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Australian Curriculum

MathsWorld 8

Australian Curriculum edition

Jill Vincent Beth Price Natalie Caruso
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First published 2012 by
MACMILLAN EDUCATION AUSTRALIA PTY LTD
15–19 Claremont Street, South Yarra, VIC, 3141

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National Library of Australia
Cataloguing-in-Publication entry

Title:	Mathsworld 8 Australian curriculum edition / Jill Vincent ... [et al.].
Edition:	1st ed.
ISBN:	9781420229622 (pbk.)
Target Audience:	For secondary school age.
Subjects:	Mathematics--Textbooks. Mathematics--Problems, exercises, etc
Other Authors/ Contributors:	Vincent, Jill.
Dewey Number:	510

Publisher: Colin McNeil
Project editor: Hannah Cartmel
Editor: Monique Miotto
Illustrators: DiZign, Andrew Craig and Nives Porcellato
Cover designer: Dimitrios Frangoulis
Text designer: Dimitrios Frangoulis
Production controller: Loran McDougall
Photo research and permission clearance: Sarah Johnson
Consultant: Tracey MacBeth-Dunn
Typeset in 10pt Times Ten by DiZign and Sunset Publishing Services
Cover image: Shutterstock/tonyz20

Printed in Malaysia

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Contents

Introduction

The ***MathsWorld Australian Curriculum editions*** have been rigorously developed to cover all the content and requirements of the Australian Curriculum. They are carefully written to provide comprehensive and accessible textbooks which cater for a range of ability levels. The textbooks are accompanied by a complete teacher resource package that provides extra resources, teacher notes and further support.

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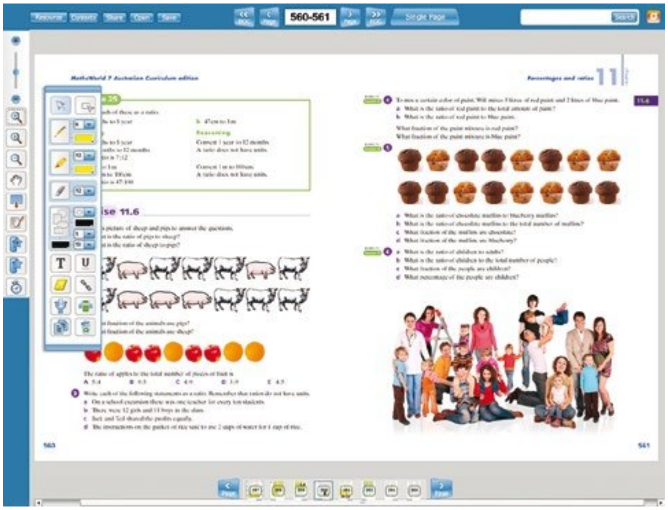

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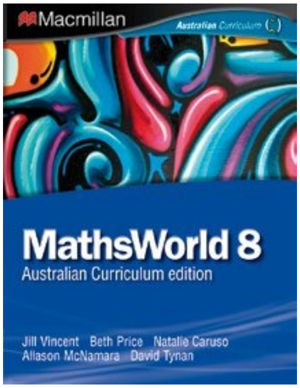
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Theory and worked examples are structured into manageable sections and are designed to illustrate the main concepts and skills for each topic. Solutions to worked examples are structured into two columns, with a working column indicating what students should write and a reasoning column showing the thought process for each major step of working.

Sets of carefully graded sets of exercise questions occur at the end of each section. Exercise questions are cross-referenced to the relevant worked examples to reinforce learning and provide links where help is needed. Each exercise contains one or two clearly marked challenge questions.

At the end of each chapter there are analysis tasks that explore and tie together concepts covered in the chapter.

Each chapter finishes with a thorough review section, consisting of a concise summary, a visual map task and sets of multiple-choice, short-answer and extended-response questions.

Practice quiz
Chapter 1

A practice quiz is included as a link from the ebook and can be used as revision for tests or as individual student assignments.

Icons throughout the book indicate links to a wide range of technology files, worksheets, templates, quizzes and class activities.

Tossing a coin

Technology files are indicated by the icon to the left and consist of GeoGebra, HTML, Excel and PowerPoint files. The icon shown here links to an Excel file that simulates the tossing of a coin 10, 100 and 1000 times.

Sieve of Eratosthenes

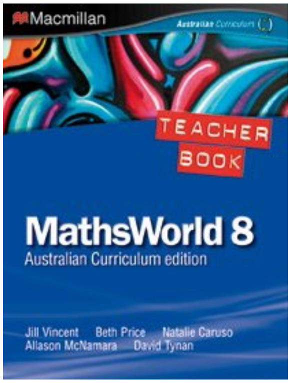
Icons for worksheets, blackline master templates and class activities are shown in the margin to the left and in the ebook these are linked directly to printable files for use as needed.

Teacher book

BLM
Fraction wall

Class activity
Tangram pieces

The teacher book provides a complete package of supplementary material and support with many time-saving and customisable resources included.



Pre-tests and answers in all chapters allow teachers to assess students' prior knowledge prior to commencing each chapter. These are also provided as PDF files linked from the student ebook.

Chapter warm-ups assist teachers with the introduction of the 'big idea' relevant to each topic. These are also provided as PDF files linked from the student ebook.

Teaching notes are provided for each chapter.

Additional worked examples are provided in the teacher book—these can be used as teaching examples during the lesson.

The teacher book also contains answers to all student exercises and all additional exercise questions along with answers for all pre-tests, chapter warm-ups, chapter tests and quizzes. The answers to the analysis tasks are provided in the teacher book, along with additional analysis tasks and answers.

Each chapter includes two tests, in editable Word format. Each chapter test includes multiple choice, short answer and extended response questions and is provided with a marking scheme.

Curriculum links and planning documents, including a suggested teaching program, are included in the teacher book.

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Integers

1

Pre-test

Warm-up

Large buildings often have one or more floors below ground level. Sometimes, negative integers are used to represent these floors, with zero representing ground level and positive integers representing the floors above ground level. The buttons on this lift panel show two floors below ground level and three floors above ground level.

1.1 Reviewing integers on the number line

Integers are whole numbers that are either *positive*, *negative* or *zero*.

In Years 6 and 7 you saw how negative integers are 'opposites' of the positive integers.

Example 1

a Write the opposite of each of these integers.

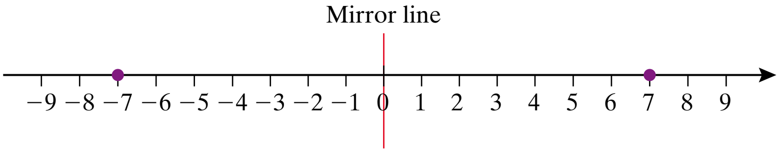
i 6 **ii** -4

b Evaluate

i $-(-5)$ **ii** $-(-(-1))$

Working	**Reasoning**
a i The opposite of 6 is -6	Each positive integer has an opposite negative integer.
ii The opposite of -4 is 4.	Each negative integer has an opposite positive integer.
b i $-(-5) = 5$	$-(-5)$ means the opposite of -5.
ii $-(-(-1)) = -1$	$-(-1)$ means the opposite of -1, which is 1. So $-(-(-1))$ is the opposite of 1.

On a horizontal number line, the positive integers are to the right of zero and the negative integers are to the left of zero. The positions of the negative integers on the number line can be thought of as reflections of their matching positive integer in an imaginary mirror line at zero. For example, on the horizontal number line -7 is the same distance to the left of zero as 7 is to the right of zero.



The further a negative number is from zero, the smaller it is. This is the opposite of positive numbers. For positive numbers, the numbers further from zero are the largest numbers.

Integers on the number line

Example 2

Show the following numbers

a on a vertical number line with a scale from −6 to 6.

i −5 **ii** 2 **iii** −3

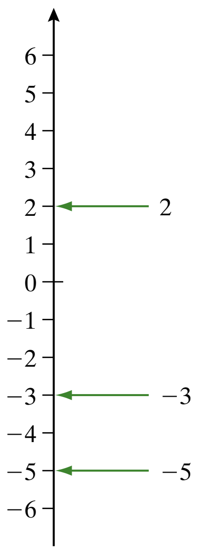
b on a horizontal number line with a scale from −6 to 6.

i 3 **ii** −4 **iii** −6

Vertical number line

Working

a



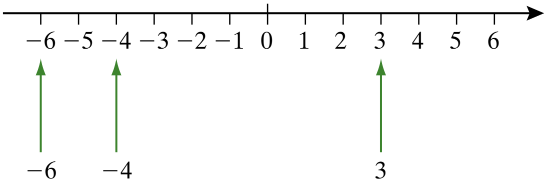
Reasoning

First draw and label the number line.
Space the scale marks equal distances apart.

0 goes in the middle.
The positive numbers go up from 0.
The negative numbers go down from zero.

Then place the numbers next to the matching scale marks.

b



First draw and label the number line.
Again, 0 goes in the middle.
The positive numbers go right from 0.
The negative numbers go left from zero.

Then place the numbers below the matching scale marks.

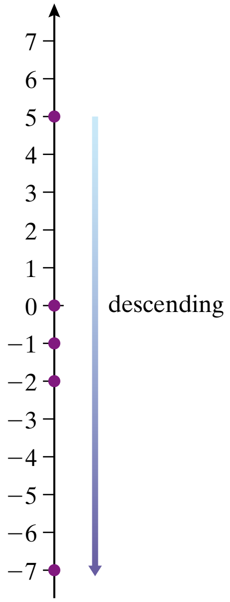
Example 3

Arrange the integers 5, −2, 0, −1, −7 in descending order.

continued

Example 3 continued

Working	Reasoning
In descending order the numbers are 5, 0, −1, −2, −7.	Descending order means from largest to smallest. The highest of the numbers is 5, then 0, −1, −2 and −7. −7 is the lowest, so it is the smallest.



Example 4

For each of these pairs of integers

- **i** put a square around the larger.
- **ii** use < or > to make a true statement.

a −5 or 4 **b** −7 or −11

Working	Reasoning
a **i** −5 or $\boxed{4}$	Positive numbers are larger than negative numbers. For example 4 °C is 'warmer' than −5 °C.
ii $-5 < 4$	The wide end of the sign faces the larger number.
b **i** $\boxed{-7}$ or −11	Negative numbers closer to zero are larger than negative numbers further away from zero.
ii $-7 > -11$	The wide end of the sign faces the larger number.

Example 5

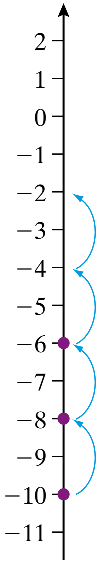
In each of these patterns the integers are going up or down by a constant amount. Write the next two integers for each pattern.

a −10, −8, −6, __, __, … **b** 9, 6, 3, __, __, …

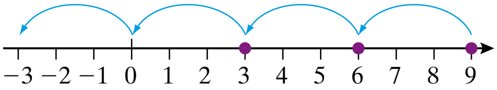
continued

Example 5 continued

Working	**Reasoning**
a The pattern with the next two integers is −10, −8, −6, **−4, −2,** …	The numbers in this pattern are going up by twos. Using a number line, the next two integers in the pattern are −4 and −2.



Working	**Reasoning**
b The pattern with the next two integers is 9, 6, 3, **0, −3,** …	The numbers in this pattern are going down by threes. Using a number line, the next two integers in the pattern are 0 and −3.



exercise 1.1

LINKS TO Example 1

● Write the opposites for each of the following.

a −1 **b** 4 **c** −11 **d** 9

LINKS TO Example 1

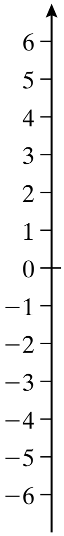
● Evaluate.

a −(−6) **b** −(−12) **c** −(−17) **d** −(−(−7))

LINKS TO Example 2a

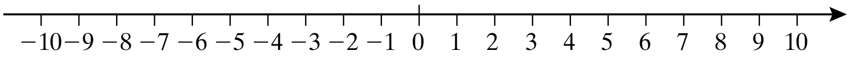
● Mark these numbers on a vertical number line with a scale from −6 to 6.

a −5 **b** 5 **c** −1 **d** −4



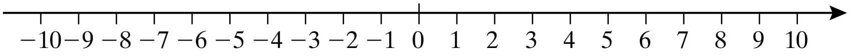
LINKS TO Example 2b

● Mark these numbers on a horizontal number line with a scale from −10 to 10.



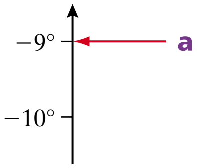
a −10 **b** 8 **c** −6 **d** −2

Mark and label each of these integers on the number line then mark and label the opposite integer.



a −5 **b** 9 **c** −3 **d** 4

Draw a vertical line 20 cm long to represent a thermometer. Carefully mark the scale from −10° C to 10 °C. On your thermometer use arrows to point to the following temperatures, as shown in this portion of the thermometer.



a −9°C **b** −2°C **c** 1°C **d** 8°C
e −3°C **f** 3°C **g** −5°C **h** −4°C

LINKS TO Example 3

Arrange these sets of integers in descending order.

a 3, −9, 0, −7, 6 **b** −1, 0, 4, −6, 5
c −8, 3, −5, −6, 8 **d** −7, 0, 1, −9, −3

Arrange these sets of integers in ascending order.

a 7, −8, 0, −4, 2 **b** −6, 5, −3, 0, −4
c −9, 1, −1, −5, −8 **d** −2, 2, −9, −3, 0

List all the integers between these integers pairs.

a −3 and 1 **b** −7 and −2 **c** 3 and −3 **d** 5 and −2

LINKS TO Example 4a

Write the larger of each of the following pairs of integers.

a −6 and 6 **b** −9 and −3 **c** 0 and −10 **d** −6 and −5

LINKS TO Example 4b

Copy these pairs of integers. Use > or < to make true statements.

a −3   − 7 **b** −9   0 **c** −2   1 **d** −4   − 9

LINKS TO Example 5

Copy these number patterns and extend to the next two numbers.

a 2, 1, 0, __, __ **b** 4, 1, −2, __, __
c −9, −5, −1, __, __ **d** 0, −5, −10, __, __
e 9, 2, −5, __, __ **f** 3, −4, −11, __, __
g 9, 5, 1, __, __ **h** −13, −8, −3, __, __

Barrie, who lives in Ontario, Canada, wrote: ‘If you count −27°C and a metre and a half of snow as mild, then we had a mild winter. Usually we get to −40°C and last year we got about five metres of snow in total.’

According to Barrie, how much warmer was this year than last year?

exercise 1.1 challenge

What number is half-way between 0 and −1. Write the number as a decimal and as a fraction and show it on a number line.

1.2 Adding and subtracting integers

In Year 7 three methods were used to help see the result of adding or subtracting integers.

- moving along the number line
- mentally counting on or back
- the annihilation model, where −1 and 1 cancel each other out (or annihilate each other) because −1 + 1 = 0

Adding and subtracting a positive integer

Adding and subtracting positive integers

If we add a positive integer, we move towards larger numbers, that is, to the right on the number line.

Example 6

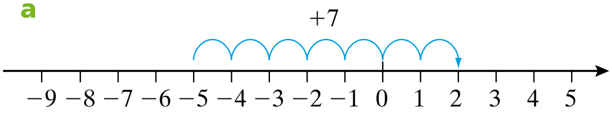
Show the following additions on a horizontal number line then write the addition to show the result.

a −5 + 7

b −8 + 5

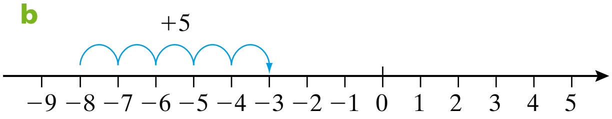
Working

a



−5 + 7 = 2

b



−8 + 5 = −3

Reasoning

To add 7 to −5, start at −5 and move 7 to the right on the number line.

To add 5 to −8, start at −8 and move 5 to the right on the number line.

If we subtract a positive integer, we move towards smaller numbers, that is, to the left on the number line.

Example 7

Show the following subtractions on a horizontal number line then write the subtraction to show the result.

a 2 − 7

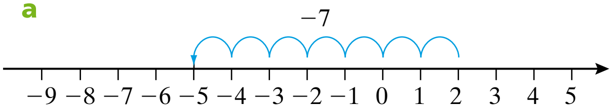
b −5 − 3

continued

Example 7 continued

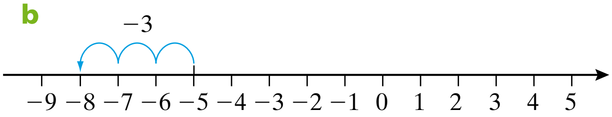
Working

a



$2 - 7 = -5$

b



$-5 - 3 = -8$

Reasoning

To subtract 7 from 2, start at 2 and move 7 to the left on the number line.

To subtract −3 from −5, start at −5 and move 3 to the left on the number line.

Example 8

Write each of the following in symbols and then evaluate.

a Ben works on the 3rd floor. He takes the lift down 5 levels to where his car is parked. What level is he on now?

b The overnight temperature is −5 °C. By lunchtime the temperature has risen by 9 degrees Celsius. What is the temperature now?

Working

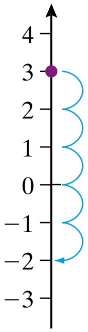
a $3 - 5 = -2$

Ben is now on level −2, that is, two levels below ground level.

Reasoning

Going down 5 levels is equivalent to subtracting 5.

Start at 3 on the number line and go down 5.

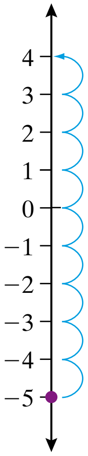


b $-5 + 9 = 4$

The temperature is now 4 °C.

Increasing by 9 degrees is equivalent to adding 9.

Start at −5 on the number line and go up 9.



Example 9

Calculate the following by mentally counting on or counting back.

a $-5 + 11$ **b** $-2 - 4$ **c** $5 - 13$

Working	**Reasoning**
a $-5 + 11 = 6$	Addition – so count on. Start at -5. Counting on 5 takes us to 0, then 6 more brings us to 6.
b $-2 - 4 = -6$	Subtraction – so count back. Start at -2 and count back 4, so $-3, -4, -5, -6$.
c $5 - 13 = -6$	Subtraction – so count back. Start at 5. Counting back 5 takes us to 0, then back 8 more, so $-1, -2, -3, -4, -5, -6, -7, -8$.

Annihilation of integers: addition

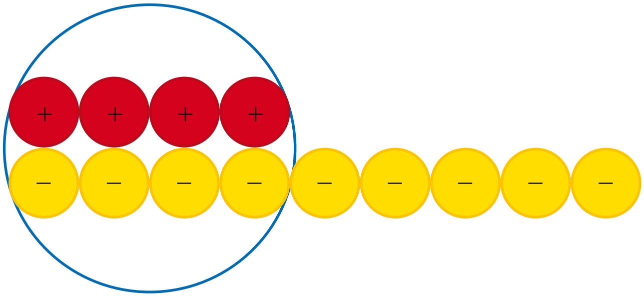
Annihilation of integers: subtraction

Example 10

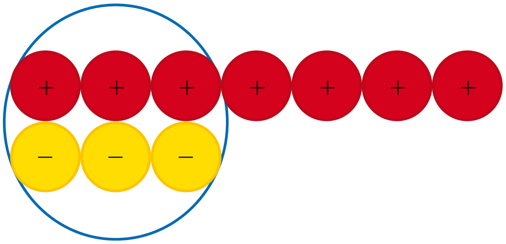
Use the annihilation model to evaluate.

a $4 - 9$ **b** $-3 + 7$

Working	**Reasoning**
a $-9 + 4$ $= -5 - \underbrace{4 + 4}_{0}$ $= -5$	Rewrite -9 as $-4 - 5$. 4 and -4 will annihilate each other, leaving -5.



Working	**Reasoning**
b $-3 + 7$ $= -3 + \underbrace{3 + 4}_{+7}$ $= \underbrace{-3 + 3}_{0} + 4$ $= 4$	Rewrite 7 as 3 + 4. -3 and 3 will annihilate each other, leaving 4.



Example 11

Evaluate each of the following.

a $8 - 14 + 2$ **b** $-3 + 7 - 12$

Working	**Reasoning**
a $8 - 14 + 2$ $= -6 + 2$ $= -4$	Work from left to right. $\underbrace{8 - 14}_{-6} + 2$
b $-3 + 7 - 12$ $= 4 - 12$ $= -8$	Work from left to right. $\underbrace{-3 + 7}_{4} - 12$

Adding and subtracting a negative integer

Adding and subtracting negative integers

In Year 7 we extended addition and subtraction number patterns to adding and subtracting a negative integer. We will look at these patterns again to recall what it means to add or subtract a negative integer.

Starting with the addition $4 + 2 = 6$ we have:

$$4 + 2 = 6$$
$$4 + 1 = 5$$
$$4 + 0 = 4$$
$$4 + (-1) =$$
$$4 + (-2) =$$

Following the pattern we see that $4 + (-1) = 3$ and $4 + (-2) = 2$. This means that

$$4 + (-1) = 4 - 1 = 3$$
$$4 + (-2) = 4 - 2 = 2$$

Adding a negative number

Adding a negative number is the same as subtracting the opposite (positive) number.

For example, $4 + (-2) = 4 - 2 = 2$.

Starting with the subtraction 4 – 2 = 2 we have:

$$4 - 2 = 2$$
$$4 - 1 = 3$$
$$4 - 0 = 4$$
$$4 - (-1) =$$
$$4 - (-2) =$$

Following the pattern we see that $4 - (-1) = 5$ and $4 - (-2) = 6$. This means that

$$4 - (-1) = 4 + 1 = 5$$
$$4 - (-2) = 4 + 2 = 6$$

Subtracting a negative number

Subtracting a negative number is the same as adding the opposite (positive) number.

For example, $4 - (-2) = 4 + 2 = 6$.

Example 12

Calculate by first replacing the two signs between the numbers with a single sign.

a $5 + (-6)$ **b** $-4 + (-5)$ **c** $1 - (-7)$ **d** $-3 - (-7)$

Working	**Reasoning**
a $5 + (-6) = 5 - 6$ $= -1$	Adding a negative integer is the same as subtracting its opposite.
b $-4 + (-5) = -4 - 5$ $= -9$	Adding a negative integer is the same as subtracting its opposite.
c $1 - (-7) = 1 + 7$ $= 8$	Subtracting a negative integer is the same as adding its opposite.
d $-3 - (-7) = -3 + 7$ $= 4$	Subtracting a negative integer is the same as adding its opposite.

Integers and algebra

In Year 7, we evaluated expressions by substituting integer values for pronumerals. When completing tables of values, negative integer values for x are often included.

Example 13

Use substitution to complete the following.

a Evaluate the expression $a + b$ when $a = -4$ and $b = 3$.

b Complete the table of values for the rule $y = x - 3$.

x	-3	-2	-1	0	1	2	3
y							

Working

a When $a = -4$ and $b = 3$,

$$\begin{aligned} a + b &= -4 + 3 \\ &= -1 \end{aligned}$$

b $y = x - 3$

x	-3	-2	-1	0	1	2	3
y	-6	-5	-4	-3	-2	-1	0

Reasoning

Substitute -4 for a and 3 for b then evaluate.

Substitute each of the values for x in the rule $y = x - 3$.

Tech tip

The TI-30XB MultiView calculator can be used to add and subtract negative integers.
For example, to find $-4 + (-5)$ (example 12 part b) type:

$\boxed{(-)}\ \boxed{4}\ \boxed{+}\ \boxed{(-)}\ \boxed{5}\ \boxed{\text{enter}}$

To find $1 - (-7)$ (example 12 part c) type:

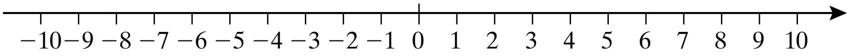
$\boxed{1}\ \boxed{-}\ \boxed{(-)}\ \boxed{7}\ \boxed{\text{enter}}$

Note: Be careful not to confuse negative $\boxed{(-)}$ with minus $\boxed{-}$. They might look similar to us, but the calculator sees them quite differently.

exercise 1.2

LINKS TO
Example 6

Evaluate. Use the horizontal number line to help you.



a $-4 + 9$

b $-6 + 8$

c $-5 + 5$

d $-1 + 8$

e $-7 + 9$

f $-3 + 4$

g $-10 + 8$
h $-2 + 7$
i $-7 + 6$
j $-5 + 4$
k $-3 + 7$
l $-5 + 9$

LINKS TO Example 7

Evaluate. Use the horizontal number line to help you.

a $4 - 9$
b $-6 - 8$
c $-5 - 5$
d $1 - 8$
e $7 - 9$
f $-3 - 4$
g $2 - 7$
h $-2 - 7$
i $-7 - 6$
j $-5 - 4$
k $-3 - 7$
l $-5 - 3$

LINKS TO Example 8

Write each of these as a number sentence then find the result.

a The temperature was 3°C then fell by 9 degrees Celsius.
b Lucy parked car on basement level 4 then went up 7 floors in the lift.
c A diver was 2 metres below the surface of the water then descended a further 5 metres.
d Andrew owed \$20 then paid back \$6.

LINKS TO Example 9

Calculate each of the following.

a $3 - 10$
b $-4 + 8$
c $-25 + 13$
d $-11 + 18$
e $-6 - 19$
f $-23 + 17$
g $14 - 36$
h $-12 + 13$
i $-24 - 16$
j $-5 + 19$
k $-12 - 13$
l $-11 - 9$

LINKS TO Example 10

Complete these additions by using either the annihilation method or counting on.

a $-9 + 7$
b $-8 + 1$
c $-13 + 8$
d $-6 + 6$
e $-10 + 4$
f $-4 + 10$
g $-8 + 7$
h $-7 + 8$
i $-3 + 6$
j $-7 + 7$
k $-5 + 12$
l $-12 + 5$

LINKS TO Example 11

Evaluate each of the following, working from left to right.

a $13 - 8 - 7$
b $-5 + 8 - 3$
c $4 - 7 + 13$
d $-7 + 11 - 6$
e $-1 - 13 - 8$
f $-19 + 17 - 7$
g $-9 - 14 + 13$
h $-4 - 4 - 4$
i $-18 + 14 - 6$
j $5 - 11 - 7$
k $-12 + 17 - 5$
l $-21 + 9 - 3$

LINKS TO Example 12a

Evaluate by first replacing the two signs between the numbers with a single sign.

a $5 + (-8)$
b $-6 + (-6)$
c $-2 + (-3)$
d $-7 + (-8)$
e $-8 + (-5)$
f $8 + (-5)$
g $12 + (-16)$
h $-12 + (-16)$
i $7 + (-7)$
j $-5 + (-20)$
k $-12 + (-13)$
l $-11 + (-9)$

LINKS TO Example 12b

Evaluate by first replacing the two signs between the numbers with a single sign.

a $7 - (-8)$
b $-4 - (-4)$
c $-2 - (-7)$
d $-3 - (-8)$
e $-9 - (-11)$
f $6 - (-6)$
g $10 - (-5)$
h $-15 - (-8)$
i $-11 - (-11)$
j $-5 - (-9)$
k $-12 - (-13)$
l $-17 - (-2)$

Evaluate by first replacing the two signs between the numbers with a single sign.

a $11 + (-4)$ **b** $-10 - (-10)$ **c** $-14 - (-7)$

d $-5 + (-2) + 3$ **e** $8 - (-8)$ **f** $-1 + (-11) + 6$

g $-12 - (-4) - 3$ **h** $-32 + (-18) - 6$ **i** $7 + (-17) + 8$

j $-14 - (-17) + 9$ **k** $-9 + (-23) - (-5)$ **l** $-21 - (-3) + (-5)$

LINKS TO Example 13

Complete each of the following tables of values for the rule shown.

a $y = x - 5$

x	-3	-2	-1	0	1	2	3
y							

b $y = -x + 2$

x	-3	-2	-1	0	1	2	3
y							

c $y = x + 5$

x	-3	-2	-1	0	1	2	3
y							

d $y = -x - 4$

x	-3	-2	-1	0	1	2	3
y							

Complete this magic square so that each row, column and diagonal add to -24.

		-5
	-8	
		-7

The highest mountain in the world is Mt Everest whose summit is 8848 m above sea level. The deepest part of the ocean is the Mariana Trench to the east of the Philippines and north of New Guinea. The Mariana Trench has a depth of 11 032 m.

a Write the height of Mt Everest as an integer.

b Write the height of Mariana Trench as an integer.

c Write a number expression to show the difference between the two heights then evaluate your expression.

exercise 1.2 challenge

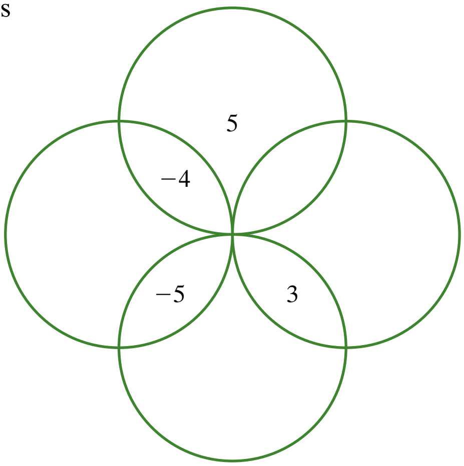
- In the game of golf, each hole has a score that experts think is the score for which a good golfer should aim. It is called 'par'. Golfers often take more shots to get the ball in the hole. Sometimes they take fewer. Instead of using positive or negative numbers to show scores above or below par, they have special words for each score. Some of these are shown in the table.

3	Triple bogey
2	Double bogey
1	Bogey
0	Par
−1	Birdie
−2	Eagle
−3	Albatross

  Carrie plays nine holes of golf. Her scores are: birdie, bogey, par, par, double bogey, birdie, eagle, par and triple bogey.

  Write this list with integers, and decide how many shots, or 'strokes', above or below par she finished the nine holes.

- Fill the gaps in this diagram so that the numbers in each circle add to zero.



1.3 Multiplying integers

Train travel multiplication

By extending multiplication number patterns we see what it means to multiply by a negative integer.

We start by looking at $4 \times 4 = 16, 4 \times 3 = 12$, and so on. As we multiply 4 by 4, 3, 2, 1 and 0 we notice that the product is decreasing by 4 each time: 16, 12, 8, 4, 0. We can then extend this to multiplying 4 by negative integers. The pattern will continue $-4, -8, -12, -16\ldots$ So we see that, logically, if a positive integer is multiplied by a negative integer, the product is negative.

We know also that the order in which we multiply two numbers does not matter, for example, $4 \times 3 = 3 \times 4$. So if $4 \times (-4) = -16$, then $-4 \times 4 = -16$.

We now look at the pattern, $-4 \times 4 = -16,\ -4 \times 3 = -12$ and so on. This time we notice that the products are increasing by 4 each time: $-16, -12, -8, -4, 0$ so logically the next products in the sequence are 4, 8, 12, 16. So the product of two negative integers is positive.

$$\begin{aligned}
4 \times 4 &= 16 \\
4 \times 3 &= 12 \\
4 \times 2 &= 8 \\
4 \times 1 &= 4 \\
4 \times 0 &= 0 \\
4 \times (-1) &= -4 \\
4 \times (-2) &= -8 \\
4 \times (-3) &= -12 \\
4 \times (-4) &= -16
\end{aligned}$$

$$\begin{aligned}
-4 \times 4 &= -16 \\
-4 \times 3 &= -12 \\
-4 \times 2 &= -8 \\
-4 \times 1 &= -4 \\
-4 \times 0 &= 0 \\
-4 \times (-1) &= 4 \\
-4 \times (-2) &= 8 \\
-4 \times (-3) &= 12 \\
-4 \times (-4) &= 16
\end{aligned}$$

Products of integers

If the signs of two integers are the *same* (both positive or both negative) the product is *positive*.

If the signs of two integers are *different* (one positive and the other negative) the product is *negative*.

Example 14

Evaluate each of the following.

a -5×9 **b** $-7 \times (-12)$ **c** $6 \times (-8)$

d $-3 \times (-4) \times (-5)$ **e** $5 \times (-4) \times (-7)$ **f** $3 \times 12 \times (-2)$

continued

Example 14 continued

Working	**Reasoning**
a $-5 \times 9 = -45$	The product of a negative integer and a positive integer is negative.
b $-7 \times (-12) = 84$	The product of two negative integers is positive.
c $6 \times (-8) = -48$	The product of a positive integer and a negative integer is negative.
d $-3 \times (-4) \times (-5) = 12 \times (-5)$ $= -60$	The product of the first two negative integers is positive. When multiplied by another negative integer, the result is negative.
e $5 \times (-4) \times (-7) = -20 \times (-7)$ $= 140$	The product of the first two integers is negative. When multiplied by another negative integer, the result is positive.
f $3 \times 12 \times (-2) = 36 \times (-2)$ $= -72$	The product of the first two integers is positive. When multiplied by a negative integer, the result is negative.

Example 15

Complete the following.

a Complete the following table of values for the rule $y = -4x$.

x	-3	-2	-1	0	1	2	3
y							

b If $a = -2$, $b = 3$ and $c = -1$, calculate

- **i** ab
- **ii** $4bc$

Working

a

x	-3	-2	-1	0	1	2	3
y	12	8	4	0	-4	-8	-12

Reasoning

Substitute each of the values of x into the rule $y = -4x$.
For example, when $x = -3$,

$$\begin{aligned} y &= -4 \times (-3) \\ &= 12 \end{aligned}$$

b i $$\begin{aligned} ab &= -2 \times 3 \\ &= -6 \end{aligned}$$

Substitute $a = -2$ and $b = 3$ then evaluate.

ii $$\begin{aligned} 4bc &= 4 \times 3 \times (-1) \\ &= -12 \end{aligned}$$

Substitute $b = 3$ and $c = -1$ then evaluate.

Raising integers to a power

When a negative integer is squared, the result is positive because two negative integers are being multiplied, for example, $(-2)^2 = (-2) \times (-2) = 4$. However, when a negative integer is raised to the power 3, that is, cubed, the result is negative, for example, $(-2)^3 = (-2) \times (-2) \times (-2) = -8$. We can see from the pattern in the following table that

- even powers of negative integers are positive
- odd powers of negative integers are negative.

	Power	Sign
$(-2)^1 = -2$	1	−
$(-2)^2 = (-2) \times (-2) = 4$	2	+
$(-2)^3 = \underbrace{(-2) \times (-2)}_{4} \times (-2) = -8$	3	−
$(-2)^4 = \underbrace{(-2) \times (-2)}_{4} \times \underbrace{(-2) \times (-2)}_{4} = 16$	4	+

Example 16

Evaluate each of the following.

a $(-7)^2$ **b** -7^2 **c** $-(-7)^2$

Working	**Reasoning**
a $(-7)^2 = 49$	$(-7)^2 = (-7) \times (-7) = 49$ The product of two negative integers is positive.
b $-7^2 = -49$	-7^2 means the negative of $7^2 = -(7 \times 7)$
c $-(-7)^2 = -49$	$-(-7)^2$ means the negative of $(-7)^2$ $= -((-7) \times (-7))$

Example 17

Evaluate each of the following.

a $(-3)^3$ **b** $(-2)^6$

Working	**Reasoning**
a $(-3)^3 = -3 \times (-3) \times (-3)$ $= -27$	The product of three negative integers is negative.
b $(-2)^6 = -2 \times (-2) \times (-2) \times$ $(-2) \times (-2) \times (-2)$ $= 64$	$\underbrace{-2 \times (-2)}_{4} \times \underbrace{(-2) \times (-2)}_{4} \times \underbrace{(-2) \times (-2)}_{4}$ When an even number of negative integers is multiplied together, the result is positive.

Do negative numbers have square roots and cube roots?

The square root of a number is the number that when multiplied by itself, the original number is obtained. The square root of 9 is 3 because 3 multiplied by itself is 9. We have already seen that when a negative number is multiplied by itself, the result is positive. Therefore there is no number that when multiplied by itself gives a negative number. So the square root of a negative number cannot be evaluated or placed on the number line. For example, $\sqrt{-9}$ cannot be placed on the number line. If you try to find the square root of $\sqrt{-9}$ with your calculator you will obtain an error message.

The cube root of a number is the number that when raised to the power of three gives the original number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2$. Although a negative integer does not have a square root, a negative integer does have a cube root. For example, the cube root of -8 is -2 because $(-2) \times (-2) \times (-2) = -8$. We write $\sqrt[3]{-8} = -2$.

Example 18

Evaluate, where possible.

a $\sqrt{81}$ **b** $\sqrt{-25}$ **c** $\sqrt[3]{27}$ **d** $\sqrt[3]{-27}$

Working	**Reasoning**
a $\sqrt{81} = 9$	The square root of a number is the number that when multiplied by itself gives the original number. $9 \times 9 = 81$
b $\sqrt{-25}$ does not exist in the real number system.	There is no real number that when multiplied by itself will give a negative result.
c $\sqrt[3]{27} = 3$	The cube root of a number is the number that when raised to the power three gives the original number.
d $\sqrt[3]{-27} = -3$	$(-3) \times (-3) \times (-3) = -27$

Tech tip

The TI-30XB MultiView calculator can be used to multiply by a negative integer. For example, to find $-3 \times (-2)$, type:

$\boxed{(-)}\ \boxed{3}\ \boxed{\times}\ \boxed{(-)}\ \boxed{2}\ \boxed{\text{enter}}$

To find $7 \times (-4)$, type:

$\boxed{7}\ \boxed{\times}\ \boxed{(-)}\ \boxed{4}\ \boxed{\text{enter}}$

Note: Be careful not to confuse negative $\boxed{(-)}$ with minus $\boxed{-}$.
They might look similar to us but the calculator sees them quite differently.

Tech tip

The TI-30XB MultiView calculator can be used to raise an integer to a power. For example, to find $(-3)^3$ (example 17 part a), type:

$\boxed{(}\ \boxed{-}\ \boxed{3}\ \boxed{)}\ \boxed{\wedge}\ \boxed{3}$ **enter**

To find -3^3, type:

$\boxed{-}\ \boxed{3}\ \boxed{\wedge}\ \boxed{3}$ **enter**

Note: -3^3 does not mean the same as $(-3)^3$.

exercise 1.3

LINKS TO Example 14

Complete this table of multiplications.

$6 \times 3 =$	$2 \times 3 =$	$-5 \times 3 =$	$-8 \times 3 =$
$6 \times 2 =$	$2 \times 2 =$	$-5 \times 2 =$	$-8 \times 2 =$
$6 \times 1 =$	$2 \times 1 =$	$-5 \times 1 =$	$-8 \times 1 =$
$6 \times 0 =$	$2 \times 0 =$	$-5 \times 0 =$	$-8 \times 0 =$
$6 \times (-1) =$	$2 \times (-1) =$	$-5 \times (-1) =$	$-8 \times (-1) =$
$6 \times (-2) =$	$2 \times (-2) =$	$-5 \times (-2) =$	$-8 \times (-2) =$
$6 \times (-3) =$	$2 \times (-3) =$	$-5 \times (-3) =$	$-8 \times (-3) =$

Complete this table to summarise what happens to the signs when multiplying integers.

×	−	+
+		
−		

LINKS TO Example 14

Evaluate each of the following.

a $4 \times (-3)$
b -2×5
c $-4 \times (-1)$
d -5×6
e $7 \times (-4)$
f $-6 \times (-5)$
g -10×7
h $-8 \times (-4)$
i $2 \times (-9)$
j $6 \times (-3)$
k -7×8
l $-9 \times (-6)$
m $-4 \times (-12)$
n $10 \times (-10)$
o -11×7
p $-12 \times (-3)$

LINKS TO Example 14

Evaluate each of the following.

a $2 \times (-3) \times 4$
b $-5 \times (-2) \times 6$
c $-7 \times 5 \times (-2)$
d $2 \times (-3) \times (-9)$
e $-5 \times 2 \times 5$
f $4 \times (-2) \times (-7)$
g $2 \times (-6) \times (-4)$
h $-3 \times (-5) \times (-6)$
i $8 \times (-1) \times (-7)$
j $-8 \times 5 \times (-7)$
k $2 \times (-2) \times (-11)$
l $4 \times 3 \times (-6)$
m $8 \times (-3) \times (-3)$
n $2 \times 6 \times (-8)$
o $-4 \times (-5) \times 7$
p $-3 \times 3 \times (-8)$
q $-6 \times (-1) \times 6$
r $-4 \times 6 \times 2$

LINKS TO Example 15

Complete each of the following tables of values for the rule shown.

a $y = 2x$

x	-3	-2	-1	0	1	2	3
y							

b $y = -x$

x	-3	-2	-1	0	1	2	3
y							

c $y = -4x$

x	-3	-2	-1	0	1	2	3
y							

d $y = -2x$

x	-3	-2	-1	0	1	2	3
y							

e $y = 3x$

x	-3	-2	-1	0	1	2	3
y							

f $y = -3x$

x	-3	-2	-1	0	1	2	3
y							

If $a = -4$, $b = 2$, $c = -3$ and $d = -1$, calculate each of the following.

a ab **b** ac **c** abc **d** $abcd$

LINKS TO Example 16

Evaluate these squares.

a 5^2 **b** $(-5)^2$ **c** -5^2 **d** $-(-5)^2$
e $(-1)^2$ **f** -3^2 **g** -7^2 **h** $-(-2)^2$
i $(-8)^2$ **j** $(-9)^2$ **k** $-(-9)^2$ **l** -9^2

LINKS TO Example 17

Evaluate these powers.

a $(-1)^3$ **b** $(-1)^4$ **c** $(-2)^5$ **d** $(-3)^4$
e $(-1)^{24}$ **f** $(-1)^{25}$ **g** $(-4)^3$ **h** $(-5)^3$

LINKS TO Example 18

Evaluate where possible.

a $\sqrt{64}$ **b** $-\sqrt{49}$ **c** $\sqrt{-49}$ **d** $\sqrt{100}$
e $\sqrt{-100}$ **f** $-\sqrt{100}$ **g** $\sqrt{36}$ **h** $-\sqrt{36}$
i $\sqrt{1}$ **j** $-\sqrt{1}$ **k** $\sqrt{25}$ **l** $-\sqrt{25}$
m $\sqrt[3]{8}$ **n** $\sqrt[3]{-8}$ **o** $-\sqrt[3]{27}$ **p** $\sqrt[3]{-64}$

exercise 1.3 challenge

● Copy and complete each of these multiplication puzzles. The integers at the left of each row must be multiplied by the integers at the top of each column to give the integers in the white squares.

a

×		−8	
		−72	45
	28		
	32		−40

b

×	−7		
		−72	
	56		16
		−54	−18

1.4 Dividing integers

Every multiplication can be rewritten as two divisions. For example:

$-4 \times (-3) = 12$ can be rewritten as $12 \div (-4) = -3$ and $12 \div (-3) = -4$.

$4 \times (-3) = -12$ can be rewritten as $-12 \div 4 = -3$ and $-12 \div (-3) = 4$.

Example 19

Find the missing number for each multiplication. Then rewrite each multiplication as a division in two different ways.

a $-4 \times \square = -28$ **b** $5 \times \square = -20$ **c** $-3 \times \square = 36$

Working	**Reasoning**
a $-4 \times \boxed{7} = -28$ $-28 \div 7 = -4$ $-28 \div (-4) = 7$	$4 \times 7 = 28$ The missing number must be positive because the product of a negative number and a positive number is negative.
b $5 \times \boxed{-4} = -20$ $-20 \div (-4) = 5$ $-20 \div 5 = -4$	$5 \times 4 = 20$ The missing number must be negative because the product of a positive number and a negative number is negative.
c $-3 \times \boxed{(-12)} = 36$ $36 \div (-12) = -3$ $36 \div (-3) = -12$	$3 \times 12 = 36$ The missing number must be negative because the product of two negative numbers is positive number.

From divisions in example 19 above, we can see that:

If numbers with the *same* sign are divided, the quotient will be a *positive* number.
If numbers with *different* signs are divided, the quotient will be a *negative* number.

Example 20

Evaluate.

a $54 \div (-6)$ **b** $\frac{-63}{9}$ **c** $-35 \div (-5)$

Working	**Reasoning**
a $54 \div (-6) = -9$	The signs of 54 and -6 are different so the quotient is negative.

continued

Example 20 continued

Working

b $\frac{-63}{9} = -7$

c $-35 \div (-5) = 7$

Reasoning

The signs of -63 and 9 are different so the quotient is negative.

The signs of -35 and -5 are the same so the quotient is positive.

Example 21

Complete the table of values for the rule $y = -\frac{x}{2}$.

x	-6	-4	-2	0	2	4	6
y							

Working

x	-6	-4	-2	0	2	4	6
y	3	2	1	0	-1	-2	-3

Reasoning

Substitute each value for x into the rule $y = -\frac{x}{2}$.

e.g. when $x = -6$

$$\begin{aligned} y &= -\frac{-6}{2} \\ &= -(-3) \\ &= 3 \end{aligned}$$

Tech tip

The TI-30XB MultiView calculator can be used to divide by a negative integer. For example, to find $-35 \div (-5)$ (example 20 part c), type:

$\boxed{(-)}$ $\boxed{3}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{(-)}$ $\boxed{5}$ $\boxed{\text{enter}}$

To find $54 \div (-6)$ (example 20 part a), type:

$\boxed{5}$ $\boxed{4}$ $\boxed{\div}$ $\boxed{(-)}$ $\boxed{6}$ $\boxed{\text{enter}}$

Note: Be careful not to confuse negative $\boxed{(-)}$ with minus $\boxed{-}$. They might look similar to us but the calculator sees them quite differently.

exercise 1.4

LINKS TO Example 19

Find the missing number for each multiplication. Then rewrite each multiplication as a division in two different ways.

a $-7 \times \square = 56$ **b** $-4 \times \square = -36$ **c** $-5 \times \square = 60$ **d** $-6 \times \square = -24$

e $-6 \times \square = 30$ **f** $-7 \times \square = 42$ **g** $-9 \times \square = -81$ **h** $-3 \times \square = -54$

i $-4 \times \square = 56$ **j** $-15 \times \square = 45$ **k** $-6 \times \square = -78$ **l** $-11 \times \square = 132$

Complete this table to summarise what happens to the signs when dividing integers.

÷	−	+
+		
−		

LINKS TO Example 20

Evaluate the following divisions.

a $8 \div (-4)$ **b** $-15 \div 3$ **c** $-20 \div (-4)$ **d** $56 \div (-8)$

e $-18 \div (-6)$ **f** $-45 \div 9$ **g** $28 \div (-4)$ **h** $-54 \div (-6)$

i $-84 \div 7$ **j** $-36 \div (-12)$ **k** $40 \div (-8)$ **l** $-42 \div 7$

Evaluate the following divisions.

a $\frac{-18}{3}$ **b** $\frac{32}{-4}$ **c** $\frac{-60}{-6}$ **d** $\frac{-30}{6}$

e $\frac{0}{-2}$ **f** $\frac{-100}{-20}$ **g** $\frac{5}{-1}$ **h** $\frac{-9}{-1}$

i $\frac{75}{-5}$ **j** $\frac{-96}{-6}$ **k** $\frac{150}{-30}$ **l** $\frac{-120}{8}$

Calculate.

a $60 \div (-2) \div 5$ **b** $-84 \div (-4) \div 7$ **c** $-200 \div 4 \div (-5)$

d $24 \div (-1) \div 3$ **e** $-54 \div (-3) \div 2$ **f** $-60 \div (-5) \div (-4)$

g $140 \div (-10) \div (-2)$ **h** $-90 \div 3 \div (-5)$ **i** $-144 \div 6 \div (-8)$

j $-72 \div 9 \div (-1)$ **k** $-240 \div (-20) \div (-1)$ **l** $243 \div (-3) \div (-9)$

LINKS TO Example 21

Complete each of the following tables of values for the rule shown.

a $y = \frac{x}{2}$

x	−6	−4	−2	0	2	4	6
y							

b $y = \frac{-x}{3}$

x	−12	−9	−6	−3	0	3	6
y							

c $y = \frac{-2x}{3}$

x	-12	-9	-6	-3	0	3	6
y							

d $y = \frac{-3x}{4}$

x	-12	-8	-4	0	4	8	12
y							

exercise 1.4 challenge

● We can write the division 17 divided by 5 in two ways.

i $17 \div 5 = 3$ remainder 2 **ii** $17 = 5 \times 3 + 2$.

a Write 17 divided by -5 in each of these ways.

b Write -17 divided by 5 in the two ways.

c Write -17 divided by 5 in the two ways but this time so there is a positive remainder. Hint: the quotient will need to be different.

The remainder when negative integers are involved depends on whether we choose to have a positive remainder or a negative remainder. This also means that there are two possible quotients. When working with numbers we would usually choose to have a positive remainder. In computer programming languages, different conventions for positive or negative remainders have been adopted by different programming languages.

d Write the following divisions in the two ways first with a negative remainder and then with a positive remainder.

- **i** $-23 \div 4$
- **ii** $-17 \div 4$
- **iii** $-47 \div 8$
- **iv** $-37 \div 9$
- **v** $-13 \div (-2)$

1.5 Order of operations

The order of operations that applies with positive numbers also applies with negative numbers.

1 **B**rackets, **P**owers and **R**oots
2 **O**f
3 **D**ivisions and
4 **M**ultiplications } are worked from left to right in the order that they occur.
5 **A**dditions
6 **S**ubtractions } are worked from left to right in the order that they occur.

Example 22

Evaluate the following.

a $3 - 36 \div 9 \times (-2) - 15$

b $-5 \times (-4 + 3 \times (-2))$

c $\dfrac{13 + 3 \times 4}{-7 + 2}$

Working	**Reasoning**
a $3 - \underbrace{36 \div 9}_{4} \times (-2) - 15$	Calculate divisions and multiplications from left to right in the order they occur.
$= 3 - \underbrace{4 \times (-2)}_{-8} - 15$	
$= 3 - (-8) - 15$	Subtracting a negative integer is equivalent to adding the corresponding positive integer.
$= 3 + 8 - 15$ $= -4$	Work additions and subtractions from left to right in the order they occur.
b $-5 \times (-4 + 3 \times (-2))$ $= -5 \times (-4 + (-6))$	Calculate brackets first. Within the bracket there is a multiplication so do this before the addition.
$= -5 \times (-4 - 6)$	Adding a negative integer is equivalent to subtracting the corresponding positive integer.
$= -5 \times (-10)$ $= 50$	The product of two negative integers is positive.
c $\dfrac{13 + 3 \times 4}{-7 + 2}$ $= \dfrac{13 + 12}{-5}$	First simplify the numerator and denominator separately. In the numerator, complete the multiplication before the addition.
$= \dfrac{25}{-5}$ $= -5$	A positive number divided by a negative number gives a negative number.

Example 23

Evaluate.

a $-2 + (-9 + 7)^3$

b $\sqrt{-6 + 2 \times 5}$

Working	**Reasoning**
a $-2 + (-9 + 7)^3$ $= -2 + (-2)^3$ $= -2 + (-8)$ $= -2 - 8$ $= -10$	Complete brackets first then the power. Adding a negative integer is equivalent to subtracting the corresponding positive integer.
b $\sqrt{-6 + 2 \times 5}$ $= \sqrt{-6 + 10}$ $= \sqrt{4}$ $= 2$	Under the square root sign, complete multiplication before addition.

exercise 1.5

LINKS TO Example 22

Evaluate.

a $6 + (-24) \div 8$

b $15 - (-2) \times 8$

c $-63 \div 7 + 2$

d $-35 \div (-5) + 8 \times (-3)$

e $-6 \times (-4 + 7) + 2 \times (-1)$

f $56 \div (-8) + (-2) \times 3$

g $-72 \div (-6) + 4 \times (-3)$

h $-54 \div 6 + 3 - 4 \times (-2)$

Evaluate the following.

a $\dfrac{8 - 12}{2}$

b $\dfrac{11 - 15}{5 - 3}$

c $\dfrac{-1 + (-5) \times 7}{4 \times (-9)}$

d $\dfrac{4 \times (-3)}{-7 + 1}$

e $\dfrac{-7 + 4 \times (-5) - 6}{3}$

f $\dfrac{4 + 48 \div (-6)}{5 - 6}$

g $\dfrac{7 + 3 \times (-6)}{3 - (-2)}$

h $\dfrac{3 \times (-5) + 6 \times (-2) + 1}{5 - 9}$

LINKS TO Example 23

Evaluate the following.

a $-3 + 5^2$

b $(-3 + 5)^2$

c $(-3)^2 + 5$

d $-(3 + 5)^2$

e $5 \times (-2)^3$

f $-2 \times 3^2 - 4$

g $(-2 \times 3)^2$

h $2^4 \div (-5 - 3)$

To evaluate $15 - 3 \times (4 - 8 \div 2^3)$ the first step is to calculate

A $15 - 3$

B 3×4

C $4 - 8$

D $8 \div 2$

E 2^3

The expression $\frac{6 - 3^2}{3}$ is equal to

A -1 **B** 1 **C** 3 **D** -3 **E** 6

This calculation has a mistake in it. Find the mistake and correct it so that the expression is evaluated correctly.

$$\begin{aligned} &-5 + 28 \div 7 \times (-2) + (-3)^2 \\ &= -5 + 28 \div 7 \times (-2) + 9 \\ &= -5 + 28 \div (-14) + 9 \\ &= -5 + (-2) + 9 \\ &= -5 - 2 + 9 \\ &= 2 \end{aligned}$$

If $a = -4$, $b = 2$, $c = -3$ and $d = -1$ calculate each of the following.

a $a(b + c)$ **b** $a(b - c)$ **c** $d(a + b)$ **d** $b(c - d)$

Complete each of the following tables of values for the rule shown.

a $y = 2x + 1$

x	-3	-2	-1	0	1	2	3
y							

b $y = 3x - 4$

x	-3	-2	-1	0	1	2	3
y							

c $y = -2x + 5$

x	-3	-2	-1	0	1	2	3
y							

d $y = -\frac{x}{2} + 3$

x	-6	-4	-2	0	2	4	6
y							

Consider the formula $T = a^2 + b^3$

a Find the value of T when

- **i** $a = 3$ and $b = 2$.
- **ii** $a = 3$ and $b = -2$.
- **iii** $a = -3$ and $b = -2$.

b Which of these are the same, and why?

exercise 1.5 challenge

- The average of six numbers is -7. If five of the numbers are $-8, 4, -19, 2$ and -11, what is the sixth number?

- It is often useful to use $+$ and $-$ signs to indicate the direction in which an object is moving. For example, an object that moves 5 m to the right has moved 5 m and an object that moves 5 m to the left has moved -5 m. Speeds can also be given $+$ or $-$ signs according to direction and they are then called velocities. The formula $s = \frac{t(u + v)}{2}$ is used to calculate the distance and direction, s metres, which an object moves in time t seconds if its starting velocity is u m/sec and its velocity at the end of t seconds is v m/sec.

  Use the formula to calculate s when:

  a $t = 5, u = -2$ and $v = 6$

  b $t = 3, u = 4$ and $v = -6$

  c $t = 1, u = -8$ and $v = -2$

  d $t = 4, u = -3$ and $v = -5$

Analysis task

Negative fortunes!

The concept of negative numbers was difficult for people to understand because numbers were associated at first with counting. In about 600 BCE, the Indian mathematician Brahmagupta explained the concept of negative numbers in terms of fortunes and debts:

> A debt minus zero is a debt.
> A fortune minus zero is a fortune.
> Zero minus zero is a zero.
> A debt subtracted from zero is a fortune.
> A fortune subtracted from zero is a debt.
> The product of zero multiplied by a debt or fortune is zero.
> The product of zero multiplied by zero is zero.
> The product or quotient of two fortunes is one fortune.
> The product or quotient of two debts is one fortune.
> The product or quotient of a debt and a fortune is a debt.
> The product or quotient of a fortune and a debt is a debt.

If we choose the integer -1 to represent a debt, we can express the first line of Brahmagupta's explanation as $-1 - 0 = -1$. If we choose the integer 1 to represent a fortune, we can express the second line of Brahmagupta's explanation as $1 - 0 = 1$.

a Using -1 for a debt and 1 for a fortune, express each of the other lines of Brahmagupta's writing as a number expression.

Many people still have difficulty understanding negative numbers. Look at the following report about a British lottery scratch card and answer the questions below.

A lottery scratch card, Cool Cash, was withdrawn in Britain because players couldn't understand whether or not they had won. To qualify for a prize, users had to scratch away a window to reveal a temperature lower than the number displayed on each card. As the game had a winter theme, the temperature was usually below freezing. But the concept of comparing negative numbers proved too difficult for some people. Dozens of complaints were received on the first day from players who could not understand how, for example, -5 is higher than -6. One 23-year-old who had bought several cards complained:

On one of my cards it said I had to find temperatures lower than -8*. The numbers I uncovered were* -6 *and* -7 *so I thought I had won, and so did the woman in the shop. But when she scanned the card the machine said I hadn't. When I phoned to complain, they fobbed me off with some story that* -6 *is higher, not lower, than* -8 *but I'm not having it. I think people are being given the wrong impression. The card doesn't say to look for a colder or warmer temperature, it says to look for a higher or lower number.* 6 *is a lower number than* 8*. Imagine how many people have been misled.*

b What misunderstanding did the 23-year old have?

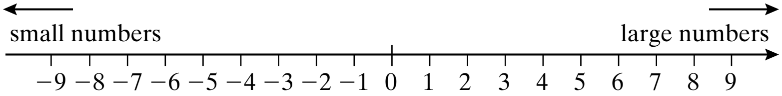
c Why do you think people have this misunderstanding?

d Explain carefully, using diagrams, how the lottery company could have helped people understand the game.

Review Integers

Summary

- The numbers below zero on a vertical number line (or to the left on a horizontal number line) are called **negative** numbers.
- Integers are whole numbers. They can be positive, negative and zero.
- To add and subtract positive integers, one way to calculate is to use a number line. To add, go towards the large numbers (usually up or to the right). To subtract, go towards the small numbers (usually down or to the left).



- Another way to add positive integers is to count on, and another way to subtract positive integers is to count back.
- The annihilation model can also help us with adding and subtracting numbers.
- Adding a negative number is the same as subtracting the opposite (positive) number. For example, $4 + (-2) = 4 - 2 = 2$.
- Subtracting a negative number is the same as adding the opposite (positive) number. For example, $4 - (-2) = 4 + 2 = 6$.
- If the signs of two integers are the *same* (both positive or both negative) the product is *positive*.
- If the signs of two integers are *different* (one positive and the other negative) the product is *negative*.
- If numbers with the *same* sign are divided, the quotient will be a *positive* number.
- If numbers with *different* signs are divided, the quotient will be a *negative* number.
- The same order of operations applies to integers as to any other numbers. (Brackets, Powers and Roots, Of, Divisions and Multiplications, Additions and Subtractions.)
- The square root of a negative integer cannot be evaluated or placed on the number line.
- The cube root of a negative integer does exist and is negative.

Visual map

annihilation
ascending order
counting back
counting on
cube
cube root
descending order
integer
negative
number line
order of operations
positive
power
product
quotient
square
square root
table of values

Revision

Multiple-choice questions

Arranged in ascending order the set of integers $-5, 4, 0, -8, 7$ is

A $-8, 7, -5, 4, 0$
B $0, 4, -5, 7, -8$
C $7, 4, 0, -5, -8$
D $-8, -5, 0, 4, 7$
E $-5, -8, 0, 4, 7$

$3 - 7 - (-10)$ is equal to

A -14 **B** 0 **C** 6 **D** -40 **E** -6

Evaluate $4 - (-3) \times 5 + 2$

A 21 **B** 37 **C** -9 **D** 49 **E** 25

The expression $\frac{-36}{-4}$ gives the same result as

A $\frac{36}{-4}$ **B** $\frac{-36}{4}$ **C** $\frac{18}{-2}$ **D** $\frac{-18}{2}$ **E** $\frac{18}{2}$

When evaluated, $-60 \div (-6) \div 2$ equals

A -20 **B** 5 **C** -5 **D** 10 **E** 20

Short-answer questions

Evaluate.

a $-11 - 5$ **b** $-7 + 15$ **c** $8 - 23$
d $-5 + (-6)$ **e** $14 - (-10)$ **f** $-3 - (-9)$

Evaluate.

a $6 \times (-5)$ **b** -7×9 **c** $-8 \times (-7)$
d $3 \times (-5) \times 4$ **e** $-6 \times (-2) \times (-9)$ **f** $2 \times (-7) \times (-3)$

Evaluate.

a $16 \div (-4)$ **b** $-28 \div 7$ **c** $-80 \div (-10)$
d $24 \div (-6) \div (-1)$ **e** $\frac{114}{-3}$ **f** $\frac{-96}{-4}$
g $\frac{-72}{9}$ **h** $\frac{-56}{6 - (-1)}$ **i** $\frac{-30}{-10 \div 2}$

Evaluate, where possible.

a $(-4)^2$ **b** $-(-7)^2$ **c** -5^2 **d** $(-2)^4$
e $-\sqrt{36}$ **f** $\sqrt{-49}$ **g** $(-3)^2 - \sqrt{4}$ **h** $(-1)^3$
i $\sqrt[3]{-64}$ **j** $\sqrt[3]{27}$ **k** $-\sqrt[3]{27}$ **l** $-\sqrt[3]{-1}$

Evaluate.

a $-3 + 8 \times (-2) + 6 \div (-3)$

b $\dfrac{15 - (-9)}{2 \times (-3)}$

c $-3 \times (-4 + 2)^3$

d $\sqrt{-2 + 3 \times 6}$

Complete the table of values for these rules.

a $y = x - 7$

x	−3	−2	−1	0	1	2	3
y							

b $y = 5x$

x	−3	−2	−1	0	1	2	3
y							

c $y = -3x$

x	−3	−2	−1	0	1	2	3
y							

d $y = -\dfrac{x}{2} - 5$

x	−6	−4	−2	0	2	4	6
y							

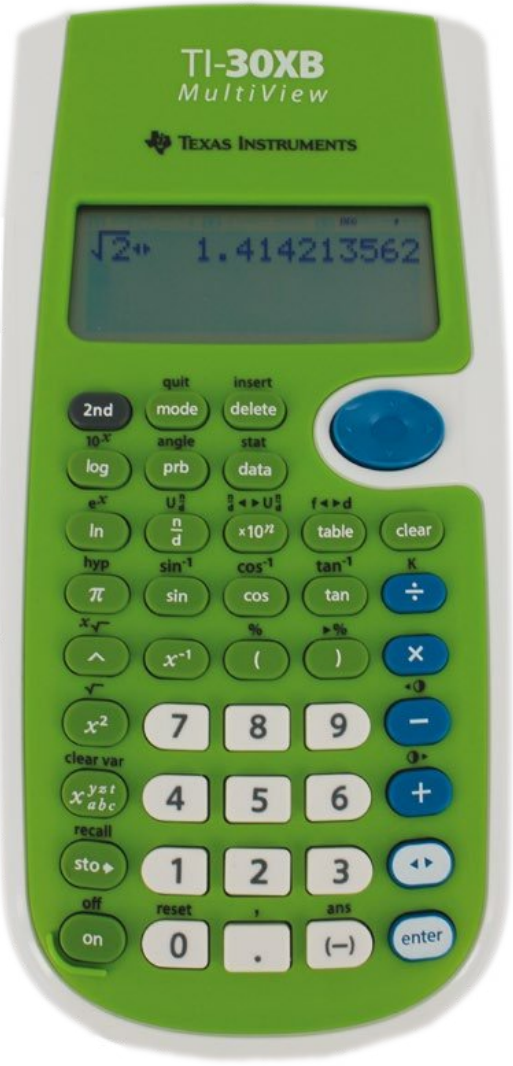
Extended-response questions

A *diabolical magic square* is a special magic square where there are also other groups of numbers that add to the same total. For example, all four numbers in the middle square, shaded yellow, add to the same as each row, column and diagonal. Fill in the gaps in this diabolical magic square.

−3	−6	1	−6
0			
	−7	2	
3			−8

Fractions and decimals

2



Pre-test

Warm-up

Working with numbers, including decimals and fractions, is an important part of everyday life. We regularly use decimal numbers when calculating with money, metric measurements and sports times. Integers, proper and improper fractions, and terminating and recurring decimals are all rational numbers. The square root of 2 shown on this calculator, is an irrational number. The decimal places of the square root of 2 go on forever, but the calculator is displaying only nine decimal places.

2.1 Comparing and ordering fractions and decimals

Negative fractions and decimals

Just as we can have negative integers, we can also have negative fractions and decimals. Each *positive fraction* has an *opposite negative fraction*. The opposite fraction for $\frac{1}{2}$ is $-\frac{1}{2}$ and the opposite for the mixed number $1\frac{3}{10}$ is $-1\frac{3}{10}$.

Example 1

Write the opposites of these fractions and decimals.

a $2\frac{4}{5}$ **b** $-\frac{3}{4}$ **c** -0.72 **d** 1.85

Working

a The opposite of $2\frac{4}{5}$ is $-2\frac{4}{5}$.

b The opposite of $-\frac{3}{4}$ is $\frac{3}{4}$.

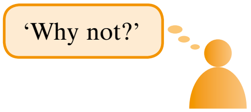
c The opposite of -0.72 is 0.72.

d The opposite of 1.85 is -1.85

Reasoning

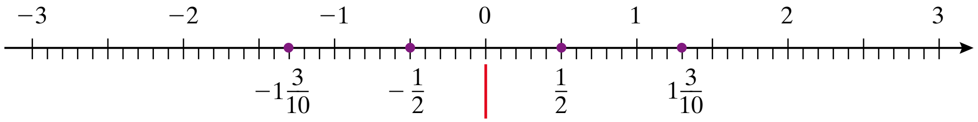
Correct mathematical setting out is important.

We do **not** write the answers as $2\frac{4}{5} = -2\frac{4}{5}$ or $-0.72 = 0.72$.



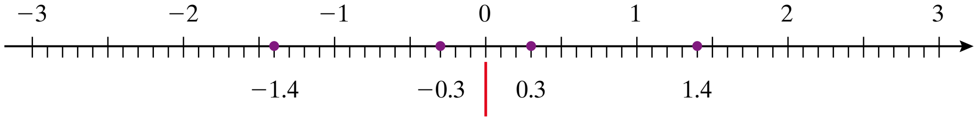
Fractions and decimals on the number line

Negative fractions and decimals, just like negative integers, can be shown on the number line to the left of zero. The positions of each number and its opposite are reflections of each other in a mirror line through zero.



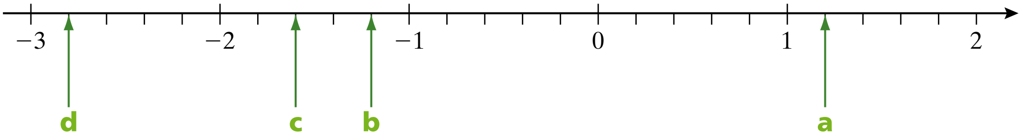
$$\frac{1}{-2} = \frac{-1}{2} = -\frac{1}{2}$$

We usually write the − sign in front of the fraction or in the numerator.



Example 2

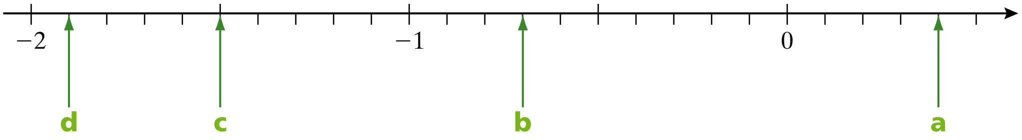
Write each fraction shown on the line below.



Working	Reasoning
a $1\frac{1}{5}$	There are 5 spaces between 0 and 1. This means that each scale mark shows $\frac{1}{5}$. Number **a** is 1 space to the right of 1 so it is $1\frac{1}{5}$.
b $-1\frac{1}{5}$	This number is 1 space, or $\frac{1}{5}$, to the left of -1 so it is $-1\frac{1}{5}$. Note that it is the mirror image of point **a**, $1\frac{1}{5}$.
c $-1\frac{3}{5}$	This number is 3 spaces, or $\frac{3}{5}$, to the left of -1 so it is $-1\frac{3}{5}$. It is also 2 spaces, or $\frac{2}{5}$, to the right of -2, which makes it $-1\frac{3}{5}$ too.
d $-2\frac{4}{5}$	This number is 4 spaces, or $\frac{4}{5}$, to the left of -2 so it is $-2\frac{4}{5}$. It is also 1 space, or $\frac{1}{5}$, to the right of -3, which makes it $-2\frac{4}{5}$ too.

Example 3

Write each decimal shown on the number line below.



Working	Reasoning
a 0.4	There are 10 divisions between -1 and 0, so each division is one tenth, that is, 0.1. Number **a** is 4 tenths to the right of 0.
b -0.7	This number is 7 tenths to the left of 0.
c -1.5	This number is 5 tenths to the left of -1 so it is 5 tenths smaller than -1. We can also see that it is half-way between -1 and -2.

continued

Example 3 continued

Working

d -1.9

Reasoning

This number is 9 tenths to the left of -1 so it is 9 tenths smaller than -1. We can also see that it is 1 tenth to the right of -2 so it is 1 tenth larger than -2.

Comparing fractions

Fractions can be placed in order of size by converting them to equivalent fractions with the same denominator and then comparing the numerators. Negative fractions are smaller than positive fractions.

BLM Fraction wall

Example 4

Write the fractions $\frac{2}{3}, -\frac{5}{8}, \frac{7}{12}, -\frac{3}{4}$ in ascending order. Show them on a number line.

Working

$\frac{2}{3}, -\frac{5}{8}, \frac{7}{12}, -\frac{3}{4}$ becomes

$\frac{16}{24}, -\frac{15}{24}, \frac{14}{24}, -\frac{18}{24}$

In order, $-\frac{18}{24}, -\frac{15}{24}, \frac{14}{24}, \frac{16}{24}$

Answer: $-\frac{3}{4}, -\frac{5}{8}, \frac{7}{12}, \frac{2}{3}$.

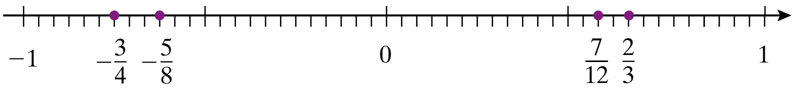
Reasoning

The largest denominator is 12.
12, $\boxed{24}$, 36, 48, 60, … … All of the denominators divide exactly into 24.
Change each fraction into 24ths.

Order the fractions from smallest to largest by looking at the numerators. (Think of the negative signs as belonging to the numerators, because $-\frac{3}{4} = \frac{-3}{4}$ etc.)

Simplify to convert back to the starting fractions.

On a number line showing 24ths:



When we compare mixed numbers it is helpful to convert them into improper fractions first.

We convert negative mixed numbers to improper fractions in the same way as for positive mixed numbers:

$$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

$$-1\frac{3}{4} = -\left(\frac{4}{4} + \frac{3}{4}\right) = -\frac{7}{4}$$

Example 5

Convert these mixed numbers into improper fractions.

a $2\frac{1}{5}$ **b** $-2\frac{1}{5}$ **c** $-4\frac{7}{8}$

Working	**Reasoning**
a $2\frac{1}{5} = \frac{11}{5}$	$2\frac{1}{5} = \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$ $= \frac{11}{5}$
b $-2\frac{1}{5} = -\frac{11}{5}$	$-2\frac{1}{5} = -\left(\frac{5}{5} + \frac{5}{5} + \frac{1}{5}\right)$ $= -\frac{11}{5}$
c $-4\frac{7}{8} = -\frac{39}{8}$	$-4\frac{7}{8} = -\left(\frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{7}{8}\right)$ $= -\frac{39}{8}$

Example 6

Insert < or > to make a true statement.

$-2\frac{5}{8}$ ___ $-2\frac{3}{4}$

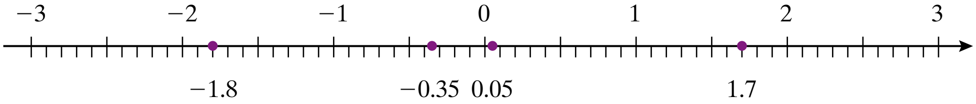
Working	**Reasoning**
$-2\frac{5}{8} = -\frac{21}{8}$	$-2\frac{5}{8} = -\left(\frac{8}{8} + \frac{8}{8} + \frac{5}{8}\right)$
$-2\frac{3}{4} = -\frac{11}{4}$ $= -\frac{22}{8}$	$-2\frac{3}{4} = -\left(\frac{4}{4} + \frac{4}{4} + \frac{3}{4}\right)$ Convert to equivalent fractions with the same denominator.
$-\frac{21}{8} > -\frac{22}{8}$	Compare the numerators, thinking of the numbers as -21 eighths and -22 eighths.
$-2\frac{5}{8} > -2\frac{3}{4}$	

Decimal numbers can be placed in order by locating them on the number line and by comparing the signs and the digits in matching place value positions. Negative decimal numbers are smaller than positive decimal numbers.

Example 7

Arrange these decimal numbers in ascending order (from smallest to largest) and show them on a number line.
0.05, −0.35, −1.8, 1.7

Working	Reasoning
−1.8, −0.35, 0.05, 1.7	−0.35 and −1.8 are negative numbers so they are smaller than 1.7 and 0.05. −0.35 is between 0 and −1, but −1.8 is between −1 and −2 so −1.8 is smaller than −0.35. 0.05 is between 0 and 1 but 1.7 is between 1 and 2 so 0.05 is smaller than 1.7.



Decimal numbers can be arranged in order of sizes by comparing the digits in matching place value positions.

Example 8

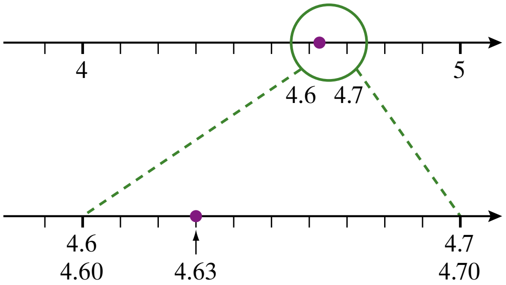
a Make 2.35 __ 2.149 a true statement by filling the gap with < or >.

b Sort these numbers into ascending order, that is, into order from smallest to largest.
5.203, 2.35, 5.32, 2.305, 5.302

Working	Reasoning
a 2.35 > 2.149	First compare the whole numbers. These are the same. Next compare the tenths. The number on the left has 3 tenths, which is bigger than the 1 tenth on the right. The symbol is >
b 5.203, 2.35, 5.32, 2.305, 5.302 In ascending order these are: 2.305, 2.35, 5.203, 5.302, 5.32	First compare the whole numbers. Those with a whole number of 2 are smaller. For these numbers, compare the tenths—the same—then the hundredths. 2.3**0**5 is smaller than 2.3**5** Next repeat the process with the numbers beginning 5. Compare the tenths. 5.**2**03 is smallest of them. For the last two, compare the hundredths. 5.3**0**2 is smaller than 5.3**2**

We often need to round decimal numbers to a given number of decimal places.

In the upper diagram of the number line here, the number shown by the dot is 5 if rounded to the nearest whole number. Correct to one decimal place it is 4.6. Zooming in to two decimal places on the lower diagram the number is 4.63. Each extra decimal place allows us to zoom in to locate a number on the number line with greater accuracy.



Example 9

Write the two 2-decimal place numbers that 4.3281 is between. Underline the number to which it is closest.

Working	**Reasoning**
4.32|81 is between 4.32 and <u>4.33</u>	First put a line after the second decimal place. The smaller of the two numbers has the digits before the line. The second has its last digit one bigger. 4.32 has 2 in the hundredths place. The next 2-digit number will have 3 in the hundredths place. 4.3281 is closer to 4.33 than to 4.32 because the thousandths digit is 8.

Decimal numbers can be rounded to a required number of decimal places by looking at the digit in the next place value.

Example 10

Round

a 8.6174 to the nearest hundredth.

b 7.12958 to 3 decimal places.

Working	**Reasoning**
a 8.61|74 $\approx$ 8.62	First put a line after the hundredths digit. Decide which two 2 decimal place numbers the given number is between. Here they are 8.61 and 8.62 Now look at the thousandths digit, 7. So 8.6174 is closer to 8.62
b 7.12958 $\approx$ 7.130	7.129|58 is between 7.129 and 7.130 The next digit is 5, so 7.12958 is closer to 7.130.

Tech tip

The TI-30XB MultiView calculator can be set to show a desired number of decimal places.

To do this, first press the mode button.

On the 3rd line down is FLOAT. If FLOAT is highlighted the calculator will give as many decimal places as it can fit on the screen.

To change this, arrow ▼ down to the 3rd line then arrow ▶ across until the desired number of decimal places is highlighted. Press enter to select this setting.

To return to the home screen, press clear or 2nd [quit].

exercise 2.1

LINKS TO Example 1

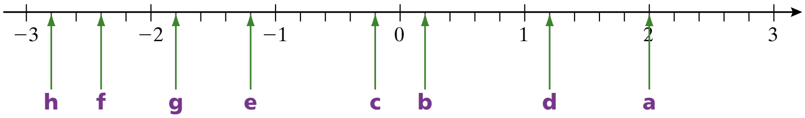
Write the opposite of each of these numbers.

a $\frac{2}{3}$ **b** $-3\frac{1}{4}$ **c** 1.34 **d** −0.02

e $-\frac{7}{8}$ **f** $1\frac{3}{7}$ **g** −2.6 **h** 0.58

LINKS TO Example 2

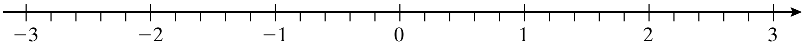
Write each fraction number shown on the number line below.



LINKS TO Example 2

BLM Number lines

Mark each of the following fractions on the number line shown.

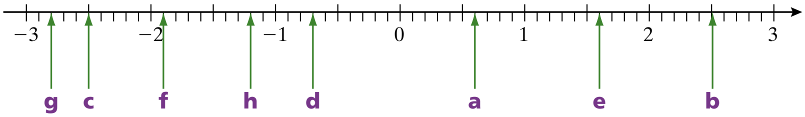


a $1\frac{3}{5}$ **b** $-1\frac{3}{5}$ **c** $2\frac{1}{5}$ **d** $-2\frac{1}{5}$

e $-1\frac{1}{5}$ **f** $2\frac{4}{5}$ **g** $-2\frac{3}{5}$ **h** $-\frac{2}{5}$

LINKS TO Example 3

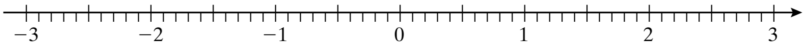
Write each decimal number shown on the number line below.



LINKS TO Example 3

BLM Number lines

Mark each of the following decimals on the number line shown.



a 1.3 **b** -1.3 **c** -2.8 **d** 2.8
e 0.6 **f** -2.2 **g** -1.9 **h** 1.5

LINKS TO Example 4

Write these fractions in ascending order, then show them on a number line.

a $-\frac{4}{5}, \frac{1}{10}, -\frac{3}{10}, -\frac{17}{20}, \frac{1}{5}$ **b** $\frac{5}{6}, -\frac{7}{12}, -\frac{1}{3}, -\frac{1}{4}, \frac{1}{12}$

LINKS TO Example 5

Convert these mixed numbers into improper fractions.

a $2\frac{1}{2}$ **b** $-2\frac{1}{4}$ **c** $2\frac{5}{6}$ **d** $-3\frac{1}{5}$
e $-2\frac{9}{16}$ **f** $-1\frac{4}{11}$ **g** $-2\frac{8}{9}$ **h** $-5\frac{1}{3}$

LINKS TO Example 6

Make true statements using $<$ or $>$.

a $2\frac{1}{8}$ ___ $1\frac{7}{8}$ **b** $-2\frac{1}{8}$ ___ $-1\frac{7}{8}$ **c** $\frac{1}{8}$ ___ $-\frac{3}{8}$ **d** $-2\frac{1}{8}$ ___ $-1\frac{6}{8}$
e $1\frac{6}{7}$ ___ $1\frac{4}{7}$ **f** $-1\frac{6}{7}$ ___ $-1\frac{4}{7}$ **g** $1\frac{4}{7}$ ___ $-1\frac{6}{7}$ **h** $-1\frac{4}{7}$ ___ $1\frac{6}{7}$

Arrange these fractions in descending order.

a $1\frac{3}{11}, -2\frac{1}{11}, -1\frac{8}{11}, 2\frac{3}{11}, -2\frac{10}{11}$ **b** $-1\frac{6}{7}, 1\frac{1}{7}, 2\frac{5}{7}, -2\frac{4}{7}, -2\frac{2}{7}$

Write the next three numbers in each of these patterns.

a $-\frac{1}{5}, 0, +\frac{1}{5}, \ldots$ **b** $-\frac{9}{13}, -\frac{10}{13}, -\frac{11}{13}, \ldots$ **c** $-\frac{4}{7}, -\frac{3}{7}, -\frac{2}{7}, \ldots$
d $1\frac{1}{3}, 1, \frac{2}{3}, \ldots$ **e** $-2\frac{7}{11}, -2\frac{8}{11}, -2\frac{9}{11}, \ldots$ **f** $-1\frac{5}{8}, -1\frac{6}{8}, -1\frac{7}{8}, \ldots$

LINKS TO Example 7

Write these decimal numbers in descending order and show them on a number line.

a $-0.8, 0.2, -1.4, 1.1, -0.5$ **b** $2.3, -3.2, 1.9, -1.9, 0.7$

LINKS TO Example 8a

Make each statement true by filling the gap with either $<$ or $>$.

a 2.8 ___ 2.6 **b** 5.08 ___ 5.13 **c** 6.02 ___ 6.2
d 3.26 ___ 3.2 **e** 0.265 ___ 0.652 **f** 9.55 ___ 7.552
g 6.19 ___ 5.47 **h** 3.462 ___ 3.46 **i** 5.7 ___ 5.777
j 8.012 ___ 8.12 **k** 2.71 ___ 2.8 **l** 1.42 ___ 0.358
m 4.8 ___ 4.73 **n** 9.351 ___ 9.62 **o** 0.33 ___ 0.572

LINKS TO Example 8a

Write the larger of each pair of decimal numbers.

a $-1.7, -0.3$ **b** $-2.1, -2.8$ **c** $+5.3, +5.9$ **d** $+0.4, -0.7$
e $-4.16, -4.6$ **f** $-7.54, -7.45$ **g** $+1.81, -1.18$ **h** $-3.3, -3.33$

LINKS TO Example 8b

Arrange in ascending order:

a 4.71, 4.62, 4.75, 4.69 **b** 6.12, 6.20, 6.02, 6.21
c 1.53, 3.15, 1.35, 3.51 **d** 8.002, 8.202, 8.022, 8.020
e 2.464, 4.26. 2.446, 4.624, 2.64 **f** 9.502, 5.029, 5.29, 9.52, 5.092
g 7.414, 7.114, 7.14, 7.11, 7.41 **h** 3.261, 3.6, 1.263, 3.26, 1.63

LINKS TO Example 8b

● Arrange in descending order:

a 3.4, 3, 4.3, 3.04
b 8, 7.78, 7, 8.7
c 5.16, 5.6, 6.105, 5, 6.5
d 2.711, 2.17, 2.171, 2.7, 2.117
e 4.064, 3.06, 2.446, 4.604, 2.64
f 1.507, 3.078, 2.78, 1.57, 2.087
g 1.939, 1.339, 1.39, 1.33, 1.93
h 6.564, 6.6, 6.546, 6.56, 6.64

● Arrange these temperatures in order from coldest to warmest.

a 1.5°C, −2.7°C, 0.4°C, −2.3°C, 0.8°C, −3.1°C
b −5.6°C, −6.2°C, −5.5°C, −6.4°C, −4.9°C, −4.3°C

● Michael did two Maths tests of the same difficulty. His score on the first was 15 out of 20. On the second test he scored 24 out of 30. Which was his better result, the first or the second? Explain why.

● Amy has done three tests. Her results were 11 out of 15 for English, 7 out of 10 for Japanese and 23 out of 30 for Maths. Arrange these results in ascending order.

LINKS TO Example 9

● For each of these numbers, write the two 2 decimal place numbers that it is between. Then underline which of these two numbers it is closest to.

a 6.472
b 2.616
c 5.5142
d 7.6392
e 21.746
f 0.0063
g 46.387
h 14.46921

LINKS TO Example 10a

● Round each of these numbers to the nearest hundredth.

a 2.763
b 0.418
c 9.325
d 6.462
e 4.0192
f 8.82371
g 3.2954
h 1.99623

LINKS TO Example 10b

● Round each of these numbers to the number of decimal places in the bracket.

a 3.1489 (1)
b 98.9054 (2)
c 7.422446 (4)
d 131.5845 (3)
e 0.0084 (1)
f 3.829 (2)
g 224.973 (1)
h 52.496 (2)
i 37.30274 (2)
j 5.69962 (3)
k 15.60823 (1)
l 28.37291 (4)
m 40.26859 (2)
n 51.99999 (3)
o 16.409 (2)
p 120.0725 (3)

● The temperature in a laboratory that is making a particular sort of crystal must be carefully controlled. State the new temperature for each of the following:

a The temperature in the lab was −2.1°C. It went down 0.6°C.
b The temperature in the lab was −3.5°C. It went up 0.2°C.
c The temperature in the lab was −4.2°C. It went up 0.4°C.
d The temperature in the lab was −2.7°C. It went down 0.6°C.

exercise 2.1 challenge

● Find three fractions between $\frac{3}{5}$ and $\frac{4}{5}$.

● Write four decimal numbers between −0.005 and −0.043.

2.2 Terminating and recurring decimals

Converting fractions into decimals

Any proper or improper fraction $\frac{a}{b}$, can be converted into a decimal by dividing the numerator, a, by the denominator, b.

To convert $\frac{a}{b}$ to a decimal, we calculate $a \div b$.

Terminating decimals

When the denominator of the fraction divides exactly into the numerator, with no remainder, the decimal number is called a **terminating decimal**. For example, when we divide 3 by 4, there is no remainder so the fraction $\frac{3}{4}$ can be expressed as the terminating decimal 0.75.

$$\begin{array}{r} 0.\,7\,5 \\ 4\overline{)\,3.{}^{3}0{}^{2}0} \end{array}$$

Example 11

Convert each of the following to a decimal number.

a $\frac{5}{8}$ **b** $2\frac{3}{4}$ **c** $-1\frac{1}{2}$

Working

a

$$\begin{array}{r} 0.\,6\,2\,5 \\ 8\overline{)\,5.{}^{5}0{}^{2}0{}^{4}0} \end{array}$$

$\frac{5}{8} = 0.625$

Reasoning

To convert $\frac{5}{8}$ to a fraction, divide 5 by 8.

Put zeros as necessary and continue until the division terminates.

$\frac{5}{8}$ is between $\frac{1}{2}$ and 1 so as a decimal it will be between 0.5 and 1.

b

$$\begin{array}{r} 0.\,7\,5 \\ 4\overline{)3.{}^{3}0{}^{2}0} \end{array}$$

$2\frac{3}{4} = 2.75$

The whole number part of the decimal number is 2.

Convert $\frac{3}{4}$ to a decimal number by dividing 3 by 4.

c $-1\frac{1}{2} = -1.5$

Convert $1\frac{1}{2}$ to a decimal number and place the negative sign to the left.

Recurring decimals

Many fractions do not convert to terminating decimals. When we divide 2 by 3, the digit 6 keeps recurring and the division never terminates. The fraction $\frac{2}{3}$ converts to the **recurring decimal** 0.6666666…, where … indicates that the digits keep going. To show the recurring digit we place a dot or bar over the 6.

$$\frac{2}{3} = 0.666666\ldots = 0.\overline{6} \text{ or } 0.\dot{6}.$$

The word recur means that something happens again or repeats.

Example 12

Express each of these fractions

i as a recurring decimal.

ii correct to two decimal places.

a $\frac{4}{9}$

b $\frac{1}{6}$

Working	**Reasoning**
a i $\begin{array}{r} 0.\,4\,4\,4\,4\ldots \\ 9\overline{)\,4.{}^40{}^40{}^40{}^40\ldots} \end{array}$ $\frac{4}{9} = 0.\overline{4}$ or $0.\dot{4}$	The digit 4 keeps recurring. Place a bar or dot over the 4 in 0.4.
ii $\frac{4}{9} = 0.44$ correct to two decimal places.	
b i $\begin{array}{r} 0.1666\ldots \\ 6\overline{)1.{}^10{}^40{}^40{}^4\ldots} \end{array}$ $\frac{1}{6} = 0.1\overline{6}$ or $0.1\dot{6}$	The digit 6 keeps recurring. Put a bar or dot over 6, but not over 1 as this is not recurring.
ii $\frac{1}{6} = 0.17$ correct to two decimal places.	

With some fractions there is more than one recurring digit. We put a bar over all the recurring digits or a dot over the first and last of the set of recurring digits. For example, $\frac{20}{99} = 0.20202020\ldots = 0.\overline{20}$ or $0.\dot{2}\dot{0}$

Example 13

Express each of these fractions

i as a recurring decimal.

ii correct to two decimal places.

a $\frac{3}{11}$ **b** $\frac{7}{22}$ **c** $1\frac{5}{11}$

Working

a

$$\begin{array}{r} 0.\ 2\ 7\ 2\ 7\ldots \\ 11\overline{)\,3.{}^{3}0{}^{8}0{}^{3}0{}^{8}0\ldots} \end{array}$$

i $\frac{3}{11} = 0.\overline{27}$ or $0.\dot{2}\dot{7}$

ii $\frac{3}{11} = 0.27$ correct to two decimal places.

Reasoning

The digits 2 and 7 keep recurring. Put a bar over 27, or put a dot over 2 and over 7.

b

$$\begin{array}{r} 0.\ 3\ 1\ \ 8\ 1\ 8\ldots \\ 22\overline{)\,7.{}^{7}0{}^{4}0{}^{18}0{}^{4}0{}^{18}\ldots} \end{array}$$

i $\frac{7}{22} = 0.3\overline{18}$ or $0.3\dot{1}\dot{8}$

ii $\frac{7}{22} = 0.32$ correct to two decimal places.

The digits 18 keep recurring. Put a line over the two recurring digits, 1 and 8. The 3 does not recur so we do not put the line over it.

c **i**

$$\begin{array}{r} 0.\ 4\ 5\ 4\ 5\ldots \\ 11\overline{)5.{}^{5}0{}^{6}0{}^{5}0{}^{6}0\ldots} \end{array}$$

$1\frac{5}{11} = 1.\overline{45}$ or $1.\dot{4}\dot{5}$

ii $1\frac{5}{11} = 1.45$ correct to two decimal places.

1 is the whole number part of the decimal number.

Convert $\frac{5}{11}$ to a decimal by dividing 5 by 11. Put a bar over the two recurring digits or put a dot over each of the recurring digits.

Sometimes, decimals have a part that does not repeat followed by a repeating part. For example,

$$\frac{3}{22} = 0.1363636\ldots$$

$$= 0.1\overline{36} \text{ or } 0.1\dot{3}\dot{6}$$

Some recurring decimals have a large number of recurring digits. The fraction $\frac{2}{57}$, for example, has a pattern of eighteen recurring digits.

$$\frac{2}{57} = 0.035087719298245614035087719298245614\ldots$$

$$= 0.\overline{035087719298245614} \text{ or } 0.\dot{0}3508771929824561\dot{4}$$

Example 14

Convert the following fractions to decimals.

a $\frac{1}{7}$ **b** $2\frac{3}{13}$

Working	**Reasoning**
a $\begin{array}{l} \quad 0.\,1\,4\,2\,8\,5\,7\,1\,4\,2\,8\,5\,7 \\ 1\ldots \\ 7\overline{)\,1.{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0\ldots} \end{array}$ $\frac{1}{7} = 0.\overline{142857}$ or $0.\dot{1}4285\dot{7}$	There is a repeating pattern of six digits 142857. Place a bar over the six recurring digits or a dot over the first and last.
b $\begin{array}{l} \quad 0.\,2\,3\,0\,7\,6\,9\,2\,\ldots \\ 13\overline{)3.{}^{3}0{}^{4}0{}^{1}00{}^{9}0{}^{12}0{}^{3}0{}^{4}\ldots} \end{array}$ $2\frac{3}{13} = 2.\overline{230769}$ or $2.\dot{2}3076\dot{9}$	2 is the whole number part of the decimal. Convert $\frac{3}{13}$ to a decimal number by dividing 3 by 13. Write 2 in front of 230769 then put a bar over the six recurring digits or put a dot over the first and last of the recurring digits.

We can tell from the denominator of a fraction whether it converts to a terminating decimal or to a recurring decimal:

- If the only prime factors of the denominator are either 2 or 5 or both, then the fraction converts to a terminating decimal. This is because 2 and 5 will always divide exactly into a multiple of 10 as we add zeros in the division of the numerator of the fraction by the denominator. Examples are the denominators 2, 4, 5, 8, 10, 16, 20
- If the denominator has prime factors other than 2 or 5, the decimal will always be recurring. Examples are the denominators 3, 6, 7, 9, 11, 12, 13, 14, 15.

Example 15

Sort these fractions into two groups according to whether they convert to terminating decimals or recurring decimals.

$\frac{4}{7}, \frac{9}{16}, \frac{11}{12}, \frac{19}{25}, \frac{17}{20}, \frac{4}{15}$

Working	**Reasoning**
Terminating: $\frac{9}{16}, \frac{19}{25}, \frac{17}{20}$ Recurring: $\frac{4}{7}, \frac{11}{12}, \frac{4}{15}$	16 has 2 as its only prime factor. 25 has 5 as its only prime factor. 20 has 2 and 5 as its only prime factors. 7 is a prime number. 12 has 3 as a prime factor as well as 2. 15 has 3 as a prime factor as well as 5.

Converting terminating decimals into fractions

When converting a decimal number to a fraction, the digits of the decimal number indicate the numerator of the fraction. The number of decimal places indicates the power of 10 in the denominator.

Example 16

Express the following as fractions.

a 0.35 **b** 2.019

Working	**Reasoning**
a 0.35 $= \frac{35}{100}$	The digits after the decimal point, 35, become the numerator. The smallest place value in 0.35 is hundredths so the denominator is 100. There are 35 hundredths. Check: 0.35 has 2 decimal places and the denominator has two zeros.
$= \frac{35 \div 5}{100 \div 5}$ $= \frac{7}{20}$	Simplify the fraction. 5 is a factor of both numerator and denominator.
b 2.019 $= 2 + 0.019$ $= 2 + \frac{19}{1000}$	Write the whole number, 2. The digits 19 become the numerator. The smallest place value in 2.019 is thousandths so the denominator is 1000. There are 19 thousandths.
$= 2\frac{19}{1000}$	Check: 2.019 has 3 decimal places and the denominator has 3 zeros. The numerator and denominator have no common factors, so this fraction cannot be simplified.

Converting recurring decimals to fractions [challenge]

All recurring decimals can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers.

To convert 0.333… into a fraction, for example, we look at the number of recurring digits. As there is only one recurring digit, we start by multiplying 0.333… by 10.

$10 \times 0.333\ldots = 3.333\ldots$

We then subtract $1 \times 0.333\ldots$ from both sides. On the left side, we obtain $9 \times 0.333\ldots$ while on the right side, we obtain 3.

$$\begin{array}{rl} & 10 \times 0.333\ldots = 3.333\ldots \\ - & \underline{1 \times 0.333\ldots = 0.333\ldots} \\ & 9 \times 0.333\ldots = 3 \end{array}$$

Dividing both sides by 9, we obtain

$$\begin{aligned} 0.333\ldots &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}$$

So the recurring decimal $0.\overline{3}$ can be converted to the fraction $\frac{1}{3}$.

If there are two recurring digits, as in 0.1818…, we multiply the recurring decimal by 100 instead of 10. If there are three recurring digits, we multiply by 1000 and so on.

Example 17

Convert the following decimals to fractions.

a 0.1818… **b** $0.\overline{16}$

Working	**Reasoning**
a $\begin{array}{rl} & 100 \times 0.1818\ldots = 18.1818\ldots \\ - & \underline{1 \times 0.1818\ldots = 0.1818\ldots} \\ & 99 \times 0.1818\ldots = 18 \end{array}$	0.1818… has two recurring digits, so multiply by 100. Write 1 × 0.1818…= 0.1818… underneath. Subtract left sides and subtract right sides.
$\begin{aligned} 0.1818\ldots &= \frac{18}{99} \\ &= \frac{2}{11} \end{aligned}$	Divide both sides by 99. Simplify the fraction.
b $\begin{array}{rl} & 100 \times 0.1616\ldots = 16.1616\ldots \\ - & \underline{1 \times 0.1616\ldots = 0.1616\ldots} \\ & 99 \times 0.1616\ldots = 16 \end{array}$ $0.1616\ldots = \frac{16}{99}$	There are two recurring digits so multiply by 100. Divide both sides by 99. The fraction cannot be simplified.

Converting between fractions and decimals can be done with a scientific calculator.

Tech tip

The TI-30XB MultiView calculator can convert a decimal to a fraction.

To do this, first type in the decimal number.

Then press 2nd table which gives [f◂▸d]. Lastly press enter.

The fraction will be given in simplest form.

If the decimal number is greater than 1, an improper fraction will be given.

To convert it to a mixed number press 2nd ×10ⁿ which gives [$\frac{n}{d}$◂▸U$\frac{n}{d}$].

Then press enter.

The TI-30XB MultiView calculator can also convert a fraction to a decimal.

First enter the fraction (proper or improper) or mixed number.

A fraction such as $\frac{2}{3}$ is entered by typing the numerator 2, then $\frac{n}{d}$ then the denominator 3 then ▶.

A mixed number such as $4\frac{2}{3}$ is entered by typing in the whole number part 4, then 2nd $\frac{n}{d}$ which gives [U$\frac{n}{d}$], then the numerator 2, ▶ then the denominator 3 ▶.

When the fraction or mixed number has been entered, press 2nd table which gives [f◂▸d] (fraction to decimal) then enter.

exercise 2.2

LINKS TO Example 11

● Express each of the fractions below as a decimal.

a $\frac{3}{10}$ **b** $\frac{1}{4}$ **c** $\frac{7}{8}$ **d** $-\frac{3}{5}$ **e** $\frac{1}{2}$

f $\frac{9}{25}$ **g** $\frac{1}{8}$ **h** $-\frac{9}{20}$ **i** $-\frac{5}{8}$ **j** $\frac{11}{50}$

LINKS TO Example 12

● Express each of these fractions

i as a recurring decimal.

ii correct to two decimal places.

a $\frac{2}{9}$ **b** $\frac{8}{9}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$ **e** $\frac{1}{12}$

f $\frac{11}{12}$ **g** $\frac{5}{9}$ **h** $\frac{7}{12}$ **i** $\frac{5}{12}$ **j** $\frac{5}{6}$

LINKS TO Example 13

Express each of these fractions

i as a recurring decimal.

ii correct to two decimal places.

a $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{3}{11}$ **d** $1\frac{4}{11}$

e $1\frac{8}{11}$ **f** $\frac{3}{22}$ **g** $\frac{9}{22}$ **h** $\frac{21}{22}$

LINKS TO Example 14

Convert $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ into decimals. What pattern do you notice in the recurring digits?

Express each of these mixed numbers as decimals. Use a bar over any recurring digits.

a $2\frac{1}{2}$ **b** $7\frac{2}{5}$ **c** $1\frac{2}{3}$ **d** $5\frac{3}{4}$ **e** $3\frac{2}{25}$

f $1\frac{4}{9}$ **g** $2\frac{1}{3}$ **h** $2\frac{13}{50}$ **i** $5\frac{1}{6}$ **j** $6\frac{2}{7}$

LINKS TO Example 15

Sort these fractions into two groups according to whether they convert to terminating decimals or recurring decimals.

$\frac{7}{12}, \frac{5}{9}, \frac{7}{8}, \frac{4}{13}, \frac{9}{20}, \frac{2}{3}, \frac{19}{50}, \frac{12}{17}, \frac{11}{25}, \frac{5}{6}, \frac{1}{4}, \frac{7}{18}, \frac{4}{15}, \frac{3}{10}, \frac{8}{21}, \frac{4}{11}, \frac{6}{7}, \frac{3}{5}, \frac{9}{16}, \frac{31}{64}$

As a decimal $1\frac{9}{11}$ is exactly equal to

A 1.8 **B** $1.\overline{81}$ **C** $1.\overline{8}$ **D** $1.\overline{18}$ **E** 1.9

LINKS TO Example 16a

Express each of the decimal numbers below as a fraction, in simplest form.

a 0.7 **b** 0.1 **c** 0.6 **d** 0.8 **e** 0.5

f 0.04 **g** 0.013 **h** 0.45 **i** 0.34 **j** 0.016

LINKS TO Example 16b

Express each of the decimal numbers below as a fraction, in simplest form.

a 8.3 **b** 4.9 **c** 3.5 **d** 5.2 **e** 2.4

f 7.03 **g** 9.35 **h** 6.022 **i** 1.204 **j** 4.024

Expressed in simplest form, 2.65 is equal to

A $26\frac{5}{10}$ **B** $\frac{65}{100}$ **C** $2\frac{13}{20}$ **D** $\frac{13}{20}$ **E** $2\frac{65}{1000}$

When comparing fractions, the method usually used is to write both fractions as equivalent fractions with a common denominator. Now that we can convert fractions to decimals, it could be quicker to compare fractions by first converting them to decimals. With each of the parts below, time yourself using each method. Then write the larger fraction.

a Which of the fractions $\frac{3}{5}$ and $\frac{2}{3}$ is larger?

b Which of the fractions $\frac{5}{8}$ and $\frac{7}{12}$ is larger?

c Which of the fractions $\frac{2}{9}$ and $\frac{3}{11}$ is larger?

d Which method was faster for you? If it was not always the same method, write down when the other method was faster for you.

exercise 2.2 challenge

LINKS TO Example 17

● Convert each of these recurring decimals to fractions.

a 0.666…
b 0.222…
c 0.555…
d 0.3636…
e 0.8282…
f 0.214214…

● Convert the recurring decimal $0.\overline{12345}$ to a fraction.

2.3 Fraction calculations

Adding and subtracting fractions

Before we add or subtract fractions, we must write them as equivalent fractions with the same denominator.

Example 18

Calculate.

a $\frac{3}{4}+\frac{2}{3}$ **b** $\frac{9}{10}-\frac{2}{5}$ **c** $\frac{1}{2}-\frac{7}{8}+\frac{1}{4}$

Working	**Reasoning**
a $\frac{3}{4}+\frac{2}{3}$	Find the lowest common multiple of the denominators.
$=\frac{3}{4}\left(\times\frac{3}{3}\right)+\frac{2}{3}\left(\times\frac{4}{4}\right)$	The smallest number that both 3 and 4 divide into exactly is 12. This will be the common denominator.
$=\frac{9}{12}+\frac{8}{12}$	Write both fractions as equivalent fractions with the same denominator.
$=\frac{17}{12}$	Add the numerators.
$=1\frac{5}{12}$	Convert the improper fraction to a mixed number.
b $\frac{9}{10}-\frac{2}{5}$	Find the lowest common multiple of the denominators.
$=\frac{9}{10}-\frac{2}{5}\left(\times\frac{2}{2}\right)$	The smallest number that both 10 and 5 divide into exactly is 10. This will be the common denominator.
$=\frac{9}{10}-\frac{4}{10}$	Write both fractions as equivalent fractions with the same denominator.
$=\frac{5}{10}$	Subtract the numerators.
$=\frac{1}{2}$	Write the answer in fraction form. Simplify the fraction.
c $\frac{1}{2}-\frac{7}{8}+\frac{1}{4}$	Express the fractions as equivalent fractions with the same denominator.
$=\frac{4}{8}-\frac{7}{8}+\frac{2}{8}$	Work from left to right.
$=-\frac{3}{8}+\frac{2}{8}$	
$=-\frac{1}{8}$	

When adding mixed numbers we can first convert the mixed numbers to improper fractions. However, it is easier to add the whole number parts and add the fraction parts. Example 19 shows the addition of two mixed numbers using each method.

Example 19

Calculate $2\frac{1}{6} + 1\frac{3}{4}$

Working	**Reasoning**
$2\frac{1}{6} + 1\frac{3}{4}$	Change the mixed numbers into improper fractions.
$= \frac{13}{6} + \frac{7}{4}$	Find the lowest common multiple of the denominators.
$= \frac{13}{6}\left(\times\frac{2}{2}\right) + \frac{7}{4}\left(\times\frac{3}{3}\right)$	The smallest number that both 6 and 4 divide into exactly is 12. This will be the common denominator.
$= \frac{26}{12} + \frac{21}{12}$	Write both fractions as equivalent fractions with the same denominator.
$= \frac{47}{12}$	Add the numerators.
$= 3\frac{11}{12}$	Convert the improper fraction to a mixed number.
Alternative method:	Add the whole numbers and add the fractions.
$2\frac{1}{6} + 1\frac{3}{4}$	Write the fractions as equivalent fractions with common denominator 12.
$= 3 + \frac{1}{6} + \frac{3}{4}$	Add the numerators then combine the whole number part and the fraction part as a mixed number.
$= 3 + \frac{2}{12} + \frac{9}{12}$	
$= 3\frac{11}{12}$	

When we subtract mixed numbers it is usually easier to convert them to improper fractions first to avoid making mistakes if we are subtracting a larger fraction part from a smaller fraction part.

Example 20

Calculate

a $2\frac{1}{4} - 1\frac{7}{10}$

b $1\frac{2}{3} - 2\frac{4}{9} + \frac{1}{6}$

continued

Example 20 continued

Working	Reasoning
a $2\frac{1}{4} - 1\frac{7}{10}$	Change the mixed numbers into improper fractions.
$= \frac{9}{4} - \frac{17}{10}$	Work out the common denominator for these fractions by finding the lowest common multiple of 4 and 10. This is 20.
$= \frac{45}{20} - \frac{34}{20}$	Write both fractions as equivalent fractions with the same denominator.
$= \frac{11}{20}$	Subtract the numerators.
b $1\frac{2}{3} - 2\frac{4}{9} + \frac{1}{6}$	Convert the mixed numbers to improper fractions. Express each fraction as an equivalent fraction with the same denominator. The lowest common multiple of 3, 9 and 6 is 18. Work from left to right.
$= \frac{5}{3} - \frac{22}{9} + \frac{1}{6}$	
$= \frac{30}{18} - \frac{44}{18} + \frac{3}{18}$	
$= -\frac{14}{18} + \frac{3}{18}$	
$= -\frac{11}{18}$	

Multiplying fractions

To multiply two fractions, we multiply the numerators and multiply the denominators. If any of the numbers in the numerators have a common factor with any of the numbers in the denominators, it is easier to cancel first before multiplying.

Example 21

Calculate

a $5 \times \frac{2}{3}$

b $\frac{2}{3}$ of $\frac{5}{6}$

Working	Reasoning
a $5 \times \frac{2}{3}$	Multiply the numerator of the fraction by the whole number, 5.
$= \frac{10}{3}$	Convert the improper fraction to a mixed number.
$= 3\frac{1}{3}$	

continued

Example 21 continued

Working

b $\frac{2}{3}$ of $\frac{5}{6} = \frac{2}{3} \times \frac{5}{6}$

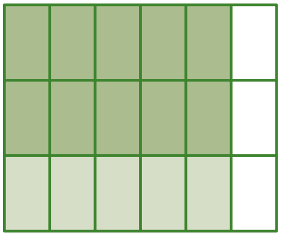
$= \frac{10}{18}$

$= \frac{5}{9}$

Reasoning

Write 'of' as ×.

We can see in the diagram that $\frac{2}{3}$ of $\frac{5}{6}$ is ten eighteenths.



Multiply the numerators and multiply the denominators.

Simplify the fraction.

The same rules for signs apply to multiplying fractions as to multiplying integers. The product of two negative fractions is positive. The product of a negative fraction and a positive fraction is negative.

Example 22

Calculate.

a $\frac{2}{3} \times \frac{4}{7}$

b $\frac{9}{14} \times \frac{21}{24}$

c $\frac{8}{15} \times \left(-\frac{3}{4}\right)$

Working

a $\frac{2}{3} \times \frac{4}{7}$

$= \frac{8}{21}$

b $\frac{9}{14} \times \frac{21}{24}$

$= \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{3}{\cancel{21}}}{\underset{8}{\cancel{24}}}$

$= \frac{9}{16}$

Reasoning

Multiply the numerators and multiply the denominators.

Simplify by cancelling.

3 is a common factor of 9 in the numerator and 24 in the denominator.

7 is a common factor of 21 in the numerator and 14 in the denominator.

Multiply the numbers in the numerator: $3 \times 3 = 9$

Multiply the numbers in the denominator: $2 \times 8 = 16$

continued

Example 22 continued

Working	Reasoning
c $\frac{8}{15} \times \left(-\frac{3}{4}\right)$	Multiplying a positive number by a negative number gives a negative number.
$= -\left(\frac{8}{15} \times \frac{3}{4}\right)$	Put the negative sign at the front. Multiply $\frac{8}{15}$ by $\frac{3}{4}$.
$= -\left(\frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\right)$	
$= -\frac{2}{5}$	

Mixed numbers must be converted to improper fractions before multiplying.

Example 23

Evaluate.

a $1\frac{4}{5} \times 2\frac{1}{3}$ **b** $2\frac{1}{3} \times \left(-1\frac{5}{7}\right)$ **c** $\frac{3}{5} \times \left(-2\frac{1}{4}\right) \times \left(-2\frac{1}{2}\right)$

Working	Reasoning
a $1\frac{4}{5} \times 2\frac{1}{3}$	Convert each mixed number into an improper fraction.
$= \frac{9}{5} \times \frac{7}{3}$	
$= \frac{\overset{3}{\cancel{9}}}{5} \times \frac{7}{\underset{1}{\cancel{3}}}$	Cancel where it is possible to do so. Here, 9 and 3 have a common factor of 3.
$= \frac{21}{5}$	Multiply the numerators and multiply the denominators.
$= 4\frac{1}{5}$	Convert the improper fraction to a mixed number.
b $2\frac{1}{3} \times \left(-1\frac{5}{7}\right)$	Multiplying a positive number by a negative number gives a negative number. Put the negative sign at the front.
$= -\left(\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{7}}}\right)$	Convert mixed numbers to improper fractions. Cancel common factors.
$= -4$	

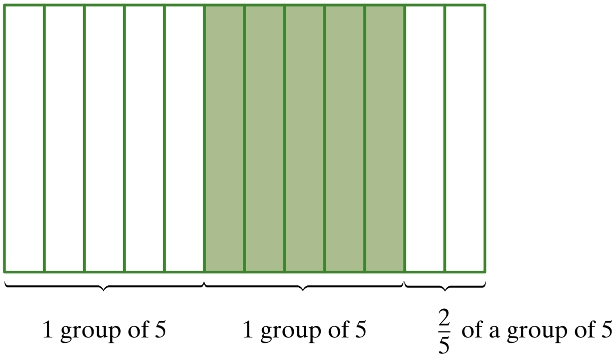
continued

Example 23 continued

Working	Reasoning
c $\frac{3}{5}\times\left(-2\frac{1}{4}\right)\times\left(-2\frac{1}{2}\right)$	Multiplying two negative numbers gives a positive number. Convert mixed numbers to improper fractions.
$=\frac{3}{\underset{1}{\cancel{5}}}\times\frac{9}{4}\times\frac{\overset{1}{\cancel{5}}}{2}$	Cancel common factors.
$=\frac{27}{8}$	Multiply the numerators and multiply the denominators.
$=3\frac{3}{8}$	Convert the improper fraction to a mixed number.

Dividing fractions

When we divide a whole number by another whole number, we can think in terms of how many groups we can make. For example, to find 12 ÷ 5 we can ask ‘how many groups of 5 can we make from 12?’.

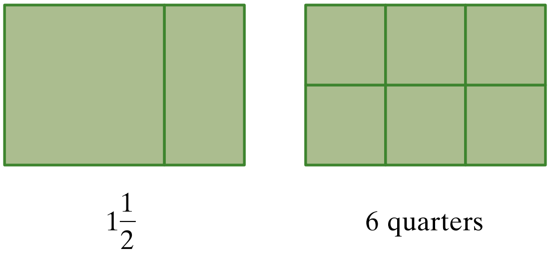


We can make $2\frac{2}{5}$ groups of 5.

Notice that $12 \div 5 = 2\frac{2}{5}$ and $2\frac{2}{5} \times 5 = 12$.

We can apply the same reasoning to the division by a fraction. This time, suppose we are finding $1\frac{1}{2} \div \frac{1}{4}$.

Suppose the square below measures 1 unit by 1 unit, so its area is 1 square unit. With the extra half square the area is $1\frac{1}{2}$ square units. Dividing the shape into quarter units, we see that there are 6 quarters.



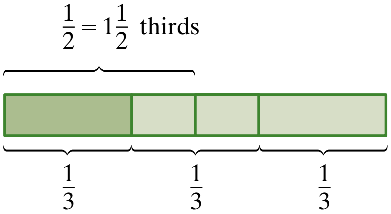
We can write $1\frac{1}{2} \div \frac{1}{4} = 6$. But we also know that $1\frac{1}{2} \times 4 = 6$.

4 is called the reciprocal of $\frac{1}{4}$ so to divide by $\frac{1}{4}$ we multiply by the reciprocal, 4.

The division $\frac{1}{2} \div \frac{1}{3}$ means 'how many thirds in one half?' If the green bar represents the whole, we can see that there are $1\frac{1}{2}$ thirds in one-half. But we know that $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$.

So $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1}$

$\frac{3}{1}$ is the reciprocal of $\frac{1}{3}$



To divide by a fraction, multiply by its reciprocal.

The same rules for signs apply to dividing fractions as to dividing integers. The quotient of two negative fractions is positive. The quotient of a negative fraction and a positive fraction is negative.

Example 24

Evaluate.

a $2 \div \frac{3}{4}$

b $\frac{7}{8} \div \left(-\frac{3}{5}\right)$

continued

Example 24 continued

Working	Reasoning
a $2 \div \frac{3}{4}$	Write 2 as $\frac{2}{1}$.
$= \frac{2}{1} \div \frac{3}{4}$	Dividing by $\frac{3}{4}$ is the same as multiplying by its
$= \frac{2}{1} \times \frac{4}{3}$	reciprocal, $\frac{4}{3}$.
$= \frac{8}{3}$	Multiply the numerators and multiply the denominators.
$= 2\frac{2}{3}$	If the answer is an improper fraction convert it into a mixed number.
b $\frac{7}{8} \div \left(-\frac{3}{5}\right)$	Dividing a positive number by a negative number gives a negative number.
$= -\left(\frac{7}{8} \div \frac{3}{5}\right)$	Dividing by $\frac{3}{5}$ is the same as multiplying by its
$= -\left(\frac{7}{8} \times \frac{5}{3}\right)$	reciprocal, $\frac{5}{3}$.
$= -\frac{35}{24}$	Multiply the numerators and multiply the denominators.
$= -1\frac{11}{24}$	If the answer is an improper fraction convert it into a mixed number.

Mixed numbers must be converted into improper fractions before dividing.

Example 25

Calculate $1\frac{5}{6} \div 1\frac{2}{3}$

Working	Reasoning
$1\frac{5}{6} \div 1\frac{2}{3}$	Convert mixed numbers into improper fractions.
$= \frac{11}{6} \div \frac{5}{3}$	Dividing by $\frac{5}{3}$ is the same as multiplying by its reciprocal, $\frac{3}{5}$.
$= \frac{11}{6} \times \frac{3}{5}$	If possible, cancel common factors. That is, divide a denominator and a numerator by a common factor, where possible. Here, 3 and 6 have a common factor of 3.
$= \frac{11}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{5}$	
$= \frac{11}{10}$	Multiply the numerators and multiply the denominators.
$= 1\frac{1}{10}$	If the answer is an improper fraction convert it into a mixed number.

Order of operations

The same order of operations applies to fraction calculations as to integer calculations. When ‘of’ occurs in a calculation, it is calculated as a multiplication, but it must be done before other multiplications and divisions. It is helpful to put brackets around the calculation involving ‘of’.

The order of operations is

1 **B**rackets, **P**owers and **R**oots
2 **O**f
3 **D**ivisions and
4 **M**ultiplications } are worked from left to right in the order that they occur.
5 **A**dditions
6 **S**ubtractions } are worked from left to right in the order that they occur.

Example 26

Calculate $\frac{4}{5} \div \frac{2}{3}$ of $3\frac{1}{2}$

Working	**Reasoning**
$\frac{4}{5} \div \frac{2}{3} \text{ of } 3\frac{1}{2}$	Put brackets as a reminder to calculate ‘of’ first.
$= \frac{4}{5} \div \left(\frac{2}{3} \text{ of } 3\frac{1}{2}\right)$	Convert $3\frac{1}{2}$ to an improper fraction.
$= \frac{4}{5} \div \left(\frac{\overset{1}{\cancel{2}}}{3} \times \frac{7}{\underset{1}{\cancel{2}}}\right)$	Calculate the multiplication in the brackets.
$= \frac{4}{5} \div \frac{7}{3}$	
$= \frac{4}{5} \times \frac{3}{7}$	Multiply by the reciprocal of $\frac{3}{7}$.
$= \frac{12}{35}$	Calculate the multiplication.

Solving problems involving fractions

The first step in solving a problem involving fractions is to decide which operations are involved.

Addition	Finding a total made up of several parts.
Subtraction	Finding the difference between two quantities or how much is left over.
‘Of’ and multiplying	Finding a fraction of a quantity or finding the total when there are several lots of a quantity.
Division	Sharing a quantity.

Example 27

Calculate each of the following.

a Jess mixed $2\frac{1}{2}$ cups of water with $\frac{3}{4}$ cup of cordial. How many cups of drink did she have altogether?

b How much cordial would Jess need to add to make up the drink to exactly 4 cups?

Working	Reasoning
a $2\frac{1}{2}+\frac{3}{4}$ $= 2+\frac{1}{2}+\frac{3}{4}$ $= 2+\frac{2}{4}+\frac{3}{4}$ $= 2+\frac{5}{4}$ $= 2+1\frac{1}{4}$ $= 3\frac{1}{4}$ Jess had $3\frac{1}{4}$ cups of drink altogether.	Two quantities are combined so the problem is an addition problem.
b $4-3\frac{1}{4}=\frac{3}{4}$ Jess needs to add another $\frac{3}{4}$ of a cup of cordial.	The question is asking for the difference between 4 cups and the amount of drink that Jess has. The problem is a subtraction problem.

Example 28

Bill is following a cupcake recipe that makes 12 cupcakes. He wants to make more cupcakes than the mixture is designed for so he is making $2\frac{1}{2}$ times the mixture.

a If the recipe was for 12 cupcakes, how many cupcakes will Bill be able to make?

b The recipe includes $1\frac{1}{3}$ cups of flour. How many cups of flour does he need?

c If Bill put chocolate icing on $\frac{3}{5}$ of the cupcakes, how many cupcakes had chocolate icing?

continued

Example 28 continued

Working

a $2\frac{1}{2} \times 12$

$= \frac{5}{\cancel{2}_{1}} \times \frac{\cancel{12}^{6}}{1}$

$= 30$

Bill will be able to make 30 cupcakes.

b $1\frac{1}{3} \times 2\frac{1}{2}$

$= \frac{\cancel{4}^{2}}{3} \times \frac{5}{\cancel{2}_{1}}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

Bill needs $3\frac{1}{3}$ cups of flour.

c $\frac{3}{5}$ of 30

$= \frac{3}{\cancel{5}_{1}} \times \frac{\cancel{30}^{6}}{1}$

$= 18$

18 cupcakes had chocolate icing.

Reasoning

Bill is making $2\frac{1}{2}$ times the recipe mixture, so he will get $2\frac{1}{2}$ times 12 cupcakes.

The problem is asking how many cups is $2\frac{1}{2}$ times $1\frac{1}{3}$ cups. The problem is a multiplication problem.

There are 30 cakes.

$\frac{3}{5}$ of them had chocolate icing.

Example 29

Nell takes $\frac{3}{4}$ hours to deliver a box of leaflets. Assuming the same rate, how many boxes of leaflets could Nell deliver in 5 hours?

Working

$5 \div \frac{3}{4}$

$= \frac{5}{1} \times \frac{4}{3}$

$= \frac{20}{3}$

$= 6\frac{2}{3}$

Nell could deliver $6\frac{2}{3}$ boxes of leaflets.

Reasoning

The question is asking 'how many three-quarters in 5?' The problem is a division problem.

Tech tip

To calculate $\frac{9}{10} - \frac{2}{5}$ (example 18 part b) using the TI-30XB MultiView calculator, type in the following:

9 $\frac{n}{d}$ 1 0 ▶ − 2 $\frac{n}{d}$ 5 **enter**

To calculate $2\frac{1}{6} + 1\frac{3}{4}$ (example 19) using the TI-30XB MultiView calculator, type in the following:

2 2nd $\frac{n}{d}$ 1 ▼ 6 ▶ + 1 2nd $\frac{n}{d}$ 3 ▼ 4 **enter**

To change the answer to a mixed number press 2nd [$\frac{n}{d}$◀ ▶U$\frac{n}{d}$] **enter**

Fraction multiplications and divisions like those in the following exercise can be done with a scientific calculator.

Tech tip

To calculate $\frac{2}{3}$ of $\frac{5}{6}$ (example 21) using the TI-30XB MultiView calculator, type in the following:

2 $\frac{n}{d}$ 3 ▶ × 5 $\frac{n}{d}$ 6 **enter**

To work $1\frac{5}{6} \div 1\frac{2}{3}$ (example 25) using the TI-30XB MultiView calculator, type in the following:

1 2nd $\frac{n}{d}$ 5 ▼ 6 ▶ ÷ 1 2nd $\frac{n}{d}$ 2 ▼ 3 **enter**

To change the answer to a mixed number press 2nd [$\frac{n}{d}$◀ ▶U$\frac{n}{d}$] **enter**

exercise 2.3

▶ LINKS TO Example 18

● Evaluate each of the following and simplify, where possible.

a $\frac{2}{7} + \frac{3}{7}$
b $\frac{7}{9} - \frac{5}{9}$
c $\frac{2}{9} + \frac{1}{9}$
d $\frac{7}{10} - \frac{3}{10}$
e $\frac{5}{6} - \frac{3}{4}$
f $\frac{7}{15} + \frac{3}{5}$
g $\frac{1}{2} - \frac{3}{8}$
h $\frac{2}{3} + \frac{1}{7}$
i $\frac{2}{9} + \frac{1}{3}$
j $\frac{3}{8} + \frac{5}{12}$
k $\frac{4}{9} + \frac{5}{6}$
l $\frac{7}{10} + \frac{8}{15}$
m $\frac{5}{7} - \frac{2}{5}$
n $\frac{1}{6} + \frac{8}{9}$
o $\frac{9}{10} - \frac{3}{4}$
p $\frac{3}{8} + \frac{5}{16}$

LINKS TO Example 19

Evaluate each of the following and simplify, where possible.

a $1\frac{2}{7} + 2\frac{3}{7}$ **b** $1\frac{8}{11} + 1\frac{2}{11}$ **c** $1\frac{4}{9} + 1\frac{3}{9}$ **d** $3\frac{1}{3} + 2\frac{1}{3}$

e $\frac{6}{7} + 1\frac{3}{7}$ **f** $2\frac{2}{3} + 1\frac{2}{3}$ **g** $1\frac{3}{5} + 1\frac{4}{5}$ **h** $2\frac{4}{9} + \frac{7}{9}$

i $1\frac{1}{2} + 2\frac{1}{3}$ **j** $1\frac{1}{3} + 1\frac{5}{9}$ **k** $1\frac{3}{4} + 1\frac{1}{6}$ **l** $2\frac{1}{2} + 1\frac{7}{10}$

m $1\frac{3}{8} + 1\frac{1}{6}$ **n** $2\frac{1}{2} + 1\frac{3}{4}$ **o** $1\frac{2}{3} + 1\frac{3}{7}$ **p** $2\frac{1}{4} + 1\frac{3}{10}$

LINKS TO Example 20

Evaluate each of the following and simplify, where possible.

a $3\frac{4}{5} - 1\frac{3}{5}$ **b** $2\frac{6}{7} - 1\frac{2}{7}$ **c** $3\frac{2}{3} - 2\frac{1}{3}$ **d** $2\frac{4}{9} - 1\frac{2}{9}$

e $2\frac{1}{3} - 1\frac{2}{3}$ **f** $3\frac{2}{5} - 1\frac{4}{5}$ **g** $3\frac{1}{9} - 1\frac{5}{9}$ **h** $2\frac{4}{7} - 1\frac{5}{7}$

i $2\frac{1}{2} - 1\frac{1}{4}$ **j** $3\frac{1}{3} - 1\frac{5}{6}$ **k** $2\frac{3}{4} - 1\frac{2}{3}$ **l** $3\frac{2}{3} - 1\frac{1}{2}$

m $1\frac{7}{10} - 1\frac{1}{4}$ **n** $3\frac{3}{4} - 2\frac{1}{8}$ **o** $1\frac{5}{6} - 1\frac{1}{9}$ **p** $3\frac{1}{3} - 1\frac{4}{5}$

LINKS TO Example 21

Evaluate each of the following.

a $3 \times \frac{2}{7}$ **b** $4 \times \frac{2}{5}$ **c** $5 \times \frac{2}{3}$ **d** $\frac{7}{8} \times 120$

e $\frac{9}{16} \times 40$ **f** $\frac{1}{3}$ of 6 **g** $\frac{2}{5}$ of 10 **h** $\frac{3}{4}$ of 20

i $\frac{1}{3}$ of $\frac{2}{7}$ **j** $\frac{1}{4}$ of $\frac{3}{5}$ **k** $\frac{2}{5}$ of $\frac{4}{5}$ **l** $\frac{3}{4}$ of $2\frac{2}{5}$

LINKS TO Example 22a

Evaluate each of the following.

a $\frac{2}{7} \times \frac{1}{3}$ **b** $\frac{3}{5} \times \frac{1}{4}$ **c** $\frac{1}{2} \times \frac{1}{5}$ **d** $\frac{1}{5} \times \frac{2}{3}$

e $\frac{4}{5} \times \frac{2}{3}$ **f** $\frac{3}{7} \times \frac{4}{5}$ **g** $\frac{3}{4} \times \frac{5}{7}$ **h** $\frac{2}{3} \times \frac{5}{11}$

i $\frac{5}{6} \times \frac{1}{4}$ **j** $\frac{4}{9} \times \frac{2}{5}$ **k** $\frac{3}{7} \times \frac{2}{7}$ **l** $\frac{2}{3} \times \frac{4}{7}$

LINKS TO Example 22b

Evaluate each of the following.

a $\frac{2}{5} \times \frac{3}{4}$ **b** $\frac{7}{10} \times \frac{5}{8}$ **c** $\frac{3}{4} \times \left(-\frac{5}{6}\right)$ **d** $\frac{4}{21} \times \frac{7}{9}$

e $\frac{6}{7} \times \left(-\frac{2}{9}\right)$ **f** $\frac{3}{25} \times \frac{10}{11}$ **g** $\frac{8}{9} \times \frac{7}{20}$ **h** $\frac{2}{21} \times \frac{14}{15}$

i $\frac{2}{9} \times \frac{3}{4}$ **j** $\frac{10}{21} \times \frac{7}{15}$ **k** $\frac{2}{25} \times \frac{15}{16}$ **l** $\frac{21}{25} \times \frac{10}{28}$

LINKS TO Example 23

Carry out these multiplications of mixed numbers.

a $1\frac{2}{3} \times 1\frac{1}{2}$ **b** $1\frac{1}{2} \times 1\frac{5}{6}$ **c** $1\frac{3}{7} \times 1\frac{2}{5}$

d $1\frac{3}{5} \times 1\frac{1}{8}$ **e** $1\frac{1}{3} \times \frac{15}{16} \times \frac{4}{5}$ **f** $\frac{5}{6} \times 1\frac{7}{8} \times \frac{2}{5}$

g $-2\frac{3}{4} \times 2\frac{2}{3}$ **h** $1\frac{1}{3} \times \left(-1\frac{2}{7}\right)$ **i** $-2\frac{2}{3} \times \left(-1\frac{1}{4}\right)$

j $1\frac{4}{9} \times \left(-1\frac{2}{7}\right)$ **k** $\frac{8}{11} \times \left(-4\frac{2}{5}\right) \times \frac{3}{4}$ **l** $-1\frac{1}{4} \times 6 \times \left(-\frac{14}{15}\right)$

LINKS TO Example 24

● Evaluate each of the following

a $4 \div \frac{1}{2}$ **b** $3 \div \frac{4}{5}$ **c** $6 \div \frac{3}{5}$ **d** $6 \div \frac{4}{5}$

e $\frac{3}{5} \div \frac{1}{3}$ **f** $\frac{7}{15} \div \frac{3}{5}$ **g** $\frac{14}{15} \div \left(-\frac{7}{10}\right)$ **h** $-\frac{5}{6} \div \frac{3}{4}$

i $-\frac{7}{12} \div \left(-\frac{3}{8}\right)$ **j** $\frac{2}{9} \div \left(-\frac{1}{6}\right)$ **k** $-\frac{21}{22} \div \left(-\frac{4}{11}\right)$ **l** $-\frac{4}{7} \div \frac{3}{4}$

LINKS TO Example 25

● Carry out these divisions of mixed numbers.

a $1\frac{1}{4} \div 1\frac{2}{3}$ **b** $2\frac{1}{3} \div 1\frac{3}{4}$ **c** $1\frac{3}{4} \div 1\frac{1}{8}$ **d** $2\frac{1}{5} \div 1\frac{3}{5}$

e $1\frac{5}{9} \div 1\frac{1}{6}$ **f** $5\frac{1}{2} \div 3\frac{2}{3}$ **g** $2\frac{1}{4} \div \left(-1\frac{7}{8}\right)$ **h** $-3\frac{1}{9} \div \left(-1\frac{1}{6}\right)$

i $-4\frac{1}{2} \div 1\frac{3}{4}$ **j** $4\frac{1}{2} \div \left(-3\frac{3}{8}\right)$ **k** $-6\frac{2}{5} \div \left(-4\frac{4}{15}\right)$ **l** $-2\frac{14}{15} \div 1\frac{1}{10}$

● Calculate each of the following.

a $\frac{3}{4} \div \frac{1}{2}$ of $2\frac{1}{4}$ **b** $1\frac{3}{4} + \frac{2}{3} \times 1\frac{1}{5}$

c $\frac{4}{5} + \frac{3}{4}$ of $2\frac{1}{2}$ **d** $\frac{5}{8}$ of $2\frac{2}{3} + 1\frac{1}{4}$

e $1\frac{3}{4} \div 1\frac{1}{3} + \frac{1}{3}$ of $4\frac{1}{2}$ **f** $2\frac{3}{4} - \frac{3}{4}$ of $5\frac{3}{5}$

g $2\frac{3}{4} - 1\frac{1}{3} \div 1\frac{7}{9}$ **h** $3\frac{3}{4} \times \frac{2}{3}$ of $\left(2\frac{1}{3} + 1\frac{1}{2}\right)$

LINKS TO Example 27

● Saide worked for $\frac{3}{4}$ of an hour on her Science assignment and then spent $\frac{1}{3}$ of an hour finishing a story for English. How many hours did these tasks take her?

● Jessica mowed $\frac{2}{3}$ of the lawn then had a drink. She mowed $\frac{1}{4}$ of the lawn then called a friend. How much of the lawn had she mown?

● On Monday the crew paved $\frac{3}{8}$ of the walkway. On Tuesday they paved $\frac{5}{12}$ of the walkway. They finished it on Wednesday. What fraction of the walkway did they pave on Wednesday?

● The nursery had $2\frac{2}{3}$ trays of blue pansies and $1\frac{1}{4}$ trays of yellow pansies. How many trays of pansies did they have altogether?

- To paint the living room it took $2\frac{1}{2}$ hours to wash the walls, $\frac{2}{3}$ hour to fill the cracks, and the rest of the time to put on the paint. If the whole job took $5\frac{1}{4}$ hours, how long did the actual painting take?
- To make 10 L of a party drink, $3\frac{5}{6}$ L of orange juice were added to $2\frac{1}{4}$ L of dry ginger ale and then water was added. How many litres of water were needed?
- Tim jogged $3\frac{1}{3}$ times around the oval, while Tara jogged $1\frac{5}{6}$ times around. How many more times around the oval did Tim jog?
- LINKS TO **Example 28a** Lisa notices that she is taking $1\frac{1}{4}$ minutes per question in her maths homework. If she has 18 questions to do, how long will it take her?
- LINKS TO **Example 28b** Gin-Li can walk $4\frac{1}{5}$ kilometres per hour. If she walks for $3\frac{1}{3}$ hours at this speed how far will she travel altogether?
- LINKS TO **Example 28c** Counting the novelty events, $\frac{3}{5}$ of the students at a particular school swam in the swimming carnival. If the school had 350 students, how many swam?
- LINKS TO **Example 29** A serve of rice to go with a stir-fry is about $\frac{1}{7}$ kg. From a 2 kg packet of rice, how many serves of rice can be obtained?
- To calculate the width of a rectangle, the area is divided by the length. What is the width of a rectangular room that has a length of $3\frac{2}{3}$ m and an area of $8\frac{1}{4}$ m^2?

exercise 2.3 challenge

- There are $7\frac{1}{5}$ litres of petrol left in the tank of a car. If the car uses $\frac{4}{5}$ litre of petrol to travel 10 km, how many kilometres can it travel with the petrol in the tank?
- Kim made 36 cupcakes. She sold $\frac{5}{9}$ of them, and gave $\frac{3}{4}$ of those left to her family. How many were left at the end?

●

a Carry out the following multiplications.

i $1\frac{1}{2} \times 1\frac{1}{3}$ **ii** $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4}$ **iii** $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5}$

b Now predict the result for $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5} \times 1\frac{1}{6} \times 1\frac{1}{7} \times 1\frac{1}{8}$.

c Check it by multiplying.

d Explain what is happening in these patterns.

e For the general product $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \ldots \times 1\frac{1}{n}$, what would be the result?

2.4 Review of decimal calculations

Addition and subtraction

When adding or subtracting decimal numbers, the decimal points and the matching place values should be lined up.

Example 30

Evaluate

a 18.92 + 5.471 + 6.9 + 21.4

b 35.9 − 6.15

c 13.6 − 18.9

Working	**Reasoning**
a $\begin{array}{r} 18.92 \\ 5.471 \\ 6.9 \\ +\ 21.4 \\ \hline 52.691 \\ \hline \end{array}$	Write the numbers under each other, lining up the digits in place value columns and lining up the decimal points. Add the digits starting from the right. Line up the decimal point in the answer. Check the answer by estimating. 20 + 5 + 7 + 20 = 52, so OK.
b $\begin{array}{r} {}^{2}\cancel{3}\,{}^{1}5.{}^{8}\cancel{9}\,{}^{1}0 \\ -\ 6.15 \\ \hline 29.75 \\ \hline \end{array}$	Write the numbers under each other, lining up the digits in place value columns and lining up the decimal points. Subtract working from right to left. Line up the decimal point in the answer. Check the answer by estimating 36 − 6 = 30, so OK.
c $\begin{array}{r} 18.9 \\ -\ 13.6 \\ \hline 5.3 \\ \hline \end{array}$ 13.6 − 18.9 = −5.3	When subtracting a larger number from a smaller number, the result will be negative. It is easier to do the subtraction the other way around, and then write the answer to include the negative sign.

Multiplication

Multiplying a decimal number by 10 simply shifts each digit up to the next higher place value column, multiplying by 100 shifts each digit up two place value columns, and so on.

Multiplying and dividing by powers of 10

Example 31

Find without using a calculator:

a 21.45×3 **b** 35.6×100

Working	**Reasoning**
a $21.45 \times 3 = 64.35$	
b 35.6×100 $= 3560$	Each digit must move up two place value columns. This is equivalent to moving the decimal point two places to the right.

Example 32

Find without using a calculator:

a 2.6×0.04 **b** 0.0048×2000 **c** 0.007×0.12

Working	**Reasoning**
a 2.6×0.04 $= \frac{26}{10} \times \frac{4}{100}$ $= \frac{104}{1000}$ $= 0.104$	Notice that the number of zeros in the denominator of $\frac{104}{1000}$ is the sum of the number of zeros in the denominators of $\frac{26}{10}$ and $\frac{4}{100}$. This also means that the number of decimal places in 0.104 is the sum of the number of decimal places in 2.6 and 0.04.
b 0.0048×2000 $= 0.0048 \times 2 \times 1000$ $= 0.0096 \times 1000$ $= 9.6$	Multiplying by 2000 can be done in two steps. Multiply 0.0048 by 2 then by 1000. To multiply by 1000, each digit must move up three place value columns. This is equivalent to moving the decimal point three places to the right.
c 0.007×0.12 $= \frac{7}{1000} \times \frac{12}{100}$ $= \frac{84}{100\,000}$ $= 0.00084$	Notice that the number of zeros in 100 000 is the sum of the number of zeros in the denominators of $\frac{7}{1000}$ and $\frac{12}{100}$. This also means that the number of decimal places in 0.00084 is the sum of the number of decimal places in 0.007 and 0.12. $0.\underbrace{007}_{3\text{ d.p.}} \times 0.\underbrace{12}_{2\text{ d.p.}} = 0.\underbrace{00084}_{5\text{ d.p.}}$

Division

Dividing a decimal number by 10 shifts each digit down to the next lower place value column, dividing by 100 shifts each digit down two place value columns, and so on.

Multiplying and dividing by powers of 10

Example 33

Calculate

a $478.32 \div 100$

b $0.075 \div 1000$

Working	**Reasoning**
a $478.32 \div 100$ $= 4.7832$	Each digit must move down two place value columns. This is equivalent to moving the decimal point two places to the left.
b $0.075 \div 1000$ $= 0.000075$	Each digit must move down three place value columns. This is equivalent to moving the decimal point three places to the left. Zeros are placed in the empty place value columns.

Example 34

Evaluate.

a $7.13 \div 4$

b $2.354 \div 9$ correct to 2 decimal places

Working	**Reasoning**
a $\begin{array}{r} 1.\,7\,8\,2\,5 \\ 4\overline{)7.{}^{3}1{}^{3}3{}^{1}0{}^{2}0} \end{array}$	Divide as usual. Continue adding zeros and dividing until there is no remainder or until a repeating pattern appears. Here there is no remainder. Check: $7 \div 4 \approx 2$, so OK.
b $\begin{array}{r} 0.\,2\,6\,1\ldots \\ 9\overline{)\,2.{}^{2}3{}^{5}5{}^{1}4} \end{array}$ $2.354 \div 9$ $= 0.26$ correct to 2 decimal places	Divide as usual. Continue putting a zero in the next place value until three decimal places (one more than needed) have been calculated. Round to two decimal places. Check: $2 \div 9 \approx 2 \div 10 \approx 0.2$, so OK.

Example 35

Evaluate.

$3.51 \div 0.08$

continued

Example 35 continued

Working

3.51 ÷ 0.08
= 351 ÷ 8

$$\begin{array}{r} 0\,4\,3.\,8\,7\,5 \\ 8\overline{)3^{3}5^{3}1.^{7}0^{6}0^{4}0} \end{array}$$

So 3.51 ÷ 0.08 = 43.875

Reasoning

Unless we are using a calculator, we rewrite the division so that the divisor is a whole number.
Here 0.08 × 100 = 8 which is a whole number.
Both parts of the division must be multiplied by 100 so that the result of the division remains the same.
(3.51 × 100) ÷ (0.08 × 100)
Do the division as usual.
Check: 320 ÷ 8 = 40, so OK.

exercise 2.4

LINKS TO Example 30a

Carry out the following additions by first lining up the digits in their columns

a 4.153 + 11.62 + 8 **b** 21.35 + 18.477 + 9.6
c 7 + 8.53 + 11.6 **d** 5.087 + 6 + 3.96
e 19.36 + 22.305 + 17.6 **f** 9.28 + 26.42 + 31.9
g 146.5 + 98.26 + 207 + 77.73 **h** 52.43 + 19.7 + 34.06 + 8.49
i 20.314 + 9.56 + 17.8 **j** 36.2 + 18.28 + 9.7 + 24.16

LINKS TO Example 30b

Carry out the following subtractions, by setting out in place value columns.

a 6.785 − 3.724 **b** 12.73 − 9.25 **c** 25.78 − 13.662 **d** 48.61 − 36.193
e 4.8 − 3.614 **f** 55.3 − 31.704 **g** 152.8 − 34.37 **h** 307.43 − 142.7
i 453.1 − 80.246 **j** 55.8 − 28.931 **k** 18.3 − 23.1 **l** 12.4 − 15.8

LINKS TO Example 31

Evaluate each of the following without using a calculator.

a 16.6 × 5 **b** 9.72 × (−8) **c** 54.6 × 7 **d** − 19.2 × 9
e 2.45 × 1000 **f** 9.6 × 5000 **g** 1.84 × 40 **h** 57.8 × 300

LINKS TO Example 32

Evaluate each of the following without using a calculator.

a 4.6 × 0.3 **b** 5.7 × 0.02 **c** 8.3 × 0.7 **d** 7.3 × 0.05
e 4.61 × 0.03 **f** 40.8 × 0.9 **g** 15.6 × 0.4 **h** 142.3 × 0.07
i 0.04 × 60 **j** 0.0075 × 300 **k** 0.0081 × 8000 **l** 0.0065 × 400
m 0.2 × 0.4 **n** 0.03 × 0.5 **o** 0.01 × 0.08 **p** −0.8 × 0.9

LINKS TO Example 33

Find each of the following without using a calculator.

a 59.3 ÷ 1000 **b** 1.86 ÷ 100 **c** 349.5 ÷ 100 **d** 2.36 ÷ 1000
e 0.0048 ÷ 100 **f** 38.9 ÷ 10000 **g** 0.01472 ÷ 100 **h** 2.006 ÷ 10

LINKS TO Example 34

Calculate the following, rounding to three decimal places where appropriate.

a 45.1 ÷ 2 **b** 27.3 ÷ 5 **c** 3.72 ÷ 8 **d** 34.5 ÷ 6
e 53.26 ÷ 4 **f** 1.017 ÷ 6 **g** 403.4 ÷ 5 **h** 0.243 ÷ 8
i 18.2 ÷ 3 **j** 25.1 ÷ 9 **k** 5.14 ÷ 6 **l** 26.3 ÷ 3
m 46.51 ÷ 11 **n** 6.26 ÷ 6 **o** 73.9 ÷ 12 **p** 3.71 ÷ 11

LINKS TO Example 35

Evaluate each of the following by first rewriting so that the divisor is a whole number.

a 5.01 ÷ 0.3
b 7.86 ÷ 0.6
c 0.1428 ÷ 0.007
d 0.612 ÷ 0.09
e 1.308 ÷ 0.04
f 3.74 ÷ 1.1
g 0.9348 ÷ 0.12
h 3.543 ÷ 0.005
i 19.2 ÷ 0.00012

Each day the amount of food eaten by a mouse is about 0.6 times its own mass. How much would be eaten by a mouse of mass 18.4 g?

To find the area of a rectangle, multiply its length by its width. What is the area of a rectangle that is 5.4 m long and 3.6 m wide?

Sally wants to split the bill for a meal equally between herself and her three friends. If the bill is $73.60, how much should each pay?

Hannah buys 4.3 kilos of tomatoes at a cost of $2.45 per kilo, for her mum to make tomato sauce. How much would the tomatoes cost?

William puts 36.3 L of petrol in his car to fill the petrol tank. Petrol costs 134 cents per litre. How much will this cost?

A motorbike can go 17.4 km on 1 L of petrol. How far can it go on a tank that holds 23.5 L?

Guy and his luggage together weigh 82.6 kg. Guy knows that his weight is 64.3 kg. How much does his luggage weigh?

In a 4 × 100 m relay, the times of the four runners were 11.82 s, 14.26 s, 13.71 s and 12.05 s. What was the total time run by the team?

A movie runs for 93.82 minutes. If 26.49 minutes has already been watched, how long will the rest take?

If a person takes 16 breaths each minute, and breathes in 0.714 L of air with each breath, how many litres of air are breathed in each minute? How many litres is this each hour?

exercise 2.4 challenge

When 6.37 is multiplied by 4.39 the answer has four decimal places. Suppose we want an answer correct to two decimal places. This can be done in two ways.

a Round each of the numbers to one decimal place before multiplying them. Then multiply the rounded numbers. The answer will have two decimal places.
b Multiply 6.37 by 4.39. Then round the result to two decimal places.
c The answers are close to each other, but they are different. Which is closer to the actual, unrounded answer? Why?

2.5 Squares of fractions and decimals

Squaring a fraction or decimal number means multiplying it by itself. Just as for integers, squaring a negative fraction or decimal number gives a positive number.

Example 36

Evaluate.

a $\left(\frac{2}{9}\right)^2$ **b** $\left(-1\frac{3}{4}\right)^2$ **c** $-\left(\frac{2}{3}\right)^2$

Working	**Reasoning**
a $\left(\frac{2}{9}\right)^2 = \frac{2}{9} \times \frac{2}{9}$	'Squared' means 'multiplied by itself'.
$= \frac{4}{81}$	Multiply the numerators, then multiply the denominators.
b $\left(-1\frac{3}{4}\right)^2 = -1\frac{3}{4} \times -1\frac{3}{4}$	'Squared' means 'multiplied by itself'.
$= -\frac{7}{4} \times -\frac{7}{4}$	Make each mixed number into an improper fraction.
$= \frac{49}{16}$	Two negative numbers multiply to give a positive number. Multiply the numerators, then the denominators.
$= 3\frac{1}{16}$	Convert the improper fraction to a mixed number.
c $-\left(\frac{2}{3}\right)^2$	The negative sign is not part of the squaring process.
$= -\frac{4}{9}$	

Example 37

Find the value of

a 0.7^2 **b** $(-0.3)^2$ **c** -0.8^2

Working	**Reasoning**
a 0.7^2	$0.7 \times 0.7 = \frac{7}{10} \times \frac{7}{10}$
$= 0.7 \times 0.7$	$= \frac{49}{100}$
$= 0.49$	$= 0.49$

continued

Example 37 continued

Working	**Reasoning**
b $(-0.3)^2$ $= -0.3 \times -0.3$ $= 0.09$	Multiplying a negative number by a negative number gives a positive number. Writing the decimal number as a fraction allows us to see where the decimal point should be. $-0.3 \times -0.3 = -\frac{3}{10} \times -\frac{3}{10}$ $= \frac{9}{100}$ $= 0.09$
c -0.8^2 $= -0.64$	The negative sign is not part of the squaring process.

Tech tip

To find $\left(-1\frac{3}{4}\right)^2$ (example 36 part b) using the TI-30XB MultiView calculator, type:

$\boxed{(}\ \boxed{(-)}\ \boxed{1}\ \boxed{\text{2nd}}\ \boxed{\text{U}\tfrac{n}{d}}\ \boxed{3}$ ▶ $\boxed{4}$ ▶ $\boxed{)}\ \boxed{x^2}$ **enter**

To change the answer to a mixed number press $\boxed{\text{2nd}}\ \boxed{\tfrac{n}{d} \blacktriangleleft \blacktriangleright \text{U}\tfrac{n}{d}}$ **enter**

To find the square of a decimal number, type the number then $\boxed{x^2}$ **enter**.

exercise 2.5

LINKS TO Example 36a,c

Evaluate the squares of the following proper fractions.

a $\left(\frac{1}{2}\right)^2$
b $\left(\frac{1}{3}\right)^2$
c $\left(\frac{2}{5}\right)^2$
d $-\left(\frac{3}{4}\right)^2$
e $\left(-\frac{2}{3}\right)^2$
f $\left(-\frac{3}{5}\right)^2$
g $-\left(\frac{4}{9}\right)^2$
h $\left(-\frac{5}{6}\right)^2$

LINKS TO Example 36b

Evaluate the squares of the following mixed number fractions.

a $\left(1\frac{1}{3}\right)^2$
b $\left(1\frac{1}{2}\right)^2$
c $-\left(1\frac{2}{3}\right)^2$
d $\left(2\frac{1}{2}\right)^2$
e $\left(-2\frac{1}{3}\right)^2$
f $-\left(2\frac{1}{3}\right)^2$
g $\left(-3\frac{1}{2}\right)^2$
h $\left(-3\frac{1}{3}\right)^2$

LINKS TO Example 37a

Calculate each of the following without using a calculator.

a 0.1^2
b 0.4^2
c 0.8^2
d 0.06^2
e 0.3^2
f 0.12^2
g 0.011^2
h 0.009^2

LINKS TO Example 37b,c

Calculate each of the following without using a calculator.

a $(-0.5)^2$ **b** $(-0.7)^2$ **c** -0.2^2 **d** $(-0.05)^2$

e -0.07^2 **f** $(-0.8)^2$ **g** $(-0.001)^2$ **h** $(-0.012)^2$

A square postage stamp has sides of $2\frac{1}{2}$ cm. Calculate its area.

A square table has sides of $1\frac{1}{2}$ m. A tablecloth covers the table top and has an overhang of $\frac{1}{6}$ m on each side.

a How long is each side of the square table cloth? **b** What is its area?

A square vegetable patch has a side length of $3\frac{1}{2}$ metres.

a What is its area?

b Fertiliser is to be spread at 40 grams per square metre. How many grams of fertiliser would be needed for this vegetable patch?

A square spa bath has sides of 1.1 m. Calculate the area of its lid which covers the top of the spa bath exactly.

A square mirror has four sides each 0.4 m long. What will be the area of the mirror?

A chess board measures 0.3 m by 0.3 m. What is its area?

exercise 2.5 challenge

a Copy and complete this table.

b When the number to be squared has ONE decimal places its square has ___ decimal places.

c When the number to be squared has TWO decimal places its square has ___ decimal places.

d When the number to be squared has THREE decimal places its square has ___ decimal places.

e When the number to be squared has FIVE decimal places its square has ___ decimal places.

Square	Meaning	Value
0.4^2	0.4×0.4	0.16
0.3^2		
0.12^2		
0.02^2		
0.09^2		
0.007^2		
		0.25
		0.0064

The perimeter of a square picnic rug is $6\frac{2}{3}$ metres. Find the area of the rug.

2.6 Square roots and irrational numbers

What are irrational numbers?

A **rational number** is a number that can be written in the form of a fraction $\frac{a}{b}$, where a and b are integers.

Rational numbers include

- all the numbers we normally think of as fractions, for example, $\frac{1}{2}$, $-\frac{3}{7}$, $\frac{8}{17}$, $2\frac{3}{8} = \frac{19}{8}$.
- integers (whole numbers), because we can write whole numbers as fractions, for example, $3 = \frac{3}{1}$, $-7 = -\frac{7}{1}$
- all terminating decimals, for example, $0.37 = \frac{37}{100}$, $2.9 = \frac{29}{10}$
- recurring decimals, because they can be written as fractions, for example, $0.333\ldots = \frac{1}{3}$.

Rational square roots

Finding the 'square root' reverses the process of 'squaring'.

If we write a square number as a square, we can easily find the square root by reversing the squaring.

$16 = 4^2$ so $\sqrt{16} = 4$

In the same way we can find the square root of a fraction if both the numerator and denominator are square numbers.

Example 38

Evaluate the following square roots.

a $\sqrt{\frac{9}{100}}$

b $\sqrt{2\frac{7}{9}}$

Working

a $\sqrt{\frac{9}{100}} = \frac{3}{10}$

Reasoning

Find the square root of the numerator, and the square root of the denominator. Note that if the square root is multiplied by itself, that is it is squared, the result is the original number.

Here $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$.

continued

Example 38 continued

Working	**Reasoning**
b $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}}$	Make the mixed number into an improper fraction.
$= \frac{5}{3}$	Find the square root of numerator and denominator.
$= 1\frac{2}{3}$	Convert the improper fraction to a mixed number.

Irrational square roots

There are many numbers that do not have exact square roots. If you use a calculator to find the square root of 2, you may find that your calculator displays the value as $\sqrt{2}$. This is because there is no decimal value that is exactly equal to $\sqrt{2}$. The calculator may display the approximate value of $\sqrt{2}$ as 1.414213562 but if we use a calculator that can display a greater number of decimal places, for example, 31 decimal places, we obtain: 1.4142135623730950488016887242097.

Unlike recurring decimals that we met earlier in this chapter, the decimal places for $\sqrt{2}$ continue forever without ever forming a repeating pattern of digits. We call the square root of 2 an **irrational number**. Irrational numbers must not be confused with recurring decimals.

An irrational number is a number that cannot be written in the form of a fraction $\frac{a}{b}$ (where a and b are integers). Square roots of most numbers are irrational numbers.

We say that the **exact value** of the square root of 2 is $\sqrt{2}$ and the **approximate value** is 1.414…, depending on how many decimal places we may require in a calculation.

Example 39

Use a calculator to find if each of these square roots is rational or irrational. For irrational square roots, give the approximate value correct to three decimal places.

a $\sqrt{169}$ **b** $\sqrt{7}$ **c** $\sqrt{10}$ **d** $\sqrt{50}$

Working	**Reasoning**
a $\sqrt{169}$ is rational. $\sqrt{169} = 13$	
b $\sqrt{7}$ is irrational. $\sqrt{7} \approx 2.646$	$\sqrt{7} = 2.64575131\ldots$ Round to 3 decimal places.

continued

Example 39 continued

Working	**Reasoning**
c $\sqrt{10}$ is irrational $\sqrt{10} \approx 3.162$	$\sqrt{10} = 3.16227766\ldots$ Round to 3 decimal places.
d $\sqrt{50}$ is irrational. $\sqrt{50} \approx 7.071$	$\sqrt{50} = 7.07106781\ldots$ Round to 3 decimal places.

The square roots of some decimal numbers are rational numbers, for example, $\sqrt{0.81} = 0.9$ whereas others have irrational square roots, for example, $\sqrt{8.1} = 2.846049894\ldots$. If we write the decimal numbers as fractions we can see that if the numerator and the denominator are both square numbers (that is, 100 or 10 000), the square root is rational. Neither 10 nor 1000 are square numbers, so $\sqrt{8.1}$ and $\sqrt{0.081}$ are both irrational.

$\sqrt{81}$	$\sqrt{9^2}$	9	Rational
$\sqrt{8.1}$	$\sqrt{\frac{81}{10}} = \sqrt{\frac{9^2}{10}}$	2.846049894…	Irrational
$\sqrt{0.81}$	$\sqrt{\frac{81}{100}} = \sqrt{\frac{9^2}{10^2}} = \frac{9}{10}$	0.9	Rational
$\sqrt{0.081}$	$\sqrt{\frac{81}{1000}} = \sqrt{\frac{9^2}{10^2 \times 10}}$	0.2846049894…	Irrational
$\sqrt{0.0081}$	$\sqrt{\frac{81}{10\,000}} = \sqrt{\frac{9^2}{100^2}} = \frac{9}{100}$	0.09	Rational

Example 40

If possible, find the exact value of the following. Otherwise find an approximate value correct to 2 decimal places.

a $\sqrt{0.0016}$ **b** $\sqrt{0.016}$

Working	**Reasoning**
a $\sqrt{0.0016}$	
$= \sqrt{\frac{16}{10\,000}}$	Express the decimal as a fraction.
$= \sqrt{\frac{4 \times 4}{100 \times 100}}$	16 and 10 000 are both square numbers.
$= \frac{4}{100}$	Find the square root of the numerator and of the denominator.
$= 0.04$	Express the result as a decimal

continued

Example 40 continued

Working	Reasoning
b $\sqrt{0.016}$	Express the decimal as a fraction.
$= \sqrt{\dfrac{16}{1000}}$	16 is a square number but 1000 is not.
$= \sqrt{\dfrac{4 \times 4}{100 \times 10}}$	Thus an exact square root cannot be found.
Using a calculator $\sqrt{0.016}$ $\approx 0.12649\ldots$ ≈ 0.13	A calculator can give an approximate value.

As we saw for integers in chapter 1, the square roots of negative fractions and negative decimals do not exist in our real number system.

Finding squares and square roots of fractions like those in this exercise can be done using a scientific calculator.

Tech tip

To find $\sqrt{2\frac{7}{9}}$ (example 38 part b) using the TI-30XB MultiView calculator,

type:

2nd [√] **2** **2nd** [U n/d] **7** ▶ **9** ▶ ▶ **enter**

To change the answer to a mixed number press **2nd** [n/d ◂ ▸ U n/d] **enter**

To find a square root, type in **2nd** x^2, that gives [√], then the decimal number.

Lastly press **enter**.

Note that the calculator finds approximate square root of a number that does not have an exact square root.

exercise 2.6

LINKS TO Example 38a

● Evaluate the square roots of the following proper fractions.

a $\sqrt{\dfrac{1}{25}}$

b $\sqrt{\dfrac{1}{36}}$

c $\sqrt{\dfrac{4}{9}}$

d $\sqrt{\dfrac{25}{49}}$

e $\sqrt{\dfrac{9}{16}}$

f $\sqrt{\dfrac{25}{36}}$

g $\sqrt{\dfrac{81}{100}}$

h $\sqrt{\dfrac{121}{144}}$

LINKS TO Example 38b

● Evaluate the square roots of the following mixed number fractions.

a $\sqrt{1\frac{7}{9}}$ **b** $\sqrt{2\frac{1}{4}}$ **c** $\sqrt{2\frac{7}{9}}$ **d** $\sqrt{6\frac{1}{4}}$

e $\sqrt{5\frac{4}{9}}$ **f** $\sqrt{12\frac{1}{4}}$ **g** $\sqrt{7\frac{1}{9}}$ **h** $\sqrt{11\frac{1}{9}}$

LINKS TO Example 39

● Sort these numbers into the two columns shown.

Rational numbers	Irrational numbers

$\frac{2}{7}$ $\sqrt{13}$ 21 0.783 -2.5 $\sqrt{7}$ $\sqrt{\frac{4}{9}}$ $0.\overline{18}$

$-\frac{5}{8}$ $\frac{3}{20}$ $\sqrt{23}$ $0.\overline{285714}$ $\sqrt{9}$ $\sqrt{40}$

LINKS TO Example 40

● Evaluate each of the following mentally and write your answers.

a $\sqrt{0.64}$ **b** $\sqrt{0.01}$ **c** $\sqrt{0.04}$ **d** $\sqrt{0.25}$

e $\sqrt{0.0049}$ **f** $\sqrt{0.0036}$ **g** $\sqrt{0.000009}$ **h** $\sqrt{1.21}$

● A square area bounded by four main roads has an area of 0.64 square kilometres. What is its side length?

● A square window has a glass area of 1.44 square metres. What is the length of each side?

● Which of these numbers have an exact square root? For those numbers which do, find the square root.

a 0.025 **b** 0.81 **c** 0.0049 **d** 0.000121 **e** 0.001 **f** 0.0001

● To create a square notice board with an area of $2\frac{1}{4}$ square metres, what would be the side length needed?

● A square tarpaulin has an area of $2\frac{7}{9}$ square metres. How long are its sides?

● The square bottom of a children's wading pool has an area of $6\frac{1}{4}$ square metres.

a How long is one side of the wading pool? **b** What is the perimeter of the pool?

exercise 2.6 challenge

● Use your calculator to evaluate each of the following then see if you can explain what you observe.

a $\sqrt{\frac{1}{3}}$ **b** $\frac{\sqrt{3}}{3}$

Analysis task

Approximate square roots

For a number that has an exact square root, when you divide the number by its square root the answer you get is the square root again. For example, $\sqrt{25} = 5$, so $25 \div 5 = 5$.

For numbers that do not have an exact square root, an *iterative* method is used. Suppose you have a simple calculator that does not have a square root function. We can use a repeated guess, check and improve method until we find the square root correct to the required number of decimal places. This process of getting closer and closer to the value we want by repeating a process is called *iteration*.

Example: Find the square root of 11 correct to two decimal places.

11 is between 9 and 16 so its square root will be between 3 and 4. Guess 3.5.

Using a calculator $11 \div 3.5 = 3.1428\ldots$

Next guess is between 3.1428… and 3.5. Guess 3.3.

It is neater to put this into a table.

Guess	**11 ÷ guess =**	**Next guess between**
3.5	11 ÷ 3.5 = 3.1428…	3.1428… and 3.5
3.3	11 ÷ 3.3 = 3.33333…	3.3 and 3.33333…
3.32	11 ÷ 3.32 = 3.31325…	3.31325… and 3.32
3.315	11 ÷ 3.315 = 3.31825…	3.315 and 3.31825…
3.316	11 ÷ 3.316 = 3.31724…	3.316 and 3.31724…
3.3165	11 ÷ 3.3165 = 3.31674…	3.3165 and 3.31674
3.3166	11 ÷ 3.3166 = 3.316649…	

So $\sqrt{11} = 3.32$ correct to two decimal places.

For each of the following, set up a table like the table above, but with appropriate headings in the middle column. For example, in part **a** the heading for the middle column will be $21 \div \text{guess} =$.

a Between what two whole numbers is $\sqrt{21}$?

b To find $\sqrt{40}$ the first guess is 6.5. Now $40 \div 6.5 = 6.1538\ldots$

  i Between what two numbers does the next guess lie?

  ii What should the next guess be?

c Find $\sqrt{7}$ correct to one decimal place.

d Find $\sqrt{55}$ correct to two decimal places.

Review Fractions and decimals

Comparing and ordering fractions

- Fractions and decimals, positive and negative, can be located on the number line.
- Equivalent fractions are fractions that can be simplified to the same fraction, for example, $\frac{4}{12}, \frac{3}{9}$ and $\frac{2}{6}$ are equivalent fractions. They can all be simplified to $\frac{1}{3}$.
- Equivalent fractions are located at the same position on the number line.
- To compare fractions, convert to equivalent fractions with the same denominator then compare the numerators. If the denominators are the same, the larger the numerator, the larger the fraction.
- Alternatively, convert the fractions to decimals by dividing the numerator by the denominator then compare the decimal numbers.
- Mixed numbers have a whole number part and a fraction part.
- Mixed numbers can be converted into improper fractions

Converting between fractions and decimals

- A fraction $\frac{a}{b}$ can be converted to a decimal by calculating $a \div b$. If there is no remainder, the decimal is a terminating decimal. If a repeating pattern occurs in the digits, then the decimal is a recurring decimal.
- Fractions with denominators that have only 2 and 5 as their prime factors give terminating decimals.
- Decimals are converted to fractions by putting the digits of the decimal number in the numerator of the fraction and a power of 10 in the denominator. The number of decimal places tells us how many zeros in the denominator. Simplify the fraction if possible. For example, $0.06 = \frac{6}{100} = \frac{3}{50}$

Adding fractions

- Proper and improper fractions: convert to equivalent fractions with the same denominator, then add the numerators. If the sum is an improper fraction convert to a mixed number.
- Mixed numbers: add the whole number parts then add the fraction parts as above.

Subtracting fractions

- Convert mixed numbers to improper fractions, convert to equivalent fractions with same denominators, then subtract the numerators. If the result is an improper fraction, convert to a mixed number.

Multiplying fractions

- Convert mixed numbers to improper fractions. Cancel where possible. Multiply the numerators. Multiply the denominators. If the product is an improper fraction convert to a mixed number.

Dividing fractions

- Convert mixed numbers to improper fractions. Multiply by the reciprocal of the fraction that comes after the division sign. Then proceed as for multiplying fractions.

Order of operations

- Powers, brackets, of.
- Division and multiplication from left to right.
- Addition and subtraction from left to right.

Squares and square roots

- Rational numbers can be written in the form of a fraction $\frac{a}{b}$, where a and b are integers. Integers, fractions, terminating decimals and recurring decimals are rational numbers.
- Irrational numbers cannot be written as a fraction $\frac{a}{b}$. Square roots of most numbers are irrational, for example, $\sqrt{2}$, $\sqrt{\frac{3}{4}}$, $\sqrt{5.1}$
- Squaring a number and finding the square root are inverse processes.
- Numbers that are square numbers have rational square roots.
- Numbers that are not square numbers have irrational square roots.
- To square fractions and mixed numbers, convert mixed numbers to improper fractions first then square the numerator and square the denominator.
- To find the square root of a fraction or mixed number, first convert mixed numbers to improper fractions. If the numerator and denominator are both square numbers, find the square root of the numerator and the square root of the denominator. Otherwise the fraction does not have a rational square root.

Cubes and cube roots

- Cubing a number and finding the cube root are inverse processes.
- The symbol for cube root is $\sqrt[3]{}$. For example $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$
- Numbers that are cube numbers have rational cube roots. For example, $\frac{8}{27} = \left(\frac{2}{3}\right)^3$ so $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$
- Cube roots of most numbers are irrational, for example, $\sqrt[3]{10}$, $\sqrt[3]{\frac{1}{2}}$, $\sqrt[3]{2.3}$

Visual map

approximate square root
ascending order
cancel
decimal
decimal place
denominator
descending order
equivalent fraction
fraction
improper fraction
integer
irrational number
lowest common denominator
mixed number
numerator
place value columns
proper fraction
rational number
reciprocal
recurring decimal
round
simplify
square
square number
square root
terminating decimal

Revision

Multiple-choice questions

● Which one of the following fractions converts to a recurring decimal?

A $\frac{2}{5}$ **B** $\frac{7}{8}$ **C** $\frac{11}{25}$ **D** $\frac{27}{32}$ **E** $\frac{8}{15}$

● As a decimal, $\frac{5}{8}$ is

A 0.625 **B** 0.58 **C** 0.85 **D** 1.3 **E** 1.6

● $2\frac{2}{5} \times 2\frac{2}{3} =$

A $\frac{2}{5}$ **B** $\frac{9}{10}$ **C** $4\frac{4}{15}$ **D** $6\frac{2}{5}$ **E** $6\frac{4}{15}$

● $0.12 \times 0.7 =$

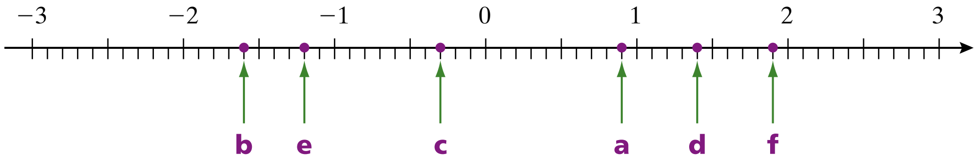
A 0.072 **B** 0.084 **C** 0.72 **D** 0.84 **E** 8.4

● $\left(1\frac{2}{3}\right)^2$ is equal to

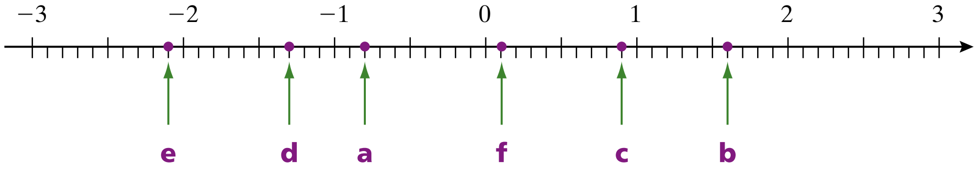
A $1\frac{4}{9}$ **B** $1\frac{5}{9}$ **C** $2\frac{2}{3}$ **D** $2\frac{4}{9}$ **E** $2\frac{7}{9}$

Short-answer questions

Write each fraction shown on the number line in its simplest form.



Write each decimal number shown on the number line.



Arrange these numbers in ascending order.

$-2\frac{1}{2}, 3.4, -0.84, 3\frac{1}{2}, 3.38, -3, -4\frac{1}{2}, -3.5, -0.746, 0.002$

Convert each of these fractions into a decimal marking any recurring digits by a bar or dots.

a $\frac{7}{8}$ **b** $\frac{6}{7}$ **c** $\frac{4}{15}$ **d** $\frac{23}{40}$

e $1\frac{2}{3}$ **f** $2\frac{4}{5}$ **g** $1\frac{4}{11}$ **h** $3\frac{17}{20}$

Convert each of these terminating decimals into fractions.

a 0.85 **b** 1.24 **c** 0.848 **d** 0.125

e 0.76 **f** 0.425 **g** 2.59 **h** 0.04

Evaluate without using a calculator.

a $\frac{7}{8} + \frac{1}{6}$ **b** $1\frac{2}{3} + 2\frac{3}{5}$ **c** $\frac{11}{12} - \frac{3}{4}$ **d** $2\frac{1}{2} - 1\frac{5}{6}$

Evaluate without using a calculator.

a $\frac{3}{5}$ of 12 **b** $\frac{2}{3}$ of $\frac{1}{2}$ **c** $\frac{3}{10} \times \frac{2}{9}$ **d** $\frac{2}{5} \times \frac{3}{8} \times \frac{5}{9}$

e $1\frac{2}{3} \times 2\frac{2}{5}$ **f** $\frac{2}{3} \div 4$ **g** $\frac{3}{5} \div \frac{2}{9}$ **h** $1\frac{2}{5} \div 3\frac{1}{2}$

Evaluate

a 1.234 + 12.34 + 123.4 **b** 56.78 – 5.678

Evaluate without using a calculator.

a 0.08 × 0.6 **b** 12.34 × 0.005 **c** 21.4 ÷ 5 **d** 38.1 ÷ 0.03

Evaluate without using a calculator.

a $\left(\frac{3}{5}\right)^2$ **b** $\left(2\frac{1}{2}\right)^2$ **c** $-\left(\frac{1}{4}\right)^2$ **d** $\left(-\frac{5}{7}\right)^2$

e $\sqrt{\frac{1}{9}}$ **f** $\sqrt{1\frac{9}{16}}$ **g** $\sqrt{2\frac{1}{4}}$ **h** $\sqrt{11\frac{1}{9}}$

i $\left(\frac{1}{4}\right)^3$ **j** $\left(-\frac{1}{2}\right)^3$ **k** $\sqrt[3]{\frac{1}{27}}$ **l** $\sqrt[3]{3\frac{3}{8}}$

Evaluate without using a calculator.

a $(0.7)^2$ **b** $(0.06)^2$ **c** $\sqrt{0.16}$ **d** $\sqrt{0.0025}$

Fleur got $\frac{17}{20}$ on one maths test, and $\frac{25}{30}$ on another. Which was her best result? Explain why.

When $2\frac{1}{3}$ litres of red paint are mixed with $1\frac{3}{4}$ litres of white paint, what is the volume of pink paint produced?

An area of bush is $12\frac{1}{2}$ hectares. If each possum requires a territory of $1\frac{3}{4}$ hectares, how many possums would be expected in this area? Give your answer as a mixed number and then round it to the nearest whole possum.

A bushwalking trek was 24.6 km long. On the first day the group walked 8.9 km, and on the second day they walked 9.2 km. How much further did they need to walk on the third day to complete the trek?

A dolphin can swim the length of a 50-metre pool in 6.7 seconds. How long would it take a dolphin swimming at this rate to swim 350 metres?

Nick has been sorting oranges into 12 kg bags. If each orange weighs 0.3 kg, how many oranges should be in each bag?

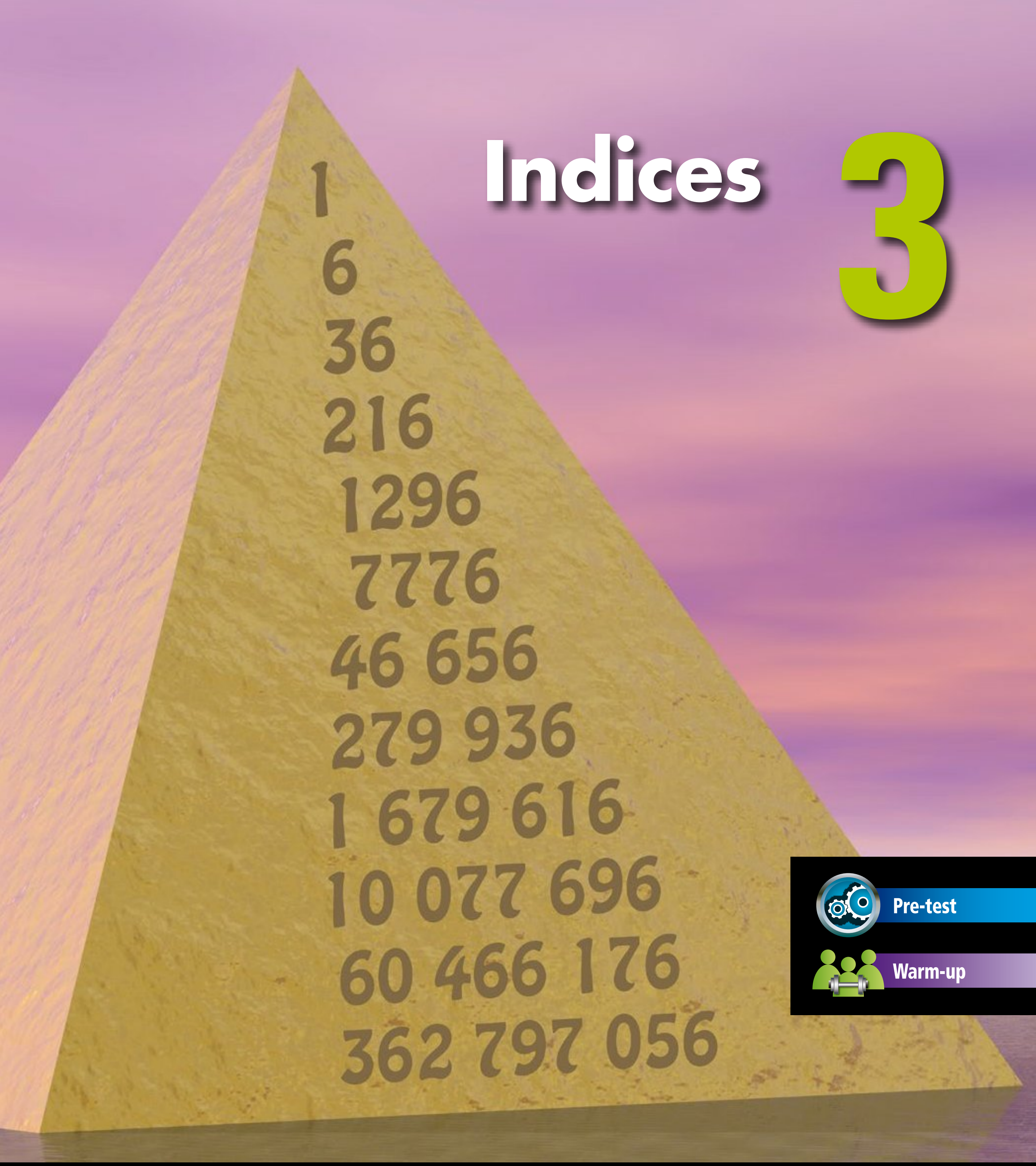
$23.60 is split equally between 4 friends. How much money does each person get?

Extended-response questions

When a whole number was divided by another whole number, the result was 3.375. Each of the whole numbers was less than 100. Give all possible answers for the pair of whole numbers.

Indices

3



Pre-test

Warm-up

This pyramid shows powers of 6. When a number is raised to a power, the value grows in size rapidly as the power increases, for example $6^1 = 6$, $6^2 = 36$, $6^3 = 216$, $6^4 = 1296$, and so on. When we write 6^2, 6 is called the base and 2 is called the index. Another name for an index is an exponent, so this type of growth is called exponential growth.

3.1 Index notation and prime factors

In Year 7, we found that products such as 2 × 2 × 2 × 2 × 2 could be written as 2^5.

5 is the index or exponent

$$\text{2 is the base} \leftarrow 2^{\overset{\uparrow}{5}} \quad = \quad \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{5 factors}} \quad = \quad 32$$

Read as: 'two to the power of five'

Index form	Factor form of 2^5	Standard numeral

The second and third powers have special names. A **second power** (that is, a number with an index of two) is read **'squared'** and a **third power** (that is, a number with an index of 3) is read **'cubed'**. For example, 3^2 is read '3 squared' and 5^3 is read '5 cubed'.

Example 1

For each of the following, state the base and the index.

a 7^4 **b** 23^4

Working	**Reasoning**
a base: 7 index: 4	The index number tells us how many factors of the base number are multiplied together.
b base: 23 index: 4	There are four 23s multiplied together.

Numbers that are written in factor form can be simplified by writing them in index form.

Example 2

Write the following in index form.

a $6 \times 6 \times 6 \times 6$ **b** $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Working	**Reasoning**
a $6 \times 6 \times 6 \times 6 = 6^4$	6 is the base. There are four factors of 6 so the index is 4.
b $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$ $= 2^2 \times 3^4 \times 5^3$	Write each of the repeated factors in index form.

The reverse can also be done – numbers or expressions that are written in index form can be written in factor form (also referred to as expanded form) to show the factors.

Example 3

Write the following in factor form.

a 10^6　　**b** $2^4 \times 5^3 \times 7^3$

Working	Reasoning
a $10 \times 10 \times 10 \times 10 \times 10 \times 10$	There are six factors of 10.
b $2^4 \times 5^3 \times 7^3$ $= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$	For each number in index form, the index shows how many of the base numbers are multiplied together.

Some numbers can be written in index form in more than one way.

For example, 64 can be written in index form with base 2, base 4 or base 8.

$$\begin{aligned} 64 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^6 \end{aligned}$$

$$\begin{aligned} 64 &= \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \\ &= 4 \times 4 \times 4 \\ &= 4^3 \end{aligned}$$

$$\begin{aligned} 64 &= \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \\ &= 8 \times 8 \\ &= 8^2 \end{aligned}$$

Example 4

Express 3^4

a in factor form.　　**b** in index form using a different base and evaluate.

Working	Reasoning
a $3^4 = 3 \times 3 \times 3 \times 3$	3^4 is the product of four threes.
b $3^4 = 9 \times 9$ $3^4 = 81$	3^4 is the product of two nines.

A calculator can be used to find the value of a power. This can be done by repeated multiplication or by using the ‘raised to a power’ button on the calculator.

Example 5

Find the value of 13^5

a by using repeated multiplication on your calculator.

b by using the 'raised to the power' key.

Working	Reasoning
a $13^5 = 13 \times 13 \times 13 \times 13 \times 13$ $= 371\,293$ $\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}$ **enter**	The index of 5 means that there are five 13s multiplied together.
b $13^5 = 371\,293$ $\boxed{1}\boxed{3}\boxed{\wedge}\boxed{5}$ **enter**	The key $\boxed{\wedge}$ on the TI calculator means 'raised to the power'.

When two or more numbers raised to a power are multiplied, the powers are evaluated first before multiplying.

Example 6

Evaluate $2^3 \times 3^2 \times 5$

Working	Reasoning
$2^3 \times 3^2 \times 5$ $= 8 \times 9 \times 5$ $= 360$	Evaluate each power first before multiplying.

Finding prime factors

You will recall from Year 7 that a **prime number** is a number that has only itself and 1 as factors. For example, 3 is a prime number because the only factors of 3 are 1 and 3. A factor of a number that is a prime number is called a **prime factor**. For example, 15 has factors 1, 3, 5 and 15. The prime factors of 15 are 3 and 5. A **composite number** can be written as the product of its prime factors. For example, 15 can be written as 3×5. Many composite numbers have repeated prime factors. For example, $32 = 2 \times 2 \times 2 \times 2 \times 2$ that can be written in index form as 2^5.

Prime factors

When completing a factor ladder, we test the number for divisibility by 2, then by 3, then by 5 and so on. We continue dividing by prime numbers in order until we end up with 1. The left hand column then gives a neat list of all the prime factors from smallest to largest.

Example 7

For each of these numbers

- **i** use a factor ladder to find the prime factors.
- **ii** list the prime factors.
- **iii** write the number as the product of its prime factors in index form.

a 4500 **b** 4950

Working

Reasoning

a i

2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

Start with the smallest prime number, 2.
4500 is divisible by 2 to give 2250.
2250 is divisible by 2 to give 1125.
1125 is not divisible by 2, so go to the next prime number, which is 3.
1125 is divisible by 3 to give 375.
375 is divisible by 3 to give 125.
125 is not divisible by 3, so go to the next prime number, which is 5.
125 is divisible by 5 to give 25.
25 is divisible by 5 to give 5.
5 is divisible by 5 to give 1.

ii The prime factors of 4500 are 2, 3 and 5.

The factors are listed on the left side of the ladder. There are three different factors.

iii $4500 = 2^2 \times 3^2 \times 5^3$

The factors listed on the left side of the ladder show how many of each factor are multiplied together.

b i

2	4950
3	2475
3	825
5	275
5	55
11	11
	1

Start with the smallest prime number, 2.
4950 is divisible by 2 to give 2475.
2475 is not divisible by 2, so go to the next prime number, which is 3.
2475 is divisible by 3 to give 825.
825 is divisible by 3 to give 275.
275 is not divisible by 3, so go to the next prime number, which is 5.
275 is divisible by 5 and 55 is divisible by 5.
11 is not divisible by 5 and it is not divisible by the next prime number, which is 7. Go to the next prime number, 11.
11 is divisible by 11 to give 1.

ii The prime factors of 4950 are 2, 3, 5 and 11

There are four different factors.

iii
$$\begin{aligned}4950 &= 2 \times 3 \times 3 \times 5 \times 5 \times 11\\ &= 2 \times 3^2 \times 5^2 \times 11\end{aligned}$$

Write 4950 as the product of the prime factors listed on the left side of the ladder.

