example the words 'events', 'independent', 'dependent', 'and', 'or', and 'both' take on new altered meanings. Some students see success when the words are defined using student-friendly language before they work on probability problems and then re-define the definitions after (Sutherland et al., 2022). While other students see success when they work on probability problems without any definitions beforehand and then learn the definitions later (Kazak et al., 2015; Chick & Baker, 2005).

It is also essential that questions are asked that relate to expectation and variation (Watson & Kelly, 2006). For example, suppose there is a bag of 100 assorted lollies which contain 30 snakes, 20 milk bottles, 10 teeth, 6 pineapples, and the remaining are red frogs. The question '*What is the probability that*…?' is a question that relates to expectation. Questions such as, '*If you pulled a handful of 10 lollies from the bag, how many are milk bottles*?' can promote a discussion about what is likely, and the 'correct' answer enhances student's appreciation for variation and expectation (Shaughnessy & Zawojewski, 1999).

This big idea is built upon a strong understanding of number, number properties, ratios and proportions. Therefore, understanding the connection between number, number properties, equality, and algebra is pivotal for future success in mathematics. Progress in this chapter and lessons such as statistics impact the big ideas of probabilistic and statistical thinking and reasoning.

Variation with informal inference

Informal inference is also a complex idea in probability. There needs to be 'both the expression of an expectation and a level of confidence expressed in the expectation because the result is not known for certain' (Watson & Fitzallen, 2022, p 531).

To develop students' informal inference it is essential that students have an opportunity to create meaning from a sample space they are unfamiliar with (Watson & Fitzallen, 2022). For example, give students a mystery die with an unknown shape and labelling system, and ask them to make predictions about what will happen after 10 tosses, 100 tosses, and 1000 tosses.

It is important to remember that student's beliefs and ideas of probability progress through stages of development. The levels of SOLO taxonomy (Biggs & Collis, 1982) is one way to describe that students start with beliefs and superstitions about probability and are eventually able to use proportional reasoning to estimate and interpret the language of probability.

Variation with distribution and expectation

This chapter will focus on students' ability to interpret, create, and organise data to represent variation that occurs within probability. Students will expand their existing knowledge of tables and tree diagrams to create and interpret two-way tables and Venn diagrams, in order to determine variation within events and experiments.

Distribution of data within various visual representations is a powerful cognitive tool that allows learners to interact with complex ideas. When given data (created or collected), students work with variation, think about the context and model to determine what is appropriate, and draw informal inferences (Pfannkuch et al., 2018). When presented with various distributions using visual representations, students can reason with variation, expectation, randomness, and the law of large numbers (Watson, 2018). For example, slight variations of box plots after 50 tosses of a 6-sided die can highlight key features of the data while avoiding some of the misconceptions when learning about box plots (Watson, 2018).

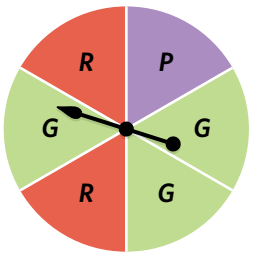
While hands-on manipulatives, and creating visual representations are relevant when students are learning probability, digital technology can aid in the understanding of distribution and expectation (Watson, 2018). For example, the digital tool TinkerPlots can create visual representations of a random experiment. Students can develop an origin story of how the results came to be and create a deeper meaning of the results, without the constraints of having to create graphs or perform the experiment themselves.

This big idea is built upon a strong understanding of number, number properties, and statistical reasoning. Therefore, understanding the connection between probabilistic and statistical reasoning is pivotal for future success in each of these topics. Progress in this chapter and lessons such as statistics impact the big ideas of probability.

Visual representations

Spinners, counters, dice

This chapter will use spinners and counters. Using these visuals and hands-on manipulatives can develop and extend mathematical knowledge (Chick, 2018), however they do have their limitations. Spinners are a great tool for introducing the concept of fairness and sample space, however it may hinder the understanding of bias (Chick, 2018). Two sided counters or coins are a great tool when investigating the real vs fake tossing sequence, however, the appreciation of randomness may be lost (Chick, 2018).



Spinners


Counters

It is important that teachers and students have a strong understanding of how visuals can illustrate certain concepts and how much evidence of these visuals is needed to demonstrate the concept (Chick, 2018).

Two-way tables

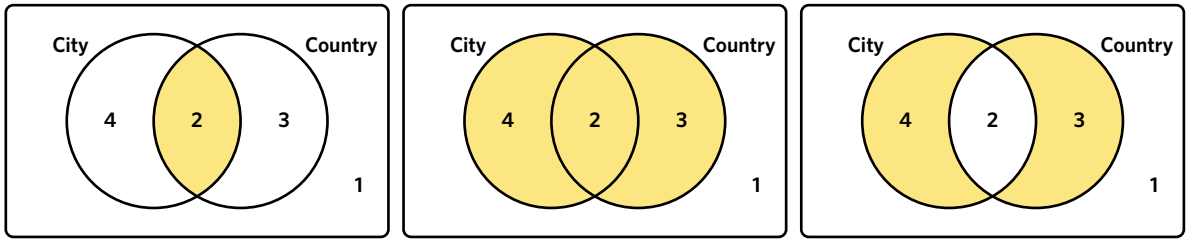
Two way tables, or two-way contingency tables display the frequency (count) of two categories, written in a table format with rows and columns.

Students sometimes struggle with the information in two-way tables by ignoring information from other cells or using an initiative approach (Obersteiner et al., 2015). One strategy that is helpful is making students aware of their initial responses and to slow down and think again before answering (Obersteiner et al., 2015).

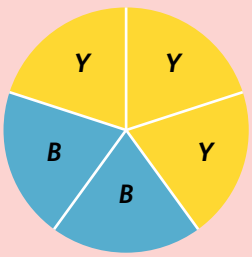
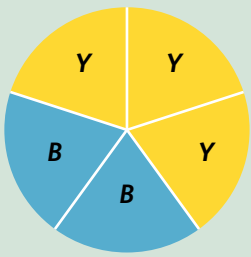
	Like country	**Dislikes country**	**Total**
Likes city	2	4	6
Dislikes city	3	1	4
Total	5	5	10

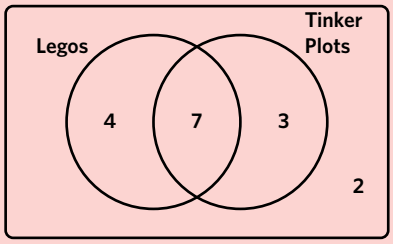
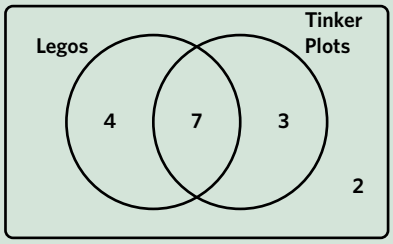
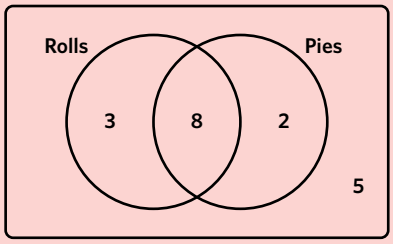
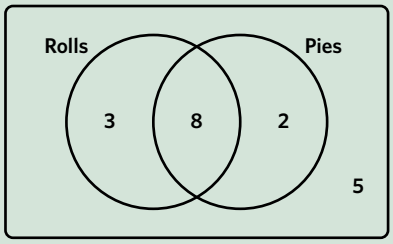
Venn diagrams

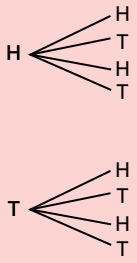
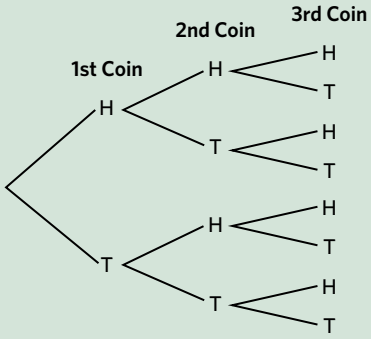
A Venn diagram provides a visual representation of similarities and differences between two or more sets of information. Venn diagrams are essential for connecting the words 'and', 'or' and 'both' in data sets. Venn diagrams are essential for developing probabilistic knowing and set notation.



Misconceptions

Misconception	Incorrect	Correct	Lesson
Students assume that outcomes of an event are equally likely to occur.	$\Pr(\text{grey}) = \frac{2}{5}$ $\Pr(\text{red}) = \frac{2}{5}$ 	$\Pr(\text{grey}) = \frac{2}{5}$ $\Pr(\text{red}) = \frac{3}{5}$ 	10A
Students repeat outcomes when listing the sample space of an experiment.	The sample space of, POOL is: S = {P, O, O, L}	The sample space of, POOL is: S = {P, O, L}	10A
Students calculate based on ratio of outcomes rather than fraction of total outcomes.	$\Pr(\text{blue}) = \frac{3}{2}$ $\Pr(\text{orange}) = \frac{2}{3}$ 	$\Pr(\text{blue}) = \frac{3}{5}$ $\Pr(\text{orange}) = \frac{2}{5}$ 	10A
Students think that the probability of an event not occurring means it might not happen.	The probability of not getting green is equal to 0. 	The probability of not getting green is equal to $\frac{3}{5}$. 	10B
Students assume that complementary events are equally likely to occur	$\Pr(\text{Y}') = \frac{3}{5}$ 	$\Pr(\text{Y}') = \frac{2}{5}$ 	10B

Misconception	Incorrect	Correct	Lesson
Students think the probability of unknown events is the sum of all other probabilities.	$\Pr(\text{green}) = \frac{2}{5}$ $\Pr(\text{yellow}) = \frac{1}{10}$ $\Pr(\text{blue}) = \frac{1}{5}$ $\Pr(\text{purple}) = ?$ $\Pr(\text{purple}) = \frac{2}{5} + \frac{1}{10} + \frac{1}{5}$ $= \frac{4}{10}$	$\Pr(\text{green}) = \frac{2}{5}$ $\Pr(\text{yellow}) = \frac{1}{10}$ $\Pr(\text{blue}) = \frac{1}{5}$ $\Pr(\text{purple}) = ?$ $\Pr(\text{purple}) = 1 - \frac{2}{5} - \frac{1}{10} - \frac{1}{5}$ $= \frac{3}{10}$	10B
Students confuse set A with only A.	 People who use Legos = 4	 People who use Legos = 11	10C
Students believe that the word 'and' requires that they add up all parts of set A and set B rather than the intersection.	 There are 13 people who like rolls and pies.	 There are 8 people who like rolls and pies.	10C
Students think that probability can be more than 1 and give the numerator of a probability only.	<table><tr><th></th><th>Plays sport</th><th>Does not play sport</th><th>Total</th></tr><tr><td>**Plays an instrument**</td><td>12</td><td>7</td><td>19</td></tr><tr><td>**Does not play and instrument**</td><td>15</td><td>8</td><td>23</td></tr><tr><td>**Total**</td><td>27</td><td>15</td><td>42</td></tr></table> $\Pr(\text{plays an instrument}) = 19$	<table><tr><th></th><th>Plays sport</th><th>Does not play sport</th><th>Total</th></tr><tr><td>**Plays an instrument**</td><td>12</td><td>7</td><td>19</td></tr><tr><td>**Does not play and instrument**</td><td>15</td><td>8</td><td>23</td></tr><tr><td>**Total**</td><td>27</td><td>15</td><td>42</td></tr></table> $\Pr(\text{plays an instrument}) = \frac{19}{42}$	10C
Students use their own meaning to understand the language of probability (eg. 'or', 'and', 'at least', etc).	Roland flips a coin and then tosses an 8-sided die. What is the probability of obtaining a head or a number less than 3, or both? $\Pr(\text{head}) = \frac{1}{2}$ $\Pr(\text{number less than 3}) = \frac{2}{8}$ $\Pr(\text{head or number less than 3})$ $= \frac{1}{2} + \frac{2}{8} = \frac{3}{4}$ or $\Pr(\text{head or number less than 3})$ $= \frac{1}{2} + \frac{2}{8} + \frac{2}{16}$ $= \frac{14}{16} = \frac{7}{8}$	Roland flips a coin and then tosses an 8-sided die. What is the probability of obtaining a head or a number less than 3, or both? $\Pr(\text{head}) = \frac{1}{2}$ $\Pr(\text{number less than 3}) = \frac{2}{8} = \frac{1}{4}$ $\Pr(\text{head OR number less than 3})$ $= [\Pr(\text{head}) + \Pr(\text{number less than 3})] - [\Pr(\text{head and less than 3})]$ $= \left[\frac{1}{2} + \frac{1}{4}\right] - \left[\frac{1}{2} \times \frac{1}{4}\right]$ $= \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$	10D

Misconception	Incorrect	Correct	Lesson
Students miss branches in the tree diagram	Draw a tree diagram and list the outcomes of flipping three two-sided coins.  There are 8 possible outcomes.	Draw a tree diagram and list the outcomes of flipping three two-sided coins.  There are 8 possible outcomes.	10D
Students think that the experimental probability will equal the theoretical probability for any number of trials.	Pick a counter out of the bag 15 times. Determine the experimental probability of getting green.  Experimental probability $= \Pr(G) = \frac{3}{5}$	Pick a counter out of the bag 15 times. Determine the experimental probability of getting green.  **Green** | **Blue** | **Number of trials** 5 | 10 | 15 Experimental probability $= \Pr(G) = \frac{5}{15} = \frac{1}{5}$	10D
Students confuse experimental probability with theoretical probability.	The results of flipping a 2-sided coin 15 times are shown below. **Outcome** | **Heads** | **Tails** **Frequency** | 6 | 9 $\Pr(\text{Heads}) = \frac{\text{heads}}{\text{number of sides}} = \frac{1}{2}$	The results of flipping a 2-sided coin 15 times are shown below. **Outcome** | **Heads** | **Tails** **Frequency** | 6 | 9 $\Pr(\text{Heads}) = \frac{\text{frequency of heads}}{\text{total number of trials}} = \frac{6}{15} = \frac{2}{5}$	10E
Students think the expected occurrence equals the frequency of an experiment.	The results of flipping a 2-sided coin 15 times are shown below. **Outcome** | **Heads** | **Tails** **Frequency** | 6 | 9 Expected occurrence of landing on heads after 50 trials? $\Pr(\text{Heads}) = \frac{2}{5} \times 15 \text{ trials} = \frac{30}{5} = 6$	The results of flipping a 2-sided coin 15 times are shown below. **Outcome** | **Heads** | **Tails** **Frequency** | 6 | 9 Expected occurrence of landing on heads after 50 trials? $\Pr(\text{Heads}) = \frac{2}{5} \times 50 \text{ trials} = \frac{100}{5} = 20$	10E

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Back to contents

10A Introduction to probability

In everyday life, predictions are made about whether certain events will occur or not. Mathematics can be used to calculate and compare probabilities to help make those predictions more accurate. Probabilities are commonly stated using percentages, but fractions and decimals can also be used.

LEARNING INTENTIONS

Students will be able to:

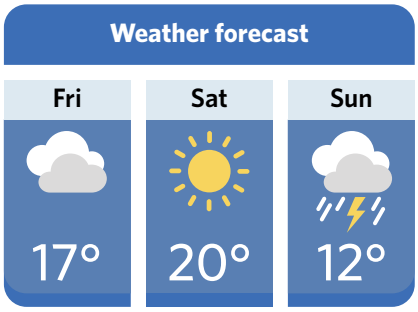
- understand that probabilities range between and including 0 and 1
- list the sample space for a given event
- calculate theoretical probability of events.

KEY TERMS AND DEFINITIONS

A **trial** is a single test in an experiment.

A **compound event** is an event that consists of multiple outcomes.

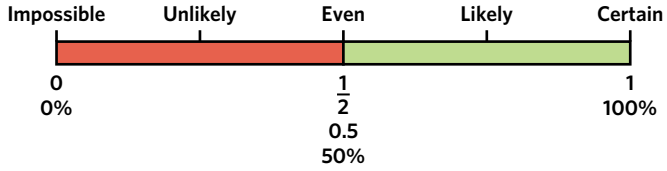
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



There are many events in life that aren't guaranteed to happen, such as whether it will rain on a given day, or how many of each type of meal a restaurant will sell. So, we think of these events in terms of their probability of occurring.

Key ideas

1. The probability of an event refers to how likely the event will occur. It can be represented numerically between 0 and 1 using decimals, fractions and percentages.



2. The terms experiment, outcome, sample space and event are used in probability.

For the counters shown above:

Term	Description	Example
Experiment	A series of trials conducted to examine the results of chance activities.	Picking a counter.
Outcome	A result of a trial.	Picking a green counter. Picking a blue counter.
Sample space	A list of all possible outcomes of an experiment.	S = {blue, green, orange}
Event	A single outcome or a group of outcomes (known as a compound event).	Picking a green or blue counter.

Continues →

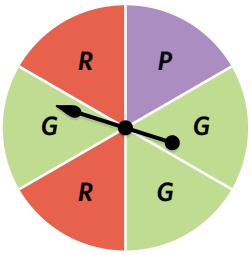
3. The **theoretical probability** states the likelihood of an event occurring in an experiment.

$$\text{Pr(event)} = \frac{\text{number of event outcomes}}{\text{total number of outcomes}}$$

Worked example 1

Applying probabilities

A spinner with coloured segments was constructed, shown below.



Each trial is the spinner being spun, and the outcome is what colour the spinner lands on.

a. List the sample space of the experiment. WE1a

Working

S = {red, green, purple}

Thinking

The sample space lists all the possible outcomes when the spinner is spun. Repeated outcomes are listed once.

b. Calculate Pr(red), the probability of landing on red written as a simplified fraction. WE1b

Working

Number of red segments: 2

Total number of segments: 6

$$\text{Pr(red)} = \frac{\text{number of red segments}}{\text{total number of segments}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Thinking

Step 1: Determine the number of event outcomes and the total number of outcomes.

Step 2: Substitute into the theoretical probability formula and simplify.

c. Calculate Pr(red or green), the probability of landing on red or green. WE1c

Working

Number of red segments: 2

Number of green segments: 3

Total number of segments: 6

$$\text{Pr(red or green)} = \frac{\text{number of red and green segments}}{\text{total number of segments}}$$

$$= \frac{5}{6}$$

Thinking

Step 1: Determine the number of event outcomes and the total number of outcomes.

Step 2: Substitute into the theoretical probability formula and simplify.

Continues →

Student practice

A standard, 6-sided die is rolled and the top face is recorded.

a. List the sample space of the experiment.

b. Calculate Pr(2), the probability of landing a 2, written as a fraction.

c. Calculate Pr(3 or 6), the probability of rolling a 3 or a 6, written as a fraction.

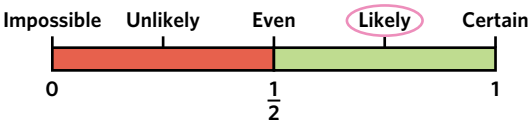
10A Questions

Understanding worksheet

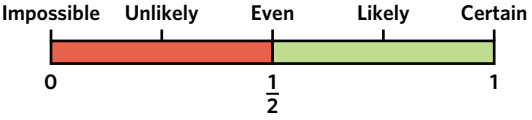
1. Circle the indicator to describe the probability of each event occurring.

Example

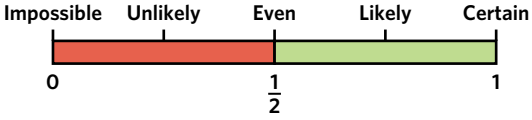
On any day, the temperature will be higher than 20 degrees in summer.



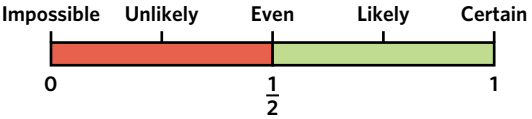
a. There is a 97% chance of a train being on time in the morning.



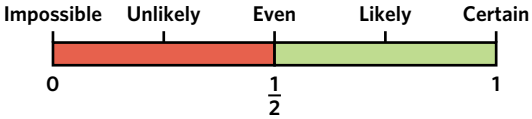
b. Choosing a red marble out of a bag only containing red marbles.



c. Rolling a 2 or 3 on a 6-sided die.



d. Flipping a head on a coin after 5 trials.



2. Tick either true or false for each statement about the coloured counters below.

Example

	True	False
You are equally likely to pick a green counter and a blue counter.	☐	☑

	True	False
a. You are twice as likely to pick a blue counter than a green counter from bucket A.	☐	☐
b. You are 4 times more likely to pick a blue counter than an orange counter.	☐	☐
c. You are less likely to pick a green counter than an orange counter.	☐	☐
d. You are equally likely to pick an orange or green counter, than a blue counter.	☐	☐

3. Fill in the blanks by using the words provided.

outcome | experiment | trial | sample space

An _____ is conducted where the letters in the word MATHS are chosen at random.

The _____ is S = {M, A, T, H, S}. The _____ of 1 _____ could be picking the letter A.

Fluency

Question working paths

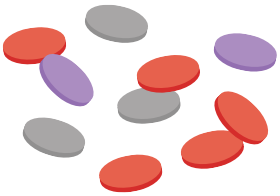
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (b,c,d,e), 5 (b,c,d,e), 6 (b,c,d,e), 7 (b,c,d,e), 8 (b,c,d,e), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9

4. The letters in the word PROBABILITY are randomly chosen in an experiment. WE1

- **a.** List the sample space of the experiment.
- **b.** Calculate Pr(O), the probability of choosing the letter O, written as a simplified fraction.
- **c.** Calculate Pr(I), the probability of choosing the letter I, written as a percentage.
- **d.** True or false: the probability of choosing any letter is equally likely.
- **e.** Calculate Pr(vowel), the probability of choosing a letter that is a vowel, written as a decimal. For the purposes of this question, Y is a consonant.
- **f.** What is an event that is less likely to occur than choosing the letter P?

---

5. The counters below will be picked out at random. WE1



- **a.** List the sample space of the experiment.
- **b.** Calculate Pr(purple), the probability of picking a purple counter, written as a simplified fraction.
- **c.** Calculate Pr(grey), the probability of picking a grey counter, written as a percentage.
- **d.** True or false: the probability of picking a grey counter is more likely than picking a purple counter.
- **e.** Calculate Pr(red or purple), the probability of picking a red or purple counter, written as a decimal.
- **f.** What is an event that is as equally likely to occur as picking a red counter?

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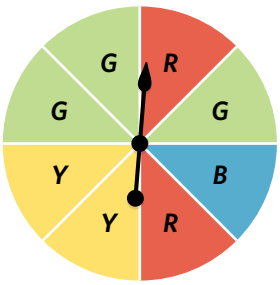
6. A spinner with coloured segments was constructed. Each trial is the spinner being spun, and the outcome is what colour the spinner lands on. WE1



- **a.** List the sample space of the experiment.
- **b.** Calculate Pr(blue), the probability of landing on blue, written as a simplified fraction.
- **c.** Calculate Pr(red), the probability of landing on red, written as a percentage.
- **d.** True or false: landing on red is 2 times as likely as landing on blue.
- **e.** Calculate Pr(green or red), the probability of landing on green or red, written as a decimal.
- **f.** Which event is equally likely to landing on yellow?

---

7. A computer is randomly generating numbers between 1 and 10 (including the numbers 1 and 10). WE1

- **a.** List the sample space of the experiment.
- **b.** Calculate Pr(7), written as a fraction.
- **c.** Calculate Pr(greater than or equal to 4), written as a percentage.

d. True or false: generating a number that is a multiple of 3 is as equally likely as generating a number that is less than 3.

e. Find Pr(prime number), the probability of landing on a prime number, written as a decimal.

f. What is an event that is half as likely to occur as generating a number that is a factor of 8.

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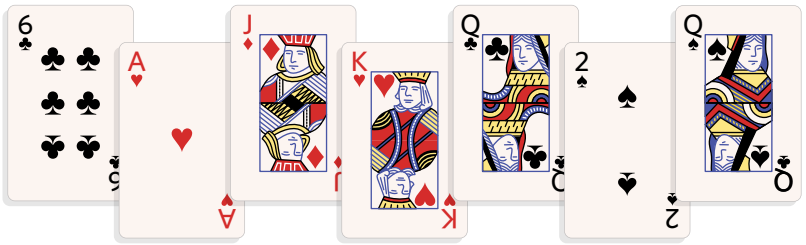
8. Zev is conducting an experiment where he is shuffling the cards below and randomly selecting 1 card. WE1



a. List the sample space of the experiment.

b. Calculate Pr(6), the probability of picking a 6 card, written as a fraction.

c. Calculate Pr(red), the probability of picking a red card, written as a percentage to the nearest whole number.

d. True or false: picking a picture card is less likely than picking a spades (♠) card?

e. Calculate Pr(even and black), the probability of picking a card that is an even number and is black, written as a fraction.

f. What is an event that is more likely than picking a clubs (♣) card?

---

9. A standard, 6-sided die is rolled and the top face recorded. What is the probability of it landing on a 1 or an even number?

A. $\frac{1}{6}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** $\frac{2}{3}$ **E.** $\frac{5}{6}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

A bag contains 3 red marbles, 4 black marbles, 2 green marbles and 1 white marble. A marble is picked out at random.

a. What is the sample space of the experiment?

Student A

$S = \{R, R, R, B, B, B, B, G, G, W\}$

Student B

$S = \{R, B, G, W\}$

b. What is Pr(red or white), the probability of picking a red or white marble?

Student A

$$\Pr(\text{red or white}) = \frac{\text{number of white marbles}}{\text{number of red marbles}}$$
$$= \frac{1}{3}$$

Student B

$$\Pr(\text{red or white}) = \frac{\text{number of red and white marbles}}{\text{total number of marbles}}$$
$$= \frac{4}{10}$$
$$= \frac{2}{5}$$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. In an experiment, a group of people are randomly surveyed and are asked for the month they were born in. List all the possible outcomes of the experiment.

12. 10 tennis balls, 4 soccer balls and 6 basketballs are in a sports bag. What is the probability of picking a soccer ball?

13. In a class of 24 students, 8 students have 1 sibling, 10 students have 2 siblings and 6 students had 3 or more siblings. Calculate the likelihood of selecting a random student who has 2 or more siblings.

14. James has 2 boxes of chocolate which contains the following chocolates:

Chocolates	Box 1	Box 2
Freddo Frogs	6	8
Caramello Koala	4	5
Mars bar	2	4

Which box is James most likely to pick a Mars bar from?

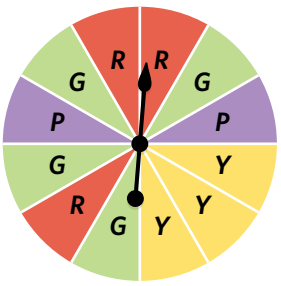
15. In a gameshow, the numbers 2, 6 and 9 are written on cards and put in a hat. 1 card is chosen and then put back in the hat, and another card is selected. The 2 numbers are added together. You win if the sum is less than 7. How many times more likely are you to lose compared to winning?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Jedda is a street magician who spins a spinner shown below to play a game with people. A game is fair if the probability of winning is equal to the probability of losing. She proposes the following rules for the game:



- Jedda wins if the spinner lands on purple or green.
- The player wins if the spinner lands on yellow or red.

a. List the sample space for the game.

b. Calculate the probability of the player winning, written as a fraction.

c. Determine if the game is fair by calculating the probability of the player losing.

d. Jedda changes the rules so that the player will now only win if the spinner lands on purple or red and loses if it lands on yellow or green. Explain whether or not this game is fair.

e. Propose a set of rules for a fair game and an unfair game.

17. An unknown number of yellow and black marbles will be put into 2 separate bags.

	Bag A	Bag B
Yellow		
Black		

Use each digit from 1 to 9 only once to fill in the boxes to make the probability of drawing:

a. a yellow marble from either bag the same.

b. a black marble from bag A twice as likely as drawing a yellow marble from bag B.

c. Use your answers to explain why it is possible to have different numbers of marbles in each bag, but the same probabilities.

Extra spicy

18. A bag contains some green marbles, blue marbles and white marbles with the following conditions:

- The probability of picking a blue marble is $\frac{1}{2}$.
- There are 5 green marbles.
- There are 4 white marbles.

How many blue marbles are there?

19. Construct a spinner with 4 colours (red, green, blue and yellow) that meets the following conditions:

- The probability of landing on blue is $\frac{1}{3}$.
- There is an even chance of landing on red and green.
- Landing on yellow is twice as likely to occur as landing on red.

20. A number of playing cards are put into a hat:

- Some are Jacks.
- There are 2 more Queens than Jacks.
- There are 3 more Kings than Queens.

Which expression represents the probability of choosing a Queen, where n is the number of Jacks?

A. $\frac{n}{8}$

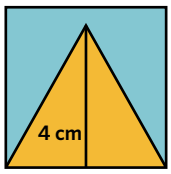
B. $\frac{2}{3n+5}$

C. $\frac{n}{3n+5}$

D. $\frac{n-2}{3n-7}$

E. $\frac{n+2}{3n+7}$

21. A dart is thrown at a dartboard shown below. It consists of a square and a triangle.



What is the probability of the dart landing in the triangular area?

A. $\frac{1}{4}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** $\frac{2}{3}$ **E.** $\frac{3}{4}$

Remember this?

22. What value of ★ makes this number sentence true?

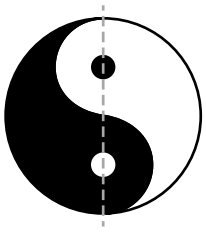
$16 < 4 \times ★$

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

23. Nick has 64 gigabytes of storage space on his phone. He has used $\frac{3}{8}$ of the storage space. How many gigabytes of storage space does he have left?

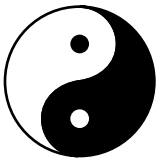
A. 8 **B.** 21 **C.** 24 **D.** 32 **E.** 40

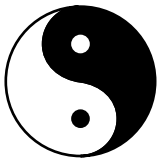
24. The following yin yang undergoes the following transformations in order:



- Rotated 90 degrees anticlockwise.
- Reflected vertically down across the dotted line.

What does the object look like after the transformations?

A. 

B. 

C. 

D. 

E. 

Back to contents

10B Complementary events

For any event with a probability of occurring there is also a probability of the event not occurring. For example, if an event is 'raining today', then there is also the event 'not raining today'. These pairs of events are called complementary events.

LEARNING INTENTIONS

Students will be able to:

- understand complementary events and that the probabilities of complementary events sum to 1
- state the complement of a given event
- calculate probabilities of complementary events.

KEY TERMS AND DEFINITIONS

An **outcome** is the result of a trial in an experiment.

An **event** is a single outcome or group of outcomes (known as a compound event).

The **sample space** of an experiment lists all the possible outcomes.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

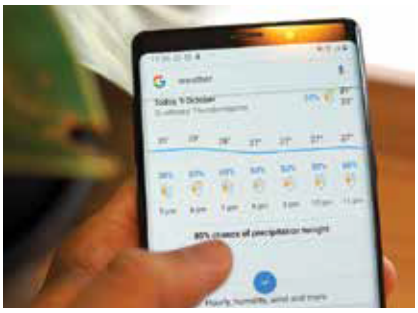


Image: TY Lim/Shutterstock.com

If the probability of it raining on a day is 80%, then we can also say that the probability of it not raining is 20%. This can help you decide whether or not you need to bring an umbrella.

Key ideas

1. The sum of the probabilities of all outcomes in an experiment equals 1.

Experiment: picking a counter

Sample space = {blue, green, orange}

$\Pr(\text{blue}) + \Pr(\text{green}) + \Pr(\text{orange})$

$= \frac{4}{9} + \frac{2}{9} + \frac{3}{9}$

$= \frac{9}{9}$

$= 1$

Continues →

2. For an event (A), the **complementary event** (written as A′) is the outcome or group of outcomes where the event does not occur. The probabilities of events and their complementary events sum to 1.

Event	**Complementary event**
Picking a blue counter 	Picking an orange or green counter 
$\Pr(\text{blue}) = \frac{4}{9}$	$\Pr(\text{blue}') = \Pr(\text{orange or green}) = \frac{5}{9}$
$\Pr(\text{blue}) + \Pr(\text{blue}') = \frac{4}{9} + \frac{5}{9} = \frac{9}{9}$ $= 1$ **or** $\Pr(\text{blue}') = 1 - \Pr(\text{blue})$ $= 1 - \frac{4}{9}$ $= \frac{5}{9}$	

Worked example 1

Determining complementary events

For each event, state the complementary event.

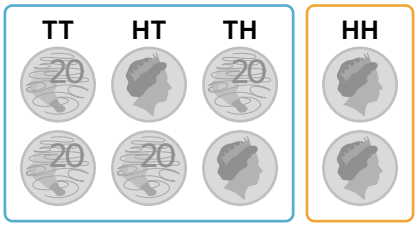
a. Experiment: the result of a chess match involving Magnus Carlsen.
Event: Magnus Carlsen winning. WE1a

Working	Thinking
$S = \{\text{win, lose, draw}\}$	**Step 1:** List the sample space of the experiment.
Lose and draw.	**Step 2:** List the outcomes that are not the event.
Magnus Carlsen loses or the match is a draw.	**Step 3:** State the complementary event.

b. Experiment: flipping a coin.
Event: landing two heads. WE1b

Working	Thinking
$S = \{\text{HH, HT, TH, TT}\}$	**Step 1:** List the sample space of the experiment.
HT, TH and TT	**Step 2:** List the outcomes that are not the event.
Landing two tails, or a tail and a head.	**Step 3:** State the complementary event.

Visual support



Continues →

Student practice

For each event, state the complementary event.

a. Experiment: picking a letter at random from the word COMPLEMENT.
Event: selecting a vowel.

b. Experiment: picking a coin from a wallet containing 5c, 10c, 50c and $1 coins.
Event: picking a 50c coin.

Worked example 2

Calculating probabilities of events and complementary events

An experiment is conducted where a spinner with three colours (purple, orange and cyan) is spun. The following probabilities are known:

- $\Pr(\text{purple}) = 0.5$
- $\Pr(\text{orange}) = 0.25$

a. Calculate $\Pr(\text{purple}')$ WE2a

Working

$\Pr(\text{purple}') = 1 - \Pr(\text{purple})$
$= 1 - 0.5$
$= 0.5$

Thinking

Subtract the probability of the event from 1.

b. Calculate $\Pr(\text{cyan})$ WE2b

Working

$\Pr(\text{cyan}) = 1 - \Pr(\text{purple}) - \Pr(\text{orange})$
$\Pr(\text{cyan}) = 1 - 0.5 - 0.25$
$= 0.5 - 0.25$
$= 0.25$

Thinking

Step 1: Find the missing probability by subtracting all other probabilities from 1.

Step 2: Simplify.

Student practice

An experiment is conducted where red, blue and green marbles are drawn from a bag. The following probabilities are known:

- $\Pr(\text{red}) = \frac{1}{3}$
- $\Pr(\text{blue}) = \frac{1}{6}$

a. Calculate $\Pr(\text{red}')$

b. Calculate $\Pr(\text{green})$

10B Questions

Understanding worksheet

1. For each pair of events, tick whether they are complementary or not.

Example

Event 1	Event 2	Complementary	Not complementary
Generating an even number between 1 and 10.	Generating an odd number between 1 and 10.	☑	☐

	Event 1	Event 2	Complementary	Not complementary
a.	Getting out of bed.	Not getting out of bed.	☐	☐
b.	Walking home from school.	Riding a bike home from school.	☐	☐
c.	Picking a picture card from a deck of 52 cards.	Picking a number card from a deck of 52 cards.	☐	☐
d.	Rolling at least a three on a die.	Rolling three or less on a die.	☐	☐

2. For each experiment and event, list the outcomes of the complementary event.

Example

Experiment	Event	Outcome(s) of complementary event
Catching public transport in Melbourne.	Taking the train.	Taking the bus. Taking the tram.

	Experiment	Event	Outcome(s) of complementary event
a.	Flipping a coin.	Flipping a head.	
b.	Rolling a six-sided die.	Rolling a 4.	
c.	Picking a red, green or blue marble out of a bag.	Picking a blue marble.	
d.	Generating a whole number between 1 and 10.	Generating an odd or prime number.	

3. Fill in the blanks by using the words provided.

probabilities | outcomes | complementary | sum

For any event in an experiment, there will be a _____ event. That event can be a single or group of _____. The _____ of this event and the original event _____ to 1.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d,e), 8	4 (e,f,g,h), 5 (c,d,e), 6 (c,d,e), 7 (c,d,e,f), 8

4. For each event, state the complementary event. WE1

- **a.** Picking an orange counter from a bag of orange and blue counters.
- **b.** Picking a watermelon icy pole from a bag containing watermelon, lime and raspberry icy poles.
- **c.** Picking a red card from a standard deck of 52 cards.
- **d.** Rolling a die and rolling an even number.
- **e.** Rolling a die and rolling the number 5.
- **f.** Rolling a die and rolling the number 1, 3 or 5.
- **g.** Randomly picking a number less than 6 from the numbers 1 to 15.
- **h.** Randomly picking a number less than 3 or greater than 10 from the numbers 1 to 15.

---

5. A spinner with three colours (black, yellow and white) is constructed. The following probabilities are given: WE2

- $\Pr(\text{black}) = 0.2$
- $\Pr(\text{yellow}) = 0.4$

Write all probabilities as decimals.

- **a.** State the event complementary to picking black.
- **b.** Calculate $\Pr(\text{black}')$, the probability of not landing on black.
- **c.** Calculate $\Pr(\text{white})$, the probability of landing on white.
- **d.** Calculate $\Pr(\text{not white or black})$, the probability of not landing on white or black.
- **e.** True or false: the probability of not selecting black is equal to the probability of selecting yellow or selecting white.

---

6. A bucket containing a number of counters with four different colours (green, blue, orange and red) are picked out of a container. The following probabilities are given: WE2

- $\Pr(\text{green}) = \frac{7}{15}$
- $\Pr(\text{blue}) = \frac{1}{5}$
- $\Pr(\text{red}) = \frac{1}{3}$

For this question, write all probabilities as fractions.

- **a.** State the event complementary to picking blue.
- **b.** Calculate $\Pr(\text{blue}')$, the probability of not picking blue.
- **c.** Calculate $\Pr(\text{orange})$, the probability of picking orange.
- **d.** Calculate $\Pr(\text{not orange or blue})$, the probability of not picking orange or blue.
- **e.** True or false: the probability of not picking red is less than the probability of picking green or orange.

---

7. A standard deck of 52 playing cards is shuffled and one card drawn at random.

- **a.** State the event complementary to picking a spade card.
- **b.** Calculate $\Pr(\text{spade}')$, written as a percentage.
- **c.** Calculate $\Pr(\text{queen})$, written as a fraction.
- **d.** Calculate $\Pr(\text{queen}')$, written as a fraction.
- **e.** Calculate $\Pr(\text{even card or a picture card})$.
- **f.** Calculate $\Pr(\text{not an even or a picture card})$.

8. A random number is generated between 1 and 20, where each number is equally likely to be generated. What is the probability of not generating a number less than 8?

A. $\frac{1}{4}$ **B.** $\frac{7}{20}$ **C.** $\frac{7}{13}$ **D.** $\frac{13}{20}$ **E.** $\frac{3}{4}$

Spot the mistake

9. Select whether Student A or Student B is incorrect.

A number of purple, cyan, yellow and magenta marbles were placed in a bag and picked out at random. The following probabilities are known:

- $\Pr(\text{cyan}) = \frac{2}{5}$
- $\Pr(\text{purple}) = \frac{1}{10}$
- $\Pr(\text{magenta}) = \frac{1}{5}$

a. Calculate $\Pr(\text{cyan}')$

Student A

$$\begin{aligned}\Pr(\text{cyan}') &= 1 - \Pr(\text{cyan}) \\ &= 1 - \frac{2}{5} \\ &= \frac{5}{5} - \frac{2}{5} \\ &= \frac{3}{5}\end{aligned}$$

Student B

$$\begin{aligned}\Pr(\text{cyan}') &= \Pr(\text{cyan}) \\ &= \frac{2}{5}\end{aligned}$$

b. Calculate $\Pr(\text{yellow})$

Student A

$$\begin{aligned}\Pr(\text{yellow}) &= 1 - \Pr(\text{cyan}) - \Pr(\text{purple}) - \Pr(\text{magenta}) \\ &= 1 - \frac{2}{5} - \frac{1}{10} - \frac{1}{5} \\ &= \frac{10}{10} - \frac{4}{10} - \frac{1}{10} - \frac{2}{5} \\ &= \frac{3}{10}\end{aligned}$$

Student B

$$\begin{aligned}\Pr(\text{yellow}) &= \frac{2}{5} + \frac{1}{10} + \frac{1}{5} \\ &= \frac{4}{10} + \frac{1}{10} + \frac{2}{10} \\ &= \frac{7}{10}\end{aligned}$$

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. On one day at a train station, a train was cancelled. State the outcomes of the complementary event.

11. A store selling PlayStation 5 consoles states that the probability of the store running out of PS5s is 0.65. What is the probability that they will not run out?

12. The probability that Max rides his bike to work is $\frac{1}{3}$. The probability that he takes the tram to work is $\frac{1}{2}$. Calculate the probability that he will neither ride his bike nor take the tram to work?

13. The Year 8 students at a school were surveyed for their blood type (A, B, AB and O). The results are shown below:

Blood type	A	B	AB	O
Number of students	76	20	6	98

If a student was chosen at random, find the probability that they are not blood type A or AB, written as a percentage.

14. Patrick's 'On repeat' playlist has songs from the following artists: Two Door Cinema Club (5), Last Dinosaurs (7), Gordi (10) and Maggie Rogers (8).

Which artist has a probability of not playing next equal to $\frac{5}{6}$?

Reasoning

Question working paths

Mild 15 (a,b,c,e) | Medium 15 (a,b,c,e), 16 (a,b) | Spicy All

15. Acacia makes and sells custom made dice online based on probabilities that the customer provides. Each die can have multiple faces with the same number. One day she receives the following orders:

Die 1	**Die 2**
Four sides Numbers: 1, 3 and 4 $\Pr(3) = \frac{1}{2}$ $\Pr(4) = 0.25$	Five sides Numbers: 1, 3, 5 and 6 $\Pr(3) = 0.20$ $\Pr(5) = \frac{1}{5}$ $\Pr(6) = 40\%$

a. For each die, state the events complementary to rolling a 3.

b. For each die, calculate $\Pr(3')$.

c. Which die has the highest probability of rolling a 1?

d. For each die, determine which number(s) have multiple faces. Explain your reasoning.

e. Name a game or activity that requires the use of dice.

16. Two events are called mutually exclusive events if they do not occur at the same time. In an experiment, two orange counters, one blue counter and three green counters are put in a bag and one is drawn at random.

a. State a pair of mutually exclusive events.

b. State a pair of non-mutually exclusive events.

c. Are complementary events always mutually exclusive? Explain your answers to parts **a** and **b**.

Extra spicy

17. In a bag containing 4 red balls and 7 blue balls, one red ball is drawn without being put back. Find the probability that the next ball drawn is not red.

A. $\frac{2}{5}$ **B.** $\frac{4}{7}$ **C.** $\frac{7}{11}$ **D.** $\frac{7}{10}$ **E.** $\frac{7}{4}$

18. Jerone is rolling two standard six-sided dice. Using complementary events, calculate the probability of rolling the two dice with different numbers on top.

19. A baker estimates that the probability of them baking a great cake is x, the probability of them baking an okay cake is twice the probability of baking a great cake, and the probability of baking a terrible cake is 0.4

What is the probability of not baking a great cake?

A. 0.2 **B.** 0.3 **C.** 0.4 **D.** 0.6 **E.** 0.8

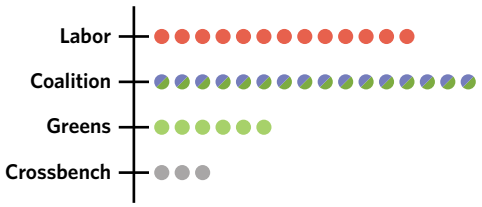
20. A basket contains the following Easter eggs: nine plain chocolate, two white chocolate and some number of caramel chocolate eggs. The probability of selecting a plain chocolate egg is $\frac{3}{5}$. What is the probability of not picking a white chocolate egg?

Remember this?

21. Which option is correct when 12 is substituted for g?

A. $13 + g = 1$ **B.** $25 = 13 - g$ **C.** $25 + g = 12$ **D.** $g + 13 = 25$ **E.** $13 - 12 = g$

22. The following graph shows the number of senators each party has in the Australian Senate in 2022.



The total number of senators is 76.

How many people does each dot on the graph represent?

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

23. Bayak is drawing cards from a deck of 52 cards. He draws five picture cards in a row, and draws a sixth card.

Which of the following is true about the sixth card?

A. It is certain to be a number card.
B. It is certain to be a picture card.
C. It is more likely to be a number card than a picture card.
D. It is more likely to be a picture card than a number card.
E. There is an even chance that it is a picture card.

Back to contents

10C Venn diagrams and two-way tables

Venn diagrams are named after the English mathematician John Venn, who was born in 1834. Venn diagrams and two-way tables provide information that is otherwise presented through words. The benefit of these methods for presenting information is that they allow people to communicate and learn content in a way that may suit them better. The ability to use all methods promotes stronger skills in understanding, applying and communicating information.

LEARNING INTENTIONS

Students will be able to:

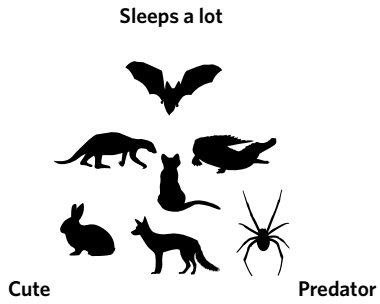
- interpret and construct Venn diagrams
- interpret and construct two-way tables
- understand the words 'and', 'or' and 'at least' in the context of probability events.

KEY TERMS AND DEFINITIONS

A **two-way table** displays the frequency (count) of two categories, written in a table format with rows and columns.

A **Venn diagram** provides a visual representation of similarities and differences between two or more sets of information.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

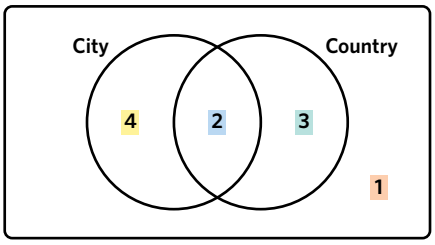


Tables and Venn diagrams are used with information relating to science, sport, psychology, business and other areas of life. Venn diagrams are also used in internet memes.

Key ideas

1. Where there is more than one category of information, Venn diagrams and two-way tables are alternative ways of displaying information.

Venn diagram representation:



Two-way table representation:

	Likes country	Dislikes country	Total
Likes city	2	4	6
Dislikes city	3	1	4
Total	5	5	10

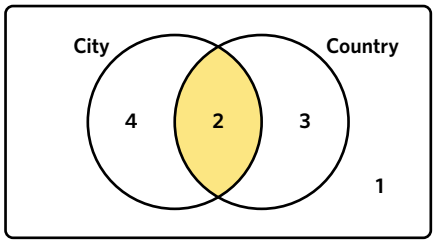
Word representation:

Ten people were surveyed on how they feel about the city and the country. Four people said that they only like the city, three people said that they only like the country, two people said they like both, and one person said they don't like either.

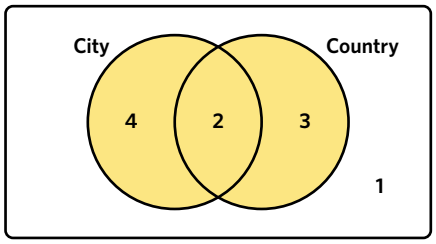
Continues →

2. In probability, language is very important.

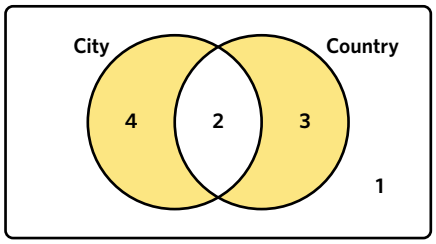
The word 'and' refers to the overlap of two or more categories of information.
For example, the yellow section below represents the number of people who like the **city and country**.



The word 'or' can be inclusive, suggesting it belongs to the two separate categories as well as both.
For example, the yellow section below represents the number of people who like the **city or country or both**.



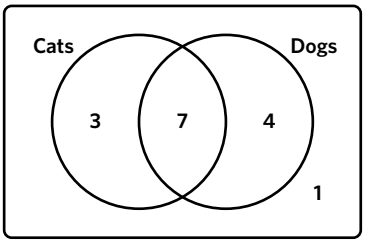
The word 'or' can be exclusive, suggesting it belongs to the two separate categories but not to both.
For example, the yellow section below represents the number of people who like the **city or country but not both**.



Worked example 1

Interpreting information from Venn diagrams

Consider the following Venn diagram showing 15 people's responses relating to how they feel about cats and dogs.



a. How many people like dogs? WE1a

Working

$7 + 4 = 11$ people

Thinking

Count the numbers within the relevant circle.
Note: Without the word 'only', both numbers are counted.

Continues →

b. How many people like cats or dogs, but not both? WE1b

Working

$3 + 4 = 7$ people

Thinking

Add up the numbers in the two circles, not including the overlap.

c. Determine Pr(only cats). WE1c

Working

People who only like cats $= 3$

$$\Pr(\text{only cats}) = \frac{3}{15}$$
$$= \frac{1}{5}$$

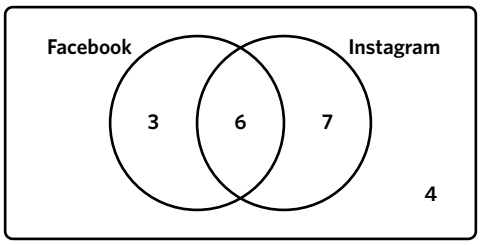
Thinking

Step 1: Determine the number of people who fit the description.

Step 2: Determine the probability and express it as a fraction in the simplest form.

Student practice

Consider the following Venn diagram showing the number of Facebook and Instagram users in a sample of 20.



a. How many people use Instagram?

b. How many people use Facebook or Instagram, but not both?

c. Determine Pr(only Instagram).

Worked example 2

Interpreting information from two-way tables

Consider the following two-way table representing the responses of 18 people and their use of pencils and textas.

	Uses textas	Does not use textas	Total
Uses pencil	7	6	13
Does not use pencil	3	2	5
Total	10	8	18

a. How many people use pencils? WE2a

Working

13 people

Thinking

Find the information in the 'total' column for the relevant category.

Visual support

	Uses textas	Does not use textas	Total
Uses pencil	7	6	13
Does not use pencil	3	2	5
Total	10	8	18

Continues →

b. How many people use textas or pencils or both? WE2b

Working

$3 + 6 + 7 = 16$ people

Thinking

Add up the totals for each category, as well as the number for both.

Visual support

	Uses textas	Does not use textas	Total
Uses pencil	7	6	13
Does not use pencil	3	2	5
Total	10	8	18

c. Determine Pr(uses textas only). WE2c

Working

People who use textas only $= 3$

$$\Pr(\text{uses textas only}) = \frac{3}{18}$$
$$= \frac{1}{6}$$

Thinking

Step 1: Determine the number of people who fit the description.

Step 2: Determine the probability and express it as a fraction in the simplest form.

Visual support

	Uses textas	Does not use textas	Total
Uses pencil	7	6	13
Does not use pencil	3	2	5
Total	10	8	18

Student practice

Consider the following two-way table showing a sample of 22 people and their buying habits for dental floss and mouthwash.

	Buys dental floss	Does not buy dental floss	Total
Buys mouthwash	4	6	10
Does not buy mouthwash	7	5	12
Total	11	11	22

a. How many people buy mouthwash?

b. How many people buy dental floss or mouthwash or both?

c. Determine Pr(buys dental floss only).

Worked example 3

Constructing Venn diagrams and two-way tables

25 people were asked whether they make phone calls and send text messages daily. 15 people said they make phone calls daily, 18 people said they text daily and 2 people said they do not do either daily.

a. Use the given information to construct a two-way table. WE3a

Working

	Text messages	No text messages	Total
Phone calls			
No phone calls			
Total			

Thinking

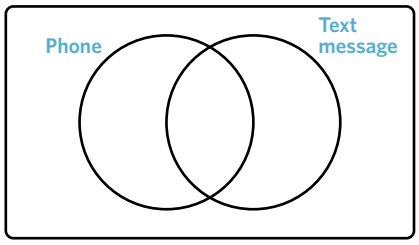
Step 1: Draw the grid to make a two-way table and fill in the headings.

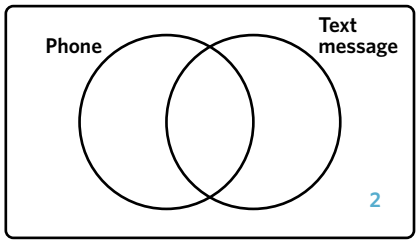
	Text messages	No text messages	Total
Phone calls	10	5	15
No phone calls	8	2	10
Total	18	7	25

Step 2: Fill in the numbers using the information provided.

b. Use the given information to construct a Venn diagram. WE3b

Working





Overlap of Phone and Text message
$= 15 + 18 - (25 - 2)$

$= 10$

Phone only $= 15 - 10 = 5$

Text message $= 18 - 10 = 8$

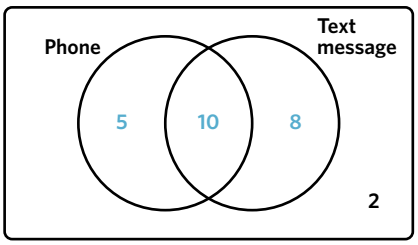
Thinking

Step 1: Draw the rectangle, circles and write the labels on the Venn diagram.

Step 2: Fill in the information that is known.

Step 3: Determine what numbers are written in the blank parts of the Venn diagram.

Continues →



Step 4: Fill in the remaining information.

Student practice

20 people who experienced a recent natural disaster in Australia were interviewed to ask whether they have enough food and water. 5 people said they had enough food, 8 people said they had enough water and 9 people said they did not have enough food or water.

a. Use the information given to construct a two-way table.

b. Use the information given to construct a Venn diagram.

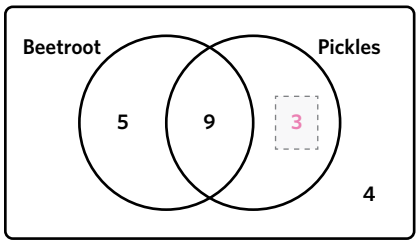
10C Questions

Understanding worksheet

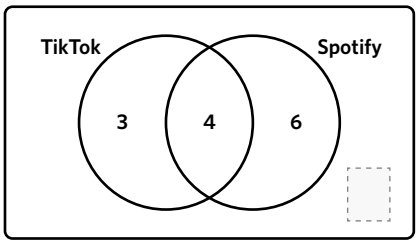
1. Using the information provided, fill in the blanks in the following Venn diagrams.

Example

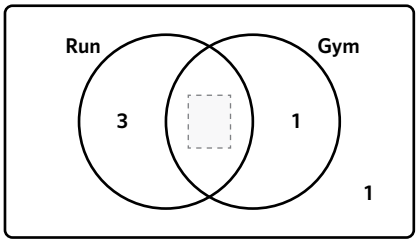
21 students are surveyed on whether they eat beetroot and pickles. It is found that 14 students eat beetroot, 12 students eat pickles, 9 students eat both and 4 do not eat either.



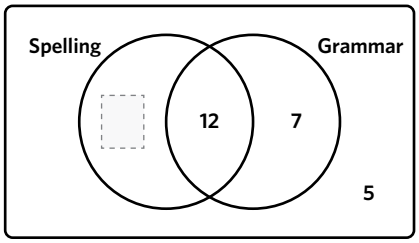
a. 15 students are asked whether they use Tiktok and Spotify. It was found that 7 use TikTok, 10 use Spotify and 2 students don't use either.



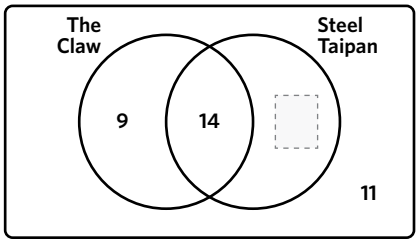
b. 30 elite athletes are asked whether they run or go to the gym as part of their training. 27 say they run, 25 say that they go to the gym and 1 person, who is injured, says that they do not do either.



c. 28 professional writers are asked whether they are confident at spelling and grammar. It is found that 12 are confident at both, 5 are not confident with either and 19 are confident at grammar.



d. 50 visitors are surveyed on their way out from Dreamworld. It is found that 23 people went on The Claw, 30 people went on the Steel Taipan and 11 people did not go on either.



2. Using the information provided, fill in the blanks in the following two-way tables.

Example

	Likes beetroot	Dislikes beetroot	Total
Likes pickles	9	4	13
Dislikes pickles	5	3	8
Total	14	7	21

a.

	Baby	Adult	Total
Female	2	4	6
Male		3	6
Total	5	7	12

b.

	Moons	No moons	Total
Rings	4	0	4
No rings		2	4
Total	6	2	8

c.

	Siblings	No siblings	Total
Single parent		1	9
Two parents	12	3	15
Total	20		24

d.

	Childcare	No childcare	Total
Working			21
Not working	2	7	9
Total	19	11	30

3. Fill in the blanks by using the words provided.

inclusive | exclusive | Venn diagram | two-way table

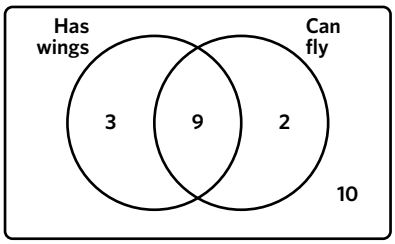
A _____ has two or more circles, which represent the different categories and the similarities and differences between them. A _____ must have exactly two categories and can be used to present the same information as in a Venn diagram. When interpreting two-way tables and Venn diagrams, the word 'or' can include the overlap of both categories, which makes it _____, or it might be _____, which does not include the overlap between the categories.

Fluency

Question working paths

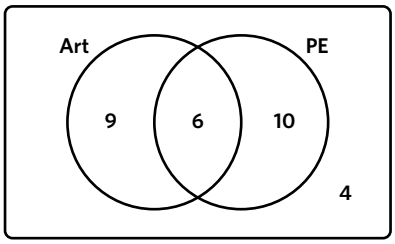
Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7 (a,b,c), 8 (a,b), 9	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c), 9	4 (c,d,e), 5 (c,d,e), 6 (c,d,e), 7 (c,d,e), 8 (c,d), 9

4. Consider the following Venn diagram of 24 animals in terms of whether they have wings and whether they are capable of flight. WE1a,b



a. How many animals have no wings and cannot fly?
b. How many animals have wings and can fly?
c. How many animals have wings or can fly but not both?
d. How many animals have wings?
e. How many animals cannot fly?

5. Consider the following Venn diagram showing 29 students and whether they take Art and Physical Education (PE). WE1c



a. Determine Pr(neither Art or PE).
b. Determine Pr(both Art and PE).
c. Determine Pr(Art).
d. Determine Pr(Not PE).
e. Determine Pr(Art or PE but not both).

6. Consider the following two-way table showing data for 18 animals in terms of the number of legs the animal walks on and whether they have a tail. WE2a,b

	Tail	No tail	Total
Two legs	3	2	5
Four legs	8	5	13
Total	11	7	18

a. How many animals walk on two legs and have a tail?
b. How many animals walk on four legs and have no tail?
c. How many animals have two legs or a tail or both?
d. How many animals have no tail?
e. How many animals have four legs or have no tail but not both?

7. Consider the following two-way table of students in year 8 who play sport and play an instrument. WE2c

	Plays sport	Does not play sport	Total
Plays an instrument	12	7	19
Does not play an instrument	15	8	23
Total	27	15	42

a. Determine Pr(plays an instrument but does not play sport).

b. Determine Pr(plays sport and plays an instrument).

c. Determine Pr(plays an instrument).

d. Determine Pr(plays sport or plays an instrument or both).

e. Determine Pr(does not play an instrument).

---

8. Construct a two-way table and a Venn diagram given the following information. WE3

a. At the gym, customers are surveyed as to whether they would take part in a yoga or meditation class. Of the people surveyed, 5 said they would only be interested in yoga, 2 said they would only be interested in meditation, 3 people were interested in both and 1 person was not interested in either.

b. 20 patients at a physiotherapy clinic are prescribed additional exercises. 14 patients are prescribed with pilates, 5 patients are prescribed with swimming, and 3 patients are prescribed with both.

c. 30 adults are asked if they are afraid of snakes and spiders. 13 people are afraid of snakes, 15 are afraid of spiders and 7 people are not afraid of either (or so they claim).

d. At Edrolo Secondary College, junior school students choose to study either Indonesian or Chinese as a second language, but not both. This is known as a mutually exclusive event, where there is no overlap between the two categories. 26 students choose Indonesian, 41 students choose Chinese and 5 students do not choose either.

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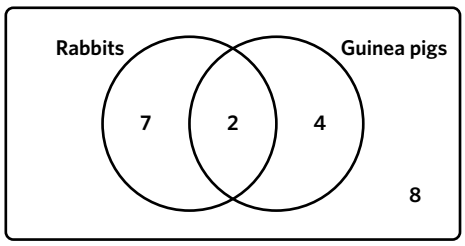
9. The following Venn diagram shows the number of people who come in with their pets to the Rabbits and Guinea Pig Specialists veterinary clinic. Which statement is correct?



A. There are 11 people who bring in rabbits and guinea pigs.

B. There are 13 people who bring in rabbits or guinea pigs or both.

C. There are two people who bring in rabbits or guinea pigs.

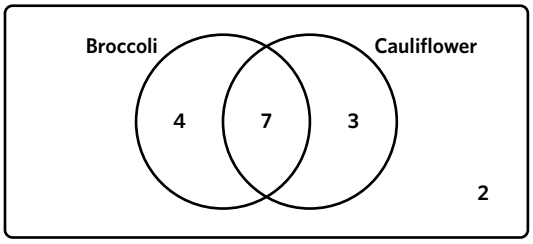
D. There are four people who own guinea pigs.

E. There are seven people who own rabbits.

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. In the following Venn diagram, determine Pr(broccoli).



Student A

People who eat broccoli $= 11$

Total number of people $= 16$

$\Pr(\text{broccoli}) = \frac{11}{16}$

Student B

People who eat broccoli $= 4$

Total number of people $= 16$

$\Pr(\text{broccoli}) = \frac{4}{16}$

$= \frac{1}{4}$

b. In the following two-way table, how many people eat chewing gum or Tic Tacs or both?

	Tic Tac	No Tic Tac	Total
Chewing gum	5	9	14
No chewing gum	3	15	18
Total	8	14	22

Student A

$3 + 9 = 12$ people

Student B

$5 + 9 + 3 = 17$ people

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Five friends go to a concert. Two people buy food and water, one person buys food only and one person does not buy anything. How many people only bought water?

12. A family of 10 go to pick strawberries and cherries together. If 4 members only picked strawberries, 2 members only picked cherries and 3 members picked both, how many family members did not pick either?

13. Leonora is the sports captain at her school and surveys students to see how many teams can be formed for basketball and netball next term. Of the 100 students surveyed, 50 said they would be interested in basketball and 45 said they would be interested in netball. If 25 students did not express interest for either, how many students said they would be interested in both?

14. In his after hours painting class, Pablo counts 4 people with short hair and 8 with long hair. For the same group of 12 people, there are 7 who bring their own paint brushes and 5 who borrow ones from the art studio. If there are 2 people with short hair who bring their own paint brushes, what is the probability that a person selected at random has long hair or borrows paint brushes from the studio, including those that identify with both?

15. Mr Bjarnelind wants to shortlist international cities to visit with his family. He looks at two factors in making his decision, the duration of the flight and whether the city is child-friendly. Mr B's research shows that of the 20 cities considered, 5 cities have flights that are ten hours or less and 13 cities are child-friendly. If there are 4 cities that are not child-friendly and have flights over ten hours, what is the probability of choosing a city that is child friendly but has a flight that is more than ten hours?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Therese collects information on 30 animals to understand if there is a link between an animal being a predator and how much they sleep. The results are recorded in the two-way table below.

Hours of sleep per day	Predator	Not a predator	Total
Less than 12 hours	3	13	16
At least 12 hours	10	4	14
Total	13	17	30

a. How many animals sleep at least 12 hours a day and are not predators?

b. What is the probability that an animal picked at random is a predator or sleeps less than 12 hours a day, but not both?

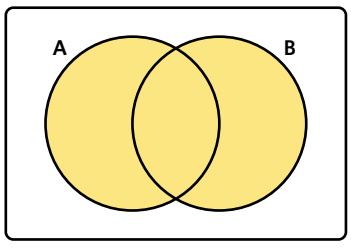
c. Convert this two-way table into a Venn diagram.

d. The probability of picking an animal that sleeps less than 12 hours is $\frac{8}{15}$. Therese's friend Taylen used the Venn diagram from part **c** and got an answer of $\frac{13}{30}$. Explain how Taylen might have calculated his incorrect answer.

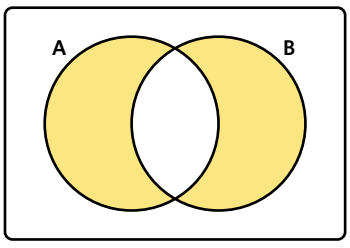
e. Why might predator animals sleep more?

17. Consider the following Venn diagrams.

Venn diagram 1:



Venn diagram 2:



a. State the Venn diagram that describes 'A or B' if the exclusive 'or' is implied.

b. State the Venn diagram that describes 'A or B' if the inclusive 'or' is implied.

c. Compare your answers for part **a** and **b**. What kind of wording can be used to define whether the inclusive or exclusive 'or' is being used in each situation?

Extra spicy

18. Ichika surveys her class to see how many people have cats and dogs at home. In a class of 25 students, 10 students have cats and 12 have dogs. What is the largest possible probability that a student owns both cats and dogs? Write your answer as a fraction in the simplest form.

19. The probability of picking out a coloured tie is three times as likely as the probability of picking out a dotted tie. Mr Spencer has 13 ties in total, where 1 tie is both dotted and coloured, and 2 ties are neither dotted or coloured. How many ties are dotted but not coloured?

20. A number is selected at random from the first 30 positive integers. What is the probability that the number is a multiple of either 3 or 13, but not both?

A. $\frac{11}{30}$ **B.** $\frac{2}{5}$ **C.** 0.45 **D.** $\frac{17}{30}$ **E.** 80%

21. In a survey of toilet paper brand purchases, where the brands are A, B, and C, the results are as follows.

- 34% buy brand A
- 46% buy brand B
- 52% buy brand C
- 17% buy brand A and B
- 19% buy brand A and C
- 20% buy brand B and C
- 15% buy only brand A

The percentage of people who only buy brand C is:

A. 10% **B.** 20% **C.** 30% **D.** 32% **E.** 52%

Remember this?

22. Samara wishes to purchase two 1.5 litre bottles of Coca Cola and three packs of crackers. One bottle of Coca Cola costs \$3.15 and one pack of crackers costs \$2.20.

If Samara pays with a \$20 note, how much change will she receive?

A. \$5.35 **B.** \$7.10 **C.** \$8.35 **D.** \$12.45 **E.** \$14.65

23. Conor has eaten $\frac{8}{25}$ of a chocolate bar.

Which decimal shows how much of the chocolate bar he has eaten?

A. 0.120 **B.** 0.230 **C.** 0.320 **D.** 0.475 **E.** 0.625

24. Agatha and Mia are buying rings.

Mia goes to buy 5 times the number of rings Agatha buys plus an additional 3 rings. However, Mia then decides that this will cost too much and so puts back 2 rings.

Let a be the number of rings Agatha buys.

Which of the following expressions represent the number of rings Mia buys?

A. $5a + 3$ **B.** $3 + (a \times 5) - 1$ **C.** $(a \div 5) + 3$ **D.** $5a + 1$ **E.** $1 - (a \div 5)$

Back to contents

10D Tree diagrams and multi-step experiments

When predicting the likelihood of events, we often wish to assess the probability of two or more outcomes occurring together. In these instances, a useful way to determine the possible outcomes is a two-way table or a tree diagram.
A two-way table is useful when there are exactly two events, whereas a tree diagram can be used for two or more probability events.

LEARNING INTENTIONS

Students will be able to:

- interpret tree diagrams and two-way tables for multi-step experiments
- determine the probability of outcomes in multi-step experiments
- construct two-way tables and tree diagrams for multi-step experiments.

KEY TERMS AND DEFINITIONS

Multi-step experiments describe a probability event that is made up of two or more actions, such as tossing three coins and looking at the different combinations of possible outcomes.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

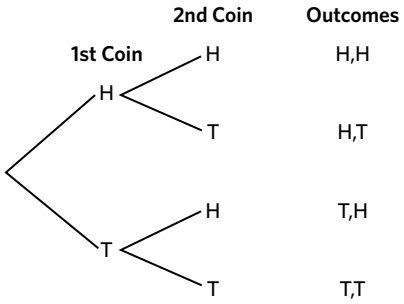
Image: Grindi/Shutterstock.com

It is common to predict outcomes of multi-step experiments in real life. For example, we may wish to know what the combined chance of rain and strong wind is. This then allows us to decide whether to bring an umbrella or a rain jacket.

Key ideas

1. The outcomes of a two-way experiment can be shown using a tree diagram or a two-way table.

Tree diagram representation



Two-way table representation

	H	**T**
H	H,H	H,T
T	T,H	T,T

Word representation

Two coins are tossed. The possible outcomes are: getting heads for both coins; getting a head on the first and a tail on the second coin; getting a tail on the first coin and a head on the second or; getting two tails.

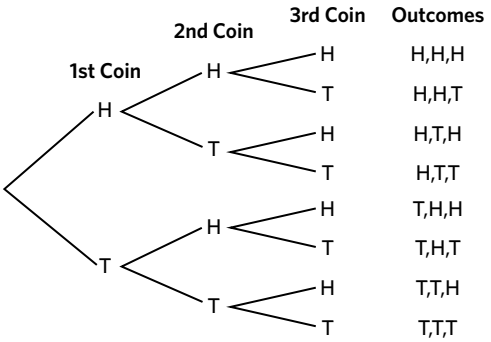
2. The probability is determined using the same formula as for one-step experiments.

$$\Pr(\text{event 1, event 2}) = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$$

Worked example 1

Interpreting tree diagrams for multi-step experiments

The following tree diagram shows the outcomes for tossing three coins.



a. How many outcomes are possible? WE1a

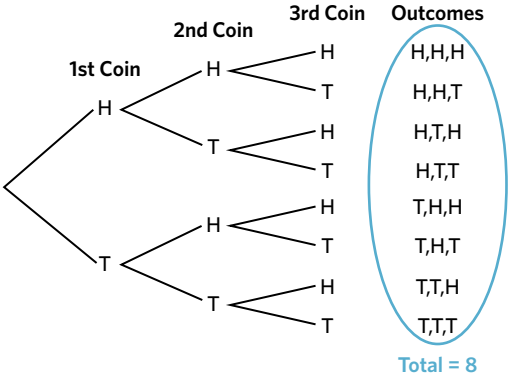
Working

There are 8 possible outcomes.

Thinking

Count the number of outcomes listed.

Visual support



b. Determine Pr(T,T,T). WE1b

Working

Outcomes with T,T,T $= 1$

Total number of outcomes $= 8$

$\Pr(\text{T,T,T}) = \frac{1}{8}$

Thinking

Step 1: Count the number of desired outcomes.

Step 2: Count the total number of outcomes.

Step 3: Write the probability as a fraction in the simplest form.

c. Determine Pr(at least one head). WE1c

Working

Outcomes with at least one head $= 7$

Total number of outcomes $= 8$

$\Pr(\text{at least one head}) = \frac{7}{8}$

Thinking

Step 1: Count the number of desired outcomes.

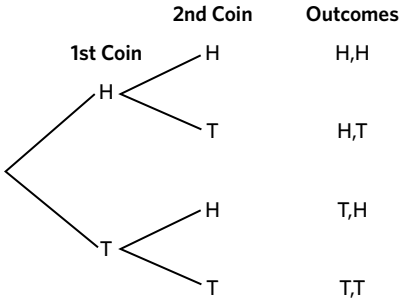
Step 2: Count the total number of outcomes.

Step 3: Write the probability as a fraction in the simplest form.

Continues →

Student practice

The following tree diagram shows the outcomes for tossing two coins.



a. How many outcomes are possible?

b. Determine Pr(H,H).

c. Determine Pr(at least one tail).

Worked example 2

Interpreting two-way tables for two-step experiments

The following two-way table shows the outcomes of throwing two 6-sided dice.

	1	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

a. How many outcomes are possible? WE2a

Working

There are 36 possible outcomes.

Thinking

Count the number of outcomes listed in the two-way table.

b. Determine Pr(same number twice). WE2b

Working

Outcomes with same number twice $= 6$

Total number of outcomes $= 36$

$$\Pr(\text{same number twice}) = \frac{6}{36}$$
$$= \frac{1}{6}$$

Thinking

Step 1: Count the number of desired outcomes

Step 2: Count the total number of outcomes.

Step 3: Write the probability as a fraction in the simplest form.

Continues →

c. Determine Pr(sum of less than 10). WE2c

Working

Outcomes with sum less than $10 = 30$

Total number of outcomes $= 36$

$\Pr(\text{sum of less than } 10) = \frac{30}{36}$

$= \frac{5}{6}$

Thinking

Step 1: Count the number of desired outcomes. Note: 'less than' does not include the number specified.

Step 2: Count the total number of outcomes.

Step 3: Write the probability as a fraction in the simplest form.

Student practice

The following two-way table shows the outcomes of tossing a coin and throwing a 6-sided die.

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

a. How many outcomes are possible?

b. Determine Pr(tail).

c. Determine Pr(head and less than 4).

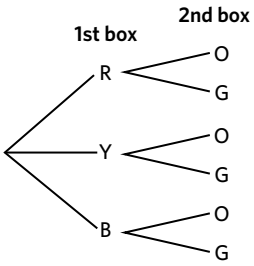
Worked example 3

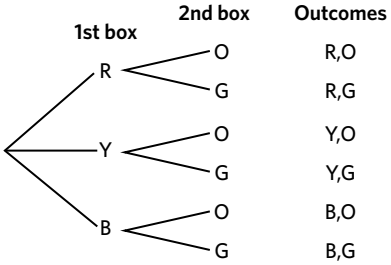
Constructing two-way tables and tree diagrams for two-step experiments

Reina puts a red (R), yellow (Y) and blue (B) Skittle into a small box, then one orange (O) and one green (G) Skittle into a second box. She picks a Skittle from the first box and then one from the second box.

a. Construct a tree diagram to show all possible outcomes. WE3a

Working





Thinking

Step 1: Draw the branches and labels for each branch.

Step 2: List the outcomes.

Continues →

b. Construct a two-way table to show all possible outcomes. WE3b

Working

		2nd box	
		Orange	**Green**
1st box	**Red**		
	Yellow		
	Blue		

		2nd box	
		Orange	**Green**
1st box	**Red**	R,O	R,G
	Yellow	Y,O	Y,G
	Blue	B,O	B,G

or

		1st box		
		Red	**Yellow**	**Blue**
2nd box	**Orange**	R,O	Y,O	B,O
	Green	R,G	Y,G	B,G

Thinking

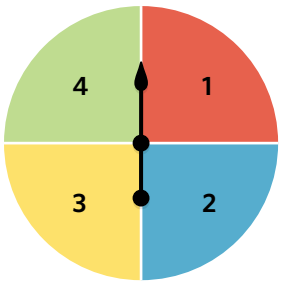
Step 1: Draw the table grid and fill in the headings.

Step 2: Fill in the table with outcomes.

Note: It does not matter whether the 1st box is a column or a row, as long as all combinations are listed correctly.

Student practice

Pico has one spin on a wheel with wedges labelled 1, 2, 3 and 4, followed by tossing a coin.



a. Construct a tree diagram to show all possible outcomes.

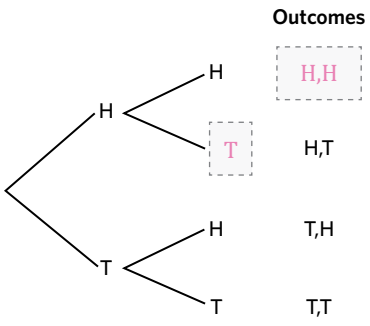
b. Construct a two-way table to show all possible outcomes

10D Questions

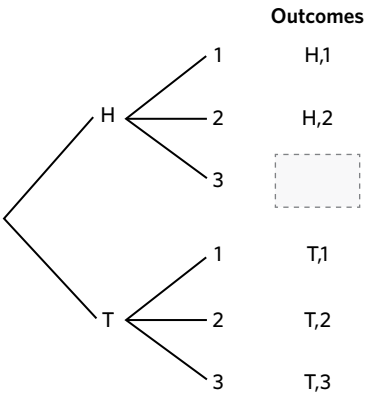
Understanding worksheet

1. Fill in the blanks.

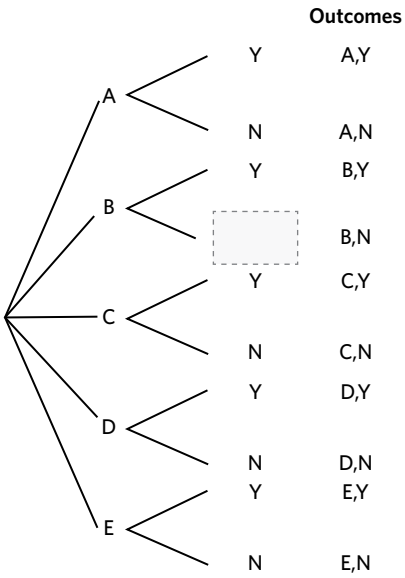
Example



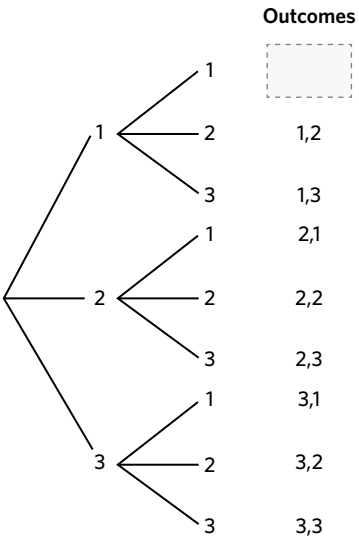
a.



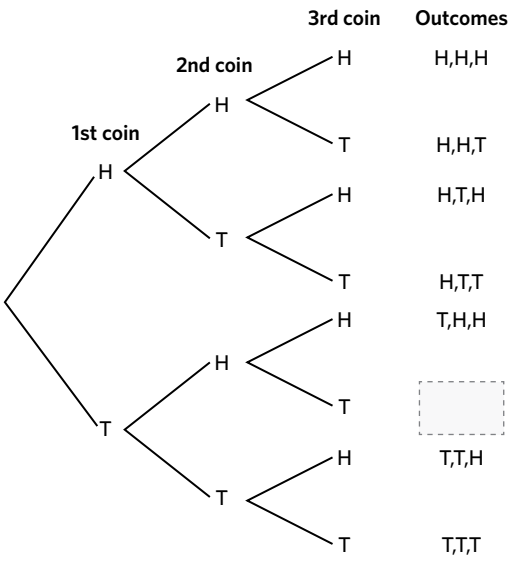
b.



c.



d.



2. Fill in the blanks.

Example

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

a.

	H	T
H	H,H	H,T
T	T,H	

b.

	1	2	3	4	5
B	B,1		B,3	B,4	B,5
R	R,1	R,2	R,3	R,4	R,5

c.

	1	2	3	4	5	6	7
1	1,1	1,2	1,3	1,4	1,5	1,6	1,7
2	2,1	2,2		2,4	2,5	2,6	2,7

d.

	R	O	C	K	S
M		M,O	M,C	M,K	M,S
A	A,R	A,O	A,C	A,K	A,S
T	T,R	T,O	T,C	T,K	T,S
H	H,R	H,O	H,C	H,K	H,S
S	S,R	S,O	S,C	S,K	S,S

3. Fill in the blanks by using the words provided.

tree diagram | two-way table | outcomes | multi-step

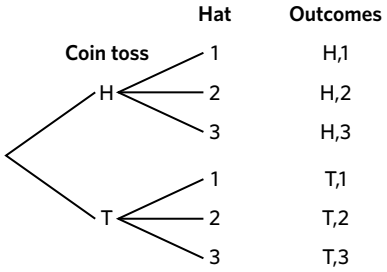
In looking at _____ experiments, there are two visuals that are helpful in displaying the full list of possible _____. The first is a _____, which shows the different branches that are possible from the first and subsequent steps. The second is a _____, which has column and row headings for the first and second steps of a two-step experiment.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b,c), 7(a,b,c), 8 (a,b), 9	4 (b,c,d), 5 (b,c,d), 6 (b,c,d), 7 (b,c,d), 8 (b,c), 9	4 (c,d,e), 5 (c,d,e), 6 (c,d,e), 7 (c,d,e), 8 (c,d), 9

4. The following tree diagram shows the outcomes for tossing a coin and then picking a number out of a hat. WE1



a. How many outcomes are possible?

b. Determine Pr(H,2).

c. Determine Pr(odd number).

d. Determine Pr(head or tail).

e. Determine Pr(tail and at least 2).

---

5. The following tree diagram shows the outcomes for two spins on a spinner. WE1

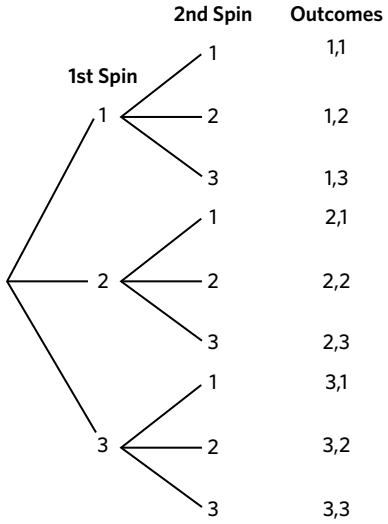
a. How many outcomes are possible?

b. Determine Pr(3,2).

c. Determine Pr(two of the same number).

d. Determine Pr(two different numbers).

e. Determine Pr(sum of 4 or less).



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6. The following two-way table shows the outcomes of picking the letters W, H, A and T out of a bag, followed by picking the letters T, H and E out of a second bag. WE2

a. How many outcomes are possible?

b. Determine Pr(T,E).

c. Determine Pr(one T and one H).

d. Determine Pr(forming a pronoun).

e. Determine Pr(at least one vowel).

	T	H	E
W	W,T	W,H	W,E
H	H,T	H,H	H,E
A	A,T	A,H	A,E
T	T,T	T,H	T,E

7. The following two-way table shows the outcomes of throwing two 6-sided dice. WE2

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

a. How many outcomes are possible?

b. Determine Pr(1,7).

c. Determine Pr(two different numbers).

d. Determine Pr(at least one '5').

e. Determine Pr(sum of no more than 9).

---

8. For the following scenarios, construct a tree diagram and a two-way table showing all possible outcomes. WE3

a. Haru tosses two coins.

b. Sukshma tosses a coin and then spins a wheel with four wedges numbered 1 to 4.

c. Fleur has an elephant (E), frog (F) and goblin (G) t-shirt in her t-shirt drawer. In her pants drawer, she has jeans (J), leggings (L) and tracksuit pants (T).

d. Elijah tosses a 5-sided die, then picks a letter out of a hat containing the letters A, B, C and D.

---

9. Roland flips a coin and then tosses an 8-sided die. What is the probability of obtaining a head or a number less than 3, or both? Select the correct response.

A. $\frac{5}{8}$ **B.** $\frac{1}{16}$ **C.** $\frac{9}{16}$ **D.** $\frac{11}{16}$ **E.** 11

Spot the mistake

10. Select whether Student A or Student B is incorrect.

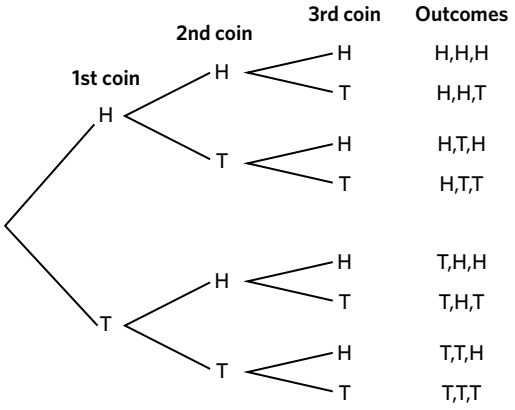
a. Mr Vincendese tosses three coins. List the outcomes.

Student A

	H	T	
H	H,H,H	H,T,H	**H**
T	T,H,T	T,T,T	**T**

There are 4 possible outcomes.

Student B



There are 8 possible outcomes.

b. Ms Lin takes two spins on a 4-wedged spinner. What is the probability of the the sum being greater than 6?

Student A

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
4	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Number of outcomes with sum greater than 6 $= 6$

Total number of outcomes $= 16$

Pr(sum greater than 6) $= \frac{6}{16}$

$= \frac{3}{8}$

Student B

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
4	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Number of outcomes with sum greater than 6 $= 3$

Total number of outcomes $= 16$

Pr(sum greater than 6) $= \frac{3}{16}$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Scar tosses two coins at the same time to determine the fate of his minions. How many possible combinations exist?

12. Nala plays two games of paper-scissors-rock with a friend. What is the probability of her friend choosing rock both times?

13. Sarabi throws two 6-sided dice in a game of Monopoly. How many possible combinations are there of getting the same number on both dice?

14. Simba plays Lucky Wheel at the school fete, where there are 10 different wedges, each with a different prize. If Simba pays for two spins, what is the probability of the wheel landing on two different wedges?

15. Pumba decides to dress up for the king's coronation. He has one red bow tie, one orange, one yellow, one white and one blue. He also has a number of hair ties to choose from. If there are 85 possible combinations of bow ties and hair ties, how many types of hair ties does Pumba have? Assume that there is one of each type of hair tie.

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Dungeons and Dragons is a fantasy roleplaying game that was first published in 1974. As of 2022, over 50 million people have played the game.
In Dungeons and Dragons, there are six types of dice, with 4, 6, 8, 10, 12 or 20 sides each. In one version of the game, there are four 6-sided dice, two 10-sided dice and one each of the other types.

Image: Undorik/Shutterstock.com

a. An 8-sided die and then a 4-sided die are rolled to determine spell damage. Determine how many possible pairs of numbers can be rolled with the two dice.

b. If two 8-sided dice are rolled, how many possible outcomes are there?

c. When two 8-sided dice are rolled, the numbers are not added together. Instead, the results form a two digit number, in the order that the dice are rolled. Given this game rule, what is the probability of rolling 23?

d. Using the same game rule, what is the probability of rolling an even number?

e. Why might some people enjoy games such as Dungeons and Dragons more than games such as Uno or Pokemon Go?

17. Consider the following.

1 coin tossed $=$ 2 outcomes

2 coins tossed $=$ 4 outcomes

a. How many possible outcomes are there when 3 coins are tossed?

b. How many possible outcomes are there when 4 coins are tossed?

c. Compare your answers for part **a** and **b**. What pattern can be identified?

Extra spicy

18. Ms Adash asks her student Michael and a second student Bena to each think of a number between 1 and 100. What is the probability that of the numbers chosen, one is odd and the other is even, in any order?

19. Standard 6-sided dice have pips (dots) where those on opposite sides always sum to seven. For the following stack of two dice, what is the probability that the number of pips on the two touching faces sum to at least seven? Express your answer as a percentage.

Image: Cordova Creative/Shutterstock.com

20. There are two types of human twins, fraternal and identical. Identical twins are always the same sex, whilst fraternal twins can be the same sex or different. If one third of twins are identical, what is the probability that any set of twins are both male and identical? Select the correct response.

A. $\frac{1}{6}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{3}$ **D.** $\frac{1}{2}$ **E.** $\frac{2}{3}$

21. A hat contains x student names from 8A and y names from 8B. One name is picked at random. What is the probability that the picked name is from 8A?

A. $\frac{x}{y}$ **B.** $\frac{x}{x+y}$ **C.** $\frac{y}{x+y}$ **D.** $\frac{x+y}{x}$ **E.** $\frac{x+y}{y}$

Remember this?

22. Paulo bought a mountain bike on sale at 20% off the original price. The original price was $450. How much did Paulo pay for the bike?

A. $90 **B.** $360 **C.** $370 **D.** $430 **E.** $540

23. Sarah's pet hamster has a hamster wheel. The wheel makes 56 full revolutions every 2 minutes. How many minutes will it take for the wheel to make 1200 revolutions? Select the correct response.

A. 21.4 **B.** 25.6 **C.** 33.8 **D.** 42.9 **E.** 56.0

24. Super SeaGear charges an hourly rate for renting marine equipment, as well as a flat fee. The overall cost C is represented by the formula $C = 35h + 80$, where h is the number of hours the gear is rented for.

Maisie rents some equipment to sail for 9 hours. How much does she pay? Select the correct response.

A. $115 **B.** $198 **C.** $255 **D.** $315 **E.** $395

Back to contents

10E Experimental probability

For some experiments, the theoretical probabilities of events can be calculated without conducting the experiments (such as rolling a die). For many other experiments (such as predicting the eye colour of students), the probabilities can only be estimated by conducting experiments. In this case the term experimental probability is used and it is an estimate of the theoretical probability.

LEARNING INTENTIONS

Students will be able to:

- understand the difference between theoretical and experimental probability
- calculate the experimental probability of an event given the results of the experiment
- calculate the expected number of occurrences given a probability and a number of trials
- understand that running more trials generally gives a better estimate of the true probability of an experiment.

KEY TERMS AND DEFINITIONS

An **experiment** is a series of trials conducted to examine the results of chance activities.

Frequency refers to how many times something occurs.

The **theoretical probability** states the likelihood of an event occurring in an experiment.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Sai Thaw Kyar/Shutterstock.com

Before medical drugs can be sold to the public, experiments (called clinical trials) need to be conducted on small samples of people to test if the drugs are safe and actually work. Scientists calculate experimental probabilities which state the likelihood of the drug working as intended.

Key ideas

1. The **experimental probability** can be used to estimate the theoretical probability of an event. It is determined by conducting multiple trials in an experiment.

$$\text{Pr(event)} = \frac{\text{frequency of event}}{\text{total number of trials}}$$

	Frequencies	Experimental probability
Experiment: flipping a coin Number of trials: 20	**Heads:** 14	$\Pr(\text{heads}) = \frac{14}{20} = 70\%$
	Tails: 6	$\Pr(\text{tails}) = \frac{6}{20} = 30\%$

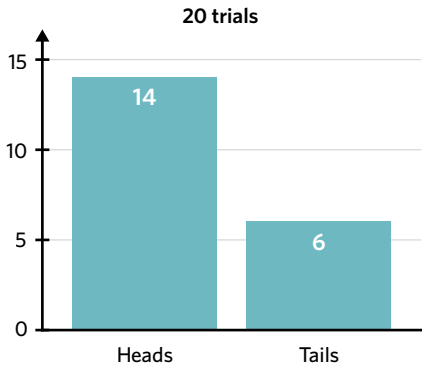
2. The value of the experimental probability approaches the value of the theoretical probability as more trials are conducted.

Theoretical probability:

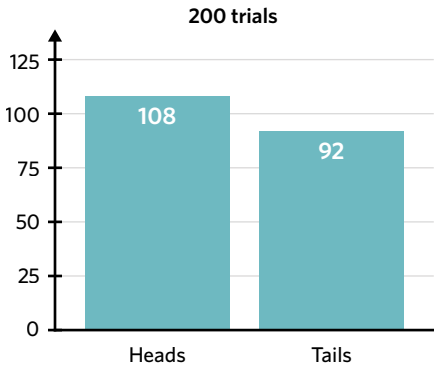
$\Pr(\text{heads}) = \frac{1}{2} = 50\%$

Continues →

Experimental probability:



$\Pr(\text{heads}) = \frac{14}{20} = 70\%$



$\Pr(\text{heads}) = \frac{108}{200} = 54\%$

3. The **expected occurrence** of an event, written as E(event), states how many times an event is expected to occur. It is calculated using probability of the event and the total number of trials conducted.

E(event) = Pr(event) × number of trials

Worked example 1

Determining experimental probabilities

Calculate the experimental probability of rolling a 3.

Outcome	1	2	3	4
Frequency	13	6	4	7

Working

Trials: 30

Frequency: 4

$$\Pr(3) = \frac{\text{frequency of 3}}{\text{total number of trials}}$$
$$= \frac{4}{30}$$
$$= \frac{2}{15}$$

Thinking WE1

Step 1: Determine the number of trials.

Step 2: Determine the frequency of the event.

Step 3: Calculate the experimental probability.

Student practice

Calculate the experimental probability of rolling a 3.

Outcome	1	2	3	4
Frequency	7	6	6	1

Worked example 2

Calculating expected occurrences

The counters below will be picked out at random.

For each event:

i. Calculate the theoretical probability.

ii. Calculate the expected occurrence for the given number of trials.

a. Picking an orange counter (50 trials). WE2a

Working

i. $\Pr(\text{orange}) = \dfrac{\text{number of orange counters}}{\text{total number of counters}}$

$= \dfrac{3}{10}$

ii. $\text{E(orange)} = \Pr(\text{orange}) \times \text{number of trials}$

$= \dfrac{3}{10} \times 50$

$= \dfrac{3}{10} \times \dfrac{50}{1}$

$= \dfrac{150}{10}$

$= 15$

Thinking

Step 1: Calculate the probability of the event.

Step 2: Multiply the probability of the event by the number of trials.

Step 3: Simplify.

b. Picking a blue or green counter (30 trials). WE2b

Working

i. $\Pr(\text{blue or green}) = \dfrac{\text{number of blue and green counters}}{\text{total number of counters}}$

$= \dfrac{7}{10}$

ii. $\text{E(blue or green)} = \Pr(\text{blue or green}) \times \text{number of trials}$

$= \dfrac{7}{10} \times 30$

$= \dfrac{7}{10} \times \dfrac{30}{1}$

$= \dfrac{210}{10}$

$= 21$

Thinking

Step 1: Calculate the probability of the event.

Step 2: Multiply the probability of the event by the number of trials.

Step 3: Simplify.

Student practice

A standard six-sided die was rolled.

For each event:

i. Calculate the theoretical probability.

ii. Calculate the expected occurrence for the given number of trials.

a. Rolling a 4 (12 trials).

b. Rolling a 2 or a 5 (30 trials).

10E Questions

Understanding worksheet

1. For each event, determine whether a theoretical probability or an experimental probability only can be determined.

Example

Event	Theoretical probability	Experimental probability only
Rolling a standard six-sided die and landing a 4 or a 5.	☑	

	Event	Theoretical probability	Experimental probability only
a.	After surveying a class, selecting a student in a class with brown eyes.	☐	☐
b.	A customer buying a cinnamon doughnut.	☐	☐
c.	Picking a red marble out of a bag containing three blue, two red and five black marbles.	☐	☐
d.	Khaby Lambe reaching 1 000 000 views on his next TikTok video.	☐	☐

2. An experiment is conducted where a spinner with three colours is spun. Use each table to determine the frequency for each event, and the total number of trials.

Example

Outcome	Red	Green	Blue
Frequency	4	5	9

Event: Landing on red

Frequency: 4

Trials: 18

a.

Outcome	Red	Green	Blue
Frequency	5	7	2

Event: Landing on green

Frequency:

Trials:

b.

Outcome	Red	Green	Blue
Frequency	4	9	4

Event: Landing on blue

Frequency:

Trials:

c.

Outcome	Red	Green	Blue
Frequency	6	2	1

Event: Landing on red or green

Frequency:

Trials:

d.

Outcome	Red	Green	Blue
Frequency	7	14	4

Event: Landing on green or blue

Frequency:

Trials:

3. Fill in the blanks by using the words provided.

trials | theoretical | experimental | frequency

When an experiment is conducted where a die is rolled a number of times, the _____ probability can be calculated by using the _____ of each event. When the probability is predicted before the experiment is conducted, this is called the _____ probability. More accurate experiments can be conducted by increasing the number of _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c), 7	4 (b,c,d,e), 5 (b,c,d,e), 6 (c,d,e), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (e,f,g), 7

4. Six experiments were conducted where a six-sided die was rolled a different number of times. WE1
For each experiment, calculate the experimental probability of rolling a 4.

a.

Outcome	1	2	3	4	5	6
Frequency	1	2	2	2	3	0

b.

Outcome	1	2	3	4	5	6
Frequency	6	5	4	5	6	4

c.

Outcome	1	2	3	4	5	6
Frequency	10	7	7	10	9	7

d.

Outcome	1	2	3	4	5	6
Frequency	5	16	24	18	23	14

e.

Outcome	1	2	3	4	5	6
Frequency	36	32	27	38	34	3

f.

Outcome	1	2	3	4	5	6
Frequency	84	92	76	89	85	74

5. A computer is randomly generating numbers between 1 and 10 (including the numbers 1 and 10). WE2

For each event:

i. Calculate the theoretical probability.
ii. Calculate the expected occurrence for the given number of trials.

a. Generating a 10 (10 trials).
b. Generating a 3 or a 5 (15 trials).
c. Generating an odd number (30 trials).
d. Generating at least a 3 (65 trials).
e. Generating a prime number (70 trials).
f. Generating a multiple of 3 or an even number (90 trials).

6. In an experiment, a spinner with four colours (red, blue, purple, green) was spun. The results are shown below:

Outcome	Red	Blue	Purple	Green
Frequency	3	6	5	2

a. How many times was the spinner spun?

b. Calculate the experimental probability of landing on red.

c. What is the expected occurrence of landing on red after 32 trials?

d. Calculate the experimental probability of landing on red or blue.

e. What is the expected occurrence of landing on red or blue after 96 trials?

f. Calculate the experimental probability of not landing on purple or red.

g. What is the expected occurrence of not landing on purple or red after 200 trials?

---

7. Three cards labelled 1, 2 and 3 are put into a hat. Over 20 trials, a card is pulled out randomly and put back. The results are shown below:

Outcome	1	2	3
Frequency	7	8	5

What is the experimental probability and theoretical probability of pulling out a 2 or a 3?

	Experimental	**Theoretical**
A.	$\frac{2}{5}$	$\frac{1}{3}$
B.	$\frac{2}{5}$	$\frac{2}{3}$
C.	$\frac{2}{3}$	$\frac{13}{20}$
D.	$\frac{13}{20}$	$\frac{2}{3}$
E.	$\frac{1}{3}$	$\frac{2}{5}$

Spot the mistake

8. Bel was drawing cards from a standard deck of 52 cards. Each time she noted down the suit of the card (club ♣, diamond ♦, heart ♥ and spade ♠). The results are shown below.

Outcome	Club	Diamond	Heart	Spade
Frequency	3	6	5	6

Select whether Student A or Student B is incorrect.

a. Calculate the experimental probability of drawing a diamond card.

Student A

Frequency of diamonds: 6

Number of trials: 20

$$\Pr(\text{diamond}) = \frac{\text{frequency of diamonds}}{\text{total number of trials}}$$

$$= \frac{6}{20}$$

$$= \frac{3}{10}$$

Student B

$$\Pr(\text{diamond}) = \frac{\text{diamonds}}{\text{number of suits}}$$

$$= \frac{1}{4}$$

b. What is the expected occurrence of drawing a diamond after 50 trials?

Student A

$$\begin{aligned} E(\text{diamond}) &= \frac{3}{10} \times 20 \\ &= \frac{60}{10} \\ &= 6 \end{aligned}$$

Student B

$$\begin{aligned} E(\text{diamond}) &= \frac{3}{10} \times 50 \\ &= \frac{150}{10} \\ &= 15 \end{aligned}$$

Problem solving

Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

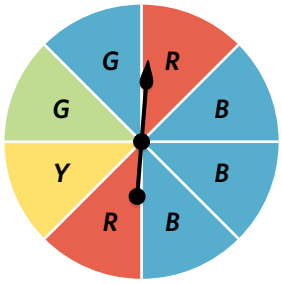
9. Cindy plays Fortnite and is checking the statistics of the games she has played in the last week. The results are shown below.

Outcome	Top 25	Between 26 and 75	Bottom 25
Frequency	5	15	10

Based on the table above, what is the experimental probability of Cindy being in the top 25 in her next match?

10. A shop sells four types of toasties: chicken schnitzel, smashed avocado, roast veggie and roast beef. Maddy predicts that the probability of a customer ordering the roast veggie toastie is $\frac{3}{10}$. If the shop has 150 toastie orders in one day, how many are expected to be roast veggie toasties?

11. Noah is spinning a spinner that he constructed.



He records the colour each time the spinner is spun. There were 10 reds, 23 blues, 4 greens and 8 yellows.

Is the experimental probability of landing on red or blue greater than, equal to or less than the theoretical probability?

12. A factory that produces computer chips for smartphones estimates that the probability of a chip being faulty is 0.002. How many chips are expected to be faulty in a day if the factory produces 2000 per day?

13. A weather reporter stated that the probability of the daily temperature being over 20 degrees in a month in Sydney was 30%. If this corresponded to 9 days, over what number of days was the temperature recorded?

Reasoning

Question working paths

Mild 14 (a,b,c,e) | Medium 14 (a,b,c,e), 15 (a,b) | Spicy All

14. Researchers have developed a drug to help reduce the severity of headaches. Two experiments were conducted to test the effectiveness of the drug. They recruited participants online. Two days after taking the drug, the patients were asked whether the drug resulted in a less severe headache, a more severe headache or had no effect. The results are shown below.

There were different patients in each experiment.

Survey response	Less severe headache	No effect	More severe headache
Experiment 1	42	6	2
Experiment 2	240	108	52

a. How many trials were conducted in each experiment?

b. Calculate the experimental probabilities of the drug resulting in a less severe headache for each experiment.

c. If 600 people with a headache took the drug, how many people would be expected to experience a more severe headache for each experiment?

d. Which experimental probability is likely to be closest to the theoretical probability? Explain.

e. List two ways that each experiment could be improved to produce more accurate results.

15. A four-sided die numbered 1 to 4 is to be rolled 20 times in an experiment.

Outcome	1	2	3	4
Frequency	☐	☐	☐	☐

a. Fill in the frequencies in the table, so that the theoretical probability of rolling a 1 is equal to or nearly equal to the experimental probability.

b. Fill in the frequencies in the table, so that the experimental probability of rolling a 1 is at least two times the theoretical probability.

c. If the experimental probability is greater than the theoretical probability, does this mean that you are more likely to roll a 1 on the 24th roll? Explain your answer.

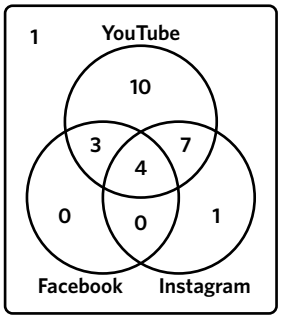
Extra spicy

16. Two coins are flipped and the outcomes are recorded. What is the expected occurrence of flipping two tails after 200 trials?

A. 25 **B.** 50 **C.** 75 **D.** 100 **E.** 150

17. During June of 2021, it rained for 18 days, and the temperature was less than 10°C for 22 days. Construct a tree diagram to represent the experimental probabilities. Assume a day's temperature is not affected by whether it rains.

18. The Venn diagram shows the results of a survey of a class asking them which social media platforms they use.



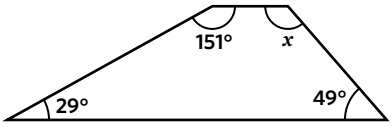
Use this information to predict how many in the entire school of 500 students are expected to use two or more social media platforms.

A. 14
B. 77
C. 192
D. 269
E. 280

19. A news site stated that out of 500 people, 120 are expected to own a Nintendo Switch and an Xbox One, 50 people own neither, and in total 300 own a Switch. Students used this to predict how many students in their class of 25 own either Nintendo Switch or an Xbox One, both or neither. How many students are expected to own a Nintendo Switch and not an Xbox One?

Remember this?

20. What is the size of the missing angle?



A. 29 **B.** 49 **C.** 131
D. 151 **E.** 180

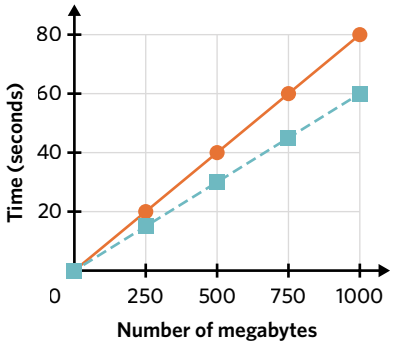
21. Two friends have different internet speed plans, plan A and plan B.

Plan A can download 750 megabytes per minute.

Plan B can download 1000 megabytes per minute.

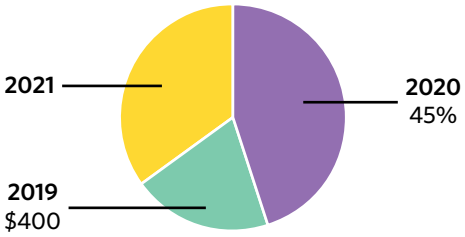
The friend with plan A downloads a 1250 megabyte file in 100 seconds.

How long (in seconds) does it take for the friend with plan B to download the same file?



A. 50 **B.** 75 **C.** 100
D. 125 **E.** 150

22. The pie chart below shows how an esports club spent a $2000 grant over a 3-year period.



How much money was spent in 2021?

A. $400 **B.** $445 **C.** $700 **D.** $900 **E.** $1555

Chapter 10 extended application

1. There are 108 cards in a game of Uno, of which 76 are number cards, 24 are action cards and 8 are wild cards. The image shows this breakdown of cards.

Image: i am Em/Shutterstock.com

a. From the deck, Kevin takes one set of red number cards, with the numbers 1 through to 9. List the sample space for the cards that Kevin takes.

b. If Kevin draws one card only from his set, what is the chance that the card will not be a '3'? Give your answer as a fraction.

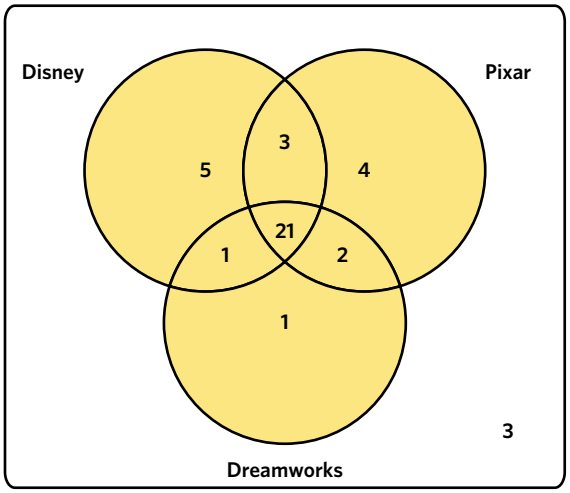
c. From the nine red number cards, Kevin draws one card and places it back, then draws a second card and places it back. Using technology or by hand, construct a two-way table showing the different combinations that the two cards could take.

d. What is the probability that the sum of the two cards is at least 16? Give your answer as a simplified fraction.

e. Kevin places his red number cards back into the deck. Kevin's classmate Keanna repeatedly draws a card from the full deck and then puts it back. If Keanna draws 45 times, how many times would you expect her to draw an action card?

f. Uno is a game that can be played by 2 to 10 players. Do you think the game is better with more players or less players? Explain your answer.

2. Zahraa conducts a survey of students in her year level, to determine whether students like movies made by Disney, Pixar and Dreamworks studios. The results of the survey are shown in the following Venn diagram.



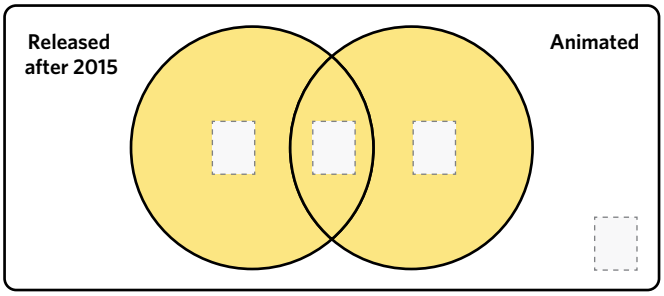
a. How many students did Zahraa survey?

b. How many students like Disney and Pixar movies, but not Dreamworks?

c. What is the probability of picking a student who enjoys movies from all three studios?

d. What is the probability of picking a student who enjoys Disney or Dreamworks or both?

e. Research 10 movies that you have watched or want to watch, to complete the following Venn diagram.



3. Countries participate in an international paper plane competition called Over The Moon. In 2022, the top contestants had the following times for their paper plane having the longest airtime.

Rank	Country	Airtime (seconds)
1	Canada	14.72
2	USA	14.01
3	Japan	13.69
4	Chile	13.51
5	Japan	13.42
6	Bosnia and Herzegovina	13.31
7	Australia	13.27
8	Japan	13.01
9	Austria	12.72
10	Andorra	12.60

Image: Pasuwan/Shutterstock.com

a. Based on the results table, what is the probability that a top 10 contestant is from Australia? Give your answer as a decimal.

b. The winning contestant conducted 300 trials and of those, 80 throws achieved airtimes of over 14 seconds. Determine the experimental probability of achieving an airtime of over 14 seconds, giving your answer as a percentage to 1 decimal place.

c. If the probability of achieving an airtime of over 13 seconds is $\frac{3}{10}$ on any particular throw and a contestant conducts 50 trials for practice, how many throws are expected to have an airtime of over 13 seconds?

d. The 2017 winner for longest airtime was Australian contestant called Kino. Kino conducted many trials in order to create the winning plane. Explain why increasing the number of trials might lead to more certainty on the effectiveness of a particular paper plane design.

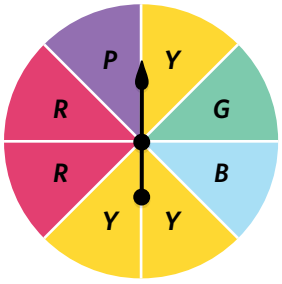
e. Other than paper plane design, what is one other factor that might contribute to winning a paper plane competition?

Chapter 10 review

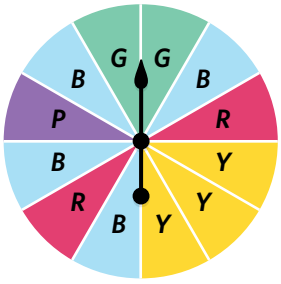
Multiple choice

1. For which of these spinners is the probability of landing on blue equal to $\frac{1}{4}$? 10A

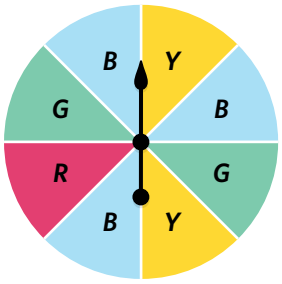
A.



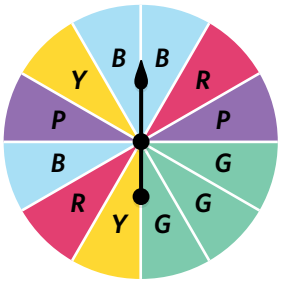
B.



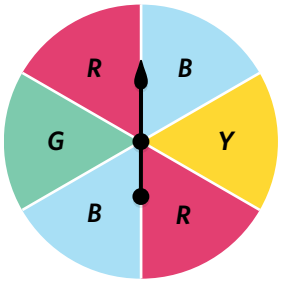
C.



D.



E.



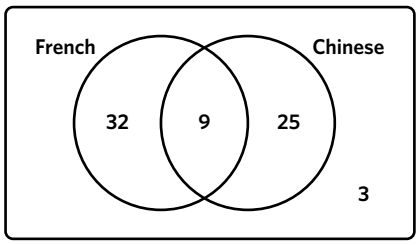
2. Determine which of the following shows two events that are correctly classified as complementary or not complementary. 10B

	Event 1	Event 2	Complementary	Not complementary
A.	Being a sibling	Being an only child	☐	☑
B.	Rolling at least a two on a die	Rolling two or less on a die	☑	☐
C.	Picking a number card from a deck of cards	Picking a picture card from a deck of cards	☑	☐
D.	All students attend class	No students attend class	☑	☐
E.	Rolling an even number on a die	Rolling an odd number on a die	☐	☑

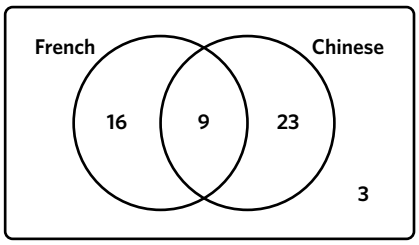
3. A group of 50 Year 8 students were asked what language they learn. It was found that 32 students learn French, 25 students learn Chinese, 9 students learn both and 3 students don't learn either. 10C

Which of the following Venn diagrams correctly depicts the information provided?

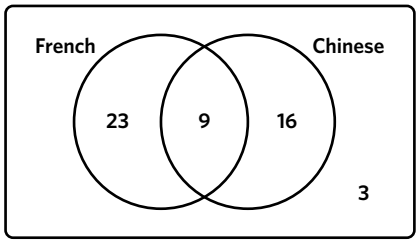
A.



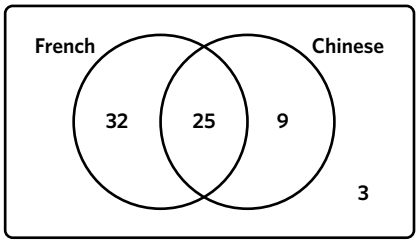
B.



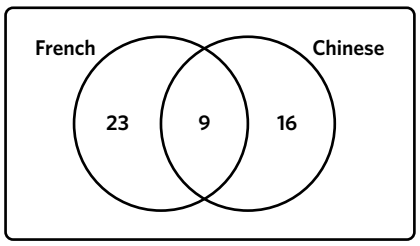
C.



D.



E.



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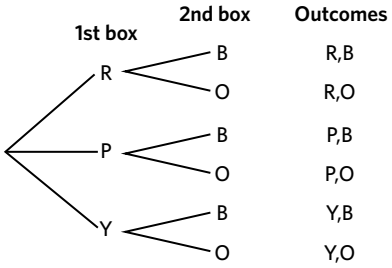
4. Sarah has two boxes of macarons, the first with red (R), pink (P) and yellow (Y) macarons, and the second with brown (B) and orange (O) macarons. She picks one macaron from the first box and another from the second. The tree diagram below shows the possible outcomes. 10D

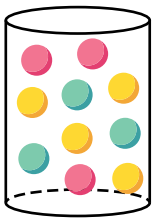


What is the probability of Sarah selecting a pink macaron from the first box and then a brown macaron from the second?

A. $\frac{1}{9}$ **B.** $\frac{1}{8}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{3}$ **E.** $\frac{2}{3}$

---

5. The following diagram shows a bucket with different coloured balls, where balls will be picked out at random. 10E



If there are 60 trials, what is the expected occurrence of picking a yellow ball?

A. 4
B. 10
C. 18
D. 20
E. 24

Fluency

6. Letters in the word OPPORTUNITIES are to be randomly selected in an experiment. 10A

a. List the sample space of the experiment.

b. Calculate Pr(consonant), the probability of choosing a letter that is a consonant.

c. True or false: the probability of choosing the letter P is twice as likely as the probability of choosing the letter R.

d. What are three events that are equally as likely as choosing the letter O?

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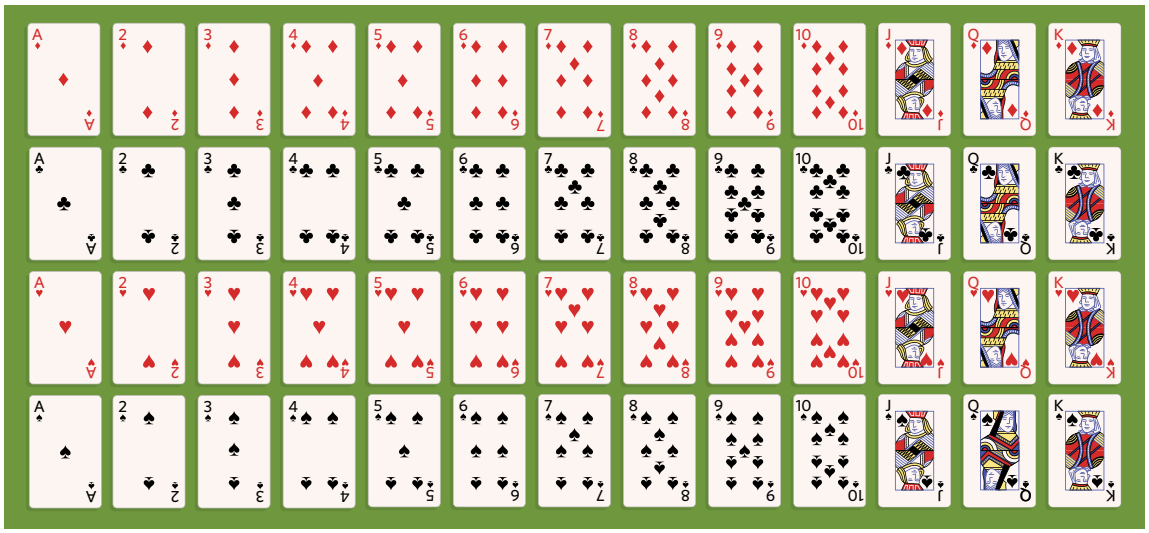
7. A card is drawn at random from a standard deck of 52 playing cards. Note that the jokers have been removed, and aces are considered to have a value of 1. 10B



Write your answer as a simplified fraction where possible.

a. State the event(s) complementary to picking a black card.

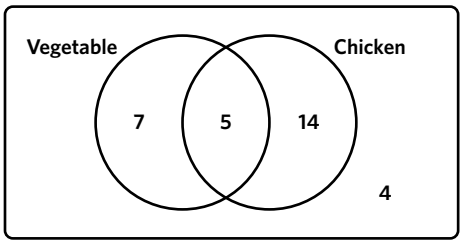
b. Calculate Pr(picture card′).

c. Calculate Pr(a number less than 5 and not red).

d. Calculate Pr(not an odd number or a picture card).

---

8. Consider the following Venn diagram showing 30 students and whether they bought a vegetable or a chicken burger. 10C



Write your answer as a simplified fraction where possible.

a. Determine Pr(neither vegetable or chicken).

b. Determine Pr(chicken).

c. Determine Pr(not vegetable).

d. Determine Pr(vegetable or chicken but not both).

9. Consider the following two-way table of students in Year 8 who use Tiktok and Instagram. 10C

	Uses TikTok	Does not use TikTok	Total
Uses Instagram	14	12	26
Does not use Instagram	18	16	34
Total	32	28	60

Write your answer as a simplified fraction where possible.

a. Determine Pr(uses Instagram but does not use TikTok).

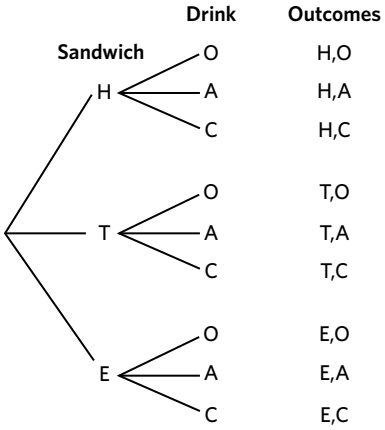
b. Determine Pr(uses TikTok).

c. Determine Pr(uses Instagram or TikTok but not both).

d. Determine Pr(does not use TikTok).

---

10. There are three sandwiches at the school cafeteria which Mia can choose from: a ham (H) sandwich, tuna (T) sandwich or an egg (E) sandwich. Mia can also choose a drink to go with her sandwich, where there is orange juice (O), apple juice (A) or coconut water (C). The following tree diagram shows the possible outcomes of this scenario. 10D



Write your answer as a simplified fraction where possible.

a. Determine Pr(E,O).

b. Determine Pr(any juice).

c. Determine Pr(not egg).

d. Determine Pr(ham or egg).

---

11. The following two-way table shows the outcomes of picking Jess (J), Erica (E) and Nick (N) to participate in a game, where each participant is to select a numbered tile from a bag containing the number 2, 5, 9 or 10. 10D

	2	**5**	**9**	**10**
J	J,2	J,5	J,9	J,10
E	E,2	E,5	E,9	E,10
N	N,2	N,5	N,9	N,10

Write your answer as a simplified fraction where possible.

a. Determine Pr(N,9).

b. Determine Pr(first letter of name is a consonant).

c. Determine Pr(at least 9).

d. Determine Pr(prime number).

12. A six-sided, standard die is rolled numerous times. 10E

For each event:

i. Calculate the experimental probability, written as a fraction.

ii. Calculate the expected occurrence for the given number of trials.

a. Rolling an even number (50 trials)

b. Rolling 4 or less (30 trials)

c. Rolling 3 or 6 (15 trials)

d. Rolling a prime number (70 trials)

---

13. Customers outside Pawtastic Pet Store were surveyed for what type of pet they own. Each customer owns one pet only. The results are shown below. 10E

Outcome	Dog	Cat	Guinea Pig	Goldfish	Bird
Frequency	10	8	6	9	3

a. How many customers were surveyed?

b. Calculate the experimental probability of a customer owning a pet that does not have four legs. Express your answer as a fraction.

c. What is the expected occurrence of a customer owning an aquatic pet if 60 customers were to be surveyed?

d. What is the expected occurrence of a customer not having a bird or a guinea pig if 140 customers were to be surveyed?

Problem solving

14. In a gameshow, a contestant is to select numbered tiles from a bag. The tiles contain odd numbers between 0 and 35. What is the probability the contestant will select a tile with a number under 10? 10A

---

15. Three houses are competing in a swimming carnival: blue, red and yellow. The probability of the blue house winning is $\frac{1}{5}$, and the red and yellow houses are both twice as likely to win as the blue house. Find the probability that the red house does not win. 10B

---

16. A travel agency has collected data on 70 families who have booked a holiday to either the Netherlands or France for next July or August. From their sample, 21 families have booked to go to the Netherlands in July, and 25 families wish to go to France in August. In total, 35 families have booked holidays in July and 31 families have booked holidays to the Netherlands. How many families want to go to the Netherlands in August or France in July? 10C

---

17. Dhileena is trying to plan her outfit for her school formal. She has eight different dresses to choose from in her wardrobe. She also has many different necklaces to choose from in her jewellery cabinet which she could wear with her dress. If there are 136 possible combinations of dresses and necklaces, how many necklaces does Dhileena have? 10D

---

18. Hannah is a call centre sales representative. The probability of her making a sale in any call is 25%. At least how many calls will Hannah need to make if she wishes to make 5 sales? 10E

Reasoning

19. A study of the services provided by widget repair persons was conducted to observe the relationship between quality of service and whether the worker was factory trained or not. The findings are summarised in the two-way table below.

Training	Satisfactory service	Unsatisfactory service
Factory trained	72	48
Not factory trained	32	64

a. Convert the table into a Venn diagram.

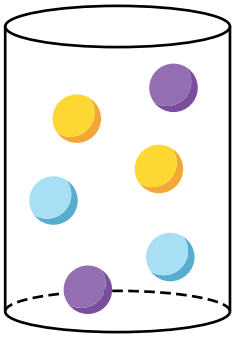
b. How many workers took part in the study?

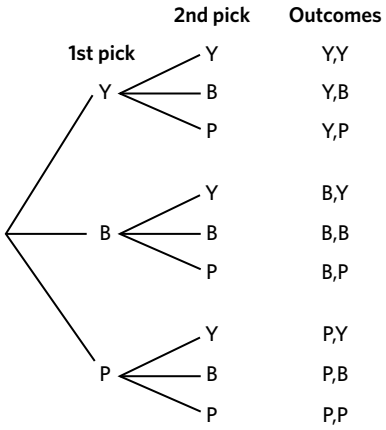
c. What is the probability that a worker chosen at random is factory trained or provides satisfactory service, but not both?

d. If a separate study was to survey 450 workers who were not factory trained, how many would be expected to not provide satisfactory service?

e. List two reasons why it might be important to conduct studies and collect data on the quality of repair services.

20. Jen is to pick out two coloured counters one after the other at random from the bucket shown below. The tree diagram depicts the list of possible outcomes, where 'Y' is Yellow, 'B' is Blue and 'P' is Purple.





Write your answer as a simplified fraction where possible.

a. Jen picks out a blue counter first, and then puts it back in the bucket. She then picks out another counter. What is the probability that this second counter is also blue?

b. Jen does the same again, this time picking out a purple counter and then putting it back. What is the probability that her next counter will be yellow?

c. Now suppose that Jen picks out a yellow counter and does not place it back in the bucket. What is the probability that the next counter she picks will be yellow?

d. Compare your answers from parts **a–c**. What happens to the probabilities if the counters are not replaced?

Back to contents

11

Rates and ratios

Number and Algebra

Research summary

11A Introduction to ratios

11B Equivalent ratios

11C Dividing a quantity in a given ratio

11D Introduction to rates

11E Solving rate problems

11F Speed

Chapter 11 extended application

Chapter 11 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 11 research summary

Rates and ratios

Big ideas

Proportional reasoning

This chapter will focus on students' ability to identify and apply their extensive understanding of proportional reasoning and fraction knowledge to understand the relationship between ratios of fractions, how to find equivalent ratios, divide quantities in a given ratio, and problem solve with rates.

Understanding proportional relationships, non-proportional relationships, and how to reason with proportional relationships is important for sensemaking and problem solving (Lamon, 2007; Weiland et al., 2020). Additionally, the concept of the unit rate should be 'interpreted as the functional, structural relationship in proportional reasoning and as the constant of proportionality in a direct variation situation' (Weiland et al., 202 p 184). Furthermore, it is essential that students view ratios as having a constant relationship between the two quantities (Weiland et al., 2020). For example, the ratio $2:1$, can be viewed as two cups of water for every 1 cup of water.

Covariational thinking (i.e. thinking about how two quantities have a constant relationship and vary together) is essential in reducing proportional reasoning misconceptions (Dougherty et al., 2016) and in understanding the third abstractable quantity or intensive quantity (Weiland et al., 2020). For example, the relationship between water and rice creates an abstractable quantity of texture, and the relationship of distance to time is speed.

Abstractable quantities underpin student's understanding of how quantities compare in a ratio, and that by changing one quantity of the ratio without similarly changing the other affects the abstractable quantity (Weiland et al., 2020). For example, increasing the amount of water at a different rate than the amount of rice will increase the softness of the rice.

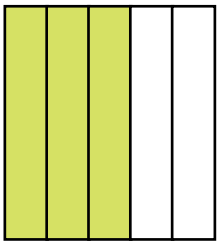
Ratios are closely linked together by the inseparable unit called the composed unit (Lobato et al., 2014). Thinking of ratios as a composed unit will strengthen students' understanding of the relationship between the two quantities (Dougherty et al., 2016). For example, rather than thinking of cooking rice as 2 part rice for every 1 part water, the composing unit is $2:1$ and not two separate quantities.

It is essential that ratios are thought of using multiplicative and proportional comparisons and not additive comparisons as this will cause misconceptions (Lamon, 2007;, Lobato et al., 2014). Using proportional and multiplicative reasoning allows ratios to be reasoned as 'having a fixed number to parts that are variable in size as well as a varying number of groups that are made up of a fixed number of units' (Weiland et al., 2020 p 184).

Lastly, fractions and ratios share similar but also different characteristics. The similarities and differences of these concepts can affect student's interpretation and calculation of ratios (Weiland et al., 2020). It is essential that students understand that part-part relationships are solely for ratios, while part-whole relationships can be represented by a ratio or a fraction. For example ratios can be represented $\frac{2}{5}$ which may represent 2 parts to 5 parts, while the fraction $\frac{2}{5}$ represents a single quantity, a quotient, or an operator.

This big idea is essential as it develops the fundamental skills needed for algebra such as understanding gradient, constant rate of change, and similar figures and underpins trigonometric relationships found in triangles. Furthermore this concept extends outside of mathematics to science when using quantities such as density, acceleration, and velocity.

Generalising



This chapter will build upon students' arithmetic and factual generalisation to create contextual and symbolic generalisation of rates, ratios, and unit rates to solve problems.

Strategies which are not context or task specific can lead to students developing a deeper contextual understanding of the concept of ratio (Cai & Wang, 2005). Therefore, students should be given the opportunity to create their own meaning of rules, relationships, and mathematical concepts (Cai & Wang, 2005). For example, give students this problem and ask them how they could see $\frac{3}{5}$ of something and $\frac{5}{3}$ of something (Sowder et al., 1998).

Due to the nature of proportional relationships and student's interesting and creative ways they invent strategies to reason with proportions, it is essential that teachers recognise and have a robust understanding of proportional reasoning and other topics of mathematics (Weiland et al., 2020). Furthermore certain known strategies such as the scalar strategy, function strategy and cross product strategy should be carefully considered when they are taught as they can indicate different levels of understanding of proportional reasoning (Lautert & Schliemann, 2020: Carney et al., 2022). For example, these two methods are correct in solving for ratios, however they require two different explanations for why they are correct.

Method 1	**Method 2**
20 : 10 ÷10 ÷10 2 : 1 ×50 ×50 100 : 50	$\frac{20}{10} = \frac{?}{50}$ $20 \times 50 = 10 \times ?$ $1000 = 10 \times ?$ $100 = ?$

Lastly, students will need to formulate generalisations for three different types of proportional problems; simple, double and multiple proportional relationships (Vergnaud, 1988). Having a meaningfully generalised understanding of multiplicative comparison relationships can increase a student's understanding of gradient in an equation as both a composed unit and a multiplicative comparison (Carney et al., 2022). Additionally, students often find it easier to understand a composed unit rather than a multiplicative comparison. For example, Table 1 describes how different statements relate to different conceptions and Figure 1 represents the item types based on the difficulties.

Item statements	**Student conception**
4 coffees for $13 dollars, how many coffees can you buy with $9?	Informal reasoning
It would cost □ to buy 15 coffees	Composed unit
Number of coffees = □ × cost	Multiplicative comparison

Table 1 Student conceptions related to item statements (adapted from Carney et al., 2022).

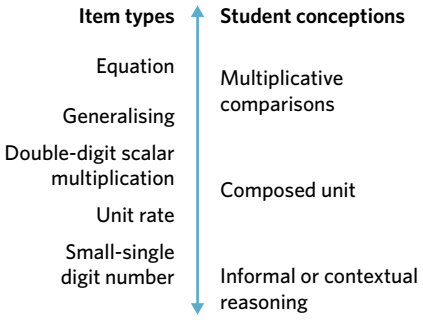


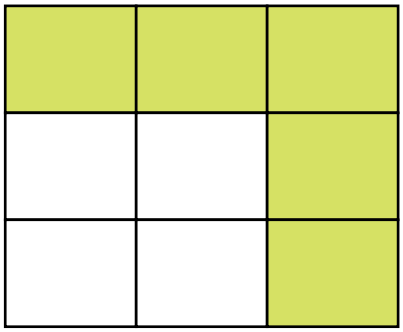
Figure 1 Item types based on difficulties (adapted form Carney et al., 2022)

This big idea is built upon a strong understanding of proportional understanding, number and number properties, patterns, and proportional reasoning. Therefore, a conceptual understanding of how to manipulate ratios, rates and unit rates is essential for success in future mathematics topics especially in functions, graphing, calculus, chemistry, and physics.

Visual representations

Bar model

Bar models, also known as tape or strip diagrams, help students construct a pictorial equation from a word problem as a whole rather than as distinct parts. The bar model is usually represented by a rectangle representing the relationship between the known and unknown quantities and can be divided to represent proportional relationships. In this chapter, bar models will aid in proportional relationships.

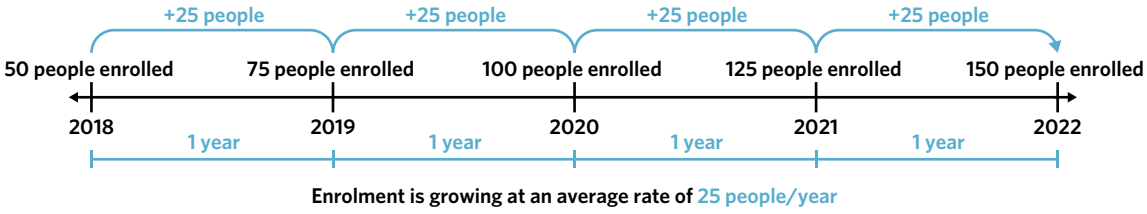


It is important to remember that representations should be presented concurrently with symbols to support students' learning (Dougherty et al., 2016). Additionally concrete drawings and strategies do provide students with an entry level of understanding and have limitations. It is important for students to use concrete drawing strategies as an entry point when learning and should move onto more algebraic generalisations to further their conceptual development (Cai & Wang, 2005).

Number lines

A number line is traditionally shown as a horizontal line or axis but can be a vertical line. A number line shows the order and size of numbers and helps deepen our understanding of mathematical concepts such as fractions, decimals, percentages, negative numbers and mathematical operations.

This chapter uses a double number line to aid in learning about proportional relationships. Double number lines can support reasoning proportionally (Lobato and ellis2010). Again, it is essential that students use double number lines as an entry point when learning, and use symbols concurrently to support their learning (Dougherty et al., 2016; Cai & Wang, 2005).



Enrolment is growing at an average rate of 25 people/year

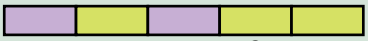
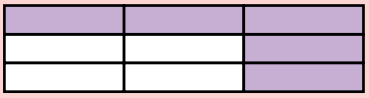
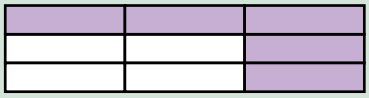
Cartesian plane

Cartesian coordinates describe a plane with two intersecting perpendicular lines, representing each dimension of the plane, that form four quadrants. Ordered pairs (points) are located on the plane by their coordinates, which provide their location within the plane. Points can be connected to create figures that lie on the plane.

In this chapter, the cartesian plane will be used to strengthen students' understanding of abstractable quantities and covariational thinking. Tables, graphs, and other representations of concepts can aid in conceptual understanding (Dougherty et al., 2016).

Additionally, the visual representation of the cartesian plane relies both as a process and an object for students. Students must be able to move fluidly from constructing and interpreting the cartesian plane and algebraic functions, and proportional reasoning as this is foundational to multiple representations within algebra (Moschkovich et al., 1993).

Misconceptions

Misconception	Incorrect	Correct	Lesson
Students apply the concepts of fractions to ratios.	 Part : Part $= \frac{2}{5}$	 Part : Part $= \frac{2}{3}$	11A
Students assume order does not matter when writing ratios.	 Shaded : Non-Shaded 4 : 5	 Shaded : Non-Shaded 5 : 4	11A 11C
Students think that proportional relationships are additive.	1 : 2 = 2 : 3	1 : 2 = 2 : 4	11A 11B
Students affect the proportion of the ratio by multiplying or dividing the ratio by different numbers.	2 : 3 ×2 ×3 4 : 9	2 : 3 ×2 ×2 4 : 6	11B 11E
Students affect the proportion of the ratio by multiplying or dividing only one part of the ratio.	4 : 10 ÷2 2 : 10	4 : 10 ÷2 ÷2 2 : 5	11B 11E
Students assume that ratios do not have a fixed number of parts and treat the ratio as separate quantities when dividing them to calculate values of parts in a ratio.	24 in 1 : 3 24 ÷ 1 = 24 24 ÷ 3 = 8	24 in 1 : 3 1 + 3 = 4 parts 24 ÷ 4 = 6 per part	11C
Students are unsure how units are included when calculating rates.	25 or 25 km or 25 km/h or 25 h	$25\frac{\text{km}}{\text{h}}$ or 25 km\h	11D
When calculating a unit rate, students assume the quantities can be divided in any order.	100 L in 2 hours $\frac{2}{100}$ = 0.02 L/h	100 L in 2 hours $\frac{100}{2}$ = 50 L/h	11D
Students leave ratios unsimplified.	72 people in 12 minutes $\frac{72}{12} = \frac{36}{6}$ people/min	72 people in 12 minutes $\frac{72}{12}$ = 6 people/min	11D
Students disregard the proportion of the ratio and use any part of the ratio to solve problems.	20 coffees in 10 minutes ? coffees in 50 minutes 20 × 50 = 1000 ∴ 1000 coffees in 50 minutes	20 coffees in 10 minutes ? coffees in 50 minutes 20 : 10 ÷10 ÷10 2 : 1 ×50 ×50 100 : 50 ∴ 100 coffees in 50 minutes	11E
Students find the unit rate only.	20 coffees in 10 minutes ? coffees in 50 minutes Unit rate: $\frac{20}{10}$ = 2 ∴ 2 coffees	20 coffees in 10 minutes ? coffees in 50 minutes Unit rate: $\frac{20}{10}$ = 2 coffees/min 2 × 50 = 100 ∴ 100 coffees	11E
Students use a different arrangement of the speed formula to calculate distance, time or speed.	s = 60 km/h t = 3 hours $d = \frac{60}{3}$ = 20 km	s = 60 km/h t = 3 hours d = 60 × 3 = 180 km	11F

Misconception	Incorrect	Correct	Lesson
Students change the proportions of the abstractable quantity, speed, when solving problems.	How long does it take to travel 180 km at 120 km every 2 hours? $\frac{180}{120} = 1.5$ hours	How long does it take to travel 180 km at 120 km every 2 hours? $\frac{120}{2} = 60$ km/h $\frac{180}{60} = 3$ hours	11F
Students assume that stationary periods are not needed when calculating the average speed.	Total distance $= 9$ km Time spent travelling $= 2$ hours Time spent stationary $= 1$ hour $\frac{9}{2} = 4.5$ km/h	Total distance $= 9$ km Time spent travelling $= 2$ hours Time spent stationary $= 1$ hour $\frac{9}{2+1} = \frac{9}{3} = 3$ km/h	11F

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Back to contents

11A Introduction to ratios

Ratios describe a proportional relationship between two or more quantities with the same units. This means if one quantity is doubled or halved, then the other quantity will also double or halve. Ratios are seen in many aspects of everyday life, such as quantities in cooking, comparing costs of supermarket items or ratio of profits to loss for a business.

LEARNING INTENTIONS

Students will be able to:

- understand the relationship between fractions, percentages and ratios
- express fractions as ratios
- write a ratio from a situation that has been described.

KEY TERMS AND DEFINITIONS

If two quantities have a **proportional relationship** they can be written as a ratio.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

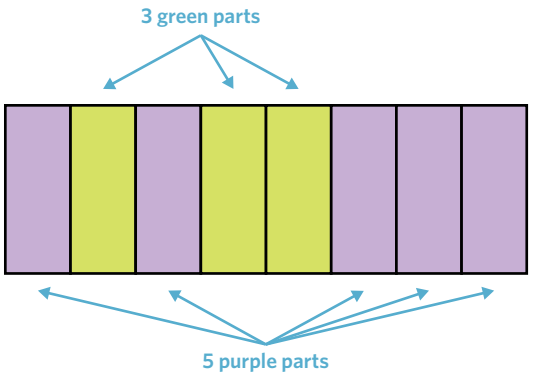
Image: GEORGII MIRONOV/Shutterstock.com

Building plans need to be drawn to scale so that they contain accurate details required by a builder. The scale is given as a ratio. For example, a scale of 1 : 100 means distance of 1 mm on the plans represents 100 mm in real life.

Key ideas

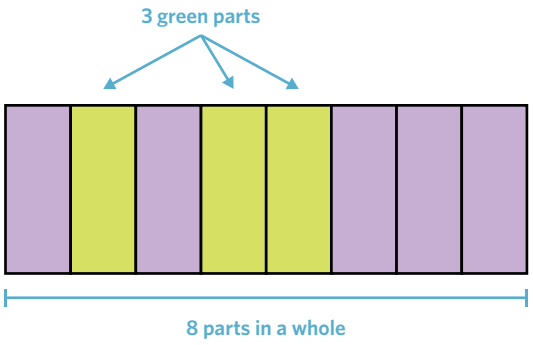
1. Ratios describe a proportional relationship between two or more quantities with the same unit. There are two types of ratios: part-part and part-whole.

A part-part ratio compares two or more individual parts in a whole.



The ratio of green parts to purple parts is 3 : 5.

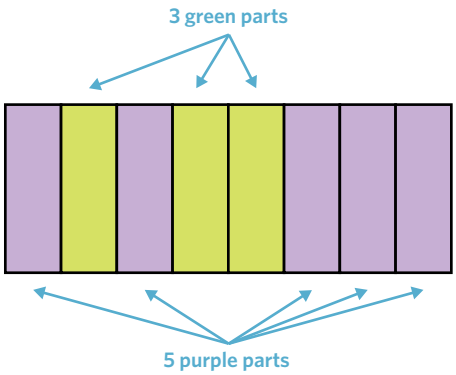
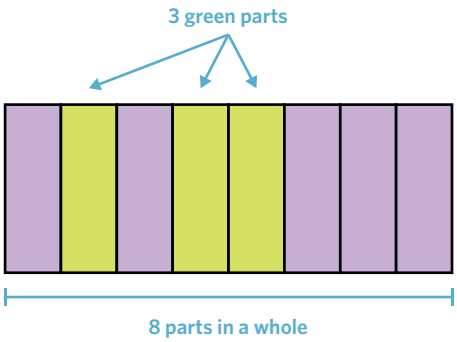
A part-whole ratio compares two or more parts to a whole.



The ratio of green parts to all parts in a whole is 3 : 8.

Continues →

2. Ratios can be represented as fractions and percentages.

Ratio	Part-part	Part-whole
	 The ratio of green parts to purple parts is 3 : 5. For every 3 green parts there are 5 purple parts.	 The ratio of green parts to all parts is 3 : 8.
Fraction	The number of green parts is $\frac{3}{5}$ of the number of purple parts. **or** The fraction of green parts to purple parts is $\frac{3}{5}$.	$\frac{3}{8}$ of the whole are green parts.
Percentage	The number of green parts as a percentage of the number of purple parts is 60%.	37.5% of the whole are green parts.

Worked example 1

Writing ratios

Write the ratio described in each scenario.

a. A pasta recipe needs 3 eggs and 2 cups of flour. What is the ratio of cups of flour to eggs? WE1a

Working

Cups of flour = 2

Eggs = 3

flour : eggs

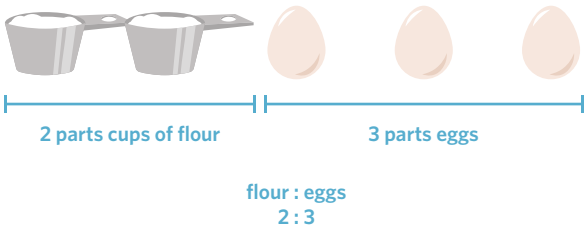
2 : 3

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-part ratio.

Visual support



Continues →

b. In a pencil case there are 7 pencils and 3 rulers. What is the ratio of rulers to all items in the pencil case? WE1b

Working

Rulers $= 3$

All items $= 3 + 7 = 10$

rulers : all items

$3 : 10$

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-whole ratio.

Student practice

Write the ratio described in each statement.

a. During one week, it rained on 4 days and was sunny on the other days. What is the ratio of sunny days to rainy days?

b. At a pet day care, there are 6 cats and 7 dogs. What is the ratio of dogs to all pets?

Worked example 2

Working with fractions and ratios

Write the fraction or ratio described in each scenario.

a. In a mini packet of Smarties, the ratio of green to purple Smarties is $3 : 4$. What is the fraction of green Smarties to all Smarties? WE2a

Working

Green Smarties $= 3$

All Smarties $= 3 + 4 = 7$

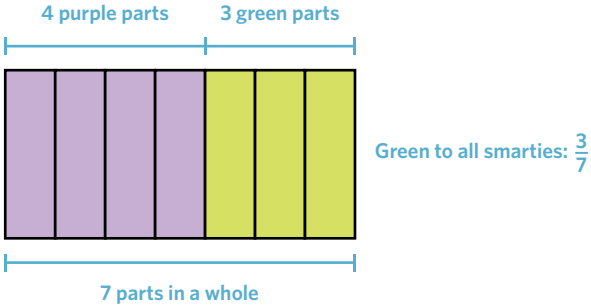
$\frac{3}{7}$

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-whole ratio.

Visual support



b. In a mini packet of Smarties, the ratio of green to purple Smarties is $3 : 4$. What is the fraction of purple Smarties to green Smarties? WE2b

Working

Purple Smarties $= 4$

Green Smarties $= 3$

$\frac{4}{3}$

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-part ratio.

Continues →

c. In a bag of red and green apples, $\frac{2}{5}$ of all apples are red. What is the ratio of red apples to green apples? WE2c

Working

Red apples $= 2$

Green apples $= 5 - 2 = 3$

red apples : green apples

$2 : 3$

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-part ratio.

Student practice

Write the fraction described in each scenario.

a. In a bowl of cereal, the ratio of oats to dried fruit is $9 : 2$. What is the fraction of oats to all parts?

b. In a bowl of cereal, the ratio of oats to dried fruit is $9 : 2$. What is the fraction of dried fruit to oats?

c. At a cafe that sells coffees and hot chocolates, $\frac{5}{6}$ of all drinks are coffees. What is the ratio of coffees to hot chocolates?

Worked example 3

Working with percentages and ratios

Write the percentage or ratio described in each scenario.

a. At a train station, the ratio of late trains to all trains (on-time and late) trains is $4 : 10$. What is the percentage of on-time trains to all trains? WE3a

Working

On-time trains $= 10 - 4 = 6$

All trains $= 10$

$\frac{6}{10}$

$\frac{6}{10} \times 100 = \frac{600}{10}$

$= 60\%$

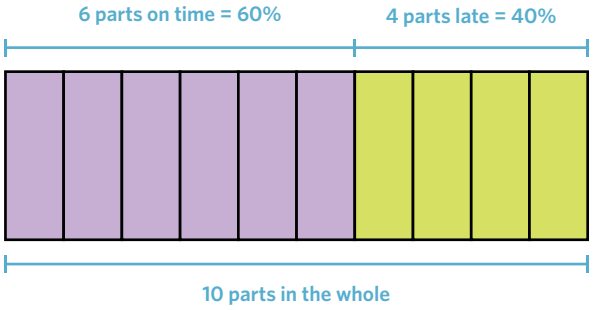
Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-whole fraction.

Step 3: Convert to a percentage.

Visual support



Continues →

b. At a train station, the ratio of late trains to late and on-time trains is 4 : 10. What is the number of on-time trains as a percentage of late trains? WE3b

Working

On-time trains $= 10 - 4 = 6$

Late trains $= 4$

$\frac{6}{4} = \frac{3}{2}$

$\frac{3}{2} \times 100 = \frac{300}{2}$

$= 150\%$

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-part fraction and simplify.

Step 3: Convert to a percentage.

c. In a class of students, 43% of all students have an Android phone and the rest have iPhones. What is the ratio of students with iPhones to students with Android phones? WE3c

Working

iPhones $= 100 - 43 = 57\%$

Android $= 43\%$

iPhones : Android phones

57 : 43

Thinking

Step 1: Determine the number of parts needed for each quantity.

Step 2: Write the part-part ratio.

Student practice

Write the percentage or ratio described in each scenario.

a. In a fridge, the ratio of soft drinks to fruit juices is 4 : 8. What is the percentage of soft drinks to all drinks? Round your answer to the nearest percent.

b. In a fridge, the ratio of soft drinks to fruit juices is 4 : 8. What is the number of soft drinks as a percentage of fruit juices?

c. 79% of plants at a nursery are native and the rest are foreign. What is the ratio of native plants to foreign plants?

11A Questions

Understanding worksheet

1. Write the ratio for each statement about the numbers below:

1, 2, 3, 4, 5 and 6.

Example

2 and 3 to all numbers 2 : 6

a. 1 to all numbers 1 : ☐

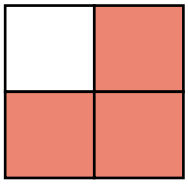
b. Even to odd numbers ☐ : 3

c. 3 to multiples of 2 ☐ : ☐

d. Prime numbers to all numbers ☐ : ☐

2. Fill in the boxes for each area model.

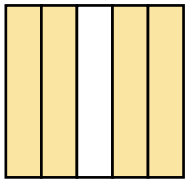
Example



Ratio of red parts to white parts: 3 : 1

Fraction of red parts to all parts: $\frac{3}{4}$

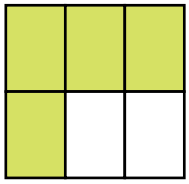
a.



Ratio of white parts to all parts:

Fraction of white parts to all parts: $\frac{1}{5}$

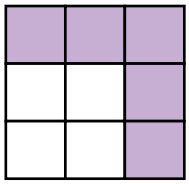
b.



Ratio of green parts to white parts: 4 : 2

Fraction of green parts to all parts:

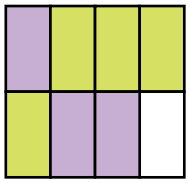
c.



Ratio of white parts to purple parts:

Fraction of purple parts to white parts:

d.



Ratio of purple parts to white parts:

Fraction of purple parts to green parts:

3. Fill in the blanks by using the words provided.

part-part | part-whole | proportional | quantities

A ratio describes a _____ relationship between _____. If a rectangle consists of 3 green parts and 4 blue parts, 3 : 7 is a _____ ratio and 4 : 3 is a _____ ratio.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b), 7 (a,b,c,d), 8 (a,b), 9	4 (c,d,e,f), 5 (b,c,d,e), 6 (b,c), 7 (b,c,d,e), 8 (b,c), 9	4 (e,f,g,h), 5 (c,d,e,f), 6 (c,d), 7 (c,d,e,f), 8 (c,d), 9

4. Write the ratio described in each scenario. WE1

a. A fruit bowl contains 6 bananas and 2 kiwi fruits. What is the ratio of kiwi fruit to all fruits in the bowl?

b. A Rocket League team has won 11 matches and lost 9. What is the ratio of wins to losses?

c. A computer monitor is 16 inches wide and 9 inches high. What is the ratio of the width to the height?

d. A family has 2 grandparents, 4 parents and 7 children. What is the ratio of children to all family members?

e. The three sides of a triangle are 2 cm, 5 cm and 4 cm. What is the ratio of the longest side length to the shortest side length?

f. A post on Instagram has 13 likes, 7 comments and 3 shares. What is the ratio of likes to comments and shares?

g. In a staffroom, there are 7 English, 5 Maths, 6 Science and 8 PE teachers. What is the ratio of Maths and Science teachers to all teachers?

h. In a packet of nuts, there are 18 peanuts, 4 macadamia nuts, 6 cashews and 11 pistachios. What is the ratio of peanuts and cashews to macadamia nuts and pistachios?

5. Write the fraction described in each scenario. WE2a,b

a. At a boarding school the ratio of boarders to non-boarders is $3:1$. What is the fraction of non-boarders to boarders?

b. The ratio of cordial to water is $1:5$. What fraction of the total volume is water?

c. The ratio of vowels to all letters in a language is $3:14$. What fraction of all letters are consonants?

d. In an alloy metal, the ratio of copper to tin to aluminium is $5:3:1$. What is the fraction of copper to aluminium?

e. The ratio of green to yellow to white in a tin of paint is $2:4:7$. What is the fraction of yellow paint to all colours?

f. The ratio of Xbox Series X to PS5 to Switch consoles sold globally is $16:22:23$. What is the fraction of Switch sales to Xbox and PS5 sales?

---

6. Write the ratio described in each scenario. WE2c

a. $\frac{1}{10}$ of the world's population is left-handed. What is the ratio of left-handers to the whole population?

b. US dollars as a fraction of Australian dollars on one day was $\frac{5}{7}$. What is the ratio of US dollars to Australian dollars?

c. At a school, the fraction of those who study a language to those who don't is $\frac{4}{9}$. What is the ratio of those who study a language to the whole school?

d. At an election, the fraction of all voters who voted for a major party was $\frac{27}{29}$. What is the ratio of those who voted for a major party to those who voted for other parties?

---

7. Write the percentage described in each scenario, rounded to the nearest whole number where needed. WE3a,b

a. In a team, the ratio of employees who prefer to work from home to those who prefer to work at the office is $7:8$. What is the percentage of the team who prefer to work at the office?

b. The ratio of land to the whole Earth is $3:10$ and the rest is sea. What percentage of the whole earth is sea?

c. The ratio of savings to expenses for a person is $3:5$. What is the amount of savings as a percentage of the amount of expenses?

d. In a party mix of chocolates, the ratio of milk to white to dark chocolate is $5:1:3$. What percentage of the whole mix is dark chocolate?

e. In a cookie dough recipe, the ratio of flour to butter to sugar is $3:2:1$. What is the amount of butter as a percentage of the amount of flour?

f. The ratio of air to water to soil in a plant pot is $3:7:10$. What is the percentage of the whole pot that is water and soil?

---

8. Write the ratio described in each scenario. WE3c

a. 3% of a petrol mix is ethanol and the rest is octane. What is the ratio of ethanol to octane?

b. At a clothes store, 60% of all shirts sold were medium t-shirts and the rest were small shirts. What is the ratio of medium t-shirts sold to all t-shirts sold?

c. The length of a leaf of a plant is 11% the height of the plant. What is the ratio of leaf length to plant height?

d. After a piece of wood was cut, the length of the shortest piece as percentage of the longer piece was 20%. What is the ratio of the longest piece to the original piece of wood?

---

9. The fraction of all games that are multiplayer is $\frac{7}{15}$. What is the ratio of multiplayer to single player games?

A. $7:15$ **B.** $8:15$ **C.** $7:8$ **D.** $8:7$ **E.** $8:22$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. The ratio of humans to sheep in New Zealand is $5 : 28$.

What fraction of the total population of humans and sheep are sheep?

Student A

Sheep $= 28$

Human $= 5$

$\frac{5}{28}$

Student B

Sheep $= 28$

Humans and sheep $= 5 + 28 = 33$

$\frac{28}{33}$

b. The ratio of headphones sold in the US to those sold outside the US is $1 : 3$.

What percentage of all headphones sold are sold outside the US, rounded to a whole number?

Student A

Sold outside the US $= 3$

Sold outside and in the US $= 1 + 3 = 4$

$\frac{3}{4} \times 100 = \frac{300}{4}$

$= 75\%$

Student B

Sold in the US $= 1$

Sold outside the US $= 3$

$\frac{1}{3} \times 100 = \frac{100}{3}$

$= 33\%$

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. A farmer's paddock has a width of 301 m and a length of 452 m. Write the ratio of the paddock's width to its perimeter.

12. Two painters, Julia and Brianca, divided up payment for their day's work based on the ratio of hours that they worked. Julia worked from 7 am to 3 pm and Brianca worked from 9 am to 2 pm. What fraction of the total pay does Julia receive?

13. Coles reported that their ratio of revenue to expenses was $14 : 9$ and Woolworths reported a ratio of $5 : 3$. Which business had the greatest revenue as a percentage of their expenses?

14. Jenny is making her world famous cordial but forgot the exact ratio of water to sugar to lemon juice she needed. She filled $\frac{1}{3}$ of the bowl with water, $\frac{2}{5}$ with sugar and $\frac{1}{5}$ with lemon juice. She later found that the ratio she was meant to use was $5 : 6 : 4$. Did she get the ratio right?

15. It is recommended that vegetarians have a $5 : 1 : 1$ ratio of legumes (such as soybeans and lentils) to eggs to nuts as part of their diet. Nick made a meal where 65% of the meal was legumes and the rest made equally up of eggs and nuts. Does this meal satisfy the recommendation?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. To buy a house worth $800 000, Ying pays a $100 000 deposit and takes out a loan for the rest. The Loan to Value Ratio (LVR) is used by banks when someone is taking out a loan. It is a ratio of the loan amount to the price of the house. If the LVR of a loan is higher than 80%, the buyer usually needs to also pay for Lenders Mortgage Insurance (LMI).

a. Calculate how much money Ying will need to borrow from the bank.

b. Write the ratio of the loan amount to the price of the house.

c. Write the ratio found in part **b** as a percentage to show that the LVR of Ying's loan is higher than 80%.

d. To decrease the loan amount, Ying can put more money on the deposit. Calculate by how much the loan amount should increase or decrease so that Ying does not have to pay the LMI.

e. Discuss two reasons why a bank will make a buyer pay for insurance if they have an LVR of higher than 80%.

17. Fill in the boxes using the digits 1 to 9 once each to create a part-part ratio where the first part represents the:

□□ : □

a. smallest possible fraction of the whole.

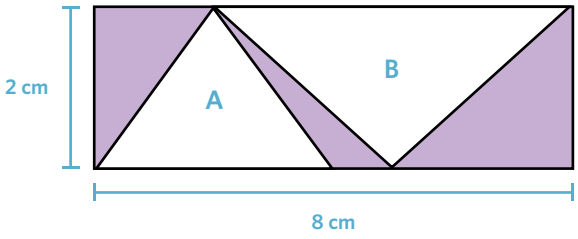
b. largest possible fraction of the whole.

c. Using how you approached parts **a** and **b**, explain how you could compare two ratios to determine which is the greater ratio.

Extra spicy

18. In the diagram below, the base of

- triangle A is half the length of the rectangle
- triangle B is three quarters the length of the rectangle.



What is the ratio of the unshaded area to the shaded area?

A. 10 : 6 **B.** 10 : 16 **C.** 8 : 2 **D.** 8 : 16 **E.** 6 : 4

19. Ash and Joey collect Pokémon cards and Yu-Gi-Oh cards. Altogether, the number of Pokémon cards equals the number of Yu-Gi-Oh cards.

The ratio of Pokémon cards Ash has to Pokémon cards Joey has is 3 : 7.

The ratio of Yu-Gi-Oh cards Ash has to Yu-Gi-Oh cards Joey has is 1 : a.

What is the smallest value of a, so that Joey has more Yu-Gi-Oh cards than he has Pokémon cards?

20. For two whole numbers, the ratio of their difference to their sum is $7:11$. What are the two smallest positive whole numbers for which this is true?

21. If the ratio $p:q$ is equal to $6:14$, write the formula in terms of p and q where q is the subject.

A. $q = \frac{7}{3}p$ **B.** $p = \frac{7}{3}q$ **C.** $q = \frac{3}{7}p$ **D.** $q = \frac{7}{10}p$ **E.** $q = \frac{3}{4}p$

Remember this?

22. The table below shows the fractions of the favourite colours in a classroom.

Colour	Fraction of the children
Purple	$\frac{1}{25}$
Dark blue	$\frac{1}{5}$
Red	$\frac{1}{10}$
Pink	$\frac{1}{20}$
Green	$\frac{1}{50}$

Which of these colours is most favourited within the classroom?

A. Purple **B.** Dark blue **C.** Red **D.** Pink **E.** Green

23. Renae was pouring tea into her 1 litre bottle, she only filled a quarter of the bottle.

How many millilitres of tea did she pour?

A. 5 **B.** 50 **C.** 250 **D.** 2000 **E.** 25 000

24. The fraction $\frac{1}{6}$ equals 0.166 as a decimal.

What does $\frac{1}{12}$ equal as a decimal.

A. 0.166 **B.** 0.138 **C.** 0.066 **D.** 0.83 **E.** 0.083

Back to contents

11B Equivalent ratios

Just like equivalent fractions, equivalent ratios represent the same values. Equivalent ratios will have different quantities, but the relationship between the quantities will be identical. Simplifying ratios involves finding equivalent ratios and allows the same quantities from different scenarios to be compared.

LEARNING INTENTIONS

Students will be able to:

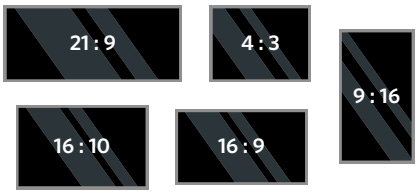
- write equivalent ratios using multiplication and division
- simplify ratios involving fractions and decimals.

KEY TERMS AND DEFINITIONS

Highest common factor (HCF) is the largest number that is a factor of two or more numbers.

A **ratio** is a proportional relationship between two or more quantities with the same unit.

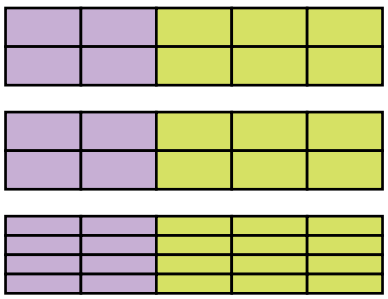
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



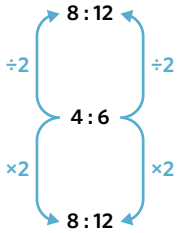
Aspect ratios are ratios of the width to the lengths of computer monitors, TV screens and smartphones. Movies are typically filmed with an aspect ratio of 16 : 9. If the ratio of the width to the height simplifies to this ratio, the movie will fit exactly on the screen without distorting the picture.

Key ideas

1. **Equivalent ratios** are formed when each part in a ratio is multiplied or divided by the same number.



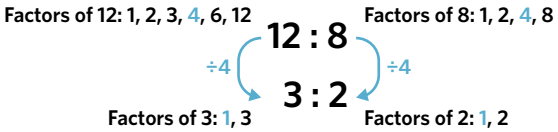
Purple parts to green parts



2. Two or more ratios are equivalent if the same parts are **in proportion**.

Equivalent	Not equivalent
$\frac{15}{5} = 3$ $\quad 15 : 18 \quad 5 : 6 \quad \frac{18}{6} = 3$	$\frac{15}{5} = 3$ $\quad 15 : 18 \quad 5 : 9 \quad \frac{18}{9} = 2$

3. Ratios can be simplified if all parts have a highest common factor other than 1.



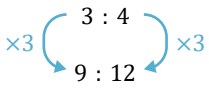
Worked example 1

Making equivalent ratios

For each ratio, make an equivalent ratio by performing the operation in the brackets.

a. 3 : 4 (×3) WE1a

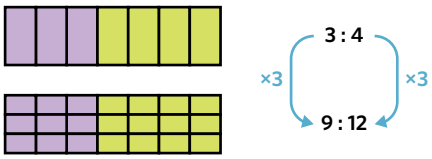
Working



Thinking

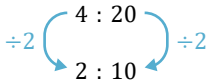
Multiply each part by the same number.

Visual support



b. 4 : 20 (÷2) WE1b

Working



Thinking

Divide each part by the same number.

Student practice

For each ratio, make an equivalent ratio by performing the operation in the brackets.

a. 2 : 7 (×2) **b.** 18 : 12 (÷3)

Worked example 2

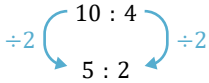
Simplifying ratios

Write each ratio in its simplest form.

a. 10 : 4 WE2a

Working

The HCF of 10 and 4 is 2.

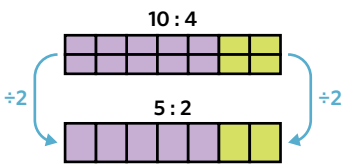


Thinking

Step 1: Determine the HCF.

Step 2: Divide each part by the HCF.

Visual support

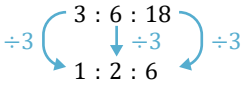


Continues →

b. 3 : 6 : 18 WE2b

Working

The HCF of 3, 6 and 18 is 3.



Thinking

Step 1: Determine the HCF.

Step 2: Divide each part by the HCF.

Student practice

Write each ratio in its simplest form.

a. 4 : 16 **b.** 15 : 5 : 20

Worked example 3

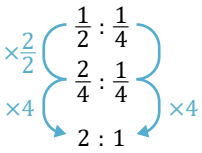
Simplifying ratios involving fractions and decimals

Write each ratio in its simplest form.

a. $\frac{1}{2} : \frac{1}{4}$ WE3a

Working

The LCD of 2 and 4 is 4.



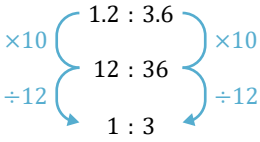
Thinking

Step 1: Convert each part into equivalent fractions with the lowest common denominator (LCD).

Step 2: Multiply each part by the LCD.

b. 1.2 : 3.6 WE3b

Working



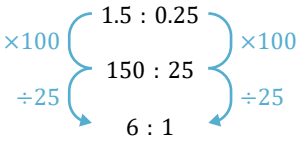
Thinking

Step 1: Multiply each part by the power of 10 that will make both parts whole numbers.

Step 2: Simplify by dividing each part by the HCF.

c. 1.5 : 0.25 WE3c

Working



Thinking

Step 1: Multiply each part by the power of 10 that will make both parts whole numbers.

Step 2: Simplify by dividing each part by the HCF.

Student practice

Write each ratio in its simplest form.

a. $\frac{2}{3} : \frac{1}{6}$ **b.** 3.3 : 2.2 **c.** 0.24 : 0.48

Worked example 4

Expressing quantities with the same units

For each statement, change the quantities to the same unit in brackets and write the ratio in its simplest form.

a. 4 cm to 25 mm (mm) WE4a

Working

1 cm = 10 mm

4 cm = 40 mm

40 : 25

÷5 (40 : 25 → 8 : 5) ÷5

Thinking

Step 1: Convert all parts to the same unit.

Step 2: Write the ratio.

Step 3: Simplify by dividing each part by the HCF.

b. 2 years to 8 months (months) WE4b

Working

1 year = 12 months

2 years = 24 months

24 : 8

÷8 (24 : 8 → 3 : 1) ÷8

Thinking

Step 1: Convert all parts to the same unit.

Step 2: Write the ratio.

Step 3: Simplify by dividing each part by the HCF.

Student practice

For each statement, change the quantities to the same unit and write the ratio in its simplest form.

a. 30 cm to 2 m (cm) **b.** 7 days to 2 weeks (days)

11B Questions

Understanding worksheet

1. Fill in the blank to make each pair of ratios equivalent.

Example

6 : 5 = 24 : 20

a. 3 : 5 = 6 : ☐

b. 9 : 12 = ☐ : 6

c. ☐ : 15 = 7 : 3

d. 4 : ☐ = 12 : 21

2. Decide whether the following ratios are equivalent.

Example

	Equivalent	Not equivalent
10 : 7 and 20 : 14	☑	☐

	Equivalent	Not equivalent
a. 5 : 7 and 10 : 15	☐	☐
b. 6 : 8 and 3 : 4	☐	☐
c. 16 : 9 and 8 : 3	☐	☐
d. 1 : 7 and 4 : 11	☐	☐

3. Fill in the blanks by using the words provided.

highest common factor | equivalent ratio | simplest form | whole number

An _____ is formed by multiplying or dividing all parts in a ratio by the same number. Ratios are in their _____ when each part is a _____ and the _____ is 1.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. For each ratio, make an equivalent ratio by performing the operation in the brackets. WE1

a. 5 : 9 (×2) **b.** 8 : 3 (×10) **c.** 20 : 12 (÷4) **d.** 15 : 30 (÷3)

e. 5 : 14 : 4 (×5) **f.** 56 : 24 : 8 (÷8) **g.** 14 : 22 $\left(\times\frac{1}{2}\right)$ **h.** 6 : 5 $\left(\div\frac{1}{4}\right)$

5. Write each ratio in its simplest form. WE2

a. 15 : 10 **b.** 30 : 20 **c.** 6 : 24 **d.** 21 : 49

e. 130 : 70 : 20 **f.** 10 : 14 : 8 **g.** 63 : 18 : 9 **h.** 12 : 42 : 66

6. Write each ratio in its simplest form. WE3a

a. $\frac{3}{5} : \frac{3}{10}$ **b.** $\frac{5}{6} : \frac{1}{2}$ **c.** $\frac{2}{3} : 3$ **d.** $\frac{2}{5} : \frac{1}{4}$

e. $6 : 1\frac{3}{8}$ **f.** $7\frac{3}{4} : \frac{7}{8}$ **g.** $6\frac{4}{5} : 3\frac{3}{10}$ **h.** $\frac{3}{4} : 2\frac{1}{4} : \frac{3}{2}$

7. Write each ratio in its simplest form. WE3b,c

a. 1.2 : 3 **b.** 3.2 : 1.8 **c.** 4.8 : 5.6 **d.** 0.13 : 0.45

e. 7.2 : 1.44 **f.** 2.1 : 0.4 : 3.0 **g.** 1.6 : 3.2 : 8.8 **h.** 2.68 : 5.16 : 0.12

8. For each statement, change the quantities to the same unit in brackets and write the ratio in its simplest form. WE4

a. 450 cm to 9 metres (cm) **b.** 4 minutes to 300 seconds (seconds)

c. 4400 mL to 20 L (mL) **d.** $4.50 to 85 cents (cents)

e. $2\frac{1}{3}$ years to 20 months (months) **f.** 40 cm^2 to 300 mm^2 (mm^2)

g. 5400 seconds to 6 hours (seconds) **h.** 6.2 kg to 2400 g to 4 kg (g)

9. Which of the following is the simplest form of the ratio 36 : 22?

A. 11 : 18 **B.** 36 : 11 **C.** 14 : 1 **D.** 18 : 11 **E.** 12 : 11

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. For the ratio 9 : 4, write the equivalent ratio multiplied by 6.

Student A

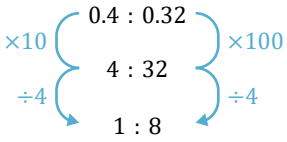
$9 \times 6 : 4 = 54 : 4$

Student B

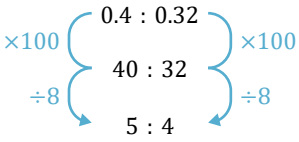
$9 \times 6 : 4 \times 6 = 54 : 24$

b. Write the ratio 0.4 : 0.32 in simplest form.

Student A



Student B



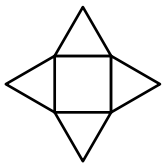
Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. A shade of purple is made by combining red and blue paint tins in the ratio 2 : 3. This is enough paint to cover 20 m^2. How many tins of each colour is required to cover 60 m^2?

12. Matej is making multiple tiles of the pattern shown below. It consists of 4 equilateral triangles and 1 square.



If Matej has 48 triangles, what number of squares does he need so that he only has completed tiles?

13. At his cafe, Hugh makes milkshakes which need milk and ice cream. He uses $\frac{1}{3}$ of a container of ice cream and $\frac{2}{5}$ of a carton of milk for a large milkshake. What is the fully simplified ratio of milk to ice cream for each large milkshake?

14. Jemima is making concrete for a building project where the ratio of cement to sand to aggregate should be 1 : 2 : 4. She mixes 5 bags of cement, 10 bags of sand and 15 bags of aggregate. Calculate the simplified ratio to determine if Jemima has used the correct quantities for her mix.

15. Alex is making her own soil mix for her indoor plants. To make the soil, she mixes a ratio of 2 parts peat moss, 1 part perlite and $\frac{1}{2}$ part compost. She has 8 bags of peat moss but needs to buy perlite and compost. If bags of perlite cost $29.98 and bags of compost cost $9.46, how much will it cost her to buy enough perlite and compost?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Daniel and Joyce are planning activities to do on an overseas trip to Ireland. After some research, Daniel writes an itinerary for one of the days spent in Dublin:

Time	Activity
9:30 to 10:00	Taking the bus
10:00 to 12:30	Visiting Trinity College
12:30 to 1:00	Walking
1:00 to 4:00	National Museum of Ireland
4:00 to 5:00	Taking the bus
5:00 to 9:00	Relaxing at the hotel

Joyce wants to ensure that they balance the time spent doing various activities. She proposes a ratio of 2 hours relaxing to 3 hours sightseeing to 0.5 hours travel time between locations.

a. Write Joyce's proposed ratio in simplest form.

b. Write the ratio of hours spent relaxing to sightseeing to travel time for Daniel's itinerary in simplest form.

c. Determine if the percentage of travel time to all activities on the day is more than, less than or the same as the percentage of travel time to all activities proposed by Joyce.

d. Daniel is planning a fourth day in Ireland and has allocated 6 hours sightseeing in the city of Galway. How many hours should be allocated to time relaxing and time travelling according to Joyce's proposed ratio?

e. Is it better to ensure that the time spent doing activities every day satisfies Joyce's proposed ratio, or is it better that all days combined satisfy this ratio? Provide an advantage and a disadvantage for each option.

17. For the ratio $23 : 11$ make:

a. two different ratios by adding or subtracting a number from both parts.

b. two different ratios by multiplying or dividing both parts by a number.

c. Compare one ratio from each of part **a** and part **b** to the original ratio. Determine which operations can be used to make equivalent ratios and which cannot.

Extra spicy

18. Which of the following is the fully simplified form of the ratio $30ab : 75bc$?

A. $a : 45c$ **B.** $2a : 5c$ **C.** $2b : 5b$ **D.** $2ab : 5bc$ **E.** $30 : 75$

19. If $a : b = 6 : 5$ and $b : c = 2 : 3$, find the ratio $a : b : c$.

A. $6 : 2 : 3$ **B.** $6 : 5 : 3$ **C.** $6 : 7 : 3$ **D.** $12 : 10 : 15$ **E.** $12 : 20 : 15$

20. Three points on a Cartesian plane can be connected by a straight line. The points have coordinates $(2,4)$, $(8,a)$ and $(b,2)$. The ratio of the y-coordinate to the x-coordinate of all points are equivalent ratios. What are values of a and b?

21. Using the numbers 1 to 9 only once each, fill in the blanks to create three equivalent ratios.

$\square : \square = \square : \square = \square : \square$

Remember this?

22. A stadium wants to carry out a survey about the tennis game using a sample of individuals that attended the last game.

They want the survey to be representative of adults and children that attend.

At the last game, there were twice as many children as adults.

Which of the samples would the stadium get the most representative results?

A sample with:

A. 50 adults and 50 children
B. 150 adults and 300 children
C. 300 adults and 150 children
D. 300 adults and 200 children
E. None of the above

23. Zac is saving for a new computer.

He has created a table to help keep track of his savings.

Number of months (x)	3	5	8	10
Amount in Zac's savings account (y)	135	255	435	555

What would be the rule for the linear relationship between the amount of money in Zac's savings account and the number of months saving?