Using index notation to find highest common factors

When two numbers are expressed in index notation, it is easy to find their highest common factor. The highest common factor is found by:

- Step 1: Look for the prime factors that are common to both numbers.
- Step 2: For each common prime factor, find the power that is common to both numbers, that is, the lower of the two powers.
- Step 3: Multiply the lowest of the two powers of each common prime factor.

If we consider the two numbers 108 and 96, for example, $108 = 2^2 \times 3^3$ and $96 = 2^5 \times 3$.

- Step 1: Prime factors common to both numbers are 2 and 3.
- Step 2: The power of 2 common to both numbers is 2^2 and the power of 3 common to both numbers is 3^1 (if we took a higher power it would not be common to both 108 and 96).
- Step 3: The highest common factor of 108 and 96 is $2^2 \times 3^1 = 12$.

Example 8

Find the highest common factor of each of these pairs of numbers by first writing each number in index notation.

a 24 and 90

b 4500 and 4950

Working	**Reasoning**
a $24 = 2^3 \times 3$ $90 = 2 \times 3^2 \times 5$ The highest common factor of 24 and 90 is $2^1 \times 3^1$, that is, 6.	The power of 2 common to both numbers is 2^1. The power of 3 common to both numbers is 3^1. 5 is not a factor of 24. Multiple the common factors 2^1 and 3^1.
b $4500 = 2^2 \times 3^2 \times 5^3$ $4950 = 2 \times 3^3 \times 5^2 \times 11$ The highest common factor of 4500 and 4950 is $2^1 \times 3^2 \times 5^2$, that is, 450.	See example 7 for prime factors of 4500 and 4950. The power of 2 common to both numbers is 2^1. The power of 3 common to both numbers is 3^2. The power of 5 common to both numbers is 5^2. 11 is not a factor of 4500. Multiply the common factors 2^1, 3^2 and 5^2.

Using index notation to find lowest common multiples

When two numbers are expressed in index notation, it is easy to find their lowest common multiple.

- Step 1: Look for all the prime factors that occur in the numbers.
- Step 2: For each prime factor choose the higher of two powers.
- Step 3: Multiply the higher of the powers of each prime factor occurring in either number.

If we consider the two numbers 48 and 90, for example, $48 = 2^4 \times 3$ and $90 = 2 \times 3^2 \times 5$:

- Step 1: Prime factors that occur in the numbers are 2, 3 and 5.
- Step 2: The highest powers are 2^4, 3^2 and 5^1. (The lowest common multiple must include 2^4 so that it is a multiple of 48 and it must include 3^2 and 5 so that it is a multiple of 90.)
- Step 3: The lowest common multiple of 48 and 90 is therefore $2^4 \times 3^2 \times 5 = 16 \times 9 \times 5 = 720$.

Example 9

Find the lowest common multiple of each of these pairs of numbers by first writing each number in index notation.

a 24 and 54

b 96 and 175

Working	**Reasoning**
a $24 = 2^3 \times 3$ $54 = 2 \times 3^3$ The prime factors are 2 and 3. The highest powers are 2^3 and 3^3. The lowest common multiple of 24 and 54 is $2^3 \times 3^3$, that is, 216.	The lowest common multiple must include 2^3 as a factor so it is a multiple of 24. It must include 3^3 as factor so it is a multiple of 54.
b $96 = 2^5 \times 3$ $175 = 5^2 \times 7$ The prime factors are 2, 3, 5 and 7. The highest powers are $2^5, 3^1, 5^2$ and 7^1. The lowest common multiple is $2^5 \times 3^1 \times 5^2 \times 7^1$, that is, 16 800	The lowest common multiple must include 2^5 and 3 as factors so it is a multiple of 96. It must include 5^2 and 7 as factors so it is a multiple of 175.

Tech tip

The TI-30XB MultiView calculator can be used to show repeated multiplication by the same factor.

For example, to find 13 × 13 ×13 ×13 ×13, type:

$\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}\boxed{\times}\boxed{1}\boxed{3}$ **enter**

To raise a number to a power, for example, 13^5, type:

$\boxed{1}\boxed{3}\boxed{\wedge}\boxed{5}$ ▷ **enter**

To show the multiplication of two or more numbers each raised to a power, for example, $3 \times 2^4 \times 3^3$, type:

$\boxed{3}\boxed{\times}\boxed{2}\boxed{\wedge}\boxed{4}$ ▷ $\boxed{\times}\boxed{3}\boxed{\wedge}\boxed{3}$ ▷ **enter**

exercise 3.1

LINKS TO Example 1

For each of the following state the base and the index.

		Base	Index
a	3^6		
b	6^3		
c	7^4		
d	10^5		
e	2^{12}		

LINKS TO Example 2a

Write the following in index form.

a 3 × 3 × 3 × 3 × 3

b 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

c 10 × 10 × 10 × 10 × 10 × 10

d 5 × 5 × 5 × 5 × 5

e four squared

f seven cubed

g six raised to the power four

h four raised to the power six

i seven raised to the power five

j two raised to the power nine

k the eighth power of twelve

l thirty-six cubed

LINKS TO Example 2b

Write each of the following in index form.

a 2 × 2 × 5 × 5 × 5

b 2 × 2 × 7 × 7 × 7 × 7

c 2 × 2 × 2 × 2 × 3 × 3 × 3

d 3 × 3 × 3 × 3 × 5 × 5

e 2 × 2 × 2 × 7 × 7

f 3 × 5 × 5 × 5 × 7 × 7

g $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$ **h** $2 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7$

i $5 \times 7 \times 7 \times 7 \times 7 \times 11 \times 11$ **j** $2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

LINKS TO Example 3a

Expand by writing in factor form.

a 3^6 **b** 2^7 **c** 13^3 **d** 4^4

e 5^5 **f** 10^7 **g** 11^4 **h** 6^8

LINKS TO Example 3b

Write each of the following in factor form.

a $2^2 \times 5^3$ **b** $2^3 \times 3^2$ **c** $3^4 \times 5^2 \times 7^2$ **d** $2^4 \times 3^2 \times 5 \times 11^2$

e $2^5 \times 7^3$ **f** $2^4 \times 3^4 \times 5$ **g** $2 \times 3^2 \times 5^3$ **h** $2 \times 3 \times 7^3$

i $3^2 \times 5^4 \times 13$ **j** $2^5 \times 5^2 \times 17$ **k** $3^4 \times 7^3 \times 11^3$ **l** $13^2 \times 17^3$

LINKS TO Example 4

Example 4 showed that $3^4 = 81 = 9^2$, that is, 81 can be written in index form in two different ways. Write each of the following numbers in index form in two different ways.

a 16 **b** 64 **c** 625 **d** 256

LINKS TO Example 5

Do repeated multiplication on your calculator to evaluate each of the following.

a 8^5 **b** 12^7

c 34^6 **d** 42^5

e the fifth power of 6 **f** the third power of 17

g the fourth power of 24 **h** the seventh power of 15

Copy and complete the following table of squared and cubed numbers.

Words	Index form	Factor form	Value
four squared			
	8^3		
		$14 \times 14 \times 14$	
			324
	28^2		
fifty-two cubed			

LINKS TO Example 5

Use your calculator to evaluate these powers.

a 2^{10} **b** 2^{20} **c** 3^8 **d** 3^{12}

e 6^{12} **f** 7^7 **g** 12^8 **h** 18^5

LINKS TO Example 6

Use your calculator to evaluate.

a $2^4 \times 3^5$ **b** $2^3 \times 5^4$

c $2^7 \times 3^8$ **d** $3^4 \times 5^3$

e $2^2 \times 3^3 \times 5^4$ **f** $2^3 \times 3^2 \times 5^3 \times 7$

g $2^7 \times 3^4 \times 7^2$ **h** $3^5 \times 5^2 \times 7^3 \times 11^2$

LINKS TO Example 7

For each of these composite numbers

- **i** use a factor ladder to find the prime factors
- **ii** write the number as the product of its prime factors

a 1512
b 2025
c 1728
d 392
e 3969
f 2200
g 7875
h 5625
i 1350
j 3528
k 12 960
l 121 275

LINKS TO Example 8

For each of these pairs of numbers,

- **i** write each number in index form (using a factor ladder to find the prime factors).
- **ii** use the index form to find the highest common multiple of the two numbers.

a 30 and 54
b 56 and 96
c 42 and 126
d 27 and 162
e 98 and 378
f 144 and 256
g 512 and 198
h 75 and 255
j 78 and 234
k 147 and 504
l 1400 and 2940

LINKS TO Example 9

Find the lowest common multiple for each of the pairs of numbers in question 11.

exercise 3.1 challenge

Find the smallest number that is divisible by all the numbers from 2 to 15 inclusive.

3.2 Multiplying and dividing numbers in index form

If we expand $2^3 \times 2^4$ by writing each index form in its factor form, we see that we can take a shortcut to the answer by adding the indices 3 and 4 to give 7.

$$\begin{aligned} 2^3 \times 2^4 &= \underbrace{2 \times 2 \times 2}_{\textit{three factors}} \times \underbrace{2 \times 2 \times 2 \times 2}_{\textit{four factors}} \\ &= \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{\textit{seven factors}} \\ &= 2^7 \end{aligned}$$

Multiplication index law

To multiply powers of the same base, add the indices.

For example, $2^3 \times 2^5 = 2^{3+5} = 2^8$

Example 10

Simplify $3^4 \times 3^7$.

Working	**Reasoning**
$3^4 \times 3^7$	3^4 means $3 \times 3 \times 3 \times 3$
$= 3^{4+7}$	3^7 means $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
	$\underbrace{3 \times 3 \times 3 \times 3}_{4\ \textit{factors}} \times \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}_{7\ \textit{factors}}$
	$= \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}_{11\ \textit{factors}}$
$= 3^{11}$	Simplify using the multiplication index law.

Example 11

Find the missing index that makes each equation true.

a $7^{\square} = 7^{5+3}$

b $4^9 = 4^3 \times 4^{\square}$

continued

Example 11 continued

Working

a $7^{\square} = 7^5 \times 7^3$

$7^{\square} = 7^{5+3}$

$7^{\square} = 7^8$

The missing index is 8.

Reasoning

7^5 is the product of five sevens.

7^3 is the product of three sevens.

$$7^{\square} = \underbrace{7 \times 7 \times 7 \times 7 \times 7}_{5\ factors} \times \underbrace{7 \times 7 \times 7}_{3\ factors}$$

$$7^{\square} = \underbrace{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}_{8\ factors}$$

Altogether there are eight sevens.

Working

b $4^9 = 4^3 \times 4^{\square}$

$4^9 = 4^{3+\square}$

$4^9 = 4^{3+6}$

The missing index is 6.

Reasoning

4^9 is the product of nine fours.

4^3 is the product of three fours.

$4^{\square}$ must be the product of six fours.

$$\underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}_{9 \text{ factors}}$$

$$\underbrace{4 \times 4 \times 4}_{3 \text{ factors}} \times \underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{6 \text{ factors}}$$

Division index law

When dividing numbers that have the same base, subtract the indices.

For example, $2^5 \div 2^3 = 2^{5-3} = 2^2$

Example 12

Simplify.

a $8^9 \div 8^5$

b $\dfrac{3^4}{3^3}$

Working

a $\dfrac{8^9}{8^5}$

$= 8^{9-5}$

$= 8^4$

Reasoning

Cancelling down as shown below shows why we obtain the same result using the division index law.

$$\frac{\overbrace{\overset{1}{8} \times \overset{1}{8} \times \overset{1}{8} \times \overset{1}{8} \times \overset{1}{8} \times 8 \times 8 \times 8 \times 8}^{9\ factors}}{\underbrace{\overset{1}{8} \times \overset{1}{8} \times \overset{1}{8} \times \overset{1}{8} \times \overset{1}{8}}_{5\ factors}}$$

$= 8 \times 8 \times 8 \times 8 = 8^4$

Simplify using the division index law

continued

Example 12 continued

Working	Reasoning
b $\frac{3^4}{3^3}$ $= 3^{4-3}$ $= 3^1$ $= 3$	$\frac{\overbrace{\overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times 3}^{4\ \textit{factors}}}{\underbrace{\underset{1}{3} \times \underset{1}{3} \times \underset{1}{3}}_{3\ \textit{factors}}}$

Example 13

Find the missing index that makes each of these equations true.

a $4^{\square} = \frac{4^{13}}{4^8}$

b $5^3 = 5^{\square} \div 5^4$

Working	Reasoning
a $4^{\square} = \frac{4^{13}}{4^8}$ $4^{\square} = 4^{13-8}$ $4^{\square} = 4^5$	Simplify $\frac{4^{13}}{4^8}$ using the division index law.
The missing index is 5.	
b $5^3 = 5^{\square} \div 5^4$ $5^3 = 5^{\square-4}$ $5^3 = 5^{7-4}$ The missing index is 7.	Use the division index law to write $5^{\square} \div 5^4$ as $5^{\square-4}$ 3 is equal to $\square - 4$

Example 14

Simplify $\frac{3^5 \times 3^7}{3^2 \times 3^6}$

Working	Reasoning
$\frac{3^5 \times 3^7}{3^2 \times 3^6} = \frac{3^{5+7}}{3^{2+6}}$	Simplify the numerator and simplify the denominator by applying the multiplication index law.
$= \frac{3^{12}}{3^8}$ $= 3^{12-8}$ $= 3^4$	Simplify using the division index law.

The zero index

In chapter 1 we saw how extending number patterns was helpful in understanding negative integers. We now see how extending number patterns in powers can help us understand the meaning of a zero power.

Powers of				
Two	Three	Four	Five	Six
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$	$10^4 = 10\,000$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$10^3 = 1000$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$10^2 = 100$
$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$	$10^1 = 10$
$2^0 = ?$	$3^0 = ?$	$4^0 = ?$	$5^0 = ?$	$10^0 = ?$

Starting at the top of the powers of 2 column, as we go down the column, each number is half the number in the previous row (16, 8, 4, 2), so logically we would expect the next number to be 1. In the powers of 3 column, each number is one third of the number in the previous row (81, 27, 9, 3), so again we would expect 1 to be the next number. A similar pattern occurs in all the other columns. This suggests that any number raised to the power 0 is equal to 1.

If we take an example such as $\frac{3^4}{3^4}$, we can simplify this in two ways.

By the division index law, $\frac{3^4}{3^4} = 3^{4-4} = 3^0$

But $\frac{3^4}{3^4} = \frac{\overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times \overset{1}{3}}{\underset{1}{3} \times \underset{1}{3} \times \underset{1}{3} \times \underset{1}{3}} = 1$

So we have obtained two different expressions for $\frac{3^4}{3^4}$.

The two expressions are both correct so this means that $3^0 = 1$

Zero index

Any non-zero number raised to the power zero is equal to 1.

For example, $3^0 = 1$

Example 15

Evaluate 7^0 using a calculator.

Working

$\boxed{7}\,\boxed{\wedge}\,\boxed{0}$ **enter**

$7^0 = 1$

Example 16

Simplify $4^0 + 5 - 3^0$

Working	Reasoning
$4^0 + 5 - 3^0$	$4^0 = 1$
$= 1 + 5 - 1$	$3^0 = 1$
$= 5$	

exercise 3.2

LINKS TO Example 10

Write each of these in simplest index form.

a $2^3 \times 2^4$ **b** $5^6 \times 5^7$ **c** $8^6 \times 8$ **d** $10^3 \times 10^3$ **e** $2^5 \times 2^6$

f 3×3^4 **g** $11^2 \times 11^3$ **h** 10×10^4 **i** $7^4 \times 7^2$ **j** $9^3 \times 9^4$

In simplest index form, $3^4 \times 3^5$ is equal to

A 3^9 **B** 3^{20} **C** 6^6 **D** 6^9 **E** 9^9

LINKS TO Example 11

Find the missing index that makes the equation true.

a $2^3 \times 2^7 = 2^{\square}$ **b** $3^{\square} \times 3^5 = 3^8$ **c** $5^6 \times 5^{\square} = 5^{15}$

d $7^2 \times 7^7 = 7^{\square}$ **e** $2^8 \times 2^{\square} = 2^{12}$ **f** $4^6 \times 4^{\square} = 4^{11}$

g $10^3 \times 10^{13} = 10^{\square}$ **h** $6^8 \times 6^{\square} = 6^9$ **i** $10^6 \times 10^{\square} = 10^{23}$

The missing index that makes the equation $12^{\square} \times 12^3 = 12^6$ true is

A 2 **B** 3 **C** 9 **D** 18 **E** 36

LINKS TO Example 12

Write each of these in simplest index form.

a $\frac{2^{12}}{2^6}$ **b** $3^8 \div 3^3$ **c** $\frac{5^{13}}{5^9}$ **d** $\frac{10^{20}}{10^{10}}$ **e** $7^7 \div 7$ **f** $\frac{8^5}{8}$

g $3^{12} \div 3^4$ **h** $\frac{5^{15}}{5^3}$ **i** $\frac{6^3}{6}$ **j** $\frac{4^8}{4^4}$ **k** $\frac{3^7}{3^4}$ **l** $10^{18} \div 10^{12}$

LINKS TO Example 13

Find the value of the missing index.

a $\frac{5^{\square}}{5^8} = 5^4$ **b** $\frac{3^{15}}{3^{\square}} = 3^9$ **c** $\frac{4^{12}}{4^{\square}} = 4^2$ **d** $5^7 \div 5^4 = 5^{\square}$

e $3^{\square} \div 3^{11} = 3^5$ **f** $2^{21} \div 2^{\square} = 2^{14}$ **g** $\frac{5^{14}}{5^6} = 5^{\square}$ **h** $10^{17} \div 10^{13} = 10^{\square}$

i $\frac{2^{20}}{2^{\square}} = 2^{10}$ **j** $\frac{6^{15}}{6^{\square}} = 6^3$ **k** $\frac{7^{24}}{7^{\square}} = 7^{12}$ **l** $10^{16} \div 10^8 = 10^{\square}$

The missing index that makes the equation $\frac{6^{\square}}{6^4} = 6^3$ true is

A 7 **B** 12 **C** 1 **D** 6 **E** 3

Simplify each of the following, expressing your answers in index form.

a $3^3 \times 2^4 \times 3^2 \times 2^8$ **b** $5^4 \times 2^5 \times 5^2 \times 2$ **c** $2^9 \times 7^3 \times 2^2 \times 7^5$

d $\frac{4^3 \times 9^6}{4 \times 9^4}$ **e** $\frac{2^7 \times 7^6}{2^4 \times 7^3}$ **f** $\frac{3^9 \times 11^3}{3^5 \times 11}$

LINKS TO Example 14

Simplify each of the following in index form, and then evaluate. Hint: simplify the numerator and /or denominator first.

a $\frac{3^4 \times 3^7}{3^8}$ **b** $\frac{5^6 \times 5^3}{5^5 \times 5^2}$ **c** $\frac{8^5}{8^2 \times 8}$ **d** $\frac{6^4 \times 6^5}{6^7}$

LINKS TO Examples 15, 16

Simplify each of the following.

a 5^0 **b** 20^0 **c** 7^0 **d** 1^0 **e** 17^0

f 100^0 **g** $4^0 + 3^0$ **h** $5^0 + 13^0$ **i** 7×6^0 **j** $3^0 \times 3^2$

The value of $2^0 + 6 \times 5^0 - 4^0$ is

A 1 **B** 4 **C** 0 **D** 6 **E** 3

exercise 3.2 challenge

The prefix ‘kilo’ normally means one thousand (10^3), ‘mega’ means one million (10^6) and ‘giga’ means one billion (10^9). When we use these prefixes for the amount of computer memory, they do not mean quite the same.

1 kilobyte $= 2^{10}$ bytes
1 megabyte $= 2^{10}$ kilobytes
1 gigabyte $= 2^{10}$ megabytes

a Use your calculator to evaluate 2^{10}
b Using index notation, how many bytes in a megabyte?
c Use your calculator to evaluate your answer from part **b**.
d Using index notation, how many bytes in a gigabyte?
e Use your calculator to evaluate your answer from part **d**.

Evaluate each of the following.

a $\frac{2^3}{2^4}$ **b** $\frac{3^8}{3^{10}}$

3.3 Powers with brackets

Consider a power of a number raised to a power, for example, $(2^3)^4$. If we use the multiplication index law to simplify $(2^3)^4$ we find that it equals 2^{12}.

$$\begin{aligned}(2^3)^4 &= 2^3 \times 2^3 \times 2^3 \times 2^3\\ &= 2^{3+3+3+3} \qquad \text{(Multiplication index law)}\\ &= 2^{3\times 4}\\ &= 2^{12}\end{aligned}$$

We can see that there is a shortcut here. We can simply multiply the two indices.

Power of a power law

To raise a power to another power, multiply the indices.

For example, $(3^2)^4 = 3^{2\times 4} = 3^8$

Example 17

Simplify.

a $(2^3)^2$ **b** $(10^2)^3$

Working	Reasoning
a $(2^3)^2$ $= 2^{3\times 2}$ $= 2^6$	When a power is raised to another power, multiply the indices. We can obtain the same result by using the multiplication index law. $(2^3)^2 = 2^3 \times 2^3$ $= 2^{3+3}$ $= 2^6$
b $(10^2)^3$ $= 10^{2\times 3}$ $= 10^6$	We can obtain the same result by using the multiplication index law. $(10^2)^3 = 10^2 \times 10^2 \times 10^2$ $= 10^{2+2+2}$ $= 10^6$

Example 18

Find the missing index that makes the equation $(10^{\square})^6 = 10^{30}$ true.

continued

Example 18 continued

Working

$(10^{\square})^6 = 10^{30}$

$10^{\square \times 6} = 30$

$10^{5 \times 6} = 30$

$\square = 5$

The missing index is 5.

Reasoning

When a power is raised to another power, multiply the indices.

A product inside brackets may be raised to a power, for example, $(2 \times 3)^4$. If we write $(2 \times 3)^4$ as the product of all its factors, we see that it can be written as $2^4 \times 3^4$.

$$\begin{aligned}(2 \times 3)^4 &= 2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= 2^4 \times 3^4\end{aligned}$$

Power of a product law

A power of a product is equal to the product of the powers.

For example, $(3 \times 4)^2 = 3^2 \times 4^2$

Example 19

Evaluate $4^3 \times 5^3$ without a calculator by first writing it as the power of a product.

Working

$$\begin{aligned}4^3 \times 5^3 &= (4 \times 5)^3 \\ &= 20^3 \\ &= 20 \times 20 \times 20 \\ &= 8000\end{aligned}$$

Reasoning

The product of 4^3 and 5^3 equals the product (4×5) raised to the power 3.

Power of a fraction or quotient law

A power of a fraction or quotient is equal to the quotient of the powers.

For example, $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$

Example 20

Expand the brackets.

a $\left(\frac{3}{4}\right)^3$ **b** $\left(1\frac{1}{4}\right)^2$

Working

a $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3}$

$= \frac{27}{64}$

Reasoning

3 and 4 are both raised to the power 3.

b $\left(1\frac{1}{4}\right)^2 = \left(\frac{5}{4}\right)^2$

Convert the mixed number to an improper fraction.

$= \frac{25}{16}$

5 and 4 are both squared.

$= 1\frac{9}{16}$

Convert the improper fraction to a mixed number.

Tech tip

The TI-30XB MultiView calculator can be used to show the raising of a power to a power. For example, to find $(2^3)^2$ (example 16 part a), type:

(2 ^ 3) ▶ ^ 2 ▶ enter

exercise 3.3

LINKS TO Example 17

Write each of these in simplest index form.

a $(2^3)^2$ **b** $(2^4)^3$ **c** $(3^2)^2$ **d** $(4^2)^2$ **e** $(5^3)^2$

f $(10^5)^3$ **g** $(6^2)^4$ **h** $(10^8)^2$ **i** $(7^2)^2$ **j** $(3^5)^4$

$(5^4)^3$ can be simplified to

A 60 **B** 5^{12} **C** 5^7 **D** 12 **E** 20^3

LINKS TO Example 18

Find the missing index that makes each equation true.

a $(3^2)^3 = 3^{\square}$ **b** $(2^{\square})^6 = 2^{18}$ **c** $(5^4)^{\square} = 5^{12}$ **d** $(3^{\square})^7 = 3^{21}$

e $(10^{\square})^5 = 10^{25}$ **f** $(2^6)^{\square} = 2^{12}$ **g** $(6^6)^6 = 6^{\square}$ **h** $(3^4)^{\square} = 3^{12}$

i $(2^{11})^3 = 2^{\square}$ **j** $(4^{\square})^3 = 4^9$ **k** $(10^6)^{\square} = 10^{48}$ **l** $(3^2)^3 = 3^{\square}$

The missing index that makes the equation $(5^{\square})^6 = 5^{18}$ true is

A 12 **B** 3 **C** 15 **D** 60 **E** 6

LINKS TO Example 19

Expand these expressions but do not evaluate.

a $(2 \times 4)^3$ **b** $(3 \times 5)^2$ **c** $(3 \times 7)^4$ **d** $(2 \times 5)^7$

e $(4 \times 7)^5$ **f** $(2 \times 6)^0$ **g** $(4 \times 5)^{10}$ **h** $(2 \times 11)^3$

i $(5 \times 7)^0$ **j** $(7 \times 10)^2$ **k** $(3 \times 8)^3$ **l** $(3 \times 11)^0$

LINKS TO Example 19

Evaluate each of the following products of powers without using a calculator by first writing each as the power of a product.

a $2^4 \times 5^4$ **b** $4^2 \times 5^2$ **c** $2^2 \times 25^2$ **d** $5^3 \times 6^3$

e $4^5 \times 25^5$ **f** $4^2 \times 15^2$ **g** $16^2 \times 5^2$ **h** $8^2 \times 125^2$

LINKS TO Example 20

Evaluate each of the following.

a $\left(\frac{1}{2}\right)^2$ **b** $\left(\frac{2}{5}\right)^2$ **c** $\left(\frac{2}{3}\right)^3$ **d** $\left(\frac{3}{4}\right)^2$ **e** $\left(\frac{5}{8}\right)^2$

f $\left(\frac{3}{10}\right)^2$ **g** $\left(\frac{1}{3}\right)^3$ **h** $\left(1\frac{1}{2}\right)^2$ **i** $\left(2\frac{1}{2}\right)^3$ **j** $\left(2\frac{1}{3}\right)^2$

Evaluate

a $(3^5)^0$ **b** $(7^0)^4$

exercise 3.3 challenge

Evaluate

a $\frac{(5^2)^3}{5^4}$ **b** $\frac{(10^4)^5}{(10^6)^3}$ **c** $\frac{(2^4)^2 \times 2^5}{(2^2)^6}$ **d** $\frac{(3^2)^5}{(3^0)^2 \times (3^2)^0}$

Find the value of each pronumeral.

a $(2^4)^a = 4096$ **b** $(3^b)^5 = 59\,049$

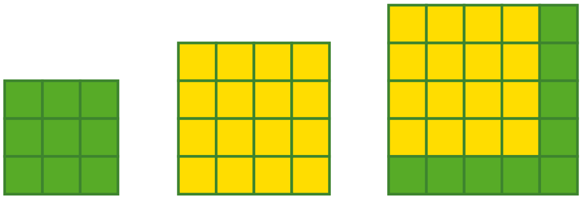
Write each of these products of powers as a power of a product.

a $3^2 \times 4^3$ **b** $9^3 \times 5^2$

3.4 Exploring sums of squares and cubes

Mathematicians have known for over 2500 years that certain square numbers add to give another square number. For example, $9 + 16 = 25$, that is, $3^2 + 4^2 = 5^2$. The set of three whole numbers 3, 4 and 5 are known as a **Pythagorean triad**. The word *triad* means a set of three. The numbers are called Pythagorean triads because the Greek mathematician Pythagoras is reputed to have investigated these numbers, although it is believed that Babylonian mathematicians were aware of these triads about 1500 years earlier.

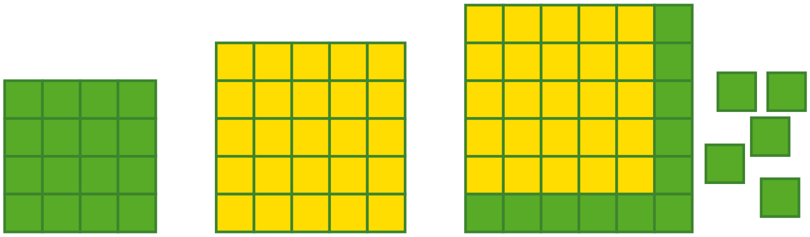
Finding a Pythagorean triad is like finding two squares that can be rearranged to make a third, larger square. For example:



$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

Only certain sets of three whole numbers are of this type. If, for example, we add 4^2 and 5^2, that is 16 and 25, we obtain 41, which is not a square number. As shown below, we cannot arrange the two squares to make a third square without having some leftover pieces.



$$4^2 + 5^2$$

$$16 + 25 = 41$$

A Pythagorean triad is a set of three whole numbers in which the sum of the squares of the two smaller numbers is equal to the square of the third number.

Example 21

Find whether each of these sets of three numbers is a Pythagorean triad.

a 3, 7, 8 **b** 5, 12, 13

Working	Reasoning
a $3^2 + 7^2 = 9 + 49$ $= 58$ But $8^2 = 64$, so $3^2 + 7^2 \neq 8^2$	If a set of three numbers is a Pythagorean triad, the sum of the squares of the two smaller numbers must equal the square of the third number.
The set of numbers 3, 7 and 8 is not a Pythagorean triad.	The sum of the squares of 3 and 7 is not equal to the square of 8.
b $5^2 + 12^2 = 25 + 144$ $= 169$ $13^2 = 169$ $5^2 + 12^2 = 13^2$ The set of numbers 5, 12, 13 is a Pythagorean triad.	The sum of the squares of 5 and 12 is equal to the square of 13.

Mathematicians tried to discover whether it was possible to find the sum of two cubes that would equal another cube. For example, $6^3 + 8^3$ is almost equal to 9^3 because $6^3 + 8^3 = 216 + 512 = 728$ and $9^3 = 729$. However, no-one was ever able to find two cube numbers whose sum was exactly equal to another cube number. In fact, the 17th century French mathematician Pierre de Fermat claimed that he had proved it was not possible to find any numbers that satisfied the equation $x^n + y^n = z^n$ if n was greater than 2.

Unfortunately Fermat died without leaving any written proof. For over 350 years mathematicians tried to prove whether Fermat was correct, but it was not until 1994 that the English mathematician Andrew Wiles produced a proof that Fermat was in fact correct. Wiles worked on his proof of Fermat's conjecture for eight years!

So, although we can find many whole numbers that satisfy the Pythagorean triad equation, $x^2 + y^2 = z^2$, there are no whole numbers that satisfy the equations $x^3 + y^3 = z^3$ or $x^4 + y^4 = z^4$ and so on. In 2000, which was the International Year of Mathematics, the Czech Republic honoured Pierre de Fermat and Andrew Wiles on the stamp shown here.

exercise 3.4

LINKS TO Example 21

Which of these sets of three numbers is a Pythagorean triad?

a 8, 15, 17 **b** 6, 11, 15 **c** 7, 24, 25

d 9, 13, 16 **e** 9, 17, 20 **f** 9, 40, 41

LINKS TO Example 21

We have seen that the set of numbers 3, 4 and 5 is a Pythagorean triad. Each of the following sets of numbers are multiples of 3, 4 and 5. Find whether each of these sets of numbers is also a Pythagorean triad.

a 6, 8, 10 **b** 9, 12, 15 **c** 12, 16, 20 **d** 15, 20, 25

LINKS TO Example 21

Using what you found in question 2, complete the following Pythagorean triads.

a 18, ___, 30 **b** ___, 28, 35 **c** ___, 36, 45 **d** 24, ___, 40

a The set of numbers 5, 12, 13 is a Pythagorean triad. Choose a set of multiples of 5, 12 and 13, for example, 10, 24, 26 and find if they are also a Pythagorean triad. Repeat for two other multiples of 5, 12 and 13.

b Repeat part **a** for the Pythagorean triad 8, 15, 17.

c Write a sentence describing what you have found about multiples of Pythagorean triads.

a Find the squares of 1, 11 and 111.

b Using the pattern you observe in 1^2, 11^2 and 111^2, find 111111^2 and 111111111^2.

50 can be written as $1^2 + 7^2$ or as $5^2 + 5^2$. Write each of the following numbers as the sum of two squares in two different ways. Hint: make a list of square numbers up to $25^2 = 625$.

a 65 **b** 85 **c** 130 **d** 265 **e** 290 **f** 650

The digits 1, 6 and 9 can be arranged to form three different square numbers. Write each of the three square numbers in index form.

In this question a spreadsheet is used to explore the sum of the cubes of whole numbers.

a In a new spreadsheet put a list of whole numbers in column A. To do this, start by putting 1 in cell A1. Then in cell A2, enter the formula: **=A1+1**. Drag the formula in cell A2 down to cell A40. You should now have the numbers 1, 2, 3 … 40 in column A.

b Now produce a set of ‘cube’ numbers in column B. What formula will you need to put in cell B1?

c Look for a pattern in the sum of the cube numbers in column B. Write each sum in index form.

d Notice that the sums of the cubes are the squares of the numbers 1, 3, 6, 10, 15. Investigate the pattern in these numbers. Hint: double each of the numbers 1, 3, 6, 10, 15 then look at the factors. Can you write a general expression in terms of n for the sum of the cubes of the first n whole numbers?

exercise 3.4 challenge

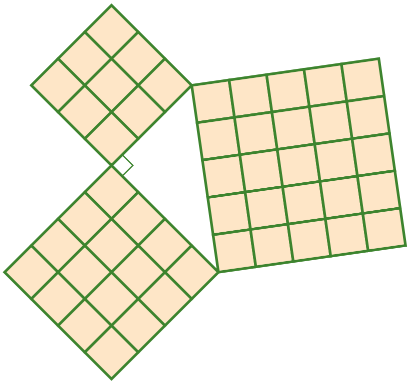
● Write each of the following numbers as the sum of three cubes.

a 73 **b** 134 **c** 92 **d** 99

● **a** Evaluate $10^2 + 10^2 + 15^2$ then rewrite the number as the sum of *two* squares.

b Evaluate the squares of 10, 11, 12, 13 and 14 then use + and = signs to write a relationship between the five squares.

● The longest side of a right-angled triangle, that is, the side opposite the right angle, is called the **hypotenuse**. In the diagram, the three sides of the triangle are 3, 4 and 5 units long. By constructing a square on each side, we can see the relationship $3^2 + 4^2 = 5^2$. In other words, the sum of the squares of the two shorter sides is equal to the square of the hypotenuse.

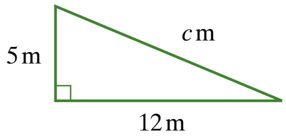


This relationship applies only to **right-angled triangles** and is known as **Pythagoras' Theorem**, although it may have been discovered much earlier by Babylonian mathematicians.

We can write it in the form $a^2 + b^2 = c^2$, where a and b are the lengths of the two shorter sides and c is the length of the hypotenuse.

Using the rule $a^2 + b^2 = c^2$, calculate the lengths of the hypotenuse for each of these right-angled triangles. In each case, write an equation in the form $c^2 = \ldots$ When you have found c^2, you will need to take the square root to find c. Sometimes c^2 will not be a square number so you will need to use your calculator to find the square root of c^2 and round it to a sensible number of decimal places.

a



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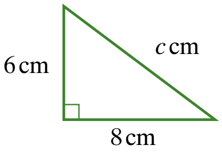

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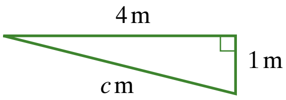

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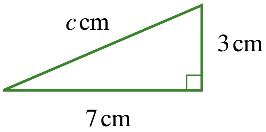
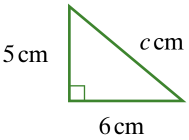
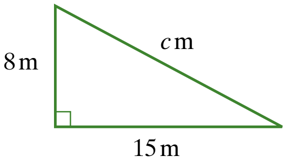


e



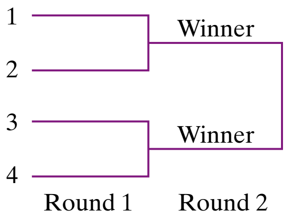
f



Analysis task

Knockout tournaments

Four teams, A, B, C and D, were playing against each other in a knockout tournament. In the first round, A and B played against each other and C and D played against each other. The winners from this first round then played against each other in the second round as shown in the diagram on the right. Note that there are two matches in round 1 and one match in round 2.



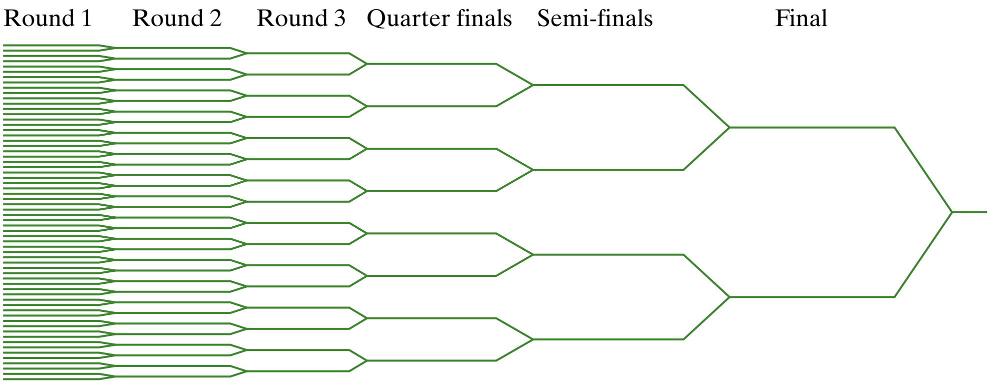
a Draw a similar diagram to show the matches and rounds for eight teams.

b Show why this sort of knockout tournament would not work with six teams.

c What is the next number after 8 for which a knockout tournament of this type would work?

d Copy and complete the following table to show how many matches and rounds would be needed for 4 and 8 teams.

Number of teams	Number of matches				Number of rounds
	Round 1	Round 2	Round 3	Total	
4 (= 2^2)					
8 (= 2^3)					



e How many teams are represented on this knockout diagram?

f What is the total number of matches played?

g The Australian Tennis Open normally has 128 players. How many rounds would be needed? Explain.

h List the number of matches in each round for the Australian Tennis Open.

i What is the total number of matches for the Australian Tennis Open?

Review Indices

Summary

4^3 ← Index or exponent
↑
Base

x^4 ← Index or exponent
↑
Base

$4^5 = 4 \times 4 \times 4 \times 4 \times 4$

Index form — Factor form

- A composite number can be written as a product of its prime factors; for example, $72 = 2^3 \times 3^2$.
- Composite numbers can be broken down into prime factors; that is, factors that are prime numbers. 72 can be written as $2 \times 2 \times 2 \times 3 \times 3$, so the prime factors of 72 are 2 and 3.
- A factor ladder can be used to find the prime factors of a composite number.

Using index notation to find highest common factors

- Step 1: Look for the prime factors that are common to both numbers.
- Step 2: For each common prime factor, find the power that is common to both numbers, that is, the lower of the two powers.
- Step 3: Multiply the lower of the two powers of each common prime factor.

Using index notation to find lowest common multiples

- Step 1: Look for all the prime factors that occur in the numbers.
- Step 2: For each prime factor choose the higher of two powers.
- Step 3: Multiply the higher of the powers of each prime factor occurring in either number.

Multiplication index law

- To multiply powers of the same base, add the indices.
  For example, $2^3 \times 2^5 = 2^{3+5} = 2^8$

Division index law

- To divide powers of the same base, subtract the indices.
  For example, $3^7 \div 3^2 = 3^{7-2} = 3^5$

Zero index

- Any non-zero number or pronumeral raised to the power zero is equal to 1.
  For example, $3^0 = 1$

Power of a power law

- To raise a power to a power, multiply the indices.
  For example, $(3^2)^4 = 3^{2\times 4} = 3^8$

Power of a product law

- A power of a product is equal to the product of the powers.
  For example, $(3 \times 4)^2 = 3^2 \times 4^2$

Power of a fraction or quotient law

- A power of a fraction or quotient is equal to the quotient of the powers.
  For example, $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

base
composite number
exponent
factor form
factor ladder
hypotenuse
index
index form
power
prime factor
prime number
Pythagorean triple

Revision

Multiple-choice questions

● The prime factors of 1764 are

A 2, 3 and 5 **B** 3, 5 and 11 **C** 2, 3, 7 and 11
D 2, 3 and 7 **E** 2, 3, 7 and 13

● Written as the product of its prime factors, 4752 is equal to

A $2^3 \times 3^2 \times 7$ **B** $2^3 \times 3^3 \times 13$ **C** $2^4 \times 3^3 \times 11$
D $2^4 \times 3^3 \times 7^2$ **E** $2^3 \times 3^4 \times 7 \times 11$

● $3^2 \times 4^3 \times 3 \times 4^2$ is equal to

A 144^7 **B** 144^6 **C** 14^7 **D** $9^3 \times 16^5$ **E** $3^3 \times 4^5$

● The highest common factor of $2^3 \times 3^2 \times 5 \times 7^2$ and $2^2 \times 3 \times 5 \times 7^3$ is

A 210 **B** $2 \times 3 \times 5 \times 7$ **C** $2^3 \times 3^2 \times 5 \times 7^3$
D $2^2 \times 3 \times 5 \times 7^2$ **E** $2^5 \times 3^3 \times 5^2 \times 7^5$

● The lowest common multiple of $2^3 \times 3^2 \times 5 \times 7^2$ and $2^2 \times 3 \times 5 \times 7^3$ is

A 210 **B** $2 \times 3 \times 5 \times 7$ **C** $2^3 \times 3^2 \times 5 \times 7^3$
D $2^2 \times 3 \times 5 \times 7^2$ **E** $2^5 \times 3^3 \times 5^2 \times 7^5$

Short-answer questions

● Write in index notation.

a $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ **b** $3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4$

● Write in factor (expanded) form.

a 5^4　　**b** $2^4 \times 5^3 \times 7^2$

● For each of these numbers

i use a factor ladder to find the prime factors.

ii write the number as the product of its prime factors in index form.

a 1800　　**b** 2160　　**c** 8250　　**d** 12 000

● Use your answers to question 8 to find the highest common factor of

a 1800 and 2160　　**b** 2160 and 8250

● Use your answers to question 8 to find the lowest common multiple of

a 1800 and 2160　　**b** 1800 and 8250

● Simplify, leaving your answers in index form.

a $3^7 \times 3^4$　　**b** $8^{10} \div 8^5$　　**c** $(3^2)^3$　　**d** $\dfrac{5^3 \times 5^2}{5^4}$

e $(5^2)^2$　　**f** $4^5 \times 4^7 \div 4^9$　　**g** $3^3 \times 5^2 \times 3 \times 5^3$　　**h** $2^4 \times 3^4$

● Evaluate.

a $\left(\dfrac{1}{4}\right)^2$　　**b** $\left(\dfrac{2}{7}\right)^2$　　**c** $\left(1\dfrac{3}{4}\right)^2$　　**d** $\left(\dfrac{2}{3}\right)^3$

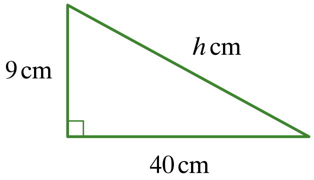
● Evaluate.

a 5^0　　**b** $3^0 + 7^0$　　**c** 3×2^0

● Which of these sets of numbers are Pythagorean triplets? Justify your answers.

a 6, 8, 10　　**b** 4, 5, 7　　**c** 10, 24, 26　　**d** 8, 15, 17

● Find the length of the hypotenuse of this right angled triangle.



Extended-response question

● Lucy was organising a reunion of the 166 other students who had been in her year level at primary school. She decided to phone five students and ask each of them to phone five other students, who in turn would phone five more students. Each student who received a phone call was given a list of five names so that no student received more than one phone call.

a After the three rounds of calls how many students altogether would have received phone calls about the reunion?

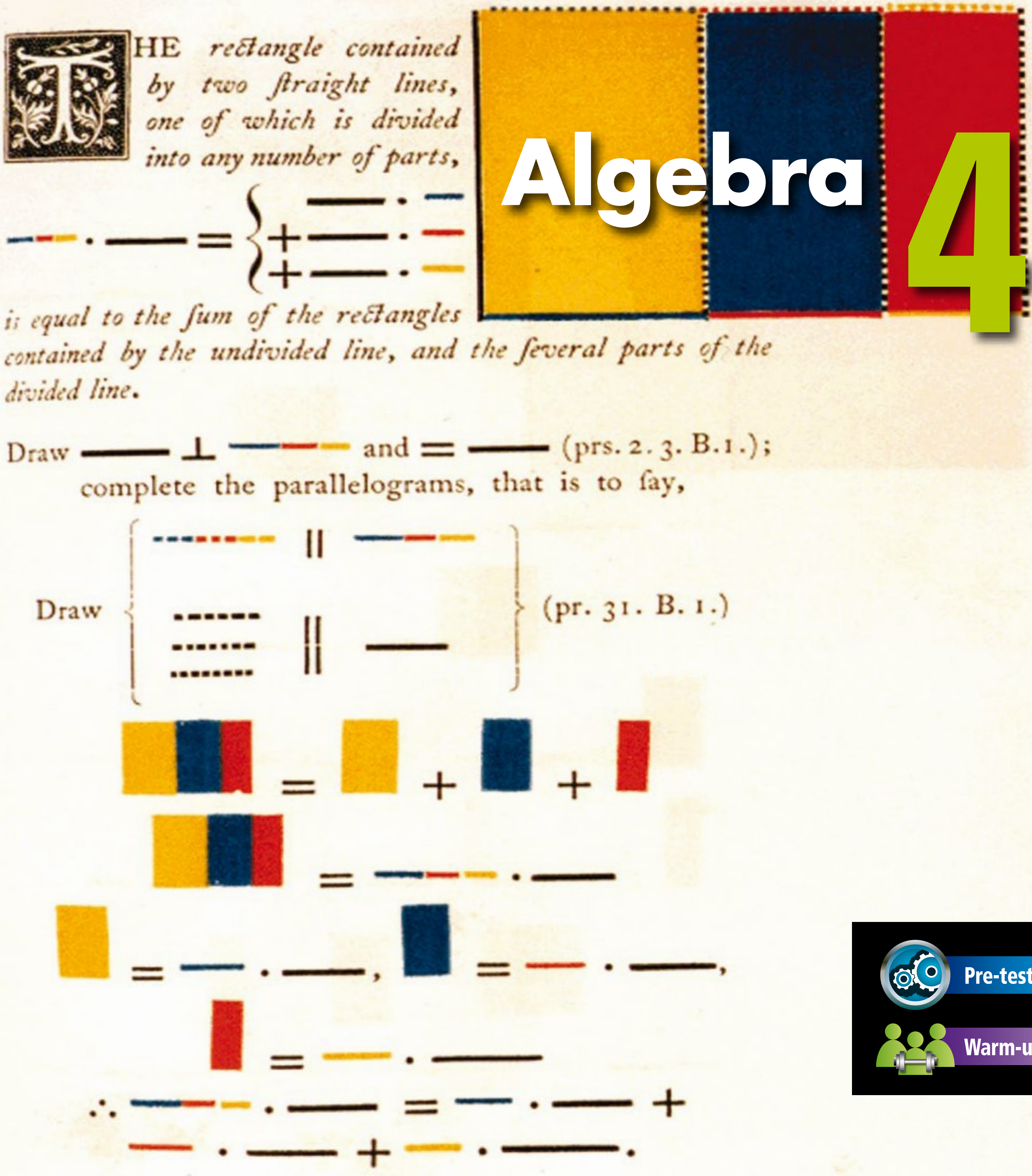
b After the three rounds of calls, how many students would still need to be contacted?

c If Jake was using the same method to contact 400 people, how many people should each person call assuming that there were three rounds of calls and that as many as possible of the 400 people would be contacted?

d How many people would still need to be contacted?

Algebra

4



THE rectangle contained by two ſtraight lines, one of which is divided into any number of parts,

= + +

is equal to the ſum of the rectangles contained by the undivided line, and the ſeveral parts of the divided line.

Draw ⊥ and = (prs. 2. 3. B. 1.); complete the parallelograms, that is to ſay,

Draw || || (pr. 31. B. 1.)

= + +

=

= , =

=

∴ = + + .

Pre-test

Warm-up

The page shown here is from a 19th century illustrated version of Euclid's famous set of 13 books titled *The Elements*. Euclid was an ancient mathematician who lived about 2500 years ago in the city of Alexandria. The page shows a proof of the distributive law using areas of rectangles. The whole area is made up of the yellow rectangle plus the dark blue rectangle plus the red rectangle. The area of each rectangle is found by multiplying its length by its width. In the diagram above a multiplication sign is represented by a dot. In algebra we could write $a(b + c + d) = ab + ac + ad$

4.1 Algebraic terms and expressions

Algebra is a way of using pronumerals to generalise number patterns or to represent a relationship between quantities that can vary. Algebra allows us to solve real-world problems but first we have to learn the techniques of algebra. A pronumeral is a letter that we use to stand for a number.

The word pronumeral comes from the Latin ‘pro’ meaning ‘for’ and ‘numeralis’ meaning ‘number’.

In Year 7 several words that are used in algebra were introduced.

A term may be

- a number, for example, 24
- a pronumeral, for example, t
- pronumerals and a number multiplied or divided, for example, $4t$ or $-6ab$ or $\frac{h}{3}$

Note that terms can be positive or negative. In Year 7 we worked mainly with positive terms. This year we will work with negative terms as well.

A term that is a number is called a **constant term** or just a **constant**. This is because its value stays the same. For example, the term 24 is a constant.

For terms with pronumerals, the number at the beginning of each term is called its **coefficient**.

The coefficient of the term $\frac{2x}{3}$ is a fraction because x has been multiplied by 2 and divided by 3. We can also think of this as $\frac{2}{3}$ of x so we could write the term as $\frac{2}{3}x$.

Just as with multiplication of numbers, the **Commutative law** applies to terms in algebra, that is, the order does not matter when pronumerals are multiplied. For example, $ab = ba$.

If there is more than one pronumeral in a term, these are written in alphabetic order, for example, $5ab$.

A coefficient may be positive or negative and may be a whole number, a fraction or a decimal. For example, the coefficient of the term $-3x$ is -3.

Example 1

State the coefficient of each of the following terms.

a $4t$ **b** x **c** $-6ab$ **d** $\frac{h}{3}$

continued

Example 1 continued

Working	Reasoning
a The coefficient of $4t$ is 4.	t has been multiplied by 4.
b The coefficient of x is 1.	x is the same as $1x$.
c The coefficient of $-6ab$ is -6.	The coefficient includes the sign in front of the term.
d The coefficient of $\frac{h}{3}$ is $\frac{1}{3}$.	We could also write $\frac{h}{3}$ as $\frac{1}{3}h$ or $\frac{1h}{3}$.

Example 2

Simplify each of the following.

a $8 \times n$

b $d \times 9$

c $-4 \times c$

d $k \times j$

e $y \times y$

f $b \div (-3)$

Working	Reasoning
a $8 \times n = 8n$	The multiplication sign is not included.
b $d \times 9 = 9d$	When a pronumeral and a number are multiplied, the coefficient is written first.
c $-4 \times c = -4c$	A negative number multiplied by a positive pronumeral gives a negative term.
d $k \times j = jk$	It is the custom to write pronumerals in alphabetical order when they are multiplied.
e $y \times y = y^2$	When a pronumeral is multiplied by itself we use the 'squared' symbol.
f $b \div (-3) = \frac{-b}{3}$ or $-\frac{b}{3}$	We normally put the negative sign before the fraction or in the numerator.

Expressions

An expression is made up of terms that are added or subtracted. For example, $2m + 5$ is an expression made up of the terms $2m$ and 5.

In chapter 1 we saw that adding a negative number is equivalent to subtracting the opposite positive number. Therefore, we can think of a term with a negative coefficient as the addition of a negative term. For example, we can think of $x - 2y$ as $x + (-2y)$ so the two terms in the expression are x and $-2y$.

Example 3

Consider the expression below.

$5x - 2xy + \frac{y}{3} - 6$

a How many terms are in the expression?

b What is the coefficient of the second term?

c What is the coefficient of y?

d What is the constant term?

Working	Reasoning
a There are four terms.	The four terms are $5x, -2xy, \frac{y}{3}$ and -6.
b The coefficient of the second term is -2.	We can think of $-2xy$ as $+(-2xy)$.
c The coefficient of y is $\frac{1}{3}$.	We can think of $\frac{y}{3}$ as $\frac{1}{3}y$.
d The constant term is -6.	The term -6 does not include a pronumeral.

Writing algebraic expressions

We have already seen that multiplication signs are normally left out in algebraic terms and expressions. For example, $4 \times a$ is written as $4a$. We have also seen that divisions are written as fractions, for example, $\frac{x}{3}$ rather than $x \div 3$.

Example 4

Simplify the following expressions.

a $2 \times x + 3 \times y$ **b** $3 \times a + (-4) \times (b - 3)$ **c** $a - (-2) \times (b + 1)$

Working	Reasoning
a $2 \times x + 3 \times y$ $= 2x + 3y$	Multiplication signs are put between a coefficient and a pronumeral.
b $3 \times a + (-4) \times (b - 3)$ $= 3a - 4(b - 3)$	Multiplication signs are put between a coefficient and a pronumeral or between a coefficient and a bracketed expression. $+(-4)$ can be simplified to -4.
c $a - (-2) \times (b + 1)$ $= a + 2(b + 1)$	$-(-2)$ can be simplified to $+ 2$.

It is often necessary to include brackets in an expression to ensure that the required order of operations is carried out. For example, if the expression $x + 3$ is multiplied by 4, we write $4(x + 3)$, not $4x + 3$.

Example 5

State whether the expressions are equivalent.

a $\frac{x+3}{4}$ and $x + 3 \div 4$ **b** $x - 5 \times 2$, $x - 10$ and $2(x - 5)$

Working	**Reasoning**
a $\frac{x+3}{4}$ and $x + 3 \div 4$ are not equivalent.	$\frac{x+3}{4}$ is equivalent to $(x + 3) \div 4$. $x + 3 \div 4$ means $x + \frac{3}{4}$.
b $x - 5 \times 2$ and $x - 10$ are equivalent expressions. $2(x - 5)$ is not equivalent to $x - 5 \times 2$ and $x - 10$.	Multiplication is done before subtraction, so $x - 5 \times 2 = x - 10$. $2(x - 5)$ means $2x - 10$.

We can use algebra to turn stories or situations involving mathematical operations into algebraic expressions.

Example 6

Using n as the starting number, write an expression for each of the following.

a Increase the number by 4
b Take 3 from the number
c Take the number from 3
d Add 7 to the number then multiply the result by -2
e Multiply the number by 2 then divide the result by 5
f Subtract the number from 12 then divide the result by 3

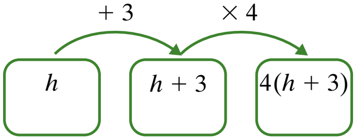
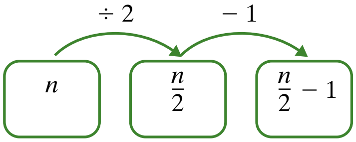
Working	**Reasoning**
a $n + 4$	Increase the number by 4 means adding 4 to the number.
b $n - 3$	3 is subtracted from the number.
c $3 - n$	This is not the same as $n - 3$.
d $-2(n + 7)$	7 is added to the number first then the result is multiplied by -2.
e $\frac{2n}{5}$	This could also be written as $\frac{2}{5}n$.
f $\frac{12 - n}{3}$	The number is subtracted from 12 first, then the result is divided by 3.

Example 7

For each of the following, define a pronumeral to stand for the number then write an algebraic expression to describe the situation.

a Choose a number. Add 3 to it, and then multiply the result by 4. What is the result?

b Andy has a number of chocolate biscuits for his brothers. He shares them equally between his two younger brothers. Then from each he takes back 1 biscuit for his friend. What is the share of one brother?

Working	**Reasoning**
a Let h be the chosen number. The result is $4(h + 3)$.	Choose any letter and explain that it stands for the chosen number. This is called ‘defining’. 
b Let n stand for the number of chocolate biscuits that Andy starts with. Each share is $\frac{n}{2} - 1$.	Define, or choose and explain, the pronumeral. Sharing between 2 brothers is written as a division by 2. This is $\frac{n}{2}$ for each brother. Taking back 1 biscuit from each share means that each share is 1 smaller, so subtract 1. 

If the situation involves units, for example, metres or litres, the units do not become part of the expression.

exercise 4.1

LINKS TO Example 1

State the coefficient of each of the following terms.

a $3a$ **b** $-2m$ **c** $5ab$ **d** $\frac{d}{4}$

e $-7xy$ **f** h **g** $-\frac{2x}{3}$ **h** $-n$

LINKS TO Example 2

Simplify the following terms using algebraic notation.

a $2 \times x$ **b** $5 \times a \times b$ **c** $1 \times y$ **d** $\frac{1}{2} \times x$

e $-1 \times m$ **f** $k \div 3$ **g** $\frac{4}{5} \times b$ **h** $a \div (-2)$

i $n \times n$ **j** $-5 \times d \div 4$ **k** $b \times a$ **l** $d \div (-4)$

LINKS TO Example 3

For the algebraic expression $3a - 7b + 8c - 1$,

a how many terms are there?
b what is the constant term?
c what is the coefficient of the second term?
d what is the pronumeral in the third term?

LINKS TO Example 3

For the algebraic expression $\frac{2f}{5} + \frac{g}{4} - \frac{3}{7}$,

a how many terms are there?
b what is the constant term?
c what is the coefficient of the second term?
d what is the pronumeral in the first term?

LINKS TO Example 3

How many terms are there in each of the following expressions?

a $3u + 11$
b $12a - 5b + 6$
c $4j + 7 + 7k$
d $8w + x - 2y + 3z$
e $n - 1$
f $2p - 3q + r$
g $1.3d - 2e + 0.5f - 2.4$
h $y^2 + 6y$
i $9 - 12m$
j $4y + 6z - 10$
k $xy + 5y - x + 7$
l $22c - 55$

LINKS TO Example 3

State the constant term in each of the following.

a $9r - 4s + 1$
b $6a - 8$
c $7 + 4f - g + 5h$
d $3b + 2 + 9c$
e $11j - 3k + 4$
f $p - 3$
g $15 - 9m$
h $pq + 8p - q + 6$
i $25h - 30$
j $b^2 + 6b - 7$
k $0.4x - 1.1y + 3.2$
l $2.5a - 1.4b + 0.3$

For each of the following expressions state the coefficient of the second term. (Remember to include the sign of negative coefficients.)

a $5d + 2e$
b $3m + n + 6o$
c $11a - 2b + 9$
d $7t + \frac{u}{6} - 2$
e $3x + \frac{5}{2}x^2 - 7$
f $36 - 24g$
g $4.1w - 2.5x + 0.3y - 1.7$
h $\frac{4a}{3} - \frac{b}{3}$
i $0.5a - 0.75b + 0.8$

In the expression $\frac{j}{3} - \frac{2k}{5} + \frac{3}{4}$, the term with the pronumeral k has the coefficient

A $\frac{1}{3}$
B 2
C -2
D $\frac{2}{5}$
E $-\frac{2}{5}$

LINKS TO Example 4

Express in simplest form.

a $3 \times a + 4 \times b$
b $-5 \times x - 3 \times y$
c $2 \times x - x \times x$
d $4 \times (x - 3)$
e $6 \times a + (-5) \times b$
f $-4 \times x + (-3) \times x \times x$
g $-3 \times m - (-2) \times (n + 1)$
h $m \div 3 + (-2) \times n \div 5$
i $c \div (-2) + (-2) \times d$
j $3 \times r \times (4 + 2 \times (-r))$

LINKS TO Example 5

Which of the following are true?

a $(x + 2) \times 3$ is the same as $3(x + 2)$
b $\frac{2}{3}x$ is the same as $\frac{2x}{3}$

c $\frac{x+2}{3}$ is the same as $x + 2 \div 3$

d $x - 3 \div 2$ is the same as $(x - 3) \div 2$

e $\frac{2x+5}{4}$ is the same as $(2x + 5) \div 4$

f $(2x + 3) \times 4$ is the same as $4(2x + 3)$

g $\frac{3}{4}x$ is the same as $\frac{3}{4x}$

h $x - 2 \times 3$ is the same as $3(x - 2)$

LINKS TO Example 6

For each of the following let n be the starting number then write each as an algebraic expression.

a Increase the number by 7.

b Take 5 from the number.

c Divide the number by 6.

d Multiply the number by 4 then subtract 6.

e Multiply the number by 3 then divide the result by 11.

f Add 5 to the number, and then multiply the result by 2.

g Subtract 9 from the number then divide the result by 10.

h Subtract the number from 9 then multiply the result by 8.

LINKS TO Example 7

I am a number, y.
Triple me.
Then add two to that.
Then divide that by seven.
Finally subtract four from that.
Write an algebraic expression for this number story.

Brad had some CDs. Choose a pronumeral for the number of CDs Brad started with.

a He had more than one of some so he gave away 4 of them. Write an expression for the number of CDs Brad then had.

b After his birthday the number of CDs Brad had was 3 times the number he had in part **a**. Write an expression for the final number of Brad's CDs in terms of the number he started with.

Wally made a number of pancakes. Choose a pronumeral for the number of pancakes Wally first made.

a He then made five more. Write an expression for the number of pancakes Wally then had in terms of the number that he first made.

b This total number of pancakes was shared evenly between three people. Write an expression for the number of pancakes in each person's share in terms of the number of pancakes Wally first made.

exercise 4.1 challenge

Write the sequence of steps used to build up this expression.

$$\frac{3(4n - 5) - 1}{5} + 7$$

4.2

Simplifying: adding and subtracting like terms

Working with like terms

As we know, $2a$ can be written as $2 \times a$ because it means 2 lots of a.

$2a$ can also be written in extended form as $a + a$.

Similarly $5a$ can be written as $a + a + a + a + a$. This means that, combining them,

$$2a + 5a = \underbrace{a + a} + \underbrace{a + a + a + a + a} = 7a$$

Similarly $3k$ can be written in extended form as $k + k + k$. Combining $2a$ and $3k$ we see the following:

$$2a + 3k = \underbrace{a + a} + \underbrace{k + k + k}.$$

The a's and k's are different and so they cannot be grouped together. $2a + 3k$ is as simple as it can be.

Only terms with identical pronumeral parts can be collected together. We call such term **like terms**.

Terms with different pronumeral parts cannot be collected together, as we saw with $2a + 3k$. We call such term **unlike terms**.

Some examples of like and unlike terms are shown below.

Like terms	**Unlike terms**
$6x$ and $-5x$	$6x$ and $-5y$
2 and 7	$2m$ and 7
$8xy$ and $4xy$ and $3yx$	$8x$ and $4xy$
$3y^2$ and $2y^2$	$3y$ and $2y^2$

Example 8

For each pair of terms, state whether it is like or unlike.

a 5 and 11

b $7d$ and $3e$

c $2k$ and $-9k$

d $4ab$ and $6ba$

e $8x$ and $5x^2$

continued

Example 8 continued

Working	Reasoning
a 5 and 11 are like terms.	Both of these terms are constants.
b $7d$ and $3e$ are unlike terms.	The pronumerals in each term are different.
c $2k$ and $-9k$ are like terms.	The pronumeral, k, is the same for both terms.
d $4ab$ and $6ba$ are like terms.	$4ab$ means $4 \times a \times b$ and $6ba$ means $6 \times b \times a$. The order in which numbers and pronumerals are multiplied does not matter.
e $8x$ and $5x^2$ are unlike terms.	In like terms the powers of pronumerals must be the same.

Example 9

Simplify the following expressions.

a $x + x$ **b** $x - x$

c $3x - 5x$ **d** $-7x - 4x$

Working	Reasoning
a $x + x = 2x$	There are two x's.
b $x - x = 0$	If you subtract x from x you will get zero.
c $3x - 5x = -2x$	Three x minus five x equals negative two x.
d $-7x - 4x = -11x$	Negative seven x minus four x equals negative eleven x.

Example 10

Where possible, simplify the following expressions.

a $8 + 12$ **b** $12d + 9e$ **c** $9k + (-11k)$

d $4ab + 6ba$ **e** $14xy + 22x$ **f** $8x^2 + 15x^2 - x^2$

Working	Reasoning
a $8 + 12 = 20$	Constants can be added.
b $12d + 9e$ cannot be made simpler.	$12d$ and 9e are unlike terms. They cannot be collected together.
c $9k + (-11k) = 9k - 11k = -2k$	Both terms have k as the pronumeral.

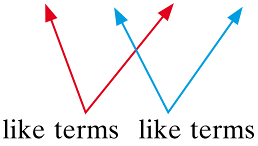
continued

Example 10 continued

Working	**Reasoning**
d $4ab + 6ba = 10ab$	$4ab$ and $6ba$ are like terms.
e $14xy + 22x$ cannot be simplified.	$14xy$ and $22x$ are unlike terms.
f $8x^2 + 15x^2 - x^2 = 22x^2$	Each of the terms is an x^2 term. Remember that the $-x^2$ term means $-1x^2$. To find the coefficient of x^2 we look at the coefficients: $8 + 15 - 1 = 22$.

The **Associative law** that we have seen in arithmetic applies also to algebraic expressions. Where there is a group of more than two terms, we can rearrange them so that the like terms are next to each other, then simplify each group of like terms.

$$5a + 2b - 3a - 7b = \underbrace{5a - 3a}_{2a} + \underbrace{2b - 7b}_{-5b}$$



$$= 2a - 5b$$

Move the sign in front of each term with it.

Example 11

Simplify each of the following expressions by gathering like terms.

a $8a + 5 + 3a - 12$

b $7b - 2c + 9c + b$

c $3x - 5y + 7x + 2y - x$

d $4x^2 + 3x - 4x - 6 + 5x^2 + 8$

Working	**Reasoning**
a $8a + 5 + 3a - 12$ $= 8a + 3a + 5 - 12$ $= 11a - 7$	Rewrite the expression with the like terms next to each other. Move the sign in front of each term with it.
b $7b - 2c + 9c + b$ $= 7b + b - 2c + 9c$ $= 8b + 7c$	Rewrite the expression with the like terms next to each other. Move the sign in front of each term with it.
c $3x - 5y + 7x + 2y - x$ $= 3x + 7x - x - 5y + 2y$ $= 9x - 3y$	Collect like terms. Simplify the expression. $3x + 7x - x = 9x$ $-5y + 2y = -3y$
d $4x^2 + 3x - 4x - 6 + 5x^2 + 8$ $= 4x^2 + 5x^2 + 3x - 4x - 6 + 8$ $= 9x^2 - x + 2$	Collect like terms. Simplify the expression. $4x^2 + 5x^2 = 9x^2$ $3x - 4x = -x$ $-6 + 8 = 2$

exercise 4.2

LINKS TO Example 8

State whether each pair of terms is like or unlike.

a 3 and 4 **b** m and $3m$ **c** $-5b$ and $2b$ **d** $-2x$ and $3y$
e $2de$ and $3d$ **f** $2a^2$ and $4a^2$ **g** $5u$ and $2u$ **h** $3f^2$ and $3f$
i de and $2de$ **j** -5 and 1 **k** $-2x$ and $-2x$ **l** $-4k$ and k

LINKS TO Example 8

Which of the following expressions does not contain like terms?

A $4x + x$ **B** $3t^2 - 2t$ **C** $7mn + 3mn$ **D** $12 - 8$ **E** $-6k + 4k$

LINKS TO Example 8

Arrange the following into groups of like terms.

a $2x, 5, 3y, -7, 4y, -3x$ **b** $4t, t^2, -3, 5t^2, 6, -8t$

LINKS TO Example 9

Simplify the following expressions by collecting like terms.

a $x + 2x$ **b** $4a + 6a$ **c** $3n + 4n$ **d** $7b - 6b$
e $-5x + 8x$ **f** $-3a + 3a$ **g** $-5x + 9x$ **h** $-2x - 2x$

LINKS TO Example 10

Where possible, simplify the following expressions.

a $6 + 9$ **b** $-3 + 7$ **c** $5 + -8 + 2$
d $12d + 9e$ **e** $6h + 5h + h$ **f** $9k + (-11k)$
g $4ab + 6ab$ **h** $3xy + 8x$ **i** $6m^2 + 2m^2$
j $8x^2 + 5x^2 + x^2$ **k** $-4de + 6de + (-2de)$ **l** $8v + 5v + 4w$
m $8m - 6n$ **n** $7h - h$ **o** $4k - 10k$
p $12r - 12r$ **q** $11gh - 2gh$ **r** $-4ab - 6ab$
s $-8x - 5x^2$ **t** $4j + 8j - 5j$ **u** $5h^2 - 6h^2 + h^2$

LINKS TO Example 11

For the expression $7a - 3b + 4a + 10b$, which of the following is *not* a way of writing like terms side by side?

A $7a + 4a - 3b + 10b$ **B** $7a + 4a + 10b - 3b$
C $10b - 3b + 7a + 4a$ **D** $-3b + 10b + 4a + 7a$
E $7a - 4a + 3b + 10b$

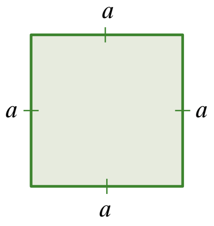
LINKS TO Example 11

Simplify each of the following expressions by first collecting like terms.

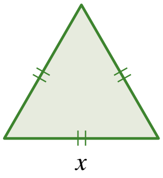
a $3x + 5x + 7 + 4$ **b** $4d + d + 2e + 7e$ **c** $5c + 3c -4s + 6s$
d $8b + 2 + 3b + 9$ **e** $2m + 4n + 6n +5m$ **f** $12 + 4k -4k + 6$
g $7b^2 + 4b + 2b^2 + 5b$ **h** $3xy + 7 + xy - 2xy$ **i** $p + 6p^2 - 8p + 2p^2$
j $6st + 4st + 3 + 2st$ **k** $9mn + 11 - 2mn - 11$ **l** $8n^2 + 6 -2n^2 + 5n$

Write an expression for the perimeter of each of the shapes shown below.

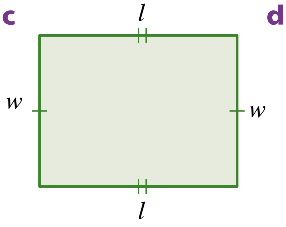
a



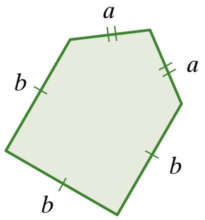
b



c



d



Sarah saved a number of dollars, represented by the pronumeral x.

a Her brother, Sam, saved $5 more than Sarah. Write an expression for the amount of money that Sam saved.

b By adding the amount that Sarah saved to the amount that Sam saved, write an expression for the total amount saved by the two of them together.

c Write the expression for the total amount in its simplest form.

A school group went on a three day hike. On the first day they walked a number of kilometres represented by the pronumeral d.

a On the second day they walked 4 kilometres further than on the first day. Write an expression for the distance walked on the second day.

b On the third day they walked 2 kilometres less than on the first day. Write an expression for the distance walked on the third day.

c By adding the distances walked on each of the three days write an expression for the total distance walked.

d Write the expression for the total distance in its simplest form.

Don't include units in expressions

Simplify each of the following expressions by first writing like terms side by side.

a $9a - 5a + 5 - 4$ **b** $6j - j + 3k - 8k$ **c** $7h - 2h - 6u + 6u$

d $-5w + 6 + 2w - 4$ **e** $2c - 3d + 8d - 7c$ **f** $4 + 6m - m - 4$

g $6a^2 - 3a + 4a^2 - a$ **h** $5cd - 4 - 2cd + cd$ **i** $v^2 - 6v - 8v^2 + 3v$

j $8y - 5y + 2 - 3y$ **k** $4pq + 6 - 3pq - 10$ **l** $3n^2 - 2n + 7n - 9$

The simplest form of the expression $2d - 6e - d + 3e$ is

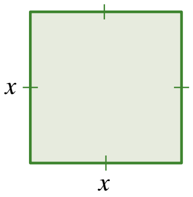
A $d - 9e$ **B** $d - 3e$ **C** $2 - 3e$ **D** $2 - 9e$ **E** $3d + 9e$

For the shapes below

a find the area of each.

b find the total area of all four shapes.

i



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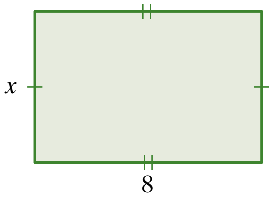

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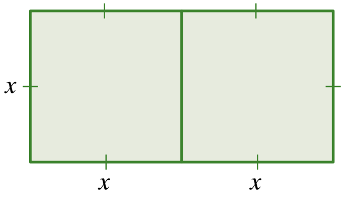

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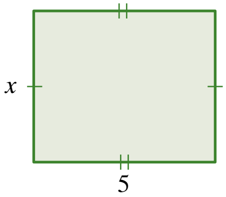


exercise 4.2 challenge

● Gino's Uncle Dom will not say how old he is. Uncle Dom does say that Gino's Grandpa Tony is twice Uncle Dom's age. Also Aunt Rosa is 3 years younger than Uncle Dom, and Uncle Frank is 5 years older than Uncle Dom.

a Define a pronumeral to represent Uncle Dom's age.

b Find the simplest possible expression, in terms of uncle Dom's age, for the total ages of Uncle Dom, Aunt Rosa, Uncle Frank and Grandpa Tony.

c Will this expression still be true for the total of their ages at the same time next year?

4.3 Simplifying: multiplying terms

Algebraic expressions are simplified using the same rules as numbers.

When dealing with positive and negative pronumerals the same rules apply as for numbers; for example, $-a \times b = -ab$ and $-a \times (-b) = ab$.

When multiplying numbers, the **Associative law** tells us that we can rearrange the numbers in a different order. The Associative law also applies when multiplying algebraic terms. For example,

$$\begin{aligned} 3a \times 4 &= 3 \times a \times 4 \\ &= \underbrace{3 \times 4}_{12} \times a \\ &= 12a \end{aligned}$$

Example 12

Carry out each of the following multiplications.

a $3 \times 4k$ **b** $-4 \times (-2h)$ **c** $5a \times 10b$

d $6e \times (-2e) \times 5$ **e** $0.2x \times 0.7y$ **f** $3 \times a \times b \times 4 \times a$

Working	Reasoning
a $3 \times 4k = 12k$	$3 \times 4k = \underbrace{3 \times 4}_{12} \times k$
b $-4 \times (-2h) = 8h$	$-4 \times (-2h) = \underbrace{-4 \times (-2)}_{8} \times h$
c $5a \times 10b = 50ab$	$5a \times 10b = 5 \times a \times 10 \times b$ $= \underbrace{5 \times 10}_{50} \times \underbrace{a \times b}_{ab}$
d $6e \times (-2e) \times 5 = -60e^2$	$6e \times (-2e) \times 5 = 6 \times e \times (-2) \times e \times 5$ $= \underbrace{6 \times (-2)}_{-12} \times 5 \times \underbrace{e \times e}_{e^2}$ $= -60e^2$

continued

Example 12 continued

Working	Reasoning
e $0.2x \times 0.7y = 0.14xy$	$0.2x \times 0.7y = 0.2 \times x \times 0.7 \times y$ $= \underbrace{0.2 \times 0.7}_{0.14} \times \underbrace{x \times y}_{xy}$ $= 0.14xy$
f $3 \times a \times b \times 4 \times a$ $= 3 \times 4 \times a \times a \times b$ $= 12 \times a^2 \times b$ $= 12a^2b$	Put the like terms together. $3 \times 4 = 12$ $a \times a = a^2$

exercise 4.3

LINKS TO Example 12

● Simplify each of the following multiplications.

a $3 \times 2x$ **b** $4m \times 6$ **c** $11 \times 7v$ **d** $5 \times (-4g)$
e $-7 \times (-3w)$ **f** $-10 \times 8t$ **g** $2h \times (-6k)$ **h** $5u \times 9u$
i $2b \times 5a$ **j** $-7a \times (-3b)$ **k** $-4k \times 4j$ **l** $-9w \times (-2w)$

LINKS TO Example 12

● Simplify each of the following multiplications.

a $2 \times 5a \times 3$ **b** $4w \times 2w \times 5$ **c** $5u \times 4 \times t$
d $3 \times 2g \times 11f$ **e** $-2e \times (-2e) \times 7$ **f** $-6m \times 2 \times (-3n)$
g $-5x \times (-2) \times (-3x)$ **h** $-3w \times 4u \times 2v$ **i** $-3f \times 6e \times (-5d)$

LINKS TO Example 12

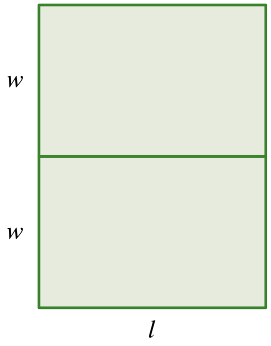
● Simplify each of the following multiplications.

a $4 \times 0.3g$ **b** $0.5 \times 8z$ **c** $-2 \times 0.4y$ **d** $0.3a \times 0.2$
e $7 \times (-0.2k)$ **f** $-0.6m \times (-0.3n)$ **g** $\frac{1}{2} \times 4b$ **h** $\frac{3}{4} \times 8d$

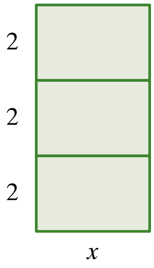
● Each diagram shows a large rectangle made up of identical smaller rectangles. Answer all three of the following questions for each of the diagrams below.

- **i** What is the width of the large rectangle?
- **ii** What is its length?
- **iii** What is the area of the large rectangle?

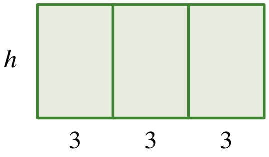
a



b

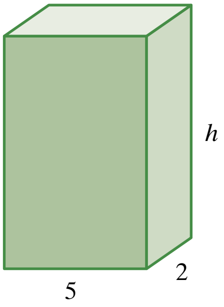


c

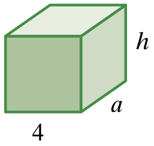


Write a term to describe each of the following volumes.

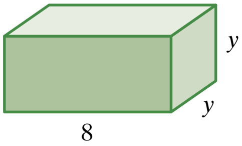
a



b



c

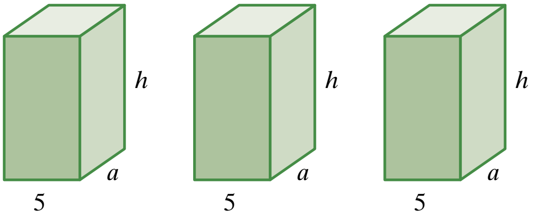


Don't forget that for a rectangular prism the volume is
length × width × height.

Each morning x trains, each with 6 carriages, travel to the city. There is seating for y people in each carriage. Write an expression for the total number of people that could be seated?

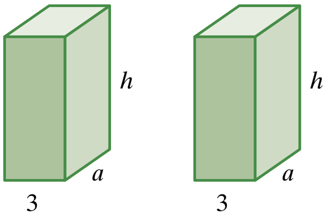
At the warehouse are n boxes, each holding t egg cartons. Each egg carton contains 12 eggs. Write an expression for the total number of eggs in the boxes.

exercise 4.3 challenge



Consider the group of blocks above.

- **a** What is the total volume of this group of blocks? Write the multiplication that you carried out to produce this.
- **b** What is the total volume of these two blocks?



- **c** What is the total volume of all five blocks?

4.4 Simplifying: dividing terms

In section 4.1 we saw that when using algebra, divisions are written as fractions.

Expressions can be simplified if the numerator and denominator have a common factor. The process is the same as simplifying fractions in arithmetic by cancelling down.

Example 13

Simplify the following.

a $-8x \div 2$ **b** $\frac{14x^2y}{7x}$ **c** $-10c \div (-15)$

Working	**Reasoning**
a $-8x \div 2$ $= \frac{-8x}{2}$ $= -4x$	First write the division as a fraction. -8 divided by 2 equals -4
b $\frac{14x^2y}{7x} = \frac{\overset{2}{\cancel{14}}\overset{1}{\cancel{x}}y}{\underset{1}{\cancel{7}}\underset{1}{\cancel{x}}}$ $= 2xy$	7 and x are both common factors in the numerator and denominator.
c $-10c \div (-15)$ $= \frac{-\overset{2}{\cancel{10}}c}{-\underset{3}{\cancel{15}}}$ $= \frac{2c}{3}$	First write the division as a fraction. 5 is a common factor of 10 and 15. A negative number divided by a negative number gives a positive number.

exercise 4.4

LINKS TO Example 13

Simplify each division.

a $10n \div 5$ **b** $9a \div 3$ **c** $22m \div 11$ **d** $\frac{14v}{2}$

e $\frac{20k}{4}$ **f** $\frac{18e}{6}$ **g** $12w \div 48$ **h** $20c \div 90$

i $8u \div 24$ **j** $\frac{15h}{25}$ **k** $\frac{22t}{77}$ **l** $\frac{63x}{45}$

In simplest form the division $8n \div 14$ is

A $\dfrac{n}{6}$ **B** $\dfrac{8}{14n}$ **C** $\dfrac{4n}{7}$ **D** $\dfrac{4}{7n}$ **E** $\dfrac{n}{3}$

Simplify each division.

a $\dfrac{3n}{n}$ **b** $\dfrac{u}{4u}$ **c** $\dfrac{6h}{9h}$ **d** $\dfrac{a^2}{7a}$

e $\dfrac{de}{e^2}$ **f** $\dfrac{m^2}{mn}$ **g** $\dfrac{8k^2}{12k}$ **h** $5t \div 9t$

i $\dfrac{6c}{24c^2}$ **j** $21r \div 3r^2$ **k** $\dfrac{10a^2}{35ab}$ **l** $10w \div 12w^2$

There are 3 cartons. Inside each carton are 20 boxes. Inside each box are x jars of jam. All of these jars of jam are shared equally among 18 people. Write an expression for the share of one of those people.

For a New Year's party there are 4 boxes each containing 15 bags of n fortune cookies. These fortune cookies are shared equally among 100 people. Write an expression for the number of fortune cookies for each person.

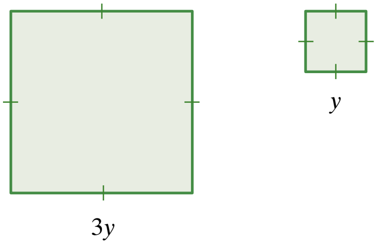
exercise 4.4 challenge

Consider the squares at right.

a Write a term for the area of the small green square.

b Write a term for the area of the large empty square.

c Divide the area of the large empty square by the area of the small green square.



d What does the answer to part **c** tell you about the number of green squares that will fit inside the large empty square?

Small blocks are being packed into a large box.

a What is the volume of one small block of length a, width b and height 5?

b What is the volume of a large box with length $8a$, width $5b$ and height c?

c Find an expression for the number of identical small blocks that will fit inside the large box.

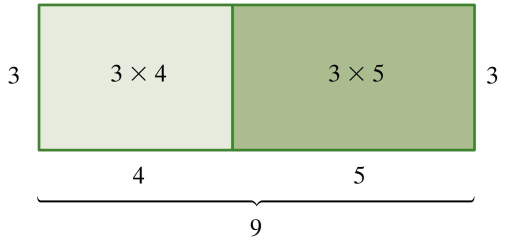
d If the value of c is 15, how many of the small blocks will fit in the box?

4.5 Expanding brackets: the distributive law

Expanding brackets 1

Expanding brackets 2

In chapter 1 we saw how the **distributive law** states that multiplication is distributed over a sum of numbers in brackets. An area model is useful to illustrate the distributive law.



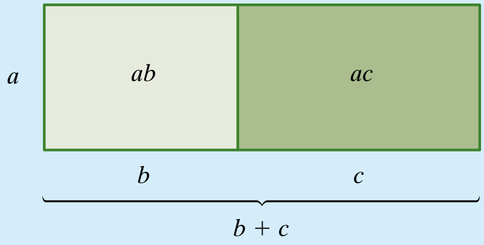
$$\begin{aligned} &3 \times (4 + 5) \\ &= 3 \times 9 \\ &= 27 \end{aligned}$$

$$\begin{aligned} &3 \times (4 + 5) \\ &= 3 \times 4 + 3 \times 5 \\ &= 12 \quad + 15 \\ &= 27 \end{aligned}$$

We can use algebra with the area model to generalise the distributive law.

Distributive law

$$\begin{aligned} a(b + c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$



Compare the diagrams above with Oliver Byrne's diagram of Euclid's explanation at the beginning of this chapter. We can see how algebra makes the explanation of the areas much simpler than Euclid's explanation.

The number before the brackets is distributed to the terms inside the brackets.
In the following example, the 3 is 'distributed' to both the a and the 7.

$$\begin{aligned} &3\,(a + 7) \\ &= 3 \times a + 3 \times 7 \\ &= 3a + 21 \end{aligned}$$

Removing brackets in an expression by applying the distributive law is called **expanding**.

Example 14

Expand each of the following expressions. For each, show the area model.

a $4 \times (3 + 2)$ **b** $4(m + 2)$

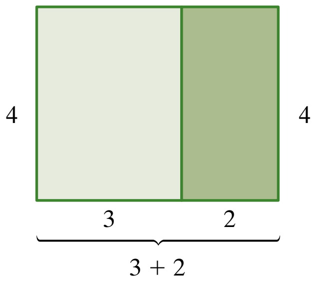
Working

a $4 \times (3 + 2)$

$= 4 \times 3 + 4 \times 2$

$= 12 + 8$

$= 20$



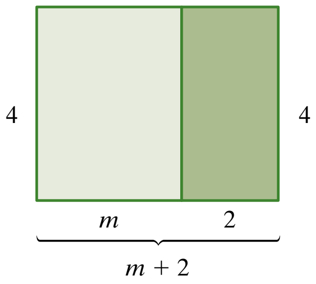
Reasoning

This is 4 lots of 3 and 4 lots of 2.
Using the model, the left area is 4×3 and the right area is 4×2.

b $4(m + 2)$

$= 4 \times m + 4 \times 2$

$= 4m + 8$



This is 4 lots of m and 4 lots of 2.
Using the model, the left area is $4 \times m$ and the right area is 4×2.
Don't worry about not knowing what m stands for. Just mark one length m.

Example 15

Expand the following expressions.

a $4(a + 6)$ **b** $5(2x - 3y)$

c $b(a + 8b)$ **d** $-3(4s + 2t)$

Working

a $4(a + 6)$

$= 4 \times a + 4 \times 6$

$= 4a + 24$

b $5(2x - 3y)$

$= 5 \times 2x - 5 \times 3y$

$= 10x - 15y$

c $b(a + 8b)$

$= b \times a + b \times 8b$

$= ab + 8b^2$

Reasoning

Multiply a by 4 and multiply 6 by 4.

Multiply $2x$ by 5 and multiply $-3y$ by 5.
Multiplying a negative term by a positive term gives a negative term.

Multiply a by b and multiply $8b$ by b.

continued

Example 15 continued

Working	Reasoning
d $-3(4s + 2t)$ $= -3 \times 4s + (-3) \times 2t$ $= -12s - 6t$	Multiply $4s$ by -3 and multiply $2t$ by -3. Multiplying a negative term by a positive term gives a negative term.

To multiply the contents of a bracket by a negative term, special care must be taken with the signs.

Example 16

Expand each of the following expressions.

a $-(3x - 7)$ **b** $-x(x - 7)$

Working	Reasoning
a $-(3x - 7)$ $= -1(3x - 7)$ $= -3x + 7$	The negative sign without a number before the brackets means -1. Multiply $3x$ by -1 and multiply -7 by -1. The product of two negative numbers is positive.
b $-x(x - 7)$ $= -x^2 + 7x$	Multiply x by $-x$ and multiply -7 by $-x$. The product of two negative numbers is positive.

Special care must be taken with signs if the term before the brackets is negative and there is also a negative sign inside the brackets.

Example 17

Expand the following and hence simplify.

a $4(x + 3) + 8$ **b** $3(x + 5) + 7x + 8$ **c** $3(2x + 1) - 4(2x - 3)$

Working	Reasoning
a $4(x + 3) + 8$ $= 4x + 12 + 8$ $= 4x + 20$	Expand the brackets. Collect like terms: $12 + 8 = 20$.
b $3(x + 5) + 7x + 8$ $= 3x + 15 + 7x + 8$ $= 3x + 7x + 15 + 8$ $= 10x + 23$	Expand the brackets. Group like terms and simplify. $3x + 7x = 10x \quad 15 + 8 = 23$

continued

Example 17 continued

Working

c $3(2x + 1) - 4(2x - 3)$
$= 6x + 3 - 8x + 12$
$= -2x + 15$

Reasoning

Expand the brackets, taking special care with the double negative signs:
$-4 \times (-3) = +12$
Collect like terms and simplify.

exercise 4.5

LINKS TO **Example 14a**

● Use the distributive law to evaluate each of the following by expanding the brackets.

a $5 \times (6 + 2)$ **b** $3 \times (5 + 1)$ **c** $8 \times (5 - 3)$
d $4 \times (7 - 5)$ **e** $2 \times (2 + 5)$ **f** $10 \times (7 - 2)$

LINKS TO **Example 14b**

● Expand each of the following expressions. For each, draw an area model.

a $2(a + 3)$ **b** $3(a + 5)$ **c** $5(x + 6)$ **d** $4(h + 7)$
e $3(2x + 5)$ **f** $4(2m + 3)$ **g** $7(2n + 1)$ **h** $8(3y + 5)$

LINKS TO **Example 15**

● Expand each of the following expressions.

a $3(m + 5)$ **b** $8(d + 4)$ **c** $11(h - 2)$
d $7(2 + b)$ **e** $9(3 - a)$ **f** $4(a + b + 3)$
g $4(2g + 3)$ **h** $3(11 + 5t)$ **i** $2(6 - 4x)$
j $6(4e + 5f)$ **k** $5(3a - 2b)$ **l** $10(5d + 4 - 3e)$
m $x(y + 7)$ **n** $a(4 + b)$ **o** $d(e - 8)$
p $g(g + 5)$ **q** $k(3 - k)$ **r** $w(2 + w)$

● A garden is 2 metres longer than its width. Let w metres represent its width.

a Write an expression for the length of the garden in terms of its width, w.
b Write an expression that uses brackets for the area of the garden.
c By expanding, write the area expression without brackets.
d If the width of the garden is 5 metres, find its length and its area.

● The width of a sheet of paper is 11 cm less than its length. Let L cm represent its length.

a Write an expression for the width of the sheet of paper in terms of its length, L.
b Write an expression that uses brackets for the area of the sheet of paper.
c By expanding, write the area expression without brackets.
d If the length of the sheet of paper is 32 cm, find its width and its area.

● Calculate the value of each expression by first expanding the bracket. Take special care with the signs. Compare your answers with those from question 1

a $-5 \times (6 + 2)$ **b** $-3 \times (5 + 1)$ **c** $-8 \times (5 - 3)$
d $-4 \times (7 - 5)$ **e** $-2 \times (2 + 5)$ **f** $-10 \times (7 - 2)$

LINKS TO Example 16

Expand each of the following expressions.

a $-2(h + 6)$
b $-6(d + 9)$
c $-4(e - 6)$
d $-(4 + r)$
e $-3(8 - p)$
f $-(f - g + 3)$
g $-8(2c + 5)$
h $-(3 + 2q)$
i $-9(10 - 3m)$
j $-11(3m + 7n)$
k $-6(5a - 3b)$
l $-3(4j + 1 - k)$
m $-c(d + 3)$
n $-m(7 + n)$
o $-2g(h - 5)$
p $-t(t + 4)$
q $-5x(9 - x)$
r $-4a(3 + a)$

LINKS TO Example 17a

Expand the following expressions and hence simplify.

a $4(x + 7) + 9$
b $3(x + 10) + 11$
c $2(x - 5) + 12$
d $4(x - 16) - 24$
e $5(2x - 3) + 11$
f $-7(3x - 1) + 6$
g $-2(x + 8) - 14$
h $-6(9 - x) + 18$
i $6(2x + 9) - 13$
j $-5(8x + 12) + 13$
k $-10(5 - 9x) - 16$
l $-15(3x - 11) + 35$

LINKS TO Example 17b

Expand the following expressions and then simplify.

a $4(x + 3) + 2x + 6$
b $-9(x + 7) - 5x + 13$
c $-3(x - 5) + 4x - 2$
d $-5(2x + 12) + 13$
e $7(3x - 7) - 2x + 14$
f $8(5x - 11) + 15x - 18$
g $20 + 5x(x + 3) - 4x$
h $7r + 34 - 3(9r + 5)$

LINKS TO Example 17c

Expand the following expressions and then simplify.

a $2(x + 5) + 3(x + 6)$
b $3(2 + x) + 5(4 + x)$
c $12(x - 3) - 7(x + 9)$
d $5(2x - 3) + 6(3x - 7)$
e $3(6x - 7) - 2(5x - 8)$
f $14(2x + 5) + 5x(x + 4)$
g $12x(5x - 7) + 2x(61 - 8x)$
h $12(2 + x) + 13(2 + x)$
i $11(x - 3) - 8(x - 3)$
j $-3(5 - 2x) + (5 - 2x)$
k $-11(3x - 4) - 5(3x - 4)$
l $-2(9 - 2x) - 13(9 - 2x)$

exercise 4.5 challenge

Fill the boxes in each expression on the left so that, when expanded, it will give the expression on the right.

a $\square(a + 3) = 7a + 21$
b $9(\square - 4) = 27g - 36$
c $\square(\square - 8) = 3y^2 - 24y$
d $\square(3m - \square) = -6mn + 8n^2$

4.6

Factorising algebraic terms and expressions

In the last section we used the distributive law to expand brackets according to the pattern

$$\begin{aligned} a(b + c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$

In this section we will learn to use the distributive law in reverse to write expressions with brackets. This is called **factorising**.

For example we can expand $5(m + 3)$

$$\begin{aligned} &= 5 \times m + 5 \times 3 \\ &= 5m + 15 \end{aligned}$$

The starting expression with the brackets is called the **factorised form**.

The finishing expression without the brackets is called the **expanded form**.

$$\begin{tikzcd} 5(m+3) \arrow[r, bend left, "\text{Expand}"] & 5m+15 \arrow[l, bend left, "\text{Factorise}"] \end{tikzcd}$$

Expanding and factorising are inverse processes because each undoes the other.

Factors of numbers

From working with whole numbers we already know that

- a factor of a whole number is a whole number that will divide exactly into the first number, that is, without leaving a remainder.
- for every number, 1 and the number itself are factors. For prime numbers those are the only factors. For example, the factors of the prime number 7 are 1 and 7.
- for numbers that are not prime, apart from 1, there are also other factors. These numbers can be written as
  - a product of prime factors, for example $12 = 2 \times 2 \times 3 = 2^2 \times 3$.
  - a list of all of the possible factors. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12.
- if two numbers have the same factor it is called a **common factor**.
- the largest factor that two numbers have in common is called their **highest common factor**.

In Year 7, we used factor ladders to find the prime factors of composite numbers. Example 18 reviews this method.

Example 18

Express 12 as the product of its prime factors using a factor ladder.

Working

2	12
2	6
3	3
	1

As a product of its prime factors 12 is $2 \times 2 \times 3 = 2^2 \times 3$.

Reasoning

Start with the smallest prime number, 2. Continue to divide by 2 as long as there are even numbers to divide into.

Then, if possible divide by the prime numbers 3, 5, 7, and so on.

Stop when the result 1 is reached. The divisors down the left side are the prime factors.

The following example shows a method for finding common factors of numbers that we can then apply to terms in algebra. Two factor ladders, one for each number, are written alongside each other.

Example 19

Find the highest common factor of 18 and 24 using a double factor ladder.

Working

2	18	24
3	9	12
	3	4

The highest common factor is $2 \times 3 = 6$.

Reasoning

Write the two numbers side by side.

Find a number that is a factor of both numbers. Divide it into both of them.

Find another number that is a factor of both and divide again.

Keep dividing until there is no other number that is a factor of both.

The highest common factor is found by multiplying the divisors from the left side.

Factors of terms

Factors of terms include any whole number that divides exactly into the term, and any pronumerals that divide exactly into the term.

For example, $2ab$ has factors 1, 2, a, b, $2a$, $2b$, ab and $2ab$ because each of these will divide exactly into $2ab$ without a remainder.

We can find common factors and the highest common factor using factor ladders in the same way as we did for numbers. If we are trying to find the highest common factor of three terms, we have three factor ladders alongside each other.

Example 20

Find the highest common factor of $6a$, $4ab$ and $2a^2$ using a triple factor ladder.

Working

2	$6a$	$4ab$	$2a^2$
a	$3a$	$2ab$	a^2
	3	$2b$	a

The highest common factor is $2 \times a = 2a$.

Reasoning

Write the three terms side by side.

Find a number or pronumeral that is a factor of both numbers. Divide it into all of them.

Find another number or pronumeral that is a factor of both and divide again.

Keep dividing until there is no other number or pronumeral that is a factor of all of them.

The highest common factor is found by multiplying the divisors from the left side.

Factorising expressions

When we factorise an expression, we must find the highest common factor of each term in the expression.

Example 21

Factorise each of the following expressions

a $10y + 25$ **b** $ab + a$ **c** $n^2 - 4n$ **d** $6x^2 - 3x$

Working

a

5	$10y + 25$
	$2y + 5$

$10y + 25 = 5(2y + 5)$

Reasoning

Write the terms side by side.
5 divides exactly into both terms.
There is no number or pronumeral that is a factor of both $2y$ and 5.

So the highest common factor is 5.

To write $10y + 25$ in factorised form, we write the common factor 5 outside the brackets, then write $2y + 5$ inside the brackets. Note that this is the distributive law in reverse.

We can check our factorisation by expanding.

continued

Example 21 continued

Working | **Reasoning**

b

$$\begin{array}{c|c} a & ab + a \\ \hline & b + 1 \end{array}$$

$$ab + a = a(b + 1)$$

a is a common factor, so divide both terms in the expression by a.
Note that a divided by a is 1.

The expression $b + 1$ goes inside the brackets.
Check by expanding:
$a(b + 1) = ab + a$

c

$$\begin{array}{c|c} n & n^2 - 4n \\ \hline & n - 4 \end{array}$$

$$n^2 - 4n = n(n - 4)$$

n is a common factor, so divide both terms in the expression by n.
Note that n^2 divided by n is n.

The expression $n - 4$ goes inside the brackets.
Check by expanding:
$n(n - 4) = n^2 - 4n$

d

$$\begin{array}{c|c} 3 & 6x^2 - 3x \\ \hline x & 2x^2 - x \\ \hline & 2x - 1 \end{array}$$

$$6x^2 - 3x = 3x(2x - 1)$$

Divide both terms by the common factor 3.
Then divide the result by the common factor x.
There are no more common factors so stop.
The highest common factor is $3 \times x$ which is equal to $3x$.

The expression $2x - 1$ goes inside the brackets.
Check by expanding:
$3x(2x - 1) = 6x^2 - 3x$

Special care is needed if the first term in the expression is negative. It is the custom in this case to take a negative common factor. This means that the sign of the terms inside the brackets change.

Example 22

Factorise each of the following expressions

a $-4x + 12$ **b** $-5x - 10$ **c** $-8x^2 - 6x$

continued

Example 22 continued

Working

a

$$\begin{array}{c|c} -4 & -4x + 12 \\ \hline & x - 3 \end{array}$$

b

$$\begin{array}{c|c} -5 & -5x - 10 \\ \hline & x + 2 \end{array}$$

$-5x - 10 = -5(x + 2)$

c

$$\begin{array}{c|c} -2 & -8x^2 - 6x \\ \hline x & 4x^2 + 3x \\ \hline & 4x + 3 \end{array}$$

$-8x^2 - 6x = -2x(4x + 3)$

Reasoning

When the coefficient of the first term is negative, write the first common factor with a negative sign. Here that is -4.

When dividing by a negative number or pronumeral, the effect is to change the signs of each term.

$12 \div (-4) = -3$

Instead of writing $x + (-3)$ we write $x - 3$ in the brackets. Check by expanding:

$-4(x - 3) = -4x + 12$

The common factor is -5.

$-10 \div (-5) = +2$

The expression $x + 2$ goes inside the brackets.

Check by expanding:

$-5(x + 2) = -5x - 10$

Again, the expression begins with a negative term so the first common factor is written with a negative sign.

Dividing by a negative number or pronumeral changes the sign of each term.

Check by expanding:

$-8x^2 - 6x = -2x(4x + 3)$

Once you understand how highest common factors are found, you may choose to leave out the step with the factor ladder.

Example 23

Factorise the following expressions.

a $5a + 15ab$

b $6x^2 + 9xy$

Working

a $5a + 15ab$

$= 5a(1 + 3b)$

b $6x^2 + 9xy$

$= 3x(2x + 3y)$

Reasoning

The highest common factor of $5a$ and $15ab$ is $5a$.

The highest common factor of $6x^2$ and $9xy$ is $3x$.

exercise 4.6

LINKS TO Example 18,19

Using common factor ladders, find the highest common factor (HCF) for each pair or group of numbers.

a 18 and 24 **b** 32 and 40 **c** 16 and 120
d 150 and 120 **e** 75 and 125 **f** 32, 40 and 48

LINKS TO Example 20

Using common factor ladders, write the highest common factor (HCF) for each pair or group of terms.

a $7p$ and 21 **b** $6a$ and $9b$ **c** $15ef$ and $10de$
d $12uv$ and $8uw$ **e** $15ab$ and $5b^2$ **f** $12k$, $18k^2$ and $6jk$

The highest common factor of $16a^2$ and $24ab$ is

A 2 **B** 4 **C** $4a$ **D** $8a$ **E** $16a$

LINKS TO Example 20

State the highest common factor of each of these pairs or groups of terms.

a $6x$ and 15 **b** $3ab$ and $15a^2$
c $24xy$ and $32y$ **d** $18m^2$ and $54mn$
e $-x^2$ and $-3x$ **f** $-8ab$ and $-20a$
g $14x$, $21xy$ and $35x^2$ **h** $24a^2b$, $18ab$ and $12ab^2$

LINKS TO Example 21

Factorise each of the following expressions.

a $2n + 6$ **b** $3y - 12$ **c** $7k + 14$ **d** $6p + 10$
e $4 + 14s$ **f** $10t - 5$ **g** $3 + 15f$ **h** $8 - 20m$
i $14 + 21h$ **j** $4c - 6e$ **k** $14a + 21b$ **l** $18w - 12x$
m $6 + 2x$ **n** $9x + 12$ **o** $5a - 5$ **p** $7m + 21$
q $12b - 16$ **r** $6 - 12x$ **s** $21y - 14$ **t** $27 - 36y$

Factorise each of the following expressions.

a $ac + ae$ **b** $de - e$ **c** $5h + gh$ **d** $n^2 - 2n$
e $y^2 + y$ **f** $7t - t^2$ **g** $8mn + 14np$ **h** $12uv - 18u$
i $25q + 20pq$ **j** $y^2 + 2yz$ **k** $6x^2 - 3x$ **l** $3k - 9k^2$
m $6a + 15b - 9\mathrm{c}$ **n** $de - df + dg$ **o** $a^2 + 4a - 3ab$ **p** $3x^2 - 12x + 15$

LINKS TO Example 22

Factorise each of the following expressions.

a $-2a + 8$ **b** $-3g + 15$ **c** $-7j + 7$ **d** $-5p - 25$
e $-11r - 33$ **f** $-9w - 9$ **g** $-4 + 6t$ **h** $-6m + 21$
i $-10 + 25h$ **j** $-12q - 8$ **k** $-33 - 77r$ **l** $-18a - 24$

LINKS TO Example 23

Fill the gaps in the following to make each equation true.

a $3 + 6x = 3(1 + \_\_)$ **b** $16 + 20x = \_\_(4 + 5x)$
c $15x + 45 = \_\_(\_\_ + 3)$ **d** $28 - 8x = \_\_(7 - \_\_)$
e $36x + 9 = 9(\_\_ + 1)$ **f** $100 + 25x = \_\_(\_\_ + 5x)$
g $96 -24x = \_\_(\_\_ - x)$ **h** $104 -143x = 13(\_\_ - \_\_)$

LINKS TO Example 23

Factorise each of the following expressions.

a $-3a + 6$
b $9x^2 + 27$
c $12a + 16a^2$
d $-4x + 12$
e $6x + 18x^2$
f $b^2 - 47b$
g $-12m - 20$
h $-24x + 32y$
i $32x^2 + 16$
j $25x^2 + 30x$
k $-16 + 24x$
l $2x^2 + 12x$
m $-54c + 60d$
n $-11a - 44b$
o $48a^2 + 24a$
p $7x^2 - 14x$
q $55x^2 - 22x$
r $-3xy + 6y$
s $-20 - 15m$
t $99a^2 - 33a$
u $-15y - 40$
v $4a^2 - 12a$
w $-8x - 12y$
x $-9x + 15y$

Find the common factor inside the brackets then completely factorise the expression.

a $5(2x - 4)$
b $2(6d + 9e)$
c $a(10 - 25b)$
d $y(5y - 20)$
e $-3(ab - ac)$
f $6(-m + 3n)$

Simplify the expression $\dfrac{8b - 14}{2}$ by first factorising the numerator.

exercise 4.6 challenge

Here is a 'magic' trick.

1 Think of a number.
2 Add 5 to the number.
3 Double the answer.
4 Subtract 8.
5 Divide the answer by 2.
6 Subtract the original number.

a What happens when you carry out these steps?
b Show algebraically that this will always work. Hint: let n stand for the chosen number.

Let n be an integer.

a What is the next integer?
b What is the next integer after that?
c Write an expression for the sum of these three consecutive integers.
d Factorise the expression you wrote in part **c**.
e Explain how you know that the sum of three consecutive integers is always divisible by 3.

4.7 Substitution of values for pronumerals

Substituting into expressions

We can evaluate expressions by substituting values for the pronumerals. For example, $2a + b$ can be evaluated for the particular values of a and b. If $a = 4$ and $b = 3$, then $2a + b = 2 \times 4 + 3 = 8 + 3 = 11$. The rules for directed numbers and the order of operations apply when evaluating expressions.

Example 24

Evaluate the following expressions if $a = 5$ and $b = 6$.

a $2a - 3b$ **b** $-5a(2 + 3b)$

Working	**Reasoning**
a $2a - 3b$	Substitute 5 for a and 6 for b.
$= 2 \times 5 - 3 \times 6$	Complete the multiplication before the subtraction.
$= 10 - 18$	
$= -8$	
b $-5a(2 + 3b)$	There is no need to expand the brackets. Substitute 5 for a and 6 for b.
$= -5 \times 5 \times (2 + 3 \times 6)$	
$= -25 \times (2 + 18)$	Complete the multiplications.
$= -25 \times 20$	Add the numbers in the brackets.
$= -500$	Complete the multiplication.

Example 25

Evaluate each of the following expressions when $m = -10$ and $n = 2$.

a $-4mn$ **b** $-\frac{7m}{n}$ **c** $\frac{10 + m}{n}$ **d** $m^2(3n - 8)$

Working	**Reasoning**
a $-4mn$	First write in the × signs that are omitted in algebra.
$= -4 \times m \times n$	Replace each pronumeral by its given value.
$= -4 \times (-10) \times 2$	Calculate the value of the expression.
$= 80$	

continued

Example 25 continued

Working	Reasoning
b $-\frac{7m}{n}$	Replace each pronumeral by its given value, writing the × sign that is omitted in algebra.
$= \frac{-7 \times (-10)}{2}$	Evaluate.
$= \frac{70}{2}$	
$= 35$	
c $\frac{10 + m}{n}$	
$= \frac{10 + (-10)}{2}$	
$= \frac{0}{2}$	0 divided by any number is 0.
$= 0$	
d $m^2(3n - 8)$	Again, write the × signs that are omitted in algebra, then substitute and evaluate.
$= (-10)^2 \times (3 \times 2 - 8)$	
$= 100 \times (6 - 8)$	Evaluate the expressions in brackets first.
$= 100 \times (-2)$	
$= -200$	

Substitution can be used to check whether an expression has been simplified, expanded or factorised correctly. When the same number is substituted into both the original expression and the final expression, the values of both expressions should be the same. Different values for original and final expressions mean that there is a mistake. It may be either in the simplifying or in the substitution and evaluation.

Example 26

Check whether the factorisation $-d^2 - 5d = -d(d - 5)$ is correct.

Working	Reasoning
Let $d = 3$	Choose a number for the pronumeral.
$\text{LS} = -d^2 - 5d$	Substitute it into the original expression, and evaluate.
$= -3^2 - 5 \times 3$	
$= -9 - 15$	
$= -24$	

continued

Example 26 continued

Working	Reasoning
$\text{RS} = -d(d - 5)$ $= -3 \times (3 - 5)$ $= -3 \times (-2)$ $= 6$	Substitute the same value for the pronumeral into the simplified, factorised or expanded expression and evaluate. If the two values are identical, the two expressions may be equivalent. If the two values are different, the two expressions are not equivalent.
LS $\neq$ RS so the factorisation is incorrect.	The factorisation was incorrect because it should have been $-d(d + 5)$

exercise 4.7

LINKS TO Example 24

Evaluate the following expressions if

i $a = 4$ and $b = 7$ **ii** $a = 3$ and $b = 6$. **iii** $a = 10$ and $b = 11$.

a $3a + 9b$ **b** $-4a + 8b$ **c** $6a - 7b$ **d** $-2a - 6b$

Evaluate each of the following expressions when $a = 5$ and $b = 3$.

a $8a$ **b** $7b$ **c** $10ab$ **d** $2ab$

e $6b^2$ **f** a^2b **g** $a - b$ **h** $b - a$

i $4a + b$ **j** $b + 4a$ **k** $a + 4b$ **l** $a^2 - 6b$

If $t = 3$ and $y = 7$, the value of $t^2 + 2y$ is

A 20 **B** 23 **C** 33 **D** 36 **E** 58

LINKS TO Example 25

Given that $d = 6$ and $e = -2$, evaluate each of the following expressions.

a $\frac{d}{3}$ **b** $\frac{d}{e}$ **c** $\frac{e}{8}$ **d** $\frac{8}{e}$

e $\frac{4d}{3}$ **f** $\frac{de}{4}$ **g** $\frac{d}{3e}$ **h** $\frac{8}{de}$

i $\frac{5d}{e}$ **j** $\frac{d + e}{4}$ **k** $\frac{3d}{2e}$ **l** $\frac{d + 3e}{d - 3e}$

Given that $x = 10$ and $y = 5$, the value of $\frac{x + 2y}{10}$ is

A 2 **B** 3.5 **C** 10 **D** 11 **E** 25

LINKS TO Example 25

If $g = 3$ and $h = 6$, find the value of each of the following expressions.

a $5(g + 1)$ **b** $2(h + 3)$ **c** $9(6 - h)$ **d** $-3(g + h)$

e $7(2g - h)$ **f** $4(2h - g)$ **g** $h(g - 4)$ **h** $g(h - 5)$

i $g(h - 5g)$ **j** $2h(1 + g)$ **k** $g^2(h - 4)$ **l** $gh(2 - g)$

When $a = 4$ and $b = 3$, $ab(b + 7)$ has the value

A 43 **B** 17 **C** 120 **D** 132 **E** 430

LINKS TO Example 25

Evaluate each of the following expressions when $m = 6$ and $n = -2$.

a $11m$ **b** $3mn$ **c** $m - n$ **d** $m + 3n$

e $\frac{m}{2}$ **f** $\frac{10}{n}$ **g** $\frac{3m - 2n}{2}$ **h** $\frac{2m + n}{5}$

i $4(m + 2)$ **j** $7(m - 2n)$ **k** $n(m + 3)$ **l** $2m(n^2 + 5)$

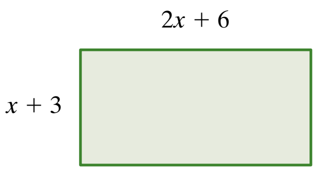
LINKS TO Example 26

Substitute $a = 3$ and $b = 5$ into the expressions each side of the 'equals' sign in $3a + 4b - a + 2b = 4a + 2b$.

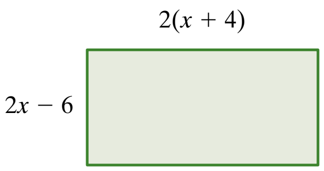
a Are the two sides equal?

b Correct the expression on the right side.

exercise 4.7 challenge



Rectangle A



Rectangle B

a Write an expression for the perimeter of rectangle A. Simplify your expression.

b Write an expression for the perimeter of rectangle B. Simplify your expression.

c Write an expression for the sum of the perimeters of rectangles A and B, giving the expression in simplified form.

d Factorise your expression from part **c**.

e Write an expression to show the difference in length of the perimeters of rectangles A and B.

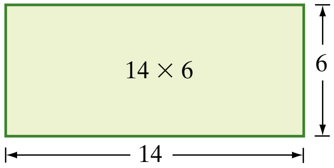
f Factorise your expression from part **e**.

Analysis task

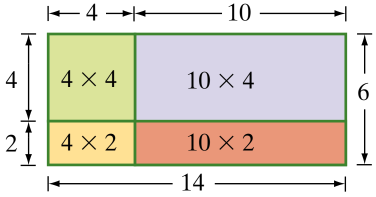
Expanding areas

Expanding areas

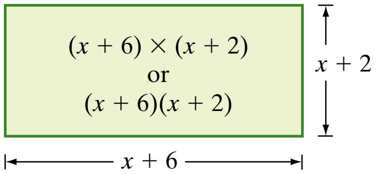
In the following analysis task all the lengths are in the same units, for example metres, and all the areas are in square metres.



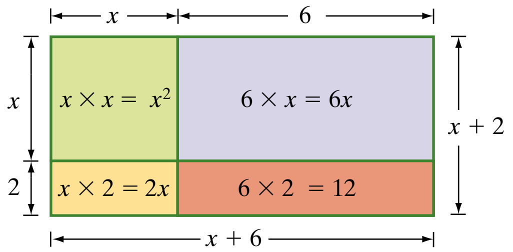
The area of this rectangle is 14 × 6, but we can also break the rectangle into four parts and add the areas of each part. From this we can see that
$14 \times 6 = 4 \times 4 + 10 \times 4 + 4 \times 2 + 10 \times 2$



The same method can now be used to investigate algebraic expressions.
In the this diagram, the area of the rectangle is $(x + 6)(x + 2)$.



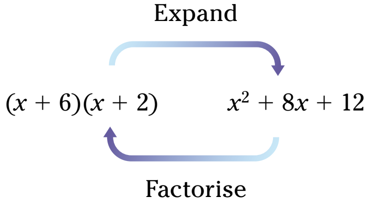
We can also calculate the area of the rectangle by dividing it into four smaller rectangles as shown. The area is then equal to the sum of the four separate areas.



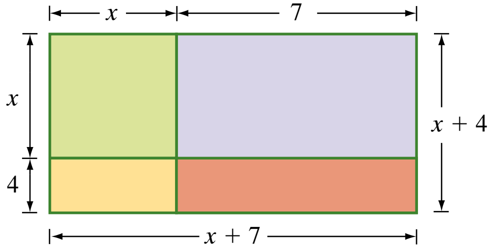
We know that the areas must be the same when calculated in the two different ways, so we can write:

$$\begin{aligned}(x + 6)(x + 2) &= x^2 + 2x + 6x + 12\\ &= x^2 + 8x + 12\end{aligned}$$

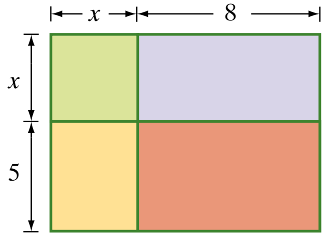
We have expanded $(x + 6)(x + 2)$ but looking at this the other way around we can also see that $(x + 6)$ and $(x + 2)$ are the factors of $x^2 + 8x + 12$.



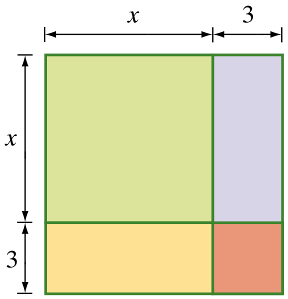
a Copy the diagram on the right then, on your diagram, write an expression for the area of each of the four rectangles inside the large rectangle. Now add the four expressions and simplify to obtain an expansion of $(x + 4)(x + 7)$ in the same way as in the example above.

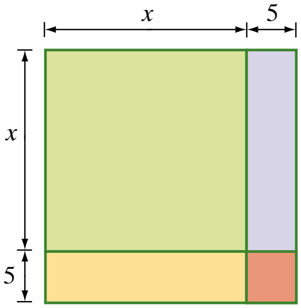


b Use the same method to find the expansion of $(x + 5)(x + 8)$.

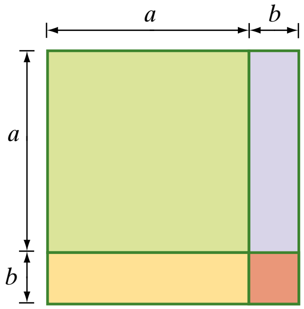


c Can you write $(x + 3)(x + 3)$ and $(x + 5)(x + 5)$ in a shorter way? Now use the same method as in parts **a** and **b** to find expansions for these two expressions.





d Copy the diagram on the right and use it to help you find an expansion for $(a + b)^2$.



e Now try the following expansions. Use a diagram if you need to.

i $(m + 4)(m + 7)$

ii $(b + 11)(b + 6)$

iii $(x + 9)(x + 5)$

iv $(a + 4)^2$

v $(k + 9)^2$

Review Algebra

Summary

- Algebraic expressions are made up of terms, for example $5x^2 - 3xy - 2$.
- Each term is either made up of a coefficient with a sign, and one or more pronumerals, for example $-3xy$, or it is a constant with no pronumeral, for example -2.
- Like terms have the same pronumerals and their powers must be the same, for example $3bc$ and $-5cb$ are like terms but $3x$ and $4x^2$ are unlike terms.
- Expressions can be simplified by collecting like terms, for example, $2x - 5y + 7x - 11y$ can be simplified to $9x - 16y$.
- Brackets can be expanded by applying the distributive law. Every term inside the brackets is multiplied by the term in front of the brackets. For example, $-3x(x - 4) = -3x \times x - (-3x) \times 4 = -3x^2 + 12x$.
- If the terms in an expression have a common factor, the expression can be factorised by taking out the common factor. For example, $12a^2 + 16ab = 4a(3a + 4b)$.
- Expanding and factorising are opposite processes.
- Numbers can be substituted for pronumerals in an expression. The expression can then be evaluated. For example, substituting $x = 2$ and $y = -1$ in the expression $3x - 2y$ gives $3 \times 2 - 2 \times (-1) = 6 + 2 = 8$.

Visual map

Using the following terms (and others if you wish), construct a mind map that illustrates your understanding of the key issues covered in this chapter.

associative law
coefficient
commutative law
constant
define
distributive law
evaluate
expand
expression
factors
factorise
like terms
pronumeral
simplify
substitute
term
unlike terms

Revision

Multiple-choice questions

The coefficient of the third term in the expression $5x^2 - 3xy - y^2$ is

A -2 **B** -1 **C** 0 **D** 1 **E** 2

The like terms in the expression $3a^2 + 4ab + 7ba + b^2 + 3$ are

A $3a^2$ and 3 **B** $3a^2$ and b^2 **C** $3a^2$ and $4ab$ **D** $4ab$ and $7ba$ **E** $7ba$ and b^2

In simplest form, $\dfrac{18a^2b}{12a}$ is equal to

A $6ab$ **B** $6a^2b$ **C** $\dfrac{3a^2b}{2a}$ **D** $\dfrac{3ab}{2}$ **E** $\dfrac{3b}{2}$

When expanded, $d(d + 7)$ becomes

A $2d + 7$ **B** $d^2 + 7$ **C** $2d + 7d$ **D** $9d$ **E** $d^2 + 7d$

The highest common factor of $18a^2$ and $45ab$ is

A 3 **B** 9 **C** $3a$ **D** $9ab$ **E** $9a$

Short-answer questions

For the expression $3y^2 + 6y - 5xy - 10$, state

a the number of terms.

b the pronumeral in the second term.

c the coefficient of the third term.

d the constant term.

Express in simplest form

a $3 \times e \div 8$ **b** $4 \times c + -7 \times d$ **c** $(g + 4) \div 5 - 3$

Simplify each of the following expressions by gathering like terms.

a $4h + 1 + 3h - 9$ **b** $3j - 2k + 7k + j$ **c** $3t^2 + 5t - t - 8$

Express each of the following in simplest form.

a $2m \times -5m \times 3n$ **b** $30p \div 5$ **c** $\dfrac{6q^2}{33qr}$

Expand each of the following:

a $5(t - 4)$ **b** $u(v + 3w)$ **c** $-2x(4x - 5y)$

Factorise each of the following expressions.

a $3a + 21$ **b** $5 - 10b$ **c** $-6c^2 - 8c$

Given that $d = 6$ and $e = -3$, evaluate each of the following expressions.

a $-2de$ **b** $2d - 3e$ **c** $e^2(d - 7)$

Extended-response questions

● A cross-country team had three practice runs before a race. On the first run they ran a number of kilometres represented by the pronumeral a.

- **a** On the second run they ran 1 kilometre further than on the first run. Write an expression for the distance of the second run.
- **b** The third run was 2 kilometres less than the first run. Write an expression for the distance of the third run.
- **c** By adding the distances for each of the three runs write an expression for the total distance run.
- **d** Write the expression for the total distance in its simplest form.

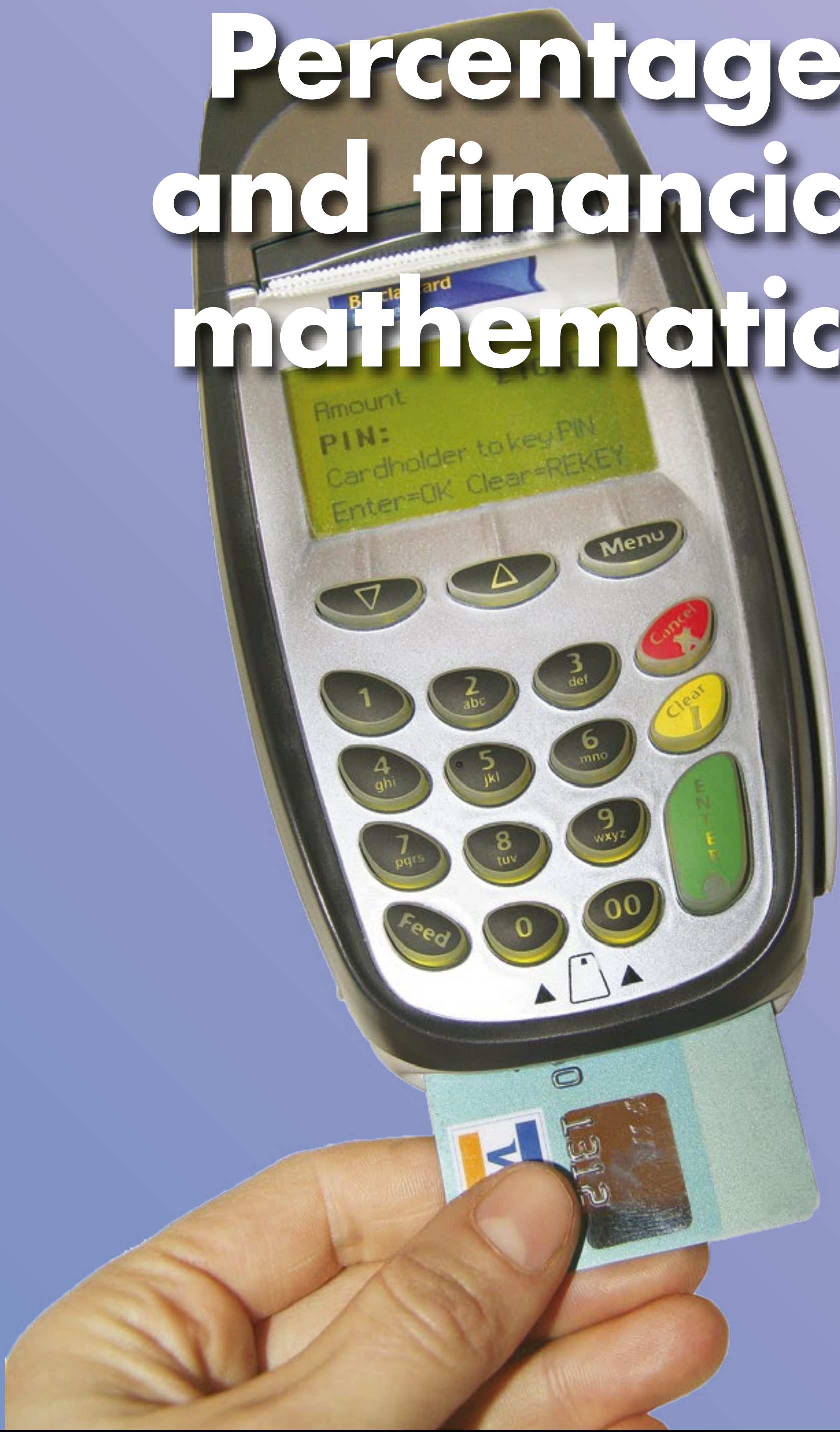
Practice quiz
Chapter 4

● A floor rug is 1 metre longer than its width. Let w metres represent its width.

- **a** Write an expression for the length of the rug in terms of its width, w.
- **b** Write an expression that uses brackets for the area of the rug.
- **c** By expanding, write the area expression without brackets.
- **d** If the width of the rug is 3 metres, find its length and its area.

5

Percentages and financial mathematics



Pre-test

Warm-up

Percentages play an important part in our everyday lives as consumers. Shops apply percentage mark-ups to the goods they sell and advertise percentage discounts. We pay 10% GST on most of the things we buy. Changes in property prices in cities and towns are often reported as percentage increases or decreases.

5.1 Fractions, decimals and percentages

What is a percentage?

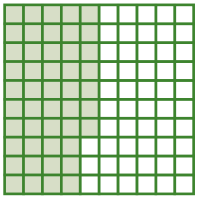
Percentages express a quantity as a number of parts out of 100 parts. ‘Per cent’ means ‘out of 100’.

The whole amount is called 100 per cent, written as 100%.

Percentages are used widely. Because percentages all have a denominator of 100, it is easier to compare percentages than it is to compare fractions with different denominators.

Example 1

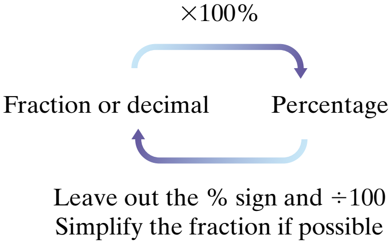
What percentage of this grid of 100 squares is shaded, and what percentage is unshaded?



Working	**Reasoning**
47% of the grid is shaded. 100% – 47% = 53% 53% of the grid is not shaded.	There is a total of 100 small squares. 47 of these are shaded. The whole grid is 100%. Subtract 47% from 100%.

Decimal fraction percentage

Because a percentage is ‘out of 100’, it is very easy to convert a percentage to a fraction or decimal. We write the percentage as a fraction with a denominator of 100. Note that this is the same as dividing the percentage by 100.



Example 2

Complete this table of fractions, decimals and percentages.

Fraction	Decimal	Percentage
$\frac{3}{4}$		
	0.48	
		54%
$1\frac{5}{8}$		
		$8\frac{1}{2}$

Working and reasoning

Fraction	Decimal	Percentage
$\frac{3}{4}$	$3 \div 4 = 0.75$	$0.75 = 0.75 \times 100\% = 75\%$
$0.48 = \frac{48}{100} = \frac{12}{25}$	0.48	$0.48 = 0.48 \times 100\% = 48\%$
$54\% = \frac{54}{100} = \frac{27}{50}$	$54\% = \frac{54}{100} = 0.54$	54%
$1\frac{5}{8}$	$1\frac{5}{8} = \frac{13}{8} = 1.625$	$1\frac{5}{8} = 1.625 \times 100\% = 162.5\%$
$8\frac{1}{2}\% = \frac{17}{2}\% = \frac{17}{200}$	$8\frac{1}{2}\% = 8.5\% = \frac{8.5}{100} = 0.085$	$8\frac{1}{2}\%$

Example 3

35% of Year 8 students at Broadfield State High School play soccer. Write this

a as a fraction. **b** as a decimal.

Working	Reasoning
a $35\% = \frac{35}{100} = \frac{7}{20}$	35 out of every 100 students play soccer.
b $35\% = 0.35$	$35 \div 100 = 0.35$

Tech tip

The TI-30XB MultiView calculator can be used to convert a fraction to a percentage. For example, to convert $\frac{5}{8}$ to a percentage, type:

$\boxed{5}\,\boxed{\frac{n}{d}}\,\boxed{8}\,\boxed{\text{2nd}}\,\boxed{\blacktriangleright}\,\boxed{\text{enter}}$.

The TI-30XB MultiView calculator can be used to convert decimals to percentages. For example, to convert 0.875 to a percentage, type:

$\boxed{0}\,\boxed{\cdot}\,\boxed{8}\,\boxed{7}\,\boxed{5}\,\boxed{\text{2nd}}\,\boxed{\blacktriangleright}\,\boxed{\text{enter}}$.

Note that $\boxed{\text{2nd}}\,\boxed{\blacktriangleright}$ gives $[\blacktriangleright\%]$.

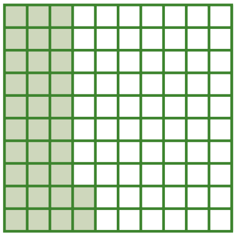
exercise 5.1

LINKS TO Example 1

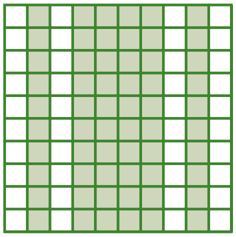
For each of the following grids split into one hundred equal squares, find:

i the percentage shaded. **ii** the percentage not shaded.

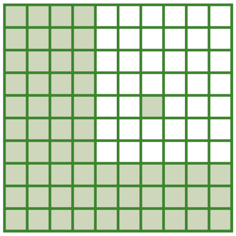
a



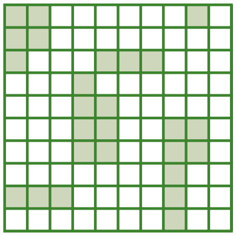
b



c



d



LINKS TO Example 2

Complete this table of fractions, decimals and percentages.

	Fraction	Decimal	Percentage
a	$\frac{5}{8}$		
b		0.36	
c			70%
d	$2\frac{3}{4}$		

	Fraction	Decimal	Percentage
e			$12\frac{1}{2}$
f	$\frac{2}{3}$		
g		0.84	
h			95%
i	$1\frac{2}{5}$		
j			16.4%
k		2.8	
l	$1\frac{9}{20}$		
m		2.56	
n			212%
o	$\frac{17}{40}$		
p			125%

0.37 as a percentage is

A 0.037% **B** 0.37% **C** 3.7% **D** 37% **E** 370%

45% as a fraction is

A $\frac{4}{9}$ **B** $\frac{5}{9}$ **C** $\frac{5}{20}$ **D** $\frac{45}{1}$ **E** $\frac{9}{20}$

$3\frac{1}{4}\%$ as a decimal is

A 3.25 **B** 32.50 **C** 325 **D** 0.325 **E** 0.0325

LINKS TO Example 3

Abdul pays 42% of his income on rent. Write this as a fraction in its simplest form.

Alex was solving an equation and obtained the result $x = \frac{3}{16}$. Write this as a decimal.

● A salesperson is paid $7\frac{1}{2}\%$ commission for the goods that she sells. Write this as a decimal.

● The following table gives approximate percentages of nitrogen, oxygen and carbon dioxide in air. Complete the table.

	Percentage	**Fraction**	**Decimal**
Nitrogen	80%		
Oxygen	20%		
Carbon dioxide	0.04%		

● The weather forecast predicts that there is a 15% chance of rain. Write this percentage as a fraction and as a decimal.

● Australian coins are made of mixtures of metals called alloys. Convert each of the following percentages to fractions.

a 5c, 10c, 20c and 50c coins are made of cupro nickel. This is 75% copper and 25% nickel.

b $1 and $2 coins are made of aluminium bronze. This is 92% copper, 6% aluminium and 2% nickel.

exercise 5.1 challenge

● Many metals are not pure, but are mixtures called alloys. Even metals like gold and silver that are used in jewellery are mixtures.

Pure gold is said to be 24 carat gold. Gold and precious stones used to be weighed in a balance with seeds on the other side of the balance. 'Carat' comes from the Arabic word for 'seed'.

a What fraction of 18 carat gold is actually gold?

b What percentage is this?

c What percentage of 9 carat gold is actually gold?

Other metals added to gold can make it harder and give different colours.

d White 9 carat gold is made of a mixture of gold and silver. What percentage of it is silver?

e Red 18 carat gold has 4.5% silver and the rest is copper, which gives the red colour. What is the percentage of copper?

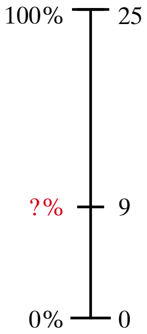
f Sterling silver is 92.5% silver and the rest is copper. Colour a 10 cm strip to show the percentages of silver and copper.

5.2 Expressing a part as a percentage of the whole

Problems involving percentages can be of several different sorts. It is important to understand what needs to be calculated. Compare the three different problems shown below. In each case a double number line has been drawn showing percentages on the left and the actual numbers on the right. The double number line helps us to see what is missing, that is, what we need to calculate.

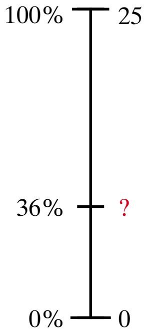
1 Expressing a part as a percentage of the whole
9 students in a class of 25 travel to school by bus. What percentage of the students travel by bus?

What percentage is 9 of 25?



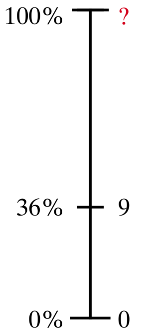
2 Finding a percentage of the whole
36% of students in a class of 25 travel to school by bus. How many students travel to school by bus?

What number is 36% of 25?



3 Finding the whole
36% of students in a class travel to school by bus. If 9 students travel to school by bus, how many students are there in the class?

What is the whole if 9 is 36% of the whole?



In this section we will look at the first type of problem: expressing one quantity or part as a percentage of the whole.

Example 4

Express these as percentages.

a 13 out of 20

b 64 out of 72.

Working

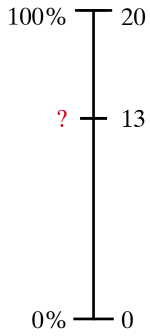
Reasoning

a

$$\frac{13}{\underset{1}{\cancel{20}}} \times \frac{\overset{5}{\cancel{100}}\%}{1}$$

$$= 13 \times 5\%$$

$$= 65\%$$



Write 13 out of 20 as a fraction.

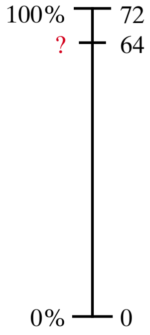
Multiply by 100%.

b

$$\frac{\overset{8}{\cancel{64}}}{\underset{9}{\cancel{72}}} \times \frac{100\%}{1}$$

$$= \frac{800}{9}\%$$

$$= 88.\bar{8}\%$$



Write 64 out of 72 as a fraction.

Multiply by 100%.

Convert the improper fraction to a mixed number.

Example 5

Complete the following.

a Ranee obtained 53 out of 60 for her mathematics test. What was this result as a percentage?

b Express $16 as a percentage of $75.

continued

Example 5 continued

Working

Reasoning

a $\frac{53}{60}$

$= \frac{53}{\underset{3}{60}} \times \overset{5}{100}\%$

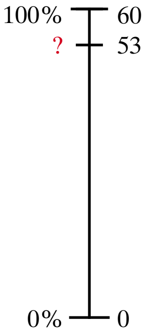
$= \frac{53 \times 5}{3}\%$

$= \frac{265}{3}\%$

$= 88\frac{1}{3}\%$

or $88.\overline{3}\%$

Ranee obtained $83\frac{1}{3}\%$ for her maths test.



Write 53 out of 60 as a fraction.
Multiply by 100%.

Cancel out common factors in the numerator and denominator.

b $\frac{16}{75}$

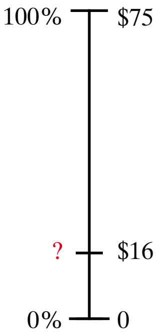
$= \frac{16}{\underset{3}{75}} \times \overset{4}{100}\%$

$= \frac{16 \times 4}{3}\%$

$= \frac{64}{3}\%$

$= 21\frac{1}{3}\%$

\$16 is $21\frac{1}{3}\%$ of \$75.



\$16 and \$75 are in the same unit.
Write as a fraction without the \$ signs.
Multiply by 100%.

Cancel out common factors in the numerator and denominator.

Quantities must be in the same unit before we calculate a percentage.

Example 6

Calculate 4 cm as a percentage of 1.75 m, correct to one decimal place.

continued

Example 6 continued

Working

4 cm, 1.75 m

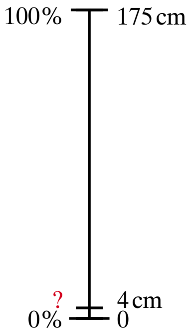
$\frac{4}{175}$

$= \frac{4}{175} \times 100\%$

$= 2.29\%$

4 cm is approximately 2.3% of 1.75 m.

Reasoning



The amounts must be in the same unit.
Convert 1.75 m to 175 cm.
Write 4 as a fraction of 175.
Multiply by 100%.

Example 7

At Greenbank College, 37 of the 98 Year 8 students play soccer. Express this as a whole number percentage.

Working

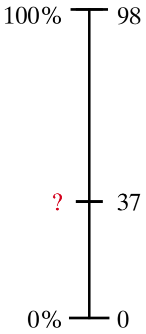
$\frac{37}{98}$

$= \frac{37}{98} \times 100\%$

$= 37.76\%$

38% of students play soccer.

Reasoning



Write 37 out of 98 as a fraction.
Multiply by 100%.

Example 8

350 people were surveyed about their attitude to creating a second pedestrian crossing near a school. Of those surveyed, 324 people were in favour of the new crossing, 15 were not in favour because they thought the existing crossing was adequate and the remainder were unsure.

To the nearest whole number percentage, what percentage

a of people were in favour of the new crossing?

b thought the existing crossing was adequate?

c were unsure?

continued

Example 8 continued

Working	Reasoning
a $\frac{324}{350} \times 100\%$ $= 92.6\%$ 93% of people were in favour.	The fraction of people in favour is $\frac{324}{350}$ Multiply by 100%.
b $\frac{15}{350} \times 100\%$ $= 4.3\%$ 4% of people thought the existing crossing was adequate.	The fraction of people not in favour is $\frac{15}{350}$ Multiply by 100%.
$92.6\% + 4.3\% = 96.9\%$ $100\% - 96.9\% = 3.1\%$ 3% of people were unsure.	The remainder of the people were unsure, so the percentages who were in favour and not in favour are subtracted from 100%. When using results from previous calculations it is better to work with the unrounded values then round the final value. In this case we would still obtain 3% using the rounded values, but with some calculations using the rounded values could affect the final value.
Alternatively: $350 - 324 - 15 = 11$ 11 people were unsure. $\frac{11}{350}$ $= \frac{11}{\underset{7}{350}} \times \overset{2}{100}\%$ $= \frac{22}{7}\%$ $= 3.1\%$	We can also find the percentage of people who were unsure by first finding the actual number of people who were unsure. We can then calculate 11 as a percentage of 350 to obtain 3.1% as before.

Tech tip

The TI-30XB MultiView calculator can be used to express one quantity as a percentage of another.

For example, to express 37 as a percentage of 98 (example 6), type:

3 7 $\frac{n}{d}$ 9 8 2nd ▶ enter.

Note that 2nd ▶ gives [▸%].

exercise 5.2

LINKS TO Example 4

Calculate each of the following as a percentage, rounding to the nearest percent.

a 12 out of 60
b 27 out of 65
c 146 out of 400
d 472 out of 1650
e 14.5 out of 20
f 87.5 out of 125
g 49 g out of 172 g
h \$785 out of \$1600
i \$2394 out of \$8700
j 19 mL out of 55 mL
k 7 cm out of 250 cm
l 147 min out of 180 min

15 students in class 8B were on an excursion. The other 8 were at school. The percentage of the class on the excursion was closest to

A 53%
B 47%
C 65%
D 35%
E 15%

In a school, 48 of the Year 8 students learn Indonesian and the other 36 learn Italian. The percentage of students learning Italian is closest to

A 75%
B 57%
C 43%
D 25%
E 33%

LINKS TO Example 5

A packet of breakfast cereal stated the following percentages for 30 g of cereal:

Protein	4.0 g
Fat	0.5 g
Carbohydrate	19.5 g
Fibre	3.4 g

a Calculate the percentage of each.
b The ‘missing’ percentage is probably mainly water. What is the size of this missing percentage?

LINKS TO Example 6

Chris found that there was an error of 14 cm in the measurement of the length of a wall. If the length of the wall was measured accurately as 4.26 m, what was the percentage error? Give the error correct to the nearest percent.

LINKS TO Example 7

In a delivery of 4800 bathroom tiles to a warehouse, 284 were cracked. What percentage is this, correct to the nearest whole number?

LINKS TO Example 8

Out of 147 Year 8 students, 65 chose to go on camp and the rest chose a city week experience. What percentage of students chose each option? Give your answer to the nearest percent.

exercise 5.2 challenge

Australia has an area 7 686 850 square kilometres. If 3 074 750 square kilometres is covered by sand dunes, what percentage of Australia’s area is covered by sand dunes? Give your answer to the nearest percent.

The population of the world is approximately 6.8 billion, that is, 6.8 thousand million. Australia’s population is approximately 22.5 million. Correct to two decimal places, what percentage of the world’s population lives in Australia?

In a sample of 400 people, 18% said they regularly rode a bicycle to work. Of those who regularly rode a bicycle to work, 75% were male. How many people who rode to work were male?

5.3

Finding a percentage of the whole

In this section we look at the second type of problem: finding a percentage of the whole.

Data is often given as percentages, for example, 35% of the 160 Year 8 students at Broadfield State High school play soccer. We may be more interested in knowing the actual number who play soccer.

Example 9

35% of the 160 Year 8 students at Broadfield State High school play soccer. How many students play soccer?

Working

35% of 160

= 0.35 × 160

= 56

56 students play soccer.

Reasoning

$$35\% = \frac{35}{100} = 0.35$$

Example 10

Calculate the following.

- **a** 80% of 560 mL.
- **b** 48% of $2750 using a calculator.

continued

Example 10 continued

Working

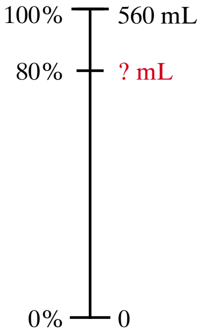
Reasoning

a 80% of 560

= 0.8 × 560

= 448

80% of 560 mL is 448 mL.

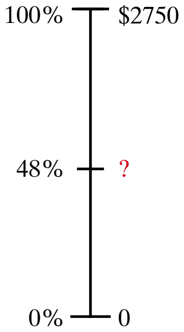


$80\% = \frac{80}{100} = 0.8$

b 48% of \$2750

= 0.48 × 2750

= \$1320



Use a calculator to find 0.48 × 2750. Because we are finding a percentage of a quantity, the answer will have the same unit as the original quantity, in this case, dollars.

Tech tip

The TI-30XB MultiView calculator can find a percentage of a quantity.

For example, to find 80% of 560 mL (example 9 part a), type:

8 0 2nd (× 5 6 0 enter.

Note that 2nd (gives [%].

exercise 5.3

LINKS TO Example 9

Calculate the following quantities.

a 35% of 240

b 21.5% of 800

c 24% of 175 g

d 5.4% of \$1200

e 0.4% of 640 mL

f 200% of \$1800

g 140% of \$24 000

h 13.6% of 745 kg

i 125% of 560 000

j 1.45% of 1600 **k** $1\frac{3}{4}\%$ of 2500 **l** 0.15% of 90

m 148% of 6000 **n** $12\frac{1}{2}\%$ of 7200 **o** 0.4% of 254 500

28% of 360 is

A 100.8 **B** 12.9 **C** 129 **D** 1290 **E** 108

0.03% of 48 is

A 1.44 **B** 0.0144 **C** 1600 **D** 16 **E** 0.16

LINKS TO Example 10b

A water reservoir can hold 24 868 megalitres of water. If it is 82.9% full, how many megalitres of water does it contain?

A 148 megabytes file was being downloaded from the internet. The progress bar on the computer screen showed that the file was 72% downloaded.

a How many megabytes had been downloaded?
b How many more megabytes remained to be downloaded?

George obtained a score of 85% for a test. If the test was marked out of 70, how many marks did George lose on the test?

Rosy earns $14 250. She does not have to pay tax on the first $6000 but must pay 15% tax on the part of her income that is over $6000.

a What income will she pay tax on?
b How much tax does she pay?
c What percentage of her whole income does this tax represent?

exercise 5.3 challenge

Of the students in Year 8 at a particular school, 120 are boys and 80 are girls. 15% of the boys and 9% of the girls played interschool volleyball. What percentage of the Year 8 students played interschool volleyball?

5.4 Finding the whole

So far we have looked at two different types of percentage problem:

- Expressing a quantity as a percentage of the whole.
- Finding a percentage of the whole

In this section we look at the third type of percentage problem: finding the whole when we know a certain percentage of the whole.

Example 11

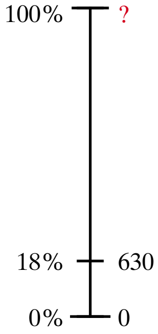
18% of a number is 630. What is the number?

Working

$$\begin{aligned}
18\% \text{ of the number} &= 630 \\
1\% \text{ of the number} &= \frac{630}{18} \\
&= 35 \\
100\% \text{ of the number} &= 35 \times 100 \\
&= 3500
\end{aligned}$$

The number is 3500.

Reasoning



Find 1% by dividing by 18.

Multiply by 100 to find 100%.

Example 12

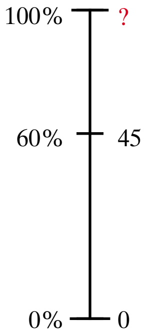
45 of the Year 8 students at Alpha Secondary College play soccer. This represents 60% of the Year 8 students. How many Year 8 students are there altogether at Alpha Secondary College?

Working

$$\begin{aligned}
60\% \text{ of Year 8 students} &= 45 \\
1\% \text{ of Year 8 students} &= \frac{45}{60} \\
100\% \text{ of Year 8 students} &= \frac{45}{60} \times \frac{100}{1} \\
100\% \text{ of Year 8 students} &= 75
\end{aligned}$$

There are 75 Year 8 students at Alpha Secondary College.

Reasoning



Find 1% by dividing by 60.

Multiply by 100 to find 100%.

exercise 5.4

LINKS TO Example 11

Find the whole quantity represented by each of these by first finding 1% then finding 100%.

a 30% of a quantity is 240
b 80% of a quantity is 720
c 45% of a quantity is 765
d 16% of a quantity is 560
e 32% of a quantity is 2368
f 9% of a quantity is 504
g 7% of a quantity is 1015
h 14% of a quantity is 7336

LINKS TO Example 12

Find the whole quantity represented by each of these by first finding 1% then finding 100%.

a 24% of houses in a suburb is 3168 houses.
b 35% of students at a university is 2590
c 18% of the population of a town is 5148 people.
d 75% of the cost of a car is \$17 700
e 41% of the cost of a cruise is \$3649
f 17% of Tom's salary is \$7004
g 12% of a bank loan is \$6816
h 48% of the number of spectators is 34 176

If 27% of a quantity is 62 100 then the whole quantity is

A 2300 **B** 230 000 **C** 16 767 **D** 1 674 000 **E** 62 173

In Solar Street, 105 houses had solar electricity. This represented 84% of the houses. How many houses were there altogether in Solar Street?

In a class of Year 8 students, 18 came to school by bus. This represented 75% of the students in the class. How many students were in the class?

A water reservoir contained 117 670 megalitres of water. This was 41% of its maximum capacity. What was the maximum capacity of the reservoir?

When Year 8 students at Highfield College were surveyed about how they travelled to school each day, it was found that 25% walk, 9% ride bikes, 59% come by bus and the remainder are driven to school.

a What percentage of students are driven to school?
b If 39 students walk to school, how many Year 8 students are there at Highfield College?
c Calculate the number of students who
  i ride bikes **ii** come by bus **iii** come by car

11.2% of a downloaded file is 2.8 MB. What is the total size of the file?

1.7% of a batch of batteries were found to be faulty. If there were 51 faulty batteries, what was the total number of batteries in the batch?

exercise 5.4 challenge

- ● In a particular region there are 97 100 people aged 0 to 14 years. This is 19.6% of the population. What is the population of the region?
- ● The percentage of girls in a youth club was 44%. If there were 14 boys in the group, how many more girls would need to join to make the percentages equal at 50% each?

5.5 Percentage increase and decrease

When quantities such as populations, house prices or occurrence of diseases change, the changes are usually expressed as percentage increases or decreases.

Percentage increase or decrease

Percentage increase or decrease is expressed as a percentage of the original value.

$$\text{Percentage increase or decrease} = \frac{\text{Change in value}}{\text{Original value}} \times 100\,\%$$

$$\text{where, change in value} = \text{larger value} - \text{smaller value}$$

If a quantity increases, there is a **percentage increase**.

Example 13

Over the last five years the average house price in Boyesville has increased from \$247 000 to \$385 000.

a What is the average increase in price?

b What is the percentage increase? Give the percentage to the nearest whole number.

Working

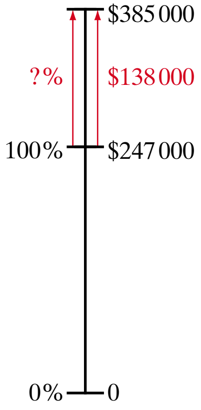
a Increase

= \$385 000 − \$247 000

= \$138 000

The average increase in house price is \$138 000.

Reasoning



Subtract the smaller value from the larger value.

continued

Example 13 continued

Working

Reasoning

b Percentage increase

$$= \frac{\text{change in value}}{\text{old value}}$$

$$= \frac{138\,000}{247\,000} \times 100\%$$

$$\approx 55.9\%$$

There has been a 56% increase in average house price.

Express the change as a percentage of the original (or old) value.

If a quantity decreases, there is a **percentage decrease**.

Example 14

The amount of water in a reservoir decreased over summer from 542 000 megalitres to 531000 megalitres.

a What was the decrease in the amount of water?

b What is the percentage decrease? Give the percentage to the nearest whole number.

Working

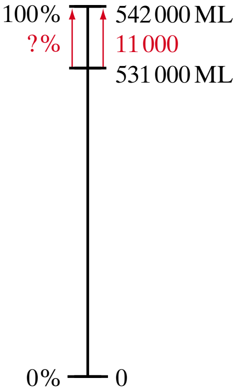
Reasoning

a Decrease

$$= 542\,000 - 531\,000$$

$$= 11\,000$$

The decrease in the amount of water was 11 000 megalitres.



Subtract the smaller value from the larger value.

b Percentage decrease

$$= \frac{\text{change in value}}{\text{old value}}$$

$$= \frac{11\,000}{542\,000} \times 100\%$$

$$\approx 2.0\%$$

There has been a 2% decrease in the amount of water.

Express the change as a percentage of the original (or old) value.

When percentage changes are reported we may want to calculate a new value, or we may want to know the old value.

Increasing an amount by a given percentage

If a quantity is increased by a percentage, we can calculate the size of the increase and add it to the original value. For example, if a membership fee of $100 is increased by 10%, the increase is $10, giving the new value of $110. We can see that the new value is 110% of the old value. We can calculate 110% by multiplying the old price by 1.10.

Percentage increase

Increased amount = (100% + percentage increase) × original amount

Example 15

Increase these amounts by the percentages shown.

a If the cost of a $4000 car is increased by 15%, find the new value of the car.

b If the price of a $650 bicycle is increased by 24%, find the new value of the bicycle.

Working

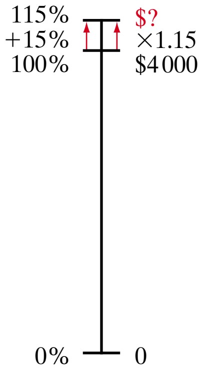
a 100% + 15% = 115%

= 1.15

4000 × 1.15 = 4600

The new value of the car is $4600.

Reasoning



Add the percentage increase to 100%.

Convert 115% to the decimal 1.15.

Multiply 4000 by 1.15.

continued

Example 15 continued

Working

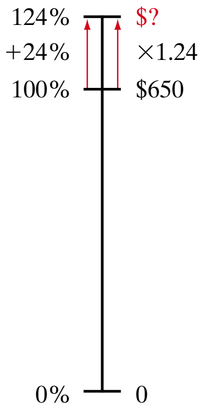
b $100\% + 24\% = 124\%$

$= 1.24$

$650 \times 1.24 = 806$

The new value of the bicycle is $806.

Reasoning



Add the percentage increase to 100%.

Convert 124% to the decimal 1.24.

Multiply 650 by 1.24.

Decreasing an amount by a given percentage

If a quantity is decreased by a percentage, we can calculate the size of the decrease and subtract it from the original quantity. For example, if $100 is decreased by 10%, the decrease is $10, giving the new value of $90. We can see that the new value is 90% of the old value. We can calculate 90% by multiplying the old price by 0.90.

Percentage decrease

Decreased amount = (100% − percentage decrease) × original amount

Example 16

Decrease these quantities by the percentages shown.

a If the price of a $385 camera decreases by 10%, find the new cost.

b If 7200 litres of water in a tank decreases by 16%, find the new volume of water in the tank.

continued

Example 16 continued

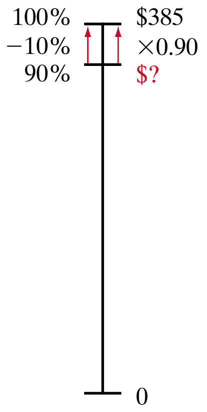
Working **Reasoning**

a 100% − 10% = 90%

= 0.9

385 × 0.9 = 346.50

The new price of the camera is $346.50



Subtract the percentage decrease from 100%.

Convert 90% to the decimal 0.9.

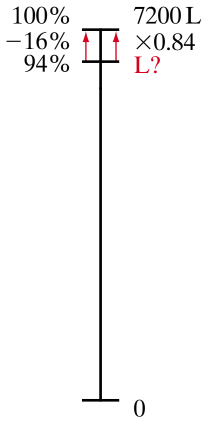
Multiply 385 by 0.9.

b 100% − 16% = 84%

= 0.84

7200 × 0.84 = 6048

The new volume of water is 6048 litres.



Subtract the percentage decrease from 100%.

Convert 84% to the decimal 0.84.

Multiply 7200 by 0.84.

exercise 5.5

LINKS TO Example 13

The population of a town increased from 14 652 to 16 014. What was the percentage increase? Give your answer to one decimal place.

The price of apples increased from $2.85 per kilo to $3.15 per kilo. What was the percentage decrease? Give your answer to one decimal place.

LINKS TO Example 14

The price of gold fell from $US1217 per ounce to $US1119. What was the percentage fall? Give your answer to one decimal place.

LINKS TO Examples 13, 14

Calculate each of these increases or decreases as a percentage of the original value, stating whether it is a percentage increase or a percentage decrease. Round the percentage to one decimal place where appropriate.

	Original value	New value	% increase or decrease
a	400	450	
b	2700	2754	
c	4200	3500	
d	180 000	150 000	
e	276	248	
f	895	964	
g	147 560	154 984	
h	16 873	14 965	
i	4596	6873	
j	565 000	875 000	
k	17 480	13 240	
l	1158	1295	

● The wheat harvest in a region increased from 5.8 million tonnes to 8.6 million tonnes. What was the percentage increase? Give your answer to one decimal place.

● A barley crop decreased from 9.3 million tonnes to 8.1 million tonnes. What was the percentage decrease? Give your answer to one decimal place.

LINKS TO Example 15

● Increase each of these amounts by the percentage shown.

a 1800 by 35%
b 480 by 16%
c 9500 by 25%
d 285 kg by 40%
e 4800 L by 1.5%
f $1300 by 8%
g 60 cm by 2%
h $2800 by 12%
i 1500 students by 75%
j 4500 jobs by 60%
k 15 000 people by 4.5%
l 12 500 tonne by 1.8%

LINKS TO Example 16

● Decrease each of these amounts by the percentage shown.

a 1500 by 65%
b 120 by 30%
c 7600 by 18%
d 590 students by 2%
e $8650 by 70%
f $1200 by 5.4%
g 9000 L by 2.5%
h $1600 by 12.5%
i 3600 people by 7.5%
j 4500 by 8%
k 48 000 kangaroos by 2.4%
l 12 500 penguins by 1.8%

● Police in a particular region reported a 15.8% increase in thefts from parked cars over the past year. If the number of thefts in the region was previously 38, what was the new number of thefts? Round to the nearest whole number.

● The population of a town of 34 800 increased by 0.4%. The population would then be closest to

A 87 000
B 48 720
C 36 192
D 34 939
E 13 920

- Chris has had a salary increase of 8%. If his current salary is \$54 000 per year, what is his new salary?
- Angie paid \$23 750 for her new car one year ago. After one year of use, its value had dropped by 15%. What is the value of the car now? Give the value to the nearest dollar.
- A company's profit rose by 86% since the previous year. If the profits in the previous year were \$2 100 000, what was the new profit?
- Heavy rainfall increased the water supplies by 2.3%. If the total amount of water in the reservoirs was 1 374 000 mL before the rain, what is the new amount?
- There was a 1.5% increase in the number of students at Greenbank High School. If the number was previously 952, what is the new number of students? Round to the nearest whole number.

exercise 5.5 challenge

- A town population of 9000 increased by 4% during 2010 then increased a further 5% in 2011. What was the overall percentage increase?
- In July, a car factory exceeded its production target by approximately 5%. If the factory produced 1200 cars in July, what was its production target?

5.6 Mark-ups and discounts

Mark-ups

A retail shop buys stock from a wholesaler at a certain price, and then sells the stock at a higher price. The amount that the retailer adds to the **wholesale price** to set the **retail price** is called the **mark-up**.

$$\text{Retail price (or selling price)} = \text{wholesale price} + \text{mark-up}$$

The mark-up allows the retailer to pay expenses such as shop rental and wages for sales staff as well as providing the retailer's own income.

Different shops may purchase from the same wholesaler, but use different mark-ups on the purchase price, so retail price of a particular item may vary from shop to shop.

The **percentage mark-up** is normally calculated as a percentage of the wholesale price.

For example, applying a 40% mark-up to a \$30 pair of jeans, adds 40% of 30 or $0.4 \times \$30$ to the price of the jeans. This is a mark-up of \$12 and the retail price of the jeans is $\$30 + \$12 = \$42$. Another way to find the retail price of the jeans would be find 140% of \$30 or $1.4 \times \$30 = \$42$. This is the method we used in section 5.5 to calculate percentage increases.

Mark-up and retail price

Mark-up = percentage mark-up × wholesale price

Retail price = wholesale price + mark-up, or

Retail price = (100% + percentage mark-up) × wholesale price

A retailer decides on a percentage mark-up then needs to calculate this percentage of the wholesale price to work out the retail price.

In this section we assume that the retail price is the price before GST is added.

Example 17

A shop puts a 70% mark-up on T-shirts bought from a wholesaler at \$12. What is the retail price?

continued

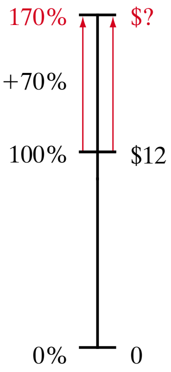
Example 17 continued

Working

Method 1:

Mark-up = 70% of $12
= 0.7 × $12
= $8.40
Retail price = $12 + $8.40
= $20.40
The retail price is $20.40.

Reasoning



Mark-up is 70% of the wholesale price
Add the mark-up to the wholesale price.

Method 2:

Retail price
= (100 + percentage mark-up) % × wholesale price
= (100 + 70) % × $12
= 170% × $12
= 1.70 × $12
= $20.40

Add the percentage mark-up to 100%.
Convert to a decimal.
Multiply the wholesale price by this decimal number.

Example 18

A shop puts a 60% mark-up on goods it buys from a wholesaler.

a What is the selling price as a percentage of the wholesale price?

b If the shop pays the wholesaler $6 for DVDs what is the retail price?

c A retailer put a 60% mark-up on a brand of MP3 player so that the retail price was $120. What was the wholesale price?

Working

a Selling price
= 100% of wholesale price + 60% of wholesale price
= 160% of wholesale price.

Reasoning

Add the percentage mark-up to 100% of the wholesale price.

continued

Example 18 continued

Working

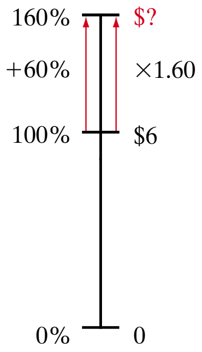
b Retail price = 160% of \$6

= 1.6 × \$6

= \$9.60

The retail price of a DVD is \$9.60.

Reasoning



The selling price is 160% of the wholesale price.

c 160% of wholesale price = \$120

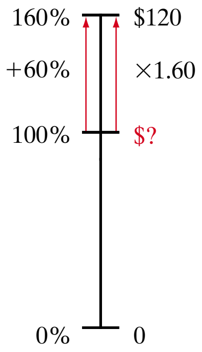
$$1\% \text{ of wholesale price} = \frac{\$120}{160}$$

= \$0.75

100% of wholesale price = 0.75 × 100

= \$75

The wholesale price is \$75.



Divide \$120 by 160% to find 1% of wholesale price. Multiply by 100 to find 100%.

Example 19

A shop buys jeans from a wholesaler for \$55 and marks them up to \$99.

a What is the mark-up?

b What is the percentage mark-up?

Working

a Mark-up = selling price – wholesale price

= \$99 – \$55

= \$44

The mark-up is \$44.

Reasoning

The mark-up is the amount that the shop adds to the wholesale price.

continued

Example 19 continued

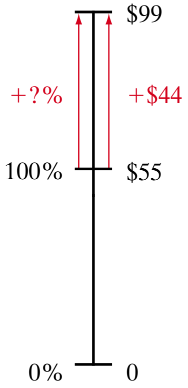
Working

b $\%\text{ mark-up} = \frac{44}{55} \times 100\%$

$= 80\%$

The percentage mark-up is 80%.

Reasoning



Write the mark-up as a fraction of the wholesale price. Multiply by 100%.

Finding the wholesale price

If we know the percentage mark-up a retailer has applied, we may wish to work backwards to find the wholesale price. Supposing the retailer has applied a 70% mark-up, the retail price is 170% of the wholesale price.

$$\begin{aligned} 170\% \text{ of wholesale price} &= \text{retail price} \\ 1\% \text{ of wholesale price} &= \frac{\text{retail price}}{170} \\ 100\% \text{ of wholesale price} &= \frac{\text{retail price}}{170} \times 100 \end{aligned}$$

To find the wholesale price, find 1% of the wholesale price then multiply by 100.

Discounts

When a retailer wants to clear stock quickly—for example, to provide income for purchasing new season's fashions—prices may be reduced. The reduction in price is called a **discount**.

For example, applying a 25% discount to a $42 pair of jeans reduces the price of the jeans by 25% of $42 or 0.25 × $42. This is a discount of $10.50 and the retail price of the jeans is $42 − $10.50 = $31.50.

If we think of the normal retail price as 100% then applying a 25% discount means that the sale price is (100 – 25)% or 75% of the normal retail price. So $31.50 is 75% of $42.

This is the method we used in section 5.5 to calculate percentage decreases.

Discounts

Discount = percentage discount × normal price

Discounted price = normal price − discount, or
Discounted price = (100% − percentage discount) × normal price

Example 20

Jeans marked $99 are offered for sale at a percentage discount of 30%.

a What is the discount?

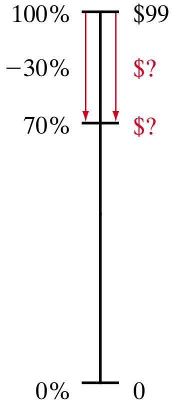
b What is the discount price of the jeans?

Working

a Discount = 30% of $99
= 0.3 × $99
= $29.70

The discount is $29.70.

Reasoning



A percentage discount is a percentage of the original price.

b Discount price = original price – discount
= $99 – $29.70
= $69.30

The discount price of the jeans is $69.30.

Subtract the discount from the original price.

If we think of the normal retail price as 100% then applying a 30% discount (as we did in example 19) means that the sale price is 70% or (100 – 30)% of the normal retail price. So $69.30 is 70% of $99. In general, we can say that:

Sale price

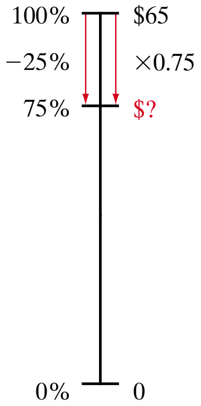
Sale price = (100% − percentage discount) × normal retail price

Example 21

A shop advertises that shoes are reduced by 25%.

a What is the sale price as a percentage of the original price?

b What is the sale price of a pair of shoes that normally sells at $65?

Working	Reasoning
a Selling price = 100% of normal price – 25% of normal price = 75% of normal price	 Subtract the percentage discount from 100% of the normal price.
b Sale price $= 75\%$ of $65 $= 0.75 \times \$65$ $= \$48.75$ The sale price is $48.75.	Find 75% of the normal price.

When an item is discounted we can work out the percentage discount by expressing the discount as a percentage of the normal price.

Percentage discount

$$\text{Percentage discount} = \frac{\text{discount}}{\text{normal price}} \times 100\%$$

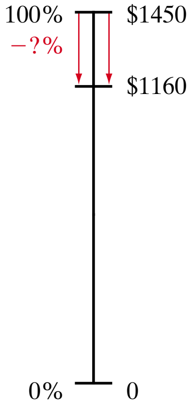
Example 22

A computer that normally sells for \$1450 is advertised for sale at \$1160. What is the percentage discount?

Working

$$\begin{aligned}\text{Discount} &= \$1450 - \$1160\\ &= \$290\end{aligned}$$

Reasoning



The discount is the amount by which the price has been reduced.

$$\begin{aligned}\text{Percentage discount} &= \frac{\text{Discount}}{\text{Normal retail price}} \times 100\%\\ &= \frac{290}{1450} \times 100\%\\ &= 20\%\end{aligned}$$

Percentage discount is calculated as a percentage of normal retail price.

Finding the normal price of discounted goods

If we see a discounted price we may be interested to know what the normal price was. By calculating 1% of the normal price we can then easily find 100% of the normal price.

Finding the normal retail price

To find the normal retail price, find 1% of normal retail price and then multiply by 100.

Example 23

A refrigerator was advertised in a sale for \$675. If the shop had reduced all goods by 25%, what was the normal price of the refrigerator before the sale?

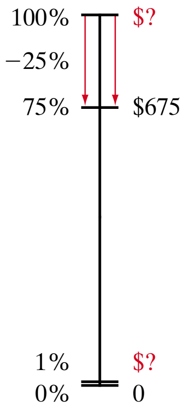
continued

Example 23 continued

Working

75% of the normal price = $675

Reasoning



If the price has been reduced by 25%, the sale price is (100 – 25)%, that is, 75% of the normal price.

$$
\begin{aligned}
1\% \text{ of the normal price} &= \frac{\$675}{75} \\
&= \$9 \\
100\% \text{ of the normal price} &= \$9 \times 100 \\
&= \$900
\end{aligned}
$$

The normal price of the refrigerator was $900.

Dividing $675 by 75 gives the value of 1% of the normal price. It is then easy to multiply by 100 to find 100% of the normal price.

exercise 5.6

LINKS TO Example 17

Complete this table.

	Wholesale price	% mark-up	Mark-up	Retail price (before GST)
a	$36	50%		
b	$250	35%		
c	$15	60%		
d	$90	100%		
e	$12	120%		
f	$940	30%		

LINKS TO Example 18

A retailer puts a 40% mark-up on goods bought for $65. What is the retail price?

A retailer buys DVDs for $6.50 and puts a 70% mark-up on them. What is the retail price?

LINKS TO Example 19

A computer desk is bought from a wholesaler for \$35 and marked up to \$89.

a What is the mark-up?

b What is the percentage mark-up correct to one decimal place?

The wholesale price of a particular model of washing machine is \$560. The retailer marks up the washing machine to \$820. What is the percentage mark-up correct to one decimal place?

A mobile phone was bought from a wholesaler at \$18 and marked up by the retailer to \$60. The percentage mark-up was

A 30% **B** $42\frac{6}{7}\%$ **C** 70% **D** $2\frac{1}{3}\%$ **E** $233\frac{1}{3}\%$

A retailer puts a 75% mark-up on toys bought from a wholesaler at \$16. What is the retail price?

A retailer buys shirts from a wholesaler for \$24 and marks them up to \$60.

a What is the mark-up? **b** What is the percentage mark-up?

LINKS TO Example 20

Complete this table.

	Normal price	% discount	Discount	Discounted price
a	\$18	25%		
b	\$55	10%		
c	\$150	$33\frac{1}{3}\%$		
d	\$24	40%		
e	\$1250	20%		
f	\$27 000	15%		

LINKS TO Example 21

T-shirts normally selling at \$18 were advertised in a sale at 40% discount. What is the sale price?

LINKS TO Example 21

A cinema gives a 10% discount to students in the school holidays. If the tickets are normally \$8, what is the discounted price?

A department store offers a 5% discount on all goods on a particular day. What is the saving on purchases totalling \$246?

An airline advertises a 15% discount for airfares.

a What was the original price of a fare that is reduced to \$680?

b By how much has the fare been reduced?

A shop advertises a sale with 20% discount on all goods. What was the original price of a refrigerator that has been reduced to \$880?

A supermarket advertises a special on 4 L containers of ice-cream. If the ice-cream is now selling at \$4.60 and the discount is 8%, what was the original price?

LINKS TO Example 22

A television was advertised in a sale at \$500. The original price was \$750. The percentage discount was

A 50% **B** $33\frac{1}{3}\%$ **C** $66\frac{2}{3}\%$ **D** 15% **E** 150%

LINKS TO Example 23

During a sale the price of a shirt was reduced by 15%. If the sale price was \$42.50, what was the original price?

exercise 5.6 challenge

A shop offers a discount of 10% on all goods then offers a further discount of 20% of the reduced price.

a Is this the same as a 30% discount? Use the example of an item originally costing \$100 to justify your answer.

b Supposing the 20% discount had been offered first, then the 10% discount. Would this have made any difference?

5.7 Profit and loss

Profit

If an item is sold for more than the cost price, the seller has made a **profit**.

Profit

Profit = selling price − cost price

The **percentage profit** is usually calculated as a percentage of the cost price.

Percentage profit

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

Example 24

A painting was bought for \$350 and sold a year later for \$580.
Calculate

- **a** the profit.
- **b** the percentage profit to the nearest whole number percentage.

Working	Reasoning
a $\text{profit} = \$580 - \$350$ $= \$230$	Subtract the cost price from the selling price.
b $\%\ \text{profit} = \frac{230}{350} \times 100\%$ $\approx 65.7\%$ The percentage profit was 66%.	$\%\ \text{profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$

Loss

If an item is sold for less than the cost price, the seller has made a **loss**.

Loss

Loss = cost price – selling price

The **percentage loss** is usually calculated as a percentage of the cost price.

Percentage loss

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

Example 25

A Sydney house bought for $1 635 500 in 2007 recently sold for $1.25 million.

a What was the loss?

b What was the percentage loss, correct to one decimal place?

Working	Reasoning
a Loss = cost price – selling price = $1 635 500 – $1 250 000 = $385 500 The loss was $385 500.	The loss is the difference between the price paid for the house and the selling price.
b $\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$ $= \frac{385\,000}{1\,635\,500} \times 100\%$ $= 23.6\%$ The percentage loss was 23.6%.	The percentage loss is calculated as a percentage of the cost price.

exercise 5.7

For questions 1–9, round your answers to the nearest whole number percentage.

Note: in each of the questions in this exercise assume that the selling price is the retailer's price before GST is added.

● Complete the following table.

	Cost price	Selling price	Profit or loss (state amount and whether profit or loss)	Percentage profit or loss
a	$18	$45		
b	$120	$180		
c	$32	$24		
d	$8.50	$6.00		

	Cost price	Selling price	Profit or loss (state amount and whether profit or loss)	Percentage profit or loss
e	$165 000	$240 000		
f	$16	$40		
g	$800	$960		
h	$54	$48		
i	$25	$95		
j	$1500	$600		
k	$22.50	$67.50		
l	$750	$480		

LINKS TO Example 24

Jodie makes jewellery and sells it on a craft market. It costs her $4.80 to make each bracelet and she sells the bracelets for $8.50 each.

a What profit does Jodie make on each bracelet?

b What is the percentage profit?

Chloe bought a second-hand bicycle for $85. She restored it and later sold it for $135.

a What profit did Chloe make on the bicycle?

b What was the percentage profit?

LINKS TO Example 25

Eric makes furniture. It costs him $220 to make a table which he sells for $650.

a What profit does Eric make on the table?

b What is the percentage profit?

The Smith family paid $285 000 for their house two years ago. They are moving interstate and sold the house recently for $267 000.

a What loss did they make on the price of the house?

b What percentage loss is this? Give the percentage correct to one decimal place.

The Tan family buy a house for $265 000 and spend $15 000 renovating it. They sell it after three years for $315 000. What percentage of profit did they make on the house?

A greengrocer buys a 20 kg box of apples for $35 and sells the apples for $4.20 per kilogram. What percentage of profit did she make on the apples?

Karl paid $49.50 for a textbook and sold it secondhand for $15.

a What loss did Karl make on the book? **b** What was the percentage loss?

Aron bought a surfboard for $195. After a year he sold it for $160. What loss did he make?

exercise 5.7 challenge

- A man buys a pony for $1000. He sells it to a woman and makes a profit of 10%. The woman later sells the pony back to the man and makes a 10% loss. Overall, what percentage profit does the man make?

- A furniture salesman sold two sets of tables and chairs for $990 per set. Based on their cost when he bought them from the manufacturer, he made a 10% profit on one set, and a 10% loss on the other set.
  - **a** Did the salesman make a profit or a loss overall?
  - **b** Calculate the amount of the profit or loss.

5.8 Goods and Services Tax (GST)

In Australia, a government tax of 10% is added to most goods and services. Certain items are exempt from GST, including unprocessed food such as fresh fruit and vegetables. The displayed price of goods must include GST.

When goods are sold by a manufacturer to a wholesaler then to a retail shop, all the businesses are able to claim back the GST costs. The only GST that is actually paid is by the end-user/consumer of the product.

Calculating the price including GST

A retailer must set the price of goods then add a further 10%. This means that the final price is 110% of the retailer's price.

Calculating GST

GST = 10% of price pre-GST price

Price including GST = 110% of pre-GST price

Example 26

The cost of a packet of A4 paper before GST is added is \$4.50. What is the cost when GST of 10% is added?

Working	Reasoning
Method 1:	
GST = 10% of \$4.50 = 45 cents	Calculate the GST.
Price including GST = \$4.50 + \$0.45 = \$4.95	Add the GST to the cost before GST.
Method 2:	
Price with GST = 110% of pre-GST = 110% of \$4.50 = 1.1 × \$4.50 = \$4.95	Add 10% GST to 100% to give 110%. Calculate 110% of price before GST.

Example 27

The retail price of a laptop computer before GST is added is $1270. Calculate the price including GST.

Working

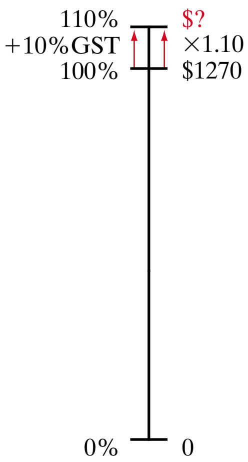
Price including GST

$= \$1270 \times 110\%$

$= \$1270 \times 1.10$

$= \$1397$

Reasoning



Price including GST

$= 110\%$ of pre-GST price.

$110\% = 1.10$

Calculating the price before GST is added

The amount of GST included in a retail price can be calculated by dividing by 110 to find 1% of the retail price and then multiplying by 10 to find 10% of the retail price. This means that GST $= \frac{10}{110}$ of the retail price, which simplifies to $\frac{1}{11}$.

$\frac{1}{11}$ of the retail price represents GST.

Example 28

The cost of hiring a hall is $286 including GST.

a How much of the $286 is GST? **b** What is the cost before GST is added?

Working	Reasoning
a GST $= \frac{1}{11}$ of $\$286$ $= \$26$	$\frac{10}{110} = \frac{1}{11}$ of the cost is GST.
b Cost before GST is added $= \$286 - \$26$ $= \$260$	Cost before GST $= \frac{10}{11} \times \$286$ $= \$260$

Example 29

The tickets for a concert cost \$16.50 inclusive of GST. What was the pre-GST price?

Working

$$\text{Pre-GST price} = \$16.50 \times \frac{10}{11}$$
$$= \$15$$

Reasoning

\$16.50 is 110% of the pre-GST price.
Divide \$16.50 by 110%.
This is the same as multiplying by $\frac{10}{11}$.

exercise 5.8

LINKS TO Example 26

Complete this table.

	Pre-GST price	GST	Price including GST
a	\$4.50		
b	\$12.40		
c	\$118.90		
d	\$560		
e	\$72.50		
f	\$22 400		

'Pre-GST' means 'before GST is added'

LINKS TO Example 26

A pair of jeans has a pre-GST price of \$57.50.

a How much GST will be added?
b What is the price including GST?

The pre-GST price of muesli bars at a supermarket is \$4.80?

a How much GST must be added?
b What is the price after GST is added?

A retailer has marked up the price of T-shirts to \$12.

a How much GST must now be added?
b What is the price inclusive of GST?

LINKS TO Example 27

A printer charges \$28 per book to print a special order for a customer. GST must then be added. How much will the books cost?

A supermarket sets a price of \$7.40 on 4 litre containers of ice-cream. What is the price after GST is added?

LINKS TO Example 28a

Cinema tickets cost \$14.30 inclusive of GST. How much of this price is GST?

A family spent \$38.50 at a restaurant. How much of this price was GST?

LINKS TO Example 27

Concert tickets are \$30.80 each, including GST.

a How much of this price is GST?
b What was the price before GST was added?

- A computer was advertised at $1485 including GST. The price before GST was added was
  A $1475 **B** $1336.5 **C** $1350 **D** $1320 **E** $1385

- A clinting store manager buys T-shirts from a wholesaler for $5 and sells them for $9.90 including GST.
  - **a** How much of the $9.90 is GST?
  - **b** What is the clothing store's price before GST is added?
  - **c** What is the percentage mark-up by the clothing store?

LINKS TO Example 29

- What was the pre-GST price of a pair of shoes that is advertised for $49.50?

- What was the amount of a gas bill before GST was added if the amount including GST is $204.16?

exercise 5.8 challenge

- A retailer buys DVDs from a wholesaler at $12. The retailer puts a mark-up of 48% then adds GST and rounds down to the nearest 5 cents. In a sale, the DVDs are discounted by 20%. What is the final sale price of the DVDs?

Analysis task

Pricing a T-shirt

A wholesaler imports T-shirts for $4 each. The wholesaler marks them up by 180% before selling them to retailers.

a What price does the retailer pay for each T-shirt?

A retailer who buys the T-shirts marks them up by 240% before adding GST.

b What is the selling price of the T-shirts?

c How much of the selling price is GST?

The retail shop has a sale where all items are reduced by 30%.

d What is the discount price of the T-shirts?

e By how much have the T-shirts been reduced?

f How does this discount price compare with the wholesale price that the retailer originally paid?

g What percentage mark up on the wholesale price does this discount price represent?

After the sale is finished, the retail shop puts the price back to what it was before the sale. In another sale the shop advertises ‘3 for the price of 2’.

h Suggest why the retailer advertises ‘3 for the price of 2’ rather than stating the percentage discount.

i Calculate the cost per T-shirt with this offer.

j What discount is this on each T-shirt?

k What percentage discount is this?

The retailer sold 1200 T-shirts in the ‘3 for the price of 2 sale’.

l How much money would be received from the sale?

m Compare the income from the sale of 1200 T-shirts with the amount the retailer would have paid the wholesaler for the 1200 T-shirts.

Review Percentages and financial mathematics

Summary

Percentage

- To change a decimal or a fraction to a percentage, find that fraction or decimal 'of ' 100%. Remember that 'of ' is calculated by multiplying.
- To change a percentage to a fraction, write it over 100, then cancel if possible.
- To change a percentage to a decimal, write it over 100, then write in decimal form.

$$\begin{tikzcd}
\text{Fraction or decimal} \arrow[r, bend left, "\times 100\%"] & \text{Percentage} \arrow[l, bend left, "\text{Leave out the \% sign and } \div 100"]
\end{tikzcd}$$

Expressing one quantity as a percentage of another

- Write the quantities as a fraction then convert to a percentage by multiplying by 100.

Finding a percentage of a quantity

- Multiply the quantity by the percentage.

Percentage increase and decrease

- Increased amount = (100% + percentage increase) × original amount
- Decreased amount = (100% + percentage decrease) × original amount

Mark-ups

- Marked-up price = (100% + percentage mark-up) × wholesale price
- Percentage mark-up $= \dfrac{\text{mark-up}}{\text{wholesale price}} \times 100\%$

Discounts

- Discount price = (100% − percentage discount) × normal price
- Percentage discount $= \dfrac{\text{discount}}{\text{normal price}} \times 100\%$

Profit

- Profit = selling price − cost price
- Percentage profit = $\dfrac{\text{profit}}{\text{cost price}} \times 100\%$

Loss

- Loss = cost price − selling price
- Percentage loss = $\dfrac{\text{loss}}{\text{cost price}} \times 100\%$

Calculating GST

- GST = 10% of pre-GST price
- Price including GST = 110% of pre-GST price
- $\dfrac{1}{11}$ of retail price is GST

Visual map

cost price
discount
GST
loss
mark-up
percentage
percentage decrease
percentage increase
pre-GST price
profit
retailer
retail price
whole
wholesaler
wholesale price

Revision

Multiple-choice questions

- In a particular library, there are four fiction books for every five nonfiction books. The percentage of fiction books in this library is approximately

  A 40% **B** 44.4% **C** 50% **D** 55.5% **E** 80%

- 16% of a number is 3680. The number is

  A 3696 **B** 3764 **C** 4268.8 **D** 5888 **E** 23 000

- A retailer puts a 30% mark-up on computers. What was the wholesale price of a computer that had been marked up to $1560?

  A $468 **B** $1092 **C** $1200 **D** $1260 **E** $2028

A retailer puts a mark-up of 25% on watches then adds 10% GST. If the wholesale price of a watch is $60, what is the selling price of the watch inclusive of GST?

A $81 **B** $82.50 **C** $85 **D** $83.50 **E** $95

The price of a TV after GST is added is $990. The price before GST is added is

A $99 **B** $880 **C** $891 **D** $900 **E** $980

Short-answer questions

Express the first quantity as a percentage of the second quantity correct to one decimal place.

a 45, 72 **b** 240 g , 400 g **c** $7.50, $48 **d** 15 mL, 180 mL

Calculate the following.

a 32% of 400 students **b** 4.6% of $3500

c 125% of $340 000 **d** 0.3% of 24 g

Calculate the following

a 15% of a number is 825. What is the number?

b 45 students at a school belonged to a swimming club. If this represented 18% of students, how many students were there at the school?

c Jason paid $5700 deposit on a new car. If this represented 15% of the cost, what was the cost of the car?

d 2603 people who lived in Harryville went to the local agricultural show. If this represented 38% of people in the town, what was the population of Harryville?

Increase the following amounts by the percentage shown.

a 32 cm by 45% **b** $165 by 16%

Decrease the following amounts by the percentage shown.

a 22.5 minutes by 2% **b** $318 000 by 4.3%

a MP3 players are bought from a wholesale importer for $75 and marked up 60%. What is the marked-up price?

b Wetsuits are bought from a wholesaler for $140 and marked up to $245. What is the percentage mark up?

During a sale, a shop advertises 15% discount on all goods.

a If the normal price of a CD is $8.00, what is the discount price?

b If the discount price of a digital camera is $178.50, what is the normal price?

a Calculate the GST payable on a car for which the price before GST is $28 500.

b A computer is advertised for sale at $1815. What was the price before GST?

A jeweller buys earrings for $15 per pair. She marks them up by 80% and then adds GST. What is the selling price?

Extended-response questions

● Jane buys chocolates in bulk at $80 for 15 kg. She pays 5 cents each for small bag to put the chocolates in. She sells each of the 200 g bags of chocolates for $2.50 plus GST.

a How many bags does Jane need?

b How much does she pay for the bags?

c What is the total amount Jane spends on the chocolates and the bags?

d How much GST must Jane add?

e How much does Jane receive if she sells all the bags?

f How much of this is GST?

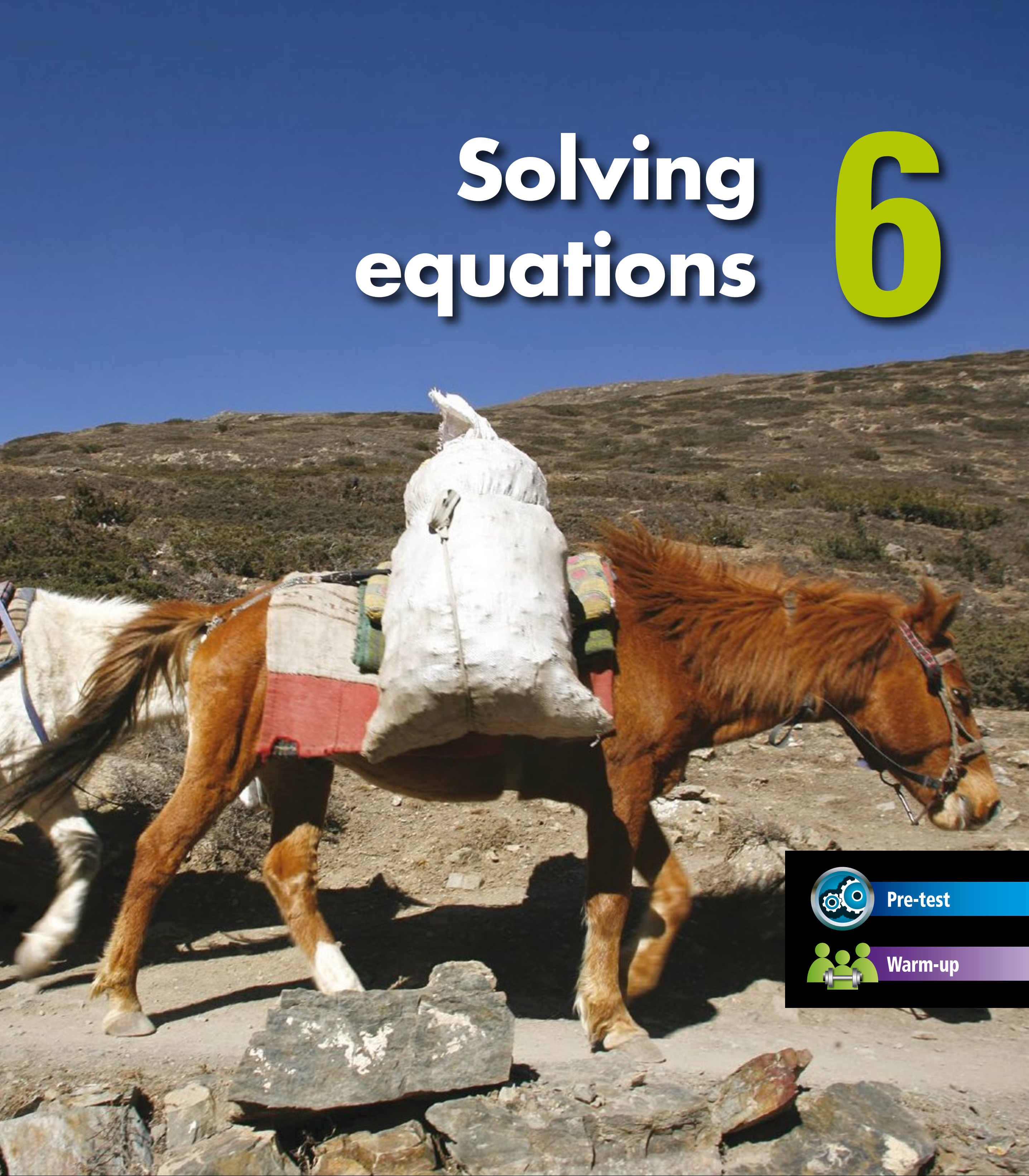
g How much profit does she make?

h Calculate her percentage profit as a percentage of her cost price. Give your answer correct to one decimal place.

Practice quiz
Chapter 5

Solving equations

6



Pre-test

Warm-up

The Donkey and Mule Problem: The mule says to the donkey, “If you give me one of your sacks, I would have as many as you.” The donkey says to the mule, “If you give me one of your sacks, I would have twice as many as you.” How many sacks do they have?

6.1 Solving equations with arithmetic

An **equation** shows that two expressions are equal to each other. The equation $3x + 4 = 10$ tells us that the expression $3x + 4$ has the value 10.

We can solve this equation to find the value of x. The value of x which makes this equation true is 2.

When $x = 2$, the left side is equal to $3 \times 2 + 4$, which is 10, so the left side (LS) of the equation $3x + 4 = 10$ equals the right side (RS). Therefore, the **solution** to the equation is $x = 2$.

Solving by inspection

Many simple equations can be solved just by inspection. For example, if $x + 9 = 12$ we can see that x must have the value 3.

Example 1

Solve the following by inspection then check by substitution.

a $x + 2 = 8$ **b** $x - 3 = -5$ **c** $-4x = 20$ **d** $\frac{x}{3} = 8$

Working	**Reasoning**
a $x + 2 = 8$ $x = 6$ Check: $6 + 2 = 8$	If 2 is added to 6 the result is 8.
b $x - 3 = -5$ $x = -2$ Check: $-2 - 3 = -5$	If 3 is subtracted from -2 the result is -5.
c $-4x = 20$ $x = -5$ Check: $-4 \times (-5) = 20$	If -4 is multiplied by -5 the result is 20.
d $\frac{x}{3} = 8$ $x = 24$ Check: $\frac{24}{3} = 8$	If 24 is divided by 3 the result is 8.

Solving by backtracking

We can represent an equation as a **flow chart** to show the operations that have been done on the unknown to build up the equation.

We can then reverse the operations to find the value of the unknown, that is, the solution to the equation. We call this process **backtracking**.

Example 2

Convert each of the following equations to a flow chart, and then use backtracking to find the value of x.

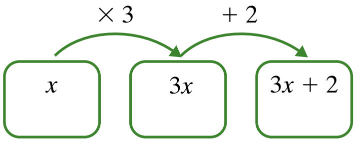
a $3x + 2 = 11$ **b** $\frac{3x + 1}{2} = 5$ **c** $\frac{2n - 7}{3} + 8 = 13$

Working

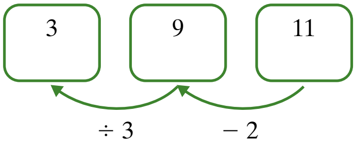
Reasoning

a $3x + 2 = 11$

This is a two-step process.



x has been multiplied by 3 and then 2 has been added to $3x$.



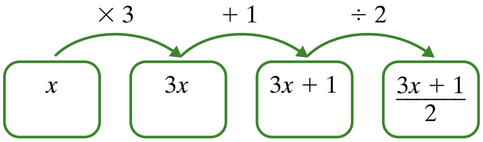
Backtrack to find the starting number. In backtracking, reverse the operations. Subtract 2 from 11 to get 9 and then divide 9 by 3 to get 3.

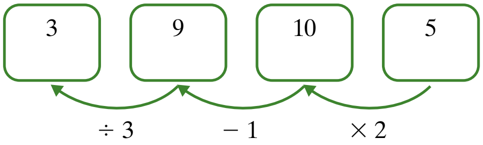
$x = 3$

Write the solution.

b $\frac{3x + 1}{2} = 5$

x has been multiplied by 3, 1 has been added to $3x$ and $3x + 1$ has been divided by 2.





Backtrack to find the starting number. In backtracking, reverse the operations. Multiply 5 by 2 to get 10, subtract 1 from 10 to get 9 and then divide 9 by 3 to get 3.

$x = 3$

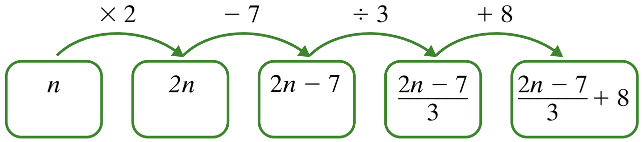
Write the solution.

continued

Example 2 continued

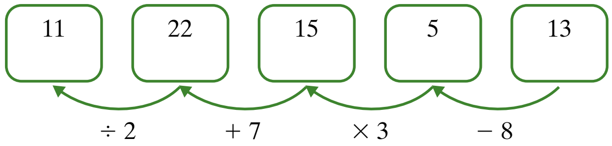
Working

c $\frac{2n-7}{3} + 8 = 13$



Reasoning

This is a four-step process. First, n is multiplied by 2, then 7 is subtracted, then the result is divided by 3 and finally, 8 is added.



Back track to find the starting number.
In the backtracking process, each step is reversed.

$n = 11$

Write the solution.

exercise 6.1

LINKS TO Examples 1a, b

Solve the following equations by inspection and then check by substitution.

a $a + 3 = 5$
b $b + 11 = 23$
c $14 + x = 21$
d $d - 8 = 3$
e $x - 12 = 8$
f $15 - x = 9$
g $x + 6 = 4$
h $h + 15 = 7$
i $35 - x = 32$
j $43 - x = 50$
k $k - 10 = -12$
l $x - 16 = -20$

The solution to $3 - x = -5$ is

A 2
B −2
C 8
D −8
E 15

LINKS TO Example 1c

Solve the following equations by inspection and then check by substitution.

a $3x = 12$
b $8y = 24$
c $7m = 56$
d $-2n = -12$
e $-5x = 25$
f $-6x = 42$
g $28x = 14$
h $24x = 4$
i $4h = 4.8$
j $3x = 6.6$
k $2x = -2.4$
l $4k = 0$

LINKS TO Example 1d

Solve the following equations by inspection and then check by substitution.

a $\frac{x}{3} = 12$
b $\frac{b}{5} = 100$
c $\frac{x}{9} = -9$
d $\frac{d}{11} = -8$
e $\frac{-x}{8} = 2$
f $\frac{-m}{7} = 4$
g $\frac{x}{-4} = -7$
h $\frac{h}{-5} = -60$
i $\frac{x}{3} = 0$
j $\frac{-y}{5} = 4$
k $\frac{-x}{7} = -8$
l $\frac{x}{7} = 0$

LINKS TO Example 2

● Convert each of the following equations to a flow chart, and then use backtracking to find the value of x.

a $3x + 2 = 15$ **b** $5 - x = 12$

c $\frac{2x}{3} = 6$ **d** $\frac{x}{5} + 7 = 18$

e $\frac{x + 1}{4} = 7$ **f** $9 - 2x = 11$

g $\frac{2x - 5}{5} = 3$ **h** $2(x + 1) - 5 = 15$

exercise 6.1 challenge

● For which of these equations would backtracking not be a suitable strategy? Explain.

a $x - 3 = 7$ **b** $2x + 5 = 5x$

c $3(4x - 1) = 2x + 7$ **d** $\frac{x}{5} + 7 = 18$

6.2 Solving equations with algebra: doing the same to both sides

The aim in solving an equation is to obtain the unknown by itself. This can be done by doing the same to both sides. To solve the equation $x - 6 = 7$, for example, we add 6 to both sides, giving the solution $x = 13$.

$$x - 6 = 7$$
$$x \underbrace{- 6 + 6}_{0} = 7 + 6$$
$$x = 13$$

One-step equations

One-step equations can be solved using one of these operations:

- adding the same to both sides
- subtracting the same from both sides
- multiplying both sides by the same number
- dividing both sides by the same number.

Example 3

Solve the following equations for x.

a $x + 5 = 6$ **b** $7 = x - 3$ **c** $2r = 32$ **d** $\frac{y}{3} = -4$

Working	**Reasoning**
a $x + 5 = 6$ $x + 5 - 5 = 6 - 5$ $x = 1$	Subtract 5 from both sides of the equation.
Check: LS $= x + 5$ $= 1 + 5$ $= 6$ = RS	Check the solution by substitution.

continued

Example 3 continued

Working	**Reasoning**
b $7 = x - 3$ $x - 3 = 7$ $x - 3 + 3 = 7 + 3$ $x = 10$	Swap the right hand side of the equation with the left hand side. Add 3 to both sides of the equation.
Check: $\text{RS} = x - 3$ $= 10 - 3$ $= 7$ $= \text{LS}$	Check the solution by substitution.
c $2r = 32$ $\frac{2r}{2} = \frac{32}{2}$ $r = 16$	Divide both sides of the equation by 2.
Check: $\text{LS} = 2r$ $= 2 \times 16$ $= 32$ $= \text{RS}$	Check the solution by substitution.
d $\frac{y}{3} = -4$ $\frac{y}{3} \times 3 = -4 \times 3$ $y = -12$	Multiply both sides of the equation by 3.
Check: $\text{LS} = \frac{y}{3}$ $= -\frac{12}{3}$ $= -4$ $= \text{RS}$	Check the solution by substitution.

Two-step equations

Two-step equations need two operations to solve them. Once again, whatever you do to the left side of the equation you must do to the right side.

Example 4

Solve the following equations for x.

a $2x + 3 = 7$ **b** $3x - 8 = 32$

Working	**Reasoning**
a $2x + 3 = 7$ $2x + 3 - 3 = 7 - 3$	In the equation, x has been multiplied by 2 then 3 has been added.
$2x = 4$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2$	Subtract 3 from both sides of the equation. Divide both sides of the equation by 2.
Check: $\text{LS} = 2x + 3$ $= 7$ $= \text{RS}$	Check the solution by substitution.
b $3x - 8 = 32$ $3x - 8 + 8 = 32 + 8$ $3x = 40$ $\frac{3x}{3} = \frac{40}{3}$	In the equation, x has been multiplied by 3 then 8 has been subtracted. Add 8 to both sides of the equation.
$x = \frac{40}{3}$	Divide both sides of the equation by 3.
$= 13\frac{1}{3}$	Convert to a mixed number.
Check: $\text{LS} = 3x - 8$ $= 40 - 8$ $= 32$ $= \text{RS}$	Check the solution by substitution.

An equation may have a common factor in every term. It may be easier to divide both sides by the common factor first, but we must be careful to divide every term by this common factor. Example 5 shows two methods for solving an equation with a common factor in every term.

Example 5

Solve $9x + 12 = 15$

a by first subtracting 12 from both sides.

b by first dividing both sides by the common factor 3.

Working	Reasoning
a $9x + 12 = 15$ $9x + 12 - 12 = 15 - 12$	Subtract 12 from both sides.
$9x = 3$ $\frac{9x}{9} = \frac{3}{9}$ $x = \frac{1}{3}$	Divide both sides by 9.
Check: $\text{LS} = 9x + 12$ $= \overset{3}{\cancel{9}} \times \frac{1}{\underset{1}{\cancel{3}}} + 12$ $= 3 + 12$ $= 15$ $= \text{RS}$	Check the solution by substitution.
b $\frac{9x}{3} + \frac{12}{3} = \frac{15}{3}$	Divide both sides by the common factor 3.
$3x + 4 = 5$ $3x + 4 - 4 = 5 - 4$	Subtract 4 from both sides.
$3x = 1$ $\frac{3x}{3} = \frac{1}{3}$ $x = \frac{1}{3}$	Divide both sides by 3.
Check: $\text{LS} = 9x + 12$ $= 3 + 12$ $= 15$ $= \text{RS}$	Check the solution by substitution.

In some equations, the unknown may have a coefficient that is a fraction. We can think of this as a two-step equation where we multiply by the denominator and divide by the numerator of the fraction. For example, for the equation $\frac{2x}{3} = 5$, we multiply both sides by 3 and divide both sides by 2.

Example 6

Solve these equations by doing the same to both sides.

a $\frac{3x}{7} = 2$

b $\frac{-5x}{8} = 3$

Working	**Reasoning**
a $\frac{3x}{7} = 2$ $\frac{3x}{7} \times 7 = 2 \times 7$ $3x = 14$ $\frac{3x}{3} = \frac{14}{3}$ $x = 4\frac{2}{3}$	In the equation x has been multiplied by 3 then divided by 7. Multiply both sides by 7 and divide both sides by 3.
Alternatively: $\frac{3x}{7} = 2$ $\frac{3x}{7} \times \frac{7}{3} = 2 \times \frac{7}{3}$ $x = \frac{14}{3}$ $x = 4\frac{2}{3}$	Alternatively, this can be done in a single step: In the equation x has been multiplied by $\frac{3}{7}$, so divide both sides by $\frac{3}{7}$. This is the same as multiplying both sides by $\frac{7}{3}$.
Check: $\text{LS} = \frac{3x}{7}$ $= 3x \div 7$ $= 3 \times 4\frac{2}{3} \div 7$ $= \frac{\overset{1}{\cancel{3}}}{1} \times \frac{14}{\underset{1}{\cancel{3}}} \div 7$ $= 14 \div 7$ $= 2$ $= \text{RS}$	Check the solution by substitution.

continued

Example 6 continued

Working

b

$$\frac{-5x}{8} = 3$$

$$\frac{-5x}{8} \times \frac{8}{-5} = 3 \times \frac{8}{-5}$$

$$x = \frac{24}{-5}$$

$$= -4\frac{4}{5}$$

Check:

$$\begin{aligned}
\text{LS} &= -\frac{5x}{8} \\
&= -5x \div 8 \\
&= -5 \times -4\frac{4}{5} \div 8 \\
&= -\frac{\cancel{5}^{1}}{1} \times -\frac{24}{\cancel{5}_{1}} \div 8 \\
&= 24 \div 8 \\
&= 3 \\
&= \text{RS}
\end{aligned}$$

Reasoning

In the equation x has been multiplied by $\frac{-5}{8}$, so divide both sides by $\frac{-5}{8}$. This is the same as multiplying both sides by $\frac{8}{-5}$.

Convert to a mixed number.

Check the solution by substitution.

When we multiply both sides of an equation by the denominator of a fraction to express the equation without fractions, we must be careful to multiply every term in the equation. Example 7 shows two methods for solving an equation where x is in the numerator of a fraction.

Example 7

Solve $\frac{x}{4} - 2 = 3$

a by first adding 2 to both sides.

b by first multiplying both sides by 4.

Working

a

$$\frac{x}{4} - 2 = 3$$

$$\frac{x}{4} - 2 + 2 = 3 + 2$$

$$\frac{x}{4} = 5$$

$$\frac{x}{4} \times 4 = 5 \times 4$$

$$x = 20$$

Reasoning

Add 2 to both sides.

Multiply both sides by 4.

continued

Example 7 continued

Working	Reasoning
Check:	Check the solution by substitution.
$\text{LS} = \frac{x}{4} - 2$	
$= \frac{20}{4} - 2$	
$= 5 - 2$	
$= 3$	
$= \text{RS}$	
b $\frac{x}{4} - 2 = 3$	Multiply both sides by 4.
$\frac{x}{4} \times \frac{4}{1} - 2 \times 4 = 3 \times 4$	On the left side $\frac{x}{4}$ and 2 both need to be
$x - 8 = 12$	multiplied by 4.
$x - 8 + 8 = 12 + 8$	Add 8 to both sides.
$x = 20$	

Example 8

Solve the equation $\frac{3x}{5} + 2 = 23$

Working	Reasoning
$\frac{3x}{5} + 2 = 23$	Subtract 2 from both sides.
$\frac{3x}{5} + 2 - 2 = 23 - 2$	
$\frac{3x}{5} = 21$	The coefficient of x is $\frac{3}{5}$.
$\frac{3x}{5} \times \frac{5}{3} = 21 \times \frac{5}{3}$	Dividing both sides by $\frac{3}{5}$ is the same as multiplying both sides by $\frac{5}{3}$.
$x = 35$	
Check:	Check the solution by substitution.
$\text{LS} = \frac{3x}{5} + 2$	
$= \frac{3 \times \cancel{35}^{7}}{\cancel{5}_{1}} + 2$	
$= 21 + 2$	
$= 23$	
$= \text{RS}$	

Note the difference between the equation in the next example and those in examples 7 and 8. In the equation in example 9, the entire left side has been divided by 5, so that the numerator of the fraction is $2x - 3$. We start by multiplying both sides by 5 to clear the fraction.

Example 9

Solve the equation $\frac{2x - 3}{5} = 3$

Working	**Reasoning**
$\frac{2x-3}{5} = 3$ $\frac{2x-3}{5} \times 5 = 3 \times 5$	Multiply both sides by 5 to clear the fraction.
$2x - 3 = 15$ $2x - 3 + 3 = 15 + 3$	Add 3 to both sides.
$2x = 18$ $\frac{2x}{2} = \frac{18}{2}$ $x = 9$	Divide both sides by 2.
Check: $\text{LS} = \frac{2x-3}{5}$ $= \frac{2 \times 9 - 3}{5}$ $= \frac{18-3}{5}$ $= \frac{15}{5}$ $= 3$ $= \text{RS}$	Check the solution by substitution.

exercise 6.2

LINKS TO Examples 3a, b

Solve the following equations by doing the same to both sides.

a $x + 7 = 11$
b $x - 7 = 11$
c $a + 8 = 15$
d $a - 8 = 15$
e $b + 9 = 24$
f $p - 13.5 = 21.2$
g $y + 14 = 9$
h $d - 8 = -4$
i $x + 23 = 37$
j $b - 12 = -5$
k $t + 15 = 8$
l $x - 8 = -7$

LINKS TO Example 3c

Solve the following equations by doing the same to both sides.

a $4d = 8$
b $7m = 35$
c $3b = 9$
d $5x = 45$
e $11k = 88$
f $4a = 10$
g $\frac{m}{9} = 8$
h $\frac{x}{6} = 13$
i $\frac{a}{-2} = 4$
j $\frac{b}{5} = -11$
k $\frac{x}{-9} = -12$
l $\frac{x}{8} = -14$

LINKS TO Example 4

Solve the following two-step equations for x.

a $2x - 5 = 7$ **b** $3b + 5 = 11$ **c** $4t - 7 = 17$ **d** $7x - 8 = 27$

e $3e + 7 = 22$ **f** $5x - 6 = 24$ **g** $4x + 13 = 45$ **h** $11h + 6 = 83$

i $3a + 14 = 59$ **j** $9x - 16 = 56$ **k** $3k - 14 = 22$ **l** $6x - 7 = -43$

m $7m - 17 = 39$ **n** $4n - 7 = 15$ **o** $5x + 17 = 24$ **p** $8 - 3x = -19$

LINKS TO Example 5

Solve the following two-step equations by first dividing both sides by a common factor.

a $5x + 25 = 100$ **b** $6x - 30 = 24$ **c** $9x + 27 = 72$ **d** $12x - 60 = 96$

e $14x + 49 = 77$ **f** $27x + 18 = 108$ **g** $132x - 88 = 22$ **h** $144x - 48 = 36$

For each of these pairs of equations, explain whether the two equations will give the same solution.

a $2x + 4 = 20$ and $x + 2 = 10$ **b** $3x + 6 = 15$ and $x + 6 = 5$

c $4x + 8 = 12$ and $x + 8 = 3$ **d** $6x + 9 = 18$ and $2x + 3 = 6$

LINKS TO Example 5

a The equation $2x + 4 = 10$ can be solved in more than one way. Solve the equation $2x + 4 = 10$ by

- **i** first subtracting 4 from both sides. **ii** first dividing both sides by 2.

b Which method do you prefer? Explain.

c If you were solving the equation $3x - 7 = 23$, would it be easier to first add 7 to both sides or to divide both sides by 3? Explain.

LINKS TO Example 5

To solve $3x + 15 = 30$

A subtract 15 from both sides of the equation and then multiply both sides by 3.

B divide both sides of the equation by 3 and then subtract 5 from both sides.

C divide both sides of the equation by 3 and then subtract 15 from both sides.

D add 15 to both sides of the equation and then divide both sides by 3.

E multiply both sides of the equation by 3 and then subtract 45 from both sides.

LINKS TO Example 6

Solve the following equations by doing the same to both sides.

a $\frac{3x}{5} = 21$ **b** $\frac{2a}{3} = 14$ **c** $\frac{4y}{5} = 12$ **d** $\frac{2x}{7} = 3$

e $\frac{5h}{8} = 2$ **f** $\frac{3a}{7} = 9$ **g** $\frac{2x}{5} = -4$ **h** $\frac{3b}{4} = -6$

i $\frac{5x}{3} = -5$ **j** $\frac{-2m}{5} = 8$ **k** $\frac{3x}{2} = 4$ **l** $\frac{-4a}{5} = 6$

LINKS TO Example 7

Solve the following equations.

a $\frac{a}{3} + 4 = 12$ **b** $\frac{r}{5} - 7 = 4$ **c** $\frac{b}{6} + 4 = 7$ **d** $\frac{k}{3} - 13 = 21$

e $\frac{x}{3} - 5 = 7$ **f** $\frac{b}{2} - 5 = 11$ **g** $\frac{a}{4} + 7 = 13$ **h** $\frac{x}{7} - 2 = 9$

i $\frac{p}{6} - 7 = -9$ **j** $\frac{y}{3} - 4 = -2$ **k** $\frac{x}{5} - 8 = -7$ **l** $8 + \frac{m}{3} = 16$

m $9 - \frac{b}{6} = 5$ **n** $\frac{x}{3} + 4.5 = 12.7$ **o** $\frac{x}{5} + 11.8 = 14.6$ **p** $8 + \frac{y}{5} = 17$

LINKS TO Example 7

The equation $\frac{x}{3} + 4 = 5$ can be solved in more than one way.

a Solve the equation $\frac{x}{3} + 4 = 5$

- **i** by first subtracting 4 from both sides
- **ii** by first multiplying both sides by 3.

b Which method do you prefer? Explain.

LINKS TO Example 7

For each of these pairs of equations, explain whether the two equations will give the same solution.

a $\frac{x}{4} - 8 = 3$ and $x - 32 = 12$

b $\frac{x}{5} + 3 = 11$ and $x + 15 = 11$

c $\frac{x}{7} - 3 = 9$ and $x - 21 = 63$

d $\frac{x}{4} - 11 = -2$ and $x - 44 = -2$

LINKS TO Example 8

Solve the following equations.

a $\frac{2a}{5} + 11 = 15$ **b** $\frac{3x}{4} - 7 = 8$ **c** $\frac{4m}{3} + 12 = 24$ **d** $\frac{3b}{7} + 5 = 17$

e $\frac{4x}{3} - 9 = 23$ **f** $\frac{7x}{2} - 11 = 24$ **g** $\frac{2x}{5} + 7 = 8$ **h** $\frac{3x}{5} - 8 = -11$

i $\frac{5m}{2} + 7 = 12$ **j** $\frac{3x}{7} + 5.3 = 11.9$ **k** $\frac{2x}{5} - 6 = 3$ **l** $8.3 - \frac{k}{4} = 3.5$

LINKS TO Example 9

Solve the following equations.

a $\frac{x + 5}{7} = 3$ **b** $\frac{x - 6}{4} = 13$ **c** $\frac{a - 4}{11} = 11$

d $\frac{m + 3}{5} = 7$ **e** $\frac{k + 13}{5} = -2$ **f** $\frac{2a + 3}{17} = 3$

g $\frac{3b - 8}{4} = 7$ **h** $\frac{7x - 11}{4} = 6$ **i** $\frac{3b - 7}{5} = -5$

j $\frac{2y - 5}{7} = 13$ **k** $\frac{4 - 5d}{3} = 8$ **l** $\frac{11 - 3h}{13} = -4$

Which one of the following does not have $x = 10$ as a solution?

A $2x + 5 = 25$ **B** $\frac{x}{2} - 5 = 0$ **C** $30 - 2x = 10$

D $\frac{6x}{10} - 12 = 6$ **E** $\frac{4x}{5} - 8 = 0$

For each of the following equations, find the value of x when $y = 13$.

a $y = 2x - 5$ **b** $y = 3x + 7$ **c** $y = 7x + 6$ **d** $y = 4x + 13$

exercise 6.2 challenge

Solve the following equations for x in terms of the given pronumerals.

a $a + b + x = c$ **b** $\frac{ax}{b} = c$ **c** $\frac{ax + b}{c} = d$

6.3 Solving equations with brackets

When equations include an expression in brackets, we normally expand the brackets as the first step.

Example 10

Solve the equation $3(x + 4) = 17$

Working	**Reasoning**
$3(x + 4) = 17$	Expand the brackets.
$3x + 12 = 17$	
$3x + 12 - 12 = 17 - 12$	Subtract 12 from both sides.
$3x = 5$	
$\frac{3x}{3} = \frac{5}{3}$	Divide both sides by 3.
$x = \frac{5}{3}$	
$x = 1\frac{2}{3}$	
Check: LS $= 3(x + 4)$ $= 3\left(\frac{5}{3} + 4\right)$ $= \overset{1}{\cancel{3}} \times \frac{5}{\underset{1}{\cancel{3}}} + 3 \times 4$ $= 5 + 12$ $= 17$ = RS	Check the solution by substitution.

Sometimes there may be a common factor on both sides of the equation so it may be easier to divide both sides by this common factor first. Example 11 shows two different methods for solving an equation that has a common factor on both sides.

Example 11

Solve the equation

a $3(x + 4) = 18$

b $3(2a - 3) = 0$

continued

Example 11 continued

Working	Reasoning
Method 1:	
$3(x+4)=18$	
$\frac{3(x+4)}{3}=\frac{18}{3}$	Divide both sides by the common factor, 3.
$x+4=6$	
$x+4-4=6-4$	Subtract 4 from both sides.
$x=2$	
Method 2:	
$3(x+4)=18$	
$3x+12=18$	Expand the brackets.
$3x+12-12=18-12$	Subtract 12 from both sides.
$3x=6$	
$\frac{3x}{3}=\frac{6}{3}$	Divide both sides by 3.
$x=2$	
Check:	Check the solution by substitution.
LS $=3(x+4)$ $=3(2+4)$ $=3\times 6$ $=18$ $=$ RS	
b $3(2a-3)=0$	
$\frac{\overset{1}{\cancel{3}}(2a-3)}{\underset{1}{\cancel{3}}}=\frac{0}{3}$	Divide both sides by 3. Zero divided by any number is zero.
$2a-3=0$	
$2a-3+3=0+3$	Add 3 to both sides.
$2a=3$	
$\frac{\overset{1}{\cancel{2}}a}{\underset{1}{\cancel{2}}}=\frac{3}{2}$	Divide both sides by 2.
$a=1\frac{1}{2}$	
Check:	Check the solution by substitution.
LS $=3(2a-3)$ $=3\left(\overset{1}{\cancel{2}}\times\frac{3}{\underset{1}{\cancel{2}}}-3\right)$ $=3\times 3-3\times 3$ $=0$ $=$ RS	

When brackets are expanded, the equation might be simplified by collecting like terms.

Example 12

Solve the following equations.

a $5(x + 3) - 8x = 11$

b $10(2y - 5) - 4(3y - 2) = 8$

Working

Reasoning

a

$$5(x + 3) - 8x = 11$$

$$5x + 15 - 8x = 11$$

Expand the brackets from the equation.
Collect like terms.

$$-3x + 15 = 11$$

$$-3x + 15 - 15 = 11 - 15$$

Subtract 15 from both sides.

$$-3x = -4$$

$$\frac{-3x}{-3} = \frac{-4}{-3}$$

Divide both sides of the equation by −3.

$$x = 1\frac{1}{3}$$

Check:

Check the solution by substitution.

$$\begin{aligned} \text{LS} &= 5\left(\frac{4}{3} + 3\right) - 8 \times \frac{4}{3} \\ &= 5 \times \frac{13}{3} - \frac{32}{3} \\ &= \frac{65}{3} - \frac{32}{3} \\ &= \frac{33}{3} \\ &= 11 \\ &= \text{RS} \end{aligned}$$

b

$$10(2y - 5) - 4(3y - 2) = 8$$

$$\frac{10(2y - 5)}{2} - \frac{4(3y - 2)}{2} = \frac{8}{2}$$

Divide both sides of the equation by 2
as it is a common factor.

$$5(2y - 5) - 2(3y - 2) = 4$$

$$10y - 25 - 6y + 4 = 4$$

Expand the brackets.

$$4y - 21 = 4$$

Collect like terms.

$$4y - 21 + 21 = 4 + 21$$

Add 21 to both sides.

$$4y = 25$$

$$\frac{4y}{4} = \frac{25}{4}$$

Divide both sides of the equation by 4.

$$-2(3y - 2) = -6y + 4$$

$$y = 6\frac{1}{4}$$

continued

Example 12 continued

Working

Check:

$$\begin{aligned} \text{LS} &= 10\left(2\times 6\frac{1}{4}-5\right)-4\left(3\times 6\frac{1}{4}-2\right)\\ &= 10\left(12\frac{1}{2}-5\right)-4\left(18\frac{3}{4}-2\right)\\ &= 10\times 7\frac{1}{2}-4\times 16\frac{3}{4}\\ &= 10\times\frac{15}{2}-4\times\frac{67}{4}\\ &= 75-67\\ &= 8\\ &= \text{RS} \end{aligned}$$

Reasoning

Check the solution by substitution.

exercise 6.3

LINKS TO Example 10

Solve the following equations by first expanding the brackets.

a $3(x + 2) = 8$ **b** $5(x - 3) = 7$ **c** $2(x - 5) = 9$ **d** $3(x - 1) = 4$

e $4(x - 5) = -6$ **f** $3(x - 4) = -5$ **g** $2(2x - 3) = 9$ **h** $3(x - 7) = 12$

i $3(2x - 1) = 7$ **j** $3(2x + 1) = 20$ **k** $2(3x - 4) = 10$ **l** $5(4x -5) = 15$

LINKS TO Example 11

Solve the following equations by dividing through by the common factor at the start.

a $4(x - 7) = 16$ **b** $2(x - 5) = 14$ **c** $7(2x - 11) = 21$ **d** $3(2x - 5) = 9$

e $2(x + 1) = 10$ **f** $3(4 + a) = 12$ **g** $-5(f + 4) = 25$ **h** $-7(4 + k) = -63$

i $-5(a - 9) = 35$ **j** $12(8 - h) = 60$ **k** $5(2x + 5) = 30$ **l** $6(7 - 3x) = 24$

The equation $3(2x + 3) = 27$ can be solved in more than one way.

a Solve the equation $3(2x + 3) = 27$ by

  i first expanding the brackets **ii** dividing both sides by 3.

b Which method do you prefer? Explain.

c Which method would you use if you were solving the equation $3(3x - 5) = 11$? Explain.

Solve the following equations, choosing the most suitable first step: either dividing both sides by a common factor or expanding the brackets.

a $4(x - 3) = 12$ **b** $4(x - 3) = 11$ **c** $-7(x + 1) = 28$ **d** $-7(x + 1) = 17$

e $4(x - 2) = 5$ **f** $4(x - 2) = 8$ **g** $3(2x - 1) = 33$ **h** $3(2x - 1) = 7$

i $2(x + 3) = 15$ **j** $2(x + 3) = 16$ **k** $5(x + 7) = -36$ **l** $5(x + 7) = -20$

Solve the following equations.

a $4(x + 3) = 0$ **b** $5(2x + 7) = 0$ **c** $7(6 - 5x) = 0$ **d** $-6(1 - 4x) = 0$

LINKS TO Example 12a

Solve the following equations.

a $2(a + 4) - 5 = 11$

b $3(b - 4) - 7 = 15$

c $3(x + 7) - x = 9$

d $5(x - 6) - 2x = 20$

e $3(m + 1) + 4m = 10$

f $-2(n - 4) + 7n = 11$

g $-3(x - 2) - 2x = 21$

h $-3(h + 4) + 7h = -16$

i $-5(2y - 6) - 3y = 4$

j $2(x - 3) + x - 7 = 11$

k $3(x + 1) + 2x - 5 = 18$

l $2(2x + 1) + 5x - 8 = 57$

LINKS TO Example 12b

Solve the following equations.

a $3(x + 2) - 5(x + 4) = 3$

b $4(2x - 1) - 3(x - 7) = 15$

c $3(3x + 1) - 2(2x - 3) = 17$

d $3(x + 1) + 2(x - 1) = 41$

e $4(x - 2) + 5(3x + 4) = -7$

f $3(x + 2) - 2(x + 1) = 9$

g $2(x + 3) - 5(x + 2) = 2$

h $4(x + 2) + 2(2x - 5) = 22$

i $3(x - 5) - 4(x - 3) = -9$

j $5(2x + 1) - 2(2x - 3) = 14$

k $4(2x + 3) - 5(x - 2) = 16$

l $3(2x - 5) - 5(2x + 1) = 4$

exercise 6.3 challenge

Solve the following equations for x in terms of the given pronumerals.

a $\dfrac{a(bx + c)}{d} = e$

b $a(x + b) = cx$

6.4 Solving equations with pronumerals on both sides

Equations with pronumerals on both sides

We can solve equations with the same pronumeral on both sides of the equation by collecting like terms. Usually the pronumerals are collected on the left side of the equation and the constants on the right side. If there are brackets in the equation, they are usually expanded first before the like terms are collected.

Example 13

Solve the following equations.

a $5x - 7 = 2x + 5$ **b** $10x + 1 = 5x$

Working	**Reasoning**
a $5x - 7 = 2x + 5$	Subtract $2x$ from both sides so that there are no x terms on the right side.
$5x - 2x - 7 = 2x - 2x + 5$	
$3x - 7 = 5$	Add 7 to both sides.
$3x = 12$	
$x = 4$	Divide both sides by 3.
Check: $\text{LS} = 5 \times 4 - 7$ $= 20 - 7$ $= 13$ $\text{RS} = 2 \times 4 + 5$ $= 13$ $\text{RS} = \text{LS}$	Check the solution by substitution.
b $10x + 1 = 5x$	Subtract $5x$ from both sides so that there are no x terms on the right side.
$10x - 5x + 1 = 5x - 5x$	
$5x + 1 = 0$	Subtract 1 from both sides.
$5x = -1$	
$x = -\frac{1}{5}$	Divide both sides by 5.
Check: $\text{LS} = 10 \times \left(-\frac{1}{5}\right) + 1$ $= -2 + 1$ $= -1$ $\text{RS} = 5 \times \left(-\frac{1}{5}\right)$ $= -1$ $\text{RS} = \text{LS}$	Check the solution by substitution.

Example 14

a $3(2x-1)=x$

b $5(2x-3)=4(x+3)$

Working	Reasoning
a $3(2x-1)=x$ $6x-3=x$	Expand the brackets on the left side of the equation.
$6x-x-3=x-x$ $5x-3=0$	Subtract x from both sides so that there are no x terms on the right side.
$5x=3$	Add 3 to both sides.
$x=\frac{3}{5}$	Divide both sides by 5.
Check: $\text{LS}=3\left(2\times\frac{3}{5}-1\right)$ $=3\left(\frac{6}{5}-\frac{5}{5}\right)$ $=3\times\frac{1}{5}$ $=\frac{3}{5}$ $\text{RS}=\frac{3}{5}$ $\text{RS}=\text{LS}$	Check the solution by substitution.
b $5(2x-3)=4(x+3)$ $10x-15=4x+12$	Expand the brackets on each side of the equation.
$10x-4x-15=4x-4x+12$ $6x-15=12$	Subtract $4x$ from both sides so that there are no x terms on the right hand side.
$6x=27$	Add 15 to both sides.
$x=\frac{27}{6}$ $x=4\frac{1}{2}$	Divide both sides by 6. Convert to a mixed number.
Check: $\text{LS}=5\times\left(2\times4\frac{1}{2}-3\right)$ $=5(9-3)$ $=5\times6$ $=30$ $\text{RS}=4\times\left(4\frac{1}{2}+3\right)$ $=4\times7\frac{1}{2}$ $=30$ $\text{RS}=\text{LS}$	Check the solution by substitution.

exercise 6.4

LINKS TO Example 13

1. Solve the following equations for x.

a $4x + 7 = 3x + 8$ **b** $7x - 11 = 2x + 9$

c $10x - 13 = 3x + 15$ **d** $3.4x + 7.1 = 2.5x + 8$

e $13x - 9 = 7x + 6$ **f** $-3x + 7 = 2x + 22$

g $13x - 5 = 11x - 8$ **h** $4x - 7 = 8x - 15$

i $6x - 11 = -3x + 7$ **j** $5 - 3x = 17 - x$

LINKS TO Example 14

2. Solve the following equations for x.

a $2(x + 3) = 5x$ **b** $5(x + 7) = -2x$

c $3(x - 4) = -x$ **d** $3(x - 2) = 2x + 1$

e $4(2x + 1) = 11x - 5$ **f** $3(4x + 5) = -7x - 4$

g $3(3x - 5) = 4(x + 5)$ **h** $2(5x - 8) = 3(3x - 4)$

i $-3(2x - 1) = 2(2x + 9)$ **j** $5(2x - 11) = 4(x - 3) + 5$

k $4(2x + 1) = 5 - 3(x - 7)$ **l** $-5(3x - 1) = 11 - 4(2x + 3)$

exercise 6.4 challenge

1. Solve these equations for x.

a $\dfrac{x + 6}{4} = \dfrac{x + 7}{3}$ **b** $\dfrac{3x + 4}{5} = \dfrac{5 - 2x}{7}$

6.5 Further practice with mixed equation types

In this section you will have an opportunity to use your skills from the previous sections in chapter 6 to decide on the most appropriate strategy for solving each equation. There are no worked examples in this section because the equations are all of types you have met in the previous sections.

In general, we solve equations by doing the same to both sides. However, depending on whether the equation has brackets or pronumerals on both sides, it is sometimes necessary to do one or more steps first.

Equations with brackets

In general, expand the brackets and collect any like terms. For example, $2(x - 3) = 5$, expand brackets to give $2x - 6 = 5$.

If there is a common factor on both sides, it may be easier to divide both sides by the common factor rather than expanding the brackets. For example, $2(x - 3) = 4$, divide both sides by 2 to give $x - 3 = 2$.

Equations with pronumerals on both sides

Collect all the terms containing the pronumeral onto the left side. For example, $2x - 7 = 5x$, subtract $5x$ from both sides.

Equations including a fraction

Multiply each term on both sides by the denominator to remove the fraction. For example, $\frac{x + 4}{3} = 7 + x$, multiply each term by 3 to give $x + 4 = 21 + 3x$.

exercise 6.5

1. Solve each of these equations using the most efficient method.

a $2(3x - 1) = 11$	**b** $4a - 3 = 3(a - 1)$
c $\frac{7x + 5}{4} = 10$	**d** $3(h - 4) = 6(2h + 1)$
e $0.5b + 13 = 2(b - 1)$	**f** $7 - m = 1 + 2m$
g $\frac{2n}{3} - 3 = 14$	**h** $0.4x + 2.3 = x - 3.4$
i $2(y - 4) + 7 = y + 11$	**j** $3(x - 4) - 5(2x - 1) = 21$
k $13 - 5x = 2(x - 4)$	**l** $1.8k + 4.5 = 18$

m $-4x + 3(x - 5) = 2(x - 3)$ **n** $\dfrac{4a - 3}{5} - 7 = 2$

o $3y - 2(3y - 5) = 5$ **p** $11.2a - 14.3 = 24.9$

q $2.4b - 1.6 = 5.4 - 1.1b$ **r** $4(2 - x) + 5 = 5(7 - 2x)$

- In designing taxi ranks, local councils often use the algebra rule $L = 5.5n + 1.5$, where n is the number of taxis and L metres is the length of the taxi rank. Silverton Council has a 73-m length of kerbside that it wants to turn into a taxi rank. How many taxis could be accommodated? Hint: Substitute 73 for L and solve the equation for n.
- Matt's weekly pay for selling washing machines and refrigerators depends on how many sales he makes. His pay, $\$P$, is given by the rule $P = 12(3n + 9)$, where n is the number of sales he makes in a week. How many sales would he need to make to earn $900 per week?
- For a plane flying at different altitudes in a particular region, the outside air temperature, t °C, at different altitudes was found to be related approximately to the altitude, a m, by the equation $t = 30 - 0.007a$. If the outside air temperature was −23 °C, at what altitude was the plane flying? Give your answer to the nearest 100 metres.

exercise 6.5 challenge

- Solve the following equations for x.

  a $\dfrac{x}{4} + \dfrac{x}{2} = 6$ **b** $\dfrac{x}{3} - \dfrac{x}{4} = 5$ **c** $\dfrac{x}{6} + \dfrac{x}{3} = -2$

  d $\dfrac{x}{5} + \dfrac{x}{10} = 2$ **e** $\dfrac{x + 5}{7} + \dfrac{x + 4}{3} = 13$ **f** $\dfrac{x - 1}{7} + \dfrac{x + 2}{2} = 21$

- Solve these equations for x.

  a $\dfrac{x - 4}{3} + \dfrac{x + 6}{2} = 10$ **b** $\dfrac{x - 3}{2} + \dfrac{x + 1}{3} = 4$

  c $\dfrac{x + 1}{2} + \dfrac{x + 2}{5} = 3$ **d** $\dfrac{2x - 3}{3} - \dfrac{x - 5}{2} = 5$

  e $\dfrac{3x - 4}{4} - \dfrac{2(x - 3)}{5} = 3$ **f** $\dfrac{7(x + 2)}{3} - \dfrac{3(x - 3)}{4} = 18$

- Solve these equations for x in terms of the given pronumerals.

  a $\dfrac{ax}{b} + \dfrac{cx}{d} = e$ **b** $\dfrac{x - a}{b} + \dfrac{x - c}{d} = e$ **c** $\dfrac{x + a}{b} - \dfrac{x - c}{d} = e$

6.6 Solving worded problems

Many problems can be solved by using algebraic techniques. The process of translating the words of a problem into algebra is called **formulation**.

As a starting point, it is important to read the question carefully and identify what you are being asked to find. Choose a pronumeral to represent the unknown, then construct an equation that can be solved.

The four-step approach to solving algebraic word problems

1. **Words into algebra.** Decide on the unknown variable and give it a letter, then formulate an equation that involves this variable.
2. **Solve the equation.** Solve for the variable by 'doing the same to both sides'.
3. **Check the solution.** Substitute your solution back into the original equation to check that the LS = RS.
4. **Algebra back to words.** Express your solution in terms of the original problem wording.

Number puzzle problems

Example 15

Fran thought of a number, doubled it and added 7. She ended up with the number 33.

- **a** Write an equation to represent the number Fran thought of.
- **b** Solve the equation to find the number.
- **c** Check your solution and make sure it fits the given information.
- **d** What was the number that Fran thought of?

Working	Reasoning
a Let n be the number that Fran thought of. $2n + 7 = 33$	*Step 1: Words into algebra* Define a pronumeral for the unknown number. The number is doubled, so write $2n$. Fran adds 7, so write $2n + 7$ Fran ended up with 33, so write $2n + 7 = 33$.

continued

Example 15 continued

Working	**Reasoning**
b $2n + 7 = 33$ $2n = 26$ $n = 13$ The number Fran thought of was 13.	*Step 2: Solve the equation* Subtract 7 from both sides. Divide both sides by 2. Write a sentence to answer the question.
c Check: $LS = 2 \times 13 + 7$ $= 33$ $= RS$	*Step 3: Check your solution* The solution fits the given information.
d The number Fran thought of was 13.	*Step 4: Algebra into words* Write a sentence to answer the question

Problems involving consecutive integers

Consecutive integers are integers that come one after the other, for example, 3 and 4 are consecutive integers, 7, 8 and 9 are consecutive integers. Notice that each consecutive integer is one more than the integer before it. If we let n stand for an integer, then the next integer will be $n + 1$, the next integer will be $n + 2$, and so on.

Consecutive even integers or **consecutive odd integers** are two apart, for example, 6, 8, 10, or 11, 13, 15. If we let n stand for an even integer, then the next even integer will be $n + 2$, the next integer will be $n + 4$, and so on, and similarly for odd integers.

Example 16

The sum of three consecutive integers is 72. Find the three integers.

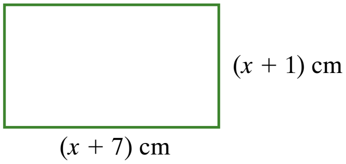
Working	**Reasoning**
Let n be the first integer, so the other integers are $n + 1$ and $n + 2$. $n + (n + 1) + (n + 2) = 72$	*Step 1: Words into algebra* Choose a letter for the unknown quantity and write an equation. Collect like terms.
$3n + 3 = 72$ $3n = 69$ $n = 23$	*Step 2: Solve the equation* Subtract 3 from both sides and divide both sides by 3.
$LS = 23 + (23 + 1) + (23 + 2)$ $= 23 + 24 + 25$ $= 72$ $= RS$	*Step 3: Check the solution* Substitute $n = 23$ in the left side of the equation.
The three numbers are 23, 24 and 25.	*Step 4: Algebra into words*

Finding unknown dimensions

Formulas for perimeter and area can be used to formulate equations to find unknown dimensions.

Example 17

A rectangle has length $(x + 7)$ cm and width $(x + 1)$ cm. If the perimeter is 28 cm, find the dimensions of the rectangle.

Working	**Reasoning**
	*Step 1: Words into algebra* Drawing a diagram helps visualise the problem.
$2(x + 7) + 2(x + 1) = 28$	The perimeter of a rectangle is twice the length plus twice the width.
$x + 7 + x + 1 = 14$ $2x + 8 = 14$	*Step 2: Solve the equation* Divide both sides by a common factor of 2 and collect like terms.
$2x = 6$	Subtract 8 from both sides.
$x = 3$	Divide both sides by 2.
$\text{LS} = 2\,(3 + 7) + 2\,(3 + 1)$ $= 28$ $= \text{RS}$	*Step 3: Check the solution* Substitute $x = 3$ into the original equation.
Length $= (x + 7)$ cm $= 3 + 7$ $= 10$ cm Width $= (x + 1)$ $= 3 + 1$ $= 4$ cm	*Step 4: Algebra into words* The question asked for the dimensions of the rectangle, not just the value of x. Substitute $x = 3$ into the given expressions for the length and width.
The dimensions of the rectangle are 10 cm by 4 cm.	

Calculating ages

In problems involving the ages of two or more people, the problem may refer, for example, to the ages of two people in 4 years time. It is important to remember that *both* people will be 4 years older in 4 years time.

Example 18

Tom's sister is 5 years older than Tom. In 3 years time Tom's sister will be twice as old as Tom.

a Define a pronumeral for Tom's age and use the given information to write an equation.

b Solve the equation.

c Check that your solution is correct and that your solution matches the given information.

d How old are Tom and his sister now?

Working	**Reasoning**
a Let t years be Tom's age now. The age of Tom's sister now is $t + 5$. In 3 year's time, Tom's age will be $t + 3$ and his sister's age will be $t + 5 + 3$. $t + 5 + 3 = 2(t + 3)$	*Step 1: Words into algebra* Use a pronumeral for Tom's age. Write an expression for Tom's sister's age. Write an equation to show that in 3 years time Tom's sister's age is twice Tom's age.
b $t + 8 = 2t + 6$ $t - t + 8 = 2t - t + 6$ $8 = t + 6$ $2 = t$ $t = 2$	*Step 2: Solve the equation.* Collect like terms (the constant terms 5 and 3). Expand the brackets. Subtract t from both sides. Subtract 6 from both sides.
c Check: $\text{LS} = 2 + 8$ $= 10$ $\text{RS} = 2(2 + 3)$ $= 10$ $= \text{LS}$	*Step 3: Check the solution* Substitute the solution into each side of the equation.
If Tom is 2 his sister must be 7. In three year's time, Tom will be 5 and his sister will be 10 so she will be twice as old as Tom.	Also check that the solution fits the given information.
d Tom is now 2 and his sister is 7.	*Step 4: Algebra into words* Write a sentence giving the ages of Tom and his sister.

Equations and angles

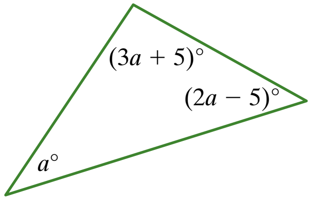
If the angles in a triangle are given as algebraic expressions we can use our knowledge that the angles of a triangle add to 180° to write an equation and solve it. We must then substitute the solution into each expression to find the size of each angle. In a similar way, if the angles in a quadrilateral are given as algebraic expressions we can use our knowledge that the angles of a quadrilateral add to 360°.

Example 19

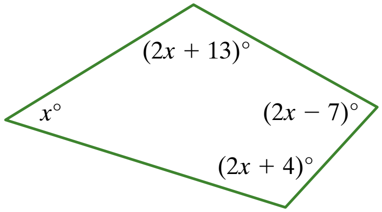
For each of the following,

- **i** write an equation.
- **ii** solve the equation.
- **iii** find the size of each angle.
- **iv** check your solution.
- **v** write a sentence giving the size of each angle.

a



b



Working	**Reasoning**
a i $a + 3a + 5 + 2a - 5 = 180$	*Step 1: Words into algebra* The three angles add to 180°. We don't include the degrees symbol in the equation.
ii $a + 3a + 5 + 2a - 5 = 180$ $6a = 180$ $a = 30$	*Step 2: Solve the equation* Collect like terms. Divide both sides by 6.
iii Check: $\text{LS} = 30 + 3 \times 30 + 5 + 2 \times 30 - 5$ $= 30 + 90 + 5 + 60 - 5$ $= 180$ $= \text{RS}$ $30 + 55 + 95 = 180$	*Step 3: Check the solution* Substitute $a = 30$ into the left side of the equation and check that it equals 180. Check that the three angles add to 180°.
iv $a = 30$ $3a + 5 = 3 \times 30 + 5$ $= 95$ $2a - 5 = 2 \times 30 - 5$ $= 55$	Calculate each of the angle sizes by substituting $a = 30$ into the expressions $3a + 5$ and $2a - 5$.

continued

Example 19 continued

Working	**Reasoning**
v The sizes of the angles are 30°, 55° and 95°.	*Step 4: Algebra into words* Write a sentence stating the size of each angle.
b i $x + 2x + 13 + 2x - 7 + 2x + 4 = 360$	*Step 1: Words into algebra* The four angles add to 360°. We don't include the degrees symbol in the equation.
ii $\begin{aligned} x + 2x + 13 + 2x - 7 + 2x + 4 &= 360 \\ 7x + 10 &= 360 \\ 7x &= 350 \\ x &= 50 \end{aligned}$	*Step 2: Solve the equation* Collect like terms. Subtract 10 from both sides. Divide both sides by 7.
Check: $\begin{aligned} \text{LS} &= 50 + 2 \times 50 + 13 + 2 \times 50 - 7 + 2 \times 50 + 4 \\ &= 50 + 100 + 13 + 100 - 7 + 100 + 4 \\ &= 360 \\ &= \text{RS} \end{aligned}$ $50 + 113 + 93 + 104 = 360$	*Step 3: Check the solution* Substitute $x = 30$ into the left side of the equation and check that it equals 360. Check that the three angles add to 360°.
iii The sizes of the angles are 50°, 113°, 95° and 104°.	*Step 4: Algebra into words* Write a sentence stating the size of each angle.
iv $x = 50$ $\begin{aligned} 2x + 13 &= 2 \times 50 + 13 \\ &= 113 \\ 2x - 7 &= 2 \times 50 - 7 \\ &= 93 \\ 2x + 4 &= 2 \times 50 + 4 \\ &= 104 \end{aligned}$	Calculate each of the angle sizes by substituting $x = 50$ into the expressions $2x + 13$, $2x - 7$ and $2x + 4$.

Comparing charges

The total charge made by many tradespeople is made up of a fixed charge plus a charge depending on the time taken to complete the work. Equations can be used to compare the amounts charged by different tradespeople.

Example 20

Sam, an electrician, charges \$79 per visit plus \$15 per 15 minutes of work. Bev, another electrician, charges \$100 per visit plus \$12 per 15 minutes of work. For what length of work would the total charge be the same for both electricians?

Working	**Reasoning**
Let x be the number of 15 minutes worked. Sam: total charge $= 79 + 15x$ Bev: total charge $= 100 + 12x$ If the total charges are equal, $79 + 15x = 100 + 12x$	*Step 1: Words into algebra* Sam charges \$15 for each 15 minutes. Bev charges \$12 for each 15 minutes. Put the two total charges equal to each other to find the value of x.
$79 + 3x = 100$ $3x = 21$ $x = 7$	*Step 2: Solve the equation* Subtract $12x$ from both sides to get the x terms onto one side. Subtract 79 from both sides. Divide both sides by 3.
$\text{LS} = 79 + 15 \times 7$ $= 184$ $\text{RS} = 100 + 12 \times 7$ $= 184$	*Step 3: Check the solution* Substitute $x = 7$ into the original equation. RS = LS
7 lots of 15 minutes = 105 minutes = 1 hour 45 minutes The total charge for each electrician would be the same if they worked for 1 hour 45 minutes.	*Step 4: Algebra into words* x represents the number of 15 minutes, so multiply 7 by 15 to find the time. The question asked for the length of time for which the charges would be the same for the two electricians.

exercise 6.6

LINKS TO Example 15

For each of the following

- **i** formulate an equation.
- **ii** solve the equation to find the original number.
- **iii** check your solution.
- **iv** write a sentence stating the number.

a Jody asked her sister to think of a number, then add 7 to it. Her sister said that her answer was 10.

b Michael asked his brother to think of a number, then subtract 8 from it. His brother said that his answer was −12.

c Lin asked her brother to think of a number, then multiply it by −8. Her brother said that his answer was 40.

d Tuan asked his brother to think of a number, then divide it by 9. His brother said that his answer was −3.

● For each of the following

- **i** formulate an equation.
- **ii** solve the equation to find the number.

a If a number is divided by 4 then 7 is added, the answer is 12.

b If a number is multiplied by −5 then 13 is added, the answer is −32.

c If I add 11 to a number then double the result, I get 46.

d If I add 17 to a number then divide the result by 5, I get 35.

● If a number is multiplied by 3 then 4 is added, the answer is the same as if the number is doubled then 8 is added.

a Let the number be n and write an equation.

b Solve the equation to find the number.

c Check that your solution fits the given information.

d Write a sentence stating the number.

● The average of three numbers x, $x + 3$ and $x + 12$ is 114.

a Write an equation.

b Find the value of x.

c Substitute to find the other two numbers.

d Check that your solution fits the given information.

e Write a sentence stating the three numbers.

▶LINKS TO
Example 16

● For each of the following

- **i** write an equation (let the smallest of the three numbers be n).
- **ii** solve the equation to find n.
- **iii** state the three numbers.
- **iv** check your solution.
- **v** write a sentence stating the three numbers.

a The sum of three consecutive integers is 177. What are the three numbers?

b The sum of three consecutive odd integers is 177. What are the three numbers?

▶LINKS TO
Example 17

● A rectangle has length 30 cm and width $(a + 3)$ cm.

a Write a rule for the perimeter, P cm, in terms of a.

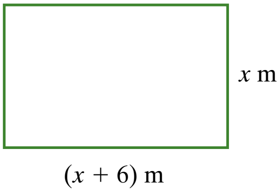
b If the perimeter is 90 cm, solve the equation to find the value of a.

A rectangular garden is 6 m longer than it is wide.

a If the perimeter is 28 m and the width is x m, write an equation for the perimeter in terms of x.

b Solve the equation to find x.

c State the length and width of the garden.



A paddock has length 80 m and width $(x + 5)$ m.

a Draw a labelled diagram of the paddock.

b Write a rule for the area, A m^2, in terms of x.

c If the area of the paddock is 2000 m^2, solve the equation to find the value of x.

A rectangle is three times as long as it is wide.

a If the width is x m, what is the length?

b Draw a diagram showing the length and width in terms of x.

c If the perimeter is 24 m, write an equation and solve it to find x.

d What are the dimensions of the rectangle?

LINKS TO Example 18

Shoshanna's sisters are twins who are 2 years older than her.

a Write an expression for the sum of the three sisters' ages, if Shoshanna's age is x years.

b The sum of their ages is 46. Write an equation and solve it to find all the girls' ages.

LINKS TO Example 18

Maria is now four times as old as Kate. Four years ago, Maria was six times as old as Kate. How old are Maria and Kate now?

LINKS TO Example 18

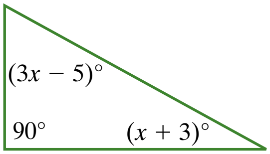
In 4 years time I will be one-third of the age my mother will be. If I am now 13 years old, how old is my mother now?

LINKS TO Example 19

For each of the following,

- **i** write an equation.
- **ii** solve the equation to find x.
- **iii** check your solution.
- **iv** write a sentence stating all angles.
- **v** check that the angles have the correct sum.

a



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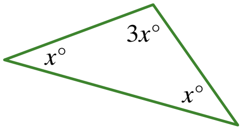

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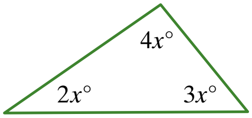

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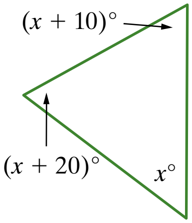

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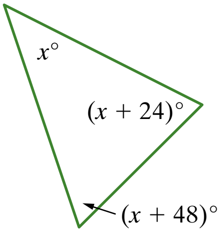

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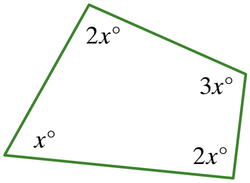

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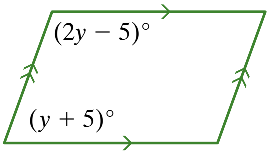

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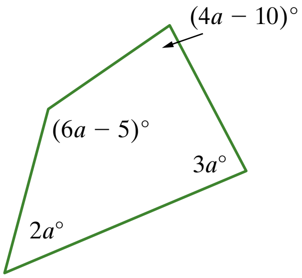
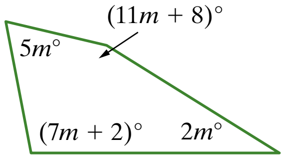


i



LINKS TO
Example 20

Harry wants to hire a bike for part of a day. Cheap Bike Hire offers two bike hire plans. With Plan A you pay a fee of $5 plus $4.00 per hour. With Plan B you pay $14 plus $1 per hour. For what numbers of hours is Plan A cheaper? If Harry hires a bike for x hours

- **a** Write an expression for the cost if he uses Plan A
- **b** Write an expression for the cost if he uses Plan B
- **c** Write an equation to show the two costs being the same.
- **d** Solve the equation to find the number of hours for which the two plans will cost the same.

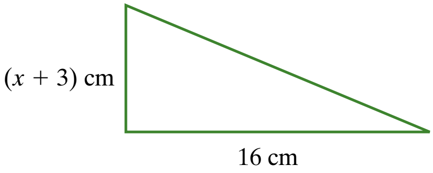
LINKS TO
Example 20

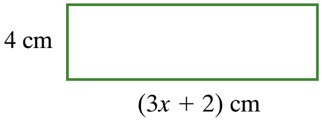
Pete's Plumbing charges a call-out fee of $40 and an hourly rate of $40. Brad's Bathrooms charges a call-out fee of $60 and an hourly rate of $30. If a plumbing job takes x hours

- **a** Write an expression for the amount charged by Pete's Plumbing
- **b** Write an expression for the amount charged by Brad's Bathrooms.
- **c** Write an equation to show the two amounts being the same.
- **d** Solve the equation to find the number of hours for which the two amounts are the same.

exercise 6.6 challenge

The following rectangle and triangle have the same area.





- **a** Write an equation.
- **b** Solve the equation to find the value of x.
- **c** Find the lengths of the unknown sides of the triangle and the rectangle.

Analysis task

Transposing a formula

The area of a rectangle is given by the formula $A = lw$. If we know the area and the length of a rectangle, we can use this formula to calculate the width.

a The area of a rectangular paddock is 288 m^2 and the width is length is 18 m. Substituting $A = 288$ and $l = 18$, we obtain $288 = 18w$. Solve this equation to find w.

Instead of substituting into the formula $A = lw$, we could rearrange the formula first.

$$A = lw$$
$$\frac{A}{l} = \frac{lw}{l}$$
$$\frac{A}{l} = w$$

We can rewrite this in the form $w = \frac{A}{l}$.

This is called **transposing** the formula to make w the **subject**. Notice that rearranging the formula is exactly like solving an equation. We do the same to both sides until we obtain the required variable on its own.

b Sam is a landscape gardener, and he is designing a garden that will include several circular flower beds. He plans to surround each flower bed with plastic garden edging that is sold in various lengths. Sam wants to know how large a flower bed he can surround for different lengths of edging.

- **i** Transpose the formula $C = \pi d$ for the circumference, C, of a circle to make d the subject (what will you divide both sides by?)
- **ii** Use the transposed formula to calculate the diameter of the flower bed that could be enclosed with 6 m of garden edging. Give the diameter in metres correct to one decimal place.

c The perimeter of a rectangular paddock is 176 m and the width is 32 m. There are two ways we could find the length of the paddock

- **i** Substitute $P = 176$ and $w = 32$ in the formula $P = 2(l + w)$ and solve the equation for l.
- **ii** Transpose the formula first to make l the subject then substitute $P = 176$ and $w = 32$ to find l.

d The formula for converting the number of Fahrenheit degrees, f, into the corresponding number of Celsius degrees, c, is $c = \frac{5}{9}(f - 32)$.

- **i** Use this formula to calculate the temperature in degrees Fahrenheit when the temperature is 25°C by substituting $c = 25$ into the formula then solving the equation for f.
- **ii** Transpose the formula $c = \frac{5}{9}(f - 32)$ to make c the subject first and then substitute $c = 25$ to find f. Is your value for f the same as you obtained in part **i**?
- **iii** If you had several temperatures to convert which method would be easier?

Review Solving equations

Summary

Equations

- Two expressions that are equal can be joined by an = sign to make an equation, for example, $3x + 4 = 13$.

Solving

- Solving an equation means finding the value of the pronumeral that makes the equation true.

Solution

- The value of the pronumeral that makes the equation true is called the solution. The solution to the equation $3x + 4 = 13$ is $x = 3$.

Arithmetic solving

- Inspection
- Guess, check and improve
- Backtracking

Algebraic solving

- Equations are solved algebraically by doing the same to both sides. Depending on whether the equation has brackets, pronumerals on both sides, or fractions, it is sometimes necessary to do one or more steps first.

Equations with brackets

- In general, expand the brackets and collect any like terms.
- If there is a common factor on both sides, it may be easier to divide both sides by the common factor rather than expanding the brackets.
  For example:
  $2(x - 3) = 5$, expand brackets to give $2x - 6 = 5$.
  $2(x - 3) = 4$, divide both sides by 2 to give $x - 3 = 2$.

Equations with pronumerals on both sides

- Collect all the terms containing the pronumeral onto the left side.
  For example:
  $2x - 7 = 5x$, subtract $5x$ from both sides to give $-3x - 7 = 0$.

Equations with a fraction

- Multiply both sides by the denominator of the fraction.
  For example:
  $\frac{x + 5}{3} = 8$, multiply both sides by 3, to give $x + 5 = 24$

Word problems solved with equations

- **Step 1: Words into algebra**. Decide on the unknown variable and give it a letter, then formulate an equation that involves this variable.
- **Step 2: Solve the equation**. Solve for the variable by ‘doing the same to both sides’.
- **Step 3: Check the solution**. Substitute your solution back into the original equation to check that the LS = RS. Also check that your solution to your equation fits the given information.
- **Step 4: Algebra back to words**. Express your solution in a sentence in terms of the original problem wording. Check that you have answered the question asked. In some word problems you may need to substitute your solution to the equation into expressions to answer the question asked.

Visual map

Using the following terms (and others if you wish), construct a mind map that illustrates your understanding of the key issues covered in this chapter.

algebraic solving
arithmetic strategies
backtracking
check
equation
evaluate
expand
expression
factors
factorise
flow chart
formulation
guess, check and improve
like terms
pronumeral
solution
solving
solving by inspection
substitute

Revision

Multiple-choice questions

● The solution to $5 - x = -8$ is

A 3 **B** -3 **C** 13 **D** -13 **E** 14

● If $\frac{x}{2} - 3 = 4$ then

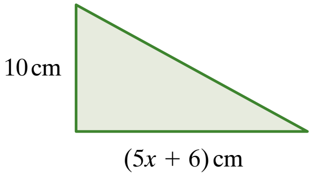
A $x - 6 = 4$ **B** $x - 3 = 8$ **C** $\frac{x}{2} = 7$ **D** $x - 6 = 2$ **E** $\frac{x}{2} = 1$

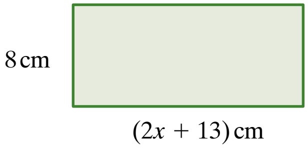
The solution to the equation $\frac{2x + 5}{3} = 7$ is

A $x = 3$ **B** $x = 4\frac{1}{2}$ **C** $x = -\frac{1}{2}$ **D** $x = 8$ **E** $x = 18$

The solution to the equation $2(3x + 5) - 3(x - 4) = 16$ is

A $x = -\frac{1}{2}$ **B** $x = -2$ **C** $x = 2$ **D** $x = 5$ **E** $x = 6$





If the above shapes have the same area, then

A $10(5x + 6) = 8(2x + 13)$

B $10(5x + 6) = 4(2x + 13)$

C $5(5x + 6) = 4(2x + 13)$

D $5(5x + 6) = 8(2x + 13)$

E $5x + 16 = 2x + 21$

Short-answer questions

Solve the following equations.

a $x + 4 = 8$

b $8 - a = 16$

c $3d + 9 = 11$

d $15 = 12n - 6$

e $-x = 5$

f $-3x = 3$

g $-3a = 0$

h $\frac{2h}{3} = 14$

i $\frac{-3x}{5} = 12$

j $\frac{x}{3} + 7 = 13$

k $5 - \frac{3k}{4} = 8$

l $3(x - 4) = 21$

m $5(m + 2) = 13$

n $-2(2n + 5) = 0$

o $\frac{x + 6}{5} = 11$

p $\frac{2x - 3}{7} = -3$

q $4 - 3k = 11$

r $2(x - 1) - 5 = 12$

s $5(2x - 1) - 2(x - 1) = 7$

t $18 = 7 - 5y$

u $-3(3x + 1) - 7x = 7$

v $4a - 3(2a + 5) = -22$

Solve the following equations for x.

a $3x - 17 = 22$

b $\frac{x}{5} + 9 = 23$

c $\frac{3x - 7}{4} = 8$

d $\frac{2(4x + 5)}{7} = 5$

e $11x - 8 = 4(3x + 4)$

f $\frac{3.2x - 7.7}{3} = 8.1$

g $5x + 14 = -9$ **h** $\frac{4x}{7} + 13 = 25$

i $5(2x + 5) = 3(4x - 1) + 13$ **j** $\frac{3x}{2} - 8 = x + 13$

k $\frac{3(5x + 1)}{7} = -6$ **l** $\frac{6.5x}{4} + 2.7 = 7.9$

Extended-response questions

For questions 8–10 follow these steps.

- Define a pronumeral to represent the unknown.
- Formulate an equation.
- Solve the equation.
- Write a sentence giving the answer to the question.

Write equations for the following and solve.

a Tim asked his brother to think of a number, then multiply it by −11. His brother said that the answer was −99. What was his brother's original number?

b Betty asked her sister to think of a number, then divide it by 12. Her sister said that the answer was −14. What was her sister's original number?

c I am 6 years younger than my sister and the sum of our ages multiplied by 5 is 150. How old is my sister?

d I had $\$x$ in my bank account and after I withdrew two-fifths of it, there was \$90.75 left. How much did I have in my account originally?

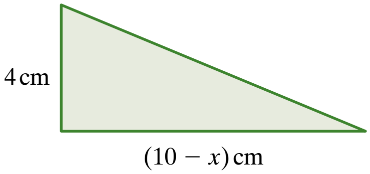
A rectangle has length $20a$ cm and width $(a + 2)$ cm.

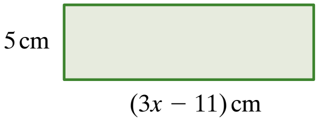
a Write a rule for the perimeter, P cm, of the rectangle in terms of a.

b Find the area, $A\text{ cm}^2$, of the rectangle in terms of a.

c If the perimeter is 88 cm, what is the area of the rectangle?

The following rectangle and triangle have the same area.





Practice quiz
Chapter 6

a Write an equation.

b Solve the equation to find the value of x.

c Check your solution.

d Find the length of the unknown sides of the triangle and the rectangle.

7

Congruency and quadrilateral properties

Pre-test
Warm-up

Lines, angles, triangles and quadrilaterals play an important part in our everyday lives in art, architecture, tool design and sport. Triangles are used in the structure of bridges and roof trusses because they are rigid. Quadrilaterals have many uses because they are *not* rigid shapes. Parallelograms and rhombuses formed by hinged bars, such as in this scissor lift, are found in many tools and everyday objects.

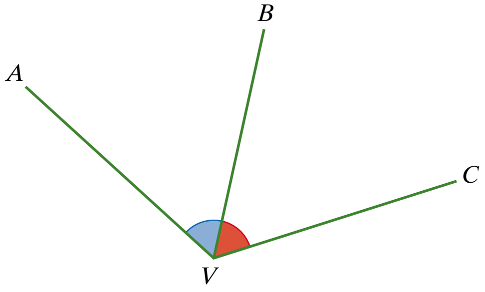
7.1 Reviewing angles and parallel lines

Adjacent angles

Umbrella

Expanding toolbox

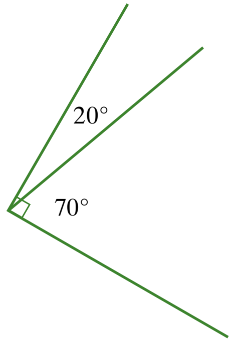
The point where two straight lines or segments intersect is called a **vertex**. **Adjacent angles** are angles that are next to each other. They share a common vertex and a common arm. In the diagram, $\angle AVB$ and $\angle BVC$ are adjacent angles. They share the common vertex V and the common arm VB.



Complementary angles

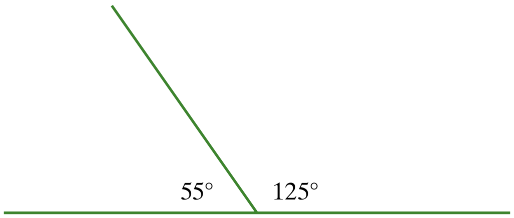
Complementary and supplementary angles

Angles that add to 90° are called **complementary angles**. The two angles 70° and 20° are complementary angles. We say that 70° is the **complement** of 20°. Two adjacent angles that are complementary form a **right angle**.



Supplementary angles

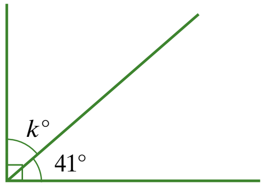
Supplementary angles are angles that add to 180°. The two angles 55° and 125° are supplementary angles. We say that 55° is the **supplement** of 125°. Two adjacent angles that are supplementary form a straight line.



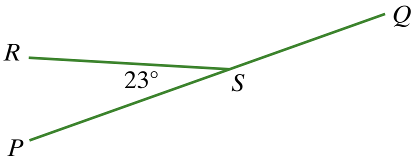
Example 1

Find the following.

a Find the value of *k*.



b *PSQ* is a straight line. Find the size of ∠*RSQ*.



Working	**Reasoning**
a $k = 90 - 41$ (complementary angles) $= 49$	$k + 41 = 90$ (complementary angles)
b $\angle RSQ = 180 - 23$ (supplementary angles) $= 157$	∠*RSQ* and ∠*RSP* are supplementary angles because *PSQ* is a straight line.

Example 2

Find the following.

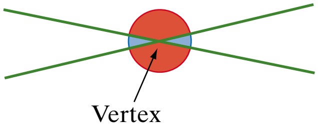
a What is the complement of 43°?

b What is the supplement of 122°?

Working	**Reasoning**
a Complement of 43° = 47°	90° – 43° = 47°
b Supplement of 122° = 58°	180° – 122° = 58°

Vertically opposite angles

Vertically opposite angles



When two straight lines intersect the angles that are on opposite sides of the vertex are called **vertically opposite angles**.

The two angles marked in red are vertically opposite angles.

The two angles marked in blue are vertically opposite angles.

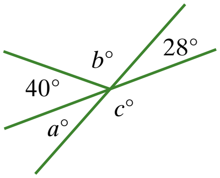
Vertically opposite angles are equal.

Angles at a point

Angles meeting at a point form a complete revolution, that is, they add to 360°.

Example 3

Find the value of each pronumeral then show that the five angles add to 360°.

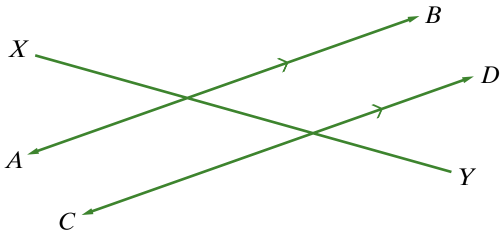


Working	**Reasoning**
$a = 28$ (vertically opposite angles)	
$a + b + 40 = 180$ (supplementary angles) $28 + b + 40 = 180$ $b = 180 - 68$ $b = 112$	A straight angle is 180°
$28 + c = 180$ (supplementary angles) $c = 152$	A straight angle is 180° To find c we could also have added the other four angles at the point and subtracted from 360°. $c = 360 - (28 + 40 + 112 + 28)$ $= 360 - 208$ $c = 152$
Check: $28 + 40 + 112 + 28 + 152 = 360$	The five angles meet at a point so they add to 360°

Parallel lines and transversals

Parallel lines

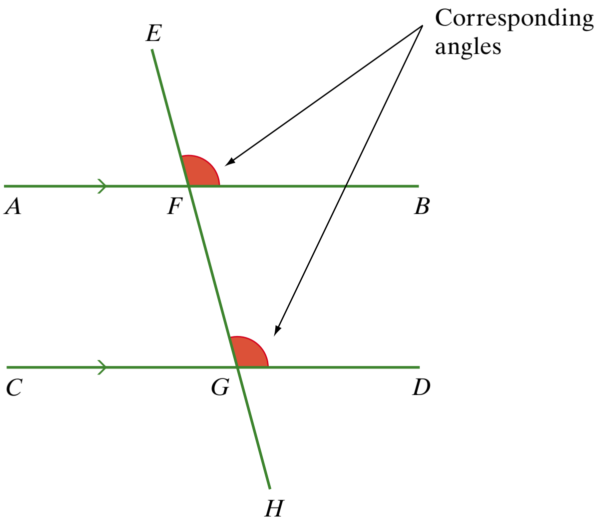
A line or line segment that cuts across a set of parallel lines is called a **transversal**. In this figure, the line segment XY is a transversal, cutting across the parallel lines AB and CD.



Corresponding angles

When parallel lines are crossed by a transversal, **corresponding angles** are angles that are in corresponding positions on the same side of the transversal.

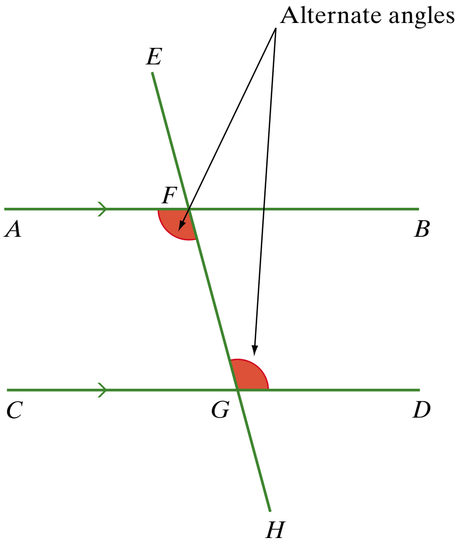
Corresponding angles are equal.



Alternate angles

Alternate angles are angles that are between a pair of parallel lines, but on opposite sides of the transversal.

Alternate angles are equal.

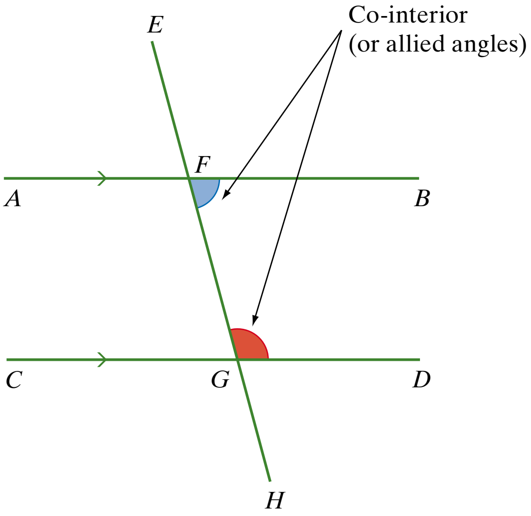


Co-interior angles (or allied angles)

Co-interior angles are sometimes called allied angles. They are angles that are between a pair of parallel lines, but on the same side of the transversal.

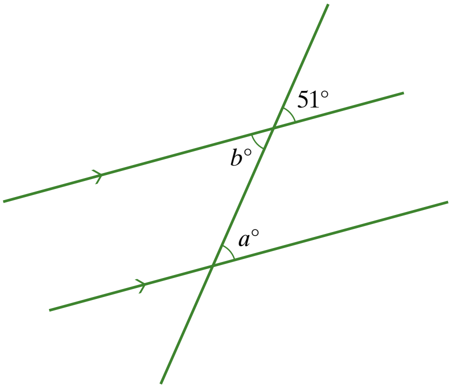
Co-interior angles are supplementary.

Supplementary angles add to 180°.



Example 4

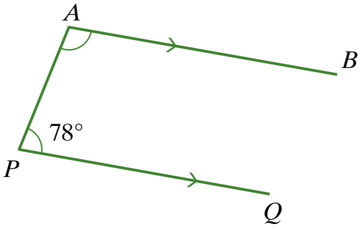
Find the values of the pronumerals.



Working	**Reasoning**
$a = 51$ (corresponding angles)	The angles marked $a°$ and 51° are corresponding angles. As the lines are parallel, the corresponding angles are equal.
$b = 51$ (alternate angles)	The angles marked $a°$ and $b°$ are alternate angles. As the lines are parallel, the alternate angles are equal.
Or, $b = 51$ (vertically opposite angles)	The angles marked $b°$ and 51° are vertically opposite angles, so they are equal.

Example 5

Find the size of $\angle PAB$.



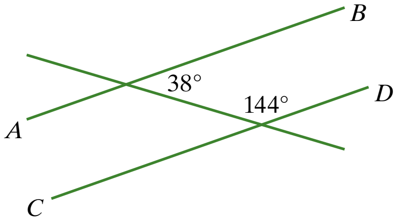
Working	**Reasoning**
$\angle PAB = 180° - 78°$ $= 102°$ (co-interior angles)	$\angle PAB$ and $\angle APQ$ are co-interior angles. As AB and PQ are parallel, the marked angles add to 180°.

Lines and transversals

We know that when parallel lines are cut by a transversal, corresponding angles are equal but the reverse is also true. If the corresponding angles are equal then the lines are parallel. If they are not equal then the lines are not parallel. The same applies with alternate and co-interior angles.

Example 6

Are line segments *AB* and *CD* parallel?



Working

38° and 144° are allied angles.
The allied angles do not add to 180°.

Line segments *AB* and *CD* are not parallel.

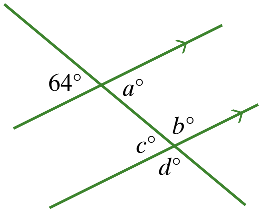
Reasoning

38° and 144° are allied angles.

Allied angles formed by a transversal cutting across two line segments add to 180° if the line segments are parallel.

Example 7

Find the value of *a*, *b*, *c* and *d*.

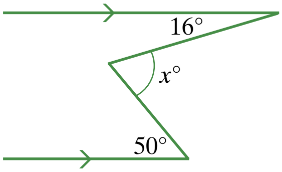


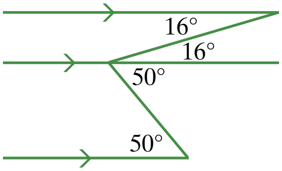
Working	Reasoning
$a = 64$ (vertically opposite angles)	The angles marked $a°$ and 64° are vertically opposite angles so they are equal.
$b = 116$ (co-interior angles)	The angles marked $a°$ and $b°$ are co-interior angles so they are supplementary. $180° - 64° = 116°$
$c = 64$ (corresponding angles)	The angles marked 64° and $c°$ are corresponding angles so they are equal.
$d = 116$ (vertically opposite angles)	The angles marked $b°$ and $d°$ are vertically opposite angles so they are equal.

Sometimes we need to draw extra lines to help us find an unknown angle. In the diagram in the next example, drawing a line through the vertex of the unknown angle parallel to the pair of parallel lines allows us to make use of alternate angles. The required unknown angle is then the sum of the two alternate angles.

Example 8

Find the value of the pronumeral.

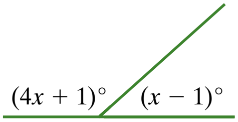


Working		**Reasoning**
$x = 16 + 50$ $x = 66$		Draw a line through the vertex of the unknown angle parallel to the pair of parallel lines. Label the alternate angles. Add the sizes of the two alternate angles to find the unknown angle.

Example 9

For the angles below,

- **a** use your knowledge of angles to write an equation.
- **b** solve the equation to find the value of x.
- **c** find the size of each of the labelled angles.



Working	**Reasoning**
a $4x + 1 + x - 1 = 180$ (supplementary angles)	The angles form a straight angle so they add to 180°.
b $5x = 180$ $x = 36$	Collect like terms. Divide both sides by 5
c $x - 1 = 36 - 1$ $= 35$ $4x + 1 = 4 \times 36 + 1$ $= 145$ The angle sizes are 35° and 145°.	Substitute $x = 36$ into the expressions for the angles.

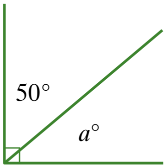
exercise 7.1

In this exercise each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

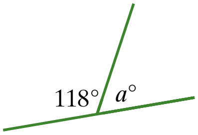
LINKS TO Examples 1, 3

Find the unknown angles in each of the following diagrams.

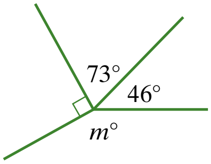
a



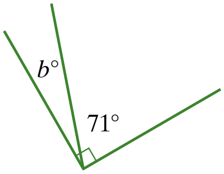
b



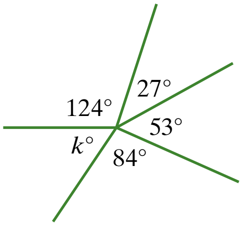
c



d



e



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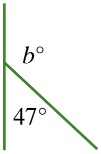
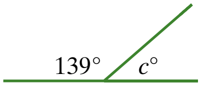


g



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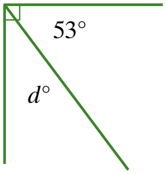

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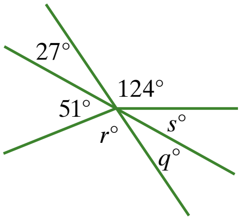


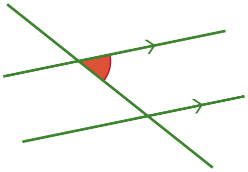
LINKS TO Example 2

Copy and complete the following table where possible. There are two angles for which you will not be able to write the complement.

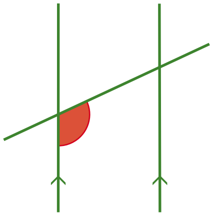
Angle	Complement	Supplement
13°		
88°		
129°		
54°		
142°		

Each of the following diagrams has one marked angle. Copy each diagram and mark with a cross the angle indicated, for example in part **a**, use a cross to mark the alternate angle to the marked angle.

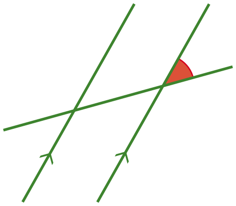
a alternate angle



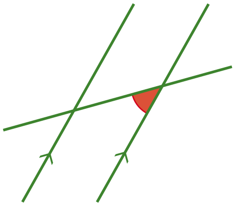
b co-interior angle



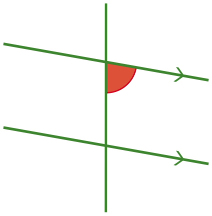
c vertically opposite angle



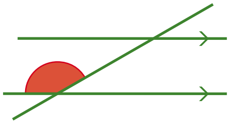
d alternate angle



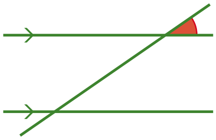
e corresponding angle



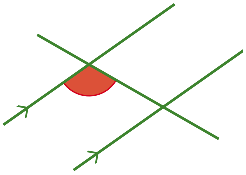
f alternate angle



g corresponding angle



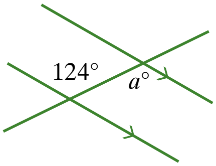
h co-interior angle



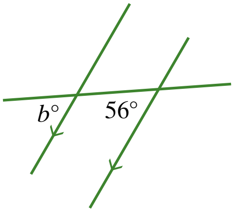
LINKS TO Examples 4, 5

Find the unknown angles in each of the following diagrams.

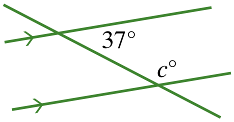
a



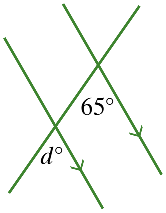
b



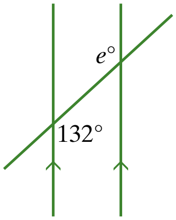
c



d



e



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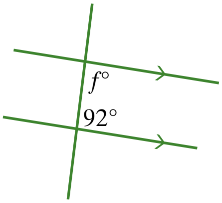
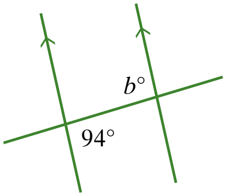


g



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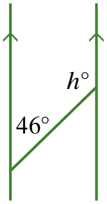

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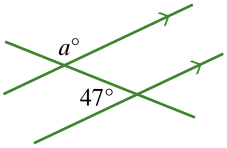
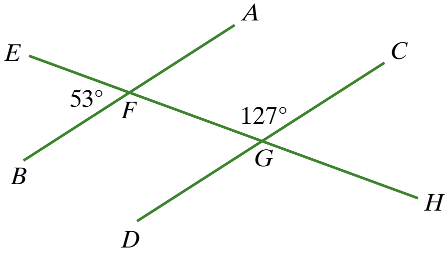


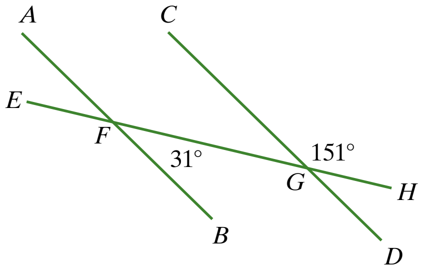
LINKS TO
Example 6

State if the line segments AB and CD are parallel in each of the following diagrams. Justify each answer.

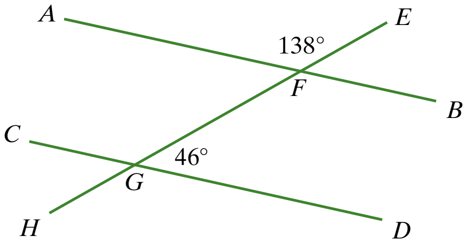
a



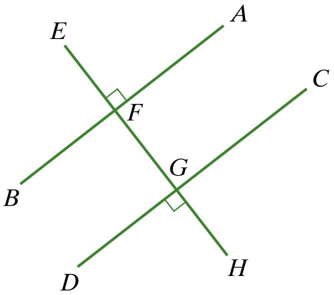
b



c



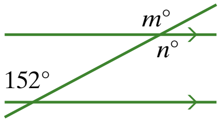
d



LINKS TO
Example 7

Find the unknown angles in each of the following diagrams.

a



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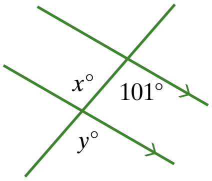

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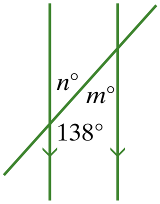

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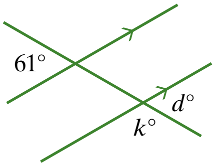

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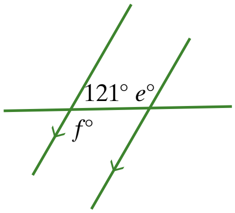

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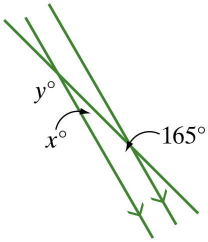

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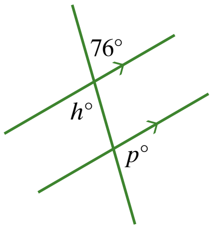

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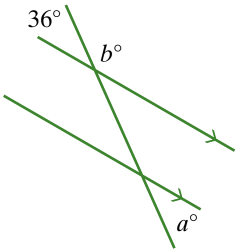

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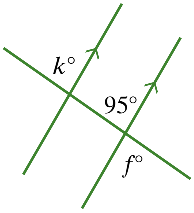

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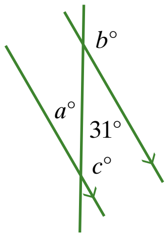

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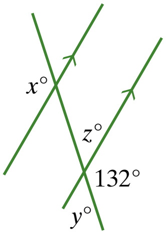

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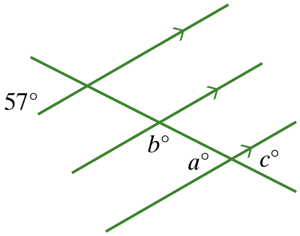

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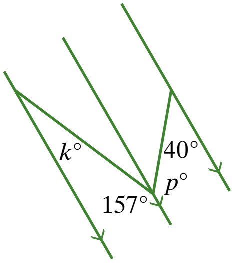

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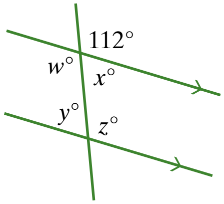

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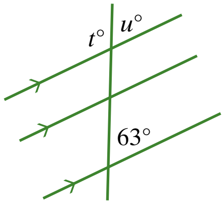


Name each of the following angles.

a the corresponding angle to ∠ABN

b the allied angle to ∠MBC

c the alternate angle to ∠NBC

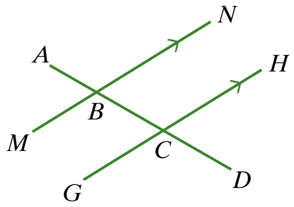
d an angle that is adjacent to ∠DCH and is the supplement of ∠DCH

e the corresponding angle to ∠GCB

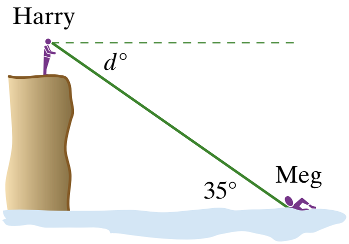
f the allied angle to ∠HCB

g the angle vertically opposite ∠GCD

h all the angles that are the supplement of ∠NBC.



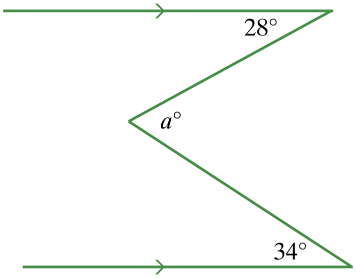
Harry is standing on top of a cliff, looking downwards at Meg who is swimming in the sea below. Meg looks up at Harry through an angle of 35°. Through what angle, d°, does Harry look down from the horizontal as he watches Meg? Justify your answer.



LINKS TO
Example 8

Find the value of each pronumeral.

a



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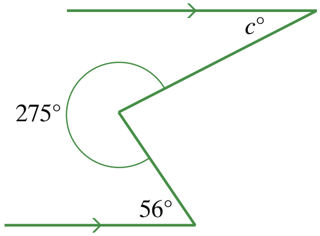

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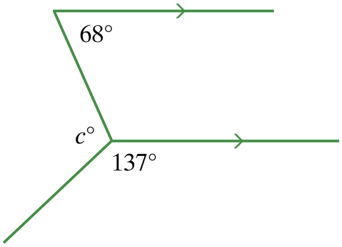

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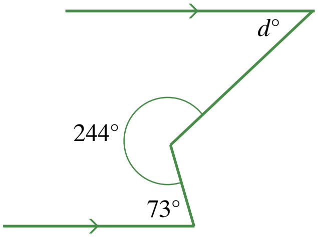


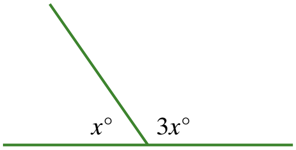
exercise 7.1 challenge

LINKS TO Example 9

For each of the following

- **i** use your knowledge of angles to write an equation (do not put degrees symbols in the equations).
- **ii** solve the equation to find the value of x.
- **iii** find the size of each of the labelled angles.

a



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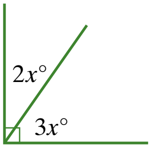

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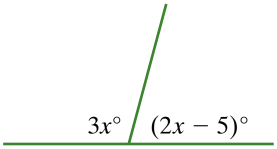

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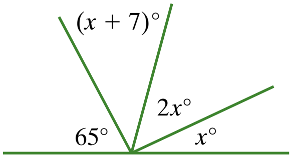

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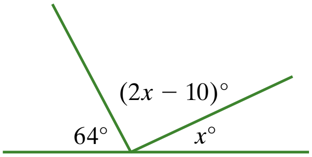

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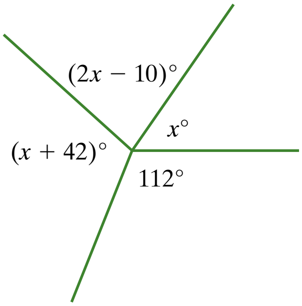

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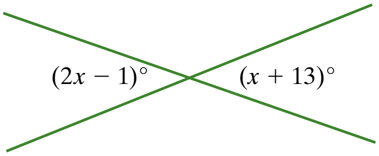

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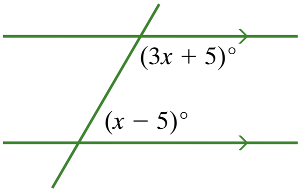
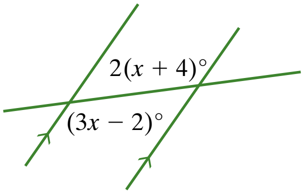
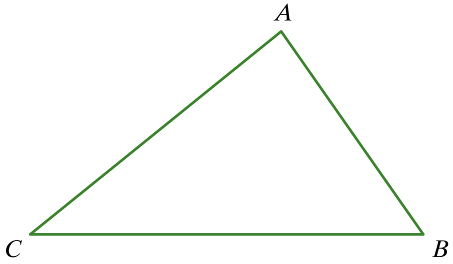


i

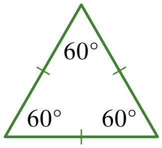
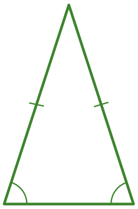
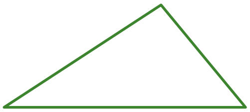
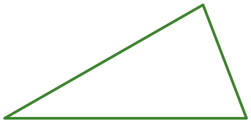
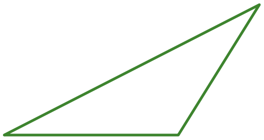
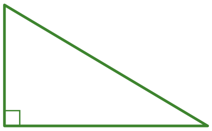


7.2 Reviewing triangles

A triangle is a plane shape with three straight sides. Each 'corner' of the triangle is called a vertex. Triangles are named according to the letters at the vertices. We can start at any vertex, for example, $\triangle ABC$ or $\triangle CAB$.



Types of triangles

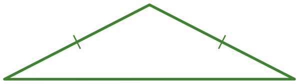
According to sides	
Equilateral triangle Three sides are equal in length. (Each angle is 60°.)	
Isosceles triangle Two sides are equal. (The angles opposite the equal sides are equal.)	
Scalene triangle The sides are all of different lengths. (The angles are all different.)	
According to angles	
Acute-angled triangle All three angles are acute.	
Obtuse-angled triangle One angle is obtuse.	
Right-angled triangle One angle is a right angle.	

Example 10

Identify each of these triangles according to

i the sides. **ii** the angles.

a



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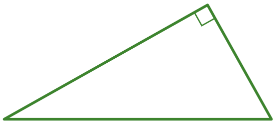


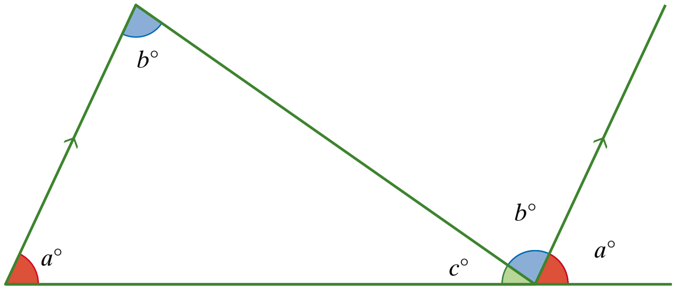
Working	**Reasoning**
a **i** isosceles triangle	The triangle has two equal sides.
ii obtuse-angled triangle	The triangle has one obtuse angle.
b **i** scalene triangle	The three sides have different lengths.
ii right-angled triangle	The triangle has a right-angle.

Interior angles of triangles

Angle sum of a triangle

The three interior angles of a triangle add to 180°.

We can prove this by extending one side of the triangle then drawing a line at this vertex of the triangle so that it is parallel to the opposite side.



The two red angles (marked $a°$) are equal because they are corresponding angles.

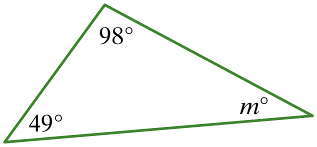
The two blue angles (marked $b°$) are equal because they are alternate angles between parallel lines.

Together the red, blue and green angles make a straight line, that is $a° + b° + c° = 180°$. So the three angles of the triangle must also make 180°.

Example 11

Find the value of each pronumeral.

a



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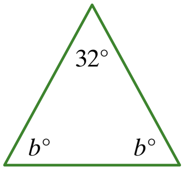


Working	**Reasoning**
a $180 - 98 - 49 = 33$ So, $m = 33$	The three interior angles of a triangle add to 180°.
b $180° - 32° = 148°$ $148° \div 2 = 74°$	The three angles add to 180° The two angles labelled $b°$ are equal.

Algebra can be used to find unknown angles. We can use our knowledge that the three angles of the triangle add to 180° to write an equation, then solve it to find the unknown angles.

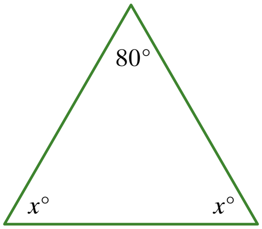
Example 12

Consider the triangle shown at right.

a Write an equation to show the three angles of the triangle adding to 180°.

b Solve the equation to find the value of x.

c State the sizes of each of the angles in the triangle.

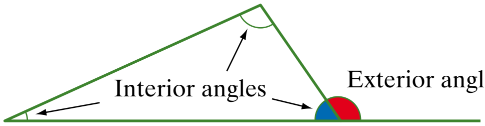


Working	**Reasoning**
a $x + x + 80 = 180$ $2x + 80 = 180$	The three angles add to 180°. $x + x = 2x$
b $2x + 80 = 180$ $2x + 80 - 80 = 180 - 80$ $2x = 100$ $x = 50$	Subtract 80 from both sides. Divide both sides by 2.
c The three angle sizes are 50°, 50°, 80°.	One angle is 80°. The other two angles are $x°$ and $x°$, that is 50° and 50°.

Exterior angles of a triangle

Exterior angle of a triangle

When the side of a triangle is extended, the angle that is formed outside the triangle is called an **exterior angle**. The three angles inside the triangle are **interior angles** of the triangle.

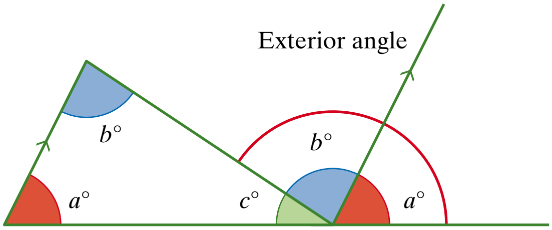


Notice that the exterior angle makes a straight line angle (180°) with the adjacent interior angle marked in blue.

Exterior angle rule

An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

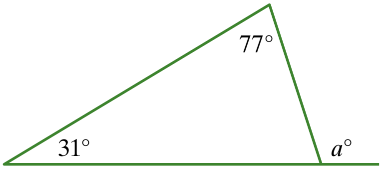
We can prove this using the same diagram that we used to prove that the three angles of a triangle add to 180°.



We can see that the exterior angle is equal to $a° + b°$, that is it is equal to the sum of the two angles $a°$ and $b°$ inside the triangle.

Example 13

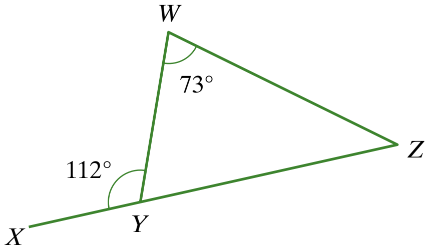
Find the value of a.



Working	**Reasoning**
$a = 31 + 77$ $= 108$	An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
or,	
Size of missing interior angle: $= 180° - 77° - 31°$ $= 72°$	The three angles of the triangle add to 180°.
$a + 72 = 180$ $a = 108$	The exterior angle and the adjacent interior angle are supplementary (add to 180°).

Example 14

Find the size of $\angle WZY$.



Working

$$\begin{aligned}\angle WZY + 73° &= 112° \\ \angle WZY &= 112° - 73° \\ \angle WZY &= 39°\end{aligned}$$

Reasoning

An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Finding angles

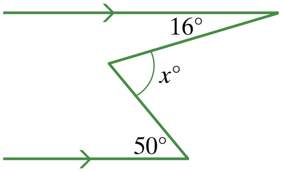
When working out the size of unknown angles in a diagram, we sometimes need to find the size of other angles in the diagram before we get to the angle we want. Questions such as the following enable us to identify clues that might help us in this angle chase.

- Are there any parallel lines cut by transversals?
- Are there any angles that together make up a straight line?
- Are there any exterior angles?
- Is there an isosceles triangle?
- Do we already know two of the three angles of a triangle?

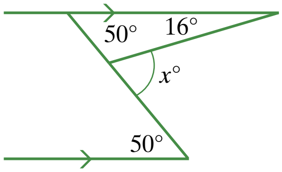
It is often possible to find more than one way of solving an angle problem. In example 15, we return to the angle problem in example 8 and use a different method to find the unknown angle.

Example 15

Use the fact that the exterior angle of a triangle is equal to the sum of the interior opposite angles to find the value of x.



Working



$$\begin{aligned}x &= 16 + 50 \\ &= 66\end{aligned}$$

Reasoning

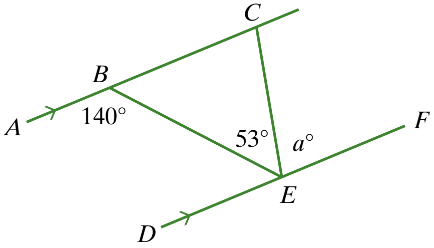
Continue the line segment as shown to make a triangle.

Using alternate angles between the parallel lines we know that the angle in the triangle is 50°.

The angle labelled $x°$ is an exterior angle to the triangle. The exterior angle is equal to the sum of the two interior opposite angles.

Example 16

Find the value of a.

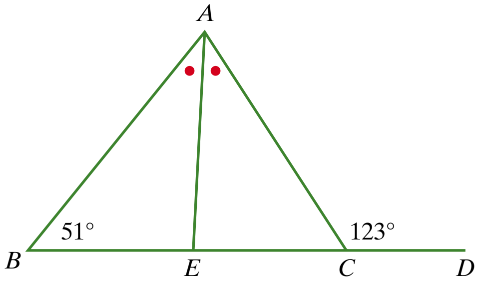


Working	**Reasoning**
$\angle BED = 40^\circ$	$\angle ABE$ and $\angle BED$ are allied angles so they are supplementary.
$\angle BED + \angle BEC + \angle CEF = 180^\circ$	$\angle BED$, $\angle BEC$ and $\angle CEF$ form a straight line.
$40 + 53 + a = 180$ $93 + a = 180$ $a = 180 - 93$ $a = 87$	We don't write 87° because a represents a number.

Remember that supplementary angles add to 180°.

Example 17

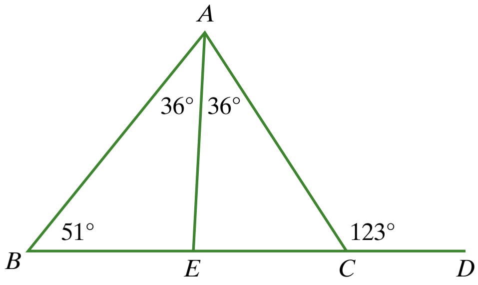
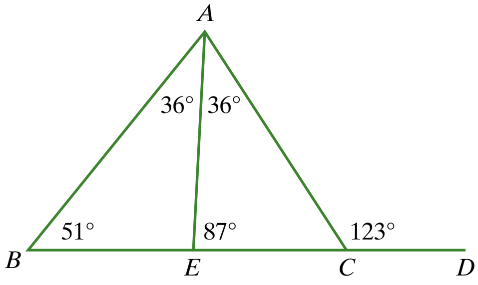
In the following diagram, AE bisects $\angle BAC$. Find $\angle AEC$.



Bisect means to cut exactly in half.

continued

Example 17 continued

Working	Reasoning
$\angle BAC + 51^\circ = 123^\circ$ Another way would be to find $\angle ACE$ first ($\angle ACE$ and $\angle ACD$ form a straight line). Then $\angle BAC$ can be found.	The exterior angle of a triangle is equal to the sum of the two interior opposite angles. $\angle ACD$ is an exterior angle to $\triangle ABC$, so $\angle BAC + \angle ABC = \angle ACD$.
$\angle BAC = 72^\circ$ $\angle EAC = 36^\circ$	$123^\circ - 51^\circ = 72^\circ$ AE bisects $\angle BAC$ so $\angle EAC = 72^\circ \div 2$ 
$\angle AEC + 36^\circ = 123^\circ$ $\angle AEC = 87^\circ$	The exterior angle of a triangle is equal to the sum of the two interior opposite angles. $\angle ACD$ is an exterior angle to $\triangle AEC$, so $\angle AEC + \angle EAC = \angle ACD$. 

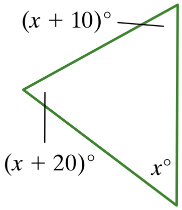
Example 18

For the triangle shown

a write an equation.

b solve the equation to find the value of x.

c find the size of the three angles in the triangle.



Working	**Reasoning**
a $x + x + 10 + x + 20 = 180$	The three angles of the triangle add to 180°.
b $3x + 30 = 180$ $3x = 150$ $x = 50$	Simplify the left side then solve the equation to find the value of x.
c $x + 10 = 50 + 10$ $= 60$ $x + 20 = 50 + 20$ $= 70$ The three angles are 50°, 60° and 70°.	Substitute $x = 50$ into the expressions for the angles.

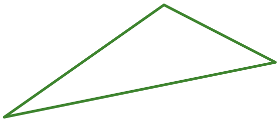
exercise 7.2

LINKS TO **Example 10**

● Classify each of these triangles according to its

- **i** sides.
- **ii** angles.

a



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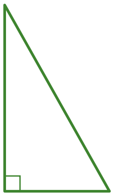

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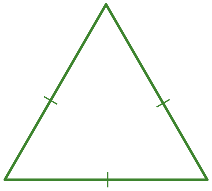

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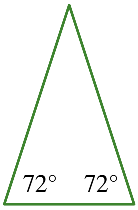

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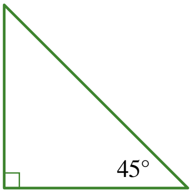

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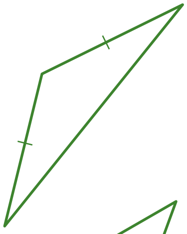

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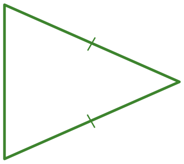

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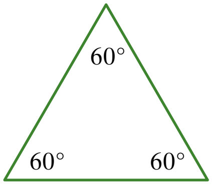

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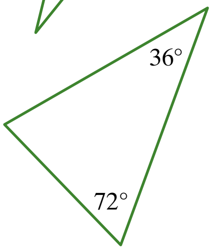
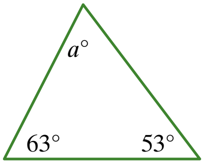


LINKS TO
Example 11

Find the value of the pronumerals in each of the following. Give reasons.

a



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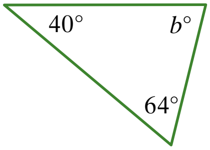

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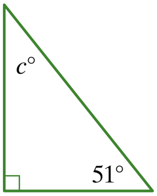

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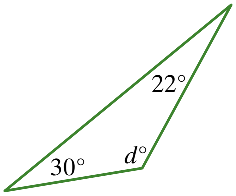

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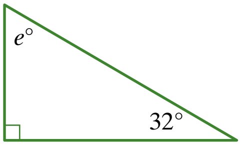

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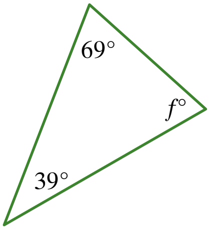
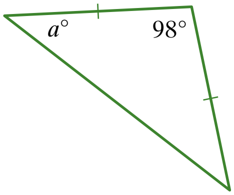


LINKS TO
Example 11

Find the value of the pronumerals in each of the following.

a



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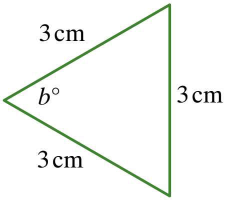

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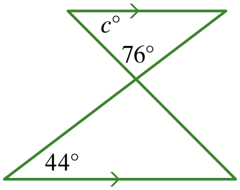

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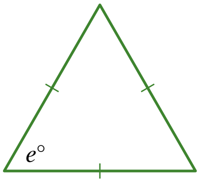

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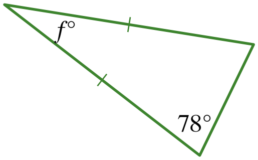

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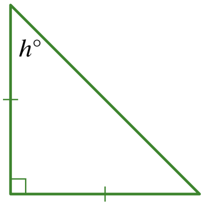

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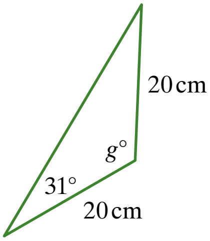

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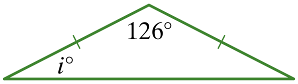

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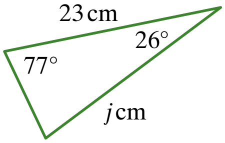

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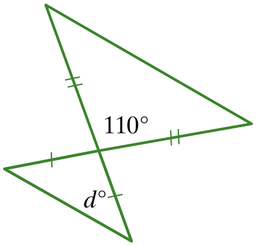
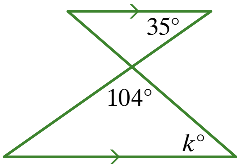


k



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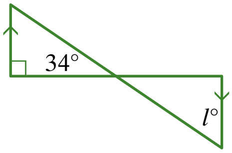
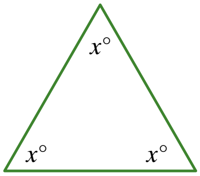


LINKS TO Example 12

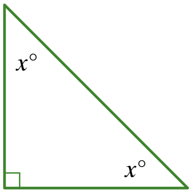
For each of the following triangles

- **i** write an equation showing that the three angles add to 180°. Do not use degrees signs in your equation.
- **ii** solve the equation to find the value of the pronumeral.
- **iii** state the sizes of the three angles of the triangle. (Check that the angles add to 180°.)

a



b



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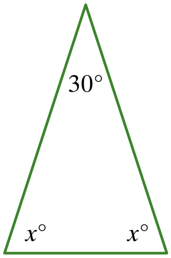

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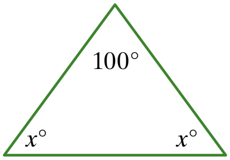
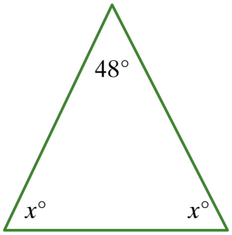


e



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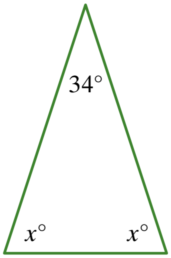


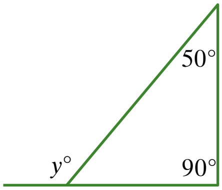
If two angles of a triangle are complementary, what do we know about the third angle?

Can a triangle have two obtuse angles? Explain.

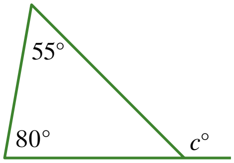
LINKS TO Example 13

Find the value of the pronumeral in each of the following.

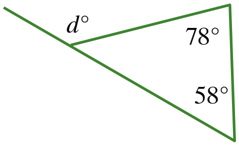
a



b



c



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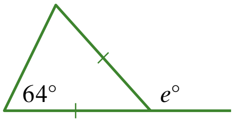

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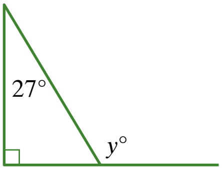

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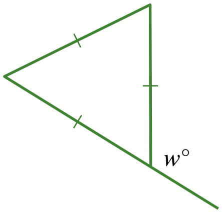
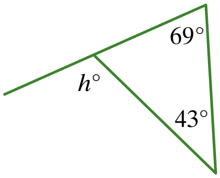


g



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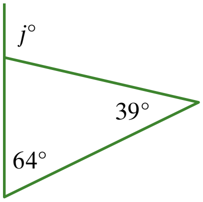

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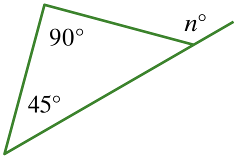
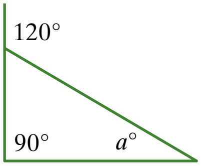


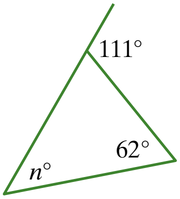
LINKS TO **Example 14**

Find the value of the pronumeral.

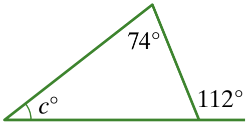
a



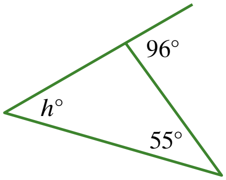
b



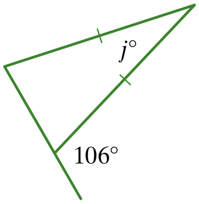
c



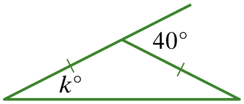
d



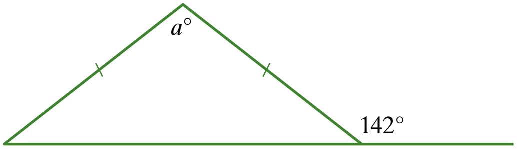
e



f

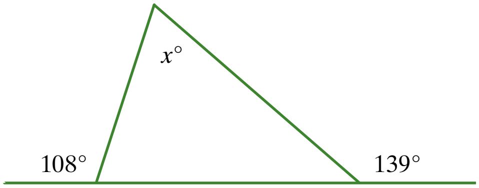


The value of *a* is



A 38° **B** 66° **C** 76° **D** 104° **E** 142°

The value of *x* is



A 27° **B** 41° **C** 67° **D** 72° **E** 113°

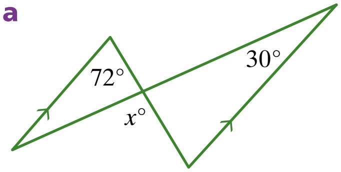
LINKS TO **Example 15**

Find the unknown angles in question 9 from exercise 7.1 by using the angles in a triangle.

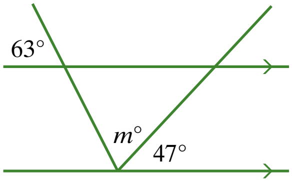
LINKS TO **Example 16**

Copy each of the following diagrams. Label on your drawings the sizes of all the angles that you had to find in order to calculate the value of the pronumeral, then write the value of the pronumeral.

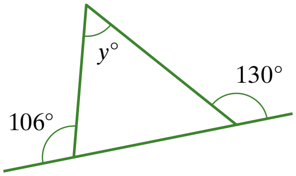
a



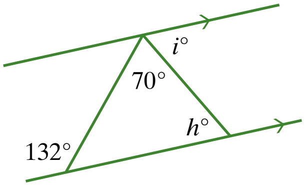
b



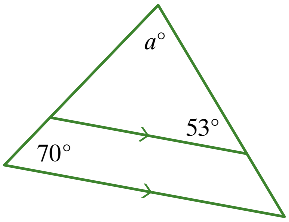
c



d



e



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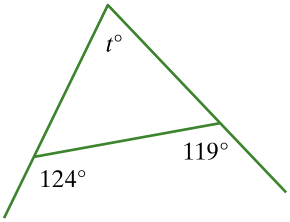
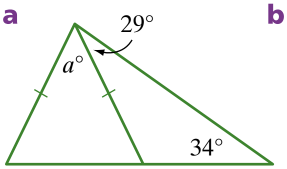


exercise 7.2 challenge

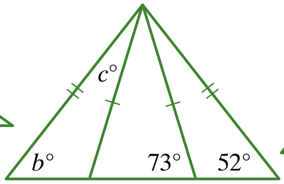
LINKS TO Example 17

Copy each of the following diagrams and find the value of each pronumeral. Hint: you may need to find the size of other angles in the figure before you find the value of the pronumeral. If so, label the size of each angle on your diagram.

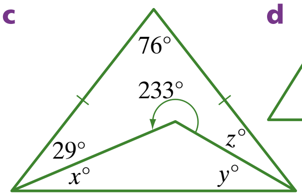
a



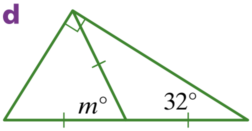
b



c



d



LINKS TO Example 18

Before the development of satellite communication and global positioning systems, sailors used angle measurements to locate their position. One technique used to find the distance from a landmark is called 'doubling the angle'. This involves measuring the angle between the ship's path and an object such as a lighthouse, and then sailing towards the object until the angle is double the previous angle measurement. These angle measurements are referred to as 'the angle on the bow'.

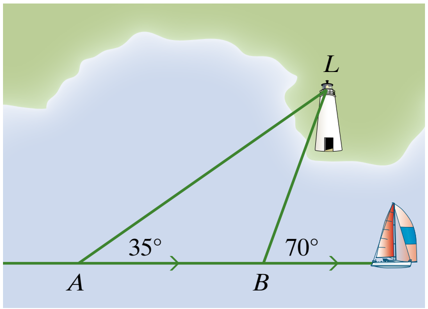
In the diagram, the angle on the bow was 35° at A and 70° at B.

a Explain why $\triangle ABL$ must be an isosceles triangle.

b The boat is travelling at a speed of 8 knots and takes 30 minutes to travel from A to B. Calculate the distance in nautical miles from A to B.

c How far is the boat from the lighthouse when it is at B?

d If 1 nautical mile is equivalent to 1.852 km, what is the distance from the lighthouse in kilometres?



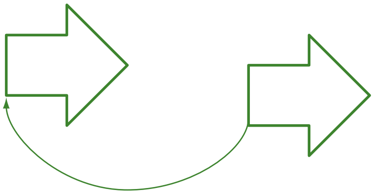
1 knot is a speed of 1 nautical mile per hour.

7.3 Congruency and isometric transformations

If two shapes are **congruent**, one can be placed exactly over the other.

Congruent comes from a Latin word meaning *to come together* or *agree*.

These two shapes are congruent because we can place one exactly over the other.



If two or more shapes are congruent, both of the following conditions must be true.

- Matching angles must be equal.
- Matching side lengths must be equal.

We use the symbol ≡ to indicate that two shapes are congruent.

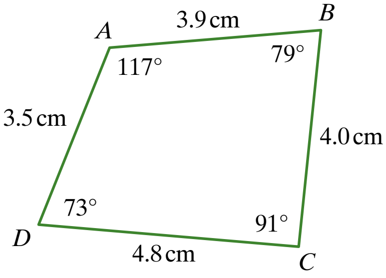
Example 19

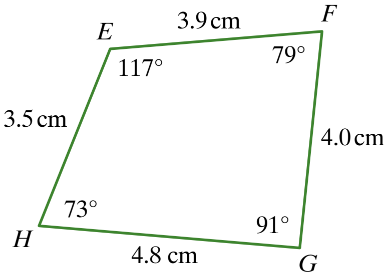
Are the following pairs of quadrilaterals congruent? Explain.

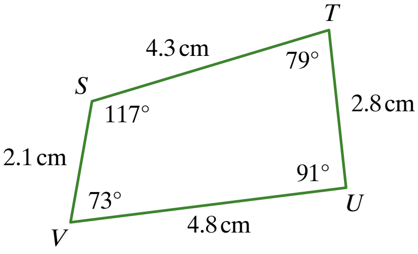
a *ABCD* and *EFGH*

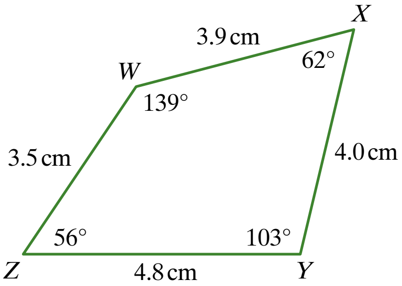
b *ABCD* and *STUV*

c *ABCD* and *WXYZ*









continued

Example 19 continued

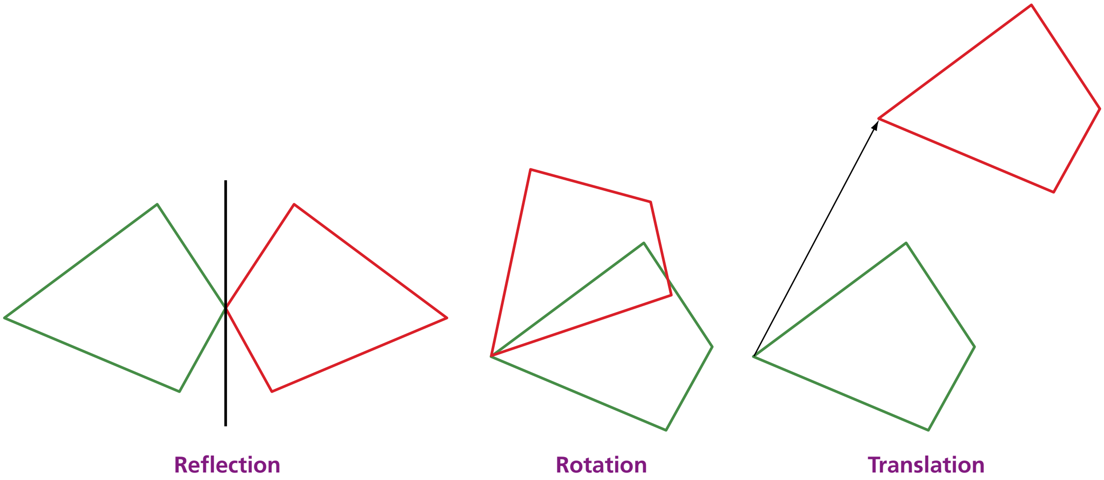
Working	Reasoning
a The angles of quadrilateral *ABCD* are the same as the angles of quadrilateral *EFGH*. The matching sides have the same length. So *ABCD* ≡ *EFGH*	For shapes to be congruent the matching angles of one must be the same as the angles of the other *and* the matching sides must be the same length. The quadrilaterals must be named according to the matching sides. To match the sides of quadrilateral *ABCD*, the second quadrilateral must be named *EFGH*, not, for example, *GHEF*.
b Quadrilaterals *ABCD* and *STUV* are not congruent. Although their angles are the same, the sides are different.	Matching angles must be the same *and* matching sides must be the same.
c Quadrilaterals *ABCD* and *WXYZ* are not congruent. Although their sides are the same, the angles are different.	Matching angles must be the same *and* matching sides must be the same.

You will recall from Year 7 that a change in the size or position of a figure is called a **transformation**. An **isometric transformation** is where the position of a geometric figure is changed but its shape and size stay the same, that is, a congruent figure is produced.

Isometric means *the same measure.*

Three different types of isometric transformation are

- reflection
- rotation
- translation.



Reflection Rotation Translation

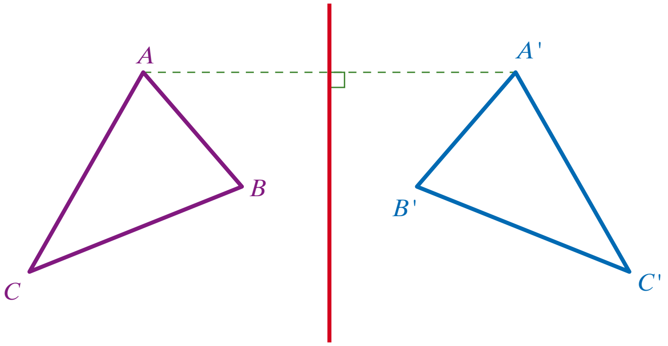
Isometric transformations produce congruent shapes

A shape that is produced by a transformation is called the **image** of the original shape. If we use the letters A, B, C, and so on to label the vertices of the original shape, we normally use A', B', C', and so on to label the vertices of its image. For example, if a triangle is labelled ABC, its image would be labelled $A'B'C'$. We write A'. We say 'A dash'.

Reflection

Reflection

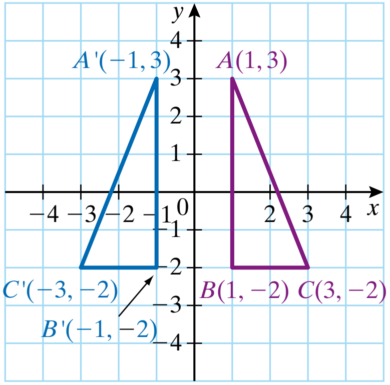
Reflection of a shape produces a mirror image. Each point and its image can be joined by a line that is perpendicular to the mirror line. Each point and its image are the same distance from the mirror line. Reflection is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.



Reflection on the Cartesian plane

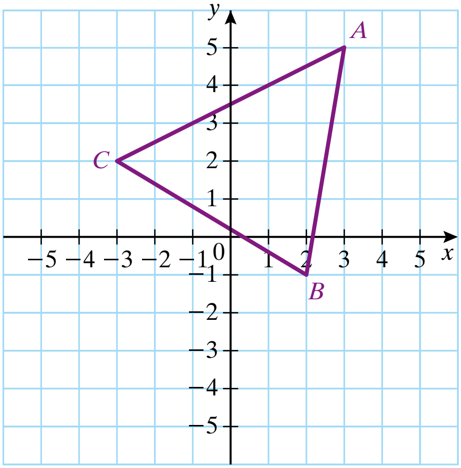
When a shape is reflected on the Cartesian plane

- reflection in the y-axis changes the sign of the x-coordinate of each vertex but the y-coordinates remain unchanged.
- reflection in the x-axis changes the sign of the y-coordinate of each vertex but the x-coordinates remain unchanged.



Example 20

Reflect triangle ABC in the x-axis then complete the table to show the coordinates of each vertex of the triangle and its image. Each grid square is 1 unit.

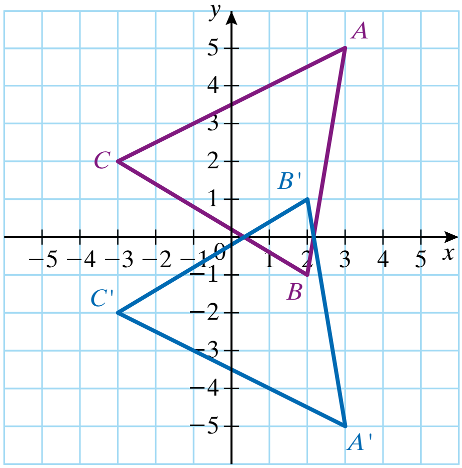


A: (,)	A': (,)
B: (,)	B': (,)
C: (,)	C': (,)

continued

Example 20 continued

Working



A: (3, 5)	*A*′: (3, −5)
B: (2, −1)	*B*′: (2, 1)
C: (−3, 2)	*C*′: (−3, −2)

Reasoning

The sign of each *x*-coordinate stays the same. The sign of each *y*-coordinate changes. Reflection is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

Rotation

Rotation

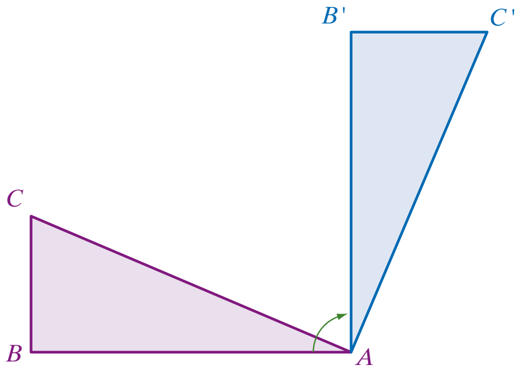
Rotation on the Cartesian plane

Rotation occurs when a shape is turned about a point. In describing a rotation we must state

- the angle of rotation.
- the direction of rotation (clockwise or anticlockwise).
- the point about which the shape is to be rotated.

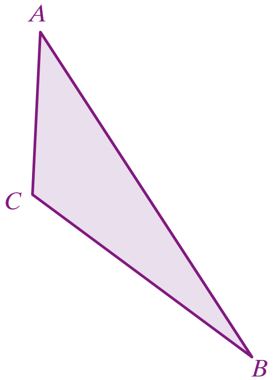
If a shape is rotated through 360° it will end up back in its original position.

Triangle *ABC* has been rotated through 90° about point *A* in a clockwise direction to produce the image *A*′*B*′*C*′. Rotation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

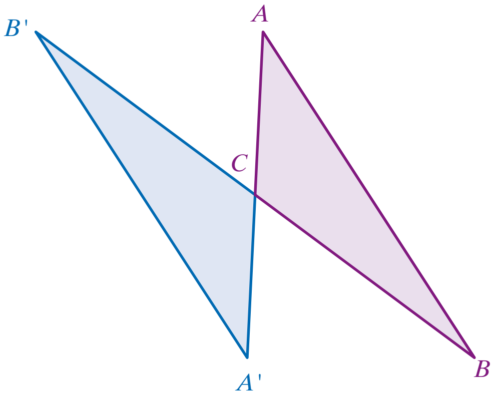


Example 21

Rotate triangle ABC through 180° about point C.



Working



Reasoning

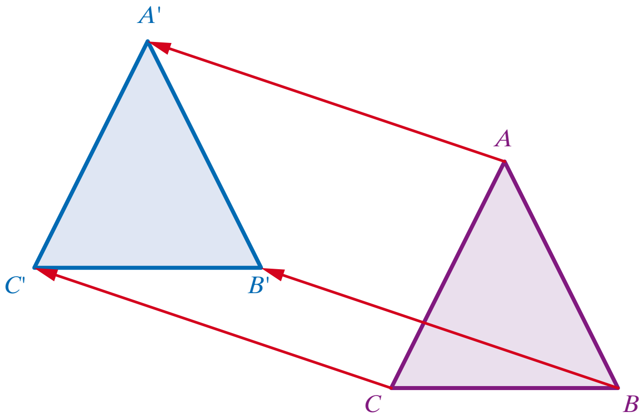
Each side of the triangle is rotated through 180°.
Rotation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

Translation

Translation

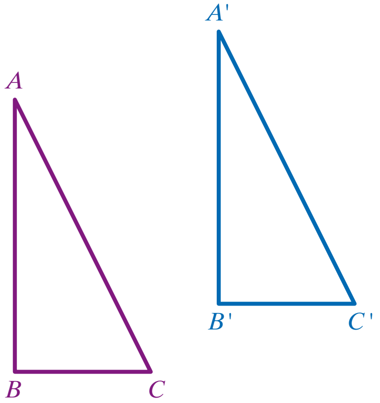
Translation on the Cartesian plane

Translation occurs when a shape slides into a different position without being turned. Each point on the object is translated the same distance. Translation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

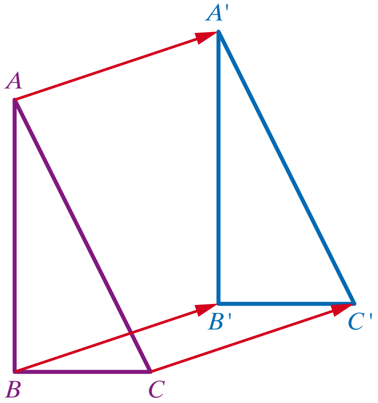


Example 22

Draw arrows at A, B and C to show how each point has been translated.



Working

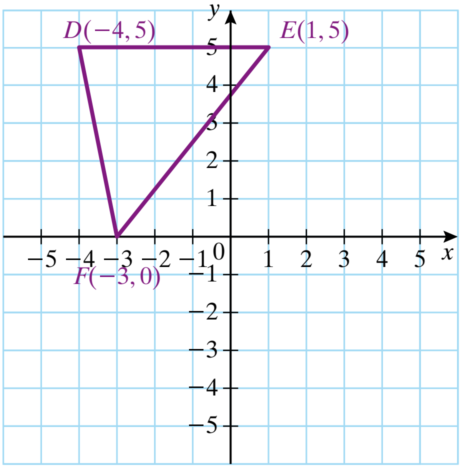


Reasoning

Each point is translated the same distance in the same direction. $\triangle A'B'C'$ is exactly the same shape and size as $\triangle ABC$.
Translation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

Example 23

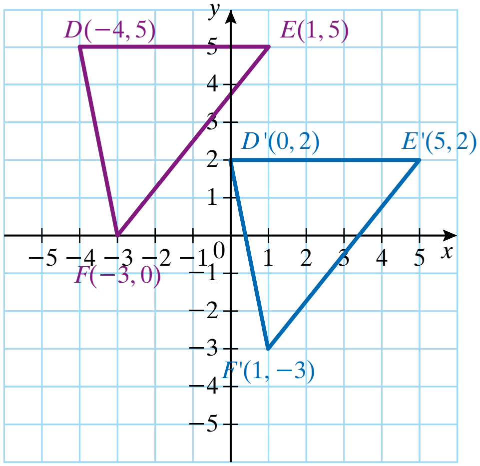
Translate $\triangle DEF$ 4 units to the right and 3 units down.



continued

Example 23 continued

Working



Reasoning

Translating 4 units to the right and 3 units down adds 4 to each *x*-coordinate and subtracts 3 from each *y*-coordinate. Translation is an isometric transformation so $\triangle D'E'F' \equiv \triangle DEF$.

If two triangles are congruent, we can use one or more isometric transformations to place one exactly over the other.

Example 24

For each of the following pairs of congruent triangles,

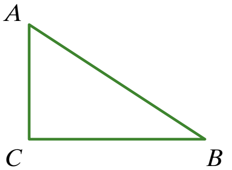
i state the transformations that would be needed to place triangle 2 exactly over triangle 1.

ii list the matching vertices.

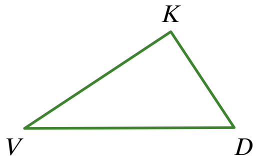
iii list the matching sides.

iv name triangle 2 to match triangle 1.

a

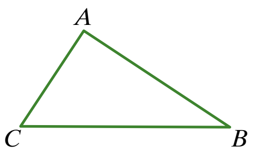


Triangle 1

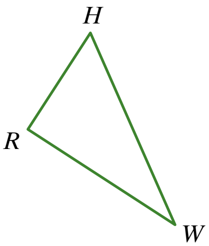


Triangle 2

b



Triangle 1



Triangle 2

continued

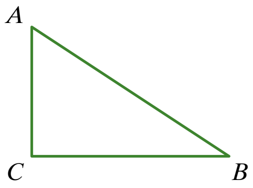
Example 24 continued

Working

Reasoning

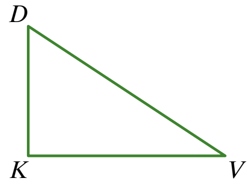
a **i** Rotation and translation

ii $A \leftrightarrow D$
$B \leftrightarrow V$
$C \leftrightarrow K$



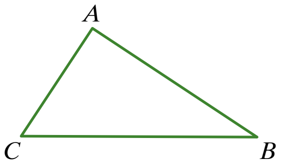
iii $AB \leftrightarrow DV$
$BC \leftrightarrow VK$
$CA \leftrightarrow KD$

iv $\triangle DVK$



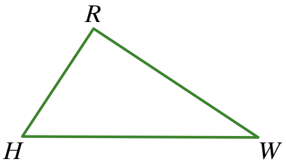
b **i** Rotation, reflection and translation

ii $A \leftrightarrow R$
$B \leftrightarrow W$
$C \leftrightarrow H$



iii $AB \leftrightarrow RW$
$BC \leftrightarrow WH$
$CA \leftrightarrow HR$

iv $\triangle RWH$



exercise 7.3

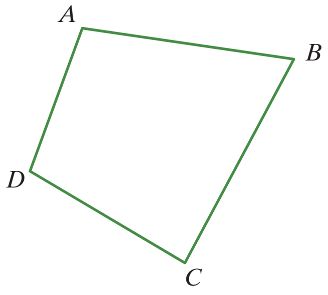
LINKS TO Example 19

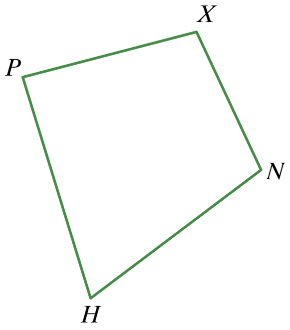
These two quadrilaterals are congruent. Name

a the matching vertices.

b the matching sides.

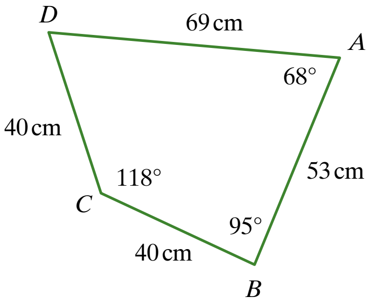
c the quadrilateral on the right to match the name of quadrilateral *ABCD*.

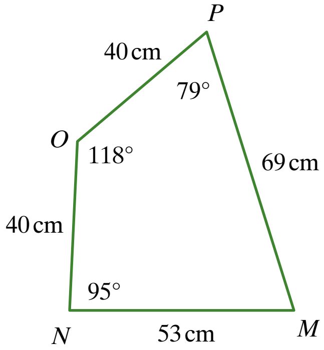




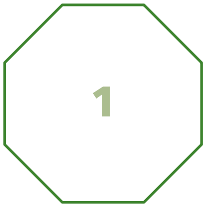
LINKS TO **Example 19**

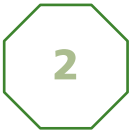
Is quadrilateral $ABCD$ congruent to quadrilateral $MNOP$? Explain.

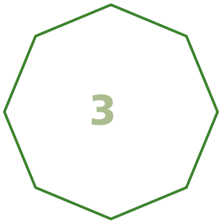


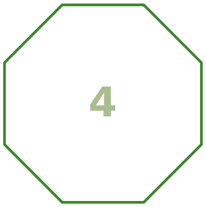


Which of the following shapes are congruent?









A 1 and 2 **B** 1 and 3 **C** 1 and 4 **D** 1, 2 and 4 **E** 1, 3 and 4

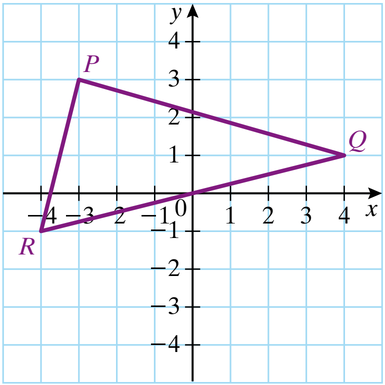
LINKS TO **Example 20**

Exercise 7.3 question 4

$\triangle PQR$ is shown at right.

a Reflect $\triangle PQR$ in the x-axis. Each grid square is 1 unit.

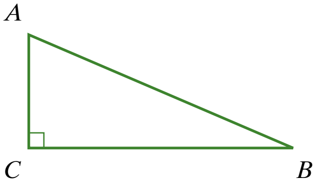
b Make a table to show the coordinates of P, Q and R and their images.



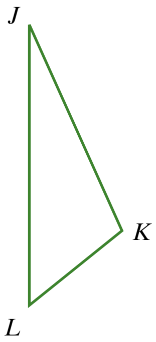
LINKS TO **Example 21**

Copy each of these triangles and rotate it through the given angle.

a Rotate $\triangle ABC$ through 90° anticlockwise about point A.



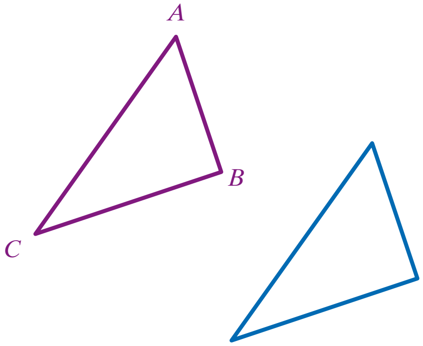
b Rotate $\triangle JKL$ through 180° about point L.



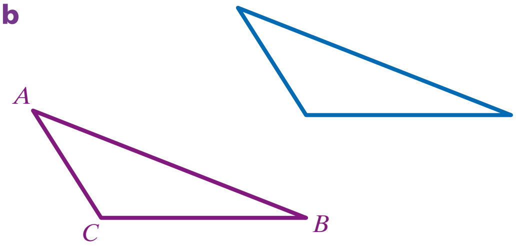
LINKS TO Example 22

Draw translation arrows to show the direction of translation and label the vertices of the translated triangle as A', B' and C'.

a



b

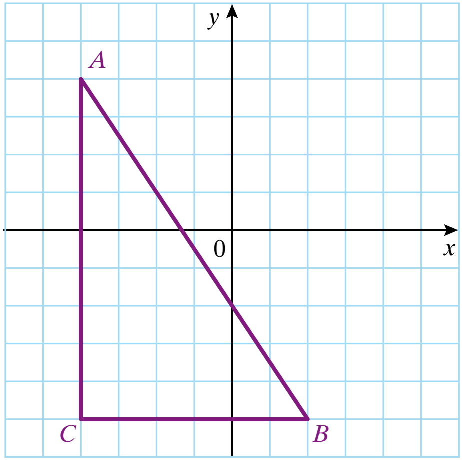


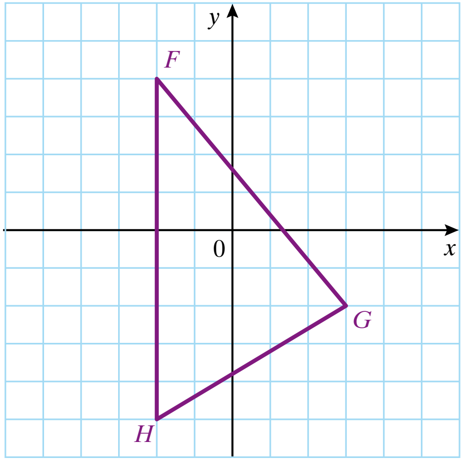
LINKS TO Example 23

Translate each of these triangles in the direction indicated. Each grid square is 1 unit. In each case make a table to show the coordinates of each vertex and its image.

a Translate $\triangle ABC$ 3 units to the right and 1 unit up.

b Translate $\triangle FGH$ 2 units to the left and 1 unit down.





$\triangle ABC$ has vertices $A(-3,-5)$, $B(-1,6)$ and $C(4,3)$. On graph paper, carefully draw $\triangle ABC$ and then translate it 4 units to the left and 3 units up. Label the vertices A', B' and C' and show their coordinates.

LINKS TO Example 24

For each of the following pairs of congruent triangles

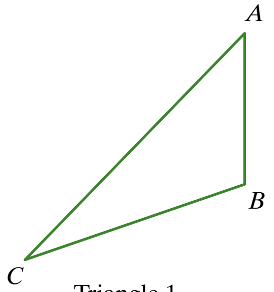
i state the transformations that would be needed to place triangle 2 exactly over triangle 1.

ii List the matching vertices.

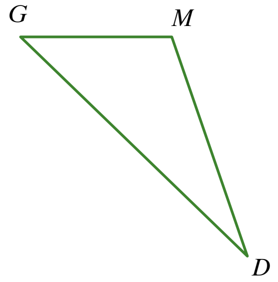
iii List the matching sides.

iv Name triangle 2 to match triangle 1.

a

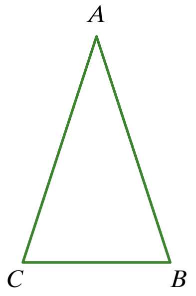


Triangle 1

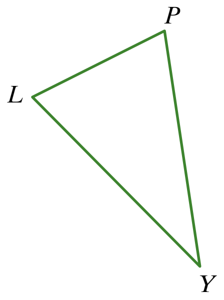


Triangle 2

b

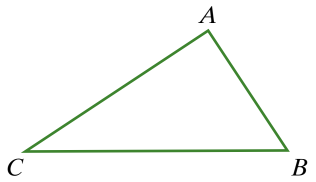


Triangle 1

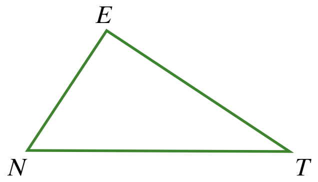


Triangle 2

c

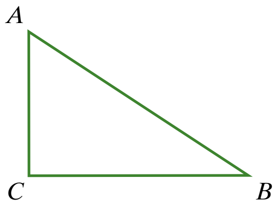


Triangle 1

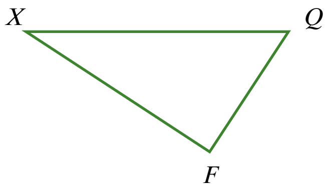


Triangle 2

d



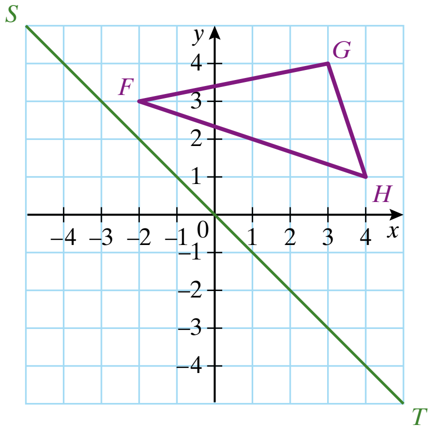
Triangle 1



Triangle 2

exercise 7.3 challenge

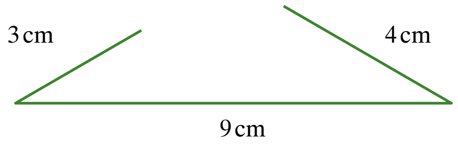
Reflect △*FGH* in the mirror line *ST* and label the vertices of the image triangle using the correct notation.



7.4 Triangle construction and congruency

Constructing triangles

The sum of the two shortest sides of a triangle must be greater than the third side. For example, a triangle cannot be constructed if the side lengths are 3 cm, 4 cm and 9 cm as the two shortest sides will not meet.



Example 25

For which of these sets of lengths is it possible to construct a triangle?

a 3 cm, 5 cm, 11 cm　**b** 4 cm, 6 cm, 7 cm　**c** 2.5 cm, 3 cm, 5.5 cm

Working	**Reasoning**
a $3\text{ cm} + 5\text{ cm} < 11\text{ cm}$ A triangle cannot be constructed.	The sum of the two shortest sides is less than the third side, so the sides will not meet.
b $4\text{ cm} + 6\text{ cm} > 7\text{ cm}$ A triangle can be constructed.	The sum of the two shortest sides is greater than the third side, so the sides will meet.
c $2.5\text{ cm} + 3\text{ cm} = 5.5\text{ cm}$ A triangle cannot be constructed.	The sum of the two shortest sides is equal to the third side. Although they meet to make the length of the third side, they cannot make a triangle.

Every triangle has three sides and three angles so there are six pieces of information about a triangle that we could be given: the lengths of the three sides and the sizes of the three angles. The question is: do we actually need all six pieces of information or is there a smaller number of pieces of information that will allow us to accurately identify the triangle? In the following examples, six different sets of information are considered.

Side-Side-Side (SSS)

In Year 7 you saw how a compass and ruler could be used to accurately draw a triangle if you were given the three sides. An animation showing this construction can be seen on the website www.mathopenref.com/consttrianglesss.html.

Given the three side lengths of a triangle, the size and shape of the triangle is determined.

Example 26

Construct a triangle with sides 6 cm, 4.5 cm and 3.5 cm

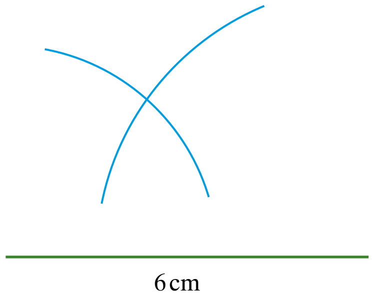
a using pencil, compass and ruler.

b using GeoGebra.

Working

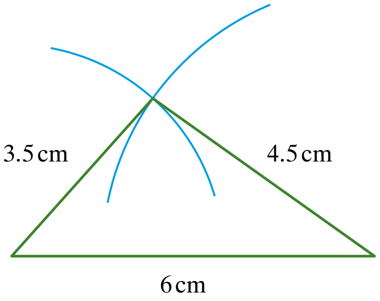
Reasoning

a



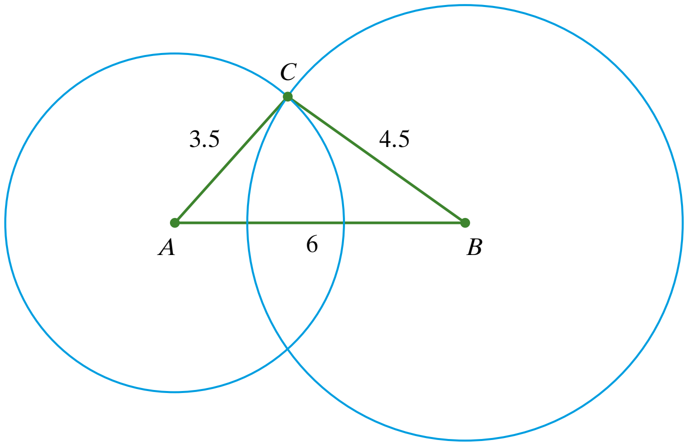
Draw a line segment 6 cm long.

With a compass opened to exactly 4.5 cm, place the compass point at one end of the segment and draw an arc as shown. Then with the compass opened at exactly 3.5 cm, place the compass point on the other end of the 6 cm line segment and draw another arc.



Where the two arcs intersect represents the third vertex of the triangle. Draw a line segment to join the three vertices.

b

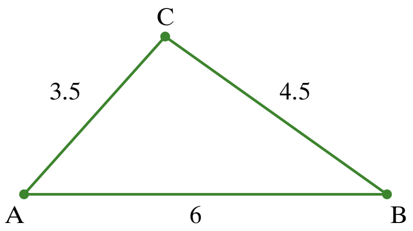


Using GeoGebra, we start by constructing a line interval (line segment) of length 6 cm. Construct circles of radius 3.5 and 4.5 at points *A* and *B* respectively. Construct an intersection point *C* at the intersection of the two circles, then construct line intervals *AC* and *BC*.

continued

Example 26 continued

Working

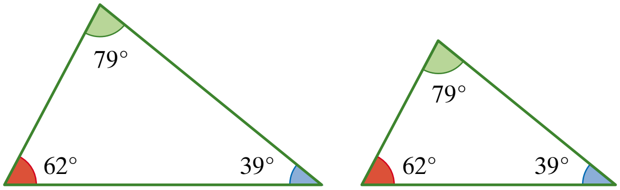


Reasoning

Finally, *hide* the circles by using the **Show/hide** tool. Note that we do not *delete* the circles because this would cause all those parts of the construction that depended on the two circles to be deleted.

Angle-Angle-Angle (AAA)

The two triangles below both have angles of 62°, 79° and 39°. Although they are the same shape, they are different sizes. Note that we actually need to know only two of the angle sizes because we can calculate the size of the third angle.

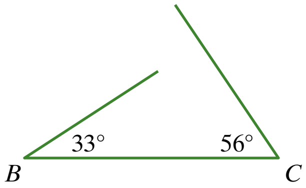


Given the three angles of a triangle, the *shape* of the triangle is determined, but not its *size*.

Example 27

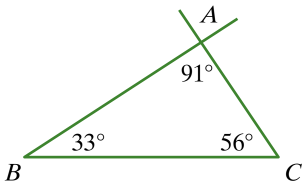
Construct a triangle ABC with angles 33°, 56° and 91° at B, C and A respectively.

Working



Reasoning

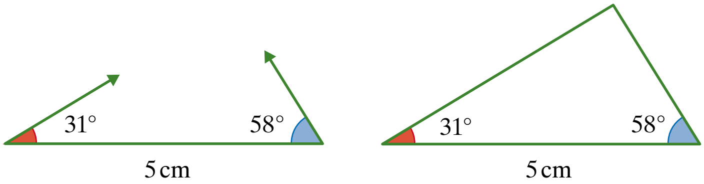
Draw a line segment any length and label it BC. Use a protractor to construct an angle of 33° at B and an angle of 56° at C.



Extend the arms of the angles to meet at A. The third angle will be 91°.
The shape of the triangle is fixed by the angle sizes, but the size will depend on how long we make the first line segment.

Angle-Side-Angle (ASA)

If we know the sizes of two angles and the length of the side joining them, there is only one possible triangle we can draw. An animation showing this construction can be seen on the website www.mathopenref.com/consttriangleasa.html.

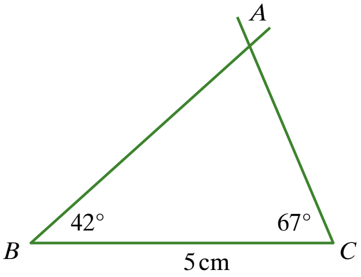


Given two angles of a triangle and the length of the side between them, the shape and size of the triangle is determined.

Example 28

Construct a triangle ABC in which $BC = 5\,\text{cm}$, $\angle ABC = 42°$ and $\angle ACB = 67°$.

Working

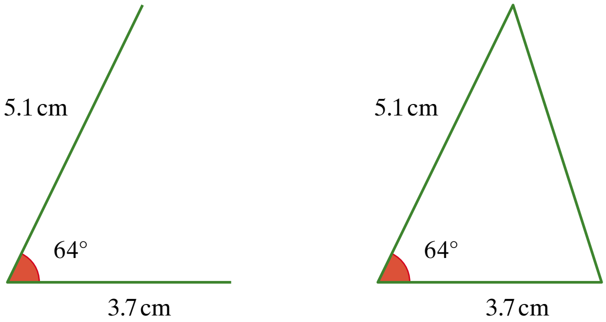


Reasoning

Draw a line segment, BC, 5 cm long. Use a protractor to construct an angle of 42° at B and an angle of 67° at C. Extend the arms of the angles to meet at A.

Side-Angle-Side (SAS)

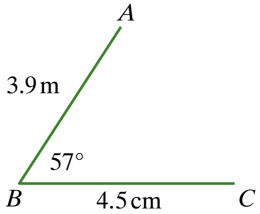
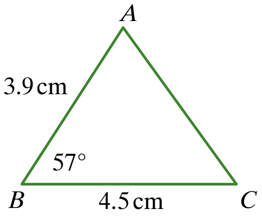
If we know the lengths of two sides and the size of the angle between them, there is only one possible triangle we can draw. An animation showing this construction can be seen on the website www.mathopenref.com/consttrianglesas.html.



Given two sides of a triangle and the size of the angle between them, the shape and size of the triangle is determined.

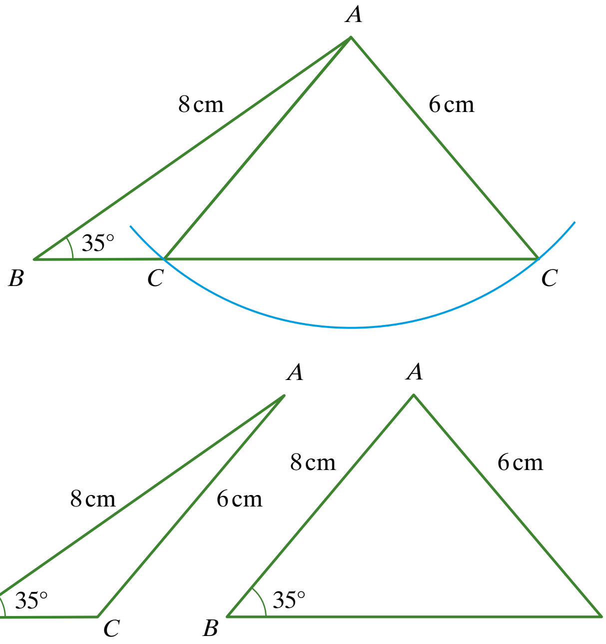
Example 29

Construct a triangle ABC in which $BC = 4.5\,\text{cm}$, $AB = 3.9\,\text{cm}$ and $\angle ABC = 57°$.

Working	**Reasoning**
	Draw a line segment, BC, 4.5 cm long. Use a protractor to construct an angle of 57° at B, with $BA = 3.9\,\text{cm}$.
	Use a ruler to join A and C.

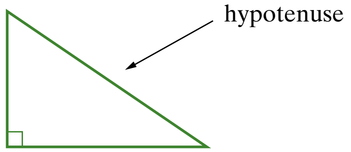
Side-Side-Angle (SSA)

If we are given the lengths of two sides and an angle, but the angle is not between the two given sides, then two different triangles are possible, as shown below. So knowing two sides and an angle that is not the included angle is not sufficient to fix the shape of the triangle.



Right angle-Hypotenuse-Side (RHS)

In any right-angled triangle the side that is opposite the right angle is always the longest side of the triangle. This side of a right-angled triangle has a special name – it is called the **hypotenuse**.



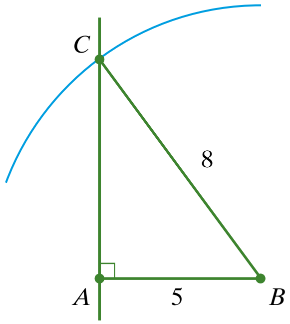
Knowing the length of the hypotenuse and one other side of a right-angled triangle is sufficient to fix the shape of the triangle.

Example 30

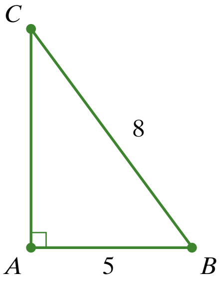
Use interactive geometry software such as GeoGebra to construct a triangle ABC in which $\angle CAB = 90°$, $AB = 5$ cm and $BC = 8$ cm.

Working

Reasoning



Start by constructing a line interval (line segment) AB with length 5. Use the **perpendicular line** tool to construct a line through point A perpendicular to AB. Then construct a circle with centre B and radius 8. Construct an intersection point C where the circle crosses the perpendicular line through A. Join BC and join AC with line intervals.



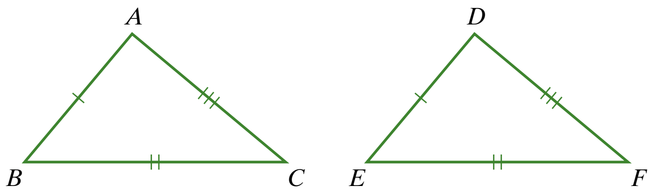
Hide the circle and the perpendicular line.

Conditions for triangles to be congruent

We have seen that certain sets of information about a triangle ensure that only one shape is possible for the triangle. These are also the sets of information that allow us to decide if two triangles are congruent.

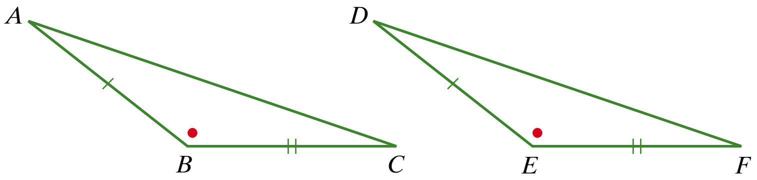
Side–Side–Side (SSS)

The three sides of one triangle are equal in length to the three sides of the other triangle.



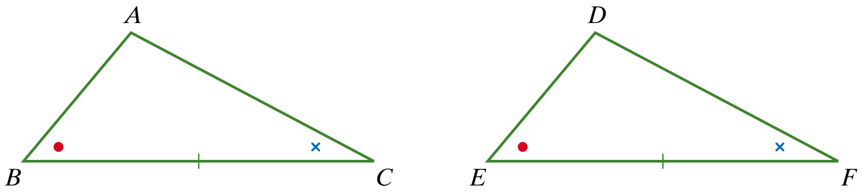
Side–Angle–Side (SAS)

Two sides of one triangle are equal to the corresponding two sides of the other triangle, and the angles in between these two sides are equal.



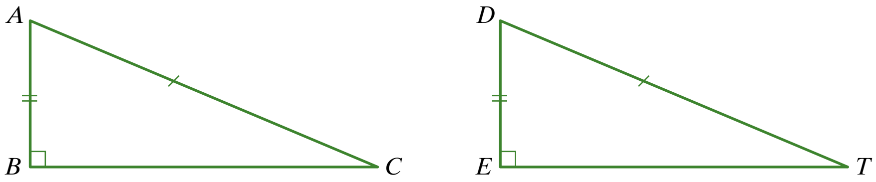
Angle–Side–Angle (ASA)

Two angles and a side of one triangle are equal to two angles and the matching side in the other triangle.



Right angle–Hypotenuse–Side (RHS)

Both triangles are right-angled and the hypotenuse and another side of one triangle are equal to the hypotenuse and the matching side of the other triangle.



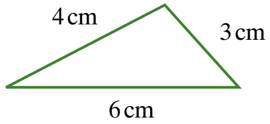
The sets of conditions that do *not* guarantee that two triangles are congruent are AAA and ASS.

We can use the conditions for congruency to see if triangles are congruent. Usually, as in example 31 part **a**, the given information is sufficient to tell whether or not two triangles are congruent. However, sometimes, as in example 31 part **b**, we may not have enough information to be certain that the two triangles are not congruent.

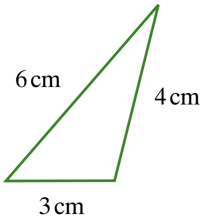
Example 31

Find if these pairs of triangles are congruent. Give your reasons.

a

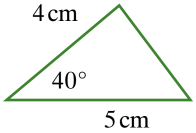


Triangle 1

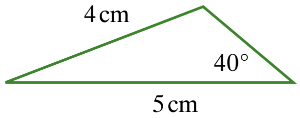


Triangle 2

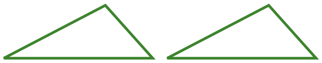
b



Triangle 3



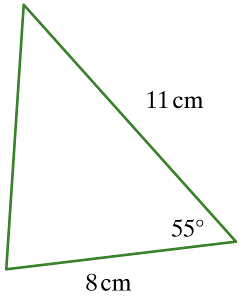
Triangle 4

Working	**Reasoning**
a Triangles 1 and 2 are congruent because the sides of triangle 1 are the same length as the sides of triangle 2. (SSS)	Rotate Triangle 2 so it is in the same orientation as Triangle 1.  Triangles are congruent if the three sides of one are equal in length to the three sides of the other.
b Triangles 3 and 4 are not necessarily congruent because in Triangle 4 the angle of 40° is not between the matching sides of 5 cm and 4 cm.	Triangles are congruent if two sides of one triangle and the size of the angle between them are the same as the matching two sides of the other triangle and the angle between them. We need to say that the triangles are not necessarily congruent because it is possible that the triangles are isosceles so that the other angles at the end of the 5 cm side are also 40°.

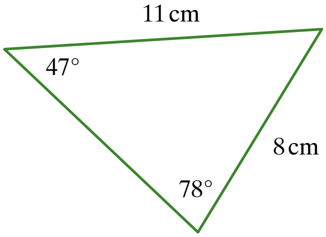
In some cases we may need to calculate a missing angle size in one triangle before we can decide if the triangles are congruent.

Example 32

Find if these pairs of triangles are congruent. Give your reasons.



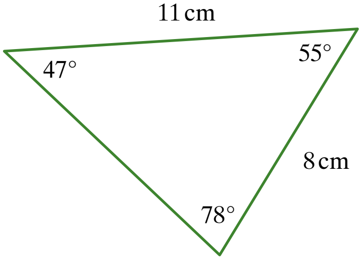
Triangle A



Triangle B

Working

In triangle B, the angle between the two given sides is 55°.



So two sides and the angle between them of triangle E are equal to two sides and the angle between them of triangle F.

The triangles are congruent (SAS).

Reasoning

The third angle of triangle B

$= 180° - (47° + 78°)$

$= 55°$

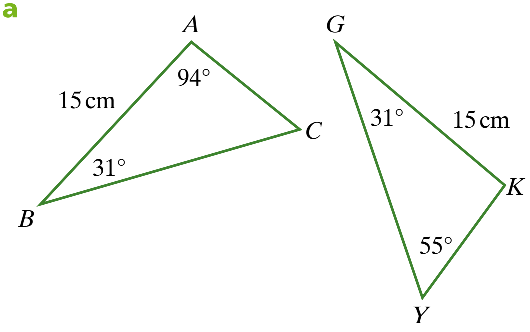
If two sides and the angle between them of one triangle are equal to two sides and the angle between them of the other triangle, the triangles are congruent.

Example 33 is another example where we need to calculate missing angles sizes before deciding if the two triangles are congruent.

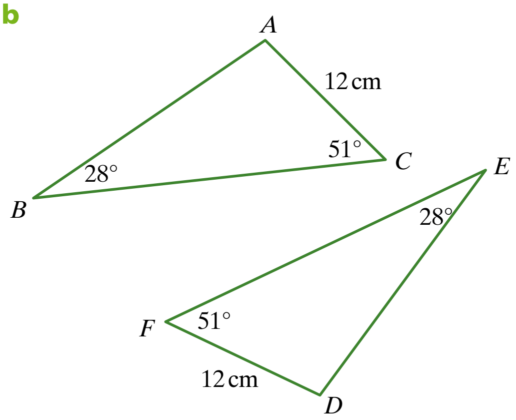
Example 33

Find if these pairs of triangles are congruent. Give your reasons.

a



b



Working

a $\angle GKY = 180° - 55° - 31°$

$\angle GKY = 94°$

Both triangles have angles of 31° and 94° joined by a side of length 15 cm.

$\triangle ABC \equiv \triangle KGY$ (ASA)

Reasoning

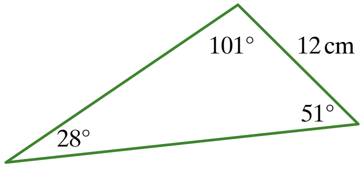
Calculate the missing angle $\angle GKY$.

Two triangles are congruent if two angles and the side joining them in one triangle are equal to two angles and the side joining them in the other triangle.

We name the congruent triangles by matching the vertices. If we rotate the second triangle so that it is exactly on top of $\triangle ABC$, K corresponds to A, G to B and Y to C.

So we say that $\triangle ABC \equiv \triangle KGY$.

b In each triangle, the missing angle size is $180° - (28° + 51°) = 101°$.

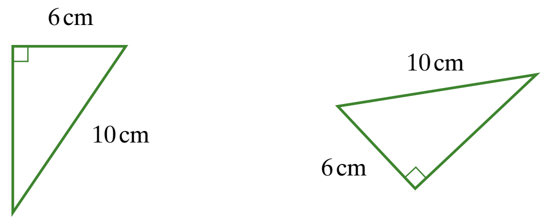


$\triangle ABC \equiv \triangle DEF$ (ASA)

By calculating the missing angle size in each triangle we can see that two angles and the side joining them in $\triangle ABC$ are equal to the two matching angles and side in $\triangle DEF$.

Example 34

Find if this pair of triangles are congruent. Give your reasons.



Triangle A Triangle B

Working

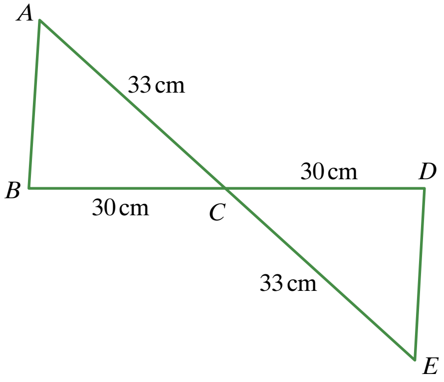
Both triangles are right-angled.
The hypotenuse is the same length in each triangle.
Both triangles have another pair of matching sides equal.
The triangles are congruent. (RHS)

Reasoning

Two right-angled triangles are congruent if the hypotenuse and another side of one triangle are equal to the hypotenuse and another side of the other triangle.

Example 35

Find if this pair of triangles are congruent. Give your reasons.



Working

$BC = DC$
$AC = EC$

$\angle ACB = \angle ECD$ (vertically opposite angles)
Two sides and the included angle of $\triangle ABC$ are equal to the matching two sides and the included angle of $\triangle EDC$.
$\triangle ABC \equiv \triangle EDC$ (SAS)

Reasoning

Two matching sides of each triangle are given. The included angles are equal because they are vertically opposite angles.

When we name the congruent triangles, we put the letters in matching order.

Showing that two triangles are congruent enables us to show many properties in geometry to be true.

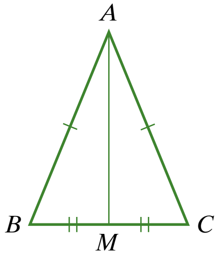
An isosceles triangle is defined as a triangle that has two equal sides. We know that isosceles triangles also have two equal angles, but this is not part of the definition of an isosceles triangle. Example 36 shows how we can use congruent triangles to prove that the base angles of the isosceles triangle must be equal.

Example 36

In isosceles $\triangle ABC$, $AB = AC$ and M is the midpoint of BC.

a Explain (with reasons) why $\triangle AMB \equiv \triangle AMC$.

b Explain why this tells us that $\angle ABC = \angle ACB$.



Working

a In $\triangle AMB$ and $\triangle AMC$
$AB = AC$ (definition of isosceles triangle)
$BM = CM$ (M is midpoint of BC)
AM is common to both triangles.
$\triangle AMB \equiv \triangle AMC$ (SSS)

b If the triangles are congruent, their matching angles must be the same.
So, $\angle ABC = \angle ACB$

Reasoning

Two triangles are congruent if three sides of one triangle are equal to three sides of the other triangle.

exercise 7.4

LINKS TO Example 25

1 For which of these sets of lengths is it possible to construct a triangle? Explain.

a 8 cm, 4 cm, 7 cm
b 3 cm, 8 cm, 12 cm
c 5 cm, 8 cm, 6 cm
d 6 cm, 7 cm, 13 cm
e 5 cm, 6 cm, 7 cm
f 6.5 cm, 3.4 cm, 2.9 cm
g 5.6 cm, 2.9 cm, 4.3 cm
h 4.8 cm, 2.4 cm, 2.4 cm
i 17 cm, 23.5 cm, 45 cm

In each of the questions 2–6, compare your triangles with those of other students in the class. Are your triangles the same shape and size?

LINKS TO Example 26

2 Use ruler, pencil and compass to construct each of these triangles. Label the vertices A, B and C and label the side lengths.

a $AB = 7$ cm, $AC = 6$ cm, $BC = 3$ cm
b $AB = 8$ cm, $AC = 6$ cm, $BC = 4$ cm

LINKS TO Example 27

3 Use ruler, pencil and protractor to construct each of these triangles. Label the vertices A, B and C and label the angle sizes.

a 40° at vertex A, 60° at vertex B, 80° at vertex C
b 55° at vertex A, 35° at vertex B, 90° at vertex C

LINKS TO Example 28

Use ruler, pencil and protractor to construct each of these triangles. Label the vertices A, B and C and label the length of AB and the angle sizes.

a $AB = 7\,\text{cm}$, 40° at vertex A, 30° at vertex B

b $AB = 6\,\text{cm}$, 25° at vertex A, 45° at vertex B

LINKS TO Example 29

Use interactive geometry software such as GeoGebra to construct each of these triangles.

a $\triangle ABC$ in which $AB = 6$ cm, $BC = 7$ cm, $\angle ABC = 60°$

b $\triangle ABC$ in which $AB = 8$ cm, $BC = 6$ cm, $\angle ABC = 45°$

LINKS TO Example 30

Use interactive geometry software such as GeoGebra to construct each of these triangles.