A. $y = 60x - 45$
B. $y = 75x + 135$
C. $y = 135x + 255$
D. $y = 255x + 60$
E. $y = 435x + 135$

24. Which of the following options represents a certain event?

A. $\frac{1}{8}$ **B.** $\frac{2}{10}$ **C.** $\frac{3}{10}$ **D.** $\frac{4}{8}$ **E.** 1

Back to contents

11C Dividing a quantity in a given ratio

Quantities can be shared based on a ratio. This is achieved using the unitary method, which involves finding an equivalent ratio based on how big each part is.
A second method of doing this involves converting ratios to fractions and multiplying by the quantity being shared.

LEARNING INTENTIONS

Students will be able to:

- divide quantities into a given ratio using fractions or the unitary method
- find total quantities given a ratio.

KEY TERMS AND DEFINITIONS

The **unitary method** is a process of calculation where the first step is to determine the value of one unit.

A **part-whole** ratio compares two or more parts to a whole.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Bro Crock/Shutterstock.com

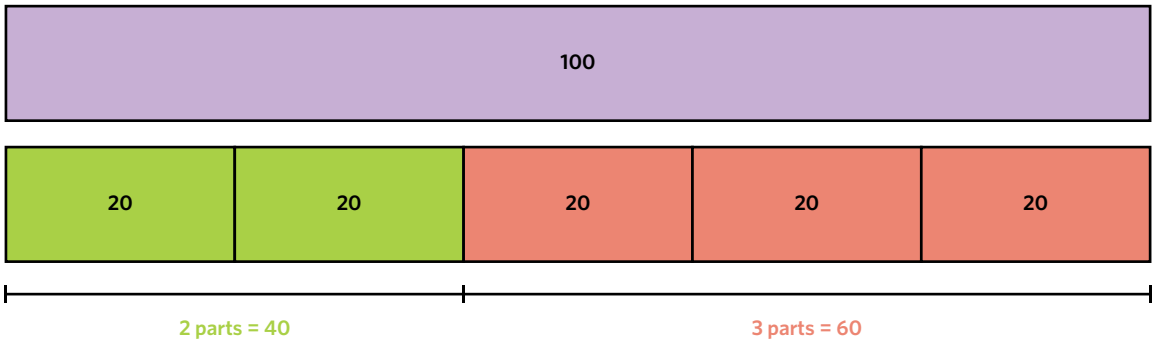
Shareholders of a company receive some portion of the profit that a company makes in a year, called a dividend. How much a person receives depends on how many shares they have in the company, which can be written as a ratio to or percentage of the total number of shares in the company. This ratio can be used to split the profit for each shareholder.

Key idea

1. Quantities can be shared into smaller quantities determined by a ratio. This can be achieved using the unitary method or part-whole fractions.

100 shared in the ratio 2 : 3

1 part = 20



Worked example 1

Calculating values of parts in a ratio

For each ratio and quantity to be shared, calculate the value of 1 part.

a. 20 in the ratio 4 : 1 WE1a

Working

$4 + 1 = 5$ parts

$20 \div 5 = 4$ per part

Thinking

Step 1: Calculate the number of parts in the whole.

Step 2: Calculate the value of 1 part by dividing the total quantity by the number of parts.

Visual support

b. 18 in the ratio 1 : 4 : 1 WE1b

Working

$1 + 4 + 1 = 6$ parts

$18 \div 6 = 3$ per part

Thinking

Step 1: Calculate the number of parts in the whole.

Step 2: Calculate the value of 1 part by dividing the total quantity by the number of parts.

Student practice

For each quantity and ratio, calculate the value of 1 part.

a. 35 in the ratio 3 : 4

b. 50 in the ratio 3 : 5 : 2

Worked example 2

Sharing a quantity in a given ratio

Share each quantity into the given ratio.

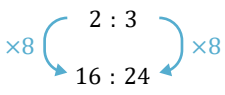
a. 40 in the ratio 2 : 3 WE2a

Working

Method 1

$2 + 3 = 5$ parts

$40 \div 5 = 8$ per part



40 shared in the ratio 2 : 3 is 16 and 24.

Thinking

Method 1

Step 1: Calculate the number of parts in the whole.

Step 2: Calculate the value of 1 part.

Step 3: Make an equivalent ratio by multiplying the ratio by the value of 1 part.

Step 4: State the value of each share.

Continues →

Working

Method 2

$2 + 3 = 5$ parts

$\frac{2}{5}$ and $\frac{3}{5}$

$$\frac{2}{5} \times 40 = \frac{2}{5} \times \frac{40}{1}$$
$$= \frac{80}{5}$$
$$= 16$$
$$\frac{3}{5} \times 40 = \frac{3}{5} \times \frac{40}{1}$$
$$= \frac{120}{5}$$
$$= 24$$

40 shared in the ratio 2 : 3 is 16 and 24.

Thinking

Method 2

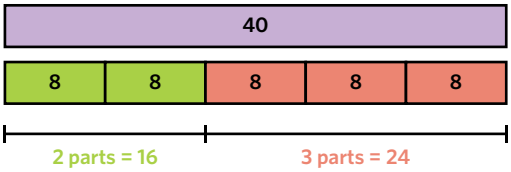
Step 1: Calculate the number of parts in the whole.

Step 2: Write the part-whole fraction for each share of the ratio.

Step 3: Multiply each fraction by the quantity to be shared and simplify.

Step 4: State the value of each share.

Visual support



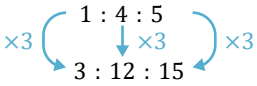
b. 30 in the ratio 1 : 4 : 5 WE2b

Working

Method 1

$1 + 4 + 5 = 10$

30 ÷ 10 = 3 per part



30 shared in the ratio 1 : 4 : 5 is 3, 12 and 15.

Method 2

$1 + 4 + 5 = 10$

$\frac{1}{10}$

$\frac{4}{10} = \frac{2}{5}$

$\frac{5}{10} = \frac{1}{2}$

$\frac{1}{10} \times 30 = \frac{30}{10} = 3$

$\frac{2}{5} \times 30 = \frac{60}{5} = 12$

$\frac{1}{2} \times 30 = \frac{30}{2} = 15$

30 shared in the ratio 1 : 4 : 5 is 3, 12 and 15.

Thinking

Method 1

Step 1: Calculate the number of parts in the whole.

Step 2: Calculate the value of 1 part.

Step 3: Make an equivalent ratio by multiplying the ratio by the value of 1 part.

Step 4: State the value of each share.

Method 2

Step 1: Calculate the number of parts in the whole.

Step 2: Write the part-whole fraction for each share of the ratio and simplify.

Step 3: Multiply each fraction by the quantity to be shared and simplify.

Step 4: State the value of each share.

Continues →

Student practice

Share each quantity into the given ratio.

a. 18 in the ratio 1 : 2

b. 60 in the ratio 2 : 7 : 3

Worked example 3

Finding quantities using ratios

For each scenario, determine the quantity that was shared in the given ratio.

a. Jude and Jarrod shared some money in the ratio 1 : 3. Jarrod received $12. What is the total amount of money shared? WE3a

Working

Method 1

3 parts = $12

$12 ÷ 3 = $4 per part

1 + 3 = 4

4 × 4 = $16

Method 2

3 parts = $12

12 ÷ 3 = 4 per part

Jude's share = 1 × 4

= 4

12 + 4 = $16

Thinking

Method 1

Step 1: Calculate the value of 1 part.

Step 2: Calculate the number of parts in the whole.

Step 3: Multiply the number of parts in the whole by the value of 1 part.

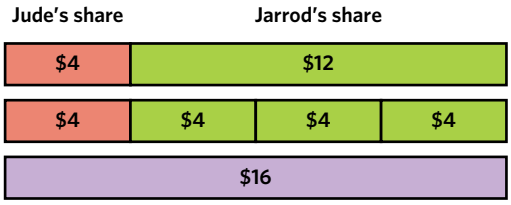
Method 2

Step 1: Calculate the value of 1 part.

Step 2: Multiply the number of parts in the other share by the value of 1 part.

Step 3: Add together the values of each share.

Visual support



b. Harry, Ron and Hermione shared books in the ratio 2 : 1 : 5. Harry received 6 books. WE3b

Working

Method 1

2 parts = 6 books

6 ÷ 2 = 3 books per part

2 + 1 + 5 = 8

8 × 3 = 24 books

Thinking

Method 1

Step 1: Calculate the value of 1 part.

Step 2: Calculate the number of parts in the whole.

Step 3: Multiply the number of parts in the whole by the value of 1 part.

Continues →

Working	Thinking
Method 2	Method 2
2 parts = 6 books	**Step 1:** Calculate the value of 1 part.
6 ÷ 2 = 3 books per part	
Ron's share = 1 × 3 = 3 books	**Step 2:** Multiply the number of parts in the other share by the value of 1 part.
Hermione's share = 5 × 3 = 15 books	
6 + 3 + 15 = 24 books	**Step 3:** Add together the values of each share.

Student practice

For each scenario, determine the quantity that was shared in the given ratio.

a. Miranda and Michael shared pencils in the ratio 1 : 5. Miranda had 2 pencils.
What is the total number of pencils?

b. Bianca, Brian and Bernadette shared bags of flour in the ratio 3 : 2 : 9. Bianca had 150 g of flour.
What is the total weight of flour?

11C Questions

Understanding worksheet

1. For each ratio, calculate the number of parts in the whole.

Example

2 : 9 =
11 parts

a. 7 : 5 = ☐ parts

b. 10 : 1 = ☐ parts

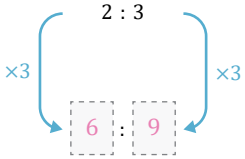
c. 12 : 13 = ☐ parts

d. 5 : 9 : 4 = ☐ parts

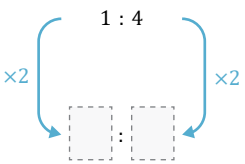
2. Find the value of each share in the ratio given the value of 1 part.

Example

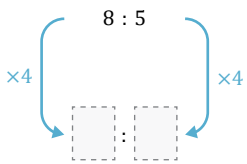
1 part = 3



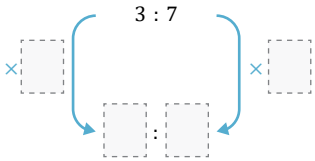
a. 1 part = 2



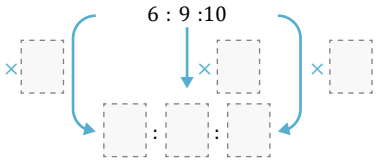
b. 1 part = 4



c. 1 part = 6



d. 1 part = 5



3. Fill in the blanks by using the words provided.

part-whole | equivalent | value | parts

Sharing quantities in a ratio first requires determining how many _____ there are in the whole.

The unitary method involves making _____ ratios based on the _____

of each part. Each share can also be calculated by multiplying the quantity by _____ fractions.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7

4. For each ratio and quantity to be shared, calculate the value of 1 part. Round to two decimal places where appropriate. WE1

a. 25 in the ratio 2 : 3
b. 20 in the ratio 3 : 1
c. 14 in the ratio 5 : 2
d. 60 in the ratio 3 : 6 : 1
e. 156 in the ratio 7 : 4 : 1
f. 31 in the ratio 9 : 6
g. 48 in the ratio 11 : 8 : 3
h. 230 in the ratio 5 : 8 : 2

5. Share each quantity into the given ratio. Round to two decimal places where appropriate. WE2

a. 10 in the ratio 3 : 2
b. 32 in the ratio 7 : 1
c. 65 in the ratio 8 : 5
d. 45 in the ratio 3 : 10 : 2
e. 41 in the ratio 4 : 1
f. 161 in the ratio 8 : 6 : 9
g. 62 in the ratio 6 : 7 : 5
h. 34 in the ratio $\frac{1}{4} : \frac{3}{5}$

6. For each scenario, determine the quantity that was shared in the given ratio. Round to two decimal places where appropriate. WE3

a. Yindi and Kora shared some money in the ratio 4 : 3. Kora received $18.
What was the total amount of money?

b. Ariel spent his time relaxing and studying in the ratio 2 : 1. He spent 6 hours relaxing.
What was the total amount of time?

c. A piece of wood is cut into two pieces in the ratio 4 : 5. The length of the smaller piece is 20 cm.
What is the total length of wood, in centimetres?

d. In an online game, the ratio of knights to mages to defenders is 5 : 2 : 3. The number of mages is 500. What is the total number of players?

e. Concrete is made by mixing cement, sand and aggregate in the ratio 1 : 2 : 4. There is 200 kg of cement. What is the total mass of concrete, in kilograms?

f. Some motorbike fuel contains octane and oil in a ratio of 5 : 1. The amount of octane is 1 L.
What is the total amount of fuel, in litres?

g. A volume of paint was made by mixing yellow, red and white paint in the ratio $3 : 2 : 4$. The volume of yellow paint was 1.2 L. What is the total volume of paint, in litres?

h. A large plastic water bottle is made out of new and recycled plastic in the ratio $\frac{1}{4} : \frac{5}{14}$. There is 20 g of new plastic. What is the total weight of plastic, in grams?

---

7. What are the values of each share when 360 is shared in the ratio $4 : 2$?

A. 90 and 180 **B.** 120 and 240 **C.** 120 and 480 **D.** 180 and 180 **E.** 240 and 120

Spot the mistake

8. Select whether Student A or Student B is incorrect.

a. 48 is shared in the ratio $3 : 5$.

Student A

$3 + 5 = 8$ parts in the whole

$\frac{3}{8} \times 48 = \frac{3 \times 48}{8}$

$= \frac{144}{8}$

$= 18$

$\frac{5}{8} \times 48 = \frac{5 \times 48}{8}$

$= \frac{240}{8}$

$= 30$

Student B

$48 \div 3 = 16$

$48 \div 5 = 9.6$

$= 10$

b. Tori and Bela shared some money in the ratio $9 : 13$. Tori received \$27. Determine the quantity that was shared.

Student A

$\$27 \div 13 = \$2.08$ per part

Number of parts in the whole $= 9 + 13$

$= 22$

Quantity that was shared $= 22 \times \$2.08$

$= \$46$

Student B

$\$27 \div 9 = \$3$ per part

Number of parts in the whole $= 9 + 13$

$= 22$

Quantity that was shared $= 22 \times \$3$

$= \$66$

Problem solving

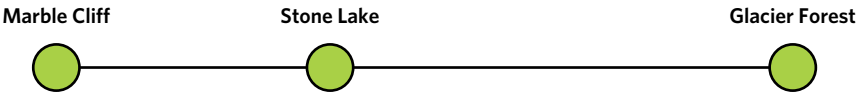
Question working paths

Mild 9, 10, 11	Medium 10, 11, 12	Spicy 11, 12, 13

9. In the world's population, the ratio of people with brown eyes to other colours is $3 : 2$. In a year level of 60 students, what is the expected number of students with brown eyes?

10. Two friends, Dain and Felicity, are playing an online RPG (role playing game) and have just defeated a boss. They will share the loot received, 45 000 gold pieces, according to the ratio of damage they each dealt. Dain dealt 4000 damage and Felicity dealt 5000 damage. How much gold should Felicity receive?

11. The ratio of the distance between Marble Cliff and Stone Lake to the distance between Stone Lake and Glacier Forest is 3 : 4.



The distance between Marble Cliff and Stone Lake is 21 km. Find the distance between Marble Cliff and Glacier Forest.

12. The angles of a triangle are in the ratio 2 : 5 : 1. Calculate the sizes of the angles.

13. Beatrix knows that on average the ratio of muffins to pastries sold at her bakery is 4 : 11. Muffins cost $6.50 and pastries cost $4.70. Today, she is expecting 210 customers to come in and buy one item each. How much revenue is she expecting to make from sales today, to the nearest cent?

Reasoning

Question working paths

Mild 14 (a,b,c,e)	Medium 14 (a,b,c,e), 15 (a,b)	Spicy All

14. Kurt runs his own plants store and wants to start offering custom made soil mixes containing fertiliser for different types of plants. Fertiliser contains various amounts of the elements nitrogen (N), phosphorus (P) and potassium (K). The ratio of these elements is called the NPK ratio. Different NPK ratios are needed for different needs and types of plants.

Soil type	NPK ratio
Indoor plant	3 : 2 : 1
Flowering and fruiting	4 : 5 : 3

a. How many parts in total are there in the NPK ratio for indoor plants?

b. Kurt plans to offer bags of indoor plant soil that contain 12 kg of fertiliser. Calculate the amount of each element he should put in the bags.

c. Kurt wants to offer a large container of soil to help plants produce more flowers and fruits. If there is 9 kg of potassium (K), how much fertiliser in total will it need to contain?

d. For a new type of soil designed for succulents and cacti, fertiliser needs to have an NPK ratio of 1 : 2 : 2. This product will be a smaller 10 kg bag. If there can be no more than 4 kg of fertiliser in the bag, what is the maximum amount of each element that he can put in?

e. Kurt needs to figure out how much each of his products will cost. List at least two things he should consider when determining the prices.

15. Using the numbers 1 to 9 only once each, fill in the blanks to share the quantity (a two digit number) in the ratio which gives the:

Ratio ☐ : ☐

Quantity ☐ ☐

a. smallest whole number difference between the two numbers representing the ratio.

b. largest whole number difference between the two numbers representing the ratio.

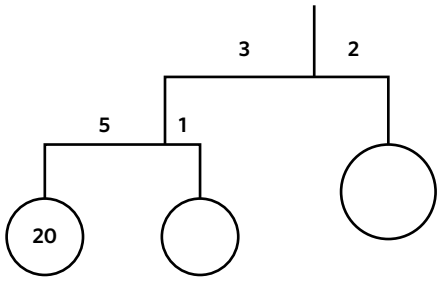
c. When is it possible for a quantity to be shared into whole number shares?

Extra spicy

16. Inge and Jeremy share some money in the ratio 5 : 3. Inge received $12 more than Jeremy. How much money was shared overall?

A. $24 **B.** $32 **C.** $48 **D.** $72 **E.** $96

17. In the diagram below, the numbers on each horizontal bar represents the ratio of the weights attached to the bar that keep the bars balanced.



Calculate the missing weights.

18. Mason and Reuben share 320 playing cards in the ratio 9 : 7. Mason gave half of his share to Rochelle. Reuben gave a fifth of his share to Rochelle. What fraction of the total amount did Rochelle receive?

A. $\frac{1}{16}$ **B.** $\frac{53}{160}$ **C.** $\frac{59}{160}$ **D.** $\frac{9}{16}$ **E.** $\frac{7}{10}$

19. Some coins were shared between Alex and Clare in the ratio 13 : 12. Clare gave 10 coins to Alex, and the ratio of coins they have is now 3 : 2. What is the total amount of coins that was initially shared?

Remember this?

20. A hat contains 25 lollies that are either blue or red.

The probability of picking a random blue lolly is 60%.

What is the probability of picking a red lolly?

A. 20% **B.** 40% **C.** 50% **D.** 60% **E.** 70%

21. The table below shows the attendance for a carnival over 3 nights.

Night	Number of people
Friday	987
Saturday	1035
Sunday	867

The cost of each ticket was $15.

What is the mean amount of money collected from ticket sales over the three nights?

A. $10 000 **B.** $14 350 **C.** $14 445 **D.** $15 604 **E.** $15 708

22. $50 + \frac{87 + x}{2} = 125$

What is the value of x?

A. 50 **B.** 63 **C.** 87 **D.** 98 **E.** 105

Back to contents

11D Introduction to rates

Rates are ratios between two quantities with different units. Unlike ratios, rates must be given with units that define the two quantities measured against each other. Rates are used in a variety of contexts, from everyday life to scientific applications, such as physics.

LEARNING INTENTIONS

Students will be able to:

- understand what a rate is
- calculate and simplify rates
- calculate average rates.

KEY TERMS AND DEFINITIONS

Units of measurement define the magnitude and type of quantity. Unit rates are always expressed as quantities per one unit of the other quantity, and must include units of measurement.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: BallBall14/Shutterstock.com

Heart rates measure how many times a heart beats per minute. Different animals, including humans, have different heart rates at various stages in their lives. Heart rates may also change due to atmospheric conditions, fitness levels, and stress factors. Deviations from the expected heart rate for an individual can be used by health professionals to establish a diagnosis.

Key ideas

1. Rates are ratios of two quantities with different units.

As a ratio = quantity 1 : quantity 2	As a rate = quantity 1/quantity 2
metres : seconds	m/s
kilometres : hours	km/h
cents : metres	cents/m
dollars : kilograms	$/kg

2. Rates in their simplest form are written as quantities per one unit.

As a ratio:

80 km : 4 hours → (÷4) → 20 km : 1 hours

As a rate:

20 km per 1 hours = 20 km/h

As a ratio:

$50 : 2 kg → (÷2) → $25 : 1 kg

As a rate:

$20 per 1 kg = 25 $/kg

Worked example 1

Simplifying rates

Write the following as rates in their simplest form.

a. 36 tourists for 3 tour guides. WE1a

Working

36 tourists : 3 guides

12 tourists : 1 guide

12 tourists/guide

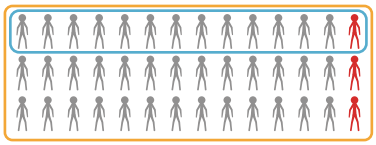
Thinking

Step 1: Write the first given quantity as a ratio of the second given quantity.

Step 2: Simplify the ratio.

Step 3: Write the ratio as a rate with units.

Visual support



b. 100 km for 2 hours. WE1b

Working

100 km : 2 hours

50 km : 1 hour

50 km/h

Thinking

Step 1: Write the first given quantity as a ratio of the second given quantity.

Step 2: Simplify the ratio.

Step 3: Write the ratio as a rate with units.

Student practice

Write the following as rates in their simplest form.

a. 35 children for 7 parents.

b. $56 for 8 kilograms.

Worked example 2

Calculating average rates

Calculate the average rates for each of the following.

a. The enrolment numbers in a university course grew from 50 people in 2018 to 150 people in 2022. What is the average rate of growth of enrolment numbers in the course between 2018 and 2022 WE2a

Working

$150 - 50 = 100$ people

$2022 - 2018 = 4$ years

100 people : 4 years

25 people : 1 year

Average rate of growth = 25 people/year.

Thinking

Step 1: Calculate the changes in both given quantities.

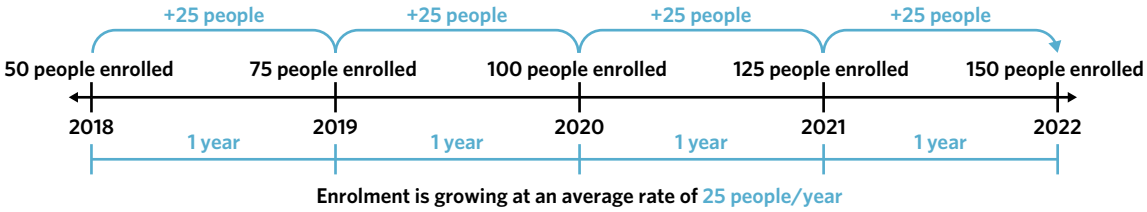
Step 2: Write the changes in quantities as a ratio.

Step 3: Simplify the ratio.

Step 4: Write the ratio as a rate, with units.

Continues →

Visual support



b. A bonsai tree is 5 cm tall when it is first planted in a pot. When the tree is 3 years old, it is 18 cm tall. What is the average growth rate of the tree over the 3 years? Round the answer to 2 decimal places. WE2b

Working

$18 - 5 = 13$ cm

13 cm : 3 years

$\frac{13}{3} = 4.33$

Average rate of growth $= 4.33$ cm/year

Thinking

Step 1: Calculate the change in growth over the given time period.

Step 2: Write the change in growth as a ratio of time.

Step 3: Convert the ratio to a division and calculate.

Step 4: Write the ratio as a rate, with units.

Student practice

Calculate the average rates for each of the following.

a. The number of people attending the museum yearly has increased from 20 000 to 34 000 between 2018 and 2022. What is the average rate of growth of museum attendance between 2018 and 2022, in people/year?

b. When Fabian was 14 years old, his hair was 10 cm long. Now that he is 21 years old, his hair is 61 cm long. What is the average rate of Fabian's hair growth, between the ages of 14 and 21 in cm/year? Round the answer to 2 decimal places.

11D Questions

Understanding worksheet

1. Write the given ratios as rates.

Example

As a ratio	As a rate
dollars : grams	$/g

	As a ratio	As a rate
a.	students : teachers	
b.	dollars : metre	

c.	kilometres : second	
d.	people : square metre	

2. Match each description with the given rates.

Example

Description

Speed of a car ● ——— ● 71 km/h

Description		Rate
a. Running speed of a person	●	● $21.38/h
b. Cost of petrol	●	● 10 km/h
c. Minimum wage	●	● 1 m/day
d. Bamboo growth rate	●	● $1.71/L

3. Fill in the blanks by using the words provided.

average | units | different | quantities | one

Rates are used to compare two quantities with _____ units. Rates are usually written in their simplest form, as quantities per _____ unit. It is important to include _____ with rates, as this distinguishes rates from ratios and fractions. The units also contain information about the _____ that are being compared. _____ rates measure how much a rate has changed per unit, over a finite interval.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

4. Write the following as rates in their simplest form. WE1a

a. 15 waiters for 5 tables
b. 18 patients for 3 doctors
c. 25 customers for 5 checkouts
d. 28 children for 4 kindergarten teachers
e. 190 guests in 95 hotel rooms
f. 480 sandwiches for 120 guests
g. 312 students in 6 buses
h. 510 points in 3 games of Scrabble

5. Write the following as rates in their simplest form. Round the answers to 2 decimal places, where applicable. WE1b

a. $12 for 3 kilograms
b. $420 for 6 grandchildren
c. 800 km in 5 hours
d. 25 L of water for 4 people
e. 11 kg of soil for 5 planters
f. 160 m^2 for 100 chickens
g. $1500 in 7 weeks
h. 70 L of paint for 6 square metres

6. Calculate the average rates for each of the following. WE2a

- **a.** The number of customers increased from 25 to 100 over 5 years.
- **b.** The number of rabbits increased from 4 to 28 over 8 months.
- **c.** The number of roses on a bush increased from 3 to 23 over 4 weeks.
- **d.** The number of goals increased from 10 to 55 between the second and seventh game.
- **e.** The number of qualified hairdressers increased from 120 to 300 between 2018 and 2020.
- **f.** The number of pages read in a book increased from 12 to 150 in 3 days.
- **g.** The population of a town increased from 1500 to 2430 between 1950 and 1956.
- **h.** The number of bacteria in a Petri dish increased from 11 million to 139 million between 12:00 pm and 12:08 pm.

7. Calculate the average rates for each of the following. Round the answers to 2 decimal places, where applicable. WE2b

- **a.** The price of a subscription increased from $15 to $20 in 2 years.
- **b.** The time Manny spends on homework has increased from 20 minutes to 70 minutes in 10 days.
- **c.** Johnny's height increased from 140 cm to 165 cm in 3 years.
- **d.** The distance increased from 0 km to 20 km in 6 minutes.
- **e.** The area of a mould spot grew from 10 cm^2 to 45 cm^2 in 6 days.
- **f.** The cost of rent increased from $380 to $440 per week between 2018 and 2022.
- **g.** The depth of rainfall in Melbourne increased from 5 mm to 12 mm between 12:00 pm and 5:00 pm.
- **h.** The mass of a watermelon increased from 50 grams to 1.5 kilograms in 90 days.

8. Which of the following options shows the simplified rate, rounded to 2 decimal places, for using 17 litres of paint to cover 25 metres squared?

A. 0.68 L **B.** 0.68 L/m^2 **C.** 1.47 m^2/L **D.** 1.47 L/m^2 **E.** 17 L/m^2

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Write $21 for 7 metres as a simplified rate.

Student A

$21 : 7 metres

$\frac{21}{7} = 3$

$3/m

Student B

7 metres : $21

$\frac{7}{21} = \frac{1}{3} \approx 0.33$

$0.33/m

b. Write 30 km for 2 litres of petrol as a simplified rate.

Student A

30 km : 2 L

$\frac{30}{2} = 15$

15 km

Student B

30 km : 2 L

$\frac{30}{2} = 15$

15 km/L

Problem solving

Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Joey pays $6.30 for 3 mangoes. Write the price per mango as a simplified rate.

11. Enyd is learning Japanese using an app. After 20 days of using the app, her vocabulary grew from 30 words to 150 words. What is the average rate at which Enyd is expanding her Japanese vocabulary, in words/day?

12. It takes Marlene 16 hours to crochet a square blanket 2 m long. Write the rate at which Marlene can crochet, in m^2/h.

13. Grandma gives Lottie and Oscar $2.50 for every year that makes up each of their ages. Lottie is 3 years older than Oscar, and she gets $55 from grandma. How old is Oscar?

14. After working at the same factory for 5 years, Missy's salary increased from $2500 per month to $3200 per month. What is the average rate at which Missy's salary increased over the 5 years, in dollars/month per year?

Reasoning

Question working paths

Mild 15 (a,b,c,e) | Medium 15 (a,b,c,e), 16 (a,b) | Spicy All

15. The following table shows the target typing speeds for a variety of levels. Most young people and adults are expected to be able to type at a standard speed. For more specialised jobs and purposes, these expectations may be higher.

Level	Words per minute (wpm)	Minimum rate
Standard	40–<50	40 words/minute
Above standard	50–<60	50 words/minute
Productive	60–<70	60 words/minute
High	70–<120	70 words/minute
Competitive	120+	120 words/minute

a. What is the minimum expected typing speed for a young adult?

b. A student improves their typing level from the minimum of standard to the minimum of high over 10 days of practice. Calculate the average rate of increase in their typing speed.

c. Sumit typed up an essay 2475 words long in 45 minutes. Calculate Sumit's typing rate in wpm and determine his current typing level.

d. In 2019, Barbara Blackburn hit a peak typing rate of 212 wpm during a competition. At this rate, how long would it take her to type up Sumit's essay from part c, to the nearest minute?

e. Typists are advised to prioritise accuracy over speed when in the process of improving their typing skills. State two possible reasons why accuracy is considered to be more important than speed when it comes to typing.

16. Determine the simplified rates described in parts **a** and **b**.

a. 100 km in 4 hours.

b. $50 for 2 kilograms.

c. Compare your answers for parts **a** and **b** and note their similarities and differences. What makes it possible to distinguish between the two rates?

Extra spicy

17. Emma invests some money and doubles it at the end of 5 years. If the investment grows at a rate of $250 per year, then how much money did Emma invest in the first place?

18. Nancy and Amber decide to meet at the park. Nancy walks at a rate of 2.4 km/h, while Amber walks at a rate of 4.2 km/h. They both leave at the same time and will walk towards each other until they meet. What is the simplified ratio of the distance covered by Amber compared to the distance covered by Nancy, by the time they meet?

A. 2.4 : 4.2 **B.** 4.2 : 2.4 **C.** 4 : 7 **D.** 7 : 4 **E.** 24 : 42

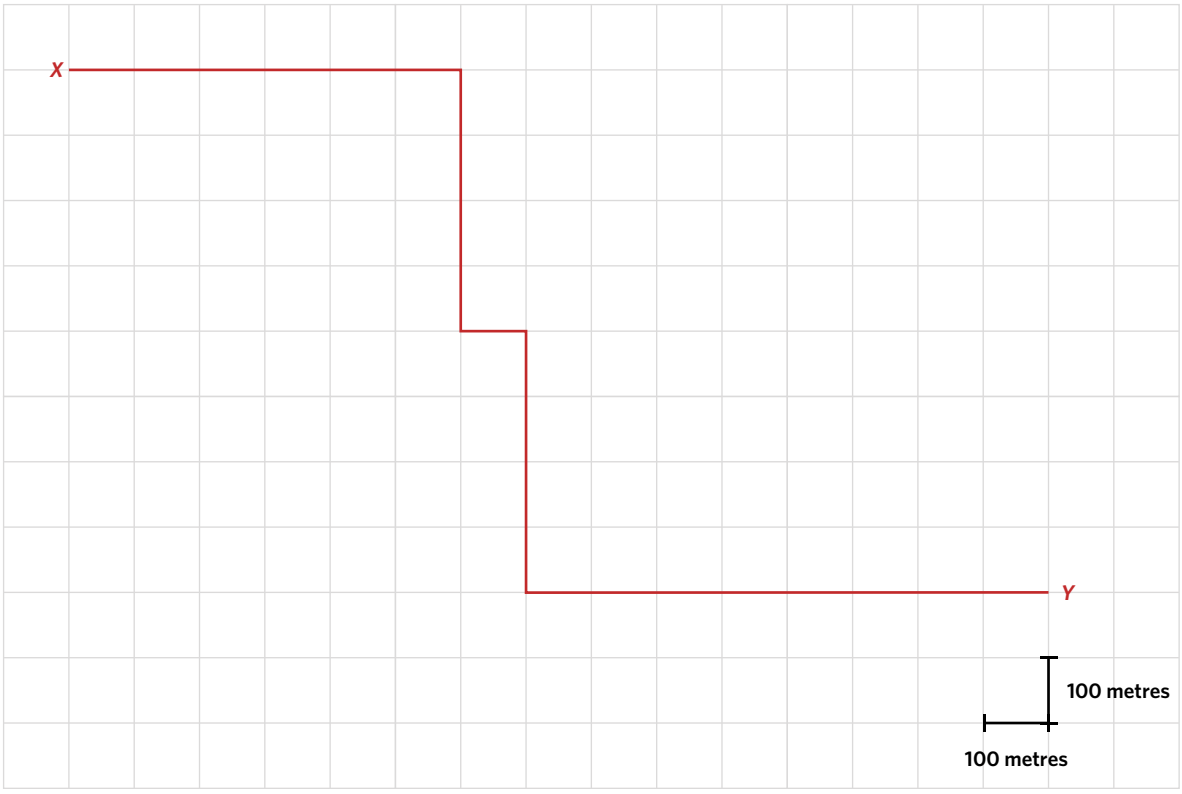
19. Ross needs to read a book for his English class. The book is 230 pages long and Ross normally reads at a rate of 15 pages per day. If Ross starts reading the book on 14 April 2022, 5 days out of every week, on which date will he finish reading the book?

A. 28 April **B.** 29 April **C.** 4 May **D.** 5 May **E.** 6 May

20. 2 lumberjacks have to cut down 100 trees. 1 of the lumberjacks can cut at a rate of 3 trees per hour and the other at a rate of 2 trees per hour. How many trees will each of them cut and how long will it take, if they work simultaneously, starting on opposite corners of the plantation?

Remember this?

21. James walks through a garden from *X* to *Y*.



How far did he walk in **kilometres**?

A. 0.023 **B.** 0.23 **C.** 2.3 **D.** 23 **E.** 2300

22. How many degrees does the **minute hand** of a clock turn in 45 minutes?

A. 27° **B.** 90° **C.** 190° **D.** 270° **E.** 360°

23. Marcus rolls a standard 6-sided die once.

Which of these is Marcus most likely to roll?

A. The number 2
B. A number less than 3
C. A number greater than 2
D. A even number
E. An odd number

Back to contents

11E Solving rate problems

Solving problems involving rates requires proportional reasoning. Firstly, it is important to interpret the rate correctly, in context. Following this, a variety of skills may need to be applied, from simplifying ratios to combining rates using fraction addition.

LEARNING INTENTIONS

Students will be able to:

- solve problems involving rates
- calculate and compare rates
- combine rates.

KEY TERMS AND DEFINITIONS

In a rate, the quantities are **proportional** to each other. This means that the ratio between the two quantities must always remain the same.

Unit rates are rates in their simplest form, per one unit.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: jamesteohart/Shutterstock.com

Rates at their very base allow us to navigate the world and make decisions about things like choosing the most fuel efficient car, renting a financially suitable property, or choosing an appropriately paid job. At their highest level, rates allow us to compare and derive meaning from complex data. For example, in theory, a quantum computer will take 200 seconds to solve a problem which would take a supercomputer 10 000 years.

Key ideas

1. In a rate, a change in one quantity requires the same change to occur in the other quantity.

As a rate:

$32/2 kg

×2 ↑ ↑ ×2

$16/1 kg

÷4 ↓ ↓ ÷4

$4/0.25 kg

Always simplifies to the unit rate or ratio

As a ratio:

32 : 2

×2 ↑ ↑ ×2

16 : 1

÷4 ↓ ↓ ÷4

4 : 0.25

2. Rates can be combined using fraction addition.

$$\text{If rate 1} = \frac{\text{amount 1}}{\text{units of time}} \text{ and rate 2} = \frac{\text{amount 2}}{\text{units of time}}$$

$$\text{then rate 1} + \text{rate 2} = \text{combined rate}$$

$$\therefore \text{combined rate} = \frac{\text{amount 1}}{\text{units of time}} + \frac{\text{amount 2}}{\text{units of time}}$$

Worked example 1

Solving rate problems

Calculate.

a. A snail can travel at a speed of 13 mm/second. How far can a snail travel in 30 seconds? WE1a

Working

13 mm : 1 second

30 × 13 mm : 30 × 1 second

390 mm : 30 seconds

A snail can travel 390 mm in 30 seconds.

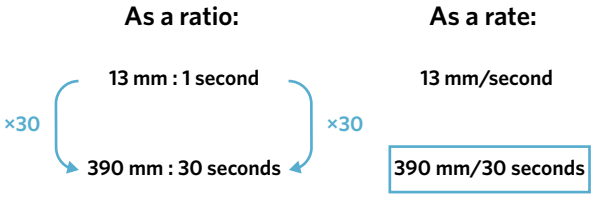
Thinking

Step 1: Write the rate as a ratio of time.

Step 2: Multiply the ratio to the required interval of time.

Step 3: Convert the ratio to a rate in context.

Visual support



b. Teo is a postal worker, who can deliver mail to an average of 80 houses every 20 minutes. How long will it take Teo to deliver mail to 600 houses? WE1b

Working

80 houses : 20 minutes

80 ÷ 20 houses : 20 ÷ 20 minutes

4 houses : 1 minute

4 × 150 houses : 1 × 150 minutes

600 houses : 150 minutes

It will take Teo 150 minutes to deliver to 600 houses.

Thinking

Step 1: Write the rate as a ratio of time.

Step 2: Convert to a unit ratio.

Step 3: Multiply the ratio to the required number.

Step 4: Convert the ratio to a rate in context.

Student practice

Calculate.

a. Seb can make coffees at an average rate of 2 cups/minute. How many cups of coffee can Seb make in 15 minutes?

b. The queue at a bakery moves up at a rate of 14 people every 7 minutes. How long will it take the bakery to serve all 224 customers in the queue?

Worked example 2

Comparing rates

Compare the rates.

a. Sonny can complete a 24 km hike in 10 hours, whereas Sheila can complete a 20 km hike in 8 hours. Determine who is faster and calculate the difference between Sonny and Sheila's hiking speeds. WE2a

Working	Thinking
Sonny: 24 km : 10 hours	**Step 1:** Convert the first given rate to a unit rate.
$24 \div 10$ km : $10 \div 10$ hours	
2.4 km : 1 hour	
∴ Sonny's hiking rate = 2.4 km/h	
Sheila: 20 km : 8 hours	**Step 2:** Convert the second given rate to a unit rate.
$20 \div 8$ km : $8 \div 8$ hours	
2.5 km : 1 hour	
∴ Sheila's hiking rate = 2.5 km/h	
2.5 km/h > 2.4 km/h	**Step 3:** Determine which rate shows a larger quantity per unit and calculate the difference.
Sheila > Sonny	
2.5 − 2.4 = 0.1 km/h	
∴ Sheila is faster than Sonny by 0.1 km/h.	

b. Eddie pays $640 in rent every 2 weeks for his apartment, and Seamus pays $870 in rent every 3 weeks for his. Determine whose rent is higher and calculate the difference in the amount of rent paid by each person in 23 weeks. WE2b

Working	Thinking
Eddie: $640 : 2 weeks	**Step 1:** Convert the first given rate to a unit rate.
$640 \div 2 : 2 \div 2$ weeks	
$320 : 1 week	
∴ Eddie's rent = $320/week	
Seamus: $870 : 3 weeks	**Step 2:** Convert the second given rate to a unit rate.
$870 \div 3 : 3 \div 3$ weeks	
$290 : 1 week	
∴ Seamus' rent = $290/week	
$320/week > $290/week	**Step 3:** Determine which rate shows a larger quantity per unit.
Eddie's rent > Seamus' rent	
Eddie's rent in 23 weeks = $320 \times 23 = 7360$	**Step 4:** Calculate the totals for the given time period for each rate.
Seamus' rent in 23 weeks = $290 \times 23 = 6670$	
$7360 − 6670 = 690$	**Step 5:** Calculate the difference between the totals.
∴ The difference in rent over 23 weeks is $690.	

Continues →

Student practice

Compare the rates.

a. Donnie walks to a park 4.5 km away in 1.2 hours, whereas Terry walks to a park 5.4 km away in 1.5 hours. Determine who is faster and calculate the difference between Donnie's and Terry's walking speeds.

b. Layla spends an average of 4.5 hours commuting every 3 days, while Josiah spends an average of 3.5 hours commuting every 2 days. Determine who spends more time commuting daily and calculate the difference in time spent commuting over 30 days.

Worked example 3

Combining rates

It takes Sean 20 hours to paint a house, while it would take Marty 25 hours to paint the same house. How many hours will it take Sean and Marty to paint the house together, to the nearest hour? WE3

Working	Thinking
Sean: 1 house/20 hours Marty: 1 house/25 hours	**Step 1:** Write both rates as quantities per units of time.
Combined rate $= \frac{1}{20} + \frac{1}{25} = \frac{5}{100} + \frac{4}{100} = \frac{9}{100}$	**Step 2:** Rewrite the rates as fractions and add them together.
9 houses : 100 hours $9 \div 9$ houses : $100 \div 9$ hours 1 house : $\frac{100}{9}$ hours	**Step 3:** Rewrite the combined rate as a ratio and simplify per one required unit.
$100 \div 9 = 11.11 \approx 11$ It will take approximately 11 hours.	**Step 4:** Complete the division.

Student practice

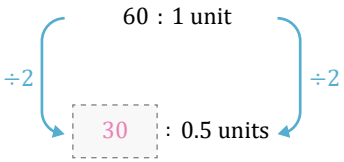
It takes Phillipa 4 hours to mow the lawn, while it would take Ronny 5 hours to mow the same lawn. How many hours will it take Phillipa and Ronny to mow the lawn together, to the nearest hour?

11E Questions

Understanding worksheet

1. Determine the missing values.

Example



a. 30 : 1 unit

÷2 ÷2

[ ] : 0.5 units

b. 36 : 1 unit

÷4 ÷4

[ ] : 0.25 units

c. 100 : 1 unit

÷5 ÷5

[ ] : 0.2 units

d. 44 : 2 units

÷4 ÷4

[ ] : 0.5 units

2. Determine the equivalent operations performed on the quantities.

Example

÷ 4 — 20 people : 1 hour — ÷ 4

5 people : 0.25 hour

a. 40 people : 1 hour

÷ [ ] ÷ [ ]

20 people : 0.5 hours

b. $12 : 1 m

÷ [ ] ÷ [ ]

$3 : 0.25 m

c. 3 kg : $1

÷ [ ] ÷ [ ]

0.3 kg : $0.1

d. 24 hours : 4 days

÷ [ ] ÷ [ ]

3 hours : 0.5 days

3. Fill in the blanks by using the words provided.

equal | proportional | addition | operations

The quantities in a rate must remain [ ]. In order to accomplish this, identical [ ] must be performed on both quantities when one of them is multiplied or divided. To compare rates, the quantities must be expressed per [ ] unit(s). Combining rates requires knowledge of fraction [ ] and proportional reasoning.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c), 5 (a,b,c), 6 (a,b), 7 (a,b), 8 (a,b), 9	4 (c,d,e), 5 (c,d,e), 6 (b,c), 7 (b,c) 8 (b,c), 9	4 (e,f,g), 5 (e,f,g), 6 (c,d), 7 (c,d) 8 (c,d), 9

4. Calculate. WE1a

a. Daria can type at a rate of 75 words/minute. How many words can Daria type in 45 minutes?

b. Mr Wong marks tests at a rate of 4 tests/hour. How many tests can Mr Wong mark in 5.5 hours?

c. Michael scored an average of 5 goals every 2 matches. How many goals did Michael score over 34 matches?

d. Ms Lin teaches an average of 130 children every 2 years. How many children has Ms Lin taught over 13 years of teaching?

e. Isha buys an average of 7 souvenirs for every 3 countries that she visits. How many souvenirs has Isha bought after visiting 51 countries?

f. Wyatt finds an error on every third page of a manuscript. How many errors has Wyatt found in a manuscript which is 340 pages long?

g. Morgan's car uses 5 litres of petrol for every 100 km of travel. How many litres of petrol will Morgan need to travel a distance of 360 km?

---

5. Calculate. WE1b

a. 240 g of squash is enough to make soup for 3 people. How many people can soup be made for, with 720 g of squash?

b. A tutor spends 45 minutes with every student. How many students can the tutor see in 270 minutes?

c. A honey bee can pollinate 180 flowers in 2 foraging flights. How many foraging flights will it take 1 single bee to pollinate 4500 flowers?

d. A diver ascends a distance of 24 metres every 3 minutes. How long will it take the diver to ascend a distance of 40 metres?

e. Cheyenne answered 2 work calls every 9 minutes. How long was Cheyenne's work day if she answered a total of 67 calls, to the nearest hour?

f. It takes 24 trees to make 200 000 sheets of paper. How many sheets of paper can be made from 162 trees?

g. The electricity output of 10 lemons is equivalent to 7 volts. What is the electricity output of 153 lemons?

---

6. Compare the rates. WE2a

a. Wendy serviced 16 computers in a 4 hour work day and Tim serviced 21 computers in a 7 hour work day. Determine who is working faster and calculate the difference between the number of computers serviced per hour.

b. Lester gets paid $500 for 8 hours of work and Bobbie gets $600 for 10 hours of work. Determine who is paid more and calculate the difference between Lester and Bobbie's pay per hour.

c. Gianni used 7 m of material to make 3 dresses and Coco used 11 m to make 5 dresses. Determine who used more material per dress and calculate the difference in number of metres used per dress, rounded to 2 decimal places.

d. Carlo saved $370 in 2 weeks and Nora saved $460 in 20 days. Determine who is saving money at a faster rate and calculate the difference between Carlo and Nora's savings per day, to the nearest cent.

---

7. Compare the rates. WE2b

a. Mark serves 14 customers every 2 hours, while Nick serves 24 customers every 4 hours. Determine who serves more customers and calculate the difference between the number of customers served by each person in an 8 hour work day.

b. The fuel efficiency of Cindy's car is 4 L per 100 km, while the fuel efficiency of Tahlia's car is 8.5 L per 200 km. Determine whose car is more efficient and calculate the difference in the amount of fuel used by each person to complete a 250 km trip.

c. Arlene takes 5 hours to knit a 2 m^2 blanket, while Darlene takes 6 hours to knit a 2.5 m^2 blanket. Determine who is the faster knitter and calculate the difference in the amount of time it would take each person to knit a 3 m^2 blanket.

d. A three bedroom house in Melbourne can be rented for $1400 per fortnight, while a similar house in Sydney can be rented for $3350 per month (equivalent to 30.4 days). Determine which city has the higher rent and calculate the difference between the total amount of rent paid during a non-leap year.

8. Combine the rates. WE3

a. It takes 2 rowers 3 and 4 minutes respectively to row a distance of 1 km. How long will it take both of the rowers to row 1 km together, to the nearest minute?

b. It takes 2 movers 6 and 5 hours respectively to move a one-bedroom apartment. How long will it take both of them to move the apartment together, to the nearest hour?

c. It takes 2 students 15 and 18 hours respectively to complete an assignment. How long will it take both of them to complete the assignment together, to the nearest hour?

d. 3 librarians take 8, 10, and 12 hours to organise the non-fiction section of the library. How long will it take all 3 of them to organise the section together, to the nearest hour?

---

9. Jack earned $140.50 for 3 hours of work. How much will Jack get paid for 5 hours of work?

A. $46.83 **B.** $234.17 **C.** $421.50 **D.** $702.50 **E.** $3372

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. In air, sound can travel around 1 km in 3 seconds. How far can sound travel in a minute?

Student A

1 km : 3 seconds

1×20 km : 3×20 seconds

20 km : 60 seconds

Sound can travel 20 km per minute.

Student B

1 km : 3 seconds

$1 \div 20$ km : 3×20 seconds

0.05 km : 60 seconds

Sound can travel 0.05 km per minute.

b. Conrad gathered 2.5 kg of blueberries in 2 hours and Hania gathered 3 kg in 3 hours. Determine who gathered blueberries at a faster rate.

Student A

Conrad: 2.5 kg : 2 hours

Hania: 3 kg : 3 hours

$2.5 \text{ kg} < 3 \text{ kg}$

$\therefore$ Hania gathered berries at a faster rate.

Student B

Conrad: 2.5 kg : 2 hours

$2.5 \div 2$ kg : $2 \div 2$ hours

1.25 kg : 1 hour

$\therefore$ Conrad's rate = 1.25 kg/hour

Hania: 3 kg : 3 hours

$3 \div 3$ kg : $3 \div 3$ hours

1 kg : 1 hour

$\therefore$ Hania's rate = 1 kg/hour

1.25 kg/hour > 1 kg/hour

$\therefore$ Conrad gathered berries at a faster rate.

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. On a particular day, Dr Rosanna starts seeing patients at 8:30 am and finishes by 5:30 pm. If she saw 40 patients that day, then what is the rate at which Dr Rosanna saw her patients, in patients/hour, to the nearest patient?

12. Ruth's tropical fish are breeding at a rate of 8 new fry every 2 days per female fish. If Ruth has a total of 9 female fish, then how long will it take them to birth a total of 90 new fry?

13. One small ink cartridge contains enough ink for 600 pages of printing, while a medium cartridge is twice as efficient as the small cartridge. If a large ink cartridge is enough for 1800 pages, then how much ink does it contain, in terms of the small and medium cartridges combined?

14. 7 robots at a car factory can assemble 336 cars in 1 day. How many days will it take 10 robots to assemble 1920 cars?

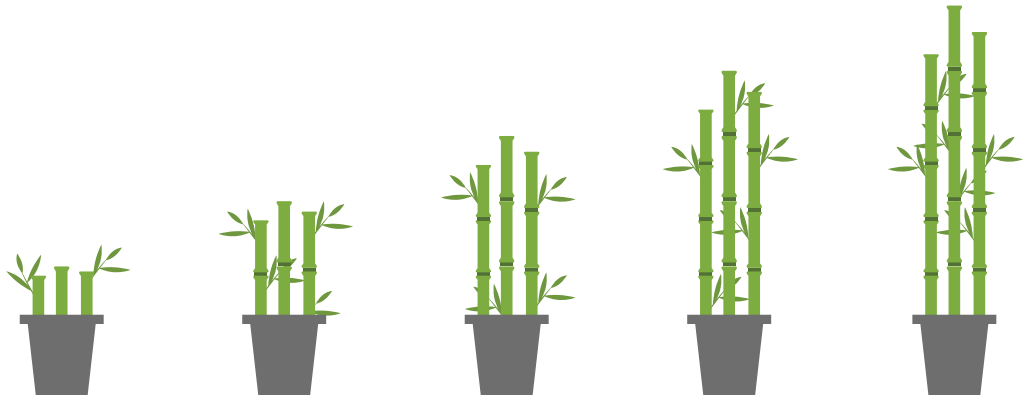
15. Bo and Maira own a joinery business. Bo can assemble a regularly sized kitchen in 7 hours and Maira can assemble it in 8 hours. How long will it take both of them to assemble a kitchen which is twice the size of a regular kitchen, in hours, correct to one decimal place?

Reasoning

Question working paths

Mild 16 (a,b,d)	Medium 16 (a,b,d), 17 (a,b)	Spicy All

16. Bamboo is one of the fastest growing plants in the world. It is the largest member of the grass family.



a. Certain species of bamboo can grow up to 1 metre every day during peak growth. What is the total increase in height over a one week period at this growth rate?

b. Moso bamboo can increase in height up to 3 m every 21 days. At this rate, how many weeks will it take the moso bamboo to grow to its full height of 12 m, from 0 m?

c. Golden bamboo grows at a rate of 1 m every year. Calculate how much faster moso bamboo grows compared to golden bamboo, in mm/day, to the nearest mm.

d. Bamboo is a versatile, strong, and biodegradable material which has a number of uses. Name two everyday items made from bamboo.

17. For parts **a** and **b**, compare the rates without completing any calculations.

a. \$45 per 2 kg (\$45/2 kg) and \$50 per 2 kg (\$50/2 kg).

b. 15 km per 3 hours (15 km/3 hours) and 15 km per 2 hours (15 km/2 hours).

c. Using the answers from parts **a** and **b**, explain how fractions can be used to compare rates.

Extra spicy

18. One train carriage can transport up to 150 passengers at a time. Every day, 226 six-carriage trains operate across all of Melbourne. What is the minimum number of days it will take to transport all 5 078 000 people living in Melbourne, if each person travels via train at least once?

A. 1 **B.** 2 **C.** 3 **D.** 17 **E.** 25

19. A pool with a volume of 375 m^3 is being filled at a rate of 0.25 L/s. If 1 cm^3 = 1 mL, then how many days will it take to fill the pool?

20. The probability of randomly selecting 2 blue jelly beans from 2 identical packets of candy is 36%. How many blue jelly beans are there per packet of 100 candies?

A. 9 **B.** 18 **C.** 36 **D.** 60 **E.** 72

21. It takes 10 people 16 hours to finish a project. How long will it take 4 people to finish half of the project?

Remember this?

22. 17 children at a carnival were given the choice of 4 snacks they could have. They each voted for 1 snack.

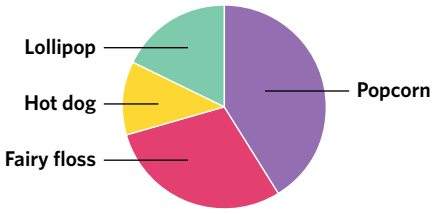
7 children voted for popcorn.

5 children voted for fairy floss.

2 children voted for a hot dog.

The rest voted for the lollipop.

How many children voted for the lollipop?



A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

23. An unknown number is subtracted from 23.

The result is multiplied by 4 to give an answer of 68.

Which of these is the unknown?

A. 2 **B.** 4 **C.** 6 **D.** 8 **E.** 10

24. A taxi company wants to estimate the amount of money they make for each trip, they use the formula

$m = 3 + 6d$

where m is the amount of money made per trip and d is the distance travelled in kilometres.

Select the statement that is not true.

A. A 5 kilometre trip costs $43.

B. A 10 kilometre trip costs $63.

C. Every kilometre $6 is added to the trip total.

D. The base rate for trips is $3.

E. A 23 kilometre trip costs $141.

Back to contents

11F Speed

Speed is one of the most commonly used rates. It is a ratio of the distance travelled by an object to the time taken. Speeds of everyday objects, like cars and aeroplanes, are generally given in 'kilometres per hour' or km/h. However, according to the International System of Units or SI, the units for speed are 'metres per second' or m/s.

LEARNING INTENTIONS

Students will be able to:

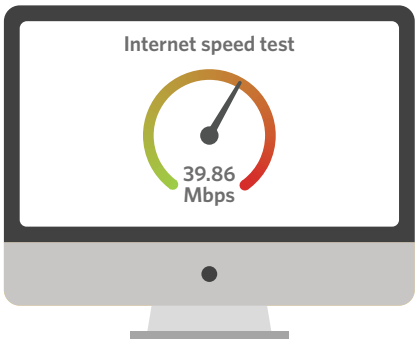
- understand the relationship between speed, distance, and time
- calculate average speed
- solve distance-time problems.

KEY TERMS AND DEFINITIONS

Constant speed occurs when the speed of an object remains the same and does not increase or decrease.

An object is **stationary** when distance does not change over a time interval and the speed is equal to zero.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Much like the physical speed of an object, internet speed, measured in 'megabits per second' or Mbps, shows us how fast digital information can be transferred from one point to another. As a general rule, we prefer our internet speeds to be as fast as possible. This allows for uninterrupted streaming and gaming, as well as the connection of multiple devices on the same network.

Key ideas

1. Speed is the rate of distance travelled per unit of time. The formula can be rearranged to calculate time and distance.

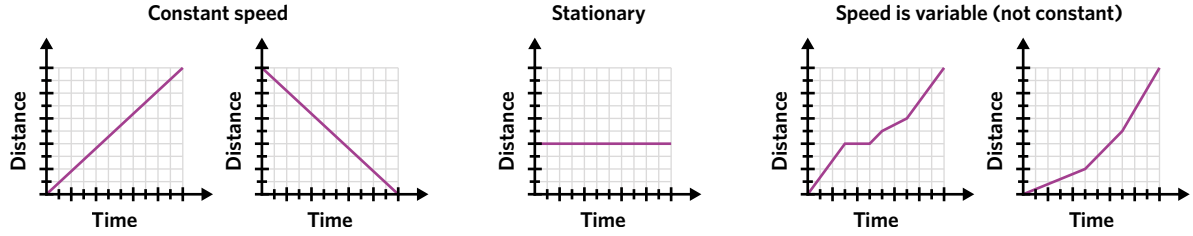
$$s = \frac{d}{t} \Rightarrow t = \frac{d}{s} \Rightarrow d = st$$

Where s = speed, d = distance, and t = time

2. When an object is not travelling at constant speed over a time interval, **average speed** can be calculated.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

3. Distance-time graphs show the change in distance relative to time. They are a visual representation of the speed of an object.



Worked example 1

Calculating speed, time and distance

Calculate.

a. The speed of a car that travelled 430 km in 4 hours. WE1a

Working

$s = \frac{d}{t} = \frac{430}{4}$

$= 430 \div 4 = 107.5$ km/h

Thinking

Step 1: Substitute the given values into the formula for speed.

Step 2: Calculate the speed.

b. The time taken to cycle a distance of 24 km at a speed of 10 km/h. WE1b

Working

$t = \frac{d}{s} = \frac{24}{10}$

$= 24 \div 10 = 2.4$ hours

$0.4 \times 60 = 24$ minutes

$\therefore 2.4$ hours $= 2$ hours and 24 minutes.

Thinking

Step 1: Substitute the given values into the formula for time.

Step 2: Calculate the time.

c. The distance travelled in 5 minutes at a speed of 32 m/minute. WE1c

Working

$d = st = 32 \times 5$

$= 160$ m

Thinking

Step 1: Substitute the given values into the formula for distance.

Step 2: Calculate the distance.

Student practice

Calculate.

a. The speed of a motorbike that travelled 370 km in 4 hours.

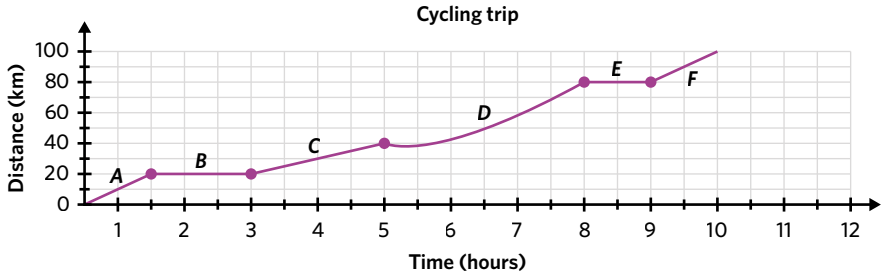
b. The time taken to jog 14 km at a speed of 8 km/h.

c. The distance travelled in 3 hours at a speed of 52 km/h.

Worked example 2

Interpreting distance-time graphs

Use the cyclist's distance-time graph to identify the following.



a. The parts of the trip during which the cyclist was stationary. WE2a

Working

The cyclist stopped during parts *B* and *E* of the trip.

Thinking

Identify the part(s) of the distance-time graph where the plotted line is horizontal.

b. The parts of the trip during which the cyclist was travelling at constant speed. WE2b

Working

The cyclist travelled at constant speed during parts *A*, *C*, and *F* of the trip.

Thinking

Identify the part(s) of the distance-time graph where the plotted line has a constant gradient and is not horizontal or curved.

c. The part of the trip during which the cyclist was travelling at variable speed. WE2c

Working

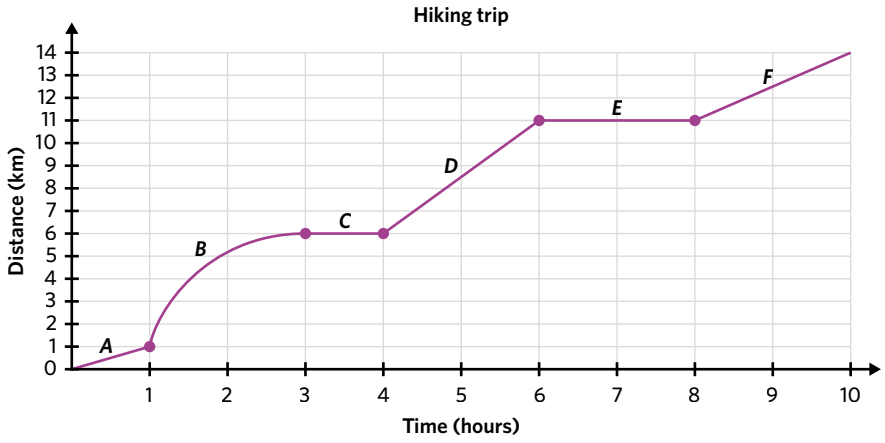
The cyclist travelled at variable speed during part *D* of the journey.

Thinking

Identify the part(s) of the distance-time graph where the plotted line is curved.

Student practice

Use the hiker's distance-time graph to identify the following.



a. The parts of the trip during which the hiker was stationary.

b. The parts of the trip during which the hiker was travelling at constant speed.

c. The part of the trip during which the hiker was travelling at a variable speed.

Worked example 3

Solving distance-time problems

Use the cyclist's distance time-graph from worked example **2** to answer the questions.

a. Determine how many hours the cyclist was stationary during the entire trip. WE3a

Working

Part B is between 1.5 and 3 hours:
$3 - 1.5 = 1.5$ hours

Part E is between 8 and 9 hours:
$9 - 8 = 1$ hour

$1.5 + 1 = 2.5$ hours

Thinking

Step 1: Determine the duration of each stationary part by comparing the start and end times along the horizontal axis.

Step 2: Calculate the total time spent stationary by adding the durations of the parts together.

b. Calculate the speed of the cyclist during part C of the trip. WE3b

Working

$d = 40 - 20 = 20$ km

$t = 5 - 3 = 2$ hours

$s = \frac{d}{t} = \frac{20}{2} = 10$ km/h

Thinking

Step 1: Identify the distance and duration of the specified part of the journey.

Step 2: Substitute the distance and time into the formula for speed and calculate.

c. Calculate the average speed of the cyclist during the entire trip. WE3c

Working

Total distance $= 100$ km

Time taken $= 10$ hours

Average speed $= \frac{100}{10} = 10$ km/h.

Thinking

Step 1: Determine the total distance covered over the entire trip and the total time taken.

Step 2: Substitute the values into the formula for average speed and calculate.

Student practice

Use the hiker's distance-time graph from worked example **2** to answer the questions.

a. Determine how many hours the hiker was stationary during the entire trip.

b. Calculate the speed of the hiker during part D of the trip.

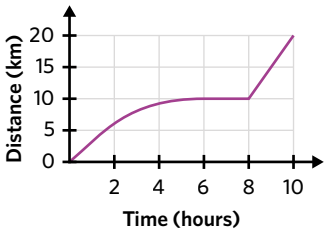
c. Calculate the average speed of the hiker during the entire trip.

11F Questions

Understanding worksheet

1. Circle the correct description for each graph.

Example

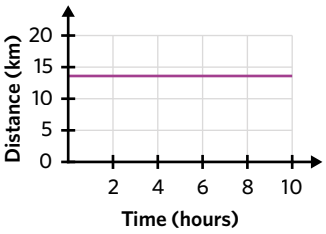


Constant speed

Stationary

Variable speed

a.

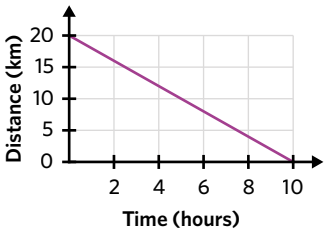


Constant speed

Stationary

Variable speed

b.

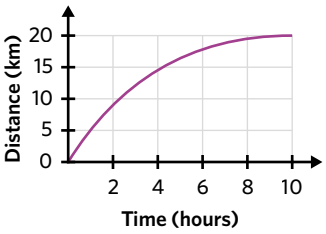


Constant speed

Stationary

Variable speed

c.

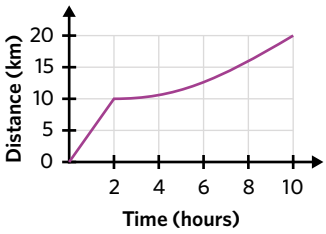


Constant speed

Stationary

Variable speed

d.

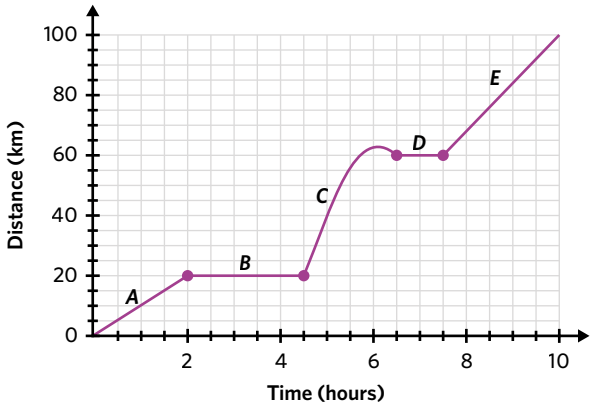


Constant speed

Stationary

Variable speed

2. Match the description of the speed of the object during each part of the journey with one of the given words.



Constant

Stationary

Variable

Example

Part *E* description of speed: Constant

a. Part *A* description of speed: _____

b. Part *B* description of speed: _____

c. Part *C* description of speed: _____

d. Part *D* description of speed: _____

3. Fill in the blanks by using the words provided.

hour | time | second | distance

Speed is a rate that shows the relationship between distance and _____. The speeds of objects can be expressed as _____ in any units per any units of time. Kilometres per _____ are often used to describe speeds of everyday objects, whereas scientific applications require speeds of all objects to be expressed in metres per _____.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Calculate the speed of: WE1a

a. a cyclist who cycled 26 km in 2 hours.
b. a runner who ran 400 m in 50 seconds.
c. a car that travelled 110 km in 2.5 hours.
d. a bird that flew 50 km in 1.6 hours.
e. a spacecraft that travelled 1 000 000 km in 3.2 hours.
f. a flea that jumped 30 cm in 0.25 minutes.
g. an asteroid that travelled 39 km in 2.4 seconds.
h. a cheetah that ran 18 km in 0.15 hours.

5. Calculate the time taken to: WE1b

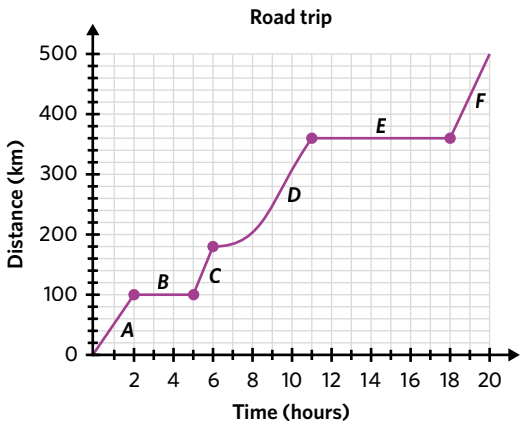
a. drive a distance of 110 km at a speed of 55 km/h.
b. swim a distance of 1 km at a speed of 2 km/h.
c. walk a distance of 24 km at a speed of 5 km/h.
d. run a distance of 150 m at a speed of 8 m/second.
e. fly a distance of 6105 km at a speed of 740 km/h.
f. climb a height of 120 m at a speed of 1.6 m/s.
g. run a distance of 21 km at a speed of 4.2 m/s.
h. walk a distance of 450 m at a speed of 3.6 km/h.

6. Calculate the distance travelled in: WE1c

a. 25 minutes at a speed of 3 km/minute.
b. 6 hours at a speed of 48 km/h.
c. 30 minutes at a speed of 5.6 m/minute.
d. 4.5 hours at a speed of 34 km/h.
e. 20 minutes at a speed of 84 km/h.
f. 34 seconds at a speed of 24 m/minute.
g. 93 seconds at a speed of 36 km/minute.
h. 430 seconds at a speed of 342 km/h.

7. Use the family car's distance-time graph to identify the following. WE2

a. The parts of the trip during which the car was stationary.
b. The parts of the trip during which the car was travelling at constant speed.
c. The part of the trip during which the car was travelling at variable speed.
d. The part of the trip which took the least amount of time.
e. The part of the trip which took the most amount of time.
f. The part of the trip during which the family was resting at a motel overnight, assuming that they left for the trip at 10 am.
g. The part of the trip during which the car was travelling at the fastest constant speed.
h. The part of the trip during which the car was travelling at the slowest constant speed.



8. Use the family car's distance-time graph from question **7** to answer the questions. WE3

- **a.** Determine how many hours the car was stationary during the entire trip.
- **b.** Calculate the speed of the car during part *C* of the trip.
- **c.** Calculate the speed of the car during part *A* of the trip.
- **d.** Calculate the speed of the car during part *F* of the trip.
- **e.** Calculate the average speed of the car during the entire trip.
- **f.** Calculate the average speed of the car during part *D* of the trip.
- **g.** Calculate the average speed of the car during travel times only, excluding stopping time.
- **h.** Calculate the time it would have taken for the trip at an average speed without any stops, as in part **g**.

---

9. Calculate the average speed of a hiker who spent 3 hours walking and 1.8 hours stationary during a 15 km trip.

A. 0.32 km/h **B.** 3.125 km/h **C.** 5 km/h **D.** 12.5 km/h **E.** 72 km/h

Spot the mistake

10. Select whether Student A or Student B is incorrect.

- **a.** Calculate the time taken to travel a distance of 45 km at a speed of 30 km/h.

Student A

$d = 45$ km

$s = 30$ km/h

$t = \frac{d}{s}$

$t = \frac{45}{30} = 1.5$ hours

Student B

$d = 45$ km

$s = 30$ km/h

$t = \frac{s}{d}$

$t = \frac{s}{d} = \frac{30}{45} = 0.\overline{6} \approx 0.7$ hours $= 42$ minutes.

- **b.** Calculate the distance travelled in 15 minutes at a speed of 20 km/h.

Student A

$s = 20$ km/h

$t = 15$ minutes

$d = st$

$d = 20 \times 15 = 300$ km

Student B

$s = 20$ km/h

$t = 15$ minutes $= 0.25$ hours

$d = st$

$d = 20 \times 0.25 = 5$ km

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Herbie walked to the beach from his house at an average speed of 3 km/h for 45 minutes. If Herbie also stopped for a coffee for 15 minutes duing this time, then how far away is the beach from his house?

12. Renae walks from her house to the river 5 km away. She jogs back at a speed of 7 km/h. How long will it take Renae to get home at this speed, to the nearest minute?

13. Ismael and Astrid both competed in the 1 km race. Ismael finished the race in 17 minutes and Astrid finished it in three tenths of an hour. Determine who won the race and by how much.

14. On the way to the supermarket from her house, Sana's average walking speed was 4 km/h. She walked back the same way, at an average speed of 3 km/h. If the supermarket is 3.5 km away from Sana's house, then how much longer did it take her to get home compared to how long it took to get there, to the nearest minute?

15. A car leaves a street corner and travels at a constant speed of 60 km/h. A drone leaves the same corner 30 minutes after the car and follows it at a speed of 120 km/h. How long will it take the drone to catch up to the car?

Reasoning

Question working paths

Mild 16 (a,b,d)	Medium 16 (a,b,d), 17 (a,b)	Spicy All

16. Thrust SSC or Thrust SuperSonic Car is the fastest land vehicle in the world. Developed by British engineers, it uses jet engines as opposed to petrol or diesel, which allows it to reach phenomenal speeds.

Image: Dmitry Eagle Orlov/Shutterstock.com

Image: patruflo/Shutterstock.com

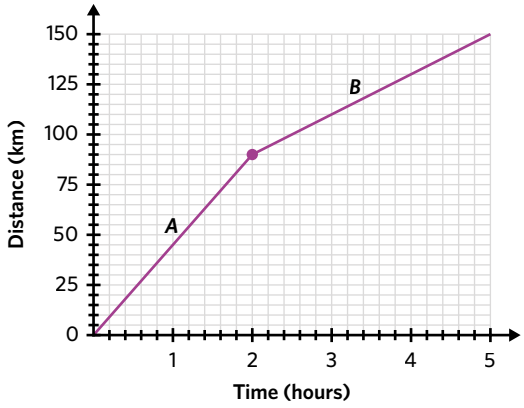
a. The first automobile, developed in 1886, could reach a speed of 16 km/h. Calculate the time taken to travel 1228 km at this speed, to the nearest hour.

b. On 15 October 1997, Thrust SSC reached a speed of 1228 km/h. Calculate the time taken to travel 16 km at this speed, to the nearest second.

c. Breaking the sound barrier requires an object to travel faster than sound. Using 343 m/s for the speed of sound in air, verify whether Thrust SSC broke the sound barrier by comparing the speeds.

d. The record setting driver of the Thrust SSC, Andy Green, stated that driving at very fast speeds means that the driver has far less control of the vehicle compared to driving at a slower speed. Name another reason for imposing speed limits on roads.

17. Use the distance-time graph to answer the questions.



a. Calculate the speed during part *A*.

b. Calculate the speed during part *B*.

c. Using your answers for parts **a** and **b**, explain whether it is possible to determine the average speed of an object using fraction addition of component speeds.

Extra spicy

18. The distance between Melbourne and Darwin by plane is 3124 km. A plane leaves at 6:45 am local time in Melbourne and arrives at 12:25 pm local time in Darwin. If Melbourne is 1 hour and 30 minutes ahead of Darwin, then what was the average speed of the plane during the journey, in km/h, to the nearest km?

19. Callan can run 400 m in 55 seconds and Bernice can run 400 m in 50 seconds. They want to start the race at a fair distance from each other, so that in theory, each athlete can finish the 400 m race at the same time. If Callan starts at 400 m from the finish line, how far behind her should Bernice start?

A. 5 m **B.** 36 m **C.** 40 m **D.** 50 m **E.** 100 m

20. A bus that is 20 m long is travelling at a constant speed. It takes the bus 2 minutes to completely clear a bridge 700 m long. What is the speed of the bus, in km/h?

A. 21 km/h **B.** 21.6 km/h **C.** 22.2 km/h **D.** 35 km/h **E.** 36 km/h

21. Piper leaves her house in the morning and travels at a constant speed of 50 km/h for an hour. When she is 12 km into her trip, she notices that the minute hand is directly over the hour hand on her watch. What time did Piper leave her house, to the nearest minute?

Remember this?

22. The fraction of monkey species that are considered to be under threat of extinction is $\frac{262}{334}$.

What percentage of monkey species are considered to be under threat of extinction?

A. 0.784% **B.** 0.885% **C.** 78.4% **D.** 88.5% **E.** 92%

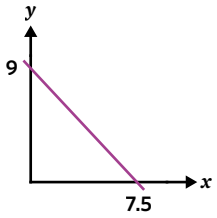
23. Maria uses the same rule to change each ♥ into a ●.

♥	●
3	11
4	16
8	36

Which of these rules did Maria use?

A. ● = ♥ × 3 − 2
B. ● = ♥ × 4 − 5
C. ● = ♥ × 5 − 3
D. ● = ♥ × 2 − 5
E. None of the above

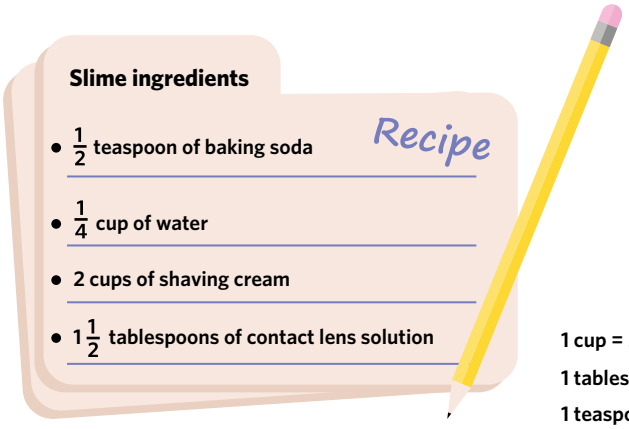
24. Which of these equations represents the line in the graph?



A. $y = 9 - 1.2x$
B. $y = 9 - 7.5x$
C. $y = 9 + 1.2x$
D. $y = 9 + 7.5x$
E. $y = 9 + 9x$

Chapter 11 extended application

1. A Year 8 science class used the following recipe and conversion chart to make slime. Answer the questions using the information given in the image.



1 cup = 240 mL
1 tablespoon = 20 mL
1 teaspoon = 5 mL

a. Simplify the ratio of water to shaving cream.

b. Convert all the quantities in the recipe to mL.

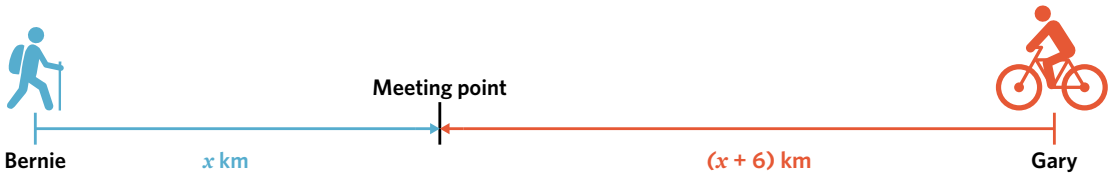
c. Simplify the ratio of baking soda to water.

d. Simplify the ratio of water to contact lens solution.

e. Simplify the ratio of all four ingredients, i.e. baking soda : water : shaving cream : contact lens solution.

f. Describe a fun experiment that you would like to do or have done in a science classroom.

2. Bernie is walking down a country road to meet his friend Garry, who is cycling along the same road, but in the opposite direction. By the time they do meet, Garry has cycled 6 km more compared to the distance walked by Bernie.



a. Bernie starts walking at 8:00 am and he meets Garry by 10:30 am. Using x to represent the total distance covered, write an expression for Bernie's average speed.

b. Garry starts cycling at 8:30 am and also arrives at the meeting point at 10:30 am. Write an expression for Garry's average speed.

c. Using the expression for Bernie's speed in part **a**, write another expression to show Bernie's distance from his starting point at 9:00 am.

d. Using the expression for Garry's speed in part **b**, write another expression to show Garry's distance from his starting point at 9:00 am.

e. At 9:00 am, Bernie and Garry were exactly the same distance away from each of their respective starting points. What is the total distance covered by each person?

f. In the country, phone reception can often be weak or disappear completely. For this reason, people walk up to higher ground to improve their signal. Explain why this method can work in some cases.

3. To go on the Ghost Train ride, a person must be at least 12 years old and be of a minimum height. Further to this, if the person is under 14 years old, they must be accompanied by an adult.

Ghost train rules:

.You must be at least 12 years old
. Children under 14 must be accompanied by an adult
. Height restrictions apply

a. In the line to ride the Ghost Train, there are 3 times as many children under 14 as there are adults. Write a simplified ratio of adults to children in the line.

b. Of the children under 14, half do not meet the minimum height requirements. Write a simplified ratio of children who do and do not meet the height requirements.

c. In total, there were 400 people in the line, including those who did not meet requirements. How many people in the line were accepted on the ride?

d. Write a simplified ratio of adults to children under 14 who were accepted on the ride.

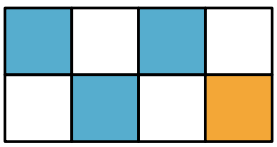
e. On another day, there are 20 more adults and 50 more children riding the Ghost Train. Originally, a third of the children in the line were not accepted on the ride. How many people were in the line in the first place?

f. Give a reason for height restrictions on amusement rides.

Chapter 11 review

Multiple choice

1. Which of the following correctly represent the ratio of blue parts to orange parts for the area model? 11A



A. 1 : 3
B. 1 : 4
C. 4 : 3
D. 3 : 4
E. 3 : 1

---

2. Which of the following operations complete the equivalent ratio? 11B

□ (13 : 17 ⇄ 65 : 85) □

A. +52, +68 **B.** −52, −68 **C.** ×5, ×5 **D.** ÷5, ÷5 **E.** +5, +5

---

3. Which of the following correctly represents the share in the ratio 3 : 7 if 1 part = 2. 11C

A. ×1 (3 : 7 ⇄ 3 : 7) ×1

B. ×2 (3 : 7 ⇄ 6 : 8) ×2

C. ÷2 (3 : 7 ⇄ 6 : 14) ÷2

D. ÷2 (3 : 7 ⇄ 1.5 : 7.5) ÷2

E. ×2 (3 : 7 ⇄ 6 : 14) ×2

---

4. Determine the equivalent operations performed on the quantities. 11E

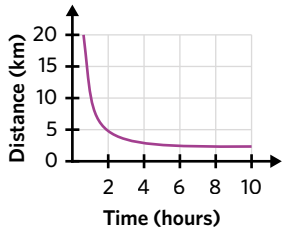
□ (40 km : 5 hours ⇄ 8 km : 1 hour) □

A. ÷8, ÷8 **B.** ×8, ×8 **C.** ×5, ×5 **D.** ÷5, ÷5 **E.** ÷40, ÷40

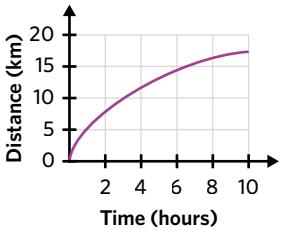
---

5. Which of the following graphs correctly represents constant speed? 11F

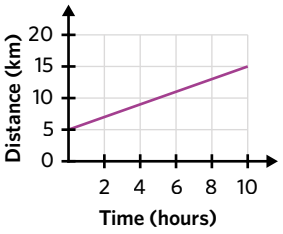
A.



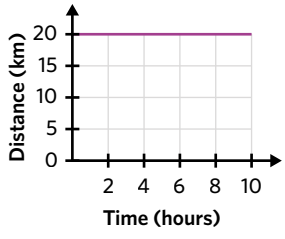
B.



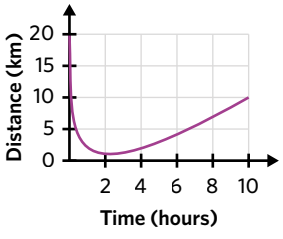
C.



D.



E.



Fluency

6. Write the ratio described in each scenario. 11A

a. A glucose molecule contains 6 carbon atoms, 12 hydrogen atoms and 6 oxygen atoms.
What is the ratio of oxygen atoms to all the atoms in a glucose molecule?

b. $\frac{3}{7}$ of the fish in a fish tank are goldfish.
What is the ratio of goldfish to other types of fish?

c. The height of a truck is 20% the truck's length.
What is the ratio of the truck's height to its length?

d. The recipe of a dish calls for 3 teaspoons of salt, 0.5 teaspoons of pepper, 2 teaspoons of sugar and 1 teaspoon of chilli flakes.
What is the ratio of salt and pepper to sugar and chilli flakes?

---

7. For each ratio, make an equivalent ratio by performing the operation in the brackets. 11B

a. $3 : 4$ $(\times 4)$

b. $16 : 32$ $(\div 8)$

c. $3 : 7 : 11$ $(\times 5)$

d. $12 : 22 : 32$ $\left(\times \frac{3}{2}\right)$

---

8. Write each ratio in its simplest form. 11B

a. $12 : 21$

b. $\frac{5}{7} : \frac{9}{14}$

c. $\frac{1}{3} : \frac{3}{5} : 2\frac{5}{6}$

d. $2.3 : 4.7 : 6.78$

---

9. For each ratio and quantity to be shared, calculate the value of 1 part. Round to 2 decimal places where appropriate. 11C

a. 30 in the ratio $3 : 7$

b. 45 in the ratio $2 : 9$

c. 68 in the ratio $4 : 5 : 8$

d. 171 in the ratio $1 : 4 : 8$

---

10. For each scenario, determine the quantity that was shared in the given ratio. Round to 2 decimal places where appropriate. 11C

a. Leonardo and Michaelangelo split a box of pizza into the ratio $2 : 3$. Michelangelo had 9 slices of pizza. How many slices of pizza were there in the box?

b. In a classroom, the ratio of tables to chairs is $1 : 6$. There are 12 tables in the classroom. What is the total number of furniture?

c. A sporting event hosts basketball, football and volleyball games in the ratio of $3 : 7 : 2$. 8 volleyball games were run during the event. What is the total number of games that took place during the event?

d. In a game of tug of war, the ratio of teachers and students per team is $1 : 4$. There are 5 teachers in 1 of the teams. Assuming both teams have the same number of teachers and students, what is the total number of participants in the game?

---

11. Calculate the average rates for each of the following. Round the answers to 2 decimal places, where applicable. 11D

a. The bank balance rose from \$1500 to \$2500 over 5 months.

b. The number of customers in a restaurant increased from 6 to 90 between 5:40 pm to 6:08 pm.

c. The volume of water in a fish tank increased from 10 litres to 27 litres in 8 minutes.

d. The distance walked increased from 500 m to 1.2 km between 4:18 pm to 4:35 pm.

12. Compare the rates. 11E

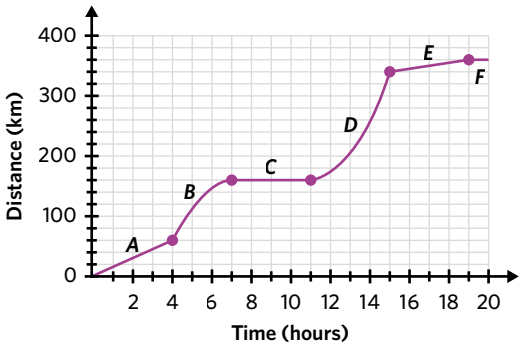
- **a.** Charles collects 800 gold coins for completing 5 quests and Boyle collects 1400 gold coins for completing 8 quests. Determine who collected more gold coins per quest and calculate the difference in the number of gold coins collected per quest.
- **b.** Dom ran 5 km in 40 minutes and Nak ran 9 km in 70 minutes. Determine who is a faster runner and calculate the difference between the distance run, in metres, per minute, rounded to 2 decimal places.
- **c.** Socheata takes 20 minutes to fold 8 origami birds while Sorya takes 35 minutes to fold 15 origami birds. Determine who is the faster folder and calculate the difference in the number of origami birds folded in one hour.
- **d.** Kathy takes 3 hours to bake 40 cupcakes while Kai takes 6 hours to bake 110 cupcakes. Determine who is the more efficient baker and calculate the difference in the amount of time it would take each person to bake 200 cupcakes to the nearest hour.

13. Combine the rates. 11E

- **a.** It takes two students 10 hours and 12 hours respectively to complete a project individually. How long will it take both students to complete the project together, to the nearest hour?
- **b.** It takes two farmers 45 minutes and 60 minutes respectively to move 50 bags of wheat. How long will it take both farmers to move 50 bags of wheat together, to the nearest minute?
- **c.** It takes two kitchen hands 55 minutes and 75 minutes respectively to clean a sink full of dishes. How long will it take both kitchen hands to clean the dishes together, to the nearest minute?
- **d.** Three gardeners take 35, 32 and 44 minutes respectively to mow the lawn of a house. How long does it take all three to mow the lawn together, to the nearest minute?

14. Use the truck's distance-time graph to identify the following. 11F

- **a.** The parts of the trip during which the car was travelling at constant speed.
- **b.** The parts of the trip during which the car was travelling at variable speed.
- **c.** The part of the trip during which the car was travelling at the fastest constant speed.
- **d.** The part of the trip during which the car covers the longest distance.



Problem solving

15. In a shopping mall's parking lot, there are 350 cars and 500 motorbikes. Write the ratio of motorbikes to the total number of vehicles in the parking lot. 11A

16. When Kris does his shopping for clothes, he buys 3 pairs of trousers for every 5 t-shirts. If Kris bought 9 pairs of trousers, how many t-shirts did he buy? 11B

17. The ratio of the width to length of a swimming pool is 5 : 12. Find the dimensions of the swimming pool if it has an area of 1500 m^2. 11C

18. Over the first 3 days of Lunar New Year, Wang collected 51 red packets. What is the average rate at which Wang collects red packets? 11D

19. Sok and Stein live in a house that has 3 identical bedrooms. They are trying to repaint the walls of the bedrooms. Sok takes 3 hours to paint 1 room while Stein only takes 2 hours. How long will it take both of them to paint all the bedrooms? 11E

20. In a training session for a cycling race, Chloe leaves the starting line at 150 km/h. After 15 minutes, Rachel leaves the starting line at 170 km/h. How long will it take, in hours, for Rachel to pass Chloe by 2 km? 11F

Reasoning

21. Chef Pham is an executive chef of a two Michelin star restaurant. To earn her third Michelin star, she needs to develop a legendary dish to impress the Michelin inspectors.

a. Chef Pham decides to create a spice mix for her dish. She mixes cinnamon, star anise and cardamom in the ratio of 4 : 3 : 7. What is the ratio for 24 servings of her new dish if the base ratio is enough for 4 servings?

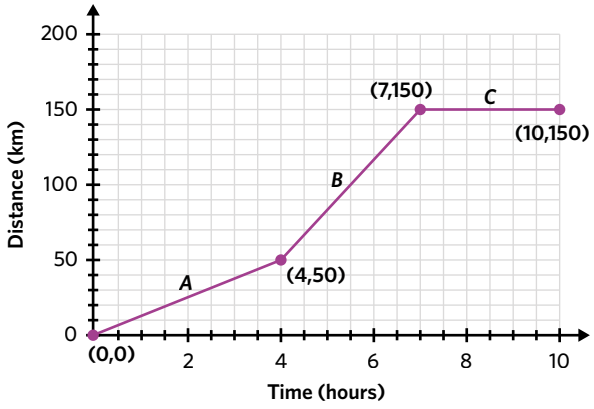
b. Chef Pham now tries to work out the time it takes to prepare this dish. She knows that the ratio of blending and rubbing spices to preparing vegetables and garnishes to cooking is 4 : 5 : 3. If the dish takes 60 minutes to finish, calculate the time required for each process.

c. During the first night that this dish is served, the kitchen sent out 50 servings throughout the dinner service, which took 2 hours and 5 mins. Find the rate of servings/minute, and hence find the ratio a servings : b minutes, where a, b are both the smallest whole number possible.

d. Two new chefs are trying to improve the efficiency of preparing the dish. Individually, Chef Dau takes 72 minutes to prepare 4 servings of the dish from start to finish while Chef Nguyen takes 66 minutes. The two chefs decide to combine their skills and cook together. How long does it take the two chefs to cook 48 servings of the dish? Round your answer to the nearest minute.

e. Besides great food, what other aspects of a meal makes for a great dining experience?

22. Use the distance-time graph to answer the following questions.

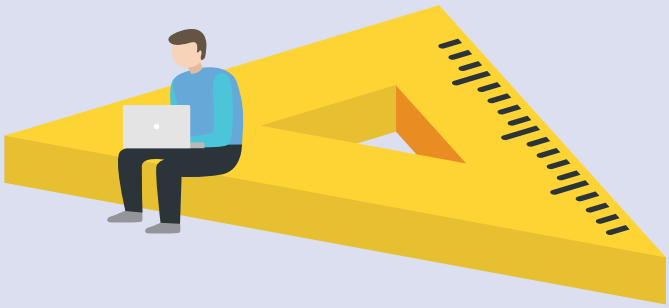


a. Calculate the difference in speeds between lines B and A.

b. Calculate the difference in speeds between lines C and B.

c. With reference to parts **a** and **b**, explain why the difference in part **b** is equal to the speed of line B, however the difference in part **a** is not equal to the speed of either line B or line A.

Back to contents



12

Measurement

Measurement and Geometry

Research summary

12A Perimeter and units of length *(Revision)*

12B Circumference of a circle

12C Area of squares, rectangles and triangles

12D Area of special quadrilaterals

12E Area of a circle

12F Composite shapes with sectors *(Extension)*

12G Volume of prisms and cylinders

12H Time

Chapter 12 extended application

Chapter 12 review

Image: Marish/Shutterstock.com

Back to contents

Chapter 12 research summary

Measurement

Big ideas

Hierarchy and properties

In this chapter, students will apply their understanding of the hierarchy and properties of 2D figures and shapes to calculate the area and perimeter of quadrilaterals, circles and composite shapes. Additionally students will apply hierarchy and properties of 2D shapes to 3D shapes to find the volume of prims, cylinders, and composite figures.

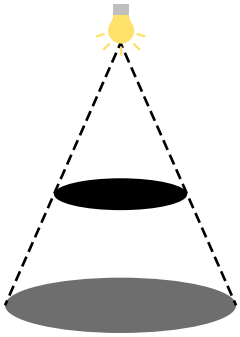
When 3D objects are not in their 'typical' position, students may have difficulty accurately identifying them (Sinclair et al., 2016). Students sometimes have trouble understanding how 3D shapes can be represented onto a 2D space. One strategy to promote 3D shapes is for students to draw 3D shapes from different perspectives (Sinclair et al., 2016).

It is essential that students understand why area, perimeter, and circumference formulas work rather than memorisation so they may develop their conceptual understanding (O'Dell et al., 2016). For example, relating the area of a circle through triangles students are able to relate the area formula for a circle to the area formula for a triangle (O'Dell et al., 2016).

Students can struggle with the understanding of π (Leung, 2014). It may be one of the first times students are working with an idea that is constant. They often think that π changes depending on the size of the circle, or is a magical number, or is a variable.

One method to prove the constant nature of π is to use linearity (Leung, 2014). Students measure the circumference and diameter, and plot these points on a cartesian plane (circumference, diameter). The slope of a straight line is constant which leads to the constant nature of π (Leung, 2014). While there are measurement errors, this provides an opportunity for teachers and students to engage in rich discussions why errors would exist in an experiment like this which improves students' reasoning skills (Leung, 2014).

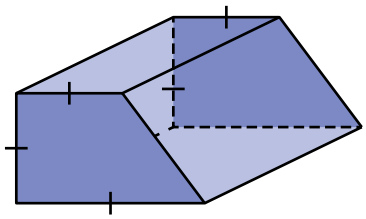
Another strategy to prove the constant nature of π that increases student's algebraic and geometric connections of circles is to use similar circles (Leung, 2014). Students can view a bulb that projects light onto a circular disc and the shadow that is created (Leung, 2014). By knowing that two circles are similar, the ratios of their corresponding sides are equal, and therefore the ratio π is constant, regardless of the size of the circle (Leung, 2014).



Transformations of relationships

In this chapter, students will need to understand how certain shapes can transform by composing and decomposing them into different shapes to calculate their perimeter, circumference, sectors, area and volume. Additionally students will transform time measurements using a base 60 system, and transform time using various time zones across Australia and the world.

Students will need to recognise certain transformations of 2D and 3D shapes. Spatial thinking, and mental rotation of shapes are beneficial for proving symmetrying, understanding area measurement tasks, as well as creating and decomposing 2D and 3D figures (Bruce & Hawes, 2014). For example, students should be able to look at this trapezoid prism and recognise that it can be composed of a triangular prism and a cube prism, or decomposed of a rectangular prism and triangular prism.



Having strong spatial thinking is essential for further geometric topics such as isometric drawings, trigonometry, and algebraic topics such as transformations of functions. Additionally, having a strong understanding of how time can transform is essential for everyday life.

Geometric measurement

In this chapter, students will use measurement to calculate the perimeter, area, circumference, sectors, and volume of shapes. Additionally students will use various time measurements to determine time in various time zones across Australia and the world, and convert between 12 and 24 hour time.

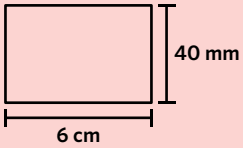
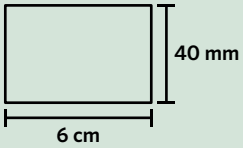
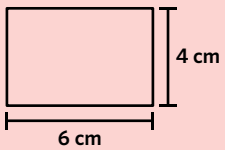
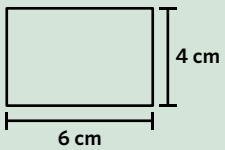
Concrete artefacts can help students understand geometric conceptions. For example, a rectangular prism is 'like a long box' or 'a 3D rectangle with some sides that are equal sides'. Hands-on learning and task -based interviews can improve students' reasoning about 3D objects and geometric reasoning with prisms (Downton & Livy, 2021).

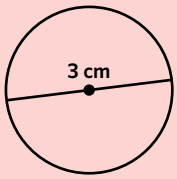
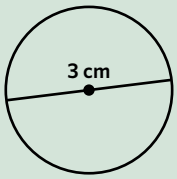
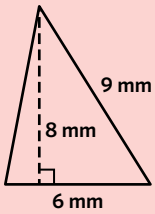
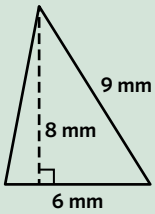
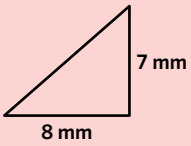
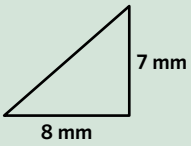
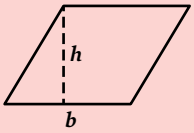
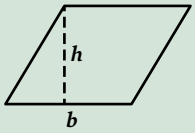
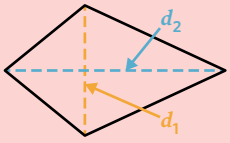
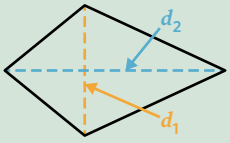
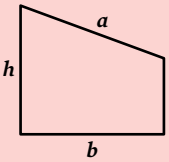
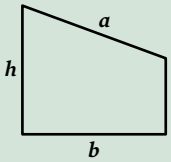
Additionally manipulatives improve students' understanding of the area of circles (Arnigo et al., 2018). For example, give students a cup and ask various questions such as how do you know where the centre is, and to measure the circumference and diameter with yarn and a ruler (Arnigo et al., 2018).

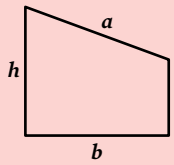
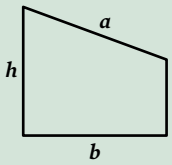
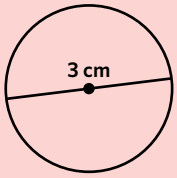
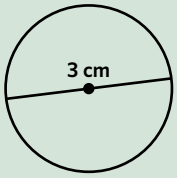
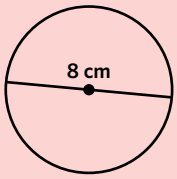
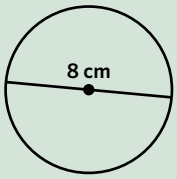
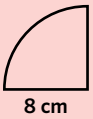
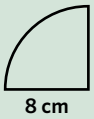
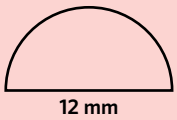
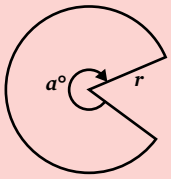
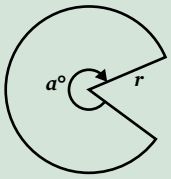
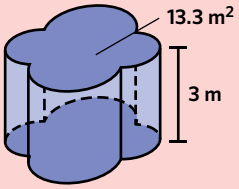
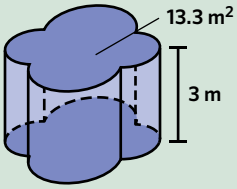
Virtual manipulatives can help students understand the relationship between perimeter, area, volume and surface area of 2D and 3D objects (Sinclair et al., 2016). For example, students often think that 3D shapes must have different surface areas and volumes. A simple strategy is to challenge students to create various prisms that have the same volume and same surface area, same volume different surface area, different volume and same surface area, and different volumes and different surface areas.

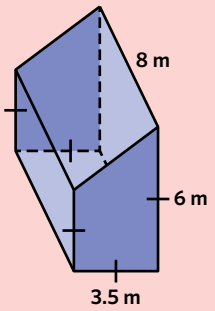
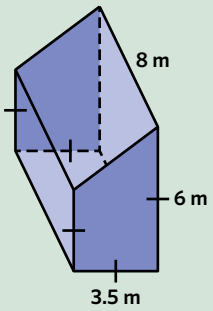
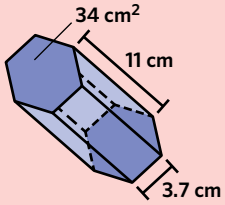
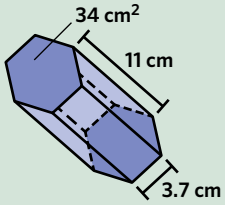
Geometric measurement is an essential part of mathematics and also various other parts of students' learning journey.

Misconceptions

Misconception	Incorrect	Correct	Lesson
Students use different units when calculating the perimeter of a shape.	Perimeter $P = (2 \times 40) + (2 \times 6) = 92$ cm 	Perimeter $P = (2 \times 4) + (2 \times 6) = 20$ cm 	12A
Students calculate the area when calculating the perimeter and vice versa.	Perimeter $P = 4 \times 6 = 24$ cm 	Area $A = 4 \times 6 = 24$ cm^2 	12A 12C

Misconception	Incorrect	Correct	Lesson
Students use various ratios to get the diameter or radius of a circle.	 Diameter = 1.5 cm Radius = 6 cm	 Diameter = 3 Radius = 1.5 cm	12B
Students think that π is an exact distance.	$\pi = 3.14$	$\pi \approx 3.14$	12B
Students think that π is a variable and/or a 'magical' number.	π varies and is a magical number that we use when working with circles.	π is a constant number that occurs when we divide the circumference by the diameter.	12B
When calculating the area of a triangle, students use slant height as the perpendicular height.	 Area of triangle $= \frac{9 \times 6}{2} = 27$ mm^2	 Area of triangle $= \frac{9 \times 8}{2} = 36$ mm^2	12C
Students use the area of a rectangle to find the area of the triangle.	 Area of triangle $= 7 \times 8 = 56$ mm^2	 Area of triangle $= \frac{7 \times 8}{2} = 28$ mm^2	12C
When converting units for area, students convert using the same ratio for lengths.	55 mm$^2 \rightarrow$ cm^2 $= \frac{55}{10} = 5.5$ cm^2	55 mm$^2 \rightarrow$ cm^2 $= \frac{55}{100} = 0.55$ cm^2	12C
When calculating the area of parallelograms, students see the triangles and use the area of a triangle formula.	 Area of parallelogram $= \frac{b \times h}{2}$	 Area of parallelogram $= b \times h$	12D
When calculating the area of a kite, students multiply the diagonals only.	 Area of a kite $= d_1 \times d_2$	 Area of a kite $= \frac{d_1 \times d_2}{2}$	12D
Students use any side length to calculate the area of a trapezium.	 Area of trapezium $= \frac{a + h}{2} \times b$	 Area of trapezium $= \frac{a + b}{2} \times h$	12D

Misconception	Incorrect	Correct	Lesson
Students multiply two sides together to find the area of a trapezium.	 Area of trapezium $= \frac{b \times h}{2}$ or $= \frac{a \times h}{2}$	 Area of trapezium $= \frac{a + b}{2} \times h$	12D
When calculating the area of circles, students calculate the circumference.	 Area $= 2\pi r = 2 \times 1.5 \times \pi = 3\pi \text{ cm}^2$ Or Area $= \pi \times d = \pi \times 3 = 3\pi \text{ cm}^2$	 Area $= \pi r^2 = \pi \times 1.5^2 = 2.25\pi \text{ cm}^2$	12E
When calculating the area of a circle using a diameter, students divide the squared diameter by 2.	 Area $= \frac{8^2}{2} \times \pi$ $= \frac{64}{2}\pi = 32\pi \text{ cm}^2$	 Area $= \left(\frac{8}{2}\right)^2 \times \pi$ $= (4)^2\pi = 16\pi \text{ cm}^2$	12E
Students divide the area of quadrants and semi circles by the radius before squaring the radius.	 Area $= \pi\left(\frac{8}{2}\right)^2$ $= \pi(4)^2 = 16\pi \text{ mm}^2$	 Area $= \frac{\pi(4)^2}{2}$ $= \frac{16\pi}{2} = 8\pi \text{ mm}^2$	12E
When finding the area of a semicircle students use the diameter.	 Area $= \frac{\pi(12)^2}{2}$ $= \frac{144\pi}{2} = 72\pi \text{ mm}^2$	 Area $= \frac{\pi(6)^2}{2}$ $= \frac{36\pi}{2} = 18\pi \text{ mm}^2$	12E 12F
Students multiply the radius by the fraction of the sector before squaring the radius.	 Area $= \pi\left(r \times \frac{a}{360}\right)^2$	 Area $= \pi r^2 r \times \frac{a}{360}$	12F
When given the area of a cross section, students square the cross section to calculate the volume of a prism.	 $V = (13.2)^2 \times 3 = 530.67 \text{ m}^3$	 $V = 13.2 \times 3 = 39.9 \text{ m}^3$	12G

Misconception	Incorrect	Correct	Lesson
Students ignore that irregular-3D shapes can be decomposed or composed of various regular 3D shapes and apply various methods to calculate the volume.	 Volume $= (3.5 \times 6 \times 8) = 160\ \text{m}^3$	 The shape is a rectangular prism decomposed of a square prism $V = (3.5 \times 6 \times 8)$ $-\left(\frac{3.5(6-3.5)}{2} \times 8\right) = 133\text{m}^3$ Or The shape is a square prism composed of a triangular prism. $V = (3.5 \times 6 \times 8)$ $+\left(\frac{(3.5)(6-3.5)}{2} \times 8\right) = 133\text{m}^3$ Or The shape is a trapezium prism $V = \left(\frac{6+3.5}{2} \times 3.5\right) \times 8 = 133\text{m}^3$	12G
Students use a measurement that is not perpendicular to the cross section to calculate volume.	 Volume $= 34 \times 3.7 = 125.8\ \text{cm}^3$	 Volume $= 34 \times 11 = 374\ \text{cm}^3$	12G
Students think that time is a base 10 measurement.	3 hours and 30 minutes $= 3.3$ hours	3 hours and 30 minutes $= 3.5$ hours	12H
When calculating time zones of two cities, students think that one city is equal to UTC +0.	What time is it in Melbourne, if it is 1 pm in Rome? Melbourne is UTC +10 1 pm + 10 = 11 pm	What time is it in Melbourne, if it is 1 pm in Rome? Melbourne is UTC +10 Rome is UTC +3 $10 - 3 = 7$ Melbourne is 7 hours ahead 1 pm + 7 = 8 pm	12H

References and additional readings

Arnigo, J. C., Lim, E., Manalo, M. J., Pacheco, V. L., Secosana, B., Sy, A., & Elipane, L. (2018). Utilizing Lesson Study in Teaching Circles. *Advanced Science Letters*, *24*(11), 7876–7879. https://doi.org/10.1166/asl.2018.12447

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Fujita, T., Doney, J., & Wegerif, R. (2019). Students' collaborative decision-making processes in defining and classifying quadrilaterals: a semiotic/dialogic approach. *Educational Studies in Mathematics*, *101*(3), 341–356. https://doi.org/10.1007/s10649-019-09892-9

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Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: an ICME-13 survey team report. *ZDM*, *48*(5), 691–719. https://doi.org/10.1007/s11858-016-0796-6

Back to contents

12A Perimeter and units of length

The first and most important step to any measurement is describing a length. When describing a distance between two objects or the perimeter of an object, most countries will use the metric system. This lesson will revise conversion between different metric units as well as the rules and techniques used to measure the perimeter of shapes.

LEARNING INTENTIONS

Students will be able to:

- convert between units of length
- determine the perimeter of a shape
- determine the perimeter of shapes with missing side lengths.

KEY TERMS AND DEFINITIONS

A **metre (m)** is a standardised unit measuring length.

A **centimetre (cm)** is one-hundredth of a metre.

A **millimetre (mm)** is one-thousandth of a metre.

A **kilometre (km)** is one thousand metres.

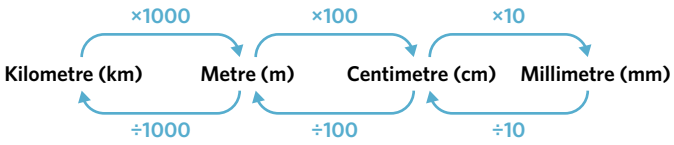
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: ZHMURCHAK/Shutterstock.com

Determining the distance for travel or measuring objects around us informs many important decisions in our daily life. From planning distances for travel to measuring lengths of building material to construct a house, measurement is an integral life skill.

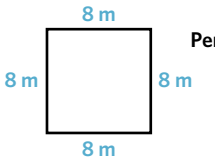
Key ideas

1. We can convert different units of length by multiplying or dividing by powers of 10.



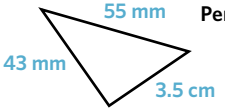
2. The perimeter of a shape is the sum of its side lengths.

Same unit of measurement



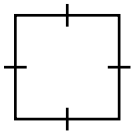
Perimeter = 8 m + 8 m + 8 m + 8 m
= 32 m

Different unit of measurement

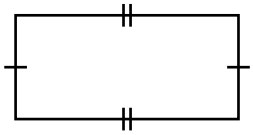


Perimeter = 43 mm + 55 mm + 3.5 cm
= 43 mm + 55 mm + 35 mm
= 133 mm

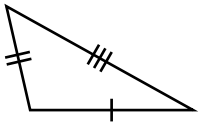
3. Sides with the same marking are the same length.



The side lengths are all equal.



All the widths are the same size.
All the lengths are the same size.
The widths and lengths are different sizes.



None of the side lengths are equal.

Worked example 1

Converting units of length

Convert the following.

a. 13.8 cm to mm

WE1a

Working

$13.8 \text{ cm} = (13.8 \times 10) \text{ mm}$

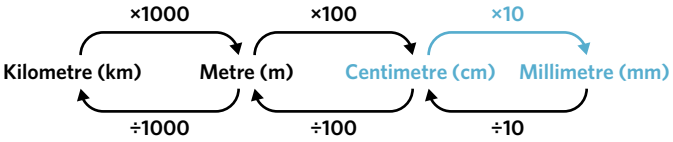
$= 138 \text{ mm}$

Thinking

Determine the multiplication or division required.

1 cm = 10 mm

Visual support



b. 185 000 cm to km

WE1b

Working

$185\,000 \text{ cm} = (185\,000 \div 100) \text{ m}$

$= 1850 \text{ m}$

$185\,000 \text{ m} = (185\,000 \div 1000) \text{ km}$

$= 1.850 \text{ km}$

Thinking

Step 1: Determine the multiplication or division required.

100 cm = 1 m

Step 2: Determine the multiplication or division required.

1000 m = 1 km

Student practice

Convert the following.

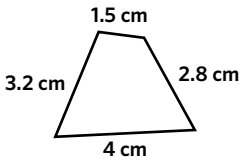
a. 11.6 cm to mm **b.** 255 000 cm to km

Worked example 2

Calculating the perimeter

Determine the perimeters of the following shapes.

a.



WE2a

Working

$P = (1.5 + 4 + 3.2 + 2.8) \text{ cm}$

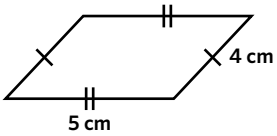
$= 11.5 \text{ cm}$

Thinking

Add the lengths if they are all the same unit of measurement.

Continues →

b.



WE2b

Working

$P = (5 + 5 + 4 + 4)$ cm

$= 18$ cm

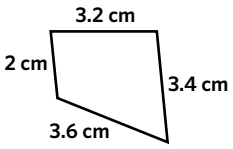
Thinking

Add the lengths if they are all the same unit of measurement. Take note of equal side lengths.

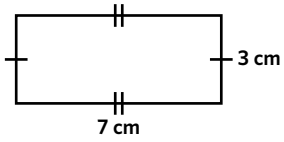
Student practice

Determine the perimeters of the following shapes.

a.



b.



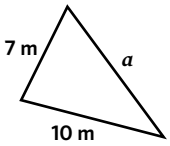
Worked example 3

Calculating the missing value of a perimeter

Determine the unknown values.

a. Perimeter $= 28$ m

WE3a



Working

$P = 28$ m

$28 = 7 + 10 + a$

$28 - 7 - 10 = a$

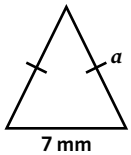
$a = 11$ m

Thinking

Subtract known side lengths from the perimeter.

b. Perimeter $= 23$ mm

WE3b



Working

$P = 23$ mm

$23 = 7 + a + a$

$23 - 7 = 2a$

$16 = 2a$

$a = 8$ mm

Thinking

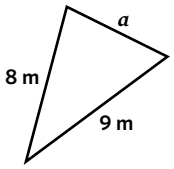
Subtract known side lengths from the perimeter.

Continues →

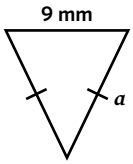
Student practice

Determine the unknown values.

a. Perimeter = 22 m



b. Perimeter = 31 mm



12A Questions

Understanding worksheet

1. Circle appropriate units of measurement for the following.

Example

Distance between cities: mm cm m (km)

a. Distance between classrooms: mm cm m km

b. Width of a blade of grass: mm cm m km

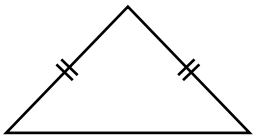
c. Length of a shoe: mm cm m km

d. High jump results: mm cm m km

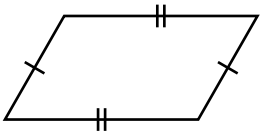
2. Determine the number of measurements needed in order to find the perimeters of the following shapes.

Example

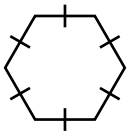
2 different measurements



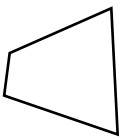
a. ☐ different measurements



b. ☐ different measurements



c. ☐ different measurements



d. ☐ different measurements

3. Fill in the blanks by using the words provided.

convert | perimeter | length | measurement

When measuring the length of a shape's _____, we must add all of the side lengths together. If two sides have the same marking then they have the same _____. Metric units of _____ are all related by powers of ten. When calculating the perimeter of a shape that has different units of measurement for different sides it is important to _____ all measurements to the same unit of length.

Fluency

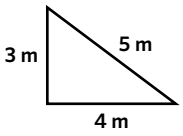
Question working paths

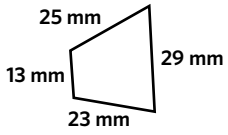
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

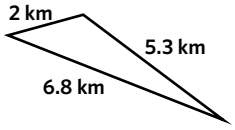
4. Convert. WE1

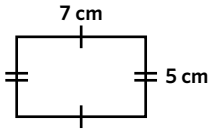
a. 200 cm to m **b.** 1.3 cm to mm **c.** 1200 m to km **d.** 7 m to mm
e. 73.5 cm to m **f.** 1456 cm to km **g.** 0.0263 m to mm **h.** 28 560 045 mm to km

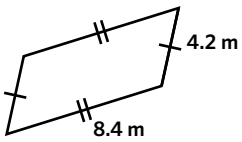
5. Determine the perimeters of the following shapes. WE2

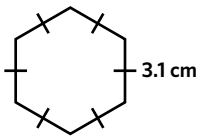
a.


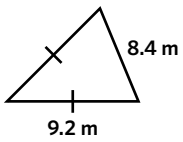
b.


c.


d.


e.


f.


g.


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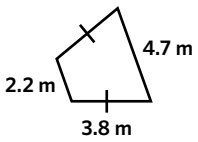
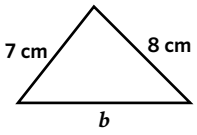
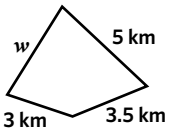
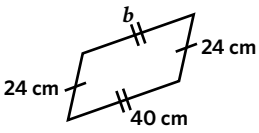
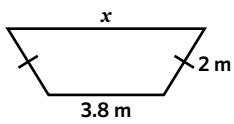


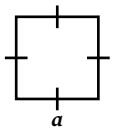
6. Determine the unknown values. WE3

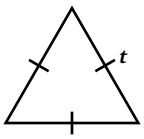
a. Perimeter = 25 cm


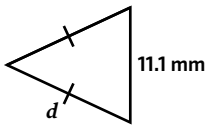
b. Perimeter = 16.1 km


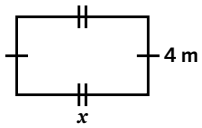
c. Perimeter = 128 cm


d. Perimeter = 13.8 m


e. Perimeter = 20 km


f. Perimeter = 72 cm


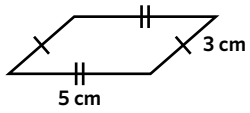
g. Perimeter = 37.5 mm


h. Perimeter = 22 m


7. Calculate the lengths of the following perimeters in the specified units.

a. 4 m + 200 cm (m) **b.** 15 cm + 70 mm (cm) **c.** 6 cm + 25 mm (mm) **d.** 350 cm + 7 m (cm)
e. 1 m + 875 mm (mm) **f.** 15 800 cm + 42 m (m) **g.** 3 km + 95 m (km) **h.** 8 m + 38 cm (mm)

8. Calculate the perimeter of the following shape.

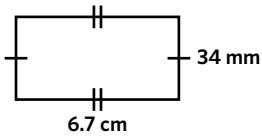


A. 8 cm **B.** 11 cm **C.** 13 cm **D.** 15 cm **E.** 16 cm

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the perimeter.



Student A

$P = 2 \times 6.7 \text{ cm} + 2 \times 34 \text{ mm}$

$P = 2 \times 6.7 \text{ cm} + 2 \times 34 \text{ mm}$

$= 13.4 \text{ cm} + 68 \text{ mm}$

$= 81.4 \text{ cm}$

Student B

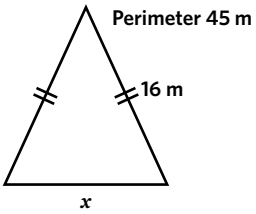
$P = 2 \times 6.7 \text{ cm} + 2 \times 34 \text{ mm}$

$P = 2 \times 6.7 \text{ cm} + 2 \times 3.4 \text{ cm}$

$= 13.4 \text{ cm} + 6.8 \text{ cm}$

$= 20.2 \text{ cm}$

b. Calculate the unknown length.



Student A

$45 \text{ m} = 16 \text{ m} + x$

$45 \text{ m} - 16 \text{ m} = x$

$x = 29 \text{ m}$

Student B

$45 \text{ m} = 16 \text{ m} + 16 \text{ m} + x$

$45 \text{ m} - 32 \text{ m} = x$

$x = 13 \text{ m}$

Problem solving

Question working paths

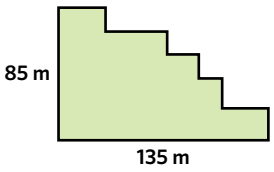
Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Eliud runs 3 km to warm up for his training. How many laps of a standard 400 m athletics track does he need to complete?

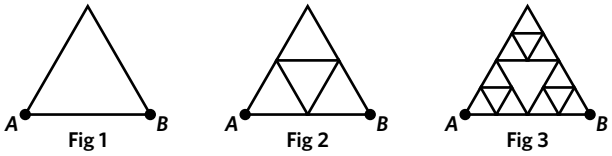
11. Robert has a table that is 80 cm tall. If he stacks fifteen 78 mm high cubes on top of each other on top of the table, what is the total height of the table and stack, in cm?

12. Hannah wants to make a square on the ground with her 12 m rope. Determine the length of one of the sides.

13. Phemines has bought a new block of land as displayed in the diagram. If it costs $12 for 1 metre of fencing, determine how much it will cost her to build a fence at the perimeter of her new block of land?



14. Wacław loves geometric patterns. He began by drawing an equilateral triangle with each side of length s, like the one in Fig 1. He then created Fig 2 and 3 by drawing smaller equilateral triangles inside the same triangle. Determine the side lengths of each of the smaller triangles shown in Fig 2 and Fig 3 below, in terms of s.

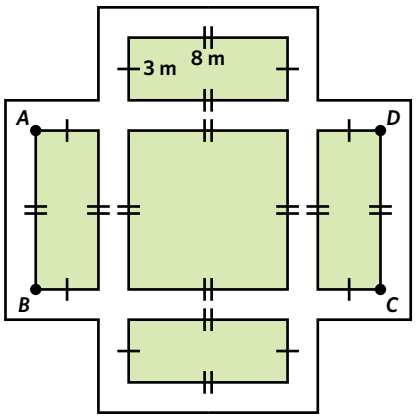


Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

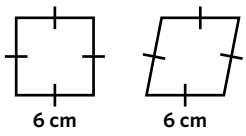
15. Sonya has created a walking track for her athletes, so they can warm up before training. The track is 1 m wide. With this design, Sonya can get the athletes to run various distances as warm up exercises.



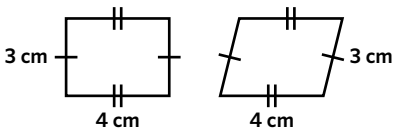
- **a.** Determine the length of the inner square's perimeter.
- **b.** Determine the perimeter of one of the rectangles.
- **c.** Determine the perimeter of the rectangle created with the points A, B, C and D.
- **d.** Determine the length of the perimeter of the outside edge of the walking track.
- **e.** What are some good ways to warm up before training?

16. Use the diagrams to determine the length of the perimeters in parts **a** and **b**.

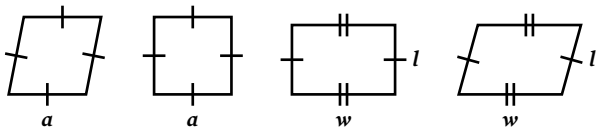
a.



b.

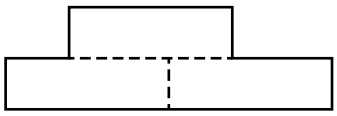


c. Use your answers to parts **a** and **b**, and the diagram above, to establish a general rule for calculating the perimeter of the special quadrilaterals in the diagram.



Extra spicy

17. The shape in the diagram is made up of three rectangles, each having side lengths of 3 cm and 1 cm.
What is the perimeter of the shape?



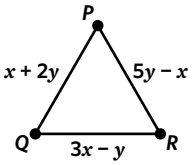
18. 8 km is approximately 5 miles. Which of the following distances is closest to 1.2 km?

A. 0.75 miles **B.** 1 mile **C.** 1.2 miles **D.** 1.6 miles **E.** 1.9 miles

19. Which of the following is equal to one million millimetres?

A. 1 metre **B.** 10 metres **C.** 100 metres **D.** 1 kilometre **E.** 10 kilometres

20. $\triangle PQR$ is an equilateral triangle with side lengths shown in the diagram.



What is the value of x and y if the perimeter is equal to 42.

Remember this?

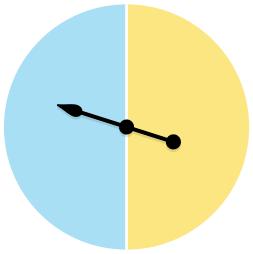
21. Tri writes the following number sentence, where the stars represent the same number.

$15 - \bigstar = \bigstar - 9$

What number does $\bigstar$ represent?

A. 9 **B.** 10 **C.** 11 **D.** 12 **E.** 13

22. Michael spins the following spinner three times.



It lands on yellow each time.

If he spins it a fourth time, which of the following is true?

A. It will certainly land on blue.
B. It is more likely to land on blue than yellow.
C. It has an equal chance of landing on yellow and blue.
D. It is more likely to land on yellow than blue.
E. It will certainly land on yellow.

23. Alistair is negotiating for a $1000 mattress. Initially, the salesperson agrees to reduce the price by 50%. The next day, he comes back and negotiates to receive a further 25% discount on this already discounted price.

How much did Alistair pay for the mattress?

A. $250 **B.** $375 **C.** $475 **D.** $875 **E.** $925

Back to contents

12B Circumference of a circle

Circles are a fundamental shape and many objects in the natural world are formed with or create circles. For this reason they have been studied by mathematicians for thousands of years. The irrational number π is fundamental to circles. Archimedes of Syracuse famously discovered the estimation 22 ÷ 7 for π over 2000 years ago.

LEARNING INTENTIONS

Students will be able to:

- identify the diameter, radius and circumference of a circle
- understand that the circumference divided by the diameter of any circle is equal to π
- calculate the circumference of a circle (in terms of π) using a formula (calculator)
- calculate the circumference of a circle to an approximate value of π.

KEY TERMS AND DEFINITIONS

The **circumference** is the perimeter of a circle.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

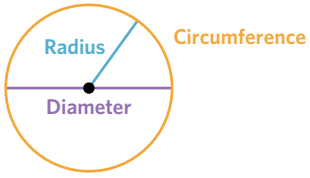
Image: Iri_sha/Shutterstock.com

All objects in the universe move around other, larger objects. Tracking celestial bodies from Earth using a telescope and photography produces visible paths that closely resemble circles. We call these paths orbits and their properties depend on the masses of the orbiting bodies.

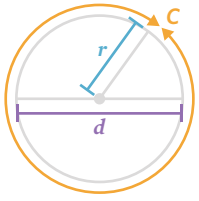
Key ideas

1. The **radius** is the direct distance from the centre of a circle to any point on the circumference.

The **diameter** is a straight line between two points on the circumference of a circle that passes through the centre.



2. The length of the circumference divided by the diameter is always equal to the number π.

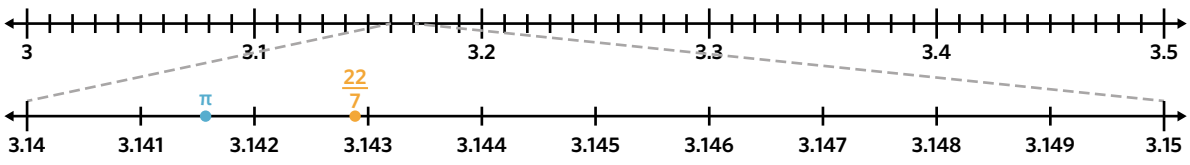


$\frac{C}{d} = \pi$ $\frac{C}{2r} = \pi$

$C = \pi d$ $C = 2\pi r$

3. π is an irrational number. Rounded to 8 decimal places, $\pi \approx 3.14159265\ldots$

π can be approximated by rounding or with fractions:



Worked example 1

Calculating the circumference of a circle from the diameter or radius

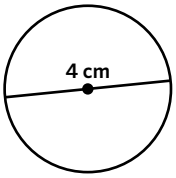
Calculate the circumference in:

i. terms of π.

ii. and correct to 3 decimal places.

a.

WE1a



Working

$C = \pi d$

$= 4\pi$ cm

$= 4 \times \pi$

≈ 12.566 cm

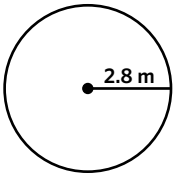
Thinking

i. Apply the formula and express circumference in terms of π.

ii. Calculate the circumference and round the value to 3 decimal places.

b.

WE1b



Working

$C = 2\pi r$

$= 2 \times \pi \times 2.8$

$= 5.6\pi$ m

$= 5.6 \times \pi$

≈ 17.593 m

Thinking

i. Apply the formula and express circumference in terms of π.

ii. Calculate the circumference and round the value to 3 decimal places.

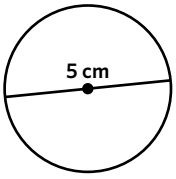
Student practice

Calculate the circumference in:

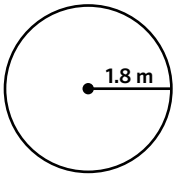
i. terms of π.

ii. and correct to 3 decimal places.

a.



b.

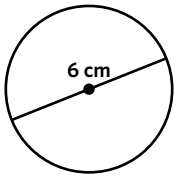


Worked example 2

Estimating the circumference (without a calculator)

Determine the circumference using the given estimations of π.

a. $\pi \approx \frac{22}{7}$ WE2a



Working

$C = \pi d$

$= 6\pi$ cm

$\approx 6 \times \frac{22}{7}$

$= \frac{132}{7}$

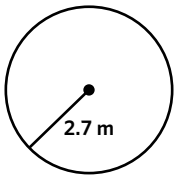
$= 18\frac{6}{7}$ cm

Thinking

Step 1: Apply the formula and express circumference in terms of π.

Step 2: Substitute in the given estimation of π and evaluate. Write the value of the circumference as a mixed number.

b. $\pi \approx 3.14$ WE2b



Working

$C = 2\pi r$

$= 2 \times \pi \times 2.7$

$= 5.4\pi$ m

$\approx 5.4 \times 3.14$

$= 16.956$ m

Thinking

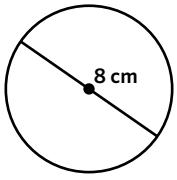
Step 1: Apply the formula and express circumference in terms of π.

Step 2: Substitute in the given estimation of π and evaluate. Write the value of the circumference correct to three decimal places.

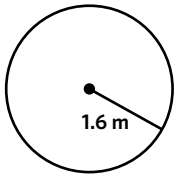
Student practice

Determine the circumference using the given estimations of π.

a. $\pi \approx \frac{22}{7}$



b. $\pi \approx 3.14$



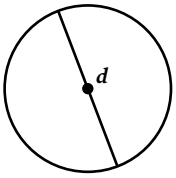
Worked example 3

Calculating the radius and diameter with the circumference

Determine the values of the unknowns, correct to 3 decimal places.

a. $C = 9$ cm

WE3a



Working

$C = \pi d$

$d = \frac{C}{\pi}$

$= \frac{9}{\pi}$ cm

≈ 2.865

Thinking

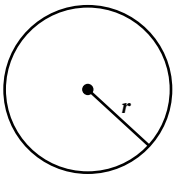
Step 1: Rearrange the formula, with the unknown value as the subject.

Step 2: Apply the formula and express the unknown in terms of π.

Step 3: Solve using a calculator and round to 3 decimal places.

b. $C = 12.8$ m

WE3b



Working

$C = 2\pi r$

$r = \frac{C}{2\pi}$

$= \frac{12.8}{2\pi}$ m

≈ 2.037 m

Thinking

Step 1: Rearrange the formula, with the unknown value as the subject.

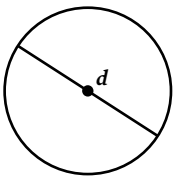
Step 2: Apply the formula and express the unknown in terms of π.

Step 3: Solve using a calculator and round to 3 decimal places.

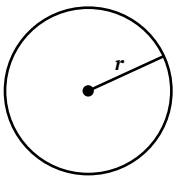
Student practice

Determine the values of the unknowns, correct to 3 decimal places.

a. $C = 11$ mm



b. $C = 10.6$ cm

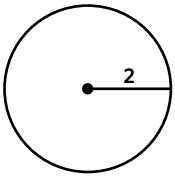


12B Questions

Understanding worksheet

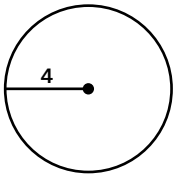
1. Determine the length of the radius and diameter of the following circles.

Example



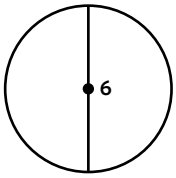
$r =$ 2 , $d =$ 4

a.



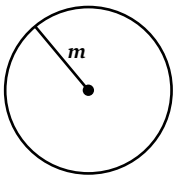
$r = 4, d =$ ☐

b.



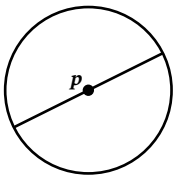
$r =$ ☐ , $d = 6$

c.



$r =$ ☐ , $d =$ ☐

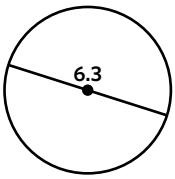
d.



$r =$ ☐ , $d =$ ☐

2. Determine the number that is multiplied by π to calculate the circumference.

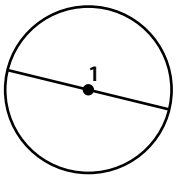
Example



Circumference: 19.79

$19.79 =$ 6.3 $\times \pi$

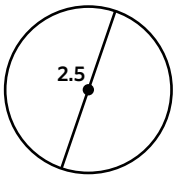
a.



Circumference: 3.14159

$3.14159 =$ ☐ $\times \pi$

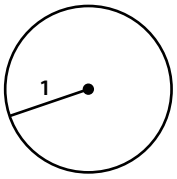
b.



Circumference: 7.85

$7.85 =$ ☐ $\times \pi$

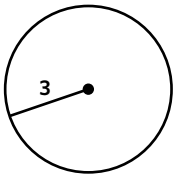
c.



Circumference: 6.283185

$6.283185 =$ ☐ $\times \pi$

d.



Circumference: 18.8596

$18.8596 =$ ☐ $\times \pi$

3. Fill in the blanks by using the words provided.

diameter | circumference | radius | π

All circles contain three main properties. The perimeter of a circle is commonly referred to as the _____.

The _____ of a circle is the straight line joining its centre to any point on the circumference.

The _____ is a straight line which crosses the centre of a circle and joins any two points on the circumference. The ratio of the circumference to the diameter of any circle is always equal to _____.

Fluency

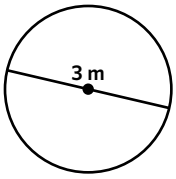
Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (b,c,d,e), 5 (b,c,d,e), 6 (b,c,d,e) 7 (b,c,d,e), 8 (c,d,e,f), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (e,f,gh), 9

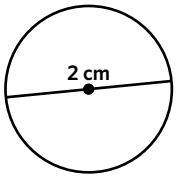
4. Calculate the circumference in: WE1a

i. terms of π.
ii. and correct to 3 decimal places.

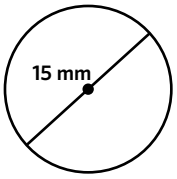
a.



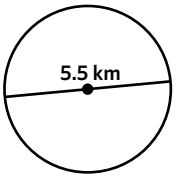
b.



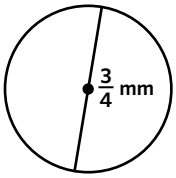
c.



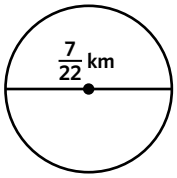
d.



e.



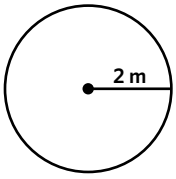
f.



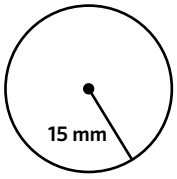
5. Calculate the circumference in: WE1b

i. terms of π.
ii. and correct to 3 decimal places.

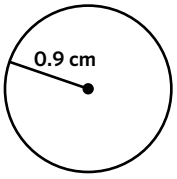
a.



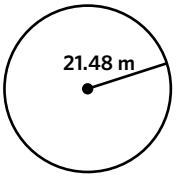
b.



c.

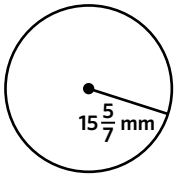


d.



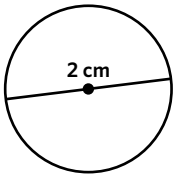
e.

f.

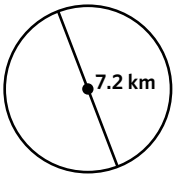


6. Determine the circumference using the given estimations of π, correct to 3 decimal places. WE2a

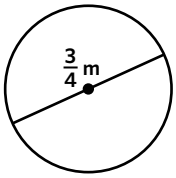
a. $\pi \approx \frac{22}{7}$



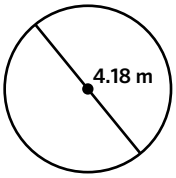
b. $\pi \approx 3.14$



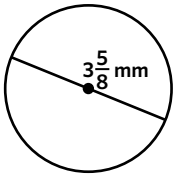
c. $\pi \approx \frac{22}{7}$



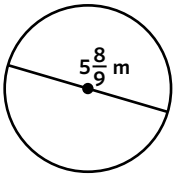
d. $\pi \approx 3.14$



e. $\pi \approx \frac{22}{7}$

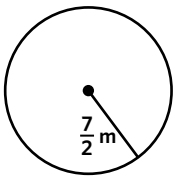


f. $\pi \approx \frac{333}{106}$



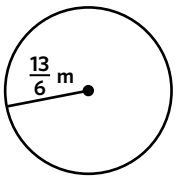
7. Determine the circumference using the given estimations of π, correct to 3 decimal places. WE2b

a. $\pi \approx \frac{22}{7}$



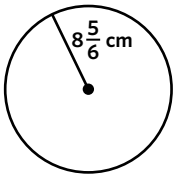
b. $\pi \approx 3.14$

c. $\pi \approx \frac{22}{7}$



d. $\pi \approx 3.14$

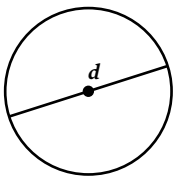
e. $\pi \approx \frac{333}{106}$



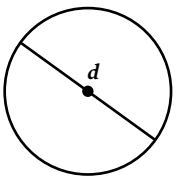
f. $\pi \approx 3.14159$

8. Determine the values of the unknowns, correct to 3 decimal places. WE3

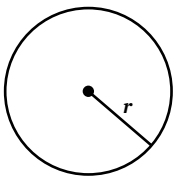
a. $C = 5$ m



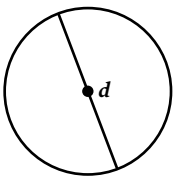
b. $C = 15.3$ cm



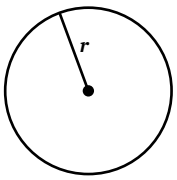
c. $C = 6$ km



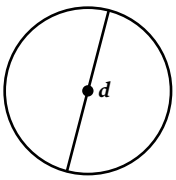
d. $C = \frac{3\pi}{2}$ cm



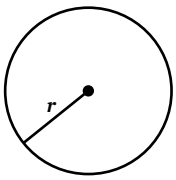
e. $C = 7.6$ mm



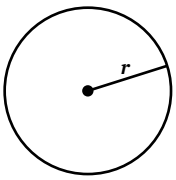
f. $C = 1345$ mm



g. $C = 0.23$ km

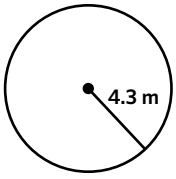


h. $C = 4\frac{5}{8}$ m



9. Determine the circumference, correct to 3 decimal places.

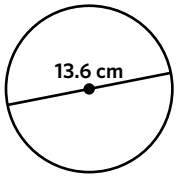
A. 4.3 m
B. 8.6 m
C. 13.509 m
D. 27.018 m
E. 27.019 m



Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Find the circumference of the circle, correct to 3 decimal places.



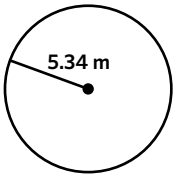
Student A

$C = 2\pi r$
$= 2 \times 13.6 \times \pi$
$= 27.2 \times \pi$
≈ 85.451

Student B

$C = \pi d$
$= 13.6 \times \pi$
≈ 42.726

b. Find the circumference of the circle, correct to 2 decimal places.



Student A

$C = 2\pi r$
$= 2 \times 5.34 \times \pi$
$= 10.68 \times \pi$
≈ 33.55

Student B

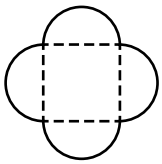
$C = 2\pi r$
$= 2 \times 5.34 \times 3.14$
rounded to 2dp
≈ 33.5352

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Pietro created a walking path with semi-circles around a square for his race horses. The square has side lengths of ten metres. How long is his new walking track? Round your answer correct to 2 decimal places.



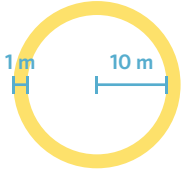
12. The Earth is almost a perfect circle at the equator. The distance around the equator is 40 075 km. What is the direct distance from the surface of the Earth to its centre, to the nearest kilometre?

13. Newtown is planning to install a new Ferris wheel as a feature of its tourism advertising campaign. The wheel, excluding the carriages, has a total length of 80 m. The pods can carry 10 passengers each, but need to be spaced out by 10 m around the Ferris wheel. How many passengers can be on the Ferris wheel at the one time?

14. Guiseppe has measured his pizzas to have a circumference of 94 cm. To the nearest centimetre, what is the minimum width of the pizza boxes Giuseppe needs to order?

15. Xavier is designing circular walkways over the local wetlands. The tradesmen need the plans to be correct to three decimal places. Which of the following estimates for π will produce an estimate for the exterior circumference of the walkway?

$\pi \approx \frac{22}{7}$, $\pi \approx \frac{333}{106}$ and $\pi \approx \frac{355}{113}$

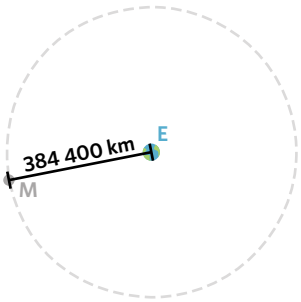


Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. An object in space moves around another object in space on a path that is very close to a circle. It is possible to use the facts about circles to understand their movement. Provide all of your answers rounded to the nearest whole number.



a. The Moon is approximately 384 400 km from the Earth. Calculate the distance the Moon travels in one orbit of the Earth.

b. The Moon takes 27.32 days to orbit the Earth. Calculate how many kilometres it travels per hour.

c. The Earth is approximately 150 million km from the Sun. It orbits the Sun in 365.256 days. Calculate how many kilometres it travels around the Sun each hour.

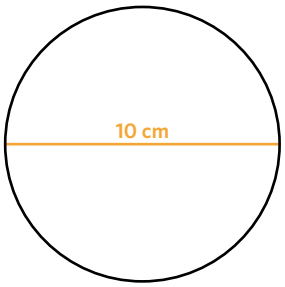
d. Below is a table of planets with their orbital distance from the Sun and the time of one orbit. Calculate the speed that each of them is moving at.

Planet	Orbital distance from the Sun	Time of one orbit (to nearest Earth day)	Speed (km/h)
Mercury	59 million km	88 days	
Venus	108 million km	225 days	
Mars	228 million km	687 days	
Jupiter	778 million km	4330 days	

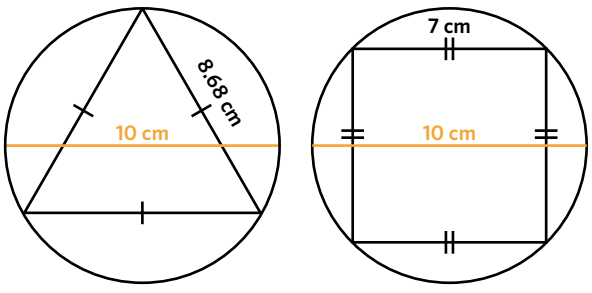
e. Is it possible for humans to colonise other planets?

17. For the following.

a. Calculate the circumference of the circle below, correct to 2 decimal places.



b. Calculate the perimeter of the triangle and square below, correct to 2 decimal places, where applicable.

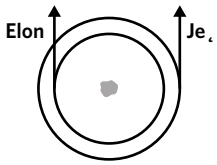


c. Compare the circumference from part **a** to the perimeters from part **b**. What will happen to the difference between the circumference of a circle and the perimeter of a regular shape inside of it if the number of sides of the shape is increased?

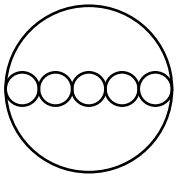
Extra spicy

18. Elon orbits an asteroid in a clockwise direction, completing one orbit every 5 minutes. Jeff orbits the same asteroid in a counter clockwise direction, completing an orbit every 6 minutes. If they are directly opposite each other, how many times will they pass each other in the next hour?

A. 11
B. 15
C. 22
D. 30
E. 60

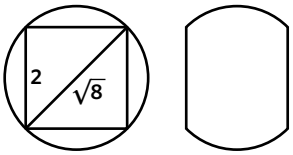


19. Calculate the difference between the circumference of the large circle and the sum of the circumferences of all the small circles.



20. A perfect circle is drawn around a square that has side lengths of 2 units and a diagonal of $\sqrt{8}$ units. Two curves from the circle and two sides from the square are used to create the second shape in the diagram. Which is the closest approximation of the perimeter of the second shape?

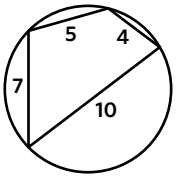
A. 5.111
B. 6.221
C. 8.443
D. 9.657
E. 12.886



21. Brahmagupta (598–670 AD) discovered the formula for the area of a cyclic quadrilateral, all four vertices on the circumference of a circle. Where a, b, c, d and the four side lengths and s is equal to half of the total perimeter. The area is equal to:

$\sqrt{(s-a)(s-b)(s-c)(s-d)}$

What is the area of the cyclic quadrilateral in the diagram?

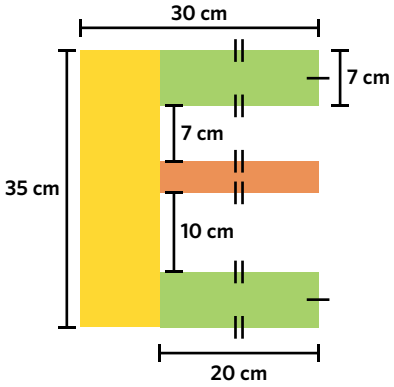


Remember this?

22. In Ms R's prep class, students are making letters using wooden blocks.

Eleanor makes an uppercase 'E' as shown below.

What is the perimeter of the shape in cm?



A. 130 cm **B.** 189 cm **C.** 210 cm **D.** 227 cm **E.** 270 cm

23. Tori goes on holiday to the Gold Coast for three nights. She goes out for dinner each night. On the first night, she pays $41 for her meal. On the second night, she pays $36. On the third night, she pays $28.

What is the mean cost per meal?

A. $31 **B.** $32 **C.** $33 **D.** $34 **E.** $35

24. Stuart has a system where he rewards himself for study using Tim Tams.

For each hour he studies Maths, he earns one Tim Tam.

For each hour he studies English, he earns two Tim Tams.

For each hour he studies French, he earns three Tim Tams.

He records the hours of study he does in one week in a table.

Subject	Number of hours
Maths	𝍸 I
English	III
French	I

If he does the same amount of study for each subject the next two weeks, how many Tim Tams will he collect?

A. 14 **B.** 15 **C.** 28 **D.** 30 **E.** 32

Back to contents

12C Area of squares, rectangles and triangles

Calculating areas has been an important skill ever since civilisations began. Official Egyptian surveying of farmland around the Nile dates back to 1400 BC. Communities used area measurements to manage and distribute the land around them. Area is measured in unit squares, and in this lesson we revise the methods of determining how many units are contained in particular squares, rectangles and triangles.

LEARNING INTENTIONS

Students will be able to:

- convert between units of area
- determine the area of squares and rectangles
- determine the area of triangles.

KEY TERMS AND DEFINITIONS

The **area** is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.

A **hectare** is a metric unit of area equal to 10 000 square metres.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

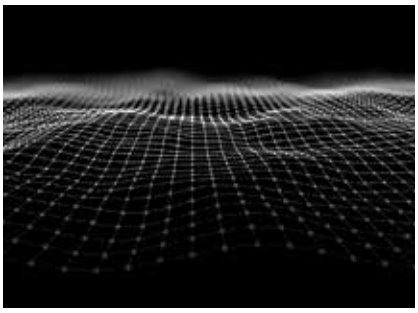
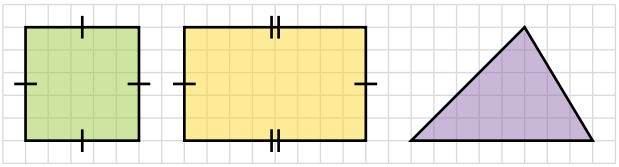


Image: SkillUp/Shutterstock.com

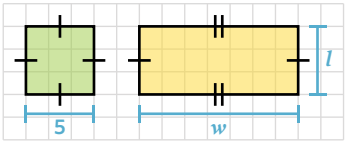
Very complicated surfaces are always reduced to many small squares and triangles to simplify them, which allows better measurement and understanding of them.

Key ideas

1. The area of a 2D shape can be found by counting the number of unit squares the shape covers on a grid.

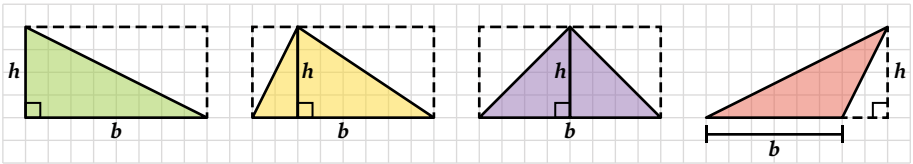


2. The area of a square and rectangle is equal to one side length multiplied by a perpendicular side length.



$A = s \times s = s^2$ $A = l \times w$

The area of a triangle is equal to half of the rectangle created by the base and height of the triangle.

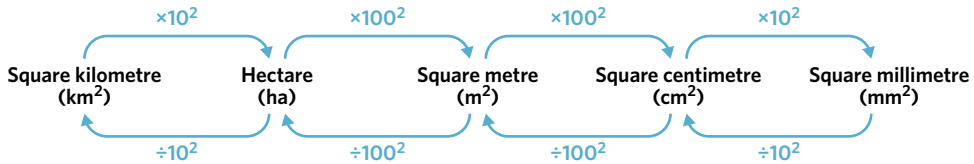


$A = \frac{b \times h}{2}$

Note: The base is always a side of the triangle and the height is the line that is perpendicular to the base and passes through the vertex opposite to the base.

Continues →

3. We can convert between different units of area measurement by multiplying or dividing by powers of 10.



Worked example 1

Converting between units of area measurement

Convert.

a. $560\ mm^2$ to cm^2 WE1a

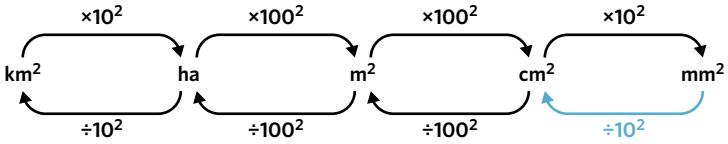
Working

$560 \div 10^2 = 5.6\ cm^2$

Thinking

Complete the necessary multiplication or division and state the value, with units.

Visual support



b. $0.75\ km^2$ to m^2 WE1b

Working

$0.75 \times 10^2 \times 100^2 = 750\ 000\ m^2$

Thinking

Complete the necessary multiplication or division and state the value, with units.

Student practice

Convert.

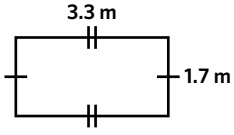
a. $380\ mm^2$ to cm^2 **b.** $0.88\ km^2$ to m^2

Worked example 2

Using formulas to calculate area

Determine the areas of the shapes.

a. WE2a



Working

$A = 3.3 \times 1.7$

$= 5.61\ m^2$

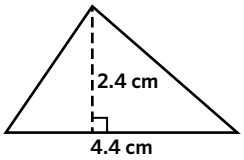
Thinking

Apply the formula (Area = width × length) to side lengths and calculate the area, with units.

Continues →

b.

WE2b



Working

$A = \frac{4.4 \times 2.4}{2}$

$= \frac{10.56}{2}$

$= 5.28 \text{ cm}^2$

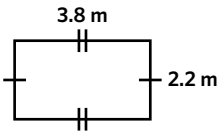
Thinking

Apply the formula Area $= \frac{\text{base} \times \text{height}}{2}$ to the given measurements and calculate the area, with units.

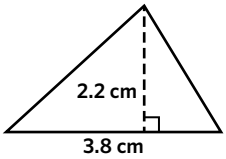
Student practice

Determine the areas of the shapes.

a.



b.



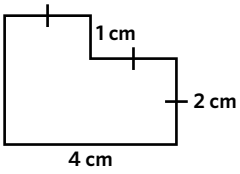
Worked example 3

Calculating areas of composite shapes

Determine the areas of the following composite shapes.

a.

WE3a



Working

Total area = area of large rectangle + area of small rectangle

$A = 4 \times 2 + 1 \times 2$

$= 8 \text{ cm}^2 + 2 \text{ cm}^2 = 10 \text{ cm}^2$

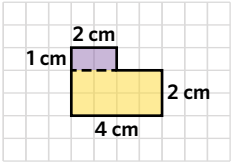
Thinking

Step 1: Write a word equation to describe the composite area, using component shapes.

Step 2: Apply the formulas for the areas of each component shape.

Step 3: Calculate the composite area, with units.

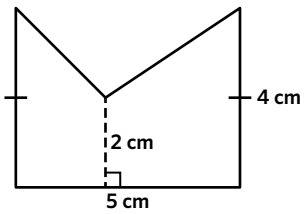
Visual support



Continues →

b.

WE3b



Working

A = rectangle − triangle

$A = 4 \times 5 - \frac{2 \times 5}{2}$

$= 20 - 5 = 15 \text{ cm}^2$

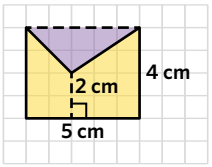
Thinking

Step 1: Write a word equation to describe the composite area, using component shapes.

Step 2: Apply the formulas for the areas of each component shape.

Step 3: Calculate the composite area, with units.

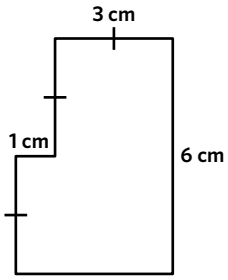
Visual support



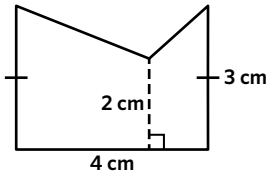
Student practice

Determine the areas of the following composite shapes.

a.



b.



12C Questions

Understanding worksheet

1. Fill in the missing unit of measurement.

Example

$328.9864 \text{ m}^2 = 32\,898\,640$ [mm²]

a. $15 \text{ cm}^2 = 1500$ [ ]

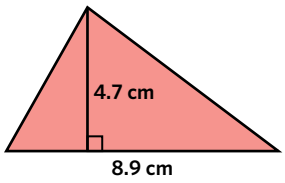
b. $11 \text{ ha} = 110\,000$ [ ]

c. $11 \text{ m}^2 = 0.0011$ [ ]

d. $419\,007\,000 \text{ m}^2 = 419.007$ [ ]

2. Fill in the blanks.

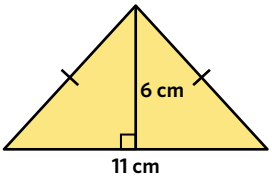
Example



Base of a triangle: 8.9

Height of a triangle: 4.7

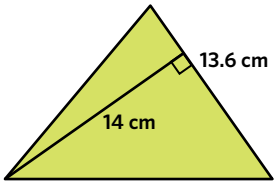
a.



Base of a triangle:

Height of a triangle:

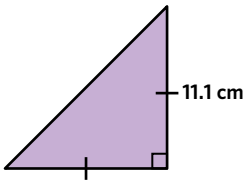
b.



Base of a triangle:

Height of a triangle:

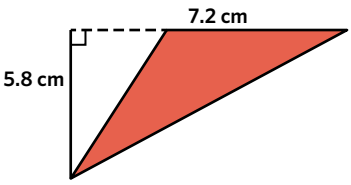
c.



Base of a triangle:

Height of a triangle:

d.



Base of a triangle:

Height of a triangle:

3. Fill in the blanks by using the words provided.

multiplying | formula | squares | perpendicular

One way to find the area of triangles, squares, rectangles or any other 2D shapes is by counting the number of unit _____ the shapes cover on a grid. However, a more efficient way is by using a _____ or equation. The formula to find the area of a triangle is equal to $\frac{b \times h}{2}$, where b is equal to the base and h is equal to the _____ height. We can also convert 1 square metre to 1 square centimetre by _____ by 100^2.

Fluency

Question working paths

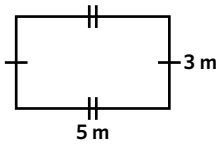
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8

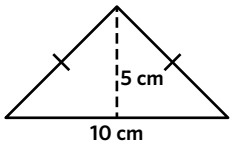
4. Convert. WE1

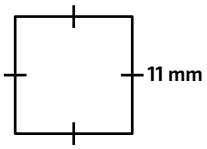
a. 12 cm^2 to mm^2 **b.** 3000 cm^2 to m^2 **c.** 37 m^2 to cm^2 **d.** 2 ha to km^2

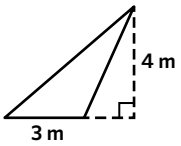
e. 845 000 mm^2 to m^2 **f.** 1.8 ha to cm^2 **g.** 0.089 km^2 to m^2 **h.** 92 050 000 mm^2 to km^2

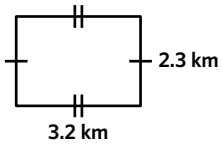
5. Determine the areas of the shapes. WE2

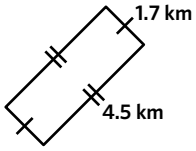
a.


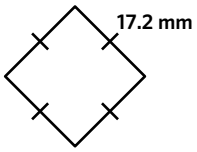
b.


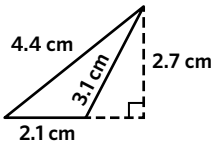
c.


d.


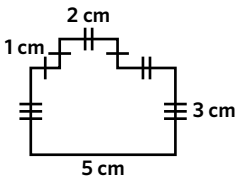
e.


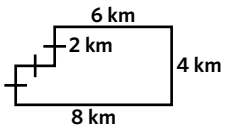
f.


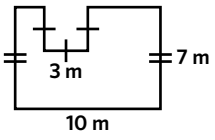
g.


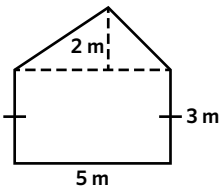
h.


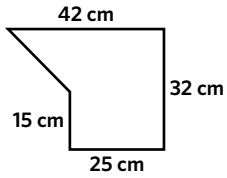
6. Determine the areas of the composite shapes. WE3

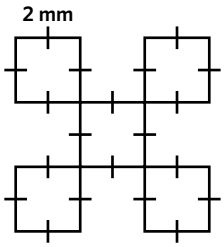
a.


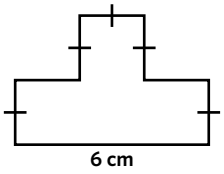
b.


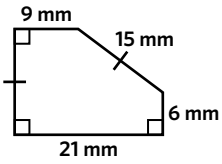
c.


d.


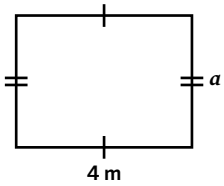
e.


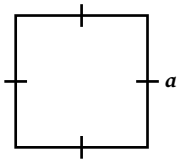
f.


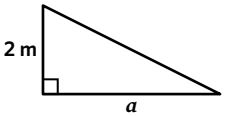
g.


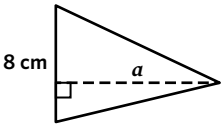
h.


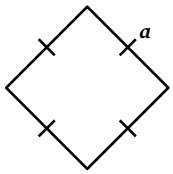
7. Determine the value of the unknown.

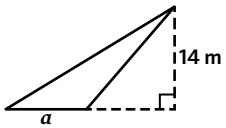
a. Area = 12 m^2


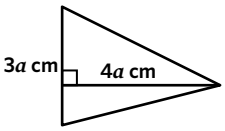
b. Area = 16 cm^2


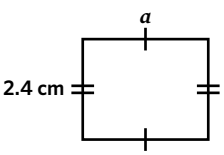
c. Area = 4 m^2


d. Area = 45 cm^2


e. Area = 1.44 km^2


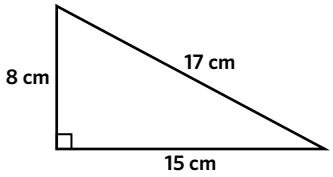
f. Area = 35 m^2


g. Area = 54 cm^2


h. Area = 720 mm^2


8. Calculate the area of this shape.

A. 40 cm^2
B. 60 cm^2
C. 68 cm^2
D. 120 cm^2
E. 127.5 cm^2



Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Convert 250 cm^2 to m^2.

Student A

$250 \div 100 = 2.5$

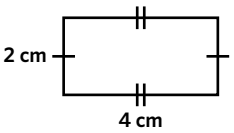
$250\ cm^2 = 2.5\ m^2$

Student B

$250 \div 100^2 = 0.025$

$250\ cm^2 = 0.025\ m^2$

b. Calculate the area of this shape.



Student A

$A = 2 \times 2 + 2 \times 4$

$= 10\ cm^2$

Student B

$A = 2 \times 4$

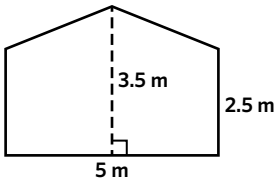
$= 8\ cm^2$

Problem solving

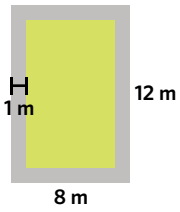
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Mark is planning to paint the side of his barn. He has measured it to be 5 m wide, while the right and left sides are both 2.5 m high. The barn reaches 3.5 m at its highest point perpendicular to the ground. How many square metres does he need to paint?

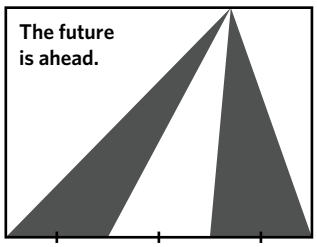


11. Dean's yard was 8 m wide and 12 m long. If he created a 1 m wide path around the edge of his yard, what area did the path cover?



12. The regulations of a soccer pitch are that it must be a rectangular field with length from 100 m to 120 m and width from 50 m to 100 m. What is the difference in area between the smallest and largest possible fields?

13. Fran's company is trying to position itself as a futurist start-up. They have created a design on their business cards to give the impression of heading to the future. If their business cards are 8 cm wide and 6 cm high, then what area of the card is painted black, excluding the writing?



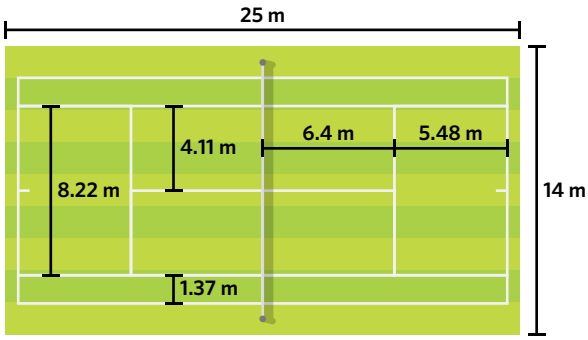
14. Angelo has put up his 5-acre rectangular block of land for sale. The advertising states that the property has 125 m of road frontage. Considering that a hectare equals 2.5 acres, how far back from the road does his property go, in metres?

Reasoning

Question working paths

Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Warren's tennis club has decided to install a hardcourt as an alternative to its plethora of clay courts. The measurements of the court area and lines are provided in the diagram.



a. What is the total area of the court?

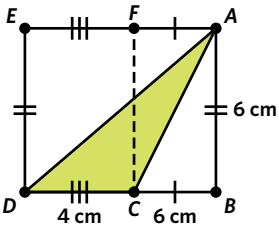
b. Determine the total length of all the white lines.

c. If the lines are 5 cm wide, then how many square metres of white paint will need to be applied?

d. If Warren wanted the total area of the white lines to be 10 m^2, then how wide should he make the lines, in cm to two decimal places?

e. Can local and public sporting club facilities survive without volunteers?

16. Use the diagram to answer the questions.



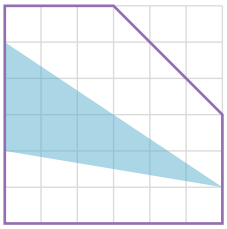
a. Calculate the areas of $\triangle ABC$ and $\triangle AED$.

b. Calculate the area of the shaded triangle $\triangle ADC$ using your answer to part **a** and the area of the quadrilateral $AEDB$.

c. Use your answer to part **b** to explain how the area of the quadrilateral $EFCD$ is related to the area of $\triangle ADC$.

Extra spicy

17. What fraction of the area contained by the purple pentagon is covered by the blue triangle?



A. $\frac{2}{7}$

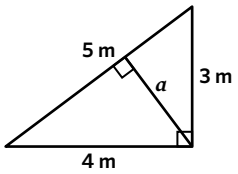
B. $\frac{1}{3}$

C. $\frac{2}{5}$

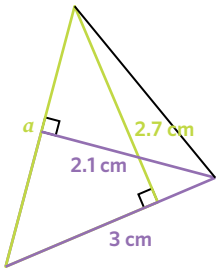
D. $\frac{1}{4}$

E. $\frac{2}{9}$

18. Determine the value of a.

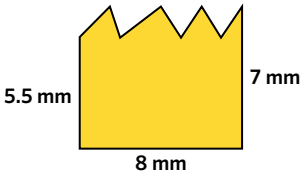


19. Determine the value of a.



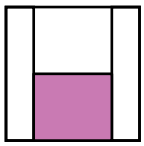
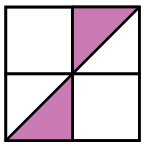
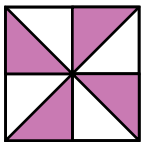
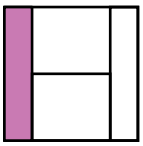
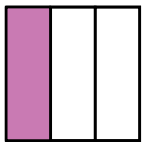
A. 1.93 cm **B.** 1.89 cm **C.** 2.33 cm **D.** 3.86 cm **E.** 7.72 cm

20. Claude has created the following design for a crown sprite for the king to wear in his video game. The crown consists of a rectangle 5.5 mm and 8 mm with four triangles that have the same height. If a square pixel has a width of 0.25 mm and a height of 0.25 mm. How many pixels covers the same amount of area as the crown?



Remember this?

21. Which square has one quarter shaded?

A.  **B.**  **C.**  **D.**  **E.** 

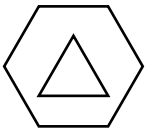
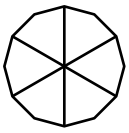
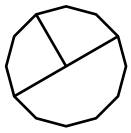
22. Claire makes \$96 selling 24 scrunchies at the school fete.

All her scrunchies are the same price.

How much money will she make selling 30 scrunchies?

A. \$110 **B.** \$120 **C.** \$130 **D.** \$140 **E.** \$150

23. Which of these images looks the same after one sixth of a turn?

A.  **B.**  **C.**  **D.**  **E.** 

Back to contents

12D Area of special quadrilaterals

The formulas for finding the areas of triangles and rectangles are used to find the areas of special quadrilaterals. Special quadrilaterals have properties that lead to consistent area formulas, if the measurements of particular properties are known.

LEARNING INTENTIONS

Students will be able to:

- understand that special quadrilaterals can be formed by rectangles and triangles
- calculate the areas of rhombuses, kites and trapeziums using formulas.

KEY TERMS AND DEFINITIONS

A **parallelogram** is a quadrilateral with two opposite pairs of parallel sides.

A **trapezium** is a quadrilateral with one pair of parallel sides.

A **kite** is a quadrilateral with reflection symmetry across a diagonal.

A **rhombus** is a quadrilateral with four equal sides.

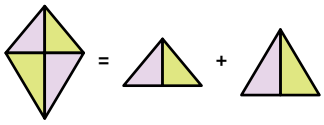
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Michael Rosebrock/Shutterstock.com

Modern building design incorporates diverse shapes. Knowing the area of a parallelogram helps to understand the amount of windows required in a building like the one at the Port of Hamburg.

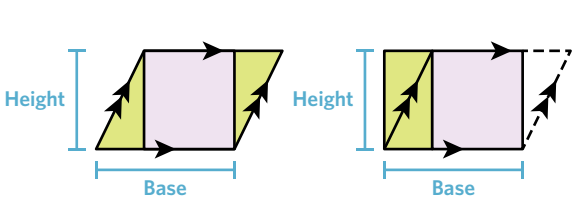
Key ideas

1. The area of a larger shape is equal to the sum of the area of the smaller shapes contained within it.

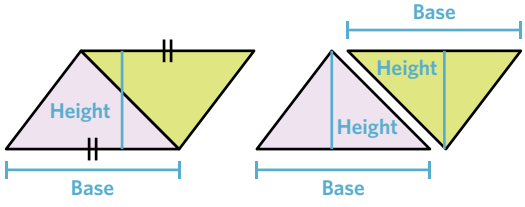


2. The area of special quadrilaterals can be found using formulas.

The area of a parallelogram is equal to the length of its base multiplied by the length of its height perpendicular to the base.

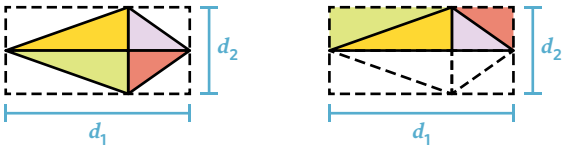


Area = base × height



$$\text{Area} = \frac{\text{base} \times \text{height}}{2} + \frac{\text{base} \times \text{height}}{2} = \text{base} \times \text{height}$$

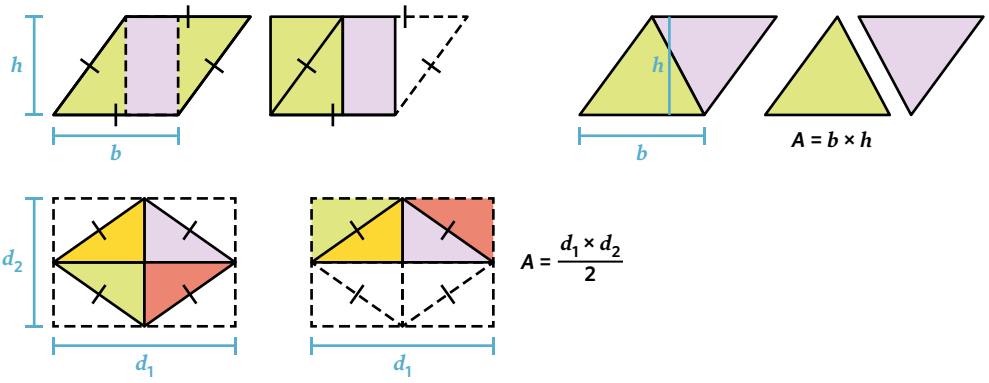
The area of a kite is equal to half of the rectangle created by its perpendicular diagonals.



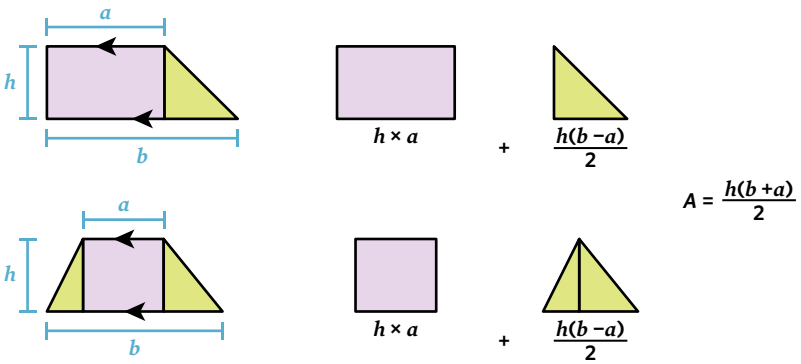
$$A = \frac{d_1 \times d_2}{2}$$

Continues →

The area of a rhombus can be calculated using either the formula of a parallelogram or a kite.



The area of a trapezium is equal to the average of the lengths of the parallel sides multiplied by the perpendicular distance between them.



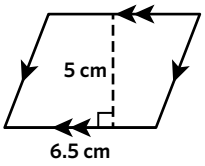
Worked example 1

Determining the area of a parallelogram

Determine the areas of the following shapes.

a.

WE1a



Working

Parallelogram

Area = base × height

= 6.5 × 5

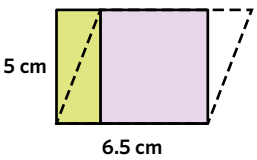
= 32.5 cm^2

Thinking

Step 1: Recognise the shape and formula that can be used with measurements given.

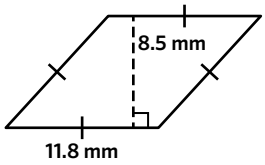
Step 2: Apply the formula to the measurements given.

Visual support



Continues →

b.



WE1b

Working

A rhombus is a parallelogram.

Area = base × height

$= 11.8 \times 8.5$

$= 100.3 \text{ mm}^2$

Thinking

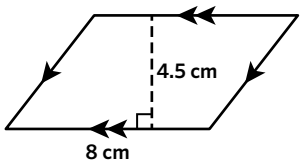
Step 1: Recognise the shape and formula that can be used with measurements given.

Step 2: Apply the formula to the measurements given.

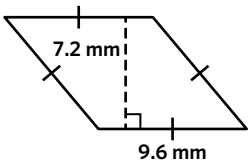
Student practice

Determine the areas of the following shapes.

a.



b.

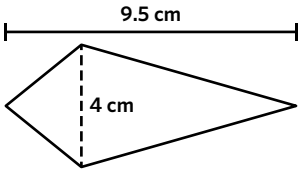


Worked example 2

Determining the area of a kite

Determine the areas of the following shapes.

a.



WE2a

Working

Kite

$\text{Area} = \frac{d_1 \times d_2}{2}$

$= \frac{4 \times 9.5}{2}$

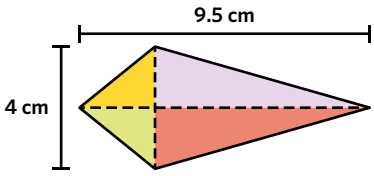
$= 19 \text{ cm}^2$

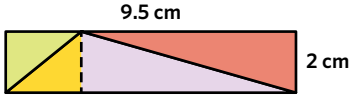
Thinking

Step 1: Recognise the shape and formula that can be used with measurements given.

Step 2: Apply the formula to the measurements given.

Visual support

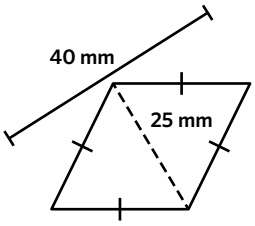




Continues →

b.

WE2b



Working

Kite

$$\text{Area} = \frac{d_1 \times d_2}{2}$$

$$= \frac{40 \times 25}{2}$$

$$= 500 \text{ mm}^2$$

Thinking

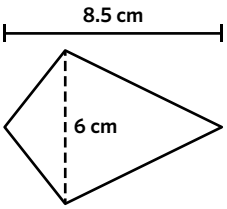
Step 1: Recognise the shape and formula that can be used with measurements given.