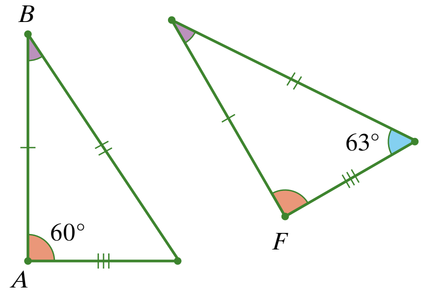
a $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 6$ cm and $BC = 10$ cm

b $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 5$ cm and $BC = 7$ cm

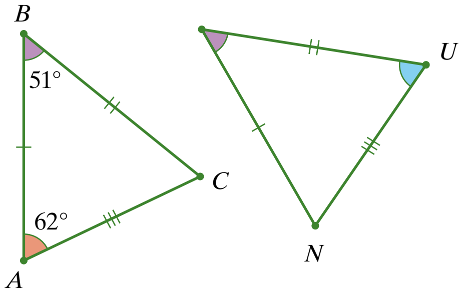
LINKS TO Examples 31, 32

Each of these pairs of triangles are congruent. Name the triangle that is congruent to $\triangle ABC$, making sure the order of letters is correct.

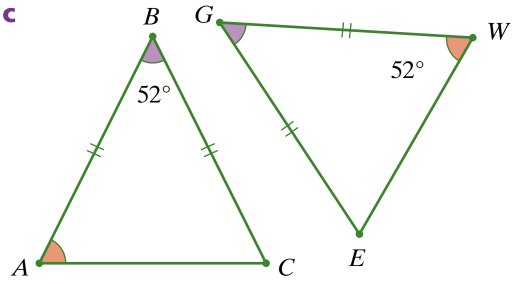
a



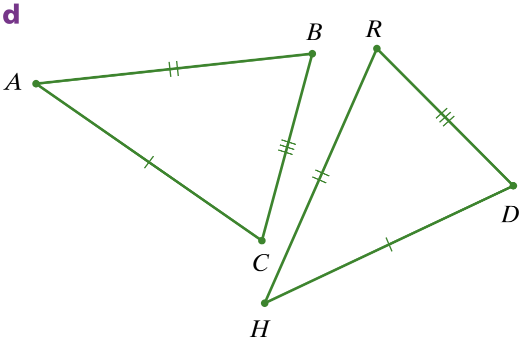
b



c



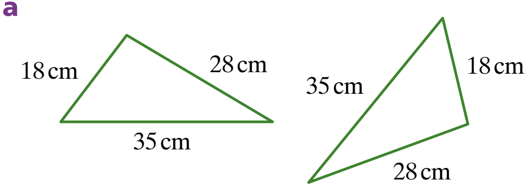
d



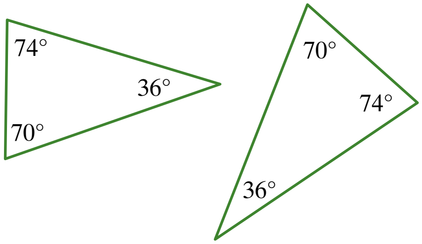
LINKS TO Examples 31–35

State whether the following pairs of triangles are congruent, giving your reasons.

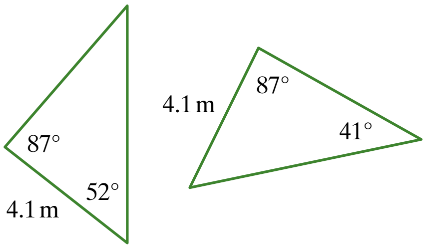
a



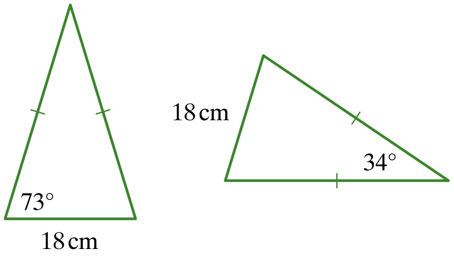
b



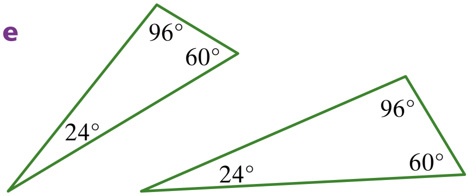
c


87°
52°
4.1 m
4.1 m
87°
41°

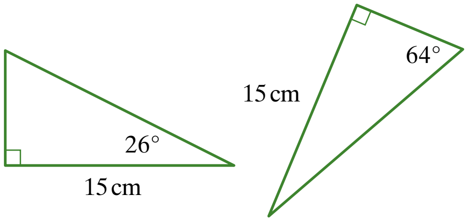
d


73°
18 cm
18 cm
34°

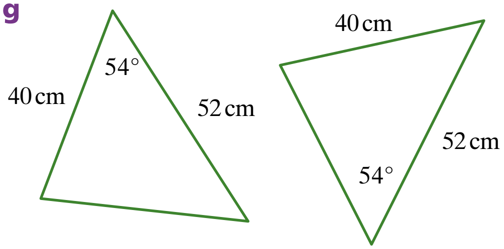
e


96°
60°
24°
96°
24°
60°

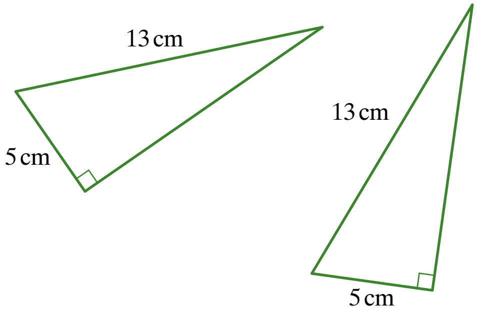
f


26°
15 cm
15 cm
64°

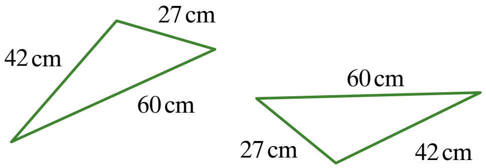
g


40 cm
54°
52 cm
40 cm
54°
52 cm

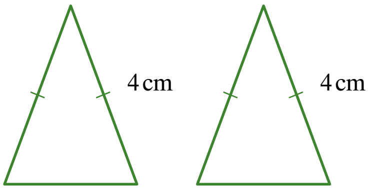
h


13 cm
5 cm
13 cm
5 cm

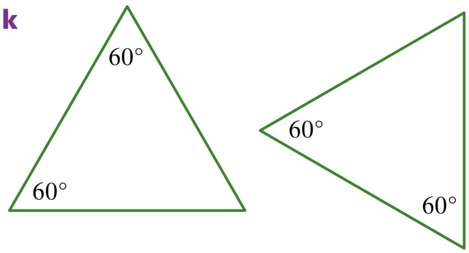
i


42 cm
27 cm
60 cm
60 cm
27 cm
42 cm

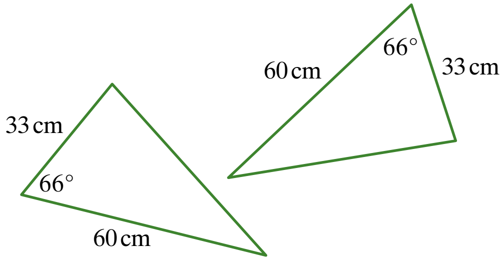
j


4 cm
4 cm

k

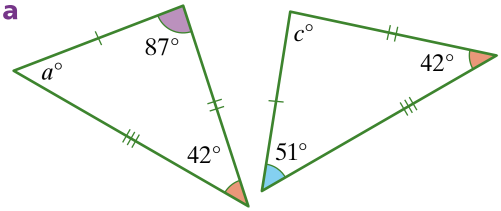

60°
60°
60°
60°

l

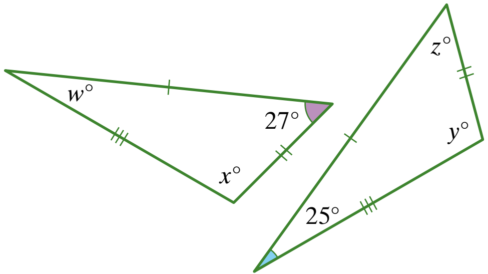

33 cm
66°
60 cm
60 cm
66°
33 cm

Find the values of the pronumerals in these pairs of congruent triangles.

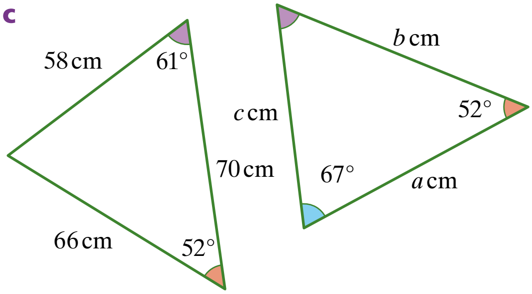
a



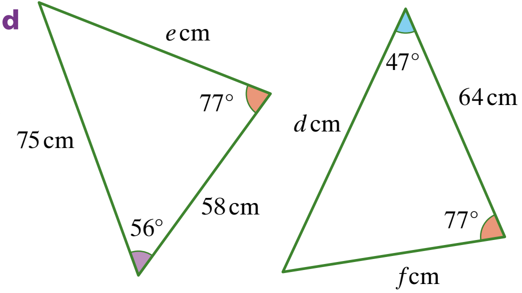
b



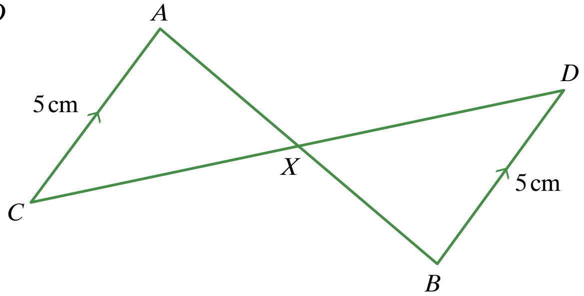
c



d

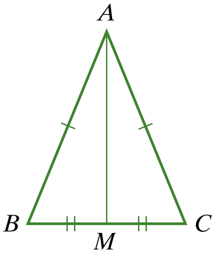


Show that triangles $\triangle AXC$ and $\triangle BXD$ are congruent.



LINKS TO Example 36

In example 36, it was shown that $\triangle AMB \equiv \triangle AMC$. Show that the line segment from the vertex A of an isosceles triangle to the midpoint M of the base is perpendicular to the base.



Complete the following.

- **a** Construct $\triangle PQR$ where $\angle PQR$ is 40°, $PR = 6$ cm and $QR = 8$ cm.
- **b** Construct $\triangle KLM$ where $\angle KLM$ is 40°, $KL = 6$ cm and $LM = 8$ cm.
- **c** Are triangles PQR and KLM congruent?

exercise 7.4 challenge

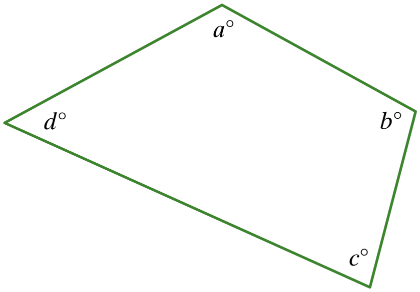
● Andy and Josh are working with a set of plastic geometry strips with the following lengths: 7.5 cm, 11 cm, 15 cm and 22 cm. There are three strips of each length. Triangles can be made by joining the strips together with paper fasteners. For example, a triangle can be made by joining 11 cm, 11 cm and 15 cm strips. What is the total number of possible triangles that Andy and Josh could make? Assume that they pull each triangle apart and reuse the strips. Hint: make a systematic list.

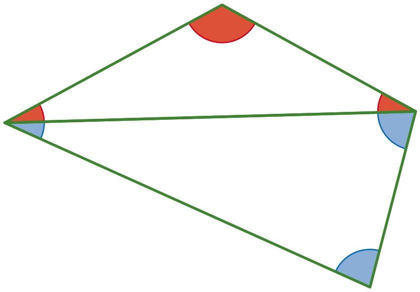
7.5 Quadrilaterals and their properties

Angle sum of a quadrilateral

A quadrilateral is a plane figure with four straight sides.

The sum of the four angles of any quadrilateral is 360°.





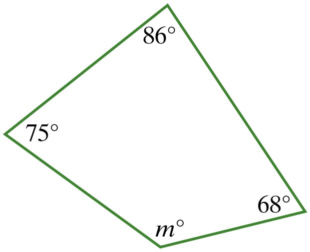
$$a + b + c + d = 360$$

We can easily prove this by dividing the quadrilateral into two triangles. The three angles of each triangle have a sum of 180°. (The three angles shown in red add to 180° and the three angles shown in blue add to 180°.) This makes a total of 360° for the angles of the quadrilateral.

We can use the property that the angles of a quadrilateral add to 360° to calculate unknown angles in quadrilaterals.

Example 37

Find the value of the pronumerals in this quadrilateral.



Working

$$\begin{aligned} m + 75 + 86 + 68 &= 360 \\ m + 229 &= 360 \\ m &= 360 - 229 \\ m &= 131 \end{aligned}$$

Reasoning

The four angles of a quadrilateral add to 360°. Write an equation and solve it.

Special quadrilaterals

Some quadrilaterals have special properties.

Squares, rhombuses and rectangles are all special parallelograms because they all share the properties of parallelograms, that is, both pairs of opposite sides are parallel.

A square is a special rhombus that has right angles.

A square is also a special rectangle that has four equal sides.

We can use the properties of special quadrilaterals to identify their type.

Example 38

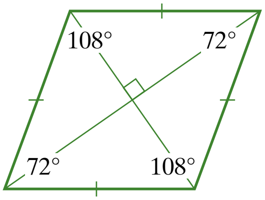
Identify each of these quadrilaterals. Draw a diagram for each to show the given information.

a Both pairs of opposite sides of the quadrilateral are parallel. The four angles of the quadrilateral are 72°, 108°, 72°, 108°. The diagonals intersect at right angles.

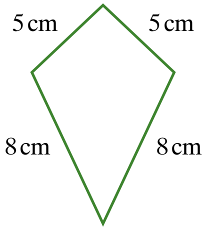
b The quadrilateral has one pair of adjacent sides with length 8 cm and the other pair with length 5 cm. There are no parallel sides.

Working

a The quadrilateral is a rhombus.



b The quadrilateral is a kite.



Reasoning

The first sentence tells us that the quadrilateral belongs to the family of parallelograms.

The second sentence tells us that it is not a square or a rectangle.

The diagonals of squares, rhombuses and kites intersect at right angles.

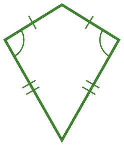
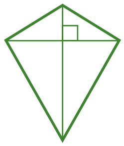
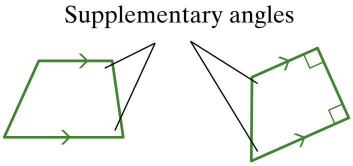
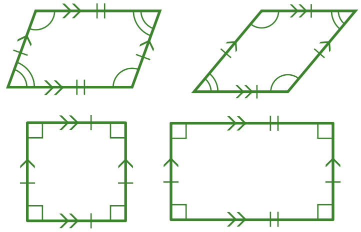
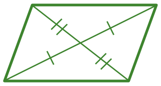
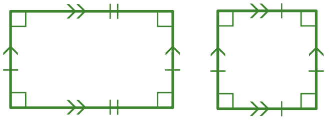
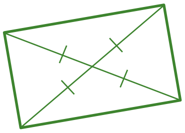
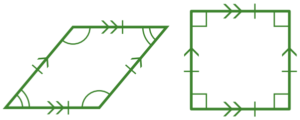
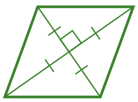
The quadrilateral must be a rhombus as we know it is not a kite or a square.

The quadrilateral is not a parallelogram or a trapezium. Kites have two pairs of adjacent sides equal.

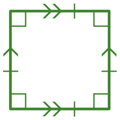
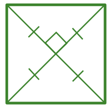
We can use the properties of special quadrilaterals to find unknown angles.

Properties of Special quadrilaterals

Parallelogram family

Name	Properties	Examples
Kite	■ Two pairs of adjacent (next to each other) sides are equal. ■ One pair of opposite angles are equal. ■ Diagonals are perpendicular. ■ One diagonal bisects the unequal angles.	 
Trapezium	■ One pair of opposite sides are parallel. ■ Two pairs of angles are supplementary.	
Parallelogram	■ Both pairs of opposite sides are parallel. ■ Both pairs of opposite sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other.	 
Rectangle	■ Both pairs of opposite sides are parallel and equal. ■ All angles are right angles. ■ Diagonals are equal in length. ■ Diagonals bisect each other.	 
Rhombus	■ Both pairs of opposite sides are parallel. ■ All four sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other. ■ Diagonals are perpendicular. ■ Diagonals bisect the angles of the rhombus.	 

continued

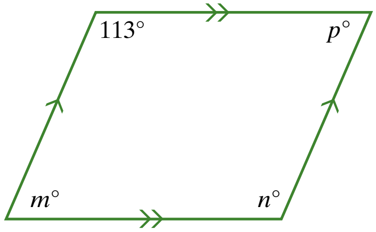
Name	**Properties**	**Examples**
Square	- Both pairs of opposite sides are parallel. - All four sides are equal. - All angles are right angles. - Diagonals are equal in length. - Diagonals are perpendicular. - Diagonals bisect each other. - Diagonals bisect the right angles of the square.	  A square is a special rectangle and a special rhombus.

Interior angles of quadrilaterals

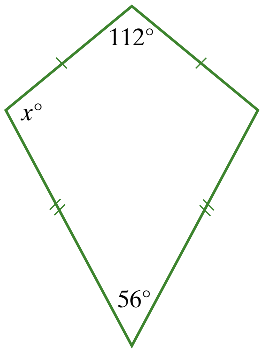
Example 39

Find the value of the pronumerals in these quadrilaterals.

a



b



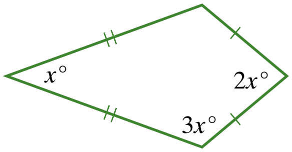
Working	**Reasoning**
a $m = 180 - 113$ $m = 67$	Adjacent angles of a parallelogram are supplementary.
$n = 113$	Opposite angles of a parallelogram are equal.
$p = 67$	Opposite angles of a parallelogram are equal.
b $360 - (112 + 56)$ $= 360 - 168$ $= 192$ $x = \frac{1}{2} \text{ of } 192$ $x = 96$	The quadrilateral is a kite. The other unknown angle is also x°. Add the two given angles and subtract from 360°. Halve the result.

Example 40

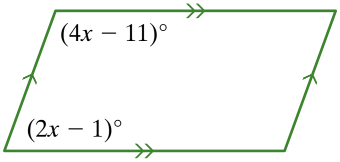
For each of the following figures

i write an equation.
ii find the value of x.
iii find the sizes of the four angles.

a



b



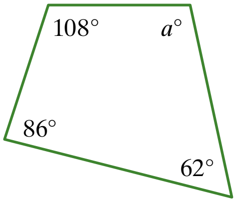
Working	**Reasoning**
a i $3x + 3x + 2x + x = 360$ **ii** $9x = 360$ $x = 40$	The figure is a kite, so the angle opposite the angle marked $3x°$ is also $3x°$. The four angles add to 360°.
iii The four angles are 40°, 80°, 120°, 120°.	Substitute $x = 40$ into the expression for each angle.
b i $2x - 1 + 4x - 11 = 180$ **ii** $6x - 12 = 180$ $6x = 192$ $x = 32$	Adjacent angles in a parallelogram are supplementary.
iii The four angles are 63°, 63°, 117°, 117°.	Substitute $x = 32$ into the expressions $2x - 1$ and $4x - 11$.

exercise 7.5

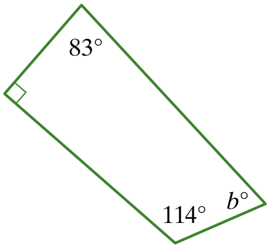
LINKS TO Example 37

Find the value of each of the pronumerals in the following quadrilaterals.

a



b



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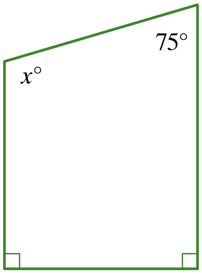
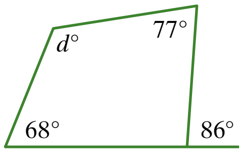
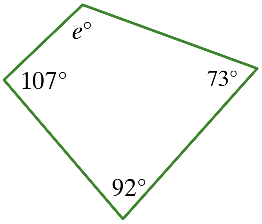


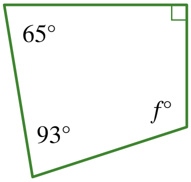
d



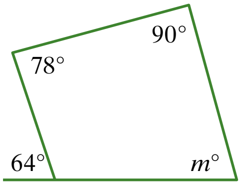
e



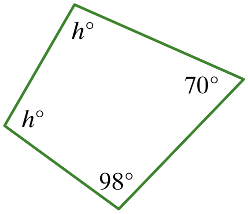
f



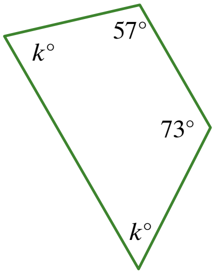
g



h



i



In each of the following, three angles of a quadrilateral are given. Calculate the size of the fourth angle for each.

a 73°, 115°, 100° **b** 127°, 53°, 90° **c** 29°, 143°, 97° **d** 156°, 37°, 19°

e 48°, 67°, 140° **f** 110°, 38°, 105° **g** 150°, 75°, 90° **h** 108°, 72°, 72°

Parallelogram diagonals

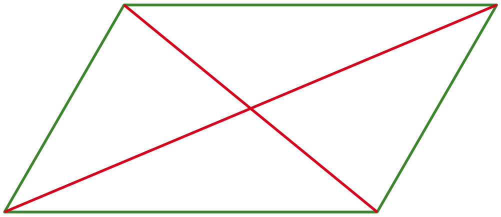
Investigate the diagonals of the six special quadrilaterals shown in the diagrams on the next page. Record your answers to the following questions (Yes or No) in a table as shown.

a For which quadrilaterals are the two diagonals equal in length?

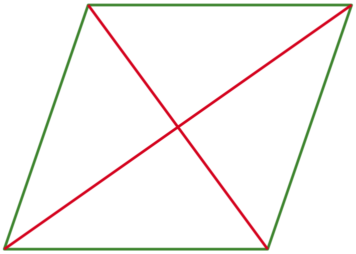
b For which quadrilaterals do the two diagonals bisect each other?

c For which quadrilaterals do the two diagonals intersect at right angles?

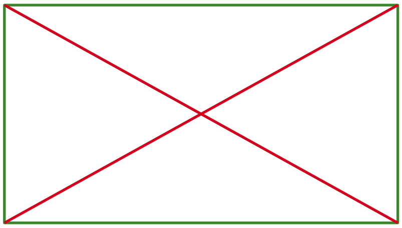
Special quadrilateral	**Diagonals equal in length?**	**Diagonals bisect each other?**	**Diagonals intersect at right angles?**
Parallelogram			
Rhombus			
Rectangle			
Square			
Trapezium			
Kite			

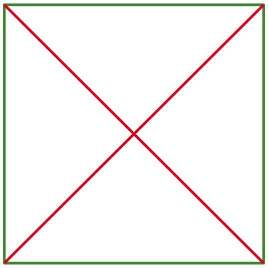


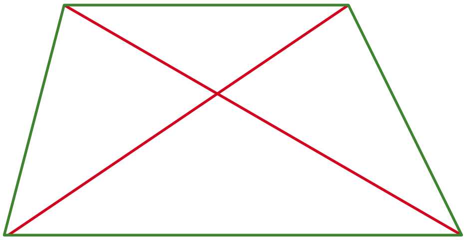
Parallelogram



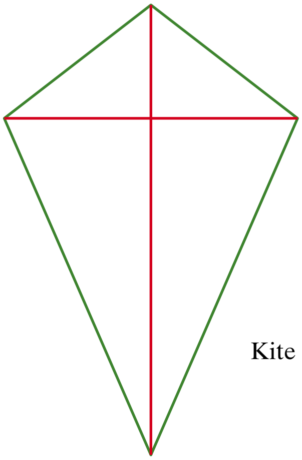
Rhombus







Trapezium



Kite

List the special quadrilaterals that match the following descriptions.

- **a** Both pairs of opposite angles are equal.
- **b** One pair of opposite angles is equal.
- **c** All angles are right angles.

Copy and complete the following table.

a For each property, list all the special quadrilaterals with that property.

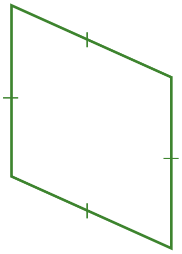
Property	**Special quadrilaterals**
Both pairs of opposite sides equal	
All sides equal	
Two pairs of adjacent sides equal but opposite sides not equal	
Two pairs of parallel sides	
One pair of parallel sides	

b Which special quadrilaterals fit the definition of a parallelogram and can therefore be regarded as belonging to the family of parallelograms?

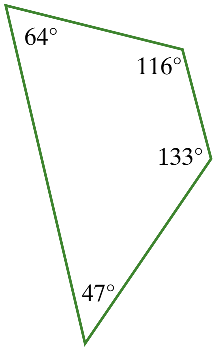
LINKS TO Example 38

Identify each of these quadrilaterals, justifying your answer.

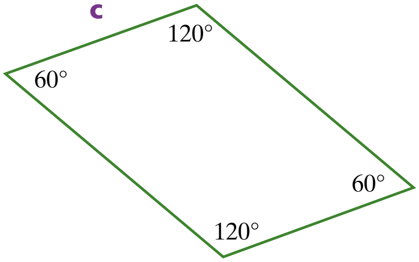
a



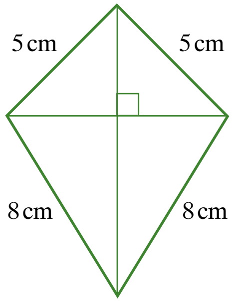
b



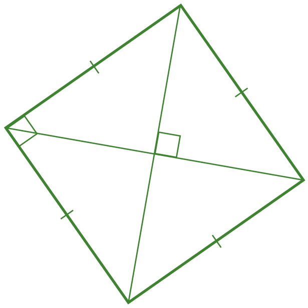
c



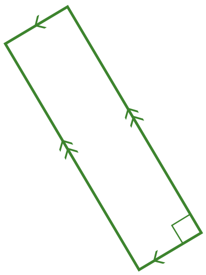
d



e



f



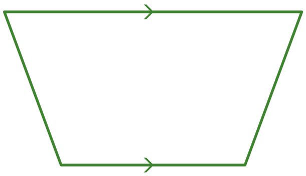
A quadrilateral has both pairs of opposite sides parallel. The diagonals are equal in length and intersect at right angles. The quadrilateral is a

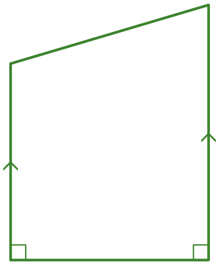
A rhombus **B** square **C** kite **D** rectangle **E** parallelogram

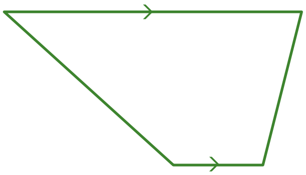
A quadrilateral has two pairs of equal sides. The diagonals are different lengths and intersect at right angles. One of the diagonals is bisected by the other. The quadrilateral is a

A trapezium **B** rectangle **C** parallelogram **D** kite **E** rhombus

a To which special group of quadrilaterals do each of these quadrilaterals belong?





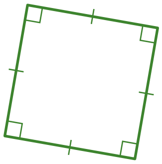


b What do they all have in common?

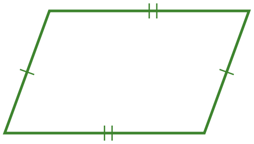
For each of the following figures,

- **i** list the properties marked on the figure.
- **ii** identify the shape.

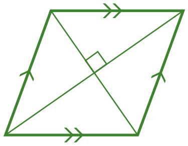
a



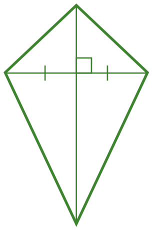
b



c



d



Identify each of the following quadrilaterals and make a drawing of each to show the properties described.

a All sides are equal and the diagonals are of different lengths but intersect at right angles.

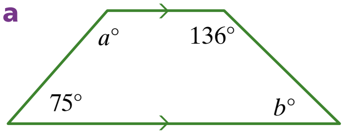
b Both pairs of opposite sides are parallel and the diagonals are equal in length and bisect each other but do not intersect at right angles.

c The sides are all of different lengths and two pairs of adjacent angles are supplementary.

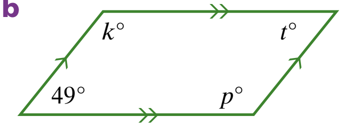
LINKS TO Example 39

Find the value of each of the pronumerals. You may find it easier to copy each diagram and label the size of each angle as you find it.

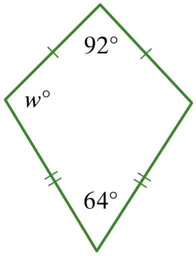
a



b



c



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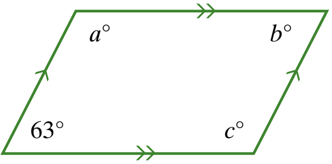

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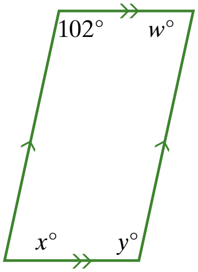

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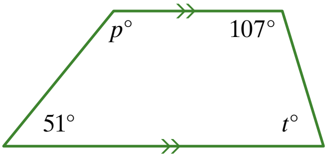
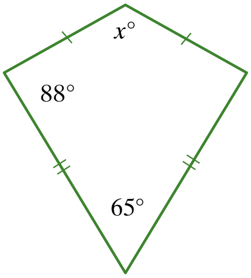


g



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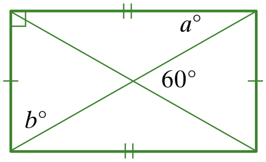
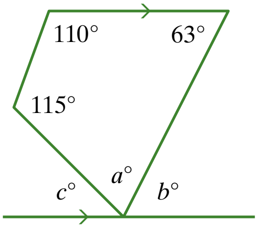


i



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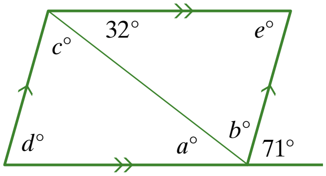
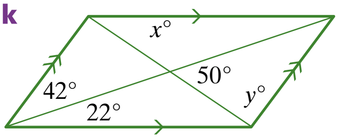
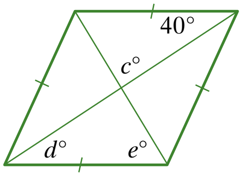


k



l



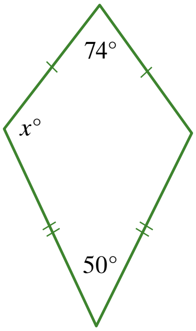
LINKS TO
Example 39

A quadrilateral has angles 115° and 47° with the other two angles equal to each other.

- **a** What is the size of each of these angles? Explain how you worked out your answer.
- **b** In a quadrilateral one angle is 57° and the other three angles are equal to each other. What is the size of each of these angles? Explain how you worked out your answer.

The value of x, in the diagram on the right, is

A 56 **B** 112 **C** 118 **D** 124 **E** 236



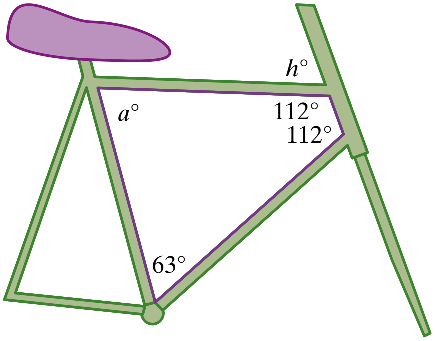
A quadrilateral has both pairs of opposite sides parallel and has four right angles. The quadrilateral is

- **A** a rectangle.
- **B** a square.
- **C** a rhombus.
- **D** a rectangle or a square.
- **E** a rectangle, a square or a rhombus.

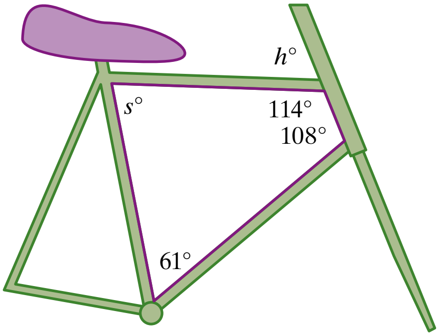
The diagonals of a quadrilateral are equal in length and intersect at right angles. Which of the special quadrilaterals (square, rectangle, rhombus, kite or trapezium) could the quadrilateral be? Is there more than one possibility? Justify your answer.

Part of the frame of the bicycle shown below forms a quadrilateral. The angle, $h°$, which the head tube makes with the horizontal is called the *head angle*. The angle, $s°$, which the seat tube makes with the horizontal is called the *seat angle*. Slight changes in these angles are important. Triathlon bikes generally have steeper seat angles than road bikes. Calculate h and s for the two bicycle frames shown below.

a Road bicycle



b Triathlon bicycle

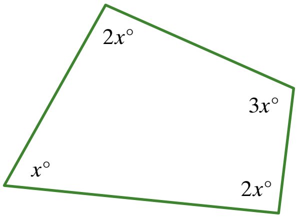


LINKS TO Example 40

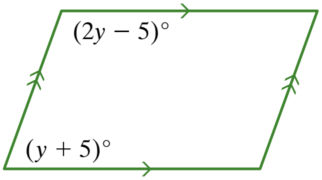
For each of the following

- **i** write an equation.
- **ii** solve the equation for x.
- **iii** find the size of each of the angles.

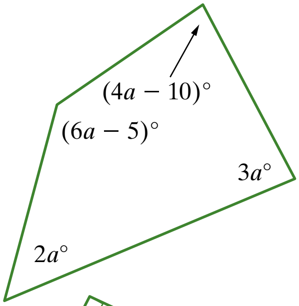
a



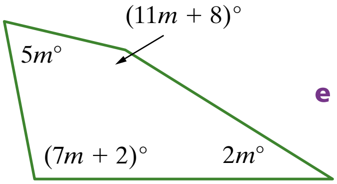
b



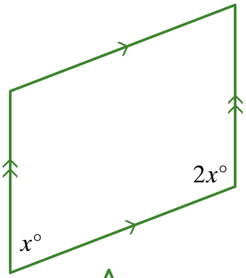
c



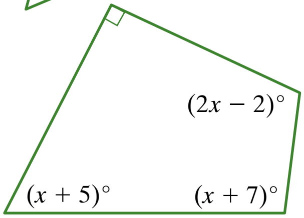
d



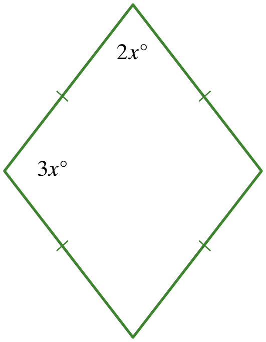
e



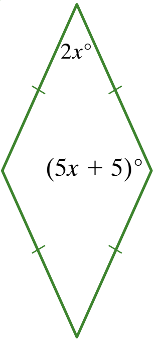
f



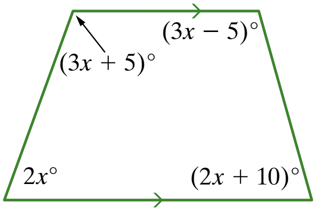
g



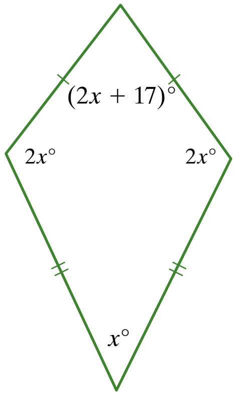
h



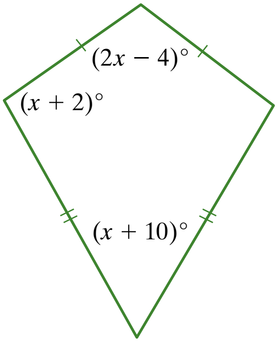
i



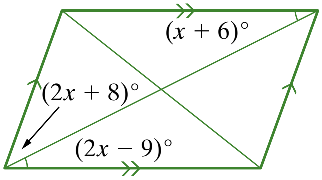
j



k

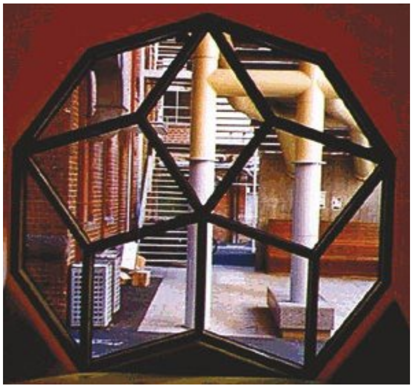


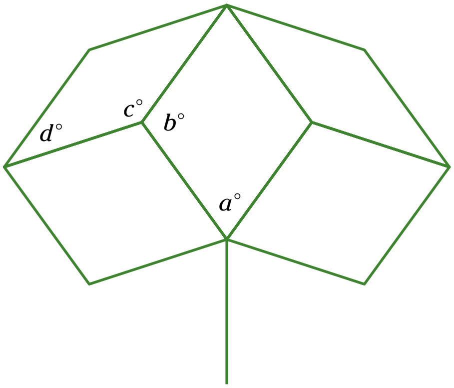
l



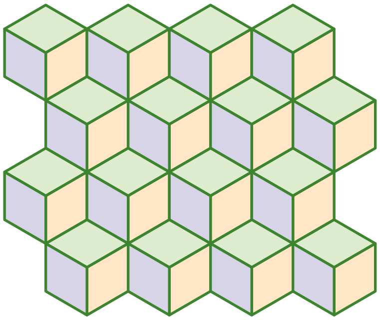
exercise 7.5 challenge

- There are two different shaped rhombuses in the window below, one with angles labelled $a°$ and $b°$ and the other with angles $c°$ and $d°$. By looking at the way the rhombuses fit together at the centre, calculate the sizes of the angles in each of the two rhombuses.





- In the patchwork quilt design shown here, three congruent rhombuses combine to form each hexagon.
  - **a** By looking at the way the rhombuses fit together, calculate the sizes of the angles in each rhombus.
  - **b** Make a careful, accurate drawing of one of the rhombuses. Label the angle sizes.



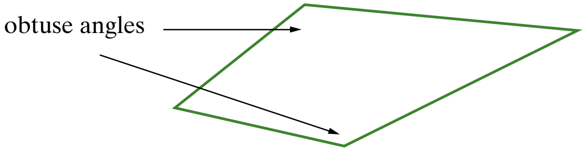
7.6 Using congruent triangles to explain quadrilateral properties

How do we know if a mathematical statement is true or false?

- **Definitions**

  Some statements are true 'by definition'. For example, both pairs of opposite sides of a parallelogram are parallel. This is true because this is the definition of a parallelogram.
- **Counterexamples**

  If we can find one example of a statement which is not true, then the statement is false. The example is called a counterexample. If a student claimed that a quadrilateral cannot have two obtuse angles, we could draw an example of a quadrilateral with two obtuse angles to show that the student's claim was wrong.

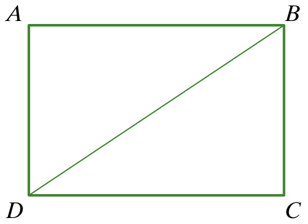
  

  Although finding a counterexample can show that a statement is false, we cannot claim that a statement is true just because we haven't found any counterexamples.
- **Deductive reasoning**

  We start with something we already know is true. We then develop a logical sequence of statements, using evidence to support our reasoning. The evidence we use must be based on definitions or statements that have already been shown to be true—either by mathematicians in the past or by ourselves. This type of reasoning is called deductive reasoning.

Example 41

Show that the diagonal BD of rectangle $ABCD$ divides the rectangle into two congruent triangles.



continued

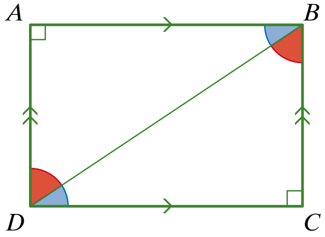
Example 41 continued

Working

In $\triangle ABD$ and $\triangle CDB$,
1. $\angle ABD = \angle CDB$ (alternate angles, $AB \parallel DC$)
2. $\angle ADB = \angle CBD$ (alternate angles, $AB \parallel DC$)
3. BD is shared by both triangles.

So $\triangle ABD \equiv \triangle CDB$ (ASA)

Reasoning



Recall from Year 7 that || is the symbol for 'is parallel to'.

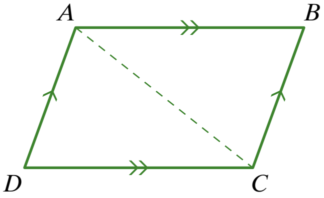
We define a parallelogram as 'a quadrilateral with opposite sides parallel'.

We can use this definition to show other properties of a parallelogram.

So, the diagonal BD of rectangle $ABCD$ divides the rectangle into two congruent triangles.

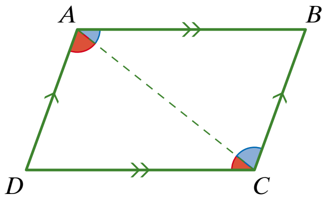
Example 42

In this figure, $ABCD$ is a parallelogram and AC is a diagonal. From the definition of a parallelogram, we know that $AB \parallel DC$ and $AD \parallel BC$. Using this information and your knowledge of angles associated with parallel lines and transversals, show that $\triangle ABC$ is congruent to $\triangle CDA$. Justify each statement that you make.



Working

In $\triangle ABC$ and $\triangle CDA$,
1. AC is common to both triangles.
2. $\angle BCA = \angle DAC$ (alternate angles, $AD \parallel BC$)
3. $\angle CAB = \angle ACD$ (alternate angles, $AB \parallel DC$)



So, $\triangle ABC \equiv \triangle CDA$ (ASA)
Hence, we have shown that $\triangle ABC$ is congruent to $\triangle CDA$.

Reasoning

Look for clues: $\triangle ABC$ and $\triangle CDA$ share the side AC.

Diagonal AC is a transversal cutting across the parallel sides. This suggests that we could find equal angles in $\triangle ABC$ and $\triangle CDA$.

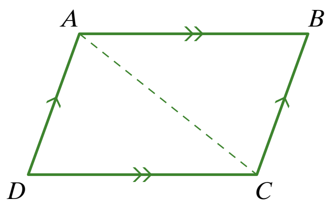
Two triangles are congruent if two angles of one triangle are equal to two angles of the other triangles and the sides joining the two equal angles are equal.

Parallelogram sides

Example 43

Using what was shown in example 42, show that both pairs of opposite sides of a parallelogram are equal.

Working



$\triangle ABC \equiv \triangle CDA$ (shown in example 42)

So,

$AB = CD$

$BC = DA$

So both pairs of opposite sides of a parallelogram are equal.

Reasoning

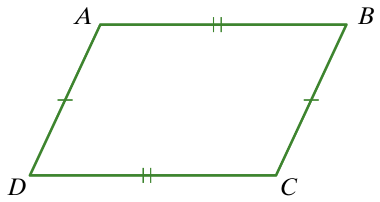
Matching sides of congruent triangles are equal.

We know that a quadrilateral with both pairs of opposite sides parallel is a parallelogram because this is the definition of a parallelogram. We have also shown that the opposite sides of a parallelogram are equal. However, can we assume that the converse is true? In other words, is a quadrilateral with both pairs of opposite sides equal necessarily a parallelogram?

Example 44 shows that a quadrilateral with both pairs of opposite sides equal is a parallelogram.

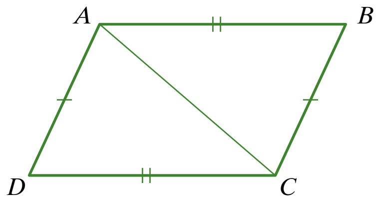
Example 44

Show that quadrilateral $ABCD$ is a parallelogram.



Working

Draw the diagonal AC.



Reasoning

We want to show that $\triangle ABC$ is congruent to $\triangle CDA$. Then we can see which angles are equal. If alternate angles are equal then opposite sides are parallel and $ABCD$ is a parallelogram.

continued

Example 44 continued

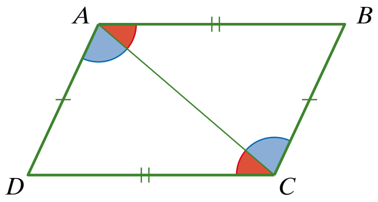
Working

In $\triangle ABC$ and $\triangle CDA$:
$BC = DA$ (given)
$AB = CD$ (given)
AC is common to both triangles.
So, $\triangle ABC \equiv \triangle CDA$ (SSS)

So, $\angle BCA = \angle DAC$ (matching angles in congruent triangles).
But $\angle BCA$ and $\angle DAC$ are alternate angles.
So, $AD \parallel CB$
Similarly, $\angle BAC = \angle DCA$
But $\angle BAC$ and $\angle DCA$ are alternate angles.
So, $AB \parallel CD$
So, quadrilateral $ABCD$ is a parallelogram.

Reasoning

We could have drawn the diagonal BD instead and worked with $\triangle AB$ and $\triangle CDB$.

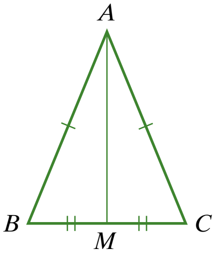


So, opposite sides are parallel and quadrilateral $ABCD$ is a parallelogram.

exercise 7.6

LINKS TO
Example 41

In example 36 it was shown that $\triangle AMB \equiv \triangle AMC$. Use this to show that $AM \perp BC$.

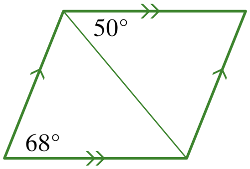


Recall from Year 7 that $\perp$ is the symbol for 'is perpendicular to'.

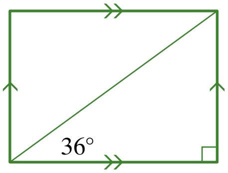
These special quadrilaterals have all been divided into two triangles by a diagonal.

a Use your knowledge of parallel lines and special quadrilaterals to write the angle sizes in each triangle. Use symbols to mark the equal sides.

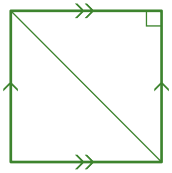
i Parallelogram



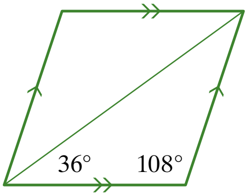
ii Rectangle



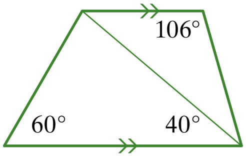
iii Square



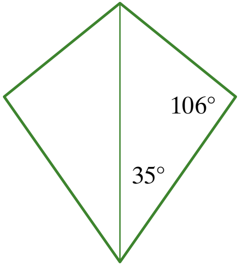
iv Rhombus



v Trapezium



vi Kite



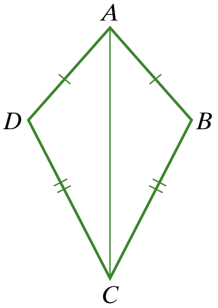
b In which of the quadrilaterals are two congruent triangles formed?

LINKS TO Examples 42, 43, 44

ABCD is a kite.

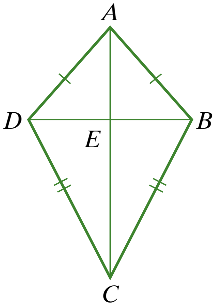
a Show that $\triangle ADC \equiv \triangle ABC$

b Hence show that $\angle ADC = \angle ABC$

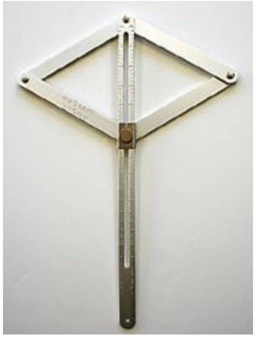


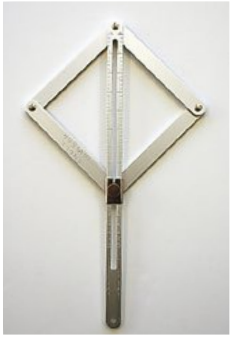
Kite diagonals

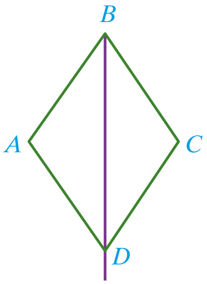
Once we have shown that $\triangle ADC \equiv \triangle ABC$ in question 3, we can show that other properties of the kite are true. Show that the diagonal *AC* bisects $\angle DAB$ and $\angle DCB$.



The tool shown below is an angle bisector. It consists of a rhombus hinged at the four vertices. One vertex of the rhombus can slide along the vertical slotted bar. As it does so, the length of *BD* changes.







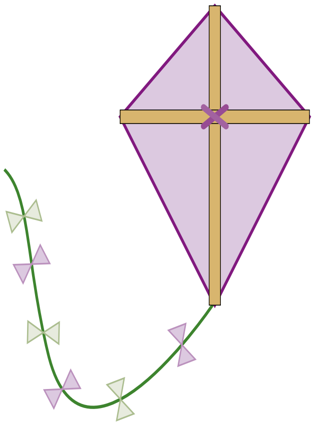
a What name do we give to *BD* in the rhombus?

b What can you say about $\triangle ABD$ and $\triangle CBD$? Explain why.

c What does this tell you about $\angle ABD$ and $\angle CBD$?

d Explain why the tool is called an angle bisector.

e Is there any other special quadrilateral that would work in the same way?



Kenji made the frame of a kite by tying two sticks together as shown at right. What properties of the mathematical kite was Kenji making use of in constructing the frame for his kite from the two sticks?

Car jack

The photograph below shows a common car jack for lifting a car to change a wheel. The top of the jack is placed underneath the wheel axle and the horizontal screw is turned to raise or lower the jack.

a On which special quadrilateral shape is the jack based?

b Which property of this quadrilateral shape ensures that the car goes vertically upwards as the screw is turned?

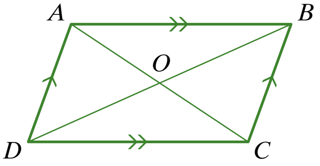
The photograph shows temporary barriers at a railway station.

a On which special quadrilateral is the design of these barriers based?

b Which properties of this special quadrilateral make it particularly suitable for using in the barriers?

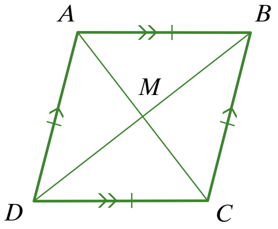
exercise 7.6 challenge

- Congruent triangles can be used to show that the diagonals of a parallelogram bisect each other.
  - **a** Show that $\triangle AOB$ and $\triangle COD$ are congruent.
  - **b** Hence show that $AO = OC$ and $BO = OD$
  - **c** What can you now say about the two diagonals?



Rhombus diagonals

- Congruent triangles can be used to show that the diagonals of a rhombus intersect at right angles. Using the property you have shown in question 9,
  - **a** Show that $\triangle ABM \equiv \triangle CDM \equiv \triangle ADM \equiv \triangle CBM$.
  - **b** Show that $\angle AMB = \angle CMD = \angle CMB = \angle AMD$ and hence show that the four angles are right angles.



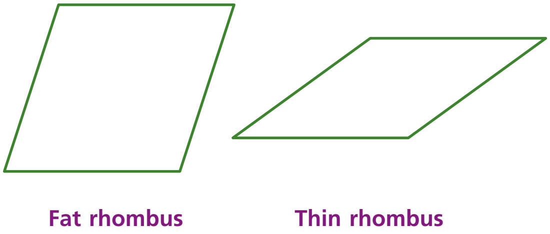
Analysis task

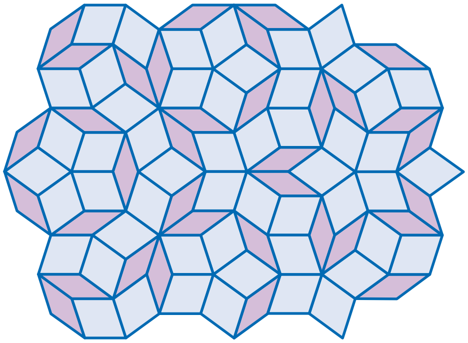
Penrose tiles

Most tessellations form a repeating pattern. This means it is possible to copy part of the tessellation and slide (translate) the copy (without rotation) to match up exactly with another region of the tessellation.

Certain tile shapes, however, can tessellate in such a way that there is no repeating pattern. Sir Roger Penrose, a mathematician at the University of Oxford in England, was investigating shapes to see if he could find shapes that would form non-repeating patterns. In 1974 he realised that two simple rhombuses were able to tessellate for ever without ever repeating exactly the same arrangement. The two rhombuses became known as the fat and thin rhombuses and are also known now as Penrose tiles.

The way the tiles go on forever without repeating exactly the same pattern is like the decimal places of an irrational number, such as $\sqrt{5}$.





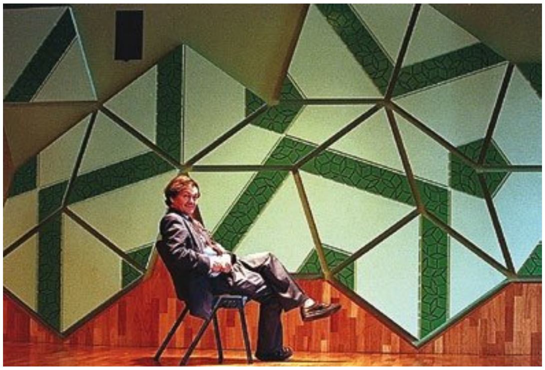
a By looking at the way the rhombuses fit together, work out the angle sizes of the fat and thin rhombuses.

b Using ruler and protractor or computer drawing tools, carefully construct a fat rhombus and a thin rhombus with sides 4 cm long and the angle sizes you have calculated.

Penrose tiles have been used in several paving and building designs in different places in the world. The photograph on the left side of the next page shows a pavement of Penrose rhombuses at the University of Oxford in England. The arcs on each rhombus serve the same purpose as the bumps and dents on the pieces of a jigsaw puzzle and must be matched when the rhombuses are fitted together. The fat and thin rhombuses can then fit together in only one way, resulting in the non-repeating tessellation. Without these bands, the rhombuses could, of course, be arranged to make a repeating tessellation.

c Make a drawing to show how the fat and thin rhombuses could make a repeating pattern.

In Australia, Penrose tiles have been used in the design of Storey Hall at RMIT University in Melbourne, and in the floor of the Chemistry building at the University of Western Australia in Perth. Sir Roger Penrose is shown in the photograph on the right on the next page when he visited Storey Hall in 2000. The green bands on the rhombuses serve the same purpose as the arcs on the paving stones: to ensure that a non-repeating tessellation is produced.



Penrose found that there were $\frac{1+\sqrt{5}}{2}$ times as many fat rhombuses as thin rhombuses.

The ancient Greeks called this number the *golden ratio* because they believed that a rectangle shape was most pleasing to the eye if the length was $\frac{1+\sqrt{5}}{2}$ the width.

d Use your calculator to find the approximate value of $\frac{1+\sqrt{5}}{2}$ correct to one decimal place.

BLM
Penrose tile template

e Using the Penrose tile templates provided in the student ebook, cut out about 32 fat rhombuses and 20 thin rhombuses. Explain why these numbers of each tile have been suggested.

f Fit the tiles together, making sure that the bands match as shown above. Paste your tessellation onto a sheet of A3 paper.

Review Congruency and quadrilateral properties

Summary

Angle properties

- Adjacent angles share a common vertex and a common arm.
- Complementary angles add to 90°.
- Supplementary angles add to 180°.
- Vertically opposite angles are equal.

Angle properties for parallel lines cut by a transversal

- Alternate angles are equal.
- Corresponding angles are equal.
- Co-interior (allied) angles are supplementary.

Triangle properties

- The three angles of any triangle add to 180°.
- An exterior angle of a triangle is equal to the sum of the two interior opposite angles.
- The base angles of an isosceles triangle are equal.

Congruency

Congruent figures must satisfy *both* of these conditions:

- matching angles must be equal
- matching sides must be equal in length.

Isometric transformations

- Isometric transformations produce congruent shapes.
    - reflection
    - rotation
    - translation

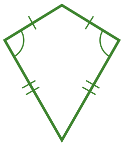
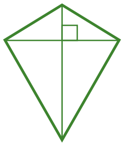
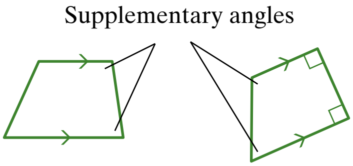
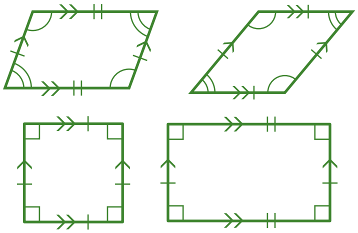
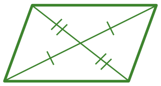
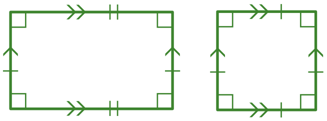
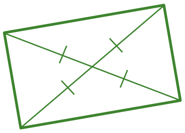
Congruent triangles

- SSS (three sides of one triangle equal to three sides of the other)
- SAS (two sides of one triangle equal to two sides of the other triangle and the angle between them the same)
- ASA (two angles of one triangle equal to the two angles of the other triangle and the side joining them equal)

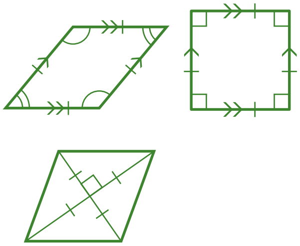
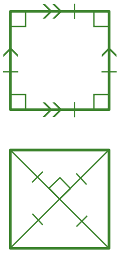
- RHS (both triangles right-angled, hypotenuse and another side of one triangle equal to the hypotenuse and another side of the other triangle)

Quadrilateral properties

The four angles of a quadrilateral add to 360°.

Name	Properties	Examples
Kite	■ Two pairs of adjacent (next to each other) sides are equal. ■ One pair of opposite angles are equal. ■ Diagonals are perpendicular. ■ One diagonal bisects the unequal angles.	 
Trapezium	■ One pair of opposite sides are parallel. ■ Two pairs of angles are supplementary.	
Parallelogram	■ Both pairs of opposite sides are parallel. ■ Both pairs of opposite sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other.	 
Rectangle	■ Both pairs of opposite sides are parallel and equal. ■ All angles are right angles. ■ Diagonals are equal in length. ■ Diagonals bisect each other.	 

continued

Name	Properties	Examples
Rhombus	■ Both pairs of opposite sides are parallel. ■ All four sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other. ■ Diagonals are perpendicular. ■ Diagonals bisect the angles of the rhombus.	
Square	■ Both pairs of opposite sides are parallel. ■ All four sides are equal. ■ All angles are right angles. ■ Diagonals are equal in length. ■ Diagonals are perpendicular. ■ Diagonals bisect each other. ■ Diagonals bisect the right angles of the square.	 A square is a special rectangle and a special rhombus.

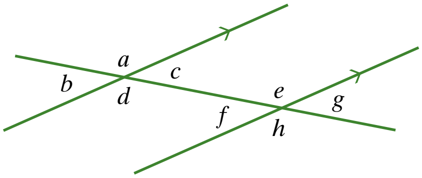
Visual map

alternate angles
co-interior angles
complementary angles
congruent
corresponding angles
diagonal
equilateral triangle
isometric transformation
kite
parallel
parallelogram
perpendicular
isosceles triangle
quadrilateral
rectangle
reflection
rhombus
rotation
scalene triangle
square
supplementary angles
transformation
translation
transversal
trapezium
vertically opposite angles

Revision

Multiple-choice questions

Use the following diagram for questions 1 and 2.



The angle that is alternate to angle c is

A angle a **B** angle b **C** angle d **D** angle e **E** angle f

The angle that is corresponding to angle h is

A angle b **B** angle d **C** angle e **D** angle f **E** angle g

If the angle at the vertex of an isosceles triangle is 46°, then each of the other two angles is

A 22° **B** 67° **C** 72° **D** 134° **E** 144°

The value of x is

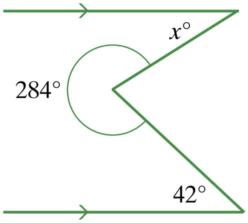
A 34

B 42

C 76

D 16

E 118



If triangle PQR is translated 4 units to the right and 3 units down, the coordinates of the image $P'Q'R'$ will be

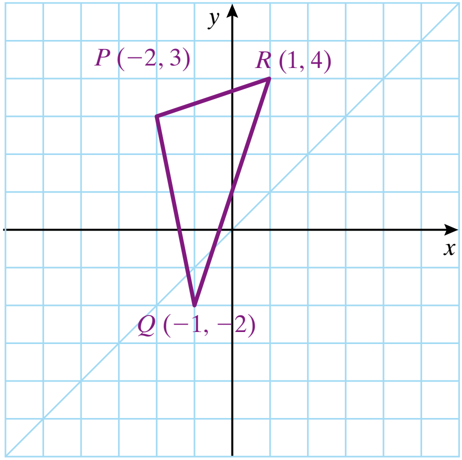
A $P'(-6, 0)$, $Q'(-5, -5)$, $R'(-3, 1)$

B $P'(-5, 7)$, $Q'(-4, 2)$, $R'(-2, 8)$

C $P'(2, 0)$, $Q'(3, -5)$, $R'(5, 1)$

D $P'(2, 6)$, $Q'(3, 1)$, $R'(5, 7)$

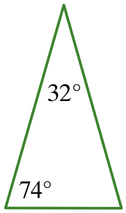
E $P'(-3, 2)$, $Q'(-2, 2)$, $R'(0, 3)$



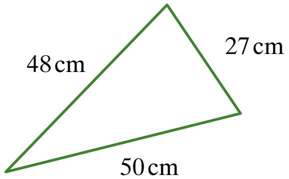
Short-answer questions

Identify each of these triangles according to their sides.

a

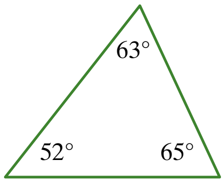


b

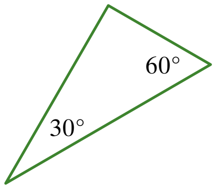


Identify each of these triangles according to their angles.

a

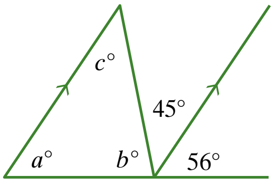


b

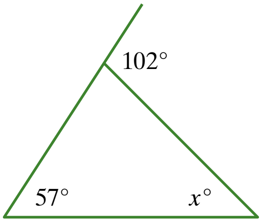


Find the value of the pronumerals.

a



b



Construct the following triangles, in each case labelling the given side lengths and angle sizes.

a Two sides are 4.5 cm and 6.0 cm and the angle between them is 60°.

b The three sides are 5 cm, 8 cm and 9 cm.

c Two angles are 40° and 70° and the side joining them is 5 cm.

Is it possible to construct triangles with the following side lengths? Explain.

a 5 cm, 8 cm, 13 cm **b** 7 cm, 12 cm, 13 cm **c** 8 cm, 9 cm, 20 cm

State whether each of the following statements is true or false.

a If the three angles of one triangle are exactly the same sizes as the three matching angles of another triangle, then the triangles must be congruent.

b If the three sides of one triangle are exactly the same lengths as the three matching sides of another triangle, then the triangles must be congruent.

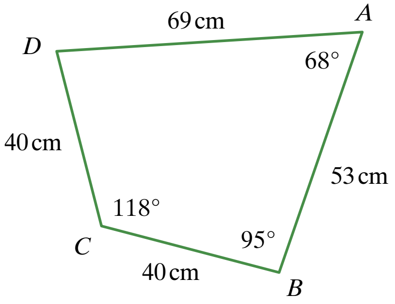
Quadrilateral 1 and Quadrilateral 2 are shown below.

a Name the matching pairs of sides in these two quadrilaterals.

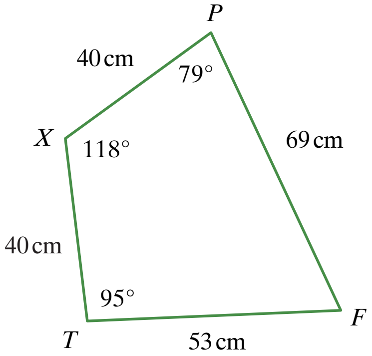
b Calculate the size of $\angle ADC$ and $\angle PFT$

c Are the quadrilaterals congruent?

d If Quadrilateral 1 is called $ABCD$, how do we name Quadrilateral 2?



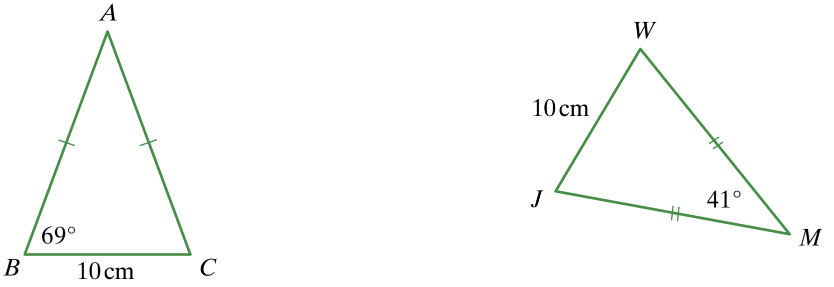
Quadrilateral 1



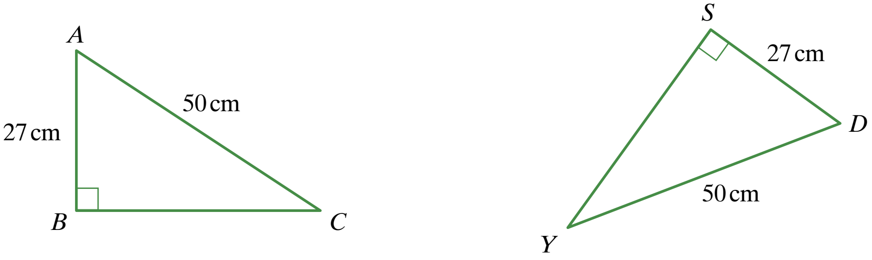
Quadrilateral 2

For each of these pairs of triangles, decide if the two triangles are congruent, justifying your answer in each case. For those pairs that you decide are congruent, name the triangles according to their matching vertices.

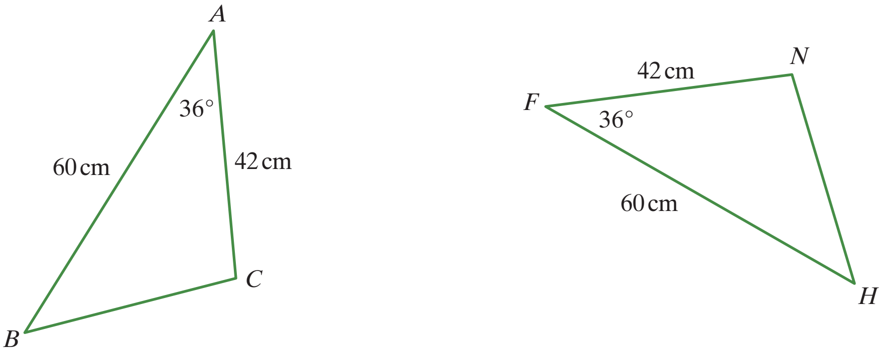
a



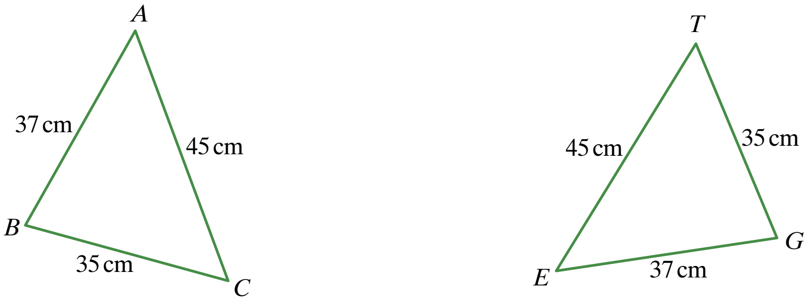
b



c

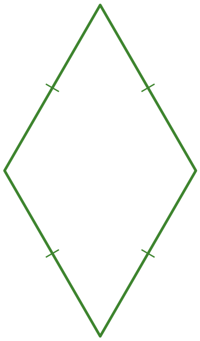


d

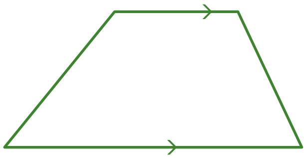


Identify these quadrilaterals.

a

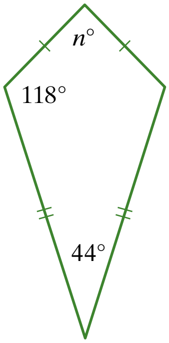


b

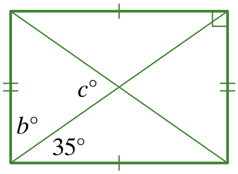


Find the value of the pronumerals.

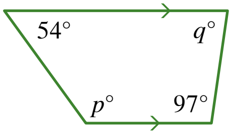
a



b



c



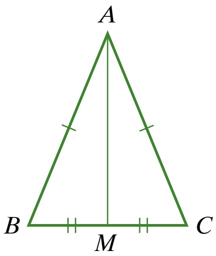
Practice quiz
Chapter 7

Extended-response questions

$\triangle ABC$ is an isosceles triangle and M is the midpoint of AB.

a Show that $\triangle AMB \equiv \triangle AMC$, giving reasons.

b Use what you have shown in part **a** to show that $AM \perp BC$, explaining your reasoning.



Ratios, rates and time

8

Pre-test

Warm-up

Ratios can be used to compare groups of people, for example, the ratio of girls to boys in a class, or the student-teacher ratio in a school. Ratios compare quantities that are measured in the same unit so they do not have a unit. Rates compare different quantities and must have a unit. We use many rates in everyday life such as runs per over in cricket, kilobytes per second in downloading internet files, kilometres per hour, litres per 100 kilometres and heart beats per minute.

8.1 Ratios

A ratio compares two or more groups or values that are measured in the same unit. Because the unit is the same for each group, ratios do not have units. For example, the ratio of consonants to vowels in our alphabet is 21:5. Notice that the order of the numbers in the ratio must be the same as the order in which the two quantities are given. If we wrote 5:21, this would be the ratio of vowels to consonants.

Simplifying ratios

Ratios can be simplified in a similar way to simplifying fractions. We look for a common factor in the two parts of the ratio.

Example 1

Simplify these ratios.

a 16:24

b 45:18

Working	**Reasoning**
a 16:24 = 2:3	16 and 24 have a common factor of 8.
b 45:18 = 5:2	45 and 18 have a common factor of 9.

Part:part and part:whole ratios

When we compare quantities, we may compare part of the quantity to another part (a part-part comparison) or we may compare part of the quantity to the whole (a part-whole comparison).

Part:part	one part of a group is compared with another part
Part:whole	one part of the group is compared with the whole group

Example 2

A hair salon proprietor found that 85% of customers who had their hair dyed were female. Express the ratio of female to male customers in simplest form.

Working	**Reasoning**
females to males 85% to 15%	If 85% were female then 15% were male.
85:15 17:3	85 and 15 have a common factor of 5.

In the following example, both part-part and part-whole ratios are calculated.

Part:part	Australians born overseas:those born in Australia
Part:whole	Australians born overseas:all Australians

Example 3

Census data showed that 1 out of 5 Australians were born overseas.

a Write this as a ratio.

b What fraction of Australians were born overseas?

c What percentage of Australians were born overseas?

d What percentage of Australians were born in Australia?

e What is the ratio of Australians born overseas to those born in Australia?

Working	Reasoning
a 1:5	We read this as '1 to 5'.
b $\frac{1}{5}$	1 out of every 5 Australians were born overseas.
c $\frac{1}{5}$ $= \frac{1}{5} \times \frac{100}{1}\%$ $= 20\%$	A percentage is a fraction out of 100. Write as a fraction and multiply by 100% to find the percentage. 20 out of every 100 Australians were born overseas.
d 80%	If 20% were born overseas, the other 80% were born in Australia.
e 1:4	For every 1 Australian born overseas, there were 4 born in Australia.

Example 4

In a particular plane there are 14 seats in first class, 70 in business class and 266 seats in economy class.

a Express these numbers as a ratio in simplest form.

b What percentage of the seating is in business class?

Working	Reasoning
a first:business:economy	
14:70:266	14 is a common factor of 14, 70 and 266.
1:5:19	

continued

Example 4 continued

Working

b Percentage of business class seats

$= \frac{5}{25} \times 100\%$

$= 20\%$

Reasoning

Instead of working with the actual number of seats, it is easier to calculate the percentage from the simplified ratio.

$1 + 5 + 19 = 25$

5 out of 25 seats are business class.

Write as a fraction and multiply by 100% to find the percentage.

Comparing ratios

It is often useful to compare part:whole ratios for two different groups. For example we may want to compare the ratio of girls to boys in two different classes. One way of doing this is to convert each part:whole ratio to a percentage.

Example 5

a Class A has a ratio of 5 girls to 4 boys and class B has a ratio of 13 girls to 11 boys. In which class are there more girls to boys?

b Shawn mixes a drink with 50 mL of cordial to 180 mL of water. Jane mixes her drink with 60 mL of cordial to 150 mL of water. Which drink has the higher concentration of cordial to water?

Working

a Class A

girls:boys

5:4

$5 + 4 = 9$

Out of every 9 students, 5 are girls.

Percentage of girls

$= \frac{5}{9} \times 100\%$

$\approx 55.5\%$

Class B

girls:boys

13:11

$13 + 11 = 24$

Out of every 24 students, 13 are girls.

Percentage of girls

$= \frac{13}{24} \times 100\%$

$\approx 54.2\%$

Reasoning

Add the two parts of the ratio to find the total number of parts.

$5 + 4 = 9$

Write as a fraction and multiply by 100% to convert to a percentage.

Add the two parts of the ratio to find the total number of parts.

Write as a fraction and multiply by 100% to convert to a percentage.

continued

Example 5 continued

Working	Reasoning
Class A has a slightly higher percentage of girls and as such it has more girls to boys.	Compare the percentages and write a sentence to answer the question.
b Shawn cordial:water	
50:180 $50 + 180 = 230$ Out of every 230 mL of drink, 50 mL is cordial.	Add the two parts of the ratio to find the total number of parts.
Percentage of cordial $= \frac{50}{230} \times 100\%$ $\approx 21.7\%$	Write as a fraction and multiply by 100% to convert to a percentage.
Jane cordial:water 60:150	
$60 + 150 = 210$ Out of every 210 mL of drink, 60 mL is cordial. Percentage of cordial	Add the two parts of the ratio to find the total number of parts.
$= \frac{60}{210} \times 100\%$ $\approx 28.6\%$	Write as a fraction and multiply by 100% to convert to a percentage.
Jane has a higher concentration of cordial to water in her drink.	Compare the percentages and write a sentence to answer the question.

Unit ratios

Another way of comparing two ratios is to convert them both to unit ratios.

To convert a ratio to a **unit ratio** we divide both numbers in the ratio by the smaller of the two numbers so that we can compare the larger number with 1. For example the ratio 3:2 can be expressed as a unit ratio as 1.5:1 by dividing each number in the ratio by the smaller number, 2.

We can use the unit ratios method to compare either part:part or part:whole ratios. When comparing two ratios using the method of unit ratios, we must be careful to make the same quantity become 1 in each ratio. Example 7 shows how the two problems in Example 5 can be solved using the unit ratio method.

Example 6

Express these ratios as unit ratios.

a 13:5 **b** 4:7

Working	**Reasoning**
a 13:5 $= \frac{13}{5}:\frac{5}{5}$ $= 2.6:1$	Divide both numbers in the ratio by the smaller of the two numbers, 5.
b 4:7 $= \frac{4}{4}:\frac{7}{4}$ $= 1:1.75$	Divide both numbers in the ratio by the smaller of the two numbers, 4.

Example 7

a Class A has a ratio of 5 girls to 4 boys and class B has a ratio of 13 girls to 11 boys. In which class are there more girls to boys?

b Shawn mixes a drink with 50 mL of cordial to 180 mL of water. Jane mixes her drink with 60 mL of cordial to 150 mL of water. Which drink has the higher concentration of cordial to water?

Working		**Reasoning**
a Class A girls:boys 5:4 $\frac{5}{4}:\frac{4}{4}$ 1.25:1	Class B girls:boys 13:11 $\frac{13}{11}:\frac{13}{11}$ $1.\overline{18}:1$	Divide each number in the first ratio by the smaller number in that ratio. In this case we divide 5 and 4 by 4 because 4 is the smaller number. We must make sure that the same quantity (boys) becomes 1 in the other ratio. We divide both 13 and 11 by 11.
Class A has a higher ratio of girls to boys.		
b Shawn cordial:water 50:180 $= \frac{50}{180}:\frac{180}{180}$ $\approx 0.28:1$	Jane cordial:water 60:150 $= \frac{60}{150}:\frac{150}{150}$ $= 0.4:1$	Compare the amounts of cordial for the same amount of water. Jane's drink has 0.4 mL of cordial for every 1 mL of water.
Jane's drink has the higher concentration of cordial to water.		Shawn's drink has only 0.28 mL of cordial for every mL of water.

Map scales

A map scale is a ratio of the distance on a map to the actual distance on the ground. Distances must be expressed in the same unit. For example, a map scale of 1:100 000 means that 1 cm on the map represents 100 000 cm or 1 km on the ground.

Example 8

Consider these map scale problems.

a A distance of 5 km is represented on a map by 2 cm. Express the scale of the map as a ratio.

b The scale on a map is 1:20 000. What distance on the ground is represented by 1 cm on the map?

Working	**Reasoning**
a 2 cm represents 5 km, so 2 cm represents 500 000 cm.	Remember that the quantities must be in the same unit before they can be written as a ratio.
2:500 000	Once the quantities are in the same unit, the ratio is written without units.
1:250 000 The scale of the map is 1:250 000.	2 is a factor of both parts of the ratio.
b Scale 1:20 000 1 cm to 20 000 cm	Express both numbers in the ratio in the same unit.
1 cm to 200 m 1 cm on the map represents 200 m on the ground.	Convert 20 000 cm to a more appropriate unit for distance.

Tech tip

The TI-30XB MultiView calculator can be used to simplify ratios using the fractions key. To simplify 56:20, type:

5 6 $\frac{n}{d}$ 2 0 ▶ enter

exercise 8.1

LINKS TO Example 1

● Simplify each of the following ratios.

a 6:14
b 24:52
c 15:72
d 49:28
e 13:65
f 18:45
g 56:32
h 45:27
i 19:57
j 64:36
k 15:110
l 72:45
m 63:56
n 48:120
o 34:85

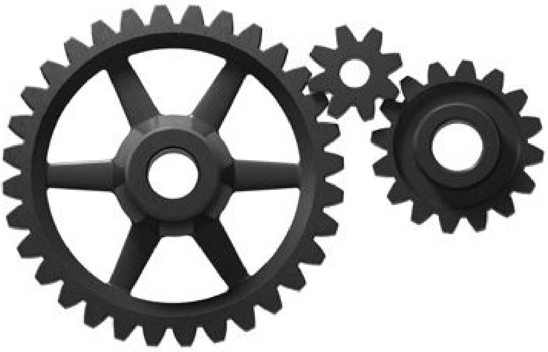
LINKS TO Example 2

Consider the two gear wheels at right.

- **a** What is the number of teeth on the large gear wheel?
- **b** What is the number of teeth on the small gear wheel?
- **c** What is the ratio of teeth on the large wheel to teeth on the small wheel? Give the ratio in its simplest form.

Three gear wheels are shown on the righ

- **a** What is the ratio of teeth on the three wheels (from left to right)?
- **b** Give the ratio in its simplest form.



LINKS TO Example 2

A survey showed that on a particular day about 55% of 11–13 year olds played some kind of sport.

- **a** What percentage of children in this age group did not play any sport on that day?
- **b** What is the ratio of children who played sport to the total number of children in this age group? Write the ratio in its simplest form.
- **c** What is the ratio of children who played sport to children who did not play sport? Write the ratio in its simplest form.

LINKS TO Example 3

In a particular class of 24 students, 9 played instruments in the school band.

- **a** Write the ratio in its simplest form of students in the class who play in the band to the total number of students.
- **b** Write the ratio in its simplest form of students in the band to students not in the band.
- **c** What percentage of students play an instrument in the school band?

LINKS TO Example 3

At a particular school, 28 out of a total of 140 Year 8 students ride bikes to school. In simplest form, find

- **a** the ratio of students who ride bikes to the total number of students.
- **b** the ratio of students who ride bikes to those who do not ride bikes.
- **c** the percentage of students who ride bikes to school.

84% of parking spaces in a car park are long-stay parking spaces.

a What percentage of the parking spaces are short-stay?

b What is the ratio of long-stay to short-stay parking spaces? Write the ratio in its simplest form.

c What is the ratio of long-stay parking spaces to the total number of parking spaces? Write the ratio in its simplest form.

LINKS TO Example 4

18 carat gold is 18 parts gold out of 24 parts and 6 parts other metals.

a What is the fraction of gold in 18 carat gold? Write the fraction in its simplest form.

b Write the fraction of gold as a percentage.

c What is the fraction of other metals?

d Write the fraction of other metals as a percentage.

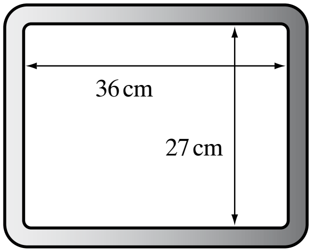
e What is the ratio in its simplest form of gold to other metals?

f What is the ratio of gold to the total number of parts.

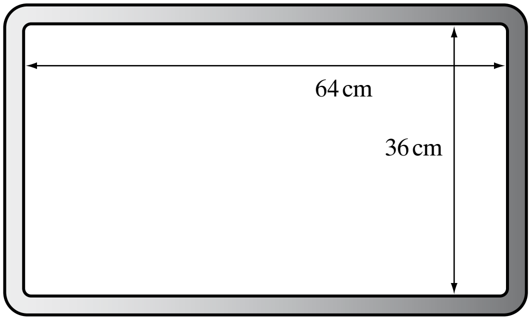
g What is the percentage gold in 18 carat gold?

The ratio of width to height of a rectangle is called its aspect ratio. Calculate the aspect ratio of the following television screens. Give your answers in simplest form.

a



b



LINKS TO Example 5

A recipe for a cake has 150 g of flour and 90 g of butter. Another recipe has 200 g of flour and 150 g of butter. Which recipe, the first or the second, has the greater proportion of flour compared with butter? Show your reasoning.

LINKS TO Example 5

Which of these classes has the highest proportion of girls to boys?

A 18 girls to 13 boys **B** 14 girls to 10 boys **C** 13 girls to 9 boys
D 15 girls to 12 boys **E** 16 girls to 11 boys

LINKS TO Example 6

Write each of these as a unit ratio. Round to three decimal places where appropriate.

a 5:4 **b** 7:5 **c** 5:13 **d** 20:57 **e** 8:23
f 6:29 **g** 12:13 **h** 15:7 **i** 3:8 **j** 14:9
k 9:4 **l** 10:3 **m** 21:13 **n** 27:8 **o** 24:17

The ratio of students to teachers varies in different schools.

a Woodside High School has 1320 students and 60 teachers. Express the student:teacher ratio as a unit ratio.

b Mountville High School has 940 students and 41 teachers. Express the student:teacher ratio as a unit ratio.

c Which school—Woodside High School or Mountville High School—has the lower student:teacher ratio?

LINKS TO Examples 6, 7

Pink paint is made by mixing white and red paints. The ratio of white paint to red paint will determine whether the paint is pale pink or a deeper pink.

a Convert each of these ratios into a unit ratio. Use three decimal places.

- **i** 7 parts white to 5 parts red
- **ii** 5 parts white to 3 parts red
- **iii** 10 parts white to 9 parts red
- **iv** 9 parts white to 7 parts red
- **v** 5 parts white to 4 parts red
- **vi** 14 parts white to 11 parts red

b Which of the mixtures will give the palest pink?

c Which of the mixtures will give the deepest pink?

LINKS TO Example 7

Alice mixes a drink with 30 mL of cordial to 170 mL of water. Liam mixes his drink with 50 mL of cordial to 200 mL of water. Which drink has the higher proportion of cordial? Show your reasoning.

LINKS TO Example 8

A distance of 1 cm on a map represents an actual distance of 20 km. Express the scale of the map as a ratio.

The scale on a map is 1:25 000. What distance on the ground is represented by 1 centimetre on the map? Give the distance in metres.

The scale on a map is 1:400 000. A distance of 1 cm on the map represents a distance on the ground of

A 400 000 km **B** 4000 km **C** 400 km **D** 40 km **E** 4 km

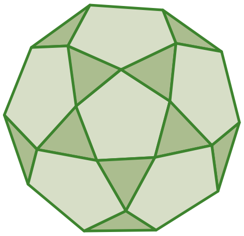
exercise 8.1 challenge

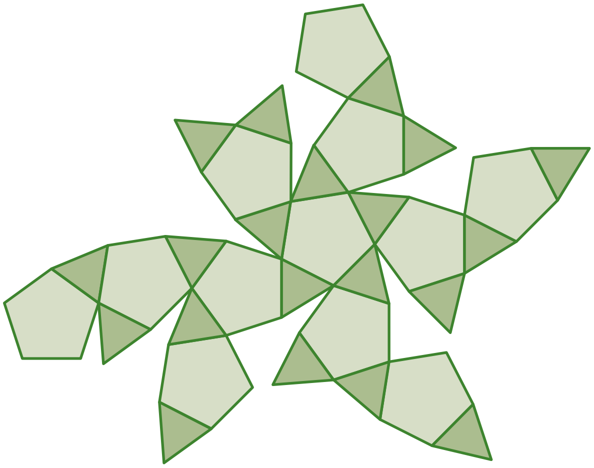
The polyhedron shown here with its net is called an icosidodecahedron.

a What is the ratio of pentagons to equilateral triangles?

b What fraction of the polygons are pentagons?

c What percentage of the polygons are pentagons?





8.2 Dividing quantities in given ratios

There are often situations where we need to divide a quantity into parts in a particular ratio. Consider the following problem.

Students at Highcrest Secondary College are planning to sell T-shirts as a fundraiser for buying sporting equipment for the school. The T-shirts come in two sizes—medium and large. A survey of a sample of students shows that sizes are needed in the ratio 7 large to 3 medium. If the fundraising committee plans to have 350 T-shirts made, how many of each size should there be?

To divide the T-shirts in the ratio 7:3, we need a total of 7 + 3 = 10 parts. Each part is 350 ÷ 10, that is, 35. So we need 7 lots of 35 large T-shirts and 3 lots of 35 medium T-shirts, that is 245 large and 105 medium T-shirts. This method of calculating the size of each part is called the **unitary method** because we start by finding the size of one part or unit.

A very similar method uses fractions. We again start by finding the total number of parts. We then say that $\frac{7}{10}$ of the T-shirts must be large and $\frac{3}{10}$ medium.

$\frac{7}{10}$ of 350 = 245 and $\frac{3}{10}$ of 350 = 105

Notice that the two methods are equivalent. In the unitary method we find the number of large T-shirts by dividing 350 by 10 and multiplying by 7. This is equivalent to finding $\frac{7}{10}$ of 350.

Example 9

State the total number of parts when a quantity is divided in these ratios.

a 3:8

b 15:13:12

Working	Reasoning
a 3 parts + 8 parts = 11 parts	Add the numbers in the ratio to find the total number of parts to divide the quantity into.
b 15 parts + 13 parts + 12 parts = 40 parts	

Example 10 shows the unitary method and the fraction method for dividing a quantity in a given ratio.

Example 10

Divide 240 in the ratio 5:7.

Working	Reasoning
Unitary method:	
5 + 7 = 12 parts	Find the number of parts.
240 ÷ 12 = 20	Find the size of 1 part.
1 part = 20	
5 parts = 5 × 20 = 100	Find the size of 5 parts and 7 parts.
7 parts = 7 × 20 = 140	
240 divided in the ratio 5:7 is 100:140.	Check: 100 + 140 = 240
Fraction method:	Write the part to whole ratios as fractions $\frac{5}{12}, \frac{7}{12}$
5 + 7 = 12 parts	
$\frac{5}{12} \times 240 = 100$	Find $\frac{5}{12}$ of 240.
$\frac{7}{12} \times 240 = 140$	Repeat for $\frac{7}{12}$ of 240 or simply subtract 100 from 240.

240 divided in the ratio 5:7 is 100:140.

Example 11

$3000 is divided between Jack, Fred and Bella in the ratio 7:5:3. Use the unitary method to find how much each person receives.

Working			Reasoning
7 + 5 + 3 = 15 parts			Find the total number of parts.
Each part = $3000 ÷ 15 = $200			Find the size of each part.
Jack	Fred	Bella	Multiply the number of parts for each person by the size of each part.
7 parts	5 parts	3 parts	
7 × $200	5 × $200	3 × $200	Check that the amounts, $1400, $1000 and $600 add to the total $3000.
= $1400	= $1000	= $600	
Check: $1400 + $1000 + $600 = $3000			
Jack receives $1400, Fred receives $1000 and Bella receives $600.			Answer the question.

exercise 8.2

LINKS TO Example 9

State the total number of parts in each of the following ratios.

a 7:8 **b** 5:11 **c** 16:9 **d** 14:13 **e** 12:11
f 7:13 **g** 15:13 **h** 9:5 **i** 3:5:7 **j** 4:7:13

LINKS TO Example 10

Divide the following.

a 18 in the ratio 5:4
b 500 in the ratio 7:3
c 240 in the ratio 5:7
d 300 in the ratio 12:13
e 45 in the ratio 2:7
f 640 in the ratio 7:9
g 3500 in the ratio 9:5
h 840 in the ratio 4:1
i 2000 in the ratio 7:3
j 480 in the ratio 5:1
k 3000 in the ratio 7:8
l 1200 in the ratio 7:17
m 720 in the ratio 5:4
n 560 in the ratio 4:3
o 175 in the ratio 5:2
p 810 in the ratio 13:5
q 504 in the ratio 4:17
r 4500 in the ratio 12:13

5040 divided in the ratio 9:5 is

A 560:1008 **B** 3240:1800 **C** 2835:1575 **D** 7840:14112 **E** 3024:1689

Divide

a 960 in the ratio 3:4:5
b 260 in the ratio 2:4:7
c 540 in the ratio 2:3:4
d 270 in the ratio 2:3:4
e 450 in the ratio 1:2:2
f 3600 in the ratio 1:2:3
g 6600 in the ratio 1:5:6
h 1232 in the ratio 2:3:6
i 6300 in the ratio 2:5:8
j 5200 in the ratio 1:5:7
k 4848 in the ratio 2:5:5
l 2665 in the ratio 4:4:5

6720 divided in the ratio 2:5:5 is

A 1920:4800:4800 **B** 1344:3360:3360 **C** 960:2400:2400
D 562:565:565 **E** 1120:2800:2800

Divide

a \$375 in the ratio 2:1
b \$60 in the ratio 2:3
c 450 m in the ratio 7:8
d 560 g in the ratio 9:7
e \$8000 in the ratio 13:3
f \$9000 in the ratio 4:11
g 4500 L of in the ratio 2:7
h \$520 in the ratio 8:5
i 35 lollies in the ratio 1:2:4
j \$1800 in the ratio 2:3:5

LINKS TO Example 11

A bag contains red, green and yellow jelly snakes in the ratio 7:3:2. If there are 24 jelly snakes in the bag, how many of each colour are there?

There are 450 students at a sports event. The ratio of girls to boys at the event is 8 to 7. How many girls are at the event?

Nancy, Evan and Jay put money into a business in the ratio 2:3:4. They share the profits in the same ratio. If the profit is \$54 000, how much will Jay receive?

A \$6000 **B** \$13 500 **C** \$24 000 **D** \$49 000 **E** \$30 000

- George and Anton won $120 in a poster design competition. They estimated that George spent 2 hours and Anton spent 3 hours working on the design, so they decided it was fair to divide their prize money in the same proportion. How much would each receive?
- Consider these rectangles.
  - **a** The perimeter of a rectangle is 160 cm. If the length and width are in the ratio 5:3, find the length and width of the rectangle. Think carefully about your answer.
  - **b** The perimeter of a rectangle is 36 m. If the length and width are in the ratio 7:2, find the length and width of the rectangle.
- The ratio of boys to girls in a school is 5:6.
  - **a** If there are 913 students at the school, find how many boys and how many girls there are.
  - **b** Find how many more girls there are than boys.
  - **c** Approximately what percentage of the students are boys to the nearest tenth of a percent?
- Great Aunt Ermyntrude is giving $1700 to her great nieces and nephew in proportion to their ages. Nancy is 5, Jack is 9 and Emily is 11. How much will they each receive?
- Nina, Phuong and George put a total of $9000 into a business in the ratio 5:4:3.
  - **a** How much did each person contribute?
  - **b** If the business made a profit of $22 560, how should the profit be divided between Nina, Phuong and George?
- The fund-raising committee at Valley Secondary College is ordering 300 T-shirts. The committee has decided to limit the colours to four. They have surveyed students and found that the colour preferences are blue:green:yellow:pink in the ratio 5:4:2:1. How many of each colour should be ordered?
- Consider these angle problems.
  - **a** The angles of a triangle are in the ratio 2:3:4. What are the sizes of the three angles?
  - **b** The angles of a quadrilateral are in the ratio 1:2:4:5. What are the sizes of the four angles?
- An inheritance of $112 000 was shared in the ratio 1:2:4 by Angie, Will and Jess, in that order. How much did each receive?
- Year 8 students raised $752 from lunchtime food stalls. The money was divided between the Red Cross and the Children's Hospital in the ratio 5:3. How much did each organisation receive?
- A piece of wire 72 cm long is bent into a triangle so that the three sides are in the ratio 4:3:2. How long are the three sides of the triangle?
- A small business allocates $12 950 to building maintenance and advertising in the ratio 5:2. How much will be spent on each?

- Three children in a family were given pocket money each month in the ratio of their ages. The children were 5, 6 and 7 years old. If $27 was divided between them, how much would each child receive each month?
- A concrete mixture contains cement, sand and gravel in the ratio 2:4:5. If 100 kg of concrete is to be mixed, what masses of cement, sand and gravel are required? Give each quantity to the nearest half kilogram.

exercise 8.2 challenge

- A circle is divided into four sectors, with the sector angles in the ratio 1:2:3:4. What are the sizes of each of the sector angles?
- The sum of the angles of a pentagon is 540°. Is it possible for a pentagon to have five angles that are in the ratio 1:2:3:4:5? Explain your reasoning.

8.3 Proportion

When two ratios are equivalent, for example, 2:3 and 4:6, we say that they are **in proportion**. We can write this as a proportion statement as shown:

$$2:3 = 4:6$$

We sometimes need to calculate the unknown value in a proportion statement, for example,

$$a:16 = 9:8$$

In this example it is easy to see that $a = 18$. This is because 16 is twice 8, so a must be twice 9. However, it is not always easy to see at a glance the required value, so we need a method that will apply in all situations.

Using the same example again, we can write the second ratio under the first:

a:16

9:8

This can then be written as an equation: $\frac{a}{9} = \frac{16}{8}$

Solving this equation, we obtain: $\frac{a}{9} \times \frac{9}{1} = \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}}} \times \frac{9}{1}$ (Multiplying both sides by 9 to obtain a on its own.)

$$a = 2 \times 9$$
$$= 18$$

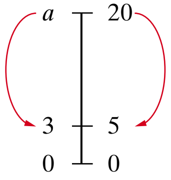
Example 12

The ratios in each pair are in proportion. Write a proportion equation for each pair of ratios. In each case, write your equation with the pronumeral in the numerator.

a a:20 and 3:5

b 4:7 and 12:b

Working	Reasoning
a a:20 3:5	Write the ratios one under the other.
$\frac{a}{3} = \frac{20}{5}$	Put left side over left side and right side over right side.

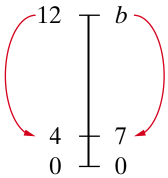


continued

Example 12 continued

Working

b 12:b
4:7



$$\frac{12}{4} = \frac{b}{7}$$

Reasoning

Write the ratios one under the other with the ratio containing the pronumeral on top.

Put left side over left side and right side over right side.

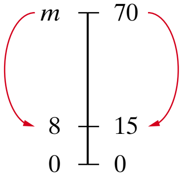
Example 13

Find the value of the pronumeral in each of these proportion statements.

a 8:15 = m:70 **b** 6:x = 21:35

Working

a 8:15 = m:70
m:70
8:15



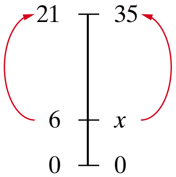
$$\frac{m}{8} = \frac{70}{15}$$

$$\frac{m}{\cancel{8}} \times \frac{\cancel{8}}{1} = \frac{70}{15} \times \frac{8}{1}$$

$$m = \frac{70}{15} \times \frac{8}{1}$$

$$m = 37\frac{1}{3}$$

b 6:x = 21:35
6:x
21:35



$$\frac{6}{21} = \frac{x}{35}$$

$$\frac{6}{21} \times \frac{35}{1} = \frac{x}{\cancel{35}} \times \frac{\cancel{35}}{1}$$

$$x = \frac{6}{21} \times \frac{35}{1}$$

$$x = 10$$

Reasoning

a Write the ratios one under the other, making sure that the ratio that includes the unknown is on top.

Write an equation.

Multiply both sides by $\frac{8}{1}$ to obtain m on its own.

Evaluate.

b Write the ratios one under the other, making sure that the ratio that includes the unknown is on top.

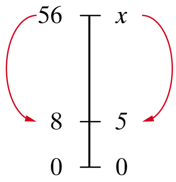
Write an equation.

Multiply both sides by $\frac{35}{1}$ to obtain x on its own.

Evaluate.

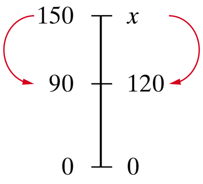
Example 14

The ratio of students in a school playing soccer to students playing Australian Rules football is 8:5. 56 students in the school play soccer. Write a proportion equation and solve to find the number of students who play Australian Rules.

Working	**Reasoning**
Let x be the number of students playing Australian Rules. $56{:}x = 8{:}5$ $56{:}x$ $8{:}5$ 	Write the ratios underneath each other.
$\frac{56}{8} = \frac{x}{5}$	Write the proportion equation with x in the numerator.
$\frac{56}{8} \times \frac{5}{1} = \frac{x}{\cancel{5}} \times \frac{\cancel{5}}{1}$	
$x = \frac{56}{8} \times \frac{5}{1}$ $x = 35$	Solve the equation for x.
35 students play Australian Rules football.	Answer the question asked. Note that in this example it is easy to see that 56 is 7 times 8, so x must be 7 times 5.

Example 15

The ingredients for a cake include 120 g of flour and 90 g of butter. Mathew is making a cake with 150 g of butter. Write a proportion equation for the ingredients of the cake and solve to find how much flour Mathew should use.

Working	**Reasoning**
Let the amount of flour be x g. 	Use a pronumeral to stand for the number of grams of flour.
$x{:}150 = 120{:}90$ $x{:}150$ $120{:}90$	Write a proportion statement and write the ratios one under the other.

continued

Example 15 continued

Working	Reasoning
$\frac{x}{120} = \frac{150}{90}$	Write an equation with the pronumeral in the numerator.
$\frac{x}{\cancel{120}} \times \frac{\cancel{120}}{1} = \frac{150}{90} \times \frac{120}{1}$	Solve the equation.
$x = \frac{150}{90} \times \frac{120}{1}$ $= 200$	Evaluate $\frac{150}{90} \times \frac{120}{1}$
Mathew should use 200 g of flour.	Answer the question asked.

exercise 8.3

LINKS TO Example 12

The ratios in each pair are in the same proportion. Write a proportion equation for each pair of ratios. In each case, write your equation with the pronumeral in the numerator.

a a:64 and 3:4
b b:35 and 3:7
c c:13 and 5:65
d 11:5 and d:45
e 7:e and 91:52
f 5:7 and 90:f
g 2:9 and 16:g
h 7:8 and h:568
i 4:i and 68:102
j 3:8 and 267:j
k 5:4 and 3.5:k
l 7:m and 98:42

LINKS TO Example 13

Find the value of the pronumeral in each of the following proportion equations.

a $\frac{a}{3} = \frac{28}{42}$
b $\frac{b}{7} = \frac{52}{91}$
c $\frac{125}{150} = \frac{c}{6}$
d $\frac{12}{13} = \frac{d}{156}$
e $\frac{e}{9} = \frac{8}{12}$
f $\frac{4}{7} = \frac{f}{84}$
g $\frac{21}{119} = \frac{g}{17}$
h $\frac{h}{5} = \frac{39}{65}$
i $\frac{i}{27} = \frac{8}{9}$
j $\frac{4}{9} = \frac{j}{207}$
k $\frac{k}{9} = \frac{84}{36}$
l $\frac{m}{5} = \frac{585}{13}$
m $\frac{2.4}{7.5} = \frac{m}{3.4}$
n $\frac{4}{1\frac{3}{4}} = \frac{n}{3\frac{1}{2}}$
o $\frac{o}{11.2} = \frac{14.7}{3}$
p $\frac{p}{5} = \frac{58.5}{13}$

LINKS TO Example 14

The ratio of children to adults at a children's party was 7 to 2. There were 21 children and n adults. Write a proportion equation and solve it to find the number of adults, n.

The ratios 15:a and 80:54 are in the same proportion. A correct proportion equation is

A $\frac{15}{a} = \frac{54}{80}$
B $\frac{a}{15} = \frac{80}{54}$
C $a = \frac{54}{80} \div 15$
D $a = \frac{80}{54} \times \frac{15}{1}$
E $\frac{a}{54} = \frac{15}{80}$

- The ratio of adults to children in a cinema was 5 to 8. If there were 96 children, how many adults were there? (Hint: Let n = number of adults. Write a proportion equation and solve it for n.)

- The ratio of fiction books to reference books in a library is 4:3.
  a If there are 412 fiction books, how many reference books are there?
  b How many books are there altogether?

LINKS TO Example 15

- The ingredients for a cake include 150 g of flour and 90 g of butter.
  a What is the ratio of flour to butter? Give your answer in its simplest form.
  b If Danny wanted to make a smaller cake using 60 g of butter, how much flour would he need to use?

- Cordial is to be mixed with water in the ratio 1 part cordial to 6 parts water. The amount of water to be added to 450 mL of cordial would be
  A 456 mL **B** 7 mL **C** 75 mL **D** 2.7 L **E** 3.15 L

- Joe mixed paint, thinner and hardener in the ratio 5:2:1 to paint his car. If he used a 2.4 L tin of paint, how much thinner and hardener did he use? Give the amounts in millilitres.

- The list of ingredients for a recipe for 4 people includes 180 g of cous cous. How much cous cous would be needed for 7 people?

- A jam recipe has 3.5 kg of sugar for 4 kilograms of strawberries. How much sugar should be used with 7.5 kg of strawberries? Give the amount of sugar correct to two decimal places.

- At the end of a European holiday Jason exchanged 74 euros for Australian dollars and received $96.15. How many Australian dollars would be received for 123 euros?

- The ratio of width to height for a cinema screen is 1.83:1. If the screen is 5 metres wide, how high will it be correct to two decimal places (nearest centimetre)?

- A metal factory produces bronze metal by mixing copper and tin in the ratio 3:2. If 7.5 tonnes of copper are used, how much tin will be needed?

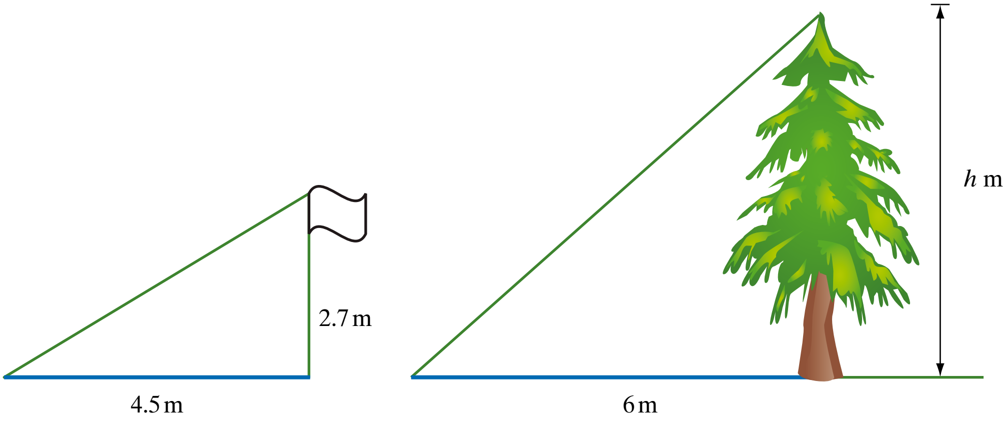
- A concrete mixture contains cement, sand and gravel in the ratio 2:4:5. A 50 kg bag of cement is used to make concrete. How much sand and how much gravel will be required?

- A car can travel 92 km on 8 L of petrol. Assuming it used petrol at the same rate, how far could it travel on the following amounts of petrol?
  a 1 L **b** 5 L **c** 10 L **d** 50 L **e** 45 L **f** 32 L

exercise 8.3 challenge

A flagpole 2.7 m high casts a shadow 4.5 m long. At the same time, a nearby tree casts a shadow 6 m long.

If the height of the tree is h m, write a proportion equation and solve it to find the height of the tree.



The instructions on a bottle of carpet cleaner state that one part of carpet cleaner should be mixed with four parts of water.

a If Bill measures 20 mL of carpet cleaner, how much water should he mix with it?

b If Jade wants 500 mL of mixture, how much carpet cleaner and how much water would she mix?

8.4 Rates

A **rate** makes a comparison between two different quantities that are measured in different units. That is, a rate is one quantity measured in relation to another quantity. Some examples of rates are as follows.

Birth rate	Number of births per thousand people
Cricketer's run rate	Number of runs per over
Average speed	Number of kilometres per hour
Oil production	Number of barrels per day
Water pollution	Number of microorganisms per 10 mL
Concentration of salt solution	Number of grams per litre

Notice that each of these rates includes the word **per**, which we often represent with the division symbol **/**.

Because rates compare quantities measured in different units, units are always specified. Examples are \$/week, g/cm^3, runs/over, km/h and \$/kg.

Calculating with rates

The unit of a rate provides a guide for calculating that rate. For example, if the unit is \$/week, then we know that we must divide the number of dollars by the number of weeks.

Example 16

The total for runs for the 50 overs in a 1-day cricket match is 220 runs.
Calculate the run rate.

Working

$$\text{Run rate} = \frac{\text{number of runs}}{\text{number of overs}}$$

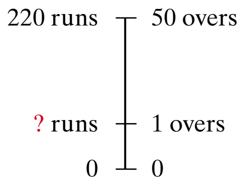
$$= \frac{220}{50} \text{ runs/over}$$

$$= 4.4 \text{ runs/over}$$

The run rate is 4.4 runs/over.

Reasoning

The unit is runs/over.
So divide the number of runs by the number of overs.



Example 17

2.5 kg of apples cost $8.75. Express this as a rate in $/kg.

Working

$$\text{Cost rate} = \frac{\text{Cost}}{\text{Quantity of apples}}$$

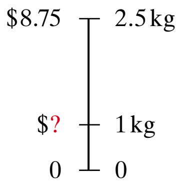
$$= \frac{\$8.75}{2.5\text{ kg}}$$

$$= 3.50\ \$/\text{kg}$$

Reasoning

The unit is $/kg, so divide the cost in dollars by the mass in kilograms.

We would normally write this as $3.50/kg.



Using rates to calculate quantities

To find a quantity given a rate we need to look at how the rate is written.

Using rates to calculate quantities

To *find the quantity that is on the top* of a rate fraction *multiply* the other quantity by the rate.

For example, the cost of 2 kg of apples at a price of 5 $/kg is 2 × 5 = 10 or $10.

To *find the quantity that is on the bottom* of a rate fraction *divide* the other quantity by the rate.

For example, the number of kilogram of apples we could buy for $10 at a price of 5 $/kg is $\frac{10}{5} = 2$ or 2 kg.

Example 18

Using a HappyChat phone card, the cost of telephone calls to Canada is 2.4 cents/minute. What is the cost of a 16-minute call?

Working

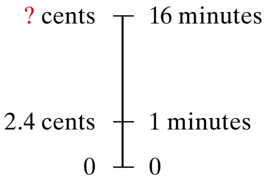
Rate = 2.4 cents/minute

$$\text{Cost of call} = 16\text{ minutes} \times 2.4\text{ cents/minute}$$

$$= 38.4\text{ cents}$$

Reasoning

'Cents' is on top of the rate fraction so multiply the number of minutes by the rate.



Example 19

Tuan typed 2400 words in 48 minutes.

a What was Tuan's typing rate?

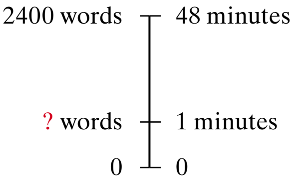
b Assuming Tuan typed at the same rate, how long would it take him to type 6000 words?

Working

a

$$\text{Typing rate} = \frac{\text{Number of words}}{\text{Time}} = \frac{2400 \text{ words}}{48 \text{ minutes}} = 50 \text{ words/minute}$$

Reasoning

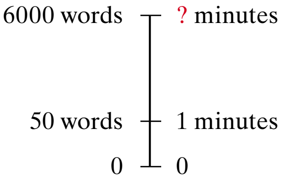


b

$$\text{Time} = \frac{\text{Number of words}}{\text{Rate}} = \frac{6000 \text{ words}}{50 \text{ words/minute}} = 120 \text{ minutes}$$

'Minute' is on the bottom of the rate fraction so divide the other quantity (6000 words) by the rate (50 words/minute).

Every 50 words takes Tuan 1 minute. Find how many lots of 50 words there are in 6000 words.



Petrol consumption

The petrol consumption of cars is usually stated as the average number of litres of petrol per 100 km. Cars use petrol at a greater rate during city driving, where they are continually changing speed and stopping at traffic lights, than when they are travelling at a fairly uniform speed on country roads.

Example 20

A car uses 29.4 litres of petrol in travelling 350 km.

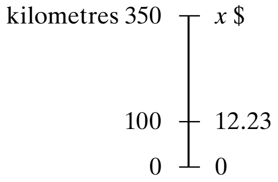
a What is the petrol consumption in L/100 km?

b If the cost of petrol is 145.6 cents/L, what would be the cost of petrol for travelling 100 km?

c What would the petrol cost be for a 350 km journey?

continued

Example 20 continued

Working	Reasoning
a $29.4 \times \frac{100}{350} = 8.4$ 8.4 L/100 km	29.4 L are used to travel 350 km so multiply 29.4 by $\frac{100}{350}$ to get the consumption in L/100 km.
b Travelling 100 km would use 8.4 L of petrol Cost = 8.4 L × 145.6 cents/L = 1223.04 cents =$12.23	'Cents' is on the top of the rate fraction so multiply the other quantity (8.4 L) by the rate (145.6 cents/L).
The cost of petrol is $12.23 for 100 km	Write a sentence to answer the question.
c Cost of petrol is $12.23/100 km = 1223 cents for each 100 km = 12.23 cents/km	It is easier to convert the rate to cents/km. Rate = $12.23/100 km. = 1223 cents/100 km = 12.23 cents/km
Cost for 350 km = 12.23 × 350 = 4280.5 cents = $42.81	'Cents' is on top of the rate fraction so multiply the other quantity (350 km) by the rate (12.23 cents/km).
The cost for 350 km is $42.81.	Write a sentence to answer the question.
Alternative method using proportion: $\$12.23$ for 100 km $\$x$ for 350 km $\frac{x}{12.23} = \frac{350}{100}$ $x = \frac{350}{100} \times 12.23$ ≈ 42.81	 Write the two rates underneath each other. Write a proportion equation. Solve the equation.
The cost for 350 km is $42.81.	Write a sentence to answer the question.

Density

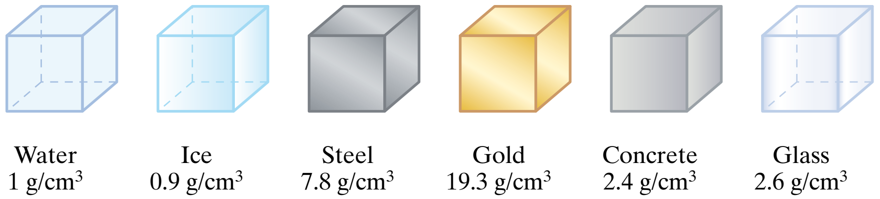
The mass of an object depends on its size and the material it is made from. To compare different types of materials, we can compare the masses of samples that have the same volume.

The mass of a unit volume of a material is called the **density** of the material. Density is measured in units such as grams per cubic centimetre or kilograms per cubic metre.

Calculating density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

The densities of some materials are shown below. From this information we can see that 1 cm^3 of ice has a mass slightly less than the mass of 1 cm^3 of water. Gold is much denser, as 1 cm^3 of gold has a mass about 19 times the mass of 1 cm^3 of water.



If we know the volume of an object and the density of the material it is made of, we can calculate its mass.

We know, for example, that 1 cm^3 of water has a mass of 1 g, so 20 cm^3 of water will have a mass of 20 g.

Calculating mass

Mass = Density × Volume

Example 21

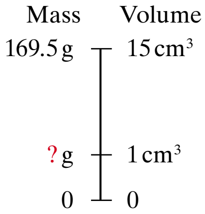
A piece of lead was found to have a mass of 169.5 g and a volume of 15 cm^3. Calculate the density of lead.

Working

$$\begin{aligned}\text{Density} &= \frac{\text{Mass}}{\text{Volume}}\\ &= \frac{169.5}{15}\ \text{g/cm}^3\\ &= 11.3\ \text{g/cm}^3\end{aligned}$$

Reasoning

Density is a rate. The unit is g /cm^3 so divide 169.5 g by 15 cm^3.



Example 22

The volume of a concrete block is 2000 cm^3. Find the mass of the block if the density of concrete is 2.4 g/cm^3. Give your answer in the most suitable unit.

Working

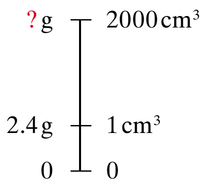
Mass of 1 cm^3 of concrete = 2.4 g

Mass of 2000 cm^3 of concrete
= 2000 × 2.4 g
= 4800 g
= 4.8 kg

Reasoning

The rate is g /cm^3. Grams is on top of the rate fraction, so multiply cubic centimetres (volume) by therate (density).

1000 g = 1 kg



exercise 8.4

LINKS TO Examples 16, 17

Calculate the cost per kilogram if

a a 2.5 kg bag of oranges costs $5.00
b a 5 kg bag of potatoes costs $6.00
c 1.5 kg of onions costs $3.36
d 1.5 kg of apples costs $3.99

Fifty-eight square metres of instant lawn cost $406. What is the cost per square metre?

A netball team scored a total of 614 goals in 11 matches. What was their scoring rate?

Carol earned $261 after working for 18 hours. What was Carol's hourly rate of pay?

Pulse rate is measured as the number of heartbeats per minute. After participating in a sprint race, Jodie counted 38 heartbeats in 20 seconds. What was Jodie's pulse rate?

A cricketer's strike rate is the number of runs per 100 balls faced.

a Calculate Hilary's strike rate (to the nearest run per ball) if she scored 48 runs off 75 balls.

b Calculate Andy's strike rate (to the nearest run per ball) if he scored 74 runs off 124 balls.

A family of four used 54 kilolitres of water during the three months July–September. Find, in litres per day to the nearest litre,

a the average daily water use for the family.

b the average daily water use per person.

LINKS TO Example 18

The carbon dioxide emissions for Australia average about 20 kg per person. If the population of Australia is approximately 22 million, the total carbon dioxide emissions for Australia each year are closest to

A 44 tonnes.
B 1100 tonnes.
C 440 000 tonnes.
D 440 tonnes.
E 1100 million tonnes.

LINKS TO Example 18

For every litre of petrol used, a car emits about 2.4 kg of carbon dioxide in the exhaust. Calculate the carbon dioxide emissions from a car that uses 50 litres of petrol.

Lee uses a phone card to phone China at the rate of 2.3 cents per minute. What is the cost of a call lasting 1 hour 16 minutes?

An internet cafe charges 11 cents per minute. What is the cost for 35 minutes?

LINKS TO Example 19

Juan ironed 10 shirts in 4 minutes.

a What was Juan's ironing rate?

b How long would it take him to iron 12 shirts?

LINKS TO Example 19

Craig earns $15 per hour working in a garden centre. How many hours would Craig need to work to earn $180?

LINKS TO Example 19

Nick was practising to improve his typing speed. He could type an average of 48 words per minute. If Nick types at this rate, how long would it take him to type 1200 words?

LINKS TO **Example 20**

● Trinh used 55 litres of petrol in travelling 585 km. Calculate the petrol consumption of Trinh's car in L/100 km to one decimal place.

● A car manufacturer stated that the petrol consumption for a particular model of car was 9.7 litres/100 km. How many kilometres would the car be able to travel on 1 litre of petrol? Give your answer to one decimal place.

● A car used 19.2 L in travelling 240 km. The fuel efficiency of the car in litres per 100 km is

A 12.5 **B** 8 **C** 46.08 **D** 26.88 **E** 5.2

● Car A travelled 240 km on 20.88 L of petrol. Car B travelled 180 km on 16.56 L of petrol.

a Calculate the petrol consumption of each car in litres per 100 km.

b Which car is the most efficient in terms of petrol used?

c If petrol costs 145.6 cents/L, compare the cost for travelling 100 km for car A and car B.

LINKS TO **Example 21**

● A small car has a fuel consumption rate of 8.6 L/100 km.

a If the car travels 18 000 km in a year, how many litres of petrol would be used?

b The carbon dioxide emission rate for a car is approximately 2.4 kg carbon dioxide per litre of petrol. Calculate the carbon dioxide emissions for the amount of petrol used by the car in part **a**. Give the amount in tonnes.

● The following table shows the mass and volume of four different objects.

a Copy the table and then calculate the density of each object. Use the densities shown on page 350 to suggest what each object might be made from.

Mass (g)	Volume (cm^3)	Density (g/cm^3)	Possible material
246	94.6		
24.9	1.29		
4250	1770		
394	50.5		

b If the density of aluminium is 2.7 g/cm^3, calculate the mass of a piece of aluminium that has a volume of 245 cm^3.

LINKS TO **Example 22**

● If the density of water is 1 g/cm^3, what is the mass of 1 litre of water? (Hint: 1 cm^3 = 1 mL)

exercise 8.4 challenge

● An internet file of 9.6 MB takes 12 seconds to download. What is the rate at which the file is being downloaded? Give your answer in kilobytes per second.
(1 megabyte = 1024 kilobytes)

● Anita's computer was downloading a 4.6 MB internet file at the rate of 340 KB/s. How long would it take to download the file? Give your answer to the nearest second.

8.5 Average speed

Speed is a rate because it is a measure of the distance travelled in a certain time; for example, kilometres per hour or metres per second. The relationship between speed, distance and time can be expressed in three different ways.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Speed usually varies over different parts of a journey. A car travelling along a highway, for example, may be travelling at 100 km/h but slows down when it catches up to a slow vehicle, and speeds up as it passes. In many situations we use the **average speed** for the whole journey rather than the actual speed at any particular time.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Example 23

Calculate each of these average speeds in kilometres per hour.

a A truck travels 240 km in 3 hours.

b A car travels 210 kilometres in 2.4 hours.

c Mei walks 14 km in $3\frac{1}{2}$ h.

Working	**Reasoning**
a $\text{Average speed} = \frac{\text{distance}}{\text{time}}$ $= \frac{240\text{ km}}{3\text{ h}}$ $= 80\text{ km/h}$ The average speed is 80 km/h.	The truck travels 240 km in 3 hours so we divide 240 by 3 to find how far the truck travels in 1 hour.

continued

Example 23 continued

Working

b $$\begin{aligned}\text{Average speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{210\,\text{km}}{2.4\,\text{hours}}\\ &= 87.5\,\text{km/h}\end{aligned}$$

The average speed is 87.5 km/h.

Reasoning

Average speed is calculated by dividing the total distance travelled by the total time taken.

Working

c $$\begin{aligned}\text{Average speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{14\text{ km}}{3\frac{1}{2}\text{ h}}\\ &= \frac{14\text{ km}}{3.5\text{ h}}\\ &= 4\text{ km/h}\end{aligned}$$

The average speed is 4 km/h.

Reasoning

Mei walks 14 km in $3\frac{1}{2}$ hours so we divide 14 km by $3\frac{1}{2}$ to find how far she walks in 1 hour.

Convert $3\frac{1}{2}$ h to 3.5 h
or

$$\begin{aligned}&14 \div 3\frac{1}{2}\\ &= 14 \div \frac{7}{2}\\ &= \overset{2}{\cancel{14}} \times \frac{2}{\underset{1}{\cancel{7}}}\\ &= 4\end{aligned}$$

Example 24

a Gemma drives at an average speed of 85 km/h for 2 hours. How far does she travel?

b Harry cycles at an average speed of 16 km/h for 3.2 hours. How far does he travel?

Working

a $$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 85\text{ km/h} \times 2\text{ h}\\ &= 170\text{ km}\end{aligned}$$

Gemma travels 170 km.

Alternatively:

Gemma travels 85 km in 1 hour so she will travel 2 × 85 = 170 km in 2 hours.

Reasoning

Every hour Gemma travels 85 km, so multiply the number of hours by 85.

Working

b $$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 16\,\text{km/h} \times 3.2\,\text{h}\\ &= 51.2\,\text{km}\end{aligned}$$

Harry travels 51.2 km.

Reasoning

Every hour, Harry will travel 16 km. In 3.2 hours he will travel 3.2 times 16 km.

The answer is the distance Harry travels, so the unit will be kilometres.

Example 25

A truck is travelling at a steady speed of 90 km/h. How long would it take the truck to travel 36 km? Give your answer in the most appropriate units.

Working	**Reasoning**
$\text{Distance} = \text{speed} \times \text{time}$ $\text{time} = \text{distance} \div \text{speed}$ $= 36 \div 90$ $= 0.4$ It takes the truck 0.4 h to travel 36 km.	Every hour the truck travels 90 km so we need to know how many lots of 90 there are in 36. So we divide 36 by 90.

exercise 8.5

LINKS TO Example 23

● Calculate the following speeds in kilometres per hour.

a A truck travels 600 km in 8 hours.

b A plane flies 2400 km in 8 hours.

c A high speed train travels 140 km in $\frac{1}{2}$ h.

d Fred walks 2.1 km in $\frac{1}{2}$ h.

e Kirsty jogs 3.8 km in 0.75 h.

f Justin cycles 18 km in 1.2 h.

g An elite swimmer swims 1500 m in 0.4 h.

h A bus travels 160 km in 1.6 h.

● The Indian-Pacific train travels from Sydney to Perth, a distance of 4352 km, in 57 hours 30 minutes. What is the average speed? Give your answer to one decimal place.

LINKS TO Example 24

● Calculate the following distances travelled.

a A car travels at 90 km/h for 4 hours

b A train travels at 120 km/h for 5 hours

c Emma walks at 4 km/h for 3 hours

d A plane flies at 805 km/h for 21 hours

e A truck travels at 85 km/h for $3\frac{1}{2}$ hours

f Aron jogs at 7 km/h for 2.4 hours

g Miki cycles at 15 km/h for 1.2 hours

h A car travels at 96 km/h for $2\frac{3}{4}$ hours

● Anna jogs at an average speed of 9 km/h for 48 minutes. She jogs

A 5.3 km. **B** 18.75 km. **C** 11.25 km. **D** 7.2 km. **E** 8.4 km.

● Jeremy runs 5 km from his home to school at an average speed of 12 km/h. It takes him

A 25 minutes **B** 32 minutes **C** 60 minutes **D** 12 minutes **E** 144 minutes

LINKS TO Example 25

Calculate the following times in hours.

- **a** Lin cycles 28 km at 14 km/h.
- **b** A car travels 360 km at 90 km/h.
- **c** Maryam walks 14 km at 3.5 km/h.
- **d** Sacha jogs 5.6 km at 7 km/h.
- **e** A plane flies 8800 km at 640 km/h.
- **f** A truck drives 570 km at 95 km/h.
- **g** A car travels 72 km at 80 km/h.
- **h** A train travels 240 km at 150 km/h.

Complete this table using the given units.

Distance	Time	Speed
70 km	5 h	
	$2\frac{1}{3}$ h	75 km/h
120 km		90 km/h
15 km	$\frac{3}{4}$ h	
10 km		4 km/h
	$1\frac{1}{4}$ h	28 km/h

On August 16, 2008, Usain Bolt ran 100 m in 9.69 seconds. Correct to one decimal place, what was his average speed in m/s?

On April 26, 2009, French swimmer Frédérick Bousquet set a new world swimming speed record in the 50 m freestyle with a time of 20.94 seconds at the French Championships in Montpelier, France. Correct to one decimal place, what was his average speed in m/s?

A snail crawls 21 mm in 15 seconds. What is the speed in mm/s?

Police in England were surprised when a hamster (an animal about the size of a small rat) was handed in to the police station in the seaside town of Cleveleys, near Blackpool. The hamster, nicknamed Speedy by police, was found cruising along the esplanade in a toy racing car powered by a treadmill. Police officers described the car as ‘a model hot-rod racing car with large wheels at the back and small ones at the front’. They removed Speedy from his car after he made several attempts to escape from their desk.

- **a** If Speedy travelled 60 cm in 4 seconds, what was his speed in cm/sec?
- **b** How many metres per second is this?
- **c** At this speed how many metres would Speedy travel in 1 minute?
- **d** How many metres would he travel in 1 hour?
- **e** How many kilometres per hour is this?

- On a test run in 2001, the French TGV (Train à Grande Vitesse, French for 'high-speed train') travelled 1067 km from Calais to Marseille at an average speed of 317.5 km/h. How long did the journey take? Give the time in hours correct to one decimal place.

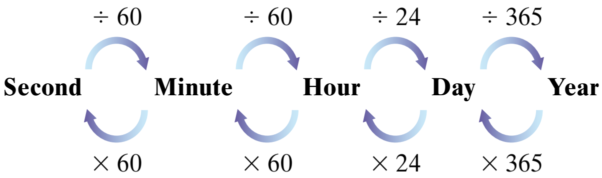
exercise 8.5 challenge

- Mike cycled at an average speed of 30 km/h for $1\frac{1}{2}$ hours then for a further $2\frac{1}{4}$ hours at an average speed of 25 km/h. What was Mike's average speed for the whole journey?
- The distance between two towns is 90 km. A truck averages 60 km/h in one direction, in heavy traffic, and 85 km/h on the return journey.
  - **a** Find the total time for the journey. Give your answer
    - **i** in hours as a decimal, to two decimal places.
    - **ii** in hours and minutes, to the nearest minute.
  - **b** Calculate the average speed for the whole journey. Give your answer to the nearest kilometre per hour.

8.6 Calculating with time

Units of time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
14 days = 1 fortnight
12 months = 1 year
365 days = 1 year
(366 days = 1 leap year)
52 weeks and 1 day = 1 year
10 years = a decade
100 years = a century
1000 years = a millennium



	Hour	**Minute**	**Second**
Symbol	h	min	s

Example 26

Express these times using the correct symbols.

a 45 minutes **b** 3 hours and 17 minutes

c 4 minutes and 45 seconds.

continued

Example 26 continued

Working	**Reasoning**
a 45 min	'min' is a symbol, not an abbreviation so we do not write 'mins'.
b 3 h 17 min	We do not write 'and' between the hours and minutes.
c 4 min 45 s	

Example 27

Consider the following.

a How many seconds are in 1 hour and 12 minutes?

b Convert 5000 seconds to hours, minutes and seconds. Express the times using the correct symbols.

Working	**Reasoning**
a 1 hour and 12 minutes = 72 minutes = 72 × 60 seconds = 4320 seconds	1 hour = 60 minutes 1 minute = 60 seconds
b 5000 seconds = (5000 ÷ 60) minutes = 83 min 20 s = 1 h 23 min 20 s	60 seconds = 1 minute We divide by 60 to find how many lots of 60 seconds. There are 83 whole minutes and 20 seconds left over. 60 minutes = 1 hour 83 minutes = 1 hour and 23 minutes

24-hour time

Many electronic clocks display 24-hour time. After midday, instead of starting at 1 o'clock again, the time continues as 13:00. Midnight is then 24:00 hours.

Although we call 13:00 hours 'thirteen hundred hours' it does not really mean this. When we get to 13:59 hours, the next minute takes us to the next hour, that is, 14:00 hours.

It is the custom to express 24-hour time with four digits, so 4.48 am would be written as 04:48.

After midday, we turn 12-hour times into 24-hour times by adding 12 to the hour. So 4.48 pm becomes 16:48 hours.

12-hour time	24-hour time
12 midnight	00:00
1 am	01:00
2.40 am	02:40
12 noon	12:00
1.10 pm	13:10
7.15 pm	19:15
11.59 pm	23:59

Example 28

Convert

a 3:45 am into 24-hour time.

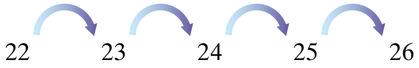
b 10:18 pm into 24-hour time.

c 17:58 into 12-hour time.

Working	Reasoning
a 3:45 am = 03:45	AM time so put zero in front of 3 to make four digits.
b 10:18 + 12:00 = 22:18	PM time so add 12:00 hours
c 17:58 − 12:00 = 5:58 pm	17:58 is PM time because it is greater than 12:00. Subtract 12:00

Intervals of time

When finding time intervals we subtract the start time from the end time unless it is stated that the interval is inclusive of both the start and finish times. For example, ‘January 22 to January 26 inclusive’ means all five days: January 22, 23, 24, 25, 26. However, if it is January 22 today and we ask ‘how many days until January 26’, the answer is four days.



26 − 22 = 4 days

Example 29

Jane was on holiday from 24 March until 7 June inclusive (that is, count both 24 March and 7 June). How many days was Jane on holiday?

Working	Reasoning
24 March to 31 March	8 days We count 24 March as part of Jane's holiday.
1 April to 30 April	30 days
1 May to 31 May	31 days
1 June to 7 June	7 days We count 7 June as part of Jane's holiday.
Total days = 8 + 30 + 31 + 7 = 76	76 days
Jane was on holiday for 76 days.	

Example 30

How long is it between 8:12 am and 3:35 pm?

Working

Interval	Hours	Minutes
8:12 am to 9:00 am		48 min
9:00 am to 12 noon	3 h	
12 noon to 3:00 pm	3 h	
3:00 pm to 3:35 pm		35 min
Total time	6 h	83 min
	7 h	23 min

The time between 8:12 am and 3:35 pm is 7 hours and 23 minutes.

Reasoning

60 minutes in 1 hour
60 − 12 = 48

Add the hours and add the minutes

83 min = 1 h 23 min

Example 31

A movie lasts for 3 hours and 12 minutes. The movie has been going for 1 hour 43 minutes. How long is it before the movie finishes?

Working

Interval	Hours	Minutes
1 h 43 min to 2 h		17 min
2 h to 3 h	1 h	
3 h to 3 h 12 min		12 min
Total time left	1 h	29 min

It is 1 h 29 min before the movie finishes.

Reasoning

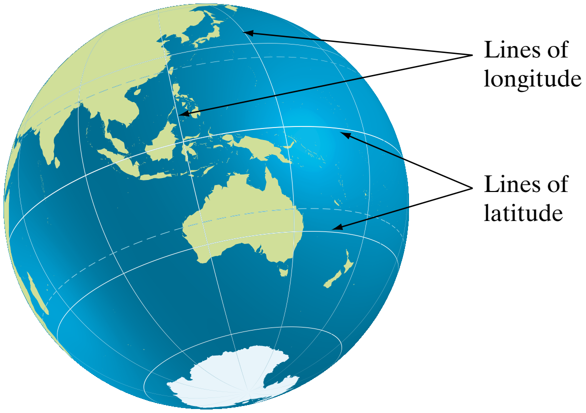
Find many minutes until the next whole hour.
Find the number of whole hours.
Find how many extra minutes till finish time.
Add the hours and add the minutes. Check that the number of minutes is not more than 1 hour.

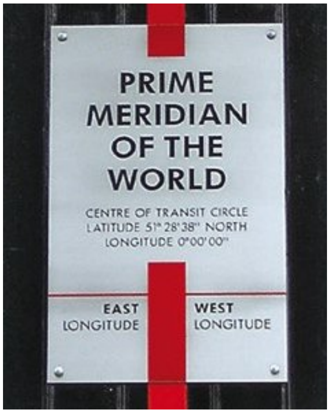
Time zones

The Earth rotates through 360° every 24 hours, that is, 15° per hour. This means that the sun rises at different times in different regions of the Earth. **Lines of longitude** mark 15° intervals around the Earth. Lines of longitude are also called **meridians**.

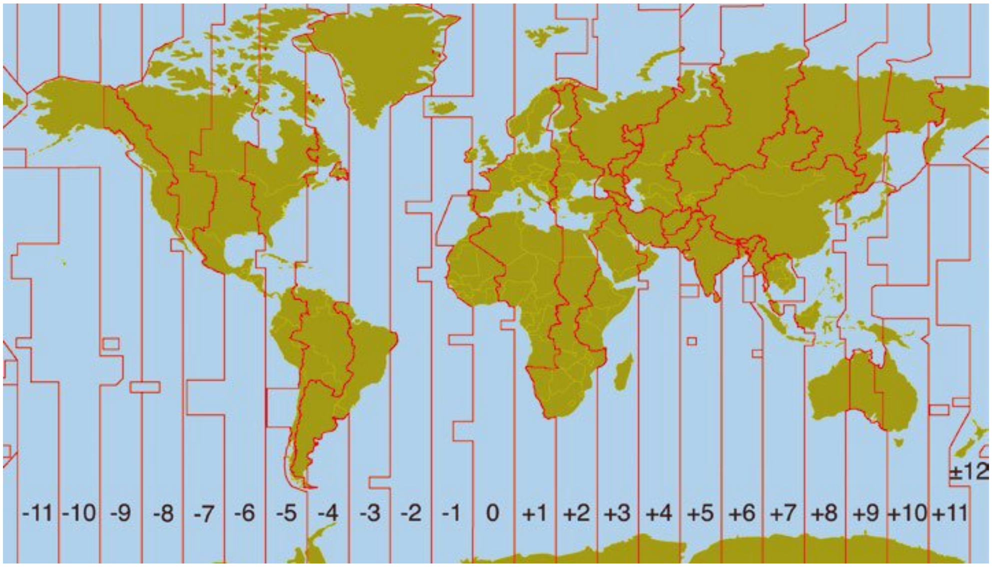
The 0° meridian (called the Prime Meridian) passes through Greenwich, near London. Time at the 0° meridian was previously called *Greenwich Mean Time* (GMT) but is now called *Coordinated Universal Time* (UTC).

The photo below shows the prime meridian passing down the wall to meet the line on the ground at the Greenwich Observatory.





When it is midday at places along the 0° Meridian (for example, Greenwich, Iceland, Morocco, and Ghana), it will be 11 am at places 15° west of the 0° Meridian and 1 pm at places 15° east of the 0° Meridian. The map below shows the time zones and how much each zone is ahead of or behind UTC.



International time zones

The International Date Line passes through the Pacific Ocean between Australia and the west coast of the United States of America. It roughly corresponds with the 180° meridian, but it bends around some of the Pacific islands to avoid confusion. The International Date Line is the line where the date changes. For example, at 1 minute before midnight on Monday in the time zone to the west of the International Date Line it will be 1 minute before midnight on Sunday in the time zone to the east of the International Date Line.

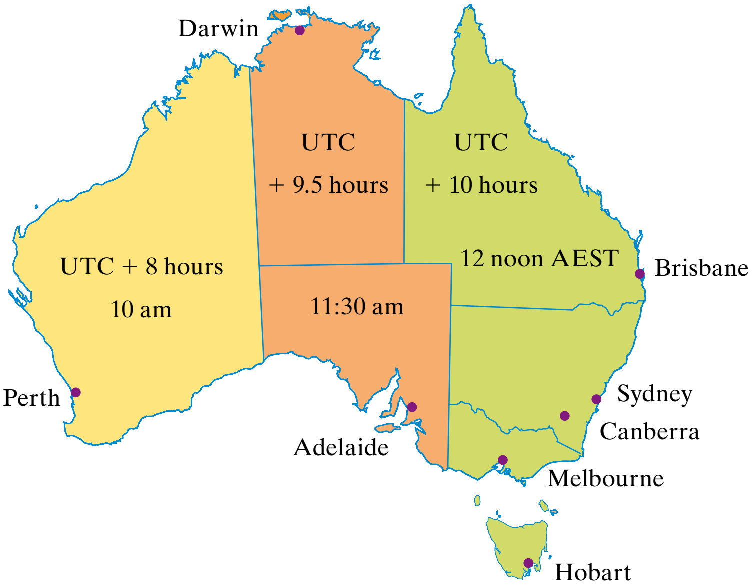
Example 32

Juan is working in Brisbane in July and wishes to phone his family in Buenos Aires. Buenos Aires is 3 hours behind UTC and Brisbane is 10 hours ahead of UTC. If it is 10 pm in Brisbane what time is it in Buenos Aires?

Working	Reasoning
10 pm − 13 hours = 9 am	Buenos Aires time is 3 hours behind UTC, and Brisbane is 10 hours ahead of UTC, so Buenos Aires is 13 hours behind Brisbane.

Australian time zones

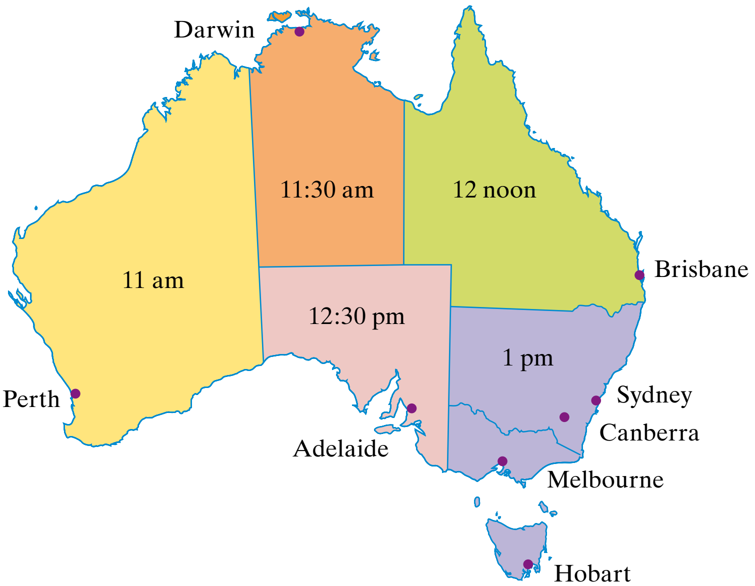
The Australian time zones are shown on this map. The Eastern states are on Australian Eastern Standard Time (AEST). South Australia and Northern Territory are half an hour behind AEST and Western Australia is 2 hours behind AEST. For example, if it is 12 noon in Brisbane in July it will be 11.30 am in Adelaide and 10 am in Perth.



Daylight saving (summer time)

Daylight saving allows us to make better use of the longer hours of daylight in summer. Putting the time one hour forward in the summer, so that 6 am becomes 7 am, is equivalent to starting work or school an hour earlier so that people have an extra hour of daylight left for enjoying outside activities in the evenings.

Daylight saving makes the time zones more complicated in summer. Northern Territory, Queensland and Western Australia do not adopt daylight saving. From the end of October to the end of March, ACT, New South Wales, South Australia, Tasmania and Victoria set their clocks forward one hour, so that 1 am becomes 2 am. Queensland remains on Australian Eastern Standard Time so Queensland is one hour behind the other eastern states for the summer months. In summer, Northern Territory is one hour behind South Australia.



Example 33

Calculate these times.

a In August, when it is 9 am in New South Wales, what time is it in Western Australia?

b In January, when it is 3 pm in Darwin, what time is it in Hobart?

Working	Reasoning
a 9 am − 2 hours = 7 am	In August, New South Wales is on Australian Eastern Standard Time. Western Australia is two hours behind AEST.
b 3 pm + $1\frac{1}{2}$ hours = 4:30 pm	In January, Hobart is on Eastern Summer Time. Darwin is one and a half hours behind Eastern Summer Time, so Hobart is one and a half hours ahead of Darwin.

exercise 8.6

LINKS TO Example 26

BLM Exercise 8.6

1 Express these times using the correct symbols.

a 54 seconds **b** 5 hours 17 minutes

c 27 minutes 15 seconds **d** 2 hours 43 minutes 18 seconds

2 Complete each of the following time conversions.

a $5\frac{1}{2}$ minutes = 330 ________ **b** 120 minutes = 2 ________

c 6 ________ = 360 seconds **d** $\frac{1}{2}$ hour = 1800 ________

e 90 seconds = 1.5 ________

f 4 minutes and 37 seconds = 277 ________

g 2 ________ = 7200 seconds

h $\frac{1}{2}$ ________ = 30 seconds

i $\frac{1}{2}$ day = 12 ________

j $\frac{1}{4}$ hour = 900 ________

k 4 hours and 18 minutes = 15480 ________

l 1 year = 8760 ________

LINKS TO Example 27

Complete each of the following time conversion statements.

a 300 seconds = ________ minutes

b 72 hours = ________ days

c 730 days = ________ years

d $3\frac{1}{2}$ years = ________ months

e 98 days = ________ weeks

f 3 years and 40 days = ________ days

g 210 seconds = ________ minutes

h $\frac{3}{4}$ hour = ________ minutes

i 2 years = ________ hours

j 4 minutes and 55 seconds = ________ seconds

k $2\frac{1}{2}$ hours = ________ minutes

l 1 hour and 13 minutes = ________ seconds

m $5\frac{1}{2}$ days = ________ hours

n 800 minutes = ________ hours and ________ minutes

o 15 minutes = ________ seconds

p 270 hours = ________ days and ________ hours

LINKS TO Example 28

Show these analogue clock times as 24-hour clock times.

a 5 pm

b 10:15 am

c 1:45 pm

d 7:55 pm

e 3:22 am

f 8:17 am

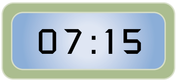
g 6:49 am

h 10:57 pm

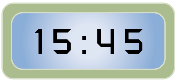
Match these 24-hour clock times with the activities shown below.

a

b



c



d

Wake up Bedtime Lunchtime Go home from school

Write two different 24-hour times that would correspond to 4:42.

Write these times in 24-hour time.

a half past seven in the morning

b one minute to midnight

c quarter to ten in the evening

d twenty minutes past eight in the morning

e twenty minutes past eight in the evening

f quarter past nine in the evening

g five minutes to eleven in the evening

h eighteen minutes past midnight

i exactly one hour past midnight

j one minute past midnight.

LINKS TO Example 29

The number of days from 11 May to 23 August inclusive is

A 34 days **B** 43 days **C** 95 days **D** 105 days **E** 103 days

What will be the date 90 days after 17 September?

A 17 December **B** 16 December **C** 15 December
D 12 December **E** 11 December

Amin was born on 23 March and Nada was born in the same year on 21 July.

a How many days older than Nada is Amin?
b How many weeks is this?
c If Amin was born on a Tuesday, on what day of the week was Nada born?

LINKS TO Example 30

Find the length of time from

a 8:15 am to 11:45 am
b 3:20 pm to 7:30 pm
c 7:25 am to 9:55 am
d 8:10 am to 2:35 pm
e 10:17 am to 4:32 pm
f 8:25 am to 5:15 pm
g 2:40 am to 8:15 pm
h 7:50 am to 3:45 pm
i 9:30 pm Monday to 7:15 pm Tuesday
j 5:25 pm Friday to 8:10 am Monday.

LINKS TO Example 31

Daniel is recording several TV programs. He has a 3-hour videotape and the lengths of the programs are 1 hour 5 minutes, 35 minutes and 70 minutes. Will the three programs fit on the videotape?

The times for ten songs on a CD album are shown below in minutes and seconds.

3:08 3:45 6:06 11:36 3:47 6:20 5:06 3:07 6:38 7:49

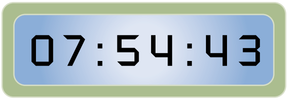
a Calculate the total playing time for the ten songs.
b If the CD plays for one hour, what is the maximum length an eleventh song could have been?

Halil arrives home from school at 6.09 pm. He is very hungry! A note says that the meal in the oven needs 2 hours and 20 minutes to cook, and it went into the oven at 4.15 pm. How long does Halil have to wait until the casserole is done?

Kerensa flies from Melbourne to Sydney. How many minutes is the flight?

Depart	Melbourne	06:30
Arrive	Sydney	07:50

A station clock shows the following time:
If the next train is at 8:07, how many minutes and seconds until the train departs?



● This timetable for the Ghan train that runs between Adelaide and Darwin is given below in 12-hour time.

Adelaide – Alice Springs – Darwin	
Sun	
Depart Adelaide	12:20 pm
Mon	
Arrive Alice Springs*	1:45 pm
Depart Alice Springs	6:00 pm
Tues	
Arrive Katherine	9:00 am
Depart Katherine	1:00 pm
Arrive Darwin	5:30 pm

*Arrives Alice Springs 1 hour earlier during daylight saving time.

a Calculate how long the train takes to travel from
- **i** Adelaide to Alice Springs?
- **ii** Alice Springs to Katherine.
- **iii** Katherine to Darwin.

b How long does the train stop in Katherine?

c What is the total length of time from leaving Adelaide to arriving in Darwin?

d Copy the timetable and add another column. Write the times in 24-hour time.

LINKS TO Example 32

Chris flies from Brisbane to Auckland, New Zealand, leaving at 08:30 Australian Eastern Standard Time. New Zealand is 2 hours ahead of Australian Eastern Standard Time.

a If the flight takes 2 hours and 55 minutes, what is the time in Brisbane when Chris arrives in Auckland?

b What is the time in Auckland when Chris arrives?

LINKS TO Example 33

Copy and complete this table for January if it is 7 pm Australian Eastern Standard Time.

State/Territory	Time
ACT	
New South Wales	
Northern Territory	
Queensland	
South Australia	
Tasmania	
Victoria	
Western Australia	

Jeff lives in Brisbane.

a If he wants to phone his brother in Perth on August 20th at 9 am Perth time, what time does he need to phone from Brisbane?

b What time in Brisbane would Jeff need to phone a friend in Melbourne in January, at 9 am Melbourne time?

Kylie leaves Sydney at 09:40 on 20th November to fly to Adelaide.

a If the flight takes 1 hour and 20 minutes, what is the time in Sydney when Kylie arrives in Adelaide?

b What is the time in Adelaide when Kylie arrives?

Jack flies from Perth to Brisbane in July on a flight that leaves Perth at 16:45.

a What time is it in Brisbane when Jack's flight leaves Perth?

b The length of the flight is 4 h 30 min. What time is it in Brisbane when Jack arrives?

exercise 8.6 challenge

Natasha lives in Tasmania. She wants to phone a friend in Jakarta (in Indonesia) in summer at 11 am. Jakarta is 3 hours behind Australian Eastern Standard Time. At what time in Tasmania should Natasha phone?

Jason is flying from Melbourne to Sydney to connect with his flight to Singapore at 11:20 on 13 July. He wants to check in at Sydney for his Singapore flight at least $2\frac{1}{2}$ hours before departure. Flight times from Melbourne to Sydney are shown below.

Depart Melbourne	06:00	06:30	07:00	07:15	07:30	08:00	08:30
Arrive Sydney	07:20	07:50	08:20	08:35	08:50	09:20	09:50

a On which flight would you recommend Jason to travel? Justify your answer.

b Jason's flight arrives in Singapore at 19:50 Singapore time. If Singapore is 2 hours behind Sydney time, how long does Jason's flight take?

Analysis task

Possum forest park

A small forest park is home to two kinds of possums. Population counts began in 1993, when researchers carried out a full count (that is, a *census*) of all possums. They counted 140 mountain brushtails and 60 ringtails. In 1995, the number of mountain brushtails had gone up to 175 and there were 75 ringtails.

a What was the ratio of mountain brushtails to ringtails

i in 1993? **ii** in 1995?

Give each ratio in simplest terms. What do you notice?

b In 1997, the total population of possums was 270. Estimate how many possums of each type there were.

A full count is expensive to carry out, and the ratio of possum types appeared to stay the same. Therefore, in some of the following years, the researchers decided to save time and money by counting only one type of possum and estimating the population of the other type.

c In 1999, there were 102 ringtails. What was the estimate for mountain brushtails?

d The count of mountain brushtails in 2001 was 294. What was the estimate for ringtails that year?

In 2003, a full census was conducted again, and researchers counted 280 mountain brushtails and 160 common ringtails. They found that there were now also 40 common brushtail possums in the park.

Brushtail possum

e What was the ratio of mountain brushtails to ringtails to common brushtails?

f Obviously there were now more common brushtails in 2003 than in 1993. But how had the populations of the other two types of possum changed as percentages of the total possum population of the park? Had both percentages decreased, or had one percentage stayed the same—or even increased?

In 2005, only ringtails and common brushtails were counted. There were 176 ringtails and 44 common brushtails. This gave the same ratio as 2003, so the researchers assumed that the 2003 ratio had also remained steady for mountains brushtails.

g How many mountain brushtails did they estimate the park contained in 2005?

Ringtail possum

When another census was conducted in 2007, the researchers realised that the total population of possums in the park had stabilised at 480, the figure they had counted in the 2003 census. They had been right about the ratio of ringtails to common brushtails staying the same, but the number of mountain brushtails had been in decline. There were now 53 common brushtails in the park.

h Assuming a stable population of 480, revise your estimate of the number of mountain brushtails in the park in 2005.

i Find the number of possums of each type in 2007 and give each number as an approximate percentage of the park's total possum population.

Review Ratios rates, and time

Summary

Ratio

- A ratio is a comparison of two or more quantities with the same unit. A ratio does not have units.
- Part:part ratio compares two parts of the whole.
- Part:whole ratio compares a part to the whole quantity.

Equivalent ratios

- Ratios are equivalent if they can be simplified to the same ratio. For example 4:7 and 8:14 are equivalent ratios.

Proportion

- Equivalent ratios are in proportion. For example 4:7 and 8:14 are in the same proportion.

Proportion equation

- If two ratios are in proportion, we can write a proportion equation and solve it.

  For example if x:24 = 13:6 then writing the ratios under each other gives

  x:24

  13: 6

  The proportion equation is $\frac{x}{13} = \frac{24}{6}$ so $x = \frac{24}{6} \times \frac{13}{1} = 52$

Rate

- A comparison of two quantities to show how one quantity varies with the other
- A rate needs units, for example, km/h, % per annum, g/L.
- To find the quantity that is on the top of a rate fraction multiply the other quantity by the rate.

  For example, the cost of 2 kg of apples at a price of 5 $/kg is 2 x 5 = 10 or $10.
- To find the quantity that is on the bottom of a rate fraction divide the other quantity by the rate.

  For example, the number of kilogram of apples we could buy for $10 at a price of 5 $/kg is $\frac{10}{5} = 2$ or 2 kg.

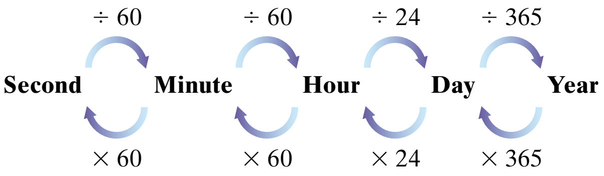
Density

- Density $= \frac{\text{mass}}{\text{volume}}$

Speed

- Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$

Converting units of time



10 years = 1 decade   100 years = 1 century   1000 years = 1 millennium

24-hour time

Hours are measured continuously from midnight, so that 1 pm becomes 13:00 hours; for example, 3 o'clock in the afternoon is 15:00 hours.

Time zones

See maps on page 363 and 364.

Visual map

AEST
average speed
constant rate
daylight saving
density
equivalent ratios
gradient
part : part ratio
part : whole ratio
percentage
proportion
proportion equation
rate
ratio
scale
speed
time interval
time zonE
UTC
12-hour time
24-hour time

Revision

Multiple-choice questions

In a school of 1224 students, girls and boys are in the ratio 9:8. The number of girls is

A 72   **B** 576   **C** 648   **D** 1088   **E** 1377

In a cycling competition prizes were awarded to the first four competitors in the ratio 7:4:3:1. Lee came second and won $120. The total amount of prize money was

A \$480   **B** \$450   **C** \$330   **D** \$1440   **E** \$1800

If 2:3 = x:24, then x is equal to

A 8   **B** 9.6   **C** 14.4   **D** 16   **E** 36

● A recipe contains flour and sugar in the ratio 5:3. If 240 g of flour is used, the amount of sugar needed will be:

A 48 g **B** 80 g **C** 90 g **D** 144 g **E** 150 g

● A car uses 59.5 litres of petrol on a 640 kilometre journey. The fuel consumption rate in L/100 km is closest to

A 10.8 **B** 0.09 **C** 9.3 **D** 38 **E** 108

Short-answer questions

● In a box of pens there are 14 red pens and 6 blue pens.

a What is the ratio in simplest form of red pens to blue pens?

b What is the ratio in simplest form of red pens to the total number of pens?

c What percentage of the pens are blue?

● Simplify the following ratios.

a 16:24 **b** 21:42 **c** 87:72 **d** 91:26

● In a school of 1250 students, the ratio of boys to girls is 3:2. How many girls are there at the school?

● The ratio of fiction books to reference books in a library is 7:12. In total, there are 4617 books in the library.

a How many of the books are fiction books?

b How many are reference books?

● Complete the following.

a Divide 300 in the ratio 5:7.

b Divide $480 in the ratio 7:5:4.

c Ada and Nancy win $450 in a competition. They decide to divide their prize money in the ratio 2:3. How much will Ada receive and how much will Nancy receive?

● Tom, Hui and Carla shared the profits from their business in the ratio 3:4:8. If Carla's share was $12 000, how much each did Tom and Hui receive?

● In school A there were 760 students and 33 teachers. In school B there were 426 students and 21 teachers. Which school had a smaller number of students per teacher? Show your reasoning.

● Erica and Nina work.

a Eric earned $425.50 after working for 23 hours. What was Eric's hourly rate of pay?

b Nita earns $13.50/h working in a garden centre. How many hours would she need to work before earning $540?

● Sean walks 2.8 km in $\frac{2}{3}$ h.

a What is Sean's average walking speed in km/h?

b How long would it take Sean to walk 8.5 km at the same speed to the nearest minute?

Water is being pumped out of a pool at the rate of 1500 L/day. If the pool contains 12 000 L, how long will it take to empty the pool?

Car A travelled 185 km on 15.35 L of petrol.

a Calculate the petrol consumption of the car in litres per 100 km to one decimal place.

b If petrol costs 145.6 cents/litre, what is the cost of the petrol used?

A car travels 225 km on 24 L of petrol.

a How far could the car travel on 50 L of petrol assuming it used petrol at the same rate?

b What is the fuel consumption rate in L/100 km to one decimal place?

The scale on a map is 1:20 000. If the distance between two points on the map is 32 mm, find the actual distance on the ground between these two points in metres.

Express these times using the symbols for the units.

a 6 minutes 48 seconds
b 3 hours 55 minutes

Convert each of these times into the unit shown in brackets.

a 300 minutes (h)
b 1 minute 38 seconds (s)
c 180 seconds (min)
d 2 hours (s)

Convert these 12-hour times into 24-hour times.

a 7:34 am
b 5:48 pm

Convert these times into 12-hour times, indicating whether the time is am or pm.

a 14:30
b 09:24

Starting at March 12, how many days until May 20?

How many days from June 4 to June 18 inclusive?

A station clock shows this time:
If the next train is at 10:14, how long is it until the train departs? Give the time to the nearest minute.

Adam is flying in January from Sydney to Perth on a flight which leaves Sydney at 08:25. The flight time is 5 hours 5 minutes.

a What is the time in Sydney when Adam's flight arrives in Perth? Give the time in 24-hour time.

b What is the time in Perth when Adam's flight arrives?

Extended response questions

The Murraytown Annual Show committee has decided to limit the number of showbags to four different types: Flying Frogs, Glowing Grubs, Crunchy Critters and Bouncing Beetles. They have surveyed local children and found that the popularity of the four bags is 6:4:3:1. If the committee is ordering 3500 show bags, how many of each type should be ordered?

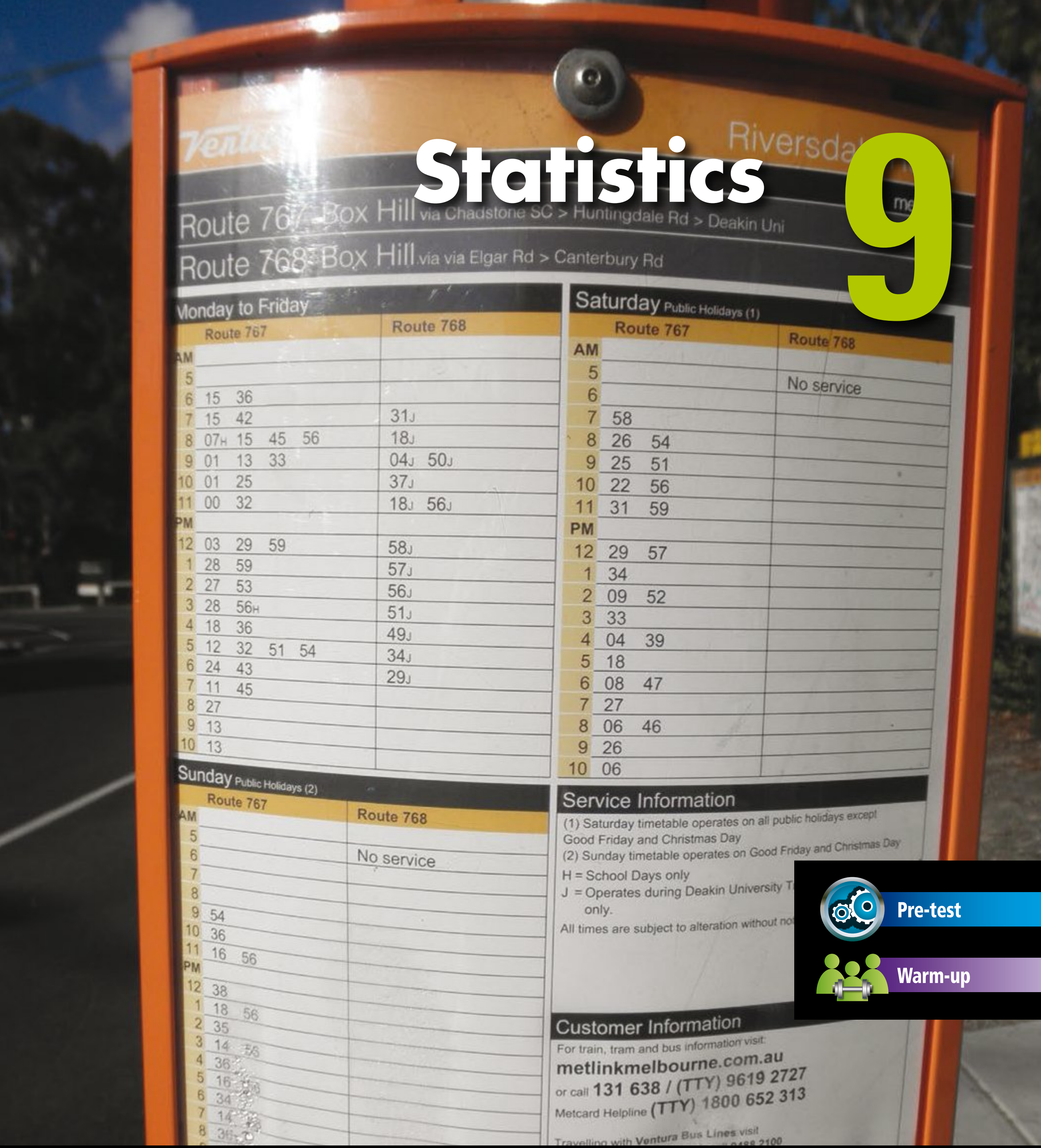
● The Hillworth Show committee has decided on the same four showbags for their annual show. However a survey in the area showed that the popularity of the four types: Flying Frogs, Glowing Grubs, Crunchy Critters and Bouncing Beetles is 7:3:3:2. If the committee is ordering 560 Flying Frogs bags, how many of the other three types will they order, and what is the total number of bags they will order?

● Kenji was finding the density of a piece of metal with an irregular shape. He placed some water in a beaker which had millilitre markings on the side, and observed how much the water rose when he placed the piece of metal in the water. The two water levels in the beaker were 120 mL and 198 mL. Kenji also found that the mass of the piece of metal was 210.6 g.

a Find the density of the metal.

b Another piece of the same metal was found to cause an increase of 84 mL in the level of water in the beaker. Calculate the mass of the piece of metal.

9 Statistics



Ventu

Riversda

Route 76 Box Hill via Chadstone SC > Huntingdale Rd > Deakin Uni

Route 768 Box Hill via via Elgar Rd > Canterbury Rd

Monday to Friday

	Route 767	Route 768
AM		
5		
6	15 36	
7	15 42	31J
8	07H 15 45 56	18J
9	01 13 33	04J 50J
10	01 25	37J
11	00 32	18J 56J
PM		
12	03 29 59	58J
1	28 59	57J
2	27 53	56J
3	28 56H	51J
4	18 36	49J
5	12 32 51 54	34J
6	24 43	29J
7	11 45	
8	27	
9	13	
10	13	

Saturday Public Holidays (1)

	Route 767	Route 768
AM		
5		No service
6		
7	58	
8	26 54	
9	25 51	
10	22 56	
11	31 59	
PM		
12	29 57	
1	34	
2	09 52	
3	33	
4	04 39	
5	18	
6	08 47	
7	27	
8	06 46	
9	26	
10	06	

Sunday Public Holidays (2)

	Route 767	Route 768
AM		
5		
6		No service
7		
8		
9	54	
10	36	
11	16 56	
PM		
12	38	
1	18 56	
2	35	
3	14 56	
4	36	
5	16 [illegible]	
6	34	
7	14 [illegible]	
8	36	

Service Information

(1) Saturday timetable operates on all public holidays except Good Friday and Christmas Day

(2) Sunday timetable operates on Good Friday and Christmas Day

H = School Days only

J = Operates during Deakin University T only.

All times are subject to alteration without no

Customer Information

For train, tram and bus information visit:

metlinkmelbourne.com.au

or call **131 638 / (TTY) 9619 2727**

Metcard Helpline **(TTY) 1800 652 313**

Travelling with **Ventura Bus Lines** visit

Pre-test

Warm-up

This bus timetable shows one way of representing data. For each hour through the day, the bus departure times are shown as minutes past the hour. In this respect, the design of the timetable resembles a stem-and-leaf plot. At what time of the day are buses most frequent on this route? How do the frequencies of buses relate to people's daily activities?

9.1 Reviewing types of data

Data is a collection of unprocessed information. The characteristic that is being measured, counted or observed is often referred to as a **variable** as it is likely to vary from one individual to another. By categorising, calculating, and displaying data in more useful ways, statistics helps us transform data into useful information.

Categorical and numerical data

Categorical data is data that can be grouped into categories, such as favourite colours, types of cars, country of birth, postcode.

Numerical data is data that is obtained by measuring or counting, such as height, number of flights per day from an airport. Numerical data can be added and averaged.

Example 1

Ten students were asked the following three questions.

Question	**Response data collected**
1 What is your least favourite season?	winter, summer, summer, spring, autumn, summer, autumn, spring, summer, summer
2 What year are you in at school?	year 8, year 7, year 8, year 9, year 7, year 8, year 7, year 7, year 10, year 7
3 How many people are in your family?	4, 3, 2, 3, 5, 7, 2, 7, 6, 3

Classify the responses to the questions as categorical data or numerical data.

Working	**Reasoning**
Categorical data	The responses are categories of seasons of the year.
Categorical data	The responses include numbers, but it would not make sense to add or average this data. Each year level is a category.
Numerical data	The responses are numbers which could be added (e.g. to find the total people in all 10 homes) and averaged (to find the mean number of people per home).

Example 2

For each of the following variables, give three examples of categories that would be possible.

a your mother's country of birth

b highest assignment grade obtained in year 7

c student's year of birth

d year level

Working	Reasoning
a Australia, Vietnam, Lebanon	Names of countries are categories.
b B+, A, C	Letter grades are categories, but if the assignment marks were given as numbers they would be numerical data.
c 1998, 1999, 2000	Even though years are numbers, it does not make sense to add or average them.
d year 7, year 12, year 3	Note that categories can include numbers. Even though year levels include numbers, it does not make sense to add or average them.

Categorical data

In example 2, the categories for mother's country of birth were just names (for example, Australia or Vietnam). This type of categorical data is called **nominal** categorical data. However, the categories for year level could be ordered and the assignment grade could be ranked. Categorical data that can be ordered or ranked is called **ordinal** categorical data.

Example 3

For each of the statistical variables in example 2, state whether they are nominal or ordinal categorical variables.

Working	Reasoning
a Nominal	Individual countries are just sub-groups of the country variable.
b Ordinal	Grades can be ranked.
c Ordinal	Year of birth can be ranked.
d Ordinal	Year level can be ranked.

Numerical data

It is helpful to think of numerical data as a variable that is counted or measured. For example, if you asked a group of students how many brothers they had, or how long their hand span was, it would be possible to collect data that you could add, rank, or average to provide more information.

If the numerical data can take only take particular values within a range, it is termed **discrete** numerical data. Discrete data is counted. For example, the number of cars passing per hour, children in a family, goals scored and shoe size are examples of discrete data.

A video store keeps a record of the number of new release DVDs loaned in a week.

Day	Number of new release DVDs on loan
Monday	23
Tuesday	47
Wednesday	32
Thursday	26
Friday	53
Saturday	67
Sunday	45

This data set is an example of discrete data. The units of measurement are DVDs and these can only take whole number values: 0, 1, 2, 3, … There is no possible value between 1 and 2 or 2 and 3.

If the data can take all values within a range it is referred to as **continuous** numerical data. Continuous data is measured. Height, weight, temperature and time elapsed are examples of continuous data.

The results of the final of the men's 400 m freestyle final at the 2000 Sydney Olympics are shown below.

Name	Time (minutes)
I. Thorpe (Aus)	3:40.59 (world record)
M. Rosolino (It)	3:43.40
K. Keller (US)	3:47.00
E. Brembilla (It)	3:47.01
D.C. Coman (Rom)	3:47.38
C. Carvin (US)	3:47.58
G. Hackett (Aus)	3:48.22
R. Neethling (SA)	3:48.52

This means 3 minutes and 43.40 seconds. Times are rounded to the nearest hundredth of a second.

We can think of continuous data as data that are measured and that still have meaning when broken down into smaller and smaller units.

Example 4

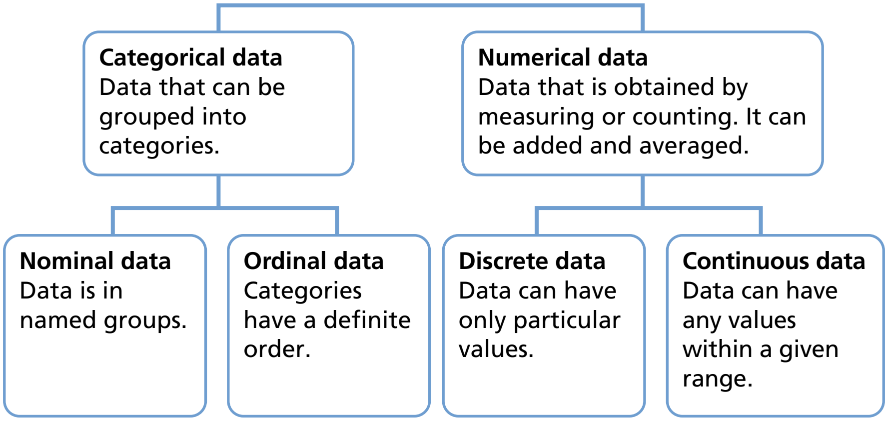
Classify these as discrete numerical or continuous numerical data.

a foot length	**b** shoe size

Working	**Reasoning**
a Foot lengths are continuous numerical data.	The length of a person's foot can vary continuously within a range, for example, 18.76 cm, 19.4 cm, 19.43 cm, and so on.
b Shoe sizes are discrete numerical data.	There are certain shoe sizes, for example, 5, 6, 7, sometimes with half sizes in between.

Summary of data types

The types of data are summarised in the following diagram.



- **Categorical data** Data that can be grouped into categories.
  - **Nominal data** Data is in named groups.
  - **Ordinal data** Categories have a definite order.
- **Numerical data** Data that is obtained by measuring or counting. It can be added and averaged.
  - **Discrete data** Data can have only particular values.
  - **Continuous data** Data can have any values within a given range.

Example 5

Classify the following as either categorical or numerical data. If it is numerical data, decide whether it is continuous or discrete. If it is categorical data, decide whether it is nominal or ordinal.

a students' ages in years	**b** length of forearm
c time waiting for a bus	**d** suburb in which student lives
e house number in street address	

Working	**Reasoning**
a Numerical (discrete)	Only whole year values possible.
b Numerical (continuous)	Any length measurement possible within reasonable limits.

continued

Example 5 continued

Working	Reasoning
c Numerical (continuous)	Any time measurement between reasonable limits.
d Categorical (nominal)	Each suburb is a category.
e Categorical (ordinal)	There are numbers with an order, but it makes no sense to add or average them.

exercise 9.1

LINKS TO **Example 1**

● Classify the following data as either categorical (C) or numerical (N).

- **a** age of a parent
- **b** points scored by a volleyball team
- **c** styles of dance
- **d** makes of cars
- **e** subjects offered at Year 12 at your school
- **f** lengths of shoes
- **g** responses to the survey question ‘What is your favourite icecream flavour?’
- **h** number of cars in car parks.

LINKS TO **Example 2**

● Which one of the following is an example of three categories that would be possible for the variable ‘year of birth’?

- **A** January, February, July
- **B** Australia, New Zealand, China
- **C** 1991, 2001, 2009
- **D** Monday, Wednesday, Sunday
- **E** Year 8 Blue, Year 8 Gold, Year 8 Red

● For the following survey questions, find a way of rewording the question so that the responses would contain numerical data rather than categorical data.

- **a** Do you watch television each night?
- **b** What is your favourite fruit?
- **c** Which one of the following phrases best describes your height relative to your classmates?

  Taller than most   *Similar to most*   *Shorter than most*

● For the following survey question, find a way of rewording the question so that responses would contain categorical data rather than numerical data.

- **a** How many types of plants do you have in your garden?
- **b** What is the total number of minutes that you spend travelling on public transport each week?
- **c** What is your age?

LINKS TO Example 3

Which one of the following is *not* an example of an ordinal categorical variable?

A Year level at school
B Pizza sizes (small, medium, etc)
C Letter grade on a maths test
D Favourite colour
E Year of birth

LINKS TO Example 4

Imagine that you are explaining to a student, who was absent, the difference between continuous numerical and discrete numerical data. How would you explain it to them?

LINKS TO Examples 3, 4

For each of the following survey items, decide whether the response data would be nominal categorical (NC), ordinal categorical (OC), discrete numerical (DN) or continuous numerical (CN). In each case, also give a sample response.

a How many children are there in your family?
b What is the length of your bike?
c List your three favourite colours.
d How often do you go to the city (never/sometimes/often)?
e What colour is your hair?
f How many times have you travelled out of your state?
g Do you own a mobile phone?
h How far did you travel to come to school today?
i How many emails have you sent this week?
j In what year were you born?

Give three examples, not already mentioned in this exercise, which are

a categorical but not nominal
b numerical and discrete
c numerical and continuous.

exercise 9.1 challenge

Edith lives close to the city and each morning many people park their cars in her street in order to be closer to public transport. Edith is interested in the cars that park in her street. One morning she collects data about the cars parked in the street. Give examples of two variables related to the cars that are

a nominal categorical.
b ordinal categorical.
c discrete numerical.
d continuous numerical.

9.2 Frequency tables and grouped data

A **frequency table** can be used to organise raw data that has been collected. It has a column representing every possible data value and one to display the frequency of each of these data values, which is the number of times each data value occurs. An additional column headed 'Tally' is often included to assist with organising the data.

Remember that numerical data can be discrete (counted) or continuous (measured). If discrete numerical data is collected then a frequency table can be constructed simply by tallying how many times each data value occurs.

Example 6

For a maths project, Carla conducts a survey to determine the number of children living in each house in her street. The raw data is given below.
2, 3, 2, 2, 2, 4, 1, 3, 3, 0, 0, 0, 2, 1, 2, 4, 2, 2, 0, 3, 2, 0, 1, 2

a Display the data using a tally column and a frequency column.

b How many houses were included in Carla's survey?

c What is the most common number of children?

d How many houses had no children living there?

Working

Reasoning

a

Number of children	Tally	Frequency
0	𝍸	5
1	|||	3
2	𝍸 𝍸	10
3	||||	4
4	||	2
Total		24

The first column has the number of children per house which can take values 0 to 4.

In the tally column | represents one observation of that value. Use 𝍸 to bundle the data into groups of 5.

The frequency column is filled in by counting the tallies for each value.

b There were 24 houses included in Carla's survey.

This is the total for the frequency column.

c The most common number of children living in a house is 2.

This is the value with the highest frequency.

d There were 5 houses with no children living there.

This is the frequency for the value 0.

Using intervals

It is often convenient to display data in intervals, particularly if the data is continuous; for example, if you are recording student heights correct to the nearest centimetre.

When representing a set of discrete data in intervals of 5, we would use intervals such as 130–134, 135–139, etc. With continuous data, we use intervals such as 130–<135, 135–<140, and so on, as shown in the following example where 130-<135 means 'from 130 up to but not including 135', and so on.

Example 7

The following data gives the height of 30 Year 8 students measured correct to the nearest centimetre.

144, 144, 150, 150, 152, 152, 153, 155, 156, 158, 159, 160, 160, 161, 163,
163, 163, 163, 164, 164, 168, 168, 169, 169, 169, 170, 172, 173, 178, 187

a What is the shortest height measurement for a Year 8 student?

b What is the tallest height measurement for a Year 8 student?

c If the data is to be recorded in a frequency table using class intervals of 5 cm,

  i what is the first class interval? **ii** what is the last class interval?

d Display the data in a frequency table using columns with headings height interval, tally and frequency.

e Which height interval has the greatest frequency?

Working	Reasoning
a The smallest value of the data is 144 cm. The shortest height of a Year 8 student is 144 cm.	It is easier to prepare the frequency table if the data is sorted into order first.
b The largest value of the data is 187 cm. The tallest height of a Year 8 student is 187 cm.	
c i The first class interval is 140−<145 cm	Since the smallest value of the data is 144 and when working in class intervals of 5 cm it is logical to start the interval at a multiple of 5.
ii The last class interval is 185−<190 cm	185−<190 means from 185 up to but not including 185. This is as high as we have to go to get to 187 cm.

continued

Example 7 continued

Working

d

Height interval	Tally	Frequency
140–<145	||	2
145–<150		0
150–<155	𝍸	5
155–<160	||||	4
160–<165	𝍸 ||||	9
165–<170	𝍸	5
170–<175	|||	3
175–<180	|	1
180–<185		0
185–<190	|	1

e The height interval with the greatest frequency is 160–<165 cm.

Reasoning

Remember the use of 𝍸 which helps to bundle the data into groups of 5.

The frequency column is completed by working out the total of the tally for each of the height intervals.

This heights of 9 students are in the interval 160–<165 cm.

Example 8

At a paper-plane competition, the following flight distances (in centimetres) were recorded by the judges.

167, 145, 206, 190, 182, 250, 177, 196, 287, 200, 280, 174, 218, 295, 227,
265, 238, 197, 318, 249, 188, 252, 259, 231, 268, 270, 206, 239, 222, 241

Organise this data into a frequency table with class intervals of 30 centimetres.

Working

Distances travelled by paper planes (cm)	
Class interval (cm)	**Frequency**
140–<170	2
170–<200	7
200–<230	6
230–<260	8
260–<290	5
290–<320	2

Reasoning

When working in class intervals of 30, it is logical to start each interval at a multiple of 10. The smallest value of the data is 145, so make the first class interval 140−<170.

The last class interval is 290−<320, since the largest value of the data is 318.

Although a tally column is useful to ensure that we do not miss any data values, it is not necessary to include the tally column.

exercise 9.2

LINKS TO Examples 6, 7

An English teacher gave a spelling test marked out of 10 to her class. The results are shown below.

8, 6, 10, 9, 8, 6, 5, 10, 4, 10, 8, 6, 7, 9, 9, 10, 8, 6, 5, 8, 10, 7, 9, 10, 6, 7, 6, 10, 9, 7

a Use a frequency table to organise this data.
b How many students sat the test?
c Which test score was scored the most?
d What is the difference between the highest and lowest test score?
e What fraction of the test scores were more than 8?

LINKS TO Examples 6, 7

In an archery tournament an archer fires 30 arrows at a target in one round. 10 is the top score and 1 is the lowest score you can get with an arrow that hits the target. Elio's scores for two rounds are listed below.

10, 9, 7, 2, 1, 10, 8, 7, 6, 6, 5, 2, 1, 7, 6, 7, 3, 10, 6, 2, 4, 8, 5, 6, 3, 10, 8, 4, 3, 2,
8, 8, 7, 6, 8, 9, 1, 2, 4, 6, 5, 9, 3, 10, 8, 4, 3, 8, 7, 7, 10, 9, 7, 4, 9, 4, 8, 5, 9, 9

a Tally the results and the frequency for each score.
b What was Elio's most common score?
c What was Elio's least common score?
d Consider the distribution of Elio's scores and comment on his overall performance.

Which one of the following statements is *not* correct about the frequency table at right?

A The frequency of the value 5 is 1.
B The most commonly occurring value is 4.
C There were 13 observations in total.
D The value 1 had no observations.
E The value 3 had a frequency lower than the frequency of the value 4.

Value	Frequency
0	1
1	4
2	0
3	2
4	5
5	1

LINKS TO Example 8

Twelve students entered a long jump competition. Their best jumps (in centimetres) were as follows.

171, 184, 191, 173, 182, 176, 178, 182, 190, 183, 179, 174

Copy and complete the frequency table.

Class interval (cm)	Frequency
170−<175	3
175−<180	
180−<185	
185−<190	
190−<195	
Total	

A survey was conducted where students were asked to estimate how many hours in one week they would do some form of exercise. The results of the survey are summarised in the frequency table below.

Number of hours	Frequency
0–2	3
3–5	7
6–7	18
8–9	9
≥10	3
Total	

a How many students were surveyed?
b How many students did less than 3 hours of exercise each week?
c How many students did 8 hours or more of exercise each week?
d What percentage of the total number of students surveyed did 6–7 hours of exercise per week?

LINKS TO Example 8

Fifty people were surveyed and asked to take their own pulse by counting the number of beats in one minute. Their pulse rates in beats per minute are shown below.

63, 72, 58, 67, 58, 69, 80, 80, 90, 82, 84, 76, 93, 76, 78, 60, 68, 75, 84, 89, 92, 98, 103, 97, 68, 76, 80, 82, 80, 78, 94, 95, 87, 96, 93, 85, 76, 87, 92, 68, 76, 84, 108, 96, 75, 73, 65, 67, 62, 74

a What were the maximum and minimum pulse rates?
b Tally the data into class intervals of 5 beats per minute and calculate the frequency of each interval.
c What was the most common class interval for pulse rate?

Year 8 students at a school were asked how time long it took them to travel to school on a particular day. Their times, in minutes, are given below.

31, 32, 45, 26, 36, 37, 41, 24, 26, 23,
24, 45, 36, 67, 35, 38, 43, 44, 14, 52,
43, 35, 23, 62, 23, 55, 33, 44, 55, 25,
22, 35, 65, 69, 25, 42, 56, 13, 34, 28,
65, 52, 33, 75, 76, 43, 34, 22, 47, 71

Construct a grouped frequency table using class intervals of 10 to display this data.

A class of 30 students were surveyed about the number of hours of TV they watched during the last week of term. The raw data is given below.

25, 26, 3, 8, 14, 22, 15, 5, 9, 18, 20, 12, 14, 31, 23, 9, 8,
22, 20, 9, 7, 21, 12, 10, 0, 27, 16, 12, 21, 7

a Claire wants to construct a frequency table using class intervals of 5 to display the data. She sets up her table using the intervals 1–5, 6–10 and so on. Explain what is wrong with her choice of intervals.

b Construct a frequency table using class intervals of 5 to display the data.

The data below are the lengths of 45 randomly selected leaves from branches of a palm tree measured correct to the nearest centimetre.

81, 72, 68, 73, 35, 27, 90, 51, 84, 48, 50, 43, 52, 58, 47, 76, 29, 34, 92, 97, 46, 78, 65, 69, 66, 93, 82, 47, 72, 75, 73, 28, 45, 47, 46, 51, 43, 85, 84, 29, 47, 82, 71, 58, 76

a Complete the following frequency table for this data.

Length interval (cm)	Tally	Frequency
20–<30		
30–<40		
40–<50		
50–<60		
60–<70		
70–<80		
80–<90		
90–<100		

b Complete another frequency table for this data using intervals 20–<40, 40–<60, and so on.

c Complete another frequency table for this data using intervals 20–<60 and 60–<100.

d Which size of interval do you think best represents the length of a leaf of a palm tree?

exercise 9.2 challenge

The frequency table below summarises the data obtained for one fisherman in a fishing competition. The lengths of 30 fish caught by the fisherman were measured correct to the nearest centimetre.

Write down a set of 30 lengths of fish that could be represented by this frequency table.

Length interval (cm)	Frequency
0–<10	9
10–<20	13
20–<30	4
30–<40	3
40–<50	1

9.3 Summarising data: measures of centre and spread

There are several ways of describing the centre or middle of a set of data.

- using the mean
- using the median
- using the mode.

Mean

The **mean** of a set of data values is calculated by adding together all the data values then dividing by the total number of data values.

$$\text{Mean} = \frac{\text{sum of all values}}{\text{number of values}}$$

Example 9

Calculate the mean of these five heights.
148 cm, 154 cm, 152 cm, 159 cm, 151 cm

Working	Reasoning
$\text{Mean} = \frac{148 + 154 + 152 + 159 + 151}{5}$	Add the five heights. Divide by 5.
$= \frac{764}{5}$	
$= 152.8$	

The mean height is 152.8 cm.

Calculating the mean for data in a frequency table

If the data is presented in a frequency table, then the mean can be calculated by multiplying each value, x, of the data by its frequency, f, adding these results then dividing by the total number of data values.

$$\text{Mean} = \frac{\text{sum of all values}}{\text{number of values}} = \frac{\text{sum of all } (x \times f)}{\text{number of values}}$$

Example 10

Visitors to a library were surveyed about the number of fiction books they had read in the past four weeks. The results are summarised in this frequency table.
Calculate the mean number of fiction books read by the 60 people surveyed.

Number of fiction books read in four weeks (x)	Number of people (f)
0	4
1	5
2	8
3	22
4	15
5	6

Working

Number of fiction books read in four weeks (x)	Number of people (f)	$x \times f$
0	4	$0 \times 4 = 0$
1	5	$1 \times 5 = 5$
2	8	$2 \times 8 = 16$
3	22	$3 \times 22 = 66$
4	15	$4 \times 15 = 60$
5	6	$5 \times 6 = 30$
Total	60	177

$$\text{Mean} = \frac{177}{60} = 2.95$$

The mean number of fiction books read is 2.95.

Reasoning

4 people read no books, 5 people each read 1 book, 8 people read 2 books, and so on. Multiply each value of the data by its frequency.
Add these results then divide by the total number of observations.
Note that the mean is not always a whole number.

Example 11

A Year 8 class of 25 students was surveyed regarding the number of siblings they had. The results of the survey are summarised in the frequency table below.

Find the mean number of siblings for this class.

Siblings are brothers and sisters.

Number of siblings (x)	Frequency (f)
0	3
1	7
2	8
3	4
4	2
5	1

continued

Example 11 continued

Working

Number of siblings (x)	Frequency (f)	$x \times f$
0	3	0
1	7	7
2	8	16
3	4	12
4	2	8
5	1	5
Total	25	48

Reasoning

Add a column to the table and fill this in by multiplying each number of siblings (x) by its frequency (f).
Add a row for the totals.
Add the values in the new column to give the total.

$$\begin{aligned}\text{mean} &= \frac{\text{sum of all values}}{\text{number of values}}\\ &= \frac{48}{25}\\ &= 1.92\end{aligned}$$

The mean number of siblings is 1.92.

Use the formula to calculate the mean. The sum of all values has been found in the table and there were 25 data values because 25 students were surveyed.

Median

The **median** of a set of data is the middle value of the data when it is ordered. The data can either be arranged in ascending order (smallest to largest) or descending order (largest to smallest).

Finding the median

For an *odd* number of data values, the median is the middle value.

For an *even* number of data values, the median is half-way between the two middle values.

Example 12

Find the median.

a Find the median of the following set of test scores out of 45.

15, 42, 38, 26, 44, 36, 31, 29, 33, 26, 36, 40, 36, 42, 33, 25, 27, 34

b One student was away on the day of the test and sat it on the day she returned to school. Her score was 38. Find the median of the test scores if this student's score is now included in the data.

continued

Example 12 continued

Working	**Reasoning**
a 15, 25, 26, 26, 27, 29, 31, 33, 33, 34, 36, 36, 36, 38, 40, 42, 42, 44	Arrange the data in order.
15, 25, 26, 26, 27, 29, 31, 33, 33, $\mid$ 34, 36, 36, 36, 38, 40, 42, 42, 44	The median is the middle value. There is an even number of data values, so the median is half-way between the two middle values.
Median $= \dfrac{33 + 34}{2}$ $= 33.5$	The median occurs half way between the 9th and 10th values of the data. Average the two values to find the number half way between them.
b 15, 25, 26, 26, 27, 29, 31, 33, 33, 34, 36, 36, 36, 38, $\underline{38}$, 40, 42, 42, 44	Include the new value in the correct place so that the data remains in order.
15, 25, 26, 26, 27, 29, 31, 33, 33, $\boxed{34}$, 36, 36, 36, 38, 38, 40, 42, 42, 44 Median = 34	The median is the middle value. There are now 19 values. With an odd number of data values, the median is the middle value.

Mode

The **mode** is the most frequently observed or most common value of the data. Some sets of data may have more than one mode. Data that has two modes is said to be **bimodal**.

Example 13

Find the mode for each of the following sets of test scores.

a 15, 42, 38, 26, 44, 36, 31, 29, 33, 26, 36, 40, 36, 42, 33, 25, 27, 34.

b 21, 32, 33, 45, 48, 45, 29, 32, 33, 41, 43, 49, 26, 19, 33, 38, 45, 18, 22

Working	**Reasoning**
a The mode is 36.	This is the most commonly occurring value.
b The modes are 33 and 45	Both the values 33 and 45 occur three times. This data set is bimodal because the most frequent scores, 33 and 45, both occur three times.

Comparing the measures of centre

Measure of centre	Advantages	Disadvantages
Mean	Every piece of data is used in calculating the mean. If one data value changes, the mean will also change. In this way, the mean represents all the data.	The mean can be affected by extreme values in the data. For example, adding one extremely high house price to a set of data can make it appear that the mean cost of housing has risen significantly.
Median	When there are a few extreme values in a set of data, the median can be a fairer measure of centre than the mean. For example, is the mean average salary a good measure of centre if the managing director is the only person in a company who earns more than that amount?	Although all of the data values are put in order to find the median, changing one or more of the values might not affect the median. Therefore the median does not 'include' each individual value in the way that the mean does.
Mode	The mode is useful for analysing data that cannot be 'averaged', such as postcodes, eye colour or yes–no responses to a question.	When a set of data has more than one mode, this might not provide a useful measure of centre. The mode does not 'include' each individual value in the way that the mean does.

Measuring spread

Range

The **range** of a set of data is the difference between the highest and lowest values of the data.

Range = highest data value – lowest data value

Example 14

Felicity has a worm farm and is investigating the length of the worms. She collects 20 worms from the farm and measures their length. The results, in centimetres, are

6, 11, 10, 9, 11, 16, 8, 15, 7, 8, 12, 10, 7, 9, 10, 5, 3, 7, 12, 8

Find

a the mean worm length

b the median worm length

c the mode worm length

d the range

Working	**Reasoning**
a Mean $= \dfrac{\text{sum of all values}}{\text{number of values}} = \dfrac{184}{20} = 9.2$ The mean length is 9.2 cm.	Add all the data values and divide by 20.
b 3, 5, 6, 7, 7, 7, 8, 8, 8, 9, 9, 10, 10, 10, 11, 11, 12, 12, 15, 16 The median length is 9 cm.	Arrange the data values in order. The median is the middle value. There is an even number of values so the median is halfway between the two middle values. In this case the two middle values are both 9.
c The modal lengths are 7 cm, 8 cm and 10 cm.	There are three modes because 7 cm, 8 cm and 10 cm each occur 3 times.
d Range = highest value – lowest value = 16 cm – 3 cm = 13 cm	When the data values are arranged in order it is easy to see the highest and lowest values.

Example 15

Find the range for the following sets of test scores.

a 15, 42, 38, 26, 44, 36, 31, 29, 33, 26, 36, 40, 36, 42, 33, 25, 27, 34

b

score	*f*
5	4
6	2
7	3
8	4
9	2
10	3

continued

Example 15 continued

Working	Reasoning
a The largest value is 44. The smallest value is 15. $\text{Range} = 44 - 15$ $= 29$	Find the largest and smallest values. The range is the difference between the largest value and the smallest value.
b The largest value is 10. The smallest value is 5. $\text{Range} = 10 - 5$ $= 5$	Find the largest and the smallest values. These are in the score column. The range is the difference between the largest value and the smallest value.

Outliers and their effect on measures of centre

Extremely small and extremely large data values that lie beyond the rest of the data values are called **outliers**. There may be more than one outlier in a data set.

Because the median is the middle value of a data set, changing the size of the smallest or largest value so that it becomes an outlier will not have any effect on the median. However, changing the size of the smallest or largest data value will change the sum of the data values and, hence, change the mean. Removing outliers from a data set will, therefore, affect the mean more than it affects the median. Removing outliers will not affect the mode.

Example 16

Students participating in the CensusAtSchool project were asked to complete a concentration task (unassisted, under supervision) and to record the time they took. The times (in seconds) for a sample of 30 students were: 40, 42, 129, 98, 68, 40, 43, 66, 57, 35, 51, 46, 76, 68, 37, 56, 46, 51, 23, 44, 34, 64, 52, 33, 50, 33, 42, 50, 42, 33.

a Arrange the times in ascending order.

b Calculate the mean time to one decimal place.

c What is the median time?

d Which data value is an outlier?

e What are the mean and median if this outlier is removed?

f How does the range change if the outlier is removed?

Working	Reasoning
a 23, 33, 33, 33, 34, 35, 37, 40, 40, 42, 42, 42, 43, 44, 46, 46, 50, 50, 51, 51, 52, 56, 57, 64, 66, 68, 68, 76, 98, 129	Ascending order is from smallest to largest.
b $\text{Mean} = \frac{1549}{30}$ $\approx 51.633\ldots$ The mean time is 51.6 seconds.	The mean is found by adding all the data values and dividing by the number of data values.

continued

Example 16 continued

Working	Reasoning
c The median time is 46 seconds.	The two middle values are 46 seconds.
d The time of 129 seconds is an outlier.	The time of 129 seconds is 31 seconds longer than the next longest time.
e Mean $= \frac{1420}{29}$ $\approx 48.965\ldots$	When the outlier is removed there are 29 data values.
When the outlier is removed, the mean time is 49.0 but the median time is still 46 seconds.	The middle value is 46 seconds.
f Range when outlier is included $= 129 - 23$ $= 106$	The range is the difference between the largest data value and the smallest data value.
Range when outlier is removed $= 98 - 23$ $= 75$	
The range becomes smaller if outliers are removed.	Answer the question.

Tech tip

Summarising data with spreadsheets

Most spreadsheet software includes features for summarising statistical data. For example, Microsoft Excel has a range of inbuilt functions for calculating **summary statistics** (numbers that summarise a set of data).

Suppose the data were entered into cells A6 to A17. The following formulas can be entered to calculate summary statistics for this data.

Summary statistic	Spreadsheet formula used
Number of data values (n)	=COUNT(A6:A17)
Total	=SUM(A6:A17)
Mean	=AVERAGE(A6:A17)
Median	=MEDIAN(A6:A17)
Mode	=MODE(A6:A17)
Minimum (smallest value)	=MIN(A6:A17)
Maximum (largest value)	=MAX(A6:A17)

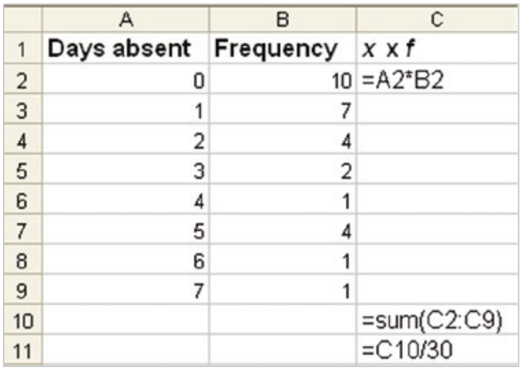
Example 17

This frequency table shows the number of days absent in term 2 for a class of 30 students.

Use a spreadsheet to find the mean number of days absent per student.

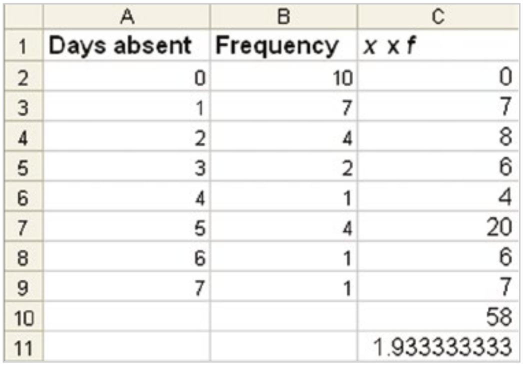
Days absent	Frequency
0	10
1	7
2	4
3	2
4	1
5	4
6	1
7	1

Working



	A	B	C
1	Days absent	Frequency	x x f
2	0	10	=A2*B2
3	1	7	
4	2	4	
5	3	2	
6	4	1	
7	5	4	
8	6	1	
9	7	1	
10			=sum(C2:C9)
11			=C10/30

Used with permission from Microsoft



	A	B	C
1	Days absent	Frequency	x x f
2	0	10	0
3	1	7	7
4	2	4	8
5	3	2	6
6	4	1	4
7	5	4	20
8	6	1	6
9	7	1	7
10			58
11			1.933333333

Used with permission from Microsoft

The mean number of days absent is 1.9.

Reasoning

Enter the data in columns A and B.
In cell C2 enter the formula =A2*B2 to calculate $x \times f$. Copy the formula down to cell C9.
In cell C10 enter the formula =sum(C2:C9) to calculate the sum of the $x \times f$ values.
In cell C11, enter the formula =C10/30 to find the mean number of days absent for the 30 students.

Example 18 uses the same data as example 14 to show how a spreadsheet can be used to calculate the mean, median and mode.

Example 18

Felicity has a worm farm and is investigating the length of the worms. She collects 20 worms from the farm and measures their length. The results, in centimetres, are

6, 11, 10, 9, 11, 16, 8, 15, 7, 8, 12, 10, 7, 9, 10, 5, 3, 7, 12, 8

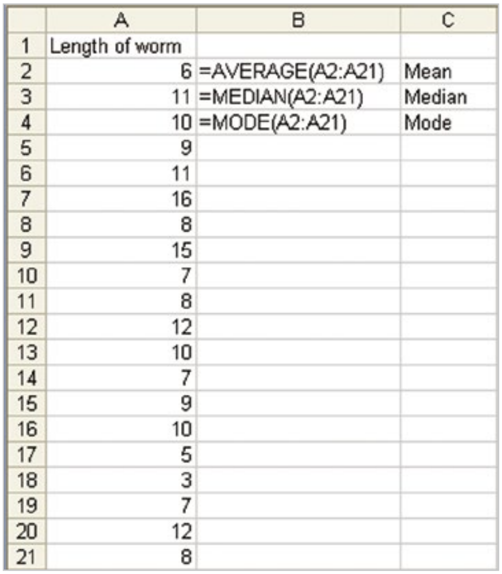
Use a spreadsheet to find

a the mean worm length

b the median worm length

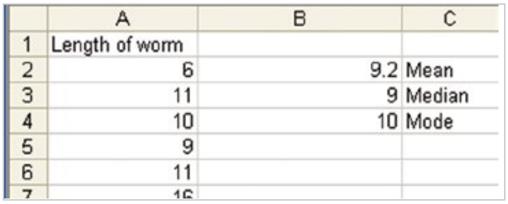
c the mode worm length

Working



	A	B	C
1	Length of worm		
2	6	=AVERAGE(A2:A21)	Mean
3	11	=MEDIAN(A2:A21)	Median
4	10	=MODE(A2:A21)	Mode
5	9		
6	11		
7	16		
8	8		
9	15		
10	7		
11	8		
12	12		
13	10		
14	7		
15	9		
16	10		
17	5		
18	3		
19	7		
20	12		
21	8		

Used with permission from Microsoft



	A	B	C
1	Length of worm		
2	6	9.2	Mean
3	11	9	Median
4	10	10	Mode
5	9		
6	11		
7	16		

Used with permission from Microsoft

Reasoning

Enter the data in column A, cells A2 to A21 then use the formulas given to find the required statistics.

Tech tip

The TI-30XB MultiView calculator can be used to find the mean and median of a data set.

Press **data** then enter the data in L1. Type one data value then press **enter** and continue this process until all data values are entered in the list. Press **2nd** **data** for [**stat**] and select 1-Var Stats by pressing **1**. The data is in L1 so make sure that L1 is highlighted by pressing ▶ if it is not already highlighted then press **enter**. For FRQ make sure that ONE is highlighted then press **enter**. CALC will now be highlighted in the bottom right of the screen. Press **enter**. The mean is $\bar{x}$. Use the ▼ to find the median which is denoted by Med.

exercise 9.3

LINKS TO Example 9

Calculate the mean of these sets of numbers.

a 21, 24, 35, 18, 41, 19, 27, 26, 31, 36, 16, 24

b 7.2, 3.4, 5.3, 6.4, 3.7, 4.5, 5.9, 1.2, 6.8, 5.6, 7.4, 4.1, 5.7, 2.9, 6.7

LINKS TO Examples 10, 11

Find the mean for each of the following. Give answers correct to two decimal places.

a

x	f
1	4
2	2
3	3
4	4
5	2

b

x	f
0	3
1	1
2	5
3	2
4	3

c

x	f
10	5
11	3
12	2
13	4
14	1

LINKS TO Examples 10, 11

The table at right shows the number of pets owned by a group of 30 people surveyed at the pet shop.
What is the mean number of pets, correct to one decimal place?

Number of pets	Frequency
1	9
2	5
3	6
4	4
5	3
6	3
Total	30

LINKS TO Example 12

Find the median of each of the following data sets.

a 7, 16, 24, 37, 72, 84, 95
b 2, 1, 5, 7, 8, 3, 2, 5, 6, 9, 3
c 113, 112, 115, 118, 117, 114, 111, 112, 119
d 18.3, 14.7, 16.6, 12.5, 15.4, 20.3, 19.7, 13.2

LINKS TO Example 13

The mode for the data set 9, 1, 7, 5, 8, 9, 8, 3, 9 is

A 5 **B** 6 **C** 7 **D** 8 **E** 9

For the data set, 18, 13, 15, 17, 18, 15, 13, 18, 16, 13, 11 the mode is

A 13 **B** 15 **C** 18 **D** 13 and 18 **E** 13 and 15

LINKS TO Example 14

Find the mean, median and mode for each of the following sets of data to one decimal place, where appropriate. Use Excel to check your answers.

a 3, 4, 5, 5, 6, 7, 9, 10, 10, 10, 12, 15, 17, 20
b 17, 13, 38, 22, 15, 20, 45, 62, 18, 37, 56, 45, 61, 24, 17
c 35.8, 47.2, 29.7, 36.5, 45.8, 31.3, 21.1, 43.7, 39.6, 24.5, 22.3
d −5, 6, −3, 3, 2, 4, −7, −5, 2, 10, −7, 9, −5, 3, 2, 6, −5, 7, 1, 0, 3

LINKS TO Example 14

The weekly salaries of six casual employees at a department store are $160, $220, $100, $180, $160, $220.

Find the mean, median and mode for this set of data. (Find the mean to the nearest cent.)

In eight games of netball, Bianca scored 13, 28, 14, 13, 15, 12, 14 and 14 points.

a Find the mean, median and mode.
b She wants to convince her coach that she should be made goal shooter for the team. Which measure of centre should she choose? Explain.

Carlo's wages for 6 weeks of working at McDonald's were: $140, $90, $260, $135, $90, $125.

a Find the mean, median and mode.
b He wants to convince his parents that he should still receive pocket money. Which measure of centre should he choose? Explain.

Clare phoned seven friends on Tuesday night on her mobile phone. The number of minutes each call lasted was 25, 32, 9, 4, 16, 25, 3.

a Find the mean, median and mode to one decimal place, where appropriate.
b She wants to convince her parents that she didn't spend too much time on the phone on Tuesday. Which measure of centre should she choose? Explain.

The shoe sizes for a group of primary school children are shown in this table.

Shoe size	Frequency
3	3
4	8
5	5
6	4
7	3
8	1

a Use Excel to find the mean shoe size to one decimal place.
b Find the median and modal shoe size.
c If Danny wants to make the point that this group of students have small feet, should he use the mean, the median or the mode? Explain.

LINKS TO Example 15

Find the range of each of the following data sets.

- **a** 3, 4, 5, 5, 6, 7, 9, 10, 10, 10, 12, 15, 17, 20
- **b** 17, 13, 38, 22, 15, 20, 45, 62, 18, 37, 56, 45, 61, 24, 17
- **c** 35.8, 47.2, 29.7, 36.5, 45.8, 31.3, 21.1, 43.7, 39.6, 24.5, 22.3
- **d** 18.3, 14.7, 16.6, 12.5, 15.4, 20.3, 19.7, 13.2, 21.4, 15.6
- **e** 415, 624, 1036, 571, 348, 179, 916, 1357, 1468, 1521, 529, 631
- **f** −5, 6, −3, 3, 2, 4, −7, −5, 2, 10, −7, 9, −5, 3, 2, 6, −5, 7, 1, 0, 3

The Stayhere hotel chain owns a variety of hotels which are rated from 1 star to 5 star. The ratings of their hotels are summarised in the following frequency table.

Star rating	Frequency
1	2
2	4
3	8
4	10
5	2

- **a** What is the mean star rating of a Stayhere hotel to one decimal place?
- **b** Find the median and mode star ratings.
- **c** If the hotel chain was advertising, which of the mean, median or mode would give the most favourable representation of the star ratings of the hotels in the chain? Explain.

The mean of a group of six numbers is 8. What might the numbers be?

Write down a group of seven different numbers that have a median of 7.

A group of eight numbers have a median of 10. What might the numbers be?

Marika scored grades of 85, 64 and 77 on three tests. What grade must she achieve on the next test to have an average grade of exactly 80?

The average height of a group of students is 148 cm. If the sum of their heights is 740 cm, how many students are in the group?

exercise 9.3 challenge

- **a** Find a set of eight numbers that have a mean of 10, a median of 12 and a range of 20.
- **b** Find the effect on the mean when a set of scores has each score doubled.
- **c** Find the effect on the mean when a set of scores has each score increased by 20%.

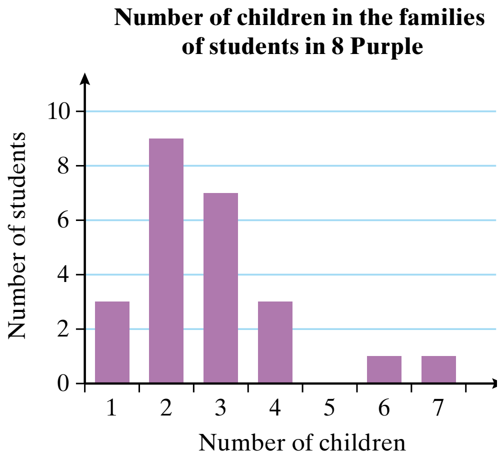
9.4 Displaying data

Data can be displayed in a table or as a graph. Graphs have the advantage that they are visual displays where we can immediately see patterns in the data. The type of graph we choose must be suitable for the type of data. A summary of visual displays and the types of data for which they are suitable is given below. Included are bar and column graphs, dot plots, pie charts and stem-and-leaf plots that you have used in Year 7. Another type of graph, the histogram, is also included. You will meet histograms again in section 9.5.

Bar graph or column graph

- Often the best choice for *categorical* variables. Also appropriate for *discrete numerical* variables.
- Each category has its own labelled 'bar' or column.
- It is usual to show the *category* along the horizontal axis and the *frequency* along the vertical axis.

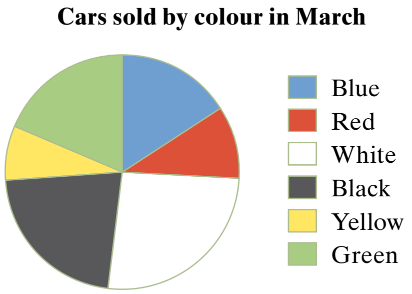
Example



Pie chart

- Pie charts are suitable for visually comparing categorical data. Discrete numerical data can also be displayed in a pie chart.
- They are used to display each value or category of the data as part of a whole. This makes it easy for comparisons to be made.
- The sectors of the pie chart may be labelled with actual numbers or with percentages.
- If there are too many data categories then the pie chart becomes too complex to interpret easily.
- If the data include some very small values, for example, 0.5%, it will be difficult to show these.

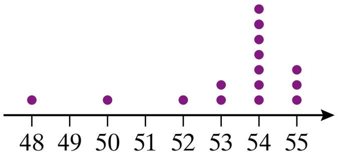
Example



Dot plot

- Best for a small set of *discrete numerical* data where the values are reasonably close together in size.
- Provides a quick view of the 'shape' of a distribution.
- Makes it easy to identify any clusters or outliers.
- Dot plots show each individual data value.

Example



Number of matches in a match box

Stem-and-leaf plot

- Appropriate for *discrete numerical* data, where the values are reasonably close together in size.
- The data is 'grouped' (into rows) but each of the actual values can still be read from the plot.
- Provides a quick view of the 'shape' of a distribution; looks a little like a column graph or dot plot on its side.
- Show each individual data value.
- Use split stems to provide more detail.
- To show large or small data values in a stem-and-leaf plot, the values may first need to be rounded.

Example

Number of words per sentence

Stem	Leaf
1	0 1 3 4 4
1	5 6 6 8 9
2	2 3 3 4 4
2	5 7 7
3	2 2 3
3	7
4	0

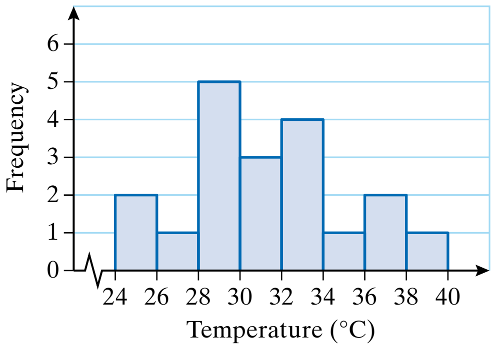
(Key: 1|5 means 15)

Histogram

- Appropriate for *continuous numerical* data.
- The horizontal axis has a continuous numerical scale. The vertical axis shows the *frequency*.
- The data is grouped into *intervals* (for example 24−<26, 26−<28, 28−<30, and so on).
- The numbers along the horizontal axis label the marks *between* intervals, not the columns themselves.
- To display more detail, decrease the size of the class interval.

Example

January temperatures in Year 10C's classroom



Example 19 shows the use of a pie chart to display categorical data.

Example 19

The results of a survey in which students were asked 'What is your favourite type of movie?' are shown in the table.

a Construct a pie chart to display this data.

b Suggest why a pie chart is suitable for displaying this data.

c Construct a different type of display that would be suitable for this data

Favourite type of movie	**Number of students**
Drama	14
Comedy	26
Kids	12
Horror	10
Action	18
Other	10

Working

a Calculating pieces of the pie:
There is a total of 90 observations.

Drama: $\frac{14}{90} \times 360^\circ = 56^\circ$

Comedy: $\frac{26}{90} \times 360^\circ = 104^\circ$

Kids: $\frac{12}{90} \times 360^\circ = 48^\circ$

Horror: $\frac{10}{90} \times 360^\circ = 40^\circ$

Action: $\frac{18}{90} \times 360^\circ = 72^\circ$

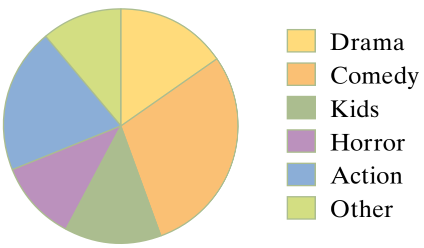
Other: $\frac{10}{90} \times 360^\circ = 40^\circ$

Reasoning

Calculate the fraction of the pie that each category contributes. This fraction of 360° gives the proportion of the total circle area that should be used for each category.

Use a protractor to mark the angles needed for each part of the pie.

Favourite type of movie



Make sure the pie chart is labelled with a title and a legend.

You can either have the legend at the side, as shown here or you can label each piece of the pie individually.

continued

Example 19 continued

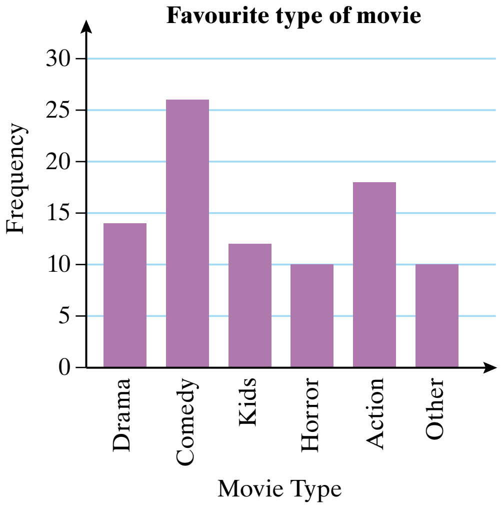
Working

b The frequencies for the different types of movie are similar, but with enough difference to show up in the pie graph. There are no extremely small data values that would have been difficult to show on the pie graph.

Reasoning

Pie graphs are not suitable for data where there are too many different categories or where there are extremely small data values.

c A column graph would be suitable.



Like the pie chart, the column graph is suitable for categorical data, particularly when there is not a large variation between frequencies. Both types of display clearly show the most popular movie types.

Outliers and graph types

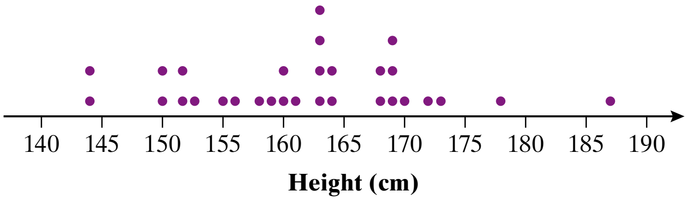
Sometimes outliers occur as a result of an error and should be ignored. Perhaps an incorrect measurement was made, a data value was recorded incorrectly or a person misinterpreted a survey question. Sometimes, though, outliers are important and should not be ignored. Investigating outliers may lead to interesting new information.

Pie charts and column graphs do not display individual data values, so they do not show whether there are any outliers. Dot plots and stem-and-leaf plots, which display each data value, clearly show if there are any outliers. Example 20 shows the use of a dot plot to display the heights of a group of students. A dot plot also shows the 'shape' of the data, that is, how the data values are spread out or bunched up.

Example 20

The following data shows the height in centimetres of 30 Year 8 students.
144, 144, 150, 150, 152, 152, 153, 155, 156, 158, 159, 160, 160, 161, 163,
163, 163, 163, 164, 164, 168, 168, 169, 169, 169, 170, 172, 173, 178, 187

a Construct a dot plot of the data.
b Are there any data values that seem to be outliers?
c What height seems to be a typical height for this group of students?
d Describe the shape of the data.

Working	**Reasoning**
a 	Each data value is represented as a dot at the appropriate height.
b The heights 144 cm and 187 cm seem to be outliers.	144 cm and 187 cm seem quite different from most of the heights.
c About 163 cm is a typical height.	We could also express the typical height as being in the range 160 cm–170 cm.
d The heights are distributed over a wide range, from 144 cm to 187 cm, with most students having heights between 150 cm and 170 cm. Half of the heights are between 160 cm and 170 cm.	The heights are very spread out and do not show a clear clustering around any particular height.

Example 21

Construct a stem-and-leaf plot of the data in example 20 using

a one stem for each multiple of 10.
b two stems for each multiple of 10.

144, 144, 150, 150, 152, 152, 153, 155, 156, 158, 159, 160, 160, 161, 163,
163, 163, 163, 164, 164, 168, 168, 169, 169, 169, 170, 172, 173, 178, 187

continued

Example 21 continued

Working

a **Heights of 30 Year 8 students**

Stem	Leaf
14	4 4
15	0 0 2 2 3 5 6 8 9
16	0 0 1 3 3 3 3 4 4 8 8 9 9 9
17	0 2 3 8
18	7

(Key: 14|4 means 144)

Reasoning

Split each piece of data into a stem and a leaf, starting with 144 having stem 14 and leaf 4.

b **Heights of 30 Year 8 students**

Stem	Leaf
14	4 4
14	
15	0 0 2 2 3
15	5 6 8 9
16	0 0 1 3 3 3 3 4 4
16	8 8 9 9 9
17	0 2 3
17	8
18	
18	7

(Key: 14|4 means 144)

Where two stems are used for each multiple of 10 then the digits 0, 1, 2, 3 and 4 are placed with the first stem and the digits 5, 6, 7, 8 and 9 are placed with the second stem.

Using two stems for each multiple of 10 shows the outliers more clearly.

exercise 9.4

● Dante attends a large school. He conducts a survey regarding students' preferences for snack foods. His data are displayed in the graph on the next page.

a What comparison can be made from this graph?

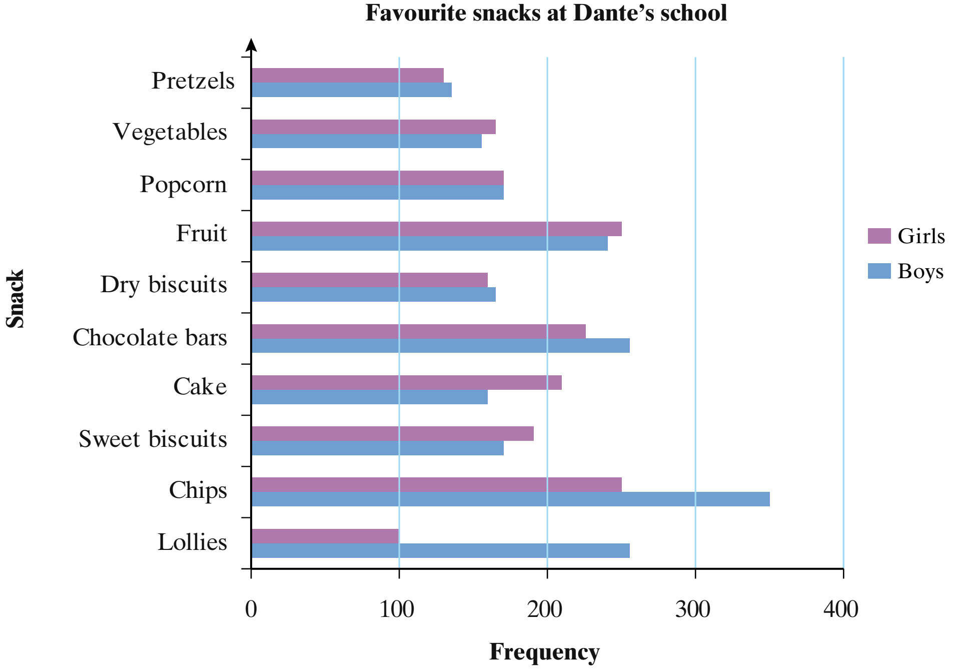
b Which snack food was least preferred by the girls surveyed?

c Which snack foods were preferred by substantially more boys than girls?

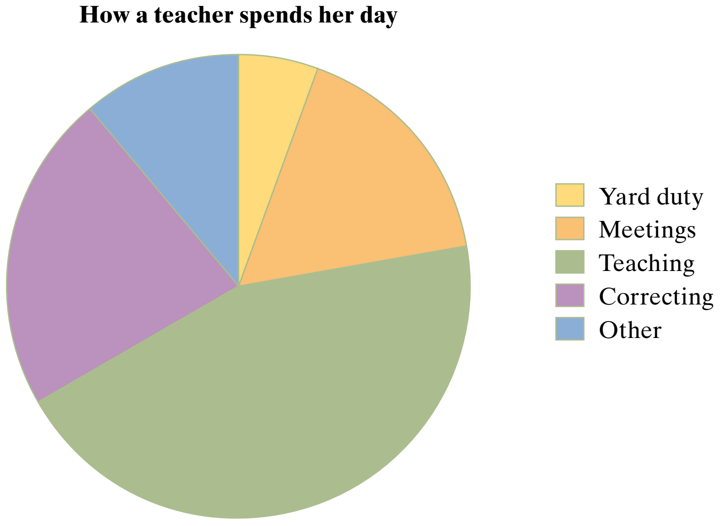
d Which snack foods were preferred by more girls than boys?

e Which snack food was preferred by equal numbers of girls and boys?

f Do you think a pie chart would be suitable for displaying Dante's data?



The pie chart below shows how a teacher spends her day. She arrives at work at 8 am and leaves at 5 pm.



a Use a protractor to measure the angle at the centre of each sector, then complete the table.

	Angle	Fraction of time	Percentage of time (to nearest whole number)
Yard duty			
Meetings			
Teaching			
Correcting			
Other			

b Construct a column graph for the same data.

● Julia kept a record of how she spent her time one day.

a Fill in the angle column with the angle for each activity.

b Construct a pie chart to represent this information.

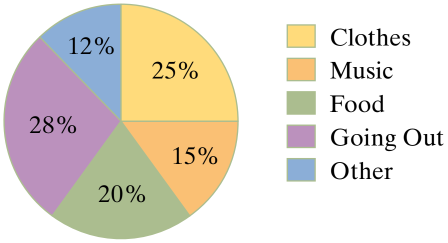
c Use the same information to construct a column graph.

d Which graph do you prefer as a visual display of how Julia's time was spent? Explain your reasoning.

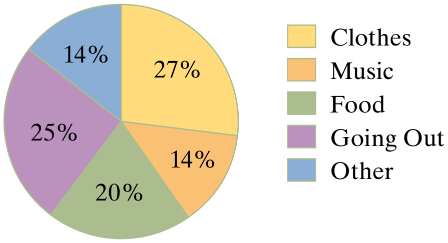
Activity	Hours	Angle
Sleeping	7	
Working	8	
Travelling	1	
Eating	3	
Sport	2	
TV	1	
Other	2	

● The two graphs below represent how two friends Jane and Priyanka spent their monthly income from their part-time jobs.

How Jane spent her income this month



How Priyanka spent her income this month



a What important information is not given in the graphs that would enable you to make comparisons of the amount each girl spends on each category?

b Is it reasonable to conclude that Jane spent more on going out than Priyanka? Why?

● Fifty people were surveyed and asked to take their own pulse by counting the number of beats in one minute. Their pulse rates in beats per minute are shown below.

63, 72, 58, 67, 58, 69, 80, 80, 90, 82, 84, 76, 93, 76, 78, 60, 68, 75, 84, 89, 92, 98, 103, 97, 68, 76, 80, 82, 80, 78, 94, 95, 87, 96, 93, 85, 76, 87, 92, 68, 76, 84, 108, 96, 75, 73, 65, 67, 62, 74.

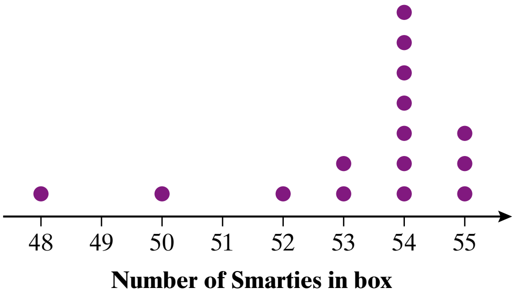
a Construct a dot plot for this set of data. **b** Describe the shape of the distribution.

c Construct an ordered stem-and-leaf plot for the data set using
  i one stem for each multiple of 10. **ii** two stems for each multiple of 10.

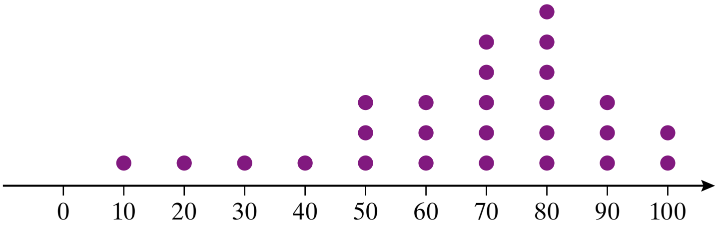
d Which of the pulse rates seem to be outliers?

e Which plot best shows the outliers?

● Huan opens 15 boxes of Smarties and counts the number of Smarties in each box. He displays his results from this experiment using the dot plot below. How many Smarties might he expect to find in the next packet he opens?



● The dot plot shows the test scores from a group of Year 8 students.



Find the range.

● For each of the data sets represented by these stem-and-leaf plots find:

- **i** the mean (to one decimal place)
- **ii** the median
- **iii** the mode
- **iv** the range

a

Stem	Leaf
3	2 3 8
4	4 5 5
5	1 6 7

(Key: 3|2 means 32)

b

Stem	Leaf
6	1 5 8
7	1 4 5 6
8	2 2 2

(Key: 6|1 means 61)

c

Stem	Leaf
2	4 5 7
3	1 1 1 2 4 6 6
4	0 2

(Key: 2|4 means 24)

d

Stem	Leaf
5	9
6	1 3 4 8 9
7	0 4
8	4 4 6
9	5 6 8

(Key: 5|9 means 5.9)

e

Stem	Leaf
0	3
0	5 7 7 8
1	1 1 3 4 4 5 5 5
1	5 6 6 6 7 9
2	0 1 1 1 2 3 3 4
2	5 6 6 9

(Key: 1|1 means 11)

f

Stem	Leaf
0	5 7 9
1	2 8 8
2	1 6 6 6 7
3	0 3 4 7 8
4	2 6
5	1 2

(Key: 1|2 means 12)

● Students in a Year 8 class were asked to monitor the number of mobile phone text messages they sent in a particular day. The stem-and-leaf plot displays the results.

Stem	Leaf
0	0 0 0 0 0 0 0 0 4 4 4
0	5 8
1	0 1 2 2 4 4
1	6 6 7
2	0 0 2
2	8 8
3	0
3	9

(Key: 2|8 means 28)

a What was the least number of messages sent?

b What was the most number of messages sent?

c How many students sent between 10 and 19 messages inclusive?

d An amended plot is shown on the right which excludes data from students who do not own a mobile phone. One of the zeros remains. How is this possible?

e How many students did not own a mobile phone?

f Use the amended plot to calculate the mean number of messages sent.

g What was the median number of messages sent?

h What was the range?

i Were there any outliers?

Stem	Leaf
0	0 4 4 4
0	5 8
1	0 1 2 2 4 4
1	6 6 7
2	0 0 2
2	8 8
3	0
3	9

(Key: 2|8 means 28)

exercise 9.4 challenge

● The results for two classes on their statistics tests are shown below. The results are rounded off to the nearest 10%.

8 Blue: 20, 30, 40, 50, 50, 50, 60, 60, 60, 60, 70, 70, 70, 70, 70, 70, 80, 80, 80, 80, 80, 80, 90, 90, 90, 90, 100

8 Gold: 10, 30, 30, 40, 50, 50, 50, 50, 60, 60, 60, 60, 60, 60, 70, 70, 70, 70, 70, 80, 80, 80, 90, 90, 90, 100, 100

a Use separate dot plots to display these sets of data.

b Which class had the greatest range of results?

c Which class had more results above 80%?

d Which class performed better on the test? Why?

9.5 Histograms and frequency polygons

The shape of a distribution

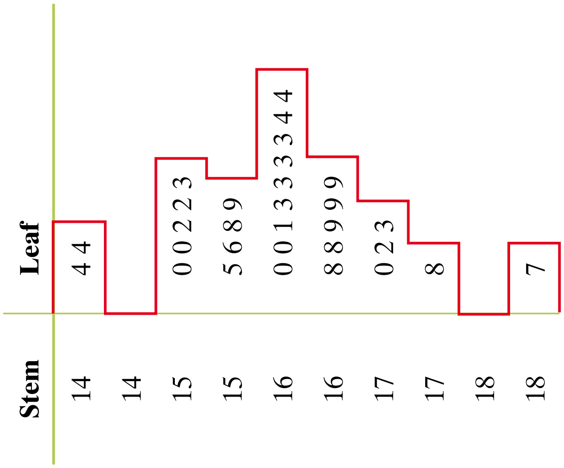
In section 9.4 the heights of 30 Year 8 students were displayed as a stem-and-leaf plot.

Heights of 30 Year 8 students

Stem	Leaf
14	4 4
14	
15	0 0 2 2 3
15	5 6 8 9
16	0 0 1 3 3 3 3 4 4
16	8 8 9 9 9
17	0 2 3
17	8
18	
18	7

(Key: 14|4 means 144)

If we turn this stem-and-leaf plot on its side, we are able to see the 'shape' of the distribution. We can easily see that the heights are clustered around 160 to 164 cm and that heights in the intervals 140 to 144 cm and 185 to 190 cm are outliers.



Histograms

A **histogram** is used to display numerical data that has been divided into class intervals. Histograms are usually used for continuous data, but can also be used for discrete data that has been grouped in class intervals.

The shape obtained by turning the stem and leaf plot on its side resembles a histogram. In appearance, a histogram is similar to a bar chart or column graph but there are no gaps between the columns. This is because the class intervals are designed with no gaps between them; for example, 140 – < 145, 145 – < 150, 150 – < 155 and so on. The horizontal axis is a continuous numerical scale. The height of each column shows the frequency of the class interval.

The numbers on the horizontal axis label the *divisions between intervals*. This is different from a column graph, where the *columns* are labelled rather than the divisions between the columns.

Example 22

The frequency table of grouped data shows the heights of 30 Year 8 students.

Class interval (cm)	Frequency
140–<145	2
145–<150	
150–<155	5
155–<160	4
160–<165	9
165–<170	5
170–<175	3
175–<180	1
180–<185	0
185–<190	1
Total	30

a Construct a histogram.

b How can we tell the number of students in the class from the histogram?

c Which class interval contained the greatest number of heights?

d How many students had heights less than 160 cm?

e How many students had heights greater than or equal to 170 cm?

f Are there any outliers?

continued

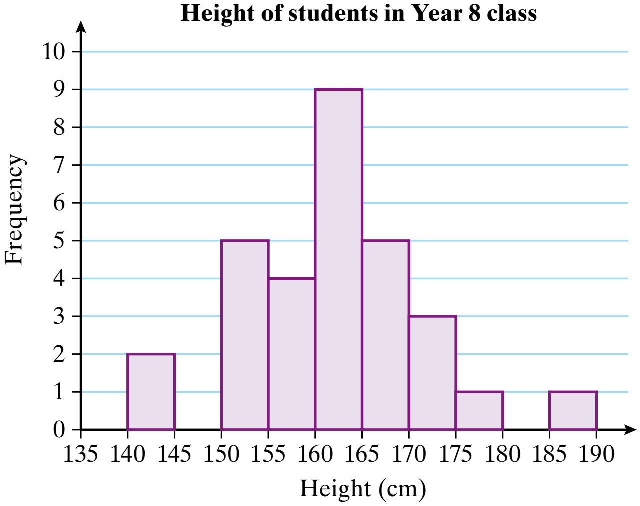
Example 22 continued

Working

Reasoning

a

Height of students in Year 8 class



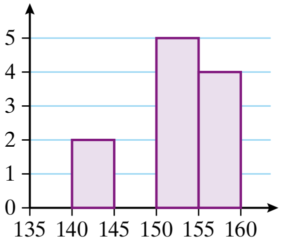
Each class interval is represented by a column. For example, the column between 150 and 155 is 5 units high to indicate that the heights of 5 students were in the interval 150 cm up to but not including 155 cm.

b The sum of the heights of all the columns gives the total number of students.

The column heights represent the frequency of each class interval.
$2 + 5 + 4 + 9 + 5 + 3 + 1 + 1 = 30$ students

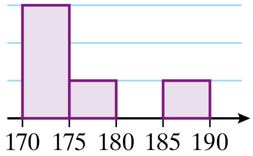
c The class interval 160–<165 cm contained the greatest number of heights.

d Eleven students had heights less than 160 cm.



$2 + 5 + 4 = 11$ students

e Five students had heights greater than or equal to 170 cm.



$3 + 1 + 1 = 5$ students

f Heights in the intervals 140–<145 cm and 185–<190 cm are outliers.

These class intervals are outside the range of the other class intervals.

Frequency polygons

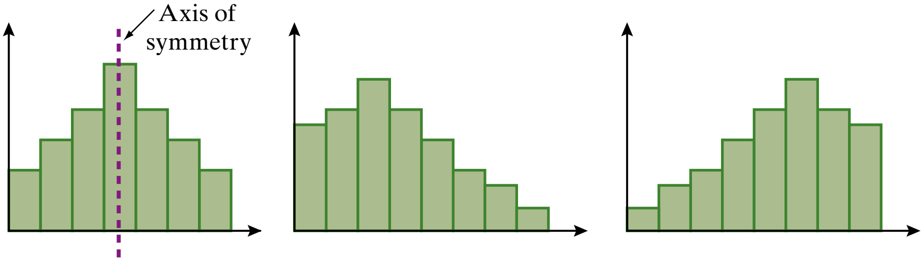
A **frequency polygon** is constructed by joining the midpoints of all the columns of a histogram. The graph is extended to finish on the horizontal axis, half a column width to the left of the leftmost column and half a column width to the right of the rightmost column.

The overall shape of a histogram and its frequency polygon help us analyse the distribution of the data.

When the data has a symmetrical distribution, the histogram has a vertical axis of symmetry. A distribution that is asymmetrical—that is, not symmetrical—is described as having

- positive skew if the data is spread out in the positive direction.
- negative skew if the data is spread out in the negative direction.

The shape of a distribution



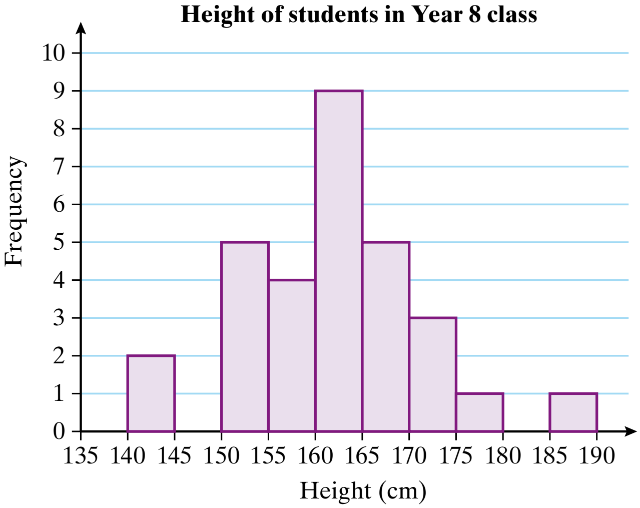
Symmetrical distribution

Asymmetrical distribution: positive skew

Asymmetrical distribution: negative skew

Example 23

The histogram below shows the heights of 30 Year 8 students.



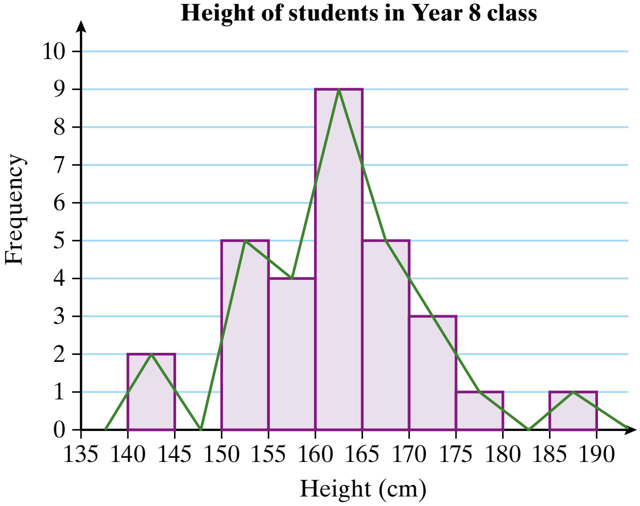
a Add a frequency polygon to the histogram. **b** Comment on the shape of the distribution.

continued

Example 23 continued

Working

a



b The heights are all in the range 140–<190 cm. Over half of the heights are clustered in the interval 150–<170 cm. The data is slightly positively skewed.

Reasoning

Mark the midpoint of each interval at the top of each column in the histogram. Join these midpoints from left to right using straight lines.
Finish the polygon by drawing straight lines to meet the horizontal axis at a distance of half a column width to the left of the first column and to the right of the last column.

The histogram is not symmetrical.

Small intervals in grouped data give a more detailed picture of the data, whereas larger intervals consolidate and simplify the data. In example 21, for example, using intervals of 5 rather than 10 allowed the outliers to be seen. In example 24 we see how the interval size affects the shape of the histogram and frequency polygon.

Example 24

The frequency tables shown below for the distances travelled by paper planes use different class intervals.

i Construct a histogram with frequency polygon for each of the frequency tables.
ii Describe the shape of the distribution.

a

Class interval (cm)	Frequency
140–<170	2
170–<200	7
200–<230	6
230–<260	8
260–<290	5
290–<320	2
Total	30

continued

Example 24 continued

b

Class interval (cm)	Frequency
140–<155	1
155–<170	1
170–<185	3
185–<200	4
200–<215	3
215–<230	3
230–<245	4
245–<260	4
260–<275	3
275–<290	2
290–<305	1
305–<320	1
Total	30

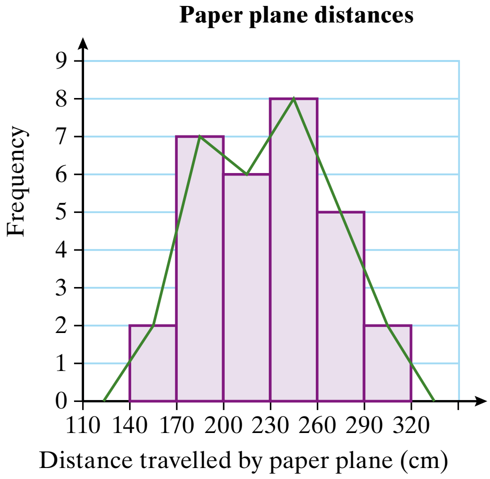
c

Class interval (cm)	Frequency
100–<200	9
200–<300	20
300–<400	1
Total	30

Working

Reasoning

a i



ii Most distances are in the interval 170–<290 cm. The data distribution is reasonably symmetrical. The centre of the data seems to be about 230 cm.

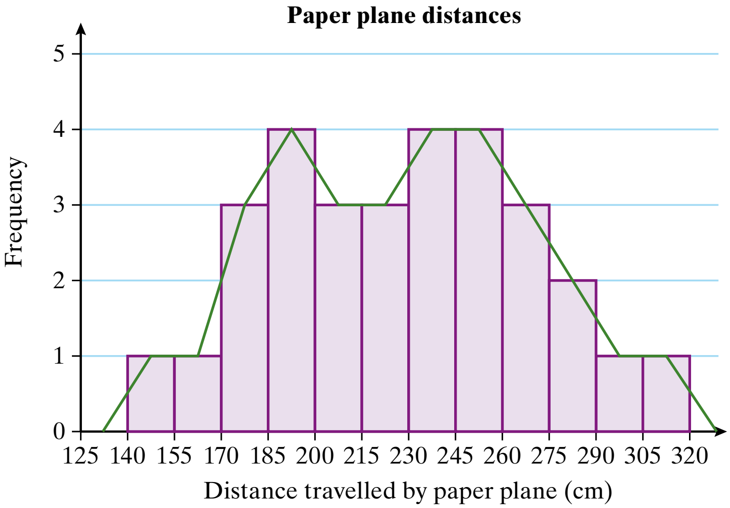
There is no strong skewing of the data towards high or low values.

continued

Example 24 continued

Working **Reasoning**

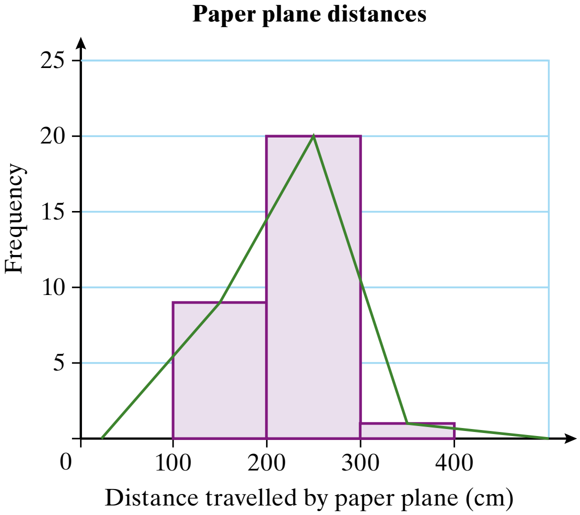
b i



ii The histogram with class intervals of 30 cm clearly shows the shape of the data. with clustering in the interval 170–<290 cm. The histogram with intervals of 15 cm spreads the data into columns with more even frequencies and does not really provide greater information about the distribution.

Intervals that are too small give a large number of classes and we lose the 'shape' of the 'distribution', that is, it is harder to see where data values are clustered.

c i



ii The histogram with class intervals of 100 cm bunches up the data too much to clearly see where the centre of the data is and it is harder to see extreme values.

Intervals that are too large give a small number of classes, so the smallest and largest classes tend to 'hide' any extreme values.

exercise 9.5

The frequency table shows the time in minutes for students to complete a task.

a How many students were there?

b Construct a histogram of the data.

Time (minutes)	Frequency
0–<2	4
2–<4	3
4–<6	11
6–<8	7
8–<10	4
Total	

The cost per week of renting a house was surveyed and the results are shown in the table below.

a How many houses were surveyed?

b Construct a histogram to show the data.

c Add a frequency polygon to the histogram.

d Describe the shape of the distribution.

e Draw another histogram with the data grouped in intervals of \$100 instead of \$50.

Weekly rent ($)	Frequency
0–<50	0
50–<100	1
100–<150	2
150–<200	4
200–<250	5
250–<300	7
300–<350	3
350–<400	3
400–<450	2
450–<500	1
Total	

The frequency table show the number of minutes that members of a gym spent on a treadmill.

a Construct a histogram.

b Add a frequency polygon.

c Describe the distribution, including whether the data is positively or negatively skewed.

Time (minutes)	Frequency
20–<30	9
30–<40	9
40–<50	5
50–<60	6
60–<70	1
Total	30

- Year 8 students at a school were asked how long it took them to travel to school on a particular day. Their times, in minutes, are given below.

  13, 14, 22, 22, 23, 23, 23, 24, 24, 25, 25, 26, 26, 28, 31, 32, 33, 33, 34, 34, 35, 35, 35, 36, 36, 37, 38, 41, 42, 43, 43, 43, 44, 44, 45, 45, 47, 52, 52, 55, 55, 56, 62, 65, 65, 67, 69, 71, 75, 76

  a Construct a frequency table using class intervals of 10 minutes.
  b Construct a histogram.
  c Describe the distribution.

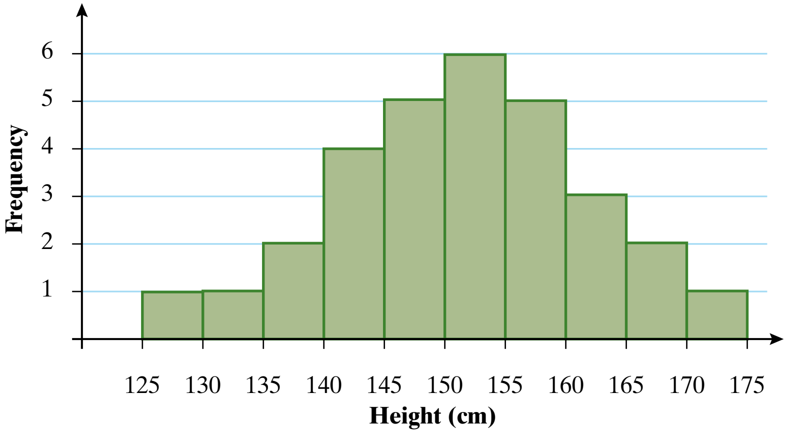
- The frequency table shows the number of hours of TV watched by the students in a class during the last week of term.

  a Construct a histogram.
  b Add a frequency polygon.
  c Describe the distribution, including whether the data is positively or negatively skewed.

Time (hours)	**Frequency**
0–<5	2
5–<10	8
10–<15	6
15–<20	3
20–<25	7
25–<30	3
30–<35	1
Total	

- The histogram shows the heights of students in a class.

  a How many students are in the class?
  b Which 5 cm class interval contains the greatest number of heights?
  c How many students have heights less than 145 cm?
  d Is the distribution positively or negatively skewed?



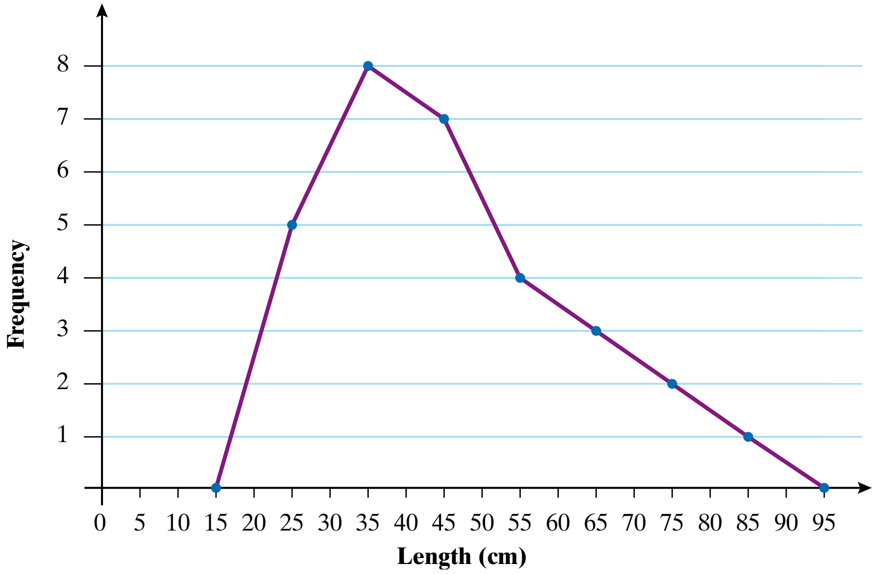
● Fifty people were surveyed and asked to take their own pulse by counting the number of beats in one minute. Their pulse rates in beats per minute are shown below.

58, 58, 60, 62, 63, 65, 67, 67, 68, 68, 68, 69, 72, 73, 74, 75, 75, 76, 76, 76, 76, 76, 78, 78, 80, 80, 80, 80, 82, 82, 84, 84, 84, 85, 87, 87, 89, 90, 92, 92, 93, 93, 94, 95, 96, 96, 97, 98, 103, 108

- **a** Construct a frequency table using class intervals of 10 beats per minute, starting with the interval 50–<60 beats per minute.
- **b** Construct a histogram.
- **c** In which interval of 10 beats per minute were most of the pulse rates?
- **d** Describe the distribution.
- **e** Construct another histogram using intervals of 5 beats per minute.
- **f** Which histogram shows the shape of the distribution more effectively? Explain.

exercise 9.5 challenge

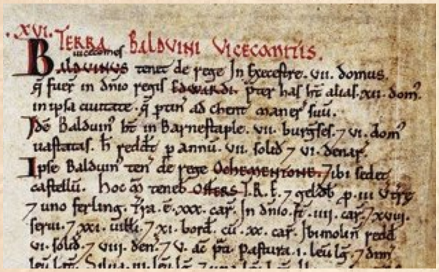
● The graph below is a frequency polygon for a set of data.



- **a** What is the size of the class interval?
- **b** Add a histogram to the graph.
- **c** Describe the distribution.

9.6 Collecting and reporting population data

The collection of statistical data has been a part of human culture over many centuries. The famous Domesday Book is one example.



In 1066, England was invaded and conquered by William of Normandy (now part of France). William became the king of England and was known as William the Conqueror. In 1085, he ordered a survey of the whole of England to determine the amount of land and resources owned at the time, and hence determine how much money he could raise in taxes.

Some examples of the questions landholders were asked include the following.

- How much is the manor worth?
- How many ploughs are there in the manor?
- How many mills and fishponds are there?
- How many freemen, villagers and slaves are there in the manor?

The data from this large-scale survey was hand-written into a book which became known as the Domesday Book. It is regarded as one of the most important records of life in the Middle Ages.

You can find more about the Domesday Book at the website www.domesdaybook.co.uk.

Every day we see media reports that include statistical data. This data may come from a variety of sources.

Census data

The process of obtaining information about every member of a population is called a **census**. The Australian Bureau of Statistics carries out a census every five years. The year 2011 was a census year.

Opinion polls based on samples of the population

It is not always convenient to survey or question the whole population. Often, only part of the population is questioned and conclusions about the whole population are based on the responses from that group. We refer to this as taking a **sample** of the population.

It is important to choose a sample that represents the population fairly. If a sample does not represent the population fairly, we say that it is **biased**.

Opinion polls are commonly used to make predictions about the beliefs of a population with respect to a certain issue. A well-conducted opinion poll should seek the opinions of a large enough sample of the population to confidently predict that the results represent the opinions of the population. The sample should include appropriate proportions of different groups within the population. The Lowy Institute describes how their opinion polls are conducted:

> For this opinion poll, Field Works Market Research conducted 1,001 interviews between 6 and 21 March 2010. Survey interviews were conducted by telephone. The sample was designed to be nationally representative of all Australians 18 years and older. Quotas were set for each state and territory, with broad age-group and gender quotas. Interviewers continued making calls until each quota was filled.

Note how the sample included

- people from each state and territory
- people in each broad age group
- males and females

and that telephone calls were continued until the correct number of people in each group was reached.

When an issue is controversial, some people will strongly oppose it, others will strongly support it, whilst others may not have such strong feelings or may not be willing to express an opinion either way. People's opinions may also reflect the culture in which they live. City people who are concerned about the environment are more likely to oppose logging in native forests than country people whose see their jobs at risk if logging is stopped.

Opinion polls are often conducted by inviting people to phone in to a magazine or radio/TV station give their opinion on a current issue. The results of these surveys are often extreme, with a very large proportion of people who respond supporting one particular viewpoint. This is because usually only a very small proportion of the public actually bother to respond, and those who do are often those with very strong views. The results of surveys of this type need to be treated with caution as they may not express the views of the wider population.

A questionnaire consists of a set of questions, usually focusing on a particular topic. Questionnaires can be administered by an interviewer, or respondents can fill in the questionnaire themselves.

Statistics in the media

Media reports often state statistics such as percentages. When we read these reports we should consider the following questions.

- Is the report based on a sample of people or the whole population?
- If the report is based on a sample, how representative is this sample of the whole population?
- Does the report provide details of the sample, for example, number of people, age-group, how they were selected?
- Does the report state the source of the data?

Example 25

For each of the following questions, decide whether it would be more appropriate to collect data for the whole population or to use a sample.

a How many schools in New South Wales teach Chinese?

b What proportion of paper clips produced by the machines in a large factory are faulty?

c What proportion of children in Tasmania like liquorice?

d How many Australians were born overseas?

Working	**Reasoning**
a Whole population	This data would best obtained by surveying all schools. In this case 'whole population' refers to all the schools, rather than all the people in New South Wales.
b Sample	It would be too expensive to check every paper clip. A sample of paper clips from each machine would be appropriate.
c Sample	This data would be best obtained from a sample of children of different ages.
d Whole population	This data would be obtained from census data for the whole population.

Designing a questionnaire

As we saw in Year 7, questionnaires need to be designed carefully. The following guidelines will help you in preparing a survey.

- Questions should be unbiased.
- Questions should be easy to understand.
- Questions should not be upsetting or embarrassing.
- Questions should be easy to analyse.

Example 26

Suggest what is wrong with each of these interview questions.

a Smart people pay their bills using the internet. Do you pay your bills using the internet?

b What is your weekly income?

c How old are you?

d Tell me how many times you shopped in a supermarket last year and how many items did you buy each time and where do you prefer to go for your holidays?

Working	**Reasoning**
a The question implies that you are not smart if you don't pay your bills on the internet. If you want to be considered smart you would probably say that you do.	The first sentence could be left out. People will give more honest answers if they are not made to feel they should answer in a particular way.
b People often don't like telling their personal information.	People are often reluctant to give personal information such as weight, age, salary.
c If there are a lot of people being interviewed it would be much easier to ask for age categories, for example, 15 – 19, 20 – 25.	It is much easier to sort a large amount of data if the data are already grouped in some way.
d This is a very complicated question. It needs to be broken into separate questions.	Short simple questions are more likely to be answered in a straightforward way.

exercise 9.6

LINKS TO Example 1

For each of the following questions, decide whether it would be more appropriate to use a sample or a census to collect the relevant data.

a What proportion of households in Queensland were tuned in to a particular television program?

b What proportion of tennis players in Australia are left-handed?

c How many students are enrolled at each year level of non-government schools in South Australia?

d What proportion of elastic bands from a given machine in a certain factory are faulty?

LINKS TO Example 2

● Suggest what is wrong with these interview questions.

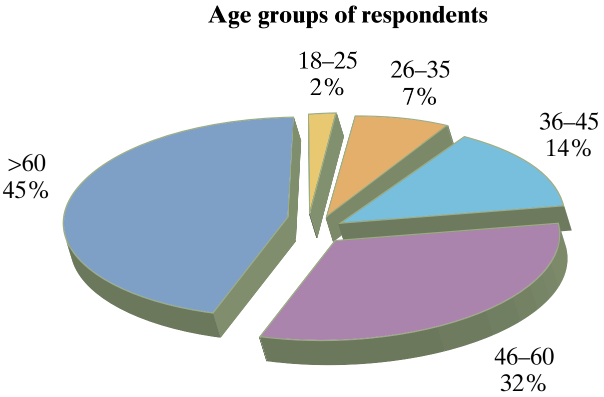
a Are you tall?

b Most people think the town hall is a very ugly building. What do you think?

c Do you play sport?

d How much pocket money do you get each week?

● A local shire posted 200 surveys to all residents and rate payers within the shire asking about their opinions on various local issues. A total of 56 completed surveys were returned. The report on the survey presented by the shire council included the following pie graph showing the percentage of respondents in each age group.



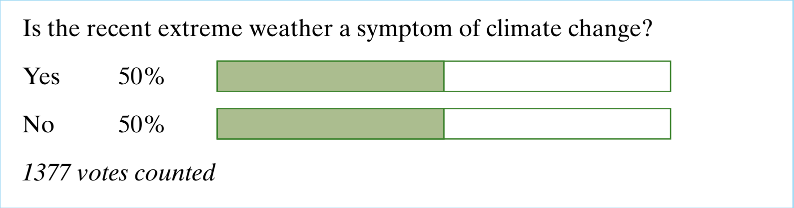
a Calculate the percentage response to the survey.

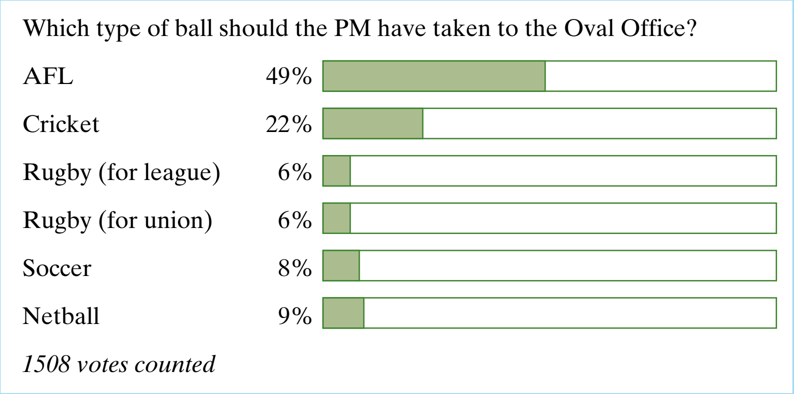
b Calculate the number of respondents in each age group and complete the following table by calculating the percentage of the 56 respondents for each age group. Round to the nearest whole number.

Age group	% of respondents	Number of respondents
18–25		
26–25		
35–45		
46–60		
> 60		

c Comment on how typical of the whole population of the shire the results of the survey questions might be.

The ABC conducts online surveys on issues that have been in the news. In each case two or more choices are given and respondents click on their choice. The following survey questions related to the 2010/2011 weather events in Australia and to Prime Minister Julia Gillard's first meeting with President Obama in the Oval Office in Washington.





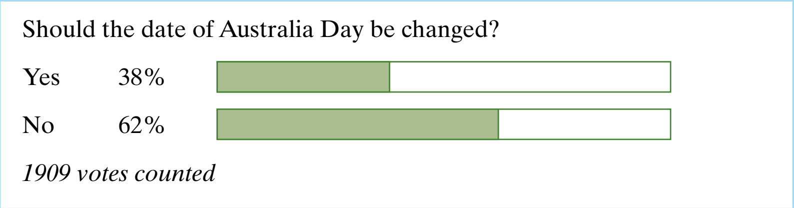
www.abc.net.au/thedrum/polls/

a Do you think the people who responded to these questions would be a representative sample of all Australians? Give reasons for your answer.

b Why do you think almost half the people chose an AFL ball.

c Suggest why the pattern of responses is so different for the two questions.

Comment on the design of the following opinion poll question.



Find a newspaper report (for example, from a national or local newspaper) that includes statistics about people and answer the following questions.

a Is the report based on a sample of people or the whole population?

b If the report is based on a sample, how representative is this sample of the whole population?

c Does the report provide details of the sample, for example, number of people, age-group, how they were selected?

d Does the report state the source of the data?

Decide on a place (for example, a city, holiday resort or country) where you would like to go for a holiday.

a Use one item of statistical information to support your choice.

b Suggest how this item of statistical information could be used by someone else to decide that they did not want to go to this place for a holiday.

The following opinion polls taken in Japan and in Australia in 2008 show the different cultural attitudes to whaling.

Ministry of Foreign Affairs Japan 2008

In a Japanese public opinion survey conducted at the beginning of February by a major newspaper, 65% of respondents expressed support for the continuation of whaling research, while 56% expressed approval for the consumption of whales as food. These results revealed a large difference in public opinion between Australians and Japanese.

www.mofa.go.jp/dispatches/vol02/perspectives.html

The 2008 Lowy Institute Poll

JAPANESE WHALING

Thinking about the Australian government's efforts to stop Japanese whaling, please say which of the following statements comes closest to your own view.

The Australian government should do more to pressure Japan to stop all whaling even if we risk losing valuable trade deals	58%
The Australian government's response is about right	33%
The Australian government should not be involved because we risk jeopardising our commercial relationship with Japan	4%
The Australian government should not be involved because Japanese whaling should not be stopped	3%
Don't know/refuse	1%

www.lowyinstitute.org/Publication.asp?pid=895

a What information is included in the Australian opinion poll results that is not included in the Japanese poll results?

b Suggest reasons for the support for whaling expressed by the majority of Japanese people.

c Why do you think Australians in general have very strong anti-whaling opinions?

● The following data shows the results of two questions in the 2008 Lowy poll relating to Australia exporting uranium.

ATTITUDES TOWARDS URANIUM

Climate change and energy needs have increased international interest in nuclear power and it seems likely that Australia will be exporting more uranium in coming years. Please say whether you strongly agree, agree, disagree or strongly disagree that Australia should:

	Strongly agree	Agree	Disagree	Strongly disagree	Don't know
Take responsibility for the nuclear waste from the uranium it exports by storing it in Australia	12%	30%	24%	32%	2%
Only export uranium to countries which have signed the global Nuclear Non-proliferation Treaty	53%	35%	6%	3%	2%

a To which of the two questions did people express the strongest opinions?

b Complete the following table by combining the percentages for *agree* and *strongly agree*, and for *disagree* and *strongly disagree*.

	Agree or strongly agree	Disagree or strongly disagree	Don't know
Take responsibility for the nuclear waste from the uranium it exports by storing it in Australia			
Only export uranium to countries which have signed the global Nuclear Non-proliferation Treaty			

c Can you suggest any reasons for the different opinions expressed for the two questions?

● Labels such as Generation X and Baby Boomers are associated with people of different age groups. Public opinion stereotypes suggest that younger people (Generation X) are less motivated and less committed to their jobs than the older Baby Boomer group. A research study of 1123 employees in five different organisations showed that these stereotypes were incorrect. There was no significant differences between the two age groups.

If a company was about to appoint new employees, what age group would they be more likely choose if they believed

a the stereotypes.
b the facts.

exercise 9.6 challenge

● In October 2007 Roy Morgan Research conducted a survey of a sample of 637 Victorians to find out their opinions on banning the shooting of native water birds for recreational purposes. The sample represented a cross-section of Victorians aged 14 years and over. The survey showed that a large majority of Victorians (75%) thought that the shooting of native water birds should be banned in Victoria. Twenty percent thought it should not be banned, and 5% were undecided. Participants in the survey were then given further information. They were told that drought and climate change had caused the numbers of native water birds across eastern Australia to drop by over 80%, that at least one in four native water birds shot at are wounded, and that duck shooting had been banned in WA, NSW and Queensland. The proportion of Victorians who wanted to see the activity banned then increased to 87%, only 10% thought it should not be banned, and 3% remain undecided.

a What percentage of Victorians believed that the shooting of native water birds should be banned?
b How did the percentage of people who opposed the ban change when they were given further information about shooting of native water birds?
c Suggest how you might expect opinions on this issue to vary amongst different groups of people in the population.

Analysis task

The bottled water debate

The following media reports relate to bottled water.

Monte Sant' Angelo first to ban bottled water

by Boel Eriksson

A NORTH Sydney school is the first in Australia to ban bottled water.

Monte Sant' Angelo Mercy College will launch the bottled water ban and their new water stations today, and is hoping other schools will follow suit.

The water stations are similar to those used in Bundanoon, which last year became the world's first town to boycott bottled water in an effort to reduce the town's carbon footprint.

The school has started off by banning sales of bottled water in the school canteen and will also ask students not to bring in disposable plastic bottles.

They will also sell stainless steel water containers in the college shop, and next year try to make all year 7 students buy one as part of their compulsory uniform purchase.

Do Something founder Jon Dee, and eco-adventurer David de Rothschild launched the ban with the school's environmental captain, year 12 student Claudia Saunders.

Claudia said the student-led ban, which had been planned for a year, was designed to cut plastic bottle waste, reduce the negative environmental impacts of bottled water and save students money.

"The response has been really good so far," she said.

"This is a really simple way of breaking behavioural patterns. Now we're putting the challenge to other student leaders."

Mr Dee, who worked with Monte on the initiative, said the school had set a good example.

"This water won't just save them money it's also better for their teeth than bottled water," he said.

The Mosman Daily, 28 July 2010

The following is a media release from the Australian Bottled Water Institution on July 28, 2010.

> The bottled water industry welcomes debate around litter and recycling but says that outright bans limit choice for consumers.
>
> The Australasian Bottled Water Institute, representing the nation's large and small bottlers, has today welcomed recent initiatives to raise heightened awareness of the importance of recycling and reducing litter.
>
> However, following the announcement that a Sydney school is to ban bottled water, the industry says that so-called bans on the sale of bottled water are not effective solutions to litter. "Students should have access to as many sources of water as possible," said the Institute's CEO, Mr Geoff Parker.
>
> "To eliminate bottled water from a school seems illogical. If the school was serious about tackling environmental issues, they'd look at providing infrastructure like proper recycling bins and education to tackle the broader issues of litter and waste across all types of packaging. There are a lot of different food and beverages in the canteen and vending machines that are packaged in recyclable paper and plastic. Eliminating litter and improving recycling requires us all to take action and we believe this is where the focus of education should be" Mr Parker said.
>
> Australia already leads the rest of the world in recycling of plastic which is supported by over 90% of homes having access to kerbside recycling. Recycling rates have almost doubled in Australia in the past 10 years and the country is well ahead of other key markets such as the EU, UK and US. Soft drink and water plastic bottles made from PET have the highest recycling rate of all plastic packaging in Australia at more than 43%*. By way of comparison, US and UK PET bottle recycling rates sit at around 27%*.
>
> Additionally the industry continues to invest millions in improving designs to reduce the amount of packaging required to make bottles and to improve water and energy efficiency in the manufacture of all beverages.
>
> "Bottled water provides a convenient option for those looking for a healthy choice when out and about. Drinking more water, whether from tap, bubbler or bottle, is a positive step forward towards improving the health of our nation and taking one of these choices away is a step backwards." added Mr Parker.
>
> Research conducted by Auspoll shows 75% of the Australian public support the availability of bottled water and of those that didn't, all supported a right to choose.
>
> *December 2009 National Plastics Recycling Survey, Hyder Consulting Pty Ltd

September 7, 2010

The aquabubbler drinking fountain blog lists the following facts:

- Producing and delivering a litre of bottled water can emit hundreds of times more greenhouse gases than a litre of tap water.
- Australians spend more that half a billion dollars a year on bottled water.
- Last year the sale of bottled water increased by 10 percent.
- According to British research, drinking one bottle of water has the same environmental impact as driving a car for a kilometre.

- In many cases, a litre of bottled water is more expensive than a litre of petrol.
- Less than 1 percent of all plastics is recycled. Therefore, almost all plastics are incinerated or end up in a landfill.
- Nearly eight out of every 10 bottles will end up in a landfill.
- Recycling a single plastic bottle can conserve enough energy to light a 60-watt light bulb for up to six hours.

www.aquabubbler.com.au/blog/

a What are the reasons the Sydney school give for banning bottled water?

b List the statistics that Geoff Parker uses to oppose the action of the school.

c Do Geoff Parker's statistics address the reasons the school give for banning bottled water?

d Geoff Parker quotes the Auspoll results that 75% of Australians support the availability of bottled water. If you wished to find out if this result was representative of the whole population of Australia, what questions would you want to ask Auspoll?

e Claudia Saunders states "the response has been really good so far." What statistics could Claudia have given to support her claim?

The aquabubbler drinking fountain blog includes eight different statistics about bottled water.

f Which of the statistics quoted on the aquabubbler blog would have been obtained from sample data?

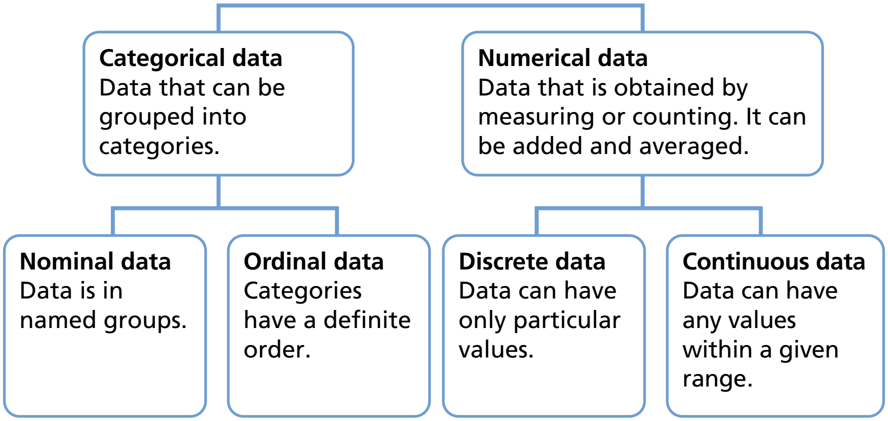
g Do any of the statistics quoted on the aquabubbler blog relate to the whole population?

h Which of the quoted statistics are based on experimental data?

Review Statistics

Summary

Types of data



Categorical data
Data that can be grouped into categories.

Numerical data
Data that is obtained by measuring or counting. It can be added and averaged.

Nominal data
Data is in named groups.

Ordinal data
Categories have a definite order.

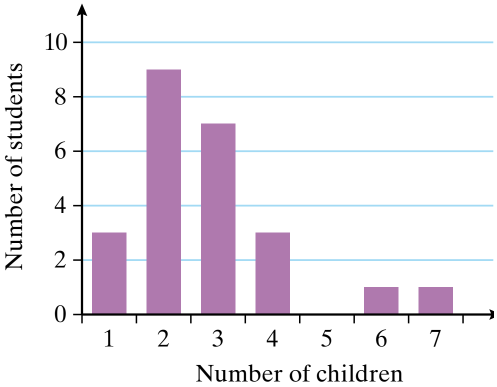
Discrete data
Data can have only particular values.

Continuous data
Data can have any values within a given range.

Collecting population data

- Census data is collected from the whole population.
- Sample data is collected from a sample of the population.
- Questionnaires and opinion polls can be used to collect specific information from a sample.

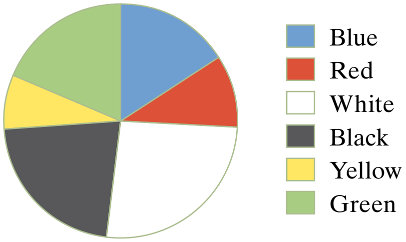
Displaying data

Bar graph or column graph	**Example**
- Often the best choice for *categorical* variables. Also appropriate for *discrete numerical* variables. - Each category has its own labelled ‘bar’ or column. - It is usual to show the *category* along the horizontal axis and the *frequency* along the vertical axis.	**Number of children in the families of students in 8 Purple** 

<table>
<tr><th>Pie chart</th><th>Example</th></tr>
<tr><td>

Pie charts are suitable for visually comparing categorical data. Discrete numerical data can also be displayed in a pie chart.
They are used to display each value or category of the data as part of a whole. This makes it easy for comparisons to be made.
The sectors of the pie chart may be labelled with actual numbers or with percentages.
If there are too many data categories then the pie chart becomes too complex to interpret easily.
If the data include some very small values, for example, 0.5%, it will be difficult to show these.

</td><td>
Cars sold by colour in March



</td></tr>
<tr><th>Dot plot</th><th>Example</th></tr>
<tr><td>

Best for a small set of <i>discrete numerical</i> data where the values are reasonably close together in size.
Provides a quick view of the ‘shape’ of a distribution.
Makes it easy to identify any clusters or outliers.
Dot plots show each individual data value.

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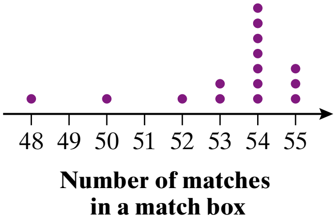
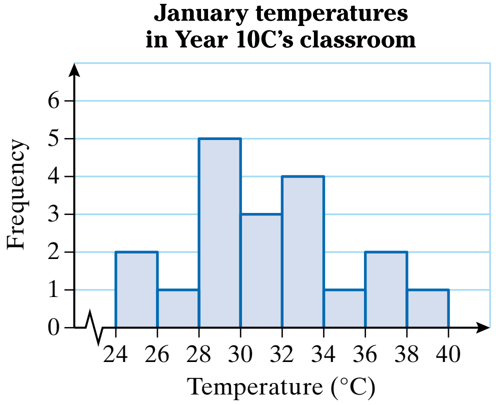


</td></tr>
<tr><th>Stem-and-leaf plot</th><th>Example</th></tr>
<tr><td>

Appropriate for <i>discrete numerical</i> data, where the values are reasonably close together in size.
The data is ‘grouped’ (into rows) but each of the actual values can still be read from the plot.
Provides a quick view of the ‘shape’ of a distribution; looks a little like a column graph or dot plot on its side.
Show each individual data value.
Use split stems to provide more detail.
To show large or small data values in a stem-and-leaf plot, the values may first need to be rounded.

</td><td>
Number of words per sentence
<table>
<tr><th>Stem</th><th>Leaf</th></tr>
<tr><td>1</td><td>0 1 3 4 4</td></tr>
<tr><td>1</td><td>5 6 6 8 9</td></tr>
<tr><td>2</td><td>2 3 3 4 4</td></tr>
<tr><td>2</td><td>5 7 7</td></tr>
<tr><td>3</td><td>2 2 3</td></tr>
<tr><td>3</td><td>7</td></tr>
<tr><td>4</td><td>0</td></tr>
</table>
(Key: 1|5 means 15)
</td></tr>
</table>

Histogram	**Example**
■ Appropriate for *continuous numerical* data. ■ The horizontal axis has a continuous numerical scale. The vertical axis shows the *frequency*. ■ The data is grouped into *intervals* (for example 24–<26, 26–<28, 28–<30, and so on). ■ The numbers along the horizontal axis label the marks *between* intervals, not the columns themselves. ■ To display more detail, decrease the size of the class interval.	

Summarising data: measuring centre and spread

- Centre refers to the location of the middle of a data set.
- The mean, median and mode are helpful measures of where the centre (or middle) of a set of numerical data is located.
  - Mean: the arithmetic average of the data. To find the mean of a set of data values add all the data values together and divide by the total number of data values.
  - Median: For an *odd* number of data values, the median is the middle value. For an *even* number of data values, the median is half-way between the two middle values.
  - Mode: the most frequent data value (or interval).
- Spread is the amount of variability in a data set.
- Range is a measure of spread. To find the range subtract the smallest data value from the largest data value.

Visual map

bar graph
categorical data
census
column graph
continuous numerical data
discrete numerical data
dot plot
frequency
frequency polygon
frequency table
grouped data
histogram
mean
median
mode
nominal categorical data
numerical data
ordinal categorical data
opinion poll
outlier
pie chart
population
questionnaire
range
sample
stem-and-leaf plot
tally mark

Revision

Multiple-choice questions

Which one of the following is not categorical data?

A red, green, black, green, blue, orange, black, red
B 1st, 2nd, 3rd, 1st, 3rd, 2nd, 4th, 5th, 3rd, 2nd
C 3, 12, 13, 12, 7, 18, 8, 19
D cat, dog, mouse, dog, cat, fish
E high, medium, low, high, very high, low, medium

Which of the following numerical variables is continuous?

A number of children in a family
B cost of 1 kg of apples at a greengrocer
C length of a caterpillar
D quantity of leaves on a tree
E age at next birthday

The data set below shows the mass in grams of eight eggs from a carton.

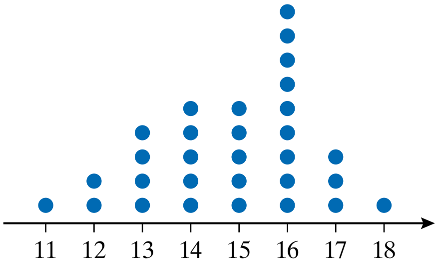
61, 55, 61, 68, 51, 53, 69, 54

The mean mass of the eggs is

A 11 **B** 58 **C** 59 **D** 59.5 **E** 61

Which one of the following statements is correct about the dot plot shown?

A The youngest member of the club is aged 12.
B Eight members of the club are aged 16.
C The number of members of the club aged less than 13 is the same as the number aged more than 16.
D There are twice as many 16 year olds as 13 year olds in the club.
E There are the same number of 14 year olds and 15 year olds in the club.



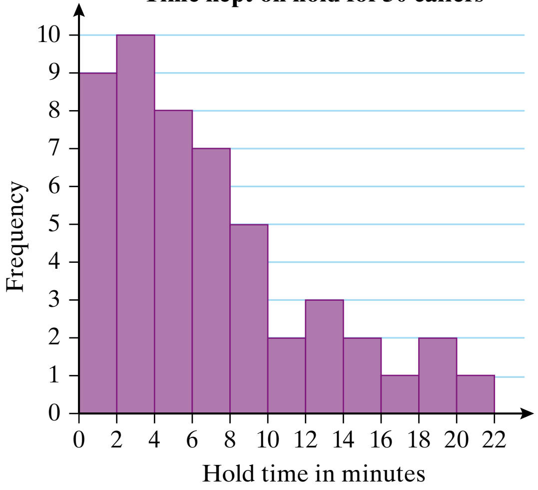
Ages of members of a school chess club

The time, in minutes, spent on hold by 50 callers to an insurance company was recorded and summarised in the form of a histogram as shown.

The median time spent on hold was somewhere between

A 2 and 4 minutes
B 4 and 6 minutes
C 6 and 8 minutes
D 8 and 10 minutes
E 10 and 12 minutes.

Time kept on hold for 50 callers



Short-answer questions

Classify the following as either categorical or numerical data. If it is numerical data, decide whether it is continuous or discrete. If it is categorical data, decide whether it is nominal or ordinal.

a The number of front windows in houses.
b The colour of letterboxes.
c Course being studied at university.
d The area of students' bedrooms.
e The number of days until your birthday.
f The letter grade received on a science project.

Supposing Ella pours a packet of M&Ms into a paper cup and as she eats each of them she records the colour of coating on each one. Using the abbreviations Br for Brown, R for Red, O for Orange, B for Blue, Y for Yellow and G for Green, Ella's data are shown below.

G, R, B, O, G, R, Y, Br, R, Br, B, R, O, Br, G, Br, O, R, R, B,

Y, B, G, Br, Y, R, R, B, O, B, G, R, G, Br, Br, Y, G, B, B, Br,

R, Br, R, R, B, O, G, R, B, Br, Y, Br, Br, O, R, Y, G, Y, B, Br

a Why is the data categorical nominal data?
b Display the data in a frequency table using columns with headings Colour, Tally and Frequency.
c Which was the most common colour of the M&Ms in the packet?
d What fraction of the M&Ms in this packet are orange?

The data showing the number of minutes that 30 members of a gym spent on a treadmill are listed below.

24, 49, 54, 21, 24, 57, 28, 33, 27, 34, 22, 32, 38, 43, 28, 26, 23, 46, 31, 60, 51, 53, 39, 52, 36, 38, 40, 50, 37, 46

Use the data to complete the grouped frequency table.

Time (minutes)	Tally	Frequency
20–29		
30–39		
40–49		
50–59		
60–69		
Total		

The average rainfall in millimetres for each month from January to December in Darwin is listed below.

421, 355, 320, 101, 21, 1, 1, 6, 16, 71, 141, 246

a Find the mean monthly rainfall for Darwin. Answer correct to one decimal place.
b Find the median monthly rainfall.
c What is the range?
d Would a pie chart be suitable for displaying this data? Explain.

The table on the right shows the number of dogs owned by people attending a dog show.

a How many people attended the show?

b State the mode.

c Calculate the mean.

d Determine the median.

Number of dogs	Frequency
1	20
2	24
3	15
4	10
5	6

The following table shows the scores out of 20 obtained by students on a spelling test.

Score	20	19	18	17	16
Number of students	12	14	18	10	6

a How many students did the test?

b What percentage of students obtained a score higher than 18?

c Use a dot plot to represent this data.

A school librarian records the number of books borrowed by each of 25 students over lunchtime one day. This frequency table summarises the data.

What is the mean number of books borrowed?

Number of books	Frequency
1	7
2	9
3	5
4	2
5	2

There are 480 members of a hockey club. The members are men, women and children.

a In a pie chart showing the different groups of members, if 150° is the angle of the sector representing men, how many men are in the club?

b There are 160 women in the club. What angle should the sector representing the women contain?

c Construct a pie chart representing the membership of the hockey club.

Extended-response questions

The final scores, out of 100, for 20 teams at a club trivia night are listed below.

72, 70, 76, 80, 84, 84, 88, 69, 64, 87, 91, 96, 94, 65, 78, 82, 93, 77, 69, 86

a Display this data using an ordered stem-and-leaf plot.

b What was the median score?

c Describe the shape of the data.

The ages of mothers of 30 Year 8 students are listed below.

39, 44, 55, 48, 35, 46, 46, 42, 41, 42, 31, 52, 41, 38, 34, 40, 39, 54,
37, 56, 42, 41, 57, 60, 33, 46, 44, 42, 37, 50

a Construct a stem-and-leaf plot starting at 30 and using two stems for each interval of 10 years.

b What is the median age?

c What is the range?

d Do any of the ages seem to be outliers?

e Describe the shape of the distribution.

The data showing the number of minutes that 30 members of a gym spent on a treadmill are listed below.

24, 49, 54, 21, 24, 57, 28, 33, 27, 34, 22, 32, 38, 43, 28, 26, 23,
46, 31, 60, 51, 53, 39, 52, 36, 38, 40, 50, 37, 46

Time (minutes)	Tally	Frequency
20–<30		
30–<40		
40–<50		
50–<60		
60–<70		

a Use the data to complete the grouped frequency table.

b Construct a histogram representing this data.

c Add a frequency polygon to your histogram.

d Describe the shape of the distribution.

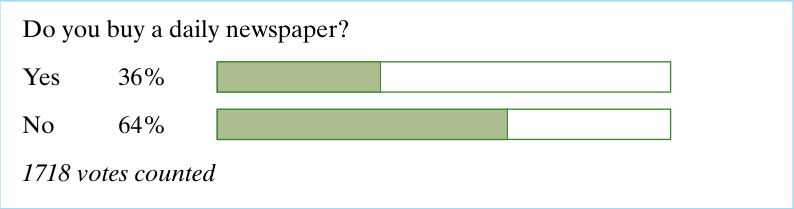
e What was the median time spent on the treadmill?

The following poll question is from the Australian Broadcasting Corporation (ABC) website.

a What number of people out of the 1718 people who responded did *not* buy a daily newspaper? Give your answer to the nearest person.

b What is the ratio in simplest form of people who said they did buy a daily newspaper to those who said they did not.

c Do you think these proportions would be representative of the whole population of Australia? Justify your answer with reasons.



Do you buy a daily newspaper?

Yes 36%

No 64%

1718 votes counted

10

Perimeter, area and volume

Pre-test

Warm-up

Penny farthing bicycles date back to about 1870. They were named after two coins of the time. The farthing, worth one quarter of a penny, was a very small coin compared with the large penny coin. Evandale in Tasmania hosts the annual National Penny Farthing Championship. The photograph shows the 2010 men's and women's winners, Huw Morgan and Amy Vestr. Estimate the diameter of each of the wheels. What is the advantage of the large front wheel? Would there be a disadvantage in having both wheels the size of the front wheel? How many turns would the rear wheel have to make for each turn of the front wheel?

10.1 Reviewing length and perimeter

Units of length

When comparing measurements we need to make sure they are in the same unit. For example, to compare a measurement in metres with another measurement in millimetres, we could convert both to millimetres or both to metres.

Converting between length units

$$\begin{tikzcd}
\mathbf{mm} \arrow[r, bend left, "\div 10"] & \mathbf{cm} \arrow[l, bend left, "\times 10"] \arrow[r, bend left, "\div 100"] & \mathbf{m} \arrow[l, bend left, "\times 100"] \arrow[r, bend left, "\div 1000"] & \mathbf{km} \arrow[l, bend left, "\times 1000"]
\end{tikzcd}$$

Example 1

Convert each of these measurements into the unit shown in brackets.

a 2.8 m (cm) **b** 5970 mm (m) **c** 3.69 cm (mm) **d** 287 m (km)

Working	**Reasoning**
a $2.8\text{ m} = 2.8 \times 100$ $= 280\text{ cm}$	$1\text{ m} = 100\text{ cm}$ Larger unit to smaller unit, so more of them. Multiply.
b $5970\text{ mm} = 5970 \div 1000$ $= 5.970\text{ m}$	$1000\text{ mm} = 1\text{ m}$ Smaller unit to larger unit, so fewer of them. Divide.
c $3.69\text{ cm} = 3.69 \times 10$ $= 36.9\text{ mm}$	$1\text{ cm} = 10\text{ mm}$ Larger unit to smaller unit, so more of them. Multiply.
d $287\text{ m} = 287 \div 1000$ $= 0.287\text{ km}$	$1\text{ km} = 1000\text{ m}$ Smaller unit to larger unit, so fewer of them. Divide.

Calculating perimeter

Perimeter is the length of the boundary around the outside of a shape. For example, the perimeter of a sports ground, a swimming pool, a house or a farm is simply the distance all the way around the outside.

Perimeter

Perimeter = the sum of the lengths of all the sides

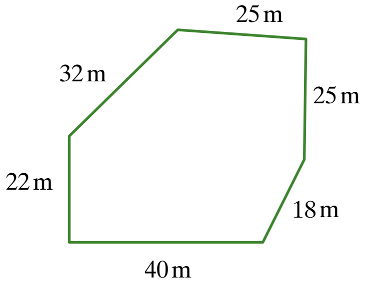
For an irregular shape we use the method of adding the lengths of all the sides.

Perimeter is a measurement of length, so we use the units millimetre, centimetre, metre and kilometre.

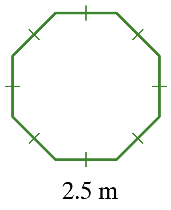
Example 2

Find the perimeter of these shapes.

a



b



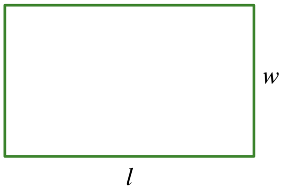
Working	**Reasoning**
a Perimeter = 25 + 25 + 18 + 40 + 22 + 32 = 162 m	Add the lengths of the six sides.
b Perimeter = 8 × 2.5 = 20 cm	The eight sides are equal in length.

Rectangle and square

Both pairs of opposite sides of a rectangle are equal, so we can simplify the calculation of the perimeter:

Perimeter of rectangle = 2 × (length + width)

$$P = 2(l + w)$$



For a square, all four sides are equal.

Perimeter of a square = 4 × length

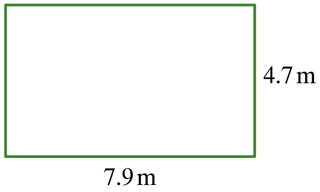
$$P = 4l$$

Example 3

Find the perimeter of

a a rectangle with dimensions 4.7 m by 7.9 m.

b a square with side length 5.7 cm.

Working	**Reasoning**
a 	Perimeter is the total distance around the rectangle.
Perimeter $= 2(l + w)$ Perimeter $= 2(4.7 + 7.9)$ $= 2 \times 12.6$ $= 25.2$ m	Perimeter = 2 × (length + width)
b Perimeter $= 4l$ $= 4 \times 5.7$ $= 22.8$ cm	The four sides of a square are each the same length, so multiply the side length by 4 to find the perimeter.

If we know the perimeter of a rectangle and either the length or the width, we can use the formula and work backwards to find the missing dimension.

Example 4

The perimeter of a rectangle is 38.6 m. The width of the rectangle is 7.2 m. What is the length?

Working	**Reasoning**
$P = 2(l + w)$	
$38.6 = 2(l + 7.2)$	Substitute 38.6 for P and 7.2 for w.
$19.3 = l + 7.2$	Divide both sides by 2.
$l = 19.3 - 7.2$	Subtract 7.2 from both sides.
$l = 12.1$	
The length of the rectangle is 12.1 m.	

exercise 10.1

LINKS TO Example 1

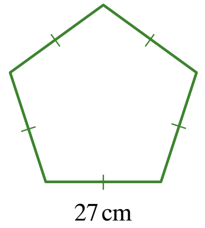
Convert each of these measurements of length into the units shown in brackets.

a 8.5 m (millimetres)
b 0.42 cm (millimetres)
c 4.81 m (centimetres)
d 0.4 cm (metres)
e 155 m (kilometres)
f 1.3 m (millimetres)
g 0.5 mm (centimetres)
h 24 250 mm (metres)
i 1450 cm (metres)

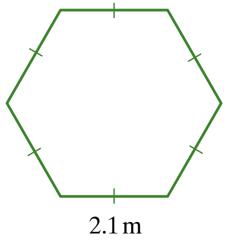
LINKS TO Example 2

Find the perimeter of each of these shapes.

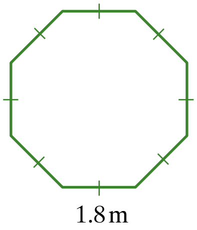
a



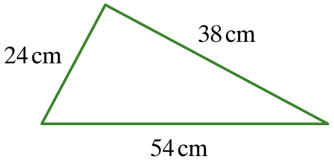
b



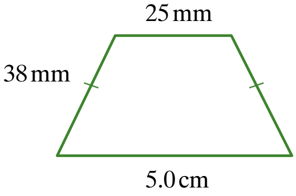
c



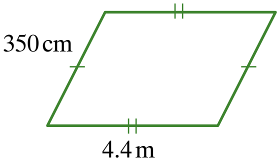
d



e

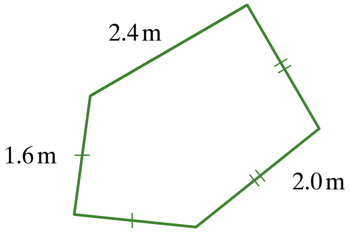


f



The perimeter of this shape is

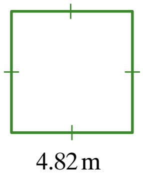
A 7.6 m
B 6.0 m
C 9.6 m
D 8.0 m
E 7.2 m



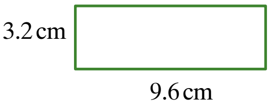
LINKS TO Example 3

Find the perimeter of each of these rectangles.

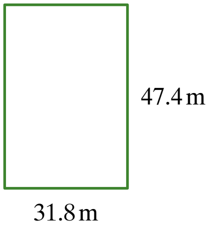
a



b



c



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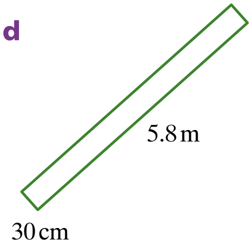

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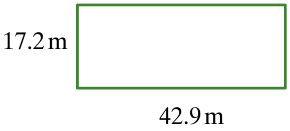

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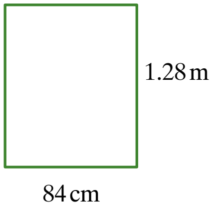

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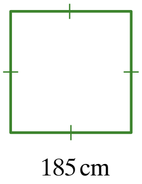
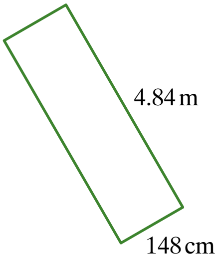


h



A rectangle has length 29 cm and breadth 17 cm. The perimeter is

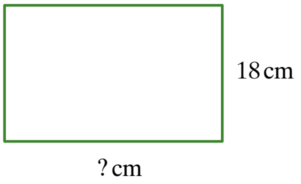
A 46 cm **B** 63 cm **C** 75 cm **D** 92 cm **E** 493 cm

LINKS TO Example 4

The perimeter of this rectangle is 56 cm. What is the missing side length?

A 38 cm **B** 19 cm **C** 20 cm

D 10 cm **E** 3.1 cm



The perimeter of a rectangle is 110 cm. One of the dimensions is 22 cm. What is the other dimension?

A 5 cm **B** 33 cm **C** 44 cm **D** 66 cm **E** 88 cm

The perimeter of a rectangle is 1.26 m. The length is 29 cm. How wide is the rectangle?

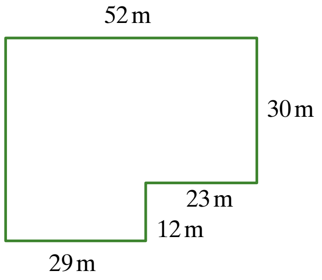
The perimeter of a rectangle is 5.86 m. If the width is 124 cm, how long is the rectangle?

The perimeter of a rectangle playing field is 380 m. If the length of the field is 120 m, what is the width?

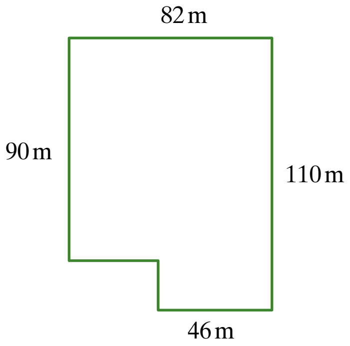
Molly has 5 m of edging to go around a garden. She wants the garden to be 1.4 m long. How wide can the garden be?

Find the perimeter of each of these shapes. In each case you will need to find any missing side lengths first.

a



b



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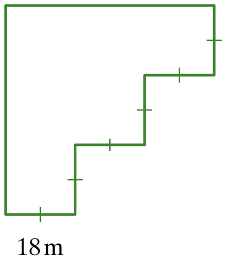

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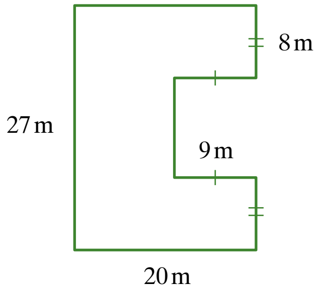
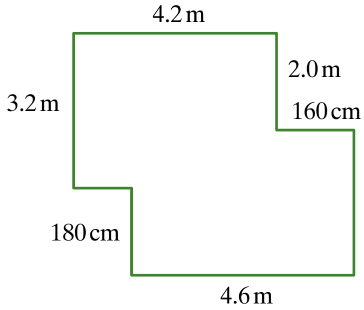


e



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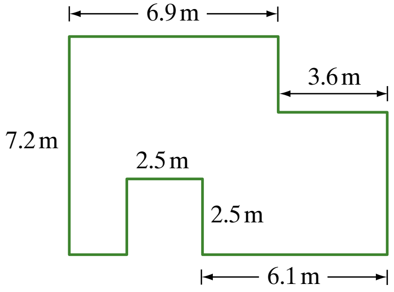
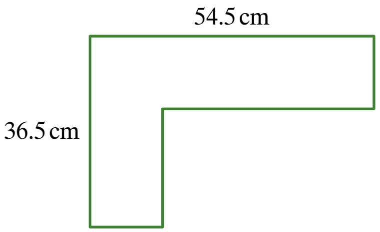


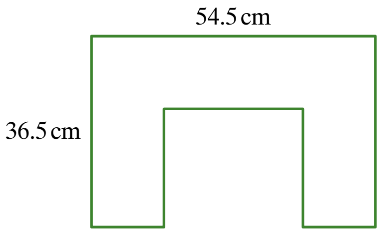
exercise 10.1 challenge

Is there enough information given for each of the following shapes to calculate the perimeter? If so, find the perimeter. If there is insufficient information, show what additional information would be needed.

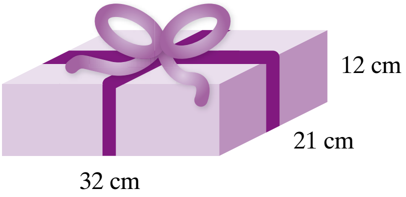
a



b



Dawn has wrapped a birthday present and is going to tie a piece of ribbon around it as shown. She estimates that she will need 25 cm for the bow. What is the total length of ribbon that Dawn is going to need?



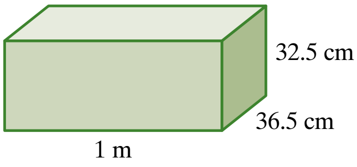
Australia Post gives the following information about parcel sizes.

Maximum weight and size of a parcel accepted over the counter:

Maximum weight	***20 kg***
Maximum length	***The greatest linear dimension must not exceed 105 cm***
Maximum girth	***The girth of a parcel (measured around the other two dimensions) must not exceed 140 cm.***

Which of the following parcels would be accepted? Explain.

a



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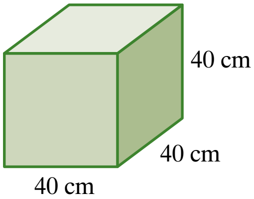


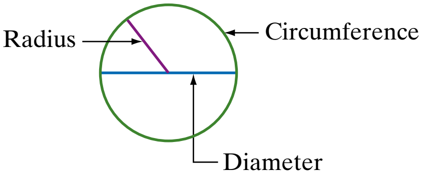
10.2 Circumference of a circle

The perimeter of a circle has a special name: the **circumference**.

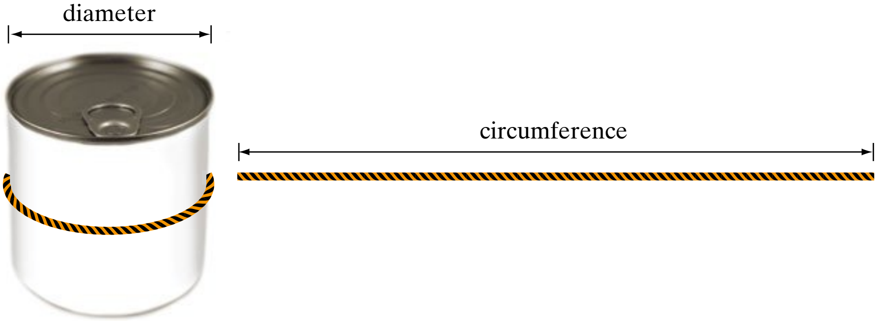
The **radius** of a circle is the distance from the centre of the circle to the circumference.

The **diameter** of a circle is the distance across the circle passing through the centre of the circle.

The diameter is twice the length of the radius.



We can estimate the circumference of a circle by wrapping a piece of string around a cylindrical can and then measuring the length of the string.



Estimating pi

If we measure accurately, we find that, for any circle, the circumference is approximately 3.14 times the diameter.

We give the name π to this number; π is a Greek letter which is pronounced 'pie'.

This means that if we know the diameter or radius of a circle we can calculate its circumference.

Estimating pi

Circumference

$C = \pi d$ or $C = 2\pi r$ and $d = 2r$

where C is the circumference, d is the diameter and r is the radius.

The history of π

It has been known for at least 4000 years that the ratio of the circumference to the diameter of a circle is the same for all circles. It was not until the 18th century that the Greek letter π was used for this ratio.

About 4000 years ago, the ancient Babylonians used the value $3\frac{1}{8}$ and the Egyptians used $\frac{256}{81}$ for the ratio of circumference to diameter. Archimedes, a Greek mathematician who lived from 287 – 212 BCE, found that the value was less than $3\frac{1}{7}$ but greater than $3\frac{10}{71}$. The 5th century Chinese astronomer, Tsu Ch'ung-chih, and his son, Tsu Keng-chih, calculated that π was approximately $\frac{355}{113}$.

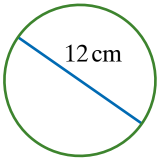
For the past few hundred years, though, mathematicians have known that there is no fraction that is exactly equal to π. The decimal places of π go on forever, with no repeating pattern. As we saw in chapter 2, another name for a fraction is a **rational number**. There is no fraction that is exactly equal to π so π is an **irrational number**. Computers have calculated π to over 2 billion decimal places. Correct to 25 decimal places, the value of π is 3.1415926535897932384626434.

In calculations involving π we use the π button on a calculator. We need to round answers to a sensible number of decimal places. For example, if the diameter has been given as a whole number, then it makes sense to round the circumference to no more than one or two decimal places.

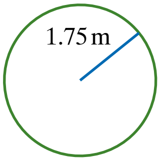
Example 5

Calculate the circumference of these circles correct to one decimal place. Use the π button on your calculator.

a



b



Working	**Reasoning**
a $C = \pi d$ $= \pi \times 12$ $\approx 37.699\ldots$ The circumference is 37.7 cm, correct to one decimal place.	The diameter is given so use $C = \pi d$. The calculator uses an approximate value for π so we put an 'approximately equals' sign in front of 37.699. The answer is rounded to the required number of decimal places. Check: π is a little more than 3, so the answer should be a little more than 3×12.

continued

Example 5 continued

Working	Reasoning
b $C = 2\pi r$ $= 2\pi \times 1.75$ $= 3.5\pi$ $\approx 10.99\ldots$	The radius is given so use $C = 2\pi r$. The calculator uses an approximate value for π so we put an 'approximately equals' sign in front of 10.99…
The circumference is 11.0 m correct to one decimal place.	Check: π is a little more than 3, so the answer should be a little more than 3×3.5; that is, more than 9 but less than 12.

If we know the circumference of a circle we can work backwards to find the diameter and the radius.

Example 6

A circle has circumference 5.8 m. Calculate the following to the nearest 10 centimetres.

a the diameter

b the radius.

Working	Reasoning
a $C = \pi d$ $5.8 = \pi d$	The diameter is wanted so use $C = \pi d$. Substitute in 5.8.
$d = \dfrac{5.8}{\pi}$ $\approx 1.846\ldots \text{ m}$	Divide both sides by π.
Diameter is approximately 180 cm.	184.6 cm is rounded to the nearest 10 centimetres.
b $r = d \div 2$ $= 1.846\ldots \div 2$ $= 90 \text{ cm}$	Use the unrounded value for diameter from part **a**.

Tech tip

The TI-30XB MultiView calculator can be used to calculate the circumference of a circle.

If the diameter is a decimal number, for example, 1.75 m (example 5 part b), type:
$\boxed{\pi}\boxed{\times}\boxed{1}\boxed{\cdot}\boxed{7}\boxed{5}$ **enter** .

If the diameter is a whole number or fraction, for example, 12 cm (example 5 part a), type:
$\boxed{\pi}\boxed{\times}\boxed{1}\boxed{2}$ **enter** **◂▸**

(Typing **◂▸** gives a rational approximation.)

Tech tip

The TI-30XB MultiView calculator can also be used to calculate the radius or diameter of a circle, given the circumference.

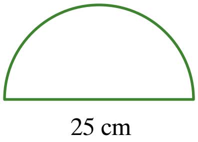
If the circumference of a circle is 5.8 m (example 6), type:

$\boxed{5}\,\boxed{\cdot}\,\boxed{8}\,\boxed{\div}\,\boxed{\pi}$ **enter** then $\boxed{\text{ans}}\,\boxed{\div}\,\boxed{2}$ **enter** or $\boxed{\div}\,\boxed{2}$ **enter**

Some shapes are parts of a circle, for example a half circle (semicircle) or quarter circle. The perimeter of these shapes are made up of straight sides and parts of the circumference.

Example 7

Find the perimeter of this semicircle correct to 1 decimal place.



Working

Diameter = 25 cm

$$\text{Perimeter} = 25 + \frac{\pi \times 25}{2}$$
$$\approx 25 + 39.27$$
$$= 64.27 \text{ cm}$$

The perimeter is 64.3 cm.

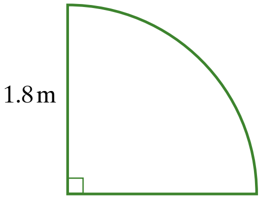
Reasoning

The straight side of the shape = 25 cm
The curved part is half the circumference of a circle with diameter 25 cm.
Add the two parts of the perimeter.

Round to 1 decimal place.

Example 8

Find the perimeter of each of this shape correct to one decimal place.
quadrant (quarter circle)



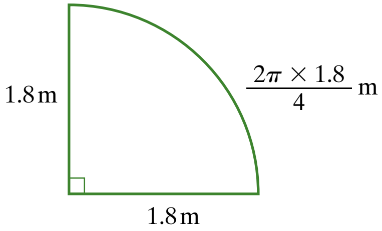
Working

$$\text{Perimeter} = 1.8 + 1.8 + \frac{2\pi \times 1.8}{4}$$
$$= 3.6 + 0.9\pi$$
$$\approx 6.427$$

The perimeter is 6.4 m correct to one decimal place.

Reasoning

The radius of the quarter circle is 1.8 m.

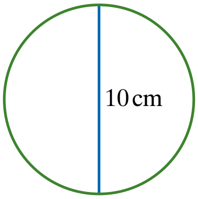


exercise 10.2

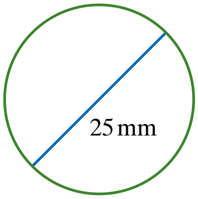
LINKS TO Example 5a

Find the circumference of the circles shown below. Use the π button on your calculator. Round your answers to one decimal place.

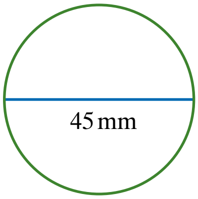
a



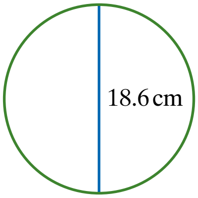
b



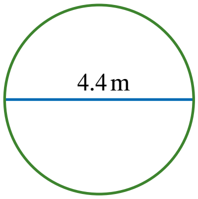
c



d



e



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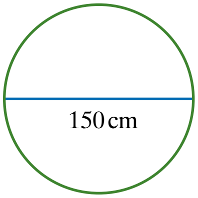
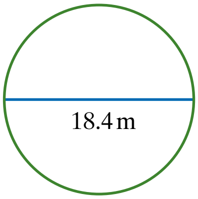


g



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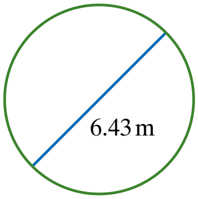
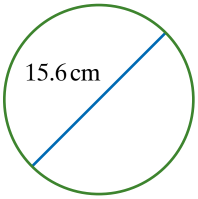


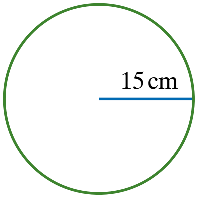
i



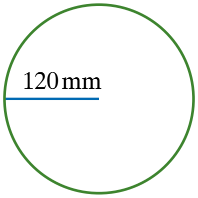
LINKS TO Example 5b

Find the circumference of the circles shown below. Use the π button on your calculator. Round your answers to two decimal places.

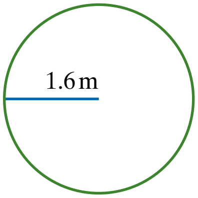
a



b



c



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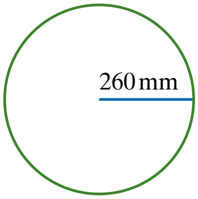

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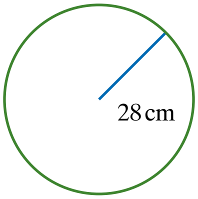

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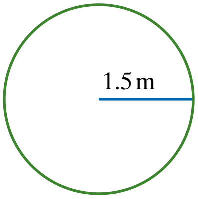

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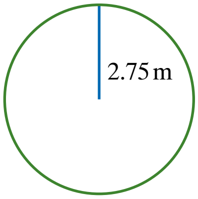

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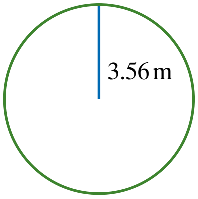
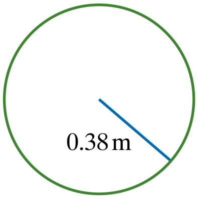


i



Find the circumference of each of the following circles. Use the π button on your calculator. Round your answers to one decimal place.

a radius 5.3 cm
b diameter 12 mm
c diameter 18.5 cm
d diameter 2.5 m
e radius 78 mm
f radius 1.75 m
g diameter 4.5 m
h diameter 15.0 cm
i radius 22.5 mm
j radius 27 cm
k radius 8.6 m
l diameter 1.4 m

A circular running track has a diameter of 120 m.

a How far is one lap of the track? Give your answer to the nearest metre.

b If Abdul runs six laps of the track, how far does he run? Give your answer in kilometres, correct to 2 decimal places.

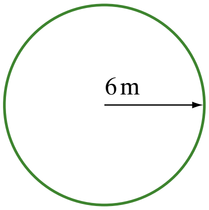
A circular garden bed has a radius of 1.4 m.

a Hannah is putting edging around the outside of the garden.
How much edging will she need correct to two decimal places?

b If the edging is sold in 6 m packs, how many packs will Hannah need to buy?

The circumference of this circle is closest to

A 12 m
B 19 m
C 36 m
D 38 m
E 113 m



The diameter of a circle is 40 cm. The circumference is closest to

A 43 cm
B 63 cm
C 126 cm
D 252 cm
E 1600 cm

LINKS TO Example 6

The circumference of a circle is 54 cm. The diameter is closest to

A 27 cm
B 170 cm
C 108 cm
D 17 cm
E 8.5 cm

The circumference of a circle is 11.5 m. The radius is closest to

A 23 m
B 1.8 m
C 36 m
D 3.7 m
E 7.4 m

LINKS TO Example 6

Calculate the diameter of circles that have the following circumferences. Give your answers correct to one decimal place.

a 6.28 m
b 19.6 cm
c 125.5 cm
d 16.5 m
e 108 cm
f 140 mm
g 13.4 m
h 56.8 cm

Find the radius of circles that have the following circumferences. Give the radius correct to one decimal place.

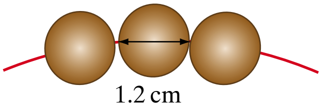
a 17 cm
b 85.6 cm
c 2.7 m
d 48.6 cm
e 140 cm
f 78 cm
g 1.86 m
h 7.5 m

- Sophia has 4.5 m of braid to go around the edge of a circular rug. What is the diameter of the largest rug that the braid will go around? Round to the nearest centimetre.

- Alex is making a circular tablecloth which has a diameter of 2 m.
  - **a** How much binding will be needed to go around the edge of it? Give your answer in metres correct to two decimal places.
  - **b** If the binding is sold in packets of 5 m, how many packets will Alex need to buy?
  - **c** If a factory is binding the edge of 2 m diameter circular tablecloths, how many could be made using a 1000 m roll of binding?

- A bicycle wheel has a diameter of 60 cm.
  - **a** What is the circumference of the wheel? Give your answer correct to one decimal place.
  - **b** How far does the bicycle move forward each time the wheels turn through a complete revolution?
  - **c** Approximately how many revolutions would each wheel complete if the bicycle travelled a distance of 1 km?

- Sally and Nicholas are decorating a birthday cake for their sister. They are starting to put a ring of chocolate buttons around the edge as shown.
  - **a** The diameter of the cake is 21 cm. Find the circumference to one decimal place.
  - **b** The diameter of each chocolate button is 1.2 cm. How many chocolate buttons will Sally and Nicholas need?



- The circumference of the circle made by these children's feet is 12 m. What is the diameter of the circle to two decimal places?

The circumference of this tree trunk is approximately circular and is 190 cm. What is the radius of the trunk to the nearest centimetre?

William has a model railway with a circular track. If it is 5 m around the track, what is the diameter of the railway track? Give your answers correct to one decimal place.

A trundle wheel is a device for measuring distances. As the wheel is pushed along, it clicks each time the wheel completes a full revolution. If each revolution of the trundle wheel represents 1 m along the ground, what would the diameter of the trundle wheel have to be? Give your answer in centimetres correct to one decimal place.

Brett Richardson from Brisbane built this penny-farthing bicycle in 1988. In 2005, he rode it 480 km from Cooper Creek to Cunnamulla with a group of 40 other cyclists (on conventional bicycles) to raise money for the Royal Flying Doctor Service.

a If the diameter of the large wheel of a penny-farthing bicycle is 150 cm, how far will the bicycle move forward for each revolution of the large wheel? Give your answer to the nearest centimetre.

b How many revolutions would the large wheel make in 480 km?

c A standard bicycle wheel has a diameter of 26 inches (66 cm). Compare the distance moved during each revolution of the large wheel of the penny-farthing bicycle with the distance moved for each revolution of a standard bicycle wheel.

d If the diameter of the small wheel of the penny-farthing bicycle is 50 cm, how many times (to the nearest turn) will the small wheel turn for each revolution of the large wheel?

The following rational numbers are all numbers that have been calculated in the past for the ratio of the circumference to diameter of a circle.

a Use your calculator to convert each fraction into a decimal.

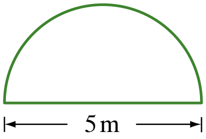
i $3\frac{1}{8}$ **ii** $\frac{256}{81}$ **iii** $3\frac{1}{7}$ **iv** $3\frac{10}{71}$ **v** $\frac{355}{113}$

b We of course know now that π is not a rational number. Correct to 25 decimal places, π equals 3.1415926535897932384626434. Which of the five fractions in part **a** is closest to the value of π?

LINKS TO Example 7

Find the perimeter of each of these shapes based on semicircles. Round your answer to two decimal places.

a



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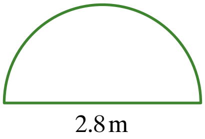

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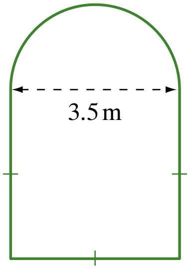

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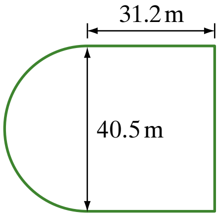

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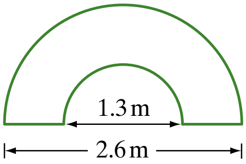

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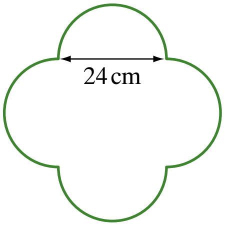
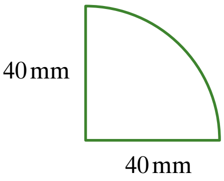


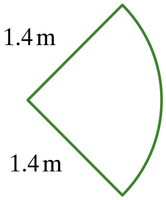
LINKS TO Example 8

Find the perimeter of each of these shapes based on quarter circles. Round your answer to one decimal place.

a



b



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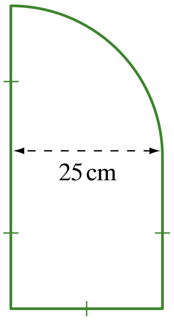

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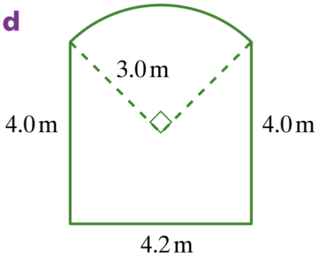

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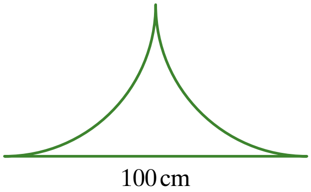

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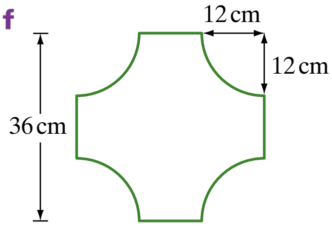
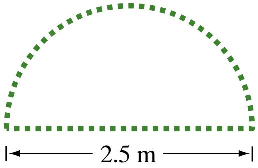
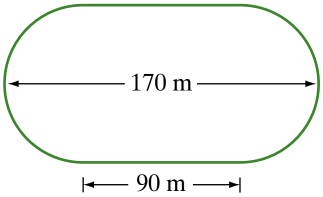


How many metres of edging are needed to put around the outside of this flower garden? Give your answer in metres correct to one decimal place.

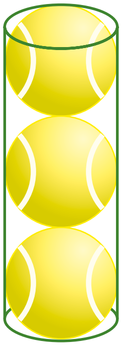


The racetrack shown has semicircular ends. Find the perimeter of the track. Give your answer to the nearest metre.



exercise 10.2 challenge

Tennis balls are packaged in threes in cylinder-shaped containers as shown. Which would be greater: the height of the cylinder or the circumference? Justify your answer.



10.3

Area: rectangles, triangles and parallelograms

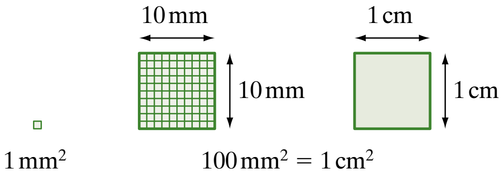
Units of area

The relationship between square millimetres, square centimetres and square kilometres is shown below.

$100\,\text{mm}^2 = 1\,\text{cm}^2$

$10\,000\,\text{cm}^2 = 1\,\text{m}^2$

$1\,000\,000\,\text{m}^2 = 1\,\text{km}^2$

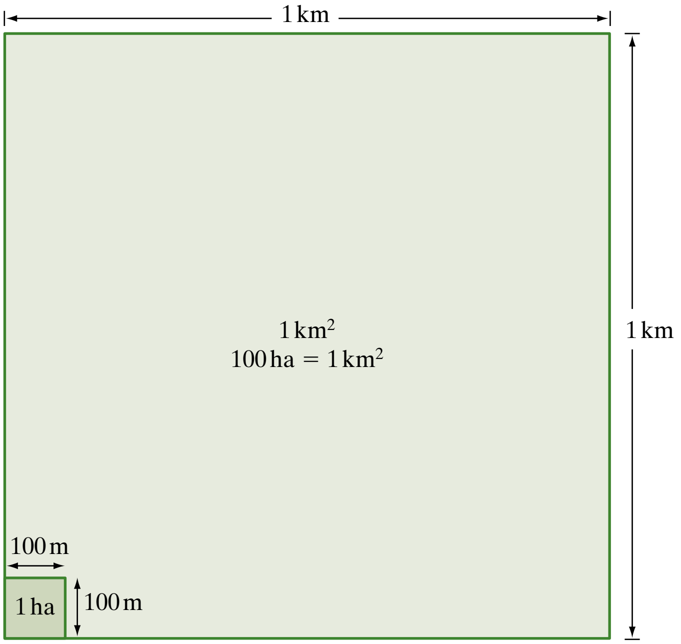


Areas of farms and parks are often measured in hectares. The abbreviation for hectare is ha.

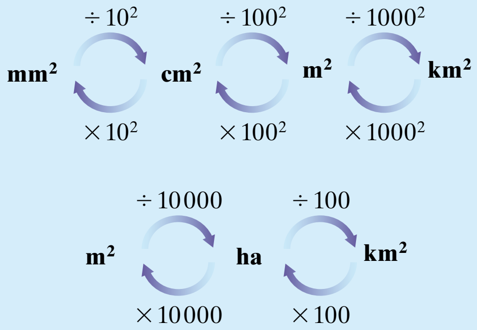
$1\text{ ha} = 10\,000\,\text{m}^2$

A square paddock 100 m by 100 m would have an area of 1 hectare.

$100\text{ ha} = 1\,\text{km}^2$



Converting between units of area



Example 9

Convert

a 18.5 cm² into square millimetres.

b 0.45 km² into square metres.

c 5.7 square kilometres into hectares.

Working	**Reasoning**
a $18.5\text{ cm}^2 = 18.5 \times 10^2 \times 10$ $= 1850\text{ mm}^2$	To convert cm² to mm² multiply by 10^2.
b $0.45\text{ km}^2 = 0.45 \times 1000^2$ $= 450\,000\text{ m}^2$	To convert km² to m² multiply by 1000^2.
c $5.7\text{ km}^2 = 5.7 \times 100$ $= 570\text{ ha}$	$1\text{ km}^2 = 100\text{ ha}$

Example 10

Convert

a 572 000 cm² to square metres.

b 2450 mm² into square centimetres.

Working	**Reasoning**
a $572\,000\text{ cm}^2 = 572\,000 \div 100^2$ $= 57.2\text{ m}^2$	To convert cm² to m² divide by 100^2.

continued

Example 10 continued

Working

b $2450\text{ mm}^2 = 2450 \div 10^2$
$= 24.5\text{ cm}^2$

We say 'square centimetres', not 'centimetres squared'.

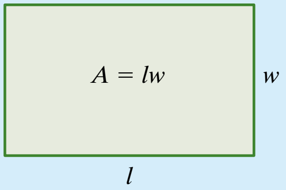
Reasoning

To convert mm^2 to cm^2 divide by 10^2.

Area of a rectangle and square

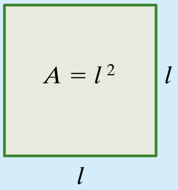
Area of rectangle $= lw$

where l is the length and w is the width



Area of square $= l^2$

where l is the length of each side



The words height and breadth are also used to describe the dimensions of a rectangle. Height would normally be used if the rectangle was vertical. The important thing is that we know which sides we are talking about.

Example 11

Find the area of a rectangular piece of land 18.5 m wide and 24.6 m long.

Working

Area of rectangle $= lw$
Area $= 18.5 \times 24.6$
$= 455.1\text{ m}^2$
The area of the land is 455.1 m^2.

Reasoning

Area of rectangle is length times width.

When calculating areas it is important to convert all measurements to the same unit.

Example 12

A rectangular farm has length 0.6 km and width 450 m. Find the area in

i square metres. **ii** hectares.

Working	Reasoning
i Area of rectangle $= lw$ $= 600 \times 450$ $= 270000\,\text{m}^2$ The area of the farm is 270 000 square metres.	Since we know the relationship between square metres and hectares, convert 0.6 km to 600 m.
ii Area $= 270000\,\text{m}^2$ $= 270000 \div 10000$ $= 27$ ha The area of the farm is 27 hectares.	1 hectare $= 10000\,\text{m}^2$ i.e., every $10000\,\text{m}^2$ is 1 hectare, so divide by 10 000 to find the number of hectares.

Finding an unknown side length in a rectangle or square involves working backwards using the area formula.

Example 13

Calculate the missing side lengths.

a A rectangle has area $540\,\text{cm}^2$ and length 27 cm. Find the width.

b A square has area $441\,\text{cm}^2$. Find the length of the sides.

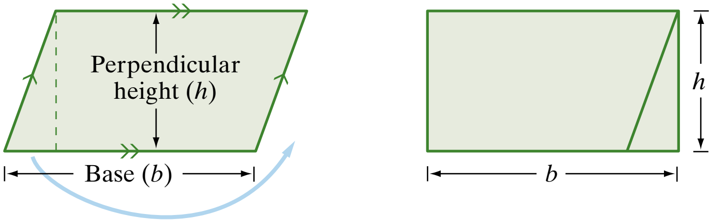
Working	Reasoning
a $A = lw$ $540 = 27 \times w$	Substitute 540 for A and substitute 27 for l.
$w = \frac{540}{27}$ $= 20$	Solve the equation for w. Divide both sides by 27.
The width of the rectangle is 20 cm.	Answer the question.
b $A = l^2$ $441 = l^2$ $l = \sqrt{441}$ $l = 21$	Substitute 441 for A. Solve the equation for l. Take the square root of both sides. $\boxed{\sqrt{\ }}\ \boxed{4}\ \boxed{4}\ \boxed{1}$ enter
The length of each side of the square is 21 cm.	Answer the question.

Parallelogram area

Area of parallelogram Power Point

Area of a parallelogram

The area of a parallelogram is the same as the area of a rectangle with the same base and the same perpendicular height.



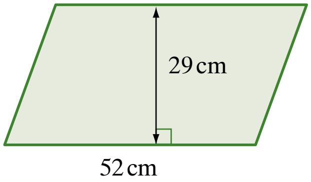
Area of parallelogram = Base × Perpendicular height

$$A = bh$$

where *b* is the base and *h* is the perpendicular height.

Example 14

Find the area of this parallelogram.

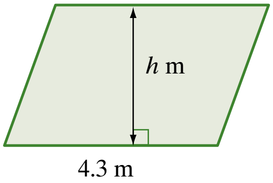


Working	**Reasoning**
$A = bh$ $= 52 \times 29$ $A = 1508$	Multiply the length of the base by the perpendicular height.
The area is 1508 cm^2.	Answer the question.

Finding an unknown side length in a parallelogram involves working backwards using the area formula.

Example 15

The area of this parallelogram is 15.05 m^2. Find the perpendicular height.



continued

Example 15 continued

Working

$$A = bh$$

$$15.05 = 4.3 \times h$$

$$h = \frac{15.05}{4.3}$$

$$h = 3.5$$

The perpendicular height is 3.5 m.

Reasoning

Write the area formula.

Substitute $A=15.05$ and $b=4.3$.

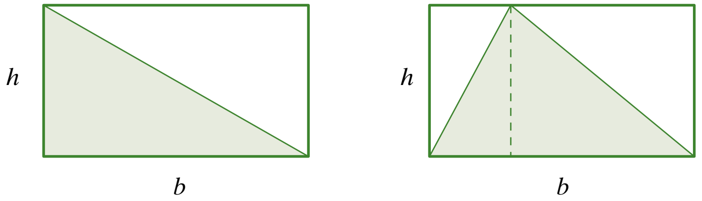
Solve the equation. Divide both sides by 4.3.

Area of a triangle

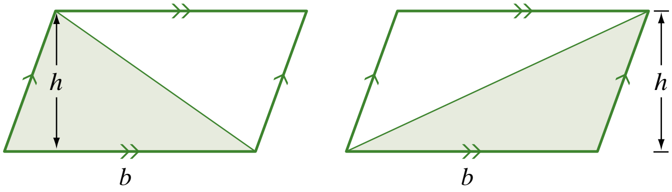
Area of a triangle

Area of a triangle Power Point

It is easy to see that the area of a right-angled triangle is half the area of a rectangle with the same base and height. For other triangles this also applies, where the two parts of the triangle are each half a rectangle.



It is also easy to see that the area of a triangle is half the area of a parallelogram with the same base and height. Each diagonal of a parallelogram divides it into two congruent triangles. The area of each triangle is, therefore, half the area of the parallelogram.



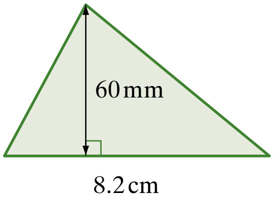
Area of triangle = Area of parallelogram ÷ 2

$$A = \frac{bh}{2} \text{ or } A = \frac{1}{2}bh$$

where b is the base and h is the perpendicular height.

Example 16

Find the area of this triangle.



Working

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 8.2 \times 6.0$$
$$A = 24.6\ \text{cm}^2$$

The area is 24.60 cm^2.

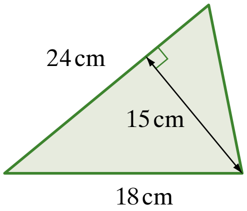
Reasoning

Convert 60 mm to 6.0 cm.
The area of a triangle is half the base times the perpendicular height.
Substitute values for *b* and *h*.

When calculating the area of a triangle, the side we call the base is the side that is shown perpendicular to the given height.

Example 17

Calculate the area of this triangle.



Working

$$A = \frac{bh}{2}$$
$$= \frac{24 \times 15}{2}$$
$$A = 180$$

The area is 180 cm^2.

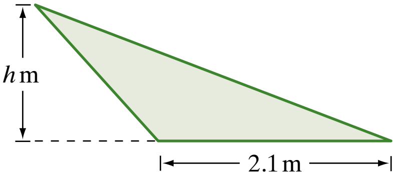
Reasoning

The given height is perpendicular to the side labelled 24 cm. So the side labelled 24 cm is the base. We do not need the side labelled 18 cm to calculate the area.
Write the area formula.
Substitute b=24 and h=15. Solve the equation.
Answer the question.

If we know the area and either the base or perpendicular height of a triangle we can use the area formula to work backwards to find the missing dimension.

Example 18

The triangle on the right has an area of 3.4 m^2. If the base of the triangle is 2.1 m, find the perpendicular height, h m correct to one decimal place.



Working

$$\begin{aligned} \text{Area of triangle} &= \frac{bh}{2} \\ 3.4 &= \frac{2.1 \times h}{2} \\ 2 \times 3.4 &= 2.1h \\ h &= \frac{2 \times 3.4}{2.1} \\ &\approx 3.24 \end{aligned}$$

Perpendicular height is approximately 3.2 m.

Reasoning

Write the area formula.

Substitute $A=3.4$ and $b=2.1$. Solve the equation.

Multiply both sides by 2.

Divide both sides by 2.1.

exercise 10.3

LINKS TO Examples 9, 10

● Convert these areas into the units shown in brackets.

a 274 mm^2 (cm^2)
b 35.2 cm^2 (mm^2)
c 670 000 cm^2 (m^2)
d 0.00095 m^2 (cm^2)
e 3780 ha (km^2)
f 56 000 000 m^2 (km^2)
g 4 520 000 m^2 (ha)
h 1.8 km^2 (ha)
i 0.7 km^2 (ha)
j 34 m^2 (cm^2)
k 18 000 mm^2 (cm^2)
l 32 000 000 m^2 (km^2)
m 25 600 000 mm^2 (m^2)
n 120 ha (m^2)
o 0.5 ha (m^2)

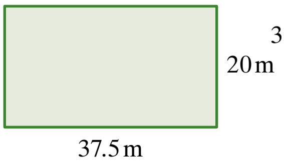
● 28 000 cm^2 is equal to

A 28 m^2 **B** 280 m^2 **C** 2.8 m^2 **D** 28 mm^2 **E** 280 mm^2

LINKS TO Example 11

● Find the area of each of the following rectangles.

a



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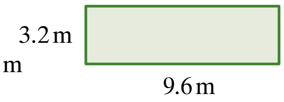

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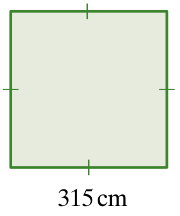

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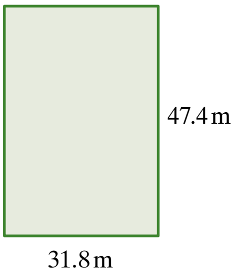

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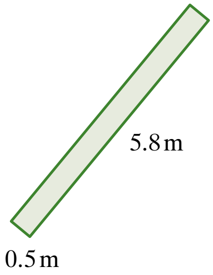

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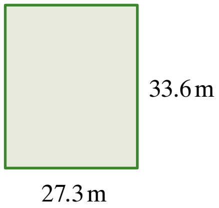

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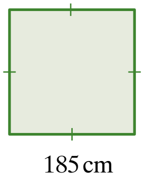

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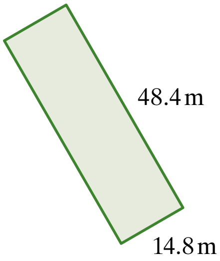


Find the area of the floor of a room that is 5.2 m long and 3.4 m wide.

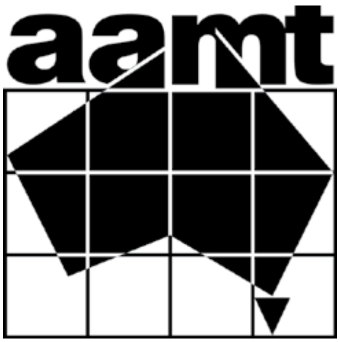
LINKS TO Example 12

A rectangular farm property has a length of 1.2 km and a width of 870 m. Find the area in hectares.

Kakadu National Park has an area of approximately 19 800 km^2. How many hectares is this?

The Victorian Alpine National Park has an area of approximately 650 000 hectares. How many square kilometres is this?

The logo of the Australian Association of Mathematics Teachers is shown here. A 1000 km × 1000 km square grid has been superimposed over a diagrammatic map of Australia. Estimate the area of Australia in square kilometres.



LINKS TO Example 13

Find the missing side length for each of these rectangles.

a area 480 cm^2, length 32 cm
b area 552 cm^2, width 23 cm
c area 35.1 m^2, width 4.5 m
d area 1963 m^2, length 60.4 m

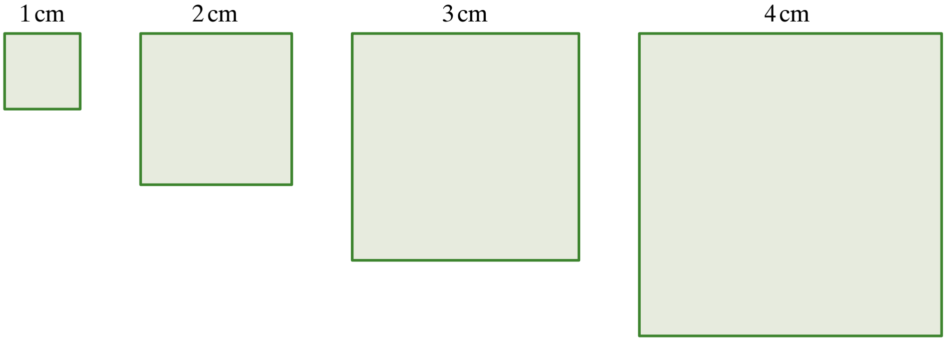
Find the side length of each of these squares.

a area 784 cm^2
b area 1225 cm^2
c area 31.36 m^2
d area 342.25 m^2

The perimeter of a square is 96 cm. What is the area of the square?

The length of a rectangle is 11.45 m and the perimeter is 37.70 m. Find the area of the rectangle.

Consider the squares below.



a Calculate the area of each square.

b Complete the following sentences.

  i When the lengths of the sides of a square are doubled, the area is multiplied by ______

  ii When the lengths of the sides of a square are multiplied by 3, the area is multiplied by ______

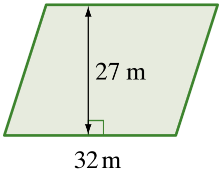
  iii The area of a square with sides of length 5.6 cm would be ______ times the area of a square with sides of length 2.8 cm.

  iv The area of a square with sides of length 7.2 cm would be ______ times the area of a square with sides of length 2.4 cm.

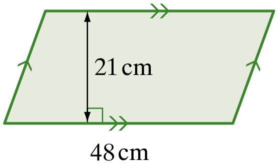
LINKS TO Example 14

Find the area of each of these parallelograms. If the dimensions are given in two different units, convert to the larger of the two units before calculating the area.

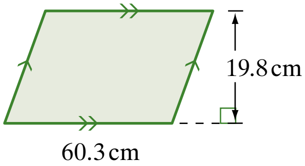
a



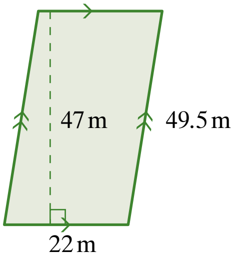
b



c



d



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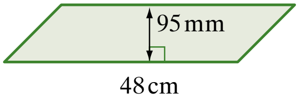

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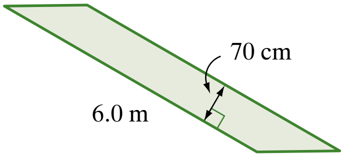

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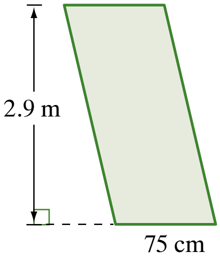

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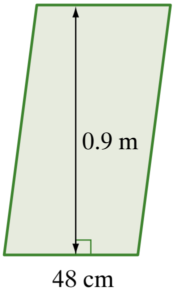

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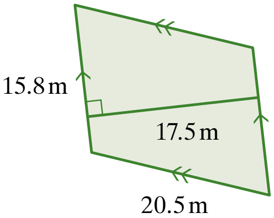

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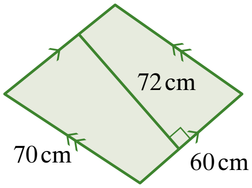

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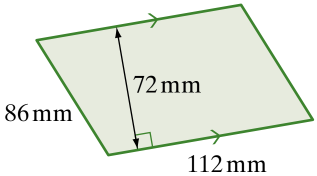

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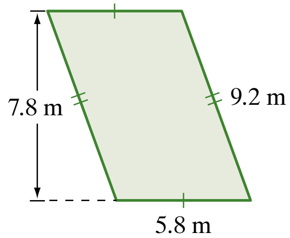
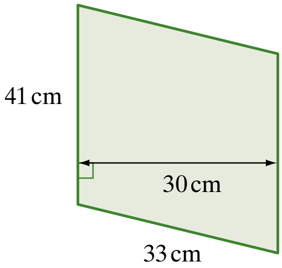


The area of this parallelogram is

A 1353 cm^2 **B** 1230 cm^2 **C** 990 cm^2

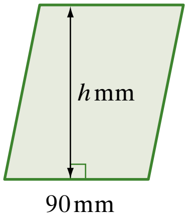
D 615 cm^2 **E** 2220 cm^2



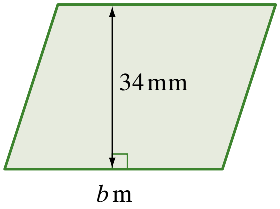
LINKS TO Example 15

Find the missing dimension for these parallelograms.

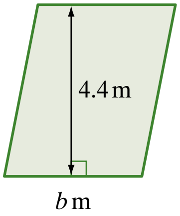
a Area = 3960 mm^2



b Area = 952 m^2



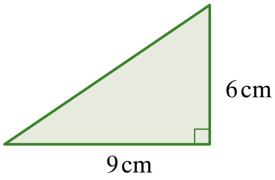
c Area = 9.68 m^2



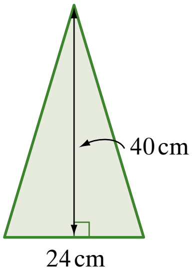
LINKS TO Examples 16, 17

Find the area of each of these triangles.

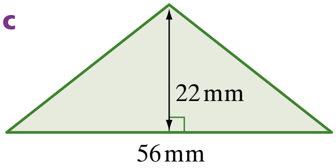
a



b



c



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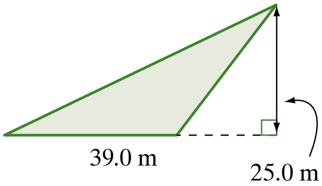

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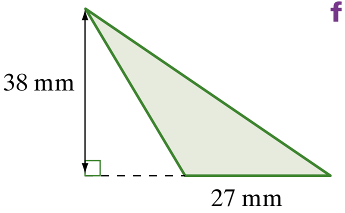

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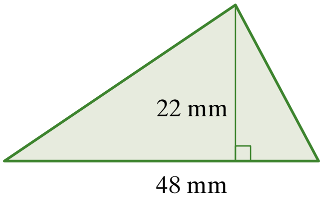

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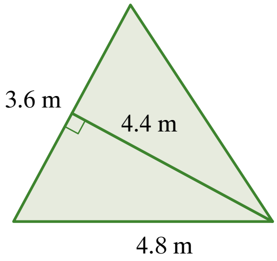

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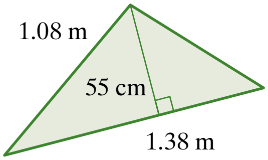

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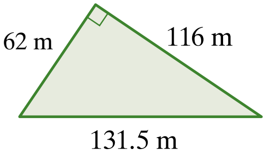

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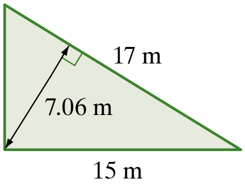

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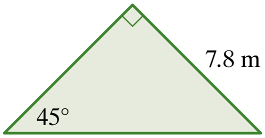

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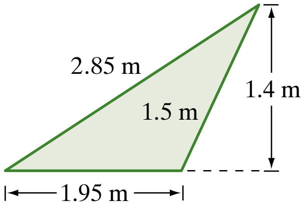

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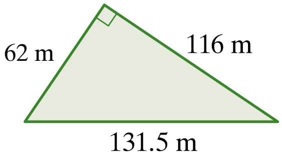
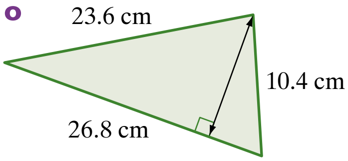


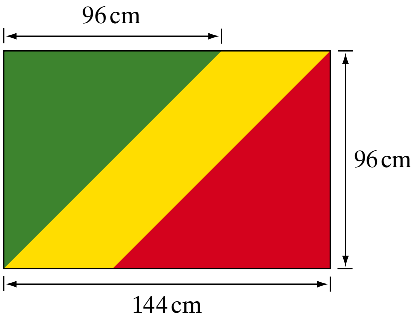
o

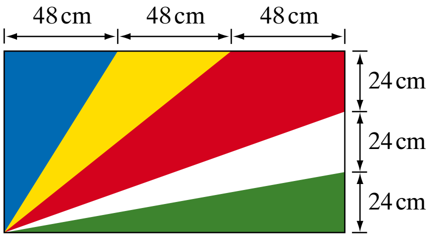


Calculate the area of each of the colours that make up the flags of the countries below.

a Republic of Congo (in Africa)

b Seychelles (a group of islands off the east coast of Africa)

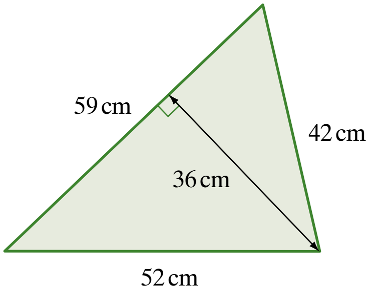




The area of this triangle is

A 756 cm^2 **B** 936 cm^2 **C** 1062 cm^2

D 1872 cm^2 **E** 2124 cm^2

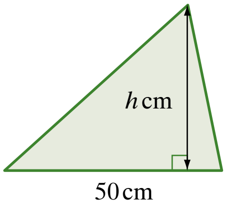


Find the area of a triangle with base 48 cm and perpendicular height 65 cm.

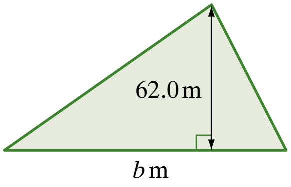
LINKS TO Example 18

Find the missing dimension for each of these triangles.

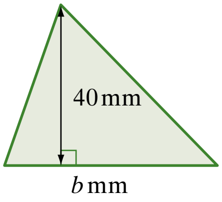
a area = 950 cm^2



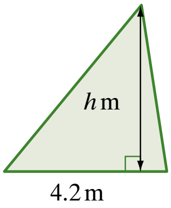
b area = 3720 m^2



c area = 1060 mm^2



d area = 9.03 m^2



e area 3240 cm^2, base 90 cm

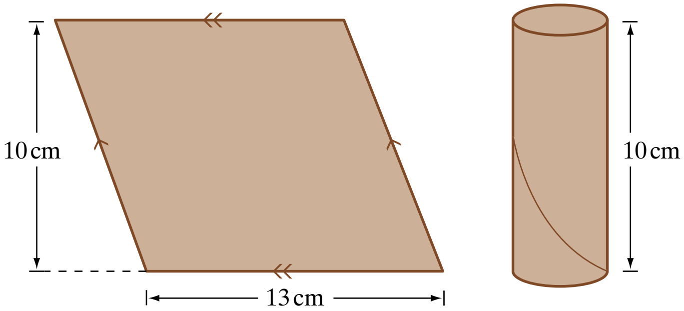
f area 408 m^2, height 16 m

g area 1500 mm^2, base 40 mm

h area 806.88 m^2, height 49.2 m

exercise 10.3 challenge

A cardboard toilet roll tube is made by rolling a parallelogram-shaped piece of cardboard into a cylinder shape.



a What is the area of the piece of cardboard?

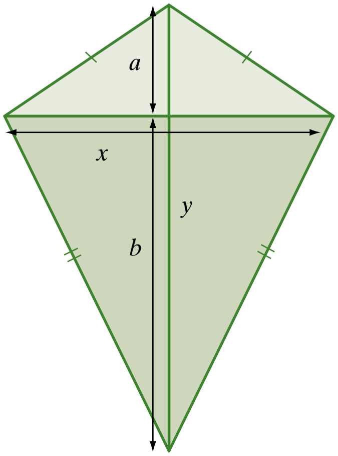
b What is the diameter of the toilet roll tube in centimetres (to two decimal places)? (The cardboard does not overlap but is held in place by another layer of thinner card around the outside.)

10.4 Area: rhombuses, kites and trapezia

Area of a kite

Area of a kite

We can see that the kite is made up of two isosceles triangles. In the following diagram, x and y are the lengths of the two diagonals of the kite.



The short diagonal x cuts the long diagonal y into two parts, a and b.

$a + b = y$

The diagonals of a kite intersect at right angles, so the two parts, x and y are the perpendicualr heights of the two triangles.

Area of the pale green triangle $= \frac{1}{2}x \times a$

Area of the dark green triangle $= \frac{1}{2}x \times b$

We can combining the areas of the two triangles to find the area of the kite.

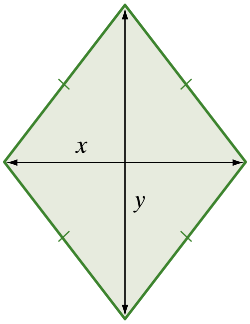
$$\text{Area of kite} = \frac{1}{2}xa + \frac{1}{2}xb$$
$$= \frac{1}{2}x(a + b)$$
$$= \frac{1}{2}xy$$

We have taken out the common factor $\frac{1}{2}x$

Area of a kite is half the product of the lengths of the diagonals.

Area of a rhombus

The rhombus can be thought of as a special case of the kite where all four sides are equal.



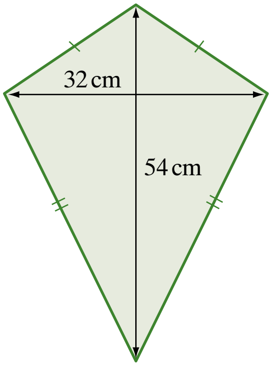
Area of a rhombus is half the product of the lengths of the diagonals.

$$A = \frac{1}{2}xy$$

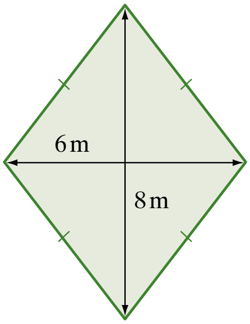
Example 19

Calculate the area of these shapes.

a Kite



b Rhombus



Working

a $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 32 \times 54$

$A = 864$

The area is of the kite is 864 cm^2.

Reasoning

Area of a kite is half the product of the two diagonals.

continued

Example 19 continued

Working

b $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 6 \times 8$

$A = 24$

The area of the rhombus is 24 m^2.

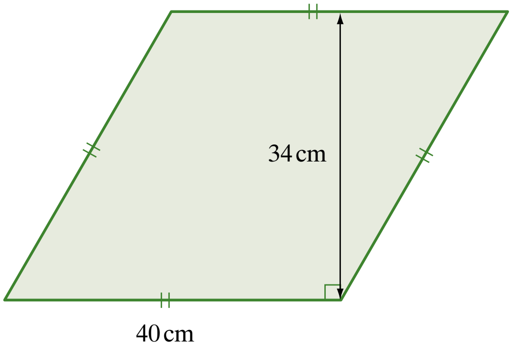
Reasoning

Area of a rhombus is half the product of the two diagonals.

As well as being a special kite, the rhombus is also a special parallelogram. If we know the length of the one side and the perpendicular height of the rhombus then we can use the formula for the area of a parallelogram.

Example 20

Calculate the area of this rhombus.



Working

$A = bh$

$= 40 \times 34$

$= 1360\text{ cm}^2$

The area of the rhombus is 1360 cm^2.

Reasoning

Area of a parallelogram is base times height.

Area of a square

The area of a square can of course be found by squaring the length of a side. However, the square is a special rhombus, where the lengths of the two diagonals are equal, that is $x = y$.

Area of a square $= \frac{1}{2}y^2$, where y is the length of each diagonal.

Example 21

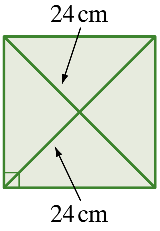
The length of each diagonal of a square is 24 cm. Calculate the area of the square.

Working

$$\begin{aligned} A &= \frac{1}{2}y^2 \\ &= \frac{1}{2} \times 24^2 \\ &= \frac{1}{2} \times 576 \\ A &= 288 \end{aligned}$$

The area is 288 cm^2.

Reasoning



Area of a trapezium

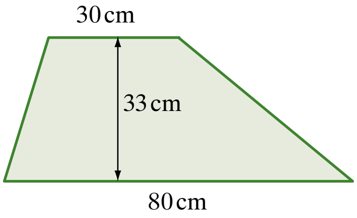
Area of a trapezium

Rather than trying to memorise another formula, it is easier to cut a trapezium into simpler shapes. The following three examples show the calculation of the area of the same trapezium by three different methods.

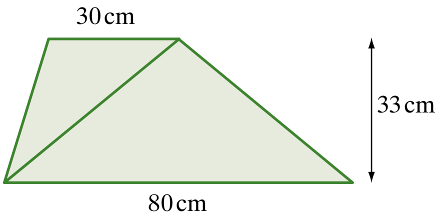
In example 22, a diagonal divides the trapezium into two triangles.

Example 22

Find the area of this trapezium by dividing it into two triangles and calculating the areas of these triangles.



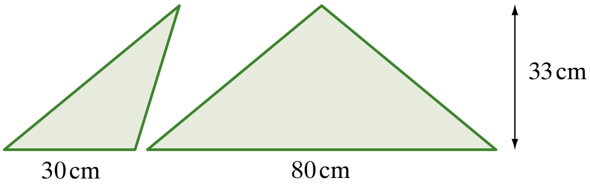
Working



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 80 \times 33 + \frac{1}{2} \times 30 \times 33 \\ &= 1320 + 495 \\ &= 1815\ \text{cm}^2 \end{aligned}$$

The area of the trapezium is 1815 cm^2.

Reasoning

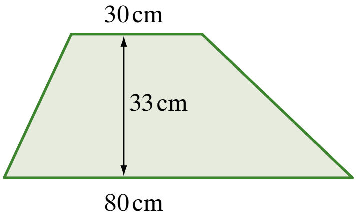


The trapezium can be divided into two triangles with bases 30 cm and 80 cm. Both triangles have the same perpendicular height, 33 cm.
Add the areas of the two triangles.

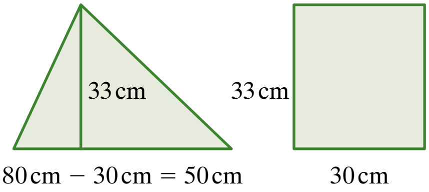
In example 23, the trapezium is divided into two triangles and a rectangle.

Example 23

Find the area of this trapezium by dividing and rearranging it into a rectangle and a triangle.



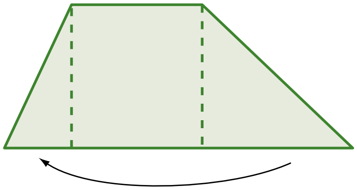
Working



$$\text{Area} = \frac{1}{2} \times 50 \times 33 + 30 \times 33$$
$$= 825 + 990$$
$$= 1815 \text{ cm}^2$$

The area of the trapezium is 1815 cm^2.

Reasoning

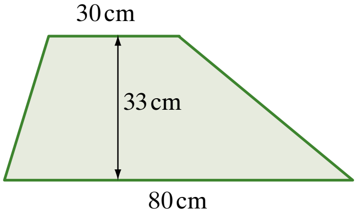


Cut the rectangle from the middle and slide the two triangles together.
The base of this new triangle is
80 cm − 30 cm = 50 cm

Another way of finding the area of a trapezium is to arrange two congruent trapeziums together so that they make a parallelogram. The area of each trapezium is then half the area of the parallelogram.

Example 24

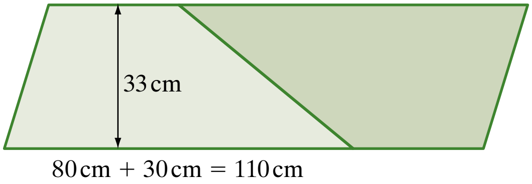
Find the area of this trapezium by arranging another congruent trapezium to form a parallelogram and calculating the area of this parallelogram.



continued

Example 24 continued

Working



Area of parallelogram

= bh

= 110×33

= 3630 cm^2

Area of trapezium = $3630 \div 2$

= 1815 cm^2

Reasoning

The supplementary angles of the trapezium mean that the two trapeziums fit together exactly to make a parallelogram. The length of the parallel sides is the sum of the lengths of the parallel sides of the trapezium.

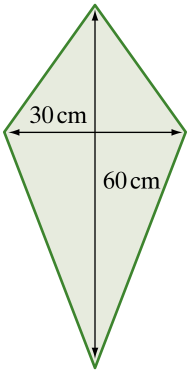
Halve area of parallelogram.

exercise 10.4

LINKS TO Examples 19,20,21

Use the lengths of the diagonals to calculate the area of each kite and rhombus.

a



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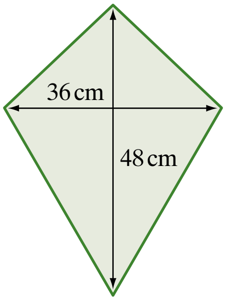

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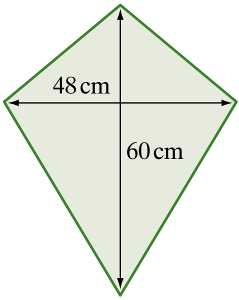

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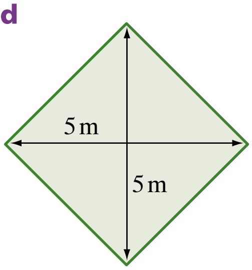

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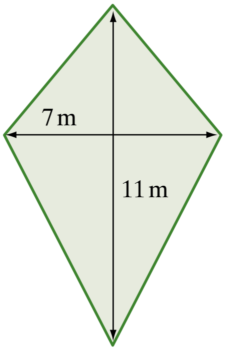

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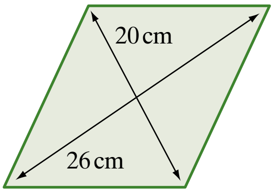

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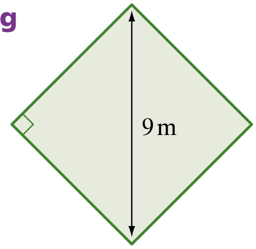

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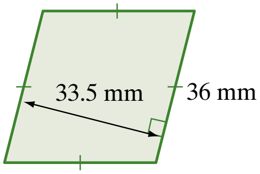
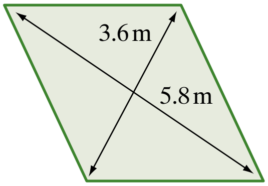


i



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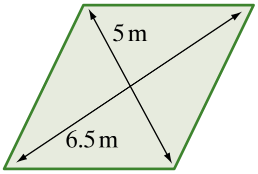

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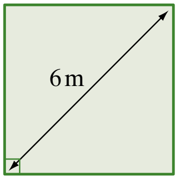

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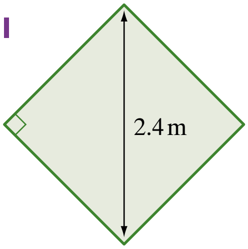
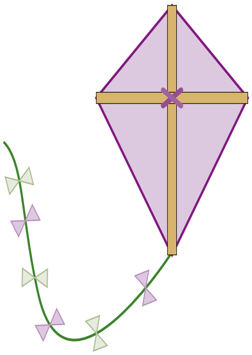


Kenji made the frame of a kite by tying two sticks together as shown to form the diagonals of the kite. If the lengths of the sticks were 60 cm and 90 cm, what area of material would be needed for the kite?



The diagonals of these rhombus floor tiles are 52 cm and 30 cm.

a Calculate the area of each tile.

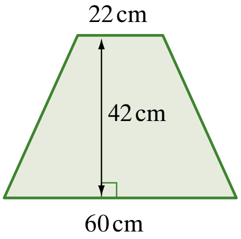
b Approximately how many tiles would be needed to cover an area of 60 m^2?

www.AlibertiArtTile.com

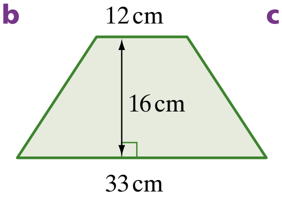
LINKS TO Example 22

Find the area of each of these trapeziums by dividing each trapezium into two triangles.

a



b



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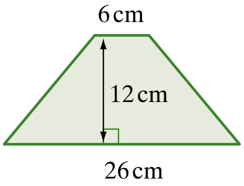
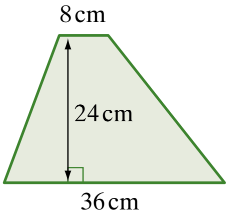


d

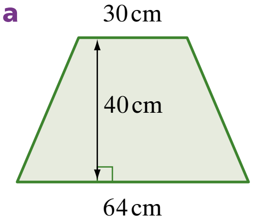


LINKS TO **Example 23**

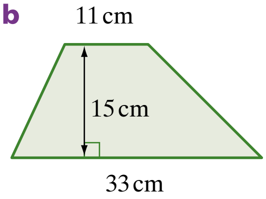
For each of these trapeziums,

- **i** Make a diagram to show each trapezium divided and rearranged into a rectangle and a triangle.
- **ii** Calculate the area by adding the areas of the rectangle and triangle.

a



b



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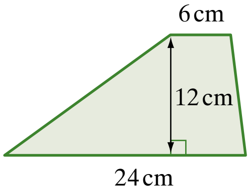
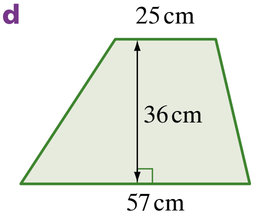


d



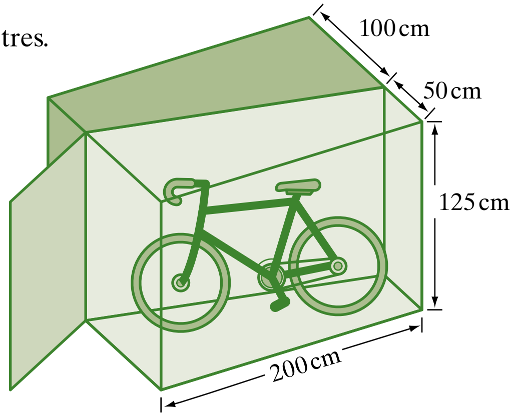
LINKS TO **Example 24**

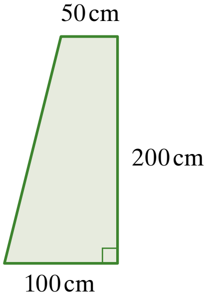
Calculate the area of each of the trapeziums in questions 4 and 5 using the method in example 24.

This rhombus-shaped mirror is 90 cm high and 70 cm wide. Calculate its area.

The bike lockers shown here have a trapezium-shaped floor. Note how two of the lockers fit together to make a rectangular floor. Find the area of the concrete bottom of each bike locker

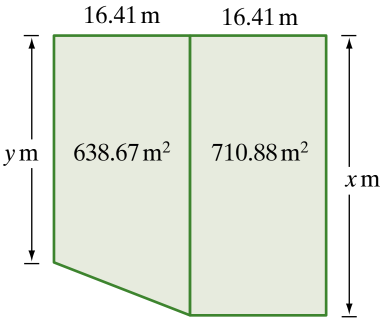
- **a** in square centimetres.
- **b** in square metres.





exercise 10.4 challenge

Two blocks of land were advertised for sale. One block was a rectangle and the other was a trapezium. The widths and areas of the blocks were given in the advertisement, but the dimensions x metres and y metres were not shown. Find the values of x and y to two decimal places.

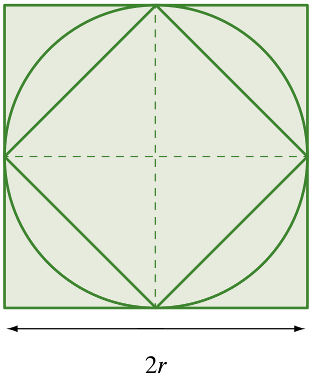


10.5 Area: circles

Estimating circle area

In the diagram below, squares have been drawn outside and inside a circle. If the radius of the circle is r, the side length of the outside square is $2r$.

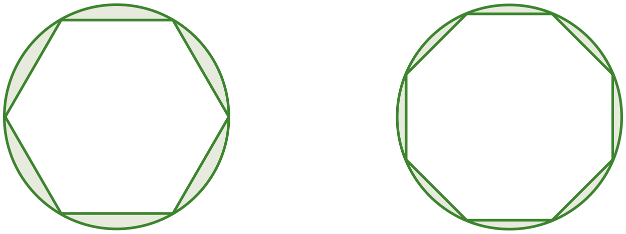
The area of the outside square is $(2r)^2$, that is, $4r^2$.



We can see from the diagram that the area of the inside square is half the area of the outside square, so the inside square must have area $2r^2$. (Each side of the inside square cuts one of the small squares in half).

This means that the area of the circle is somewhere between $4r^2$ and $2r^2$.

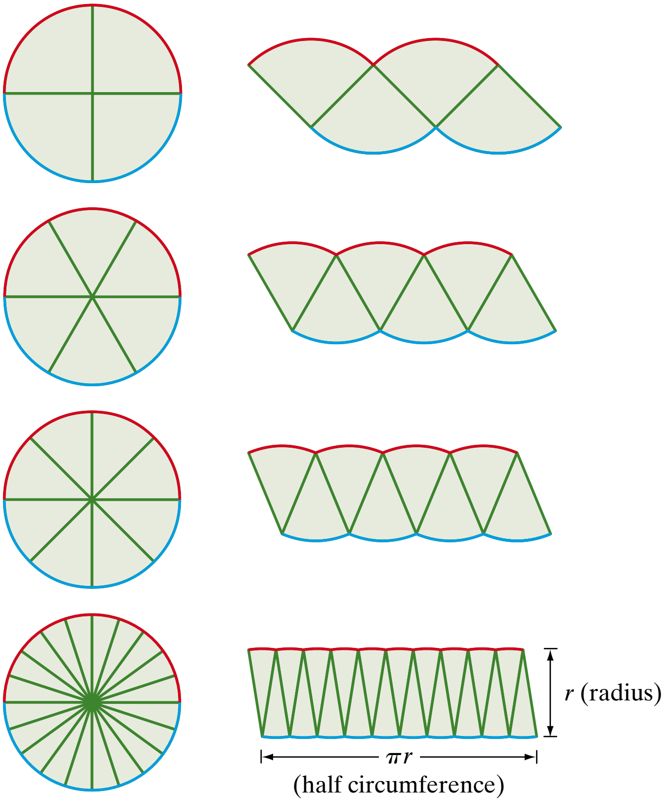
Archimedes estimated the area of a circle by drawing polygons inside the circle. By calculating the areas of polygons with more and more sides, he was able to obtain a very close approximation of the area of the circle.



Another method for considering the area of a circle is to divide it into a number of equal sectors. In the following diagrams, circles have been divided into sectors and arranged as shown. As the number of sectors increases, each sector becomes thinner. As the sectors become thinner, the shape looks more and more like a parallelogram. We can imagine that if we divided the circle into more and more sectors, we could eventually regard the shape as a parallelogram.

Area of a circle

The PowerPoint file **Area of a circle** in the student ebook is an animated diagram showing the sectors arranged to form a parallelogram.



The base of the 'parallelogram' is equal to half the circumference of the circle, that is, πr. The perpendicular height of this 'parallelogram' shape is equal to the radius of the circle, r.

$$\begin{aligned}\text{Area of this parallelogram shape} &\approx \text{base} \times \text{perpendicular height}\\ &\approx \pi r \times r\\ &\approx \pi r^2\end{aligned}$$

It can be shown that eventually, as the sectors become infinitely thin, the area is equal to πr^2.

But the area of this parallelogram is the same as the area of the circle because it is simply a different arrangement of the same set of sectors.

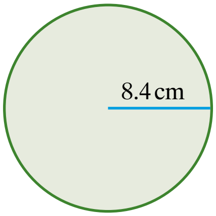
Area of a circle

Area of circle $= \pi r^2$

where r is the radius of the circle.

Example 25

Find the area of this circle. Give your answer correct to one decimal place.



Working	Reasoning
$A = \pi r^2$ $= \pi \times 8.4^2$ $= 221.67\ldots$ The area of the circle is 221.7 cm^2 correct to one decimal place.	Calculate the answer to one more decimal place than required.

If the diameter is given, this must first be halved to find the radius.

Example 26

Find the area of a circle with diameter 2.75 m. Give your answer correct to two decimal places.

Working	Reasoning
Radius $= 2.75 \div 2$ $= 1.375$ m	The radius of a circle is half of the diameter.
$A = \pi r^2$ $= \pi \times 1.375^2$ $= 5.939\ldots$	Calculate the answer to one more decimal place than required.
The area of the circle is 5.94 m^2 correct to two decimal places.	Answer the question.

Finding an unknown radius and diameter of a circle involves working backwards using the circle area formula.

Example 27

a Find the radius of a circle in centimetres, correct to one decimal place, if the circle has an area of 10 cm^2.

b What is the diameter of the circle correct to one decimal place?

continued

Example 27 continued

Working	Reasoning
a Area of circle $= \pi r^2$	
$10 = \pi \times r^2$	Write the area formula.
$r^2 = 10 \div \pi$	Substitute $r = 10$.
$r = \sqrt{10 \div \pi}$	Divide both sides by π.
≈ 1.78	If we know the value of r^2, we must take the square root to find the value of r. Calculate answer to one more decimal place than required.
Radius is 1.8 cm to one decimal place.	Round the value of the radius to the required number of decimal places.
b $d = 2r$	The diameter is twice the radius.
$= 2 \times \sqrt{10 \div \pi}$	Use the value for the radius obtained using your calculator. Round after multiplying by 2.
$d \approx 3.568$	
The diameter is 3.6 cm correct to one decimal place.	