Step 2: Apply the formula to the measurements given.

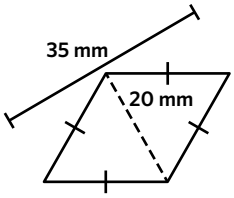
Student practice

Determine the areas of the following shapes.

a.



b.



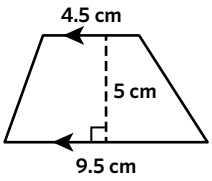
Worked example 3

Determining the area of a trapezium

Determine the areas of the following shapes.

a.

WE3a



Working

Trapezium

$$\text{Area} = \frac{h \times (a + b)}{2}$$

$$= \frac{5 \times (4.5 + 9.5)}{2}$$

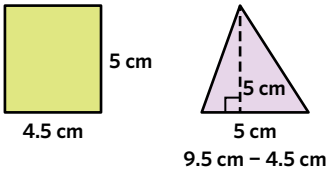
$$= 35 \text{ cm}^2$$

Thinking

Step 1: Recognise the shape and formula that can be used with measurements given.

Step 2: Apply the formula to the measurements given.

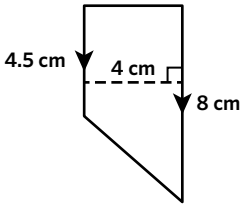
Visual support



Continues →

b.

WE3b



Working

Trapezium

$$\text{Area} = \frac{h \times (a + b)}{2}$$

$$= \frac{4 \times (4.5 + 8)}{2}$$

$$= 25 \text{ cm}^2$$

Thinking

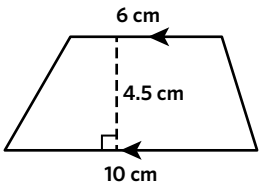
Step 1: Recognise the shape and formula that can be used with measurements given.

Step 2: Apply the formula to the measurements given.

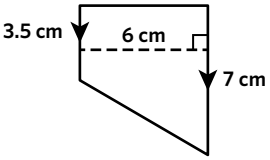
Student practice

Determine the areas of the following shapes.

a.



b.

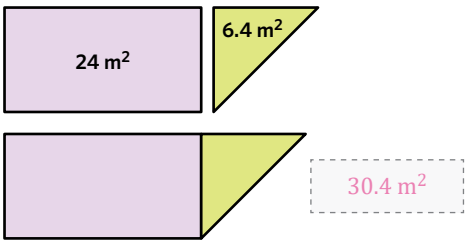


12D Questions

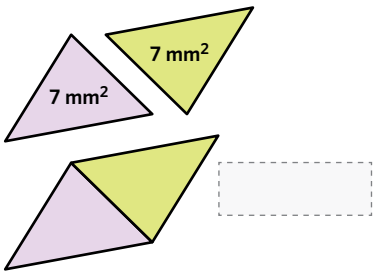
Understanding worksheet

1. Add the areas of the smaller shapes to find the area of the larger shape.

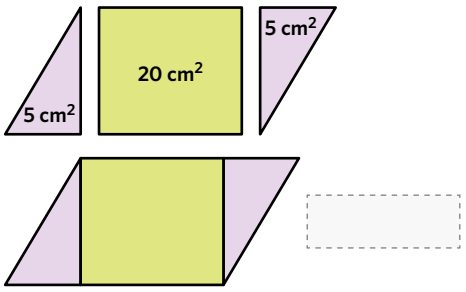
Example



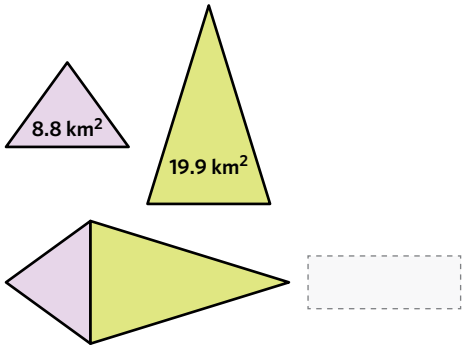
a.



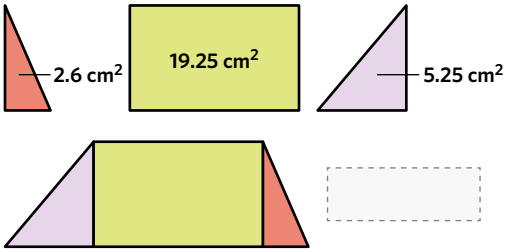
b.



c.

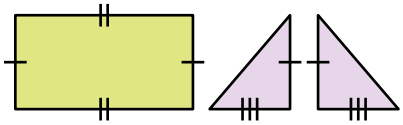


d.



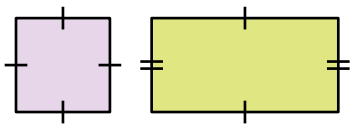
2. Circle all possible shapes that could be created by transforming and joining the triangles and rectangles.

Example



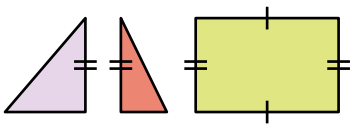
Trapezium Kite Parallelogram
Rectangle Rhombus Square

a.



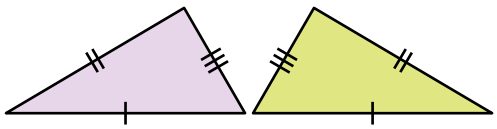
Trapezium Kite Parallelogram
Rectangle Rhombus Square

b.



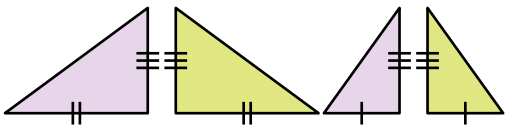
Trapezium Kite Parallelogram
Rectangle Rhombus Square

c.



Trapezium Kite Parallelogram
Rectangle Rhombus Square

d.



Trapezium Kite Parallelogram
Rectangle Rhombus Square

3. Fill in the blanks by using the words provided.

rhombus	parallel	parallelogram	diagonals

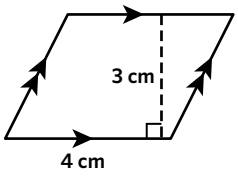
The area of a _____ is equal to the area of a rectangle that has the same base length and height perpendicular to the base. The area of a kite is equal to half of the area of the rectangle created by its _____. The area of a _____ can be found by using the formula for the area of a parallelogram or a kite. The area of a trapezium is found by adding the lengths of both _____ sides, multiplying this sum by the distance between them, and dividing the product by two.

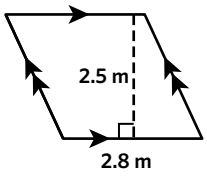
Fluency

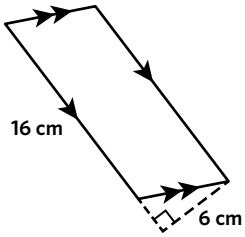
Question working paths

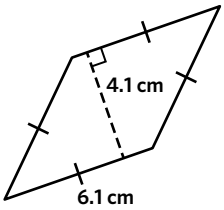
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d,e), 8 (b,c,d,e), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (c,d,e,f), 8 (c,d,e,f), 9

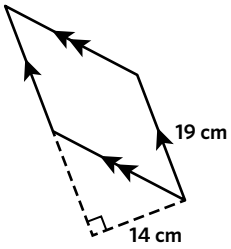
4. Determine the areas of the following shapes. WE1

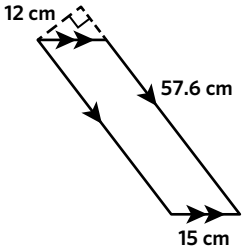
a.


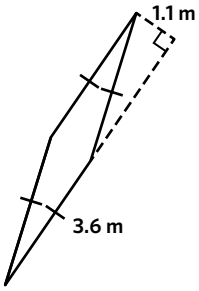
b.


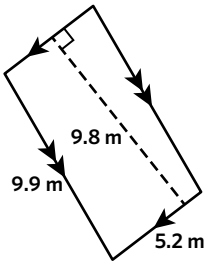
c.


d.


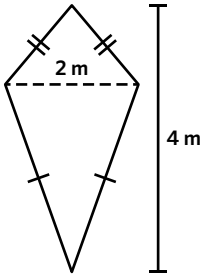
e.


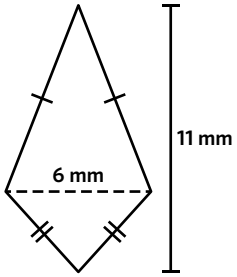
f.


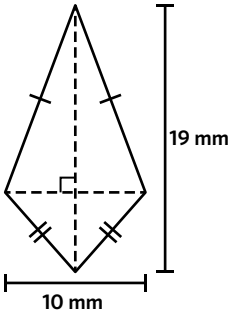
g.


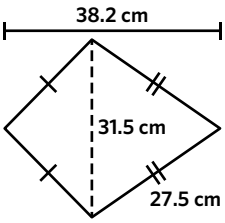
h.


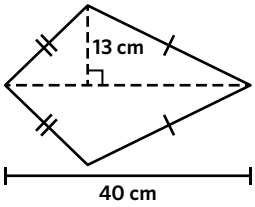
5. Determine the areas of the following shapes. WE2

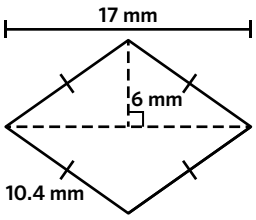
a.


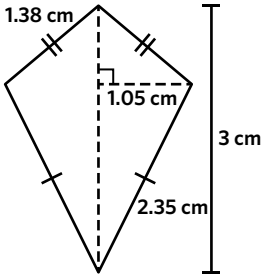
b.


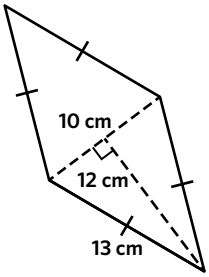
c.


d.


e.


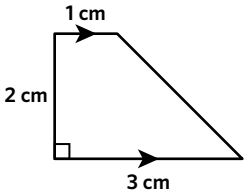
f.


g.


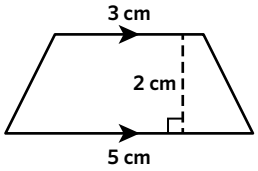
h.


6. Determine the areas of the following shapes. WE3

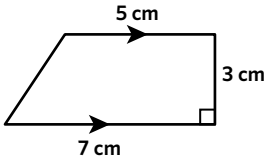
a.



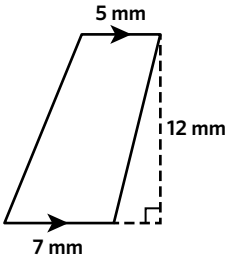
b.



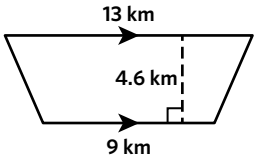
c.



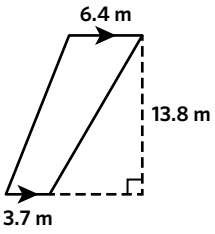
d.



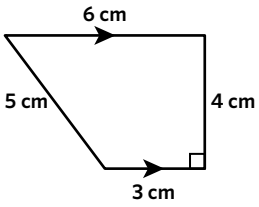
e.



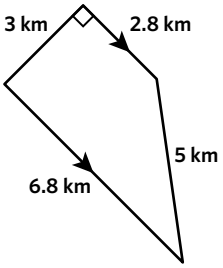
f.



g.



h.



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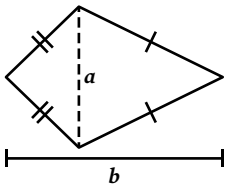
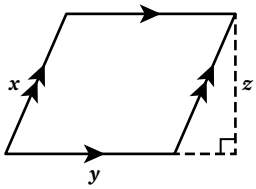
7. Describe the areas of the following shapes with an algebraic expression. You do not need to calculate the areas.

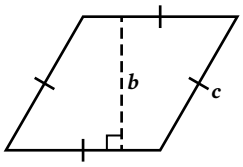
a.



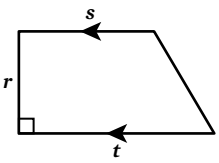
b.



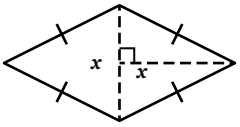
c.



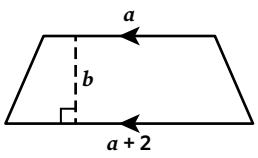
d.



e.



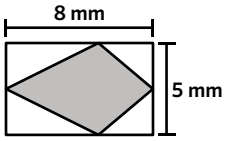
f.



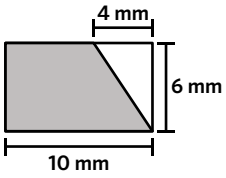
---

8. Determine the areas that are shaded in the following diagrams.

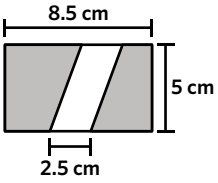
a.



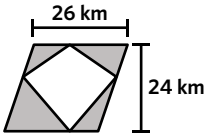
b.



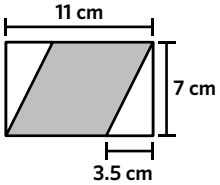
c.



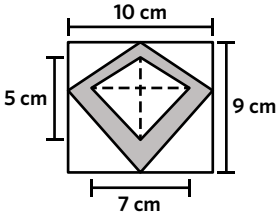
d.



e.



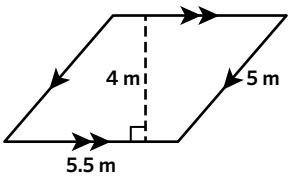
f.



---

9. Determine the area of the following shape.

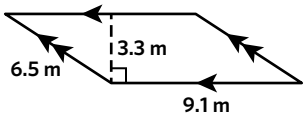
A. 9.5 m^2
B. 10.5 m^2
C. 20 m^2
D. 22 m^2
E. 27.5 m^2



Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Find the area of the following shape.



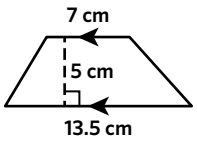
Student A

$6.5 \times 9.1 = 59.15\ \text{m}^2$

Student B

$9.1 \times 3.3 = 30.03\ \text{m}^2$

b. Find the area of the following shape.



Student A

$$\frac{5 \times (7 + 13.5)}{2}$$

$$= \frac{102.5}{2} = 51.75\ \text{cm}^2$$

Student B

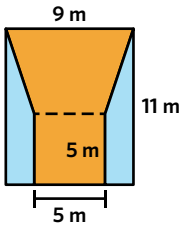
$5 \times (13.5 - 7) = 32.5\ \text{cm}^2$

Problem solving

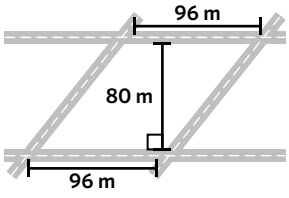
Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

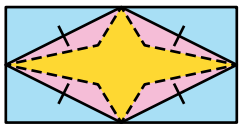
11. Bobby has built some water barriers to help funnel his sheep into a smaller area before he shears them. The piece of land he's working with is a 9 m by 11 m rectangle. The area he's trying to funnel them into is a 5 m by 5 m square. Use the diagram to determine the area of land in this rectangle that is not covered by water. Water is coloured blue.



12. Strawberry Fields have re-zoned some farmland on the outskirts of their town. Roads are running parallel to each other in an east-west direction and streets are running parallel to each other in northeast-southwest direction. If the roads are 80 m apart and the intersections between roads and streets are 96 m away from each other, then what is the area of the block of land enclosed by the streets and roads?

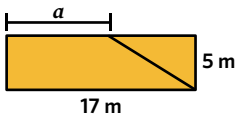


13. Joshua created a pattern mural on a wall that was 4 m high and 7 m wide. He started by drawing a rhombus that had a vertex on each side of the wall and painting a star pattern inside the rhombus. He then coloured the wall outside of the rhombus blue. What is the size of the area that is covered by blue paint?



14. Tori has created a kite that has a total surface area of 1400 cm^2. What is the area of the smallest rectangle she could place around it?

15. Ravi wants to place sound proofing foam along a 17 m by 5 m wall to reduce feedback when recording his sitar. Unfortunately, he only has enough to cover 65 m^2 so he decides to compromise by covering as much as he can with a trapezium shape. Determine the value of a in the following diagram.

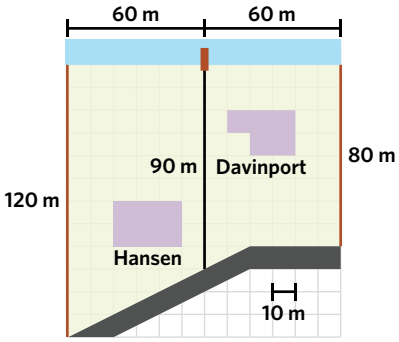


Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. The Hansen and Davinport families bought two blocks of land alongside each other. Both were promised even access to the river and an equal amount of land, so a fence line was built to satisfy all of these conditions and the families then proceeded to build their homes. The diagram shows their houses as purple and has the lengths of fence lines.

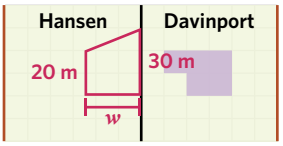


a. Calculate the total area of the Hansen family's property.

b. Calculate the total area of the Davinport family's property.

c. Calculate the combined area of Hansen's and Davinport's blocks of land and determine what area each family should have, if they are to have an equal share.

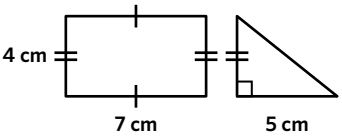
d. The Hansen family offers the Davinport family a trapezium piece of land that has sides of 20 m and 30 m that are parallel to the current fence line. How wide does the trapezium need to be so that both families have an equal amount of land in total?



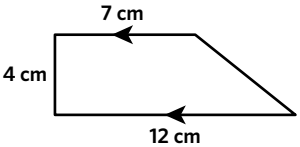
e. Does the amount of river shore line improve a property?

17. For the following.

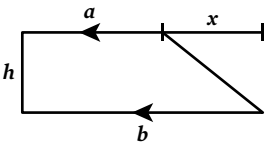
a. Calculate the areas of the rectangle and triangle and add them together.



b. Calculate the area of the trapezium.

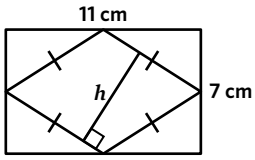


c. Use your answers in parts **a** and **b** and the diagram below to explain how the formula for the area of a trapezium must be $\frac{h(a + b)}{2}$.



Extra spicy

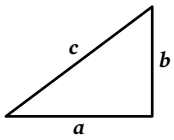
18. A rhombus with a perimeter of 26 cm sits inside a rectangle with side lengths 11 cm and 7 cm. Determine the length of h, rounded to 2 decimal places.



19. A solid white plastic cube is painted blue on the outside. The cube is then cut into eight smaller cubes of equal size. What fraction of the total surface area of the new cubes is blue?

A. $\frac{1}{8}$ **B.** $\frac{1}{3}$ **C.** $\frac{3}{8}$ **D.** $\frac{1}{2}$ **E.** $\frac{3}{4}$

20. Pythagoras's theorem states that for a right angled triangle:



$c^2 = a^2 + b^2$

or

$c = \sqrt{a^2 + b^2}$

Use Pythagoras' theorem to calculate the perimeter of a rhombus that has diagonals 10 cm and 12 cm long, to the nearest cm.

21. The diagram shows an equilateral triangle with its corners at midpoints of alternate sides of a regular hexagon. What fraction of the hexagon is shaded?

A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{3}{8}$ **D.** $\frac{4}{9}$ **E.** $\frac{7}{12}$

Remember this?

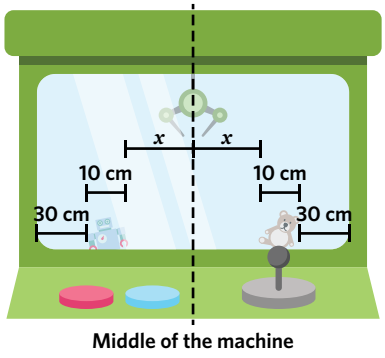
22. What number does the heart represent in the following number sentence? Round your answer to 1 decimal place if required.

♥ ÷ (230.391 + 18.129) = 2.5

A. −3600.8 **B.** −530.7 **C.** 0.1 **D.** 621.3 **E.** 3600.8

23. Merna is playing with a claw machine at the mall. The claw starts in the centre of the machine.

There are two prizes left.



If the machine is 150 cm wide, determine the distance between the two prizes.

A. 35 cm **B.** 50 cm **C.** 65 cm **D.** 70 cm **E.** 80 cm

24. The following table shows the weights of some pets coming into a veterinary clinic.

Name	Weight	Species
Spike	5 kg	Dog
Freddo	0.1 kg	Lizard
Cleopatra	0.25 kg	Snake
Rosie	15 kg	Dog
Sprout	2 kg	Cat
Fluffy	61 kg	Dog
Bonnie	3 kg	Cat

How much heavier is the heaviest dog than the heaviest cat?

A. 2 kg **B.** 12 kg **C.** 13 kg **D.** 58 kg **E.** 59 kg

Back to contents

12E Area of a circle

Just like the circumference, the area of a circle has a direct relationship with the radius, diameter and the number π. The area of a circle can be expressed in exact form as a number multiplied by π or as an estimate by rounding.

LEARNING INTENTIONS

Students will be able to:

- determine the area of a circle given its radius or diameter using a calculator
- calculate the area of a circle using an approximate value of π
- calculate the area of a semicircle and quadrant.

KEY TERMS AND DEFINITIONS

A **semicircle** is exactly half of a circle.

A **quadrant** is exactly one quarter of a circle.

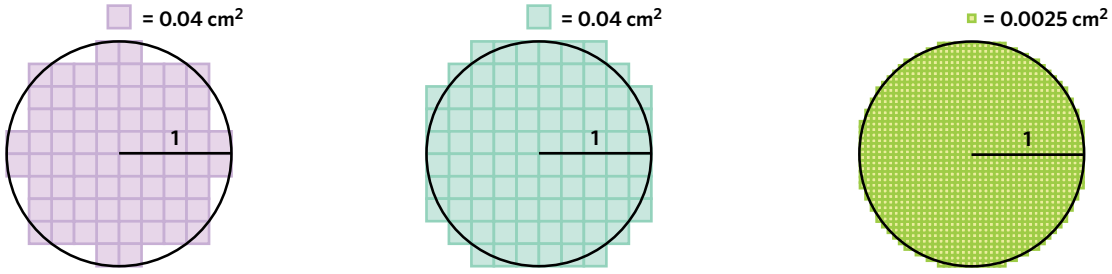
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Kent Raney/Shutterstock.com

Some farmers create circles when watering their crops from a sprinkler. To help farmers know how many crops they can grow, they can calculate the area covered by the sprinklers by using the distance the sprinkler sprays water as the radius of the circle.

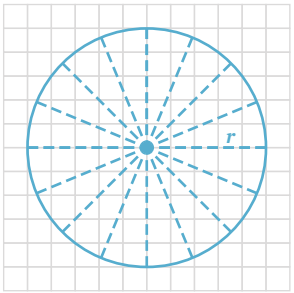
Key ideas

1. The area of a circle can be estimated by counting the number of squares that fit inside it.



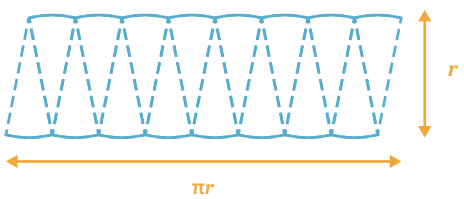
2. The formula to determine the area of a circle is $A = \pi r^2$.

The formula can be derived by dividing a circle into multiple sectors and placing them together into a shape that resembles a rectangle. Smaller sectors will produce a more accurate estimation for the area of a circle.



Area of the rectangle = area of the original circle

$l \times w = \pi r \times r = \pi r^2$



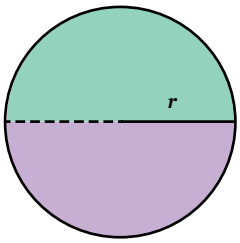
The **length** of the rectangle can be estimated by half of the circumference of the circle so $l = \frac{2\pi r}{2} = \pi r$.

The **width** of the rectangle is the radius of the original circle so $w = r$.

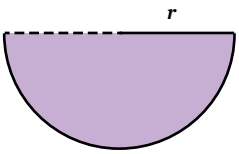
Continues →

3. The area of a semicircle is equal to half of the area of a circle and the area of a quadrant is equal to a quarter of the area of a circle.

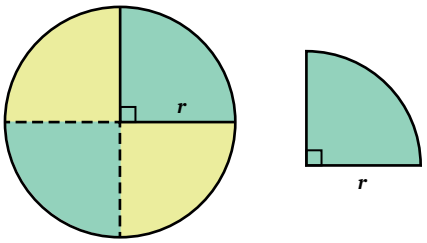
Area of circle = πr^2



Area of semicircle = $\frac{\pi r^2}{2}$



Area of quadrant = $\frac{\pi r^2}{4}$



Worked example 1

Calculating the area of a circle from the diameter or radius

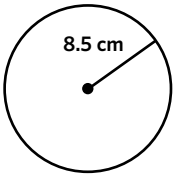
Calculate the area:

i. in terms of π.

ii. correct to 3 decimal places.

a.

WE1a



Working

$A = \pi \times r^2$

$= \pi \times (8.5)^2$

$= 72.25\pi \text{ cm}^2$

$\approx 226.980 \text{ cm}^2$

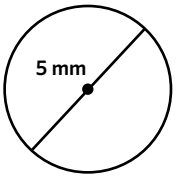
Thinking

i. Apply the formula and express area in terms of π.

ii. Calculate the area and round the value to 3 decimal places.

b.

WE1b



Working

$r = \frac{d}{2} = 2.5 \text{ mm}$

$A = \pi \times r^2$

$= \pi \times (2.5)^2$

$= 6.25\pi \text{ mm}^2$

$\approx 19.635 \text{ mm}^2$

Thinking

Step 1: Convert the diameter into a radius.

Step 2:

i. Apply the formula and express area in terms of π.

ii. Calculate the area and round the value to 3 decimal places.

Continues →

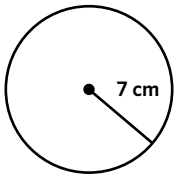
Student practice

Calculate the area:

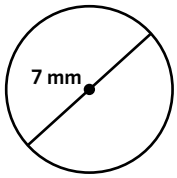
i. in terms of π

ii. correct to 3 decimal places.

a.



b.

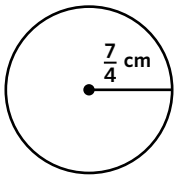


Worked example 2

Estimating the area of a circle from the diameter or radius

Determine the area of the circles using the given estimations of π.

a. $\pi \approx \frac{22}{7}$ WE2a



Working

$A = \pi \times r^2$

$= \pi \times \left(\frac{7}{4}\right)^2$

$= \frac{49}{16}\pi \text{ cm}^2$

$\approx \frac{49}{16} \times \frac{22}{7} = \frac{77}{8}$

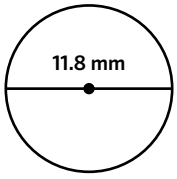
$= 9\frac{5}{8} \text{ cm}^2$

Thinking

Step 1: Apply the formula and express the area in terms of π.

Step 2: Substitute in the given estimation of π and evaluate. Write the value of the area as a mixed number.

b. $\pi \approx 3.14$ WE2b



Working

$r = \frac{d}{2} = 5.9 \text{ mm}$

$A = \pi \times r^2$

$= \pi \times 5.9^2$

$= 34.81\pi \text{ mm}^2$

$\approx 34.81 \times 3.14$

$= 109.30 \text{ mm}^2$

Thinking

Step 1: Convert the diameter into a radius.

Step 2: Apply the formula and express the area in terms of π.

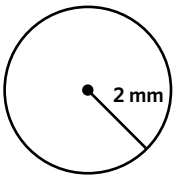
Step 3: Substitute in the given estimation of π and evaluate.

Continues →

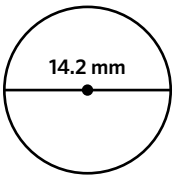
Student practice

Determine the area of the circles using the given estimations of π.

a. $\pi \approx \frac{22}{7}$



b. $\pi \approx 3.14$



Worked example 3

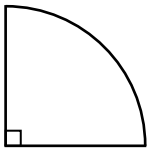
Calculating the area of semicircles and quadrants

Calculate the area of the semicircles and quadrants:

i. in terms of π.

ii. correct to 3 decimal places.

a.



WE3a

Working

$$A = \frac{\pi \times r^2}{4}$$

$$= \frac{\pi \times 6^2}{4}$$

$$= \frac{\pi \times 36}{4}$$

$$= 9\pi \text{ cm}^2$$

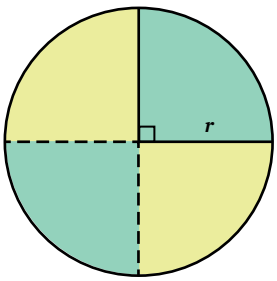
$$\approx 28.274 \text{ cm}^2$$

Thinking

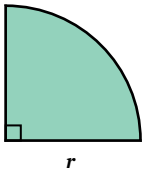
i. Apply the formula and express area in terms of π.

ii. Calculate the area and round the value to 3 decimal places.

Visual support



Area = $\pi \times r^2$

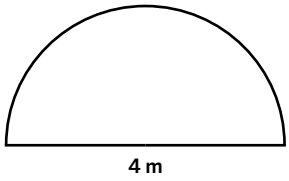


Area = $\frac{\pi \times r^2}{4}$

Continues →

b.

WE3b



Working

$r = \frac{d}{2} = 2$ m

$A = \frac{\pi \times r^2}{2}$

$= \frac{\pi \times 2^2}{2}$

$= 2\pi$ m^2

≈ 6.283 m^2

Thinking

Step 1: Convert the diameter into a radius.

Step 2:

i. Apply the formula and express area in terms of π.

ii. Calculate the area and round the value to 3 decimal places.

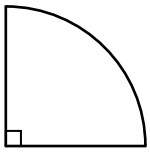
Student practice

Calculate the area of the semicircles and quadrants in:

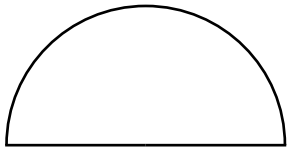
i. terms of π

ii. and correct to 3 decimal places.

a.



b.



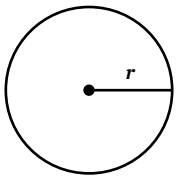
Worked example 4

Calculating the radius and diameter using the area of a circle

Determine the values of the unknowns, correct to 3 decimal places.

a. $A = 25$ cm^2

WE4a



Working

$A = \pi \times r^2$

$25 = \pi \times r^2$

$r^2 = \frac{25}{\pi}$

$r = \sqrt{\frac{25}{\pi}}$

≈ 2.821 cm

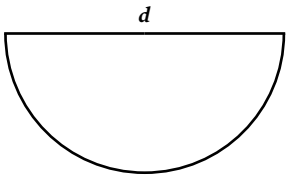
Thinking

Step 1: Rearrange the formula, with the unknown value as the subject.

Step 2: Solve using a calculator and round to 3 decimal places.

Continues →

b. $A = 25\ \text{m}^2$ WE4b



Working

$$A = \frac{\pi \times r^2}{2}$$

$$= \frac{\pi \times \left(\frac{d}{2}\right)^2}{2}$$

$$\frac{\pi \times \left(\frac{d}{2}\right)^2}{2} = 25$$

$$\left(\frac{d}{2}\right)^2 = \frac{2 \times 25}{\pi}$$

$$\frac{d}{2} = \sqrt{\frac{50}{\pi}}$$

$$d = 2 \times \sqrt{\frac{50}{\pi}}$$

$$\approx 7.979\ \text{m}$$

Thinking

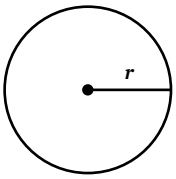
Step 1: Rearrange the formula, with the unknown value as the subject and express in terms of π.

Step 2: Solve using a calculator and round to 3 decimal places.

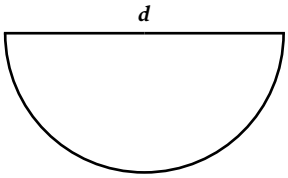
Student practice

Determine the values of the unknowns, correct to 3 decimal places.

a. $A = 20\ \text{cm}^2$



b. $A = 20\ \text{m}^2$

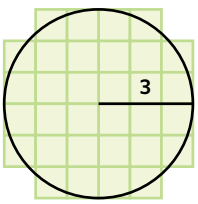


12E Questions

Understanding worksheet

1. Count the squares to estimate the area of the circles.

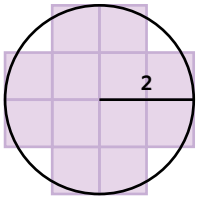
Example



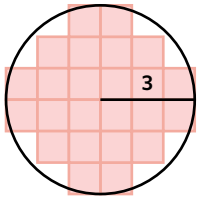
= 1 cm²

Area of squares is

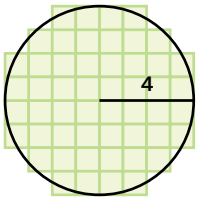
= 32 cm²

a. 

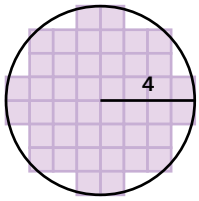
= 1 cm²
Area of squares is
= ☐ cm²

b. 

= 1 cm²
Area of squares is
= ☐ cm²

c. 

= 1 cm²
Area of squares is
= ☐ cm²

d. 

= 1 cm²
Area of squares is
= ☐ cm²

2. Square the numbers.

Example

$11^2\pi =$ 121 π

a. $4^2\pi =$ ☐ π

b. $17^2\pi =$ ☐ π

c. $2.3^2\pi =$ ☐ π

d. ☐ $^2\pi = 81\pi$

3. Fill in the blanks by using the words provided.

radius | exact | irrational | area

The ☐ of a circle is equal to π times the ☐ squared.

Because π is an ☐ number the answer a calculator gives is always a rounded estimate.

The ☐ solution is always a number multiplied by π.

Fluency

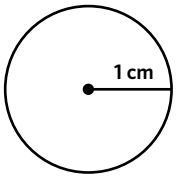
Question working paths

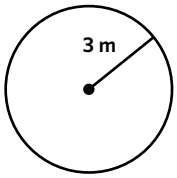
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

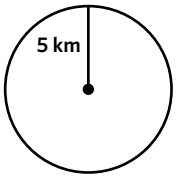
4. Calculate the area of the circles: WE1a

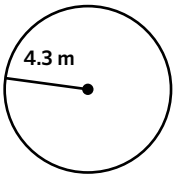
i. in terms of π.

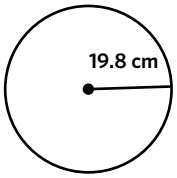
ii. correct to 3 decimal places.

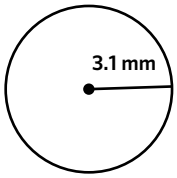
a. 

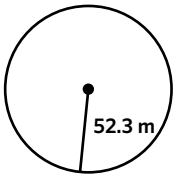
b. 

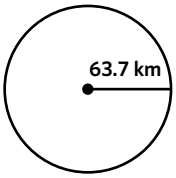
c. 

d. 

e. 

f. 

g. 

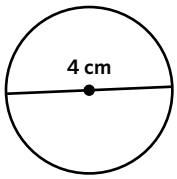
h. 

5. Calculate the area of the circles: WE1b

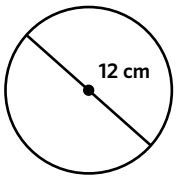
i. in terms of π.

ii. correct to 3 decimal places.

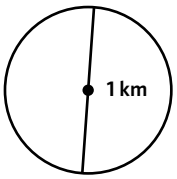
a.



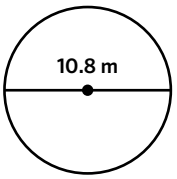
b.



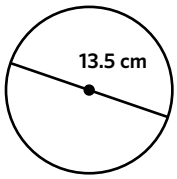
c.



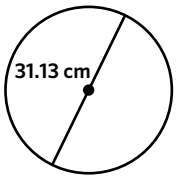
d.



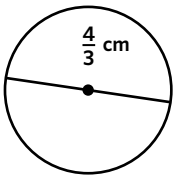
e.



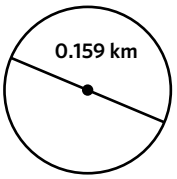
f.



g.



h.



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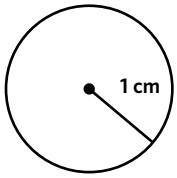
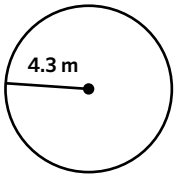
6. Determine the area of the circles using the given estimations of π, correct to 3 decimal places, as needed. WE2

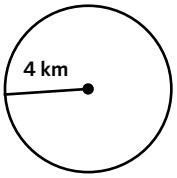
a. $\pi \approx \frac{22}{7}$



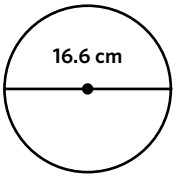
b. $\pi \approx 3.14$



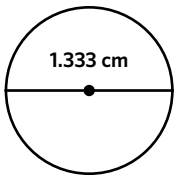
c. $\pi \approx \frac{22}{7}$



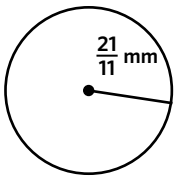
d. $\pi \approx 3.14$



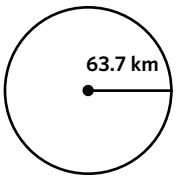
e. $\pi \approx 3.14$



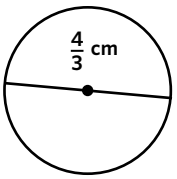
f. $\pi \approx \frac{22}{7}$



g. $\pi \approx 3.14$



h. $\pi \approx \frac{333}{106}$



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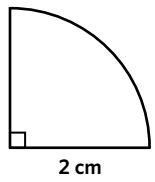
7. Calculate the area of the semicircles and quadrants: WE3

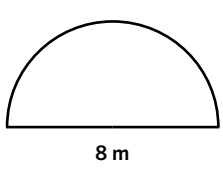
i. in terms of π.

ii. correct to 3 decimal places.

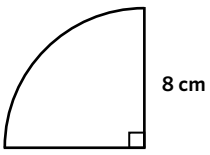
a.



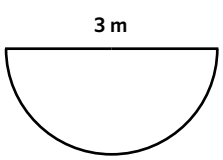
b.



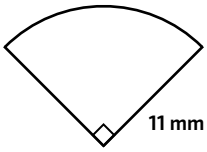
c.



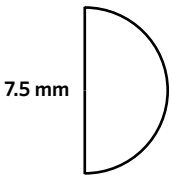
d.



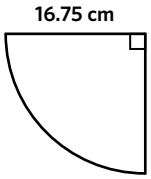
e.



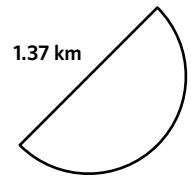
f.



g.



h.



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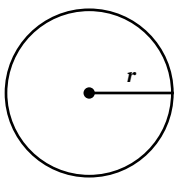
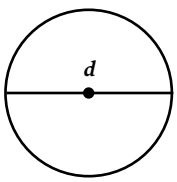
8. Determine the values of the unknowns, correct to 3 decimal places. WE4

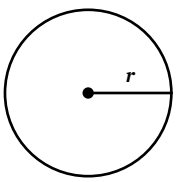
a. $A = 6\ \text{m}^2$



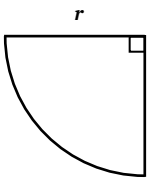
b. $A = 12\ \text{cm}^2$



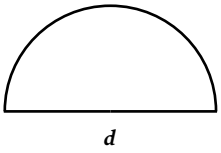
c. $A = 14.75\ \text{cm}^2$



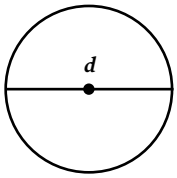
d. $A = 3.8\ \text{km}^2$



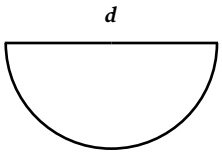
e. $A = 15.5 \text{ mm}^2$



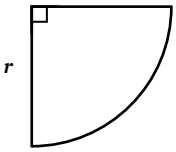
f. $A = 25\pi \text{ m}^2$



g. $A = 14.137 \text{ mm}^2$

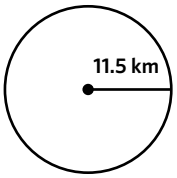


h. $A = 12.566 \text{ mm}^2$



9. Calculate the area of the circle, correct to 3 decimal places.

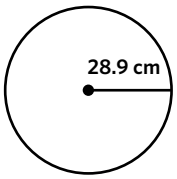
A. 72.256 km^2
B. 103.869 km^2
C. 415.476 km^2
D. 830.951 km^2
E. 1305.255 km^2



Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the area of the circle, correct to 3 decimal places.



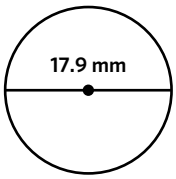
Student A

$A = \pi \times 28.9^2$
$= \pi \times 835.21$
$\approx 2623.89 \text{ cm}^2$

Student B

$A = (\pi \times 28.9)^2$
$\approx 90.792^2$
$= 8243.1873 \text{ cm}^2$

b. Calculate the area of the circle, correct to 3 decimal places



Student A

$A = \pi \times \left(\frac{17.9}{2}\right)^2$
$= \pi \times 80.1025$
$\approx 251.649 \text{ mm}^2$

Student B

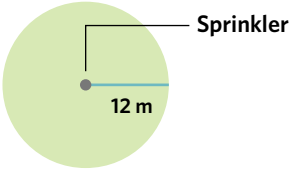
$A = \pi \times 17.9^2$
$= \pi \times 320.41$
$\approx 1006.598 \text{ mm}^2$

Problem solving

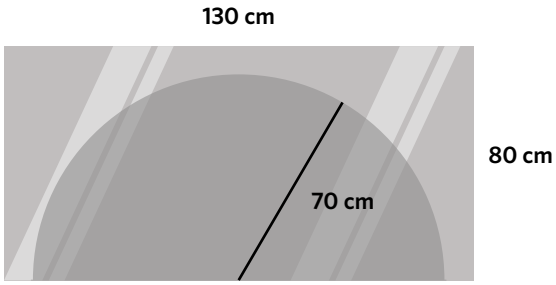
Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Melanie is watering her crops with a sprinkler that rotates 360° and sprays water in every direction. How much area does the sprinkler water?



12. Henry has a windscreen that is 80 cm high and 130 cm wide. If he attaches a single windscreen wiper that is 70 cm long, what is the area of the windscreen that is untouched by the windscreen wiper, correct to 3 decimal places?



13. Annunziata sells pizzas for $24. Her pizzas have a radius of 18 cm. She is creating a new pizza size that has a radius of 21 cm. How much should she sell the new size of pizza so her customers pay exactly the same amount per cm^2?

14. Ferdinand measures the distance around his circular plate to be 20 cm. What is the area that his plate covers?

15. Archibald has a 20 m rope that is 3 cm thick on his ship. When he is at port, his ropes are rolled up and placed on the dock, as shown in the diagram. What is the radius of a circle that has the same amount of area that Archibald's rope would cover?

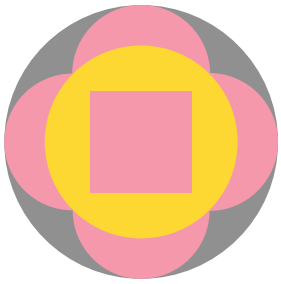
Image: Ivan Marc/Shutterstock.com

Reasoning

Question working paths

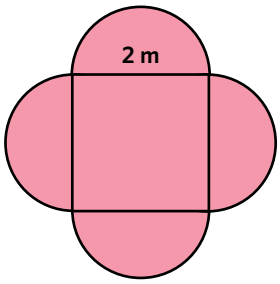
Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Preston is planning to paint his logo on the brick wall at the side of his flower shop. He needs to calculate how much grey, pink and yellow paint he will require.



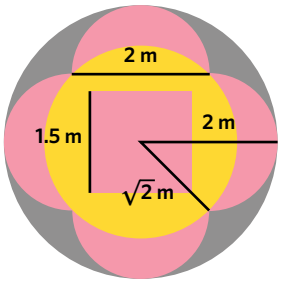
a. Determine the area of a circle with a radius of 2 m.

b. Use the area of a semicircle with a diameter of 2 m to determine the area of the following shape.



c. Determine the area of a circle with radius $\sqrt{2}$ m.

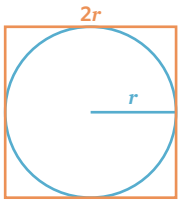
d. Use the diagram below and your answers to parts **a**, **b** and **c** to determine the area of the logo that is grey, pink and yellow.



e. Which company has a logo that makes you connect with their product?

17. For the following.

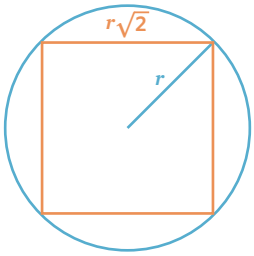
a. Divide the area of a circle by the area of the square that it fits perfectly in to see what fraction of the area of the square is covered by the circle. See measurements in the diagram.



Area of the square = $4r^2$

Area of the circle = πr^2

b. Divide the area of the square that fits perfectly inside a circle by the area of the circle to determine what fraction of the circle is covered by the square.



Area of the circle = πr^2

Area of the square = $2r^2$

c. Use your answers to parts **a** and **b** to determine what is the best fit; a round peg in a square hole or a square peg in a round hole?

Extra spicy

18. What is the area of the triangle ABC?

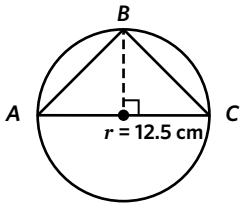
A. 37.5 cm^2

B. 78.125 cm^2

C. 156.25 cm^2

D. 490.87 cm^2

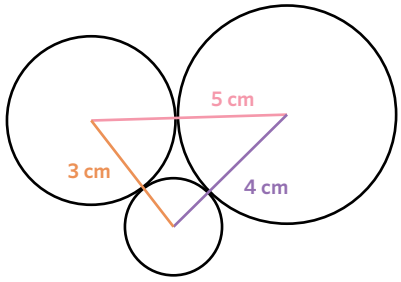
E. More information required



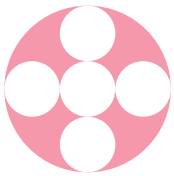
19. A circle with diameter x cm has a circle with a radius $\frac{x}{6}$ cm removed. What is the total area remaining?

A. $\pi\frac{x^2}{6}\text{ cm}^2$ **B.** $\pi\frac{2x^2}{9}\text{ cm}^2$ **C.** $\pi\frac{x^2}{3}\text{ cm}^2$ **D.** $\pi\frac{7x^2}{9}\text{ cm}^2$ **E.** $\pi\frac{x^2}{9}\text{ cm}^2$

20. What is the sum of the areas of the circles in the diagram?

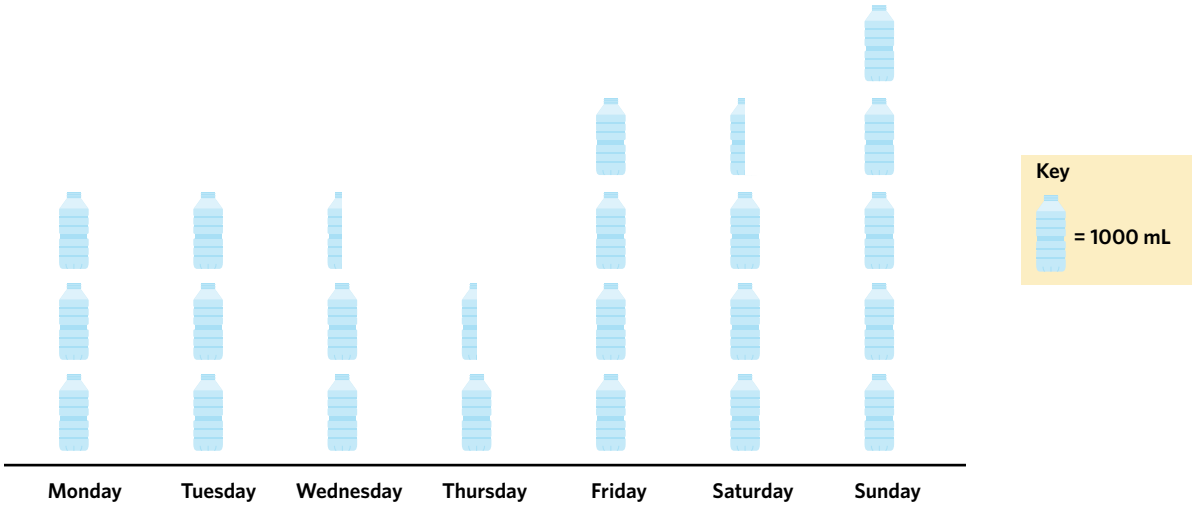


21. What fraction of the circle is shaded?



Remember this?

22. Jolene tracked the number of bottles of water she drank every day for one week in the following picture graph:



What is the difference between the amount of water she drank on her best and worst water-drinking days?

A. 1500 mL **B.** 2000 mL **C.** 2500 mL **D.** 3000 mL **E.** 3500 mL

23. The fraction $\frac{1}{4}$ equals 0.25 as a decimal. What is $\frac{1}{8}$ as a decimal?

A. 0.045 **B.** 0.08 **C.** 0.12 **D.** 0.125 **E.** 0.16

24. Farai's age is three times Rufus's age plus an additional 2 years.

Let m be Rufus' age.

Which of the following expressions is equal to Farai's age?

A. $3m + 2$ **B.** $3 + 2m$ **C.** $\frac{m}{3} - 2$ **D.** $3 - \frac{m}{2}$ **E.** $5m$

Back to contents

12F Composite shapes with sectors

When dealing with circular shapes, there is often only a part of the circle that is observed or of interest. By recognising that there are 360 degrees in a full revolution of a circle, the area and perimeter of a sector can be determined. Fractions can be used to show the proportion of the whole circle represented by the sector.

LEARNING INTENTIONS

Students will be able to:

- calculate the perimeter of composite shapes that contain sectors
- calculate the area of composite shapes that contain sectors.

KEY TERMS AND DEFINITIONS

Highest common factor (HCF) is the largest number that is a factor of two or more numbers.

A **sector** comprises two radii and an arc that is a part of a circle with the same radius.

A fraction is in its **simplest form** when the numerator and denominator have no common factors other than 1.

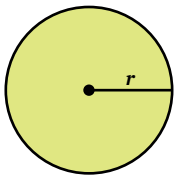
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Tobik/Shutterstock.com

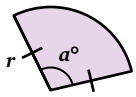
A slice of pizza is made with two straight cuts from the centre of a circle. The mathematical rules of sectors are used to determine total surface area or the amount of crust per slice.

Key ideas

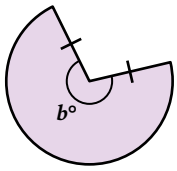
1. The area of a sector is equal to a fraction of the area of a circle with the same radius.



Area of circle = πr^2

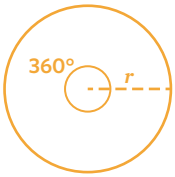


Area of sector = $\frac{a°}{360°} \times \pi r^2$

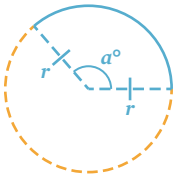


Area of sector = $\frac{b°}{360°} \times \pi r^2$

2. The arc length of a sector is found by multiplying the circumference by the angle of the sector divided by 360°.



Circumference = $2\pi r$



Arc of sector = $\frac{a}{360} \times 2\pi r$

Worked example 1

Finding fractions with angles

Determine the value of the unknown in simplest form.

a. $x = \frac{50°}{360°}$ WE1a

Working

$x = \frac{50}{360}$

$= \frac{5}{36}$

Thinking

Simplify the fraction.

b. $\frac{3}{10} = \frac{a}{360°}$ WE1b

Working

$\frac{3 \times 360°}{10} = a$

$a = 3 \times 36°$

$= 108°$

Thinking

Solve for the unknown.

Student practice

Determine the value of the unknown in simplest form.

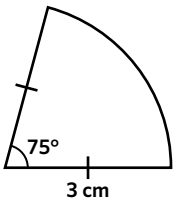
a. $x = \frac{60°}{360°}$ **b.** $\frac{1}{9} = \frac{a}{360°}$

Worked example 2

Finding the area of shapes involving circular sectors

Find the areas of the following shapes, correct to 3 decimal places.

a. WE2a



Working

$\frac{a°}{360°} \times \pi r^2$

$= \frac{75°}{360°} \times \pi \times 3^2$

$= \frac{5}{24} \times 9 \times \pi$

$\approx 5.890 \text{ cm}^2$

Thinking

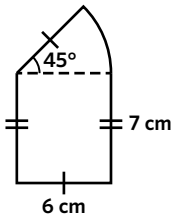
Step 1: Substitute the angle and radius into the formula.

Step 2: Complete the calculation and round to 3 decimal places.

Continues →

b.

WE2b



Working

Total area = sector area + rectangle area

$= \frac{45°}{360°} \times \pi \times 6^2 + 7 \times 6$

$= \frac{1}{8} \times \pi \times 36 + 42$

$\approx 56.137 \text{ cm}^2$

Thinking

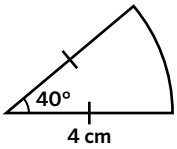
Step 1: Break the shape up into smaller components.

Step 2: Complete the calculation and round to 3 decimal places.

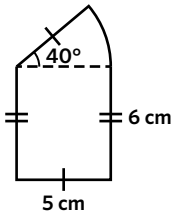
Student practice

Find the areas of the following shapes, correct to 3 decimal places.

a.



b.



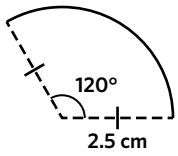
Worked example 3

Finding the perimeter of shapes involving circular sectors

Calculate the following, correct to 3 decimal places.

a. The arc length.

WE3a



Working

$= \frac{a°}{360°} \times 2\pi r$

$= \frac{120°}{360°} \times 2\pi \times 2.5$

$= \frac{1}{3} \times 5 \times \pi$

$\approx 5.236 \text{ cm}$

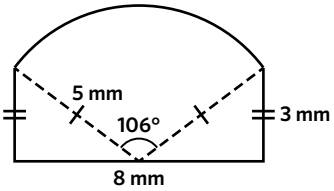
Thinking

Step 1: Substitute the angle and radius of the sector into the formula.

Step 2: Complete the calculation and round to 3 decimal places.

Continues →

b. The perimeter. WE3b



Working

Perimeter = arc length + straight lengths

$= \left(\frac{106°}{360°} \times 2\pi \times 5\right) + (2 \times 3 + 8)$

$= \frac{53}{180} \times 10 \times \pi + 14$

≈ 23.25 mm

Thinking

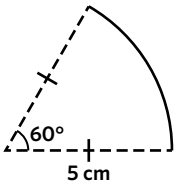
Step 1: Break the shape up into smaller components.

Step 2: Complete the calculation and round to 3 decimal places.

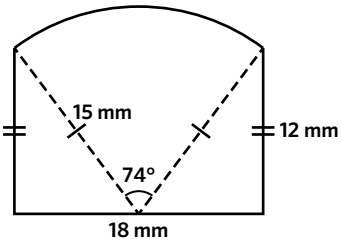
Student practice

Calculate the following, correct to 3 decimal places.

a. The arc length.



b. The perimeter.

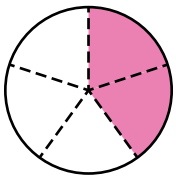


12F Questions

Understanding worksheet

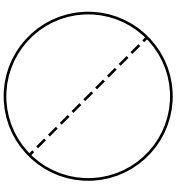
1. Colour in the fraction of the circle for a sector of the given angle.

Example



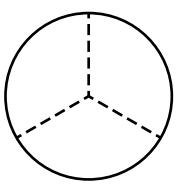
$\frac{144°}{360°} = \frac{2}{5}$

a.



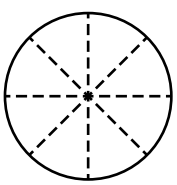
$\frac{180°}{360°} = \frac{1}{2}$

b.



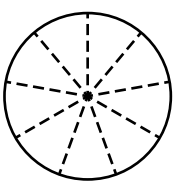
$\frac{240°}{360°} = \frac{2}{3}$

c.



$\frac{135°}{360°} = \frac{3}{8}$

d.

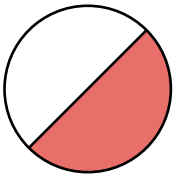


$\frac{160°}{360°} = \frac{4}{9}$

2. Find the area of the shaded sections.

Example

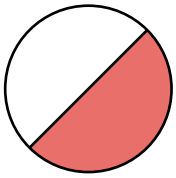
Area of circle $= 16\pi$



Fraction of that circle that is shaded $\frac{3}{8}$

Area of shaded sector $= \frac{3}{8} \times 16\pi = 6\pi$

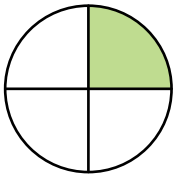
a. Area of circle $= 4\pi$



Fraction of that circle that is shaded $\frac{\quad}{\quad}$

Area of shaded sector $= \frac{\quad}{\quad} \times 4\pi = 2\pi$

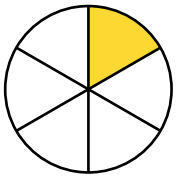
b. Area of circle $= 8\pi$



Fraction of that circle that is shaded $\frac{\quad}{\quad}$

Area of shaded sector $= \frac{\quad}{\quad} \times 8\pi = 2\pi$

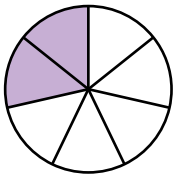
c. Area of circle $= 18\pi$



Fraction of that circle that is shaded $\frac{\quad}{\quad}$

Area of shaded sector $= \frac{\quad}{\quad} \times 18\pi = 3\pi$

d. Area of circle $= 21\pi$



Fraction of that circle that is shaded $\frac{\quad}{\quad}$

Area of shaded sector $= \frac{\quad}{\quad} \times 21\pi = 6\pi$

3. Fill in the blanks by using the words provided.

radii | circle | multiplying | circumference

The fraction of a _____ that represents a sector is determined by dividing the angle of the sector by 360°. The area of a sector is calculated by _____ the area of the circle with the same radius by the fraction of the circle that is equivalent to the sector. When calculating the perimeter of a sector, the _____ is multiplied by the fraction of the circle represented by the sector to determine the arc length. The length of the two _____ must then be added to the arc length to find the total perimeter of the sector.

Fluency

Question working paths

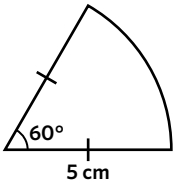
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

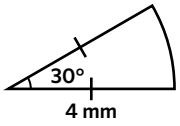
4. Determine the value of the unknown WE1

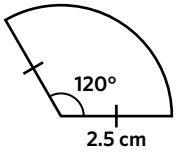
a. $\frac{1}{2} = \frac{a}{360°}$ **b.** $x = \frac{120°}{360°}$ **c.** $\frac{3}{4} = \frac{a}{360°}$ **d.** $x = \frac{132°}{360°}$

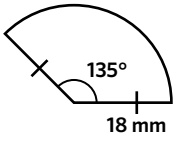
e. $\frac{3}{8} = \frac{a}{360°}$ **f.** $x = \frac{135°}{360°}$ **g.** $\frac{7}{18} = \frac{a}{360°}$ **h.** $x = \frac{114°}{360°}$

5. Find the areas of the following shapes, correct to 3 decimal places. WE2a

a.


b.


c.


d.


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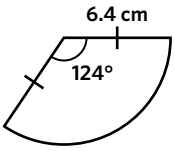

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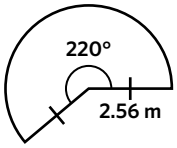

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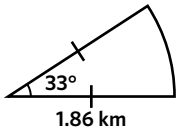

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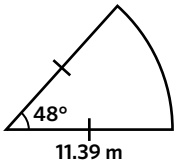
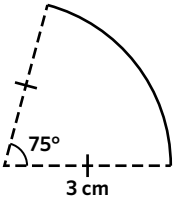
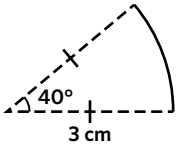


6. Calculate the following arc lengths, correct to 3 decimal places. WE3a

a.


b.


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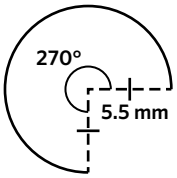
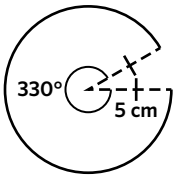


d.


e.


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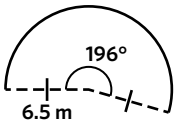

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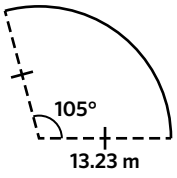

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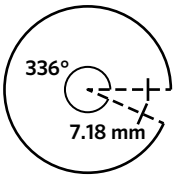
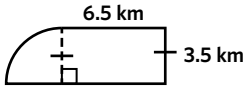
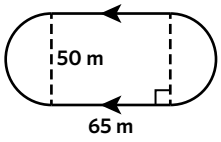
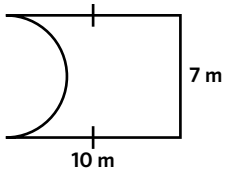
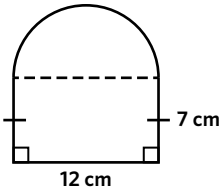


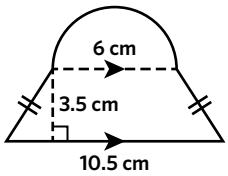
7. Find the areas of the following shapes, correct to 3 decimal places. WE2b

a.


b.


c.


d.


e.


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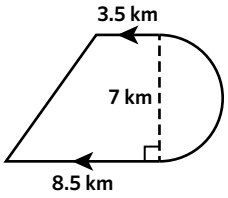

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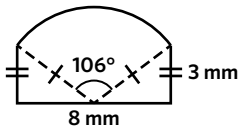

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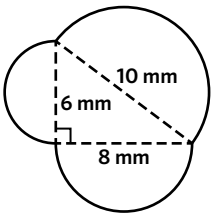
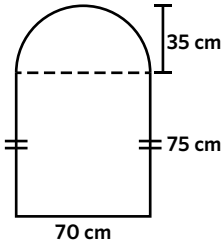


8. Calculate the following perimeters, correct to 3 decimal places. WE3b

a.


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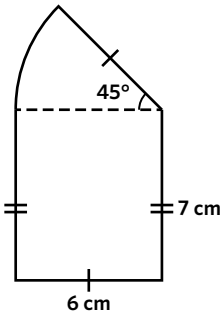

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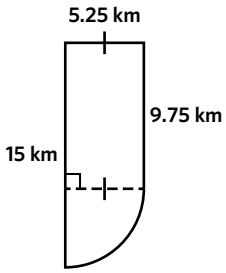

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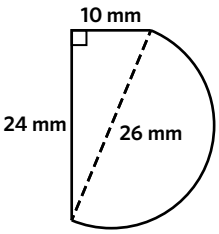

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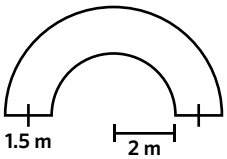

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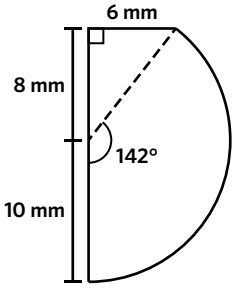
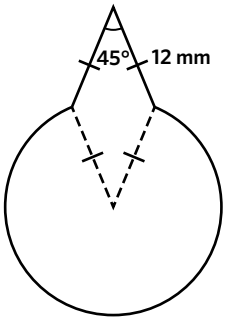


g.


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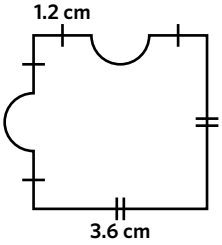
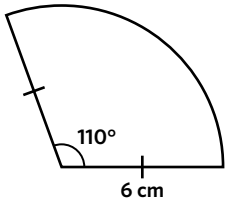


9. Find the area of the following shape, correct to 3 decimal places.

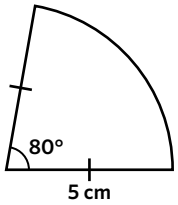
A. 5.76 cm^2
B. 23.519 cm^2
C. 34.558 cm^2
D. 108.566 cm^2
E. 113.097 cm^2



Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Find the area of the following shape, correct to 3 decimal places.



Student A

$= \frac{80°}{360°} \times \pi \times 5^2$

$= \frac{2}{9} \times \pi \times 5^2$

$\approx 17.453 \text{ cm}^2$

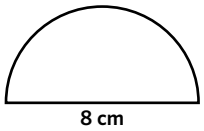
Student B

$= \frac{80°}{360°} \times (\pi \times 5)^2$

$= \frac{2}{9} \times (\pi \times 5)^2$

$\approx 54.831 \text{ cm}^2$

b. Find the perimeter of the following shape, correct to 3 decimal places.



Student A

$= \frac{180°}{360°} \times 2\pi \times 8 + 2 \times 8$

$= \frac{1}{2} \times 16 \times \pi + 16$

$\approx 41.133 \text{ cm}$

Student B

$= \frac{180°}{360°} \times 2\pi \times 4 + 2 \times 4$

$= \frac{1}{2} \times 8 \times \pi + 8$

$\approx 20.566 \text{ cm}$

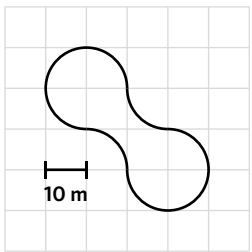
Problem solving

Question working paths

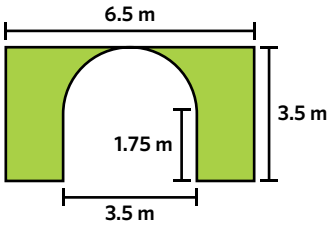
Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Jeremy has eight friends over for his birthday party. To ensure he and all of his friends have an equal share of the cake he needs to cut nine equal slices. If each is a sector of his cake, how many degrees should there be between each cut?

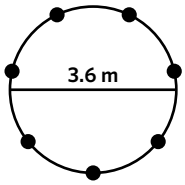
12. Bernie is planning a new go-kart track that is made up of identical arcs joined together. He connects a series of arcs from 8 quadrants and draws them on a grid. If the grid represents 10 m intervals, then how long is one lap of Bernie's track, correct to 3 decimal places.



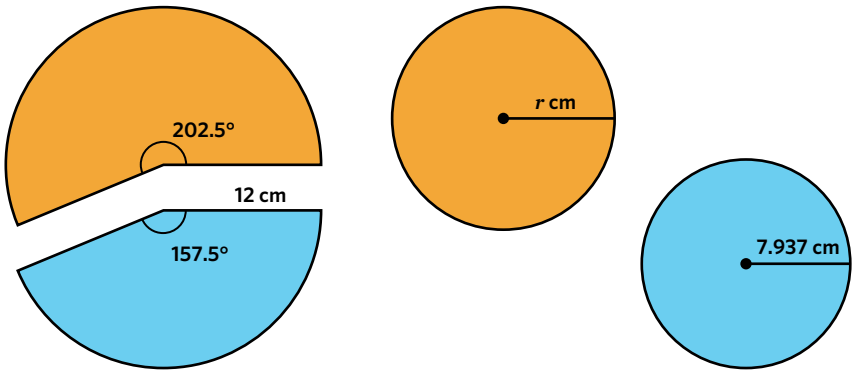
13. Tui has a wall between her lounge and dining room that has a height of 3.5 m and a width of 6.5 m. She has cut an arch into the wall that consists of a semicircle with a radius of 3.5 m and a rectangle that is 1.75 m high. Calculate the area of the wall she has left, correct to 3 decimal places.



14. Campbell has 7 players on his basketball team. He wants them to be equally spaced around the centre circle for a passing drill. What should the circular distance between each player be, correct to 3 decimal places?



15. Sean's frisbee has a radius of 12 cm. He cuts it into sectors of 202.5° and 157.5° and turns both of them into smaller frisbees. What is the radius of the bigger of the two new frisbees?



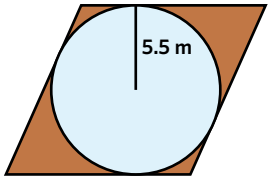
Reasoning

Question working paths

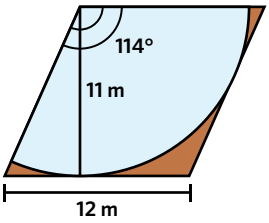
Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Bonnie has constructed a rhombus shaped garden bed that has side lengths of 12 m and a height of 11 m. One pair of internal angles is 114° and the others are 66°. Bonnie often spends long periods away from home and needs to set up an automatic watering system. Round all answers to 3 decimal places.

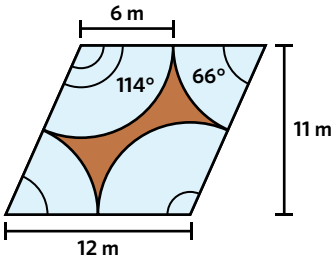
a. Calculate the area covered by a sprinkler with a radius of 5.5 m, if it rotated through a 360° angle.



b. Calculate the area a sprinkler with radius of 11 m could water, if it turned through a 114° angle.



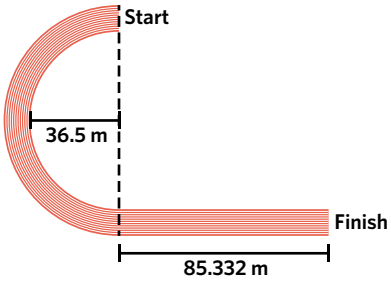
c. Calculate the total area that all four sprinklers in the diagram below could water.



d. Which set up of sprinklers covers the largest area, without overwatering the plants by overlapping?

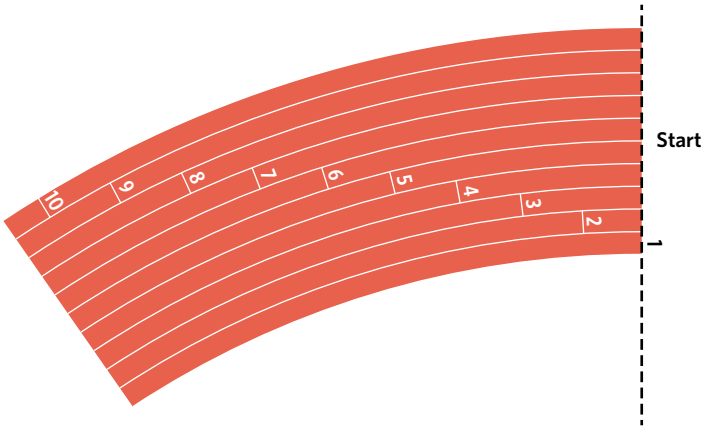
e. Do you prefer to grow edible plants or pretty flowers?

17. One lap of an athletics track is 400 m. One of the most exciting races at the Olympics is the 200 m.



a. Show that a semicircle with a radius of 36.5 m and a straight line of 85.332 m has a total length of 200 m, correct to 3 decimal places.

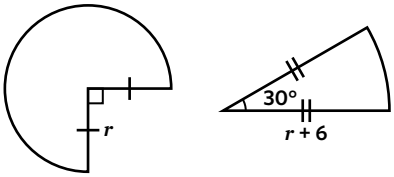
b. If each lane is one metre wide, that implies that the second lane is a semicircle with a radius of 37.5 m and a straight line of 85.332 m. Calculate the distance of the second lane.



c. To ensure that each athlete runs the same distance, the 200 m has a staggered start. Calculate how far the stagger is for the first 3 lanes. What do you notice about this number?

Extra spicy

18. The shapes in the diagram below have equal areas. Determine the value of r.



A. $\frac{3}{4}$

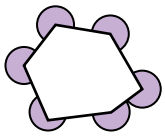
B. $\frac{4}{3}$

C. 3

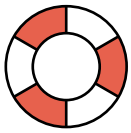
D. 4

E. 9

19. What is the total area of the six sectors formed with the external angles of the hexagon below, assuming that each sector has a radius of 1 cm.

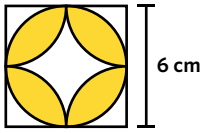


20. The diagram shows two circles with the same centre. The radius of the outer circle is equal to the diameter of the inner circle. If the region between the two circles is divided into six equal segments, what fraction of the outer circle is shaded in the diagram?



A. $\frac{3}{7}$ **B.** $\frac{3}{8}$ **C.** $\frac{3}{9}$ **D.** $\frac{3}{10}$ **E.** $\frac{3}{11}$

21. The shape in the diagram is bounded by 4 quadrants and a circle inside a square that has side lengths of 6 cm. What is the total area of the shape that is shaded in the diagram?



Remember this?

22. Evaluate $\frac{3}{4} + \frac{8}{9}$.

A. $\frac{11}{15}$ **B.** $1\frac{23}{36}$ **C.** $1\frac{25}{36}$ **D.** $1\frac{3}{4}$ **E.** $\frac{36}{15}$

23. Sam has six red scarves, three yellow scarves and one pink scarf in a bag.

Each day, she randomly selects a scarf to wear, then places it back into the bag at the end of the day.

She does this every day for 20 days.

How many days should Sam expect to wear a yellow scarf?

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

24. Dylan and Cleo like to play a computer game. They record their scores over the course of one week and compare.

Dylan

Stem	Leaf
1	5 8
2	1 5 5 8 9
3	0 1 2 3 3 5
4	1 1 5 5 6
5	0 9
6	

Key: 1 | 2 = 12

Cleo

Stem	Leaf
1	9
2	2 3 8
3	3 3 6 7
4	0 4 4 6 8 9
5	3 3 5 5 9
6	1 2

Key: 1 | 2 = 12

Which of the following statements is true?

A. Cleo played the lowest-scoring game.
B. Dylan played the highest-scoring game.
C. Dylan played more games than Cleo with a score over 45 points.
D. The median score for Cleo is higher than the median score for Dylan.
E. The range of scores for Cleo is greater than the range of scores for Dylan.

Back to contents

12G Volume of prisms and cylinders

The volumes of prisms and cylinders are calculated using the same concept. Their uniform cross sections allow for both volumes to be calculated using the same formula. The rule for volume of a prism can also be applied to prisms with varying shapes as their base as long as the area of the base represents the cross section of the solid prism.

LEARNING INTENTIONS

Students will be able to:

- convert between units of volume
- identify prisms and cylinders based on their cross-sections
- calculate the volumes of rectangular prisms, triangular prisms and cylinders using formulas.

KEY TERMS AND DEFINITIONS

A **cross section** is a surface that is created when making a straight cut through a 3D shape.

A **cylinder** is a 3-dimensional object that has 2 flat, circular bases that are connected by a curved surface.

A **prism** is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.

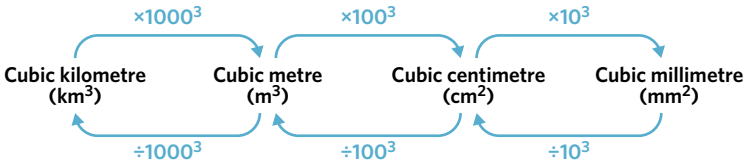
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: alexandre zveiger/Shutterstock.com

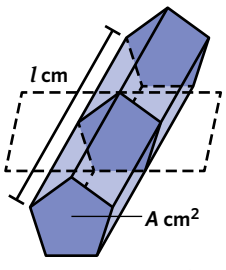
Calculating the volume or capacity of three dimensional shapes is an important skill used everyday. For example, determining the amount of water needed to fill a pool or energy required to heat a room.

Key ideas

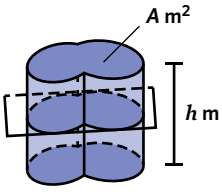
1. We can convert between units of measurement by multiplying powers of ten.



2. The volume of a shape with a uniform cross section is equal to the area of the cross section multiplied by the length, width or depth depending which is perpendicular to the cross section.



Volume = $l \times A$ cm³



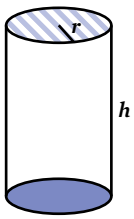
Volume = $h \times A$ m³

Continues →

3. The formula for the volume of cylinders and prisms includes the formula for the area of the cross section.

Cylinder

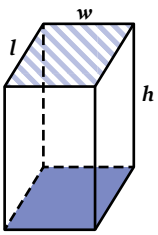
Area of cross section = πr^2



Volume = $h \times \pi r^2$

Rectangular prism

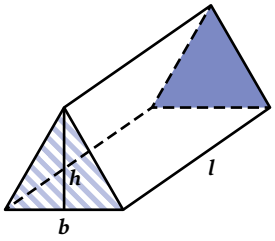
Area of cross section = $w \times l$



Volume = $l \times w \times h$

Triangular prism

Area of cross section = $\frac{b \times h}{2}$



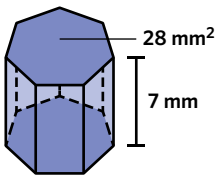
Volume = $l \times \frac{b \times h}{2}$

Worked example 1

Finding the volume of shapes with uniform cross-sections

Calculate the volumes of the following shapes.

a. WE1a



Working

Volume = area of cross section × height

= 28 × 7

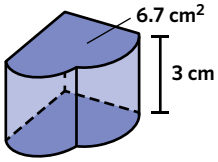
= 196 mm^3

Thinking

Step 1: Multiply the area of the cross section and the height to determine the volume.

Step 2: Give your answer with units.

b. WE1b



Working

Volume = area of cross section × height

= 6.7 × 3

= 20.1 cm^3

Thinking

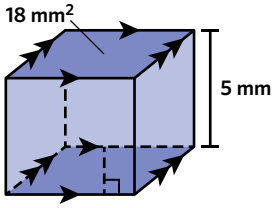
Step 1: Multiply the area of the cross section and the height to determine the volume.

Step 2: Give your answer with units.

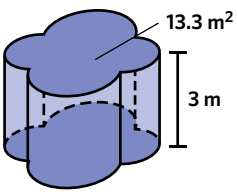
Student practice

Calculate the volumes of the following shapes.

a.



b.

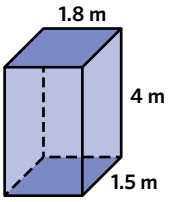


Worked example 2

Finding the volume of prisms and cylinders

Calculate the volumes of the following shapes, correct to 3 decimal places as needed.

a. WE2a



Working

Volume = area of cross section × height

$= 1.8 \times 1.5 \times 4$

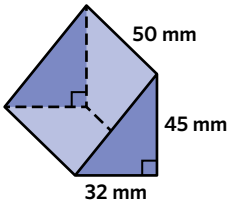
$= 10.8 \text{ m}^3$

Thinking

Step 1: Multiply the area of the cross section and the height to determine the volume.

Step 2: Give your answer with units.

b. WE2b



Working

Volume = area of cross section × length

$= \frac{32 \times 45}{2} \times 50$

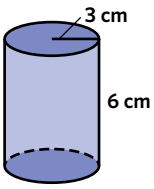
$= 36\,000 \text{ mm}^3$

Thinking

Step 1: Multiply the area of the cross section and the length to determine the volume.

Step 2: Give your answer with units.

c. WE2c



Working

Volume = area of cross section × height

$= \pi \times 3^2 \times 6$

$= 54\pi$

$\approx 169.646 \text{ cm}^3$

Thinking

Step 1: Multiply the area of the cross section and the height to determine the volume in exact form.

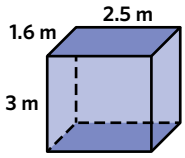
Step 2: Round to 3 decimal places and give your answer with units.

Continues →

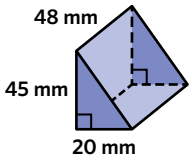
Student practice

Calculate the volumes of the following shapes, correct to 3 decimal places as needed.

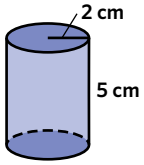
a.



b.



c.

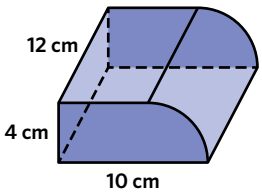


Worked example 3

Finding the volume of composite shapes

Calculate the volumes of the following shapes, correct to 3 decimal places as needed.

a.



WE3a

Working

Volume = area of cross section × width

= (rectangle + quadrant) × width

$= \left(4 \times 6 + \frac{\pi \times 4^2}{4}\right) \times 12$

$= (24 + 4\pi) \times 12$

$= 288 + 48\pi$

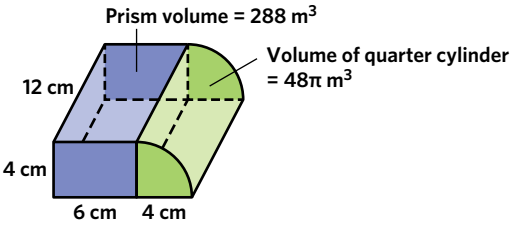
$\approx 438.796 \text{ cm}^3$

Thinking

Step 1: Multiply the area of the cross section and the width to determine the volume in exact form.

Step 2: Round to 3 decimal places and give your answer with units.

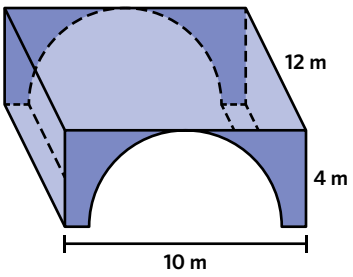
Visual support



Continues →

b.

WE3b



Working

Volume = area of cross section × length

= (rectangle − semicircle) × length

$= \left(4 \times 10 - \frac{\pi \times 4^2}{2}\right) \times 12$

$= (40 - 8\pi) \times 12$

$= 480 - 96\pi$

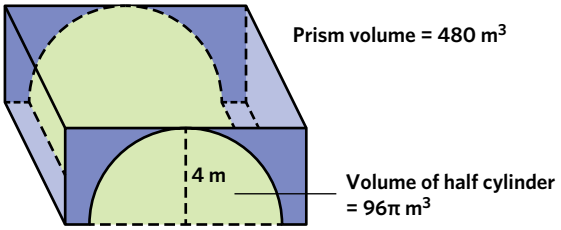
$\approx 178.407 \text{ m}^3$

Thinking

Step 1: Multiply the area of the cross section and the width to determine the volume in exact form.

Step 2: Round to 3 decimal places and give your answer with units.

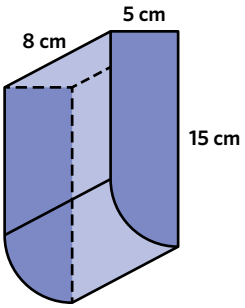
Visual support



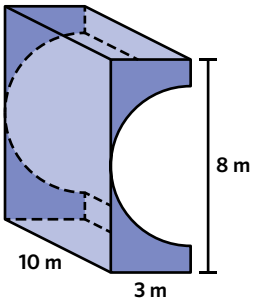
Student practice

Calculate the volumes of the following shapes, correct to 3 decimal places as needed.

a.



b.



12G Questions

Understanding worksheet

1. Complete the following conversions between units of volume.

Example

$0.0021\ km^3 =$ 2100 m^3

a. $35\ 000\ mm^3 =$ _____ cm^3

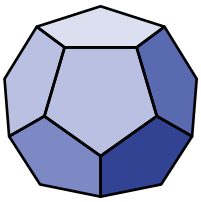
b. $0.04\ m^3 =$ _____ cm^3

c. $58\ 000\ m^3 =$ _____ km^3

d. $670\ 000\ mm^3 =$ _____ m^3

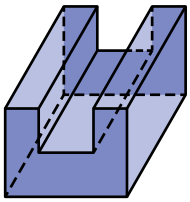
2. Do the following shapes have a consistent cross section?

Example



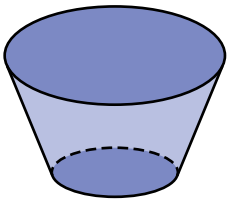
Yes (No)

a.



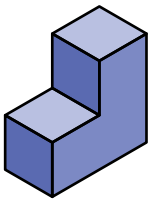
Yes No

b.



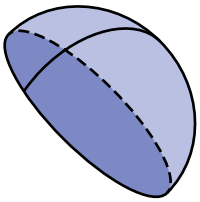
Yes No

c.



Yes No

d.



Yes No

3. Fill in the blanks by using the words provided.

prism | cylinder | volume | perpendicular

We can calculate the _____ of a 3D shape with a consistent cross section by finding the area of the cross section and multiplying it by the length that is _____ to the cross section.

A _____ is a 3D shape that has a polygon for the cross section. A _____ is a 3D shape with a circle for the cross section.

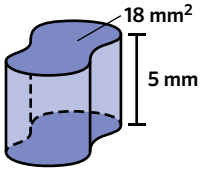
Fluency

Question working paths

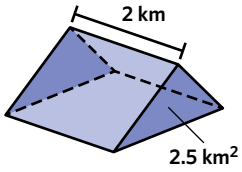
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8	4 (b,c,d,e), 5 (c,d,e,f), 6 (b,c,d,e), 7 (b,c,d,e), 8	4 (c,d,e,f), 5 (e,f,g,h), 6 (c,d,e,f), 7 (c,d,e,f), 8

4. Calculate the volumes of the following shapes. WE1

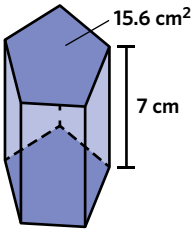
a.



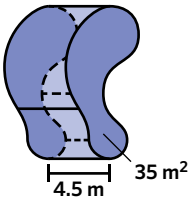
b.



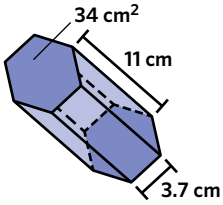
c.



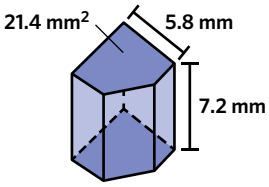
d.



e.

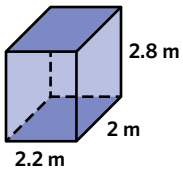


f.

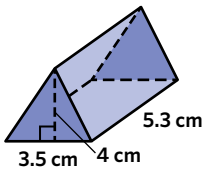


5. Calculate the volumes of the following shapes. WE2a,b

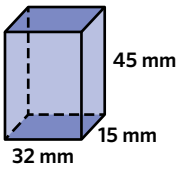
a.



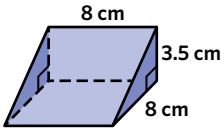
b.



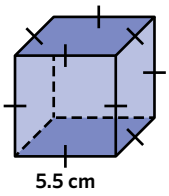
c.



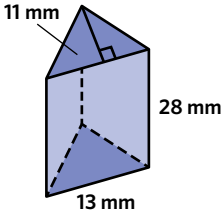
d.



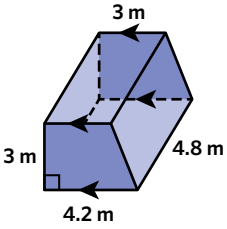
e.



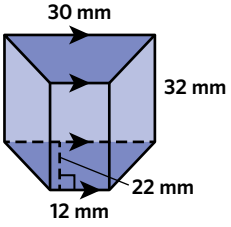
f.



g.

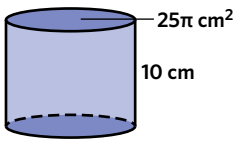


h.

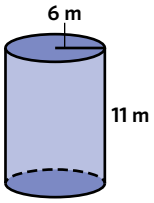


6. Calculate the volumes of the following shapes, correct to 3 decimal places. WE2c

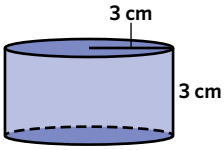
a.



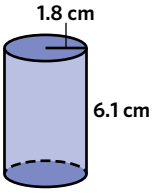
b.



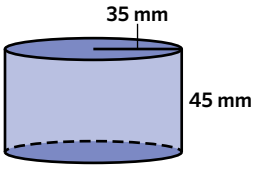
c.



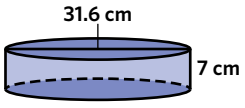
d.



e.

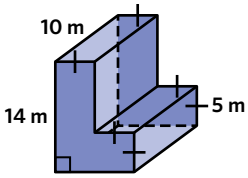


f.

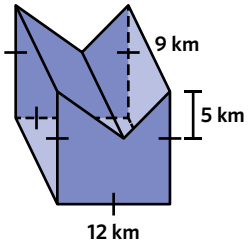


7. Calculate the volumes of the following shapes, correct to 3 decimal places as needed. WE3

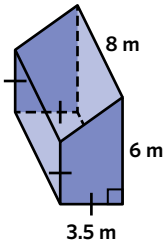
a.



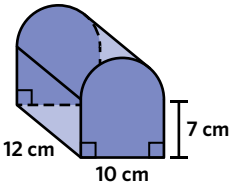
b.



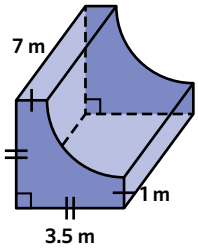
c.



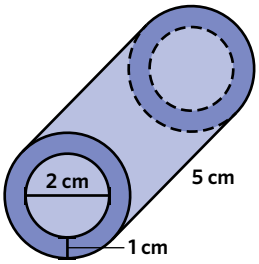
d.



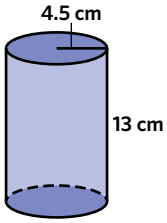
e.



f.



8. Calculate the volume of the following shape.

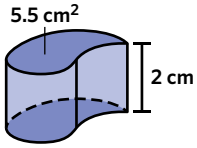


A. 117 cm^3
B. 263.25 cm^3
C. 367.566 cm^3
D. 827.024 cm^3
E. 1053 cm^3

Spot the mistake

9. Select whether Student A or Student B is incorrect.

a. Calculate the volume.



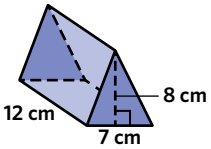
Student A

$5.5 \times 2 = 11\ cm^3$

Student B

$5.5^2 \times 2 = 60.5\ cm^3$

b. Calculate the volume.



Student A

$7 \times 12 \times 8 = 672\ cm^3$

Student B

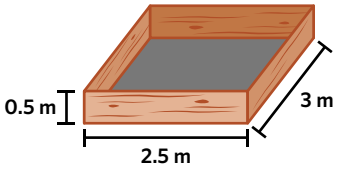
$\frac{7 \times 8}{2} \times 12 = 336\ cm^3$

Problem solving

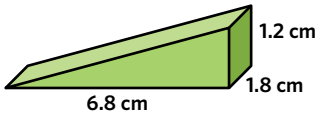
Question working paths

Mild 10, 11, 12	Medium 11, 12, 13	Spicy 12, 13, 14

10. Ali's garden bed has a width of 2.5 metres, a length of 3 metres and a height of 0.5 metres. What volume of soil can it hold?



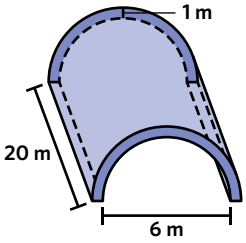
11. Charya is going to make 100 door stops for his school with recycled plastic. The door stops will be in the shape of a triangular prism with height of 1.2 cm, width of 1.8 cm and length of 6.8 cm. What volume of plastic will he need to make all 100 door stops?



12. Coryn needs to fill his water tank because it is completely empty. It is circular with a width of 3 metres and a height of 1.8 metres. A cubic metre is equivalent to 1000 litres. How much water does Coryn need to fill his tank, rounded to the nearest litre?

13. Melbourne covers an area of 9993 km^2. If 40 mm depth of rain fell in a day, what is the total volume of rain? Give your answer in m^3.

14. Enzo has been tasked with building a 20 m long tunnel that has a 6 m wide semicircle for a cross section. The tunnel is to be built using concrete that needs to be 1 m thick. How many cubic metres of concrete does Enzo need to build the tunnel, correct to 3 decimal places?



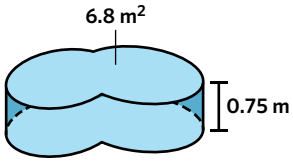
Reasoning

Question working paths

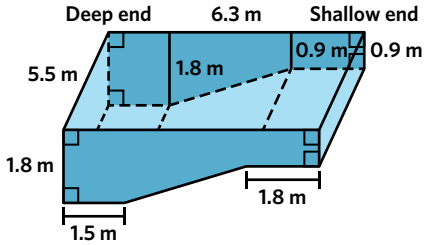
Mild 15 (a,b,c,e)	Medium 15 (a,b,c,e), 16 (a,b)	Spicy All

15. Leslie has built an inground pool and spa in her backyard for summer entertaining. She needs to calculate the volume of both the spa and pool to plan for the amount of water she will use.

a. Her spa is 0.75 m deep and has a surface area of 6.8 m^2. Calculate the volume of her spa.



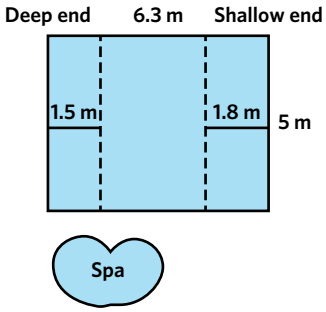
b. Her pool is 5.5 metres wide and 6.3 metres long. Calculate the volumes of the deep and shallow ends of the pool, using the given information.



c. Calculate the total volume of the pool by first finding the volume of the trapezoid prism between the shallow and deep ends of the pool.

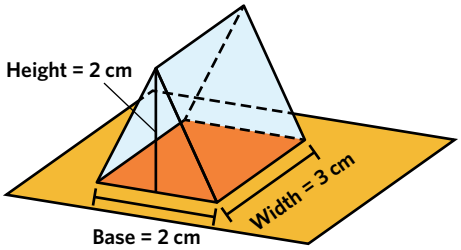
d. Evaporation and the filtering process causes 5% of the water to be lost each week. A litre of water is equivalent to 1000 cm^3. How many litres of water does Leslie need to replace each week?

e. Draw a top view of the pool and spa with any other features you would like to have around the pool to increase the fun Leslie and her friends can have.

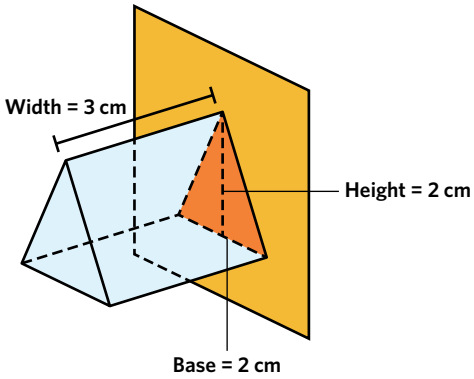


16. For the following.

a. Find the area of the rectangular cross section by multiplying the base and width. Multiply the area by the height to find the volume of the triangular prism.



b. Use the height and base measurements to find the area of the triangular cross section. Multiply the area by the width to find the volume of the triangular prism.



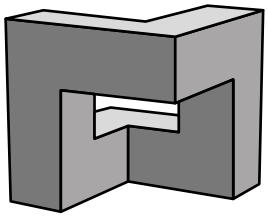
c. Explain why the cross section in part **b** gives the correct volume of the triangular prism and why the cross section in part **a** does not.

Extra spicy

17. Box A has a width of 10 cm, a length of 12 cm and a height of 9 cm. Box B has a width of 5 cm and length of 4 cm. What must the height of Box B be to have the same volume as Box A?

18. A 1 m × 1 m × 1 m cube has three $\frac{1}{3}$ m × $\frac{1}{3}$ m × 1 m rectangular prisms removed from it, as demonstrated in the diagram. What is the volume of the remaining shape?

19. The sculpture Cubo Vazado by Franz Weissmann is made by removing cubical blocks from a solid cube to leave the symmetrical shape shown. If the edges of the sculpture have lengths of 1 m, 2 m or 3 m, what is the volume of the sculpture?



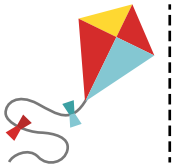
A. 9 m^3 **B.** 11 m^3 **C.** 12 m^3 **D.** 14 m^3 **E.** 18 m^3

20. Yareli's water container is 80% full. After she removes 20% of the water in her container she has 1152 ml of water left. What volume of water does Yareli's water container hold when it is full?

A. 1400 ml **B.** 1600 ml **C.** 1700 ml **D.** 1800 ml **E.** 2000 ml

Remember this?

21. The following image is reflected over the dotted line. It is then rotated 180 degrees anticlockwise. Which of the following shows the final position of the image?



A.

B.

C.

D.

E.

22. During flu season, the fraction of children across a school district who have been exposed to the flu is $\frac{130}{1000}$. What percentage of students have not been exposed?

A. 0.13% **B.** 0.87% **C.** 13% **D.** 87% **E.** $\frac{130}{1000}$%

23. Ben is helping his younger sister learn her times tables. Every week he tests her on all of her times tables.

At first, she got 30 correct answers.

After two weeks, she got three times as many correct answers.

Two weeks after that, Ben tested her when she was tired so she scored half of what she scored on the previous test.

Which number sentence represents the number of correct answers on the third test?

A. $30 \div 3 \times \frac{1}{2} = 5$

B. $30 \div 3 \div \frac{1}{2} = 20$

C. $30 + 3 - \frac{1}{2} = 32\frac{1}{2}$

D. $30 \times 3 \times \frac{1}{2} = 45$

E. $30 \times 3 \div \frac{1}{2} = 180$

Back to contents

12H Time

Time can be measured in different lengths of duration and expressed with a 12-hour or 24-hour clock. As the world becomes more and more connected the different time of day in different locations needs to be taken into account.

LEARNING INTENTIONS

Students will be able to:

- convert between units of time
- convert between 24-hour and 12-hour time
- calculate duration using 12-hour and 24-hour time
- convert times between different time zones across Australia or the world.

KEY TERMS AND DEFINITIONS

Coordinated Universal Time or **UTC** is the standard by which the world regulates clocks and time.

Daylight saving time is a practice that occurs in some states of Australia where clocks are advanced 1 hour to increase the amount of daylight after work in summer months.

Duration is the time that passes between two moments.

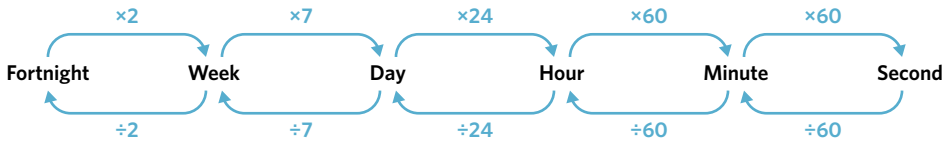
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Undrey/Shutterstock.com

Time is part of the planning process for events or travel. International travel planning requires the traveller to consider the time the journey takes and what timezone they are travelling to.

Key ideas

1. Different units of time can be converted into other units of time.



Some units of time have a more complicated relationship.

A month can be 28, 29, 30 or 31 days.

1 year is 365 days, except for a leap year which has 366 days.

1 year $\approx$ 52 weeks.

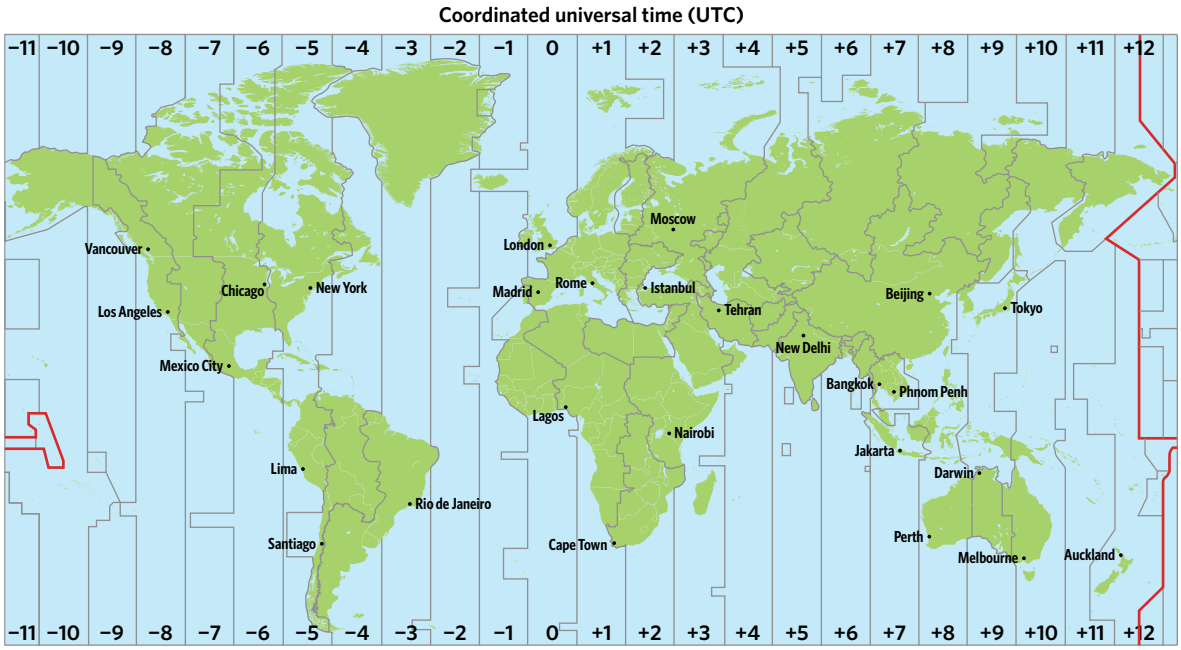
2. The time of the day can be expressed in 12-hour and 24-hour time. In 24-hour time the first two digits are counting the hours after midnight and the second two digits count the minutes after the hour.

12 hour time	12 am (midnight)	12:30 am	9:15 am	10:55 am	12 pm (midday)	1:24 pm	7:08 pm	8:05 pm	11:59 pm
24 hour time	0000	0030	0915	1055	1200	1324	19 08	2005	2359

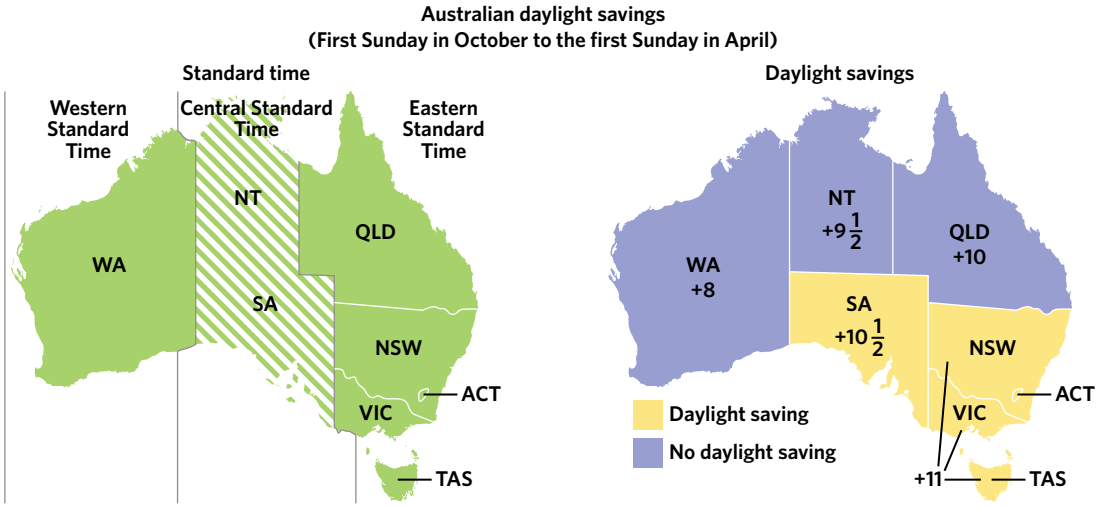
19 hours after midnight. 8 minutes after the hour.

Continues →

3. The world is separated into different time zones to account for the rotation of the earth. Coordinated Universal Time or UTC, is the time in Greenwich UK. Anywhere else in the world is either somewhere between 12 hours earlier or 12 hours later.



Each region is in a particular time zone and can also have daylight savings. In Australia, we have 3 different time zones and different states and territories start daylight saving time on different days of the year; some do not have daylight saving time at all.



Worked example 1

Converting between different units of time

Convert.

a. $5\frac{1}{2}$ hours to minutes. WE1a

Working

$5\frac{1}{2}$ hours $= 5\frac{1}{2} \times 60$ minutes

$= 330$ minutes

Thinking

Determine the multiplication or division required.

1 hour $= 60$ minutes

Continues →

b. 50 months to years. WE1b

Working

$50 \text{ months} = \frac{50}{12} \text{ years}$

$= \frac{25}{6} \text{ years}$

$= 4\frac{1}{6} \text{ years}$

Thinking

Determine the multiplication or division required.

12 months = 1 year

Student practice

Convert.

a. $3\frac{1}{2}$ hours to minutes.

b. 42 months to years.

Worked example 2

Convert between 24-hour and 12-hour time

Convert.

a. 9:23 am to 24-hour time. WE2a

Working

9 am = hour 09

9:23 am = 0923

Thinking

Convert the hour and write the time.

b. 1438 to 12-hour time. WE2b

Working

hour 14 = 2 pm

1438 = 2:38 pm

Thinking

Convert the hour and write the time.

Student practice

Convert.

a. 6:45 am to 24-hour time.

b. 1615 to 12-hour time.

Worked example 3

International time of the day

Determine the time in the following locations.

a. Moscow when UTC is 8:30 am. WE3a

Working

Moscow time = UTC +3

8:30 am + 3 hours = 11:30 am

Thinking

Determine the difference between Moscow time and UTC. Add the hours to UTC.

Continues →

b. Mexico City when the time in Rome is 5:20 pm. WE3b

Working

Mexico City time = UTC −6

Rome time = UTC +1

Mexico City time = Rome time − 7 hours

5:20 pm − 7 hours = 10:20 am

Thinking

Step 1: Determine the difference in time between both locations.

Step 2: Add or subtract the difference in time.

Student practice

Determine the time in the following locations.

a. Beijing when UTC is 10:20 am.

b. Nairobi when the time in Bangkok is 3:30 pm.

Worked example 4

Duration of time

Determine the duration between the given times.

a. 6:40 am and 11:30 am. WE4a

Working

11 hours − 6 hours = 5 hours

30 min − 40 min = −10 min

5 hours − 10 min = 4 hours 50 minutes

Thinking

Step 1: Subtract the hours and minutes of start time from the hours and minutes of end time.

Step 2: Add or subtract the minutes to find the duration.

b. 1625 Monday and 0910 Tuesday. WE4b

Working

24 − 16 = 8 hours

0 − 25 = −25 minutes

8 hours − 25 min = 7 hours 35 minutes.

7 + 9 = 16 hours

35 + 10 = 45 minutes

16 hours 45 minutes

Thinking

Step 1: Taking into account there are 24 hours in a day, calculate the difference in time between start time and end of the same day.

Step 2: Add the duration of the second day.

Student practice

Determine the duration between the given times.

a. 8:40 am and 10:10 am.

b. 1930 Sunday and 0835 Monday.

12H Questions

Understanding worksheet

1. Colour in the digital clock so it shows the same time as the analogue clock.

Example


a.


b.


c.


d.


2. Draw the hands of the analogue clock so that it shows the same time as the digital clock.

Example


a.


b.


c.


d.


3. Fill in the blanks by using the words provided.

24-hour | 12-hour | daylight saving | UTC

Time of the day is recorded in both _____ time, where am and pm are used to indicate if it is before or after midday, and _____ time. The time of the day is influenced by location due to the Earth rotating on its axis. Clocks around the world are synchronised using Coordinated Universal Time (_____). Another factor to take into account is the practice of _____, moving the clock 1 hour forward to increase the amount of sunlight after work.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Convert. WE1

a. 2 hours to minutes.

b. $1\frac{1}{2}$ days to hours.

c. $3\frac{3}{4}$ years to months.

d. 1 week to hours.

e. 45 minutes to seconds.

f. 585 minutes to hours and minutes.

g. $3\frac{13}{36}$ days to hours.

h. $6\frac{17}{24}$ years to months.

5. Convert. WE2

a. 12:30 pm to 24-hour time.

b. 1315 to 12-hour time.

c. 7:20 am to 24-hour time.

d. 12:18 am to 24-hour time.

e. 2323 to 12-hour time.

f. 5:55 pm to 24-hour time.

g. 0921 to 12-hour time.

h. 4:16 am to 24-hour time.

6. Determine the time in the following locations. WE3a

a. Melbourne when UTC is 2:00 am.

b. Madrid when UTC is 1:30 pm.

c. Lima when UTC is 10:40 am.

d. New Delhi (UTC +5:30) when UTC is 3:15 pm.

e. Cape Town when UTC is 11:45 pm.

f. Adelaide when UTC is 4:55 pm.

g. Daylight saving time in Sydney when UTC is 11:38 am.

h. Los Angeles when UTC time is 7:12 am.

7. Determine the time in the following locations. WE3b

a. Tokyo when it is 4:30 pm in Beijing.

b. Perth when it is 3:00 pm in Sydney.

c. Rio de Janeiro when it is 10:40 am in Mexico City.

d. Brisbane when it is 5:15 pm in daylight saving time in Adelaide.

e. Darwin when it is 9:30 am in New Delhi.

f. London when it is 5:00 am in Jakarta.

g. Auckland when it is 6:35 pm in Los Angeles.

h. Vancouver when it is 11:43 am daylight saving time in Melbourne.

8. Determine the duration between the given times. WE4

a. 3:00 pm and 5:30 pm.
b. 9:45 am and 11:10 am.
c. 1650 and 1738.
d. 10:27 am and 1:35 pm.
e. 12:58 am and 4:12 pm.
f. 11:23 pm Monday and 2:28 am Tuesday.
g. 8:50 pm Friday and 6:45 pm Saturday.
h. Wednesday 1:35 pm and Friday 10:45 am.

---

9. Determine the time in Sydney when it is 8:30 am in Tehran (UTC +3:30).

A. 2:00 am **B.** 2:30 am **C.** 8:30 am **D.** 2:30 pm **E.** 3:00 pm

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Convert 252 minutes to hours.

Student A

$252 \div 60 = 4.2$

254 minutes = 4 hours and 20 minutes

Student B

$4 \times 60 = 240$ and

$252 = 240 + 12$

254 minutes = 4 hours and 12 minutes

b. How much time passes between 12:40 am and 1:30 pm?

Student A

12:40 to 1:40 is 1 hour

1:30 is 10 min earlier than 1:40

Duration = 50 min

Student B

12:40 am to 1:40 pm is 13 hours

1:30 is 10 min earlier than 1:40

Duration = 12 hours and 50 min

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Patrick has gone for a jog at lunchtime. He started running at 12:45 pm and finished running at 1:25 pm. How long did he run for?

12. Vutha is visiting family in Phnom Penh but will continue to work remotely for his company in Brisbane. If he's scheduled to have a meeting with Tori at 10:30 am Brisbane time, what time will this be in Phnom Penh?

13. Perth is in the UTC +8 time zone and Sydney is in the UTC +10 time zone. Perth doesn't use daylight saving time, but Sydney does. Play in the test match in Perth is scheduled to start at 10:30 pm. Richie lives in Sydney and wants to watch the start of play. What time will it be in Sydney when play starts?

14. Rebecca's flight to Sydney leaves Los Angeles at 5:20 pm on Thursday. The total duration of the flight is 15 hrs and 55 mins. What day and time does she arrive in Sydney?

15. Kathy is going to meet with six of her students at lunchtime today. Southey High School's lunchtime is from 12:18 pm to 1:20 pm. If she plans to have a 2-minute gap between each meeting, how much time will she have per student in minutes and seconds if each student has an equal share of her time?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Matthew lives in Melbourne which is in the time zone UTC +10 but has two brothers who live overseas. Andrew lives in London which is UTC +0 and Dylan lives on Vancouver Island which is UTC −8. All of the brothers work from 9:00 am to 5:00 pm Monday to Friday and sleep between 11:00 pm and 7:00 am the following day, every night of the week.

a. What day of the week and what time is it on Vancouver Island and in London when Matthew goes to bed on Friday night and wakes up on Saturday morning?

b. What day of the week and what time is it on Vancouver Island and in Melbourne when Andrew starts his work week Monday morning and finishes his work week on Friday night?

c. What day of the week and what time is it in London and Melbourne when Dylan wakes up on Saturday morning and goes to bed on Sunday night?

d. Matthew wants to have a video call with both of his brothers on Sunday. What time of the day in Melbourne could they schedule the call?

e. What are some of the greatest benefits of people being able to video call anyone across the world?

17. The time that the Earth orbits around the Sun is called a year. The distance that the Earth travels in a single orbit around the Sun is approximately 939 950 000 km. Scientists have measured the speed the Earth is orbiting the Sun at approximately 107 225 km every hour.

a. Calculate the distance the Earth travels in one day.

b. Calculate the distance the Earth travels around the Sun in 365 days.

c. Compare your answer in part **b** to the total distance the Earth orbits around the Sun to explain why leap years are necessary.

Extra spicy

18. What fraction of a day is spent at school if school starts at 8:30 am and finishes at 3:15 pm?

A. $\frac{25}{96}$ **B.** $\frac{9}{32}$ **C.** $\frac{31}{96}$ **D.** $\frac{13}{48}$ **E.** $\frac{18}{32}$

19. A train was scheduled to leave the station at 1740 and scheduled to arrive at its destination at 1820. It arrived 5 minutes late to the station and the journey then took 42 minutes. What time did it arrive at its destination?

20. The following clock shows the time in the morning. The current time can be calculated by rotating the hour hand 108°. What is the current time?

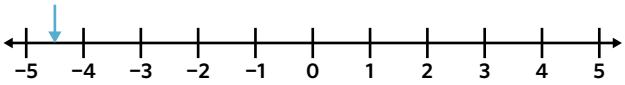
21. In 2010 John Isner and Nicolas Mahut set the record for the longest tennis match in history while playing at Wimbledon. The duration of the match was 11 hours and 5 minutes. The fifth set lasted 8 hours and 11 minutes. Approximately what fraction of the whole match was taken up by the fifth set?

A. $\frac{1}{5}$ **B.** $\frac{2}{5}$ **C.** $\frac{3}{5}$ **D.** $\frac{3}{4}$ **E.** $\frac{9}{10}$

Remember this?

22. What number is at the position the arrow is pointing to?

A. −5.5
B. −4
C. −4.5
D. −3
E. −3.5



23. A square has an area of 81 m^2. What is its side length?

A. 7 m **B.** 8 m **C.** 9 m **D.** 10 m **E.** 11 m

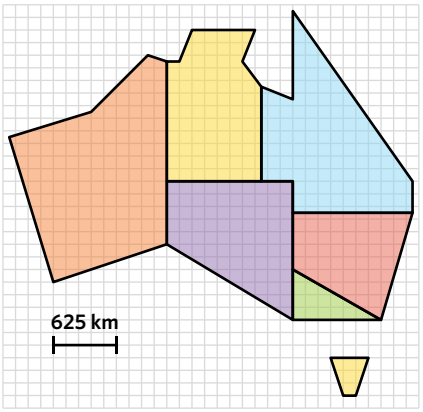
24. Numbers such as 1, 3, 5 and 13 are called prime numbers.

Which of the following statements is always true about prime numbers?

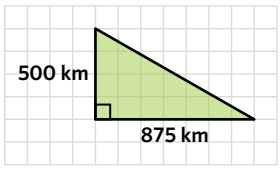
A. A prime number multiplied by a prime number is a prime number.
B. A prime number divided by a prime number is a prime number.
C. Fractions are prime numbers since their only factors are 1 and itself.
D. Prime numbers are always odd.
E. The next prime year is 2027.

Chapter 12 extended application

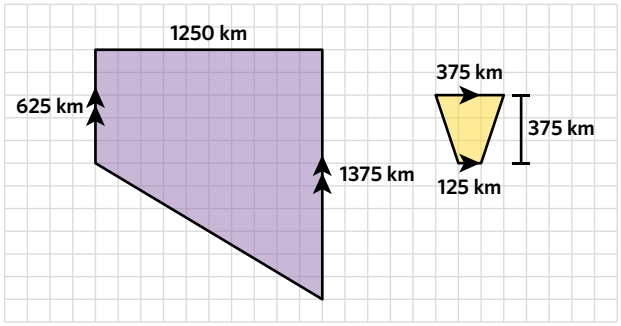
1. To the right is an estimated map of Australia. The grid lines in both directions are scaled to represent a distance of 125 km. This means that each square grid is equal to 125^2 km^2 or 15 625 km^2.



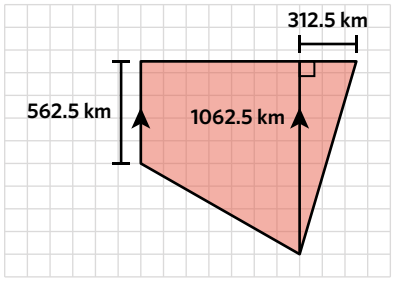
a. Use the right-angle triangle representation of Victoria to estimate the area included in Victoria.



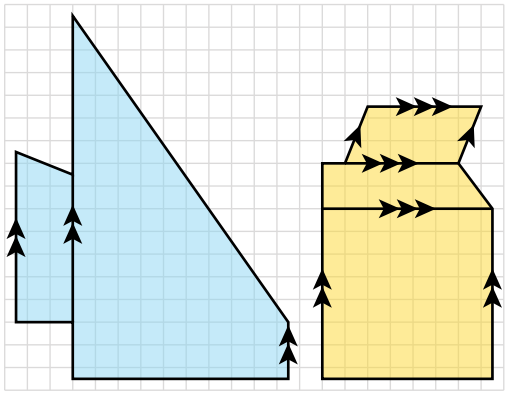
b. South Australia and Tasmania both form trapeziums. Determine the area covered by them both.



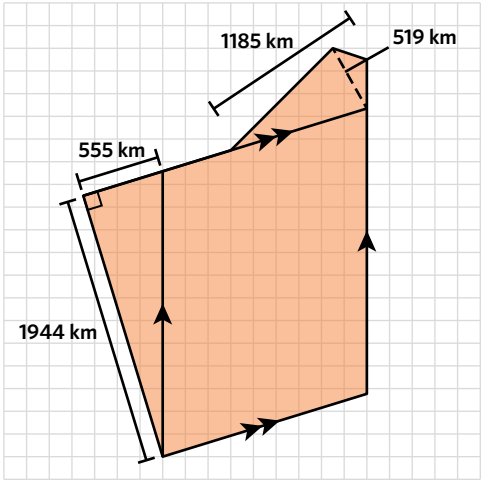
c. NSW and the ACT combined form a composite shape. Notice that the borders here do not sit exactly on a grid line. Use the measurements given and the grid lines to estimate the area covered by NSW and ACT.



d. Calculate the area of the composite shapes that form Queensland and Northern Territory. If a border doesn't sit on the grid lines, assume that they are in the middle of them.



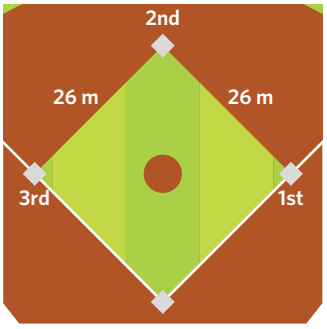
e. To determine the total land area of Australia, first calculate the area of the composite shape below that forms Western Australia. Combine the estimated areas of all states to calculate an estimate for the total area of Australia.



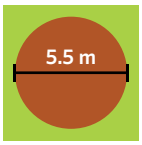
f. Which states in Australia have you travelled to and which one is your favourite?

2. Patrick is building an indoor baseball stadium. The playing surface will be constructed with synthetic grass and dirt. Before he can establish what the cost to build and maintain the playing surface will be, he needs to determine the total area, the area of synthetic grass, and the area of dirt.

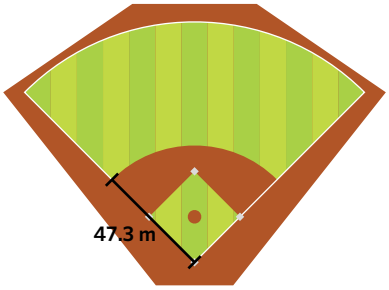
a. Calculate the area of the square that is created by straight lines between the bases.



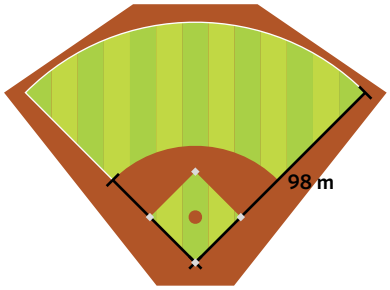
b. The pitcher's mound is a circle with a width of 5.5 m. Calculate the area of this circle, accurate to 3 decimal places.



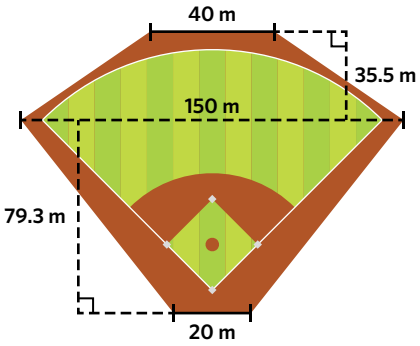
c. The infield is all of the playing area inside a 90° sector with a radius of 47.3 m. Calculate the area of the infield, accurate to 3 decimal places.



d. The infield and outfield is contained in a 90° sector with a radius of 98 m. Calculate this area, accurate to 3 decimal places.



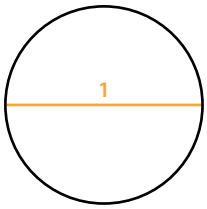
e. The total playing surface is defined as a composite shape of two trapeziums. Calculate their area and then determine the total area of synthetic grass and the total area of dirt.



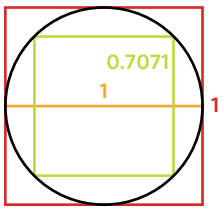
f. What are the positives and negatives of building indoor stadiums?

3. π is an irrational number. Mathematicians, scientists and engineers find ways to approximate π to as many decimal places as possible. Archimedes used a method that was based on the realisation that increasing the number of sides of a polygon causes it to become more like a circle.

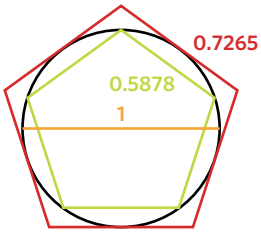
a. A circle with a diameter of 1 has a circumference of π. Use a calculator to find the value of π and round it to 5 decimal places.



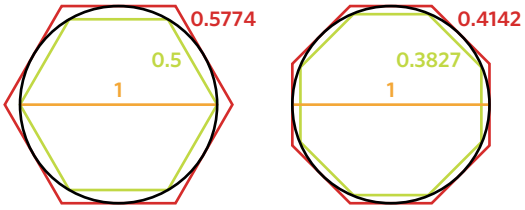
b. Notice the diagram. It shows the largest square that fits inside the circle and the smallest square that fits outside the circle. Determine the perimeters of both squares and compare them to the circumference of the circle.



c. This diagram shows the largest pentagon that fits inside a circle and the smallest pentagon that fits outside the circle. Determine the perimeters of both pentagons and compare them to the circumference of the circle.



d. This diagram continues the pattern of polygons from parts **a** and **b** with hexagons and octagons. Determine the perimeters of the hexagons and octagons and compare them with the circumference of the circle.



e. The table below shows a list of polygons with different numbers of sides. The 'inside' and 'outside' measurements refer to the side length of the largest and smallest polygon that fits inside and outside of the circle, as in parts **b** to **d** above. Determine the perimeters of the polygons and compare them with the circumference of the circle.

Sides	10	20	30	40	50	60	70	80
Inside	0.30902	0.15643	0.10453	0.07846	0.06279	0.05234	0.04486	0.03926
Outside	0.32492	0.15838	0.10510	0.07870	0.06291	0.05241	0.04491	0.03929

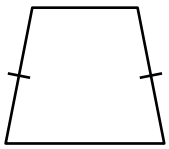
f. How many decimal places are accurately calculated with an 80-sided polygon? Is this a better estimate than Archimedes' $\frac{22}{7}$ estimation?

Chapter 12 review

Multiple choice

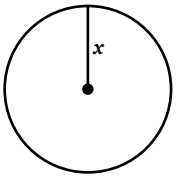
1. How many numbers of measurements are needed in order to find the perimeter of the following shape? 12A

A. 1
B. 2
C. 3
D. 4
E. 5



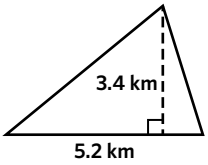
2. What is the length of the radius and diameter of the following circle? 12B

A. radius $= x$, diameter $= x$
B. radius $= 2x$, diameter $= x$
C. radius $= x$, diameter $= 2x$
D. radius $= 2x$, diameter $= 2x$
E. radius $= 3x$, diameter $= 3x$



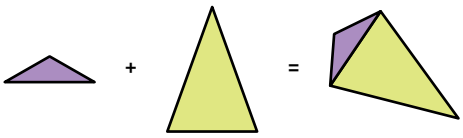
3. What is the base and height of the following triangle? 12C

A. base $= 5.2$ km, height $= 5.2$ km
B. base $= 5.2$ m, height $= 3.4$ m
C. base $= 3.4$ km, height $= 5.2$ km
D. base $= 3.4$ m, height $= 3.4$ m
E. base $= 5.2$ km, height $= 3.4$ km



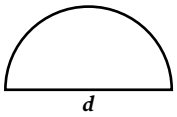
4. What shape could be created by transforming and joining the following shapes together? 12D

A. Rectangle
B. Triangle
C. Rhombus
D. Kite
E. Trapezium



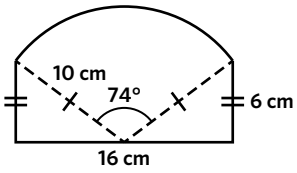
5. What is the correct value of the unknown diameter to two decimal places if the area of this shape is $A = 22.7$ m^2? 12E

A. 3.60 m
B. 3.80 m
C. 4.77 m
D. 7.60 m
E. 14.45 m

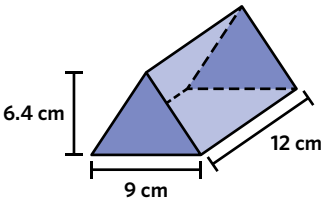


6. What is the correct perimeter for this shape to two decimal places? 12F

A. 33.30 cm
B. 40.92 cm
C. 48.21 cm
D. 52.66 cm
E. 60.82 cm



7. What is the correct volume for the following shape? 12G



A. 150.8 cm^3
B. 194.8 cm^3
C. 257.6 cm^3
D. 345.6 cm^3
E. 583.2 cm^3

8. Which of the following analogue clocks correctly matches the digital? 12H

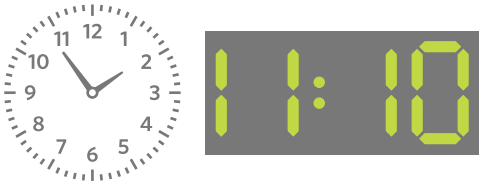
A.

B.

C.

D.

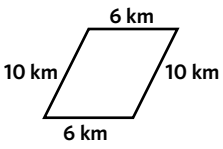
E.



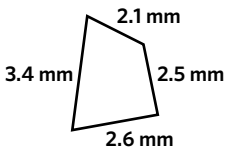
Fluency

9. Determine the perimeters of the following shapes. 12A

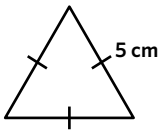
a.



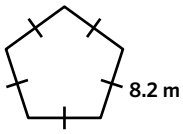
b.



c.



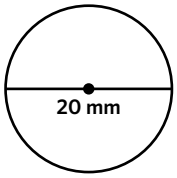
d.



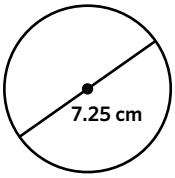
10. Calculate the circumference of the following circles: 12B

i. in terms of π
ii. correct to 3 decimal places.

a.

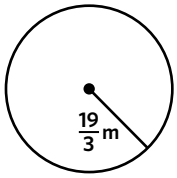


b.



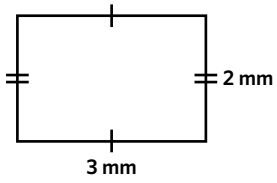
c.

d.

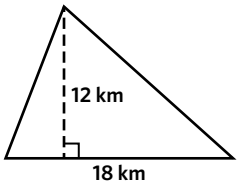


11. Determine the area of the following shapes. 12C

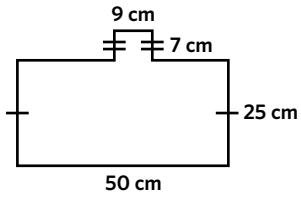
a.



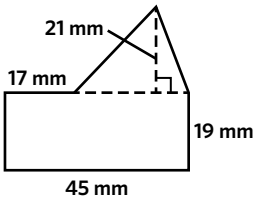
b.



c.

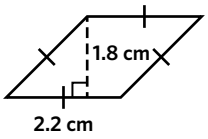


d.

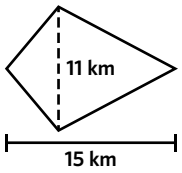


12. Determine the area of the following shapes. 12D

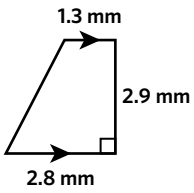
a.



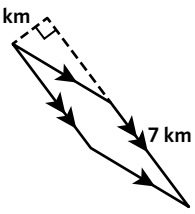
b.



c.



d.

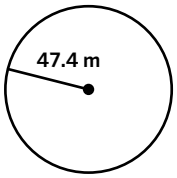


13. Determine the areas of the following shapes: 12E

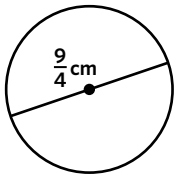
i. in terms of π

ii. correct to 3 decimal places.

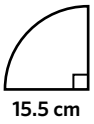
a.



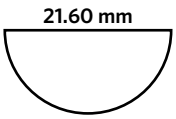
b.



c.

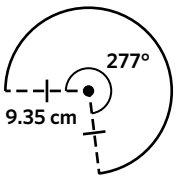


d.

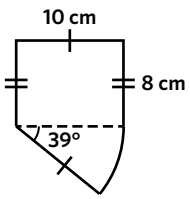


14. Calculate, and round your answer to 3 decimal places. 12F

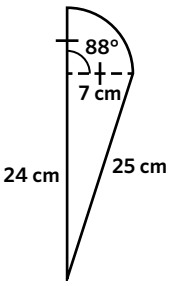
a. The perimeter of:



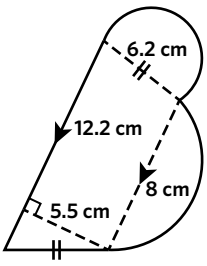
b. The area of:



c. The perimeter of:

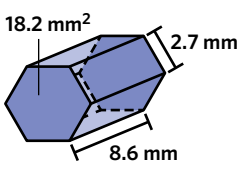


d. The area of:

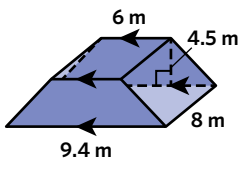


15. Calculate the volumes of the following shapes, correct to 3 decimal places. 12G

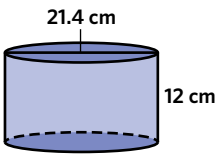
a.



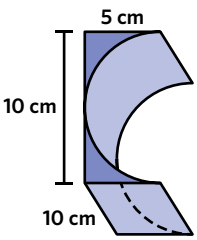
b.



c.



d.



16. Convert or determine the time in the following locations. 12H

a. Convert $4\frac{5}{16}$ days to hours.

b. Convert 21:13 to 12-hour time.

c. Determine the time in Los Angeles when UTC is 8:35 am.

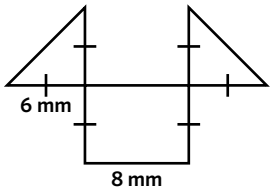
d. Determine the time in Vancouver when it is 5:00 am in New York.

Problem solving

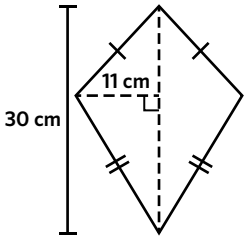
17. Victoria wants to build a square-shaped wooden fence around her garden. The perimeter of her garden is 5 m. What is the length of each side of the garden in cm? 12A

18. Quin has a hula hoop that has a circumference of 1.2 m. Using an estimated value of $\pi \approx \frac{22}{7}$, what is the radius of Quin's hula hoop? 12B

19. Sheridan's favourite clothing brand is Cathnandu. Cathnandu's logo is in the following shape. What is the area of this logo on Sheridan's Cathnandu jacket? 12C

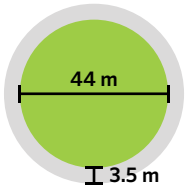


20. Jess wants to give her friend a kite-shaped box filled with sweets as a gift for her birthday. The top view of the box is shown in the diagram below. She also wishes to cover the top of the box with photos of them, where each photo will be 3 cm by 5 cm. How many photos will she be able to fit on the top of the box? 12D



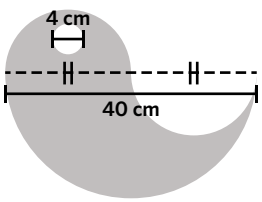
21. A circular garden bed at a popular tourist attraction has a diameter of 44 metres and is surrounded by a dog walking path of 3.5 metres. 12E

In order to protect dogs' paws from the heat of the pavement during summer, the city council has decided to paint the path with a reflective grey material in order to reduce road surface temperatures. One can of this paint will cover approximately 60 m^2. How many cans of paint are needed to paint the path above? Round your answer to the nearest can.

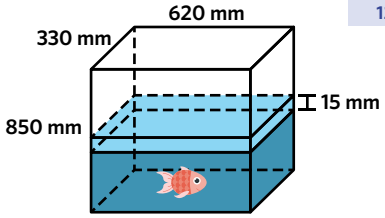


22. Alea is President for a student society and wishes to design a logo for the club and then stitch it onto members' uniforms. The logo is shown below. 12F

At the local craft store, thread costs \$0.50 per cm^2. By calculating the area of the logo, determine how much the thread will cost for Alea to two decimal places.



23. Malia is conducting a science experiment on displacement of water and volumes of objects. She learns that the volume of displaced liquid should approximately equal the volume of the object that is placed. When Malia puts a small fish into a rectangular fish tank (shown below), she notices that the water level rises by 15 mm. What is the volume of the fish she put in? Express your answer in cm^3.



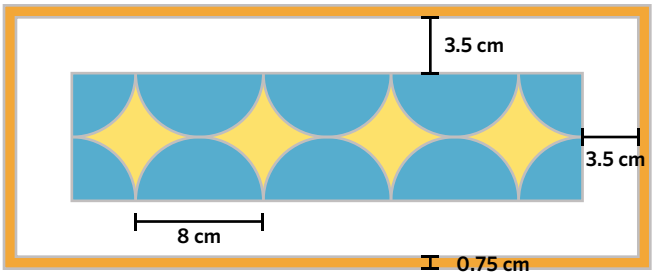
12G

24. Will's journey from Singapore to Melbourne has two stopovers in Kuala Lumpur and Bali, with the whole journey taking 34 hours and 30 mins. If Will leaves Singapore at 9:15 pm on Thursday, what would the local time and day be in Melbourne when he arrives? Singapore time is UTC +8 while Melbourne time is UTC +11.

12H

Reasoning

25. Ryan wishes to paint the following image from a painting kit he has purchased and frame it onto his bedroom wall. The reference image in the kit has the following given dimensions, with an outer frame of width 0.75 cm. Each of the semicircles are exactly equal.



Ryan measures his room wall and finds that he has enough space for a painting 75 cm long and 12 cm wide. Ryan is still unsure whether he has enough wall space and paint, he wishes to find more exact dimensions of the image.

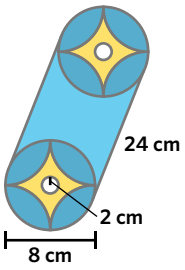
a. Calculate the area of the available wall space Ryan has.

b. Determine the total area of blue in the painting given that all the semicircles and quadrants have the same radius. Express your answer in exact form in terms of π.

c. Ryan does some research and finds that wall art should cover 60%–75% of available wall space. By finding the total area of the painting, determine whether Ryan has enough wall space.

d. The frame and canvas cost $34.95 and $22.50 respectively. The blue paint costs $2.35 per cm^2 while the yellow paint costs $1.20 per cm^2. How much will the painting cost Ryan in total? Write your answer to 2 decimal places.

e. Ryan likes the design of the painting so much that he decides to use it to make a cylindrical kaleidoscope. He also cuts out a circle of radius 2 cm in the front and back face for viewing purposes. The kaleidoscope is shown below.

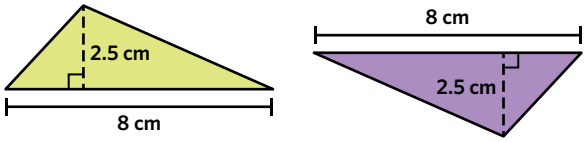


Find the volume of the cylindrical kaleidoscope, correct to 2 decimal places.

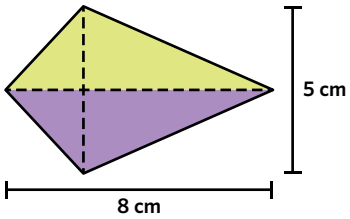
f. What are some different materials that kaleidoscopes could be made out of?

26. For the following.

a. Calculate the areas of the triangles below and add them together.

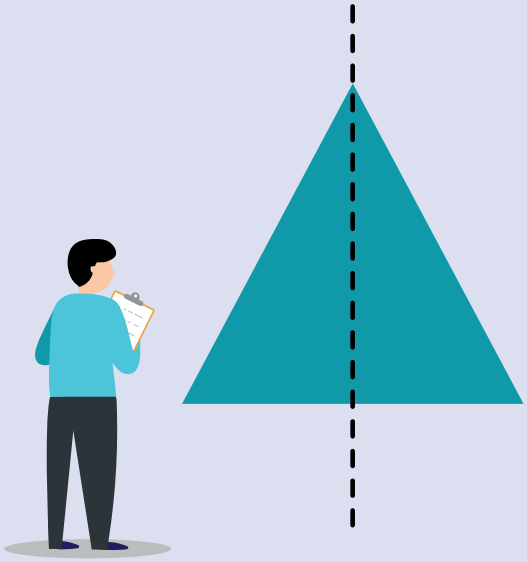


b. Calculate the area of the kite.



c. Use your answers in parts **a** and **b** to explain why the formula for the area of a kite is $\frac{d_1 \times d_2}{2}$.

Back to contents



13

Pythagoras' theorem

Measurement and Geometry

Research summary

13A Introduction to Pythagoras *(Extension)*

13B Calculating the hypotenuse *(Extension)*

13C Calculating the shorter side *(Extension)*

Chapter 13 extended application

Chapter 13 review

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Back to contents

Chapter 13 research summary

Pythagoras' theorem

Big ideas

Generalising

This chapter will focus on combining multiplicative thinking and proportional reasoning skills with building conceptual and procedural knowledge of Pythagoras' theorem and applying the Pythagoras' formula to special right-angled triangles such as the $30° - 60° - 90°$ and the $45° - 45° - 90°$. In this chapter, students must arrange familiar and foreign mathematical symbols to work with various numbers and manoeuvre properties to conclude evidence or assumptions about right-angled triangles.

This chapter will ask students to solve and rearrange Pythagoras' formula, $a^2 + b^2 = c^2$, where c is the hypotenuse of a right-angled triangle. While showing students the other forms of this formula is quicker, students can flex and build their algebraic thinking and reduce misconceptions about the formula. For example, have students use the formula $a^2 + b^2 = c^2$ to determine the shorter sides of the triangle.

The success of this chapter is underpinned by students' conceptual understanding of variables. For example, the formula Pythagoras' formula, $a^2 + b^2 = c^2$, where c is the hypotenuse of a right-angled triangle does not mean that variable c is always the hypotenuse for all right-angled triangles.

Using the phrase 'cancelling' or 'goes away' may cause misconceptions in algebra and when solving equations. So instead, students and teachers are encouraged to use phrases such as 'sums to zero' and 'simplifies to 1'. Replacing the word cancel with mathematical language will help strengthen students' connection to number properties in algebra and avoid confusion. For example, $3^2 - 3^2$ sums to zero and $\frac{4c^2}{4c^2}$ simplifies to 1.

This big idea is built upon a strong understanding of number, number properties, pattern, and equality. Therefore, a conceptual understanding of variables and the relationship of words, equations, number, number properties, equality, and algebra is pivotal for future success in mathematics.

Hierarchy and properties

This chapter will focus on the properties of right-angled triangles in two-dimensions and three-dimensions. Additionally, in this chapter, students will conclude and apply assumptions about right-angled triangles to special right-angled triangles such as the $30° - 60° - 90°$ and the $45° - 45° - 90°$, primitive Pythagorean triads, and Pythagorean triples.

Students should be offered the opportunity to experience new mathematical concepts, theorems and techniques to discover and solve Pythagorean theorems so that they may be able to apply and operate at a high cognitive level (Wittmann, 2021). Several visual proofs can be easily understood (Nelsen, 1993) and are effective in building conceptual relationships (Wittmann, 2021), which involve student's pre-existing knowledge of triangles, angles, area and algebra. For example, give students four congruent right-angled triangles on cardboard or use a geometry applet like GeoGebra and ask if they can make a square shape with a c side.

This big idea is built upon a strong understanding of number, number properties, patterns, and equality. Therefore, a conceptual understanding of variables and the relationship and connection of words, equations, number, number properties, equality, and algebra is pivotal for future success in mathematics.

Geometric measurement

This chapter will apply students' understanding of geometric measurement, reasoning with measurement, and understanding of length, area and volume, and units of measurement.

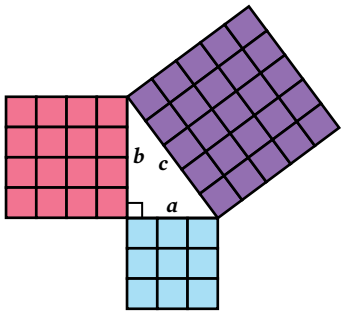
Students must have a strong understanding of measurement when completing this chapter so that when they solve problems involving Pythagoras' theorem, they may reason with the measurements, understand scale, use measuring, and choose appropriate units.

This big idea is built on students' understanding of the properties of shape and measurement. Understanding measurements and relating numbers to units and other measurements are essential when problem-solving and using measurements in topics such as similar triangles, mass, volume, and scale.

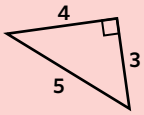
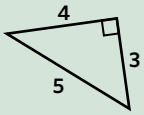
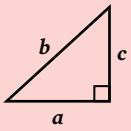
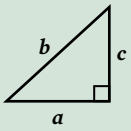
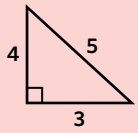
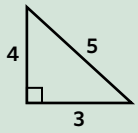
Visual representations

Pythagoras' theorem

The Pythagoras theorem states 'The square on the hypotenuse of a right triangle is equal to the sum of the squares on the two legs'. The theorem is one that represents squares that build a right triangle. This visual is one that draws from students' understanding of area and squaring numbers. However, there are other visual proofs that can be easily understood for students that build on their pre-existing knowledge of area, perimeter and algebra such as the Bhaskara's Proof (Nelsen, 1993: Whittmann, 2021).



Misconceptions

Misconception	Incorrect	Correct	Lesson
Students identify any side of a right-angled triangle as the hypotenuse.	 The hypotenuse is 3 and 4 because they are closest to the 90° angle.	 The hypotenuse is 5, because it is directly opposite of the 90° angle.	13A
When using pythagorean triples, students identify any number in the set as the hypotenuse.	(3,4,5) The hypotenuse is 3 because it is the first number in the set.	(3,4,5) The hypotenuse is 5 because the triad is written as (a, b, c) where c is the hypotenuse.	13A
Students assume that the variables a and b always denote the shorter sides of a right-angled triangle and the variable c, is always the hypotenuse.	 $a^2 + b = c^2$	 $c^2 + a = b^2$	13A
When using Pythagoras' theorem, students double the side lengths.	$6^2 + 8^2 = 10^2$ $12 + 16 = 20$ $28 \neq 20$ $\therefore$ (6,8,10) is not a right-angled triangle.	$6^2 + 8^2 = 10^2$ $36 + 64 = 100$ $100 = 100$ $\therefore$ (6,8,10) is a right-angled triangle	13A 13B
When using or solving for Pythagoras' theorem, students assume that the side length of the triangle is squared.	$c^2 = 100$ The side length is 100.	$c^2 = 100$ The side length is 10.	13A 13B
Students think the inverse operation of an exponent is to divide.	$25 + 144 = c^2$ $169 = c^2$ $\frac{169}{2} = \frac{c}{2}$ $84.5 = c$	$25 + 144 = c^2$ $169 = c^2$ $\sqrt{169} = c^2$ $13 = c^2$	13B 13C
Students think Pythagoras' theorem, $a^2 + b^2 = c^2$, can only be used to solve for the hypotenuse.	The equation $a^2 + b^2 = c^2$ is only used to determine the hypotenuse.	The equation $a^2 + b^2 = c^2$ can be rearranged to determine any side length of a right-angled triangle.	13C
Students think that a is always the shortest side when using Pythagoras' theorem.	 $a = 3$ because it is the shortest side of the right-angled triangle.	 $a = 3$ or 4 because these are the shorter sides of the right-angled triangle.	13C

Misconception	Incorrect	Correct	Lesson
When solving for a shorter side length using Pythagoras' theorem, students do not consider order of operations.	$a^2 = 10^2 - 8^2$ $a^2 = 2^2$	$a^2 = 10^2 - 8^2$ $a^2 = 100 - 64$	13C
When solving for a shorter side length using Pythagoras' theorem, students do not consider inverse operations.	$a^2 + 8^2 = 10^2$ $a^2 = 10^2 + 8^2$	$a^2 + 8^2 = 10^2$ $a^2 = 10^2 - 8^2$	13C

References and additional readings

Güner, N. (2018). *How to teach the Pythagorean theorem: An analysis of lesson plans.* Ankara University Journal of Faculty of Educational Sciences, 51(1), 119–141. https://doi.org/10.30964/auebfd.405041

Nelsen, R. B. (1993). *Proofs without words: exercises in visual thinking.* The Mathematical Association Of America.

Wittmann, E. C. (2021). *Designing Teaching: The Pythagorean Theorem.* Connecting Mathematics and Mathematics Education, 95–160. https://doi.org/10.1007/978-3-030-61570-3_7

Back to contents

13A Introduction to Pythagoras

In a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. This proof was credited to a Greek philosopher named Pythagoras, although it was used by many civilizations and centuries before his time. There are now over 340 proofs that prove Pythagoras' theorem.

LEARNING INTENTIONS

Students will be able to:

- label the hypotenuse and shorter sides of right-angled triangles
- understand that Pythagoras' Theorem can only be used for right-angled triangles
- analyse if three numbers form a Pythagorean triad.

KEY TERMS AND DEFINITIONS

A **right-angled triangle** contains one interior angle that is 90°.

A **Pythagorean triad** is a set of three positive integers represented by (a,b,c), that satisfies the rule $a^2 + b^2 = c^2$.

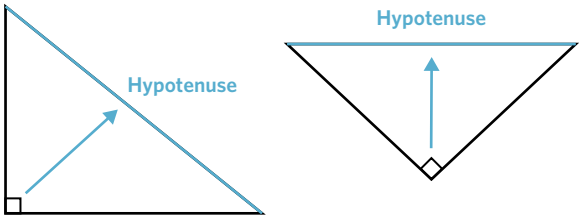
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Image: Curioso.Photography/Shutterstock.com

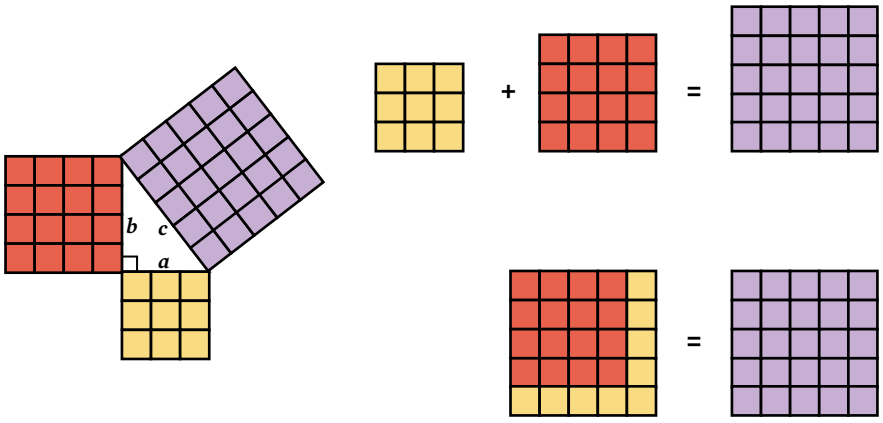
A right-angled triangle is one of the most fundamental shapes that shape our world. We can use the theorem for construction, engineering, navigation, architecture, design, the size of TVs, and even medicine.

Key ideas

1. The hypotenuse is the longest side of a right-angled triangle and is always the side opposite the right angle.



2. Pythagoras' theorem states the square of the hypotenuse is equal to the sum of the squares of the other two shorter sides which can also be written as: $a^2 + b^2 = c^2$, where c is the hypotenuse. Note: If any triangle has side lengths that form a Pythagorean triad, then the triangle must be a right-angled triangle.



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

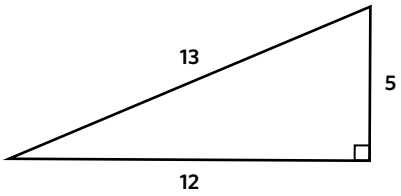
Worked example 1

Applying Pythagoras' theorem

Generate the equation that proves that the following triangles are right-angled triangles using Pythagoras' theorem, $a^2 + b^2 = c^2$.

a.

WE1a



Working

$c = 13$

$a = 5$

$b = 12$

$a^2 + b^2 = c^2$

$5^2 + 12^2 = 13^2$ or $12^2 + 5^2 = 13^2$

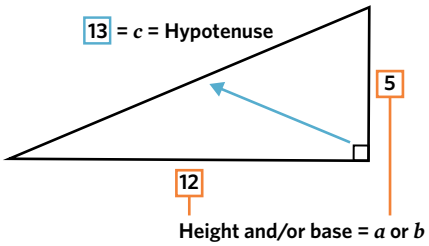
Thinking

Step 1: Identify the hypotenuse of the triangle by determining the longest side and/or the side which is opposite to the right angle.

Step 2: Identify the shorter sides of the triangle.

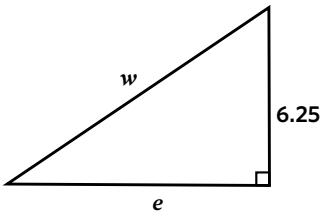
Step 3: Substitute the values for Pythagoras' theorem.

Visual support



b.

WE1b



Working

$c = w$

$a = e$

$b = 6.25$

$a^2 + b^2 = c^2$

$e^2 + 6.25^2 = w^2$ or $6.25^2 + e^2 = w^2$

Thinking

Step 1: Identify the hypotenuse of the triangle by determining the longest side and/or the side which is opposite to the right angle.

Step 2: Identify the shorter sides of the triangle.

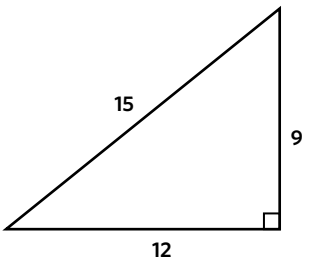
Step 3: Substitute the values for Pythagoras' theorem.

Continues →

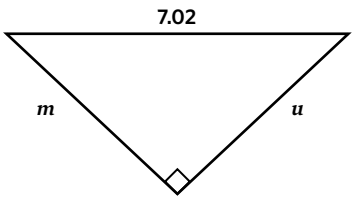
Student practice

Generate the equation that proves that the following triangles are right-angled triangles using Pythagoras' theorem, $a^2 + b^2 = c^2$.

a.



b.



Worked example 2

Determining Pythagorean triads

Determine if the following are Pythagorean triads.

a. (9, 10, 11) WE2a

Working

$a = 9$

$b = 10$

$c = 11$

$11^2 = 121$

$9^2 + 10^2 = 81 + 100 = 181$

$181 \neq 121$

$\therefore a^2 + b^2 \neq c^2$ hence, (9, 10, 11) is not a Pythagorean triad.

Thinking

Step 1: Identify the values inside the Pythagorean triad as (a, b, c).

Step 2: Determine the square of the largest number and the sum of the squares of the two smaller numbers.

Step 3: Compare $a^2 + b^2$ with c^2 to determine if (a, b, c) is a Pythagorean triad.

b. (15, 20, 25) WE2b

Working

$a = 15$

$b = 20$

$c = 25$

$25^2 = 625$

$15^2 + 20^2 = 225 + 400 = 625$

$625 = 625$

$\therefore a^2 + b^2 = c^2$ hence, (15, 20, 25) is a Pythagorean triad.

Thinking

Step 1: Identify the values inside the Pythagorean triad as (a, b, c).

Step 2: Determine the square of the largest number and the sum of the squares of the two smaller numbers.

Step 3: Compare $a^2 + b^2$ with c^2 to determine if (a, b, c) is a Pythagorean triad.

Student practice

Determine if the following are Pythagorean triads.

a. (4, 6, 11)

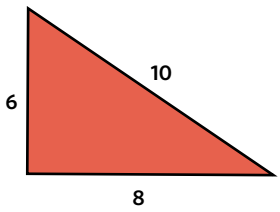
b. (8, 15, 17)

Worked example 3

Determining right-angled triangles

Determine if the following are right-angled triangles.

a. WE3a



Working

$c = 10$

$a = 6$

$b = 8$

$a^2 + b^2 = c^2$

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$

∴ The triangle forms a right-angled triangle.

Thinking

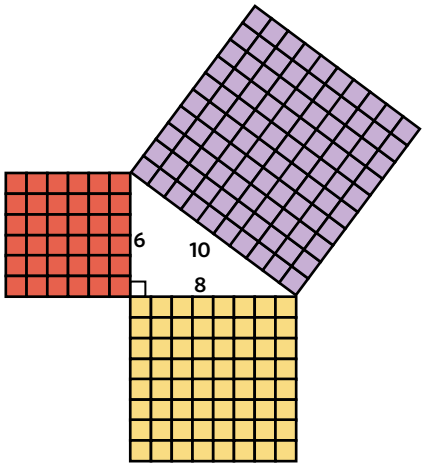
Step 1: Identify the longest side of the triangle.

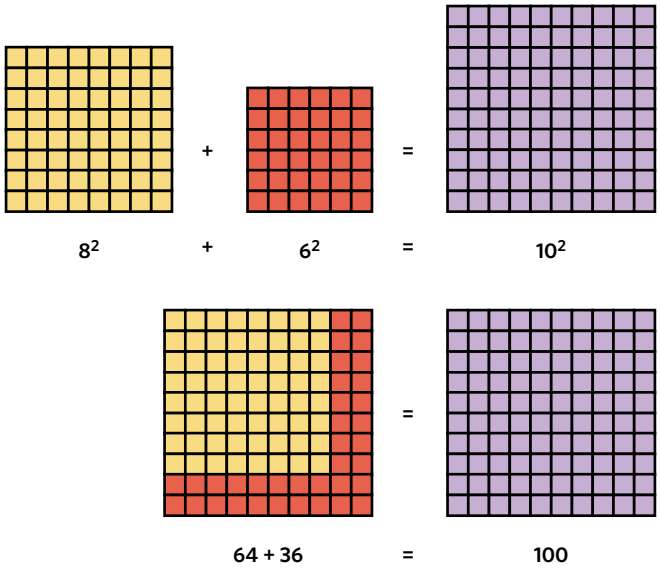
Step 2: Identify the shorter sides of the triangle.

Step 3: Substitute the values for Pythagoras' theorem.

Step 4: If $a^2 + b^2 = c^2$, then the Pythagorean triad does form a right-angled triangle.

Visual support





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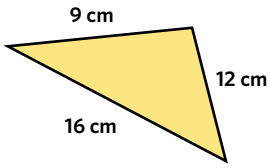
b.



WE3b

Working

$c = 16$

$a = 9$

$b = 12$

$a^2 + b^2 = 9^2 + 12^2$

$= 81 + 144 = 225$

$c^2 = 16^2 = 256$

$225 \neq 256$

$\therefore$ This is not a right-angled triangle.

Thinking

Step 1: Identify the longest side of the triangle.

Step 2: Identify the shorter sides of the triangle.

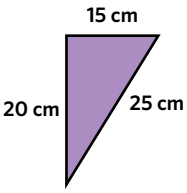
Step 3: Substitute the values for Pythagoras' theorem.

Step 4: If $a^2 + b^2 \neq c^2$, then the Pythagorean triad does not form a right-angled triangle.

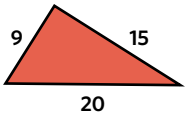
Student practice

Determine if the following are right-angled triangles.

a.



b.

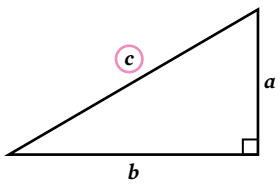


13A Questions

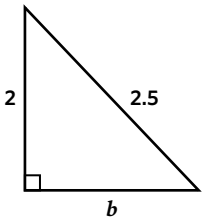
Understanding worksheet

1. Circle the variable or number which describes the hypotenuse.

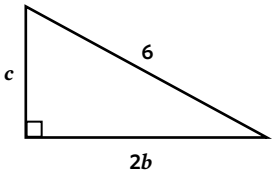
Example



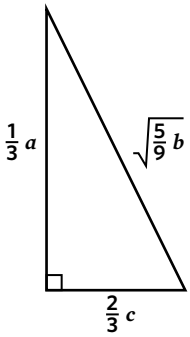
a.



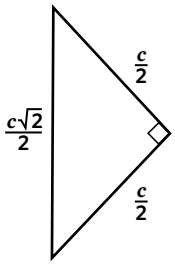
b.



c.

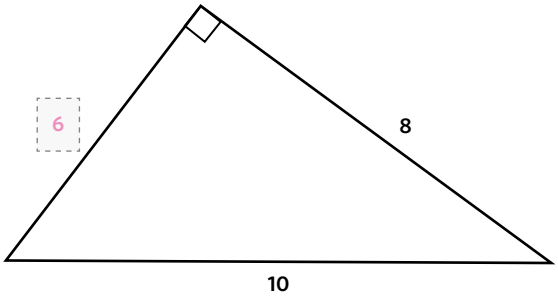


d.



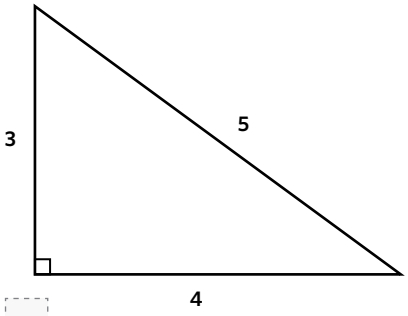
2. Use Pythagoras' theorem to fill in the boxes.

Example



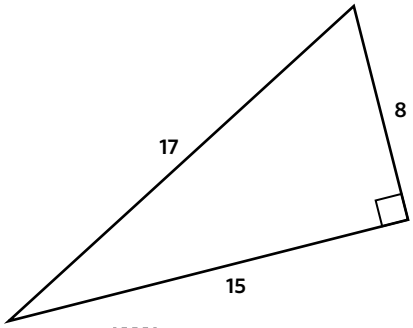
$6^2 + \boxed{8^2} = 5^2$

a.



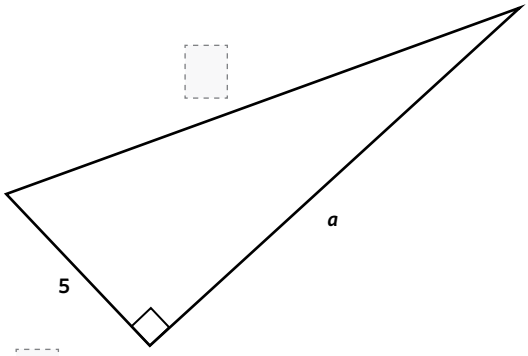
$\square + 4^2 = 5^2$

b.



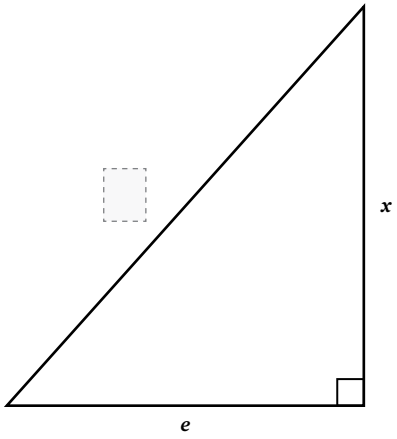
$8^2 + 15^2 = \square$

c.



$\square + 5^2 = 13^2$

d.



$r^2 = \square + x^2$

3. Fill in the blanks by using the words provided.

$a^2 + b^2 = c^2$	90°	longest side	(a, b, c)

A Pythagorean triad is represented by $\square$, and c represents the $\square$ which is called a hypotenuse in a right-angled triangle. To prove a Pythagorean triad is a right-angled triangle or if any triangle contains an interior angle that is $\square$ the formula $\square$ can be used.

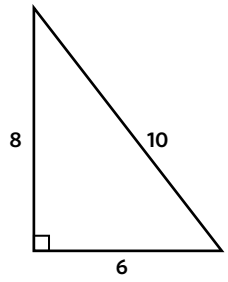
Fluency

Question working paths

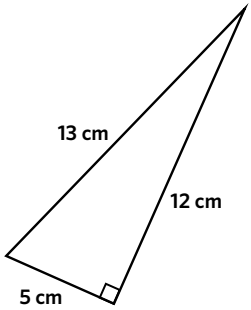
Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (c,d,e,f), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (e,f,g,h), 8 (e,f,g,h), 9

4. Generate the equation that proves that the following triangles are right-angled triangles using Pythagoras' theorem, $a^2 + b^2 = c^2$. WE1

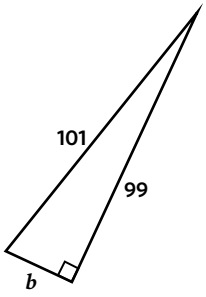
a.



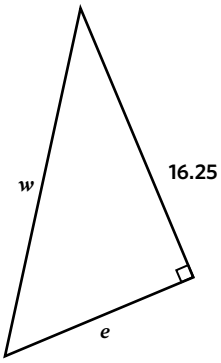
b.



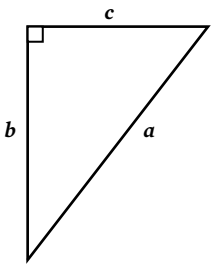
c.



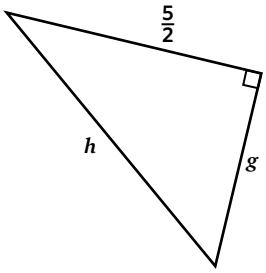
d.



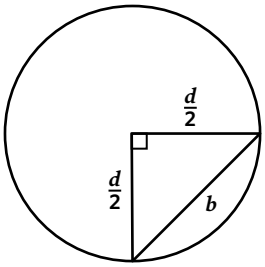
e.



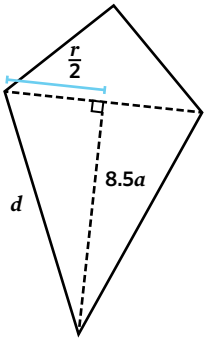
f.



g.



h.



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5. Fill in the missing values to make the working out of Pythagoras' theorem true.

a. $4^2 + 3^2 = 5^2$

$16 + \square = 25$

$\square = 25$

b. $8^2 + 6^2 = 10^2$

$64 + \square = 100$

$\square = 100$

c. $9^2 + (\square)^2 = 15^2$

$\square + 144 = 225$

$\square = 225$

d. $1.5^2 + 2^2 = 2.5^2$

$\square + \square = \square$

$\square = \square$

e. $6.5^2 = (\square)^2 + 2.5^2$

$\square = 36 + \square$

$\square = \square$

f. $25^2 = (\square)^2 + 24^2$

$625 = 49 + \square$

$625 = \square$

g. $(\square)^2 + 12^2 = (\sqrt{169})^2$

$25 + \square = \square$

$\square = \square$

h. $(\square)^2 + (\sqrt{3})^2 = 2^2$

$\square + \square = \square$

$\square = \square$

---

6. Determine if the following are Pythagorean triads. WE2

a. (3, 4, 5)

b. (6, 8, 10)

c. (5, 12, 13)

d. (10, 8, 18)

e. (9, 12, 15)

f. (15, 20, 25)

g. (9, 40, 42)

h. (20, 21, 29)

7. Determine if the dimensions are correct for a right-angled triangle, where c is the hypotenuse and a and b are the shorter sides of a triangle. WE2

a. $a = 8, b = 6, c = 10$

b. $a = 3, b = 5, c = 4$

c. $a = 24, b = 7, c = 25$

d. $a = 1.5, b = 2, c = 2.5$

e. $a = \frac{9}{2}, b = 20, c = \frac{41}{2}$

f. $a = 3, b = 4, c = \sqrt{25}$

g. $a = 6, b = \sqrt{16}, c = 10$

h. $a = 1, b = 1, c = \sqrt{3}$

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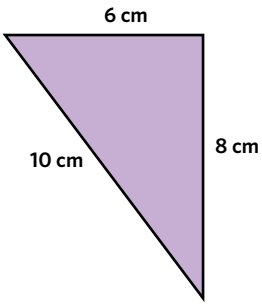
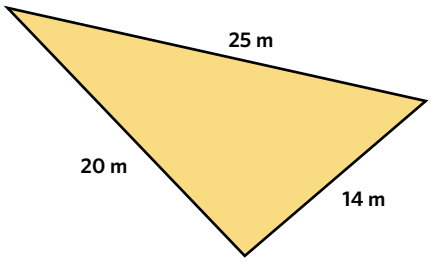
8. Determine if the following shaded figures are right-angled triangles. WE3

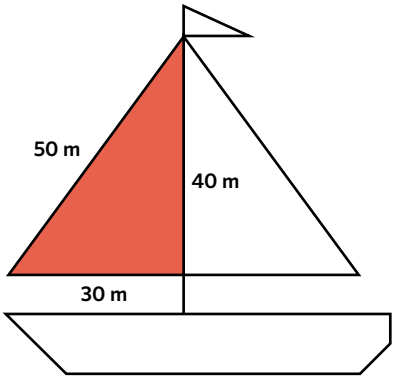
a.



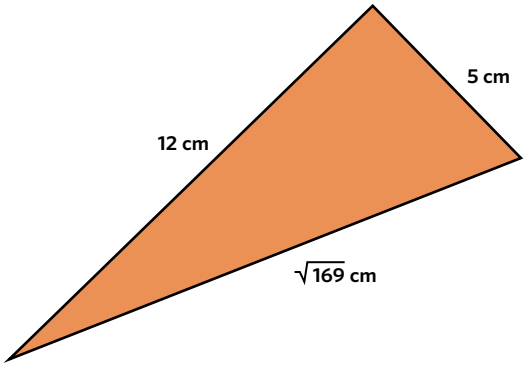
b.



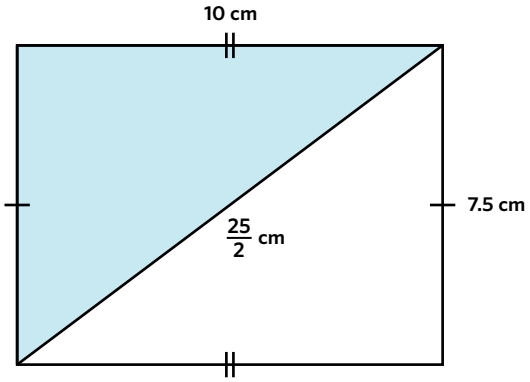
c.



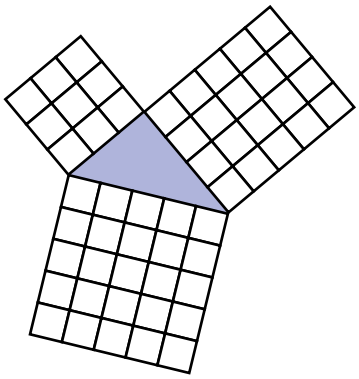
d.



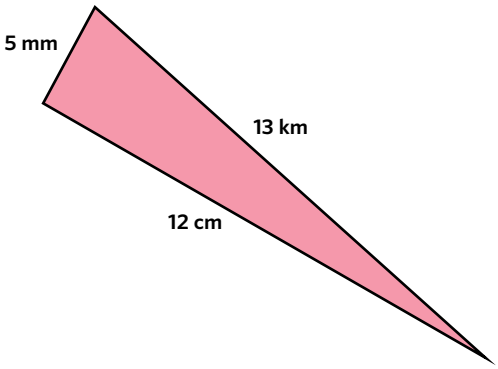
e.



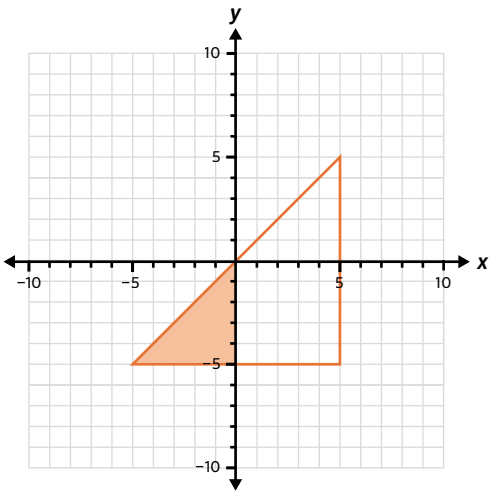
f.



g.



h.



9. What are the triangle's dimensions based on the working out?

Step 1: $a^2 + b^2 = c^2$

Step 2: $81 + 144 = 225$

Step 3: $225 = 225$

A. $a = 81, b = 144, c = 225$

B. $a = 9, b = 12, c = 15$

C. $a = \frac{81}{2}, b = 72, c = \frac{225}{2}$

D. $a = 9^2, b = 12^2, c = 15^2$

E. $a = \sqrt{9}, b = \sqrt{12}, c = \sqrt{15}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Determine if the following is a Pythagorean triad.

(50,120,130)

Student A

$50^2 + 120^2 = 130^2$

$100 + 240 = 260$

(50,120,130) is a Pythagorean triad.

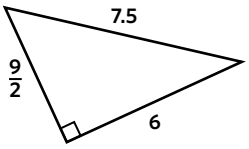
Student B

$50^2 + 120^2 = 130^2$

$2500 + 14\,400 = 16\,900$

(50,120,130) is a Pythagorean triad.

b. Determine if the following is a right-angled triangle



Student A

$\left(\frac{9}{2}\right)^2 + 7^2 = 7.5^2$

$\left(\frac{18}{4}\right) + 49 = 56.28$

$4.5 + 49 = 56.28$

$53.5 \neq 56.28$

Student B

$\left(\frac{9}{2}\right)^2 + 7^2 = 7.5^2$

$\left(\frac{81}{4}\right) + 49 = 56.28$

$20.25 + 49 = 56.28$

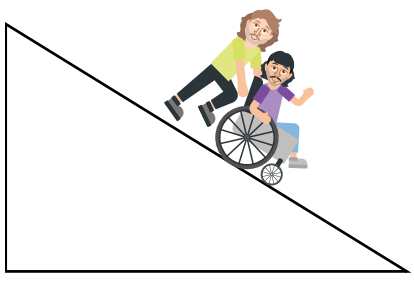
$69.25 \neq 56.28$

Problem solving

Question working paths

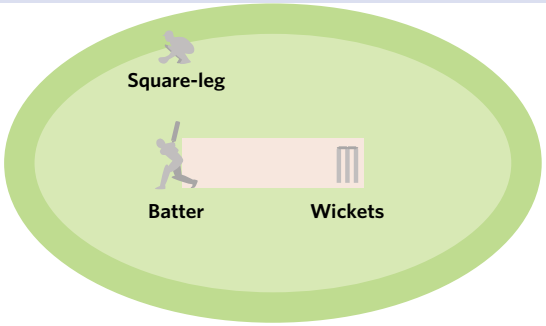
Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. The local council wants to build a ramp that has a length of 800 cm, a horizontal distance of 700 cm and a vertical distance of 50 cm. Determine if this style of ramp is possible.



12. In a game of cricket, a news anchor tells a story of a batter who hits a ball at a 90° degree angle and is caught by their opponent at the Square Leg position from 20 metres away.

The batter runs 20 metres straight down to the wickets while the opponent throws 25 metres to the wickets and gets the batter out. Was this story possible?

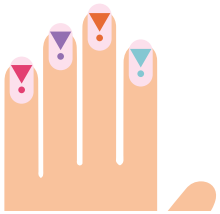


13. Ritchie leaves his house with his dog Daisy. They walk 600 metres south down Beach Street, then 450 metres east along Neptune Street. They plan on finishing up by walking diagonally all the way home for 750 metres. Will Ritchie and Daisy be able to get all the way back home based on these directions?

14. NailMood is celebrating Pythagoras Day on the 345th day of the year with a special nail design. The nail design is an equilateral triangle with side length s that was formed by placing two right-angled triangles together that have a height of h.

Clients who can correctly rewrite Pythagoras' theorem, $a^2 + b^2 = c^2$, in terms of s and h will get their nails done for free.

What is the Pythagoras' formula in terms of s and h?