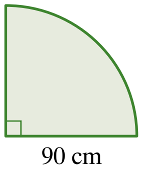
Area of parts of circles

Some shapes are parts of a circle, for example a half circle (semicircle) or quarter circle (quadrant). For a semicircle, we calculate the area of the whole circle then divide it by 2. For a quarter circle we divide the area of the whole circle by 4.

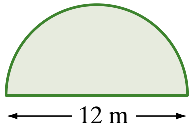
Example 28

Find the area of these shapes correct to the nearest whole number.

a



b



Working	Reasoning
a $A = \dfrac{\pi r^2}{4}$	The shape is one quarter of a circle. The radius is 90 cm. Divide the area of a whole circle by 4.
$= \dfrac{\pi \times 90^2}{4}$	
≈ 6361.7	
The area is 6362 cm^2.	Round to the nearest whole number.

continued

Example 28 continued

Working

b $A = \dfrac{\pi r^2}{2}$

$= \dfrac{\pi \times 6^2}{2}$

≈ 56.5

The area is 57 m^2.

Reasoning

The shape is half a circle.

Divide the area of a whole circle by 2.

$r = d \div 2$

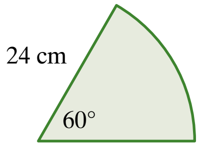
$= 12 \div 2$

$r = 6$

To find the area of a sector, we need to know the radius of the sector and the sector angle. We calculate the area as a fraction of the whole circle. For example, if the sector angle is 60°, the sector is $\dfrac{60°}{360°}$, that is $\dfrac{1}{6}$, of the whole circle. A quadrant of a circle is, of course, $\dfrac{90°}{360°}$, that is $\dfrac{1}{4}$, of the whole circle.

Example 29

Find the area of this sector correct to the nearest square centimetre.



Working

$\text{Area} = \pi \times 24^2 \times \dfrac{60}{360}$

≈ 301.6

The area of the sector is 302 cm^2 correct to the nearest square centimetre.

Reasoning

The sector is $\dfrac{60°}{360°}$ of the whole circle with radius 24 cm.

Tech tip

The TI-30XB MultiView calculator can be used to calculate the area of a circle.

If the radius is a decimal number, for example, 8.4 cm, press the following keys, $\boxed{\pi}\boxed{\times}\boxed{8}\boxed{\cdot}\boxed{4}\boxed{x^2}$ **enter** (example 25).

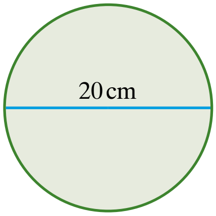
If the radius is a whole number or fraction, for example, 12 cm, press the following keys, $\boxed{\pi}\boxed{\times}\boxed{1}\boxed{2}\boxed{x^2}$ **enter** then ◀▶ to give a rational approximation of the area.

exercise 10.5

LINKS TO Examples 25, 26

Find the area of each of these circles. Use the π button on your calculator. Give your answers correct to the nearest whole number.

a



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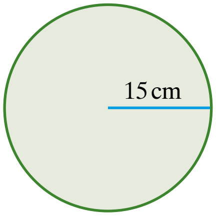
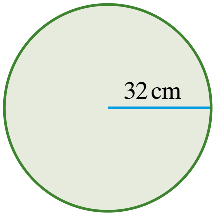
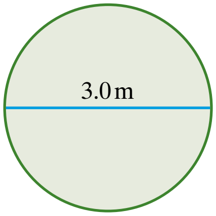


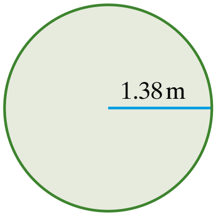
c



d



e



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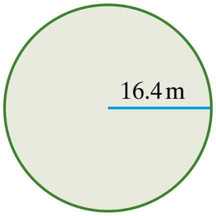

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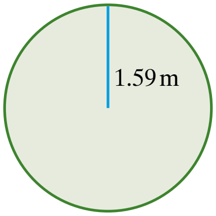

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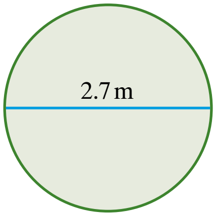
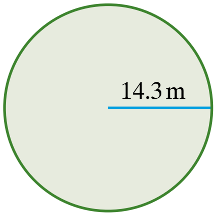


i



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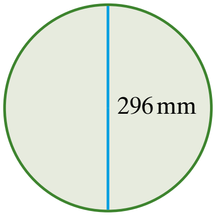

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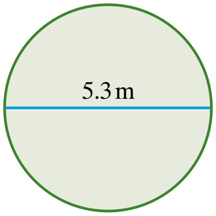

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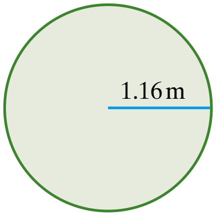


The diameter of a circle is 12 cm. The area of the circle is approximately

A 38 cm^2 **B** 113 cm^2 **C** 355 cm^2 **D** 452 cm^2 **E** 1421 cm^2

Find the area of the following circles. Give your answers correct to one decimal place. Calculate or use 3.142. Round your answers to one decimal place.

a radius 5.3 cm **b** diameter 12 mm **c** diameter 18.5 cm **d** diameter 2.5 m
e radius 78 mm **f** radius 1.75 m **g** diameter 4.5 m **h** diameter 15 cm
i radius 22.5 mm **j** radius 27 cm **k** radius 8.6 m **l** diameter 1.4 m

A circular swimming pool has diameter 11.2 m. What is the area of the pool? Give your answer correct to one decimal place.

LINKS TO Example 27

For each of these circles find

i the radius.
ii the diameter.

Give your answers correct to one decimal place.

a Area = 450 cm^2 **b** Area = 240 cm^2 **c** Area = 75 cm^2 **d** Area = 8.5 cm^2
e Area = 140 cm^2 **f** Area = 6530 cm^2 **g** Area = 18.4 cm^2 **h** Area = 90 cm^2

The area of a circle is 500 mm^2. The radius of the circle is closest to

A 160 mm **B** 80 mm **C** 13 mm **d** 26 mm **E** 7 mm

The area of the water exposed to sunlight in these water treatment ponds is important because the sunlight plays a part in purifying the water. The diameter of each pond is 24 m. Calculate the area of water exposed to sunlight in each pond in square metres, correct to one decimal place.

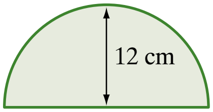
This giant dish of paella was in Madrid, Spain. Paella is a traditional Spanish food that contains rice, meat and vegetables. There was enough paella to feed 100 000 people. The diameter of the paella pan was 21 m.

a Calculate the circumference of the paella pan, correct to one decimal place.
b What is the area of the top of the pan, correct to one decimal place?

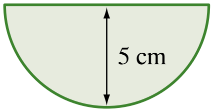
LINKS TO Example 28

Calculate the area of each of the following correct to one decimal place.

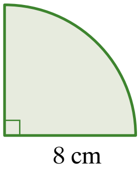
a



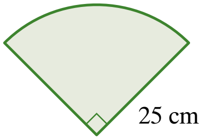
b



c

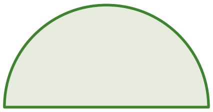


d

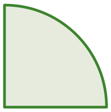


Each of the following shapes is part of a circle with radius 10 cm. Calculate each area correct to one decimal place.

a



b



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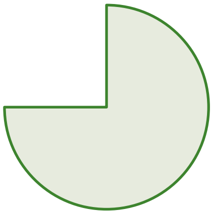
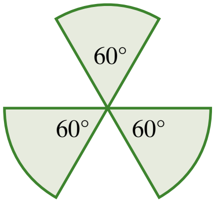


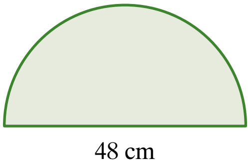
d



The area of this semicircle is closest to

A 452 cm^2 **B** 905 cm^2 **C** 1810 cm^2

D 3619 cm^2 **E** 7238 cm^2

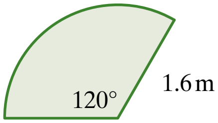


A garden is in the shape of a semicircle with radius 7 m. What is the area of the garden? Give your answer correct to one decimal place.

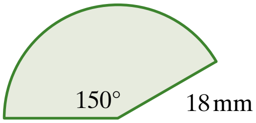
LINKS TO Example 29

Calculate the area of each of these sectors correct to one decimal place.

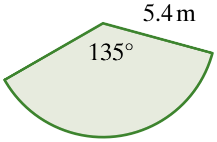
a



b



c



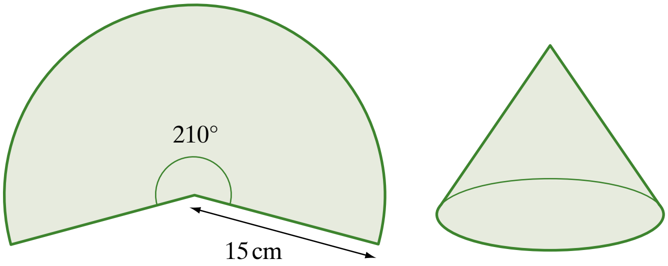
exercise 10.5 challenge

Matt made a cake in a round tin that had a diameter of 18 cm.

a Calculate the area of the top of the cake to the nearest square cm.

b Matt wants to make a square cake with the same amount of mixture. What length of sides of a square tin would have approximately the same area as Matt's round tin?

Josie was making party hats for her little sister's birthday party. She cut sectors from circles of coloured card then glued them into cone shapes as shown. She was then going to sprinkle glitter glue on the outside of each hat. To find out how much glitter glue to buy, Josie needed to know the area of each sector.



a Work out a method for calculating the area of each hat to be covered with glitter glue.

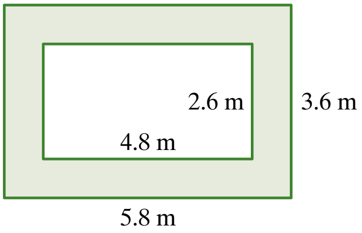
b Use your method to calculate the area to the nearest square centimetre.

10.6 Finding a region: subtracting areas

Sometimes the required area is an area surrounding other areas that are not required—one or more regions appear as 'holes' within an outer region. In these cases we subtract the 'holes' from the outer region.

Example 30

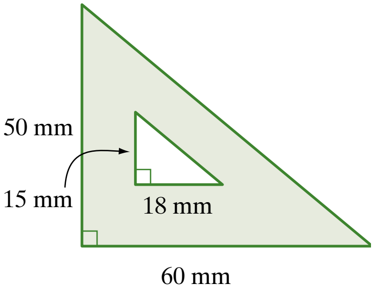
Find the area of the shaded region.



Working	**Reasoning**
Area of outer rectangle $= 5.8 \times 3.6$ $= 20.88\text{ m}^2$	Area of rectangle is found by multiplying length by width.
Area of inner rectangle $= 4.8 \times 2.6$ $= 12.48\text{ m}^2$	
Shaded region $= 20.88 - 12.48$ $= 8.40\text{ m}^2$	Shaded region is found by subtracting area of inner rectangle from area of outer rectangle.

Example 31

Find the shaded area.



continued

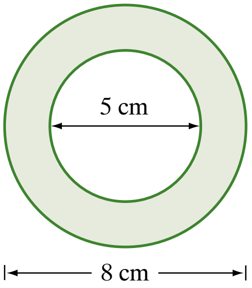
Example 31 continued

Working	**Reasoning**
Outside triangle $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 60 \times 50$ $= 1500 \text{ mm}^2$	Calculate the area of the outside triangle. Area is half base times height.
Inside triangle $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 18 \times 15$ $= 135 \text{ mm}^2$	Calculate the area of the inside triangle. Area is half base times height.
Shaded area $= 1500 - 135$ $= 1365 \text{ mm}^2$	Subtract the area of the inside triangle from the area of the outside triangle.

When a circle is cut from another circle, the area of the remaining ring is calculated by subtracting the area of the smaller circle from the area of the larger circle.

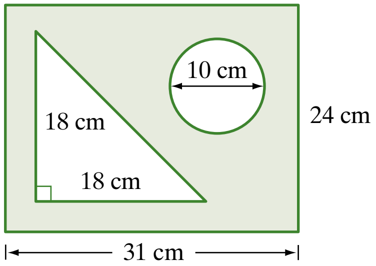
Example 32

Find the area of a ring that has an outside diameter of 8 cm and an inside diameter of 5 cm. Give your answer correct to one decimal place.

Working	**Reasoning**
	The radius of a circle is half of the diameter. Shaded area = outside area − inside area
Radius of outer circle = 4 cm Radius of inner circle = 2.5 cm Area $= \pi \times 4^2 - \pi \times 2.5^2$ $\approx 30.63 \text{ cm}^2$ Area of ring is 30.6 cm^2 correct to one decimal place	Calculate answer to one more decimal place than required. Round to required number of decimal places.

Example 33

Find the area of the shaded region in the figure below. Give your answer correct to one decimal place.

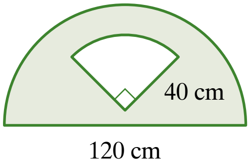


Working	Reasoning
Shaded area = Area of rectangle − area of circle − area of triangle $= lw - \pi r^2 - \frac{1}{2}bh$ $= 31 \times 24 - \pi \times 5^2 - \frac{1}{2} \times 18 \times 18$	Required region does not include the circle and triangle, so subtract the areas of these from the area of the rectangle.
$\approx 744 - 78.54 - 162$ $= 503.46$ $= 503.5$ cm^2 (to one decimal place)	Radius of circle = 5 cm Calculate answer to one more decimal place than needed.

Regardless of the shape of the cut-outs, the remaining area is always calculated by subtracting the area of the inner shapes from the entire area.

Example 34

Find the shaded area correct to the nearest whole number.



Working	Reasoning
Semicircle $A = \frac{1}{2}\pi r^2$ $= \frac{1}{2} \times \pi \times 60^2$ $A = 5654.9$ cm^2	The shape is half a circle. Divide the area of a whole circle by 2. $r = d \div 2$ $= 60 \div 2$ $r = 30$

continued

Example 34 continued

Working

Quarter circle

$$A = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \pi \times 40^2$$

$$A = 1256.6 \text{ cm}^2$$

$$\text{Shaded area} = 5654.9 - 1256.6$$
$$= 4398.3 \text{ cm}^2$$

The shaded area correct to the nearest whole number is 4398 cm^2.

Reasoning

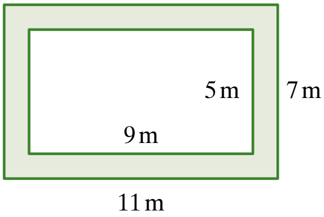
The shape is one quarter of a circle.
The radius is 40 cm.
Divide the area of a whole circle by 4.

exercise 10.6

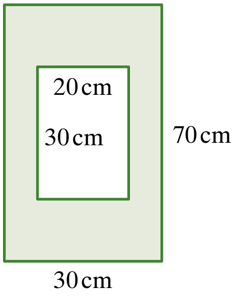
LINKS TO Example 30

Find the area of each of the shaded regions.

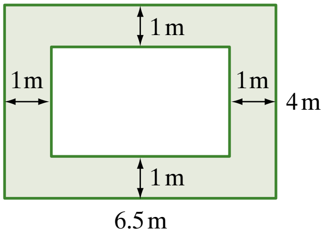
a



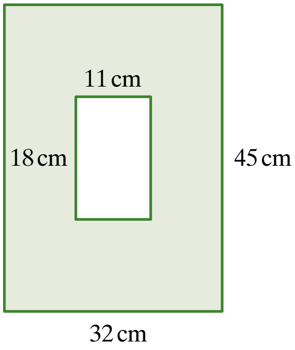
b



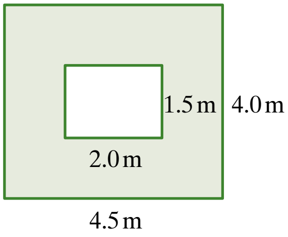
c



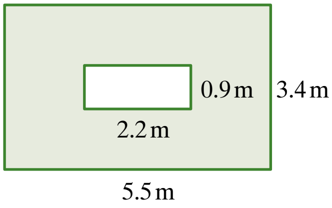
d



e

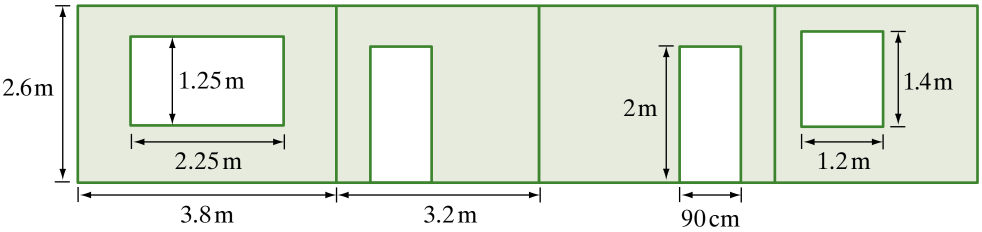


f



The four walls of a room have the dimensions shown on the diagram below. The two doors are the same size.

a Calculate the area of the walls (shaded area).



b If the walls are to be painted with two coats of paint, and 1 litre of paint covers 14 m^2, is a 5 L tin of paint enough to paint the walls? Explain.

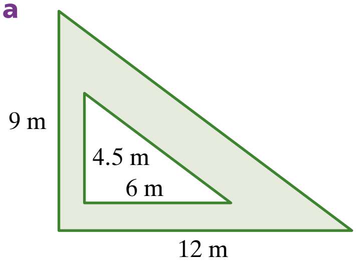
c Calculate the area of the ceiling of the room.

d If the ceiling is to have one coat of paint, will a 1 L tin be sufficient? Explain.

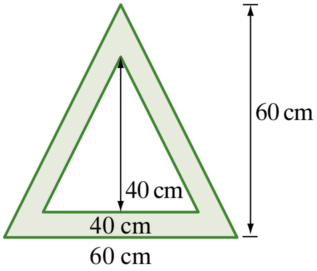
LINKS TO Example 31

Find the area of each of the shaded regions.

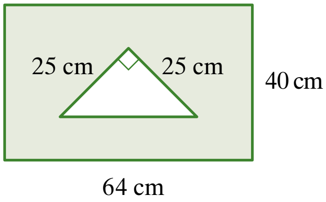
a



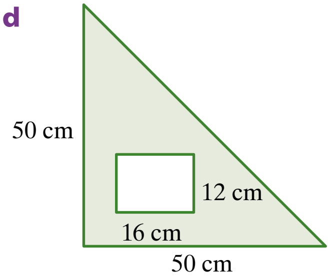
b



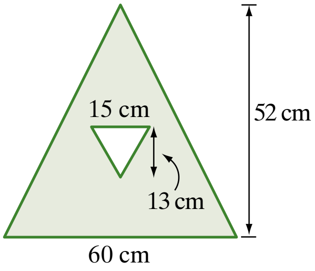
c



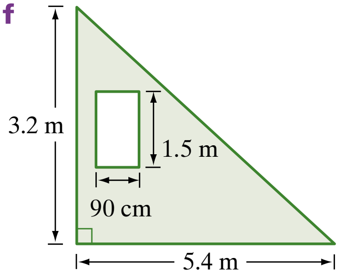
d



e

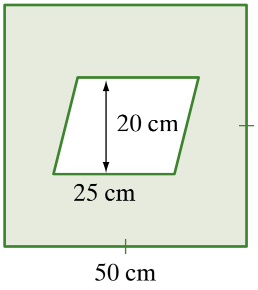


f

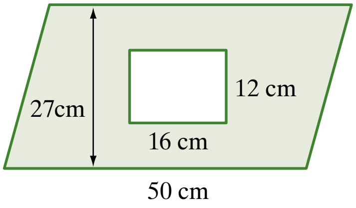


Find the area of each of the shaded regions based on rectangles and parallelograms.

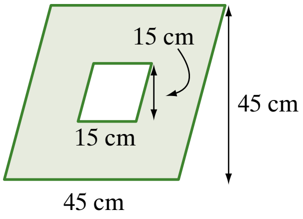
a



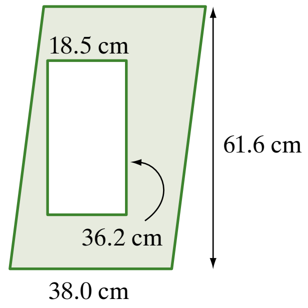
b



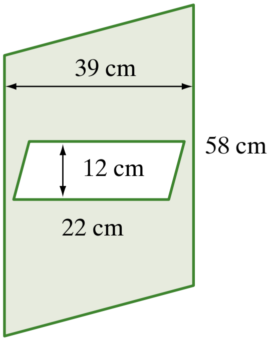
c



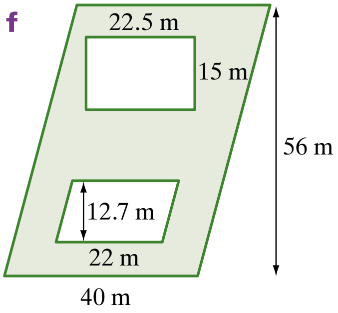
d



e

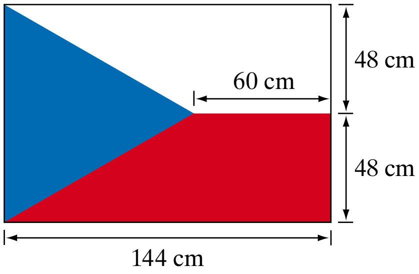


f

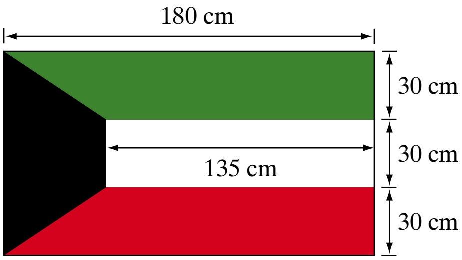


Calculate the area of each coloured section in these flags.

a Czech Republic (in Europe)

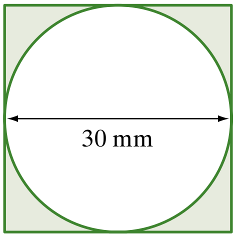


b Kuwait (in the Middle East)

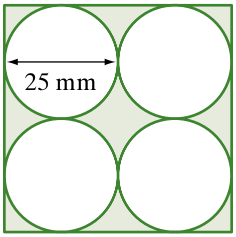


Calculate the shaded area in each of the following figures. Give your answers correct to two decimal places.

a



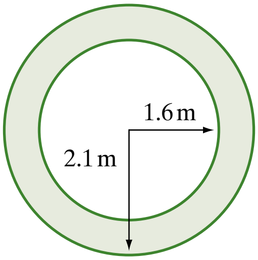
b



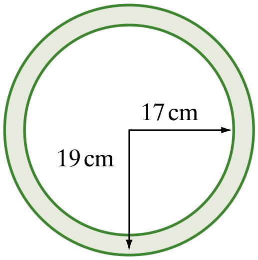
LINKS TO Example 32

Find the area of the shaded region in each of the following. Give your answers correct to the nearest whole number.

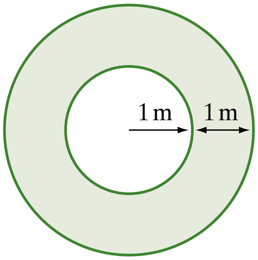
a



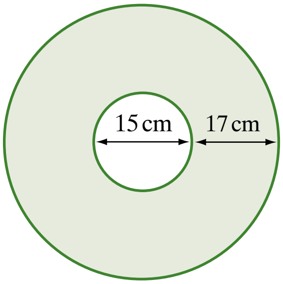
b



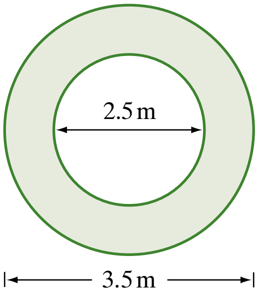
c



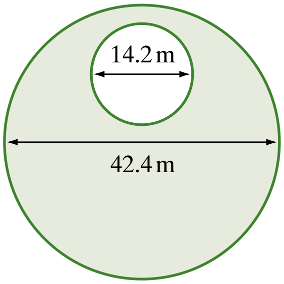
d



e



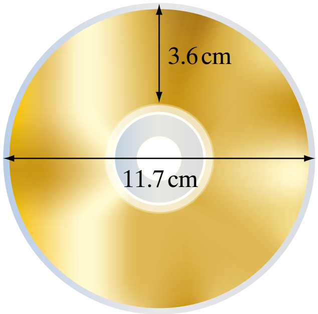
f



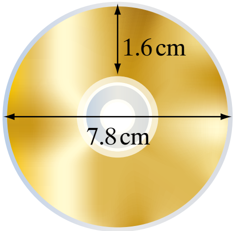
Two CDs are shown below.

a Calculate the recording areas for each CD, correct to one decimal place.

i



ii

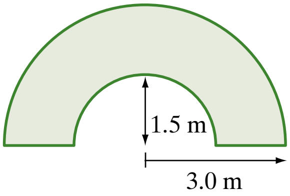


b Approximately how many times larger is the recording area of the large CD compared with the small CD?

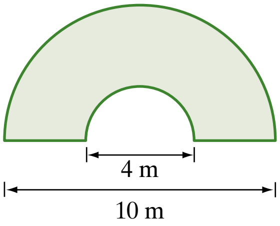
exercise 10.6 challenge

Find the area of each of the shaded regions. Give your answers correct to one decimal place.

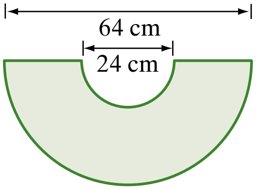
a



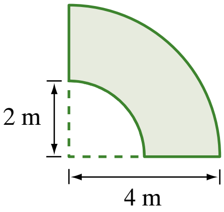
b



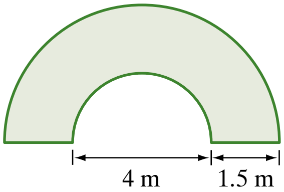
c



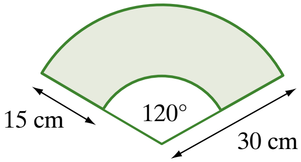
d



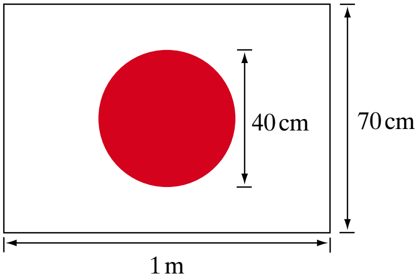
e



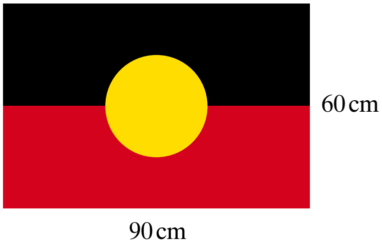
f



Calculate the area of red and the area of white in this Japanese flag to the nearest square centimetre.



The official Aboriginal flag of Australia is divided across the centre into red and black regions. The gold circle on the flag below has a diameter of 30 cm. Calculate the areas of red, black and gold to the nearest square centimetre.



A circular pool has a diameter of 5.6 m. A path 1.2 m wide surrounds the pool.

a Draw a clear labelled diagram to show the diameter of the pool and the width of the path.

b What is the area of the pool to one decimal place?

c What is the radius of the outer edge of the path?

d What is the area of the path to one decimal place?

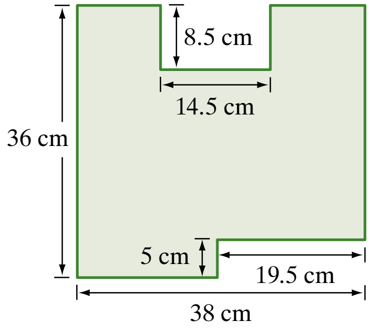
10.7 Composite shapes: adding areas

We often need to find areas of shapes that are made up of more than one simple shape. If the required area is based on rectangles, it can be divided into two or more rectangles. Example 33 shows the calculation of the area of a composite shape that can be divided into several rectangles. In this example, we could also calculate the area by calculating the area of the rectangle that would enclose the shape, then subtract the 'missing' rectangles. Both methods of calculating the area are shown.

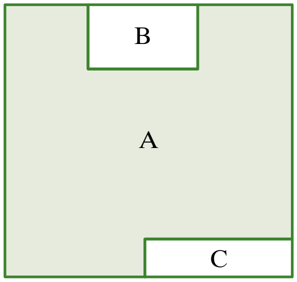
Example 35

Working

Method 1: Subtracting areas



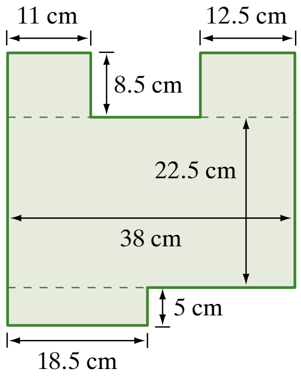
Reasoning

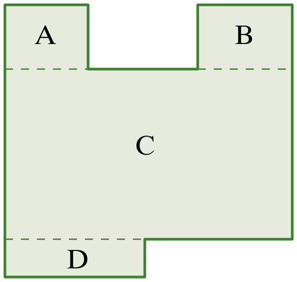


Area
$= 38 \times 36 - 5 \times 19.5 - 8.5 \times 14.5 \text{ cm}^2$
$= 1368 - 97.5 - 123.25 \text{ cm}^2$
$= 1147.25 \text{ cm}^2$

Shaded area
= Area A − Area B − Area C

Method 2: Adding areas





continued

Example 35 continued

Working

Area

$= 11 \times 8.5 + 12.5 \times 8.5 + 38 \times 22.5 + 18.5 \times 5$

$= 93.5 + 106.25 + 855 + 92.5$

$= 1147.25\text{ cm}^2$

Reasoning

Shaded area

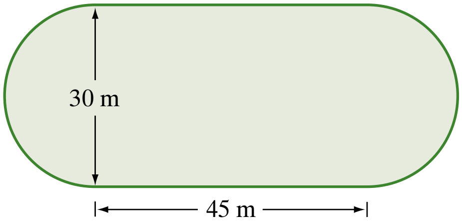
= Area A + Area B + Area C + Area D

Which method do you prefer?

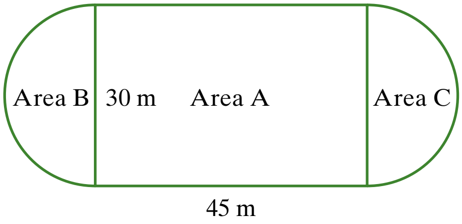
Compound shapes may sometimes include circles or parts of circles.

Example 36

The sportsground on the right has semicircular ends. Find the area to the nearest square metre.



Working



Area $= \text{Area A} + \text{Area B} + \text{Area C}$

$= l \times w + \pi r^2$

$= 30 \times 45 + \pi \times 15^2$

$\approx 1350 + 706.8$

$\approx 2056.8\text{ m}^2$

Area is 2057 m^2 to the nearest m^2.

Reasoning

The area is made up of a rectangle and two half circles.
Area B + Area C make up a whole circle with radius 15 m.
The radius of the half circles is 15 m.

The separate areas are added.

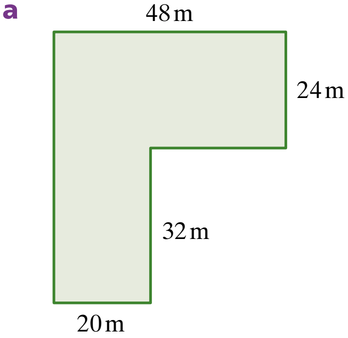
Calculate answer to one more decimal place than needed.

exercise 10.7

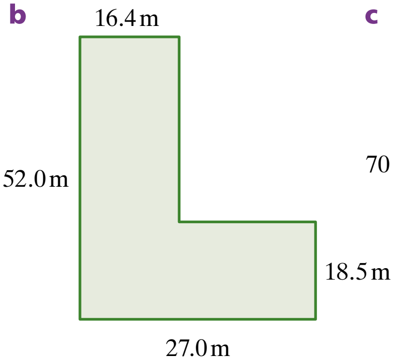
LINKS TO Example 35

Calculate the area of each of the following floor plans of houses.

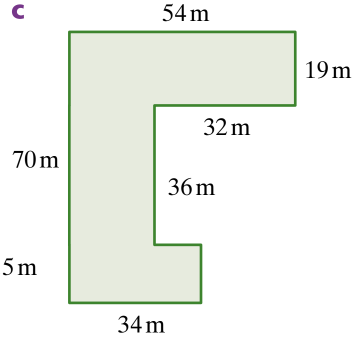
a



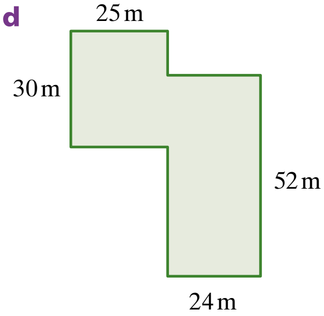
b



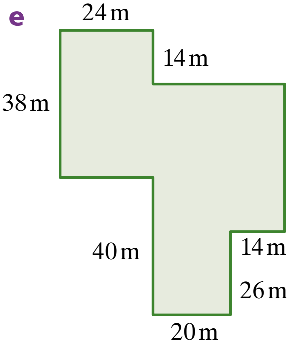
c



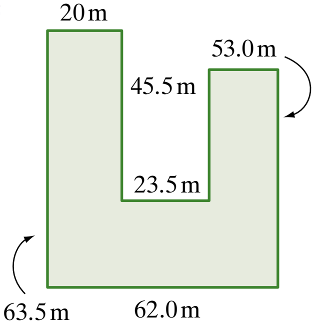
d



e

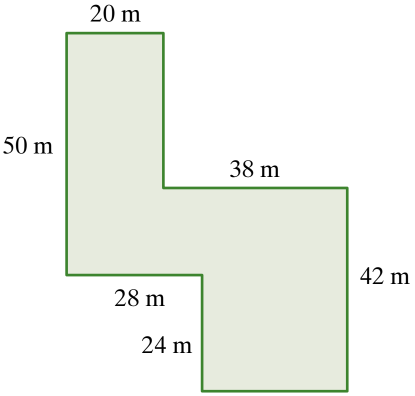


f



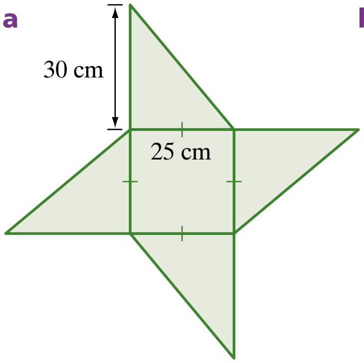
The area of this shape in square metres is

A 2660
B 2260
C 1568
D 2596
E 2404

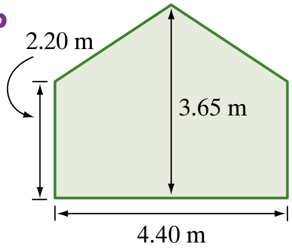


Calculate the area of each of the following shapes.

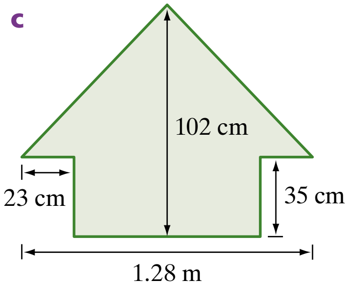
a



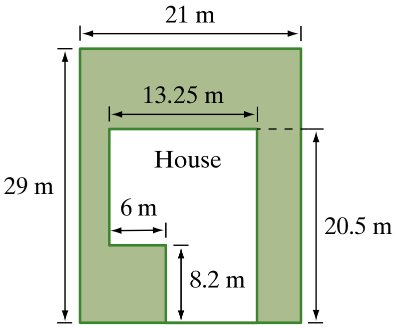
b



c



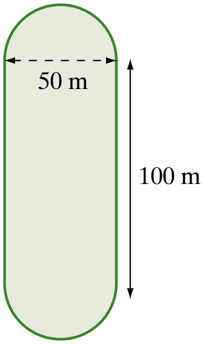
How many square metres of instant lawn would be needed for the following garden? Give your answer to the nearest square metre.



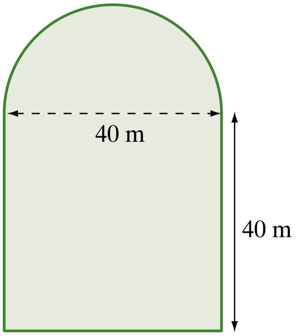
LINKS TO Example 36

Calculate the area of each of the following shapes based on rectangles and semicircles correct to the nearest whole number.

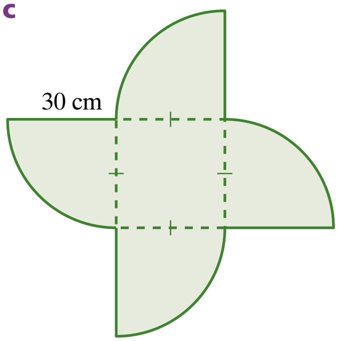
a



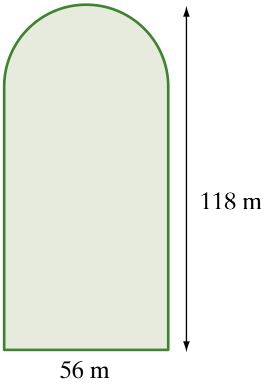
b



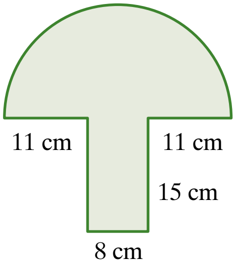
c



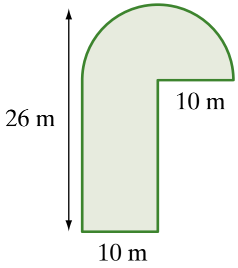
d



e



f

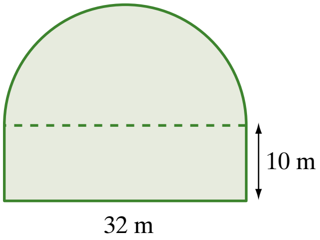


The area of this shape in square metres is closest to

A 420 **B** 562

C 722 **D** 1124

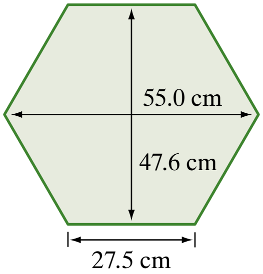
E 1928



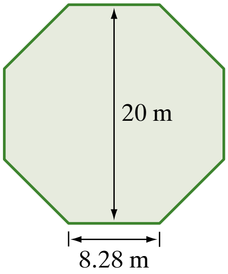
exercise 10.7 challenge

Find the area of these regular polygons by dividing each polygon into congruent isosceles triangles. Draw dotted lines to show an isosceles triangle on each polygon.

a Find the area of this regular hexagon.

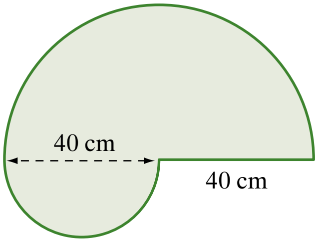


b Find the area of this regular octagon.

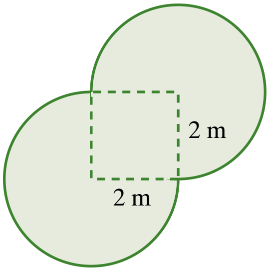


Calculate the area of these shapes. Give your answers correct to one decimal place.

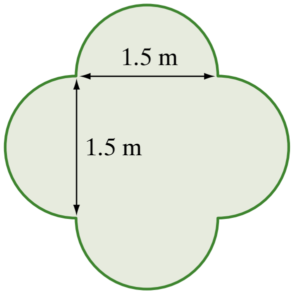
a



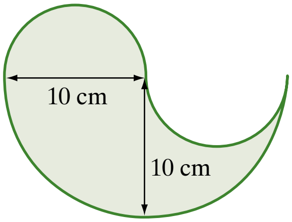
b



c



d



10.8 Volume: rectangular and triangular prisms

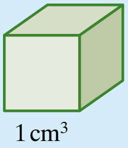
Units of volume and capacity

We say ‘cubic centimetres’, not ‘centimetres cubed’.

The volume of a 3-dimensional object is the amount of space it occupies. Volume is measured in cubic units, for example, cubic centimetres.

$10\,\text{mm} \times 10\,\text{mm} \times 10\,\text{mm} = 1000\,\text{mm}^3 = 1\,\text{cm}^3$

$100\,\text{cm} \times 100\,\text{cm} \times 100\,\text{cm} = 1\,000\,000\,\text{cm}^3 = 1\,\text{m}^3$

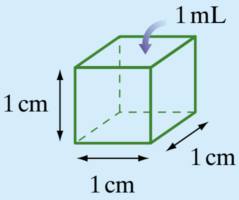


Converting units of volume

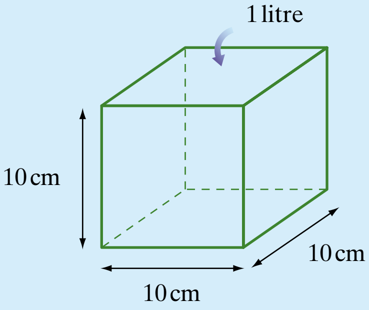
$$\begin{tikzcd}
\text{mm}^3 \arrow[r, bend left, "\div 10^3"] & \text{cm}^3 \arrow[l, bend left, "\times 10^3"] \arrow[r, bend left, "\div 100^3"] & \text{m}^3 \arrow[l, bend left, "\times 100^3"] \arrow[r, bend left, "\div 1000^3"] & \text{km}^3 \arrow[l, bend left, "\times 1000^3"]
\end{tikzcd}$$

The volumes of liquids and gases are often measured in litres (or millilitres, kilolitres, megalitres or gigalitres). These units are usually used when we are talking about the **capacity** of a container, that is, how much liquid or gas the container will hold. Garden supplies such as bags of mulch are also often measured in litres.

A volume of 1 cubic centimetre is equal to a volume of 1 millilitre.



Volume unit	Capacity unit
$1\,\text{cm}^3$	1 millilitre (1 mL)
$1000\,\text{cm}^3$	1 litre (1 L)
$1\,\text{m}^3$	1 kilolitre (1 kL)



$$\begin{tikzcd}
\text{mL} \arrow[r, bend left, "\div 1000"] & \text{L} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \text{kL} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \text{ML} \arrow[l, bend left, "\times 1000"]
\end{tikzcd}$$

1 litre = 1000 mL or $1000\,\text{cm}^3$

continued

When measuring large quantities of water, such as the amount of water used in a city in a day or the amount of water in a reservoir, megalitres or gigalitres are usually used.

1000 kilolitres = 1 megalitre

1000 megalitres = 1 gigalitre

Example 37

Convert each of these volumes into the unit shown in brackets.

a 1 460 000 cm^3 (cubic metres)

b 0.00379 cm^3 (cubic millimetres)

Working	**Reasoning**
a $1\,460\,000\,\text{cm}^3 = 1\,460\,000 \div 100^3$ $= 1.46\,\text{m}^3$	To convert cm^3 to m^3 divide by 100^3.
b $0.00379\,\text{cm}^3 = 0.00379 \times 10^3$ $= 3.79\,\text{mm}^3$	To convert cm^3 to mm^3 multiply by 10^3.

Example 38

Convert these volumes into the units shown in brackets.

a 3750 mL (L)

b 150 kL (L)

Working	**Reasoning**
a 3750 mL = 3750 ÷ 1000 = 3.750 L	1000 mL = 1 L
b 150 kL = 150 × 1000 = 150 000 L	1 kL = 1000 L

Example 39

Express these volumes in the units shown in brackets.

a 3260 cm^3 (L)

b 4.8 mL (mm^3)

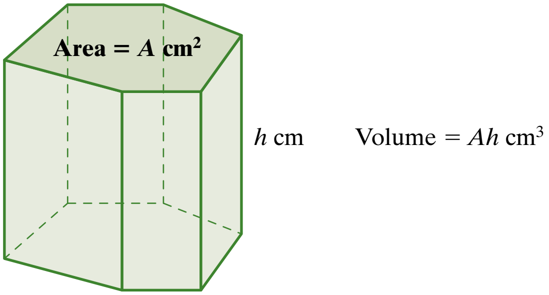
Working	**Reasoning**
a $3260\,\text{cm}^3 = 3260\,\text{mL}$ $= 3.26\,\text{L}$	$1\,\text{cm}^3 = 1\,\text{mL}$ 1000 mL = 1 L
b $4.8\,\text{mL} = 4.8\,\text{cm}^3$ $= 4.8 \times 10^3$ $= 4800\,\text{mm}^3$	$1\,\text{mL} = 1\,\text{cm}^3$ To convert cm^3 to mm^3 multiply by 10^3.

Volume of a prism

Volume of prism = Area of cross-section × Height

$$V = Ah$$

This formula applies to any prism.



Volume of a rectangular prism

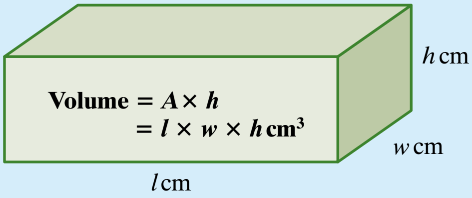
To calculate the volume of a rectangular prism we need to know the length, l, the width, w, and the height, h.

Volume of a rectangular prism

$V = Ah$

$V = lwh$

where l is the length, w is the width and h is the height of the prism.



Example 40

Find the volume of a rectangular prism with length 43 mm, width 19.5 mm and height 11 mm. Give your answer in cubic centimetres correct to one decimal place.

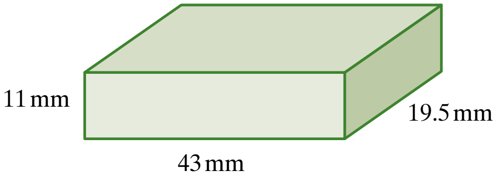
Working

Length = 43 mm = 4.3 cm
Width = 19.5 mm = 1.95 cm
Height = 11 mm = 1.1 cm

$$\begin{aligned} V &= Ah \\ &= lwh \\ &= 4.3 \times 1.95 \times 1.1 \\ &= 9.2235 \\ &= 9.2\text{ cm}^3 \text{ to one decimal place.} \end{aligned}$$

Reasoning

It is easier to convert the dimensions of the prism to centimetres first.
1 cm = 10 mm
To convert millimetres to centimetres, divide by 10.



Volume of a cube

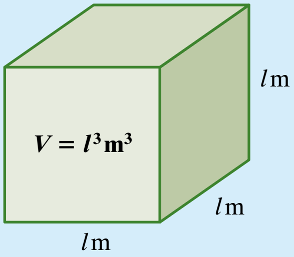
A cube is a special case of a rectangular prism where the length, width and height are all equal.

Volume of a square

$V = Ah$

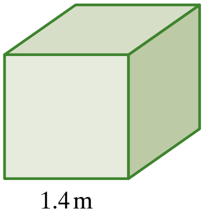
$V = l^3$

where V is the volume and l is the length of each side.



Example 41

Find the volume of this cube.



Working

$V = Ah$

$V = l^3$

$= 1.4^3$

$= 2.744$

The volume is 2.744 m^3.

Reasoning

Volume $= 1.4 \times 1.4 \times 1.4$

$= 1.4^3 \text{ m}^3$

Using the calculator:

1 · 4 ^ 3 enter

If we know the volume of a rectangular prism, we can calculate the height for a given area of the base.

Example 42

A container in the shape of a rectangular prism has a capacity of 1.5 L. The base measures 15 cm by 20 cm. What is the height of the container?

Working

1.5 L = 1500 mL

= 1500 cm^3

Reasoning

Convert 1.5 litres to millilitres then to cubic centimetres.

continued

Example 42 continued

Working

$$\begin{aligned} V &= Ah \\ V &= lwh \\ 1500 &= 15 \times 20 \times h \\ h &= \frac{1500}{15 \times 20} \\ h &= 5 \end{aligned}$$

The height of the container is 5 cm.

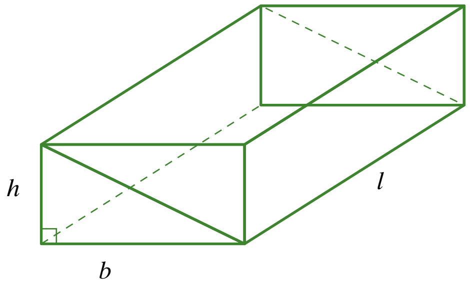
Reasoning

Substitute for the length and width of the base and for the volume to find the height.

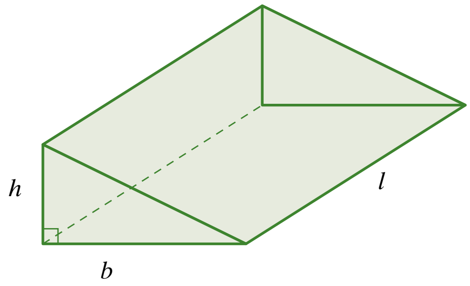
Volume of a triangular prism

A triangular prism has triangular ends joined by rectangular faces. Anywhere along the length of the prism, the triangular cross-section is the same.

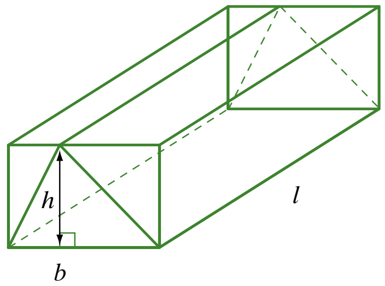
We saw in section 10.3 that the area of a triangle is half the area of a rectangle with the same base and perpendicular height. Using this understanding we can reason that the volume of a triangular prism is half the volume of the corresponding rectangular prism.

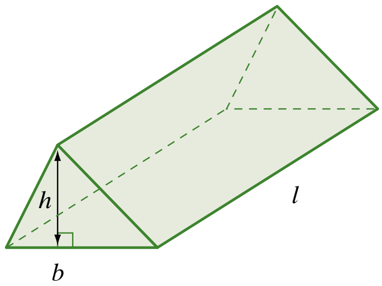


Volume of a rectangular prism $= Al = bhl$

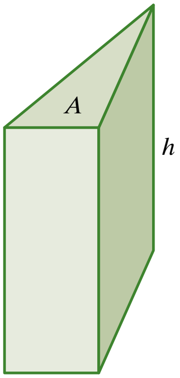


Volume of a triangular prism $= Al = \frac{1}{2}bhl$





To find the volume of any prism we multiply the area of the end, A, by the length of the prism, l. For a triangular prism, the end is a triangle and $A = \frac{1}{2}bh$. So the volume is given by $\frac{1}{2}bhl$.



Volume of a triangular prism

The volume of a triangular prism , V, is given by:

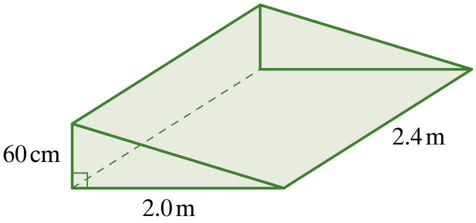
$$V=\frac{1}{2}bhl$$

where b is the length of the base of the triangular end, h is the perpendicular height of the triangular end and l is the length of the prism.

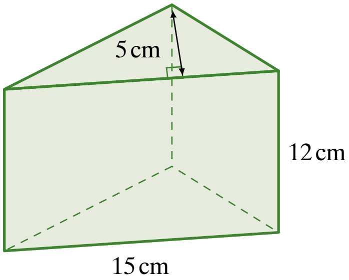
Example 43

Calculate the volume of these triangular prisms.

a



b



Working

a $V = Al$

$= \frac{1}{2}bhl$

$= \frac{1}{2} \times 2.0 \times 0.6 \times 2.4$

$= 1.44$

The volume is $1.44\,\text{m}^3$.

Reasoning

Convert 60 cm to 0.6 m.
The base of the triangular end is 2.0 m and the perpendicular height of the triangular end is 0.6 m.
Area of the triangular end

$= \frac{1}{2} \times 2 \times 0.6$

continued

Example 43 continued

Working

b $V = Al$

$$= \frac{1}{2}bhl$$

$$= \frac{1}{2} \times 15 \times 5 \times 12$$

$$= 450$$

The volume is 450 cm^3.

Reasoning

The base of the triangular end is 15 m and the perpendicular height of the triangular end is 5 cm.

Area of triangular end

$$= \frac{1}{2} \times 15 \times 5$$

exercise 10.8

Convert these volumes into the units shown in brackets.

a 5960 mm^3 (cm^3) **b** 18 cm^3 (mm^3) **c** 560 000 cm^3 (m^3)

d 0.0037 m^3 (cm^3) **e** 2 954 000 000 mm^3 (m^3) **f** 0.00009438 m^3 (cm^3)

g 0.0762 cm^3 (mm^3) **h** 3 780 000 000 cm^3 (m^3) **i** 55 cm^3 (mm^3)

j 1800 cm^3 (m^3) **k** 250 mm^3 (cm^3) **l** 160 000 mm^3 (cm^3)

m 1.6 m^3 (mm^3) **n** 450 cm^3 (mm^3) **o** 1.8 m^3 (cm^3)

75 600 cm^3 is equal to

A 756 m^3 **B** 7.56 m^3 **C** 756 000 mm^3 **D** 0.0756 m^3 **E** 7 560 000 mm^3

LINKS TO Example 38

Convert these volumes into the units shown in brackets.

a 27 000 mL (L) **b** 150 mL (L) **c** 2.5 L (mL)

d 40 kL (L) **e** 48 000 L (kL) **f** 0.003 kL (mL)

LINKS TO Example 39

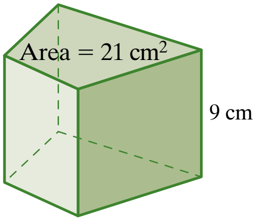
Convert these volumes into the units shown in brackets.

a 270 cm^3 (mL) **b** 3.5 m^3 (kL) **c** 0.15 m^3 (mL)

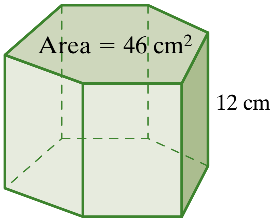
d 0.0036 m^3 (L) **e** 1600 mm^3 (mL) **f** 5 L (cm^3)

Find the volume of these prisms.

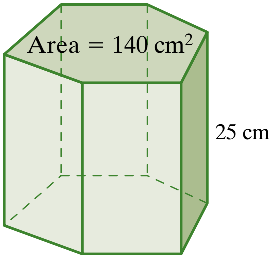
a



b



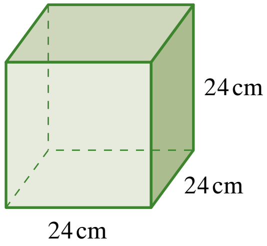
c



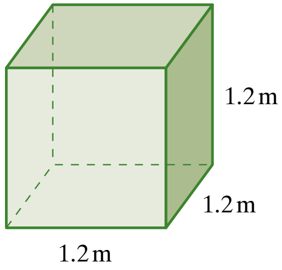
LINKS TO Examples 40, 41

Find the volume of these rectangular prisms (parts **a** and **b** are cubes).

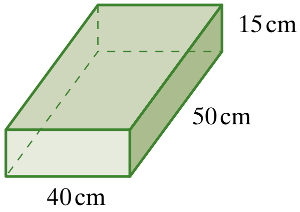
a



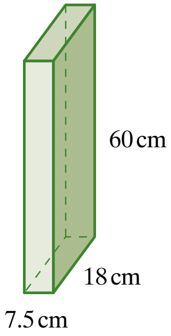
b



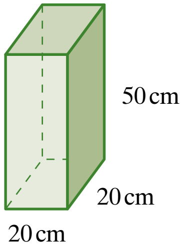
c



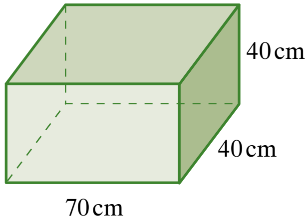
d



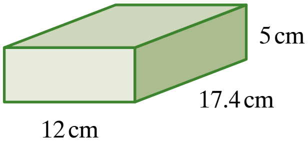
e



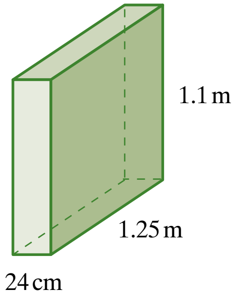
f



g



h



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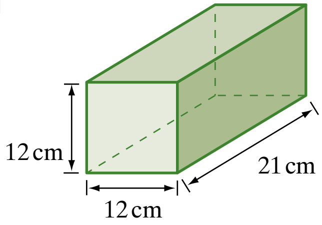

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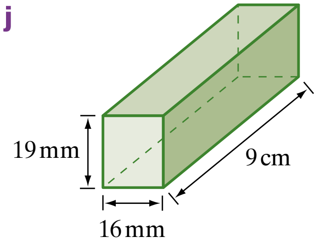

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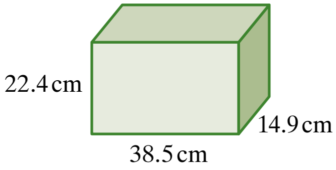

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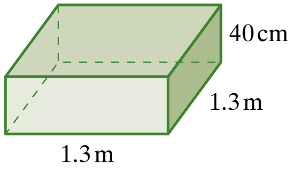


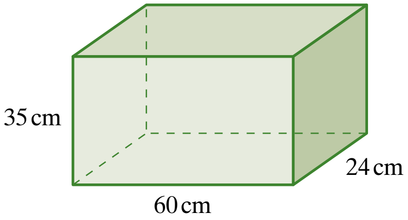
A rectangular prism has a square base with side length 8 cm and a height of 12 cm. The volume of the prism is

A 72 cm^3 **B** 96 cm^3 **C** 192 cm^3 **D** 768 cm^3 **E** 1152 cm^3

Find the volume of this fish tank

a in cubic centimetres.

b in millilitres.

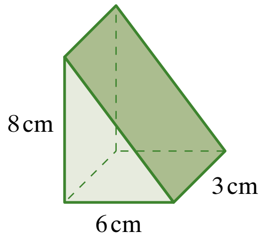


A new breakfast cereal is to be packaged in rectangular prism packets having a volume of 6000 cm^3. Suggest four possible sets of dimensions (length, width and height) for the packet. Which of your four sets of dimensions do you think is the most convenient shape for the packet? Explain.

LINKS TO Example 43

Find the volume of these prisms.

a



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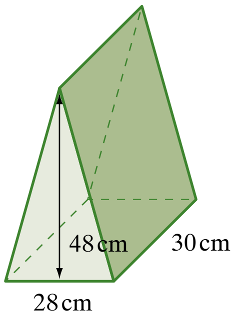

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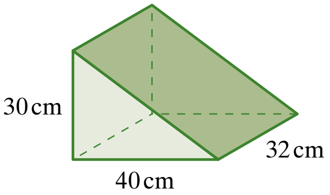

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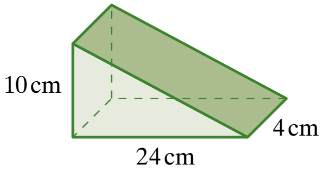

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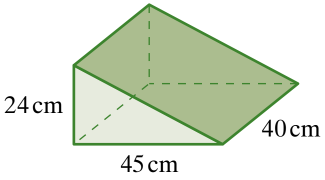

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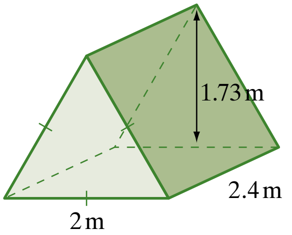

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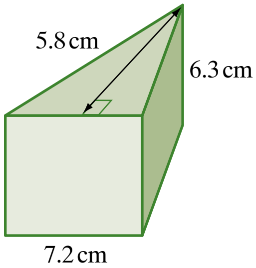

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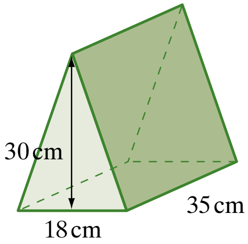

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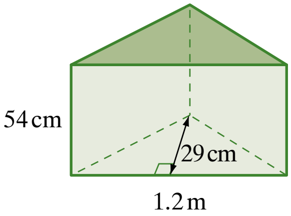

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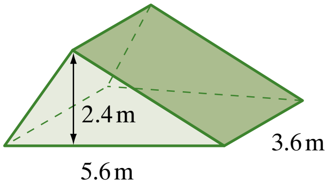

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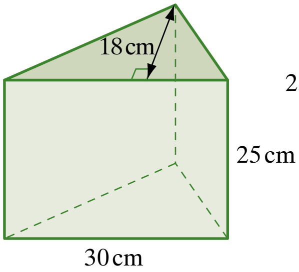

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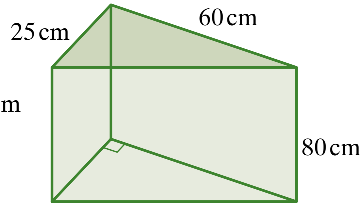


The volume of this triangular prism is

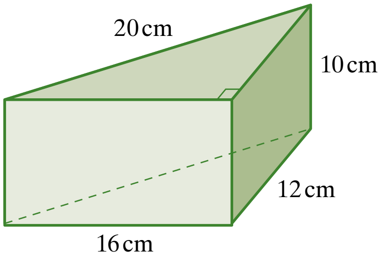
A 1600 cm^3

B 1200 cm^3

C 960 cm^3

D 1920 cm^3

E 19 200 cm^3



● The volume of this prism in cubic centimetres is

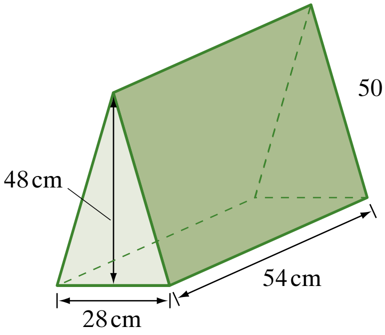
A $\frac{1}{2} \times 28 \times 48 \times 50 \times 54$

B $\frac{1}{2} \times 28 \times 48 \times 50$

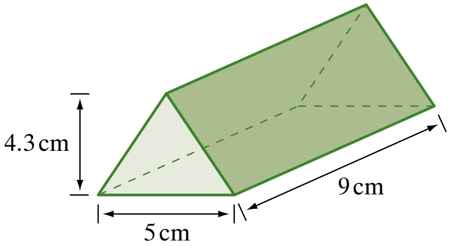
C $\frac{1}{2} \times 28 \times 48 \times 54$

D $28 \times 54 \times 50$

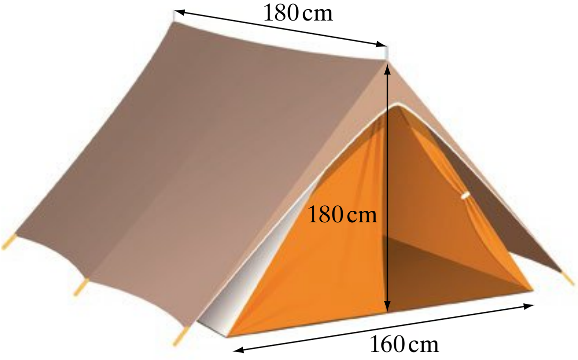
E $28 \times 48 \times 50 \times 54$



● Find the volume of chocolate bar in the triangular prism chocolate on the right.



● Calculate the volume of the inside of this tent in cubic metres.



exercise 10.8 challenge

● Josie makes candles by melting candle wax and pouring it into hollow metal moulds. One of Josie's moulds is a triangular prism. The triangular ends have base 7 cm and perpendicular height 6 cm. The height of the candle mould is 11.4 cm.

a What is the volume of the mould?

b How many candles could Josie make if she had a rectangular block of candle wax 20 cm long, 40 cm wide and 10 cm high?

10.9 Volume of cylinders

Volume of cylinders

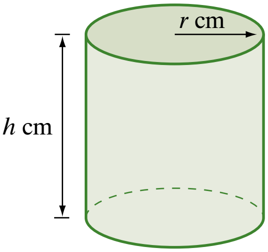
As for prisms, the volume of a cylinder is found by multiplying the cross-sectional area by the height.

$$\text{Volume} = \text{Area of cross-section} \times \text{Height}$$

$$V = Ah$$

$$V = \pi r^2 h$$

where r is the radius of the cylinder and h is the height.



Volume = $\pi r^2 h$ cm^3

Example 44

A cylindrical can has a diameter of 8 cm and a height of 8.5 cm.

a Find the volume of the can. Give your answer to the nearest cubic centimetre.

b How many millilitres of juice will the can hold?

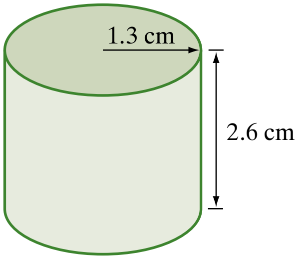
Working	Reasoning
a $V = \pi r^2 h$	
$V = \pi \times 4^2 \times 8.5$	Radius is 4 cm
≈ 427.25	
$= 427$ cm^3 to the nearest cm^3	
b The can will hold 427 ml of juice	1 cm^3 = 1 mL

exercise 10.9

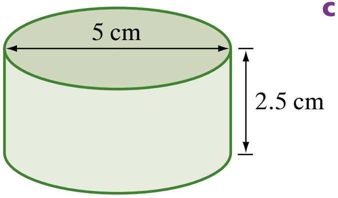
LINKS TO Example 44

Find the volume of these cylinders. Be careful to check whether it is the radius or diameter of the cylinder that has been given. Give the volumes to the nearest whole number.

a



b



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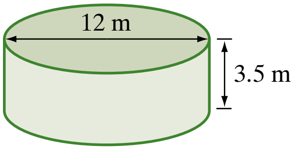

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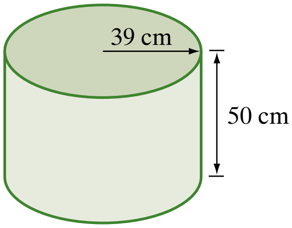

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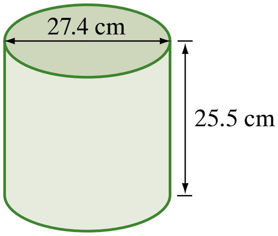

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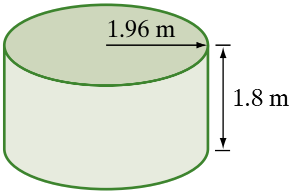

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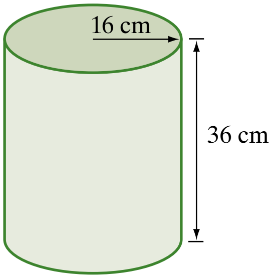

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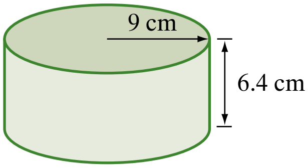

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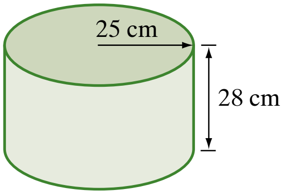


The volume of this cylinder is given by

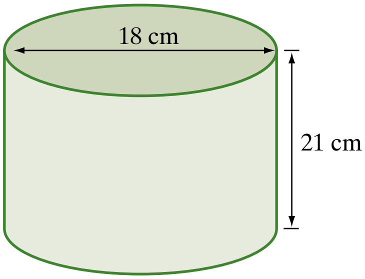
A $\pi \times 18 \times 21$

B $\pi \times 9 \times 21$

C $\pi \times 9^2 \times 21$

D $\pi \times 18^2 \times 21$

E $\pi \times (9 \times 21)^2$



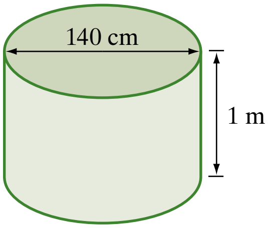
A rainwater tank is 1 metre high and has a diameter of 140 cm.

Find the volume of the tank

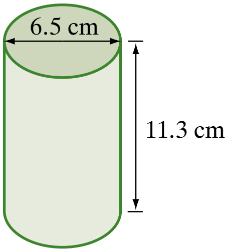
a in cubic metres.

b in kilolitres.

Give the volume correct to one decimal place.



exercise 10.9 challenge



A 375 mL can of lemonade has a diameter of 6.5 cm and a height of 11.3 cm.

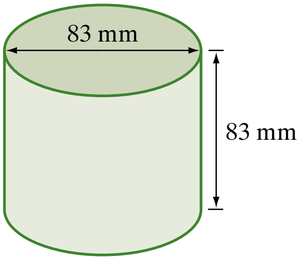
a Calculate the volume

  i in cubic centimetres and

  ii in millilitres.

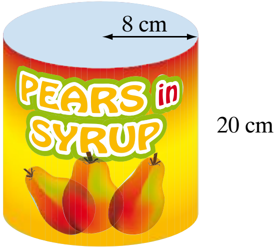
Give your answers correct to the nearest whole number.

b Compare your calculated volume with the labelled volume of 375 mL.



A 450 g can of sliced pineapple has a diameter of 83 mm and a height of 83 mm. Give the volume correct to the nearest cubic centimetre.

Calculate the volume of this can correct to the nearest cubic centimetre.

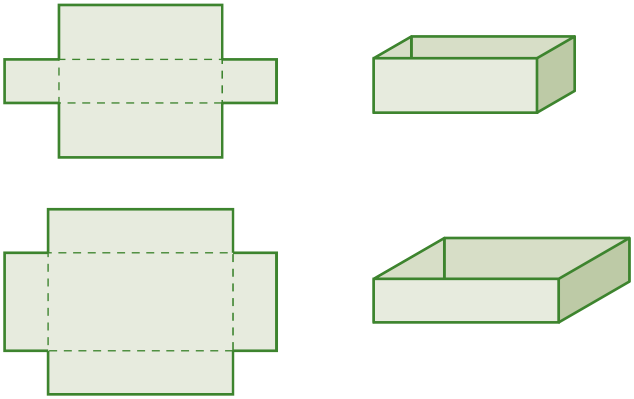


Analysis task

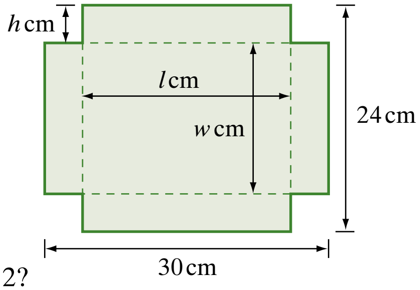
Cake boxes

Cake boxes

A cake shop has cardboard shapes which can be folded to make a box for cakes. The pieces of cardboard are rectangles from which squares have been cut from the corners. The cardboard is folded along the dotted lines shown to form an open box. The shape of the box depends on the size of the square cut outs.



A piece of cardboard 30 cm by 24 cm has squares of side length h cm cut from each corner, as shown in the diagram. The base of the box which is formed is l cm and the width is w cm.



a Squares of side length 2 cm are cut from the corners, that is, $h = 2$. What are the values of l and w?

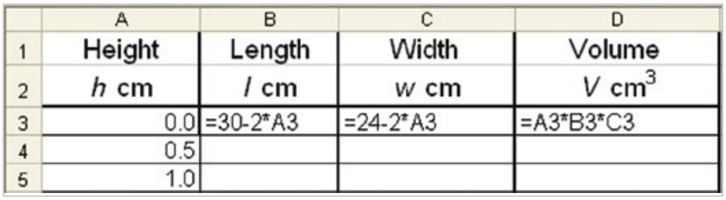
b What is the area of the base of the box when $h = 2$?

c What is the height of the box when $h = 2$?

d What is the volume of the box when $h = 2$?

e What would happen if $h = 12$?

Open the spreadsheet *Cake boxes* in the student ebook. Drag all three formulas down to row 27.



	A	B	C	D
1	Height	Length	Width	Volume
2	h cm	l cm	w cm	V cm^3
3	0.0	=30-2*A3	=24-2*A3	=A3*B3*C3
4	0.5			
5	1.0			

Used with permission from Microsoft

f Explain how the formula in column B is calculating the length of the box.

g Explain how the formula in column C is calculating the width of the box.

h Explain how the formula in column D is calculating the volume of the box.

i What is the largest value for the volume in your spreadsheet?

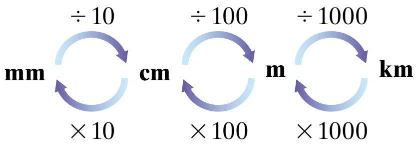
j Which values of h, l and w in your spreadsheet give the largest volume for the box?

Review Perimeter, area and volume

Summary

Length

- Converting units of length

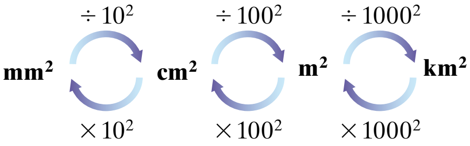


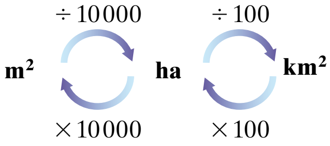
- Length formulas

Perimeter of rectangle	$P = 2(l + w)$	l is the length and w is the width
Circumference of circle	$C = \pi d = 2\pi r$	d is the diameter, r is the radius

Area

- Converting units of area





- Area formulas

Rectangle	$A = lw$	where l is the length and w is the width.
Parallelogram	$A = bh$	where b is the base and h is the perpendicular height.
Triangle	$A = \frac{1}{2}bh$	where b is the base and h is the perpendicular height.
Rhombus and kite	$A = \frac{1}{2}xy$	where x and y are the lengths of the diagonals.
Square	$A = l^2$ or $A = \frac{1}{2}d^2$	where l is the side length and d is the diagonal length.
Circle	$A = \pi r^2$	where r is the radius

Volume

- Converting units of volume and capacity

$$\begin{tikzcd}
\mathbf{mm^3} \arrow[r, bend left, "\div 10^3"] & \mathbf{cm^3} \arrow[l, bend left, "\times 10^3"] \arrow[r, bend left, "\div 100^3"] & \mathbf{m^3} \arrow[l, bend left, "\times 100^3"] \arrow[r, bend left, "\div 1000^3"] & \mathbf{km^3} \arrow[l, bend left, "\times 1000^3"]
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{mL} \arrow[r, bend left, "\div 1000"] & \mathbf{L} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \mathbf{kL} \arrow[l, bend left, "\times 1000"] \arrow[r, bend left, "\div 1000"] & \mathbf{ML} \arrow[l, bend left, "\times 1000"]
\end{tikzcd}$$

Volume unit	Capacity unit
1 cm^3	1 millilitre (1 mL)
1000 cm^3	1 litre (1 L)
1 m^3	1 kilolitre (1 kL)

- Volume formulas

Prism	$V = Ah$	A is the area of the cross-section and h is the height of the prism.
Triangular prism	$\text{V} = \frac{1}{2}bhl$	b is the base and h is the perpendicular height of the triangular cross-section, and l is the height of the prism.
Rectangular prism	$V = lwh$	l is the length, w is the width and h is the height of the prism.
Cylinder	$V = \frac{1}{2}\pi r^2 h$	r is the radius of the circular cross-section and h is the height of the prism.

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

area
capacity
circle
circumference
composite shape
cross-section
cube
cylinder
diagonals
diameter
dimensions
kite
parallelogram
perimeter
perpendicular height
prism
radius
rectangle
rectangular prism
rhombus
square
trapezium
triangle
triangular prism
volume

Revision

Multiple-choice questions

The circumference of a circle with radius 12 cm is closest to

A 19 cm **B** 38 cm **C** 75 cm **D** 113 cm **E** 452 cm

An area of 0.046 m^2 is equal to

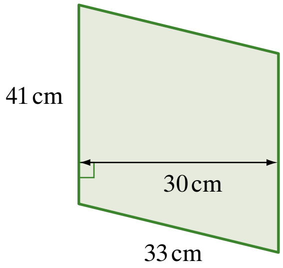
A 0.46 cm^2 **B** 4.6 cm^2 **C** 46 cm^2 **D** 460 cm^2 **E** 4600 cm^2

The lengths of the diagonals of a kite are 16 cm and 24 cm. The area of the kite is

A 20 cm^2 **B** 40 cm^2 **C** 96 cm^2 **D** 192 cm^2 **E** 768 cm^2

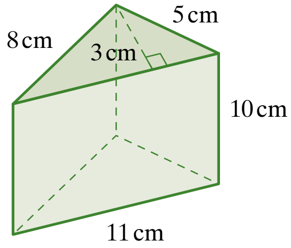
The area of this parallelogram is

A 1353 cm^2 **B** 1230 cm^2 **C** 990 cm^2 **D** 615 cm^2 **E** 2220 cm^2



The volume of this prism is

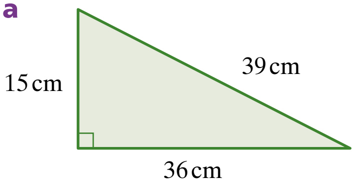
A 330 cm^3 **B** 13 200 cm^3 **C** 4400 cm^3 **D** 270 cm^3 **E** 165 cm^3



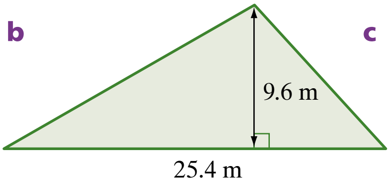
Short-answer questions

Calculate the area of each of these shapes.

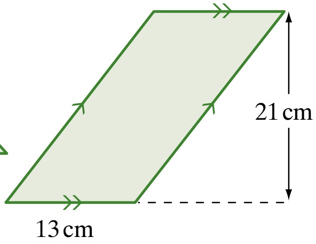
a



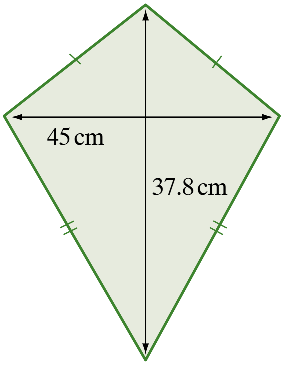
b



c



d



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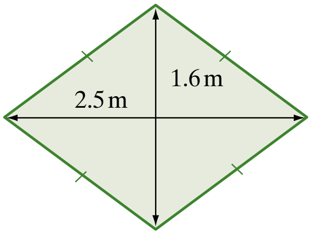
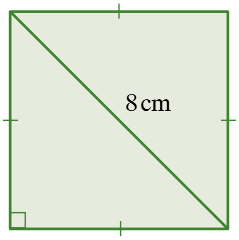


f



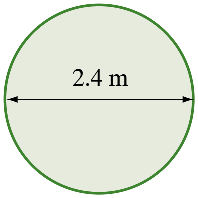
For each of these circles calculate

i the circumference.

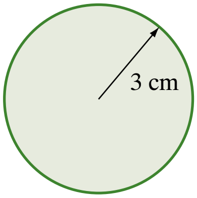
ii the area.

Give your answers correct to one decimal place.

a



b



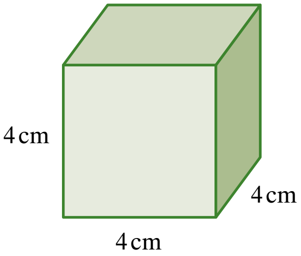
The circumference of a circle is 54 cm. Calculate the following correct to one decimal place.

a the diameter

b the radius

Calculate the volume of these prisms.

a



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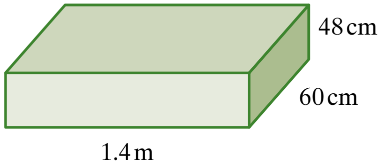

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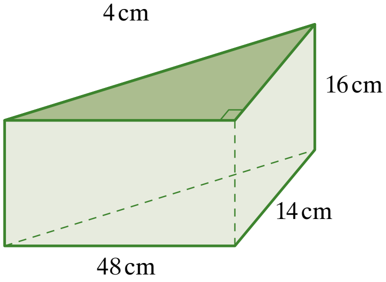

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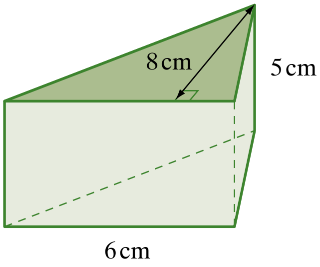
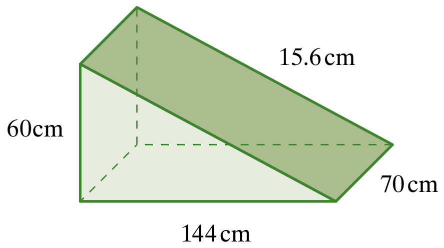
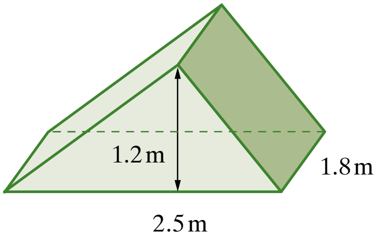


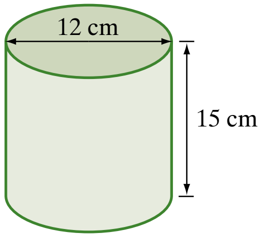
e



f



Calculate the the volume of this cylinder correct to the nearest cubic centimetre.



Extended-response questions

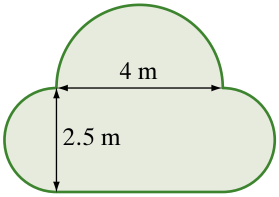
A circular garden has diameter 2.5 m.

- **a** Calculate the length of garden edging needed to surround the garden. Give the length correct to one decimal place.
- **b** Calculate the area of the garden correct to one decimal place.
- **c** Seedlings are planted in the circular garden. Each seedling needs a space of about 50 cm^2. Approximately how many seedlings are needed?

A rectangular swimming pool is 7.5 m long and 3.5 m wide.

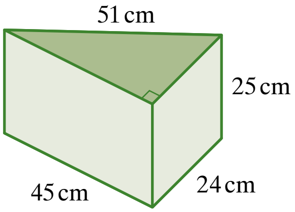
- **a** What is the area of the pool?
- **b** The pool has a 1 metre wide path all around it. Draw a careful diagram of the pool and the path and label it to show the dimensions of the pool and the outside dimensions of the path.
- **c** Find the area of the path.

A garden is based on a rectangle and semicircles as shown in the figure on the right.



- **a** A border is to be placed around the entire edge of the garden. How much edging will be required?
- **b** The garden is to be planted with flowers. What is the total area of the garden?
- **c** If each plant requires 100 cm^2, how many plants will be required? Give your answer to the nearest 10 plants.

● A fish tank is in the shape of a triangular prism with the inside dimensions as shown on the diagram.



a Calculate the volume of the tank in cubic centimetres.

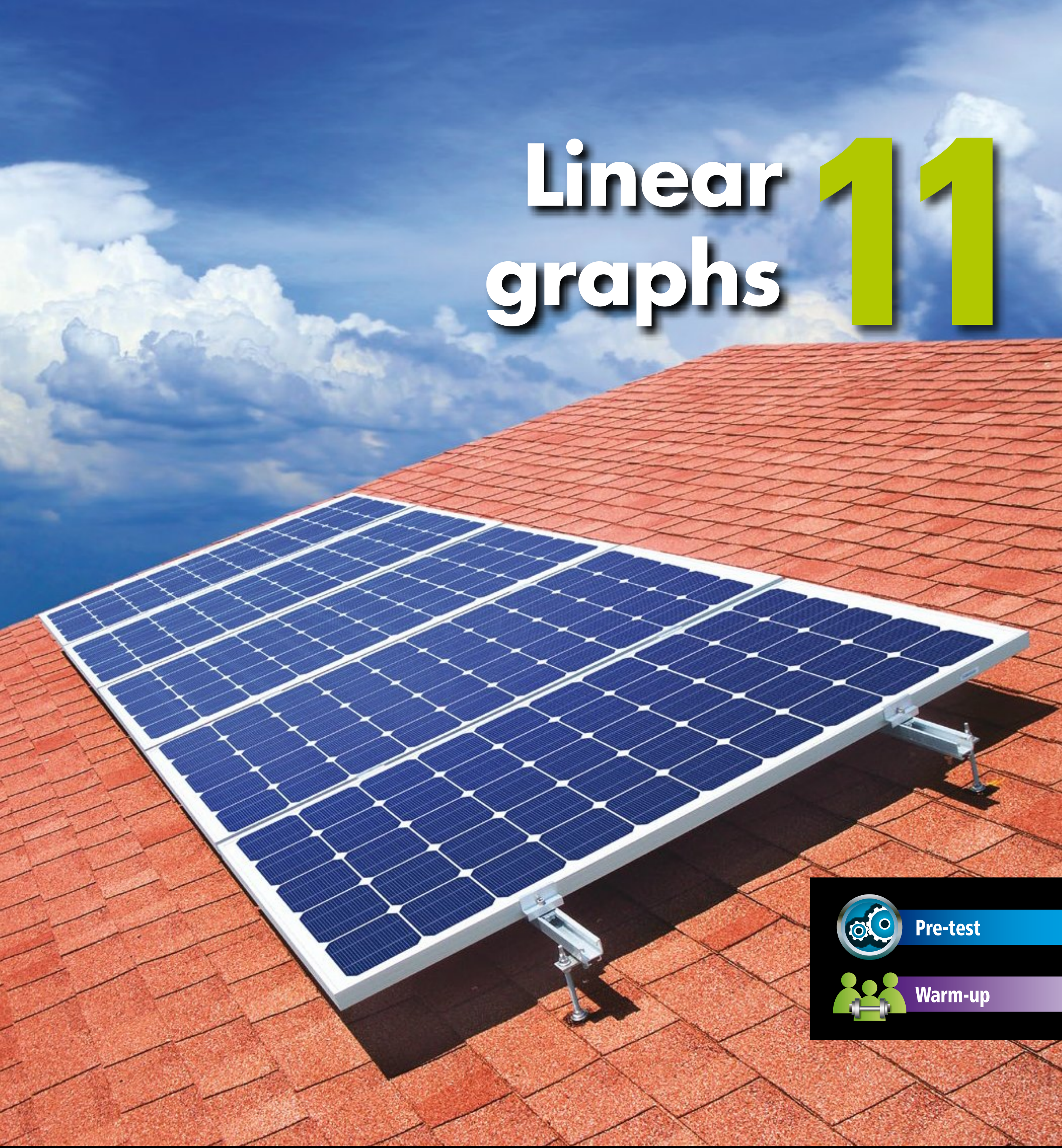
b Find the capacity of the tank

i in millilitres. **ii** in litres.

c The tank is to be filled up to 5 cm from the top. How much water is required?

Linear graphs

11



Pre-test

Warm-up

The solar panels on this roof generate electricity as sunlight shines on them. Each panel generates the same amount of electricity, so there is a direct relationship between the number of panels and the total amount of electricity generated. For example, six panels produce twice as much electricity as three panels. This is called a linear relationship. If we plotted a graph of the amount of electricity produced against the number of panels, the graph would be a straight line.

11.1 The Cartesian plane

Coordinates

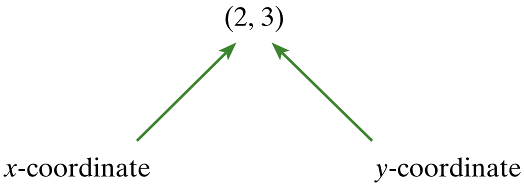
Cartesian coordinates

In Year 7 you saw that points on the **Cartesian plane** can be represented by pairs of numbers in brackets called **Cartesian coordinates**, for example (2, 5). You may remember that the word *Cartesian* comes from the name of the 17th century French mathematician, René Descartes, who first introduced the idea of specifying points by horizontal and vertical coordinates.

The horizontal position of the point on the Cartesian plane is referred to as the ***x*-coordinate** of the point. The vertical position of the point is called the ***y*-coordinate**. We write the *x*- and *y*-coordinates in brackets as an **ordered pair**, for example, (2, 3).

René Descartes (1596–1650)

The first number in the brackets is always the *x*-coordinate and the second number is the *y*-coordinate.

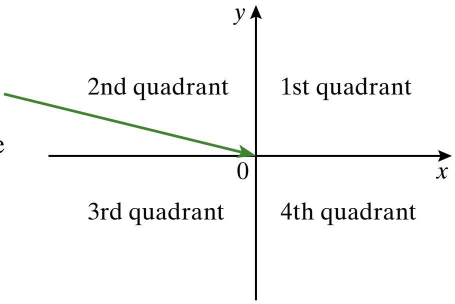


The order of the coordinates in the brackets is easy to remember because the coordinates are in alphabetical order; that is, *x*, then *y*.

The *x*-axis and the *y*-axis include negative numbers as well as positive numbers. The *x*-axis and the *y*-axis cross at the point (0, 0). This point is called the **origin**.

Anywhere along the *x*-axis, $y = 0$. Anywhere along the y-axis, $x = 0$.

The two axes divide the Cartesian plane into four sections called **quadrants**.

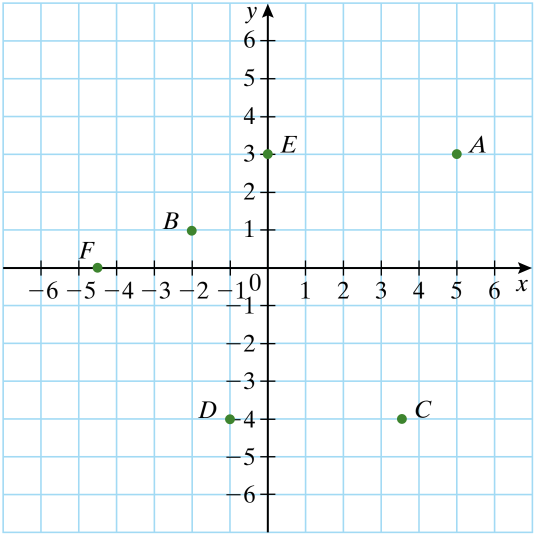


Example 1

Plot the following ordered pairs as points on the Cartesian plane.

A (5, 3) *B* (−2, 1) *C* $(3\frac{1}{2}, -4)$ *D* (−1, −4) *E* (0, 3) *F* $(-4\frac{1}{2}, 0)$

Working



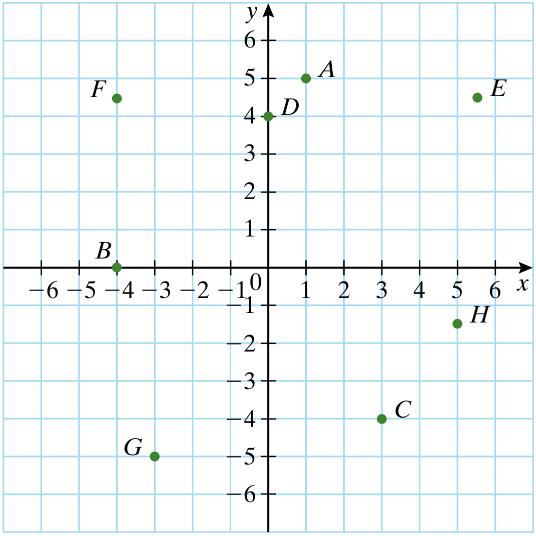
Reasoning

A: 5 units to the right, 3 units up.
B: 2 units to the left, 1 unit up.
C: $3\frac{1}{2}$ units to the right, 4 units down.

D: 1 unit to the left, 4 units down.
E: on the *y*-axis, 3 units up.
F: $4\frac{1}{2}$ units to the left on the *x*-axis.

Example 2

Write the coordinates of the points labelled *A* to *H* as ordered pairs.



continued

Example 2 continued

Working	Reasoning
A $(1, 5)$	Right 1, up 5.
B $(-4, 0)$	Left 4, on the x-axis
C $(3, -4)$	Right 3, down 4
D $(0, 4)$	On the y-axis, up 4
E $(5\frac{1}{2}, 4\frac{1}{2})$	Right $5\frac{1}{2}$, up $4\frac{1}{2}$
F $(-4, 4\frac{1}{2})$	Left 4, up $4\frac{1}{2}$
G $(-3, -5)$	Left 3, down 5
H $(5, -1\frac{1}{2})$	Right 5, down $1\frac{1}{2}$

exercise 11.1

LINKS TO Example 1

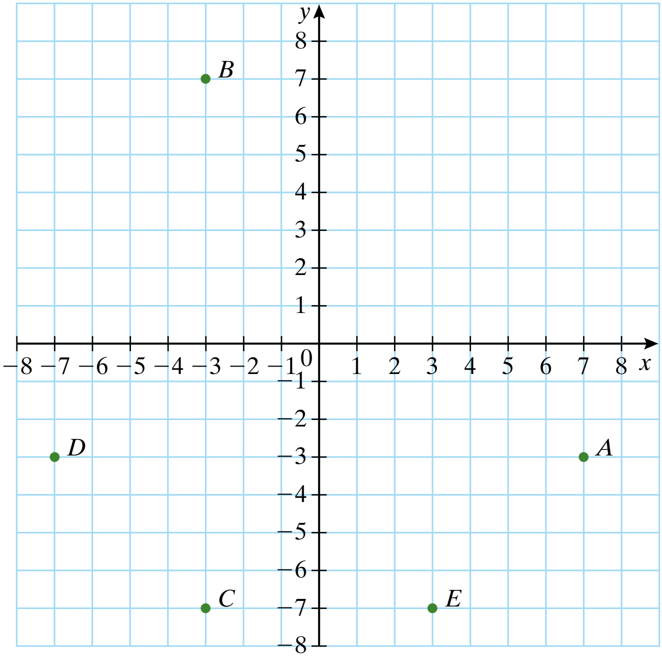
BLM Cartesian plane

Plot these points on the Cartesian plane.

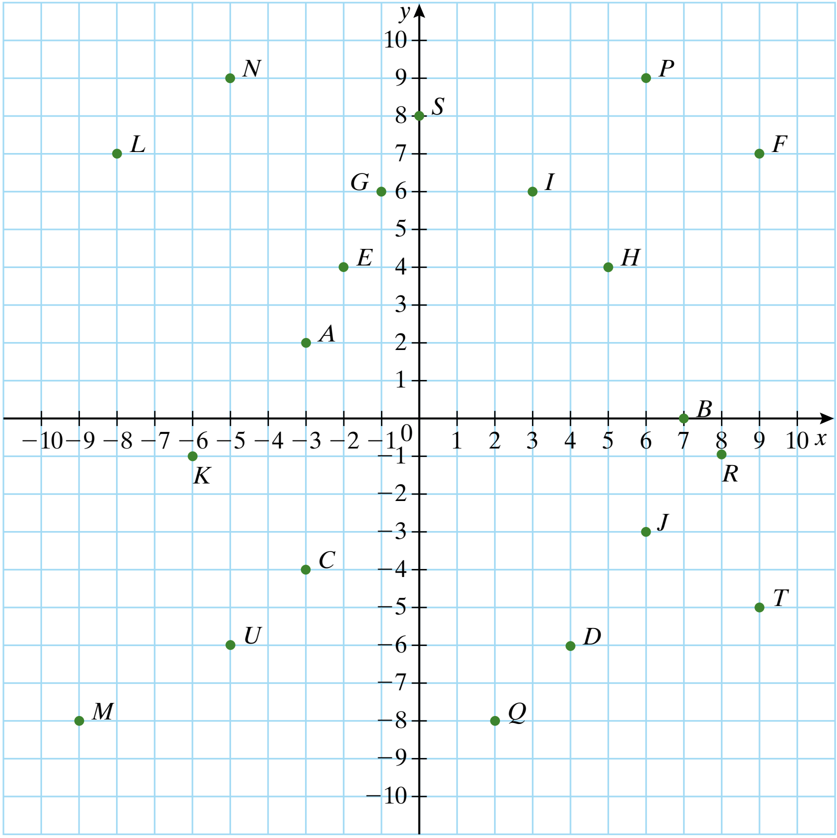
A $(4, 1)$	B $(6, 0)$	C $(-9, 0)$	D $(-5, -6)$	E $(-6, 6)$
F $(0, -8)$	G $(-5, -7)$	H $(7, -4)$	I $(5, -4)$	J $(0, 5)$
K $(-6, -8)$	L $(-5, 0)$	M $(-3, 4)$	N $(3, -9)$	P $(0, -2)$
Q $(4, 6)$	R $(-7, 2)$	S $(9, 0)$	T $(3, 0)$	U $(-8, -2)$

LINKS TO Example 2

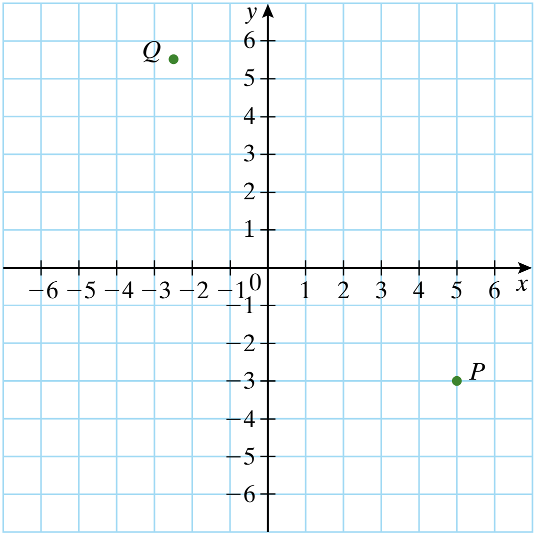
Which one of the points correctly shows the point $(-3, 7)$?



Write the coordinates of each of these points.



Use this Cartesian plane for questions 4 and 5.



Which is the ordered pair corresponding to the point P?

A $(5, 3)$ **B** $(3, 5)$ **C** $(5, -3)$ **D** $(3, -5)$ **E** $(-3, -5)$

Which is the ordered pair corresponding to the point Q?

A $(-3\frac{1}{2}, 5\frac{1}{2})$ **B** $(-3\frac{1}{2}, 5\frac{1}{2})$ **C** $(-2\frac{1}{2}, -4\frac{1}{2})$ **D** $(-2\frac{1}{2}, 5\frac{1}{2})$ **E** $(-3\frac{1}{2}, -5\frac{1}{2})$

The point $(-6, 5)$ is

A 6 units down and 5 units to the right. **B** 6 units to the left and 5 units up.
C 6 units up and 5 units to the left. **D** 6 units down and 5 units to the left.
E 6 units to the right and 5 units up.

Which of these ordered pairs is 7 units to the left and 8 units down?

A $(7, 8)$ **B** $(8, -7)$ **C** $(-8, -7)$ **D** $(-7, -8)$ **E** $(7, -8)$

The following ordered pairs tell us the position of points on the Cartesian plane.

A $(5, 4)$
B $(0, 3)$
C $(5, 5)$
D $(2\frac{1}{2}, 0)$
E $(0, 3.5)$
F $(2, 7)$
G $(1, 5)$
H $(6, 0)$

a Which points are on the x-axis?
b Which points are on the y-axis?
c Which point has the same y-coordinate as point C?

exercise 11.1 challenge

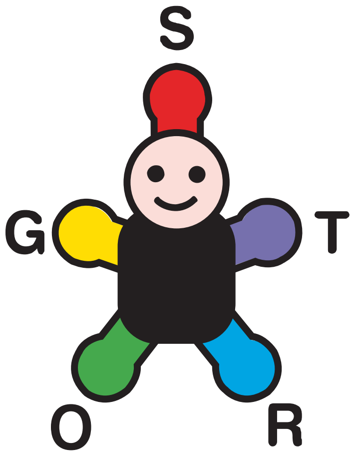
What shape is formed by the following set of four points?

$(-1, -2\frac{1}{2}), (-\frac{1}{2}, 2\frac{1}{2}), (3\frac{1}{2}, -2\frac{1}{2}), (6\frac{1}{2}, 2\frac{1}{2})$

11.2 *Linear number patterns*

In *MathsWorld 7* and in chapter 4 of *MathsWorld 8*, we saw that number patterns could be expressed in different ways:

- story or situation
- table of values
- rule
- ordered pairs
- graph



Words to rule

Expressing a number pattern as an algebra rule is an important part of using algebra to solve problems.

Example 3

Write each of the following as an algebra rule.

a y is always 1 more than twice x.

b y is always 3 less than -2 times x.

Working	**Reasoning**
a $y = 2x + 1$	'twice x' is $2x$. One more than this is $2x + 1$
b $y = -2x - 3$	'-2 times x' is $-2x$. Three less than this is $-2x - 3$

Rule to table of values and lists of ordered pairs

Consider the number pattern with the rule $y = -2x + 3$. To construct a table of values, we obtain the y values by multiplying each x value by -2 and adding 3, for example, $-2 \times (-2) + 3 = 7$, $-1 \times (-2) + 3 = 5$, $0 \times (-2) + 3 = 3$ and so on.

x	-2	-1	0	1	2
y	7	5	3	1	-1

We can then express the values in the table as a list of ordered pairs.

$(-2, 7), (-1, 5), (0, 3), (1, 1), (2, -1)$

Example 4

Using integer values of x from -3 to 3, construct a table of values for each of the rules in example 3 then express as a list of ordered pairs.

Working

a $y = 2x + 1$

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

$(-3, -5), (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5), (3, 7)$

b $y = -2x - 3$

x	-3	-2	-1	0	1	2	3
y	3	1	-1	-3	-5	-7	-9

$(-3, 3), (-2, 1), (-1, -1), (0, -3), (1, -5), (2, -7), (3, -9)$

Reasoning

Substitute each value for x into the rule to obtain the corresponding y value.

Ordered pairs to graph

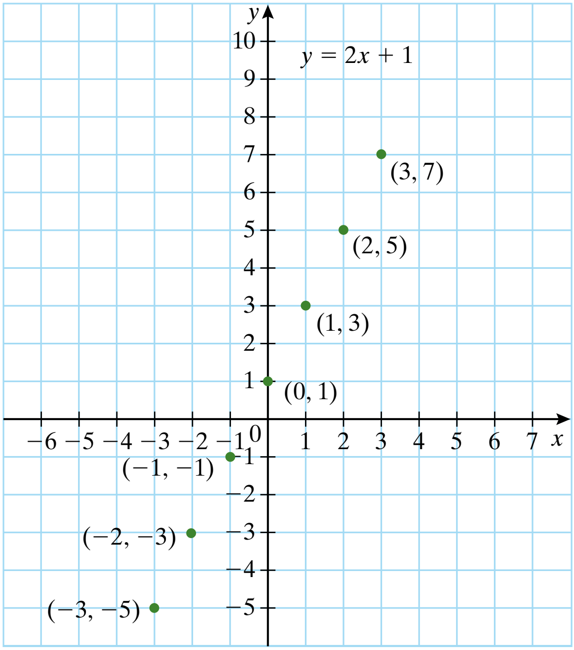
Example 5

For each of the sets of ordered pairs in example 4, rule and label a set of axes on graph paper then plot the ordered pairs.

Working

a $y = 2x + 1$

$(-3, -5), (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5), (3, 7)$



Reasoning

The first number in each ordered pair is the x-coordinate.

The second number in each ordered pair is the y-coordinate.

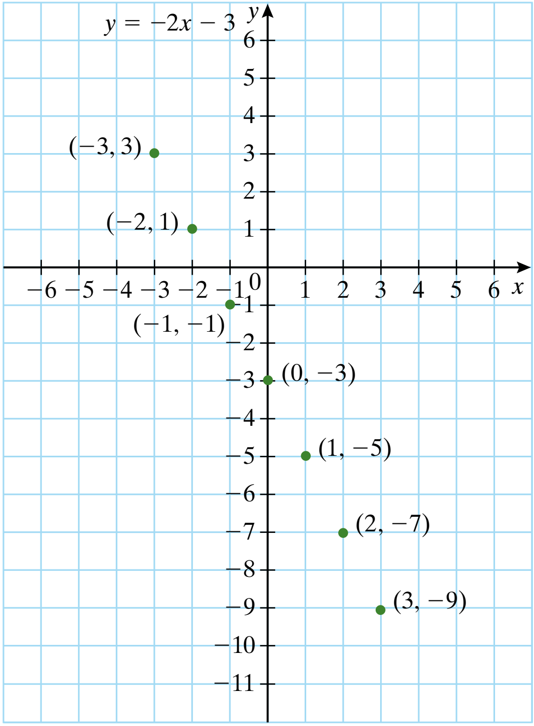
continued

Example 5 continued

Working

b $y = -2x - 3$

$(-3, 3), (-2, 1), (-1, -1), (0, -3), (1, -5), (2, -7), (3, -9)$



Reasoning

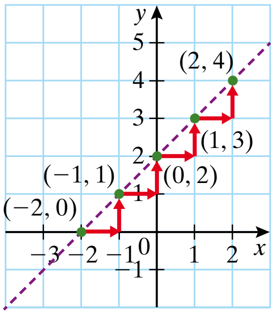
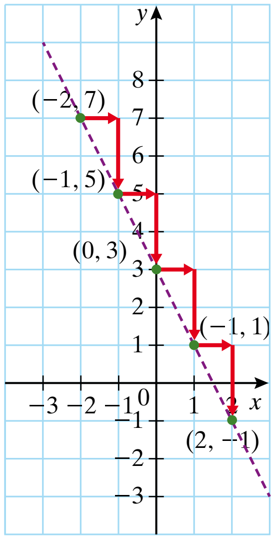
The first number in each ordered pair is the *x*-coordinate.
The second number in each ordered pair is the *y*-coordinate.

How does *y* change as *x* increases in equal steps of 1 unit?

When we construct a table of values we are looking at the relationship between each *x* value and its corresponding *y* value. For example, for the rule $y = 2x + 1$, when $x = -3$, $y = -5$. We will now we turn our attention to how the *y* values change as the *x* values increase in equal steps of 1 unit.

Look at the tables of values on the next page for the rules $y = x + 2$ and $y = -2x + 3$. We see that for the rule $y = x + 2$, as the *x* values increase in equal steps of 1 unit, the *y* values also increase in equal steps of 1 unit. For the rule $y = -2x + 3$, as the *x* values increase in equal steps of 1 unit, the *y* values decrease in equal steps of 2 units.

When the ordered pairs are plotted on the Cartesian plane, we find that each set of points lies along a straight line. Notice how the *y* values on the graph are changing in equal steps as the *x* values increase in steps of 1.

<table>
<tr><td>Words</td><td>y is always 2 more than x</td><td>y is always 3 more than negative two times x</td></tr>
<tr><td>Rule</td><td>$y = x + 2$</td><td>$y = -2x + 3$</td></tr>
<tr><td>Table of values</td><td>+ 1 + 1 + 1 + 1
<table>
<tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
</table>
+ 1 + 1 + 1 + 1</td><td>Note that a table of values may be horizontal or vertical.
<table>
<tr><td></td><td>x</td><td>y</td><td></td></tr>
<tr><td></td><td>−2</td><td>7</td><td></td></tr>
<tr><td>+ 1</td><td>−1</td><td>5</td><td>− 2</td></tr>
<tr><td>+ 1</td><td>0</td><td>3</td><td>− 2</td></tr>
<tr><td>+ 1</td><td>1</td><td>1</td><td>− 2</td></tr>
<tr><td>+ 1</td><td>2</td><td>−1</td><td>− 2</td></tr>
</table></td></tr>
<tr><td></td><td>The x values are increasing in equal steps of 1 unit.
The y values are also <i>increasing in equal steps</i> of 1 unit.</td><td>The x values are increasing in equal steps of 1 unit.
The y values are <i>decreasing in equal steps</i> of 2 units.</td></tr>
<tr><td>Ordered pairs</td><td>(−2, 0), (−1, 1), (0, 2), (1, 3), (2, 4)
+ 1 + 1 + 1 + 1</td><td>(−2, 7), (−1, 5), (0, 3), (1, 1), (2, −1)
− 2 − 2 − 2 − 2</td></tr>
<tr><td>Graph</td><td>
</td><td>
</td></tr>
<tr><td></td><td>The x values are increasing in equal steps of 1 unit.
The y values are also <i>increasing in equal steps</i> of 1 unit</td><td>The x values are increasing in equal steps of 1 unit.
The y values are <i>decreasing in equal steps</i> of 2 units.</td></tr>
</table>

Linear relationships

The relationships represented in these rules, tables of values, lists of ordered pairs and graphs in examples 3, 4 and 5 are called **linear relationships** because

- as the x values increase in steps of 1 unit, the y values increase or decrease in equal steps.
- the points lie along a straight line.

Linear relationships

If the y values increase or decrease in equal steps as the x values increase in steps of 1 unit, then the relationship is called a linear relationship. A linear relationship has a straight line (linear) graph.

Example 6

For each of these tables of values, use arrows to show the increase or decrease in the y values as the x values increase in steps of 1.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b

x	0	1	2	3	4
y	7	4	1	−2	−5

Working

a

+ 1 + 1 + 1 + 1

x	−2	−1	0	1	2
y	0	2	4	6	8

+ 2 + 2 + 2 + 2

Reasoning

The x values are increasing in steps of 1.
The y values are increasing in steps of 2.

b

+ 1 + 1 + 1 + 1

x	0	1	2	3	4
y	7	4	1	−2	−5

− 3 − 3 − 3 − 3

The x values are increasing in steps of 1.
The y values are decreasing in steps of 3.

Example 7

Decide if each of these tables of values represents a linear relationship.

a

x	-1	0	1	2	3
y	-3	-6	-9	-12	-15

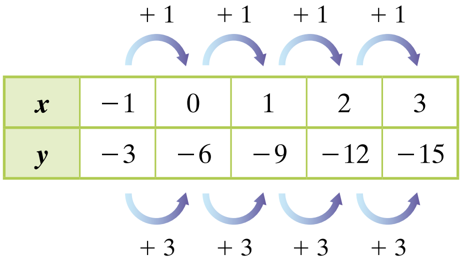
b

x	-3	-2	-1	0	1
y	5	0	-3	-4	-3

Working

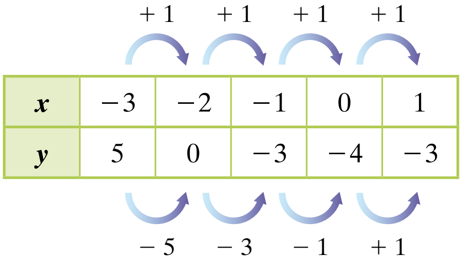
a The relationship is linear because the y values are decreasing in equal steps of 3 as the x values are increasing in steps of 1.

Reasoning



x	-1	0	1	2	3
y	-3	-6	-9	-12	-15

b The relationship is not linear because the y values are not increasing or decreasing in equal steps as the x values are increasing in steps of 1.



x	-3	-2	-1	0	1
y	5	0	-3	-4	-3

Example 8

For each of these lists of ordered pairs

- **i** describe the change in y values as the x values increase in steps of 1.
- **ii** decide if the list of ordered pairs represents a linear relationship.

a $(-1, 2), (0, -3), (1, -8), (2, -13), (3, -18)$

b $(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)$

Working

a **i** $(-1, 2), (0, -3), (1, -8), (2, -13), (3, -18)$

As the x values increase in steps of 1, the y values decrease in equal steps of 5.

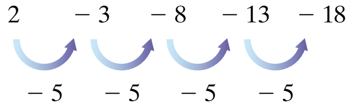
ii The relationship is linear.

Reasoning

The x values are increasing in steps of 1.

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

The y values are decreasing in equal steps.



continued

Example 8 continued

Working

b **i** $(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)$

As the x values increase in steps of 1, the y values are not changing in equal steps.

ii The relationship is not linear.

Reasoning

The x values are increasing in steps of 1.

$-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

The y values are not increasing in equal steps.

$-8 \quad -1 \quad 0 \quad 1 \quad 8 \quad 27$

$+7 \quad +1 \quad +1 \quad +7 \quad +19$

Example 9

For each of these tables of values

- **i** write a list of ordered pairs.
- **ii** plot the points on the Cartesian plane.
- **iii** decide if the relationship is linear.

a

x	y
−2	4
−1	1
0	0
1	1
2	4
3	9

b

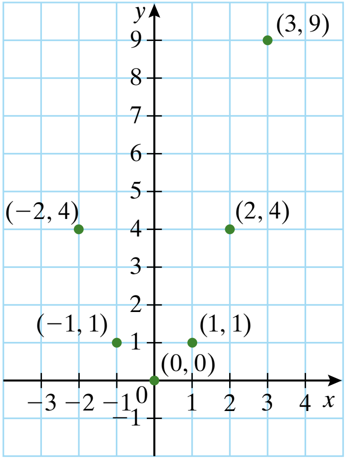
x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7

Working

Reasoning

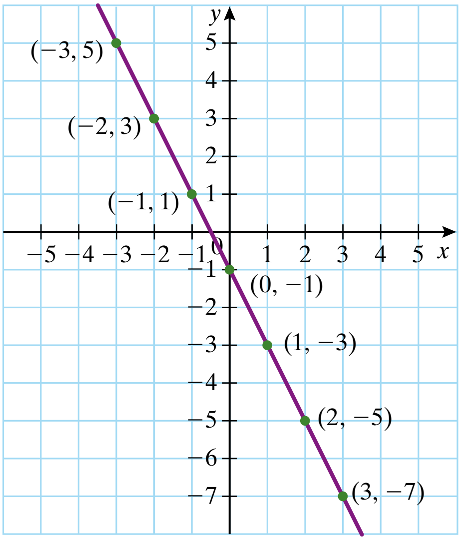
a **i** $(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)$

ii



continued

Example 9 continued

Working	**Reasoning**
iii The relationship is not linear.	The y values are not increasing in equal steps. The points are not in a straight line.
b i $(-3, 5), (-2, 3), (-1, 1), (0, -1), (1, -3), (2, -5), (3, -7)$	The y values are decreasing in equal steps as the x values increase in steps of 1. The points lie along a straight line.
ii 	
iii The relationship is linear.	

When we know that a relationship is linear, we can use the table of values or list of ordered pairs to find the value of other ordered pairs that satisfy the relationship.

Example 10

Give the next three pairs of integer values in these linear relationships.

a

x	−1	0	1	2	3			
y	−4	−1	2	5	8			

b $(-2, 2), (-1, 0), (0, -2), (1, -4), (2, -6)$

continued

Example 10 continued

Working

Reasoning

a

x	-1	0	1	2	3	4	5	6
y	-4	-1	2	5	8	11	14	17

As the x values increase in steps of 1, the y values are increasing in steps of 3.

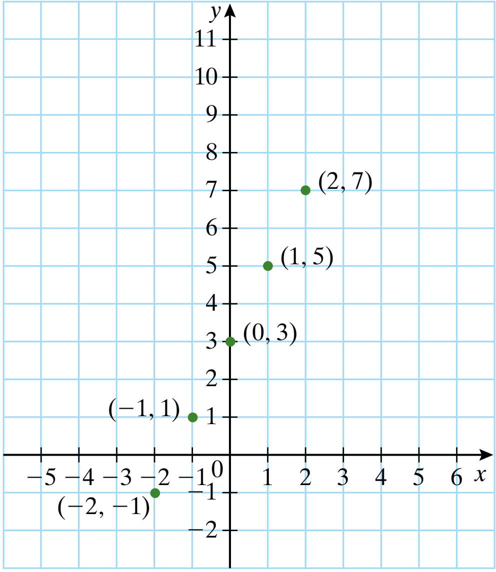
b $(3, -8)$, $(4, -10)$, $(5, -12)$

As the x values increase in steps of 1, the y values are decreasing in steps of 2.

We can also use the graph to find other ordered pairs that satisfy the linear relationship.

Example 11

Extend the pattern in the linear graph to find the value of y when $x = 4$. Show your reasoning.



continued

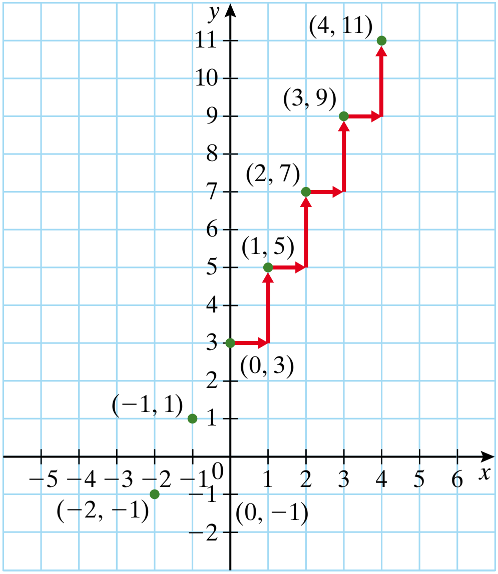
Example 11 continued

Working

As x increases in steps of 1 unit, y is increasing in steps of 2 units. So the ordered pairs $(3, 9)$ and $(4, 11)$ satisfy the linear relationship.

When $x = 4$, $y = 11$

Reasoning



exercise 11.2

LINKS TO Example 3

1 Write each of these linear patterns as a rule.

- **a** y is always 3 less than x.
- **b** y is always 2 more than x.
- **c** y is always 3 more than twice x.
- **d** y is always 4 less than 3 times x.
- **e** y is always 1 more than negative x.
- **f** y is always eight times x.
- **g** y is always 5 less than twice x.
- **h** y is always 3 times x.
- **i** y is always negative 6 times x.
- **j** y is always 3 more than half of x.

2 y is always 4 less than negative 5 times x. This can be written as

- **A** $y = 4 - 5x$
- **B** $y = -5x + 4$
- **C** $y = -4x - 5$
- **D** $y = -5x - 4$
- **E** $y = 5 - 4x$

3 The rule $y = -2x - 3$ can be written in words as

- **A** y is always 3 less than negative two times x
- **B** y is always 2 more than negative three times x
- **C** y is always 3 less than twice x
- **D** y is always 2 less than negative three times x
- **E** y is always 3 more than twice x

y is always 7 more than negative 5 times x. This can be written as

A $y = -5x - 7$
B $y = -7x + 5$
C $y = -7x - 5$
D $y = -5x + 7$
E $y = 7 + 5x$

LINKS TO Example 4

For each of these rules complete the table of values.

a $y = 7x$

x	−3	−2	−1	0	1	2	3
y							

b $y = -4x$

x	−3	−2	−1	0	1	2	3
y							

c $y = 2x + 7$

x	−3	−2	−1	0	1	2	3
y							

d $y = 2x + 5$

x	1	2	3	4	5	6	7
y							

e $y = -x + 4$

x	−3	−2	−1	0	1	2	3
y							

f $y = 5x + 3$

x	1	2	3	4	5	6	7
y							

For each of these rules

i construct a table of values with values from −3 to 3.
ii Make a list of ordered pairs.

a $y = x + 1$ **b** $y = x - 2$ **c** $y = x + 5$ **d** $y = x - 3$
e $y = 2x + 1$ **f** $y = 2x$ **g** $y = 3x - 2$ **h** $y = 3x + 1$

LINKS TO Examples 6, 7

Look at each of these tables of values and decide if the ordered pairs form a linear pattern.

a

x	y
1	−6
2	−5
3	−4
4	−3
5	−2

b

x	−8	−7	−6	−5	−4
y	8	7	6	5	4

LINKS TO Examples 6, 7

Look at each of these tables of values and decide if the ordered pairs form a linear pattern.

a

x	y
1	2
2	4
3	8
4	16
5	32

b

x	y
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5

c

x	y
1	2
2	3
3	5
4	8
5	12

d

x	1	2	3	4	5
y	−2	0	2	4	6

e

x	1	2	3	4	5
y	−2	1	6	13	22

f

x	1	2	3	4	5
y	4	7	10	13	16

g

x	−5	−4	−3	−2	−1
y	25	16	9	4	1

h

x	1	2	3	4	5
y	2	6	12	20	30

i

x	−3	−2	−1	0	1
y	−15	−11	−7	−3	1

j

x	0	1	2	3	4
y	1	3	9	27	81

k

x	−10	−9	−8	−7	−6
y	−21	−17	−13	−9	−5

LINKS TO Example 8

Decide if each of these sets of ordered pairs form a linear pattern.

a (−2, 2), (−1, 1), (0, 0), (1, −1), (2, −2)

b (1, 2), (2, 5), (3, 8), (4, 11), (5, 14)

c (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)

d (−2, 11), (−1, 9), (0, 7), (1, 5), (2, 3)

LINKS TO Examples 5, 6, 9

BLM Cartesian plane

Each of these lists of ordered pairs and tables of values represents a linear relationship.

- **i** use graph paper to draw and label the *x*-axis and *y*-axis then plot the points on the Cartesian plane
- **ii** State the change in the *y* values as the *x* values increase in steps of 1 unit.

a (0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)

b (−3, −6), (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4), (3, 6)

c (−3, −4), (−2, −3), (−1, −2), (0, −1), (1, 0), (2, 1), (3, 2)

d (−3, −7), (−2, −5), (−1, −3), (0, −1), (1, 1), (2, 3), (3, 5)

e

x	−3	−2	−1	0	1	2	3
y	4	5	6	7	8	9	10

f

x	0	1	2	3	4	5	6
y	−5	−3	−1	1	3	5	7

g

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

h

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2

i

x	−3	−2	−1	0	1	2	3
y	−2	0	2	4	6	8	10

j

x	0	1	2	3	4	5	6
y	−10	−7	−4	−1	2	5	8

LINKS TO Example 10

Each of these lists of ordered pairs or tables of values represents a linear pattern. Give the next three ordered pairs in each pattern.

a

x	0	1	2	3	4	5	6
y	0	4	8	12	16	20	24

b

x	0	1	2	3	4	5	6
y	−3	−2	−1	0	1	2	3

c

x	0	1	2	3	4	5	6
y	2	4	6	8	10	12	14

d

x	0	1	2	3	4	5	6
y	7	10	13	16	19	22	25

e

x	0	1	2	3	4	5	6
y	4	2	0	−2	−4	−6	−8

f

x	−3	−2	−1	0	1	2	3
y	−2.8	−1.4	0	1.4	2.8	4.2	5.6

g (−3, 3), (−2, 4), (−1, 5), (0, 6), (1, 7), (2, 8), (3, 9)

h (−3, 2), (−2, 1), (−1, 0), (0, −1), (1, −2), (2, −3), (3, −4)

i (−3, −3), (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7), (3, 9)

j (−3, −8), (−2, −5), (−1, −2), (0, 1), (1, 4), (2, 7), (3, 10)

For the linear pattern (−3, −13), (−2, −8), (−1, −3), (0, 2) the next three ordered pairs will be

A (1, 3), (2, 4), (3, 5)

B (1, 4), (2, 6), (3, 8)

C (1, 0), (2, 10), (3, 15)

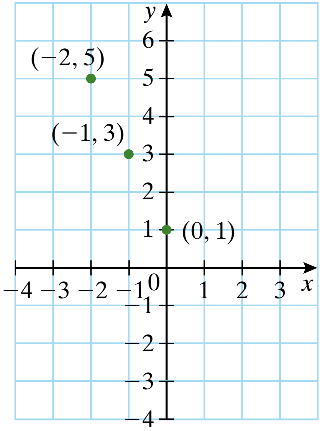
D (1, 7), (2, 12), (3, 17)

E (1, 3), (2, 8), (3, 13)

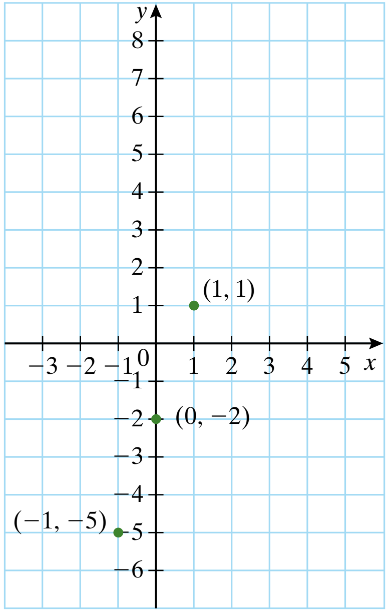
LINKS TO Example 11

Predict the value of y when $x = 3$ for the following linear graphs.

a



b



exercise 11.2 challenge

Consider the pattern (−3, 11.1), (−2, 8.1), (−1, 5.1), (0, 2.1).

a Describe this linear pattern in words.

b Write the next three ordered pairs.

11.3 Finding linear rules

Graphing linear rules

In the previous section we looked at how the y values changed as the x values increased in steps of 1. In this section we will see how to find a linear rule from a table of values, list of ordered pairs or a graph. Consider the linear rule $y = 2x + 4$. As the x values increase in equal steps of 1 unit, the y values increase in equal steps of 2 units.

- the coefficient of x in the rule is the same as the change in the y values as the x values increase in steps of 1.
  $y = 2x + 4$

+ 1 + 1 + 1 + 1

x	−2	−1	0	1	2
y	0	2	4	6	8

+ 2 + 2 + 2 + 2

- the constant term in the rule is the value of y when $x = 0$
  $y = 2x + 4$

+ 1 + 1 + 1 + 1

x	−2	−1	0	1	2
y	0	2	4	6	8

+ 2 + 2 + 2 + 2

The general rule for a linear relationship: $y = mx + b$

All linear relationships can be expressed in the form $y = mx + b$.

- m is equal to the change in the y values as the x values increase in steps of 1.
- b is the value of y when $x = 0$

To find the rule $y = mx + b$ for a table of values:

- the change in the y values as the x values increase in steps of 1 tells us the value of m.
- the value of y when $x = 0$ tells us the value of b.

Example 12

Write the rule for the linear relationship shown in each of these tables of values.

a

x	−2	−1	0	1	2
y	−4	−1	2	5	8

b

x	y
−3	0
−2	−1
−1	−2
0	−3
1	−4

Working

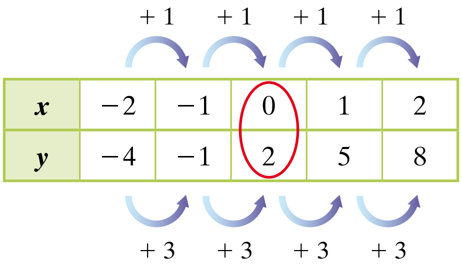
a $m = 3$

$b = 2$

The rule is $y = 3x + 2$

Reasoning

As the x values increase in steps of 1, the y values are increasing in steps of 3. When $x = 0$, $y = 2$.



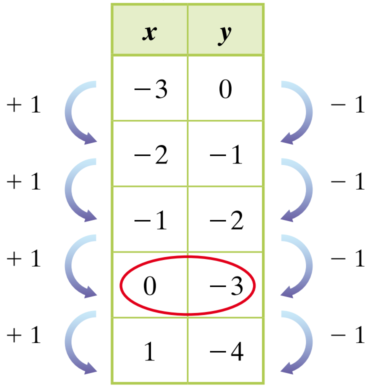
x	−2	−1	0	1	2
y	−4	−1	2	5	8

b $m = -1$

$b = -3$

The rule is $y = -x - 3$

As the x values increase in steps of 1, the y values are decreasing in steps of 1. When $x = 0$, $y = -3$.



x	**y**
−3	0
−2	−1
−1	−2
0	−3
1	−4

The rule for a linear relationship represented as a list of ordered pairs can be found in the same way as for a table of values.

Example 13

Write the rule for the linear relationship shown in each of these lists of ordered pairs.

a $(-2, 2), (-1, 0), (0, -2), (1, -4), (2, -6)$

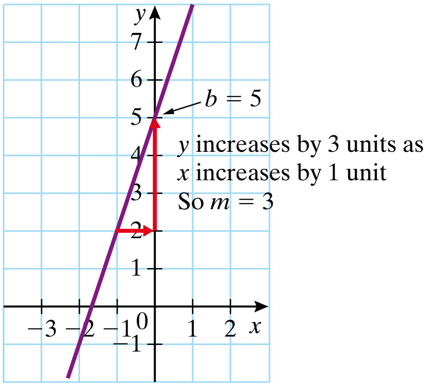
b $(-2, -1), (-1, 2), (0, 5), (1, 8), (2, 11)$

c $(-2, -1), (-1, -0.5), (0, 0), (1, 0.5), (2, 1)$

Working	Reasoning
a $m = -2$ $b = -2$ The rule is $y = -2x - 2$.	As the x values increase in steps of 1, the y values are decreasing in steps of 2. When $x = 0, y = -2$.
b $m = 3$ $b = 5$ The rule is $y = 3x + 5$.	As the x values increase in steps of 1, the y values are increasing in steps of 3. When $x = 0, y = 5$.
c $m = 0.5$ $b = 0$ The rule is $y = 0.5x$.	As the x values increase in steps of 1, the y values are increasing in steps of 0.5. When $x = 0, y = 0$.

Finding the rule for a linear graph

To find a rule from a linear graph, we again need the values of m and b. To find the value of m we choose convenient points where the graph crosses grid lines then draw a right-angled triangle on the graph as shown. We can then see the number of units that y increases or decreases as x increases by steps of 1 unit. The value of b is found by looking at the point where the graph crosses the y-axis. For the graph shown, $m = 3$ and $b = 5$ so the rule is $y = 3x + 5$.



Example 14

For each of these linear graphs,

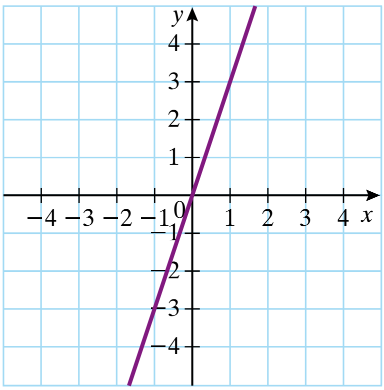
i find the values of m and b.

ii state the rule.

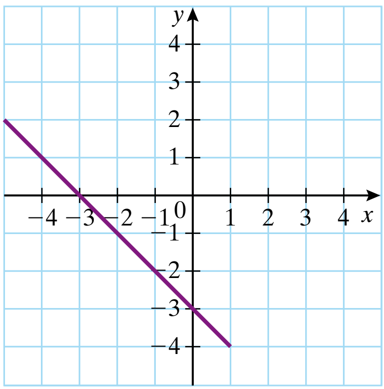
continued

Example 14 continued

a



b

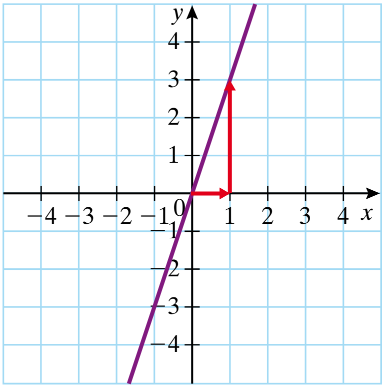


Working

a **i** $m = 3, b = 0$

ii $y = 3x$

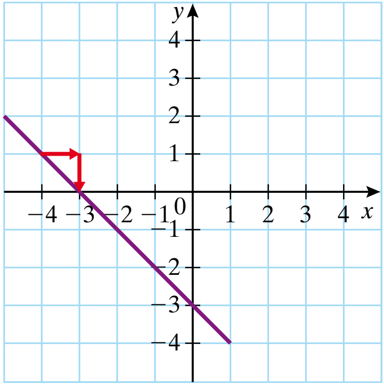
Reasoning



As x increases in steps of 1 unit, y increases in steps of 3 units.
The graph crosses the y-axis at $y = 0$.

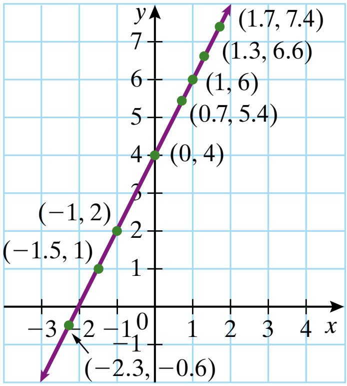
a **i** $m = -1, b = -3$

ii $y = -x - 3$



As x increases in steps of 1 unit, y decreases in steps of 1 unit.
The graph crosses the y-axis at $y = -3$.

Linear graphs as continuous lines



So far we have considered only integer values for *x*. We know that a line consists of an infinite set of points. This means that there is an infinite number of ordered pairs along a linear graph that satisfy the linear relationship. If we know the rule for a linear relationship we can substitute any value for *x*, including fractions and decimals, to find the corresponding value of *y*. For example, each of the ordered pairs shown on the graph satisfies the linear relationship $y = 2x + 4$. Between each of the points shown there will be other ordered pairs that also satisfy $y = 2x + 4$. This is why we are able to draw a continuous line for the graph of the linear relationship. Arrow heads are often put on the ends of the line to show that the graph extends infinitely in both directions.

The minimum number of points needed to draw a linear graph

When we have two points, there is only one straight line that can be drawn through them. For example, for the two points shown below, only one line is possible.

When we construct a table of values before drawing a graph of a linear relationship we actually need only two values for *x* in the table of values but a third value should be used as a check. We have seen that the value of *y* when $x = 0$ is useful as this tells us where the graph crosses the *y*-axis. So the values $x = 0$, $x = 1$ and $x = 2$ are suitable values to use when constructing a table of values.

Example 15

Construct a table of values for the rule $y = 4x - 5$ for the values $x = 0$, $x = 1$ and $x = 2$ then draw the graph.

Working

x	0	1	2
y	−5	−1	3

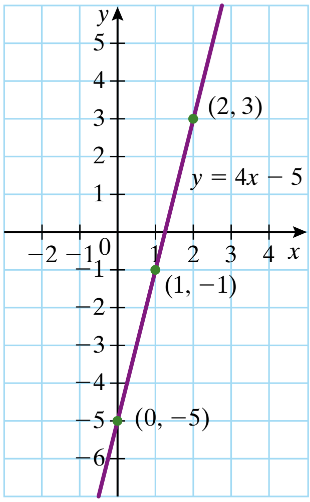
Reasoning

$4 \times 0 - 5 = -5$
$4 \times 1 - 5 = -1$
$4 \times 2 - 5 = 3$

continued

Example 15 continued

Working

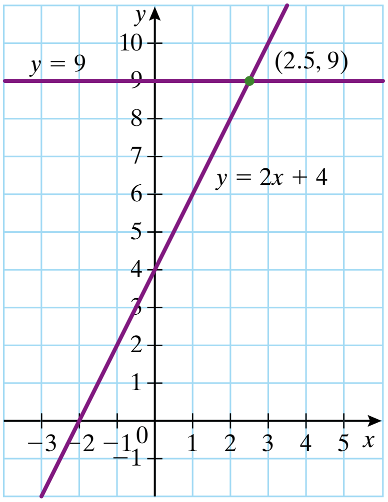


Reasoning

Plot the three points and rule a line through them.

Using graphs to solve linear equations

The graph of a linear relationship can be used to solve certain linear equations. For example, if we have the graph of $y = 2x + 4$, we can use the graph to solve an equation such as $2x + 4 = 9$ by finding the point on the graph where $y = 9$. After drawing the graph of $y = 2x + 4$, we rule a horizontal line through $y = 9$. The point where the two lines intersect $(2.5, 9)$ is the point that satisfies the equation $2x + 4 = 9$. This tells us that the solution to the equation $2x + 4 = 9$ is $x = 2.5$.



Example 16

Solve the following equations by first drawing the graph of $y = 3x - 1$.

a $3x - 1 = 8$

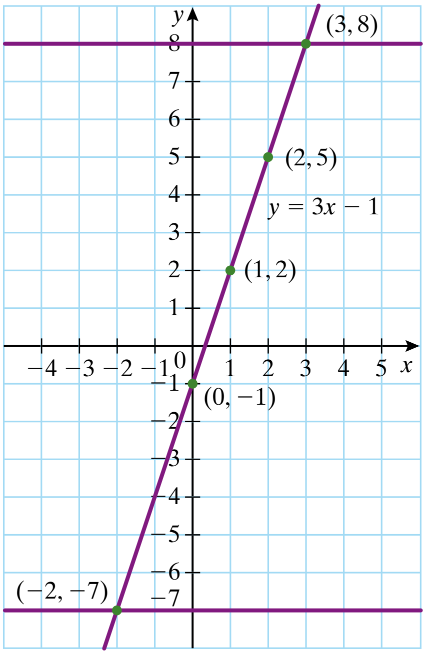
b $3x - 1 = -7$

continued

Example 16 continued

Working

x	0	1	2
y	−1	2	5



a The solution to the equation $3x - 1 = 8$ is $x = 3$.

b The solution to the equation $3x - 1 = -7$ is $x = -2$.

Reasoning

Construct a table of values using $x = 0$, $x = 1$, $x = 2$.
Draw and label the axes.
Plot the points.

Draw a horizontal line through $y = 8$. The intersection of this line with the line $y = 3x - 1$ is at $x = 3$.

Draw a horizontal line through $y = -7$. The intersection of this line with the line $y = 3x - 1$ is at $x = -2$.

exercise 11.3

LINKS TO Example 12

● For each of these linear relationships

i state the values of m and b

ii write the rule

a

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

b

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

c

x	−3	−2	−1	0	1	2	3
y	2	3	4	5	6	7	8

d

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

e

x	−3	−2	−1	0	1	2	3
y	−1	1	3	5	7	9	11

f

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

g

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1

h

x	−3	−2	−1	0	1	2	3
y	−17	−11	−5	1	7	13	19

i

x	−3	−2	−1	0	1	2	3
y	15	10	5	0	−5	−10	−15

j

x	−3	−2	−1	0	1	2	3
y	−1.5	−1	−0.5	0	0.5	1	1.5

Which of these rules matches the table of values.

x	−3	−2	−1	0	1	2	3
y	17	13	9	5	1	−3	−7

A $y = 5x + 2$

B $y = 5x + 4$

C $y = 4x + 5$

D $y = -4x + 5$

E $y = 4x + 5$

LINKS TO Example 13

For each of these linear relationships

- **i** state the values of m and b.
- **ii** write the rule.

a $(-2, 4), (-1, 2), (0, 0), (1, -2), (2, -4)$

b $(-2, 0), (-1, 2), (0, 4), (1, 6), (2, 8)$

c $(-3, -5), (-2, -4), (-1, -3), (0, -2), (1, -1)$

d (−2, 10), (−1, 7), (0, 4), (1, 1), (2, −2)

e (−2, −13), (−1, −7), (0, −1), (1, 5), (2, 11)

f (−2, 11), (−1, 7), (0, 3), (1, −1), (2, −5)

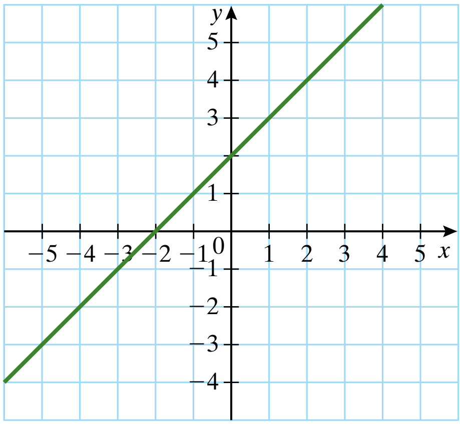
LINKS TO Example 14

For each of these linear graphs

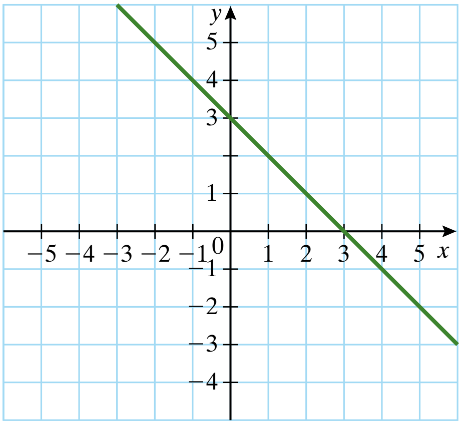
i state the values of m and b.

ii write the rule.

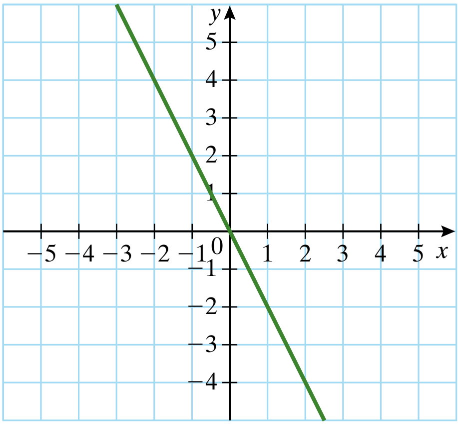
a



b



c



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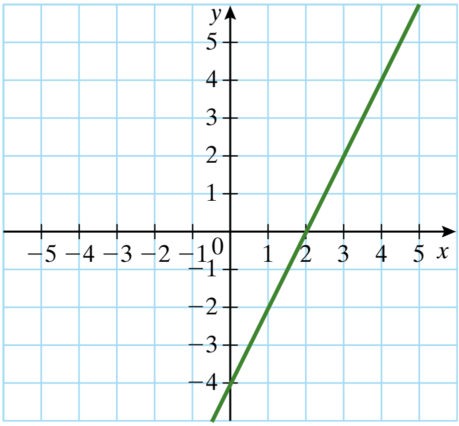

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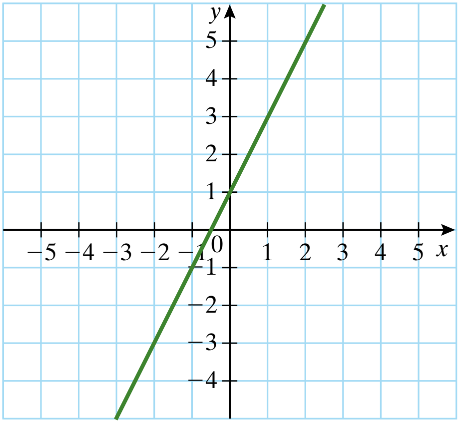

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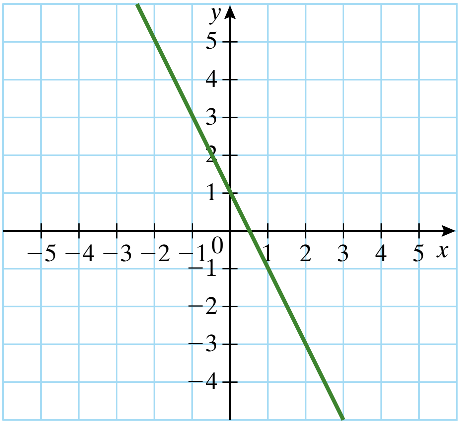

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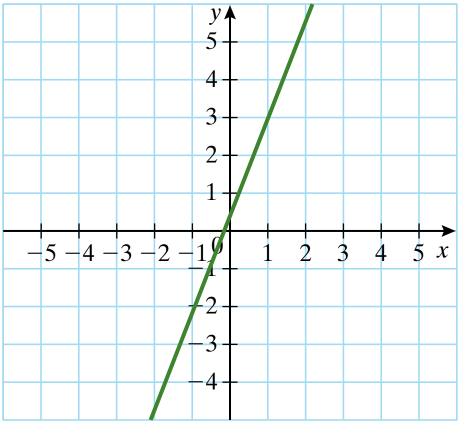

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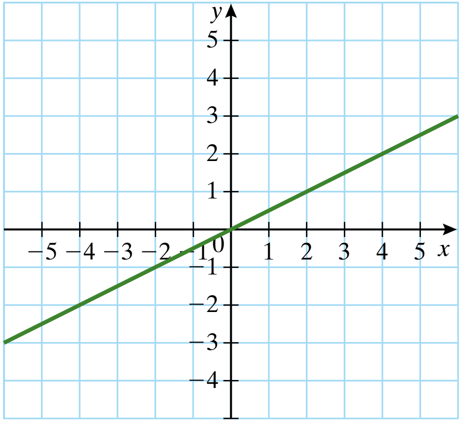

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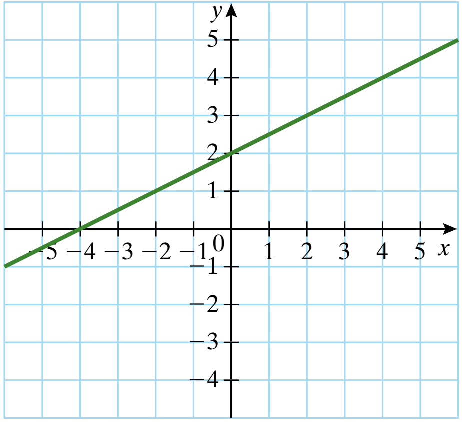

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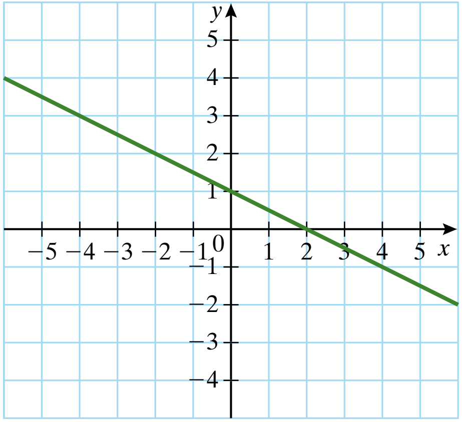


LINKS TO Example 15

Construct a table of values for each of these linear relationships using the values $x = 0$, $x = 1$, $x = 2$ then use graph paper to draw the graphs.

a $y = 2x + 7$ **b** $y = -x - 5$ **c** $y = -2x + 5$ **d** $y = 3x - 8$

LINKS TO Example 16

Solve the following equations by first drawing the graph of $y = -2x + 3$.

a $-2x + 3 = 7$ **b** $-2x + 3 = 9$

Solve the following equations by first drawing the graph of $y = -3.5x + 2$.

a $-3.5x + 2 = 12.5$ **b** $-3.5x + 2 = -5$

exercise 11.3 challenge

Find the rule for this linear pattern.

x	3	4	5	6	7	8
y	2.0	1.4	0.8	0.2	−0.4	−1

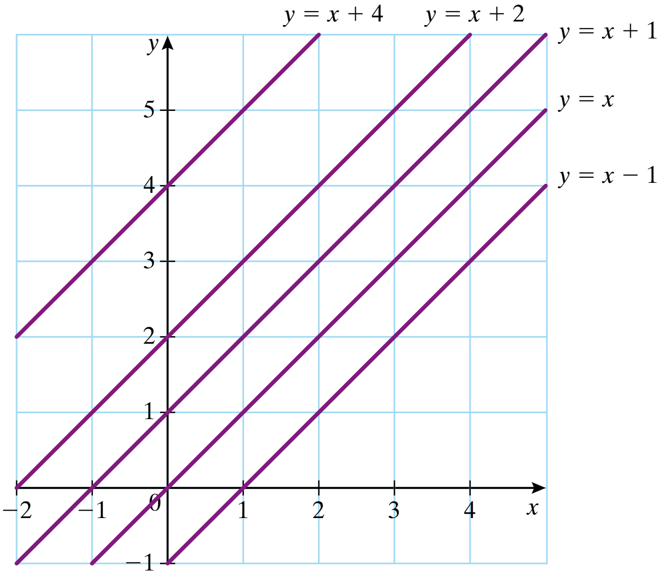
11.4 Families of linear relationships

Families of linear graphs

We have seen how a linear relationship can be represented by the rule $y = mx + b$ where m is equal to the change in the y values as the x values increase in steps of 1 and b is the value of y when $x = 0$. In this section we look at how the value of m affects the graph of a linear relationship.

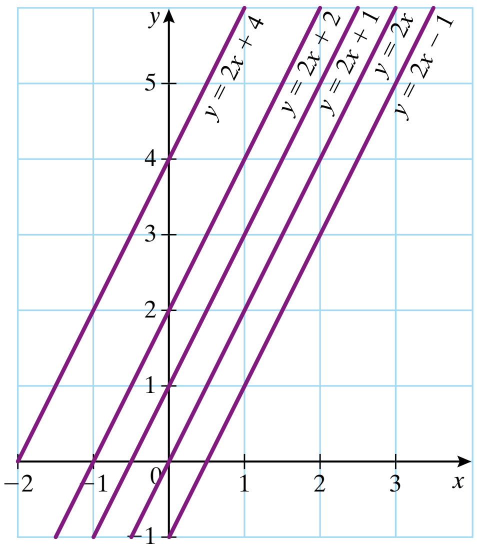
We start by looking at the graphs of $y = x$, $y = x + 1$, $y = x + 2$, $y = x + 4$ and $y = x - 1$.

- The graphs are a set of parallel lines. This is because for each of the linear rules, the coefficient of the x term is 1, so in each case the y values increase in steps of 1 as the x values increase in steps of 1.
- The position where each graph crosses the y-axis (that is, the value of y when $x = 0$) is determined by constant term in the rule. For example, the graph of $y = x + 4$ crosses the y-axis at $y = 4$.



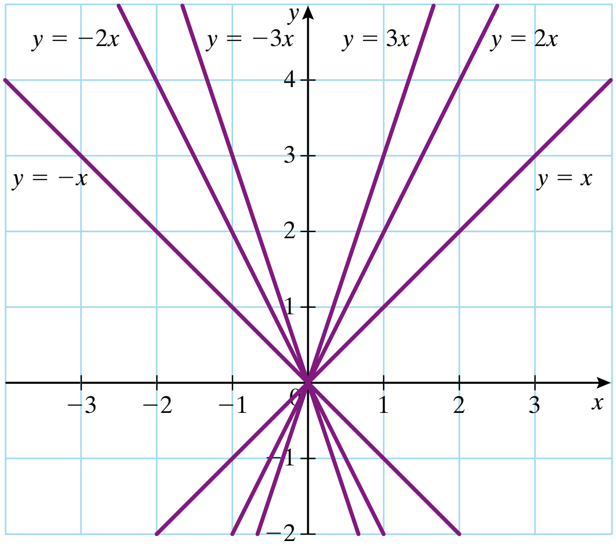
We now consider the graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, $y = 2x + 4$ and $y = 2x - 1$.

- The graphs are again a set of parallel lines.
- The graphs are steeper than the previous set of graphs. As the x values increase in steps of 1, the y values are increasing in steps of 2.
- We again see that the value of y where each graph crosses the y-axis is the constant term in the rule.



We now consider the graphs of $y = x$, $y = 2x$, $y = 3x$, $y = -x$, $y = -2x$, $y = -3x$.

- The graphs of $y = x$, $y = 2x$, $y = 3x$ slope upwards to the right because the y values are increasing as x increases.
- The graphs of $y = -x$, $y = -2x$, $y = -3x$ slope downwards to the right because the y values are decreasing as x increases.
- All the graphs pass through the point $(0, 0)$ because the constant term in the rule is zero.



Gradient and *y*-intercept

The slope of a linear graph is called the **gradient** of the graph. Graphs that slope upwards to the right have a positive gradient and graphs that slope downwards to the right have a negative gradient. The gradient, m, is given by the coefficient of the x term in the linear rule. For example, the graph of $y = -2x + 1$ has gradient -2 and the graph of $y = 3x - 2$ has gradient 3.

The position where the graph crosses the y-axis is called the ***y*-intercept** and is given by the value of b in the linear rule $y = mx + b$. For example, the graph of $y = -2x + 1$ has its y-intercept at $y = 1$ and the graph of $y = 3x - 2$ has its y-intercept at $y = -2$.

Gradient and *y*-intercept

- m is the gradient, that is, the change in the y values as the x values increase in steps of 1.
- b is the y-intercept, that is, the value of y when $x = 0$
- linear graphs with the same gradient are parallel lines
- If the gradient is positive, the graph slopes upwards to the right
- If the gradient is negative, the graph slopes downwards to the right
- If $b = 0$, the linear graph has the rule $y = mx$ and the graph passes through (0, 0)

Example 17

For this set of linear rules, state which graphs

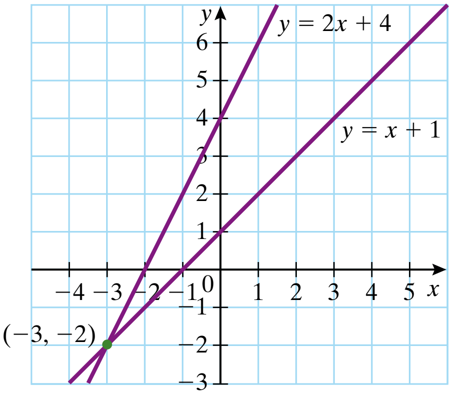
a will be parallel lines
b pass through the origin, that is, the point (0, 0)
c have their y-intercept at $y = 2$
d slope downwards to the right

$y = 3x$	$y = -2x + 3$	$y = 3x + 1$	$y = -x + 2$	$y = -x$
$y = 4x + 2$	$y = 7x$	$y = x + 2$	$y = -3x + 5$	$y = -4x$

Working	**Reasoning**
a The graphs of $y = 3x$ and $y = 3x + 1$ are parallel. The graphs of $y = -x + 2$ and $y = -x$ are parallel.	The slope of the graph is determined by m, the coefficient of the x term.
b The graphs of $y = 3x$, $y = -x$, $y = 7x$ and $y = -4x$ pass through (0, 0).	In the rule $y = mx + b$, b is the value of y where the graph crosses the y-axis. If $b = 0$, the graph passes through (0, 0).
c The graphs of $y = -x + 2$, $y = x + 2$ and $y = 4x + 2$ have their y-intercept at $y = 2$.	In the rule $y = mx + b$, b is the y-intercept, that is, the value of y when $x = 0$.
d The graphs of $y = -2x + 3$, $y = -x + 2$, $y = -x$, $y = -3x + 5$ and $y = -4x$ slope downwards to the right.	In the rule $y = mx + b$, if the gradient, m, is negative, the graph slopes downwards to the right.

Intersecting lines

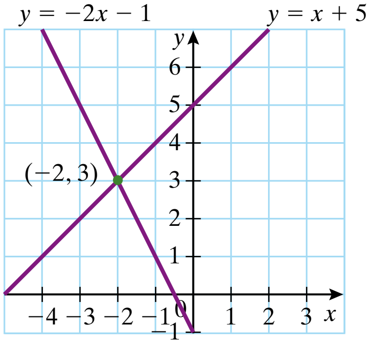
Linear graphs that are not parallel will intersect at a point. We can tell from the linear rules whether a pair of linear graphs will intersect. We have already seen that parallel lines have the same gradient, that is, the value of m is the same. If two linear rules have *different* values for m, then they will intersect. For example, we know that the graphs of $y = 2x + 4$ and $y = x + 1$ will intersect because they have different gradients. Graphing the two linear relationships on the same set of axes shows that the graphs intersect at the point $(-3, -2)$.



Example 18

Use computer graphing software or a graphing calculator to find the point where the graphs of $y = -2x - 1$ and $y = x + 5$ intersect.

Working



The graphs intersect at the point $(-2, 3)$.

Reasoning

Draw the graphs of $y = -2x - 1$ and $y = x + 5$ on the same set of axes. Find the coordinates of the point where the graphs intersect.

exercise 11.4

LINKS TO Example 17

1. For this set of linear rules, state which graphs
   a will be parallel lines
   b pass through the origin, that is, the point (0, 0)
   c have their y-intercept at $y = -3$
   d slope downwards to the right

$y = -2x$ $\quad$ $y = -4x + 1$ $\quad$ $y = -3x + 3$ $\quad$ $y = -x - 3$ $\quad$ $y = -x$

$y = 4x + 2$ $\quad$ $y = x$ $\quad$ $y = x - 3$ $\quad$ $y = -7x$ $\quad$ $y = -4x - 3$

BLM Cartesian plane

2. Consider these equations.

$y = 2x$
$y = 2x + 3$
$y = 2x - 4$

a For each of these equations construct a table of values with x values 0, 1 and 2.
b Plot all the graphs on the same Cartesian plane and label each line with its equation.
c Describe any patterns you notice in the graphs.

3. Consider these equations.

$y = -2x$
$y = -2x + 5$
$y = -2x - 3$

a For each of these equations construct a table of values with x values 0, 1 and 2.
b Plot all the graphs on the same Cartesian plane and label each line with its equation.
c Describe any patterns you notice in the graphs.

LINKS TO Example 18

4. Use computer graphing software or a graphing calculator to find the point where the graphs of the following linear relationships intersect.

a $y = 2x + 3$ and $y = x - 2$
b $y = x + 3$ and $y = 2x + 7$
c $y = -3x + 5$ and $y = x - 11$
d $y = -2x + 7$ and $y = x + 4$
e $y = -2x - 1$ and $y = -x + 2$
f $y = 5x - 3$ and $y = 3x + 7$

exercise 11.4 challenge

1. Consider these equations.

$y = 2x$

$$y = -\frac{1}{2}x$$

a For each of these equations construct a table of values with x values 0, 1 and 2.
b Plot the graphs on the same Cartesian plane and label each line with its equation.
c Describe any pattern you notice in the graphs.

11.5 Application of linear graphs

There are many real world situations where the relationship between two variables is linear. Tables of values, lists of ordered pairs, graphs and algebra rules help us investigate these relationships.

Dependent and independent variables

In real-world situations where there is a relationship between two variables, one variable may depend on the other. For example, as rainwater fills a tank, the amount of water in the tank depends on the amount of rain. In this example, we refer to the amount of rain as the **independent variable** and the amount of water in the tank as the **dependent variable**. We show the independent variable on the horizontal axis (x-axis) and the dependent variable on the vertical axis (y-axis).

Example 19

For each of the following, state which is the independent variable and which is the dependent variable.

a The height, h metres, up a mountain and the temperature T°C.

b An engineer took n minutes to repair a washing machine and charged \$$C$.

Working	**Reasoning**
a Height, h m, is the independent variable and temperature, T°C, is the dependent variable.	The temperature depends on the height up the mountain. The height does not depend on the temperature.
b The independent variable is the number of minutes, n. The dependent variable is the cost \$$C$.	The cost depends on the number of minutes taken to repair the washing machine.

Choosing suitable values

When we are working with real-world examples, we are sometimes interested only in certain values of x; for example, positive integer values of x, or values of x between 1 and 10. At other times we may be interested in all values of x, including zero, positive and negative numbers, and fractions as well as whole numbers.

We always need to think about which values of x are sensible to use. For example, if we are drawing a graph to show the number of computers sold each day, we would use only positive whole numbers.

Should the graph be a continuous line?

Drawing a line through a set of points implies that any ordered pair satisfies the linear relationship. However, this would not make sense if the independent variable could have only certain values. For example, if the independent variable is the number of computers sold, only positive integer values would be possible. In this case we would show the graph as a set of points, but it would not make sense to join them with a continuous line.

Should the points be joined?

When we draw a line through a set of points, this implies that that any points along the line, for example, $x = 1.4$, are included. This would not make sense if the x-values represented, for example, the number of computers sold. We do not draw a line through the points unless any point along the line is possible in the situation we are looking at.

Example 20

In which of these situations explain whether it be appropriate to join the points on the graph with a line.

a $\$C$ is the amount earned by a car salesperson who sells n cars in a week.

b V litres is the amount of water left in a tank after h hours as water is being pumped out.

Working	Reasoning
a Only whole numbers of cars can be sold so the points should not be joined.	We only join the points with a line if all the decimal values in between are possible in the situation. In this case, the in-between values are not possible.
b Water is continuously being pumped out so that h could be 1.4 or 3.75 for example.	We can join the points with a line if all the decimal points in between are possible in the situation. In this case, all the in-between values are possible.

Pronumerals stand for numbers, not objects

When we use linear functions in real-world situations, instead of using x and y as the pronumerals, we often use pronumerals related to the variables. For example, we might use h to stand for a *number* of hours, or c cents for the *number* of cents for a phone call. However, we must remember that the pronumeral is always standing in place of a *number*.

Example 21

Using a Go Chat phone card, the cost of phone calls to China is a fixed 20 cents plus 6 cents per minute. Calls are rounded up to the nearest whole number of minutes.

a Which is the independent variable and which is the dependent variable?

b Make a table to show the cost, c cents, of calls of length t minutes, where t is a whole number. Use whole number values of t from 1 to 6.

c Use the table to make a list of ordered pairs.

d Draw a graph of the data. Should the points be joined?

e Write the rule for c in terms of t.

f Describe the connection between c and t in words.

g Use the rule to find the cost of a call of length 23 minutes.

Working

a The number of minutes, t, is the independent variable. The cost, c cents, is the dependent variable.

Reasoning: The cost, c cents, depends on the number of minutes in the call.

b

t	1	2	3	4	5	6
c	26	32	38	44	50	56

Reasoning: The independent variable goes on the top row of the table.

c (1, 26), (2, 32), (3, 38), (4, 44), (5, 50), (6, 56)

Reasoning: The independent variable is the first number in the ordered pair.

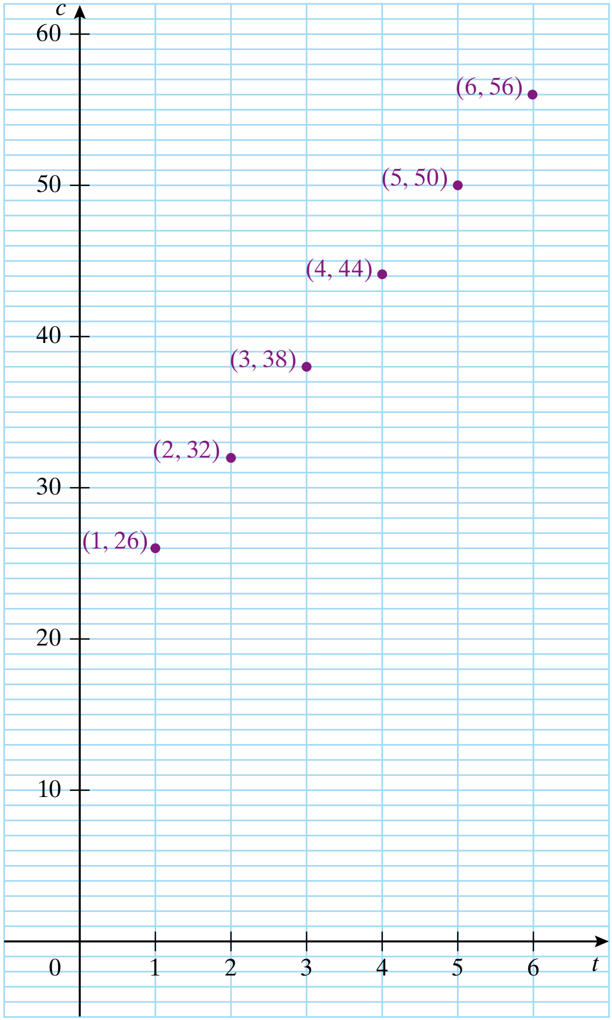
continued

Example 21 continued

Working

Reasoning

d



The points should not be joined because calls are charged for whole numbers of minutes.

continued

Example 21 continued

Working	Reasoning
e $c = 6t + 20$	The general rule for a linear relationship is $y = mx + b$ where m is the gradient and b is the y-intercept. In this example, t corresponds with x and c corresponds with y. Looking at the graph we can see that the ordered pair $(0, 20)$ satisfies the linear relationship. At the start of the phone call, when $t = 0$, the cost would be the fixed charge of 20 cents, that is, $c = 20$. So $m = 6$ and $b = 20$
f The cost in cents is 6 times the length of the number of minutes in the call plus 20 cents.	Each minute costs 20 cents, then the fixed charge of 20 cents is added.
g $c = 6t + 20$ $= 6 \times 23 + 20$ $= 138 + 20$ $= 158$	Substitute $t = 23$ in the rule.

The cost of the call would be 158 cents or \$1.58.

exercise 11.5

LINKS TO Example 19

For each of these situations state the
  i the independent variable.
  ii the dependent variable.

a the length, L cm, of a candle after burning for t hours

b the number of sheets of A4 paper in a bundle and the mass of the bundle of paper in grams

c the number of text messages sent and the total cost of sending the text messages

d the time taken to heat some muffins in a microwave and the number of muffins.

LINKS TO Examples 20, 21

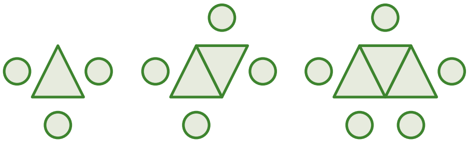
Kindergarten tables in the shape of equilateral triangles can be put together as shown. Suppose there are n tables and c children who can be seated.

a What is the independent variable?

b What is the dependent variable?

c Make a table of values to show the number of children who can be seated at 1, 2, 3, 4 and 5 tables.

d A graph of the ordered pairs is to be plotted. Is it appropriate to join the points with a line?



The rule connecting c and n in question 2 is

A $n = c + 2$ **B** $n = c + 1$ **C** $c = n + 2$ **D** $c = n + 1$ **E** $c = 3n$

BLM

5 mm graph paper

Alice found a magic cake cut into equal-sized pieces. Alice's height before she ate any cake was exactly 1 m. Each time she ate a piece of cake, her height increased by 1 metre.

a Complete the table to show Alice's height, h m, after eating n pieces of cake.

n	0	1	2	3	4	5	6
h							

b Write a list of ordered pairs.

c Which is the independent variable?

d Which is the dependent variable?

e Construct a graph to show the values of n and h.

f Do the points lie on a straight line?

g Write a rule for h in terms of n.

h Describe the rule in words.

i Use your rule to predict Alice's height after eating 13 pieces of cake.

Sarah is jogging at a constant speed of 140 metres per minute. Complete the table to show the distance, d m, that Sarah has jogged after t minutes.

a

t	0	1	2	3	4	5	6
d							

b What is the independent variable?

c What is the dependent variable?

d Use graph paper, a graphing calculator or spreadsheet to draw a graph of the data.

e What is the gradient? (Include the unit.)

f What does the gradient of the graph represent?

g What is the y-intercept (in this case the d-intercept)?

h Write a rule for d in terms of t.

The weekly earnings, \$$W$, of a computer salesperson working for Hi-tech Computers depends on how many computers, n, are sold. Each salesperson receives a basic amount of \$300 and a bonus of \$50 for each computer sold.

a Copy and complete the table.

n	0	1	2	3	4	5	6
W	300						

b What is the independent variable?

c What is the dependent variable?

d Use graph paper or a spreadsheet to draw a graph of the data. Is it appropriate to join the points? Explain.

e Calculate the gradient of the graph. What does the gradient represent?

f What is the y-intercept (in this case the W-intercept)? What does the W-intercept represent?

g Write a rule for W in terms of n.

h Use your rule to calculate the weekly earnings of a salesperson who sold 11 computers.

exercise 11.5 challenge

● Trentville City Council Planning Department is deciding the length of roadway for a taxi rank outside the railway station. It has been recommended that 5.5 metres should be allowed for each taxi, with an extra 1.5 metres at the end.

a Complete this table of values for the number of taxis, n, and the length of road, L m.

n	1	2	3	4	5	6
L	7					

b Use graph paper or a spreadsheet to construct a graph from your table of values.

c Why is it not appropriate to join the points?

d Write a rule for L in terms of n.

● Water is being pumped out of a 24 kilolitre tank so that the volume is decreasing by 2.4 kL/day.

a Copy and complete the table to show the volume, V kilolitres, remaining in the tank after n days.

n	0	1	2	3	4	5	6
V	24						

b What is the independent variable?

c What is the dependent variable?

d Plot a graph of the n and V values.

e What is the gradient ?

f Is the gradient positive or negative? Explain what this means.

g What is the V intercept? What does the V intercept represent?

h Write a rule for V in terms of n.

i Use your rule to predict the number of kilolitres of water remaining in the tank after 9 days.

j How many days will it take for the tank to empty?

Analysis task

Powered by the sun

Electricity is measured in a unit called a kilowatt-hour which is abbreviated to kWh. At a particular location the following data was collected to show the average daily generation of electricity by systems of solar panels. A house could typically have about 12 or 16 panels. Schools and other public buildings could have 60 or more panels.

Number of solar panels	Average electricity produced per day
P	***E* (kWh)**
12	8.3
16	11.1
20	13.8
24	16.6
30	20.8
36	24.9
42	29.1
48	33.2
54	37.4
60	41.5

a Which is the independent variable and which is the dependent variable?

b Use graph paper or a spreadsheet to graph the data.

c If 13.8 kWh of electricity is produced by 20 panels, how much would be produced by 1 panel? Round to one decimal place.

d What is the E-intercept? Is this what you expected? Explain.

e Write a rule for E in terms of P.

f Use your rule to calculate the average electricity produced per day by 200 panels.

g Try to find out your average daily electricity usage at home. (Your electricity bill should provide this information.)

h Based on the data in the table, how many solar panels would you need to supply this average daily electricity?

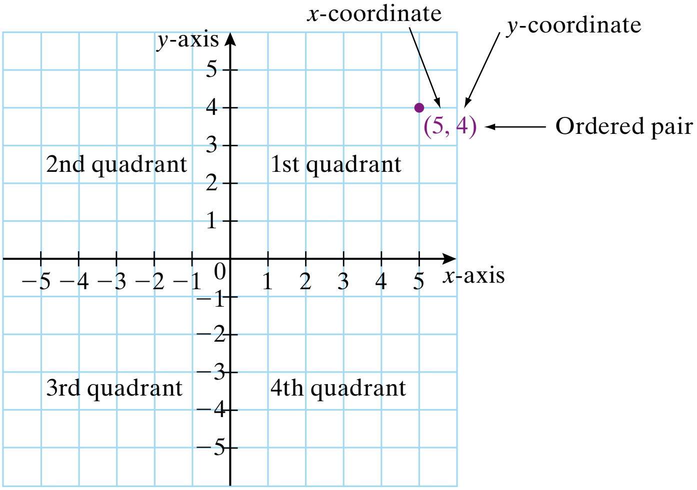
i What assumption are you making here?

Review Linear graphs

Summary

Cartesian coordinates

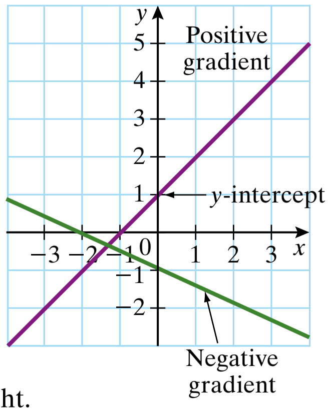
Points on the Cartesian plane have an *x*-coordinate and a *y*-coordinate. The coordinates give the horizontal and vertical distance from the origin (0, 0). For example, the point with coordinates (5, 4) is 5 units to the right and 4 units up.



Ordered pair

An ordered pair is an *x*-coordinate and a *y*-coordinate that together specify the position of a point on the coordinate plane; for example, (5, 4).

Linear relationships



- A relationship is linear if the *y*-values go up or down in equal steps when the *x* values go up in equal steps.
- The ordered pairs of a linear relationship lie in a straight line, that is, the graph is linear.
- A linear relationship has the rule $y = mx + b$.
- m is the gradient, that is, the change in the y values as the x values increase in steps of 1.
- b is the *y*-intercept, that is, the value of y when $x = 0$.
- Linear graphs with the same gradient are parallel lines.
- If the gradient is positive, the graph slopes upwards to the right.
- If the gradient is negative, the graph slopes downwards to the right.
- If $b = 0$, the linear graph has the rule $y = mx$ and the graph passes through (0, 0).

Visual map

Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key issues covered in this chapter.

Cartesian coordinates
Cartesian plane
dependent variable
gradient
graph
independent variable
intercept
intersection
linear relationship
ordered pair
origin
rule
table of values
x-axis
x-coordinate
y-axis
y-coordinate
y-intercept

Revision

Multiple-choice questions

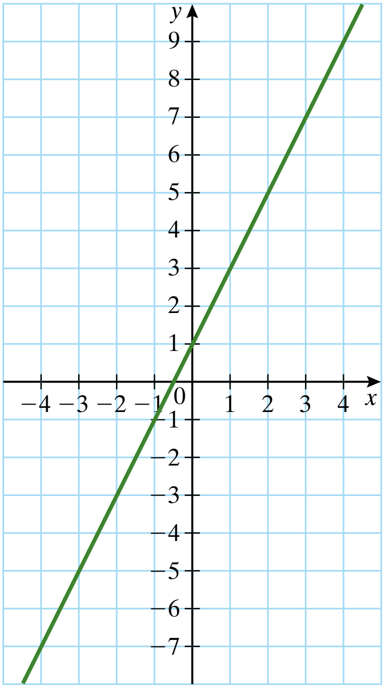
- Which one of these points is not on one of the axes?
  A (4, 0) **B** (0, 5) **C** (1, 2) **D** (8, 0) **E** (0, 1)

- Which of these lists of ordered pairs represents a linear relationship?
  A (0, 1), (1, 2), (2, 4), (3, 6), (4, 8)
  B (1, −1), (2, 2), (3, 5), (4, 8), (5, 11)
  C (−1, 1), (0, 0), (1, −1), (2, 0), (3, 1)
  D (−3, 1), (−2, 1), (−1, 0), (1, −2), (2, −3)
  E (−3, 1), (−2, 3), (−1, 6), (1, 10), (2, 15)

- The rule for this linear relationship is
  A $y = x + 2$
  B $y = 2x + 3$
  C $y = x + 3$
  D $y = 3x + 2$
  E $y = -x$

x	−1	0	1	2	3
y	−1	2	5	8	11

- The rule for the relationship represented by this graph is
  A $y = x$
  B $y = x + 1$
  C $y = 2x + 1$
  D $y = 3x - 2$
  E $y = 3x + 1$



The graph of a linear relationship has gradient 3 and y-intercept -4. The rule for the function is

A $y = 3x + 4$
B $y = 4x - 3$
C $y = -4x + 3$
D $y = 3x - 4$
E $y = -3x - 4$

Short-answer questions

Write these linear patterns as a rule.

a y is always three more than negative four times x.
b y is always four less than negative x.

Write the rule for each of the following linear relationships.

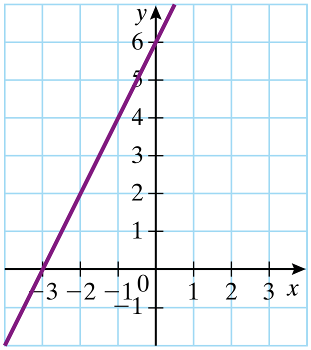
a

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1

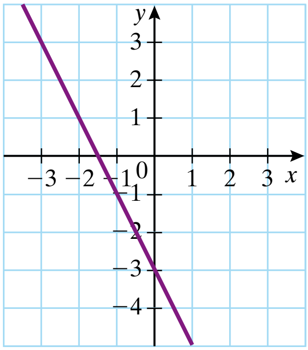
b $(-2, 13), (-1, 9), (0, 5), (1, 1), (2, -3)$

Write the rule for each of the following linear graphs.

a



b



a Complete this table of values for the rule $y = 2x + 5$

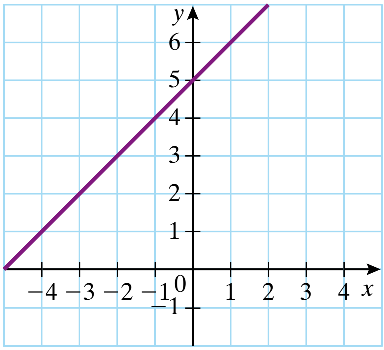
x	-3	-2	-1	0	1	2	3
y							

b Plot the graph.
c What is the gradient?
d What is the y-intercept?

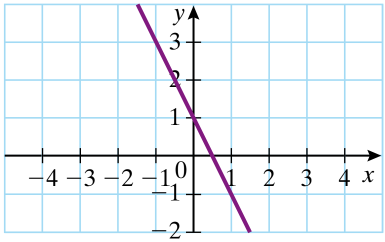
For each of these linear graphs,

i find the values of m and b. **ii** state the rule.

a



b



For this set of linear rules, state which graphs

a will be parallel lines.

b pass through the origin, that is, the point (0, 0).

c have their y-intercept at $y = -2$.

d slope upwards to the right.

$y = -x$ $\quad y = -2x - 1$ $\quad y = -3x - 2$ $\quad y = -x - 2$ $\quad y = x + 2$

$y = 4x + 2$ $\quad y = x$ $\quad y = x - 3$ $\quad y = -7x$ $\quad y = -4x - 3$

Solve the following equations by first drawing a graph of $y = -2x + 7$.

a $-2x + 7 = -3$ **b** $-2x + 7 = 5$

Use computer graphing software or a graphing calculator to draw the following graphs on the same set of axes.

$y = -2x$ $\quad y = -2x + 1$ $\quad y = -2x + 3$ $\quad y = -2x - 3$

a What is similar about the graphs?

b What is different about them?

Use computer graphing software or a graphing calculator to draw the following graphs on the same set of axes.

$y = x + 4$ $\quad y = -2x + 4$ $\quad y = 3x + 4$ $\quad y = -x + 4$

a What is similar about the graphs?

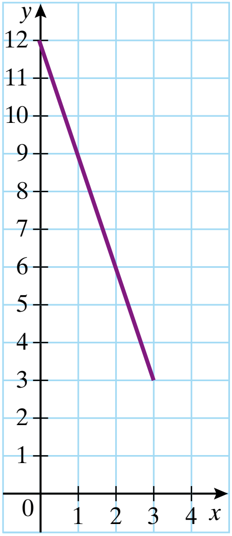
b What is different about them?

Use computer graphing software or a graphing calculator to find the point where these graphs intersect.

$y = -x - 1$ and $y = 2x + 5$

Extended-response questions

● The following graph shows the length, y cm,of a burning candle x hours after it was lit.



a What is the independent variable? What is the independent variable?

b What was the length of the candle when it was lit?

c What is the value of the gradient, m?

d What is the value of the y-intercept, b?

e Write a rule for y in terms of x.

f Use your rule to calculate the length of the candle 3.5 hours after it was lit.

g Use the rule to calculate the length of the candle 4 hours after it was lit.

Practice quiz
Chapter 11

Probability 12

Pre-test

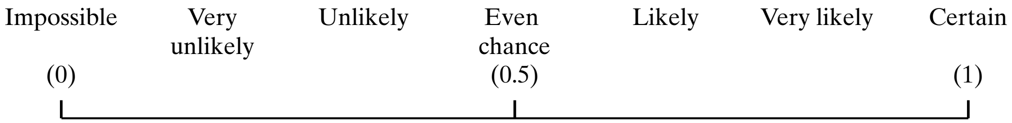
Warm-up

If you were to close your eyes and select a lolly from this jar, what colour would you select? Would your chances of selecting a red lolly be the same as your chances of selecting a purple one? How could you calculate the theoretical probability of selecting a particular colour? Would these theoretical probabilities allow you to know the colour of your next lolly? Probability and chance affect our lives in many ways. What is the chance that it will rain today? What is the chance there will be someone I know at the party? What is the chance that it will be hot if I plan to go to the beach?

12.1 Review of probability

Probability is measure of how likely something is to happen.

A scale from 0 to 1 can be used to represent probabilities with 0 representing impossible and 1 representing certain. The closer a probability of an event is to zero, the less likely the event, and the closer a probability is to 1, the more likely the event.



Example 1

For each of the following events, assign a ‘chance’ word to indicate the likelihood of the event using the words *impossible, unlikely, possible, likely* or *certain*.

a An eight is thrown on a standard six-sided die.
b Ten cards are drawn from a shuffled standard pack of playing cards and they are all hearts.
c It will be raining somewhere in the world today.
d In a family of two children, there will be one girl and one boy.
e The year 2020 will be a leap year.

Working	Reasoning
a impossible	Only the numbers 1 to 6 are possible.
b unlikely	It can happen, but it would not happen very often.
c likely	With different seasons and climates in different parts of the world, it will probably be raining somewhere.
d possible	Boys and girls are born in roughly equal numbers, so in a family of two children, there could be one girl and one boy.
e certain	If the year is divisible by 4 it is a leap year, except for years that are divisible by 100 (but years that are divisible by 400 are leap years).

Example 2

A weather report gave the chance of rain as 30%.

a Mark this chance on a probability scale.
b Describe in words the chance of rain.

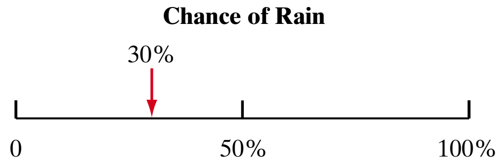
continued

Example 2 continued

Working

Reasoning

a



b There is approximately a 1 in 3 chance of it raining and 2 in 3 chance of it not raining, so it is more likely that it will not rain.

Outcomes, sample space and events

When we consider probabilities, we refer to each thing that can happen as an **outcome**. The set of all possible outcomes is called the **sample space,** S. For example, if we roll a standard die, the sample space, S, is the set of numbers $\{1, 2, 3, 4, 5, 6\}$. Each of the six possible numbers is an outcome.

We use the word **event** to describe the particular outcome or set of outcomes that we are interested in. If we roll a die, there are many different events we may be interested in, such as

- getting a 6
- getting an odd number
- getting a number greater than 2.

Notice that for the event ‘getting a 6’ there is only one possible outcome. In this case the words outcome and event have the same meaning. For the event ‘getting an odd number’ there are three possible outcomes: 1, 3 and 5.

Equally likely events have the same chance of occurring. For example, the chances of a coin landing head up or landing tail up are equally likely.

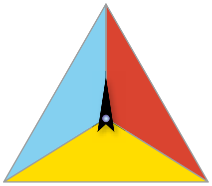
Example 3

For each of these spinners

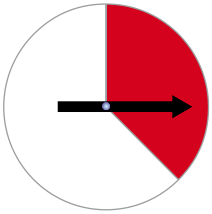
i list the sample space

ii state whether or not the events in the sample space are equally likely.

a



b



continued

Example 3 continued

Working	**Reasoning**
a i S = {red, yellow, blue}	If you spin this spinner there are three possible outcomes: spinning yellow, spinning red or spinning blue.
ii The events 'spinning yellow', 'spinning red' and 'spinning blue' are equally likely.	The chance of spinning yellow with this spinner is the same as the chance of spinning red or spinning blue as each colour has the same area.
b i S ={white, red}	If you spin this spinner there are two possible outcomes: spinning white or spinning red.
ii The events 'spinning white' and 'spinning red' are not equally likely.	With this spinner you are more likely to spin white than red because there is more white area than red area.

Theoretical probability

The probability that an event occurs is given by

$$\Pr(\text{Event}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Probabilities can be given as fractions, decimals or percentages. If probabilities are given as fractions, the fractions should be simplified if possible. For example, the probability of drawing a black card from a pack of 52 playing cards is $\frac{26}{52} = \frac{1}{2}$, or 0.5 or 50%.

Example 4

One card is drawn at random from a standard deck of 52 playing cards. Determine the probability of

a drawing an ace.

b drawing a heart.

c drawing a picture card.

Clubs	Spades	Diamonds	Hearts
♣K	♠K	♦K	♥K
♣Q	♠Q	♦Q	♥Q
♣J	♠J	♦J	♥J
♣10	♠10	♦10	♥10
♣9	♠9	♦9	♥9
♣8	♠8	♦8	♥8
♣7	♠7	♦7	♥7
♣6	♠6	♦6	♥6
♣5	♠5	♦5	♥5
♣4	♠4	♦4	♥4
♣3	♠3	♦3	♥3
♣2	♠2	♦2	♥2
♣A	♠A	♦A	♥A

Working	Reasoning
a $\Pr(\text{ace}) = \frac{4}{52} = \frac{1}{13}$	A deck has 52 playing cards. 4 cards in the deck are aces. Simplify the fraction.
b $\Pr(\text{heart}) = \frac{13}{52} = \frac{1}{4}$	A deck has 52 playing cards. 13 cards in the deck are hearts. Simplify the fraction.
c $\Pr(\text{picture card}) = \frac{12}{52} = \frac{3}{13}$	A deck has 52 playing cards. There are 12 picture cards: king, queen and jack in each of the four suits.

Sometimes we are interested in the probability of one event *or* another. We then need to look at all the outcomes and list those that are favourable to one event or to the other. For example, suppose we roll a die and we are interested in the probability of 'a prime number *or* a 6'. The five favourable outcomes out of the total of six possible outcomes are 2, 3, 4, 5, 6.

$$\Pr(\text{prime number or } 6) = \frac{4}{6} = \frac{2}{3}$$

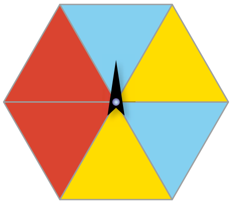
Example 5

Find the probability of the following events with this spinner.

a landing on red

b landing on blue

c landing on red *or* on blue.



Working

a $$\Pr(\text{red}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Reasoning

There are 6 possible sides the spinner could land on. Two of these are red.

b $$\Pr(\text{blue}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$

There are 6 possible sides the spinner could land on. Two of these are blue.

c $$\Pr(\text{red or blue}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{4}{6} = \frac{2}{3}$$

There are 6 possible sides the spinner could land on. Two of these are red and two are blue. There are 4 favourable outcomes for ‘red *or* blue’.

Example 6

A prize wheel has the numbers 1 to 36 arranged at random around the edge as shown.

a List the sample space for one spin of the wheel.

b With one spin of the wheel, find the probability that the winning number is

- **i** 17
- **ii** even.
- **iii** a prime number.
- **iv** a number greater than 20
- **v** a multiple of 4 or 6

continued

Example 6 continued

Working	Reasoning
a $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36\}$	The sample space is a list of all possible outcomes. In this case, with any one spin, the list of all possible outcomes is the numbers from 1 to 36 inclusive.
b i $\text{Pr}(17) = \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ $= \dfrac{1}{36}$	The number 17 appears only once on the wheel.
ii The even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36. $\text{Pr(even)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ $= \dfrac{18}{36}$	The even numbers are the favourable outcomes.
$= \dfrac{1}{2}$	Simplify the fraction.
iii The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31. $\text{Pr(prime)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ $= \dfrac{11}{36}$	The prime numbers are the favourable outcomes. Note: 1 is not prime.
iv The numbers greater than 20 are 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. Pr(number greater than 20) $= \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ $= \dfrac{16}{36}$ $= \dfrac{4}{9}$	The numbers greater than 20 are the favourable outcomes.
v The multiples of 4 or 6 are 4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36. Pr(multiple of 4 or 6) $= \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$ $= \dfrac{12}{36}$	In probability the statement ‘a multiple of 4 or 6’ means a multiple of 4 or 6 or both 4 and 6.
$= \dfrac{1}{3}$	Simplify the fraction.

Complementary events

If a die is rolled and the event you are interested in is 'rolling a 6' then the event 'not rolling a 6' is referred to as the complement of this event. The events 'rolling a 6' and 'not rolling a 6' are referred to as **complementary events**.

The probabilities of complementary events always add to 1 because it is certain that one or the other must occur.

Complementary events

The complement of an event A is the set of outcomes that are not A.

$$\Pr(\text{event } A) + \Pr(\text{complement of } A) = 1$$

Example 7

The probability that Fui is late to school on Tuesday is 0.4. What is the probability that Fui is not late to school on Tuesday?

Working

$$\begin{aligned}\Pr(\text{Fui not late}) &= 1 - 0.4 \\ &= 0.6\end{aligned}$$

Reasoning

The events 'Fui is late' and 'Fui is not late' are complementary and, therefore, their probabilities add to 1. It is certain that he is either late or not late.

Example 8

A spinner has unequal sections numbered 1 to 6. The probability that Oscar spins a 5 is $\frac{2}{7}$. What is the probability that Oscar does not spin a 5?

Working

$$\begin{aligned}\Pr(\text{Oscar does not spin 5}) &= 1 - \frac{2}{7} \\ &= \frac{5}{7}\end{aligned}$$

Reasoning

The events 'Oscar spins a 5' and 'Oscar does not spin a 5' are complementary and, therefore, their probabilities add to 1.
A common denominator is required for subtraction.

'Oscar does not spin a 5' is equivalent to saying 'Oscar spins 1, 2, 3, 4, or 6' so there are five possible outcomes for 'Oscar does not spin a 5'.

exercise 12.1

Simplify fractions whenever possible.

LINKS TO Example 2

Draw a probability scale from zero to one and mark where you think each of the following would lie.

- **a** A new car will break down in the next year.
- **b** Snow will be found on the sun.
- **c** You will be allowed to stay out after midnight on a school night.
- **d** It will rain in winter somewhere in Victoria.
- **e** In a family of three children, exactly one will be a girl.

LINKS TO Example 2

A weather report gave the chance of snow in alpine areas as 80%.

- **a** Mark this chance on a probability scale.
- **b** Describe in words the chance of snow.

Explain what is wrong with each of the following statements.

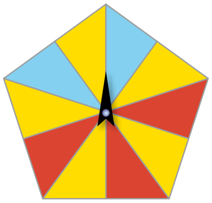
- **a** Isabella says that the probability she will get an A on her maths test is −0.5
- **b** Jordan says that the probability he will get into the school hockey team is 1.5

LINKS TO Example 3

For each of these spinners

- **i** list the sample space.
- **ii** state whether or not the outcomes in the sample space are equally likely.

a



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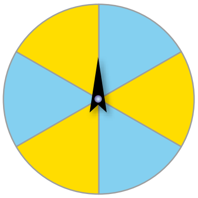
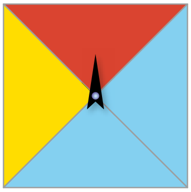


LINKS TO Example 3

Of the spinners below

- **i** which give equally likely events?
- **ii** determine the probability of the event ‘spinning yellow’.

a



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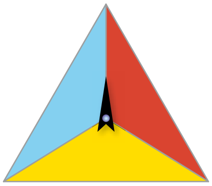

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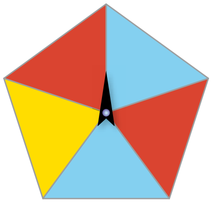

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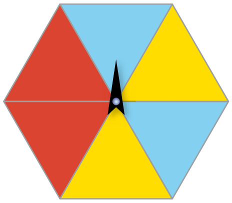

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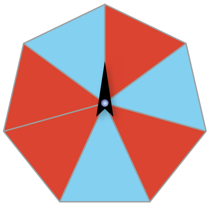
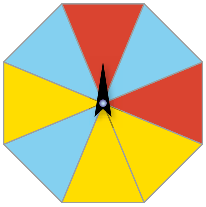


f



LINKS TO Example 3

List the sample space for each of the following events.

a rolling a single die

b spinning the spinner in question 4 part **a i**

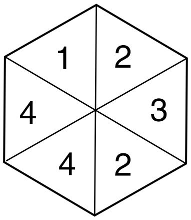
c tossing a coin.

LINKS TO Example 4

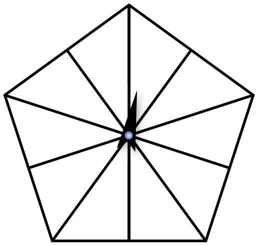
Consider the spinner on the right. In one spin, find the probability of spinning:

a a 4?

b an odd number?



Copy the spinner shown twice and colour it in two different ways so that Pr(yellow) = 0.4, Pr(green) = 0.2, Pr(red) = 0.1 and Pr(black) = 0.3.



LINKS TO Example 5

One card is selected at random from a deck of 52 playing cards. Find the probability that the card is

a a Jack.

b a club.

c a red queen.

d a black even number.

e an ace or a diamond.

f a number greater than 5 but less than 10.

g an even number or a multiple of 3.

h not a spade.

i neither a heart nor a king.

j an even number but not a diamond.

Clubs	Spades	Diamonds	Hearts
♣K	♠K	♦K	♥K
♣Q	♠Q	♦Q	♥Q
♣J	♠J	♦J	♥J
♣10	♠10	♦10	♥10
♣9	♠9	♦9	♥9
♣8	♠8	♦8	♥8
♣7	♠7	♦7	♥7
♣6	♠6	♦6	♥6
♣5	♠5	♦5	♥5
♣4	♠4	♦4	♥4
♣3	♠3	♦3	♥3
♣2	♠2	♦2	♥2
♣A	♠A	♦A	♥A

LINKS TO Example 6

Manu spins the wheel shown.

a List the sample space.

Find the probability that the number he spins is

b an even number.

c a multiple of 3 or an even number.

d a multiple of 5 and an odd number.

e a prime number or a purple number.

f not a green number.

g neither an even number nor a yellow number.

LINKS TO Example 7

The probability that it rains on the day of Madeleine's party is 0.3. What is the probability that it does not rain?

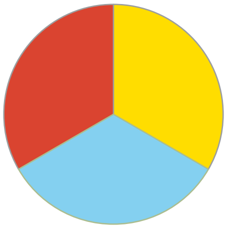
LINKS TO Example 8

Maggie selects a lolly from a lolly jar at random. The probability that the lolly she selects is a snake is $\frac{3}{8}$. What is the probability that the lolly she selects is not a snake?

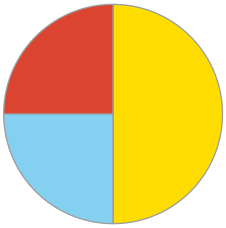
The night before his party, Henri listens to the weather forecast. The weather reporter says there is a 20% chance of rain. But Henri is optimistic and likes to look on the bright side. Suggest what he might conclude from this weather report.

For which spinner are the events 'spinning red' and 'spinning yellow' complementary?

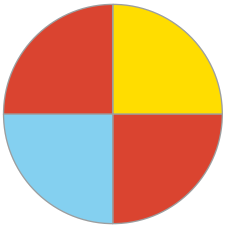
A



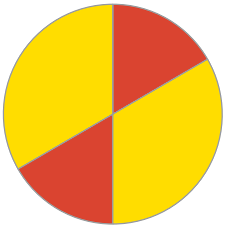
B



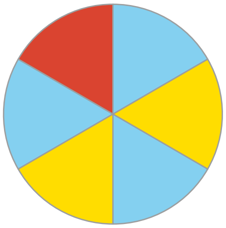
C



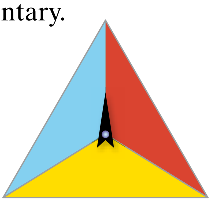
D



E



In each of the following, state whether or not the events are complementary.



- **a** tossing a head with one toss of a coin and tossing a tail with one toss of a coin
- **b** rolling an even number with one roll of a die and rolling an odd number with one roll of a die
- **c** spinning red with one spin of this spinner and spinning yellow with one spin of this spinner
- **d** drawing a club from a standard deck of playing cards and drawing a heart from a standard deck of playing cards
- **e** rolling a 5 or greater with one roll of a die and rolling a 4 or less with one roll of a die
- **f** rolling a prime number with one roll of a die and rolling an even number with one roll of a die.

The Budgerigars can either win, lose or draw their next netball match. The probability that they win is $\frac{1}{2}$ and the probability of a draw is $\frac{1}{10}$. What is the probability that they will lose?

Carmela selects one card at random from a standard deck of 52 playing cards. Determine the probability that the card selected is not an ace.

● One card is selected at random from a standard pack of 52 playing cards. Find each of the following probabilities.

a Pr(heart) **b** Pr(red or black)

c Pr(not a club) **d** Pr(picture card)

The picture cards are the Jack, Queen and King in each suit.

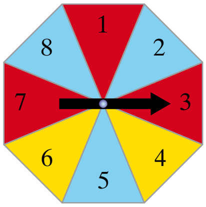
● For this spinner, find the following probabilities.

a Pr(blue)

b Pr(odd number)

c Pr(blue and odd)

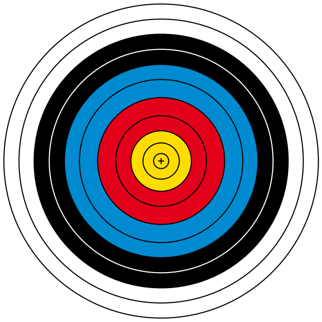
d Pr(blue or odd).



exercise 12.1 challenge

● Is it possible to create a spinner with only the colours blue, green, yellow and red with Pr(green) $= \frac{1}{3}$, Pr(blue) $= \frac{2}{5}$, Pr(red) $= \frac{1}{4}$ and Pr(yellow) $= \frac{1}{10}$? Explain.

● In an archery tournament, competitors fire arrows at the target shown. The diameters of each of the coloured regions and their scores are shown in the table below. The chance of hitting each coloured region depends on the area of the region. Jing fires one arrow and hits the target.



a For each of the five coloured regions

- **i** calculate the area (use your understanding from chapter 10 of calculating the area of a ring).
- **ii** calculate the area of each coloured region as a fraction of the total area of the target.

b Estimate the probability that Jing's score is

i 1 **ii** 2 **iii** 3

iv 4 **v** 5

Colour	Diameter (cm)	Score
Yellow	12	5
Red	24	4
Blue	36	3
Black	48	2
White	60	1

● A survey of about 800 Australian school students asked the students to consider the roll of a single die and to say whether there were any numbers that were harder or easier to get. Approximately one quarter (26%) of the students said that some numbers were harder to get, and a similar percentage (27%) said that some numbers were easier to get. Of the students who thought that some numbers were harder or easier to get, 55% said that a 6 was hardest to get and 49% thought that 1 and 2 were easiest to get.

a Were the students who said that a 6 is harder to get correct in their thinking?

b Explain why people might think a 6 is harder to get.

c Why might people think a 1 or a 2 are easier to get.

12.2 Estimating probabilities

Probability experiments

In section 12.1 we were considering **theoretical probability**. It is also possible to estimate probabilities using data from a **probability experiment**. Tossing a coin or rolling a die a number of times and recording the outcomes are examples of probability experiments. The data from a probability experiment can be used to calculate the **relative frequency** of an event. In a probability experiment such as tossing a coin, each repeated toss is called a **trial**. We normally collect data for a large number of trials. To calculate relative frequency, we express the frequency of the outcome as a fraction of the total number of trials.

Relative frequency (or experimental probability)

$$\text{Relative frequency of an outcome} = \frac{\text{frequency of the outcome}}{\text{number of trials}}$$

Example 9

A coin was tossed 40 times and 23 heads were obtained. Calculate the relative frequency of obtaining a head.

Working

Relative frequency of heads

$$= \frac{\text{number of heads}}{\text{total number of trials}}$$

$$= \frac{23}{40}$$

$$= 0.575$$

Reasoning

Express the frequency of heads as a fraction of the total number of trials.

The greater the number of trials, the closer we expect the relative frequency to come to the theoretical probability. For example, if we toss a coin 10 times we could obtain 5 heads and 5 tails, but we could obtain, for example, 8 heads and 2 tails. However, if we toss a coin 10 000 times, we would expect to obtain close to equal numbers of heads and tails. We can use a relative frequency as an estimate of probability.

Estimating probabilities from survey data

In many real-world situations it is not possible to calculate theoretical probabilities. We can use a survey to collect data then calculate relative frequencies. From the relative frequencies we can estimate probabilities. For example, supposing the colours of cars passing a busy intersection were recorded and 246 out of a total of 920 were found to be black.

$$\begin{aligned}\text{Relative frequency of black} &= \frac{\text{number of black cars}}{\text{total number of cars}}\\ &= \frac{246}{920}\\ &\approx 0.27\end{aligned}$$

We can then say that if we pick a car at random passing that intersection, there is a probability of 0.27 that it will be black.

Example 10

In a class of 26 students, 9 students played in the school band. What is the probability that a student selected at random plays in the school band?

Working	**Reasoning**
$\Pr(\text{band}) = \frac{9}{26}$	The relative frequency of students who play in the band is $\frac{9}{26}$. We can use the relative frequency as an estimate of probability.

exercise 12.2

LINKS TO Example 9

● Polly tossed a coin 25 times. She obtained 16 heads and 9 tails. Find the relative frequency (as a decimal) of

a heads **b** tails.

LINKS TO Example 9

● Guy rolled a die 80 times and obtained the following results.

Number on die	Frequency
1	10
2	16
3	11
4	17
5	12
6	14

Find the relative frequency as a decimal of getting

a a 6

b a number greater than 3

c either a 1 or a 6

d an even number.

▶ LINKS TO Example 10

A group of 200 secondary school students were surveyed about their usual method of transport to school. The results are shown.

One student is selected at random from the group. Find the probability that the student comes to school by

a bus.
b bicycle.
c on foot.
d either by train or by car.

Method of transport	Number of students
Car	24
Bus	84
On foot	28
Train	19
Bicycle	45
Total	200

Chloe writes each of the letters of the word MATHEMATICS on separate pieces of paper and places them in a bag. She selects one letter at random from the bag. Determine the probability that the letter is

a M
b C
c a consonant
d a vowel
e a letter in the second half of the alphabet
f not an M or an S.

The table below shows the data from a survey of 300 people about blood type.

a Use the results of the survey to determine the probability that a randomly selected person has blood type
  i O^+
  ii B^+
  iii AB^+ or AB^-.
b What is the probability that a randomly selected person has negative blood type?

Blood type	Number of people
O^+	111
A^+	105
B^+	27
AB^+	9
O^-	21
A^-	18
B^-	6
AB^-	3

exercise 12.2 challenge

Tran's money box contains two \$2 coins, five \$1 coins, 15 fifty cent coins, 20 twenty cent coins and 28 ten cent coins. Tran turns his money box upside down and shakes it until one coin falls out. Determine the probability that the coin is

a ten cents **b** gold **c** more than twenty cents in value?

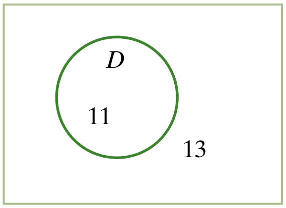
12.3 Venn diagrams and probability

Organising data with Venn diagrams

In section 12.2 we saw how probabilities can be estimated from experimental data and from data collected in a survey. We now look at how data can be organised using a **Venn diagram**. Venn diagrams are named after the English mathematician John Venn, who introduced this type of diagram in 1880.

In a Venn diagram, a rectangle represents the total number of data values. Inside the rectangle, data for an event is shown in a circle.

In the Venn diagram shown, the rectangle represents all the students in a particular class of 24 students. The circle represents 'students who have a dog'. The complement, 'students who do not have a dog', is shown in the rectangle outside the circle.



$n = 24$

Example 11

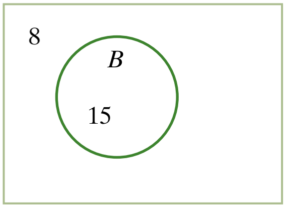
15 students in a class of 23 come to school by bus.

a Show the information on a Venn diagram.

b What is the complement of the set of students who come to school by bus?

c If a student from this class is selected at random, what is the probability that they come to school by bus?

d What is the probability of the complement of part **c**?

Working	Reasoning
a  $n = 23$	The rectangle represents the 23 students in the class. The set of 15 students who come to school by bus is represented by the circle. The 8 students who do not come to school by bus are shown outside the circle.
b The complement of the set of students who come to school by bus is the set of 8 students who do not come to school by bus.	$23 - 15 = 8$

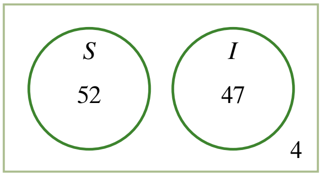
continued

Example 11 continued

Working	**Reading**
c $\Pr(\text{bus}) = \frac{15}{23}$	The probability that a randomly selected student comes to school by bus is equal to the relative frequency of students coming to school by bus.
d $\Pr(\text{not bus}) = \frac{8}{23}$	The probability that a randomly selected student does not come to school by bus is the complement of Pr(bus).

Mutually exclusive and non-mutually exclusive events

Mutually exclusive events are events that cannot occur at the same time. One event automatically excludes the other. Suppose a school offers students a choice of Spanish or Indonesian but not both. Because students cannot study both languages, being in the Spanish class and being in the Indonesian class are mutually exclusive. In the Venn diagram here we show this as two separate circles, where S represents the 52 students who choose Spanish and I represents the 47 students who choose Indonesian. There are 4 students who study neither language.



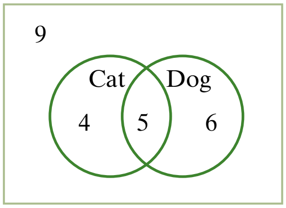
$n = 103$

In many situations, events are **not mutually exclusive**. In these cases, the circles in the Venn diagram overlap.

The Venn diagram below represents a class of 24 students. The students who have a dog as a pet and the students who have a cat as a pet are shown as overlapping circles. This overlapping region represents students who have a cat *and* a dog. The rest of the rectangle outside the circles represents the 9 students who have neither a dog nor a cat.

We can see from the Venn diagram

- 4 students have a cat only
- 6 have a dog only
- 5 students have both a cat and a dog
- 9 students have neither a cat nor a dog
- a total of 4 + 5 = 9 students have a cat
- a total of 5 + 6 = 11 students have a dog.



$n = 24$

The four numbers 9, 4, 5, and 6 must add to the total number of students in the class: 9 + 4 + 5 + 6 = 24.

Example 12

State whether the following are mutually exclusive or not mutually exclusive.

a selecting a student with brown hair and who wears glasses when selecting a single student from a school

b selecting a person born in July and in September when selecting a single person at a party.

Working	**Reasoning**
a Not mutually exclusive	A person can have brown hair and wear glasses.
b Mutually exclusive	If you are born in July you cannot be born in September too.

Venn diagrams with 'and', 'or' and 'not'

We often use the words 'and', 'or' and 'not' when we are making selections. These words are important when we are doing internet searches. Most internet search engines provide an advanced search option where you can list words related to the topic you are researching. For example, an advanced search option may provide the following options.

Find web pages with

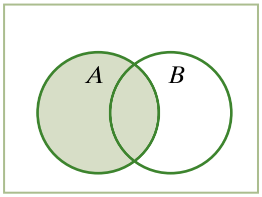
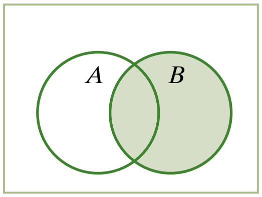
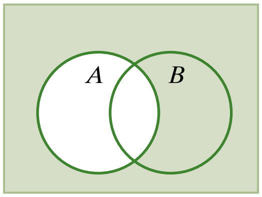
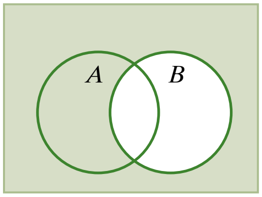
- all of these words
- any of these words
- none of these words.

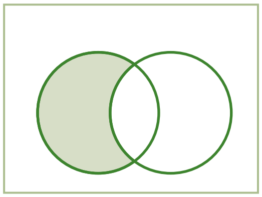
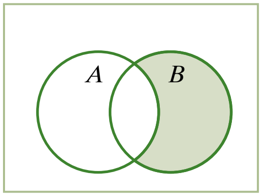
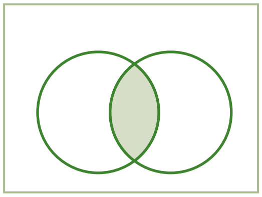
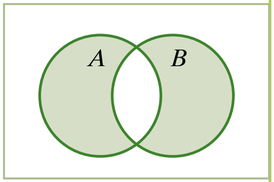
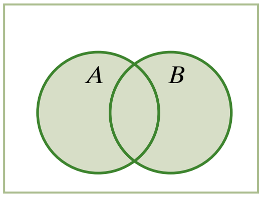
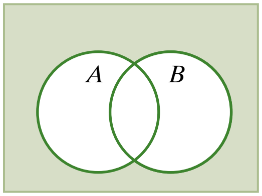
'All of these words' is equivalent to putting *and* between your search words.

'Any of these words' is equivalent to putting *or* between your search words.

'None of these words' means *not* to include any of these words.

In the Venn diagrams below, the circles represent data for two categorical variables, A and B. The shading in the diagrams shows how the different parts of the Venn diagram relate to the words 'and', 'or' and 'not'.

A		B	
Not A (that is, the complement of A)		Not B (that is, the complement of B)	

A but not *B*		*B* but not *A*	
A and *B*		*A* or *B* but not both	
A or *B* or both		Neither *A* nor *B* nor both	

Example 13

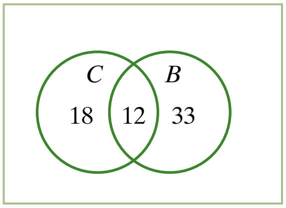
A chess club has 30 members and a bushwalking club has 45 members. There are 12 people who are in both clubs.

a Draw a Venn diagram to show this information. Use *C* to represent the set of chess club members and *B* to represent the set of bushwalking club members.

b How many people are in either the chess club or the bushwalking club or both.

Working

a



$n = 63$

b $18 + 12 + 33 = 63$

63 people are in either the chess club or the bushwalking club or both.

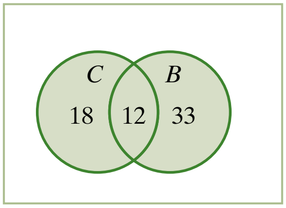
Reasoning

$30 - 12 = 18$

18 people are in the chess club but not both.

$45 - 12 = 33$

33 people are in the bushwalking club but not both.



$n = 63$

Example 14

Of a group of 30 students, 15 play tennis, 20 play cricket and 5 play neither sport.

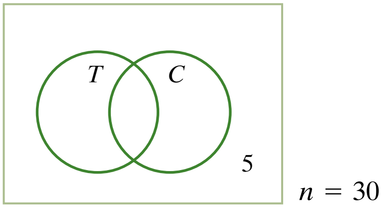
a Show the information on a Venn diagram. Use T to represent the set of tennis players and C to represent the set of cricket players.

b How many students play both sports?

c How many students play tennis or cricket but not both?

Working

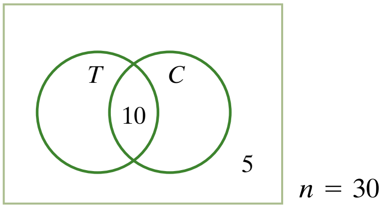
a



Reasoning

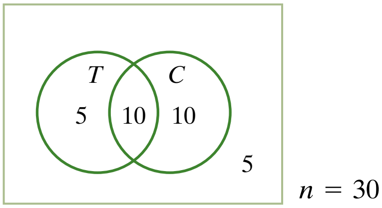
Draw a Venn diagram with two intersecting circles labelled T and C, representing the students who play tennis and cricket. It is helpful to record the total number of students.

Five students play neither sport so write 5 in the space outside both sets.



15 play tennis and 20 play cricket, and 5 play neither, making a total of 40 students.

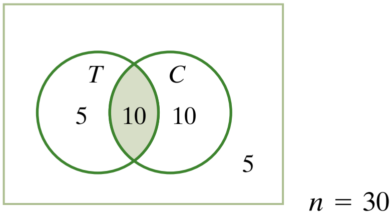
As there are only 30 students in the group, 10 students must play both sports. Write this in the intersection.



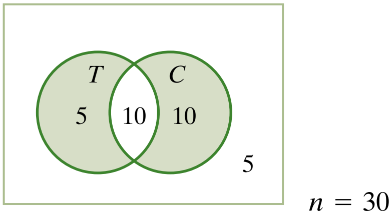
If 15 students play tennis but 10 of them play both sports, then 5 students must play tennis only.

If 20 students play cricket but 10 of them play both sports, then 10 students must play cricket only.

b 10 students play both sports.



c 15 students play tennis or cricket but not both.



Example 15

Using the information in the Venn diagram from Example 14, find the probability that a student selected at random from the group of 30 students

a plays tennis.

b plays tennis or cricket or both.

c plays tennis or cricket but not both.

d plays tennis and cricket.

e does not play tennis.

f plays tennis but not cricket.

g plays neither tennis not cricket.

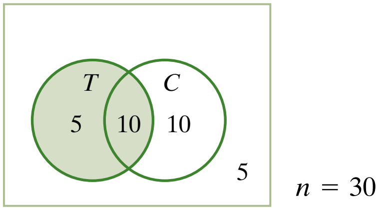
Working

a $\Pr(T) = \frac{15}{30}$

$= \frac{1}{2}$

Reasoning

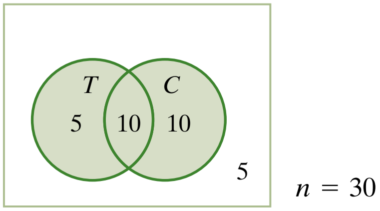
15 students play tennis.
The total number of students in the group is 30.



b $\Pr(T \text{ or } C \text{ or both}) = \frac{25}{30}$

$= \frac{5}{6}$

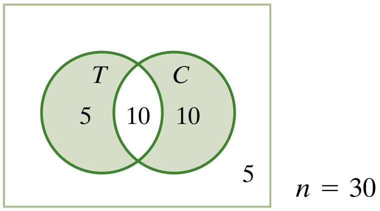
$5 + 10 + 10 = 25$ students play tennis or cricket or both.



c $\Pr(T \text{ or } C \text{ but not both}) = \frac{15}{30}$

$= \frac{1}{2}$

5 students play tennis only and 10 students play cricket only.



continued

Example 15 continued

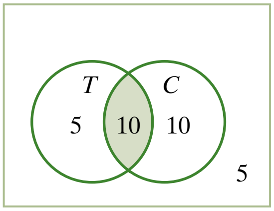
Working

Reasoning

d $\Pr(T \text{ and } C) = \frac{10}{30}$

$= \frac{1}{3}$

10 students play tennis *and* cricket.

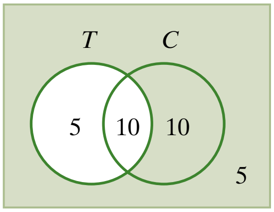


$n = 30$

e $\Pr(\text{not } T) = \frac{15}{30}$

$= \frac{1}{2}$

15 students play tennis. The complement of these 15 students is the 15 students who do not play tennis.

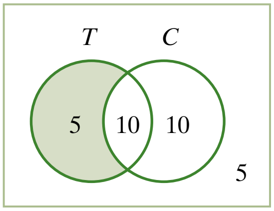


$n = 30$

f $\Pr(T \text{ but not } C) = \frac{5}{30}$

$= \frac{1}{6}$

5 of the 15 students who play tennis do not play cricket.

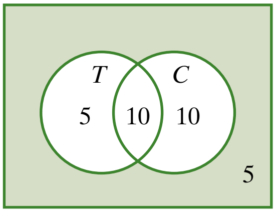


$n = 30$

g $\Pr(\text{neither } T \text{ nor } C) = \frac{5}{30}$

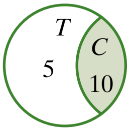
$= \frac{1}{6}$

5 students out of 30 play neither sport.



$n = 30$

Sometimes we are interested in the probability of an event *given that* another event occurs. For example, using the Venn diagram from Example 15, suppose we wish to know the probability that a randomly selected student plays cricket *given that we know they play tennis*. In this case, we are looking at part of the Venn diagram shown:

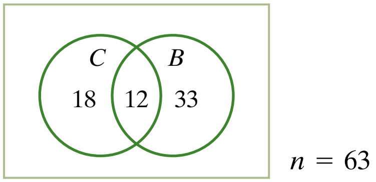


Of the 15 students who play tennis, 5 students play cricket.

$$\begin{aligned}\Pr(C \text{ given that we know } T) &= \frac{5}{15}\\ &= \frac{1}{3}\end{aligned}$$

Example 16

Using the Venn diagram from example 13, what is the probability that a randomly selected person belongs to the bushwalking club, given that we know they belong to the chess club?

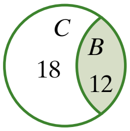


Working

A total of 30 people belong to the chess club and 12 of these belong to the bushwalking club.

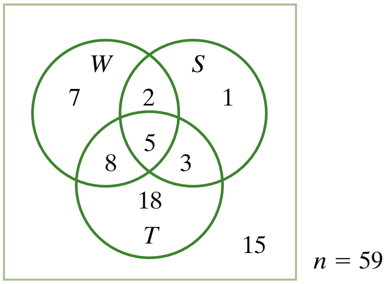
$$\begin{aligned}\Pr(W \text{ given that we know } C) &= \frac{12}{30}\\ &= \frac{2}{5}\end{aligned}$$

Reasoning



Example 17

Students at a camp could participate in water polo (*W*), trampolining (*T*) or skateboarding (*S*) or choose a different activity. They had the opportunity to do more than one of the sports if they wished. Their choices are represented in the Venn diagram shown. State the number of students who participated in each of the following.



a trampolining
b both water polo and skateboarding
c all three activities
d both water polo and trampolining but not skateboarding
e did not participate in skateboarding
f did not participate in water polo, trampolining or skateboarding
g trampolining but not water polo
h at least two of water polo, trampolining or skateboarding

continued

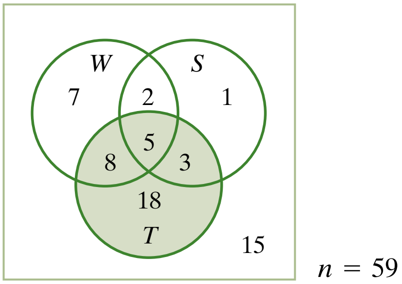
Example 17 continued

Working

Reasoning

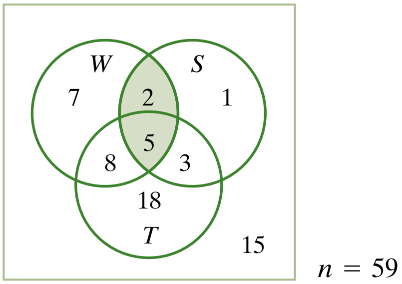
a $18 + 8 + 5 + 3 = 34$

34 students participated in trampolining.

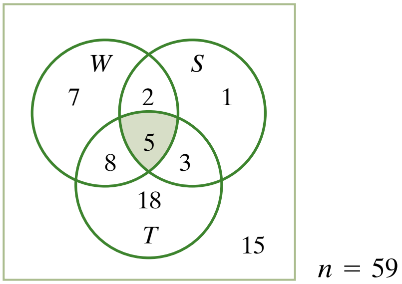


b $2 + 5 = 7$

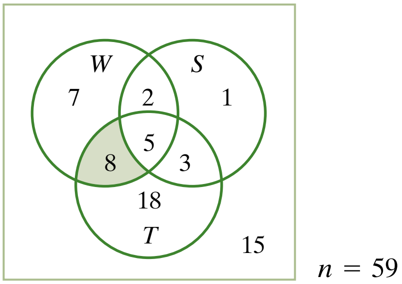
7 students participated in both water polo and skateboarding.



c 5 students participated in all three activities.

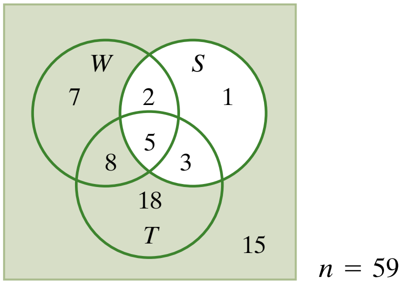
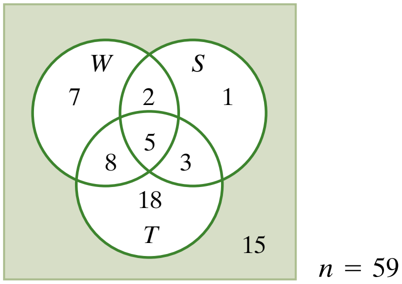
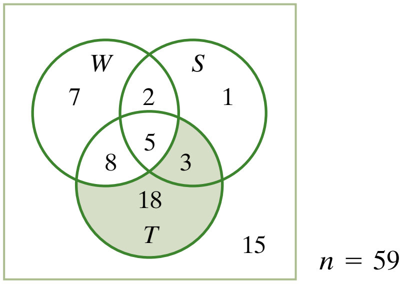
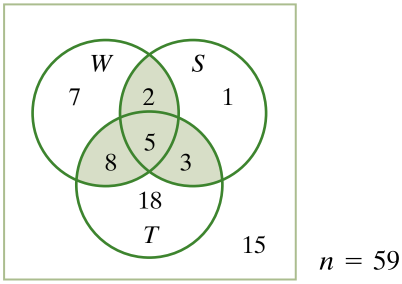


d 8 students participated in both water polo and trampolining but not skateboarding.



continued

Example 17 continued

Working	**Reasoning**
e $7 + 8 + 18 + 15 = 48$ 48 students did not participate in skateboarding.	
f 15 students did not participate in water polo, trampolining or skateboarding.	
g $18 + 3 = 21$ 21 students participated in trampolining but not water polo.	
h $2 + 8 + 3 + 5 = 18$ 18 students participated in at least two of the three activities.	

exercise 12.3

LINKS TO **Example 11**

● In a class of 22 students, 9 students played a musical instrument.

a Show this information on a Venn diagram.

b What is the complement of the set of students who play a musical instrument?

c What is the probability that a student selected at random from this class plays a musical instrument?

LINKS TO **Example 11**

● A survey of 115 Year 11 students at a school showed that 27 of them had part-time jobs.

a Show this information on a Venn diagram.

b What is the complement of the set of students who have part-time jobs?

c What is the probability that a Year 11 student selected at random from this school does not have a part-time job?

LINKS TO **Example 12**

● State whether the following are mutually exclusive or not mutually exclusive.

a rolling a prime number and rolling a composite number on a single roll of a standard six-sided die

b selecting a King and selecting a spade from a pack of cards

c selecting a student who rides a bike to school and selecting a student who has a brother or sister when selecting a single student from a school

d selecting a student who was born in Australia and selecting a student who was not born in Australia when selecting a single student from a school.

LINKS TO **Example 12**

● Hayden surveyed all the lights in his house. He found that 5 lights were switched on and 11 lights were switched off.

a Explain why this is an example of mutually exclusive events.

b Show the data on a Venn diagram.

LINKS TO **Example 12**

● Year 9 at Belleville High School can choose between camp and a city week experience but cannot choose both. Out of 147 students, 65 chose to go on camp and the rest chose the cityweek experience. Show the information on a Venn diagram.

LINKS TO **Example 13**

● Twelve friends meet for a movie night. Six brought corn chips, 5 brought sweets and 5 forgot to bring any snacks.

a Show the information on a Venn diagram.

b How many brought snacks?

c How many brought corn chips and sweets?

d How many brought sweets only?

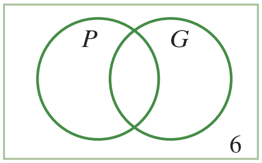
LINKS TO **Example 13**

● In a group of 15 students, 9 play the piano, 5 play guitar and 6 play neither instrument, as shown on the Venn diagram.

a How many play both?

b How many play the guitar only?

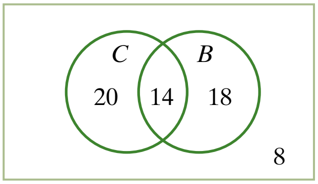
c Complete the Venn diagram.



$n = 15$

LINKS TO Example 13

In a survey, people were asked whether they owned a bike (B) or a car (C). The results are shown in the Venn diagram.



How many people

a owned only a bike?

b owned both a bike and a car?

c owned neither a bike nor a car?

d were surveyed?

LINKS TO Example 13

In a puppy class of 25 puppies, 16 have mastered the command to sit, 12 have learned to heel and 6 can do neither.

a Show this information on a Venn diagram.

b How many puppies can both sit and heel?

c How many can sit but not heel?

d How many can heel but not sit?

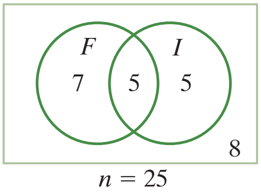
e How many can do one or the other but not both?

LINKS TO Example 13

In a survey of 120 television viewers, 82 viewers said they like to watch football, 38 like to watch motor racing and 24 like watching both. Draw a Venn diagram and find how many of the viewers like watching neither football nor motor racing.

LINKS TO Examples 14, 15

The Venn diagram shows the number of students who study French or Indonesian. Use the diagram to find the probability that a student selected at random



a studies French

b studies French and Indonesian

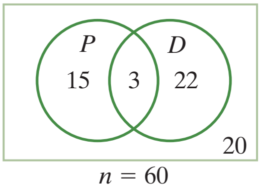
c studies French or Indonesian

d does not study Indonesian

e studies Indonesian but not French.

LINKS TO Examples 14, 15

The Venn diagram shows the number of people in a park who are walking a dog or power walking. Use the diagram to find the probability that a person selected at random is



a power walking

b power walking with a dog

c power walking without a dog

d walking a dog but not power walking.

LINKS TO Examples 14, 15

A survey of 120 students shows that 35 of them catch a bus to school. Five of these bus travellers are among the 47 students who catch a train for at least part of their journey. If one of the surveyed students is chosen at random, use a Venn diagram to help find the probability that this student

a catches a bus but not a train

b does not catch either a bus or a train.

LINKS TO Examples 14, 15

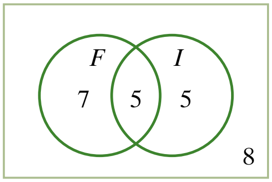
There are 150 Year 8 students at a school. Eighty-five of them are in a sports team, and 50 of the sports team members are in a music group. Altogether, there are 90 year 8 students in music groups. If one of the Year 8 students is chosen at random, use a Venn diagram to help find the probability that this student

a is in a sports team but not a music group.

b is in neither a sports team or a music group.

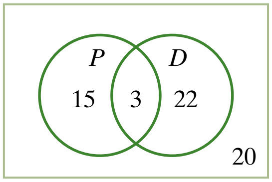
LINKS TO Example 16

The following Venn diagram shows the numbers of students in a class who study French and Indonesian. What is the probability that a randomly selected student studies Indonesian given that we know they study French?



LINKS TO Example 16

The following Venn diagram shows the numbers of people in a park who are power walking or walking a dog. What is the probability that a randomly selected person is power walking given that we know they are walking a dog?



LINKS TO Example 17

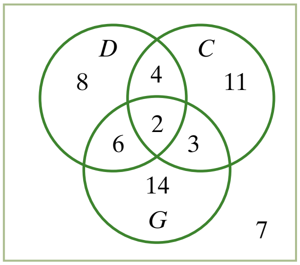
The Venn diagram at the right shows the results of a survey in which customers in a pet shop were asked if they had a dog (D), a cat (C) or a goldfish (G). How many people owned

a a dog? **b** a cat?

c only a goldfish? **d** both a cat and a goldfish?

e neither a dog nor a cat? **f** none of these pets?

g at least one pet? **h** at least two pets?



$n = 55$

LINKS TO Example 17

Of 75 customers who made purchases from a newspaper stand, 50 bought a newspaper, 53 bought sweets, 44 bought a magazine and 30 bought all three. There were 39 customers who purchased both a newspaper and sweets, 34 who bought both sweets and a magazine and 36 who bought both a newspaper and a magazine.

a Draw a Venn diagram to show the information.

b How many people purchased neither a newspaper nor a magazine nor sweets?

c How many people purchased at least one of the three items?

d How many people purchased at least two of the three items?

LINKS TO Example 17

In a survey, 55 students were asked whether they play football, cricket or hockey. The results showed that 37 students play football, 20 play cricket and 15 play hockey. The survey also showed that 13 play both football and cricket, 4 play both cricket and hockey and 7 play both football and hockey. Only 3 students play all three sports.

a Draw a Venn diagram to show the information.

b How many of the students surveyed did not play any of the three sports?

c How many students played at least one of the three sports?

d How many students played at least two of the three sports?

LINKS TO Example 17

Some students participate in various extra-curricular activities, as shown in the Venn diagram.

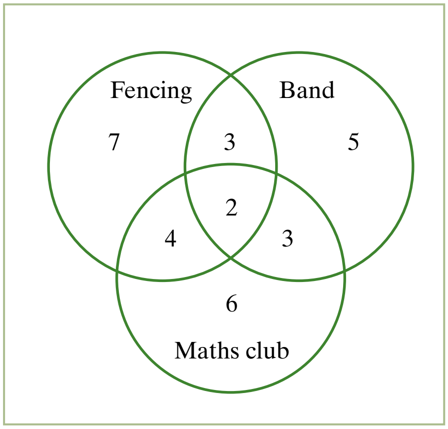
One of the students is chosen at random. What is the probability that the student participates in

a fencing only?

b fencing and band only?

c maths club but not fencing?

d all three activities?



$n = 30$

exercise 12.3 challenge

LINKS TO Example 17

A bushwalking club with 80 members and a camping club with 137 members decided to join together to make a wilderness club. They merged their membership lists and found that the new club had 181 members.

a Draw a Venn diagram to summarise this information.

b Find the probability that a member of the wilderness club chosen at random used to belong to

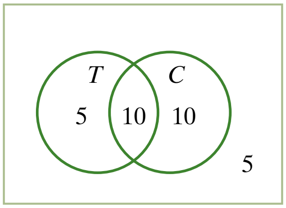
- **i** the bushwalking club
- **ii** both clubs
- **iii** the camping club but not the bushwalking club.

12.4 Two-way tables

We have seen how Venn diagrams are useful for representing data from surveys and probability experiments. Two-way tables provide another useful method for organising and displaying this data.

'Two-way' refers to the two directions, columns and rows, in which variables are displayed. The rows represent one variable, for example, *boy* or *girl*, and the columns represent another variable, for example, *play tennis* or *don't play tennis*.

In example 14 in section 12.3, a Venn diagram was used to display the information that in a group of 30 students, 15 play tennis, 20 play cricket and 5 play neither sport.



$n = 30$

This same information can be shown in a two-way table. The columns show how many students *play tennis* or *do not play tennis*. The rows show how many *play cricket* or *do not play cricket*.

	Play tennis	Do not play tennis	Total
Play cricket	10	10	20
Do not play cricket	5	5	10
Total	15	15	30

The two-way table can be used to calculate probabilities in the same way that we did with Venn diagrams.

Example 18

A group of 60 students from Years 7 and 8 were asked whether they prefer to study Mathematics or English. From a total of 34 Year 7 students, 20 said that they prefer to study Mathematics. In total, 28 students prefer to study English.

a Use a two-way table to represent the results of the survey.

b What is the probability that a randomly selected student from the group is in Year 8 and prefers Mathematics?

continued

Example 18 continued

Working

Reasoning

a

	Mathematics	English	Total
Year 7	20		34
Year 8			
Total		28	60

Construct a two-way table and enter the information provided.

	Mathematics	English	Total
Year 7	20	14	34
Year 8	12	14	26
Total	32	28	60

Work out each of the missing numbers.

The total number who prefer mathematics $= 60 - 28$
$= 32$

Total number in Year 8 $= 60 - 34$
$= 26$

Number in Year 8 who prefer mathematics $= 32 - 20$
$= 12$

Number in Year 8 who prefer English
$= 26 - 12$
$= 14$

Number in Year 7 who prefer English
$= 34 - 20$
$= 14$

b Pr(Year 8 and prefers English) $= \frac{12}{60}$

$= \frac{1}{5}$

	Mathematics	English	Total
Year 7	20	14	34
Year 8	12	14	26
Total	32	28	60

12 out of the 60 students are in Year 8 and prefer mathematics. This fraction can be simplified.

A two-way table can provide a useful way of recording the data for an experiment involving two events.

Example 19

Peter conducts an experiment where he tosses a coin and rolls a die. He does this 100 times, recording the results in a two-way table.

	Odd	Even	Total frequency
Heads	22	27	49
Tails	26	25	51
Total frequency	48	52	100

Use Peter's data to estimate the probability of

a tossing a head.

b rolling an odd number.

c tossing tails and rolling an odd number.

d tossing tails or rolling an even number.

Working

a $\text{Pr(head)} = \dfrac{49}{100}$

Reasoning

Use $\dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$.

There were 49 heads out of 100 trials.

	Odd	Even	Total frequency
Heads	22	27	49
Tails	26	25	51
Total frequency	48	52	100

b
$$\begin{aligned}\text{Pr(odd)} &= \frac{48}{100}\\ &= \frac{12}{25}\end{aligned}$$

There were 48 odds out of 100 trials.

	Odd	Even	Total frequency
Heads	22	27	49
Tails	26	25	51
Total frequency	48	52	100

c
$$\begin{aligned}\text{Pr(tails and odd)} &= \frac{26}{100}\\ &= \frac{13}{50}\end{aligned}$$

26 out of the 100 outcomes were both tails and odd.

	Odd	Even	Total frequency
Heads	22	27	49
Tails	26	25	51
Total frequency	48	52	100

continued

Example 19 continued

Working

d Pr(tails or even)

$$= \frac{26 + 25 + 27}{100}$$

$$= \frac{78}{100}$$

$$= \frac{39}{50}$$

Reasoning

The favourable outcomes are all those in the Tails row and Even column, that is,
26 (tails, odd) + 25 (even, tails) + 27 (even, heads).

Don't count the entry in the Even column and the Tails row twice.

	Odd	Even	Total frequency
Heads	22	27	49
Tails	26	25	51
Total frequency	48	52	100

Example 20

Ashleigh conducts an experiment where she tosses a coin and spins a spinner, 300 times.

She records the results of her experiment in a two-way table.

	Red	Yellow	Blue	Total frequency
Heads	55	48	50	153
Tails	50	47	50	147
Total frequency	105	95	100	300

Use Ashleigh's data to estimate the probability of

a tossing a head.

b spinning yellow.

c tossing a tail and spinning blue.

d tossing a head or spinning red.

continued

Example 20 continued

Working

a Relative frequency of a head $= \dfrac{153}{300}$

$= 0.51$

An estimate of Pr(head) = 0.51

Reasoning

Relative frequency gives an estimate of the theoretical probability.

	Red	Yellow	Green	Total
Heads	55	48	50	153
Tails	50	47	50	147
Total	105	95	100	300

b Relative frequency of spinning yellow

$= \dfrac{95}{300}$

$= 0.32$

An estimate of Pr(yellow) = 0.32

	Red	Yellow	Green	Total
Heads	55	48	50	153
Tails	50	47	50	147
Total	105	95	100	300

c Relative frequency of tail and green

$= \dfrac{50}{300}$

$= 0.17$

An estimate of Pr(tail and green) = 0.17

	Red	Yellow	Green	Total
Heads	55	48	50	153
Tails	50	47	50	147
Total	105	95	100	300

d Relative frequency of head or red

$= \dfrac{55 + 48 + 50 + 50}{300}$

$= \dfrac{203}{300}$

$= 0.68$

An estimate of Pr(head or red) = 0.68

Don't count the entry in the Red column and the Heads row twice.

	Red	Yellow	Green	Total
Heads	55	48	50	153
Tails	50	47	50	147
Total	105	95	100	300

exercise 12.4

LINKS TO Example 18

1 The two-way table below shows the people in a cinema at one session of a movie.

A spot prize is given to one randomly selected person in the cinema. What is the probability that the person who receives the prize is

	Male	Female	Total
Adult	22	28	50
Child	36	34	70
Total	58	62	120

a a child.
b a male.
c a female child.
d an adult male.
e not a female adult?
f not a male child.

LINKS TO Examples 19, 20

2 The results of a survey of Year 8 students about ear piercing are shown below in a two-way table.

a Fill in the missing boxes in the table.

	Boy	Girl	Total
Ear piercing	20	38	
No ear piercing		4	
Total	38		

b If one student from the group is selected at random, what is the probability that
- **i** they have an ear piercing?
- **ii** they are a girl?
- **iii** they are a boy with no ear piercing?
- **iv** they are a girl with an ear piercing?

LINKS TO Examples 19, 20

3 A survey of 1000 people revealed the following information.

	Blue eyes	Brown eyes
Dark hair	294	271
Light hair	239	196

One of these people is chosen at random. What is the probability that the person has

a light hair?
b blue eyes?
c light hair and blue eyes?
d light hair or blue eyes?

LINKS TO Examples 19, 20

4 A group of 50 students were asked whether they prefer individual sports or team sports. Of the 27 boys surveyed, 21 preferred team sports. In total, 16 students preferred individual sports.

a Use this information to complete the two-way table below.

	Boy	Girl	Total
Team sports			
Individual sports			
Total			

b If one student is selected at random from this group, what is the probability that the student
- **i** is a girl?
- **ii** prefers individual sports?
- **iii** is a girl who prefers team sports?
- **iv** is a boy who prefers individual sports?

In a survey, 200 people were asked if they approved of longer lessons. The results are shown in the table.

	Student	Teacher	Total
Approved	44	45	
Disapproved	86	25	
Total			200

a How many teachers were surveyed?

b How many people approved?

c One of the people surveyed was chosen at random.
What is the probability that this person
  i approved?
  ii was a student?
  iii was a student who disapproved?

d One of the students was chosen at random. What is the probability that he approved?

e One of the teachers was chosen at random. What is the probability that she approved?

One month, there were 90 babies born in a hospital. A total of 44 baby girls were born and of these, 31 weighed less than 4 kg. In total, 70 of the babies born weighed less than 4 kg.

a Use this information to complete the two-way table below.

	Less than 4 kg	More than 4 kg	Total
Boy			
Girl			
Total			

If one of the babies born in this month were selected at random, what is the probability that the baby

b is a boy?

c weighs more than 4 kg?

d is a boy weighing less than 4 kg?

A school offered a maths competition and a French competition, and 35 students entered. There were 25 entrants for the maths competition and 15 for the French competition. 17 students entered the maths competition but not the French competition. One of the 35 students is chosen at random. What is the probability that the student

a entered both competitions?

b entered the French competition but not the maths competition?

Sonja collects data about cars in a car yard. For each car she records whether the car has two doors (2) or four doors (4) and whether it is a sedan (S) or a station wagon (W). Her data is shown below.

2S 4S 4S 2W 4S 2S 2W 2S 4S 4W 2S 2S 4S 4W 4S
2W 4S 4S 2S 2S 4S 2S 4W 2S 4W 2S 2S 4W 2S 4S

a Organise this data using a two-way table.

b A vehicle inspector randomly selects one car from the yard to be checked. What is the probability that the car is

- **i** a sedan?
- **ii** a sedan with two doors?
- **iii** a station wagon with four doors?

exercise 12.4 challenge

A questionnaire is given to 260 students from Years 7, 8 and 9. Each student was asked for their year level at school and whether they travel to school by train, bus or using other means of transport. There were 102 Year 7 students of whom 37 travelled to school by train and 20 of whom travelled to school by bus. One hundred and fifty of the students questioned travelled to school using other means of transport. Fifty-three of these students were in Year 9. Sixty-two of the students questioned came to school by train and 12 of these were in Year 8. In total, 73 Year 8 students were questioned.

a Complete the two-way table below.

	Year 7	**Year 8**	**Year 9**	**Total**
Train				
Bus				
Other				
Total				

If one student is selected at random from the group, what is the probability that the student

b travels to school by train?

c is in Year 8?

d is in Year 7 and travels to school by bus?

e is in Year 9 and doesn’t travel to school by bus or train?

f is in Year 8 or travels to school by train?

Analysis task

What's the probability?

The data in the table below shows the responses of a sample of 40 Year 8 students to three CensusAtSchool questions: What is your favourite music? Does your shower at home have a water-saving showerhead? Does your family have a water tank?

Gender	Favourite music	Water-saving showerhead (S)	Water tank (T)
Female	Rock	Yes	Yes
Male	Rap/Hip Hop	No	No
Female	Other	No	No
Male	Punk	No	Yes
Female	Other	No	No
Male	Rap/Hip Hop	Yes	No
Female	Pop	Yes	No
Male	Rap/Hip Hop	No	No
Male	Techno/Electronic	No	Yes
Female	Pop	Yes	Yes
Female	Pop	Yes	No
Female	Rap/Hip Hop	Yes	No
Male	Pop	No	No
Female	Other	Yes	No
Male	Metal	No	No
Female	Pop	Yes	Yes
Female	Rap/Hip Hop	No	No
Female	Rhythm and Blues (R&B)	Yes	No
Female	Rhythm and Blues (R&B)	Yes	No
Female	Pop	Yes	Yes
Male	Rap/Hip Hop	No	Yes
Female	Rhythm and Blues (R&B)	Yes	Yes
Female	Rap/Hip Hop	No	No
Female	Pop	Yes	Yes

Gender	Favourite music	Water-saving showerhead (S)	Water tank (T)
Male	Rap/Hip Hop	Yes	No
Female	Pop	No	No
Male	Rap/Hip Hop	No	No
Male	Rap/Hip Hop	No	Yes
Female	Other	No	No
Male	Pop	No	No
Male	Rock	Yes	No
Female	Pop	No	No
Female	Pop	Yes	No
Male	Techno/Electronic	Yes	No
Female	Pop	Yes	No
Male	Techno/Electronic	Yes	Yes
Male	Rock	Yes	No
Male	Pop	No	Yes
Male	Metal	No	No
Female	Pop	No	Yes

a Sort the data in the first two columns (gender and favourite music) into a two-way table as shown below.

	Pop	Rap/Hip Hop	Other	Total
Female				
Male				
Total				

b A student is selected at random from the 40 students. Calculate the probability that

i the student is female.

ii the student's favourite music is Pop.

iii the student's favourite music is neither Pop nor Rap/Hip Hop.

iv the student is male and his favourite music is Rap/Hip Hop.

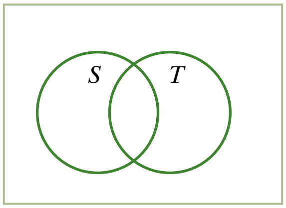
v the student is female and her favourite music is Rap/Hip Hop.

vi the student is female and her favourite music is not Rap/Hip Hop.

c Sort the data in columns 3 and 4 into a two-way table as shown below.

	Water saving showerhead (S)	**No water saving showerhead (S')**	**Total**
Water tank (T)			
No water tank (T')			
Total			

d Use your sorted data in the two-way table in part **c** to construct a Venn diagram.



e A student is selected at random from the 40 students. Calculate the following probabilities.

- **i** The student's family has a water-saving showerhead.
- **ii** The student's family has a water-saving showerhead and a water tank.
- **iii** The student's family has a water tank but does not have a water-saving showerhead.
- **iv** The student's family has neither a water tank nor a water-saving showerhead.
- **v** The student's family has at least one of a water tank and a water-saving showerhead.
- **vi** The student's family has either a water tank or a water-saving showerhead but not both.

Review Probability

Summary

- Probability is the mathematical study of chance.
  - Pr(Event) is always a number between 0 and 1 (inclusive).
  - If Pr(Event) = 0, this means that it is impossible.
  - If Pr(Event) = 1, this means that it is certain.
  - The closer the probability is to 1, the more likely it is that that event will occur.
- The complement of an event A is the set of outcomes that are not A.
- Pr(event A) + Pr(complement of A) = 1
- The sample space lists all possible outcomes for an event.
- If all outcomes are known, and equally likely, the theoretical probability of an event occurring can be calculated as the number:

$$\text{Pr(Event)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

- Venn diagrams and two-way tables are useful for displaying and organising data from probability experiments.
- Events can be mutually exclusive.
- Venn diagrams and two-way tables are useful for calculating probabilities associated with ‘and’, ‘or’ and ‘not’.
- Relative frequency (or experimental probability) can be calculated from data from probability experiments.

$$\text{Relative frequency of an outcome} = \frac{\text{frequency of the outcome}}{\text{number of trials}}$$

Visual map

certain
chance
complementary events
event
expected frequency
frequency
impossible
mutually exclusive
outcome
probability
probability experiment
random
relative frequency
sample space
theoretical probability
trial
two-way table
Venn diagram

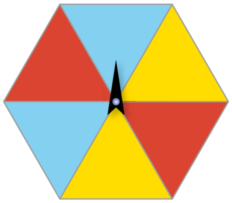
Revision

Multiple-choice questions

Which one of the following could not be a probability?

A 40% **B** 0.2 **C** $\frac{3}{8}$ **D** $0.\dot{3}$ **E** $\frac{5}{4}$

For the spinner shown, the theoretical probability of spinning blue is



A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{2}{3}$ **D** $\frac{1}{6}$ **E** $\frac{1}{4}$

With one roll of a fair die, the probability of getting a number greater than 2 showing uppermost is

A $\frac{1}{6}$ **B** $\frac{5}{6}$ **C** $\frac{2}{3}$ **D** $\frac{1}{3}$ **E** $\frac{1}{2}$

A bag contains twelve marbles. Six of the marbles are blue, four are yellow and two are red. Daniel selects one marble from the bag. The probability that the marble is not red is

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** $\frac{5}{6}$

A card is drawn at random from a pack of 52 playing cards. The probability that it is an ace or a club is

A $\frac{1}{4}$ **B** $\frac{1}{13}$ **C** $\frac{1}{52}$ **D** $\frac{4}{13}$ **E** $\frac{17}{52}$

Short-answer questions

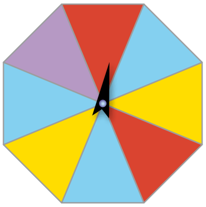
If the probability that it will rain tomorrow is 0.3, what is the probability that it will not rain?

A card is drawn at random from a pack of 52 playing cards. Determine the probability that it is

a black. **b** not a club. **c** a picture card.

d a queen or a king. **e** a picture card and a heart.

A spinner is shown at right.



a List the possible outcomes for this spinner.

b Are all the outcomes equally likely? Explain.

c Find the probability of spinning

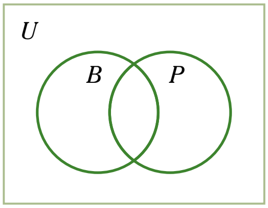
- **i** yellow.
- **ii** red.
- **iii** blue.
- **iv** not blue.

d In 300 spins, how many times would you expect to spin yellow?

State whether the following are mutually exclusive or not mutually exclusive.

- **a** rolling a 3 and rolling an even number with a single roll of a standard six-sided die
- **b** selecting a jack and selecting a heart when a single card is selected from a standard pack of cards
- **c** selecting a 5 and selecting a Queen when a single card is selected from a standard pack of cards
- **d** choosing a vowel and choosing a consonant when selecting a letter at random from the letters of the word PROBABILITY.

In a survey of 50 people, 28 people liked broccoli and 36 people liked peas. Six of the people surveyed liked neither vegetable.



- **a** Complete this Venn diagram to show this information.
- **b** Use the Venn diagram to determine the probability that a person selected at random from the group
  - **i** likes broccoli.
  - **ii** likes both broccoli and peas.

In a class of 32 students, 12 study Mathematics and History, 8 study History only and 3 study neither History nor Mathematics.

- **a** Show this information on a Venn diagram.
- **b** How many students study Mathematics?
- **c** How many students study Mathematics but not History?
- **d** How many students study at least one of Mathematics or History

Extended-response questions

A garage keeps records of the cars it repairs each year. Last year the garage repaired 1500 cars. The two-way table below shows the records of the cars repaired by the garage.

	Made before 2010	**Made in 2010 or after**	**Total**
Small vehicle	768	392	
Large vehicle	192	148	
Total			

- **a** Fill in the totals in this table.
- **b** The repair record for one car is chosen at random. What is the probability that it is for a car
  - **i** made before 2007?
  - **ii** in the large vehicle category?
  - **iii** made after 2007 and in the small vehicle category?

Forty students were questioned about the type of computer they owned. Of these students, 30 owned a desktop computer (D), 25 had a laptop (L) and 5 students had neither.

a Represent this information using a Venn diagram.

b Copy this table, show the information above, and then find the missing numbers.

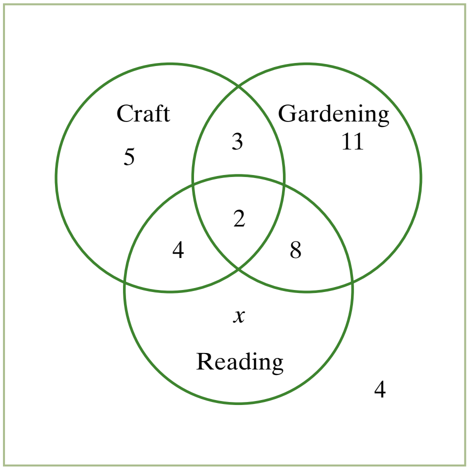
	D	D'	Total
L			
L'			
Total			40

c Find the probability that a randomly selected student has

- **i** a laptop only.
- **ii** both a laptop and a desktop computer.
- **iii** at least a laptop or a desktop computer.

d Given that we know a student has a desktop computer, what is the probability that they also have a laptop?

The members of a senior citizens' group were asked which of three activities they enjoyed. The results of the survey are shown in the Venn diagram below.



a How many people said they enjoy all three activities?

b How many members enjoy reading and craft but not gardening?

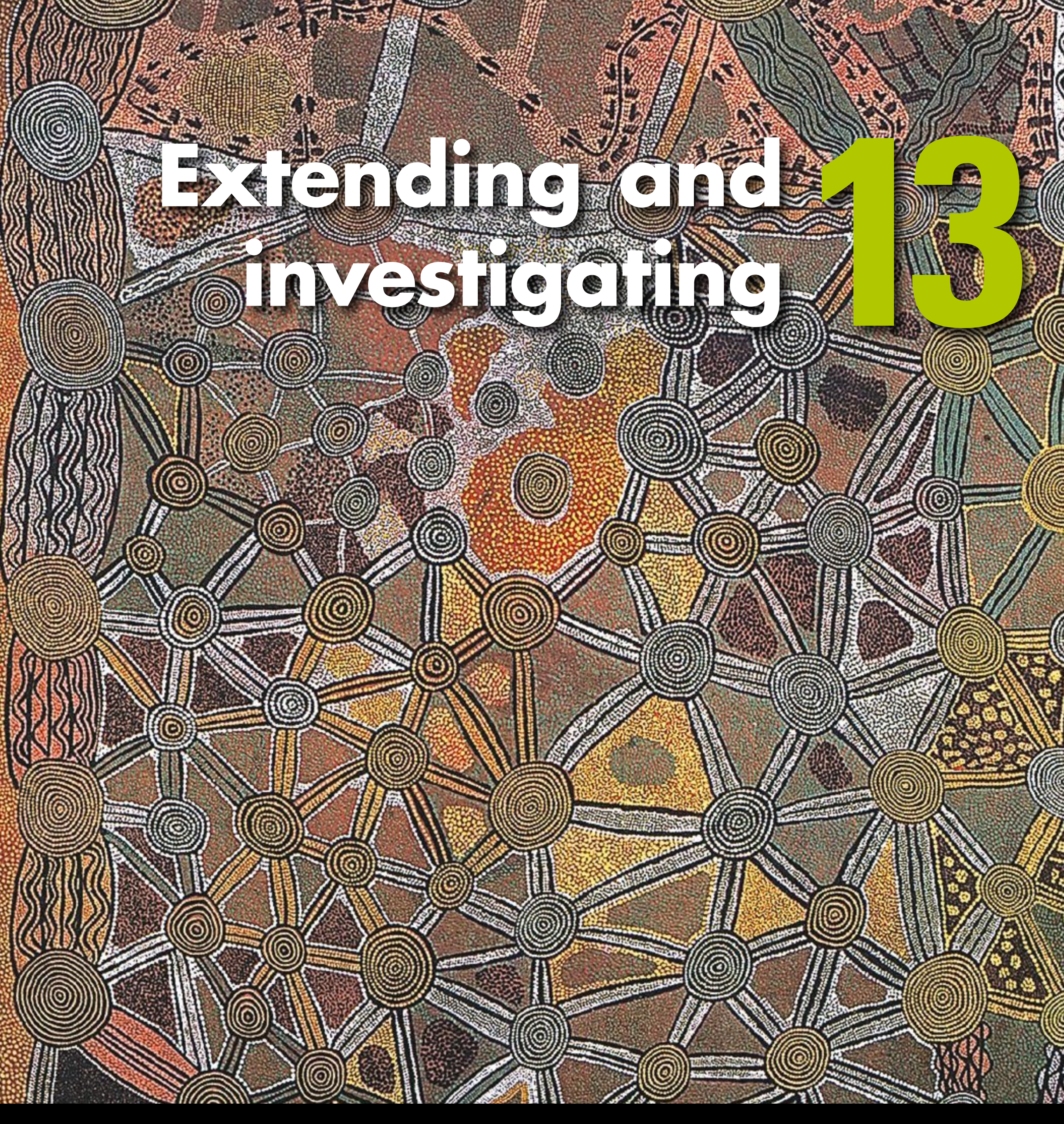
c Given that 50 people were surveyed, how many people like reading only?

d One member is chosen at random. What is the probability that they

- **i** enjoy craft?
- **ii** enjoy at least two of the activities?

e A garden visit is planned for people who enjoy gardening.

- **i** How many people are in this group?
- **ii** What is the probability that a person from this group also enjoys craft?



Extending and investigating 13

Some of the investigations in this chapter extend topics included in *MathsWorld 8* whilst others introduce new topics. One of these new topics, investigation 13.13, is an investigation of network graphs. A network is a pattern of connections, for example, between people or between places. The artwork shown here is called *Kangaroo and Shield People Dreaming* at Lake Mackay and was painted by Tim Payungka Tjapangati, an Indigenous artist from the Western Desert region of Central Australia. Aboriginal family groups are connected either by kinship (relationship) or by overlapping land areas, resulting in a complex network of relationships over time and space.

13.1 Odds and evens

You have probably noticed that when you add two even integers or two odd integers, the answer is always even. When you add an odd integer and an even integer, the answer is always odd. Although we have observed this and we have never found any counter-examples, algebra provides us with a way of proving that it is true for all cases.

a If we let m and n be any integers, explain how we known that $2m$ and $2n$ will always be even integers.

We can now show that if two even integers are added, the answer is always even.

$$2m + 2n = 2(m + n)$$

Because 2 is a factor of $2(m + n)$, we know that $2(m + n)$ must be an even integer.

b Explain how we know that $2m + 1$ and $2n + 1$ will always be odd integers.

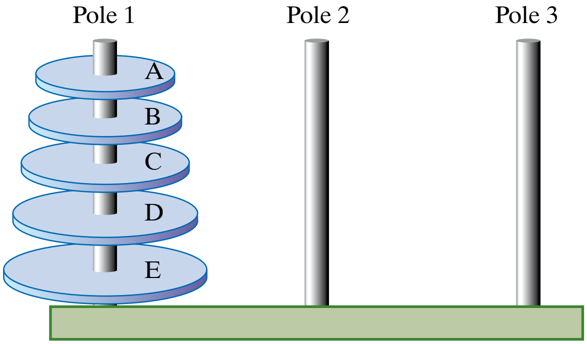
c Using a similar method as that used for even integers, show that the sum of two odd integers must be even.

d Using a similar method, show that the sum of an even integer and an odd integer must be odd.

13.2 Tower of Hanoi

This puzzle was invented by the French mathematician Edouard Lucas in 1883, and is often called the Tower of Hanoi puzzle. Discs of different sizes are stacked from smallest to largest on Pole 1. The discs are to be moved to Pole 3.

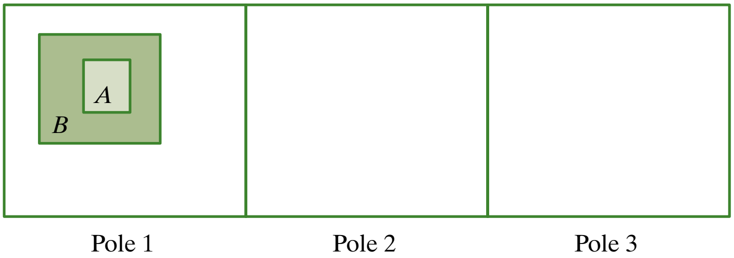
Only one disc can be moved at a time by placing it on a pole. A larger disc cannot be placed on a smaller disc. In the case shown, there are five discs.



In this investigation we are going to discover the smallest number of moves required to transfer the discs from Pole 1 to Pole 3.

- Cut five rectangular pieces of paper of different sizes to represent five discs, labelling the pieces A, B, C, D and E in order from smallest to largest as shown on the diagram below. Draw three blank adjoining squares in a row to represent the three poles.
- Start with just two discs A and B, and place them on Pole 1. Move the pieces according to the rules to find the smallest number of moves required to move them both to Pole 3. (You can also do this on a computer by going to the website www.dynamicdrive.com/dynamicindex12/towerhanoi.htm)

a How many moves are needed?



b Repeat using three discs A, B and C. How many moves are needed?

c The diagrams below record the number and type of moves needed when starting with two discs and three discs on Pole 1. Examine the diagrams and then make a similar diagram for the moves when starting with four discs on Pole 1.

Two discs

Move	Pole 1	Pole 2	Pole 3	Piece moved
	A, B			
1	B	A		A
2		A	B	B
3			A, B	A

Three discs

Move	Pole 1	Pole 2	Pole 3	Piece moved
	A, B, C			
1	B, C		A	A
2	C	B	A	B
3	C	A, B		A
4		A, B	C	C
5	A	B	C	A
6	A		B, C	B
7			A, B, C	A

d Based on patterns for the 'piece moved' in the diagrams from part **c**, predict the sequence of pieces moved and hence the number of moves when starting with five discs A, B, C, D and E on Pole 1.

e Make a copy of the table below. Use the given diagrams and your diagram from part **c** to find the numbers to complete the first four rows of the table. Then use observed patterns in the table to help complete the remaining rows.

Number of discs	Total number of moves	Number of moves for disc				
		A	B	C	D	E
1						
2						
3						
4						
5						

f Notice the pattern:

$$1 = 2 - 1 = 2^1 - 1$$
$$3 = 4 - 1 = 2^2 - 1$$
$$7 = 8 - 1 = 2^3 - 1$$

Use this pattern to predict how many moves would be needed for

i 6 discs.

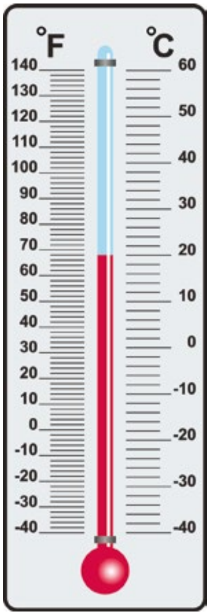
ii 20 discs.

g How many moves would be needed for n discs?

13.3 Celsius and Fahrenheit temperatures

The thermometer shows two different temperature scales—the Celsius scale which is used in Australia and most other countries, and the Fahrenheit scale which is still officially used in the USA.

Use the thermometer to answer questions **a** to **e**.



a What temperature is the thermometer showing on the Celsius scale?

b What temperature is this on the Fahrenheit scale?

c On the Celsius scale 40°C would be a very hot day. What temperature does this correspond to on the Fahrenheit scale? (Make sure you are looking at 40 degrees above zero on the Celsius scale!)

d What Celsius temperature is equivalent to 60° Fahrenheit?

e Is there a temperature which is the same on both scales?

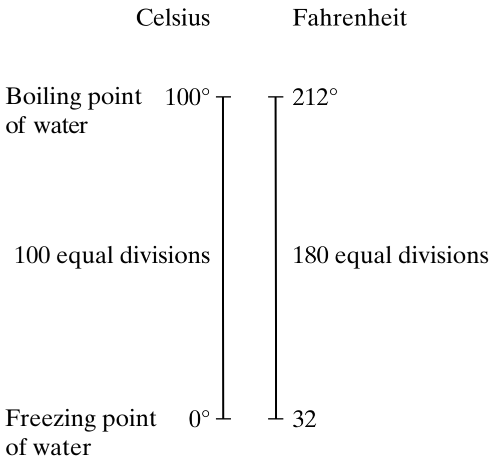
The Celsius and Fahrenheit temperature scales were both defined in terms of the freezing point and boiling point of water. For the Celsius scale it was decided to call the freezing point of water 0°C and the boiling point 100°C, so that there were 100 equal divisions representing whole Celsius degrees in between.

For the Fahrenheit scale, the freezing point of water was chosen as 32° and the boiling point as 212°, so that there were 180 equal divisions representing whole Fahrenheit degrees in between.

From this we can see that 100 Celsius degrees is equivalent to 180 Fahrenheit degrees.

This means that each Fahrenheit degree is equal to $\frac{100}{180}$ Celsius degree, that is,

1 Fahrenheit degree $= \frac{5}{9}$ Celsius degree.



To convert a Fahrenheit temperature into the equivalent Celsius temperature, we must first subtract the difference of 32 degrees to allow for the different starting points of the two scales, then multiply the Fahrenheit temperature by $\frac{5}{9}$.

f Convert 78 °F into the equivalent Celsius temperature.

g Convert −40 °F into the equivalent Celsius temperature. Does this agree with what you gave as your answer to part **e**?

h Write a set of instructions for converting Celsius temperatures into the equivalent Fahrenheit temperatures.

i Use your instructions to convert 15 °C into the equivalent Fahrenheit temperature.

Class activity
Celsius and Fahrenheit

j Type the headings Celsius and Fahrenheit in cells A1 and B1 (respectively) of a spreadsheet. In column A, below the heading enter the numbers −40 to 100 to represent Celsius temperatures from −40 °C to 100°C. (An easy way to do this is to type −40 in cell A2, then type the formula = A2+5 in cell A3. By dragging the formula down to row 30, you should obtain the numbers −40 to 100 in 5 degree intervals) In cell B2, type a formula which, when dragged down, will convert the Celsius temperatures in column A into Fahrenheit temperatures in column B.

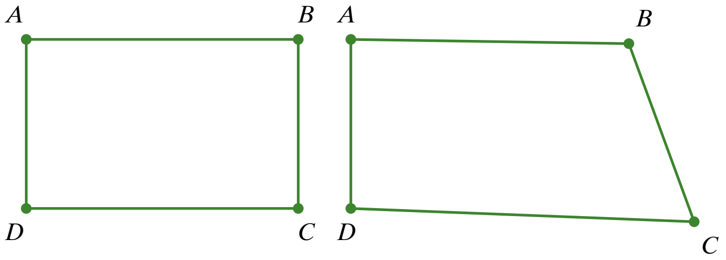
k Using an X-Y scatter graph with the points joined, construct a conversion graph for Celsius and Fahrenheit temperatures. Label the axes and put a title on your graph. Print your graph and paste it into your book.

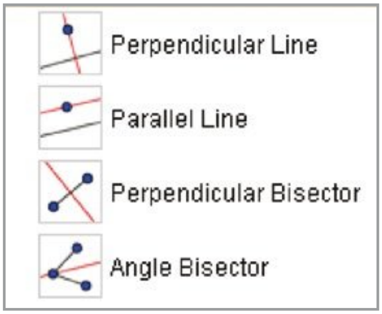
13.4 Constructing drag-resistant quadrilaterals

The geometry software *GeoGebra* allows you to drag shapes on the screen so that angles and lengths may change.

In this investigation you will use GeoGebra to construct some geometric shapes, including a rectangle, that will keep their special shapes when you drag them. You can download GeoGebra at www.geogebra.org.

Edward drew a rectangle using GeoGebra but when he dragged one corner, the shape was no longer a rectangle.





However, if you use the special geometry tools in the software to construct the rectangle, it will stay a rectangle when it is dragged, even though the length of the sides may change.

a Look at the list of tools in the drop-down menu for the fourth icon from the left in the toolbar at the top of the screen. Which of these tools could you use to construct a rectangle that would stay a rectangle when dragged around the screen? (Hint: Think about the properties of rectangles.)

b Using GeoGebra, construct a rectangle that will stay a rectangle when you drag it. When you are satisfied that your rectangle is correct, hide your construction lines. Note that we do not delete construction lines because that would delete everything that depended on them. So we use the **Show/Hide** tool which you will find in the drop-down menu for the last icon on the right of the toolbar. Insert text to put a heading on your screen page. Save your construction with the name *Rectangle*. Ask another student to test your shape by dragging the vertices to see if your construction is correct.

c Construct each of these other special quadrilaterals.

- **i** Trapezium
- **ii** Parallelogram
- **iii** Rhombus
- **iv** Square
- **v** Kite

Each quadrilateral must keep its special properties when you drag it around the screen. For example, the parallelogram may change size as it is dragged, but its opposite sides must stay parallel. In each case you will need to think about the special properties of the quadrilateral, then explore the software tools to find suitable tools for completing your construction. The **Circle with center and radius** tool is useful for obtaining equal length sides, for example in the square and rhombus constructions. For symmetrical shapes such as the kite, the **Mirror object at line** tool is useful.

13.5 Polygons

The word polygon literally means ‘many angles’, but of course a polygon has many sides too. A polygon is defined as a plane figure with straight sides. The family of polygons includes triangles and quadrilaterals, which used to be called trigons and tetragons respectively.

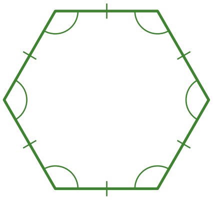
Regular and irregular polygons

If all the *angles* of a polygon are equal, the polygon is called an *equiangular polygon*.

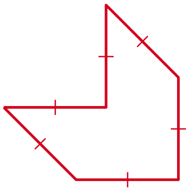
If all the *sides* of a polygon are equal, the polygon is called an *equilateral polygon*.

Regular polygons have all their angles equal *and* all their sides equal; that is, they are *equiangular* and *equilateral*. Polygons that are not regular are called irregular polygons.

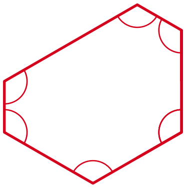
Different types of hexagons, for example, are shown below.



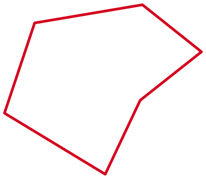
Regular hexagon



Equilateral hexagon



Equiangular hexagon

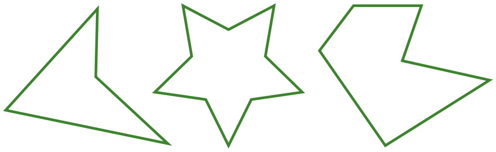


Neither equilateral nor equiangular

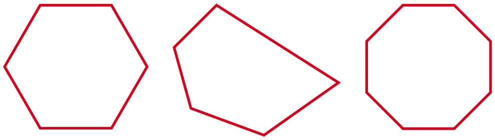
Irregular hexagons

Concave and convex polygons

Concave polygons are polygons with one (or more) reflex angle. *Convex polygons* have no reflex angles. All regular polygons are convex polygons.



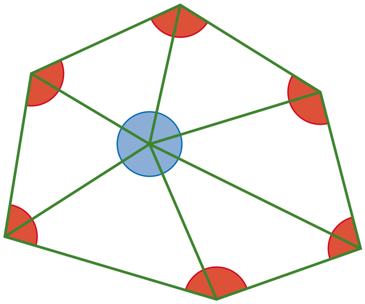
Concave polygons



Convex polygons

By choosing any point inside the polygon, we can connect the point to each vertex and divide the polygon into triangles. The number of triangles is the same as the number of sides. The hexagon below, for example, is divided into six triangles. The interior angles of the hexagon are the red angles, so to find their sum we need to subtract the blue angles at the point, that is 360°, from the sum of the angles of the six triangles.

Sum of the angles of the hexagon $= (6 \times 180°) - 360°$
$= 1080° - 360°$
$= 720°$



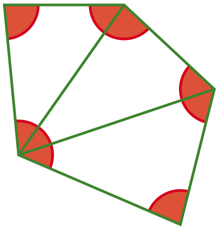
For a polygon with n sides,

Sum of the angles of the polygon $= (n \times 180°) - 360°$

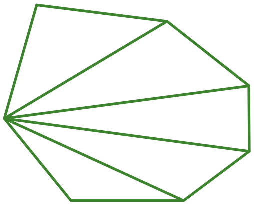
Another method for finding the sum of the interior angles of a polygon

We saw in section 7.5 that the sum of the interior angles of any quadrilateral is 360° because a quadrilateral can be divided into two triangles, and $2 \times 180° = 360°$. The same method may be extended to other polygons. A pentagon, for example, may be divided into three triangles by drawing diagonals from one vertex, so the sum of the interior angle measures of any pentagon is $3 \times 180°$, that is, 540°.

Similarly, a hexagon can be divided into four triangles, so the sum of the interior angle measures of any hexagon is $4 \times 180°$, that is, 720°.



Pentagon
5 sides
3 triangles
Angle sum $= 3 \times 180°$



Hexagon
6 sides
4 triangles
Angle sum $= 4 \times 180°$

Using this diagonal method, we can see a simple relationship between the number of triangles and the number of sides.

The number of triangles is always two less than the number of sides.

So it is easy to calculate the sum of the interior angle measures for any polygon by subtracting 2 from the number of sides, then multiplying by 180°.

Sum of the interior angles of a polygon

For a polygon with n sides, the diagonals divide the polygon into $(n - 2)$ triangles, so the sum of the interior angles is $(n - 2) \times 180°$.

a Find the sum of the interior angles of a heptagon. Draw a diagram to illustrate your method.

We have now found two expressions for the sum of the interior angles of a polygon with n sides: $(n \times 180°) - 360°$ and $(n - 2) \times 180°$.

b Show that these two expressions are equivalent.

BLM Polygon angles

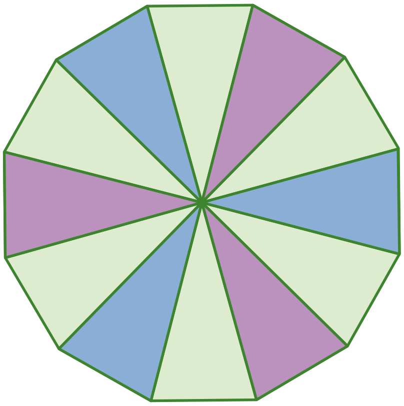
c Copy and complete the following table.

Number of sides	Name of polygon	Number of triangles	Sum of the interior angles	Size of each interior angle in the regular polygon
3	Triangle	1	1 × 180° = 180°	180° ÷ 3 = 60°
4	Quadrilateral	2	2 × 180° = 360°	360° ÷ 4 = 90°
5		3		
6		4		
7				
8				
9				
10				
11	Undecagon			
12	Dodecagon			
15	15-sided polygon			
24	24-sided polygon			
30	30-sided polygon			
36	36-sided polygon			

d A 50 cent piece is in the shape of a regular polygon.

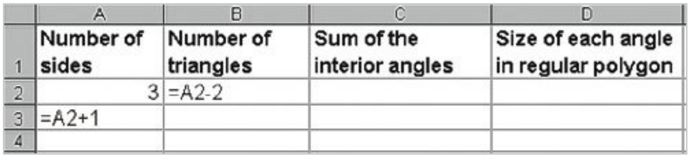
- **i** What is the name of the regular polygon?
- **ii** What is the size of each interior angle?

e A beach umbrella is made from 12 pieces of fabric sewn together as shown. Each piece is an isosceles triangle. Calculate the sizes of the three angles in each isosceles triangle. Can you find two different ways of working out the sizes of the angles?



Class activity
Polygons

f Use a spreadsheet to calculate the sum of the interior angles of regular polygons, and the size of each interior angle. The method is the same as that used in parts **b** to **d**. Enter appropriate formulas in cells C2 and D2 and drag down the four formulas. The sum of the interior angles and the size of each angle in regular polygons will then be automatically calculated. Drag each of the formulas down to about row 10 000.



	A	B	C	D
1	Number of sides	Number of triangles	Sum of the interior angles	Size of each angle in regular polygon
2	3	=A2-2		
3	=A2+1			
4				

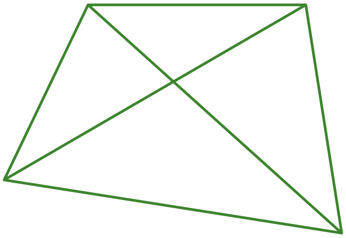
Used with permission from Microsoft

i What happens to the size of each of the interior angles as the number of sides becomes very large? What size does the angle seem to be approaching?

ii Could the angle size ever reach this value? Explain.

13.6 Polygon diagonals

In a quadrilateral, one diagonal can be drawn from each of the four vertices, but because each diagonal is connected to two vertices, there are only two diagonals.



a How many diagonals can be drawn from each vertex of a pentagon?

b How many diagonals does a pentagon have?

c How many diagonals can be drawn from each vertex of a hexagon?

d How many diagonals does a hexagon have?

e Can you see a pattern in how many diagonals can be drawn from each vertex?

f If a polygon has n sides, how many diagonals can be drawn from each vertex?

g How many diagonals would an n-sided polygon have? Write an expression in terms of n.

h Use your expression to calculate the number of diagonals for a 20-sided polygon.

13.7 Why is sugar called a carbohydrate?

Ordinary cane sugar, or sucrose, belongs to a group of chemical compounds called **carbohydrates**. Sucrose consists of molecules that are made up of just three elements: carbon, hydrogen and oxygen. However, many other chemical substances are also composed only of carbon, hydrogen and oxygen. What is it that determines the differences between sugar and those other substances?

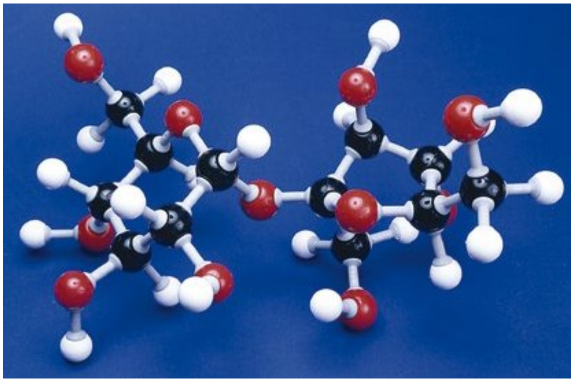
When scientists analyse chemical substances such as sucrose, they use a variety of methods to determine what a molecule of the substance looks like. One of the first steps is to determine the ratio of atoms of the different elements in each molecule.

Water molecules, for example, are made up of hydrogen and oxygen atoms in the ratio 2:1.

hydrogen:oxygen
2:1

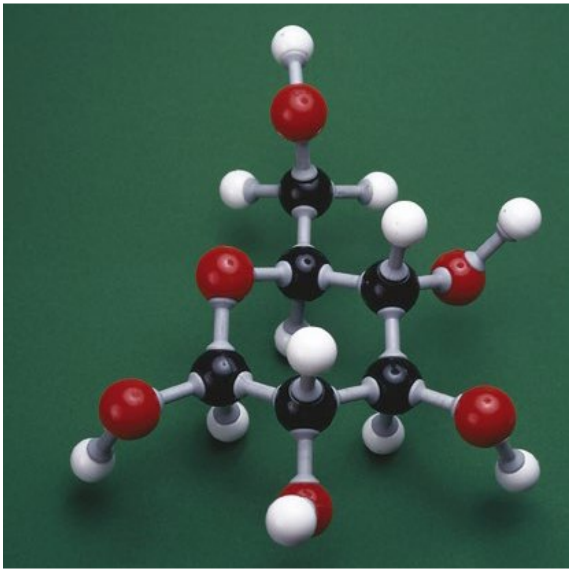
We show this ratio by writing the chemical formula for water as H_2O.
The chemical formula for a molecule of sucrose is $C_{12}H_{22}O_{11}$.
The black, red and white balls in this model of a sucrose molecule represent carbon, oxygen and hydrogen respectively.

An important sugar in our bodies is glucose. Glucose, like sucrose, is a carbohydrate. Using only the ratio of atoms, the formula for glucose would be CH_2O. However, scientists have shown that each glucose molecule is made up of six carbon atoms, 12 hydrogen atoms and six oxygen atoms. Therefore, the chemical formula of glucose is $C_6H_{12}O_6$.


Sucrose molecule

a What is the ratio of hydrogen to oxygen in a molecule of sucrose?
b Write this ratio in its simplest form.
c Write the ratio of hydrogen to oxygen in a molecule of glucose in its simplest form.
d Can you see why sucrose and glucose are called carbohydrates?

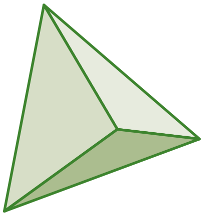
Once scientists have determined the number and type of atoms that make up the molecules of a substance, they can use other techniques to explore the arrangement of atoms in the molecule. The following model shows how the carbon, hydrogen and oxygen atoms are joined in a molecule of glucose. The black, red and white balls represent carbon, oxygen and hydrogen respectively.


Glucose molecule

13.8 The five regular polyhedra

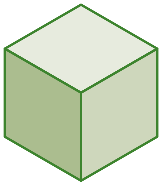
BLM
Regular polyhedral nets

There are only five different regular polyhedra.



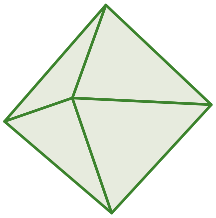
Regular tetrahedron

4 equilateral triangle faces
3 triangles meet at each vertex



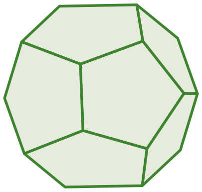
Cube

6 square faces
3 squares meet at each vertex



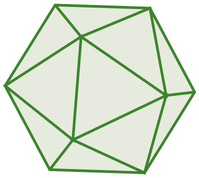
Regular octahedron

8 equilateral triangle faces
4 triangles meet at
each vertex



Regular dodecahedron

12 regular pentagon faces
3 pentagons meet at
each vertex

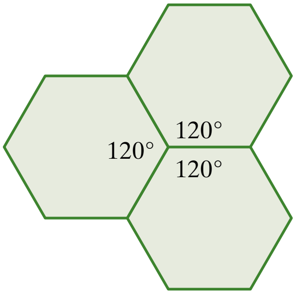


Regular icosahedron

20 equilateral triangle faces
5 triangles meet at
each vertex

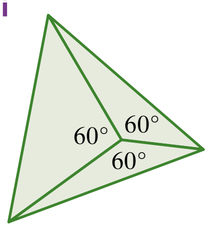
a What is the minimum number of polygon faces that would have to meet at each vertex to form a solid shape?

b Could a solid shape be formed if three regular hexagons met at each vertex? Explain.



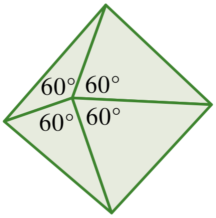
c Calculate the sum of the angles at each vertex in the three regular polyhedra that are made up of equilateral triangles.

i



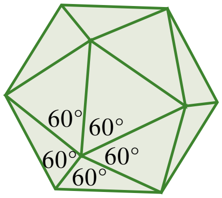
Regular tetrahedron

ii



Regular octahedron

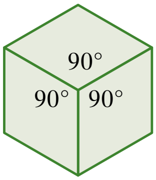
iii



Regular icosahedron

d Would it be possible to have a solid shape formed if six equilateral triangles met at each vertex? Explain.

e Calculate the sum of the angles at each vertex in the cube.



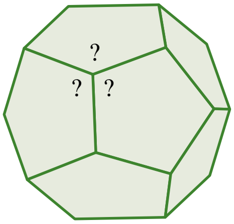
f Would it be possible to have a solid shape formed if four squares met at each vertex? Explain.

g What is the sum of the angles at each vertex in a dodecahedron?

h Why is it not possible for more than three pentagons to meet at a vertex?

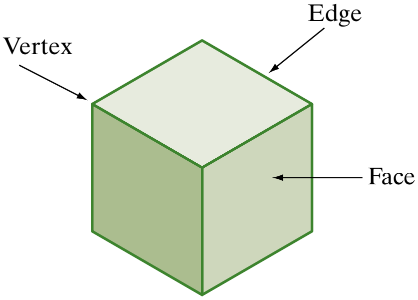
i Supposing we tried to make a regular polyhedron from regular heptagons or regular octagons. Explain why this would not be possible.

j Can you now see why the tetrahedron, cube, octahedron, dodecahedron and icosahedron are the only possible regular polyhedra? Explain.



13.9 Polyhedra and Euler's rule

A three-dimensional shape where all the faces are polygons is called a polyhedron (the plural of polyhedron is *polyhedra* or *polyhedrons*). The word polyhedron comes from Greek words meaning *many bases*. The line segment where two faces of a polyhedron meet is called an edge, and each point where edges meet is called a vertex (plural *vertices*).

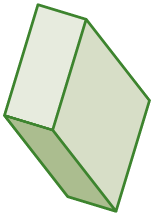


Where polygons meet on the surface of a polyhedron, the sum of the angles must be less than 360°. If the sum of the angles were to equal 360°, the polygons would form a flat surface and therefore could not form a polyhedron.

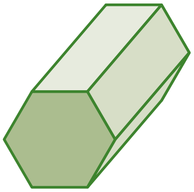
There are many different polyhedra. Three common groups of polyhedra are:

- prisms
- pyramids
- regular polyhedra.

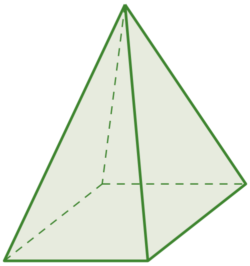
Investigation 13.8 shows the five regular polyhedra.



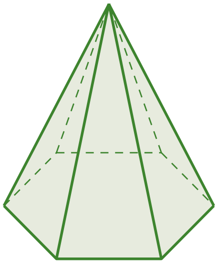
Rectangular prism



Hexagonal prism



Right square pyramid
Square base
4 isosceles triangle faces



Right hexagonal pyramid
Hexagon base
6 isosceles triangle faces

The 18th century Swiss mathematician Leonhard Euler (pronounced ‘oiler’) discovered a relationship between the number of faces, vertices and edges for any polyhedron.

For the following activity you will need a collection of polyhedra. Chocolate packets, for example, are useful for some of the prisms, and you may find pyramid-shaped candles. You may also construct your own models of the five regular polyhedra using the nets provided in the student ebook. Cut out each net carefully then fold it and tape it together.

a Copy and complete the following table and try to find the relationship that Euler discovered. Include any other polyhedra for which you have models.

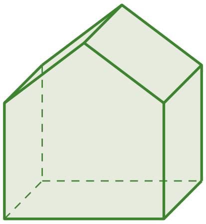
Name of polyhedron	Faces (*F*)	Vertices (*V*)	Edges (*E*)
Tetrahedron			
Cube			
Octahedron			
Square pyramid			
Triangular prism			
Hexagonal prism			

b Using the letters F, V and E, write an expression for the relationship between the numbers of faces, vertices and edges. This expression is known as **Euler’s rule**.

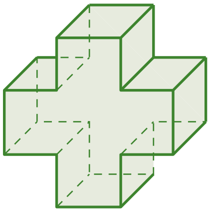
c Copy and complete the following table to show the number of faces, vertices and edges for each of the following prisms. Does Euler's rule apply in each case?

Shape	Faces	Vertices	Edges	Does Euler's rule apply?
A				
B				
C				

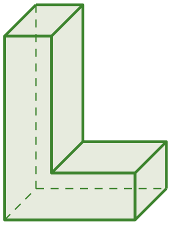
A



B



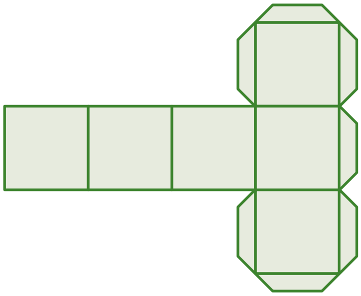
C



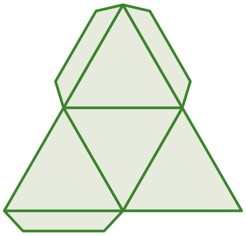
d Andrea counted the faces, vertices and edges of three polyhedra. Her results are shown in the table below. Could these all have been correct? Explain.

Shape	Faces	Vertices	Edges
X	10	14	22
Y	18	24	36
Z	16	24	38

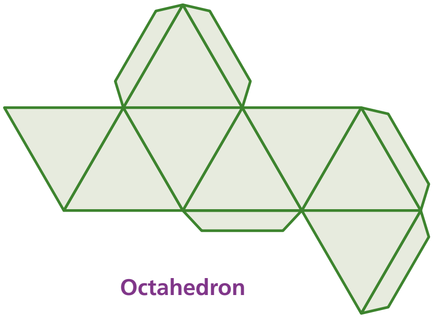
e When designing a net for a polyhedron, we can include tabs for sticking the net together after it has been folded into the polyhedron shape. The net for the cube shown below has seven tabs.



- **i** How many edges does a cube have?
- **ii** In this net, how many of the edges are already joined?
- **iii** If T is the number of tabs, E is the total number of edges and J is the number of edges already joined in the net, make up a rule for the number of tabs needed to stick the net together.
- **iv** Look at these nets for a tetrahedron and an octahedron. Does your rule work for these?



Tetrahedron



Octahedron

13.10 Playing soccer with a truncated icosahedron

Balls for various sports are made in different ways according to how they are used in the game. Tennis balls, for example, need to bounce and are moulded in one piece from rubber. Soccer balls and basketballs are sometimes made from pieces of synthetic leather stitched together to make the outer covering, although they are often moulded to look as if they are made from joined pieces. They have a bladder of air inside to force them into a spherical shape.

The diagram on the left in part **d** shows a regular icosahedron. 'Icosa' means 20. There are 20 equilateral triangles making up the surface of the regular icosahedron.

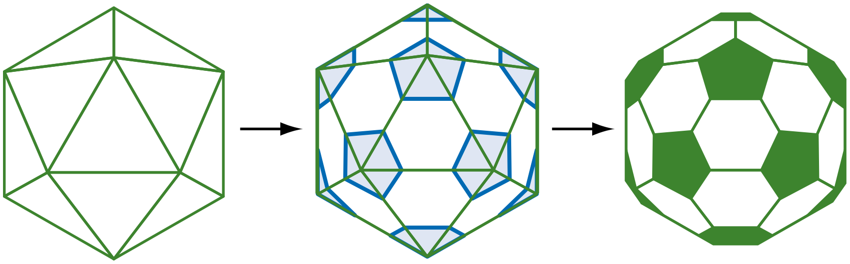
a How many equilateral triangles meet at each vertex?

b How many faces does the regular icoshedron have?

The other two diagrams show how each vertex can be sliced off to make new faces (shown in green). 'Truncated' means 'cut off' so the new shape is called a truncated icoshedron instead of a regular icoshedron.

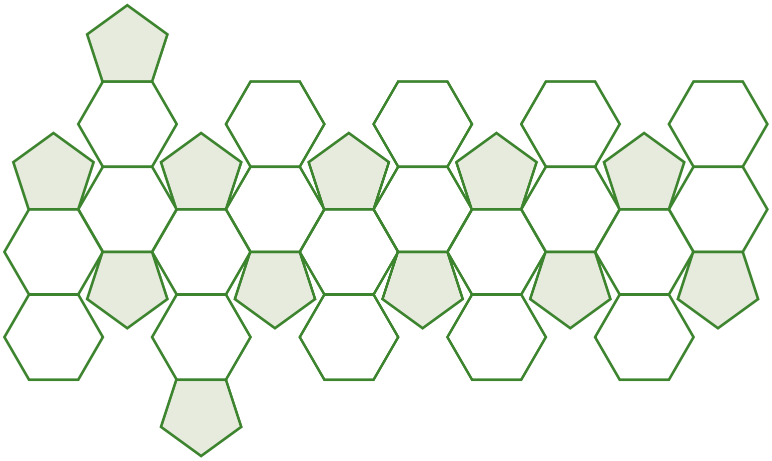
c What shape are the new (green) faces formed by slicing off each vertex?

d What is left of each equilateral triangle?



BLM Truncated icosahedron net

The diagram below shows the net for a truncated icosahedron. This is what the truncated icosahedron would look like if we could open it out flat.



e How many regular hexagons are there?

f How many regular pentagons are there?

g What is the total number of faces of the truncated icosahedron?

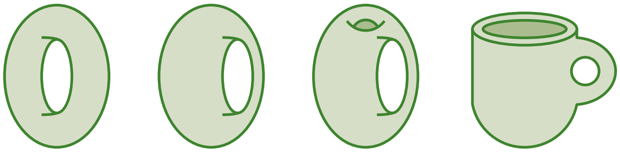
h Work out how many edges the truncated icosahedron has. Hint: first calculate the total number of sides in the hexagons and pentagons.

i Work out how many vertices the truncated icosahedron has. Hint: first calculate the total number of vertices in the hexagons and pentagons.

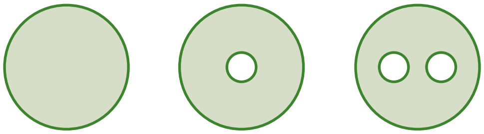
j Show that Euler's rule applies to the truncated icosahedron.

13.11 Topology

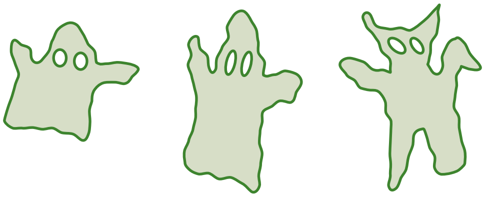
Can you turn a one-holed doughnut into a coffee mug? If you want to prove that this can be done, take a lump of playdough about the size of your fist and work it into a doughnut shape with one hole. Without making any more holes in the dough, shape one side into a handle as shown. Now push a hollow into the top to make the dough into a mug shape.



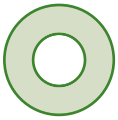
Topology is the branch of mathematics that deals with surfaces when they are deformed by smoothly shaping or bending without making any extra holes in them. If we stretch an elastic band or flatten a lump of playdough the new shapes will look different, but will be topologically the same. But as soon as we make a hole in the elastic band or dough, it is topologically different. The shapes shown below are all topologically different.

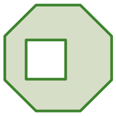


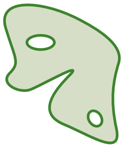
The shapes on the right are all topologically the same because even though they have been stretched into different shapes, they all have two holes.



a Which of the following shapes are topologically equivalent?

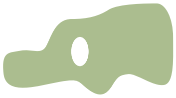

A


B


C

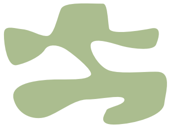
b Which of these three objects—key, button and CD—are topologically equivalent? Explain.

c Match the shapes that are topologically the same.

i


ii


iii


iv


v


vi


d Some of these capital letters are topologically equivalent to each other.

A B C D E F G H

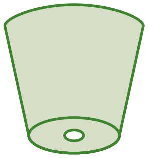
- **i** Which letter(s) is (are) topologically equivalent to A?
- **ii** Which letter(s) is (are) topologically equivalent to B?
- **iii** Which letter(s) is (are) topologically equivalent to C?

e For which of these letters are the capital (upper case) letters topologically equivalent to the lower case letters?

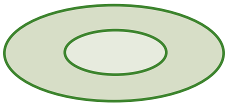
a b c d e f g h
ABCDEFGH

f Consider the digits and item below.

0 1 2 3 4 5 6 7 8 9


Flower pot

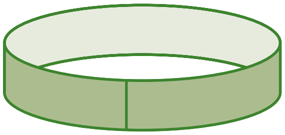

Vase



Plate

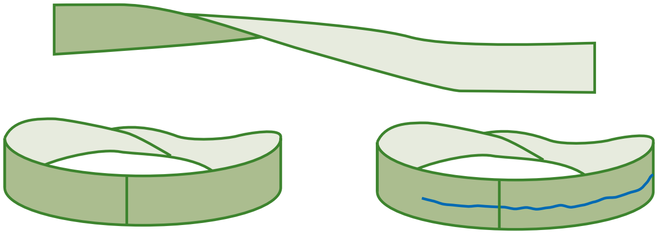
- **i** Which of the ten digits is (are) topologically equivalent to the flower pot?
- **ii** Which of the ten digits is (are) topologically equivalent to the vase?
- **iii** Which of the ten digits is (are) topologically equivalent to the plate?

g Cut a strip of paper about 2 cm wide and 30 cm long. Carefully bend the strip around, being careful not to twist it, and overlap the ends slightly then glue them together.



Notice that the paper ring has an inside and an outside; that is, it has two surfaces.

Cut another strip of paper. This time put one twist in the strip before gluing the ends together.

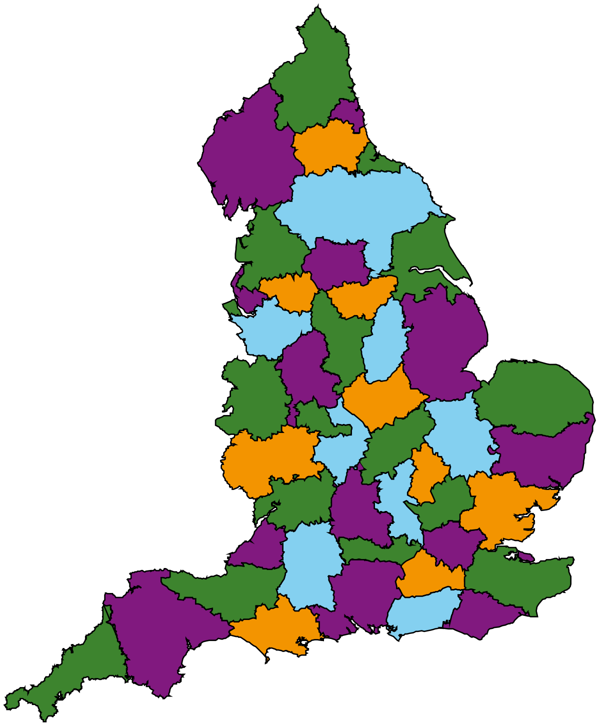


Using a felt pen, draw a continuous line around the loop until you end up back where you started. What do you notice? How many surfaces does the loop have? This twisted loop is known as a Moebius strip. The lampshade shown here is twisted like a Moebius strip.

13.12 Map colouring

There may not seem to be anything mathematical about colouring a map, and the problem may not appear to be connected with topology. However, when we colour a map, the size or shape of each region is not important. What is important is how many other regions each region touches.

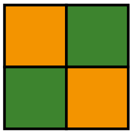
The problem of how many colours are needed to colour a map goes back to 1852. Francis Guthrie, a student in London, was colouring a map of all the counties of England so that no two neighbouring counties were the same colour. He noticed that he needed only four different colours, as shown in the map of the counties of England below.

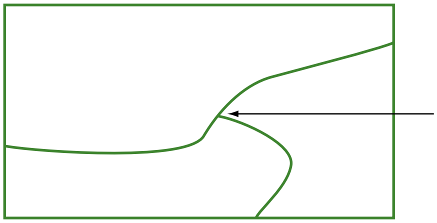


Francis Guthrie found that only four colours were needed to colour a map of the counties of England.

Francis wondered if this was true for any map. He asked his brother Frederick, who was studying mathematics, but Frederick didn't know either. Neither did Frederick's teacher, who was a celebrated mathematician. For 120 years many people tried unsuccessfully to find an answer to the problem of whether four colours were enough to colour any map. It wasn't until 1976 that two mathematicians, Wolfgang Haken and Kenneth Appel developed a computer proof of the four-colour problem. Although at first people were reluctant to accept a proof by a computer, the proof is now accepted and the problem is known as the four-colour theorem.

In the case of the simple 'maps' on the right, only two colours are needed. Notice that regions that meet at a single point are allowed to be the same colour.



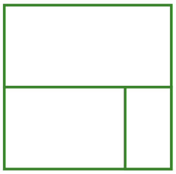


Because three different regions meet at this point, three colours are needed.

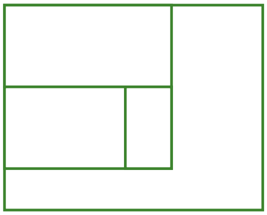
BLM
Map colouring

a Find the minimum number of colours required for each of these maps.

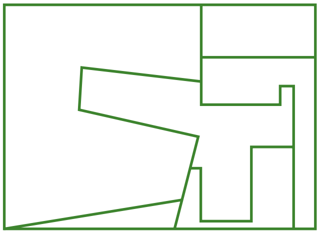
i



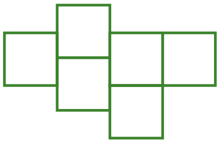
ii

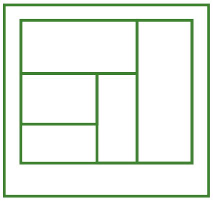


b What is the minimum number of colours needed to colour the following drawing?

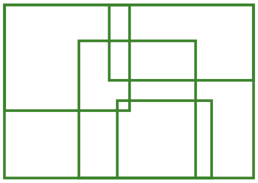


c What is the minimum number of colours needed to colour each of these figures if no adjacent areas are to be the same colour?





iii



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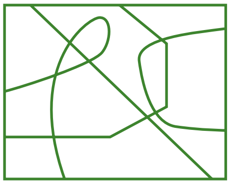
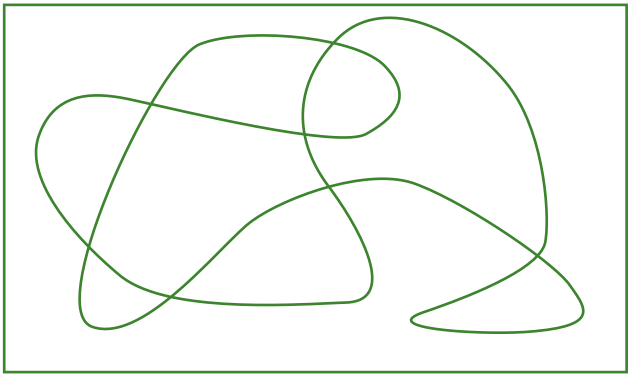
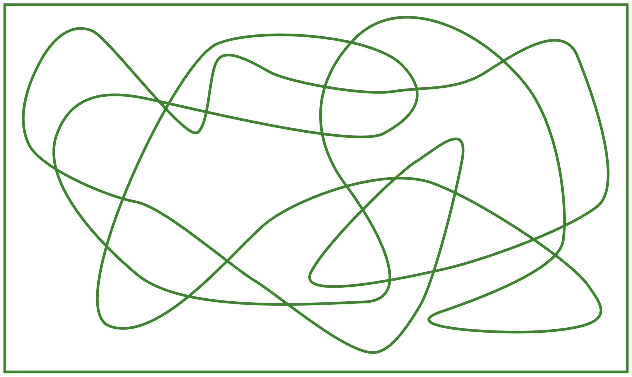


d **i** Draw a rectangle about 6 cm by 8 cm as the border for a 'map'. Now place your pencil somewhere inside the border and draw a continuous curve without lifting your pencil until you have returned to your starting point. You can cross over your line as many times as you wish (see, for example, the drawing below). Colour each of the regions of your map, using the smallest possible number of colours.



ii How many colours do you need? Compare your findings with those of other students in your class.

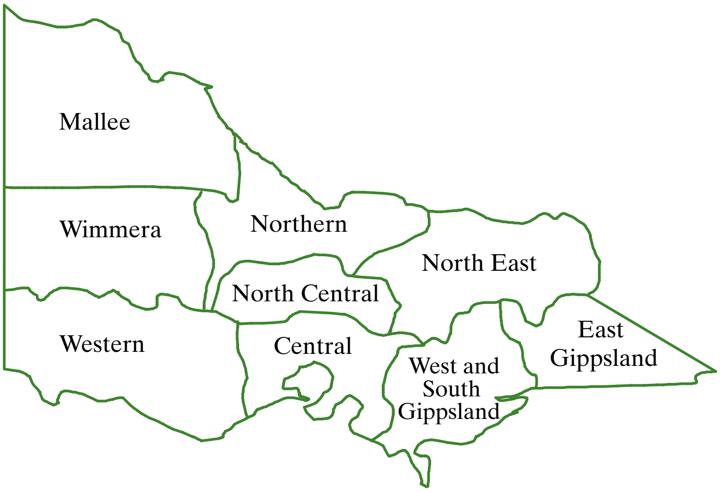
iii Draw another map in the same way, but before you colour it, draw another continuous curve on top of the first one as shown here. Now colour your map, again seeing what is the smallest number of colours you need.



iv Can you explain your findings?

e What is the minimum number of colours needed to colour the Australian states and territories?

f The map below shows the regions of Victoria. What is the smallest number of colours needed to colour this map so that neighbouring regions are different colours?



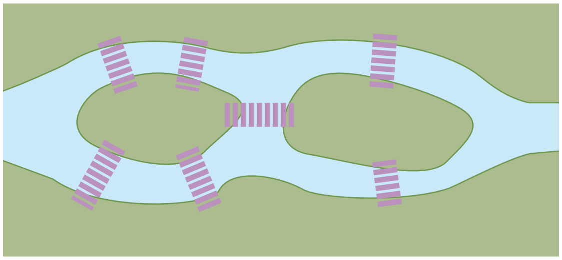
g Find a black and white map divided into regions as in parts **d** and **e**, for example, a map of regions of your own state, metropolitan cities within your capital city, or a map of the United States of America. Show that only four colours are needed to colour the regions.

13.13 Networks

A network is a pattern of connections, for example, between people or between places.

The Indigenous artwork at the beginning of this chapter is an example of a network. The Australian Aboriginal people have a strong connection with the land, and their concept of dreaming represents a long relationship over many generations between Aboriginal family groups, adjacent tribal groups and physical features of the country, such as waterholes. These connections are represented in the painting and have similarities with the networks in this investigation.

a List some of the different situations in everyday life where we use the term 'network'.



A famous network problem originated in the eighteenth century in the town of Königsberg (now called Kaliningrad) in Europe. Two branches of a river joined in the town, and there were two islands where the rivers joined. Seven bridges enabled the people of Königsberg to get around the town, and the townspeople often wondered if it was possible to walk around the town so that each bridge was crossed exactly once.

BLM
Networks

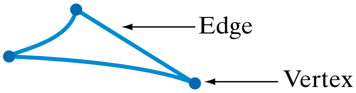
b Copy the drawing and then try to work out a continuous route that takes you over every bridge just once without lifting your pencil. Is it possible?

In 1736, the mathematician Leonhard Euler became interested in the problem. Euler found that it was impossible to cross every bridge just once, but it led him to develop a theory of networks. Network problems are actually topology problems, because the network is still the same if it is stretched.

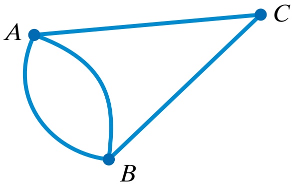
Before we investigate networks in more detail, we need to know some of the terms that are used in the study of networks.

Edges and vertices

Network graphs are made up of points called vertices, joined by curves or straight lines which we call edges.



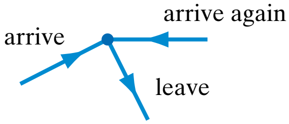
An odd vertex is a vertex that has an odd number of edges connected to it and an even vertex is one that has an even number of edges connected to it. For example, in this diagram, vertices A and B are odd because they both have three edges connected to them, whereas vertex C is even because it has two edges connected to it.



Euler paths

Euler discovered that if there were more than two odd vertices in a network it was not possible to have a continuous path that passes along every edge just once. We say that the network is not traversable.

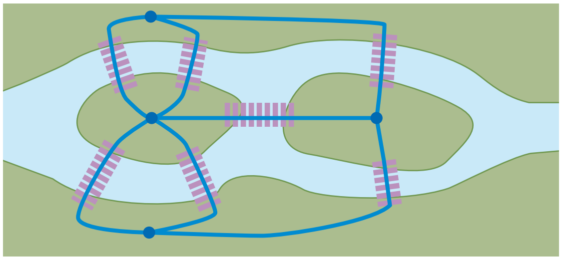
If you arrive at an odd vertex with three edges connected to it, you have two other paths to leave by, but if you arrive at the same vertex again, you have no more paths to leave by.



So the only way of dealing with odd vertices is to either start at them or finish at them. If there are only two odd vertices, you can start at one of them and finish at the other one, so it is possible to have a continuous path, that is, the network is traversable.

A continuous path around a network is possible only if the network has no more than two odd vertices. This continuous path is called a **Euler path** and the network is said to be *traversable*.

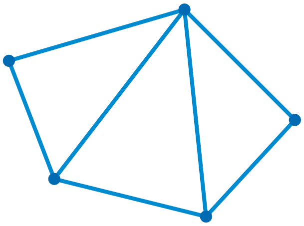
We can draw the Königsberg bridges as a network with each bridge represented as an edge, and the different regions of the city as vertices.



All four vertices are odd with five edges connected to the vertex at the centre, and three edges connected to the other vertices. So according to Euler's theory, it is impossible to have a continuous route that passes along every edge just once, that is, the network does not have an Euler path.

c What would happen if one of the bridges was removed? Would it matter which one?

d Does this network have an Euler path, that is, is the network traversable?

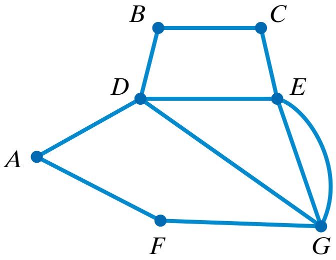


e Draw the Euler path for this network.

Euler circuits

If a network is traversable, it is possible to start and finish at the same vertex only if the network has no odd vertices. A path that starts and finishes at the same vertex is called an **Euler circuit**.

f Show that the following network has an Euler circuit.



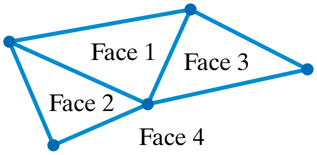
Network graphs and Euler's rule

Investigation looked at the relationship between the number of faces (F), vertices (V) and edges (E) in a polyhedron.

Euler's rule which applies to polyhedra also applies to network graphs:

$$E = F + V - 2$$

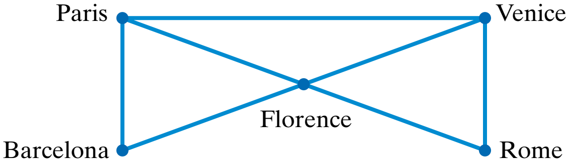
A network graph divides the plane (flat surface) into regions. These regions are called faces. Notice that the region outside the boundaries of the edges is also called a face.



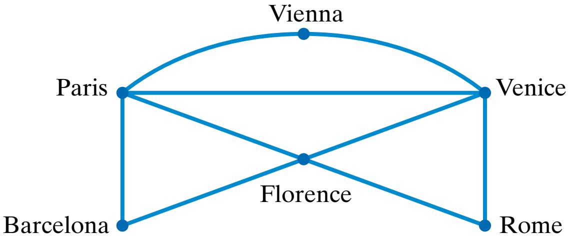
Vertices	Faces	Edges
5	4	7

g Show that Euler's rule applies to the network graph in part **f**.

h Marco and Paola are planning a European holiday. They have worked out a route starting in Paris and visiting Venice, Rome, Florence and Barcelona as shown in this network diagram.



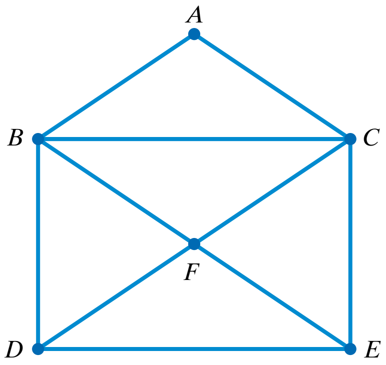
- **i** How many odd vertices are there?
- **ii** Is it possible for Marco and Paola to travel along each route only once and visit each of the five cities?
- **iii** If Marco and Paola start in Paris, where will they finish?
- **iv** Marco and Paola want to start and finish in Paris. They decide to include Vienna in their tour, as shown. How many odd vertices are there now?



- **v** Copy the network and show that it is now possible to travel along every route only once and to start and finish in Paris.
- **vi** How many times will Marco and Paolo visit Paris?

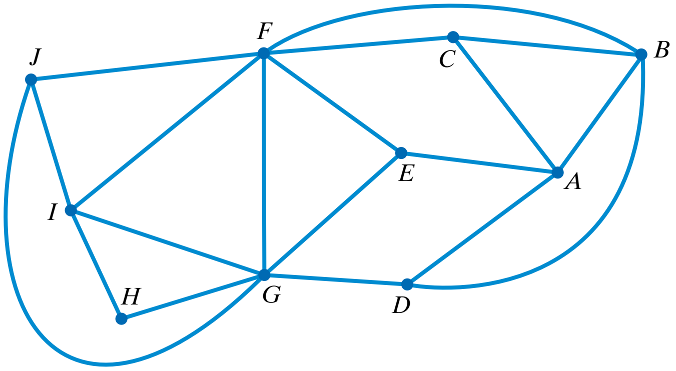
i **i** For the network on the right, list the number of edges at each vertex.

- **ii** Which vertices are odd?
- **iii** Copy the network and show how you can trace over the network without lifting your pencil or going over an edge more than once; that is, show that the network is a traversable Euler path. Explain where you can start and where you finish.
- **iv** Does the network have an Euler path or an Euler circuit?



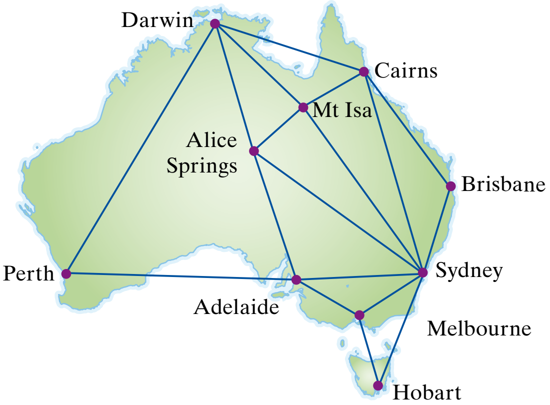
- **v** Show where you could add another edge linking two of the vertices so that the network would have an Euler circuit.

j



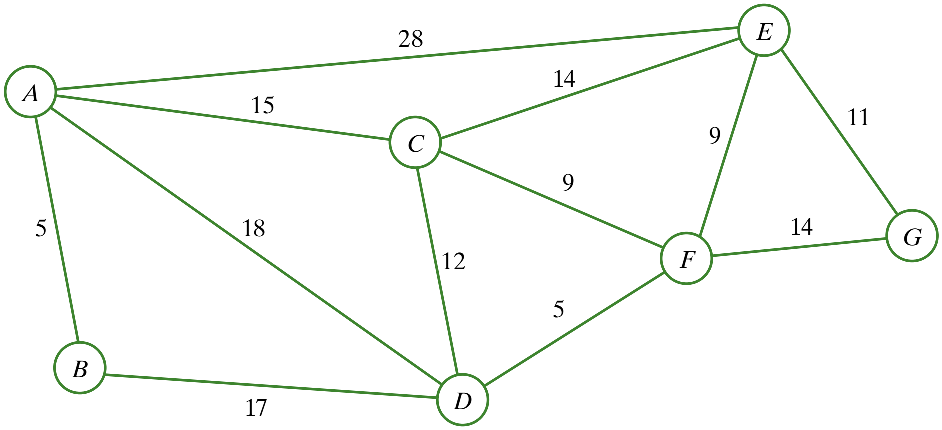
- **i** How many faces, vertices and edges does this network have?
- **ii** Show that Euler's rule applies to the network.
- **iii** Which vertices are odd?
- **iv** Does the network have an Euler path? Explain.
- **v** If vertex A and all the connections to it are removed, will there be an Euler path? Explain.
- **vi** If vertex A and all the connections to it are removed, will there be an Euler circuit? Explain.

k The Nguyen family, who live in Melbourne, have earned enough frequent flyer points to fly around Australia for a holiday. They have planned a route that takes them to all the places shown on the map.



- **i** Make a table showing the name of each place and whether the vertex is odd or even.
- **ii** How many odd vertices are there?
- **iii** Can the Nguyen family fly via every one of the routes shown without flying back over the same route?
- **iv** If they start in Melbourne, where will they finish?
- **v** The Nguyens decide to leave out one flight so that they can start and finish in Melbourne. Which flight must they leave out?
- **vi** Can you work out a route they could take, starting and finishing in Melbourne?

l The following map shows the roads connecting seven towns: *A*, *B*, *C*, *D*, *E*, *F* and *G*, and the distances in kilometres between the towns.



- **i** Mel wants to travel from *A* to *G* by the shortest route. Which route should she take and how far is it?
- **ii** Mel works for the local council and is checking road signs. Without working out a route, decide if it is possible for her to travel along each road only once. Explain your reasoning.
- **iii** If Mel starts at *A*, where will she finish?
- **iv** Work out a route for Mel and give the order of towns that she will pass through in order to travel over every road.
- **v** Give the number of edges, vertices and faces and show that Euler's rule applies.

13.14 How long is a chain?

In the Imperial System of length measurements:

12 inches = 1 foot

3 feet = 1 yard
22 yards = 1 chain
80 chains = 1 mile

The illustration on the next page shows the measuring tool known as *Gunter's Chain* that was used by surveyors until the early 20th century. It is made up of 100 links, with markers every 10 links. When stretched out it is exactly 1 chain or 22 yards long. This is the length of a cricket pitch.

- **a** Using the conversion that 1 inch is approximately 2.54 cm, approximately how many centimetres are in a yard?
- **b** Which metric unit of length is nearest to a yard?
- **c** Using your answer to part **a**, convert 1 chain to metres.

d Using your answer to part c and the relationship 80 chains = 1 mile, how many metres are in 1 mile?

e How many kilometres in 1 mile?

f The perambulator shown dates back to about 1880. It was used by early surveyors in Queensland and is now in the Museum of Lands, Mapping and Surveying in Brisbane. The circumference of the perambulator is equal to one-tenth of a chain. How many yards is this?

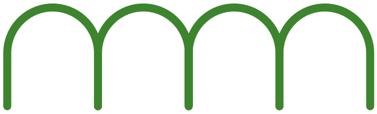
g Convert this circumference to inches.

h Using the conversion 1 inch is approximately 2.54 cm, calculate the circumference in centimetres.

i What is the diameter of the wheel? Give your answer to the nearest millimetre.

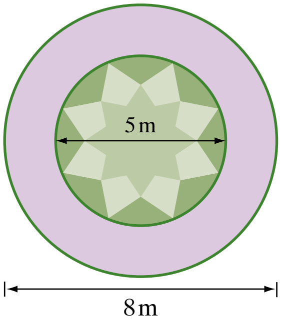
13.15 Floral clock

You may have seen the floral clock in Melbourne's St Kilda Road. The photograph below shows the clock after it has been dug up to prepare it for planting a new design. It has an inner circle that is planted with several colours of flowers to form a design. Around the outside is a wide border of a different colour. The diameter of the inner circle is 5 metres and the outside diameter of the clock is 8 metres.



a Around the outside of the clock is a metal edging. Calculate the distance around the edge of the clock to the nearest centimetre.

The illustration below shows a geometric design of flowers in the 5 m diameter inner circle, surrounded by a ring of purple flowers.



b Calculate the area of the inner circle design of the floral clock correct to two decimal places.

c If the approximate number of plants required for the inner circle is 4000, how many *plants per square metre does* this represent?

d How many *square centimetres per plant does* this represent?

e Calculate the area of the ring of purple flowers around the outside of the design of the clock to two decimal places.

f Based on the number of plants per square metre that you calculated in part **c**, estimate the number of plants required for the outer ring.

g What is the total number of plants needed?

h Suppose the cost of the plants is $25 per 100 plants. Calculate the approximate cost of planting the entire clock.

i It takes four gardeners about four days to replant the clock. This includes pulling out the old design, replacing the soil, designing and pegging out the new design and planting. If each gardener works 7.5 hours each day, how many hours does it take to replant the clock?

j The minute hand is 2.1 m long. How far does the tip of the minute hand travel in a year? Give your answer to the nearest kilometre.

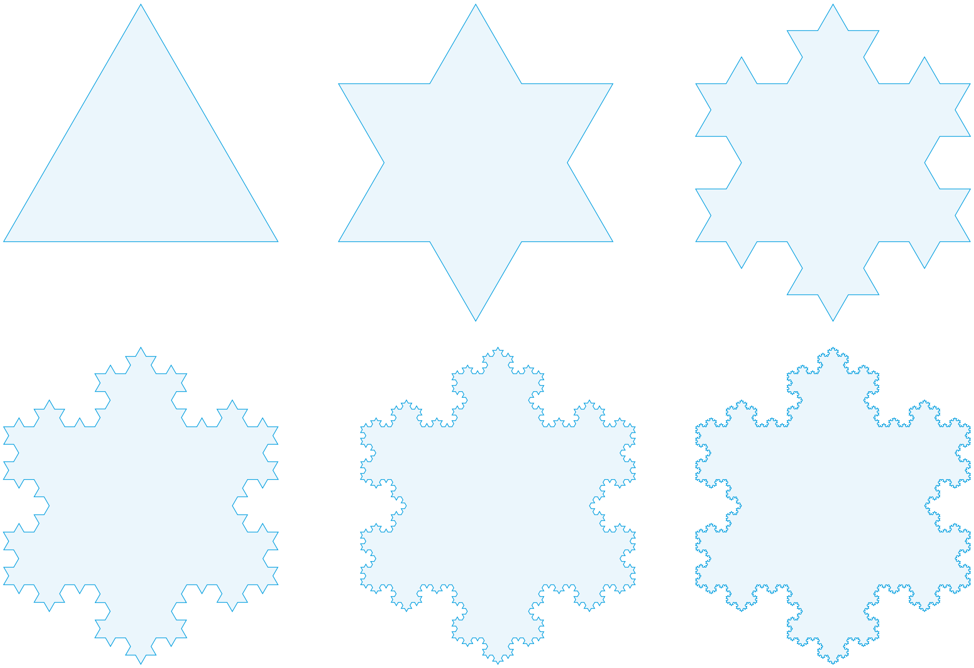
13.16 Snowflake fractal

Snowflake

Many mathematicians have explored fractals, and computer graphics have enabled fractals to be produced easily. One of the properties of fractals is that they are self-replicating, that is, they are copies of each other at different scales. We can zoom into a fractal and always find another identical (or approximately identical), but smaller, version of it.

The growth of many plants is fractal-like. This photograph shows a type of broccoli plant known as Romanesco broccoli. Notice how each part of the broccoli is like a smaller version of the whole broccoli. Similarly, if you look at a fern frond you will notice that each structure of the frond is repeated on a different scale.

One well-known fractal is the *Von Koch Snowflake*. We start with an equilateral triangle then divide each side into three equal parts. In the middle section we construct two more sides of an equilateral triangle. The figures below show what happens if we continue this process several times.



Suppose the original equilateral triangle has sides of length 243 cm.

a Calculate the perimeter of the equilateral triangle.

b Calculate the perimeter of the second figure, that is, the six-pointed star.

c Calculate the perimeter of the third figure.

d Can you see a pattern in the length of the perimeters? Can you work out a shorthand way of expressing the perimeter of each fractal in terms of the fractal before it?

e Calculate the perimeters of the other three fractal figures.

f Theoretically we could continue this process over and over again. What is happening to the perimeter each time? What do you think the perimeter would eventually be?

13.17 Winning at the fair

People at a school fair can play a game of chance. It costs $1 for each game. A player rolls two dice.

- If the two dice show the same number then the player wins a fluffy toy worth $2.
- If the two dice show different numbers then the player adds those numbers. If the sum of the two numbers is 7, then the player receives a chocolate bar worth $1. Otherwise the player loses their $1.

a Design a simulation of 50 trials of this game. Explain the design you used for your experiment.

b Use the results of the simulation of 50 trials to determine how many players could expect to win a fluffy toy if 50 people played the game.

c Comment on how reliable you think this estimate is.

d How could you improve this estimate?

e Use the method you have described in part **d** to come up with a better estimate of how many players could expect to win a fluffy toy if 50 people played the game.

f If 1200 games are played at the school fair, how many fluffy toys are likely to be won?

g How many chocolate bars are likely to be won?

h If 1200 games are played at the fair, how much profit could the school expect to make? Explain how you determined your answer.

13.18 Scrabble

Some letters occur more frequently than others in the English language. The letters A and E, for example, occur in many words, whereas J, X and *Z* are less common. In the word game Scrabble there are different numbers of each letter and different letters are worth different scores.

The table below shows the frequency of the letters and their scores for Scrabble.

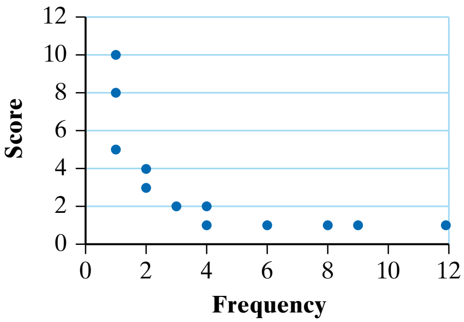
Letter	Frequency	Score	Letter	Frequency	Score
A	9	1	N	6	1
B	2	3	O	8	1
C	2	3	P	2	3
D	4	2	Q	1	10
E	12	1	R	6	1
F	2	4	S	4	1
G	3	2	T	6	1
H	2	4	U	4	1
I	9	1	V	2	4
J	1	8	W	2	4
K	1	5	X	1	8
L	4	1	Y	2	4
M	2	3	Z	1	10

There are also two blank tiles.

We might expect that the frequency of the letters would be related to the frequency of the letters in English words. We might also expect that the least common letters would be worth the highest score and the most common letters would be worth the lowest scores.

a The following X-Y scatter graph shows the relationship between the frequency of the Scrabble letters and the score for the letters. What does this graph tell us?

Relationship between frequency and scores for Scrabble letters



b Take two different paragraphs of written English each of about 100 words from different sources; for example, from a daily newspaper, a magazine article, a page in one of your textbooks, or a novel. Prepare a table with four columns as shown below. Make the Tally column the widest column. Count the numbers of each of the 26 letters of the alphabet and use tally marks to record the occurrence of the letters. Record the frequencies in the third column. Convert the frequencies into percentages and round to the nearest whole number percentage.

Letter	Tally	Frequency	Percentage frequency
A			

c Enter the first and last columns of the table (Letter and Percentage frequency) into columns A and B of a spreadsheet, then enter the frequencies of each letter in Scrabble into column C. Construct an X-Y scatter graph of the data in columns B and C.

d What does the graph tell you? Does it support the conjecture that the frequency of the letters in Scrabble would be related to their frequency in English words?

e Find some printed text in another language which uses the same alphabet as English; for example, Spanish, French, Italian. Repeat parts **a** to **d** for your chosen language. (Ignore any accents on letters.)

f Describe any major differences in frequencies of certain letters between English and the language you have selected.

g Design a Scrabble game for your selected language, making sure that the 100 tiles are divided as closely as possible in the correct proportions according to their frequencies.

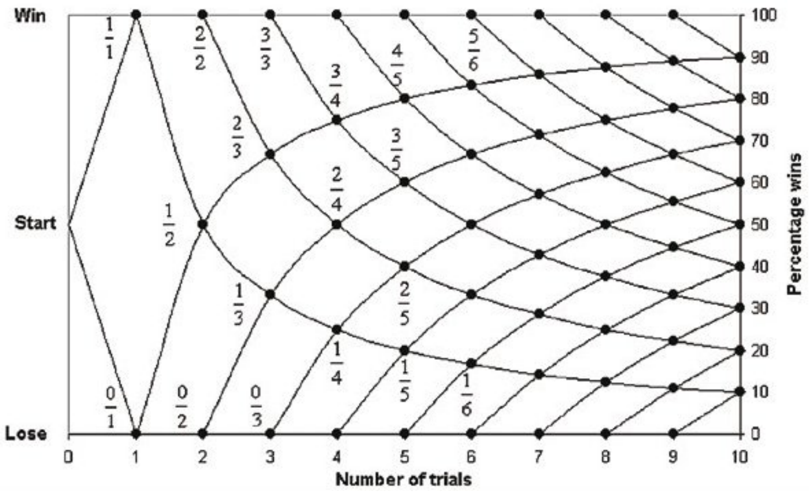
h Make a table to show the scores you would give to each letter in this language.

13.19 Probability charts

10 trial probability simulator

50 trial probability simulator

The probability chart below is a way of keeping a tally of the outcomes for 10 trials of an experiment such as tossing a coin.



Look at the patterns of fractions (not all of the fractions have been labelled on the chart). These represent all the possible fractions of heads as the coin is tossed a greater number of times, up to 10 times.

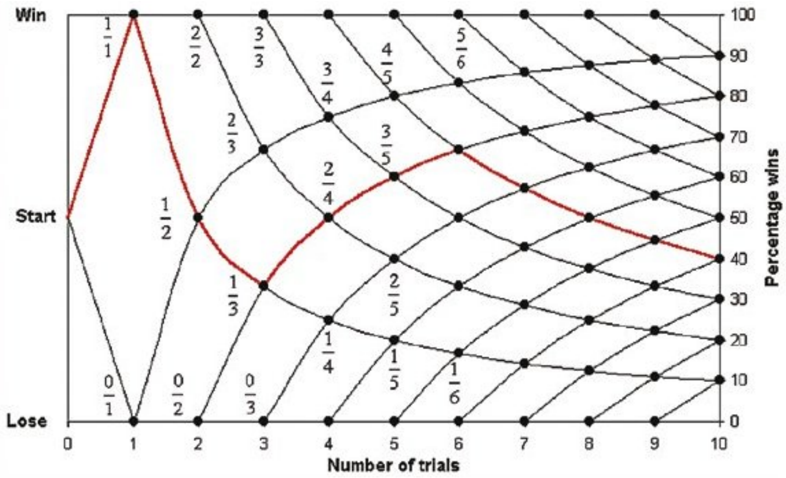
Supposing we choose heads to be a win. We toss the coin and if it lands heads, we draw a line on the graph from the start position up to the point labelled $\frac{1}{1}$ to show that 1 out of 1 of our tosses is a head. However, if the coin lands tails, we draw a line from the start position to the point labelled $\frac{0}{1}$. We then toss the coin again. Depending on whether the coin lands heads or tails, we then move in the following way:

Result	**Move**
Heads the first time, heads the second time	Move up to $\frac{1}{1}$ then along the top to the point labelled $\frac{2}{2}$ as we now have 2 heads out of a total of 2 tosses.
Heads the first time, tails the second time	Move up to $\frac{1}{1}$ then down the line to the point labelled $\frac{1}{2}$ because we have 1 head out of a total of 2 tosses.

Result	**Move**
Tails the first time, heads the second time	Move down the line to the point labelled $\frac{0}{1}$ then up the line to $\frac{1}{2}$ because we have 1 head out of a total of 2 tosses.
Tails the first time, tails the second time	Move down to $\frac{0}{1}$ then across the bottom to the point labelled $\frac{0}{2}$ because we have 0 heads out of a total of 2 tosses.

We continue in this way, moving up along a line to the next point if the coin lands heads and down along a line to the next point if the coin lands tails. When we have reached 10 tosses, the position we end at on the right side of the chart indicates the percentage of wins—in this case, the percentage of heads.

a List the order of heads and tails in the sequence of 10 trials shown in the following probability chart.



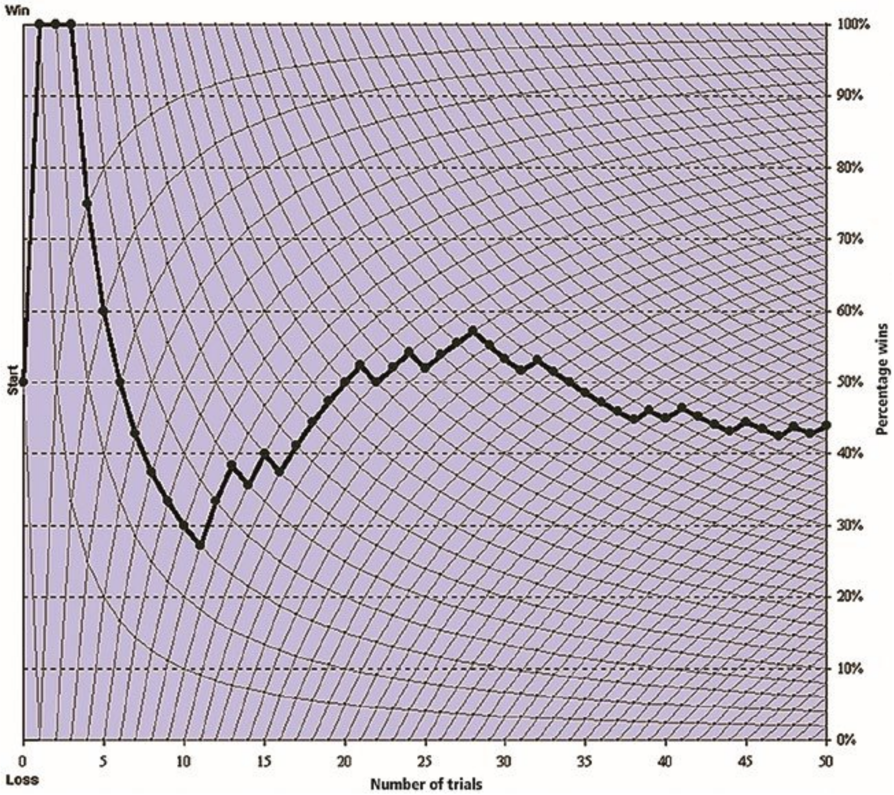
b What percentage of wins (that is, percentage of heads) was obtained in the ten trials?

c What was the longest run of heads in the ten trials shown in this probability chart? What was the longest run of tails?

d Using a 10-trial probability chart (in the student ebook), record your results for tossing a coin 10 times. Paste your chart into your mathematics workbook. What percentage of wins did you obtain?

e Compare your probability chart with those of other students in your class. Would you expect them to be the same? Explain.

f The following 50-trial probability chart shows the results from 50 tosses of a coin. What percentage of heads was obtained?



g What was the longest run of heads?

h What was the longest run of tails?

i Use a 50-trial probability chart (in the student e-book), toss a coin 50 times and record your results.

j What percentage of wins did you obtain?

k Compare your chart with those of other students and with the 10-trial chart above. Describe any differences between the pattern of heads and tails on the 10-trial and 50-trial charts.

Answers

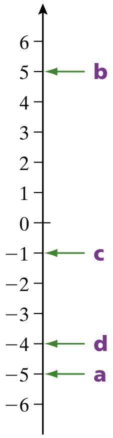
Chapter 1

exercise 1.1

1 **a** 1 **b** −4 **c** 11 **d** −9

2 **a** 6 **b** 12 **c** 17 **d** −7

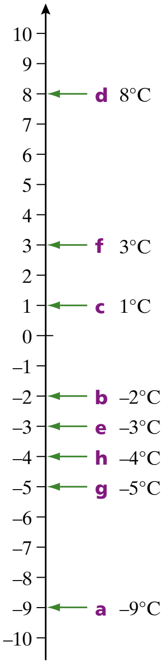
3



4 see below

5 see below

6



7 **a** 6, 3, 0, −7, −9 **b** 5, 4, 0, −1, −6
c 8, 3, −5, −6, −8 **d** 1, 0, −3, −7, −9

8 **a** −8, −4, 0, 2, 7 **b** −6, −4, −3, 0, 5
c −9, −8, −5, −1, 1 **d** −9, −3, −2, 0, 2

9 **a** −2, −1, 0 **b** −6, −5, −4, −3
c 2, 1, 0, −1, −2 **d** 4, 3, 2, 1, 0, −1

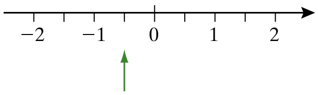
10 **a** 6 **b** −3 **c** 0 **d** −5

11 **a** $-3 > -7$ **b** $-9 < 0$
c $-2 < 1$ **d** $-4 > -9$

12 **a** 2, 1, 0, −1, −2 **b** 4, 1, −2, −5, −8
c −9, −5, −1, 3, 7 **d** 0, −5, −10, −15, −20
e 9, 2, −5, −12, −19 **f** 3, −4, −11, −18, −25
g 9, 5, 1, −3, −7 **h** −13, −8, −3, 2, 7

13 13°C warmer

14 $-0.5, -\frac{1}{2}$



exercise 1.2

1 **a** 5 **b** 2 **c** 0 **d** 7
e 2 **f** 1 **g** −2 **h** 5
i −1 **j** −1 **k** 4 **l** 4

2 **a** −5 **b** −14 **c** −10 **d** −7
e −2 **f** −7 **g** −5 **h** −9
i −13 **j** −9 **k** −10 **l** −8

3 **a** $3 - 9 = -6$ **b** $-4 + 7 = 3$
c $-2 - 5 = -7$ **d** $-20 + 6 = -14$

4 **a** −7 **b** 4 **c** −12 **d** 7
e −25 **f** −6 **g** −22 **h** 1
i −40 **j** 14 **k** −25 **l** −20

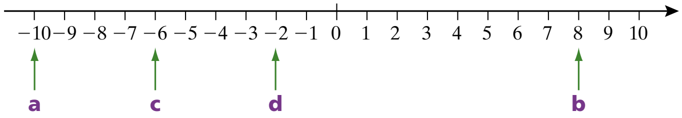
5 **a** −2 **b** −7 **c** −5 **d** 0
e −6 **f** 6 **g** −1 **h** 1
i 3 **j** 0 **k** 7 **l** −7

6 **a** −2 **b** 0 **c** 10 **d** −2
e −22 **f** −9 **g** −10 **h** −12
i −10 **j** −13 **k** 0 **l** −15

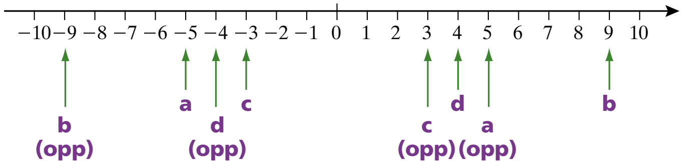
7 **a** $5 - 8 = -3$ **b** $-6 - 6 = -12$
c $-2 - 3 = -5$ **d** $-7 - 8 = -15$
e $-8 - 5 = -13$ **f** $8 - 5 = 3$
g $12 - 16 = -4$ **h** $-12 - 16 = -28$
i $7 - 7 = 0$ **j** $-5 - 20 = -25$
k $-12 - 13 = -25$ **l** $-11 - 9 = -20$

8 **a** $7 + 8 = 15$ **b** $-4 + 4 = 0$
c $-2 + 7 = 5$ **d** $-3 + 8 = 5$
e $-9 + 11 = 2$ **f** $6 + 6 = 12$
g $10 + 5 = 15$ **h** $-15 + 8 = -7$
i $-11 + 11 = 0$ **j** $-5 + 9 = 4$
k $-12 + 13 = 1$ **l** $-17 + 2 = -15$

4



5



9 **a** $11 - 4 = 7$ **b** $-10 + 10 = 0$
c $-14 + 7 = -7$ **d** $-5 - 2 + 3 = -4$
e $8 + 8 = 16$ **f** $-1 - 11 + 6 = -6$
g $-12 + 4 - 3 = -11$ **h** $-32 - 18 - 6 = -56$
i $7 - 17 + 8 = -2$ **j** $-14 + 17 + 9 = 12$
k $-9 - 23 + 5 = -27$ **l** $-21 + 3 - 5 = -23$

10 **a**

x	−3	−2	−1	0	1	2	3
y	−8	−7	−6	−5	−4	−3	−2

b

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1

c

x	−3	−2	−1	0	1	2	3
y	2	3	4	5	6	7	8

d

x	−3	−2	−1	0	1	2	3
y	−1	−2	−3	−4	−5	−6	−7

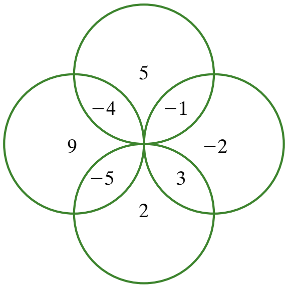
11

−9	−10	−5
−4	−8	−12
−11	−6	−7

12 **a** 8848 **b** −11 032
c $8848 - (-11\,032) = 19\,880$

13 $-1 + 1 + 0 + 0 + 2 + (-1) + (-2) + 0 + 3 = 2$, two shots over par

14



exercise 1.3

1

$6 \times 3 = 18$	$2 \times 3 = 6$	$-5 \times 3 = -15$	$-8 \times 3 = -24$
$6 \times 2 = 12$	$2 \times 2 = 4$	$-5 \times 2 = -10$	$-8 \times 2 = -16$
$6 \times 1 = 6$	$2 \times 1 = 2$	$-5 \times 1 = -5$	$-8 \times 1 = -8$
$6 \times 0 = 0$	$2 \times 0 = 0$	$-5 \times 0 = 0$	$-8 \times 0 = 0$
$6 \times (-1) = -6$	$2 \times (-1) = -2$	$-5 \times (-1) = 5$	$-8 \times (-1) = 8$
$6 \times (-2) = -12$	$2 \times (-2) = -4$	$-5 \times (-2) = 10$	$-8 \times (-2) = 16$
$6 \times (-3) = -18$	$2 \times (-3) = -6$	$-5 \times (-3) = 15$	$-8 \times (-3) = 24$

2

×	−	+
+	−	+
−	+	−

3 **a** −12 **b** −10 **c** 4 **d** −30
e −28 **f** 30 **g** −70 **h** 32
i −18 **j** −18 **k** −56 **l** 54
m 48 **n** −100 **o** −77 **p** 36

4 **a** −24 **b** 60 **c** 70 **d** 54
e −50 **f** 56 **g** 48 **h** −90
i 56 **j** 280 **k** 44 **l** −72
m 72 **n** −96 **o** 140 **p** 72
q 36 **r** −48

5 **a**

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

b

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

c

x	−3	−2	−1	0	1	2	3
y	12	8	4	0	−4	−8	−12

d

x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

e

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

f

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

6 **a** −8 **b** 12 **c** 24 **d** −24

7 **a** 25 **b** 25 **c** −25 **d** −25
e 1 **f** −9 **g** −49 **h** −4
i 64 **j** 81 **k** −81 **l** −81

8 **a** −1 **b** 1 **c** −32 **d** 81
e 1 **f** −1 **g** −64 **h** −125

9 **a** 8 **b** −7
c does not exist in the real number system
d 10
e does not exist in the real number system
f −10 **g** 6 **h** −6 **i** 1
j −1 **k** 5 **l** −5

10 **a**

×	−4	−8	5
9	−36	−72	45
−7	28	56	−35
−8	32	64	−40

b

×	−7	−6	−2
12	−84	−72	−24
−8	56	48	16
9	−63	−54	−18

exercise 1.4

1 **a** $-7\times\boxed{-8}=56$

$$\frac{56}{-7}=-8 \qquad \frac{56}{-8}=-7$$

b $-4\times\boxed{9}=-36$

$$\frac{-36}{9}=-4 \qquad \frac{-36}{-4}=9$$

c $-5\times\boxed{-12}=60$

$$\frac{60}{-12}=-5 \qquad \frac{60}{-5}=-12$$

d $-6\times\boxed{4}=-24$

$$\frac{-24}{-6}=4 \qquad \frac{-24}{4}=-6$$

e $-6\times\boxed{-5}=30$

$$\frac{30}{-6}=-5 \qquad \frac{30}{-5}=-6$$

f $-7\times\boxed{-6}=42$

$$\frac{42}{-7}=-6 \qquad \frac{42}{-6}=-7$$

g $-9\times\boxed{9}=-81$

$$\frac{-81}{9}=-9 \qquad \frac{-81}{-9}=9$$

h $-3\times\boxed{18}=-54$

$$\frac{-54}{-3}=18 \qquad \frac{-54}{18}=-3$$

i $-4\times\boxed{-14}=56$

$$\frac{56}{-14}=-4 \qquad \frac{56}{-4}=-14$$

j $-15\times\boxed{-3}=45$

$$\frac{45}{-3}=-15 \qquad \frac{45}{-15}=-3$$

k $-6\times\boxed{13}=-78$

$$\frac{-78}{13}=-6 \qquad \frac{-78}{-6}=-13$$

l $-11\times\boxed{-12}=132$

$$\frac{132}{-12}=-11 \qquad \frac{132}{-11}=-12$$

2

÷	−	+
+	−	+
−	+	−

3 **a** −2 **b** −5 **c** 5 **d** −7
e 3 **f** −5 **g** −7 **h** 9
i −12 **j** 3 **k** −5 **l** −6

4 **a** −6 **b** −8 **c** 10 **d** −5
e 0 **f** 5 **g** −5 **h** 9
i −15 **j** 16 **k** −5 **l** −15

5 **a** −6 **b** 3 **c** 10 **d** −8
e 9 **f** −3 **g** 7 **h** 6
i 3 **j** 8 **k** −12 **l** 9

6 **a**

x	−6	−4	−2	0	2	4	6
y	−3	−2	−1	0	1	2	3

b

x	−12	−9	−6	−3	0	3	6
y	4	3	2	1	0	−1	−2

c

x	−12	−9	−6	−3	0	3	6
y	8	6	4	2	0	−2	−4

d

x	−12	−8	−4	0	4	8	12
y	9	6	3	0	−3	−6	−9

7 **a** $17\div(-5)=-3$ remainder 2
$17=-5\times(-3)+2$
b $-17\div5=-3$ remainder -2
$-17=5\times(-3)+(-2)$
c $-17\div5=-4$ remainder 3
$17=5\times(-4)+3$
d **i** $-23\div4=-5$ remainder -3
$-23=4\times(-5)+(-3)$
$-23\div4=-6$ remainder 1
$-23=4\times(-6)+1$
ii $-17\div4=-4$ remainder -1
$-17=4\times(-4)+(-1)$
$-17\div4=-5$ remainder 3
$-17=4\times(-5)+3$
iii $-47\div8=-5$ remainder -7
$-47=8\times(-5)+(-7)$
$-47\div8=-6$ remainder 1
$-47=8\times(-6)+1$
iv $-37\div9=-4$ remainder -1
$-37=9\times(-4)+(-1)$
$-37\div9=-5$ remainder 8
$-37=9\times(-5)+8$
v $-13\div(-2)=6$ remainder -1
$-13=-2\times6+(-1)$
$-13\div(-2)=7$ remainder 1
$-13=-2\times7+1$

exercise 1.5

1 **a** 3 **b** 31 **c** −7 **d** −17
e −20 **f** −13 **g** 0 **h** 2

2 **a** −2 **b** −2 **c** 1 **d** 2
e −11 **f** 4 **g** −2.2 **h** 6.5

3 **a** 22 **b** 4 **c** 14 **d** −64
e −40 **f** −22 **g** 36 **h** −2

4 E

5 A

6 $-5 + 28 \div 7 \times (-2) + (-3)^2$
$= -5 + 28 \div 7 \times (-2) + 9$
$= -5 + 4 \times (-2) + 9$
$= -5 + (-8) + 9$
$= -5 - 8 + 9$
$= -4$

7 **a** 4 **b** −20 **c** 2 **d** −4

8 **a**

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

b

x	−3	−2	−1	0	1	2	3
y	−13	−10	−7	−4	−1	2	5

c

x	−3	−2	−1	0	1	2	3
y	11	9	7	5	3	1	−1

d

x	6	4	2	0	2	4	6
y	6	5	4	3	2	1	0

9 **a** **i** 17 **ii** 1 **iii** 1
b ii and iii are the same because $3^2 = (-3)^2$

10 the sixth number is −10

11 **a** 10 **b** − 3 **c** − 5 **d** − 16

Revision

1 D **2** C **3** A **4** E **5** B

6 **a** −16 **b** 8 **c** −15
d −11 **e** 24 **f** 6

7 **a** −30 **b** −63 **c** 56
d −60 **e** −108 **f** 42

8 **a** −4 **b** −4 **c** 8
d 4 **e** −38 **f** 24
g −8 **h** −8 **i** 6

9 **a** 16 **b** −49 **c** −25 **d** 16 **e** −6

f does not exist in the real number system
g 7 **h** −1
i − 4 **j** 3
k − 3 **l** 1

10 **a** −21 **b** −4 **c** 24 **d** 4

11 **a**

x	−3	−2	−1	0	1	2	3
y	−10	−9	−8	−7	−6	−5	−4

b

x	−3	−2	−1	0	1	2	3
y	−15	−10	−5	0	5	10	15

c

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

d

x	6	4	2	0	2	4	6
y	2	3	4	5	6	7	− 8

12

−3	−6	1	−6
0	−5	−4	−5
−14	−7	2	5
3	4	−13	−8

Chapter 2

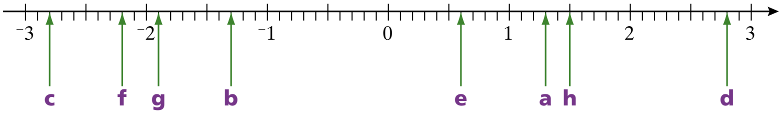
exercise 2.1

1 **a** $-\frac{2}{3}$ **b** $3\frac{1}{4}$ **c** −1.34 **d** 0.02
e $\frac{7}{8}$ **f** $-1\frac{3}{7}$ **d** 2.6 **h** −0.58

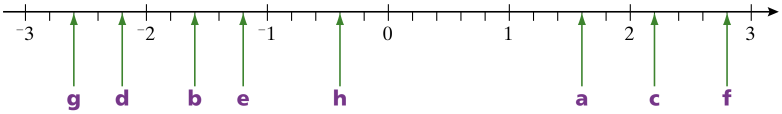
2 **a** 2 **b** $\frac{1}{5}$ **c** $-\frac{1}{5}$ **d** $1\frac{1}{5}$
e $-1\frac{1}{5}$ **f** $-2\frac{2}{5}$ **g** $-1\frac{4}{5}$ **h** $-2\frac{4}{5}$

4 **a** 0.6 **b** 2.5 **c** −2.5 **d** −0.7
e 1.6 **f** −1.9 **g** −2.8 **h** −1.2

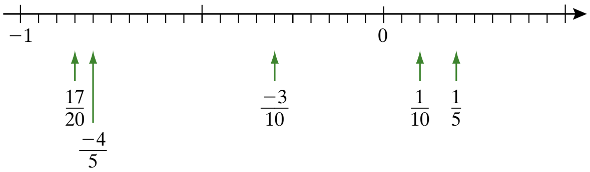
3



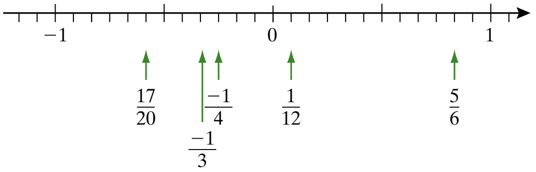
5



6 **a** $-\frac{17}{20}, -\frac{4}{5}, -\frac{3}{10}, \frac{1}{10}, \frac{1}{5}$



b $-\frac{7}{12}, -\frac{1}{3}, -\frac{1}{4}, \frac{1}{12}, \frac{5}{6}$



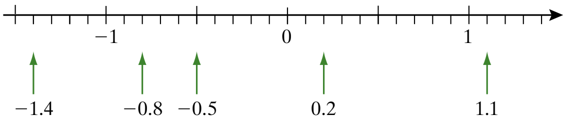
7 **a** $\frac{5}{2}$ **b** $-\frac{9}{4}$ **c** $\frac{23}{6}$ **d** $-\frac{16}{5}$
e $-\frac{41}{16}$ **f** $-\frac{15}{11}$ **g** $-\frac{26}{9}$ **h** $-\frac{16}{3}$

8 **a** $2\frac{1}{8} > 1\frac{7}{8}$ **b** $-2\frac{1}{8} < -1\frac{7}{8}$
c $\frac{1}{8} > -\frac{3}{8}$ **d** $-2\frac{1}{8} < -1\frac{6}{8}$
e $1\frac{6}{7} > 1\frac{4}{7}$ **f** $-1\frac{6}{7} < -1\frac{4}{7}$
g $1\frac{4}{7} > -1\frac{6}{7}$ **h** $-1\frac{4}{7} < 1\frac{6}{7}$

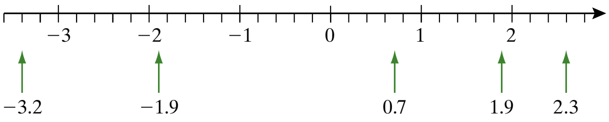
9 **a** $2\frac{3}{11}, 1\frac{3}{11}, -1\frac{8}{11}, -2\frac{1}{11}, -2\frac{10}{11}$
b $2\frac{5}{7}, 1\frac{1}{7}, -1\frac{6}{7}, -2\frac{2}{7}, -2\frac{4}{7}$

10 **a** $+\frac{2}{5}, +\frac{3}{5}, +\frac{4}{5}$ **b** $-\frac{12}{13}, -1, -1\frac{1}{13}$
c $-\frac{1}{7}, 0, \frac{1}{7}$ **d** $\frac{1}{3}, 0, -\frac{1}{3}$
e $-2\frac{10}{11}, -3, -3\frac{1}{11}$ **f** $-2, -2\frac{1}{8}, -2\frac{2}{8}$

11 **a** 1.1, 0.2, −0.5, −0.8, −1.4



b 2.3, 1.9, 0.7, −1.9, −3.2



12 **a** $2.8 > 2.6$ **b** $5.08 < 5.13$ **c** $6.02 < 6.2$
d $3.26 > 3.2$ **e** $0.265 < 0.652$ **f** $9.55 > 7.552$
g $6.19 > 5.47$ **h** $3.462 > 3.46$ **i** $5.7 < 5.777$
j $8.012 < 8.12$ **k** $2.71 < 2.8$ **l** $1.42 > 0.358$
m $4.8 > 4.73$ **n** $9.351 < 9.62$ **o** $0.33 < 0.572$

13 **a** −0.3 **b** −2.1 **c** +5.9 **d** +0.4
e −4.16 **f** −7.45 **g** +1.81 **h** −3.3

14 **a** 4.62, 4.69, 4.71, 4.75
b 6.02, 6.12, 6.20, 6.21
c 1.35, 1.53, 3.15, 3.51
d 8.002, 8.020, 8.022, 8.202
e 2.446, 2.464, 2.64, 4.26. 4.624
f 5.029, 5.092, 5.29, 9.502, 9.52
g 7.11, 7.114, 7.14, 7.41, 7.414
h 1.263, 1.63, 3.26, 3.261, 3.6

15 **a** 4.3, 3.4, 3.04, 3
b 8.7, 8, 7.78, 7
c 6.5, 6.105, 5.6, 5.16, 5
d 2.711, 2.7, 2.171, 2.17, 2.117
e 4.604, 4.064, 3.06, 2.64, 2.446
f 3.078, 2.78, 2.087, 1.57, 1.507
g 1.939, 1.93, 1.39, 1.339, 1.33
h 6.64, 6.6, 6.564, 6.56, 6.546

16 **a** −3.1 °C, −2.7 °C, −2.3 °C, 0.4 °C, 0.8 °C, 1.5 °C
b −6.4 °C, −6.2 °C, −5.6 °C, −5.5 °C, −4.9 °C, −4.3 °C

17 the second—putting both marks in 60ths they are $\frac{45}{60}$ and $\frac{48}{60}$

18 Japanese, English, Maths

19 **a** 6.47, 6.48 **b** 2.61, 2.62 **c** 5.51, 5.52
d 7.63, 7.64 **e** 21.74, 21.75 **f** 0.00, 0.01
g 46.38, 46.39 **h** 14.46, 14.47

20 **a** 2.76 **b** 0.42 **c** 9.33 **d** 6.46
e 4.02 **f** 8.82 **g** 3.30 **h** 2.00

21 **a** 3.1 **b** 98.91 **c** 7.4224 **d** 131.585
e 0.0 **f** 3.83 **g** 225.0 **h** 52.50
i 37.30 **j** 5.700 **k** 15.6 **l** 28.3729
m 40.27 **n** 52.000 **o** 16.41 **p** 120.073

22 **a** −2.7 °C **b** −3.3 °C **c** −3.8 °C **d** −3.3 °C

23 Answers will vary. $\frac{13}{20}, \frac{7}{10}, \frac{3}{4}$ are possibilities.

24 Answers will vary. −0.018, −0.022, −0.039 and −0.041 are possibilities.

exercise 2.2

1 **a** 0.3 **b** 0.25 **c** 0.875 **d** −0.6
e 0.5 **f** 0.36 **g** 0.125 **h** −0.45
i −0.625 **j** 0.22

2 **a** **i** $0.\overline{2}$ **ii** 0.22
b **i** $0.\overline{8}$ **ii** 0.89
c **i** $0.\overline{7}$ **ii** 0.78
d **i** $0.\overline{6}$ **ii** 0.67
e **i** $0.\overline{083}$ **ii** 0.08
f **i** $0.\overline{916}$ **ii** 0.92
g **i** $0.\overline{5}$ **ii** 0.56
h **i** $0.58\overline{3}$ **ii** 0.58
i **i** $0.\overline{416}$ **ii** 0.42
j **i** $0.8\overline{3}$ **ii** 0.83

3 **a** **i** $0.\overline{09}$ **ii** 0.09
b **i** $0.\overline{18}$ **ii** 0.18
c **i** $0.\overline{27}$ **ii** 0.27
d **i** $1.\overline{36}$ **ii** 1.36
e **i** $1.\overline{72}$ **ii** 1.73
f **i** $0.1\overline{36}$ **ii** 0.14
g **i** $0.4\overline{09}$ **ii** 0.41
h **i** $0.9\overline{54}$ **ii** 0.95

4 $\frac{1}{7} = 0.\overline{142857}$, $\frac{2}{7} = 0.\overline{285714}$, $\frac{3}{7} = 0.\overline{428571}$, $\frac{4}{7} = 0.\overline{571428}$, $\frac{5}{7} = 0.\overline{714285}$, $\frac{6}{7} = 0.\overline{857142}$. The recurring digits are always the same, 1, 2, 4, 5, 7 and 8, but reordered.

5 **a** 2.5 **b** 7.4 **c** $1.\overline{6}$ **d** 5.75
e 3.08 **f** $1.\overline{4}$ **g** $2.\overline{3}$ **h** 2.26
i $5.1\overline{6}$ **j** $6.\overline{285714}$

6 terminating: $\frac{7}{8}, \frac{9}{20}, \frac{19}{50}, \frac{11}{25}, \frac{1}{4}, \frac{3}{10}, \frac{3}{5}, \frac{9}{16}, \frac{31}{64}$
recurring: $\frac{7}{12}, \frac{5}{9}, \frac{4}{13}, \frac{2}{3}, \frac{12}{17}, \frac{5}{6}, \frac{7}{18}, \frac{4}{15}, \frac{8}{21}, \frac{4}{11}, \frac{6}{7}$

7 B

8 **a** $\frac{7}{10}$ **b** $\frac{1}{10}$ **c** $\frac{3}{5}$ **d** $\frac{4}{5}$
e $\frac{1}{2}$ **f** $\frac{1}{25}$ **g** $\frac{13}{1000}$ **h** $\frac{9}{20}$
i $\frac{17}{50}$ **j** $\frac{2}{125}$

9 **a** $8\frac{3}{10}$ **b** $4\frac{9}{10}$ **c** $3\frac{1}{2}$ **d** $5\frac{1}{5}$
e $2\frac{2}{5}$ **f** $7\frac{3}{100}$ **g** $9\frac{7}{20}$ **h** $6\frac{11}{500}$
i $1\frac{51}{250}$ **j** $4\frac{3}{125}$

10 C

11 **a** $\frac{2}{3}$ **b** $\frac{5}{8}$ **c** $\frac{3}{11}$
d Answers will vary.

12 **a** $\frac{2}{3}$ **b** $\frac{2}{9}$ **c** $\frac{5}{9}$
d $\frac{4}{11}$ **e** $\frac{82}{99}$ **f** $\frac{214}{999}$

13 $\frac{12\,345}{99\,999}\left(= \frac{4115}{33\,333}\right)$

exercise 2.3

1 **a** $\frac{5}{7}$ **b** $\frac{2}{9}$ **c** $\frac{1}{3}$ **d** $\frac{2}{5}$
e $\frac{1}{12}$ **f** $1\frac{1}{15}$ **g** $\frac{1}{8}$ **h** $\frac{17}{21}$
i $\frac{5}{9}$ **j** $\frac{19}{24}$ **k** $1\frac{5}{18}$ **l** $1\frac{7}{30}$
m $\frac{11}{35}$ **n** $1\frac{1}{18}$ **o** $\frac{3}{20}$ **p** $\frac{11}{16}$

2 **a** $3\frac{5}{7}$ **b** $2\frac{10}{11}$ **c** $2\frac{7}{9}$ **d** $5\frac{2}{3}$
e $2\frac{2}{7}$ **f** $4\frac{1}{3}$ **g** $3\frac{2}{5}$ **h** $3\frac{2}{9}$
i $3\frac{5}{6}$ **j** $2\frac{8}{9}$ **k** $2\frac{11}{12}$ **l** $4\frac{1}{5}$
m $2\frac{13}{24}$ **n** $4\frac{1}{4}$ **o** $3\frac{2}{21}$ **p** $3\frac{11}{20}$

3 **a** $2\frac{1}{5}$ **b** $1\frac{4}{7}$ **c** $1\frac{1}{3}$ **d** $1\frac{2}{9}$
e $\frac{2}{3}$ **f** $1\frac{3}{5}$ **g** $1\frac{5}{9}$ **h** $\frac{6}{7}$
i $1\frac{1}{4}$ **j** $1\frac{1}{2}$ **k** $1\frac{1}{12}$ **l** $2\frac{1}{6}$
m $\frac{9}{20}$ **n** $1\frac{5}{8}$ **o** $\frac{13}{18}$ **p** $1\frac{8}{15}$

4 **a** $\frac{6}{7}$ **b** $1\frac{3}{5}$ **c** $3\frac{1}{3}$ **d** 105
e $22\frac{1}{2}$ **f** 2 **g** 4 **h** 15
i $\frac{2}{21}$ **j** $\frac{3}{20}$ **k** $\frac{8}{25}$ **l** $1\frac{4}{5}$

5 **a** $\frac{2}{21}$ **b** $\frac{3}{20}$ **c** $\frac{1}{10}$ **d** $\frac{2}{15}$
e $\frac{8}{15}$ **f** $\frac{12}{35}$ **g** $\frac{15}{28}$ **h** $\frac{10}{33}$
i $\frac{5}{24}$ **j** $\frac{8}{45}$ **k** $\frac{6}{49}$ **l** $\frac{8}{21}$

6 **a** $\frac{3}{10}$ **b** $\frac{7}{16}$ **c** $-\frac{5}{8}$ **d** $\frac{4}{27}$
e $-\frac{4}{21}$ **f** $\frac{6}{55}$ **g** $\frac{14}{45}$ **h** $\frac{4}{45}$
i $\frac{1}{6}$ **j** $\frac{2}{9}$ **k** $\frac{3}{40}$ **l** $\frac{3}{10}$

7 **a** $2\frac{1}{2}$ **b** $2\frac{3}{4}$ **c** 2 **d** $1\frac{4}{5}$
e 1 **f** $\frac{5}{8}$ **g** $-7\frac{1}{3}$ **h** $-1\frac{5}{7}$
i $3\frac{1}{3}$ **j** $-1\frac{6}{7}$ **k** $-2\frac{2}{5}$ **l** 7

8 **a** 8 **b** $3\frac{3}{4}$ **c** 10 **d** $7\frac{1}{2}$
e $1\frac{4}{5}$ **f** $\frac{7}{9}$ **g** $-1\frac{1}{3}$ **h** $-1\frac{1}{9}$
i $1\frac{5}{9}$ **j** $-1\frac{1}{3}$ **k** $2\frac{5}{8}$ **l** $-\frac{16}{21}$

9 **a** $\frac{3}{4}$ **b** $1\frac{1}{3}$ **c** $1\frac{5}{9}$ **d** $1\frac{3}{8}$
e $1\frac{1}{3}$ **f** $1\frac{1}{2}$ **g** $-1\frac{1}{5}$ **h** $2\frac{2}{3}$
i $-2\frac{4}{7}$ **j** $-1\frac{1}{3}$ **k** $1\frac{1}{2}$ **l** $-2\frac{2}{3}$

10 **a** $3\frac{3}{8}$ **b** $2\frac{11}{20}$ **c** $2\frac{27}{40}$ **d** $2\frac{11}{12}$
e $2\frac{13}{16}$ **f** $-1\frac{9}{20}$ **g** 2 **h** $9\frac{7}{12}$

11 $1\frac{1}{12}$ hours

12 $\frac{11}{12}$

13 $\frac{5}{24}$

14 $3\frac{11}{12}$ trays

15 $2\frac{1}{12}$ hours

16 $3\frac{11}{12}$ litres

17 $1\frac{19}{11}$

18 $22\frac{1}{2}$ minutes

19 14 km

20 210

21 14

22 $2\frac{1}{4}$ m

23 90 km

24 4

25 **a** **i** 2 **ii** $2\frac{1}{2}$ **iii** 3
b $4\frac{1}{2}$
c Correct
d Most of the numerators and denominators cancel leaving only the first denominator and the last numerator.
e $\frac{n+1}{2}$

exercise 2.4

1 **a** 23.773 **b** 49.427 **c** 27.13 **d** 15.047
e 59.265 **f** 67.60 **g** 529.49 **h** 114.68
i 47.674 **j** 88.34

2 **a** 3.061 **b** 3.48 **c** 12.118 **d** 12.417
e 1.186 **f** 23.596 **g** 118.43 **h** 164.73
i 372.854 **j** 26.869 **k** −4.8 **l** −3.4

3 **a** 83 **b** −77.76 **c** 382.2 **d** −172.8
e 2450 **f** 48 000 **g** 73.6 **h** 17 340

4 **a** 1.38 **b** 0.114 **c** 5.81 **d** 0.365
e 0.1383 **f** 36.72 **g** 6.24 **h** 9.961
i 2.4 **j** 2.25 **k** 64.8 **l** 2.6
m 0.08 **n** 0.015 **o** 0.0008 **p** 0.72

5 **a** 0.0593 **b** 0.0186 **c** 3.495
d 0.00236 **e** 0.000048 **f** 0.00389
g 0.0001472 **h** 0.2006

6 **a** 22.55 **b** 5.46 **c** 0.465 **d** 5.75
e 13.315 **f** 0.1695 **g** 80.68 **h** 0.030375
i $6.0\overline{6}$ **j** $2.7\overline{8}$ **k** $0.85\overline{6}$ **l** $8.7\overline{6}$
m $4.22\overline{81}$ **n** $1.04\overline{3}$ **o** $6.158\overline{3}$ **p** $0.33\overline{72}$

7 **a** $50.1 \div 3 = 16.7$ **b** $78.6 \div 6 = 13.1$
c $142.8 \div 7 = 20.4$ **d** $61.2 \div 9 = 6.8$
e $130.8 \div 4 = 32.7$ **f** $37.4 \div 11 = 3.4$
g $93.48 \div 12 = 7.79$ **h** $3543 \div 5 = 708.6$
i $1\,920\,000 \div 12 = 160\,000$

8 11.04 g

9 19.44 m^2

10 \$18.40

11 \$10.54

12 \$48.64

13 408.9 km

14 18.3 kg

15 51.84 s

16 67.33 minutes

17 11.424 L, 685.44 L

18 **a** $6.4 \times 4.4 = 28.16$
b $6.37 \times 4.39 = 27.9643 \approx 27.96$
c Answer **b** is closer to the true answer. By rounding early, accuracy is lost.

exercise 2.5

1 **a** $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{4}{25}$ **d** $\frac{9}{16}$
e $\frac{4}{9}$ **f** $\frac{9}{25}$ **g** $\frac{16}{81}$ **h** $\frac{25}{36}$

2 **a** $1\frac{7}{9}$ **b** $2\frac{1}{4}$ **c** $2\frac{7}{9}$ **d** $6\frac{1}{4}$
e $5\frac{4}{9}$ **f** $7\frac{1}{9}$ **g** $12\frac{1}{4}$ **h** $11\frac{1}{9}$

3 **a** 0.01 **b** 0.16 **c** 0.64 **d** 0.0036
e 0.09 **f** 0.0144 **g** 0.000121 **h** 0.000081

4 **a** 0.25 **b** 0.49 **c** 0.04 **d** 0.0025
e 0.0049 **f** 0.64 **g** 0.000001 **h** 0.000144

5 $6\frac{1}{4}$ cm^2

6 **a** $1\frac{5}{6}$ m **b** $3\frac{13}{36}$ m^2

7 **a** $12\frac{1}{4}$ m^2 **b** 490 g

8 1.21 m^2

9 0.16 m^2

10 0.09 m^2

11

Square	Meaning	Value
$(0.4)^2$	0.4×0.4	0.16
$(0.3)^2$	0.3×0.3	0.09
$(0.12)^2$	0.12×0.12	0.0144
$(0.02)^2$	0.02×0.02	0.0004
$(0.09)^2$	0.09×0.09	0.0081
$(0.007)^2$	0.007×0.007	0.000049
$(0.5)^2$	0.5×0.5	0.25
$(0.08)^2$	0.08×0.08	0.0064

12 $2\frac{7}{9}$ m^2

exercise 2.6

1 **a** $\frac{1}{5}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $\frac{5}{7}$
e $\frac{3}{4}$ **f** $\frac{5}{6}$ **g** $\frac{9}{10}$ **h** $\frac{11}{12}$

2 **a** $1\frac{1}{3}$ **b** $1\frac{1}{2}$ **c** $1\frac{2}{3}$ **d** $2\frac{1}{2}$
e $2\frac{1}{3}$ **f** $3\frac{1}{2}$ **g** $2\frac{2}{3}$ **h** $3\frac{1}{3}$

3

Rational numbers	Irrational numbers
$\frac{2}{7}$	$\sqrt{13}$
21	$\sqrt{7}$
0.783	$\sqrt{23}$
−2.5	$\sqrt{40}$
$\sqrt{\frac{4}{9}}$	
$0.\overline{18}$	
$-\frac{5}{8}$	
$\frac{3}{20}$	
$\sqrt{9}$	
$0.\overline{285714}$	

4 **a** 0.8 **b** 0.1 **c** 0.2 **d** 0.5
e 0.07 **f** 0.06 **g** 0.003 **h** 1.1

5 0.8 km

6 1.2 m

7 **a** no exact value **b** 0.9 **c** 0.07
d 0.011 **e** no exact value **f** 0.01

8 $1\frac{1}{2}$ m

9 $1\frac{2}{3}$ m

10 **a** $2\frac{1}{2}$ m **b** 10 m

11 **a** 0.577350269… **b** 0.577350269…

They are equivalent expressions.