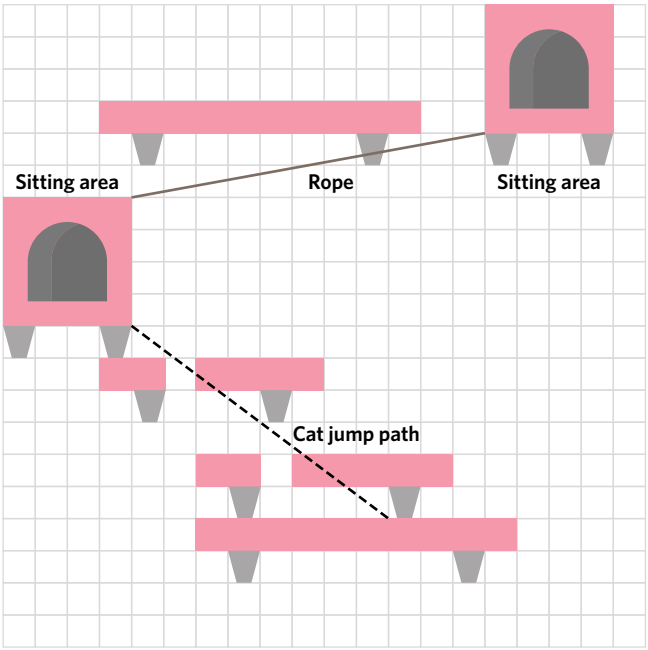
15. Grace is comparing three televisions, a 75 inch 66 inch or a 55 inch from Aldi's special buys to place inside her entertainment unit. Her entertainment unit is 60 inches wide and 36 inches high. Grace wants to centre the television so that there is no more than four inches of space between the television and the entertainment unit on either side. If televisions are measured by their diagonal and in inches, which one should Grace buy?

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Rahul is building a cat play centre. He mocks up a design for his contractor, Natalya, and uses grid paper where each square represents a length 0.5 m.



a. Rahul wants his cats to be able to jump through floating platforms to reach a sitting area. What formula can Natayla use to calculate the total distance of the jump path?

b. Natayla calculates that the cat jump path will be a maximum of 5 m. Show how Natayla determined this length.

c. Rahul also wants to connect the sitting areas with rope. He tells Natayla that the rope will be 5.5 metres long. Determine if Rahul's calculations are correct.

d. Using part **c**, determine the length of the rope. Round your answer to one decimal point.

e. Rahul wants to use this design but scale it by a fourth so it will fit in his sisters apartment. What will be the length of the new cat jump path and do you think this scale is appropriate for a fully grown cat?

17. Use the Pythagorean triad (3,4, 5), to determine if the following equations can be used to calculate a right angled triangle.

a. $b^2 + a^2 = c^2$

b. $c^2 - a^2 = b^2$

c. How are these two formulas related to the Pythagorean theorem formula $a^2 + b^2 = c^2$?

Extra spicy

18. A total of 50 square tiles with dimensions 1 cm by 1 cm are used to create three squares. What are the dimensions and area of each of the three squares?

19. Which of the following is not a rearrangement of Pythagoras' theorem, where p is the hypotenuse and m and n are the shorter sides?

A. $\sqrt{m^2 + n^2} = p$

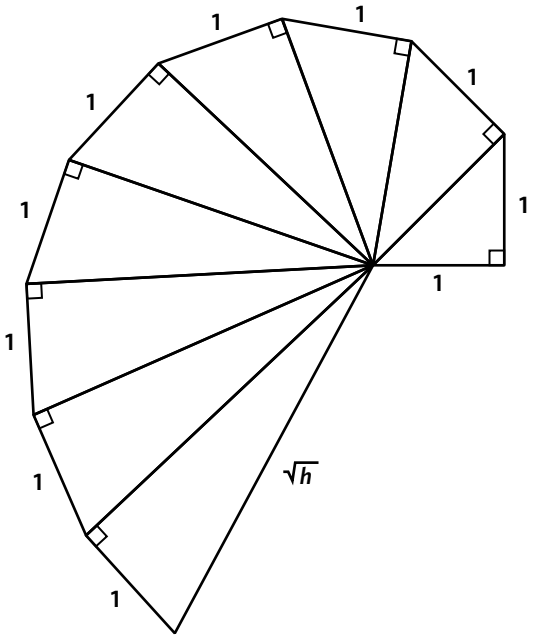
B. $\sqrt{p^2 - m^2} = \sqrt{n^2}$

C. $\sqrt{p^2 - m^2} = n$

D. $\sqrt{m^2 + n^2} = \sqrt{p^2}$

E. $\sqrt{m^2 + n^2} = \sqrt{p}$

20. Solve for h.



A. $h = \sqrt{2}$ **B.** $h = \sqrt{9}$ **C.** $h = 9$ **D.** $h = \sqrt{10}$ **E.** $h = 10$

21. Figure 2 was made using the triangles from Figure 1. Express the areas of sections p,y, t, h, $\triangle$, and g in terms of a, b, and c.

Figure 1

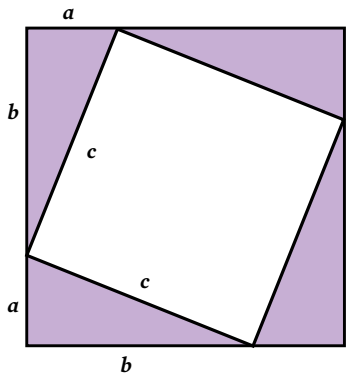
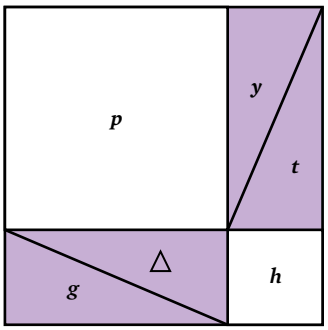
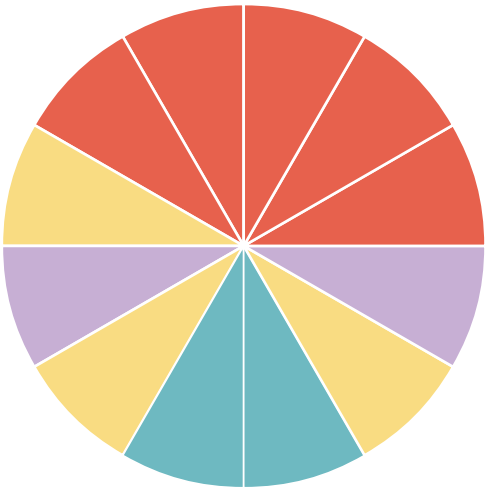


Figure 2



Remember this?

22. A wildlife shelter recorded data about the different species of wildlife it rescued in a fortnight. This is displayed in the following graph:



12 more possums than kangaroos were rescued.

How many animals were rescued in total?

A. 24 **B.** 30 **C.** 36 **D.** 42 **E.** 48

23. Emily put together some lolly bags to give to her classmates at the end of the year.

Each lolly bag had three sour straps and four banana lollies. She used 52 banana lollies altogether. How many sour straps did she use?

A. 33 **B.** 36 **C.** 39 **D.** 42 **E.** 45

24. Sarah bought two identical cartons of milk. After she used $\frac{1}{3}$ of one carton of milk, she had a total of 5L left. How many L were in one full carton of milk?

A. 1.5 L **B.** 2 L **C.** 2.25 L **D.** 2.5 L **E.** 3 L

Back to contents

13B Calculating the hypotenuse

The hypotenuse is the longest side of a right-angled triangle, or the side opposite the right angle. We can use algebra to determine the hypotenuse, if given the shorter sides of the triangle. Determining the hypotenuse is an important step in solving many problems in geometry.

LEARNING INTENTIONS

Students will be able to:

- use Pythagoras' theorem to solve for the length of the hypotenuse in right angled triangles
- understand how to solve equations with rational numbers and surds
- rearrange Pythagoras' theorem formula so that the hypotenuse, c, is the subject.

KEY TERMS AND DEFINITIONS

Exact form is when a number cannot be simplified to a whole number or rational number and left as an irrational number.

Surds are the square roots of numbers that cannot be simplified into a whole number.

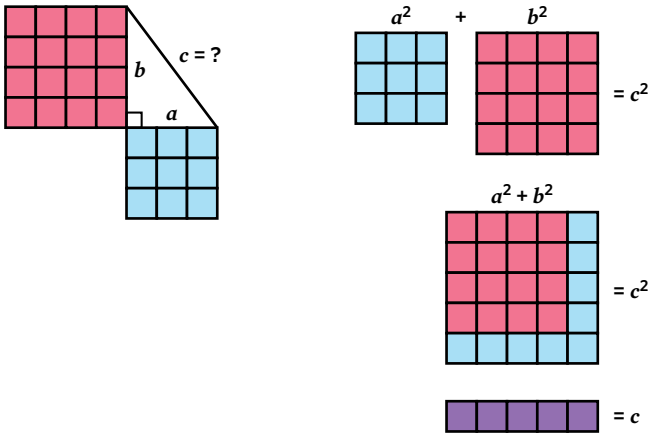
A **rational** number is a number that can be expressed as a ratio of two integers, whereas an **irrational** number cannot.

WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?

Calculating the hypotenuse of a triangle can be applied to many important aspects of our daily lives. For example, knowing the hypotenuse can be very useful in making decisions ranging from what size TV you should buy to determining what size window to build in a house.

Key ideas

1. Pythagoras' theorem can be used to find missing side lengths of right-angled triangles using the formula: $a^2 + b^2 = c^2$, where c is the hypotenuse.



2. The Pythagoras' theorem formula $a^2 + b^2 = c^2$, where c is the hypotenuse, can be rearranged to help solve for missing side lengths.

$a^2 + b^2 = c^2$

$\sqrt{a^2 + b^2} = \sqrt{c^2}$

$\sqrt{a^2 + b^2} = c$

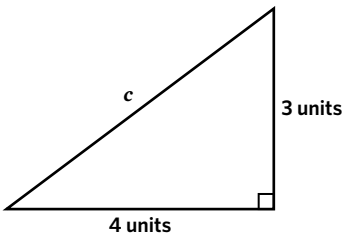
Worked example 1

Calculating the hypotenuse

Calculate the length of the hypotenuse for the right-angled triangle. Give your answer in exact form.

a.

WE1a



Working

$a = 4$

$b = 3$

$c = c$

$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

$25 = c^2$

$\sqrt{25} = \sqrt{c^2}$

$5 \text{ units} = c$

Thinking

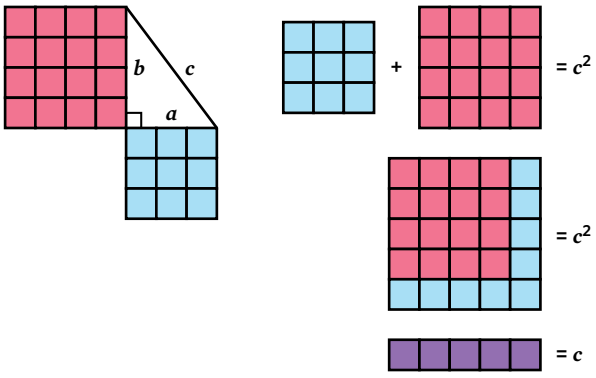
Step 1: Substitute the values in Pythagoras' theorem.

Step 2: Simplify both sides of the equation to find the solution for the hypotenuse.

Step 3: Isolate the hypotenuse by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 4: Identify if the answer is a perfect square, otherwise, leave your answer as a surd.

Visual support



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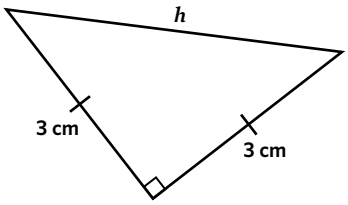
b.



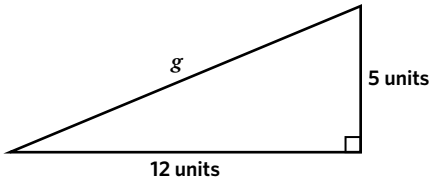
WE1b

Working	Thinking
$a = 3$	
$b = 3$	
$c = h$	
$3^2 + 3^2 = h^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$9 + 9 = h^2$	**Step 2:** Simplify both sides of the equation to find the solution for the hypotenuse.
$18 = h^2$ $\sqrt{18} = \sqrt{h^2}$	**Step 3:** Isolate the hypotenuse by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{18}$ cm $= h$	**Step 4:** Identify if the answer is a perfect square, otherwise, leave your answer as a surd.

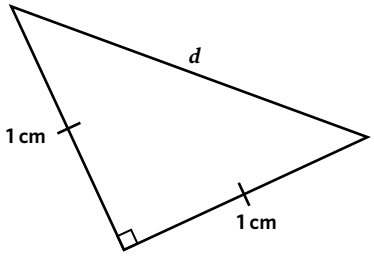
Student practice

Calculate the length of the hypotenuse for the right-angled triangle. Give your answer in exact form.

a.



b.



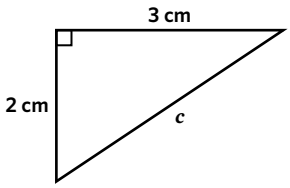
Worked example 2

Using a calculator to calculate the hypotenuse

Use a calculator to calculate the length of the hypotenuse for the right-angled triangle. Round your answer to two decimals places.

a.

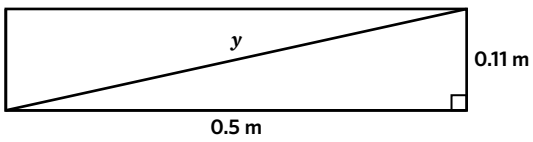
WE2a



Working	Thinking
$c = g$	
$a = 3$	
$b = 2$	
$3^2 + 2^2 = c^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$9 + 4 = c^2$	**Step 2:** Simplify both sides of the equation to find the solution for the hypotenuse.
$13 = c^2$ $\sqrt{13} = \sqrt{c^2}$	**Step 3:** Isolate the hypotenuse by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$3.605... \approx c$	**Step 4:** Use a calculator to calculate the square root.
$3.61 \approx c$	**Step 5:** Round the answer to the specified number of decimal places.

b.

WE2b



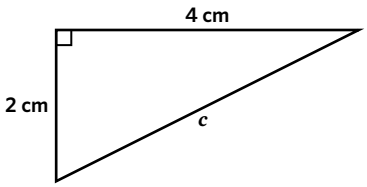
Working	Thinking
$a = 0.11$	
$b = 0.5$	
$c = y$	
$y^2 = (0.11)^2 + (0.5)^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$y^2 = (0.0121) + (0.25)$	**Step 2:** Simplify both sides of the equation to find the solution for the hypotenuse.
$y^2 = 0.2621$ $\sqrt{y^2} = \sqrt{0.2621}$	**Step 3:** Isolate the hypotenuse by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$y \approx 0.511...$	**Step 4:** Use a calculator to calculate the square root.
$y \approx 0.51$ m	**Step 5:** Round the answer to the specified number of decimal places.

Continues →

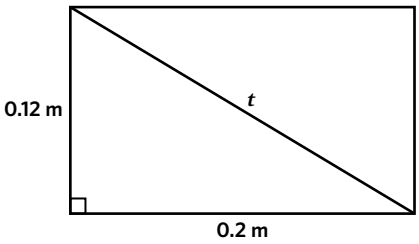
Student practice

Use a calculator to calculate the length of the hypotenuse for the right-angled triangle. Round your answer to two decimal places.

a.



b.



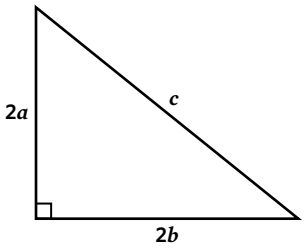
Worked example 3

Manipulating Pythagoras' theorem

Rearrange the equation so that the hypotenuse, c, is the subject.

a.

WE3a



Working	Thinking
$c = c$	
$a = 2a$	
$b = 2b$	
$(2a)^2 + (2b)^2 = c^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$4a^2 + 4b^2 = c^2$	**Step 2:** Simplify both sides of the equation.
$\sqrt{4a^2 + 4b^2} = \sqrt{c^2}$	**Step 3:** Isolate c by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{4a^2 + 4b^2} = c$	**Step 4:** Leave your answer in terms of a, b and c, where c is the subject.

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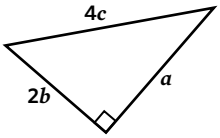
b.



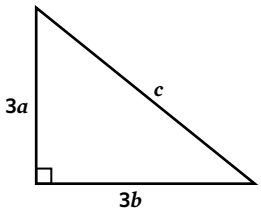
WE3b

Working	Thinking
$c = 4c$	
$a = a$	
$b = 2b$	
$(4c)^2 = a^2 + (2b)^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$(4c)^2 = a^2 + 4b^2$	**Step 2:** Simplify both sides of the equation.
$\sqrt{(4c)^2} = \sqrt{a^2 + b^2}$ $4c^2 = \sqrt{a^2 + b^2}$	**Step 3:** Isolate c by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$c = \frac{\sqrt{a^2 + 4b^2}}{4}$	**Step 4:** Leave your answer in terms of a, b and c, where c is the subject.

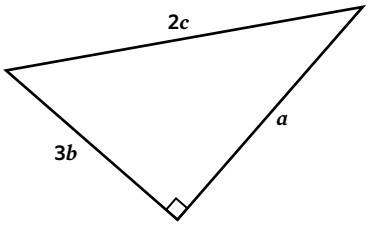
Student practice

Rearrange the equation so that the hypotenuse, c, is the subject.

a.



b.



13B Questions

Understanding worksheet

1. Determine if the surd simplifies to a whole number.

Example

	Yes	No
$\sqrt{5}$	☐	☑

		Yes	No			Yes	No
a.	$\sqrt{1}$	☐	☐	**b.**	$\sqrt{2}$	☐	☐
c.	$\sqrt{4}$	☐	☐	**d.**	$\sqrt{9}$	☐	☐

2. Fill in the missing parts to make the equation true.

Example

$\sqrt{c^2} = \sqrt{16 + 9}$

$c = \sqrt{25}$

a. $\sqrt{81 + 20} = \sqrt{c^2}$

$\sqrt{} = c$

b. $\sqrt{c^2} = \sqrt{44 + 100}$

$c = \sqrt{}$

$c = 12$

c. $k^2 = 13^2 + ()^2$

$k = \sqrt{} + 121$

$k = \sqrt{290}$

d. $\sqrt{} + 144 = \sqrt{n^2}$

$\sqrt{} = n$

$13 = n$

3. Fill in the blanks by using the words provided.

irrational | $a^2 + b^2 = c^2$ | opposite | c

When determining side lengths of right-angled triangles we can use the formula _____, where _____ is the hypotenuse. This is also the longest side of a right-angled triangle and is _____ of the right angle. Sometimes the side lengths are left in exact form or rounded because the side lengths can be an _____ number.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d,e), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (c,d,e,f), 8 (e,f,g,h), 9

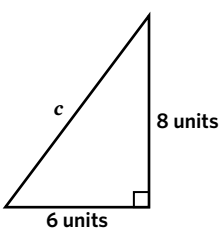
4. Simplify the surds to a whole number or leave in exact form.

a. $\sqrt{1}$ **b.** $\sqrt{2}$ **c.** $\sqrt{4}$ **d.** $\sqrt{10}$

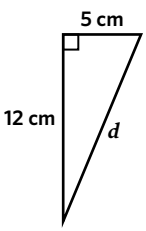
e. $\sqrt{81}$ **f.** $\sqrt{142}$ **g.** $\sqrt{169}$ **h.** $\sqrt{0.25}$

5. Calculate the length of the hypotenuse for the right-angled triangle. Give your answer in exact form. WE1a

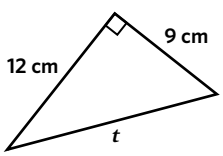
a.



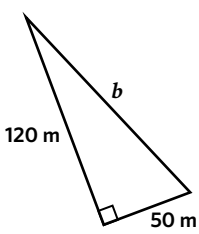
b.



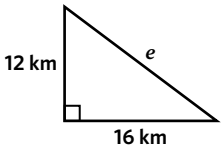
c.



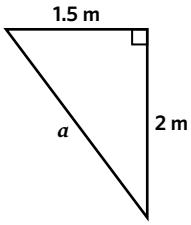
d.



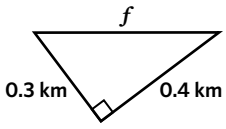
e.



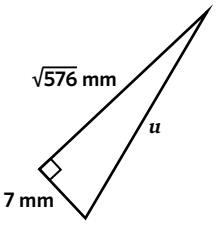
f.



g.

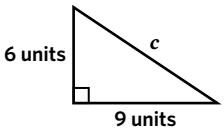


h.

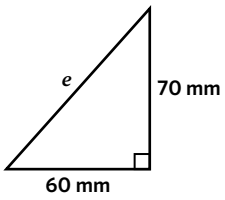


6. Calculate the length of the hypotenuse for the right-angled triangle. Give your answer in exact form. WE1b

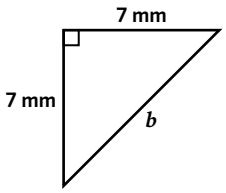
a.



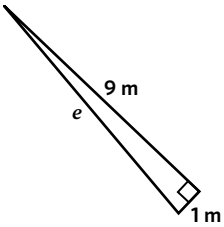
b.



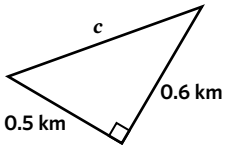
c.



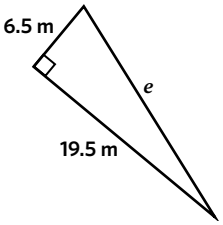
d.



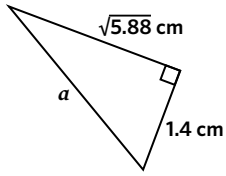
e.



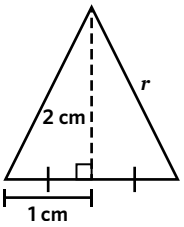
f.



g.

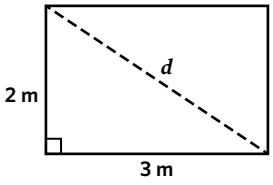


h.

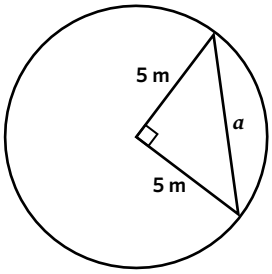


7. Use a calculator to calculate the length of the hypotenuse for the right-angled triangle. Round your answer to two decimals places. WE2b

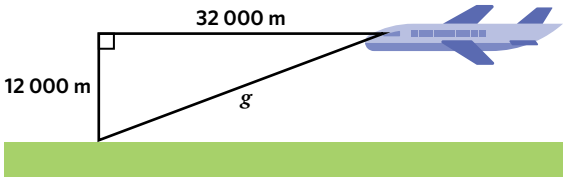
a.



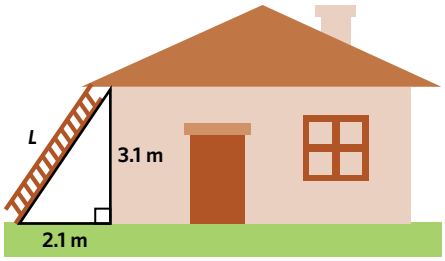
b.



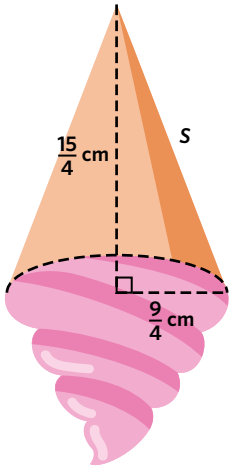
c.



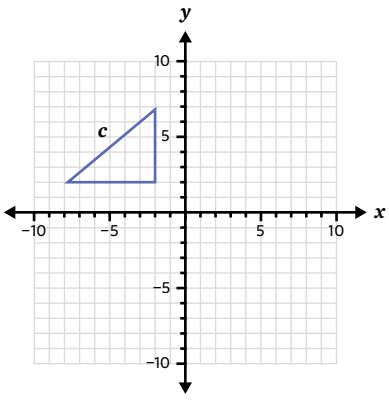
d.



e.

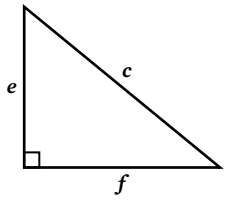


f.

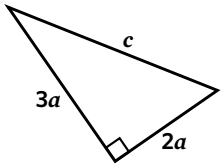


8. Rearrange the equation so that the hypotenuse, c, is the subject. WE3

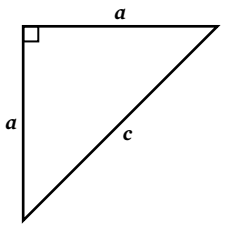
a.



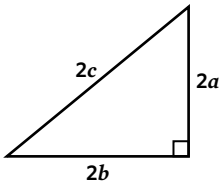
b.



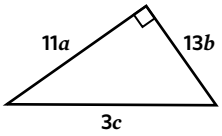
c.



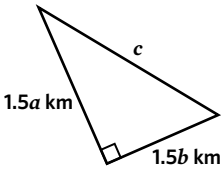
d.



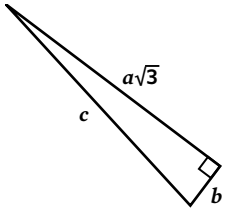
e.



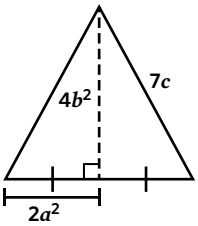
f.



g.



h.



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9. Determine the value of c.

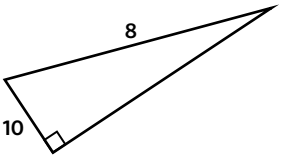
$12^2 + 14^2 = c^2$

A. $\sqrt{340}$ **B.** 26 **C.** $\sqrt{26^2}$ **D.** 170 **E.** 340

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Use a calculator to calculate the length of the hypotenuse for the right-angled triangle. Round your answer to two decimals places.



Student A

$\sqrt{10^2 + 8^2} = c$

$\sqrt{10^2} + \sqrt{8^2} = c$

$10 + 8 = c$

$18 = c$

Student B

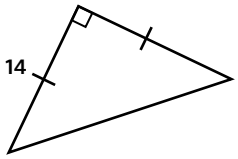
$\sqrt{10^2 + 8^2} = c$

$\sqrt{100 + 64} = c$

$\sqrt{164} = c$

$12.81 \approx c$

b. Calculate the length of the hypotenuse for the right-angled triangle. Give your answer in exact form.



Student A

$c^2 = 14^2 + 14^2$

$c = 392$

Student B

$c^2 = 14^2 + 14^2$

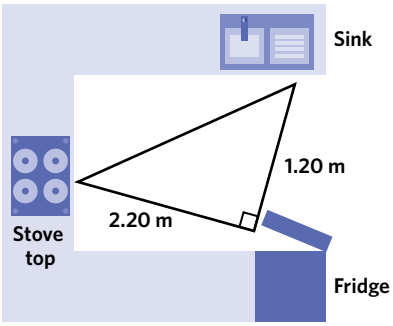
$c = \sqrt{392}$

Problem solving

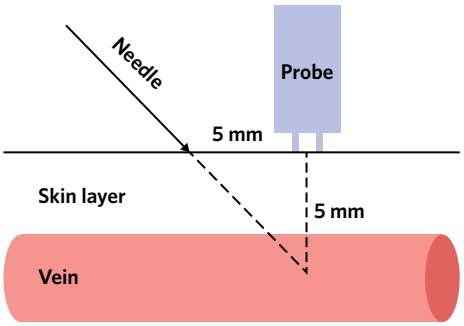
Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

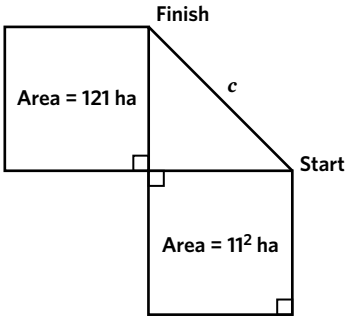
11. The kitchen work triangle determines how functional it is to use the cook top, sink and fridge. No part of the triangle should be less than 1.2 metres or more than 2.7 metres. The diagram shows Hymie's kitchen. Determine the distance from the sink to the stove top and if Hymie's kitchen work triangle meets the criteria for a functional kitchen space. Round your answer to 2 decimal places.



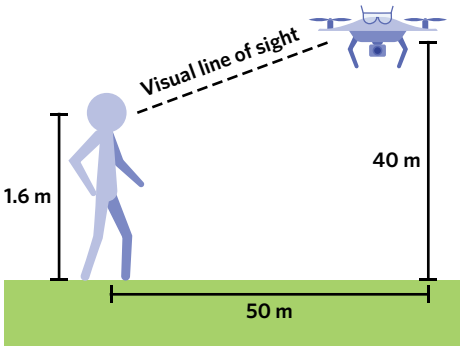
12. Darina is studying medicine and learns that she must use Pythagoras' theorem to determine how long a needle needs to be so that it meets the sound waves from the ultrasound probe in a vein. The needle will be placed 5 mm from the probe, and the sound waves need to reach 5 mm to the vein. What is the minimum length the needle needs to be so that it reaches the sound waves in the vein? Leave your answer in exact form.



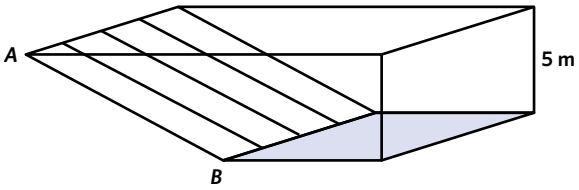
13. ZipIt, a zipline company, is installing a new zipline that will connect two corners of their properties together using a cable. ZipIt knows the area of the two properties is measured in hectares (ha), and 1 ha $= 10\,000\ \text{m}^2$. Determine the length of the zipline rope that will connect the two corners of the property together. Give your answer in exact form.



14. Jarrod is flying his drone but lost his glasses and can only see distances less than 50 metres away. According to drone regulations, he must keep his drone in his visual line of sight. He flies his drone directly upwards, 40 metres from the ground, and 50 metres horizontally away from him. The distance from the ground to his eye is 1.6 metres. Determine by how much the drone is in or out of his line of sight. Round your answer to a whole number.



15. The Redland City council is building a new Olympic size indoor lap and diving pool that will be 50 m by 25 m. The pool will gradually increase in depth from point A until it reaches a diving depth of 5 m at point B. The bottom of the diving area is 500 m^2. There will be vertical tiled lines to denote the gradual increase in depth of the pool, and the diving area will be painted a solid colour. What is the length of the pool's slant? Round your answer to 3 decimal places.

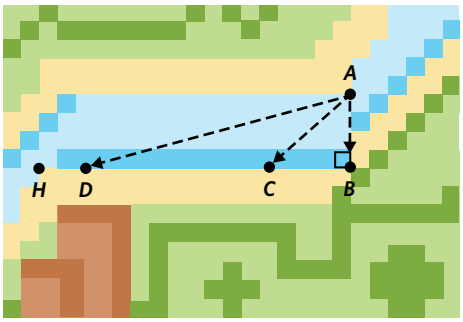


Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Petra creates a Minecraft world and puts their shelter at point H. $\overline{BH}$ is 36 block units, the river is 12 block units wide, and $\overline{BC}$ is a $\frac{1}{4}$ of $\overline{BH}$. Petra wants to prepare routes back to the shelter in case they are chased by a skeleton in the game.



a. The distance of $\overline{DH}$ is two block units less than $\overline{BC}$. Determine the amount of block units there are in the distances of $\overline{BC}$, $\overline{CD}$ and $\overline{DH}$.

b. Determine how many block units it will take Petra if they decide to swim the river in a straight line from point A to C.

c. Determine how many block units it will take Petra if they decide to swim the river in a straight line from point A to D and then walk from point D to the shelter. Round your answer to a whole block unit.

d. If it takes 2 block units per second to swim and 5.5 block units per second to sprint, determine which route, $\overline{AB}$ to H or $\overline{AC}$ to H or $\overline{AD}$ to H will be the quickest for Petra to take and by how many seconds than the other routes. Round your answer to 2 decimal places.

e. If Petra has extra building blocks, determine another route or option they could do to protect themselves from the skeleton.

17. Use the Pythagorean triple (3,4,5), to determine if the following expressions, where a and b are the shorter sides of the triangle, can be used to calculate a hypotenuse.

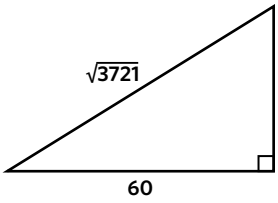
a. $\sqrt{a^2 + b^2}$

b. $\sqrt{b^2 + a^2}$

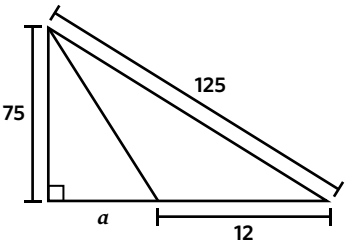
c. Give at least one reason why it does not matter how the sides a and b of the triangle are labelled and why the formula can be be written in the form $c = \sqrt{a^2 + b^2}$?

Extra spicy

18. Calculate the missing side length of the triangle.

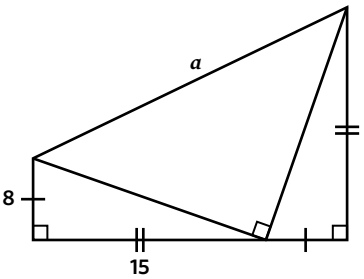


19. What is the value of a?



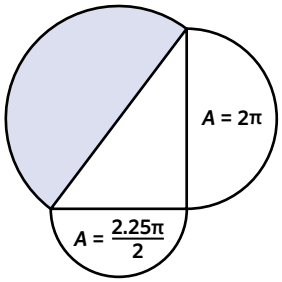
A. 88 **B.** 99.93 **C.** 100 **D.** 112 **E.** 9988

20. What is the value of a?



A. $\sqrt{17}$ **B.** 17 **C.** $\sqrt{578}$ **D.** $\sqrt{1156}$ **E.** 289

21. Three semicircles form a right-angled triangle. What is the area of the shaded semi-circle?



Remember this?

22. Which of these is equal to $48 \times 3 + 2$?

A. $50 - 6$ **B.** $48 + 50$ **C.** $50 \times 2 + 5$ **D.** 48×5 **E.** $40 \times 3 + 26$

23. Sarah counted the stationery in her pencil case.

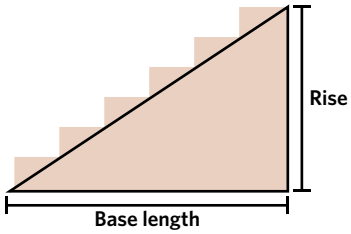
$\frac{1}{4}$ were pens. $\frac{1}{5}$ were pencils.

She had 3 more pens than pencils.

How many items of stationery did Sarah count?

A. 20 **B.** 30 **C.** 40 **D.** 50 **E.** 60

24. Joanne is rebuilding the staircase at her investment property. She requires that the stairs have a rise of 1 m for every 1.75 m of base length of the staircase.



What is the base length for a staircase with a rise of 1.8m?

A. 3 m **B.** 3.15 m **C.** 3.3 m **D.** 3.45 m **E.** 3.6 m

Back to contents

13C Calculating the shorter side

Applying and rearranging the Pythagoras' theorem formula relies on the ability to solve equations, algebra skills, and understanding the shape of right-angled triangles. This lesson will explore rearranging Pythagoras' theorem to find the third side when given any two sides of a triangle.

LEARNING INTENTIONS

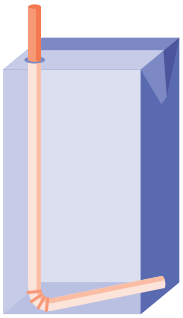
Students will be able to:

- use Pythagoras' theorem to solve for the length of the shorter sides in right angled triangles
- understand how to solve equations with rational numbers and surds
- rearrange Pythagoras' theorem formula so that the shorter length, a or b, is the subject.

KEY TERMS AND DEFINITIONS

The **subject** of a formula is the variable isolated on one side of the equal sign.

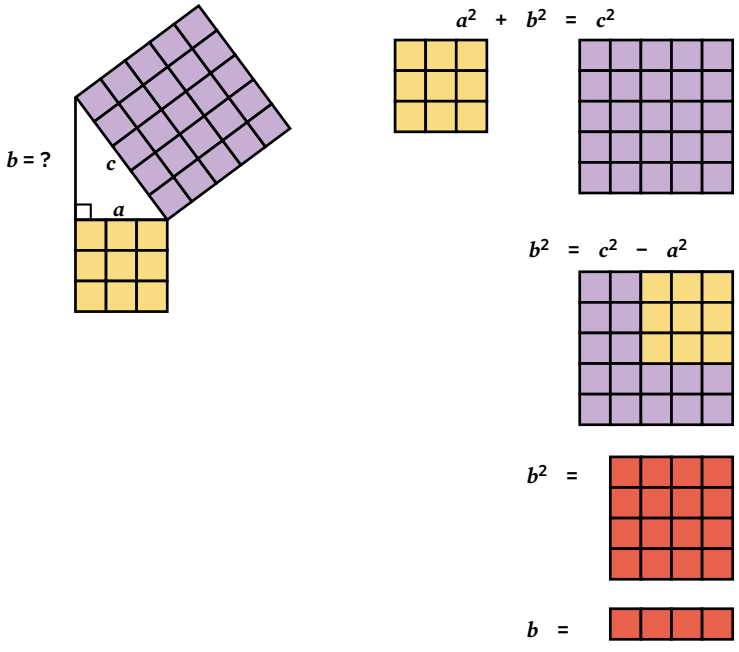
WHERE DO WE SEE THIS MATHS IN THE REAL WORLD?



Calculating the shorter lengths of a right-angled triangle can be applied to many important aspects of our daily lives. For example, product designers can use Pythagoras' theorem to determine the maximum and minimum lengths of bendy straws so the straw can be used for drinking with an optimal and aesthetic bend.

Key idea

1. The Pythagoras' theorem formula $a^2 + b^2 = c^2$, where c is the hypotenuse, can be rearranged to help solve for missing side lengths.

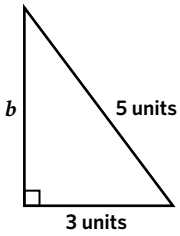


Worked example 1

Calculating the shorter side

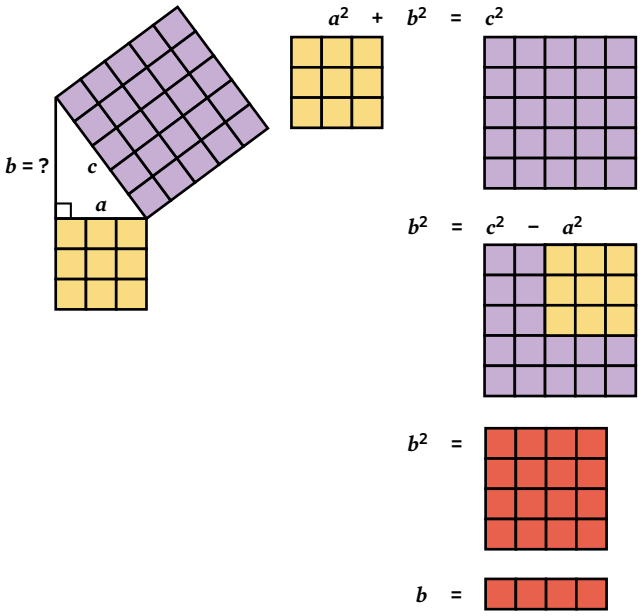
Calculate the missing side lengths. Give your answer in exact form.

a. WE1a



Working	Thinking
$a = 3$	
$b = b$	
$c = 5$	
$3^2 + b^2 = 5^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$9 + b^2 = 25$	**Step 2:** Simplify both sides of the equation to find the solution for the shorter side.
$-9 + 9 + b^2 = 25 - 9$	**Step 3:** Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{b^2} = \sqrt{16}$	**Step 4:** Identify if the answer is a perfect square, otherwise, leave your answer as a surd.
$b = 4$ units	

Visual support



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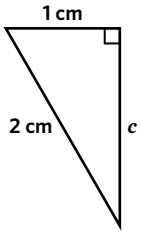
b.



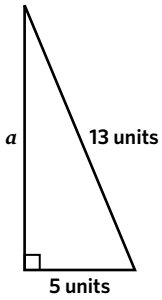
WE1b

Working	Thinking
$a = d$	
$b = 1$	
$c = 2$	
$2^2 = d^2 + 1^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$4 = d^2 + 1$	**Step 2:** Simplify both sides of the equation to find the solution for the shorter side.
$4 - 1 = d^2 + 1 - 1$	**Step 3:** Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{3} = \sqrt{d^2}$ $\sqrt{3}$ cm $= d$	**Step 4:** Identify if the answer is a perfect square, otherwise, leave your answer as a surd.

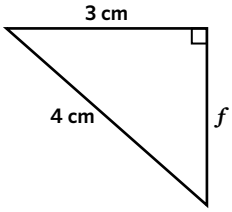
Student practice

Calculate the missing side lengths. Give your answer in exact form.

a.



b.



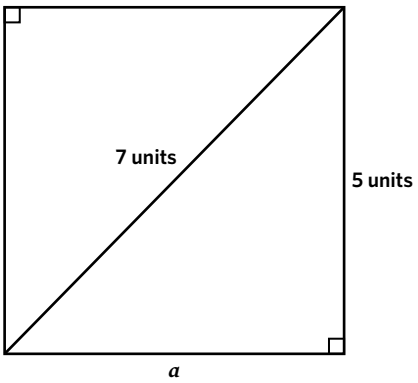
Worked example 2

Using a calculator to calculate the shorter side

Use a calculator to calculate the missing side lengths. Round your answer to two decimal places.

a.

WE2a



Working

$a = a$

$b = 5$

$c = 7$

$a^2 + 5^2 = 7^2$

$a^2 + 25 = 49$

$a^2 + 25 - 25 = 49 - 25$

$\sqrt{a^2} = \sqrt{24}$

$a \approx 4.898...$

$a \approx 4.90$ units

Thinking

Step 1: Substitute the values in Pythagoras' theorem.

Step 2: Simplify both sides of the equation to find the solution for the shorter side.

Step 3: Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 4: Use a calculator to calculate the square root.

Step 5: Round the answer to the specified number of decimal places.

Visual support

$(a)^2 + (5)^2 = (7)^2$

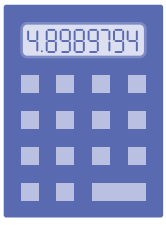
$a^2 + 25 = 49$

-25 -25

$a^2 = 24$

$\sqrt{}$ $\sqrt{}$

$a = \sqrt{24}$



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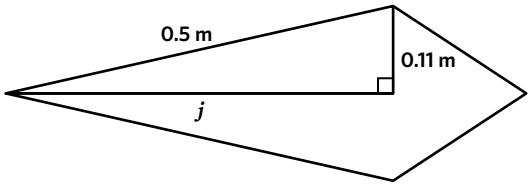
b.



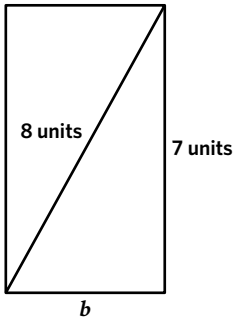
WE2b

Working	Thinking
$a = 0.11$	
$b = j$	
$c = 0.5$	
$(0.5)^2 = (0.11)^2 + j^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$0.25 = 0.0121 + j^2$	**Step 2:** Simplify both sides of the equation to find the solution for the shorter side.
$0.25 - 0.0121 = 0.0121 - 0.0121 + j^2$	**Step 3:** Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{0.2379} = \sqrt{j^2}$	**Step 4:** Use a calculator to calculate the square root.
$j \approx 0.4877...$	**Step 5:** Round the answer to the specified number of decimal places.
$j \approx 0.49$ m	

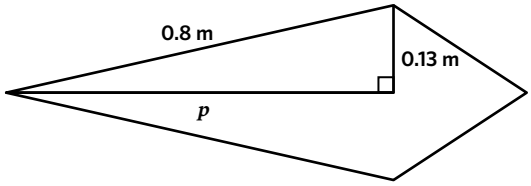
Student practice

Use a calculator to calculate the missing side lengths. Round your answer to two decimals places.

a.



b.



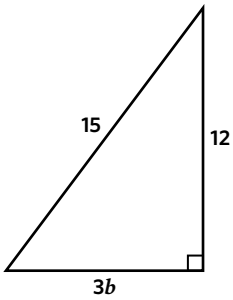
Worked example 3

Using Pythagoras' theorem to solve equations

Use algebra to determine the value of the variable. Round your answer to two decimal places.

a.

WE3a



Working

$a = 12$

$b = 3b$

$c = 15$

$(15)^2 = (12)^2 + (3b)^2$

$225 = 144 + 9b^2$

$225 - 144 = 144 - 144 + 9b^2$

$\frac{81}{9} = \frac{9b^2}{9}$

$\sqrt{9} = \sqrt{b^2}$

$3.00 = b$

Thinking

Step 1: Substitute the values in Pythagoras' theorem.

Step 2: Simplify both sides of the equation to find the solution for the shorter side.

Step 3: Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.

Step 4: Use a calculator to calculate the square root.

Step 5: Round the answer to the specified number of decimal places.

Visual support

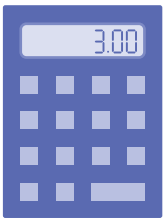
$(15)^2 = (12)^2 + (3b)^2$

$225 = 144 + 9b^2$ (−144 / −144)

$81 = 9b^2$ (÷9 / ÷9)

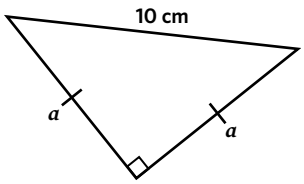
$9 = b^2$ (√ / √)

$3.00 = b$



Continues →

b.



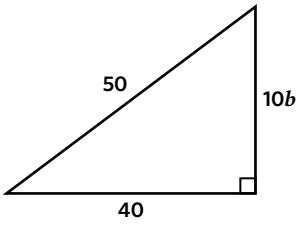
WE3b

Working	Thinking
$a = a$	
$b = a$	
$c = 10$	
$a^2 + a^2 = 10^2$	**Step 1:** Substitute the values in Pythagoras' theorem.
$2a^2 = 100$	**Step 2:** Simplify both sides of the equation to find the solution for the shorter side.
$\frac{2a^2}{2} = \frac{100}{2}$	**Step 3:** Isolate the shorter side by applying the inverse operations. To keep the equation true, apply the same operations on both sides of the equal sign.
$\sqrt{a^2} = \sqrt{50}$	**Step 4:** Use a calculator to calculate the square root.
$a \approx 7.071...$	**Step 5:** Round the answer to the specified number of decimal places.
$a \approx 7.07$ cm	

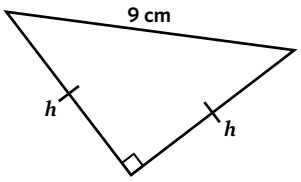
Student practice

Use algebra to determine the value of the variable. Round your answer to two decimal places.

a.



b.



13C Questions

Understanding worksheet

1. Determine the values.

Example

$-9^2 =$ -81

a. $4 - 3^2 = 4 -$ ☐

b. $4 + (-3)^2 = 4 +$ ☐

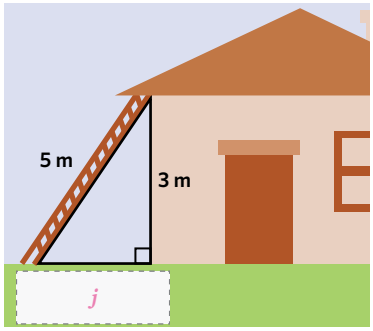
c. $-3^2 =$ ☐

d. $(-3)^2 =$ ☐

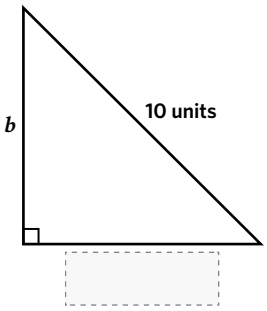
2. Fill in the missing values in the diagram and/or the scenario so they match.

Example

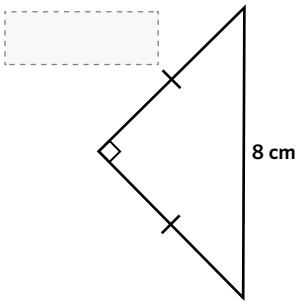
A ladder leans on a building j metres away. The ladder is 5 metres long and the building is [3] metres high.



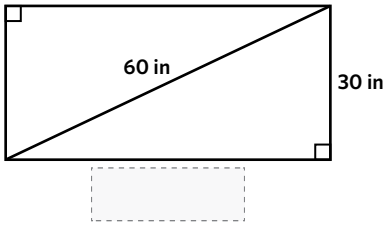
a. A right-angled triangle has a hypotenuse of 10 units and the shorter sides are [ ] units and 7 units.



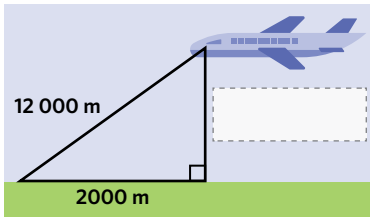
b. An isosceles right-angled triangle has side lengths k cm and a hypotenuse of [ ] cm.



c. A TV is measured by its diagonal which is 60 inches. The width of the TV is [ ] inches and the length is L inches.



d. A pilot is 2000 m away from the runway, their altitude from the ground is a metres, and their visual line of sight to the runway is [ ] m.



3. Fill in the blanks by using the words provided.

rational | $a = \sqrt{c^2 - b^2}$ | b | surd

We can use algebra to manipulate Pythagoras' formula to solve for a shorter side or use the formula _____, where a and _____ are the shorter sides and c is the hypotenuse.

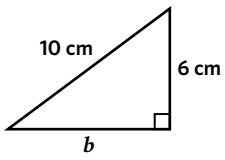
When calculating side lengths of triangles, the answer may be given in exact form or rounded to a specified number of decimal places. The $\sqrt{4}$ is a _____ number and can be simplified to 2, but $\sqrt{2}$ is a _____ or an irrational number and can be left in its exact form of $\sqrt{2}$.

Fluency

Question working paths

Mild	Medium	Spicy
4 (a,b,c,d), 5 (a,b,c,d), 6 (a,b,c,d), 7 (a,b,c,d), 8 (a,b,c,d), 9	4 (c,d,e,f), 5 (c,d,e,f), 6 (c,d,e,f), 7 (b,c,d,e), 8 (c,d,e,f), 9	4 (e,f,g,h), 5 (e,f,g,h), 6 (e,f,g,h), 7 (c,d,e,f), 8 (e,f,g,h), 9

4. Substitute the side lengths of the given right-angled triangles into Pythagoras' theorem, $a^2 + b^2 = c^2$.

a.


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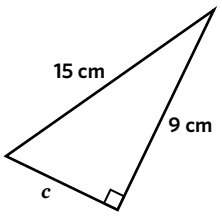
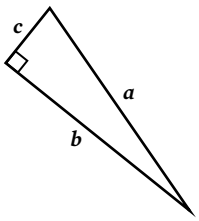


c.


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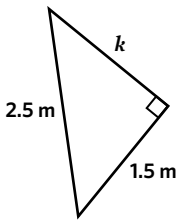

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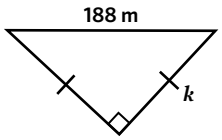

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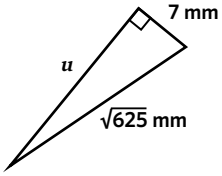

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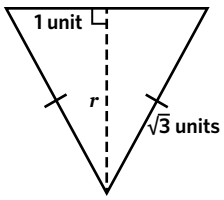

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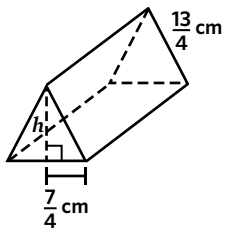
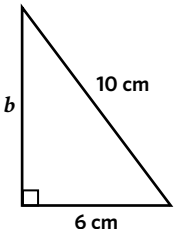


5. Calculate the missing side lengths. Give your answers in exact form. WE1a

a.


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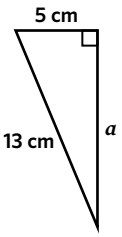

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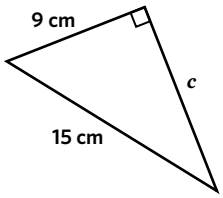

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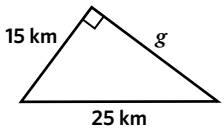

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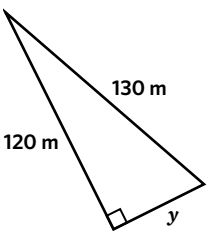

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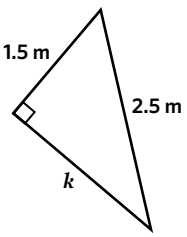

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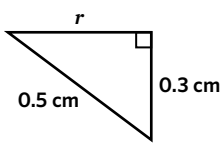

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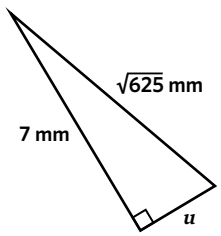
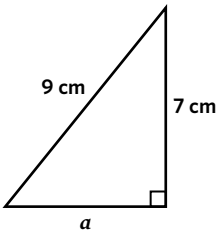
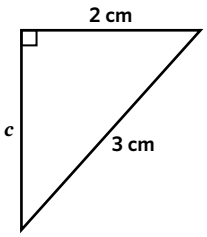


6. Calculate the missing side lenths. Give your answer in exact form. WE1b

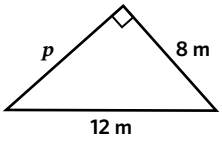
a.



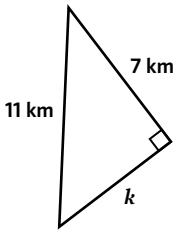
b.



c.



d.



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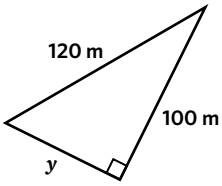

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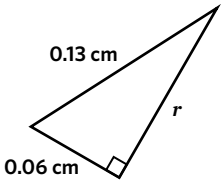

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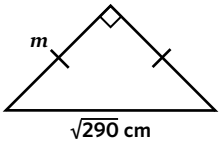

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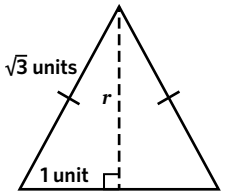

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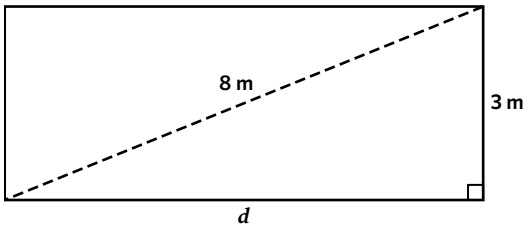
7. Use a calculator to calculate the missing side lenths. Round your answer to two decimal places. WE2

a.



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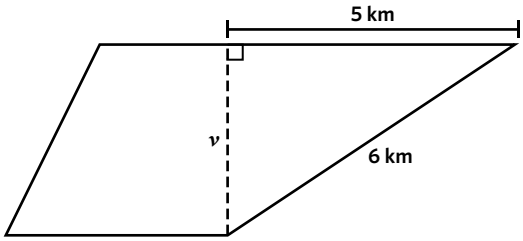

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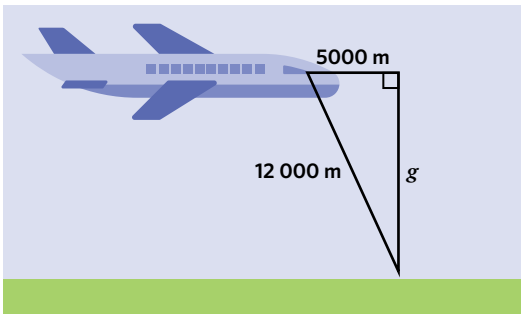

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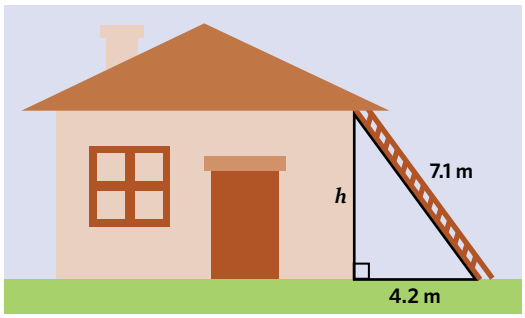

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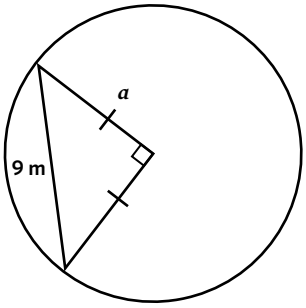

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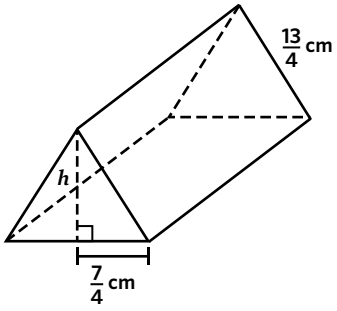
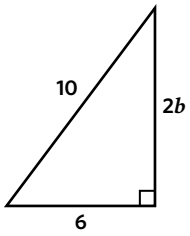
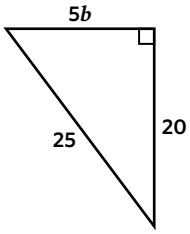


8. Use algebra to determine the value of the variable. Round your answer to two decimal places. WE3

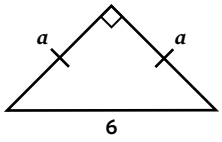
a.



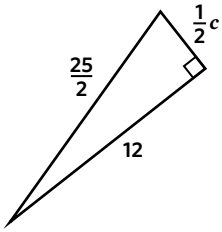
b.



c.



d.



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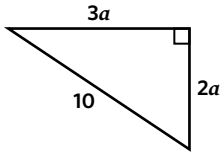

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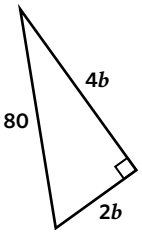
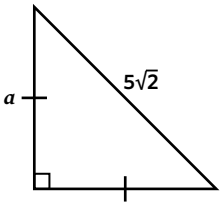


g.



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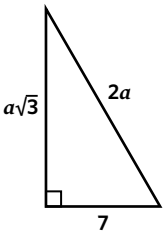


9. Which of the given options is equivalent to step 3 of the problem below?

Step 1: $a^2 + b^2 = c^2$

Step 2: $6^2 + b^2 = 9^2$

Step 3: $\square$

Step 4: $b = \sqrt{45}$

A. $b = \sqrt{81 + 36}$

B. $b^2 = \sqrt{81 + 36}$

C. $b = \sqrt{6^2 - 9^2}$

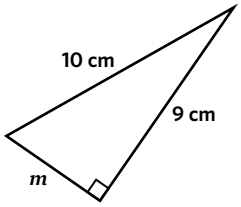
D. $b = \sqrt{9^2 - 6^2}$

E. $b = \sqrt{18 - 12}$

Spot the mistake

10. Select whether Student A or Student B is incorrect.

a. Calculate the missing side length for the right-angled triangle. Give your answer in exact form.



Student A

$9^2 + m^2 = 10^2$

$m = \sqrt{10^2 + 9^2}$

$m = \sqrt{181}$ cm

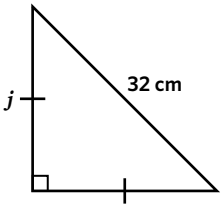
Student B

$10^2 = 9^2 + m^2$

$\sqrt{10^2 - 9^2} = m$

$\sqrt{19}$ cm $= m$

b. Use algebra to determine the value of the variable. Round your answer to two decimal places.



Student A

$j^2 + j^2 = 32^2$

$2j^4 = 32^2$

$j^4 = \frac{1024}{2}$

$\sqrt[4]{j^4} = \sqrt[4]{512}$

$j = 4.76$ cm

Student B

$j^2 + j^2 = 32^2$

$2j^2 = 1024^2$

$j^2 = \frac{1024}{2}$

$\sqrt{j^2} = \sqrt{512}$

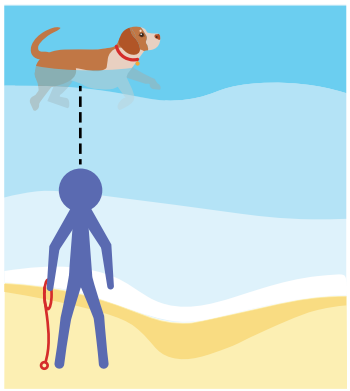
$j = 22.63$ cm

Problem solving

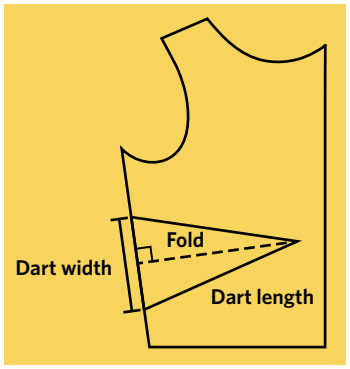
Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

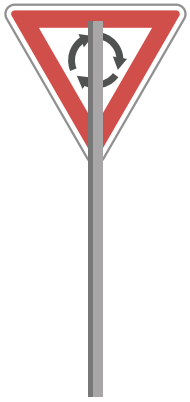
11. Schnitzel von Crumb has been taken off his leash and runs straight to the water d metres away. He paddles in a straight line parallel to the beach for 5 metres, then travels diagonally back to his owner on the beach who is standing 8 metres away. What is the total distance Schnitzel von Crumb travelled? Round your answer to the nearest metre.



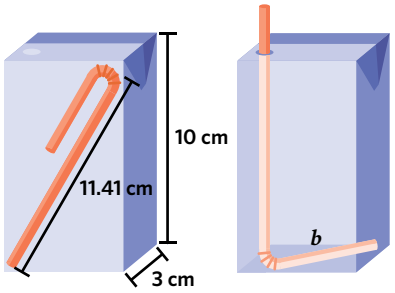
12. Sophie is making a dress for her fashion design final project. She uses sewing darts to create a curved shape from her fabric. Darts are isosceles triangles that are folded down the middle and sewn on the dart length. Determine the dart width if the length of the dart is 11 mm and the fold is 7.5 mm. Round your answer to one decimal place.



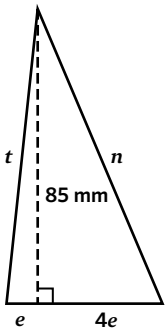
13. A new roundabout sign needs to be installed with a metal pole. The roundabout sign is an equilateral triangle with dimensions 900 mm. To meet regulations, the distance from the bottom of the roundabout sign to the ground must be 2000 mm, the metal pole does not extend past the sign, and the sign is centred on the pole. Determine the maximum length of the metal pole. Round your answer to two decimal places.



14. A hack to drink a juice box is to place the bendy straw upside down. The bendy part of the straw must form a 90° angle while lying inside of the box at a diagonal. Currently the bendy straw is folded on itself and is the same length as the diagonal length of the front of the box, 11.41 cm.
Determine the maximum length of b, correct to two decimal places.



15. Calculate the missing side lengths, t, e, n of the triangle if the area of the triangle is 1840 mm^2.
Round the lengths to the nearest mm.

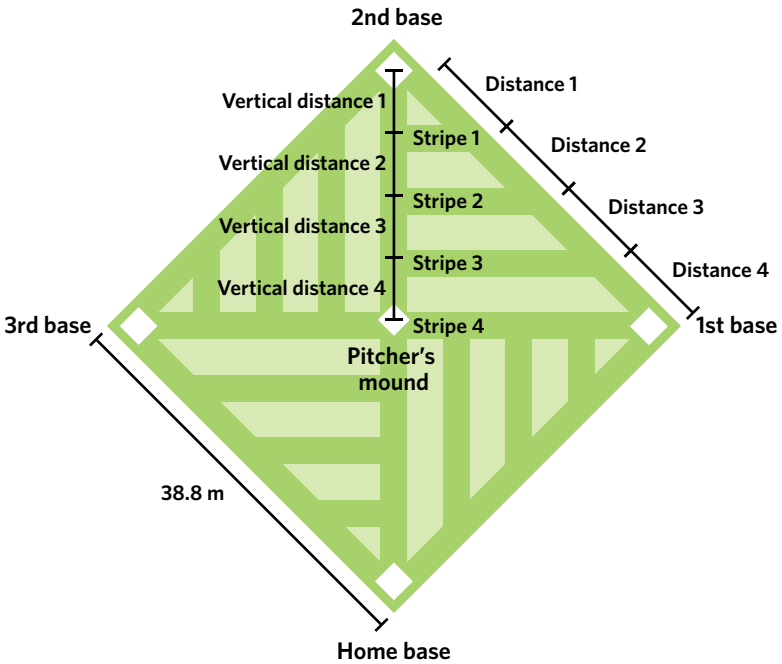


Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. The Brisbane Bandits baseball team wants to transform the look of their baseball field to celebrate their four championship wins by placing four equally spaced stripes in their field. The baseball field is shaped like a square where all the sides have equal lengths and all interior angles are equal to 90°.



a. Calculate the vertical distance between home base and 2nd base. Round your answer to the nearest whole metre.

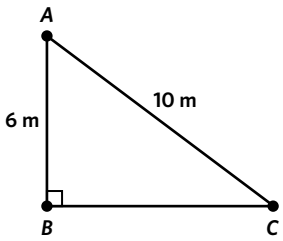
b. Using your rounded answer from part **a**, determine the vertical distance between each of the four stripes from 2nd base to the mound and the distance between each of the four stripes from 2nd base to 1st base. Round your answer to the nearest whole metre.

c. Using your rounded answer from part **b**, calculate the length of stripe 1. Give your answer in exact form.

d. Determine the length of stripes 2, 3 and 4. Round your answer to the nearest whole metre.

e. The Brisbane bandits will need to create a new design in a few weeks so that the grass does not grow in odd directions affecting play. Sketch two different designs they could consider that will celebrate their 4 championship wins.

17. Use the right-angled triangle to answer the following questions.



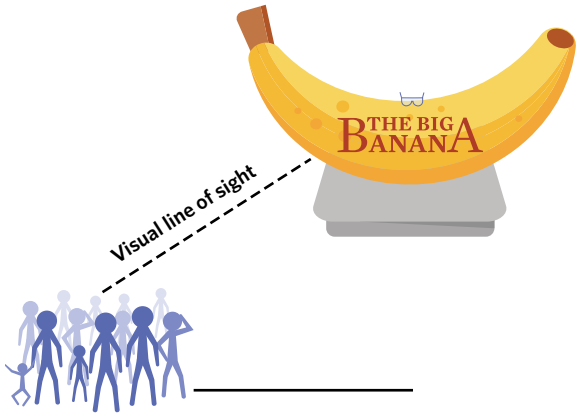
a. What is the length of $\overline{BC}$?

b. What is the total length of $\overline{AB}$ and $\overline{BC}$?

c. Determine why it is faster to walk along the hypotenuse rather than the two shorter sides of a right-angled triangle?

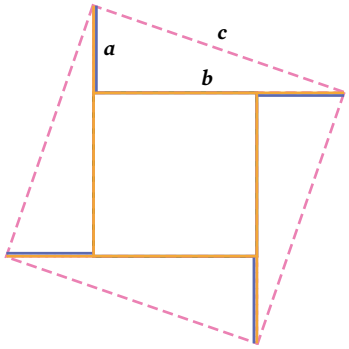
Extra spicy

18. Jarrod lost his glasses again while waiting in a straight line to view the iconic Big Banana statue in Coffs Harbour. He can see things within 50 m and is 1.6 m tall. The Big Banana statue is 5 m high and 11 m long. From what distance will he be able to start seeing the Big Banana?

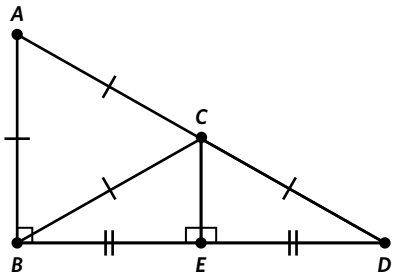


A. 37.72 m **B.** 49.56 m **C.** 49.75 m **D.** 49.88 m **E.** 49.97 m

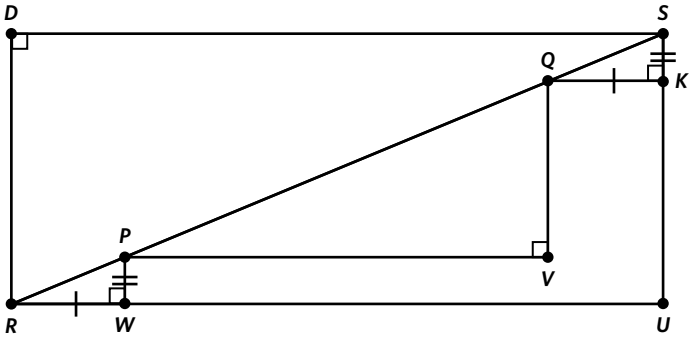
19. Four congruent triangles make a square with side lengths equal to c. Show that the combined area of the four triangles and the inner square equals c^2.



20. The side length $\overline{AB} = x$ and the side length $\overline{BE} = x - 1$, and the distance $\overline{CE} = 4$. Determine the distance of $\overline{AB}$, $\overline{BE}$, $\overline{AD}$ and $\overline{BD}$.



21. What additional information is needed to determine the length of $\overline{PQ}$, if the the rectangle $SURD$ has an area of 17 cm^2 and width 7 cm and the length of $\overline{RP} = \overline{SQ}$.



A. The area of the triangle RPW.
B. The length of $\overline{RS}$ or the diagonal length of $\overline{DU}$.
C. The area of the triangle PVQ.
D. The length of $\overline{RW}$ and the length of $\overline{SK}$.
E. The area of the triangle SQK.

Remember this?

22. At Mount Buller, it is 4°C right now. The coldest ever temperature at Mount Buller is −11.4°C. What is the difference between the temperature right now and the coldest ever temperature?

A. −15.4°C **B.** −7.4°C **C.** 7.4°C **D.** 8°C **E.** 15.4°C

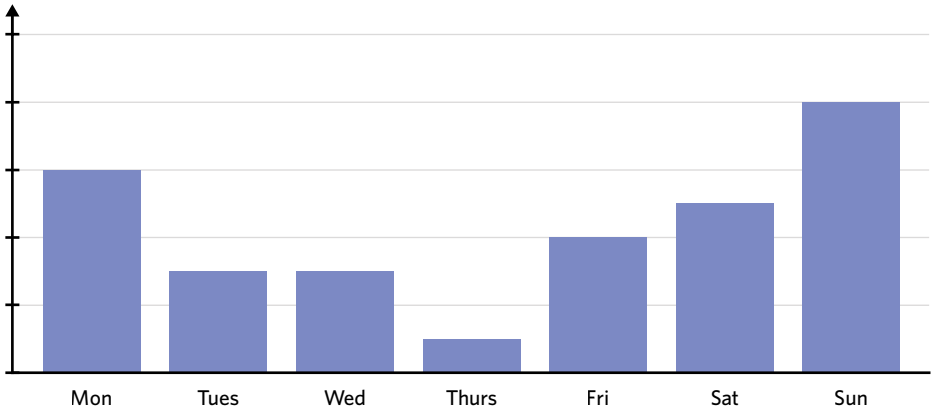
23. Before Anne can paint a canvas, she needs to cover it in gesso. She used 0.5 L of gesso to cover a 3 m by 2 m canvas. How many litres of gesso would be required to cover a canvas that is 2.5 m by 6 m?

A. 0.75 L **B.** 1 L **C.** 1.25 L **D.** 1.5 L **E.** 1.75 L

24. Claire recorded the number of hours she watched TikTok each day for a week.

She created a column graph but forgot to label some parts of it.

In total, she watched TikTok for 30 hours.



How many hours did she watch TikTok on Saturday?

A. 4 **B.** 5 **C.** 6 **D.** 7 **E.** 8

Chapter 13 extended application

1. Breeshey is building a foldable table for camping and found a design online. She currently has 3 wooden table tops that have a thickness of 20 mm. These 3 tops will be placed on top of the trestle legs' horizontal slats that she will build. Before building, Breeshey wants to make sure that the measurements in her design will be able to fit in her car and with her existing camping equipment. Her measurements are in millimetres.

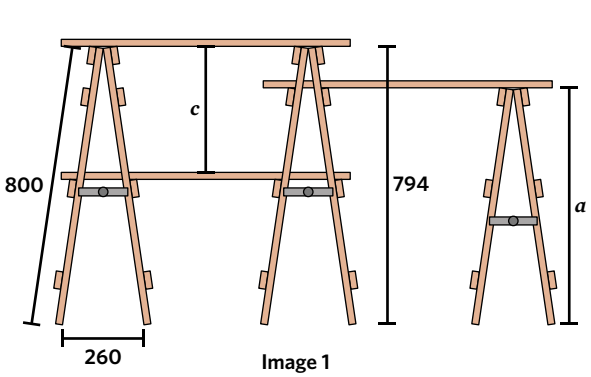


Image 1

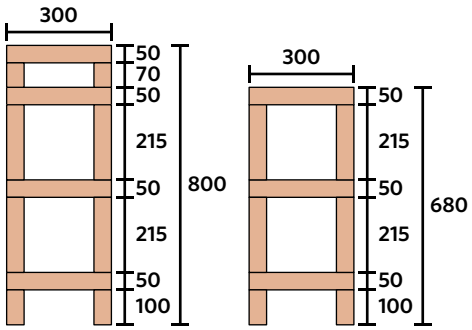


Image 2

a. The height of a is 674 mm and length of the shorter trestle legs are 680 mm, calculate the distance between these 2 trestle legs to the nearest millimetre.

b. Currently the design states that when the legs open to a certain degree, the height will be 794 mm and the gap between the 2 longer trestle legs will be 260 mm forming an isosceles triangle. Determine if these measurements are correct.

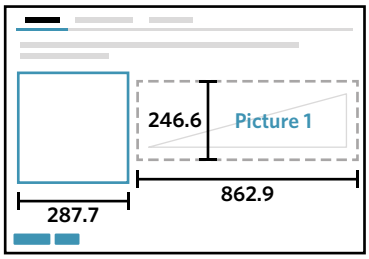
c. The angle between the 2 trestle legs in part **b** is 20°. Determine the new height and distance between the longer trestle legs if the angle is increased to 60°.

d. Given that the angle between the longer trestle legs is 60° and the measurements of Image 2, determine the height of c to the nearest whole number.

e. The type of wood that Breeshay wants to use is merbau wood which comes in 1.8 m lengths that cost $11.20 each. Assume that the wood cannot be reconnected once it is cut. Determine the total number of 1.8 m lengths of timber she will need to build her trestle legs and calculate their total price.

f. A brand new plastic trestle table that folds up can range from $30 − $250. What are the pros and cons of building a wooden table instead of buying a plastic trestle table?

2. Chris and Kai are engineers at Edrolo and are redesigning the Edrolo platform for new question types for the online platform. The engineers need to code in such a way so that when the pictures are dragged and dropped into the blue square, the pictures will shrink without causing distortion. The pictures are measured in pixels.



Images waiting to be dragged and dropped.

What happens when you drag the image.

Ideal look for platform.

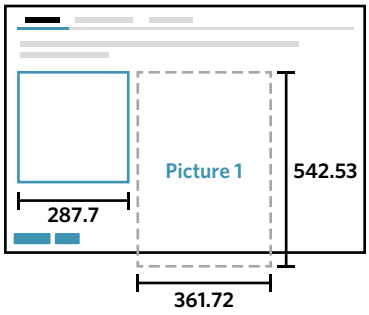
a. Determine the diagonal length in pixels of the drop zone. Give your answer to 1 decimal place.

b. Determine the diagonal length in pixels of Picture 1. Give your answer to 1 decimal place.

c. Chris says that the code is dependent on diagonals. He says the length and width of Picture 1 will need to be shrunk by using the following percentage $\left(\frac{\text{diagonal of dropzone}}{\text{diagonal of picture}}\right) \times 100\%$. Determine the new length and width of Picture 1 and if the new scaled picture will fit in the drop zone. Give your answer to the nearest whole number.

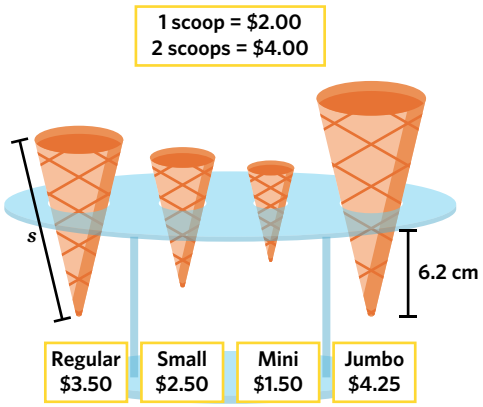
d. Kai says that the code is to shrink the length and width of Picture 1 using the following percentage $\left(\frac{\text{diagonal of dropzone}}{\text{diagonal of picture}}\right)^2 \times 100\%$. Determine the new length and width of Picture 1 and if her method will shrink Picture 1 to fit inside the dropzone. Give your answer to 1 decimal place.

e. Determine if the formula used in part **d** will work for Picture 2.



f. One method Chris and Kai could use is trial and error when finding ways to shrink the pictures. Give pros and cons to using more advanced maths techniques to scale images.

3. Agusta is customising a new stand to showcase her gelato cones in her gelato store. She sells mini, small, regular, and jumbo. The size of the mini cone is $\frac{1}{4}$ the size of the jumbo cone, and the size of the small cone is $\frac{2}{3}$ the size of the regular cone, and the size of the jumbo cone is $\frac{4}{3}$ the size of the regular cone. The size of each cone refers to its diameter and height.



a. Determine the diameter of the other 3 ice cream cones if the regular size cone diameter is 6 cm.

b. Determine the height of the 4 cones if the slant height of the regular cone is 12 cm to 2 decimal places.

c. The diameter of the holes that the cones will go through on the stand can be calculated using the ratio cone height : cone diameter. If the jumbo cone sticks out 6.2 cm from the hole, determine the diameter of the hole. Round your answer to 1 decimal place.

d. The diameter of the hole for each cone and the height that sticks out from the hole is proportional. Determine the size of each of the holes that are needed for the cones to be placed into the stand. Round your answer to 1 decimal place.

e. The surface area of the ice cream cones can be calculated using the formula πrs, where r is the radius, and s is the slant height. Augusta wants to make sure her customers are getting the better buy based on the surface area of the cone. Determine the surface area for the mini, small, regular, and jumbo cone to determine which one cone is the better buy.

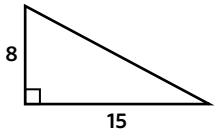
f. Draw another option of how Agusta could present her cones to customers that doesn't require a stand.

Chapter 13 review

Multiple choice

1. What is the missing value in the box for the equation below the right-angled triangle? 13A

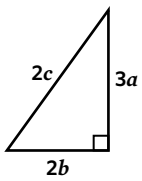
$\square^2 + 15^2 = 17^2$



A. 2
B. 4
C. 6
D. 8
E. 17

---

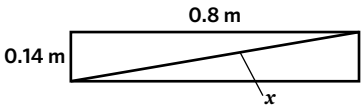
2. Which of the following is the correct equation to determine the length of the hypotenuse, in terms of c? 13B



A. $c = \frac{3a^2 + 2b^2}{2}$
B. $c = \frac{\sqrt{3a^2 + 2b^2}}{4}$
C. $c = \frac{\sqrt{9a^2 + 4b^2}}{2}$
D. $c = \frac{9a^2 + 4b^2}{4}$
E. $c^2 = \frac{\sqrt{9a^2 + 4b^2}}{4}$

---

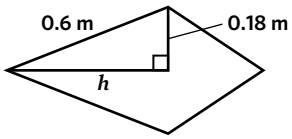
3. What is the value for x in the diagram below, correct to 3 decimal places? 13B



A. 0.657 m
B. 0.812 m
C. 0.854 m
D. 0.970 m
E. 1.034 m

---

4. What is the value for h in the diagram below, correct to 3 decimal places? 13C

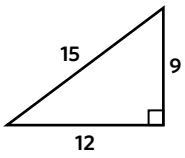


A. 0.164 m
B. 0.300 m
C. 0.328 m
D. 0.572 m
E. 0.626 m

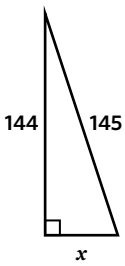
Fluency

5. Generate the equation that proves that the following triangles are right-angled triangles using Pythagoras' theorem, $a^2 + b^2 = c^2$. 13A

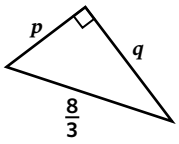
a.



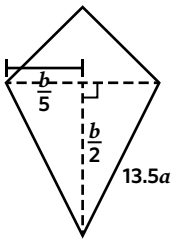
b.



c.



d.



6. Determine if the dimensions are correct for a right-angled triangle, where c is the length of the longest side, and a and b are the lengths of the shorter sides of a triangle. 13A

a. $a = 12, b = 5, c = 13$

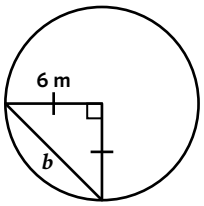
b. $a = 63, b = 16, c = 64$

c. $a = 16, b = \sqrt{225}, c = 34$

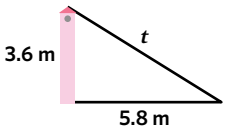
d. $a = \frac{11}{2}, b = 30, c = \frac{61}{2}$

7. Use a calculator to calculate the lengths of the hypotenuse for the right-angled triangles. Round your answers to two decimal places. 13B

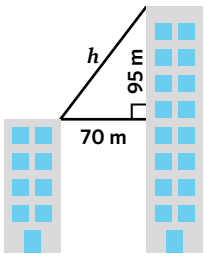
a.



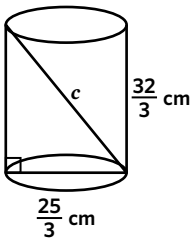
b.



c.

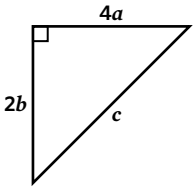


d.

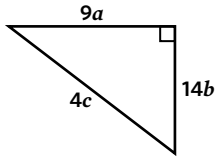


8. Rearrange Pythagoras' formula so that c is the subject. 13B

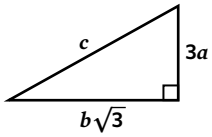
a.



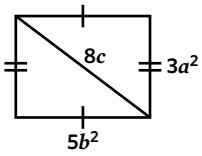
b.



c.

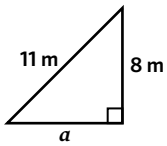


d.

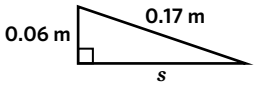


9. Calculate the missing side lengths. Give your answers in exact form. 13C

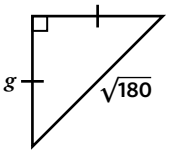
a.



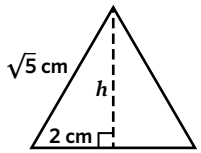
b.



c.

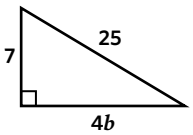


d.

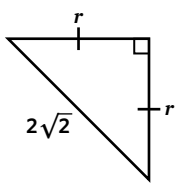


10. Use algebra to determine the values of the variables. Round your answers to two decimal places where necessary. 13C

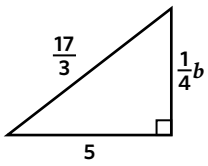
a.



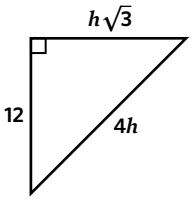
b.



c.

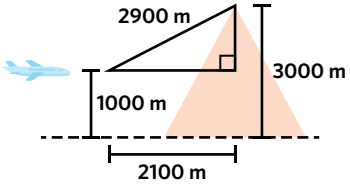


d.

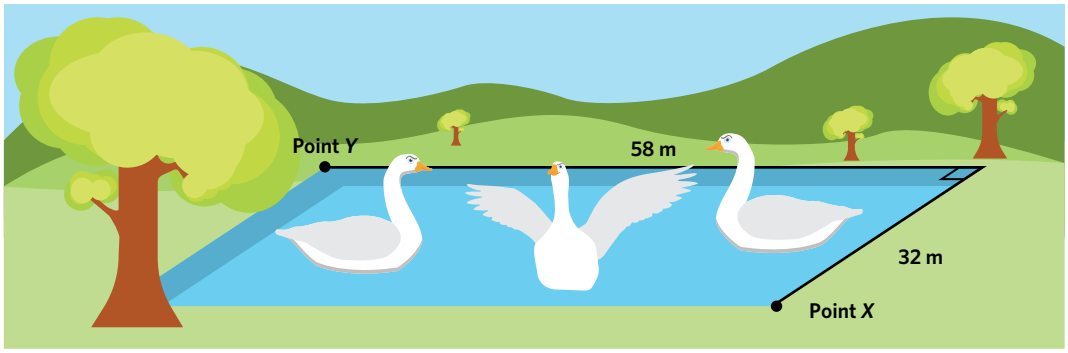


Problem solving

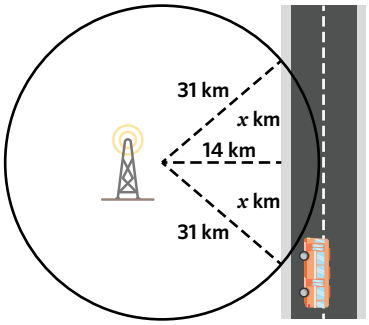
11. A plane is flying at a horizontal distance of 2100 m away from the base of a mountain and at an altitude of 1000 m in the air. It is due to fly 2900 m in the trajectory shown below, towards a mountain peak 3000 m high. Determine if the flight path is possible. 13A



12. Rachel is currently at point *X* and wants to cross to Point *Y*. There is a rectangular pond, however she cannot walk through it as it is filled with mischievous geese. Therefore, Rachel must walk 32 metres directly north and 58 metres directly west. How much further is this compared to if Rachel could walk straight through the pond? Round your answer to the nearest metre. 13B

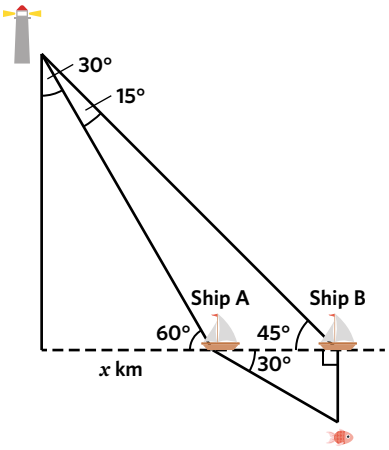


13. Tourists travel in a shuttle bus from the airport to their respective hotels in a rural neighbourhood. There is a cell tower on the route, and there is signal coverage as far as 31 km away from the tower. The bus passes the tower through the road shown in the diagram below, where its closest point to the tower is 14 km away. For how many kilometres will people on the bus have signal? Round your answer to two decimal places. 13C



Reasoning

14. A lighthouse sits on top of a hill and overlooks two fishing boats (Ship A and Ship B) in the water. The two fishing boats are also trying to fish for the same fish as it is a rare species. This is shown in the diagram below.



a. The crew on ship A look up at the lighthouse at a 60° angle and the viewers of the lighthouse look down at Ship A at a 30° angle. Determine the height of the hill and length of the line of sight between the lighthouse and Ship A in terms of x.

b. Assume that $x = 4.2$ km. What is the height of the hill? Round your answer to two decimal places.

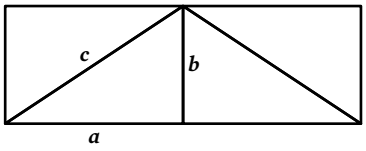
c. How far is Ship B from the lighthouse? Round your answer to two decimal places.

d. Find the distance between Ship A and Ship B.

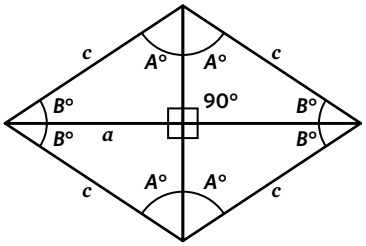
e. There are 388 working lighthouses in Australia and over 18 600 lighthouses worldwide. Why are lighthouses essential when we have modern day navigation technology?

---

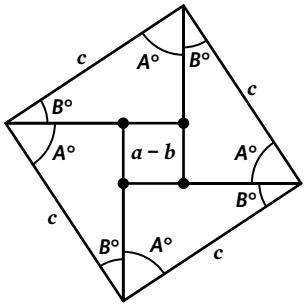
15. Four congruent triangles are cut out of a rectangular piece of cardboard. Two students determine they have made a square with side length c using the four triangles.



Student A



Student B



a. Determine the combined angle measurement of $A + B$.

b. Determine if Student A and/or Student B has made a square with side length c.

c. Explain how it is possible to determine which of the students made a square with the given information only.

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Back to contents

Answers

Image: Marish/Shutterstock.com

1A Adding and subtracting whole numbers

Student practice

Worked example 1

a. 45 **b.** 211

Worked example 2

a. 658 **b.** 1753

Understanding worksheet

1. **a.** = **b.** ≠ **c.** = **d.** ≠

2. **a.** = **b.** ≠ **c.** ≠ **d.** =

3. commutative; associative; subtraction; partitioning; compensation

Fluency

4. **a.** 17 **b.** 19 **c.** 74 **d.** 193
e. 211 **f.** 389 **g.** 273 **h.** 661

---

5. **a.** 68 **b.** 489 **c.** 9 **d.** 152
e. 11 086 **f.** 887 **g.** 384 **h.** 53 293

---

6. **a.** 776 **b.** 5986 **c.** 4421 **d.** 170
e. 869 **f.** 9066 **g.** 8929 **h.** 6235

---

7. **a.** 399 **b.** 859 **c.** 321 **d.** 655
e. 6193 **f.** 1386 **g.** 5591 **h.** 4359

---

8. **a.** 41 **b.** 654 **c.** 208 **d.** 4691
e. 6713 **f.** 3496 **g.** 9028 **h.** 5290

---

9. B

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

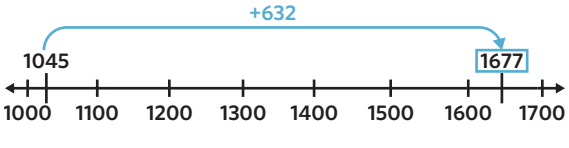
11. Tao is an Uber driver. Over two weeks he earned $1045 and $632. How much did Tao earn in total?

Key points

- Over two weeks Tao earned $1045 and $632.
- How much did Tao earn in total?

Explanation

Tao's total earnings = earnings for week 1 + earnings for week 2
= $1045 + $632
= $1677



1045 + 632 = 1677

Answer

Tao earned $1677 in total over two weeks.

12. There are 1213 students that attend Plight Street secondary school. 801 of the students travel to school by public transport. How many students do not travel by public transport to get to Plight Street secondary school?

Key points

- There are 1213 students that attend Plight Street secondary school.
- 801 of the students travel to school by public transport.
- How many students do not travel by public transport to get to Plight Street secondary school?

Explanation

Number of students who do not use public transport = total number of students − number of students who use public transport
= 1213 − 801
= 412

Answer

412 students do not travel by public transport to get to Plight Street secondary school.

13. Steven got a new job and needs to buy a whole new outfit. He has a budget of $900. He buys a $557 suit, a $125 tie, and matching $212 shoes. How much money does Steven have remaining after all his purchases?

Key points

- Steven has a $900 budget to buy a new outfit.
- He buys a $557 suit, a $125 tie, and matching $212 shoes.
- How much money does Steven have remaining after all his purchases?

Explanation

Money remaining after purchases = budget − money used for purchases
= 900 − 557 − 125 − 212
= 6

Answer

Steven has $6 remaining after all his purchases.

14. When Charlotte started her bike ride her bike monitor already displayed 147 km. Halfway through her ride the monitor showed 169 km. By the end of her ride, how far had Charlotte travelled?

Key points

- When Charlotte started her bike ride her bike monitor already displayed 147 km.
- Halfway through her ride the monitor showed 169 km.
- By the end of her ride, how far had Charlotte travelled?

Explanation

Total distance travelled = distance displayed halfway through Charlotte's ride − distance displayed at start of bike ride, multiplied by two
= 169 − 147
= 22 × 2
= 44 km

Answer

Charlotte has travelled a total of 44 km.

15. Winston spent 450 minutes in total working in his garden over four different Sundays. In the first week he spent two hours gardening, in the second week 118 minutes and in the third week an hour and a half. How much time did Winston spend gardening in the fourth week?

Key points

- Winston spent 450 minutes in total working in his garden over four different Sundays.
- In the first week he spent two hours gardening.
- In the second week 118 minutes and in the third week an hour and a half.
- How much time did Winston spend gardening in the fourth week?

Explanation

Time spent gardening in the fourth week
= total time spent over 4 weeks − time spent in week 1
− time spent in week 2 − time spent in week 3

$= 450 - 2 \times 60 - 118 - 60 \times 1.5$

$= 450 - 120 - 118 - 90$

$= 122$ minutes

Answer

Winston spent 122 minutes gardening in the fourth week.

Reasoning

16. a. The balance of the account on the 1st of August is $1595.

b. They have $805 remaining in their account after paying rent.

c. They can expect to have $90 in the account at the end of August.

d. If Matthew contributed that same amount of money at the start of August, the balance would have been $235 at the end of August.

e. Suggested option 1: Tori and Matthew can use a budgeting application to keep track of their expenses.

Suggested option 2: Tori and Matthew can plan to set aside $200 every month for unexpected expenses.

Note: There are other possible options.

17. a. $=$ **b.** $\neq$

c. The answer for part **a** is $=$, but the answer for part **b** is $\neq$. They are different because the commutative law applies to addition and not subtraction.

Extra spicy

18. 13, 48 and 39

19. D

20. C

21. 149 and 67

Remember this?

22. C **23.** B **24.** A

1B Multiplying and dividing whole numbers

Student practice

Worked example 1

a. 87 000 **b.** 270

Worked example 2

a. 78 **b.** 13

Worked example 3

a. 5145 **b.** 223 r 5 or $233\frac{5}{12}$

Understanding worksheet

1. a. Commutative law **b.** Associative law

c. Associative law **d.** Commutative law

2. a.

5	10	3
	50	15

b.

6	50	8
	300	48

c.

5	50	9
	250	45

d.

6	40	7
	240	42

3. dividend; divisor; quotient; product

Fluency

4. a. 60 **b.** 4100

c. 12 300 **d.** 257 000

e. 650 000 **f.** 8 109 000

g. 2 400 000 **h.** 7 550 000

5. a. 10 **b.** 32 **c.** 8 **d.** 49

e. 53 **f.** 707 **g.** 200 **h.** 81 000

6. a. 48 **b.** 135 **c.** 112 **d.** 432

e. 192 **f.** 363 **g.** 1008 **h.** 2150

7. a. 16 **b.** 19 **c.** 14 **d.** 13

e. 14 **f.** 103 **g.** 102 **h.** 1002

8. a. 203 **b.** 524

c. 319 **d.** 1728

e. 3344 **f.** 9460

g. 136 952 **h.** 74 806 277

9. a. 18 **b.** 12 r 1 or $12\frac{1}{8}$

c. 26 r 2 or $26\frac{2}{5}$ **d.** 13 r 2 or $13\frac{2}{11}$

e. 22 r 2 or $22\frac{1}{6}$ **f.** 211 r 7 or $211\frac{7}{16}$

g. 218 r 2 or $218\frac{2}{19}$ **h.** 1115 r 6 or $1115\frac{2}{9}$

10. B

Spot the mistake

11. a. Student B is incorrect. **b.** Student B is incorrect.

Problem solving

12. Holly counts lollies to place into party bags for her birthday. If there are 120 lollies and a total of 10 party bags, how many lollies should she put into each bag?

Key points

- There are 120 lollies.
- There are 10 party bags.
- How many lollies should she put into each bag?

Back to contents

Explanation

Lollies per bag = total lollies ÷ total bags

= 120 ÷ 10

When dividing by 10, move the numbers one place to the right.

120 ÷ 10 = 12

Answer

12 lollies should go into each bag.

13. Alex receives \$5 pocket money each week and saves it all up for a \$448 Xbox Series S Console. How many weeks would Alex need to save for?

Key points

- Alex receives \$5 each week.
- Alex is saving up \$448.
- How many weeks does Alex need to save for?

Explanation

Number of weeks = total money needed ÷ money received each week

= 448 ÷ 5

```
  0  8  9  r 3
5)4 ⁴4 ⁴8
```

So 448 ÷ 5 = 89 r 3

89 r 3 > 89, so 89 weeks will not be enough to save \$448.

So round up to 90 weeks.

Answer

Alex would need to save for 90 weeks.

14. The current time is 14:00. How many minutes have passed since midnight?

Key points

- The current time is 14:00.
- Midnight in 24-hour time is 0:00.
- There are 60 minutes in 1 hour.
- How many minutes have passed since midnight?

Explanation

Number of hours passed = 14:00 − 0:00

= 14 hours

Number of minutes passed = number of hours passed × number of minutes in one hour

= 14 × 60

= (10 + 4) × 60

= (10 × 60) + (4 × 60)

= 600 + 240

= 840 minutes

Answer

840 minutes have passed since midnight.

15. Cassie finds a cookie recipe that makes 10 cookies. The recipe requires 75 g flour, 50 g butter, 50 g condensed milk and 20 g caster sugar. How many grams of each ingredient does Cassie need to make 40 cookies?

Key points

- The recipe makes 10 cookies.
- The recipe requires 75 g of flour.
- The recipe requires 50 g of butter.
- The recipe requires 50 g of condensed milk.
- The recipe requires 20 g of caster sugar.
- How many grams of each ingredient does Cassie need to make 40 cookies?

Explanation

Calculate how many batches of the recipe Cassie needs to make 40 cookies.

Number of batches of recipe required = number of cookies required ÷ number of cookies recipe makes

= 40 ÷10

When dividing by 10, move the numbers one place to the right.

40 ÷10 = 4

To calculate the number of grams of each ingredient Cassie needs, multiply each quantity in the recipe by 4.

75 × 4 = 300 g, so 300 g of flour required.

50 × 4 = 200 g, so 200 g of butter required, and 200 g of condensed milk required.

20 × 4 = 80 g, so 80 g of caster sugar required.

```
   +2
   7 5       5 0       2 0
+    4    ×    4    ×    4
 -----     -----     -----
 3 0 0     2 0 0       8 0
```

Answer

Cassie needs 300 g flour, 200 g butter, 200 g condensed milk and 80 g caster sugar.

16. Exercise books are on sale for 35c each. Karylle requires 2 books per subject and studies 7 subjects at school. How much will it cost Karylle to buy exercise books for every subject? Give your answer in dollars and cents.

Key points

- Exercise books cost 35c.
- Karylle requires 2 books per subject.
- Karylle studies 7 subjects at school.
- How much will it cost Karylle to buy exercise books for every subject? Give your answer in dollars and cents.

Explanation

Total books required = books per subject × number of subjects studied

= 2 × 7

= 14 books

Total cost = total books required × cost per book

= 14 × 35

= (10 × 35) + (4 × 35)

= 350 + 140

= 490 cents

There are 100c in \$1. To find the number of dollars, divide the number of cents by 100.

```
    0  0  4  r 90
100)4 ⁴9 ⁴⁹0
```

So the cost is 4 dollars and 90 cents.

Answer

It will cost Karylle 4 dollars and 90 cents to buy the exercise books.

Reasoning

17. **a.** Charlotte would earn \$120.

b. Charlotte needs to work 167 hours to save up for the car.

c. It would take Charlotte 16 weeks to save up for the car.

d. Suggested option 1: Charlotte can increase her working hours each week so that her total pay increases.

Suggested option 2: Charlotte could find a different job that has a higher hourly rate.

Note: There are other possible options.

18. **a.** 10 **b.** 100
c. The product of multiplying by 100 is 10 times the product of multiplying by 10.

Extra spicy

19. 5 pm **20.** 189 000 mm^2

21. C **22.** C

Remember this?

23. C **24.** E **25.** E

1C Adding and subtracting integers

Student practice

Worked example 1

a. 2 **b.** −3

Worked example 2

a. −6 **b.** 2

Understanding worksheet

1. **a.** 6 − 8 **b.** −3 + 7
c. 4 + (−3) **d.** −1 − 5

2. **a.** + **b.** − **c.** − **d.** +

3. whole; zero; negative; subtracting; positive; increase; decrease

Fluency

4. **a.** 2 **b.** −8 **c.** −6 **d.** 9
e. −13 **f.** 53 **g.** −106 **h.** 129

5. **a.** 3 **b.** −4 **c.** 7 **d.** −16
e. 4 **f.** −143 **g.** −31 **h.** −305

6. **a.** −2 **b.** −12 **c.** −12 **d.** −25
e. −5 **f.** −100 **g.** −66 **h.** −401

7. **a.** 8 **b.** −6 **c.** 20 **d.** −11
e. 0 **f.** 105 **g.** −85 **h.** 14

8. **a.** −4 **b.** 10 **c.** −12 **d.** −14
e. 9 **f.** 10 **g.** 25 **h.** −17

9. C

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. An iceberg floating in the Arctic Ocean is submerged to −18 metres. If 2 metres of the iceberg is sticking out of the water, how tall is the iceberg altogether?

Key points

- An iceberg floating in the Arctic Ocean is submerged to −18 metres.
- 2 metres of the iceberg is sticking out of the water.
- How tall is the iceberg altogether?

Explanation

To find the total height of the iceberg, add the height of the iceberg above water level and the height below water level.

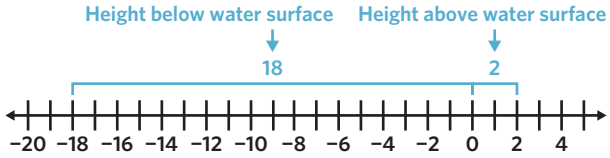
The height above water level is 2 metres.

The height below water level is 18 metres.

2 + 18 = 20 metres

Alternatively:

height of the iceberg = highest point − lowest point
= 2 − (−18)
= 2 + 18
= 20 metres



Answer

The iceberg is 20 metres long.

12. Mikko's bank account has a balance of −$50. $67 is withdrawn from his account. What is the balance of Mikko's account after the withdrawal?

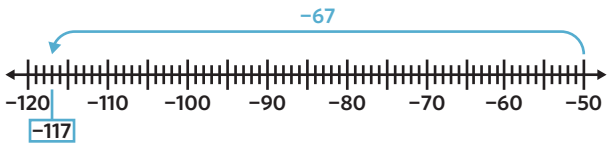
Key points

- Mikko's bank account has a balance of −$50.
- $67 is withdrawn from his account.
- What is the balance of Mikko's account after the withdrawal?

Explanation

Withdrawing $67 means that there is $67 less in Mikko's bank balance.

−50 − 67 = −117



Answer

Mikko's account balance is −$117.

13. A scuba diver dives to 21 metres below sea level. She must make a safety stop at 5 metres below sea level on her way back up to the surface to decompress her body. How far towards the surface did the diver have to swim before making her safety stop?

Key points

- A scuba diver dives to 21 metres below sea level.
- She must make a safety stop at 5 metres below sea level.
- How far towards the surface did the diver have to swim before making her safety stop?

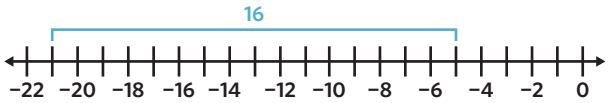
Explanation

Find the distance the diver swims before making her safety stop.

The diver's initial position is −21 m.

The diver's final position is −5 m.

Distance = final position − initial position
= −5 − (−21)
= −5 + 21
= 16 m



Answer

The diver had to swim 16 metres before making her safety stop.

14. In Australia, the highest temperature ever recorded was 51°C in Western Australia in 2022. The lowest temperature ever recorded was −23°C in New South Wales in 1994. What is the difference between the highest and lowest temperatures ever recorded in Australia?

Key points

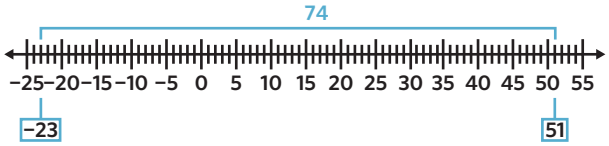
- The highest temperature ever recorded was 51°C.
- The lowest temperature ever recorded was −23°C.
- What is the difference between the highest and lowest temperatures?

Explanation

Difference = highest temperature − lowest temperature

$= 51 - (-23)$

$= 74$°C



Answer

The difference between the highest and lowest temperatures is 74°C.

15. An ion is a charged atom because there are different numbers of positive protons and negative electrons inside of it. If an ion has a charge of −2 and contains 8 protons, how many electrons does it contain?

Key points

- An ion has a charge of −2.
- The ion contains 8 protons.
- How many electrons does it contain?

Explanation

Let the number of electron be x.

There are 8 positive protons.

The overall charge is −2.

$x + 8 = -2$

$x = -10$

Answer

The ion contains 10 electrons.

Reasoning

16. a. The distance to the motel from the camp is 11 km.

b. The distance to the car park from the camp is −11 km.

c. The distance to the waterfall from the camp is 5 km.

d. Paulie and Christopher drove 1 km further compared to how far they walked.

e. Suggested option 1: They could have parked closer to where they set up camp.

Suggested option 2: They could park their car at the motel and camp between the waterfall and the motel.

Note: There are other possible options.

17. a. $x = 9, y = -9, x - y = 18$

b. $x = -9, y = 9, x - y = -18$

c. Subtraction only decreases a value if the integer being subtracted is positive. Subtraction increases a value if the integer being subtracted is negative.

Extra spicy

18. A **19.** E **20.** 1993 **21.** −33

Remember this?

22. C **23.** D **24.** C

1D Multiplying and dividing integers

Student practice

Worked example 1

a. −30 **b.** 28 **c.** 60

Worked example 2

a. −3 **b.** 8

Understanding worksheet

1. a. Positive integer **b.** Negative integer

c. Negative integer **d.** Positive integer

2. a. − **b.** − **c.** + **d.** +

3. direction; left; right; positive; opposite

Fluency

4. a. −8 **b.** −14 **c.** 0 **d.** 6

e. 27 **f.** 60 **g.** −84 **h.** 96

5. a. −6 **b.** −14 **c.** −16 **d.** 0

e. 40 **f.** 60 **g.** 48 **h.** −84

6. a. −3 **b.** 2 **c.** −4 **d.** −5

e. 7 **f.** 2 **g.** −16 **h.** 19

7. a. −52 **b.** −3 **c.** 0 **d.** 0

e. −40 **f.** 2070 **g.** 54 **h.** 241

8. A

Spot the mistake

9. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. Tyson pays $180 every week for his room in a sharehouse. This includes his rent and $30 for electricity, gas and internet access. Over 20 weeks, how much did Tyson pay for only his rent?

Key points

- Tyson pays $180 every week for his room.
- The $180 he pays includes rent as well as $30 for electricity, gas and internet access.
- How much did Tyson pay only for rent over 20 weeks?

Explanation

We first find how much Tyson pays only in rent every week. As the $180 includes the $30 he pays for electricity, gas and internet, we need to deduct $30 from $180 to find out how much he pays purely in rent.

$180 - 30 = 150$

This means that Tyson pays $150 in rent every week.

We then find how much he pays over 20 weeks.
To do this,we multiply $150 by 20.

$150 \times 20 = \$3000$

Therefore, Tyson pays $3000 in rent over 20 weeks.

Answer

Tyson pays $3000 in rent over 20 weeks.

11. Jan is competing on a trivia game show. $250 is deducted every time a contestant gives an incorrect answer. If Jan has earned $2500 and then incorrectly answers three questions, how much money will she be taking home?

Key points

- An incorrect answer leads to a deduction of $250.
- So far, Jan has earned $2500.
- Jan incorrectly answers 3 questions.
- How much money will Jan bring home?

Explanation

To find how much money Jan takes home, we need to find how much is deducted from her earned amount of $2500 because of her incorrect answers.

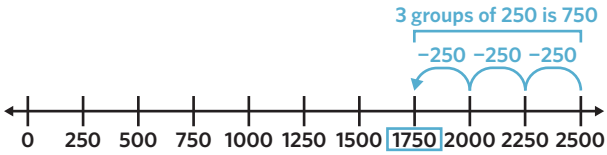
As Jan incorrectly answered 3 questions, and each incorrect answer leads to a deduction of $250, this means her total amount deducted will be $250 multiplied by 3.

$250 \times 3 = 750$

This means that we need to deduct $750 from the total earned amount of $2500.

$2500 - 750 = \$1750$

Jan will bring home $1750.



Answer

Jan will take home $1750.

12. On a school camp, a box of apples is put out for the students. Throughout the day, a total of 266 students came and took apples out of the box until there were only 2 left. If there were 800 apples to begin with, then how many apples on average did each student take?

Key points

- 266 students took apples out of the box throughout the day.
- Only 2 apples were left.
- There were 800 apples to begin with.
- How many apples did each student take on average?

Explanation

If there were 800 apples to begin with at the beginning of the day and then only 2 left by the end, this means a total of 798 apples were taken by students.

As it says that 266 students took apples out of the box, this means that the 798 apples were taken by 266 students.

To find how many apples each student took on average, we divide the number of apples taken by the number of students:

$768 \div 298 = 3$

Therefore, each student took 3 apples on average.

Answer

Each student took 3 apples.

13. Jez is playing a computer game where he can respawn after failing a mission. Every time he respawns, it costs him 1200 coins in the game. Jez has already collected 34 000 coins. How many times can he respawn before he runs out of coins and has to restart the mission?

Key points

- Each respawn costs Jez 1200 coins.
- So far, Jez has collected 34 000 coins.
- How many times can Jez respawn before he runs out of coins?

Explanation

To find how many times Jez can respawn, we divide the number of coins he has collected by the number of coins it costs him each time he respawns.

$34\,000 \div 1200 = 28.333$

Since the number of times Jez can respawn cannot be a decimal or fraction, the maximum number of times he can respawn before he runs out of coins is 28.

Answer

Jez can respawn 28 times before he runs out of coins.

14. Gary is paying for a new gaming console in weekly instalments of $140. If Gary had −$40 in his account after 4 weeks of regular payments, then what was the balance of Gary's account to begin with?

Key points

- Gary pays weekly instalments of $140 for his console.
- After 4 weeks of payments, Gary had −$40 in his account.
- What was Gary's starting account balance?

Explanation

To find how much was in Gary's account to begin with, we need to add back the amount that Gary paid over the 4 weeks to the ending balance of −$40.

As Gary paid weekly instalments of $140, over 4 weeks he would have paid $140 \times 4 = \$560$.

We then add $560 to the ending balance of −$40:

$-40 + 560 = 520$

Therefore, Gary's account balance was $520 to begin with.

Answer

Gary had $520 in his account to begin with.

Reasoning

15. **a.** The depth of the Mariana Trench is −11 000 m.

b. Light can travel an additional 800 m from the Sunlight Zone.

c. 25 Sunlight zones can fit inside the Hadal Zone.

d. There are 1101 atm at the deepest part of the ocean.

e. Suggested option 1: It may be dangerous for explorers and the potential loss of life outweighs the benefits of gathering new data.

Suggested option 2: The cost of the expeditions may be less than the potential future value of the discoveries that marine biologists make.

Note: There are other possible options.

16. a.

$x = 1$
$x \times (-1) = -1$
$x \times (-1) \times (-1) = 1$
$x \times (-1) \times (-1) \times (-1) = -1$

b.

$y = -1$
$y \times (-1) = 1$
$y \times (-1) \times (-1) = -1$
$y \times (-1) \times (-1) \times (-1) = 1$

c. The results alternate between positive and negative. This shows that multiplying by a negative changes the direction of the result.

Note: Your answers will vary depending on what value is substituted for x and y. The signs should be the same as shown in the solutions for parts **a** and **b**.

Extra spicy

17. E **18.** D **19.** 94

20. 2 cm, 6 cm, 10 cm

Remember this?

21. B **22.** D **23.** D

1E Order of operations

Student practice

Worked example 1

a. -54 **b.** 3

Worked example 2

a. 1 **b.** -8

Worked example 3

a. $7 - (24 \div 2) = -5$ **b.** $12^2 \div (-3 \times 6) = -8$

Understanding worksheet

1. a. $13 + (12 \div 6) - 4$ **b.** $12 \times (3^3) \div 10$ **c.** $(18 \div 3) \times 13 \div 13$ **d.** $-8 \times [19 - (3 \times 42)]$

2. a. $(3 - 6) - 9$ **b.** $\frac{3 \times 30}{2}$ **c.** $3 + (-9 - -4)$ **d.** $(-12 \div 2)^2 - 8$

3. operations; order; inverse; brackets

Fluency

4. a. 33 **b.** -8 **c.** -16 **d.** 12
e. -29 **f.** -71 **g.** 16 **h.** 12

5. a. -8 **b.** -6 **c.** -13 **d.** -36
e. 9 **f.** -1276 **g.** 7 **h.** -10

6. a. -6 **b.** 0 **c.** -1 **d.** -25
e. 26 **f.** 19 **g.** 12 **h.** 6

7. a. -1 **b.** -6 **c.** 7 **d.** 27
e. -5 **f.** 28 **g.** 6 **h.** 5

8. a. $(2 + 2) \times 4 = 16$
b. $-15 - (10 \times -8) = 65$
c. $(-4 - 13) \times -2 = 34$
d. $(6 - 4^2) \times -2 = 20$
e. $(5 - (-3)^2) \times -7 = 28$
f. $-36 + \left(\frac{\sqrt{144}}{2}\right) = -30$
g. $-3 \times [2 \times (5 - 3)] = -12$
h. $\frac{-30}{60 \times 2 \div (3 \times 4)} = -3$

9. a. $5 - \frac{20}{2} = -5$ or $5 - 20 \div 2 = -5$
b. $(-20 + 13) - 9 = -16$ or $-20 + 13 - 9 = -16$
c. $-5 \times 4 + 33 = 13$ or $(-5 \times 4) + 33 = 13$
d. $3 \times (-8 + -2) = -30$
e. $\frac{15 \times 3}{-5} = -9$ or $(15 \times 3) \div -5$ or $15 \times 3 \div -5$
f. $(-2 \times 3) \times (-66 \div 11) = 36$
or $-2 \times 3 \times (-66 \div 11) = 36$
g. $\frac{-8 \times 2}{4^2} = -1$ or $(-8 \times 2) \div 4^2$
h. $\left(\frac{6}{2}\right)^2 = 9$ or $(6 \div 2)^2 = 9$

10. E

Spot the mistake

11. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

12. In a game of AFL, each goal is worth six points and each behind is worth one point. Hawthorn scored 79 points in total. If they scored 11 goals, how many behinds did they kick?

Key points

- Each goal is worth six points.
- Each behind is worth one point.
- Hawthorn scored 79 points in total.
- They scored 11 goals.
- How many behinds did they kick?

Explanation

Since points are scored from points and behinds,

Points from behinds = total points − points from goals

= total points − (number of goals × points per goal)

= 79 − (11 × 6)

= 13

Each behind scores 1 point, so Hawthorn kicked 13 behinds.

79 | 79 | 11 × 6 number of goals × points per goal

Total points − **points from goals**

Answer

Hawthorn kicked 13 behinds.

13. Collyn is saving money for a new VR headset which costs $529.00. She has saved $33 a week over twelve weeks. How much more money does she need to save in order to purchase the VR headset?

Key points

- Collyn wants to buy a new VR headset that costs $529.
- She has saved $33 a week for 12 weeks.
- How much more money does she need to save to be able to purchase the VR headset?

Explanation

As Collyn has saved $33 a week over 12 weeks, the total amount she has saved after the 12 weeks is $33 × 12.

However, the VR headset that she wants to purchase is $529. Therefore, we need to subtract the total amount she has saved from $529 to find how much more money she needs.

$529 - 33 \times 12$

Using the order of operations, we evaluate multiplication first before subtraction.

$33 \times 12 = 396$

$529 - 396 = 133$

Therefore, Collyn needs to save $133 more to be able to buy the headset.

Answer

Collyn needs to save $133 more.

14. Niamh has collected 20 Pop vinyl bobble head figures, half of them have been kept in their original packaging. Her dad gave her 152 more figures. Niamh noticed that only an eighth of the figures from her dad were in their original packaging. How many figures in original packaging does Niamh have?

Key points

- Niamh has 20 figures.
- Half are in their original packaging.
- Her dad gave her 152 more figures.
- Only one eighth of the figures from her dad are in their original packaging.
- How many figures in original packaging does Niamh have?

Explanation

The figures in original packaging in total is the sum of the figures in original packaging from Niamh's initial collection and the figures in original packaging from her dad.

So total figures in original packaging
= (original collection ÷ 2) + (gifted figures ÷ 8)

$= (20 \div 2) + (152 \div 8)$

Using order of operations, brackets are evaluated first.

Total figures in original packaging $= 10 + 19$

$= 29$

Answer

Niamh has 29 figures in the original packaging.

15. Mateo wants to knit five jumpers and needs 1496 metres of yarn for each jumper. A single ball of yarn is 170 metres long and costs $5. A four pack of yarn is on sale for $16. How much will Mateo save by purchasing all the yarn for the jumpers when the price is discounted?

Key points

- Mateo wants to knit five jumpers.
- He needs 1496 metres of yarn for each jumper.
- One ball of yarn is 170 metres long and costs $5.
- A four pack of yarn is on sale for $16.
- How much will Mateo save by purchasing all the yarn for the jumpers when the price is discounted?

Explanation

To find how much Mateo will save, we can compare the price of purchasing all the yarn without a discount to the price with one and find the difference.

As Mateo wants to knit five jumpers, he needs $1496 \times 5 = 7480$ metres of yarn in total. One ball of yarn is 170 metres, and so he needs $7480 \div 170 = 44$ balls of yarn. However, he would only need 11 four-packs of yarn as $44 \div 4 = 11$.

Price without discount $= 44 \times \$5$

Price with discount $= 11 \times \$16$

Subtracting the price with the discount from the price without:

$44 \times 5 - 11 \times 16$

$= 220 - 176$

$= 44$

Answer

Mateo would save $44 by buying the discounted wool.

16. Zom-beads make necklaces to sell. A 40 cm necklace has fifty beads, and a 70 cm necklace has twice the amount of beads as the 40 cm. A 20 cm necklace is made with a quarter of the beads of a 70 cm necklace. How many beads are needed to make three 40 cm, five 70 cm and two 20 cm necklaces.

Key points

- A 40 cm necklace has fifty beads.
- A 70 cm necklace has twice the amount of beads as the 40 cm.
- A 20 cm necklace is made with a quarter of the beads of a 70 cm necklace.
- How many beads are needed to make three 40 cm, five 70 cm and two 20 cm necklaces.

Explanation

Total beads required = (3 × number of beads for 40 cm necklace)
+ (5 × number of beads for 70 cm necklace)
+ (2 × number of beads for 20 cm necklace)

A 40 cm necklace has 50 beads.

A 70 cm necklace has $2 \times 50 = 100$ beads.

A 20 cm necklace has $100 \div 4 = 25$ beads.

So total beads required $= (3 \times 50) + (5 \times 100) + (2 \times 25)$.

Brackets are simplified first according to order of operations.

$= 150 + 500 + 50$

$= 700$ beads

Answer

700 beads are needed to make the necklaces.

Reasoning

17. **a.** One crown-of-thorns sea star can destroy 250 square kilometres of coral over 25 years.

b. $\frac{1000}{50} = 20$. A scuba diver can inject 20 sea stars within 1 minute.

c. The dive was 48 minutes altogether.

d. The diver injected 277 sea stars in the last 20 minutes.

e. Suggested option 1: We can reduce the fishing of the predator of crown-of-thorns sea stars.

Suggested option 2: We can recruit more divers as part of government funding to inject crown-of-thorns sea stars.

Note: There are other possible options.

18. a. $3 \times (2 + 0 - 4 \div 1) = -6$

b. $4 \times (3 + 2 - 0 \div 1) = 20$

c. Given that one answer is negative and the other is positive, the results are possible as the brackets can either result in a positive or negative integer.

Extra spicy

19. $(-9 + 4)^3 \times 6$

20. Casey made $6576 more profit this year.

21. E **22.** D

Remember this?

23. D **24.** A

25. E

Chapter 1 extended application

1. a. −6371 km

b. −2890 km

c. Outer core, 2260 km

d. 1221 km

e. Suggested option 1: They are scarce and highly valued.

Suggested option 2: They are popular for their utility (E.g. to be used for jewellery).

Note: There are other possible options.

2. a. 3

b. Win: 3

Draw: 1

Loss: −1

c. 9

d. Koalas and Wombats

e. Suggested option 1: Participants should ensure they shake hands at the end of a game.

Suggested option 2: Participants could praise their fellow competitors.

Note: There are other possible options.

3. a. 160 hours

b. 4 weeks

c. 2206 weeks

d. Let C be the cost of a jumper, h be the number of hours

Equation: $C = 21h + 36$

Solid coloured jumpers: $540

Striped jumpers: $708

e. Suggested option 1: They could hire more workers.

Suggested option 2: They could outsource some of their manufacturing/knitting.

Note: There are other possible options.

Chapter 1 review

Multiple choice

1. B **2.** A **3.** D **4.** C **5.** A

Fluency

6.	**a.** 36	**b.** 67	**c.** 414	**d.** 329
7.	**a.** 99	**b.** 171	**c.** 123	**d.** 99 222
8.	**a.** 304	**b.** 713	**c.** 40 000	**d.** 40 003
9.	**a.** 49	**b.** 10 201	**c.** 16 652	**d.** 15
10.	**a.** −88	**b.** −186	**c.** −147	**d.** −351
11.	**a.** 180	**b.** −5	**c.** 34 155	**d.** −91
12.	**a.** 131	**b.** −361	**c.** 3	**d.** 38

13. a. $17 \times 70 = 1190$ **b.** $\frac{132}{-12} - 3 = -14$

c. $\frac{-92}{2} + (-22) = -68$ **d.** $\frac{4}{5} \times \sqrt{5 \times 17 - 4} = \frac{36}{5}$

Problem solving

14. Vutha has received his maths test paper. Vutha sees that he has got 18 marks out of 20 for multiple choice questions, 31 marks out of 40 for fluency questions, 17 marks out of 20 for problem solving questions and 6 marks out of 10 for reasoning questions. What is Vutha's overall mark out of 90, for this test?

Key points

- Vutha has got 18 marks for multiple choice questions, 31 marks for fluency questions, 17 marks for problem solving questions, and 6 marks for reasoning questions.
- What is Vutha's overall mark out of 90, for this test?

Explanation

Vutha's overall mark = Sum of marks Vutha got for each section of the test

$= 18 + 31 + 17 + 6$

$= 72$ marks

Answer

Vutha's overall mark for this test is 72 out of 90.

15. The pyramids were built in 2490 B.C., and the Luxor Temple was built in 1400 B.C.. Given the following timeline, where B.C. years are represented as negative integers, write an equation and calculate the number of years there are between the building of the pyramids and the Luxor Temple.

Key points

- The pyramids were built in 2490 B.C., and the Luxor Temple was built in 1400 B.C..
- On the timeline, B.C. years are represented as negative integers.
- Write an equation and calculate the number of years there are between the building of the pyramids and the Luxor Temple.

Explanation

Number of years between the building of the pyramids and the Luxor Temple = year Luxor Temple was built − year pyramids was built

$= -1400 - (-2490)$

$= 1090$

Answer

$-1400 - (-2490) = 1090$

There are 1050 years between the building of the pyramids and the Luxor Temple.

16. On Monday, Becky checked the weather and it forecasted 30°C. To calculate the degrees in Fahrenheit (°F), the degrees in Celsius are multiplied by 1.8 then 32 is added to the result. What is the temperature on Monday in °F, if Becky rounds 1.8 to the nearest whole number to complete the conversion?

Key points

- On Monday, Becky checked the weather and it forecasted 30°C.
- To calculate the degrees in Fahrenheit (°F), the degrees in Celsius are multiplied by 1.8 then 32 is added to the result.
- What is the temperature on Monday in °F, if Becky rounds 1.8 to the nearest whole number to complete the conversion?

Explanation

Because $8 > 5$, 1.8 rounded to the nearest whole number would be 2.

Degrees in Fahrenheit (°F) = Degrees in Celsius (°C) $\times$ 2 + 32

$= 30 \times 2 + 32$

$= 92$°F

Answer

Monday's temperature is 92°F.

17. During mental health awareness month, St. Rose College decides to start a fundraising event to raise money for mental health research by selling chocolate. One chocolate company will be providing twenty boxes of chocolate, each containing fifty bars. If each bar of chocolate costs $2, how much money will St. Rose College raise if the students sell all the chocolates?

Key points

- One chocolate company will be providing twenty boxes of chocolate, each containing fifty bars.
- Each bar of chocolate costs $2.
- How much money will St. Rose College raise if the students sell all the chocolates?

Explanation

Money raised by St. Rose College = Cost of each bar of chocolate $\times$ number of bars of chocolate in a box $\times$ number of boxes

$= 2 \times 50 \times 20$

$= \$2000$

Answer

St. Rose College will raise $2000.

18. Kirra wants to buy a new phone plan. Telstar offers her the following monthly deal:

- $20 for the phone

For the first 2000 MB

- $5 for every 500MB of data used

After the first 2000 MB

- $14 for every 1000 MB of data used

In March Kirra used a total of 3500 MB of data. How much can Kirra expect to pay in March?

Key points

- Telstar offers Kirra a deal; Kirra needs to pay $20 monthly for the phone.
- For the first 2000 MB data that Kirra uses, she will need to pay $5 for every 500MB of data used.
- After the first 2000 MB, Kirra needs to pay $14 for every 1000 MB of data used.
- How much can Kirra expect to pay if she used 3500?

Explanations

There are $2000 \div 500 = 4$ lots of 500MB in the first 2000 MB.

For the first 2000 MB, Kirra pays $5 \times 4 = \$20$.

Kirra uses $3500 - 2000 = 1500$ MB after the first 2000 MB.

There are $1500 \div 1000 = 1.5$ lots of 1000MB in the next 1500 MB.

For the next 1500 MB, Kirra pays $14 \times 1.5 = \$21$.

In total, Kirra pays $20 + 20 + 21 = \$61$ for her phone plan in March.

Answer

Kirra can expect to pay $61 in March.

Reasoning

19. a. The optometrist should record 18 points.

b. Jedda should interpret negative powers as near-sightedness.

c. Jedda should buy contact lenses. They are $29 cheaper than glasses for one year's supply.

d. Plan B is cheaper for Jedda.

e. Suggested option 1: Yes, as it reduces the cost if something goes wrong.
Suggested option 2: No, because you're paying money to protect yourself against something that might never happen.

Note: There are other possible options.

20. a. $x = 9, y = -9, x - y = 18$

b. $x = 9, y = 8, x \times y = 72$ or $x = 8, y = 9, x \times y = 72$ or $x = -9, y = -8, x \times y = 72$ or $x = -8, y = -9, x \times y = 72$

c. Part **a** has one solution because only the difference between the largest possible positive number and the smallest possible negative number will make the largest result.

Part **b** has four solutions because both two largest possible positive numbers and two smallest possible negative numbers multiply to the largest result.

2A Equivalent fractions

Student practice

Worked example 1

a. $\frac{17}{6}$ **b.** $2\frac{2}{5}$

Worked example 2

a. $\frac{6}{8} = \frac{12}{16} = \frac{30}{40}$ **b.** $\frac{12}{48} = \frac{6}{24} = \frac{1}{4}$

Worked example 3

a. $\frac{2}{3}$ **b.** $\frac{8}{3}$

Understanding worksheet

1. **a.** $\frac{5}{2}$ **b.** $2\frac{2}{4}$ **c.** $\frac{8}{5}$ **d.** $1\frac{4}{7}$

2. **a.** $\frac{2}{3} = \frac{4}{6}$ **b.** $\frac{3}{7} = \frac{6}{14}$ **c.** $1\frac{2}{6} = 1\frac{3}{9}$ **d.** $\frac{8}{14} = \frac{12}{21}$

3. improper; mixed number; equivalent; simplest

Fluency

4. **a.** $\frac{7}{4}$ **b.** $\frac{12}{5}$ **c.** $\frac{23}{4}$ **d.** $\frac{23}{7}$
e. $\frac{33}{4}$ **f.** $\frac{68}{5}$ **g.** $\frac{122}{7}$ **h.** 63

---

5. **a.** $2\frac{1}{2}$ **b.** $1\frac{1}{4}$ **c.** $2\frac{2}{3}$ **d.** $3\frac{2}{5}$
e. $4\frac{1}{4}$ **f.** $6\frac{5}{6}$ **g.** $11\frac{3}{11}$ **h.** $4\frac{5}{17}$

---

6. **a.** $\frac{7}{10} = \frac{14}{20} = \frac{21}{30}$ **b.** $\frac{2}{5} = \frac{6}{15} = \frac{12}{30}$
c. $\frac{100}{80} = \frac{25}{20} = \frac{10}{8}$ **d.** $\frac{84}{48} = \frac{42}{24} = \frac{21}{12}$
e. $\frac{7}{4} = \frac{21}{12} = \frac{28}{16}$ **f.** $\frac{13}{6} = \frac{39}{18} = \frac{78}{36}$
g. $\frac{48}{32} = \frac{24}{16} = \frac{6}{4}$ **h.** $\frac{15}{36} = \frac{10}{24} = \frac{5}{12}$

---

7. **a.** $\frac{12}{48}$ **b.** $\frac{72}{48}$ **c.** $\frac{44}{48}$ **d.** $\frac{64}{48}$
e. $3\frac{18}{48}$ **f.** $5\frac{84}{48}$ **g.** $\frac{80}{48}$ **h.** $\frac{26}{48}$

---

8. **a.** $\frac{2}{3}$ **b.** $\frac{1}{2}$ **c.** $\frac{6}{5}$ **d.** $\frac{3}{2}$
e. $\frac{7}{8}$ **f.** $5\frac{2}{3}$ **g.** $\frac{11}{18}$ **h.** $\frac{9}{7}$

---

9. E

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. If six friends share four cantaloupes, what fraction of a cantaloupe does each person receive? Give your answer in its simplest form.

Key points

- Six friends share four cantaloupes.
- Find the fraction of a cantaloupe each person receives in simplest form.

Explanation

$$\text{Cantaloupe each person receives} = \frac{\text{all cantaloupes}}{\text{number of people}}$$
$$= \frac{4}{6}$$

Simplify $\frac{4}{6}$.

HCF of 4 and 6 is 2.

$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

$4 \div 6 = \frac{2}{3}$

Answer

Each person receives $\frac{2}{3}$ of a cantaloupe.

12. How long are each of Carlos' steps if he took 12 equal steps and crossed his 15 m front garden? Give your answer in its simplest form.

Key points

- Carlos took 12 equal steps.
- Carlos crossed his 15 m front garden.
- Find the fraction of the length of each of Carlos' steps in simplest form.

Explanation

$$\text{Length of each step} = \frac{\text{length of front garden}}{\text{number of steps}}$$
$$= \frac{15}{12}\text{ m}$$

Simplify $\frac{15}{12}$.

HCF of 15 and 12 is 3.

$\frac{15 \div 3}{12 \div 3} = \frac{5}{4}$

Answer

Each of Carlos' step is $\frac{5}{4}$ m long.

13. Papa Joe's Pizza cuts large pizzas into eighths. Mrs. Gooley wants to buy pizza for her 24 students. What is the minimum number of pizzas she needs to order so that each student receives three slices?

Key points

- Papa Joe's Pizza cuts large pizzas into eighths.
- Mrs. Gooley wants to buy pizza for her 24 students.
- Each student receives three slices.
- What is the minimum number of pizzas she needs to order?

Explanation

Number of slices needed
= Number of slices for each student × number of students
= 3 × 24
= 72 slices

$$\text{Number of pizzas needed} = \frac{\text{number of slices for each student}}{\text{number of slices in one pizza}}$$
$$= \frac{72}{8}$$

Simplify $\frac{72}{8}$.

HCF of 72 and 8 is 8.

$\frac{72 \div 8}{8 \div 8} = \frac{9}{1}$
$= 9$

Answer

Mrs. Gooley needs to order a minimum of 9 pizzas.

14. Yoanna's car drives 600 km on a tank of fuel. How many tanks of fuel will she need to drive 2600 km from Bendigo to Alice Springs?

Key points

- Yoanna's car drives 600 km on a tank of fuel.
- She needs to drive 2600 km from Bendigo to Alice Springs.
- How many tanks of fuel will she need?

Explanation

Number of tanks of fuel needed

$= \frac{\text{distance}}{\text{distance that each tank of fuel covers}}$

$= \frac{2600}{600}$

$= \frac{26}{6}$ tanks of fuel

Simplify $\frac{26}{6}$.

HCF of 26 and 6 is 2.

$\frac{26 \div 2}{6 \div 2} = \frac{13}{3}$

$13 \div 3 = 4$ remainder 1

$\frac{13}{3} = 4\frac{1}{3}$

Answer

Yoanna will need $4\frac{1}{3}$ tanks of fuel to drive from Bendigo to Alice Springs.

15. 300 students were surveyed about school camps and 80 chose to camp at Gariwerd (Grampians National Park). If half of the remaining students chose to travel to the Gold Coast, express the fraction of students who wanted to travel to the Gold Coast in the simplest form.

Key points

- 300 students were surveyed about school camps.
- 80 chose to camp at Gariwerd (Grampians National Park).
- Half of the remaining students chose to travel to the Gold Coast.
- Express the fraction of students who wanted to travel to the Gold Coast in the simplest form.

Explanation

Number of students who chose Gold Coast

$= \frac{\text{total number of students} - \text{number of students who chose Gariwerd}}{2}$

$= \frac{300 - 80}{2}$

$= \frac{220}{2}$ students

$= 110$ students

Fraction of students who chose Gold Coast

$= \frac{\text{number of students who chose Gold Coast}}{\text{total number of students}}$

$= \frac{110}{300}$

HCF of 110 and 300 is 10.

$\frac{110 \div 10}{300 \div 10} = \frac{11}{30}$

Answer

$\frac{11}{30}$ of the students wanted to travel to the Gold Coast.

Reasoning

16. a. Each of the organisers take home $1\frac{3}{4}$ cakes.

b. Each person receives 14 slices.

c. It took them 215 minutes to set up the hall.

d. The minimum number of cuts is 4.

e. Suggested option 1: They can make videos about the effects of climate change and post it online.

Suggested option 2: They can host a mini concert fundraising event.

Note: There are other possible options.

17. a. $\frac{165}{200} = \frac{33}{40}$ **b.** $\frac{504}{567} = \frac{16}{18}$

c. The answer in part **a** is in its simplest form whereas the answer in part **b** is not. To determine if a fraction is in its simplest form, find the highest common factor of the numerator and the denominator. The fraction is in simplest form if the highest common factor is 1.

Extra spicy

18. C **19.** 10 km **20.** $\frac{1}{3}$ **21.** C

Remember this?

22. E **23.** A **24.** C

2B Adding and subtracting fractions

Student practice

Worked example 1

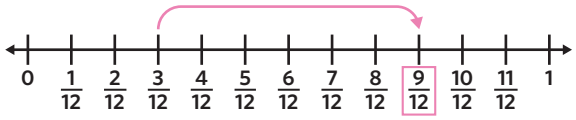
a. $\frac{8}{3} = 2\frac{2}{3}$ **b.** $\frac{4}{5}$ **c.** $3\frac{5}{6}$ **d.** $3\frac{1}{3}$

Worked example 2

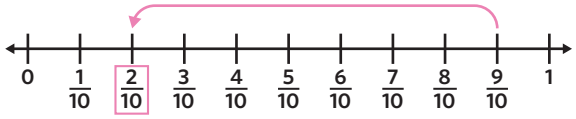
a. $\frac{5}{4} = 1\frac{1}{4}$ **b.** $\frac{31}{35}$ **c.** $4\frac{4}{15}$ **d.** $\frac{1}{3}$

Understanding worksheet

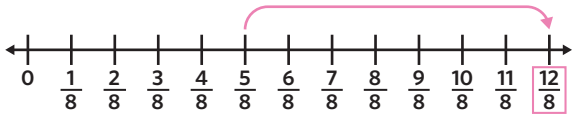
1. a. $\frac{3}{12} + \frac{6}{12} = \frac{9}{12}$



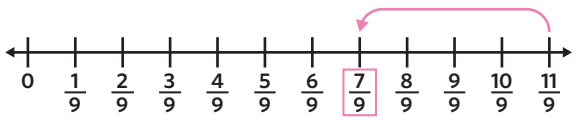
b. $\frac{9}{10} - \frac{7}{10} = \frac{2}{10}$



c. $\frac{5}{8} + \frac{7}{8} = \frac{12}{8}$



d. $1\frac{2}{9} - \frac{4}{9} = \frac{11}{9} - \frac{4}{9} = \frac{7}{9}$



2. a. $\frac{2}{14}$ **b.** $\frac{6}{9}$ **c.** $\frac{1}{2}$ **d.** $\frac{10}{15}$

3. subtract; denominator; lowest; common; simplest

Fluency

4. **a.** $\frac{1}{7}$ **b.** $\frac{1}{2}$
c. 2 **d.** 2
e. $\frac{25}{8} = 3\frac{1}{8}$ **f.** $\frac{5}{3} = 1\frac{2}{3}$
g. $\frac{1}{5}$ **h.** $\frac{3}{2} = 1\frac{1}{2}$

5. **a.** $\frac{3}{4}$ **b.** $3\frac{4}{7}$ **c.** $2\frac{3}{11}$ **d.** $2\frac{9}{13}$
e. $7\frac{1}{4}$ **f.** $1\frac{4}{11}$ **g.** $5\frac{5}{6}$ **h.** $1\frac{1}{3}$

6. **a.** The LCD is 24. **b.** The LCD is 15.
c. The LCD is 27. **d.** The LCD is 18.
e. The LCD is 16. **f.** The LCD is 15.
g. The LCD is 24. **h.** The LCD is 15.

7. **a.** $\frac{1}{6}$ **b.** $\frac{19}{12} = 1\frac{7}{12}$
c. 3 **d.** $\frac{19}{12} = 1\frac{7}{12}$
e. $\frac{11}{24}$ **f.** $\frac{23}{12} = 1\frac{11}{12}$
g. $\frac{13}{15}$ **h.** $\frac{11}{4} = 2\frac{3}{4}$

8. **a.** $2\frac{2}{3}$ **b.** 4
c. $\frac{2}{9}$ **d.** $6\frac{1}{6}$
e. $\frac{8}{21}$ **f.** $8\frac{5}{6}$
g. $\frac{46}{45} = 1\frac{1}{45}$ **h.** $8\frac{11}{18}$

9. D

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. The junior boys' basketball team won six games, lost three games, and drew two games. What fraction of all the games did the basketball team win or draw?

Key points

- The team won six games, lost three games, and drew two games.
- What fraction of all the games did the basketball team win or draw?

Explanation

fraction of games the team wins or draws
$= \frac{\text{games the team wins or draws}}{\text{total games played}}$

games the team wins or draws $= 6 + 2$
$= 8$

games played $= 6 + 3 + 2$
$= 11$

So $\frac{\text{games the team wins or draws}}{\text{total games played}} = \frac{8}{11}$

Answer

The basketball team won or drew $\frac{8}{11}$ of their games.

12. Dusty made breakfast by combining $\frac{2}{5}$ of a bowl of Weet-Bix with $\frac{1}{3}$ of a bowl of chopped bananas and $\frac{3}{15}$ of a bowl of milk. What fraction of the bowl is filled with Weet-Bix and bananas?

Key points

- Dusty combines $\frac{2}{5}$ of a bowl of Weet-Bix with $\frac{1}{3}$ of a bowl of chopped bananas and $\frac{3}{15}$ of a bowl of milk.
- What fraction of the bowl is filled with Weet-Bix and bananas?

Explanation

The fraction of the bowl filled with Weet-Bix and bananas is the sum of the fraction filled with Weet-Bix and the fraction filled with bananas.

Fraction filled with Weet-Bix and bananas $= \frac{2}{5} + \frac{1}{3}$

LCD is 15. Write equivalent fractions with equal denominators.

$\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15}$
$= \frac{11}{15}$

Answer

$\frac{11}{15}$ of the bowl is filled with Weet-Bix and bananas.

13. During a training session, Marta spent $\frac{2}{8}$ practising free kicks, $\frac{3}{8}$ doing fitness drills and $\frac{2}{16}$ kicking penalties. If Marta was in the gym for the rest of her training session, what fraction of the entire training session did she spend in the gym?

Key points

- Marta spent $\frac{2}{8}$ of her session practising free kicks, $\frac{3}{8}$ doing fitness drills and $\frac{2}{16}$ kicking penalties.
- She was in the gym for the rest of her training session.
- What fraction of the entire training session did she spend in the gym?

Explanation

Time in the gym $=$ total time $-$ time not in gym

The fraction of the time not spent in the gym is the fraction of the time she practiced free kicks, did fitness drills and kicked penalties.

Fraction of time not in gym $= \frac{2}{8} + \frac{3}{8} + \frac{2}{16}$

LCD is 8. Write equivalent fractions with equal denominators.

$\frac{2}{8} + \frac{3}{8} + \frac{2}{16} = \frac{2}{8} + \frac{3}{8} + \frac{1}{8}$

Add the numerators.

$\frac{2}{8} + \frac{3}{8} + \frac{1}{8} = \frac{6}{8}$
$= \frac{3}{4}$

So fraction of time in gym $= 1 - \frac{3}{4}$
$= \frac{1}{4}$

Answer

Marta spent $\frac{1}{4}$ of her training session in the gym.

14. Students in the year 8 cohort either play the guitar, the piano, or do not play an instrument. What fraction of the students do not play any instrument if $\frac{3}{7}$ of the students play guitar and $\frac{6}{14}$ play the piano?

Key points

- Students play the guitar, the piano, or do not play an instrument.
- $\frac{3}{7}$ of the students play guitar and $\frac{6}{14}$ play the piano.
- What fraction of the students do not play any instrument?

Explanation

Students not playing an instrument
= total students − guitar players − piano players

$\frac{3}{7}$ of the students play guitar and $\frac{6}{14}$ play the piano, so

Fraction of students not playing an instrument $= 1 - \frac{3}{7} - \frac{6}{14}$

LCD is 7. Write equivalent fractions with equal denominators.

$1 - \frac{3}{7} - \frac{6}{14} = \frac{7}{7} - \frac{3}{7} - \frac{3}{7}$

Subtract the numerators.

$\frac{7}{7} - \frac{3}{7} - \frac{3}{7} = \frac{7 - 3 - 3}{7}$

$= \frac{1}{7}$

Answer

In the year 8 cohort, $\frac{1}{7}$ of the students do not play an instrument.

15. Cheryl baked a small cake for the Edrolo Christmas party. Meeky ate $\frac{3}{20}$ of the cake, Bretty ate $\frac{2}{5}$ and also took some home for his children. How much cake did Bretty take home for his children, if there was $\frac{1}{10}$ of the cake remaining?

Key points

- Meeky ate $\frac{3}{20}$ of the cake.
- Bretty ate $\frac{2}{5}$.
- Bretty also took some home for his children.
- There was $\frac{1}{10}$ of the cake remaining.
- What fraction of the cake did Bretty take home?

Explanation

Amount taken home = whole cake − Meeky's share − Bretty's share − remaining cake

$= 1 - \frac{3}{20} - \frac{2}{5} - \frac{1}{10}$

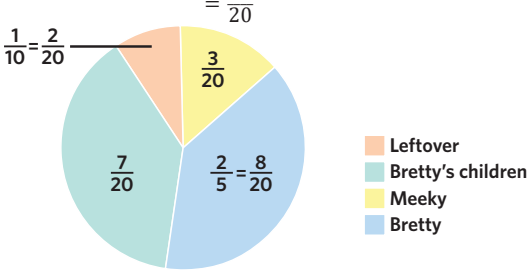
LCD is 20. Write equivalent fractions with equal denominators.

$1 - \frac{3}{20} - \frac{2}{5} - \frac{1}{10} = \frac{20}{20} - \frac{3}{20} - \frac{8}{20} - \frac{2}{20}$

Subtract the numerators.

$\frac{20}{20} - \frac{3}{20} - \frac{8}{20} - \frac{2}{20} = \frac{20 - 3 - 8 - 2}{20}$

$= \frac{7}{20}$



Answer

Bretty took $\frac{7}{20}$ of the cake back home to his children.

Reasoning

16. a. Over the two days, Julian ran a total of $\frac{3}{2}$ or $1\frac{1}{2}$ km.

b. Julian would have to run another $\frac{1}{12}$ km to match his record.

c. Ambrosia ran more. She ran $\frac{1}{10}$ km more than Julian.

d. Suggested option 1: Julian and Ambrosia can go for training runs together and encourage each other to run further each time.

Suggested option 2: Julian and Ambrosia can set personal weekly goals that increase by 1 km each week.

Note: There are other possible options.

17. a. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

b. $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{db}$

c. Fractions need common denominators when adding. If fractions have different denominators, the denominators can be multiplied to find a common denominator.

Extra spicy

18. B

19. $\frac{5}{4}$ or $1\frac{1}{4}$ cups

20. $\frac{2}{15}$ metres

21. C

Remember this?

22. B

23. C

24. E

2C Multiplying and dividing fractions

Student practice

Worked example 1

a. $\frac{5}{21}$ **b.** 6 **c.** $2\frac{2}{3}$ or $\frac{8}{3}$

Worked example 2

a. 10 **b.** $\frac{5}{48}$ **c.** $1\frac{7}{20}$ or $\frac{27}{20}$

Worked example 3

a. $\frac{5}{28}$ **b.** $6\frac{2}{3}$ or $\frac{20}{3}$

Understanding worksheet

1. a. $\frac{10}{4}$ **b.** $\frac{4}{3}$ **c.** $\frac{6}{5}$ **d.** $\frac{6}{2}$

2. a. $\frac{10}{7}$ **b.** $\frac{2}{5}$ **c.** $\frac{4}{1}$ **d.** $\frac{1}{3}$

3. denominators; reciprocal; cross-simplifying; improper

Fluency

4. a. $\frac{3}{8}$ **b.** $\frac{7}{3}$ **c.** 3 **d.** $\frac{10}{21}$

e. $\frac{18}{5}$ **f.** 5 **g.** 7 **h.** $\frac{160}{9}$

5. a. 4 **b.** $\frac{8}{5}$ **c.** $\frac{4}{21}$ **d.** $\frac{11}{5}$

e. $\frac{10}{7}$ **f.** $\frac{44}{81}$ **g.** $\frac{112}{121}$ **h.** $\frac{17}{130}$

6. **a.** $\frac{5}{21}$ **b.** $\frac{16}{7}$ **c.** $\frac{10}{3}$ **d.** $\frac{1}{4}$
e. $\frac{95}{68}$ **f.** $\frac{49}{24}$ **g.** 45 **h.** $\frac{110}{21}$

7. **a.** $\frac{28}{45}$ **b.** $\frac{47}{72}$ **c.** $\frac{74}{45}$ **d.** $\frac{19}{70}$
e. $\frac{27}{10}$ **f.** $\frac{64}{15}$ **g.** $\frac{21}{20}$ **h.** $\frac{61}{40}$

8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. Hawi initially charged the battery of her electric car to $\frac{4}{5}$ of its full capacity. After driving for a few days, she saw that she used $\frac{3}{7}$ of the initial charge. What fraction of a full charge did she use?

Key points

- Hawi initially charged her electric car to $\frac{4}{5}$ of its full capacity.
- She used $\frac{3}{7}$ of the initial charge.
- What fraction of a full charge did she use?

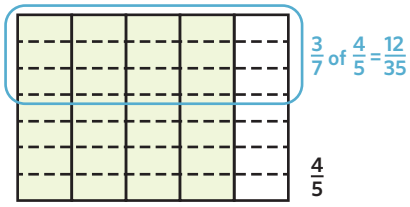
Explanation

Hawi used $\frac{3}{7}$ of $\frac{4}{5}$

$\frac{3}{7}$ of $\frac{4}{5} = \frac{3}{7} \times \frac{4}{5}$

$= \frac{3 \times 4}{7 \times 5}$

$= \frac{12}{35}$



Answer

Hawi used $\frac{12}{35}$ of a full charge.

11. A group of students are planning a 24-hour treadmill challenge to raise money for charity. If each person runs for two-fifths of an hour, how many students are needed for the full 24 hours?

Key points

- The treadmill challenge is 24 hours long.
- Each student runs for two-fifths of an hour.
- How many students are needed to complete 24 hours?

Explanation

Number of students = total time ÷ time per student

$= 24 \div \frac{2}{5}$

$= 24 \times \frac{5}{2}$

$= \frac{24 \times 5}{1 \times 2}$

$= \frac{120}{2}$

$= 60$

Answer

60 students are needed for the full 24 hours.

12. Georgia buys cans of dog food in boxes of 10 cans. She feeds her dog three-quarters of a can every day. If she has three-fifths of a full box left, how many days' worth of food does she have for her dog?

Key points

- A box of dog food has 10 cans.
- Georgia feeds her dog three-quarters of a can every day.
- She has three-fifths of a full box left.
- How many days' worth of food does she have for her dog?

Explanation

Find how many cans of dog food Georgia has left.

$\frac{3}{5}$ of 10 cans $= \frac{3}{5} \times 10$

$= \frac{30}{5}$

$= 6$

Find the number of days' worth of food.

Number of days' worth of food
= number of cans ÷ amount used per day

$= 6 \div \frac{3}{4}$

$= 6 \times \frac{4}{3}$

$= 8$

Answer

Georgia has 8 days' worth of food for her dog.

13. The floors in Harish's rectangular dining room need to be covered in varnish. The room has a length of $6\frac{2}{3}$ m and a width of $3\frac{2}{7}$ m. If $4\frac{8}{21}$ tins were needed, how much area did each tin cover?

Key points

- The length of the room is $6\frac{2}{3}$ m.
- The width of the room is $3\frac{2}{7}$ m.
- $4\frac{8}{21}$ tins of varnish were used.
- How much area did each tin cover?

Explanation

Find the area of the dining room.

Area = length × width

$= 6\frac{2}{3} \times 3\frac{2}{7}$

$= \frac{20}{3} \times \frac{23}{7}$

$= \frac{460}{21}$ m^2

Find the area each tin covers.

Area each tin covers
= Area of the dining room ÷ number of tins used

$= \frac{460}{21} \div 4\frac{8}{21}$

$= \frac{460}{21} \div \frac{92}{21}$

$= \frac{460}{21} \times \frac{21}{92}$

$= \frac{460}{92}$

$= 5$ m^2

Answer

Each tin covers 5 m^2.

14. Maddie and Hugh are playing the board game Blokus that involves placing differently shaped tiles on the board. $\frac{3}{8}$ of the board is covered in tiles, and $\frac{2}{5}$ of those are Maddie's red tiles.
What fraction of the board does Maddie still need to cover so that her tiles take up at least half of the board?

Key points

- $\frac{3}{8}$ of the board is covered in tiles.
- $\frac{2}{5}$ of those are Maddie's red tiles.
- Find the fraction of the board Maddie needs to cover to take up at least half of the board.

Explanation

Find the fraction of the board Maddie's tiles currently cover.

$\frac{2}{5}$ of $\frac{3}{8} = \frac{2}{5} \times \frac{3}{8}$

$= \frac{6}{40}$

$= \frac{3}{20}$

Find the fraction of the board Maddie still needs to cover.

$\frac{1}{2} - \frac{3}{20} = \frac{10}{20} - \frac{3}{20} = \frac{7}{20}$

Answer

Maddie still needs to cover $\frac{7}{20}$ of the board.

Reasoning

15. a. 180 people bought tickets for the first session.

b. $\frac{1}{5}$ of the total available seats were occupied by people who bought snacks.

c. 3 sessions will run between 5:30 pm and 11 pm.

d. The cinema will make $16 020 revenue for the night.

e. Suggested option 1: The cinema could increase the popcorn and drink price.

Suggested option 2: The cinema could advertise so more people would fill up the theatre.

Note: There are other possible options.

16. a. $\frac{1}{9} \div \frac{9}{1}$ **b.** $\frac{9}{1} \div \frac{1}{9}$

c. Part **a** uses a small fraction to divide by a bigger valued fraction to obtain a smaller fraction. Part **b** uses a large fraction to divide by a smaller valued fraction to obtain a larger fraction. A fraction divided by a smaller valued fraction will result in a larger fraction.

Extra spicy

17. E **18.** D **19.** ♥ $= \frac{9}{2}$ **20.** $\frac{24}{125}$

Remember this?

21. E **22.** D **23.** C

2D Working with negative fractions

Student practice

Worked example 1

a. $-\frac{1}{4}$ **b.** $\frac{7}{6} = 1\frac{1}{6}$ **c.** $-\frac{27}{20} = -1\frac{7}{20}$

Worked example 2

a. $\frac{10}{33}$ **b.** $-\frac{18}{25}$ **c.** $-\frac{14}{9} = -1\frac{5}{9}$

Understanding worksheet

1. a. $-\frac{3}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{3}{4}$ **d.** $\frac{1}{5}$

2. a. − **b.** + **c.** + **d.** −

3. positive; direction; negative; reciprocal

Fluency

4. a. $-\frac{3}{8}$ **b.** $\frac{7}{11}$ **c.** $-\frac{1}{5}$ **d.** $-\frac{1}{9}$
e. $-\frac{1}{8}$ **f.** $\frac{1}{15}$ **g.** $\frac{24}{35}$ **h.** $\frac{5}{24}$

5. a. $\frac{7}{2} = 3\frac{1}{2}$ **b.** $\frac{-14}{11} = -1\frac{3}{11}$
c. $\frac{-4}{3} = -1\frac{1}{3}$ **d.** $\frac{7}{9}$
e. $-\frac{7}{8}$ **f.** $\frac{19}{15} = 1\frac{4}{15}$
g. $-\frac{55}{18} = -3\frac{1}{18}$ **h.** $-\frac{125}{28} = -4\frac{13}{28}$

6. a. $\frac{-3}{10}$ **b.** $\frac{5}{24}$ **c.** $-\frac{18}{25}$ **d.** $-\frac{2}{3}$
e. $-\frac{6}{25}$ **f.** $\frac{16}{45}$ **g.** $-\frac{15}{16}$ **h.** $\frac{3}{16}$

7. a. $-\frac{25}{24} = -1\frac{1}{24}$ **b.** $-\frac{15}{4} = -3\frac{3}{4}$
c. $-\frac{33}{80}$ **d.** -58
e. -24 **f.** $\frac{76}{45} = 1\frac{31}{45}$
g. $-\frac{110}{133}$ **h.** $-\frac{216}{875}$

8. a. $\frac{5}{24}$ **b.** $\frac{28}{45}$
c. $\frac{7}{24}$ **d.** $-\frac{2}{7}$
e. $-\frac{37}{36} = -1\frac{1}{36}$ **f.** $\frac{19}{70}$
g. $\frac{1}{8}$ **h.** $-\frac{133}{72} = -1\frac{61}{72}$

9. B

Spot the mistake

10. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. Ciara needs $3\frac{1}{5}$ rolls of fabric to create her lounge room curtains and half as much for her bedroom curtains. If she already has 4 rolls of fabric, what fraction of another roll does she need to complete the curtains for both rooms?

Key points

- Ciara needs $3\frac{1}{5}$ rolls of fabric to create her lounge room curtains.
- She needs half as many rolls of fabric for her bedroom curtains.
- She already has 4 rolls of fabric.
- What fraction of another roll does she need to complete the curtains for both rooms?

Explanation

Ciara needs $3\frac{1}{5} \div 2$ rolls of fabric for her bedroom curtains.

$$= 3\frac{1}{5} \times \frac{1}{2}$$
$$= \frac{16}{5} \times \frac{1}{2}$$
$$= \frac{16}{10}$$
$$= \frac{8}{5}$$

Therefore, Ciara needs $\frac{8}{5} + \frac{16}{5} = \frac{24}{5}$ rolls of fabric for both her loungeroom and bedroom curtains.

$$\frac{24}{5} = 4\frac{4}{5}$$

Ciara needs another $4\frac{4}{5} - 4 = \frac{4}{5}$ of a roll.

Answer

Ciara needs $\frac{4}{5}$ of another roll to complete the curtains for both rooms.

12. Sarah brought $\frac{3}{4}$ of a large block of chocolate to class, and Nadia brought $\frac{3}{5}$ of the same size block. They combined their chocolate and gave their teacher Mr Ellis the equivalent of half of a large block of chocolate. What fraction of a large block do Sarah and Nadia have left?

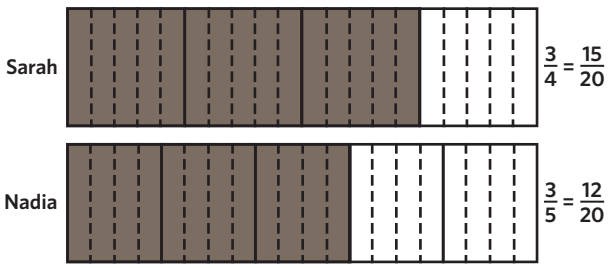
Key points

- Sarah brought $\frac{3}{4}$ of a large block of chocolate to class.
- Nadia brought $\frac{3}{5}$ of the same size block of chocolate.
- Sarah and Nadia combined their chocolate and gave Mr Ellis half of a large block.
- What fraction of a large block do Sarah and Nadia have left?

Explanation

Sarah and Nadia have

$$\frac{3}{4} + \frac{3}{5} = \frac{15}{20} + \frac{12}{20}$$
$$= \frac{27}{20} \text{ large blocks of chocolate combined.}$$



Sarah and Nadia gave Mr Ellis half of a block.

$$\frac{27}{20} - \frac{10}{20} = \frac{17}{20}$$

Therefore, Sarah and Nadia have $\frac{17}{20}$ of a large block left.

Answer

Sarah and Nadia have $\frac{17}{20}$ of a large block left.

13. Mackenzie spends three eighths of her monthly salary on bills. If she buys christmas gifts with a third of the remaining amount, what fraction of her salary does she have left to save?

Key points

- Mackenzie spends three eighths of her monthly salary on bills.
- She buys Christmas gifts with a third of the remaining amount.
- What fraction of her salary does Mackenzie have left to save?

Explanation

Let Mackenzie's whole monthly salary be 1. She spent $\frac{3}{8}$ of her salary on bills. Therefore, she has $1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8}$

$$= \frac{5}{8} \text{ of her salary left over.}$$

Mackenzie then spends $\frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$ of her salary on Christmas gifts.

Therefore, Mackenzie has $\frac{5}{8} - \frac{5}{24} = \frac{15}{24} - \frac{5}{24}$

$$= \frac{10}{24}$$
$$= \frac{5}{12} \text{ of her salary left to save.}$$

Answer

Mackenzie has $\frac{5}{12}$ of her salary left to save.

14. Patrick is learning the waltz. After every two steps forwards he takes one step backwards. Each of his forwards dancing steps are four fifths of a metre long. If the size of his backwards steps are three quarters of the length of a forwards step, how far forward has he moved after 12 steps in total?

Key points

- After every two steps forward, Patrick takes one step backwards.
- Each of Patrick's forward steps are four fifths of a metre long.
- Patrick's backwards steps are three quarters the length of his forwards steps.
- How far forward has he moved after 12 steps in total?

Explanation

Represent forwards steps as *F* and backwards steps as *B*, the pattern of steps Patrick takes is:

F F B F F B F F B F F B

Therefore, he takes 8 steps forward and 4 steps backward

Each forward step is $\frac{4}{5}$ m long.

Therefore, each backward step is $\frac{4}{5} \times \frac{3}{4} = \frac{12}{20}$

$$= \frac{3}{5} \text{ m long.}$$

After 12 steps taken, Patrick has moved $\frac{4}{5} \times 8 = \frac{32}{5}$ m forwards and $\frac{3}{5} \times 4 = \frac{12}{5}$ m backwards.

Therefore, overall, Patrick moves $\frac{32}{5} - \frac{12}{5} = \frac{20}{5}$

$$= 4 \text{ m forwards.}$$

Answer

Patrick has moved forward 4 metres in total.

15. Liza fills an empty bucket for $2\frac{3}{5}$ minutes at a rate of $12\frac{1}{2}$ litres per minute. Henry then puts a hole in the bucket. How much water is left in the bucket if it was leaking for $4\frac{1}{3}$ minutes at a rate of $7\frac{1}{6}$ litres per minute?

Key points

- Liza fills an empty bucket for $2\frac{3}{5}$ minutes at a rate of $12\frac{1}{2}$ litres per minute.
- Henry puts a hole in the bucket.
- How much water is left in the bucket if it was leaking for $4\frac{1}{3}$ minutes at a rate of $7\frac{1}{6}$ litres per minute?

Explanation

Amount of water filled in = number of minutes × rate per minute

$$= 2\frac{3}{5} \times 12\frac{1}{2}$$
$$= \frac{13}{5} \times \frac{25}{2}$$
$$= \frac{325}{10}$$
$$= \frac{65}{2}$$
$$= 32\frac{1}{2} \text{ litres.}$$

Amount of water leaked out = number of minutes × rate per minute

$$= \frac{13}{3} \times \frac{43}{6}$$
$$= \frac{559}{18}$$
$$= 31\frac{1}{18} \text{ litres.}$$

Amount of water left in bucket = amount filled in − amount leaked out

$$= 31\frac{1}{2} - 31\frac{1}{18}$$
$$= 1\frac{8}{18}$$
$$= 1\frac{4}{9} \text{ or } \frac{13}{9} \text{ litres.}$$

Answer

There is $1\frac{4}{9}$ or $\frac{13}{9}$ litres of water left in the bucket.

Reasoning

16. a. $\frac{1}{8}$ kg of flour is left after the demonstration pizzas and the students' pizzas have been made.

b. $\frac{27}{8}$ kg of flour is left after the demonstration pizzas and the students' pizzas have been made.

c. 27 more pizzas can be made with the remaining flour.

d. 200 slices of salami and $\frac{125}{2}$ or $62\frac{1}{2}$ anchovies were available for the class to use.

e. Suggested option 1: The cooking method that makes pizza the quickest would win my vote.

Suggested option 2: The ingredient that makes the crust fluffy and light would win my vote.

Note: There are other possible options.

17. a. The missing value in the blue square is $\frac{49}{4}$ or $12\frac{1}{4}$

b. The missing value in the blue square is $\frac{49}{4}$ or $12\frac{1}{4}$

c. Multiplying two negative fractions is the same as multiplying two positive ones in the sense that both yield positive answers. As the two fractions being multiplied have the same directions in both parts **a** and **b**, the answers are equal.

Extra spicy

18. A

19. Vutha is required to pay $\frac{2}{9}$ of the bill.

20. Henry can put in $\frac{63}{100}$ of Ronald's soybeans.

21. E

Remember this?

22. C **23.** 42 green tiles **24.** B

2E Four operations with decimals

Student practice

Worked example 1

a. 0.0374 **b.** 253 900

Worked example 2

a. 9.311 **b.** 7.296

Worked example 3

a. 9.36 **b.** 98.25

Understanding worksheet

1. a. $4.58 \times 100 = 458$
$458 \div 100 = 4.58$

b. $5.87 \times 10 = 58.7$
$58.7 \div 10 = 5.87$

c. $3.25 \times 100 = 325$
$325 \div 100 = 3.25$

d. $62.5 \times 1000 = 62\,500$
$62500 \div 1000 = 62.5$

2. a. $4.81 \times 4 = 19.24$ **b.** $1.46 \times 2 = 2.92$

c. $2.34 \times 3 = 7.02$ **d.** $2.89 \times 6 = 17.34$

3. subtract; dividing; multiplying; whole

Fluency

4. a. 90.521 **b.** 6547.7
c. 0.53434 **d.** 73434
e. 0.035719 **f.** 4 252 600
g. 0.007583 **h.** 788 730

5. a. 237.8 **b.** 420.6
c. 39.926 **d.** 10.28
e. 577.285 **f.** 26.2118
g. 345.9776 **h.** 930.014

6. a. 485.5 **b.** 698.6
c. 34.36 **d.** 1188.2
e. 816 **f.** 6.696
g. 638.64 **h.** 567.196

7. a. 1.51 **b.** 1.12 **c.** 35.1 **d.** 39.6
e. 593 **f.** 2.2275 **g.** 87 **h.** 2.04

8. C

Spot the mistake

9. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. Carlos is at JB-HiFi with $120 to spend. He wants to buy a wireless headset for $32.87, a power bank for $48.19 and a mouse for $37.51. How much change should Carlos expect?

Key points

- Carlos has $120 to spend.
- He wants to buy a wireless headset for $32.87, a power bank for $48.19 and a mouse for $37.51.
- How much change should Carlos expect?

Explanation

Change = total money − cost of products

The cost of the products Carlos wants to buy is the sum of each price.

Cost of products = $32.87 + $48.19 + $37.51

Evaluate using the vertical algorithm.

$$\begin{array}{rrrrr} & {\scriptstyle +1} & {\scriptstyle +1} & {\scriptstyle +1} & \\ & 3 & 2.8 & 7 & \\ & 4 & 8.1 & 9 & \\ + & 3 & 7.5 & 1 & \\ \hline 1 & 1 & 8.5 & 7 & \end{array}$$

Change = $120 − $118.57

Evaluate using the vertical algorithm.

$$\begin{array}{rrrrr} & 1 & 9 & 9 & 1 \\ 1 & \not{2} & 0.\not{0} & \not{0} & 0 \\ -\,1 & 1 & 8.5 & 7 & \\ \hline & & 1.4 & 3 & \end{array}$$

Answer

Carlos should expect $1.43 in change.

11. Macca had a drum of cable that was 100.50 metres, which he cut into three pieces. One of the pieces was 22.34 metres and another was 58.89 metres. What was the length of the final cut?

Key points

- The cable is 100.50 metres.
- He cut the cable into three pieces.
- One of the pieces was 22.34 metres and another was 58.89 metres.
- What was the length of the final cut?

Explanation

Third piece length = total length − length of first two pieces

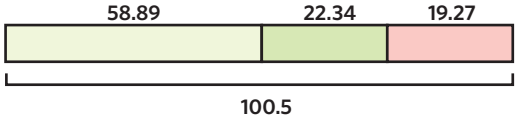
Length of first two pieces = 22.34 + 58.89

Evaluate using the vertical algorithm.

$$\begin{array}{rrrrr} & {\scriptstyle +1} & {\scriptstyle +1} & {\scriptstyle +1} & \\ & 2 & 2.3 & 4 & \\ + & 5 & 8.8 & 9 & \\ \hline & 8 & 1.2 & 3 & \end{array}$$

So third piece length = 100.5 − 81.23

$$\begin{array}{rrrrr} & & 9 & 1 & 4 & 1 \\ & \not{1} & \not{0} & 0.\not{5} & \not{0} & 0 \\ - & & 8 & 1.2 & 3 & \\ \hline & & 1 & 9.2 & 7 & \end{array}$$



Answer

The length of the final cut was 19.27 metres.

12. Annora jogs to work and Surendra drives to work. Surendra's average driving speed is 10 times faster than Annora's average jogging pace. Annora's average pace is 10.24 km/h. What is Surendra's average driving speed in km/h?

Key points

- Surendra's average driving speed is 10 times faster than Annora's average jogging pace.
- Annora's average pace is 10.24 km/h.
- What is Surendra's average driving speed in km/h?

Explanation

Surendra's average driving speed is 10 times faster than Annora's average jogging pace, so driving speed = 10 × jogging speed

= 10 × 10.24

To multiply by 10, move the decimal one place to the left.

10 × 10.24 = 102.40

Answer

Surendra's average driving speed is 102.4 km/h.

13. André's car has a total fuel capacity of 45.5 litres. When Andre goes to fill up his tank the price of each litre of petrol is $2.05. Assuming that his tank is completely empty, how much will it cost André to fill the tank? Provide your final answer to the nearest cent.

Key points

- The total fuel capacity of 45.5 litres.
- The price of each litre of petrol is $2.05.
- The tank is completely empty.
- How much will it cost to fill the tank? Provide your final answer to the nearest cent.

Explanation

Cost = number of litres × price per litre

= 45.5 × 2.05

Calculate using the vertical algorithm.

$$\begin{array}{rrrrr} & & {\scriptstyle 1} & {\scriptstyle 1} & \\ & & {\scriptstyle 2} & {\scriptstyle 2} & \\ & & 4 & 5 & 5 \\ \times & & 2 & 0 & 5 \\ \hline & 2 & 2 & 7 & 5 \\ & 0 & 0 & 0 & 0 \\ + & 9 & 1 & 0 & 0 & 0 \\ \hline & 9 & 3 & 2 & 7 & 5 \end{array}$$

There are three decimal places in the multiplication, so the answer must have three decimal places.

So 45.5 × 2.05 = 93.275

Now round to the nearest cent (hundredth). The critical digit is 5, so increase the next digit to the left.

Answer

It will cost André $93.28 to fill the tank.

14. The Melbourne Cricket Ground (MCG) is 172.8 metres long. Max calculates that he can kick the ball six times in order for the ball to traverse the total length of the MCG. If all of his kicks have the same length, what is the length of one of Max's kicks?

Key points

- The MCG is 172.8 metres long.
- Six kicks equal the length of the MCG.
- All of his kicks have the same length.
- What is the length of one of Max's kicks?

Explanation

Length of one kick = Length of MCG ÷ 6

= 172.8 ÷ 6

Use short division.

$$\begin{array}{r} 0\ \ 2\ \ 8\ .\ 8 \\ 6\overline{)1\ ^{1}7\ ^{5}2\ .\ ^{4}8} \end{array}$$

Answer

The length of one of Max's kicks is 28.8 metres.

Reasoning

15. a. Nazar walked 5.925 km in total on Monday.

b. Ari's bus journey to school is 53.5 km.

c. Jola rode 18.66 km on Friday.

d. The average distance of Nazar's walk home is 2.087 km.

e. Suggested option 1: They could make an informative video on the impact of cars on the environment.

Suggested option 2: They could present the benefits of cycling and/or walking to school as a form of exercise.

Note: There are other possible options.

16. a. 571.141 **b.** 5711.41

c. The answers from part **a** and **b** consist of the same numbers but differ by a power of 10. When multiplying one of the numbers in a decimal multiplication by a power of 10, it has the same effect of multiplying the overall result by a power of 10.

Extra spicy

17. Width = 1.5 metres; height = 0.5 metres

18. E **19.** E **20.** 6 minutes

Remember this?

21. B **22.** B **23.** C

2F Terminating and recurring decimals

Student practice

Worked example 1

a. 0.3125 **b.** $0.583\dot{3}$ or $0.58\overline{3}$

Worked example 2

a. Rational **b.** Rational

Understanding worksheet

1. a. Recurring **b.** Non-recurring

c. Recurring **d.** Terminating

2. a. 2.666666... $2.\dot{6}$ $2.\overline{6}$

b. 78.41254125... $78.\dot{4}12\dot{5}$ $78.\overline{4125}$

c. 0.13257132571... $0.\dot{1}3257\dot{1}$ $0.\overline{13257}$

d. 0.2307923079... $0.\dot{2}3079\dot{2}$ $0.\overline{23079}$

3. finite; infinite; non-recurring; irrational

Fluency

4. a. $1.\dot{3}$ or $1.\overline{3}$ **b.** $0.\dot{5}$ or $0.\overline{5}$

c. $0.\dot{7}\dot{2}$ or $0.\overline{72}$ **d.** $0.\dot{3}\dot{9}$ or $0.\overline{39}$

e. $0.30\dot{5}$ or $0.30\overline{5}$ **f.** $0.5\dot{2}$ or $0.5\overline{2}$

g. $0.\dot{7}14285\dot{7}$ or $0.\overline{714285}$ **h.** $0.6\dot{1}5384\dot{6}$ or $0.\overline{615384}$

---

5. a. 0.7 **b.** $0.\dot{1}$ or $0.\overline{1}$

c. 0.375 **d.** $1.8\dot{3}$ or $1.8\overline{3}$

e. 0.44 **f.** $1.41\dot{6}$ or $1.41\overline{6}$

g. $0.7\dot{2}$ or $0.7\overline{2}$ **h.** $1.\dot{1}42857\dot{1}$ or $1.\overline{142857}$

---

6. a. Rational **b.** Irrational

c. Rational **d.** Rational

e. Rational **f.** Irrational

g. Rational **h.** Irrational

---

7. D

Spot the mistake

8. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

9. Rob is making jelly cups for his daughter's birthday. He has 4 litres of jelly mix that he will divide equally into 25 cups. In litres, how much jelly will each cup contain?

Key points

- Rob has 4 litres of jelly mix.
- Rob will equally divide the jelly mix into 25 cups.
- In litres, how much jelly will each cup contain?

Explanation

Litres per cup = total jelly mix ÷ number of cups

= 4 ÷ 25

Calculate 4 ÷ 25 using short division.

$$\begin{array}{r} 0\ .\ 1\ \ 6 \\ 25\overline{)4\ .\ ^{4}0^{15}0} \end{array}$$

Answer

Each cup will contain 0.16 litres of jelly.

10. Sahir is running a half-marathon, which is 21 km long. He ran $\frac{4}{9}$ of the marathon along a beach. How many kilometres, written as a decimal, did Sahir run along the beach?

Key points

- The half-marathon is 21 km long.
- He ran $\frac{4}{9}$ of the marathon along a beach.
- How many kilometres, written as a decimal, did Sahir run along the beach?

Explanation

The distance along the beach is $\frac{4}{9}$ of 21.

$\frac{4}{9} \times 21 = \frac{84}{9}$

$= 84 \div 9$

Calculate 84 ÷ 9 using short division.

$$\begin{array}{r} 0\ \ 9\ .\ 3\ \ 3 \\ 9\overline{)8\ ^{8}4\ .\ ^{3}0\ ^{3}0} \end{array}$$

Answer

Sahir ran $9.\dot{3}$ (or $9.\overline{3}$) kilometres along the beach.

11. There are 99 students in the Year 8 class at McCrawley Grammar. 47 students voted in the school election. What portion of the Year 8 students did not vote in the election? Express your answer as a decimal.

Key points

- There are 99 students.
- 47 students voted.
- What portion of the Year 8 students did not vote in the election? Express your answer as a decimal.

Explanation

The number of students who did not vote is the difference between the number of students and the number of voting students.

$99 - 47 = 52$ students.

Divide the number of non-voting students by the number of all students to calculate the portion of Year 8 students who did not vote.

$52 \div 99$

Evaluate the division using short division

$$99\overline{)\begin{array}{cccccccc} 0 & 0 & . & 5 & 2 & 5 & 2 \\ 5 & 2 & . & 0 & 2 & 0 & 2 & 0 \\ {\scriptstyle 52} & & {\scriptstyle 52} & & {\scriptstyle 25} & & {\scriptstyle 25} & \end{array}}$$

Answer

$0.5\dot{2}\dot{2}$ (or $0.\overline{52}$) of the Year 8 students did not vote.

12. Rowena is packaging 200 colouring pencils into bundles to sell on her online store. Explain whether Rowena should divide the pencils into groups of four or six so that every bundle has the exact same number of pencils.

Key points

- Rowena is packaging 200 colouring pencils into bundles.
- Every bundle has the exact same number of pencils.
- Explain whether Rowena should divide the pencils into groups of four or six

Explanation

To divide the pencils into identical bundles, the result of $200 \div$ pencils per bundle must be a whole number.
Substitute pencils per bundle $= 4$ and 6 respectively.

$200 \div 4 = 50$

$$99\overline{)\begin{array}{ccccc} 0 & 5 & 0 \\ 2 & 0 & 0 \\ {\scriptstyle 2} & {\scriptstyle 0} & \end{array}}$$

$200 \div 6 = 33.\dot{3}$

$$6\overline{)\begin{array}{ccccccc} 0 & 3 & 3 & . & 3 & 3 \\ 2 & 0 & 2 & 0 & . & 2 & 0 \\ {\scriptstyle 2} & & {\scriptstyle 2} & & {\scriptstyle 2} & & {\scriptstyle 2} \end{array}}$$

Bundles of 4 pencils produce an integer answer, so dividing the pencils into identical bundles is possible.

Answer

Rowena should divide the pencils into groups of four.

13. Warren just started a new book. He read $\frac{13}{20}$ of it in one night and then read half of the rest of the book the next night. A week later, he read a third of the pages he had not yet read. Written as a decimal, how much of the book does he have left to read?

Key points

- Warren read $\frac{13}{20}$ of the book.
- He then read half of the rest of the book.
- He then read a third of the pages he had not yet read.
- Written as a decimal, how much of the book does he have left to read?

Explanation

Calculate the fraction of the book he read at each stage.

He first read $\frac{13}{20}$ of the book.

Calculate how much of the book was left.

$1 - \frac{13}{20} = \frac{7}{20}$

He then read half of this amount.

$\frac{7}{20} \times \frac{1}{2} = \frac{7}{40}$

Subtract this from $\frac{7}{20}$ to calculate how much of the book was then left.

$\frac{7}{20} - \frac{7}{40} = \frac{7}{40}$

He then read one third of the pages left.

$\frac{7}{40} \times \frac{1}{3} = \frac{7}{120}$

Subtract this from $\frac{7}{40}$ to calculate how much of the book was then left.

$\frac{7}{40} - \frac{7}{120} = \frac{14}{120} = \frac{7}{60}$

Convert $\frac{7}{60}$ to a decimal using short division.

$$60\overline{)\begin{array}{cccccc} 0 & . & 1 & 1 & 6 & 6 \\ 7 & . & 0 & 0 & 0 & 0 \\ {\scriptstyle 70} & & {\scriptstyle 100} & & {\scriptstyle 400} & {\scriptstyle 400} \end{array}}$$

Answer

Warren has $0.11\dot{6}$ of the book to read.

Reasoning

14. a. The side length of the veggie patch is 8.60 m.

b. The area each plant will have is 3.08 m^2.

c. An exact area of 74 m^2 requires a side length of $\sqrt{74}$ m. This is a non-recurring decimal which is an irrational number. It can't be measured.

d. Suggested option 1: Side length of 9 m which can plant 27 vegetables.

Suggested option 2: Side length of 10 m which can plant 33 vegetables.

Note: There are other possible options.

15. a. $\frac{4}{5}, \frac{3}{10}, \frac{7}{8}$ **b.** $\frac{1}{6}, \frac{5}{9}, \frac{7}{15}, \frac{11}{12}$

c. If the prime factorisation of a denominator contains only 2s or 5s, then the fraction represents a terminating decimal. Otherwise, the fraction represents a recurring decimal.

Extra spicy

16. $7.3947\dot{2}$ or $7.3947\overline{2}$ hours

17. C **18.** B **19.** 0.293 mm

Remember this?

20. C **21.** A **22.** D

2G Rounding and estimating with decimals

Student practice

Worked example 1

a. 64.3 **b.** 7.90

Worked example 2

a. 4200 **b.** 10

Understanding worksheet

1. **a.** 0.9 **b.** 3 **c.** 7.26 **d.** −1.9

2. **a.** $53 - 29 \approx 50 - 30 = 20$

b. $762 - 23 \approx 800 - 20 = 16\,000$

c. $11.63 - 67.4 \approx 10 - 70 = 700$

d. $\frac{2823}{613} \approx \frac{3000}{600} = 5$

3. critical; lead; estimate; approximately; equal

Fluency

4. **a.** 0.23 **b.** 3.88 **c.** 6.300 **d.** 1.7269
e. 4.511 **f.** 14.5110 **g.** 23.500 **h.** 20.00

---

5. **a.** 180 **b.** 950 **c.** 4 **d.** 24 000
e. 700 **f.** 7.5 **g.** $\frac{1}{4}$ or 0.25 **h.** 1.6

---

6. **a.** 2410 **b.** 195 **c.** −440 **d.** 8
e. 2 **f.** 3 600 000 **g.** 31 500 **h.** 20 000

---

7. **a.** Greater than **b.** Greater than
c. Greater than **d.** Less than
e. Greater than **f.** Less than
g. Less than **h.** Greater than

---

8. **a.** 0.13 **b.** 0.333 **c.** 0.3333 **d.** 0.286
e. 0.417 **f.** 1.1429 **g.** 1.4444 **h.** 0.6923

---

9. E

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. According to her GPS, Tracey needs to drive west for 28.74 km to arrive at her desired destination. If her speedometer says she has already driven 17.6 km, how much further does she need to drive, if she rounded the remaining distance to the nearest km?

Key points

- Tracey needs to drive west for 28.74 km.
- Tracey has already driven 17.6 km.
- How much further does she need to drive, if she rounded the remaining distance to the nearest km?

Explanation

Distance Tracey still needs to drive = total distance − distance already driven

$= 28.74 - 17.6$

$= 11.14$ km

Now round to the nearest kilometre.

Draw a line between the digit being rounded and the critical digit.

11.|14

The critical digit is less than 5, so the digit being rounded stays the same.

Write the number with no decimal places.

$11.14 \approx 11$ km

Answer

Tracey needs to drive a further 11 km.

12. Aaron is standing next to a stack of 1.9 cm thick books and is as tall as 87 of them. How tall is Aaron, if he rounded his height to the nearest cm?

Key points

- The books are 1.9 cm thick.
- Aaron is as tall as 87 books.
- How tall is Aaron, if he rounded his height to the nearest cm?

Explanation

Height = thickness of each book × number of books

$= 1.9 \times 87$

$= 165.3$ cm

Now round to the nearest cm.

Draw a line between the digit being rounded and the critical digit.

165.|3

The critical digit is less than 5, so the digit being rounded stays the same. Write the number with no decimal places.

Answer

Aaron is 165 cm tall.

13. 29 of Keenan's students have booked a parent teacher interview with him. Keenan has 5 hours to complete the interviews. If Keenan uses lead digit rounding, how much time does he approximately have for each student interview?

Key points

- Keenan has 29 parent teacher interviews.
- He has 5 hours to complete the interviews.
- There are 60 minutes in 1 hour.
- If Keenan uses lead digit rounding, how much time does he approximately have for each student interview?

Explanation

Convert the time to minutes. There are 60 minutes in 1 hour, so multiply the number of hours by 60.

$5 \times 60 = 300$ minutes.

Time per interview = total time ÷ number of interviews

Round the total time and the number of interviews using lead digit rounding before dividing.

Consider the total time. Draw a line between the digit being rounded and the critical digit.

3|00

The critical digit is less than 5, so round all numbers in the calculation to the lead digit.

So 300 stays the same.

Consider the number of interviews. Draw a line between the digit being rounded and the critical digit.

2|9

The critical digit is more than 5, so increase the digit being rounded by 1.

So $29 \approx 30$

Complete the calculation using the rounded numbers.

Time per interview $\approx 300 \div 30$

≈ 10 minutes

Answer

Keenan has approximately 10 minutes for each student interview.

14. Radio signals and other electromagnetic waves travel at 299 792.458 km per second. The distance from Earth to the Moon is 384 399 km. Use lead digit rounding to estimate the time it takes for electromagnetic waves to travel from the Earth to the Moon. Round your answer to two decimal places.

Key points

- Electromagnetic waves travel at 299 792.458 km per second.
- The distance from Earth to the Moon is 384 399 km.
- Use lead digit rounding to estimate the time it takes for electromagnetic waves to travel from the Earth to the Moon. Round your answer to two decimal places.

Explanation

Total seconds taken = total distance ÷ distance travelled per second

Round the total distance and distance travelled per second using lead digit rounding before calculating.

Consider the distance travelled per second. Draw a line between the digit being rounded and the critical digit.

2|99792.458

The critical digit is more than 5, so the digit being rounded increases.

$299\,792.458 \approx 300\,000$

Consider the total distance. Draw a line between the digit being rounded and the critical digit.

3|84 399

The critical digit is more than 5, so increase the digit being rounded by 1.

So

$384\,399 \approx 400\,000$

Complete the calculation using the rounded numbers.

Total seconds taken $\approx 400\,000 \div 300\,000$

≈ 1.33

Answer

It takes approximately 1.33 seconds for electromagnetic waves to travel from the Earth to the Moon.

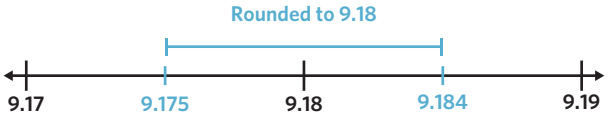
15. Esther and Katie had a very close finish in the 70 m sprint. Their times were both rounded to the nearest hundredth of a second. They both finished in 9.18 seconds, but the photo finish showed that Esther won. What is the largest possible time difference between Esther and Katie's race times, in thousandths of a second?

Key points

- Esther and Katie's times were both rounded to the nearest hundredth of a second.
- They both finished in 9.18 seconds.
- What is the largest possible time difference between Esther and Katie's race times, in thousandths of a second?

Explanation

To maximise the time difference, find the slowest and fastest times that would be rounded to 9.18.



Slowest possible time: 9.184

Fastest possible time: 9.175

Calculate the difference between these times.

$9.184 - 9.175 = 0.009$ seconds.

Answer

The largest possible time difference between Esther and Katie's race times is 0.009 seconds.

Reasoning

16. a. There are approximately 45 000 people in the Shane Warne stand.

b. There are approximately 15 000 people in the Members Reserve stand.

c. There are approximately 16 000 people in the Ponsford stand.

d. Approximately 86 000 people are attending the game.

e. Suggested option 1: Look at how full the stadium is overall, and estimate the proportion of occupied seats to total seats.

Suggested option 2: Count the number of people who enter the stadium from a particular entrance and assume the same number of people entering from all the other entrances.

Note: There are other possible options.

17. a. 40 030 km

b. 40 035 km

c. The estimate in part **a** is closer to the exact result than the answer in part **b**. This shows that the more decimal places included, the more accurate an estimation. When rounding to a smaller decimal place, the estimation becomes less accurate.

Extra spicy

18. D

19. 44.48 and 44.53

20. C

21. $74

Remember this?

22. D **23.** C **24.** D

Chapter 2 extended application

1. a.

Usain BOLT	1.04 m
Tyson GAY	1.03 m
Yohan BLAKE	1.03 m
Asafa POWELL	1.03 m
Justin GATLIN	1.03 m

b.

Usain BOLT	100 m
Tyson GAY	98.86 m
Yohan BLAKE	98.86 m
Asafa POWELL	98.56 m
Justin GATLIN	98.36 m

c.

Athlete	Distance run	How much Bolt beat them by
Tyson GAY	9886 cm	114 cm
Yohan BLAKE	9886 cm	114 cm
Asafa POWELL	9856 cm	144 cm
Justin GATLIN	9836 cm	164 cm

d. 10 mm

e. Suggested option 1: We have a better knowledge of sports science, so athletes receive better training.

Suggested option 2: We have made many important medical advances, so sports injuries receive much better treatment.

Note: There are other possible options.

2. a. $\frac{7}{24}$ of a standard circuit board.

b. $\frac{7}{11}$ g

c. Gold: $\frac{7}{66}$ g

Silver: $\frac{7}{33}$ g

Copper: $\frac{7}{22}$ g

d. 48 circuit boards.

e. $311.41

f. Suggested option 1: Launch an advertising campaign raising awareness of the trade-in service and the environmental importance of recycling metals.

Suggested option 2: Offer trade-in services at a more diverse range of locations such as supermarkets.

Note: There are other possible options.

3. a. $\frac{13}{16}$ of the maximum altitude.

b. $5\frac{1}{3}$ sandbags.

c. 4 sandbags.

d. $1\frac{1}{4}$ sandbags.

e. $\frac{49}{64}$ of the maximum altitude.

f. Suggested option 1: In Luxor, Egypt above the Valley of the Kings.

Suggested option 2: In Queenstown, New Zealand.

Note: There are other possible options.

Chapter 2 review

Multiple choice

1. B **2.** D **3.** D **4.** C **5.** D

Fluency

6. a. $\frac{21}{10}$ **b.** $\frac{17}{3}$ **c.** $7\frac{1}{2}$ **d.** $7\frac{11}{12}$

7. a. $\frac{1}{5} = \frac{2}{10} = \frac{10}{50}$ **b.** $\frac{6}{18} = \frac{12}{36} = \frac{2}{6}$

c. $2\frac{3}{4} = 2\frac{36}{48} = \frac{11}{4}$ **d.** $\frac{54}{30} = \frac{45}{25} = 1\frac{4}{5}$

8. a. $\frac{1}{13}$ **b.** $\frac{9}{4} = 2\frac{1}{4}$ **c.** $\frac{4}{7}$ **d.** $\frac{47}{6} = 7\frac{5}{6}$

9. a. $\frac{12}{18} = \frac{2}{3}$ **b.** $\frac{3}{13}$ **c.** $\frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$ **d.** $\frac{14}{9} = 1\frac{5}{9}$

10. a. $\frac{19}{30}$ **b.** $\frac{10}{67}$ **c.** $\frac{97}{280}$ **d.** $\frac{31}{5} = 6\frac{1}{5}$

11. a. $-\frac{11}{30}$ **b.** $-\frac{4}{9}$ **c.** $-\frac{13}{24}$ **d.** $-\frac{8}{11}$

12. a. 17.9 **b.** 25 **c.** 5.88 **d.** 5.7708

13. a. 42.2 **b.** 205.025 **c.** 114.956 **d.** 24

14. a. 0.375 **b.** 0.028 **c.** $1.\overline{6}$ **d.** $3.\overline{384615}$

15. a. 639.2 **b.** 5.00 **c.** 500.00 **d.** 81

16. a. 15 **b.** 600 **c.** 25.5 **d.** 6.975

Problem solving

17. Sienna spends $4 per day on chai lattes. How many chai lattes will she drink to have spent $24 on chai lattes?

Key points

- Sienna spends $4 per day on chai lattes.
- How many chai lattes will she have to drink to have spent $24?

Explanation

Each chai latte costs $4, so to have spent $24 Sienna will need to drink $24 \div 4$ chai lattes.

$$24 \div 4 = \frac{24}{4}$$
$$= \frac{6}{1}$$
$$= 6$$

Sienna will need to drink 6 chai lattes to have spent $24.

Answer

Sienna will need to drink 6 chai lattes to have spent $24.

18. Clelia has written a short story for a competition, and is checking the word count at the beginning, middle and end. The word count at the beginning and middle sum to $\frac{4}{5}$ of the whole story. The middle and ending sum to $\frac{2}{3}$ of the whole story. What fraction of the whole story was the middle?

Key points

- Clelia has written a short story for a competition, and is checking the word count at the beginning, middle and end.
- The word count at the beginning and middle sum to $\frac{4}{5}$ of the whole story.
- The middle and ending sum to $\frac{2}{3}$ of the whole story.
- What fraction of the whole story was the middle?

Explanation

The whole story consists of beginning + middle + end.

The beginning and middle sum to $\frac{4}{5}$, so beginning + middle $= \frac{4}{5}$.

The whole story is 1, so ending $= 1 - \frac{4}{5} = \frac{1}{5}$.

The middle and ending sum to $\frac{2}{3}$, so middle + ending $= \frac{2}{3}$.

Therefore, middle $= \frac{2}{3} - \frac{1}{5} = \frac{7}{15}$

Answer

The middle was $\frac{7}{15}$ of the whole story.

19. A group of friends are comparing their hourly rates at their part-time jobs. John makes $\frac{2}{5}$ of what Sam makes. Sam makes $\frac{7}{6}$ of what Tracey makes. Ishir makes $\frac{4}{5}$ of what Tracey makes. If Ishir makes $24 per hour, how much does John make per hour?

Key points

- John makes $\frac{2}{5}$ of what Sam makes.
- Sam makes $\frac{7}{6}$ of what Tracey makes.
- Ishir makes $\frac{4}{5}$ of what Tracey makes.
- Ishir makes $24 per hour.
- How much does John make per hour?

Explanation

Ishir makes $24 per hour and $\frac{4}{5}$ of what Tracey makes, so $\frac{4}{5} \times$ Tracey's rate $= 24$.

Tracey's hourly rate is therefore $24 \div \frac{4}{5}$, which is:

$= \frac{24}{1} \times \frac{5}{4}$

$= 6 \times 5$

$= 30$

Therefore, Tracey makes $30 per hour. Sam makes $\frac{7}{6}$ of what Tracey makes, so Sam's hourly rate is $\frac{7}{6} \times 30$.

$= \frac{7}{6} \times \frac{30}{1}$

$= 7 \times 5$

$= 35$

Sam makes $35 per hour. John makes $\frac{2}{5}$ of what Sam makes, so John's hourly rate is $\frac{2}{5} \times 35$.

$= \frac{2}{5} \times \frac{35}{1}$

$= 2 \times 7$

$= 14$

John's hourly rate is $14.

Answer

John's hourly rate is $14.

20. On a particular day, the temperature in the South Pole in Antarctica was $-62\frac{4}{10}$°C and in Mount Buller it was $-1\frac{4}{5}$°C. Find how much warmer Mount Buller was than the South Pole, as a mixed number.

Key points

- The temperature in the South Pole in Antarctica was $-62\frac{4}{10}$°C.
- The temperature in Mount Buller was $-1\frac{4}{5}$°C.
- How much warmer is Mount Buller compared to the South Pole, as a mixed number?

Explanation

It was $-62\frac{4}{10}$°C in Antarctica and $-1\frac{4}{5}$°C in Mount Buller, so the difference can be shown by subtracting the lower temperature from the higher temperature.

$$-1\tfrac{4}{5} - \left(-62\tfrac{4}{10}\right) = -\tfrac{9}{5} + \tfrac{624}{10}$$
$$= -\tfrac{18}{10} + \tfrac{624}{10}$$
$$= \tfrac{606}{10}$$
$$= 60\tfrac{6}{10}$$
$$= 60\tfrac{3}{5}\text{°C}$$

Answer

Mount Buller was $60\frac{3}{5}$°C or warmer than the South Pole.

21. At the beginning of last year, one ounce of silver was $15.65. At the end of last year, one ounce of silver was $17.90. If Nadia bought 0.4 ounces of silver at the beginning of last year and sold it at the end of last year, what profit did she make?

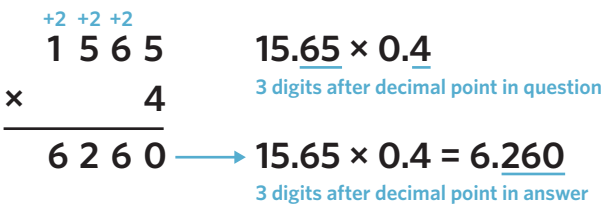
Key points

- At the beginning of last year, one ounce of silver was $15.65.
- At the end of last year, one ounce of silver was $17.90.
- Nadia bought 0.4 ounces of silver at the beginning of last year and sold it at the end of last year.
- What profit did she make?

Explanation

Nadia's profit is the difference between her purchase price at the beginning of last year and her selling price at the end of last year.

She bought 0.4 ounces of silver, and one ounce of silver was $15.65 at the beginning of last year, so this cost her $15.65 \times 0.4 = $6.26. We can calculate this using the vertical algorithm.



If her selling price was $17.90 per ounce, she would have made $17.90 \times 0.4 = $7.16.

Therefore, Nadia's profit is $7.16 - 6.26 = $0.9 or 90 cents.

Answer

Nadia's profit was $0.9 or 90 cents.

22. In a laboratory, Ellett is measuring the speed of a snail using very precise equipment. If the distance a snail travels in two minutes is $1.\overline{3}$ m, calculate its speed in metres per minute. Write your answer as a decimal using appropriate notation.

Key points

- Ellett is measuring the speed of a snail using very precise equipment.
- The distance a snail travels in two minutes is $1.\overline{3}$ m.
- Calculate the snail's speed in metres per minute, writing your answer as a decimal.

Explanation

If a snail travels $1.\overline{3}$ metres in two minutes, converting this to a fraction this means it travels $\frac{4}{3}$ m in two minutes. In one minute, it must travel $\frac{4}{3} \div 2$ metres.

$\frac{4}{3} \div 2 = \frac{4}{3} \times \frac{1}{2}$

$= \frac{4}{6} = \frac{2}{3}$

$= 0.\overline{6}$

The snail's speed is $0.\overline{6}$ metres per minute.

Answer

The snail's speed is $0.\overline{6}$ metres per minute.

23. A doctor is prescribing some medication for a patient. The dosage depends on the weight of the patient, and is described by the following rule:

$\text{dosage} = 1.105 \times 80.54 - 1.0418$ mg

It is important to ensure that a patient has the correct dosage, so after performing the calculation, she performs lead digit rounding to check the reasonableness of her answer. What answer does lead digit rounding obtain?

Key points

- A doctor is prescribing some medication for a patient.
- The dosage depends on the weight of the patient, and is described by the following rule: $\text{dosage} = 1.105 \times 80.54 - 1.0418$ mg.
- She performs lead digit rounding to check the reasonableness of her answer as each patient needs the correct dosage.
- What answer does lead digit rounding obtain?

Explanation

Performing lead digit rounding on the dosage equation, we obtain:

$\text{dosage} = 1 \times 80 - 1$

This gives a dosage of 79 mg.

Answer

Lead digit rounding gives an answer of 79 mg.

Reasoning

24. a.

	Per 100 mL (to 7 decimal places)	Per 100 mL (to 1 decimal place)
Fat (total)	0.1705473 g	0.2 g
Saturated fat	0.0058101 g	0.0 g
Carbohydrate (total)	14.2188915 g	14.2 g
Sugar	14.0611112 g	14.1 g
Protein	0.0000101 g	0.0 g
Salt	0.0000001 g	0.0 g

b. Fat: 0.5 g

Saturated fat: 0 g

Carbohydrate: 35.5 g

Sugar: 35.25 g

Protein: 0 g

Salt: 0 g

c. $\frac{1}{3}$ of the initial amount of the drink is left.

d. $0.\overline{3}$

e. The volume of Berry Blitz is 200 cm^3.

f. Suggested option 1: The school could offer freshly pressed juices instead of soft drinks.

Suggested option 2: The school could offer flavoured carbonated water instead of soft drinks.

Note: There are other possible options.

25. a. $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$\frac{998}{999} \times \frac{999}{1000} = \frac{998}{1000} = \frac{499}{500}$

b. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

$\frac{997}{998} \times \frac{998}{999} \times \frac{999}{1000} = \frac{997}{1000}$

c. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{999}{1000} = \frac{1}{1000}$

The same strategies were used to solve parts **a** and **b**, which involve simplifying out numbers that appear in both the numerator of one fraction and the denominator of another in the series. The numbers simplify because they divide into each other (numbers divided by themselves always equal 1). In the above series, this leaves $\frac{1}{1000}$ as the final answer as all other numbers in the middle are cancelled.

3A Percentages, decimals, and fractions

Student practice

Worked example 1

a. 0.27 **b.** $2\frac{1}{2}$

Worked example 2

a. 1% **b.** $\frac{3}{4}$

Worked example 3

a. 0.8 **b.** $66\frac{2}{3}\%$

Understanding worksheet

1. **a.** Fraction **b.** Decimal
c. Percentage **d.** Percentage

2. **a.** 40% **b.** $\frac{83}{100}$ **c.** 0.15 **d.** $\frac{11}{20}$

3. fraction; convert; decimal; percentage

Fluency

4. **a.** 0.25 **b.** 0.78 **c.** 1.23 **d.** 0.04
e. 0.391 **f.** 6.027 **g.** 0.053 **h.** 0.0002

5. **a.** $\frac{13}{100}$ **b.** $\frac{7}{100}$ **c.** $\frac{2}{5}$ **d.** $1\frac{3}{10}$
e. $\frac{2}{25}$ **f.** $2\frac{1}{4}$ **g.** $\frac{7}{1000}$ **h.** $\frac{1}{40}$

6. **a.** 25% **b.** 160%
c. 47.3% **d.** 8%
e. 5602% **f.** 11090%
g. 0.5% **h.** 1.04%

7. **a.** $\frac{9}{10}$ **b.** $\frac{4}{5}$ **c.** $\frac{29}{50}$ **d.** $1\frac{1}{10}$
e. $3\frac{9}{100}$ **f.** $\frac{1}{20}$ **g.** $18\frac{13}{50}$ **h.** $\frac{9}{125}$

8. **a.** 0.5 **b.** 0.7 **c.** $0.\overline{3}$ **d.** 0.625
e. 1.25 **f.** 2.45 **g.** $0.\overline{4}$ **h.** $10.\overline{6}$

9. **a.** 25% **b.** 2% **c.** 99% **d.** 37.5%
e. 180% **f.** $366\frac{2}{3}\%$ **g.** 720% **h.** $77\frac{7}{9}\%$

10. A

Spot the mistake

11. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

12. Bela estimates that three-quarters of the Australian population own a mobile phone. What percentage does this represent?

Key points

- Bela estimates that three-quarters of the Australian population own a mobile phone.
- What percentage does this represent?

Explanation

Three-quarters represented as a fraction is $\frac{3}{4}$.

We can convert $\frac{3}{4}$ to a percentage by first multiplying by 100.

$\frac{3}{4} \times 100 = \frac{300}{4}$

We can then set out the fraction as short division, and then add a percentage sign to the answer.

$$\begin{array}{r} 0\ \ 7\ \ 5\ \ r\ \ 0 \\ 4\overline{)3\ \ {}^{2}0\ \ {}^{0}0} \end{array}$$

$= 75\%$

Answer

75%

13. It takes $5\frac{1}{2}$ minutes for Matchoo to heat up his frozen lasagne for lunch. Express the time as a decimal.

Key points

- It takes $5\frac{1}{2}$ minutes for Matchoo to heat up his frozen lasagne for lunch.
- Express the time as a decimal.

Explanation

We first convert the fractional portion of the time $\left(\frac{1}{2}\right)$ to a decimal.

$\frac{1}{2} = 1 \div 2$

$$2\overline{)1.0}\ \ 0.5$$

$\frac{1}{2} = 0.5$, $5\frac{1}{2} = 5.5$.

$5\frac{1}{2}$ represented as a decimal is 5.5.

Answer

5.5 minutes

14. In its simplest form, what fraction of Tori's hair is straight if 55% of it is curly?

Key points

- In its simplest form, what fraction of Tori's hair is straight if 55% of it is curly?

Explanation

If 55% of Tori's hair is curly, that means that $100 - 55 = 45\%$ of it is straight.

To find 45% in simplest form, we need to convert it to a fraction.

As % means out of 100, by definition, $45\% = \frac{45}{100}$.

Simplifying this, we get:

$\frac{45}{100} = \frac{9}{20}$

Answer

$\frac{9}{20}$ of Tori's hair is straight.

15. Kathy cuts her pizza into six equal pieces. Write down the value that one piece represents, as a fraction, decimal, and percentage.

Key points

- Kathy cuts her pizza into six equal pieces.
- Write down the value that one piece represents, as a fraction, decimal, and percentage.

Back to contents

Explanation

If Kathy cuts her pizza into six equal pieces, the value of one piece must be 1 out of 6 or one-sixth.

We can represent one-sixth as a fraction, decimal and percentage.

Fraction: $\frac{1}{6}$

Decimal: $0.1\overline{6}$

$$\begin{array}{r} 0\,.\,1\,\overline{6} \\ 6\overline{)1\,.\,{}^{4}0} \end{array}$$

Percentage: $0.1\overline{6} \times 100 = 16.\overline{6}\%$

$0.1\overline{6} = 16\frac{2}{3}\%$

Answer

Fraction: $\frac{1}{6}$

Decimal: $0.1\overline{6}$

Percentage: $16\frac{2}{3}\%$

16. Patrick has 0.8 hours left to finish a science experiment. Rewrite the amount of time as a fraction out of 60, so that Patrick knows how many minutes he has left.

Key points

- Patrick has 0.8 hours left to finish a science experiment.
- Rewrite the amount of time as a fraction out of 60, so that Patrick knows how many minutes he has left.

Explanation

As 1 hour = 60 minutes, 0.8 hours is 0.8×60 minutes:

$$\begin{aligned} 0.8 \times 60 &= \frac{8}{10} \times 60 \\ &= \frac{480}{10} \\ &= 48 \end{aligned}$$

Therefore, 0.8 hours = 48 minutes.

Writing this as a fraction out of 60 gives us $\frac{48}{60}$.

Answer

$\frac{48}{60}$

Reasoning

17. **a.** 20% **b.** $\frac{9}{50}$ **c.** $12.64

d. Vutha needs 125 grams of plain flour, 112.5 grams of rolled oats, 100 grams of desiccated coconut, 187.5 grams of unsalted butter, and 100 grams of golden syrup.

e. Suggested option 1: Vutha can make something with ingredients he already has at home so he doesn't need to buy any extra ingredients.

Suggested option 2: Vutha can write notes to his friends with a kind message.

Note: There are other possible options.

18. **a.** 8% **b.** $\frac{2}{3}$

c. In order to compare all the values, they must be all the same type. Convert all of the values to a percentage, then compare.

Note: Other possible solutions include converting all to fractions or all to decimals to compare.

Extra spicy

19. B **20.** D **21.** 24% **22.** $\frac{5}{32}$

Remember this?

23. E **24.** B **25.** B

3B Expressing as a percentage and calculating percentages of quantities

Student practice

Worked example 1

a. 65% **b.** $66\frac{2}{3}\%$

Worked example 2

a. 20% **b.** 18%

Worked example 3

a. 30 **b.** 39.1

Understanding worksheet

1. **a.** 80 **b.** 24 **c.** 43 **d.** 36

2. **a.** 12 **b.** 24 **c.** 30 **d.** 100

3. percentages; out of; fractions; multiplication

Fluency

4. **a.** 28% **b.** 50% **c.** 21% **d.** 150% **e.** 75% **f.** 60% **g.** 40% **h.** 26%

5. **a.** $37\frac{1}{2}\%$ **b.** $12\frac{1}{2}\%$ **c.** $58\frac{1}{3}\%$ **d.** $137\frac{1}{2}\%$ **e.** $86\frac{2}{3}\%$ **f.** $42\frac{6}{7}\%$ **g.** $108\frac{1}{3}\%$ **h.** $81\frac{9}{11}\%$

6. **a.** 25% **b.** 10% **c.** 32% **d.** 125% **e.** 35% **f.** 250% **g.** 60% **h.** 14%

7. **a.** 6 **b.** 15 **c.** 28 **d.** 65 **e.** 27 **f.** 105 **g.** 2401 **h.** 1386

8. **a.** 2.5 **b.** 22.1 **c.** 49.8 **d.** 92.9 **e.** 106.9 **f.** 424.4 **g.** 16.7 **h.** 437.2

9. E

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. Hunter has 1 yellow, 2 green and 3 pink chocolate clinkers. What percentage of clinkers are pink?

Key points

- Hunter has 1 yellow, 2 green and 3 pink chocolate clinkers.
- What percentage of clinkers are pink?

Explanation

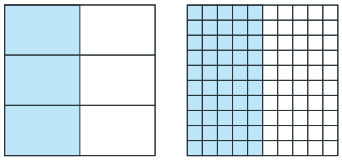
Total number of clinkers = yellow clinkers + green clinkers + pink clinkers

$$\begin{aligned} &= 1 + 2 + 3 \\ &= 6 \text{ clinkers.} \end{aligned}$$

3 out of 6 clinkers are pink. Express this as a percentage.

$$\begin{aligned} 3 \text{ out of } 6 &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

$$\frac{1}{2} \times \frac{100}{1}\% = \frac{1}{1} \times \frac{50}{1}\%$$
$$= \frac{50}{1}\%$$
$$= 50\%$$



$$\frac{3}{6} = \frac{50}{100}$$

Answer:

50% of the clinkers are pink

12. Casio piano mini-keyboards have a marked price of $110 but are discounted by 20% as part of an end of financial year (EOFY) sale. What dollar amount is the discount?

Key points

- The mini-keyboards normally cost $110.
- This is discounted by 20%.
- What dollar amount is the discount?

Explanation

Calculate 20% of $110.

Write the percentage and whole number as fractions. Replace 'of' with a multiplication sign.

$$\frac{20}{100} \times \frac{110}{1} = \frac{1}{5} \times \frac{110}{1}$$
$$= \frac{1}{1} \times \frac{22}{1}$$
$$= \frac{22}{1}$$
$$= \$22$$

Answer

The discount is $22.

13. A standard running track is 400 m. What percentage of the track does Boyd run, if the race is 5 km?

Key points

- The track is 400 m.
- The race is 5 km.
- There are 1000 m in 1 km.
- What percentage of the track does Boyd run?

Explanation

Convert the length of the track and the race to the same units.

Multiply 5 km by 1000 to calculate the distance of the race in m.

$5 \times 1000 = 5000$ m.

Calculate 5000 m as a percentage of 400 m.

Write the number sentence as a fraction, then multiply by 100% and simplify.

$$\frac{5000}{400} \times \frac{100}{1}\% = \frac{5000}{4} \times \frac{1}{1}\%$$
$$= \frac{1250}{1}\%$$
$$= 1250\%$$

Answer

Boyd runs 1250% of the track.

14. 23 minutes have elapsed in a 90-minute multiple choice exam, where each question is worth one mark. If Cas has completed 8 out of 40 questions, will they need to go faster or slower in order to finish the exam in time?

Key points

- 23 minutes have elapsed.
- The exam is 90 minutes long.
- Each question is worth one mark.
- Cas has completed 8 out of 40 questions.
- Will Cas need to go faster or slower in order to finish the exam in time?

Explanation

Calculate the percentage of the exam's time that has elapsed, and the percentage of the exam that has been completed. Compare the two percentages.

Calculate 23 minutes as a percentage of 90 minutes.

$$\frac{23}{90} \times \frac{100}{1}\% = \frac{23}{9} \times \frac{10}{1}\%$$
$$= \frac{230}{9}\%$$
$$\approx 25.6\% \text{ to 1 d.p.}$$

Calculate 8 questions as a percentage of 40 questions

$$\frac{8}{40} \times \frac{100}{1}\% = \frac{1}{5} \times \frac{100}{1}\%$$
$$= \frac{1}{1} \times \frac{20}{1}\%$$
$$= \frac{20}{1}\%$$
$$= 20\%$$

$25.6\% > 20\%$

That is, the time elapsed is greater than the portion of questions answered.

Therefore, Cas should go faster in order to finish in time.

Answer

Cas should go faster.

15. Sally-Anne owns a veterinary business and pays 25% company tax, down from the old tax rate of 27.5%. If the business had taxable income of 1.78 million dollars, how much tax is saved at the new tax rate, compared to the old one? Give your answer in dollars.

Key points

- Sally-Anne pays 25% company tax.
- The old tax rate was 27.5%.
- The business had taxable income of 1.78 million dollars.
- How much tax is saved at the new tax rate, compared to the old one? Give your answer in dollars.

Explanation

Calculate the tax Sally-Anne would have paid at this year's rate and the previous rate.

Tax paid at the new rate:

$$\frac{25}{100} \times \$1\,780\,000 = \$445\,000$$

Tax paid at old rate:

$$\frac{27.5}{100} \times \$1\,780\,000 = \$489\,500$$

Subtract the tax paid at this year's rate from the tax paid at the previous year's rate to calculate the tax saved.

$\$489\,500 - \$445\,000 = \$44\,500$

Answer

Sally-Anne saves $44 500 at the new tax rate compared to the old rate.

Reasoning

16. a. The minimum wage for a 15-year-old employee is $9.35.

b. The minimum wage of a 17-year-old casual employee as a percentage of the wage of an 18-year-old casual employee is 84.62%.

c. Tajah-Rose needs to save 68% of pay.

d. Jordan is spending $156.26 on food.

e. Suggested option 1: One way to better manage money in order to save for these items is to record and track our expenses to prevent overspending.

Suggested option 2: One way to better manage money in order to save for these items is to set up automatic savings in our bank account.

Note: There are other possible options.

17. a. 48

b. 24

c. When we multiply 120 by 20% instead of 40%, the answer is halved, in line with the percentage being halved.

Extra spicy

18. 1.52 m **19.** $16\frac{2}{3}\%$ **20.** C **21.** A

Remember this?

22. B **23.** D **24.** D

3C Increasing and decreasing by a percentage

Student practice

Worked example 1

a. $100 **b.** $40

Worked example 2

a. $13.75 **b.** $5.60

Worked example 3

a. $70 **b.** $9.09

Understanding worksheet

1. a. $45 **b.** $9 **c.** $35 **d.** $12

2. a. $40 **b.** $16 **c.** $2.50 **d.** $300

3. discount; cost price; mark-up; GST

Fluency

4. a. $120 **b.** $30
c. $82.50 **d.** $100
e. $475 **f.** $1.31
g. $7.49 **h.** $1555.20

---

5. a. $36 **b.** $45 **c.** $48 **d.** $0
e. $168 **f.** $1.92 **g.** $2.33 **h.** $30.03

---

6. a. $3 **b.** $6 **c.** $30 **d.** $6.50
e. $2.38 **f.** $32.90 **g.** $6.87 **h.** $3.28

---

7. a. $8 **b.** $18 **c.** $60 **d.** $1.90
e. $17 **f.** $46.50 **g.** $5.09 **h.** $18

---

8. a. $1.10 **b.** $7.50 **c.** $9 **d.** $9.96
e. $21 **f.** $60.01 **g.** $90 **h.** $299.99

---

9. a. $7.27 **b.** $13.64
c. $27.27 **d.** $3.18
e. $10.90 **f.** $127.18
g. $227.23 **h.** $1090.90

---

10. C

Spot the mistake

11. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

12. Lorelai buys a monkey lamp online that is usually $30. If there is a 10% storewide sale and free delivery, how much does she pay for the lamp?

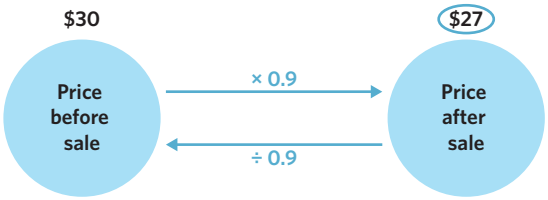
Key points

- The lamp originally cost $30.
- There is a 10% discount and no extra delivery charge.
- What is the new price of the lamp?

Explanation

Find the dollar amount of the discount.

$$10\% \text{ of } \$30 = \frac{10}{100} \times \frac{30}{1}$$
$$= \frac{1}{10} \times \frac{30}{1}$$
$$= \$3$$
$$\text{New price} = \$30 - \$3$$
$$= \$27$$



Answer

Lorelai pays $27 for the lamp.

13. Luke owns a cafe and sells sprinkled and glazed donuts at a 100% mark-up. If the retail price per donut is $4, what is the cost price?

Key points

- Luke sells his products at a 100% mark-up.
- The retail price per donut is $4.
- What is the cost price?

Explanation

$$\text{Retail price} = \text{cost price} + \text{mark-up}$$
$$\$4 = \text{cost price} + \text{cost price} \times 100\%$$
$$\$4 = 2 \times \text{cost price}$$
$$\text{Cost price} = \$2$$

Answer

The cost price is $2.

14. Rory wants to earn $50 per hour for writing magazine articles. Including GST, how much would clients need to pay per hour?

Key points

- Rory wants to earn $50 per hour for writing magazine articles.
- How much do clients need to pay per hour after GST?

Explanation

Price after GST = Price before GST × 1.1
= $50 × 1.1
= $55

Answer

Clients would need to pay $55 per hour.

15. Ceol buys gravy cheese chips at a chip shop in St Kilda for $12. What is the price before GST?

Key points

- Ceol buys gravy cheese chips for $12.
- Calculate the price before GST.

Explanation

Price before GST = Price after GST ÷ 1.1
= $12 ÷ 1.1
= $10.9090...

Answer

The price before GST is $10.91.

16. Dashy sells personalised mugs online. The wholesale price of each mug is $5. If she puts a mark-up of 120% and then GST of 10% onto the marked-up price, what is the price that customers are paying for the mug?

Key points

- The price of each mug is $5.
- A mark-up of 120% is added to the cost price.
- A GST of 10% is added onto the marked-up price.
- What is the retail price?
- Retail price = price after GST

Explanation

Calculate the retail price.

Marked-up price = percentage mark-up × cost price
= 220% × 5
$= \frac{220}{100} \times \frac{5}{1}$
= $11

Calculate the price after GST.

Price after GST = Price before GST × 1.1
= $11 × 1.1
= $12.10

Answer

The price that customers are paying for the mug is $12.10.

Reasoning

17. **a.** The discounted price of a pack of candles is $4.

b. Beatrix sells each blackforest cake for $66.

c. The price of a vanilla slice after GST is $6.50.

d. The price of a $50 cake before GST is $45.45.

e. Suggested option 1: Some businesses sell at a loss because they have excess stock.

Suggested option 2: Other businesses might do it to attract more customers.

Note: There are other possible options.

18. **a.** 159.20 **b.** 191.04

c. The answer for **b** is different from the original number. In order to obtain the same answer, it would be necessary to decrease by 20% of 199 and then increase by 20% of 199, as opposed to increasing by 20% of the lower value 159.20.

Extra spicy

19. $84 222 **20.** 31.6% **21.** C **22.** D

Remember this?

23. D **24.** C **25.** B

3D Percentage change

Student practice

Worked example 1

a. 20% increase **b.** 11% decrease

Worked example 2

a. 75% profit **b.** 60% loss

Worked example 3

a. 16.7% **b.** 11.4%

Understanding worksheet

1. **a.** profit **b.** loss
c. profit **d.** break-even

2. **a.** 500 **b.** 800 **c.** 250 **d.** 900

3. profit; retail price; break-even; loss

Fluency

4. **a.** 50% increase **b.** 25% increase
c. 20% decrease **d.** 400% increase
e. 50% increase **f.** 5% decrease
g. 125% increase **h.** 60% decrease

---

5. **a.** 25% increase **b.** 9.1% decrease
c. 6.4% decrease **d.** 42.9% increase
e. 0.2% decrease **f.** 1.2% increase
g. 0.6% decrease **h.** 0.2% increase

---

6. **a.** 20% profit **b.** 10% profit
c. 50% loss **d.** 25% loss
e. 10% profit **f.** 5% loss
g. 175% profit **h.** 15% loss

---

7. **a.** 67% profit **b.** 100% profit
c. 20% loss **d.** 42% loss
e. 300% profit **f.** 38% loss
g. 33% loss **h.** 44% loss

---

8. **a.** 11.76% **b.** 5.26% **c.** 15.38% **d.** 2.20%
e. 4.04% **f.** 1.49% **g.** 1.23% **h.** 95.10%

9. D

Spot the mistake

10. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. The height of a tree is measured each year. If it measured 2.5 m in 2022 and 3 m in 2023, by what percentage did the tree's height increase?

Key points

- The height of a tree is measured each year.
- It measured 2.5 m in 2022 and 3 m in 2023.
- By what percentage did the tree's height increase?

Explanation

We can calculate the percentage increase of the tree from 2022 to 2023.

$$\text{Increase} = 3 - 2.5$$
$$= 0.5$$
$$\text{Percentage increase} = \frac{0.5}{2.5} \times 100\%$$
$$= 20\%$$

Answer

The tree's height increased by 20%.

12. Raoul owns a furniture store and sells a particular type of rug for \$150. If the cost price is \$60, what is Raoul's percentage profit?

Key points

- Raoul owns a furniture store and sells a particular type of rug for \$150.
- The cost price is \$60.
- What is Raoul's percentage profit?

Explanation

We can calculate Raoul's percentage profit by finding the difference between the cost price and selling price, dividing that by the cost price and then multiplying by 100.

$$\text{Profit} = 150 - 60$$
$$= \$90$$

$150 - 60 = \$90$

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

$\$60$

$$\text{Percentage profit} = \frac{90}{60} \times 100\%$$
$$= 150\%$$

Raoul made a 150% profit.

Answer

Raoul made a 150% profit.

13. At the local supermarket, pastries that are close to their use by date are heavily reduced. If the cost price of a croissant is \$1.83 and the discounted price is \$0.95, what is the percentage loss? Round your answer to the nearest percentage.

Key points

- Pastries that are close to their use by date are heavily reduced.
- The cost price of a croissant is \$1.83.
- The discounted price is \$0.95.
- What is the percentage loss? Round your answer to the nearest percentage.

Explanation

Percentage loss is calculated as $\frac{\text{loss}}{\text{cost}} \times 100\%$.

If the cost price of the croissant is \$1.83 and the discounted price is \$0.95, the loss is calculated as:

$$\text{Loss} = 1.83 - 0.95$$
$$= \$0.88$$
$$\text{Percentage loss} = \frac{0.88}{1.83} \times 100\%$$
$$= 48.1\%$$
$$\approx 48\%$$

The percentage loss is 48%.

Answer

The percentage loss is 48%.

14. Alanna thinks that she will get $\frac{39}{50}$ on her Physical Education test, but her actual mark is $\frac{43}{50}$. What was the percentage error in Alanna's prediction? Give your answer to 1 decimal place.

Key points

- Alanna thinks that she will get $\frac{39}{50}$ on her Physical Education test.
- Alanna's actual mark is $\frac{43}{50}$.
- What was the percentage error in Alanna's prediction? Give your answer to 1 decimal place.

Explanation

Calculate the difference between Alanna's predicted mark and actual mark, as percentages.

Predicted: $\frac{39}{50} = 78\%$

Actual: $\frac{43}{50} = 86\%$

Difference: $86 - 78 = 8\%$

$$\text{Percentage error} = \frac{\text{difference}}{\text{actual}} \times 100\%.$$
$$= \frac{8}{86} \times 100\%$$
$$= 9.3\%$$

Alanna's prediction had a 9.3% error.

Answer

Alanna's prediction had a 9.3% error.

15. The maximum temperature in Melbourne on a winter day was 13°C, but decreased by 115% overnight. What was the overnight temperature, rounded to the nearest degree Celsius?

Key points

- The maximum temperature in Melbourne on a winter day was 13°C.
- The temperature decreased by 115% overnight.
- What was the overnight temperature, rounded to the nearest degree Celsius?

Explanation

We can calculate the overnight temperature by finding 115% of 13°C, and then subtracting the result from 13°C.

$$13 \times 115\%$$
$$= 13 \times \frac{115}{100}$$
$$= 14.95$$

Subtracting 14.95 from 13 gives:

$$13 - 14.95$$
$$= -1.95$$

Therefore, the overnight temperature was −1.95°C.
When rounded to the nearest degree, the temperature is −2°C.

Answer

The overnight temperature was −2°C.

Reasoning

16. a. Rounded to the nearest million, the population was 22 million in 2010 and 26 million in 2020.

b. The percentage increase in population between 2010 and 2020 was 18.2%.

c. The population increased by 104% between 1970 and 2020.

d. Bingley's prediction had a 1.29% error.

e. Suggested option 1: Human overpopulation may cause serious harm to the environment and worsen climate change in future.

Suggested option 2: Underpopulation could result in less production of goods in the economy and under-utilisation of resources.

Note: There are other possible options.

17. a. $33\frac{1}{3}$% increase **b.** 25% decrease

c. The percentage change for part **b** is lower than that in part **a**. The order does not matter in terms of the numerator in the percentage change calculation, where we simply find the difference between the two numbers. However, the order matters for the denominator in terms of which number is the original value, as a higher original value results in a lower percentage change (and vice versa).

Extra spicy

18. 57% **19.** 5% **20.** C **21.** B

Remember this?

22. D **23.** C **24.** E

3E The unitary method

Student practice

Worked example 1

a. Two jam donuts for $3 is the best buy.

b. 300 g of coconut sugar for $6.90 is the best buy.

Worked example 2

a. $15 **b.** $8

Worked example 3

a. 5 **b.** $2

Understanding worksheet

1. a. 6 for $8.80; $1.47 each

b. 120 g for $4; $3.33 per 100 g

c. 200 g for $2.75; $1.38 per 100 g

d. 1.4 kg for $17.50; $1.25 per 100 g

2. a. 120 **b.** 130 **c.** 60 **d.** 90

3. unitary method; unit price; one; best buy

Fluency

4. a. Two packs of Shape biscuits for $5.

b. Three avocados for $7.

c. A six-pack of highlighters for $9.99.

d. A 20-pack of toilet paper for $11.

5. a. A 500 g bag of frozen peas for $2.30.

b. A 1.25 L bottle of spring water for $1.65.

c. A 140 g tube of toothpaste for $1.02.

d. A 330 mL bottle of kombucha for $3.20.

6. a.	$4	**b.**	$25	**c.**	$32	**d.**	$15
e.	$60	**f.**	$108	**g.**	$73	**h.**	$123
7. a.	$2	**b.**	$9	**c.**	$18.75	**d.**	$9
e.	$25.60	**f.**	$22	**g.**	$6.80	**h.**	$3.15
8. a.	6	**b.**	5	**c.**	4	**d.**	10
e.	20	**f.**	15	**g.**	160	**h.**	50
9. a.	$18	**b.**	$3	**c.**	$3.57	**d.**	$14.79
e.	$39.95	**f.**	$1.75	**g.**	$88.96	**h.**	$83.84

10. E

Spot the mistake

11. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

12. Prasad is looking to buy chew toys for his guinea pigs. He can buy a twin pack for $4 or a four-pack for $7. Which is less expensive per chew toy?

Key points

- Prasad is choosing between a twin pack chew toys for $4 or a four-pack for $7.
- Which of the two is the best buy?

Explanation

Find the unit price for the twin pack.

Two chew toys cost $4.

1 chew toy costs $4 ÷ 2 = $2

Find the unit price for the four-pack.

Four chew toys cost $7.

1 chew toy costs $7 ÷ 4 = $1.75

Answer

A four-pack for $7 is the less expensive chew toy.

13. Emile creates handmade face cream at home and sells it online. If the mark-up is 150% and the retail price is $30, what is Emile's cost price?

Key points

- The mark-up price is 150% and the retail price is $30.
- What is the cost price?

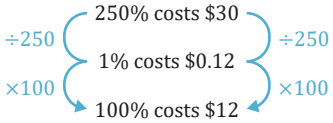
Explanation

Percentage after mark-up $= 100\% + 150\%$
$= 250\%$

250% costs \$30

$1\% = \$30 \div 250$
$= \$0.12$

$100\% = \$0.12 \times 100$
$= \$12$



Answer

Emile's cost price is \$12.

14. Brian drinks 40% of the water from his drink bottle and there is 300 mL left. How much water was in the drink bottle to begin with?

Key points

- There is a 40% decrease of water and the final volume is 300 mL.
- What is the initial volume of water?

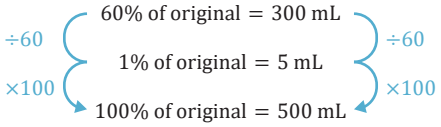
Explanation

Final value percentage $= 100\% - 40\%$
$= 60\%$

60% of original $= 300$ mL

1% of original $= 300 \div 60$
$= 5$ mL

100% of original $= 5 \times 100$
$= 500$ mL



Answer

There was originally 500 mL of water in the drink bottle.

15. Dexter buys ingredients to bake cookies for friends. He is choosing between a 200 g bag of chocolate chips for \$3 and a 375 g bag for \$5.50. Which option is the best buy?

Key points

- Dexter is choosing between a 200 g bag of chocolate chips for \$3 and a 375 g bag for \$5.50.
- Which option is the best buy?

Explanation

Find the unit price per 100 g for the 200 g bag.

200 g costs \$3.

100 g costs $\$3 \div 2 = \$1.5$

Find the unit price per 100 g for the 375 g bag.

375 g costs \$5.50.

100 g costs $\$5.50 \div 3.75 \approx \$1.47$

Answer

A 375 g bag of chocolate chips for \$5.50 is the best buy.

16. Diep buys a portable speaker during a 10% storewide sale. She then decides to buy one for her sister the following year for \$103.95, even though the sale has ended and the retail price has increased by 5%. What price did Diep pay for her own speaker?

Key points

- A portable speaker was bought with a 10% discount.
- The same speaker was bought without discount and with a 5% increase from the original price for \$103.95.
- What is the price of the speaker with a 10% discount?

Explanation

Find the original price of the speaker.

Percentage after price increase $= 100\% + 5\%$
$= 105\%$

105% costs \$103.95

$1\% = \$103.95 \div 105$
$= \$0.99$

$100\% = \$0.99 \times 100$
$= \$99$

Find the discounted price.

Percentage after discount $= 100\% - 10\% = 90\%$

Discounted price $= 90\%$ of \$99

$$= \frac{90}{100} \times \frac{99}{1}$$
$$= \frac{9}{10} \times \frac{99}{1}$$
$$= \$89.10$$

Answer

Diep paid \$89.10 for her own speaker.

Reasoning

17. a. Larks Shoes is the best buy.

b. Larks Shoes is (still) the best buy.

c. Their cost price is \$42.14.

d. Suggested option 1: How many pairs of shoes are needed.

Suggested option 2: The quality of the shoes.

Note: There are other possible options.

18. a. $x = 3.84$

b. $x = 3.84$

c. The two solutions are exactly the same. The method used in part **a** requires one less step than the method used in part **b**. However, each individual will have their own preference as to which is easier to understand or easier to use.

Extra spicy

19. C **20.** 59.9 hours **21.** \$2.75 **22.** E

Remember this?

23. C **24.** B **25.** B

Chapter 3 extended application

1. a. $\frac{49}{58}$

b. Decimal: 0.84

Percentage: 84%

c. \$41.65

d. Plan 3 is the best buy.

e. Suggested option 1: International calling features.

Suggested option 2: Length of contract.

Note: There are other possible options.

f. Suggested option:

Phone company: Phonehome

Monthly fee: $45

Data inclusion: 40 GB

Why the plan is right for me: It is affordable, allows for 40 GB of data a month which is more than enough.

Note: There are other possible options.

2. a. The Razor RX-3 would cost $30 975 next year.

b. The dealership's percentage profit was 8%.

c. The Vroom Vell was $28 455 before GST.

d. The cost price of the Haiku HR-V is $32 768.

e. The lower the better for fuel economy.

The hybrid Motor Moon Cross has the best fuel economy.

f. Suggested option: Ms Muscat might choose the Motor Moon Cross as it has the best fuel economy and is also a hybrid vehicle, meaning it is better for the environment. The cost of the car itself is somewhat on the high end when comparing all four cars, so the decision may depend on whether Ms Muscat can afford the car.

Note: There are other possible options.

3. a. There was a 1.7% decrease between 2017 and 2022.

b. Mr Robertson's index rating is 7.198.

c. Miss Qin's percentage error was 0.92%.

d. The 2016 happiness index rating was 7.255.

e. Suggested option: There are many people who experienced genuine hardship during the Coronavirus pandemic, including those that lost loved ones to the virus, suffered long term illness, lost all income, or were in some other way affected. Without minimising what any person went through, there is cause to argue that hardship and suffering can give rise to kindness and gratitude. If Dr Frankl was able to survive three years in concentration camps and find meaning in life and in his suffering, perhaps it is possible for others to do the same.

Note: There are other possible options.

Chapter 3 review

Multiple choice

1. B **2.** C **3.** D **4.** B **5.** E

Fluency

6. a. 75% **b.** 95% **c.** 197% **d.** 3.5%

7. a. $\frac{9}{20}$ **b.** $2\frac{1}{2}$ **c.** $308\frac{3}{10}$ **d.** $\frac{84}{125}$

8. a. 35.8% **b.** 29.2% **c.** 216.7% **d.** 63.0%

9. a. 16.3 **b.** 107.9 **c.** 87.6 **d.** 63.5

10. a. $69 **b.** $80 **c.** $4.68 **d.** $124.2

11. a. $86.36 **b.** $34.09 **c.** $1164.54 **d.** $38.95

12. a. 10% increase **b.** 20% decrease

c. 400% increase **d.** 32% decrease

13. a. 25% profit **b.** 50% profit

c. 44% loss **d.** 40% loss

14. a. Five cans for $3.90

b. Five-pack of pens for $3.20

c. 3 L of milk for $4.35

d. 250 mL bottle of Fanta for $2.24

15. a. 30 **b.** 25 **c.** 60 **d.** 150

Problem solving

16. Jimmy ate three-eighths of a pizza and saved the rest for the next day. What percentage of pizza did Jimmy save for the next day?

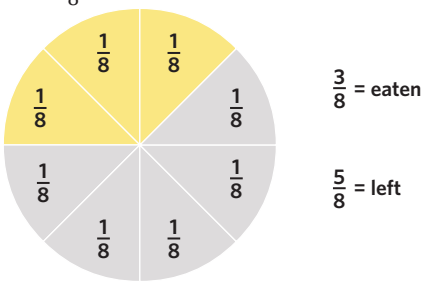
Key points

- Jimmy ate three-eighths of a pizza and saved the rest for the next day.
- What percentage of pizza did Jimmy save for the next day?

Explanation

Jimmy saved $1 - \frac{3}{8} = \frac{5}{8}$ of the pizza for the next day.

This is $\frac{5}{8} \times 100\% = 62.5\%$.



Answer

Jimmy saved 62.5% of the pizza for the next day.

17. Daisy was aiming to do 15 laps at the swimming pool. She was only able to complete 60% of her goal. How many laps did she swim in total?

Key points

- Daisy was aiming to do 15 laps at the swimming pool.
- She was only able to complete 60% of her goal.
- How many laps did she swim in total?

Explanation

$$\text{Daisy swam } 15 \times 60\% = 15 \times \frac{60}{100}$$
$$= \frac{900}{100}$$
$$= 9 \text{ laps in total.}$$

Answer

Daisy swam 9 laps in total.

18. Berty has a budget of $500 and wants to buy a new phone. The one he wants costs $756. He waits for a couple of months and the seller has discounted the phone by 25%. Will Berty now be able to afford the phone?

Key points

- Berty has a budget of $500 and wants to buy a new phone.
- The one he wants costs $756.
- He waits for a couple of months and the seller has discounted the phone by 25%.
- Will Berty now be able to afford the phone?

Explanation

The discounted price of the phone is $756 \times (1 - 25\%)$
$= 756 \times (1 - 0.25)$
$= 756 \times 0.75$
$= \$567$

Berty would not be able to afford the phone now because $\$567 > \$500$.

Answer

Berty would not be able to afford the phone now.

19. Lilly bought all of the art supplies to create a painting. The resources were $95 in total. Lilly then had to pay $12 to advertise her artwork online. She sold the painting for $115. Calculate the percentage profit or loss. Give your answer to one decimal place.

Key points

- Lilly bought all of the art supplies to create a painting. The resources were $95 in total.
- Lilly then had to pay $12 to advertise her artwork online.
- She sold the painting for $115.
- Calculate the percentage profit or loss. Give your answer to one decimal place.

Explanation

Lilly's profit = selling price − cost of resources − cost of advertising

$= 115 - 95 - 12$
$= \$8$ profit

Percentage profit = (profit ÷ (cost of resource + advertising)) × 100%

$= \left(\frac{8}{95 + 12}\right) \times 100\%$
$= \frac{8}{107} \times 100\%$
$= 7.5\%$ profit

Answer

Lilly's percentage profit is 7.5%.

20. Morgan needs fresh fruit for a grazing box. He looks at two different stalls at a market. Stall A has a 1 kg bag of grapes for $15.20 and 250 g of strawberries for $5.25. Stall B has a 500 g bag of grapes for $11 and 500 g of strawberries for $8.50. Which stall has the best buy for grapes and which stall has the best buy for strawberries?

Key points

- Stall A has a 1 kg bag of grapes for $15.20 and 250 g of strawberries for $5.25.
- Stall B has a 500 g bag of grapes for $11 and 500 g of strawberries for $8.50.
- Which stall has the best buy for grapes and which stall has the best buy for strawberries?

Explanation

Stall A:

Price for a 1 kg bag of grapes $= \$15.20$

Price for a 1 kg bag of strawberries $= 5.25 \times \frac{1000}{250} = 5.25 \times 4$
$= \$21$

Stall B:

Price for a 1 kg bag of grapes $= 11 \times \frac{1000}{500} = 11 \times 2$
$= \$22$

Price for a 1 kg bag of strawberries $= 8.50 \times \frac{1000}{500} = 8.50 \times 2$
$= \$17$

Therefore, stall A sells cheaper grapes and stall B sells cheaper strawberries.

Answer

Stall A has the best buy for grapes and stall B has the best buy for strawberries.

Reasoning

21. a. Ava sold 15 pairs of socks.

b. The percentage of button up shirts Ava sold is 25%.

c. The new price of the bucket hats are $18.25 each.

d. The profit percentage when she sells the shirts in her shop is 25%.

e. Suggested option 1: Ava could sell jackets at her stall.

Suggested option 2: Ava could sell jeans at her stall.

Note: There are other possible options.

22. a. 156

b. 109.2

c. The answer in part **b** is smaller than 120. This is because in part **b,** we decreased 30% of 156, which is greater than 120. If we had decreased 30% of 120 in part **b**, the answer would be 120.

4A Index notation

Student practice

Worked example 1

a. Base: 2; Index: 7 **b.** Base: z; Index: 4

Worked example 2

a. $8^4 \times m^2$ **b.** $2 \times 2 \times 2 \times 2 \times 2 \times y \times y$

Worked example 3

a. 20 **b.** 213

Understanding worksheet

1. **a.** 4^6 **b.** 7^2 **c.** x^3 **d.** 12^b

2. **a.** $2 \times 2 \times 2 \times 2 = 2^4$
b. $9 \times 9 \times 9 = 9^3$
c. $y \times y \times y \times y \times y \times y \times y = y^6$
d. $3 \times 3 \times 5 \times 5 \times 5 = 3^2 \times 5^3$

3. base; factors; expanded form; power

Fluency

4. **a.** Base: 7; Index: 5 **b.** Base: 2; Index: 3
c. Base: 1; Index: 8 **d.** Base: 12; Index: 3
e. Base: 9; Index: 11 **f.** Base: p; Index: 2
g. Base: y; Index: 15 **h.** Base: a; Index: c

---

5. **a.** 8^4
b. 3^3
c. 1^7
d. $5^2 \times 6^1$ or $5^2 \times 6$
e. $0.2^2 \times 4^3 \times 7^1$ or $0.2^2 \times 4^3 \times 7$
f. $k^4 \times 9^2$
g. $3^2 \times x^2 \times y^1 \times z^1 \times 4^1$ or $3^2 \times x^2 \times y \times z \times 4$ or $36x^2yz$
h. $m^2 \times 5.3^3 \times 1 \times n^3$

---

6. **a.** $6 \times 6 \times 6$
b. $4 \times 4 \times 4 \times 4 \times 4 \times 4$
c. $2 \times 2 \times 9 \times 9 \times 9 \times 9$
d. $a \times a \times a \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
e. $1.7 \times 1.7 \times 1.7 \times 1.7 \times 10 \times 5 \times 5 \times 5 \times 5$
f. $a \times a \times a \times 8 \times 8 \times 13.2 \times x \times x \times x \times y \times y \times y \times y \times y$
g. $q \times q \times q \times r \times r \times r \times r \times s$
h. $8 \times n \times n \times n \times m \times m \times 3 \times 3 \times 3 \times 3 \times k \times k \times k \times k$

---

7. **a.** 67 **b.** 11 **c.** -2 **d.** 20
e. 5 **f.** -16 **g.** 14 **h.** 51

---

8. E

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. Julia received 5 Amazon gift cards for Christmas. If she can buy 5 movies with each card, how many movies can she buy in total?

Key points

- Julia received 5 Amazon gift cards for Christmas.
- She can buy 5 movies with each card.
- How many movies can she buy in total?

Explanation

Number of movies Julia can buy in total
= Number of gift cards × Number of movies Julia can buy with each gift card
$= 5 \times 5$
$= 5^2$
$= 25$

Base — 5^2 — Index

2 factors of 5
$5^2 = 5 \times 5$

Answer

Julia can buy 25 movies in total.

11. Fatima is stacking LEGO blocks. Every day, the number of blocks she stacks is equal to double the number of blocks she had the previous day. On day 1, she stacks 2 blocks. Write an expression in expanded form to represent the number of blocks she stacks on day 4.

Key points

- Every day, the number of blocks Fatima stacks is equal to double the number of blocks she had the previous day.
- On day 1, she stacks 2 blocks.
- Write an expression in expanded form to represent the number of blocks she stacks on day 4.

Explanation

Day 1 Fatima stacks 2 blocks.
Day 2 Fatima stacks 2 × day 1 $= 2 \times 2 = 2^2 = 4$ blocks
Day 3 Fatima stacks 2 × day 2 $= 2 \times 4 = 2^3 = 8$ blocks
Day 4 Fatima stacks 2 × day 3 $= 2 \times 8 = 2^4 = 16$ blocks

Answer

In day 4, Fatima stacks $2 \times 2 \times 2 \times 2 = 16$ blocks.

12. When bread is left out at room temperature, it begins to spoil and mould begins to grow. The initial area of the mould is 2 cm^2 and multiplies by 1.5 every day. How big is the area of mould on a piece of bread after 3 days?

Key points

- The initial area of the mould on a piece of bread is 2 cm^2.
- The area of the mould multiplies by 1.5 every day.
- How big is the area of mould on a piece of bread after 3 days?

Explanation

Initial area of mould $= 2$ cm^2

Area of mould on day 1
$= 1.5 \times$ initial area of mould $= 1.5 \times 2 = 3$ cm^2

Area of mould on day 2
$= 1.5^2 \times$ initial area of mould $= 1.5^2 \times 2 = 4.5$ cm^2

Area of mould on day 3
$= 1.5^3 \times$ initial area of mould $= 1.5^3 \times 2 = 6.75$ cm^2

Answer

After 3 days, the area of mould on a piece of bread will be 6.75 cm^2.

13. John receives $10 of pocket money every week. In the first week that Tanaka was given pocket money she received $2. Afterwards, her parents gave double the amount from the previous week. Assuming that they both started receiving pocket money in the same week, after seven weeks, who has more money and by how much?

Key points

- John receives $10 of pocket money every week.
- In the first week that Tanaka was given pocket money she received $2.
- Afterwards, her parents gave double the amount from the previous week.
- John and Tanaka both started receiving pocket money in the same week.
- After seven weeks, who has more money and by how much?

Explanation

John's money after 7 weeks = John's pocket money each week × 7

$= 10 \times 7$

$= \$70$

For Tanaka:

Week 1 Tanaka receives $2

Week 2 Tanaka receives $2 \times$ Week $1 = 2 \times 2 = 2^2 = \$4$

Week 3 Tanaka receives $2 \times$ Week $2 = 2 \times 4 = 2^3 = \$8$

Week 4 Tanaka receives $2 \times$ Week $3 = 2 \times 8 = 2^4 = \$16$

Week 5 Tanaka receives $2 \times$ Week $4 = 2 \times 16 = 2^5 = \$32$

Week 6 Tanaka receives $2 \times$ Week $5 = 2 \times 32 = 2^6 = \$64$

Week 7 Tanaka receives $2 \times$ Week $6 = 2 \times 64 = 2^7 = \$128$

Tanaka's total money on week 7

$= 2 + 4 + 8 + 16 + 32 + 64 + 128 = \$254$

In week 7, Tanaka will have more money. This is $254 - 70 = \$184$

Answer

After 7 weeks, Tanaka will have $184 more pocket money than John.

14. At 1 pm, Kareem, Geoff and Bela, were told a secret. One hour later, they each told the secret to three of their own friends. Every hour each person who hears the secret tells three other people. At what time were 243 people told the secret, excluding Kareem, Geoff, and Bela?

Key points

- At 1 pm, Kareem, Geoff and Bela, were told a secret.
- One hour later, they each told the secret to three of their own friends.
- Every hour each person who hears the secret tells three other people.
- At what time were 243 people told the secret, excluding Kareem, Geoff, and Bela?

Explanation

Number of people who were told this secret by Kareem, Geoff and Bela:

At 2 pm there are $3 \times 3 = 3^2 = 9$ people who knew the secret

At 3 pm there are $3 \times 9 = 3^3 = 27$ people who knew the secret

At 4 pm there are $3 \times 27 = 3^4 = 81$ people who knew the secret

At 5 pm there are $3 \times 81 = 3^5 = 283$ people who knew the secret

Answer

At 5 pm, 243 people told the secret, excluding Kareem, Geoff, and Bela.

Reasoning

15. a. 40×2^3

b. Megan would expect to count 640 *E. coli* bacteria after 4 hours.

c. The index of the base 2 represents the number of hours passed after first counting the bacteria.

d. Megan would expect 2600 more more *D. coli* compared to *E. coli* after 4 hours.

e. Suggested option 1: Megan could measure the area of the bacteria growth and approximate the number based on the area.

Suggested option 2: Megan could weigh the bacteria and approximate the number of bacteria based on the weight.

Note: There are other possible options.

16. a. 81

b. -27

c. Negative bases with odd powers give negative results. Negative bases with even powers give positive results.

Extra spicy

17. B **18.** A

19. $2^6 + 2^4 + 2^3 + 2^1$ **20.** 9000

Remember this?

21. D **22.** C **23.** D

4B Multiplying indices

Student practice

Worked example 1

a. 3^9 **b.** y^5

Worked example 2

a. $2^3 \times 5^8$ **b.** $u^{10}v^{10}$ **c.** $40t^8r^8$

Understanding worksheet

1. a. Law applies **b.** Law does not apply

c. Law does not apply **d.** Law applies

2. a. $4^2 \times 4^5 = 4^7$ **b.** $7^4 \times 7 = 7^5$

c. $(-3)^4 \times (-3)^2 = (-3)^6$ **d.** $5y^3 \times 2y^9 = 10y^{12}$

3. simplify; First; index; bases; added

Fluency

4. a. 6^6 **b.** 1^{12} **c.** 10^{11} **d.** t^{13}

e. $(-4)^8$ **f.** 5^{21} **g.** y^{15} **h.** $(-z)^{13}$

---

5. a. $2^7 \times 4^9$ **b.** $3^{14} \times 8^8$

c. $5^4 \times 7^6$ **d.** $10^8 \times 4^{17}$

e. $12^{14} \times 8^7$ **f.** $(-11)^{12} \times 6^{20}$

g. $(-7)^{10} \times (-8)^{16}$ **h.** $4 \times 5^{15} \times (-3)^{24}$

6. **a.** x^5y^8 **b.** $t^{11}r^{12}$
c. $u^{11}v^9$ **d.** a^2c^2
e. $2x^8y^5$ **f.** $12x^{12}y^{12}$
g. $630g^{13}h^9$ **h.** $-60d^{11}f^{12}$

---

7. **a.** $28x^8$ **b.** $4a^5b^4$
c. $2^7 \times x^6$ **d.** $-125j^{13}$
e. $3^6 \times 15y^8$ **f.** $-18t^{15} \times 5^2 \times 2^7$
g. $-10k^{21}$ **h.** $24m^7n^{10}$

---

8. D

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

10. A sunflower's height increases by a factor of six every month. If the height of a fully grown sunflower is 216 cm, then how many months have passed since the sunflower was 1 cm tall?

Key points

- A sunflower's height increases by a factor of six every month.
- The height of a fully grown sunflower is 216 cm.
- How many months have passed since the sunflower was 1 cm tall?

Explanation

Let the number of months passed since the sunflower was 1 cm tall be x.

$1 \times 6^x = 216$

$6^x = 216$

$x = 3$ as $6 \times 6 \times 6 = 216$

$$6^3 = \overbrace{6 \times 6 \times 6}^{\text{3 factors of 6}}$$

Answer

3 months have passed since the sunflower was 1 cm tall.

11. At Papa's Gelateria, there are five flavours of gelato available. A customer can also choose to add one of three types of sauces and one of three types of sprinkles. Express the number of combinations of ice cream, sauce, and sprinkles in index notation.

Key points

- At Papa's Gelateria, there are five flavours of gelato available.
- A customer can also choose to add one of three types of sauces and one of three types of sprinkles.
- Express the number of combinations of ice cream, sauce, and sprinkles in index notation.

Explanation

Number of combination of ice cream, sauce and sprinkles
= Number of ice cream flavours × number of types of sauces × number of types of sprinkles

$= 5 \times 3 \times 3$

$= 5 \times 3^2$

$$5 \times 3^2 = 5 \times \overbrace{3 \times 3}^{\text{2 factors of 3}}$$

Answer

There are 5×3^2 combinations of ice cream, sauce and sprinkles.

12. A single dandelion can spread up to 200 seeds. If each one of these seeds grows and produces its own seeds, then what is the expected number of seeds produced by the second generation of one plant? Express your answer in index notation.

Key points

- A single dandelion can spread up to 200 seeds.
- Each one of these seeds grows and produces its own seeds.
- What is the expected number of seeds produced by the second generation of one plant? Express your answer in index notation.

Explanation

First generation of one dandelion produces 200 seeds.

Second generation of one dandelion produces $200 \times 200 = 200^2$ seeds.

$$200^2 = \overbrace{200 \times 200}^{\text{2 factors of 200}}$$

Answer

The expected number of seeds produced by the second generation is given by 200^2.

13. Simon posts a funny picture on his personal social media page. Forty of his friends proceed to share the picture with four of their own friends each. Then all of these people share the post with another four friends each. Express the number of times Simon's post has been shared by friends of friends, in index notation.

Key points

- Forty of his friends share Simon's picture with four of their own friends each.
- All of these people share the post with another four friends each.
- Express the number of times Simon's post has been shared by friends of friends, in index notation.

Explanation

40 friends shared this picture to 4 friends each,
$\therefore 40 \times 4$ times in total.

40×4 friends shared this picture to 4 friends each,
$\therefore 40 \times 4 \times 4 = 40 \times 4^2$ times in total.

$$40 \times 4^2 = 40 \times \overbrace{4 \times 4}^{\text{2 factors of 4}}$$

Answer

Simon's post has been shared by friends of friends 40×4^2 times.

14. All of Mrs Doubtfire's offspring, including children and grandchildren have had two kids of their own each. If Mrs Doubtfire has 12 great-grandchildren altogether, then how many of her own children did she have?

Key points

- All of Mrs Doubtfire's offspring, including children and grandchildren have had two kids of their own each.
- Mrs Doubtfire has 12 great-grandchildren altogether.
- How many of her own children did she have?

Explanation

Suppose Mrs Doubtfire has x children.

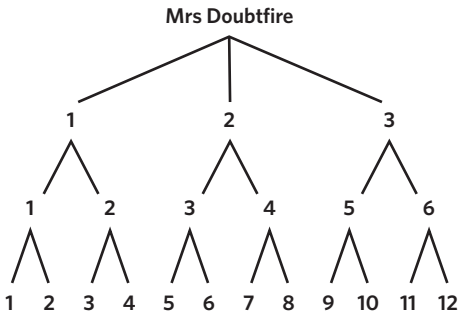
She would have $2x$ grandchildren because each of her children have 2 children of their own.

She would then have $2x \times 2 = 4x$ great-grandchildren.

$4x = 12$

$4x \div 4 = 12 \div 4$

$x = 3$



Answer

Mrs Doubtfire had 3 of her own children.

Reasoning

15. a. There are 1000 bytes in one kB.

b. Three kB can be multiplied together to make one GB.

c. One TB is kB^4.

d. Amber can store 50 movies onto a one TB hard drive.

e. Suggested option 1: Digital data storage is used to store our photos, music, movies and more for personal entertainment.

Suggested option 2: Digital data storage is used to store personal medical information, such as vaccination status etc.

Note: There are other possible options.

16. a. Substituting 2 as x yields $16 \times 4 = 64$.

b. Substituting 2 again for x yields $16 \times -8 = -128$.

c. The index can change the sign of the result. Raising a negative number to an even power (e.g. $(-2)^2$) yields a positive result, but raising it to an odd power (e.g. $(-2)^3$) yields a negative result.

Extra spicy

17. C

18. B

19. $a = 2, b = 4$

20. $-12x^{3y}y^{7x}$

Remember this?

21. E

22. D

23. D

4C Dividing indices

Student practice

Worked example 1

a. 3^6 **b.** z^8

Worked example 2

a. $5t^5$ **b.** $2^8 \times 5^7$ **c.** $4u^3v^8$

Understanding worksheet

1. a. Law does not apply **b.** Law applies

c. Law does not apply **d.** Law applies

2. a. 2 **b.** 9 **c.** 7 **d.** 5

3. Second; common; subtract; dividend; zero

Fluency

4. a. 3^4 **b.** 9^5 **c.** 1 **d.** $(-2)^2$

e. $(-x)^8$ **f.** 7^5 **g.** 8^7 **h.** y^6

5. a. $2x^6$ **b.** $8v^7$ **c.** $\frac{y^4}{3}$ **d.** $\frac{t^3}{4}$

e. $\frac{3r^4}{2}$ **f.** $-2w^3$ **g.** $\frac{4}{3}$ **h.** $\frac{3d^{13}}{7}$

6. a. $2^3 \times 3^5$ **b.** $9^6 \times 7^5$

c. 5×8^2 **d.** $(-4)^6 \times 10^6$

e. $6^2 \times t^3$ **f.** $f \times 15^6$

g. $9x^7$ **h.** 1

7. a. b^3c^7 **b.** x^3y^3 **c.** $2t^2s^2$ **d.** $2u^{11}v^7$

e. $2x^{10}y^2$ **f.** $\frac{n^{12}m^{16}}{13}$ **g.** $\frac{k^{12}p^{15}}{7}$ **h.** $\frac{-v^{14}w^{11}}{5}$

8. E

Spot the mistake

9. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. A competition starts with a group of 64 players. The final round occurs when only two players are left. If in each round, half of the players are eliminated, then how many rounds does it take to get to the final two players?

Key points

- The competition starts with a pool of 64 players.
- The final occurs when only two players are left.
- In each round, half of the players are eliminated.
- How many rounds does it take to get to the final two players?

Explanation

Calculate the number of people left after each round.

Number of players after 1 round $= \frac{64}{2} = 32$

Number of players after 2 rounds $= \frac{64}{2^2} = 16$

Number of players after 3 rounds $= \frac{64}{2^3} = 8$

Number of players after 4 rounds $= \frac{64}{2^4} = 4$

Number of players after 5 rounds $= \frac{64}{2^5} = 2$

So there are 2 players left after 5 rounds.

Answer

It takes 5 rounds to get to the final 2 players.

11. The number of people employed at a company has grown from just four to well over 200 staff. If staff numbers quadrupled every year, then what is the minimum number of years that the company has been around for?

Key points

- Originally, there were four staff.
- Now there are over 200 staff.
- Staff numbers quadrupled every year.
- What is the minimum number of years that the company has been around for?

Explanation

Number of staff after 1 year $= 4^2 = 16 < 200$

Number of staff after 2 years $= 4^3 = 64 < 200$

Number of staff after 3 years $= 4^4 = 256 > 200$

So the company has been around for at least three years.

Answer

The company has been around for at least three years.

12. A small piece of algae with an area of 1 cm^2 doubles in size every day for 28 days until it completely covers the whole lake. When copper is added to the water to remove the algae, it halves in size every day until it disappears completely. What was the area of the algae patch on the 15th day of treatment? Express your answer in index notation.

Key points

- Originally, the algae had an area of 1 cm^2.
- The algae doubles in size every day for 28 days until it completely covers the whole lake.
- The copper then halves the algae in size every day until it disappears completely.
- What was the area of the algae patch on the 15th day of treatment? Express your answer in index notation.

Explanation

Calculate the area of the algae patch after 28 days of growth.

Algae patch after 28 days of growth $= 1 \times 2^{28} = 2^{28}$

Calculate the area of the algae patch on the 15th day of copper treatment.

Algae patch area after 15 days of copper treatment

$$= \frac{\text{algae patch area after 28 days of growth}}{2^{15}}$$

$$= \frac{2^{28}}{2^{15}}$$

$$= 2^{28-15}$$

$$= 2^{13}$$

Answer

The area of the algae patch on the 15th day of treatment is 2^{13} cm^2.

13. When Tori opened up an online tutoring service, she had only two students. This number has been increasing steadily by a factor of five every six months. If at the beginning of January 2022, the number of students subscribing to Tori's service has grown to 250, then when did she first open her business?

Key points

- Originally, Tori had two students.
- This number has increased by a factor of five every six months.
- At the beginning of January 2022, the number of students subscribing to Tori's service was 250.
- When did she first open her business?

Explanation

Calculate the number of students at the end of each 6 month period until the number of students is 250.

Number of students after six months $= 2 \times 5 = 10$

Number of students after twelve months $= 2 \times 5^2 = 50$

Number of students after eighteen months $= 2 \times 5^3 = 250$

So she first opened her business 18 months before the beginning of January 2022.

Counting back 18 months, we find that Tori first opened her business at the beginning of July 2020.

Answer

Tori first opened her business at the beginning of July 2020.

14. When Gavin was born, his parents invested $1000 for him. The investment tripled every four years and reduced by a factor of three every five years. Including the initial investment amount, express how much money Gavin has accumulated after 60 years, in index form.

Key points

- When Gavin was born, his parents invested $1000 for him.
- The investment tripled every four years.
- The investment reduced by a factor of three every five years.
- Including the initial investment amount, express how much money Gavin has accumulated after 60 years, in index form.

Explanation

Calculate the number of times the investment tripled, given the investment tripled every four years over 60 years.

Number of times the investment tripled $= 60 \div 4 = 15$ times

Calculate the number of times the investment is reduced by a factor of three, given the investment reduced by a factor of three every five years over 60 years.

Number of times the investment reduced by a factor of $3 = 60 \div 5 = 12$ times

Now express how much money Gavin has accumulated after 60 years in index form.

Money accumulated

$$= \text{original amount} \times \frac{3^{\text{number of times investment tripled}}}{3^{\text{number of times investment reduced by a factor of 3}}}$$

$$= 1000 \times \frac{3^{15}}{3^{12}}$$

$$= 1000 \times 3^{15-12}$$

$$= 1000 \times 3^3$$

Answer

Gavin has accumulated $\$1000 \times 3^3$ after 60 years.

Reasoning

15. a. The area of the brownie is 256 cm^2.

b. The dimensions of one square piece is 4 cm by 4 cm.

c. The area of the whole brownie is 4^4 cm^2.

d. Eight people helped themselves to the brownie.

e. Suggested option 1: Talk with his family and propose that all treats are split equally amongst his family members.

Suggested option 2: Model this behaviour himself.

Note: There are other possible options.

16. a. $\frac{3^p}{3^q} = 3^{(p-q)} = 9$ for $p = 4$ and $q = 2$

b. $\frac{3^p}{3^q} = 3^{(p-q)} = \frac{1}{9}$ for $p = 2$ and $q = 4$

c. We can evaluate negative integer indices by calculating the positive index, then taking the reciprocal.

So where a positive index means how many times to multiply the base, a negative index means how many times to divide by the base.

Extra spicy

17. E **18.** E **19.** $p = 2$ **20.** $\frac{3}{4}$

Remember this?

21. C **22.** D **23.** E

4D Raising powers

Student practice

Worked example 1

a. 5^{24} **b.** 4^5p^{35}

Worked example 2

a. 2^2x^{28} **b.** 10^4b^2

Understanding worksheet

1. a. $(3^8)^3 = 3^8 \times 3^8 \times 3^8$

b. $(x^{4)5} = x^4 \times x^4 \times x^4 \times x^4 \times x^4$

c. $(5t^5)^2 = 5t^5 \times 5t^5$

d. $(4^3z^6)^4 = 4^3z^6 \times 4^3z^6 \times 4^3z^6 \times 4^3z^6$

2. a. $8^2 \times 8^2 \times 8^2 = (8^2)^3$

b. $y^5 \times y^5 \times y^5 \times y^5 \times y^5 \times y^5 \times y^5 = (y^5)^7$

c. $3d^6 \times 3d^6 \times 3d^6 \times 3d^6 = (3d^6)^4$

d. $9^4f^3 \times 9^4f^3 \times 9^4f^3 \times 9^4f^3 \times 9^4f^3 = (9^4f^3)^5$

3. third; multiplied; coefficients; simplify; indices

Fluency

4. a. 3^8 **b.** 9^{50} **c.** 13^{24} **d.** a^{18}
e. x^{36} **f.** p^{100} **g.** 1 **h.** 1

---

5. a. 2^3b^{12} **b.** 7^4x^{20} **c.** 10^8p^{16} **d.** 9^9u^{72}
e. $6^{10}s^{70}$ **f.** 1 **g.** $5^{14}t^7$ **h.** 3^9r^{33}

---

6. a. 5^{14} **b.** 4^{20} **c.** y^{11} **d.** x^{42}
e. 2^{27} **f.** v^{21} **g.** 4^5a^{27} **h.** 2^4w^{28}

---

7. a. 7^5 **b.** 6^9 **c.** x^9 **d.** y^{12}
e. 2^4p^{25} **f.** $\frac{u}{4^2}$ **g.** $12k^7$ **h.** 8^8

---

8. a. 3^2 **b.** x^8 **c.** 5^7q^{27} **d.** $2^46^2c^{46}$
e. 7^3v^{26} **f.** 2^5q^{35} **g.** $3^{19}n^{20}$ **h.** $8^{31}x^{64}$

---

9. D

Spot the mistake

10. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. Genna shares a link to her online store with 10^2 people. In a few days, the number of people who have the link to Genna's store has increased by a power of three. Express the number of people who received the link in index form.

Key points

- Genna shares the link with 10^2 people.
- The number of people with the link will increase by a power of 3.
- Express the number of people who received the link in index form.

Explanation

Raise 10^2 to a power of 3 and simplify.

$(10^2)^3 = 10^6$

Answer

10^6 people have the link.

12. Alex's current phone contains 2^4 gigabytes (GB) of storage space. The store manager says she can offer her more than double her storage if she upgrades to the new model which has 2^8 GB of storage space. Calculate and state whether the manager is correct or incorrect.

Key points

- Alex's phone contains 2^4 GB of storage space.
- The store manager claims she can offer her more than double this if she upgrades to a phone with 2^8 GB of storage space.
- Calculate and state whether the manager is correct or incorrect.

Explanation

Calculate twice the storage of the current phone.

$2^4 \times 2 = 2^5$

$2^5 < 2^8$, so the new phone has more than double the storage space of Alex's current phone.

Answer

Yes, the store manager is correct, since the new phone has more than double the storage of Alex's current phone.

13. A meal deal at the Double R Diner consists of a hot dog with a piece of cake. A customer can have one of five different types of toppings on their hot dog. There are five types of cake available, which comes with one of five types of ice cream. In index form how many possible combinations of hot dogs with toppings and cakes with ice cream are possible.

Key points

- There are 5 possible hot dog toppings.
- There are 5 possible types of cake.
- There are 5 possible types of ice cream.
- In index form, how many possible combinations of hot dogs with toppings and cakes with ice cream are possible.

Explanation

Write the number of possible meal deal combinations in the basic version in index form.

There are 5 possible hot dog toppings, 5 possible types of cakes and 5 possible types of ice cream, so the number of possible combinations will be $5 \times 5 \times 5$.

$5 \times 5 \times 5 = 5^3$

Answer

There are 5^3 choices for the meal deal.

14. The basic version of a computer game has four different types of helmets, armour and boots that the player can mix and match to dress their character. Downloading extra content will raise the number of available combinations of outfits by a power of two. Express this number in index form.

Key points

- The basic version has four different types of helmets, armour and boots that the player can mix and match.
- Downloading extra content raises the number of combinations of outfits by a power of two.
- Express this number in index form.

Explanation

Write the number of possible outfit combinations in the basic version in index form.

There are 4 helmets, 4 different armours, and 4 pairs of boots, so the number of possible combinations will be $4 \times 4 \times 4$.

$4 \times 4 \times 4 = 4^3$

Downloading extra content raises the number of combinations of outfits by a power of two, so raise 4^3 to a power of 2 and simplify.

$(4^3)^2 = 4^{3\times2} = 4^6$

Answer

There are 4^6 possible outfits.

15. Shelby breeds tropical fish. Over three months, the number of fish Shelby has has increased by a power of three. When Shelby bought another fish tank and more fish, the total number of fish increased by a power of two. If Shelby initially had three fish, then how many fish does she have now?

Key points

- Shelby breeds tropical fish.
- Over three months, the number of fish Shelby has increased by a power of three.
- Shelby bought another fish tank and more fish and now the total number of fish has increased by a power of two.
- Shelby initially had 3 fish.
- How many fish does she have now?

Explanation

Let the number of fish she has now be x.

Over 3 months, the number of fish she has has increased by a power of three, so we represent this as x^3.

After Shelby bought another fish tank and more fish, the total number of fish increased by a power of two, so we raise x^3 to a power of 2.

$(x^3)^2 = x^6$

If Shelby initially had 3 fish, we substitute 3 for x.

$x = 3$

$(3^3)^2 = 3^6$

$3^6 = 729$

Answer

Shelby has 3^6 or 729 fish now.

Reasoning

16. a. Anna cleared nine platforms.

b. Oleg cleared the barriers once.

c. Rebecca's final score was $(3^6)^4$.

d. Anna got $(3^9)^4$, which was the highest score.

e. Suggested option 1: Chat with classmates I am not close with.

Suggested option 2: Ask my friends to introduce me to friends they have that I haven't met yet.

Note: There are other possible options.

17. a. If $x = 3$, $6(x^2)^3 = 4374$

b. If $x = 3$, $(6x^2)^3 = 157\,464$

c. Parts **a** and **b** have different answers, so $6(x^2)^3 \neq (6x^2)^3$ for all values of x. All terms inside the brackets were raised to a power of 3. In part **b**, this included 6. In part **a**, it did not include 6, and 6 was multiplied by $(x^2)^3$ instead.

Extra spicy

18. C **19.** C **20.** $x = 12$ **21.** 3^2 cm^2

Remember this?

22. D **23.** B **24.** C

Chapter 4 extended application

1. a. Base = 10, index = 8

b. The kilometre distance between Earth and Venus in scientific notation is 4.1×10^7 km.

c. 5 048 800 000

d. The distance in kilometres between Earth and Atlas is 2.7×10^7 km.

e. Suggested option 1: Humans might have invented the measurement of 1 foot based on a foot size.

Suggested option 2: Humans might invent a unit of measurement based on an average human's arm lengths.

Note: There are other possible options.

2. a. 4000 fry would be birthed in an 8-week period.

b. This will result in 4^5 babies.

c. 3^4 babies would survive from 3^4 clutches.

d. 2^{15} babies would be birthed over the lifespan of one male seahorse.

e. Suggested option 1: We can stop buying seahorse souvenirs to protect seahorse from being endangered.

Suggested option 2: We can ask the government to regulate wildlife trade more heavily to protect seahorse from being endangered.

Note: There are other possible options.

3. a. The value of a three year old sedan is \$39 366. It has reduced in price by \$14 634.

b. The value of the car when it is 10 years old is \$18 829.

c. Andrew will have saved \$13 000 in 4 years time.

d. Andrew can afford an Electro car sedan that is 14 years old.

e.

Year	Savings	Age of cars (years)	Year car made
2031	\$5500	22 years	2009
2032	\$8000	19 years	2013
2033	\$10 500	16 years	2017
2034	\$13 000	14 years	2020
2035	\$15 500	12 years	2023
2036	\$18 000	11 years	2025

f. Suggested option 1: Andrew should buy a car at 18 because in 2034, Andrew will be able to afford a 14-year-old car which is cost efficient.

Suggested option 2: Andrew should wait a few years before buying a car because in 2036, Andrew will be able to afford an 11-year-old car which will be better than a 14-year-old car.

Note: There are other possible options.

Chapter 4 review

Multiple choice

1. C **2.** B **3.** B **4.** D **5.** E

Fluency

6. **a.** 4^4 **b.** $8^3 \times 7^2$
c. $5^2 \times 2 \times a^2 \times b^2 \times c$ **d.** $-2.3^2 \times x^2 \times y^3 \times z$

7. **a.** 30 **b.** 4 **c.** 128 **d.** 40

8. **a.** $3^6 \times 5^{10}$ **b.** $12^3 \times 8^{10}$
c. $(-9)^{12} \times 6^{17}$ **d.** $15^7 \times (-2)^{13} \times 4^5$

9. **a.** $5^2 \times a^6 \times b^6$ **b.** $3^4 \times (-p)^{13}$
c. $-21a^{18} \times 4^2 \times 2^5$ **d.** $-5x^{12} \times 24y^{13}$

10. **a.** $5x^3$ **b.** $\frac{p^3}{8}$ **c.** $\frac{-w^8}{4}$ **d.** $\frac{5c^{21}}{8}$

11. **a.** a^7b^4 **b.** $\frac{4p^9}{3}$ **c.** $\frac{-k^{17}r^8}{6}$ **d.** $\frac{24v^2w^{13}}{35}$

12. **a.** 8^{10} **b.** $3^5 \times 2^7 \times a^{46}$
c. 4^3v^5 **d.** 5^4

13. **a.** 6^9a^{48} **b.** 2^5
c. $12^{19}v^{134}$ **d.** 4^2n^{28}

Problem solving

14. The force of gravity on the Sun's surface is 3^3 times stronger than it is on Earth. If the force of gravity on Earth is 9.8 Newtons (N), then what is the force of gravity on the Sun's surface, written in index and expanded form?

Key points

- The force of gravity on the Sun's surface is 3^3 times stronger than it is on Earth.
- The force of gravity on Earth is 9.8 N.
- Write the force of gravity on the Sun in index and expanded form.

Explanation

As the force of gravity on the Sun is 3^3 times stronger than Earth's, it can be expressed by the following:

force of gravity on the Sun $=$ force of gravity on Earth $\times 3^3$.

Substituting the given value for Earth's gravity we get (9.8×3^3) N $=$ force of gravity on the Sun.

In expanded form, force of gravity on the Sun $= (9.8 \times 3 \times 3 \times 3)$ N.

Answer

The force of gravity on the Sun is 9.8×3^3 or $9.8 \times 3 \times 3 \times 3$ Newtons.

15. Maria is visiting her local Mexican restaurant Flexica Mexicana. There are four different types of meals available (tacos, quesadilla, burrito, nachos). She can choose to add onto those meals one of four different types of meat, one of three different types of fillings and one of three different types of sauces. What are the numbers of combinations of meals, meat, fillings and sauces that she can have? Express in index notation.

Key points

- There are four different types of meals available (tacos, quesadilla, burrito, nachos).
- Maria can choose to add onto those meals one of four different types of meat, one of three different types of filling and one of three different types of sauces.
- What are the combinations of meals, meat, filling and sauce that she can have? Express in index notation.

Explanations

Number of combinations $=$ types of meals $\times$ types of meat $\times$ types of fillings $\times$ types of sauces

$$= 4 \times 4 \times 3 \times 3$$
$$= 4^2 \times 3^2$$

Answer

Maria can have $4^2 \times 3^2$ types of combinations of meals, meat, fillings and sauces.

16. Tom's parents invested \$10 000 into a savings fund the day Tom was born. Every five years, the investment would quadruple, but then would be reduced by a factor of four every two years. How much money would Tom have on his 60th birthday, including the initial investment amount? Express in index form.

Key points

- Tom's parents invested \$10 000 into a savings fund the day Tom was born.
- Every five years, the investment would quadruple.
- Every two years, the investment would be reduced by a factor of four.
- How much money would Tom have on his 60th birthday, including the initial investment amount? Express in index form.

Explanation

On Tom's 60th birthday, the investment would have quadrupled $60 \div 5 = 12$ times.

It would have reduced by a factor of four for $60 \div 6 = 10$ times.

10 000 quadrupled 12 times and reduced by a factor of four 10 times. Using index notation this can be shown by $10\,000 \times \frac{4^{12}}{4^{10}} = 10\,000 \times 4^2$.

Answer

Tom would have an investment amount of $10\,000 \times 4^2$ dollars on his 60th birthday.

17. Alannah is trying to grow her Tiktok account by posting more videos. She initially had three followers. After one week, her number of followers increased by a power of two. Two days later, Alannah posted a video of herself singing, and the total number of followers increased by a power of six. Express how many followers Alannah has now in index form.

Key points

- Alannah initially had three followers.
- After one week, her number of followers increased by a power of two.
- Two days later, Alannah posted a video of herself singing, and the total number of followers increased by a power of six.
- Express how many followers Alannah has now in index form.

Explanation

After one week, Alannah would have 3^2 followers.

Two days later, Alannah would have $(3^2)^6 = 3^{12}$ followers.

Answer

Alannah now has 3^{12} followers in index form.

Reasoning

18. a. There are 10^2 tiles in one section.

b. In total, Brad has to paint $3 \times 10^2 = 300$ tiles.

c. The entire paint job will take Brad $5 \times 3 \times 10^2 = 1500$ minutes to complete.

d. No, Brad cannot finish the entire paint job within 20 hours. He would need 5 more hours to complete it.

e. Suggested option 1: Brad can choose a water-based paint instead of oil-based so that the paint would not need chemical solvents to clean up.

Suggested option 2: Brad can choose to use painting kits that are not made of plastic, so that they can be reused.

Note: There are other possible options.

19. a. $(2^3)^4 = 4096$

b. $(2^4)^3 = 4096$

c. The answer from part **a** is equal to the answer from part **b**. This is because the Third Index Law states to multiply the indices when raising a term in index form to another power. Both 3×4 and 4×3 multiply to 12 as multiplication is commutative, so the answers for both parts are equal to each other.

5A Introduction to algebra

Student practice

Worked example 1

a. **i.** Expression

ii. The terms are $7a, -2b, 9$

iii. The coefficient of b is -2

iv. The constant is 9

b. **i.** Equation

ii. The terms are $4a, -b, -161, 43, -c$

iii. The coefficient of b is -1

iv. The constants are -161 and 43

Worked example 2

a. $-5e + 12$

b. $-\frac{m}{2} + 4 = 35$

Worked example 3

a. b is subtracted from eighteen and equals nine.

b. A quarter of e is added to two groups of t.

Understanding worksheet

1. **a.** $8y + 7 = 3$

b. $3 - 9a$

c. $6y - 0.5m$

d. $\frac{1}{3}at = 3hs$

2. **a.** $y^2 + 5$

b. $-x^2 - 3$

c. $-2 + x + y$

d. $xy + y - x$

3. 1; constant; variable; term; equation

Fluency

4. **a.** Expression

b. Equation

c. Variable

d. Constant

e. Variable

f. Term

g. Coefficient

h. Term

---

5. **a.** **i.** Expression

ii. The terms are $2b, 3u, -g$

iii. The coefficient of b is 2

iv. No constant or the constant is 0

b. **i.** Equation

ii. The terms are $\frac{1}{2}a, -7b, -2.5, 5$

iii. The coefficient of b is -7

iv. The constants are $-2.5, 5$

c. **i.** Equation

ii. The terms are $c^2, -a^2, 3^2$

iii. The coefficient of b is 0

iv. The constant is 3^2

d. **i.** Expression

ii. The terms are $-\frac{11}{6}b, -at$

iii. The coefficient of b is $-\frac{11}{6}$

iv. No constant or the constant is 0

e. **i.** Equation

ii. The terms are $ca, -0.5b, s$

iii. The coefficient of b is -0.5

iv. No constant or the constant is 0

f. **i.** Expression

ii. The terms are $\frac{b}{4}, -ak, 9e, -8r$

iii. The coefficient of b is $\frac{1}{4}$

iv. No constant or the constant is 0

g. **i.** Equation

ii. The terms are $\frac{d}{5}, 25a^2, 6, -b$

iii. The coefficient of b is -1

iv. The constant is 6

h. **i.** Expression

ii. The terms are $\frac{3b}{4}, \frac{1a}{4}$

iii. The coefficient of b is $\frac{3}{4}$

iv. No constant or the constant is 0

---

6. **a.** Terms: $y, 8, 4e$

b. Terms: $8h, -a, -56w$

c. Terms: $\frac{1}{2}, -2sp, -32a$

d. Terms: $0.5s, -2k, 78, \frac{1}{2}y$

e. Terms: $e^2, 2el, s^2, 1$

f. Terms: $\frac{a}{4}, -4rc, -100$

g. Terms: $8.9w^2, -\frac{1}{12}r, \sqrt{y}$

h. Terms: $\frac{a}{100}, -\frac{a}{b}, -\frac{b}{100}$

---

7. **a.** $a - 9$

b. $13 - (a + b)$

c. $bc = -10$

d. $\frac{-d}{8} = 32$

e. $\frac{1}{3}fgh = 90$

f. $\frac{3d}{2} + 4e$

g. $\frac{e}{4} + f^2$

h. $g^2 = \sqrt{h}$

---

8. **a.** Seven is added to a.

b. b is subtracted from a.

c. Ten is equal to the difference between seven and b.

d. The product of a and b is equal to negative fifteen.

e. Three groups of b are subtracted from two groups of a.

f. Half of a is added to three groups of y.

g. Eighty-one is divided by the difference of c and two.

h. g squared plus three groups of r equals the square root of twenty-five.

---

9. **a.** $10t = 110$

b. $18w = 234$

c. $8200 + 62p = B$

d. $st = d$

e. $2l + 2w = P$

f. $x^2 + x$

g. $3r + \frac{3r}{2} = z$ or $3r + 1.5r = z$

h. $y^3 + yep$

---

10. D

Spot the mistake

11. **a.** Student B is incorrect.

b. Student A is incorrect.

Problem solving

12. Kavitha's sister is s years old and Kavitha is currently half of her sister's age. Write an expression to represent how old Kavitha is.

Key points

- Kavitha's sister is s years old.
- Kavitha is currently half of her sister's age.
- Write an expression to represent how old Kavitha is.

Explanation

Kavitha is half of her sister's age.

So Kavitha's age = Her sister's age $\div$ 2

$= s \div 2$

$= \frac{s}{2}$

Answer

Kavitha is current $\frac{s}{2}$ years old.

13. A person's maximum heart rate can be calculated by subtracting seven tenths of their age from 207. Write an equation that describes the maximum heart rate (R) for somebody that is y years old.

Key points

- Maximum heart rate is subtracting seven tenths of their age from 207.
- Represent the maximum heart rate as R and a person's age as y.
- Write an equation for the maximum heart rate.

Explanation

Maximum heart rate equals 207 minus seven tenths of a person's age.

Seven tenths of a person age means we need to multiply $\frac{7}{10}$ and y, so $\frac{7}{10}y$.

Therefore,

$R = 207 - \frac{7}{10}y$

Answer

The equation for maximum heart rate is $R = 207 - \frac{7}{10}y$.

14. Atsuko and Lena bought art supplies. Atsuko bought seven tubes of paint cost \$$k$ each and \$72.20 worth of paintbrushes. Lena purchased three of the same tubes of paint and three canvases that cost \$$r$ each. Write an expression that represents the difference that Atsuko and Lena spent.

Key points

- Atsuko spent seven of \$$k$ and \$72.20.
- Lena spent three of \$$k$ and three of \$$r$.
- Write an expression that represents the difference that Atsuko and Lena spent.

Explanation

Write an expression for the amount that Atsuko spent.

Seven of \$$k$ and \$72.20 is $7k + 72.20$.

Write an expression for the amount that Lena spent.

Three of \$$k$ and three of \$$r$ is $3k + 3r$.

Subtract one expression from the other.

Answer

The difference that Atsuko and Lena spent is $(7k + 72.20) - (3k + 3r)$ or $(3k + 3r) - (7k + 72.20)$ or $4k - 3r + 72.20$ or $-4k + 3r - 72.20$.

15. Alex wants to improve his magic show by using maths. He tells the audience to select any number (x) and multiply it by three and then by two. Afterward he instructs them to divide the result by the number they started with and then subtract one. Without simplifying, write an expression that describes the scenario of Alex's mathematical magic act in terms of x.

Key points

- A number, represented by x, is multiplied by 3 and then by 2.
- The result is divided by the original number and then subtract one.
- Write an expression that describes the scenario.

Explanation

Multiply x by 3 and then by 2.

$x \times 3 = 3x$

$3x \times 2 = 2(3x)$

Divide the result by the original number. Then subtract 1.

$2(3x) \div x = \frac{2(3x)}{x}$

$\frac{2(3x)}{x} - 1$

Answer

The expression that describes Alex's magic mathematical act is $\frac{2(3x)}{x} - 1$.

16. A large square is made up of two rectangles each with an area of x cm^2, a small square with an area of x^2 cm^2 and a smaller square with an area equal to 1 cm^2. Express the area of the large square.

Key points

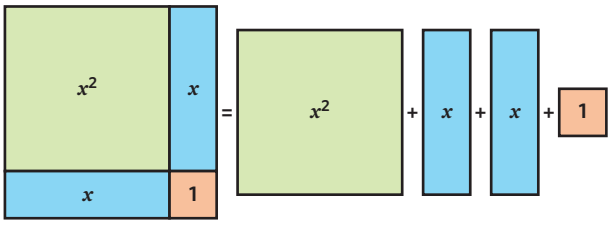
- A large square is made up of two rectangles, a small square and a smaller square.
- Each rectangle has an area of x cm^2.
- The small square has an area of x^2 cm^2.
- The smaller square has an area of 1 cm^2.
- Write an expression for the total area of the large square.

Explanation

Area of large square = sum of the area of the smaller components.

$= x^2 + x + x + 1$

$= x^2 + 2x + 1$



Answer

The area of the large square is $x^2 + 2x + 1$.

Reasoning

17. a. The equation that represents how much Kofi receives per hour is $K = 0.5r$.

b. The expression that Kofi would receive in a week is $2 \times 4 \times 0.5r + 5 \times 0.5r = 6.5r$.

c. The expression that represents the difference in Amir's pay is $0.7r - 0.6r$.

d. The equation that represents how much Hanna would receive is $H = 2 \times 6 \times (1.25 + 0.7)r = 23.4r$.

e. Suggested option 1: It is a livable wage because a person who is aged 16–20 years old is still in school or university, so there isn't much for them to spend on.

Suggested option 2: It isn't a livable wage because a person who is aged 16–20 years old might want to move out to live on their own and this is not enough to pay rent.

Note: There are other possible options.

18. a. $a - 0 \times b = a$

b. $0 \times a - b = -b$

c. A variable with no number written before it has a coefficient of 1. This is because a coefficient is a number that a variable is being multiplied by, so 1 times a variable equals the variable itself.

Extra spicy

19. C **20.** 4

21. C **22.** Perimeter $= 4s$

Remember this?

23. B **24.** E **25.** C

5B Substitution

Student practice

Worked example 1

a. 68 **b.** -11

Worked example 2

Time(t), in hours	2	4	6
Water(w), in litres	220	420	620

Understanding worksheet

1. a. 12 **b.** 8 **c.** 44 **d.** 22

2. a. $4 \times 2 = 8$ **b.** $2 + 3 \times 3 = 11$

c. $2 + 5 - 3 = 4$ **d.** $-5 - 3 \times 5 = -20$

3. variable; substitute; value; operations

Fluency

4. a. 10 **b.** 35 **c.** -13 **d.** -72

e. 12 **f.** 21 **g.** 78 **h.** -1.2

5. a. -11 **b.** -5 **c.** 32 **d.** -22

e. -11 **f.** 7.5 **g.** -8 **h.** -10.5

6. a. \$80 **b.** \$200 **c.** \$230 **d.** \$650

e. \$1820 **f.** 2 **g.** 5 **h.** 8

7. C

Spot the mistake

8. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. Zora wants to print out the photos from her thirteenth birthday. The cost of printing out her photos from Officeworks costs $\$C = 1.5 + 0.25p$, where p is the number of photos and C is the cost. How much should Zora expect to pay for 50 photos?

Key points

- The cost of printing out her photos from Officeworks costs $\$C = 1.5 + 0.25p$, where p is the number of photos and C is the cost.
- How much should Zora expect to pay for 50 photos?

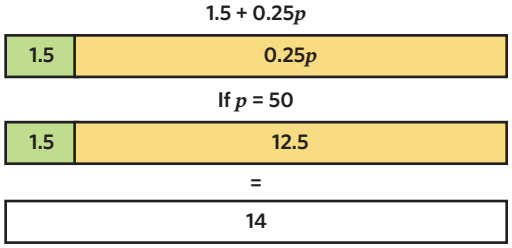
Explanation

Substitute $p = 50$ and simplify.

$\$C = 1.5 + (0.25 \times 50)$

$= 1.5 + 12.5$

$= \$14$



Answer

Zora is expected to pay \$14.

10. Arnie is focusing on her fitness skills and wants to make sure that every time she attends the gym she reaches her maximum heart rate. This is calculated using $R = 207 - 0.7a$, where R is heart rate and a is age. Arnie is 29 years old, what is her maximum heart rate?

Key points

- This is calculated using $R = 207 - 0.7a$, where R is heart rate and a is age.
- Arnie is 29 years old.
- What is her maximum heart rate?

Explanation

Substitute $a = 29$ and simplify.

$R = 207 - (0.7 \times 29)$

$= 207 - 20.3$

$= 186.7$

Answer

Arnie's maximum heart rate is 186.7.

11. Kathy takes an Uber home from work everyday. The total cost is calculated by adding \$2.50 per kilometre travelled plus an additional \$0.75 per minute of idle time in traffic. How much will Kathy's total ride cost if she travelled 7.25 km and spent 12.5 minutes sitting idly in traffic?

Key points

- The total cost is calculated by adding \$2.50 per kilometre travelled plus an additional \$0.75 per minute of idle time in traffic.
- Kathy travelled 7.25 km.
- Kathy spent 12.5 minutes sitting idly in traffic.
- How much will Kathy's total ride cost?

Explanation

Let total cost be \$$C$, k be the kilometres travelled and i be the minutes of idle time spent in traffic.

The total cost is calculated by adding \$2.50 per kilometre travelled plus an additional \$0.75 per minute of idle time in traffic, so $\$C = 2.5k + 0.75i$

Substitute $k = 7.25$ and $i = 12.5$

$\$C = (2.5 \times 7.25) + (0.75 \times 12.5)$

$= 18.125 + 9.375$

$= \$27.50$

Answer

Kathy's ride cost \$27.50.

12. Claire went skiing at Mt. Hotham. She went down the Big D slope at a constant speed of 18 km/h. The rule used is $k = 18x$, where k is total kilometres and x is time, in hours. How many kilometres can Claire expect to have skied after 45 minutes?

Key points

- $k = 18x$, where k is total kilometres and x is time, in hours.
- There are 60 minutes in 1 hour.
- How many kilometres can Claire expect to have skied after 45 minutes?

Explanation

Convert 45 minutes to hours by dividing by 60.

$45 \div 60 = 0.75$ hours.

Substitute $x = 0.75$ into $k = 18x$.

$k = 18 \times 0.75$

$= 13.5$ km

Answer

Claire will ski 13.5 km.

13. Cindy is baking some cupcakes in an oven.

t = time in minutes

c = number of cupcakes

She has two ovens:

Convection oven: $t = 15 + 7.5c$

Fan forced oven: $t = 25 + 5c$

How much quicker is it to cook 10 cupcakes in a fan forced oven?

Key points

- t = time in minutes
- c = number of cupcakes
- Convection oven: $t = 15 + 7.5c$
- Fan forced oven: $t = 25 + 5c$
- How much quicker is it to cook 10 cupcakes in a fan forced oven?

Explanation

Substitute $c = 10$ into the equations for both ovens.

Convection oven:

$t = 15 + (7.5 \times 10)$

$= 15 + 75$

$= 90$ minutes

Fan forced oven:

$t = 25 + (5 \times 10)$

$= 25 + 50$

$= 75$ minutes

Find the difference between the two times.

$90 - 75 = 15$ minutes

Answer

The cupcakes cook 15 minutes faster in the fan forced oven.

Reasoning

14. a. Izzy and Collette will have an equal number of points after 1 goal.

b. Izzy and Isaiah will both have 9 points.

c. The difference in total points is 15 points.

d. Suggested option 1: They could do a set number of practice sessions a week.

Suggested option 2: They could record themselves to see how they can improve their skills.

Note: There are other possible options.

15. a. If $a = 1, b = 2$, then $(a + b)^2 = 9$. Answers will vary.

b. If $a = 1, b = 2$, then $a^2 + b^2 = 5$. Answers will vary.

c. $(a + b)^2 \neq a^2 + b^2$.

$a^2 + b^2$ is the sum of the squares of the two numbers, whereas $(a + b)^2$ is the square of the sum of the two numbers.

Extra spicy

16. $r = \frac{63}{2}$ or 31.5

17. B **18.** $\frac{28}{5}$ **19.** B

Remember this?

20. E **21.** D **22.** D

5C Adding and subtracting terms

Student practice

Worked example 1

a. $7f$ and $3f$ are like terms.

b. $-2qw$, $5qw$ and $8wq$ are like terms.

Worked example 2

a. $10a - b$ **b.** $17pq + 2p - 2q$

Worked example 3

a. -7 **b.** -16

Understanding worksheet

1. a. $2a + 4b + a$ **b.** $10p - 2p + 3n - 7p$

c. $4km + 6k + 2m + 8mk$ **d.** $x^2 + 6x - 9x^2 + 2$

2. a. $4p + 10$ **b.** $6p - 3$ **c.** $-p$ **d.** $3p + 4$

3. simplify; commutative; constants; powers

Fluency

4. a. $3x$ and $2x$ are like terms

b. 1 and 4 are like terms

c. $7m$ and $-3m$ are like terms

d. $-6p$ and p are like terms

4 and 2 are like terms

e. $8x$ and $3x$ are like terms

f. $10xy$, $-7xy$ and $-8yx$ are like terms

g. $10x^2$ and $4x^2$ are like terms

h. $6ab$ and $9ab$ are like terms

5. **a.** $15x$ **b.** $2p + 5$
c. $5u + 2j$ **d.** $q + 10$
e. $4a + 4.2$ **f.** $\frac{3}{5}b + \frac{3}{5}$
g. $7x + 5y$ **h.** $-\frac{7}{3}m + \frac{1}{6}n$

6. **a.** $5jk$ **b.** $4pr + 2p$
c. $2a^2 + a$ **d.** $2k^2 + 8$
e. $4h^3 - 4h^2 - h$ **f.** $m + n + 7mn$
g. $17ab - 3c - 4$ **h.** $mn + 2l^2m + 5m^2l$

7. **a.** 45 **b.** 36 **c.** 56 **d.** 18
e. 54 **f.** 90 **g.** 26 **h.** −40

8. B

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. Vasili and Ivy went shopping at Officeworks. Vasili bought 4 pencils that cost $\$n$ each and 5 exercise books that cost $\$m$ each. Ivy bought 2 of the same pencils and 1 of the same exercise book. Write an expression to represent the total amount of money they both spent together.

Key points

- Vasili bought 4 pencils that cost $\$n$ each.
- Vasili bought 5 exercise books that cost $\$m$ each.
- Ivy bought 2 of the same pencils.
- Ivy bought one of the same exercise books.
- Write an expression to represent the total amount of money they both spent together.

Explanation

The amount of money spent in total = money spent on pencils + money spent on exercise books

Vasili bought 4 pencils and Ivy bought 2.

$2 + 4 = 6$ pencils, so 6 pencils were bought in total. Each pencil cost $\$n$, therefore they spent $\$6n$ on pencils.

Vasili bought 5 exercise books and Ivy bought 1.

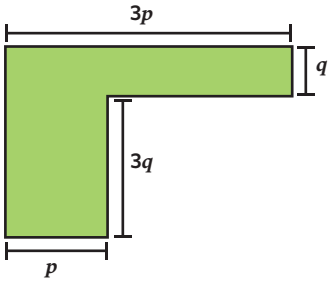
$5 + 1 = 6$ books, so 6 books were bought in total. Each book cost $\$m$, therefore they spent $\$6m$ on books.

So the amount of money spent in total $= \$(6m + 6n)$.

Answer

The total amount of money Vasili and Ivy spent together is $\$(6m + 6n)$

11. What expression can be written for the perimeter of the shape below?



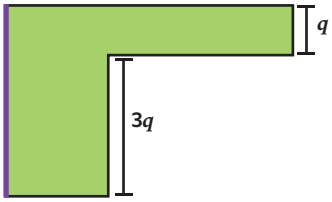
Key points

- What expression can be written for the perimeter of the shape?

Explanation

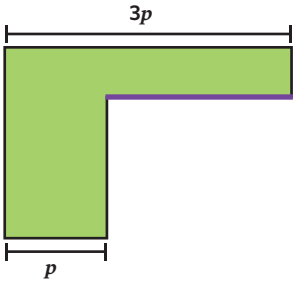
To find the perimeter, add all the side lengths together.

There are two unknown sides. Find the length of the unknown sides.



From the diagram, we can see the unknown side is the sum of q and $3q$.

$3q + q = 4q$



From the diagram, we can see that the unknown side is the difference between $3p$ and p.

$3p - p = 2p$

Now add together all of the side lengths.

$3p + p + 2p + 3q + q + 4q = 6p + 8q$

Answer

The perimeter of the shape is $6p + 8q$.

12. The Miranda family has a shared data pool for their mobile internet plans. Fahrad used p gigabytes of data. Jina used 5 gigabytes less than Fahrad. Garath used 2 times as many gigabytes as Fahrad. What is the simplified expression for the total amount of data the family used?

Key points

- Fahrad used p gigabytes of data.
- Jina used 5 gigabytes less than Fahrad.
- Garath used 2 times as many gigabytes as Fahrad.
- What is the simplified expression for the total amount of data the family used?

Explanation

Jina used 5 gigabytes less than Fahrad. This is 5 less than p, which can be written as $p - 5$.

Garath used 2 times as many gigabytes as Fahrad. This is double p, which can be written as $2p$.

Add together the three amounts of data used to find the total amount of data used.

$p + p - 5 + 2p = 4p - 5$

Answer

The simplified expression for the total amount of data the family used is $4p - 5$ gigabytes.

13. Naheed took an Uber home from a concert. The Uber costs $1.45 per kilometre ($d$), plus a $0.55 booking fee. If the Uber distance is 4.5 km, how much will it cost Naheed to get home?

Key points

- The Uber costs \$1.45 per kilometre (d), plus a \$0.55 booking fee.
- The Uber distance is 4.5 km.
- How much will it cost Naheed to get home?

Explanation

Calculate the cost of the Uber by writing an expression for the cost per ride, and substituting in the specific information about Naheed's ride.

The cost = (number of kilometres cost per kilometre) + booking fee

The booking fee is \$0.55. Naheed travelled 4.5 km.

Therefore, the total cost $= (4.5 \times \$1.45) + \$0.55$

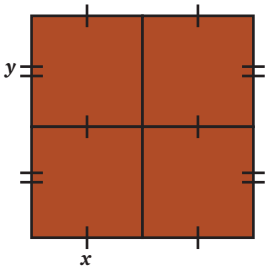
$= \$7.075$

$\approx \$7.08$

Answer

It costs Naheed \$7.08 to get home.

14. Rebecca has drawn a design for a wooden roof of an outdoor area in her backyard. It consists of four brown rectangles.



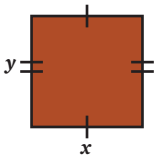
Write a simplified expression that represents the total area of the roof.

Key points

- The roof consists of 4 brown rectangles.
- Write an expression that represents the total area of the roof.

Explanation

Area of one rectangle $= x \times y = xy$ square units.



There are 4 identical rectangles.

Total area of the roof is $xy + xy + xy + xy = 4xy$ square units.

Answer

The total area of the roof is $4xy$ square units.

Reasoning

15. a. The total number of points Kai scored in the game is $3a + 5b + 4c$.

b. The total number of points Kai scored in the game is $7a + 5b$ or $7c + 5b$.

c. Kai scored 53 points.

d. Suggested option 1: The game designer should consider how difficult or how likely it is to score the goal.

Suggested option 2: The game designer should consider the distance from the point of shot to the goal.

Note: There are other possible options.

16. a. $a + b + 2a + 3b - 2; -5b + 10a - 2 + 9b - 7a$

Note: There are other possible options.

b. $3 + 6a - b + 5b - 3a - 5; 2a + 9b - 5a - 2 - 5b + 6a$

Note: There are other possible options.

c. -1. All the equivalent expressions have the same value for any given a and b as they all simplify to the same expression.

Extra spicy

17. $10x + 20$ **18.** D **19.** $j^2 - 4k^2$ **20.** E

Remember this?

21. B **22.** C **23.** D

5D Multiplying and dividing terms

Student practice

Worked example 1

a. $40ab$ **b.** $12def$

Worked example 2

a. $\frac{4t}{3}$ **b.** $\frac{-3b}{d}$

Worked example 3

a. $2a^2bc$ **b.** $\frac{7b^2}{c}$

Understanding worksheet

1. a. $6h$ **b.** $12xy$
c. $8p^2$ **d.** $2ab + 2a$

2. a. $3 \times h \times k$ **b.** $2 \times 3 \times b \times c$
c. $2 \times b \times b \times a$ **d.** $2 \times 3 \times x \times x \times x \times y$

3. variables; coefficient; factors; exponent

Fluency

4. a. $10g$ **b.** $21r$ **c.** $8jk$ **d.** $6xy$
e. $15ax$ **f.** $36xyz$ **g.** $9sty$ **h.** $8def$

5. a. $\frac{3x}{5}$ **b.** $\frac{2}{5p}$ **c.** $\frac{2}{3h}$ **d.** $\frac{2x}{yz}$
e. $\frac{r}{4}$ **f.** $\frac{2w}{3z}$ **g.** $\frac{17g}{9bx}$ **h.** $\frac{1}{2t}$

6. a. $-15ef$ **b.** $-4j$ **c.** $6apx$ **d.** $-\frac{6qp}{k}$
e. $\frac{4}{3k}$ **f.** $-16.1rt$ **g.** $-12xz$ **h.** $\frac{20b}{q}$

7. a. $9x^2$ **b.** $9b^2z$ **c.** $2q$ **d.** $20xy^3z$
e. $\frac{d^2}{c}$ **f.** $8k^5p^3$ **g.** $\frac{5u}{3}$ **h.** $\frac{13r}{9s}$

8. a. $18c^2x^2$ **b.** $\frac{a^4b}{3}$
c. $9a^6b^2$ **d.** $\frac{x^3y^2}{2}$
e. $-\frac{3a}{4b}$ **f.** $\frac{20a}{9b^2cd}$
g. $77q^3w + 105hqw^2$ **h.** $\frac{9x + 15f}{2y}$

5D ANSWERS

9. D

Spot the mistake

10. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. Hera wants to calculate the area of her cargo bay. She uses the arm spans of Ezra and Kanan. Write an equation for the area if the width is three times Kanan's arm span (k) and the length is four times Ezra's arm span (e).

Key points

- The width is three times Kanan's arm span (k).
- The length is four times Ezra's arm span (e).
- The area of a rectangle is width $\times$ length.
- Write an equation for the area.

Explanation

The area of a rectangle is width $\times$ length.

The width is $3 \times k = 3k$ and the length is $4 \times e = 4e$.

So area $= 3k \times 4e$

$= 12ek$

Answer

The area is $12ek$.

12. Maxine's Hardware shop wanted to keep track of the number of nails that arrive in a particular order. Write an expression to determine how many nails are in an order that arrives with (c) amount of crates. The crates contain (b) amount of boxes. The boxes contain (n) amount of nails.

Key points

- An order arrives with (c) amount of crates.
- Each crate contains (b) amount of boxes.
- Each box contains (n) amount of nails.
- Write an expression for the amount of nails in an order.

Explanation

The number of nails in an order is the the number of nails in a box (n) multiplied by the number of boxes (b) in a crate, and multiplied by the number of crates (c)

So number of nails $= n \times b \times c$

$= nbc$

Answer

The number of nails in this order is nbc.

13. Freeda was organising fruit snacks for her childcare centre. Each day a number of parent volunteers (v) would cut the same amount of pieces (p) each and Freeda would share them equally among the children (c). Write an expression that would allow Freeda to determine how many pieces each child would receive once she knew how many parents and how many children attended that day.

Key points

- Each day a number of parent volunteers (v) would cut the same amount of pieces (p) each.
- Freeda would share them equally among the children (c).
- Write an expression that would allow Freeda to determine how many pieces each child would receive.

Explanation

Since the pieces are shared equally amongst the children,

Number of pieces each child receives $=$ number of pieces produced $\div$ number of children

v parent volunteers cut p pieces each,

Number of pieces produced $= v \times p = vp$

There are c children.

So each child receives $\frac{vp}{c}$ pieces.

Answer

Each child receives $\frac{vp}{c}$ pieces.

14. How many tiles would Dimitry need for the great dining hall in his palace if it is x metres wide and y metres long and the tiles are a millimetres wide and b millimetres long?

Key points

- The great dining hall is x metres wide and y metres long.
- Area of a rectangle $=$ length $\times$ width
- The tiles are a millimetres wide and b millimetres long.
- How many tiles would Dimitry need for the great dining hall?

Explanation

Convert the measurements in metres to millimetres.

x m $= x \times 1000$ mm $= 1000x$ mm

y m $= y \times 1000$ mm $= 1000y$ mm

Calculate the area of the dining hall and the area of a single tile.

Area of dining hall $=$ length $\times$ width

$= 1000x \times 1000y$

$= 1\,000\,000xy$ mm^2

Area of tile $=$ length $\times$ width

$= a \times b$

$= ab$ mm^2

Number of tiles $=$ area of dining hall $\div$ area of tile

$= 1\,000\,000xy \div ab$

$= \frac{1\,000\,000xy}{ab}$

Answer

Dimitry needs $\frac{1\,000\,000xy}{ab}$ tiles for the great dining hall.

15. Dylan has enough lawn seed for 16 m^2 and his back fence is y metres long. Write an algebraic expression to describe the distance (d) away from the fence he can plant the seed for a rectangular lawn.

Key points

- Dylan has enough lawn seed for 16 m^2.
- His back fence is y metres long.
- The lawn is rectangular.
- length $\times$ width $=$ area of rectangle
- Write an algebraic expression to describe the distance (d) away from the fence he can plant the seed.

Explanation

The distance away from the fence (d) is the width of the rectangle. The length of the rectangle is the length of the fence (y).

Rearrange the formula for the area of a rectangle to make the width the subject.

Length × width ÷ length = area of a rectangle ÷ length

So width = area of rectangle ÷ length

Substitute in the known values and variables for width, length and area of the rectangle.

$d = \frac{16}{y}$

Answer

The distance away from the fence Dylan can plant the seed is described by $d = \frac{16}{y}$.

Reasoning

16. a. The volume of one container is b^2h cm^3.

b. The volume of membrane required is $0.4wh$ cm^3.

c. $\frac{0.4w}{b^2}$ containers are required.

d. Suggested option 1: Pay more upfront as this saves money in the long run.

Suggested option 2: Maintain regular waterproofing as the big building project was probably expensive, so it is probably important to save money now.

Note: There are other possible options.

17. a. $\frac{3x}{20yz}$ **b.** $\frac{15xz}{4y}$

c. Brackets are important because they tell us which operations to perform first. The order in which we perform operations determines the result we get. For example, when we divided $3x$ by $4y$ first, then divided this quotient by $5z$, we got a different result to when we divided $4y$ by $5z$ first and then divided $3x$ by this quotient.

Extra spicy

18. E **19.** 55 m^2 **20.** 336 girls **21.** B

Remember this?

22. D **23.** D **24.** C

5E Algebraic fractions

Student practice

Worked example 1

a. $\frac{5s}{7}$ **b.** $\frac{5d}{12}$

Worked example 2

a. $\frac{12}{gh}$ **b.** $\frac{4q}{3}$

Worked example 3

a. $\frac{2c}{d}$ **b.** $\frac{k}{j}$

Understanding worksheet

1. a. $6a$ **b.** $6t$ **c.** $6a$ **d.** 39

2. a. 8 **b.** $10p$ **c.** 2 **d.** $2n$

3. algebraic; denominator; reciprocal; product

Fluency

4. a. $\frac{4l}{5}$ **b.** $\frac{3a}{7}$ **c.** $\frac{t}{3}$ **d.** $\frac{3y}{16}$

e. $\frac{13b}{8}$ **f.** $\frac{g}{2}$ **g.** e **h.** $\frac{t}{20}$

5. a. $\frac{cd}{35}$ **b.** $\frac{rs}{20}$ **c.** $\frac{m}{2n}$ **d.** $\frac{15}{c}$

e. $\frac{8}{d}$ **f.** $\frac{a^2}{3}$ **g.** $\frac{2n}{m}$ **h.** $\frac{x^3}{4}$

6. a. $\frac{ax}{6}$ **b.** $\frac{5}{b}$ **c.** $\frac{n}{5m}$ **d.** $\frac{21}{t^2}$

e. $\frac{12d}{c}$ **f.** $\frac{4}{9z}$ **g.** $\frac{14a}{15c}$ **h.** $\frac{5}{8b^2}$

7. C

Spot the mistake

8. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. The Muscat and Axiak family are having pizza for dinner. The bill for the pizza comes to a total of $\$p$ and each familypays half. Write an algebraic fraction for the amount the Muscat family needs to pay.

Key points

- The bill for the pizza is $\$p$.
- The bill is to be split in half.
- Write an algebraic fraction for the amount the Muscat family needs to pay.

Explanation

Since the bill is to be halved, each family pays half of the bill.

The bill is $\$p$, therefore each family pays $\$\frac{p}{2}$.

Answer

The Muscat family needs to pay $\$\frac{p}{2}$.

10. Lewis has gone to the shops. He spent half of his money on Christmas gifts and a third of his money on a pair of shoes. Let m be the total amount of Lewis' money. Write an expression to show how much money Lewis has left.

Key points

- Lewis spent half of his money on Christmas gifts and a third on a pair of shoes.
- Let m represent Lewis' money.
- Write an expression for how much money Lewis has left.

Explanation

Lewis spent $\frac{m}{2}$ on Christmas gifts and $\frac{m}{3}$ on a pair of shoes.

The amount of money he has left is:

$$m - \frac{m}{2} - \frac{m}{3} = \frac{6m}{6} - \frac{3m}{6} - \frac{2m}{6}$$

$$= \frac{6m - 3m - 2m}{6}$$

$$= \frac{m}{6}$$

$$\frac{6m}{6} - \frac{3m}{6} - \frac{2m}{6} = \frac{6m - 3m - 3m}{6} = \frac{m}{6}$$

Answer

Lewis has $\frac{m}{6}$ left.

5E ANSWERS

11. Blossom Collective buys flowers from the local flower market. Let d represent the total money Marielle spent at the market. She spent one eighth on tulips, one quarter on dahlias and one half on ranunculus. Write an expression to show how much money Marielle has spent on flowers.

Key points

- Let d represent the total money Marielle spent.
- She spent one eighth, one quarter and one half of her money on three different flowers.
- Write an expression for the total amount of money Marielle spent.

Explanation

Marielle spent $\frac{d}{8}$ on tulips, $\frac{d}{4}$ on dahlias and $\frac{d}{2}$ on ranunculus.

The total amount of money she spent is:

$$\begin{aligned}\frac{d}{8}+\frac{d}{4}+\frac{d}{2} &= \frac{d}{8}+\frac{2d}{8}+\frac{4d}{8}\\ &= \frac{d+2d+4d}{8}\\ &= \frac{7d}{8}\end{aligned}$$

Answer

Marielle has spent $\frac{7d}{8}$.

12. Jaxson has a number, represented as h, written on a piece of paper. On a different piece of paper he has a number written that is three times h. The sum of their reciprocals is unknown. Construct an algebraic expression to represent this scenario and simplify it.

Key points

- Jaxon represents a number as h.
- He represents another number as $3w$.
- He wants to find the sum of their reciprocal.
- Write an algebraic expression to represent the sum and simplify it.

Explanation

The reciprocal of h is $\frac{1}{h}$; the reciprocal of $3h$ is $\frac{1}{3h}$.

Their sum is $\frac{1}{h}+\frac{1}{3h}$.

Simplify:

$$\begin{aligned}\frac{1}{h}+\frac{1}{3h} &= \frac{3}{3h}+\frac{1}{3h}\\ &= \frac{3+1}{3h}\\ &= \frac{4}{3h}\end{aligned}$$

Answer

$\frac{1}{h}+\frac{1}{3h}=\frac{4}{3h}$

13. In a particular odd taxi, Odokawa the driver charges $\$\frac{7d}{2}$ per kilometre. There is also a base fee of $\$\frac{c}{3}$ regardless of the distance travelled. If Shirakawa takes this taxi and travels $\frac{n}{6}$ kilometres. How much does Shirakawa have to pay Odokawa, in terms of c, d and n?

Key points

- Odokawa charges $\$\frac{7d}{2}$ per kilometre.
- There is a base fee of $\$\frac{c}{3}$.
- Shirakawa travels $\frac{n}{6}$ kilometres.
- Write an expression for the total cost of the ride in terms of c, d and n.

Explanation

Find the cost from distance travelled.

Cost from distance travelled = cost per kilometres × kilometres travelled

$$\begin{aligned}&= \frac{7d}{2}\times\frac{n}{6}\\ &= \frac{7dn}{12}\end{aligned}$$

Find the total cost.

Total cost = Cost from distance travelled + base fee

$$\begin{aligned}&= \frac{7dn}{12}+\frac{c}{3}\\ &= \frac{7dn}{12}+\frac{4c}{12}\\ &= \frac{7dn+4c}{12}\end{aligned}$$

Answer

Shirakawa has to pay Odokawa $\$\frac{7dn+4c}{12}$.

Reasoning

14. a. The area of the senior netball court is given by lw.

b. The length of the junior netball court is given by $\frac{l}{2}$;

The width of the junior netball court is given by $\frac{2w}{3}$.

c. The area of the junior netball court is $\frac{l}{2}\times\frac{2w}{3}=\frac{lw}{3}$.

d. The difference between the senior netball court and the junior netball court is $lw-\frac{lw}{3}=\frac{2lw}{3}$.

e. Suggested option 1: Have a consistent exercise routine to maintain a good endurance.

Suggested option 2: Increase the intensity and difficulty of their drills.

Note: There are other possible options.

15. a. $\frac{x}{y}$ **b.** $\frac{y}{x}$

c. 1 multiplied by a fraction will give the same fraction. However, 1 divided by a fraction will give the reciprocal of the fraction as dividing a fraction is the same as multiplying by the reciprocal of the divisor.

Extra spicy

16. $\frac{7e}{10}$

17. C

18. $\frac{(x-1)(x+2)}{18z^2}$

19. D

Remember this?

20. E **21.** C **22.** B

5F Expanding brackets

Student practice

Worked example 1

a. $12m - 18$ **b.** $8b + 16b^2$

Worked example 2

a. $7v - 10$ **b.** $7j + 14$

Understanding worksheet

1. **a.** $2(2a + 1) = 4a + 2$ **b.** $-3(k - 2) = -3k + 6$
c. $-2(-2p - 3) = 4p + 6$ **d.** $2(-3n + 2) = -6n + 4$

2. **a.**

	+h	+5
4	4h	20

$4(h + 5) = 4h + 20$

b.

	−4	3l
3	−12	9l

$3(-4 + 3l) = 9l - 12$

c.

	+b	+7
f	fb	7f

$f(b + 7) = fb + 7f$

d.

	+3m	+2j	−1
5	15m	10j	−5

$5(3m + 2j - 1) = 15m + 10j - 5$

3. expanding; distributive law; coefficient; simplifying

Fluency

4. **a.** $6f + 6$ **b.** $3c - 12$
c. $-16 + 8x$ **d.** $-10p + 80$
e. $5s + 20$ **f.** $k^2 + 3k$
g. $3z + 3g - 15$ **h.** $20 + 2h + 6w$

---

5. **a.** $x^2 + 5x$ **b.** $8u^2 + 20u$
c. $4k + 2$ **d.** $-12g + 2g^2$
e. $0.18hv + 0.36fv$ **f.** $27hr - 18hs$
g. $-\frac{3}{2}dp - 3p$ **h.** $15w^2c - 30w^2 + 20fw$

---

6. **a.** $10v + 14$ **b.** $9w - 17$
c. $7f + 20$ **d.** $-2b + 14$
e. $24 + 6y^2 - 18y$ **f.** $12h + 5n^2 - 6n$
g. $3r - 2rg - 32g$ **h.** $-2q^2 + 19qm + 2$

---

7. **a.** $2(a + 4)$ **b.** $6(p - 1)$
c. $3(9 - r)$ **d.** $3(m - 8)$
e. $8\left(\frac{f}{2} + 3\right)$ **f.** $\frac{1}{4}(10 - 4g)$
g. $w(-4 + y)$ **h.** $\frac{1}{2}\left(\frac{5}{2} - 2k\right)$

---

8. D

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. Stacey spends $15 flat rate for parking at uni from 9 am to 3 pm. The fee for parking is an additional $5 an hour for every hour h after 3 pm. Write an expression with brackets to represent how much Stacey pays for parking for 5 days.

Key points

- Stacey spends $15 flat rate for parking at uni from 9 am to 3 pm.
- The fee for parking is an additional $5 an hour for every hour h after 3 pm.
- Write an expression with brackets to represent how much Stacey pays for parking for 5 days.

Explanation

Write an expression to represent how much Stacey pays for parking every day.

Stacey pays $5 per hour for every hour h after 3 pm. This can be represented as $5h$.

Add the $15 flat rate for parking between 9 am and 3 pm.

So the amount paid for parking is $15 + 5h$ every day.

Multiply this by 5 days using brackets

$5 \times (15 + 5h) = 5(15 + 5h)$

Answer

Stacey pays $5(15 + 5h)$ over 5 days.

11. Cindy is building a table that has a width of 0.5 m. She doesn't know how long it will be, but it will have a length of at least 1.5 m. What expanded expression represents the area of this table?

Key points

- The table's width is 0.5 m.
- The table's length is at least least 1.5 m.
- Area of rectangle = length × width
- What expanded expression represents the area of this table?

Explanation

The length of the table is $1.5 + x$, where x is equal to either zero or a positive number.

Multiply the length by the width to find the area of the rectangle.

Area of rectangle $= (1.5 + x) \times 0.5$

Expand this expression.

$(1.5 + x) \times 0.5 = 0.75 + 0.5x$

Answer

The area of the table is $0.75 + 0.5x$.

12. Bihai is creating a footpath with rectangular tiles. Each tile has a length of $a + 3$ m and a width of $a - 2$ m. He puts three together, as shown below.

Write an expanded and fully simplified expression for the total perimeter of this shape.

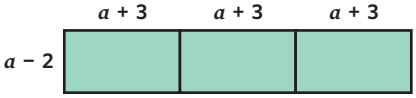
Key points

- Each tile has a length of $a + 3$ m and a width of $a - 2$ m.
- Bihai puts three tiles together.
- Write an expanded and fully simplified expression for the total perimeter of this shape.

5F ANSWERS

Explanation

Consider the rectangle the three tiles form.



The width of this rectangle is $a - 2$. The length is $3(a + 3)$.

The perimeter of a rectangle $= 2 \times$ width $+ 2 \times$ length

So the perimeter of the rectangle formed by three tiles $= 2(a - 2) + 2(3(a + 3))$.

Expand this expression.

$$\begin{aligned} 2(a - 2) + 2(3(a + 3)) &= 2a - 4 + 2(3a + 9) \\ &= 2a - 4 + 6a + 18 \\ &= 8a + 14 \end{aligned}$$

Answer

The total perimeter of the shape is $2(a - 2) + 2(3(a + 3)) = 8a + 14$.

13. Tickets for a theme park cost $\$x$ per adult ticket and $\$y$ per child ticket. A family package consists of two adult tickets and two children's tickets and is \$5 less than buying the tickets individually. A group of adults and kids buy 3 family packages, and two extra adult tickets. Find an expression that represents the total cost of all the tickets.

Key points

- The cost for an adult ticket is $\$x$.
- The cost for a child ticket is $\$y$.
- A family package consists of two adult tickets and two children's tickets.
- The family package is \$5 less than buying the tickets individually.
- A group of adults and kids buy 3 family packages, and two extra adult tickets.
- Find an expression that represents the total cost of all the tickets.

Explanation

Find an expression for the cost of one family package.

A family package consists of two adult tickets and two children's tickets. Ordinarily this would cost $\$(2x + 2y)$. However, the package is \$5 less than buying the tickets individually, so subtract \$5.

So the cost of a family package is $\$(2x + 2y - 5)$.

This group purchases three of these. Multiply this expression by 3 to find the amount spent on family packages. Then add the cost of two adult tickets to find the total cost.

$3 \times (2x + 2y - 5) + 2x = 3(2x + 2y - 5) + 2x$

Answer

The total cost of all tickets is represented by $3(2x + 2y - 5) + 2x$.

14. John, Jeremy, Melissa and Fatima are running a relay marathon. At different intervals, a new runner takes over. John is the first runner. Jeremy is the second runner, and thinks he can run 5 km more than John. Melissa is next, and thinks she can run twice as far as Jeremy. Fatima is the last runner, who can run 3 times as far as John. Construct a fully simplified expression to represent the total distance run by all runners, where x is the distance run by John.

Key points

- John is the first runner.
- Jeremy is the second runner, and thinks he can run 5 km more than John.
- Melissa is next, and thinks she can run twice as far as Jeremy.
- Fatima is the last runner, who can run 3 times as far as John.
- Construct a fully simplified expression to represent the total distance run by all runners, where x is the distance run by John.

Explanation

Construct an expression for how far each runner will run, then add them together.

The distance John runs is x.

Jeremy can run 5 km more than John, which is $x + 5$.

Melissa can run 2 times as far as Jeremy, which is $2 \times (x + 5) = 2(x + 5)$.

Fatima can run 3 times as far as John, which is $3 \times x = 3x$.

The sum of all their distances is $x + x + 5 + 2(x + 5) + 3x$

$= 2x + 5 + 2x + 10 + 3x$

$= 7x + 15$

Answer

The total distance run by all runners is $7x + 15$.

Reasoning

15. a. Let the selling price be represented by s. The profit is represented by $s - 3$.

b. Let the selling price be represented by s. The profit is represented by $5(s - 3) - 8 = 5s - 23$

c. The price should be \$15.

d. Suggested option 1: How will he market his pottery?

Suggested option 2: Will he use an online marketplace like eBay or Etsy, or set up his own website?

Note: There are other possible options.

16. a. $20 + 10n$ **b.** $6 - 3p$

c. They are equal. If there are two sets of brackets with identical expressions, the whole expressions can be treated as like terms, where the coefficients of the brackets are added or subtracted.

Extra spicy

17. E **18.** D

19. $3(5n - 2) - 3(3n + 4) = 6n - 18$

$= 6(n - 3)$

n is an integer, so $n - 3$ is also an integer. Any integer multiplied by 6 is a multiple of 6, so $3(5n - 2) - 3(3n + 4)$ is always a multiple of 6.

20. $k(50 - 2k)^2$

Remember this?

21. B **22.** D **23.** D

5G Factorising expressions

Student practice

Worked example 1

a. 2 **b.** $9w$

Worked example 2

a. $8(e - 2)$ **b.** $-9(9n - 2)$

Worked example 3

a. $-e(9n + d)$ **b.** $a + 2$

Understanding worksheet

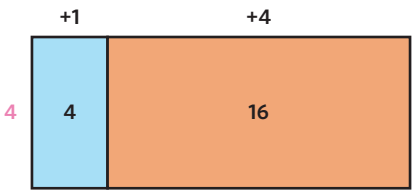
1. a. 4 groups of $2 = 4 \times 2$

b. 4 groups of $t - 2 = 4(t - 2)$

c. 3 groups of $h - 2 = 3(h - 2)$

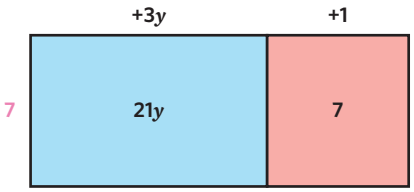
d. 4 groups of $2e + 1 = 4(2e + 1)$

2. a.



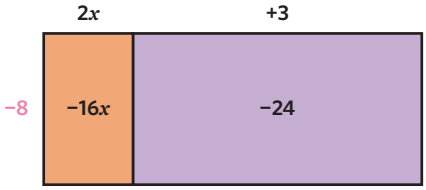
$4 + 16 = 4(1 + 4)$

b.



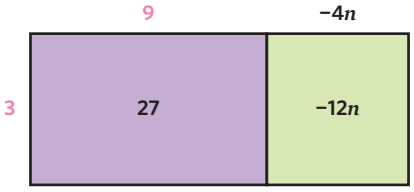
$21y + 7 = 7(3y + 1)$

c.



$-16x - 24 = -8(2x + 3)$

d.



$27 - 12n = 3(9 - 4n)$

3. -4; multiplied; expanded; 8

Fluency

4. a. 4 **b.** 9 **c.** 5 **d.** 7

e. $36b$ **f.** $2er$ **g.** $2a$ **h.** 4

5. a. $2(u + 2)$ **b.** $7(7n - 2)$

c. $60(1 - 2p)$ **d.** $9(u - 1)$

e. $8(4 - g)$ **f.** $4(18L + 35)$

g. $9(7 + 6g)$ **h.** $9(9e - 11d + 5)$

6. a. $-6(t + 1)$ **b.** $-4(1 + e)$

c. $-(n + 1)$ **d.** $-25(c - 3)$

e. $-2(14 - 23h)$ **f.** $-2(22 + 23i)$

g. $-5(23 - 36q)$ **h.** $-2(18u - 17e + 11)$

7. a. $m(5 + 6u)$ **b.** $4d(1 + 3a)$

c. $9s(7i - 3)$ **d.** $-3t(6o - 1)$

e. $-2e(5k + p)$ **f.** $-e(1 + ls)$

g. $-2s(3a - 4i - t)$ **h.** $4a(d + 4ta)$

8. a. $2(5a + 7)$ **b.** $9(n - 2)$

c. $-3(s - 1)$ **d.** $-14(w + 1)$

e. $5r(2e + 1)$ **f.** $2e(6 + 2r + i)$

g. $tu(t + 14)$ **h.** $2e(3e + 5p + 1)$

9. a. $x + 1$ **b.** $2x - 4$

c. $\frac{2}{x + 4}$ **d.** 3

e. $-\frac{1}{2}$ **f.** -3

g. $\frac{3(12 + b)}{2(b + 9)}$ **h.** $\frac{10(5r + 1)}{r - 2}$

10. E

Spot the mistake

11. a. Student B is incorrect. **b.** Student B is incorrect.

Problem solving

12. A Year 8 class is planning where to sit for a play. Currently there are 8 rows with s students in each row. An additional 16 students want to watch the play and sit with their classmates. If the Year 8 class wants to sit together and divide themselves equally, write an expression that represents the total number of rows and students that are now in each row.

Key points

- There are 8 rows of seats with s students in each row.
- 16 additional students want to join.
- All students are to be divided equally across the rows.
- Write an expression that shows the number of rows and the number of students in each row.

Explanation

Currently there are 8 rows with s students in each row.

So the current total number of student is:

$8 \times s = 8s$

An additional 16 students join in, so the total number of student now is:

$8s + 16$

Factorise.

$8s + 16 = 8(s + 2)$

There are 8 rows with $s + 2$ students in each row.

Answer

The expression that shows the number of rows and the number of students in each row is $8(s + 2)$.

13. The number of family membership plans sold by the Sydney Living Museum can be represented by the expression $x + 4y$, where $\$x$ is the cost per adult ticket and $\$y$ is the cost per child ticket. How many family membership plans did the museum sell if they sold 4 adult tickets and 16 children tickets?

Key points

- Sydney Living Museum sells family membership plans for $x + 4y$.
- $\$x$ is the cost for each adult ticket and $\$y$ is the cost for each child ticket.
- The museum sold 4 adult tickets and 16 children tickets.
- How many family membership plans did the museum sell?

Explanation

Express the number of tickets sold in terms of each family membership plan by factorising.

$4x + 16y = 4(x + 4y)$

Four of the family membership plans were sold.

Answer

The museum sold 4 family membership plans.

14. Scientists are studying ancient life in Australia. They dig in two different sites and their total findings can be represented by the expression $6p + 4a$, where p is the number of fossilised plants, and a is the number of fossilised animals. Write an expression to represent the number of fossilised plants and animals if two of the scientists split the findings equally between them.

Key points

- The total findings from the two dig sites is represented by $6p + 4a$.
- p is the number of fossilised plants and a is the number of fossilised animals.
- Write an expression to show the total findings split in two equal groups.

Explanation

Factorise the total findings, $6p + 4a$ by dividing the expression by 2.

$6p + 4a = 2(3p + 2a)$

The total number of fossilised plants and animals found by the scientists can be represented by $2(3p + 2a)$ when split in two equal parts.

Answer

Total findings split in two equal parts can be represented by $2(3p + 2a)$.

15. Riley forgot their measuring tape and used the length of their shoe (s) in cm and the width of their phone (p) in cm to calculate the length and width of a rectangular room. The area equaled $144s^2 + 12sp$. Factorise the expression so that area represents the multiplication of the length and width of the room in terms of s and p.

Key points

- The area of a rectangular room is measured using a shoe with length s and a phone with width p.
- The area of the room is $144s^2 + 12sp$.
- Find the area of the room as a multiplication of the length and width by factorising the expression above.

Explanation

Determine the HCF of $144s^2 + 12sp$.

HCF $= 12s$

Factorise the HCF from all the terms.

$144s^2 + 12sp = 12s(12s + p)$

As $12s + p > 12s$

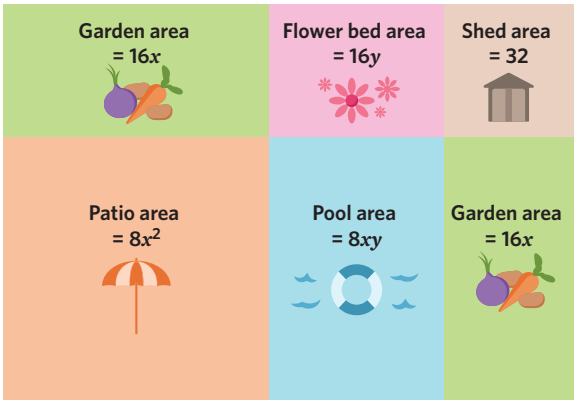
Length of the room $= 12s + p$

Width of the room $= 12s$

Answer

The area of the room is $12s(12s + p)$.

16. Emmanuelle is re-designing her rectangular backyard and only knows the area of the shed which is 32 m^2. She mocks up a design that includes 2 rectangular garden beds with the same areas, a rectangular flower bed, a rectangular pool, and a square shaped patio.



What is the length and width of their backyard in terms of x and y?

Key points

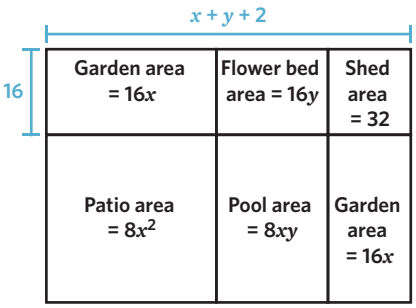
- The area of the shed is 32 m^2.
- The areas of the garden beds, the flower bed, the pool and the patio are $16x$ m^2, $16y$ m^2, $8xy$ m^2 and $8x^2$ m^2 respectively.
- Find the dimension of the backyard in terms of x and y.

Explanation

Consider the top portion of the backyard.

The area of the top portion is $16x + 16y + 32$. This can be factorised into $16(x + y + 2)$.

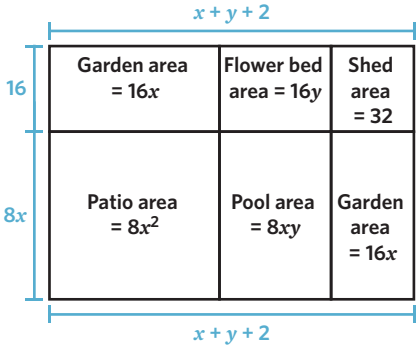
This means the top portion has a length of $(x + y + 2)$ m and a width of 16 m.



Now consider the bottom portion of the backyard.

The area of the bottom portion is $8x^2 + 8xy + 16x$. This can be factorised into $8x(x + y + 2)$.

This means the bottom portion has a length of $(x + y + 2)$ m and a width of $8x$ m.



The total width of the backyard = the width of the top portion + the width of the bottom portion

$= 8x + 16$

Answer

The length of the backyard is $(x + y + 2)$ m and the width is $(8x + 16)$ m.

Reasoning

17. a. The total cost of their combined bills is $8r + 10g$.

b. They will each pay $4r + 5g$.

c. Each friend will pay $4r + 6p + 2g$ or $2(2r + 3p + g)$.

d. The bill for each group will be $8r + 12p + 4g$.

e. Suggested option 1: It is not fair to split bills equally because some people may eat more than others.

Suggested option 2: It is fair to split bills equally because it is difficult to keep track of how much each person eats.

Note: There are other possible options.

18. a. -11 **b.** -3

c. Expressions from part **a** and part **b** are equivalent because they are rearrangements of one another. Part **a** factors out -11 so that the first term, 3, is positive. Part **b** factors out 11 and rearranges to make x in front so that the first term is positive. Upon expanding, both expressions equal $-33 + 11x$.

Extra spicy

19.

$5e - 2$	2	$10e - 4$
3	$4e + 3$	$12e + 9$
$15e - 6$	$8e + 6$	

20. C **21.** E

22. Length $= 2x - 3y$

Width $= 2y - x$

Remember this?

23. E **24.** C **25.** A

Chapter 5 extended application

1. a. $C = 4.71d$

b. The total cost that they will each pay on the first day is \$39.75.

c. The total cost that they will each pay over the next 6 days is \$172.25.

d. Rebecca's plan saves them more money. The cost difference between Rebecca and Dion's plan is \$45.02.

e. Suggested option 1: Rebecca and Dion could consider catching the train/rail.

Suggested option 2: Rebecca and Dion could also consider walking.

Note: There are other possible options.

2. a. The perimeter of the garden is $96s$ units. The area of the garden is $576s^2$ units.

b. The width of each of these rows will be $2s$ units.

c. The total perimeter of the 7 rectangular beds is $276s$ units.

d. The number of crops she can grow with these requirements is 36.

e. Bernadine should plant two swiss chards plants in each of the two rectangular plots.

f. No, Bernadine will not be able to reach her arm across all the garden beds to tend to her produce.

Suggested option 1: Bernadine could hire another farmer to stay on the other side of the garden so that they can both tend to all the produce.

Suggested option 2: Bernadine could redesign her garden to include paths which she can walk on.

Note: There are other possible options.

3. a. The symbol '?' represents the width of the box, and the symbol '??' represents the length of the box. 'h' represents the height of the box.

b. '5 cm' represents the thickness of the box.

c. The length of the garden plot is $240 = d + e + 70$ cm, and the width of the garden plot is $240 = a + b + c + 130$ cm. Note: The equations can be simplified further.

d. Yes, these dimensions will meet the criteria of the garden design, because the walking path could be wider than 50 cm.

e. Box 1 length = 240 cm; width = 51.25 cm

Box 2 length = 137.9 cm; width = 37.5 cm

Box 3 length = 51.10 cm; width = 37.5 cm

Box 4 length = 240 cm; width = 51.25 cm

f. Suggested option 1: Using the entire length of railway sleepers reduces costs.

Suggested option 2: Using the entire length of railway sleepers makes construction easier.

Note: There are other possible options.

Chapter 5 review

Multiple choice

1. A **2.** B **3.** C **4.** E **5.** B

Fluency

6. a. $x + 12$ **b.** $v + 4u$

c. $\frac{5a}{2} = bcd$ **d.** $m^2 = \frac{\sqrt{n}}{3}$

7. a. 16 **b.** 6 **c.** -13 **d.** -0.5

8. a. $9 + 9y$ **b.** $m^3 + 12 - 8m + 3m^2$

c. $4cd + 4c^2d$ **d.** $-\frac{1}{4}a + \frac{5}{4}b$

9. a. $15ab$ **b.** $\frac{5}{y}$ **c.** $9m^2n$ **d.** $\frac{27p}{10q}$

10. a. $12a$ **b.** $\frac{p}{2}$ **c.** $-4a$ **d.** $4fg$

11. a. $\frac{3x}{7}$ **b.** $\frac{m}{3}$ **c.** $\frac{55a}{24}$ **d.** $\frac{2q}{15}$

12. a. $\frac{ab}{12}$ **b.** $\frac{6}{d}$ **c.** $\frac{5y}{x}$ **d.** $\frac{27}{32u^2}$

13. **a.** $5x + 10$ **b.** $-90 + 10y$
c. $d^2 - 3d$ **d.** $15 + 3f + 6g$

14. **a.** $49w^2 + 14w$ **b.** $-20x + 4x^2$
c. $12xy - 15xz$ **d.** $10f^2g - 16f^2 + 2fg$

15. **a.** $10(1 + 2x)$ **b.** $-(f + 4)$
c. $7(2x + 7y - 3z)$ **d.** $2m(1 - no - o)$

16. **a.** $a + 2$ **b.** $\frac{-1}{x + 20}$
c. $\frac{-1}{5}$ **d.** $\frac{1}{2}$

Problem solving

17. Elsa bought x cupcakes for \$20 from the supermarket to decorate and sell at her school fete. She aims to make a profit of \$$y$ per cupcake. Find an expression for the selling price of each cupcake.

Key points

- Elsa bought x cupcakes for \$20.
- She aims to make a profit of \$$y$ per cupcake.
- Find an expression for the selling price of each cupcake.

Explanation

$$\text{Cost per cupcake} = \frac{\text{cost for all cupcakes}}{\text{number of cupcakes}}$$
$$= \frac{20}{x}$$

To calculate the selling price per cupcake, add the cost per cupcake to the profit per cupcake:

$$\text{Selling price per cupcake} = \frac{20}{x} + y$$

Answer

The selling price per cupcake is $\frac{20}{x} + y$.

18. At an animal shelter, there are birds, cats and dogs. Bill collects food for all the animals each week. The kilograms of food every week is $0.01b + 1.4c + 2d$, where b is the number of birds, c is the number of cats and d is the number of dogs. If there are 10 birds, 20 cats and 30 dogs, what is the weight of food that Bill is collecting?

Key points

- The kilograms of food every week is $0.01b + 1.4c + 2d$.
- b is the number of birds, c is the number of cats and d is the number of dogs.
- There are 10 birds, 20 cats and 30 dogs.
- What is the weight of food that Bill is collecting?

Explanation

Multiply the amount of food per species by the number of that species:

Birds: $0.01 \times 10 = 0.1$

Cats: $1.4 \times 20 = 28$

Dogs: $2 \times 30 = 60$

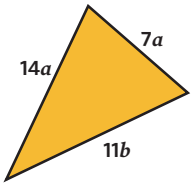
Add all amounts of food per species together to calculate the total kilograms of food collected.

$0.1 + 28 + 60 = 88.1$ kg

Answer

88.1 kilograms of food is collected.

19. What expression can be written for the perimeter of the shape below?



Key points

- What expression can be written for the perimeter of the shape below?

Explanation

To calculate the perimeter, all sides need to be added together.

$P = 14a + 7a + 11b$

$14a$ and $7a$ are like terms, so add them together to simplify the expression.

$= 14a + 7a + 11b$

$= 21a + 11b$

Answer

The expression for the perimeter of the shape is $21a + 11b$.

20. A recipe CJ wants to make requires x grams of sugar and twice as much flour. The sugar costs \$$y$ per gram. The flour is half this price per gram. How much will CJ pay for sugar and flour respectively?

Key points

- A recipe CJ wants to make requires x grams of sugar and twice as much flour.
- The sugar costs \$$y$ per gram.
- The flour is half this price per gram.
- How much will CJ pay for sugar and flour respectively?

Explanation

CJ wants to purchase x grams of sugar and $2 \times x = 2x$ grams of flour.

$$\text{Cost of sugar} = \text{cost per gram of sugar} \times \text{number of grams of sugar}$$
$$= y \times x$$
$$= xy$$
$$\text{Cost of flour} = \text{cost per gram of flour} \times \text{number of grams of flour}$$
$$= \frac{y}{2} \times 2x$$
$$= xy$$
$$\text{Total cost} = \text{cost of sugar} + \text{cost of flour}$$
$$= xy + xy$$
$$= 2xy$$

Answer

The total cost of sugar and flour is \$$2xy$.

21. Amaja queues up with an empty plate at an all-you-can-eat buffet restaurant. Let p represent the area of the plate. She fills one third of her plate with a steak and one quarter with a salad. She then fills half of the remaining space with garlic bread. Write an expression to show how much of Amaja's plate is left to fill up with dessert.

Key points

- Let p represent the area of the plate.
- She fills one third of her plate with a steak.
- She then fills one quarter with a salad.
- She then fills half of the remaining space with garlic bread.
- Write an expression to show how much of Amaja's plate is left to fill up with dessert.

Explanation

Calculate the area Amaja used, $\frac{p}{3}$ on steak and $\frac{p}{4}$ on salad.

$$p - \frac{p}{3} - \frac{p}{4} = \frac{12p - 4p - 3p}{12}$$
$$= \frac{5p}{12}$$

Half of the remaining area will be filled with garlic bread. So half of the area will remain to be filled with dessert.

$$\frac{5p}{12} \times \frac{1}{2} = \frac{5p}{24}$$

Answer

The amount of room left for dessert is $\frac{5p}{24}$.

22. Dr W's Year 8 mathematics class sits an algebra test. The average score per student can be represented by the expression $5x + 20$. Find the sum of all scores in Dr W's class of 25 students, in terms of x.

Key points

- Dr W's Year 8 mathematics class sits an algebra test.
- The average score per student can be represented by the expression $5x + 20$.
- Find the sum of all scores in Dr W's class of 25 students, in terms of x.

Explanation

$$\text{Average score of Dr W's class} = \frac{\text{Sum of all scores in Dr W's class}}{\text{No. of students in Dr W's class}}$$

Calculate the sum of scores in Dr W's class:

$$\text{sum of scores in Dr W's class} = \text{average score} \times \text{number of students}$$
$$= (5x + 20) \times 25$$
$$= 125x + 500$$

Answer

The sum of the scores in Dr W's class is $125x + 500$.

23. The area of Vicki's rectangular canvas is $13x + 26xy + 39xz$ m^2. The canvas does not have a length or width of 1 m. Vicki's assistant attempts to express this area as the product of its dimensions and writes down three potential solutions. Use factorisation to determine these three potential solutions.

Key points

- The area of Vicki's rectangular canvas is $13x + 26xy + 39xz$ m^2.
- The canvas does not have a length or width of 1 m.
- There are three ways to express this area as the product of its dimensions.
- Use factorisation to determine these three potential solutions.

Explanation

To find three potential solutions, find three different common factors of the terms to factorise out.

Find three common factors of $13x$, $26xy$ and $39xz$.

$13x = 13 \times x$

$26xy = 13 \times 2 \times x \times y$

$39xz = 13 \times 3 \times x \times z$

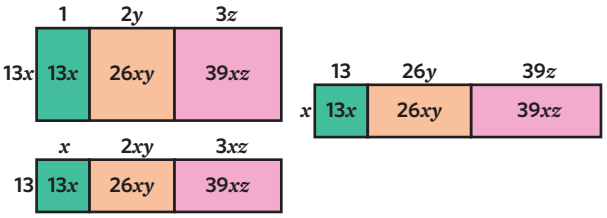
Common factors are 13, x and $13x$.

Factorise out each common factor to find the possible solutions.

$13x + 26xy + 39xz = 13x(1 + 2y + 3z)$

$13x + 26xy + 39xz = 13(x + 2xy + 3xz)$

$13x + 26xy + 39xz = x(13 + 26y + 39z)$



Answer

The three solutions Vicki's assistant wrote down are:
$13x(1 + 2y + 3z)$, $13(x + 2xy + 3xz)$ and $x(13 + 26y + 39z)$.

Reasoning

24. a. $x - 4$; 8 cm

b. $4(x - 4) = 4x - 16$

c. Design 1: $4xz - 16z$
Design 2: $4xz^2 - 16z^2$

d. $\$\frac{31c}{2}$

e. Suggested option 1: We could make book stands at home from recycled materials.

Suggested option 2: We could make picture frames at home from recycled materials.

Note: There are other possible options.

25. a. $\frac{3}{10}$

b. $\frac{4}{45}$

c. Both parts **a** and **b** need to have a common denominator. Multiply the numerator and denominator of each fraction such that the denominators are equal, then perform the subtraction. This is the same method we can use for all fractions with different denominators.

6A Introduction to equations

Student practice

Worked example 1

a. True **b.** True

Worked example 2

a. $p = 5$ **b.** $p = 5$

Worked example 3

a. $p + 9 = 15$ **b.** $4 + 2d = 18$

Understanding worksheet

1. **a.** b **b.** c **c.** b **d.** a

2. **a.** False **b.** True **c.** True **d.** False

3. equations; false; balanced; substituted

Fluency

4. **a.** False **b.** True **c.** True **d.** False
e. False **f.** False **g.** True **h.** False

---

5. **a.** $a = 5$ **b.** $a = 12$ **c.** $a = 8$ **d.** $a = 8$
e. $a = 12$ **f.** $a = 5$ **g.** $a = 12$ **h.** $a = 5$

---

6. **a.** $x = 3$ **b.** $x = 3$ **c.** $x = 3$ **d.** $x = 5$
e. $x = 8$ **f.** $x = 8$ **g.** $x = 5$ **h.** $x = 8$

---

7. **a.** $3m = 12$ **b.** $v + 2 = 9$
c. $q - 4 = 16$ **d.** $7 = 3k + 1$
e. $2(a + 4) = 8$ **f.** $9 = 3(j - 6)$
g. $2v + 3 = v$ **h.** $\frac{9 + p}{7} = 6$

---

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Mary's height is h cm and her sister Katie is 23 cm taller. Katie's height is 161 cm. Write an equation to show Katie's height.

Key points

- Mary's height is h cm.
- Her sister Katie is 23 cm taller.
- Katie's height is 161 cm.
- Write an equation to show Katie's height.

Explanation

If Katie is 23 cm taller than Mary, and Mary's height is h cm, we can represent Katie's height as $h + 23$ cm.

However, we also know that Katie's height is 161 cm. Therefore, we can represent Katie's height with the following equation:

$h + 23 = 161$

Answer

$h + 23 = 161$

11. Robert buys k kg of dried apricots at \$2.40 per kg. He spends a total of \$4.80. Write an equation to describe this situation.

Key points

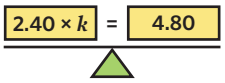
- Robert buys k kg of dried apricots at \$2.40 per kg.
- He spends a total of \$4.80.
- Write an equation to describe this situation.

Explanation

As Robert buys k kg of apricots at \$2.40 per kg, we can represent the amount he buys as $2.40k$.

However, we also know that Robert spends \$4.80. Therefore, we can equate \$4.80 with $2.40k$ and write the following equation:

$2.40k = 4.80$



Answer

$2.40k = 4.80$

12. Rebecca is working on a group project and buys everyone in her group a hot chocolate. Construct an equation for an order of six large hot chocolates and five medium hot chocolates which costs a total of \$60, if a large hot chocolate costs \$$l$ per cup and a medium hot chocolate costs \$$m$.

Key points

- Rebecca is working on a group project and buys everyone in her group a hot chocolate.
- Six large hot chocolates and five medium hot chocolates cost a total of \$60.
- A large hot chocolate costs \$$l$ per cup and a medium hot chocolate costs \$$m$.
- Construct an equation for this situation.

Explanation

Six large hot chocolates can be represented as $6l$, whereas five medium hot chocolates can be represented as $5m$.

As all of the hot chocolates combined cost \$60, this can be represented with the following equation:

$6l + 5m = 60$

Answer

$6l + 5m = 60$

13. Joanne wants to hire a bus to go to Phillip Island. There is a fixed rate of \$10 and then it costs \$1.20 per km. At the end of the trip she is charged \$370. Write an equation that displays Joanne's bus fees.

Key points

- Joanne wants to hire a bus to go to Phillip Island.
- There is a fixed rate of \$10 and then it costs \$1.20 per km.
- At the end of the trip she is charged \$370.
- Write an equation that displays Joanne's bus fees.

Explanation

Let k be the distance travelled in km and F be the fee.

If there is a fixed rate of \$10 and then a variable rate of \$1.20 per km, this can be represented as $1.2k + 10$.

As the total fee came to \$370, we can equate \$370 with the above expression to form this equation:

$1.2k + 10 = 370$

Answer

$1.2k + 10 = 370$

14. Construct an equation for the area of a trapezium (A), where the sum of the parallel sides(a and b) is halved and then multiplied by the height (h).

Key points

- The sum of the parallel sides(a and b) of a trapezium is halved.
- After halving, the result is then multiplied by the height (h).
- Construct an equation for the area of this trapezium (A).

Explanation

If the sum of a and b is halved, this can be represented as $\frac{a+b}{2}$.

As we then multiply this by the height, the expression becomes $\frac{a+b}{2} \times h$.

Therefore, the area of the trapezium is:

$A = \frac{a+b}{2} \times h$

Answer

$A = \frac{a+b}{2} \times h$

Reasoning

15. a. $6c + 25 = \$m$

b. $\$115 > \$100$ ∴ Danielle will have enough money for a ticket.

c. $6l - 10 = \$m$

d. $\$92 < \$100$ ∴ Rose will not have enough money.

e. Suggested option 1: Danielle and Rose could use a keep cup to reduce their usage of disposable coffee cups.

Suggested option 2: Danielle and Rose could ensure that they recycle their cup after they finish their coffee.

Note: There are other possible options.

16. a. True **b.** False

c. When $p = 16$, the equation in part **a** is true but the equation in part **b** is false. Although both equations are equal to 6, this difference is ultimately due to the 2 being on the numerator of the fraction for part **b**, whereas for part **a** it is a standalone constant. This changes what p should be equal to.

Extra spicy

17. $n = 6$ **18.** B **19.** A **20.** $y = 2$

Remember this?

21. C **22.** B **23.** B

6B Solving one-step and multi-step equations

Student practice

Worked example 1

a. $a = 12$ **b.** $b = 5$

Worked example 2

a. $c = 3$ **b.** $f = 5$

Worked example 3

a. $a = 5$ **b.** $y = 95$

Understanding worksheet

1. a. $a = 4$ **b.** $p = 2$ **c.** $x = 2$ **d.** $m = 3$

2. a. $\div 3, \div 3$ **b.** $+1, +1$ **c.** $\times 9, \times 9$ **d.** $\div 5, \div 5$

3. inverse; multiplying; subtracting; undo

Fluency

4. a. $z = 15$ **b.** $m = 26$ **c.** $x = 2$ **d.** $x = 16$
e. $x = 9$ **f.** $h = -16$ **g.** $c = -2$ **h.** $b = -3$

5. a. $x = 10$ **b.** $x = 3$ **c.** $e = 1$ **d.** $x = 24$
e. $x = -6$ **f.** $x = 1$ **g.** $x = -5$ **h.** $x = \pm 6$

6. a. $c = 5$ **b.** $c = 17$ **c.** $a = 4$ **d.** $s = -6$
e. $v = 3$ **f.** $b = 0$ **g.** $k = 6$ **h.** $m = 9$

7. a. $k = 7$ **b.** $x = 1$ **c.** $x = 1$ **d.** $h = 3$
e. $f = 0$ **f.** $b = \frac{1}{3}$ **g.** $v = -0.6$ **h.** $e = 0.1$

8. a. $l = 9$ **b.** $x = 20$ **c.** $j = -4$ **d.** $a = 3$
e. $h = 8$ **f.** $h = -42$ **g.** $a = 1$ **h.** $x = -\frac{13}{2}$

9. B

Spot the mistake

10. a. Student B is incorrect. **b.** Student B is incorrect.

Problem solving

11. Sally wants to buy doughnuts for her colleagues. She has $15. The price ($p$) of doughnuts is given by the equation $p = 1.2b + 3$. Where b is the number of doughnuts being bought. How many doughnuts can Sally expect to purchase?

Key points

- Sally has $15.
- The price of doughnuts is given by $p = 1.2b + 3$, where b is the number of doughnuts bought.
- How many doughnuts can Sally expect to purchase?

Explanation

To find how many doughnuts Sally can expect to purchase, solve for b where $p = 15$ as she has $15.

Substituting $p = 15$, gives the equation:

$15 = 1.2b + 3$

Solve for b:

$15 = 1.2b + 3$

-3 -3

$12 = 1.2b$

$\div 1.2$ $\div 1.2$

$10 = b$

Therefore, Sally can expect to purchase 10 doughnuts with the amount of money she has.

Answer

Sally can expect to purchase 10 doughnuts.