This can be seen as:

$$\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}, \text{ so } \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Revision

1 E **2** A **3** D **4** B **5** E

6 **a** $\frac{9}{10}$ **b** $-1\frac{3}{5}$ **c** $-\frac{3}{10}$
d $1\frac{2}{5}$ **e** $-1\frac{1}{5}$ **f** $1\frac{9}{10}$

7 **a** −0.8 **b** 1.6 **c** 0.9
d −1.3 **e** −2.1 **f** 0.1

8 $-4\frac{1}{2}, -3.5, -3, -2\frac{1}{2}, -0.84,$
$-0.746, 0.002, 3.38, 3.4, 3\frac{1}{2}$

9 **a** 0.875 **b** $0.\overline{857142}$ **c** $0.2\overline{6}$ **d** 0.575
e $1.\overline{6}$ **f** 2.8 **g** $1.\overline{36}$ **h** 3.85

10 **a** $\frac{17}{20}$ **b** $1\frac{6}{25}$ **c** $\frac{106}{125}$ **d** $\frac{1}{8}$
e $\frac{19}{25}$ **f** $\frac{17}{40}$ **g** $2\frac{59}{100}$ **h** $\frac{1}{25}$

11 **a** $1\frac{1}{24}$ **b** $4\frac{4}{15}$ **c** $\frac{1}{6}$ **d** $\frac{2}{3}$

12 **a** 3, 6, 7 and 9 **b** $\frac{2}{3}, \frac{4}{9}, \frac{1}{6}, \frac{2}{7}$

12 **a** $7\frac{1}{5}$ **b** $\frac{1}{3}$ **c** $\frac{1}{15}$ **d** $\frac{1}{12}$
e 4 **f** $\frac{1}{6}$ **g** $2\frac{7}{10}$ **h** $\frac{2}{5}$

13 **a** 136.974 **b** 51.102

14 **a** 0.048 **b** 0.0617 **c** 4.28 **d** 1270

15 **a** $\frac{9}{25}$ **b** $6\frac{1}{4}$ **c** $-\frac{1}{16}$ **d** $\frac{25}{49}$
e $\frac{1}{3}$ **f** $1\frac{1}{4}$ **g** $1\frac{1}{2}$ **h** $3\frac{1}{3}$

16 **a** 0.49 **b** 0.0036 **c** 0.4 **d** 0.05

17 $\frac{17}{20}$ was the best result. If we convert both marks to a common denominator we get $\frac{17}{20} = \frac{51}{60}$ and $\frac{25}{30} = \frac{50}{60}$, so we can see that $\frac{17}{20}$ is greater than $\frac{25}{30}$.

18 $4\frac{1}{12}$ litres

19 $7\frac{1}{7}$, so 8 possums would be expected in this area

20 6.5 km

21 46.9 seconds

22 40

23 $5.90

24 27 and 8, 54 and 16, 81 and 24

Chapter 3

exercise 3.1

1

		Base	Index
a	3^6	3	6
b	6^3	6	3
c	7^4	7	4
d	10^5	10	5
e	2^{12}	2	12

2 **a** 3^5 **b** 2^8 **c** 10^6 **d** 5^5 **e** 4^2 **f** 7^3
g 6^4 **h** 4^6 **i** 7^5 **j** 2^9 **k** 12^8 **l** 36^3

3 **a** $2^2 \times 5^3$ **b** $2^2 \times 7^4$
c $2^4 \times 3^3$ **d** $3^4 \times 5^2$
e $2^3 \times 7^2$ **f** $3 \times 5^3 \times 7^2$
g $2^4 \times 5^2 \times 7$ **h** $2 \times 5^4 \times 7^2$
i $5 \times 7^4 \times 11^2$ **j** $2 \times 3^2 \times 5^4$

4 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 3$
b $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
c $13 \times 13 \times 13$
d $4 \times 4 \times 4 \times 4$
e $5 \times 5 \times 5 \times 5 \times 5$
f $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
g $11 \times 11 \times 11 \times 11$
h $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

5 **a** $2 \times 2 \times 5 \times 5 \times 5$
b $2 \times 2 \times 2 \times 3 \times 3$
c $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7$
d $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11 \times 11$
e $2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$
f $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$
g $2 \times 3 \times 3 \times 5 \times 5 \times 5$
h $2 \times 3 \times 7 \times 7 \times 7$
i $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 13$
j $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 17$
k $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11$
l $13 \times 13 \times 17 \times 17 \times 17$

6 Answers may vary, but sample correct answers are shown.
a $16 = 4^2 = 2^4$ **b** $64 = 8^2 = 2^6$
c $625 = 25^2 = 5^4$ **d** $256 = 4^4 = 2^8$

7 **a** 32 768 **b** 35 831 808
c 1 544 804 416 **d** 130 691 232
e 7776 **f** 4913
g 331 776 **h** 170 859 375

8

Words	Index Form	Factor Form	Value
four squared	4^2	4×4	16
eight cubed	8^3	$8 \times 8 \times 8$	512
fourteen cubed	14^3	$14 \times 14 \times 14$	2744
eighteen squared	18^2	18×18	324
twenty eight squared	28^2	28×28	784
fifty-two cubed	52^3	$52 \times 52 \times 52$	14 068

9 **a** 1024 **b** 1 048 576
c 6561 **d** 531 441
e 2 176 782 336 **f** 823 543
g 429 981 696 **h** 1 889 568

10 **a** 3888 **b** 5000
c 839 808 **d** 10 125
e 67 500 **f** 63 000
g 508 032 **h** 252 130 725

11 **a** **i**

2	1512
2	756
2	378
3	189
3	63
3	21
7	7
	1

ii $2^3 \times 3^3 \times 7$

b **i**

3	2025
3	675
3	225
3	75
5	25
5	5
	1

ii $3^4 \times 5^2$

c **i**

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

ii $2^6 \times 3^3$

d **i**

2	392
2	196
2	98
7	49
7	7
	1

ii $2^3 \times 7^2$

e **i**

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

ii $3^4 \times 7^2$

f **i**

2	2200
2	1100
2	550
5	275
5	55
11	11
	1

ii $2^3 \times 5^2 \times 11$

g **i**

3	7875
3	2625
5	875
5	175
5	35
7	7
	1

ii $3^2 \times 5^3 \times 7$

h **i**

3	5625
3	1875
5	625
5	125
5	25
5	5
	1

ii $3^2 \times 5^4$

i **i**

2	1350
3	675
3	225
3	75
5	25
5	5
	1

ii $2 \times 3^3 \times 5^2$

j **i**

2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

ii $2^3 \times 3^2 \times 7^2$

k **i**

2	12 960
2	6480
2	3240
2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

ii $2^5 \times 3^4 \times 5$

l **i**

3	121 275
3	40 425
5	13 475
5	2695
7	539
7	77
11	11
	1

ii $3^2 \times 5^2 \times 7^2 \times 11$

12 **a** **i** $30 = 2 \times 3 \times 5,\ 54 = 2 \times 3^3$
ii 6
b **i** $56 = 2^3 \times 7,\ 96 = 2^5 \times 3$
ii 8
c **i** $42 = 2 \times 3 \times 7,\ 126 = 2 \times 3^2 \times 7$
ii 42
d **i** $27 = 3^3,\ 162 = 2 \times 3^4$
ii 27
e **i** $98 = 2 \times 7^2,\ 378 = 2 \times 3^3 \times 7$
ii 14
f **i** $144 = 2^4 \times 3^2,\ 256 = 2^8$
ii 16
g **i** $512 = 2^9,\ 198 = 2 \times 3^2 \times 11$
ii 2
h **i** $75 = 3 \times 5^2,\ 255 = 3 \times 5 \times 17$
ii 15
i **i** $78 = 2 \times 3 \times 13,\ 234 = 2 \times 3^2 \times 13$
ii 78
j **i** $147 = 3 \times 7^2,\ 504 = 2^3 \times 3^2 \times 7$
ii 21
k **i** $1400 = 2^3 \times 5^2 \times 7,\ 2940 = 2^2 \times 3 \times 5 \times 7^2$
ii 140
l **i** $116 = 2^2 \times 29,\ 725 = 5^2 \times 29$
ii 29

13 **a** 270 **b** 672 **c** 126 **d** 162
e 2646 **f** 2304 **g** 50 688 **h** 1275
i 234 **j** 3528 **k** 29 400 **l** 2900

14 360 360

exercise 3.2

1 **a** 2^7 **b** 5^{13} **c** 8^7 **d** 10^6 **e** 2^{11}
f 3^5 **g** 11^5 **h** 10^5 **i** 7^6 **j** 9^7

2 A

3 **a** 10 **b** 3 **c** 9 **d** 9 **e** 4
f 5 **g** 16 **h** 1 **i** 17

4 B

5 **a** 2^6 **b** 3^5 **c** 5^4 **d** 10^{10}
e 7^6 **f** 8^4 **g** 3^8 **h** 5^{12}
i 6^2 **j** 4^4 **k** 3^3 **l** 10^6

6 **a** 12 **b** 6 **c** 10 **d** 3
e 16 **f** 7 **g** 8 **h** 4
i 10 **j** 12 **k** 12 **l** 8

7 A

8 **a** $3^5 \times 2^{12}$ **b** $5^6 \times 2^6$ **c** $2^{11} \times 7^8$
d $4^2 \times 9^2$ **e** $2^3 \times 7^3$ **f** $3^4 \times 11^2$

9 **a** $3^3 = 27$ **b** $5^2 = 25$
c $8^2 = 64$ **d** $6^2 = 36$

10 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1
f 1 **g** 2 **h** 2 **i** 7 **j** 9

11 D

12 **a** $2^{10} = 1024$
b 1 megabyte $= 2^{10} \times 2^{10} = 2^{20}$ bytes.
c 1 megabyte $= 2^{20}$ bytes = 1 048 576 bytes
d 1 gigabyte $= 2^{10} \times 2^{10} \times 2^{10} = 2^{30}$ bytes
e 1 gigabyte $= 2^{30}$ bytes = 1 073 741 824 bytes

13 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$

exercise 3.3

1 **a** 2^6 **b** 2^{12} **c** 3^4 **d** 4^4 **e** 5^6
f 10^{15} **g** 6^8 **h** 10^{16} **i** 7^4 **j** 3^{20}

2 B

3 **a** 6 **b** 3 **c** 3 **d** 3
e 5 **f** 2 **g** 36 **h** 3
i 33 **j** 3 **k** 8 **l** 6

4 B

5 **a** $2^3 \times 4^3$ **b** $3^2 \times 5^2$ **c** $3^4 \times 7^4$
d $2^7 \times 5^7$ **e** $4^5 \times 7^5$ **f** $2^0 \times 6^0$
g $4^{10} \times 5^{10}$ **h** $2^3 \times 11^3$ **i** $5^0 \times 7^0$
j $7^2 \times 10^2$ **k** $3^3 \times 8^3$ **l** $3^0 \times 11^0$

6 **a** $(2 \times 5)^4 = 10^4 = 10\,000$
b $(4 \times 5)^2 = 20^2 = 400$
c $(2 \times 25)^2 = 50^2 = 2500$
d $(5 \times 6)^3 = 30^3 = 27\,000$
e $(4 \times 25)^5 = 100^5 = 10\,000\,000\,000$
f $(4 \times 15)^2 = 60^2 = 3600$
g $(16 \times 5)^2 = 80^2 = 6400$
h $(8 \times 125)^2 = 1000^2 = 1\,000\,000$

7 **a** $\frac{1}{4}$ **b** $\frac{4}{25}$ **c** $\frac{8}{27}$ **d** $\frac{9}{16}$ **e** $\frac{25}{64}$
f $\frac{9}{100}$ **g** $\frac{1}{27}$ **h** $2\frac{1}{4}$ **i** $15\frac{5}{8}$ **j** $5\frac{4}{9}$

8 **a** 1 **b** 1

9 **a** 5^2 **b** 10^2 **c** 2 **d** 3^{10}

10 **a** 3 **b** 2

11 **a** $(3 \times 2^3)^2 = (3 \times 8)^2$ **b** $(3^3 \times 5)^2 = (27 \times 5)^2$

exercise 3.4

1 **a** Yes **b** No **c** Yes
d No **e** No **f** Yes

2 Yes, all are Pythagorean triads.

3 **a** 18, 24, 30 **b** 21, 28, 35
c 27, 36, 45 **d** 24, 32, 40

4 **a** Yes **b** Yes
c Multiples of Pythagorean triads are also Pythagorean triads.

5 **a** 1, 121, 12 321
b 12 345 654 321, 12 345 678 987 654 321

6 **a** $1^2 + 8^2, 4^2 + 7^2$ **b** $2^2 + 9^2, 6^2 + 7^2$
c $7^2 + 9^2, 3^2 + 11^2$ **d** $11^2 + 12^2, 3^2 + 16^2$
e $1^2 + 17^2, 11^2 + 13^2$ **f** $5^2 + 25^2, 17^2 + 19^2$

7 $13^2 = 169, 14^2 = 196, 31^2 = 961$

8 **b** =A1^3
c $1^3 = 1^2, 1^3 + 2^3 = 3^2, 1^3 + 2^3 + 3^3 = 6^2, 1^3 + 2^3 + 3^3 + 4^3 = 10^2$
d $\left(\frac{n(n+1)}{2}\right)^2$

9 **a** $1^3 + 2^3 + 4^3$ **b** $1^3 + 2^3 + 5^3$
c $1^3 + 3^3 + 4^3$ **d** $2^3 + 3^3 + 4^3$

10 **a** $20^2 + 5^2$ **b** $10^2 + 11^2 + 12^2 = 13^2 + 14^2$

11 **a** 13 m **b** 10 cm **c** 4.1 m
d 7.6 m **e** 7.8 cm **f** 17 m

Revision

1 D

2 C

3 E

4 D

5 C

6 **a** 7^8 **b** $3^3 \times 4^3$

7 **a** $5 \times 5 \times 5 \times 5$
b $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7$

8 **i** **a**

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

b

2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

c

2	8250
3	4125
5	1375
5	275
5	55
11	11
	1

d

2	12 000
2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

ii **a** $2^3 \times 3^2 \times 5^2$ **b** $2^4 \times 3^3 \times 5$
c $2 \times 3 \times 5^3 \times 11$ **d** $2^5 \times 3 \times 5^3$

9 **a** 360 **b** 30

10 **a** 10 800 **b** 99 000

11 **a** 3^{11} **b** 8^5 **c** 3^6 **d** 5
e 5^4 **f** 4^3 **g** $3^4 \times 5^5$ **h** 6^4

12 **a** $\frac{1}{16}$ **b** $\frac{4}{49}$ **c** $3\frac{1}{16}$ **d** $\frac{8}{27}$

13 **a** 1 **b** 2 **c** 3

14 **a** Pythagorean triplet
b not a Pythagorean triplet
c Pythagorean triplet
d Pythagorean triplet

15 41 cm

16 **a** 125 **b** 41 **c** 7 **d** 57

Chapter 4

exercise 4.1

1 **a** 3 **b** -2 **c** 5 **d** $\frac{1}{4}$
e -7 **f** 1 **g** $-\frac{2}{3}$ **h** -1

2 **a** $2x$ **b** $5ab$ **c** y **d** $\frac{x}{2}$
e $-m$ **f** $\frac{k}{3}$ **g** $\frac{4b}{5}$ **h** $-\frac{a}{2}$
i n^2 **j** $-\frac{5d}{4}$ **k** ab **l** $-\frac{d}{4}$

3 **a** 4 **b** -1 **c** -7 **d** c

4 **a** 3 **b** $-\frac{3}{7}$ **c** $\frac{1}{4}$ **d** f

5 **a** 2 **b** 3 **c** 3 **d** 4
e 2 **f** 3 **g** 4 **h** 2
i 2 **j** 3 **k** 4 **l** 2

6 **a** 1 **b** -8 **c** 7 **d** 2
e 4 **f** -3 **g** 15 **h** 6
i -30 **j** -7 **k** 3.2 **l** 0.3

7 **a** 2 **b** 1 **c** -2 **d** $\frac{1}{6}$ **e** $\frac{5}{2}$
f -24 **g** -2.5 **h** $-\frac{1}{3}$ **i** -0.75

8 E

9 **a** $3a + 4b$ **b** $-5x - 3y$
c $2x - x^2$ **d** $4(x - 3)$
e $6a - 5b$ **f** $-4x - 3x^2$
g $-3m + 2(n + 1)$ **h** $\frac{m}{3} - \frac{2n}{5}$
i $-\frac{c}{2} - 2d$ **j** $3r(4 - 2r)$

10 **a**, **b**, **e** and **f** are true

11 **a** $n + 7$ **b** $n - 5$ **c** $\frac{n}{6}$
d $4n - 6$ **e** $\frac{3n}{11}$ **f** $2(n + 5)$
g $\frac{n - 9}{10}$ **h** $8(9 - n)$

12 $\frac{3y + 2}{7} - 4$

13 **a** $c - 4$ **b** $3(c - 4) = 3c - 12$

14 **a** $p + 5$ **b** $\frac{p + 5}{3}$

15 Multiply the number by 4, subtract 5, multiply by 3, subtract 1, divide by 5, add 7

exercise 4.2

1 **a** like **b** like **c** like **d** unlike
e unlike **f** like **g** like **h** unlike
i like **j** like **k** like **l** like

2 B

3 **a** $5, -7$ **b** $-3, 6$
$2x, -3x$ $-8t, 4t$
$3y, 4y$ $t^2, 5t^2$

4 **a** $3x$ **b** $10a$ **c** $7n$ **d** b
e $3x$ **f** 0 **g** $4x$ **h** $-4x$

5 **a** 15 **b** 4 **c** -1
d $12d + 9e$ **e** $12h$ **f** $-2k$
g $10ab$ **h** $3xy + 8x$ **i** $8m^2$
j $14x^2$ **k** 0 **l** $13v + 4w$
m $8m - 6n$ **n** $6h$ **o** $-6k$
p 0 **q** $9gh$ **r** $-10ab$
s $-8x - 5x^2$ **t** $7j$ **u** 0

6 E

7 **a** $8x + 11$
b $5d + 9e$
c $8c + 2s$
d $8b + 3b + 2 + 9 = 11b + 11$
e $2m + 5m + 4n + 6n = 7m + 10n$
f $12 + 6 + 4k - 4k = 18$
g $7b^2 + 2b^2 + 4b + 5b = 9b^2 + 9b$
h $3xy + xy - 2xy + 7 = 2xy + 7$
i $6p^2 + 2p^2 + p + 8p = 8p^2 - 7p$
j $6st + 4st + 2st + 3 = 12st + 3$
k $9mn - 2yz + 11 - 11 = 7mn$
l $8n^2 - 2n^2 + 5n + 6 = 6n^2 + 5n + 6$

8 **a** $4a$ **b** $3x$ **c** $2l + 2w$ **d** $2a + 3b$

9 **a** $x + 5$ **b** $x + x + 5$ **c** $2x + 5$

10 **a** $d + 4$ **b** $d - 2$
c $d + d + 4 + d - 2$ **d** $3d + 2$

11 **a** $4a + 1$
b $5j - 5k$
c $5h$
d $-5w + 2w + 6 - 4 = -3w + 2$
e $2c - 7c - 3d + 8d = -5c + 5d$
f $4 - 4 + 6m - m = 5m$
g $6a^2 + 4a^2 - 3a - a = 10a^2 - 4a$
h $5cd - 2cd + cd - 4 = 4cd - 4$
i $v^2 - 8v^2 - 6v + 3v = -7v^2 - 3v$
j $8y - 5y - 3y + 2 = 2$
k $4pq - 3pq + 6 - 10 = pq - 4$
l $3n^2 + 5n - 9$

12 B

13 **a** **i** x^2 **ii** $8x$ **iii** $2x^2$ **iv** $5x$
b $3x^2 + 13x$

14 **a** Uncle Dom's age $= a$
b $5a + 2$
c no

exercise 4.3

1 **a** $6x$ **b** $24m$ **c** $77v$ **d** $-20g$
e $21w$ **f** $-80t$ **g** $-12hk$ **h** $45u^2$
i $10ab$ **j** $21ab$ **k** $-16jk$ **l** $18w^2$

2 **a** $30a$ **b** $40w^2$ **c** $20tu$
d $66fg$ **e** $28e^2$ **f** $36mn$
g $-30x^2$ **h** $-24uvw$ **i** $90def$

3 **a** $1.2g$ **b** $4z$ **c** $-0.8y$ **d** $0.06a$
e $-1.4k$ **f** $0.18mn$ **g** $2b$ **h** $6d$

4 **a** **i** width $= l$, length $= 2w$
ii width $= x$, length $= 6$
iii width $= 9$, length $= h$
b **i** $2lw$ **ii** $6x$ **iii** $9h$

5 **a** $10h$ **b** $4ah$ **c** $8y^2$

6 $6xy$

7 $12nt$

8 **a** $3(5 \times a \times h) = 15ah$
b $6ah$
c $21ah$

exercise 4.4

1 **a** $2n$ **b** $3a$ **c** $2m$ **d** $7v$
e $5k$ **f** $3e$ **g** $\frac{w}{4}$ **h** $\frac{2c}{9}$
i $\frac{u}{3}$ **j** $\frac{3h}{5}$ **k** $\frac{2t}{7}$ **l** $\frac{7x}{5}$

2 C

3 **a** 3 **b** $\frac{1}{4}$ **c** $\frac{2}{3}$ **d** $\frac{a}{7}$
e $\frac{d}{e}$ **f** $\frac{m}{n}$ **g** $\frac{2k}{3}$ **h** $\frac{5}{9}$
i $\frac{1}{4c}$ **j** $\frac{7}{r}$ **k** $\frac{2a}{7b}$ **l** $\frac{5}{6w}$

4 $\frac{60x}{18} = \frac{10x}{3}$

5 $\frac{60n}{100} = \frac{3n}{5}$

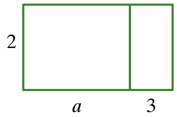
6 **a** y^2 **b** $9y^2$ **c** $\frac{9y^2}{y^2} = 9$
d It tells us that nine of the small green squares will fit within the large empty square.

7 **a** $5ab$ **b** $40abc$
c $\frac{40abc}{5ab} = 8c$ **d** 120 blocks

exercise 4.5

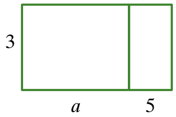
1 **a** 40 **b** 18 **c** 16 **d** 8 **e** 14 **f** 50

2 **a**



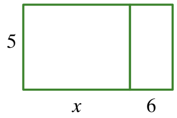
$2a + 6$

b



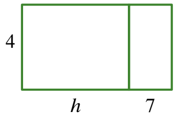
$3a + 15$

c



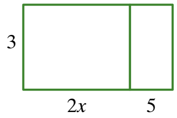
$5x + 30$

d



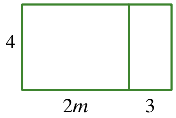
$4h + 28$

e



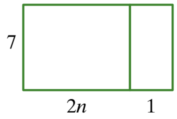
$6x + 15$

f



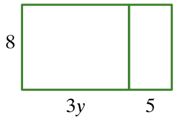
$8m + 12$

g



$14n + 7$

h



$24y + 40$

3 **a** $3m + 15$ **b** $8d + 32$
c $11h - 22$ **d** $14 + 7b$
e $27 - 9e$ **f** $4a + 4b + 12$
g $8g + 12$ **h** $33 + 15t$
i $12 - 8x$ **j** $24e + 30f$
k $15a - 10b$ **l** $50d + 40 - 30e$
m $xy + 7x$ **n** $4a + ab$
o $de - 8d$ **p** $g^2 + 5g$
q $3k - k^2$ **r** $2w + w^2$

4 **a** $w + 2$ **b** $w(w + 2)$ **c** $w^2 + 2w$
d length = 7 m and area = 35 m^2

5 **a** $L - 11$ **b** $L(L - 11)$ **c** $L^2 - 11L$
d width = 21 cm and area = 672 cm^2

6 **a** -40 **b** -18 **c** -16
d -8 **e** -14 **f** -50

7 **a** $-2h - 12$ **b** $-6d - 54$
c $-4e + 24$ **d** $-4 - r$
e $-24 + 3p$ **f** $-f + g - 3$
g $-16c - 40$ **h** $-3 - 2q$
i $-90 + 27m$ **j** $-33m - 77n$
k $-30a + 18b$ **l** $-12j - 3 + 3k$
m $-cd - 3c$ **n** $-7m - mn$
o $-2gh + 10g$ **p** $-t^2 - 4t$
q $-45x + 5x^2$ **r** $-12a - 4a^2$

8 **a** $4x + 28 + 9 = 4x + 37$
b $3x + 30 + 11 = 3x + 41$
c $2x - 10 + 12 = 2x + 2$
d $4x - 64 - 24 = 4x - 88$
e $10x - 15 + 11 = 10x - 4$
f $-21x + 7 + 6 = -21x + 13$
g $-2x - 16 - 14 = -2x - 30$
h $-54 + 6x + 18 = -36 + 6x$
i $12x + 54 - 13 = 12x + 41$
j $-40x - 60 + 13 = -40x - 47$
k $-50 + 90x - 16 = -66 + 90x$
l $-45x + 165 + 35 = -45x + 200$

9 **a** $4x + 12 + 2x + 6 = 6x + 18$
b $-9x - 63 - 5x + 13 = -14x - 50$
c $-3x + 15 + 4x - 2 = x + 13$
d $-10x - 60 + 13 = -10x - 47$
e $21x - 49 - 2x + 14 = 19x - 35$
f $40x - 88 + 15x - 18 = 55x - 106$
g $20 + 5x^2 + 15x - 4x = 5x^2 + 11x + 20$
h $7r + 34 - 27r - 15 = -20r + 19$

10 **a** $2x + 10 + 3x + 18 = 5x + 28$
b $6 + 3x + 20 + 5x = 26 + 8x$
c $12x - 36 - 7x - 63 = 5x - 99$
d $10x - 15 + 18x - 42 = 28x - 57$
e $18x - 21 - 10x + 16 = 8x - 5$
f $28x + 70 + 5x^2 + 20x = 5x^2 + 48x + 70$
g $60x^2 - 84x + 122x - 16x^2 = 44x^2 + 38x$
h $24 + 12x + 26 + 13x = 50 + 25x$
i $11x - 33 - 8x + 24 = 3x - 9$
j $-15 + 6x + 5 - 2x = -10 + 4x$
k $-33x + 44 - 15x + 20 = -48x + 64$
l $-18 + 4x - 117 + 26x = -135 + 30x$

11 **a** $7(a + 3)$ **b** $9(3g - 4)$
c $3y(y - 8)$ **d** $-2n(3m - 4n)$

exercise 4.6

1 **a**

2	18	24
3	9	12
	3	4

The highest common factor is 6.

b

2	32	40
2	16	20
2	8	10
	4	5

The highest common factor is 8.

c

2	16	120
2	8	60
2	4	30
	2	15

The highest common factor is 8.

d

2	150	120
3	75	60
5	25	20
	5	4

The highest common factor is 30.

e

5	75	125
5	15	25
	3	5

The highest common factor is 25.

f

2	32	40	48
2	16	20	24
2	8	10	12
	4	5	6

The highest common factor is 8.

2 **a**

7	$7p$	21
	p	3

The highest common factor is 7.

b

3	$6a$	$9b$
	$2a$	$3b$

The highest common factor is 3.

c

5	$15ef$	$10de$
e	$3ef$	$2de$
	$3f$	$2d$

The highest common factor is $5e$.

d

2	$12uv$	$8uw$
2	$6uv$	$4uw$
u	$3uv$	$2uw$
	$3v$	$2w$

The highest common factor is $4u$.

e

5	$15ab$	$5b^2$
b	$3ab$	b^2
	$3a$	b

The highest common factor is $5b$.

f

2	$12k$	$18k^2$	$6jk$
3	$6k$	$9k^2$	$3jk$
k	$2k$	$3k^2$	jk
	2	$3k$	j

The highest common factor is $6k$.

3 D

4 **a** 3 **b** $3a$ **c** $8y$ **d** $18m$
e $-x$ **f** $-4a$ **g** $7x$ **h** $6ab$

5 **a** $2(n+3)$ **b** $3(y-4)$ **c** $7(k+2)$
d $2(3p+5)$ **e** $2(2+7s)$ **f** $5(2t-1)$
g $3(1+5f)$ **h** $4(2-5m)$ **i** $7(2+3h)$
j $2(2c-3e)$ **k** $7(2a+3b)$ **l** $6(3w-2x)$
m $2(3+x)$ **n** $3(3x+4)$ **o** $5(a-1)$
p $7(m+3)$ **q** $4(3b-4)$ **r** $6(1-2x)$
s $7(3y-2)$ **t** $9(3-4y)$

6 **a** $a(c+e)$ **b** $e(d-1)$
c $h(5+g)$ **d** $n(n-2)$
e $y(y+1)$ **f** $t(7-t)$
g $2n(4m+7p)$ **h** $6u(2v-3)$
i $5q(5+4p)$ **j** $y(y+2z)$
k $3x(2x-1)$ **l** $3k(1-3k)$
m $3(2a+5b-3c)$ **n** $d(e-f+g)$
o $a(a+4-3b)$ **p** $3(x^2-4x+5)$

7 **a** $-2(a-4)$ **b** $-3(g-5)$ **c** $-7(j-1)$
d $-5(p+5)$ **e** $-11(r+3)$ **f** $-9(w+1)$
g $-2(2-3t)$ **h** $-3(2m-7)$ **i** $-5(2-5h)$
j $-4(3q+2)$ **k** $-11(3+7r)$ **l** $-6(3a+4)$

8 **a** $3(1+2x)$ **b** $4(4+5x)$ **c** $15(x+3)$
d $4(7-2x)$ **e** $9(4x+1)$ **f** $5(20+5x)$
g $24(4-x)$ **h** $13(8-11x)$

9 **a** $-3(a-2)$ **b** $9(x^2+3)$
c $4a(3+4a)$ **d** $-4(x-3)$
e $6x(1+3x)$ **f** $b(b-47)$
g $-4(3m+5)$ **h** $-8(3x-4y)$
i $16(2x^2+1)$ **j** $5x(5x+6)$
k $-8(2-3x)$ **l** $2x(x+6)$
m $-6(9c-10d)$ **n** $-11(a+4b)$
o $24a(2a+1)$ **p** $7x(x-2)$
q $11x(5x-2)$ **r** $-3y(x-2)$
s $-5(4+3m)$ **t** $33a(3a-1)$
u $-5(3y+8)$ **v** $4a(a-3)$
w $-4(2x+3y)$ **x** $-3(3x-5y)$

10 **a** $10(x-2)$ **b** $6(2d+3e)$ **c** $5a(2-5b)$
d $5y(y-4)$ **e** $-3a(b-c)$ **f** $-6(m-3n)$

11 $\frac{2(4b-7)}{2} = 4b - 7$

12 **a** The final number is always 1.

b $\frac{2(n+5)-8}{2} - n = \frac{2n+10-8}{2} - n$

$= \frac{2n+2}{2} - n$

$= \frac{2(n+1)}{2} - n$

$= n + 1 - n$

$= 1$

13 **a** $n+1$
b $n+2$
c $n+n+1+n+2$
d $n+n+1+n+2 = 3n+3 = 3(n+1)$
e Dividing the factorised expression in part **d** by three gives $n+1$, so the sum of three consecutive integers can always be divided by three.

exercise 4.7

1 **a** **i** 75 **ii** 63 **iii** 129
b **i** 40 **ii** 36 **iii** 48
c **i** -25 **ii** -24 **iii** -17
d **i** -50 **ii** -42 **iii** -86

2 **a** 40 **b** 21 **c** 150 **d** 30
e 54 **f** 75 **g** 2 **h** -2
i 23 **j** 23 **k** 17 **l** 7

3 B

4 **a** 2 **b** -3 **c** $-\frac{1}{4}$ **d** -4 **e** 8 **f** -3
g -1 **h** $-\frac{2}{3}$ **i** -15 **j** 1 **k** $-4\frac{1}{2}$ **l** 0

5 A

6 **a** 20 **b** 18 **c** 0 **d** -27 **e** 0 **f** 36
g -6 **h** 3 **i** -27 **j** 48 **k** 18 **l** -18

7 C

8 **a** 66 **b** -36 **c** 8 **d** 0 **e** 3 **f** -5
g 11 **h** 2 **i** 32 **j** 70 **k** -18 **l** 108

9 **a** $3a + 4b - a + 2b = 9 + 20 - 3 + 10 = 36$, $4a + 2b = 12 + 10 = 22$, so not equal
b $2a + 6b$

10 **a** $2(2x+6) + 2(x+3)$
$6x + 18$
b $2(2x-6) + 2 \times 2(x+4)$
$8x + 4$
c $14x + 22$
d $2(7x+11)$
e $8x + 4 - (6x + 18)$ or $6x + 18 - (8x + 4)$
f $2(x-7)$ or $-2(x-7)$

Revision

1 B **2** D **3** D **4** E **5** E

6 **a** 4 **b** y **c** -5 **d** -10

7 **a** $\frac{3e}{8}$ **b** $4c - 7d$ **c** $\frac{g+4}{5} - 3$

8 **a** $7h - 8$ **b** $4j + 5k$ **c** $3t^2 + 4t - 8$

9 **a** $-30m^2n$ **b** $6p$ **c** $\frac{2q}{11r}$

10 **a** $5t - 20$ **b** $uv + 3uw$ **c** $-8x^2 + 10xy$

11 **a** $3(a+7)$ **b** $5(1-2b)$ **c** $-2c(3c+4)$

12 **a** 36 **b** 21 **c** -9

13 **a** $a+1$ **b** $a-2$
c $a + a + 1 + a - 2$ **d** $3a - 1$

14 **a** $w+1$ **b** $w(w+1)$ **c** $w^2 + w$
d length = 4 m and area = 12 m^2

Chapter 5

exercise 5.1

1 **a** **i** 32% **ii** 68%
b **i** 60% **ii** 40%
c **i** 59% **ii** 41%
d **i** 26% **ii** 74%

2

	Fraction	Decimal	Percentage
a	$\frac{5}{8}$	0.625	62.5%
b	$\frac{9}{25}$	0.36	36%
c	$\frac{7}{10}$	0.7	70%
d	$2\frac{3}{4}$	2.75	275%
e	$\frac{1}{8}$	0.125	$12\frac{1}{2}$%
f	$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}$%
g	$\frac{21}{25}$	0.84	84%
h	$\frac{19}{20}$	0.95	95%
i	$1\frac{2}{5}$	1.4	140%
j	$\frac{41}{250}$	0.164	16.4%
k	$2\frac{4}{5}$	2.8	280%
l	$1\frac{9}{20}$	1.45	145%
m	$2\frac{14}{25}$	2.56	256%
n	$2\frac{3}{25}$	2.12	212%
o	$\frac{17}{40}$	0.425	42.5%
p	$1\frac{1}{4}$	1.25	125%

3 D **4** E **5** E **6** $\frac{21}{50}$ **7** 0.1875

8 0.075

9 **a**

	Percentage	Fraction	Decimal
Nitrogen	80%	$\frac{4}{5}$	0.8
Oxygen	20%	$\frac{1}{5}$	0.2
Carbon dioxide	0.04%	$\frac{1}{2500}$	0.0004

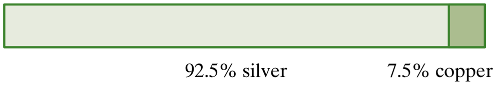
10 $\frac{3}{20}$, 0.15

11 **a** $\frac{3}{4}, \frac{1}{4}$ **b** $\frac{23}{25}, \frac{3}{50}, \frac{1}{50}$

12 **a** $\frac{18}{24} = \frac{3}{4}$ **b** 75% **c** 37.5%

d 62.5% **e** 20.5%

f



exercise 5.2

1 **a** 20% **b** 42% **c** 37% **d** 29%
e 73% **f** 70% **g** 28% **h** 49%
i 28% **j** 35% **k** 3% **l** 82%

2 C

3 C

4 **a** protein $13.\overline{3}\%$; fat $1.\overline{6}\%$; carbohydrate 65%; fibre $11.\dot{3}\%$
b $8.\dot{6}\%$

5 3%

6 6%

7 Camp 44%; city week experience 56%

8 40%

9 0.33%

10 54

exercise 5.3

1 **a** 84 **b** 172 **c** 42 g
d \$64.80 **e** 2.56 mL **f** \$3600
g \$33 600 **h** 101.32 kg **i** 700 000
j 23.2 **k** 43.75 **l** 0.135
m 8880 **n** 900 **o** 1018

2 A

3 B

4 20 615.572 ML

5 **a** 106.56 **b** 41.44

6 $10\frac{1}{2}$

7 **a** \$8250 **b** \$1237.50 **c** 8.7%

8 13.5%

exercise 5.4

1 **a** 800 **b** 900 **c** 1700 **d** 3500
e 7400 **f** 5600 **g** 14 500 **h** 52 400

2 **a** 13 200 **b** 7400 **c** 28 600 **d** 23 600
e 8900 **f** 41 200 **g** 56 800 **h** 71 200

3 B

4 125

5 24

6 287 000 ML

7 **a** 7%
b 156
c **i** 14 **ii** 92 **iii** 11

8 25 MB

9 3000

10 495 408

11 3

exercise 5.5

1 9.3%

2 10.5%

3 8.1%

4 **a** 12.5% increase **b** 2% increase
c 16.7% decrease **d** 16.7% decrease
e 10.1% decrease **f** 7.7% increase
g 5.0% increase **h** 11.3% decrease
i 49.5% increase **j** 54.9% increase
k 24.3% decrease **l** 11.8% increase

5 48.3%

6 12.9%

7 **a** 2430 **b** 556.8 **c** 11 875 **d** 399 kg
e 4872 L **f** \$1404 **g** 61.2 cm **h** \$3136
i 2625 **j** 7200 **k** 15 675 **l** 12 725

8 **a** 525 **b** 84 **c** 6232 **d** 578.2
e \$2595 **f** \$1135.20 **g** 8775 L **h** \$1400
i 3330 **j** 4140 **k** 46 848 **l** 12 275

9 44

10 D

11 \$58 320

12 \$20 188

13 \$3 906 000

14 1 405 602 ML

15 966

16 9.2%

17 1143 cars

exercise 5.6

1

	Wholesale price	% markup	Mark-up	Retail price (before GST)
a	$36	50%	$18	$54
b	$250	35%	$87.50	$337.50
c	$15	60%	$9	$24
d	$90	100%	$90	$180
e	$12	120%	$14.40	$26.40
f	$940	30%	$282	$1222

2 $91
3 $11.05
4 **a** $54 **b** 154.3%
5 46.4%
6 E
7 $28
8 **a** $36 **b** 150%
9

	Normal price	% discount	Discount	Discounted price
a	$18	25%	$4.50	$13.50
b	$55	10%	$5.50	$49.50
c	$150	$33\frac{1}{3}\%$	$50	$100
d	$24	40%	$9.60	$14.40
e	$1250	20%	$250	$1000
f	$27 000	15%	$4050	$22 950

10 $10.80
11 $7.20
12 $12.30
13 **a** $800 **b** $120
14 $1100
15 $5
16 B
17 $50
18 **a** no; it is equivalent to 28% discount.
$100 − 10% of $100 = $90
$90 − 20% of $90 = $72
b no

exercise 5.7

1

	Cost price	Selling price	Profit or loss (state amount and whether profit or loss)	Percentage profit or loss
a	$18	$45	$27 profit	150%
b	$120	$180	$60 profit	50%
c	$32	$24	$8 loss	25%
d	$8.50	$6.00	$2.50 loss	29%
e	$165 000	$240 000	$75 000 profit	45%
f	$16	$40	$24 profit	150%
g	$800	$960	$160 profit	20%
h	$54	$48	$6 loss	11%
i	$25	$95	$70 profit	280%
j	$1500	$600	$900 loss	60%
k	$22.50	$67.50	$45 profit	200%
l	$750	$480	$270 loss	36%

2 **a** $3.70 **b** 77%
3 **a** $50 **b** 59%
4 **a** $430 **b** 195%
5 **a** $18 000 **b** 6%
6 13%
7 140%
8 **a** $34.50 **b** 70%
9 $35
10 11%
11 **a** loss **b** $20

exercise 5.8

1 **a** $0.45; $4.95 **b** $1.24; $13.64
c $11.89; $130.79 **d** $56; $616
e $7.25; $79.75 **f** $2240; $24 640
2 **a** $5.75 **b** $63.25
3 **a** $0.48 **b** $5.28
4 **a** $1.20 **b** $13.20
5 $30.80
6 $8.14
7 $1.30
8 $3.50
9 **a** $2.80 **b** $28.00
10 C
11 **a** $0.90 **b** $9 **c** 80%
12 $45.00
13 $185.60
14 $15.60

Revision

1 B **2** E **3** C **4** B **5** D

6 **a** 62.5% **b** 60.0% **c** 15.6% **d** 8.3%

7 **a** 128 students **b** \$161
c \$425 000 **d** 0.072 g

8 **a** 5500 **b** 250 **c** \$38 000 **d** 6850

9 **a** 46.4 cm **b** \$191.40

10 **a** 22.05 minutes **b** \$304 326

11 **a** \$120 **b** 75%

12 **a** \$6.80 **b** \$210

13 **a** \$2850 **b** \$1650

14 \$29.70

15 **a** 75 **b** \$3.75 **c** \$83.75 **d** \$0.25
e \$206.25 **f** \$18.75 **g** \$103.75 **h** 123.9%

Chapter 6

exercise 6.1

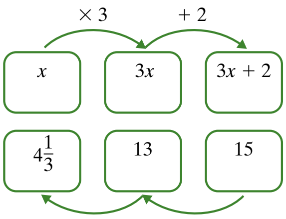
1 **a** $a = 2$ **b** $b = 12$ **c** $x = 7$ **d** $d = 11$
e $x = 20$ **f** $x = 6$ **g** $x = -2$ **h** $h = -8$
i $x = 3$ **j** $x = -7$ **k** $k = -2$ **l** $x = -4$

2 C

3 **a** $x = 4$ **b** $y = 3$ **c** $m = 8$
d $n = 6$ **e** $x = -5$ **f** $x = -7$
g $x = \frac{1}{2}$ **h** $x = \frac{1}{6}$ **i** $n = 1.2$
j $x = 2.2$ **k** $x = -1.2$ **l** $k = 0$

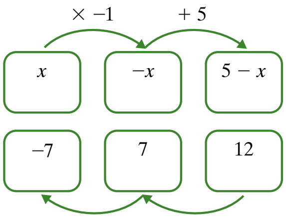
4 **a** $x = 36$ **b** $b = 500$ **c** $x = -81$
d $d = -88$ **e** $x = -16$ **f** $m = -28$
g $x = 28$ **h** $h = 300$ **i** $x = 0$
j $y = -20$ **k** $x = 56$ **l** $x = 0$

5 **a**



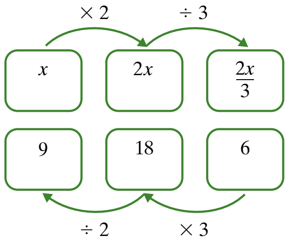
$x = 4\frac{1}{3}$

b



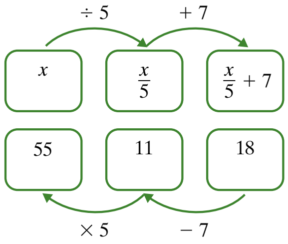
$x = -7$

c



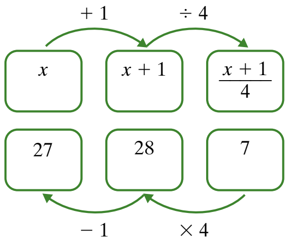
$x = 9$

d



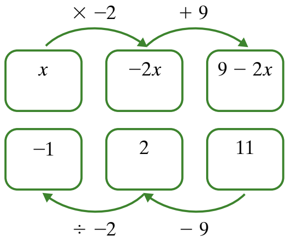
$x = 55$

e



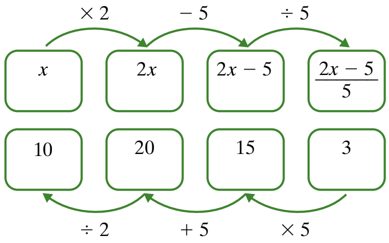
$x = 27$

f



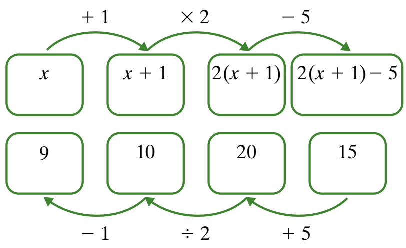
$x = -1$

g



$x = 10$

h



$x = 9$

6 **b** and **c**, they have pronumerals on both sides of the equation

exercise 6.2

1 **a** $x = 4$ **b** $x = 18$ **c** $a = 7$
d $a = 23$ **e** $b = 15$ **f** $p = 34.7$
g $y = -5$ **h** $d = 4$ **i** $x = 14$
j $b = 7$ **k** $t = -7$ **l** $x = 1$

2 **a** $d = 2$ **b** $m = 5$ **c** $b = 3$
d $x = 9$ **e** $k = 8$ **f** $a = 2.5$
g $m = 72$ **h** $x = 78$ **i** $a = -8$
j $b = -55$ **k** $x = 108$ **l** $x = -112$

3 **a** $x = 6$ **b** $b = 2$ **c** $t = 6$
d $x = 5$ **e** $e = 5$ **f** $x = 6$
g $x = 8$ **h** $h = 7$ **i** $a = 15$
j $x = 8$ **k** $k = 12$ **l** $x = -6$
m $x = 8$ **n** $x = 5.5$ **o** $x = 1\frac{2}{5}$
p $x = 9$

4 **a** $x = 15$ **b** $x = 9$ **c** $x = 5$ **d** $x = 13$
e $x = 2$ **f** $x = 3\frac{1}{3}$ **g** $x = \frac{5}{6}$ **h** $x = \frac{7}{12}$

5 **a** Yes, the second equation is equal to the first divided by 2
b No, the 6 in the second equation would need to equal 2 for the two equations to give the same solution
c No, the 8 in the second equation would need to equal 2 for the two equations to give the same solution
d Yes, the second equation is equal to the first equation divided by 3

6 **a** **i** $2x + 4 = 10$
$2x = 6$
$x = 3$
ii $2x + 4 = 10$
$x + 2 = 5$
$x = 3$
b answers will vary
c Easier to add 7 to both sides because 3 is not a factor of 7 or 23

7 B

8 **a** $x = 35$ **b** $a = 21$ **c** $y = 15$
d $x = 10.5$ **e** $h = 3.2$ **f** $a = 21$
g $x = -10$ **h** $b = -8$ **i** $x = -3$
j $m = -20$ **k** $x = 2\frac{2}{3}$ **l** $a = 7\frac{1}{2}$

9 **a** $a = 24$ **b** $r = 55$ **c** $b = 18$
d $k = 102$ **e** $x = 36$ **f** $b = 32$
g $a = 24$ **h** $x = 77$ **i** $p = -12$
j $y = 6$ **k** $x = 5$ **l** $m = 24$
m $b = 24$ **n** $x = 24.6$ **o** $x = 14$
p $y = 45$

10 **a** **i** $\frac{x}{3} + 4 = 5$
$\frac{x}{3} = 1$
$x = 3$
ii $\frac{x}{3} + 4 = 5$
$x + 12 = 15$
$x = 3$
b answers will vary

11 **a** Yes, the second equation is equal to the first multiplied by 4
b No, the 11 in the second equation would need to equal 55 to give the same solution
c Yes, the second equation is equal to the first multiplied by 7
d No, the -2 in the second equation would need to equal -8 to give the same solution

12 **a** $a = 10$ **b** $x = 20$ **c** $m = 9$
d $b = 28$ **e** $x = 24$ **f** $x = 10$
g $x = 2.5$ **h** $x = -5$ **i** $m = 2$
j $x = 15.4$ **k** $x = 22.5$ **l** $k = 19.2$

13 **a** $x = 16$ **b** $x = 58$ **c** $a = 125$
d $m = 38$ **e** $k = -23$ **f** $a = 24$
g $b = 12$ **h** $x = 5$ **i** $b = -6$
j $y = 48$ **k** $d = -4$ **l** $h = 21$

14 D

15 **a** $x = 9$ **b** $x = 2$ **c** $x = 1$ **d** $x = 0$

16 **a** $x = -a - b + c$ **b** $x = \frac{bc}{a}$
c $x = \frac{dc - b}{a}$

exercise 6.3

1 **a** $x = \frac{2}{3}$ **b** $x = 4\frac{2}{5}$ **c** $x = 9\frac{1}{2}$ **d** $x = 2\frac{1}{3}$
e $x = 3\frac{1}{2}$ **f** $x = 2\frac{1}{3}$ **g** $x = 3\frac{3}{4}$ **h** $x = 11$
i $x = 1\frac{2}{3}$ **j** $x = 5\frac{2}{3}$ **k** $x = 3$ **l** $x = 2$

2 **a** $x = 11$ **b** $x = 12$ **c** $x = 7$ **d** $x = 4$
e $x = 4$ **f** $a = 0$ **g** $f = -1$ **h** $k = 5$
i $a = 2$ **j** $h = 3$ **k** $x = \frac{1}{2}$ **l** $x = 1$

3 **a** **i** $3(2x + 3) = 27$
$6x + 9 = 27$
$6x = 18$
$x = 3$
ii $3(2x + 3) = 27$
$2x + 3 = 9$
$2x = 6$
$x = 3$
b answers will vary
c Expand the brackets first because 3 is not a common factor of both sides

4 **a** $x = 6$ **b** $x = 5\frac{3}{4}$ **c** $x = -5$
d $x = -3\frac{3}{7}$ **e** $x = 3\frac{1}{4}$ **f** $x = 4$
g $x = 6$ **h** $x = 1\frac{2}{3}$ **i** $x = 4\frac{1}{2}$
j $x = 5$ **k** $x = -14.2$ **l** $x = -11$

5 **a** -3 **b** $\frac{-7}{2} = -3\frac{1}{2}$

c $\frac{6}{5} = 1\frac{1}{5}$ **d** $\frac{1}{4}$

6 **a** $a = 4$ **b** $b = 11\frac{1}{3}$ **c** $x = -6$

d $x = 16\frac{2}{3}$ **e** $m = 1$ **f** $n = \frac{3}{5}$

g $x = -3$ **h** $h = -1$ **i** $y = 2$

j $x = 8$ **k** $x = 4$ **l** $x = 7$

7 **a** $x = 8\frac{1}{2}$ **b** $x = -\frac{2}{5}$ **c** $x = 1\frac{3}{5}$

d $x = 8$ **e** $x = -1$ **f** $x = 5$

g $x = -2$ **h** $x = 3$ **i** $x = 6$

j $x = \frac{1}{2}$ **k** $x = -2$ **l** $x = -6$

8 **a** $x = \frac{de - ac}{ab}$ **b** $x = \frac{-ab}{a - c}$

exercise 6.4

1 **a** $x = 1$ **b** $x = 4$ **c** $x = 4$

d $x = 1$ **e** $x = 2\frac{1}{2}$ **f** $x = -3$

g $x = -1\frac{1}{2}$ **h** $x = 2$ **i** $x = 2$

j $x = -6$

2 **a** $x = 2$ **b** $x = -5$ **c** $x = 3$

d $x = 7$ **e** $x = 3$ **f** $x = -1$

g $x = 7$ **h** $x = 4$ **i** $x = -1\frac{1}{2}$

j $x = 8$ **k** $x = 2$ **l** $x = \frac{6}{7}$

3 **a** $x = -10$ **b** $x = \frac{-3}{31}$

exercise 6.5

1 **a** $x = 2\frac{1}{6}$ **b** $a = 0$ **c** $x = 5$ **d** $h - 2$

e $b = 10$ **f** $m = 2$ **g** $n = 25\frac{1}{2}$ **h** $x = 9.5$

i $y = 12$ **j** $x = -4$ **k** $x = 3$ **l** $k = 7.5$

m $x = -3$ **n** $a = 12$ **o** $y = 1\frac{2}{3}$ **p** $a = 3.5$

q $b = 2$ **r** $x = 3\frac{2}{3}$

2 13 taxis

3 22 sales

4 7600 m

5 **a** $x = 8$ **b** $x = 60$ **c** $x = -4$

d $x = 6\frac{2}{3}$ **e** $x = 23$ **f** $x = 31\frac{1}{3}$

6 **a** $x = 10$ **b** $x = 6\frac{1}{5}$ **c** $x = 3$

d $x = 21$ **e** $x = 8$ **f** $x = 7$

7 **a** $x = \frac{bde}{ad + bc}$ **b** $x = \frac{ad + bc + bde}{b + d}$

c $x = \frac{bde - ad - bc}{d - b}$

exercise 6.6

1 let x be the number

a **i** $x + 7 = 10$ **ii** $x = 3$

iii $\text{LS} = 3 + 7$

$= 10$

$= \text{RS}$

iv original number was 3

b **i** $x - 8 = -12$ **ii** $x = -4$

iii $\text{LS} = -4 - 8$

$= -12$

$= \text{RS}$

iv original number was -4

c **i** $-8x = 40$ **ii** $x = -5$

iii $\text{LS} = -8 \times (-5)$

$= 40$

$= \text{RS}$

iv original number was -5

d **i** $\frac{x}{9} = -3$ **ii** $x = -27$

iii $\text{LS} = -\frac{27}{9}$

$= -3$

$= \text{RS}$

iv original number was -27

2 let n be the number

a **i** $\frac{n}{4} + 7 = 12$ **ii** number is 20

b **i** $-5n + 13 = -32$ **ii** number is 9

c **i** $2(n + 11) = 46$ **ii** number is 12

d **i** $\frac{n + 17}{5} = 35$ **ii** number is 158

3 **a** $3n + 4 = 2n + 8$ **b** $n = 4$

c If 4 is multiplied by 3 then 4 added = 16
If 4 is doubled then 8 added = 16
So, 4 fits given information.

d number is 4

4 **a** $\frac{3x + 15}{3} = 114$ **b** $x = 109$

c 109, 112, 121

d $\frac{109 + 112 + 121}{3} = 114$

e The numbers are 109, 112 and 121.

5 **a** **i** $3n + 3 = 177$ **b** **i** $3n + 6 = 177$

ii $n = 58$ **ii** $n = 57$

iii 58, 59, 60 **iii** 57, 59, 61

iv $58 + 59 + 60 = 177$ **iv** $57 + 59 + 61 = 177$

v The numbers are 58, 59 and 60. **v** The numbers are 57, 59 and 61.

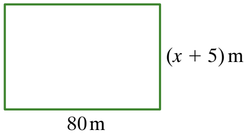
6 **a** $P = 60 + 2(a + 3)$ **b** $a = 12$

$P = 2a + 66$

7 **a** $28 = 4x + 12$ **b** $x = 4$

c length = 10 m, width = 4 m

8 **a**

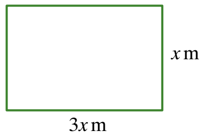


b $A = 80(x + 5)$

c $x = 20$

9 **a** $3x$

b



c $24 = 8x$
$x = 3$

d the dimensions are 9 m by 3 m

10 **a** $3x + 4$

b $3x + 4 = 46$
$x = 14$
ages are 14, 16 and 16

11 Maria is 40 and Kate is 10

12 47

13 **a** **i** $4x + 88 = 180$
ii $x = 23$
iii $\text{LS} = 4 \times 23 + 88$
$= 180$
$= \text{RS}$
iv the angles are 90°, 64° and 26°
v $90 + 64 + 26 = 180°$

b **i** $5x = 180$
ii $x = 36$
iii $\text{LS} = 5 \times 36$
$= 180$
$= \text{RS}$
iv the angles are 108°, 36° and 36°
v $108 + 36 + 36 = 180°$

c **i** $9x = 180$
ii $x = 20$
iii $\text{LS} = 9 \times 20$
$= 180$
$= \text{RS}$
iv the angles are 40°, 60° and 80°
v $40 + 60 + 80 = 180°$

d **i** $3x + 30 = 180$
ii $x = 50$
iii $\text{LS} = 3 \times 50 + 30$
$= 180$
$= \text{RS}$
iv the angles are 70°, 60° and 50°
v $70 + 60 + 50 = 180°$

e **i** $3x + 72 = 180$
ii $x = 36$
iii $\text{LS} = 3 \times 36 + 72$
$= 180$
$= \text{RS}$
iv the angles are 84°, 60° and 36°
v $84 + 60 + 36 = 180°$

f **i** $8x = 360$
ii $x = 45$
iii $\text{LS} = 8 \times 45$
$= 360$
$= \text{RS}$
iv the angles are 135°, 135°, 90° and 45°
v $135 + 90 + 90 + 45 = 360°$

g **i** $3y = 180$
ii $y = 60$
iii $\text{LS} = 3 \times 60$
$= 180$
$= \text{RS}$
iv the angles are 115°, 65°, 115° and 65°
v $115 + 65 = 180°$

h **i** $15a - 15 = 360$
ii $a = 25$
iii $\text{LS} = 15 \times 25 - 15$
$= 360$
$= \text{RS}$
iv the angles are 145°, 90°, 75° and 50°
v $145 + 90 + 75 + 50 = 360°$

i **i** $25m + 10 = 360$
ii $m = 14$
iii $\text{LS} = 25 \times 14 + 10$
$= 360$
$= \text{RS}$
iv the angles are 162°, 100°, 70° and 28°
v $162 + 100 + 70 + 28 = 360°$

14 **a** $\text{cost} = 5 + 4x$
b $\text{cost} = 14 + x$
c $5 + 4x = 14 + x$
d 3 hours

15 **a** $\text{cost} = 40 + 40x$
b $\text{cost} = 60 + 30x$
c $40 + 40x = 60 + 30x$
d 2 hours

16 **a** $8(x + 3) = 4(3x + 2)$
b $x = 4$
c triangle 7 cm, rectangle 14 cm

Revision

1 C **2** C **3** D **4** B **5** D

6 **a** $x = 4$ **b** $a = -8$ **c** $d = \frac{2}{3}$
d $n = 1\frac{3}{4}$ **e** $x = -5$ **f** $x = -1$
g $a = 0$ **h** $h = 21$ **i** $x = -20$
j $x = 18$ **k** $k = -4$ **l** $x = 11$
m $m = \frac{3}{5}$ **n** $n = -2\frac{1}{2}$ **o** $x = 49$
p $x = -9$ **q** $k = -2\frac{1}{3}$ **r** $x = 9\frac{1}{2}$
s $x = 1\frac{1}{4}$ **t** $y = -2\frac{1}{5}$ **u** $x = -\frac{5}{8}$
v $a = 3\frac{1}{2}$

7 **a** $x = 13$ **b** $x = 70$ **c** $x = 13$
d $x = 3\frac{1}{8}$ **e** $x = -24$ **f** $x = 10$
g $x = -4\frac{3}{5}$ **h** $x = 21$ **i** $x = 7\frac{1}{2}$
j $x = 42$ **k** $x = -3$ **l** $x = 3.2$

8 **a** 9 **b** −168
c 18 **d** $151.25

9 **a** $P = 2(20a + a + 2) = 42a + 4$
b $A = 20a(a + 2)$ **c** area is 160 cm^2

10 **a** $4(10 - x) = 5(3x - 11)$
b $x = 5$
c LS $= 4(10 - 5) = 20$
RS $= 5(3 \times 5 - 11) = 20$
$\therefore$ LS = RS
d triangle 5 cm, rectangle 4 cm

Chapter 7

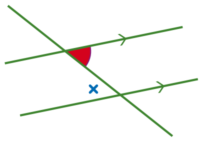
exercise 7.1

1 **a** $a = 40$ **b** $a = 62$ **c** $m = 151$ **d** $b = 19$
e $k = 72$ **f** $b = 133$ **g** $c = 41$ **h** $d = 37$
i $q = 27$ $r = 102$ $s = 29$

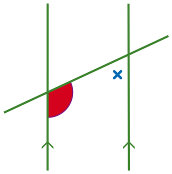
2

Angle	Complement	Supplement
13°	77°	167°
88°	2°	92°
129°	–	51°
54°	36°	126°
142°	–	38°

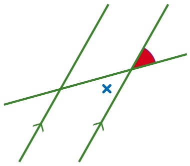
3 **a**



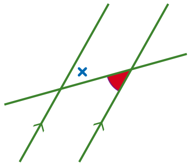
b



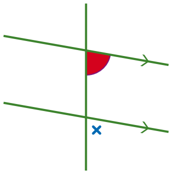
c



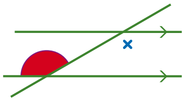
d



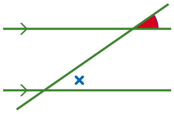
e



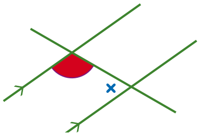
f



g



h



4 **a** $a = 124$ **b** $b = 56$ **c** $c = 143$
d $d = 65$ **e** $e = 132$ **f** $f = 88$
g $b = 94$ **h** $h = 134$ **i** $a = 133$

5 **a** 53° and 127° are supplementary, so *AB* and *CD* are parallel.
b 151° and 31° are not supplementary, so *AB* and *CD* are not parallel.
c 138° and 46° are not supplementary, so *AB* and *CD* are not parallel.
d All angles are right angles, so allied angles are supplementary, so *AB* and *CD* are parallel.

6 **a** $m = 152, n = 152$ **b** $x = 101, y = 101$
c $m = 42, n = 42$ **d** $d = 61, k = 119$
e $e = 48$ **f** $x = 15, y = 165$
g $h = 76, p = 104$ **h** $a = 36, b = 144$
i $f = 95, k = 95$
j $a = 31, b = 149, c = 149$
k $x = 132, y = 48, z = 48$
l $a = 57, b = 123, c = 57$
m $k = 23, p = 140$
n $w = 112, x = 68, y = 68, z = 112$
o $t = 117, u = 63$

7 **a** $\angle BCH$ **b** $\angle BCG$
c $\angle BCG$ **d** $\angle GCD$ or $\angle BCH$
e $\angle MBA$ **f** $\angle NBC$
g $\angle BCH$ **h** $\angle BCH, \angle ABN, \angle MBC, \angle GCD$

8 Line of sight from Harry's eye is horizontal, and the surface of the water is horizontal, so the two lines are parallel. $d = 35$ (alternate angles).

9 **a** $a = 62$ **b** $c = 29$ **c** $c = 111$ **d** $d = 43$

10 **a** **i** $4x = 180$ **ii** $x = 45$
iii 45°, 135°
b **i** $5x = 90$ **ii** $x = 18$
iii 36°, 54°
c **i** $5x - 5 = 180$ **ii** $x = 37$
iii 69°, 111°
d **i** $4x + 72 = 180$ **ii** $x = 27$
iii 27°, 34°, 54°, 65°
e **i** $3x + 54 = 180$ **ii** $x = 42$
iii 42°, 74°, 64°
f **i** $4x + 144 = 360$ **ii** $x = 54$
iii 54°, 96°, 98°, 112°
g **i** $2x - 1 = x + 13$ **ii** $x = 14$
iii 27°, 27°
h **i** $4x = 180$ **ii** $x = 45$
iii 40°, 140°
i **i** $2(x + 4) = 3x - 2$ **ii** $x = 28$
iii 64°, 82°

exercise 7.2

1 **a** **i** scalene **ii** obtuse-angled
b **i** scalene **ii** right-angled
c **i** equilateral **ii** acute-angled
d **i** isosceles **ii** acute-angled
e **i** isosceles **ii** right-angled
f **i** isosceles **ii** obtuse-angled
g **i** isosceles **ii** acute-angled
h **i** equilateral **ii** acute-angled
i **i** isosceles **ii** acute-angled

2 **a** $a = 64$ **b** $b = 76$ **c** $c = 39$ **d** $d = 128$
e $e = 58$ **f** $f = 72$

3 **a** $a = 40.5$ **b** $b = 60$ **c** $c = 60$ **d** $e = 60$
e $f = 24$ **f** $h = 45$ **g** $g = 118$ **h** $h = 45$
i $j = 23$ **j** $d = 35$ **k** $k = 41$ **l** $l = 56$

4 **a** **i** $x + x + x = 180$ or $3x = 180$
ii $x = 60$ **iii** 60°, 60°, 60°
b **i** $x + x + 90 = 180$ or $2x + 90 = 180$
ii $x = 45$ **iii** 45°, 45°, 90°
c **i** $2x + 30 = 180$
ii $x = 75$ **iii** 30°, 75°, 75°
d **i** $2x + 100 = 180$
ii $x = 40$ **iii** 40°, 40°, 100°
e **i** $2x + 48 = 180$
ii $x = 66$ **iii** 48°, 66°, 66°
f **i** $2x + 34 = 180$
ii $x = 73$ **iii** 34°, 73°, 73°

5 Other angle must be a right angle.

6 No, because two obtuse angles would add to more than 180°.

7 **a** $y = 140$ **b** $c = 135$ **c** $d = 136$
d $e = 128$ **e** $y = 117$ **f** $w = 120$
g $h = 112$ **h** $j = 103$ **i** $n = 135$

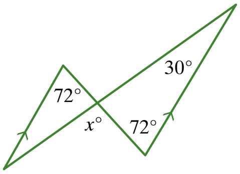
8 **a** $a = 30$ **b** $n = 49$ **c** $c = 38$
d $h = 41$ **e** $j = 32$ **f** $k = 20$

9 D

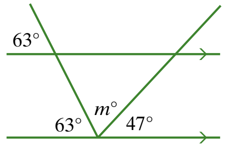
10 C

11 **a** $a = 62$ **b** $c = 29$ **c** $c = 111$ **d** $d = 43$

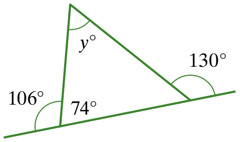
12 **a** $x = 102$



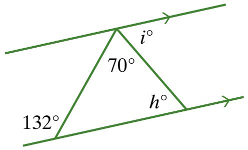
b $m = 70$



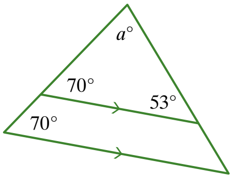
c $y = 56$



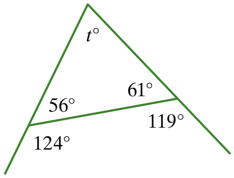
d $h = 62, i = 62$



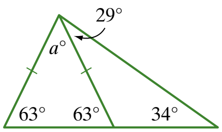
e $a = 57$



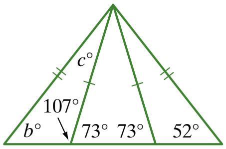
f $t = 63$



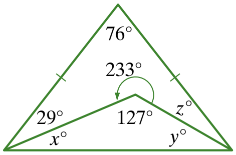
13 **a** $a = 54$ spaces



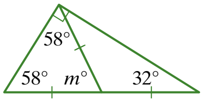
b $b = 52, c = 21$



c $x = 23, y = 30, z = 22$



d $m = 64$



14 **a** $\angle ABL = 110°$; $\angle ALB = 35°$.
So $\triangle ABL$ isosceles because it has two equal angles.
b Distance = speed × time
$= 8 \times 0.5$
$= 4$ nautical miles
c 4 nautical miles
d 7.41 km

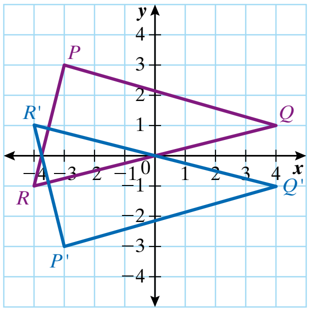
exercise 7.3

1 **a** A and N, B and H, C and P, D and X
b $AB = NH$, $BC = HP$, $CD = PX$ and $DA = XN$
c $NHPX$

2 The angles of each quadrilateral add to 360°, so $\angle D = 79°$ and $\angle M = 68°$. The angles of quadrilateral $ABCD$ are the same as the angles of quadrilateral $MNOP$. The matching sides are equal. Quadrilateral $ABCD$ is congruent to quadrilateral $MNOP$.

3 E

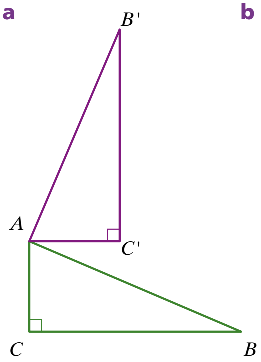
4 **a**



b

$P:(-3, 3)$	$P':(-3, -3)$
$Q:(4, 1)$	$Q':(4, -1)$
$R:(-4, -1)$	$R':(-4, 1)$

5 a



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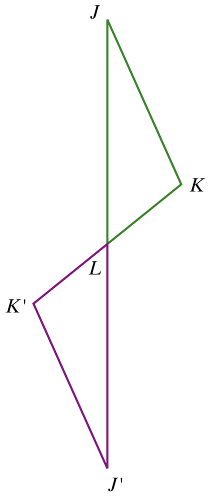
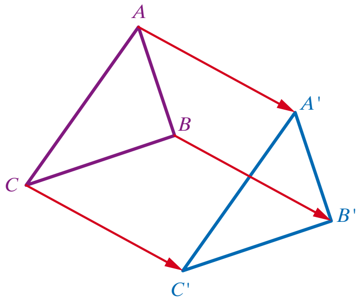


6 a



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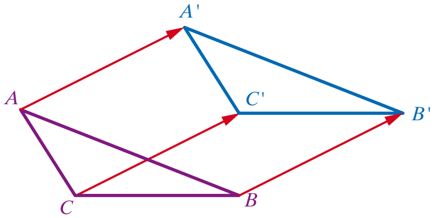
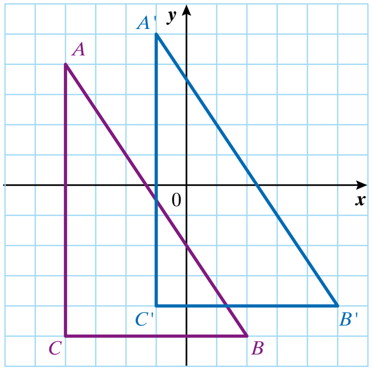
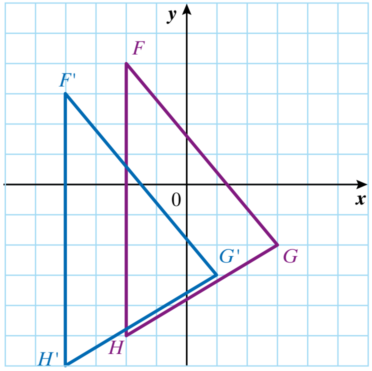


7 a



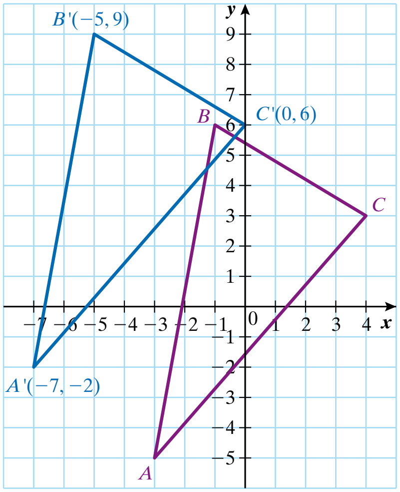
A:$(-4, 4)$	A':$(-1, 5)$
B:$(2, -5)$	B':$(5, -4)$
C:$(-4, -5)$	C':$(-1, -4)$

b



F:$(-2, 4)$	F':$(-4, 3)$
G:$(3, -2)$	G':$(1, -3)$
H:$(-2, -5)$	H':$(-4, -6)$

8



9 a i rotation and translation

ii $A \leftrightarrow G$, $B \leftrightarrow M$, $C \leftrightarrow D$

iii $AB \leftrightarrow GM$, $BC \leftrightarrow MD$, $CA \leftrightarrow DG$

iv GMD

b i rotation and translation

ii $A \leftrightarrow Y$, $B \leftrightarrow L$, $C \leftrightarrow P$

iii $AB \leftrightarrow YL$, $BC \leftrightarrow LP$, $CA \leftrightarrow PY$

iv YLP

c i reflection

ii $A \leftrightarrow E$, $B \leftrightarrow N$, $C \leftrightarrow T$

iii $AB \leftrightarrow EN$, $BC \leftrightarrow NT$, $CA \leftrightarrow TE$

iv ENT

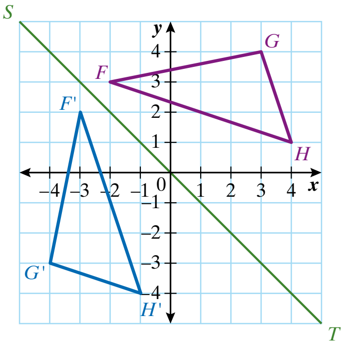
d i reflection, rotation and translation

ii $A \leftrightarrow Q$, $B \leftrightarrow X$, $C \leftrightarrow F$

iii $AB \leftrightarrow QX$, $BC \leftrightarrow XF$, $CA \leftrightarrow FQ$

iv QXF

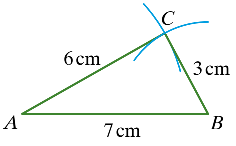
10



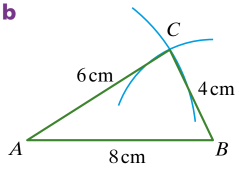
exercise 7.4

1 **a** $4 + 7 > 8$, so a triangle can be constructed
b $3 + 8 < 12$, so a triangle cannot be constructed
c $5 + 6 > 8$, so a triangle can be constructed
d $6 + 7 = 13$, so a triangle cannot be constructed
e $5 + 6 > 7$, so a triangle can be constructed
f $2.9 + 3.4 < 6.5$, so a triangle cannot be constructed
g $2.9 + 4.3 > 5.6$, so a triangle can be constructed
h $2.4 + 2.4 = 4.8$, so a triangle cannot be constructed
i $17 + 23.5 < 45$, so a triangle cannot be constructed

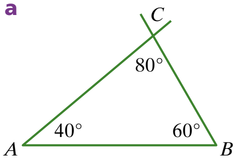
2 **a**



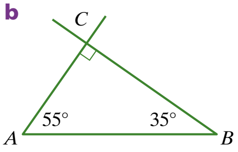
b



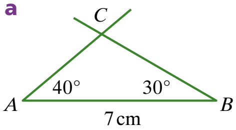
3 **a**



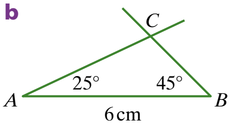
b



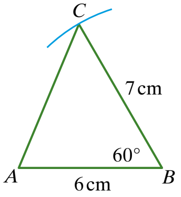
4 **a**



b



5 **a**



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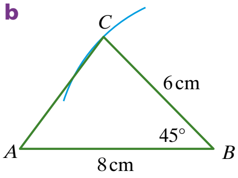
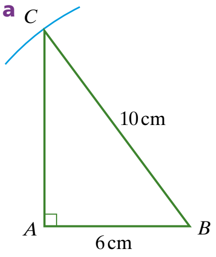
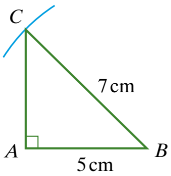


6 **a**



b



7 **a** $\triangle FDJ$ **b** $\triangle NSU$
c $\triangle WGE$ (or $\triangle EGW$) **d** $\triangle HRD$

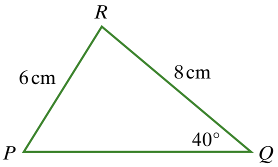
8 **a** congruent (SSS—all matching sides equal)
b not necessarily congruent as we don't know of any equal sides
c congruent (ASA—all three matching angles are equal and the side between the 87° and 52° angles are equal)
d congruent (ASA—all three matching angles are equal and the side between the two 73° angles are equal)
e not necessarily congruent as don't know of any equal sides
f congruent (ASA—all three matching angles are equal and the side between the 90° and 26° angles are equal)
g not necessarily congruent as the 54° angle is not between the matching sides of 40 cm and 52 cm
h congruent (RHS—both triangles have a right angle, equal hypotenuse and matching side equal)
i congruent (SSS—all matching sides equal)
j not necessarily congruent as we only know that two sides are equal
k not necessarily congruent as we don't know of any equal sides
l congruent (SAS—two matching sides and the angles between them are equal)

9 **a** $a = 51, c = 87$
b $w = 25, x = 128, y = 128, z = 27$
c $a = 66, b = 70, c = 58$
d $d = 75, e = 64, f = 58$

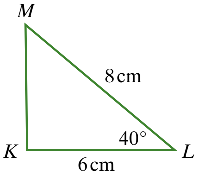
10 In $\triangle AXC$ and $\triangle BXD$,
$AC = BD$ (given)
$\angle CAX = \angle DBX$ (alternate angles)
$\angle AXB = \angle BXD$ (vertically opposite angles)
So, $\triangle AXC \equiv \triangle BXD$ (ASA)

11 As $\triangle AMB \equiv \triangle AMC$,
$\angle AMB = \angle AMC$ (matching angles in congruent triangles)
As BMC is a straight line then both $\angle AMB$ and $\angle AMC = 90°$
So AM is perpendicular to BC
So, the line segment from the vertex A of an isosceles triangle to the midpoint M of the base is perpendicular to the base.

12 **a**



b



c Not necessarily congruent as the 40° angle is not between the sides of 6 cm and 8 cm in $\triangle PQR$ but it is in $\triangle KLM$

13 16

exercise 7.5

1 **a** $a = 104$ **b** $b = 73$ **c** $x = 115$
d $d = 121$ **e** $e = 88$ **f** $f = 112$
g $m = 76$ **h** $h = 96$ **i** $k = 115$

2 **a** 72° **b** 90° **c** 91° **d** 148°
e 105° **f** 107° **g** 45° **h** 108°

3

Special quadrilateral	Diagonals equal in length?	Diagonals bisect each other?	Diagonals intersect at right angles?
Parallelogram	No	Yes	No
Rhombus	No	Yes	Yes
Rectangle	Yes	Yes	No
Square	Yes	Yes	Yes
Trapezium	No	No	No
Kite	No	No	Yes

4 **a** parallelogram, rectangle, rhombus, square
b kite
c rectangle, square

5 **a**

	Special quadrilaterals
Both pairs of opposite sides equal	parallelogram, rectangle, rhombus, square
All sides equal	rhombus, square
Two pairs of adjacent sides equal but opposite sides not equal	kite
Two pairs of parallel sides	parallelogram, rectangle, rhombus, square
One pair of parallel sides	trapezium

b parallelogram, rectangle, rhombus, square

6 **a** rhombus, four equal sides
b trapezium, two pairs of angles are supplementary
c parallelogram, opposite angles are equal and adjacent angles are supplementary
d kite, two pairs of adjacent sides are equal and diagonals are perpendicular
e square, four equal sides, one right angle and perpendicular diagonals
f rectangle, opposite sides are parallel and one right angle

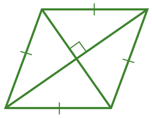
7 B

8 D

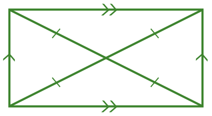
9 **a** trapezium
b one pair of opposite sides are parallel

10 **a** **i** four equal sides, four 90° angles
ii square
b **i** two pairs of opposite sides equal
ii parallelogram
c **i** two pairs of opposite sides parallel, diagonals are perpendicular
ii rhombus
d **i** diagonals are perpendicular, one diagonal bisects the other
ii kite

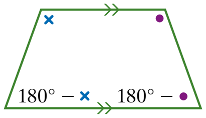
11 **a** rhombus



b rectangle



c trapezium



12 **a** $a = 105, b = 44$
b $k = 131, p = 131, t = 49$
c $w = 102$
d $a = 117, b = 63, c = 117$
e $w = 78, x = 78, y = 102$
f $p = 129, t = 73$
g $x = 119$
h $a = 30, b = 60$
i $a = 72, b = 63, c = 45$
j $a = 32, b = 77, c = 77, d = 71, e = 71$
k $x = 28, y = 88$
l $c = 90, d = 40, e = 50$

13 **a** 99°
Subtracting 115° and 47° from 360° leaves 198°, which is then halved to get 99°.
b 101°
Subtracting 57° from 360° leaves 303°, which is then divided by three to get 101°.

14 C

15 D

16 square, the only special quadrilaterals with equal diagonals are the square and rectangle and of these only the square has diagonals that intersect at right angles

17 **a** $h = 68, s = 73$
b $h = 66, s = 77$

18 **a** **i** $8x = 360$
ii $x = 45$
iii 45°, 90°, 90°, 135°

b **i** $6y = 360$
ii $y = 60$
iii 65°, 115°, 65°, 115°

c **i** $15a - 15 = 360$
ii $a = 25$
iii 50°, 75°, 90°, 145°

d **i** $25m + 10 = 360$
ii $m = 14$
iii 28°, 70°, 100°, 162°

e **i** $6x = 360$
ii $x = 60$
iii 60°, 120°, 60°, 120°

f **i** $4x + 100 = 360$
ii $x = 65$
iii 70°, 72°, 90°, 128°

g **i** $10x = 360$
ii $x = 36$
iii 72°, 108°, 72°, 108°

h **i** $14x + 10 = 360$
ii $x = 25$
iii 50°, 130°, 50°, 130°

i **i** $7x = 360$
ii $x = 51\frac{3}{7}$
iii $61\frac{3}{7}°, 102\frac{6}{7}°, 102\frac{6}{7}°, 92\frac{6}{7}°$

j **i** $7x + 17 = 360$
ii $x = 49$
iii 49°, 98°, 115°, 98°

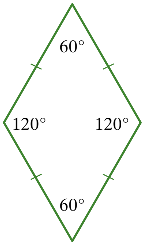
k **i** $5x + 10 = 360$
ii $x = 70$
iii 72°, 80°, 72°, 136°

l **i** $2x - 9 = x + 6$
ii $x = 15$
iii 21°, 38°, 121°, 21°, 38°, 121°

19 $a = 72, b = 108, c = 144, d = 36$

20 **a** the rhombuses have angles of 60°, 120°, 60° and 120°

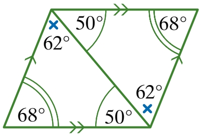
b



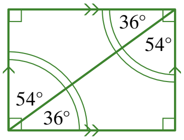
exercise 7.6

1 As $\triangle AMB \equiv \triangle AMC$, $\angle AMB = \angle AMC$ (matching angles in congruent triangles). Also BC is a straight line, so $\angle AMB$ and $\angle AMC$ are 90°. Therefore, AM meets BC at 90°, so $AM \perp BC$.

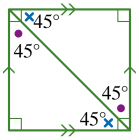
2 **a** **i**



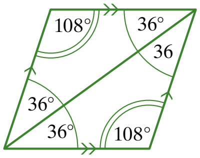
ii



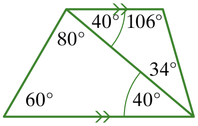
iii



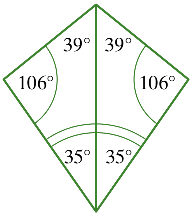
iv



v



vi



b parallelogram, rectangle, square, rhombus, kite

3 **a** In $\triangle ADC$ and $\triangle ABC$,
AC is common to both triangles.
$AD = AB$ (given)
$DC = BC$ (given)
So, $\triangle ADC \equiv \triangle ABC$ (SSS)

b $\angle ADC = \angle ABC$ (matching angles in congruent triangles)

4 As $\triangle ADC \equiv \triangle ABC$,
$\angle DAC = \angle BAC$ (matching angles)
and $\angle DCA = \angle BCA$ (matching angles in congruent triangles)
Hence, AC bisects $\angle DAB$ and $\angle DCB$

5 **a** diagonal
b $\triangle ABD \equiv \triangle CBD$ (SSS)
c $\angle ABD = \angle CBD$
d cuts $\angle ABC$ and $\angle ADC$ in half
e kite

6 diagonals of a kite are perpendicular, one diagonal bisects the other

7 **a** rhombus
b diagonals are perpendicular

8 **a** rhombus
b diagonals are perpendicular

9 In $\triangle AOB$ and $\triangle COD$,
$AB = CD$ (opposite sides of a parallelogram are equal)
$\angle BAO = \angle DCO$ (alternate angles, $AB \| CD$)
$\angle ABO = \angle CDO$ (alternate angles, $AB \| CD$)
So, $\triangle AOB \equiv \triangle COD$ (ASA)
Hence, $AO = CO$ and $BO = DO$ (matching sides in congruent triangles)
Hence the diagonals of a parallelogram bisect each other

10 In question 9 it was shown that the diagonals of a parallelogram bisect each other, so $DM = BM$
In $\triangle ADM$ and $\triangle ABM$,
AM is common to both triangles
$DM = BM$ (proven above)
$AD = AB$ (all sides of a rhombus are equal)
So, $\triangle ADM \equiv \triangle ABM$ (SSS)

Hence, $\angle AMD = \angle AMB$ (matching angles in congruent triangles)
As BD is a straight line, $\angle AMD$ and $\angle AMB$ are $90°$
So, AM is perpendicular to BD
So, the diagonals of a rhombus intersect at right angles

Revision

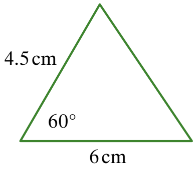
1 E **2** B **3** B **4** A **5** C

6 **a** isosceles triangle **b** scalene triangle

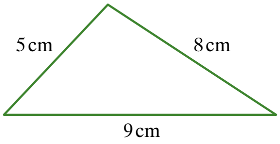
7 **a** acute-angled triangle **b** right-angled triangle

8 **a** $a = 56, b = 79, c = 45$
b $x = 45$

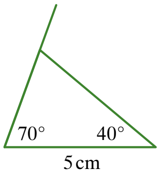
9 **a**



b



c



10 **a** no, $5 + 8 = 13$ **b** yes, $7 + 12 > 13$
c no, $8 + 9 < 20$

11 **a** false **b** true

12 **a** $AB \leftrightarrow FT, BC \leftrightarrow TX, CD \leftrightarrow XP, DA \leftrightarrow PF$
b $\angle ADC = 79°, \angle PFT = 68°$
c yes
d $FTXP$

13 **a** not congruent as matching angles are not equal
b $\triangle ABC \equiv \triangle DSY$ (RHS)
c $\triangle ABC \equiv \triangle FHN$ (SAS)
d $\triangle ABC \equiv \triangle EGT$ (SSS)

14 **a** rhombus **b** trapezium

15 **a** $n = 80$ **b** $b = 55, c = 70$
c $p = 126, q = 83$

16 **a** In $\triangle AMB$ and $\triangle AMC$,
1 AM is common
2 $AB = AC$ (given)
3 $BM = CM$ (given)
So, $\triangle AMB \equiv \triangle AMC$ (SSS)
b $\angle AMB = \angle AMC$ (matching angles in congruent triangles)
As BMC is a straight line this means that $\angle AMB = \angle AMC = 90°$
Hence, $AM \perp BC$

Chapter 8

exercise 8.1

1 **a** 3:7 **b** 6:13 **c** 5:24 **d** 7:4
e 1:5 **f** 2:5 **g** 7:4 **h** 5:3
i 1:3 **j** 16:9 **k** 3:22 **l** 8:5
m 9:8 **n** 2:5 **o** 2:5

2 **a** 36 **b** 15 **c** 12:5

3 **a** 32:8:16 **b** 4:1:2

4 **a** 45% **b** 11:20 **c** 11:9

5 **a** 3:8 **b** 3:5 **c** 37.5%

6 **a** 1:5 **b** 1:4 **c** 20%

7 **a** 16% **b** 21:4 **c** 21:25

8 **a** $\frac{3}{4}$ **b** 75% **c** $\frac{1}{4}$ **d** 25%
e 3:1 **f** 3:4 **g** 75%

9 **a** 4:3 **b** 16:9

10 mix 1 is 1.667 flour:1 butter, mix 2 is 1.333 flour:1 butter, so mix 1 has the greater proportion of flour compared to butter

11 E

12 **a** 1.25:1 **b** 1.4:1 **c** 1:2.6 **d** 1:2.85
e 1:2.875 **f** 1:4.833 **g** 1:1.083 **h** 2.143:1
i 1:2.667 **j** 1.556:1 **k** 2.25:1 **l** 3.333:1
m 1.615:1 **n** 3.375:1 **o** 1.412:1

13 **a** 22:1 **b** 940:41 **c** Woodside

14 **a** **i** 1.4:1 **ii** 1.667:1
iii 1.111:1 **iv** 1.286:1
v 1.25:1 **vi** 1.273:1
b ii **c** iii

15 Alice 1 cordial:5.667 water, Liam 1 cordial:4 water so Liam has the higher proportion of cordial (i.e. less water)

16 1:2 000 000

17 250 m

18 E

19 **a** 3:5 **b** $\frac{3}{8}$ **c** $37\frac{1}{2}\%$

exercise 8.2

1 **a** 15 **b** 16 **c** 25 **d** 27
e 23 **f** 20 **g** 28 **h** 14
i 15 **j** 24

2 **a** 10:8 **b** 350:150 **c** 100:140
d 144:156 **e** 10:35 **f** 280:360
g 2250:1250 **h** 672:168 **i** 1400:600
j 400:80 **k** 1400:1600 **l** 350:850
m 400:320 **n** 320:240 **o** 125:50
p 585:225 **q** 96:408 **r** 2160:2340

3 B

4 **a** 240:320:400 **b** 40:80:140
c 120:180:240 **d** 60:90:120

e 90:180:180 **f** 600:1200:1800
g 550:2750:3300 **h** 224:336:672
i 840:2100:3360 **j** 400:2000:2800
k 808:2020:2020 **l** 820:820:1025

5 E

6 **a** \$250:\$125 **b** \$24:\$36
c 210 m:240 m **d** 315 g:245g
e \$6500:\$1500 **f** \$2400:\$6600
g 1000 L:3500 L **h** \$320:\$200
i 5 lollies:10 lollies:20 lollies
j \$360:\$540:\$900

7 14 red, 6 green, 4 yellow

8 240

9 C

10 George \$48, Anton \$72

11 **a** length = 50 cm, width = 30 cm
b length = 14 m, width = 4 m

12 **a** 415 boys, 498 girls **b** 83
c approximately 45.5%

13 Nancy \$340, Jack \$612, Emily \$748

14 **a** Nina \$3750, Phuong \$3000, George \$2250
b Nina \$9400, Phuong \$7520, George \$5640

15 125 blue, 100 green, 50 yellow and 25 pink

16 **a** 40°, 60°, 80° **b** 30°, 60°, 120°, 150°

17 Angie \$16 000, Will \$32 000, Jess \$64 000

18 Red Cross \$470, Children's Hospital \$282

19 32 cm, 24 cm, 16 cm

20 building maintenance \$9250, advertising \$3700

21 5-year-old \$7.50, 6-year-old \$9, 7-year-old \$10.50

22 cement 18 kg, sand 36.5 kg, gravel 45.5 kg

23 36°, 72°, 108°, 144°

24 No. the last angle would then have to be $36 \times 5 = 180°$, but this would then be a straight line and the shape would not be a pentagon.

exercise 8.3

1 **a** $\frac{a}{3} = \frac{64}{4}$ **b** $\frac{b}{3} = \frac{35}{7}$ **c** $\frac{c}{5} = \frac{13}{65}$
d $\frac{d}{11} = \frac{45}{5}$ **e** $\frac{7}{91} = \frac{e}{52}$ **f** $\frac{90}{5} = \frac{f}{7}$
g $\frac{16}{2} = \frac{g}{9}$ **h** $\frac{h}{7} = \frac{568}{8}$ **i** $\frac{4}{68} = \frac{i}{102}$
j $\frac{267}{3} = \frac{j}{8}$ **k** $\frac{3.5}{5} = \frac{k}{4}$ **l** $\frac{7}{98} = \frac{m}{42}$

2 **a** $a = 2$ **b** $b = 4$ **c** $c = 5$
d $d = 144$ **e** $e = 6$ **f** $f = 49$
g $g = 3$ **h** $h = 3$ **i** $i = 24$
j $j = 92$ **k** $k = 21$ **l** $m = 225$
m $i = 1.088$ **n** $j = 8$ **o** $k = 54.88$
p $m = 22.5$

3 $\frac{21}{7} = \frac{n}{2}$
$n = 6$

4 E

5 60

6 **a** 309 **b** 721

7 **a** 5:3 **b** 100 g

8 D

9 thinner 960 mL, hardener 480 mL

10 315 g

11 6.56 kg

12 \$159.82

13 2.73 m

14 5 tonnes

15 sand 100 kg, gravel 125 kg

16 **a** 11.5 km **b** 57.5 km **c** 115 km
d 575 km **e** 517.5 km **f** 368 km

17 $\frac{h}{2.7} = \frac{6}{4.5}$, height of tree is 3.6 m

18 **a** 80 mL
b 100 mL carpet cleaner, 400 mL water

exercise 8.4

1 **a** \$2/kg **b** \$1.20/kg **c** \$2.24/kg **d** \$2.66/kg

2 \$7/m^2

3 55.8 goals/match

4 \$14.50/hour

5 114 beats/minute

6 **a** 64 runs/100 balls **b** 60 runs/100 balls

7 **a** 587 L/day **b** 147 L/day

8 C

9 120 kg

10 174.8c = \$1.75

11 \$3.85

12 **a** 2.5 shirts/minute **b** 4.8 minutes

13 12 hours

14 25 minutes

15 9.4 L/100 km

16 10.3 km

17 B

18 **a** car A 8.7 L/100 km, car B 9.2 L/100 km
b car A

19 **a** 1548 L **b** 3.7152 tonnes

20 **a**

Mass (g)	Volume (cm^3)	Density (g/cm^3)	Possible material
246	94.6	2.6	glass
24.9	1.29	19.3	gold
4250	1770	2.4	concrete
394	50.5	7.8	steel

b 661.5 g

21 1 kg

22 819.2 KB/s

23 14 seconds

exercise 8.5

1 **a** 75 km/h **b** 300 km/h **c** 280 km/h
d 4.2 km/h **e** 191.73 km/h **f** 15 km/h
g 3.75 km/h **h** 100 km/h

2 75.9 km/h

3 **a** 360 km **b** 600 km **c** 12 km **d** 16 905 km
e 297.5 km **f** 16.8 km **g** 18 km **h** 264 km

4 D

5 A

6 **a** 2 h **b** 4 h **c** 4 h **d** 0.8 h
e 13.75 h **f** 6 h **g** 0.9 h **h** 1.6 h

7 **a** 14 km/h **b** 175 km **c** $1\frac{1}{3}$ h
d 20 km/h **e** $2\frac{1}{2}$ h **f** 35 km

8 10.3 m/s

9 2.4 m/s

10 1.4 mm/s

11 **a** 15 cm/s **b** 0.15 m/s **c** 9 m
d 540 m **e** 0.54 km/h

12 3.4 hours

13 27 km/h

14 **a** **i** 2.56 hours **ii** 2 hours 43 minutes
b 70 km/h

exercise 8.6

1 **a** 54 s **b** 5 h 17 s
c 27 min 15 s **d** 2 h 43 min 18 s

2 **a** seconds **b** hours **c** minutes **d** seconds
e minutes **f** seconds **g** hours **h** minutes
i hours **j** seconds **k** seconds **l** hours

3 **a** 5 **b** 3 **c** 2 **d** 42 **e** 14
f 1135 **g** 3.5 **h** 45 **i** 17 520 **j** 295
k 150 **l** 4380 **m** 132 **n** 13, 20 **o** 900
p 11, 6

4 **a** 17:00 **b** 10:15 **c** 13:45 **d** 19:55
e 03:22 **f** 08:17 **g** 06:49 **h** 22:57

5 **a** lunchtime **b** wake up
c go home from school **d** bedtime

6 04:42 and 16:42

7 **a** 07:30 **b** 23:59 **c** 21:45 **d** 08:20
e 20:20 **f** 21:15 **g** 22:55 **h** 00:18
i 01:00 **j** 00:01

8 D

9 B

10 **a** 120 days **b** $17\frac{1}{7}$ weeks **c** Wednesday

11 **a** 3 hours 30 minutes **b** 4 hours 10 minutes
c 2 hours 30 minutes **d** 6 hours 25 minutes
e 6 hours 15 minutes **f** 8 hours 50 minutes
g 17 hours 35 minutes **h** 7 hours 55 minutes
i 21 hours 45 minutes **j** 62 hours 45 minutes

12 yes

13 **a** 57 minutes 22 seconds
b 2 minutes 38 seconds

14 26 minutes

15 1 hour 20 minutes

16 12 minutes 17 seconds

17 **a** **i** 25 hours 25 minutes **ii** 15 hours
iii 4 hours 30 minutes
b 4 hours **c** 53 hours 10 minutes
d

Adelaide–Alice Springs–Darwin		
Sunday		
Depart Adelaide	12:20 pm	12:20
Monday		
Arrive Alice Springs	1:45 pm	13:45
Depart Alice Springs	6:00 pm	18:00
Tuesday		
Arrive Katherine	9:00 am	09:00
Depart Katherine	1:00 pm	13:00
Arrive Darwin	5:30 pm	17:30

18 **a** 11:25 **b** 13:25

19

State/Territory	Time
ACT	8:00 pm
New South Wales	8:00 pm
Northern Territory	6:30 pm
Queensland	7:00 pm
South Australia	7:30 pm
Tasmania	8:00 pm
Victoria	8:00 pm
Western Australia	6:00 pm

20 **a** 11:00 am **b** 8:00 am

21 **a** 11:00 **b** 10:30

22 **a** 18:45 **b** 23:15

23 3:00 pm

24 **a** 07:15 from Melbourne
b $10\frac{1}{2}$ hours

Revision

1 C **2** B **3** D **4** D **5** C

6 **a** 7:3 **b** 7:10 **c** 30%

7 **a** 2:3 **b** 1:2 **c** 29:24 **d** 7:2

8 500

9 **a** 1701 **b** 2916

10 **a** 125:175 **b** $210:$150:$120
c Ada $180, Nancy $270

11 Tom $4500, Hui $6000

12 school A student to teacher ratio = 23.0:1
school B student to teacher ratio = 20.3:1
therefore school B had a lower student to teacher ratio

13 **a** $18.50/h **b** 40 hours

14 **a** 4.2 km/h **b** 2 hours 1 minute

15 8 days

16 **a** 8.3 L/100 km **b** $22.35

17 **a** 468.75 km **b** $10\frac{2}{3}$ L/100 km

18 640 m

19 **a** 6 min 48 s **b** 3 h 55 min

20 **a** 5 h **b** 98 s **c** 3 min **d** 7200 s

21 **a** 07:34 **b** 17:48

22 **a** 2:30 pm **b** 9:24 am

23 69

24 15

25 25 minutes

26 **a** 13:30 **b** 11:30

27 Flying Frogs 1500, Glowing Grubs 1000, Crunchy Critters 750, Bouncing Beetles 250

28 Glowing Grubs 240, Crunchy Critters 240, Bouncing Beetles 160, total 1200

29 **a** 2.7 g/cm^3 **b** 226.8 g

Chapter 9

exercise 9.1

1 **a** N **b** N **c** C **d** C
e C **f** N **g** C **h** N

2 C

3 **a** How long do you watch television for each night?
b How many pieces of fruit do you eat each day?
c What is your height?

4 **a** What types of plants do you have in your garden?
b Do you catch public transport?
c Which one of the following best describes your age: child, young adult, middle aged, senior?

5 D

6 Check with your teacher. Continuous numerical data is data that can be measured, while discrete numerical data is data that can be counted.

7 **a** DN, sample response: 4
b CN, sample response: 1.4 m
c NC, sample response: blue, green, red
d OC, sample response: never
e NC, sample response: blond
f DN, sample response: 2
g NC, sample response: yes
h CN, sample response: 3.4 km
i DN, sample response: 6
j OC, sample response: 2001

8 answers will vary, samples given below
a military rank, starting position in a race, socio-economic status
b number of pets, number of pens in pencil case, number of pairs of shoes owned
c length of hair, time you have been alive, volume of water drunk

9 answers will vary, samples given below
a colour of car, make of car
b year manufactured, motor size
c number of doors, seating capacity
d length of car, width of tyres

exercise 9.2

1 **a**

Score	Tally	Frequency
4	|	1
5	||	2
6	𝍸 |	6
7	||||	4
8	𝍸	5
9	𝍸	5
10	𝍸 ||	7

b 30 **c** 10 **d** 6 **e** $\frac{6}{15}$

2 **a**

Score	Tally	Frequency
1	|||	3
2	𝍸	5
3	𝍸	5
4	𝍸 |	6
5	||||	4
6	𝍸 ||	7
7	𝍸 |||	8
8	𝍸 ||||	9
9	𝍸 ||	7
10	𝍸 |	6

b 8 **c** 1
d Elio's scores were spread amongst all values but there were more of the higher scores, so his overall performance was good.

3 D

4

Class interval (cm)	Frequency
170–<175	3
175–<180	3
180–<185	4
185–<190	0
190–<195	2

5 **a** 40 **b** 3 **c** 12 **d** 45%

6 **a** maximum = 108, minimum = 58

b

Number of beats	Tally	Frequency
55–59	||	2
60–64	|||	3
65–69	卌 ||	7
70–74	|||	3
75–79	卌 ||||	9
80–84	卌 ||||	9
85–89	||||	4
90–94	卌 |	6
95–99	卌	5
100–104	|	1
105–109	|	1

c 75–79 and 80–84

7

Time (min)	Tally	Frequency
10–19	||	2
20–29	卌 卌 ||	12
30–39	卌 卌 |||	13
40–49	卌 卌	10
50–59	卌	5
60–69	卌	5
70–79	|||	3

8 **a** her choice of intervals doesn't include 0 and there is a data value 0

b

TV watched (hours)	Tally	Frequency
0–4	||	2
5–9	卌 |||	8
10–14	卌 ||	7
15–19	|||	3
20–24	卌 ||	7
25–29	|||	3
30–34	|	1

9 **a**

Length interval (cm)	Tally	Frequency
20–<30	||||	4
30–<40	||	2
40–<50	卌 卌	10
50–<60	卌 |	6
60–<70	||||	4
70–<80	卌 ||||	9
80–<90	卌 |	6
90–<100	||||	4

b

Length interval (cm)	Frequency
20–<40	6
40–<60	16
60–<80	13
80–<100	10

c

Class interval (cm)	Frequency
20–<60	22
60–<100	23

d answers will vary

10 answers will vary

exercise 9.3

1 **a** 26.5 **b** 5.12

2 **a** 2.87 **b** 2.07 **c** 11.53

3 2.9

4 **a** 37 **b** 5 **c** 114 **d** 16

5 E

6 D

7 **a** mean 9.5, median 9.5, mode 10
b mean 32.7, median 24, mode 17 and 45
c mean 34.3, median 35.8, all are modes
d mean 1, median 2, mode −5

8 mean $173.33, median $170, mode $160 and $220

9 **a** mean 15.375, median 14, mode 14
b mean

10 **a** mean $140, median $130, mode $90
b mode

11 **a** mean 16.3, median 16, mode 25
b median

12 **a** 5.0 **b** median 5, mode 4
c mode

13 **a** 17 **b** 49 **c** 26.1
d 8.9 **e** 1342 **f** 17

14 **a** 3.2 **b** median 3, mode 4
c mode

15 answers will vary, one possible set is 3, 4, 6, 7, 9, 19

16 answers will vary, one possible set is 1, 2, 4, 7, 16, 48, 51

17 answers will vary, one possible set is 1, 3, 3, 6, 14, 17, 18, 20

18 94

19 5

20 **a** answers will vary, one possible set is 2, 3, 3, 11, 13, 13, 13, 22
b mean would double
c mean would increase by 20%

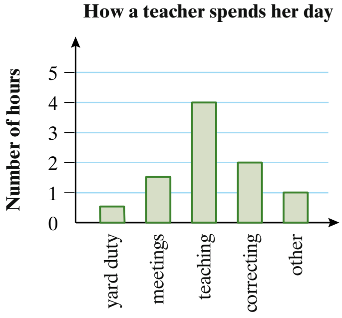
exercise 9.4

1 **a** favourite snacks of boys and girls
b lollies
c chips and lollies
d vegetables, fruit, cake and sweet biscuits
e popcorn
f A pie chart wouldn't allow for the easy comparison between boys and girls that this graph gives.

2 **a**

	Angle	Fraction of time	Percentage of time (to nearest whole number)
Yard duty	20°	$\frac{1}{18}$	6%
Meetings	60°	$\frac{1}{6}$	17%
Teaching	160°	$\frac{4}{9}$	44%
Correcting	80°	$\frac{2}{9}$	22%
Other	40°	$\frac{1}{9}$	11%

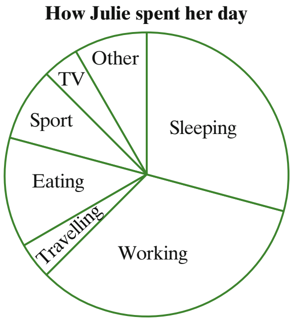
b



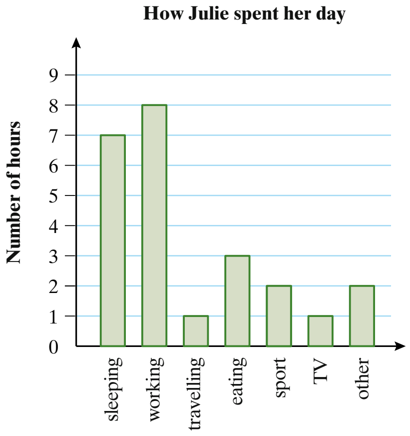
3 **a**

Activity	Hours	Angle
Sleeping	7	105°
Working	8	120°
Travelling	1	15°
Eating	3	45°
Sport	2	30°
TV	1	15°
Other	2	30°

b



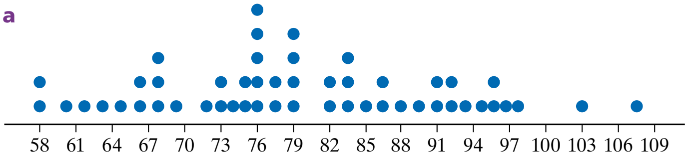
c



d answers will vary

4 **a** their incomes
b No, it depends on how much they earn. If Priyanka earns more than Jane then her 25% spent on going out may be greater than Jane's 28%.

5 **a**



b The pulse rates are distributed over a wide range from 58 beats per minute to 108 beats per minute. The pulse rates were quite evenly distributed except for two that were a fair bit higher. Half of the pulse rates were between 70 and 89.

c **i**

stem	leaf
5	8 8
6	0 2 3 5 7 7 8 8 8 9
7	2 3 4 5 5 6 6 6 6 6 8 8
8	0 0 0 0 2 2 4 4 4 5 7 7 9
9	0 2 2 3 3 4 5 6 6 7 8
10	3 8

Key 5|8 = 58

ii

stem	leaf
5	8 8
6	0 2 3
6	5 7 7 8 8 8 9
7	2 3 4
7	5 5 6 6 6 6 6 8 8
8	0 0 0 0 2 2 4 4 4
8	5 7 7 9
9	0 2 2 3 3 4
9	5 6 6 7 8
10	3
10	8

d 103 and 108 **e** the dot plot

6 54

7 90

8

a	**i** 44.6	**ii** 45	**iii** 45	**iv** 25
b	**i** 73.6	**ii** 74.5	**iii** 82	**iv** 21
c	**i** 32.4	**ii** 31.5	**iii** 31	**iv** 18
d	**i** 7.7	**ii** 7.2	**iii** 8.4	**iv** 3.9
e	**i** 16.6	**ii** 16	**iii** 14	**iv** 26
f	**i** 27.9	**ii** 26.5	**iii** 26	**iv** 47

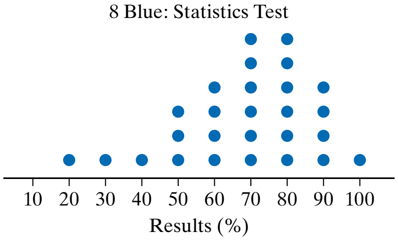
9 **a** 0 **b** 39 **c** 9

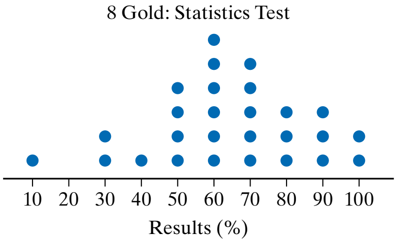
d one of the students owns a phone but didn't send any text messages

e 7 **f** 50 **g** 8

h 7 and 9 **i** 0, 1 and 5

10 **a**





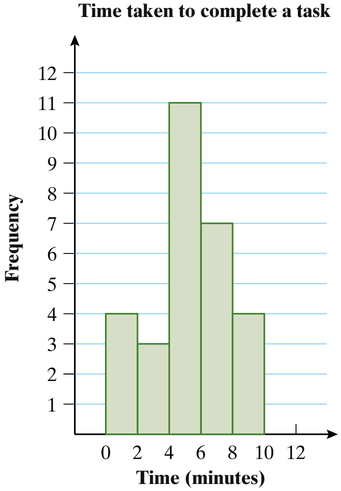
b 8 Gold **c** 8 Blue

d 8 Blue, you can see their scores are grouped together around 70 and 80 while 8 Gold's scores are grouped around 50 to 70

exercise 9.5

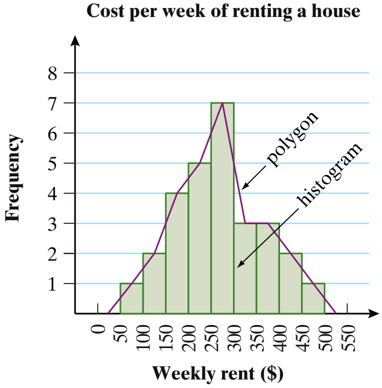
1 **a** 29

b



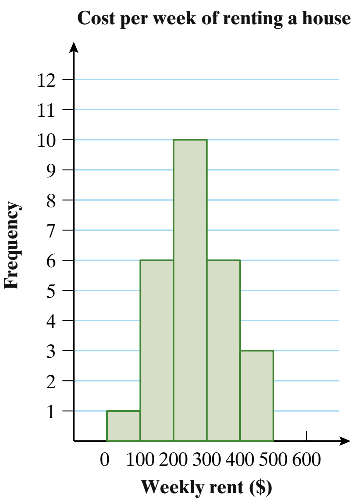
2 **a** 28

b, c

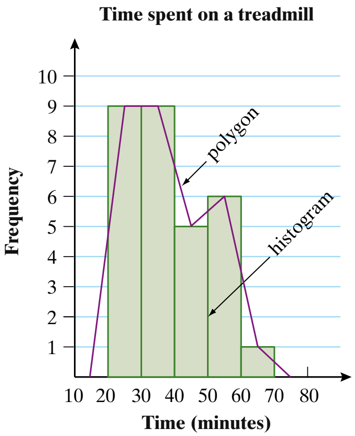


d The date is all in the range \$50–<\$500. It is reasonably symmetrical about the \$250–<\$300 interval.

e



3 **a, b**

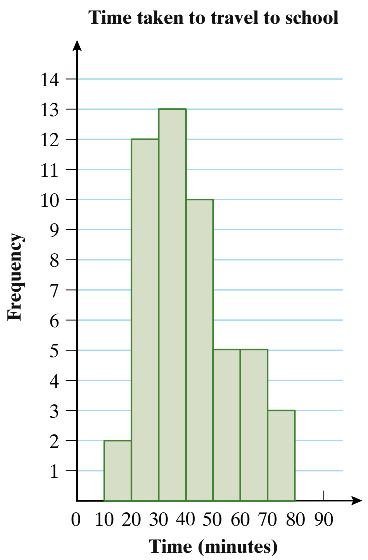


c The data is nearly all in the range 20–<60 minutes. The times are positively skewed with over half below 40 minutes.

4 **a**

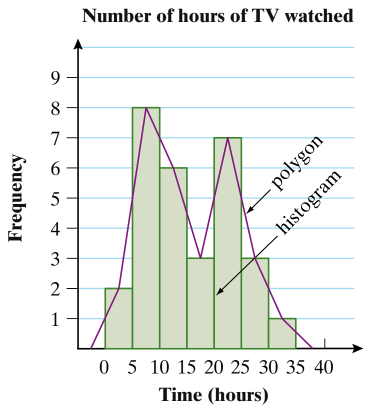
Time (minutes)	Frequency
10–<20	2
20–<30	12
30–<40	13
40–<50	10
50–<60	5
60–<70	5
70–<80	3
Total	50

b



c The data is all in the range 10–<80 minutes. The data distribution is slightly positively skewed. Half of the data values are from 20–<40 minutes.

5 **a, b**



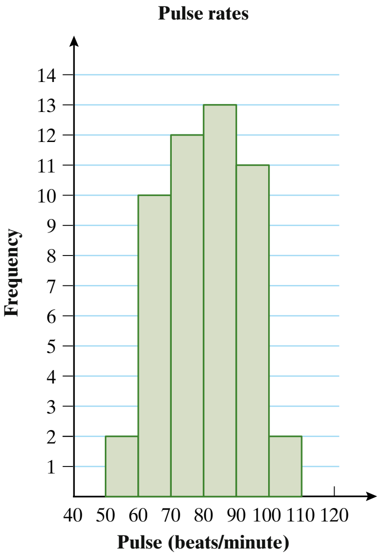
c The data is all in the range 0–<35 hours. Nearly half of the data values are in the 5–<15 hours interval. It is slightly positively skewed.

6 **a** 30
b 150–<155 cm
c 8
d no, it is close to symmetrical

7 **a**

Pulse (beats/minute)	Frequency
50–<60	2
60–<70	10
70–<80	12
80–<90	13
90–<100	11
100–<110	2
Total	50

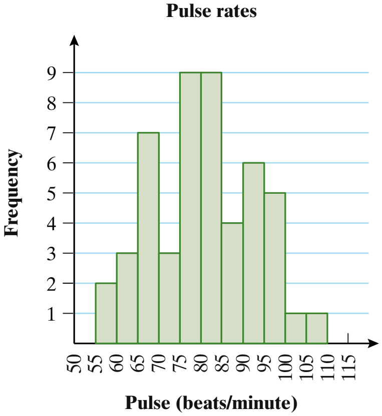
b



c 80–<90

d The data is all between 50–<110 beats per minute. The data distribution is symmetrical and the centre of the data seems to be about 80 beats/minute.

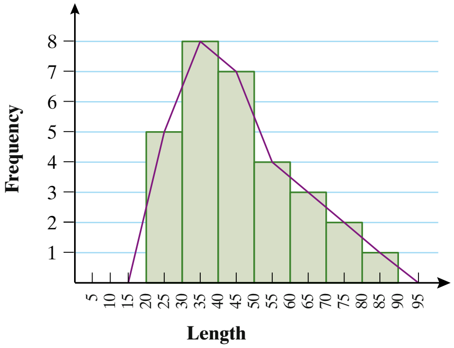
e



f The histogram with class intervals of 10 beats/minute clearly shows the shape of the data. The histogram with class intervals of 5 beats/minute spreads the data out but does not really provide greater information about the distribution.

8 **a** 10

b



c The data is all between 20–<90. The data is positively skewed. The data is clustered around 30–<40 with two-thirds of the data in the 20–<50 intervals.

exercise 9.6

1 **a** sample **b** sample
c census **d** sample

2 **a** Depends on the person's idea of tall.
b Leading them to answer and say it is ugly.
c Depends on their idea of sport and also the length of time.
d Open ended, so difficult to interpret data. Also a personal question so people may feel uncomfortable answering.

3 **a** 28%

b

Age group	% of respondents	Number of respondents
18–25	2%	1
26–35	7%	4
36–45	14%	8
45–60	32%	18
>60	45%	25

c With only a 28% response rate it is not a large sample. The major concern is that the representation of people in the younger age groups 18–45 is much less than those in the two older age groups.

4 **a** no, the results will reflect the views of readers of the ABC's website and will not be a representative sample of all Australians
b because Australian Rules is a uniquely Australian sport
c there are only two choices in the first question, but six in the second so you would expect more variety

5 the question is unbiased, easy to understand and the answers are easy to analyse

6, 7 answers will vary, discuss with your teacher

8 **a** the possible impact on trade deals between Australia and Japan

b whale meat is seen as a food source and part of Japanese culture

c not part of Australian culture and reaction to whaling methods

9 **a** the second question

b

	Agree or strongly agree	Disagree or strongly disagree	Don't know
Take responsibility for the nuclear waste from the uranium it exports by storing it in Australia	42%	56%	2%
Only export uranium to countries which have signed the global Nuclear Non-proliferation Treaty	88%	9%	2%

c first question may have a direct impact on people, so more likely to disagree

10 **a** Baby Boomers **b** Generation X

11 **a** 75% **b** increased to 87%

c answers will vary, but you would expect people involved in the sport and their friends or family to be less supportive of a ban

Revision

1 C **2** C **3** C **4** E **5** B

6 **a** numerical, discrete **b** categorical, nominal

c categorical, nominal **d** numerical, continuous

e numerical, discrete **f** categorical, ordinal

7 **a** separated into colours so it is categorical and there is no obvious order so it is nominal

b

Colour	Tally	Frequency
Brown	𝍸 𝍸 |||	13
Red	𝍸 𝍸 ||||	14
Orange	𝍸 |	6
Blue	𝍸 𝍸 |	11
Yellow	𝍸 ||	7
Green	𝍸 ||||	9

c red **d** $\frac{1}{10}$

8

Time (minutes)	Tally	Frequency
20–29	𝍸 ||||	9
30–39	𝍸 ||||	9
40–49	𝍸	5
50–59	𝍸 |	6
60–69	|	1

9 **a** 141.7 mm **b** 86 mm **c** 420 mm

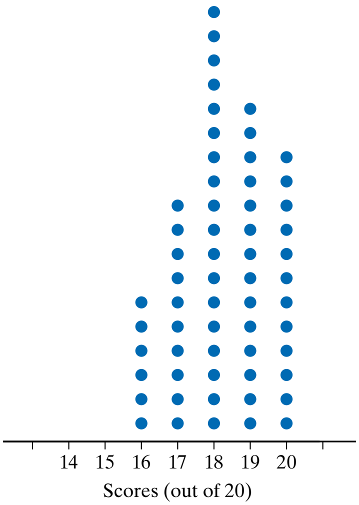
d no, there is such a big variation in values and some really small ones so it would be hard to draw and see it properly in a pie chart

10 **a** 75 **b** 2 **c** 2.44 **d** 2

11 **a** 60 **b** $43\frac{1}{3}\%$

c

Scores in Spelling Test

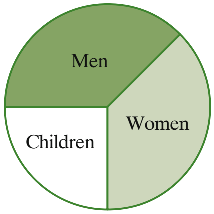


12 2.32

13 **a** 200 **b** 120°

c

Hockey Club Members



14 **a**

stem	leaf
6	4 5 9 9
7	0 2 6 8 7
8	0 2 4 4 6 7 8
9	1 3 4 6

Key 6|9 = 69

b 81
c the data is quite evenly spread out but with a small peak in the 80s

15 **a**

stem	leaf
3	1 3 4
3	5 7 7 8 9 9
4	0 1 1 1 2 2 2 2 4 4
4	6 6 6 8
5	0 2 4
5	5 6 7
6	0

Key 3|1 = 31

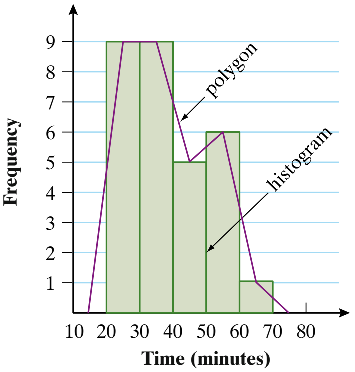
b 42
c 29
d no
e The data is all between 31 to 60 years old. The data is clustered around 35 to 48 with two thirds of the data values in this interval. The data is positively skewed.

16 **a**

Time (minutes)	Tally	Frequency
20–<29	𝍸 ||||	9
30–<39	𝍸 ||||	9
40–<49	𝍸	5
50<59	𝍸 |	6
60–<69	|	1

b, c

Time on treadmill



d The data is all in the range 20–<60 minutes. The times are positively skewed with over half below 40 minutes.
e approximately 36 minutes

17 **a** 1100 **b** 9:16
c no, the people who use the ABC website are not representative of the whole population of Australia and so these proportions wouldn't necessarily be representative either

Chapter 10

exercise 10.1

1 **a** 8500 mm **b** 4.2 mm **c** 481 cm
d 0.004 m **e** 0.155 km **f** 1300 mm
g 0.05 cm **h** 24.25 m **i** 14.5 m

2 **a** 135 cm **b** 12.6 m **c** 14.4 m
d 116 cm **e** 15.1 cm **f** 15.8 m

3 C

4 **a** 19.28 m **b** 25.6 cm **c** 158.4 m **d** 12.2 m
e 120.2 m **f** 4.24 m **g** 740 cm **h** 12.64 m

5 D

6 D

7 B

8 34 cm

9 1.69 m

10 70 m

11 1.1 m

12 **a** 188 m **b** 384 m **c** 216 m
d 112 m **e** 21.6 m **f** 40.4 m

13 **a** 182 cm **b** insufficient information

14 179 cm

15 **a** 1m < 105 cm. Girth around the other two dimensions is 138 cm, so this parcel would be accepted.
b 40 cm < 105 cm, but girth is 160 cm so this parcel would not be accepted.

exercise 10.2

1 **a** 31.4 cm **b** 78.5 mm **c** 141.4 mm
d 58.4 cm **e** 13.8 m **f** 471.2 cm
g 57.8 m **h** 20.2 m **i** 49.0 cm

2 **a** 94.25 cm **b** 753.98 mm **c** 10.05 m
d 1633.63 mm **e** 175.93 cm **f** 9.42 m
g 17.28 m **h** 22.37 m **i** 2.38 m

3 **a** 33.3 cm **b** 37.7 mm **c** 58.1 cm
d 7.9 m **e** 490.1 mm **f** 11.0 m
g 14.1 m **h** 47.1 cm **i** 141.4 mm
j 169.6 cm **k** 54.0 m **l** 4.4 m

4 **a** 377 m **b** 2.26 km

5 **a** 8.80 m **b** 2

6 D

7 C

8 D

9 B

10 **a** 2.0 m **b** 6.2 cm **c** 39.9 cm **d** 5.3 m
e 34.4 cm **f** 44.6 mm **g** 4.3 m **h** 18.1 cm

11 **a** 2.7 cm **b** 13.6 cm **c** 0.4 m **d** 7.7 cm
e 22.3 cm **f** 12.4 cm **g** 0.3 m **h** 1.2 m

12 1.43 m

13 **a** 6.28 m **b** 2 **c** 159

14 **a** 188.5 cm **b** 188.5 cm **c** 530

15 **a** 66.0 cm **b** 55

16 3.82 m

17 30 cm

18 1.6 m

19 31.8 cm

20 **a** 471 cm **b** 101 859
c the penny farthing moves more than double the distance of a standard bicycle wheel
d 3

21 **a** **i** 3.125 **ii** 3.16049… **iii** 3.14285…
iv 3.14084… **v** 3.14159…
b v

22 **a** 12.9 m **b** 7.2 m **c** 16.0 m
d 166.5 m **e** 7.4 m **f** 150.8 cm

23 **a** 142.8 mm **b** 5.0 m **c** 139.3 cm
d 16.9 m **e** 257.1 m **f** 123.4 cm

24 6.4 m

25 431 m

26 the circumference—the height is 3 times the diameter, but the circumference is 3.14… times the diameter

exercise 10.3

1 **a** 2.74 cm^2 **b** 3520 mm^2 **c** 67 m^2
d 9.5 cm^2 **e** 37.8 km^2 **f** 56 km^2
g 452 ha **h** 180 ha **i** 70 ha
j 340 000 cm^2 **k** 180 cm^2 **l** 32 km^2
m 25.6 m^2 **n** 1 200 000 m^2 **o** 5000 m^2

2 C

3 **a** 750 m^2 **b** 30.72 m^2 **c** 99 225 cm^2
d 1507.32 m^2 **e** 2.9 m^2 **f** 917.28 m^2
g 34 225 cm^2 **h** 716.32 m^2

4 17.68 m^2

5 104.4 ha

6 1 980 000 ha

7 6500 km^2

8 approximately 7 km^2

9 **a** 15 cm **b** 24 cm **c** 7.8 m **d** 32.5 m

10 **a** 28 cm **b** 35 cm **c** 5.6 m **d** 18.5 m

11 576 cm^2

12 84.73 m^2

13 **a** **i** 1 cm^2 **ii** 4 cm^2 **iii** 9 cm^2 **iv** 16 cm^2
b **i** 4 **ii** 9 **iii** 4 **iv** 9

14 **a** 864 m^2 **b** 1008 cm^2 **c** 264 cm^2
d 1034 m^2 **e** 456 cm^2 **f** 4.2 m^2
g 2.175 m^2 **h** 4320 cm^2 **i** 8064 mm^2
j 4320 cm^2 **k** 8064 mm^2 **l** 45.24 m^2

15 B

16 **a** $h = 44$ **b** $b = 28$ **c** $b = 2.2$

17 **a** 27 cm^2 **b** 480 cm^2 **c** 616 mm^2
d 487.5 m^2 **e** 513 mm^2 **f** 5.88 m^2
g 528 mm^2 **h** 798.25 cm^2 **i** 3795 cm^2
j 3596 m^2 **k** 60.01 m^2 **l** 60.84 m^2
m 1.365 m^2 **n** 3596 m^2 **o** 139.36 cm^2

18 **a** green = 4608 cm^2, yellow = 4608 cm^2, red = 4608 cm^2
b blue = 1728 cm^2, yellow = 1728 cm^2, red = 3456 cm^2, white = 1728 cm^2, green = 1728 cm^2

19 C

20 1560 cm^2

21 **a** $h = 38$ **b** $b = 120$ **c** $b = 53$
d $h = 4.3$ **e** $h = 72$ cm **f** $b = 51$ m
g $h = 75$ mm **h** $b = 32.8$ m

22 **a** 130 cm^2 **b** 4.14 cm

exercise 10.4

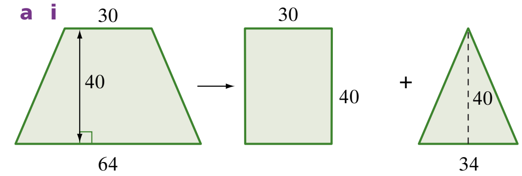
1 **a** 900 cm^2 **b** 864 cm^2 **c** 1440 cm^2 **d** 12.5 m^2
e 38.5 m^2 **f** 260 cm^2 **g** 40.5 m^2 **h** 1206 mm^2
i 10.44 m^2 **j** 16.25 m^2 **k** 18 m^2 **l** 2.88 m^2

2 2700 cm^2

3 **a** 780 cm^2 **b** 770

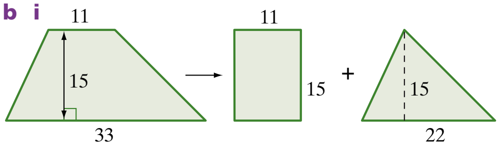
5 **a** 1722 cm^2 **b** 360 cm^2
c 192 cm^2 **d** 528 cm^2

5 **a** **i**



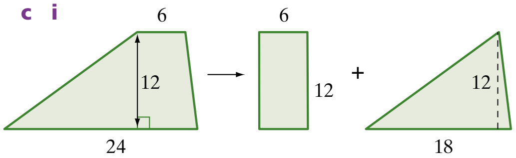
ii 1880 cm^2

b **i**



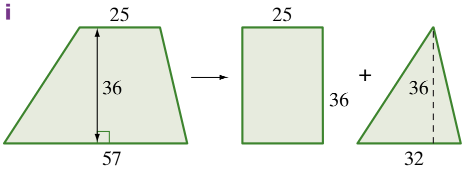
ii 330 cm^2

c **i**



ii 180 cm^2

d **i**



ii 1476 cm^2

6 **a** 1722 cm^2 **b** 360 cm^2 **c** 192 cm^2 **d** 528 cm^2
e 1880 cm^2 **f** 330 cm^2 **g** 180 cm^2 **h** 1476 cm^2

7 3150 cm^2

8 **a** 15 000 cm^2 **b** 1.5 m^2

9 $x = 43.32$, $y = 34.52$

exercise 10.5

1 **a** 314 cm^2 **b** 707 cm^2 **c** 3217 cm^2
d 7 m^2 **e** 6 m^2 **f** 845 m^2
g 8 m^2 **h** 6 m^2 **i** 642 m^2
j 68 813 mm^2 **k** 22 m^2 **l** 4 m^2

2 B

3 **a** 88.3 cm^2 **b** 113.1 mm^2 **c** 268.8 cm^2
d 4.9 m^2 **e** 19113.4 mm^2 **f** 9.6 m^2
g 15.9 m^2 **h** 176.7 cm^2 **i** 1590.4 mm^2
j 2290.2 cm^2 **k** 232.4 m^2 **l** 1.5 m^2

4 98.5 m^2

5 **a** **i** 12.0 cm **ii** 23.9 cm
b **i** 8.7 cm **ii** 17.5 cm
c **i** 4.9 cm **ii** 9.8 cm
d **i** 1.6 cm **ii** 3.3 cm
e **i** 6.7 cm **ii** 13.4 cm
f **i** 45.6 cm **ii** 91.2 cm
g **i** 2.4 cm **ii** 4.8 cm
h **i** 5.4 cm **ii** 10.7 cm

6 C

7 452.4 m^2

8 **a** 66.0 m **b** 346.4 m^2

9 **a** 226.2 cm^2 **b** 39.3 cm^2 **c** 50.3 cm^2 **d** 490.1 cm^2

10 **a** 157.1 cm^2 **b** 78.5 cm^2 **c** 235.6 cm^2 **d** 157.1 cm^2

11 E

12 77.0 m^2

13 **a** 2.7 m^2 **b** 424.1 mm^2 **c** 34.4 m^2

14 **a** 254 cm^2 **b** 16 cm

15 **a** $\frac{210}{360} \times \pi \times 15^2$ **b** 412 cm^2

exercise 10.6

1 **a** 32 m^2 **b** 1500 cm^2 **c** 17 m^2
d 1242 cm^2 **e** 15 m^2 **f** 16.72 m^2

2 **a** 28.3075 m^2
b area to be painted is 56.615 m^2, a 5 L tin will cover 70 m^2, so is enough to paint the walls twice
c 12.16 m^2
d yes, a 1 L tin will cover 14 m^2, so is enough to paint the ceiling once

3 **a** 40.5 m^2 **b** 1000 cm^2 **c** 2247.5 cm^2
d 1058 cm^2 **e** 1462.5 cm^2 **f** 7.29 m^2

4 **a** 2000 cm^2 **b** 1158 cm^2 **c** 1800 cm^2
d 1671.1 cm^2 **e** 1998 cm^2 **f** 1623.1 m^2

5 **a** red = 4896 cm^2; white = 4896 cm^2; blue = 4032 cm^2
b black = 2700 cm^2; green = 4725 cm^2; white = 4050 cm^2, red = 4725 cm^2

6 **a** 193.14 mm^2 **b** 536.50 mm^2

7 **a** 5.8 m^2 **b** 226.2 cm^2 **c** 9.4 m^2
d 1709.0 cm^2 **e** 4.7 m^2 **f** 1253.6 m^2

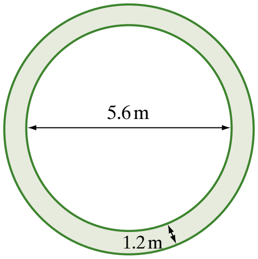
8 **a** **i** 91.6 cm^2 **ii** 31.2 cm^2
b approximately 3 times

9 **a** 10.6 m^2 **b** 33.0 m^2 **c** 1382.3 cm^2
d 9.4 m^2 **e** 13.0 m^2 **f** 706.9 cm^2

10 red: 1257 cm^2; white: 5743 cm^2

11 red area 2347 cm^2, black area 2347 cm^2, gold area 707 cm^2

12 **a**



b 24.6 m^2
c 4 m
d 25.6 m^2

exercise 10.7

1 **a** 1792 m^2 **b** 1048.9 m^2 **c** 2328 m^2
d 1998 m^2 **e** 2724 m^2 **f** 2673.5 m^2

2 E

3 **a** 2125 cm^2 **b** 12.87 m^2 **c** 7158 cm^2

4 386.575 m^2

5 **a** 6963 m^2 **b** 2228 m^2 **c** 3727 cm^2
d 6272 m^2 **e** 473 cm^2 **f** 317 m^2

6 C

7 **a** 1963.5 cm^2 **b** 331.2 cm^2

8 **a** 3141.6 cm^2 **b** 22.8 m^2
c 5.78 m^2 **d** 157.08 cm^2

exercise 10.8

1 **a** 5.96 cm^3 **b** 18 000 mm^3 **c** 0.56 m^3
d 3700 cm^3 **e** 2.954 m^3 **f** 94.38 cm^3
g 76.2 mm^3 **h** 3780 m^3 **i** 55 000 mm^3
j 0.0018 m^3 **k** 0.25 cm^3 **l** 160 cm^3
m 1 600 000 mm^3 **n** 450 000 mm^3 **o** 1 800 000 cm^3

2 D

3 **a** 27 L **b** 0.15 L **c** 2500 mL
d 40000 L **e** 48 kL **f** 3000 mL

4 **a** 270 mL **b** 3.5 kL **c** 150000 mL
d 3.6 L **e** 1.6 mL **f** 5000 cm^3

5 **a** 189 cm^3 **b** 552 cm^3 **c** 3500 cm^3

6 **a** 13 824 cm^3 **b** 1.728 m^3 **c** 30 000 cm^3
d 8100 cm^3 **e** 20 000 cm^3 **f** 112 000 cm^3

g 1044 cm^3 **h** 0.33 m^3 **i** 3024 cm^3
j 27 360 mm^3 **k** 12849.76 cm^3 **l** 0.676 m^3

7 D

8 **a** 50 400 cm^3 **b** 50 400 mL

9 answers will vary, one possible set is
125 cm × 6 cm × 8 cm, 60 cm × 10 cm × 10 cm,
30 cm × 20 cm × 10 cm, 30 cm × 25 cm × 8 cm
the last one is the most convenient shape as it will fit best on cupboard shelves

10 **a** 72 cm^3 **b** 20 160 cm^3 **c** 19 200 cm^3
d 480 cm^3 **e** 21 600 cm^3 **f** 4.152 m^3
g 131.544 cm^3 **h** 9450 cm^3 **i** 93 960 cm^3
j 24.192 m^3 **k** 6750 cm^3 **l** 60 000 cm^3

11 C

12 C

13 96.75 cm^3

14 2.592 m^3

15 **a** 239.4 cm^3 **b** 33 candles

exercise 10.9

1 **a** 14 cm^3 **b** 49 cm^3 **c** 396 m^3
d 238 918 cm^3 **e** 15 036 cm^3 **f** 22 m^3
g 28953 cm^3 **h** 1629 cm^3 **i** 54978 cm^3

2 C

3 **a** 1.5 m^3 **b** 1.5 kL

4 **a** 375 cm^3 **b** 375 mL

5 449 cm^3

6 4021 cm^3

Revision

1 C

2 D

3 D

4 B

5 E

6 **a** 270 cm^2
b 121.92 m^2
c 273 cm^2
d 850.5 cm^2
e 2 m^2
f 32 cm^2

7 **a** **i** 7.5 m
ii 4.5 m^2
b **i** 18.8 cm
ii 28.3 cm^2

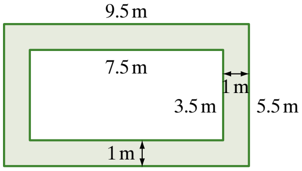
8 **a** 17.2 cm **b** 8.6 cm

9 **a** 64 cm^3 **b** 403 200 cm^3 **c** 5376 cm^3
d 120 cm^3 **e** 302 400 cm^3 **f** 2.7 m^3

10 1696 cm^3

11 **a** 7.9 m **b** 4.9 m^2
c approximately 981

12 **a** 26.25 m^2 **b**
c 26 m^2



13 **a** 22.1 m **b** 21.2 m^2 **c** 2120 plants

14 **a** 13 500 cm^3
b **i** 13 500 mL **ii** 13.5 L
c 10.8 L

Chapter 11

exercise 11.1

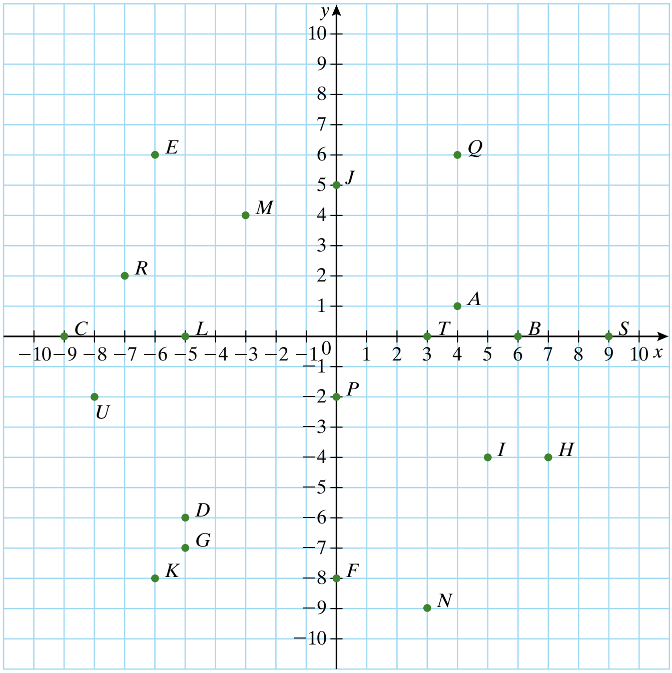
1 see below

2 B

3 $A(-3, 2)$, $B(7, 0)$, $C(-3, -4)$, $D(4, -6)$, $E(-2, 4)$, $F(9, 7)$, $G(-1, 6)$, $H(5, 4)$, $I(3, 6)$, $J(6, -3)$, $K(-6, -1)$, $L(-8, 7)$, $M(-9, -8)$, $N(-5, 9)$, $P(6, 9)$, $Q(2, -8)$, $R(8, -1)$, $S(0, 8)$, $T(9, -5)$, $U(-5, -6)$

4 C

1



5 D

6 B

7 D

8 **a** D, H **b** B, E **c** G

9 trapezium

exercise 11.2

1 **a** $y = x - 3$ **b** $y = x + 2$
c $y = 2x + 3$ **d** $y = 3x - 4$
e $y = -x + 1$ **f** $y = 8x$
g $y = 2x - 5$ **h** $y = 3x$
i $7 = -6x$ **j** $y = \frac{1}{2}x + 3$

2 D

3 A

4 D

5 **a**

x	−3	−2	−1	0	1	2	3
y	−21	−14	−7	0	7	14	21

b

x	−3	−2	−1	0	1	2	3
y	12	8	4	0	−4	−8	−12

c

x	−3	−2	−1	0	1	2	3
y	1	3	5	7	9	11	13

d

x	1	2	3	4	5	6	7
y	5.5	7.5	9.5	11.5	13.5	15.5	17.5

e

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

f

x	1	2	3	4	5	6	7
y	7.8	12.8	17.8	22.8	27.8	32.8	37.8

6 **a** **i**

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

ii (−3, −2), (−2, −1), (−1, 0), (0, 1), (1, 2), (2, 3), (3, 4)

b **i**

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1

ii (−3, −5), (−2, −4), (−1, −3), (0, −2), (1, −1), (2, 0), (3, 1)

c **i**

x	−3	−2	−1	0	1	2	3
y	2	3	4	5	6	7	8

ii (−3, 2), (−2, 3), (−1, 4), (0, 5), (1, 6), (2, 7), (3, 8)

d **i**

x	−3	−2	−1	0	1	2	3
y	−6	−5	−4	−3	−2	−1	0

ii (−3, −6), (−2, −5), (−1, −4), (0, −3), (1, −2), (2, −1), (3, 0)

e **i**

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

ii (−3, −5), (−2, −3), (−1, −1), (0, 1), (1, 3), (2, 5), (3, 7)

f **i**

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

ii (−3, −6), (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4), (3, 6)

g **i**

x	−3	−2	−1	0	1	2	3
y	−11	−8	−5	−2	1	4	7

ii (−3, −11), (−2, −8), (−1, −5), (0, −2), (1, 1), (2, 4), (3, 7)

h **i**

x	−3	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7	10

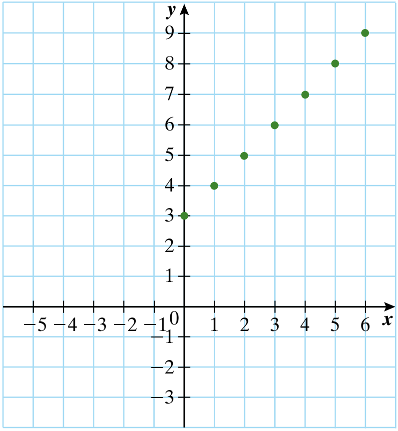
ii (−3, −8), (−2, −5), (−1, −2), (0, 1), (1, 4), (2, 7), (3, 10)

7 **a** linear, because the y values are increasing in equal steps of 1 as the x value increases by 1
b linear, because the y values are decreasing in equal steps of 1 as the x value increases by 1

8 **a** not a linear pattern **b** linear pattern
c not a linear pattern **d** linear pattern
e not a linear pattern **f** linear pattern
g not a linear pattern **h** not a linear pattern
i linear pattern **j** not a linear pattern
k linear pattern

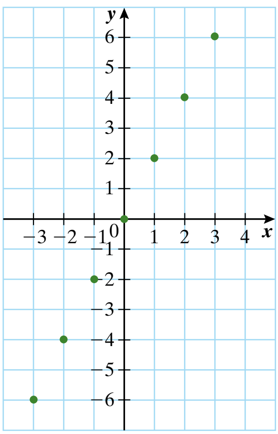
9 **a** linear pattern **b** linear pattern
c not a linear pattern **d** linear pattern

10 **a** **i**



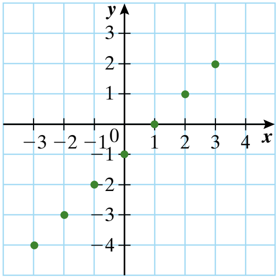
ii the y-values increase in steps of 1

b **i**



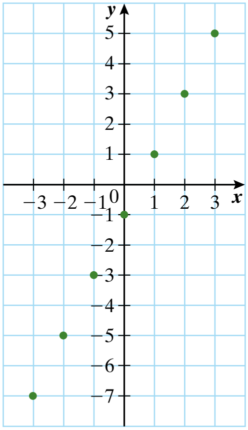
ii the y-values increase in steps of 2

c **i**



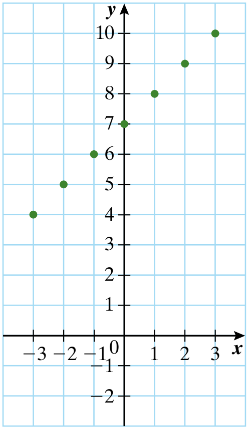
ii the y-values increase in steps of 1

d **i**



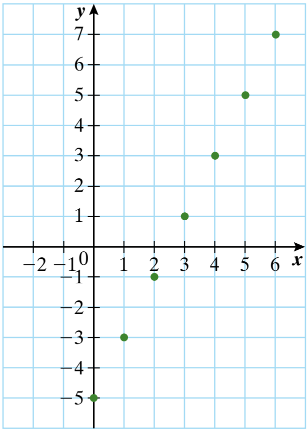
ii the y-values increase in steps of 2

e **i**



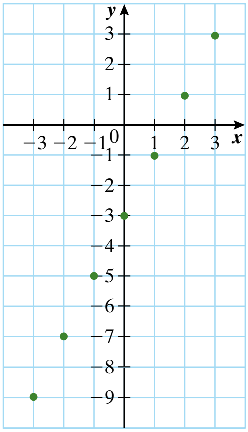
ii the y-values increase in steps of 1

f **i**



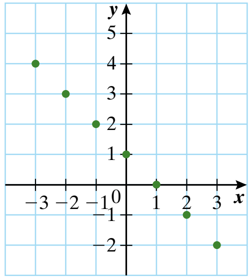
ii the y-values increase in steps of 2

g **i**



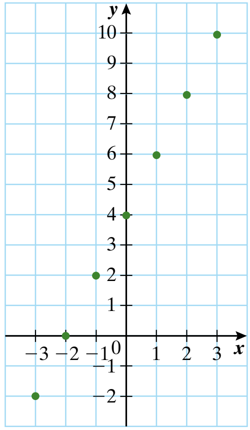
ii the y-values increase in steps of 2

h **i**



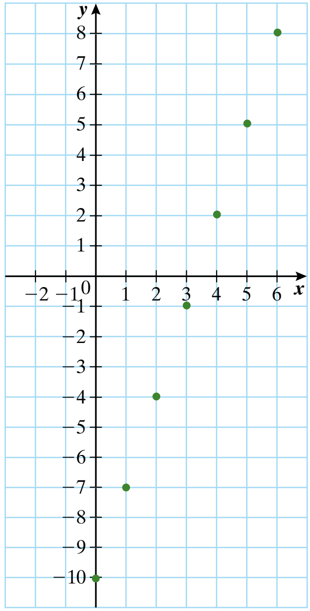
ii the *y*-value decreases in steps of 1

i **i**



ii the *y*-value increases in steps of 2

j **i**



ii the *y*-coordinate is always ten less than three times the *x*-coordinate

11 **a**

x	7	8	9
y	28	32	36

b

x	7	8	9
y	4	5	6

c

x	7	8	9
y	16	18	20

d

x	7	8	9
y	28	31	34

e

x	7	8	9
y	−10	−12	−14

f

x	4	5	6
y	7	8.4	9.8

g (4, 10), (5, 11), (6, 12)
h (4, −5), (5, −6), (6, −7)
i (4, 11), (5, 13), (6, 15)
j (4, 13), (5, 16), (6, 19)

12 D

13 **a** $y = -5$ **b** $y = 7$

14 **a** the *y*-coordinate is always 2.1 minus triple the *x*-coordinate
b (1, −0.9), (2, −3.9), (3, −6.9)

exercise 11.3

1 **a** **i** $m = 1, b = -1$
ii $y = x - 1$
b **i** $m = 2, b = 0$
ii $y = 2x$
c **i** $m = 1, b = 5$
ii $y = x + 5$
d **i** $m = 4, b = 0$
ii $y = 4x$
e **i** $m = 2, b = 5$
ii $y = 2x + 5$
f **i** $m = -1, b = 0$
ii $y = -x$
g **i** $m = -1, b = 2$
ii $y = -x + 2$
h **i** $m = 6, b = 1$
ii $y = 6x + 1$
i **i** $m = -5, b = 0$
ii $y = -5x$
j **i** $m = 0.5, b = 0$
ii $y = 0.5x$

2 D

3 **a** **i** $m = -2, b = 0$
ii $y = -2x$
b **i** $m = 1, b = 4$
ii $y = x + 4$
c **i** $m = 1, b = -2$
ii $y = x - 2$
d **i** $m = -3, b = 4$
ii $y = -3x + 4$
e **i** $m = 6, b = -1$
ii $y = 6x - 1$

f **i** $m = -4, b = 3$

ii $y = -4x + 3$

4 **a** **i** $m = 1, b = 2$

ii $y = x + 2$

b **i** $m = -1, b = 3$

ii $y = -x + 3$

c **i** $m = -2, b = 0$

ii $y = -2x$

d **i** $m = 2, b = -4$

ii $y = 2x - 4$

e **i** $m = 2, b = 1$

ii $y = 2x + 1$

f **i** $m = -2, b = 1$

ii $y = -2x + 1$

g **i** $m = 2\frac{1}{2}, b = \frac{1}{2}$

ii $y = 2\frac{1}{2}x + \frac{1}{2}$

h **i** $m = \frac{1}{2}, b = 0$

ii $y = \frac{1}{2}x$

i **i** $m = \frac{1}{2}, b = 2$

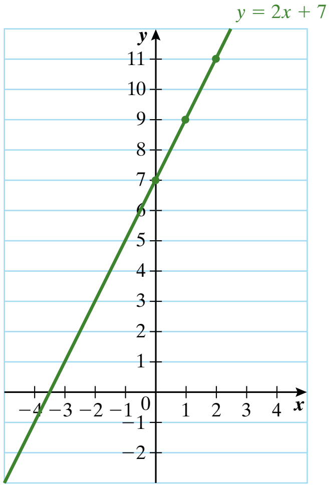
ii $y = \frac{1}{2}x + 2$

j **i** $m = -\frac{1}{2}, b = 1$

ii $y = -\frac{1}{2}x + 1$

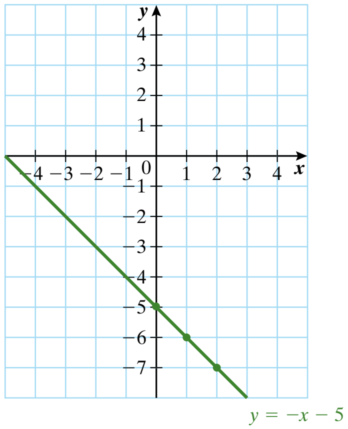
5 **a**

x	0	1	2
y	7	9	11



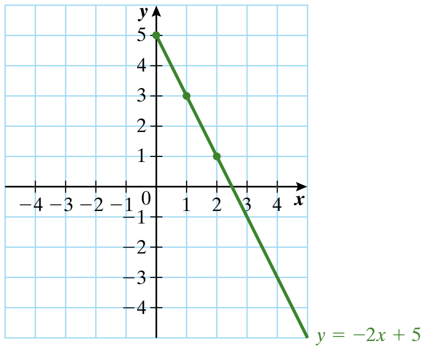
b

x	0	1	2
y	-5	-6	-7



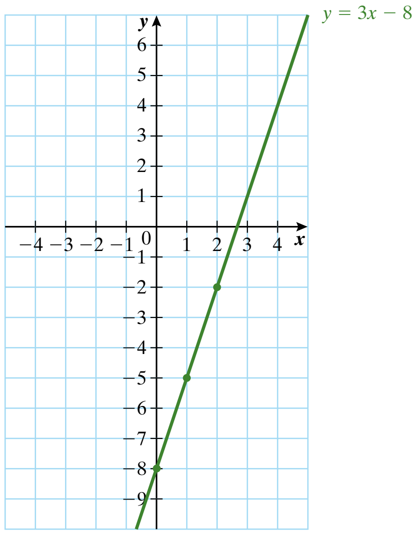
c

x	0	1	2
y	5	3	1

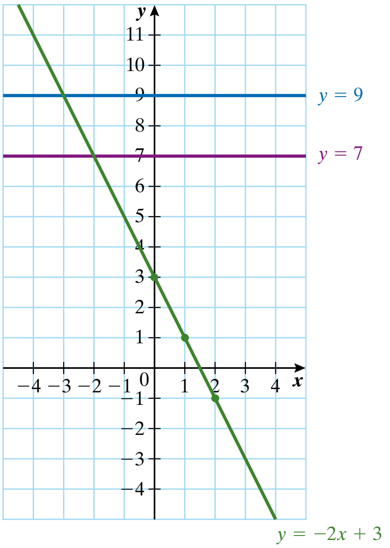


d

x	0	1	2
y	-8	-5	-2



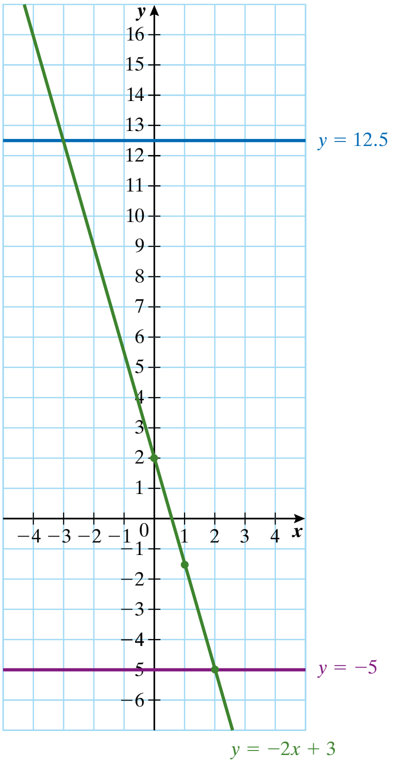
6



a $x = -2$

b $x = -3$

7



a $x = -3$

b $x = 2$

8 $y = -0.6x + 3.8$

exercise 11.4

1 **a** $y = -4x + 1$ is parallel to $y = -4x - 3$, $y = -x - 3$ is parallel to $y = -x$, $y = x$ is parallel to $y = x - 3$

b $y = -2x$, $y = -x$, $y = x$, $y = -7x$

c $y = -x - 3$, $y = x - 3$, $y = -4x - 3$

d $y = -2x$, $y = -4x + 1$, $y = -3x + 3$, $y = -x - 3$, $y = -x$, $y = -7x$, $y = -4x - 3$

2 **a** $y = 2x$

x	0	1	2
y	0	2	4

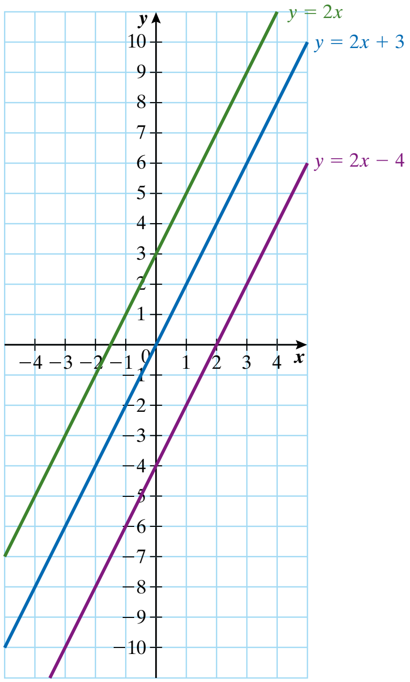
$y = 2x + 3$

x	0	1	2
y	3	5	7

$y = 2x - 4$

x	0	1	2
y	−4	−2	0

b



c the lines are all parallel, the line $y = 2x + 3$ is above $y = 2x$ and the line $y = 2x - 4$ is below $y = 2x$

3 **a**

$y = -2x$

x	0	1	2
y	0	−2	−4

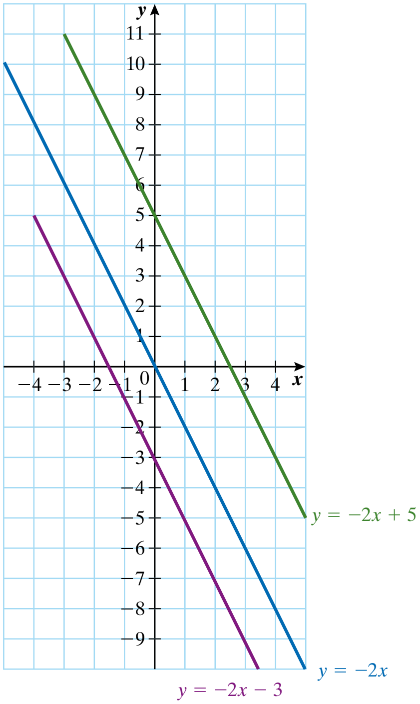
$y = -2x + 3$

x	0	1	2
y	5	3	1

$y = -2x - 3$

x	0	1	2
y	−3	−5	−7

b



c the lines are all parallel, the line $y = -2x + 5$ is above $y = -2x$ and the line $y = -2x - 3$ is below $y = -2x$

4 **a** (−5, −7)
b (1, 5)
c (−4, −1)
d (1, 5)
e (−3. 5)
f (5, 22)

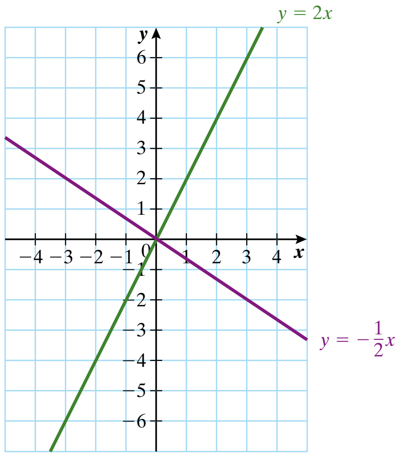
5 **a** $y = 2x$

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

$y = -\frac{1}{2}x$

x	−3	−2	−1	0	1	2	3
y	$1\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-1\frac{1}{2}$

b



c the lines are perpendicular (they meet at right angles)

exercise 11.5

1 **a** **i** time, t **ii** length, L
b **i** number of sheets **ii** mass
c **i** number of text messages **ii** total cost
d **i** number of muffins **ii** time taken

2 **a** children **b** tables

c

n	1	2	3	4	5
c	3	4	5	6	7

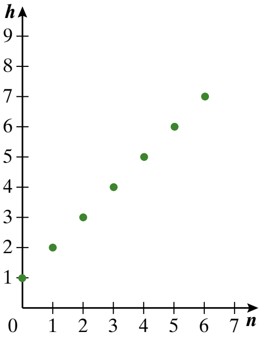
d no

3 C

4 **a**

n	0	1	2	3	4	5	6
h	1	2	3	4	5	6	7

b (0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)
c number of pieces of cake eaten (n)
d height (h)
e



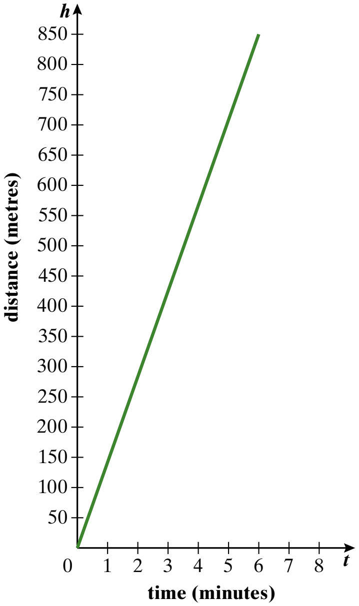
f yes **g** $h = n + 1$
h Alice's height is one more than the number of pieces of cake eaten
i 14 m

5 **a**

t	0	1	2	3	4	5	6
d	0	140	280	420	560	700	840

b time jogged (t) **c** distance travelled (d)

d



e 140 metres per minute **f** speed

g 0 **h** $d = 140t$

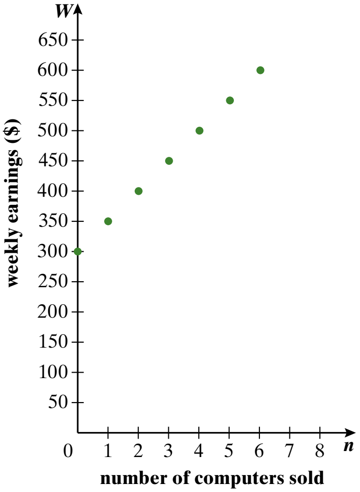
6 **a**

n	0	1	2	3	4	5	6
W	300	350	400	450	500	550	600

b number of computers sold (n)

c weekly earnings (W)

d not appropriate to join the dots as you can't sell part of a computer



e 50, the amount earned per computer sold

f 300, the amount earned even if no computers are sold

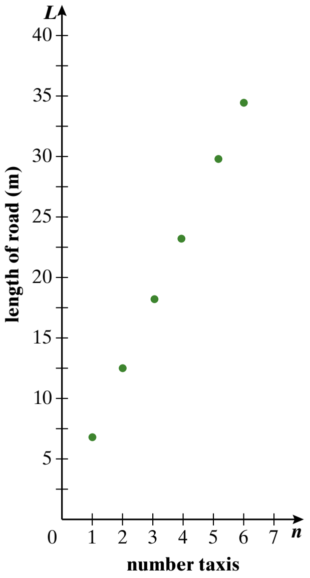
g $W = 50n + 300$

h $850

7 **a**

n	1	2	3	4	5	6
L	7	12.5	18	23.5	29	34.5

b



c you cannot have part of a taxi

d $L = 5.5n + 1.5$

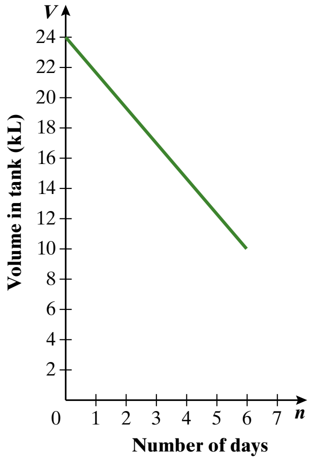
8 **a**

n	0	1	2	3	4	5	6
V	24	21.6	19.2	16.8	14.4	12	9.6

b number of days (n)

c volume of water tank (V)

d



e −2.4

f negative, the volume is decreasing as the days go on
g 24, the amount of water in the tank initially
h $V = -2.4n + 24$
i 2.4 kL
j 10

Revision

1 C **2** B **3** D **4** C **5** D

6 **a** $y = -4x + 3$
b $y = -x - 4$

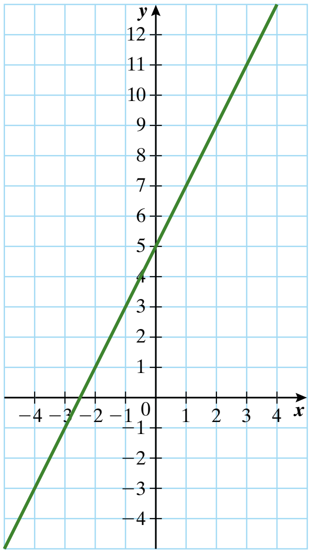
7 **a** $y = 3x - 5$
b $y = -4x + 5$

8 **a** $y = 2x + 6$
b $y = -2x - 3$

9 **a**

x	−3	−2	−1	0	1	2	3
y	−1	1	3	5	7	9	11

b

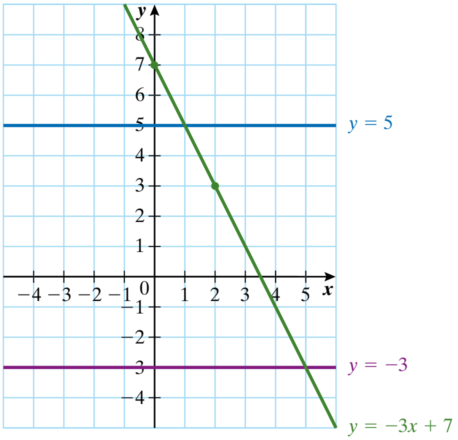


c 2 **d** 5

10 **a** **i** $m = 1, b = 5$
ii $y = x + 5$
b **i** $m = -2, b = 1$
ii $y = -2x + 1$

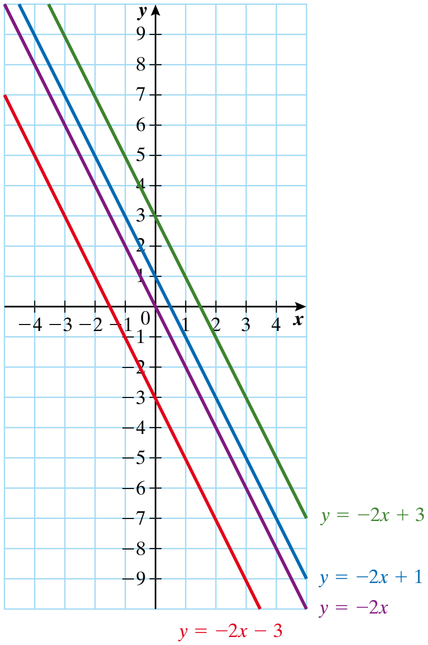
11 **a** $y = -x$ is parallel to $y = -x - 2$ and $y = x + 2$ is parallel to $y = x$ and $y = x - 3$
b $y = -x, y = x, y = -7x$
c $y = -3x - 2, y = -x - 2$
d $y = x + 2, y = 4x + 2, y = x, y = x - 3$

12



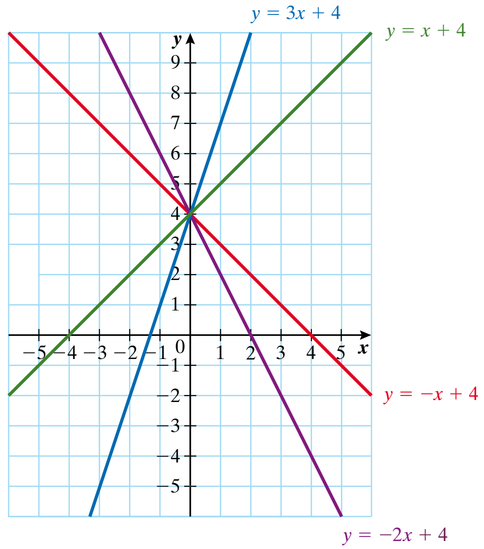
a $x = 5$
b $x = 1$

13



a they are all parallel
b they all have a different y-intercept

14



a they are all have a y-intercept of 4
b they all have different gradients

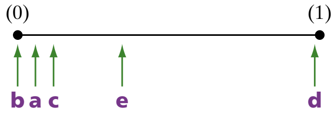
15 (−2. 1)

16 **a** independent variable is the number of hours lit, x, dependant variable is the length of the candle, y
b 12 cm
c −3
d 12
e $y = -3x + 12$
f 1.5 cm
g 0 cm

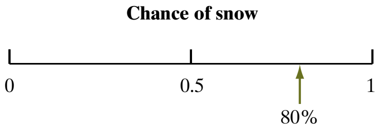
Chapter 12

exercise 12.1

1 answers will vary, sample shown below



2



b it is very likely that it will snow

3 **a** probability cannot be less than zero
b probability cannot be greater than one

4 **a** **i** $S = \{\text{blue,red,yellow}\}$
ii not equally likely
b **i** $S = \{\text{blue,yellow}\}$ **ii** equally likely

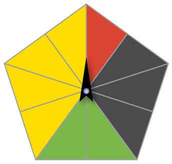
5 **i** **b** and **d**
ii **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{5}$
d $\frac{1}{3}$ **e** 0 **f** $\frac{3}{8}$

6 **a** $S = \{1, 2, 3, 4, 5, 6\}$ **b** $S = \{\text{blue,red,yellow}\}$
c $S = \{\text{head,tail}\}$

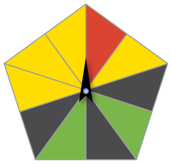
7 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$

8 answers will vary, samples shown below

a



b



9 **a** $\frac{1}{13}$ **b** $\frac{1}{4}$ **c** $\frac{1}{26}$ **d** $\frac{5}{26}$
e $\frac{4}{13}$ **f** $\frac{4}{13}$ **g** $\frac{7}{13}$ **h** $\frac{3}{4}$
i $\frac{9}{13}$ **j** $\frac{15}{52}$

10 **a**
$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\ 29, 30\}$
b $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{3}{5}$
e $\frac{1}{2}$ **f** $\frac{4}{5}$ **g** $\frac{2}{5}$

11 0.7

12 $\frac{5}{8}$

13 that it is very unlikely it will rain

14 D

15 **a** complementary **b** complementary
c not complementary **d** not complementary
e complementary **f** not complementary

16 $\frac{2}{5}$

17 $\frac{12}{13}$

18 **a** $\frac{1}{4}$ **b** 1 **c** $\frac{3}{4}$ **d** $\frac{3}{13}$

19 **a** $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{8}$ **d** $\frac{3}{4}$

20 no, the probabilities add up to $1\frac{1}{12}$ and the total probability cannot be greater then 1

21 **a** **i** yellow = 113.1 cm^2, red = 339.3 cm^2, blue = 565.5 cm^2, black = 792.1 cm^2, white = 1017.4 cm^2
ii yellow = 0.04, red = 0.12, blue = 0.20, black = 0.28, white = 0.36

b **i** 0.36
ii 0.28
iii 0.20
iv 0.12
v 0.04

22 **a** no
b answers will vary
c answers will vary

exercise 12.2

1 **a** 0.64 **b** 0.36

2 **a** 0.175 **b** 0.5375 **c** 0.3 **d** 0.5875

3 **a** $\frac{21}{50}$ **b** $\frac{9}{40}$ **c** $\frac{7}{50}$ **d** $\frac{43}{200}$

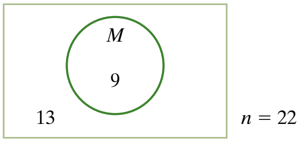
4 **a** $\frac{2}{11}$ **b** $\frac{1}{11}$ **c** $\frac{7}{11}$ **d** $\frac{4}{11}$
e $\frac{3}{11}$ **f** $\frac{8}{11}$

5 **a** **i** $\frac{111}{300} = 0.37$
ii $\frac{27}{300} = 0.09$
iii $\frac{12}{300} = 0.04$
b $\frac{48}{300} = 0.16$

6 **a** $\frac{28}{70} = 0.4$ **b** $\frac{7}{70} = 0.1$ **c** $\frac{22}{70} = 0.32$

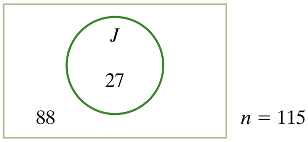
exercise 12.3

1 **a**



b the set of students who do not play a musical instrument
c $\frac{9}{22}$

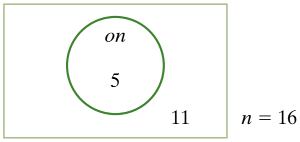
2 **a**



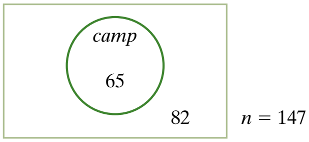
b the set of students who don't have part-time jobs
c $\frac{88}{115}$

3 **a** mutually exclusive
b not mutually exclusive
c not mutually exclusive
d mutually exclusive

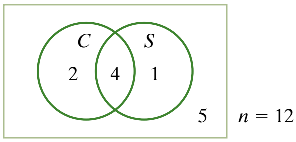
4 **a** a light can't be switched on and be switched off at the same time
b



5

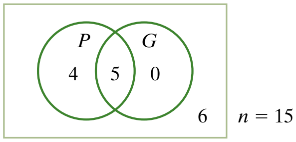


6 **a** $n = 12$



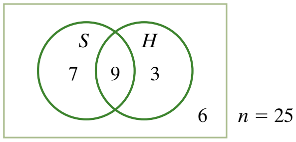
b 7 **c** 4 **d** 1

7 **a** 5
b 0
c



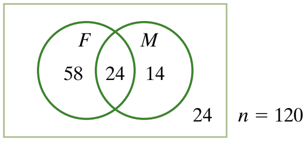
8 **a** 18 **b** 14 **c** 8 **d** 60

9 **a** $n = 25$



b 9 **c** 7 **d** 3 **e** 10

10 **a** $n = 120$

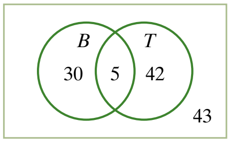


24 liked watching neither football nor motor racing

11 **a** $\frac{12}{25}$ **b** $\frac{1}{5}$ **c** $\frac{17}{25}$ **d** $\frac{3}{5}$ **e** $\frac{1}{5}$

12 **a** $\frac{3}{10}$ **b** $\frac{1}{20}$ **c** $\frac{1}{4}$ **d** $\frac{11}{30}$

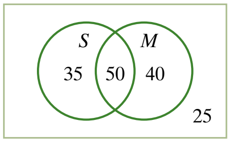
13



$n = 120$

a $\frac{1}{4}$ **b** $\frac{43}{120}$

14



$n = 150$

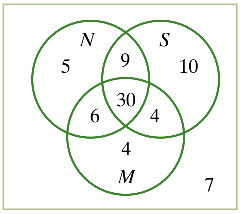
a $\frac{7}{30}$ **b** $\frac{1}{6}$

15 $\frac{5}{12}$

16 $\frac{3}{25}$

17 **a** 20 **b** 20 **c** 14 **d** 5
e 21 **f** 7 **g** 48 **h** 15

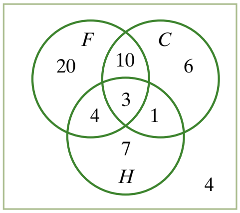
18 **a** $n = 75$



$n = 75$

b 7 **c** 68 **d** 49

19 **a** $n = 55$

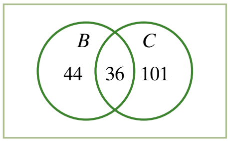


$n = 55$

b 4 **c** 51 **d** 18

20 **a** $\frac{7}{30}$ **b** $\frac{1}{10}$ **c** $\frac{3}{10}$ **d** $\frac{1}{15}$

21 **a** $n = 181$



$n = 181$

b **i** $\frac{80}{181}$ **ii** $\frac{36}{181}$ **iii** $\frac{101}{181}$

exercise 12.4

1 **a** $\frac{7}{12}$ **b** $\frac{29}{60}$ **c** $\frac{17}{60}$ **d** $\frac{11}{60}$
e $\frac{23}{30}$ **f** $\frac{7}{10}$

2 **a**

	Boy	Girl	Total
Ear piercing	20	38	58
No ear piercing	18	4	22
Total	38	42	80

b **i** $\frac{29}{40}$ **ii** $\frac{21}{40}$ **iii** $\frac{9}{40}$ **iv** $\frac{19}{40}$

3 **a** $\frac{87}{200}$ **b** $\frac{533}{1000}$ **c** $\frac{239}{1000}$ **d** $\frac{729}{1000}$

4 **a**

	Boy	Girl	Total
Team sports	21	13	34
Individual sports	6	10	16
Total	27	23	50

b **i** $\frac{23}{50}$ **ii** $\frac{8}{25}$ **iii** $\frac{13}{50}$ **iv** $\frac{3}{25}$

5 **a** 70 **b** 89
c **i** $\frac{89}{200}$ **ii** $\frac{13}{20}$ **iii** $\frac{43}{100}$
d $\frac{22}{65}$ **e** $\frac{9}{14}$

6 **a**

	Less than 4 kg	More than 4 kg	Total
Boy	39	7	46
Girl	31	13	44
Total	70	20	90

b $\frac{23}{45}$ **c** $\frac{2}{9}$ **d** $\frac{13}{30}$

7 **a** $\frac{13}{35}$ **b** $\frac{2}{35}$

8 **a**

	Two doors	Four doors	Total
Sedan	12	10	22
Station wagon	3	5	8
Total	15	15	30

b **i** $\frac{11}{15}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{6}$

9 **a**

	Year 7	Year 8	Year 9	Year 10
Train	37	12	13	62
Bus	20	9	19	48
Other	45	52	53	150
Total	102	73	85	260

b **i** $\frac{31}{130}$ **ii** $\frac{73}{260}$ **iii** $\frac{1}{13}$ **iv** $\frac{53}{260}$ **v** $\frac{123}{260}$

Revision

1 E **2** B **3** C **4** E **5** D

6 0.7

7 **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{13}$ **d** $\frac{2}{13}$ **e** $\frac{3}{52}$

8 **a** $S = \{\text{blue, purple, red, yellow}\}$

b no, there is three times more chance of blue than purple as three blue but only one purple

c **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** 0

iv $\frac{5}{8}$

d 75

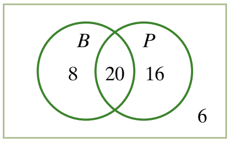
9 **a** mutually exclusive

b not mutually exclusive

c mutually exclusive

d mutually exclusive

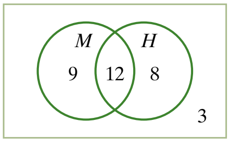
10 **a**



$n = 50$

b **i** $\frac{14}{25}$ **ii** $\frac{2}{5}$

11 **a** $n = 32$



$n = 32$

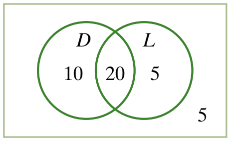
b 21 **c** 9 **d** 29

12 **a**

	Made before 2007	Made in 2007 or after	Total
Small vehicle	768	392	1160
Large vehicle	192	148	340
Total	960	540	1500

b **i** $\frac{16}{25}$ **ii** $\frac{17}{75}$ **iii** $\frac{98}{375}$

13 **a** $n = 40$



$n = 40$

b

	D	D'	Total
L	20	5	25
L'	10	5	15
Total	30	10	40

c **i** $\frac{1}{8}$ **ii** $\frac{1}{2}$ **iii** $\frac{7}{8}$

d $\frac{2}{3}$

14 **a** 2 **b** 4 **c** 13

d **i** $\frac{7}{25}$ **ii** $\frac{17}{50}$

e **i** 24 **ii** $\frac{5}{24}$

Chapter 13

Answers to all the extending and investigating problems are contained in the Teacher book.

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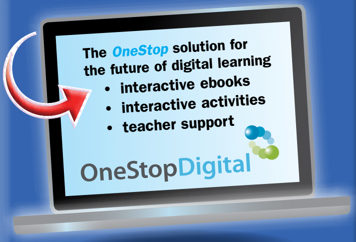

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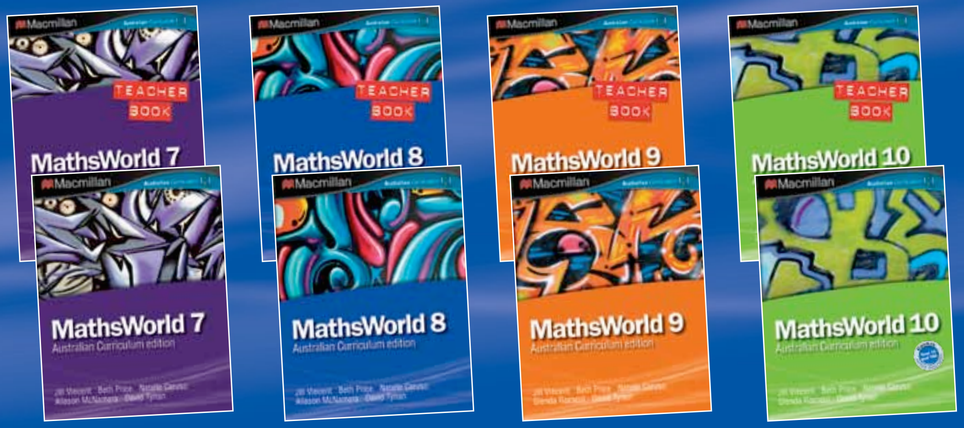

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