12. Joanne had l number of lollies. She divided these lollies amongst eight people. Her nanna gave her an additional 10 lollies. She was left with 14 lollies. How many lollies (l) did Joanne have to begin with?

Key points

- Joanne divided the lollies amongst eight people.
- Her nanna gave her an additional 10 lollies.
- Joanne had 14 lollies left.
- How many lollies did Joanne have to begin with?

Explanation

Construct an equation to solve for the number of lollies l by representing the information as an algebraic expression.

Joanne divided the lollies amongst eight people, is represented as $\frac{l}{8}$.

Joanne's nanna gave her an additional 10 lollies is represented as $\frac{l}{8} + 10$.

Joanne was left with 14 lollies at the end is represented as $\frac{l}{8} + 10 = 14$.

Solve for l in the above equation.

$$\frac{l}{8} + 10 = 14$$
$$\frac{l}{8} = 4 \quad (-10, \times 4)$$
$$l = 32$$

Therefore, Joanne had 32 lollies to begin with.

Answer

Joanne had 32 lollies to begin with.

13. 32 is two times the difference between Jonah's age (j)and 36. Assume Jonah is older than 36. Write an equation and solve for Jonah's age, j.

Key points

- 32 is two times the difference between Jonah's age j and 36.
- Jonah is older than 36.
- Write an equation and solve for Jonah's age.

Explanation

Construct an equation to solve for Jonah's age j using the given information.

32 equals two times the difference between j and 36, this can be represented as:

$32 = 2(j - 36)$

Solve for j in the above equation:

$$32 = 2(j - 36)$$
$$16 = j - 36 \quad (\div 2, +36)$$
$$52 = j$$

Therefore, Jonah is 52.

Answer

Jonah is 52.

14. Amber is participating in a star jump challenge and does a total of 16 star jumps at the start of each day. She wants to increase the number of star jumps she does each day by four until she reaches 40 star jumps each day. Using d to represent the number of days since Amber started her star jump challenge, what day can Amber expect to reach her goal?

Key points

- Amber does 16 star jumps at the start of each day.
- She wants to increase the number of star jumps each day by 4.
- Amber's goal is to reach 40 star jumps each day.
- What day can Amber expect to reach her goal?

Explanation

Construct an equation to solve for d to find what day Amber can expect to reach her goal of 40 star jumps each day.

As Amber does 16 star jumps each day and wants to increase the number of star jumps each day by 4, represent this as:

$16 + 4d$

where d is the number of days since she started the challenge.

As Amber's goal is 40 star jumps per day, this means that our equation is $16 + 4d = 40$.

Solve for d in the above equation:

$$4d + 16 = 40$$
$$4d = 24 \quad (-16, \div 4)$$
$$d = 6$$

Amber can expect to reach 40 star jumps per day 6 days after she started the challenge.

Answer

Amber can expect to reach her goal on the 6th day.

15. Moana and Billy both ran 12 kilometres together. Moana then ran twice as far as Billy. Altogether she ran 35 km. Let Billy's total distance be represented as b. Create and solve an equation for the number of kilometres Billy ran on his own.

Key points

- Moana and Billy ran 12 km together.
- Moana ran twice as far as Billy.
- Altogether she ran 35 km in total.
- Create and solve an equation for the number of kilometres Billy ran on his own.

Explanation

Construct an equation using the first three pieces of information, n. As Billy's total distance is represented as b, Moana running twice as far as Billy can be represented as $2b$.

$12 + 2b = 35$

To solve this:

$$12 + 2b = 35$$
$$2b = 23 \quad (-12, \div 2)$$
$$b = 11.5$$

Therefore, Billy ran 11.5 km on his own.

Answer

Billy ran 11.5 km on his own.

Reasoning

16. a. It will take 2.5 days for the sunflower to have a height of 15 cm.

b. It will take 12.5 days for the sunflower to have double the height given in part **a**.

c. $d = \frac{2}{3}h - \frac{15}{2}$

d. The sunflower is 11.25 cm on day 0.

e. Suggested option 1: Ensure that the sunflower has access to lots of direct sunlight.

Suggested option 2: Water the sunflower regularly.

Note: There are other possible options.

17. **a.** $a = 2$

b. $a = 8$

c. Although the values of a in parts **a** and **b** are similar in that they are both positive integers, the values themselves are different. The negative sign in front of the left hand side of the equation in part **a** makes the value of a in part **a** less than in part **b**.

Extra spicy

18. $h = 5$ **19.** D **20.** C

21. 25.33 kilometres

Remember this?

22. A **23.** B **24.** E

6C Solving equations with unknowns on both sides

Student practice

Worked example 1

a. $x = 3$ **b.** $k = -4$ **c.** $k = 13$

Understanding worksheet

1. **a.**

y
3

$y = 3$

b.

g
3

$g = 3$

c.

u	u	u	u	u
4				1

$4u = 4$
$u = 1$

d.

6		3
m	m	m

$6 = 2m$
$3 = m$

2. **a.** $+2, +2$ **b.** $-1, -1$ **c.** $-4a, -4a,$ **d.** $-5t, -5t,$

3. variables; Inverse; constants; expanded

Fluency

4. **a.** $m = 5$ **b.** $e = 4$ **c.** $g = 11$ **d.** $r = 3$
e. $y = 4$ **f.** $w = 4$ **g.** $j = -2$ **h.** $t = 15$

5. **a.** $w = -1$ **b.** $v = 3$ **c.** $j = 10$ **d.** $d = -9$
e. $z = 2$ **f.** $x = \frac{3}{2}$ **g.** $e = 8$ **h.** $y = -2$

6. **a.** $a = \frac{5}{2}$ **b.** $j = \frac{1}{2}$ **c.** $c = \frac{8}{3}$ **d.** $n = \frac{1}{5}$
e. $z = -\frac{4}{5}$ **f.** $r = -\frac{3}{4}$ **g.** $b = -1$ **h.** $s = 1$

7. E

Spot the mistake

8. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

9. An equilateral triangle has side lengths $x + 1$, and a rectangle has width x and length $x + 1$.

For what value of x are the perimeters of both shapes the same?

Key points

- An equilateral triangle has side lengths $x + 1$.
- A rectangle has width x and length $x + 1$.
- For what value of x are the perimeters of both shapes the same?

Explanation

If an equilateral triangle has side lengths $x + 1$, we can represent its perimeter as $x + 1 + x + 1 + x + 1$ or $3x + 3$.

If a rectangle has width x and length $x + 1$, we can represent its perimeter as $x + x + 1 + x + x + 1$ or $4x + 2$.

To find the value of x where the perimeters of both shapes are the same, we set the two above expressions equal to each other and solve for x.

$$3x + 3 = 4x + 2$$
$$-3x \quad -3x$$
$$3 = x + 2$$
$$-2 \quad -2$$
$$1 = x$$

Therefore, when $x = 1$ the perimeters of both shapes are the same.

Answer

The perimeters of both shapes are the same when $x = 1$.

10. Get Fit! Gym charges \$20 per month plus a \$80 sign-up fee. Just Do It! Gym charges \$40 per month but charges no sign-up fee. After how many months will the cost of gym membership be equal at both locations?

Key points

- Get Fit! Gym charges \$20 per month plus a \$80 sign-up fee.
- Just Do It! Gym charges \$40 per month but charges no sign-up fee.
- After how many months will the cost of gym membership be equal at both locations?

Explanation

We can write equations for both gyms' membership fees. Let C equal the cost of membership and m equal the number of months.

As Get Fit! Gym charges \$20 per month plus a \$80 sign-up fee, we can represent this as $C = 20m + 80$.

As Just Do It! Gym charges \$40 per month and has no sign-up fee, we can represent this as $C = 40m$.

To find the number of months where the cost of membership is equal at both locations, we set both equations equal to each other and solve for m.

$$20m + 80 = 40m$$
$$20m + 80 - 20m = 40m - 20m$$
$$80 = 20m$$
$$\frac{80}{20} = \frac{20m}{20}$$
$$4 = m$$

Therefore, after 4 months the cost of gym membership will be equal at both locations.

Answer

After 4 months the cost of gym membership will be equal at both locations.

11. Two 2000 L water tanks are being filled. Tank 1 was initially one-quarter full and is filled at a rate of 200 L per minute. Tank 2 was initially half-full and is filled at a rate of 100 L per minute. After how many minutes will the water tanks have the same amount of water?

Key points

- Two 2000 L water tanks are being filled.
- Tank 1 was initially one-quarter full and is filled at a rate of 200 L per minute.
- Tank 2 was initially half-full and is filled at a rate of 100 L per minute.
- After how many minutes will the water tanks have the same amount of water?

Explanation

Let W equal the amount of water and m equal the number of minutes. We can now write equations for both water tanks.

As Tank 1 was initially one-quarter full, this means it had $2000 \times \frac{1}{4} = 500$ L of water to start with. If it is filled at a rate of 200 L per minute, we can represent this as:

$W = 500 + 200m$

As Tank 2 was initially half full, this means it had $2000 \times \frac{1}{2} = 1000$ L of water to start with. If it is filled at a rate of 100 L per minute, we can represent this as:

$W = 1000 + 100m$

To find after how many minutes the water tanks will have the same amount of water, we set both equations above equal to each other and solve for m.

$$\begin{aligned} 500 + 200m &= 1000 + 100m \\ 500 + 200m - 500 &= 1000 + 100m - 500 \\ 200m &= 500 + 100m \\ 200m - 100m &= 500 + 100m - 100m \\ 100m &= 500 \\ \frac{100m}{100} &= \frac{500}{100} \\ m &= 5 \end{aligned}$$

After 5 minutes, the water tanks will have the same amount of water.

Answer

After 5 minutes, the water tanks will have the same amount of water.

12. In 3 years' time, Joe's current age will double. What is Joe's current age?

Key points

- In 3 years' time, Joe's current age will double.
- What is Joe's current age?

Explanation

Let Joe's current age be j.

In 3 years' time, Joe's age will be $j + 3$.

However, his age will also be double his current age. This can be represented as $2j$.

Therefore, as Joe's age in 3 years' time is equal to double his current age, we set the two above expressions equal to each other and solve for j.

$$\begin{aligned} 2j &= j + 3 \\ 2j - j &= j + 3 - j \\ j &= 3 \end{aligned}$$

Therefore, Joe is currently 3.

Answer

Joe is 3 years old.

13. In 2019, a school had the same number of Year 7 boys and girls. The following year, 50 female students joined in Year 8 which meant there were now twice as many girls compared to boys. How many students were there in Year 7?

Key points

- In 2019, a school had the same number of Year 7 boys and girls.
- The following year, 50 female students joined in Year 8.
- There are now twice as many girls compared to boys.
- How many students were there in Year 7?

Explanation

Let the number of students in Year 7 be x.

Since there were the same number of Year 7 boys as there were girls in 2019, we can represent the number of Year 7 boys (or girls) as $\frac{x}{2}$.

As 50 female students joined in the following year, the total number of students in Year 8 can be represented as $x + 50$.

However, as there were twice as many girls compared to boys in Year 8, the total number of girls is now $2 \times \frac{x}{2} = x$, and the total number of boys remains the same at $\frac{x}{2}$. Therefore, the total number of students in Year 8 can also be represented as $x + \frac{x}{2}$.

To find x, we equate the above two expressions that represent the total number of students in Year 8 and solve for x.

$$\begin{aligned} x + 50 &= x + \frac{1}{2}x \\ 50 &= \frac{1}{2}x \\ 50 \div \frac{1}{2} &= \frac{1}{2}x \div \frac{1}{2} \\ 50 \times 2 &= x \\ 100 &= x \end{aligned}$$

There were 100 students in Year 7.

Answer

There were 100 students in Year 7.

Reasoning

14. a. Let C = cost

$C = 1.2d + 5.20$

b. $C = 1.43d + 2.50$

c. $1.2d + 5.20 = 1.43d + 2.50$

$d = \frac{270}{23} = 11.74$

The distance where the cost is the same in both cities is 11.74 km.

d. It costs \$19.29 to travel the distance found in part **c** in Sydney and Melbourne.

e. Suggested option 1: Cost per minute could result in the higher dollar value due to stopping at traffic lights and/or traffic jams as the car is not travelling any distance but time is still going by.

Suggested option 2: However, driving long distances on highways could result in a lower cost per minute but a higher cost per kilometre.

Note: There are other possible options.

15. a. E.g. $3x + 4 = 4x + 2$

b. E.g. $3x + 4 = 3x + 5$

c. In part **a**, the coefficients of the unknowns were not equal, whereas in part **b** they were. Therefore, if the coefficients of the unknowns are the same on both sides, the sides will not be equal and there will be no possible solution.

Extra spicy

16. $x = 31$ **17.** C **18.** $y = \frac{1}{3}$ **19.** D

Remember this?

20. A **21.** A **22.** C

6D Formulas and applications of equations

Student practice

Worked example 1

a. $A = 3\text{ m}^2$ **b.** $a = 5\text{ m/s}^2$

Worked example 2

a. $p = 15w$ **b.** $t = 5c + 50$

Worked example 3

a. i. Let n be the number of video games.
ii. $60n = 360$
iii. $n = 6$
iv. Dion can buy 6 video games.

b. i. Let c be the cost of each bag of chips.
ii. $6c + 7 = 37$
iii. $c = 5$
iv. Each bag of chips costs $5.

Understanding worksheet

1. a. $6r = 36$ **b.** $8 = h - 4$
c. $9k - 2 = 4$ **d.** $\frac{g}{2} + 4 = 10$

2. a. Number of books
b. Original number of house plants
c. Total time taken to fill tank
d. Weekly salary

3. variables; subject; substitute; rule

Fluency

4. a. $A = 60\text{ m}^2$ **b.** $v = 5\text{ km/h}$
c. $V = 3.33\text{ mm}^3$ **d.** $A = 10\text{ cm}^2$
e. $P = \$111.11$ **f.** $x = 40\text{ m}$
g. $C = -20.56°\text{C}$ **h.** $r = 2.37\text{ m}$

---

5. a. $c = 3b$ **b.** $d = 25t$
c. $v = 50n$ **d.** $c = 24a + 2$
e. $c = 1.50l - 20$ **f.** $A = 6 + 2x$
g. $l = 2c + 4s$ **h.** $t = 2.9p + 2s$

---

6. a. i. Let d be the number of days.
ii. $4d = 80$
iii. $d = 20$
iv. Feliks will take 20 days to reach his goal.

b. i. Let v be the number of views.
ii. $0.20v = 3000$
iii. $v = 15\,000$
iv. The video game studio gets 15 000 views.

c. i. Let c be the number of customers that came to the coffee shop.
ii. $\frac{2}{3}c = 50$
iii. $c = 75$
iv. 75 customers came to the coffee shop.

d. i. Let n be the number of students in the class with more students. The other class has $n - 5$ students.
ii. $43 = 2n - 5$
iii. $24 = n$
iv. One class has 24 students, the other class has 19 students.

e. i. Let n be the number of PS5 controllers Surae bought.
ii. $75n + 158 = 383$
iii. $n = 3$
iv. Surae bought 3 PS5 controllers.

---

7. C

Spot the mistake

8. a. Student B is incorrect. **b.** Student B is incorrect.

Problem solving

9. A bag of 12 hockey balls cost $120. Individually each ball costs $12. How much cheaper is it to buy the bag of hockey balls compared to buying 12 individual balls?

Key points

- A bag of 12 hockey balls cost $120.
- Individually each ball costs $12.
- How much cheaper is it to buy the bag than to buy it individually?

Explanation

Find the cost of 12 individual balls.

$12 \times 12 = 144$

Find the difference between the cost of a bag of balls and 12 individual balls.

$\$144 - \$120 = \$24$

Answer

It is $24 cheaper to buy a bag of hockey balls than to buy them individually.

10. A basketball team played 21 games. They lost twice as many games as they won. How many games did the team lose?

Key points

- A basketball team played 21 games.
- They lost twice as many games as they won.
- How many games did the team lose?

Explanation

Let n be the number of games they won. The number of games lost would then be $2n$.

$2n + n = 21$
$3n = 21$
$n = 7$
$2n = 14$

n = games won, $2n$ = games lost

$21 = n + 2n$

$21 = 3n$

$n = 7$

Answer

The team lost 14 games.

11. Lena is paving a rectangular area of her backyard with stone slabs. The width of the area is 3.5 metres and she has budgeted $2000 for the slabs. If the stone slabs cost $90 per square metre, what is the maximum length of the area she can pave?

Key points

- The width of the area is 3.5 metres.
- The total cost is $2000.
- The stone slabs cost $90 per square metre.
- What is the maximum length of the rectangular area Lena can pave?

Explanation

Let l be the length of the area.

Total cost = area × cost per square metre

$2000 = l \times 3.5 \times 90$

$2000 = l \times 315$

$l = 6.35$ m

Answer

Lena can pave a maximum length of 6.35 m.

12. Janine is a sheep farmer, and she knows she can shear 6 kg of wool per sheep. Usually, 5 kg of wool is wasted from a flock of sheep because it is low quality and cannot be sold. How many sheep does Janine need if she wants to sell 350 kg of wool?

Key points

- Each sheep provides 6 kg of wool.
- 5 kg of wool is wasted for each flock of sheep.
- How many sheep are needed to produce 350 kg of wool?

Explanation

Let n be the number of sheep needed.

$6n - 5 = 350$

$6n - 5 + 5 = 350 + 5$

$6n \div 6 = 355 \div 6$

$n = 59.17$

$n \approx 60$

Answer

Janine needs 60 sheep to sell 350 kg of wool.

13. Daniel owns a car washing business. The cost of detergent and labour is $3 per car. The buckets, hoses, sponges and brushes cost $24 for a day's work. If Daniel charges $6 per car, what is the minimum amount of cars they need to clean to make a profit?

Key points

- The cost of washing each car is $3.
- There is an additional cost of $24 per work day.
- Daniel earns $6 for washing each car.
- How many cars need to be washed to make a profit?

Explanation

Let n be the number of cars washed.

Find the number of cars that need to be washed to make zero profit.

$6n - 3n - 24 = 0$

$3n - 24 + 24 = 0 + 24$

$3n \div 3 = 24 \div 3$

$n = 8$

To make a profit, at least 1 more car needs to be washed.

Answer

Daniel needs to clean at least 9 cars to make a profit.

Reasoning

14. a. The expression for the time travelled by Car A is $t + 1$.

b. The expression for the distance travelled by Car A is $60(t + 1)$

The expression for the distance travelled by Car B is $80t$.

c. The equation that can be solved to find when Car B catches up to Car A is $60(t + 1) = 80t$.

d. It takes 3 hours for Car B to catch up to Car A.

e. Suggested option 1: Taking two cars allows passengers to have more sitting room.

Suggested option 2: Taking two cars would consume twice as much fuel.

Note: There are other possible options.

15. a. $v = 120$ km/h

b. $v = 30$ km/h

c. When the numerator is doubled, the result is doubled. When the denominator is doubled, the result is halved.

Extra spicy

16. $g = -2$ **17.** C **18.** B **19.** $6625

Remember this?

20. C **21.** A **22.** C

6E Inequalities

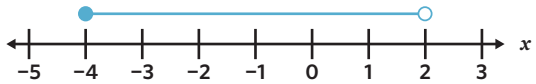
Student practice

Worked example 1

a.

b.

c.



Worked example 2

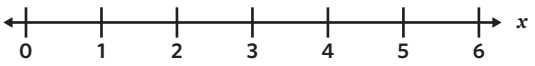
a. $3x < 10$ **b.** $x \geq 155$ **c.** $40 \leq x \leq 55$

Worked example 3

a. $x > 6$ **b.** $x \leq 7$ **c.** $x < 3$

Understanding worksheet

1. **a.**



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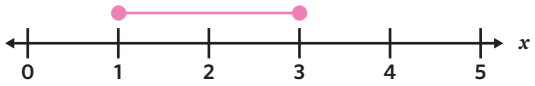

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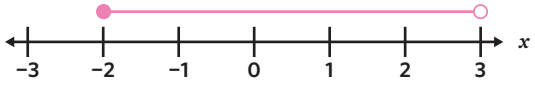

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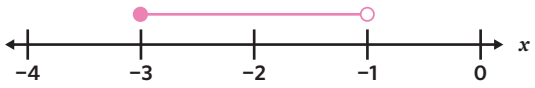


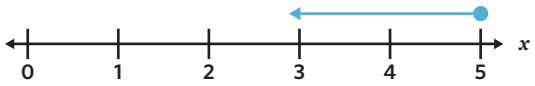
2. **a.** Greater than or equal to **b.** Less than
c. Less than or equal to **d.** Greater than

3. symbols; inequality; number line; included

Fluency

4. **a.**

b.



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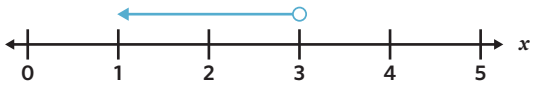

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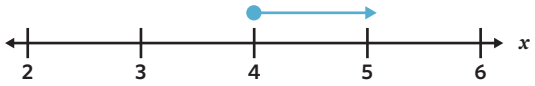

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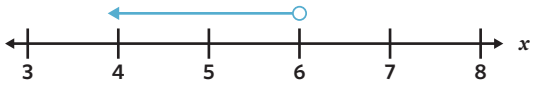

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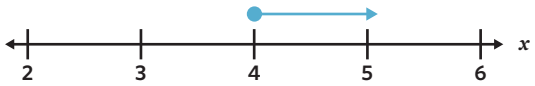

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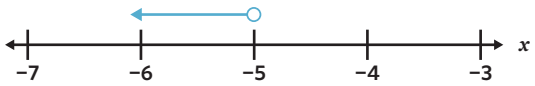

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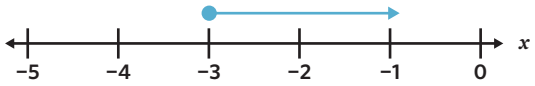
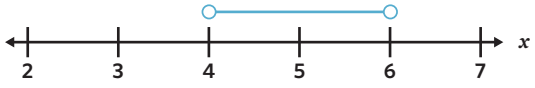
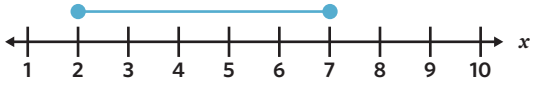


5. **a.**



b.



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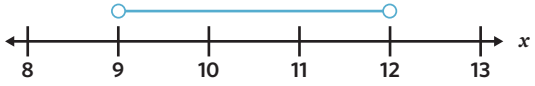

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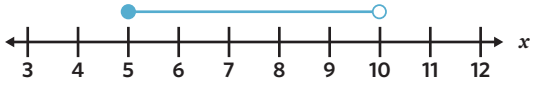
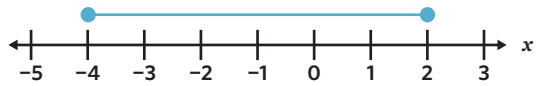
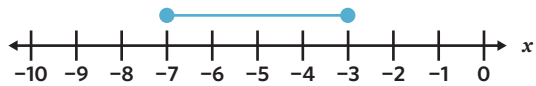


e.

f.



g.



h.

6. **a.** $x + 5 > 10$ **b.** $x - 3 < 5$
c. $x + 6 \leq 11$ **d.** $x < 14$
e. $x > 27$ **f.** $x \leq 45$
g. $-10 \leq x \leq 4$ **h.** $28 < x < 34$

7. **a.** $x < 6$ **b.** $x > 10$ **c.** $x \geq 3$ **d.** $x \geq 12$
e. $x > 7$ **f.** $x < 6$ **g.** $x \geq 4$ **h.** $x \leq 14$

8. **a.** $x \leq 8$ **b.** $x > 90$ **c.** $x > 9$ **d.** $x \geq 13$
e. $x \leq -1$ **f.** $x < -1$ **g.** $x < \frac{3}{10}$ **h.** $x \leq -2$

9. C

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

11. Ingrid buys a new set of airpods for less than $350. Write this as an inequality where the price of the airpods is represented as p.

Key points

- Ingrid buys a new set of airpods for less than $350.
- Write this as an inequality where the price of the airpods is represented as p.

Explanation

A closed circle is used to indicate that $350 is not included.

The arrow points to the left as the price of the airpod is less than $350.

Answer

$p < 350$

12. The price of a new electric scooter (s) is $2500 or more. Write this as an inequality.

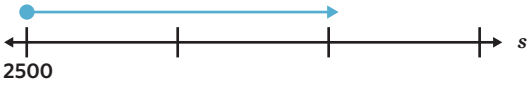
Key points

- The price of a new electric scooter (s) is $2500 or more.
- Write this as an inequality.

Explanation

An open circle is used to indicate that $2500 is included.

The arrow points to the right as the price of the scooter is greater than $2500.



Answer

$s \geq 2500$

13. Form an inequality in terms of s. Julia has saved \$150. She went online and purchased four pairs of shoes and a dress. The dress cost \$45. Each pair of shoes cost \$$s$.

Key points

- Julia has saved \$150.
- Julia purchased four pairs of shoes, each or which costs \$$s$.
- Julia purchased a dress that cost \$45.
- Form an inequality in terms of s.

Explanation

Cost of shoes = pairs of shoes that Julia has purchased × cost of each pair of shoes

$$= 4 \times s$$
$$= \$4s$$

Cost of dress = \$45

Total cost = $4s + 45$. The total cost needs to be smaller than or equal to the amount of money that Julia has saved.

$4s + 45 \leq 150$

Answer

$4s + 45 \leq 150$

14. Mike has \$600 in a savings account at the beginning of Winter. He wants to have at least \$900 in the account by the end of Winter. He adds \$30 each week. Solve and calculate how many weeks Mike is expected to add \$30 into his account in order to reach his goal.

Key points

- Mike has \$600 in a savings account at the beginning of Winter.
- He wants to have at least \$900 in the account by the end of Winter.
- He adds \$30 each week.
- Solve and calculate how many weeks Mike is expected to add \$30 into his account in order to reach his goal.

Explanation

Let the number of weeks Mike needs to save be x.

$$600 + 30x \geq 900$$
$$600 + 30x - 600 \geq 900 - 600$$
$$30x \geq 300$$
$$30x \div 30 \geq 300 \div 30$$
$$x \geq 10$$

Answer

Mike is expected to add \$30 for at least 10 weeks in order to reach his goal.

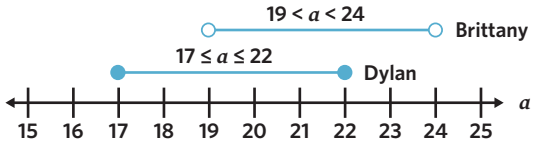
15. Dylan and Brittany are twins. It is known that Dylan is between 17 and 22, inclusive and Brittany is between 19 and 24, exclusive. Generate two inequalities on the same number line, for both Dylan and Brittany to evaluate what the range of their age is.

Key points

- It is known that Dylan is between 17 and 22, inclusive.
- Brittany is between 19 and 24, exclusive.
- Generate two inequalities on the same number line, for both Dylan and Brittany to evaluate what the range of their age is.

Explanation

Dylan's age is between 17 to 22, inclusive. Brittany's age is between 19 to 24, exclusive. Let Dylan and Brittany's age be represented as a.



Answer

The range of Dylan and Brittany's age is greater than 19 and less than or equal to 22.

Reasoning

16. **a.** $2.50 + 1.50k \leq 500$

b. $3.00 + k \leq 500$

c. Jessica can travel 331 km with Uber without exceeding her budget.

d. Jessica can expect to travel 166 more km using Yellow Taxi compared to Uber.

e. Suggested option 1: Jessica can ride a bike during her holiday.

Suggested option 2: Jessica could go jogging during her holiday.

Note: There are other possible options.

17. **a.** $-2 < x \leq 2$ **b.** $-2 \leq x \leq 2$

c. Similarity: In both parts **a** and part **b**, x is inclusive of 2. In both parts, x is 'less than or equal to' 2.

Difference: In part **a**, x is not inclusive of -2, while in part **b** it is. This is because in part **a**, x is 'greater than' -2 while in part **b**, x is 'greater than or equal to' -2.

Extra spicy

18. $x > 4$ **19.** D

20. $x = 7$, 11 or 13 **21.** C

Remember this?

22. E **23.** C **24.** E

Chapter 6 extended application

1. **a.** $x + 2y = 850$

b. The value of one console is \$692.

c. The value of Bundle 1 is \$771.

d. The value of the game in Bundle 3 is \$78.95.

e. The amount of money saved by buying the bundle instead of each included item individually is \$69.05.

f. Suggested option 1: Businesses can mass-produce products so it lowers the production of cost for each product.

Suggested option 2: Businesses can offer discounts and sell substantially more products to increase their profit.

Note: There are other possible options.

2. **a.** Michael's speed is 10 km/h.

b. Wayne lives 3 km from the skate park.

c. It takes Wayne 20 minutes to get to the skatepark from his house.

d. It takes Wayne 33 minutes to skate to Michael's house without making any stops.

e. Wayne has to leave 2 minutes early in order to meet Michael on time.

f. Suggested option 1: The local government can ensure there are regular cleaners to clean up skateparks.

Suggested option 2: The skaters should make sure that they leave the communal skate park in a reasonably tidy manner.

Note: There are other possible options.

3. a. $x + y + 5 = 20$

b. $3x + 0.5y + 5 \times 2 = 20$

c. \$10 of the total amount paid can be attributed to adults and children.

d. $x \leq 3$

e. There is one adult and 14 children in the group.

f. Suggested option 1: Tickets for children are cheaper because this event is a school fair.

Suggested option 2: Tickets for children and senior citizens are cheaper because they do not have an income.

Note: There are other possible options.

Chapter 6 review

Multiple choice

1. B **2.** A **3.** B **4.** A **5.** D

Fluency

6. a. $x = 8$ **b.** $x = 1$ **c.** $x = 10$ **d.** $x = 10$

7. a. $x = 5$ **b.** $y = 2$ **c.** $k = 25$ **d.** $x = 1$

8. a. $u = 17$ **b.** $x = 6$ **c.** $a = 0$ **d.** $b = 1$

9. a. $k = 3$ **b.** $j = 4$ **c.** $x = 6$ **d.** $z = 2$

10. a. $a = 7$ **b.** $b = 3$ **c.** $c = 2$ **d.** $d = 1$

11. a. i. Let x be the price for each pencil.

ii. $4x = 12$

iii. $x = 3$

iv. Each pencil costs \$3.

b. i. Let a be the amount of money CJ gets paid each week.

ii. $\frac{1}{3}a = 120$

iii. $a = 360$

iv. The amount of money CJ got paid each week is \$360.

c. i Let h be the number of hours Madison has been drawing for.

ii. $h + h + 3 = 8$

iii. $h = 2.5$

iv. Madison has been drawing for 2.5 hours.

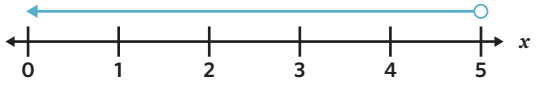
d. i Let t be the number of tests Clement scored 40 marks in.

ii. $40t + 2 \times 45 = 210$

iii. $t = 3$

iv. Clement scored 40 marks in 3 tests.

12. a.



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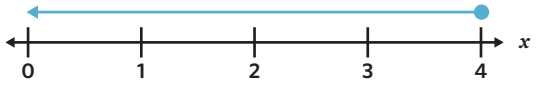

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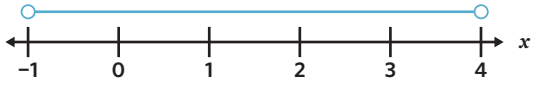

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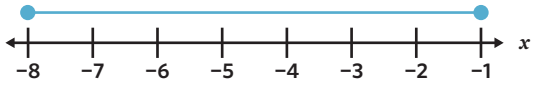


13. a. $x > 9$ **b.** $x \geq 6$ **c.** $x > 50$ **d.** $x \geq 3$

Problem solving

14. Mandy spends \$200 on x books. Each book costs \$16. Write an equation to describe this situation.

Key points

- Mandy spends \$200 on x books.
- Each book costs \$16.
- Write an equation to describe this situation.

Explanation

Total cost of x books = cost of 1 book $\times$ x

$$200 = 16x$$

Answer

The equation to describe Mandy's situation is $200 = 16x$.

15. Damien bought some cupcakes from a local bakery to share with his x students. Every student receives three cupcakes. Damien found that there were six leftover cupcakes. If there are 30 cupcakes in total, how many students does Damien have?

Key points

- Damien bought some cupcakes from a local bakery to share with his x students.
- Every student receives three cupcakes.
- Damien found that there were six leftover cupcakes.
- If there are 30 cupcakes in total, how many students does Damien have?

Explanation

Number of cupcakes each student gets $\times$ number of students + number of leftovers = total number of cupcakes

$$3x + 6 = 30$$

-6 / -6

$$3x = 24$$

$\div 3$ / $\div 3$

$$x = 8$$

Answer

Damien has 8 students.

16. Daniel did $2t$ push-ups on one day, and on the next day Daniel did four push-ups. Tasha did t push-ups each day for 4 days. Overall, Daniel and Tasha did the same number of push-ups. How many push-ups does t represent?

Key points

- Daniel did $2t$ push-ups on one day, and on the next day Daniel did four push-ups.
- Tasha did t push-ups each day for 4 days.
- Overall, Daniel and Tasha did the same number of push-ups.
- How many push-ups does t represent?

Explanation

Total number of push-ups Daniel did = total number of push-ups that Tasha did

$$2t + 4 = 4t$$
$$-2t \quad -2t$$
$$4 = 2t$$
$$\div 2 \quad \div 2$$
$$2 = t$$

Answer

t represents 2 push-ups.

17. David spent \$33 dollars in total to prepare for his party. He purchases a \$25 jumbo pack of lollies, then purchases mesh bags to divide up the lollies so that each of his guests gets one mesh bag. The cost for each mesh bag is 50 cents. How many guests are there at the party?

Key points

- David spent \$33 dollars in total to prepare for his party.
- He purchases a \$25 jumbo pack of lollies, then purchases mesh bags to divide up the lollies so that each of his guests gets one mesh bag.
- The cost for each mesh bag is 50 cents.
- How many guests are there at the party?

Explanation

Let g be the number of guests at the party.

Price of jumbo pack + (price of each mesh bag × number of guests) = total cost

$$25 + 0.5g = 33$$
$$-25 \quad -25$$
$$0.5g = 8$$
$$\div 0.5 \quad \div 0.5$$
$$g = 16$$

Answer

There are 16 guests at the party.

18. Vutha and Jayden are stacking lego blocks. It is known that Jayden has between 90 and 95 lego blocks, inclusive, and Vutha has between 87 and 93 lego blocks, exclusive. Generate two inequalities on the same number line, for both Vutha and Jayden to evaluate what the range of their number of lego blocks are.

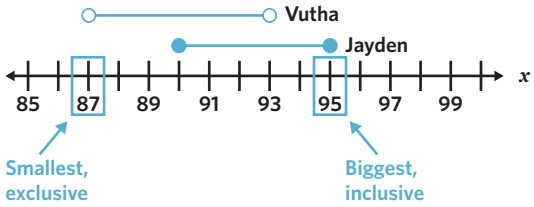
Key points

- Vutha and Jayden are stacking lego blocks. It is known that Jayden has between 90 and 95 lego blocks, inclusive.
- Vutha has between 87 and 93 lego blocks, exclusive.
- Generate two inequalities on the same number line, for both Vutha and Jayden to evaluate what the range of their number of lego blocks are.

Explanation

Jayden's range of lego blocks is $90 \leq x \leq 95$.

Vutha's range of lego blocks is $87 < x < 93$.



Therefore, the range of their number of lego blocks are $87 < x \leq 95$.

Answer

The range of their number of lego blocks are $87 < x \leq 95$.

Reasoning

19. a. The equation for James' proposal will be $3j + 2j = 300$.

b. $j = 60$. The first prize winner receives \$180 under James' proposal.

c. The equation for Bethany's proposal will be $2b + b = 300 - b$.

d. $b = 75$. Bethany proposes to spend \$75 on advertising.

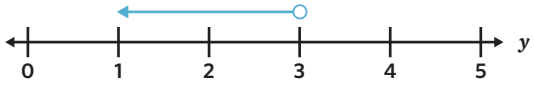
e.

f. Suggested option 1: James' proposal is better because more prize money would attract more students to participate in the fete.

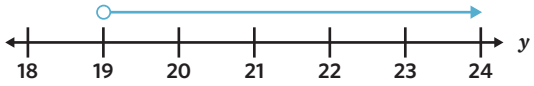
Suggested option 2: Bethany's proposal is better because the advertising poster would attract more students to participate in the fete.

Note: There are other possible options.

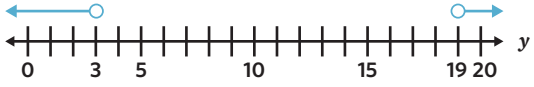
20. a. $y < 3$



b. $y > 19$



c. Both inequalities cannot be true at the same time because there is no overlap between the two number lines.



7A Angles and parallel lines

Student practice

Worked example 1

a. $a° = 65°$ as $a°$ and $25°$ are complementary angles, $b° = 270°$ as $a°$, $b°$ and $25°$ form a full revolution.

b. $q° = 35°$ as $q°$ and $35°$ are vertically opposite angles, $p° = 145°$ as $p°$ and $q°$ are supplementary angles.

Worked example 2

a. $a° = 48°$ as $a°$ and $48°$ are alternate angles.

b. $a° = 56°$ as $a°$ and $56°$ are corresponding angles, $b° = 124°$ as $b°$ and $a°$ are supplementary angles.

Worked example 3

a. $a° = 55°$ as $a°$ and $55°$ are alternate angles, $b° = 125°$ as $b°$ and $a°$ are co-interior angles.

b. $a° = 65°$ as $a°$ and $65°$ are alternate angles, $b° = 45°$ as $a°$, $b°$ and $70°$ are supplementary angles.

Understanding worksheet

1. **a.** Vertically opposite **b.** Supplementary
c. Complementary **d.** Vertically opposite

2. **a.** Alternate **b.** Corresponding
c. Co-interior **d.** Alternate, corresponding

3. complementary; supplementary; alternate; co-interior

Fluency

4. **a.** $a° = 70°$ as $a°$ and $20°$ are complementary angles.
b. $a° = 28°$ as $a°$ and $62°$ are complementary angles.
c. $p° = 72°$ as $p°$ and $18°$ are complementary angles.
d. $a° = 45°$ as $a°$, $25°$ and $20°$ are complementary angles.
e. $b° = 47°$ as $b°$ and $43°$ are complementary angles, $a° = 270°$ as $a°$ and $90°$ form a full revolution.
f. $a° = 57°$ as $a°$ and $33°$ are complementary angles.
g. $a° = 65°$ as $a°$ and $25°$ are complementary angles.
h. $a° = 51°$ as $a°$, $90°$ and $39°$ are supplementary angles.

---

5. **a.** $a° = 120°$ as $a°$ and $60°$ are supplementary angles.
b. $b° = 144°$ as $b°$ and $36°$ are supplementary angles.
c. $a° = 50°$ as $a°$ and $50°$ are vertically opposite angles.
d. $a° = 102°$ as $a°$, $22°$ and $56°$ are supplementary angles.
e. $a° = 25°$ as $a°$, $a°$ and $130°$ are supplementary angles.
f. $a° = 85°$ as $a°$, $21°$ and $74°$ are supplementary angles.
g. $a° = 45°$ as $a°$ and $45°$ are vertically opposite angles, $b° = 105°$ as $30°$, $45°$ and $b°$ are supplementary angles.
h. $a° = 110°$ as $a°$ and $70°$ are supplementary angles, $b° = 85°$ as $b°$, $70°$ and $25°$ are supplementary angles.

---

6. **a.** $a° = 59°$ as $a°$ and $59°$ are corresponding angles.
b. $a° = 67°$ as $a°$ and $67°$ are corresponding angles, $b° = 67°$ as $b°$ and $67°$ are alternate angles.
c. $a° = 127°$ as $a°$ and $127°$ are corresponding angles, $b° = 53°$ as $a°$ and $b°$ are supplementary angles.
d. $a° = 40°$ as $a°$ and $40°$ are alternate angles.
e. $a° = 122°$ as $a°$ and $58°$ are co-interior angles.
f. $a° = 59°$ as $a°$ and $59°$ are alternate angles.
g. $a° = 70°$ as $a°$ and $110°$ are co-interior angles.
h. $a° = 67°$ as $a°$ and $113°$ are supplementary angles.

---

7. **a.** $a° = 110°$ as $a°$ and $70°$ are co-interior angles, $b° = 110°$ as $a°$ and $b°$ are corresponding angles.
b. $a° = 50°$ as $a°$ and $50°$ are alternate angles, $b° = 50°$ as $a°$, $b°$ and $80°$ are supplementary angles.
c. $a° = 85°$ as $a°$ and $85°$ are corresponding angles, $b° = 95°$ as $b°$ and $a°$ are supplementary angles, $c° = 95°$ as $c°$ and $b°$ are corresponding angles, $d° = 85°$ as $d°$ and $c°$ are supplementary angles.
d. $b° = 119°$ as $b°$ and $61°$ are co-interior angles, $a° = 61°$ as $a°$ and $b°$ are co-interior angles.
e. $a° = 55°$ as $a°$ and $55°$ are corresponding angles, $b° = 74°$ as $b°$ and $74°$ are alternate angles.
f. $a° = 64°$ as $a°$ and $64°$ are alternate angles, $b° = 47°$ as $b°$ and $47°$ are alternate angles. $c° = 69°$ as $a°$, $b°$ and $c°$ are supplementary angles.
g. $a° = 113°$ as $a°$ and $113°$ are alternate angles, $b° = 45°$ as $b°$ and $45°$ are alternate angles. $c° = 22°$ as $a°$, $b°$ and $c°$ are supplementary angles.
h. $a° = 72°$ as $a°$, $59°$ and $49°$ sum to $180°$ as they are supplementary angles.

---

8. A

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. Nadia and Lena wanted to insert their tent pegs at an angle of 40° to the ground. What angle could they measure above the ground to ensure they had placed them correctly?

Key points

- Nadia and Lena wanted to insert their tent pegs at an angle of 40° to the ground.
- What angle could they measure above the ground to ensure they had placed them correctly?

Explanation

Looking at the peg line, $a°$ and $40°$ are supplementary, therefore they sum to $180°$.

$a° + 40° = 180°$

$a° = 140°$

Answer

They could measure a 140° angle above the ground to ensure they had placed them correctly.

11. Kathy is renovating her house and needs to place a wedge in the door to keep it in place. What angle (a) should she cut the bottom of her door so the wedge fits perfectly.

Key points

- Kathy has a wedge to jam her door into place.
- What angle should she cut the bottom of her door so the wedge fits in perfectly.

Explanation

Looking at the visual, $a°$ and 40° are complementary, therefore they sum to 90°.

$a° + 40° = 90°$

$a° = 50°$

Answer

She should cut the bottom of her door at a 50° angle so the wedge fits in perfectly.

12. Mr Morrow built a two-turn track for BMX sprint racing as a lunch time activity. He wanted one easy turn and one sharp turn. If the start and finish straights were parallel to each other, and the easy turn was 148°, what was the angle of the sharp turn?

Key points

- Mr Morrow built a two-turn track for BMX sprint racing as a lunch time activity.
- He wanted one easy turn and one sharp turn.
- If the start and finish straights were parallel to each other, and the easy turn was 148°.
- What was the angle of the sharp turn?

Explanation

$a°$ and 148° are co-interior, therefore they sum to 180°.

$a° + 148° = 180°$

$a° = 32°$

Answer

The angle of the sharp turn is 32°.

13. An ancient trident of Neptune had parallel prongs. If the outside angle was measured at 215° what is the measurement of angle a?

Key points

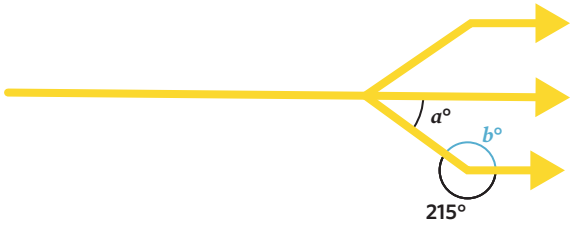
- An ancient trident of Neptune had parallel prongs.
- If the outside angle was measured at 215°.
- What is the measurement of angle a?

Explanation

$b°$ and 215° form a full revolution, therefore they sum to 360°.

$b° + 215° = 360°$

$b° = 145°$



$a°$ and $b°$ are co-interior, therefore they sum to 180°.

$a° + 145° = 180°$

$a° = 35°$

Answer

The measurement of the angle $a°$ is 35°.

14. Tony had eight friends visit for his birthday. He decided to cut the cake into nine equal slices. What angle is each slice?

Key points

- Tony had eight friends visit for his birthday.
- He decided to cut the cake into nine equal slices.
- What angle should he measure for each slice?

Explanation

A full revolution is equal to 360°. Tony will need to divide the cake nine times into nine equal slices. So the angle for each slice is $360° \div 9 = 40°$.

Answer

40° angle for each slice.

Reasoning

15. a. $a° = 52°$ as $a°$ and 128° are co-interior angles.

b. $b° = 32°$ as $b°$, $a°$ and 96° are supplementary angles.

c. $c° = 148°$ as $c°$ and 32° are co-interior angles.

d. $d° = 70°$.

e. Suggested option 1: We interact with network wiring at school.

Suggested option 2: We interact with the electoral grid network.

Note: There are other possible options.

16. a. $a°$ and $a°$ are alternate angles, therefore they are equal. $b°$ and $b°$ are alternate angles, therefore they are equal.

b. $a° + b° + c° = 180°$ as $a°$, $b°$ and $c°$ are supplementary angles.

c. From part **a**, two transversals cross a pair of parallel lines, resulting in two pairs of equal alternate angles, $a° = a°$ and $b° = b°$.

From part **b**, $a°$, $b°$ and $c°$ are supplementary angles and sum to 180°.

The two transversals crossing any parallel lines will form alternate angles $a° = a°$ and $b° = b°$.

Given that $a° + b° + c° = 180°$, we can prove that for all triangles, the sum of three internal angles is 180°.

Extra spicy

17. B **18.** $a° = 24°$ **19.** $a° = 101°$ **20.** D

Remember this?

21. D **22.** D **23.** B

7B Triangles and quadrilaterals

Student practice

Worked example 1

a. $a° = 78°$ **b.** $a° = 46°$

Worked example 2

a. $a° = 127°$ **b.** $a° = 38°$

Worked example 3

a. $a° = 112°$

b. $a° = 58°$, $b° = 122°$

Worked example 4

a. $a = 18$ mm **b.** $a = 10$ m

Understanding worksheet

1. **a.** Scalene, right **b.** Equilateral, acute
c. Isosceles, acute **d.** Isosceles, obtuse

2. **a.** Quadrilateral, trapezium
b. Quadrilateral, kite
c. Quadrilateral
d. Quadrilateral, parallelogram, rectangle

3. side lengths; internal angles; rhombuses; parallelogram

Fluency

4. **a.** $a° = 52°$ **b.** $a° = 65°$
c. $a° = 38°$ **d.** $a° = 92°$
e. $a° = 36°$ **f.** $a° = 61°$
g. $a° = 45°$ **h.** $a° = 36°$

---

5. **a.** $a° = 98°$ **b.** $a° = 52°$
c. $a° = 125°$ **d.** $a° = 66°$
e. $a° = 20°$ **f.** $a° = 135°$
g. $a° = 113°$ **h.** $a° = 20°$

---

6. **a.** $a° = 114°$ **b.** $a° = 126°$
c. $a° = 115°$ **d.** $a° = 97°$
e. $a° = 68°$ **f.** $a° = 32°$
g. $a° = 100°$ **h.** $a° = 68°$

---

7. **a.** $a° = 123°$ **b.** $a° = 76°$
c. $a° = 144°$ **d.** $a° = 42°$
e. $a° = 127°$ **f.** $a° = 67°$
g. $a° = 123°$ **h.** $a° = 99°$

---

8. **a.** $w = 10$ cm **b.** $h = 12$ cm
c. $x = 34$ mm **d.** $b = 8$ mm
e. $a = 17$ m, $b = 17$ m **f.** $d = 11$ m
g. $h = 3.8$ km **h.** $a° = 108°$, $w = 11$ cm

---

9. B

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

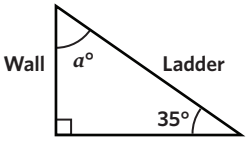
11. Nillah leaned a ladder up against a wall. If the ladder was leaning at an angle of 35° to the ground, what angle would it make at the wall?

Key points

- Nillah leaned a ladder up against a wall.
- The ladder was leaning at an angle of 35° to the ground.
- What angle would it make at the wall?

Explanation

The ladder leaning against the wall can be represented as a right-angled triangle:



As the ladder is perpendicular to the ground, we can find the angle $a°$ it would make at the wall as: $a° = 180° - 35° - 90° = 55°$.

Answer

The ladder would make a 55° angle at the wall.

12. Dylan and Andrew are racing on the parallelogram track. Dylan starts from the starting point, travels up and then across. Andrew starts from the starting point, travels across and then up. Who travels the furthest?

Key points

- Dylan and Andrew are racing on the parallelogram track.
- Dylan starts from the starting point, travels up and then across.
- Andrew starts from the starting point, travels across and then up.
- Who travels the furthest?

Explanation

As the track is a parallelogram, both pairs of opposite sides are equal in length. Therefore, travelling 'up' on either side of the track would be the same distance, and traveling 'across' on either side would be the same. As both Dylan and Andrew travel up and across once each, they effectively travel the same distance.

Answer

Dylan and Andrew travel the same distance.

13. Phillipa is making kites for her friends. She is using fibreglass sticks as structural support. What angle does she need to ensure they join at?

Key points

- Phillipa is making kites for her friends.
- She is using fibreglass sticks as structural support.
- What angle does she need to ensure they join at?

Explanation

As the diagonals of a kite are perpendicular, Phillipa needs to ensure that she joins the fibreglass sticks at an angle of 90°, otherwise the shape will not be a kite.

Answer

Phillipa needs to make sure the sticks join at a 90° angle.

14. Dorian and Raja are deciding what shape to frame their business logo. Dorian wants the frame to have four equal side lengths. Raja wants the pairs of opposite interior angles of the frame to be equal. What shape can satisfy both of them?

Key points

- Dorian and Raja are deciding what shape to frame their business logo with.
- Dorian wants four equal side lenghts.
- Raja wants two pairs of equal opposite interior angles.
- What shape can satisfy both of them?

Explanation

A rhombus has four sides that are equal in length, and would therefore satisfy Dorian's conditions. It also has two separate equal pairs of opposite interior angles, and would therefore satisfy Raja's conditions.

Answer

A rhombus would satisfy both of them.

15. Sameera is building a triangular frame for ivy to grow along her wall. She has two beams that are three metres long. She has placed one at an angle of 50° to the ground. What angle to the ground should Sameera set the other beam at to meet the current beam at the tip?

Key points

- Sameera is building a triangular frame for ivy to grow along her wall.
- She has two beams that are three metres long.
- She has placed one at an angle of 50° to the ground.
- What angle to the ground should Sameera set the other beam at to meet the current beam at the tip?

Explanation

The two beams of the frame are both three metres, which makes the triangular frame isosceles. As it is isosceles, the two bottom angles of the triangle would therefore be equal. Sameera would have to place the other beam at a 50° angle in order for them to join at the end.

Answer

Sameera has to place the other beam at a 50° angle.

Reasoning

16. a. $a° = 72°$.

b. $b° = 54°$.

c. $c° = 72°$.

d. $d° = 126°$.

e. Suggested option 1: Cindy could use plants that require less water.

Suggested option 2: Cindy could look for alternative sources of water such as grey water recycling or capturing rainwater.

Note: There are other possible options.

17. a. $a° + b°$ equals 180°.

b. $b° + c°$ equals 180°.

c. As $a° + b°$ equals 180° and $b° + c°$ also equals 180°, this implies that $a° = c°$ as they are interchangeable in both equations. The same technique can be repeated with all pairs of opposite angles using the angle that is adjacent to them both.

Extra spicy

18. D **19.** C **20.** $a° = 129°$ **21.** $a° = 43°$

Remember this?

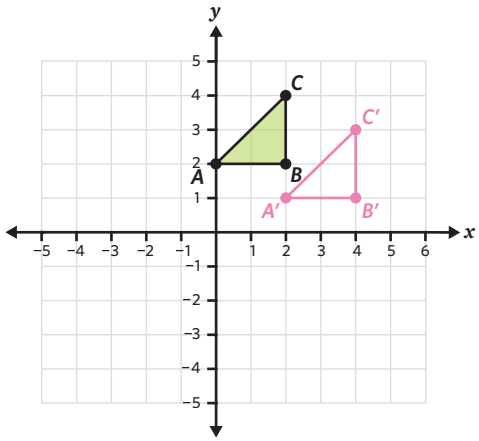
22. D **23.** E **24.** C

7C Transformations on the Cartesian plane

Student practice

Worked example 1

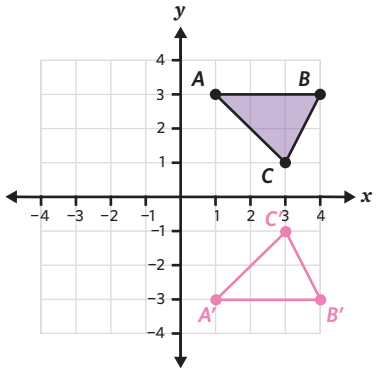
a.



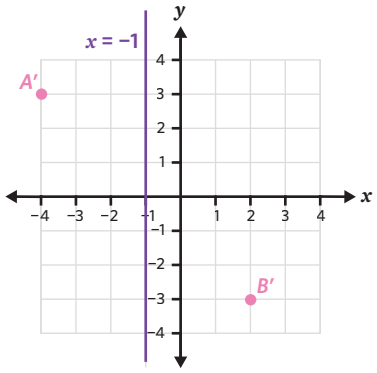
b. $B' = (2,-2)$

Worked example 2

a.

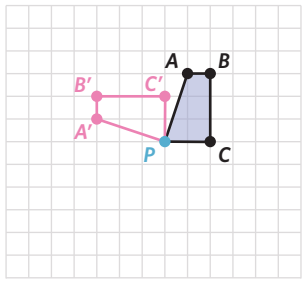


b.

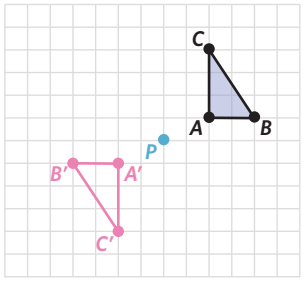


Worked example 3

a.



b.



Understanding worksheet

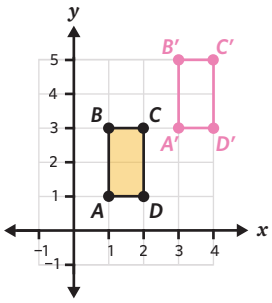
1. **a.** $(-3,-2)$ **b.** $(3,-2)$ **c.** $(2,5)$ **d.** $(3,-6)$

2. **a.** Translation **b.** Reflection in x–axis
c. Reflection in y–axis **d.** Rotation about origin

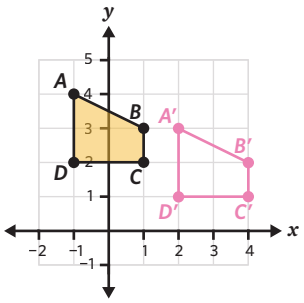
3. translation; reflection; rotation; vector

Fluency

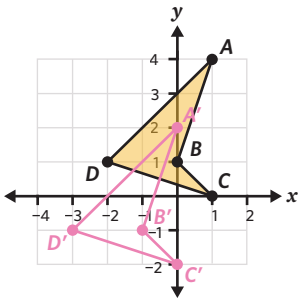
4. **a.**



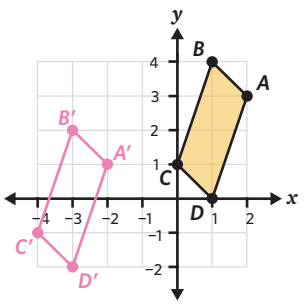
b.



c.

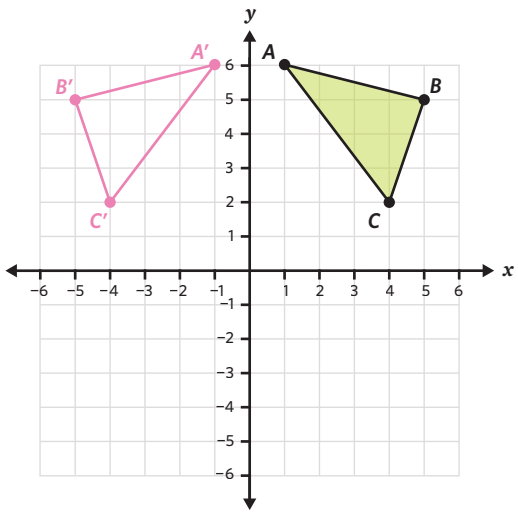


d.

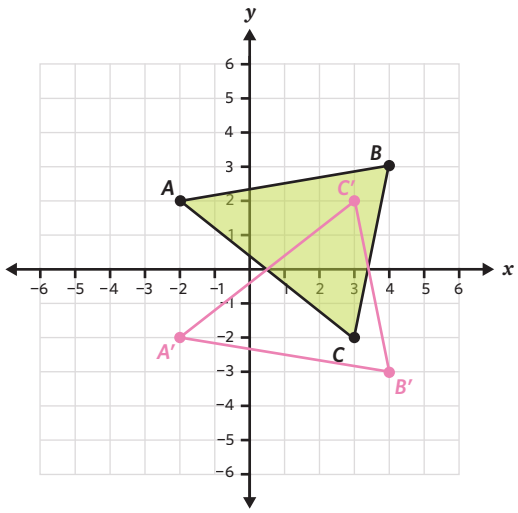


5. **a.** $A' = (3,5)$ **b.** $B' = (6,5)$
c. $C' = (6,5)$ **d.** $D' = (-6,4)$
e. $E' = (-4,9)$ **f.** $F' = (-38,-3)$
g. $G' = (15,-37)$ **h.** $H' = (21,-18)$

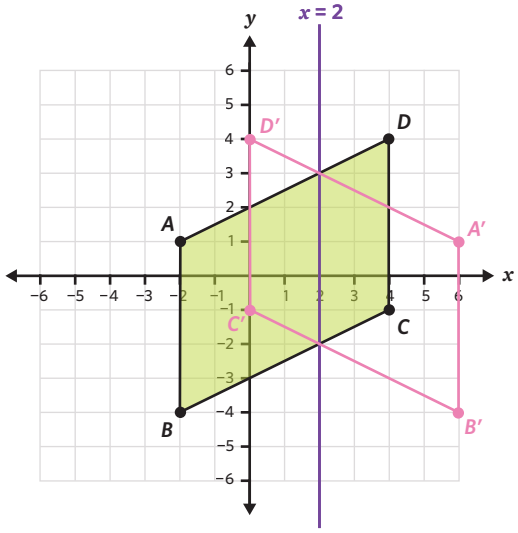
6. **a.**



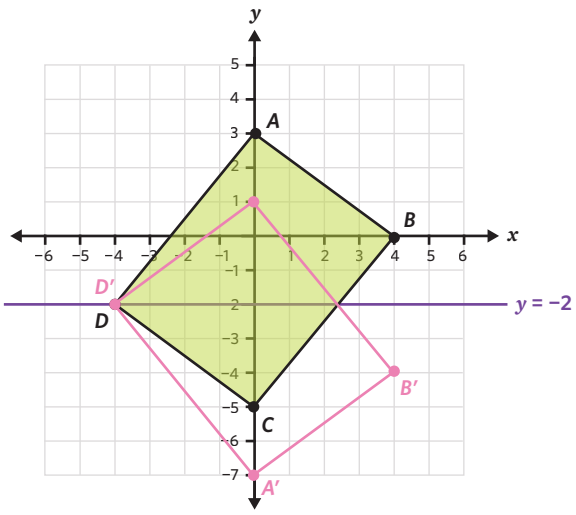
b.



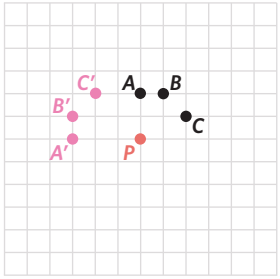
c.



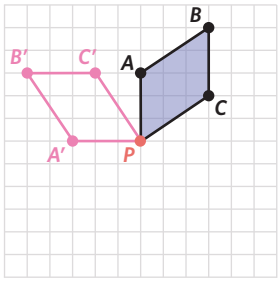
d.



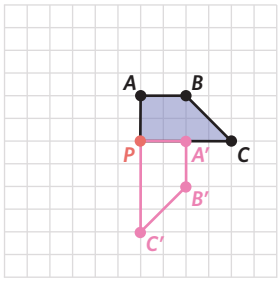
7. a.



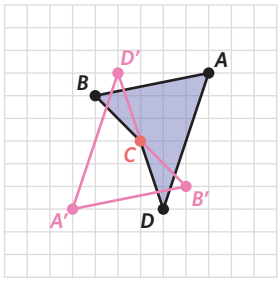
b.



c.



d.



8. E

Spot the mistake

9. a. Student B is incorrect.

b. Student B is incorrect.

Problem solving

10. Matthew is studying graphic design. He makes a design by reflecting triangle *ABC* over the mirror line. What shape has Matthew made?

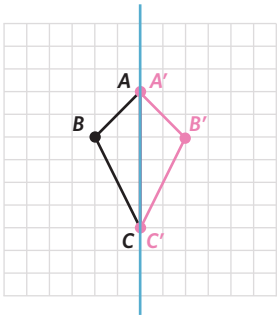
Key points

- Matthew makes a design by reflecting triangle *ABC* over the mirror line.
- What shape has Matthew made?

Explanation

Reflect points *A*, *B* and *C* in the *x*–axis.

Connect the image of the reflected points to create the reflected triangle.



Answer

The design that Matthew made is in the shape of a kite.

11. Grid town's major delivery company is called Vectors and Rogers. Drivers are given vectors to describe the journey from one stop to the next. A driver leaves the depot with the instructions: (3,5), (−2,−2) and (1,3). What is the delivery vector that goes directly from the depot to the final delivery

Key points

- Drivers are given vectors to describe the journey from one stop to the next.
- A driver leaves the depot with the instructions: (3,5), (−2,−2) and (1,3).
- What is the delivery vector that goes directly from the depot to the final delivery?

Explanation

To go directly from the depot to the final delivery, the driver could move 2 units to the right and 6 units up. The vector would therefore be (2,6).

Answer

The delivery vector that goes directly from the depot to the final delivery is (2,6).

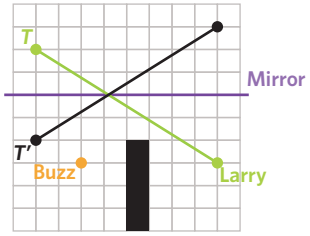
12. Larry and his friend Buzz are playing Laser Tag. Buzz is hiding behind a wall. Larry thinks he can tag his friend buzz by reflecting a laser off the mirror labeled in the diagram below. If *T* represents Larry's aim through the mirror, will Larry be able to tag Buzz?

Key points

- Buzz is hiding behind a wall. Larry thinks he can tag his friend buzz by reflecting a laser off the mirror labeled in the diagram below.
- If *T* represents Larry's aim through the mirror.
- Will Larry be able to tag Buzz?

Explanation

Draw the reflection of Larry's line over the mirror line. This line does not touch Buzz, therefore Larry is unable to tag Buzz.



Answer

Larry will not be able to tag Buzz.

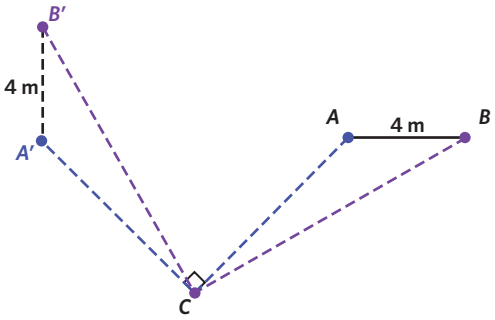
13. The points A and B are 4 m apart. If both points are rotated 90° around point C, how far apart are their images A' and B'?

Key points

- The points A and B are 4 m apart.
- Both points are rotated 90° around point C.
- How far apart are their images A' and B'?

Explanation

Rotating both points A and B by the same degree will not change the distances between the points. Therefore the images A' and B' are still 4 m apart.



Answer

The images A' and B' are 4 m apart.

14. Name the transformations from Fig1 to Fig 2 to Fig 3 to Fig 4 that Patrick used to create the following pattern so that Kysaia can replicate it.

Key points

- Name the transformations from Fig1 to Fig 2 to Fig 3 to Fig 4.

Explanation

To go from Fig 1 to Fig 2, we rotate the triangle ABC 90° anticlockwise about the origin.

To go from Fig 2 to Fig 3, we reflect the triangle $A'B'C'$ in the x–axis.

To go from Fig 3 to Fig 4, we rotate the triangle $A''B''C''$ 90° anticlockwise about the origin.

Answer

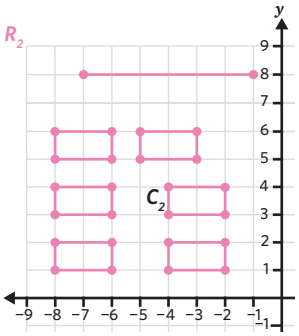
Fig 1 to Fig 2 is a rotation 90° anticlockwise about the origin.

Fig 2 to Fig 3 is a reflection in the x–axis.

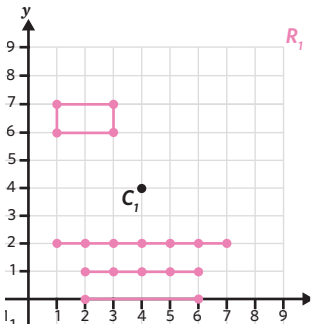
Fig 3 to Fig 4 is a rotation 90° anticlockwise about the origin.

Reasoning

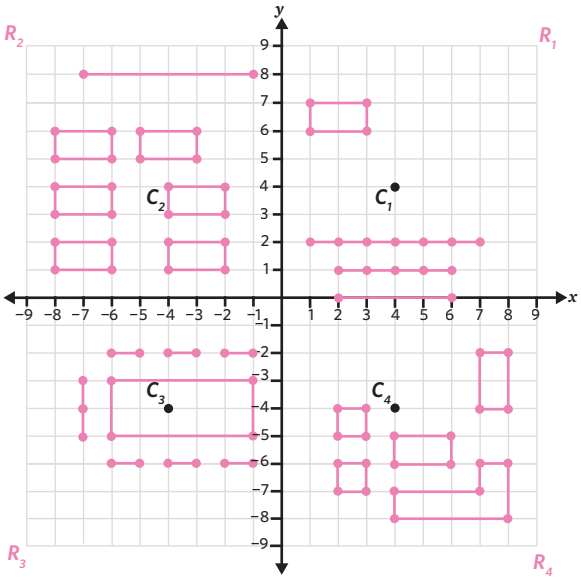
15. a.



b.



c.

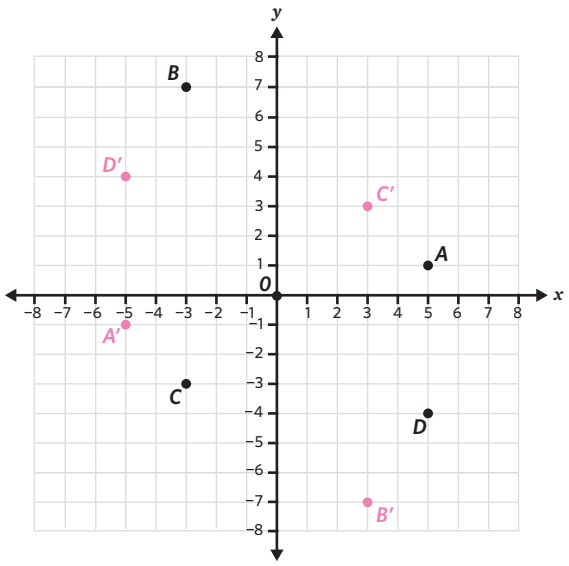


d. Suggested option 1: Yes, altering the layout of a learning space helps students be more engaged with their lessons because they can collaborate with different classmates.

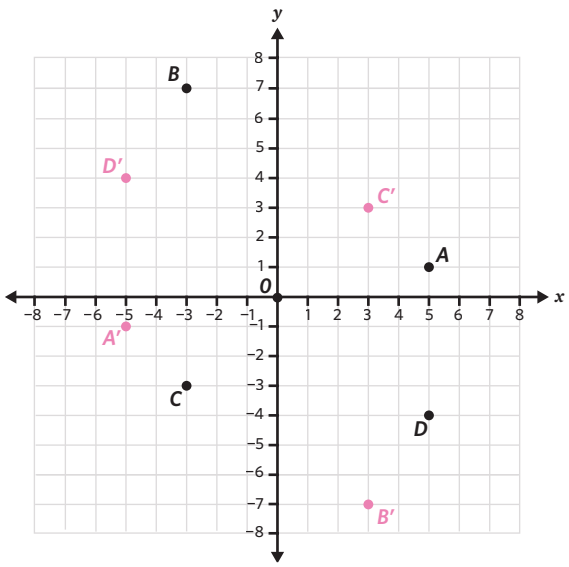
Suggested option 2: No, altering the layout of a learning space does not help students be more engaged with their lessons because they can be distracted by new classmates.

Note: There are other possible options.

16. a.



b.



c. A rotation of (x,y) anticlockwise 180° will have coordinates $(-x,-y)$.

Extra spicy

17. B 18. 11 19. $(-2,-4)$ 20. B

Remember this?

21. C 22. C 23. B

7D Congruent 2D shapes

Student practice

Worked example 1

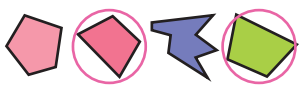
a. $\overline{CB}$ b. D

Worked example 2

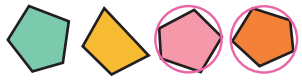
a. 1 b. 5 c. 97°

Understanding worksheet

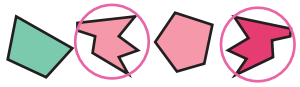
1. a.



b.



c.



d.

2. a. 2 b. 2 c. 2 d. 1

3. congruent; equal; reflected; isometric

Fluency

4. a. $\overline{AC}$ b. W
   c. $\overline{AD}$ d. Q
   e. Q f. B
   g. $\overline{TS}$ and $\overline{VU}$ h. G

5. a. 3 b. 60° c. 3 d. 2
   e. 66° f. 72° g. 5 h. 63°

6. a. 4.1 b. 114° c. 5 d. 56°

7. a. 20 b. 7.5 c. 16 d. 115°
   e. 115° f. 46°

8. A

Spot the mistake

9. a. Student A is incorrect. b. Student A is incorrect.

Problem solving

10. Euclid town was planned with streets creating congruent shapes alongside each other. We can see a pair of congruent shapes in the diagram. What is the distance from the playground to Central station if the distance from the cinema to Central station is 125 m?

Key points

- Euclid town was planned with streets creating congruent shapes.
- The pair of shapes in the diagram are congruent.
- What is the distance from the playground to Central station if the distance from the cinema to central station is 125 m?

Explanation

Compare the lengths and the angles at the vertices.

The cinema to Central station is the longest side on the right hand side of the figure and the playground is the longest side on the left hand side of the figure.

Solve.

Playground to Central station = cinema to Central station

Playground to Central station = 125 m

Answer

The distance from the playground to Central station is 125 m.

11. Dakota is welding metal pieces together following the design in the diagram. She has cut a 20 cm piece and a 30 cm piece as shown in the diagram. What angle does she need to join them at?

Key points

- She is following the diagram.
- She has cut a 20 cm piece and a 30 cm piece as shown in the diagram.
- What angle does she need to join them at?

Explanation

Compare the lengths and the angles at the vertices.

In the diagram the angle between the 20 cm side and the 30 cm side is 130°, for Dakota's design to be congruent with the diagram she must have the same angle.

Answer

Dakota needs to join the metal pieces at a 130° angle.

12. Gabriel decided to paint a mural on the wall of his solar panel shop. The design was a parallelogram that has a height of 1 m and width of 0.5 m rotated repeatedly 36°. Use the rule for the area of a parallelogram, base × height, to determine the total area Gabriel will need to paint.

Key points

- The design was a parallelogram that has a height of 1 m and width of 0.5 m.
- Each parallelogram is rotated repeatedly by 36°.
- The area of a parallelogram is base × height.
- Determine the total area Gabriel will need to paint.

Explanation

Find the area of one parallelogram.

$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= 0.5 \times 1 \\ &= 0.5 \text{ m}^2\end{aligned}$$

Find the total area.

$$\begin{aligned}\text{Total area} &= \text{area of one parallelogram} \times \text{number of parallelograms} \\ &= 0.5 \times 10 \\ &= 5 \text{ m}^2\end{aligned}$$

Answer

The total area Gabriel needs to paint is 5 m^2.

13. Noah built a square box frame with 4 congruent trapeziums. If the area of the box including the frame was 3 cm^2 squared and the area inside the frame was 2.8 cm squared. What was the area of one trapezium?

Key points

- Noah built a square box frame with 4 congruent trapeziums.
- The area of the box including the frame was 3 cm squared.
- The area inside the frame was 2.8 cm squared.
- What was the area of one trapezium?

Explanation

Determine the area of the trapeziums.

$$\begin{aligned}\text{Area of trapeziums} &= \text{total area} - \text{area inside frame} \\ &= 3^2 - 2.8^2 \\ &= 9 - 7.84 \\ &= 1.16\end{aligned}$$

Congruent figures have corresponding geometric properties, so each trapezium will have the same area.

$$\begin{aligned}\text{Area of one trapezium} &= \frac{\text{area of all trapeziums}}{4} \\ &= \frac{1.16}{4} \\ &= 0.29 \text{ cm}^2\end{aligned}$$

Answer

The area of one trapezium is 0.29 cm^2.

14. Tran wants to mow a shape in the back right corner of the lawn to mirror the shape Trevor cut in the back left corner. Tran measured 3 m along the back fence and began mowing parallel to the right fence. Tran mows 4 m and then needs to turn to complete the shape. What angle does Tran need to turn to ensure they mow the same shape as Trevor?

Key points

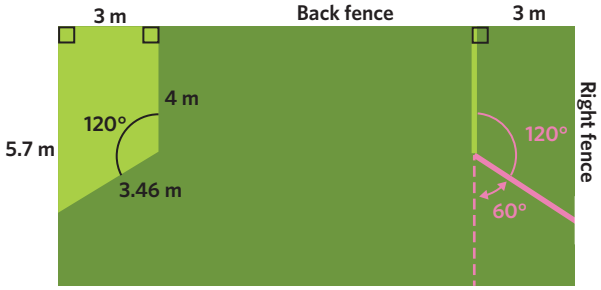
- Tran wants to copy the shape Trevor cut in the top left corner.
- Tran measured 3 m along the back fence and began mowing parallel to the right fence.
- Tran mows 4 m and then needs to turn to complete the shape.
- What angle does Tran need to turn to ensure they mow the same shape as Trevor?

Explanation

Compare the lengths and the angles at the vertices.

In Trevor's shape there is an angle of 120° after the 4 m long side.

Find the angle which Tran must turn to have an identical angle.



Tran must turn 60° to his left in order to create the correct angle.

Answer

Tran needs to turn 60° to his left.

Reasoning

15. **a.** The line segments with length 2 are $\overline{BC}$, $\overline{IJ}$, $\overline{EF}$ and $\overline{GF}$.

b. The line segments with length 4 are $\overline{AB}$, $\overline{AJ}$, $\overline{ED}$ and $\overline{GH}$.

c. The line segments with length 5 are $\overline{PA}$, $\overline{FP}$, $\overline{DP}$, $\overline{HP}$, $\overline{CP}$, and $\overline{IP}$.

d. In Toby's design the angles $\angle CPA$, $\angle IPA$, $\angle DPF$ and $\angle HPF$ are all 53°, and the angles $\angle BCP$, $\angle JIP$, $\angle EFP$ and $\angle GFP$ are all 127°

e. Suggested answer 1: Toby should make his t-shirts overseas so that those less fortunate will be able to afford them.

Suggested answer 2: Toby should make his t-shirts in Australia to create more jobs and improve the economy.

Note: There are other possible options.

16. **a.** The value of $c°$ must be 90°.

b. The angle of rotation around O is 90° anticlockwise.

c. When (3,2) was rotated 90° anticlockwise it became (−2,3). The x-value is now the negative value of what y was and the y-value is what the x-value was. The same is true when we rotated (−3,4).

Extra spicy

17. $a = 40°, b = 70°$

18. B

19. A

20. 16

Remember this?

21. A **22.** C **23.** C

7E Rules for congruent triangles

Student practice

Worked example 1

a. These triangles are congruent by ASA.

b. These triangles are congruent by SAS.

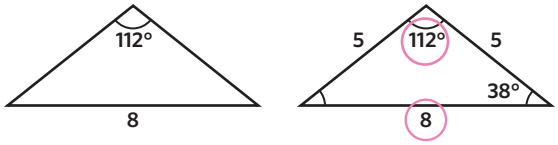
Worked example 2

a. $\triangle ADB \equiv \triangle CDB$ by SSS and $\angle DCB$ is corresponding with $\angle DAB$.
$\therefore a = 40°$

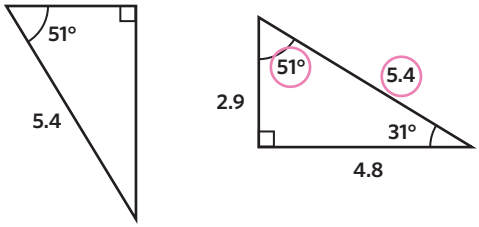
b. Triangles are congruent by AAS and the side with length x is corresponding with the side of length 9.
$\therefore x = 9$

Understanding worksheet

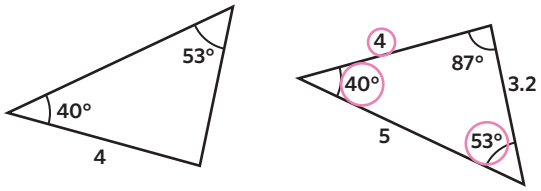
1. a.



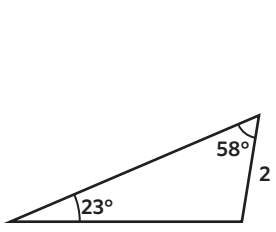
b.

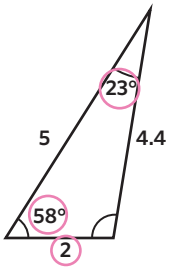


c.

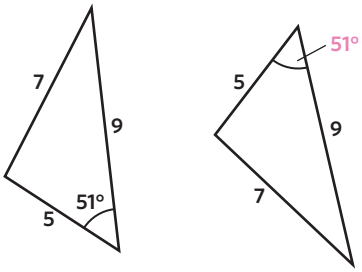


d.

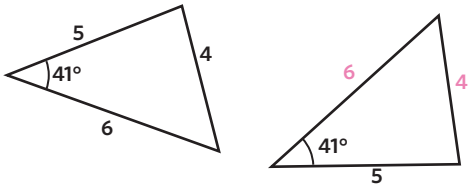




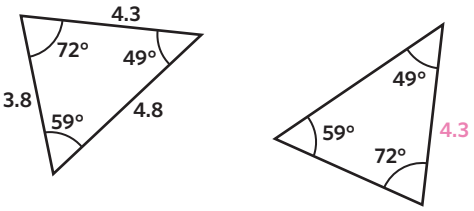
2. a.



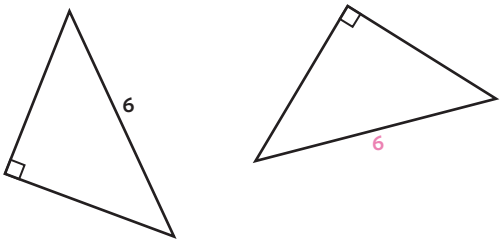
b.



c.



d.



3. equal; side lengths; adjacent; angles; hypotenuse

Fluency

4. a. These triangles are congruent by SSS.

b. These triangles are congruent by RHS.

c. These triangles are congruent by ASA.

d. These triangles are congruent by SAS.

e. These triangles are congruent by SSS.

f. These triangles are congruent by RHS.

g. These triangles are congruent by SAS.

h. These triangles are congruent by AAS.

---

5. a. These triangles do not satisfy a congruence test.

b. These triangles do not satisfy a congruence test.

c. These triangles do not satisfy a congruence test.

d. These triangles do not satisfy a congruence test.

e. These triangles do not satisfy a congruence test.

f. These triangles are congruent by SAS.

g. These triangles do not satisfy a congruence test.

h. These triangles are congruent by AAS or ASA.

---

6. a. $\triangle ABD \equiv \triangle BCD$ by RHS and $\overline{DC}$ is corresponding with $\overline{AD}$.
$\therefore x = 25$

b. $\triangle ADC \equiv \triangle ABC$ by SSS and $\angle DCA$ is corresponding with $\angle BCA$.
$\therefore a° = 40°$

c. $\triangle ABD \equiv \triangle CBD$ by RHS and $\angle ABD$ is corresponding with $\angle CBD$.
$\therefore a° = 60°$

d. Triangles are congruent by RHS and side with length h is corresponding with the side of 2.7 cm.

$\therefore h = 2.7$ cm

e. Triangles are congruent by SSS and $a°$ and $b°$ are corresponding with the angles of 75° and 30°.

$\therefore a° = 75°$ and $b° = 30°$

f. Triangles are congruent by ASA and w and l are corresponding with the side lengths 3 and 2.

$\therefore w = 3$ and $l = 2$

g. Triangles are congruent by SAS and w is corresponding with the side lengths 6.2.

$\therefore w = 6.2$

h. $\angle EGD = 120° \therefore \angle EGF = 60°$ and $\triangle EGF$ is an isosceles triangle and $\overline{EG} = \overline{EF}$.

$\triangle ABC \equiv \triangle DEF$ by AAS, so $\overline{EF} = 3$

$\therefore x = 3$

---

7. E

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. I have three logs and make a triangle. I don't like the triangle that I have made, can I make a different triangle with the same three logs? Why or why not?

Key points

- I have three logs and make a triangle.
- Can I make a different triangle with the same three logs?

Explanation

Two triangles are congruent if they satisfy any congruence test.

Using the same three logs any triangle will have the same three side lengths. Therefore these triangles satisfy the SSS test.

So all triangles made with the same three logs will be congruent.

Answer

No, since all triangles created with these logs will be congruent and therefore the same triangle.

10. Mishie and Rex are securing a pole in the ground and using ropes and pegs to secure the pole. The pole is 3 m high. Mishie places a peg at 4 m along the ground from the pole. Mishie measures the rope from the top of the pole to the peg. This has a measurement of 5 m. Rex places his peg directly opposite to Mishie's peg, also 4 m away from the pole. Will Rex also require 5 m of rope? Explain.

Key points

- The pole is 3 m high.
- Mishie places a peg at 4 m along the ground from the pole.
- The rope from the top of the pole to the peg is 5 m long.
- Rex places his peg directly opposite to Mishie's peg, also 4 m away from the pole.
- Will Rex also require 5 m of rope? Explain.

Explanation

Find corresponding features.

Both the left and right sides have corresponding side lengths of 4 m and 3 m and a corresponding angle of 90° which is adjacent to the corresponding side lengths.

Give reasons for congruent triangles.

$\therefore$ These triangles are congruent by SAS.

Since the triangles are congruent, all three sides will have corresponding side lengths.

$\therefore$ The length of Rex's rope will also be 5 m.

Answer

Rex will also require 5 m of rope because the triangles are congruent by SAS and therefore have corresponding side lengths.

11. Petro's ladder is touching the ground 4 m away from the wall. It has an angle of 35° with the ground. At what angle with the ground does Petro's ladder need to be to reach 4 m up the wall?

Key points

- Petro's ladder is touching the ground 4m away from the wall it is at an angle of 35° with the ground.
- It has an angle of 35° with the ground.
- At what angle with the ground does Petro's ladder need to be to reach 4 m up the wall?

Explanation

Find corresponding features.

Two right-angled triangles with corresponding hypotenuses and one other corresponding side length of 4m.

Give reasons for congruent triangles.

$\therefore$ These triangles are congruent by RHS.

Find the side or angle that corresponds with the pronumeral.

Corresponding angle is the angle between the ladder and the wall.

$a° = 180° - 90° - 35°$

$= 55°$

Answer

Petro's ladder needs to be at a 55° angle with the ground to reach 4 m up the wall.

12. Héritier and Alywn are playing rugby. They face off 15 m apart. Round one Héritier turned left 30° and Alywn turned right 52°. Alwyn ran 7.5 m to intercept Héritier where he would be after running 9 m. In the next round, Héritier turns right at 30° and Alwyn turns left at 52°. How far does Alwyn need to run to intercept Héritier when he runs 9 m?

Key points

- They face off 15 metres apart.
- In round one Héritier turned left 30° and Alywn turned right 52°.
- Alwyn ran 7.5 metres to intercept Héritier where he would be after running 9 metres.
- In the next round Héritier turns right 30° and Alwyn turns left 52° this time.
- How far does Alwyn need to run to intercept Héritier when he runs 9 metres?

Explanation

Find corresponding features.

Both triangles share side $\overline{HA}$ and have a corresponding side length of 9 m. Both triangles have corresponding angles of 30° and 52°, which are both adjacent to side $\overline{HA}$.

Give reasons for congruent triangles.

$\therefore$ These triangles are congruent by ASA or SAS.

Find the side or angle that corresponds with the desired value.

The corresponding side length on the lower triangle is 7.5 m.

Answer

Alywn needs to run 7.5 m to intercept Héritier.

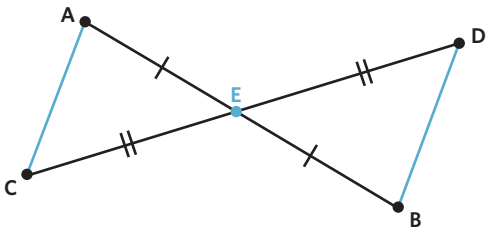
13. Alice and Bob hold a rope at either end, while Chris and Dianne hold another rope at either end. If their ropes cross exactly in the middle of each other, how do we know that there is the same distance between Alice and Charlie as there is between Bob and Dianne?

Key points

- Alice and Bob hold a rope at either end, while Chris and Dianne hold another rope at either end.
- Their ropes cross exactly in the middle of each other.
- How do we know that there is the same distance between Alice and Charlie as there is between Bob and Dianne?

Explanation

Construct two triangles denoting the centre point as E.



Find corresponding features.

$\triangle AEC$ and $\triangle BED$ have corresponding side lengths $\overline{AE} = \overline{BE}$ and $\overline{CE} = \overline{DE}$ and a corresponding angle $\angle AEC = \angle DEB$ which is adjacent to the corresponding sides.

Give reasons for congruent triangles.

$\therefore \triangle AEC \equiv \triangle BED$ by SAS

AC is corresponding with DB.

$\therefore$ They are the same length.

Answer

The triangles are congruent by SAS.

Reasoning

14. a. The side lengths $\overline{AB}$ and $\overline{AC}$ are equal and the angles $\angle ABC$ and $\angle ACB$ are also equal.

b. Maggie and Michael know that $\triangle ABD \equiv \triangle ADC$ by the SAS test.

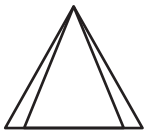
c. $\angle BAD$ and $\angle DAC$ are equal angles.

d. $\angle BDA$ and $\angle CDA$ are supplementary angles and since they are on the same line, their sum must be 180°.

$\therefore \angle BDA = \angle CDA = \frac{180}{2} = 90°$

So Maggie and Michael both have a right angled triangle.

e. Suggested option 1:



Suggested option 2:

Note: There are other possible options.

15. a. These lines can only meet at one point.

b. These lines can only meet at one point.

c. The answers from part **a** and **b** demonstrated that when a side length and two angles of a triangle are specified the remaining lines can only intersect at a single point. This means that only one triangle can be made under these conditions, therefore all triangles sharing two angles and a side length must be congruent, demonstrating why AAS as a congruence test must always be true.

Extra spicy

16. C **17.** 60° **18.** 12 **19.** D

Remember this?

20. B **21.** E **22.** D

7F Congruence and quadrilaterals

Student practice

Worked example 1

a. SSS **b.** SAS

Worked example 2

a. $\triangle ABC \equiv \triangle CDA$ by SSS

$\angle CDA$ and $\angle CBA$ are adjacent corresponding sides.

$\therefore \angle CDA = \angle CBA$

b. $\triangle ABC \equiv \triangle ADC$ by ASA

$\overline{AD}$ and $\overline{BC}$ are adjacent to corresponding angles.

$\therefore \overline{AD} = \overline{BC}$

Understanding worksheet

1. a. Supplementary angles **b.** Vertically opposite angles
c. Complementary angles **d.** Alternate angles

2. a. Quadrilateral, trapezium, parallelogram
b. Quadrilateral, kite
c. Quadrilateral, trapezium, kite, parallelogram, rhombus
d. Quadrilateral, trapezium

3. quadrilateral; triangles; transversals; intersections

Fluency

4. a. Scalene **b.** Scalene
c. Isosceles **d.** Right-angled

5. a. Vertically opposite **b.** Supplementary
c. Alternate **d.** Alternate
e. Supplementary

6. a. 67° **b.** 14 **c.** 62° **d.** 116°
e. 100° **f.** 8.7 **g.** 90° **h.** 74°

7. a. SSS **b.** ASA
c. RHS **d.** AAS
e. SAS or RHS **f.** ASA
g. SSS or RHS **h.** AAS

8. a. $\triangle ABD \equiv \triangle BCD$ by ASA

$\overline{AD}$ and $\overline{BC}$ are adjacent to corresponding angles.

$\therefore \overline{AD} = \overline{BC}$

b. $\triangle ACB \equiv \triangle ADC$ by SAS

$\overline{AC}$ and $\overline{BD}$ are opposite to corresponding angles.

$\therefore \overline{AC} = \overline{BD}$

c. $\triangle AED \equiv \triangle BEC$ by AAS

$\overline{AE}$ and $\overline{EC}$ are adjacent to corresponding angles.

$\therefore \overline{AE} = \overline{EC}$

d. $\triangle DEC \equiv \triangle DEA$ by SSS

$\angle DCE$ and $\angle DAE$ are adjacent to corresponding side lengths.

$\therefore \angle DCE = \angle DAE$

e. $\triangle BEC \equiv \triangle DEC$ by SAS

$\angle BEC$ and $\angle DEC$ are corresponding and supplementary angles.

$\angle BEC = \angle DEC = 90°$

9. E

Spot the mistake

10. a. Student A is incorrect. **b.** Student A is incorrect.

Problem solving

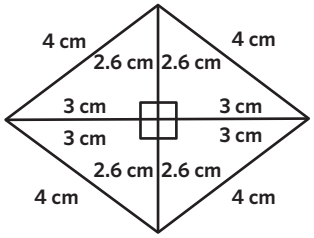
11. Makena has four right angled triangular shaped blocks, as shown in the diagram. He puts them together to make a quadrilateral. What are the names of the shapes Makena can make with the triangular blocks?

Key points

- Makena has four right angled triangular shaped blocks.
- He puts them together to make a quadrilateral.
- What are the names of the shapes Makena can make with the triangular blocks?

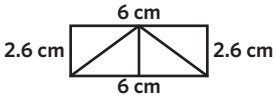
Explanation

By arranging the triangular blocks as seen below Makena can make a rhombus.



A rhombus is a particular instance of several other shapes including a parallelogram, kite, trapezium and quadrilateral.

Makena can also arrange the blocks to form a rectangle.



Answer

Rhombus, parallelogram, kite, trapezium, rectangle and quadrilateral.

12. Rebecca has two triangular stickers that are scalene and congruent. Draw a diagram of the three parallelograms Rebecca can make with the stickers, including any measurements.

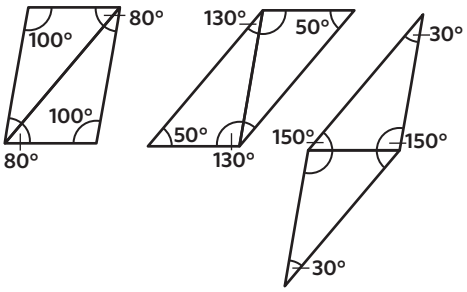
Key points

- Rebecca has two triangular stickers that are scalene and congruent.
- Include any measurements.
- Draw a diagram of the three parallelograms Rebecca can make with the stickers.

Explanation

Three separate parallelograms can be formed by connecting each of the corresponding sides of the congruent triangles.
Combine the angles at each of the vertices.

Answer



13. When Angelo is creating house frames he uses a tape measure to check for right angles in the quadrilateral shapes he creates. He measures all four sides and the diagonals. Show how Angelo can prove that all interior angles in a quadrilateral are equal to exactly 90° by using the diagram below.

Key points

- He checks for right angles in the quadrilaterals he creates.
- He measures all four sides and the diagonals.
- Including any measurements.
- Show how Angelo can prove that all interior angles in a quadrilateral are equal to exactly 90°.

Explanation

Determine the relevant triangles.

The diagonals $\overline{AC}$ and $\overline{BD}$ create 4 triangles, $\triangle ADC$, $\triangle BDC$, $\triangle DAB$ and $\triangle CBA$.

Determine if they are congruent.

$\overline{AC} = \overline{BD}, \overline{AD} = \overline{BC}$ and $\overline{AB} = \overline{DC}$

$\therefore \triangle ADC \equiv \triangle BDC \equiv \triangle DAB \equiv \triangle CBA$ by SSS

$\angle ADC$, $\angle DCB$, $\angle CBA$ and $\angle BAD$ are all corresponding as they are all adjacent to the two shortest sides of the triangle.

The sum of the internal angles of a quadrilateral is 360°.

$\therefore \angle ADC = \angle DCB = \angle CBA = \angle BAD = \frac{360}{4}$

$= 90°$

Therefore Angelo can prove the interior angles of a quadrilateral are 90°, if the diagonals have the same length.

Answer

Angelo can prove the interior angles of a quadrilateral are 90°, if the diagonals have the same length.

14. Donato has a pair of blue carpet triangles, pictured below, that he knows are congruent because they both have an angle of 70° between side lengths of 7 m and 5 m. Use the given information to determine as many of the length and angle measurements as possible in the diagram pattern below.

Key points

- The triangles are congruent.
- They both have an angle of 70°.
- They have side lengths of 7 m and 5 m.
- Use the given information to determine as many of the length and angle measurements as possible.

Explanation

The red triangles share the side lengths of $\overline{AE}$ and $\overline{BE}$.

$\therefore$ The red triangles will also have side lengths of 7 m and 5 m.

The angles between the two triangles at the centrepoint E are supplementary.

Angle $= 180° - 70°$

$= 110°$

Answer

The red triangles will have side lengths of 7 m and 5 m and an angle of 110° adjacent to these sides.

15. Toula has a kite that has diagonals with lengths 48 cm and 60 cm, pictured below. Calculate the area of Toula's kite using the formula for the area of a triangle, $\frac{\text{base} \times \text{height}}{2}$.

Key points

- Toula has a kite.
- The diagonals have lengths 48 cm and 60 cm.
- The area of a triangle is equal to $\frac{\text{base} \times \text{height}}{2}$.
- Calculate the area of Toula's kite.

Explanation

Divide the kite into two triangles along the 60 cm diagonal.

Determine the base and height of the two triangles.

The kite was divided along the 60 cm diagonal so the base of the new triangles is 60 cm. The diagonals of a kite will bisect the non equal pair of internal angles.

$\therefore$ height $= \frac{48}{2} = 24$

Calculate the area of each triangle.

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$
$$= \frac{24 \times 60}{2}$$
$$= 720 \text{ cm}^2$$

Calculate the total area of the kite.

$$\text{Total area} = 2 \times 720$$
$$= 1440 \text{ cm}^2$$

Answer

The area of Toula's kite is 1440 cm^2

Reasoning

16. a. Salvadore could make a rhombus.

b. The angle between the beams at point D is 112°.

c. The angle between these two beams will be 90°.

d. The structural supports will make a 34° angle with the ground.

e. Suggested option 1: Flowers such as orchids could be displayed in a hanging garden.

Suggested option 2: Ferns could be displayed in a hanging garden.

Note: There are other possible options.

17. a. Since the triangles are isosceles $\overline{AD} = \overline{CD}$ and $\angle DAC = \angle ACD$, therefore if $\overline{AE} = \overline{EC}$, $\triangle ADE$ and $\triangle CDE$ satisfy the SAS congruence test. The same applies for $\triangle ABC$.

b. Since $\triangle ABE \equiv \triangle BEC$ and $\triangle ADE \equiv \triangle DEC$ the pairs $\angle AED$, $\angle CED$ and $\angle AEB$ and $\angle CEB$ are both corresponding and supplementary, so they all equal 90°, showcasing that the diagonals are perpendicular to each other.

Since $\overline{AE} = \overline{EC}$, $\overline{ED}$ and $\overline{EB}$ will cut the triangles directly down the middle bisecting $\angle ABC$ and $\angle ADC$.

c. Squares and rhombuses are specific types of kites where all the sides are equal and all internal angles are equal. Unlike a normal kite, squares and rhombuses have a horizontal axis of symmetry so the diagonals will bisect all internal angles and not just two like in part **b**.

Extra spicy

18. D **19.** A **20.** 64 cm **21.** 30°

Remember this?

22. C **23.** D **24.** C

Chapter 7 extended application

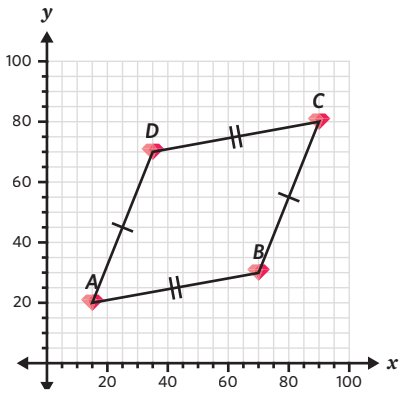
1. a. 55 units to the right and 10 units up.

b. 20 units to the right and 50 units up.

c. Type: parallelogram.

Properties: opposite sides parallel and congruent, opposite angles are equal.

d.



Opposite sides are equal and the two triangles have a common side AC and so are congruent by SSS. The distances A to B to C and A to D to C are therefore equal.

e. Method 1 (starting at A): (55,10), (−35,40), (55,10) or (20,50), (35,40), (20,50).

Method 2 (starting at C): (−55,−10), (35,−40), (−55,−10) or (−20,−50), (−35,40), (−20,−50).

f. Suggested option 1: Chess.

Suggested option 2: Sudoku.

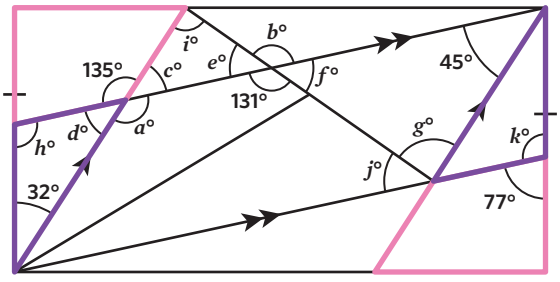
Note: There are other possible options.

2. a. 10 pieces.

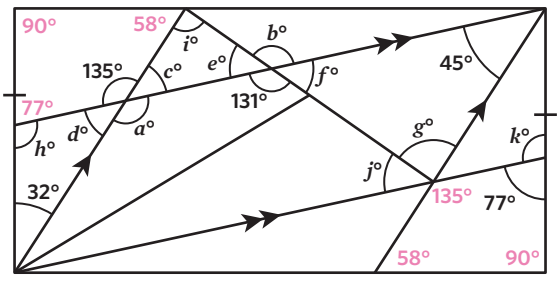
b. A: 135°, B: 131°, C: 45°, D: 45°, E: 49°, F: 49°

c. G: 86°, H: 103°, I: 86°, J: 49°, K: 103°

d.



e.



f. Suggested option 1: Handmade gifts are great because it shows that someone put time and effort into creating something for you.

Suggested option 2: Handmade gifts can be hard to keep because they are too delicate and break easily.

Note: There are other possible options.

3. a. 400 m

b. 500 m

c. $600 - y$

d. $x = 400$ m, $y = 200$ m

e. 300 m

f. Suggested option 1: Three things you should take while going camping are a tent, sleeping bags and a pillow.

Suggested option 2: Three things you should take while going camping are flashlights, camp tables and cutlery.

Note: There are other possible options.

Chapter 7 review

Multiple choice

1. B **2.** E **3.** C **4.** B **5.** A

Fluency

6. a. $m°$ and 163° are supplementary angles.

$m° + 163° = 180°$

$m° = 17°$

b. $p°$ and 90° are corresponding angles.

$p° = 90°$

c. $m°$ and 37° are alternate angles.

$m° = 37°$

d. $t°$ and 103° are vertically opposite angles.

$t° = 103°$

$t°$ and $x°$ are co-interior angles.

$103° + x° = 180°$

$x° = 77°$

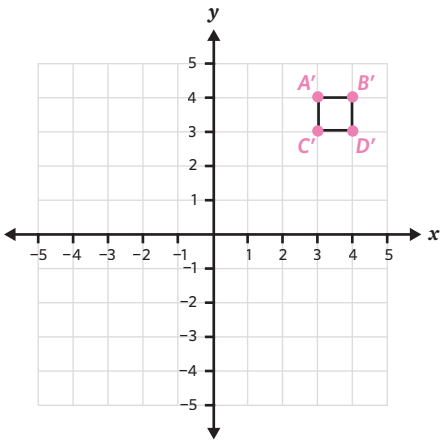
7. a. $f° = 35°$ **b.** $a° = 76°$

c. $a = 3$ **d.** $r° = 110°$

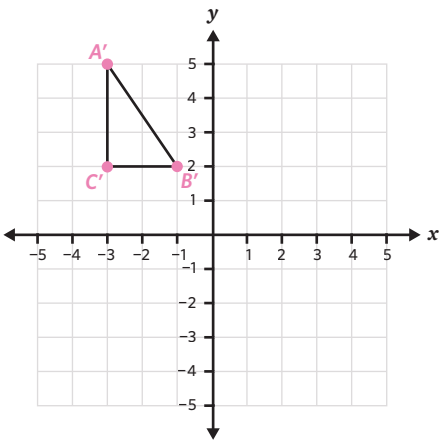
8. a. $g = 6$ **b.** $m = 8$

c. $c° = 112°$ **d.** $d° = 54°, y = 7$

9. a.

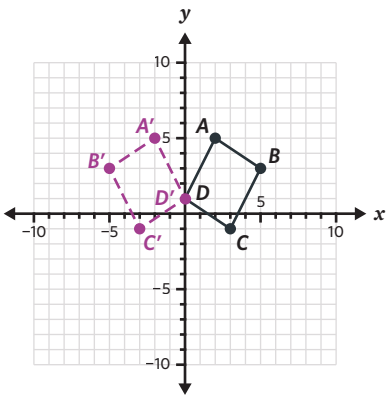


b.

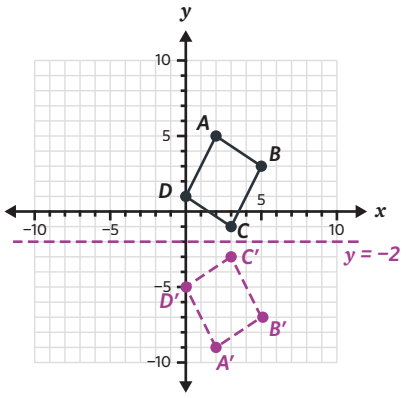


c. $B' = (9,11)$ **d.** $D' = (3,-14)$

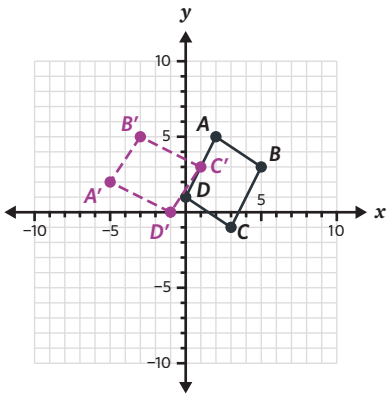
10. a.



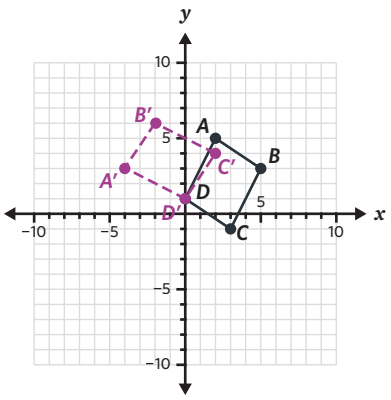
b.



c.



d.



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11. a. $\overline{CA}$ **b.** $\overline{MJ}$

c. Vertex T **d.** $\overline{ZY}, \overline{XY}, \overline{ZW}, \overline{XW}$

---

12. a. $\overline{BC} = 4$ **b.** $\angle HKJ = 67°$

c. $\overline{F'G'} = 17$ **d.** $\angle DAB = 113°$

---

13. a. These triangles are congruent by SSS.

b. These triangles are congruent by RHS.

c. These triangles are congruent by SAS.

d. These triangles are congruent by AAS.

---

14. a. These triangles are congruent by AAS. Side with length g is corresponding with the side of length 11, $\therefore g = 11$.

b. These triangles are congruent by SAS. Side with length k is corresponding with the side of length 5, $\therefore k = 5$.

c. These triangles are congruent by ASA. Side with length d is corresponding with the side of length 14.5, $\therefore d = 14.5$.

d. These triangles are congruent by SSS. $a°$ is corresponding with 60°, $\therefore a° = 60°$. Side with length z is corresponding with the side of length 4, $\therefore z = 4$.

---

15. a. $\triangle ABC \equiv \triangle ADC$ by SSS

$\angle ABC$ and $\angle ADC$ are adjacent to corresponding side lengths.

$\therefore \angle ABC = \angle ADC$

b. $\triangle LPR \equiv \triangle MOR$ by ASA or AAS

$\overline{LP}$ is corresponding with $\overline{MO}$.

$\therefore \overline{LP} = \overline{MO}$

c. $\triangle ABE \equiv \triangle CBE$ by SAS

$\overline{AB}$ is corresponding with $\overline{BC}$.

$\therefore \overline{AB} = \overline{BC}$

d. $\triangle WOZ \equiv \triangle WOX$ by RHS

$\angle ZWO$ and $\angle XWO$ are adjacent to corresponding side lengths.

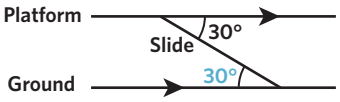
Problem solving

16. At the local playground, the slide is at an incline of 30°. If the platform at the top of the slide is parallel to the ground, what is the angle between the slide and the ground?

Key points

- At the local playground, the slide is at an incline of 30°.
- The platform at the top of the slide is parallel to the ground.
- What is the angle between the slide and the ground?

Explanation



The angle between the slide and the ground forms alternate angles with the angle between the slide and the platform.

Alternate angles are equal. Therefore, the angle we are looking for is also 30°.

Answer

The angle between the slide and the ground is 30°.

17. A pantry is being built into the corner of a kitchen as shown in the diagram. The second diagram shows the top of the pantry. The walls of the kitchen are perpendicular to one another. Inside the pantry, the pantry doors form a 45° angle with the right-hand wall. Calculate the exterior angle that the pantry doors make along the left-hand wall ($y°$).

Key points

- A pantry is being built into the corner of a kitchen as shown in the diagram. The second diagram shows the top of the pantry.
- The walls of the kitchen are perpendicular to one another.
- Inside the pantry, the pantry doors form a 45° angle with the right-hand wall.
- Calculate the exterior angle that the pantry doors make along the left-hand wall ($y°$).

Explanations

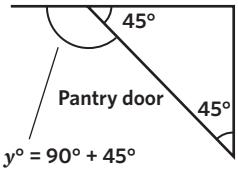
The angle that the pantry door makes along the left-hand wall ($y°$) is the opposite exterior angle of the two interior angles.

Add the two interior angles to give the value of the opposite exterior angle.

$y° = 90° + 45°$

$y° = 135°$

$y° = 135°$



$y° = 90° + 45°$

$= 135°$

Answer

The angle that the pantry doors make along the left-hand wall is 135°.

18. Alistair and Zoe are playing a game of chess, but they don't know the rules. The starting position of Alistair's knight (K) at B2 and Zoe's queen (Q) at D1 is shown below. Alistair makes two moves with his knight, which can be described by the vectors: (1,2), (−1,2). Then Zoe makes one move with her queen, which can be described by the following vectors (2, 1), (−3,4). State the final positions of the knight and queen and determine whether they are next to each other.

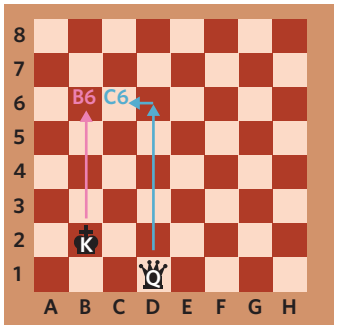
Key points

- The starting position of Alistair's knight (K) at B2 and Zoe's queen (Q) at D1 is shown below.
- Alistair makes two moves with his knight, which can be described by the vectors: (1,2), (−1,2).
- Then Zoe makes one move with her queen, which can be described by the following vectors (2,1), (−3,4).
- State the final positions of the knight and queen and determine whether they are next to each other.

Explanations

Determine Alistair's final location. Alistair's movements can be described as $(1 - 1, 2 + 2) = (0, 4)$. This means that Alistair's Knight (K) moved up by 4 units.

Determine Zoe's final location. Zoe's movements can be described as $(2 - 3, 1 + 4) = (-1, 5)$. This means that Zoe's Queen (Q) moved up by 5 units and moved to the left by 1 unit.



From the diagram, we can see that the final position of K is B6, and the final position of Q is C6. They are next to each other.

Answer

The final position of the knight is B6, and the final position of the queen is C6. They are next to each other.

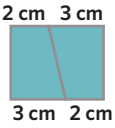
19. A decorative tiling pattern is made from congruent trapeziums. So far, the tiler has laid down sixteen tiles, as shown in the diagram below. If the tiler needs to create a row of tiles with a length of 200 cm, how many tiles do they still need to lay?

Key points

- A decorative tiling pattern is made from congruent trapeziums.
- So far, the tiler has laid down sixteen tiles, as shown in the diagram below.
- The tiler needs to create a row of tiles with a length 200 cm.
- How many tiles do they still need to lay?

Explanations

From the diagram, a pair of tiles (two congruent trapeziums form a rectangle with a length of $2 + 3 = 5$ cm.



A 200 cm row needs $200 \div 5 = 40$ of such rectangles.
Each rectangle is 2 trapezium tiles, therefore a 200 cm row of tiles require $40 \times 2 = 80$ tiles.

16 tiles have already been laid down.

$$\begin{aligned}\text{Remaining number of tiles} &= 80 - \text{existing number of tiles} \\ &= 80 - 16 \\ &= 64\end{aligned}$$

Answer

They will need to lay 64 more tiles.

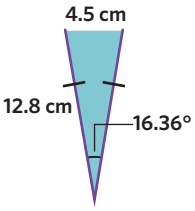
20. Mahima wishes to make the following paper fan out of blue and yellow sticks which she will need to purchase. As the blue sticks are all identical, Mahima thinks that she will need nearly 50 cm of yellow sticks in total because $4.5 \times 11 = 49.5$. Is Mahima correct? How can we verify her measurements?

Key points

- Mahima wishes to make the following paper fan out of blue and yellow sticks which she will need to purchase.
- As the blue sticks are all identical, Mahima thinks that she will need nearly 50 cm of yellow sticks in total because $4.5 \times 11 = 49.5$.
- Is Mahima correct? How can we verify her measurements?

Explanations

All individual triangles are congruent by SAS, as all blue sticks have identical length, and all angles between the blue sticks are labeled identical. Therefore, each yellow stick has an identical length of 4.5 cm.



Looking at the diagram, there are 12 blue sticks and every two blue sticks are connected with one yellow stick. Therefore, there are 11 yellow sticks.

The total length of yellow sticks Mahima needs to buy is $11 \times 4.5 = 49.5$ cm, which is nearly 50 cm. Mahima is correct.

Answer

Mahima is correct. We can verify her measurement as all individual triangles are congruent by SAS.

21. Pam has drawn a quadrilateral. Both diagonals bisect internal angles. If the diagonals are of equal length, prove that Pam's quadrilateral must be a square.

Key points

- Pam has drawn a quadrilateral.
- Both diagonals bisect internal angles.
- The diagonals are of equal length.
- Prove that Pam's quadrilateral must be a square.

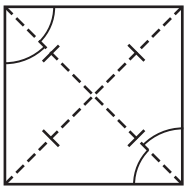
Explanations

Draw a diagram according to the information given.

A quadrilateral is a four-sided polygon.

A rhombus has both diagonals bisecting internal angles.
A rectangle has diagonals of the same length.

A square is a rectangle that is a rhombus. Therefore, Pam's quadrilateral must be a square.



Answer

A quadrilateral is a four-sided polygon. A rhombus has both diagonals bisecting internal angles. A rectangle has diagonals of the same length. A square is a rectangle that is a rhombus. Therefore, Pam's quadrilateral must be a square.

Reasoning

22. a. $x° = 135°$

b. Yes, we can conclude that all five squares have the same side length based on congruence. By definition, congruent shapes have equal side lengths and angles.

c. The leftmost triangle and the rightmost triangle are congruent by RHS.

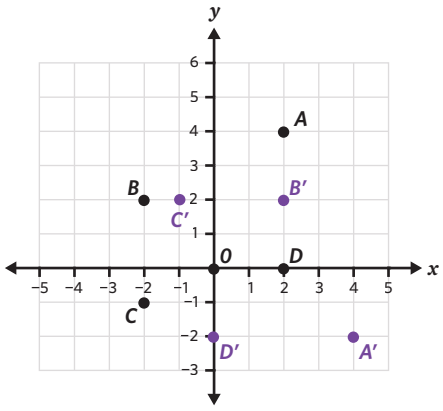
d. A square is a special rectangle, and a rectangle is a special parallelogram. A parallelogram has diagonals bisecting each other. Therefore, the square has the blue supports (diagonals) bisecting each other.

e. Suggested option 1: A bridge can be made with steel.

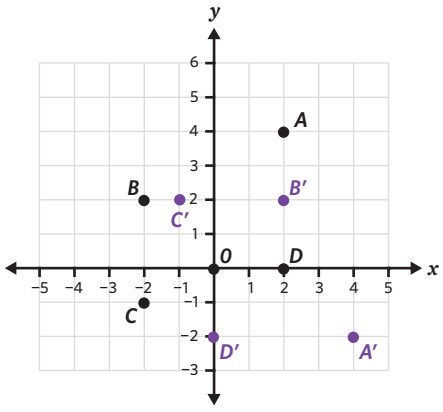
Suggested option 2: A bridge can be made with concrete.

Note: There are other possible options.

23. a.



b.



c. The answers for parts **a** and **b** are identical. This implies that rotating a point 90° clockwise is equivalent to swapping the x and y coordinates, followed by making the new y coordinate negative. When any point (x, y) is rotated 90° clockwise about the origin, the coordinates of the transformed point are given by $(y, -x)$.

8A Classifying data

Student practice

Worked example 1

a. i. A sample of 30 random people.
ii. Survey

b. i. Population of the rest home.
ii. Census

Worked example 2

a. Primary data

b. Secondary data

Worked example 3

a. i. The variable is students' arm spans, in centimetres.
ii. Numerical continuous variable.

b. i. The variable is the rate of an activity out of 5 stars.
ii. Categorical ordinal variable.

Understanding worksheet

1. a. Sample
b. Population
c. Sample
d. Population

2. a. Categorical
b. Numerical
c. Categorical
d. Numerical

3. sample; population; numerical; discrete; continuous; categorical

Fluency

4. a. i. The population of the house is being used.
ii. Census

b. i. Samples of ice cream flavours are being used.
ii. Survey

c. i. Samples of movies are being used.
ii. Survey

d. i. A sample of soil is being used.
ii. Survey

e. i. A sample of every second shopper is being used.
ii. Survey

f. i. The population of a country is being used.
ii. Census

g. i. A sample of online reviews is being used.
ii. Survey

h. i. The population of all customers is being used.
ii. Census

---

5. a. Secondary data
b. Primary data
c. Primary data
d. Primary data
e. Primary data
f. Secondary data
g. Primary data
h. Primary data

---

6. a. i. The variable is favourite food.
ii. Categorical nominal variable.

b. i. The variable is the time it takes to solve a Rubik's Cube.
ii. Numerical continuous variable.

c. i. The variable is height.
ii. Numerical continuous variable.

d. i. The variable is types of birds.
ii. Categorical nominal variable.

e. i. The variable is the number of crunches.
ii. Numerical discrete variable.

f. i. The variable is birth date.
ii. Categorical ordinal variable.

g. i. The variable is temperature.
ii. Numerical continuous variable.

h. i. The variable is car registration numbers.
ii. Categorical nominal variable.

---

7. C

Spot the mistake

8. a. Student A is incorrect.
b. Student B is incorrect.

Problem solving

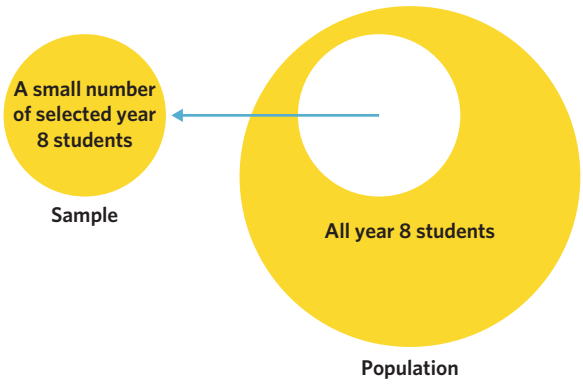
9. Mr Aykroyd is a Year 8 coordinator and wants to get feedback about recent uniform changes from the students. Should he use a sample or the population to conduct his investigation?

Key points

- Mr Aykroyd wants to get feedback about recent uniform changes from the students.
- Should he use a sample or the population to conduct his investigation?

Explanations

Uniform changes will apply to all year 8 students. The opinions on uniform changes of a selected sample of year 8 students might not be representative of the cohort. Therefore Mr Aykroyd should use a population to conduct his investigation.



Answer

Mr Aykroyd should use the population to conduct his investigation.

10. Eddie wants to investigate how long his classmates spend on homework every night. State the variable of this investigation, with appropriate units, and determine its type.

Key points

- Eddie wants to investigate how long his classmates spend on homework every night.
- State the variable of this investigation, with appropriate units, and determine its type.

Explanations

The variable is the time Eddie's classmates spend on homework every night. An appropriate unit is time in hours.

Time is a numerical continuous variable as it is a number and is measured.

Answer

The variable is the time Eddie's classmates spend on homework every night. An appropriate unit is time in hours, and the type of data is numerical continuous.

11. Ying is trying to decide what kind of outfits she needs to pack for her trip to Bali. She does some research online on the kind of weather to expect during the time of the year she will be visiting. Is Ying using primary or secondary data sources to make her decision?

Key points

- Ying is trying to decide what kind of outfits she needs to pack for her trip to Bali.
- She does some research online on the kind of weather to expect during the time of the year she will be visiting.
- Is Ying using primary or secondary data sources to make her decision?

Explanation

Research online is a secondary data source as Ying did not conduct the research herself.

Answer

Ying is using a secondary data source to make her decision.

12. A confectionery company weighs every second bag of gummy worms on the production line to make sure it matches with what it says on the packet. What is the sample and population of this process?

Key points

- A confectionery company weighs every second bag of gummy worms on the production line to make sure it matches with what it says on the packet.
- What is the sample and population of this process?

Explanation

The sample is every second bag of gummy worms selected from the population of all gummy worms produced by the confectionery company.

Answer

The sample is every second bag of gummy worms. The population is all bags of gummy worms produced by the confectionery company.

13. Donnie has to complete an assignment on the most popular music in the year he was born. To obtain the appropriate data, should Donnie just talk to his parents and relatives or go online to look up top ten hits from the year he was born?

Key points

- Donnie has to complete an assignment on the most popular music in the year he was born.
- To obtain the appropriate data, should Donnie just talk to his parents and relatives or go online to look up top ten hits from the year he was born?

Explanation

Donnie's parents and relatives are a small sample selected from the population of music listeners in the year Donnie was born.

The top ten hits from online research is obtained from the population of all music listeners in the year Donnie was born.

A population is always better than a sample as a sample might not be accurately representative of a population.

Answer

Donnie should go online to look up top ten hits from the year he was born.

Reasoning

14. a. The sample of this study is 50 Australians aged 13 to 16. The population of this study is Australian teenagers.

b. The variable of time per week spent on extracurricular activities is numerical continuous. An appropriate unit is time in hours.

c. Question 2 and 4 will result in categorical nominal data.

d. Mr Cronenberg is using secondary data.

e. Suggested option 1: Soccer

Suggested option 2: Guitar

Note: There are other possible options.

15. a. 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 7, 7, 7, 8, 9. This data could be numerical discrete.

b. Even: 2, 6, 4, 2, 8, 2, 4; Odd:1, 1, 5, 3, 7, 7, 3, 5, 1, 7, 9, 5. This data could be categorical nominal.

c. The order and organisation of data often helps with understanding what kind of data is in the set. Values that have been organised into ascending or descending order form numerical data. Data grouped into categories is categorical.

Extra spicy

16. D

17. C

18. The area of a house is numerical continuous.

19. There is no guarantee that a sample will be representative of the entire population. Using a population when possible will always be better for statistical analysis.

Remember this?

20. D **21.** A **22.** B

8B Collecting data

Student practice

Worked example 1

Method 2

Worked example 2

The voucher incentive introduces bias.

Understanding worksheet

1. a. Sample size $\approx \sqrt{(\text{population size})}$

b. Sample size ≥ 30

c. Sample size $\approx \sqrt{(\text{population size})}$

d. Sample size ≥ 30

2. a. Biased **b.** Unbiased **c.** Biased **d.** Biased

3. representative; thirty; randomly; equal

Fluency

4. **a.** Method 2 **b.** Method 1 **c.** Method 1 **d.** Method 2

5. **a.** Method 2 **b.** Method 1 **c.** Method 1 **d.** Method 2

6. **a.** Method 1 **b.** Method 2 **c.** Method 2 **d.** Method 1

7. **a.** The sample's restriction to teenage male relatives only introduces bias.

b. The sample's restriction to friends' families only introduces bias.

c. The sample's restriction to the location of Sydney city only introduces bias.

d. The sample's restriction to rescued greyhounds only introduces bias.

e. The sample's restriction to your cat only introduces bias.

f. The sample's restriction to five teachers only introduces bias.

g. The sample's size introduces bias.

h. The sample's restriction to volunteers only introduces bias.

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

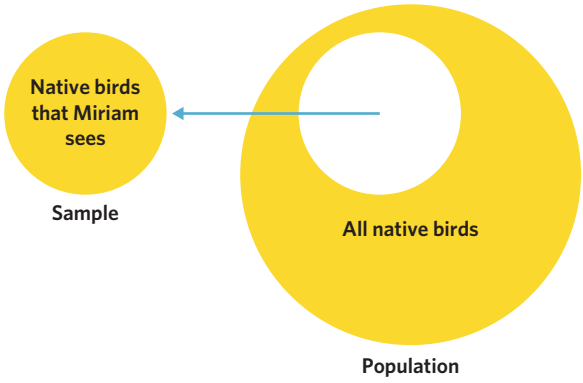
10. Miriam is a scientist who lives in New Zealand and likes to birdwatch every day. She keeps a record of all the native birds she sees when she travels around the country. What is the target population of Miriam's research?

Key points

- Miriam keeps a record of all the native birds she sees when she travels around the country.
- What is the target population of Miriam's research?

Explanation

For the sample of native birds Miriam sees when she travels across the country to be representative of the target population, the target population needs to be all native birds across the country.



Answer

The target population of Miriam's research is all native birds across New Zealand.

11. The director of a large company is organising a staff lunch. She wants to know if the employees have any dietary requirements before booking a caterer. Explain why she must use the population as opposed to a sample of her employees in order to get this data.

Key points

- She wants to know if the employees have any dietary requirements before booking a caterer.
- Explain why she must use the population as opposed to a sample of her employees in order to get this data.

Explanation

Dietary requirements of all employees need to be catered for. This is not suitable for selecting a sample as the employees not chosen in the sample are not considered. Therefore the director must use the entire population to obtain this data.

Answer

The director must use the entire population because she needs to obtain the dietary requirements of all employees.

12. Marco is planning on travelling to Thailand. He gathers data on yearly rainfall and finds that the probability of heavy rain on any day in Thailand is 34%. The majority of rainfall in Thailand occurs during the monsoon season late in the year, whereas Marco is travelling there in March. Did Marco's data provide him with a meaningful conclusion relevant to his situation? Explain your reasoning.

Key points

- He gathers data on yearly rainfall and finds that the probability of heavy rain on any day in Thailand is 34%.
- The majority of rainfall in Thailand occurs during the monsoon season late in the year, whereas Marco is travelling there in March.
- Did Marco's data provide him with a meaningful conclusion relevant to his situation? Explain your reasoning.

Explanation

Marco's method will produce a sample which is not representative of the target population of the number of rainy days on any day in Thailand. This is because the majority of rainfall in Thailand does not occur in March.

Answer

Marco's data does not provide him with a meaningful conclusion relevant to his situation because his sample was not representative of rainfall in Thailand in March.

13. Dawn has found that out of the 100 cars that passed her house on a Friday evening between 8 pm and 10 pm, two were speeding. She presents her findings to the local council and asks for speed bumps to be installed due to the high likelihood of drivers speeding on her street. Does Dawn's survey support her claim? Explain your reasoning.

Key points

- Dawn has found that out of the 100 cars that passed her house on a Friday evening between 8 pm and 10 pm, two were speeding.
- She presents her findings to the local council and asks for speed bumps to be installed due to the high likelihood of drivers speeding on her street.
- Does Dawn's survey support her claim? Explain your reasoning.

Explanation

The sample of cars between 8 pm and 10 pm on a Friday evening is not representative of all the cars driving on Dawn's street at all times. Therefore Dawn's selected sample is not representative of the population and so her survey does not support her claim.

Answer

Dawn's survey does not support her claim because her sample of cars driving between 8 pm and 10 pm on a Friday night is not representative of the targeted population of all cars on her street.

14. Callum is an engineer at a robotics company. His job is to test every third robot on the assembly line over the course of a month. If a total of 1500 robots were produced that month, find the number of robots in Callum's test sample and comment on whether it is representative of the population.

Key points

- His job is to test every third robot on the assembly line over the course of a month.
- A total of 1500 robots were produced that month.
- Find the number of robots in Callum's test sample and comment on whether it is representative of the population.

Explanation

Callum will test $1500 \div 3 = 500$ robots on the assembly line over the course of a month.

The total targeted population is 1500, which is more than 1000, so an appropriate sample size is $\sqrt{1500} \approx 39$ cars.

$500 > 39$, so the sample is representative of the population.

Answer

The number of robots in Callum's test sample is 500. This is representative of the population of all robots.

Reasoning

15. a. '2001: A Space Odyssey' is the most popular.

b. The target population of Luke's survey is all people.

c. The sample chosen by Luke is not representative of the target population because it is restricted to students at Luke's university.

d. One source of bias in Luke's sample is the restriction to only students at Luke's university. An improvement could be posting an ad online so that it reaches a wider population.

Another source of bias is that Luke chose the movies to put on his list. He could have asked people to tell him their choices and ranked them based on the number of times a movie is mentioned.

e. Suggested option 1: An advantage is the ease of access to information, such as research for a school assignment.

Suggested option 2: A disadvantage is the unknown credibility of the sources as not all sources are reliable.

Note: There are other possible options.

16. a. This sample is not representative of the target population.

b. This sample is representative of the target population.

c. The sample in part **a** is not representative of the target population because the age of pensioners is not representative of the target population of all ages.

The sample in part **b** is representative of the target population because the age of pensioners is representative of the target population of people aged 65 or above.

Extra spicy

17. C **18.** 1300 **19.** D

20. 3.3%. This is lower than 4% so he should not return the book for revisions.

Remember this?

21. C **22.** A **23.** E

8C Interpreting data displays

Student practice

Worked example 1

a. Donkey Kong = 1

Kirby = 3

Mario = 16

Pac-Man = 8

Sonic the Hedgehog = 4

b. 32

Worked example 2

a. Donkey Kong **b.** 11

Worked example 3

a. Mario **b.** 32

Worked example 4

a. 20 years **b.** 20 to 29 years

Understanding worksheet

1. a. IIII **b.** 𝍸 𝍸 **c.** 𝍸 I **d.** 𝍸 𝍸 II

2. a. Stem-and-leaf plot **b.** Dot plot

c. Pie chart **d.** Pie chart

3. frequency; dot; proportions; numerical

Fluency

4. a. Turning Red **b.** The Bad Guys

c. 72 **d.** $\frac{5}{24}$

5. a. I. The Bad Guys; II. The Bob's Burgers Movie; III. Cheaper by the Dozen; IV. Home Team; V. Marmaduke; VI. Turning Red

b. Marmaduke

c. 4

d. 26

6. a. The Loch Ness monster **b.** The Yowie

c. The Mothman **d.** 120

7. a. 104 **b.** 2 days **c.** 45 **d.** $\frac{11}{30}$

8. C

Spot the mistake

9. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

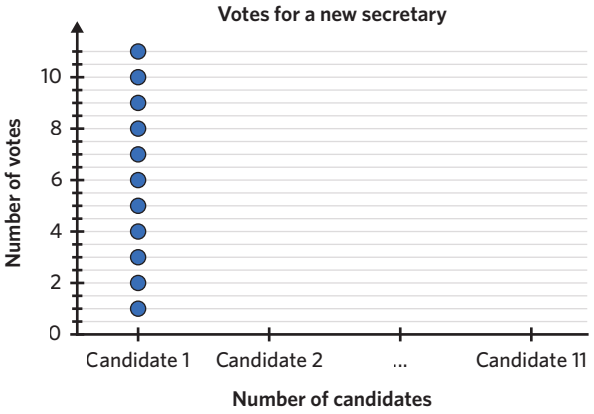
10. Zeus asks 11 members of his council to vote on a new secretary. Each member will vote only once and Zeus wants to use a dot plot to record the data. How long should Zeus make the vertical scale of his dot plot to ensure that all the potential votes can be included?

Key points

- Zeus asks 11 members of his council to vote on a new secretary.
- Each member will vote only once.
- Zeus wants to use a dot plot to record the data.
- How long should Zeus make the vertical scale of his dot plot to ensure that all the potential votes can be included?

Explanation

The longest vertical scale of his plot needs to be the highest number of potential votes for one candidate.



The highest number of potential votes for one candidate is 11, as there are 11 members in total.

The lowest number of potential votes for one candidate is 0.

Therefore, the vertical scale of Zeus' dot plot should start with 0 and go up to 11 to ensure that all the potential votes can be included.

Answer

The vertical scale of Zeus' dot plot should start with 0 and go up to 11.

11. Ms Muscat is a Year 8 coordinator who has 400 grades to analyse for an end-of-year mathematics exam. If each student received a grade from A to E for this assessment, suggest a suitable data display that Ms Muscat can use in order to compare the proportions of grades.

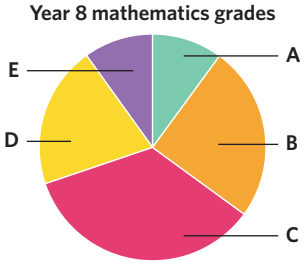
Key points

- Ms Muscat is a Year 8 coordinator who has 400 grades to analyse for an end-of-year mathematics exam.
- If each student received a grade from A to E for this assessment.
- Suggest a suitable data display that Ms Muscat can use in order to compare the proportions of grades.

Explanation

Ms Muscat should use a pie chart because the type of variable is categorical. The categories are A, B, C, D, E. The angle each sector makes within the pie chart shows the proportions of each category.

One possible pie chart for Ms Muscat is shown below.



Answer

Ms Muscat could use a pie chart to compare the proportions of grades.

12. The winner of an election must have at least half of all the votes. What is the minimum size of the angle of the sector that represents the winner in a pie chart?

Key points

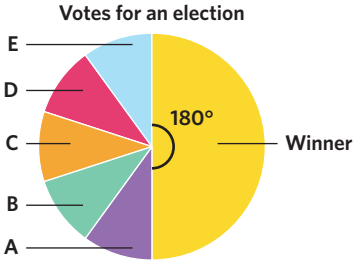
- The winner of an election must have at least half of all the votes.
- What is the minimum size of the angle of the sector that represents the winner in a pie chart?

Explanation

The minimum number of votes that a winner of an election must obtain is half of all the votes, which is 50% of the total votes.

$50\% \times 360° = 180°$

The following pie chart shows a potential spread of votes where the winner obtains 50% of the votes.



The winner of the election must have a sector with an angle of 180° or more.

Answer

The winner of the election must have a sector with an angle of 180° or more.

13. The following back-to-back stem-and-leaf plot shows the marks out of 100 for an English assignment for two classes, 8X and 8Y.

Which class received higher marks for the English assignment overall? Explain your reasoning.

Key points

- The following back-to-back stem-and-leaf plot shows the marks out of 100 for an English assignment for two classes, 8X and 8Y.
- Which class received higher marks for the English assignment overall? Explain your reasoning.

Explanation

Looking at the stem-and-leaf plot, 8Y's marks are mostly around 70 to 90, whereas 8X's are between 50 and 70. So 8Y did better overall.

8X	Stem	8Y
9 5 2 0	3	1
8	4	0
9 9 8 8 5 4 3 3	5	4
9 3 2 2	6	1 4 5 8
6 5 5 3 3	7	3 7 8 8
0 0	8	2 3 4 5 5 6 7 8 9
2	9	0 1 4 5 9

Key: 6 | 1 = 61 marks

Most frequent range of marks for 8Y

Most frequent range of marks for 8X

Answer

Class 8Y received higher marks than 8X because most of 8Y's marks are concentrated further down the stem-and-leaf plot compared to 8X's marks.

14. Sammy is a wedding planner. He asks a sample of 30 guests to try different entrees and vote for their favourite. He recorded the results in a frequency table.

There are 240 guests coming to the wedding. According to the table, at least how many servings of arancini balls should be prepared for the event?

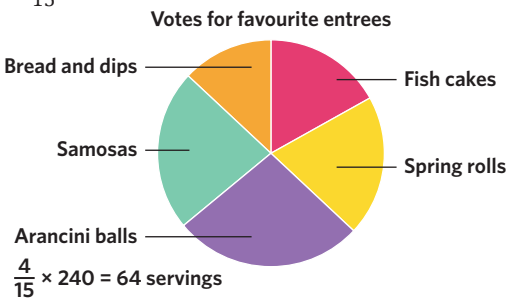
Key points

- Sammy asks a sample of 30 guests to try different entrees and vote for their favourite and records the results in a frequency table.
- There are 240 guests coming to the wedding.
- According to the table, at least how many servings of arancini balls should be prepared for the event?

Explanation

The proportion of people who voted arancini balls as their favourite is $\frac{8}{30} = \frac{4}{15}$.

If $\frac{4}{15}$ of the total 240 guests favour arancini balls, the least number of servings for arancini balls prepared should be $\frac{4}{15} \times 240 = 64$ servings.



Answer

At least 64 servings of arancini balls should be prepared for the event.

Reasoning

15. **a.** Chinese **b.** 38% **c.** 3% **d.** 12%

e. Suggested option 1: French and Indonesian could be included in the 'Other' category.

Suggested option 2: German and Khmer could be included in the 'Other' category.

Note: There are other possible options.

16. **a.** The type of variable is categorical. A dot plot is suitable to display the data.

b. The type of variable is numerical. A stem-and-leaf plot is suitable to display the data.

c. For part **a**, the variable is categorical, so it is not possible to use a stem-and-leaf plot. Stem-and-leaf plot can only be used for data that can be counted or measured with numbers.

For part **b**, the variable is numerical, a stem-and-leaf plot is suitable. For numerical data that is measured, it may not be practical to use a dot plot or pie chart, especially if each person has a distinctly different time result.

Extra spicy

17. C **18.** B **19.** 68 **20.** Foxes

Remember this?

21. B **22.** B **23.** E

8D Frequency tables and histograms

Student practice

Worked example 1

a. 0–5, 6–11, 12–17, 18–23, 24–29

Hairdresser visits in a year	Tally	Frequency
0–5	𝍸 I	6
6–11	𝍸	5
12–17	𝍸 II	7
18–23	I	1
24–29	I	1

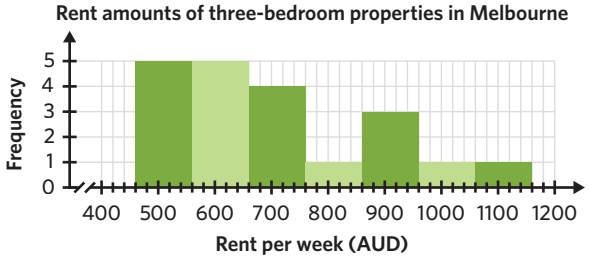
Worked example 2

a. 147–<152 cm, 152–<157 cm, 157–<162 cm, 162–<167 cm, 167–<172 cm, 172–<177 cm

Height in cm	Tally	Frequency
147–<152	II	2
152–<157	II	2
157–<162	III	3
162–<167	IIII	4
167–<172	𝍸 III	8
172–<177	I	1

Worked example 3

a.



b. Positively skewed

Understanding worksheet

1. **a.** 15–19 people **b.** 12–15 goals **c.** 46–<50 mm **d.** 550–<650 cm

2. **a.** Symmetric **b.** Bi-modal **c.** Positively skewed **d.** Negatively skewed

3. intervals; inclusive; continuous; histogram; shape

Fluency

4. **a.** 19

b. 17–22, 23–28, 29–34, 35–40

c.

Class size	Tally	Frequency
17–22	𝍸 I	6
23–28	𝍸 IIII	9
29–34	IIII	4
35–40	I	1

d. 23–28

5. a. 34

b. 136–<141 cm, 141–<146 cm, 146–<151 cm, 151–<156 cm, 156–<161 cm, 161–<166 cm, 166–<171 cm

c.

Height in cm	Tally	Frequency
136–<141	II	2
141–<146		0
146–<151	II	2
151–<156	𝍸 I	6
156–<161	III	3
161–<166	IIII	4
166–<171	III	3

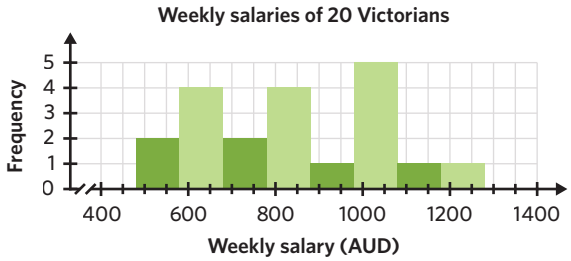
d. 151–<156 cm

6. a.

Weekly salary (AUD)	Tally	Frequency
481–<581	II	2
581–<681	IIII	4
681–<781	II	2
781–<881	IIII	4
881–<981	I	1
981–<1081	𝍸	5
1081–<1181	I	1
1181–<1281	I	1

b. 100

c.



d. $981–<$1081

e. Bi-modal

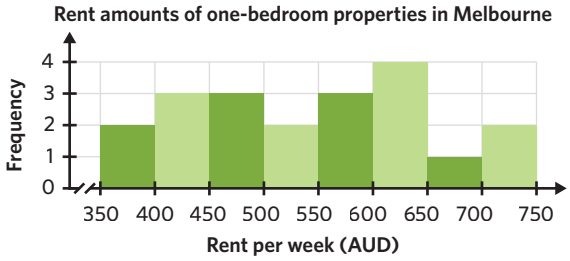
7. a. 380

b. 350–<400, 400–<450, 450–<500, 500–<550, 550–<600, 600–<650, 650–<700, 700–<750

c.

Rent per week	Tally	Frequency
350–<400	II	2
400–<450	III	3
450–<500	III	3
500–<550	II	2
550–<600	III	3
600–<650	IIII	4
650–<700	I	1
700–<750	II	2

d.



e. $600–650 per week

f. Symmetric

g. The shape of a histogram tells us where the majority of the values in the distribution lie. We can see from the above histogram that most of the rent amounts are concentrated around $400 to $650.

8. C

Spot the mistake

9. a. Student B is incorrect. b. Student A is incorrect.

Problem solving

10. Giuseppe is analysing his profits for the month of April. He collects data in Australian dollars every day and records it in a frequency table. Should Giuseppe choose inclusive or exclusive class intervals to group the data for a histogram?

Key points

- Giuseppe collects data every day, in Australian dollars.
- Should Giuseppe choose inclusive or exclusive class intervals to group the data for a histogram?

Explanation

Dollars are a continuous variable. Continuous data should be grouped in exclusive class intervals, where the maximum values of each interval are included in the following interval.

Answer

Giuseppe should choose exclusive class intervals.

11. Patrick is a teacher who wants to record the assignment results for his class of 28 students in a frequency table. The marks of the students ranged from 25 to 100, including decimals. How many exclusive class intervals of size 10 will be in Patrick's table?

Key points

- There are 28 students.
- The marks of the students ranged from 25 to 100, including decimals.
- How many exclusive class intervals of size 10 will be in Patrick's table?

Explanation

The maximum mark was 100, and the minimum mark was 25.

$\therefore$ Range $= 100 - 25 = 75$

Divide the range by 10 and round up to the nearest whole number. This will be the number of class intervals of size 10.

$75 \div 10 = 7.5 \approx 8$

Answer

There will be 8 exclusive class intervals of size 10.

12. Stacey has collected data ranging from 50 cm to 350 cm. Determine the size of each exclusive class interval if Stacey organises the measurements in 6 equal ordered groups.

Key points

- The data ranges from 50 to 350.
- Stacey organises the measurements in 6 equal ordered groups.
- Determine the size of each exclusive class interval.

Explanation

The maximum value was 350, and the minimum value was 50.

$\therefore$ Range $= 350 - 50 = 300$

Range $\div$ number of intervals $=$ size of class interval

$300 \div 6 = 50$

Answer

Each exclusive class interval is of size 50.

13. Lena is an accountant who is analysing her client's daily spendings during the previous financial year. She has to group amounts ranging from \$15 to \$500 dollars in exclusive class intervals of size 100. List the class intervals starting with the minimum of \$15 and including the maximum of \$500.

Key points

- The amounts range from \$15 to \$500.
- The exclusive class intervals are of size 100.
- List the class intervals starting with the minimum of \$15 and including the maximum of \$500.

Explanation

The maximum value was \$500, and the minimum value was \$15.

$\therefore$ Range $= 500 - 15 = 485$

Divide the range by 100 and round up to the nearest whole number to find the number of class intervals of size 100.

$485 \div 100 = 4.85 \approx 5$

Starting with the minimum value (15), group the data into exclusive class intervals of size 100. Make sure that the maximum value (500) is included in the last class interval.

The intervals will be: \$15–<\$115, \$115–<\$215, \$215–<\$315, \$315–<\$415, \$415–<\$515.

Answer

\$15–<\$115, \$115–<\$215, \$215–<\$315, \$315–<\$415, \$415–<\$515.

14. Sally asks 29 people how long it takes them to get to school in the morning. She records the results in the following table.

Predict the overall shape of the histogram Sally constructs from the data.

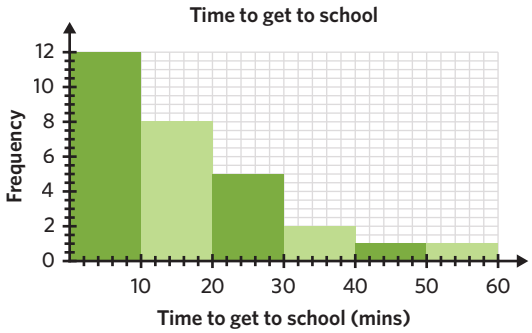
Key points

- The data is shown in the frequency table.
- Predict the overall shape of the histogram Sally constructs from the data.

Explanation

The frequency is highest in the first interval and lowest in the last interval. The frequency values decrease consistently, and this shows that the histogram will be positively skewed, with a peak on the left and tail on the right.

This prediction is correct when we create the histogram:



Answer

Positively skewed

Reasoning

15. a. 290 cents

b. 50

c. \$4.40–\$5.40

d.

Price of Big Mac	Tally	Frequency
390–<440	𝍸 III	8
440–<490	𝍸 IIII	9
490–<540	𝍸 IIII	9
540–<590	I	1
590–<640	II	2
640–<690	I	1

e. Suggested option 1: Costs of ingredients are different in different countries.

Suggested option 2: Some countries are willing to pay more for Big Macs than others.

Note: There are other possible options.

16. a. Histogram A

b. Histogram B

c. It tells you whether the temperatures tend to be warmer or colder, based on the skew. Negative skew indicates overall warmer temperatures or larger values, positive skew indicates overall colder temperatures or smaller values.

Extra spicy

17. D

18. B

19. It will make the data either positively or negatively skewed, where it may not have been before.

20. 0–<20 hours

Remember this?

21. A **22.** B **23.** E

8E Measures of centre and spread

Student practice

Worked example 1

a. Mean = 7.7; median = 8; mode = 12; range = 14.

b. Mean = 3.4; median = 3; mode = 3; range = 5.

Worked example 2

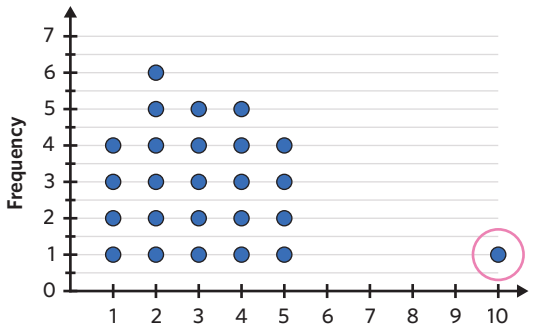
a. Outlier = 60

b. With outlier: mean = 10.1; median = 8; mode = 12; range = 59.
Without outlier: mean = 7.7; median = 8; mode = 12; range = 14.

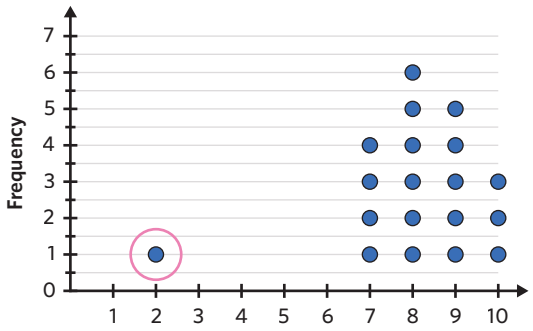
c. The mean increased from 7.7 to 10.1.
The median did not change from 8.
The mode did not change from 12.
The range increased from 14 to 59.

Understanding worksheet

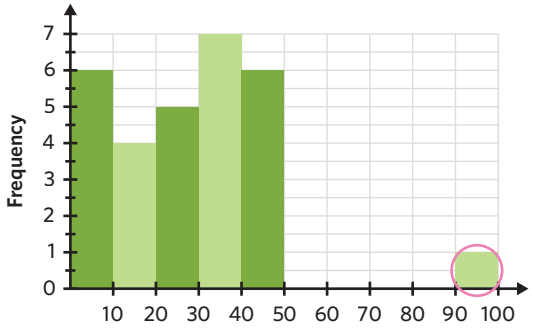
1. a.



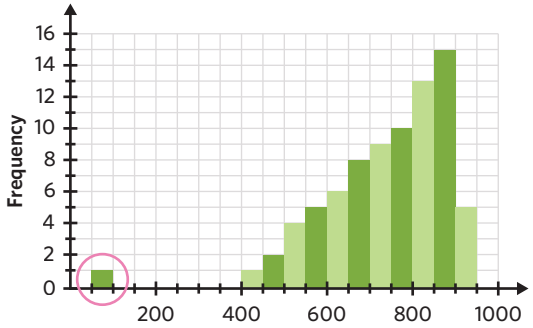
b.



c.



d.



2. a. True **b.** True **c.** True **d.** False

3. median; range; maximum; mean

Fluency

4. a. Mean = 8; median = 8; mode = 7 and 9 and 11; range = 13.

b. Mean = 10.2; median = 9; mode = 7 and 12; range = 15.

c. Mean = 19.2; median = 20.5; mode = 21 and 22; range = 19.

d. Mean = 40.3; median = 42; mode = 67; range = 59.

e. Mean = 8.3; median = 7.5; mode = 9.5 and 15.5; range = 14.5.

f. Mean = 16.5; median = 16.7; mode = 10, 14 and 18; range = 18.

g. Mean = 9.8; median = 11.25; no mode; range = 18.8.

---

5. a.

Number (n)	Frequency (f)	$n \times f$
0	3	0
1	4	4
2	6	12
3	8	24
Total	21	40

Mean = 1.9; median = 2; mode = 3; range = 3.

b.

Number (n)	Frequency (f)	$n \times f$
1	2	2
2	4	8
3	6	18
4	2	8
5	4	20
Total	18	56

Mean = 3.1; median = 3; mode = 3; range = 4.

c.

Number (n)	Frequency (f)	$n \times f$
5	1	5
6	4	24
7	4	28
8	5	40
9	3	27
10	2	20
Total	19	144

Mean = 7.6; median = 8; mode = 8; range = 5.

d.

Number (n)	Frequency (f)	$n \times f$
10	5	50
20	3	60
30	4	120
40	6	240
50	4	200
Total	22	670

Mean = 30.5; median = 30; mode = 40; range = 40.

6. **a.** Outlier = 40
b. No outliers
c. No outliers
d. Outlier = 9
e. Outlier = 0
f. Outlier = 50
g. Outlier = 138

---

7. **a.** Outlier = 130
b. With outlier: mean = 38.8; median = 30; mode = 20, 22, 23, 30 and 36; range = 129.
c. Without outlier: mean = 35.7; median = 30; mode = 20, 22, 23, 30 and 36; range = 89.
d. Median and mode are not affected by the outlier.
e. Without the outlier, the mean decreased by 3.1 seconds from 38.8 to 35.7. The range decreased by 40 seconds from 129 to 89.

---

8. D

Spot the mistake

9. **a.** Student B is incorrect.
b. Student A is incorrect.

Problem solving

10. Zoe measures the temperature inside a terrarium every day for a month. After gathering the data, she can see one outlier caused by the wrong setting on her thermometer. Should Zoe use the mean or median to estimate the average temperature inside the terrarium that month?

Key points

- Zoe measures the temperature inside a terrarium every day for a month.
- After gathering the data, she can see one outlier caused by the wrong setting on her thermometer.
- Should Zoe use the mean or median to estimate the average temperature inside the terrarium that month?

Explanation

As the outlier is caused by a wrong setting on her thermometer, Zoe would like the outlier to have the least effect on the average temperature.

Therefore, Zoe should use the median because outliers affect the mean of numerical data, and have little to no effect on the median.

Answer

Zoe should use the median to estimate the average temperature inside the terrarium that month.

11. Cindy asks 10 of her classmates how many times they have been overseas. When she organised the responses, the minimum value was 0, and the range was 7. Cindy then records one more friend's response and the range of her data increases by 5. How many times has the last friend Cindy surveyed been overseas?

Key points

- Cindy asks 10 of her classmates how many times they have been overseas.
- When she organised the responses, the minimum value was 0, and the range was 7.
- Cindy then records one more friend's response and the range of her data increases by 5.
- How many times has the last friend Cindy surveyed been overseas?

Explanation

As the range increased, the data from the last friend Cindy surveyed would be the maximum data.

New range = maximum value − minimum value

$7 + 5 =$ maximum value $- 0$

$12 =$ maximum value

Therefore, the last friend that Cindy surveyed has been overseas 12 times.

Answer

The last friend that Cindy surveyed has been overseas 12 times.

12. Jeffrey wants to become a court typist. He needs to be able to type an average of 200 spoken words per minute in order to qualify. It took Jeffrey 1.5 hours to type up a speech 19 000 words long. Does he qualify for the job?

Key points

- Jeffrey needs to be able to type an average of 200 spoken words per minute in order to qualify.
- It took Jeffrey 1.5 hours to type up a speech 19 000 words long.
- Does he qualify for the job?

Explanation

There are 60 minutes in an hour. 1.5 hours $= 1.5 \times 60 = 90$ minutes.

Mean number of words Jeffrey types per minute

$$= \frac{\text{Total number of words Jeffrey types over 90 minutes}}{90}$$

$$= \frac{19\,000}{90}$$

$\simeq 211$ words per minute

$211 > 200$. Therefore, Jeffrey qualifies for the job.

Answer

Jeffrey qualifies for the job.

13. Carol is studying for an exam every day of the school week Monday until Friday. She studied for 4 hours on Monday, 2 hours on Wednesday, 3 hours on Thursday, and 2 hours on Friday. How long did Carol study for on Tuesday if the mean number of hours she studied per day was 3 that school week?

Key points

- Carol is studying for an exam every day of the school week Monday until Friday.
- She studied for 4 hours on Monday, 2 hours on Wednesday, 3 hours on Thursday, and 2 hours on Friday.
- The mean number of hours she studied per day was 3 that school week.
- How long did Carol study for on Tuesday?

Explanation

Mean number of hours Carol studied per day

$$= \frac{\text{Total number of hours studied over the week}}{5 \text{ days}}$$

$$3 = \frac{4 + \text{number of hours studied on Tuesday} + 2 + 3 + 2}{5}$$

$$3 = \frac{11 + \text{number of hours studied on Tuesday}}{5}$$

$15 = 11 +$ number of hours studied on Tuesday

Number of hours studied on Tuesday $= 4$

Answer

Carol studied for 4 hours on Tuesday.

14. 5 friends are comparing their scores for a test. The median and mode of their scores is 75 out of 100. If the range of their scores is 50, then what is the lowest possible mark in the data set?

Key points

- 5 friends are comparing their scores for a test.
- The median and mode of their scores is 75 out of 100.
- The range of their scores is 50.
- What is the lowest possible mark in the data set?

Explanation

The lowest mark possible in the data set would be (the highest mark in the data set −50).

For the mark to be the lowest, the highest mark in the data set should be minimised.

Given the median and the mode of the data set is 75, the lowest highest possible mark would be 75.

One possible dataset could be: 25, 75, 75, 75, 75.

Note: There are other possible datasets. Datasets that satisfy: the lowest being 25, the highest being 75, 75 appearing more than once will be correct.

Answer

The lowest possible mark in the dataset is 25 out of 100.

Reasoning

15. a. 46

b. 14, 17 and 18

c. Mean = 20.1; median = 18; range = 32.

d. Mean = 18.7; median = 18; range = 13.

The mean decreased from 20.1 to 18.7.

The median did not change from 18.

The range decreased from 32 to 13.

e. Suggested option 1: The most important rule is that players must not participate in violent conduct towards the opponents.

Suggested option 2: The most important rule is that players must obey the instructions of the sports official.

Note: There are other possible options.

16. a. Mean = 5.3; median = 4.5; mode = 2 and 4; range = 8.

b. Mean = 21.1; median = 23.5 ; mode = 10; range = 48.

c. The data set in part **a** had more consistent results. The bigger the range is, the less reliable small data sets are.

Extra spicy

17. 25 **18.** B **19.** D **20.** 2, 4, 4, 4, 6

Remember this?

21. D **22.** E **23.** C

Chapter 8 extended application

1. a. $\frac{a+b+c+d+e}{5} = 17; a+b+c+d+e = 85$

b. $\frac{a+b+c}{3} = 15\frac{1}{3}; a+b+c = 46$

c. $\frac{c+d+e}{3} = 18\frac{1}{3}; c+d+e = 55$

d. Conrad's age is 16 years old.

e. Andy and Barbara are 15 years old, Donna is 18 years old, Ernie is 21 years old.

f. Suggested option 1: How to split household chores so that the house is tidy and clean.

Suggested option 2: Be considerate to other people, for example, do not spend too much time in the bathroom.

Note: There are other possible options.

2. a. The total annual income of Average Town is $170 000 000.

b. The mean annual income of Average Town after Milo moves away is $85 003.

c. The mean annual income of Sleepy Village after Milo moves there is $40 159.

d. The total annual income of Sleepy Village needs to increase by $11 250 000.

e. The annual income of Average Town after the move is $80 000.

f. Suggested option 1: Moving to the countryside could provide a quieter life if this is of the person's preference.

Suggested option 2: Moving to the city provides more career opportunities.

Note: There are other possible options.

3. a. 5854 computer games were sold in 2022.

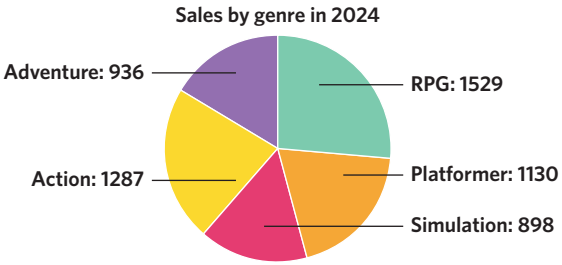
b. 950 adventure games were sold in 2023.

c. Game sales decreases by 0.9% from 2022 to 2023.

d.

Game genre	Number of sales in 2022	Number of sales in 2023	Sales increase/ decrease (%)
Action	1508	1393	−7.626%
Adventure	964	950	−1.452%
Platformer	890	1003	12.697%
RPG	1472	1500	1.902%
Simulation	1020	957	−6.176%

e.



f. Suggested option 1: Support physical stores that are slowly going obsolete.

Suggested option 2: For collection purposes.

Note: There are other possible options.

Chapter 8 review

Multiple choice

1. B **2.** D **3.** C **4.** D **5.** E

Fluency

6. a. i. Population
ii. Census

b. i. Sample
ii. Survey

c. i. Sample
ii. Survey

d. i. Population
ii. Census

7. **a.** Primary data **b.** Primary data
c. Secondary data **d.** Primary data

8. **a.** Method 2 **b.** Method 1
c. Method 2 **d.** Method 2

9. **a.** The sample is restricted to people aged 16 years only. It does not include young people of other ages who are members of the population, introducing bias.
b. The sample is too small to be representative of the population, introducing bias.
c. The sample is restricted to people at a particular desk only. It does not include people at other desks in the office who are members of the population, introducing bias.
d. The sample is restricted to people living on a particular street only. It does not include people living on other streets in Victoria who are members of the population, introducing bias.

10. **a.** Ice-cream **b.** Caramel
c. 60 **d.** $\frac{1}{5}$

11. **a.** 25 animals **b.** 16 days
c. 57 animals **d.** $\frac{5}{31}$

12. **a.** 25
b. The intervals are 10–15, 16–21, 22–27, 28–33, and 34–39 people.
c.

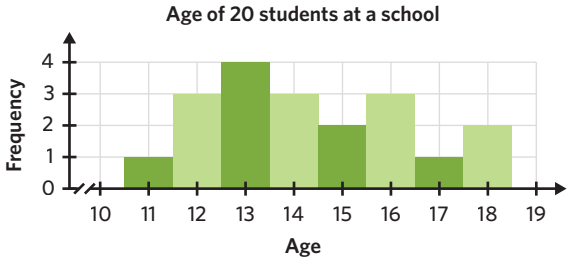
Number of people	Tally	Frequency
10–15	I	1
16–21	𝍸 IIII	9
22–27	𝍸 I	6
28–33	III	3
34–39	I	1

d. 16–21 people

13. **a.** 7
b.

Age	Tally	Frequency
11	II	2
12	III	3
13	IIII	4
14	III	3
15	II	2
16	III	3
17	I	1
18	II	2

c.



d. Symmetric

14. **a.** Mean = 15.1; median = 14; mode = 10, 12, 19; range = 13
b. Mean = 27.7; median = 25; mode = none; range = 26
c. Mean = 54.2; median = 53.5; mode = 42; range = 44
d. Mean = 89; median = 89; mode = 90; range = 23

15. **a.** 3
b. Mean = 14.2; median = 14.5; mode = 17, 18; range = 19
c. Mean = 14.5; median = 15; mode = 17, 18; range = 13
d. The mean, median, and range were affected by the outlier. The mode was not affected by the outlier.

Problem solving

16. Jessica wants to investigate how long her classmates listen to music everyday. State the variable of this investigation, with appropriate units, and determine its type.

Key points

- Jessica wants to investigate how long her classmates listen to music everyday.
- State the variable of this investigation, with appropriate units, and determine its type.

Explanation

Determine the variable, or the quantity or quality is being measured, counted, or observed.

The variable is the amount of time Jessica's classmates listen to music everyday. Appropriate units are time in minutes or time in hours.

Determine whether the variable is numerical or categorical first. Then, if the variable is numerical, determine whether it is counted or measured.

Time is a numerical continuous variable as it is a number and is measured.

Answer

The variable is the amount of time Jessica's classmates listen to music everyday. An appropriate unit is time in minutes. The type of variable is numerical continuous.

17. Lauren is a food critic and reviews food items at restaurants. She reviewed every second item on the menu at 12 restaurants. If there is an average of 36 food items on each menu, find the number of food items in Lauren's testing sample and comment on whether it is representative of the population.

Key points

- Lauren reviewed every second item on the menu at 12 restaurants.
- There are an average of 36 food items on each menu.
- Find the number of food items in Lauren's testing sample and comment on whether it is representative of the population.

Explanation

Lauren reviews $(36 \div 2) \times 12 = 216$ food items. The total population is 432 food items, so an appropriate sample size is ≥ 30.

$216 \geq 30$, so the sample size is appropriate and the sample is representative of the population.

Answer

The number of food items in Lauren's testing sample is 216. This is representative of the population of food items at the restaurants.

18. Ms Suzanna is a Year 8 English teacher for 4 classes, who has 180 grades to analyse for an essay assessment. If each student received a grade from A to E for the assessment, suggest a suitable data display that she can use in order to compare the proportions of grades.

Key points

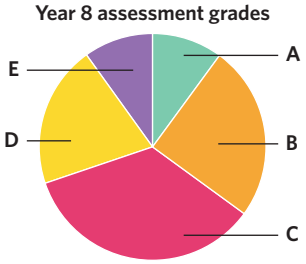
- Ms Suzanna has 180 grades to analyse for an essay assessment.
- Each student received a grade from A to E for the assessment.
- Suggest a suitable data display that she can use in order to compare the proportions of grades.

Explanation

Data displays include pie charts, dot plots, and stem-and-leaf plots.

The data that Ms Suzanna is visualising is categorical. The categories are A, B, C, D, and E. Therefore, Ms Suzanna should use a pie chart because it divides categories into sectors that represent proportions of each category.

One possible pie chart for Ms Suzanna is shown below.



Answer

Ms Suzanna could use a pie chart to compare the proportions of grades.

19. Ryan is analysing his business' profits for the month of March. He collects data in Australian dollars everyday and records it in a frequency table. Should Ryan choose inclusive or exclusive class intervals to group the data for a histogram? Explain why.

Key points

- Ryan collects data in Australian dollars everyday.
- Should Ryan choose inclusive or exclusive class intervals to group the data for a histogram? Explain why.

Explanation

Dollars are a continuous variable. Continuous data should be grouped in exclusive class intervals, where the maximum values of each interval are included in the following interval.

Answer

Ryan should choose exclusive class intervals to group the data for a histogram.

20. Jimmy is studying for a test every day of the school week, Monday until Friday. He studied for three hours on Monday, four hours on Tuesday, four hours on Wednesday and one hour on Friday. How long did Jimmy study for on Thursday night if the mean number of hours he studied in the week was three hours?

Key points

- Jimmy is studying for a test every day of the school week, Monday until Friday.
- He studied for three hours on Monday, four hours on Tuesday, four hours on Wednesday and one hour on Friday.
- The mean number of hours he studied in that week was three hours.
- How long did Jimmy study for on Thursday night?

Explanation

$$\text{Mean number of hours Jimmy studied per day} = \frac{\text{Total number of hours studied over the week}}{5 \text{ days}}$$

$$3 = \frac{3 + 4 + 4 + \text{number of hours studied on Thursday} + 1}{5}$$

$$3 = \frac{\text{number of hours studied on Thursday} + 12}{5}$$

$$15 = \text{number of hours studied on Thursday} + 12$$

$$\text{Number of hours studied on Thursday} = 15 - 12$$

$$= 3$$

Answer

Jimmy studied for 3 hours on Thursday.

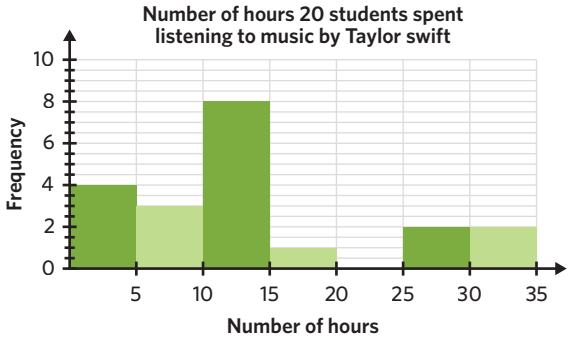
Reasoning

21. a. Primary data

b.

Number of hours	Tally	Frequency
0–<5	IIII	4
5–<10	III	3
10–<15	~~IIII~~ III	8
15–<20	I	1
20–<25		0
25–<30	II	2
30–<35	II	2

c.



d. Mean = 12.8; median = 11; range = 34

e. Suggested option 1: An advantage of having a pre-made playlist is that Tina does not have to pay money to hire a band.

Suggested option 2: A disadvantage of having a pre-made playlist is that live music is a more immersive musical experience that students may find cool and unique.

Note: There are other possible options.

22. a. The type of variable is categorical. A dot plot is a suitable data display.

b. The type of variable is numerical. A stem-and-leaf plot is a suitable data display.

c. For part **a**, the variable is categorical, so it is not possible to use a stem-and-leaf plot. A stem-and-leaf plot can only be used for data that can be counted or measured with numbers.

For part **b**, the variable is numerical. For numerical data that is measured, such as time, it may be impractical to use a dot plot or pie chart, especially if each person has a different time result.

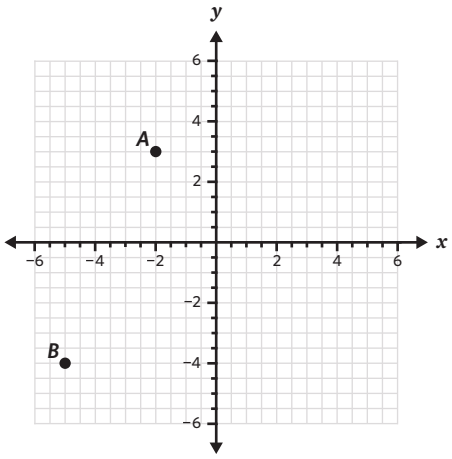
9A The Cartesian plane

Student practice

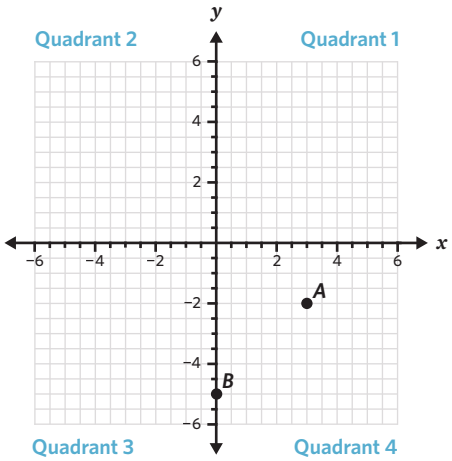
Worked example 1

a. (3,5) **b.** (5,0)

Worked example 2



Worked example 3



Point A is found in the fourth quadrant.

Point B is found on the y-axis.

Understanding worksheet

1. **a.** (2,2) **b.** (4,−4)
c. (−4,0) **d.** (−3,−3)

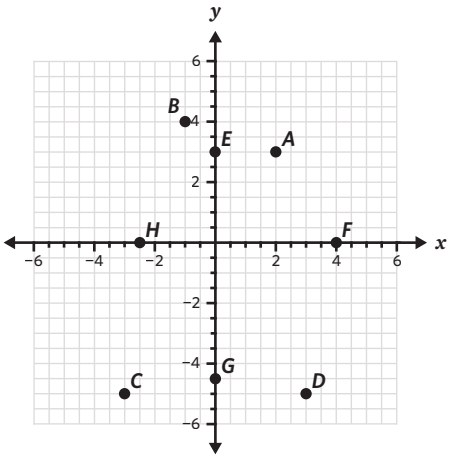
2. **a.** Quadrant 2 **b.** Quadrant 1
c. y-axis **d.** x-axis

3. Cartesian; (x,y); horizontal; four

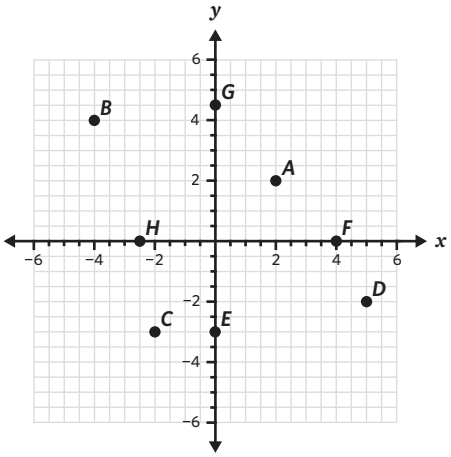
Fluency

4. **a.** $A(3,2)$ **b.** $B(-3,4)$
c. $C(0,4)$ **d.** $D(-3,0)$
e. $E(-3,-5)$ **f.** $F(4,-5)$
g. $G(2.5,-4)$ **h.** $H(5.5,0)$

5.



6.



a. Quadrant 1 **b.** Quadrant 2
c. Quadrant 3 **d.** Quadrant 4
e. y-axis **f.** x-axis
g. y-axis **h.** x-axis

7. **a.** Quadrant 1 **b.** Quadrant 1
c. Quadrant 2 **d.** Quadrant 2
e. Quadrant 3 **f.** Quadrant 4
g. y-axis **h.** x-axis

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Nora lives on 3rd street and 5th road, represented as (3,5). Her grandmother lives at 5th street and 9th road, represented as (5,9). Plot the coordinates on a Cartesian plane.

Key points

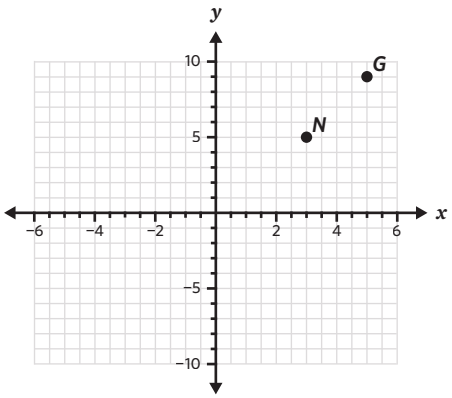
- Nora lives at (3,5).
- Nora's grandmother lives at (5,9).
- Plot the coordinates on a Cartesian plane.

Explanation

Nora lives at (3,5), which is 3 units right and 5 units up from the origin.

Nora's grandmother lives at (5,9), which is 5 units right and 9 units up from the origin.

Answer



11. Stanley is keeping track of the amount of push ups and sit ups he does per day by using a Cartesian plane. He uses the x-axis for the number of push ups and the y-axis for the number of sit ups. Plot the following: Monday (10,32), Tuesday (15,25), Wednesday (5,63) and Thursday (32,20) to then determine what day Stanley did the most push ups?

Key points

- Stanley uses the x-axis for the number of push ups and the y-axis for the number of sit ups.
- He recorded: Monday (10,32), Tuesday (15,25), Wednesday (5,63) and Thursday (32,20).
- Plot these coordinates, and then determine the day that Stanley did the most push ups.

Explanation

Plot the coordinates on the Cartesian plane.

Monday is 10 units right and 32 units up.

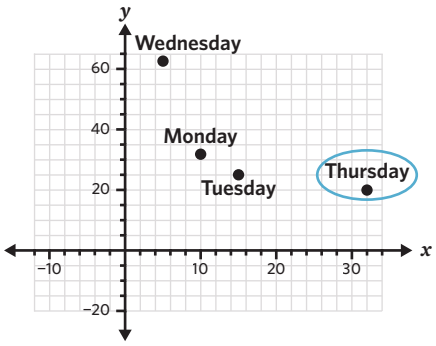
Tuesday is 15 units right and 25 units up.

Wednesday is 5 units right and 63 units up.

Thursday is 32 units right and 20 units up.

The number of push ups is represented along the x-axis.

The coordinate with the highest x-value (i.e. the coordinates furthest to the right) is (32,20), which is Thursday.



Answer

Stanley did the most push ups on Thursday.

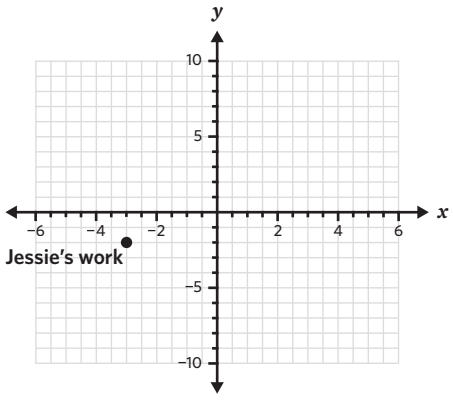
12. On a Cartesian plane, Jessie's work is located at $(-3,-2)$. Jessie lives four blocks north and five blocks east. State the ordered coordinates for Jessie's home.

Key points

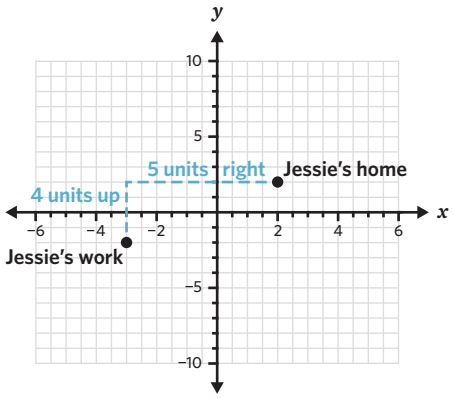
- Jessie's work is located at $(-3,-2)$.
- Jessie lives four blocks north and five blocks east.
- State the ordered coordinates for Jessie's home.

Explanation

Plot $(-3,-2)$ on the Cartesian plane. It is three units down and 2 units left of the origin.



To find Jessie's home, determine the coordinates that are four units up and five units right.



So Jessie's home is located at (2,2).

Alternatively, increase the x-coordinate by 5 and the y-coordinate by 4.

$(-3 + 5, -2 + 4) = (2,2)$

Answer

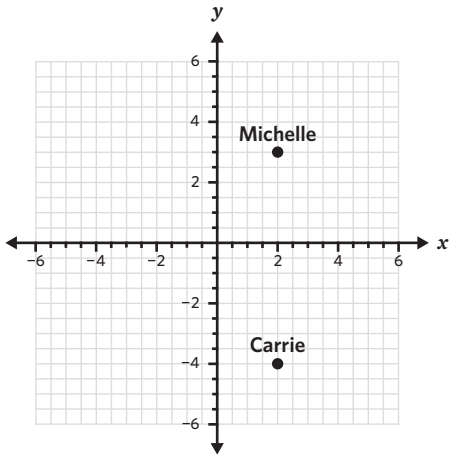
Jessie's home is located at (2,2).

13. The Cartesian plane below displays information about where seven family members live. Show that the distance between Michelle and Carrie is equal to seven units.

Key points

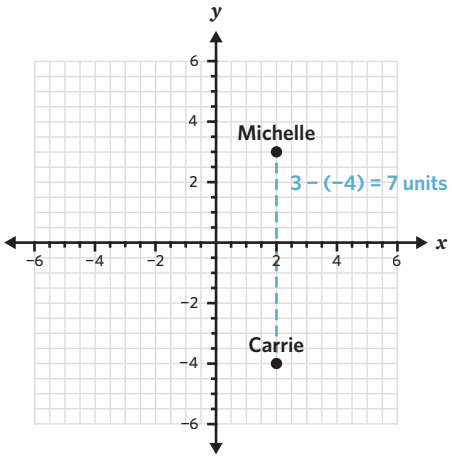
- The Cartesian plane shows where each family member lives.
- Show that the distance between Michelle and Carrie is equal to seven units.

Explanation



Michelle is located at (2,3) and Carrie is located at (2,−4).

Both family members' locations have the same x-coordinate, so the distance between them will be the difference between their y-coordinates.



$3 - (-4) = 7$

Answer

The distance between Michelle and Carrie is the difference between their y-coordinates, 7 units.

14. A particular shopping centre displays the following information on a Cartesian plane. Coles is located at the origin, Bank of Melbourne is located at $(-3,-4)$, the food court has the same x-coordinate as Bank of Melbourne and a y-coordinate of 1. The bathrooms are one unit left and three units up from the food court. The bathrooms are located in which quadrant?

Key points

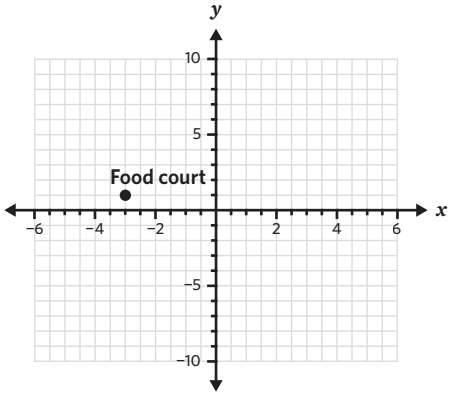
- Bank of Melbourne is located at $(-3,-4)$.
- The food court has the same x-coordinate as Bank of Melbourne and a y-coordinate of 1.
- The bathrooms are one unit left and three units up from the food court.
- The bathrooms are located in which quadrant?

Explanation

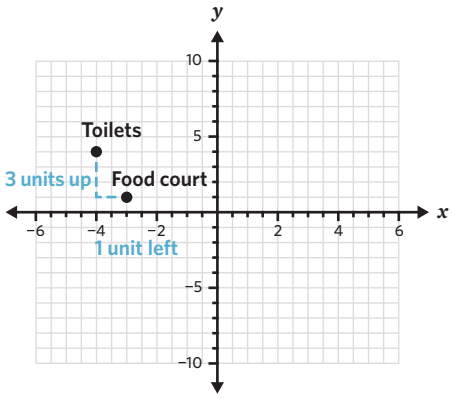
The food court has the same x-coordinate as Bank of Melbourne and a y-coordinate of 1.

The x-coordinate of the Bank of Melbourne is -3, so the food court is at $(-3,1)$.

Plot this on a Cartesian plane. $(-3,1)$ is three units left and one unit up from the origin.



To find the bathrooms, determine the coordinates that are one unit left and three units up from the food court.



So the bathrooms are located at $(-4,4)$.

Alternatively, decrease the x-coordinate by 1 and increase the y-coordinate by 3.

$(-3 - 1,1 + 3) = (-4,4)$

$(-4,4)$ is in quadrant 2.

Answer

The bathrooms are in quadrant 2.

Reasoning

15. a. $(-3,2)$ **b.** Quadrant 2

c. $(-10,0)$ **d.** $(-2.5,0)$

e. Suggested option 1: The building where classes are held could be situated halfway between the houses that are furthest away from each other.

Suggested option 2: The school could allow more time for these students to get to class.

Note: There are other possible options.

16. a. Quadrant 4 **b.** Quadrant 2

c. Since the positive and negative signs of the x- and y-coordinates are different for these two points, they are not found in the same quadrant.

Extra spicy

17. $(-1,0)$ $(0,0)$ $(1,0)$ $(2,0)$ **18.** $(1,-3.5)$

19. C **20.** B

Remember this?

21. C **22.** B **23.** C

9B Tables of values

Student practice

Worked example 1

a.

x	-3	-2	-1	0	1
y	-10	-6	-2	2	6
Coordinate	$(-3,-10)$	$(-2,-6)$	$(-1,-2)$	$(0,2)$	$(1,6)$

b.

x	-3	-2	-1	0	1
y	8	6	4	2	0
Coordinate	$(-3,8)$	$(-2,6)$	$(-1,4)$	$(0,2)$	$(1,0)$

Worked example 2

a.

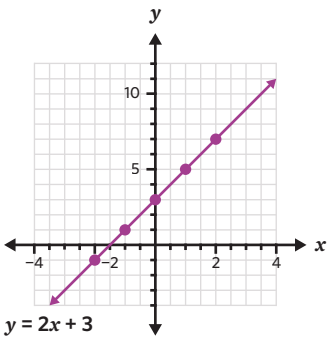
x	0	1	2	3
y	2	5	8	11

b.

x	−1	0	1	2
y	5	3	1	−1

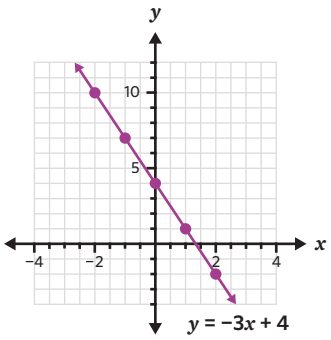
Worked example 3

a.



b.

x	−2	−1	0	1	2
y	10	7	4	1	−2



Understanding worksheet

1. a.

x	1	2	3	4
y	2	5	8	11
Coordinate	(1,2)	(2,5)	(3,8)	(4,11)

b.

x	0	1	2	3
y	1	1.5	2	2.5
Coordinate	(0,1)	(1,1.5)	(2,2)	(3,2.5)

c.

x	−1	0	1	2
y	7	5	3	1
Coordinate	(−1,7)	(0,5)	(1,3)	(2,1)

d.

x	−2	−1	0	1
y	14	10	6	2
Coordinate	(−2,14)	(−1,10)	(0,6)	(1,2)

2. **a.** 2 **b.** 2 **c.** −1 **d.** −1

3. Cartesian; x-coordinates; y-coordinates; smallest

Fluency

4. a.

x	−3	−2	−1	0	1
y	−2	−1	0	1	2
Coordinate	(−3,−2)	(−2,−1)	(−1,0)	(0,1)	(1,2)

b.

x	−2	−1	0	1	2
y	−6	−4	−2	0	2
Coordinate	(−2,−6)	(−1,−4)	(0,−2)	(1,0)	(2,2)

c.

x	−1	0	2	4	6
y	1	0	−2	−4	−6
Coordinate	(−1,1)	(0,0)	(2,−2)	(4,−4)	(6,−6)

d.

x	−2	0	2	4	6
y	4	−2	−8	−12	−16
Coordinate	(−2,4)	(0,−2)	(2,−8)	(4,−12)	(6,−16)

5. a.

x	−2	−1	0	1	2
y	−6	−3	0	3	6

b.

x	−2	−1	0	1	2
y	−10	−5	0	5	10

c.

x	−2	−1	0	1	2
y	−2	0	2	4	6

d.

x	−2	−1	0	1	2
y	−5	−2	1	4	7

e.

x	−2	−1	0	1	2
y	1	2	3	4	5

f.

x	−2	−1	0	1	2
y	−1	1	3	5	7

g.

x	−2	−1	0	1	2
y	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

h.

x	−2	−1	0	1	2
y	0.5	1.25	2	2.75	3.5

6. a.

x	−2	−1	0	1	2
y	8	4	0	−4	−8

b.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

c.

x	−2	−1	0	1	2
y	9	6	3	0	−3

d.

x	−2	−1	0	1	2
y	0	−2	−4	−6	−8

e.

x	−2	−1	0	1	2
y	2	1	0	−1	−2

f.

x	−2	−1	0	1	2
y	0	−1	−2	−3	−4

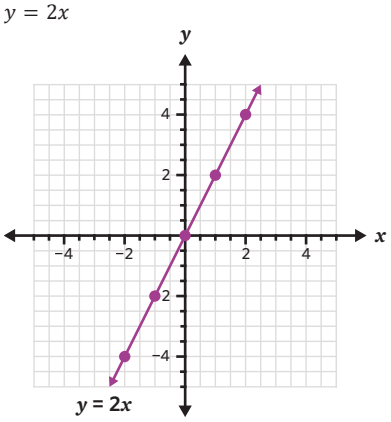
g.

x	−2	−1	0	1	2
y	$\frac{5}{2}$	$\frac{9}{4}$	2	$\frac{7}{4}$	$\frac{3}{2}$

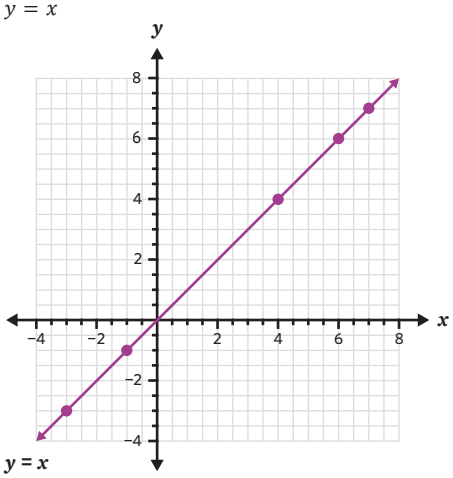
h.

x	−2	−1	0	1	2
y	4	3.5	3	2.5	2

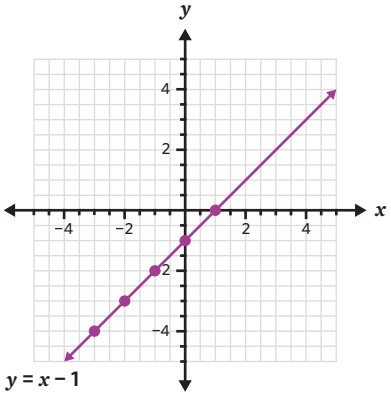
7. a. $y = 2x$



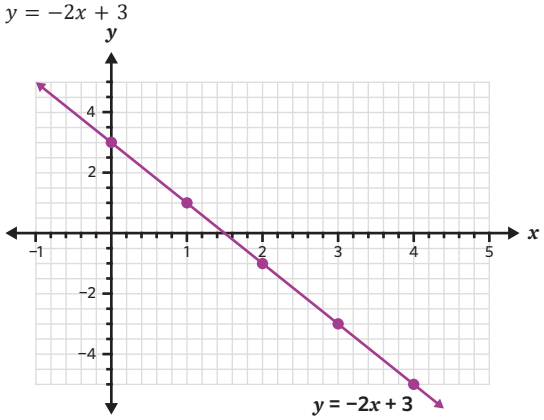
b. $y = x$



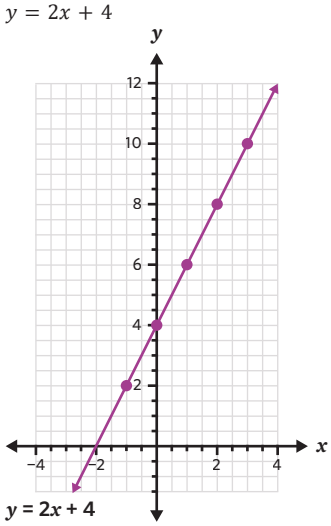
c. $y = x - 1$



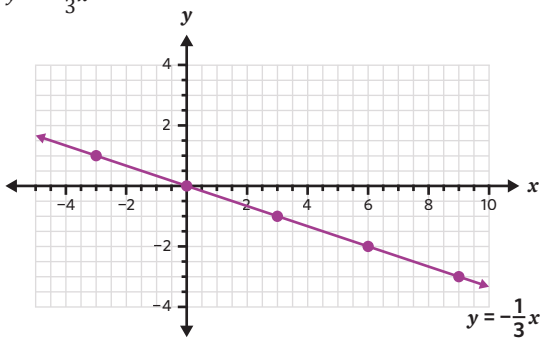
d. $y = -2x + 3$



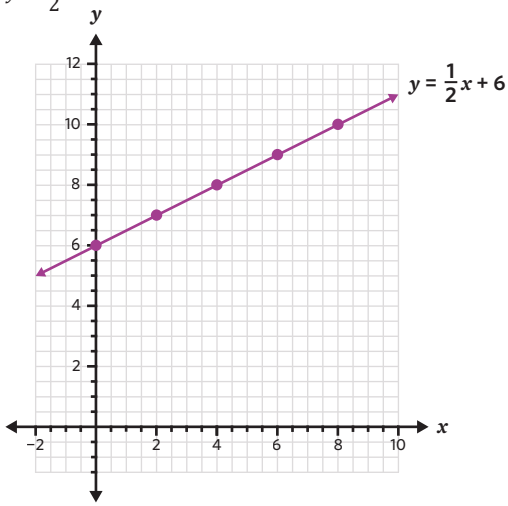
e. $y = 2x + 4$



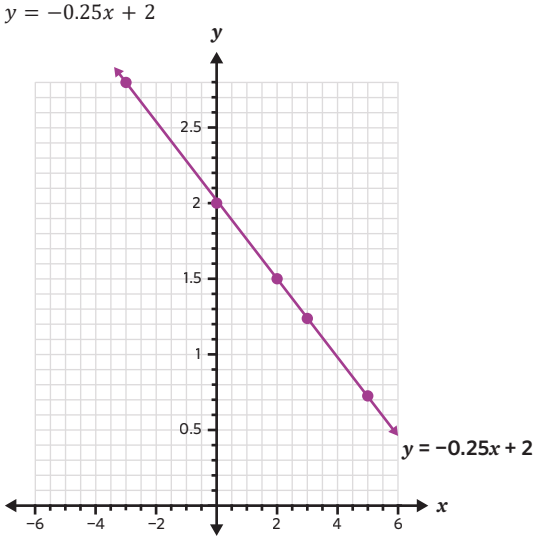
f. $y = -\frac{1}{3}x$



g. $y = \frac{1}{2}x + 6$

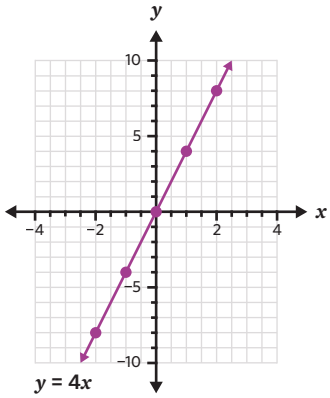


h. $y = -0.25x + 2$



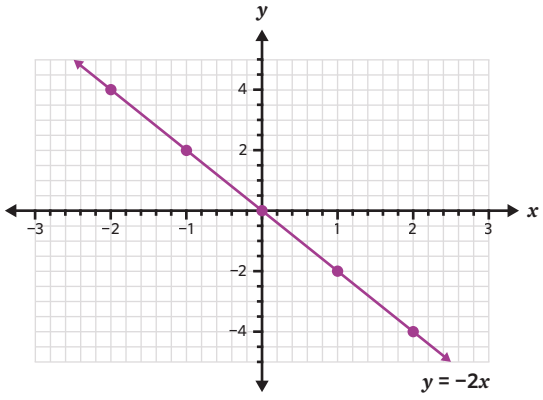
8. a.

x	−2	−1	0	1	2
y	−8	−4	0	4	8



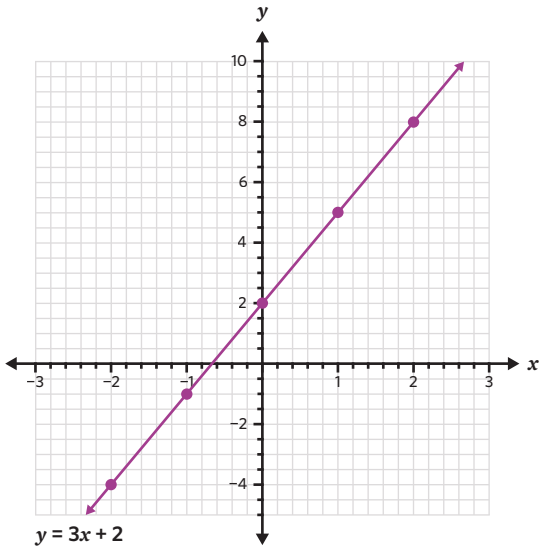
b.

x	−2	−1	0	1	2
y	4	2	0	−2	−4



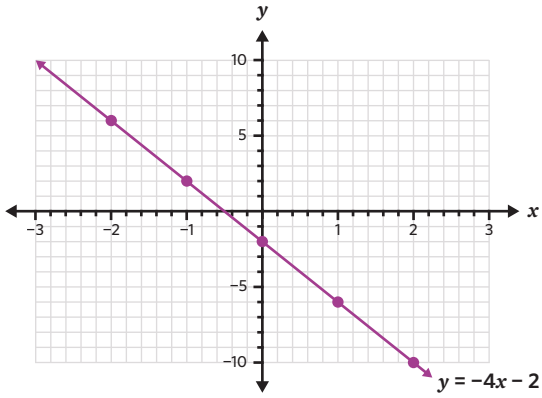
c.

x	−2	−1	0	1	2
y	−4	−1	2	5	8



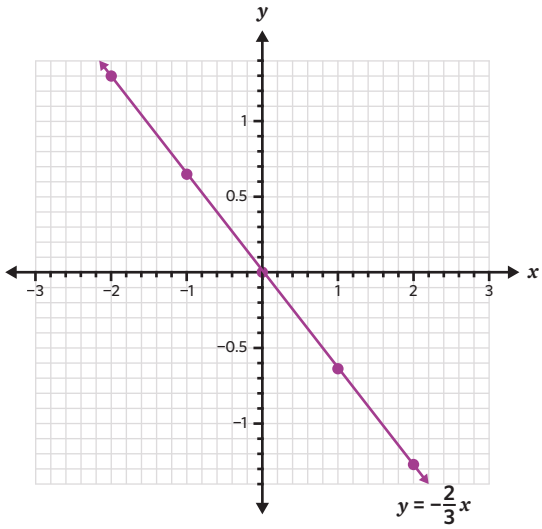
d.

x	−2	−1	0	1	2
y	6	2	−2	−6	−10



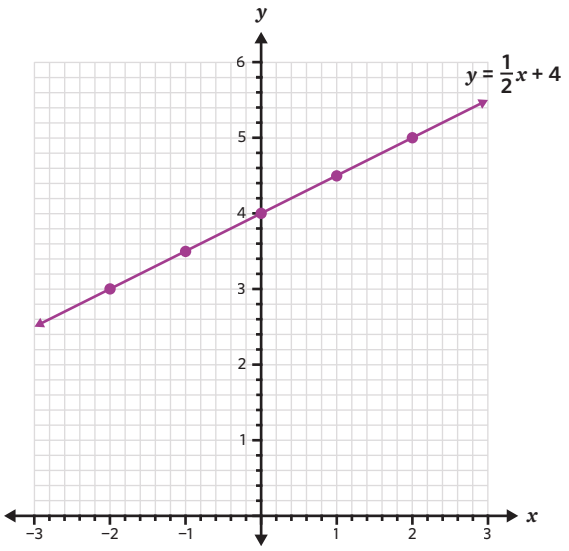
e.

x	−2	−1	0	1	2
y	$\frac{4}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{4}{3}$



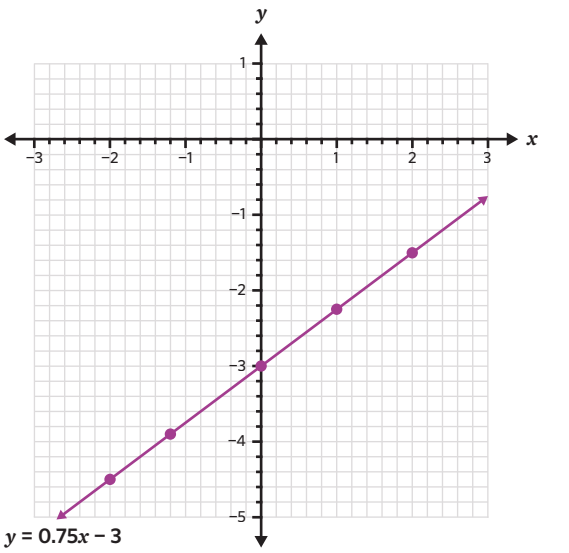
f.

x	−2	−1	0	1	2
y	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5



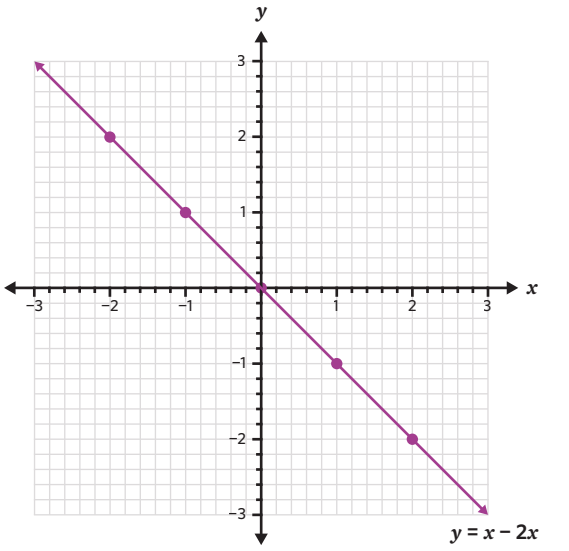
g.

x	−2	−1	0	1	2
y	−4.5	−3.75	−3	−2.25	−1.5



h.

x	−2	−1	0	1	2
y	2	1	0	−1	−2



9. E

Spot the mistake

10. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. Lester starts the Bolt for Gold running competition that his high school sports department is currently enrolled in. The distance that he has achieved over the first five days generates a linear pattern. State the coordinates of Lester's first five consecutive runs.

Key points

- Lester starts the Bolt for Gold running competition that his high school sports department is currently enrolled in.
- The distance that he has achieved over the first five days generates a linear pattern.
- State the coordinates of Lester's first five consecutive runs.

Explanation

The day (d) and distance (k) are the x and y values respectively, and so the coordinates are:

Day(d)	1	2	3	4	5
Distance, km (k)	3	3.25	4.5	5.75	7
Coordinate	(1,3)	(2,3.25)	(3,4.5)	(4,5.75)	(5,7)

Answer

(1,3); (2,3.25); (3,4.5); (4,5.75); (5,7)

12. Clinton loves to bake cookies. The time (t) it takes him to bake (c) number of cookies is found by the rule $t = 2c + 15$. Generate a table of values that displays how long it will take Clinton to bake 1, 4, 6, 10 and 20 cookies.

Key points

- Clinton loves to bake cookies.
- The time (t) it takes him to bake (c) number of cookies is found by the rule $t = 2c + 15$.
- Generate a table of values that displays how long it will take Clinton to bake 1, 4, 6, 10 and 20 cookies.

Explanation

We can construct a table of values and substitute 1, 4, 6, 10 and 20 as c to find how much time it takes:

c	1	4	6
t	2(1) + 15 = 17	2(4) + 15 = 23	2(6) + 15 = 27

c	10	20
t	2(10) + 15 = 35	2(20) + 15 = 55

Answer

c	1	4	6	10	20
t	17	23	27	35	55

13. For Christmas, Edrolo purchases travel vouchers for their employees. Each employee receives a $150 voucher, plus an additional one off $30 transaction fee. The rule that displays the cost is represented as $\$c = 150v + 30$, where v represents the number of vouchers and c represents the cost in dollars ($). Generate a table of values that displays how much it will cost Edrolo to purchase 50, 70, 90, 120 and 200 travel vouchers.

Key points

- For Christmas, Edrolo purchases travel vouchers for their employees.
- Each employee receives a $150 voucher, plus an additional one off $30 transaction fee.
- The rule that displays the cost is represented as $\$c = 150v + 30$, where v represents the number of vouchers and c represents the cost in dollars ($).
- Generate a table of values that displays how much it will cost Edrolo to purchase 50, 70, 90, 120 and 200 travel vouchers.

Explanation

We can construct a table of values and substitute 50, 70, 90, 120 and 200 as v to find each cost:

v	50	70
c ($)	$150(50) + 30 = 7530$	$150(70) + 30 = 10\,530$

v	90	120
c ($)	$150(90) + 30 = 13\,530$	$150(120) + 30 = 18\,030$

v	200
c ($)	$150(200) + 30 = 30\,030$

Answer

v	50	70	90	120	200
c ($)	7530	10 530	13 530	18 030	30 030

14. Steve loves ordering pizza from Cagney's restaurant. All medium sized pizza costs $14. Each additional topping costs $2. Generate a table of values and a graph to display this scenario. Toppings on the pizza is represented as (x) and is between 0 and 6, and total cost is represented as (y).

Key points

- Steve loves ordering pizza from Cagney's restaurant.
- All medium sized pizza costs $14.
- Each additional topping costs $2.
- Toppings on the pizza is represented as (x) and is between 0 and 6, and total cost is represented as (y).
- Generate a table of values and a graph to display this scenario.

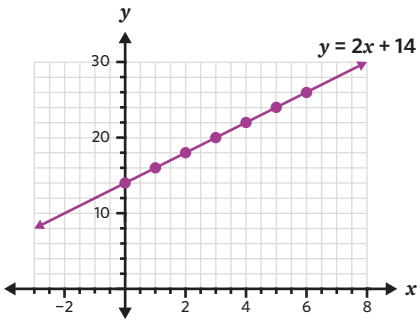
Explanation

As each pizza costs $14 but each additional topping x costs $2, this can be represented by the rule: $y = 2x + 14$

We can generate a table of values with this rule by substituting values from 0 to 6 as x:

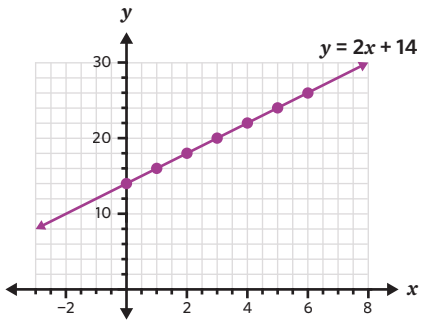
x	0	1	2	3	4	5	6
y	14	16	18	20	22	24	26

We can also construct a graph:



Answer

x	0	1	2	3	4	5	6
y	14	16	18	20	22	24	26



15. Daniel has been saving to purchase his first car. After one year he has saved $2000, after three years he has $3500. Generate a table of values displaying Daniel's savings and construct a graph using this information.

Key points

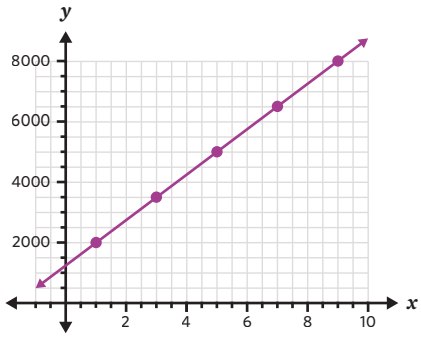
- Daniel has been saving to purchase his first car.
- After one year he has saved $2000.
- After three years he has saved $3500.
- Generate a table of values displaying Daniel's savings and construct a graph using this information.

Explanation

We can generate a table of values based on the pattern of Daniel's savings, where he saves an additional $1500 every 2 years.
Let s be savings and t be the number of years.

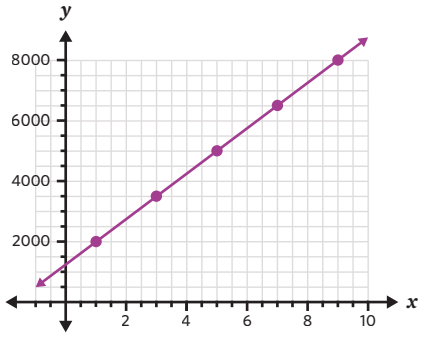
Number of years (t)	1	3	5	7	9
Savings (s)	2000	3500	5000	6500	8000

We can also construct a graph by plotting the points from the table on a Cartesian plane:



Answer

Number of years (t)	1	3	5	7	9
Savings (s)	2000	3500	5000	6500	8000



Reasoning

16. a.

Burgers (b)	15	18	20	25	40
Cost $(c)	39.25	45.1	49	58.75	88

Burgers (b)	50	60
Cost $(c)	107.5	127

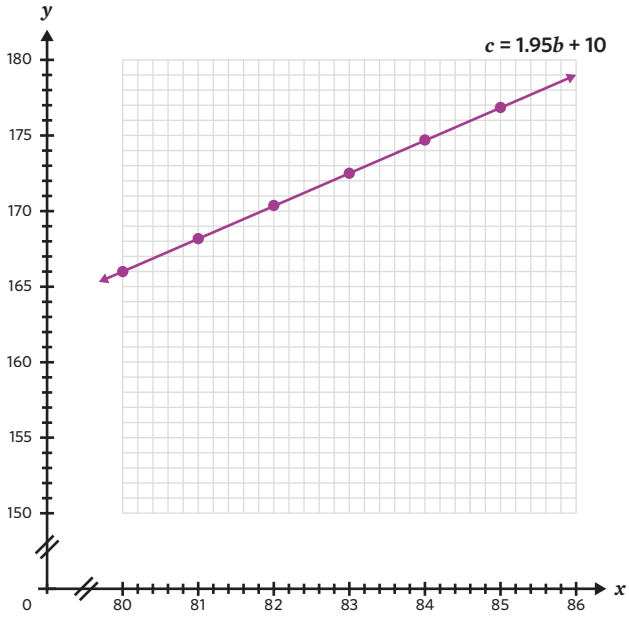
b. (15,39.25), (18,45.1), (20,49), (25,58.75), (40,88), (50,107.5), (60,127)

c. $1.95(100) + 10 = \$205$

d.

Burgers (b)	80	81	82	83
Cost $(c)	166	167.95	169.9	171.85

Burgers (b)	84	85
Cost $(c)	173.8	175.75



e. Suggested option 1: Matt could have barbeque sausages.

Suggested option 2: Matt could have barbeque chicken skewers.

Note: There are other possible options.

17. a.

x	−3	−2	−1	0	1	2	3
y	−8	−6	−4	−2	0	2	4

b.

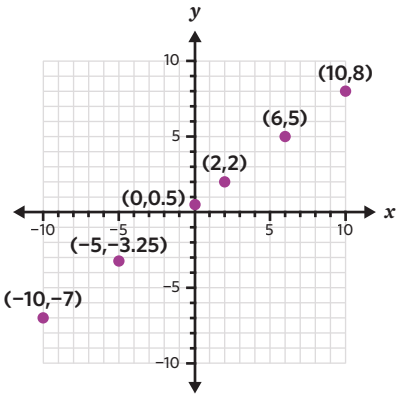
x	−3	−2	−1	0	1	2	3
y	4	2	0	−2	−4	−6	−8

c. The x-coordinates for parts **a** and **b** are the same, but the y-coordinates are different. The y-coordinates for part **b** are flipped and backwards from those in part **a**. This is due to the coefficient values being different, where the coefficient is negative in part **b** but positive in part **a**.

Extra spicy

18. Both $(-3,11)$ and $(2,-14)$ are coordinates for $y = -5x - 4$.

19.



20. A **21.** B

Remember this?

22. C **23.** A **24.** C

9C Linear rules

Student practice

Worked example 1

a. $y = 3x + 2$ **b.** $y = -2x + 3$

Worked example 2

a. $y = 2x - 2$ **b.** $y = -3x + 4$

Worked example 3

a. $y = 2x - 4$ **b.** $y = -2x + 2.25$

Understanding worksheet

1. a. 2 **b.** 5 **c.** −0.5 **d.** −0.8

2. a. +3 **b.** −3 **c.** −0.5 **d.** 2

3. equation; relationship; coordinates; coefficient

Fluency

4. a. $y = 4x + 3$ **b.** $y = 3x + 2$
c. $y = 4x$ **d.** $y = 3x + 14$
e. $y = x - 9$ **f.** $y = 2x - 6$
g. $y = 1.5x + 7$ **h.** $y = 4.5x + 18$

5. a. $y = 2x + 5$ **b.** $y = 7x + 3$
c. $y = 2$ **d.** $y = 2x - 1$
e. $y = -0.5x + 8$ **f.** $y = -11x$

6. a. $y = 4x + 3$ **b.** $y = 3x + 2$
c. $y = 4x$ **d.** $y = -4x + 5$
e. $y = 0.5x + 3$ **f.** $y = -0.2x + 3$
g. $y = -5x + 2$ **h.** $y = \frac{2}{3}x + 3$

7. a. $y = 4x + 6$ **b.** $y = 2x + 2$
c. $y = x - 3$ **d.** $y = -3x - 3$
e. $y = \frac{1}{2}x + 2$ **f.** $y = \frac{2}{3}x + 9$
g. $y = \frac{1}{5}x + 4$ **h.** $y = \frac{1}{4}x - \frac{7}{2}$

8. C

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. Georgie generates a linear equation $y = -2x + 8$. State the constant and coefficient for this linear equation.

Key points

- Georgie generates a linear equation $y = -2x + 8$.
- State the constant and coefficient for this linear equation.

Explanation

The coefficient is the number that a pronumeral is being multiplied by.

$$y = -2x + 8$$

The coefficient is -2.

The constant is a number that cannot change its value in an expression.

$$y = -2x + 8$$

The constant is 8.

Answer

The coefficient is -2 and the constant is 8.

11. Clinton is the Head of Marketing at Butter Chicken Co. He has calculated that for each new post he puts up on social media, the number of followers increases by 15. Write a rule for to show the linear relationship between the number of posts (x) and number of followers (y).

Key points

- For every new post he gains 15 followers.
- Write a rule for to show the linear relationship between the number of posts (x) and number of followers (y).

Explanation

To determine the coefficient, calculate the value that y changes by for each x-value increase of 1. For each post (x), he gains 15 followers (y). ∴ The coefficient of x is 15.

The coordinate where $x = 0$ is (0,15).
Therefore, the constant is 15.

The rule is $y = 15x + 15$.

Answer

The linear relationship between the number of posts (x) and number of followers (y) is $y = 15x + 15$.

12. Tori is currently going through physio rehab and is having an ice salt bath each night to help with muscle recovery. For each bath she applies 125 grams of ice salt. The bag has a total of 1600 grams. On the tenth night she has 350 grams remaining. Generate two known coordinates and a rule that displays how much ice salt (s, grams) is left in the packet on any given day (d).

Key points

- For each bath she applies 125 grams of ice salt.
- The bag has a total of 1600 grams.
- On the tenth night she has 350 grams remaining.
- Generate two known coordinates and a rule that displays how much ice salt (s, grams) is left in the packet on any given day (d).

Explanation

In the beginning, there were 1600 grams in the bag. ∴ (0,1600)

After 10 days, there were 350 grams remaining. ∴ (10,350)

To determine the coefficient, calculate the value that s changes by for each d-value increase of 1. For each bath Tori uses 125 grams of salt. ∴ The coefficient of d is -125.

Identify the constant by referring to the coordinate where $d = 0$. The coordinate where $d = 0$ is (0,1600). ∴ The constant is 1600.

The rule is $s = -125d + 1600$.

Answer

The two known coordinates are (0,1600) and (10,350). The rule that displays how much ice salt (s, grams) is left in the packet on any given day (d) is $s = -125d + 1600$.

13. Lenny is working on her maths homework with her sister, Penny. Lenny explains to Penny that the following graph does not represent $y = -4y + 5$. Explain where Penny has gone wrong.

Key points

- The graph does not represent $y = -4x + 5$.
- Explain where Penny has gone wrong.

Explanation

To determine the coefficient, calculate the value that y changes by for each x-value increase of 1.

y-value decreases by 4. ∴ The coefficient is -4.

Refer to the y-axis to determine the constant, this is when $x = 0$, this coordinate is (0,20). ∴ The constant is 20.

The rule is $y = -4x + 20$.

Penny has the correct value for the coefficient but an incorrect value for the constant.

Penny has taken the constant to be the x-value when $y = 0$, when it is the y-value when $x = 0$.

Answer

Penny has the incorrect value for the constant. This is due to her mistaking the constant as the x-value when $y = 0$.

14. Xavier is in grade one. Each new day he learns to spell an additional three words. The following coordinates display how many words he has learnt on three particular days in grade one. Calculate and explain what the value of w represents when $d = 0$.

Key points

- Each new day he learns to spell an additional three words.
- The coordinates display how many words he has learnt on three particular days in grade one.
- Calculate and explain what the value of w represents when $d = 0$?

Explanation

Identify any two ordered pairs of coordinates.

(20,240) and (30,270)

The w-value increases by $270 - 240 = 30$ for each d-value increase of $30 - 20 = 10$.

∴ The w-value increases by 3 for each d-value increase of 1.

∴ The coefficient is 3.

Substitute any one coordinate into the rule and solve for the constant.

(20,240)

$240 = 20(3) + \text{constant}$

$240 = 60 + \text{constant}$

$180 = \text{constant}$

The constant is 180. This represents the number of words ($w = 180$) Xavier knew when he started grade one ($d = 0$).

Answer

The constant is 180. This represents the number of words ($w = 180$) Xavier knew when he started grade one ($d = 0$).

Reasoning

15. a. This value when $x = 0$ represents the value of the motorised scooter at the start of 2018.

b. The value of the motorised scooter in the third year is $2450.

c. The value of the coefficient of x is -350.

d. In this scenario, the value of the coefficient represents the change of value of the motorised scooter per year.

e. Suggested option 1: Mobile phones may have a linear depreciation model.

Suggested option 2: A car may have a linear depreciation model.

Note: There are other possible options.

16. a. The coefficient of x is 3.

b. The coefficient of x is 4.

c. In part **a**, the value of x increases by 1 between each pair of coordinates. Therefore, the value of the coefficient of x is calculated by the difference between y-values of two adjacent pairs of coordinates.

In part **b**, the value of x increases by a different number between each pair of coordinates. Therefore, the value of the coefficient of x is calculated by dividing the difference between y-values by the difference between x-values of any two pairs of coordinates.

Extra spicy

17. B **18.** $\frac{3}{2} \neq 1$ **19.** $y = 2x + 2$ **20.** C

Remember this?

21. D **22.** E **23.** C

9D Gradient

Student practice

Worked example 1

a. Positive gradient **b.** Negative gradient

c. Undefined gradient **d.** Zero gradient

Worked example 2

a. $m = 3$ **b.** $m = -3$

Worked example 3

a. $m = 3$ **b.** $m = -2$

Understanding worksheet

1. a. Negative **b.** Positive

c. Zero **d.** Undefined

2. a. $m = \frac{8}{4}$ **b.** $m = \frac{4}{8}$ **c.** $m = \frac{-4}{2}$ **d.** $m = \frac{-4}{1}$

3. $\frac{\text{rise}}{\text{run}}$; steeper; vertical; zero

Fluency

4. a. Positive **b.** Negative **c.** Positive **d.** Negative

e. Undefined **f.** Zero

---

5. a. $m = 2$ **b.** $m = 4$

c. $m = 1$ **d.** $m = -\frac{3}{2}$

e. $m = -2$ **f.** $m = -\frac{1}{3}$

g. $m = \frac{9}{2}$ **h.** $m = \frac{73}{100}$

---

6. a. $m = 3$ **b.** $m = 4$

c. $m = 4$ **d.** $m = -1$

e. $m = -\frac{1}{4}$ **f.** $m = -\frac{2}{5}$

g. $m = -\frac{1}{3}$ **h.** $m = -1$

---

7. A

Spot the mistake

8. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

9. Antonia plays Overcooked on her new Nintendo Switch. For every hour she plays she does 30 star jumps. After three hours, Antonia has done 90 star jumps. State the gradient for this scenario.

Key points

- Antonia plays Overcooked on her new Nintendo Switch. For every hour she plays she does 30 star jumps.
- After three hours, Antonia has done 90 star jumps.
- State the gradient for this scenario.

Explanation

For the first hour Antonia does 30 start jumps.

The gradient is the number of the jumps Antonia does per hour.

∴ The gradient is 30.

Answer

The gradient for this scenario is 30.

10. Duyen has a Frank Green reusable cup. Duyen fills up her coffee and leaves it open. The coffee has an initial temperature of 90°C. The temperature decreases at a constant rate. For every one minute, it decreases by 2°C. Using the table of values, calculate the gradient.

Key points

- Duyen has a Frank Green reusable cup. Duyen fills up her coffee and leaves it open. The coffee has an initial temperature of 90°C.
- The temperature decreases at a constant rate.
- For every one minute, it decreases by 2°C.
- Using the table of values, calculate the gradient.

Explanation

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Choose two coordinates.

After 1 minute the temperature is 88°C. ∴ (1,88)

After 2 minutes the temperature is 86°C. ∴ (2,86)

Match coordinates.

(1,88) (2,86)

(x_1,y_1) (x_2,y_2)

$m = \frac{86 - 88}{2 - 1}$

$m = \frac{-2}{1}$

$m = -2$

Answer

The gradient is -2.

11. Matthew rode his bike to work and each week he measured his average time. His initial average time was 30 minutes. After week four he has an average time of 18 minutes. Calculate the gradient for Matthew's average riding time.

Key points

- Matthew rode his bike to work and each week he measured his average time. His initial average time was 30 minutes.
- After week four he has an average time of 18 minutes.
- Calculate the gradient for Matthew's average riding time.

Explanation

Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Choose two coordinates.

Matthew's initial time was 30 minutes. ∴ (0,30)

After four weeks Matthew's average time is 18 minutes. ∴ (4,18)

Match coordinates.

(0,30) (4,18)

(x_1,y_1) (x_2,y_2)

$m = \frac{18 - 30}{4 - 0}$

$m = \frac{-12}{4}$

$m = -3$

Answer

The gradient for Matthew's average riding time is -3.

12. Emilia needs to get her washing machine serviced. The technician has estimated that it will take 1–2 hours to service. For one hour they charge \$75 and for 2 hours they charge \$125. Find the gradient for this scenario and determine if it is positive or negative.

Key points

- Emilia needs to get her washing machine serviced. The technician has estimated that it will take 1–2 hours to service.
- For one hour they charge \$75 and for 2 hours they charge \$125.
- Find the gradient for this scenario and determine if it is positive or negative.

Explanation

Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Choose two coordinates.

For one hour they charge \$75. ∴ (1,75)

For two hours they charge \$125. ∴ (2,125)

Match coordinates.

(1,75) (2,125)

(x_1,y_1) (x_2,y_2)

$m = \frac{125 - 75}{2 - 1}$

$m = \frac{50}{1}$

$m = 50$

∴ The gradient is positive.

Answer

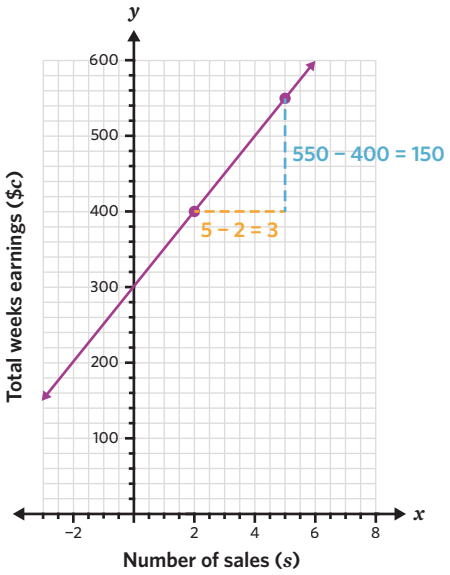
The gradient for this scenario is 50 and it is positive.

13. Henry is a car salesperson. The following graph displays Henry's sales data. Using the gradient, determine how much Henry earns per sale?

Key points

- Henry is a car salesperson. The following graph displays Henry's sales data.
- Using the gradient, determine how much Henry earns per sale?

Explanation



Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Choose two coordinates and match coordinates.

(2,400) (5,550)

(x_1,y_1) (x_2,y_2)

$m = \frac{550 - 400}{5 - 2}$

$m = \frac{150}{3}$

$m = 50$

∴ The gradient is 50, so Henry earns \$50 per sale.

Answer

Henry earns \$50 per sale.

Reasoning

14. a. The gradient for Company A is 80.

b. The daily rate for Company D is \$60.

c. Company B: (1,150),(5,350)

Gradient for Company B $= \frac{350 - 150}{5 - 1}$

$= \frac{200}{4}$

$= 50$

Company C: (1,190),(5,350)

Gradient for Company C $= \frac{350 - 190}{5 - 1}$

$= \frac{160}{4}$

$= 40$

$40 \neq 50$

Therefore, the gradients for Company B and C are not the same for the first two days.

d. Suggested option 1: Bronwen can take public transport for her trip in Queensland.

Suggested option 2: Bronwen can ride a bike for her trip in Queensland.

Note: There are other possible options.

15. a. $m = -3$ **b.** $m = -3$

c. The gradient value for line A and line B is the same. The similarity when calculating the gradient is the use of the formula, $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. The difference when calculating the gradient is the different x_1, y_1, x_2, y_2 values.

For line A, $x_1 = 3, y_1 = -6, x_2 = 6, y_2 = -15$; for line B, $x_1 = 3, y_1 = -3, x_2 = 6, y_2 = -12$.

Extra spicy

16. E **17.** $a = 3$ **18.** B **19.** $m = 2$

Remember this?

20. D **21.** E **22.** C

9E Gradient-intercept form

Student practice

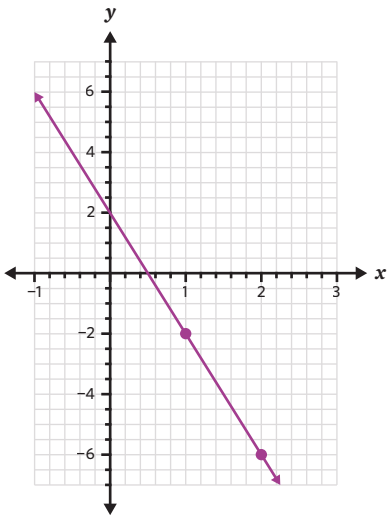
Worked example 1

a. -4 is the gradient (m)
2 is the y-intercept (c)

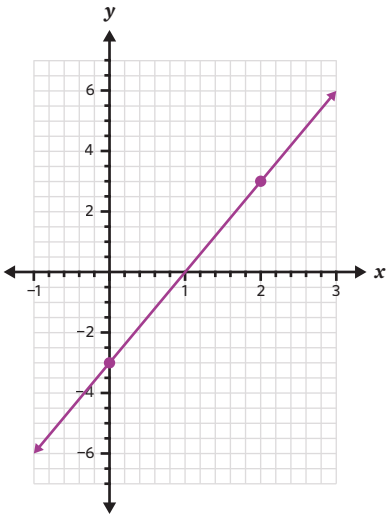
b. 3 is the gradient (m)
-3 is the y-intercept (c)

Worked example 2

a.



b.



Worked example 3

a. 5 is the y-intercept (c)
-2.5 is the x-intercept

b. -3 is the y-intercept (c)
0.75 is the x-intercept

Understanding worksheet

1. a. $y = 3x + 1$ **b.** $y = -2x + 4$

c. $y = \frac{1}{2}x + 3$ **d.** $y = -\frac{1}{3}x - 4$

2. a. $y - 4x = -1$ **b.** $x = -\frac{y}{2} + 2$

c. $y + 3 = 3x$ **d.** $x = \frac{-3y}{2} + \frac{3}{2}$

3. $y = mx + c$; y-intercept; gradient; rearranged

Fluency

4. a. $m = 3, c = 1$ **b.** $m = 5, c = -1$

c. $m = -4, c = -2$ **d.** $m = \frac{2}{3}, c = 1$

e. $m = -\frac{1}{2}, c = 3$ **f.** $m = \frac{1}{4}, c = 0$

g. $m = -1, c = 2$ **h.** $m = 0, c = 2$

---

5. a. $m = 2, c = 2$ **b.** $m = 4, c = -3$

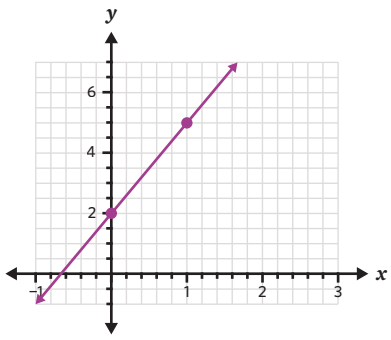
c. $m = -2, c = 2$ **d.** $m = \frac{1}{2}, c = 2$

e. $m = -\frac{3}{4}, c = -3$ **f.** $m = -\frac{1}{2}, c = 0$

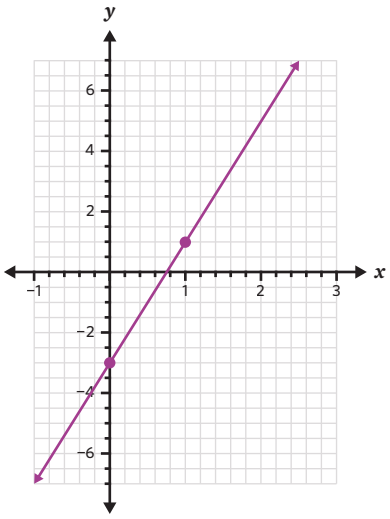
g. $m = -1, c = -3$ **h.** $m = 0, c = -4$

---

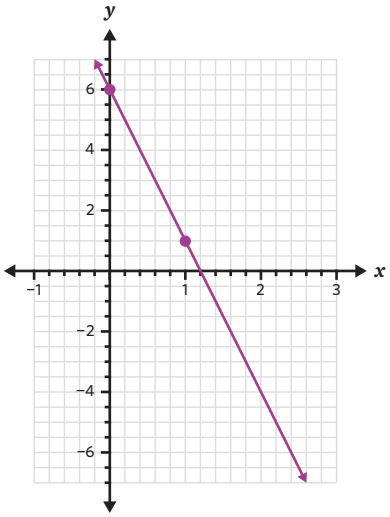
6. a.



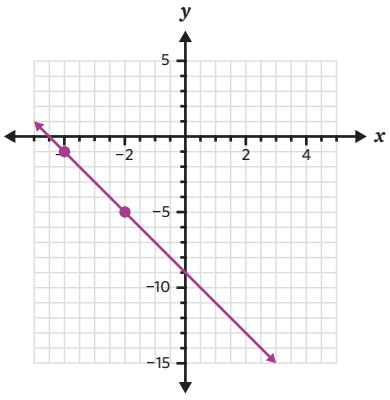
b.


y
x
6
4
2
−1
1
2
3
−2
−4
−6

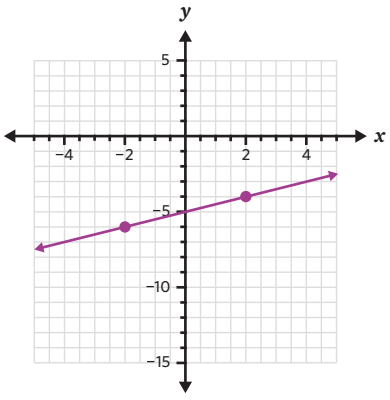
c.


y
x
6
4
2
−1
1
2
3
−2
−4
−6

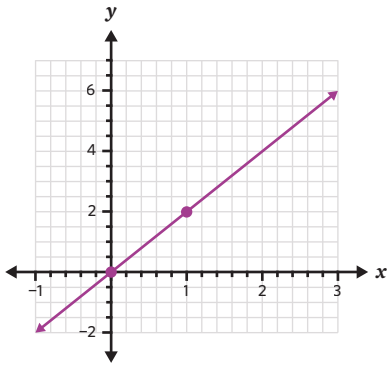
d.


y
x
5
−2
2
4
−5
−10
−15

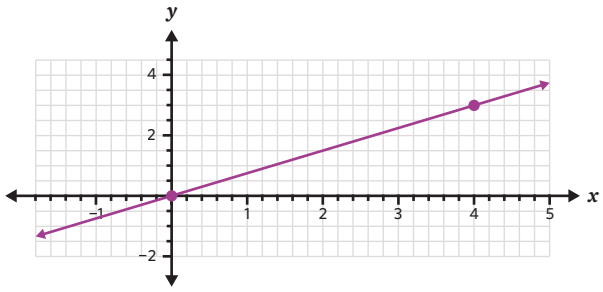
e.


y
x
5
−4
−2
2
4
−5
−10
−15

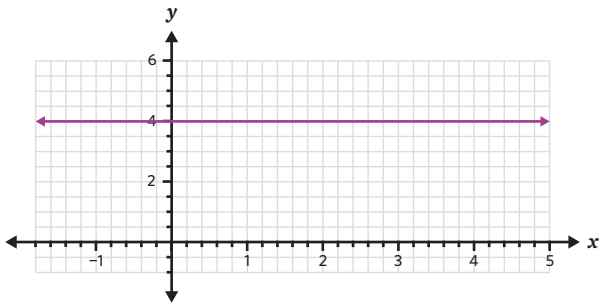
f.


y
x
6
4
2
−1
1
2
3
−2

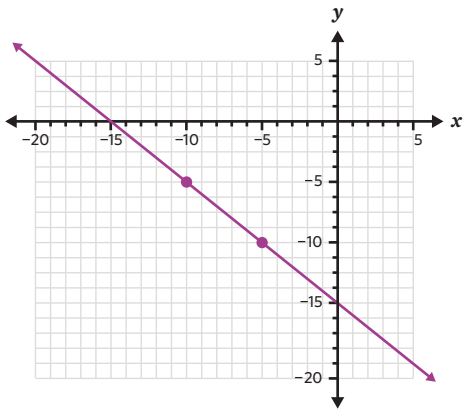
g.


y
x
4
2
−1
1
2
3
4
5
−2

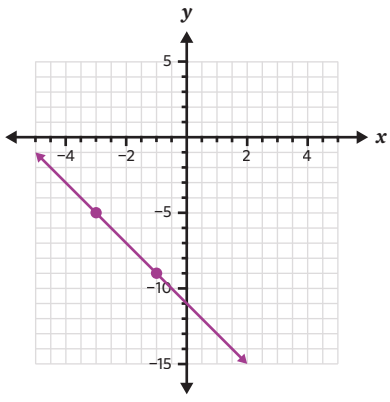
h.


y
x
6
4
2
−1
1
2
3
4
5

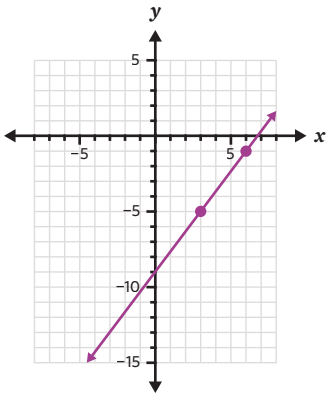
7. a.


y
x
5
−20
−15
−10
−5
5
−5
−10
−15
−20

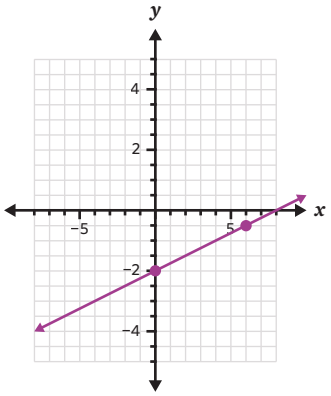
b.


y
x
5
−4
−2
2
4
−5
−10
−15

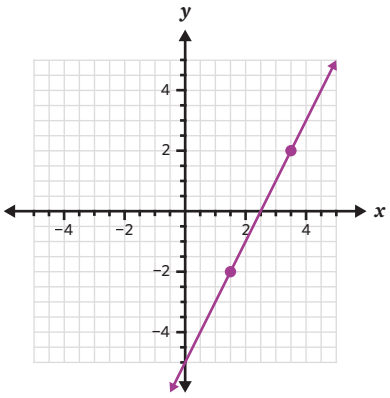
c.



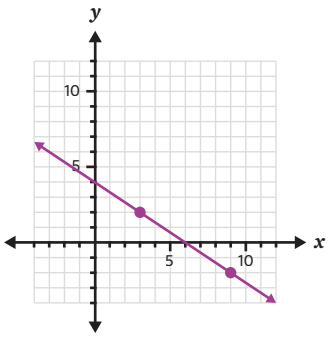
d.



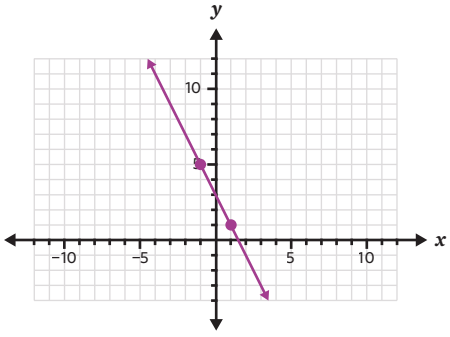
e.



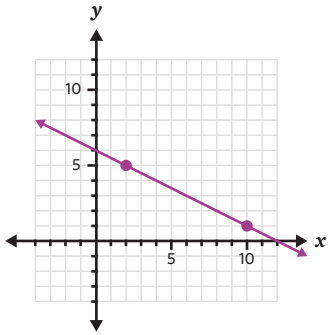
f.



g.



h.



8. **a.** -1 is the y-intercept
1 is the x-intercept

b. 5 is the y-intercept
-1.25 is the x-intercept

c. 2 is the y-intercept
0.4 is the x-intercept

d. -3 is the y-intercept
-1.5 is the x-intercept

e. 3 is the y-intercept
-2 is the x-intercept

f. 5 is the y-intercept
-45 is the x-intercept

g. 3 is the y-intercept
6 is the x-intercept

h. $\frac{42}{5}$ or 8.4 is the y-intercept
6 is the x-intercept

9. D

Spot the mistake

10. **a.** Student A is incorrect.

b. Student B is incorrect.

Problem solving

11. Vaike went on a walk and the rule linking her distance (d) and time (t) in the gradient-intercept form is $d = 4t$.
State the gradient and y-intercept of the rule.

Key points

- Distance (d) and time (t) in the gradient-intercept form is $d = 4t$.
- State the gradient and y-intercept of the rule.

Explanation

The linear equation in gradient-intercept form is denoted as $y = mx + c$.

In this case, $d = y$ and $t = x$.

The coefficient of t is the gradient. $\therefore m = 4$.

The constant is the y-intercept. $\therefore c = 0$.

Answer

The gradient is 4 and the y-intercept is 0.

12. Stefan is making a BBQ for the footy finals. His BBQ gas bottle starts with 9 kg of gas. Gas is used at a constant rate of 0.5 kg per hour. Generate an equation in the gradient intercept form to display Stefan's gas bottle in terms of mass of gas (G) and (t) time, per hour.

Key points

- His BBQ gas bottle starts with 9 kg of gas.
- Gas is used at a constant rate of 0.5 kg per hour.
- Generate an equation in the gradient intercept form to display Stefan's gas bottle in terms of mass of gas (G) and (t) time, per hour.

Explanation

Substitute the values and variables given into the linear equation in gradient-intercept form.

The gradient is 0.5, as it is consistently depreciating each hour, so $m = -0.5$.

$y = -0.5x + c$

The y is going to be replaced with G as we are calculating the mass of gas over a period of time.

$G = -0.5x + c$

The x is going to be replaced with the t.

$G = -0.5t + c$

The c is going to be replaced with +9 as it is the starting value. It is a positive, so $G = -0.5t + 9$.

Answer

$G = -0.5t + 9$

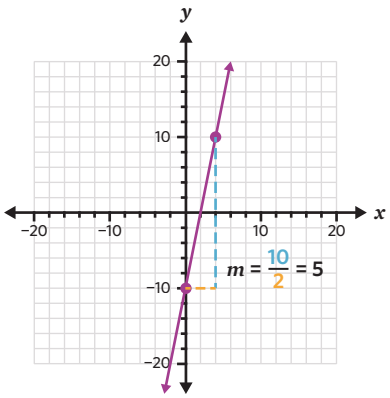
13. Bonnie is doing her maths homework and has been asked the following question: 'If the x-intercept of a line is positive and the y-intercept is negative, does the line have a positive or negative gradient?' What should Bonnie's answer to this be?

Key points

- 'If the x-intercept of a line is positive and the y-intercept is negative, does the line have a positive or negative gradient?'
- What should Bonnie's answer to this be?

Explanation

The gradient will be positive as for every unit increase on the x-axis, the y value will also increase. Both the y and x values are increasing which makes the gradient positive.



Answer

The gradient will be positive.

14. Tasha is eating an ice cream. The ice cream starts at a height of 15 cm. The ice cream is melting at a constant rate of 2.5 cm/minute. Generate an equation in the gradient-intercept form to represent this scenario.

Key points

- The ice cream starts at a height of 15 cm.
- The ice cream is melting at a constant rate of 2.5 cm/minute.
- Generate an equation in the gradient-intercept form to represent this scenario.

Explanation

Substitute the values given into the gradient-intercept form.

The gradient is −2.5, as it is consistently depreciating each minute, so $m = -2.5$.

$y = -2.5x + c$

The c is going to be replaced with 15 as it is the starting point.

$y = -2.5x + 15$

Answer

$y = -2.5x + 15$

15. A cyclist rides at a constant rate of 29 km per hour. The equation in gradient-intercept form can be written as $D = 29t$, where D represents total distance, in km and t represents time, in hours. Using the context of the question, explain why the y-intercept is equal to zero.

Key points

- A cyclist rides at a constant rate of 29 km per hour.
- The equation in gradient-intercept form can be written as $D = 29t$.
- D represents total distance, in km.
- t represents time, in hours.
- Explain why the y-intercept is equal to zero.

Explanation

The cyclist is starting the ride at 0 km, which means the starting point for this scenario is 0. This is represented as the y-intercept.

Answer

The cyclist's starting point is at 0 km, $\therefore c$ will be 0.

Reasoning

16. a. The value of the car when it was new was $23 500. This is represented as the y-intercept.

b. The equation will be $V = 23\,500 - 1250t$.

c. The m is −1250, this is how much the value of the car will depreciate each year.

d. Suggested option 1: Technology depreciates in value over time.

Suggested option 2: Jewellery can depreciate in value over time.

Note: There are other possible options.

17. a. $y = 5 - 3x$

b. $y = 5 - 3x$

c. The first set of equations given in part **a** and **b** have different coefficients of x and y. Both equations in part **a** and **b** rearranged have the same values for the gradient and the y-intercept.

Extra spicy

18. $y = -10x - 6$

19. $y = -2x + 4$

20. A

21. A

Remember this?

22. D

23. C

24. D

9F Solving linear equations graphically

Student practice

Worked example 1

a. $x = 2$

b. $x = 1$

Worked example 2

a. $x = 0$ and $y = 3$

b. $x = -2$ and $y = 1$

Understanding worksheet

1. **a.** 2 **b.** 3 **c.** −2 **d.** −4.5

2. **a.** $x = 3$ **b.** (3,5)
c. (−4,16) **d.** $x = 0.5$

3. RHS; straight; intersection; shared

Fluency

4. **a.** $x = 1$ **b.** $x = 2$
c. $x = 3$ **d.** $x = 0$
e. $x = 1.5$ **f.** $x = -0.5$
g. $x = -0.9$ **h.** $x = 0.6$

---

5. **a.** $x = 1$ **b.** $x = 2$
c. $x = 0.5$ **d.** $x = 3$
e. $x = 1.25$ **f.** $x = 3.5$
g. $x = -0.25$ **h.** $x = 4.25$

---

6. **a.** $x = 2$ and $y = 5$ **b.** $x = 2$ and $y = 7$
c. $x = 0$ and $y = -3$ **d.** $x = 4$ and $y = -9$
e. $x = -2$ and $y = -1$ **f.** $x = 1.333$ and $y = 6.333$
g. $x = 0.909$ and $y = 2.364$ **h.** No solution

---

7. **a.** $x = 9$ and $y = 1$ **b.** $x = 3$ and $y = 4$
c. $x = 1$ and $y = 5$ **d.** $x = -2$ and $y = 5$
e. $x = -2$ and $y = -3$ **f.** $x = -1$ and $y = -4$
g. $x = 0.5$ and $y = -2$ **h.** $x = -3$ and $y = 4.667$

---

8. A

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. Two friends are heading out to dinner. They are both travelling from different locations. They are also travelling at different speeds (km/h). There is a point where their travelling paths cross over. The point at which they meet on the way to dinner is mathematically referred to as what? Explain.

Key points

- The two friends cross paths at a certain point.
- What is the mathematical name of the point at which they meet?

Explanation

From key idea 2, a point at which two lines meet is referred to as the point of intersection.

Answer

The point at which their paths cross is called the point of intersection.

11. Matheus has graphed the equation $y = 3x + 2$ to represent the rate it takes to build a snowman. His friend Olaf has graphed $x = -0.5y + 3.5$ to represent the rate it takes for the snowman to melt. Graphically show the point where the rate of building a snowman equals the rate of the snowman melting.

Key points

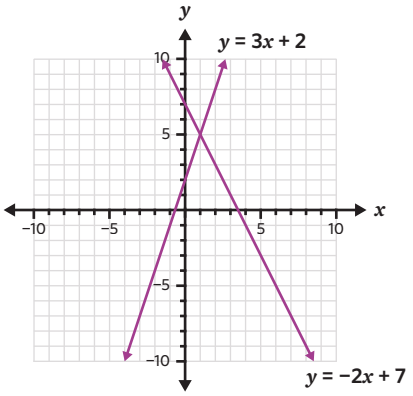
- The rate of building the snowman is represented by the equation $y = 3x + 2$.
- The rate of of the snowman melting is represented by the equation $x = -0.5y + 3.5$.
- Represent the point where both rates are equal through graphs.

Explanation

Rearrange $x = -0.5y + 3.5$ to the form $y = mx + c$.

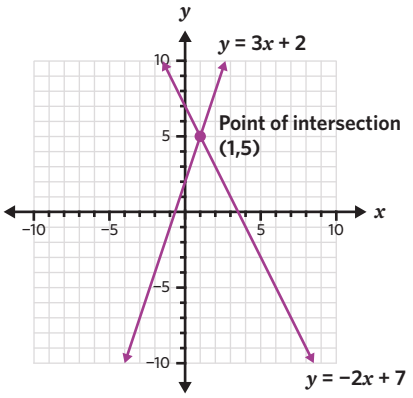
$y = -2x + 7$

Plot and label both of the graphs on the same set of axes.



Identify the point of intersection and label it.

Answer



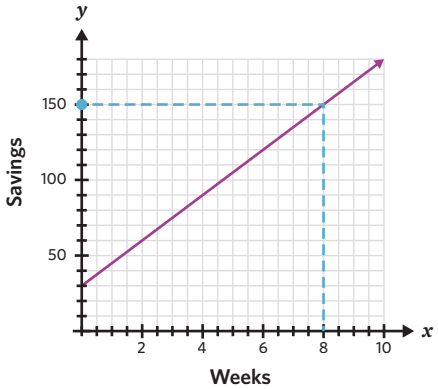
12. You are saving to go to Ed Sheeran's next music concert. Your savings are displayed in the following graph. You will need to have a total of $150 by a certain date. How many weeks will it take to save enough money to go to the concert?

Key points

- The savings is displayed in the graph.
- The goal is to reach $150.
- Find the number of weeks it takes to reach the goal.

Explanation

Locate $150 on the y-axis. Use it to locate the x-coordinate.



Answer

It takes 8 weeks to save enough money to go to the concert.

13. The following graph displays the number of hours worked (h) and the total amount saved ($\$a$) over a month for Mimi and Yuri. Evaluate how many hours it will take both Mimi and Yuri in order for them to have saved the same amount.

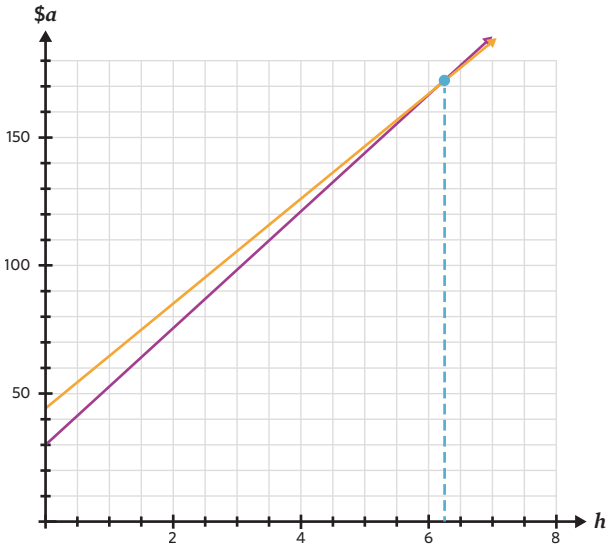
Key points

- The graph displays the number of hours worked and the total amount saved over a month for Mimi and Yuri.
- Find the number of hours it takes for Mimi and Yuri to save the same amount of money.

Explanation

The point where Mimi and Yuri save the same amount of money is when the two graphs meet, that is, the point of intersection.

Locate the point of intersection and trace to the x-axis to find the number of hours.



Answer

It takes 6.25 hours for Mimi and Yuri to have saved the same amount of money.

14. Aram knows that two lines have a point of intersection at the coordinate $(2,-2)$. The rules for the lines are:

Rule 1: $-x + 2y = -6$

Rule 2: $3x + 2y = 2$

Using the gradient-intercept method or a digital technology of your choice, graph and prove that the solution $(2,-2)$ is true for both lines.

Key points

- Two lines have a point of intersection at $(2,-2)$.
- The rules for the lines are: $-x + 2y = -6$ and $3x + 2y = 2$.
- Plot the graphs of the lines and verify that $(2,-2)$ is the point of intersection.

Explanation

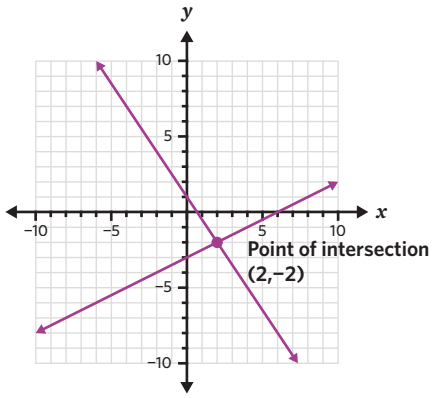
Rearrange both rules into the form $y = mx + c$.

Rule 1: $y = \frac{x}{2} - 3$

Rule 2: $y = \frac{-3x}{2} + 1$

Plot the graphs on the same set of axes and locate the point of intersection.

Answer



Reasoning

15. a. The rule that applies to Anthony is $C = 6 + 2d$; the rule that applies to Simon is $C = 4 + 3d$.

b. It takes Anthony 3 days to have 12 cookies.

c. It takes Simon 5 days to have 19 cookies.

d. It takes 2 days for Simon and Anthony to have the same amount of cookies.

e. Suggested option 1: You get to bring the snacks that you like, which may not be available at the venue.

Suggested option 2: The snacks provided at the venue are expensive, so it is cheaper to bring your own.

Note: There are other possible options.

16. a. $x = -\frac{8}{3}$ **b.** $x = \frac{8}{3}$

c. The equations in parts **a** and **b** are almost identical, only differ by a negative sign. In part **a**, the equation has a negative gradient and the solution is $x = -\frac{8}{3}$. In part **b**, the equation has a positive gradient and the solution is $x = \frac{8}{3}$.

The solutions are also almost identical, differing only by a negative sign.

Extra spicy

17. $y = \frac{2x}{3} + 1$

$\frac{2 \times 6}{3} + 1 = 4 + 1 = 5$ ✓

18. E

19. B

20. $y = 3x - 12$

$x = 3$

Remember this?

21. B, D, E **22.** E **23.** E

9G Non-linear graphs

Student practice

Worked example 1

a.

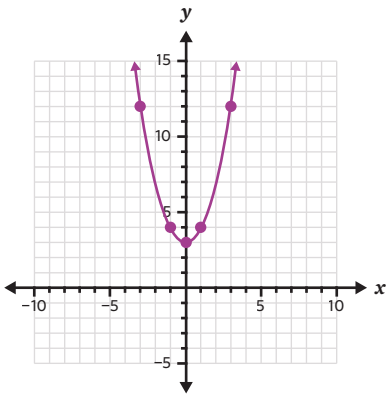
x	−3	−1	0	1	3
y	12	4	3	4	12

b.

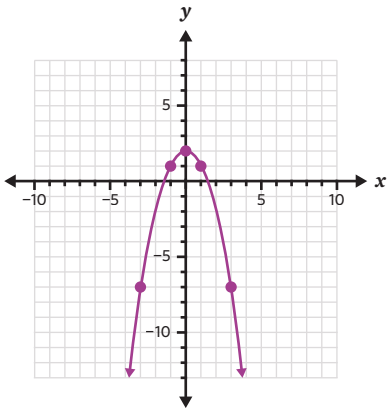
x	−3	−1	0	1	3
y	−7	1	2	1	−7

Worked example 2

a.



b.



Understanding worksheet

1. **a.** Non-linear **b.** Linear **c.** Non-linear **d.** Linear

2. **a.** $y = -1$ **b.** $y = 2$ **c.** $y = 0.25$ **d.** $y = 2$

3. non-linear; parabola; power; symmetrical

Fluency

4. **a.**

x	−3	−1	0	1	3
y	9	1	0	1	9

b.

x	−3	−1	0	1	3
y	27	3	0	3	27

c.

x	−3	−1	0	1	3
y	36	4	0	4	36

d.

x	−3	−1	0	1	3
y	2.25	0.25	0	0.25	2.25

e.

x	−3	−1	0	1	3
y	10	2	1	2	10

f.

x	−3	−1	0	1	3
y	8.5	0.5	−0.5	0.5	8.5

g.

x	−3	−1	0	1	3
y	0	−2	0	4	18

h.

x	−3	−1	0	1	3
y	15	3	0	−1	3

5. **a.**

x	−3	−1	0	1	3
y	−9	−1	0	−1	−9

b.

x	−3	−1	0	1	3
y	−18	−2	0	−2	−18

c.

x	−3	−1	0	1	3
y	−45	−5	0	−5	−45

d.

x	−3	−1	0	1	3
y	−4.5	−0.5	0	−0.5	−4.5

e.

x	−3	−1	0	1	3
y	−7	1	2	1	−7

f.

x	−3	−1	0	1	3
y	−13	−5	−4	−5	−13

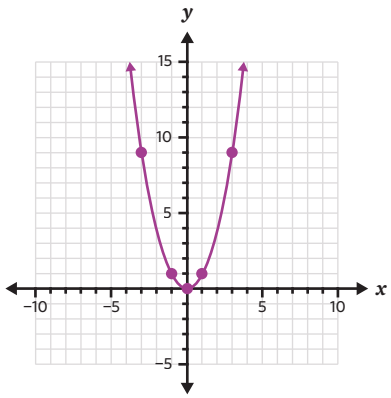
g.

x	−3	−1	0	1	3
y	−15	−3	0	1	−3

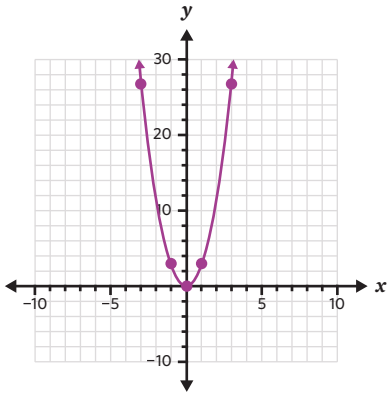
h.

x	−3	−1	0	1	3
y	0	2	0	−4	−18

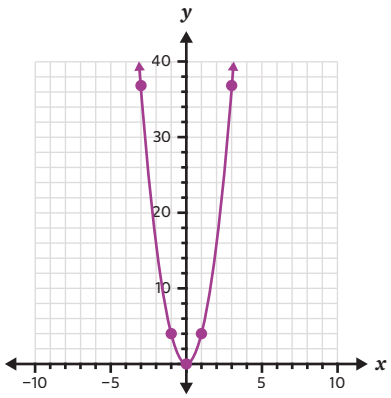
6. **a.**



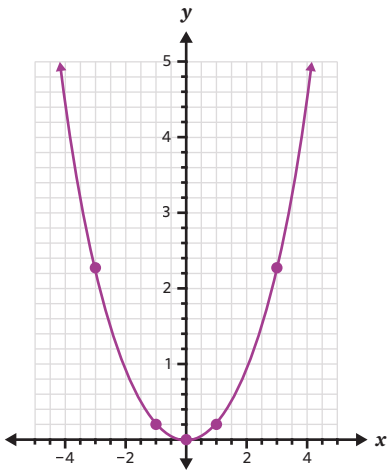
b.


y
x
30
20
10
−10
−10
−5
5
10

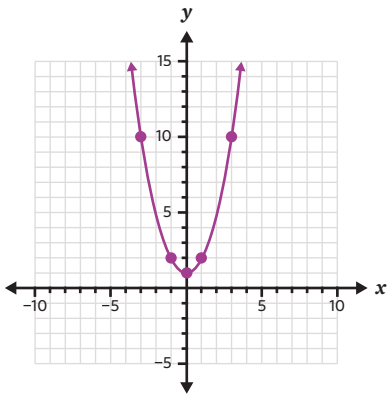
c.


y
x
40
30
20
10
−10
−5
5
10

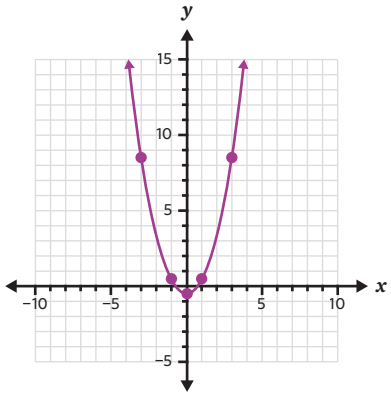
d.


y
x
5
4
3
2
1
−4
−2
2
4

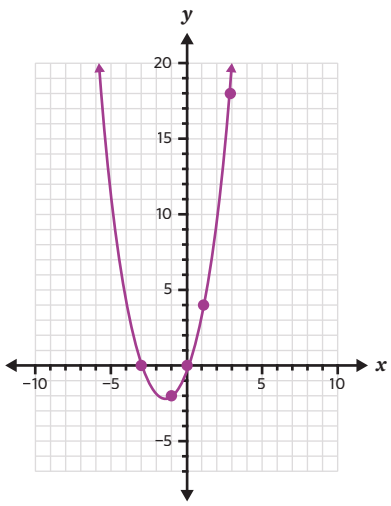
e.


y
x
15
10
5
−5
−10
−5
5
10

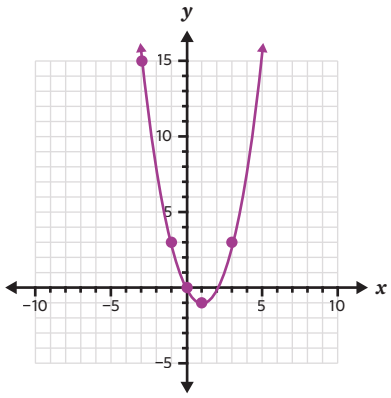
f.


y
x
15
10
5
−5
−10
−5
5
10

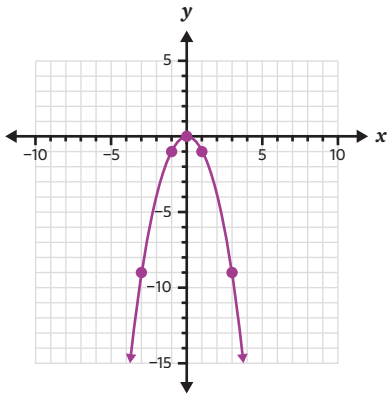
g.


y
x
20
15
10
5
−5
−10
−5
5
10

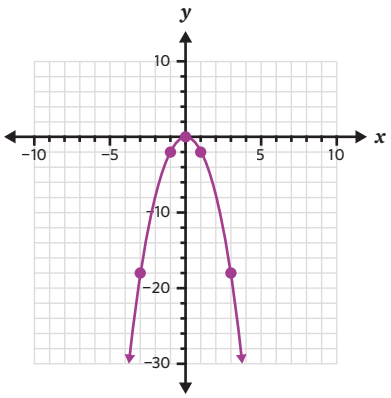
h.


y
x
15
10
5
−5
−10
−5
5
10

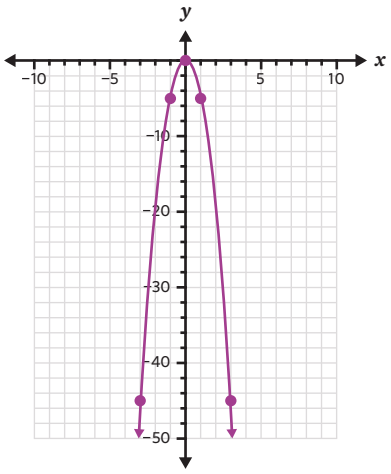
7. a.


y
x
5
−5
−10
−15
−10
−5
5
10

b.



c.



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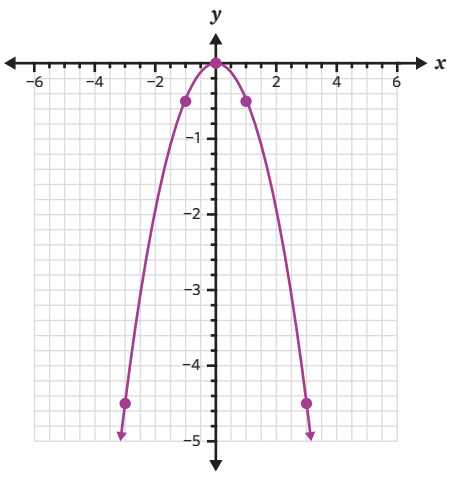
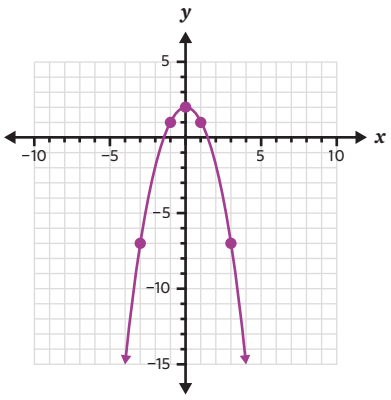
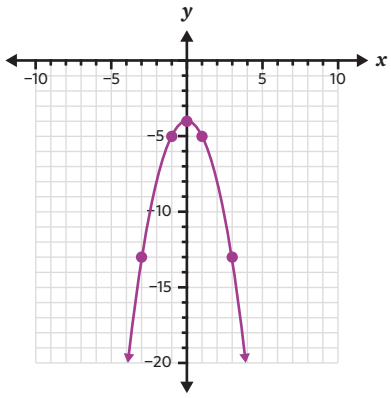


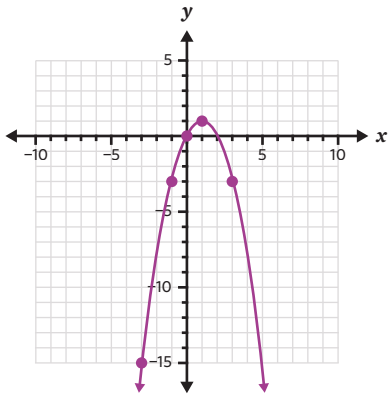
e.



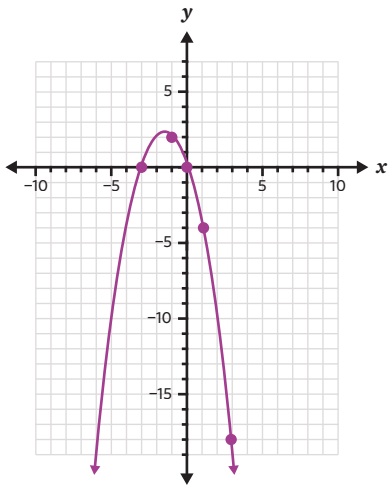
f.



g.



h.



8. A

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

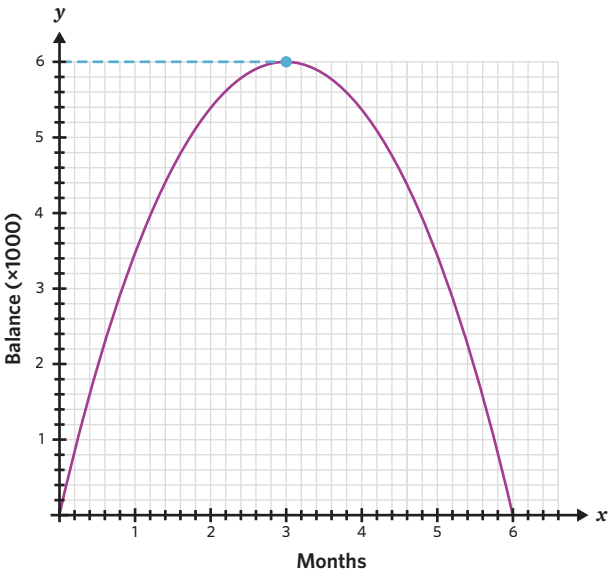
10. The following graph displays Kayla's savings over a certain period within a year. Kayla started saving at the beginning of January. Calculate Kayla's maximum savings amount.

Key points

- The graph displays Kayla's savings over 6 months.
- Find Kayla's maximum savings amount.

Explanation

Trace the maximum point of the graph to its corresponding y-axis.



The maximum point is \$6000.

Answer

Kayla's maximum savings amount is \$6000.

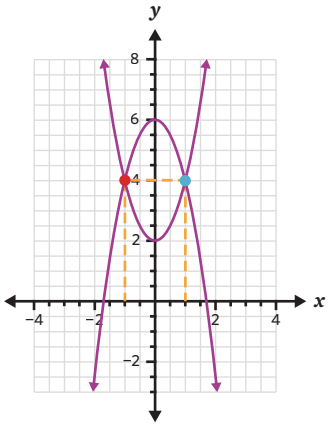
11. Ron and Kyle are running around a local football oval. They are running in opposite directions but at different speeds. During their run, they cross paths twice. The first point of intersection is marked with a red dot. The second point of intersection is marked with a blue dot. State the coordinates for both points of intersection.

Key points

- During Ron's and Kyle's run, they cross paths twice.
- The first point of intersection is marked with a red dot, and the second a blue dot.
- Find the coordinates for both points of intersection.

Explanation

Trace the dots to their respective x- and y- coordinates on the axes.



The coordinate for the red dot is $(-1,4)$, for the blue dot is $(1,4)$.

Answer

The coordinates of the points of intersection are $(-1,4)$ and $(1,4)$.

12. A tennis ball is dropped from a height of 15 metres. The equation $d = -\frac{5}{3}t^2 + 15$ provides the distance above ground, (d), of the tennis ball, after t seconds. After how many seconds does it take for the ball to hit the ground?

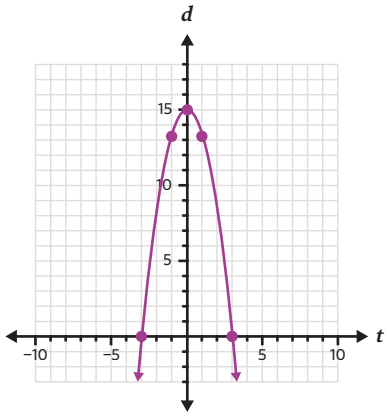
Key points

- A tennis ball is dropped from a height of 15 metres.
- The equation that describes the motion is $d = -\frac{5}{3}t^2 + 15$, where t is time in seconds, d is distance above ground in metres.
- Find the time it takes for the ball to hit the ground.

Explanation

Plot the graph $d = -10t^2 + 15$ by first generating a table of values.

t	−3	−1	0	1	3
d	0	13.33	15	13.33	0



Because time, t, must be greater than 0, we only look at the right half of the graph.

When the ball hits the ground, $d = 0$. At $d = 0$, $t = 3$.

Answer

It takes the ball 3 seconds to hit the ground.

13. Jacob threw a rock into a lake and the equation $h = -5s^2 + 40s + 1.2$ can be used to find the height (h), in centimetres of the rock at any given point. Calculate the height of the rock after 6 seconds (s).

Key points

- The equation $h = -5s^2 + 40s + 1.2$ describes the motion of the rock after being thrown into a lake, where t is time in seconds, h is height in centimetre.
- Find the height of the rock after 6 seconds.

Explanation

The equation to find height given time input is $h = -5s^2 + 40s + 1.2$. To find the height after 6 seconds, substitute 6 into s.

$$h = -5(6)^2 + 40(6) + 1.2$$
$$= -5 \times 36 + 40 \times 6 + 1.2$$
$$= -180 + 240 + 1.2$$
$$= 61.2 \text{ cm}$$

Answer

The height of the rock after 6 seconds is 61.2 cm.

14. Jenny is playing Angry Birds. She has taken a shot that will move her distance away from the point of origin by 2.83 metres, rounded to two decimal places. The new position can be represented as (2.83,0). The non-linear equation that this shot creates is $y = -0.25x^2 + 2$, where x represents the distance in metres and y represents the height in metres. Calculate the distance the bird has travelled when the bird is at a height of 1.4 m, correct to one decimal place.

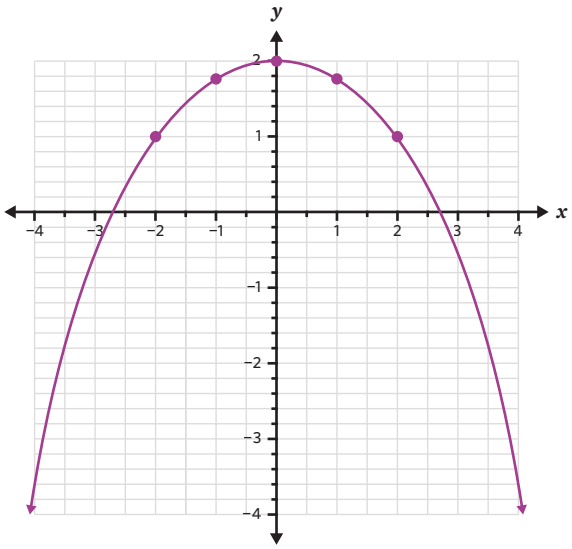
Key points

- The shot takes the bird 2.83 metres from the point of origin, with a coordinate of (2.83,0).
- The equation that describes the motion is $y = -0.25x^2 + 2$, where x is the distance in metres and y is the height in metres.
- Find the distance the bird has travelled when it is at a height of 1.4 m to one decimal place.

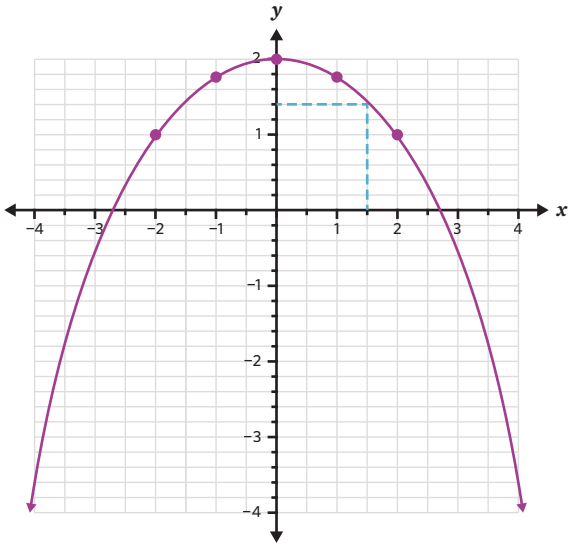
Explanation

Plot the graph of $y = -0.25x^2 + 2$ by first generating a table of values.

x	-2	-1	0	1	2
y	1	1.75	2	1.75	1



Trace the graph to find the corresponding x value of $y = 1.4$.



When $y = 1.4$, $x \approx 1.5$.

Answer

The bird has travelled approximately 1.5 m when it is at a height of 1.4 m.

Reasoning

15. a. The coordinate for Arlo's starting position is (0,0).

b. When $x = 2$, $y = 0$.

c. The maximum height of Arlo's jump is 1 m.

d. When $x = 1.5$, $y = 0.75$. This value represents Arlo's height above ground, that is, Arlo is 0.75 m high when he is 1.5 m away from his original position (0.5 m away from the bird).

e. Suggested option 1: Cubic graph.

Suggested option 2: Exponential graph.

Note: There are other possible options.

16. a.

x	-2	-1	0	1	2
y	10	1	-2	1	10

b.

x	-2	-1	0	1	2
y	-10	-1	2	-1	-10

c. The values generated in the tables in parts **a** and **b** are almost identical, namely they are the numbers 10, 1 and 2. The only difference is in their signs, the values in part **b** are the negative of the values in part **a**.

Extra spicy

17. $x = -1$ and $y = -3$ or $x = 3$ and $y = 5$

18. $x = -2$ or $x = 2$

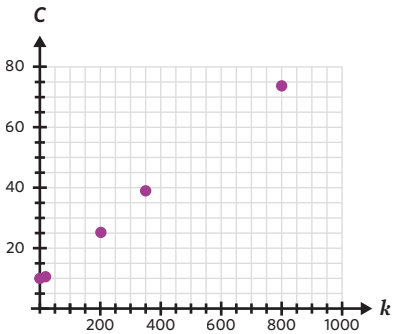
19. B **20.** A

Remember this?

21. B **22.** E **23.** C

Chapter 9 extended application

1. a.



All points are located in quadrant 1.

b. The constant value is \$10.00. This is the supply charge, it means that regardless of the kWh consumed, a \$10.00 charge will be placed on the consumer.

c. Megawatt charges \$0.12 per kWh while Electabuzz charges \$0.08 per kWh. As the cost of Megawatt increases more rapidly, it will produce a steeper line.

d.

k	0	100	200	300	400
C	8.5	20.5	32.5	44.5	56.5

k	500	600	700	800
C	68.5	80.5	92.5	104.5

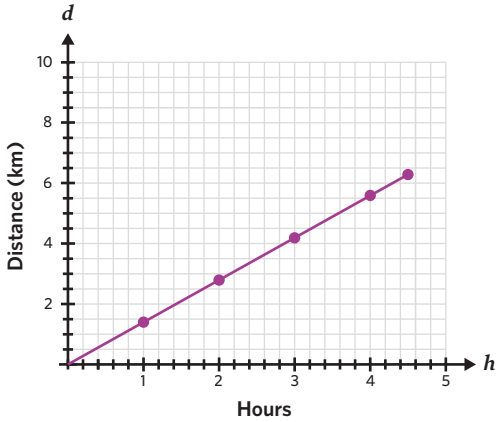
e. Megawatt is the more expensive option after 37.5 kWh.

f. Suggested option 1: Use less air-con during summer.

Suggested option 2: Minimise light usage during the day.

Note: There are other possible options.

2. a.



b. The gradient for this line has an upward trend, which displays a positive gradient. The gradient of 0.7, also indicates that it has a positive gradient.

c. Average pace on day 1 $= m_1 = \frac{6.3 - 1.4}{4.5 - 1} = 0.7$ km/h

Average pace on day 2 $= m_2 = \frac{9.8 - 7}{5 - 1} = 1.4$ km/h

$0.7 = \frac{1}{2} \times 1.4 \therefore m_1 = \frac{m_2}{2}$

d. 2 hours and 45 minutes $= 2.75$ hours.

$d = 1.4 \times 2.75 = 3.85$ km.

e. The equation for day 1 has a y-intercept of 0 because the travellers had not travelled any distance at the moment of starting. The equation for day 2 has a y-intercept of 6.3 because the travellers had already travelled 6.3 km by the end of day 1.

f. Suggested option 1: Plenty of drinking water.

Suggested option 2: Sunglasses, cap and sunscreen to prevent sunburn.

Note: There are other possible options.

3. a. Tank one: $y = -95x + 100\,000$

Tank two: $y = -75x + 95\,000$

b. The point of intersection represents the time it takes for the two tanks to contain the same amount of water, that is it takes 250 minutes for both tanks to contain 76 250 L of water.

c.

Minutes (x)	50	250	700	750	1050
Litres (y)	95 250	76 250	33 500	28 750	250

d. It will take approximately 1053 minutes for tank one to reach zero litres. This is the x-intercept at (1053,0).

e. Using the point (1000,20 000), $20\,000 = -75 \times 1000 + c$; $c = 95\,000$.

f. Suggested option 1: Plant the crops where they would be exposed to a lot of sunlight.

Suggested option 2: Use the right kind and amount of fertiliser.

Note: There are other possible options.

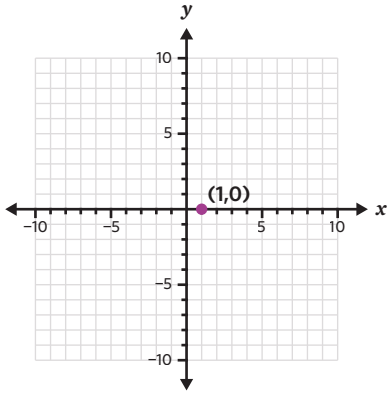
Chapter 9 review

Multiple choice

1. B **2.** D **3.** A **4.** E **5.** E

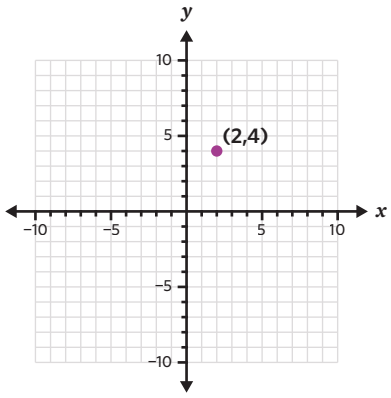
Fluency

6. a.



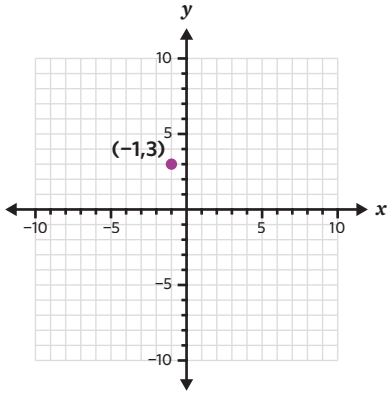
(1,0) is on the x-axis

b.



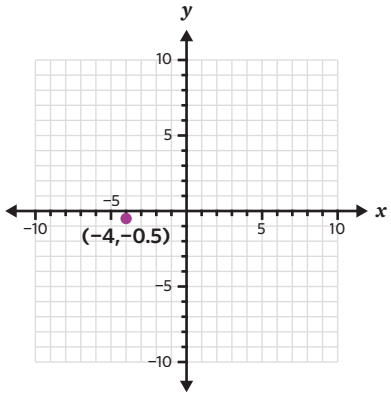
(2,4) is in Quadrant 1

c.



(−1,3) is in Quadrant 2

d.



(−4,−0.5) is in Quadrant 3

7. a.

x	-2	-1	0	1	2
y	0	2	4	6	8

b.

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5

c.

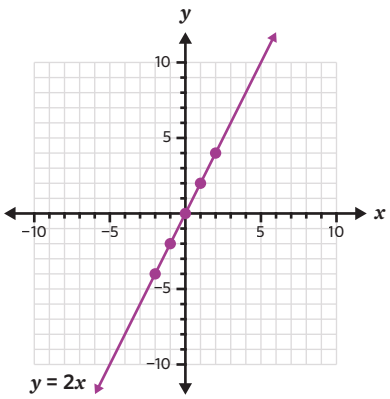
x	-2	-1	0	1	2
y	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4

d.

x	-2	-1	0	1	2
y	$5\frac{1}{3}$	$4\frac{2}{3}$	4	$3\frac{1}{3}$	$2\frac{2}{3}$

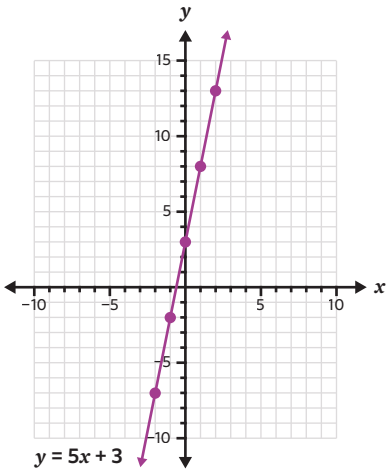
8. a.

x	-2	-1	0	1	2
y	-4	-2	0	2	4



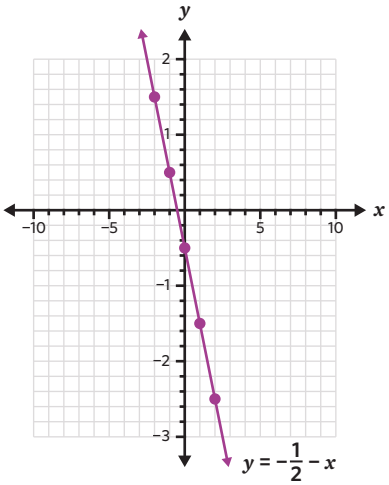
b.

x	-2	-1	0	1	2
y	-7	-2	3	8	13



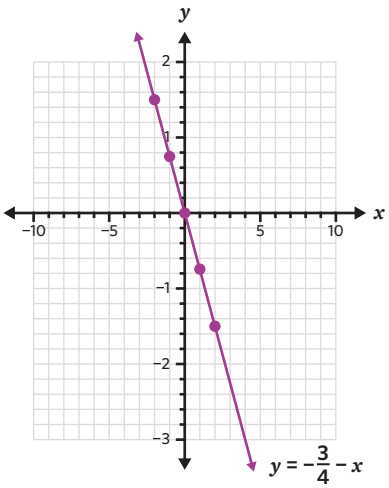
c.

x	-2	-1	0	1	2
y	$1\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-1\frac{1}{2}$	$-2\frac{1}{2}$



d.

x	-2	-1	0	1	2
y	$\frac{3}{2}$	$\frac{3}{4}$	0	$-\frac{3}{4}$	$-\frac{3}{2}$



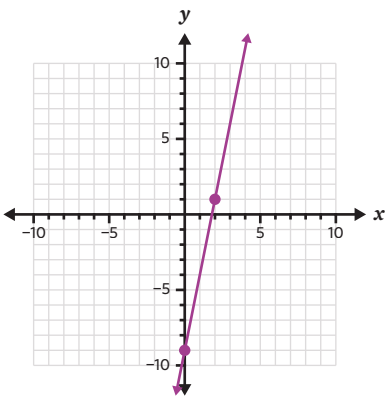
9. a. $y = 2x + 1$ **b.** $y = \frac{2}{3}x - 4$

c. $y = -\frac{1}{2}x - 3$ **d.** $y = -3x + 5$

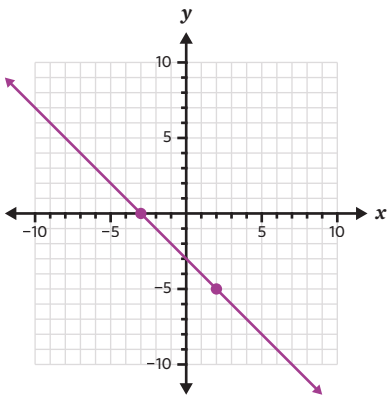
10. a. 0 **b.** 4 **c.** $\frac{3}{5}$ **d.** $-\frac{3}{7}$

11. a. 2 **b.** $\frac{1}{3}$ **c.** -1 **d.** $-\frac{10}{9}$

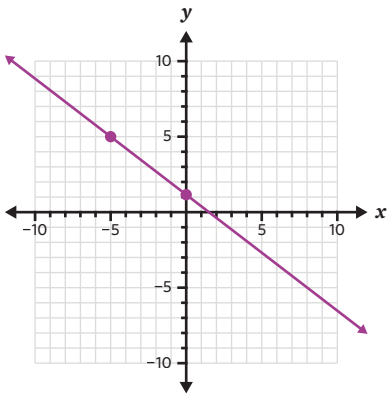
12. a.



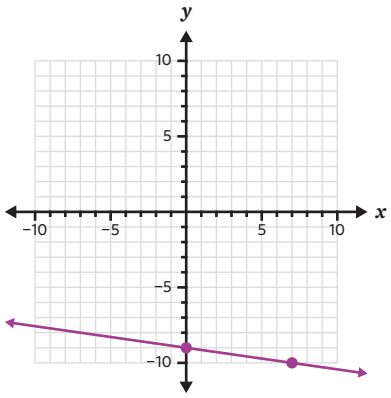
b.



c.



d.



13. a. x-intercept: $\left(\frac{2}{7},0\right)$, y-intercept: $(0,-2)$

b. x-intercept: $\left(\frac{8}{3},0\right)$, y-intercept: $(0,8)$

c. x-intercept: $(91,0)$, y-intercept: $(0,-13)$

d. x-intercept: $\left(-\frac{17}{57},0\right)$, y-intercept: $\left(0,\frac{17}{19}\right)$

14. a. $x = 2$ **b.** $x = -2$

c. $x = 0.5$ **d.** $x = -1.5$

15. a.

x	−3	−1	0	1	3
y	18	2	0	2	18

b.

x	−3	−1	0	1	3
y	−27	−3	0	−3	−27

c.

x	−3	−1	0	1	3
y	21	5	0	−3	−3

d.

x	−3	−1	0	1	3
y	−30	−8	0	6	12

Problem solving

16. A battleship game is represented on a Cartesian plane. A submarine is located at (7,11). An enemy submarine is located seven units west and five units south. State the ordered coordinates for the enemy submarine.

Key points

- A battleship game is represented on a Cartesian plane.
- A submarine is located at (7,11).
- An enemy submarine is located seven units west and five units south.
- State the ordered coordinates for the enemy submarine.

Explanation

Seven units west and five units south from the coordinate (7,11) yields the new coordinate $(7 - 7, 11 - 5) = (0,6)$.

Therefore, the enemy submarine is located at (0,6).

Answer

(0,6)

17. A large cup of boba from Ling Ling's Tea costs $6.80. A customer may choose to add up to six toppings, where each topping costs $0.50. Generate a table of values and a graph to display the cost of a cup of boba, C, with any given number of toppings, t,where $0 \leq t \leq 6$.

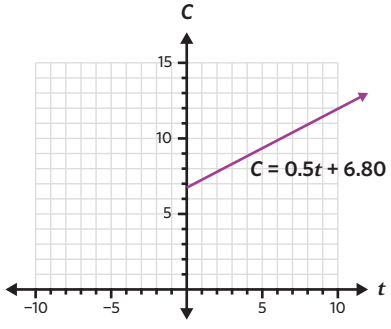
Key points

- A large cup of boba from Ling Ling's Tea costs $6.80.
- A customer may choose to add up to six toppings, where each topping costs $0.50.
- Generate a table of values and a graph to display the cost of a cup of boba, C, with any given number of toppings, t.

Explanation

As one cup of boba with no toppings costs $6.80, this means that when $t = 0$, $C = \$6.80$. We can generate a table of values as well as a graph as more toppings are added, which adds an extra $0.5t$ to the cost each time.

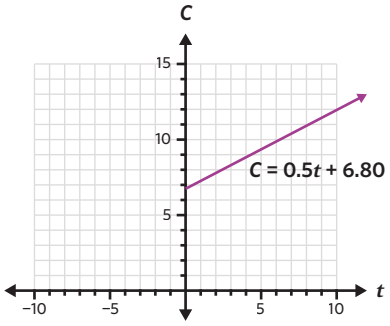
t	C
0	6.80
1	6.80 + 0.5(1) = 7.3
2	6.80 + 0.5(2) = 7.8
3	6.80 + 0.5(3) = 8.3
4	6.80 + 0.5(4) = 8.8
5	6.80 + 0.5(5) = 9.3
6	6.80 + 0.5(6) = 9.8



The function displayed in the graph is $C = 0.5t + 6.80$, where the gradient is 0.5 (the cost of a topping) and the y-intercept is 6.80 (the cost with no toppings).

Answer

t	C
0	6.80
1	$6.80 + 0.5(1) = 7.3$
2	$6.80 + 0.5(2) = 7.8$
3	$6.80 + 0.5(3) = 8.3$
4	$6.80 + 0.5(4) = 8.8$
5	$6.80 + 0.5(5) = 9.3$
6	$6.80 + 0.5(6) = 9.8$



18. Joey and Jack are keeping track of the velocity, in cm/s, of a projectile in a physics experiment. The following table displays their record of the projectile's instantaneous velocity, v, at a given time, t.

Determine v when $t = 0$ and explain what the value represents.

Key points

- Joey and Jack are keeping track of the velocity, in cm/s, of a projectile in a physics experiment.
- The following table displays their record of the projectile's instantaneous velocity, v, at a given time, t.
- Determine v when $t = 0$ and explain what the value represents.

Explanation

Identify the gradient between the two points (3,42) and (6,54) in the table. As $\frac{54 - 42}{6 - 3} = \frac{12}{3} = 4$, the gradient is 4.

Solve for the unknown value v_0 when $t = 0$, using the two coordinates $(0,v_2)$ and (3,42), and the fact that the gradient between two points is 4:

$$\frac{42 - v_0}{3 - 0} = 4$$

$$42 - v_0 = 12$$

$$v_0 = 30$$

Therefore, when $t = 0$, $v = 30$. This means that the projectile has an initial velocity of 30 cm/s.

Answer

$v = 30$ cm/s, which represents the initial velocity of the projectile when Joey and Jack first begin their experiment.

19. Poh and Lee are competing in a dumpling eating competition. The number of dumplings eaten for each of them are displayed in the following graph. Determine the gradient of each line and explain what they represent.

Key points

- Poh and Lee are competing in a dumpling eating competition.
- The number of dumplings eaten for each of them are displayed in the following graph.
- Determine the gradient of each line and explain what they represent.

Explanation

To find the gradient of Poh's line, we use the coordinates (1,4) and (2,8). The gradient is calculated by $\frac{8 - 4}{2 - 1} = 4$. This means that for each additional minute, Poh eats 4 dumplings.

To find the gradient of Lee's line, we use the coordinates (2,6) and (3,9). The gradient is calculated by $\frac{9 - 6}{3 - 2} = 3$. This means that for each additional minute, Lee eats 3 dumplings.

Answer

Poh's line: gradient $= 4$, and he consumes 4 dumplings every minute.

Lee's line: gradient $= 3$, and he consumes 3 dumplings every minute.

20. A tub of water initially has 5 L of water in it. Additional water is poured into it at a rate of $\frac{1}{5}$ L per second. Generate an equation in the gradient-intercept form to represent the volume of water in the tub at a given time, represented as t.

Key points

- A tub of water initially has 5 L of water in it.
- Additional water is poured into it at a rate of $\frac{1}{5}$ L per second.
- Generate an equation in the gradient-intercept form to represent the volume of water in the tub at a given time, represented as t.

Explanation

Let V be the volume and t be the time in seconds.

The y-intercept of the equation is 5 as this is the volume of water in litres at time $t = 0$.

The gradient is $\frac{1}{5}$ as for every second t that passes, $\frac{1}{5}$L is poured in.

Therefore, the equation in gradient-intercept form is $V = \frac{1}{5}t + 5$.

Answer

$V = \frac{1}{5}t + 5$

21. The neighbourhood around Khemarak Phoumin Street and Hun Sen Street is represented on a Cartesian plane. The two streets are represented by the following equations:

Khemarak Phoumin Street: $-7x + y = 13$

Hun Sen Street: $-3x + 6y = 0$

Using the gradient-intercept method, graph the two lines to determine where the two boulevards intersect.

Key points

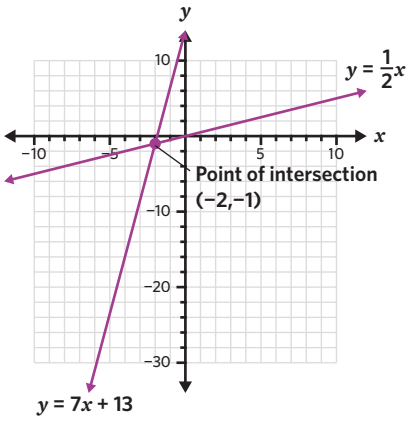
- The neighbourhood around Khemarak Phoumin Street and Hun Sen Street is represented on a Cartesian plane.
- The two streets are represented by the following equations: Khemarak Phoumin Street: $-7x + y = 13$, Hun Sen Street: $-3x + 6y = 0$.
- Using the gradient-intercept method, graph the two lines to determine where the two streets intersect.

Explanation

Convert the Khemarak Phoumin Street equation to gradient-intercept form by adding $7x$ to both sides, obtaining $y = 7x + 13$.

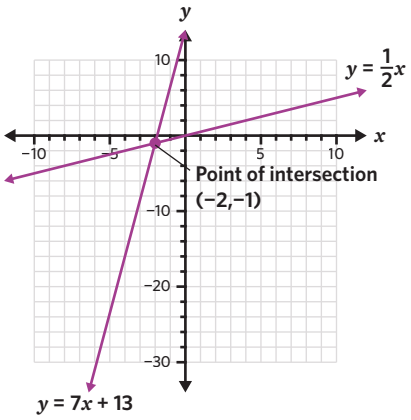
Similarly, the Hun Sen Street equation is converted by adding $3x$ to both sides and dividing every term by 6, obtaining $y = \frac{3}{6}x$ or $y = \frac{1}{2}x$.

The lines are displayed in the following graph:



The graph shows that both lines meet at the point $(-2,-1)$.

Answer



The streets meet at $(-2,-1)$.

22. Jerry is trying to send his cousin, Muscle Mouse, who lives next door, a letter that he had stored in a glass bottle. To send the letter, Jerry has to slingshot the bottle so that it flies over the fence and lands on the other side. Tom, who is 0.7 m tall, is standing 5 m east of Jerry next to a fence. Using the diagram below, determine how high Tom needs to jump in order to catch the bottle.

Key points

- To send the letter, Jerry has to slingshot the bottle so that it flies over the fence and lands on the other side.
- Tom, who is 0.7 m tall, is standing 5 m east of Jerry next to a fence.
- Using the diagram below, determine how high Tom needs to jump in order to catch the bottle.

Explanation

Looking at the diagram, Jerry's position is (0,0) on the Cartesian plane. Since Tom is standing 5 m east of Jerry next to a fence, Tom's position will be (5,0) on the x-axis.

Looking at the trajectory of the graph, when $x = 5, y = 1$, meaning that at Tom's position, the bottle will be 1 m above the ground.

Knowing that Tom is 0.7 m tall, he will need to jump $1 - 0.7 = 0.3$ m in order to catch the bottle.

Answer

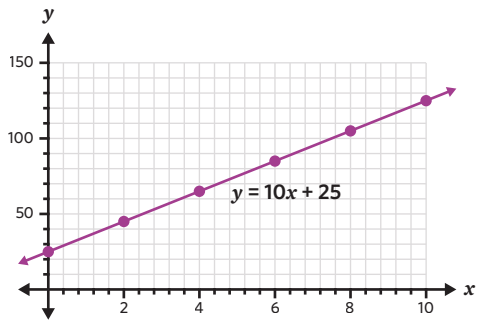
Tom needs to jump 0.3 m in order to catch the bottle.

Reasoning

23. a.

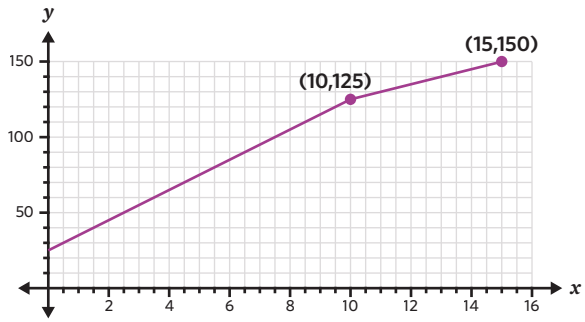
Time (m)	Temperature (T)
0	25
1	35
2	45
3	55
4	65
5	75
6	85
7	95
8	105
9	115
10	125

b.



The rule is $y = 10x + 25$.

c.



The gradient between the two points is 5.

The gradients found in part **b** and part **c** represent the change in internal temperature of the oven as each minute passes. The discrepancy in the gradient indicates that the internal temperature did not rise consistently.

d. It takes 20 minutes for the oven to return to room temperature.

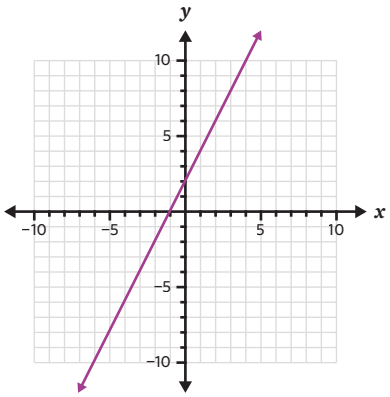
e. Suggested option 1: It is important to test the internal temperature of an oven for better, more consistent results when cooking.

Suggested option 2: It is important to test the internal temperature of an oven because if the temperature is inaccurate, this can result in food being undercooked which may be dangerous to consume.

Note: There are other possible options.

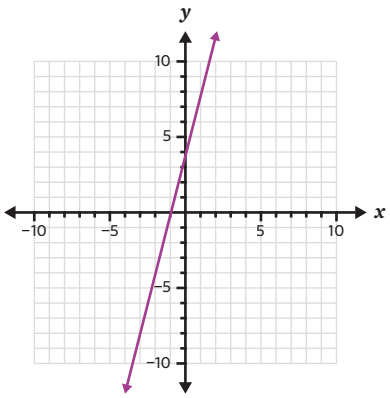
24. a.

x	−2	−1	0	1	2
y	−2	0	2	4	6



b.

x	−2	−1	0	1	2
y	−8	−3	2	7	12



c. The gradients in the graphs increase going from parts **a** to **b**. Comparing the graphs, we can see that as the gradient increases, the slope of the linear line also increases and becomes steeper.

10A Introduction to probability

Student practice

Worked example 1

a. S = {1, 2, 3, 4, 5, 6}

b. $\frac{1}{6}$ **c.** $\frac{1}{3}$

Understanding worksheet

1. **a.** Likely **b.** Certain **c.** Unlikely **d.** Even

2. **a.** True **b.** False **c.** True **d.** False

3. experiment; sample space; outcome; trial

Fluency

4. **a.** S = {P, R, O, B, A, I, L, T, Y}

b. $\frac{1}{11}$ **c.** $18.\overline{18}\%$ **d.** False **e.** $0.\overline{36}$

f. Choosing the letter X.

Note: There are other possible options.

---

5. **a.** S = {purple, grey, red}

b. $\frac{1}{5}$ **c.** 30% **d.** True **e.** 0.7

f. Picking a grey or purple counter.

---

6. **a.** S = {green, red, blue, yellow}

b. $\frac{1}{8}$ **c.** 25% **d.** True **e.** 0.625

f. Landing on red.

---

7. **a.** S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

b. $\frac{1}{10}$ **c.** 70% **d.** False **e.** 0.4

f. Generating a number that is a factor of 7.

Note: There are other possible options.

---

8. **a.** S = {6, A, J, K, Q, 2}

b. $\frac{1}{7}$ **c.** 43% **d.** False **e.** $\frac{2}{7}$

f. Picking a picture card.

Note: There are other possible options.

---

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. In an experiment, a group of people are randomly surveyed and are asked for the month they were born in. List all the possible outcomes of the experiment.

Key points

- A group of people are randomly surveyed and are asked for the month they were born in.
- List all the possible outcomes of the experiment.

Explanation

The possible outcomes are the 12 months of the year.

The possible outcomes are: January, February, March, April, May, June, July, August, September, October, November and December.

Answer

S = {January, February, March, April, May, June, July, August, September, October, November, December}

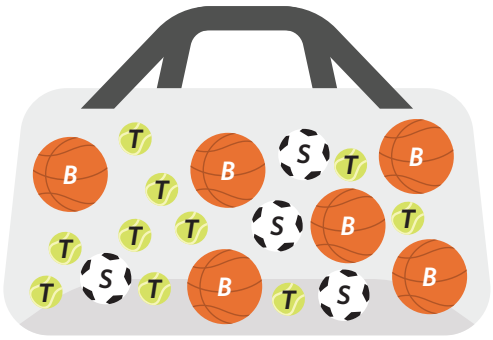
12. 10 tennis balls, 4 soccer balls and 6 basketballs are in a sports bag. What is the probability of picking a soccer ball?

Key points

- A sports bag has 10 tennis balls.
- It also has 4 soccer balls.
- It also has 6 basketballs.
- What is the probability of picking a soccer ball?

Explanation

Determine the number of soccer balls and the total number of balls in the sports bag.



Number of soccer balls = 4

Total number of balls in a sports bag = 10 + 4 + 6 = 20

Calculate the probability:

$$\Pr(\text{soccer ball}) = \frac{\text{number of soccer balls}}{\text{total number of balls}}$$
$$= \frac{4}{20}$$
$$= \frac{1}{5}$$

Answer

$\Pr(\text{soccer ball}) = \frac{1}{5}$

13. In a class of 24 students, 8 students have 1 sibling, 10 students have 2 siblings and 6 students had 3 or more siblings. Calculate the likelihood of selecting a random student who has 2 or more siblings.

Key points

- The class has 24 students in total.
- 8 students have 1 sibling.
- 10 students have 2 siblings.
- 6 students have 3 or more siblings.
- Calculate the likelihood of selecting a random student who has 2 or more siblings.

Explanation

Determine the number of students with 2 or more siblings and the total number of students.

The number of students with 2 or more siblings includes those with 2 siblings and those with 3 or more.

Number of students with 2 or more siblings = 10 + 6 = 16

Total number of students = 24

Calculate the probability:

Pr(2 or more siblings)

$= \frac{\text{number of students with 2 or more siblings}}{\text{total number of students}}$

$= \frac{16}{24}$

$= \frac{2}{3}$

Answer

$\Pr(\text{2 or more siblings}) = \frac{2}{3}$

14. James has 2 boxes of chocolate contain the following chocolates:

Chocolates	Box 1	Box 2
Freddo Frogs	6	8
Caramello Koala	4	5
Mars bar	2	4

Which box is James most likely to pick a Mars bar?

Key points

- There are 6 Freddo Frogs in box 1 and 8 in box 2.
- There are 4 Caramello Koalas in box 1 and 5 in box 2.
- There are 2 Mars bars in box 1 and 4 in box 2.
- Which box is James most likely to pick a Mars bar?

Explanation

Calculate the probabilities of picking a Mars bar in box 1 and box 2.

Box 1:

Number of Mars bars in box 1: 2

Total number of chocolates in box 1: $2 + 4 + 6 = 12$

$\Pr(\text{Mars bar from box 1}) = \frac{2}{12} = \frac{1}{6}$

Box 2:

Number of Mars bars in box 2: 4

Total number of chocolates in box 2: $8 + 5 + 4 = 17$

$\Pr(\text{Mars bar from box 2}) = \frac{4}{17}$

Compare the 2 probabilities:

$\frac{4}{17}$ is greater than $\frac{1}{6}$. So, James is more likely to pick a Mars bar from box 2.

Answer

James is more likely to pick a Mars bar from box 2.

15. In a gameshow, the numbers 2, 6 and 9 are written on cards and put in a hat. 1 card is chosen and then put back in the hat, and another card is selected. The 2 numbers are added together. A winning score is a sum less than 7. How many times are you more likely to lose compared to winning?

Key points

- In a gameshow, the numbers 2, 6 and 9 are written on cards and put in a hat.
- 1 card is chosen and then put back in the hat, and another card is selected.
- The 2 numbers are added together.
- You win if the sum is less than 7.
- How many times more likely are you to lose compared to winning?

Explanation

The table shows all 9 possible outcomes of adding 2, 6, and 9.

+	2	6	9
2	4	8	11
6	8	12	15
9	11	15	18

There is only 1 outcome out of the 9 possible, where the sum is less than 7, i.e. $2 + 2 = 4$.

∴ The probability of winning is:

$\Pr(\text{winning}) = \frac{\text{sum less than 7}}{\text{total possible outcomes}} = \frac{1}{9}$

The probability of losing is:

$\Pr(\text{losing}) = \frac{\text{sum greater than 8}}{\text{total possible outcomes}} = \frac{8}{9}$

As $\frac{8}{9} = 8 \times \frac{1}{9}$, it is 8 times more likely to lose than win.

Answer

The probability of losing is 8 times that of winning.

Reasoning

16. a. S = {green, yellow, purple, red}

b. The probability of the player winning is $\frac{1}{2}$.

c. The probability of the player losing is $\frac{1}{2}$. Therefore the game is fair.

d. In this game, the probability of winning is now $\frac{5}{12}$ and the probability of losing is $\frac{7}{12}$. As there is not an equal chance to win, this game is not fair.

e. Fair game: the player wins if the spinner lands on yellow, and loses if it lands on green.

Unfair game: the player wins if the spinner lands on purple, and loses if it lands on yellow, green or red.

Note: There are other possible options.

17. a.

	Bag A	Bag B
Yellow	2	3
Black	6	9

Note: There are other possible options.

b.

	Bag A	Bag B
Yellow	6	4
Black	3	8

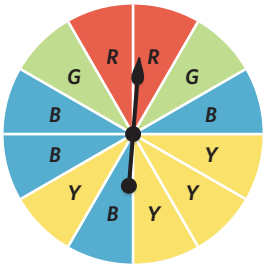
Note: There are other possible options.

c. Probabilities are fractions of the total number of marbles, rather than how many of each marble there is.

Extra spicy

18. There are 9 blue marbles.

19.



20. E **21.** B

Remember this?

22. E **23.** E **24.** E

10B Complementary events

Student practice

Worked example 1

a. Picking out letters that are not vowels (C, M, P, L, N, T).

b. Picking out coins in the wallet that are 5c, 10c, or $1.

Worked example 2

a. $\Pr(\text{red}') = \frac{2}{3}$ **b.** $\Pr(\text{green}) = \frac{1}{2}$

Understanding worksheet

1. **a.** Complementary **b.** Not complementary

c. Complementary **d.** Not complementary

2. **a.** Flipping a tail.

b. Rolling a 1, 2, 3, 5, 6.

c. Picking a red or green marble out of a bag.

d. Generating 4, 6, 8, or 10.

3. complementary; outcomes; probabilities; sum

Fluency

4. **a.** Picking a blue counter.

b. Picking a lime or raspberry icy pole from the bag of icy poles.

c. Picking a black card from a standard deck of 52 cards.

d. Rolling a die and rolling an odd number.

e. Rolling a die and rolling 1, 2, 3, 4, 6.

f. Rolling a die and rolling 2, 4, 6.

g. Randomly picking a number greater than or equal to 6 and less than or equal to 15.

h. Randomly picking a number greater than or equal to 3 and less than or equal to 10.

---

5. **a.** Picking yellow or white.

b. $\Pr(\text{black}') = 0.8$

c. $\Pr(\text{white}) = 0.4$

d. $\Pr(\text{not white or black}) = 0.4$

e. True

---

6. **a.** Picking green, red or orange.

b. $\Pr(\text{blue}') = \frac{4}{5}$

c. $\Pr(\text{orange}) = 0$

d. $\Pr(\text{not orange or blue}) = \frac{4}{5}$

e. False

---

7. **a.** Picking a heart, club or diamond.

b. $\Pr(\text{spade}') = 75\%$

c. $\Pr(\text{queen}) = \frac{1}{13}$

d. $\Pr(\text{queen}') = \frac{12}{13}$

e. $\Pr(\text{even or picture card}) = \frac{8}{13}$

f. $\Pr(\text{not even or a picture card}) = \frac{5}{13}$

---

8. D

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student B is incorrect.

Problem solving

10. On one day at a train station, a train was cancelled. State the outcomes of the complementary event.

Key points

- On one day at a train station, a train was cancelled.
- State the outcomes of the complementary event.

Explanation

S = {train cancelled, train not cancelled}

The complementary event of the train being cancelled is the train not being cancelled.

Answer

The complementary event is the train not being cancelled.

11. A store selling PlayStation 5 consoles states that the probability of the store running out of PS5s is 0.65. What is the probability that they will not run out?

Key points

- The probability of the store running out of PS5s is 0.65.
- What is the probability that they will not run out?

Explanation

S = {store running out of PS5, store not running out of PS5}

The complementary event of the store running out of PS5 is the event of the store not running out of PS5.

The probability of two complementary events sum to 1.

$\Pr(\text{store not running out of PS5}) = 1 - 0.65 = 0.35$

Answer

The probability that the store will not run out of PS5 is 0.35.

12. The probability that Max rides his bike to work is $\frac{1}{3}$. The probability that he takes the tram to work is $\frac{1}{2}$. Calculate the probability that he will neither ride his bike nor take the tram to work.

Key points

- The probability that Max rides his bike to work is $\frac{1}{3}$.
- The probability that he takes the tram to work is $\frac{1}{2}$.
- Calculate the probability that he will neither ride his bike nor take the tram to work.

Explanation

S = {bike, tram, neither bike nor tram}

$$\begin{aligned}\Pr(\text{neither bike nor tram}) &= 1 - \Pr(\text{bike}) - \Pr(\text{tram}) \\ &= 1 - \frac{1}{3} - \frac{1}{2} \\ &= \frac{1}{6}\end{aligned}$$

Answer

The probability that he will neither ride his bike nor take the tram to work is $\frac{1}{6}$.

13. The Year 8 students at a school were surveyed for their blood type (A, B, AB and O). The results are shown below:

If a student was chosen at random, find the probability that they are not blood type A or AB, written as a percentage.

Key points

- The Year 8 students at a school were surveyed for their blood type (A, B, AB and O). The results are shown in the table.
- If a student was chosen at random, find the probability that they are not blood type A or AB, written as a percentage.

Explanation

The total number of students is $76 + 20 + 6 + 98 = 200$.

$\Pr(\text{type A}) + \Pr(\text{type AB}) = \frac{76}{200} + \frac{6}{200} = \frac{82}{200}$

$\Pr(\text{not type A or type AB}) = 1 - \frac{82}{200} = \frac{118}{200} = \frac{59}{100} = 59\%$

Answer

The probability that the student is not blood type A or AB is 59%.

14. Patrick's 'On repeat' playlist has songs from the following artists: Two Door Cinema Club (5), Last Dinosaurs (7), Gordi (10) and Maggie Rogers (8).

Which artist has a probability of not playing next equal to $\frac{5}{6}$?

Key points

- Patrick's 'On repeat' playlist has songs from the following artists: Two Door Cinema Club (5), Last Dinosaurs (7), Gordi (10) and Maggie Rogers (8).
- Which artist has a probability of not playing next equal to $\frac{5}{6}$?

Explanation

The probabilities of each event are:

$\Pr(\text{Two Door Cinema Club}) = \frac{5}{30} = \frac{1}{6}$

$\Pr(\text{Last Dinosaurs}) = \frac{7}{30}$

$\Pr(\text{Gordi}) = \frac{10}{30} = \frac{1}{3}$

$\Pr(\text{Maggie Rogers}) = \frac{8}{30} = \frac{4}{15}$

$\Pr(\text{playing next}) = 1 - \Pr(\text{not playing next})$

$= 1 - \frac{5}{6}$

$= \frac{6}{6} - \frac{5}{6}$

$= \frac{1}{6}$

Two Door Cinema Club has a probability of playing next equal to $\frac{1}{6}$.

So the probability of Two Door Cinema Club not playing next is $\frac{5}{6}$.

Answer

Two Door Cinema Club.

Reasoning

15. a. For die 1, the event complementary to rolling a 3 is rolling a 1 or a 4. For die 2, the event complementary to rolling a 3 is rolling a 1, 5 or a 6.

b. For die 1, $\Pr(3') = \frac{1}{2}$. For die 2, $\Pr(3') = 0.80$

c. Die 1

d. For die 1, there are four sides. So the probability of rolling one side is $\frac{1}{4} = 0.25$. $\Pr(3)$ is $\frac{1}{2}$ which is twice the probability of rolling one side. So 3 is on two sides.

For die 2, there are 5 sides. So the probability of rolling one side is $\frac{1}{5} = 0.2 = 20\%$. $\Pr(6)$ is 40% which is twice the probability of rolling one side. So 6 is on two sides.

e. Suggested answer 1: Dungeons and Dragons.

Suggested answer 2: Monopoly.

Note: There are other possible options.

16. a. Picking an orange counter or a green counter, and picking a blue counter.

Note: There are other possible options.

b. Picking an orange counter or a green counter, and picking an orange counter or a blue counter.

Note: There are other possible options.

c. Complementary events are always mutually exclusive. For example, the complementary event to picking an orange or a green counter is picking a blue counter. The first event only has the outcomes picking an orange or green counter, and the second event only has the outcome picking a blue counter. There is no overlap, so the events are mutually exclusive.

Extra spicy

17. D **18.** $\frac{5}{6}$ **19.** E **20.** $\frac{13}{15}$

Remember this?

21. D **22.** B **23.** C

10C Venn diagrams and two-way tables

Student practice

Worked example 1

a. 13 **b.** 10 **c.** $\frac{7}{20}$

Worked example 2

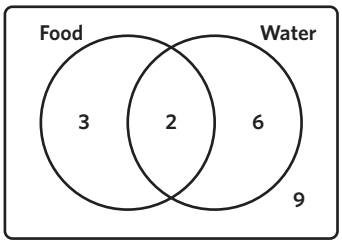
a. 10 **b.** 17 **c.** $\frac{7}{22}$

Worked example 3

a.

	Enough water	Not enough water	Total
Enough food	2	3	5
Not enough food	6	9	15
Total	8	12	20

b.



Understanding worksheet

1. **a.** 2 **b.** 24 **c.** 4 **d.** 16

2. **a.** 3 **b.** 2 **c.** 8; 4 **d.** 17; 4

3. Venn diagram; two-way table; inclusive; exclusive

Fluency

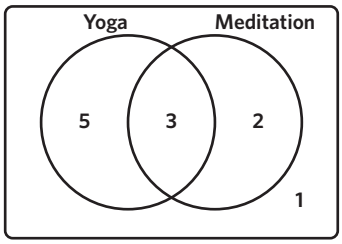
4. **a.** 10 **b.** 9 **c.** 5 **d.** 12
e. 13

5. **a.** $\frac{4}{29}$ **b.** $\frac{6}{29}$ **c.** $\frac{15}{29}$ **d.** $\frac{13}{29}$
e. $\frac{19}{29}$

6. **a.** 3 **b.** 5 **c.** 13 **d.** 7
e. 10

7. **a.** $\frac{1}{6}$ **b.** $\frac{2}{7}$ **c.** $\frac{19}{42}$ **d.** $\frac{17}{21}$
e. $\frac{23}{42}$

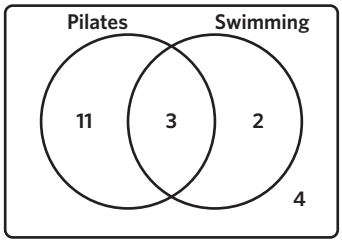
8. **a.** Venn diagram



Two-way table

	Meditation	Not meditation	Total
Yoga	3	5	8
Not yoga	2	1	3
Total	5	6	11

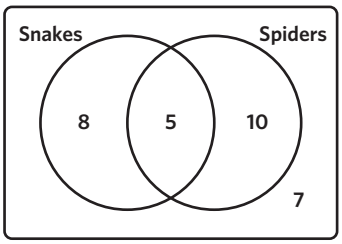
b. Venn diagram



Two-way table

	Swimming	Not swimming	Total
Pilates	3	11	14
Not pilates	2	4	6
Total	5	15	20

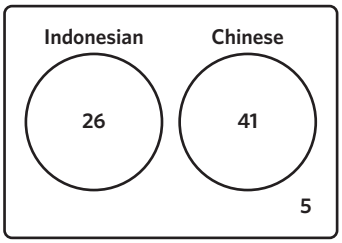
c. Venn diagram



Two-way table

	Spiders	Not spiders	Total
Snakes	5	8	13
Not snakes	10	7	17
Total	15	15	30

d. Venn diagram



Two-way table

	Chinese	Not Chinese	Total
Indonesian	0	26	26
Not Indonesian	41	5	46
Total	41	31	72

9. B

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

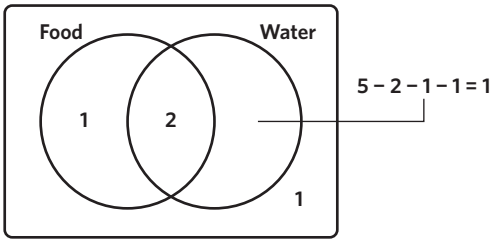
11. Five friends go to a concert. Two people buy food and water, one person buys food only and one person does not buy anything. How many people only bought water?

Key points

- 5 friends go to a concert.
- 2 people buy food and water, 1 person buys food only and 1 person does not buy anything.
- How many people only bought water?

Explanation

Draw a Venn diagram to show the given information.



Answer

One person only bought water.

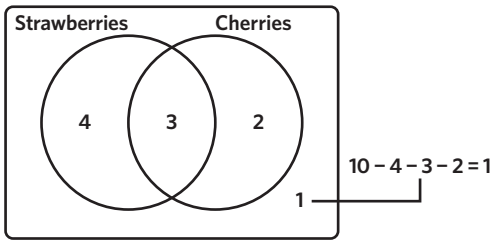
12. A family of 10 go to pick strawberries and cherries together. If four members only picked strawberries, two members only picked cherries and three members picked both, how many family members did not pick either?

Key points

- A family of 10 go to pick strawberries and cherries together.
- Four members only picked strawberries, two members only picked cherries and three members picked both.
- How many family members did not pick either?

Explanation

Draw a Venn diagram to show the given information.



Answer

One family member did not pick strawberries or cherries.

13. Leonora is the sports captain at her school and surveys students to see how many teams can be formed for basketball and netball next term. Of the 100 students surveyed, 50 said they would be interested in basketball and 45 said they would be interested in netball. If 25 students did not express interest for either, how many students said they would be interested in both?

Key points

- Leonora is the sports captain at her school and surveys students to see how many teams can be formed for basketball and netball next term.
- Of the 100 students surveyed, 50 said they would be interested in basketball and 45 said they would be interested in netball.
- 25 students did not express interest for either.
- How many students said they would be interested in both?

Explanation

Draw a two-way table to show the given information and fill in the missing boxes.

	Netball	**Not netball**	**Total**
Basket-ball	$45 - 25 = 20$	$50 - 20 = 30$	50
Not basket-ball	$50 - 25 = 25$	25	$100 - 50 = 50$
Total	45	$30 + 25 = 55$	100

Answer

20 students said they would be interested in both basketball and netball.

14. In his after hours painting class, Pablo counts 4 people with short hair and 8 with long hair. For the same group of 12 people, there are 7 who bring their own paint brushes and 5 who borrow ones from the art studio. If there are 2 people with short hair who bring their own paint brushes, what is the probability that a person selected at random has long hair or borrows paint brushes from the studio, including those that identify with both?

Key points

- In his after hours painting class, Pablo counts 4 people with short hair and 8 with long hair.
- For the same group of 12 people, there are 7 who bring their own paint brushes and 5 who borrow ones from the art studio.
- There are 2 people with short hair who bring their own paint brushes.
- What is the probability that a person selected at random has long hair or borrows paint brushes from the studio, including those that identify with both?

Explanation

Draw a two-way table to show the given information and fill in the missing boxes.

	Bring brushes	**Borrows brushes**	**Total**
Long hair	$7 - 2 = 5$	$5 - 2 = 3$	8
Short hair	2	$4 - 2 = 2$	4
Total	7	5	12

There are $5 + 3 + 2 = 10$ people who have long hair or borrow brushes.

$$\Pr(\text{long hair or borrows brushes or both}) = \frac{10}{12} = \frac{5}{6}$$

Answer

The probability that a person selected at random has long hair or borrows paint brushes from the studio, including those that identify with both is $\frac{5}{6}$.

15. Mr Bjarnelind wants to shortlist international cities to visit with his family. He looks at two factors in making his decision, the duration of the flight and whether the city is child-friendly. Mr B's research shows that of the 20 cities considered, 5 cities have flights that are ten hours or less and 13 cities are child-friendly. If there are 4 cities that are not child-friendly and have flights over ten hours, what is the probability of choosing a city that is child friendly but has a flight that is more than ten hours?

Key points

- Mr Bjarnelind wants to shortlist international cities to visit with his family. He looks at two factors in making his decision, the duration of the flight and whether the city is child-friendly.
- Mr B's research shows that of the 20 cities considered, 5 cities have flights that are ten hours or less and 13 cities are child-friendly.
- There are 4 cities that are not child-friendly and have flights over ten hours.
- What is the probability of choosing a city that is child friendly but has a flight that is more than ten hours?

Explanation

Draw a two-way table to show the given information and fill in the missing boxes.

	Child-friendly city	**Non-child-friendly**	**Total**
Flights that are 10 hours or less	$5 - 3 = 2$	$7 - 4 = 3$	5
Flights more than 10 hours	$13 - 2 = 11$	4	$20 - 5 = 15$
Totals	13	$20 - 13 = 7$	20

There are $5 + 3 + 2 = 10$ people who have long hair or borrow brushes.

Pr(child-friendly city that has a flight more than 10 hours) $= \frac{11}{20}$

Answer

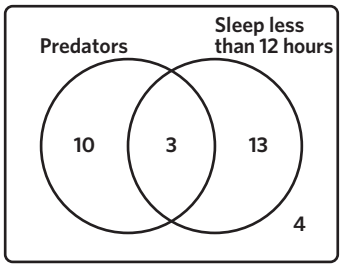
The probability of choosing a city that is child friendly but has a flight that is more than 10 hours is $\frac{11}{20}$.

Reasoning

16. a. 4 animals sleep at least 12 hours a day and are not predators.

b. The probability is $\frac{23}{30}$.

c.



d. Taylen might have calculated his incorrect answer because he counted the animals that only sleep less than 12 hours. Instead he should have counted both the '3' and the '13' within the circle that is labelled 'sleep less than 12 hours'.

e. Suggested answer 1: Predators use a lot of energy when hunting for prey, and may need more time to recover from this.

Suggested answer 2: Predators being asleep for more hours may give prey animals a better chance at survival.

Note: There are other possible options.

17. a. Venn diagram 2 shows the exclusive 'or'.

b. Venn diagram 1 shows the inclusive 'or'.

c. Venn diagram 1 shows 'A or B'.

Venn diagram 2 shows 'only A or only B' or 'A or B but not both'.

Extra spicy

18. $\frac{2}{5}$ **19.** 2 **20.** B **21.** C

Remember this?

22. B **23.** C **24.** D

10D Tree diagrams and multi-step experiments

Student practice

Worked example 1

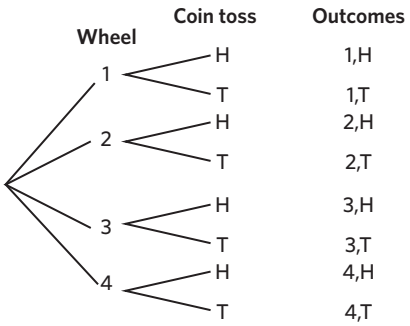
a. 4 **b.** $\frac{1}{4}$ **c.** $\frac{3}{4}$

Worked example 2

a. 12 **b.** $\frac{1}{2}$ **c.** $\frac{1}{4}$

Worked example 3

a.



b.

Wheel spin \ Coin toss	H	T
1	1,H	1,T
2	2,H	2,T
3	3,H	3,T
4	4,H	4,T

Understanding worksheet

1. a. H,3 **b.** N **c.** 1,1 **d.** T,H,T

2. a. T,T **b.** B,2 **c.** 2,3 **d.** M,R

3. multi-step; outcomes; tree diagram; two-way table

Fluency

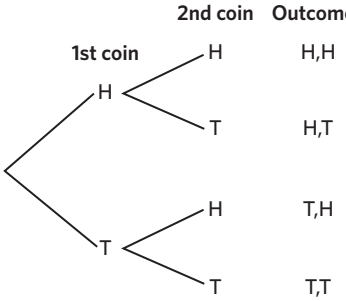
4. a. 6 **b.** $\frac{1}{6}$ **c.** $\frac{2}{3}$ **d.** 1 **e.** $\frac{1}{3}$

5. a. 9 **b.** $\frac{1}{9}$ **c.** $\frac{1}{3}$ **d.** $\frac{2}{3}$ **e.** $\frac{2}{3}$

6. a. 12 **b.** $\frac{1}{12}$ **c.** $\frac{1}{6}$ **d.** $\frac{1}{6}$ **e.** $\frac{1}{2}$

7. a. 36 **b.** 0 **c.** $\frac{5}{6}$ **d.** $\frac{11}{36}$ **e.** $\frac{5}{6}$

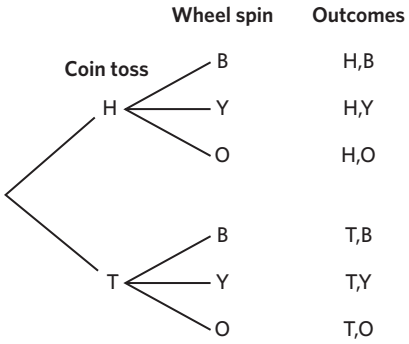
8. a. Tree diagram



Two-way table

	H	T
H	H,H	H,T
T	T,H	T,T

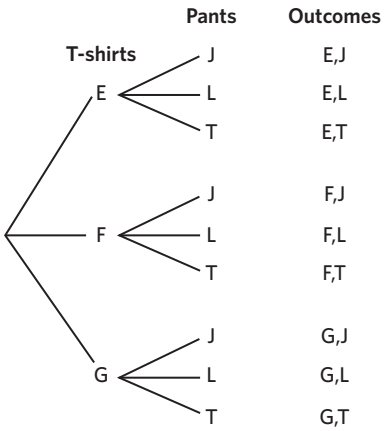
b. Tree diagram



Two-way table

	Blue (B)	Yellow (Y)	Orange (O)
H	H,B	H,Y	H,O
T	T,B	T,Y	T,O

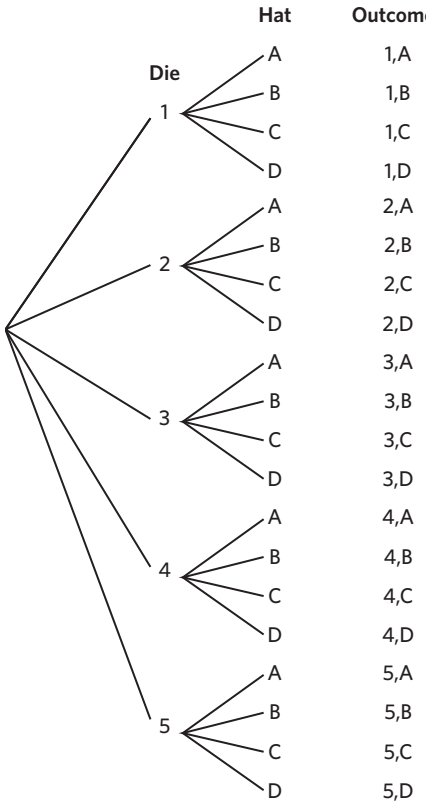
c. Tree diagram



Two-way table

	J	L	T
E	E,J	E,L	E,T
F	F,J	F,L	F,T
G	G,J	G,L	G,T

d. Tree diagram



Two-way table

	A	B	C	D
1	1,A	1,B	1,C	1,D
2	2,A	2,B	2,C	2,D
3	3,A	3,B	3,C	3,D
4	4,A	4,B	4,C	4,D
5	5,A	5,B	5,C	5,D

9. A

Spot the mistake

10. a. Student A is incorrect. **b.** Student A is incorrect.

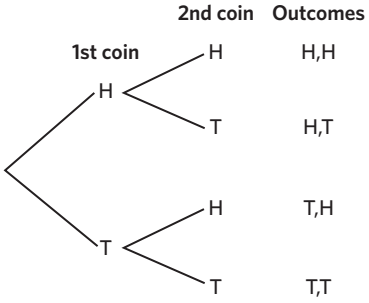
Problem solving

11. Scar tosses two coins at the same time to determine the fate of his minions. How many possible combinations exist?

Key points

- Scar tosses two coins at the same time to determine the fate of his minions.
- How many possible combinations exist?

Explanation



10D ANSWERS

Answer

4 combinations exist.

12. Nala plays two games of paper-scissors-rock with a friend. What is the probability of her friend choosing rock both times?

Key points

- Nala plays two games of paper-scissors-rock with a friend.
- What is the probability of her friend choosing rock both times?

Explanation

	S	P	R
S	S,S	S,P	S,R
P	P,S	P,P	P,R
R	R,S	R,P	R,R

Number of desired outcomes (R,R) $= 1$

Total number of outcomes $= 9$

$\Pr(\text{R,R}) = \frac{1}{9}$

Answer

The probability is $\frac{1}{9}$.

13. Sarabi throws two 6-sided dice in a game of Monopoly. How many possible combinations are there of getting the same number on both dice?

Key points

- Sarabi throws two 6-sided dice in a game of Monopoly.
- How many possible combinations are there of getting the same number on both dice?

Explanation

The desired outcomes include (1,1), (2,2), (3,3), (4,4), (5,5), (6,6).

$\therefore$ Number of desired outcomes $= 6$

Answer

There are 6 combinations possible.

14. Simba plays Lucky Wheel at the school fete, where there are 10 different wedges, each with a different prize. If Simba pays for two spins, what is the probability of the wheel landing on two different wedges?

Key points

- Simba plays Lucky Wheel at the school fete where there are 10 different wedges.
- Each wedge has a different prize.
- Simba pays for two spins.
- What is the probability of the wheel landing on two different wedges?

Explanation

Number of desired outcomes (two different wedges) $= 90$

Total number of outcomes $= 100$

$\Pr(\text{two different wedges}) = \frac{90}{100} = \frac{9}{10}$

Answer

The probability is $\frac{9}{10}$.

15. Pumba decides to dress up for the king's coronation. He has one red bow tie, one orange, one yellow, one white and one blue. He also has a number of hair ties to choose from. If there are 85 possible combinations of bow ties and hair ties, how many types of hair ties does Pumba have? Assume that there is one of each type of hair tie.

Key points

- Pumba decides to dress up for the king's coronation.
- He has one red bow tie, one orange, one yellow, one white and one blue.
- He also has a number of hair ties to choose from.
- There are 85 possible combinations of bow ties and hair ties.
- How many types of hair ties does Pumba have? Assume that there is one of each type of hair tie.

Explanation

There are 5 types of bow ties.

There are a total of 85 possible outcomes.

Total number of outcomes = types of bow ties $\times$ types of hair ties

Let x be the number of types of hair ties.

$5 \times x = 85$

$x = 17$

Answer

Pumba has 17 types of hair ties.

Reasoning

16. **a.** 32 **b.** 64 **c.** $\frac{1}{64}$ **d.** $\frac{1}{2}$

e. Suggested option 1: Dungeons and Dragons is a game that involves more skill than Uno or Pokemon Go which are games more based on luck.

Suggested option 2: Dungeons and Dragons has role-playing whereas Uno or Pokemon Go do not.

Note: There are other possible options.

17. **a.** There are 8 possible outcomes.

b. There are 16 possible outcomes.

c. The pattern is that each time, the number of outcomes doubles when an extra coin is tossed.

Extra spicy

18. $\frac{1}{2}$ **19.** 50% **20.** A **21.** B

Remember this?

22. B **23.** D **24.** E

10E Experimental probability

Student practice

Worked example 1

$\frac{3}{10}$

Worked example 2

a. **i.** $\frac{1}{6}$ **ii.** 2

b. **i.** $\frac{1}{3}$ **ii.** 10

Understanding worksheet

1. **a.** Experimental probability only

b. Experimental probability only

c. Theoretical probability

d. Experimental probability only

2. **a.** Frequency: 7; Trials: 14 **b.** Frequency: 4; Trials: 17
c. Frequency: 8; Trials: 9 **d.** Frequency: 18; Trials: 25

3. experimental; frequency; theoretical; trials.

Fluency

4. **a.** $\frac{1}{5}$ **b.** $\frac{1}{6}$ **c.** $\frac{1}{5}$ **d.** $\frac{9}{50}$
e. $\frac{19}{85}$ **f.** $\frac{89}{500}$

5. **a.** **i.** $\frac{1}{10}$ **ii.** 1 **b.** **i.** $\frac{1}{5}$ **ii.** 3 **c.** **i.** $\frac{1}{2}$ **ii.** 15 **d.** **i.** $\frac{4}{5}$ **ii.** 52
e. **i.** $\frac{2}{5}$ **ii.** 28 **f.** **i.** $\frac{7}{10}$ **ii.** 63

6. **a.** 16 **b.** $\frac{3}{16}$ **c.** 6 **d.** $\frac{9}{16}$
e. 54 **f.** $\frac{1}{2}$ **g.** 100

7. D

Spot the mistake

8. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. Cindy plays Fortnite and is checking the statistics of the games she has played in the last week. The results are shown below.

Outcome	Top 25	Between 26 and 75	Bottom 25
Frequency	5	15	10

Based on the table above, what is the experimental probability of Cindy being in the top 25 in her next match?

Key points

- The table shows the frequency of game outcomes.
- What is the experimental probability of Cindy being in the top 25 in her next match?

Explanation

Total number of games $= 5 + 15 + 10 = 30$

Experimental probability $= \frac{\text{number of top 25 games}}{\text{total number of games}}$

$= \frac{5}{30}$

$= \frac{1}{6}$

Outcome	Top 25
Frequency	5
Experimental probability	$\frac{1}{6}$

Answer

The experimental probability of Cindy being in the top 25 is $\frac{1}{6}$.

10. A shop sells four types of toasties: chicken schnitzel, smashed avocado, roast veggie and roast beef. Maddy predicts that the probability of a customer ordering the roast veggie toastie is $\frac{3}{10}$. If the shop has 150 toastie orders in one day, how many are expected to be roast veggie toasties?

Key points

- Maddy predicts that the probability of a customer ordering the roast veggie toastie is $\frac{3}{10}$.
- The shop has 150 toastie orders.
- How many toasties are expected to be roast veggie toasties?

Explanation

Theoretical probability $= \frac{3}{10}$

Number of trials $= 150$

Expected occurrence $=$ theoretical probability $\times$ number of trials

$= \frac{3}{10} \times 150$

$= 45$

Answer

45 toasties are expected to be roast veggie toasties.

11. Noah is spinning a spinner that he constructed.

He records the colour each time the spinner is spun. There were 10 reds, 23 blues, 4 greens and 8 yellows.

Is the experimental probability of landing on red or blue greater than, equal to or less than the theoretical probability?

Key points

- He records the colour each time the spinner is spun.
- There were 10 reds, 23 blues, 4 greens and 8 yellows.
- Is the experimental probability of landing on red or blue greater than, equal to or less than the theoretical probability?

Explanation

Calculate the theoretical probability of landing on red or blue using the spinner.

The spinner is made up of 8 equal sectors.

2 of these are red, and 4 of these are blue.

So $2 + 4 = 6$ are red or blue.

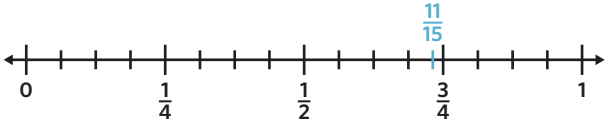
There are 8 sectors, so the theoretical probability is $\frac{6}{8} = \frac{3}{4}$.

Calculate the experimental probability using the recorded data.

10 reds and 23 blues were spun, so $10 + 23 = 33$ trials were red or blue.

The number of trials is $10 + 23 + 4 + 8 = 45$.

So the experimental probability is $\frac{33}{45} = \frac{11}{15}$.



	Experimental probability	**<, = or >**	**Theoretical probability**
Red or blue	$\frac{11}{15}$	$<$	$\frac{3}{4}$

$\frac{11}{15} < \frac{3}{4}$, so the experimental probability is less than the theoretical probability.

Answer

The experimental probability is less than the theoretical probability.

12. A factory that produces computer chips for smartphones estimates that the probability of a chip being faulty is 0.002. How many chips are expected to be faulty in a day if the factory produces 2000 per day?

Key points

- The theoretical probability of a chip being faulty is 0.002.
- The factory produces 2000 chips per day.
- How many chips are expected to be faulty in a day?

Explanation

Expected occurrence = theoretical probability × number of trials

$= 0.002 \times 2000$

$= 4$

Answer

4 chips

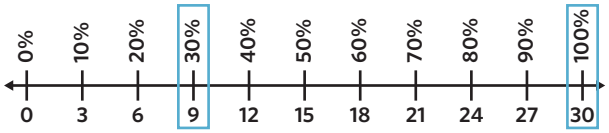
13. A weather reporter stated that the probability of the daily temperature being over 20 degrees in a month in Sydney was 30%. If this corresponded to 9 days, over what number of days was the temperature recorded?

Key points

- The probability of the daily temperature being over 20 degrees in a month in Sydney was 30%.
- The expected number of days is 9.
- Over what number of days was the temperature recorded?

Explanation

The theoretical probability is the probability given by the reporter, and the expected occurrence is the number of days it is expected to be over 20 degrees.



Theoretical probability is 30% = 0.3.

Expected occurrence = 9

Expected occurrence = theoretical probability × number of trials

$$\text{So number of trials} = \frac{\text{expected occurrence}}{\text{theoretical probability}}$$

$$= \frac{9}{0.3}$$

$= 30$

Answer

The temperature was recorded over 30 days.

Reasoning

14. a. 50 trials in Experiment 1

400 trials in Experiment 2

b. $\frac{21}{25}$ in Experiment 1

$\frac{3}{5}$ in Experiment 2

c. 24 people in Experiment 1

78 people in Experiment 2

d. Experiment 2, since it had more trials

e. Suggested option 1: Increase the number of trials.

Suggested option 2: Ensure the experiment contains an accurate cross-section of the population.

Note: There are other possible options.

15. a.

Outcome	1	2	3	4
Frequency	5	5	5	5

Note: There are other possible options.

b.

Outcome	1	2	3	4
Frequency	10	3	3	4

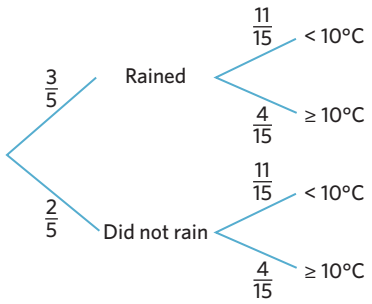
Note: There are other possible options.

c. You are not more likely to roll a 1 on the 24th roll; each roll is independent of the rolls before it and thus independent of the experimental probability.

Extra spicy

16. B

17.



18. D **19.** 9

Remember this?

20. C **21.** B **22.** C

Chapter 10 extended application

1. a. S = {red 1, red 2, red 3, red 4, red 5, red 6, red 7, red 8, red 9}

b. $\frac{8}{9}$

c.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9
2	2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9
3	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9
4	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9
5	5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9
6	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9
7	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9
8	8,1	8,2	8,3	8,4	8,5	8,6	8,7	8,8	8,9
9	9,1	9,2	9,3	9,4	9,5	9,6	9,7	9,8	9,9

c. $\frac{2}{27}$

d. 10 times

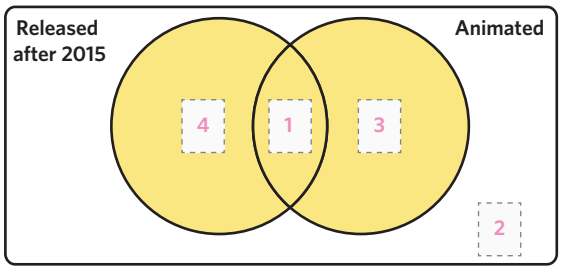
e. Suggested option 1: Uno is better with less players because it is easier to follow what is happening in the game.

Suggested option 2: Uno is better with more players, as it is more fun to play with a large group of people.

Note: There are other possible options.

2. a. 40 **b.** 3 **c.** $\frac{21}{40}$ **d.** $\frac{33}{40}$

e. Sample response:



Note: There are other possible options.

3. a. 0.1

b. 26.7%

c. 15 times

d. When more trials are conducted, there is more certainty around the most likely outcome in terms of testing how long a particular design might last in the air.

e. Suggested option 1: The technique of throwing the plane.
Suggested option 2: Conditions on the day if throwing planes outside (E.g. wind).

Note: There are other possible options.

Chapter 10 review

Multiple choice

1. D **2.** C **3.** A **4.** C **5.** E

Fluency

6. a. S = {O, P, R, T, U, N, I, E, S}

b. $\frac{7}{13}$

c. True

d. Choosing the letter P, T and I

7. a. Picking a red card **b.** $\frac{10}{13}$

c. $\frac{2}{13}$ **d.** $\frac{5}{13}$

8. a. $\frac{2}{15}$ **b.** $\frac{19}{30}$ **c.** $\frac{3}{5}$ **d.** $\frac{7}{10}$

9. a. $\frac{1}{5}$ **b.** $\frac{8}{15}$ **c.** $\frac{1}{2}$ **d.** $\frac{7}{15}$

10. a. $\frac{1}{9}$ **b.** $\frac{1}{3}$ **c.** $\frac{2}{3}$ **d.** $\frac{2}{3}$

11. a. $\frac{1}{12}$ **b.** $\frac{2}{3}$ **c.** $\frac{1}{2}$ **d.** $\frac{1}{2}$

12. a. **i.** $\frac{1}{2}$ **ii.** 25 **b.** **i.** $\frac{2}{3}$ **ii.** 20 **c.** **i.** $\frac{1}{3}$ **ii.** 5 **d.** **i.** $\frac{1}{2}$ **ii.** 35

13. a. 36 **b.** $\frac{1}{3}$ **c.** 15 **d.** 105

Problem solving

14. In a gameshow, a contestant is to select numbered tiles from a bag. The tiles contain odd numbers between 0 and 35. What is the probability the contestant will select a tile with a number under 10?

Key points

- In a gameshow, a contestant is to select numbered tiles from a bag.
- The tiles contain odd numbers between 0 and 35.
- What is the probability the contestant will select a tile with a number under 10?

Explanation

A tile with an odd number under 10: 1, 3, 5, 7, 9. There are five tiles in total that match this criteria.

There are $\frac{35+1}{2} = \frac{36}{2} = 18$ odd number tiles in the bag.

Probability of selecting a tile with a number under 10 $= \frac{5}{18}$.

Answer

The probability of the contestant selecting a tile with a number under 10 is $\frac{5}{18}$.

15. Three houses are competing in a swimming carnival: blue, red and yellow. The probability of the blue house winning is $\frac{1}{5}$, and the red and yellow houses are both twice as likely to win as the blue house. Find the probability that the red house does not win.

Key points

- Three houses are competing in a swimming carnival.
- The three houses are blue, red and yellow.
- The probability of the blue house winning is $\frac{1}{5}$, and the red and yellow houses are both twice as likely to win as the blue house.
- Find the probability that the red house does not win.

Explanation

Probability that the red house does not win = probability that the blue house wins + probability that the yellow house wins

$$= \frac{1}{5} + \frac{1}{5} \times 2$$
$$= \frac{1}{5} + \frac{2}{5}$$
$$= \frac{3}{5}$$

Answer

The probability that the red house does not win is $\frac{3}{5}$.

16. A travel agency has collected data on 70 families who have booked a holiday to either the Netherlands or France for next July or August. From their sample, 21 families have booked to go to the Netherlands in July, and 25 families wish to go to France in August. In total, 35 families have booked holidays in July and 31 families have booked holidays to the Netherlands. How many families want to go to the Netherlands in August or France in July?

Key points

- A travel agency has collected data on 70 families who have booked a holiday to either the Netherlands or France for next July or August.
- From their sample, 21 families have booked to go to the Netherlands in July, and 25 families wish to go to France in August.
- In total, 35 families have booked holidays in July and 31 families have booked holidays to the Netherlands.
- How many families want to go to the Netherlands in August or France in July?

Explanation

Convert the information given into a two-way table and fill in the two way table using the information given.

	Netherlands	France	Total
July	21	$35 - 21 = 14$	35
August	$31 - 21 = 10$	25	$25 + 10 = 35$
Total	31	$14 + 25 = 39$	70

There are 10 families that want to go to the Netherlands in August, and 14 families that want to go to France in July. This is $10 + 14 = 24$ families in total.

Answer

24 families want to go to the Netherlands in August or to France in July.

17. Dhileena is trying to plan her outfit for her school formal. She has eight different dresses to choose from in her wardrobe. She also has many different necklaces to choose from in her jewellery cabinet which she could wear with her dress. If there are 136 possible combinations of dresses and necklaces, how many necklaces does Dhileena have?

Key points

- Dhileena is trying to plan her outfit for her school formal. She has eight different dresses to choose from in her wardrobe.
- She also has many different necklaces to choose from in her jewellery cabinet which she could wear with her dress.
- There are 136 possible combinations of dresses and necklaces.
- How many necklaces does Dhileena have?

Explanation

Number of possible combinations of dresses and necklaces
$$= \text{number of dresses} \times \text{number of necklaces}$$
$$136 = 8 \times \text{number of necklaces}$$
$$136 \div 8 = 8 \times \text{number of necklaces} \div 8$$
$$\text{number of necklaces} = 17$$

Answer

Dhileena has 17 necklaces.

18. Hannah is a call centre sales representative. The probability of her making a sale in any call is 25%. How many calls would Hannah need to make if she wishes to make 5 sales?

Key points

- Hannah is a call centre sales representative. The probability of her making a sale in any call is 25%.
- How many calls would Hannah need to make if she wishes to make 5 sales?

Explanation

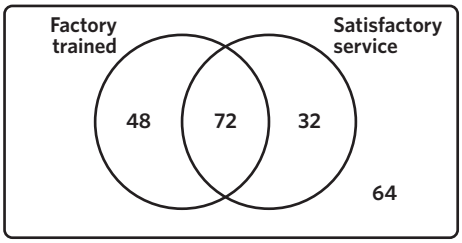
Number of sales made = number of calls made × probability of making a sale in a call
$$5 = \text{number of calls made} \times 25\%$$
$$5 \div 25\% = \text{number of calls made} \times 25\% \div 25\%$$
$$20 = \text{number of calls made}$$

Answer

Hannah would need to make 20 calls if she wishes to make 5 sales.

Reasoning

19. a.



b. 216 workers took part in the study.

c. $\frac{10}{27}$

d. 300 workers are expected to not provide satisfactory service.

e. Suggested option 1: Conducting studies and collecting data on the quality of repair services shows that the business is striving for high quality customer services which could increase the number of customers.

Suggested option 2: Conducting studies and collecting data on the quality of repair services could ensure better training for the employees and therefore higher employee satisfaction.

Note: There are other possible options.

20. a. The probability that the second counter is also blue is $\frac{1}{9}$.

b. The probability that her next counter will be yellow is $\frac{1}{9}$.

c. The probability that her next counter will be yellow is $\frac{1}{15}$.

d. In parts **a** and **b**, the counters are replaced before picking the next and so the total number of counters in the bucket does not change between trials. As there are the same number (two) of each counter in the bucket the numerator also does not change.

In part **c**, the counter was not replaced and so the denominator of the fraction for the second pick changes as the total number of counters changes between trials.

11A Introduction to ratios

Student practice

Worked example 1

a. 3 : 4 **b.** 7 : 13

Worked example 2

a. $\frac{9}{11}$ **b.** $\frac{2}{9}$ **c.** 5 : 1

Worked example 3

a. 33% **b.** 50% **c.** 79 : 21

Understanding worksheet

1. **a.** 6 **b.** 3 **c.** 1 : 3 **d.** 3 : 6

2. **a.** 1 : 5 **b.** $\frac{4}{6}$ **c.** 4 : 5, $\frac{5}{4}$ **d.** 3 : 1, $\frac{3}{4}$

3. proportional; quantities; part-whole; part-part

Fluency

4. **a.** 2 : 8 **b.** 11 : 9 **c.** 16 : 9 **d.** 7 : 13
e. 5 : 2 **f.** 13 : 10 **g.** 11 : 26 **h.** 24 : 15

---

5. **a.** $\frac{1}{3}$ **b.** $\frac{5}{6}$ **c.** $\frac{11}{14}$ **d.** $\frac{5}{1}$
e. $\frac{4}{13}$ **f.** $\frac{23}{38}$

---

6. **a.** 1 : 10 **b.** 5 : 7 **c.** 4 : 13 **d.** 27 : 2

---

7. **a.** 53% **b.** 70% **c.** 60% **d.** 33%
e. 67% **f.** 85%

---

8. **a.** 3 : 97 **b.** 3 : 5 **c.** 11 : 100 **d.** 5 : 6

---

9. C

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. A farmer's paddock has a width of 301 m and a length of 452 m. Write the ratio of the paddock's width to its perimeter.

Key points

- The paddock has a width of 301 m.
- The paddock has a length of 452 m.
- What is the ratio of the width to its perimeter?

Explanation

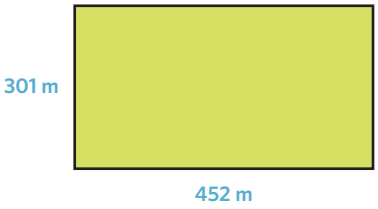
Need the ratio width : perimeter

Perimeter = 2 × width + 2 × length

Perimeter = 2 × 301 + 2 × 452

Perimeter = 1506

so width : perimeter = 301 : 1506



Answer

301 : 1506

12. Two painters, Julia and Brianca, divided up payment for their work on a day based on the ratio of hours that they worked. Julia worked from 7 am to 3 pm and Brianca worked from 9 am to 2 pm. What fraction of the total pay does Julia receive?

Key points

- Julia and Brianca divide their payment based on the ratio of hours that they worked.
- Julia worked from 7 am to 3 pm.
- Brianca worked from 9 am to 2 pm.
- What fraction of the total pay does Julia receive?

Explanation

Determine the number of parts needed for each quantity.

Julia worked 7 am–3 pm = 8 hours

Brianca worked 9 am–2 pm = 5 hours

Determine the number of parts in the whole

Total number of hours worked = Julia's hours + Brianca's hours
= 8 + 5
= 13 hours

Determine the part-whole fraction.

$\frac{8}{13}$

Answer

$\frac{8}{13}$

13. Coles reported that their ratio of revenue to expenses was 14 : 9 and Woolworths reported a ratio of 5 : 3. Which business had the greatest revenue as a percentage of their expenses?

Key points

- Coles' ratio of revenue to expenses was 14 : 9.
- Woolworths ratio was 5 : 3.
- Which business had the greatest revenue as a percentage of their loss?

Explanation

Write the part-part fractions for each business.

Coles = $\frac{14}{9}$ Woolworths = $\frac{5}{3}$

Convert the part-part fractions to percentages.

Coles: $\frac{14}{9} \times 100 = 156\%$ Woolworths: $\frac{5}{3} \times 100 = 167\%$

Compare the two percentages.

Woolworths has the greatest revenue as a percentage of their loss.

Answer

Woolworths

14. Jenny is making her world famous cordial but forgot the exact ratio of water to sugar to lemon juice she needed. She filled $\frac{1}{3}$ of the bowl with water, $\frac{2}{5}$ with sugar and $\frac{1}{5}$ with lemon juice. She later found that the ratio she was meant to use was 5 : 6 : 4. Did she get the ratio right?

Key points

- Jenny has forgotten the ratio of water to sugar to lemon juice for her lemonade.
- She filled $\frac{1}{3}$ of the bowl with water, $\frac{2}{5}$ with sugar and $\frac{1}{5}$ with lemon juice.
- The correct ratio is 5 : 6 : 4.
- Did she get the ratio right?

Explanation

Modify the part-whole fractions so that they share a common denominator, the lowest common multiple of 3 and 5 is 15, so this will be the denominator.

$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$ $\frac{1}{5} = \frac{3}{15}$

Using the new part-whole fraction, write the part-part-part ratio.

5 : 6 : 3

Compare this to the correct ratio.

They are not the same.

Answer

No

15. It is recommended that vegetarians have a 5 : 1 : 1 ratio of legumes (such as soybeans and lentils) to eggs to nuts as part of their diet. Nick made a meal where 65% of the meal was legumes and the rest made up equally of eggs and nuts. Does this meal satisfy the recommendation?

Key points

- Vegetarians should have a 5 : 1 : 1 ratio of legumes to eggs to nuts.
- Nick's meal is 65% legumes and the rest is made up of eggs and nuts.
- Does this meal satisfy the recommendation?

Explanation

Write the part-part-part ratio as a part-whole fraction for the legumes.

Number of parts in the whole $= 5 + 1 + 1$
$= 7$

Part-whole fraction $= \frac{5}{7}$

Convert the part-whole fraction to a percentage.

$\frac{5}{7} \times 100 = \frac{500}{7}$
$= 71\%$

As the part-whole percentage of the legumes is not 65%, Nick's meal does not satisfy the recommendation.

Answer

No

Reasoning

16. a. Ying will need to borrow $700 000 from the bank.

b. The ratio of the loan amount to the price of the house is 700 000 : 800 000.

c. The percentage of the loan amount to the price of the house is 87.5%

d. The loan amount needs to decrease by $60 000 in order for Ying not to pay LMI.

e. Suggested option 1: When banks give out larger loans they place themselves at risk and LMI ensures the bank will be protected.

Suggested option 2: When someone takes a mortgage with a deposit of less than 20%, they are more likely to be unable to make their mortgage payments, LMI protects the bank from losing their money.

Note: There are other possible options.

17. a. The ratio where the left side represents the smallest portion of the whole is 12 : 9.

b. The ratio where the left side represents the largest portion of the whole is 98 : 1.

c. To compare two ratios it is helpful to evaluate them as fractions by dividing the first part of the ratio by the second part, whichever of the two ratios has the higher value for this is the greater ratio.

Extra spicy

18. A **19.** 4 **20.** 9 and 2 **21.** A

Remember this?

22. B **23.** C **24.** E

11B Equivalent ratios

Student practice

Worked example 1

a. 4 : 14 **b.** 6 : 4

Worked example 2

a. 1 : 4 **b.** 3 : 1 : 4

Worked example 3

a. 4 : 1 **b.** 3 : 2 **c.** 1 : 2

Worked example 4

a. 3 : 20 **b.** 1 : 2

Understanding worksheet

1. a. 10 **b.** 4.5 **c.** 35 **d.** 7

2. a. Not equivalent **b.** Equivalent
c. Not equivalent **d.** Not equivalent

3. equivalent ratio; simplest form; whole; highest common factor

Fluency

4. a. 10 : 18 **b.** 80 : 30
c. 5 : 3 **d.** 5 : 10
e. 25 : 70 : 20 **f.** 7 : 3 : 1
g. 7 : 11 **h.** 24 : 20

5. a. 3 : 2 **b.** 3 : 2
c. 1 : 4 **d.** 3 : 7
e. 13 : 7 : 2 **f.** 5 : 7 : 4
g. 7 : 2 : 1 **h.** 2 : 7 : 11

6. **a.** 2 : 1 **b.** 5 : 3
c. 2 : 9 **d.** 8 : 5
e. 48 : 11 **f.** 62 : 7
g. 68 : 33 **h.** 1 : 3 : 2

7. **a.** 2 : 5 **b.** 16 : 9
c. 6 : 7 **d.** 13 : 45
e. 5 : 1 **f.** 21 : 4 : 30
g. 2 : 4 : 11 **h.** 67 : 129 : 3

8. **a.** 1 : 2 **b.** 4 : 5
c. 11 : 50 **d.** 90 : 17
e. 7 : 5 **f.** 40 : 3
g. 1 : 4 **h.** 31 : 12 : 20

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. A shade of purple is made by combining red and blue paint tins in the ratio 2 : 3. This is enough paint to cover 20 m^2. How many tins of each colour is required to cover 60 m^2?

Key points

- A shade of purple is made by combining red and blue paint tins in the ratio 2 : 3.
- This is enough paint to cover 20 m^2.
- How many tins of each colour is required to cover 60 m^2?

Explanation

To cover 20 m^2, we need 2 tins of red paint and 3 tins of blue paint.

To cover $20 \times 3 = 60$ m^2, we need $2 \times 3 = 6$ tins of red paint and $3 \times 3 = 9$ tins of blue paint.

$2 : 3$ (×3) → $6 : 9$ (×3)

Answer

To cover 60 m^2, we need 6 tins of red paint and 9 tins of blue paint.

12. Matej is making multiple tiles of the pattern shown below.
It consists of 4 equilateral triangles and 1 square.

If Matej has 48 triangles, what number of squares does he need so that he only has completed tiles?

Key points

- The pattern consists of 4 equilateral triangles and 1 square.
- Matej has 48 triangles.
- What number of squares does he need so that he only has completed tiles?

Explanation

The ratio of triangle : square needed to complete 1 tile is 4 : 1.

48 triangles would make $48 \div 4 = 12$ tiles.

12 patterns would need $12 \times 1 = 12$ squares.

$4 : 1$ (×12) → $48 : 12$ (×12)

Answer

Matej needs 12 squares so that he only has completed tiles.

13. At his cafe, Hugh makes milkshakes which need milk and ice cream. He uses $\frac{1}{3}$ of a container of ice cream and $\frac{2}{5}$ of a carton of milk for a large milkshake. What is the fully simplified ratio of milk to ice cream for each large milkshake?

Key points

- He uses $\frac{1}{3}$ of a container of ice cream and $\frac{2}{5}$ of a carton of milk for a large milkshake.
- What is the fully simplified ratio of milk to ice cream for each large milkshake?

Explanation

Simplify the milk : ice cream ratio, $\frac{2}{5} : \frac{1}{3}$.

$\frac{2}{5} : \frac{1}{3}$ (×15) → $\frac{6}{15} : \frac{5}{15}$ (×15) → $6 : 5$

Answer

The fully simplified ratio of milk to ice cream for each large milkshake is 6 : 5.

14. Jemima is making concrete for a building project where the ratio of cement to sand to aggregate should be 1 : 2 : 4. She mixes 5 bags of cement, 10 bags of sand and 15 bags of aggregate. Calculate the simplified ratio to determine if Jemima has used the correct quantities for her mix.

Key points

- The ratio of cement to sand to aggregate should be 1 : 2 : 4.
- She mixes 5 bags of cement, 10 bags of sand and 15 bags of aggregate.
- Calculate the simplified ratio to determine if Jemima has used the correct quantities for her mix.

Explanation

Simplify the ratio 5 : 10 : 15.

$5 : 10 : 15$ (÷5) → $1 : 2 : 3$ (÷5)

The simplified ratio is 1 : 2 : 3. This is not equal to 1 : 2 : 4.

Therefore, Jemima has used the incorrect quantities for her mix.

Answer

Jemima has used the incorrect quantities for her mix.

15. Alex is making her own soil mix for her indoor plants. To make the soil, she mixes a ratio of 2 parts peat moss, 1 part perlite and $\frac{1}{2}$ part compost. She has 8 bags of peat moss but needs to buy perlite and compost. If each bag of perlite cost $29.98 and each bag of compost cost $9.46, how much will it cost her to buy enough perlite and compost?

Key points

- To make the soil, Alex mixes a ratio of 2 parts peat moss, 1 part perlite and $\frac{1}{2}$ part compost.
- She has 8 bags of peat moss but needs to buy perlite and compost.
- One bag of perlite costs $29.98 and one bag of compost costs $9.46.
- How much will it cost her to buy enough perlite and compost?

Explanation

The ratio of peat moss : perlite : compost is $2 : 1 : \frac{1}{2}$.

Make an equivalent ratio to represent 8 bags of peat moss.

$2 : 1 : \frac{1}{2}$ (×4, ×4, ×4) → $8 : 4 : 2$

Therefore, to use up 8 bags of peat moss, Alex will need 4 bags of perlite and 2 bags of compost.

Cost of 4 bags of perlite = Cost of 1 bag of perlite × 4
= 29.98 × 4
= $119.92

Cost of 2 bags of compost = Cost of 1 bag of compost × 2
= 9.46 × 2
= $18.92

Total cost for perlite and compost = 119.92 + 18.92
= $138.84

Answer

It would cost Alex $138.84 to buy enough perlite and compost.

Reasoning

16. a. Joyce's proposed ratio is 4 : 6 : 1.

b. The ratio of hours spent relaxing to sightseeing to travelling for Daniel's itinerary is 8 : 11 : 4.

c. The percentage of time travelling to time for all activities for Daniel's itinerary is more than the percentage proposed by Joyce.

d. 4 hours relaxing and 1 hour travelling.

e. Suggested option 1: An advantage of ensuring every day satisfies Joyce's proposed ratio is that it ensures that not too much time is spent travelling or relaxing instead of sightseeing. A disadvantage is that this may limit the amount of time spent sightseeing when more might be needed.

Suggested option 2: An advantage of satisfying the ratio over every day of the trip is that the same amount of time is spent on each activity but each day can vary depending on what sights they want to see. A disadvantage is that they might spend a whole day relaxing or travelling and not seeing any interesting sights.

Note: There are other possible options.

17. a. $23 + 1 : 11 + 1 = 24 : 12$

$23 - 10 : 11 - 10 = 13 : 1$

b. $23 \times 2 : 11 \times 2 = 46 : 22$

$23 \div 3 : 11 \div 3 = \frac{23}{3} : \frac{11}{3}$

c. In part **a**, the ratio 24 : 12 is not equivalent to the original ratio 23 : 11. In part **b**, the ratio 46 : 22 is equivalent to the original ratio 23 : 11. Therefore, multiplying or dividing both parts by a number can be used to make equivalent ratios, and adding or subtracting a number from both parts cannot be used to make equivalent ratios.

Extra spicy

18. B

19. D

20. $a = 16$ and $b = 1$

21. $1 : 2 = 3 : 6 = 4 : 8$

Remember this?

22. B **23.** A **24.** E

11C Dividing a quantity in a given ratio

Student practice

Worked example 1

a. 5 per part **b.** 5 per part

Worked example 2

a. 6 and 12 **b.** 10, 35 and 15

Worked example 3

a. 12 pencils **b.** 700 g of flour

Understanding worksheet

1. a. 12 **b.** 11 **c.** 25 **d.** 18

2. a. $1 : 4$ (×2, ×2) → $2 : 8$

b. $8 : 5$ (×4, ×4) → $32 : 20$

c. $3 : 7$ (×6, ×6) → $18 : 42$

d. $6 : 9 : 10$ (×5, ×5, ×5) → $30 : 45 : 50$

3. parts; part-whole; value; equivalent

Fluency

4. a. 5 **b.** 5 **c.** 2 **d.** 6
e. 13 **f.** 2.07 **g.** 2.18 **h.** 15.33

5. a. 6 : 4 **b.** 28 : 4
c. 40 : 25 **d.** 9 : 30 : 6
e. 32.80 : 8.20 **f.** 56 : 42 : 63
g. 20.67 : 24.11 : 17.22 **h.** 10 : 24

6. a. $42 **b.** 9 hours
c. 45 cm **d.** 2500 players
e. 1400 kg **f.** 1.2 L
g. 3.6 L **h.** 48.57 g

7. E

Spot the mistake

8. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

9. In the world's population, the ratio of people with brown eyes to other colours is 3 : 2. In a year level of 60 students, what is the expected number of students with brown eyes?

Key points

- The ratio of people with brown eyes to other colours is 3 : 2.
- A year level has 60 students.
- What is the expected number of students with brown eyes?

Explanation

60 into the ratio 3 : 2. The number of parts in the whole is $3 + 2 = 5$.

The value of 1 part is $60 \div 5 = 12$.

60 shared in the ratio 3 : 2 is 36 and 24.

Therefore, the expected number of students with brown eyes is 36.

Answer

The expected number of students with brown eyes is 36.

10. Two friends, Dain and Felicity, are playing an online RPG (role playing game) and have just defeated a boss. They will share the loot received, 45 000 gold pieces, according to the ratio of damage they each dealt. Dain dealt 4000 damage and Felicity dealt 5000 damage. How much gold should Felicity receive?

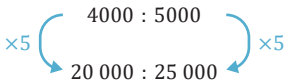
Key points

- Two friends, Dain and Felicity, are playing an online RPG (role playing game) and have just defeated a boss.
- They will share the loot received, 45 000 gold pieces, according to the ratio of damage they each dealt.
- Dain dealt 4000 damage and Felicity dealt 5000 damage.
- How much gold should Felicity receive?

Explanation

45 000 into the ratio 4000 : 5000. The number of parts in the whole is $4000 + 5000 = 9000$.

The value of 1 part is $45\,000 \div 9000 = 5$.



45 000 shared in the ratio 4000 : 5000 is 20 000 and 25 000.

Therefore, the amount of gold Felicity should receive is 25 000.

Answer

Felicity should receive 25 000 gold.

11. The ratio of the distance between Marble Cliff and Stone Lake to the distance between Stone Lake and Glacier Forest is 3 : 4.

The distance between Marble Cliff and Stone Lake is 21 km. Find the distance between Marble Cliff and Glacier Forest.

Key points

- The ratio of the distance between Marble Cliff and Stone Lake to the distance between Stone Lake and Glacier Forest is 3 : 4.
- The distance between Marble Cliff and Stone Lake is 21 km.
- Find the distance between Marble Cliff and Glacier Forest.

Explanation

3 parts = 21 km. The value of 1 part is $21 \div 3 = 7$ km.

The number of parts in total is $3 + 4 = 7$.

Distance between Marble Cliff and Glacier Forest is

$7 \times 7 = 49$ km.

Answer

The distance between Marble Cliff and Glacier Forest is 49 km.

12. The angles of a triangle are in the ratio 2 : 5 : 1. Calculate the sizes of the angles.

Key points

- The angles of a triangle are in the ratio 2 : 5 : 1.
- Calculate the sizes of the angles.

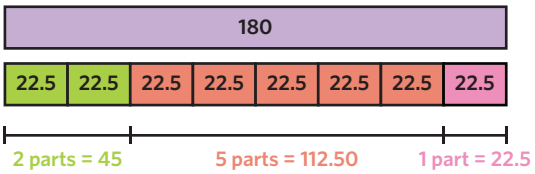
Explanation

The internal angles of a triangle sum to 180°.

180 into the ratio 2 : 5 : 1. The number of parts in the whole is $2 + 5 + 1 = 8$.

The value of 1 part is $180 \div 8 = 22.5°$.

The sizes of the angles are $2 \times 22.50 = 45°$, $5 \times 22.50 = 112.50°$ and $1 \times 22.50 = 22.50°$ respectively.



Answer

The sizes of the angles are 45°, 112.5° and 22.5°.

13. Beatrix knows that on average the ratio of muffins to pastries sold at her bakery is 4 : 11. Muffins cost $6.50 and pastries cost $4.70. Today, she is expecting 210 customers to come in and buy one item each. How much revenue is she expecting to make from sales today?

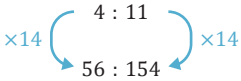
Key points

- On average the ratio of muffins to pastries sold at her bakery is 4 : 11.
- Muffins cost $6.50 and pastries cost $4.70.
- Today, she is expecting 210 customers to come in and buy one item each.
- How much revenue is she expecting to make from sales today?

Explanation

210 into the ratio 4 : 11. The number of parts in the whole is $4 + 11 = 15$.

The value of 1 part is $210 \div 15 = 14$.



Today, 56 muffins and 154 pastries are expected to be sold.

This is a revenue of $56 \times 6.5 + 154 \times 4.7 = \$1087.80$ in total.

Add together the amount earned from sales to get total revenue.

Answer

Beatrix is expecting to make $1087.80 in revenue from sales.

Reasoning

14. a. There are 6 parts in total in the NPK ratio for indoor plants.

b. Kurt should put 6 kg of Nitrogen, 4 kg of Phosphorus and 2 kg of Potassium.

c. It will need to contain 36 kg of fertiliser.

d. The maximum amount of each element Kurt can put in is 0.8 kg Nitrogen, 1.6 kg Phosphorus and 1.6 kg Potassium.

e. Suggested option 1: Kurt should consider how much people that live around his shop are willing to spend when determining the prices.

Suggested option 2: Kurt should consider the supply costs when determining the prices.

Note: There are other possible options.

15. **a.** Ratio 1 : 2, quantity 36, the whole number difference between the two shares is 1.

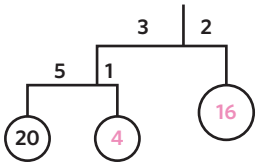
b. Ratio 1 : 8, quantity 27, the whole number difference between the two shares is 7.

c. A quantity is able to be shared into whole number shares when the total quantity is divisible by the total number of parts or the total number of parts is a factor of the quantity.

Extra Spicy

16. C

17.



18. C **19.** 125 coins

Remember this?

20. B **21.** C **22.** B

11D Introduction to rates

Student practice

Worked example 1

a. 5 children/parent **b.** $7/kg

Worked example 2

a. 3500 people/year **b.** 7.29 cm/year

Understanding worksheet

1. **a.** students/teachers **b.** $/m
c. km/s **d.** people/m^2

2. **a.** 10 km/h **b.** $1.71/L
c. $21.38/h **d.** 1 m/day

3. different; one; units; quantities; average

Fluency

4. **a.** 3 waiters/table **b.** 6 patients/doctor
c. 5 customers/checkout **d.** 7 children/teacher
e. 2 guests/room **f.** 4 sandwiches/guest
g. 52 students/bus **h.** 170 points/game

5. **a.** $4/kg **b.** $70/grandchild
c. 160 km/h **d.** 6.25 L/person
e. 2.2 kg/planter **f.** 1.6 m^2/chicken
g. $214.29/week **h.** 11.67 L/m^2

6. **a.** 15 customers/year **b.** 3 rabbits/month
c. 5 roses/week **d.** 9 goals/game
e. 90 hairdressers/year **f.** 46 pages/day
g. 155 people/year **h.** 16 million bacteria/minute

7. **a.** $2.5/year **b.** 5 minutes/day
c. 8.33 cm/year **d.** 3.33 km/minute
e. 5.83 cm^2/day **f.** $15/year
g. 1.4 mm/hour **h.** 16.11 g/day

8. B

Spot the mistake

9. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. Joey pays $6.30 for 3 mangoes. Write the price per mango as a simplified rate.

Key points

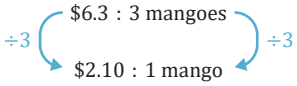
- Joey pays $6.30 for 3 mangoes.
- Write the price per mango as a simplified rate.

Explanation

$6.30 for 3 mangoes as a ratio would be $6.30 : 3.

Simplifying this ratio and dividing both sides by 3 gives $2.10 : 1 mango. As a rate, this would be $2.10/mango.

As a ratio:



As a rate:

$2.10 per mango = $2.10/mango

Answer

$2.10/mango

11. Enyd is learning Japanese using an app. After 20 days of using the app, her vocabulary grew from 30 words to 150 words. What is the average rate at which Enyd is expanding her Japanese vocabulary, in words/day?

Key points

- After 20 days of using the app, Enyd's vocabulary grew from 30 words to 150 words.
- What is the average rate at which Enyd is expanding her Japanese vocabulary, in words/day?

Explanation

The change in Enyd's vocabulary growth is $150 - 30 = 120$ words.

As she used the app for 20 days, the changes in quantities as a ratio would be 120 words : 20 days.

Simplify and divide both sides by 20 to get 6 words : 1 day.

6 words : 1 day expressed as an average rate is 6 words/day.

Answer

6 words/day

12. It takes Marlene 16 hours to crochet a square blanket 2 m long. Write the rate at which Marlene can crochet, in m^2/h.

Key points

- It takes Marlene 16 hours to crochet a square blanket 2 m long.
- Write the rate at which Marlene can crochet, in m^2/h.

Explanation

The blanket is a square, and so to convert the length 2 m to m^2, the length is squared to get $2 \times 2 = 4$ m^2.

The ratio would therefore be 4 m^2 : 16 hours. To express this ratio as a simplified rate, we divide both sides by 4 to get 0.25 m^2/h.

Answer

0.25 m^2/h

13. Grandma gives Lottie and Oscar $2.50 for every year that makes up each of their ages. Lottie is 3 years older than Oscar, and she gets $55 from grandma. How old is Oscar?

Key points

- Grandma gives Lottie and Oscar $2.50 for every year that makes up each of their ages.
- Lottie is 3 years older than Oscar.
- Lottie gets $55 from grandma.
- How old is Oscar?

Explanation

As Lottie gets $2.50 for every year of her age and she has been given $55, this means she is $55 \div 2.50 = 22$ years old.

Lottie is 3 years older than Oscar, so Oscar must be $22 - 3 = 19$ years old.

Answer

19 years old

14. After working at the same factory for 5 years, Missy's salary increased from $2500 per month to $3200 per month. What is the average rate at which Missy's salary increased over the 5 years, in dollars/month per year?

Key points

- After working at the same factory for 5 years, Missy's salary increased from $2500 per month to $3200 per month.
- What is the average rate at which Missy's salary increased over the 5 years, in dollars/month per year?

Explanation

The change in Missy's salary is $\$3200 - \$2500 = \$700$ per month.

To find the average rate, we write the increase as a ratio of the number of years, which is $700 per month : 5 years.

Simplify this ratio by dividing both sides by 5 to get $140 per month : 1 year.

This ratio expressed as an average rate would be $140 dollars/month per year.

Answer

$140 dollars/month per year

Reasoning

15. **a.** 40 words per minute **b.** 3 wpm/day

c. 55 wpm; Above standard **d.** 12 minutes

e. Suggested option 1: Accuracy is more important so that proper spelling and grammar are used and so the reader can actually understand what is being typed out.

Suggested option 2: Accuracy is more important as speed can be learnt with practice and repetition, whereas it is harder to learn accurate typing.

Note: There are other possible options.

16. **a.** 25 km/h **b.** $25/kg

c. Parts **a** and **b** are similar in terms of numerical value (i.e. both contain quantities of 25) however the units that are being represented and thus the meanings of the rates are completely different. The answer in part **a** expresses a speed (distance relative to time) as a rate of 25 kilometres per hour, whereas the answer in part **b** expresses the dollar amount of something relative to its weight as a rate of $25 per kilogram.

Extra spicy

17. Emma initially invested $1250.

18. D **19.** D

20. It will take 20 hours, where the first lumberjack will cut 60 trees and the second lumberjack will cut 40.

Remember this?

21. C **22.** D **23.** C

11E Solving rate problems

Student practice

Worked example 1

a. 30 cups of coffee **b.** 112 minutes

Worked example 2

a. Donnie is faster than Terry by 0.15 km/h.

b. In 30 days, Josiah spends 7.5 hours more commuting compared to Layla.

Worked example 3

Approximately 2 hours

Understanding worksheet

1. **a.** 15 **b.** 9 **c.** 20 **d.** 11

2. **a.** $\div 2, \div 2$ **b.** $\div 4, \div 4$ **c.** $\div 10, \div 10$ **d.** $\div 8, \div 8$

3. proportional; operations; equal; addition

Fluency

4. **a.** 3375 words **b.** 22 tests

c. 85 goals **d.** 845 children

e. 119 souvenirs **f.** 113 errors

g. 18 litres

---

5. **a.** 9 people **b.** 6 students

c. 50 foraging flights **d.** 5 minutes

e. 5 hours **f.** 1 350 000 sheets of paper

g. 107.1 volts

---

6. **a.** Wendy services 1 more computer per hour than Tim.

b. Lester gets paid $2.50 per hour more than Bobbie.

c. Gianni used 0.13 more metres of material than Coco.

d. Carlo saves $3.43 more than Nora per day.

---

7. **a.** Mark serves 8 more customers per 8 hour workday compared to Nick.

b. Cindy will need 0.625 L less petrol to complete a 250 km trip compared to Tahlia.

c. Darlene will need 0.3 hours less time to knit a 3 m^2 blanket compared to Arlene.

d. Sydney's rent is $3722 higher during a non-leap year compared to Melbourne.

8. **a.** It will take both of the rowers 2 minutes to row 1 km together, to the nearest minute.

b. It will take both of them 3 hours to move the apartment together, to the nearest hour.

c. It will take both of them 8 hours to complete the assignment together, to the nearest hour.

d. It will take all 3 of them 3 hours to organise the section together, to the nearest hour.

---

9. B

Spot the mistake

10. **a.** Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. On a particular day, Dr Rosanna starts seeing patients at 8:30 am and finishes by 5:30 pm. If she saw 40 patients that day, then what is the rate at which Dr Rosanna saw her patients, in patients/hour, to the nearest patient?

Key points

- On a particular day, Dr Rosanna starts seeing patients at 8:30 am and finishes by 5:30 pm.
- She saw 40 patients that day.
- What is the rate at which Dr Rosanna saw her patients, in patients/hour, to the nearest patient?

Explanation

From 8:30 am to 5:30 pm, there are 9 hours. Therefore, Dr Rosanna worked for 9 hours on that day.

9 hours : 40 patients

$9 \div 9$ hours : $40 \div 9$ patients

1 hour : 4 patients to the nearest patient.

Answer

Dr Rosanna saw her patients at a rate of 4 patients/hour, to the nearest patient.

12. Ruth's tropical fish are breeding at a rate of 8 new fry every 2 days per female fish. If Ruth has a total of 9 female fish, then how long will it take them to birth a total of 90 new fry?

Key points

- Ruth's tropical fish are breeding at a rate of 8 new fry every 2 days per female fish.
- Ruth has a total of 9 female fish.
- How long will it take them to birth a total of 90 new fry?

Explanation

For each female fish:

8 new fry : 2 days

$8 \div 2$ new fry : $2 \div 2$ days

4 new fry : 1 day

4×22.5 new fry : 1×22.5 days

90 new fry : 22.5 days

Therefore, for 9 female fish, it will take them $22.5 \div 9 = 2.5$ days to birth a total of 90 new fry.

Answer

It will take them 2.5 days to birth a total of 90 new fry.

13. One small ink cartridge contains enough ink for 600 pages of printing, while a medium cartridge is twice as efficient as the small cartridge. If a large ink cartridge is enough for 1800 pages, then how much ink does it contain, in terms of the small and medium cartridges combined?

Key points

- One small ink cartridge contains enough ink for 600 pages of printing.
- A medium cartridge is twice as efficient as the small cartridge.
- A large ink cartridge is enough for 1800 pages.
- How much ink does it contain, in terms of the small and medium cartridges combined?

Explanation

1 small cartridge : 600 pages

Since a medium cartridge is twice as efficient as the small cartridge:

1 medium cartridge : 600×2 pages

1 medium cartridge : 1200 pages

1 large cartridge : 1800 pages

As $1800 = 1200 + 600$, one large cartridge contains the same amount of ink as one small and one medium cartridge combined.

Answer

A large ink cartridge contains the same amount of ink as one small and one medium cartridge together.

14. 7 robots at a car factory can assemble 336 cars in 1 day. How many days will it take 10 robots to assemble 1920 cars?

Key points

- 7 robots at a car factory can assemble 336 cars in 1 day.
- How many days will it take 10 robots to assemble 1920 cars?

Explanation

In 1 day:

7 robots : 336 cars

$7 \div 7$ robots : $336 \div 7$ cars

1 robot : 48 cars

1×10 robots : 48×10 cars

10 robots : 480 cars

Therefore, it will take 10 robots $1920 \div 480 = 4$ days to assemble 1920 cars.

Answer

It will take 10 robots 4 days to assemble 1920 cars.

15. Bo and Maira own a joinery business. Bo can assemble a regularly sized kitchen in 7 hours and Maira can assemble it in 8 hours. How long will it take both of them to assemble a kitchen which is twice the size of a regular kitchen, in hours, correct to one decimal place?

Key points

- Bo can assemble a regularly sized kitchen in 7 hours.
- Maira can assemble it in 8 hours.
- How long will it take both of them to assemble a kitchen which is twice the size of a regular kitchen, in hours, correct to one decimal place?

Explanation

For regularly sized kitchen:

Bo: 1 kitchen : 7 hours. Therefore, Bo's rate is $\frac{1}{7}$ kitchen/hour.

Maira: 1 kitchen : 8 hours. Therefore, Maira's rate is $\frac{1}{8}$ kitchen/hour.

Combined rate $= \frac{1}{7} + \frac{1}{8} = \frac{8}{56} + \frac{7}{56} = \frac{15}{56}$ kitchen/hour.

15 regular kitchens : 56 hours

The large kitchen they are assembling now is twice as big as the regular kitchen.

15 × 2 regular kitchens : 56 × 2 hours

∴ 15 large kitchens : 112 hours

15 ÷ 15 large kitchens : 112 ÷ 15 hours

1 large kitchen : 7.5 hours

Answer

It takes both of them 7.5 hours to assemble a kitchen which is twice the size of a regular kitchen.

Reasoning

16. a. The total increase in height over a one week period at this growth rate is 7 metres.

b. At this rate, it will take 12 weeks for the moso bamboo to grow to its full height of 12 m, from 0 cm.

c. Moso bamboo grows 140 mm/day faster compared to golden bamboo, to the nearest mm.

d. Suggested option 1: Hairbrushes and furniture.

Suggested option 2: Toothbrushes and bedsheets.

Note: There are other possible options.

17. a. $50 per 2 kg ($50/2 kg) is a higher rate compared to $45 per 2 kg ($45/2 kg).

b. 15 km per 2 hours (15 km/2 hours) is a higher rate compared to 15 km per 3 hours (15 km/3 hours).

c. When comparing rates with fractions with the same denominator, the fraction that has the bigger numerator is the higher rate. When comparing rates with fractions with the same numerator, the fraction that has the smaller denominator is the higher rate.

Extra spicy

18. E

19. It takes 18 days to fill the pool.

20. D

21. It takes 4 people 20 hours to finish half of the project.

Remember this?

22. B **23.** C **24.** A

11F Speed

Student practice

Worked example 1

a. 92.5 km/h **b.** 1.75 hour **c.** 156 km

Worked example 2

a. Parts *C* and *E* **b.** Parts *A*, *D* and *F*

c. Part *B*

Worked example 3

a. 3 hours **b.** 2.5 km/h **c.** 1.4 km/h

Understanding worksheet

1. a. Stationary **b.** Constant speed

c. Variable speed **d.** Variable speed

2. a. Constant **b.** Stationary

c. Variable **d.** Stationary

3. time; distance; hour; second

Fluency

4. a. 13 km/h **b.** 8 m/s

c. 44 km/h **d.** 31.25 km/h

e. 312 500 km/h **f.** 120 cm/min

g. 16.25 km/s **h.** 120 km/h

---

5. a. 2 hours

b. 0.5 hours or 30 minutes

c. 4.8 hours or 4 hours and 48 minutes

d. 18.75 seconds

e. 8.25 hours or 8 hours and 15 minutes

f. 75 seconds

g. 5000 seconds or 83 minutes and 20 seconds

h. 0.125 hours or 7 minutes and 30 seconds

---

6. a. 75 km **b.** 288 km **c.** 168 m **d.** 153 km

e. 28 km **f.** 13.6 m **g.** 55.8 km **h.** 40.85 km

---

7. a. *B*, *E* **b.** *A*, *C*, *F* **c.** *D* **d.** *C*

e. *E* **f.** *E* **g.** *C* **h.** *A*

---

8. a. 10 hours **b.** 80 km/h **c.** 50 km/h **d.** 70 km/h

e. 25 km/h **f.** 36 km/h **g.** 50 km/h **h.** 10 hours

---

9. B

Spot the mistake

10. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. Herbie walked to the beach from his house at an average speed of 3 km/h for 45 minutes. If Herbie also stopped for a coffee for 15 minutes during this time, then how far away is the beach from his house?

Key points

- Herbie walked to the beach from his house at an average speed of 3 km/h for 45 minutes.
- Herbie also stopped for a coffee for 15 minutes.
- How far away is the beach from his house?

Explanation

$s = 3$ km/h

$t = 45 - 15 = 30$ minutes $= 0.5$ hours

$d = st = 3 \times 0.5 = 1.5$ km

Answer

The beach is 1.5 km away from his house.

12. Renae walks from her house to the river 5 km away in one hour. She jogs back at a speed of 7 km/h. How long will it take Renae to get home at this speed, to the nearest minute?

Key points

- Renae walks from her house to the river 5 km away in one hour.
- She jogs back at a speed of 7 km/h.
- How long will it take Renae to get home at this speed, to the nearest minute?

Explanation

$s = \frac{d}{t} = \frac{5}{7}$ hours

$= \frac{5}{7} \times 60$ minutes

$= 42.857...$

$\simeq 43$ minutes

Answer

It will take Renae 43 minutes to get home at this speed, to the nearest minute.

13. Ismael and Astrid both competed in the 1 km race. Ismael finished the race in 17 minutes and Astrid finished it in three tenths of an hour. Determine who won the race and by how much.

Key points

- Ismael and Astrid both competed in the 1 km race.
- Ismael finished the race in 17 minutes.
- Astrid finished it in three tenths of an hour.
- Determine who won the race and by how much.

Explanation

The person who wins the race is the person who took less time to finish the race.

Ismael took 17 minutes to finish the race.

Astrid took $\frac{3}{10}$ hours $= \frac{3}{10} \times 60 = 18$ minutes to finish the race.

$18 > 17, 18 - 17 = 1$ minute

$\therefore$ Ismael won the race by one minute.

Answer

Ismael won the race by one minute.

14. On the way to the supermarket from her house, Sana's average walking speed was 4 km/h. She walked back the same way, at an average speed of 3 km/h. If the supermarket is 3.5 km away from Sana's house, then how much longer did it take her to get home compared to how long it took to get there, to the nearest minute?

Key points

- On the way to the supermarket from her house, Sana's average walking speed was 4 km/h.
- She walked back the same way, at an average speed of 3 km/h.
- The supermarket is 3.5 km away from Sana's house.
- How much longer did it take her to get home compared to how long it took to get there, to the nearest minute?

Explanation

Way to supermarket:

$t = \frac{d}{s} = \frac{3.5}{4} = \frac{7}{8}$ hours

$\frac{7}{8} \times 60 = 52.5$ minutes

Way from supermarket:

$t = \frac{d}{s} = \frac{3.5}{3} = \frac{7}{6}$ hours

$\frac{7}{6} \times 60 = 70$ minutes

Time difference $= 70 - 52.5$

$= 17.5$

$\simeq 18$ minutes

Answer

It took Sana 18 more minutes to get home compared to how long it took to get there, to the nearest minute.

15. A car leaves a street corner and travels at a constant speed of 60 km/h. A drone leaves the same corner 30 minutes after the car and follows it at a speed of 120 km/h. How long will it take the drone to catch up to the car?

Key points

- A car leaves a street corner and travels at a constant speed of 60 km/h.
- A drone leaves the same corner 30 minutes after the car and follows it at a speed of 120 km/h.
- How long will it take the drone to catch up to the car?

Explanation

When the drone catches up to the car, the distance the drone has travelled would equal the distance the car travelled during that time plus the 30 minutes headstart.

$d_{drone} = s \times t = 120t$ km

$d_{car} = s \times t + s \times t_{headstart} = 120t$ km

$= 60t + 60 \times 0.5$ as 30 minutes $= 0.5$ hours

$= 60t + 30$ km

$d_{drone} = d_{car}$

$120t = 60t + 30$

$60t = 30$

$t = 0.5$ hours

Answer

It will take the drone 0.5 hour to catch up to the car.

Reasoning

16. **a.** It takes 77 hours to travel 1228 km at this speed.

b. The time taken to travel 16 km at this speed is 47 seconds.

c. Thrust SSC did not break the sound barrier record because its speed is 341 m/s, which is slower than the speed of sound in air, which is 343 m/s.

d. Suggested option 1: Imposing speed limit has positive environmental benefits as lower speed leads to lower vehicle emissions.

Suggested option 2: Imposing speed limit enables cars on the same road to all drive at a similar speed to reduce traffic jams.

Note: There are other possible options.

17. **a.** 45 km/h

b. 20 km/h

c. Average speed for entire distance $= \frac{\text{total } d}{\text{total } t}$

$= \frac{150}{5}$

$= 30$ km/h

From part **a**, the speed during part A is 45 km/h.

From part **b**, the speed during part B is 20 km/h.

If:

Average speed for both part A and B = speed for part A $\times$ fraction of hours for part A + speed for part B $\times$ fraction of hours for part B

$= 45 \times \frac{2}{5} + 20 \times \frac{3}{5}$

$= 18 + 12$

$= 30$ km/h

Therefore, it is possible to determine the average speed of an object using fraction addition of component speeds.

Extra spicy

18. 436 km/h **19.** C **20.** B **21.** 11:46 am

Remember this?

22. C **23.** E **24.** A

Chapter 11 extended application

1. a. 1 : 8

b. 2.5 mL baking soda

60 mL water

480 mL shaving cream

30 mL contact lens solution

c. 2 : 1

d. 1 : 6

e. 2 : 1 : 8 : 6

f. Suggested option 1: A fun science experiment would be making elephant's toothpaste (or foam) from soap, water, yeast and hydrogen peroxide.

Suggested option 2: A fun science experiment would be making a balloon inflate on its own with vinegar and baking soda.

Note: There are other possible options.

2. a. $\frac{x}{2.5}$ km/h

b. $\frac{x + 6}{2}$ km/h

c. $\frac{x}{2.5}$ km

d. $\frac{x + 6}{4}$ km

e. Total distance covered by Bernie: 10 km

Total distance covered by Garry: 16 km

f. Suggested option 1: This method can work as walking up to higher ground can put your phone in the line of sight with a cell tower.

Suggested option 2: Walking up to higher ground can improve signal as there are less disruptions in connections to cell towers, such as hills or buildings.

Note: There are other possible options.

3. a. 1 : 3

b. 1 : 1

c. 250 people in the line were accepted on the ride.

d. 2 : 3

e. 420 people were in line in the first place.

f. Suggested option 1: Amusement rides impose height restrictions due to some rides being potentially dangerous and small children being at risk of falling.

Suggested option 2: Amusement rides impose height restrictions due to safety fixtures of the ride possibly being ineffective if people are too short or tall.

Note: There are other possible options.

Chapter 11 review

Multiple choice

1. E **2.** C **3.** E **4.** D **5.** C

Fluency

6. a. 6 : 24 **b.** 3 : 4 **c.** 1 : 5 **d.** 7 : 6

7. a. 12 : 16 **b.** 2 : 4

c. 15 : 35 : 55 **d.** 18 : 33 : 48

8. a. 4 : 7 **b.** 10 : 9

c. 10 : 18 : 85 **d.** 115 : 235 : 339

9. a. 3 **b.** 4.09 **c.** 4 **d.** 13.15

10. a. 15 slices **b.** 84 furniture

c. 48 games **d.** 50 participants

11. a. \$200/month **b.** 3 customers/minute

c. 2.13 litres/minute **d.** 41.18 m/minute

12. a. Boyle collected 15 more gold coins per quest than Charles.

b. Nak runs 3.57 more metres per minute than Dom.

c. Socheata folds 2 more origami birds in an hour than Sorya.

d. Kai takes 4 fewer hours to bake 200 cupcakes than Kathy.

13. a. It will take both students 5 hours to complete the projects together.

b. It will take both farmers 26 minutes to move 50 bags of wheat together.

c. It takes both kitchen hands 32 minutes to clean the dishes together.

d. It takes all three gardeners 12 minutes to mow the lawn together.

14. a. *A, E* **b.** *B, D* **c.** *A* **d.** *D*

Problem solving

15. In a shopping mall's parking lot, there are 350 cars and 500 motorbikes. Write the ratio of motorbikes to the total number of vehicles in the parking lot.

Key points

- In a shopping mall's parking lot, there are 350 cars and 500 motorbikes.
- Write the ratio of motorbikes to the total number of vehicles in the parking lot.

Explanation

Number of motorbikes $= 500$.

Total number of vehicles $=$ number of cars $+$ number of motorbikes

$= 350 + 500$

$= 850$

$\therefore$ Motorbikes : total number of vehicles $= 500 : 850$.

This simplifies to 10 : 17.

Answer

The ratio of motorbikes to the total number of vehicles in the parking lot is 10 : 17.

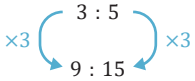
16. When Kris does his shopping for clothes, he buys 3 pairs of trousers for every 5 t-shirts. If Kris bought 9 pairs of trousers, how many t-shirts did he buy?

Key points

- When Kris does his shopping for clothes, he buys 3 pairs of trousers for every 5 t-shirts.
- Kris bought 9 pairs of trousers.
- How many t-shirts did he buy?

Explanation

Trousers : t-shirts = 3 : 5



Therefore, Kris bought 15 t-shirts along with 9 pairs of trousers.

Answer

Kris bought 15 t-shirts.

17. The ratio of the width to length of a swimming pool is 5 : 12. Find the dimensions of the swimming pool if it has an area of 1500 m^2.

Key points

- The ratio of the width to length of a swimming pool is 5 : 12.
- It has an area of 1500 m^2.
- Find the dimensions of the swimming pool.

Explanation

Let x be the value of 1 part. Width $= 5x$ m, length $= 12x$ m.

Area of swimming pool = width × length

$$1500 = 5x \times 12x$$
$$1500 = 60x^2$$
$$x^2 = 25$$
$$x = 5$$

So width $= 5 \times 5 = 25$ m, length $= 12 \times 5 = 60$ m.

Answer

The width of the swimming pool is 25 m and the length is 60 m.

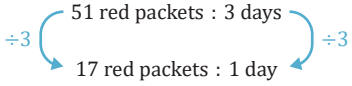
18. Over the first 3 days of Lunar New Year, Wang collected 51 red packets. What is the average rate at which Wang collects red packets?

Key points

- Over the first 3 days of Lunar New Year, Wang collected 51 red packets.
- What is the average rate at which Wang collects red packets?

Explanation

As a ratio:



As a rate:
17 red packets/day

Answer

Wang collects red packets at an average rate of 17 per day.

19. Sok and Stein live in a house that has 3 identical bedrooms. They are trying to repaint the walls of the bedrooms. Sok takes 3 hours to paint 1 room while Stein only takes 2 hours. How long will it take both of them to paint all the bedrooms?

Key points

- Sok and Stein live in a house that has 3 identical bedrooms.
- Sok takes 3 hours to paint 1 room.
- Stein only takes 2 hours to paint 1 room.
- How long will it take both of them to paint all the bedrooms?

Explanation

Calculate the combined rates of Sok and Stein.

Sok: 1 room/3 hours

Stein: 1 room/2 hours

Combined rate $= \frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$.

5 rooms : 6 hours

5 ÷ 5 rooms : 6 ÷ 5 hours

1 room : $\frac{6}{5}$ hours

6 ÷ 5 = 1.2

It will take both of them 1.2 hours to paint 1 bedroom. Therefore, to paint all 3 bedrooms they will need 1.2 × 3 = 3.6 hours.

Answer

It will take both of them 3.6 hours to paint 3 bedrooms.

20. In a training session for a cycling race, Chloe leaves the starting line at 150 km/h. After 15 minutes, Rachel leaves the starting line at 170 km/h. How long will it take for Rachel to pass Chloe by 2 km?

Key points

- In a training session for a cycling race, Chloe leaves the starting line at 150 km/h.
- After 15 minutes, Rachel leaves the startling line at 170 km/h.
- How long will it take for Rachel to pass Chloe by 2 km?

Explanations

Let t be Rachel's travel time.

Rachel's distance $= s \times t = 170t$ km

Chloe's distance $= s \times t + s \times t_{head\ start}$

$= 150t + 150 \times 0.25$ as 15 minutes $= 0.25$ hours

$= 150t + 37.5$ km

Rachel's distance = Chloe's distance + 2

$$170t = 150t + 37.5 + 2$$
$$20t = 39.5$$
$$t = 1.975 \text{ hours}$$

Answer

It will take Rachel 1.975 hours to pass Chloe by 2 km.

Reasoning

21. a. The ratio for 24 servings is 24 : 18 : 42.

b. Blending and rubbing spices requires 20 minutes, preparing vegetables and garnishes requires 25 minutes and cooking requires 15 minutes.

c. The kitchen sends out 0.4 servings/minute, and 2 servings every 5 minutes.

d. It takes the 2 chefs 413 minutes to cook 48 servings of the dish.

e. Suggested option 1: The presentation of the food makes a great dining experience.

Suggested option 2: The ambience of the restaurant makes a great dining experience.

Note: There are other possible options.

22. a. The difference in speeds between lines B and A is $\frac{125}{6}$ km/h.

b. The difference in speeds between lines C and B is equal to $\frac{100}{3}$ km/h.

c. Line C displays a horizontal line indicating the speed is at 0 km/h, therefore 0 subtracted from any number will have no change. Line A and B both display positive speeds, therefore the difference between lines B and A will not equal any of their own speeds.

12A Perimeter and units of length

Student practice

Worked example 1

a. 116 mm **b.** 2.55 km

Worked example 2

a. 12.2 cm **b.** 20 cm

Worked example 3

a. 5 m **b.** 11 mm

Understanding worksheet

1. **a.** m **b.** mm **c.** cm **d.** m

2. **a.** 2 **b.** 1 **c.** 4 **d.** 3

3. perimeter; length; measurement; convert

Fluency

4. **a.** 2 m **b.** 13 mm
c. 1.2 km **d.** 7000 mm
e. 0.735 m **f.** 0.01456 km
g. 26.3 mm **h.** 28.560045 km

---

5. **a.** 12 m **b.** 90 mm **c.** 14.1 km **d.** 24 cm
e. 25.2 m **f.** 18.6 cm **g.** 26.8 m **h.** 14.5 cm

---

6. **a.** $b = 10$ cm **b.** $w = 4.6$ km
c. $b = 40$ cm **d.** $x = 6$ m
e. $a = 5$ km **f.** $t = 24$ cm
g. $d = 13.2$ mm **h.** $x = 7$ m

---

7. **a.** 6 m **b.** 22 cm **c.** 85 mm **d.** 1050 cm
e. 1875 mm **f.** 200 m **g.** 3.095 km **h.** 8380 mm

---

8. E

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

10. Eliud runs 3km to warm up for his training. How many laps of a standard 400 m athletics track does he need to complete?

Key points

- Eliud needs to run 3 km.
- The athletics track is 400 m.
- How many laps of this athletics track does he need to run?

Explanation

Convert 3 km to m. There are 1000 m in 1 km, so multiply 3 km by 1000.

$3 \times 1000 = 3000$ m

Calculate the number of 400 m laps required to run 3 km.

$3000 \div 400 = 7.5$ laps

Answer

Eliud needs to run 7.5 laps.

11. Robert has a table that is 80 cm tall. If he stacks fifteen 78 mm high cubes on top of each other on top of the table, what is the total height of the table and stack, in cm?

Key points

- The table is 80 cm tall.
- Robert stacks fifteen 78 mm high cubes on top of each other on top of the table.
- What is the total height of the table and stack, in cm?

Explanation

Convert 78 mm to cm. There are 10 mm in 1 cm, so divide by 10.

$78 \div 10 = 7.8$ cm

Calculate how tall a stack of 15 cubes is, given each cube is 7.8 cm tall.

$15 \times 7.8 = 117$ cm

Add this to the height of the table (80 cm) to calculate the total height.

$117 + 80 = 197$ cm

Answer

The total height of the table and the stack is 197 cm.

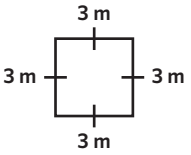
12. Hannah wants to make a square on the ground with her 12 m rope. Determine the length of one of the sides.

Key points

- Hannah wants to make a square.
- She will use 12 m of rope.
- Determine the length of one of the sides.

Explanation

A square has four equal sides. Hannah will use 12 m to make these four sides, so each side is $12 \div 4 = 3$ m.



3 m + 3 m + 3 m + 3 m = 12 m

Answer

Each side is 3 m.

13. Phemines has bought a new block of land as displayed in the diagram. If it costs \$12 for 1 metre of fencing, determine how much it will cost her to build a fence at the perimeter of her new block of land?

Key points

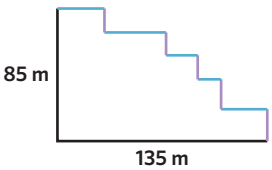
- It costs \$12 for 1 metre of fencing.
- How much will it cost her to build a fence at the perimeter of her new block of land?

Explanation

The total cost of the fencing will be \$12 per every metre of fencing required.

To calculate the amount of fencing we must first calculate the length of the perimeter.

From the diagram we can see that the blue lines are parallel to the line at the bottom of the diagram and the pink lines are parallel to the line on the left. We also see that the pink and blue lines are perpendicular to each other.



This means that the blue lines must sum to the length of the line on the bottom and the pink lines must sum to the length of the line on the left.

So the total perimeter $= 135 + 135 + 85 + 85 = 440$ m

Because the cost of the fencing is \$12 per metre the total cost $= 12 \times 440 = \$5280$.

Answer

It will cost Phemines \$5280 to build a fence at the perimeter of her new block of land.

14. Wacław loves geometric patterns. He began by drawing an equilateral triangle with each side of length s, like the one in Fig 1. He then created Fig 2 and 3 by drawing smaller equilateral triangles inside the same triangle. Determine the side lengths of each of the smaller triangles shown in Fig 2 and Fig 3 below, in terms of s.

Key points

- The large triangle is equilateral, with a side length of s.
- Waclaw created Fig 2 and 3 by drawing smaller equilateral triangles inside the same triangle.
- Determine the side lengths of the smaller triangles in Fig 2 and 3, in terms of s.

Explanation

The vertices of the centre small equilateral triangle in Figure 2 split each of the sides of the larger equilateral triangle into two. So the side length is half of s, which is $\frac{s}{2}$.

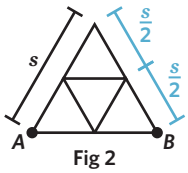


Fig 2

Now consider the equilateral triangles of side length $\frac{s}{2}$. These are further split into even smaller equilateral triangles in Figure 3.

Fig 3

The vertices of the centre triangle inscribed in this equilateral triangle splits each of the $\frac{s}{2}$ sides in two. So the side length is half of $\frac{s}{2}$, which is $\frac{s}{4}$.

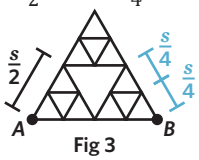


Fig 3

Answer

The largest side length is s, the second-largest side length is $\frac{s}{2}$, and the third-largest side length is $\frac{s}{4}$.

Reasoning

15. a. The length of the inner square's perimeter is 32 m.

b. The perimeter of the rectangle is 22 m.

c. The perimeter of the rectangle $ABCD$ is 48 m.

d. The length of the perimeter is 72 m.

e. Suggested option 1: You can warm up by going jogging.

Suggested option 2: You can warm up by stretching.

Note: There are other possible options.

16. a. Perimeter of square = perimeter of rhombus = 24 cm

b. Perimeter of rectangle = perimeter of parallelogram = 14 cm

c. Perimeter of a rhombus = perimeter of a square $= 4a$

Perimeter of a rectangle = perimeter of a parallelogram $= 2l + 2w$

Extra spicy

17. 16 cm

18. A

19. D

20. $x = 6$ and $y = 4$

Remember this?

21. D **22.** C **23.** B

12B Circumference of a circle

Student practice

Worked example 1

a. i. $C = 5\pi$ cm **ii.** ≈ 15.708 cm

b. i. $C = 2\pi \times 1.8$ m **ii.** ≈ 11.310 m

Worked example 2

a. $25\frac{1}{7}$ cm **b.** 10.048 m

Worked example 3

a. $d \approx 3.501$ mm **b.** $r \approx 1.687$ cm

Understanding worksheet

1. a. $d = 8$ **b.** $r = 3$

c. $r = m, d = 2m$ **d.** $r = \frac{p}{2}, d = p$

2. a. 1 **b.** 2.5 **c.** 2 **d.** 6

3. circumference; radius; diameter; π

Fluency

4. a. i. $C = 3\pi$ m **ii.** ≈ 9.425 m

b. i. $C = 2\pi$ cm **ii.** ≈ 6.283 cm

c. i. $C = 15\pi$ mm **ii.** ≈ 47.124 mm

d. i. $C = 5.5\pi$ km **ii.** ≈ 17.279 km

e. i. $C = \frac{3}{4}\pi$ mm **ii.** ≈ 2.356 mm

f. i. $C = \frac{7}{22}\pi$ km **ii.** ≈ 1.000 km

---

5. a. i. $C = 4\pi$ m **ii.** ≈ 12.566 m

b. i. $C = 30\pi$ mm **ii.** ≈ 94.248 mm

c. i. $C = 1.8\pi$ cm **ii.** ≈ 5.655 cm

d. i. $C = 42.96\pi$ m **ii.** ≈ 134.963 m

e. i. $C = \frac{50}{157}\pi$ km **ii.** ≈ 1.000 km

f. i. $C = \frac{220}{7}\pi$ mm **ii.** ≈ 98.736 mm

6. **a.** 6.286 cm **b.** 22.608 km
c. 2.357 m **d.** 13.125 m
e. 11.393 mm **f.** 18.500 m

---

7. **a.** 22.000 m **b.** 15.700 km
c. 13.619 m **d.** 38.465 m
e. 55.500 cm **f.** 1570.795 km

---

8. **a.** $d = 1.592$ m **b.** $d = 4.870$ cm
c. $r = 0.955$ km **d.** $d = 1.500$ cm
e. $r = 1.210$ mm **f.** $d = 428.127$ mm
g. $r = 0.037$ km **h.** $r = 0.736$ m

---

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student B is incorrect.

Problem solving

11. Pietro created a walking path with semi-circles around a square for his race horses. The square has side lengths of ten metres. How long is his new walking track? Round your answer correct to 2 decimal places.

Key points

- The square has side lengths of ten metres.
- How long is his new walking track?
- Round your answer to 2 decimal places.

Explanation

Calculate the circumference of two of the semicircles put together. The diameter will be 10 m.

$C = \pi d$

$C = \pi \times 10$

$C = 31.416$, rounded to 3 decimal places.

Multiply this circumference by two to calculate the other two semi-circles perimeter.

$31.416 \times 2 = 62.83185307$

Answer

The new walking track will be 62.83 m long.

12. The Earth is almost a perfect circle at the equator. The distance around the equator is 40 075 km. What is the direct distance from the surface of the Earth to its centre, to the nearest kilometre?

Key points

- The distance around the equator is 40 075 km.
- What is the direct distance from the surface of the Earth to its centre, to the nearest kilometre?

Explanation

Rearrange the equation to make the unknown, (r), is the subject.

$C = 2\pi r$

$r = \frac{C}{2\pi}$

Substitute in the circumference given into the equation to find r.

$r = \frac{40\,075}{2\pi}$

$r = 6378$, rounded to the nearest kilometre.

Answer

The distance from the surface of Earth to the centre is 6378 km.

13. Newtown is planning to install a new Ferris wheel as a feature of its tourism advertising campaign. The wheel, excluding the carriages, has a total length of 80 m. The pods can carry 10 passengers each, but need to be spaced out by 10 m around the Ferris wheel. How many passengers can be on the Ferris wheel at one time?

Key points

- The wheel, excluding the carriages, has a total length of 80 m.
- The pods can carry 10 passengers each.
- Need to be spaced out by 10 m around the Ferris wheel.
- How many passengers can be on the Ferris wheel at one time?

Explanation

Calculate the circumference of the Ferris wheel if the diameter is 80 m.

$C = \pi d$

$C = \pi \times 80$

$C = 251.327$ m rounded to 3 decimal places.

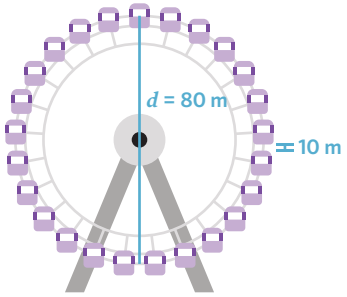
Divide the circumference by 10, to calculate how many pods can be around the wheel.

$251.327 \div 10 = 25.1327$

Round this value down, as there can not be half pods, this means there can be 25 pods around the Ferris wheel.

Multiply the amount of pods by the amount of people that can fit into each pod, to calculate the amount of people that can fit on the Ferris wheel at one time.

$25 \times 10 = 250$



Answer

250 passengers can fit on the Ferris wheel at one time.

14. Guiseppe has measured his pizzas to have a circumference of 94 cm. To the nearest centimetre, what is the minimum width of the pizza boxes Giuseppe needs to order?

Key points

- Guiseppe has measured his pizzas to have a circumference of 94 cm.
- What is the minimum width of the pizza boxes Giuseppe needs to order?
- To the nearest centimetre.

Explanation

Rearrange the equation to make the diameter, d, the subject.

$C = \pi d$

$d = \frac{C}{\pi}$

Substitute the given circumference into the equation to find d.

$d = \frac{94}{\pi}$

$d = 29.92$

$d = 30$ cm, rounded to the nearest centimetre.

Answer

Guiseppe needs to order a pizza box that has a minimum width of 30 cm.

15. Xavier is designing circular walkways over the local wetlands. The tradesmen need the plans to be correct to three decimal places. Which of the following estimates for π will produce an estimate for the exterior circumference of the walkway?

$\pi \approx \frac{22}{7}$, $\pi \approx \frac{333}{106}$ and $\pi \approx \frac{355}{113}$

Key points

- The tradesmen need the plans to be correct to three decimal places.
- Which of the following estimates for π will produce an estimate for the exterior circumference of the walkway?

Explanation

Substitute the given radius and each π estimates into the following equation. Add one onto the equation to get an accurate value for the circumference.

$C = 2\pi r$

Estimate one:

$C = 2 \times \frac{22}{7} \times 11$

$C = 69.143$ m, correct to three decimal places.

Estimate two:

$C = 2 \times \frac{333}{106} \times 11$

$C = 69.113$ m, correct to three decimal places.

Estimate three:

$C = 2 \times \frac{355}{113} \times 11$

$C = 69.115$ m, correct to three decimal places.

Calculate the circumference using π.

$C = 2\pi r$

$C = 2\pi \times 11$

$C = 69.115$ m, correct to three decimal places.

Compare which of the estimates was closest to the value.

Answer

The estimate, $\pi \approx \frac{355}{113}$, will produce the most accurate circumference correct to three decimal places.

Reasoning

16. a. The Moon will travel 2 415 256 km in one orbit around the Earth.

b. The Moon is travelling at approximately 3684 km/h around the Earth.

c. The Earth is travelling at approximately 107 513 km/h around the Sun.

d. Mercury is travelling at approximately 175 524 km/h around the Sun.

Venus is travelling at approximately 125 664 km/h.

Mars is travelling at approximately 86 885 km/h.

Jupiter is travelling at approximately 47 039 km/h.

e. Suggested option 1: Yes it is possible, due to the wide range of technology that humans have created, this could help make other planets livable.

Suggested option 2: No it is not possible, other planets have been found to not be liveable for humans, which means we would not survive on any other planet for long periods of time.

Note: There are other possible options.

17. a. 31.42 cm

b. Triangle's perimeter = 26.04 cm.
Square's perimeter = 28 cm.

c. The circumference in part **a** and the perimeters from part **b** will become more similar as more sides get added on. The difference between the two will decrease as the shapes from part **b** are getting closer to being a circle.

Extra spicy

18. C **19.** 0 **20.** E

21. $A = 36$ units squared

Remember this?

22. C **23.** E **24.** D

12C Area of squares, rectangles and triangles

Student practice

Worked example 1

a. 3.8 cm^2 **b.** 880 000 m^2

Worked example 2

a. 8.36 m^2 **b.** 4.18 cm^2

Worked example 3

a. 21 cm^2 **b.** 10 cm^2

Understanding worksheet

1. a. mm^2 **b.** m^2 **c.** ha **d.** km^2

2. a. Base of triangle: 11 cm
Height of triangle: 6 cm

b. Base of triangle: 13.6 cm
Height of triangle: 14 cm

c. Base of triangle: 11.1 cm
Height of triangle: 11.1 cm

d. Base of triangle: 7.2 cm
Height of triangle: 5.8 cm

3. squares; formula; perpendicular; multiplying

Fluency

4. a. 1200 mm^2 **b.** 0.3 m^2
c. 370 000 cm^2 **d.** 0.02 km^2
e. 0.845 m^2 **f.** 180 000 000 cm^2
g. 89 000 m^2 **h.** 0.000092 km^2

---

5. a. 15 m^2 **b.** 25 cm^2
c. 121 mm^2 **d.** 6 m^2
e. 7.36 km^2 **f.** 7.65 km^2
g. 295.84 mm^2 **h.** 2.835 cm^2

---

6. a. 17 cm^2 **b.** 28 km^2
c. 61 m^2 **d.** 20 m^2
e. 944.5 cm^2 **f.** 20 mm^2
g. 16 cm^2 **h.** 261 mm^2

---

7. a. $a = 3$ m **b.** $a = 4$ cm
c. $a = 4$ m **d.** $a = 11.25$ cm
e. $a = 1.2$ km **f.** $a = 5$ m
g. $a = 3$ cm **h.** $a = 3$ cm

8. B

Spot the mistake

9. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

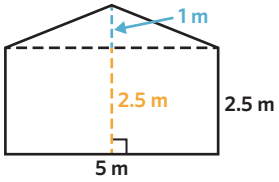
10. Mark is planning to paint the side of his barn. He has measured it to be 5 m wide, while the right and left sides are both 2.5 m high. The barn reaches 3.5 m at its highest point perpendicular to the ground. How many square metres does he need to paint?

Key points

- The barn is 5 m wide.
- The sides of the barn are 2.5 m high.
- The highest point of the barn is 3.5 m.
- Find the area of that side of the barn.

Explanation

The barn is a composite shape. Split the composite shape into a triangle and a rectangle.



Find the area of the triangle.

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 5 \times 1$$
$$= \frac{5}{2}$$
$$= 2.5 \text{ m}^2$$

Find the area of the rectangle.

$$\text{Area of rectangle} = \text{length} \times \text{width}$$
$$= 5 \times 2.5$$
$$= 12.5 \text{ m}^2$$

Find the area of the barn.

$$\text{Area of the barn} = \text{area of triangle} + \text{area of rectangle}$$
$$= 2.5 + 12.5$$
$$= 15 \text{ m}^2$$

Answer

Mark needs to paint an area of 15 square metres.

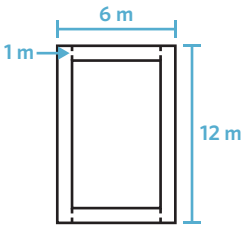
11. Dean's yard was 8 m wide and 12 m long. If he created a 1 m wide path around the edge of his yard, what area did the path cover?

Key points

- Dean's yard was 8 m wide and 12 m long.
- He creates a 1 m wide path around the edge of his yard.
- Find the area that the path covers.

Explanation

Split the path into four separate rectangles.



Find the area of the two long rectangles.

$$\text{Area of two long rectangles} = 2 \times 12 \times 1$$
$$= 24 \text{ m}^2$$

Find the area of the two short rectangles.

$$\text{Area of two short rectangles} = 2 \times 6 \times 1$$
$$= 12 \text{ m}^2$$

Find the total area.

$$\text{Total area} = 24 + 12 = 36 \text{ m}^2$$

Answer

The path covers an area of 36 m^2.

12. The regulations of a soccer pitch are that they must be a rectangular field with length from 100 m to 120 m and width from 50 m to 100 m. What is the difference in area between the smallest and largest possible fields?

Key points

- The length of the pitch is between 100 m to 120 m.
- The width of the pitch is between 50 m to 100 m.
- Find the difference in area between the smallest and largest possible fields.

Explanation

Find the area of the largest possible field.

The largest field has the maximum length and width.
So, length = 120 m and width = 100 m.

$$\text{Area} = 120 \times 100 = 12\,000 \text{ m}^2$$

Find the area of the smallest possible field.

The smallest field has the minimum length and width.
So, length 100 m and width = 50 m.

$$\text{Area} = 100 \times 50 = 5000 \text{ m}^2$$

Find the difference in the area of the largest pitch and the area of the smallest pitch.

$$12\,000 - 5000 = 7000 \text{ m}^2$$

Answer

The difference in area between the smallest and largest possible fields is 7000 m^2.

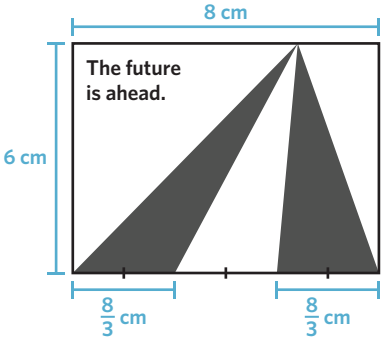
13. Fran's company is trying to position itself as a futurist start-up. They have created a design on their business cards to give the impression of heading to the future. If their business cards are 8 cm wide and 6 cm high, then what area of the card is painted black, excluding the writing?

Key points

- The business cards are 8 cm wide and 6 cm high.
- The bottom of the card is equally divided into three.
- Find the total area of the black triangle.

Explanation

Since the bottom of the card is divided equally into three, each section has a length of $\frac{8}{3}$ cm.



Each triangle has a base of $\frac{8}{3}$ cm and a height of 6 cm. So the area of the two triangles are the same. Therefore, we simply need to find the area of one triangle to double that to find the total area.

The total area of the black triangles $= 2 \times \frac{1}{2} \times \frac{8}{3} \times 6$

$= 16 \text{ cm}^2$

Answer

The total area of the card that is painted black is 16 cm^2.

14. Angelo has put up his 5-acre rectangular block of land for sale. The advertising states that the property has 125 m of road frontage. Considering that a hectare equals 2.5 acres, how far back from the road does his property go, in metres?

Key points

- Angelo's rectangular block of land is 5 acre.
- One side of the rectangle has a length of 125 m.
- A hectare equals 2.5 acres.
- Find the length of the other side of the rectangle in metres.

Explanation

Convert the area of the land from acres to hectares.

Area $= 5 \div 2.5 = 2$ hectares

Convert the area of the land from hectares to square metres.

Area $= 2 \times 100^2 = 20\,000 \text{ m}^2$

Find the other side length of the rectangle that has an area of 20 000 m^2 and one side with length 125 m.

$20\,000 \div 125 = 160$ m

Answer

The property has a length of 160 m from the road.

Reasoning

15. a. The total area of the court is 350 m^2.

b. The total length of all the white lines is 146.2 m.

c. 7.31 square metres of white needs to be applied.

d. Warren should make the line 6.84 cm wide.

e. Suggested option 1: No, it cannot survive without volunteers because the government doesn't give sufficient funding to maintain the functionality of the facilities.

Suggested option 2: Yes, it can survive without volunteers because clubbing facilities do not require high maintenance.

Note: There are other possible options.

16. a. Area of $\triangle ABC = 9 \text{ cm}^2$. Area of $\triangle AED = 21 \text{ cm}^2$.

b. Area of $\triangle ADC$ = area of quadrilateral $AEDB$ − area of $\triangle ABC$ − area of $\triangle AED$

$= 12 \text{ cm}^2$

c. The area of quadrilateral $EFCD$ is 24 cm^2. The area $\triangle ADC$ is half of the area of quadrilateral $EFCD$. This is because the base and height of $\triangle ADC$ is equal to the width and length of quadrilateral $EFCD$ respectively. Since the area of triangle $= \frac{1}{2} \times$ base $\times$ height, $\triangle ADC$ has an area that is half that of quadrilateral $EFCD$.

Extra spicy

17. A **18.** $a = \frac{12}{5}$ m **19.** D **20.** 800 pixels

Remember this?

21. B **22.** B **23.** D

12D Area of special quadrilaterals

Student practice

Worked example 1

a. 36 cm^2 **b.** 69.12 mm^2

Worked example 2

a. 25.5 cm^2 **b.** 350 mm^2

Worked example 3

a. 36 cm^2 **b.** 31.5 cm^2

Understanding worksheet

1. a. 14 mm^2 **b.** 30 cm^2
c. 28.7 km^2 **d.** 27.1 cm^2

2. a. Rectangle
b. Trapezium
c. Kite, parallelogram
d. Trapezium, kite, parallelogram, rectangle

3. parallelogram; diagonals; rhombus; parallel

Fluency

4. a. 12 cm^2 **b.** 7 m^2
c. 96 cm^2 **d.** 25.01 cm^2
e. 266 cm^2 **f.** 691.2 cm^2
g. 3.96 m^2 **h.** 50.96 m^2

---

5. a. 4 m^2 **b.** 33 mm^2
c. 95 mm^2 **d.** 601.65 cm^2
e. 520 cm^2 **f.** 102 mm^2
g. 3.15 cm^2 **h.** 120 cm^2

---

6. a. 4 cm^2 **b.** 8 cm^2
c. 18 cm^2 **d.** 72 mm^2
e. 50.6 km^2 **f.** 69.69 m^2
g. 18 cm^2 **h.** 14.4 km^2

---

7. a. $\frac{ab}{2}$ units^2 **b.** yz units^2
c. bc units^2 **d.** $\frac{r(s+t)}{2}$ units^2
e. x^2 units^2 **f.** $(a+1)b$ units^2

---

8. a. 20 mm^2 **b.** 48 mm^2
c. 30 cm^2 **d.** 312 km^2
e. 52.5 cm^2 **f.** 27.5 cm^2

---

9. D

Spot the mistake

10. a. Student A is incorrect. **b.** Student B is incorrect.

Problem solving

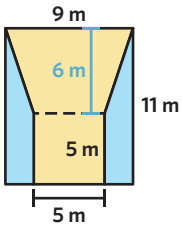
11. Bobby has built some water barriers to help funnel his sheep into a smaller area before he shears them. The piece of land he's working with is a 9 m by 11 m rectangle. The area he's trying to funnel them into is a 5 m by 5 m square. Use the diagram to determine the area of land in this rectangle that is not covered by water. Water is coloured blue.

Key points

- The piece of rectangular land is 9 m by 11 m.
- The small area is a 5 m by 5 m square.
- Water is coloured blue on the diagram.
- Determine the area of land that is not covered by water.

Explanation

The area of land not covered by water consists of a trapezium and a square.



Find the area of the trapezium.

$$\text{Area of trapezium} = \frac{6 \times (9 + 5)}{2}$$
$$= 42 \text{ m}^2$$

Find the area of the square.

$$\text{Area of square} = 5 \times 5 = 25 \text{ m}^2$$

Find the total area of the land.

$$\text{Total area} = 42 + 25 = 67 \text{ m}^2$$

Answer

The total area of the land is 67 m^2.

12. Strawberry Fields have re-zoned some farmland on the outskirts of their town. Roads are running parallel to each other in an east-west direction and streets are running parallel to each other in northeast-southwest direction. If the roads are 80 m apart and the intersections between roads and streets are 96 m away from each other, then what is the area of the block of land enclosed by the streets and roads?

Key points

- The roads and streets form a quadrilateral that have two opposite pairs of parallel sides.
- The height of the quadrilateral is 80 m and the base is 96 m.
- Find the area of the quadrilateral that the roads form.

Explanation

The road forms a parallelogram.

$$\text{Area of parallelogram} = \text{base} \times \text{height}$$
$$= 96 \times 80$$
$$= 7680 \text{ m}^2$$

Answer

The block of land enclosed by the streets and roads have an area of 7680 m^2.

13. Joshua created a pattern mural on a wall that was 4 m high and 7 m wide. He started by drawing a rhombus that had a vertex on each side of the wall and painting a star pattern inside the rhombus. He then coloured the wall outside of the rhombus blue. What is the size of the area that is covered by blue paint?

Key points

- The wall is 4 m high and 7 m wide.
- A rhombus is drawn on the wall with each vertex on the middle of each side of the wall.
- The area outside the rhombus is coloured blue.
- Find the area that's been coloured blue.

Explanation

Find the area of the rhombus.

$$\text{Area of rhombus} = \frac{d_1 \times d_2}{2}$$
$$= \frac{4 \times 7}{2}$$
$$= 14 \text{ m}^2$$

Find the area of the wall.

$$\text{Area of wall} = \text{length} \times \text{width}$$
$$= 4 \times 7$$
$$= 28 \text{ m}^2$$

$$\text{Area outside the rhombus} = \text{area of wall} - \text{area of rhombus}$$
$$= 28 - 14$$
$$= 14 \text{ m}^2$$

Answer

The area that is not covered in blue paint has a size of 14 m^2.

14. Tori has created a kite that has a total surface area of 1400 cm^2. What is the area of the smallest rectangle she could place around it?

Key points

- The kite has an area of 1400 cm^2.
- Find the area of the smallest possible rectangle that can fit around the kite.

Explanation

The area of the kite is equal to half of the rectangle created by its perpendicular diagonals.

So the area of the smallest rectangle is twice the area of the kite.

$$\text{Area of rectangle} = 1400 \times 2 = 2800 \text{ cm}^2$$

Answer

The area of the smallest rectangle that Tori could fit around the kite is 2800 cm^2.

15. Ravi wants to place sound proofing foam along a 17 m by 5 m wall to reduce feedback when recording his sitar. Unfortunately, he only has enough to cover 65 m^2 so he decides to compromise by covering as much as he can with a trapezium shape. Determine the value of a in the following diagram.

Key points

- The wall is 17 m wide and 5 m tall.
- Ravi can only cover 65 m^2 of the wall.
- He decides to cover the wall in the shape of a trapezium.
- a is the short parallel side of the trapezium.
- Find the value of a.

Explanation

The trapezium has an area of 65 m^2, a long parallel side of 17 m and a height of 5 m.

$$\text{Area of trapezium} = \frac{h(b + a)}{2}$$
$$65 = \frac{5 \times (17 + a)}{2}$$
$$130 = 5 \times (17 + a)$$
$$26 = 17 + a$$
$$a = 9 \text{ m}$$

Answer

The value of a is 9 m.

Reasoning

16. a. The total area of the Hansen family's property is 6300 m^2.

b. The total area of the Davinport family's property is 4900 m^2.

c. The combined area of the blocks of land is 11 200 m^2. Each family should have 5600 m^2 of land if they are to have an equal share.

d. The trapezium needs to be 28 m wide for both families to have an equal amount of land.

e. Suggested option 1: The amount of shore line improves a property because there would be better scenery.

Suggested option 2: The amount of shore line does not improve a property because it is irrelevant to the lives of the residents.

Note: There are other possible options.

17. a. 38 cm^2

b. 38 cm^2

c. The rectangle and triangle have the same height. The rectangle has a width of 7 and the triangle has a base of $5 = 12 - 7$.

All trapeziums will have a rectangle with area $a \times h$ and triangle with area $= \frac{h \times (b - a)}{2}$.

Therefore, total area of trapezium is:

$$a \times h + \frac{h \times (b - a)}{2}$$

$$= \frac{2 \times a \times h}{2} + \frac{h \times b - h \times a}{2}$$

$$= \frac{2 \times a \times h + h \times b - h \times a}{2} = \frac{a \times h + h \times b}{2} = \frac{h(b + a)}{2}$$

Extra spicy

18. $h = 5.92$ cm

19. D

20. $P \approx 31$ cm

21. C

Remember this?

22. D **23.** D **24.** D

12E Area of a circle

Student practice

Worked example 1

a. i. 49π cm^2

ii. 153.938 cm^2

b. i. 12.25π mm^2

ii. 38.485 mm^2

Worked example 2

a. $12\frac{4}{7}$ mm^2

b. 158.29 mm^2

Worked example 3

a. i. 4π cm^2

ii. 12.566 cm^2

b. i. $4\frac{1}{2}\pi$ m^2

ii. 14.137 m^2

Worked example 4

a. $r = 2.523$ cm

b. $d = 7.136$ m

Understanding worksheet

1. a. 12 **b.** 24 **c.** 52 **d.** 44

2. a. 16 **b.** 289 **c.** 5.29 **d.** 9

3. area; radius; irrational; exact

Fluency

4. a. i. π cm^2

ii. 3.142 cm^2

b. i. 9π m^2

ii. 28.274 m^2

c. i. 25π km^2

ii. 78.540 km^2

d. i. 18.49π m^2

ii. 58.088 m^2

e. i. 392.04π cm^2

ii. 1231.630 cm^2

f. i. 9.61π mm^2

ii. 30.191 mm^2

g. i. 2735.29π m^2

ii. 8593.167 m^2

h. i. 4057.69π km^2

ii. 12 747.609 km^2

---

5. a. i. 4π cm^2

ii. 12.566 cm^2

b. i. 36π cm^2

ii. 113.097 cm^2

c. i. $\frac{\pi}{4}$ km^2

ii. 0.785 km^2

d. i. 29.16π m^2

ii. 91.609 m^2

e. i. 45.563π cm^2

ii. 143.139 cm^2

f. i. 242.269π cm^2

ii. 761.111 cm^2

g. i. $\frac{4\pi}{9}$ cm^2

ii. 1.396 cm^2

h. i. 0.006π km^2

ii. 0.020 km^2

---

6. a. $\frac{22}{7}$ cm^2

b. 58.059 m^2

c. $\frac{352}{7}$ km^2

d. 216.315 cm^2

e. 1.395 cm^2

f. $\frac{126}{11}$ mm^2

g. 12 741.147 km^2

h. $\frac{74}{53}$ cm^2

---

7. a. i. π cm^2

ii. 3.142 cm^2

b. i. 8π m^2

ii. 25.133 m^2

c. i. 16π cm^2

ii. 20.265 cm^2

d. i. $\frac{9\pi}{8}$ m^2

ii. 3.534 m^2

e. i. $\frac{121\pi}{4}$ mm^2

ii. 95.033 mm^2

f. i. $\frac{225\pi}{32}$ mm^2

ii. 22.089 mm^2

g. i. $\frac{4489\pi}{64}$ cm^2

ii. 220.353 cm^2

h. i. $\frac{18\,769\pi}{80\,000}$ km^2

ii. 0.737 km^2

---

8. a. $r = 1.382$ m

b. $d = 3.909$ cm

c. $r = 2.167$ cm

d. $r = 2.200$ km

e. $d = 6.283$ mm

f. $d = 10.000$ m

g. $d = 6.000$ mm

h. $r = 4.000$ mm

---

9. C

Spot the mistake

10. a. Student B is incorrect.

b. Student B is incorrect.

Problem solving

11. Melanie is watering her crops with a sprinkler that rotates 360° and sprays water in every direction. How much area does the sprinkler water?

Key points

- The sprinkler rotates 360° and sprays water in every direction.
- The sprinkler can spray water up to 12 m away from itself.
- Find the area that the sprinkler waters.

Explanation

The area that the sprinkler waters is a circle with a radius of 12 m.

$$\text{Area} = \pi \times r^2$$
$$= \pi \times 12^2$$
$$= 144\pi \text{ m}^2$$

Answer

The sprinkler waters an area of 144π m^2.

12. Henry has a windscreen that is 80 cm high and 130 cm wide. If he attaches a single windscreen wiper that is 70 cm long, what is the area of the windscreen that is untouched by the windscreen wiper?

Key points

- The windscreen is 80 m high and 130 cm wide.
- The wiper is 70 cm long.
- Find the area of the windscreen that the wiper cannot wipe.

Explanation

The wind screen is a rectangle with a length and width of 130 cm and 80 cm respectively. The area of the region that the wiper can wipe is a semicircle with a radius of 70 cm.

Find the area of the windscreen.

$$A = 130 \times 80 = 10\,400 \text{ cm}^2$$

Find the area of the semicircle.

$$A = \frac{\pi \times 70^2}{2} = 2450\pi \text{ cm}^2$$

Area of region wiper cannot wipe = area of the windscreen − area of the semicircle

$$= 10\,400 - 2450\pi \text{ cm}^2$$

Answer

The area of the windscreen that is untouched by the windscreen wiper is $10\,400 - 2450\pi$ cm^2.

13. Annunziata sells pizzas for $24. Her pizzas have a radius of 18 cm. She is creating a new pizza size that has a radius of 21 cm. How much should she sell the new size of pizza so her customers pay exactly the same amount per cm^2?

Key points

- Annunziata sells pizzas for $24.
- Currently, her pizzas have a radius of 18 cm.
- The new pizza has a radius of 21 cm.
- What should the price of the new pizza be so that the customers pay exactly the same amount per cm^2?

Explanation

Find the area of the current pizza.

$$A = \pi \times 18^2 = 324\pi \text{ cm}^2$$

Find the price per cm^2 of the current pizza.

$$324\pi \div 24 = \$\frac{27\pi}{2} \text{ per cm}^2$$

Find the area of the new pizza.

$$A = \pi \times 21^2 = 441\pi \text{ cm}^2$$

Find the total price of the new pizza.

Total price = price per unit area × total area

$$= \frac{27\pi}{2} \times 441\pi$$
$$= \$\frac{11907\pi^2}{2}$$

Answer

The price of the new pizza should be $\$\frac{11907\pi^2}{2}$.

14. Ferdinand measures the distance around his circular plate to be 20 cm. What is the area that his plate covers?

Key points

- The circumference of the plate is 20 cm.
- Find the area of the plate.

Explanation

Find the radius of the plate.

Circumference of a circle $= 2\pi r$, so

$$r = \text{circumference} \div 2\pi$$
$$= 20 \div 2\pi$$
$$= \frac{10}{\pi} \text{ cm}$$

Find the area of the plate.

$$A = \pi \times \left(\frac{10}{\pi}\right)^2$$
$$= \pi \times \frac{100}{\pi^2}$$
$$= \frac{100}{\pi} \text{ cm}^2$$

Answer

The area of the plate is $\frac{100}{\pi}$ cm^2.

15. Archibald has a 20 m rope that is 3 cm thick on his ship. When he is at port, his ropes are rolled up and placed on the dock, as shown in the diagram. What is the radius of a circle that has the same amount of area that Archibald's rope would cover?

Key points

- The rope is 20 m long and 3 cm thick.
- Find the radius of a circle that has the same area as the rope when it is rolled up into a circle.

Explanation

Find the area of the rope. When stretched out, the rope can be viewed as a rectangle with lengths and widths of 20 m and 3 cm respectively. Note: 3 cm = 0.03 m.

$$A = 20 \times 0.03 = 0.6 = \frac{3}{5} \text{ m}^2$$

The area of this rectangle is the same as the area of the circle that the rope forms when rolled up.

Find the radius of the circle that has the same area.

$$A = \pi r^2$$
$$\frac{3}{5} = \pi r^2$$
$$r^2 = \frac{3}{5\pi}$$
$$r = \sqrt{\frac{3}{5\pi}} \text{ m}$$

Answer

A circle that has the same amount of area as the rope has a radius of $\sqrt{\frac{3}{5\pi}}$ m.

Reasoning

16. a. $A = 4\pi \text{ m}^2$

b. $A = 2\pi + 4 \text{ m}^2$

c. $A = 2\pi \text{ m}^2$

d. Grey area $= 2\pi - 4 \text{ m}^2$

Pink area $= \frac{7}{4} \text{ m}^2$

Yellow area $= 2\pi - \frac{9}{4} \text{ m}^2$

e. Suggested option 1: The Apple logo.

Suggested option 2: The Baskin Robbins logo.

Note: There are other possible options.

17. a. $\frac{\pi}{4}$ **b.** $\frac{2}{\pi}$

c. From part **a**, $\frac{\pi}{4} \approx 0.785$. From part **b**, $\frac{2}{\pi} \approx 0.647$. Therefore, a round peg in a square hole is a better fit than a square peg in a round hole as a round peg covers more area in a square hole than a square peg does in a round hole.

Extra spicy

18. C **19.** B **20.** $14\pi \text{ cm}^2$ **21.** $\frac{4}{9}$

Remember this?

22. E **23.** D **24.** A

12F Composite shapes with sectors

Student practice

Worked example 1

a. $x = \frac{1}{6}$ **b.** $a = 40°$

Worked example 2

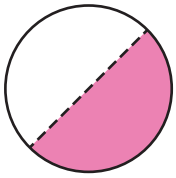
a. 5.585 cm^2 **b.** 38.727 cm^2

Worked example 3

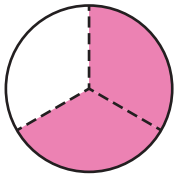
a. 5.236 cm **b.** 61.373 mm

Understanding worksheet

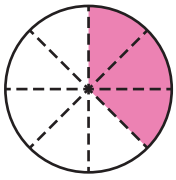
1. a.



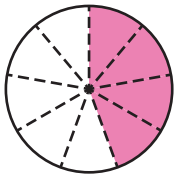
b.



c.



d.



2. a. $\frac{1}{2}, \frac{1}{2}$ **b.** $\frac{1}{4}, \frac{1}{4}$ **c.** $\frac{1}{6}, \frac{1}{6}$ **d.** $\frac{2}{7}, \frac{2}{7}$

3. circle; multiplying; circumference; radii

Fluency

4. a. $a = 180°$ **b.** $x = \frac{1}{3}$

c. $a = 270°$ **d.** $x = \frac{11}{30}$

e. $a = 135°$ **f.** $x = \frac{3}{8}$

g. $a = 140°$ **h.** $x = \frac{19}{60}$

5. a. 13.090 cm^2 **b.** 4.189 mm^2

c. 6.545 cm^2 **d.** 381.704 mm^2

e. 44.323 cm^2 **f.** 12.582 m^2

g. 0.996 km^2 **h.** 54.342 m^2

6. a. 3.927 cm **b.** 2.094 cm

c. 25.918 mm **d.** 4.712 km

e. 28.798 cm **f.** 22.235 m

g. 24.245 m **h.** 42.106 mm

7. a. 32.371 km^2 **b.** 5213.495 m^2

c. 50.758 m^2 **d.** 140.549 cm^2

e. 43.012 cm^2 **f.** 61.242 km^2

g. 35.126 mm^2 **h.** 102.540 mm^2

8. a. 329.956 cm **b.** 24.712 cm

c. 38.247 km **d.** 74.841 mm

e. 20.279 m **f.** 48.784 mm

g. 89.973 mm **h.** 15.770 cm

9. C

Spot the mistake

10. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

11. Jeremy has eight friends over for his birthday party. To ensure he and all of his friends have an equal share of the cake he needs to cut nine equal slices. If each is a sector of his cake, how many degrees should there be between each cut?

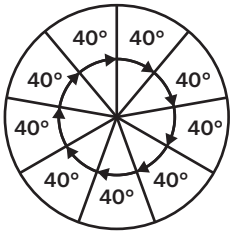
Key points

- Jeremy has eight friends over for his birthday party.
- To ensure he and all of his friends have an equal share of the cake he needs to cut nine equal slices.
- If each is a sector of his cake, how many degrees should there be between each cut?

Explanation

A full circle is 360°.

To cut the circle into 9 equal sectors, each sector would be $360° \div 9 = 40°$.



Answer

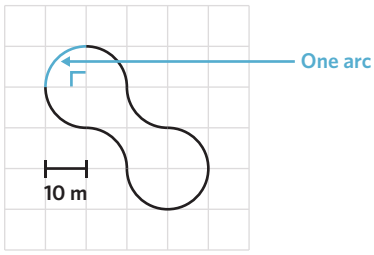
There should be 40° between each cut.

12. Bernie is planning a new go-kart track that is made up of identical arcs joined together. He connects a series of arcs from 8 quadrants and draws them on a grid. If the grid represents 10 m intervals, then how long is one lap of Bernie's track, correct to 3 decimal places.

Key points

- Bernie is planning a new go-kart track that is made up of identical arcs joined together.
- He connects a series of arcs from 8 quadrants and draws them on a grid.
- The grid represents 10 m intervals.
- How long is one lap of Bernie's track, correct to 3 decimal places?

Explanation



$$\text{The perimeter of one arc} = \frac{90°}{360°} \times 2\pi \times 10$$
$$= \frac{1}{4} \times 20 \times \pi$$
$$= 5\pi \text{ m}$$

There are 8 identical arcs in total.

$$\text{Perimeter of entire track} = \text{perimeter of one arc} \times 8$$
$$= 5\pi \times 8$$
$$= 40\pi$$
$$\simeq 125.664 \text{ m}$$

Answer

One lap of Bernie's track is 125.664 m.

13. Tui has a wall between her lounge and dining room that has a height of 3.5 m and a width of 6.5 m. She has cut an arch into the wall that consists of a semicircle with a radius of 3.5 m and a rectangle that is 1.75 m high. Calculate the area of the wall she has left, correct to 3 decimal places.

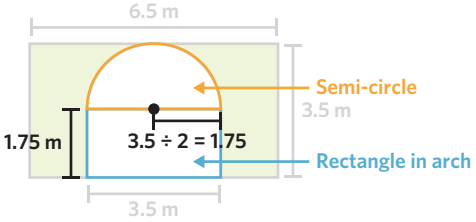
Key points

- Tui has a wall between her lounge and dining room that has a height of 3.5 m and a width of 6.5 m.
- She has cut an arch into the wall that consists of a semicircle with a radius of 3.5 m and a rectangle that is 1.75 m high.
- Calculate the area of the wall she has left, correct to 3 decimal places.

Explanation

$$\text{Area of wall Tui has left} = \text{area of big rectangle} - \text{area of arch}$$
$$\text{Area of rectangle} = l \times w$$
$$= 6.5 \times 3.5$$
$$= 22.75 \text{ m}^2$$

$$\text{Area of arch} = \text{area of semi-circle} + \text{area of rectangle in arch}$$



$$= \frac{180°}{360°} \times \pi \times (3.5 \div 2)^2 + 1.75 \times 3.5$$
$$= \frac{1.75^2\pi}{2} + 6.125$$

$$\text{Area of wall Tui has left} = 22.75 - \left(\frac{1.75^2\pi}{2} + 6.125\right)$$
$$= 11.814 \text{ m}^2$$

Answer

The area of wall Tui has left is equal to 11.814 m^2.

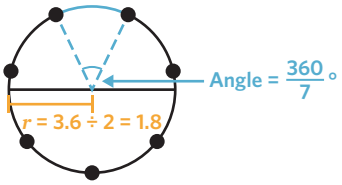
14. Campbell has 7 players on his basketball team. He wants them to be equally spaced around the centre circle for a passing drill. What should the circular distance between each player be, correct to 3 decimal places?

Key points

- Campbell has 7 players on his basketball team.
- He wants them to be equally spaced around the centre circle for a passing drill.
- What should the circular distance between each player be, correct to three decimal places?

Explanation

The distance between each player is the length of each arc of a sector.



Each arc has a radius of 1.8 m and an angle of $\left(\frac{360}{7}\right)°$.

$$\text{Length of each arc} = \frac{360}{7} \div 360 \times 2\pi \times 1.8$$
$$= \frac{360}{7} \times \frac{1}{360} \times 3.6\pi$$
$$= \frac{1}{7} \times 3.6\pi \text{ m}$$
$$= \frac{18}{35}\pi \text{ m}$$
$$\simeq 1.616 \text{ m}$$

Answer

The circular distance between each player should be 1.616 m.

15. Sean's frisbee has a radius of 12 cm. He cuts it into sectors of 202.5° and 157.5° and turns both of them into smaller frisbees. What is the radius of the bigger of the two new frisbees?

Key points

- Sean's frisbee has a radius of 12 cm.
- He cuts it into sectors of 202.5° and 157.5° and turns both of them into smaller frisbees.
- What is the radius of the bigger of the two new frisbees?

Explanation

To turn a big frisbee into two smaller ones, the total amount of material does not change.

$$\therefore \text{Area of orange sector} = \text{area of orange circle}$$
$$\text{Area of orange sector} = \frac{202.5°}{360°} \times \pi \times 12^2$$
$$= \frac{202.5°}{360°} \times 144\pi$$
$$= 81\pi \text{ cm}^2$$

Area of orange circle $= 81\pi \text{ cm}^2$

$= \pi \times r^2$

$81\pi = \pi \times r^2$

$81\pi \div \pi = \pi \times r^2 \div \pi$

$81 = r^2$

$r = 9$

The new orange frisbee has a radius of 9 cm; the new blue frisbee has a radius of 7.937 cm.

$9 > 7.937$

∴ The radius of the bigger of the two new frisbees is 9 cm.

Answer

The radius of the bigger of the two new frisbees is 9 cm.

Reasoning

16. a. The area covered by a sprinkler with a radius of 5.5 m is 95.033 m^2.

b. The area a sprinkler with radius of 11 m could water is 120.375m^2.

c. The total area that all four sprinklers could water is 113.097 m^2.

d. The sprinkler with a radius of 11 m from part **b** covers the largest area.

e. Suggested option 1: I prefer to grow edible plants because it saves time and money.

Suggested option 2: I prefer to grow pretty flowers because they make my garden look good.

Note: There are other possible options.

17. a. Total length $= 85.332 + \frac{180}{360} \times 2 \times \pi \times 36.5$

$= 85.332 + \frac{1}{2} \times 2 \times \pi \times 36.5$

$= 85.332 + 36.5\pi$

$= 85.332 + 114.6681...$

$\simeq 200$ m as required

b. The distance of the second lane is 203.142 m.

c. The stagger for each of the first three lanes is equal to π m.

Extra spicy

18. C **19.** $4\pi \text{ cm}^2$

20. B **21.** $18\pi - 36 \text{ cm}^2$

Remember this?

22. B **23.** D **24.** D

12G Volume of prisms and cylinders

Student practice

Worked example 1

a. 90 mm^3 **b.** 39.9 m^3

Worked example 2

a. 12 m^3 **b.** 21 600 mm^3 **c.** 62.832 cm^3

Worked example 3

a. 557.080 cm^3 **b.** 98.628 m^3

Understanding worksheet

1. a. 35 **b.** 40 000

c. 0.000058 **d.** 0.00067

2. a. Yes **b.** No **c.** Yes **d.** No

3. volume; perpendicular; prism; cylinder

Fluency

4. a. 90 mm^3 **b.** 5 km^3

c. 109.2 cm^3 **d.** 157.5 m^3

e. 374 cm^3 **f.** 154.08 mm^3

---

5. a. 12.32 m^3 **b.** 37.1 cm^3

c. 21 600 mm^3 **d.** 112 cm^3

e. 166.375 cm^3 **f.** 2002 mm^3

g. 51.84 m^3 **h.** 14 784 mm^3

---

6. a. 785.398 cm^3 **b.** 1244.071 m^3

c. 84.823 cm^3 **d.** 62.090 cm^3

e. 173 180.295 mm^3 **f.** 5489.870 cm^3

---

7. a. 950 m^3 **b.** 1026 km^3

c. 133 m^3 **d.** 1311.239 cm^3

e. 51.389 m^3 **f.** 47.124 cm^3

---

8. D

Spot the mistake

9. a. Student B is incorrect. **b.** Student A is incorrect.

Problem solving

10. Ali's garden bed has a width of 2.5 metres, a length of 3 metres and a height of 0.5 metres. What volume of soil can it hold?

Key points

- The rectangular garden bed has a width, length and height of 2.5 m, 3 m and 0.5 m respectively.
- Find the volume of the garden bed.

Explanation

Volume = length × width × height

$= 3 \times 2.5 \times 0.5$

$= 3.75 \text{ m}^3$

Answer

The garden bed can hold 3.75 m^3 of soil.

11. Charya is going to make 100 door stops for his school with recycled plastic. The door stops will be in the shape of a triangular prism with height of 1.2 cm, width of 1.8 cm and length of 6.8 cm. What volume of plastic will he need to make all 100 door stops?

Key points

- Each door stop is in the shape of a triangular prism with a height, width and length of 1.2 cm, 1.8 cm and 6.8 cm respectively.
- Find the volume of plastic needed for 100 door stops.

Explanation

Find the volume of each door stop.

Volume = area of cross section × width

$= \frac{1}{2} \times \text{length} \times \text{height} \times \text{width}$

$= \frac{1}{2} \times 6.8 \times 1.2 \times 1.8$

$= 7.344 \text{ cm}^3$

Find the volume of 100 door stops.

Total volume $= 100 \times 7.344$

$= 734.4 \text{ cm}^3$

Answer

Charya will need 734.4 cm^3 of plastic to make 100 door stops.

12. Coryn needs to fill his water tank because it is completely empty. It is circular with a width of 3 metres and a height of 1.8 metres. A cubic metre is equivalent to 1000 litres. How much water does Coryn need to fill his tank, rounded to the nearest litre?

Key points

- The tank has a circular cross section with a diameter of 3 metres and a height of 1.8 metres.
- $1 \text{ m}^3 = 1000$ L.
- Find the volume of the tank to the nearest litre.

Explanation

A prism with a circular cross section is a cylinder.

Volume of cylinder $= \pi \times r^2 \times h$

$= \pi \times \left(\frac{3}{2}\right)^2 \times 1.8$

$= 4.05\pi \text{ m}^3$

Convert the volume to litres.

$1 \text{ m}^3 = 1000$ L, so

$4.05\pi \text{ m}^3 = 1000 \times 4.05\pi$

$\approx 12\,723$ L

Answer

Coryn needs 12 723 L of water to fill his tank.

13. Melbourne covers an area of 9993 km^2. If 40 mm of rain fell in a day, what is the total volume of rain? Give your answer in m^3.

Key points

- Melbourne has an area of 9993 km^2.
- On a particular day, there was a 40 mm depth of rainfall.
- Find the total volume of rain in m^3, correct to 3 decimal places.

Explanation

Convert Melbourne's area to m^2.

$9993 \text{ km}^2 = 9\,993\,000\,000 \text{ m}^2$

Convert the depth of rainfall to m.

$40 \text{ mm} = 0.04 \text{ m}$

Find the volume of rainfall.

Volume = area of cross section × height

$= 9\,993\,000\,000 \times 0.04$

$= 399\,720\,000 \text{ m}^3$

Answer

The volume of rain is 399 720 000 m^3.

14. Enzo has been tasked with building a 20 m long tunnel that has a 6 m wide semicircle for a cross section. The tunnel is to be built using concrete that needs to be 1 m thick. How many cubic metres of concrete does Enzo need to build the tunnel, correct to 3 decimal places?

Key points

- The tunnel has a 6 m wide, 20 m long semi-cylinder.
- The tunnel is to be built using concrete that needs to be 1 m thick.
- Find the volume of concrete required to build the tunnel in m^3 to 3 decimal places.

Explanation

Volume = area of cross section × length

= (outer semicircle − inner semicircle) × length

$= \left(\frac{\pi \times 4^2}{2} - \frac{\pi \times 3^2}{2}\right) \times 20$

$= \frac{7\pi}{2} \times 20$

$= 70\pi \text{ m}^3$

$\approx 219.911 \text{ m}^3$

Answer

Enzo will need 219.911 m^3 of concrete to build the tunnel.

Reasoning

15. **a.** The volume of Leslie's spa is 5.1 m^3.

b. The volume of the deep end of the pool is 14.85 m^3.

The volume of the shallow end of the pool is 8.91 m^3.

c. The volume of the pool is 46.035 m^3.

d. Leslie needs to replace 2301.75 L of water each week.

e. Suggested option 1: Add a diving board at the deep end of the pool.

Suggested option 2: Add animal themed slides to the deep end of the pool.

Note: There are other possible options.

16. **a.** 12 cm^3 **b.** 6 cm^3

c. The cross-section in part **b** is a consistent cross-section, therefore, multiplying it by the perpendicular length will produce the correct area for the volume. The cross-section in part **a** is not consistent because any cross-section that is parallel to it will be a different size. Therefore it can not be used to find the volume.

Extra spicy

17. 54 cm **18.** $\frac{20}{27} \text{ m}^3$ **19.** C **20.** D

Remember this?

21. C **22.** D **23.** D

12H Time

Student practice

Worked example 1

a. 210 minutes **b.** $3\frac{1}{2}$ years

Worked example 2

a. 0645 **b.** 4:15 pm

Worked example 3

a. 6:20 pm **b.** 11:30 am

Worked example 4

a. 1 hour 30 minutes **b.** 13 hours 5 minutes

Understanding worksheet

1. **a.** 2:30 **b.** 10:40 **c.** 8:08 **d.** 4:36

2. **a.**

b.

c.

d.

3. 12-hour; 24-hour; UTC; daylight saving

Fluency

4. **a.** 120 minutes **b.** 36 hours
c. 45 months **d.** 168 hours
e. 2700 seconds **f.** 9 hours 45 minutes
g. $80\frac{2}{3}$ hours **h.** $80\frac{1}{2}$ months

---

5. **a.** 1230 **b.** 1:15 pm **c.** 0720 **d.** 0018
e. 11:23 pm **f.** 1755 **g.** 9:21 am **h.** 0416

---

6. **a.** 12:00 pm **b.** 2:30 pm
c. 5:40 am **d.** 8:45 pm
e. 1:45 am next day **f.** 1:55 am next day
g. 10:38 pm **h.** 11:12 pm previous day

---

7. **a.** 5:30 pm **b.** 1:00 pm
c. 1:40 pm **d.** 4:45 pm
e. 1:30 pm **f.** 10:00 pm previous day
g. 2:35 pm next day **h.** 4:43 pm previous day

---

8. **a.** 2 hours 30 minutes **b.** 1 hour 25 minutes
c. 48 minutes **d.** 3 hours 8 minutes
e. 3 hours 14 minutes **f.** 3 hours 5 minutes
g. 21 hours 55 minutes **h.** 45 hours 10 minutes

9. E

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. Patrick has gone for a jog at lunchtime. He started running at 12:45 pm and finished running at 1:25 pm. How long did he run for?

Key points

- Patrick started jogging at 12:45 pm and finished at 1:25 pm.
- Find the duration of Patrick's jogging session.

Explanation

Find the duration between 12:45 pm and 1:25 pm by subtracting the hours and minutes of the start time from that of the end time. This is, often, more convenient to do in a 24-hour clock.

12:45 pm = 1245

1:25 pm = 1325

13 hour − 12 hour = 1 hour

25 min − 45 min = −20 min

1 hour − 20 min = 40 minutes.

Answer

Patrick ran for 40 minutes.

12. Vutha is visiting family in Phnom Penh but will continue to work remotely for his company in Brisbane. If he's scheduled to have a meeting with Tori at 10:30 am Brisbane time, what time will this be in Phnom Penh?

Key points

- Vutha is to have a meeting with Tori at 10:30 am Brisbane time.
- Find the time that Vutha will be having his meeting in Phnom Penh.

Explanation

Brisbane time = UTC +10; Phnom Penh time = UTC +7.

So Phnom Penh time = Brisbane time − 3 hours.

10:30 am − 3 hours = 7:30 am

Answer

Vutha will have the meeting at 7:30 am in Phnom Penh.

13. Perth is in the UTC +8 time zone and Sydney is in the UTC +10 time zone. Perth doesn't use daylight saving time, but Sydney does. Play in the test match at Perth is scheduled to start at 10:30 pm. Richie lives in Sydney and wants to watch the start of play. What time will it be in Sydney when play starts?

Key points

- Perth is in the UTC +8 time zone; Sydney is in the UTC +10 time zone.
- Perth doesn't use daylight saving time but Sydney does.
- Play starts at 10:30 pm at Perth.
- Richie lives in Sydney and wants to watch the start of play.
- What time will it be in Sydney when play starts?

Explanation

Perth is in the UTC +8 time zone; Sydney is in the UTC +10 time zone.

So Sydney time = Perth time + 2 hours.

10:30 pm + 2 hours = 12:30 am next day.

Answer

It will be 12:30 am the next day in Sydney when play starts.

14. Rebecca's flight to Sydney leaves Los Angeles at 5:20 pm on Thursday. The total duration of the flight is 15 hrs and 55 mins. What day and time does she arrive in Sydney?

Key points

- Rebecca's flight leaves Los Angeles at 5:20 pm on Thursday.
- The flight takes 15 hrs and 55 mins.
- Find the time and date of Rebecca's arrival.

Explanation

Split 15 hrs 55 mins into 12 hrs + 3 hrs 55 minutes.

5:20 pm Thursday + 12 hrs = 5:20 am Friday

5:20 am Friday + 3 hrs 55 mins = 9:15 am Friday

Sydney is in the UTC +10 time zone.

Sydney time = Los Angeles time + 18 hours

9:15 am Friday + 18 hours = 3:15 am Saturday

Answer

Rebecca arrives in Sydney at 3:15 am on Saturday.

15. Kathy is going to meet with six of her students at lunchtime today. Southey High School's lunchtime is from 12:18 pm to 1:20 pm. If she plans to have a 2-minute gap between each meeting, how much time will she have per student in minutes and seconds if each student has an equal share of her time?

Key points

- Kathy is going to meet six students at lunchtime.
- Lunchtime is from 12:18 pm to 1:20 pm.
- There is a 2-minute gap between each meeting.
- Each student is to have an equal share of Kathy's time.
- Find the duration of time each student has to meet with Kathy.

Explanation

Find the duration between 12:18 pm to 1:20 pm.

12:18 pm = 1218; 1:20 pm = 1320

13 hr − 12 hr = 1 hr

20 min − 18 min = 2 mins

The duration of lunchtime is 1 hr 2 mins.

Find the available meeting time by subtracting the in between 2-min gaps.

There are six students so there are five gaps.

5 × 2 mins = 10 mins

Available time = 1 hr 2 mins − 10 mins

= 1 hr − 8 mins

= 52 mins

Divide the available time equally into six.

52 mins ÷ 6 = 8 mins 40 secs

Answer

Kathy will have 8 minutes 40 seconds per student.

Reasoning

16. a. When Matthew goes to bed on Friday night, it is 5:00 am Friday on Vancouver Island and 1:00 pm Friday in London.

When Matthew wakes up on Saturday morning, it is 1:00 pm Friday on Vancouver Island and 9:00 pm Friday in London.

b. When Andrew starts his work week Monday morning, it is 1:00 am Monday on Vancouver Island and 7:00 pm Monday in Melbourne.

When Andrew finishes his work week on Friday night, it is 9:00 am Friday on Vancouver Island and 3:00 am Saturday in Melbourne.

c. When Dylan wakes up on Saturday morning, it is 3:00 pm Saturday in London and 1:00 am Sunday in Melbourne.

When Dylan goes to bed on Sunday night, it is 7:00 am Monday in London and 5:00 pm Monday in Melbourne.

d. Matthew could schedule a video call with both of his brothers 7:00 am–9:00 am Sunday Melbourne time.

e. Suggested option 1: Being able to see and speak to one another while not being physically present.

Suggested option 2: Being able to have conferences and meetings with team members who are working overseas.

Note: There are other possible options.

17. a. The Earth travels 2 573 400 km in one day.

b. The Earth travels 939 291 000 km around the Sun in 365 days.

c. A single orbit around the Sun is approximately 939 950 000 km whereas the orbit in 365 days is 939 291 000 km. This means that the Earth doesn't complete its orbit for each calendar year and there is some distance lost. To make up for this lost distance, an extra day is added to the calendar every four years, which we call the leap year.

Extra spicy

18. B **19.** 1827 **20.** 1:24 pm **21.** D

Remember this?

22. C **23.** C **24.** E

Chapter 12 extended application

1. a. 218 750 km^2

b. South Australia: 1 250 000 km^2

Tasmania: 93 750 km^2

c. 876 953 km^2

d. Queensland: 1 646 484.375 km^2

Northern Territory: 1 285 156.25 km^2

e. 8 046 186.125 km^2

f. Suggested option 1: My favourite state in Australia is Queensland because the weather is always nice and sunny there.

Suggested option 2: My favourite state in Australia is New South Wales because Sydney is one of the most multicultural cities in Australia.

Note: There are other possible options.

2. a. 676 m^2

b. 23.758 m^2

c. 1757.163 m^2

d. 7542.964 m^2

e. Total playing area = Trapezium 1 + Trapezium 2
= 3372.5 m^2 + 6740.5 m^2 = 10 113 m^2

Total area of synthetic grass: 6438.043 m^2

Total area of dirt: 3674.957 m^2

f. Suggested option 1: An advantage of building an indoor stadium is that it is useful in any weather conditions, however a disadvantage is that there is often less space and more constraints compared to an outdoor one.

Suggested option 2: An advantage of building an indoor stadium is that it is easier to maintain due to less wear and tear, however a disadvantage is that it may be more costly to maintain due to increased use of utilities.

Note: There are other possible options.

3. a. $\pi = 3.14159$

b. Perimeter of inner square: 2.8284

Perimeter of outer square: 4

Comparison: The perimeters of the inner and outer squares are not very close to the circumference of the circle π.

c. Perimeter of inner pentagon: 2.939

Perimeter of outer pentagon: 3.6325

Comparison: Although the perimeters of the inner and outer pentagons are closer to π than the squares', they are not a very good estimation.

d. Perimeter of inner hexagon: 3

Perimeter of outer hexagon: 3.4644

Perimeter of inner octagon: 3.0616

Perimeter of outer octagon: 3.3136

Comparison: The two octagons' perimeters are closer in value to the circumference π compared to the hexagons.

e.

Sides	Perimeter (inside)	Perimeter (outside)
10	3.0902	3.2492
20	3.1286	3.1676
30	3.1359	3.153
40	3.1384	3.148
50	3.1395	3.1455
60	3.1404	3.1446
70	3.1402	3.1437
80	3.1408	3.1432

Comparison: Once the number of sides of the polygons reach 60, the inner and outer polygons correctly estimate π to 2 decimal places (3.14). However, the 80-sided polygons do not yield much better estimates compared to the 60-sided ones.

f. With an 80-sided polygon, only the first two decimal places are accurately calculated (3.14). This is not a better estimate than Archimedes' $\frac{22}{7}$ estimation as $\frac{22}{7}$ when rounded to 4 decimal places is 3.1429, and this is closest to the 5 decimal-place approximation of 3.14159.

Chapter 12 review

Multiple choice

1. C **2.** C **3.** E **4.** D

5. D **6.** B **7.** D **8.** C

Fluency

9. a. 32 km **b.** 10.6 mm **c.** 15 cm **d.** 41 m

10. a. i. 20π mm **ii.** 62.832 mm **b. i.** 7.25π cm **ii.** 22.777 cm

c. i. 8π km **ii.** 25.133 km **d. i.** $\frac{38}{3}\pi$ m **ii.** 39.794 m

11. a. 6 mm^2 **b.** 108 km^2

c. 1313 cm^2 **d.** 1149 mm^2

12. a. 3.96 cm^2 **b.** 82.5 km^2 **c.** 5.945 mm^2 **d.** 14 km^2

13. a. i. 2246.76π m^2 **ii.** 7058.405 m^2 **b. i.** $\frac{81}{64}\pi$ cm^2 **ii.** 3.976 cm^2

c. i. 60.0625π cm^2 **ii.** 188.692 cm^2 **d. i.** 58.32π mm^2 **ii.** 183.218 mm^2

14. a. 45.203 cm **b.** 114.034 cm^2

c. 59.751 cm **d.** 95.778 cm^2

15. a. 156.520 mm^3 **b.** 277.200 m^3

c. 4316.171 cm^3 **d.** 107.301 cm^3

16. a. 103.5 hours **b.** 9:13 pm

c. 12:35 am **d.** 2:00 am

Problem solving

17. Victoria wants to build a square-shaped wooden fence around her garden. The perimeter of her garden is 5 m. What is the length of each side of the garden in cm?

Key points

- The perimeter of the square-shaped garden is 5 m.
- Determine the side length of the square in cm.

Explanation

Perimeter of a square $= 4 \times$ side length

$5 = 4 \times$ side length

side length $= \frac{5}{4}$ m

$\frac{5}{4}$ m $= 125$ cm

Answer

The length of each side of the garden is 125 cm.

18. Quin has a hula hoop that has a circumference of 1.2 m. Using an estimated value of $\pi \approx \frac{22}{7}$, what is the radius of Quin's hula hoop?

Key points

- The circumference of the hula hoop is 1.2 m.
- Use the estimated value of $\pi \approx \frac{22}{7}$.
- Find the radius of the hula hoop.

Explanation

Circumference of a circle $= 2 \times \pi \times$ radius

$1.2 = 2 \times \frac{22}{7} \times$ radius

radius $= \frac{21}{110}$ m

Answer

The radius of Quin's hula hoop is $\frac{21}{110}$ m.

19. Sheridan's favourite clothing brand is Cathnandu. Cathnandu's logo is in the following shape. What is the area of this logo on Sheridan's Cathnandu jacket?

Key points

- Cathnandu's logo consists of two identical triangles and a rectangle.
- Determine the area of Cathnandu's logo.

Explanation

Find the area of each of the triangles.

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \times 6$

$= 18 \text{ mm}^2$

The area of the two triangles, then, is $2 \times 18 = 36 \text{ mm}^2$.

Find the area of the rectangle.

$\text{Area} = \text{length} \times \text{width}$

$= 8 \times 6$

$= 48 \text{ mm}^2$

Find the total area of the logo.

$\text{Total area} = 36 + 48$

$= 84 \text{ mm}^2$

Answer

The area of the logo on Sheridan's Cathnandu jacket is 84 mm^2.

20. Jess wants to give her friend a kite-shaped box filled with sweets as a gift for her birthday. The top view of the box is shown in the diagram below. She also wishes to cover the top of the box with photos of them, where each photo will be 3 cm by 5 cm. How many photos will she be able to fit on the top of the box?

Key points

- The top of the box is in the shape of a kite.
- The top of the box is to be covered with photos, each with a dimension of 3 cm by 5 cm.
- Find the number of photos that can fit on the top of the box.

Explanation

Find the area of the kite.

The kite has diagonal lengths of 30 cm and $11 \times 2 = 22$ cm.

$\text{Area} = \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 30 \times 22$

$= 330 \text{ cm}^2$

Find the area of each photo.

$\text{Area} = \text{length} \times \text{width}$

$= 5 \times 3$

$= 15 \text{ cm}^2$

Find the number of photos that can fit on the top of the box.

$\text{Number of photos} = \text{area of the top of the box} \div \text{area of each photo}$

$= 330 \div 15$

$= 22$

Answer

The top of the box can fit 22 photos.

21. A circular garden bed at a popular tourist attraction has a diameter of 44 metres and is surrounded by a dog walking path of 3.5 metres.

In order to protect dogs' paws from the heat of the pavement during summer, the city council has decided to paint the path with a reflective grey material in order to reduce road surface temperatures. One can of this paint will cover approximately 60 m^2. How many cans of paint are needed to paint the path above? Round your answer to the nearest can.

Key points

- The circular garden has a diameter of 44 m and is surrounded by a walking path of 3.5 m.
- One can of paint will cover approximately 60 m^2.
- Determine, to the nearest can, the number of cans required to paint the walking path.

Explanation

Find the area of the garden bed, the inner circle.

$\text{Area} = \pi \times r^2$

$= \pi \times \left(\frac{44}{2}\right)^2$

$= 484\pi \text{ m}^2$

Find the area of the garden bed and the walking path, the outer circle.

$\text{Area} = \pi \times \left(\frac{44}{2} + 3.5\right)^2$

$= \pi \times \left(\frac{51}{2}\right)^2$

$= \frac{2601}{4}\pi \text{ m}^2$

Find the area of the walking path.

$\text{Area of walking path} = \text{area of outer circle} - \text{area of inner circle}$

$= \frac{2601}{4}\pi - 484\pi$

$= \frac{665}{4}\pi \text{ m}^2$

Find the number of cans of paint required.

$\text{Number of cans of paint required} = \text{area of walking path} \div \text{area covered by one can of paint}$

$= \frac{665}{4}\pi \div 60$

$= \frac{133}{48}\pi$

$\approx 9 \text{ cans}$

Answer

9 cans of paint are needed to paint the walking path.

22. Alea is President for a student society and wishes to design a logo for the club and then stitch it onto members' uniforms. The logo is shown below.

At the local craft store, thread costs \$0.50 per cm^2. By calculating the area of the logo, determine how much the stitch will cost for Alea.

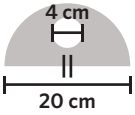
Key points

- The logo is a combination of semicircles with a circle and a semicircle cut out.
- Thread costs \$0.50 per cm^2.
- Determine the cost of thread by first determining the area of the logo.

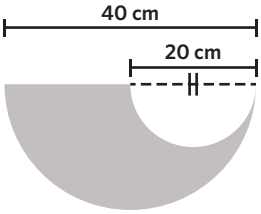
Explanation

To find the area of the logo, split the shape into two as follows.

Shape 1



Shape 2



Shape 1 is a semicircle of diameter 20 cm with a circle of diameter 4 cm cut out of it. Shape 2 is a semicircle of diameter 40 cm with a semicircle of diameter 20 cm cut out of it.

Find the area of Shape 1.

$\text{Area} = \text{area of semicircle} - \text{area of circle}$

$= \frac{1}{2} \times \pi \times \left(\frac{20}{2}\right)^2 - \pi \times \left(\frac{4}{2}\right)^2$

$= 50\pi - 4\pi$

$= 46\pi \text{ cm}^2$

Find the area of Shape 2.

Area = area of big semicircle − area of small semicircle

$$= \frac{1}{2} \times \pi \times \left(\frac{40}{2}\right)^2 - \frac{1}{2} \times \pi \times \left(\frac{20}{2}\right)^2$$

$$= 200\pi - 50\pi$$

$$= 150\pi \text{ cm}^2$$

Total area of the logo = area of Shape 1 + area of Shape 2

$$= 46\pi + 150\pi$$

$$= 196\pi \text{ cm}^2$$

Find the cost of the thread.

Cost of thread = total area × cost per unit area

$$= 196\pi \times 0.50$$

$$= 98\pi$$

$$\approx \$307.88$$

Answer

It will cost Alea $307.88 for the thread.

23. Malia is conducting a science experiment on displacement of water and volumes of objects. She learns that the volume of displaced liquid should approximately equal the volume of the object that is placed. When Malia puts a small fish into a rectangular fish tank (shown below), she notices that the water level rises by 15 mm. What is the volume of the fish she put in? Express your answer in cm^3.

Key points

- The volume of displaced liquid should equal the volume of the object that is placed.
- When a small fish is put into the tank, the water level rises by 15 mm.
- The dimension of the tank is 620 mm × 330 mm × 850 mm.
- Determine the volume of the fish that was put in, in cm^3.

Explanation

Find the volume of the displaced water. For ease of calculation, first convert all measurements to cm.

Volume = surface area × height

$$= 62 \times 33 \times 1.5$$

$$= 3069 \text{ cm}^3$$

Since the volume of object placed is equal to the volume of displace water, volume of fish is also 3069 cm^3.

Answer

The volume of the fish is 3069 cm^3.

24. Will's journey from Singapore to Melbourne has two stopovers in Kuala Lumpur and Bali, with the whole journey taking 34 hours and 30 mins. If Will leaves Singapore at 9:15 pm on Thursday, what would the local time and day be in Melbourne when he arrives? Singapore time is UTC +8 while Melbourne time is UTC +11.

Key points

- Will's journey from Singapore to Melbourne takes 34 hours and 30 mins.
- He leaves Singapore at 9:15 pm on Thursday.
- Singapore time is UTC +8 and Melbourne time is UTC +11.
- What day and time does Will arrive in Melbourne?

Explanation

Singapore time is UTC +8 and Melbourne time is UTC +11, so Melbourne time = Singapore time + 3 hours.

Will's journey takes 34 hours and 30 mins. Break 34 hours and 30 mins into 24 hours + 10 hours 30 mins.

Will leaves on Thursday at 9:15 pm. 24 hours after this is 9:15 pm Friday.

Melbourne time = 7:45 am + 3 hours = 10:45 am.

Answer

The local time and day will be 10:45 am Saturday when Will arrives in Melbourne.

Reasoning

25. a. The area of the available wall space Ryan has is 900 cm^2.

b. The total area of blue in the painting is 64π cm^2.

c. The area of the painting is 668.25 cm^2, which takes 74.25% of the available wall space. So Ryan does have enough wall space.

d. The painting will cost Ryan $595.87.

e. The volume of the cylindrical kaleidoscope is 904.79 cm^3.

f. Suggested option 1: Coloured paper.

Suggested option 2: Cardboard.

Note: There are other possible options.

26. a. 20 cm^2

b. 20 cm^2

c. The area of a kite can be thought of as the sum of the area of two identical triangles.

$$\text{Area} = \frac{1}{2} \times b \times h + \frac{1}{2} \times b \times h = b \times h$$

When stacked together to form a kite, the base, b, of the triangles is equal to one of the diagonals of the kite, say d_1. The height of one of the triangles, h, is half of the other diagonal, say d_2, where $h = \frac{d_2}{2}$. Putting it altogether,

$$\text{Area} = b \times h = d_1 \times \frac{d_2}{2} = \frac{d_1 \times d_2}{2}.$$

13A Introduction to Pythagoras

Student practice

Worked example 1

a. $12^2 + 9^2 = 15^2$ or $9^2 + 12^2 = 15^2$

b. $m^2 + u^2 = 7.02^2$ or $u^2 + m^2 = 7.02^2$

Worked example 2

a. No **b.** Yes

Worked example 3

a. Yes **b.** No

Understanding worksheet

1. **a.** 2.5 **b.** 6 **c.** $\sqrt{\frac{5}{9}b}$ **d.** $\frac{c\sqrt{2}}{2}$

2. **a.** 3^2

b. 17^2

c. Hypotenuse = 13, $a^2 + 5^2 = 13^2$

d. Hypotenuse = r, $r^2 = e^2 + x^2$

3. (a, b, c); longest side; 90°; $a^2 + b^2 = c^2$

Fluency

4. **a.** $8^2 + 6^2 = 10^2$ or $6^2 + 8^2 = 10^2$

b. $5^2 + 12^2 = 13^2$ or $12^2 + 5^2 = 13^2$

c. $b^2 + 99^2 = 101^2$ or $99^2 + b^2 = 101^2$

d. $e^2 + 16.25^2 = w^2$ or $16.25^2 + e^2 = w^2$

e. $c^2 + b^2 = a^2$ or $b^2 + c^2 = a^2$

f. $\left(\frac{5}{2}\right)^2 + g^2 = h^2$ or $g^2 + \left(\frac{5}{2}\right)^2 = h^2$

g. $\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = b^2$

h. $(8.5a)^2 + \left(\frac{r}{2}\right)^2 = d^2$ or $\left(\frac{r}{2}\right)^2 + (8.5a)^2 = d^2$

---

5. **a.** $4^2 + 3^2 = 5^2$

$16 + 9 = 25$

$25 = 25$

b. $8^2 + 6^2 = 10^2$

$64 + 36 = 100$

$100 = 100$

c. $9^2 + (12)^2 = 15^2$

$81 + 144 = 225$

$225 = 225$

d. $1.5^2 + 2^2 = 2.5^2$

$2.25 + 4 = 6.25$

$6.25 = 6.25$

e. $6.5^2 = (6)^2 + 2.5^2$

$42.25 = 36 + 6.25$

$42.25 = 42.25$

f. $25^2 = (7)^2 + 24^2$

$625 = 49 + 576$

$625 = 625$

g. $(5)^2 + 12^2 = (\sqrt{169})^2$

$25 + 144 = 169$

$169 = 169$

h. $(1)^2 + (\sqrt{3})^2 = 2^2$

$1 + 3 = 4$

$4 = 4$

---

6. **a.** Yes **b.** Yes **c.** Yes **d.** No

e. Yes **f.** Yes **g.** No **h.** Yes

---

7. **a.** Correct **b.** Incorrect

c. Correct **d.** Correct

e. Correct **f.** Correct

g. Incorrect **h.** Incorrect

---

8. **a.** Yes **b.** No **c.** Yes **d.** Yes

e. Yes **f.** No **g.** No **h.** Yes

---

9. B

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. The local council wants to build a ramp that has a length of 800 cm, a horizontal distance of 700 cm and a vertical distance of 50 cm. Determine if this style of ramp is possible.

Key points

- The local council wants to build a ramp that has a length of 800 cm, a horizontal distance of 700 cm and a vertical distance of 50 cm.
- Determine if this style of ramp is possible.

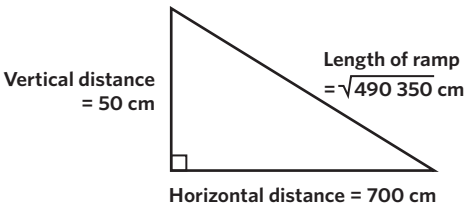
Explanation

The horizontal distance, vertical distance and the length of the ramp are pythagorean triads and should follow the formula $a^2 + b^2 = c^2$ where a is horizontal distance, b is vertical distance and c is the length of the ramp.

$700^2 + 50^2 = 800^2$

$490\,000 + 2\,500 = 800^2$

$492\,500 \neq 640\,000$



∴ This type of ramp is not possible.

Answer

This type of ramp is not possible.

12. In a game of cricket, a news anchor tells a story of a batter who hits a ball at a 90° degree angle and is caught by their opponent at the Square Leg position from 20 metres away. The batter runs 20 metres straight down to the wickets while the opponent throws 25 metres to the wickets and gets the batter out. Was this story possible?

Key points

- A batter who hits a ball at a 90° degree angle and is caught by their opponent at the Square Leg position from 20 metres away.
- The batter runs 20 metres straight down to the wickets while the opponent throws 25 metres to the wickets and gets the batter out.
- Was this story possible?

Explanation

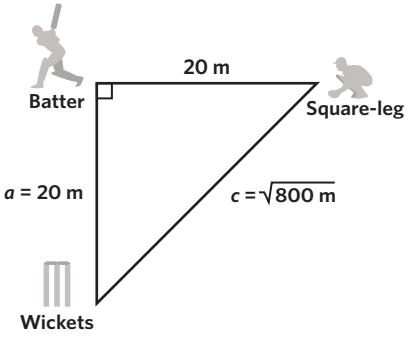
The distance between batter and square-leg ($a = 20$), between batter and wickets ($a = 20$) and between square-leg and wickets ($c = 25$) are pythagorean triads and should follow the formula $a^2 + b^2 = c^2$.

$a^2 + b^2 = c^2$

$20^2 + 20^2 = 25^2$

$400 + 400 = 625$

$800 \neq 625$



∴ This story was impossible.

Answer

This story was impossible.

13. Ritchie leaves his house with his dog Daisy. They walk 600 metres south down Beach Street, then 450 metres east along Neptune Street. They plan on finishing up by walking diagonally all the way home for 750 metres. Will Ritchie and Daisy be able to get all the way back home based on these directions?

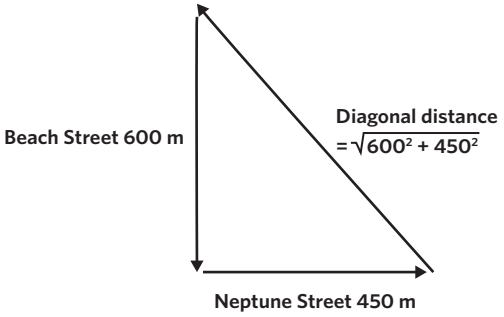
Key points

- Ritchie and Daisy walk 600 metres south down Beach Street, then 450 metres east along Neptune Street.
- They plan on finishing up by walking diagonally all the way home for 750 metres.
- Will Ritchie and Daisy be able to get all the way back home based on these directions?

Explanation

The length of Beach Street ($a = 600$), the length of Neptune Street ($b = 450$) and the diagonal distance ($c = 750$) are pythagorean triads and should follow the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$
$$600^2 + 450^2 = 750^2$$
$$360\,000 + 202\,500 = 562\,500$$
$$562\,500 = 562\,500$$



∴ Ritchie and Daisy will be able to get all the way back home based on these directions.

Answer

Yes, Ritchie and Daisy will be able to get all the way back home based on these directions.

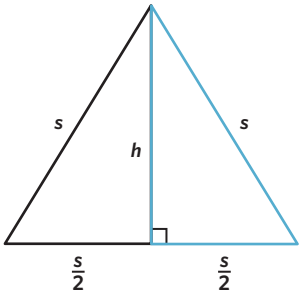
14. NailMood is celebrating Pythagoras Day on the 345th day of the year with a special nail design. The nail design is an equilateral triangle with side length s that was formed by placing two right-angled triangles together that have a height of h.

Clients who can correctly rewrite Pythagoras' theorem, $a^2 + b^2 = c^2$, in terms of s and h will get their nails done for free. What is the Pythagoras' formula in terms of s and h?

Key points

- The nail design is an equilateral triangle with side length s that was formed by placing two right-angled triangles together that have a height of h.
- Clients who can correctly rewrite Pythagoras' theorem, $a^2 + b^2 = c^2$, in terms of s and h will get their nails done for free.
- What is the Pythagoras' formula in terms of s and h?

Explanation



The nail design is an equilateral triangle, so all three sides of the triangle will have length s, which is the hypotenuse of the highlighted right-angled triangle.

The height of the right-angled triangle is h.

As the equilateral triangle is formed using two identical right-angled triangles, the base of the highlighted right-angled triangle will be half of its hypotenuse, which is $\frac{s}{2}$.

In the Pythagorean formula, $a^2 + b^2 = c^2$, a = base $= \frac{s}{2}$, b = height $= h$, c = hypotenuse $= s$.

Answer

The Pythagoras' formula in terms of s and h is $\left(\frac{s}{2}\right)^2 + h^2 = s^2$ or $h^2 + \left(\frac{s}{2}\right)^2 = s^2$.

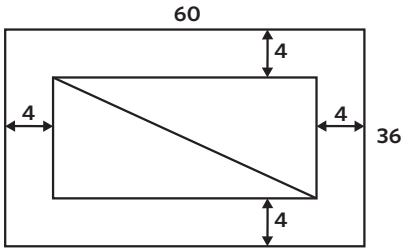
15. Grace is comparing three televisions, a 75 inch 66 inch or a 55 inch from Aldi's special buys to place inside her entertainment unit. Her entertainment unit is 60 inches wide and 36 inches high. Grace wants to centre the television so that there is no more than four inches of space between the television and the entertainment unit on either side. If televisions are measured by their diagonal and in inches, which one should Grace buy?

Key points

- Grace is comparing three televisions, a 75 inch 66 inch or a 55 inch from Aldi's special buys to place inside her entertainment unit.
- Her entertainment unit is 60 inches wide and 36 inches high.
- Grace wants to centre the television so that there is no more than four inches of space between the television and the entertainment unit on either side.
- If televisions are measured by their diagonal and in inches, which one should Grace buy?

Explanation

The TV Grace should buy should be big enough so that there are less than 4 inches of space between either side, but small enough so that it fits in the entertainment unit.



Calculate the minimum diagonal length of the TV.

Assume that the gap between the television and the entertainment unit on either side is 4 inches.

Then the dimension of the television should at least be $(60 - 4 \times 2)$ inches $\times$ $(36 - 4 \times 2)$ inches.

This is 52 inches $\times$ 28 inches.

The length ($a = 52$), width ($b = 28$) and the diagonal length (c) of the television make a Pythagorean triad and should follow the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$
$$52^2 + 28^2 = c^2$$
$$3488 = c^2$$
$$\sqrt{3488} = \sqrt{c^2}$$
$$c \approx 59 \text{ inches.}$$

$\therefore$ The minimum diagonal length of the TV is 59 inches.

Calculate the maximum diagonal length of the TV.

The diagonal length of the television should at least be 59 inches for the gap between the television and the entertainment unit on either side is less than 4 inches.

The maximum diagonal length of the television that could fit inside the entertainment unit is $\sqrt{60^2 + 36^2} \approx 70$ inches.

The only TV that has its diagonal length greater than 59 but smaller than 70 inches is LG TV. Therefore Grace should buy the LG TV.

Answer

Grace should buy the LG TV.

Reasoning

16. a. $a^2 + b^2 = c^2$

b. $(0.5 \times 8)^2 + (0.5 \times 6)^2 = 5^2$

c. Rahul's calculations are incorrect.

d. The length of the rope is 5.6 m.

e. The dimensions of the new cat jump path would be 1.25 m.

Suggested option 1: Yes, this scale is appropriate for a fully grown cat because a fully grown cat's body length is around 50 to 60 cm.

Suggested option 2: No, this scale is not appropriate for a fully grown cat because it is too short for a cat to play around with.

Note: There are other possible options.

17. a. $4^2 + 3^2 = 5^2$. Yes, this equation can be used to calculate a right angled triangle.

b. $5^2 - 3^2 = 4^2$. Yes, this equation can be used to calculate a right angled triangle.

c. The two formulas in part **a.** and part **b.** are both $a^2 + b^2 = c^2$ rearranged.

Extra spicy

18. The dimensions of each square is 3 cm $\times$ 3 cm, 4 cm $\times$ 4 cm and 5 cm $\times$ 5 cm.

The area of each square is 9 cm^2, 16 cm^2 and 25 cm^2 respectively.

19. E **20.** E

21. $p = b^2$

$y = \frac{1}{2}ab$

$t = \frac{1}{2}ab$

$h = a^2$

$\triangle = \frac{1}{2}ab$

$g = \frac{1}{2}ab$

Remember this?

22. E **23.** C **24.** E

13B Calculating the hypotenuse

Student practice

Worked example 1

a. $g = 13$ units **b.** $d = \sqrt{2}$ cm

Worked example 2

a. $c = 4.47$ cm **b.** $t = 0.23$ m

Worked example 3

a. $c = \sqrt{9a^2 + 9b^2}$ **b.** $c = \dfrac{\sqrt{a^2 + 9b^2}}{2}$

Understanding worksheet

1. a. Yes **b.** No **c.** Yes **d.** Yes

2. a.
$$\sqrt{81 + 20} = \sqrt{c^2}$$
$$\sqrt{101} = c$$

b.
$$\sqrt{c^2} = \sqrt{44 + 100}$$
$$c = \sqrt{144}$$
$$c = 12$$

c.
$$k^2 = 13^2 + (11)^2$$
$$k = \sqrt{169 + 121}$$
$$k = \sqrt{290}$$

d.
$$\sqrt{25 + 144} = \sqrt{n^2}$$
$$\sqrt{169} = n$$
$$13 = n$$

3. $a^2 + b^2 = c^2$; c; opposite; irrational

Fluency

4. a. 1 **b.** $\sqrt{2}$ **c.** 2 **d.** $\sqrt{10}$
e. 9 **f.** $\sqrt{142}$ **g.** 13 **h.** 0.5

---

5. a. $c = 10$ units **b.** $d = 13$ cm
c. $t = 15$ cm **d.** $b = 130$ m
e. $e = 20$ km **f.** $a = 2.5$ m
g. $f = 0.5$ km **h.** $u = 25$ mm

---

6. a. $c = \sqrt{117}$ units
b. $e = \sqrt{8500}$ mm or $e = 10\sqrt{85}$ mm
c. $b = \sqrt{98}$ mm or $b = 7\sqrt{2}$ mm
d. $e = \sqrt{82}$ m
e. $c = \sqrt{0.61}$ km
f. $e = \sqrt{422.5}$ m
g. $a = \sqrt{7.84}$ cm
h. $r = \sqrt{5}$ cm

7. **a.** $d = 3.61$ m **b.** $a = 7.07$ m
c. $g = 34\,176.01$ m **d.** $L = 3.74$ m
e. $s = 4.37$ cm **f.** $C = 7.81$ units

8. **a.** $c = \sqrt{e^2 + f^2}$
b. $c = \sqrt{13a^2}$ or $c = a\sqrt{13}$
c. $c = \sqrt{2a^2}$ or $c = a\sqrt{2}$
d. $c = \frac{\sqrt{4a^2 + 4b^2}}{2}$ or $c = \frac{2\sqrt{a^2 + b^2}}{2}$ or $c = \sqrt{a^2 + b^2}$
e. $c = \frac{\sqrt{121a^2 + 169b^2}}{3}$
f. $c = \sqrt{2.25a^2 + 2.25b^2}$ or $c = 1.5\sqrt{a^2 + b^2}$
g. $c = \sqrt{3a^2 + b^2}$
h. $c = \frac{\sqrt{4a^4 + 16b^4}}{7}$ or $c = \frac{2\sqrt{a^4 + 4b^4}}{7}$

9. A

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

11. The kitchen work triangle determines how functional it is to use the cook top, sink and fridge. No part of the triangle should be less than 1.2 metres or more than 2.7 metres. The diagram shows Hymie's kitchen. Determine the distance from the sink to the stove top and if Hymie's kitchen work triangle meets the criteria for a functional kitchen space. Round your answer to 2 decimal places.

Key points

- The criteria for a functional kitchen work triangle is one where no side length is less than 1.2 metres or more than 2.7 metres.
- Hymie's kitchen work triangle has shorter sides with lengths of 2.20 m and 1.20 m.
- Find the hypotenuse of Hymie's kitchen work triangle to two decimal places.
- Determine whether it meets the criteria for a functional kitchen space.

Explanation

The distance from the sink to the stove top is the hypotenuse of Hymie's kitchen work triangle. Let it be c.

$c^2 = 2.20^2 + 1.20^2$
$= 4.84 + 1.44$
$= 6.28$
$c = \sqrt{6.28}$
$= 2.51$ m

Since 2.51 m is between 1.20 m and 2.70 m, Hymie's kitchen work triangle satisfies the criteria for a functional kitchen space.

Answer

The distance from the sink to the stove top of Hymie's kitchen is 2.51 m and Hymie's kitchen work triangle satisfies the criteria for a functional kitchen space.

12. Darina is studying medicine and learns that she must use Pythagoras' theorem to determine how long a needle needs to be so that it meets the sound waves from the ultrasound probe in a vein. The needle will be placed 5 mm from the probe, and the sound waves need to reach 5 mm to the vein. What is the minimum length the needle needs to be so that it reaches the sound waves in the vein? Leave your answer in exact form.

Key points

- The needle is placed 5 mm from the probe.
- The sound waves reach 5 mm to the vein.
- Find the minimum length of the needle in exact form.

Explanation

The situation forms a right-angled triangle where the two shorter side lengths are 5 mm and 5mm and the hypotenuse is the minimum length of the needle. Let the minimum length of the needle be c.

$c^2 = 5^2 + 5^2$
$= 25 + 25$
$= 50$
$c = \sqrt{50}$ mm

Answer

The minimum length of the needle needs to be $\sqrt{50}$ mm for it to be able to reach the sound waves in the vein.

13. ZipIt, a zipline company, is installing a new zipline that will connect two corners of their properties together using a cable. ZipIt knows the area of the two properties is measured in hectares (ha) and 1 ha $= 10\,000\text{m}^2$. Determine the length of the zipline rope that will connect the two corners of the property together. Give your answer in exact form.

Key points

- The company is installing a zipline to connect two corners of their property.
- The areas of the two properties are 121 ha and 11^2 ha.
- 1 ha $= 10\,000$ m^2.
- Find the length of the zipline cable in exact form.

Explanation

Convert the area of each property into m^2.

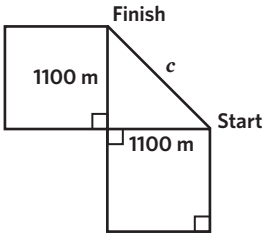
121 ha $= 1\,210\,000$ m^2

11^2 ha $= 121$ ha $= 1\,210\,000$ m^2

Find the side length of each property.

$\sqrt{1\,210\,000} = 1100$ m

Now we have a triangle with both shorter side lengths of 1100 m^2 and c as the hypotenuse.



Find c.

$c^2 = 1100^2 + 1100^2$
$= 1\,210\,000 + 1\,210\,000$
$= 2\,420\,000$
$c = \sqrt{2\,420\,000}$ m

Answer

The length of the zipline rope is $\sqrt{2\,420\,000}$ m.

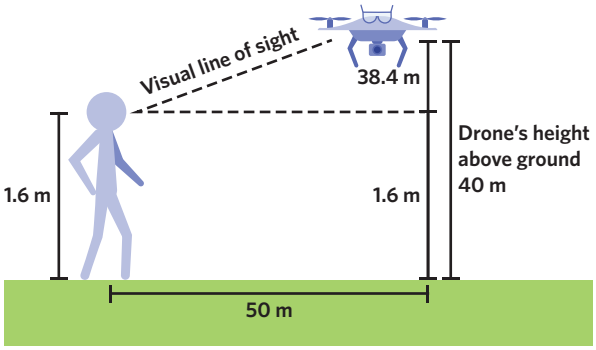
14. Jarrod is flying his drone but lost his glasses and can only see distances less than 50 metres away. According to drone regulations, he must keep his drone in his visual line of sight. He flies his drone directly upwards, 40 metres from the ground, and 50 metres horizontally away from him. The distance from the ground to his eye is 1.6 m. Determine by how much the drone is in or out of his line of sight. Round your answer to a whole number.

Key points

- Jarrod can only see distances less than 50 metres away.
- The drone is 40 metres from the ground and 50 metres horizontally away from him.
- The distance from the ground to his eye is 1.6 m.
- Determine the distance by which the drone is in or out of Jarrod's line of sight. Express in whole numbers.

Explanation

The figure below shows the schematic of the situation.



If an artificial line is drawn horizontally across from Jarrod's eye level, a right-angled triangle is formed as shown. Here, the triangle has a base of 50 m, a height of $40 - 1.6 = 38.4$ m and the visual line of sight is the hypotenuse. Let it be c.

Find c.

$c^2 = 50^2 + 38.4^2$

$= 2500 + 1474.56$

$= 3974.56$

$c = \sqrt{3974.56}$

≈ 63.04 m

Since $63.04 > 50$, the drone is out of Jarrod's line of sight.

Find the distance by which the drone is out of Jarrod's line of sight.

$63.04 - 50 \approx 13$ m

Answer

The drone is out of Jarrod's line of sight by 13 m.

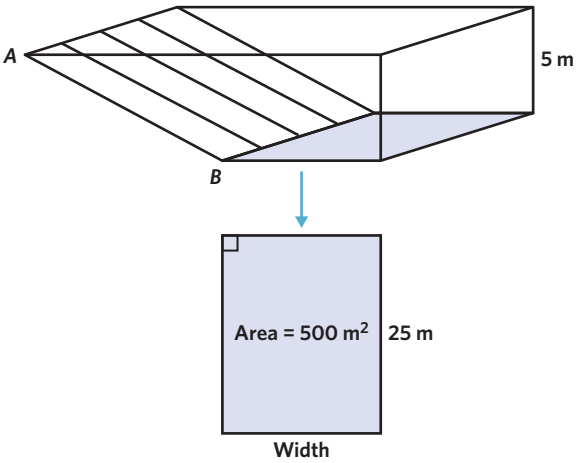
15. The Redland City council is building a new Olympic size indoor lap and diving pool that will be 50 m by 25 m. The pool will gradually increase in depth until it reaches a diving depth of 5 m, and the bottom of the diving area will be 500 m^2. There will be vertical tiled lines to denote the gradual increase in depth of the pool, and the diving area will be painted a solid colour. What is the length of one of the tile lines? Round your answer to 3 decimal places.

Key points

- The pool is 50 m by 25 m.
- The deepest part of the pool is 5 m.
- The bottom of the diving area is 500 m^2.
- Vertical tiled lines are placed to denote the gradual increase in depth of the pool.
- Find the length of one of the tile lines to 3 decimal places.

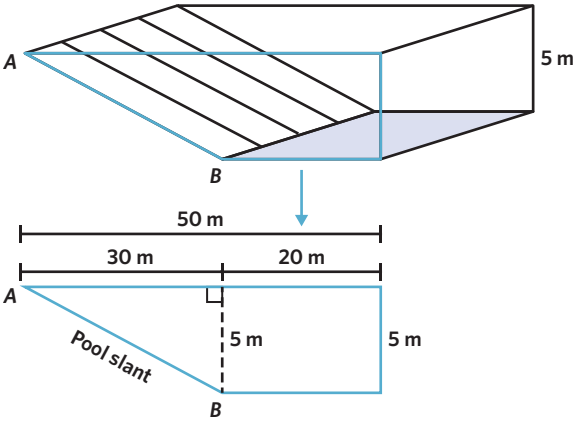
Explanation

Find the width of the bottom rectangle using the diving area.



Width $= 500 \div 25 = 20$ m

The visual below shows the front view of the pool.



There is a right-angled triangle and the three sides are:

- point A and B as the hypotenuse
- point B to the top of the pool (5 m)
- part of the length of the pool

$c^2 = 30^2 + 5^2$

$= 900 + 25$

$= 925$

$c = \sqrt{925}$

$= 30.414$ m

Answer

The length of each tile line is 30.414 m.

Reasoning

16. a. $\overline{BC}$ is 9 block units.

$\overline{CD}$ is 20 block units.

$\overline{DH}$ is 7 block units.

b. Petra will need to swim 15 block units from point A to C.

c. Petra will need to swim and walk a total of 38 block units from point A to D and from D to shelter.

d. The route $\overline{AC}$ to H is the quickest route with a time of 12.41 seconds.

It is faster than the route $\overline{AB}$ to H by 0.14 seconds, and $\overline{AD}$ to H by 4.56 seconds.

e. Suggested option 1: They could use building blocks to build a closer shelter.

Suggested option 2: Since running is faster, they could use the building blocks to make a path across the river.

Note: There are other possible options.

17. **a.** Yes, it can. **b.** Yes, it can.

c. It does not matter how the sides a and b of the triangle are labelled because of the commutative law. The formula can be be written in the form $c = \sqrt{a^2 + b^2}$ through rearranging the original formula.

$a^2 + b^2 = c^2$

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Extra spicy

18. 11

19. A

20. C

21. $A = \frac{25\pi}{8}$

Remember this?

22. E **23.** E **24.** B

13C Calculating the shorter side

Student practice

Worked example 1

a. $a = 12$ units **b.** $f = \sqrt{7}$ cm

Worked example 2

a. $b \approx 3.87$ units **b.** $p \approx 0.79$ m

Worked example 3

a. $b = 3.00$ **b.** $h \approx 6.36$ cm

Understanding worksheet

1. **a.** 9 **b.** 9 **c.** -9 **d.** 9

2. **a.** b, 7 **b.** 8, k

c. 30, L **d.** 12 000, a

3. $a = \sqrt{c^2 - b^2}$; b; rational; surd

Fluency

4. **a.** $b^2 + 6^2 = 10^2$ **b.** $c^2 + 9^2 = 15^2$

c. $b^2 + c^2 = a^2$ **d.** $k^2 + 1.5^2 = 2.5^2$

e. $k^2 + k^2 = 188^2$ **f.** $u^2 + 7^2 = (\sqrt{625})^2$

g. $r^2 + 1^2 = (\sqrt{3})^2$ **h.** $h^2 + \left(\frac{7}{4}\right)^2 = \left(\frac{13}{4}\right)^2$

---

5. **a.** $b = 8$ cm **b.** $a = 12$ cm

c. $c = 12$ cm **d.** $g = 20$ km

e. $y = 50$ m **f.** $k = 2$ m

g. $r = 0.4$ cm **h.** $u = 24$ mm

---

6. **a.** $a = \sqrt{32}$ cm **b.** $c = \sqrt{5}$ cm

c. $p = \sqrt{80}$ m **d.** $k = \sqrt{72}$ km

e. $y = \sqrt{4400}$ m **f.** $r = \sqrt{0.0133}$ cm

g. $m = \sqrt{145}$ cm **h.** $r = \sqrt{2}$ units

---

7. **a.** $d = 7.42$ m **b.** $v = 3.32$ km

c. $g = 10\,908.71$ m **d.** $h = 5.72$ m

e. $a = 6.36$ m **f.** $h = 2.74$ cm

8. **a.** $b = 4.00$ **b.** $b = 3.00$

c. $a = 4.24$ **d.** $c = 7.00$

e. $a = 2.77$ **f.** $b = 17.89$

g. $a = 5.00$ **h.** $a = 7.00$ cm

---

9. D

Spot the mistake

10. **a.** Student A is incorrect. **b.** Student A is incorrect.

Problem solving

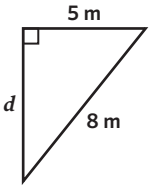
11. Schnitzel von Crumb has been taken off his leash and runs straight to the water d metres away. He paddles in a straight line parallel to the beach for 5 metres, then travels diagonally back to his owner on the beach who is standing 8 metres away. What is the total distance Schnitzel von Crumb travelled? Round your answer to the nearest metre.

Key points

- Schnitzel runs straight to the water that is d metres away from his owner.
- He paddles for 5 metres parallel to the beach.
- He then travels 8 metres diagonally back to his owner on the beach.
- Find the total distance that Schnitzel von Crumb travelled to the nearest metre.

Explanation

The path that Schnitzel von Crumb travels forms a triangle as shown below.



Find the distance d.

$d^2 + 5^2 = 8^2$

$d = \sqrt{8^2 - 5^2}$

$d = \sqrt{39}$ m

Find the total distance travelled, which is also the perimeter of the triangle.

Distance travelled $= 5 + 8 + \sqrt{39}$

≈ 19 m

Answer

The total distance that Schnitzel von Crumb travelled is 19 m.

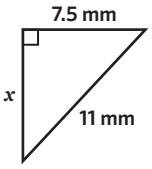
12. Sophie is making a dress for her fashion design final project. She uses sewing darts to create a curved shape from her fabric. Darts are isosceles triangles that are folded down the middle and sewn on the dart length. Determine the dart width if the length of the dart is 11 mm and the fold is 7.5 mm. Round your answer to one decimal place.

Key points

- Darts are isosceles triangles where the height is the fold and the two equal sides are the dart lengths.
- The dart length is 11 mm and the fold is 7.5 mm.
- Find the dart width to one decimal place.

Explanation

Once folded, the dart forms a right-angled triangle with one short side of 7.5 mm and a hypotenuse of 11 mm. Let the other short side be x.



Applying Pythagoras' theorem to find x.

$$x^2 + 7.5^2 = 11^2$$
$$x = \sqrt{11^2 - 7.5^2}$$
$$x = \sqrt{64.75} \text{ mm}$$

The dart width is the length before it was folded, so it is twice of what was found.

$$\text{Dart width} = 2 \times \sqrt{64.75}$$
$$\approx 16.1 \text{ mm}$$

Answer

The dart width is 16.1 mm.

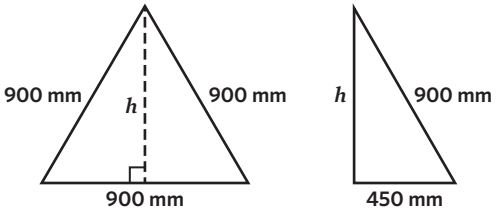
13. A new roundabout sign needs to be installed with a metal pole. The roundabout sign is an equilateral triangle with dimensions 900 mm. To meet regulations, the distance from the bottom of the roundabout sign to the ground must be 2000 mm, the metal pole does not extend past the sign, and the sign is centred on the pole. Determine the maximum length of the metal pole. Round your answer to two decimal places.

Key points

- The roundabout sign is an equilateral triangle with dimensions 900 mm.
- The distance from the bottom of the roundabout sign to the ground must be 2000 mm.
- The metal pole does not extend past the sign, and the sign is centred on the pole.
- Find the maximum length of the metal pole to two decimal places.

Explanation

Find the height of the sign. Let it be h.



When looking at half of the sign, it is a right-angled triangle with one short side of 450 mm and a hypotenuse of 900 mm.

Apply Pythagoras's theorem to find h.

$$h^2 + 450^2 = 900^2$$
$$h = \sqrt{900^2 - 450^2}$$
$$h = \sqrt{607\,500}$$

Find the maximum length of the pole.

$$\text{Maximum length of pole} = \text{required height above ground} + \text{height of the sign}$$
$$= 2000 + \sqrt{607\,500}$$
$$\approx 2779.42 \text{ mm}$$

Answer

The maximum length of the pole is 2779.42 mm.

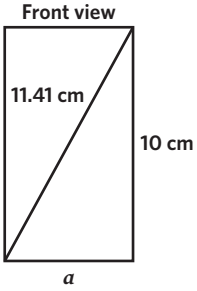
14. A hack to drink a juice box is to place the bendy straw upside down. The bendy part of the straw must form a 90° angle while lying inside of the box at a diagonal. Currently the bendy straw is folded on itself and is the same length as the diagonal length of the front of the box, 11.41 cm. Determine the maximum length of b, correct to two decimal places.

Key points

- The diagonal length of the front of the box is 11.41 cm.
- The width of the box is 3 cm and the height of the box is 10 cm.
- Find the length of b, the diagonal length of the bottom surface of the box, to two decimal places.

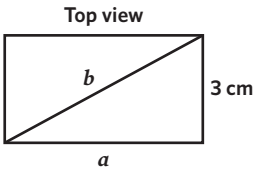
Explanation

Find the length of the box, let it be a.



$$a^2 + 10^2 = 11.41^2$$
$$a = \sqrt{11.41^2 - 10^2}$$
$$a = \sqrt{30.1881}$$
$$a = \sqrt{30.1881} \text{ cm}$$

Find b.



$$a^2 + 3^2 = b^2$$
$$b = \sqrt{a^2 + 3^2}$$
$$b = \sqrt{30.1881 + 9}$$
$$b \approx 6.26 \text{ cm}$$

Answer

The maximum length of b is 6.26 cm.

15. Calculate the missing side lengths, t, e, n of the triangle if the area of the triangle is 1840 mm^2. Round the lengths to the nearest mm.

Key points

- The area of the triangle is 1840 mm^2.
- Find the lengths t, e, n.

Explanation

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$1840 = \frac{1}{2} \times (e + 4e) \times 85$$
$$5e = \frac{1840 \times 2}{85}$$
$$e = \frac{736}{17 \times 5}$$
$$e = \frac{736}{85}$$
$$e \approx 9 \text{ mm}$$

Find the length t. Make sure to use the exact value of e and only round off at the end of the calculation.

$$e^2 + 85^2 = t^2$$
$$t = \sqrt{\left(\frac{736}{85}\right)^2 + 85^2}$$
$$t \approx 85 \text{ mm}$$

Find the length of n.

$(4e)^2 + 85^2 = n^2$

$$n = \sqrt{\left(\frac{4 \times 736}{85}\right)^2 + 85^2}$$

$n \approx 92$ mm

Answer

The side lengths t, e and n respectively are 85 mm, 9 mm and 92 mm.

Reasoning

16. a. The vertical distance between home base and 2nd base is 55 m.

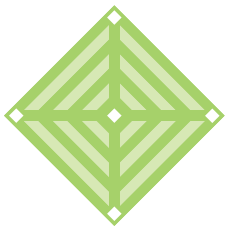
b. The distance between each of the four stripes from 2nd base to the mound is 7 m.

The distance between each of the four stripes from 2nd base to 1st base is 10 m.

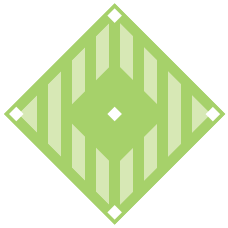
c. The length of stripe 1 is 7 m.

d. The lengths of stripes 2, 3 and 4 respectively are 14 m, 21 m and 29 m.

e. Suggested option 1:



Suggested option 2:



Note: There are other possible options.

17. a. The length of $\overline{BC}$ is 8 m.

b. The total length of $\overline{AB}$ and $\overline{BC}$ is 14 m.

c. From part **b**, we see that the total length of the two shorter sides is greater than the hypotenuse. This is always true for any right-angled triangle. Therefore, it is faster to walk along the hypotenuse as it involves less distance.

Extra spicy

18. D

19. Area of 4 triangles $= \frac{1}{2} \times a \times b \times 4 = 2ab$

Area of square $= (b - a)^2 = a^2 - 2ab + b^2$

Area of 4 triangles + square $= 2ab + a^2 - 2ab + b^2$
$= a^2 + b^2 = c^2$

20. $\overline{AB} = 8.5, \overline{BE} = 7.5, \overline{AD} = 17, \overline{BD} = 15$

21. D

Remember this?

22. E **23.** C **24.** B

Chapter 13 extended application

1. a. The distance between the 2 trestle legs is 180 mm.

b. No, the measurements are not correct. This is because the measurements do not satisfy Pythagoras's theorem. That is $130^2 + 794^2 \neq 800^2$.

c. The new height and distance between the longer trestle legs respectively are $400\sqrt{3}$ mm and the new distance is 800 mm.

d. The height of c is approximately 313 mm.

e. Breeshay will need 7 pieces of 1.8 m timber and the total price is $78.40.

f. Suggested option 1: A pro of using wooden tables is that it is generally more durable than plastic tables.

Suggested option 2: A pro of using plastic tables is that it is easier to clean and maintain.

Note: There are other possible options.

2. a. The diagonal length of the drop zone is 406.9 pixels.

b. The diagonal length of Picture 1 is 897.4 pixels.

c. The new length and width of Picture 1, using Chris's method, respectively, are 391 pixels and 112 pixels. As the length is longer than the side length of the drop zone, the new scaled picture will not fit in it.

d. The new length and width of Picture 1, using Kai's method, respectively, are 177.4 pixels and 50.7 pixels. As the length and width are both shorter than the side length of the drop zone, the new scaled picture will fit in it.

e. The formula used in part **d** will work for Picture 2, because using the formula, we get a new width of 140.8 pixels and a new length of 211.2 pixels. These are both smaller than 287.7 pixels, the side length of the drop zone.

f. Suggested option 1: A con of using maths techniques to scale images is that it can take a very long time to figure out the perfect technique to shrink all types and sizes of images.

Suggested option 2: A pro of using maths techniques to scale images is that a good formula could consistently work for many types of images.

Note: There are other possible options.

3. a. The diameter of the mini, small and jumbo cones respectively are 2 cm, 4 cm and 8 cm.

b. The height of the mini, small, regular and jumbo cones respectively are 3.87 cm, 7.75 cm, 11.62 cm and 15.49 cm.

c. The diameter of the hole for the jumbo cone will be 3.2 cm.

d. The diameter of the hole for the mini, small and regular cones respectively are 0.8 cm, 1.6 cm and 2.4 cm.

e. The surface area for the mini, small, regular and jumbo cones respectively are 4π cm^2, 16π cm^2, 36π cm^2 and 64π cm^2. Based on the surface area per unit price, the jumbo cone is the best buy.

f. Suggested option 1: She could place the cones circle side down on the counter and label them.

Suggested option 2: She could hang a poster that shows the different sized cones.

Note: There are other possible options.

Chapter 13 review

Multiple choice

1. D **2.** C **3.** B **4.** D

Fluency

5. a. $12^2 + 9^2 = 15^2$

b. $144^2 + x^2 = 145^2$

c. $p^2 + q^2 = \left(\frac{8}{3}\right)^2$

d. $\left(\frac{b}{5}\right)^2 + \left(\frac{b}{2}\right)^2 = (13.5a)^2$

---

6. a. Yes **b.** No **c.** No **d.** Yes

---

7. a. 8.49 m **b.** 6.83 m

c. 118 m **d.** 13.54 cm

---

8. a. $c = \sqrt{16a^2 + 4b^2}$ **b.** $c = \frac{\sqrt{81a^2 + 196b^2}}{4}$

c. $c = \sqrt{9a^2 + 3b^2}$ **d.** $c = \frac{\sqrt{9a^4 + 25b^4}}{8}$

---

9. a. $a = \sqrt{57}$ m **b.** $s = \sqrt{0.0253}$ m

c. $g = \sqrt{90}$ cm **d.** $h = 1$ cm

---

10. a. $b = 6$ **b.** $r = 2$

c. $b = 10.67$ **d.** $h = 3.33$

Problem solving

11. A plane is flying at a horizontal distance of 2100 m away from the base of a mountain and at an altitude of 1000 m in the air. It is due to fly 2900 m in the trajectory shown below, towards a mountain peak 3000 m high. Determine if the flight path is possible.

Key points

- A plane is flying at a horizontal distance of 2100 m away from the base of a mountain and at an altitude of 1000 m in the air.
- It is due to fly 2900 m in the trajectory shown below towards a mountain peak 3000 m high.
- Determine if the flight path is possible.

Explanation

The horizontal distance, vertical distance, and the flight path form a Pythagorean triad, and should follow the formula $a^2 + b^2 = c^2$, where a is horizontal distance, b is vertical distance, and c is the flight path.

$a = 2100$ m

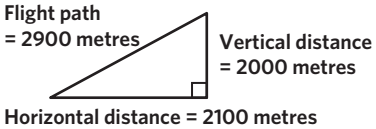
$b = 3000 - 1000 = 2000$ m

$c = 2900$ m

$2100^2 + 2000^2 = 2900^2$

$4\,410\,000 + 4\,000\,000 = 2900^2$

$8\,410\,000 = 2900^2$



This flight path is possible.

Answer

This flight path is possible.

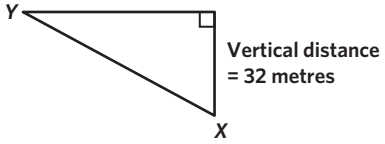
12. Rachel is currently at point X and wants to cross to Point Y. There is a rectangular pond, however she cannot walk through it as it is filled with mischievous geese. Therefore, Rachel must walk 32 metres directly north and 58 metres directly west. How much further is this compared to if Rachel could walk straight through the pond? Round your answer to the nearest metre.

Key points

- Rachel is currently at point X and wants to cross to Point Y.
- There is a rectangular pond, however she cannot walk through it as it is filled with mischievous geese.
- Therefore, Rachel must walk 32 metres directly north and 58 metres directly west.
- How much further is this compared to if Rachel could walk straight through the pond? Round your answer to the nearest metre.

Explanation

Horizontal distance = 58 metres



Calculate the distance that Rachel must walk to get from point X to point Y, given that she cannot walk through the pond.

Rachel must walk 32 metres north (vertical distance) and 58 metres west (horizontal distance).

$32 + 58 = 90$ m

Calculate the distance straight through the pond from point X to point Y, or the hypotenuse, c, of the triangle above, where $a = 32$ and $b = 58$.

$a^2 + b^2 = c^2$

$32^2 + 58^2 = c^2$

$1024 + 3364 = c^2$

$c^2 = 4388$

$c = \sqrt{4388} = 66.24$ m

Calculate how much further it is to walk around the pond than to walk straight through the pond.

Difference = Distance to walk around the pond − distance to walk directly through the pond

$= 90 - 66.24$

$= 23.76$ m

Round to the nearest metre.

$= 24$ m

Answer

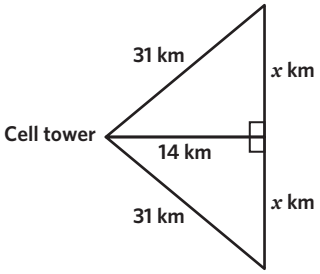
It is 24 m further for Rachel to walk around the pond compared to walking straight through the pond.

13. Tourists travel in a shuttle bus from the airport to their respective hotels in a rural neighbourhood. There is a cell tower on the route, and there is signal coverage as far as 31 km away from the tower. The bus passes the tower through the road shown in the diagram below, where its closest point to the tower is 14 km away. For how many kilometres will people on the bus have signal? Round your answer to two decimal places.

Key points

- Tourists travel in a shuttle bus from the airport to their respective hotels in a rural neighbourhood.
- There is a cell tower on the route, and there is signal coverage as far as 31 km away from the tower.
- The bus passes the tower through the road shown in the diagram below, where its closest point to the tower is 14 km away.
- For how many kilometres will people on the bus have signal? Round your answer to two decimal places.

Explanation



Calculate the length of x, where $a = 14$ km and $c = 31$ km.

$a^2 + x^2 = c^2$

$14^2 + x^2 = 31^2$

$x^2 = 31^2 - 14^2$

$x^2 = 961 - 196$

$x^2 = 765$

$x = \sqrt{765}$ km

Distance that people on the bus have signal $= 2x$

$= 2\sqrt{765}$

$= 55.32$ km

Answer

People on the bus have signal for 55.32 km.

Reasoning

14. a. Height $= x\sqrt{3}$ km, line of sight $= 2x$ km.

b. 7.27 km

c. 10.28 km

d. 3.07 km

e. Technology has the potential to fail. Lighthouses are another way of indicating where land is, which is extremely important information for ships to know in case their navigation technology fails.

15. a. 90°

b. Student B

c. In a right-angled triangle, the sum of the two non-right angles is always 90°. A square's interior angles are all equal to 90°. Orienting the angles of four congruent right-angled triangles so that two different non-right angles meet at each vertex of the quadrilateral will ensure 90° interior angles, and hence a square shape.

Back to contents

GLOSSARY

A

acute angle An acute angle is any angle that is greater than 0° but less than 90°.

adjacent angles Adjacent angles are angles that share a vertex and a common side.

algebraic fraction Algebraic fraction is a fraction using a variable in the numerator, or the denominator, or in both the numerator and the denominator.

angle The angle between line segments AB and BC is denoted $\angle ABC$.

anticlockwise Anticlockwise is the opposite direction that the hands of a clock move around.

approximately equal Approximately equal to is represented in calculations using the $\approx$ sign.

area The area is the amount of space that is contained by the boundaries of a flat, two-dimensional shape.

average A statistical average is a value which measures the central tendency of a set of numbers. It can be a mean, median, or mode.

B

best buy Best buy refers to comparing the price of two or more goods for the same quantity.

bias Statistical bias occurs when a sample is not representative of the target population.

bisect To bisect a line or an angle is to divide it into two equal parts.

C

Cartesian coordinates Cartesian coordinates are the numbers that indicate the location of a point on a Cartesian plane. The x–coordinate is written before the y-coordinate.

Cartesian plane A Cartesian plane is a set of two perpendicular number lines that intersect at the origin.

categorical data Categorical data is descriptive information and separated into groups.

centimetre (cm) A centimetre (cm) is one-hundredth of a metre.

circumference The circumference is the perimeter of a circle.

class interval A class interval defines the width of the groups into which a set of data has been organised.

clockwise Clockwise is the same direction that the hands of a clock move around.

coefficie A coefficient is the number that a pronumeral is being multiplied by.

common denominators Common denominators are when fractions have denominators that are the same.

common factors Common factors are the same factors that occur between two or more numbers.

commutative law The commutative law means that the order of the numbers in an addition and multiplication calculation can change without affecting the result.

compensation strategy The compensation strategy uses rounded numbers and then adjustments to make calculations simpler to solve.

compound event A compound event is an event that consists of multiple outcomes.

constant A constant is a number that cannot change its value in an expression or equation.

constant speed Constant speed occurs when the speed of an object remains the same and does not increase or decrease.

converting Converting is the process of rewriting a value in a different form, with no change to the value.

convex polygon All internal angles of a convex polygon are less than 180°.

Coordinated Universal Time Coordinated Universal Time or UTC is the standard by which the world regulates clocks and time.

coprime Two numbers are coprime if they have no common factors other than one.

cost price or wholesale price The cost price or wholesale price is the amount spent by businesses to make or buy an item for sale.

cross section A cross section is a surface that is created when making a straight cut through a 3D shape.

cylinder A cylinder is a 3-dimensional object that has 2 flat, circular bases that are connected by a curved surface.

D

daylight saving time Daylight saving time is a practice that occurs in some states of Australia where clocks are advanced 1 hour to increase the amount of daylight after work in summer months.

denominator The denominator is the value on the bottom of a fraction and represents the number of parts each unit is split into.

diagonal A diagonal line joins one corner to the opposite corner of a four-sided shape and is not an edge.

directed numbers Directed numbers have a direction and value, one direction is positive (+) and the other direction is negative (−).

discount A discount describes the percentage or amount that the marked price is decreased by.

distribution A statistical distribution describes how values are distributed on a scale. The shape of a distribution tells us which values are common and which are not.

Back to contents

distributive law The distributive law for multiplication means that multiplying a number by a group of numbers is the same as multiplying the number by the sum of the other numbers.

dividend A dividend is the first number in a division calculation. It is the number that is being divided by the second number.

divisor A divisor is the second number in a division calculation. It is the number by which a given value is divided.

duration Duration is the time that passes between two moments.

E

equals sign The equals sign (=) in an equation or number sentence indicates that the value on the left of the sign is the same value as on the right.

equate When you equate expressions, you make them equal to each other.

equation An equation is another name for a number sentence. An equation can contain letters, numbers and operations, as well as an equals sign. Each side of an equation is equal.

equilateral triangle An equilateral triangle contains three sides of equal length and three 60° interior angles.

estimates Estimates are approximate values that are close to the actual value.

evaluate Evaluate means to carry out mathematical operations in the correct order to find a value of an expression or equation.

event An event is a single outcome or group of outcomes (known as a compound event).

exact form Exact form is when a number cannot be simplified to a whole number or rational number and left as an irrational number.

exclusive Exclusive is excluding the value(s).

expanded form Expanded form is a way of representing repeated multiplications (factors) of all numbers in an expression.

experiment An experiment is a series of trials conducted to examine the results of chance activities.

expression An expression is a number of terms grouped together by operations.

F

factorising Factorising in algebra is the opposite of expanding brackets.

false equation A false equation is when the left-hand side and right-hand side of an equation are not equal.

fini e A finite amount can be counted, measured or given a value.

First Index Law The First Index Law states to add the indices (or powers) when multiplying terms with the same base in index form.

formula Formula is a rule written using mathematical symbols and pronumerals that are connected using an equals sign.

frequency Frequency refers to how many times something occurs.

G

gradient The gradient is the slope and direction of a line.

H

highest common factor (HCF) Highest common factor (HCF) is the largest number that is a factor of two or more numbers.

hectare A hectare is a metric unit of area equal to 10 000 square metres.

hypotenuse The hypotenuse is the longest side on a right-angled triangle; it is always opposite to the right angle.

I

improper fraction An improper fraction has a numerator greater than or equal to its denominator.

inclusive Inclusive is including the value(s).

index notation Index notation is a way of representing repeated factors of the same number.

inequality Inequality is a statement when one value or algebraic expression is less than or greater than another.

infini e An infinite amount cannot be counted, measured or given a value.

integer An integer is a whole number that is either positive, negative or zero.

inverse operations Inverse operations are mathematical operations that undo each other. The inverse of multiplication is division and the inverse of division is multiplication; the inverse of addition is subtraction and the inverse of subtraction is addition.

isometric transformation An isometric transformation does not change the size or shape of the original image after the transformation.

K

kilometre (km) A kilometre (km) is one thousand metres.

kite A kite is a quadrilateral with reflection symmetry across a diagonal.

L

lead digit rounding Lead digit rounding rounds the number based on the first digit and replaces all other digits with zeros.

line segment A line segment between vertices A and B is denoted $\overline{AB}$.

linear rule A linear rule is an equation used for a straight line.

lowest common multiple (LCM) The lowest common multiple (LCM) is the smallest number that is a multiple of two or more numbers.

M

mark-up A mark-up describes the percentage or amount that the cost price is increased by.

marked price, selling price, retail price, or recommended retail price (RRP) The marked price, also known as the selling price, retail price, or recommended retail price (RRP) is the price an item is sold for after mark-up.

metre (m) A metre (m) is a standardised unit measuring length.

millimetre (mm) A millimetre (mm) is one-thousandth of a metre.

mixed number A mixed number is a combination of a whole number and a proper fraction.

multi-step equation A multi-step equation is an equation that can be solved by applying more than two inverse operations.

multi-step experiments Multi-step experiments describe a probability event that is made up of two or more actions, such as tossing three coins and looking at the different combinations of possible outcomes.

multiplier Multiplier is the number by which a given value is multiplied.

N

number sentence A number sentence is an equation expressed using numbers and operations.

numerator The numerator is the value on the top of a fraction and represents the number of parts of a unit we are considering.

numerical data Numerical data is quantitative data that can either be discrete or continuous.

O

obtuse angle An obtuse angle is any angle greater than 90° but less than 180°.

one-step equation A one-step equation is an equation that can be solved by applying a single inverse operation.

operation An operation in maths is a mathematical process used to solve problems. The four basic operations in maths are addition, multiplication, subtraction and division.

outcome An outcome is the result of a trial in an experiment.

order of operations Order of operations is a set of rules that tells us what calculations are performed first in a number sentence or equation. The order follows brackets, indices, division or multiplication and then addition or subtraction.

P

parabola A parabola forms an equation where x is raised to the power of 2.

parallel lines Parallel lines never touch and are always the same distance apart.

parallelogram A parallelogram is a quadrilateral with two opposite pairs of parallel sides.

part-whole A part-whole ratio compares two or more parts to a whole.

partitioning strategy The partitioning strategy deconstructs larger numbers into smaller numbers to make calculations simpler to solve.

percentage A percentage represents a value out of 100 parts, where 100 parts represents one whole.

perpendicular lines Perpendicular lines meet at a right angle (90°).

population A population refers to the entire group that the conclusions of the study are applied to as opposed to part of the group.

powers of 10 Powers of 10 are the result of multiplying ten by itself a certain number of times.

prism A prism is a 3-dimensional object which has two identical polygon faces on either end, connected by rectangular faces. A prism has the same cross-section when cut anywhere along its length.

product The product is the result when two or more values are multiplied together.

proportional In a rate, the quantities are proportional to each other. This means that the ratio between the two quantities must always remain the same.

proportional relationship If two quantities have a proportional relationship they can be written as a ratio.

Pythagorean triad A Pythagorean triad is a set of three positive integers represented by (a,b,c), that satisfies the rule $a^2 + b^2 = c^2$.

Q

quadrant A quadrant is exactly one quarter of a circle.

quotient The quotient is the result of a division calculation.

R

range The range measures the spread of data. It is the difference between the maximum and minimum values.

ratio A ratio is a proportional relationship between two or more quantities with the same unit.

rational A rational number is a number that can be expressed as a ratio of two integers, whereas an irrational number cannot.

reciprocal The reciprocal of a number is 1 divided by that number.

revenue Revenue is the income that a business receives from selling goods and services.

right-angled triangle A right-angled triangle contains one interior angle that is 90°.

rhombus A rhombus is a quadrilateral with four equal sides.

rule A rule is a formula written using mathematical symbols and pronumerals to describe a pattern.

Back to contents

S

sample A sample, or sampling, refers to the collection of data from a small group that is representative of the population.

sample space The sample space of an experiment lists all the possible outcomes.

Second Index Law The Second Index Law states to subtract the index of the divisor (or denominator) from the index of the dividend (or numerator) when dividing terms with the same base in index form.

sector A sector comprises two radii and an arc that is a part of a circle with the same radius.

semicircle A semicircle is exactly half of a circle.

simplest form A fraction is in its simplest form when the numerator and denominator have no common factors other than 1.

solving equations Solving equations is a process to find the value of the unknown by performing a series of inverse operations.

stationary An object is stationary when distance does not change over a time interval and the speed is equal to zero.

subject The subject of a formula is the variable isolated on one side of the equal sign.

substitution Substitution is the process of replacing a variable or an unknown with a given value.

surds Surds are the square roots of numbers that cannot be simplified into a whole number.

T

term A term is a part of an expression or equation made up of a number and/or pronumerals. Terms are separated by addition or subtraction.

theoretical probability The theoretical probability states the likelihood of an event occurring in an experiment.

Third Index Law The Third Index Law states to multiply the indices when raising a term in index form to another power.

trapezium A trapezium is a quadrilateral with one pair of parallel sides.

trial A trial is a single test in an experiment.

true equation A true equation is when the left-hand side and right-hand side of an equation are equal.

two-step equation A two-step equation is an equation that can be solved by applying two inverse operations.

two-way table A two-way table displays the frequency (count) of two categories, written in a table format with rows and columns.

U

unit price The unit price refers to the price for when the quantity is one.

unit rates Unit rates are rates in their simplest form, per one unit.

unitary method The unitary method is a process of calculation where the first step is to determine the value of one unit.

units Units of measurement define the magnitude and type of quantity. Unit rates are always expressed as quantities per one unit of the other quantity, and must include units of measurement.

V

variable A variable is a quantity that changes and can include qualities that are given numerical values.

Venn diagram A Venn diagram provides a visual representation of similarities and differences between two or more sets of information.

vertical algorithm The vertical algorithm is a written method used to add and subtract numbers.

X

***x*-axis** The x-axis is the horizontal axis that runs left to right through zero.

***x*-intercept** The x-intercept of a straight line is where a line crosses the x-axis of a graph and when y is equal to 0.

Y

***y*-axis** The y-axis is the vertical axis that runs up and down through zero.

***y*-intercept** The y-intercept of a straight line is where a line crosses the y-axis of a graph and when x is equal to 0.